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Uganda Advanced Certificate of Education

MATHEMATICS **535/1**

Paper 1

2026

Element Of Construct

Construct	***Main focus***
Algebra	Algebraic concepts and applications
Geometry	Geometry and its applications
Calculus	Differentiation, integration and applications

© *2026* ***Mathematics*** *– Solomon Mwere F5 SPAPS*

Item 1

A food-processing company in Kampala is testing a new method of preserving fresh fruit juice. During the experiment, the concentration of a particular preservative in the juice changes with time. The research team uses the model

$$C(t) = \frac{A2^t}{\sqrt{2^{t+1}}}$$

where C(t) is the concentration of the preservative, in milligrams per litre, after t hours, and A is a positive constant.

At the beginning of the experiment, the concentration was not recorded. However, after 2 hours, laboratory measurements showed that the concentration was

$$6\sqrt{2}\, mg/litre$$

The laboratory technician needs to establish a simpler form of the model so that the concentration can easily be estimated at different times.

Later, the company determined that the juice would only meet the required preservation standard when the concentration reached 2,424 mg/litre. The technician wants to know how long after the beginning of the experiment this concentration will first be reached. On substituting the required concentration into the model, the resulting equation contains the unknown tt as an index, making it difficult for the technician to determine the time directly.

Task

a) Help the laboratory technician determine the value of A and hence express C(t) in its simplest form.

b) Help the technician determine the time, t, at which the concentration first reaches 2,424 mg/litre, giving your answer correct to 2 decimal places.

Item 2

Ssemakula runs a small carpentry workshop in Mukono. He noticed that the number of chairs he produces each week follows a certain pattern. In the first week, he made 5 chairs, and each subsequent week he increased production by 3 chairs more than the previous week. At the same time, his profit from sales grows in a different way: in the first week, he earned UGX.40,000, and each week his profit doubles compared to the previous week.

Ssemakula wants to understand these patterns mathematically so that he can plan ahead. He consults his friend, a mathematics teacher, who explains that the chair production follows an *Arithmetic Progression*, while the profit follows a *Geometric Progression*. The teacher then challenges Ssemakula to prove a general formula by induction, showing that the sum of the first n natural numbers is;

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$$\frac{1}{2}+\frac{1}{6}+\cdots+\frac{1}{n(n-1)}=1-\frac{1}{n}$$

Task

a) Determine the number of chairs Ssemakula will produce in the 10th week, and calculate the total number of chairs produced in the first 10 weeks.

b) Find the profit he will earn in the 6th week, and calculate the total profit over the first 6 weeks.

c) Using mathematical induction, prove that the sum of the first n natural numbers is $1-\frac{1}{n}$.

Item 3

Nalongo, a landscape designer in Kampala, has been contracted to create a decorative pond in the middle of a hotel garden. The pond's *length (l metres)* depends on the number of decorative tiles she arranges along its edge. The mathematical model for this is given by the quadratic relation, $L=-x^2+12x-20$, where x represents the number of tile rows.

Nalongo quickly realizes that this quadratic function is central to her design decisions. First, she must determine the *maximum possible length* of the pond, since the hotel manager insists that the pond should not exceed a certain size but should still be as large as possible for aesthetic appeal. This requires her to identify the vertex of the quadratic curve and interpret it in terms of the garden's dimensions.

However, the manager also wants to know whether the pond can actually be constructed with real dimensions. To check feasibility, Nalongo must analyze the quadratic equation,

$$-x^2+12x-20=0$$

and verify whether it has *real roots*. This involves applying the discriminant condition, ensuring that the design does not collapse into an impossible structure.

Finally, Nalongo needs to explain the *range of values of l*. Since the quadratic is concave (opening downwards), the pond's length cannot take arbitrary values. The range will tell her the minimum and maximum possible lengths, guiding the hotel in choosing a practical design.

The hotel manager is eager for a clear mathematical report that shows not only the maximum length but also the feasibility of the design and the exact range of values the pond's length can take.

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Task

a) Find the maximum possible length of the pond using the quadratic relation.

b) Determine the condition for the quadratic equation to have real roots, and check whether it is satisfied.

c) State the range of possible values of the length for the pond.

Item 4

Mr. Ssebugwawo, a civil engineer in Jinja, is designing a suspension bridge across the Nile. The curved cable of the bridge is modeled by the trigonometric function

$$y(x) = 15\cos\left(\frac{\pi x}{30}\right) + 5$$

where $y(x)$ represents the vertical height of the cable above the deck (in metres), and x is the horizontal distance from the centre of the bridge in metres. The bridge extends from $x = -30$ to $x = 30$.

