JK BEGUMISA RESOURCES

IDENTITY: INTEGRATION – REVISON QUESTIONS

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"A Physics/Mathematics Teacher"

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NOTES:

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PART I: INTEGRATION:

Algebraic and Log functions - Use of a suitable substitution: ...

i). Evaluate the following integrals:

1. $\int \frac{x}{(x^2+2)^2}\, dx$
2. $\int \left(\frac{1}{x^{\frac{1}{5}}\sqrt{1+x^{\frac{4}{5}}}} \right) dx$
3. $\int \frac{x-1}{(x+2)^3}\, dx$
4. $\int \sqrt{5+2x}\ dx$

5. $\int x(3x^2-4)^3\ dx$
6. $\int \frac{3}{\sqrt{(5-2x)^3}}\ dx$
7. $\int \frac{5}{\sqrt{1-6x}}\ dx$
8. $\int x^2\ \sqrt[3]{6-x^3}\ dx$
9. $\int \frac{\sqrt{1+In x}}{x}\ dx$
10. $\int \frac{e^{2x}}{e^{2x}-1} dx$
11. $\int \frac{\sqrt{9-x^2}}{x}\ dx$
12. $\int \frac{x}{(x^2-4)^3}\ dx$
13. $\int \frac{1}{(3x+7)^4}\ dx$
14. $\int \sqrt[5]{(-3x+8)^6}\ dx$
15. $\int \sqrt{x\sqrt{x^3}}\ dx$
16. $\int x(2x-1)^3\ dx$
17. $\int (5x-2)^3 dx$
18. $\int \frac{\sqrt{(a^2-x^3)^3}}{x^2} dx$
19. $\int \frac{1}{x}\ In^2 x\ dx$
20. $\int x^3\sqrt{1-x^2}\ dx$
21. $\int \left(1+\sqrt{x}\right)^{10}\ dx$
22. $\int \frac{18-9x}{\sqrt{3+4x-x^2}}\ dx$
23. $\int \frac{1-x}{(x+2)^5}\ dx$

ii) In each of the following, show that;

24. $\int_0^2 \frac{1}{\sqrt{4x+1}}\,dx = 1$

25. $\int \frac{2x}{(4+3x^2)^2}\,dx = -\frac{1}{3(4+3x^2)} + c$

26. $\int_0^1 5x(1-x^2)^{\frac{3}{2}}\,dx = 1$

27. $\int \frac{12x}{(1-x^2)^{\frac{3}{2}}}\,dx = \frac{12}{\sqrt{1-x^2}} + c$

28. $\int_0^1 \frac{x}{\sqrt{16-7x^2}}\,dx = \frac{1}{7}$

29. $\int \frac{4}{x(1+4Inx)^2}\,dx = -\frac{1}{(1+4Inx)} + c$

30. $\int_0^1 \frac{9}{(2x+1)^2}dx = 3$

31. $\int_4^8 \frac{6x}{\sqrt{2x-7}}dx = 68$

32. $\int_0^5 x\sqrt{3x+1}\,dx = \frac{204}{5}$

33. $\int_2^3 \frac{(1-x)^2}{1+x^2}dx = 1 - ln2$

34. $\int \frac{30x}{\sqrt{1-2x}}\,dx = -10(x+1)(1-2x)^{\frac{1}{2}} + c$

35. $\int_0^{In2} (e^x + 2e^{-x})^2 dx = 3 + 4In2$

36. $\int \frac{e^{4x}}{\sqrt[5]{e^{2x}+1}}dx$

37. $\int 6x^2\left(4^{x^3}\right)dx$

Trigonometry and Algebra.

i). Evaluate the following integrals:

38. $\int \frac{1}{x^3} sin\left(\frac{1}{x^2}\right)\,dx$

39. $\int \cot(5x-4)\,dx$

40. $\int \frac{10}{sin^2\left(\frac{x-2}{3}\right)}\,dx$

41. $\int \frac{1}{x} cos(Inx)\,dx$

42. $\int \frac{sin2x}{cosx}\,dx$

43. $\int cot^2(3x)\,dx$

44. $\int \frac{sin^3(3x)}{cos3x}\,dx$

45. $\int \frac{tan\sqrt{x}}{\sqrt{x}}\,dx$

46. $\int \frac{cosx}{\sqrt[3]{sin^2x}}\,dx$

47. $\int sin(3mx)\,dx$

48. $\int_0^{\frac{\pi}{4}} \frac{4}{1+sin2x}dx$

49. $\int \frac{sec^2t}{1+tant}dt$

50. $\int \frac{tan\theta}{cos^2\theta}d\theta$

51. $\int tan^2xsecxdx$

52. $\int \frac{tanx}{In(cosx)}dx$

53. $\int \frac{x^{\frac{1}{3}}}{\left(x^{\frac{1}{2}}+x^{\frac{1}{4}}\right)}dx$

