13 questions: 10 short answer, 3 proof/essay questions.

180 minutes. No electronic devices.

Finalists will be determined first by the scores for questions 1-10. Ranking among the finalists will take into consideration the quality of the proofs in questions 11-13.

Answers to questions 1-10 are all positive integers; enter these in the supplied answer sheet. Solutions for questions 11-13 should be written in complete sentences, starting on the back of the answer sheet and continuing on additional paper as needed.

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1. Determine the sum of all 3-digit numbers that contain only the digits 2, 3, 4, and 5.

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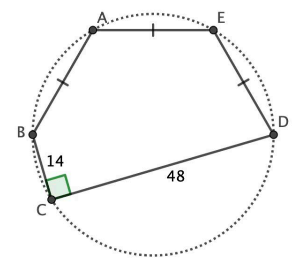
2. A pentagon inscribed in a circle has three equal length sides, a side of length 14, and a side of length 48. The sides of lengths 14 and 48 meet in a right angle, as shown in the image to the right.
   Find the perimeter of the pentagon.



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3. Write the numbers 1 to 100 on index cards, stacked in order. Now mix up these cards as follows: your friend flips a coin, and if it lands heads, deal a card from the top of your stack; if it lands tails, deal a card from the bottom of your stack. For example, if the coin landed HTTH your new stack would start 1, 100, 99, 2.

   Once you have dealt all 100 cards, look at the cards in the new order, taking five at a time, and compute the sum of each set of five. This will give you 20 sums. What is the largest number that you can be sure every sum is divisible by?

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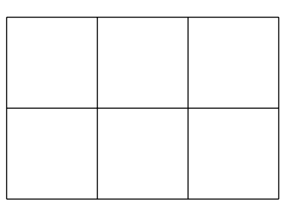
4. Place each digit 1 through 6 exactly once in the grid below, to form two 3-digit numbers (reading left-to-right) and three 2-digit numbers (reading top-to-bottom). If you do so satisfying the following rules, what will the sum of the two 3-digit numbers be?



Out of the five 2- and 3-digit numbers:

- The number of even numbers is even.
- The smallest number is the only multiple of 3.
- The median number is the only multiple of 5.
- The largest number is the only multiple of 11.
- No number is a multiple of 7.

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5. Consider the sequence $a_1 = 1$, $a_2 = 2 + 3$, $a_3 = 4 + 5 + 6$, $a_4 = 7 + 8 + 9 + 10$, $\ldots$.
   Find the value of a_{20}.

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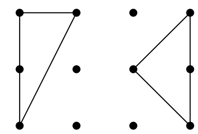
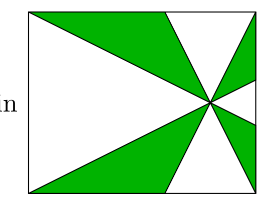
contest continues on next page

6. Lefty McGraw, a new promising right-handed pitcher, can throw curve balls or fastballs (C or F). However, he never throws three curve balls in a row. How many different sequences of 10 pitches could he throw?

7. Determine how many right triangles can be made on a 4×3 geoboard with a rubber band. That is, how many right triangles can be formed using 3 of the 12 dots as vertices arranged in a 3×4 rectangle? The dots are equally spaced horizontally and vertically. Two samples are drawn.



8. The rectangular stained-glass window shown here is 16 inches tall and 20 inches wide. The window consists of four triangles made of colored glass and four triangles made of clear glass. The four clear glass triangles are isosceles and are similar to one another. Find the total area, in square inches, of the four colored triangles.

16 in

20 in

9. Let T_n be the number of ways you can write n as a sum of one or more consecutive positive integers. For example, T_{15} is 4, since,

$$1+2+3+4+5=4+5+6=7+8=15.$$

Find the value of T_{45}.

10. Oscar and Nat like to play the game "Number Guesser." The way they play is as follows: Nat thinks of a whole number between 1 and 100, and Oscar has to figure out what number Nat is thinking of. He can only ask questions of the form "Is your number divisible by n?", where n is a whole number. When Oscar knows what number Nat is thinking of, he rings a bell and announces the number.

    One time they were playing this game, and Oscar figured out Nat's number after asking 9 questions. After announcing the answer, he correctly observed, "That felt like it took a lot of questions, but actually, I could not have figured out your number without asking at least that many."

    What was Nat's number?

11. Prove that your answer to question 3, the shuffled numbers question, is correct.

12. Explain your reasoning that led to your answer to question 7, the geoboard triangle question. Then if possible, give a generalization and justified solution.

13. Find a formula for T_{2^n}, the number of ways to write 2^n as a sum of consecutive positive integers, as in question 9. Prove your formula is correct.

END OF CONTEST