MARKING GUIDE

ITEM 2 - Graphical Analysis of a Reinforced Concrete Lintel Beam

1. The question (as set)

A 200 mm x 300 mm RC lintel beam spans 3 m between 200 mm walls (simply supported). It carries a 3 m-high block wall pressing down evenly on the beam at 6 kN/m, plus two timber roof-truss point loads of 4 kN each, sitting 1 m and 2.5 m from the left-hand end. The beam's moment capacity is known to lie between 12.5 kNm and 17.3 kNm, and each wall support can safely carry between 10 kN and 14.2 kN. Candidates must perform a graphical analysis (space diagram, vector/force diagram and funicular/link polygon - NOT the analytical/calculation method) to find the support reactions, the shear force diagram and the bending moment diagram, and use these to judge whether the beam is workable.

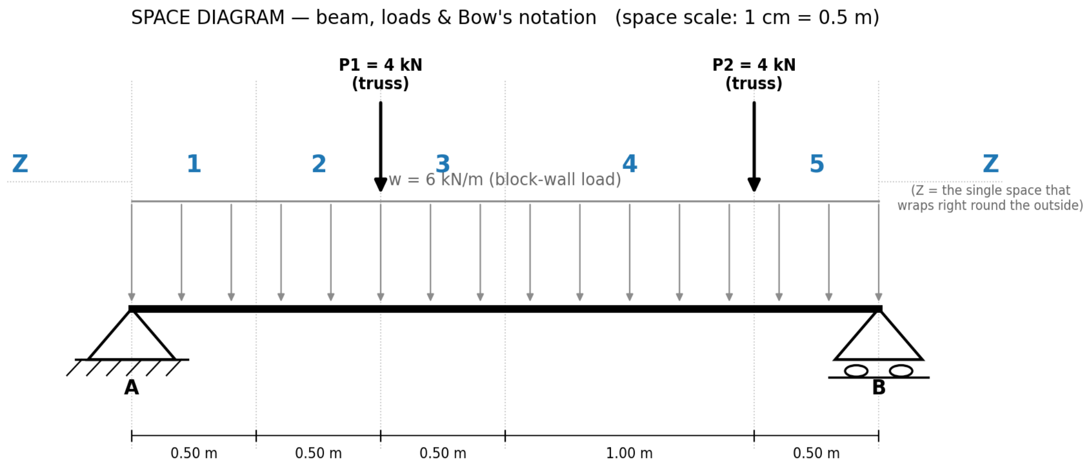
2. Idealisation used in this model answer

The UDL is split into three equal 1 m strips and each is replaced by a point load at its own centroid - this reproduces the exact total moment of the UDL regardless of how many strips are used, so it introduces no error in the reactions:

- W1 = 6 kN at 0.5 m from A
- W2 (truss P1) = 4 kN at 1.0 m from A
- W3 = 6 kN at 1.5 m from A
- W4 = the third 6 kN UDL strip (centroid at 2.5 m) combined with truss load P2 (4 kN, also at 2.5 m) = 10 kN at 2.5 m from A

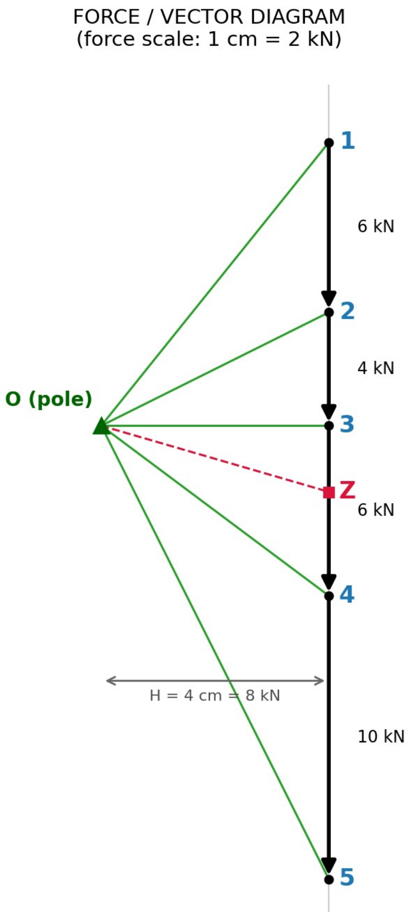
Total load = 6 + 4 + 6 + 10 = 26 kN, matching 18 kN (UDL) + 4 kN + 4 kN (trusses) exactly.