54. $\int \frac{cosx}{\sqrt{4-sin^2x}}dx$

55. $\int cosx\sqrt{4-sin^2x}\,dx$

56. $\int \frac{x}{\sqrt{4x-x^2}}dx$

57. $\int \frac{x^7}{\sqrt{1-x^4}}dx$

58. $\int \frac{dx}{x\sqrt{6x-x^2}}$

59. $\int \sqrt{x^2-9}\, dx$

60. $\int sec4wtan4wdw$

ii). In each of the following, prove that;

61. $\int_0^{\frac{\pi}{2}}(1+2cosx)^3 sinx\, dx = 10$

62. $\int_0^{\frac{\pi}{3}} sec^4x\, dx = 2\sqrt{3}$

63. $\int_0^{\frac{\pi}{3}} 15(10cosx-1)^{\frac{1}{2}} sinx\, dx = 19$

64. $\int sec^2x\{1+cot^2x\}\, dx = -cotx + tanx + C$

65. $\int \frac{sinx}{\sqrt{4cosx-1}} dx = -\frac{1}{2}\sqrt{4cosx-1} + C$

66. $\int tanxsec^4x\, dx = \frac{1}{4}sec^4x + C$

67. $\int_0^{\frac{\pi}{4}}(1-sin4x)dx = \frac{1}{4}(\pi-2)$

68. $\int_0^{\frac{\pi}{4}} \cos\left(3x+\frac{\pi}{4}\right) dx = \frac{-\sqrt{2}}{6}$

69. $\int \frac{1}{1+cosx} dx = tan\frac{x}{2} + C$

70. $\int_0^{\frac{\pi}{4}} \sin\left(2x+\frac{\pi}{4}\right) dx = \frac{\sqrt{2}}{2}$

iii) Workout the following integrals:

71. $\int \frac{tanx}{\sqrt{1+cos2x}} dx$

72. $\int sin\frac{3}{4}xcos\frac{1}{2}x\, dx$

73. $\int cosxsin3x\, dx$

74. $\int cos\frac{3}{2}xcosx\ dx$

75. $\int tan^2x\ dx$

76. $\int tan^3x\ dx$

77. $\int sin^3\frac{x}{4}\ dx$

78. $\int cosxsin^2 2xdx$

79. $\int sin^3\frac{x}{2}cos^3\frac{x}{2}\ dx$

80. $\int cosec^2\frac{1}{16}x\ dx$

81. $\int cot^2\frac{2}{3}x\ dx$

82. $\int (16+x^2)^{-\frac{3}{2}}dx$

83. $\int \frac{x}{cosec(5x^2)}dx$

84. $\int \sin^3(x)\cos^{\frac{1}{2}}(x)\ dx$

85. $\int tanxsec^6x\ dx$

86. $\int \sqrt{4-cosx}\ dx$

iv) Use suitable substitutions, to work out the following;

87. $\int \frac{3}{100+81x^2}\ dx$

88. $\int \frac{\sqrt{2}}{x^2+2}\ dx$

89. $\int_2^4 \frac{dx}{\sqrt{4-x^2}}$

90. $\int \frac{2}{\sqrt{64-4x^2}}\ dx$

91. $\int \frac{6}{(x^2-4)^{\frac{3}{2}}}\ dx$ (**use:** $x = 2sec\theta$)

92. $\int \frac{1-x}{\sqrt{x}(x+1)} dx$ (**Use**: $\sqrt{x} = tan\theta$)

93. $\int \frac{2x^2}{\sqrt{1-x^2}} dx$

94. $\int_0^1 \frac{tan^{-1}(x)}{1+x^2} dx$

95. $\int_0^{\frac{\pi}{2}} \frac{sin2x}{\sqrt{4-sin^4x}} dx$

96. $\int \frac{x+2}{\sqrt{1-4x^2}} dx$

97. By using substitution; $\boldsymbol{x = sec^2\theta}$, or otherwise, evaluate; $\int_1^2 \frac{dx}{x^2(x-1)^{\frac{1}{2}}}$

v) In each of the following, prove that;

98. $\int_0^{\frac{1}{2}} \sqrt{\frac{16x}{1-x}} dx = \pi - 2$ (**Use** $\boldsymbol{x = sin^2\theta}$)

99. $\int_1^{\sqrt{e}} \frac{1}{x\sqrt{1-(Inx)^2}} dx = \frac{\pi}{6}$

100. $\int_{\frac{4}{3}}^{\frac{5}{3}} \frac{x+1}{\sqrt{9x^2-16}} dx = \frac{1}{3}(1 - In2)$

101. $\int_0^1 \frac{x^2}{(x^2+1)^3} dx = \frac{\pi}{3}$

102. $\int_0^{\sqrt{3}} \frac{x}{x^4+9} dx = \frac{\pi}{24}$ (**Use** $\boldsymbol{x^2 = 3tan\theta}$)

103. $\int_0^1 \frac{2\sqrt{3}}{\sqrt{4\pi^2-3\pi^2x^2}} dx = \frac{2}{3}$

104. $\int_0^1 \frac{1}{(x+1)\sqrt{x}} dx = \frac{\pi}{2}$

105. $\int_0^{\frac{3}{4}} \frac{6}{\sqrt{3-4x^2}} dx = \pi$

106. $\int_0^{\frac{1}{2}} \frac{dx}{3+4x^2} = \frac{\pi\sqrt{3}}{36}$

107. $\int_{-3}^{-2} \frac{x}{\sqrt{-x^2-6x-5}} dx = \sqrt{3} + \frac{\pi}{2} - 2$

