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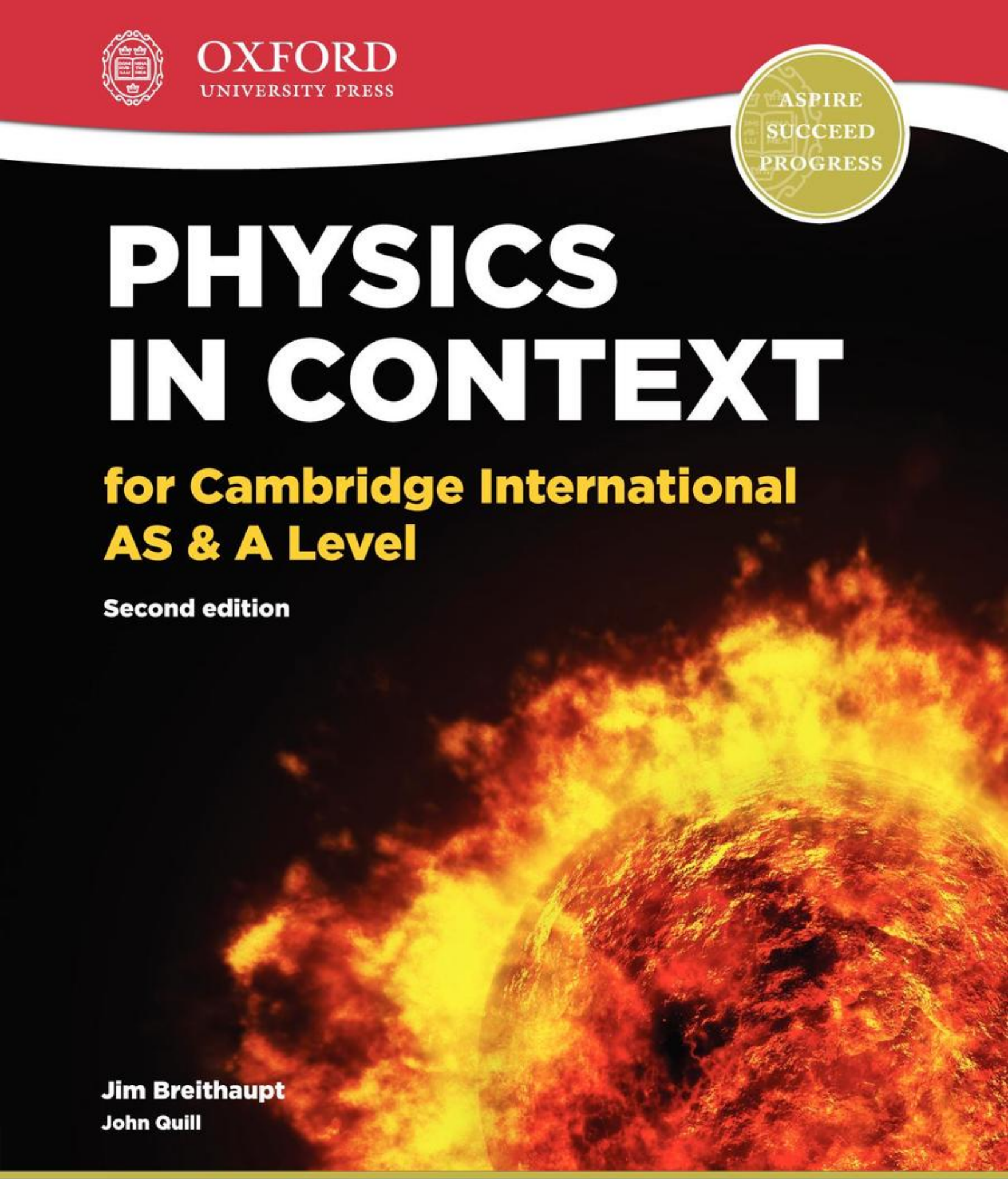
PHYSICS IN CONTEXT

for Cambridge International AS & A Level

Second edition



Jim Breithaupt
John Quill

Oxford excellence for Cambridge AS & A Level

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Extra material, including a chapter on mathematical skills and answers to end-of-chapter questions can be accessed online at: www.oxfordsecondary.com/9780198399629

Introduction

Physics in Context aims to make your study of physics successful and interesting.

It has been written specifically to meet the requirements of Supplementary Level (AS Level) and Advanced Level (A Level) Physics, and is particularly suited to the Cambridge International Examinations syllabus. The content also covers the requirements of the Caribbean Advanced Proficiency Examination (CAPE)®.

The book is divided into 25 chapters:

Chapters 1–10 cover AS Level.

Chapters 11–25 cover A Level.

New ideas are presented in the book in a careful step-by- step manner to allow you to develop a firm understanding of concepts and ideas. Key concepts that occur throughout the course are identified and discussed in detail at appropriate intervals in the book. The questions at the end of each chapter and after each key concept interval will help you to develop your grasp of the key concepts. These concepts are essential ideas, theories, principles or mental tools that help you to develop a deep understanding of their subject and make links between the different topics.

The key concepts in physics cover the following ideas.

- The development of **models of physical systems** is central to physics. Physics is the science that seeks to understand the behaviour of the Universe. Models simplify, explain and predict how physical systems behave.
- Physical models are usually based on prior observations or experiments, and their **predictions** are tested to check that they are consistent with **evidence** from further observations and experiments.
- **Mathematics** is used to express physical principles and models. It is also a **tool** to analyse theoretical models, solve quantitative problems and make predictions.
- Everything in the Universe comprises **matter** and/or **energy**. **Waves** transfer energy and are essential to many modern applications of physics.
- Matter and energy interact through **forces** and **fields**. The behaviour of the Universe is governed by fundamental forces that act over different length scales and magnitudes. These include the gravitational force, the electromagnetic force and nuclear forces.

Physics at AS and A level will require you to describe and explain facts and processes in detail and with accuracy. The course is also about developing skills so that you can apply what you have learned.

The more basic topics are covered in the early chapters of the book and each chapter contains related topics. However, the topics do not have to be studied in strict numerical order, but can be followed in any sequence which suits you or your teacher.

The layout of the book is designed to cover information in a clear way that is easy to access. Its features include:

- **Accessible language** to improve your comprehension and understanding.
- **Purple type** to highlight words that can be found in the glossary. This enables you to easily access a full explanation of important terms used in the text. The comprehensive glossary provides you with clear and concise definitions of over 250 terms used throughout the book.
- **Extension material** helps to widen your horizons and stimulate an interest in broader aspects of physics. Much of the 'Extension material' includes content that is required for the Caribbean Advanced Proficiency Examination (CAPE)®.
- **Notes** in boxes and the text to give you useful advice about certain aspects of a topic and so aid your learning. These may be helpful information or explanations to make concepts clearer.
- **In-topic questions** to provide a quick check on how well you have learnt and understood the factual content of each topic. Answers are provided at the back of the book, so that the accurately completed test gives you a concise summary of the information in each topic.
- **End of chapter questions** to effectively consolidate the information learned across all of the topics in a chapter. Answers are provided at the back of the book. Unless indicated otherwise at the end of each question, all the questions and answers in the book have been written by the author.
- **Cambridge International Examination questions** to give you practice of the type of questions you can expect in the final examination.

Cambridge International Examinations bears no responsibility for the example answers to questions taken from its past question papers which are contained in this publication.

Studying physics is an exciting experience for those prepared to probe the ideas in the subject. Discovering new ideas and applications can be very rewarding. However, finding the right answers is only part of physics though! Hitting on the right questions to ask is often the key to new discoveries. I hope this book provides all you need to do well in your exams, but more importantly, I hope it provokes many more questions from your curiosity and imagination.

Jim Breithaupt

Part 1: Skills for starting physics

On these pages you will learn to:

- Appreciate that all physical quantities consist of a numerical magnitude and a unit
- Know the unit of each physical quantity listed in the syllabus
- Make reasonable estimates of physical quantities in the syllabus
- Recall the following SI base quantities and their units: mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol) (topic 21.2)
- Express derived units in terms of SI base units
- Use SI base units to check the homogeneity of physical equations
- Use the following prefixes and their symbols to indicate decimal submultiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T)
- Use the scientific conventions for labelling graph axes and table columns

1. Using a calculator

Practice makes perfect when it comes to using a calculator. For A level Physics, you need no more than a scientific calculator. You need to make sure you master the technicalities of using a scientific calculator as early as possible in your A level Physics course. At this stage, you should be able to use a calculator to add, subtract, multiply, divide, find squares and square roots and calculate sines, cosines and tangents of angles. Further important calculator functions are described in the Mathematical skills section on the CD.

2. Making measurements

You should know at this stage how to make measurements using basic equipment such as metre rules, protractors, stopwatches, thermometers, measuring cylinders, balances (for weighing an object) and ammeters and voltmeters. During the course, you will also be expected to use micrometers, (vernier) calipers, centre-reading meters (galvanometers), oscilloscopes and data-loggers, and Hall probes for measuring magnetic fields (A2 only). The use of these items is described for reference in part 2 of this introduction, on pp.xii–xviii.

Some useful reminders about making measurements:

- Check the **zero reading** when you use an instrument to make a measurement. For example, a metre ruler worn away at one end might give a zero error or a digital meter or analogue meter (i.e. a pointer with a scale) might not read zero when the input to the meter is zero.
- When a multi-range instrument is used, start with the **highest range** and switch to a lower range if the reading is too small to measure accurately.
- Make sure you record all your measurements in a **logical order**, stating the correct unit of each measured quantity.
- Don't pack equipment away until you are sure you have enough measurements or you have checked unexpected readings. See pp.xx–xxi for **anomalous** measurements.

How to obtain accurate results from an experiment

- Make your measurements as precisely as possible and repeat them if possible to ensure they are reliable. See p.xiii for more about **precision** and **reliability**.
- Reduce experimental errors in making measurements. Such errors are either:
  1. random errors that cause repeat readings to differ at random and are caused by uncontrolled variables,
  2. systematic errors such as zero errors that cause readings to be consistently higher or lower than they should be. See p.xii for more about random and systematic errors.
- Where measurements are repeated, calculate and use the **mean values**. Assess the uncertainty in a measurement from the spread of the readings about the mean value. See p.xiii for more about how to assess and use **uncertainties** in your measurements to find out how **accurate** your results are.

3. Using measurements in calculations

When you record a measurement, you must always note the correct unit as well as the numerical value of the measurement.

The scientific system of units is called the SI system. The SI system is described in more detail later in this introduction, on p.ix. The base units of the SI system you need to remember are listed below. All other units are derived from the SI base units. The symbol for each unit is shown in brackets. Later in your course you will also meet the kelvin (K), which is the SI unit of temperature (see p.300), and the mole (mol) which is the SI unit for the amount of a substance (see pp.318–319).

- The metre (m) is the SI unit of length. Note also that 1 m = 100 cm = 1000 mm.
- The kilogram (kg) is the SI unit of mass. Note that 1 kg = 1000 g.
- The second (s) is the SI unit of time.
- The ampere (A) is the SI unit of current.

All other units are derived from the SI base units. For example,

- the SI unit of speed is the metre per second because speed = distance ÷ time. The unit is abbreviated as $m\,s^{-1}$ because it is the unit of distance (m) ÷ the unit of time (s)
- the SI unit of force is the newton (N). Because 'force = mass × acceleration' as explained in topic 2.1, the newton expressed in terms of base units is the kilogram metre per second squared ($kg\,m\,s^{-2}$).

See pages viii–ix for more about derived units.

Powers of ten and numerical prefixes are used to avoid unwieldy numerical values. For example,

- $1\,000\,000 = 10^6$ which is 10 raised to the power 6 (usually stated as '10 to the 6')
- $0.000\,000\,1 = 10^{-7}$ which is 10 raised to the power −7 (usually stated as '10 to the −7').

Prefixes are used with units as abbreviations for powers of ten. For example, a distance of 1 kilometre may be written as 1000 m or 10^3 m or 1 km. The most common prefixes are shown in Table 1.

Table 1 *Prefixes*

Prefix	pico-	nano-	micro-	milli-	kilo-	mega-	giga-	tera-
Value	10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{3}	10^{6}	10^{9}	10^{12}
Prefix symbol	p	n	μ	m	k	M	G	T

Note that the cubic centimetre (cm^3) and the gram (g) are in common use and are therefore allowed as exceptions to the prefixes in Table 1.

Standard form is usually used for numerical values smaller than 0.001 or larger than 1000.

- The numerical value is written as a number between 1 and 10 multiplied by the appropriate power of ten. For example:

$$64\,000\,\text{m} = 6.4 \times 10^4\,\text{m}$$
$$0.000\,005\,1\,\text{s} = 5.1 \times 10^{-6}\,\text{s}$$

- A prefix may be used instead of some or all of the powers of ten. For example,

$$35\,000\,\text{m} = 35 \times 10^3\,\text{m} = 35\,\text{km}$$
$$0.000\,000\,59\,\text{m} = 5.9 \times 10^{-7}\,\text{m} = 590\,\text{nm}$$

To convert a number to standard form, count how many places the decimal point must be moved to make the number between 1 and 10. The number of places moved is the power of ten that must accompany the number between 1 and 10. Moving the decimal place:

- to the left gives a positive power of ten (e.g. $64\,000 = 6.4 \times 10^4$)
- to the right gives a negative power of ten (e.g. $0.000\,005\,1 = 5.1 \times 10^{-6}$).

4. Using trigonometry

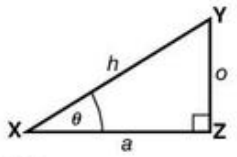


Fig 1 *A right-angled triangle*

The sine, cosine and tangent of an angle are defined from the right-angled triangle. Figure 1 shows a right-angled triangle XYZ, in which side XY is the **hypotenuse** (i.e. the side opposite the right angle), side YZ is **opposite** to angle θ and side XZ is **adjacent** to angle θ.

$$\sin\theta = \frac{\text{YZ}}{\text{XY}} = \frac{o}{h} \quad \text{where } o = \text{YZ, the side opposite angle } \theta$$

$$\cos\theta = \frac{\text{XZ}}{\text{XY}} = \frac{a}{h} \qquad h = \text{XY, the hypotenuse}$$

$$\tan\theta = \frac{\text{YZ}}{\text{XZ}} = \frac{o}{a} \qquad a = \text{XZ, the side adjacent to angle } \theta$$

Notice that when:
i $\theta = 0$, YZ = 0 so $\sin\theta = 0$ and XZ = XY so $\cos\theta = 1$
ii $\theta = 90°$, YZ = XY so $\sin\theta = 1$ and XZ = 0 so $\cos\theta = 0$

Pythagoras' theorem states that for any right-angled triangle:

The square of the hypotenuse
= the sum of the squares of the other two sides

Applying Pythagoras' theorem to the right-angled triangle XYZ gives:

$$\mathbf{XY^2 = XZ^2 + YZ^2}$$

5. Symbols in science

Symbols are used in equations and formulae to represent physical quantities. In your previous course, you may have used equations with words instead of symbols to represent physical quantities. For example , you will have met the equation 'distance moved = speed × time'. Perhaps you were introduced to the same equation in the symbolic form '$s = vt$', where s is the symbol used to represent distance , v is the symbol used to represent speed and t is the symbol used to represent time. The equation in symbolic form is easier to use because the rules of algebra are more easily applied to it than to a word equation.

If you used symbols in your GCSE course, you might have met the use of s for distance and I for current. Maybe you wondered why we don't use d for distance instead of s; or C for current instead of I. The answer is that physics discoveries have taken place in many countries. The first person to discover the key ideas about speed was Galileo, the great Italian scientist; he used the word *'scale'* from his own language for distance, and therefore assigned the symbol s to distance. Important discoveries about electricity were made by Ampère, the great French scientist; he wrote about the intensity of an electric current, so he used the symbol I for electric current. The symbols we now use are used in all countries in association with the **SI system** of units.

Table 2

Physical quantity	Symbol	Unit	Unit symbol
Distance	s	metre	m
Speed or velocity	v	metre per second	ms^{-1}
Acceleration	a	metre per second per second	ms^{-2}
Mass	m	kilogram	kg
Force	F	newton	N
Energy or work	E	joule	J
Power	P	watt	W
Density	ρ	kilogram per cubic metre	kgm^{-3}
Current	I	ampere	A
Potential difference or voltage	V	volt	V
Resistance	R	ohm	Ω

A full list of all the physical quantities, their symbols and units, that you are likely to meet in the AS or A Level Physics course is given on p.x.

6. Using equations

Equations often need to be rearranged. This can be confusing if you don't learn the following basic rules at an early stage:

- **Read an equation properly**. For example, the equation $v = 3t + 2$ is not the same as the equation $v = 3(t + 2)$. If you forget the brackets when you use the second equation to calculate v when $t = 1$, then you will get $v = 5$ instead of the correct answer $v = 9$. The first equation tells you to multiply t by 3 then add 2. The second equation tells you to add t and 2, then multiply the sum by 3.
- **Rearrange an equation properly**. In simple terms, always make the same change to both sides of an equation. For example, to make t the subject of the equation $v = 3t + 2$,

Step 1: Subtract 2 from both sides of the equation,

so $v - 2 = 3t + 2 - 2$

$= 3t$

Step 2: The equation is now $v - 2 = 3t$ and can be written

$3t = v - 2$

Step 3: Divide both sides of the equation by 3,

so $\frac{3t}{3} = \frac{v - 2}{3}$

Step 4: Cancel 3 on the top and the bottom of the left-hand side, to finish with $t = \frac{v - 2}{3}$

To use an equation as part of a calculation:

- Start by making the quantity to be calculated the **subject** of the equation.
- Write the equation out with the **numerical values** in place of the symbols.
- Carry out the calculation and make sure you give the answer with the **correct unit**.

Unless the equation is simple (e.g. $V = IR$), don't insert numerical values then rearrange the equation. Rearrange, then insert the numerical values; you are less likely to make an error if the numbers are inserted later in the process.

Estimating the result of a measurement or calculation is a useful technique to ensure big errors have not been made. For example, in a density measurement of a metal object, you would expect the result to be several times the density of water. A result of 800 kg m^{-3} compared with a value of 1000 kg m^{-3} for water means an error must have been made. Before carrying out an exact calculation of a physical quantity, a mental estimate of the approximate result of the calculation gives an 'order of magnitude' value. Combined with an awareness of the magnitudes of physical quantities, errors can thus be spotted and corrected before carrying out an accurate calculation. See the Mathematical skills section on the CD for more about signs, equations and formulae.

7. More about SI units

In the SI system of units, every derived unit can be expressed in terms of the **base units** of the SI system. Table 3, on the next page, shows how the derived units you meet at A level are related to the SI base units and, in some cases, how they relate to each other. For example, in terms of base units, the coulomb (C) is the ampere second (A s). Such knowledge can prove useful. For example, if you can't quite recall the formula for:

- Density, but you know that its unit is the kilogram per cubic metre (kg m^{-3}), you should be able to see that kg m^{-3} is the unit for mass divided by volume.
- Centripetal acceleration, remembering that the unit of acceleration (i.e. m s^{-2}) is the same as the unit of speed^2 divided by the unit of distance (i.e. m^2 s^{-2}/m) can lead you to the formula v^2/r for centripetal acceleration. See topic 11.2 if necessary.

Derived units written in terms of their base units can be used to check equations. The physical quantities on each side of an equation must match in terms of base units. If they don't match, the equation cannot be correct. For example, consider

- the equation $v = \sqrt{2gR}$, which is used to calculate the escape speed v of an object from the surface of a planet of radius R and surface gravitational field strength g
  Left-hand side base units = m s^{-1}
  Right-hand side base units = $\sqrt{\text{m s}^{-2} \times \text{m}}$ = m s^{-1}
  The equation has the same combination of base units on each side, so it is correct; we say it is **homogeneous** in terms of the base units.
- the equation $W = QV$ is used to calculate the work done W to move a charge Q through a potential difference V
  Left-hand side base units (see Table 3) = kg m^2 s^{-2}
  Right-hand side base units = (A s) × (kg m^2 s^{-3} A^{-1})
  = kg m^2 s^{-2}
  The equation has the same combination of base units on each side, so it is homogeneous. Note that for simple equations such as this, homogeneity can be checked faster by recalling basic relationships between physical quantities. In this example, one volt is one joule per coulomb, so the unit of QV is the joule per coulomb × the coulomb, which is the joule.

The links between different units do not need to be made through the SI base units. For example, the volt (V) is the joule per coulomb (J C^{-1}) which is a useful link to remember as it helps you to develop your understanding of potential difference.

There are some units in the A level specification that are not SI units but they are used in specific situations for convenience. Those listed below are in common use.

1 The **atmosphere** (atm) is a unit of pressure equal to the mean pressure of the atmosphere at sea level and is equal to 101 kPa, or 1.01 × 10^5 Pa.
2 The **electron volt** (eV) is a unit of energy defined as the work done when an electron moves through a p.d. of 1 V.
1 eV = 1.6 × 10^{-19} J.
Note that 1 MeV = 10^6 eV = 1.6 × 10^{-13} J
3 The **kilowatt hour** (kW h) is a unit of energy equal to the energy supplied to a 1 kilowatt appliance in 1 hour which is 3.6 MJ.
4 The **light year** is the distance travelled in space by light in 1 year.
5 The **litre** is a unit of volume equal to 10^{-3} m^3.

***Table 3** Links between units*

Quantity	Symbol	Unit	Unit symbol	Other forms of unit	Base unit	Chapter reference
acceleration	a	metre per second^2	ms^{-2}		ms^{-2}	1
activity of radioactive source	A	becquerel	Bq	s^{-1}	s^{-1}	23
angle	θ	radian or degree	rad or °			1,11
angular displacement	θ	radian or degree	rad or °			11
angular frequency	ω	radian per second	$rads^{-1}$			11
angular speed or velocity	ω	radian per second	$rads^{-1}$			11
capacitance	C	farad	F	CV^{-1}	$kg^{-1}m^{-2}s^4A^2$	15
charge	q, Q	coulomb	C		As	6
decay constant	λ	second $^{-1}$	s^{-1}		s^{-1}	23
density	ρ	kilogram per cubic metre	kgm^{-3}		kgm^{-3}	5
electric field strength	E	newton per coulomb	NC^{-1}	Vm^{-1}	$kgms^{-3}A^{-1}$	6
electric potential	V	volt	V	JC^{-1}	$kgm^2s^{-3}A^{-1}$	14
energy, work	E, U	joule	J		kgm^2s^{-2}	2
force	F	newton	N		$kgms^{-2}$	2
frequency	f	hertz	Hz	s^{-1}	s^{-1}	8
gravitational field strength	g	newton per kilogram	Nkg^{-1}		ms^{-2}	1,2
gravitational potential	ϕ	joule per kilogram	Jkg^{-1}		m^2s^{-2}	13
magnetic flux density	B	tesla	T	$NA^{-1}m^{-1}$ Wbm^{-2}	$kgs^{-2}A^{-1}$	18
magnetic flux	ϕ	weber	Wb	Tm^2,Vs	$kgm^2s^{-2}A^{-1}$	18
momentum	p	kilogram metre per second	$kgms^{-1}$	Ns	$kgms^{-1}$	4
permeability of free space	μ_0	henry per metre	Hm^{-1}		$kgms^{-2}A^{-2}$	18
permittivity of free space	ε_0	farad per metre	Fm^{-1}		$kg^{-1}m^{-3}s^4A^2$	14
phase difference	ϕ	radian or degree	rad or °			8
potential difference, e.m.f.	V, E	volt	V	JC^{-1}	$kgm^2s^{-3}A^{-1}$	6, 7
power	P	watt	W	Js^{-1}	kgm^2s^{-3}	2
pressure	p	pascal	Pa	Nm^{-2}	$kgm^{-1}s^{-2}$	5
resistance	R	ohm	Ω	VA^{-1}	$kgm^2s^{-3}A^{-2}$	6
resistivity	ρ	ohm metre	Ωm		$kgm^3s^{-3}A^{-2}$	6
spring constant	k	newton per metre	Nm^{-1}		kgs^{-2}	5
stress, Young modulus	σ, E	pascal	Pa	Nm^{-2}	$kgm^{-1}s^{-2}$	5
velocity, speed	v	metre per second	ms^{-1}		ms^{-1}	1
wavelength	λ	metre	m		m	8

Part 2: Practical skills

In the laboratory

The experimental skills you will develop during your course are part of the 'tools of the trade' of every physicist. Data loggers and computers are commonplace in modern physics laboratories, but awareness on the part of the user of precision, reliability, errors and accuracy are just as important as when measurements are made with much simpler equipment. Let's consider in more detail what you need to be aware of when you are working in the physics laboratory.

Safety and organisation

A science laboratory should be a safe place, but it can be dangerous if the safety rules and common sense are disregarded. Your teacher will give you a set of safety rules and should explain them to you. You must comply with them at all times. In addition, you must use your common sense and organise yourself so that you work safely. For example, if you set up an experiment with pulleys and weights, you need to ensure they are stable and will not topple over. Before you set up an electrical circuit, think about where to place the meters so you can read them easily.

Working with others

Most scientists work in teams, each person cooperating with the other team members to achieve specific objectives. In general, teamwork is effective because although each team member may have a designated part to play, the exchange of ideas within the team often gives greater insight and awareness as to how to achieve the objectives.

In your practical activities, you will often work in a small group of two or three people. In such group work, you need to cooperate with the others in the group so everyone understands the objectives of the practical activity and everyone participates in planning and carrying out the activity.

Planning

You may be asked to plan an experiment or investigation. The practical activities you carry out during your course should enable you to prepare a plan. Here are the key steps in drawing up a plan:

1 Decide in detail what you intend to investigate. Note the independent and dependent variables you intend to measure and note the variables that need to be controlled. The other variables need to be controlled to make sure they do not change. A **control variable** that can't be kept constant would cause **the dependent variable to alter**.

2 Select the equipment necessary for the measurements. Specify the range of any electrical meters you need.

3 List the key stages in the method you intend to follow and make some preliminary measurements to check your initial plans. Consider safety issues before you do any preliminary tests. If necessary, modify your plans as a result of your preliminary tests.

4 If the aim of your investigation is to test a hypothesis or theory or use the measurements to determine a physical quantity (e.g. resistivity), you need to know how to use the measurements you make.

Carrying out instructions and recording your measurements

In some investigations, you will be expected to follow instructions supplied to you either verbally or on a worksheet. You should be able to follow a sequence of instructions without guidance. However, always remember safety first and, if the instructions are not clear, ask your teacher to clarify them.

When you record your measurements, tabulate them with a column for the independent variable and one or more columns for the dependent variable. Remember to allow for repeat readings and average values, if appropriate. The table should have a clear heading for each of the measured variables, with the unit symbol shown after the heading (e.g. current/A), as below.

Single measurements of other variables (e.g. control variables) should be recorded together, immediately before or after the table. Make sure you record the unit of each measurement. In addition, you should record the precision (i.e. the least detectable reading) of each measurement. This information is important when you come to analyse and evaluate your measurements.

Table 1 *Tabulating the measurements from an investigation of p.d. against current for a wire*

potential difference/V	current/A			average current/A
	1st set	2nd set	3rd set	

length of wire/m = _____
diameter of wire/mm = _____, _____, _____
average diameter of wire/mm = _____

Making careful measurements

A measurement checklist

You should be able to:

- measure lengths using a ruler, vernier calipers and a micrometer
- weigh an object and determine its mass using a spring balance, a lever balance or a top-pan balance
- use a protractor to measure an angle and use a set square
- measure time intervals using clocks, stopwatches and the time base of an oscilloscope
- measure temperature using a thermometer
- use ammeters and voltmeters with appropriate scales
- use an oscilloscope.

In addition you should be able to:

- distinguish between systematic errors (including zero errors) and random errors
- understand what is meant by accuracy, sensitivity, linearity, reliability, precision and validity
- read analogue and digital displays.

Fig 1 Physics instruments

Measurements and errors

Measurements play a key role in science, so they must be:

1 **Reliable** – which means that a consistent value should be obtained each time the same measurement is repeated. An unreliable weighing machine in a shop would cause the customers to go elsewhere. In science, you can't go elsewhere so the measurements must be reliable. Each time a given measurement is repeated, it should give the same value within acceptable limits.
2 **Valid** – which means the measurements are of the required data or can be used to give the required data. For example, a voltmeter connected across a variable resistor in series with a lamp and a battery would not measure the potential difference across the lamp.

Errors of measurement are important in finding out how accurate a measurement is. We need to consider errors in terms of differences from the mean value. Consider the example of measuring the diameter of a uniform wire using a micrometer. Suppose the following diameter readings are taken for different positions along the wire from one end to the other:

0.34 mm, 0.33 mm, 0.36 mm, 0.33 mm, 0.35 mm

- The **range** of the measurements is 0.03 mm. This is the difference between the lowest and the highest reading. We will see later we can use this to estimate the **uncertainty** of the measurement.
- The **mean value**, $<d>$, is 0.34 mm, calculated by adding the readings together and dividing by the number of readings. If the difference between each reading and $<d>$ changed regularly from one end of the wire to the other, it would be reasonable to conclude that the wire was non-uniform. Such differences are called **systematic errors**. If no such differences can be seen in the set of readings, in other words, there is no obvious trend in the differences, the differences are said to be **random errors**.

What causes random errors? In the case of the wire, vibrations in the machine used to make the wire might have caused random variations in its diameter along its length. The experimenter might not use or read the micrometer correctly consistently. This shouldn't happen to a skilled experimenter.

The range of the diameter readings is from 0.33 mm to 0.36 mm. The readings lie within 0.015 mm (i.e. half the range) of the mean value, which we will round up to 0.02 mm. The diameter can therefore be written as 0.34 ± 0.02 mm. The diameter is accurate to ± 0.02 mm. The uncertainty in the mean value of the diameter is therefore ± 0.02 mm.

Using instruments

Instruments used in the physics laboratory range from the very basic (e.g. a millimetre scale) to the highly-sophisticated (e.g. a multichannel data recorder). Whatever type of instrument you use, you need to know what the following terms mean.

Zero error

Does the instrument read zero when it is supposed to? If not, the zero reading must be taken into account when measuring the gap width otherwise there will be a systematic error in the measurements.

Uncertainty

The uncertainty in the mean value of a measurement may be taken as half the range, expressed as a ± value (e.g. $I = 2.6 \pm 0.2$ A). If the readings are the same, the instrument precision should be used as the uncertainty.

Accuracy

An **accurate** measurement is one that has been obtained using accurately-calibrated instruments correctly and where there are no systematic errors. **Accuracy** is a measure of confidence in a measurement and is usually expressed as the uncertainty in the measurement.

Precision

The **precision** of a measurement is the degree of exactness of the measurement. The precision of an instrument is the smallest non-zero reading that can be measured using the instrument.

- If the reading of an instrument fluctuates when it is being taken, take several readings and calculate the mean value and range of the measurements. The precision of the measurement is then given by the uncertainty of the readings (i.e. half the range of the readings).
- If the reading is constant, estimate the precision of a measurement directly from the instrument (or use the stated precision from the instrument specification).

Precise readings are not necessarily accurate readings, because systematic errors could make precise readings all lower or all higher than they ought to be.

Linearity

This is a design feature of many instruments; it means the reading is directly proportional to the magnitude of the variable that causes the reading to change. For example, if the scale of a moving coil meter is linear, the reading of the pointer against the scale should be proportional to the current.

A lens may be used to read a scale with greater precision. Provided the pointer is thin compared to a scale division, using a lens enables the scale to be read to within 0.2 of a division. In Figure 2, the scale can be read to ±0.02 A (= 0.2 of a scale division × 0.1 A).

Measurement errors are caused in analogue instruments if the pointer on the scale is not observed correctly. The observer must be directly in front of the pointer when the reading is made. Figure 3 shows how a plane mirror is used for this purpose. The image of the pointer must be directly behind the pointer to ensure the observer views the scale directly in front of the pointer.

Instrument range

Multi-range instruments such as multimeters have a 'range' dial that needs to be set according to the maximum reading to be measured. For example, if the dial can be set at 0–0.10 A, 0–1.00 A or 0–10.0 A, you would use the 0–1.00 A range to measure the current through a 0.25 A torchbulb as the 0–0.10 A range is too low and the 0–1.00 A range is more sensitive than the 0–10.0 A range.

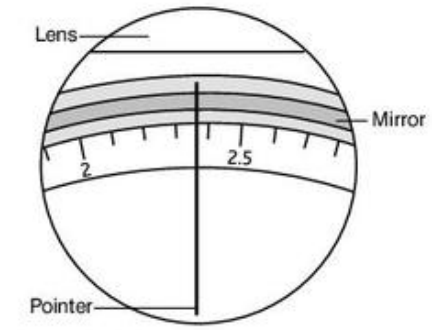


Fig 2 *Magnifying a scale*

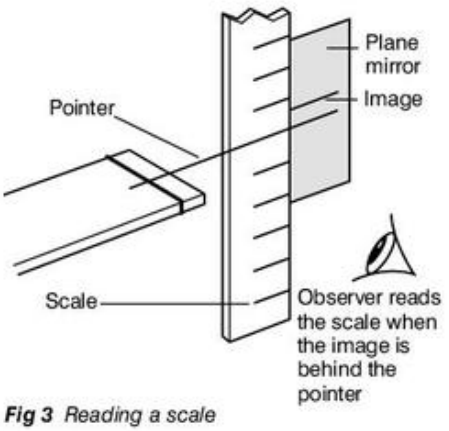
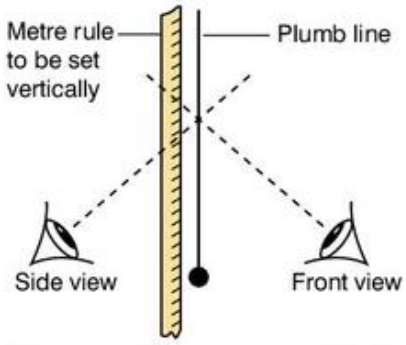


Fig 3 *Reading a scale*

Fig 4 *A multimeter*

Everyday physics instruments



If the metre rule appears parallel to the plumb line from the front and the side, the ruler must be vertical.

Fig 1 *Finding the vertical*

Rules and scales

Metre rules are often used as vertical or horizontal scales in mechanics experiments.

To set a metre rule in a vertical position:

- use a set square perpendicular to the rule and the bench, if the bench is known to be horizontal, or
- use a plumb line (a small weight on a string) to see if the rule is vertical. You need to observe the rule next to the plumb line from two perpendicular directions. If the rule appears parallel to the plumb line from both directions, then it must be vertical. See Figure 1.

To ensure a metre rule is horizontal, use a set square to align the metre rule perpendicular to a vertical metre rule.

Micrometers and vernier calipers

Micrometers give readings to within 0.01 mm. A **digital** micrometer gives a read-out equal to the width of the micrometer gap. An **analogue** micrometer has a barrel on a screw thread with a pitch of 0.5 mm. For such a micrometer:

- the edge of the barrel is marked in 50 equal intervals so each interval corresponds to changing the gap of the micrometer by 0.5/50 mm = 0.01 mm
- the stem of the micrometer is marked with a linear scale graduated in 0.5 mm marks
- the reading of a micrometer is where the linear scale intersects the scale on the barrel.

Figure 2 shows a reading of 4.06 mm. Note that the edge of the barrel is between the 4.0 mm and 4.5 mm marks on the linear scale. The linear scale intersects the 6th mark after the zero mark on the barrel scale. The reading is therefore 4.00 mm from the linear scale + 0.06 mm from the barrel scale.

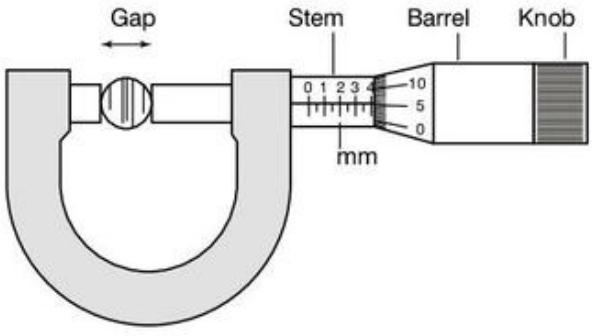


Fig 2 *Using a micrometer*

To use a micrometer correctly:

1. Check its zero reading and note the zero error if there is one.
2. Open the gap (by turning the barrel if analogue) then close the gap on the object to be measured. Turn the knob until it slips. Don't overtighten the barrel.
3. Take the reading and note the measurement after allowing if necessary for the zero error.
4. Note that the precision of the measurement is ±0.010 mm because the precision of the reading and the zero reading are both ±0.005 mm. So the difference between the two readings (i.e. the measurement) has a precision of 0.010 mm.

Vernier calipers are used for measurements of distances up to 100 mm or more. Readings can be made to within 0.1 mm. The sliding scale of an analogue vernier caliper has ten equal intervals covering a distance of exactly 9 mm so each interval of this scale is 0.1 mm less than a 1 mm interval. To make a reading:

1. The zero mark on the sliding scale is used to read the main scale to the nearest millimetre. This reading is rounded down to the nearest millimetre.

2 The mark on the sliding scale closest to a mark on the millimetre scale is located and its number noted. Multiplying this number by 0.1 mm gives the distance to be added on to the rounded-down reading.

Figure 3 shows the idea. The zero mark on the sliding scale is between 39 and 40 mm on the mm scale. So the rounded-down reading is 39 mm. The 5th mark after the zero on the sliding scale is nearest to a mark on the millimetre scale. So the extra distance to be added on to 39 mm is 0.5 mm (= 5 × 0.1 mm). Therefore, the reading is 39.5 mm.

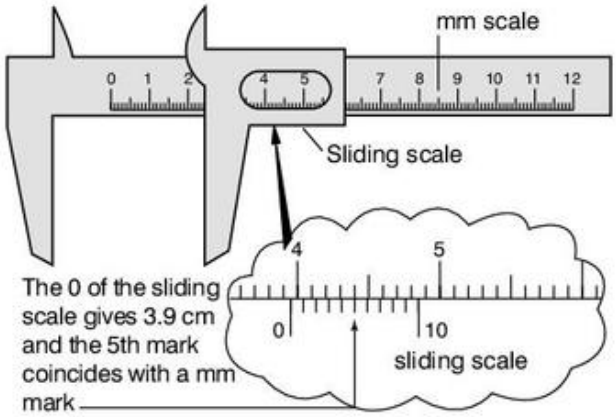


Fig 3 *Using vernier calipers*

Timers

Stopwatches used for interval timings are subject to human error because reaction time, about 0.2 s, is variable for any individual. With practice, the delays when starting and stopping a stopwatch can be reduced. Even so, the precision of a single timing is unlikely to be better than 0.1 s. Digital stopwatches usually have read-out displays with a resolution of 0.01 s but human variability makes such precision unrealistic and the precision of a single timing is the same as an analogue stopwatch.

Timing oscillations requires timing for as many cycles as possible. The timing should be repeated several times to give an average (mean) value. Any timing that is significantly different to the other values is probably due to miscounting the number of oscillations so that timing should be rejected. For accurate timings, a fiducial mark (a pointer or marker in the field of view of the experimenter that can be used as a point of reference) is essential. The mark should be lined up with the centre of the oscillations so it provides a reference position to count the number of cycles as the object swings past it each cycle.

Electronic timers use automatic switches or 'gates' to start and stop the timer. However, just as with a digital stopwatch, a timing should be repeated, if possible several times, to give an average value. Light gates may be connected via an interface unit to a computer. Interrupt signals from the light gates are timed by the computer's internal clock. A software program is used to provide a set of instructions to the computer.

Balances

A balance is used to measure the weight of an object. Spring balances are usually less precise than lever balances. Both types of balance are usually much less precise than an electronic top-pan balance. The scale or read-out of a balance may be calibrated for convenience in kilograms or grams. The accuracy of an electronic top-pan balance can easily be tested using accurately known masses.

More about measurements

You should have learnt how to:

- use the instruments described in this section
- use a calibrated Hall probe (topic 18.4)
- use a galvanometer (centre-reading meter) (topic 7.5)
- use calibration curves where appropriate.

During your course you are also expected to be able to use instruments that are more complex. Such instruments may include the oscilloscope, the travelling microscope, the data logger, sensors and light gates, the Geiger tube with a scaler counter or ratemeter, and the spectrometer. In addition, you should know how to time multiple oscillations and how to avoid parallax errors when reading a scale.

An oscilloscope

This is used to display waveforms and to measure p.d.s and time intervals. You will have probably used an oscilloscope when you studied alternating current. In topics such as capacitor discharge, you will use an oscilloscope to measure the p.d. across a discharging capacitor. When you use an oscilloscope, you should assume that the control dials for its time base and voltage gain are calibrated accurately. However:

- Always check if the oscilloscope has a variable control for either the time base or the voltage gain in addition to the fixed settings of each control dial. If so, you need to ensure that the variable control is at the correct setting (e.g. fully clockwise) for the calibration figure for each of the fixed settings to apply.

Fig 1 *The oscilloscope*

- Make sure if you are measuring direct p.d.s that the oscilloscope input is set for direct p.d. measurements rather than for a.c. measurements. Likewise, if you are measuring an a.c. waveform, you should check the input is set for a.c. measurements rather than d.c. measurements.

In addition, when measuring:

- an a.c. waveform, ensure the *y*-gain is adjusted so the vertical height of the waveform is as large as possible with the full waveform from top to bottom on the screen. When measuring a time period, ensure several cycles are displayed across the screen and that you measure across as many cycles as possible to reduce experimental uncertainty.
- d.c. potentials, for example in a capacitor discharge experiment, ensure the zero reading is correct for zero input p.d. and check it has not drifted during the investigation.

A travelling microscope

This is a microscope on an adjustable platform that can be moved vertically or horizontally by turning an adjustment screw. The microscope itself can usually be aligned vertically or horizontally, according to whether it is to be used to measure a vertical or a horizontal distance. The platform is fitted with a horizontal and a vertical vernier scale so its horizontal or vertical position can be measured to within ± 0.1 mm. A travelling microscope would be used for example to measure the internal diameter of a glass tube (e.g. 1 mm bore) as a micrometer or a conventional vernier caliper could not access the internal surface of the tube. Before use, a travelling microscope should be levelled using a spirit level in two perpendicular directions so its platform is horizontal. Otherwise, there could be a systematic error in the measurements.

A data logger

A data logger enables routine or remote measurements to be made as well as measurements over very long or short time scales. Electronic sensors connected to a data logger can be used to record the variation of a physical property such as temperature. Current and voltage sensors can be used to measure currents and potential differences.

Data loggers vary considerably in complexity and ease of use. Assuming the data logger and sensors are set up, before using a data logger, you may need to choose:

- the most appropriate time scale for the recording
- the time interval between successive recordings (or the number of recordings per second/minute/hour)
- the most appropriate range of each sensor.

If a recording is too fast or too long or the sensors are out of range, the recording should be repeated if possible.

Most data loggers will be linked to computers which are loaded with appropriate software for recording, processing and/or plotting graphs of the results. You may need to print any graphs out if you intend to use them to measure, for example, the gradient if it is a straight-line graph. However, the computer software may do such measurements for you.

Fig 2 *Using a data logger*

Light gates

These are used with a computer or a data logger or timer to remove some of the random errors associated with personal judgements when a moving object passes a certain position, for example, if you have to time an object to move from rest through a certain distance, as shown in Figure 3.

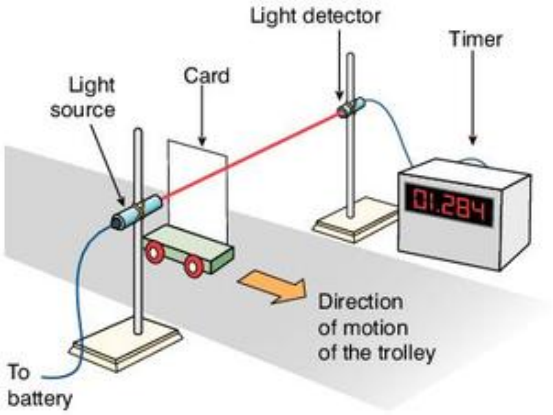


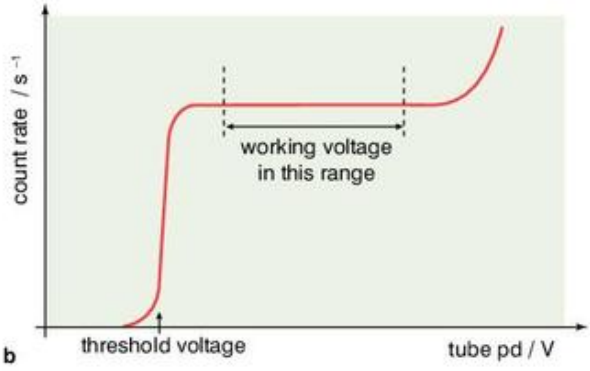
Fig 3 *Using a light gate*

The effect of using light gates should be to reduce the range of the readings for a given measurement. However, light gates may not be suitable for every experiment in which a moving object has to be timed. For example, the time period of an oscillating object that repeatedly moves backwards and forwards through a light gate could only be timed for one half cycle of the object's motion, corresponding to the object moving through the light gate in one direction to start the timing and then in the opposite direction to stop the light gate. The light gate would need to be exactly at the centre of the oscillations otherwise the timing would not be exactly one half cycle. Repeated measurements of one half cycle could be made to give a more reliable mean value and this might give better results than using a stopwatch if the oscillations are too fast to time manually.

The Geiger tube

This may be used with a scaler counter which counts the number of ionising particles that enter the tube or it may be used with a ratemeter which gives a read-out of the count rate (i.e. number of counts per unit time) of the particles entering the tube. The tube p.d. must be set at its operating p.d. which is normally in excess of 300 V. The number of counts in a certain time interval is measured by setting the counter to zero, then starting the counter and stopping it after a certain time.

a



b

Fig 4 *Using a Geiger tube* **a** *a Geiger tube connected to a scalar counter* **b** *a graph of count rate v. tube p.d.*

Figure 4b shows how the count rate varies with the tube p.d. The operating p.d. corresponds to the plateau of the graph sufficiently far from the minimum p.d. necessary for the tube to operate (i.e. the threshold p.d.) as to be unaffected by random fluctuations in the tube p.d.

- When using the tube with a scalar counter, the number of counts in a given time (e.g. 100 s) should be measured several times to give a mean value of the count rate (i.e. counts per second). The greater the total number of counts, the smaller the uncertainty in the measurement. If the time interval is too short, random errors that may occur in starting and stopping the counter could be more significant than if a longer time interval were used. If the time interval is too long, it would be difficult to tell if the activity of the source is decreasing or if an error in starting or stopping the timer has occurred.
- When using the tube with a ratemeter, ensure the ratemeter is set on the range which gives the largest reading. For example, if the range dial has three positions, 1, 10 and 100 counts per second, the range may need to be set at 1 count per second if the reading is very small on the '10' and '100' positions.

When using either a scaler counter or a ratemeter, remember to measure the background count rate and subtract it from the measurements made when the source is present.

A spectrometer

This is used to measure the wavelengths of a line emission spectrum or a line absorption spectrum. A spectrometer is fitted with a circular scale to measure the angle of diffraction of the lines of a spectrum.

Most industrial and research laboratories now use a 'spectrum analyser', which is an electronic spectrometer linked to a computer, that gives a visual display of the variation of intensity with wavelength. The spectrometer shown in Figure 5 would be used to check the calibration of the spectrum analyser. Knowledge of how a spectrometer or a spectrum analyser works or how they are used is not required in this specification.

Fig 5 *A spectrometer fitted with a diffraction grating*

Analysis and evaluation

Data processing

For a single reading, the precision of the measuring instrument determines the precision of the reading. A micrometer with a precision of 0.01 mm gives readings that each have a precision of 0.01 mm.

For several readings, the number of significant figures of the mean value should be the same as the precision of each reading. For example, consider the following measurements of the diameter of a wire: 0.34 mm, 0.33 mm, 0.36 mm, 0.33 mm, 0.35 mm. The mean value of the diameter readings works out at 0.342 mm but the third significant figure cannot be justified, as the precision of each reading is 0.01 mm. Therefore the mean value is rounded down to 0.34 mm.

Note: At the end of a calculation, don't give the answer to as many significant figures as shown on the calculator display. Give your answer to the same number of significant figures (or one more than) as the data with the least number of significant figures.

Estimating uncertainties

How confident can you be in your measurements and any results or conclusions you draw from your measurements? If you work out what each uncertainty is, as a percentage of the measurement (the percentage uncertainty), you can then see which measurement is least accurate. You can then think about how that measurement could be made more accurately.

Worked example

The mass and diameter of a ball bearing were measured and the uncertainty of each measurement was estimated.

The mass, m, of a ball bearing $= 4.85 \times 10^{-3} \pm 0.02 \times 10^{-3}$ kg

The diameter, d, of the ball bearing $= 1.05 \times 10^{-2} \pm 0.01 \times 10^{-2}$ m

Calculate and compare the percentage uncertainty of these two measurements.

Solution

The percentage uncertainty of the mass $m = 0.02/4.85 \times 100\% = 0.4\%$

The percentage uncertainty of the diameter $d = 0.01/1.05 \times 100\% = 1.0\%$

The uncertainty in the diameter measurement is therefore more than twice as much as the mass measurement.

More about uncertainties

1 **When two measurements are added or subtracted**, the uncertainty of the result is the sum of the uncertainties of the measurements. For example, the mass of a beaker is measured when it is empty and then when it contains water:

- The mass of an empty beaker $= 65.1 \pm 0.1$ g,
- The mass of the beaker and water $= 125.6 \pm 0.1$ g

Then the mass of the water could be as much as

$(125.6 + 0.1) - (65.1 - 0.1)$ g $= 60.7$ g, or as little as
$(125.6 - 0.1) - (65.1 + 0.1)$ g $= 60.3$ g.

The mass of water is therefore 60.5 ± 0.2 g.

2 **When a measurement in a calculation is raised to a power *n*,** the percentage uncertainty is increased *n* times. For example, suppose you need to calculate the area *A* of cross-section of a wire that has a diameter of 0.34 ± 0.01 mm. You will need to use the formula $A = \pi d^2/4$. The calculation should give an answer of $9.08 \times 10^{-8}\,m^2$. The percentage uncertainty of *d* is $0.01/0.34 \times 100\% = 2.9\%$. So the percentage uncertainty of *A* is 5.8% ($= 2 \times 2.9\%$). The consequence of this rule is that in any calculation where a quantity is raised to a higher power, the uncertainty of that quantity becomes much more significant.

3 **When two or more quantities are multiplied or divided by each other in a calculation,** the overall percentage uncertainty in the result is the sum of the uncertainties of each quantity. For example, if the uncertainty in a resistance *R* is 5% and in a capacitance *C* is 4%, the uncertainty in *RC* is 9%.

Percentage uncertainties

To work out the percentage uncertainty of *A*, you could calculate:

- the area of cross-section *A* where $d = 0.34 - 0.01\,mm = 0.33\,mm$
  This should give an answer of $8.55 \times 10^{-8}\,m^2$.
- the area of cross-section *A* where $d = 0.34 + 0.01\,mm = 0.35\,mm$
  This should give an answer of $9.62 \times 10^{-8}\,m^2$.

Therefore, the area lies between $8.55 \times 10^{-8}\,m^2$ and $9.62 \times 10^{-8}\,m^2$.

In other words, the area is $(9.08 \pm 0.53) \times 10^{-8}\,m^2$

(as $9.08 - 0.53 = 8.55$ and $9.08 + 0.53 = 9.62$).

The percentage uncertainty of *A* is $0.53/9.08 \times 100\% = 5.8\%$.

This is twice the percentage uncertainty of *d*.

It can be shown as a general rule that for a measurement *x*:

the percentage uncertainty in x^n is *n* times the percentage uncertainty in *x*.

Graphs and errors

Straight-line graphs are important because they are used to establish the relationship between two physical quantities. For example, consider a set of measurements of the distance fallen by an object released from rest and the time it takes. A graph of distance fallen against (time)^2 should be a straight line through the origin. If the line is straight, the theoretical equation $s = \frac{1}{2}gt^2$ (where *s* is the distance fallen, and *t* is the time taken) is confirmed. The value of *g* can be calculated as the gradient of the graph (equal to $\frac{1}{2}g$). If the straight line does not pass through the origin, there is a systematic error in the distance measurement. Even so, the gradient is still $\frac{1}{2}g$.

A best-fit test

Suppose you have obtained your own measurements for an experiment and you use them to plot a graph that is predicted to be a straight line. The plotted points are unlikely to be exactly straight in line with each other. The next stage is to draw a straight line of best fit so that the points are on average as close as possible to the line. Some problems may occur at this stage:

1 There might be a point much further than any other point from the line of best fit. The point is referred to as an **anomaly**. Methods for dealing with an anomalous point are as follows:

- If possible, the measurements for that point should be repeated and used to replace the anomalous point, if the repeated measurement is much nearer the line.
- If the repeated measurement confirms the anomaly, there could be a fault in the equipment or the way it is used. For example, in an electrical experiment, it could be caused by a change of the range of a meter to make that measurement. If no fault is found, make more measurements near the anomaly to see if these measurements lead towards the anomaly. If they do, it is no longer an anomaly and the measurements are valid.
- If a repeat measurement is not possible, the anomalous point should be ignored and a comment made in your report (or on the graph) about it.

2 The points might seem to curve away from the straight line of best fit. The uncertainty of each measurement can be used to give a small range or **error bar** for each measurement. Figure 1 shows the idea. The straight line of best fit should pass through all the error bars. If it doesn't, the following notes might be helpful. You could use the error bars to draw a straight line of maximum gradient and a straight line of minimum gradient.

- The points lie along a straight line over most of the range but curve away more and more further along the line. This would indicate that a straight-line relationship between the plotted quantities is valid only over the range of measurements which produced the straight part of the line.

- Only two or three points might seem to lie on a straight line (see Figure 2). In this case, it cannot be concluded that there is a **linear** relationship between the plotted quantities. You might need to plot further graphs to find out if a different type of graph would give a straight-line relationship. A data analysis software package on a computer could be used to test different possible relationships (or a **log graph** could be plotted).

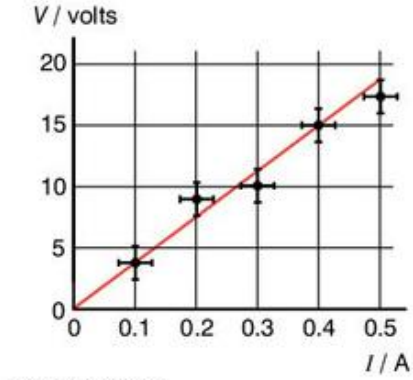


Fig 1 Error bars

Evaluating your results

You should be able to form a conclusion from the results of an investigation. This might be a final calculation of a physical quantity or property (e.g. resistivity) or a statement of the relationship established between two variables. As explained earlier, the degree of accuracy of the measurements could be used as a guide to the number of significant figures in a 'final result' conclusion. Mathematical links established or verified between quantities should be stated in a 'relationship' conclusion.

You always need to evaluate the conclusion(s) of an experiment or investigation to establish its validity. This evaluation could start with a discussion of the strength of the experimental evidence used to draw the conclusions:

- Discuss the reliability of the data and suggest improvements, where appropriate, that would improve the reliability. You may need to consider the effect of the control variables, if the experimental evidence is not as reliable as it should be.
- Discuss the methods taken (or proposed) to eliminate or reduce any random or systematic errors. Describe the steps taken to deal with anomalous results.
- Evaluate the accuracy of the results by considering the percentage uncertainty in the measurements. These can be compared to identify the most significant sources of uncertainty in the measurements, which can then lead to a discussion of how to reduce the most significant sources of uncertainty.
- Propose improvements to the strategy or experimental procedures, referring to the above discussion on validity as justification for the proposals.
- Suggest further experimental work, based on the strength of the conclusions. Strong conclusions could lead to a prediction and how to test it.

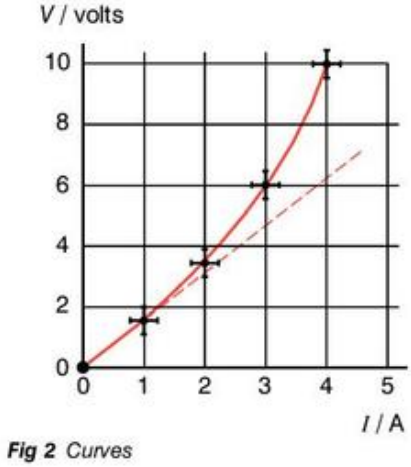


Fig 2 Curves

1 Kinematics

1.1 Vectors and scalars

On these pages you will learn to:

- Distinguish between scalar and vector quantities and give examples of each
- Add and subtract coplanar vectors
- Resolve a vector into two perpendicular components

Imagine you are planning to cycle to a friend's home several kilometres away from your home. The **distance** you travel depends on your route. However, the direct distance from your home to your friend's home is the same whichever route you choose. Distance in a given direction or **displacement** is an example of a **vector** quantity because it has magnitude and direction.

A vector is any physical quantity that has a direction as well as a magnitude.

Further examples of vectors include velocity, **acceleration**, force and **weight**.

A scalar is any physical quantity that is not directional.

For example, distance is a scalar because it takes no account of direction. Further examples of scalars include mass, density, volume and energy.

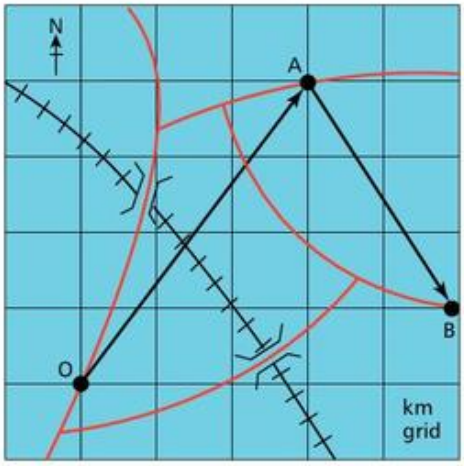
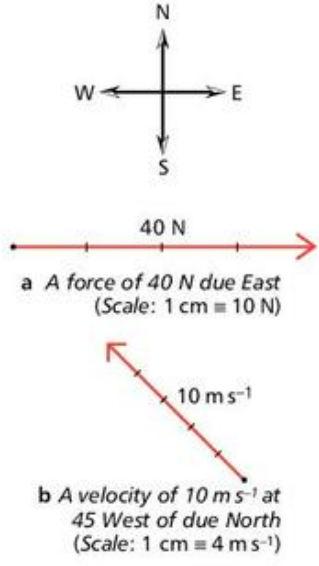


Fig 1 Map of locality

Representing vectors

Any vector can be represented as an arrow. The length of the arrow represents the **magnitude** of the vector quantity. The direction of the arrow gives the **direction** of the vector.

- **Displacement** is distance in a given direction. The displacement from one point to another can be represented on a map or a scale diagram as an arrow from the first point to the second point. The length of the arrow must be in proportion to the least distance between the two points.
- **Velocity** is **speed** in a given direction. The velocity of an object can be represented by an arrow of length in proportion to the speed pointing in the direction of motion of the object.
- **Force and acceleration** are both vector quantities and therefore can each be represented by an arrow in the appropriate direction and of length in proportion to the magnitude of the quantity.



a *A force of 40 N due East (Scale: 1 cm ≡ 10 N)*

b *A velocity of 10 m s⁻¹ at 45 West of due North (Scale: 1 cm ≡ 4 m s⁻¹)*

Fig 2 Representing a vector

On a journey

Cyclists and hill walkers should always take a map and compass to make sure they do not get lost. A compass tells the user which direction is North. A map tells the user how far he or she has gone. Consider the map shown in Figure 1. Suppose your home is at O and your friend's home is at A. Your route-plan is to cycle along the road heading North, over the railway bridge, then turn East at the next road junction.

- The **distance** to be cycled can be estimated by measuring the length of the route on the map in centimetres, then using the map scale.
- The **displacement** or direct distance from O to A is marked on the map as an arrow OA.
- Your **velocity** at any point on your journey changes, because you change direction and because your speed changes. Suppose your speed as you pass over the railway bridge is 2.0 m s^{-1}. The direction in which you are travelling as you pass over the bridge is about 10° East of due North. You can check this on

Figure 1 using a protractor. So your velocity at the railway bridge is $2.0\,\text{m s}^{-1}$ in a direction which is 10° East of due North.

Vector components

The displacement vector **OA**, shown on Figure 1, can be represented:

- as an arrow of length in proportion to the direct distance of 5.0 km from O to A. The direction of the arrow would be 53° North of due East.
- as a map reference, one part stating how far A is East or West of O and the other stating how far A is North or South of O. The two parts of the map reference, referred to as **components**, may be written as (3.0 km, 4.0 km) where East/West is first. This is the same as writing the **coordinates** of a point on a graph as (x, y), where x is the distance from the origin along the x-axis and y is the distance from the origin along the y-axis.

Resolving a vector into two perpendicular components

Resolving a vector is the process of working out the components of a vector in two perpendicular directions from the magnitude and direction of the vector. Figure 3 shows the displacement vector OA represented on a scale diagram that also shows lines due North and due East. The components of this vector along these two lines are $5.0\cos 53°$ km (= 3.0 km) along the line due East and $5.0\sin 53°$ km (= 4.0 km) along the line due North.

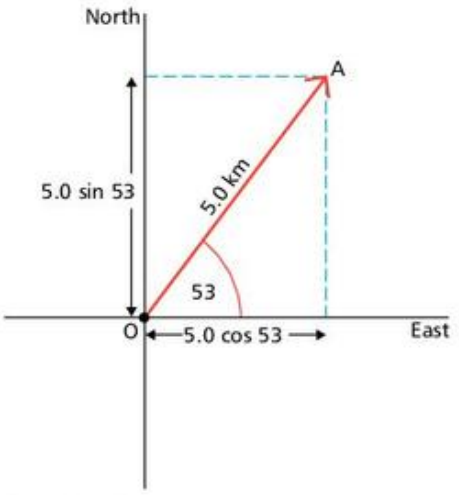


Fig 3 Resolving a vector

In general, to resolve any vector into two perpendicular components, draw a diagram showing the two perpendicular directions and an arrow to represent the vector. Figure 4 shows this diagram for a vector OP. The components are represented by the projection of the vector onto each line. If the angle θ between the vector OP and one of the lines is known:

- the component along that line = $\text{OP}\cos\theta$, and
- the component perpendicular to that line (i.e. along the other line) = $\text{OP}\sin\theta$.

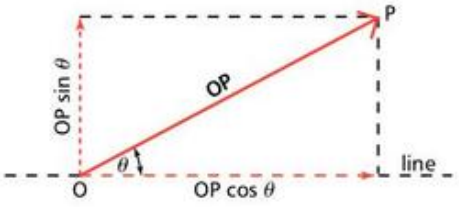


Fig 4 The general rule for resolving a vector

Example

An aircraft in level flight has a constant velocity of $50\,\text{m s}^{-1}$ in a direction of 40° North of due East, as shown in Figure 5. The angle between the direction of its velocity and due East is therefore 40°.

The components of its velocity are, in m s^{-1},

- $50\cos 40°$ due East, and
- $50\sin 40°$ due North.

