UGANDA ADVANCED CERTIFICATE OF EDUCATION

PRINCIPLE MATHEMATICS

P425/1

Paper 1

TIME: 2 HOURS 30 MINUTES

INSTRUCTIONS TO CANDIDATES

- This examination paper consists of two sections: **A** and **B**.
- It has five examination items.
- Section A has one compulsory item.
- Section B has two parts: **Part I** and **Part II**. Respond to only one item from each part.
- Respond to three items in all.
- Any additional item(s) responded to will not be scored.
- All responses must be written in the answer booklet(s) provided.
- Responses to each item should start on a new page.
- Silent, non-programmable scientific calculators and mathematical tables may be used.

SECTION A

Compulsory item

ITEM 1

A district water authority plans to establish a solar-powered water monitoring station within a circular water distribution zone located on level ground. During the site survey, three boundary points of the zone were recorded as $A(1, 2)$, $B(7, 2)$, and $C(4, 5)$. These points will be used by the engineering team to determine the equation of the circular zone and its area so that the monitoring station can be accurately positioned at the centre. The monitoring equipment is mounted on a mast represented by the vector equation $\mathbf{r} = (4, 3, 0) + \lambda(0, 1, 14)$, where the monitoring device is fixed at $\lambda = 1$. Its calibration includes a depression-angle control system governed by the trigonometric condition: $2\sin 5\theta \cos 5\theta - 2\cos^2 5\theta - 2\sin^2 5\theta + 3 = 0, \quad 0° < \theta < 90°$. A maintenance officer stationed at $D(20, 15, 12)$ carries out routine inspections of the system. For operational efficiency, The device is designed such that it can operate in more than one direction to improve coverage efficiency, the mast must placed at the centre of the circular zone, the area enclosed by the zone must not exceed $30\,\text{m}^2$ to minimise construction costs, the distance between the maintenance officer and the monitoring device must be less than $35\,\text{m}$ to ensure effective visual inspection, and the mast must be nearly vertical, making an angle between $87°$ and $90°$ with the horizontal ground.

Task

Using mathematical justification, verify whether the proposed design satisfies all operational and structural requirements.

SECTION B

Part I

Attempt one item

ITEM 2

A national infrastructure agency is evaluating three salary schemes for new technical posts in an East African infrastructure programme over 4 months. The first job pays a monthly salary following an arithmetic progression starting at 1,000,000 UGX with a common difference of 75,000 UGX, plus a bonus following a geometric progression starting at 30,000 UGX and doubling each month. The second job bases monthly pay on committee efficiency, where a committee of atleast 9 staff is chosen from 12 members. Each valid formation contributes 3,050 UGX to the compensation index. The third job uses a monthly pay of a logarithmic performance indicators given by two times log base ten of 100 plus log base ten of 10 minus log base ten of 5, and this index is scaled using a factor of 320,000 UGX to match monthly compensation levels for fair comparison. After 4 months, total earnings from all three models are computed and compared.

Task

Determine the best job to choose based on earnings.

ITEM 3

A construction company is undertaking the construction of a 12-storey smart classroom tower for a secondary school. The project manager applies mathematical modelling to plan materials, staffing, structural safety, and overall efficiency of the project. For cost balancing under engineering standards, the unit costs of the materials are constrained by two structural pricing conditions:five times the cost of cement plus four times the cost of sand plus three times the cost of gravel equals 530 million shillings, three times the cost of cement plus six times the cost of sand plus twice the cost of gravel equals 370 million shillings, and seven times the cost of cement plus four times the cost of sand plus five times the cost of gravel equals 790 million shillings. The company wishes to use 100 cement, 15 sand and 120 gravel. The total budget available for construction materials is 15,000 million shillings. The engineer uses these constraints and material quantities to check if the total cost fits the budget. After determining cost behaviour, then analyses the actual staircase design, which follows an arithmetic progression (AP), where the 4th floor requires 37 tiles and the 10th floor requires 91 tiles. Using this information, the learner determines the common difference, the first term, and finally computes the total number of tiles required for all 14 floors. The results from model is then used to check whether the available 1000 tiles. The Engineers model the position of a steel beam tip using complex numbers. The initial position is $z_1 = 3 + 3\sqrt{3}\,i$. The beam undergoes a structural transformation due to load redistribution represented by multiplication with $2 + i$. After this, a stabilizing adjustment is applied $z_2 = 3 + 3i$. The final position is used to assess stability of the monitoring system. In project scheduling, four engineers are assigned to supervise four critical floors. The number of possible inspection arrangements is determined to ensure efficient planning of supervision schedules, with a requirement of at least 15 distinct valid schedules.

Task

Using mathematical modelling, verify whether the construction project is feasible in terms of cost, design, structural stability, and scheduling efficiency.

Part II

Attempt one item

ITEM 4

Kiin Industrial Manufacturing Complex consists of two major production units: a metal fabrication section and a shoe manufacturing section. In the metal fabrication section, a worker is designing an open-top storage box from a rectangular sheet of metal measuring 4800 cm by 1300 cm. Equal squares of side x cm are cut from each of the four corners and the remaining edges are folded upward to form the sides of the box. The resulting box is intended to store finished products from the factory. In the shoe manufacturing section, Kiin StepWear Ltd has recently upgraded its machinery and trained its workforce to improve productivity. The management observes that t hours after production begins, the instantaneous rate of shoe production is given by $t^2e^{0.01t}$ shoes per hour, where $P(t)$ represents the total number of shoes produced. At the start of production, no shoes had been manufactured. The shoes produced are packed into standard storage boxes during production. After 120 hours of continuous production, the management wishes to use the model to forecast total output for planning export contracts and market expansion. Each shoe produced occupies a volume of 530 cm^3. The box is required to have sufficient capacity to store the total shoe production from the factory.

Task

Help the Kiin Industrial Manufacturing Complex find out whether the metal box has sufficient capacity to store all the shoes produced.

ITEM 5

A geological research team in western Uganda is investigating a mountain suspected to have dormant volcanic activity. The mountain profile is modelled by $y = -0.01x^2 + 100$, where y is the height (m) and x is the horizontal distance (m) from the central axis. A cross-sectional sketch of the curve is required to determine the mountain's topography and verify whether it has a single symmetrical peak suitable for magma accumulation. The mountain is assumed to form a solid of revolution when the curve is rotated about the y-axis. The resulting volume represents the volcanic rock structure used to estimate magma storage capacity. Geologists state that volcanic activity is possible only if the volume of molten magma underground is at least 1.2×10^6 m^3. The temperature of rock within the mountain varies with depth. The rate of change of temperature is proportional to the difference between the geothermal reference temperature of 900°C and the rock temperature heat. At the surface, the temperature is 60°C. At depths of 200 m and 400 m, the temperatures are 180°C and 420°C, respectively to calibrate and validate the temperature model with observed data. Geologists further state that volcanicity is possible when the temperature reaches 800°C or higher when the height is below 3000 m pressure and depth conditions must still support melting for magma generation.

Tasks:

Determine whether:

1. its symmetric central peak favours volcanic accumulation from an accurately sketch.

2. volume of the volcanic rock structure is sufficient for the volcanic activity

3. the region shows evidence of possible volcanic activity due to the temperature at 800°C

END