108. $\int_{-1}^{5} \sqrt{(1+x)(5-x)}\, dx = \frac{9\pi}{2}$

109. $\int_{-1}^{1} (x+3)\sqrt{7-6x-x^2}\, dx = 8\sqrt{3}$

110. $\int_{0}^{1} \frac{x^2-3x+1}{\sqrt{(1-x^2}}\, dx = \frac{3}{4}(\pi - 4)$

111. $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{5x^2-12x+4}{\sqrt{(1-x^2)}}\, dx = \frac{4\pi}{3}$

112. $\int_{4}^{9} \frac{1}{(4-x)\sqrt{x}}\, dx = In\sqrt{3}$

113. Using substitution; $\boldsymbol{x^3 = 9sin^2\theta}$ or otherwise, show that:

$$\int_{0}^{1} \frac{\sqrt{x}}{\sqrt{9-x^3}}\, dx = \frac{2}{3} sin^{-1}\left(\frac{1}{3}\right).$$

114. Using substitution $x^2 = u$, evaluate; $\int_{\frac{1}{2}}^{1} xsin(x^2)\cos(x^2)\, dx$, correct your answer to three significant figures.

Partial fractions:

i). Express the following in partial fractions:

115. $\frac{17x-53}{x^2-2x-15}$

116. $\frac{125+4x-9x^2}{(x-1)(x+3)(x+4)}$

117. $\frac{10x+35}{(x+4)^2}$

118. $\frac{7x^2-17x+38}{(2x+3)(x-2)^2}$

119. $\frac{4x^3+16x+7}{(x^2+4)^2}$

120. $\frac{2x^4+3x^2+1}{x^2+3x+2}$

121. $\frac{4x^3+10x+4}{x(2x+1)}$

122. $\frac{x^2-3x-7}{(x^2+x+2)(2x-1)}$

123. $\frac{13}{(2x+3)(x^2+1)}$

124. $\frac{9x^2+20x-10}{(x+2)(3x-1)}$

125. $\frac{x^3}{x^2-9}$

126. $\frac{2x^2-7x+1}{x^3-x^2+x-1}$

127. $\frac{3x^2+3x-12}{6x^3+5x^2-6x}$

ii). Evaluate the following indefinite integrals:

128. $\int \frac{dx}{2x^3+x^2-x}$

129. $\int \frac{x^2+12x-5}{(x+1)^2(x-7)} dx$

130. $\int \frac{6x^2-x-1}{3x-1} dx$

131. $\int \frac{1}{x^4-1} dx$

132. $\int \frac{x^4+x^3+x^2+1}{x^2+x-2} dx$

133. $\int \frac{x^5+1}{x^3(x+2)} dx$

134. $\int \frac{3x^3-5x^2-11x+9}{x^2-2x-3} dx$

135. $\int \frac{dx}{1+e^x}$

136. $\int \frac{x^4+1}{x^3+9x} dx$

137. $\int \frac{dx}{e^{2x}+e^x}$

138. $\int \frac{\sec(x)}{1+\sin(x)} dx$

139. $\int \sqrt{1+e^x}\, dx$

140. $\int \frac{e^x}{(e^x-1)(e^x+2)} dx$

141. $\int \frac{cos\theta d\theta}{\sin^3\theta-4sin\theta}$

142. $\int \frac{dx}{x-\sqrt[3]{x}}$

143. $\int \frac{dx}{x\sqrt{1+2x}}$

144. $\int \frac{dx}{x+\sqrt[3]{x}}$

145. $\int \frac{dx}{1+\sin(x)+\cos(x)}$

146. $\int \frac{dx}{1+sinx}$

147. $\int \frac{x^3 dx}{\sqrt[3]{x^2+1}}$

148. $\int \frac{\sqrt{x}}{x-4} dx$

149. $\int \frac{6x\, dx}{x^3-8}$

150. $\int \frac{2}{9x^2-1} dx$

151. $\int \frac{x^2+12x+12}{x^3-4x} dx$

152. $\int \frac{\cos(y)}{\sin^2(y)+\sin(y)-6} dy$

153. $\int \frac{e^{4t}+2e^{2t}-e^t}{e^{2t}+1} dt$

154. $\int \frac{\sin(\theta)d\theta}{\cos^2(\theta)+\cos(\theta)-2}$

155. $\int \frac{16x^3 dx}{4x^2-4x+1}$

156. $\int \frac{x^4}{x^2-1} dx$

157. $\int \frac{y^4+y^2-1}{y^3+y} dy$

158. $\int_5^7 \frac{x^4-7x^3+20x^2-15x-50}{(x-1)(x-2)(x-4)} dx$

159. $\int_0^1 \frac{dx}{x^3+1}$

160. $\int \frac{du}{e^{4u}+4e^{2u}+3}$

161. $\int \frac{x^2}{\sqrt[3]{2x+3}} dx$

162. $\int \frac{e^{3mx}}{1+e^{mx}} dx$

163. $\int \frac{2^x}{1-16^x} dx$

iii). Partialize each of the following integrals, and for each case, prove out the corresponding definite integral.

