

P425/2
PRINCIPAL MATHEMATICS
Paper 2
2 HOUR 30 MINUTES

UGANDA ADVANCED CERTIFICATE OF EDUCATION

PRINCIPAL MATHEMATICS

Paper 2

TIME: 2 HOUR 30 MINUTES

INSTRUCTIONS TO CANDIDATES:

*This examination paper consists of **two** Section; **A** and **B**.*

*It has **five** examination items.*

*Section **A** has **one compulsory** item.*

*Section **B** has **two** parts; I and II. Respond to only **one** item from **each** part.*

*Respond to **three** items in all.*

*Any additional item(s) responded will **not** be scored.*

***All** responses **must** be written in the answer booklet(s) provided.*

Responses to each item should start on a fresh page.

Indicate the item number you are responding to, clearly at the top of the page.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take g to be 9.8 ms^{-2} .

SECTION A:

*Item in this section is **compulsory***

Item 1

A landlord is upgrading the compound of a newly completed rental estate to improve drainage and security. The project involves paving the compound and installing a surveillance camera at a strategic location.

The paving contractor supplies rectangular concrete pavers whose advertised top surface dimensions are 10.45 cm by 3.6 cm, each measured to the nearest decimal place. Before approving the delivery, the site engineer must verify whether the pavers meet the required quality standards. According to the project specifications, the consignment should be rejected if the percentage error in the area of a paver exceeds 1.5%.

To estimate the section of the compound to be paved, the engineer models the boundary of the paved region using the curves $y = x \sin x$ and $y = e^{-2x}$, for $0 \leq x \leq 2$, where both x and y are measured in tens of metres.

For security purposes, the landlord intends to install a surveillance camera at the point where the two boundary curves intersect along the x -axis. Since an exact algebraic solution is difficult to obtain, the engineer will determine the point of the intersection graphically correct to 1 decimal place.

The landlord has allocated a budget sufficient to pave not more than 25 m² of the compound. To determine whether the planned paved section falls within the available budget, the engineer must estimate the enclosed area between the two curves using a suitable method with four equal subdivisions correct to 3 decimal places.

Task.

As a mathematics students advise the landlord

SECTION B:

Part I

*Respond to **one** item from this part*

Item 2

The sports department in a certain Radio station is preparing for the final match of a popular national football tournament involving Team A and Team B. The department is conducting a study on spectators' expectations and match outcomes to assist tournament organizers in planning security, seating arrangements, refreshments, and media coverage.

A survey was conducted among football fans, and the results are summarized below.

Predicted number of goals in the final match	0	1	2	3	4	5	6
Number of fans (in hundreds)	5	15	25	36	26	14	4

The tournament organizers would like to determine the median, mean, and standard deviation of the predicted number of goals. The organizers have agreed that if the median predicted number of goals is more than four, additional entertainment activities should be arranged during halftime because spectators are likely to be excited.

The sports analysts have also estimated that the probability of Team A scoring a goal during the match is 0.60, while the probability of Team B scoring a goal is 0.45. The organizers wish to determine the expected number of teams scoring. If the expected number of teams scoring exceeds 1.5, additional broadcasting and media resources will be allocated to cover what is anticipated to be an exciting match.

Furthermore, historical performance data indicate that the probability of Team A winning the final match is 0.73. At one regional viewing centre, 52 football fans have participated in a prediction challenge by forecasting that Team A will win the match. The organizers would like to determine the probability that at least 40 of these fans predict correctly. If this probability exceeds 0.30, the organizers will prepare recognition awards for participants who made accurate predictions.

Task.

As a sports data analyst, use the information provided to help the tournament organizers make informed decisions

Item 3:

A district health office organized a Community Health Awareness Campaign at a regional health centre. The campaign included health talks, medical consultations, nutrition demonstrations, and wellness screenings. To effectively plan refreshments and welfare services for participants, the organizing committee studied the amount of time participants spent at the event.

The time T (in hours) that a participant spends at the campaign is modeled by the probability density function

$$f(t) = \begin{cases} kt, & 0 < t < 3, \\ 3k(4 - t), & 3 < t < 4, \\ 0, & \text{otherwise.} \end{cases}$$

To improve participant welfare, the organizing committee established the following guidelines:

- Participants who stay longer than the expected (mean) time will receive two rounds of refreshments.
- Participants who stay more than 3.5 hours will receive two rounds of refreshments and a meal package.

The committee wishes to determine the percentage of participants who qualify for:

- Two rounds of refreshments only.
- Two rounds of refreshments and a meal package.