The design committee insists that the bridge must be both aesthetically pleasing and structurally safe, so Ssebugwawo has to analyze the mathematics carefully. First, he must determine the maximum and minimum heights of the cable above the deck, since these values will dictate how much tension the supporting towers must withstand. This requires applying trigonometric properties to identify the turning points of the cosine curve and interpreting them in terms of the bridge's geometry.

Later, the committee asks him to estimate the total vertical area enclosed between the cable and the deck across the span of the bridge. This is crucial for calculating the amount of steel needed to construct the cable. To do this, Ssebugwawo must integrate the function $y(x)$ over the interval $(-30, 30)$, applying calculus to find the exact area.

The committee expects a clear mathematical report that shows both the extreme values of the cable height and the total area, so they can decide whether the design is feasible and within budget.

Task

a) Determine the maximum and minimum heights of the cable above the deck using the trigonometric function.

b) Use integration to calculate the total vertical area enclosed between the cable and the deck across the span of the bridge.

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Item 5

A farmer in Luwero District is planning to establish a large vegetable farm and intends to use an irrigation channel to transport water from a nearby reservoir. He contracts an agricultural engineer to design a channel that can carry enough water to the farm while keeping the construction costs manageable.

During the initial design, the engineer proposes a rectangular channel whose width is represented by x metres. From measurements taken at the construction site and the amount of space available, he establishes that the possible width of the channel satisfies, $x^2 - 8x + 12 = 0$

However, the engineer notices that one of the possible widths would make the channel too narrow for the intended water flow. He therefore needs the farmer's mathematics student to help him establish the suitable width. The engineer further explains that the channel's length will be 4 metres greater than its width, while its depth will remain 1.5 metres. Before purchasing construction materials, he needs to know how much water the completed rectangular section would hold when it is filled to its full depth.

After the initial design, the engineer discovers that the water requirement of the farm is increasing. He therefore considers replacing part of the original channel with a specially designed 6-metre section whose cross-sectional area changes from one point to another. The cross-sectional area at a distance, x metres from one end of this section is represented by

$$A(x) = 12 + 6x - x^2, \qquad 0 \le x \le 6.$$

The engineer is now faced with a problem. At some point along the section, the channel appears wider than at other points, and he is unsure where this occurs and how large the cross-section becomes. He also cannot estimate the amount of water that the entire section can carry by simply taking one cross-sectional area and multiplying it by 6 metres because the area changes continuously along the channel. He asks the mathematics learner to help him overcome these difficulties and obtain a reliable estimate of the section's water-carrying capacity.

The contractor informs the farmer that construction of this modified section will cost UGX.100,000 for every cubic metre of water-carrying capacity provided. The farmer has only UGX.4,500,000 available for this part of the project. Before construction starts, the engineer wants to know the greatest cross-sectional area of the section, the total amount of water it can accommodate, and whether the farmer's available funds will meet the contractor's estimated cost.

Task

a) Assist the engineer with the initial channel design.

b) Assist the engineer with the modified section and advise the farmer.

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Item 6

A secondary school in Uganda plans to establish a small production unit where students will manufacture rectangular metal boxes for storing laboratory materials. The school administration has set aside a fixed amount of metal sheet for making the boxes, but the workshop instructor is concerned that the proposed design may not make efficient use of the available material.

The instructor proposes that each box should have a length that is 6 cm greater than its width, while its height is to be *half the width*. During the first trial, the students discover that the volume of the box does not match the storage requirement set by the school. The instructor records that the required volume is 1,920 cm^3 and asks the mathematics students to help establish suitable dimensions for the box before more metal sheet is cut.

After solving the problem, the instructor considers producing a second type of box. In this design, the number of boxes produced each day depends on the number of students xx assigned to the workshop. From previous observations, he models the daily production by;

$$P(x) = x^2 - 12x + 40$$

The workshop can only accommodate between 2 *and* 10 *students* at a time. However, the instructor notices that increasing the number of students does not always increase the efficiency of production. At certain levels of staffing, the production figures become equal, and he wants to understand the relationship between these values before deciding how many students should work in the unit.

The school also sells the finished boxes. The instructor finds that the price charged for each box is affected by the number of boxes produced. If q boxes are produced, the price per box, in thousands of Uganda shillings, is represented by;

$$C(q) = 50 - 2q$$

The school treasurer is interested in knowing at what production levels the income from sales would reach a specified target. However, the instructor is unsure whether the mathematical model gives one possible production level or more than one. He therefore asks the mathematics students to investigate the relationships in the production and pricing models before the school commits more resources to the project.

Task

a) Assist the workshop instructor with the design of the storage box.

b) Assist the instructor in analysing the production and pricing information.

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Item 7

It was a busy Saturday morning at Kalerwe Market when Mr. Ssemanda, a young trader dealing in fresh fruits, realised that his business had grown much faster than he had expected. For several months, he had been using a small rectangular stall, but the increasing number of customers meant that he needed more space to display his produce. He approached the market authorities to request a larger stall.