164. $f(x) = \frac{8x-1}{(2x-1)^2}$, $\int_1^{1.5} f(x)dx = \frac{3}{4} + 2In2$

165. $h(x) = \frac{3x-10}{x^2+5x-6}$, $\int h(x)dx = 4In(x+6) - In(x-1) + C$

166. $p(x) = \frac{5}{3x^2-5x}$, $\int_3^5 p(x)dx = In\left(\frac{3}{2}\right)$

167. $f(x) = \frac{2x+2}{(1-x)(1-2x)}$, $\int_{1.5}^2 f(x)dx = 7In2 - 3In3$

168. $g(x) = \frac{32-17x}{(x+1)(3x-4)^2}$, $\int_0^1 g(x)dx = 1 + 3In2$

169. $h(x) = \frac{2x^2-3}{(x-1)^2}$, $\int_2^3 h(x)dx = \frac{3}{2} + 4In2$

170. $f(x) = \frac{3x^2-2x+3}{2x(x-1)^2}$, $\int_4^9 f(x)dx = \frac{5}{24} + 2In2$

171. $m(x) = \frac{12x^2+x+3}{(6x+1)(2x^2+1)}$, $\int_0^2 m(x)dx = In\sqrt{39}$

172. $j(x) = \frac{1}{(x+x^4)}$, $\int_{\frac{1}{2}}^2 j(x)dx = In2$

173. $f(x) = \frac{70}{x(x-2)(x+5)}$, $\int_3^4 f(x)dx = 11In3 - 15In2$

174. $h(x) = \frac{x^2+3}{(x-1)}$, $\int_2^4 h(x)dx = 8 + 4In3$

175. $g(x) = \frac{x-5}{(x^2+5x+4)}$, $\int_0^2 g(x)dx = In\left(\frac{3}{8}\right)$

176. $f(x) = \frac{1}{9cos^2x - sin^2x}$, $\int_0^{\frac{\pi}{4}} f(x)dx = \frac{1}{6}In2$

177. $h(x) = \frac{(x-2)(x^2-5x-1)}{(x-1)(x-3)}$, $\int_{1.5}^2 h(x)dx = \frac{5}{4} - In6$

178. $h(x) = \frac{4x}{1-x^4}$, $\int_0^{\frac{1}{2}} h(x)dx = In\frac{5}{3}$

179. $n(x) = \frac{3x^2+1}{2x^3+x}$, $\int_1^2 n(x)dx = \frac{1}{2}In48$

180. $f(x) = \frac{x^2+3x+36}{x^3+9x^2+9x+81}$, $\int f(x)dx = In(x+9) + tan^{-1}\left(\frac{x}{3}\right) + C$

181. $f(x) = \frac{x+3}{(x+1)(x^2+4x+5)}$, $\int_0^1 f(x)dx = In\sqrt{2}$

182. $g(x) = \frac{9}{x^3+1}$, $\int_0^1 g(x)dx = 3In2 + \pi\sqrt{3}$

183. $f(x) = \frac{x+2}{(x+1)(x^2+4)}$, $\int_0^2 f(x)dx = \frac{\pi}{8} + In3$

184. $h(x) = \frac{x^2+x}{x^3-x^2+x-1}$, $\int h(x)dx = \ln(x-1) + \tan^{-1}x + c$

Integration by parts (or And Use of Basic – Technique Method): …

i). Evaluate the following;

185. $\int xe^x dx$
186. $\int xsin^2 2xdx$
187. $\int x^3 \sin(3x)\cos(2x)\,dx$
188. $\int \log(3x)\,tanxdx$
189. $\int xsec^2 xdx$
190. $\int xcot^{-1}xdx$
191. $\int xtan^{-1}(x)dx$
192. $\int \sqrt[3]{x}\,log_4^x dx$
193. $\int (x^2)\sin(x^2)\,dx$
194. $\int \cos^{-1}(\sqrt{x})dx$
195. $\int \frac{\sec^4 x}{\tan^2 x}dx$
196. $\int \frac{lnx}{x^3}dx$
197. $\int_0^3 \frac{x^2}{\sqrt{x+1}}dx$
198. $\int (2x+3)^2 sin4xdx$
199. $\int e^{2x}\sqrt{e^x+1}\,dx$

200. $\int \frac{x^3}{\sqrt{9-x^2}} dx$

201. $\int q^3\sqrt{q^2-3}\, dq$

202. $\int \frac{\ln(x-1)}{\sqrt{x-1}} dx$

203. $\int x^3 lnx dx$

204. $\int 10^{4t+1} dt$

205. $\int \sqrt{x} ln\sqrt{x}\, dx$

Note: Show that; $\int \sin^m(x)\, dx = -\frac{1}{m}\sin^{m-1}(x)\cos(x) + \frac{m-1}{m}\int \sin^{m-2}(x)\, dx$.

