

P425/1
PRINCIPAL
MATHEMATICS
Paper 1
July/August 2026
3 hours



WAKISSHA JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PRINCIPAL MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- This examination paper consists of two sections **A** and **B**. It has of **five** items.
- Section **A** has **one** compulsory item.
- Section **B** has two parts **I** and **II**, respond to one item from each part.
- Respond to **three** items in **all**.
- Any additional item(s) responded to will **not** be scored.
- Begin each item response on a **fresh** page.
- All necessary working **must** be shown clearly.
- Graph papers is provided.
- All responses must be written in the answer booklet(s)/sheets provided.
- Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.



SECTION A

The item in this section is compulsory.

Item 1

A city planning authority is optimizing its urban transport infrastructure by mapping key service hubs using a three dimensional coordinate system.

The current hubs are located at $M(2,4,3)$ km, $N(8,6,1)$ km, and $P(4,10,2)$ km. The planners have identified a site for a new logistics depot at $Q(14,8,4)$ km. The project director will only approve the depot if it lies on the straight supply line defined by hubs M and N, as this ensures efficient traffic flow.

For emergency rapid – response, a fire station is proposed at $R(2,4,8)$ km. the planners must determine the shortest distance from the fire station to the plane containing hubs M, N and P to assess the response time. The fire service response will only be confirmed adequate if this distance is less than 10 km.

During the city's expansion, engineers mapped three sensor nodes along the perimeter of a circular restricted access zone surrounding a downtown square. The three nodes were recorded at $X(16,4)$ km, $Y(4,16)$ km, and $Z(-4, 4)$ km. The city authority must determine whether a newly planned emergency access road passing through point $S(4,15)$ km lies within this restricted zone for security clearance.

Finally, during system testing, engineers discovered that the signal reach of a mobile weather station is defined by $S = (24\sin 2\theta + 7\cos 2\theta - 3)$ km. The weather bureau has a target that maximum signal reach for their stations of at least 20 km to guarantee coverage for all peripheral districts.

Task:

- Will the project director approve the new logistics depot location?
- Help the planners to check whether the fire service response distance is adequate.
- Does the emergency access road lie within the restricted security zone?
- Will the engineers achieve the required signal reach for the weather station?

SECTION B

Respond to only two items from this section, choosing one from each of parts I and II.

PART I

Item 2

A manufacturing enterprise has embarked on a strategic upgrade to optimize its industrial liquid filtration and power control systems. To ensure operational efficiency, the management team must make critical decisions regarding expansion, procurement, and system stability.

During the initial phase, the company planned to increase the number of filtration units installed annually. In the first year, 10 units were installed, and in the second year 20 units were installed. The engineering planners agreed that the annual installations would continue to grow by the same constant ratio. However, the operational budget can only support the expansion if the total number of units installed during the first ten years does not exceed 12,000.

To maintain the production line, the procurement office purchased three types of electrical components: voltage regulators, sensors, and transformers. The office received supplies from three different industrial vendors in “thousands of dollars” as shown in the table below;

Vendor	Voltage regulators	sensors	Transformers	Total cost
Alpha	2	1	3	45
Beta	3	2	1	35
Gamma	1	3	2	40

The finance department needs to determine the individual cost of each component and decide whether an allocated budget of 200,000 USD is sufficient to purchase an additional 5 voltage regulators, 4 sensors, and 6 transformers.

In the final phase of the project, a complex automation system was implemented to manage control signals. The reference signal of the system is $Z_0 = (2 + 3i)$ and the control signals are modelled by the equation $Z^3 - 3Z^2 + 7Z - 5 = 0$. The automation system will remain stable and operational if the modulus of the difference between the reference signal and each control signal, $|Z_0 - Z|$, is less than 4 units.

Task

- (a) Help the planning team establish if the installation programme is financially sustainable.
- (b) Advise the management team to take well informed decisions as they undertake the remaining two projects.

