P425/1

PRINCIPAL MATHEMATICS

Paper 1

July /August 2026

3 Hours

SECONDARY SCHOOLS JOINT MOCK EXAMINATIONS, 2026

Uganda Advanced Certificate of Education

PRINCIPAL MATHEMATICS

Paper 1

3 HOURS

INSTRUCTIONS TO CANDIDATES

This examination paper consists of two sections: A and B.

It has five examination items.

Section A has one compulsory item.

Section B has two parts: Part I and Part II. Respond to only one item from each part.

Respond to three items in all.

Any additional item(s) responded to will not be scored.

All responses must be written in the answer booklet(s) provided.

Responses to each item should start on a new page.

Silent, non-programmable scientific calculators and mathematical tables may be used.

SECTION A

Compulsory item

ITEM 1

The administration of a certain secondary school plans to install a public clock in the school's recreational park so that students and visitors can easily read the time. The park is a triangular area on a flat horizontal surface. During a survey, three vertices of the park were recorded as $P(2, 1)$, $Q(3, 5)$, and $R(1, 3)$, enabling students to determine the boundary lines, side lengths, perimeter, area, and centroid of the park for accurate clock placement.

The supporting pole is represented by the vector equation $r = (2, 3, 0) + \lambda (1, 1, 20)$, with the clock mounted at $\lambda = 1$, while a student observes it from point $A(20, 12, 0)$ outside the park.

For the engineering design, the pole must pass through the centroid of the triangular park, the area must be less than $7m^2$ to minimize land usage, the observer's distance from the clock must be less than 30m for a clear view, and the pole must be almost vertical with an inclination between 85^0 and90^0.

TASK

Analyze all design conditions and justify installation.

SECTION B

Part I

Attempt one item

ITEM 2

Jinja Industrial Manufacturing Company plans a production expansion project to meet growing demand in Uganda and East Africa. The company intends to form a production oversight committee of 11 members from 8 male engineers and 10 female engineers, requiring at least 6 men to ensure sufficient technical representation.

A newly installed machine produces 5 units initially and doubling every cycle. The management wants the cycle when total production first exceeds 400,000 units to plan production timelines. The expansion is analyzed using the binomial expansion $\left(3x + \frac{1}{x}\right)^{16}$ to determine the constant term of x, representing the optimal distribution of resources necessary for project viability.

The electrical maintenance department models machine load with $\frac{(2-i)^5\,(1-i\sqrt{3})^9}{(2+i)^7}$ to compute the modulus, ensuring the machine operates safely under stress, while the data analysis team evaluates efficiency with $2\log_2\frac{10}{9} - 3\log_2\frac{5}{2} - \log_2\frac{1}{30}$ to determine the efficiency rating index, indicating overall system performance.

Management criteria for approval are:

1. Number of ways to form committee > 8000
2. Production between 15^{th} and 20^{th} cycle
3. Load magnitude > 100
4. Efficiency index < 4
5. Constant term from binomial > 80 million

Turn Over

TASK
Using the above criteria and reasons, determine whether the expansion project should be approved.

ITEM 3

A teacher of Physical Education is preparing the school field for an inter-house athletics competition involving football, basketball, relay races, agility drills and throwing events. In order to organize the field systematically and efficiently, the teacher applies concepts from Mathematics.

First, the teacher plans to mark a rectangular training zone using field line strings and pegs. The total rope available to mark the boundary of the field is 200m. The teacher also requires the field to have an area of 2400 m^2 so that different activities can occur simultaneously. The teacher intends to determine the exact dimensions that will efficiently use the available rope while maintaining the required area.

To organize sprint drills, cones are arranged along a straight training lane such that the spacing between consecutive cones increases geometrically. The first spacing between cones is 2 m and the common ratio is 1.2. If the teacher uses 8 spacing's, the teacher needs to determine the total length covered by the cones in order to allocate adequate space on the field.

During basketball practice, the teacher studies the trajectory of the ball. After a player throws the ball toward the hoop, the height of the ball above the ground after seconds is modeled by $y = -5t^2 + 20t + 2$, where y represents the height in meters. The teacher needs to determine the maximum height reached by the ball and the time at which it occurs to ensure that the ball will not collide with nearby tents.