Hence, evaluate; $\int \sin^5 x\, dx$

206. $\int x^2 e^{-x} dx$

207. $\int 3x^3 e^{x^2}\, dx$

208. $\int y^6 \cos(10y)\, dy$

209. $\int e^{1+\ln(5t)}\, dt$

210. $\int_0^{\frac{\pi}{2}} sin\sqrt{x}\, dx$

211. $\int_0^{\frac{\pi}{2}} \frac{x}{1+cosx} dx$

212. $\int \sqrt{x}\, tan^{-1}\sqrt{x}\, dx$

213. $\int_3^9 \frac{log_{27}^{x}}{x} dx$

214. $\int \frac{In(tanx)}{sinxcosx} dx$

215. $\int sec^{-1}\sqrt{x}\ dx$

216. $\int \frac{(sin^{-1}x)^2}{\sqrt{1-x^2}} dx$

217. $\int \frac{sin^{-1}x}{x^2}\ dx$

218. $\int \frac{sin^3\theta}{cos\theta-1} d\theta$

ii). In each of the following, prove that;

219. $\int_0^{In2} 4xe^{2x}dx = -3 + 8In2$

220. $\int_0^{\frac{\pi}{2}} 4x^2cosxdx = \pi^2 - 8$

221. $\int_0^{\frac{\pi}{4}} 4xsec^2xdx = \pi - 2In2$

222. $\int \frac{4Inx}{x^3}dx = \frac{1+2Inx}{x^2} + C$

223. $\int_1^{\frac{\pi}{3}} 2sinx\{In(secx)\}dx = 1 + In\frac{1}{2}$

224. $\int x(2^x)dx = \frac{2^x}{ln2}\left(x - \frac{1}{ln2}\right) + c$

225. $\int_1^9 \frac{lnx}{\sqrt{x}}dx = 12ln3 - 8$

226. $\int x^2(2^x)dx = \frac{2^x}{ln2}\left(x^2 - \frac{2x}{ln2} + \frac{2}{\ln^2(2)}\right) + c$

227. $\int_0^{\pi} xcos\frac{1}{2}x\,dx = 2\pi - 4$

228. $\int_0^1 x^2e^xdx = e - 2$

229. $\int_1^2 x^3lnx\,dx = 4ln2 - \frac{15}{16}$

230. $\int_0^{\frac{\pi}{4}} x^2sin2xdx = \frac{\pi}{8} - \frac{1}{4}$

231. $\int_0^1 xtan^{-1}x\,dx = \frac{\pi}{4} - \frac{1}{2}$

t – substitution: ...

i). Using substitution; $t = \tan\left(\frac{1}{2}x\right)$, prove that:

232. $\int secxdx = In\{secx + tanx\} + C$

233. $\int cosecxdx = In\{cosecx - cotx\} + C$

234. $\int_0^{\frac{2\pi}{3}} \frac{1}{5+4cosx}dx = \frac{\pi}{9}$

235. $\int_0^{\frac{\pi}{2}} \frac{3\sqrt{3}}{2-cosx}dx = 2\pi$

236. $\int_0^{\frac{\pi}{2}} \frac{1}{1+sinx}dx = 1$

237. $\int_0^{\frac{\pi}{2}} \frac{1}{5+3sinx+4cosx} dx = \frac{1}{6}$

238. $\int_0^{\frac{\pi}{2}} \frac{2}{(1+sinx+2cosx)} dx = In3$

239. $\int_0^{\frac{\pi}{8}} \frac{\sqrt{3}}{2+sin4x} dx = \frac{\pi}{12}$

240. $\int_0^{\frac{\pi}{2}} \frac{1}{1+sinx+cosx} dx = In2$

ii). Use substitution; $t = tanx$, show that;

241. $\int_0^{\frac{\pi}{4}} \frac{18}{3cos^2x+sin^2x} dx = \pi\sqrt{3}$

242. $\int_0^{\frac{\pi}{4}} \frac{10}{2-tanx} dx = \pi + 3In2$

243. $\int_0^{\frac{\pi}{4}} \frac{1}{cos^2x+25sin^2x} dx = \frac{1}{5}tan^{-1}(5)$

244. $\int_0^{\frac{\pi}{3}} \frac{1}{1+8cos^2x} dx = \frac{\pi}{18}$

PART II: APPLICATIONS OF INTEGRATION:

- ### Differential Equations:

i). Form differential equations for each of the following equations, and in each case, state the order and degree of the DE formed.