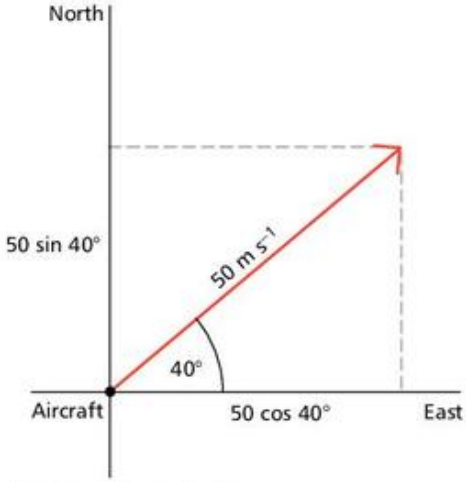


Fig 5 Resolving velocity

Addition of vectors

Using a scale diagram

Let's go back to the cycle journey in Figure 1. Suppose when you reach your friend's home at A, you then go on to another friend's home at B. Your journey is now a two-stage journey:

- **Stage 1, from O to A,** is represented by the displacement vector **OA**.
- **Stage 2, from A to B,** is represented by the displacement vector **AB**.

Figure 6 shows how the overall displacement from O to B, represented by vector **OB**, is the result of adding vector **AB** to vector **OA**. The **resultant** is the third side of a triangle, where OA and AB are the other two sides.

$$\mathbf{OB} = \mathbf{OA} + \mathbf{AB}$$

Use Figure 1 to show that the resultant displacement **OB** is 5.1 km in a direction 11° North of due East.

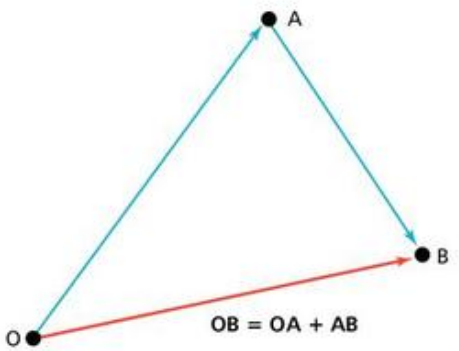


Fig 6 Displacement from O to B

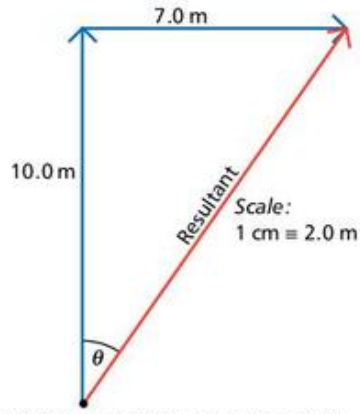
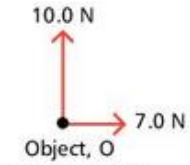
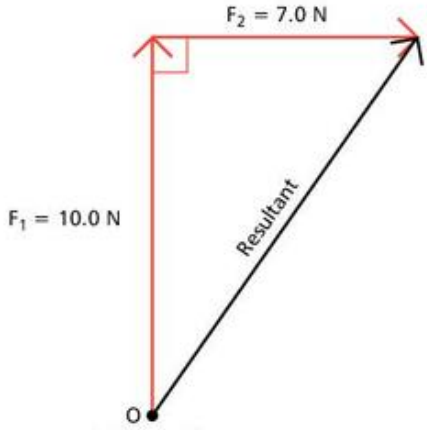


Fig 7 Adding two displacements at right angles to each other

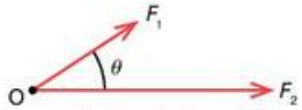


a *Forces acting*

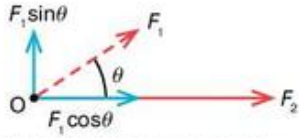


b *Vector diagram*

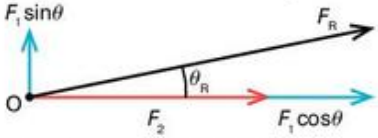
Fig 8 Two forces acting at right angles to each other



a *Vector diagram for F_1 and F_2*



b *F_1 resolved into components parallel and perpendicular to F_2*



c *using the components to find the resulting force F_1*

Fig 9 Using a calculator to find a resultant force

Using a calculator

1 Adding two perpendicular vectors

Suppose you walk 10.0 m forward then turn through exactly 90° and walk 7.0 m. At the end, how far will you be from your starting point? The vector diagram to add the two displacements is shown in Figure 7, drawn to a scale of 1 cm to 2.0 m. The two displacements form two sides of a right-angled triangle with the resultant as the hypotenuse. To find the resultant displacement,

- using Pythagoras' theorem gives 12.2 m ($= (10.0^2 + 7.0^2)^{1/2}$) for the magnitude of the resultant displacement (i.e. the distance), and
- using the trigonometry equation $\tan\theta = \frac{7.0}{10.0}$ gives 35° for the angle θ between the direction of the resultant displacement and the initial direction.

The method above can be applied to any two vectors at right angles to each other. For example,

Figure 8 shows an object, O, acted on by two forces, 7.0 N and 10.0 N perpendicular to each other. The vector diagram to add the two forces together would be as shown in Figure 7 if the labels were changed to 7.0 N and 10.0 N instead of 7.0 m and 10.0 m. As explained above, the resultant force is therefore 12.2 N at an angle of 35° to the 10 N force.

2 Adding two vectors that are at angle θ to each other

Consider an object, O, acted on by forces F_1 and F_2 at angle θ to each other, as shown in Figure 9a. The magnitude and direction of the resultant force F_R can be found by resolving one of the forces into components that are parallel and perpendicular to the other force, as shown in Figure 9b.

- Resolving F_1 parallel and perpendicular to F_2 gives $F_1\cos\theta$ for the parallel component and $F_1\sin\theta$ for the perpendicular component.
- Adding the components in each direction therefore gives the parallel component of F_R as $F_1\cos\theta + F_2$ and the perpendicular component as $F_1\sin\theta$.

Using Pythagoras' theorem to find the magnitude of the resultant force gives

$$F_R = [(F_1\cos\theta + F_2)^2 + (F_1\sin\theta)^2]^{1/2}$$

Because $\sin^2\theta + \cos^2\theta = 1$ for all angles of θ, it can be shown that

$$F_R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta.$$

Using the trigonometry rule for $\tan\theta$ to find θ_R, the angle between the resultant force and F_2 gives

$$\tan\theta_R = \frac{F_1\sin\theta}{(F_1\cos\theta + F_2)}.$$

Note The resultant of two vectors that act along the same line has a magnitude that is:

- the **sum,** if the two vectors are in the *same* direction. For example, if an object is acted on by a force of 6.0 N and a force of 4.0 N, both acting in the same direction, the resultant force is 10.0 N.
- the **difference,** if the two vectors are in *opposite* directions. For example, if an object is acted on by a 6.0 N force and a 4.0 N force in opposite directions, the resultant force is 2.0 N in the direction of the 6.0 N force.

To subtract one vector from another, the above example illustrates a general method which is to reverse one vector and then add the reversed vector to the other. To reverse a vector, point it in the opposite direction to its original direction so the sign of each component is changed from + to − or − to +.

QUESTIONS

1 A ship leaves a port, A, and travels a distance of 60 km due North; it then changes direction at B and travels a distance of 20 km due East to a second port, C. Calculate:

a the direct distance from A to C,

b the angle, θ, between due North and the line from A to C.

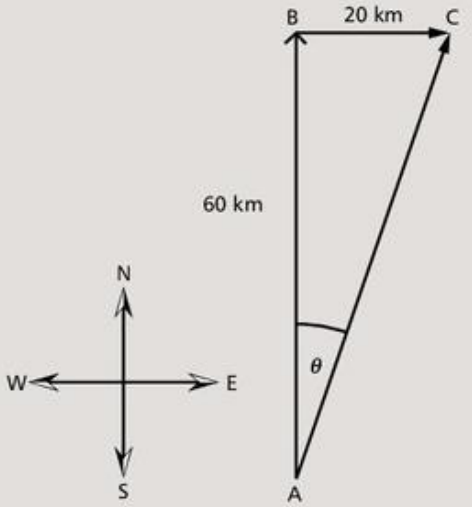


Fig 10

2 An aircraft travels on a straight flight path at a constant speed of 80 m s^{-1}, in a direction 60° North of due East:

a Calculate the components of the aircraft's velocity due North and due East.

b Calculate how far North the aircraft travels in 300 seconds.

3 A crane is used to raise one end of a steel girder off the ground, as shown in Figure 11. When the cable attached to the end of the girder is at 20° to the vertical, the force of the cable on the girder is 6.5 kN. Calculate the horizontal and vertical components of this force.

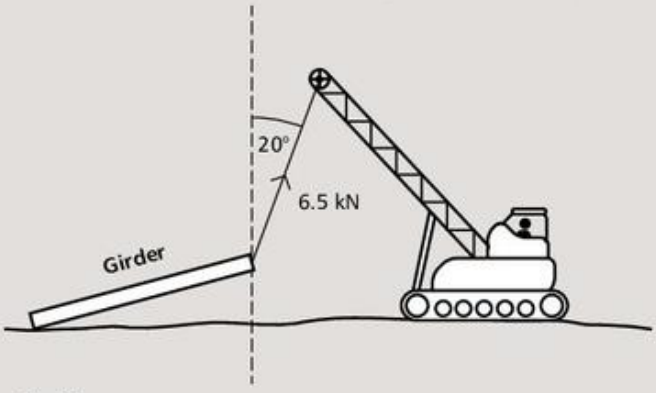


Fig 11

4 On an orienteering exercise, a team is told to go from a car park, C, to a meeting point, P, 12 km away in a direction which is 30° North of due East, as shown in Figure 12.

a Calculate how far P is: **i** due East of C, **ii** due North of C.

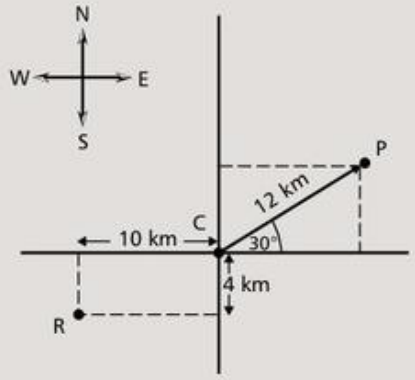


Fig 12

b A railway station, R, is 10 km due West and 4 km due South of car park C.

Calculate how far P is: **i** due East of R, **ii** due North of R.

c Calculate the direct distance from P to R.

5 Figure 13 shows three situations, **a–c**, where an object is acted on by two known forces. For each situation, calculate the magnitude and direction of the resultant force.

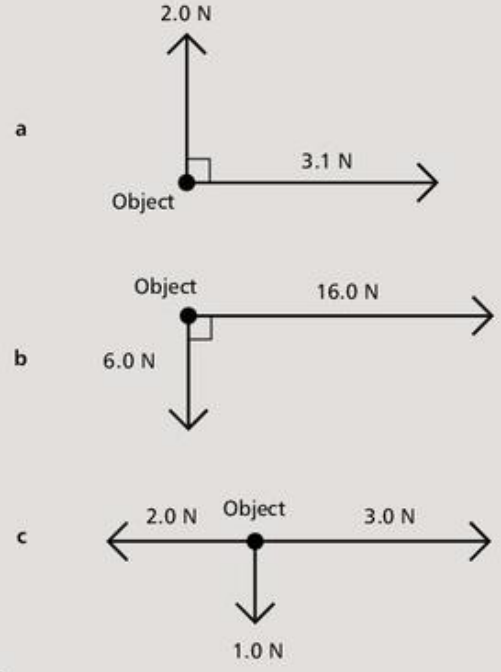


Fig 13

6 Calculate the magnitude and direction of the resultant force on an object that is acted on by a force of 4.0 N and a force of 10 N that are:

a in the same direction

b in opposite directions

c at right angles to each other

d at 35° to each other.

1.2 Speed and velocity

On these pages you will learn to:

- Define displacement, speed and velocity
- Use graphs to represent changes of displacement with time
- Determine velocity using the gradient of a displacement–time graph

Fig 1 A speeding vehicle

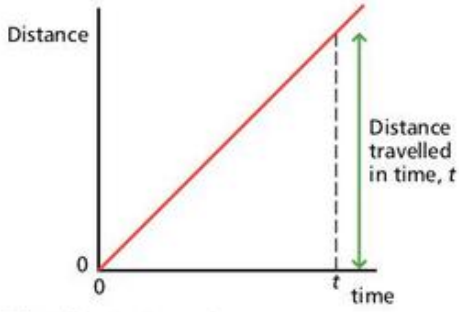
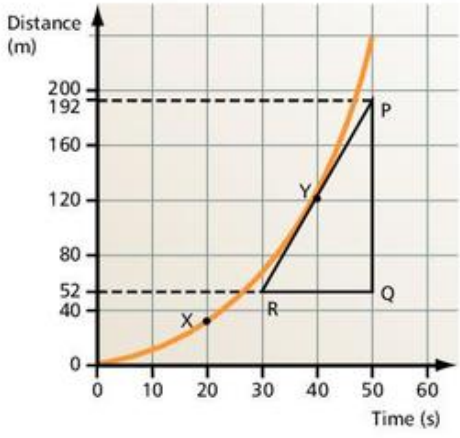


Fig 2 Constant speed



$$\text{Speed at Y} = \frac{PQ}{QR} = \frac{192-52}{20} = 7 \text{ m/s}$$

Fig 3 Changing speed

Fact file

- **Speed** is defined as change of distance per unit time.
- **Velocity** is defined as change of displacement per unit time. In other words, velocity is speed in a given direction.
- Speed is a **scalar** quantity. Velocity is a **vector** quantity.
- The unit of speed and of velocity is the **metre per second ($\mathbf{m\,s^{-1}}$)**.

Motion at constant speed

On a motorway journey, the distance a vehicle travels in a certain time can easily be worked out if the vehicle speed is constant during that time. An object moving at a constant speed travels equal distances in equal times. For example, a car travelling at a speed of $30\,m\,s^{-1}$ on a motorway travels a distance of 30 m every second or 1800 m every minute. In 1 hour, the car would therefore travel a distance of 108 000 m or 108 km.

- For an object which travels distance s, in time t at constant speed,

$$\textbf{Speed, } v = \frac{s}{t}$$

$$\textbf{Distance travelled, } s = vt$$

- For an object moving at constant speed on a circle of radius r,

$$\text{its speed } v = \frac{2\pi r}{T}$$

where T is the time to move round once and $2\pi r$ is the circumference of the circle.

Motion at changing speed

There are two types of speed cameras. One type measures the speed of a vehicle as it passes the camera. The other type is linked to a second speed camera and a computer, which works out the average speed of the vehicle between the two cameras. This will catch drivers who slow down for a speed camera and then speed up again!

- For an object that travels a distance s in time t,

$$\textbf{Average speed, } v_{av} = \frac{s}{t}$$

- For an object moving at changing speed, its distance travelled, Δs, in a short time interval, Δt, is given by:

$$\Delta s = v\,\Delta t$$

where v is the speed at that time (i.e. its instantaneous speed).

Rearranging this equation gives:

$$v = \frac{\Delta s}{\Delta t}$$

Distance–time graphs

- For an object moving at **constant** speed, the graph is **a straight line with a constant gradient** (see Figure 2).

$$\textbf{Speed of the object} = \frac{\textbf{distance travelled}}{\textbf{time taken}} = \textbf{gradient of the line}$$

- For an object moving with **changing** speed, **the gradient of the line changes**. The gradient of the line at any point can be found by drawing a tangent to the line at that point and then measuring the gradient of the tangent. This is shown in Figure 3, where PR is the tangent at point Y on the line. Show for yourself that the speed at point X on the line is $2.7\,m\,s^{-1}$.

Velocity

Take care when cycling or driving along a country lane. Such roads can be deceptive, because they wind round 'blind bends' where drivers are unable to see oncoming traffic. In such circumstances, a driver has to be ready to change speed, and change direction, very rapidly. The velocity of a car on a country road is very unlikely to be constant!

- An object moving at constant velocity moves at the same speed without changing its direction of motion. In other words, an object moving at constant velocity travels along a straight line, covering equal distances in equal times.
- If an object changes or reverses its direction of motion, its velocity changes. For example, the velocity of an object moving on a circular path at constant speed changes continuously, because its direction of motion changes continuously (Fig 4).
- An object travelling along a straight line has two possible directions of motion. To distinguish between the two directions, we need a direction code where + values are in one direction and − values in the opposite direction.

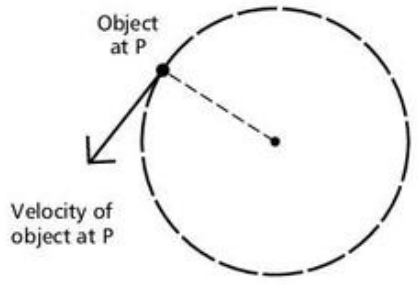


Fig 4 Circular motion

QUESTIONS

1 kilometre (km) = 1000 metres (m)

1 A car travels a distance of 60 km in 45 minutes at constant speed. Calculate its speed in:

a $km\,h^{-1}$, **b** $m\,s^{-1}$.

2 A satellite moves round the Earth at constant speed on a circular orbit of radius 8000 km, with a time period of 120 minutes. Calculate its orbital speed in:

a $km\,h^{-1}$, **b** $m\,s^{-1}$.

3 A vehicle joins a motorway and travels at a steady speed of $25\,m\,s^{-1}$ for 30 minutes, then it travels a further distance of 40 km in 20 minutes before leaving the motorway (Figure 5). Calculate:

a the distance travelled in the first 30 minutes,

b its average speed on the motorway.

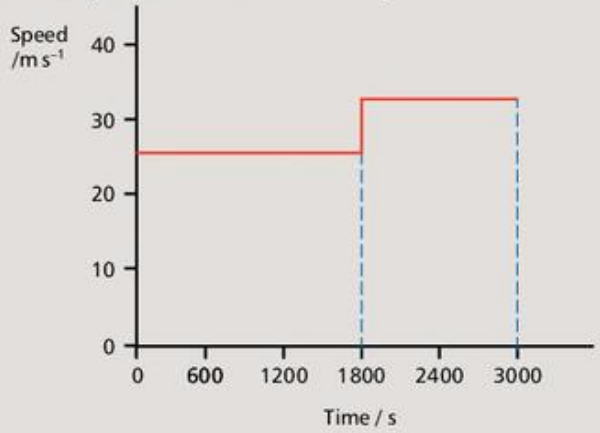


Fig 5

4 **a** Explain the difference between speed and velocity.

b A police car joins a straight motorway at Junction 4 and travels for 12 km at a constant speed for 400 s; then it leaves at Junction 5 and rejoins on the opposite side, and travels for 8 km at a constant speed for 320 s to reach an accident (Figure 6). Calculate:

i the displacement from Junction 4 to the accident,

ii the velocity of the car on each side of the motorway.

c Sketch a displacement–time graph for the journey.

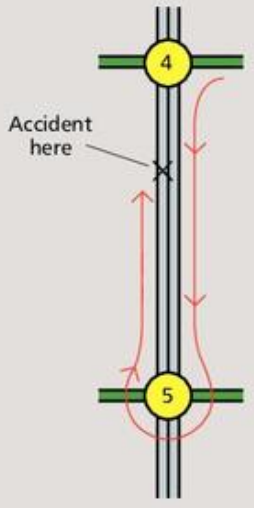


Fig 6

1.3 Acceleration

On these pages you will learn to:

- Define acceleration
- Calculate the acceleration of an object undergoing uniform acceleration
- Use graphs to represent displacement, speed, velocity and acceleration
- Determine acceleration using the gradient of a velocity–time graph

Performance tests

A car maker wants to compare the performance of a new model with the model being replaced. To do this, the velocity of each car is measured on a test track. Each vehicle accelerates as fast as possible to top velocity from a standing start. The results are listed in Table 1 below.

Table 1 *Performance of two models of a car*

Time from a standing start/s	0	2	4	6	8	10
Velocity of old model/$m\,s^{-1}$	0	8	16	24	32	32
Velocity of new model/$m\,s^{-1}$	0	10	20	30	30	30

Which car accelerates faster?

- The old model took 8 s to reach its top velocity of $32\,m\,s^{-1}$. Its velocity must have increased by $4.0\,m\,s^{-1}$ every second for 8 seconds to reach its top velocity.
- The new model took 6 s to reach its top velocity of $30\,m\,s^{-1}$. Its velocity must have increased by $5.0\,m\,s^{-1}$ every second for 6 s to reach its top velocity.

Clearly, the new model speeds up at a faster rate than the old model. In other words, its acceleration is greater.

Fig 1

The acceleration of the new model above is $5.0\,m\,s^{-2}$, because its speed increased by $5.0\,m\,s^{-1}$ every second.

The acceleration of the old model above is $4.0\,m\,s^{-2}$, because its speed increased by $4.0\,m\,s^{-1}$ every second.

Acceleration is defined as change of velocity per unit time.

- The unit of acceleration is the **metre per second per second ($m\,s^{-2}$)**.
- Acceleration is a **vector** quantity.
- **Deceleration** values are negative and signify that the velocity decreases with respect to time.

For a moving object that does not change direction, its acceleration at any point can be worked out from its rate of change of speed because there is no change of direction.

Uniform acceleration

Uniform acceleration is where the velocity of an object moving along a straight line changes at a constant rate. In other words, the acceleration is constant. Consider an object that accelerates uniformly from velocity, u, to velocity, v, in time, t, along a straight line. Figure 2 shows how its velocity changes with time.

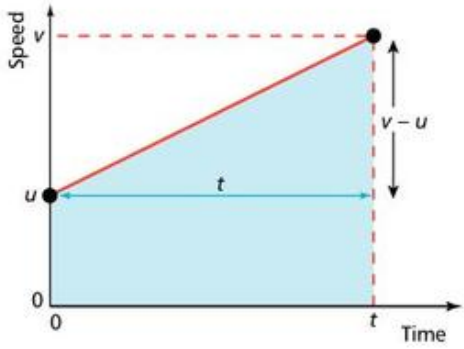


Fig 2 *Uniform acceleration*

Notice that the gradient of the line represents the acceleration $\left(= \frac{v-u}{t}\right)$

$$\text{Acceleration, } a, \text{ of the object} = \frac{\text{change of velocity}}{\text{time taken}} = \frac{v-u}{t}$$

$$a = \frac{v-u}{t}$$

where u is the initial velocity of the object and v is the velocity of the object at time t.

To calculate the velocity v at time t, rearranging this equation gives:

$$at = (v-u)$$

$$\therefore \quad v = u + at$$

Worked example

The driver of a vehicle travelling at $8\,m\,s^{-1}$ applies the brakes for 30 s and reduces the velocity of the vehicle to $2\,m\,s^{-1}$. Calculate the deceleration of the vehicle during this time.

Solution

$$u = 8\,m\,s^{-1};\ v = 2\,m\,s^{-1};\ t = 30\,s$$

$$\therefore \quad a = \frac{v-u}{t} = \frac{(2-8)}{30} = \frac{-6}{30} = -0.2\,m\,s^{-2}$$

Non-uniform acceleration

Non-uniform acceleration is where the direction of motion of an object changes, or its speed changes, at a varying rate. Figure 3 shows how the speed of an object increases for an object moving along a straight line with an increasing acceleration. This can be seen directly from the graph, because the gradient increases with time (i.e. the graph becomes steeper and steeper). The **gradient** represents the acceleration.

The acceleration at any point is the gradient of the tangent to the curve at that point. In Figure 3:

$$\text{Gradient of tangent at point P} = \frac{\text{height of gradient triangle}}{\text{base of gradient triangle}}$$

$$= \frac{4.0 - 1.0\,m\,s^{-1}}{2.0\,s} = 1.5\,m\,s^{-2}$$

Therefore, the acceleration at P is $1.5\,m\,s^{-2}$.

Acceleration = gradient of the line on the velocity–time graph

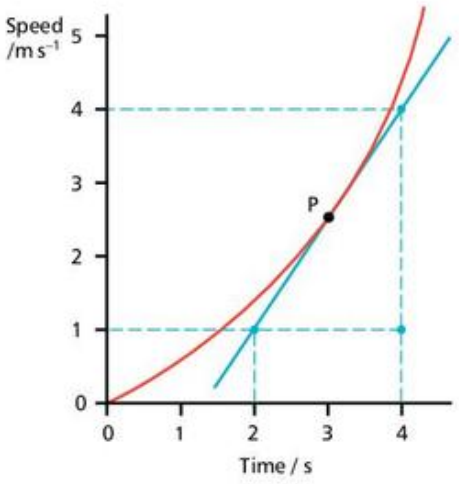


Fig 3 Non-uniform acceleration

QUESTIONS

1 **a** An aeroplane taking off accelerates uniformly on a runway from a velocity of $4\,m\,s^{-1}$ to a velocity of $64\,m\,s^{-1}$ in 40 s. Calculate its acceleration.

b A car travelling at a velocity of $20\,m\,s^{-1}$ brakes sharply to a standstill in 8.0 seconds. Calculate its deceleration, assuming its velocity decreases uniformly.

2 A cyclist accelerates uniformly from a velocity of $2.5\,m\,s^{-1}$ to a velocity of $7.0\,m\,s^{-1}$ in a time of 10 s. Calculate:

a the acceleration,

b the velocity 2.0 s later, if she continued to accelerate at the same rate.

3 A train, on a straight journey between two stations, accelerates uniformly from rest for 20 s to a velocity of $12\,m\,s^{-1}$. It then travelled at a constant velocity for a further 40 s, before decelerating uniformly to rest in 30 s.

a Sketch a velocity-time graph to represent its journey.

b Calculate its acceleration in each part of the journey.

4 The velocity of an object released in water increased as shown in Figure 4.

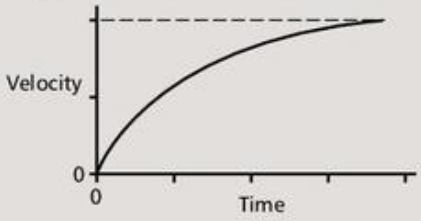


Fig 4

Describe how:

a the velocity of the object changed with time,

b the acceleration of the object changed with time.

1.4 Motion along a straight line at a constant acceleration

On these pages you will learn to:

- Derive equations for uniformly accelerated motion in a straight line
- Solve problems using equations for uniformly accelerated motion in a straight line
- Calculate the distance travelled from the area under a speed–time graph

The dynamics equations for constant acceleration

Consider an object that accelerates uniformly from initial speed, u, to final speed v, in time t without change of direction. Figure 2, p.8, shows how its speed changes with time.

1 The acceleration $a = \frac{v - u}{t}$ (as explained in unit 1.3)

As before, rearranging this equation gives:

$$v = u + at \qquad \text{(Equation 1)}$$

2 The distance moved, s = average speed × time taken

Because the acceleration is uniform, the average speed $= \frac{(u + v)}{2}$

therefore $$s = \frac{(u + v)t}{2} \qquad \text{(Equation 2)}$$

3 By combining the two equations above, to eliminate v, a further useful equation is produced.

Substitute $u + at$ in place of v in Equation 2. This gives:

$$s = \frac{(u + (u + at))t}{2} = \frac{(u + u + at)t}{2} = \frac{2ut + at^2}{2}$$

therefore $$s = ut + \tfrac{1}{2}at^2 \qquad \text{(Equation 3)}$$

4 A fourth useful equation is obtained by combining Equations 1 and 2 to eliminate t. This can be done by multiplying:

$$a = \frac{v - u}{t} \text{ and } s = \frac{(u + v)t}{2} \text{ together to give}$$

$$as = \frac{v - u}{t} \times \frac{(u + v)t}{2}$$

which simplifies to become:

$$as = \frac{(v - u)(v + u)}{2} = \frac{(v^2 - uv + uv - u^2)}{2} = \frac{(v^2 - u^2)}{2}$$

Therefore $2as = v^2 - u^2$

Rearranging this equation gives:

$$v^2 = u^2 + 2as \qquad \text{(Equation 4)}$$

The four equations, often referred to as the 'suvat' equations, are invaluable in any situation where the acceleration is constant.

WORKED EXAMPLE

A driver of a vehicle travelling at a speed of $30\,m\,s^{-1}$ on a motorway brakes sharply to a standstill in a distance of 100 m. Calculate the deceleration of the vehicle.

$u = 30\,m\,s^{-1}$, $v = 0$, $s = 100\,m$, $a = ?$

Solution

To find a, use $v^2 = u^2 + 2as$

Therefore $0 = u^2 + 2as$, because $v = 0$

Rearranging this equation gives:

$2as = -u^2$

$$a = \frac{-u^2}{2s} = \frac{-30^2}{2 \times 100} = -4.5\,m\,s^{-2}$$

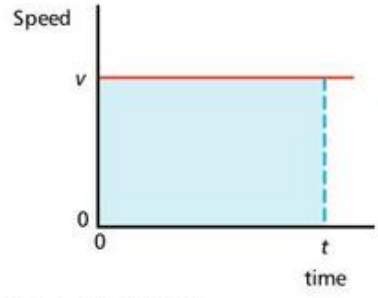


Fig 1a *Constant speed*

Using a speed–time graph to find the distance moved

An object moving at constant speed

The distance moved in time t, s = speed × time taken = vt. This distance is represented on the graph by the **area under the line between the start and time t**. This is a rectangle of height corresponding to speed v and of base corresponding to the time t (Figure 1a).

An object moving at constant acceleration

From Equation 2, the distance moved in time t, $s = \frac{(u + v)t}{2}$.

This distance is represented on the graph by the **area under the line between the start and time t**. This is a trapezium which has a base corresponding to time t and an average height corresponding to the average speed $\frac{(u + v)t}{2}$.

Therefore the area of the trapezium (= average height × base) corresponds to $\frac{(u + v)t}{2}$

which is the distance moved (Figure 1b).

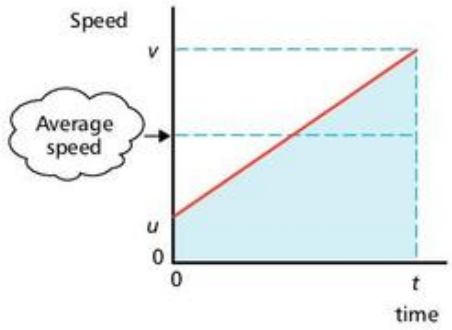


Fig 1b Constant acceleration

An object moving with a changing acceleration

Let v represent the speed at time t and $v + \delta v$ represent the speed a short time later at $t + \delta t$ (δ is pronounced 'delta' from Greek).

Because the speed change δv is small compared with the speed v (Figure 1c), the distance travelled δs in the short time interval δt is given by $\delta s = v\delta t$.

This is represented on the graph by the **area of the shaded strip under the line which has a base corresponding to δt and a height corresponding to v**. In other words, $\delta s = v\delta t$ is represented by the area of this strip.

By considering the whole area under the line in strips of similar width, the total distance travelled from the start to time t_1 is therefore represented by the sum of the areas of every strip: which is therefore the **total area** under the line.

Whatever the shape of the line of a speed–time graph:

Distance travelled = area under the line of a speed–time graph

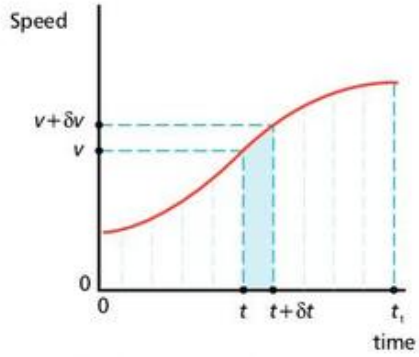


Fig 1c Changing acceleration

QUESTIONS

1 A vehicle accelerates uniformly along a straight road, increasing its speed from 4.0 m s^{-1} to 30.0 m s^{-1} in 13 s. Calculate:

a its acceleration,

b the distance it moves in this time.

2 An aircraft lands on a runway at a speed of 40 m s^{-1} and brakes to a halt in a distance of 860 m. Calculate:
a the braking time, **b** the deceleration of the aircraft.

3 A cyclist accelerates uniformly from rest to a speed of 6.0 m s^{-1} in 30 s, then brakes at uniform deceleration to a halt in a distance of 24 m.

a For the first part of the journey, calculate:
i the acceleration, **ii** the distance travelled.

b For the second part of the journey, calculate:
i the deceleration, **ii** the time taken.

c Sketch a speed–time graph for this journey.

d Use the graph to determine the average speed for the journey.

4 The speed of an athlete for the first 5 s of a sprint race is shown in Figure 2. Use the graph to determine:

a the initial acceleration of the athlete,

b the distance moved in the first 2 s,

c the distance moved in the next 2 s,

d the average speed over the first 4 s.

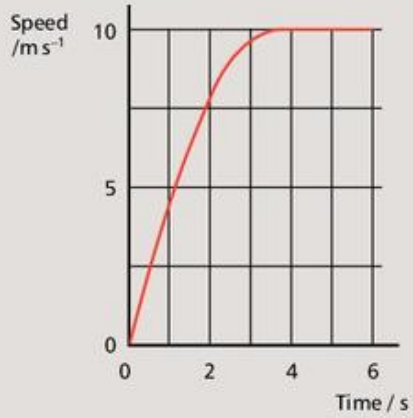


Fig 2

1.5 Free fall

On these pages you will learn to:

- Solve problems on the motion of bodies falling freely in a uniform gravitational field using equations for uniformly accelerated motion in a straight line
- Describe an experiment to determine the acceleration of free fall, g, using a falling body

Experimental tests

Does a heavy object fall faster than a lighter object?

Release a stone and a small coin at the same time. Which one hits the ground first? Galileo first discovered the answer to this question about four centuries ago. He reasoned that because any number of identical objects must fall at the same rate, then any one such object must fall at the same rate as the rest put together. So he concluded that any two objects must fall at the same rate, regardless of their relative weights. He was reported to have demonstrated the correctness of his theory by releasing two different weights from the top of the leaning Tower of Pisa.

The inclined plane test

Galileo wanted to know if a falling object speeds up as it falls. Clocks and stopwatches were devices of the future. The simplest test he could think of was to time a ball as it rolled down a plank. He devised a 'dripping water' clock, counting the number of the drips as a measure of time. He measured how long the ball took to travel equal distances down the slope from rest. His measurements showed that the ball gained speed as it travelled down the slope. In other words, he showed that the ball accelerated as it rolled down the slope. He reasoned that the acceleration would be greater the steeper the slope. So he concluded that an object falling vertically accelerates.

Acceleration due to gravity

One way to investigate the free fall of a ball is to make a multi-flash photo or video clip of the ball's flight as it falls after being released from rest. To do this, a camera is used to record the ball's descent in a dark room with a flashing light switched on. The flashing light needs to flash at a known constant rate of about 20 flashes per second. A vertical metre rule can be used to provide a scale. Figure 1 shows a possible arrangement using a steel ball and a multi-flash photo taken with this arrangement.

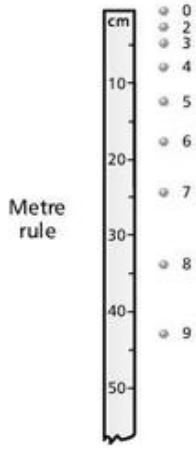


Fig 1 *Investigating free fall*

For each image of the ball on the photograph, the time of descent of the ball and the distance fallen by the ball from rest can be measured directly. The photograph shows the ball speeds up as it falls, because it travels further between successive images. Measurements from this photograph are given in Table 1 below.

Table 1 *A free-falling ball*

Number of flashes after start	0	2	3	4	5	6	7	8	9
Time of descent, t/s	0	0.06	0.10	0.13	0.16	0.19	0.23	0.26	0.29
Distance fallen, s/m	0	0.02	0.04	0.07	0.12	0.17	0.24	0.33	0.42

How can we tell if the acceleration is constant from these results?

One way is to consider how the distance fallen, s, would vary with time, t, for constant acceleration. From unit 1.4, we know that:

$$s = ut + \tfrac{1}{2}at^2, \text{ where } u = \text{the initial speed, and } a = \text{acceleration.}$$

In this experiment, $u = 0$ therefore $\boldsymbol{s = \tfrac{1}{2}at^2}$ for constant acceleration, a.

Compare this equation with the general equation for a straight-line graph $y = mx + c$, where m is the gradient and c is the y-intercept. If we let y represent s and let x represent t^2, then $m = \tfrac{1}{2}a$ and $c = 0$.

So a graph of s against t^2 should give a straight line through the origin. In addition, the gradient of the line ($= \frac{1}{2}a$) can be measured and the acceleration ($= 2 \times$ gradient) calculated (Figure 2).

As you can see, the graph is a straight line through the origin. We can therefore conclude that the equation $s = \frac{1}{2}at^2$ applies here, so the acceleration of a falling object is constant. Show for yourself that the gradient of the line is $5.0\,m\,s^{-2}$ ($\pm 0.2\,m\,s^{-2}$), giving an acceleration of $10\,m\,s^{-2}$. Because there are no external forces acting on the object, apart from its weight, this value of acceleration is known as the **acceleration of free fall** and is represented by the symbol, g. Accurate measurements give a value of $9.8\,m\,s^{-2}$ near the Earth's surface.

The 'suvat' equations on p.10 may be applied to any 'free fall' situation, where **air resistance** is negligible.

The equations can also be applied to situations where objects are thrown vertically upwards. As a general rule, apply the direction code **+ for upwards and − for downwards** when values are inserted into the 'suvat' equations.

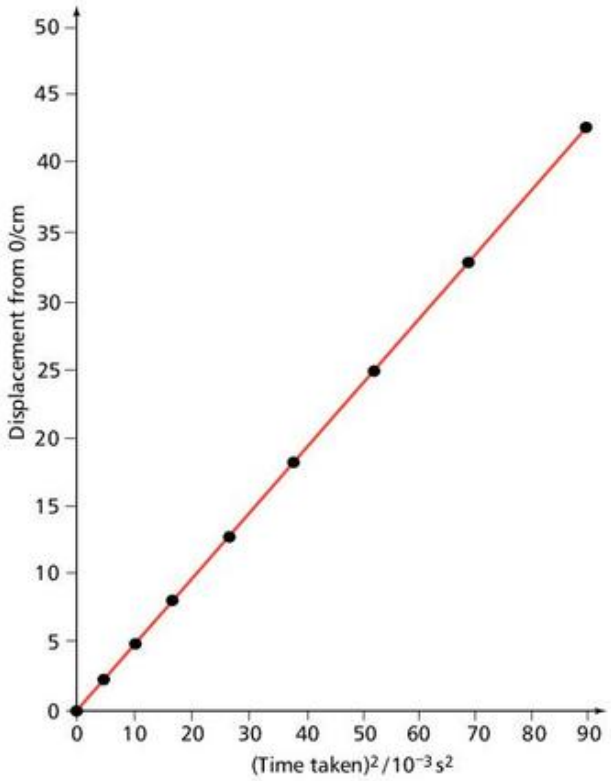


Fig 2 A graph of s against t^2

Worked example

$g = 9.8\,m\,s^{-2}$

1 A coin was released from rest at the top of a well. It took 1.6 s to hit the bottom of the well. Calculate: **a** the distance fallen by the coin, **b** its speed just before impact.

Solution

$u = 0, t = 1.6\,s, a = -9.8\,m\,s^{-2}$ (− as g acts downwards)

a To find s, use $s = \frac{1}{2}at^2$ as $u = 0$
Therefore, $s = \frac{1}{2} \times -9.8 \times 1.6^2 = -12.5\,m$ (− indicates 12.5 m downwards)

b To find v, use $v = u + at = 0 + (-9.8 \times 1.6) = -15.7\,m\,s^{-1}$ (− indicates downward velocity)

QUESTIONS

$g = 9.8\,m\,s^{-2}$

1 A pebble, released at rest from a canal bridge, took 0.9 s to hit the water.
Calculate:
- **a** the distance it fell before hitting the water,
- **b** its speed just before hitting the water.

2 A spanner was dropped from a hot air balloon, when the balloon was at rest 50 m above the ground.
Calculate:
- **a** the time taken for the spanner to hit the ground,
- **b** the speed of impact of the spanner on hitting the ground.

3 A bungee jumper jumped off a platform 75 m above a lake, releasing a small object at the instant she jumped off the platform.
- **a** Calculate **i** the time taken by the object to fall to the lake, **ii** the speed of impact of the object on hitting the water, assuming air resistance is negligible.
- **b** Explain why the bungee jumper would take longer to descend than the time taken in part **a i**.

4 An astronaut on the Moon threw an object 4.0 m vertically upwards and caught it again 4.5 s later.
Calculate:
- **a** the acceleration due to gravity on the Moon,
- **b** the speed of projection of the object,
- **c** how high the object would have risen on the Earth, for the same speed of projection.

1.6 Motion graphs

On these pages you will learn to:

- Determine displacement from the area under a velocity–time graph
- Determine velocity using the gradient of a displacement–time graph
- Determine acceleration using the gradient of a velocity–time graph

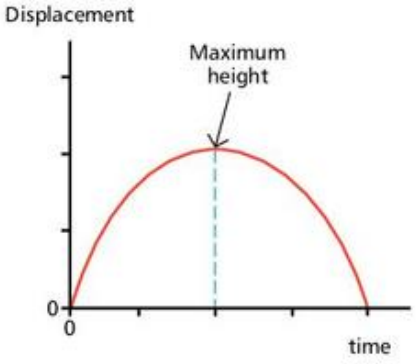


Fig 1 Displacement against time for an object projected upwards

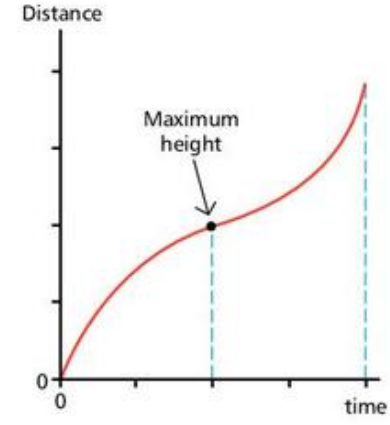


Fig 2 Distance against time for an object projected upwards

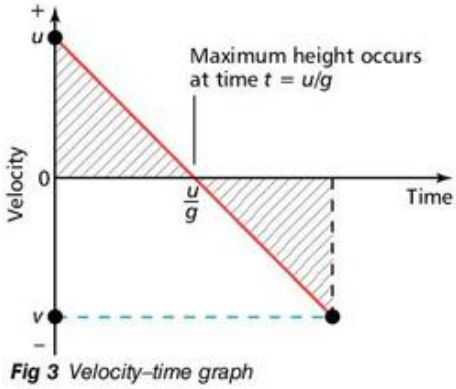


Fig 3 Velocity–time graph

The difference between a distance–time graph and a displacement–time graph

Displacement is distance in a given direction from a certain point. Consider a ball thrown directly upwards and caught when it returns. Its displacement from the instant it leaves the thrower:

- increases until it reaches maximum height,
- decreases to zero as it returns to the thrower from maximum height.

For example, if the ball rises to a maximum height of 2.0 m, on returning to the thrower its displacement from its initial position is zero. However, the distance it has travelled is 4.0 m.

The displacement of the object changes with time as shown by Figure 1. The line in this graph fits the equation $s = ut - \frac{1}{2}gt^2$, where s is the displacement, t is the time taken and u is the initial velocity of the object.

The **gradient** of the line represents the **velocity** of the object.

- Initially, the velocity is positive and large so the gradient is positive and large.
- As the ball rises, its velocity decreases so the gradient decreases.
- At maximum height, the velocity is zero so the gradient is zero.
- As the ball returns, the velocity becomes increasingly negative, corresponding to increasing speed in a downwards direction. So the gradient becomes increasingly negative.

The distance travelled by the object changes with time as shown by Figure 2. The **gradient** of this line represents the **speed**.

- From projection to maximum height, the shape is exactly the same as in Figure 1.
- After maximum height, the distance continues to increase so the line curves up, not down.

More about velocity–time graphs

Velocity is speed in a given direction. Consider how the velocity of an object thrown into the air changes with time. The object's velocity:

- decreases from its initial positive (i.e. upwards) value to zero at maximum height,
- increases in the negative (i.e. downwards) direction as it falls.

Figure 3 shows how the velocity of the object changes with time.

- **The gradient of the line is constant and negative, equal to the acceleration of free fall, g.** The acceleration of the object is the same when it descends as when it ascends, so the gradient of the line is always equal to $-9.8\,\text{m s}^{-2}$.
- **The area under the line represents the displacement of the object from its starting position.**
  - **a** The area between the **positive** section of the line and the time axis represents the **displacement** during the **ascent**.
  - **b** The area between the **negative** section of the line and the time axis represents the displacement during the **descent**.

  Taking the area for **a** as positive and the area for **b** as negative, the total area is zero. This corresponds to zero for the total displacement.

Worked example

$g = 9.8\,\text{m s}^{-2}$

A ball released from a height of 1.20 m above a concrete floor rebounds to a height of 0.82 m.

a Calculate: **i** its time of descent, **ii** the speed of the ball immediately before it hits the floor.

b Calculate: the speed of the ball immediately after it leaves the floor.

c Sketch a velocity-time graph for the ball. Assume the contact time is negligible compared with the time of descent or ascent.

Solution

a $u = 0, a = -9.8\,\text{m s}^{-2}, s = -1.2$ m

i To find t, use $s = ut + \frac{1}{2}at^2$

$$\therefore -1.2 = 0 + 0.5 \times -9.8 \times t^2$$

$$-1.2 = -4.9t^2$$

$$t^2 = \frac{-1.2}{-4.9} = 0.245$$

$$t = 0.49\,\text{s}$$

ii To find v, use $v = u + at$

$$\therefore v = 0 + -9.8 \times 0.49 = -4.8\,\text{m s}^{-1}\ (-\text{ for downwards})$$

b $v = 0, a = -9.8\,\text{m s}^{-2}, s = +0.82$ m

To find u, use $v^2 = u^2 + 2as$

$$\therefore 0 = u^2 + 2 \times -9.8 \times 0.82$$

$$u^2 = 15.7\,\text{m}^2\text{s}^{-2}$$

$$u = +4.0\,\text{m s}^{-1}\ (+\text{ for upwards})$$

c See Figure 4. Note that the line has a constant gradient equal to the acceleration due to gravity, $-9.8\,\text{m s}^{-2}$, except on impact.

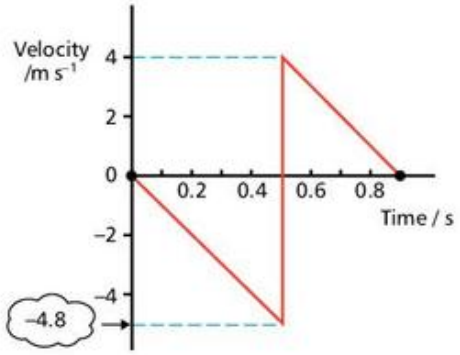


Fig 4

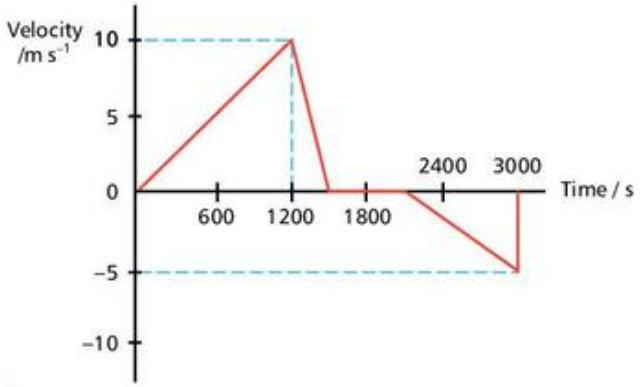


Fig 5

QUESTIONS

$g = 9.8\,\text{m s}^{-2}$

1 A swimmer swims 100 m from one end of a swimming pool to the other end at a constant speed of $1.2\,\text{m s}^{-1}$; then swims back at constant speed, returning to the starting point 210 s after starting.

a Calculate how long the swimmer takes to swim from:
i the starting end to the other end
ii back to the start from the other end.

b For the swim from start to finish, sketch:
i a displacement–time graph
ii a distance-time graph.

c Sketch a velocity–time graph for the swim.

2 A motorcyclist travelling along a straight road at a constant speed of $8.8\,\text{m s}^{-1}$ passes a cyclist travelling in the same direction at a speed of $2.2\,\text{m s}^{-1}$. After 200 s, the motorcyclist stops.

a Calculate how long the motorcyclist has to wait before the cyclist catches up.

b On the same axes, sketch a velocity–time graph for: **i** the motorcyclist, **ii** the cyclist.

3 The graph (Figure 5) shows the velocity of a train on a straight track for 50 min after it left a station.

a i Describe how the displacement of the train from the station changed with time.

ii Sketch a graph to show how the displacement in part **i** varied with time.

b i Calculate how far from the station the train was after 50 min.

ii Calculate the total distance travelled by the train in this time.

4 A ball is released from a height of 1.8 m above a level surface and rebounds to a height of 0.90 m.

a Given $g = 9.8\,\text{m s}^{-2}$, calculate: **i** the duration of its descent, **ii** its velocity just before impact, **iii** the duration of its ascent, **iv** its velocity just after impact.

b Sketch a graph to show how its velocity changes with time from release to rebound at maximum height.

c Sketch a further graph to show how the displacement of the object changes with time.

1.7 More calculations on motion along a straight line

On these pages you will learn to:

- Solve problems including those with several stages using the equations for uniformly accelerated motion in a straight line

Motion along a straight line at constant acceleration

- The 'suvat' equations for motion at constant acceleration, a, are:

$$v = u + at \quad \text{(Equation 1)}$$

$$s = \frac{(u + v)t}{2} \quad \text{(Equation 2)}$$

$$s = ut + \tfrac{1}{2}at^2 \quad \text{(Equation 3)}$$

$$v^2 = u^2 + 2as \quad \text{(Equation 4)}$$

  where s is the displacement in time t, u is the initial velocity and v is the final velocity.
- For motion along a straight line at constant acceleration, one direction along the line is 'positive' and the other direction is negative.

Fig 1 A space vehicle docking

Worked example

A space vehicle moving towards a docking station, at a speed of $2.5\,\text{m s}^{-1}$, is 26m from the docking station when its reverse thrust motors are switched on (to slow it down and stop it when it reaches the station). The vehicle decelerates uniformly until it comes to rest at the docking station, when its motors are switched off.

Calculate: **a** its deceleration, **b** how long it takes to stop, **c** its velocity, if its motors remained on for 5.0s longer than necessary.

Solution

Let the + direction represent motion towards the docking station and − away from the station.

a and **b** Initial velocity, $u = +2.5\,\text{m s}^{-1}$, final velocity, $v = 0$, displacement, $s = +26\,\text{m}$.

To find its deceleration, a, use $v^2 = u^2 + 2as$.

$$0 = 2.5^2 + 2a \times 26$$

$$-52a = 2.5^2$$

$$a = -\frac{2.5^2}{52} = -0.12\,\text{m s}^{-2}$$

To find the time taken, use $v = u + at$.

$$0 = 2.5 - 0.12t$$

$$0.12t = 2.5$$

$$t = \frac{2.5}{0.12} = 21\,\text{s}$$

c Initial velocity, $u = 2.5\,\text{m s}^{-1}$, acceleration, $a = -0.12\,\text{m s}^{-2}$, time taken $t = 26\,\text{s}$

To calculate its velocity v after 26s, use $v = u + at = 2.5 - 0.12 \times 26$

$$= -0.62\,\text{m s}^{-1}$$

The velocity of the space vehicle would be $0.62\,\text{m s}^{-1}$ away from the docking station after 26s.

Two-stage problems

Consider an object released from rest, falling then hitting a bed of sand. The motion is in two stages:

1 **Falling** motion due to gravity; acceleration = g (downwards)
2 **Deceleration** in the sand; initial velocity = velocity of object just before impact.

The acceleration in each stage is **not** the same. The link between the two stages is that the velocity at the end of the first stage is the same as the velocity at the start of the second stage.

For example, consider a ball released from a height of 0.85 m above a bed of sand that creates an impression in the sand of depth 0.025 m. For directions, let + represent upwards and − represent downwards.

Stage 1

$u = 0, s = -0.85\,\text{m}, a = -9.8\,\text{m s}^{-2}$.

To calculate the speed of impact v, use $v^2 = u^2 + 2as$

$\therefore \quad v^2 = 0^2 + 2 \times -9.8 \times -0.85 = 16.7$

$\therefore \quad v = -4.1\,\text{m s}^{-1}$

Note $v^2 = 16.7$, so $v = -4.1$ or $+4.1\,\text{m s}^{-1}$. The negative answer is chosen, as the ball is moving downwards.

Stage 2

$u = -4.1\,\text{m s}^{-1}$, $v = 0$ (as the ball comes to rest in the sand), $s = -0.025\,\text{m}$

To calculate the deceleration, a, use $v^2 = u^2 + 2as$.

$\therefore \quad 0^2 = (-4.1)^2 + 2a \times -0.025$

$\therefore \quad 2a \times 0.025 = 16.7$

$$a = \frac{16.7}{2 \times 0.025} = 334\,\text{m s}^{-2}$$

Note $a > 0$ and therefore in the opposite direction to the direction of motion, which is downwards. Thus the ball slows down in the sand with a deceleration of $334\,\text{m s}^{-2}$.

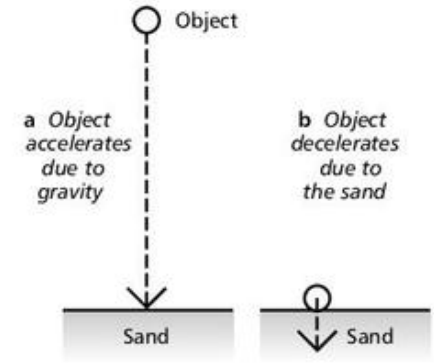


Fig 2 A two-stage problem

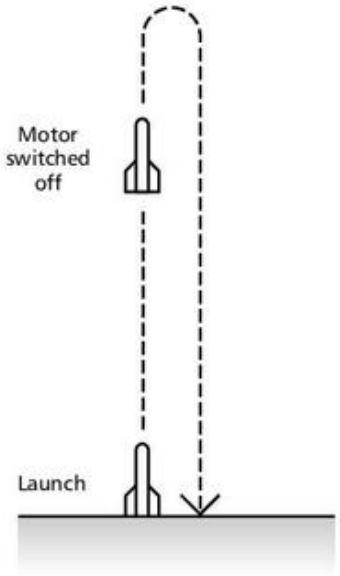


Fig 3

QUESTIONS

$g = 9.8\,\text{m s}^{-2}$

1 A vehicle on a straight downhill road accelerated uniformly from a speed of $4.0\,\text{m s}^{-1}$ to a speed of $29\,\text{m s}^{-1}$ over a distance of 850 m, when the driver braked and stopped the vehicle in 28 s.

- **a** Calculate: **i** the time taken to reach $29\,\text{m s}^{-1}$ from $4\,\text{m s}^{-1}$, **ii** its acceleration during this time.
- **b** Calculate: **i** the distance it travelled during deceleration, **ii** its average deceleration as it slowed down.

2 A rail wagon moving at a speed of $2.0\,\text{m s}^{-1}$ on a level track reached a steady incline, which slowed it down in 15.0 s and caused it to reverse. Calculate:

- **a** the distance it moved up the incline,
- **b** its acceleration on the incline,
- **c** its velocity and position on the incline after 20.0 s.

3 A cyclist accelerated from rest at a constant acceleration of $0.4\,\text{m s}^{-2}$ for 20 s, then stopped pedalling and slowed to a standstill at a constant deceleration over a distance of 260 m.

- **a** Calculate: **i** the distance travelled by the cyclist in the first 20 s, **ii** the speed of the cyclist at the end of this time.
- **b** Calculate: **i** the time taken to cover the distance of 260 m after she stopped pedalling, **ii** her deceleration during this time.

4 A rocket was launched directly upwards from rest. Its motors operated for 30 s after it left the launch pad, providing it with a constant vertical acceleration of $6.0\,\text{m s}^{-2}$ during this time. Its motors then switched off (Figure 3).

- **a** Calculate: **i** its initial velocity, **ii** its height above the launch pad when its motors switched off.
- **b** Calculate its maximum height gain after its motors switched off.
- **c** Calculate the velocity with which it would hit the ground, if it fell from maximum height without the support of a parachute.

1.8 Projectile motion 1

On these pages you will learn to:

- Describe and explain the motion due to a uniform velocity in a certain direction and a uniform acceleration in a perpendicular direction
- Solve projectile problems where projection is horizontal using the equations for uniform acceleration in a vertical direction and for uniform velocity in a horizontal direction

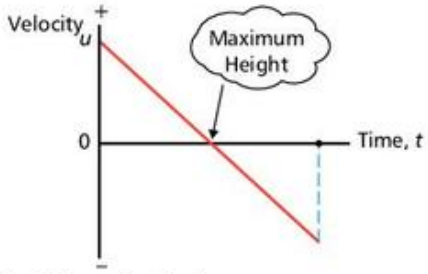


Fig 1 Upward projection

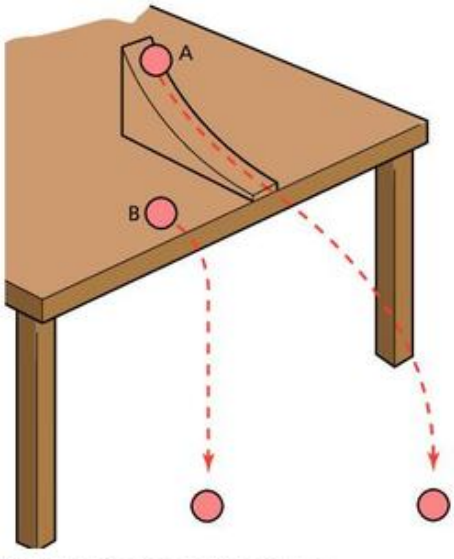


Fig 2 Testing horizontal projection

A **projectile** is any object acted upon only by the force of gravity. Air resistance is assumed to be negligible. Three key principles apply to all projectiles:

- The **acceleration** of the object is always equal to g and acts downwards, because the force of gravity acts downwards. The acceleration therefore only affects the vertical motion of the object.
- The **horizontal velocity** of the object is constant, because the acceleration of the object does not have a horizontal component.
- The motions in the horizontal and vertical directions are **independent** of each other.

Vertical projection

If an object is projected vertically, it moves vertically as it has no horizontal motion. Its acceleration is $9.8\,\text{m}\,\text{s}^{-2}$ downwards. Using the direction code '+ is upwards; − is downwards', its displacement, y, and velocity, v_y, after time t are given by:

$$v_y = u - gt$$

$$y = ut - \tfrac{1}{2}gt^2$$

where u is its initial velocity.

See topic 1.7 for more about vertical projection.

Horizontal projection

A stone thrown from a cliff top follows a curved path downwards before it hits the water. If its initial projection was horizontal:

- Its path through the air becomes steeper and steeper as it drops.
- The faster it is projected, the further away it will fall into the sea.
- The time taken for it to fall into the sea does not depend on how fast it is projected.

Suppose two balls are released at the same time above a level floor, such that one ball drops vertically and the other is projected horizontally. Which one hits the floor first? In fact, they both hit the floor simultaneously. Try it! Why should the two balls hit the ground at the same time? They are both pulled to the ground by the force of gravity which gives each ball a downward acceleration, g. The ball that is projected horizontally experiences the same downward acceleration as the other ball. This downward acceleration does not affect the horizontal motion of the ball projected horizontally; only the vertical motion is affected.

Investigating horizontal projection

A stroboscope and a camera may be used to record the motion of a projectile. Figure 3 shows a multi-flash photograph of two balls, A and B, released at the same time. B was released from rest and dropped vertically; A was given an initial horizontal projection so it followed a curved path. The stroboscope flashed at a constant rate, so images of both balls were recorded at the same time.

- **The horizontal position** of A changes by equal distances between successive flashes. This shows that the horizontal component of A's velocity is constant.
- **The vertical position** of A and B changes at the same rate. At any instant, A is at the same level as B. This shows that A and B have the same vertical component of velocity at any instant.

The projectile path of a ball projected horizontally

An object projected horizontally falls in an **arc** towards the ground. If its initial velocity is U, then at time t after projection (Figure 4):

- The **horizontal component** of its displacement,
$$x = Ut$$
(because it moves horizontally at a constant speed)
- The **vertical component** of its displacement,
$$y = \tfrac{1}{2}gt^2$$
(because it has no vertical component of its initial velocity)
- Its velocity has:
a horizontal component $v_x = U$, and
a vertical component $v_y = -gt$

Note Its speed at time $t = \sqrt{(v_x^2 + v_y^2)}$

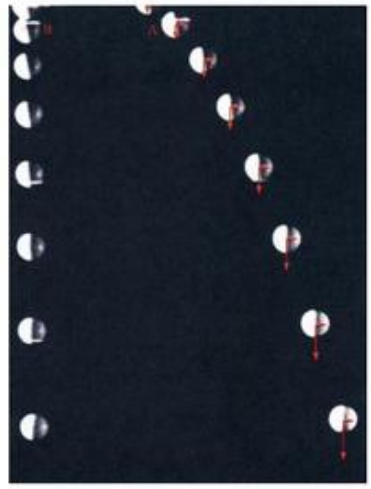


Fig 3 Multi-flash photo of two falling balls

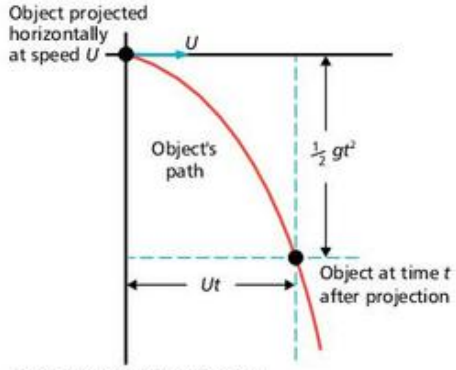


Fig 4 Horizontal projection

Worked example

$g = 9.8\,\text{m s}^{-2}$

An object is projected horizontally at a speed of $15\,\text{m s}^{-1}$ from the top of a tall tower (Figure 5) of height 35.0 m. Calculate:

a how long it takes to fall to the ground,

b how far it travels horizontally,

c its speed just before it hits the ground.

Solution

a $y = -35\,\text{m}$, $a = -9.8\,\text{m s}^{-2}$ ($-$ for downwards)

$$y = \tfrac{1}{2}gt^2$$
$$\therefore t^2 = \frac{2y}{g} = \frac{2 \times -35}{-9.8} = 7.14\,\text{s}^2$$
$$\therefore t = 2.67\,\text{s}$$

b $U = 15\,\text{m s}^{-1}$, $t = 2.67\,\text{s}$

$x = Ut = 15 \times 2.67 = 40\,\text{m}$

c Just before impact, $v_x = U = 15\,\text{m s}^{-1}$ and $v_y = -gt = -9.8 \times 2.67 = 26.2\,\text{m s}^{-1}$

$\therefore$ speed just before impact, $v = (v_x^2 + v_y^2)^{1/2} = (15^2 + 26.2^2)^{1/2} = 30.2\,\text{m s}^{-1}$

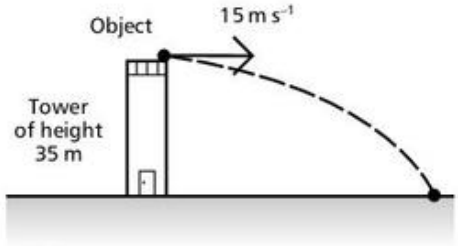


Fig 5

QUESTIONS

$g = 9.8\,\text{m s}^{-2}$

1 An object is released from a hot air balloon 50 m above the ground that is descending vertically at a speed of $4.0\,\text{m s}^{-1}$. Calculate:
- **a** the velocity of the object at the ground,
- **b** the duration of descent of the object,
- **c** the height of the balloon above the ground when the object hits the ground.