To monitor athlete performance, the teacher monitors athletes' sprint performance, modeled by $v = v_0(1.08)^t$, where v is speed after t weeks and v_0 is the initial speed. The teacher wants to determine how long it takes for an athlete's speed to double to guide training planning.

Finally, for the relay race event, the teacher has 12 athletes available and must select 5 athletes to form a training squad. The number of possible selections can be determined. After selecting 4 athletes for the final relay race, the teacher considers the different possible running orders of the athletes. The number of arrangements can be determined.

Analysis	Standard / Valid Range
Rectangular Training Zone Dimensions	30 m – 70 m
Total Length of Cones	30 m – 35 m
Sprint Lane Length	55 m – 65 m
Maximum Basket Height	Height: 20 – 25 m; Time 1.5-2.5 s
Time to Double Sprint Speed	8 – 12 Weeks
For Relay Training Squad	750 – 800 ways
For Running Order	20 – 25 ways

TASK

Using the given models, help the teacher to perform event planning, justifying all decisions.

PART II

Attempt one item

ITEM 4

Working in certain factory, Stella faces a problem of determining whether her daily routine and the factory operations are efficient, safe, and reliable.

In the early morning, she leaves home and drives 5 km to the factory. Her velocity $v(t)$ in ms^{-1} at time t seconds is modeled by $v(t) = 1.5 + 0.005t$, $t \geq 0$. For efficiency, Stella must reach the factory in less than 20 minutes so that production starts on time.

When Stella arrives at the factory, she immediately starts operating a machine that produces Panadol continuously according to the model

$f(t) = \frac{140t+460}{(t+1)(t+5)}$, $0 \leq t \leq 60$ minutes

so that the management can determine the average production rate and estimate the total number of Panadol produced in one hour for planning

Turn Over

supply and verifying whether the machine can meet the required target of at least 500 Panadol per hour.

During a short break, Stella ordered a cup of tea at an initial temperature of 98°C, with surroundings at 18°C. After 4 minutes, the tea cooled to 70°C. The cooling follows Newton's Law of Cooling, where the rate of temperature change is proportional to the difference between the tea and surroundings. Stella will drink the tea at 44°C (drinkable temperature standard). It should cool to a drinkable temperature within 15 minutes (break standard).

TASK

Assess whether Stella's travel, Panadol production, and tea cooling satisfy all factory efficiency and safety standards.

ITEM 5

Paul, an engineer, is supervising the arrangement of a football pitch at Buffa Regional Stadium. The stadium is designed to hold 6,000 spectators, and the football match is scheduled to start at 5 PM. Paul's responsibility is to ensure the pitch, seating, and crowd entry are safe, functional, and well-managed, so that all spectators are seated comfortably before kickoff and the players, especially the goalkeeper, have enough space to perform safely.

Paul is designing the turf near the goal post. This turf is slightly curved to allow smooth play, modeled by $f(x) = 4x^2 + 6x + 3, 1 \leq x \leq 5$ meters. The area of this turf must be greater than 200 m^2 to provide enough space for the goalkeeper to move safely.

He is supervising the seating arrangement which is continuous such that seats are closely packed together in columns. The number of seats in one section are modeled by $p(x) = 81 - 2x, 0 \leq x \leq 50$ meters. The stadium has four seating sections, each accommodating an equal number of spectators. Seating must ensure all 6,000 spectators can be seated safely and comfortably before the 5 PM match.

Paul needs to manage how spectators enter the stadium. The number of spectators inside at time t hours after the gates open is modeled by the logistic differential equation $\frac{dP}{dt} = 3P\left(1 - \frac{P}{6000}\right)$, $P(0) = 1$, where $P(t)$ represents the number of spectators inside at time t. This model helps Paul determine how early to open the gate, and the total time required to fill the stadium safely.

TASK

As student, help Paul to make decisions on turf area adequacy, total seating, and the time gates should be opened to have full capacity to ensure safe play and smooth crowd management.