3. Space diagram and Bow's notation



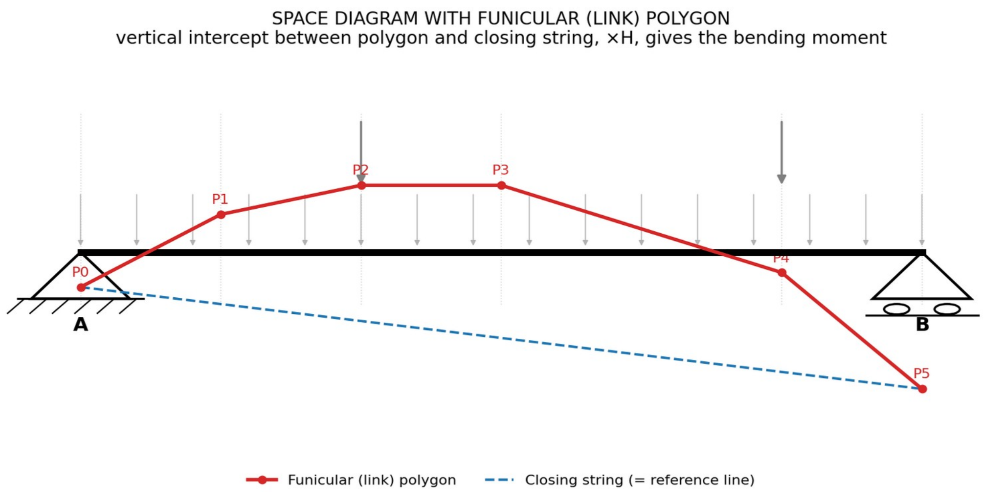
Every space between adjacent lines of action is lettered/numbered going around the diagram: Z is the single space that wraps around the outside of the whole system (left of A and right of B are the same space); 1-5 are the spaces between the load lines. A force is then named after the two spaces it separates, e.g. RA = force "Z-1", W1 = force "1-2", ... , RB = force "5-Z".

4. Force (vector) diagram



The known loads (6, 4, 6, 10 kN) are plotted as a straight vertical load line, points 1-2-3-4-5, to a force scale of 1 cm = 2 kN. A pole O is chosen at a convenient pole distance H (here H = 8 kN, drawn as 4 cm), and rays are drawn from O to each point on the load line.

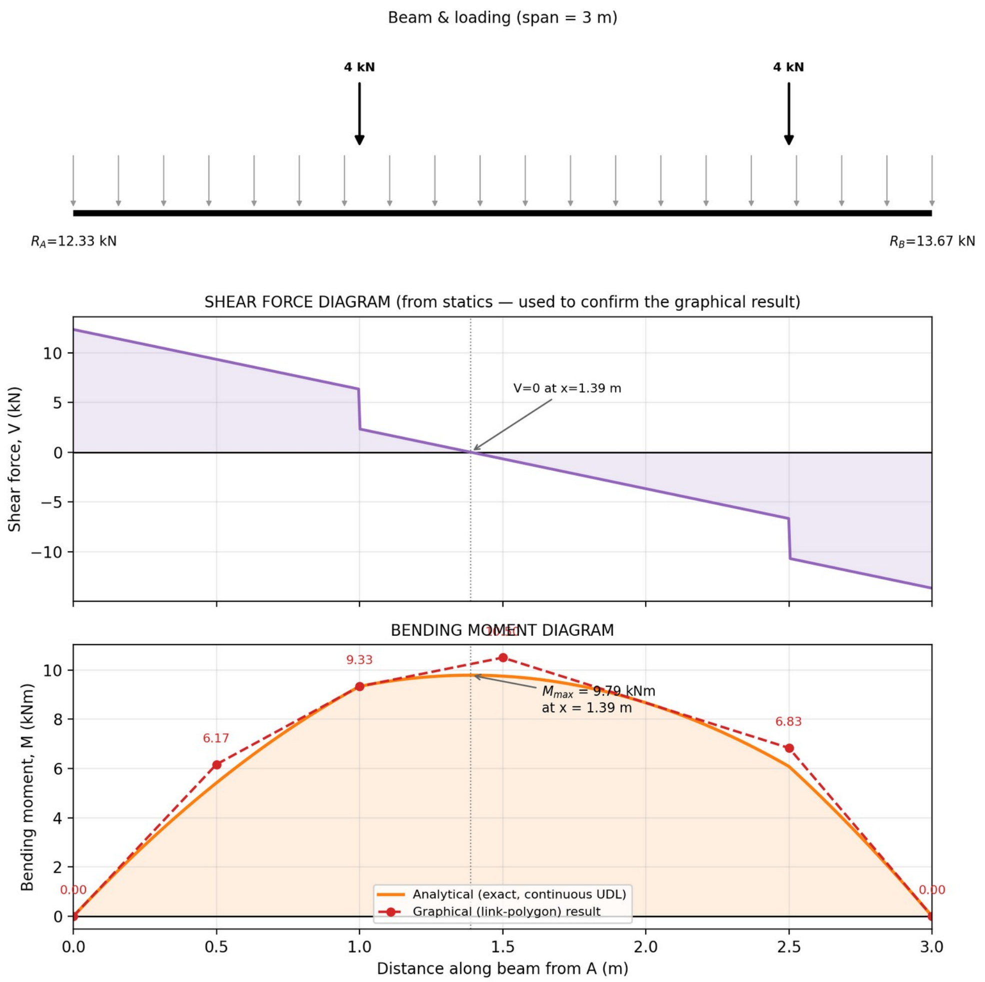
5. Funicular (link) polygon and reactions



Starting on the line of action of RA, a line is drawn parallel to ray O-1 until it meets the line of action of W1; from there, a line parallel to O-2 is drawn to meet W2's line of action, and so on through to RB. Joining the first point (P0) to the last (P5) gives the closing string. A line through the pole O, drawn parallel to this closing string, locates point Z on the load line.