2 An object is projected horizontally at a speed of $16\,\text{m s}^{-1}$ into the sea from a cliff top of height 45.0 m. Calculate:
- **a** how long it takes to reach the sea,
- **b** how far it travels horizontally,
- **c** its impact velocity.

3 A dart is thrown horizontally along a line which passes through the centre of a dartboard 2.3 m away from the point at which the dart was released. The dart hits the dartboard at a point 0.19 m below the centre. Calculate:
- **a** the time of flight of the dart,
- **b** its horizontal speed of projection.

4 A parcel is released from an aircraft travelling horizontally at a speed of $120\,\text{m s}^{-1}$ above level ground. The parcel hits the ground 8.5 s later. Calculate:
- **a** the height of the aircraft above the ground,
- **b** the horizontal distance travelled in this time by **i** the parcel, **ii** the aircraft,
- **c** the speed of impact of the parcel at the ground.

1.9 Projectile motion 2

On these pages you will learn to:

- Compare projectile motion with the motion of electrons in a uniform electric field
- Solve projectile problems where projection is not horizontal using the equations for uniform acceleration in a vertical direction and for uniform velocity in a horizontal direction

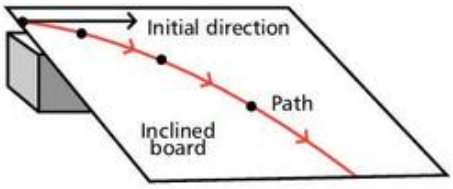


Fig 1 Using an inclined board

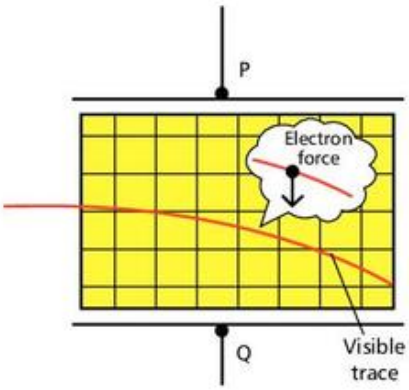


Fig 2 An electron beam on a parabolic path

(Vacuum tube used to contain apparatus not shown)

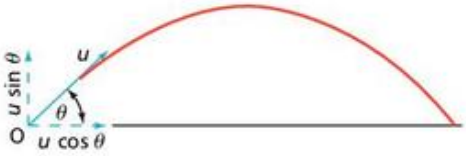


Fig 3 Projectile motion

Projectile-like motion

Any form of motion where an object experiences a constant acceleration in a different direction to its velocity will be like projectile motion. For example:

- The path of a ball rolling across an inclined board will be a projectile path. Figure 1 shows the idea. The object is projected across the top of the board from the side. Its path curves down the board and is **parabolic**. This is because the object is subjected to a constant acceleration acting down the board, and its initial velocity is across the board. The same equations as for projectile motion apply, with the motion down the board replacing the vertical motion.
- The path of a beam of electrons directed between two oppositely charged parallel plates is a **parabola**, as shown in Figure 2. Each electron in the beam is acted on by a constant force towards the positive plate, because the charge of an electron is negative. Therefore, each electron experiences a constant acceleration towards the positive plate. If its initial velocity is parallel to the plates, then its path is parabolic because its motion parallel to the plates is at **zero** acceleration; whereas its motion perpendicular to the plates is at **constant** (non-zero) acceleration.

Projection at angle θ and speed U above the horizontal

An object projected into the air follows a path that depends on its **speed** of projection and its **angle** of projection. In unit 1.8, we looked at the special cases of vertical projection and of horizontal projection. In this section, we will consider the general case of projection at angle θ to the horizontal. The analysis can be applied to the motion of any object acted on by a force that is constant in **magnitude** and **direction**.

A **positive value** of θ will signify projection at angle θ **above** the horizontal.

A **negative value** of θ will signify projection at angle θ **below** the horizontal.

For an object projected at initial speed U at an angle θ to the horizontal, as in Figure 3, its **initial velocity** has:

- a **horizontal** component $u_x = U\cos\theta$
- a **vertical** component $u_y = U\sin\theta$

If necessary, see unit 1.1 for resolving a vector into two perpendicular components.

Its **acceleration** has:

- a **horizontal** component $a_x = 0$
- a **vertical** component $a_y = -g$

For the **horizontal motion**, at time t after release:

- Its horizontal component of **velocity**, $v_x = U\cos\theta$ (unchanged), as its horizontal component of acceleration is zero.
- Its horizontal component of **displacement**, $x =$ its horizontal component of velocity $\times$ time taken $= Ut\cos\theta$

For the vertical motion,

- Its vertical component of **velocity**, $v_y = U\sin\theta - gt$
  This is obtained by applying the equation '$v = u + at$' to the vertical motion with $U\sin\theta$ for the initial speed u and $-g$ for the acceleration a.
- Its vertical component of **displacement**, $y = Ut\sin\theta - \frac{1}{2}gt^2$
  This is obtained by applying the equation '$s = ut + \frac{1}{2}at^2$' to the vertical motion with $U\sin\theta$ for the initial speed u and $-g$ for the acceleration a.

- Its speed at time $t = \sqrt{(v_x^2 + v_y^2)}$, in accordance with Pythagoras' rule for adding the two perpendicular components of a vector.

Worked example

$g = 9.8\,\text{m s}^{-2}$

An arrow is fired at a speed of $48\,\text{m s}^{-1}$ at an angle of 50° above the horizontal. Calculate:

a how long it takes to reach maximum height,
b its maximum height.

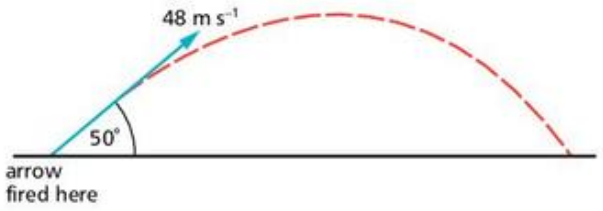


Fig 4

Solution

$U = 48\,\text{m s}^{-1}, \theta = 50°$

a The vertical component of the initial velocity,
$u_y = U\sin\theta = 48\sin 50° = 36.7\,\text{m s}^{-1}$

The vertical component of velocity at maximum height,
$v_y = 0$

Using $v_y = U\sin\theta - gt$,

$$0 = 36.7 - 9.8t$$

$$t = \frac{36.7}{9.8} = 3.74\,\text{s} = 3.7\,\text{s to 2 s.f.}$$

b The maximum height can be calculated by using $y = Ut\sin\theta - \frac{1}{2}gt^2$ with $t = 3.74\,\text{s}$,

$\therefore$ maximum height $= 36.7 \times 3.74 - (0.5 \times 9.8 \times 3.74^2)$

$= 68.7\,\text{m}$

QUESTIONS

$g = 9.8\,\text{m s}^{-2}$

1 A ball was projected horizontally at a speed of $0.52\,\text{m s}^{-1}$ across the top of an inclined board of width 600 mm and length 1200 mm (Figure 5). It reached the bottom of the board 0.90 s later.

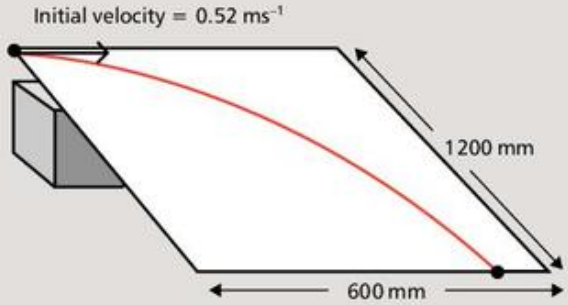


Fig 5

Calculate:

a the distance travelled by the ball across the board,
b its acceleration on the board,
c its speed at the bottom of the board.

2 An arrow is fired at a speed of $45\,\text{m s}^{-1}$ at an angle of 30° above the horizontal. Calculate:

a its maximum height,
b how long it takes to reach maximum height,
c how long it takes to return to the same horizontal level as it started at,
d the distance travelled horizontally to the point of return in **c**.

3 A cable car was travelling at a speed of $4.6\,\text{m s}^{-1}$ in a direction that was 40° above the horizontal when an object was released from the cable car (Figure 6).

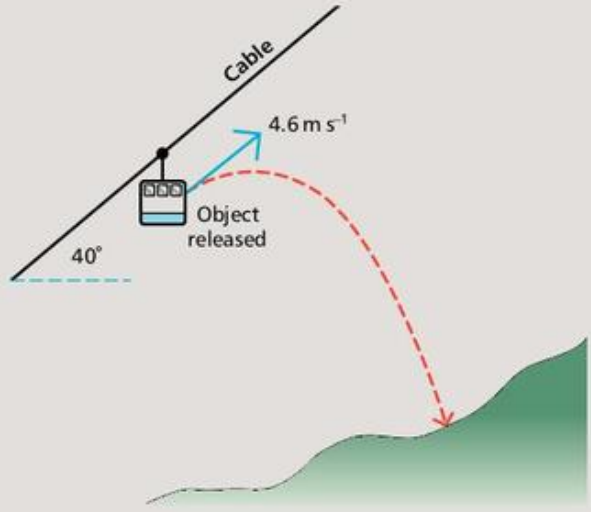


Fig 6

a Calculate the horizontal and vertical components of velocity of the object at the instant it was released.
b The object took 5.8 s to fall to the ground below. Calculate: **i** the distance fallen by the object from the point of release, **ii** the horizontal distance travelled by the object from the point of release to where it hit the ground.

4 A cannon ball was fired from the top of a tower at an angle of elevation of 25°. The ball hit the ground 2.7 s later at a distance of 58 m away from the foot of the tower.

a Calculate the horizontal and vertical components of the velocity of the cannon ball at the instant it was fired.
b Calculate the height of the tower above the ground.
c Calculate the speed of impact of the cannon ball at the ground.

1 End of chapter questions

$g = 9.81\,\text{m s}^{-2}$

1 a i State the difference between a vector and a scalar quantity.

ii Give one example of a vector quantity other than force.

iii Give an example of a scalar quantity other than speed.

b A 6.0 N force and a 5.0 N force act at right angles to each other on an object of mass 3.0 kg.

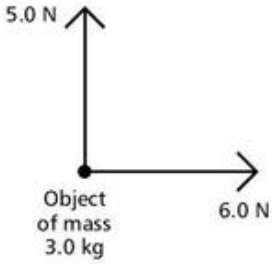


Fig 1.1

Calculate the magnitude of the resultant of these two forces.

2 A car driver joins a motorway at a speed of 22 m s^{-1} and maintains this speed for a distance of 32.5 km, when she stops for 20 min at a motorway service station. She then rejoins the motorway and travels a further distance of 47.5 km in 40 min before leaving the motorway.

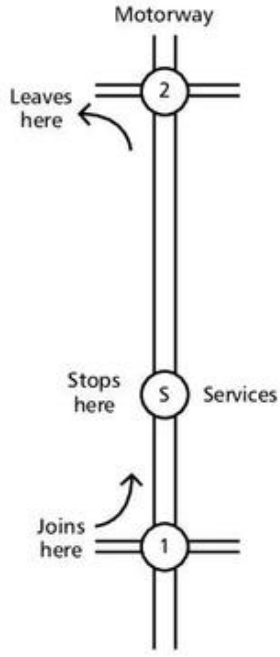


Fig 2.1

a Calculate the time taken for the first part of the journey.

b Sketch a speed–time graph for this journey on the motorway.

c Calculate the average speed of the car for the whole journey.

3 An aircraft taking off accelerated uniformly for 40 s from rest, before it left the ground after travelling a distance of 1600 m. Calculate:

a its average speed during take-off,

b its velocity at the instant it left the ground,

c its acceleration during take-off.

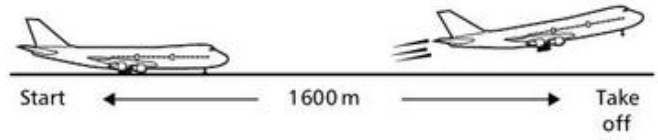


Fig 3.1

4 A train accelerated uniformly from rest for 40 s until its speed reached 15 m s^{-1}. It then travelled at a constant speed of 15 m s^{-1} for a further 60 s, before slowing down at constant deceleration and stopping 20 s later.

a Sketch a speed–time graph to represent its motion.

b Calculate its acceleration and the distance it travelled in each of the three stages of its motion.

c Calculate its average speed.

5 An object released from rest at the side of a river bridge hit the water 2.2 s later. Calculate:

a the distance fallen in air by the object,

b the speed of the object just before it hit the water.

6 A stationary railway truck on an inclined track was struck by a shunting engine moving at a constant speed of 0.6 m s^{-1}. The impact caused the truck to move up the incline at an initial speed of 2.0 m s^{-1}. The truck slowed down and stopped 10 s later.

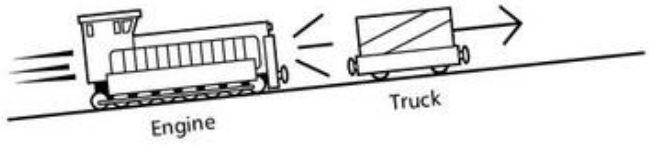


Fig 6.1

a Calculate: **i** the distance travelled by the truck before it stopped, **ii** the magnitude and direction of its acceleration.

b The truck stopped for an instant, then rolled back down the track and hit the engine again. The engine continued to move up the track at 0.6 m s^{-1}.

i Sketch a velocity–time graph on the same axes for the truck and the engine.

ii Use your graph to show that the truck hit the engine a second time 14 s after it was first hit by the engine.

iii Calculate the velocity of the truck immediately before this second impact.

7 A ball thrown vertically into the air left the thrower's hand when it was 1.6 m above the ground and hit the ground on return 3.1 s later.

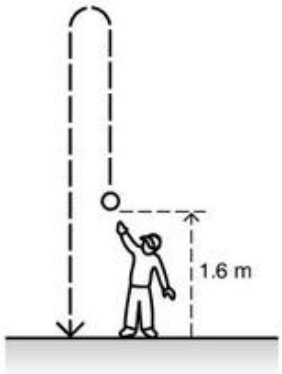


Fig 7.1

a Calculate: **i** the initial velocity of the ball, **ii** the maximum height of the ball above the ground.

b Calculate its velocity just before impact.

8 A cricketer on a level playing field threw a ball into the air at an angle of 30° above the horizontal. The ball was caught by another cricketer, 36 m away at the same level 1.8 s later.

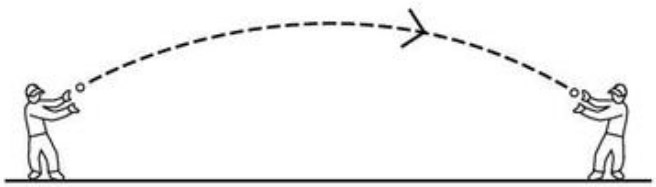


Fig 8.1

a Calculate the horizontal velocity of the ball.

b Show that the initial speed of the ball was 23 m s^{-1}.

c Calculate the maximum height reached by the ball.

9 A rocket launched vertically accelerated at a constant acceleration of 6.0 m s^{-2} for 25 s before its fuel supply ran out. It then rose to maximum height and fell back to the ground.

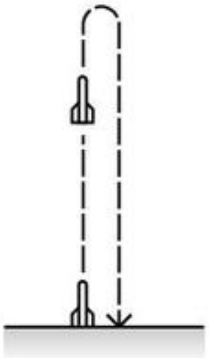


Fig 9.1

a i Calculate its velocity and its height when its fuel ran out.

ii Show that it reached a maximum height of 3.0 km.

b i Calculate how long it took to fall from maximum height to the ground.

ii Calculate its velocity immediately before impact with the ground.

iii Sketch a velocity–time graph for the motion of the rocket from launch to impact.

10 A ball is thrown horizontally from the top of a building, as shown in Figure 10.1.

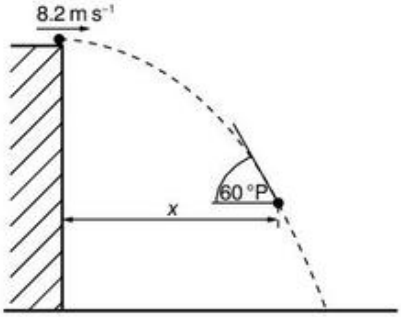


Fig 10.1

The ball is thrown with a horizontal speed of 8.2 m s^{-1}. The side of the building is vertical. At point P on the path of the ball, the ball is distance x from the building and is moving at an angle of 60° to the horizontal. Air resistance is negligible.

a For the ball at point P,

i show that the vertical component of its velocity is 14.2 m s^{-1}, *(2 marks)*

ii determine the vertical distance, in m, through which the ball has fallen, *(2 marks)*

iii determine the horizontal distance x, in m. *(2 marks)*

b The path of the ball in **a**, with an initial horizontal speed of 8.2 m s^{-1}, is shown again in Figure 10.2.

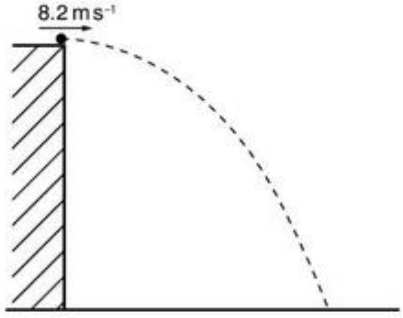


Fig 10.2

On Figure 10.2, sketch the new path of the ball for the ball having an initial horizontal speed

i greater than 8.2 m s^{-1} and with negligible air resistance (label this path G), *(2 marks)*

ii equal to 8.2 m s^{-1} but with air resistance (label this path A). *(2 marks)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 21 Q2 November 2010

2 Forces in action

2.1 Force and acceleration

On these pages you will learn to:

- Show an understanding that mass is the property of a body that resists change in motion
- State Newton's first law of motion and give examples of where it applies
- Recall and use Newton's second law of motion for constant mass in the form $F = ma$
- Describe and use the concept of weight as the effect of a gravitational field on a mass
- Recall that for a body of mass m, its weight $W = mg$, where g is the acceleration of free fall

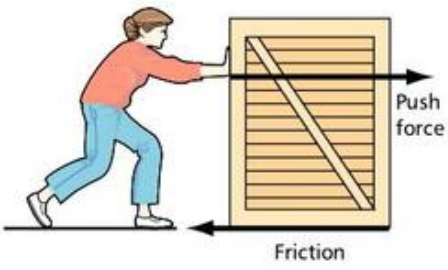


Fig 1 Overcoming friction

Motion without force

Motorists on icy roads in winter need to be very careful, because the tyres of a car have little or no 'grip' on the ice. Moving from a standstill on ice is very difficult. Stopping on ice is almost impossible, as a car moving on ice will slide when the brakes are applied. **Friction** is a hidden force that we don't usually think about until it is absent!

If you have ever tried to push a heavy crate across a rough concrete floor, you will know all about friction. The push force is opposed by friction and as soon as you stop pushing, friction stops the crate moving. If the crate had been pushed onto a patch of ice, it would have moved across the ice without any further push needed.

Figure 2 shows an air track which allows **motion** to be observed in the absence of friction. The glider on the air track floats on a cushion of air. Provided the track is level, the glider moves at a constant velocity along the track because friction is absent.

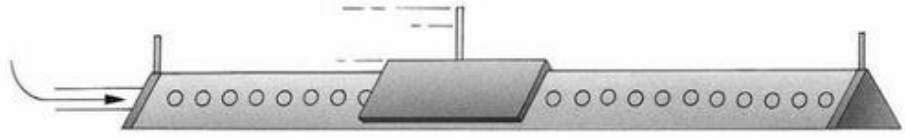


Fig 2 The linear air track

Newton's first law of motion

Objects either stay at rest, or remain in uniform motion (i.e. at constant velocity), unless acted on by a resultant force.

Sir Isaac Newton was the first person to realise that a moving object stays moving at constant velocity unless acted on by a force. He recognised that when an object is acted on by a **resultant** force, the result is to change the object's velocity. In other words, an object moving at **constant** velocity is either:

- acted on by **no forces**, or
- the forces acting on it are **balanced** (i.e. the **resultant force is zero**).

Investigating force and motion

How does the velocity of an object change if it is acted on by a constant force? Figure 3 shows how this can be investigated using a dynamics trolley and a ticker-tape timer. The timer prints dots on the tape at a constant rate of 50 dots per second; so the faster the tape is moving, the greater the spacing between adjacent dots. An electronic timer linked to a computer can be used in place of the ticker-tape timer.

The trolley is pulled along a sloped runway by means of one or more elastic bands stretched to the same length. The runway is sloped just enough to compensate for friction. To test for the correct slope, the trolley should move down the runway at constant speed after being given a brief push.

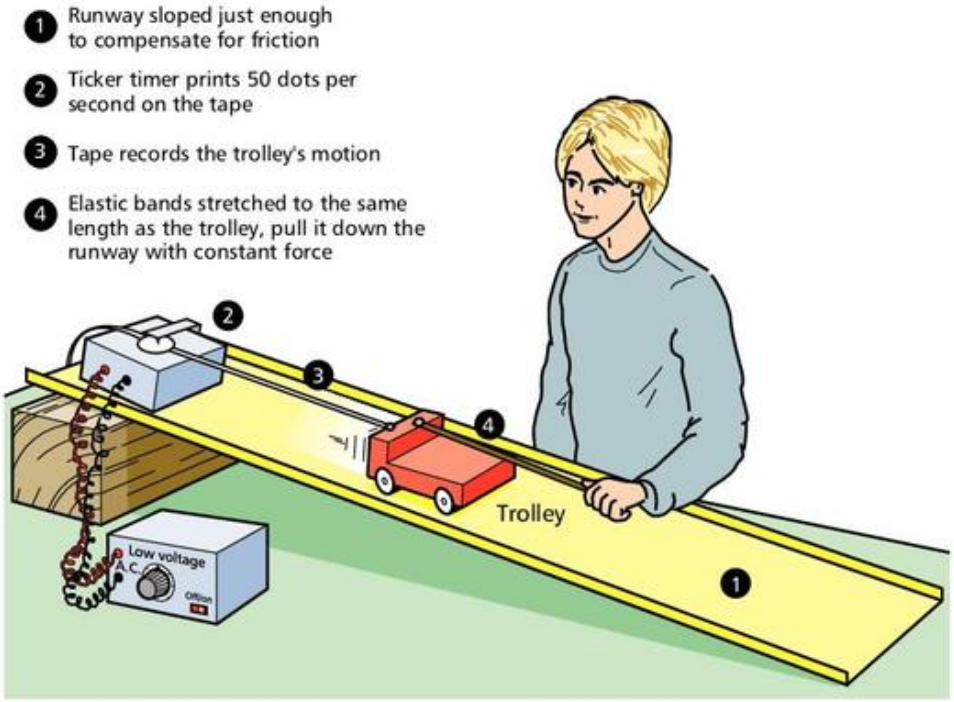


Fig 3 *Investigating force and motion*

- If a ticker-tape timer is used, the length of each 'ten dot' section of the tape gives a measure of the speed as the middle of that section went through the ticker timer. Cutting the tape into 'ten dot' sections enables a speed–time chart to be made, as shown in Figure 4.
- If an electronic timer linked to a computer is used, the measurements should be displayed directly as a speed–time graph (Figure 5).

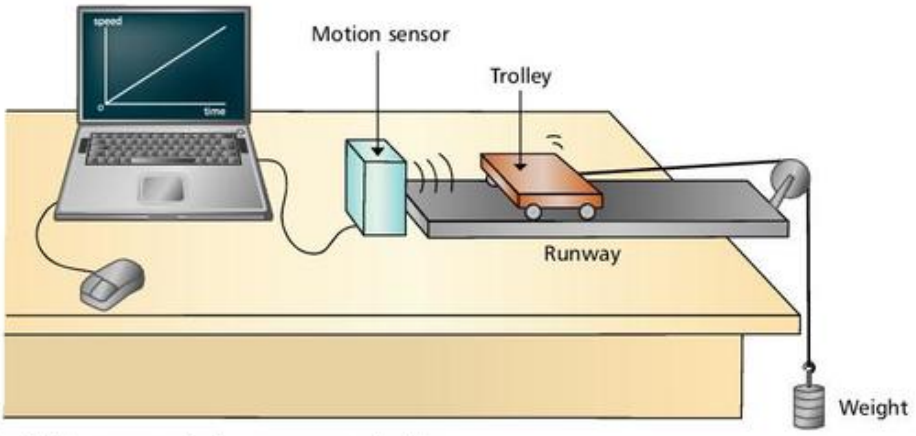


Fig 5 *Using a computer to measure acceleration*

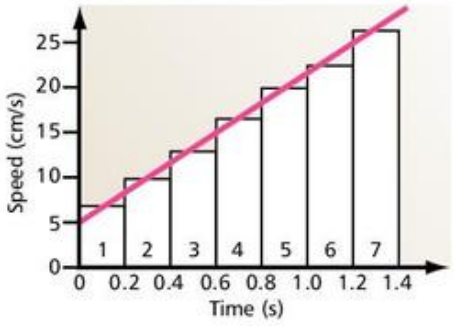


Fig 4 *Making a tape chart*

The speed–time graph should show that the speed increased at a constant rate. The **acceleration** of the trolley is therefore constant, and can be measured from the speed–time graph. Table 1 shows typical measurements using different amounts of **force** (i.e. one, or two, or three elastic bands in 'parallel') and different amounts of **mass** (i.e. a single trolley, or a double trolley, or a triple trolley).

The results in Table 1 show that the force is proportional to the mass × the acceleration. In other words, if a force F acts on an object of mass m, the object undergoes an acceleration a such that:

$$F \text{ is proportional to } ma$$

Table 1 *Varying force and mass*

Force (i.e. no. of elastic bands)	1	2	3	1	2	3
Mass (i.e. no of trolleys)	1	1	1	2	2	2
Acceleration (ms^{-2})	12	24	36	6	12	18
Mass × acceleration	12	24	36	12	24	36

The acceleration is always in the same direction as the force

For example, a projectile in motion experiences a force vertically downwards due to gravity. Its acceleration is therefore vertically downwards, no matter what its direction of motion is.

Newton's second law for constant mass

By defining the unit of force, the **newton (N)**, as the amount of force that will give an object of mass 1 kg an acceleration of $1\,m\,s^{-2}$, the proportionality statement can be expressed as an equation:

$$F = ma$$

where F = force (in N), m = mass (in kg), a = acceleration (in $m\,s^{-2}$).

This equation is known as Newton's **second law** for constant mass.

Worked example

A vehicle of mass 600 kg accelerates uniformly from rest to a speed of $8.0\,m\,s^{-2}$ in 20 s. Calculate the force needed to produce this acceleration.

Solution

Acceleration, $a = \dfrac{v-u}{t} = \dfrac{8.0-0}{20} = 0.4\,m\,s^{-2}$

Force, $F = ma = 600 \times 0.4 = 240\,N$

Weight

- The acceleration of a **falling object** acted on by gravity only is g. Because the force of gravity on the object is the only force acting on it, its weight W (in newtons) is given by:

  $$W = mg$$

  where m = the mass of the object (in kg).
- When an object is **in equilibrium**, the support force on it is equal and opposite to its weight. Therefore, an object placed on a weighing balance (e.g. a spring balance or a top-pan balance) exerts a force on the balance equal to the weight of the object. Thus the balance measures the weight of the object.
- g is also referred to as the **gravitational field strength** at a given position, as it is the force of gravity per unit mass on a small object at that position. So the gravitational field strength at the Earth's surface is $9.8\,N\,kg^{-1}$. Note that the weight of a fixed mass **depends on its location**. For example, the weight of a 1 kg object is 9.8 N on the Earth's surface and 1.6 N on the Moon's surface.
- The mass of an object is a measure of its **inertia**, which is its resistance to change of its motion. Figure 6 shows an entertaining demonstration of inertia. When the card is flicked, the coin drops into the glass because the force of friction on it due to the moving card is too small to shift it sideways.

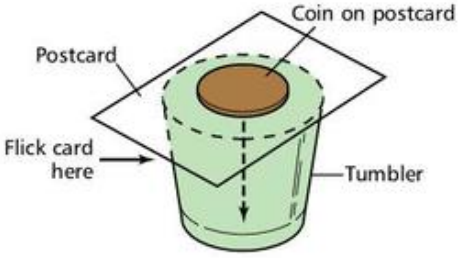


Fig 6 An 'inertia' trick

QUESTIONS

$g = 9.81\,N\,kg^{-1}$

1 A car of mass 800 kg accelerates uniformly along a straight line from rest to a speed of $12\,m\,s^{-1}$ in 50 s. Calculate:

a the acceleration of the car,

b the force on the car that produced this acceleration,

c the ratio of the accelerating force to the weight of the car.

2 An aeroplane, of mass 5000 kg, lands on a runway at a speed of $60\,m\,s^{-1}$ and stops 25 s later. Calculate:

a the deceleration of the aeroplane,

b the braking force on the aeroplane.

3 **a** A vehicle, of mass 1200 kg, on a level road accelerates from rest to a speed of $6.0\,m\,s^{-1}$ in 20 s, without change of direction. Calculate the force that accelerated the car.

b The vehicle in part **a** is fitted with a trailer of mass 200 kg. Calculate the time taken to reach a speed of $6.0\,m\,s^{-1}$ from rest for the same force as in part **a**.

4 A bullet of mass 0.002 kg, travelling at a speed of $120\,m\,s^{-1}$, hit a tree and penetrated a distance of 55 mm into the tree. Calculate:

a the deceleration of the bullet,

b the impact force of the bullet on the tree.

2.2 Using $F = ma$

On these pages you will learn to:

- Recall the relationship $F = ma$ and solve problems using it, including where more than one force acts on object
- Appreciate that acceleration and force are always in the same direction

Two forces in opposite directions

- When an object is acted on by two **unequal** forces acting in **opposite** directions, the object accelerates in the direction of the larger force. If the forces are F_1 and F_2 where $F_1 > F_2$:

$$\textbf{Resultant force, } F_1 - F_2 = ma$$

where m is the mass of the object and a is its acceleration.

- If the object is on a horizontal surface and F_1 and F_2 are **horizontal** and in opposite directions, the above equation still applies. The support force on the object is equal and opposite to its weight.

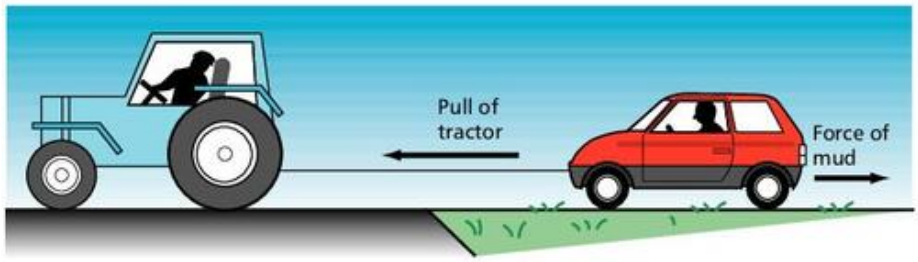


Fig 1 *Unbalanced forces*

Some examples are given below, where two forces act in different directions on an object.

Towing a trailer

Consider the example of a car of mass M fitted with a trailer of mass m on a level road. When the car and the trailer accelerate, the car pulls the trailer forward and the trailer holds the car back (Figure 2).

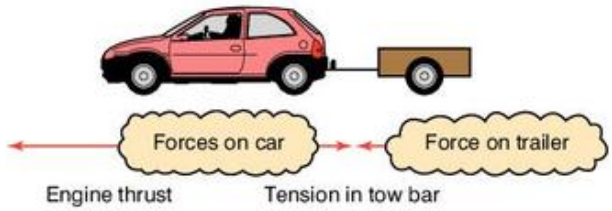


Fig 2 *Car and trailer*

- The car is subjected to a driving force F pushing it forwards (from its engine thrust) and the tension T in the tow bar holding it back. Therefore:

$$\textbf{Resultant force on car } = F - T = Ma$$

- The force on the trailer is due to the tension T in the tow bar pulling it forwards. Therefore:

$$T = ma$$

Combining the two equations gives:

$$F = Ma + ma = (M + m)a$$

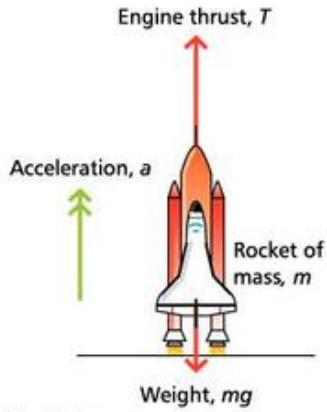


Fig 3 Rocket launch

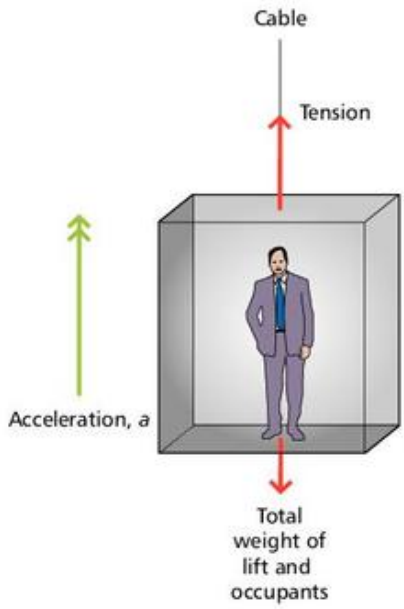


Fig 4 In a lift

Rocket problems

If T is the thrust of the rocket engine when its mass is m and the rocket is moving upwards, its acceleration a is given by:

$$T - mg = ma$$

Therefore $$\textit{Rocket thrust, } T = mg + ma$$

The rocket thrust must therefore overcome the weight of the rocket for the rocket to take off (Figure 3).

Lift problems

Using 'upwards is positive' gives the resultant force on the lift as:

$$F = T - mg$$

where T is the tension in the lift cable and m is the total mass of the lift and occupants (Figure 4).

Therefore $$T - mg = ma$$

where a = acceleration.

- If the lift is moving at a constant velocity, then $a = 0$ so $T = mg$
- If the lift is moving up and accelerating, then $a > 0$ so $T = mg + ma > mg$.
- If the lift is moving up and decelerating, then $a < 0$ so $T = mg + ma < mg$.
- If the lift is moving down and accelerating, then $a < 0$ (i.e. velocity and acceleration are both downwards and therefore negative) so $T = mg - ma < mg$.
- If the lift is moving down and decelerating, then $a > 0$ (i.e. velocity downwards and acceleration upwards and therefore positive) so $T = mg + ma > mg$.

Tension in the cable is less than the weight if:

- The lift is moving up and decelerating (i.e. velocity > 0 and acceleration < 0).
- The lift is moving down and accelerating (i.e. velocity < 0 and acceleration < 0).

Tension in the cable is greater than the weight if:

- The lift is moving up and accelerating (i.e. velocity > 0 and acceleration > 0).
- The lift is moving down and decelerating (i.e. velocity < 0 and acceleration > 0).

Worked example

$g = 9.8\,\text{m s}^{-2}$

A lift of total mass 650 kg moving downwards decelerates at $1.5\,\text{m s}^{-2}$ and stops. Calculate the tension in the lift cable during the deceleration.

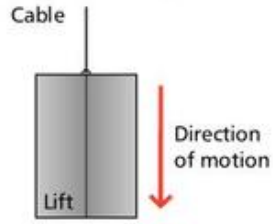


Fig 5

Solution

The lift is moving down so its velocity $v < 0$. Since it is decelerating, its acceleration a is in the opposite direction to its velocity, so $a > 0$. Therefore use $a = +1.5\,\text{m s}^{-2}$ in the equation:

$$\begin{aligned} T - mg &= ma \\ T &= mg + ma \\ &= 650 \times 9.8 + 650 \times 1.5 = 7300\,\text{N} \end{aligned}$$

Further $F = ma$ problems

Pulley problems

Consider two masses M and m (where $M > m$) attached to a thread hung over a frictionless pulley, as in Figure 6. When released, mass M accelerates downwards and mass m accelerates upwards. If a is the acceleration and T is the tension in the thread, then:

- On mass M, the resultant force $= Mg - T = Ma$.
- On mass m, the resultant force $= T - mg = ma$.

Therefore, adding the two equations gives:

$$Mg - mg = (M + m)a$$

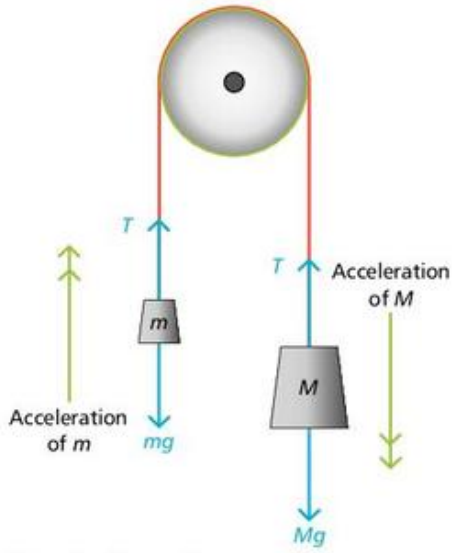


Fig 6 A pulley problem

Sliding down a slope

Consider a block of mass m sliding down a slope (Figure 7).
The component of the block's weight down the slope is $mg\sin\theta$.
If the force of friction on the block is F_0, then:

$$\text{Resultant force on the block} = mg\sin\theta - F_0$$

Therefore $$mg\sin\theta - F_0 = ma$$

where a is the acceleration of the block.

Note With the addition of an engine force F_E, the above equation can be applied to a vehicle on a downhill slope of constant gradient. Thus:

$$F_E + mg\sin\theta - F_0 = ma$$

where F_0 is the combined sum of the force of friction and the braking force.

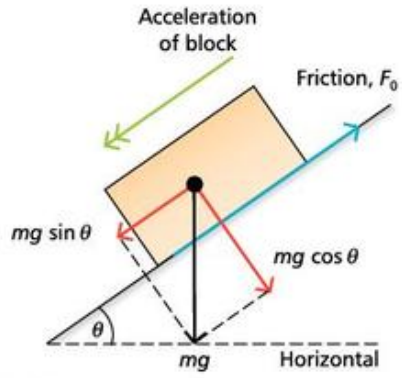


Fig 7

QUESTIONS

$g = 9.8\,\text{m s}^{-2}$

1 A rocket of mass 550 kg blasts vertically from the launch pad at an acceleration of $4.2\,\text{m s}^{-2}$. Calculate:
- **a** the weight of the rocket,
- **b** the thrust of the rocket engines.

2 A car of mass 1400 kg, pulling a trailer of mass 400 kg, accelerates from rest to a speed of $9\,\text{m s}^{-1}$ in a time of 60 s on a level road. Assuming air resistance is negligible, calculate:
- **a** the tension in the tow bar,
- **b** the engine force.

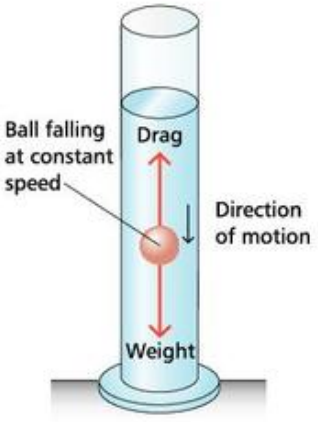
3 A lift and its occupants have a total mass of 1200 kg. Calculate the tension in the lift cable when the lift is:
- **a** stationary,
- **b** ascending at constant speed,
- **c** ascending at a constant acceleration of $0.4\,\text{m s}^{-2}$,
- **d** descending at a constant deceleration of $0.4\,\text{m s}^{-2}$.

4 A brick, of mass 3.2 kg on a sloping flat roof, at 30° to the horizontal, slides at constant acceleration 2.0 m down the roof in 2.0 s from rest.
Calculate:
- **a** the acceleration of the brick,
- **b** the frictional force on the brick due to the roof.

2.3 Terminal speed

On these pages you will learn to:

- Describe qualitatively the motion of bodies falling in a uniform gravitational field with air resistance
- Explain what is meant by terminal speed
- Explain why bodies falling in a fluid and powered vehicles reach a terminal speed and why their acceleration decreases to zero



a Falling in a fluid

b Skydiving

***Fig 1** At terminal speed*

NOTE

The acceleration at any instant is the gradient of the speed–time curve.

Drag forces

Any object moving through a fluid (something that can 'flow', i.e. a liquid or a gas) experiences a **drag force** due to the fluid. The drag force depends on:

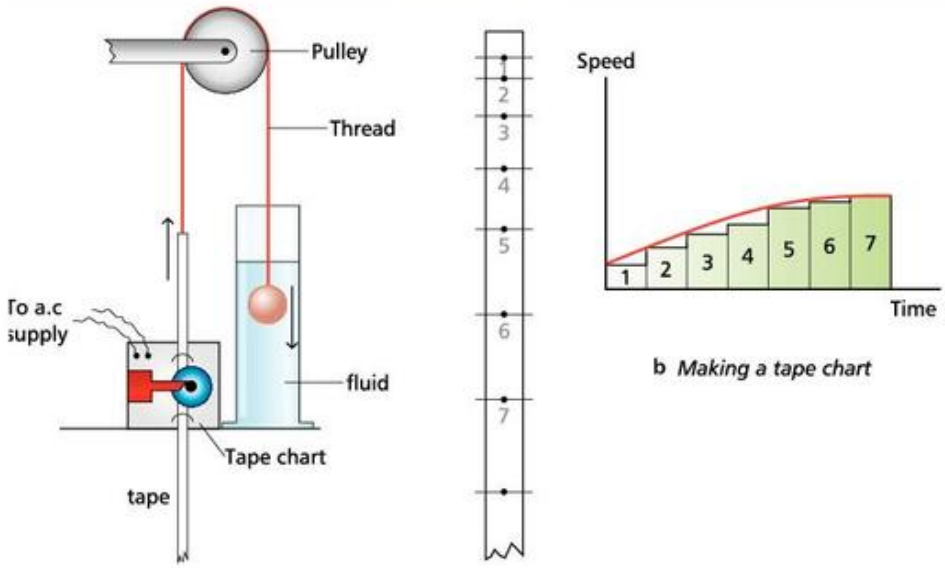
- the shape of the object,
- its speed, and
- the **viscosity** of the fluid (which is a measure of how easily the fluid flows past a surface).

The faster an object travels in a fluid, the greater the drag force on it.

Motion of an object falling in a fluid

- The speed of an object released from rest in a fluid increases as it falls, so the drag force on it due to the fluid increases. The resultant force on the object is the difference between the force of gravity on it (i.e. its weight) and the drag force. As the drag force increases, the resultant force decreases so the acceleration becomes less as it falls. If it continues falling, it attains **terminal speed** when the drag force on it is equal and opposite to its weight. Its acceleration is then zero and its speed remains constant as it falls.
- Figure 2 shows how to investigate the motion of an object falling in a fluid. When the object is released, the thread attached to the object pulls a tape through a ticker timer which prints dots on the tape. The spacing between successive dots is a measure of the speed of the object, as the dots are printed on the tape at a constant rate. A tape chart can be made from the tape to show how the speed changes with time. The results show that:

The speed increases and reaches a constant value, which is the terminal speed.



a Apparatus *b Making a tape chart*

***Fig 2** Investigating the motion of an object falling in a fluid*

- At any instant, the resultant force is $mg - F_D$, where m is the mass of the object and F_D is the drag force.

Therefore **Acceleration of the object** $= \dfrac{mg - F_D}{m} = g - \dfrac{F_D}{m}$

1 The **initial acceleration** is g, because the drag force is zero at the instant the object is released.
2 At the **terminal speed**, the potential energy lost by the object as it falls is converted to internal energy of the fluid by the drag force.

Motion of a powered vehicle

The top speed of a road vehicle or an aircraft depends on its engine power, also referred to as its **motive power**, and its shape. A vehicle with a streamlined shape can reach a higher top speed than a vehicle with the same motive power that is not streamlined.

For a powered vehicle of mass m moving on a level surface, if F_E represents the motive force of the vehicle (i.e. the engine force driving the vehicle):

$$\text{Resultant force} = F_E - F_D$$

where F_D is the combined effect of friction and the drag force due to air resistance.

Therefore, its acceleration $a = \dfrac{(F_E - F_D)}{m}$

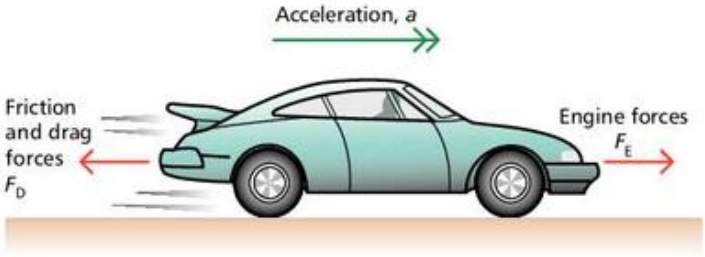


Fig 3

- Because the drag force increases with speed, the maximum speed (i.e. terminal speed) of the vehicle, v_{max}, is attained when the drag force becomes equal and opposite to the motive force.
- At maximum speed, the work done by the engine is dissipated by the drag force and becomes internal energy of the surroundings. See unit 2.6 for more about motive power.

Worked example

A car of mass 1200 kg has an engine which provides a motive force of 600 N. Calculate:
a its initial acceleration,
b its acceleration when the drag force is 400 N.

Solution

a The maximum acceleration is when the drag force is zero (i.e. when the car starts). The resultant force on the car is therefore 600 N at the start.
Therefore

$$\text{initial acceleration} = \frac{\text{force}}{\text{mass}} = \frac{600\,\text{N}}{1200\,\text{kg}} = 0.5\,\text{m s}^{-2}$$

b When the drag force = 400 N,
the resultant force = motive force − drag force
= 600 − 400 N
= 200 N

$$\therefore \text{Acceleration} = \text{force/mass} = \frac{200\,\text{N}}{1200\,\text{kg}} = 0.16\,\text{m s}^{-2}$$

Hydrofoil physics

A hydrofoil boat travels much faster than an ordinary boat, because it has a powerful jet engine that enables it to 'ski' on its hydrofoils when the jet engine is switched on.

When the jet engine is switched on and takes over from the less-powerful propeller engine, the boat speeds up and the hydrofoils are extended. The boat rides on the hydrofoils so the drag force is reduced, as its hull is no longer in the water. At top speed, the motive force of the jet engine is equal to the drag force on the hydrofoils.

Fig 4 A hydrofoil ferry

QUESTIONS

$g = 9.8\,\text{m s}^{-2}$.

1 a A steel ball of mass 0.15 kg released from rest in a liquid falls a distance of 0.20 m in 5.0 s. Assuming the ball reaches terminal speed within a fraction of a second, calculate:
i its terminal speed,
ii the drag force on it when it falls at terminal speed.

b State and explain whether or not a smaller steel ball would fall at the same rate in the same liquid.

2 Explain why a cyclist can reach a higher top speed by crouching over the handlebars instead of sitting upright while pedalling.

3 A vehicle of mass 32 000 kg has an engine which has a maximum driving force of 4.4 kN and a top speed of $36\,\text{m s}^{-1}$ on a level road.
Calculate:

a its maximum acceleration from rest,

b the distance it would travel at maximum acceleration to reach a speed of $12\,\text{m s}^{-1}$ from rest.

4 Explain why a vehicle has a higher top speed on a downhill stretch of road than on a level road.

2.4 Work and energy

On these pages you will learn to:

- Give examples of energy in different forms, its conversion and conservation, and apply the principle of conservation of energy to simple examples
- Define work as the product of a force and the displacement in the direction of the force
- Calculate the work done in a number of situations including the work done by a gas that is expanding against a constant external pressure: $W = p\Delta V$

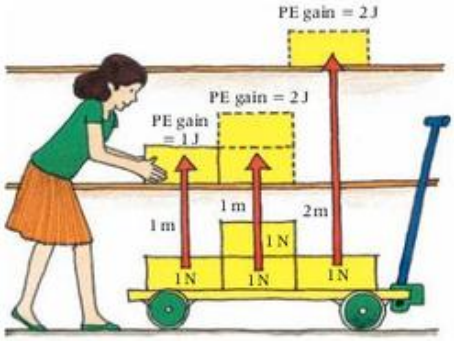


Fig 1 Using joules

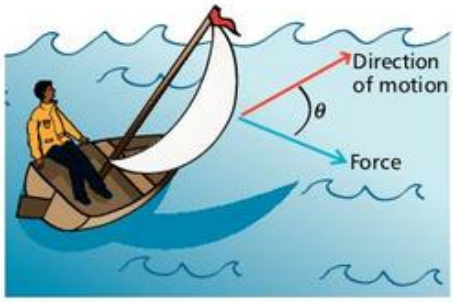


Fig 2 Force and displacement

NOTE

If $\theta = 90°$, which means that the force is perpendicular to the direction of motion, then, because $\cos 90° = 0$, the work done is zero.

Energy rules

Energy is needed to make stationary objects move, or to lift an object, or to change its shape, or to warm it up. When you lift an object, you **transfer energy** from your muscles to the object.

Objects can possess energy in **different forms**, including:

- **gravitational potential energy**, which is energy due to position in a gravitational field;
- **kinetic energy**, which is energy due to motion;
- **thermal energy**, which is energy due to the temperature of an object;
- **chemical** or **nuclear energy**, which is energy associated with chemical or nuclear reactions;
- **electrical potential energy**, which is energy of electrically charged objects;
- **elastic potential energy**, which is energy stored in an object when it is stretched or compressed.

Energy is measured in **joules (J)**. One joule is equal to the energy needed to raise a 1 N weight through a vertical height of 1 m.

Energy can be changed from one form into other forms. In any change, the total amount of energy after the change is always equal to the total amount of energy before the change. The total amount of energy is unchanged. In other words:

Energy cannot be created or destroyed.

This statement is known as the **Principle of Conservation of Energy.**

Work

Work is done on an object when a force acting on it due to another object makes it move. As a result, energy is transferred to the object by the force from the other object. The amount of work done depends on the force and the distance moved by the object. The greater the force or the further the distance, the greater the work done.

Work done = force × distance moved in the direction of the force

The **unit of work** is the joule (J).

One joule is equal to the work done when a force of 1 N moves its point of application by a distance of 1 m in the direction of the force.

Energy is also measured in joules, because the energy transferred to an object when a force acts on it is equal to the work done on it by the force. For example, a force of 2 N needs to be applied to an object of weight 2 N to raise the object steadily. If the object is raised by 1.5 m, the work done by the force is 3 J (= 2 N × 1.5 m). Therefore, the gain of potential energy of the object is 3 J.

Force and displacement

Imagine a yacht acted on by a wind force F at an angle θ to the direction in which the yacht moves, as in Figure 2. The wind force has a component $F\cos\theta$ in the direction of motion of the yacht, and a component $F\sin\theta$ at right angles to the direction of motion. If the yacht is moved a distance s by the wind, the work done on it is equal to the component of force in the direction of motion × the distance moved:

$$W = Fs\cos\theta$$

Force–distance graphs

- If a constant force F acts on an object and makes it move a distance s in the direction of the force, the work done on the object $W = Fs$. Figure 3 shows a graph of force against distance in this situation. The area under the line is a rectangle, of height representing the force and of base length representing the distance moved. Therefore **the area represents the work done**.

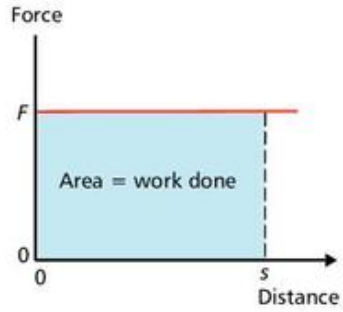


Fig 3 A force–distance graph for a constant force

- If a variable force F acts on an object and causes it to move in the direction of the force, for a small amount of distance Δs, the work done $\Delta W = F\Delta s$. This is represented on a graph of the force F against distance s by the area of a strip under the line of width Δs and height F. The total work done is therefore **the sum of the areas of all the strips** (i.e. the total area under the line). See Figure 4.

The area under the line of a force–distance graph represents the total work done.

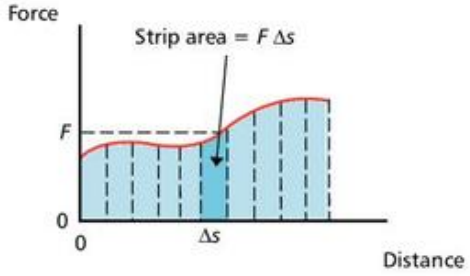


Fig 4 A force–distance graph for a variable force

Springs

For example, consider the force needed to stretch a spring. The greater the force, the more the spring is extended from its unstretched length. Figure 5 shows how the force needed to stretch a spring changes with the **extension** of the spring. The graph is a straight line through the origin. Therefore the force needed is proportional to the extension of the spring. This is known as **Hooke's law**. See topic 5.3 for more about springs.

Figure 5 is a graph of force against distance, in this case the distance the spring is extended. As explained above, the area under the line represents the work done to stretch the spring. Let F_0 represent the force needed to extend the spring to extension e_0. Therefore, because the area under the line from the origin to extension e_0 is a triangle of height F_0 and base length e_0:

$$\textbf{Area under line} = \tfrac{1}{2} \times \textbf{height} \times \textbf{base} = \tfrac{1}{2}F_0e_0$$

$$\textbf{Work done to stretch spring to extension } e_0 = \tfrac{1}{2}F_0e_0$$

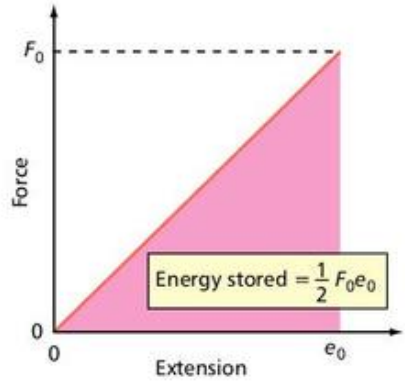


Fig 5 Force against extension for a spring

QUESTIONS

1 Calculate the work done when:
- **a** a weight of 40 N is raised by a height of 5.0 m,
- **b** a spring is stretched to an extension of 0.45 m by a force of 20 N.

2 Calculate the energy transferred by a force of 12 N when it moves an object by a distance of 4.0 m:
- **a** in the direction of the force,
- **b** in a direction at 60° to the direction of the force,
- **c** in a direction at right angles to the direction of the force.

3 A luggage trolley of total weight 400 N is pushed at a steady speed 20 m up a slope by a force of 50 N acting in the same direction as the object moves in. At the end of this distance, the trolley is 1.5 m higher than at the start. Calculate:
- **a** the work done pushing the trolley up the slope,
- **b** the gain of potential energy of the trolley,
- **c** the energy wasted due to friction.

4 A spring that obeys Hooke's law requires a force of 1.2 N to extend it to an extension of 50 mm. Calculate:
- **a** the force needed to extend it to an extension of 100 mm,
- **b** the work done when the spring is stretched to an extension of 100 mm.

2.5 Kinetic energy and potential energy

On these pages you will learn to:

- Derive, from the equations of motion, the formula for kinetic energy $E_k = \frac{1}{2}mv^2$ and recall and apply the formula
- Distinguish between gravitational potential energy and elastic potential energy
- Show an understanding of and use the relationship between force and potential energy in a uniform field to solve problems
- Derive, from the defining equation $W = Fs$, the formula $\Delta E_p = mgh$ for potential energy changes near the Earth's surface
- Recall and use the formula $\Delta E_p = mgh$ for potential energy changes near the Earth's surface

NOTE

The formula does not hold at speeds **approaching the speed of light**. Einstein's theory of special relativity tells us that the mass of an object increases with speed and that the energy of an object can be worked out from the equation $E = mc^2$, where c is the speed of light in free space and m is the mass of the object.

NOTE

The formula does not hold unless the change of height h is much smaller than the **Earth's radius**. If height h is not insignificant compared with the Earth's radius, the value of g is not the same over height h. The force of gravity on an object decreases with increased distance from the Earth.

Kinetic energy

Kinetic energy is the energy of an object due to its motion. The faster an object moves, the more kinetic energy it has. To see the exact link between kinetic energy and speed, consider an object of mass m, initially at rest, acted on by a constant force F for a time t.

Fig 1 Gaining kinetic energy

Let the speed of the object at time t be v:

$$\therefore \quad \text{Distance travelled, } s = \tfrac{1}{2}(u + v)t$$

$$= \tfrac{1}{2}vt \text{ because } u = 0$$

$$\text{Acceleration, } a = \frac{v - u}{t} = \frac{v}{t}$$

Using Newton's second law:

$$F = ma = \frac{mv}{t}$$

$\therefore$ the work done, by force F, to move the object through distance s:

$$W = Fs = \frac{mv}{t} \times \frac{vt}{2} = \tfrac{1}{2}mv^2$$

Because the gain of kinetic energy is due to the work done, then

$$\textbf{Kinetic energy, } E_K = \tfrac{1}{2}mv^2$$

Potential energy

Potential energy is the energy of an object due to its position.
If an object of mass m is raised through a vertical height h at steady speed, the force needed to raise it is equal and opposite to its weight mg. Therefore:

$$\textbf{Work done to raise the object} = \textbf{force} \times \textbf{distance moved} = mgh$$

The work done on the object increases its gravitational potential energy:

$$\textbf{Change of gravitational potential energy, } \Delta E_P = mgh$$

At the Earth's surface, $g = 9.8\,\text{m s}^{-2}$.

Energy changes involving kinetic and potential energy

An object of mass *m* released above the ground

If air resistance is negligible, the object gains speed as it falls. Its potential energy therefore decreases and its kinetic energy increases.

After falling through a vertical height h, its kinetic energy is equal to its loss of potential energy.

In other words: $\tfrac{1}{2}mv^2 = mgh$

A pendulum bob

A pendulum bob is displaced from equilibrium and then released with the thread taut. The bob passes through the equilibrium position at maximum speed, and then slows down to reach maximum height on the other side of equilibrium. If its initial height above the equilibrium position is h_0, then whenever its height above the equilibrium position is h, its speed v at this height is such that:

Its kinetic energy = its loss of potential energy from maximum height

$$\tfrac{1}{2}mv^2 = mgh_0 - mgh$$

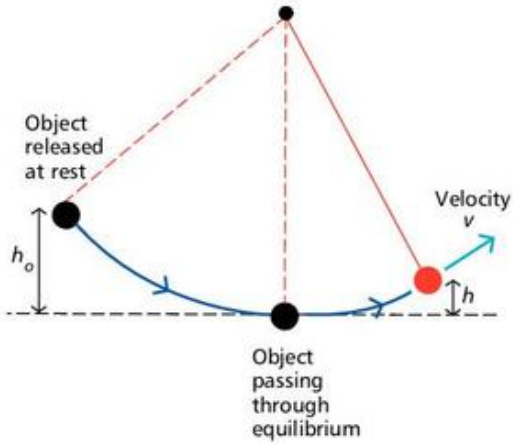


Fig 2 A pendulum in motion

A fairground vehicle of mass *m* on a downward track

If a fairground vehicle was initially at rest at the top of the track, and its speed is v at the bottom of the track, then at the bottom of the track:

- its kinetic energy = $\frac{1}{2}mv^2$,
- its loss of potential energy = mgh, where h is the vertical distance between the top and the bottom of the track,
- the work done to overcome friction and air resistance = $mgh - \frac{1}{2}mv^2$.

Worked example

$g = 9.8\,\text{m s}^{-2}$

On a fairground ride, the track descends by a vertical drop of 55 m over a distance of 120 m along the track. A train of mass 2500 kg on the track reaches a speed of $30\,\text{m s}^{-1}$ at the bottom of the descent, after being at rest at the top. Calculate:

a the loss of potential energy of the train,
b its gain of kinetic energy,
c the average frictional force on the train during the descent.

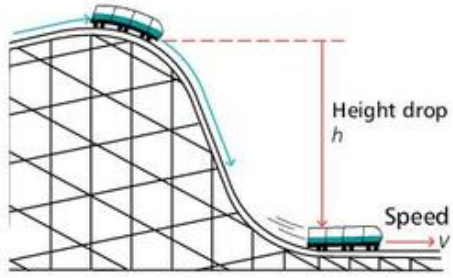


Fig 3

Solution

a Loss of potential energy = $mgh = 2500 \times 9.8 \times 55 = 1.35 \times 10^6\,\text{J}$

b Its gain of kinetic energy = $\frac{1}{2}mv^2 = 0.5 \times 2500 \times 30^2 = 1.13 \times 10^6\,\text{J}$

c Work done to overcome friction $= mg\Delta h - \frac{1}{2}mv^2$
$= 1.35 \times 10^6 - 1.13 \times 10^6$
$= 2.2 \times 10^5\,\text{J}$

Because the work done to overcome friction = frictional force × distance moved along track,

$$\text{the frictional force} = \frac{\text{work done to overcome friction}}{\text{distance moved}} = \frac{2.2 \times 10^5\,\text{J}}{120\,\text{m}} = 1830\,\text{N}$$

QUESTIONS

1 A ball of mass 0.50 kg was thrown directly up at a speed of $6.0\,\text{m s}^{-1}$. Calculate:
- **a** its kinetic energy at $6\,\text{m s}^{-1}$,
- **b** its maximum gain of potential energy,
- **c** its maximum height gain.

2 A ball of mass 0.20 kg, at a height of 1.5 m above a table, is released from rest and it rebounds to a height of 1.2 m above the table. Calculate:
- **a i** the loss of potential energy on descent,
- **ii** the gain of potential energy at maximum rebound height,
- **b** the loss of energy due to the impact.

3 A cyclist of mass 80 kg (including the bicycle) freewheels from rest 500 m down a hill. The foot of the hill is 20 m lower than the cyclist's starting point, and the cyclist reaches a speed of $12\,\text{m s}^{-1}$ at the foot of the hill. Calculate:
- **a i** the loss of potential energy,
- **ii** the gain of kinetic energy of the cyclist and cycle,
- **b i** the work done against friction and air resistance during the descent,
- **ii** the average resistive force during the descent.

4 A fairground vehicle of total mass 1200 kg, moving at a speed of $2\,\text{m s}^{-1}$, descends through a height of 50 m to reach a speed of $28\,\text{m s}^{-1}$ after travelling a distance of 75 m along the track. Calculate:
- **a** its loss of potential energy,
- **b** its initial kinetic energy,
- **c** its kinetic energy after the descent,
- **d** the work done against friction,
- **e** the average frictional force on it during the descent.

2.6 Power

On these pages you will learn to:

- Define power as work done per unit time and derive power as the product of force and velocity
- Solve problems using the relationships $W = Pt$ and $P = Fv$

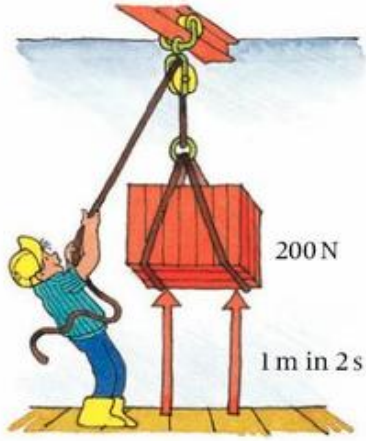


Fig 1 A 100 watt worker

Power and energy

Energy transfers

Energy can be **transferred** from one object to another by means of:

- **Work done** by a force due to one object making the other object move.
- **Heat transfer** from a hot object to a cold object. Heat transfer can be due to conduction, convection or radiation.
- In addition, **electricity, sound waves and electromagnetic radiation**, such as light or radio waves, transfer energy.