END

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ITEM ONE

for boundary lines of the triangular park.

line PQ: $\frac{y-1}{x-2} = \frac{5-1}{3-2} = 4$ M1

$y-1 = 4(x-2)$, $y = 4x-7$ B1

line QR: $\frac{y-5}{x-3} = \frac{3-5}{1-3} = 1$ M1

$y-5 = x-3$, $y = x+2$ B1

line RP: $\frac{y-3}{x-1} = \frac{1-3}{2-1} = -2$ M1

$y-3 = -2(x-1)$, $y = -2x+5$ B1

for lengths of sides of the park

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ B1

length PQ: $PQ = \sqrt{(3-2)^2 + (5-1)^2} = \sqrt{17}$ m M1 B1

length QR: $QR = \sqrt{(1-3)^2 + (3-5)^2} = \sqrt{8}$ m $= 2\sqrt{2}$ m M1 B1

length RP: $RP = \sqrt{(2-1)^2 + (1-3)^2} = \sqrt{5}$ m M1 B1

for perimeter of the park

$P = PQ + QR + RP = \sqrt{17} + \sqrt{8} + \sqrt{5} = 9.1876$ B1 M1 A1

for area of triangular park.

$A = \frac{1}{2}\left| x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2) \right|$ B1

$A = \frac{1}{2}\left| 2(5-3) + 3(3-1) + 1(1-5) \right| = \frac{1}{2}\left|4+6-4\right|$ M1

$A = \frac{1}{2}(6) = 3m^2 < 7$ A1

$\therefore$ land usage condition is satisfied. B1

for Centroid of the triangular park

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$G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ B1

$G = \left(\frac{2+3+1}{3}, \frac{1+5+3}{3}\right) = \left(\frac{6}{3}, \frac{9}{3}\right) = (2,3)$ M A1

for Verification that the pole passes through the centroid $r = (2,3,0) + \lambda(1,1,20)$ M1

at $\lambda = 0$, $r = (2,3,0)$, This point projects onto the ground at $(2,3)$ hence the pole passes through the centroid. B1

for coordinates of the clock

at $\lambda = 1$, $x = 2+1 = 3$, $y = 3+1 = 4$, $z = 20(1) = 20$

$\therefore$ clock coordinates is $C(3,4,20)$ B1

for distance of observer from the clock

$AC = \sqrt{(3-20)^2 + (4-12)^2 + (20-0)^2} = \sqrt{753} = 27.4408m < 30$ M1 A1

$\therefore$ the viewing condition is satisfied. B1

for inclination of the pole

$|d| = \sqrt{(1)^2 + (1)^2 + (20)^2} = \sqrt{402}$ B1

$\sin\theta = \frac{20}{\sqrt{402}}$, $\theta = \sin^{-1}\left(\frac{20}{\sqrt{402}}\right) = 85.95°$ M1

$85° < 85.95° < 90°$

$\therefore$ condition satisfied. B1

Therefore all engineering design conditions are satisfied and the Installation of the public clock is Justified. B1

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ITEM TWO

for formation of production oversight committee by combination model

$$\binom{8}{6}\binom{10}{5}+\binom{8}{7}\binom{10}{4}+\binom{8}{8}\binom{10}{3}=8{,}856$$ B1 M1 A1

$8{,}856 > 8{,}000$

$\therefore$ condition satisfied B1

for Geometric progression production cycle that is G.P sum formulae, $a=5$, $r=2$, $S_n=\frac{a(r^n-1)}{r-1}$ M1

$5(2^n-1) > 400{,}000$, $\ln 2^n > \ln(80{,}001)$

$$n > \frac{\ln(80{,}001)}{\ln(2)} = 16.2877 \approx 17$$ B1 A1

$\therefore$ within the range and satisfies the condition

for Binomial expansion for constant term condition

$$T_{r+1}=\binom{16}{r}(3x)^{16-r}\left(\frac{1}{x}\right)^r$$ B1

$16-2r=0$

$r=8$ A1

$$T_9=\binom{16}{8}(3x)^8\left(\frac{1}{x}\right)^8=84{,}440{,}070 > 80{,}000{,}000$$ M1 A1