245. $y = Ae^{-4t}$

246. $x = Ae^{3t} + Be^{-2t}$

247. $y = Acos3t + Bsin3t$

248. $x = Pe^{-2t} + Re^{2t}$

249. $y = Asin4t + Be^{-2t}$

ii). Solve each of the following differential Equations:

250. $x^2\frac{dy}{dx} + xy = 3x^2 + 4y^2$

251. $\frac{dy}{dx} = \frac{4x}{4x-y}$

252. $\frac{dy}{dx} - 3 = x^{-2}y^2$

253. $2x\frac{dy}{dx} = y(x^2 - y^2)^{-\frac{1}{2}}$

254. $\frac{dy}{dx} = \sqrt{\frac{y}{x+1}}, \qquad y(3) = 9$

255. $\frac{dy}{dx} = \frac{sin^2x}{y^2}, \qquad y(\pi) = 1$

256. $\frac{dy}{dx} + ycotx = cos3x, \ \ y\left(\frac{\pi}{6}\right) = 1$

257. $xdy - 3ydx = x^3dx$

258. $4\frac{dy}{dx} + 5y = e^{\frac{1}{4}x}$

259. $cosx\frac{dy}{dx} - 2ysinx = 1$

260. Show that the general solution to the differential equation;

$$\frac{dx}{dt} = \frac{1+x^2}{1+t^2} \text{ is; } x = \frac{k+t}{1-kt}, \text{ where } k \text{ is a constant.}$$

261. Find y in terms x, given that; $x\frac{dy}{dx} = cos^2y$ and $y\left(\frac{\pi}{3}\right) = 1$

262. Find the general solution of the equation: $2x\frac{dy}{dx} - (2y + 1)(x + 1) = 0$

263. Solve the differential equation;

$$\frac{dy}{dx} = x - \frac{2y}{x} \quad \text{given that } y(2) = 4$$

264. Solve the differential equation; $\frac{dy}{dx} = 4x - 7$ given that $y(2) = 3$

265. Solve the differential equation: $\frac{dy}{dx} + y = e^{-x}cos\frac{x}{2}$

266. Solve the differential equation: $y^2cos^2x = tanx\frac{dy}{dx}$

267. Solve: $2\frac{dy}{dx} = e^{x-2y}$ given that y=1 when $x = 2$

268. Solve the differential equation:

$$(x+2)\frac{dy}{dx} = (2x^2+4x+1)(y-3) \text{ given that } x = 0 \text{ when } y = 7.$$

269. The gradient function of a curve at any point (x, y) is; $\boldsymbol{\frac{dy}{dx} = 2x - \frac{16}{x^2}}$.

Given that the y −co – ordinate of the stationary point is 10, find the equation of the curve.

iii). Applications of Differential Equations – Word Problems:

270. The number x of bacterial cells in time t hours, after they were placed on a laboratory dish, is increasing at the rate proportional to the number of the bacterial cells present at that time.

a) If x_0 is the initial number of the bacterial cells and k is a positive constant, show that: $\boldsymbol{x = x_0e^{kt}}$

b) If the number of bacteria triples in 2 hours, show that: $\boldsymbol{k = In\sqrt{3}}$.

271. In a chemical reaction, the mass, m of the product after time t minutes satisfies the differential equation $\boldsymbol{\frac{dm}{dt} = \frac{m}{t(1+t^2)}}$.

Two minutes after the reaction had started, the mass of the chemical produced was 10 grammes, show that; $\boldsymbol{m = \frac{5t\sqrt{5}}{\sqrt{1+t^2}}}$.

272. The number, x thousands, of reported cases of an infectious disease, t months after it was first reported, is now dropping. The rate at which it is dropping is proportional to the square of the number of the reported cases.

a) Form a differential equation in terms of $\boldsymbol{x}$**, t** and a proportionality constant k.

Initially there were **2500** reported cases and one month later, they had dropped to **1600** cases.

b) Solve the differential equation to show that; $\boldsymbol{x = \frac{40}{9t+16}}$.

c) Find after how many months there will be **250** reported cases.

273. A machine depreciates at a rate proportional to its current value.

Initially, the machine is valued at **shs.2.5 million. 5 years later,** it was valued at **shs. 1.87 million.**

If $\boldsymbol{\theta}$ is the value of the machine after **t** years, form a differential equation and use it to find the;

i) value of the machine after **15 years**

ii) number of years it will take the machine to be valued **shs.0.5 million.**

274. Certain bacteria in a culture are thought to increase at a rate proportional to the number present in a colony.

If the number **doubles** in **10 years**, find by what factor the initial population has been multiplied after **further 20 years.**

275. The rate at which a candidate was losing support during an election campaign was directly proportional to the number of supporters he had at that time.

Initially, he had V_0 supporters and t-weeks later, He had V supporters.

i). Form a differential Equation connecting V and t

ii). Given that the supporters reduced to two thirds of the initial number in 6 weeks, solve the equation in i) above.

iii). Find how long it'll take for the candidate to remain with 20% of the initial supporters?

276. The temperature in a bathroom is maintained at the constant value of 20^0C and the water in a hot bath is left to cool down.
The rate at which the temperature of the water in the bath is cooling down is proportional to excess temperature.
Initially the bathwater had a temperature of 40^0C, and at that instant, it was cooling down a rate of 0.005^0C per second.
a) Show that: $\frac{dT}{dt} = -\frac{1}{4{,}000}(T - 20)$
b) After how long the temperature of the bathwater will drop to **36^0C**?