In any energy transfer process, the more energy transferred per second, the greater the **power** of the transfer process. For example, in a tall building where there are two elevators of the same total weight, the more powerful elevator is the one that can reach the top floor faster. In other words, its motor transfers energy from electricity at a faster rate than the motor of the other elevator. The energy transferred per second is the **power** of the motor:

Power is defined as the rate of transfer of energy.

The unit of power is the watt (W), equal to an energy transfer rate of 1 joule per second.

Note 1 kilowatt (kW) = 1000 W, and 1 megawatt (MW) = 10^6 W

If energy ΔE is transferred steadily in time t:

$$\textbf{Power, } P = \frac{E}{t}$$

Where energy is transferred by a force doing work, the energy transferred is equal to the work done by the force. Therefore, the **rate of transfer of energy** is equal to the work done per second. In other words, if the force does work W in time t:

$$\textbf{Power, } P = \frac{W}{t}$$

Power measurements

1 Muscle power

Test your own muscle power by timing how long it takes you to walk up a flight of steps. To calculate your muscle power, you will need to know your weight and the total height gain of the flight of steps:

- Your gain of potential energy = your weight × total height gain
- Your muscle power, $P = \dfrac{\text{energy transferred}}{\text{time taken}} = \dfrac{\text{weight} \times \text{height gain}}{\text{time taken}}$

Worked example

A person of weight 480 N, who climbs a flight of stairs of height 10 m in 12 s, has leg muscles of power: 480 N × 10 m/12 s = 400 W

Since each leg does work while the other is being lifted, each leg would have an output power of 400 W.

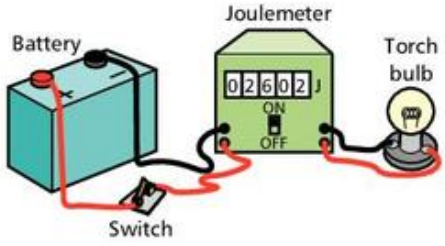


Fig 2 Using a joulemeter

2 Electrical power

The power of a 12 V light bulb can be measured using a joulemeter, as shown in Figure 2. The joulemeter is read before and after the light bulb is switched on. The difference between the readings is the energy supplied to the light bulb. If the light bulb is switched on for a measured time, the power of the light bulb can be calculated from the energy supplied to it/the time taken.

Motive power

Vehicle engines, marine engines and aircraft engines are all designed to make objects move. The output power of an engine is called its **motive** power.

When a powered object moves at a constant velocity at a constant height, the resistive forces (e.g. friction, air resistance, drag) are equal and opposite to the motive force.

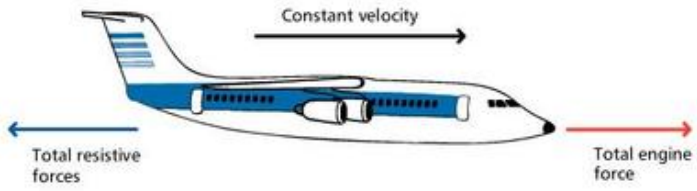


Fig 3 Engine power

The work done by the engine is converted to internal energy of the surroundings by the resistive forces.

For a powered vehicle driven by a constant force F moving at speed v:

Work done per second = force × distance moved per second

$\therefore$ Motive power of the engine, $P = Fv$

Worked example

An aircraft powered by engines that exert a force of 40 kN is in level flight at a constant velocity of 80 m s^{-1}. Calculate the motive power of the engine at this speed.

Solution

Power = force × velocity = 40000 N × 80 m s^{-1} = 3.2 × 10^6 W

When a powered object gains speed, the motive force exceeds the resistive forces on it.

Consider a vehicle that speeds up on a level road. The motive power of its engine is the work done by the engine per second. The work done by the engine increases the kinetic energy of the vehicle and enables the vehicle to overcome the resistive forces acting on it. Because the resistive forces increase the internal energy of the surroundings:

Motive power = energy per second wasted + the gain of kinetic energy per second

Juggernaut physics

The maximum mass of a truck on UK roads must not exceed 44 tonnes, which corresponds to a total mass of 44 000 kg. This limit is set so as to prevent damage to roads and bridges. European Union regulations limit the motive power of a large truck to a maximum of 6 kW per tonne. Therefore, the maximum motive power of a 44 tonne truck is 264 kW. Prove for yourself that a truck with a motive power of 264 kW moving at a constant speed of 31 m s^{-1} (= 70 miles per hour) along a level road experiences a drag force of 8.5 kN.

Fig 4 Heavy goods on the move

QUESTIONS

g = 9.8 m s^{-2}

1 A student of weight 450 N climbed 2.5 m up a rope in 18 s. Calculate:
   - **a** the gain of potential energy of the student,
   - **b** the energy transferred per second.

2 Calculate the power of the engines of an aircraft at a speed of 250 m s^{-1}, if the total engine thrust to maintain this speed is 2.0 MN.

3 A rocket of mass 5800 kg accelerates uniformly and vertically from rest to a speed of 220 m s^{-1} in 25 s. Calculate:
   - **a** its gain of potential energy,
   - **b** its gain of kinetic energy,
   - **c** the power output of its engine, assuming no energy is wasted due to air resistance.

4 Calculate the height through which a 5 kg mass would need to be dropped to lose the same energy as a 100 W light bulb would use in 1 min.

2.7 Efficiency

On these pages you will learn to:

- Recall and understand that the efficiency of a system is the ratio of useful work done by the system to the total energy input
- Show an appreciation for the implications of energy losses in practical devices and use the concept of efficiency to solve problems

Machine power

A machine that lifts or moves an object applies a force to the object to move it. If the machine exerts a force F on an object to make it move through a distance s in the direction of the force, the work done W on the object by the machine can be calculated using the equation

$$\textbf{Work done, } W = Fs$$

If the object moves at a constant velocity v due to this force being opposed by an equal and opposite force caused by friction, the object moves a distance $s = vt$ in time t.

Therefore, the output power of the machine

$$P_{OUT} = \frac{\text{work done by the machine}}{\text{time taken}} = \frac{Fvt}{t} = Fv$$

Output power, $P_{OUT} = Fv$, where F = 'output' force of the machine and v = speed of the object

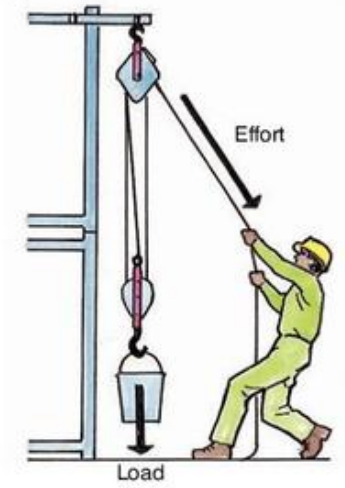


Fig 1 Using pulleys

Examples

1 An electric motor operating a sliding door exerts a force of 125 N on the door, causing it to open at a constant speed of $0.40\,m\,s^{-1}$. The output power is $125\,N \times 0.40\,m\,s^{-1} = 50\,W$. The motor must therefore transfer 50 J every second to the sliding door while the door is being opened.
Friction in the motor bearings and also electrical resistance of the motor wires means that some of the electrical energy supplied to the motor is wasted. For example, if the motor is supplied with electrical energy at a rate of $150\,J\,s^{-1}$ and it transfers $50\,J\,s^{-1}$ to the door, the difference of $100\,J\,s^{-1}$ is wasted as a result of friction and electrical resistance in the motor.

2 A pulley system is used to raise a load of 80 N at a speed of $0.15\,m\,s^{-1}$ by means of a constant '**effort**' of 30 N applied to the system. Figure 1 shows the arrangement. Note that for every metre the load rises, the effort needs to act over a distance of three metres because the load is supported by three sections of rope. The effort must therefore act at a speed of $0.45\,m\,s^{-1}$ ($= 3 \times 0.15\,m\,s^{-1}$).

- The work done on the load each second = load × distance per second $= 80\,N \times 0.15\,m\,s^{-1} = 12\,J\,s^{-1}$
- The work done by the effort each second = effort × distance each second $= 30\,N \times 0.45\,m\,s^{-1} = 13.5\,J\,s^{-1}$

The difference of $1.5\,J\,s^{-1}$ is the energy wasted each second in the pulley system. This is due to friction in the bearings and also because energy must be supplied to raise the lower pulley. For example, if the weight of the lower pulley is 6 N, the potential energy gain each second by the lower pulley would be $0.9\,J\,s^{-1}$ ($= 6\,N \times 0.15\,m\,s^{-1}$) when the load is raised at a speed of $0.15\,m\,s^{-1}$. Thus the energy wasted each second due to friction would be $0.6\,J\,s^{-1}$ ($= 1.5 - 0.9\,J\,s^{-1}$).

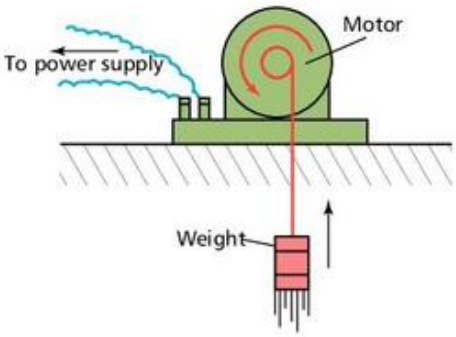


Fig 2 Efficiency

Efficiency measures

Useful energy is energy transferred for a purpose. In any machine, where friction is present, some of the energy transferred by the machine is wasted. In other words, not all the energy supplied to the machine is transferred for the intended purpose. For example, suppose a 500 W electric winch raises a weight of 150 N by 6.0 m in 10 s:

- The electrical energy supplied to the winch is $500\,W \times 10\,s = 5000\,J$

- The useful energy transferred by the machine is the potential energy gain of the load = 150 N × 6 m = 900 J.

Therefore, in this example, 4100 J of energy is wasted.

$$\text{The efficiency of a machine} = \frac{\text{useful energy transferred by the machine}}{\text{energy supplied to the machine}}$$

$$= \frac{\text{work done by the machine}}{\text{energy supplied to the machine}}$$

Notes

- **Percentage efficiency = efficiency × 100%**

In the above example, the efficiency of the machine is therefore 0.18 or 18%.

- Also: $\textbf{Efficiency} = \dfrac{\text{output power of a machine}}{\text{input power to the machine}}$

Wasting energy

In any process or device where energy is transferred for a purpose, the efficiency of the transfer process or the device is the fraction of the energy supplied which is used for the intended purpose. For example:

- A light bulb that is 10% efficient emits 10 J of energy as light for every 100 J of energy supplied to it by electricity. The rest of the energy is wasted as heat.
- An engine that is 45% efficient delivers 45 J of useful energy for every 100 J of energy supplied to it from its fuel. The rest of the energy is wasted as sound and heat.

Is it possible to stop energy being wasted as heat? In a power station, steam is used to drive turbines which turn the electricity generators. If the turbines were not kept cool, they would stop working because the **pressure** inside would build up and prevent steam entering. Stopping the heat transfer to the cooling water would stop the generators working. In general, energy tends to spread out when it is usefully used.

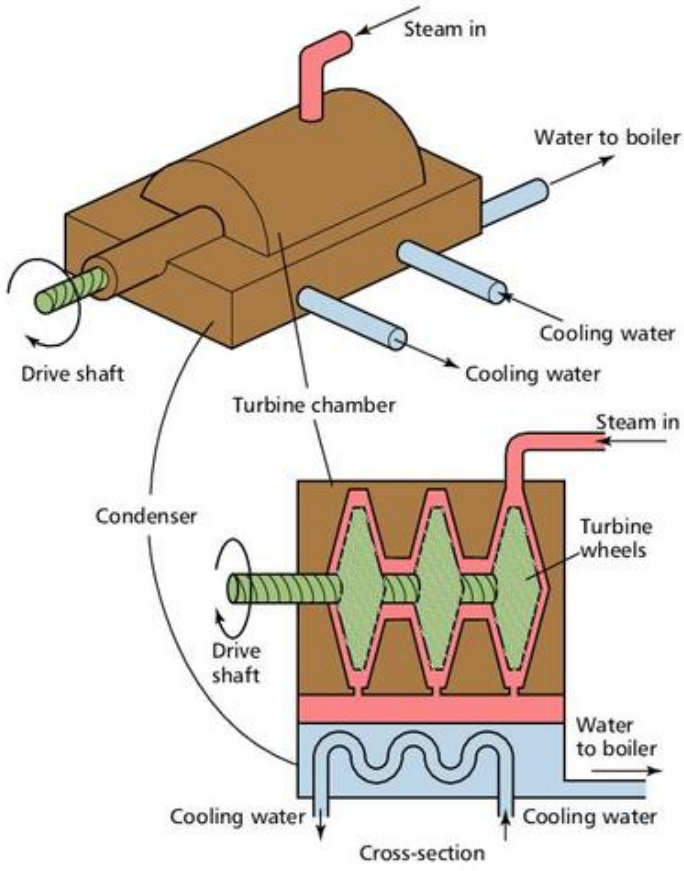


Fig 3 A power station generator

QUESTIONS

1 In a test of muscle efficiency, an athlete on an exercise bicycle pedals against a brake force of 30 N at a speed of 15 m s^{-1}.
   - **a** Calculate the useful energy supplied per second by the athlete's muscles.
   - **b** If the efficiency of the muscles is 25%, calculate the energy per second supplied to the athlete's muscles.

2 A 60 W electric motor raises a weight of 20 N through a height of 2.5 m in 8.0 s. Calculate:
   - **a** the electrical energy supplied to the motor,
   - **b** the useful energy transferred by the motor,
   - **c** the efficiency of the motor.

3 A power station has an overall efficiency of 35% and it produces 200 MW of electrical power. The fuel used in the power station releases 80 MJ/kg of fuel burned. Calculate:
   - **a** the energy per second supplied by the fuel,
   - **b** the mass of fuel burned per day.

4 A vehicle engine has a power output of 6.2 kW and uses fuel which releases 45 MJ/kg when burned. At a speed of 30 m s^{-1} on a level road, the fuel usage of the vehicle is 18 km/kg. Calculate:
   - **a** the time taken by the vehicle to travel 18 km at 30 m s^{-1},
   - **b** the useful energy supplied by the engine in this time,
   - **c** the overall efficiency of the engine.

EXTENSION

2.8 Renewable energy

On these pages you will learn to:

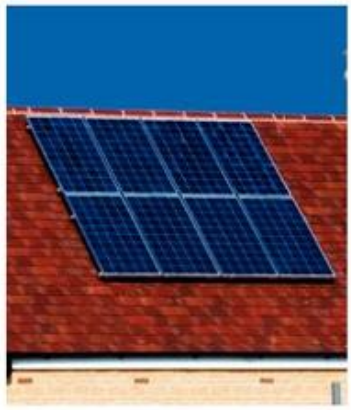
- Recognise energy transfers in renewable energy sources
- Use knowledge about power and energy to evaluate renewable energy sources

Renewable energy sources will contribute increasingly to the world's energy supplies in the future. Almost 90% of the energy we use at present is obtained from fossil fuels. Scientists think that the use of fossil fuels is causing increased global warming due to the increasing amount of carbon dioxide in the atmosphere. Increased global warming could have disastrous consequences (e.g. rising sea levels, changing weather patterns, etc.) in many regions. To cut carbon emissions, many countries are now planning to develop more renewable energy resources and to build new nuclear power stations. A nuclear power station can generate enough electricity for about 200 000 homes. In terms of physics, let's consider how much energy could be supplied from typical renewable energy resources.

Solar power

A solar panel of area 1 m^2 in space would absorb solar energy at a rate of about 1400 J s^{-1} if it absorbed all the incident solar energy. At the Earth's surface, the incident energy would be less because some would be absorbed in the atmosphere. In addition, some of the Sun's energy would be reflected by the panel itself.

Fig 1 (a) A solar heating panel



(b) A solar cell panel

- **A solar heating panel** can heat water running through it to 70°C on a hot sunny day. In hot countries in summer, a 1 m × 3 m solar heating panel can absorb up to 4000 J s^{-1} of solar energy.
- **A solar electric panel** produces electricity directly. A potential difference is produced across each solar cell when light is incident on the cell. A large array of solar cells and plenty of sunshine is necessary to produce useful amounts of power. To make a significant contribution to national needs, millions of homes and buildings would need to be fitted with solar panels.

Wind power

A wind turbine is an electricity generator on a tall tower, driven by large blades pushed round by the force of the wind. A typical modern wind turbine on a suitable site can generate about 2 MW of electrical power. Let's consider how this estimate is obtained.

- The kinetic energy of a mass m of wind moving at speed $v = \frac{1}{2}mv^2$
  $\therefore$ the kinetic energy per unit volume of air from the wind at speed $v = \frac{1}{2}\rho v^2$, where ρ is the density (i.e. mass per unit volume) of air.
- Suppose the blades of a wind turbine sweep out an area A when they rotate. For wind at speed v, a cylinder of air of area A and length v passes every second through the area swept out by the blades. So the volume of air passing per second $= vA$.
  $\therefore$ the kinetic energy per second of the wind passing through a wind turbine $= \frac{1}{2}\rho v^2 vA = \frac{1}{2}\rho v^3 A$.

For a wind turbine with blades of length 20 m, $A = \pi(20)^2 = 1300\,\text{m}^2$

The density of air, $\rho = 1.2\,\text{kg m}^{-3}$

$\therefore$ the power of the wind at $v = 15\,\text{m s}^{-1}$
$= \frac{1}{2} \times 1.2 \times 15^3 \times 1300$
$= 2.6 \times 10^6\,\text{W} = 2.6\,\text{MW}$

The calculation shows that the maximum power output of a large wind turbine at a windy site could not be more than about 2 MW. To generate the same power as a 5000 MW power station, about 2500 wind turbines would need to be constructed and connected to the electricity network.

Water power

Hydroelectricity and tidal power stations both make use of the potential energy released by water when it runs to a lower level.

- A **tidal power station** covering an area of 100 km^2 could trap a depth of 6 m of sea water twice per day. This would mean releasing a volume of $6 \times 10^8\,\text{m}^3$ of sea water over a few hours. The mass of such a volume is about 6×10^{11} kg and if its centre of mass drops through an average height of about 1 m, the potential energy released would be about 6×10^{12} J. Prove for yourself this would give an energy transfer rate of more than 250 MW if released over about 6 hours.

- A **hydroelectric power station** releases less water per second than a tidal power station but the water drops through a much greater height. However, even a large height drop of 500 m with rainfall over an area of 1000 km² at a depth of about 10 mm per day would transfer no more than about 500 MW.

Geothermal energy

Energy released by radioactive substances, deep within the Earth, heats the surrounding rock. As a result, heat or **geothermal energy** is transferred towards the Earth's surface. Geothermal power stations are located in volcanic areas or where there are hot rocks deep below the surface. Water gets pumped down to these rocks to produce steam. Then the steam produced drives electricity turbines at ground level. In some areas, buildings can be heated using geothermal energy directly. Heat flow from underground is called **ground heat**. It can be used to heat water which is then pumped round the building.

Fig 2 A geothermal power station

Biofuels and biomass

Methane from cows or animal manure and from sewage works, decaying rubbish and other sources can be used in small-scale gas-fired power stations. Methane is an example of a **biofuel** which is any fuel obtained from **biomass**, the word we use for biological material from living or recently living organisms such as animal waste. Other biofuels include ethanol from sugar cane, straw, nutshells and woodchip. Ethanol may be used as a fuel for road vehicles.

Fig 3 Using biofuel to generate electricity

Renewable energy overview

A 600 MW power station can supply electricity to about 100 000 homes. Could renewable energy be used instead?

- 25 000 homes each with a 4 kW solar panel could produce 100 MW
- 1 offshore wind farm with about 100 wind turbines could produce about 100 MW,
- 1 tidal power station or hydroelectric power station could produce about 100 MW

The renewable energy resources available in any given area vary from one area to another. Also, renewables such as wind and wave power are unreliable. However, one or more of the renewable energy resources discussed above could make a significant contribution to energy supplies in many areas. For example, solar panels, biofuels and wind farms could reduce fossil fuel dependency in hot coastal regions.

QUESTIONS

1 A solar cell panel of area 1 m² can produce 200 W of electrical power on a sunny day. Calculate the area of panels that would be needed to produce 2000 MW of electrical power.

2 The maximum power that can be obtained from a wind turbine is proportional to the cube of the wind speed. When the wind speed is 10 m s^{-1}, the power output of a certain wind turbine is 1.2 MW. Calculate the power output of this wind turbine when the wind speed is 15 m s^{-1}.

3 A hydroelectric power station produces electrical power at an overall efficiency of 25%. The power station is driven by water that has descended from an upland reservoir 650 m above the power station. Calculate the volume of water passing through the power station per second when it produces 200 MW of electrical power.

The density of water = 1000 kg m^{-3}.

4 At a tidal power station, water is trapped over an area of 200 km² when the tide is 3.0 m above the power station turbines. The trapped water is released gradually over a period of 6 hours. Calculate **a** the mass of trapped water, **b** the average loss of potential energy per second of this trapped water when it is released over a period of 6 hours. The density of sea water = 1050 kg m^{-3}.

2 End of chapter questions

1 **a** Explain why a force is needed to make an object move at a steady speed across a rough floor.

b Explain why an object sliding across ice keeps moving even though no force is pushing it.

2 A vehicle of mass 450 kg accelerates from rest to a velocity of 9.5 m s^{-1} in 38 s.

Fig 2.1

Calculate:

a the acceleration of the vehicle,

b the motive force of the vehicle engine, assuming zero resistive force,

c the ratio of the motive force to the vehicle's weight.

3 A bullet of mass 2.4 × 10^{-3} kg moving at a speed of 115 m s^{-1} hits a tree and embeds itself in the tree to a depth of 85 mm.

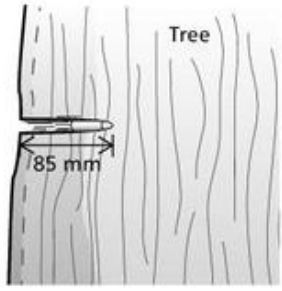


Fig 3.1

Calculate:

a the impact time,

b the deceleration of the bullet in the tree,

c the impact force on the bullet.

4 A heavy goods vehicle of total weight 320 kN has an engine which has a maximum motive force of 6 kN.

a Calculate:

i the mass of the vehicle,

ii the maximum acceleration of the vehicle,

iii the time taken for the vehicle to accelerate from rest to a speed of 20 m s^{-1},

iv the distance moved by the vehicle in this time.

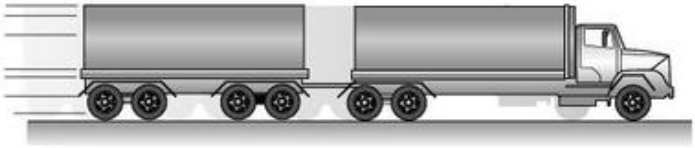


Fig 4.1

b The vehicle is used to pull a trailer of weight 60 kN. Calculate:

i the maximum acceleration of the vehicle when the trailer is attached,

ii the time taken by the vehicle to reach a speed of 20 m s^{-1} from rest with the trailer attached.

5 A steel ball bearing was released in water and fell to the bottom of the water container. The time of descent was measured at different distances as it fell. The figure below shows the measurements plotted as a graph of distance fallen against time taken.

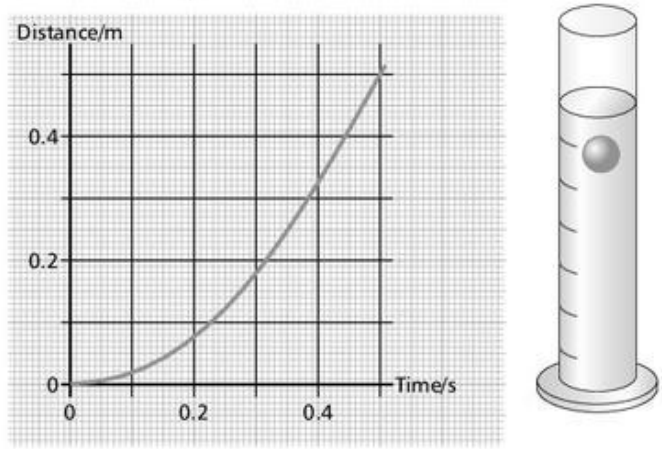


Fig 5.1

a **i** What feature of this graph represents the speed of the ball bearing at any instant?

ii Describe how the speed of the ball bearing changed as it fell.

iii Sketch a graph to show how the speed of the ball bearing changed with time during its descent.

b Explain, in terms of the forces acting on the ball, why the acceleration of the ball bearing decreased as it fell.

6 **a** Calculate the kinetic energy of a 96 kg rugby player running at a speed of 6.0 m s^{-1}.

b What speed would a 2.4 kg rugby ball need to be moving at to have the same kinetic energy as the rugby player in part **a**?

c What would be the height gain of the 2.4 kg rugby ball in part **b**, if all its kinetic energy was converted into potential energy?

7 A brick, of mass 2.5 kg, falls off the top of a wall and hits the ground 2.5 m below.
Calculate:

a the loss of potential energy of the brick,

b the kinetic energy of the brick just before impact,

c the speed of the brick just before impact.

8 A ball, of mass 0.40 kg, released from a height of 1.2 m above a horizontal concrete floor hits the floor and rebounds to a maximum height of 0.80 m.

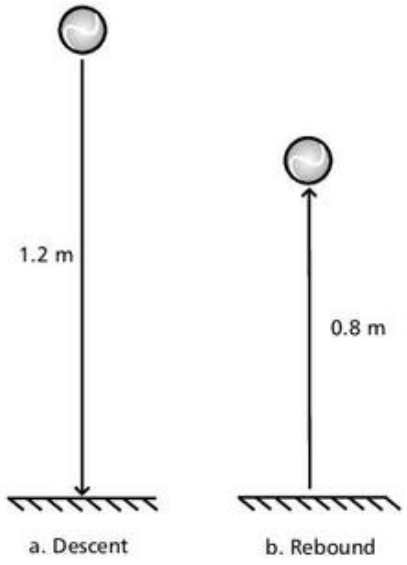


Fig 8.1

a Calculate:
- **i** the loss of potential energy of the ball just before hitting the floor,
- **ii** the kinetic energy of the ball just before impact,
- **iii** the speed of the ball just before impact.

b Calculate:
- **i** the maximum gain of potential energy of the ball after impact,
- **ii** the kinetic energy of the ball just after impact,
- **iii** the speed of the ball just after impact.

9 An aircraft, of mass 2200 kg, takes off from an aircraft carrier and reaches a speed of 85 m s^{-1} in 20 s at a height of 320 m above the aircraft carrier.

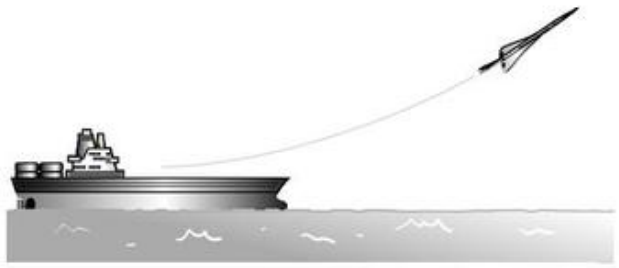


Fig 9.1

a Calculate:
- **i** the gain of potential energy of the aircraft,
- **ii** its gain of kinetic energy.

b i Estimate the average output power of the aircraft's engines in this time.

ii If the aircraft's fuel has an energy value of 50 MJ kg^{-1} and the engines have an efficiency of 30%, estimate the mass of fuel burned in the 20 s taken to reach a height of 320 m.

10 A car, of mass 1200 kg, accelerates at a steady rate of 1.5 m s^{-2} along a straight level road.

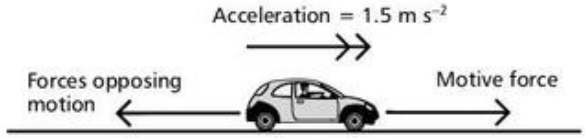


Fig 10.1

a Calculate the resultant force acting on the car.

b The total force opposing the motion of the car is 500 N when the speed of the car is 7.0 m s^{-1}. Calculate the motive force of the engine at this speed when the acceleration is 1.5 m s^{-2}.

c Calculate:
- **i** the output power of the engine,
- **ii** the power wasted due to the forces opposing the motion of the car when the car's speed is 7.0 m s^{-1}.

d Account for the difference between your answers to parts **c i** and **ii**.

11 A sky-diver jumps from a high-altitude balloon.

a Explain briefly why the acceleration of the sky-diver
- **i** decreases with time, *(2 marks)*
- **ii** is 9.8 m s^{-2} at the start of the jump. *(1 mark)*

b The variation with time t of the vertical speed v of the sky-diver is shown in Figure 11.1.

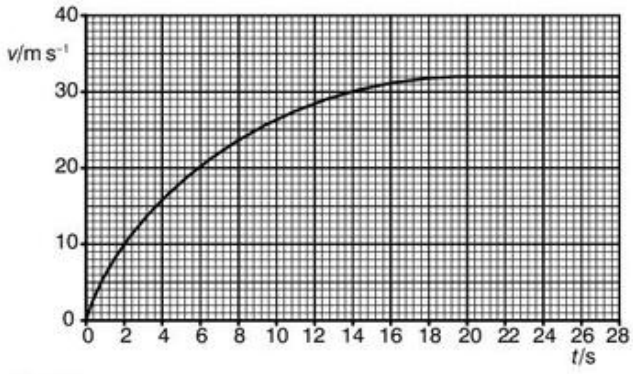


Fig 11.1

Use the diagram to determine the magnitude of the acceleration of the sky-diver at time t = 6.0 s.

(3 marks)

c The sky-diver and his equipment have a total mass of 90 kg.

i Calculate, for the sky-diver and his equipment,
- **1** the total weight, in N *(1 mark)*
- **2** the accelerating force, in N, at time t = 6.0 s. *(1 mark)*

ii Use your answers in **i** to determine the total resistive force, in N, acting on the sky-diver at time t = 6.0 s *(1 mark)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 21 Q2 November 2009

3 Forces in equilibrium

3.1 Balanced forces

On these pages you will learn to:

- Recognise different forces and represent them on free-body force diagrams
- Recognise the forces acting in simple equilibrium situations
- Resolve a force into two perpendicular components

Force as a vector

At this moment, at least two forces are acting on you or on any object near you. In addition to the force of the Earth's gravitational field, one or more support forces will be acting on you preventing you from accelerating downwards. The support force on an object may be due to other solid objects under it and/or cables suspended above it. Other types of forces that can act on objects include:

- electrostatic forces which act on charged objects in an electric field
- magnetic forces which act on magnets and electromagnets in a magnetic field
- friction forces which oppose the motion of objects when they slide over each other
- viscous forces which oppose the motion of objects moving through fluids
- upthrust forces which help objects to float.

Whatever the forces acting on an object are, they can be represented by **vectors** in a vector diagram. A vector has **magnitude** and **direction**. The two key rules for dealing with vectors are explained in topic 1.1:

- A force F can be resolved into two perpendicular components: $F\cos\theta$ and $F\sin\theta$ parallel and perpendicular to a line at angle θ to the line of action of the force.
- Two forces can be added together using a scale diagram or by using a calculator and Pythagoras' theorem after resolving one of the forces into components parallel and perpendicular to the other force. See topic 1.1 if necessary.

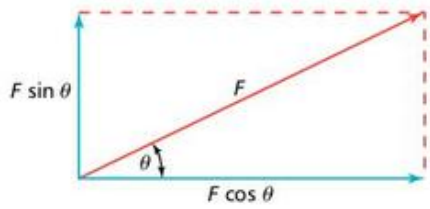


Fig 1 Resolving a force

Equilibrium of a point object

When **two forces** act on a point object, the object is in equilibrium (i.e. at rest or moving at constant velocity) only if the two forces are equal and opposite to each other. The resultant of the two forces is therefore zero. The two forces are said to be **balanced**. For example, an object resting on a surface is acted on by its weight W (i.e. the force of gravity on it) acting downwards and a support force S from the surface acting upwards. Hence S is equal and opposite to W, provided the object is at rest (or moving at constant velocity) i.e. $S = W$.

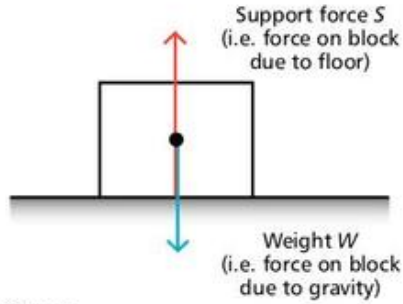


Fig 2 Balanced forces

When **three forces** act on a point object, their combined effect (i.e. resultant) is zero only if the resultant of any two of the forces is equal and opposite to the third force. One way of checking if the combined effect of the three forces is zero involves:

- resolving each force along the same parallel and perpendicular lines,
- balancing the components along each line.

Example 1

A child of weight W on a swing is at rest, due to the swing seat being pulled to the side by a horizontal force F_1. The rope is then at an angle θ to the vertical, as shown in Figure 4.

Assuming the swing seat is of negligible weight, the swing seat is acted on by three forces: the weight of the child W, the horizontal force F_1 and the tension T in the rope. Resolving the tension T vertically and horizontally gives $T\cos\theta$ for the vertical component of T (which is upwards) and $T\sin\theta$ for the horizontal component of T. Therefore, the balance of forces:

- Horizontally: $F_1 = T\sin\theta$
- Vertically: $W = T\cos\theta$

Because $\sin^2\theta + \cos^2\theta = 1$ (see p 4), then

$$F_1^2 + W^2 = T^2\sin^2\theta + T^2\cos^2\theta = T^2$$

$$\therefore \quad T^2 = F_1^2 + W^2$$

Also, because $\tan\theta = \frac{\sin\theta}{\cos\theta}$, then $\frac{F_1}{W} = \frac{T\sin\theta}{T\cos\theta} = \tan\theta$

$$\therefore \quad \tan\theta = \frac{F_1}{W}$$

Example 2

An object of weight W at rest on a rough slope is acted on by a frictional force F, which prevents it sliding down the slope, and a support force S from the slope perpendicular to the slope.

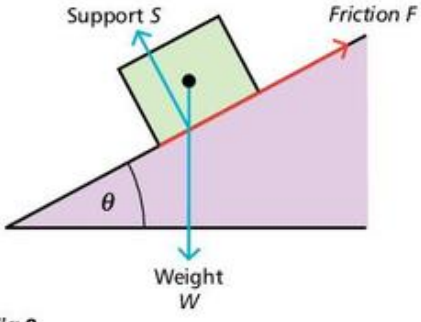


Fig 3

Resolving the three forces parallel and perpendicular to the slope gives:

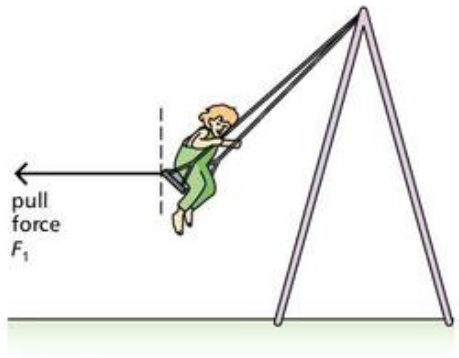
- Parallel: $F = W\sin\theta$
- Perpendicular: $S = W\cos\theta$

Because $\sin^2\theta + \cos^2\theta = 1$, then $F^2 + S^2 = W^2\sin^2\theta + W^2\cos^2\theta = W^2$

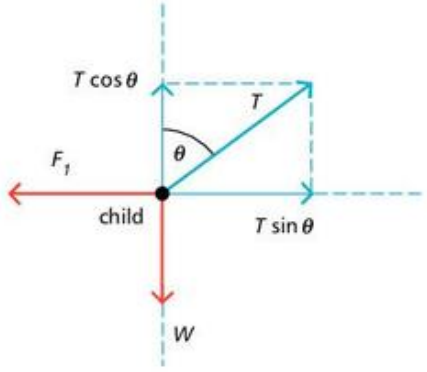
$$\therefore \quad W^2 = F^2 + S^2$$

Also, because $\tan\theta = \frac{\sin\theta}{\cos\theta}$ then $\frac{F}{S} = \frac{W\sin\theta}{W\cos\theta} = \tan\theta$

$$\therefore \quad \tan\theta = \frac{F}{S}$$



a On a swing



b Force diagram

Fig 4

QUESTIONS

1 A point object of weight 6.2 N is acted on by a horizontal force of 3.8 N:

a Calculate the resultant of these two forces.

b Determine the magnitude and direction of a third force acting on the object for it to be in equilibrium.

2 A small object of weight 5.4 N is at rest on a rough slope which is at an angle of 30° to the horizontal:

a Sketch a diagram and show the three forces acting on the object.

b Calculate: **i** the frictional force on the object, **ii** the support force from the slope on the object.

3 An archer pulled a bow string back until the two halves of the string were at 140° to each other (Figure 5). The force needed to hold the string in this position was 95 N.

Calculate:

a the tension in each part of the bow string in this position,

b the resultant force on an arrow at the instant the bow string is released from this position.

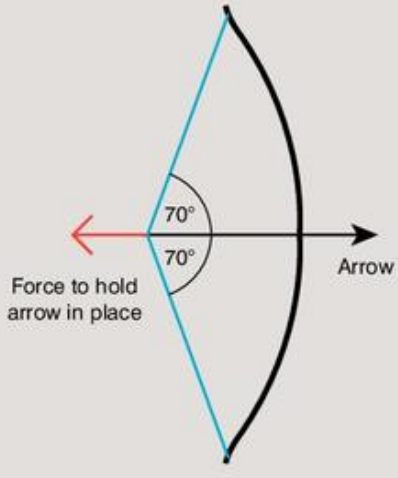


Fig 5

4 An elastic string is stretched horizontally between two fixed points, 0.80 m apart. An object of weight 4.0 N is suspended from the mid-point of the string, causing the mid-point to drop a distance of 0.12 m. Calculate:

a the angle of each part of the string to the vertical,

b the tension in each part of the string.

3.2 The Principle of Moments

On these pages you will learn to:

- Define and apply the moment of a force
- Know that the weight of a body may be taken as acting at a single point known as its centre of gravity
- State and apply the principle of moments to objects balanced on a single pivot

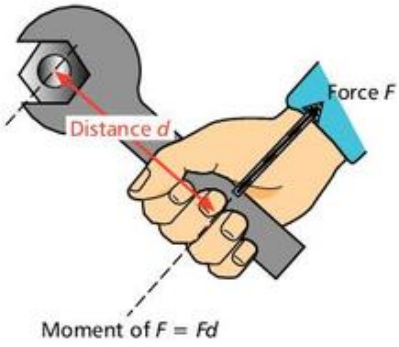


Fig 1 A turning force

Turning effects

Whenever you use a lever or a spanner, you are using a force to turn an object about a **pivot**. For example, if you use a spanner to loosen a wheel nut on a bicycle, you need to apply a force to the spanner to make it turn about the wheel axle. The effect of the force depends on how far it is applied from the wheel axle. The longer the spanner, the less force needed to loosen the nut. However, if the spanner is too long and the nut is too tight, the spanner could snap if too much force is applied to it.

The moment of a force about any point is the force × the perpendicular distance from the line of action of the force to the point. The unit of the moment of a force is the newton metre (Nm).

For a force F acting along a line of action at perpendicular distance d from a certain point:

$$\text{Moment of force} = Fd$$

Notes

- The greater the distance d, the greater the moment.
- The distance d is the **perpendicular** distance from the line of action of the force to the point (Figure 1).

Worked example

A spanner of length 0.24 m is used to tighten a wheel nut. The moment of the force must not exceed 60 N m, otherwise the wheel nut is damaged. Calculate the maximum force that should be applied to the spanner to tighten this nut.

Solution

Moment = force × distance

$$\therefore \quad F \times 0.24 = 60$$

$$F = \frac{60}{0.24} = 250\,\text{N}$$

The Principle of Moments

An object that is not a point object is referred to as a **body**. Any such object turns if a force is applied to it anywhere other than through its centre of gravity. If a body is acted on by more than one force and it is in equilibrium, the turning effects of the forces must balance out. In more formal terms, considering the moments of the forces about any point, for equilibrium:

Sum of the clockwise moments = sum of the anticlockwise moments

This statement is known as the **Principle of Moments.**

The balanced metre rule

1 Consider a uniform metre rule balanced on a pivot at its centre, supporting weights W_1 and W_2 suspended from the rule on either side of the pivot (Figure 2).

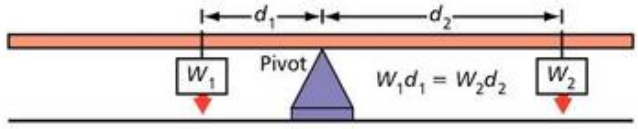


Fig 2 The Principle of Moments

- Weight W_1 provides an anticlockwise moment about the pivot of W_1d_1, where d_1 is the distance from the line of action of the weight to the pivot.
- Weight W_2 provides a clockwise moment about the pivot of W_2d_2, where d_2 is the distance from the line of action of the weight to the pivot.

For equilibrium, applying the Principle of Moments:

$$W_1d_1 = W_2d_2$$

2 If a third weight W_3 is suspended from the rule on the same side of the pivot as W_2 at distance d_3 from the pivot, then the rule can be rebalanced by increasing distance d_1.

At this new distance d_1' for W_1:

$$W_1d_1' = W_2d_2 + W_3d_3$$

Centre of gravity

A tight-rope walker and a waiter serving drinks know just how important the centre of gravity of an object can be. One slight off-balance movement can be catastrophic.

The centre of gravity of a body is the point where the weight of the body may be considered to act.

Centre of gravity tests

- Support a ruler at its centre on the end of your finger. The centre of gravity of the ruler is directly above the point of support. Tip the ruler too much and it falls off, because the centre of gravity is no longer above the point of support.
- Balance a postcard on the end of a pencil. The centre of gravity of the postcard is directly above the point of support when the card is balanced. You should find that the centre of gravity of the card is at the mid-point of the card where the two diagonals cross.
- Find the centre of gravity of a triangular card. Figure 3 shows how to do this. It should be possible to balance the card at its centre of gravity on the end of a pencil.

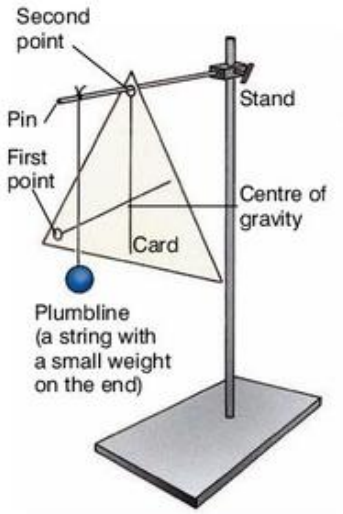


Fig 3 A centre of gravity test

Calculating the weight of a metre rule

- Locate the centre of gravity of a metre rule by balancing it horizontally on a horizontal knife edge. Note the position of the centre of gravity. The rule is said to be **uniform** if its centre of gravity is exactly at the middle of the rule.
- Balance the metre rule off-centre on a knife edge, using a known weight W_1 as shown in Figure 4. The position of the known weight needs to be adjusted gradually until the rule is exactly horizontal.

At this position:

- The known weight W_1 provides an anti-clockwise moment about the pivot of W_1d_1, where d_1 is the perpendicular distance from the line of action of W_1 to the pivot.
- The weight of the rule W_0 provides a clockwise moment of W_0d_0, where d_0 is the distance from the centre of gravity of the rule to the pivot.
- Applying the Principle of Moments:

$$W_0d_0 = W_1d_1$$

By measuring distance d_0 and d_1, the weight W_0 of the rule can therefore be calculated.

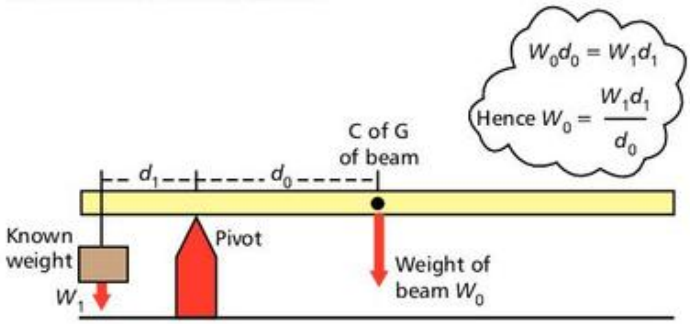


Fig 4 Finding the weight of a beam

QUESTIONS

1 A child of weight 200 N sits on a seesaw at a distance of 1.2 m from the pivot at the centre. The seesaw is balanced by a second child sitting on it at a distance of 0.8 m from the centre. Calculate the weight of the second child.

2 A uniform metre rule pivoted at its centre of gravity supports a 3.0 N weight at its 5.0 cm mark, a 2.0 N weight at its 25 cm mark and a weight W at its 80 cm mark.

a Sketch a diagram to represent this situation.

b Calculate the weight W.

3 In question **2**, the 3.0 N weight and the 2.0 N weight are swapped with each other. Sketch the new arrangement and work out the new distance of weight W from the pivot.

4 A uniform metre rule supports a 4.5 N weight at its 100 mm mark. The rule is balanced horizontally on a horizontal knife edge at its 340 mm mark. Sketch the arrangement and calculate the weight of the rule.

3.3 More on moments

On these pages you will learn to:

- State and apply the principle of moments to objects balanced on two supports
- Show an understanding that a couple is a pair of forces that tends to produce rotation only
- Define and apply the torque of a couple

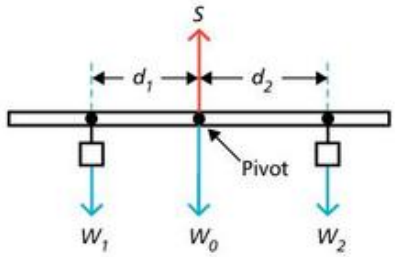


Fig 1 Support forces

Support forces

Single-support problems

When an object in equilibrium is supported at one point only, the support force on the object is equal and opposite to the total downward force acting on the object. For example, in Figure 1, a uniform rule is balanced on a knife edge pivot at its centre of gravity with two additional weights W_1 and W_2 attached to the rule. The support force S on the rule from the knife edge must be equal to the total downward weight. Therefore:

$$S = W_1 + W_2 + W_0$$

where W_0 is the weight of the rule.

As explained in topic 3.2, taking moments about the knife edge gives:

$$W_1 d_1 = W_2 d_2$$

Taking moments about a different point

Let's consider moments about the point where W_1 is attached to the rule:

$$\text{Sum of the clockwise moments} = W_0 d_1 + W_2 (d_1 + d_2)$$

$$\text{Sum of the anticlockwise moments} = S d_1 = (W_1 + W_2 + W_0) d_1$$

$$\therefore \quad (W_1 + W_2 + W_0)\, d_1 = W_0 d_1 + W_2 (d_1 + d_2)$$

Multiplying out the brackets gives:

$$W_1 d_1 + W_2 d_1 + W_0 d_1 = W_0 d_1 + W_2 d_1 + W_2 d_2$$

which simplifies to become: $\mathbf{W_1 d_1 = W_2 d_2}$

This is the same as the equation obtained by taking moments about the pivot. So moments can be taken about **any point**. It makes sense therefore to choose a point which one or more unknown forces act through, as such forces have **zero moment** about this point.

Two-support problems

Consider a uniform beam supported on two pillars X and Y, which are at distance D apart. The weight of the beam is 'shared' between the two pillars according to how far the beam's centre of gravity is from each pillar. For example:

- If the centre of gravity of the beam is mid-way between the pillars, the weight of the beam is 'shared' equally between the two pillars. In other words, the support force on the beam from each pillar is equal to half the weight of the beam.
- If the centre of gravity of the beam is at distance d_x from pillar X and distance d_y from pillar Y, as shown in Figure 2, then taking moments:

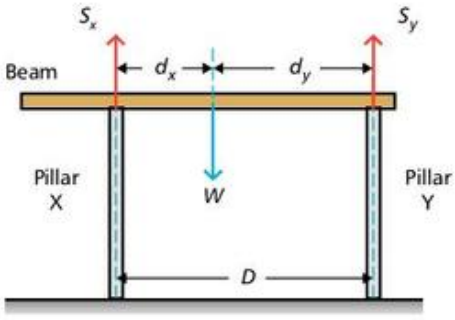


Fig 2 A two-support problem

1 Where X is in contact with the beam:
$S_y D = W d_x$, where S_y is the support force from pillar Y

$$\text{so } \mathbf{S_y} = \frac{\boldsymbol{W d_x}}{\boldsymbol{D}}$$

2 Where Y is in contact with the beam:
$S_x D = W d_y$, where S_x is the support force from pillar X

$$\text{so } \boldsymbol{S_x} = \frac{\boldsymbol{W d_y}}{\boldsymbol{D}}$$

Therefore, if the centre of gravity is closer to X than to Y, $d_x < d_y$ so $S_y < S_x$.

Worked example

A uniform beam of length 5.0 m and weight 120 N rests horizontally on the tops of two walls, X and Y (Figure 3), with its centre of gravity at a distance of 2.0 m from wall X and 1.5 m from wall Y. Calculate the support force on the beam from each wall.

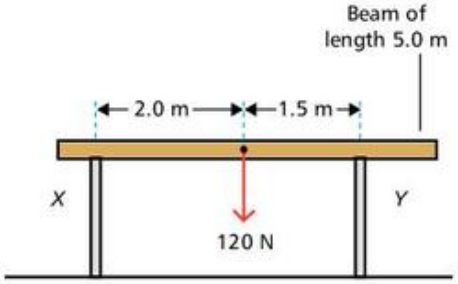


Fig 3

Solution

Let S_x and S_y represent the support forces at X and Y.

$$\therefore \quad S_x + S_y = 120\,\text{N}$$

Taking moments about X gives:

Sum of the clockwise moments
= weight of beam × distance from centre of gravity to X
= 120 N × 2.0 m = 240 N m

Sum of the anticlockwise moments
= support force S_y × distance from X to Y
= S_y × 3.5 m

∴ Applying the Principle of Moments gives:

$$3.5S_y = 240$$

$$S_y = \frac{240}{3.5} = \mathbf{69\,N}$$

Therefore $\quad S_x = W - S_y = 120 - 69 = \mathbf{51\,N}$

Couples

A **couple** is a pair of equal and opposite forces acting on a body, but not along the same line. Figure 4 shows a couple acting on a coil. The couple turns, or tries to turn, the coil.

The moment of a couple is referred to as a **torque:**

Torque of a couple = one force × perpendicular distance between lines of action of the forces

The total moment is the same, regardless of the point about which the moments are taken.

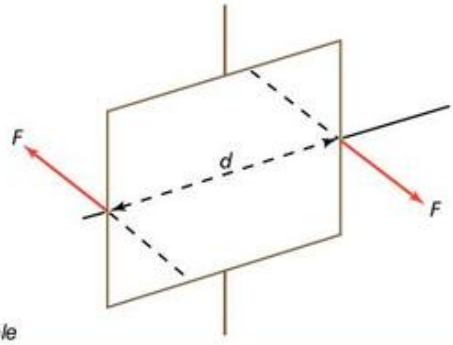


Fig 4 A couple

QUESTIONS

1 A metre rule of weight 1.2 N rests horizontally on two knife edges at the 100 mm mark and the 800 mm mark. Sketch the arrangement and calculate the support force on the rule due to each knife edge.

2 A uniform beam of weight 230 N and of length 10 m rests horizontally on the tops of two brick walls 8.5 m apart, such that a length of 1.0 m projects beyond one wall and 0.5 m projects beyond the other wall (Fig 5).

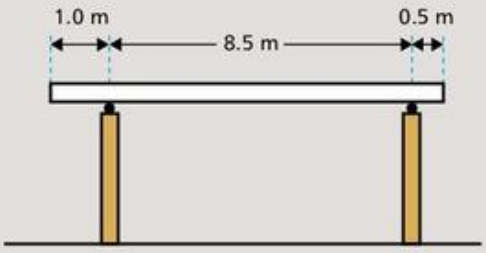


Fig 5

Calculate:

a the support force of each wall on the beam,

b the force of the beam on each wall.

3 A uniform bridge span of weight 1200 kN and of length 17.0 m rests on a support of width 1.0 m at either end. A stationary lorry of weight 60 kN is the only object on the bridge. Its centre of gravity is 3.0 m from the centre of the bridge (Fig 6). Calculate the support force on the bridge at each end.

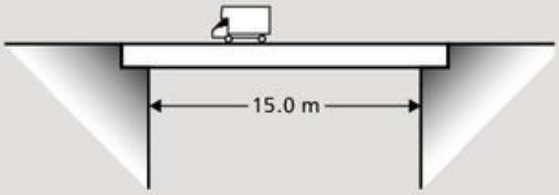


Fig 6

4 A uniform plank of weight 150 N and of length 4.0 m rests horizontally on two bricks. One of the bricks is at the end of the beam. The other brick is 1.0 m from the other end of the plank.

a Sketch the arrangement and calculate the support force on the plank from each brick.

b A child stands on the free end of the beam and just causes the other end to lift off its support.

Sketch this arrangement and calculate the weight of the child.

3.4 Stability

On these pages you will learn to:

- Recognise stable and unstable equilibrium situations
- Recognise how the stability of an object at rest on a flat surface is affected when the object or the surface is tilted

Stable and unstable equilibrium

Stable equilibrium

If a body in **stable equilibrium** is displaced then released, it returns to its equilbrium position. For example if an object such as a coat hanger hanging from a support is displaced slightly, it swings back to its equilibrium position.

Why does an object in stable equilbrium return to equilibrium when it is displaced then released?

- The reason is that the centre of gravity of the object is directly below the point of support when the object is at rest. The support force and the weight are directly equal and opposite to each other when the object is **in equilibrium**.
- However, when it is displaced then released, at the instant of release, the line of action of the weight no longer passes through the point of support so the weight **returns the object to equilibrium**.

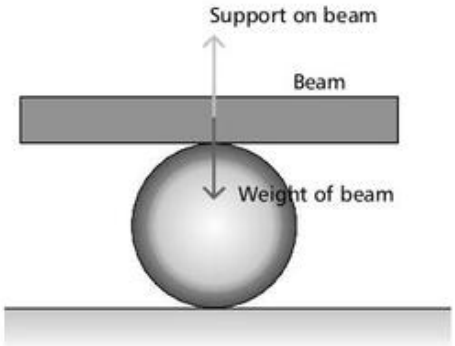


Fig 1 Unstable equilibrium

Unstable equilibrium

A plank balanced on a drum is in **unstable equilibrium**. If it is displaced slightly from equilibrium then released, the plank will roll off the drum.

- The reason is that the centre of gravity of the plank is directly above the point of support when it is **in equilibrium**. The support force is exactly equal and opposite to the weight.
- If the plank is displaced slightly, the centre of gravity is no longer above the point of support. The weight therefore acts to turn the plank **further from the equilibrium position**.

Tilting and toppling

Skittles at a bowling alley are easy to knock over because they are top-heavy. This means that the centre of gravity is too high and the base is too narrow. A slight nudge from a ball causes a skittle to tilt then tip over.

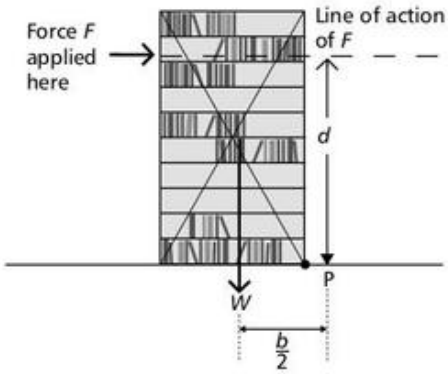


Fig 2 Tilting

Tilting

Tilting is where an object at rest on a surface is acted on by a force that raises it up on one side. For example, if a horizontal force is applied to the top of a tall free-standing bookcase, the force can make the bookcase tilt about its base along one edge.

In Figure 2, to make the bookcase tilt, the force must turn it clockwise about point P. The entire support from the floor acts at point P. The weight of the bookcase provides an anticlockwise moment about P.

- The **clockwise** moment of F about P is Fd, where d is the perpendicular distance from the line of action of F to the pivot.
- The **anticlockwise** moment of W about P is $\frac{Wb}{2}$, where b is the width of the base.

Therefore, for tilting to occur: $\boldsymbol{Fd > \frac{Wb}{2}}$

Toppling

A tilted object will **topple** over if it is tilted too far. For example, a tractor on a hill could topple over sideways if the hill is too steep. If an object on a flat surface is tilted more and more, the line of action of its weight (which is through its

centre of gravity) passes closer and closer to the 'pivot'. If the object is tilted so much that the line of action of its weight passes beyond the pivot, the object will topple over if allowed to. The position where the line of action of the weight passes through the 'pivot' is the furthest it can be tilted without toppling. Beyond this position, it topples over if it is released. See Figure 3.

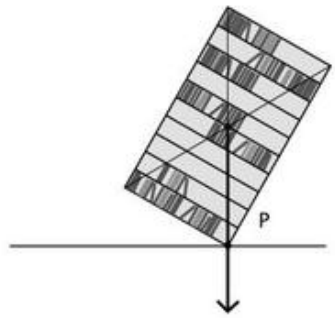


Fig 3 Toppling over

On a slope

A tall object on a slope will topple over if the slope is too great. For example, a high-sided vehicle on a road with a sideways slope will tilt over. If the slope is too great, the vehicle will topple over. This will happen if the line of action of the weight (passing through the centre of gravity of the object) lies outside the **wheel base** of the vehicle. In Figure 4, the vehicle will not topple over because the line of action of the weight lies within the wheel base.

Consider the forces acting on the vehicle on a slope when it is at rest. The sideways friction F, the support forces S_x and S_y and the force of gravity on the vehicle (i.e. its weight) act as shown in Figure 4. For equilibrium, resolving the forces parallel and perpendicular to the slope gives:

- **Parallel** to the slope:
$$F = W\sin\theta$$
- **Perpendicular** to the slope:
$$S_x + S_y = W\cos\theta$$

Note that S_x is greater than S_y because X is lower than Y.

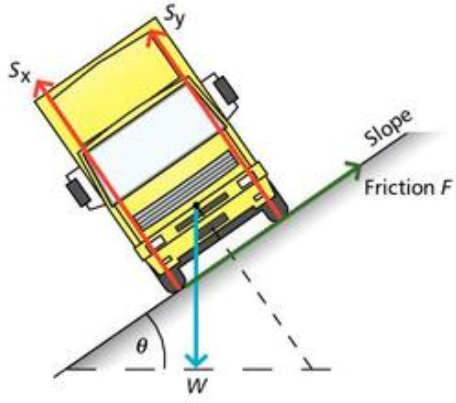


Fig 4 On a slope

QUESTIONS

1 Explain why a bookcase with books on its top shelf only is less stable than if the books were on the bottom shelf.

2 An empty wardrobe of weight 400 N has a square base 0.8 m × 0.8 m and a height of 1.8 m. A horizontal force is applied to the top edge of the wardrobe to make it tilt. Calculate the force needed to lift the wardrobe base off the floor along one side (Figure 5).

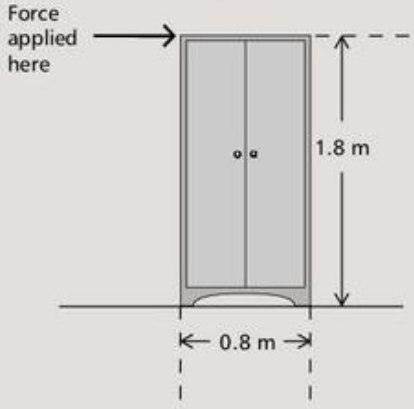


Fig 5

3 A vehicle has a wheel base of 1.8 m and a centre of gravity, when unloaded, which is 0.8 m from the ground (Figure 6).

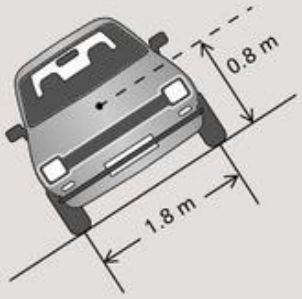


Fig 6

a The vehicle is tested for stability on an adjustable slope. Calculate the maximum angle of the slope to the horizontal if the vehicle is not to topple over.

b If the vehicle carries a full load of people, will it be more or less likely to topple over on a slope? Explain your answer.

4 Discuss whether or not a high-sided heavy goods lorry is more or less likely to be affected by strong side winds when it is fully loaded compared to when it is empty.

3.5 Equilibrium rules

On these pages you will learn to:

- Use a vector triangle to represent coplanar forces in equilibrium
- Recall that when a system is in equilibrium, there is no resultant force and no resultant torque acting on it
- Apply the above general conditions for the equilibrium of a body

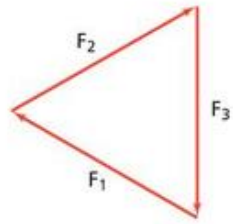


Fig 1 Triangle of forces

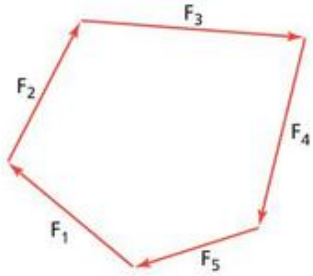


Fig 2 The closed polygon

The triangle of forces

For a point object acted on by **three forces** to be in equilibrium, the three forces must give an overall **resultant of zero**. The three forces as vectors should form a **triangle**. In other words, for three forces F_1, F_2 and F_3 to give zero resultant:

$$\textbf{Vector sum, } F_1 + F_2 + F_3 = 0$$

Any two of the forces gives a resultant which is represented by the third side of the triangle. Therefore, for equilibrium, the third force must be represented by the third side of the triangle. For example, the resultant of $F_1 + F_2$ is equal and opposite to F_3:

i.e. $$F_1 + F_2 = -F_3$$

The sine rule can be applied to the triangle of forces to find an unknown force or angle, given the other forces and angles in the triangle:

$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

where θ_1, θ_2 and θ_3 are the angles opposite sides F_1, F_2 and F_3 respectively.

The closed polygon

The triangle of forces rule can be extended for any number of forces acting on an object. If the object is in equilibrium, the force vectors drawn end-to-end must form a closed polygon. In other words, the tip of the last force vector must join the tail of the first force vector. Figure 2 shows the idea. Unfortunately, the sine rule can't be applied here; so the forces must be resolved in the same parallel and perpendicular directions to calculate an unknown force, given all the other forces.

The conditions for equilibrium of a body

Free-body force diagrams

When two objects interact, they always exert equal and opposite forces on one another. A diagram showing the forces acting on an object can become very complicated if it also shows the forces the object exerts on other objects. A **free-body force diagram** shows only the forces acting on the object.

Equilibrium

An object in equilbrium is either at rest or it moves with a constant velocity. In general, the forces acting on a body will not all act through the centre of gravity of the body. If the body is in equilibrium, the **turning effects** of the forces must balance out as well giving zero resultant.

For a body at rest:

- The **forces** must balance each other out (i.e. the resultant force must be zero).
- The Principle of Moments must apply (i.e. the **moments of the forces** about the same point must balance out).