Reading off the force diagram: RA = distance Z-1 = 6.17 cm x 2 kN/cm = 12.33 kN, and RB = distance 5-Z = 6.83 cm x 2 kN/cm = 13.67 kN. (Check by calculation: taking moments about A, RB = [18x1.5 + 4x1 + 4x2.5]/3 = 13.67 kN, and RA = 26 - 13.67 = 12.33 kN - an exact match.)

6. Shear force and bending moment diagrams



The bending moment at any section equals H times the vertical intercept, measured on the space-diagram scale, between the link polygon and its closing string. Scaling the ordinates at each load point gives the moments tabulated below, with the maximum at midspan-ish, under the second UDL strip:

- At x = 0.5 m: M = 6.17 kNm
- At x = 1.0 m: M = 9.33 kNm
- At x = 1.5 m: M = 10.50 kNm (graphical maximum)
- At x = 2.5 m: M = 6.83 kNm

An analytical check using the true, continuous UDL (rather than the 3-strip approximation used for the drawing) gives a maximum moment of 9.79 kNm at x = 1.39 m - very close to the graphical value, the small difference being purely a result of approximating the UDL by a few point loads for the drawing. Either value supports the same conclusion below.

7. Workability check

Bending moment: maximum applied moment (~9.8-10.5 kNm) is less than the beam's minimum moment capacity of 12.5 kNm, with margin to spare before the upper capacity of 17.3 kNm is reached. Bending is therefore SAFE.

Support reactions: RA = 12.33 kN and RB = 13.67 kN both fall inside the wall's supportable range of 10-14.2 kN. Bearing is therefore also SAFE.

Conclusion: the proposed 200 mm x 300 mm RC lintel beam is WORKABLE for this 3 m opening under the stated loading.

8. Marking scheme (Total = 50 marks)

No.	Marking point	Marks
1	Title block, borders, and scales clearly stated (space scale & force scale)	2
2	Correct idealisation of loading: UDL (6 kN/m) split into equal strips and replaced by point loads at their centroids; truss loads correctly placed at 1 m and 2.5 m	4
3	Space diagram drawn accurately to scale, with beam, supports and loads positioned correctly, and Bow's notation lettering applied to every space	5
4	Load (vector) line plotted to a stated force scale, loads taken in the correct sequence and to correct magnitude	6
5	Pole O sensibly selected and polar rays O-1, O-2, O-3, O-4, O-5 drawn to each load point	3
6	Funicular (link) polygon correctly constructed in the space diagram: each segment drawn parallel to its corresponding polar ray, correctly linking from support A through to support B	5
7	Closing string drawn from the first to the last side of the polygon; point Z correctly located by drawing O-Z parallel to the closing string	3
8	Reactions RA and RB scaled off the force diagram and checked against a quick calculation (accept RA ≈ 12.3 kN, RB ≈ 13.7 kN, within about ±5%)	5
9	Shear force diagram correctly derived and plotted from the reactions and loads	4
10	Bending moment diagram derived from the funicular polygon (M = H × vertical intercept); maximum moment and its position correctly identified (accept 9.5-10.5 kNm at 1.4-1.5 m from A)	5
11	Workability check completed and correctly reasoned: Mmax compared with the beam's moment capacity (12.5-17.3 kNm) and the reactions compared with the wall's bearing capacity (10-14.2 kN); correct conclusion reached	5
12	Overall accuracy, neatness, correct use of drawing instruments/scale, and clear labelling throughout	3
	TOTAL	**50**

9. Guidance for the marker

- Accept any reasonable choice of force scale, space scale and pole distance H - the resulting reactions and moments must still come out close to the values above regardless of the scales chosen; that is the property being tested.
- Accept splitting the UDL into a different number of equal strips (e.g. 6 strips of 0.5 m) provided the centroid rule is applied correctly - the reactions will still match exactly, though the peak moment reading may vary slightly (roughly 9.5-10.5 kNm is a reasonable tolerance band).
- Common errors to penalise: omitting Bow's notation; loads plotted out of sequence on the load line; funicular polygon segments not parallel to their corresponding rays; closing string omitted so reactions cannot be found; forgetting to convert the polygon ordinate to a bending moment by multiplying by H; no comparison made against the given capacity ranges; no explicit pass/fail conclusion stated.
- A candidate who reaches materially the same reactions, shear/moment values and the same overall conclusion (beam is workable) by a valid graphical construction, even with a different layout, should be awarded full marks.