P425/1
PRINCIPAL
MATHEMATICS
Paper 1
Aug,2026
2½hours

MATIGO EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PRINCIPAL MATHEMATICS

Paper 1

2hours : 30 minutes

INSTRUCTIONS TO CANDIDATES

This examination paper consists of **two** *sections;* **A** *and* **B.** *It has five examination items.*

Section A *consists of* **one compulsory** *item.*

Section **B** *has two parts,* **I** *and* **II** *each with two optional items. Respond to only two items from Section B, choosing one item from each of the parts, I and II.*

Respond to **three** *items in* **all.**

Any additional item(s) responded to will **not** *be scored.*

Begin each item on a fresh page.

All *necessary working* **must** *be shown clearly.*
Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

Turn Over

SECTION A

The item in this section is **compulsory**.

Item 1

A certain country is experiencing a crisis due to the war. The war has affected mainly three towns at locations $T_1(7,3,6), T_2(1,3,9)$, and $T_3(5,-4,3)$, with all coordinates being in kilometers. An emergency response team is to supply relief items to the affected people in the towns T_1, T_2 and T_3. The helicopter has to be stationed at location $H(10,2,16)$ km. For efficient supply of these items, the shortest distance between H and the plane containing the three towns should be at least 8km. The team has also set up its own medical camp at position $P(7,1)$ metres and the casualties are in the circular health centre with boundaries at $A(5,0), B(0,5)$, and $C(-3,4)$, with all coordinates in hundreds of metres. The casualties are to be moved from the health centre to the medical camp using a straight road which touches one of the entrances of the health centre. For the safety of the ambulance, the journey should not exceed 1200m. From the medical camp, a helicopter is to pick up these casualties and transfer them to a specialized hospital for better medical attention.

The helicopter landing is modeled by

$$cos2\theta + cos\theta = 0$$

For safe landing, the acute angle of landing should not exceed 70^0.

Task:

Using mathematical skills help the response team to know whether their plans will be successful.

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SECTION B

Answer only **two** *items from this section, choosing* **one** *item from each of the parts* **I** *and* **II**.

Part I

Item 2

A farmer is setting up a new solar pump to solve the problem of water on his farm. The pump uses a controller switch with components Z_1, Z_2 *and* Z_3 whose pumping force while operating is;

$$F = \frac{Z_1^2}{Z_2 Z_3} \text{ kN.}$$

For the safety of the pump, the size of the pumping force of the switch should be greater than 2kN and its angle when switched on must be obtuse, where $Z_1 = 2 + 3i$,$Z_2 = 1 - 2i$, and $Z_3 = 2 + i$. The pump sends water to the tank every hour such that the amount of water pumped per hour is defined by:

$$V = 100(1 + 20x)^n \text{ litres}$$

where n is the number of operational hours of the pump and the terms of the expansion represent the volume of water pumped each hour. x is the pump code which depends on the hours set for the pump to operate and on his farm, water is pumped for 10 hours and the code for this time is $\frac{1}{20}$. He will only use this pump if in the first 4 hours it pumps not less than 15000 litres

The pump sends water to the tanks using pipes of length l_1 and l_2. $l_1 = 9^w$ and $l_2 = 3^{w+1}$ where w is the width of the pipes. The total length of these pipes is 108 m, but the required pressure on the farm can be realized only if the width of the pipe is not more than 2.

Task:

Help the farmer to make valid decisions using mathematical skills.

Item 3

A certain telecom company is upgrading to 5G networks and some systems have to be checked before the upgrade is officially launched. They have two command centers whose signals are sent and move horizontally in x miles and vertically in y miles. The first command centre sends its signal of strength:

$$S_1 = \frac{2x + 4i}{2y + x}$$

and the second command centre sends its signal of strength:

$$S_2 = \frac{x}{y - i}$$

For the stability of the network, both signals must be of equal strength and the required coverage distance $D = \sqrt{x^2 + y^2}$ should be atleast 2 miles.

The primary mast sends signals hourly such that in the first three hours the frequencies of the signals sent are 400, 800 and 1600 MHz respectively. This signal sequence will be approved if the difference between the signal sent in the 7^{th} and 5^{th} hour is not less than 1900MHz.

The secondary mast produces frequencies in two phases with phase one having carrier frequencies $F_1 = f^3 - 90$ and in the second phase $F_2 = 14f^2 - 63f$. This must operate on conditions that the frequencies in both phases are equal and should have only one carrier frequency between 1 and 4.