Worked example

A uniform shelf of width 0.6 m and of weight 12 N is attached to a wall by hinges and is supported horizontally by two parallel cords attached at two of the corners of the shelf, as shown in Figure 3. The other end of each cord is fixed to the wall 0.4 m above the hinge. Calculate:

a the angle between each cord and the shelf,
b the tension in each cord.

Solution

a Let the angle between each cord and the shelf be θ.

From Figure 3, $\tan\theta = \dfrac{0.4}{0.6}$ so $\theta = 34°$

b Taking moments about the hinge gives:

- Sum of the clockwise moments
  = weight of shelf × distance from hinge to the centre of gravity of the shelf
  $= 12 \times 0.3 = 3.6\,\text{N m}$
- Sum of the anticlockwise moments $= 2Td$, where T is the tension in each cord and d is the perpendicular distance from the hinge to either cord.
  From Figure 3, it can be seen that $d = 0.6\sin\theta = 0.6\sin 34 = 0.34\,\text{m}$
  ∴ Applying the Principle of Moments gives:

$$2 \times 0.34 \times T = 3.6$$
$$T = 5.3\,\text{N}$$

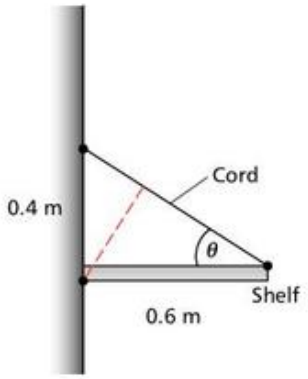


Fig 3

QUESTIONS

1 A uniform plank of length 5.0 m rests horizontally on two bricks, which are 0.5 m from either end. A child of weight 200 N stands on one end of the plank and causes the other end to lift, so it is no longer supported at that end.

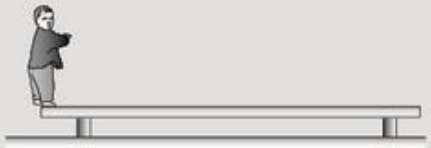


Fig 4

Calculate:

a the weight of the plank,

b the support force acting on the plank from the supporting brick.

2 A security camera is supported by a frame which is fixed to a wall and ceiling (Figure 5). The support structure must be strong enough to withstand the effect of a downward force of 1500 N acting on the camera, in case the camera is gripped by someone below it.

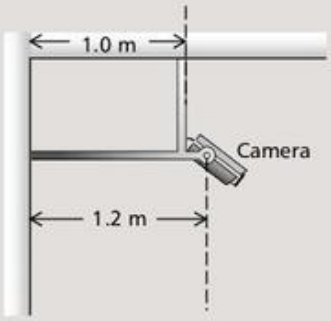


Fig 5

Calculate:

a the moment of a downward force of 1500 N on the camera about the point where the support structure is attached to the wall,

b the extra force of the vertical strut supporting the frame, when the camera is pulled with a downward force of 1500 N.

3 A crane is used to raise one end of a 15 kN girder of length 10.0 m off the ground. When the end of the girder is at rest 6.0 m off the ground, the crane cable is perpendicular to the girder (Figure 6). Calculate the tension in the cable.

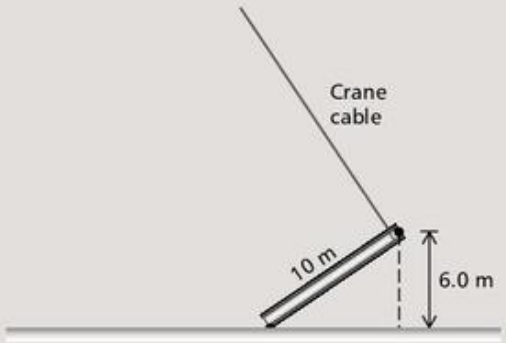


Fig 6

4 In question **3**, show that the support force on the girder from the ground has a horizontal component of 3.6 N and a vertical component of 10.2 kN. Hence calculate the magnitude of the support force.

3.6 Statics calculations

On these pages you will learn to:

- Sketch free-body force diagrams for bodies in equilibrium
- Solve problems about objects in equilibrium using the rules for equilibrium

1 Calculate the magnitude of the resultant of a 6.0 N force and a 9.0 N force acting on a point object when the two forces act:

a in the same direction,

b in opposite directions,

c at 90° to each other.

2 A point object in equilbrium is acted on by a 3 N force, a 6 N force and a 7 N force. What is the resultant force on the object if the 7 N force is removed?

3 A point object of weight 5.4 N in equilibrium is acted on by a horizontal force of 4.2 N and a second force F:

a By considering the triangle of forces rule, determine the magnitude of F.

b Calculate the angle between the direction of F and the horizontal.

4 An object of weight 7.5 N hangs on the end of a cord which is attached to the mid-point of a wire, stretched between two points on the same horizontal level. Each half of the wire is at 12° to the horizontal. Calculate the tension in each half of the wire.

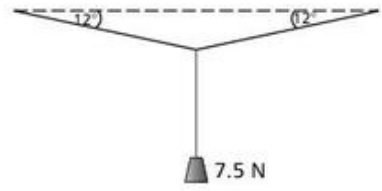


Fig 1

5 A ship is towed at constant speed by two tug boats, each pulling the ship with a force of 9 kN. The angle between the tug-boat cables is 40°.

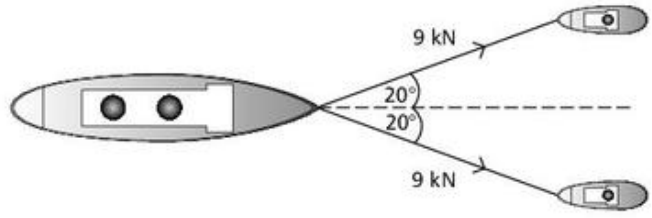


Fig 2

a Calculate the resultant force on the ship due to the two cables.

b Calculate the drag force on the ship.

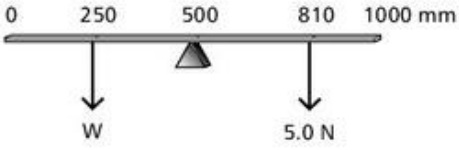


Fig 3

6 A metre rule is pivoted on a knife edge at its centre of gravity, supporting a weight of 5.0 N and an unknown weight W (as shown in Figure 3). To balance the rule horizontally with the unknown weight on the 250 mm mark of the rule, the position of the 5.0 N weight needs to be at the 810 mm mark.

a Calculate the unknown weight.

b Calculate the support force on the rule from the knife edge.

7 In Figure 3, a 2.5 N weight is also suspended from the rule at its 400 mm mark. What adjustment needs to be made to the position of the 5.0 N weight to rebalance the rule?

8 A uniform metre rule is balanced horizontally on a knife edge at its 350 mm mark by placing a 3.0 N weight on the rule at its 10 mm mark.

a Sketch the arrangement and calculate the weight of the rule.

b Calculate the support force on the rule from the knife edge.

9 A diving board has a length 4.0 m and a weight of 250 N. It is bolted to the ground at one end and projects by a length of 3.0 m beyond the edge of the swimming pool (Figure 4). A person of weight 650 N stands on the free end of the diving board. Calculate:

a the force on the bolts,

b the force on the edge of the swimming pool.

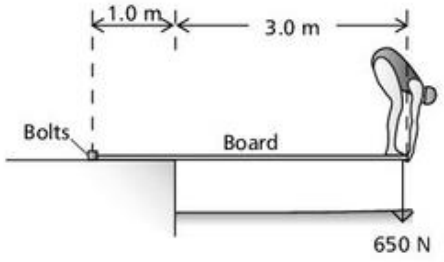


Fig 4

10 A uniform beam XY of weight 1200 N, and of length 5.0 m, is supported horizontally on a concrete pillar at each end. A person of weight 500 N sits on the beam, at a distance of 1.5 m from end X:

a Sketch a free-body force diagram of the beam.

b Calculate the support force on the beam from each pillar.

11 A bridge crane used at a freight depot consists of a horizontal span of length 12 m fixed at each end to a vertical pillar (Figure 5).

a When the bridge crane supports a load of 380 kN at its centre, a force of 1600 kN is exerted on each pillar. Calculate the weight of the horizontal span.

b The same load is moved across a distance of 2.0 m by the bridge crane. Sketch a free-body force diagram of the horizontal span, and calculate the force exerted on each pillar.

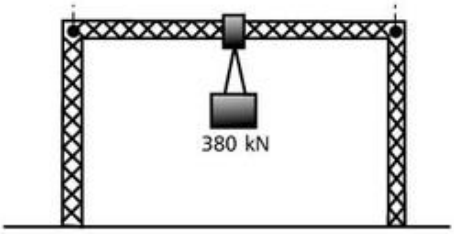


Fig 5

12 A curtain pole of weight 24 N and of length 3.2 m is supported horizontally by two wall-mounted supports X and Y, which are 0.8 m and 1.2 m from each end respectively.

a Sketch the free-body force diagram for this arrangement, and calculate the force on each support when there are no curtains on the pole.

b When the pole supports a pair of curtains of total weight 90 N drawn along the full length of the pole, what will be the force on each support?

13 A steel girder of weight 22 kN and of length 14 m is lifted off the ground at one end, by means of a crane. When the raised end is 2.0 m above the ground, the cable is vertical.

a Sketch a free-body force diagram of the girder in this position.

b Calculate the tension in the cable at this position and the force of the girder on the ground.

14 A rectangular picture of weight 24 N, hangs on a wall supported by a cord attached to the frame at each of the top corners (Figure 6). Each section of the cord makes an angle of 25° with the picture, which is horizontal along its width.

a Copy the diagram and mark the forces acting on the picture.

b Calculate the tension in each section of the cord.

Fig 6

3 End of chapter questions

1 A tug boat is used to pull a barge along a river at constant velocity, as shown below. The tug boat pulls on the barge with a force of 4.5 kN. The tug-boat engine drives the tug boat forward with a force of 7.0 kN.

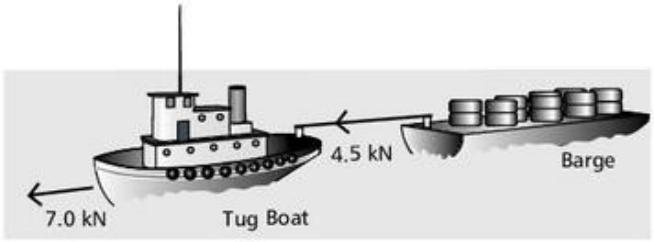


Fig 1.1

Calculate:

a the drag force on the tug boat due to the water,

b the drag force on the barge due to the water.

2 A yacht is moving due North as a result of a force, due to the wind, of 350 N in a horizontal direction of 40° East of due North, as shown below.

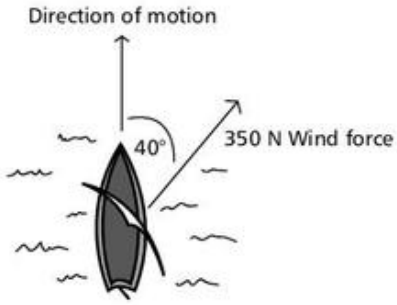


Fig 2.1

a Calculate the component of the force of the wind:
i in the direction the yacht is moving,
ii perpendicular to the direction in which the yacht is moving.

b Explain why the crew of the yacht need to lean out, as shown below, to prevent the yacht capsizing due to the force of the wind.

Fig 2.2

3 A car, of weight 6.2 kN, is towed at constant speed by a van on an uphill road as shown below. The tow rope is at an angle of 15° above the horizontal, when the road is 5° above the horizontal. The tension in the tow rope pulls the car with a force of 580 N.

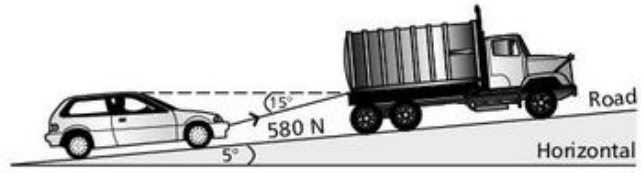


Fig 3.1

a Calculate the component of the tension in the tow rope:
i parallel to the road,
ii perpendicular to the road.

b Calculate the component of the car's weight:
i parallel to the road,
ii perpendicular to the road.

c By comparing your answers to parts **a i** and **b i**, calculate the magnitude and direction of the force of friction acting on the car.

d The van's weight is 12 kN. By considering the component of the van's weight parallel to the road and the component of the tension in the tow rope, show that the force of the van's engine must be at least 1.6 kN.

4 A uniform girder, of weight 14 kN and of length 8.0 m, rests horizontally on two wooden blocks, one at 1.0 m from one end of the girder and the other at 0.5 m from the other end (as shown below). Calculate the force of the girder on each block.

Fig 4.1

5 a i Explain what is meant by the centre of gravity of a body.
ii Define the moment of a force about a point.

b A builder needs to measure the weight of a bag of sand using a set of scales that can measure weights up to 200 N. He discovers that the bag weighs more than 200 N, so he devises the arrangement shown on p.47 to weigh the bag. This arrangement consists of a uniform plank, of length 4.5 m, resting horizontally with one end on a brick and the other end on the scales.

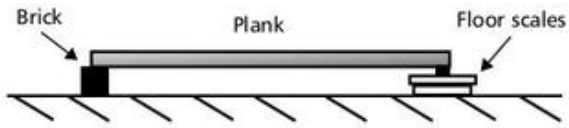


Fig 5.1 Without the bag of sand

i Without the bag of sand on the plank, the scales read 30 N. Calculate the weight of the plank.

ii With the bag of sand on the plank, the scales read 200 N when the bag is 1.5 m from the end of the plank on the scales. Show that the bag of sand has a weight of 255 N.

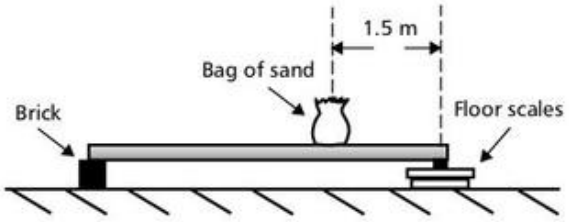


Fig 5.2 With the bag of sand

6 A flower basket, of weight 35 N, is suspended on the lower end of a rope which is tied at its upper end to a horizontal bar. The force of a horizontal wind blows the basket to one side, causing the rope to make an angle of 20° with the vertical, as shown below.

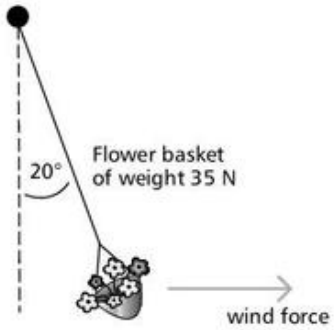


Fig 6.1

a Show that the tension in the rope in this position is 37 N.

b Calculate the force of the wind on the flower basket.

7 A paraglider, moving at constant velocity and constant height, is pulled by a cable which is at an angle of 35° to the horizontal. The force of the cable on the paraglider is 185 N.

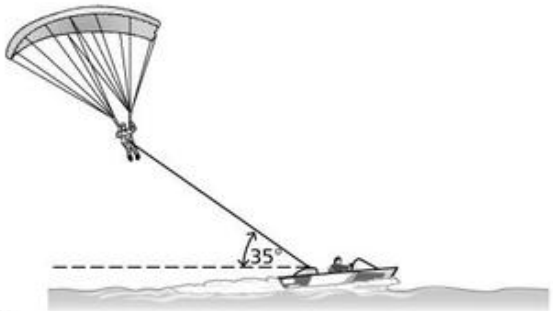


Fig 7.1

a Calculate the horizontal and vertical components of this force.

b With the aid of a force diagram, explain why the force of the parachute on the paraglider is not in the opposite direction to the force of the cable on the paraglider.

8 A bridge crane is used to transfer a container of weight 60 kN from a rail wagon onto a container lorry. The horizontal frame of the bridge crane has a length of 14 m and a total weight of 200 kN. It rests on two vertical steel pillars, as shown below.

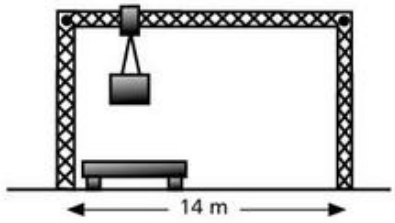


Fig 8.1

Calculate the force on each vertical pillar due to the weight of the horizontal frame and the container when:

a the container is suspended at the centre of the horizontal frame,

b the container is 4.0 m from the centre.

9 a Define the *torque* of a couple. *(2 marks)*

b A torque wrench is a type of spanner for tightening a nut and bolt to a particular torque, as illustrated in Figure 9.1.

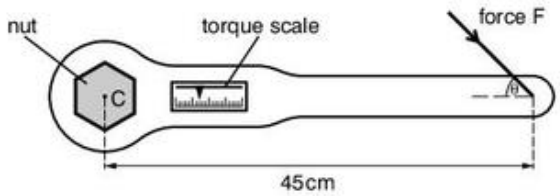


Fig 9.1

The wrench is put on the nut and a force is applied to the handle. A scale indicates the torque applied.

The wheel nuts on a particular car must be tightened to a torque of 130 N m. This is achieved by applying a force F to the wrench at a distance of 45 cm from its centre of rotation C. This force F may be applied at any angle θ to the axis of the handle, as shown in Figure 9.1.

For the minimum value of F to achieve this torque,

i state the magnitude of the angle θ, in °, that should be used, *(1 mark)*

ii calculate the magnitude of F, in N. *(2 marks)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 21 Q3 June 2009

4 Force and momentum

4.1 Momentum

On these pages you will learn to:

- Define and use linear momentum as the product of mass and velocity
- State Newton's second law in terms of momentum
- Define and use force as rate of change of momentum
- Recognise that the area under a force against time graph represents change of momentum

Fig 1 A momentum game

If you have ever run into someone on the sports field, you will know something about momentum. If the person you ran into is more massive than you, then you probably came off worse than the other person. When two bodies collide, the effect they have on each other depends not only on their initial velocities but also on the **mass** of each object. You can easily test the ideas using coins, as shown in Figure 1. You might already have developed your skill in this area! It's not too difficult to show that when a large coin and a small coin collide, the motion of the small coin is affected more.

Sir Isaac Newton was the first person to realise that a force was needed to change the velocity of an object. He realised that the effect of a force on an object depended on its mass as well as on the amount of force. He defined the momentum of a moving object as its mass multiplied by its velocity and showed how the momentum of an object changes when a force acts on it. In topic 2.1, you learned that the force needed to give an object a certain acceleration can be calculated from the equation 'force = mass × acceleration'. In this topic, we consider the ideas that Newton established in full.

Although Newton put forward his ideas over 300 years ago, his laws continue to provide the essential mathematical rules for predicting the motion of objects in any situation except inside the atom (where the rules of quantum physics apply) or at speeds approaching the speed of light (where Einstein's theory of relativity applies). For example, the launch of a satellite into orbit is carefully planned using Newton's laws of motion.

The **momentum** of an object is defined as its **mass × its velocity**.

- The unit of momentum is $\mathrm{kg\,m\,s^{-1}}$. The symbol for momentum is p.
- Momentum is a vector quantity. Its direction is the same as the direction of the object's velocity.
- For an object of mass m moving at velocity v, its momentum $p = mv$.

For example, a ball of mass 2.0 kg moving at a velocity of $10\,\mathrm{m\,s^{-1}}$ has the same amount of momentum as a person of mass 50 kg moving at a velocity of $0.4\,\mathrm{m\,s^{-1}}$.

Momentum and Newton's laws of motion

Newton's first law of motion: An object remains at rest or in uniform motion unless acted on by a resultant force.

In effect, Newton's first law tells us that a force is needed to change the momentum of an object. If the momentum of an object is constant, there is no resultant force acting on it. Clearly, if the mass of an object is constant and the object has constant momentum, it follows that the velocity of the object is also constant. However, if a moving object with constant momentum gains or loses mass, its velocity would change to keep its momentum constant. For example, a cyclist in a race who collects a water bottle as he or she speeds past a 'service' point gains mass (i.e. the water bottle) and therefore loses velocity.

Newton's second law of motion: The rate of change of momentum of an object is proportional to the resultant force on it. In other words, the resultant force is proportional to the change of momentum per second.

Consider an object of constant mass m acted on by a constant force F. Its acceleration causes a change of its speed from initial speed u to speed v in time t without change of direction. See Figure 2.

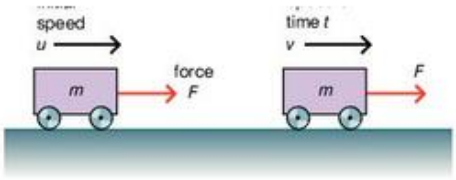


Fig 2 Force and momentum

- its initial momentum = mu, and its final momentum = mv
- change of momentum = its final momentum (mv) − its initial momentum (mu).

According to Newton's second law, the force is proportional to the change of momentum per second.

$$\text{Therefore, force } F \propto \frac{\text{change of momentum}}{\text{time taken}} = \frac{mv - mu}{t} = \frac{m(v-u)}{t} = ma$$

where $a = \dfrac{v-u}{t}$ = the acceleration of the object.

This proportionality relationship (i.e. $F \propto ma$) can be written as $F = kma$, where k is a constant of proportionality.

The value of k is made equal to 1 by defining the unit of force, **the newton**, as the amount of force that gives an object of mass 1 kg an acceleration of 1 m s^{-2} (i.e. force F = 1 N, mass m = 1 kg, acceleration, a = 1 m s^{-2} so k = 1).

Therefore, with k = 1, the equation $F = ma$ follows from Newton's second law provided the mass of the object is constant.

In general, the change of momentum of an object may be written as $\Delta(mv)$, where the symbol Δ means 'change of'. Therefore, if the momentum of an object changes by $\Delta(mv)$ in time Δt, the force F on the object is given by the equation

$$F = \frac{\Delta mv}{\Delta t}$$

1 **If m is constant**, then $\Delta(mv) = m\,\Delta v$, where Δv is the change of velocity of the object.

$$\therefore \quad F = m\frac{\Delta v}{\Delta t} = ma \text{ where acceleration } a = \frac{\Delta v}{\Delta t}$$

2 **If m changes at a constant rate** as a result of mass being transferred at constant velocity, then $\Delta(mv) = v\Delta m$, where Δm is the change of mass of the object.

$$\therefore \quad F = v\frac{\Delta m}{\Delta t} \text{ where } \frac{\Delta m}{\Delta t} = \text{change of mass per second.}$$

This form of Newton's second law is used in any situation where an object gains or loses mass continuously. For example, if a hosepipe ejects water at speed v, the force F needed to eject the water is given by $F = v\dfrac{\Delta m}{\Delta t}$ where $\dfrac{\Delta m}{\Delta t}$ = mass of water lost per second. An equal and opposite reaction force acts on the hosepipe due to the water.

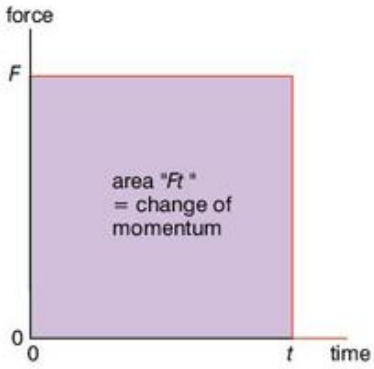


Fig 3 Force v. time for constant force

Force v. time graphs

Suppose an object of constant mass m is acted on by a constant force F which changes its velocity from initial velocity u to velocity v in time t. As explained on the previous page, Newton's second law gives

$$F = \frac{mv - mu}{t}$$

Rearranging this equation gives $Ft = mv - mu$

Figure 3 is a graph of force v. time for this situation. Because force F is constant for time t, the area under the line represents Ft which is equal to $mv - mu$. In other words, the area under the line of a force v. time graph represents the change of momentum.

Note The unit of momentum can be given as the newton second (N s) or the kilogram metre per second ($\mathrm{kg\,m\,s^{-1}}$).

Worked example

A force of 10 N acts for 20 s on an object of mass 50 kg which is initially at rest. Calculate:

a the change of momentum of the object,
b the velocity of the object at 20 s.

Solution

a Change of momentum = $Ft = 10\,\mathrm{N} \times 20\,\mathrm{s} = 200\,\mathrm{N\,s}$.
b Momentum at 20 s = 200 N s as the object was initially at rest.

$$\therefore \text{ Velocity } = \frac{\text{momentum}}{\text{mass}} = \frac{200\,\mathrm{N\,s}}{50\,\mathrm{kg}} = 4.0\,\mathrm{m\,s^{-1}}$$

QUESTIONS

1 a Calculate the momentum of
- **i** an atom of mass 4.0×10^{-25} kg moving at a velocity of $3.0 \times 10^{6}\,\mathrm{m\,s^{-1}}$,
- **ii** a pellet of mass 4.2×10^{-4} kg moving at a velocity of $120\,\mathrm{m\,s^{-1}}$,
- **iii** a bird of mass 0.56 kg moving at a velocity of $25\,\mathrm{m\,s^{-1}}$.

b Calculate:
- **i** the mass of an object moving at a velocity of $16\,\mathrm{m\,s^{-1}}$ with momentum of $96\,\mathrm{kg\,m\,s^{-1}}$,
- **ii** the velocity of an object of mass 6.4 kg that has momentum of $128\,\mathrm{kg\,m\,s^{-1}}$.

2 A train of mass 24 000 kg moving at a velocity of $15.0\,\mathrm{m\,s^{-1}}$ is brought to rest by a braking force of 6000 N. Calculate:
- **a** the initial momentum of the train,
- **b** the time taken for the brakes to stop the train.

3 An aircraft of total mass 45 000 kg accelerates on a runway from rest to a velocity of $120\,\mathrm{m\,s^{-1}}$ when it takes off. During this time, its engines provide a constant driving force of 120 kN.

Calculate:
- **a** the gain of momentum of the aircraft,
- **b** the 'take off' time.

4 The velocity of a vehicle of mass 600 kg was reduced from $15\,\mathrm{m\,s^{-1}}$ by a constant force of 400 N which acted for 20 s then by a constant force of 20 N for a further 20 s.
- **a** Sketch the force v. time graph for this situation.
- **b i** Calculate the initial momentum of the vehicle.
- **ii** Use the force v. time graph to determine the total change of momentum.
- **iii** Hence calculate the final velocity of the vehicle.

4.2 Impact force

Fig 1 A golf ball impact

On these pages you will learn to:

- Recognise how the momentum of an object changes when it undergoes an impact
- Calculate the force of an impact from the change of momentum of an object and the duration of the impact

A sports person knows that the harder a ball is hit, the further it travels. The impact changes the momentum of the ball in a very short time when the object exerting the impact force is in contact with the ball.

- If the ball is initially stationary and the impact causes it to accelerate to speed v in time t, the gain of momentum of the ball due to the impact $= mv$, where m is the mass of the ball.

  Therefore, the force of the impact $F = \dfrac{\text{change of momentum}}{\text{contact time } t} = \dfrac{mv}{t}$

- If the ball is moving with an initial velocity, u, and the impact changes its velocity to v in time t, the change of momentum of the ball $= mv - mu$.

  Therefore, the force of impact $F = \dfrac{\text{change of momentum}}{\text{contact time } t} = \dfrac{mv - mu}{t}$

Worked example

A ball of mass 0.63 kg initially at rest was struck by a bat which gave it a velocity of $35\,\text{m s}^{-1}$. The contact time between the bat and ball was 25 ms. Calculate:

a the momentum gained by the ball,

b the average force of impact on the ball.

Solution

a Momentum gained $= 0.63\,\text{kg} \times 35\,\text{m s}^{-1} = 22\,\text{kg m s}^{-1}$,

b Impact force $= \dfrac{\text{gain of momentum}}{\text{contact time } t} = \dfrac{22\,\text{kg m s}^{-1}}{0.025\,\text{s}} = 880\,\text{N}$

Vehicle safety reminders

Vehicle safety features such as crumple zones, seatbelts, collapsible steering wheels, and airbags are all designed to lessen the effect of an impact on people in the vehicle. The essential idea is to increase the time taken by an impact so the acceleration or deceleration is less and therefore the impact force is less. The result is the same using the idea of momentum; for a given change of momentum, the force is reduced if the impact time is increased. However, as explained in topic 4.3, the ideas can be developed much further by using the concept of momentum.

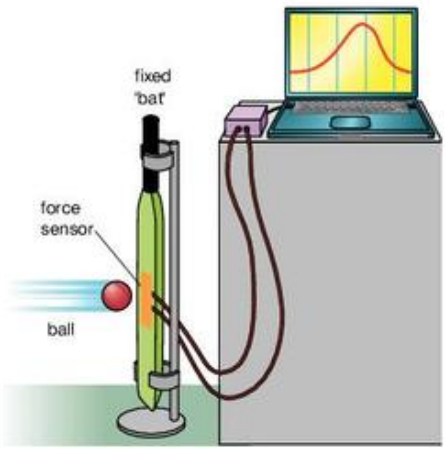
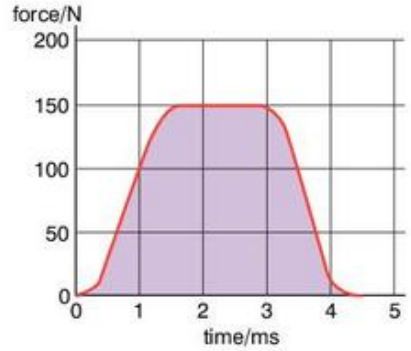


Fig 2 *Investigating an impact force on a ball*

Force v. time graphs for impacts

The variation of an impact force with time on a ball can be recorded using a force sensor connected via suitably long wires or a radio link to a computer. The force sensor is attached to the object (e.g. a bat) that causes the impact. Because equal and opposite forces act on the ball, the force on the ball due to the bat varies in exactly the same way as the force on the bat due to the ball. The variation of force with time is displayed on the computer screen.

Figure 3 shows a typical force v. time graph for an impact. The graph shows that the impact force increases then decreases during the impact. As explained on p.60, the area under the graph is equal to the change of momentum. The average force of impact can be worked out from the change of momentum divided by the contact time.



area under curve = 9 blocks.
Ft for 1 block = 50 N × 1 ms
= 5.0×10^{-2} N s.
change of momentum
= $9 \times 5.0 \times 10^{-2}$
= 0.45 N s.

Fig 3 *Force v. time for an impact*

Rebound impacts

When a ball hits a wall and rebounds, its momentum changes direction due to the impact. If the ball hits the wall normally, it rebounds normally so the direction of its momentum is reversed. The velocity and, therefore, the momentum after the impact are in the opposite direction to the velocity before the impact and therefore have the opposite sign. Figure 4 shows the idea.

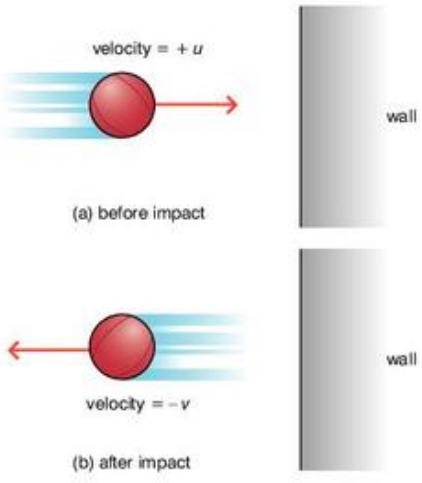


Fig 4 *A rebound*

Suppose the ball hits the wall normally with an initial speed u and it rebounds at speed v in the opposite direction. Since its direction of motion reverses on impact, a sign convention is necessary to represent the two directions. Using + for 'towards the wall' and − for 'away from the wall', its initial momentum = $+mu$, and its final momentum = $-mv$.

Therefore, its change of momentum = final momentum − initial momentum
$= (-mv) - (mu)$

The impact force $F = \dfrac{\text{change of momentum}}{\text{contact time } t} = \dfrac{(-mv) - (mu)}{t}$

Note If there is no loss of speed on impact, then $v = u$

so the impact force $F = \dfrac{(-mu) - (mu)}{t} = -\dfrac{2mu}{t}$

Worked example

A tennis ball of mass 0.20 kg moving at a speed of $18\,m\,s^{-1}$ was hit by a bat, causing the ball to go back in the direction it came from at a speed of $15\,m\,s^{-1}$. The contact time was 0.12 s. Calculate:

a the change of momentum of the ball,
b the impact force on the ball.

Solution

a Mass of ball $m = 0.20\,kg$, initial velocity $u = +18\,m\,s^{-1}$, final velocity $= -15\,m\,s^{-1}$,
Change of momentum $= mv - mu = (0.20 \times -15) - (0.20 \times 18)$
$= -3.0 - 3.6 = -6.6\,kg\,m\,s^{-1}$

b Impact force $= \dfrac{\text{change of momentum}}{\text{time taken}} = \dfrac{-6.6\,kg\,m\,s^{-1}}{0.12\,s} = -55\,N$

The minus sign indicates the force on the ball is in the same direction as the velocity after the impact.

QUESTIONS

1 A 2000 kg lorry reversing at a speed of $0.8\,m\,s^{-1}$ backs accidentally into a steel fence. The fence stops the lorry 0.5 s after the lorry first makes contact with the fence. Calculate:

a the initial momentum of the lorry,

b the force of the impact.

2 A car of mass 600 kg travelling at a speed of $3.0\,m\,s^{-1}$ is struck from behind by another vehicle. The impact lasts for 0.4 s and causes the speed of the car to increase to $8.0\,m\,s^{-1}$. Calculate:

a the change of momentum of the car due to the impact,

b the impact force.

3 A molecule of mass 5.0×10^{-26} kg moving at a speed of $420\,m\,s^{-1}$ hits a surface at right angles to the surface and rebounds at the same speed in the opposite direction in an impact lasting 0.22 ns. Calculate:

a the change of momentum,

b the force on the molecule.

4 Repeat the calculation in question **3** for a molecule of the same mass at the same speed which hits the surface at 60° to the normal and rebounds without loss of speed at 60° to the normal, as shown in Figure 5. You will need to work out the component of the molecule's velocity parallel to the normal before and after the impact. Assume the contact time is the same.

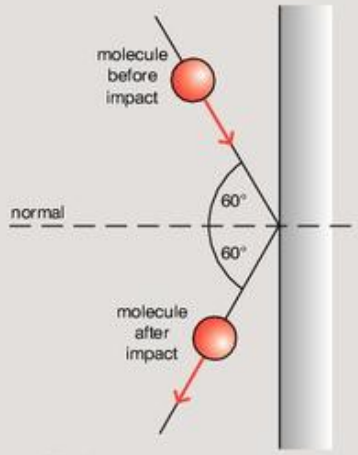


Fig 5

4.3 Conservation of momentum

On these pages you will learn to:

- State Newton's third law of motion
- State the principle of conservation of momentum
- Apply the principle of conservation of momentum to solve simple problems

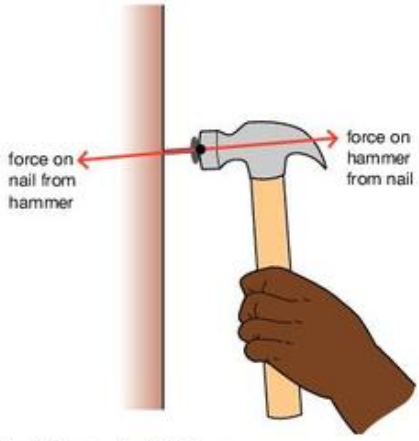


Fig 1 Newton's third Law

Newton's third law of motion

When two objects interact, they exert equal and opposite forces on each other.

In other words, if object A exerts a force on object B, there must be an equal and opposite force acting on object A due to object B.

For example,

- an object resting on a table exerts a force on the table which exerts an equal and opposite force on the object;
- a person leaning against a wall exerts a force on the wall which exerts an equal and opposite force on the person;
- a hammer hitting a nail exerts a force on the nail which exerts an equal and opposite force on the hammer;
- the Earth exerts a force due to gravity on an object which exerts an equal and opposite force on the Earth;
- a jet engine exerts a force on hot gas in the engine to expel the gas; the gas being expelled exerts an equal and opposite force on the engine.

The Principle of Conservation of Momentum

When an object is acted on by a resultant force, its momentum changes. If there is no change of its momentum, there can be no resultant force on the object. Now consider several objects which interact with each other. If no external resultant force acts on the objects, the total momentum does not change. However, interactions between the objects can transfer momentum between them. But the total momentum does not change.

The Principle of Conservation of Momentum states that for a system of interacting objects, the total momentum remains constant, provided no external resultant force acts on the system.

Consider two objects that collide with each other then separate. As a result, the momentum of each object changes. They exert equal and opposite forces on each other when they are in contact. So the change of momentum of one object is equal and opposite to the change of momentum of the other object. In other words, if one object gains momentum, the other object loses an equal amount of momentum. So the total amount of momentum is unchanged.

Let's look in detail at the example of two snooker balls A and B in collision, as shown in Figure 2.

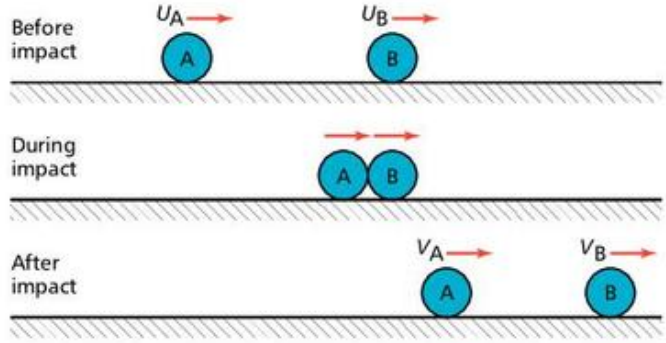


Fig 2 Conservation of momentum

- The impact force F_1 on ball A changes the velocity of A from u_A to v_A in time t

  Therefore, $F_1 = \dfrac{m_A v_A - m_A u_A}{t}$,

  where t = the time of contact between A and B,

  and m_A = the mass of ball A.

- The impact force F_2 on ball B changes the velocity of B from u_B to v_B in time t

  Therefore, $F_2 = \dfrac{m_B v_B - m_B u_B}{t}$,

  where t = the time of contact between A and B,

  and m_B = the mass of ball B.

Because the two forces are equal and opposite to each other, $F_2 = -F_1$

Therefore,

$$\frac{m_B v_B - m_B u_B}{t} = -\frac{m_A v_A - m_A u_A}{t}$$

Cancelling t on both sides gives

$$m_B v_B - m_B u_B = -m_A v_A + m_A u_A$$

Rearranging this equation gives

$$m_B v_B + m_A v_A = m_A u_A + m_B u_B$$

Therefore,

the total final momentum = the total initial momentum

In other words, the total momentum is unchanged by this collision.

Note If the colliding objects stick together as a result of the collision, they have the same final velocity. The above equation may therefore be written

$$(m_B + m_A)V = m_A u_A + m_B u_B$$

Testing conservation of momentum

Figure 3 shows an arrangement that can be used to test conservation of momentum using a motion sensor linked to a computer. The mass of each trolley is measured before the test. With trolley B at rest, trolley A is given a push so it moves towards trolley B at constant velocity. The two trolleys stick together on impact. The computer records and displays the velocity of trolley A throughout this time. The computer display shows that the velocity of trolley A dropped suddenly when the impact took place. The velocity of trolley A immediately before the collision, u_A, and after the collision (V) can be measured. The measurements should show that the total momentum of both trolleys after the collision is equal to the momentum of trolley A before the collision. In other words,

$$(m_B + m_A)V = m_A u_A$$

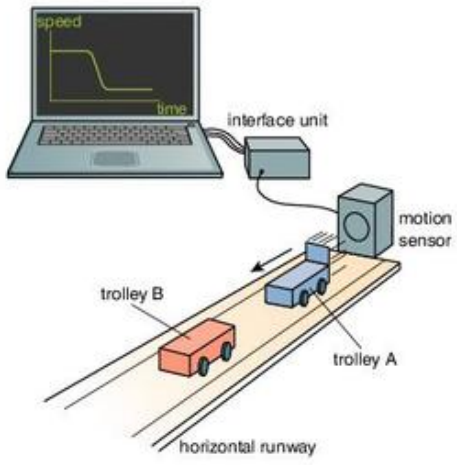


Fig 3 *Testing conservation of momentum*

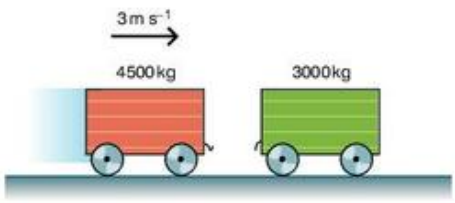


Fig 4 Colliding wagons

Worked example

A rail wagon of mass 4500 kg moving along a level track at a speed of 3.0 m s^{-1} collides with and couples to a second rail wagon of mass 3000 kg which is initially stationary. Calculate the speed of the two wagons immediately after the collision.

Solution

Total initial momentum

= initial momentum of A + initial momentum of B

= $(4500 \times 3.0) + (3000 \times 0) = 13\,500$ kg m s^{-1}

Total final momentum

= total mass of A and B × velocity V after the collision

= $(4500 + 3000)V = 7500V$

Using the Principle of Conservation of Momentum,

$$7500V = 13\,500$$

$$V = \frac{13\,500}{7500} = 1.8 \text{ m s}^{-1}$$

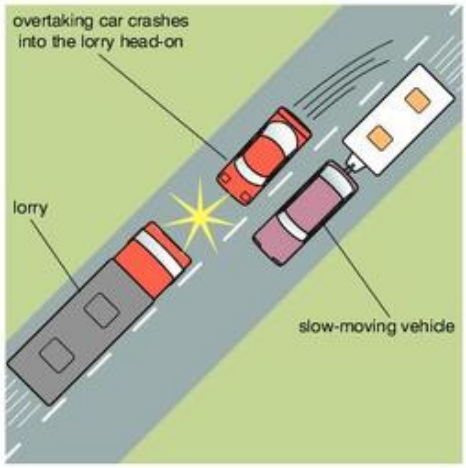


Fig 5 A crash calculation

Head-on collisions

Consider two objects moving in opposite directions that collide with each other. Depending on the masses and initial velocities of the two objects, the collision could cause them both to stop. The momentum of the two objects after the collision would then be zero. This could only happen if the initial momentum of one object was exactly equal and opposite to that of the other object. In general, if two objects move in opposite directions before a collision, then the vector nature of momentum needs to be taken into account by assigning numerical values of velocity + or − according to the direction.

For example, if a car of mass 600 kg travelling at a velocity of 25 m s^{-1} collides head-on with a lorry of mass 2400 kg travelling at a velocity of 10 m s^{-1} in the opposite direction, the total momentum before the collision is 9000 kg m s^{-1} in the direction the lorry was moving in. As momentum is conserved in a collision, the total momentum after the collision is the same as the total momentum before the collision. Prove for yourself that if the two vehicles stick together after the collision, they must have had a velocity of 3.0 m s^{-1} (= 9000 kg m s^{-1}/3000 kg) immediately after the impact in the direction the lorry was moving in.

QUESTIONS

1 A rail wagon of mass 3000 kg moving at a velocity of 1.2 m s^{-1} collides with a stationary wagon of mass 2000 kg. After the collision, the two wagons couple together. Calculate their speed immediately after the collision.

2 A rail wagon of mass 5000 kg moving at a velocity of 1.6 m s^{-1} collides with a stationary wagon of mass 3000 kg. After the collision, the 3000 kg wagon moves away at a velocity of 1.5 m s^{-1}. Calculate the speed and direction of the 5000 kg wagon after the collision.

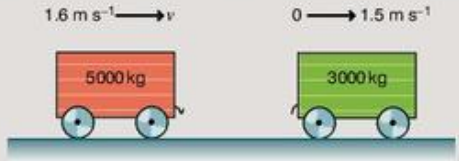


Fig 6

3 In a laboratory experiment, a trolley of mass 0.50 kg moving at a speed of 0.25 m s^{-1} collided with a trolley of mass 1.0 kg moving in the opposite direction at a speed of 0.20 m s^{-1}. The two trolleys coupled together on collision. Calculate their speed and direction immediately after the collision.

4 A ball of mass 0.80 kg moving at a speed of 2.5 m s^{-1} along a straight line collided with a ball of mass 2.5 kg which was initially stationary. As a result of the collision, the 2.5 kg ball was given a velocity of 1.0 m s^{-1} along the same line. Calculate the speed and direction of the 0.80 kg ball immediately after the collision.

4.4 Elastic and inelastic collisions

Drop a bouncy rubber ball from a measured height onto a hard floor. The ball should bounce back almost to the same height. Try the same with a cricket ball and there will be very little bounce! A **perfectly elastic** ball would be one that bounces back to exactly the same height. Its kinetic energy just after impact must equal its kinetic energy just before impact. Otherwise, it cannot regain its initial height. There is no loss of kinetic energy in a perfectly elastic collision.

A perfectly elastic collision is one where there is no loss of kinetic energy.

- A squash ball hitting a hard surface bounces off the surface with little or no loss of speed. If the ball is perfectly elastic, there is no loss of speed on impact and no loss of kinetic energy.
- A very low speed impact between two cars is almost perfectly elastic, provided no damage is done. Some of the initial kinetic energy of the two vehicles may be converted to sound. However, if the collision causes damage to the vehicles, the kinetic energy after the collision is less than before so the collision is not elastic.

A totally inelastic collision is one where the colliding objects stick together.

- A railway wagon that collides with and couples to another wagon is an example of a totally inelastic collision. Some of the initial kinetic energy is converted to other forms of energy.
- A vehicle crash in which the colliding vehicles lock together is another example of a totally inelastic collision. The total kinetic energy after the collision is less than the total kinetic energy before the collision.

A partially inelastic collision is where the colliding objects:
1 move apart, and
2 have less kinetic energy after the collision than before.

To work out if a collision is elastic or inelastic, the kinetic energy of each object before and after the collision must be worked out.

For a ball of mass m falling in air from a measured height H and rebounding to a height h, as shown in Figure 3:

- the kinetic energy immediately before impact = loss of potential energy through height $H = mgH$,
- the kinetic energy immediately after impact = gain of potential energy through height $h = mgh$.

So the height ratio $\frac{h}{H}$ gives the fraction of the initial kinetic energy that is recovered as kinetic energy after the collision.

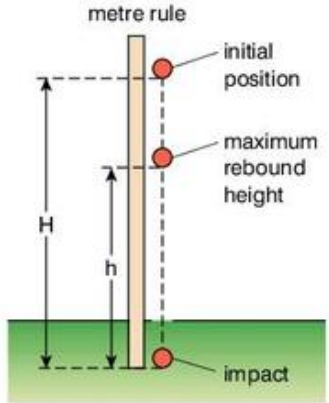


Fig 3 Testing an impact

Collisions between two objects

When two objects with known masses and initial velocities collide, if the final velocity of one of the objects is known, the final velocity of the other object can be determined using the Principle of Conservation of Momentum. The kinetic energy

On these pages you will learn to:

- Distinguish between an elastic and an inelastic collision
- Recognise that momentum is always conserved in any collision, whereas kinetic energy is only conserved in an elastic collision
- Apply the principle of conservation of momentum to elastic and inelastic interactions between bodies in both one and two dimensions
- Recognise that, for a perfectly elastic collision, the relative speed of approach is equal to the relative speed of separation

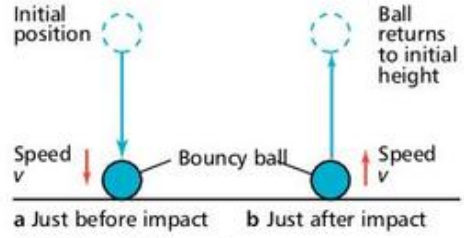


Fig 1 An elastic impact

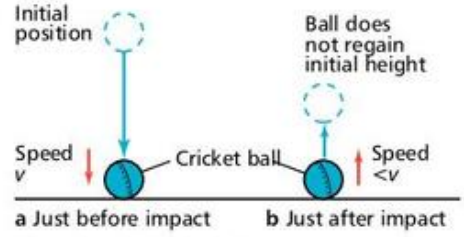


Fig 2 A partially elastic impact

For a perfectly elastic collison between two objects, using the principle of conservation of momentum and the rule that the total kinetic energy is unchanged in an elastic collision, it can be shown that their relative speed of separation = their relative speed of approach.

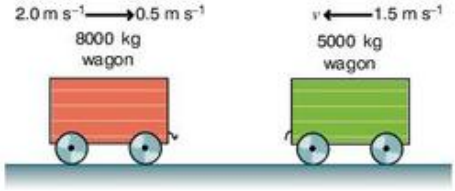


Fig 4

Fig 5 This dramatic collision was used to demonstrate that special casks used to transport radioactive materials by rail could withstand high speed impacts. In this collision, a remotely controlled diesel locomotive was driven into a stationary cask. Even though the locomotive was destroyed, the cask itself was intact afterwards.

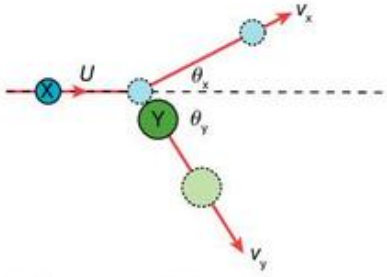


Fig 6 An oblique collision

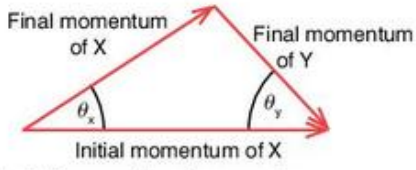


Fig 7 Conservation of momentum

of each object before or after the collision can then be calculated using the kinetic energy formula $E_k = \frac{1}{2}mv^2$. If the collision is elastic, the total kinetic energy of the objects before the collision is equal to their total kinetic energy after the collision.

- **For a collision in which the objects move along the same straight line before and after the collision** (e.g. railway wagons on a straight track), velocities in one direction are assigned positive values and velocities in the opposite direction are assigned negative values, because velocity is a vector quantity.

Worked example

A railway wagon of mass 8000 kg moving at a speed of 2.0 m s^{-1} collides with a wagon of mass 5000 kg moving at a speed of 1.5 m s^{-1} in the opposite direction. The collision reduces the speed of the 8000 kg wagon to 0.5 m s^{-1} without changing its direction. Calculate the speed V and direction of the second wagon after the collision.

Solution

$$\text{The total initial momentum} = (8000 \times 2.0) + (5000 \times -1.5)$$
$$= 16000 - 7500 = 8500\,\text{kg m s}^{-1}$$

(note the negative value of the initial velocity of the 5000 kg wagon)

$$\text{The total final momentum} = (8000 \times 0.5) + (5000 \times V)$$

Using the Principle of Conservation of Momentum

$$(8000 \times 0.5) + (5000 \times V) = 8500$$
$$5000 \times V = 8500 - 4000 = 4500$$
$$V = \frac{4500}{5000} = +0.9\,\text{m s}^{-1}$$

Prove for yourself that the collision is inelastic and that 18.6 kJ of kinetic energy is transferred to the surroundings (as heat and sound).

- **In an 'oblique' collision, two objects collide then move away in different directions from their initial directions**. Figure 6 shows an oblique collision between two spheres X and Y, where X is initially moving at velocity V towards Y, which is initially stationary. After the collision, the two objects move away at velocities v_x and v_y in directions at angles θ_x and θ_y to the initial direction of X. If the initial and final velocity of X is known and the masses of the two spheres are known, the momentum of Y after the collision can be determined using the Principle of Conservation of Momentum:
  - **either by drawing a vector diagram**, as shown in Figure 7
  - **or by calculation,** as follows.
  1. Resolve the velocities v_x and v_y parallel and perpendicular to the initial direction of X to give $v_x\cos\theta_x$ and $v_x\sin\theta_x$ for the parallel and perpendicular components of X and $v_y\cos\theta_y$ and $v_y\sin\theta_y$ for the parallel and perpendicular components of Y.
  2. Apply the Principle of Conservation of Momentum in each direction:
     - In the parallel direction: $m_xU = m_xv_x\cos\theta_x + m_yv_y\cos\theta_y$
     - In the perpendicular direction: $0 = m_xv_x\sin\theta_x + m_yv_y\sin\theta_y$, where m_x and m_y are the masses of X and Y, respectively.
  3. Insert the known values into the above equations to give a value of $v_y\cos\theta_y$ from the first equation and a value of $v_y\sin\theta_y$ from the second equation. Note that if the perpendicular component of X is assigned a positive value then the perpendicular component of Y needs to be assigned a negative value (because they are in opposite directions).
  4. The values of v_y and θ_y can then be found using $v_y^2 = (v_y\cos\theta_y)^2 + (v_y\sin\theta_y)^2$ and $\tan\theta_y = v_y\sin\theta_y / v_y\cos\theta_y$.

Worked example

A sphere X of mass 0.10 kg moving at a speed of 2.0 m s^{-1} collides with a sphere Y of mass 0.20 kg, which is initially at rest. As a result, X is deflected by 30° at a speed of 1.6 m s^{-1}. Show that Y rebounds at a speed of 0.50 m s^{-1} in a direction at 53° to the initial direction of X.

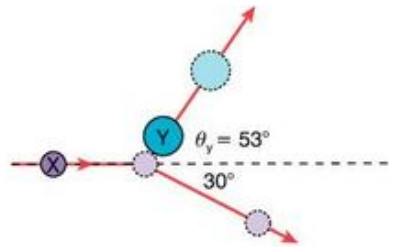


Fig 8

Solution 1: Vector diagram method

Initial momentum of X = $0.10 \times 2.0 = 0.20\,\text{kg m s}^{-1}$

Final momentum of X = $0.10 \times 1.6 = 0.16\,\text{kg m s}^{-1}$ at 30° to the initial direction of X

Final momentum of Y = $0.20 \times 0.50 = 0.10\,\text{kg m s}^{-1}$ at cos 53° to the initial direction of X

Figure 9 shows the momentum vectors for the above three momentum values. The diagram shows that the vector sum of the final momentum vector for X and the final momentum vector for Y is equal to the initial momentum vector for X.

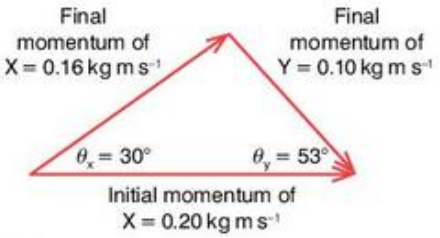


Fig 9

Solution 2: Calculation method

Apply the Principle of Conservation of Momentum parallel and perpendicular to the initial direction of X.

In the parallel direction:

$$m_x U = m_x v_x \cos\theta_x + m_y v_y \cos\theta_y$$
$$(0.10 \times 2.0) = (0.10 \times 1.6 \times \cos 30°) + (0.20 v_y \times \cos\theta_y)$$
$$(0.20 v_y \times \cos\theta_y) = (0.10 \times 2.0) - (0.10 \times 1.6 \times \cos 30°) = 0.20 - 0.139 = 0.061$$
$$v_y \times \cos\theta_y = 0.061 \div 0.20 = 0.305$$

In the perpendicular direction:

$$0 = m_x v_x \sin\theta_x + m_y v_y \sin\theta_y$$
$$0 = (0.10 \times 1.6 \times \sin 30°) + (0.20\, v_y \times \sin\theta_y)$$
$$0.20 v_y \times \sin\theta_y = -0.10 \times 1.6 \times \sin 30° = -0.080$$
$$v_y \times \sin\theta_y = -0.080 \div 0.20 = -0.400$$
$$\text{Therefore } v_y = [(v_y \cos\theta_y)^2 + (v_y \sin\theta_y)^2]^{0.5} = [0.305^2 + 0.400^2]^{0.5}$$
$$= 0.50\,\text{m s}^{-1} \text{ to 2 significant figures}$$

$$\tan\theta_y = \frac{v_y \sin\theta_y}{v_y \cos\theta_y} = \frac{-0.400}{0.305} = -1.31$$

Therefore, $\theta_y = 53°$

QUESTIONS

1 a A squash ball is released from rest above a flat surface. Describe how its energy changes if:
- **i** it rebounds to the same height,
- **ii** it rebounds to a lesser height.

b In **a ii**, the ball is released from a height of 1.2 m above the surface and it rebounds to a height of 0.9 m above the surface. Show that 25% of its kinetic energy is lost in the impact.

2 A vehicle of mass 800 kg moving at a speed of 15.0 m s^{-1} collided with a vehicle of mass 1200 kg moving in the same direction at a speed of 5.0 m s^{-1}. The two vehicles locked together on impact. Calculate:
- **a** the velocity of the two vehicles immediately after impact,
- **b** the loss of kinetic energy due to the impact.

3 An ice puck of mass 1.5 kg moving at a speed of 4.2 m s^{-1} collides head on with a second ice puck of mass 1.0 kg moving in the opposite direction at a speed of 4.0 m s^{-1}. After the impact, the 1.5 kg ice puck continues in the same direction at a speed of 0.8 m s^{-1}. Calculate:
- **a** the speed and direction of the 1.0 kg ice puck after the collision,
- **b** the loss of kinetic energy due to the collision.

4 Bumper cars at fairgrounds are designed to withstand low-speed impacts without damage. A bumper car of mass 250 kg moving at a velocity of 0.9 m s^{-1} collides elastically with a stationary car of mass 200 kg. Immediately after impact, the 250 kg car has a velocity of 0.1 m s^{-1} in the same direction as it was initially moving.
- **a i** Calculate the velocity of the 200 kg car immediately after the impact.
- **ii** Show that the collision was an elastic collision.
- **b i** What was the relative velocity of approach?
- **ii** What was the relative velocity of separation?

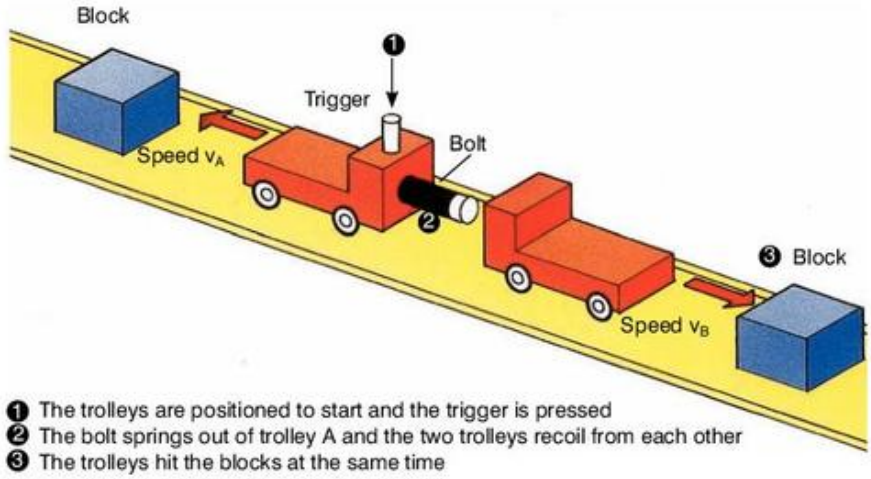
4.5 Explosions

On these pages you will learn to:

- Apply the principle of conservation of momentum to explosions and other situations where two objects initially at rest push each other away
- Recognise that two such objects initially at rest move apart with equal and opposite amounts of momentum

Fig 1 *The gun barrel recoils when the shell is fired. Large springs fitted to the barrel absorb the recoil energy of the barrel*

When two objects fly apart after being initially at rest, they recoil from each other with equal and opposite amounts of momentum. Consider Figure 2 where a trolley of mass m_A and a trolley of mass m_B, initially at rest, move away at speeds v_A and v_B respectively.



❶ The trolleys are positioned to start and the trigger is pressed
❷ The bolt springs out of trolley A and the two trolleys recoil from each other
❸ The trolleys hit the blocks at the same time

Fig 2 *Flying apart*

The total initial momentum $= 0$

The total momentum immediately after the explosion = momentum of A + momentum of B

$$= m_A v_A + m_B v_B$$

Using the Principle of Conservation of Momentum, $m_A v_A + m_B v_B = 0$

$$\therefore m_B v_B = -m_A v_A$$

The minus sign means that the two masses move away from each other in opposite directions. For example, if $m_A = 1.0\,\text{kg}$, $v_A = 2\,\text{m s}^{-1}$ and $m_B = 0.5\,\text{kg}$, then $v_B = -m_A v_A / m_B = -4.0\,\text{m s}^{-1}$. So A and B move away at speeds of $2\,\text{m s}^{-1}$ and $4\,\text{m s}^{-1}$ in opposite directions.

Testing a model explosion

In Figure 2, when the spring is released from one of the trolleys, the two trolleys, A and B, push each other apart. The bricks are positioned so that the trolleys hit the bricks at the same moment. The distance travelled by each trolley to the point of impact with the brick is equal to its speed multiplied by the time taken to travel that distance. Because the time taken is the same for the two trolleys, the distance ratio is the same as the speed ratio. Because the trolleys have equal (and opposite) amounts of momentum, the ratio of their speeds is the inverse of the mass ratio. The distance ratio should therefore be equal to the inverse of the mass ratio. In other words, if trolley A travels twice as far as trolley B, then the mass of A must be half the mass of B (so they carry away equal amounts of momentum).

QUESTIONS

1 A shell of mass 2.0 kg is fired at a speed of $140\,m\,s^{-1}$ from an artillery gun of mass 800 kg. Calculate the recoil velocity of the shell.

2 In a laboratory experiment to measure the mass of an object X, two identical trolleys A and B, each of mass 0.50 kg, were initially stationary on a track. Object X was fixed to trolley A. When a trigger was pressed, the two trolleys moved apart in opposite directions at speeds of $0.30\,m\,s^{-1}$ and $0.25\,m\,s^{-1}$.

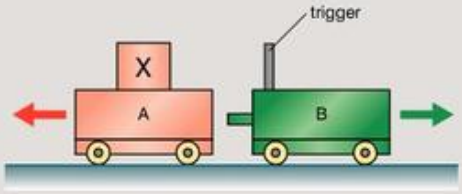


Fig 3

a Which of the two speeds given above was the speed of trolley A? Give a reason for your answer.

b Show that the mass of X must have been 0.10 kg.

3 Two trolleys, X of mass 1.2 kg and Y of mass 0.8 kg, are initially stationary on a level track.

a When a trigger is pressed on one of the trolleys, a spring pushes the two trolleys apart. Trolley Y moves away at a velocity of $0.15\,m\,s^{-1}$.

i Calculate the velocity of X.

ii Calculate the total kinetic energy of the two trolleys immediately after the impact.

b If the test had been carried out with trolley X held firmly, calculate the speed at which Y would have recoiled, assuming the energy stored in the spring before release is equal to the total kinetic energy calculated in **a ii**.

4 A person in a stationary boat of total mass 150 kg throws a rock of mass 2.0 kg out of the boat. As a result, the boat recoils at a speed of $0.12\,m\,s^{-1}$. Calculate:

a the speed at which the rock was thrown from the boat,

b the kinetic energy gained by:
i the boat, **ii** the rock.

Chapter summary

- Momentum = mass × velocity
- $\text{Force} = \dfrac{\text{change of momentum}}{\text{time taken}}$
- The Principle of Conservation of Momentum states that when two or more bodies interact, the total momentum is unchanged, provided no external forces act on the bodies.
- An elastic collision is one in which the total kinetic energy after the collision is the same as before the collision. For such a collision between two objects, the relative speed of approach is equal to the relative speed of separation.
- A totally inelastic collision is one in which the colliding objects stick together.
- In an explosion where two objects fly apart, the two objects carry away equal and opposite momentum.

