

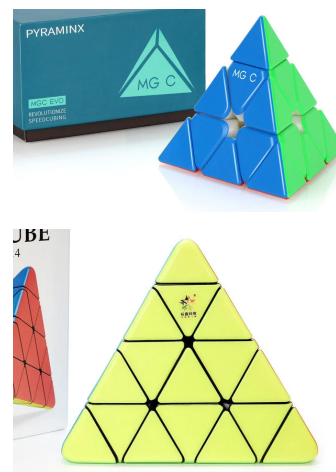
33rd Annual
UNC Math Contest
First Round – Fall 2024

10 questions. 90 minutes. No electronic devices.

All answers are positive whole numbers (do not include units).

Enter answers on separate sheet before transferring them to the online form.

1. Pyraminx puzzles are similar to Rubik's Cubes, but they are made in the shape of a tetrahedron (a triangular pyramid) rather than a cube. Each face of a Pyraminx puzzle is a triangle subdivided into smaller triangles. Pyraminx puzzles are made in a variety of sizes. The size of a Pyraminx puzzle is determined by the number of smaller triangles that can be found along an edge of the tetrahedron. The top photo on the right shows a size 3 Pyraminx puzzle, and the bottom photo shows a size 4 puzzle. As you can see from the photo, each face of the size 4 Pyraminx puzzle is subdivided into 6 downward-pointing triangles and 10 upward-pointing triangles, for a total of 16 triangles. If you could manufacture a size 30 Pyraminx puzzle, how many triangles would each face be subdivided into?



2. The great Ocardini is designing a card trick. He will end the trick by handing the volunteer a stack of 13 cards, one of which is the volunteer's selected card. He will have the volunteer pass the top card in the stack to the bottom of the stack, and then discard the next top card; and again, pass the top card in the stack to the bottom, and then discard the next top card; and continue in that fashion until only one card is left in the stack, which will be the selected card! Numbering the top card in the original pile as card 1, the next one as card 2, and so on, what position should the volunteer's card be in for the trick to work?
3. Complete this "cross-number puzzle" by placing one digit in each box. Since the answers are all 3-digit numbers, no entry should begin with a zero. What is the prime number that is the answer to 1 Across?

1	2	3
4		
5		

Across

1. A prime number.
4. A composite number.
5. A square number.

Down

1. A power of 5.
2. A power of 2.
3. A power of 3.

4. How many degrees is the smaller angle between the hour and minute hands of an analog clock at 4:52?
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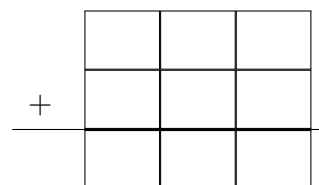
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5. A sequence is defined as follows: $a_0 = 1$, $a_1 = 2$, and $a_{n+2} = \frac{a_{n+1}}{a_n} + \frac{1}{a_n}$ for $n \geq 0$. For example, $a_2 = \frac{2}{1} + \frac{1}{1} = 3$. Find the value of a_{2024} .
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6. There is a maze in the computer game “Euler’s Magical Mystery Tour.” In the maze there are four rooms that are identical except for the colors of the walls. The four rooms are painted green, blue, purple, and yellow. Each room contains an entrance and two separate exit doors. One of the two doors is painted with the picture of a tiger; the other is painted with a picture of a princess. Each door leads through a maze of twisty little passages back to the entrance of one of the four rooms. The rooms are connected as follows:

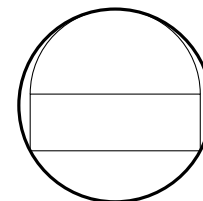
From the green room, the tiger door leads back to the green room, and the princess door leads to the purple room. From the blue room, the tiger door leads to the green room, and the princess door leads to the purple room. From the purple room, the tiger door leads to the blue room, and the princess door leads to the yellow room. From the yellow room, the tiger door leads to the blue room, and the princess door leads to the yellow room.

To win the game, players must find a path that starts and ends in the green room, and passes through each exit door exactly once. How many such paths are there?

7. Place each of the digits 1 to 9 in one of the nine boxes in the diagram to the right so that each digit is next to the succeeding digit, either horizontally or vertically, and so the whole diagram forms a true statement about the addition of three-digit numbers. What is the three-digit sum on the bottom line?



8. In Calvinball, the free throw lane is marked out by a semicircle on the longer side of a 10 ft by 30 ft rectangle. The Perimeter of Wisdom is the circle that circumscribes the free throw lane, as shown by the dark line in the diagram to the right. What is the radius of the Perimeter of Wisdom?



9. Susie’s Awesome Math Girls club has taken over Calvin’s treehouse fort. In order to keep Calvin out, they’ve decided to require a password to get into the fort. In order to get in, you have to say a three-digit whole number that is divisible by 4, 5, or 11, and you can’t use one that someone has already used. How many possible passwords are there?
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10. Let p_n be the number of sets of four (distinct) positive integers $\{a, b, c, d\}$ such that $a + b + c + d = n$. For example, $p_{13} = 3$, since the only choices for your sets are $\{1, 2, 3, 7\}$, $\{1, 2, 4, 6\}$ and $\{1, 3, 4, 5\}$. Find p_{17} .
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