The authorities informed him that a new rectangular space was available. Its length was 5 metres greater than its width, and the total area available was 84 square metres. Mr. Ssemanda immediately calculated that the space would be enough, but when he tried to mark out the dimensions on the ground, he became confused because two different sets of measurements seemed possible from the information he had been given. He called his nephew, a mathematics student, to help him settle the matter before he paid for the space.

A few weeks later, Mr. Ssemanda decided to introduce baskets for packaging his fruits. He wanted the number of oranges placed in each basket to follow a particular pattern so that customers buying larger quantities would receive progressively more oranges. On the first day, he placed 3 oranges in the first basket, 6 in the second, 9 in the third, and continued in the same manner. His assistant, however, suggested that the pattern could instead be arranged in a way where each new basket contained twice as many oranges as the previous basket. The two workers disagreed about which arrangement would provide more oranges after a certain number of baskets, and they asked the mathematics student to settle their argument.

Meanwhile, the market treasurer introduced a new pricing system. A customer buying only a few baskets would pay a higher price per basket, while customers buying many baskets would receive a discount. From the records, Mr. Ssemanda noticed that when the number of baskets sold increased, the total amount of money collected did not always increase at the same rate. He represented the total revenue, in thousands of Uganda shillings, by

$$R(x) = -2x^2 + 40x + 150,$$

where x represents the number of baskets sold.

At the end of the day, Mr. Ssemanda wanted to know whether there was a particular number of baskets at which his earnings would be greatest. He also wanted to compare this with the amount he expected to earn if he sold 10 baskets. Unfortunately, he was not familiar with the mathematical behaviour of the expression representing his revenue.

Task

a) Help Mr. Ssemanda settle the issues concerning the size of his stall and the arrangement of oranges in the baskets.

b) Help him interpret the revenue information and make an appropriate decision about his sales.

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Item 8

A group of young entrepreneurs in Mbarara has established a small factory for processing maize flour. As demand for their product increases, they plan to install a larger storage facility and improve the way maize is supplied to the factory.

During the first few weeks, the manager records the quantity of maize delivered to the factory each day. He notices that the quantities follow a regular pattern. On the first day, 120 kg are delivered, and the quantity increases by 30 kg each day. However, because of limited storage space, the manager wants to know how much maize will have been delivered altogether by the end of the first 15 days. He is also considering a second supplier whose deliveries increase by a constant ratio rather than by a constant amount. The manager is finding it difficult to compare the two supply arrangements because the records from the second supplier are expressed in terms of successive multiples.

The company is also designing a rectangular storage shed. The available floor has an area of $480\,m^2$, but a walkway of uniform width is to be left around the inside of the shed. After allowing for the walkway, the usable rectangular storage area is expected to have dimensions involving an expression of the form;

$$x + \frac{480}{x}$$

The engineer suspects that there is a particular value of xx that gives the most efficient dimensions, but he is unsure how to establish it mathematically. He has tried several values and obtained different results, making it difficult for him to recommend a final design to the company.

Before the construction contract is awarded, the project manager asks a mathematics student to examine both the supply records and the proposed storage design and provide recommendations based on the mathematical evidence.

Task

a) Assist the manager with the comparison and interpretation of the two maize-supply arrangements.

b) Assist the engineer with the storage-shed design and advise the project manager on the most efficient dimensions.

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Item 9

A logistics company in Kampala has introduced a new system for moving packages between different sections of its warehouse. During a trial, two automated carts are used to transport packages from a collection point to different storage areas. The movement of each cart is represented by a vector, with the horizontal and vertical components showing its displacement from the starting point.

During one of the trials, the supervisor discovers that the recording system failed to capture the individual displacements of the two carts. The only reliable information available shows that the combined horizontal displacement of the carts was 18 metres, while their combined vertical displacement was 12 metres.

From observations made during the trial, the supervisor also knows that the horizontal displacement of the first cart was twice its vertical displacement, while the second cart travelled 3 metres further horizontally than vertically. He is finding it difficult to reconstruct the two displacement vectors from these records and wants to establish whether the information recorded is sufficient to determine the actual movements.

In a second trial, the company changes the loading arrangement. The two carts now carry different numbers of packages. The supervisor records that the total number of packages transported by the two carts is 46, while the first cart carries 4 more packages than twice the number carried by the second cart. Before the system is approved, he wants to know exactly how many packages each cart should be assigned so that the loading instructions can be entered correctly.

The warehouse manager has asked a mathematics student to examine the trial records and help resolve the problems before the automated system is put into full operation.