$\therefore$ Condition satisfied. B1

for complex number modulus

$$\left|\frac{(2-i)^5(1-i\sqrt{3})^9}{(2+i)^7}\right|=\frac{|2-i|^5|1-i\sqrt{3}|^9}{|2+i|^7}$$ M1

$=102.4 > 100$ B1

$\therefore$ condition satisfied. A1

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for logarithm efficiency Index that is logarithm laws

$2\log_2\left(\frac{10}{9}\right) = \log_2\left(\frac{10}{9}\right)^2, \; 3\log_2\left(\frac{5}{3}\right) = \log_2\left(\frac{5}{3}\right)^3$ M1 M1

$= 3 < 4$ A1

$\therefore$ Condition satisfied. B1

Final decision is project approved. B1

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ITEM THREE

for rectangular training zone dimensions

$2(L+W) = 200$, $L+W=100$ B1

$LW = 2,400$ B1

$L(100-L) = 2,400$, $L^2 - 100L + 2,400 = 0$ M1

$(L-60)(L-40) = 0$

$\therefore L = 60$, $W = 40$ A1

$40, 60 \in [30, 70]$

$\therefore$ condition Satisfied. B1

for total length of cones using Geometric progression

$a = 2$, $r = 1.2$, $n = 8$

$S_n = \frac{a(r^n - 1)}{r-1}$, $S_8 = \frac{2(1.2^8 - 1)}{0.2} \approx 33m$ M1 B1 A1

$33 \in [30, 35]$

$\therefore$ condition Satisfied.

cone length $\approx 33m$

Including buffer $\approx 60m$ B1

$60 \in [55, 65]$

$\therefore$ condition Satisfied. B1

for maximum basketball height

$y = -5t^2 + 20t + 2$

$t = -\frac{b}{2a} = -\frac{20}{2(-5)} = 2$ B1

$y = f(2) = -5(2)^2 + 20(2) + 2 = 22m$ A1

$22 \in [20, 25]$, $2 \in [1.5, 2.5]$

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∴ condition satisfied. B1

for time to double sprint speed. M1

$2 = (1.08)^t$

$t = \frac{\log 2}{\log 1.08} \approx 9$ A1

$9 \in [8, 12]$

for relay Training squad selection. M1

$\binom{12}{5} = 792$

$792 \in [750, 800]$ B1

∴ condition satisfied.

for running order M1

$4! = 24$

$24 \in [20, 25]$

∴ condition satisfied. B1

Final project approval decision.

All conditions are satisfied.

∴ the athletics field planning should be approved. B1

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ITEM FOUR

for travel time to the factory

$V(t) = 1.5 + 0.005t$

Distance travelled (s) $= \int V(t)dt = \int (1.5 + 0.005t)\, dt$

$s = 1.5t + 0.0025t^2 + c$ M1

at $t=0$, $S=5000$, $\therefore c = 5000$, $5000 = 1.5t + 0.0025t^2$ B1

$0.0025t^2 + 1.5t - 5000 = 0$ A1

$t = \dfrac{-1.5 \pm \sqrt{(1.5)^2 - 4(0.0025)(-5{,}000)}}{2(0.0025)}$ M1

$t_1 = 1{,}145.68325$ or $t_2 = -1{,}745.6832$ B1

$\therefore t = 1{,}145.68325 \approx 19.0947$ minutes < 20 A1

$\therefore$ Travel standard satisfied. B1

for panadol production in one hour.

$f(t) = \dfrac{140t + 460}{(t+1)(t+5)}$

Total production (P) $= \displaystyle\int_0^{60} \frac{140t + 460}{(t+1)(t+5)}\, dt = \frac{80}{t+1} + \frac{60}{t+5}$

from partial fractions.