4 End of chapter questions

1 An object of mass 7.0 kg, initially at rest, is acted on by a force of 14 N for 10 s. Calculate:

a the gain of momentum of the object,

b the velocity of the object after 10 s.

2 A vehicle of mass 600 kg travelling at a velocity of $15\,m\,s^{-1}$ is acted on by a braking force of 150 N. Calculate:

a the momentum of the object at $15\,m\,s^{-1}$,

b the time taken to stop the object.

3 An object of mass 5.0 kg, initially at rest, was acted on by a force of 8.0 N in 12 s then brought to rest by a different force in 20 s.

a Show that the change of momentum of the object due to the 8.0 N force is 96 N s.

b Calculate the speed of the object after 12 s.

c Calculate the force needed to bring the object to rest in 20 s.

4 A stream of identical atoms of mass 1.0×10^{-25} kg hit a vertical surface normally at a speed of $750\,m\,s^{-1}$ at a rate of 2000 atoms per second. The atoms stick to the surface on impact.

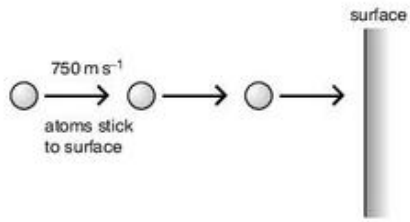


Fig 4.1

Calculate:

a the loss of momentum of a single atom,

b the average force of the atoms on the surface.

5 A molecule of mass 2.5×10^{-26} kg moving at a speed of $520\,m\,s^{-1}$ collides normally with a wall and rebounds normally without loss of speed.

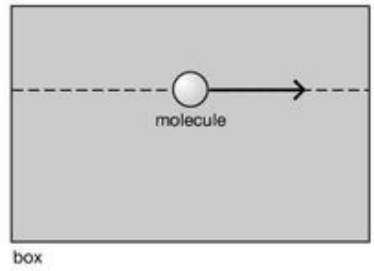


Fig 5.1

a Calculate the change of momentum of the molecule.

b The molecule is in a rectangular box and collides repeatedly with the same side of the box every 2.0 ms. Calculate the average force of impact of the molecule on the box.

6 A railway wagon of mass 2500 kg moving at a speed of $2.4\,m\,s^{-1}$ collides with a stationary wagon of mass 1500 kg on a level track. The two wagons couple together and move away at the same velocity after the impact.

a Calculate their velocity after the impact.

b **i** Calculate the loss of kinetic energy due to the impact.

ii Discuss the energy changes that take place as a result of the impact.

7 In a road accident, a van of mass 1500 kg moving at a speed of $28\,m\,s^{-1}$ ran into the back of a car of mass 900 kg moving in the same direction at a speed of $11\,m\,s^{-1}$. As a result of the impact, the car was pushed forward at a speed of $18\,m\,s^{-1}$.

a Calculate the velocity of the van immediately after the impact.

b Calculate:

i the loss of kinetic energy of the van,

ii the gain of kinetic energy of the car,

iii the total change of kinetic energy of the two vehicles.

c Discuss the effect of the impact on a person in the car.

8 In a radioactive decay, a nucleus of mass 4.0×10^{-25} kg initially at rest emitted an α-particle of mass 6.7×10^{-27} kg with a velocity of 1.5×10^{7} m s^{-1}.

a Calculate the velocity of recoil of the nucleus.

b Calculate the kinetic energy of:
i the recoil nucleus,
ii the α-particle.

9 A ball B of mass 1.2 kg travelling at constant velocity collides head-on with a stationary ball S of mass 3.6 kg, as shown in Figure 9.1.

Fig 9.1

Frictional forces are negligible.
The variation with time t of the velocity v of ball B before, during and after colliding with ball S is shown in Figure 9.2.

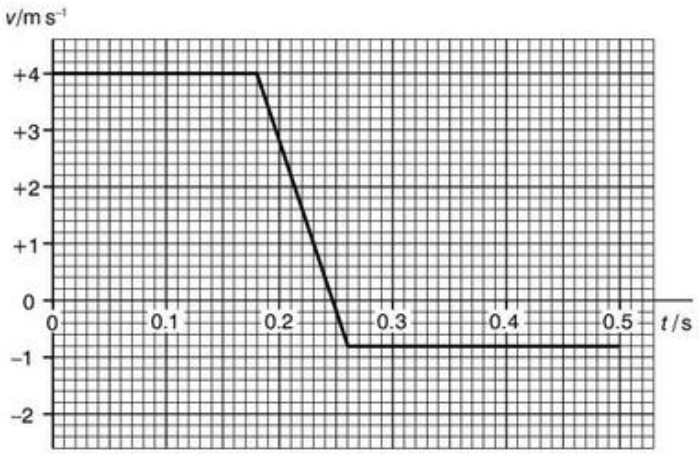


Fig 9.2

a State the significance of positive and negative values for v in Figure 9.2. *(1 mark)*

b Use Figure 9.2 to determine, for ball B during the collision with ball S,
i the change in momentum of ball B, in N s *(3 marks)*
ii the magnitude of the force acting on ball B in N *(3 marks)*

c Calculate the speed of ball S after the collision in m s^{-1} *(2 marks)*

d Using your answer in (c) and information from Figure 9.2, deduce quantitatively whether the collision is elastic or inelastic. *(2 marks)*

Adapted from Cambridge International AS and A Level Physics 9702
Paper 21 Q2 June 2009

10 A stationary nucleus of mass 220u undergoes radioactive decay to produce a nucleus D of mass 216u and an α-particle of mass 4u, as illustrated in Figure 10.1.

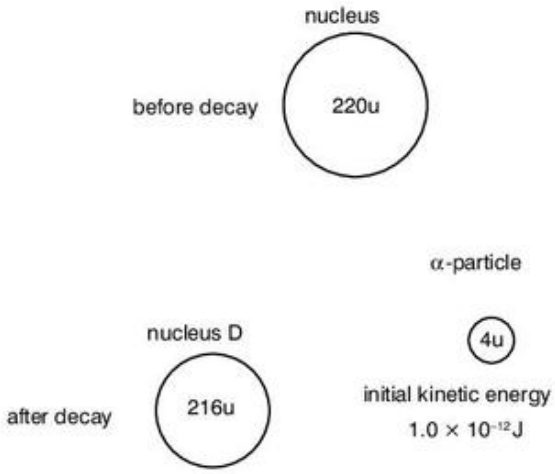


Fig 10.1

The initial kinetic energy of the α-particle is 1.0×10^{-12} J.

a i State the law of conservation of linear momentum. *(2 marks)*

ii Explain why the initial velocities of the nucleus D and the α-particle must be in opposite directions. *(2 marks)*

b i Show that the initial speed of the α-particle is 1.7×10^{7} m s^{-1}. *(2 marks)*

ii Calculate the initial speed, in m s^{-1}, of nucleus D. *(2 marks)*

c The range in air of the emitted α-particle is 4.5 cm.
Calculate the average deceleration, in m s^{-2}, of the α-particle as it is stopped by the air. *(2 marks)*

Adapted from Cambridge International AS and A Level Physics 9702
Paper 21 Q3 November 2009

5 Forces and materials

5.1 Density

On these pages you will learn to:

- Define density and use the density equation $\rho = m/V$ in calculations
- Measure the density of liquids and regular and irregular solids

Density and its measurement

Lead is much more dense than aluminium. Sea water is more dense than tap water. To compare how dense one substance is compared with another, we need to measure the mass of equal volumes of the two substances. The substance with the greater mass in the same volume is more dense. For example, a lead sphere of volume 1 cm^3 has a mass of 11.3 g; whereas an aluminium sphere of the same volume has a mass of 2.7 g.

The density of a substance is defined as its mass per unit volume.

For a certain amount of a substance of mass m and volume V, its density, ρ (pronounced 'rho'), may be calculated using the equation:

$$\text{Density, } \rho = \frac{m}{V}$$

- The unit of density is the **kilogram per metre^3 (kg m^{-3})**.
- Rearranging the above equation gives: $m = \rho V$

$$\text{or } V = \frac{m}{\rho}$$

***Table 1** Density*

Substance	Density/kg m^{-3}
Air	1.2
Aluminium	2700
Copper	8900
Gold	19 300
Hydrogen	0.083
Iron	7900
Lead	11 300
Oxygen	1.3
Silver	10 500
Water	1000

More about units

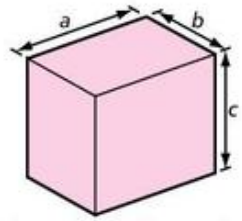
Mass 1 kg = 1000 g

Length 1 m = 100 cm = 1000 mm

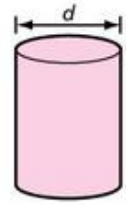
Volume 1 m^3 = 10^6 cm^3

Density $1000\,\text{kg m}^{-3} = \dfrac{10^6\,\text{g}}{10^6\,\text{cm}^3} = 1\,\text{g cm}^{-3}$

Table 1 shows the density of some common substances in kg m^{-3}. You can see that gases are much less dense than solids or liquids.



i Volume of cuboid = $a \times b \times c$



ii Volume of cylinder = $\dfrac{\pi d^2}{4} \times h$

***Fig 1** Volume formulae*

Worked example

Using the data above, calculate:
a the mass, in kilograms, of a piece of aluminium of volume 3.6×10^{-5} m^3,
b the volume, in m^3, of a mass of 0.50 kg of iron.

Solution

a $\rho = 2700\,\text{kg m}^{-3}$; mass $m = \rho V = 2700\,\text{kg m}^{-3} \times 3.6 \times 10^{-5}\,\text{m}^3 = 9.7 \times 10^{-2}\,\text{kg}$

b $\rho = 7900\,\text{kg m}^{-3}$; volume $= \dfrac{m}{\rho} = \dfrac{0.50\,\text{kg}}{7900\,\text{kg m}^{-3}} = 6.3 \times 10^{-5}\,\text{m}^3$

Density measurements

An unknown substance can often be identified if its density is measured and compared with the density of known substances. The following procedures may be used to measure the density of a substance.

- **A regular solid.** Measure its mass using a top-pan balance; measure its dimensions using Vernier calipers or a micrometer. Calculate its volume using the appropriate formula (e.g. volume of a sphere of radius r is $\frac{4}{3}\pi r^3$; volume of a cylinder of radius r and length L is $\pi r^2 L$). Calculate the density from $\dfrac{\text{mass}}{\text{volume}}$.

- **A liquid.** Measure the mass of an empty measuring cylinder. Fill the cylinder with the liquid and measure the volume of the liquid directly. Measure the mass of the cylinder and liquid to enable the mass of the liquid to be calculated. Calculate the density from $\frac{\text{mass of liquid}}{\text{volume}}$.

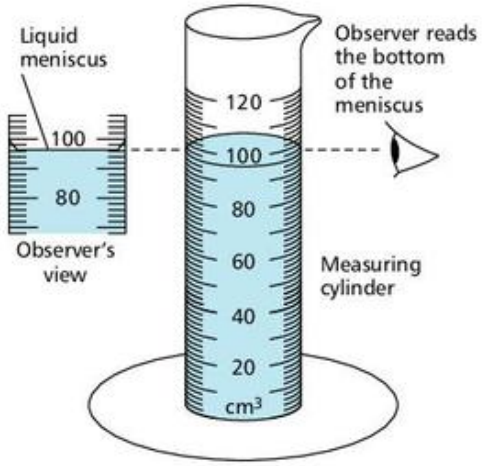


Fig 2 Using a measuring cylinder

- **An irregular solid.** Measure the mass of the object. Fill a displacement can with water up to the spout, as shown in Figure 3. Place a beaker of known mass under the spout. Immerse the object on a thread in the liquid and collect the overflow. Measure the mass of the beaker and overflow. Hence determine the mass of the overflow water and calculate its volume, given the density of water is $1000\,\text{kg m}^{-3}$.
  Calculate the density of the object from $\frac{\text{its mass}}{\text{the overflow volume}}$.

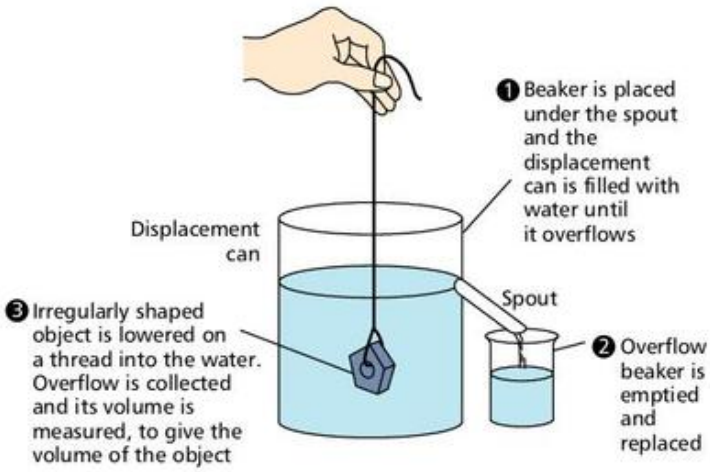


Fig 3 Measuring the volume of an irregularly shaped object

Density of alloys

An **alloy** is a solid mixture of two or more metals. For example, **brass** is an alloy of copper and zinc which has good resistance to corrosion and wear.

For an alloy that consists of two metals A and B, then for volume V of the alloy:

- If the volume of metal A is v_A, the mass of metal A is $\rho_A v_A$, where ρ_A is the density of metal A.
- If the volume of metal B is v_B, the mass of metal B $= \rho_B v_B$, where ρ_B is the density of metal B.

$$\therefore \quad \textbf{Mass of the alloy, } m = \rho_A v_A + \rho_B v_B$$

$$\text{Hence the density of the alloy, } \rho = \frac{m}{V} = \frac{\rho_A v_A + \rho_B v_B}{V} = \frac{\rho_A v_A}{V} + \frac{\rho_B v_B}{V}$$

Worked example

A brass object consists of $3.3 \times 10^{-5}\,\text{m}^3$ of copper and $1.7 \times 10^{-5}\,\text{m}^3$ of zinc. Calculate the mass and the density of this object. The density of copper is $8900\,\text{kg m}^{-3}$; the density of zinc is $7100\,\text{kg m}^{-3}$.

Solution

Mass of copper = density of copper × volume of copper
$= 8900\,\text{kg m}^{-3} \times 3.3 \times 10^{-5}\,\text{m}^3 = 0.29\,\text{kg}$

Mass of zinc = density of zinc × volume of zinc
$= 7100\,\text{kg m}^{-3} \times 1.7 \times 10^{-5}\,\text{m}^3 = 0.12\,\text{kg}$

Total mass, $m = 0.29 + 0.12 = 0.41\,\text{kg}$

Total volume, $V = 5.0 \times 10^{-5}\,\text{m}^3$

Density of alloy, $\rho = \frac{m}{V} = \frac{0.41\,\text{kg}}{5.0 \times 10^{-5}\,\text{m}^{-3}} = 8200\,\text{kg m}^{-3}$

QUESTIONS

1 A rectangular brick of dimensions 5.0 cm × 8.0 cm × 20.0 cm has a mass of 2.5 kg. Calculate:
a its volume,
b its density.

2 An empty paint tin of diameter 0.150 m and of height 0.120 m has a mass of 0.22 kg. It is filled with paint to within 7 mm of the top. Its total mass is then 6.50 kg. Calculate, for the paint in the tin:
a the mass, **b** the volume, **c** the density.

3 A solid steel cylinder has a diameter of 12 mm and a length of 85 mm. Calculate:
a its volume (in m^3),
b its mass (in kg); the density of steel is $7800\,\text{kg m}^{-3}$.

4 An alloy tube of volume $1.8 \times 10^{-4}\,\text{m}^3$ consists, by volume, of 60% aluminium and 40% magnesium.
a Calculate the mass, in the tube, of:
i aluminium, **ii** magnesium.
b Calculate the density of the alloy; the density of aluminium is $2700\,\text{kg m}^{-3}$; the density of magnesium is $1700\,\text{kg m}^{-3}$.

5.2 Pressure

On these pages you will learn to:

- Define and use the pressure equation $p = F/A$
- Derive and use the equation $\Delta p = \rho g \Delta h$
- Describe how to measure gas pressure
- Estimate the work done by a gas from a pressure against volume diagram

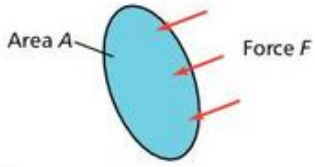


Fig 1 Pressure

Fig 2 A snowmobile

In climates where snow falls, caterpillar tracks on snow vehicles allow the vehicle to travel across snow without sinking into the snow. The tracks have a much greater contact area with the ground than ordinary tyres. Therefore, the pressure of the tracks on snow is much less than the pressure of a tyre would be if the vehicle was fitted with tyres.

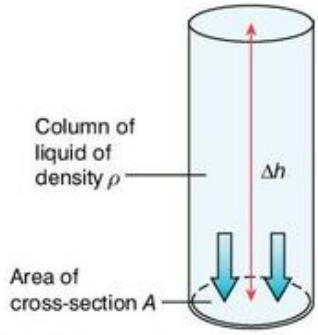


Fig 3 Calculating liquid pressure

Pressure and force

Lots of people need to measure **pressure**. For example, nurses measure blood pressure, motor vehicle technicians measure tyre pressure and gas engineers measure the pressure of the gas supply. In these examples, a liquid or a gas inside a container presses on the container surface wherever the liquid or gas is in contact with the surface. An example of pressure due to a solid is where a brick rests on a surface. The weight of the brick presses down on the surface.

Pressure is defined as the force per unit area acting on a surface perpendicular to the surface.

- The pressure of a force F acting at right angles to a surface of area A is given by the equation:

$$\textbf{Pressure, } p = \frac{F}{A}$$

- The unit of pressure is the **pascal (Pa)**, which is equal to $1\,\text{N m}^{-2}$.

Solid pressure

When a force acts on an object, the smaller the area over which the force acts, the greater the pressure of the force on a surface.

Liquid pressure

For any fluid at rest:

- the pressure at any point acts equally in all directions
- the pressure increases with depth.

The **upthrust** on an object in a fluid is due to the increase of pressure with depth in the liquid. Imagine a ball in a liquid. The average force due to the liquid pressure on its lower half acts upwards. This is greater than the average force on its upper half which acts downwards. The ball therefore experiences an upward resultant force or upthrust due to the pressure of the liquid.

The greater the density of a liquid is, the greater its pressure is at any given depth. Consider the column of liquid in the container shown in Figure 3. The pressure caused by the liquid column on the bottom of the container is due to the weight of the liquid.

For a column of height Δh and area of cross-section A,

- the volume of liquid in the container $= \Delta h A$
- the mass of liquid = its volume × density of the liquid $= \Delta h A \rho$ where ρ is the density of the liquid
- the weight of the liquid = mass × $g = \Delta h A \rho g$ where the **gravitational field strength** $g = 9.8\,\text{N kg}^{-1}$

$$\text{The pressure } p \text{ at the base of the liquid column} = \frac{\text{weight}}{\text{area of cross-section } A}$$

$$p = \frac{h A \rho g}{A} = \boldsymbol{h \rho g}$$

Worked example

$g = 9.8\,\text{N kg}^{-1}$

Calculate the pressure due to sea water of density $1050\,\text{kg m}^{-3}$ at a depth of 200 m.

Solution

pressure $p = h\rho g = 200\,\text{m} \times 1050\,\text{kg m}^{-3} \times 9.8\,\text{N kg}^{-1} = 2.1 \times 10^{6}\,\text{Pa}$

Gas pressure

As explained in topic 5.2, the pressure of a gas is due to molecules repeatedly undergoing elastic collisions with the surface of the container. The pressure of the impacts of the gas molecules on the surface is constant, assuming that the temperature of the gas is constant, and that there are a large number of gas molecules in the container.

The pressure of a gas in a container can be measured using an electronic pressure gauge or a U-tube manometer, as shown in Figure 4. The pressure of the gas forces the liquid in the manometer up the open side of the U-tube until it is at rest.

Because atmospheric pressure acts on the liquid on the 'open' side of the manometer, the gas pressure = the pressure due to the difference in liquid levels ($h\rho g$) + atmospheric pressure, where ρ is the liquid density.

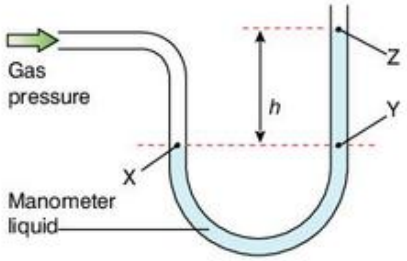


Fig 4 The U-tube manometer

- The 'excess' pressure of the gas (i.e. its pressure above atmospheric pressure) is equal to the pressure due to the difference in the level of liquid on each side.

Therefore, the pressure of the gas relative to atmospheric pressure = $h\rho g$

Atmospheric pressure varies slightly from day to day, changing with the local weather conditions. The mean pressure of the atmosphere at sea level is 101 kPa. Atmospheric pressure decreases with height. The Earth's atmosphere extends more than 100 km into space.

Work done by a gas

In a car engine, the pressure of the hot air in each cylinder forces the piston out of the cylinder. This happens each time the petrol vapour in the air in the cylinder is ignited.

When the piston moves a small distance Δs outwards, as shown in Figure 5,

- the work done W by the air in the cylinder is given by $W = F\Delta s$ where F is the force of the air on the piston,
- the increase of volume of the air in the cylinder, $\Delta V = A\Delta s$ where A is the cross-sectional area of the piston.

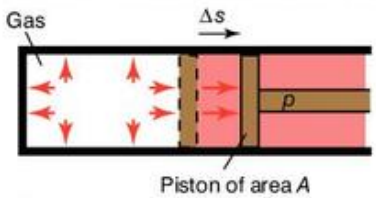


Fig 5 Work done by a gas

Since pressure = force per unit area, then $F = pA$ where p is the air pressure (assumed constant as the movement of the piston is small). Hence the work done $W = F\Delta s = pA\Delta s = p\Delta V$.

More generally, when a gas expands by a volume ΔV at constant pressure p,

the work done by the gas, $W = p\Delta V$

Notes

1. The unit of $p\Delta V$ is the joule, which is equal to the unit of pressure (Pa or $N\,m^{-2}$) multiplied by the unit of volume (m^3).
2. Figure 6 shows a pressure–volume graph for a gas when it expands at constant pressure. The area under the line is a rectangle. This area represents the work done by the gas because the height of the rectangle represents the pressure of the gas and the width represents the change of volume.

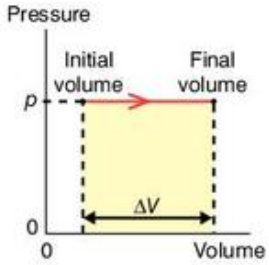


Fig 6

QUESTIONS

$g = 9.8\,m\,s^{-2}$

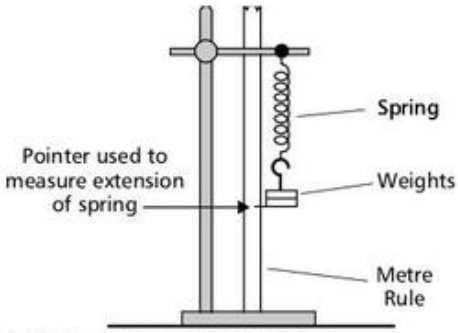
1. Calculate the pressure exerted by a paving stone of density $2500\,kg\,m^{-3}$ and of dimensions $0.80\,m \times 0.80\,m \times 0.05\,m$ when the stone rests flat on a smooth horizontal surface.
2. Calculate the force due to the atmosphere on the panes of a sealed double-glazed window of dimensions $1.5\,m \times 0.80\,m$ which has a vacuum between the two panes. The mean value of atmospheric pressure = 101 kPa.
3. The pressure in the tyres of a vehicle is 280 kPa. The contact area of each of the four tyres on the ground is $0.012\,m^2$. Calculate the weight of the vehicle.
4. The mean value of atmospheric pressure is 101 kPa.
   - **a** The density of water is $1000\,kg\,m^{-3}$. Calculate the depth of water that will give a pressure of 101 kPa.
   - **b** Domestic gas is supplied to homes at a pressure which is normally 2.5% above atmospheric pressure.

     A gas engineer used a water-filled U-tube manometer to measure the gas pressure at a house and observed a difference of 0.15 m between its levels.
     - **i** Calculate the pressure of the gas supply in this house.
     - **ii** State and explain whether or not the gas pressure at the house is normal.

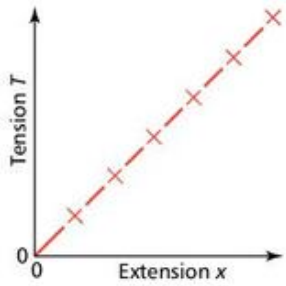
5.3 Springs

On these pages you will learn to:

- State Hooke's law for springs and explain what is meant by the elastic limit and the spring constant of a spring
- Use Hooke's law to solve problems involving springs, including springs in parallel and in series
- Calculate the energy of a stretched spring



a Testing the extension of a spring



b Hooke's law

Fig 1

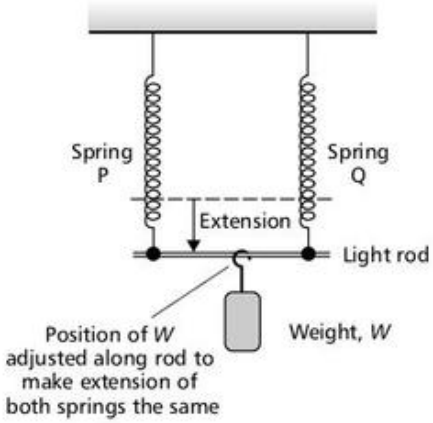


***Fig 2** Two springs in parallel*

Hooke's law

A stretched spring exerts a pull on the object holding each end of the spring. This pull, referred to as the **tension** in the spring, is equal and opposite to the force needed to stretch the spring. The more a spring is stretched, the greater the tension in it. Figure 1a shows a stretched spring at rest supporting a **load** consisting of some weights. This arrangement may be used to investigate how the tension in a spring depends on its extension from its unstretched length. The measurements may be plotted on a graph of tension v. extension (Figure 1b). The graph shows that the force needed to stretch a spring is proportional to the extension of the spring. This is known as **Hooke's law**, after Robert Hooke, a 17th-century scientist.

Hooke's law states that the force needed to stretch a spring is proportional to the extension of the spring from its natural length.

Hooke's law may be written as:

$$\textbf{Force, } F = kx$$

where k is the spring constant (sometimes referred to as the stiffness constant) and x is the extension.

- The greater the value of k, the stiffer the spring is. The unit of k is $\mathrm{N\,m^{-1}}$.
- The graph of F against x is a straight line of gradient k through the origin.
- If a spring is stretched beyond its **elastic limit**, it does not regain its initial length when the force applied to it is removed.
- A level maths students may meet Hooke's law in the form $F = \frac{\lambda x}{L}$ where L is the unstretched length of the spring and λ $(= kL)$ is the spring modulus. λ is not in the specification for A level physics.

Worked example

A vertical steel spring fixed at its upper end has an unstretched length of 300 mm. Its length is increased to 385 mm when a 5.0 N weight attached to the lower end is at rest. Calculate:

a the spring constant,
b the length of the spring when it supports an 8.0 N weight at rest.

Solution

a Use $F = kx$ with $F = 5.0\,\mathrm{N}$ and $x = 385 - 300\,\mathrm{mm} = 85\,\mathrm{mm} = 0.085\,\mathrm{m}$.

Therefore $k = \frac{F}{x} = \frac{5.0\,\mathrm{N}}{0.085\,\mathrm{m}} = 59\,\mathrm{N\,m^{-1}}$

b Use $F = kx$ with $F = 8.0\,\mathrm{N}$ and $k = 59\,\mathrm{N\,m^{-1}}$ to calculate x:

$x = \frac{F}{k} = \frac{8.0\,\mathrm{N}}{59\,\mathrm{N\,m^{-1}}} = 0.136\,\mathrm{m}$

Therefore the length of the spring $= 0.300\,\mathrm{m} + 0.136\,\mathrm{m} = 0.436\,\mathrm{m}$

Springs in parallel

Figure 2 shows a weight W supported by means of two springs, P and Q, in parallel with each other. The extension, x, of each spring is the same. Therefore:

- The force needed to stretch P, $F_\mathrm{P} = k_\mathrm{P}x$
- The force needed to stretch Q, $F_\mathrm{Q} = k_\mathrm{Q}x$, where k_P and k_Q are the spring constants of P and Q respectively.

The weight W is supported by both springs, $W = F_\mathrm{P} + F_\mathrm{Q} = k_\mathrm{P}x + k_\mathrm{Q}x = k_\mathrm{eff}x$ where the **effective spring constant,** $k_\mathrm{eff} = k_\mathrm{P} + k_\mathrm{Q}$

Springs in series

Figure 3 shows a weight W supported by means of two springs joined end-on in 'series' with each other. The tension in each spring is the same and is equal to the weight W. Therefore:

- Extension of spring P, $x_P = \frac{W}{k_P}$
- Extension of spring Q, $x_Q = \frac{W}{k_Q}$, where k_P and k_Q are the spring constants of P and Q respectively.

$$\textbf{Total extension, } x = x_P + x_Q = \frac{W}{k_P} + \frac{W}{k_Q} = \frac{W}{k_{eff}}$$

where k_{eff}, the effective spring constant, is given by the equation $\frac{1}{k_{eff}} = \frac{1}{k_P} + \frac{1}{k_Q}$

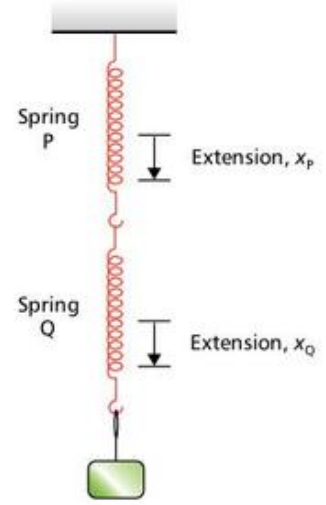


Fig 3 Two springs in series

The energy stored in a stretched spring

Elastic potential energy is stored in a stretched spring. If the spring is suddenly released, the elastic energy stored in it is suddenly converted to kinetic energy of the spring. As explained in topic 2.4, the work done to stretch a spring by extension x_0 from its unstretched length is $\frac{1}{2}F_0x_0$, where F_0 is the force needed to stretch the spring to extension x_0. The work done on the spring is stored as elastic potential energy. Therefore the elastic potential energy E_p in the spring is $\frac{1}{2}F_0x_0$. Also, since $F_0 = kx_0$, where k is the spring constant, then $E_p = \frac{1}{2}kx_0^2$.

Elastic potential energy stored in a stretched spring, $E_p = \frac{1}{2}F_0x_0 = \frac{1}{2}kx_0^2$

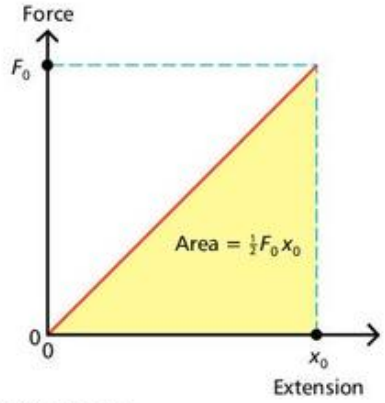


Fig 4 Energy stored in a stretched spring

QUESTIONS

$g = 9.8\,m\,s^{-2}$

1 A steel spring has a spring constant of $25\,N\,m^{-1}$. Calculate:

a the extension of the spring when the tension in it is equal to 10 N,

b the tension in the spring when it is extended by 0.50 m from its unstretched length.

2 Two identical steel springs of length 250 mm are suspended vertically side-by-side from a fixed point. A 40 N weight is attached to the ends of the two springs. The length of each spring is then 350 mm. Calculate:

a the tension in each spring,

b the extension of each spring,

c the spring constant of each spring.

3 Repeat question **2a** and **b** for the two springs in 'series' and vertical.

4 An object of mass 0.150 kg is attached to the lower end of a vertical spring of unstretched length 300 mm, which is fixed at its upper end. With the object at rest, the length of the spring becomes 420 mm as a result. Calculate:

a the spring constant,

b the energy stored in the spring,

c the weight that needs to be added to extend the spring to 600 mm.

5.4 Deformation of solids

On these pages you will learn to:

- Appreciate that deformation is caused by a force and it can be tensile or compressive
- Define stress, strain and the Young modulus and carry out calculations involving these quantities
- Describe an experiment to determine the Young modulus of a metal in the form of a wire
- Use a stress–strain graph to calculate the Young modulus of a material
- Distinguish between elastic and plastic deformation of a material
- Compare the properties of different materials in terms of their stress–strain curves

Force and solid materials

Look around at different materials and think about the effect of force on each material. To stretch, or twist, or compress the material, a pair of forces is needed. For example, stretching a rubber band requires the rubber band to be pulled by a force at either end. Some materials, such as rubber, bend or stretch easily. The **elasticity** of a solid material is its ability to regain its shape after it has been deformed or distorted, and the forces that deformed it have been released. Deformation that stretches an object is **tensile**, whereas deformation that compresses an object is **compressive**.

The arrangement shown in Figure 1a p.78 may be used to test different materials to see how easily they stretch. In each case, the material is held at its upper end and loaded by hanging weights at its lower end. The position of the pointer is measured as the weight of the load is increased in steps then decreased to zero. The extension of the strip of material at each step is its increase of length from its unloaded length. The tension in the material is equal to the weight. The measurements may be plotted as a graph of tension v. extension, as shown in Figure 1.

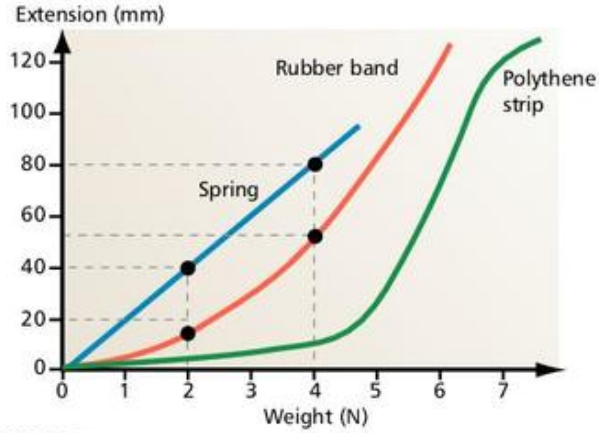


Fig 1 Typical curves

- A steel spring gives a straight line, in accordance with Hooke's law (see topic 5.3).
- A rubber band at first extends easily when it is stretched. However, it becomes 'fully stretched' and very difficult to stretch further when it has been lengthened considerably.
- A polythene strip 'gives' and stretches easily after its initial stiffness is overcome. However, after 'giving', it extends little and becomes difficult to stretch.

Stress and strain

The extension of a wire under tension may be measured using Searle's apparatus (Figure 2). A micrometer attached to the control wire is adjusted so the spirit level between the control and test wire is horizontal. When the test wire is loaded, it extends slightly causing the spirit level to drop on one side. The micrometer is then readjusted to make the spirit level horizontal again. The change of the micrometer reading is therefore equal to the extension. The extension may be measured for different values of tension by increasing the load (i.e. the test weights) in steps. At each step, the tension is equal to the total weight of the load.

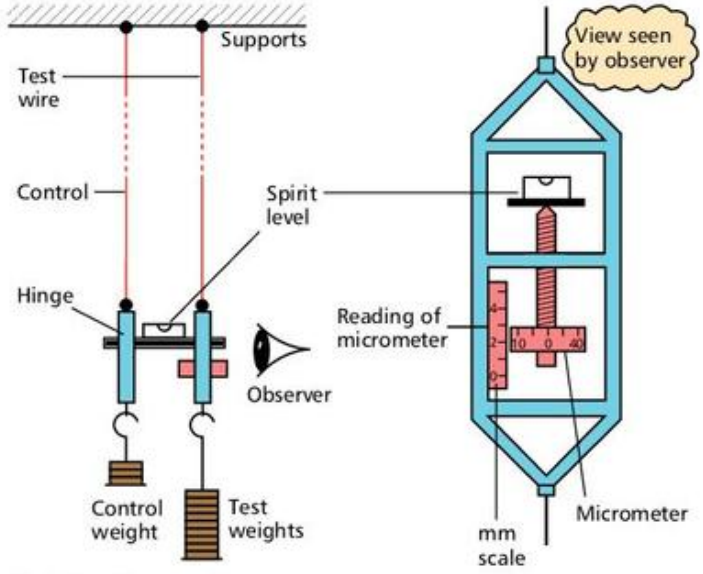


Fig 2 Searle's apparatus

For a wire of length L and area of cross-section A under tension:

- The **tensile stress** in the wire, $\sigma = \frac{F}{A}$, where F is the tension.
  The unit of stress is the **pascal (Pa)**, equal to $1\,\mathrm{N\,m^{-2}}$.
- The **tensile strain** in the wire, $\varepsilon = \frac{e}{L}$, where e is the extension of the wire (i.e.change of length, ΔL). Strain is a ratio and therefore has **no unit**.

Note To calculate the stress in the wire, the diameter d of the wire should be measured then used in the formula $A = \frac{1}{4}\pi d^2$ to calculate A. The stress can then be calculated using $\sigma = F/A$.

Stress strain graphs

Figure 3 shows how the stress in a wire varies with strain.

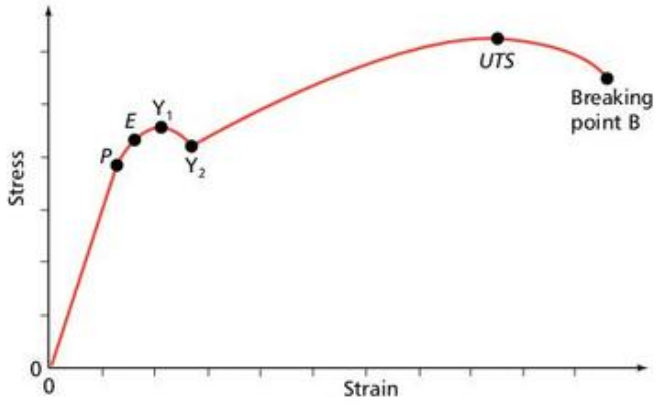


Fig 3 Stress v. strain for a metal wire

- **From 0 to the limit of proportionality P**, the stress is proportional to the strain.

The value of stress/strain is a constant, known as the **Young modulus** of the material:

$$\text{Young modulus, } E = \frac{\text{stress, } \sigma}{\text{strain, } \varepsilon} = \frac{\frac{F}{A}}{\frac{e}{L}} = \frac{FL}{Ae}$$

The unit of E is the pascal (Pa).

Because the line OP is straight and it passes through the origin, the gradient of this section is equal to stress ÷ strain. Hence the Young modulus is equal to the gradient of OP.

To determine the Young modulus of a metal in the form of a wire, the diameter and initial length of the wire are measured using a micrometer and metre ruler. The extension of the wire for different loads is then measured as described opposite. The corresponding stress and strain values can then be calculated and plotted as in Figure 3. The value of E is equal to the gradient of the initial straight section of the graph corresponding to section OP of Figure 3.

- **Beyond P**, the line curves and continues beyond the **elastic limit** E to the **yield point** Y_1, which is where the wire weakens temporarily. The elastic limit is the point beyond which the wire is permanently stretched and suffers **plastic deformation**.
- **Beyond** Y_2, a small increase in the stress causes a large increase in strain as the material of the wire undergoes **plastic flow**. Beyond maximum stress, or the **Ultimate Tensile Stress** (UTS), the wire loses its strength, extends and becomes narrower at its weakest point. Increase of stress occurs at this point due to the reduced area of cross-section, until the wire breaks at B. The stress at B is called the **breaking stress**.

Extension

Stress and strain curves for different materials

- The **stiffness** of different materials can be compared using the **gradient** of the stress-strain line which is equal to the Young modulus of the material. Thus steel is stiffer than copper (Figure 4).
- The **strength** of a material is its Ultimate Tensile Stress (UTS), which is its maximum stress. Steel is stronger than copper because its maximum stress is greater.
- A **brittle** material snaps without any noticeable yield. For example, glass breaks without any 'give'.
- A **ductile** material can be drawn into a wire. Copper is more ductile than steel.

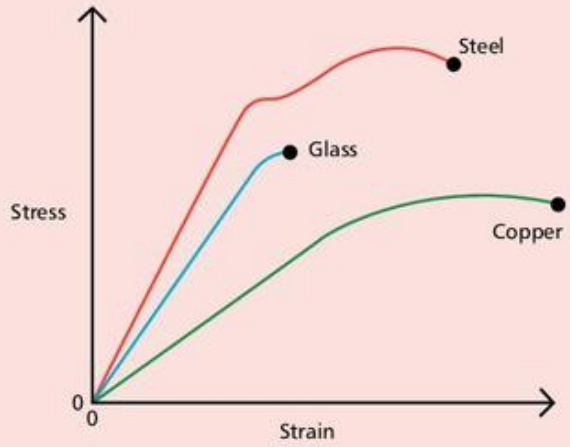


Fig 4 Stress–strain curves

QUESTIONS

1 Calculate the stress in a wire of diameter 0.25 mm when the tension in the wire is 50 N.

2 A metal wire of diameter 0.23 mm and of unstretched length 1.405 m was suspended vertically from a fixed point. When a 40 N weight was suspended from the lower end of the wire, the wire stretched by an extension of 10.5 mm. Calculate the Young modulus of the wire material.

3 A vertical steel wire of length 2.5 m and diameter 0.35 mm supports a weight of 90 N. Calculate:

a the stress in the wire,

b the extension of the wire;
Young modulus of steel = 2.0×10^{11} Pa.

4 Use Figure 4 to decide

a if copper is stronger than glass,

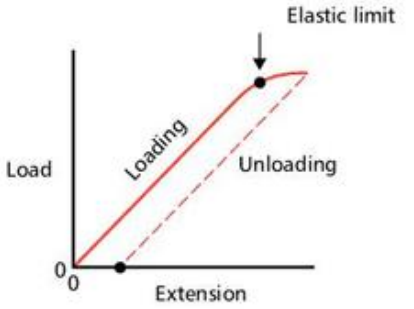
b if glass is stiffer than steel,

c if copper is more ductile than glass.
Give a reason for each answer.

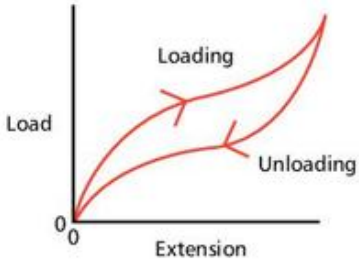
5.5 More about stress and strain

On these pages you will learn to:

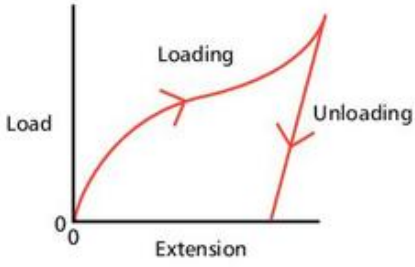
- Compare loading and unloading curves for a material under stress
- Deduce the strain energy in a deformed material from the area under the force–extension graph
- Calculate the elastic energy stored per unit volume in a material when its elastic limit is not exceeded



a *Metal wire*



b *Rubber*



c *Polythene*

Fig 1 *Loading and unloading curves*

Investigating loading and unloading of different materials

How does the **strength** of a material change as a result of being stretched? To investigate this question, the tension in a strip of material is increased by increasing the weight it supports in steps. At each step, the extension of the material is measured. Typical results for different materials are shown in Figure 1. For each material, the loading curve and the subsequent unloading curve are shown.

- For a metal wire, its extension at any given tension when it is being unloaded is exactly the same as when it was being loaded (Figure 1a). The unloading curve and the loading curve are the same straight line. Provided its elastic limit is not exceeded, the wire returns to the same length when it has been completely unloaded as it had before it was loaded. If the wire is stretched beyond its elastic limit, the unloading line is parallel to the loading line. In this case, when it is completely unloaded, the wire is slightly longer so it has a **permanent extension**.
- For a rubber band (Figure 1b), the extension during unloading at any given tension is greater than during loading. The rubber band returns to the same unstretched length, but the unloading curve is below the loading curve except at zero extension and maximum extension. The rubber band remains elastic as it regains its initial length, but it has a **low limit of proportionality**.
- For a polythene strip (Figure 1c), the extension during unloading is also greater than during loading. However, the strip does not return to the same initial length when it is completely unloaded. The polythene strip has a low limit of proportionality and suffers **plastic deformation**.

Strain energy

The area under a graph of force v. extension is equal to the work done to stretch the wire. The work done to deform an object is referred to as **strain energy**. Consider the energy changes for each of the three materials in Figure 1 when each material is loaded then unloaded.

Metal wire (or spring)

Provided the limit of proportionality is not exceeded, the work done to stretch a wire to extension x is $\frac{1}{2}Fx$, where F is the tension in the wire at this extension. Because the elastic limit is not reached, the work done is stored as elastic energy in the wire. Therefore:

$$\textbf{Elastic energy stored in a stretched wire} = \tfrac{1}{2}Fx$$

Because the graph of tension against extension is the same for unloading as for loading, all the energy stored in the wire can be recovered when the wire is unloaded.

Note Since the volume of the wire $= AL$, where A is its cross-sectional area and L is its length, then:

$$\textbf{Elastic energy stored per unit volume} = \frac{1}{2}\frac{Fx}{AL} = \tfrac{1}{2} \times \textbf{stress} \times \textbf{strain}$$

Worked example

A steel wire of uniform diameter 0.35 mm and of length 810 mm is stretched to an extension of 2.5 mm. Calculate: **a** the tension in the wire, **b** the elastic energy stored in the wire.

The Young modulus for steel $= 2.1 \times 10^{11}$ Pa

Solution

a Extension, $x = 2.5\,\text{mm} = 2.5 \times 10^{-3}\,\text{m}$

$$\text{Area of cross-section of wire} = \frac{\pi(0.35 \times 10^{-3})^2}{4}$$

$$= 9.6 \times 10^{-8}\,\text{m}^2$$

To find the tension, rearranging the Young modulus equation $E = \frac{FL}{Ax}$ gives

$$F = \frac{EAx}{L} = \frac{2.1 \times 10^{11} \times 9.6 \times 10^{-8} \times 2.5 \times 10^{-3}}{0.810} = 62\,\text{N}$$

b Elastic energy stored in the wire
$= \frac{1}{2}Fx = 0.5 \times 62 \times 2.5 \times 10^{-3} = 7.8 \times 10^{-2}\,\text{J}$

Rubber band

The work done to stretch the rubber band is represented in Figure 1b by the area under the loading curve. The work done by the rubber band when it is unloaded is represented by the area under the unloading curve. The area between the loading curve and the unloading curve therefore represents the difference between energy stored in the rubber band when it is stretched and useful energy recovered from it when it unstretches. The difference is because some of the energy stored in the rubber band becomes internal energy of the molecules when the rubber band unstretches. Figure 2 shows how the area between the loading and unloading curve can be used to determine the internal energy retained when the rubber band is stretched then released.

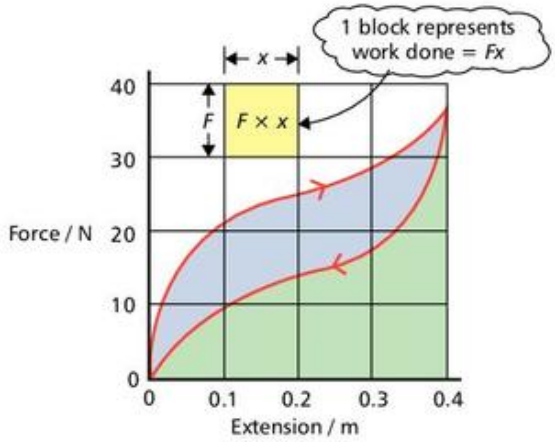


Fig 2 *Energy changes when loading and unloading rubber*

Polythene

As it does not regain its initial length, the area between the loading and unloading curve represents work done to deform the material permanently, as well as internal energy retained by the polythene when it unstretches.

The plastic behaviour of polythene is because polythene is a polymer, so each molecule is a long chain of atoms. Before being stretched, the molecules are tangled together or folded against each other. Weak bonds referred to as **cross-links** form between atoms where molecules are in contact with each other. When placed under tension, a thin sample of polythene easily stretches as the original weak cross-links break and the molecules align parallel to each other. New weak cross-links form in the stretched state and, when the tension is removed, the polythene strip remains stretched.

Extension

Notes Rubber is also a polymer, but its molecules are curled up and tangled together when it is unstretched. When placed under tension, its molecules are straightened out as its length increases more and more. When the tension is removed, its molecules curl up again and it regains its initial length.

A metal in the solid state is **crystalline** because it consists of tiny crystals or **grains** packed together. The atoms in each grain are arranged in layers in a regular pattern as in any crystal. When a metal is stretched beyond its elastic limit the layers of atoms in some of the grains slide past each other. When the distorting forces are removed, the displaced atoms do not return to their original places.

Solids such as glass are **amorphous**, which is neither crystalline nor polymeric. The atoms in an amorphous solid are randomly arranged without any regular pattern. Glass is brittle because stress causes any tiny cracks at its surface to propagate through the material.

QUESTIONS

Young modulus for steel $= 2.1 \times 10^{11}$ Pa; Young modulus for copper $= 1.3 \times 10^{11}$ Pa

1 A vertical steel cable of diameter 24 mm and of length 18 m supports a weight of 1500 N attached to its lower end. Calculate:

- **a** the tensile stress in the cable,
- **b** the extension of the cable,
- **c** the elastic energy stored in the cable, assuming its elastic limit has not been reached.

2 A vertical steel wire of diameter 0.28 mm and of length 2.0 m is fixed at its upper end and has a weight of 15 N suspended from its lower end. Calculate:

- **a** the extension of the wire,
- **b** the elastic energy stored in the wire.

3 A steel bar of length 40 mm and cross-sectional area $4.5 \times 10^{-4}\,\text{m}^2$ is placed in a vice, and compressed by 0.20 mm when the vice is tightened. Calculate:

- **a** the compressive force exerted on the bar,
- **b** the work done to compress it.

4 Figure 2 shows a force v. extension curve for a strip of rubber. Use the graph to determine:

- **a** the work done to stretch the rubber to an extension of 0.40 m,
- **b** the internal energy retained by the rubber when it unstretches.

5 End of chapter questions

$g = 9.81\,\text{m s}^{-2}$

1 A uniform copper wire of length 2.50 m has a diameter of 0.32 mm. Calculate:

a the volume of the wire,

b the mass of the wire,

c the mass per unit length of the wire.

Density of copper = $8900\,\text{kg m}^{-3}$

2 A hydraulic lift is used in a garage to raise a vehicle to enable its underside to be repaired. The figure below shows how such a lift works. The maximum pressure of the fluid in the hydraulic lift is 1.2 MPa.

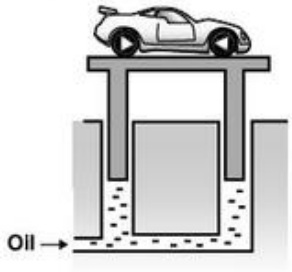


Fig 2.1

a The area of cross-section of each of the four 'legs' of the lift is $9.0 \times 10^{-3}\,\text{m}^2$. Calculate the maximum safe load the lift can raise if the lift platform has a weight of $1.5 \times 10^4\,\text{N}$.

b The lift is used to raise a vehicle of mass 2200 kg through a height of 2.4 m.

i Calculate the gain of potential energy of the vehicle.

ii Calculate the work done by the hydraulic system to lift the vehicle by 2.4 m.

iii Account for the difference between your answers to **i** and **ii**.

3 A steel spring of length 300 mm fixed at its upper end hangs vertically. When a 4.0 N weight is suspended from its lower end, the spring extends to an equilibrium length of 420 mm.

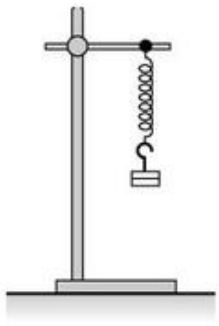


Fig 3.1

a Calculate:

i the spring constant of this spring,

ii the length of the spring when it supports a weight of 6.0 N.

b A second identical spring is suspended from the lower end of the first spring. Calculate the weight that would need to be suspended on the end of the lower spring to give a total extension for both springs equal to 90 mm.

4 **a** With the aid of a suitable example, explain what is meant by:

i a brittle material,

ii a ductile material.

b A steel guitar wire of diameter 0.28 mm and of length 800 mm is tightened by turning its tension key by 2 turns, each turn increasing the length of the wire by 1.8 mm. Calculate the increase of tension in the wire as a result.
Young modulus of steel = $2.1 \times 10^{11}\,\text{N m}^{-2}$

5 **a** Explain what is meant by the following terms:

i the elastic limit of a strip of material,

ii plastic behaviour.

b **i** Sketch a graph to show how the extension of a rubber band varies with the force used to stretch it.

ii When a car is in motion, each part of its tyres that make contact with the road is squashed and stretched as it passes through the point of contact with the road. In energy terms, explain why this repeated squashing and stretching causes the tyre to become warm.

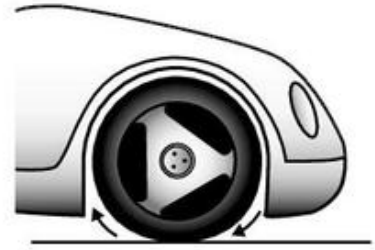


Fig 5.1

6 A crane is designed to lift a load of maximum weight of 9000 N to a maximum height of 65 m. The crane cable has an area of cross-section of $4.5 \times 10^{-4}\,\text{m}^2$ and is attached to a pulley block and hook of weight 600 N, so that the load is supported by two lengths of the crane cable (as shown in the figure below).

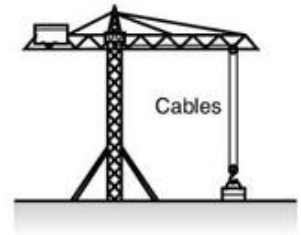


Fig 6.1

a Calculate the maximum weight of the suspended crane cable, pulley block and hook when the hook is just off the ground.

b **i** Calculate the increase of stress in the crane cables when a load of 9000 N is lifted off the ground.

ii Show that the extension of each of the two lengths of the crane cable when the load is raised off the ground is 3.3 mm. Density of steel = 8000 kg m^{-3}; Young modulus of steel = 2.0×10^{11} N m^{-2}

iii Calculate the elastic energy stored in the two crane cables due to this extension.

7 The figure below shows a graph of the force used to stretch a metal wire against the extension of the wire for a wire of length 1620 mm and of cross-sectional area 1.40×10^{-7} m^2.

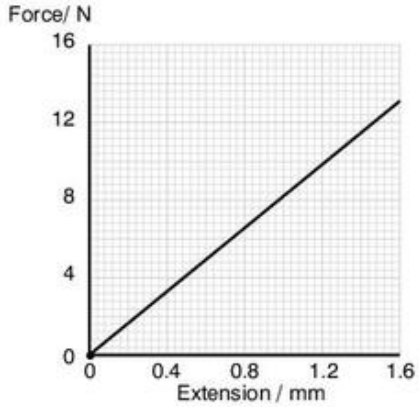


Fig 7.1

a Calculate the Young modulus of the wire.

b Calculate the energy stored in the wire when it extended from its unstretched length by 1.60 mm.

8 A spring having spring constant k hangs vertically from a fixed point. A load of weight L, when hung from the spring, causes an extension e. The elastic limit of the spring is not exceeded.

a State

i what is meant by an *elastic deformation*, *(2 marks)*

ii the relation between k, L and e. *(1 mark)*

b Some identical springs, each with spring constant k, are arranged as shown in Figure 8.1.

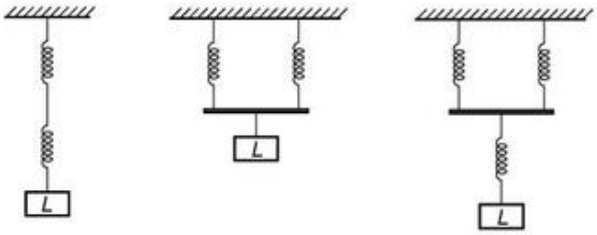


Fig 8.1

The load on each of the arrangements is L.

For each arrangement in Figure 8.1, determine

i the total extension in terms of e,

ii the spring constant in terms of k. *(5 marks)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 21 Q4 June 2009

9 a The variation with extension x of the tension F in a spring is shown in Figure 9.1.

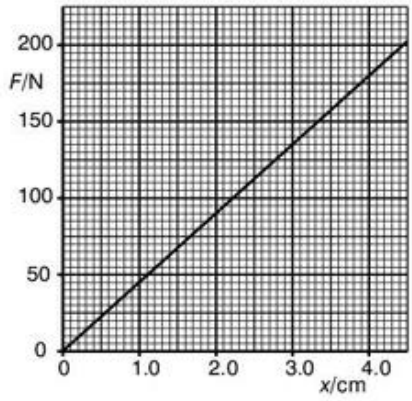


Fig 9.1

Use Figure 9.1 to calculate the energy, in J, stored in the spring for an extension of 4.0 cm. Explain your working. *(3 marks)*

b The spring in **a** is used to join together two frictionless trolleys A and B of mass M_1 and M_2 respectively, as shown in Figure 9.2.

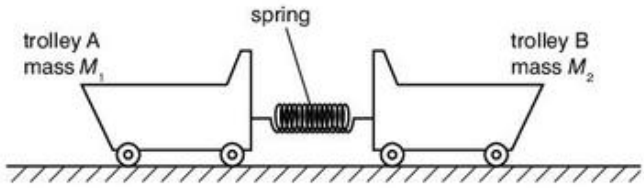


Fig 9.2

The trolleys rest on a horizontal surface and are held apart so that the spring is extended.

The trolleys are then released.

i Explain why, as the extension of the spring is reduced, the momentum of trolley A is equal in magnitude but opposite in direction to the momentum of trolley B. *(2 marks)*

ii At the instant when the extension of the spring is zero, trolley A has speed V_1 and trolley B has speed V_2. Write down

1. an equation, based on momentum, to relate V_1 and V_2, *(1 mark)*

2. an equation to relate the initial energy E stored in the spring to the final energies of the trolleys. *(1 mark)*

iii 1. Show that the kinetic energy E_k of an object of mass m is related to its momentum p by the expression

$$E_k = \frac{p^2}{2m}. \qquad \textit{(1 mark)}$$

2. Trolley A has a larger mass than trolley B. Use your answer in **ii part 1** to deduce which trolley, A or B, has the larger kinetic energy at the instant when the extension of the spring is zero. *(1 mark)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 21 Q3 June 2010

Key concepts: Forces

In studying chapters 1 to 5, you will have gained a wide appreciation of what forces do, as well as a deeper understanding of how physical principles relating to forces are combined with mathematical methods to predict and test their effects. The effects can then be compared with the predictions to find out if the mathematical methods give results consistent with observations. Such methods, developed by Galileo, Newton and other scientists, form the basis of many branches of physics and design engineering, from tunnels under the ground to robot vehicles on Mars. In addition, you should know now what we mean by 'conservation of energy', how we can use equations to calculate kinetic energy and potential energy changes and what we mean by 'efficiency' in relation to energy transfers. Momentum is another physical quantity you should now be familiar with – in particular, how you can use the relationship between force and momentum to calculate impact forces and what is meant by 'conservation of momentum' and how this important principle is used to work out what happens when objects collide or explode.

As you progressed through chapters 1–5, you will have developed your mathematical skills to a level where you can use equations and graphs to solve physical problems. In addition, you should now be able to carry out calculations accurately and express your answers in standard form in the correct units. You should also know that numerical answers should not be expressed to more significant figures than can be justified by the data provided.

Throughout the first five chapters, you will have had opportunities to develop your practical skills through measuring time intervals, distance, mass, weight and volume. Some practical experiments will have required you to use your measured data to calculate physical quantities such as density, enabling you to compare the accuracy of your results with accepted values. In addition, you should have become aware that each measurement you make has a degree of uncertainty and that uncertainties need to be estimated and used to give an overall uncertainty in a calculated quantity. Also, you will have plotted graphs, in particular straight-line graphs that can be linked to theoretical equations, from which the links can be tested. In addition, straight-line graphs can be used to determine physical quantities with less uncertainty than if a single set of measurements were used.

The following table provides an overview of the topics in chapters 1 to 5, showing where the key concepts have been developed.

Chapters 1–5: Key concepts

	Topic		Key concepts	
1.1	introduction to vectors	use of vectors to add displacement and to add forces	use of maths	problem-solving
1.2–1.7	motion along a straight line	use of dynamics equations and graphs to solve mechanics problems involving motion along a straight line	use of maths	problem-solving, including the physical significance of gradients and areas under graphs
1.5	free fall	Galileo's inclined plane test	testing	concept of acceleration
		measurement of g	forces and fields	graph of $s = \frac{1}{2}gt^2$ to determine g
1.8–1.9	projectile motion	investigating projectile paths	testing	concept of vertical and horizontal motion
		calculations of vertical and horizontal motion	use of maths	problem-solving
2.1–2.3	force and motion	investigating force and acceleration	testing	verification of $F = ma$
		calculations using $F = ma$	use of maths	problem-solving using relevant equations
		resistive forces and terminal speed	forces and fields	concept of terminal speed
2.4–2.6	energy and power	calculations using the equations for kinetic energy, potential energy, power and work	energy and matter	conservation of energy
2.7–2.8	efficiency	causes of inefficiency and energy losses		concept of dissipative forces
3.1–3.6	forces in equilibrium	principle of moments	forces and fields	problem-solving using key principles regarding equilibrium
		conditions for equilibrium	use of maths	
4.1–4.5	momentum	Newton's second law of motion	forces and fields	problem-solving using $Ft = \Delta mv$
		momentum and impact forces conservation of momentum	use of maths	problem-solving using conservation of momentum
5.1–5.3	properties of materials	density	energy and matter	practical skills, density measurements,
		pressure, Hooke's law	testing	Hooke's law
		calculations and graphs involving density, pressure and Hooke's law	use of maths	problem-solving
5.4–5.5	deformation of solids	stress, strain	energy and matter	practical skills
		Young modulus	testing	measurement of Young modulus
		calculations and graphs involving Young modulus	use of maths	problem-solving

Question

This question is about an investigation into the flight path of a ball projected horizontally from the edge of a table. It tests how we use models to make predictions about a physical system and how we then test these predictions with an experimental investigation. Notice also the use of mathematics to express the relationship between two perpendicular distances, x *and* y*. In this case, the predicted relationship is of the form* $y = kx^2$*. So by plotting a graph of* y *against* x^2*, we can tell if the relationship is valid by looking at whether or not the graph is linear (i.e. a straight-line graph) and passes through the origin. If the graph is linear, we can also determine the constant* k*, as it is equal to the gradient of the graph. You will notice as well that this investigation is about the behaviour of a particle in a field: in this case a uniform gravitational field. The path of such a particle is similar to that of a charged particle in a uniform electric field, because in both cases the acceleration of the particle is constant in magnitude and direction.*

Before answering the question, students should be given the opportunity to do this or a similar investigation.

1 A student wanted to measure the path of a small steel ball projected horizontally. Figure 1 shows the arrangement he used. The thin metal bar B was positioned at different horizontal distances from the edge of a flat laboratory bench. For each distance, the vertical position of the bar was adjusted until it was positioned so the ball hit the bar after being released from rest inside the top of the cardboard tube T.