277. At time t hours, the rate of decay of the mass, x kg, of a radioactive substance is directly proportional to the mass present at that time. Initially the mass of the radioactive substance is x_0.
a) Show that: $x = x_0 e^{-kt}$, where k is a positive constant.
b) Find the value of t when; $x = \frac{1}{2}x_0$

278. A small forest with an area of **25km^2** has caught fire and it burns at a rate proportional to the difference between the total area of the forest squared, and the area of the forest destroyed squared.
When the fire was first noticed **7km^2** of the forest had been destroyed and at that instant the rate at which the area of the forest was destroyed was **7.2km^2 per hour**.
Show clearly that;
a) $\frac{dA}{dt} = \frac{1}{80}(625 - A^2)$, where A is the area of the forest destroyed by the fire, t hours after the fire was first noticed.

b) **14km²** of the forest will be destroyed, approximately **66 minutes** after the fire was first noticed.

279. A variable x decreases with time t, both in suitable units, at a rate directly proportional to the value of $\boldsymbol{x^3}$ at that time.
If the value of x is half of its initial value when **t = 3**, determine the value of t when x has reduced to **20%** of its initial value. (**Ans**: t=24)

280. A plant grows in a pot which contains volume, V of the soil.
At a time, t, the mass of the plant is m and the volume of the soil utilized by the roots is $\boldsymbol{\alpha m}$ where α is a constant. The rate of increase of the mass of the plant is proportional to the mass of the plant times the volume of the soil not yet utilized by the roots.

a) Obtain a differential equation and verify that it can be written in the form;
$\boldsymbol{V\beta\frac{dt}{dm} = \frac{1}{m} + \frac{\alpha}{V-\alpha m}}$ where β is also a constant.

b) Find, in terms of V and β, the time taken for the plant to double its mass, if initially, mass of the plant is; $\boldsymbol{\frac{V}{4\alpha}}$.

281. An Epidermic is spreading through a community at a rate which is proportional to the product of number of people who have contracted it and those who have not yet contracted it. Given that x is the portion of the people who have contracted the epidemic at time, t,

a) Form a differential equation relating x, t and a constant, k

b) Show that, if initially, a portion P_0 of the population had contracted the epidermic, then; $\boldsymbol{x = \frac{P_0}{P_0+(1-P_0)e^{-kt}}}$.

282. According to Newton's law of cooling, the rate at which a hot object cools is proportional to the difference between the temperature of the body and that of the surrounding air (assumed constant).

If an object cools from 100^0C to 80^0C in 10 minutes and from 80^0C to 65^0C in another 10 minutes, find the temperature of the;

a) surrounding.
b) object after a further 10 Minutes.

283. A liquid cools in an environment of a constant temperature of 21^0C at a rate proportional to the excess temperature.

Initially, the temperature of the liquid is 100^0C and after 10 minutes, the temperature drops by 16^0C.

Find how long it takes for the temperature of the liquid to be 70^0C.

284. A body which is at a higher temperature than its surroundings cools according to the Newton's law of cooling $\boldsymbol{\theta = \theta_0 e^{-kt}}$, where $\boldsymbol{\theta_0}$ is the original excess of temperature and $\boldsymbol{\theta}$ is the excess temperature after t minutes.

i) Show that; $\frac{d\theta}{dt}$ is proportional to θ

ii) If the original temperature of the body is 100^0C, the temperature of the surrounding is 20^0C and the body cools to 70^0C in 10 minutes, find the value of k

iii) At what rate is the temperature decreasing after 20 minutes?

285. A laboratory investigation was carried out to investigate the effect of a newly produced drug on a certain poultry bacteria.

It was revealed that the rate at which the bacteria is killed is directly proportional to the population present at that time.

Initially, the population was $\mathbf{P_0}$ and after **t** months, it was found to be **P**.

i). Obtain the differential equation connecting **P** and **t**, and solve it.

ii). If bacterial population reduced to **two thirds** of the initial population **6 months** later, solve the equation in i) above.

iii). Find how long it'll take for only **10%** of the original population to remain.

286. The mass m grammes of a burning candle, t-hours after it was lit, satisfies the differential equation: $\frac{dm}{dt} = -k(m - 10)$, where k is a positive constant.

If the total mass of the candle was initially 120 grammes, and three hours later, its mass had halved, show that;

a) $k = \frac{1}{3} In\left(\frac{11}{5}\right)$

b) Mass of the candle after further period of **3 hours** had elapsed is approximately **32.7 grammes.**

287. A cylindrical tank of height 150cm is full of oil which started leaking from a small hole at the side of the tank.

If the leaking of the tank is defined by a differential equation:

$$\frac{dh}{dt} = \frac{1}{4}(h - 6)^{\frac{3}{2}}$$

a) Show that: $t = \frac{8}{\sqrt{h-6}} - \frac{2}{3}$

b) State how high is the hole from the bottom of the tank.

Hence, show further that it takes 200 seconds for the oil level to reach 4cm above the hole level. (**Ans:** 6cm)

288. Mould is spreading on a wall of area $20m^2$.

When it was first noticed $2m^2$ of the wall was already covered by this mould.