During the campaign, a temporary access road leading to the health centre experienced heavy traffic. For public safety, the district traffic authorities monitored the speeds of vehicles entering the venue.

The speeds of vehicles are assumed to be normally distributed, with the following information:

- 5% of the vehicles travel at speeds more than 85 m/s.
- 10% of the vehicles travel at speeds less than 25 m/s.

To ensure the safety of pedestrians and patients, a speed limit of 60 m/s was imposed within the campaign zone. The authorities agreed that if more than 25% of the vehicles exceed this speed limit, additional traffic-control measures such as speed humps and traffic marshals would be introduced.

A random sample of 10 vehicles was observed, and the organizing committee wishes to determine the probability of at least 8 vehicles driving between 20 m/s and 45 m/s.

Tasks

As a mathematics student, use the information provided to assist the;

- (a) Organizing committee plan for rounds of refreshment and a meal package
- (b) Authorities to determine whether to add traffic control measures
- (c) Organizing committee determine the required probability.

Part II

*Respond to **one** item from this part*

Item 4

A transport company has been contracted to deliver essential supplies from Masaka to a warehouse on an island in Lake Victoria. The delivery must be completed within a strict schedule because the supplies are required for a community development project.

Early in the morning, a truck driver departs from a distribution centre in Masaka carrying the supplies. The driver has been informed that the ferry connecting the mainland to the island is scheduled to depart from the port in 10 minutes' time. Missing the ferry would delay the delivery by several hours and increase transportation costs. The truck is initially 20 km from the ferry port. It has a mass of 3.5 tonnes and moves along a rough horizontal road. The engine exerts a constant driving force of 1.5 kN, while resistive forces due to friction and air resistance amount to 0.25 N per kilogram of the truck's mass. The transport manager wishes to determine whether the truck can reach the port before the ferry departs.

Upon arrival at the port, the truck is loaded onto the ferry for transportation to the island. The ferry captain must navigate directly to an island located due west of the port. However, the captain knows that the lake has a current flowing at 3 km h^{-1} in the direction $S28^\circ E$. The ferry is capable of moving at 10 km h^{-1} in still water. To ensure that the ferry reaches the island without drifting off course, the captain must determine the direction in which the ferry should be steered.

After reaching the island, the truck is driven to a warehouse where the supplies are unloaded. Some packages are discharged from the back of the truck by allowing them to leave the loading platform horizontally at a speed of 10 m s^{-1} from a height of 2 m above the ground. To prevent damage to the supplies, safety guidelines require that the packages should not strike the ground with a speed exceeding 15 m/s. The warehouse supervisor therefore wishes to assess whether the unloading procedure satisfies the safety requirements.

Tasks

As a mathematic student help the;

- (a) transport manager to determine whether the truck can reach the port before the ferry departs.
- (b) captain determine the direction in which the ferry should be steered.
- (c) warehouse supervisor to assess whether the unloading is safe

Item 5:

A large logistics company operates a fleet of cargo vehicles transporting goods between major distribution centres in Uganda. To improve efficiency and ensure the safety of goods in transit, the company has installed a real-time vector-based tracking system that monitors the positions and velocities of vehicles moving in three-dimensional space.

At 12 noon on a particular day:

- A cargo trailer departs from the company's central warehouse from position $(\tilde{i} + 7\tilde{j})km$, moving with constant velocity $(6\tilde{i} + 2\tilde{j})kmh^{-1}$
- A delivery truck departs from a regional distribution centre from position $(6\tilde{i} + 4\tilde{j})km$ with constant velocity $(-4\tilde{i} + 8\tilde{j})kmh^{-1}$

To prevent delays and possible loss of goods, the logistics control centre must determine whether the two vehicles will occupy the same position at the same time. If this occurs, the control centre must identify the exact location so that alternative routing arrangements can be made.

Later that week, one of the company's loaded trailers developed a mechanical fault while transporting goods to a distribution centre. A recovery vehicle was dispatched to tow the trailer to the warehouse.

The trailer has a mass of 8 tonnes, while the recovery vehicle has a mass of 2.5 tonnes. The recovery vehicle exerts a driving force of 4 kN while moving on horizontal a road. The coefficient of friction between the vehicles and the road surface is 0.2.

To ensure the safety of personnel and cargo, the logistics manager wishes to determine the tension in the towing system. The recovery operation can be carried out efficiently if the tension is more than 3 kN.

Tasks

- (a) Use information above to determine whether the two vehicles occupy the same position at the same time and find the exact position and time at which this occurs.
- (b) Justify the effectiveness of the recovery operation.