

Pure Mathematics.
Revision Series.
EXERCISE 1.

SOME USEFUL TOPICS IN ALGEBRA.

- Given that α and β are the roots of the equation $x^2 + px + q = 0$ express $(\alpha^2 - \beta^2)$ and $(\alpha^3 + \beta^3)$ in terms of p and q . {Ans: $-p^3 + 3pq$ }
- (a) The function $f(x) = x^3 + px^2 - 5x + q$ has a factor $(x-2)$ and has a value 5 when $x = -3$ find p and q {Ans: $p = 3, q = -10$ }
- (b) The roots of the equation $ax^2 + bx + c = 0$ are α and β . Form the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$
{Ans: $acx^2 - (b^2 - 2ac)x + ac = 0$ }
- Given the polynomial $f(x) = Q(x)g(x) + R(x)$ where $Q(x)$ is the quotient, $g(x) = (x-\alpha)(x-\beta)$ and $R(x)$ the remainder show that :
 $R(x) = \frac{(x-\beta)f(\alpha) + (\alpha-x)f(\beta)}{\alpha-\beta}$ hence find the remainder when $f(x)$ is divided by $x-2$ is 2 and when divided by $x+3$ is -3
{Ans: $\frac{5x-3}{6}$ }
- Solve for x in the equation $\log_4(6-x) = \log_2 x$ {Ans: $x = 2$ }
- Solve the equation $\log_2 x - \log_x 8 = 2$. {Ans: $x = 8, \frac{1}{2}$ }
- Given that $\log_3 x = p$ and $\log_{18} x = q$ show that $\log_6 3 = \frac{q}{p-q}$
- (a) Given that one of the roots of the equation $x^2 + px + q = 0$ is twice the other root. Show that $2p^2 = 9q$. hence or other wise, find the value of k , if the equation $x^2 - 2(k+2)x + (k^2 + 3k + 2) = 0$ has one root twice the other. {Ans, $K = -7$ or 2 }
- (b) If $a^2 + b^2 = 23ab$, show that $\log a + \log b = 2 \log(\frac{a+b}{5})$.
- Prove that $\log_b N = \frac{\log_a N}{\log_a b}$ hence solve the equation
 $\log_{10} x + \log_x 100 = 3$ {Ans: $x = 10, 100$ }
- If α and β are the roots of the equation $ax^2 + bx + c = 0$, find the equation whose roots are $(\alpha^2 + \beta^2)$ and $(\alpha^{-2} + \beta^{-2})$
{Ans: $(ac)^2 x^2 - (b^2 - 2ac)(c^2 + a^2)x + (b^2 - 2ac) = 0$ }

10. Solve the equation. $\log_x 64 + \log_4 x^2 = 7$ {Ans. $x = 2, 64$ }
11. (a) When the polynomial $P(x)$ is divided by $x - 3$ the remainder is 2 and when divided by $x + 3$ the remainder is -3, find the remainder when $P(x)$ is divided by $x^2 - 9$ {Ans: $R=3$ }
- (b) If α and β are the roots of the equation $ax^2 + bx + c = 0$ find the equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$
{Ans: $c^2x^2 - (b^2 - 2ac)x + a^2 = 0$ }
12. The polynomial $P(x)$ is divided by $(x-2)$ the remainder is 7 and when divided by $(x-2)$ the remainder is 9 when divided by x the remainder is 5. Find the remainder when the polynomial $P(x)$ is divided $x(x-2)(x+2)$. {Ans: $\frac{3}{4}x^2 - \frac{1}{2}x + 5$ }
13. The roots of the equation $px^2 + qx + r = 0$ are α and β . find the equation whose roots are $(\alpha-2)$ and $(\beta-2)$.
{Ans. $px^2 + (q+4p)x + (r+2q+4p) = 0$ }
14. If α and β are roots of the equation $ax^2 + bx + c = 0$. find the value of $\alpha^2 - \beta^2$ {Ans: $\frac{\pm b\sqrt{b^2-4ac}}{a^2}$ }
15. If the equation $x^2 + px + q = 0$ has roots α and β . Find the value of
(i) $\alpha^3\beta + \alpha\beta^3$. {Ans: $q(p^2 - 2q)$ }
- (ii) $\alpha^4 + \alpha^2\beta^2 + \beta^4$. {Ans: $((p - 2q)^2 - 2q^2)$ }
16. When the polynomial $f(x) = x^7 - ax^3 + 4b$ is divided by $x-2$ the remainder is 8 and $x-1$ is a factor find the value of a and b . {Ans: $a = 17, b = 16$ }
17. If α and β are roots of the equation $ax^2 + px + q = 0$ find the equation P whose roots are α^2 and β^2 . {Ans: $a^2x^2 - (p^2 - 2aq)x + q^2 = 0$ }
18. Show that $\log_b a = \frac{\log_c a}{\log_c b}$ hence solve the simultaneous equations
 $x + y = 20, \log_3 x = \log_9 y$. {Ans. $(4,16)(-5,25)$ }
19. If $\log_2 x = p$ and $\log_6 x = q$, Show that $2^{(p-q)} = 3^q$.
20. The function, $f(x) = x^3 + px^2 - 5x + q$ has a factor $(x - 2)$ and has a remainder 5 when divided by $(x + 3)$. Find p and q .
{Ans: $p = 3, q = -10$ }
21. Given that the roots of the equation $ax^2 + bx + c = 0$ are β and $n\beta$ show that $(n+1)^2 ac = nb^2$
22. Solve the equation $\log_3 x - 4 \log_x 3 + 3 = 0$. {Ans: $x = \frac{1}{81}, 3$ }

23. Given that $\log_3 6 = m$ and $\log_6 5 = n$ express $\log_3 10$ in terms of m and n {Ans, $nm + m - 1$ }

24. Without using tables or a calculator, solve $\log_8 \left\{ \frac{x}{2} \right\} = \frac{\log_8 x}{\log_8 2}$. {Ans. $\frac{1}{\sqrt{2}}$ }

25. Simplify. $\frac{\sqrt{2}}{\sqrt{2}-\sqrt{3}-\sqrt{5}} + \frac{\sqrt{3}}{\sqrt{2}-\sqrt{3}+\sqrt{5}}$. {Ans: $\frac{1}{12}(\sqrt{6} + 3\sqrt{10} - 2\sqrt{15})$ }