Item 3

In a rural village, the village development committee agreed to collect funds every month from households to install a community solar power grid. They decided to contribute UGX. 20,000 in the first month, and the monthly contribution then increases by UGX 5,000 each month thereafter for 20 months. The committee chair wants to determine whether the planned contributions will be sufficient to meet the budget target for the solar equipment of UGX 50 million, given a total of 400 households participating in the scheme.

The local cooperative has also introduced a productivity bonus for the solar technicians based on their hours worked per week. The total number of hours(x), accumulated by one technician in a week must satisfy the conditions: "the square of the hours plus 4,000 must be less than or equal to 125 times the number of hours" for the technician to qualify for the performance bonus. A senior technician has worked a total of 55 hours in a certain week and is unsure whether he qualifies for the bonus.

The maintenance crew consists of 6 junior technicians and 5 senior technicians. The supervisor wants to create a daily maintenance team of 5 technicians with an equal number of junior and senior technicians (or as close as possible to maintain balance). This is intended to ensure proper mentorship and will only be effective if the total number of possible districts teams exceeds 100.

To improve the safety of the remote monitoring software, the I.T department is installing a new encryption system. The strength of the system depends on a security key represented by x units, where for a strong security key, x must be at least 2 units. During testing, the software engineer finds that the overall security rating is obtained by combining two separate measurements: the first measurement is $\log_x 9$; the second measurement is $2\log_3 x$. The system will function correctly only when the total security rating is equal to 3 units.

Task:

- (a) Perform a financial feasibility study for the village development committee to determine if they will meet the set budget target.
- (b) Advise the maintenance supervisor on the effectiveness of the team selection process based on the number of district teams possible.
- (c) Determine the eligibility of the senior technician for the performance bonus.
- (d) Evaluate the technical viability of the new encryption system.

PART II

Item 4

A local cooperative is establishing an apiary project and intends to use 100 metres of protective netting to enclose a rectangular area for the bee hives. The project requires a minimum area of 500 m^2 for the hives and pollinator gardens to be effective. Additionally, the cooperative has designed a central honey-processing facility with a cylindrical storage vat.

The vat is to be constructed such that the sum of its height and its base radius is 15 metres. The facility requires a storage capacity of at least 400 m^3 for the harvest to be considered viable. To monitor hive health, the cooperative recorded an initial bee colony population of 2,000. The population changes over time due to seasonal growth and natural attrition, modelled by

$$\frac{dp}{dt} = 0.05p - 150, \text{ where } P \text{ is bee population after time } (t \text{ years}).$$

For the project to be deemed sustainable, the colony population must reach at least 4,000 within 10 years.

Task

Help the cooperative establish whether;

- (a) the available netting is sufficient to enclose the required area for the apiary.
- (b) the proposed storage vat design, can provide enough capacity to sustain the processing facility.
- (c) the colony population will remain sustainable after 10 years.

Item 5

A local paint company manufactures specialized industrial coatings, and its production profit function over time (t years) is modelled by $\frac{dp}{dt} = 0.04p(1 - 0.02t)$. After the first year of operation, the company recorded a profit of 500,000. Management forecasts that in five years, the profit will reach Ugx 2 million. The board requires an assessment to determine if this growth target is realistic.

To package the paint, the company designs a container made from a square sheet of reinforced plastic measuring 60 cm x 60 cm. The container is formed by cutting squares of side y cm from each corner and folding the sides upward. For efficient shipping, the company requires a container with maximum volume; however, only containers with a volume between $15,000 \text{ cm}^3$ and $18,000 \text{ cm}^3$ meet the durability standards for high-pressure storage.

The company also contracts a specialized firm to produce small, spherical metallic canisters for sample batches. During the manufacturing process, a 1.5% measurement error occurred in the radius of the canisters. According to the industry safety guidelines, any canister with a volume error exceeding 5% will not be certified for distribution.

Task

- (a) Help the company know whether;
 - (i) The projected profit growth target is achievable.
 - (ii) The container design meets the required volume standards for durability.
- (b) Will the spherical metallic canisters be certified for distribution?

END