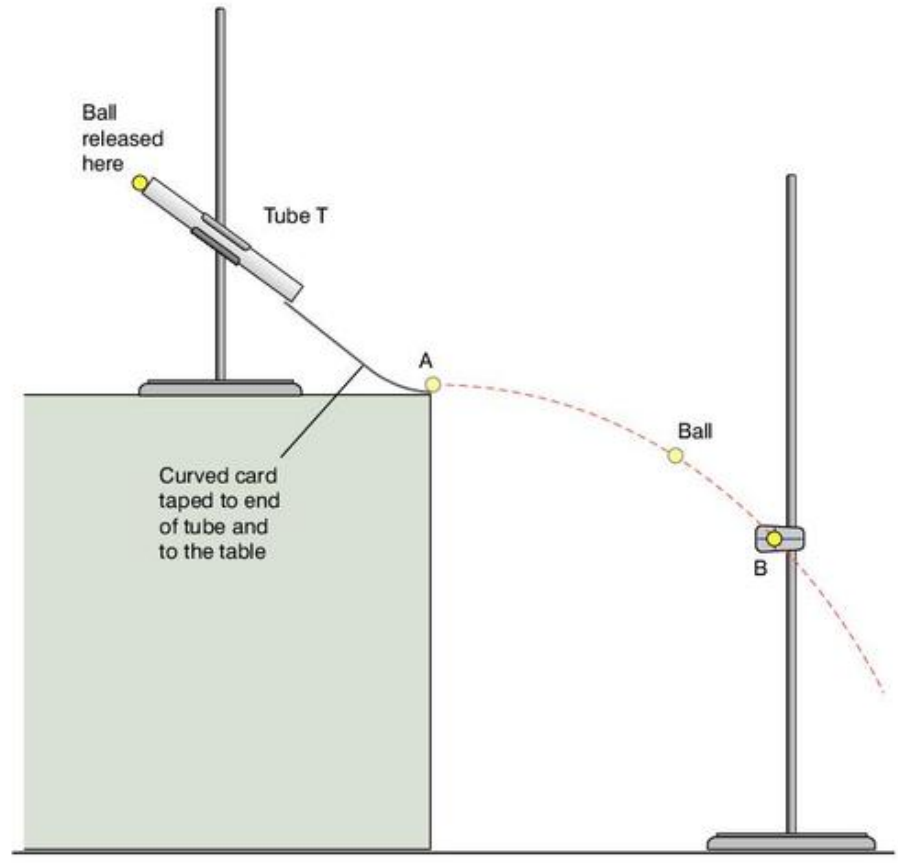


Fig 1

a i Why was it important to ensure the ball was released at rest from the same position each time the test was carried out?

ii Give two reasons why a large steel ball would have been unsuitable.

(3 marks)

b For each horizontal distance x from B to the point A where the ball left the bench, x was measured and the corresponding vertical distance y was measured in three separate tests. The results obtained are shown below.

x / m	0	0.205	0.402	0.596	0.803	0.995	1.205
y / mm	0	19	78	155	272	432	643
	0	20	65	164	277	438	631
		22	72	158	286	429	639
mean value of y / mm							
x^2 / m^2							

i Copy and complete the table of results above.

ii The student reckoned that at time t after the ball left the bench, $y = \frac{1}{2} g t^2$ and $x = Ut$ where U is the horizontal speed of projection of the ball. Use these equations to show that $y = kx^2$, where k is a constant, and derive an expression for k in terms of U and g.

iii Plot a graph of y against x^2 to find out if the student's hypothesis is correct.

iv Use the graph to determine U, the horizontal speed of projection of the ball. *(13 marks)*

c The student reckoned the distances were measured to within 2 mm. Discuss this claim in terms of the above measurements. *(4 marks)*

6 Electric current

6.1 Electric charge

On these pages you will learn to:

- Explain how to create and detect static charge
- Explain what is meant by an electric field and a field line

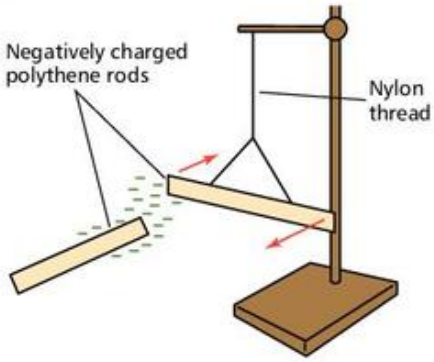


Fig 1 Electrostatic forces

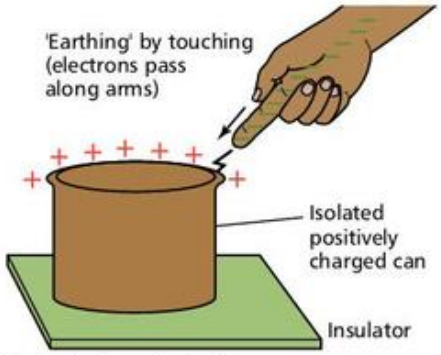


Fig 2 Discharge to Earth

Fig 3 The electroscope

Static electricity

Most plastic materials can be charged quite easily by rubbing with a dry cloth. When charged, they attract small bits of paper. Do charged pieces of plastic material attract one another? Figure 1 shows an arrangement to test for attraction. A charged perspex ruler will attract a charged polythene comb, but two charged rods of the same material always repel one another.

There are just two types of charge which are referred to as positive and negative charge. A charged object will always:

- repel another charged object that has the same type of charge (i.e. like charge)
- attract another charged object that has the other type of charge (i.e. unlike charge).

Like charges repel; Unlike charges attract

Electrons are responsible for charging in most situations. An uncharged atom contains equal numbers of **protons** and electrons. An electron carries a tiny negative charge. A proton carries a tiny positive charge equal in magnitude to that of the electron.

- Add one or more electrons to an uncharged atom and it becomes negatively charged.
- Remove one or more electrons from an uncharged atom and it becomes positively charged.

An uncharged solid contains equal numbers of electrons and protons. When an uncharged perspex rod is rubbed with an uncharged dry cloth, electrons transfer from the rod to the cloth so the rod becomes positively charged and the cloth becomes negatively charged.

- Electrical conductors such as metals contain lots of free electrons. These are electrons which move about inside the metal and are not attached to any one atom. To charge a metal up, it must first be isolated from the Earth. Otherwise, any charge given to it is neutralised by electrons transferring between the conductor and the Earth. Then the isolated conductor can be charged by direct contact with any charged object. If an isolated conductor is charged positively then 'earthed', electrons transfer from the Earth to the conductor to neutralise or discharge it. See Figure 2.
- Insulating materials do not contain free electrons. All the electrons in an insulator are attached to individual atoms. Some insulators, such as perspex or polythene, are easy to charge because their surface atoms easily gain or lose electrons.

The gold leaf electroscope is used to detect charge. If a charged object is in contact with the metal cap of the electroscope, some of the charge on the object transfers to the electroscope. As a result, the gold leaf and the metal stem which is attached to the cap gain the same type of charge and the leaf rises because it is repelled by the stem.

If another object with the same type of charge is brought near the electroscope, the leaf rises further because the object forces some charge on the cap to transfer to the leaf and stem.

Field lines and patterns

Any two charged objects exert equal and opposite forces on each other without being directly in contact. An **electric field** is said to surround each charge. Suppose a small positive charge is placed as a test charge near a body with a much bigger charge which is also positive. If the test charge is free to move, it will follow a path away from the body with the bigger charge. The path a free test charge follows is called a line of force or a field line.

The direction of an electric field line is the direction a positive test charge would move along. Figure 4 shows the patterns of fields around different charged objects. Each pattern is produced by semolina grains sprinkled on oil. An electric field is set up across the surface of the oil by connecting two metal conductors in the oil to the output terminals of a high voltage supply unit. The grains line up along the field lines, like plotting compasses in a magnetic field.

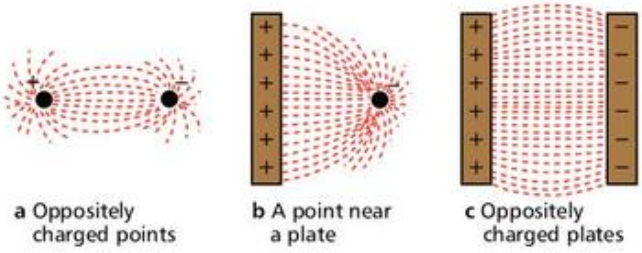


Fig 4 Electric field patterns

- Oppositely charged point objects create a field as shown in Figure 4a. The field lines become concentrated at the points. A positive test charge released from an off-centre position would follow a curved path to the negative point charge.
- A point object near an oppositely charged flat plate produces a field as shown in Figure 4b. The field lines are concentrated at the point object but they meet the plate at right angles to it.
- Two oppositely charged plates create a field as shown in Figure 4c. The field lines run parallel from one plate to the other, meeting the plates at right angles. The field is **uniform** between the plates as the field lines are parallel to each other.

Devices that use electric fields directly include:

- electrostatic precipitators which extract ash, dust and other fine particles from gases released in power stations and factories. The gases pass through a grid of wires at high voltage between earthed metal plates fixed vertically, as shown in Figure 5. The particles become charged as they pass through the grid and are then attracted onto the plates by the electric field between the grid wires and the plates.
- electrostatic sprays such as airborne crop sprays which produce charged droplets that spread out as they repel each other and are attracted onto the ground. Industrial paint sprays are used to coat metal panels with an even film of paint.

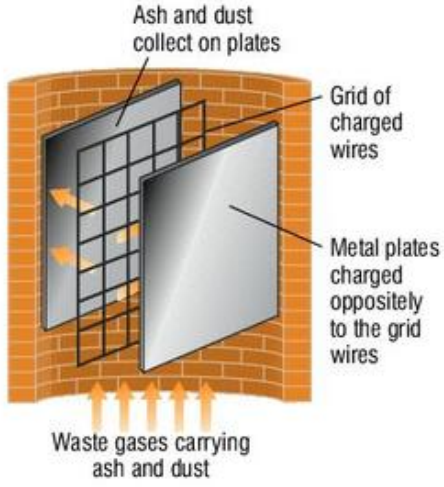


Fig 5 An electrostatic precipitator

QUESTIONS

1 Explain each of the following observations in terms of transfer of electrons:
   - **a** An insulated metal can is given a positive charge by touching it with a positively charged rod.
   - **b** A negatively charged metal sphere suspended on a thread is discharged by connecting it to the ground using a wire.

2 An insulated metal conductor is earthed before a negatively charged object is brought near to it.
   - **a** Explain why the free electrons in the conductor move as far away from the charged object as they can.
   - **b** The conductor is then briefly earthed. The charged object is then removed from the vicinity of the conductor. Explain why the conductor is left with an overall positive charge.

3 A positively charged point object is placed near an earthed metal plate, as shown in Figure 6.

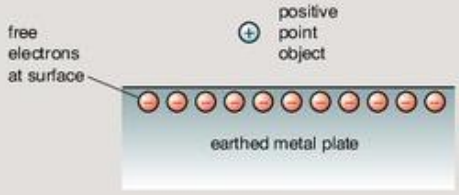


Fig 6

   - **i** Explain why electrons gather at the surface of the metal plate near the object.
   - **ii** Explain why there is a force of attraction between the object and the metal plate.

4 Sketch the pattern of the field lines of the electric field:
   - **i** between two oppositely charged parallel plates,
   - **ii** between a positively charged point object and an earthed metal plate.

6.2 Current and charge

On these pages you will learn to:

- Recognise that an electric current is a flow of charge due to the passage of charged particles
- Know that the charge on a charge carrier is quantised and that the charge carriers in a metal are electrons
- Distinguish between a conductor, an insulator and a semiconductor
- State and use the relationship between the current and the charge flow in a certain time
- Derive and use $I = Anvq$, where n is the number density of charge carriers

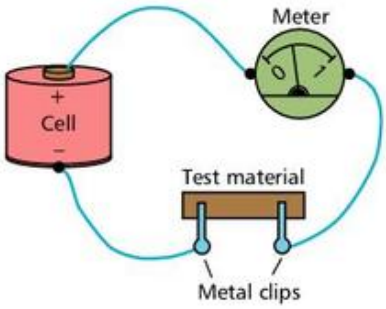


Fig 1 *Testing for conduction*

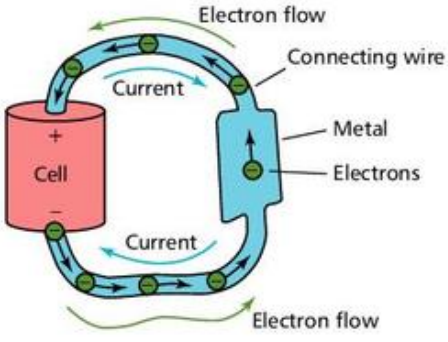


Fig 2 *Convention for current*

Electrical conduction

An electric current is a flow of charge due to the passage of charged particles. These charged particles are referred to as **charge carriers**.

- In metals, the charge carriers are **conduction electrons**. They move about inside the metal, repeatedly colliding with each other and the fixed positive ions in the metal.
- In comparison, when an electric current is passed through a salt solution, the charge is carried by ions which are charged atoms or molecules.
- Charge is quantised, which means it is only found in whole number multiples of 1.6×10^{-19}C. This basic amount is referred to as the 'quantum' of charge, denoted by the symbol e. For example, the charge on a calcium Ca^{2+} ion is equal to $+2e$.

A simple test for conduction of electricity is shown in Figure 1. If the test material is a metal, electrons pass round the circuit: leaving the battery at its negative terminal, then passing though the metal, then re-entering the battery at its positive terminal.

The convention for the direction of current in a circuit is from + **to** −, as shown in Figure 2. The convention was agreed long before the discovery of electrons. When it was set up, it was known that an electric current is a flow of charge one way round a circuit. However, it was not known if the current was due to positive charge flowing round the circuit from + to −, or if it was due to negative charge flowing from − to +.

- The unit of current is the **ampere (A)**, which is defined in terms of the force between two parallel wires when they pass the same current. See topic 18.3.
- The unit of charge is the **coulomb (C)**, equal to the charge flow in one second when the current is one ampere. The magnitude of the charge of the electron, e, is 1.6×10^{-19}C. This is sometimes referred to as the **elementary charge**.
- For a current I, the **charge flow** ΔQ in time Δt is given by the equation:

$$\Delta Q = I\Delta t$$

- For charge flow ΔQ in a time interval Δt, the current I is given by:

$$I = \frac{\Delta Q}{\Delta t}$$

The equation shows that a current of 1 A is due to a flow of charge of 1 coulomb per second. As the charge of the electron is 1.6×10^{-19}C, a current of 1 A along a wire must be due to 6.25×10^{18} electrons passing along the wire each second.

More about charge carriers

Conductors, insulators and semiconductors

Materials can be classified in electrical terms as conductors, insulators or semiconductors.

- In an **insulator**, each electron is attached to an atom and cannot move away from the atom. When a voltage is applied across an insulator, no current passes through the insulator because no electrons can move through the insulator.
- In a **metallic conductor**, most electrons are attached to atoms; but some are not and these are the **charge carriers** in the metal. When a voltage is applied across the metal, these **conduction electrons** move towards the positive terminal of the metal.
- In a **semiconductor**, the number of charge carriers increases with increase of temperature. The resistance of a semiconductor therefore decreases as its

temperature is raised. A pure semiconducting material is referred to as an **intrinsic semiconductor**, because conduction is due to electrons that break free from the atoms of the semiconductor.

- In an **electrolyte**, the charge carriers are **positive and negative ions**. When a voltage is applied to the electrodes, the positive ions are attracted to the **cathode** (i.e. the negative electrode) and the negative ions are attracted to the **anode** (i.e. the positive electrode). The ions are discharged on reaching the relevant electrode. The process is known as **electrolysis**.

Drift velocity

The charge carriers in a conductor or a semiconductor move about at random inside the material. When a p.d. is applied across the material, in addition to their random motion, the charge carriers 'drift' to the positive end. Their mean **displacement** per unit time is their **drift velocity**. The current through a material depends on the drift velocity v of the charge carriers.

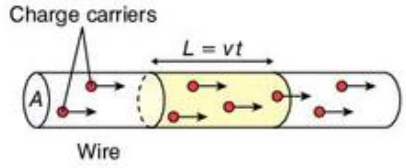


Fig 3 Drift velocity

Consider Figure 3, which shows charge carriers moving through a section of wire of cross-sectional area A and length L. The average time taken, t, for each charge carrier to move through it $= L \div v$.

If there are n charge carriers per unit volume in the wire,

- the number of charge carriers in this section $= n(A \times L)$ since the section volume $= A \times L$
- their total charge $Q =$ number of charge carriers $\times$ charge of each carrier (q)
  $= n(A \times L)q$

Therefore, the current through the section, $I = \dfrac{Q}{t} = \dfrac{nALq}{L/v} = nAvq$

For electrons, $q = e$ gives $\boldsymbol{I = nAve}$

Physics and the Human Genome Project

To map the human genome, fragments of DNA are tagged with a C, or G, or A, or T base containing a dye. Each tagged fragment carries a negative charge. A voltage is applied across a strip of gel containing a spot of liquid containing tagged fragments. The fragments are attracted to the positive electrode. The smaller the fragment, the faster it moves; so the fragments separate out according to size, as they move to the positive electrode. The fragments pass through a spot of laser light, which causes the dye attached to each tag to **fluoresce** as it passes through the laser spot. Light sensors linked to a computer detect the glow from each tag. The computer is programmed to work out and display the sequence of bases in the DNA fragments.

Fig 4 Mapping the human genome

QUESTIONS

$e = 1.6 \times 10^{-19}$ C

1 **a** The current in a certain wire is 0.35 A. Calculate the charge passing a point in the wire:
i in 10 s, **ii** in 10 min.

b Calculate the average current in a wire through which a charge of 15 C passes in:
i 5 s, **ii** 100 s.

2 Calculate the number of electrons passing a point in the wire in 1 min when the current is:
a 1 μA **b** 5.0 A

3 A certain type of rechargeable battery is capable of delivering a current of 0.2 A for 4000 s, before its voltage drops and it needs to be recharged. Calculate:
a the total charge the battery can deliver before it needs to be recharged,
b the maximum time it could be used for without being recharged if the current through it were:
i 0.5 A **ii** 0.1 A

Extension

4 Estimate the drift velocity of the conduction electrons in a copper wire of diameter 0.4 mm when a current of 2.0 A passes through it. Assume copper contains 10^{29} electrons per m^3.

6.3 Potential difference and power

On these pages you will learn to:

- Define potential difference (p.d.) and the volt
- Define the electromotive force (e.m.f.) of an electrical source
- Distinguish between e.m.f. and p.d. in terms of energy considerations
- Recall and use $V = W/Q$
- Recall and use $P = VI$

Energy and potential difference

When a torch bulb is connected to a battery, electrons deliver energy from the battery to the torch bulb. Each electron which passes through the bulb takes a fixed amount of energy from the battery and delivers it to the bulb. After delivering energy to the bulb, each electron re-enters the battery via the positive terminal to be resupplied with more energy to deliver to the bulb.

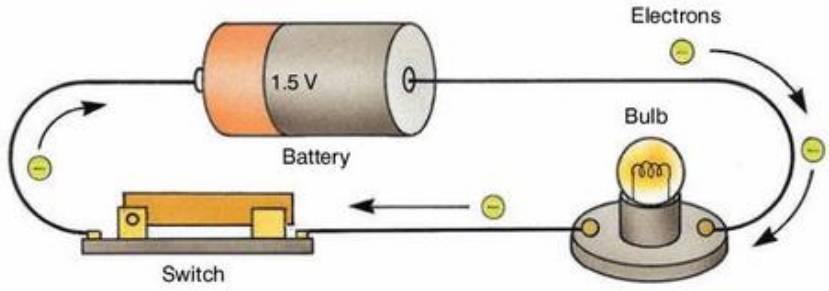


Fig 1 *Energy transfer by electrons*

Each electron in the battery has the **potential** to deliver energy, even if the battery is not part of a complete circuit. In other words, the battery supplies each electron with **electrical potential energy**. When the battery is in a circuit, each electron passing through a circuit component does work to pass through the component and therefore uses some or all of its electrical potential energy. The **work done** by an electron is equal to its loss of potential energy. The work done per unit charge is defined as the **potential difference** (abbreviated as p.d.) or **voltage**, ***V***, across the component.

Potential difference is defined as the work done (or energy transfer) per unit charge.

The unit of p.d. is the **volt (V),** which is equal to 1 joule per coulomb.

If work W is done when charge Q flows through the component, the p.d. across the component, V, is given by:

$$V = \frac{W}{Q}$$

Rearranging this equation gives $W = QV$ for the work done or energy transfer when charge Q passes through a component which has a p.d. V across its terminals.

Examples

- If 30 J of work is done when 5 C of charge passes through a component, the p.d. across the component must be 6 V $\left(= \frac{30\,\text{J}}{5\,\text{C}}\right)$.
- If the p.d. across a component in a circuit is 12 V, then 3 C of charge passing through the component would transfer 36 J of energy from the battery to the component.

Fig 2 *Electrical devices*

Energy transfer in different devices

An electric current has a **heating effect** in a component with **resistance**. It also has a **magnetic effect**, which is made use of in electric motors and loudspeakers.

- In a device that has resistance, such as an electrical heater, the work done on the device is transferred as **thermal energy**. This happens because the charge carriers repeatedly collide with atoms in the device and transfer energy to them, so the atoms **vibrate more** and the resistor becomes hotter.
- In an electric motor, the work done on the motor is transferred as **kinetic energy** of the motor. The charge carriers are electrons that need to be forced through the wires of the **spinning motor coil**, against the opposing force on the electrons due to the motor's magnetic field.
- For a loudspeaker, the work done on the loudspeaker is transferred as **sound energy**. Electrons need to be forced through the wires of the **vibrating loudspeaker coil**, against the force on them due to the loudspeaker magnet.

Electromotive force (e.m.f)

The e.m.f., E, of a source of electricity is defined as the electrical energy per unit charge produced inside the source.

The unit of e.m.f. is the **volt (V)**, the same as the unit of p.d., equal to 1 joule per coulomb.

For a source of e.m.f., E, in a circuit, the electrical energy produced when charge Q passes through the source is QE. This energy is transferred to other parts of the circuit, and some may be dissipated in the source itself due to the source's internal resistance (see topic 7.3 for internal resistance).

Fig 3 Power supplies

Electrical power and current

Consider a component, or device, which has a potential difference V across its terminals and a current I passing through it. In time Δt:

- The charge flowing through it, $Q = I\Delta t$
- The work done by the charge carriers, $W = QV = (I\Delta t)V = IV\Delta t$

Work done, $W = IV\Delta t$

The energy transfer ΔE in the component or device is equal to the work done W. Therefore, because power $= \frac{\text{energy}}{\text{time}}$, the electrical power P supplied to the device is given by:

$$P = \frac{IV\Delta t}{\Delta t} = IV$$

Electrical power, $P = IV$

Notes

- This equation can be rearranged to give: $I = \frac{P}{V}$ or $V = \frac{P}{I}$
- The unit of power is the **watt (W)**. Therefore one volt is equal to one watt per ampere. For example, if the p.d. across a component is 4 V, then the power delivered to the component is 4 W per ampere of current.
- Energy supplied by mains electricity is measured in **kilowatt hours (kW h)**, usually referred to as 'units'. One kilowatt hour is the energy transfer when 1 kW of power is supplied for exactly 1 h.

Therefore 1 kW h = 3.6 MJ (= 1000 W × 3600 s).

- The correct value of a **fuse** for an electrical appliance can be worked out using the equation $I = \frac{P}{V}$ if the power and voltage of the appliance are known.
  For example, a 230 V, 2.5 kW electric kettle would take a current of 11 A $\left(= \frac{2500\text{ W}}{230\text{ V}}\right)$. Therefore, given the choice between a 5 A fuse or a 13 A fuse, a 13 A fuse should be used.

Worked example

A 12 V, 48 W electric heater is connected to a 12 V battery. Calculate:

a the heater current, **b** the energy transfer in 300 s.

Solution

a Rearrange $P = IV$ to give $I = \frac{P}{V} = \frac{48\text{ W}}{12\text{ V}} = 4\text{ A}$

b $\Delta E = IV\Delta t = 4\text{ A} \times 12\text{ V} \times 300\text{ s} = 14\,400\text{ J}$

QUESTIONS

1 Calculate the energy transfer in 1200 s in a component when the p.d. across it is 12 V and the current is:
   a 2 A **b** 0.05 A

2 A 6 V, 12 W light bulb is connected to a 6 V battery. Calculate:
   a the current in the light bulb,
   b the energy transfer to the light bulb in 1800 s.

3 A 230 V electrical appliance has a power rating of 800 W.
   a Calculate:
   i the energy transfer in the appliance in 1 min,
   ii the current in the appliance.
   b Which of the following fuse values would be suitable for this appliance: 3 A, 5 A, or 13 A?

4 A battery has an e.m.f. of 9 V and a negligible internal resistance. It is capable of delivering a total charge of 1350 C. Calculate:
   a the maximum energy the battery could deliver,
   b the power it would deliver to the components of a circuit if the current in it was 0.5 A,
   c how long the battery would last for if it was to supply power at the rate calculated in part **b**.

6.4 Resistance

On these pages you will learn to:

- Define resistance and the ohm
- Recall and use $V = IR$
- Sketch and discuss the I–V characteristics of a metallic conductor at constant temperature, a semiconductor diode and a filament lamp
- State Ohm's law
- Recall and use $R = \rho L/A$ in resistivity calculations

Table 1 *Reminder about prefixes*

Prefix	Symbol	Value
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
kilo	k	10^{+3}
mega	M	10^{+6}
giga	G	10^{+9}

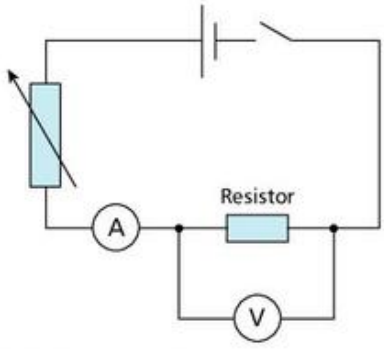


Fig 1 *Measuring resistance*

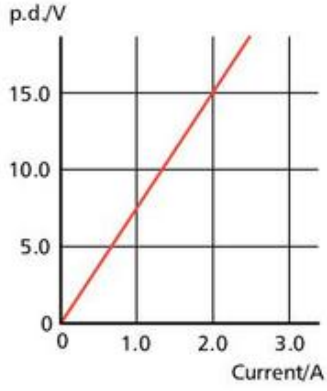


Fig 2 *Potential difference v. current for a resistor*

Definitions and laws

The **resistance** of a component in a circuit is a measure of the difficulty of making current pass through the component. Resistance is caused by the repeated collisions between the charge carriers in the material with each other and with the fixed positive ions of the material.

$$\textbf{Resistance of any component} = \frac{\textbf{p.d. across the component}}{\textbf{current through it}}$$

For a component in which the current is I when the p.d. across it is V, its resistance R is given by the equation:

$$R = \frac{V}{I}$$

The unit of resistance is the **ohm (Ω)** which is equal to 1 volt per ampere.

Rearranging the above equation gives $V = IR$ or $I = \frac{V}{R}$.

Worked example

The current through a component is 2.0 mA when the p.d. across it is 12 V. Calculate:

a its resistance at this current,
b the p.d. across the component when the current is 50 μA, assuming its resistance is unchanged.

Solution

a $R = \frac{V}{I} = \frac{12}{2.0 \times 10^{-3}} = 6000\,\Omega$

b $V = IR = 50 \times 10^{-6} \times 6000 = 0.30\,\text{V}$

Measurement of resistance

A **resistor** is a component designed to have a certain resistance which is the same regardless of the current through it. The resistance of a resistor can be measured using the circuit shown in Figure 1.

- The **ammeter**, A, is used to measure the current in the resistor. The ammeter must be in **series** with the resistor so the same current passes through both the resistor and the ammeter.
- The **voltmeter**, V, is used to measure the p.d. across the resistor. The voltmeter must be in **parallel** with the resistor so that they have the same p.d. Also, **no current** should pass through the voltmeter, otherwise the ammeter will not record the exact current in the resistor. In theory, the voltmeter should have **infinite resistance**. In practice, a voltmeter with a sufficiently high resistance would be satisfactory.
- The **variable resistor** is used to adjust the current and p.d. as necessary. To investigate the variation of current with p.d., the variable resistor is adjusted in steps. At each step, the current and p.d. are recorded from the ammeter and voltmeter respectively. The measurements can then be plotted on a graph of p.d. against current, as shown in Figure 2.

Ohm's law

The graph for a resistor is a straight line through the origin. The resistance is the same, regardless of the current. The resistance is equal to the gradient of the graph, because the gradient is constant and at any point is equal to p.d./current.

The discovery that the potential difference across a metal wire is proportional to the current through it was made by Georg Ohm in 1826 and is known as Ohm's law.

Ohm's law states that the potential difference across a metallic conductor is proportional to the current through it, provided the physical conditions do not change.

Notes

Ohm's law is equivalent to the statement that the resistance of a metallic conductor under **constant physical conditions** (e.g. temperature) is constant.

For an **ohmic conductor**, $V = IR$ where R is constant.

Resistivity

For a conductor of length L and uniform cross-sectional area A, as shown in Figure 3, its resistance R is:

- proportional to L,
- inversely proportional to A.

Hence $R = \frac{\rho L}{A}$, where ρ is a constant for that material known as its **resistivity**.

Rearranging this equation gives the following equation, which can be used to calculate the resisitivity of a sample of material of length L and uniform cross-sectional area A:

$$\text{Resistivity, } \rho = \frac{RA}{L}$$

Notes

- The unit of resisitivity is the **ohm metre (Ωm)**.
- For a conductor with a circular cross-section of diameter d:

$$A = \frac{\pi d^2}{4} \left(= \pi r^2 \text{ where radius } r = \frac{d}{2} \right)$$

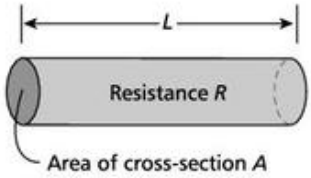


Fig 3 Resistivity

Table 2 Resistivity values of different materials at room temperature

Material	Resistivity/Ωm
Copper	1.7×10^{-8}
Constantan	5.0×10^{-7}
Carbon	3×10^{-5}
Silicon	2300.00
PVC	about 10^{14}

QUESTIONS

1 a Complete the table by calculating the missing value for each resistor:

	1	2	3	4	5
Current	2.0A	0.45A		5.0mA	
Potential difference	12.0V		5.0V	0.80V	50kV
Resistance		22Ω	40kΩ		20MΩ

b Use Figure 2 to find the resistance of the resistor that gave the results shown.

2 Calculate the resistance of a uniform wire of diameter 0.32mm and length 5.0m. The resistivity of the material = 5.0×10^{-7}Ωm.

3 Calculate the resistance of a rectangular strip of copper of length 0.08m, thickness 15mm and width 0.80mm. The resistivity of copper = 1.7×10^{-8}Ωm.

4 A wire of uniform diameter 0.28mm and length 1.50m has a resistance of 45Ω.
Calculate:

a its resistivity,

b the length of this wire that has a resistance of 1.0Ω.

6.5 Components and their characteristics

On these pages you will learn to:

- Recall the function and circuit symbols for commonly-used components, including a diode, an LED, a thermistor and an LDR
- Draw and interpret circuit diagrams with the above components in them
- Describe how a potential divider is used to supply a variable p.d.
- Describe how the resistance of a metal and a semiconductor thermistor changes with temperature

Circuit diagrams

Each type of component has its own **symbol**, which is used to represent the component in a **circuit diagram**. You need to recognise the symbols for different types of components to make progress – just like a motorist needs to know what different road signs mean. Note that, on a circuit diagram, the direction of the current is always shown from + **to** − round the circuit.

The function of each of the components shown in Figure 1 is given in the list below:

- An **ammeter** measures the current in a circuit or a branch of a circuit.
- A **voltmeter** measures the p.d. between two points in a circuit.
- A **cell** is a source of electrical energy. Note that **a battery** is a combination of cells.
- The symbol for an **indicator** or any **light source** (including a filament lamp), except a light-emitting diode, is the same.
- A **diode** allows current in one direction only. A **light-emitting diode** (or LED) emits light when it conducts. The direction which the diode conducts in is referred to as its 'forward' direction. The opposite direction is referred to as its 'reverse' direction.
- A **resistor** is a component designed to have a certain resistance.
- A **variable resistor** is used to control the current in a circuit.
- The resistance of a **thermistor** decreases with increase of temperature, if the thermistor is an intrinsic semiconductor such as silicon. Such a thermistor is described as a **negative temperature coefficient** (n.t.c.) thermistor.
- The resistance of a **light-dependent resistor** decreases with increase of light intensity.
- A **heater** is designed to transform electrical energy to heat.
- An **electric motor** is designed to transform electrical energy into kinetic energy.

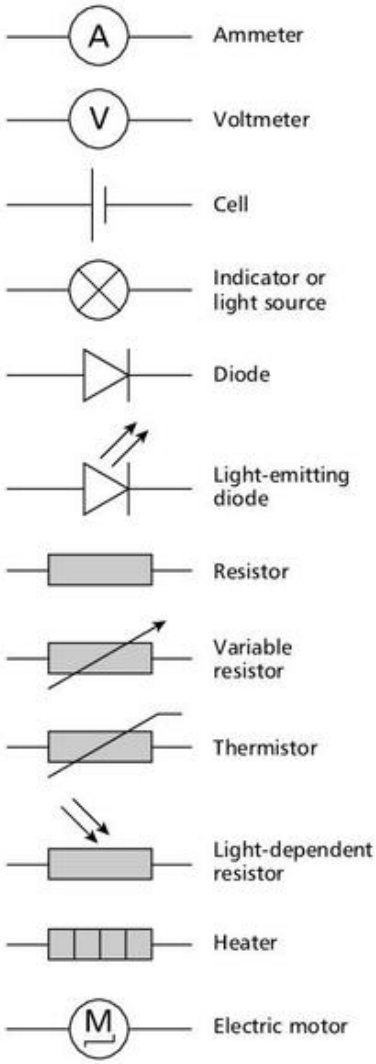
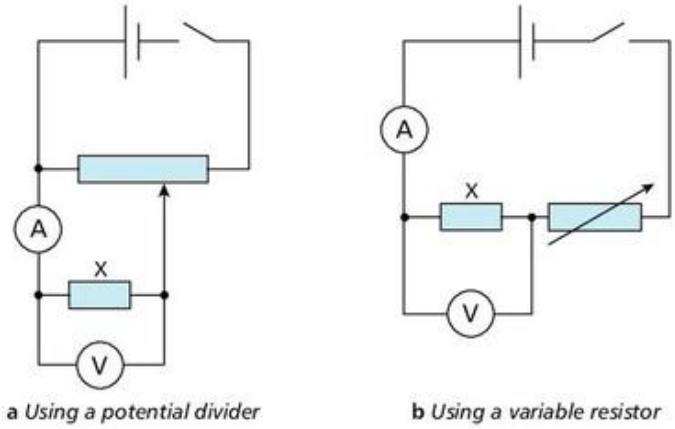


Fig 1 *Circuit components*

Investigating the characteristics of different components

To measure the variation of current with p.d. for a component, use either:

- a potential divider to vary the p.d. from zero (Figure 2a), or
- a variable resistor to vary the current to a minimum (Figure 2b).



a *Using a potential divider* **b** *Using a variable resistor*

Fig 2 *Investigating component characteristics*

The advantage of using a potential divider in comparison with a variable resistor is that the current in the component and the p.d. across it can be reduced to zero. However, in the variable resistor circuit, when the resistance of the variable resistor is at a maximum, the current cannot be reduced any further unless a further resistor (not shown) is connected in series with the variable resistor.

The measurements for each type of component may be plotted on a graph of current (on the *y*-axis) against p.d. (on the *x*-axis). Typical graphs for a wire, a filament lamp, and a thermistor are shown in Figure 3. Note that the measurements are the same, regardless of which way the current passes through each of these components.

- A **wire** gives a straight line, with a constant gradient equal to 1/resistance *R* of the wire. Any resistor at constant temperature would give a straight line.
- A **filament bulb** gives a curve with a decreasing gradient, because its resistance increases as it becomes hotter.
- A **thermistor** at constant temperature gives a straight line. The higher the temperature, the greater the gradient of the line as the resistance falls with increase of temperature. The same result is obtained for a light-dependent resistor (LDR) in respect of light intensity.

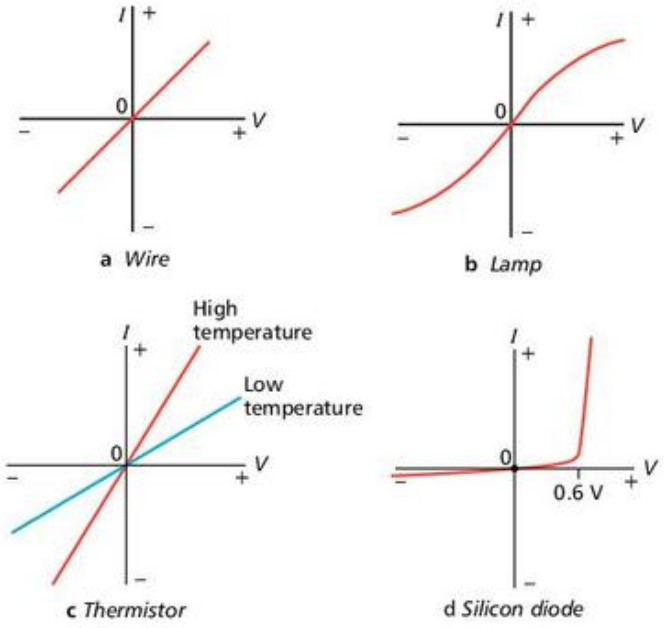


Fig 3 *Current v. potential difference for various components*

The diode

To investigate the characteristics of the **diode**, one set of measurements needs to be made with the diode in its 'forward direction' (i.e. forward biased) and another set with it in its 'reverse direction' (i.e. reverse biased). The current is very small when the diode is reverse biased and can only be measured using a milliammeter.

Typical results for a silicon diode are shown in Figure 3d. A silicon diode conducts easily in its 'forward' direction above a p.d. of about 0.6 V, and hardly at all below 0.6 V or in the opposite direction.

Resistance and temperature

- **The resistance of a metal** increases with increase of temperature. This is because the positive ions in the conductor vibrate more when its temperature is increased. The charge carriers (i.e. conduction electrons) therefore cannot pass through the metal as easily when a p.d. is applied across the conductor. A metal is said to have a **positive temperature coefficient** because its resistance increases with increase of temperature.
- **The resistance of a thermistor made from an intrinsic semiconductor** decreases with increase of temperature, as shown in Figure 4. This is because the number of charge carriers (i.e. conduction electrons) increases when the temperature is increased. A thermistor made from an intrinsic semiconductor therefore has a **negative temperature coefficient**.

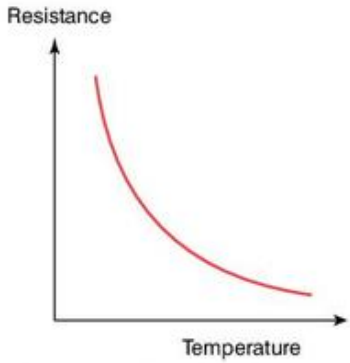


Fig 4 *Resistance v. termperature for a thermistor*

QUESTIONS

1 A filament bulb is labelled '3.0 V, 0.75 W'.
- **a** Calculate its current and its resistance at 3.0 V.
- **b** State and explain what would happen to the filament bulb if the current was increased from the value in part **a**.

2 A certain thermistor has a resistance of 50 000 Ω at 20 °C and a resistance of 4000 Ω at 60 °C. It is connected in series with an ammeter and a 1.5 V cell. Calculate the ammeter reading when the thermistor is:
- **a** at 20 °C
- **b** at 60 °C

3 A silicon diode is connected in series with a cell and a torch bulb:
- **a** Sketch the circuit diagram showing the diode in its 'forward' direction.
- **b** Explain why the torch bulb would not light if the polarity of the cell was reversed in the circuit.

4 The resistance of a certain metal wire increased from 25.3 Ω at 0 °C to 35.5 Ω at 100 °C. Assuming the resistance over this range varies linearly with temperature, calculate:
- **a** the resistance at 50 °C,
- **b** the temperature when the resistance is 30.0 Ω.

6.6 Electric field strength

On these pages you will learn to:

- Define electric field strength and use $F = QE$
- Recall and use $E = V/d$ to calculate the field strength of the uniform field between charged parallel plates

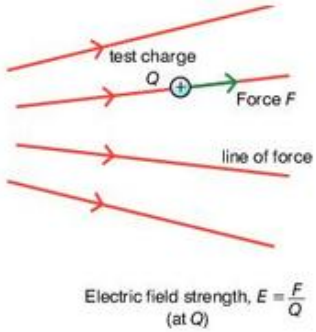


Fig 1 *Electric field strength*

Fig 2 *Lightning*

NOTE

The unit of *E* may be written as the newton per coulomb ($N\,C^{-1}$) or the volt per metre ($V\,m^{-1}$).

The link between the two can be seen because $F = QE = \frac{QV}{d}$.

Rearranging this equation gives $\frac{F}{Q} = \frac{V}{d}$ $(= E)$. Therefore, the newton per coulomb and the volt per metre are both acceptable as the unit of E.

Inside an electric field

A charged object in an electric field experiences a force due to the field. Provided the object's size and charge are both sufficiently small, the object may be used as a 'test' charge to measure the strength of the field at any position in the field.

> The electric field strength, ***E***, at a point in the field is defined as the force per unit charge on a positive test charge placed at that point.

The unit of E is the newton per coulomb ($N\,C^{-1}$).

In Figure 1, a positive test charge Q at a certain point in an electric field is acted on by force F due to the electric field. The electric field strength, E, at that point is given by the equation

$$E = \frac{F}{Q}$$

Notes

1 Rearranging this equation gives $F = QE$ for the force F on a test charge Q at a point in the electric field where the electric field strength is E.

2 Electric field strength is a vector which is in the same direction as the force on a positive test charge. In other words, the direction of a field line at any point is the direction of the electric field strength at that point. The force on a small charge in an electric field is
   - in the same direction as the electric field if the charge is positive,
   - in the opposite direction to the electric field if the charge is negative.

The lightning conductor

In a thunderstorm, a lightning strike to the ground occurs when a cloud becomes more and more charged and the electric field in the air becomes stronger and stronger. The insulating property of air suddenly breaks down and a massive discharge of electric charge occurs between the cloud and the ground.
A lightning conductor is a metal rod at the top of a tall building. The rod is connected to the ground by means of a thick metal conductor. When a charged cloud is overhead, it creates a very strong electric field near the tip of the conductor. The field ionises molecules near the tip by pulling electrons out of the molecules. The ions discharge the thundercloud so no lightning strike occurs.

The electric field between two parallel plates

Figure 4c in topic 6.1 shows that the field lines (mapped out by semolina grains) between two oppositely charged flat conductors are parallel to each other and at right angles to the plates. The field pattern for two oppositely charged flat plates is similar, as shown in Figure 3 opposite. The field lines are:

- parallel to each other,
- at right angles to the plates,
- from the positive plate to the negative plate.

The field between the plates is **uniform**. This is because the electric field strength has the same magnitude and direction everywhere between the plates. The electric field strength E can be calculated from the potential difference V between the plates and their separation d using the equation

$$E = \frac{V}{d}$$

To prove this equation, consider a small charge Q between the plates, as in Figure 3.

1 The force F on a small charge Q in the field is given by $F = QE$, where E is the electric field strength between the plates.
2 If the charge is moved from the positive to the negative plate, the work done W by the field on Q is given by W = force $F \times$ distance moved, $W = QEd$
3 By definition, the potential difference between the plates, V is the work done per unit charge when a small charge is moved through potential difference V.

Therefore, $V = \frac{W}{Q} = \frac{QEd}{Q} = Ed$ so rearranging $V = Ed$ gives $E = \frac{V}{d}$

Notes

A charged particle in a uniform electric field experiences a constant acceleration. Its motion is similar to that of a small object in a small region of the Earth's gravitational field where g is constant. The projectile equations from topic 1.8 can be used, provided g is replaced by acceleration $a = QE/m$ where m is the mass of the object and Q is its charge.

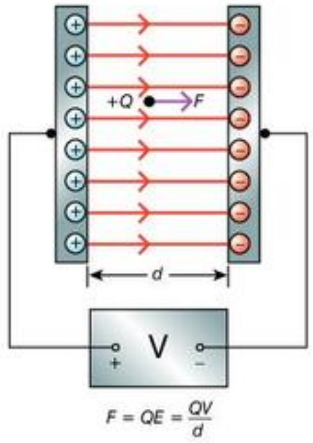


Fig 3 The electric field strength between two parallel plates

QUESTIONS

$e = 1.6 \times 10^{-19}$ C

1 A +40 nC point charge Q_1 is placed in an electric field.
  a Calculate the magnitude of the force on Q_1 if the electric field strength where Q_1 is placed is 3.5×10^4 V m^{-1}.
  b Q_1 is moved to a different position in the electric field. The force on Q_1 at this position is 1.6×10^{-3} N. Calculate the magnitude of the electric field strength at this position.

2 Figure 4 shows the path of a charged dust particle in an electric field.
  a The electric field strength at X is 65 kV m^{-1}. The force due to the field on the particle when it is at X is 8.2×10^{-3} N towards the metal surface.
    i What type of charge does X carry?
    ii Calculate the charge carried by the particle.
  b i Calculate the magnitude of the force on the particle when it is at Y where the electric field strength is 58 kV m^{-1}.
    ii State the direction of the force on the particle when it is at Y.

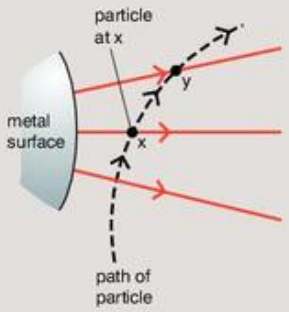


Fig 4

3 A high voltage supply unit is connected across a pair of parallel plates which are at a separation of 50 mm.
  a The voltage is adjusted to 4.5 kV. Calculate:
    i the electric field strength between the plates,
    ii the electric force on a droplet in the field that carries a charge of 8.0×10^{-19} C.
  b The separation between the plates is altered without changing the p.d. between the plates. The droplet in **a** is now acted on by a force of 4.5×10^{-14} N. Calculate the new separation between the plates.

4 A certain gas in a tube is subjected to an electric field of increasing strength. The gas becomes conducting when the electric field reaches a strength of 35 kV m^{-1}.

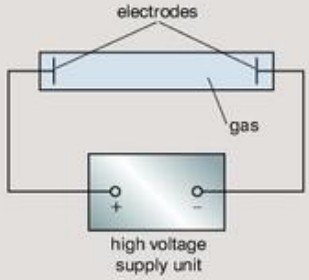


Fig 5

  a The electrodes in the tube are at a spacing of 84 mm. Assuming the field between the electrodes is uniform before the gas conducts, calculate the potential difference between the electrodes that is necessary to produce an electric field of strength 35 kV m^{-1} in the tube.
  b i Calculate the force on an electron in the tube when the electric field strength is 35 kV m^{-1}
    ii Explain why the gas becomes conducting only when the electric field strength in the tube reaches a certain value.

6 End of chapter questions

1 A rechargeable battery is capable of delivering a current of 0.25 A for 1800 s at a potential difference of 1.5 V. Calculate:

a the total charge the battery is capable of delivering without being recharged,

b the energy transferred from the battery as a result of delivering the charge calculated in part **a**.

2 A torch bulb is marked with the label '0.5 W, 1.5 V'. Calculate:

a the current in the torch bulb when it operates at normal brightness,

b the energy transferred to the torch bulb in 5 min when it operates at normal brightness.

3 a With the aid of a circuit diagram, describe how you would investigate the variation of p.d. with current for a 12 V filament light bulb.

b i Sketch a graph to show how the p.d. varies with current for a filament light bulb.

ii Describe how the resistance of a filament light bulb changes with brightness.

iii Explain why the resistance of a filament light bulb changes in the way described in part **ii**.

4 A 3.0 V battery, a diode, a switch, an ammeter and a 200 Ω resistor are connected in series with each other.

a Sketch the circuit diagram for this arrangement with the diode in the forward direction.

b When the switch is closed, the ammeter records a current of 0.012 A. Calculate:

i the p.d. across the resistor,

ii the p.d. across the diode.

c The 200 Ω resistor was replaced by a 50 Ω resistor. Calculate the ammeter reading when the switch is closed.

5 a With the aid of a suitable circuit diagram, describe how you would measure the resistivity of a uniform wire of known cross-sectional area.

b A wire of length 1.20 m and area of cross-section $6.0 \times 10^{-7}\,m^2$ has a resistance of 1.1 Ω.

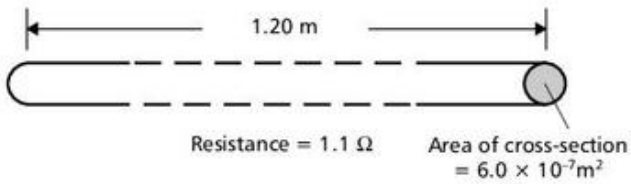


Fig 5.1

Calculate:

i the resistivity of the material of this wire,

ii the resistance of a wire consisting of this material of length 2.40 m and area of cross-section $3.0 \times 10^{-7}\,m^2$.

6 A thermistor, a 100 Ω resistor, a 9.0 V battery and a 0–100 mA ammeter are connected in series:

a Sketch the circuit diagram for this arrangement.

b i The ammeter reads 50 mA. State and explain how the ammeter reading would change if the temperature of the thermistor, which has a negative temperature coefficient, was raised.

ii What change would need to be made to the circuit to reduce the current to 20 mA without changing the temperature of the thermistor?

7 A copper wire in a domestic electric circuit has a diameter of 1.1 mm and a length of 12 m. The resistivity of copper = $1.7 \times 10^{-8}\,\Omega m$.

a Calculate:

i the resistance of this wire,

ii the p.d. across the ends of the wire when the current through it is 13 A,

iii the power dissipated by the wire due to its resistance when a current of 13 A is in it.

b Electrical safety regulations require that the p.d. across the ends of the wire should not exceed 6.0 V. Calculate the maximum safe current through the wire.

8 a Calculate the resistance of a 230 V, 100 W filament light bulb.

b The filament of the light bulb in part **a** consists of a coiled tungsten wire of diameter 0.05 mm. Calculate the total length of the wire of this filament. The resistivity of tungsten = $5.6 \times 10^{-8}\,\Omega m$

9 A light-dependent resistor has a resistance of 650 Ω in darkness. It is connected in series with a 4.5 V battery, a resistor, a milliammeter and a switch, as shown below.

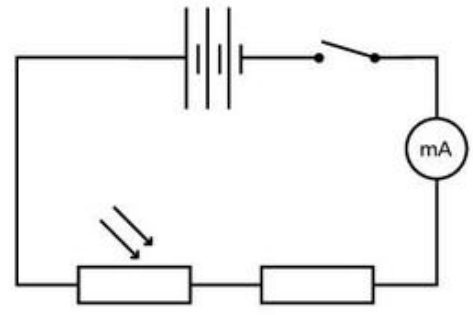


Fig 9.1

a The milliammeter reads 5.0 mA when the switch is closed and the LDR is in darkness. Calculate:
i the p.d. across the LDR,
ii the p.d. across the resistor,
iii the resistance of the resistor.

b Describe how, and explain why, the milliammeter reading changes when the LDR is exposed to light.

10 The following measurements were made in an investigation to measure the resisitivity of the material of a certain wire:

P.d. across the wire/V	0.00	2.0	4.0	6.0	8.0	10.0
Current through the wire/A	0.00	0.15	0.31	0.44	0.62	0.74

Length of wire = 1.60 m; Diameter of wire = 0.28 mm

a Plot a graph of the p.d. against the current.

b Use the graph to calculate the resistivity of the material of the wire.

11 Two insulated horizontal parallel plates at a separation of 50 mm are at a fixed p.d. of 3.5 kV, as shown in the diagram below.

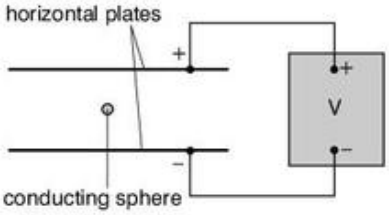


Fig 11.1

a Calculate the electric field strength between the plates.

b A small conducting sphere of mass 1.7×10^{-4} kg placed on the lower plate before the p.d. was applied was held stationary in the field between the plates when the plate p.d. was 3.5 kV.
i Explain why the sphere was held in equilibrium at this p.d.
ii Calculate the charge on the sphere and state whether the charge is positive or negative.

12 A small charged conducting ball is suspended on an insulated thread between two vertical parallel plates. When the plates are at a separation of 60 mm, a p.d. of 4200 V is applied between the plates. As a result, the ball moves to a new equilibrium position at which the thread is at an angle of 7° to the vertical, as shown in the diagram on the right.

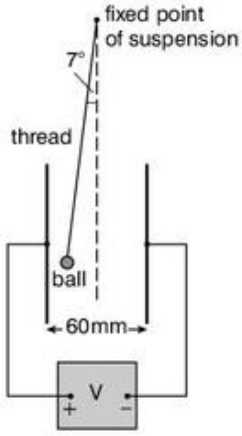


Fig 12.1

a Show that the ratio of the electric force on the ball/the weight of the ball = tan 7°.

b The mass of the ball is 5.1×10^{-4} kg. Calculate the charge on the ball.

13 a State what is meant by an *electric field*. *(1 mark)*

b The electric field between an earthed metal plate and two charged metal spheres is illustrated in Figure 13.1.

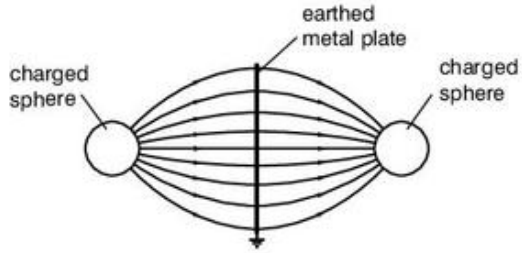


Fig 13.1

i On Figure 13.1, label each sphere with (+) or (−) to show its charge. *(1 mark)*

ii On Figure 13.1, mark a region where the magnitude of the electric field is
1. constant (label this region C), *(1 mark)*
2. decreasing (label this region D). *(1 mark)*

c A molecule has its centre P of positive charge situated a distance of 2.8×10^{-10} m from its centre N of negative charge, as illustrated in Figure 13.2.

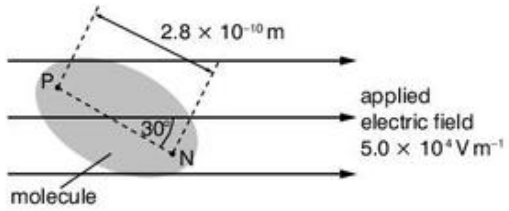


Fig 13.2

The molecule is situated in a uniform electric field of field strength 5.0×10^{4} V m^{-1}. The axis NP of the molecule is at an angle of 30° to this uniform applied electric field.

The magnitude of the charge at P and at N is 1.6×10^{-19} C.

i On Figure 13.2, draw an arrow at P and an arrow at N to show the directions of the forces due to the applied electric field at each of these points. *(1 mark)*

ii Calculate the torque, in N m, on the molecule produced by the forces in **i**. *(2 marks)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 21 Q5 June 2010

7 D.C. circuits

7.1 Circuit rules

On these pages you will learn to:

- Recall Kirchhoff's first law and appreciate the link to conservation of charge
- Recall Kirchhoff's second law and appreciate the link to conservation of energy
- Apply Kirchhoff's laws to solve simple circuit problems

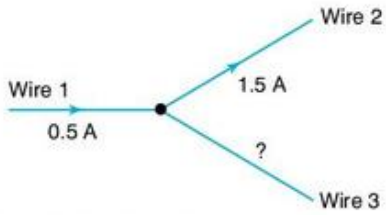


Fig 1 Kirchhoff's first law

Currrent rules

Kirchhoff's first law

At any junction in a circuit, the total current leaving the junction is equal to the total current entering the junction.

For example, Figure 1 shows a junction of three wires where the current in two of the wires (Wire 1 and Wire 2) is given. The current in Wire 3 must be 1.0 A into the junction, because the total current into the junction (1.0 A along Wire 3 + 0.5 A along Wire 1) is the same as the total current out of the junction (1.5 A along Wire 2).

Kirchhoff's first law follows from the **conservation of charge**, as charge flow in and charge flow out of a junction are always equal. The current along a wire is the charge flow per second. In Figure 1, the charge entering the junction each second is 0.5 C along Wire 1 and 1.0 C along Wire 3. The charge leaving the junction each second must therefore be 1.5 C, as the junction does not retain charge.

Components in series

- **The current entering a component is the same as the current leaving the component.** In other words, components do not use up current. At any instant, the charge entering a component each second is equal to the charge leaving it because the same number of charge carriers enter and leave the component each second. In Figure 2, A_1 and A_2 show the same reading because they are measuring the same current.
- **The current passing through two or more components in series is the same through each component.** This is because each charge carrier passes through every component and the same number of charge carriers pass through each component each second. At any instant, charge flows at the same rate through each component.

Potential difference rules

Energy and potential difference

The potential difference, or voltage, between any two points in a circuit is defined as the energy transfer per coulomb of charge that flows from one point to the other.

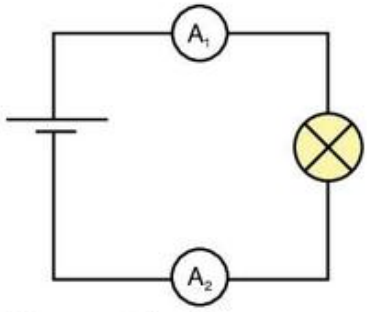


Fig 2 Components in series

If the charge carriers lose energy, the potential difference is a potential drop. If the charge carriers gain energy, which happens when they pass through a battery or cell, the potential difference is a potential rise equal to the e.m.f. of the battery or cell. The rules for potential differences are listed below with an explanation of each rule in energy terms.

- **For two or more components in series, the total p.d. across all the components is equal to the sum of the potential differences across each component.**

  Figure 3 shows a circuit consisting of a battery and three resistors in series. The p.d. across the battery terminals is equal to the sum of the potential

differences across the three resistors. This is because each coulomb of charge from the battery delivers energy to each resistor as it flows round the circuit. The p.d. across each resistor is the energy delivered per coulomb of charge to that resistor. So the sum of the potential differences across the three resistors is the **total energy** delivered to the resistors per coulomb of charge that passes through them, which is the p.d. across the battery terminals.

- **The p.d. across components in parallel is the same.**
  In Figure 4, charge carriers can pass through either of the two resistors in parallel. The same amount of energy is delivered by a charge carrier, regardless of which of the two resistors it passes through.

  If the variable resistor is adjusted so the p.d. across it is 4 V, and if the battery p.d. is 12 V, the p.d. across the two resistors in parallel is 8 V (12 V − 4 V). This is because each coulomb of charge leaves the battery with 12 J of electrical energy, and uses 4 J on passing through the variable resistor. Therefore, each coulomb of charge has 8 J of electrical energy to deliver to either of the two parallel resistors.

- **For any complete loop of a circuit, the sum of the e.m.f.s round the loop is equal to the sum of the potential drops round the loop.**
  This statement is known as **Kirchhoff's second law**. It follows from the fact that the total e.m.f. in a loop is the total electrical energy per coulomb produced in the loop, and the sum of the potential drops is the electrical energy per coulomb delivered round the loop. The above statement follows, therefore, from the **conservation of energy**.

  For example, Figure 5 shows a 9 V battery connected to a 6 V light bulb in series with a variable resistor. If the variable resistor is adjusted so that the p.d. across the light bulb is 6 V, the p.d. across the variable resistor must be 3 V (9 V − 6 V). The only source of electrical energy in the circuit is the battery, so the sum of the e.m.f.s in the circuit is 9 V. This is equal to the sum of the p.ds round the circuit (3 V across the variable resistor + 6 V across the light bulb). In other words, the battery forces charge round the circuit. Every coulomb of charge leaves the battery with 9 J of electrical energy and supplies 3 J to the variable resistor and 6 J to the light bulb.

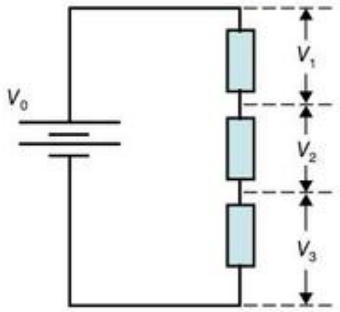


Fig 3 Adding potential differences

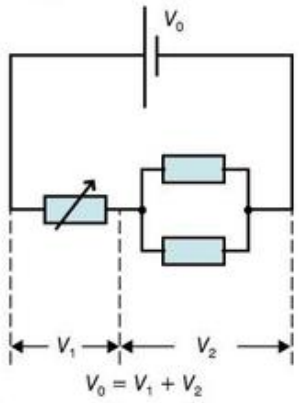


Fig 4 Components in parallel

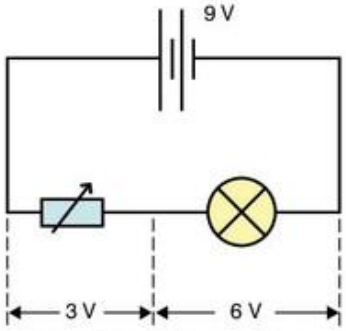


Fig 5 Applying Kirchhoff's second law

QUESTIONS

1 A battery, which has an e.m.f. of 6 V and negligible internal resistance, is connected to a 6 V, 6 W light bulb in parallel with a 6 V, 24 W light bulb, as shown in Figure 6.

Calculate:

a the current in each light bulb,

b the current in the battery,

c the power supplied by the battery.

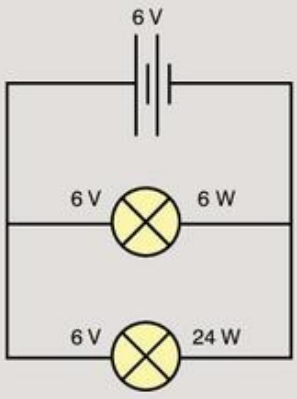


Fig 6

2 A 4.5 V battery is connected in series with a variable resistor and a 2.5 V, 0.5 W torch bulb.

a Sketch the circuit diagram for this circuit.

b The variable resistor is adjusted so that the p.d. across the torch bulb is 2.5 V. Calculate:

i the p.d. across the variable resistor,

ii the current in the torch bulb.

3 A 6.0 V battery is connected in series with an ammeter, a 20 Ω resistor and an unknown resistor R.

a Sketch the circuit diagram.

b The ammeter reads 0.20 A. Calculate:

i the p.d. across the 20 Ω resistor,

ii the p.d. across R,

iii the resistance of R.

4 In question **3**, when the unknown resistor is replaced with a torch bulb, the ammeter reads 0.12 A. Calculate:

a the p.d. across the torch bulb,

b the resistance of the torch bulb.