Task: Using your mathematical skills help the telecom company to make valid recommendations.

A telecom company is upgrading to a 5G network and running pre-launch stability checks on the following station components.

The engineering team is balancing the complex signal vectors coming from two independent transmitter sub-circuits. The first sub-circuit generates a complex signal value of $S_1 = \frac{2x+4i}{2y+x}$, and the second sub-circuit generates $S_2 = \frac{x}{y-i}$. For optimal modulation stability, both complex signal vectors must be equal. Furthermore, the total combined signal amplitude, calculated as $A = \sqrt{x^2 + y^2}$, must be at least 2 to clear the receiver's noise floor.

The primary antenna mast undergoes a stress test where its allocated aggregate channel bandwidth scales exponentially every hour. In the first three hours, the allocated bandwidth blocks are 400 MHz, 800 MHz, and 1600 MHz respectively. The automated scaling sequence passes verification if the

difference in spectrum capacity between the 7th hour and the 5th hour is at least 1900 MHz.

A secondary mast utilizes a dual-phase local oscillator to generate carrier frequencies. The frequency profile (in GHz) for Phase 1 is governed by $F_1 = f^3 - 61$, and Phase 2 is governed by $F_2 = 15f^2 - 74f + 59$. For stability, the frequencies of both phases must be equal and to comply with international regulations, the system must yield exactly one valid operational frequency between 1 GHz and 4 GHz.

Task: Using your mathematical skills help the telecom company to make valid recommendations.

Part II

Item 4

A fresh graduate in fisheries has decided to set up a fish pond to solve the problem of unemployment. He plans to construct a hemispherical pond whose radius is 5m and fill it with water to full capacity. The volume of the water in this pond is expected to be:

$V = \frac{1}{3}\pi h^2(3r - h)\, m^3$ where r is the radius of the pond and h is the depth of the water. Experts told him that in such a pond, water evaporates at a constant rate of $4\pi\ m^3$ per day, and he needs to always top up the water whenever the depth is 2m. But for the safety of the fish, the pond should be topped up with water at a time when the rate at which the depth is falling is not less than 0.2 metres per day. The fish species he plans to take care of feed at a rate modeled by:

$$\frac{dM}{dt} + M = t,$$

where M is the mass of the feeds in Kilograms remaining in the pond after t days. He plans to initially put 100 kg of feeds and add the same amount after every two days. To ensure that fish don't starve, the amount of the feeds remaining in the pond after two days should not go below 10 kgs.

The fish species he plans to care for increase at a rate proportional to the number of fish present in the pond at any time. He plans to initially place 200 fish in the pond and after 1 month he expects this number to double. He plans to start harvesting the fish after 6 months and if at the time the fish population is more than 1300 fish, then the quality of fish won't meet market standards.

Task:

Using mathematical skills, help the fresh graduate make valid decisions on his fish pond plans.

Item 5

An agriculture student doing industrial training on a certain farm has discovered some challenges on the farm and come up with the following recommendations.
The cows on the farm drink water from a cylindrical trough whose volume is

$$V = 9\pi h^3\, m^3$$

When the cows start drinking, the depth h of water after t hours is
$h = In(10 - t)$ metres.
The farm manager has been filling the trough with water to a maximum capacity at 6:00am but the water has always not been enough for the cows for a full day.
The student has recommended that they always top up thus water at noon but this will only be possible if at this time the volume of water is reducing at a rate not less than $10\pi m^3$ per hour to ensure sufficient supply.
He also recommended that they add vitamin mixture to the water to boost the growth and help in digestion. The vitamin mixture that he recommended boosts growth at a rate

$$\frac{dm}{dt} + 80t = 17$$

where m is the mass in grams of the vitamin mixture remaining in the water after t hours.
Initially 1kg of the mixture is added to the water and the booster can only have the required effect if after 5 hours the mass remaining is not less than 50g.
He also discovered that the cows on the farm feed on grass provided at a rate proportional to the square root of the amount of grass remaining. He recommended that if the cows are given 900kg of grass in the morning, they can eat 500kg of this grass in 4 hours and at this rate he thinks that this amount is enough for the cows for atleast 10 hours.

Task:

Help the agriculture student to make a valid report on his recommendations using your knowledge on calculus.

END