TASK

a) Assist the supervisor with the reconstruction and interpretation of the carts' movements.

b) Assist the warehouse manager with the package-allocation problem and advise him on the appropriate loading arrangement.

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Item 10

A construction company in Entebbe is using a drone to inspect a newly constructed suspension bridge. The engineers want the drone to follow a predetermined flight path so that it can photograph specific sections of the bridge without interfering with construction activities.

The drone's position relative to a fixed point on the bridge is described by

$$x = 2t + 1, \qquad y = t^2 - 4t + 3$$

where t represents time in minutes and *x* and *y* are measured in metres. The control engineer needs to understand the shape of the path traced by the drone, but the monitoring software only provides the coordinates in terms of *t*. He is finding it difficult to determine the equation of the actual path on the coordinate plane and to identify where the drone crosses a particular section of the bridge.

During the inspection, the engineers are particularly interested in the point where the drone's path is momentarily horizontal because photographs taken there are expected to give the clearest view of the bridge supports. The engineer also wants to know the gradient of the flight path at another specified position so that he can adjust the drone's direction if necessary.

The project manager has asked a mathematics student to analyse the flight data before the drone is allowed to carry out the full inspection.

Task

a) Assist the control engineer with the analysis of the drone's flight path and its position on the bridge.

b) Assist the project manager with the investigation of the drone's direction of motion at the important points along its flight path.

Answers

1. (a) The value of the constant is $A = 6$, and the simplified concentration model is $C(t) = 6 \cdot 2^{\frac{t-1}{2}}$. **(b)** The preservative concentration first reaches 24 mg/litre after 5.00 hours.

2. (a) Ssemakula will produce 32 chairs in the 10th week, and his total production for the first 10 weeks will be 185 chairs. **(b)** Ssemakula will earn UGX 1,280,000 in the 6th week, and his total profit over the first six weeks will be UGX 2,520,000. **(c)** The required identity has been successfully proved by mathematical induction.

3. (a) The maximum possible length of the pond is 16 metres, achieved when $x = 6$ tile rows are used. **(b)** The discriminant condition is satisfied because the discriminant is $64 > 0$. Therefore, the quadratic equation has two real roots, $x = 2$ and $x = 10$, confirming that the design is feasible. **(c)** The range is $L \leq 16$. Practical range for a physical pond: $0 < L \leq 16\, m$.

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4. (a) The maximum height of the cable is 20 m at the centre of the bridge $(x = 0)$, while the minimum is $-10\,m$ at $x = \pm 30$. **(b)** The total signed area between the curve and the deck over $[-30,30]$ is $300\,m^2$. The model is not physically consistent with a cable that must remain above the deck, because it gives negative heights at the ends.

5. (a) The appropriate channel dimensions are; $Width = 6\,m, Length = 10\,m, Depth = 1.5\,m$. When completely filled, the channel can hold 90 m^3 of water. **(b)** The modified section has a maximum cross-sectional area of $21\,m^2$, which occurs at $x = 3\,m$. Its total water-carrying capacity is $108\,m^3$. At UGX.100,000 per cubic metre, the estimated construction cost is UGX.10,800,000. Since the farmer has only UGX.4,500,000, the available funds are not sufficient. The farmer would need an additional UGX.6,300,000 to construct the entire modified section at the quoted rate.

6. (a) The suitable dimensions of the storage box are approximately; $Width = 13.86\,cm$, $Length = 19.86\,cm$, $Height = 6.93\,cm$. These dimensions give a volume of approximately $1920\,cm^3$, satisfying the school's storage requirement. **(b)** Production is lowest at 6 students (4 boxes), while 2 or 10 students produce 20 boxes. The maximum practical income is UGX.312,000 at 12 or 13 boxes. Therefore, careful planning of staffing and production is necessary.

7. (a) The stall should measure 12 m by 7 m. The geometric arrangement of oranges, where each basket doubles the previous one, will eventually provide more oranges than the arithmetic arrangement. **(b)** Mr. Ssemanda should sell 10 baskets, since this gives the maximum revenue of UGX.350,000 according to the given model.

8. (a) The first supplier will deliver 4,950 kg of maize in 15 days. The second supplier follows a geometric pattern, but more information is needed to make an exact comparison. **(b)** The most efficient shed dimensions are approximately 21.91 m × 21.91 m, giving a floor area of 480 m^2.

9. (a) The information is sufficient: Cart 1 = (6, 3) m, Cart 2 = (12, 9) m. **(b)** The correct allocation is 32 packages for Cart 1 and 14 packages for Cart 2.

10. (a) The drone follows a parabolic path and crosses the x-axis at (3, 0) and (7, 0). **(b)** The drone is momentarily horizontal at (5, −1), where the gradient is 0.

"Where there is a problem, there is a solution waiting to be discovered."

END

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