$P = \displaystyle\int_0^{60} \left(\frac{80}{t+1} + \frac{60}{t+5}\right) dt = \Big|80\ln(t+1) + 60\ln(t+5)\Big|_0^{60}$ M1 B1

$= 80\ln 61 + 60\ln 65 - 60\ln 5 = 482.78$ panadol / hour < 500 A1

$\therefore$ production standard not satisfied. B1

for Tea cooling using Newton's law

$\dfrac{dT}{dt} \propto (T - T_s)$ M1

$\dfrac{dT}{dt} = -k(T - 18)$

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$\frac{dT}{T-18} = -k\,dt$

Integrating both Sides $\int \frac{dT}{(T-18)} = \int -k\,dt$ M1

$\ln|T-18| = -kt + c$

at $t=0$, $T=98$, $\ln|98-18| = 0 + c$ M1

$\therefore c = \ln 80$

$\ln|T-18| = -kt + \ln 80$

$\ln\left|\frac{T-18}{80}\right| = -kt$

after 4 minutes, $T = 70$ B1

by substituting in the model, $\ln\left|\frac{70-18}{80}\right| = -4k$ M1

$k = -\frac{1}{4}\ln\left(\frac{52}{80}\right)$ A1

$\ln\left|\frac{T-18}{80}\right| = -\frac{t}{4}\ln\left(\frac{52}{80}\right)$

Drinkable temperature $T = 44°C$

$\ln\left(\frac{44-18}{80}\right) = -\frac{t}{4}\ln\left(\frac{52}{80}\right)$ M1

$t = \frac{-4\ln\left(\frac{26}{80}\right)}{\ln\left(\frac{52}{80}\right)} = 10.44$ minutes. < 15 A1

$\therefore$ drinkable temperature standard satisfied.

Travel and tea cooling standards we satisfied but the panadol production target was not satified.
$\therefore$ not all operational standards were satisfied. B1

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ITEM FIVE

for turf area adequacy.

$$A=\int_1^5 (4x^2+6x+3)\,dx = \left|\frac{4x^3}{3}+3x^2+3x\right|_1^5 = \quad M1$$

$$\left(\frac{500}{3}+75+15\right)-\left(\frac{4}{3}+3+3\right) = 249.33 > 200 \quad M1 \; A1$$

$\therefore$ Condition satisfied.

for total seating capacity B1

$$\text{Seats per section} = \int_0^{50}(81-2x)\,dx = \left|81x-x^2\right|_0^{50} \quad M1$$

$$= 4{,}050 - 2{,}500 = 1{,}550 \quad A1$$

Total seats $4\times 1{,}550 = 6200 > 6000$

seating capacity condition satisfied. B1

for crowd entry logistic model

$$\frac{dP}{dt} = 3P\left(1-\frac{P}{6000}\right)$$

Separating variables $\frac{6000}{P(6000-P)}\,dt = 3\,dt$ M1

$$\frac{6000}{P(6000-P)} = \frac{A}{P} + \frac{B}{6000-P}$$ in partial fractions M1

$$6000 = A(6000-P) + BP \quad M1$$

$\therefore$ $A=1$ and $B=1$ M1

$$\int\left(\frac{1}{P}+\frac{1}{6000-P}\right)dP = \int 3\,dt$$

$$\ln|P| - \ln|6000-P| = 3t + c \quad M1$$

for $P(0)=1$, $\ln\left(\frac{1}{6000-1}\right) = 3(0) + c$ M1

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$$c = \ln\left(\frac{1}{5999}\right)$$ B1

$$\ln\left(\frac{P}{6000-P}\right) = 3t + \ln\left(\frac{1}{5999}\right)$$ M1

$$t = \frac{1}{3}\ln\left(\frac{5999P}{6000-P}\right)$$ M1

for near full capacity $P = 5{,}900$ to $5{,}999$

$$t = \frac{1}{3}\ln\left(\frac{5999 \times 5999}{6000-5999}\right) = 5.7996$$ M1

$$t = \frac{1}{3}\ln\left(\frac{5999 \times 5900}{6000-5900}\right) = 4.25896$$ M1

$\therefore$ time is from 4.25896 to 5.7996 hours

Gates open $\approx$ 11.20 a.m to 12.44 p.m M1

crowd entry timing ensures full capacity before kickoff.

$\therefore$ the stadium arrangements should be approved B1

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