Let A, in m^2 represent the area of the wall covered by the mould, after time t weeks.

The rate at which A is changing is proportional to the product of the area covered by the mould and the area of the wall not yet covered by the mould.

After a further period of 2 weeks the area of the wall covered by the mould is $4m^2$.

Show that:

$$A = \frac{20}{1+9\left(\frac{2}{3}\right)^t}.$$

289. The mass of a radioactive isotope decays at a rate proportional to the mass of the isotope present. The half-life of the isotope is 80 years.

Determine the percentage of the original amount which remains after 50 years.

(**Ans:** 64.8%)

290. A moth ball evaporates at a rate proportional to its volume, losing a half of its volume every **4 weeks**.

If the volume of the moth ball is initially **15cm^3** and becomes ineffective when its volume reaches **1cm^3,** how long is the moth ball effective?

291. A container in the shape of an inverted right circular cone, the height of the cone is 12cm and its vertical angle is 60^0.

A tap delivers water into the container at a rate which is directly proportional to the depth of water collected at any time t.

a) Show that the rate at which the water level is rising is inversely proportional to the depth of the water collected.
b) If it takes one (01) minute to collect a depth of 6cm of water, calculate the time in minutes it takes to fill the whole container.

292. In a certain type of a chemical reaction, a substance A is continuously transformed into a substance B. Throughout the reaction, the sum of masses of A and B remains constant and equal to m.
The mass of B present at time, t after the commencement of the reaction is denoted by x. At any instant, the rate of increase of the mass of B is k times the mass of A where k is a positive constant.
a) Form a differential equation relating x and t.
Hence, solve the equation, given that; $x = 0$ when $t = 0$.
b) Given also that $x = \frac{1}{2}m$ when $t = In2$, determine the value of k, and show that at time t, $\boldsymbol{x = m(1 - e^{-t})}$.
c) Hence, find the value of;
i) x (in terms of m) when $t = 3In2$.
ii) t when $x = \frac{3}{4}m$.

293. A plague wipes out a community at a rate proportional to population present at any time t.
If the original population is **4millions**, and the population reduces from **2.5** to $\boldsymbol{\frac{1}{5}}$ millions in **5 years**, find how long it takes to reduce the original population to **01million**.

294. In Mitooma town, the rate at which buildings are collapsing is proportional to

those that have already collapsed.

If initially the number of buildings that had already collapsed is N_0,

a) Show that; $\boldsymbol{N = N_0 e^{kt}}$ where k is a constant.

b) If the number of collapsed buildings doubled the initial number in **10 years**, find the value of k

c) By what factor of N_0 will the buildings have collapsed after **30 years**?

295. A man starts to climb a mountain whose height is **1000m** above its foot.

He notices that the rate at which the temperature, $\boldsymbol{\theta^0 C}$ drops with height is directly proportional to the height, $\boldsymbol{h}$ risen above the foot of the mountain.

The temperature is **16⁰C** at the foot and drops to **-9⁰C** at the top of the mountain.

a). Set up a differential equation that satisfies the above word problem.

b). Calculate the;

i). height at which the temperature reaches the freezing point of water.

ii). temperature at the foot of the mountain if the temperature at the top of the same mountain is $\boldsymbol{0^0 C}$

296. The population x of a certain town follows a mathematical model; $\frac{dx}{dt} = \frac{x}{100} - \frac{x^2}{10^8}$, where t is recorded in years.

Given that the population of that town was 100,000 in 1990.

a). Determine the population of the town as a function of t (i.e t in terms of x).

b). In what year will the population of the town double that of 1990?

297. Lumbers jacks start to cut down 100 mature trees at a rate which is directly proportional to the square root of the number of trees remaining uncut.

After 10 days, 36 trees have been cut down.

a). Form a differential equation and solve it.

b). Calculate the number of days taken to cut down the remaining trees, and state the number of trees being cut down per day by the end of 25^{th} day.

298. In a rat – breeding center, the population of the rats grow in such a way that at time, t months, the rate at which the population is growing is directly proportional to the number of rats, R present at that time.
If the rat population doubles in the first two months, when will the population of rats quadruple the initial population?

299. A forensic investigator is called to a crime scene where a body has been discovered in a hotel room.
At 09:27 AM, the investigator measures the body's temperature and finds it to be 32.5^0C.
The hotel's thermostat is set at constant 20^0C. An hour later, the investigator takes the temperature again, and it has dropped to 30.9^0C.
It's assumed that the victim had a normal body temperature of 37^0Cat the time of death.

a). Using Newton's law of cooling, form a differential equation and solve it.

b). Hence, estimate the time the murder occurred.

300. The body of a murdered victim was discovered at 11PM.
The police doctor on call arrived at 11:30 PM and immediately measured the temperature of the body which was 34.78^0C.
He again took the temperature after one and a half hours and it had dropped to 34.11^0C, and noted that the room temperature was constant at 25^0C.
Estimate the time of death, assuming the victim's normal body temperature was 37^0C.

END

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