7.2 More about resistance

On these pages you will learn to:

- Derive, using Kirchhoff's laws, the formula for the combined resistance of:
  - two or more resistors in series
  - two or more resistors in parallel
- Solve problems using the formula for the combined resistance of:
  - two or more resistors in series
  - two or more resistors in parallel
- Apply Kirchhoff's laws to solve simple circuit problems
- Recall that rate of heat transfer in a resistor = I^2R

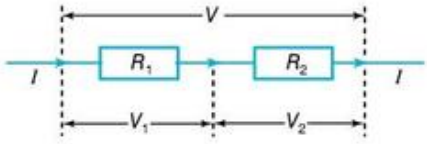


Fig 1 Resistors in series

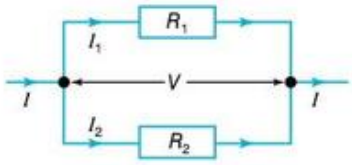


Fig 2 Resistors in parallel

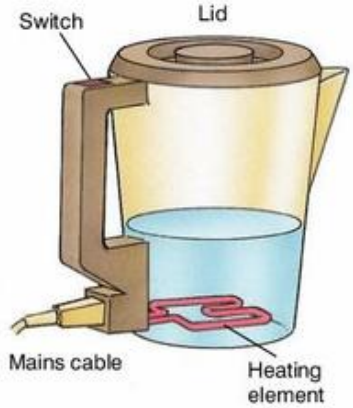


Fig 3 Heating element in a kettle

Resistor combination rules

Resistors in series

Resistors in series pass the same current. The total p.d. is equal to the sum of the individual potential differences.

- For two resistors R_1 and R_2 in series, as in Figure 1, when current I passes through the resistors:
  p.d. across R_1, $V_1 = IR_1$
  p.d. across R_2, $V_2 = IR_2$
  The total p.d. across the two resistors, $V = V_1 + V_2 = IR_1 + IR_2$
  therefore, the total resistance $R = \frac{V}{I} = \frac{IR_1 + IR_2}{I} = R_1 + R_2$
- For two or more resistors R_1, R_2, R_3, etc. in series, the theory can easily be extended to show that the **total resistance is equal to the sum of the individual resistances**:

$$R = R_1 + R_2 + R_3 + \ldots$$

Resistors in parallel

Resistors in parallel have the same p.d. The current through a parallel combination of resistors is equal to the sum of the individual currents.

- For two resistors R_1 and R_2 in parallel, as in Figure 2, when the p.d. across the combination is V,

  the current through resistor R_1, $I_1 = \frac{V}{R_1}$

  the current through resistor R_2, $I_2 = \frac{V}{R_2}$

  The total current through the combination, $I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2}$

  Since the total resistance $R = \frac{V}{I}$, then the total current $I = \frac{V}{R}$

  therefore $\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$

  Cancelling V from each term gives the following equation, which is used to calculate the total resistance R:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

- For two or more resistors R_1, R_2, R_3, etc. in parallel, the theory can easily be extended to show that the total resistance R is given by:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Resistance heating

The heating effect of an electric current in any component is due to the resistance of the component. As explained in topic 6.3, the charge carriers repeatedly collide with the positive ions of the conducting material. There is a **net transfer of energy** from the charge carriers to the positive ions as a result of these collisions. After a charge carrier loses kinetic energy in such a collision, the force due to the p.d. across the material accelerates it until it collides with another positive ion.

For a component of resistance R, when current I passes through it:

$$\text{p.d. across the component, } V = IR$$

Therefore the power supplied to the component, $P = IV = I^2R \left(= \frac{V^2}{R}\right)$

Hence the **energy per second** transferred to the component as thermal energy $= I^2R$

- If the component is at constant temperature, heat transfer to the surroundings takes place at the same rate. Therefore:

Rate of heat transfer = I^2R

- If the component heats up, its temperature rise depends on the power supplied to it (i.e. I^2R), the rate of heat transfer to the surroundings and the heat capacity of the component.
- The energy transfer per second to the component (i.e. the power supplied to it) does not depend on the direction of the current. For example, the heating effect of an alternating current at a given instant depends **only on the magnitude of the current** not on the direction of the current.

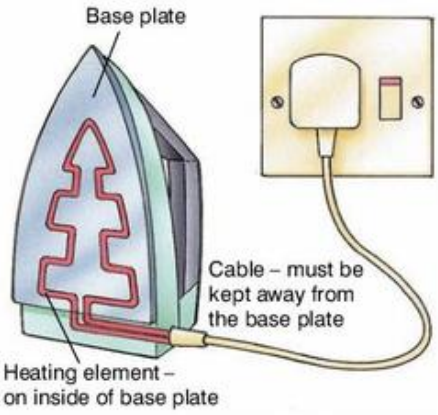


Fig 4 Heating element in an iron

QUESTIONS

1 Calculate the total resistance of each of the resistor combinations in Figure 5.

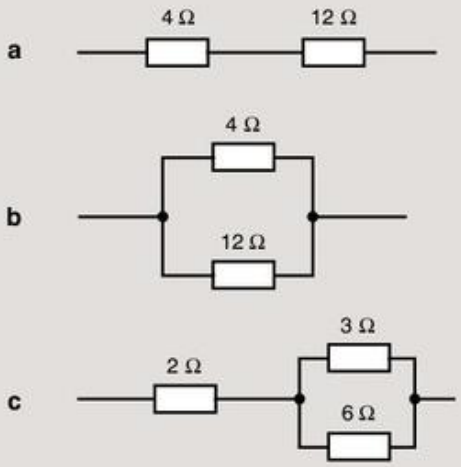


Fig 5

2 A 3 Ω resistor and a 6 Ω resistor are connected in parallel with each other. The parallel combination is connected in series with a 6 V battery of negligible internal resistance and a 4 Ω resistor, as shown in Figure 6.

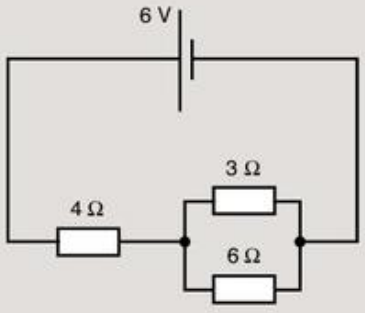


Fig 6

Calculate:

a the combined resistance of the 3 Ω resistor and the 6 Ω resistor in parallel,

b the total resistance of the circuit,

c the battery current,

d the power supplied to the 4 Ω resistor.

3 A 2 Ω resistor and a 4 Ω resistor are connected in series with each other. The series combination is connected in parallel with a 9 Ω resistor and a 3 V battery of negligible internal resistance, as shown in Figure 7.

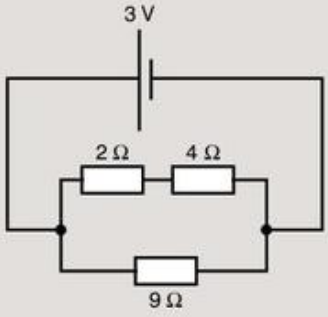


Fig 7

Calculate:

a the total resistance of the circuit,

b the battery current,

c the power supplied to each resistor,

d the power supplied by the battery.

4 Calculate:

a the power supplied to a 10 Ω resistor when the p.d. across it is 12 V,

b the resistance of a heating element designed to operate at 60 W and 12 V.

7.3 E.m.f. and internal resistance

On these pages you will learn to:

- Explain the effect of the internal resistance of a source of e.m.f. in a circuit on the terminal p.d. of the source
- Solve circuit problems in which circuits include sources of e.m.f. with internal resistance

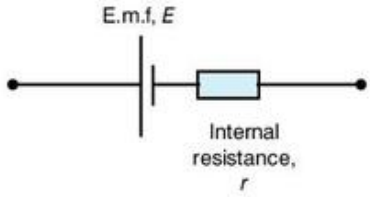


Fig 1 Internal resistance

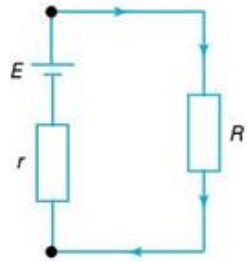


Fig 2 E.m.f. and internal resistance

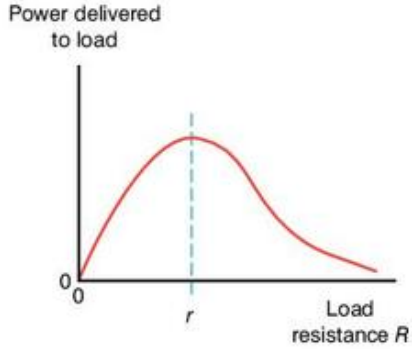


Fig 3 Power delivered to a load v. load resistance

Internal resistance

The **internal resistance** of a source of electricity is due to opposition to the flow of charge through the source. This causes electrical energy produced by the source to be dissipated inside the source when charge flows through it.

The **electromotive force (e.m.f.)** of the source is the electrical energy per unit charge produced by the source.

The **p.d. across the terminals** of the source is the electrical energy per unit charge that can be delivered by the source when it is in a circuit.

The **terminal p.d.** is less than the e.m.f. whenever current passes through the source. The difference is due to the internal resistance of the source.

The internal resistance of a source is the loss of potential difference per unit current in the source, when current passes through the source.

In circuit diagrams, the internal resistance of a source may be shown as a resistor (labelled 'internal resistance') in series with the usual symbol for a cell or battery, as in Figure 1. If no internal resistance is shown, the symbol for the cell or battery should be labelled with symbols for its e.m.f. and its internal resistance.

When a cell of e.m.f. E and internal resistance r is connected to an external resistor of resistance R, as shown in Figure 2, all the current through the cell passes through its internal resistance and the external resistor. So the two resistors are in series, which means that the total resistance of the circuit is $r + R$. Therefore, the current through the cell:

$$I = \frac{E}{R + r}$$

In other words, the cell e.m.f., $E = I(R + r) = IR + Ir$ = the cell p.d. + the 'lost' p.d.

$$\boldsymbol{E = IR + Ir}$$

The 'lost' p.d. inside the cell (i.e. the p.d. across the internal resistance of the cell) is equal to the difference between the cell e.m.f. and the p.d. across its terminals. In energy terms, the 'lost' p.d. is the energy per coulomb dissipated or wasted inside the cell due to its internal resistance.

Power

Multiplying each term of the above equation by the cell current I gives:

$$\textbf{Power supplied by the cell, } \boldsymbol{IE = I^2R + I^2r}$$

In other words,

Power supplied by the cell = power delivered to R + power wasted in the cell due to its internal resistance

The power delivered to R $= I^2R = \dfrac{E^2R}{(R + r)^2}$ since $I = \dfrac{E}{R + r}$

Figure 3 shows how the power delivered to R varies with the value of R.
It can be shown that the peak of this power curve is at $R = r$. In other words, when a source delivers power to a 'load', **maximum power is delivered to the load when the load resistance is equal to the internal resistance of the source.** The load is then said to be 'matched' to the source.

Measurement of internal resistance

The potential difference across the terminals of a cell when the cell is in a circuit can be measured by connecting a high-resistance voltmeter directly across the terminals of the cell. Figure 4 shows how the cell p.d. can be measured for different values of current. The current is changed by adjusting the variable resistor. The lamp limits the maximum current that can pass through the cell. The ammeter is used to measure the cell current.

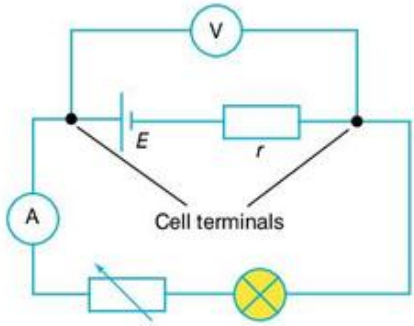


Fig 4 Measuring internal resistance

Graph of p.d. v. current

The measurements of cell p.d. and current for a given cell may be plotted on a graph, as shown in Figure 5.

The cell p.d. decreases as the current increases. This is because the 'lost' p.d. increases as the current increases.

- **The cell p.d. is equal to the cell e.m.f. at zero current**. This is because the 'lost' p.d. is zero at zero current.
- **The graph is a straight line with a negative gradient**. This can be seen by rearranging the equation:

$$E = IR + Ir \text{ to become } IR = E - Ir$$

Because IR represents the cell p.d. V, then $V = E - Ir$. By comparison with the standard equation for a straight line $y = mx + c$, a graph of V on the y-axis against I on the x-axis gives a straight line with a gradient $-r$ and a y-intercept E. See Mathematical skills.

Figure 5 shows the gradient triangle ABC, in which AB represents the lost p.d. and BC represents the current.

So the gradient $\frac{AB}{BC} = \frac{\text{lost voltage}}{\text{current}}$ = internal resistance r.

Note The internal resistance and the e.m.f. of a cell can be calculated, if the cell p.d. is measured for two different values of current. A pair of simultaneous equations can therefore be written, as follows:

- For current I_1, the cell p.d. $V_1 = E - I_1 r$
- For current I_2, the cell p.d. $V_2 = E - I_2 r$

Subtracting the first equation from the second gives:

$$V_1 - V_2 = (E - I_1 r) - (E - I_2 r) = I_2 r - I_1 r = (I_2 - I_1) r$$

Therefore, $r = \frac{(V_1 - V_2)}{(I_1 - I_2)}$

So r can be calculated from the above equation and then substituted into either equation for the cell p.d. to enable E to be calculated.

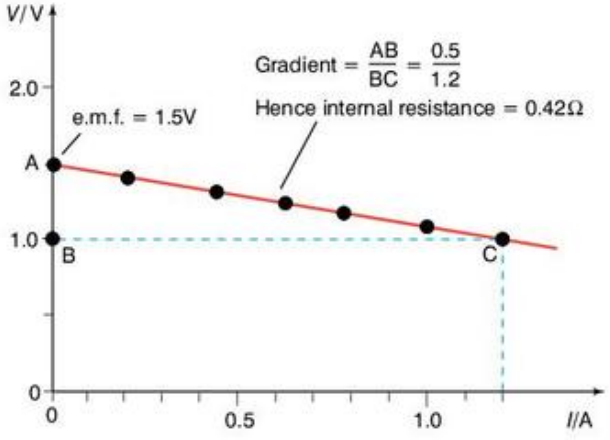


Fig 5 A graph of cell p.d. v. current

QUESTIONS

1 A battery of e.m.f. 12 V and internal resistance 1.5 Ω was connected to a 4.5 Ω resistor. Calculate:

a the total resistance of the circuit,

b the current through the battery,

c the lost p.d.,

d the p.d. across the cell terminals.

2 A cell of e.m.f. 1.5 V and internal resistance 0.5 Ω is connected to a 2.5 Ω resistor. Calculate:

a the current,

b the terminal p.d.,

c the power delivered to the 2.5 Ω resistor,

d the power wasted in the cell.

3 The p.d. across the terminals of a cell was 1.1 V when the current from the cell was 0.20 A and 1.3 V when the current was 0.10 A. Calculate:

a the internal resistance of the cell,

b the cell's e.m.f.

4 A battery of unknown e.m.f., E, and internal resistance, r, is connected in series with an ammeter and a resistance box, R. The current was 2.0 A when $R = 4.0\,\Omega$, and 1.5 A when $R = 6.0\,\Omega$. Calculate E and r.

7.4 More circuit calculations

On these pages you will learn to:

- Solve circuit problems in which circuits include two or more sources of e.m.f. in series or in parallel
- Solve circuit problems in which circuits include diodes

Circuits with a single cell and one or more resistors

Here are some rules:

- Sketch the **circuit diagram** if it is not drawn.
- To calculate the **current** passing through the cell, calculate the total circuit resistance using the resistor combination rules. Don't forget to add on the internal resistance of the cell, if that is given:

$$\text{Cell current} = \frac{\text{cell e.m.f.}}{\text{total circuit resistance}}$$

- To work out the **current and p.d.** for each resistor, start with the **resistors in series** with the cell which pass the same current as the cell current:

$$\text{P.d. across each resistor in series with cell} = \text{current} \times \text{the resistance of each resistor}$$

- To work out the p.d. across the **parallel resistors**, find the difference between the cell e.m.f. and the sum of the p.d.s across the series resistors and the internal resistance.

  The current through any parallel resistor is then given by dividing the p.d. across the parallel resistors by the resistance of that resistor.

Circuits with two or more cells in series

The same rules as above apply, except the current through the cells is calculated by dividing the **overall (i.e. net) e.m.f.** by the total resistance.

- If the cells are connected in the **same** direction in the circuit (Figure 1a), the net e.m.f. is the **sum** of the individual e.m.fs. For example, in Figure 1a the net e.m.f. is 3.5 V.
- If the cells are connected in **opposite** directions to each other in the circuit (Figure 1b), the net e.m.f. is the **difference** between the e.m.fs in each direction. For example, in Figure 1b, the net e.m.f. is 0.5 V in the direction of the 2.0 V cell.

Fig 1 Cells in series

- The **total internal resistance** is the **sum** of the individual internal resistances. This is because the cells, and therefore the internal resistances, are in series.

Circuits with cells in parallel

Kirchhoff's laws are used to analyse circuits where cells and batteries are in parallel. For example, consider the circuit in Figure 2.

To determine the current through the 2.0 Ω resistor, let x and y represent the current through the 2.0 V and 1.5 V cells, which have internal resistances of 6.0 Ω and 4.0 Ω, respectively. The current through the 2.0 Ω resistor is therefore $x + y$.

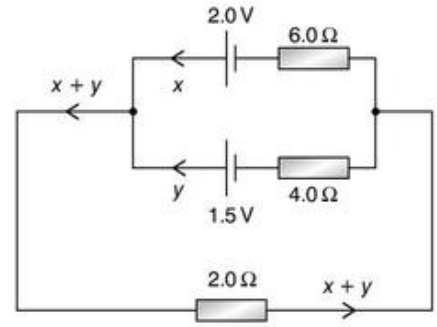


Fig 2

Applying Kirchhoff's second law

- to the outer loop (consisting of the 2.0 V cell with its internal resistance of 6.0 Ω and the 2.0 Ω resistor) gives
  $2.0 = 6.0x + 2.0(x + y)$ or $2.0 = 8.0x + 2.0y$
- to the middle loop (consisting of the 1.5 V cell with its internal resistance of 4.0 Ω and the 2.0 Ω resistor) gives
  $1.5 = 4.0y + 2.0(x + y)$ or $1.5 = 2.0x + 6.0y$

Solving these two simultaneous equations (see Mathematical skills) gives $x = 0.205$ A and $y = 0.182$ A. Hence the current through the 2.0 Ω resistor $= x + y =$ 0.387 A.

Note In such calculations, a negative value for a current would mean the current is in the opposite direction to that assumed initially.

Diodes in circuits

Assume that a semiconductor diode has:

- **zero resistance in the forward direction** when the p.d. across it is 0.6 V or greater,
- **infinite resistance in the reverse direction** or at p.ds less than 0.6 V in the forward direction.

Therefore, in a circuit with one or more diodes as above:

- a p.d. of 0.6 V exists across a diode that is forward-biased and passing a current,
- a diode that is reverse-biased has infinite resistance.

For example, suppose a diode is connected in its forward direction in series with a 1.5 V cell and a 1.5 kΩ resistor, as in Figure 3.

The p.d. across the diode is 0.6 V, because it is forward-biased. Therefore, the p.d. across the resistor is 0.9 V (= 1.5 V − 0.6 V). The current through the resistor is therefore 6.0×10^{-4} A $\left(= \frac{0.9\,\text{V}}{1500\,\Omega}\right)$.

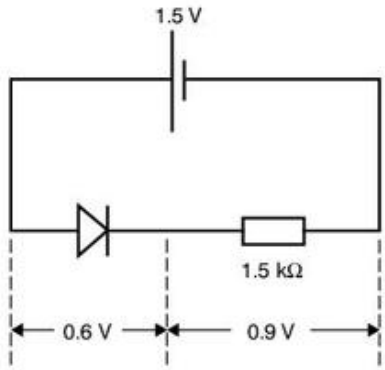


Fig 3 Using a diode

QUESTIONS

1 A cell of e.m.f. 3.0 V, and negligible internal resistance, is connected to a 4.0 Ω resistor in series with a parallel combination of a 24.0 Ω resistor and a 12.0 Ω resistor (Figure 4).

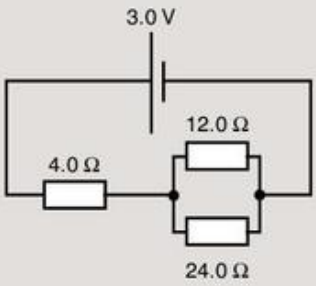


Fig 4

Calculate:

a the total resistance of the circuit,

b the cell current,

c the current and p.d. for each resistor.

2 A battery of e.m.f. 12.0 V, with an internal resistance of 3.0 Ω, is connected in series with a 15.0 Ω resistor and a battery of e.m.f. 9.0 V, which has an internal resistance of 2.0 Ω (Figure 5).

a Calculate:

i the total resistance of the circuit,

ii the battery current,

iii the current and p.d. across the 15 Ω resistor.

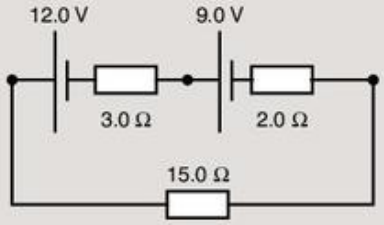


Fig 5

Extension

b A 1.0 Ω resistor is connected in parallel with the 12.0 V battery and the 3.0 Ω resistor. Calculate the current through **i** the 1.0 Ω resistor, **ii** the 12.0 V battery.

3 a Two 8 Ω resistors and a battery of e.m.f. 12.0 V, and internal resistance 8 Ω, are connected in series with each other. Sketch the circuit diagram and calculate:

i the power delivered to each external resistor,

ii the power wasted due to internal resistance.

b The two 8 Ω resistors in part **a** are reconnected in parallel with each other and then connected to the same battery. Sketch the circuit diagram and calculate:

i the power delivered to each external resistor,

ii the power wasted due to internal resistance.

4 a For the circuit shown in Figure 6, calculate the p.d. and current for each resistor and diode.

b In the circuit (Figure 6), both diodes are reversed. Sketch the new circuit and calculate the p.d. and current for each resistor and diode for this new arrangement.

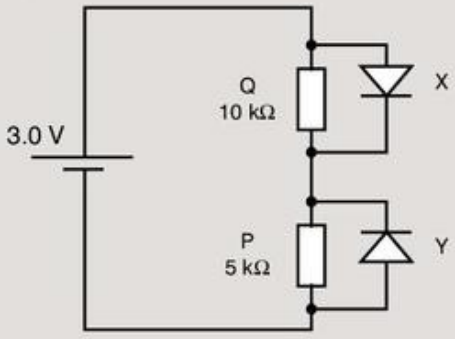


Fig 6

7.5 The potential divider

On these pages you will learn to:

- Explain the principle of a potential divider circuit as a source of fixed p.d. or of variable p.d.
- Recall and solve problems using the principle of the potentiometer as a means of comparing potential differences
- Explain the operation of an electronic sensor consisting of a sensing device and a circuit that provides a voltage output
- Explain the use of thermistors and light-dependent resistors in potential dividers

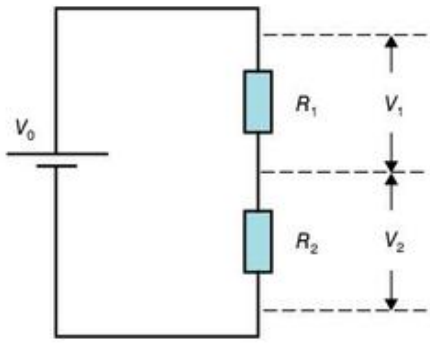


Fig 1 A potential divider

The theory of the potential divider

A **potential divider** consists of two or more resistances in series connected to a source of fixed potential difference. The source p.d. is divided beween the resistors, as they are in series with each other. A potential divider can be used to supply a p.d. of any value between zero and the source p.d.

Figure 1 shows a potential divider consisting of two resistors R_1 and R_2, in series connected to a source of fixed p.d. V_0.

Total resistance of the combination $= R_1 + R_2$

Therefore, current through the resistors, $I = \dfrac{\text{p.d. across the resistors}}{\text{total resistance}} = \dfrac{V_0}{R_1 + R_2}$

so the p.d. across resistor R_1, $V_1 = IR_1 = \dfrac{V_0 R_1}{R_1 + R_2}$

and the p.d. across resistor R_2, $V_2 = IR_2 = \dfrac{V_0 R_2}{R_1 + R_2}$

Dividing the equation for V_1 by the equation for V_2 gives:

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

This equation shows that:

The ratio of the p.ds across each resistor is equal to the resistance ratio of the two resistors.

To supply a variable p.d.

The source p.d. is connected to a fixed length of uniform resistance wire. A sliding contact on the wire can then be moved along the wire, as illustrated in Figure 2, giving a variable p.d. between the contact and one end of the wire. A uniform track of a suitable material may be used instead of resistance wire. The track may be linear or circular (Figure 2a,b). The circuit symbol for a variable potential divider which is also known as a **potentiometer** is shown in Figure 2c.

The variable potential divider in Figure 2c can be used to vary the brightness of a bulb by connecting the bulb between C and B. In contrast with using a variable resistor in series with the light bulb and the source p.d., the use of a potential divider enables the current through the light bulb to be reduced to zero. With a variable resistor at maximum resistance, there is a current through the light bulb.

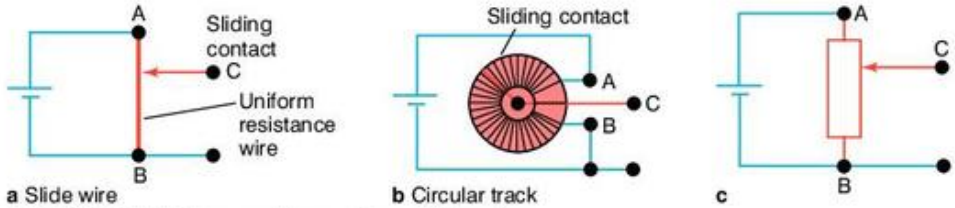


Fig 2 Potential dividers used to supply a variable p.d.

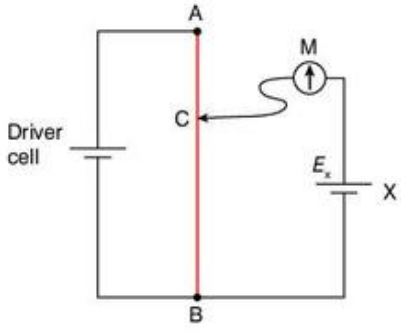


Fig 3 Comparing cell e.m.f.s

To compare potential differences

The slide-wire potentiometer can be used to compare cell e.m.f.s and potential differences. Figure 3 shows the circuit used to compare the e.m.f.s of two cells, X and Y. One of the cells (X) is shown connected in series with a centre-reading meter M between end B of the wire and the sliding contact C on the wire. The driver cell provides a constant p.d. across the wire.

The position of the sliding contact on the wire is adjusted until the meter reads zero. The cell e.m.f. E_x is then opposed equally (or 'balanced') by the p.d. between B and C. The length L_x of the wire from B to C is then measured.

The procedure is repeated using the other cell (Y). The ratio of the cell e.m.f.s, E_x/E_y is equal to the ratio of their balance lengths, L_x/L_y.

Sensor circuits

A **sensor circuit** produces an output p.d. which changes as a result of a change of a physical variable, such as temperature or pressure.

A temperature sensor

This consists of a potential divider made using a **thermistor** and a variable resistor (Figure 4).

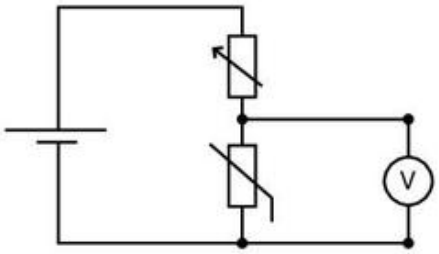


Fig 4 A temperature sensor

With the temperature of the thermistor constant, the source p.d. is divided between the thermistor and the variable resistor. By adjusting the variable resistor, the p.d. across the thermistor can then be set at any desired value. When the temperature of the thermistor changes, its resistance changes so the p.d. across it changes. For example, suppose the variable resistor is adjusted so that the p.d. across the thermistor at 20 °C is exactly half the source p.d.; if the temperature of the thermistor is then raised, its resistance falls so the p.d. across it falls.

A light sensor

The circuit is similar to Figure 4 except a light-dependent resistor (LDR) is used instead of a thermistor. The p.d. across the LDR changes when the incident light intensity on the LDR changes. If the light intensity increases, the resistance of the LDR falls and the p.d. across the LDR falls.

The Wheatstone bridge

A Wheatstone bridge can be used to compare two resistances. In Figure 5, the two resistances, R_1 and R_2, provide a fixed p.d. which is compared with a variable p.d. across a uniform resistance wire AB. The sliding contact C is adjusted until the centre-reading meter reads zero. For this 'null' reading,

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$

where l_1 and l_2 are the lengths of AC and BC respectively.

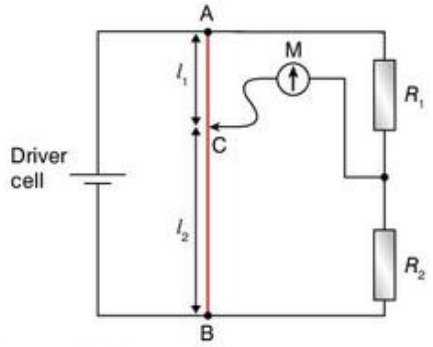


Fig 5 The Wheatstone bridge

QUESTIONS

1 A potential divider consists of a 1.0 kΩ resistor in series with a 5.0 kΩ resistor, and a battery of e.m.f. 4.5 V and negligible internal resistance.

a Sketch the circuit and calculate the p.d. across each resistor.

b A second 5.0 kΩ resistor is connected in the above circuit in parallel with the first 5.0 kΩ resistor. Calculate the p.d. across each resistor in this new circuit.

2 A 12 V battery, of negligible internal resistance, is connected to the fixed terminals of a variable potential divider (which has a maximum resistance of 50 Ω). A 12 V light bulb is connected between the sliding contact and the negative terminal of the potential divider. Sketch the circuit diagram and describe how the brightness of the light bulb changes when the sliding contact is moved from the negative to the positive terminal of the potential divider.

3 **a** A potential divider consists of an 8.0 Ω resistor in series with a 4.0 Ω resistor and a 6.0 V battery. Calculate:
i the current,
ii the p.d. across each resistor.

b In the circuit in part a, the 4 Ω resistor is replaced by a thermistor with a resistance of 8 Ω at 20 °C and a resistance of 4 Ω at 100 °C. Calculate the p.d. across the fixed resistor at: **i** 20 °C, **ii** 100 °C.

4 A light sensor consists of a 5.0 V cell, an LDR and a 5.0 kΩ resistor in series with each other. A voltmeter is connected in parallel with the resistor. When the LDR is in darkness, the voltmeter reads 2.2 V.

a Calculate:
i the p.d. across the LDR,
ii the resistance of the LDR when the voltmeter reads 2.2 V.

b Describe and explain how the voltmeter reading would change if the LDR was exposed to daylight.

7 End of chapter questions

1 Two resistors of resistances 6.0 Ω and 12.0 Ω, in parallel, are connected to a 2.0 Ω resistor and a 6.0 V battery. The diagram below shows the circuit diagram.

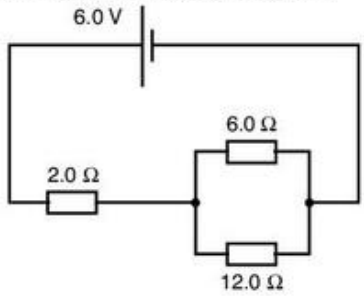


Fig 1.1

a Calculate the combined resistance of the three resistors.

b For each resistor, calculate:
- **i** the current,
- **ii** the p.d.,
- **iii** the power dissipated.

2 A light bulb is connected in series with a resistor and a battery, of negligible internal resistance, as shown below. A variable resistor is connected in parallel with the light bulb.

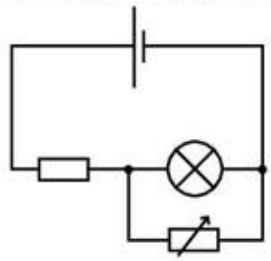


Fig 2.1

a Describe how the brightness of the light bulb changes as the resistance of the variable resistance is increased from zero.

b Explain why it is possible to reduce the bulb current to zero by adjusting the variable resistor.

3 A 15.0 Ω resistor and a 3.0 Ω resistor are connected in series with each other and with a 3.0 V battery, which has an internal resistance of 2.0 Ω, as shown below.

a Calculate:
- **i** the total resistance of the circuit,
- **ii** the p.d. across each of the external resistors,
- **iii** the p.d. across the battery terminals.

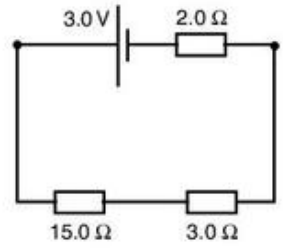


Fig 3.1

b Calculate the power dissipated:
- **i** in each resistor,
- **ii** in the battery due to its internal resistance.

4 The circuit diagram below is for the rear windscreen heater of a car. It consists of 4 heating elements, each of resistance 6.0 Ω, connected to a 12.0 V battery of internal resistance 2.0 Ω.

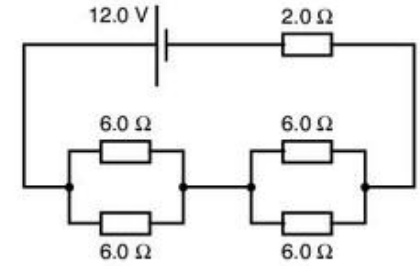


Fig 4.1

a Calculate:
- **i** the current,
- **ii** the p.d. across each heating element.

b Calculate the power dissipated:
- **i** in each heating element,
- **ii** in the battery due to its internal resistance.

c How would the operation of the heater differ if the heating elements were in parallel with each other, with the battery connected across the parallel combination?

5 A 6.0 V battery of unknown internal resistance was connected in series with a switch, a resistor of resistance 4.0 Ω and an ammeter. When the switch was closed, the ammeter read 1.0 A.

a Calculate the internal resistance of the battery.

b A second 4.0 Ω resistor was connected in parallel with the first resistor. Calculate the ammeter reading with this second resistor in the circuit.

6 A hand-held hair dryer has two 570 Ω identical heating elements, each in series with a switch. The heating elements are connected in parallel with each other and a 230 V mains electricity supply. A 12 W, 230 V electric fan in the hair dryer is used to blow air over the heating elements.

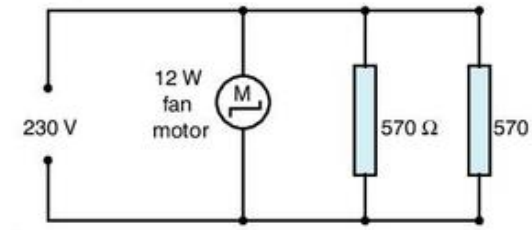


Fig 6.1

a When both heaters are being used, and the electric fan is on, calculate:
- **i** the current passing through each heating element,
- **ii** the power supplied by the electricity supply,
- **iii** the total current passing through the hair dryer.

b When one heater only is on, calculate:
i the total current passing through the heater,
ii the power supplied by the electricity supply.

7 The diagram shows a potential divider circuit consisting of a 10 kΩ resistor R connected in series with an n.t.c. thermistor and a 6.0 V battery. A high resistance voltmeter is connected across the thermistor.

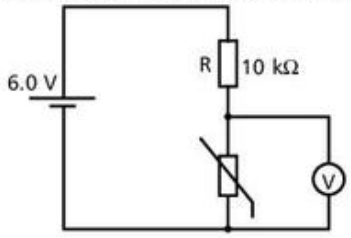


Fig 7.1

a The voltmeter reads 2.0 V when the thermistor's temperature is 20 °C. Calculate the resistance of the thermistor at this temperature.

b A second 10 kΩ resistor was connected in parallel with R. Calculate the voltmeter reading with this second resistor in the circuit when the thermistor is at the same temperature.

c Describe and explain how the voltmeter reading would change if the the temperature of the thermistor were increased.

8 The diagram shows a circuit in which two 5.0 kΩ resistors are connected in series with each other and a 5.0 V battery, of negligible internal resistance. A diode, is connected in parallel with each resistor, as shown.

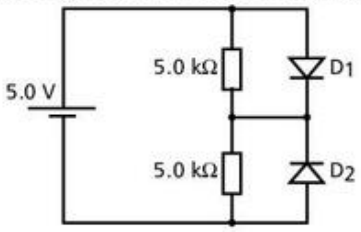


Fig 8.1

a Calculate:
i the p.d.,
ii the current through each resistor.

b Calculate the current through each resistor if the 5.0 V battery was replaced with a 9.0 V battery, also of negligible internal resistance.

9 In a potentiometer experiment using a slide-wire potentiometer as shown in topic 7.5 Figure 3, a cell X is compared with a standard cell S of e.m.f. 1.50 ± 0.01 V. The balance length for S was 735 ± 3 mm and for X was 530 ± 3 mm.

a Calculate the e.m.f. of cell X.

b Calculate the percentage uncertainty in
i the e.m.f. of S, **ii** each balance length.

c Hence calculate the uncertainty in the e.m.f. of X.

10 In a Wheatstone bridge experiment, a 5.00 Ω resistor and a slide-wire potentiometer were used as shown in topic 7.5 Figure 5 to determine the resistance R of a resistor. Length of wire = 1.00 m.

a Draw a circuit diagram to show the Wheatstone bridge circuit.

b In such an experiment, the balance point on the wire was found to be 420 mm from the end of the wire which the 5.00 Ω resistor was connected to.
i Calculate R.
ii If the precision of the balance point was ± 4 mm and the percentage uncertainty in the 5.00 Ω resistor was 2.0%, show that the percentage uncertainty in R is 3.6%.

11 A network of resistors, each of resistance R, is shown in Figure 11.1.

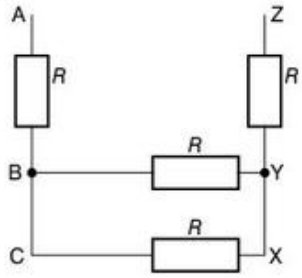


Fig 11.1

a Calculate the total resistance, in terms of R, between points
i A and C, *(1 mark)*
ii B and X, *(1 mark)*
iii A and Z. *(1 mark)*

b Two cells of e.m.f. E_1 and E_2 and negligible internal resistance are connected into the network in **a**, as shown in Figure 11.2.

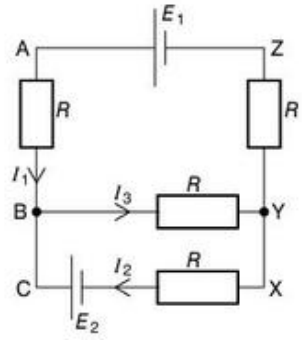


Fig 11.2

The currents in the network are as indicated in Figure 11.2.

Use Kirchhoff's laws to state the relation
i between currents I_1, I_2 and I_3, *(1 mark)*
ii between E_2, R, I_2 and I_3 in loop BCXYB, *(1 mark)*
iii between E_1, E_2, R, I_1 and I_2 in loop ABCXYZA. *(1 mark)*

Cambridge International AS and A Level Physics 9702
Paper 21 Q7 June 2009

8 Waves

8.1 Waves and vibrations

On these pages you will learn to:

- Recall that energy is transferred by a progressive wave
- Describe wave motion in ropes and springs
- Compare transverse and longitudinal waves
- Explain polarisation as a property of transverse waves only

Types of wave

Waves that pass through a substance are **vibrations** of the particles of the substance. For example, sound waves in air are created by a surface that vibrates, sending compression waves through the surrounding air. Sound waves, seismic waves and waves on strings are examples of waves that pass through a substance. These types of wave are often referred to as **mechanical waves**. When waves progress through a substance, the particles of the substance vibrate in a certain way which makes other particles vibrate in the same way, and so on.

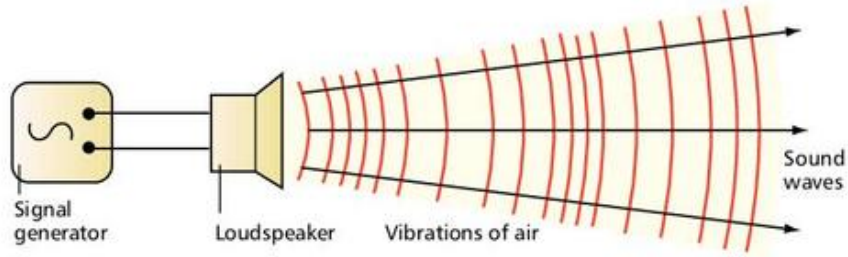


Fig 1 Creating sound waves in air

- **Electromagnetic waves** are vibrating electric and magnetic fields that progress through space, without the need for a substance. The vibrating electric field generates a vibrating magnetic field, which generates a vibrating electric field further away, and so on. Electromagnetic waves include radio waves, microwaves, infrared radiation, light, ultraviolet radiation, X-rays and gamma radiation. All electromagnetic waves travel with the same speed through free space. The full spectrum of electromagnetic waves is listed in Table 1.

***Table 1** The electromagnetic spectrum*

Type	Wavelength range
radio	>0.1 m
microwave	0.1 m to 1 mm
infrared	1 mm to about 650 nm
visible	about 650 nm to 350 nm
ultraviolet	about 350 nm to 1 nm
X-rays	<1 nm
gamma rays	<1 nm

Longitudinal and transverse waves

Longitudinal waves

Longitudinal waves are waves in which the direction of vibration is **parallel** to (i.e. along) the direction in which the wave travels. Sound waves, primary seismic waves and compression waves on a slinky are all longitudinal waves. Figure 2 shows how to send longitudinal waves along a 'slinky'. When one end of the slinky is moved to and fro repeatedly, each 'forward' movement causes a compression wave to pass along the slinky as the coils push into each other. Each 'reverse' movement causes the coils to move apart, so an 'expansion' wave passes along the slinky.

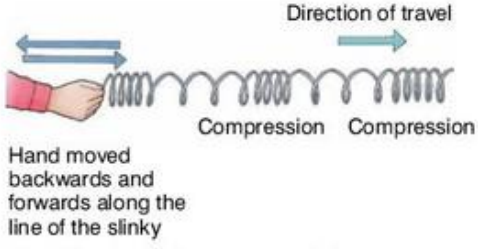


Fig 2 Longitudinal waves on a slinky

- **Sound waves** in air are created when a surface in contact with air vibrates. When the vibrating surface pushes on the air, the air molecules near the surface are pushed away from the surface, pushing on adjacent molecules which push on adjacent molecules. This creates a wave of 'high density' air (i.e. a compression) that passes through the air. When the vibrating surface 'retreats', the air molecules near the surface move back into the space vacated by the surface, allowing adjacent air molecules to fall back and so on. This creates a wave of 'low density' air (referred to as a 'rarefaction') that passes through the air behind the compression wave. The vibrating surface therefore creates a series of **compressions** and **rarefactions** that pass through the air. The air molecules are therefore repeatedly pushed to and fro along the direction in which the sound travels.

Transverse waves

Transverse waves are waves in which the direction of vibration is **perpendicular** to the direction in which the wave travels. Electromagnetic waves, secondary seismic waves and waves on a string or a wire are all transverse waves. Figure 3 shows transverse waves travelling along a rope. When one end of the rope is moved from side to side repeatedly, these sideways movements travel along the rope: each unaffected part of the rope is pulled sideways when the part next to it moves sideways.

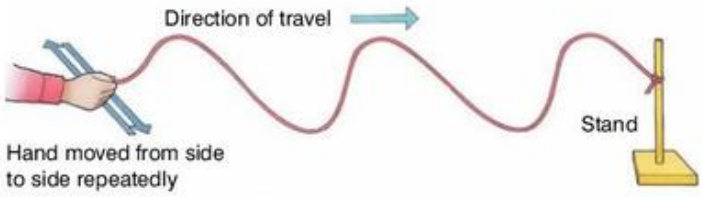


Fig 3 Making rope waves

Extension

- Transverse waves are **polarised** if the vibrations are in one plane only. If the vibrations change from one plane to another, the waves are **unpolarised**. Longitudinal waves cannot be polarised.
- Figure 4 shows **unpolarised** waves being created on a rope. Contrast this with the waves in Figure 3, which are polarised because they vibrate in one plane only.
- Light from a filament lamp or a candle is unpolarised. If unpolarised light is passed through a **polaroid filter**, the transmitted light is polarised: the filter only allows through light which vibrates in a certain direction, according to the alignment of its molecules. This is like using a 'letter box' in a board to polarise unpolarised waves on a rope, as shown in Figure 5.
- If unpolarised light is passed through two polaroid filters, the transmitted light intensity changes if one polaroid is turned relative to the other one. The filters are said to be 'crossed' when the transmitted intensity is a minimum. At this position, the polarised light from the first filter cannot pass through the second filter, as the alignment of molecules in the second filter is at 90° to the alignment in the first filter. This is like passing rope waves through two 'letter boxes' at right angles to each other, as shown in Figure 5.
- Light is part of the spectrum of electromagnetic waves. The **plane of polarisation** of an electromagnetic wave is defined as the plane in which the electric field oscillates.

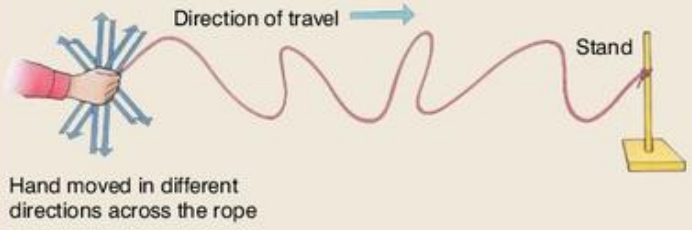


Fig 4 Unpolarised waves on a rope

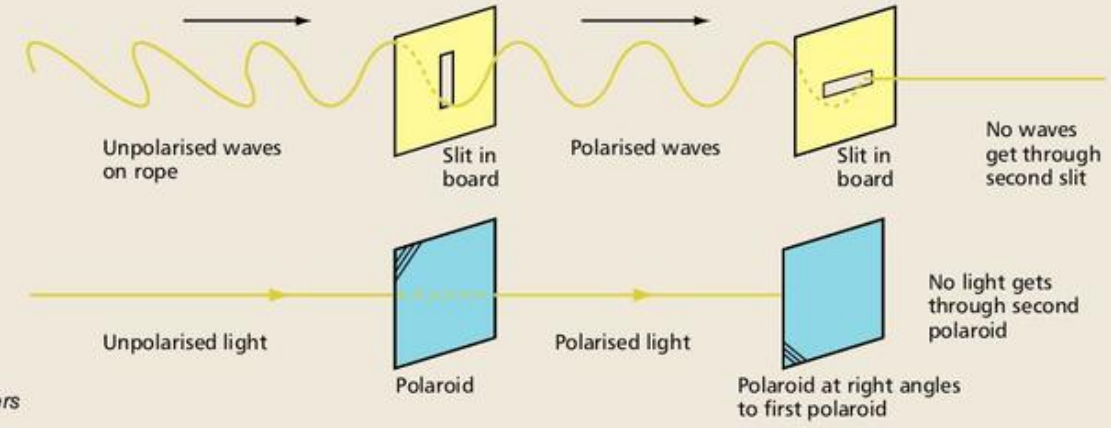


Fig 5 Polarisers

QUESTIONS

1 Classify the following types of wave as either longitudinal or transverse:

radio waves	sound waves
secondary seismic waves	microwaves

2 Sketch a snapshot of a longitudinal wave travelling on a slinky coil, indicating the direction in which the waves are travelling and areas of compression and expansion.

3 Sketch a snapshot of a transverse wave travelling along a rope, indicating the direction in which the waves are travelling and the direction of motion of the particles at the peaks and troughs.

4 **a** What is meant by a 'polarised' wave?

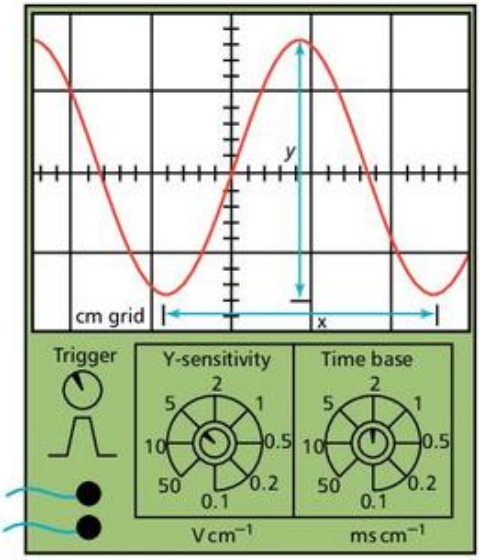
b A light source is observed through two pieces of polaroid. Describe and explain what you would expect to observe when one of the polaroids is rotated through 360°.

8.2 Measuring waves

On these pages you will learn to:

- Explain and use the terms displacement, amplitude, phase difference, period, frequency, wavelength and speed
- Deduce, recall and use the wave equation $v = f\lambda$
- Recall and use the relationship intensity $\propto$ (amplitude)^2
- Measure the frequency of sound using a calibrated cathode-ray oscilloscope

Fig 1 Measuring electrical waves



a Trace height y = 32 mm
$\therefore$ Trace amplitude = 16 mm = 1.6 cm
Given Y-sensitivity = 5 V cm^{-1}
Voltage amplitude = 5 × 1.6 = 8.0 volts

b x-distance from peak to peak = 32 mm = 3.2 cm
Given time base 2 ms cm^{-1}
Time period = 2 × 3.2 = 6.4 ms

$$\text{Frequency } f = \frac{1}{\text{Time period}} = \frac{1}{6.4 \times 10^{-3}\,\text{s}}$$

$f = 1.56 \times 10^2$ Hz = 156 Hz

Fig 3 Measuring a waveform

When an intercontinental phone call is made, sound waves are converted to electrical waves. These waves are carried by electromagnetic waves from ground transmitters to satellites in space and back to receivers on Earth, where they are converted back to electrical waves, then back to sound waves. The engineers who design and maintain communications systems need to measure the different types of wave at different stages to make sure the waves are not distorted.

Key terms

The following terms, illustrated in Figure 2, are used to describe waves:

- The **displacement** of a vibrating particle is its distance and direction from its equilibrium position.
- The **amplitude** of a wave is the maximum displacement of a vibrating particle. For a transverse wave, this is the height of a wave crest or the depth of a wave trough from the middle.
- The **intensity** of a wave is a measure of its power. It can be shown that the intensity of a wave is proportional to the square of its amplitude.
- The **wavelength** of a wave is the least distance between two adjacent vibrating particles with the same displacement and velocity at the same time (e.g. distance between adjacent crests).
- **One complete cycle of a wave** is from a maximum displacement to the next maximum displacement (e.g. from one wave peak to the next).
- The **period of a wave** is the time for one complete wave to pass a fixed point.
- The **frequency** of a wave is the number of cycles of vibration of a particle per unit time. The unit of frequency is the **hertz (Hz)**. For waves of frequency f:

$$\textbf{Period of the wave} = \frac{1}{f}$$

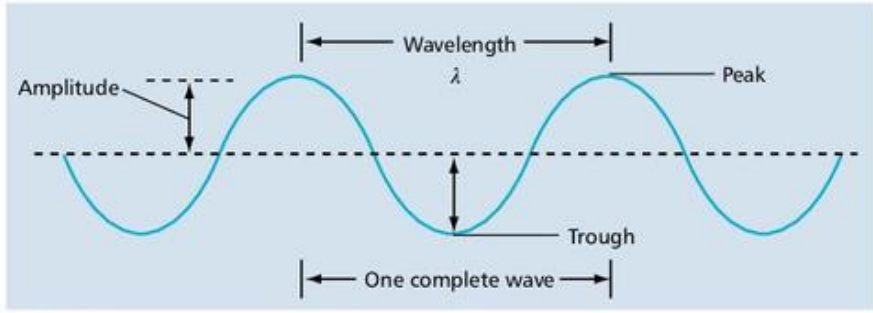


Fig 2 Parts of a wave

Using an oscilloscope to measure sound waves

The oscilloscope shown in Figure 3 is connected to a **microphone**, which detects sound waves emitted by a loudspeaker connected to a signal generator. The trace on the oscilloscope screen is the **waveform** of the sound waves produced by the loudspeaker. Figure 3 shows how to measure the amplitude and the frequency of this waveform using the oscilloscope controls. Note that the waveform **amplitude** is the distance **from the middle to the top**.

Wave speed

The higher the frequency of a wave, the shorter its wavelength. For example, if waves are sent along a rope, the higher the frequency at which they are produced, the closer together their wave peaks. The same effect can be seen in a ripple tank, when straight waves are produced at a constant frequency. If the

frequency is raised to a higher value, the waves are closer together. Consider Figure 4, which represents the crests of straight waves in a ripple tank travelling at constant speed.

- Each wave crest travels a distance equal to one wavelength (λ) in the time for one cycle.
- The time taken for one cycle $= \frac{1}{f}$, where f is the frequency of the waves.

Therefore, the speed of the waves:

$$v = \frac{\text{distance travelled in one cycle}}{\text{time taken for one cycle}} = \frac{\lambda}{1/f} = f\lambda$$

For waves of frequency f and wavelength λ:

$$\textbf{Wave speed, } v = f\lambda$$

Note The symbol c is used for the speed of electromagnetic waves.

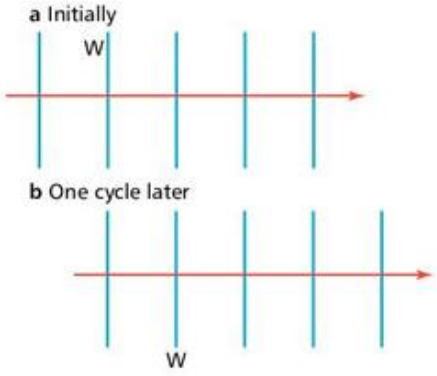


Fig 4 Wave speed

Phase difference

The **phase difference** between two vibrating particles is the fraction of a cycle between the vibrations of the two particles, measured either in **degrees or radians** where:

$$1 \text{ cycle} = 360^\circ = 2\pi \text{ radians}$$

For two points at distance d apart along a wave of wavelength λ,

$$\textbf{Phase difference, in radians} = \frac{2\pi d}{\lambda}$$

Figure 5 shows three successive snapshots of the particles of a transverse wave that progresses from left to right across the diagram. Particles O, P, Q, R and S are spaced approximately $\frac{1}{4}$ of a wavelength apart. Table 1 shows the phase difference between O and each of the other particles.

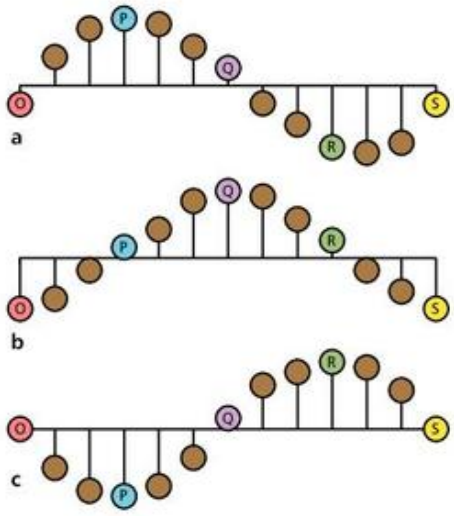


Fig 5 Progressive waves

***Table 1** Phase differences*

	P	Q	R	S
Distance from O	$\frac{1}{4}\lambda$	$\frac{1}{2}\lambda$	$\frac{3}{4}\lambda$	λ
Phase difference relative to O/radians	$\frac{1}{2}\pi$	π	$\frac{3}{2}\pi$	2π

QUESTIONS

1 Sound waves, in air, travel at a speed of 340 m s^{-1} at 20 °C. Calculate the wavelength of sound waves, in air, which have a frequency of:

a 3400 Hz **b** 18 000 Hz

2 Electromagnetic waves, in air, travel at a speed of 3.0×10^8 m s^{-1}. Calculate the frequency of electromagnetic waves of wavelength:

a 0.030 m **b** 600 nm

3 Figure 6 shows a waveform on an oscilloscope screen, when the y-sensitivity of the oscilloscope was 0.50 V cm^{-1} and the time base was set at 0.5 ms cm^{-1}. Determine the amplitude and the frequency of this waveform.

4 For the waves in Figure 5:

a determine:

i the amplitude if the wavelength is 55 mm,

ii the phase difference between P and R,

iii the phase difference between P and S.

b what would be the displacement and direction of motion of Q three-quarters of a cycle after the last snapshot?

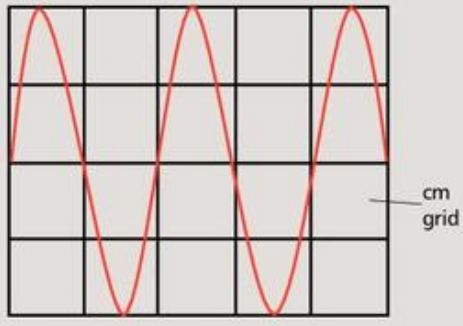


Fig 6

8.3 Wave properties 1

On these pages you will learn to:

- Know what is meant by reflection and refraction of waves
- Describe examples of reflection and refraction
- Explain what is meant by diffraction
- Show an understanding of experiments that demonstrate diffraction, including the diffraction of water waves in a ripple tank with both a wide gap and a narrow gap
- Describe wave motion in ripple tanks

Wave properties, such as reflection, refraction and diffraction, occur with many different types of wave. A **ripple tank** may be used to study these wave properties. The tank is a shallow transparent tray of water with sloping sides. The slopes prevent waves reflecting off the sides of tank. If they did reflect, it would be difficult to see the waves to be observed.

- The waves observed in a ripple tank are referred to as '**wave fronts**', which are lines of constant phase (e.g. crests).
- The direction in which a wave travels is at right angles to the wave front.

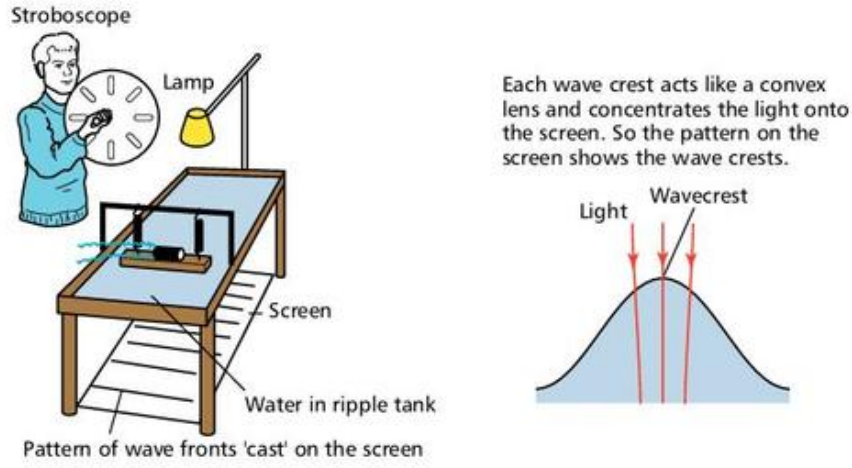


Fig 1 *The ripple tank*

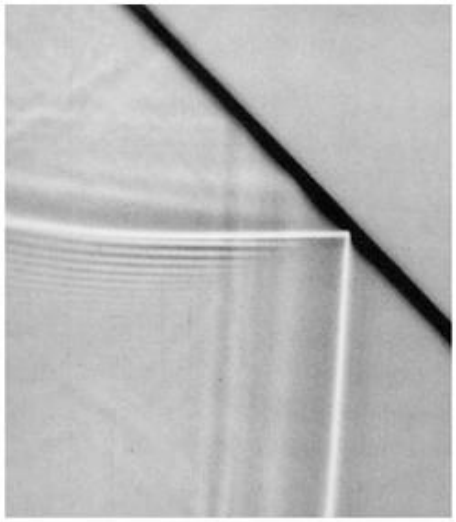


Fig 2 *Reflection of plane waves*

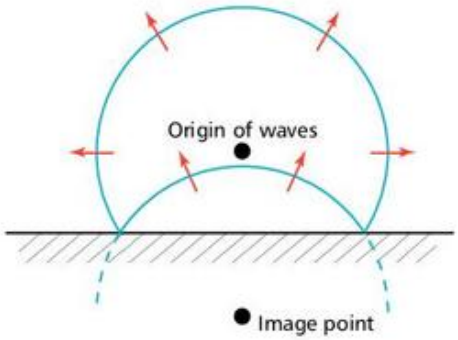


Fig 3 *Reflection of circular waves*

Reflection

- **Straight waves** directed at a certain angle to a hard flat surface (the '**reflector**') reflect off at the same angle, as shown in Figure 2. The angle between the reflected wave front and the surface is the **same** as the angle between the incident wave front and the surface. Therefore the direction of the reflected wave is at the same angle to the reflector as the direction of the incident wave. This same effect is observed when a light ray is directed at a plane mirror. The angle between the incident ray and the mirror is the **same** as the angle between the reflected ray and the mirror.
- **Circular waves** from a point source spread out equally in all directions. Figure 3 shows what happens when circular waves hit a flat reflector. The reflected waves appear to come from a point image the same distance behind the reflector as the point source is in front of it. This is why the image of an object observed in a plane mirror is the same distance behind the mirror as the object is in front of the mirror. Reflected light from each point of the object appears to come from a corresponding **image point** behind the mirror at the same distance. See Figure 3.

Refraction

When waves pass across a boundary at which the wave speed changes, the wavelength also changes. If the wave fronts are at a non-zero angle to the boundary, they change direction as well as changing speed. This effect is known as **refraction**.

Figure 4 shows the refraction of water waves in a ripple tank when they pass across a boundary, from deep to shallow water at a non-zero angle to the boundary.

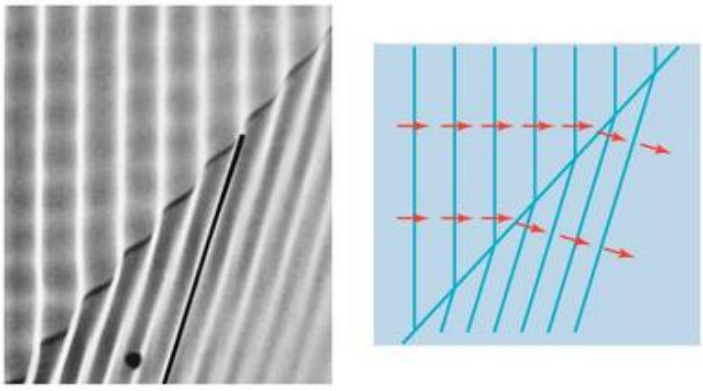


Fig 4 Refraction

Because they move more slowly in the shallow water, the wavelength is smaller in the shallow water and therefore they change direction.

The direction of the refracted waves is closer to the **normal**, the line perpendicular to the boundary, than the direction of the incident waves. Refraction of light is observed when a light ray is directed into a glass block non-normally. The light ray changes direction when it crosses the glass boundary. This happens because light waves travel more slowly in glass than in air. (See topic 9.4.)

Diffraction

Diffraction occurs when waves spread out after passing through a gap, or round an obstacle. The effect can be seen in a ripple tank when straight waves are directed at a gap, as shown in Figure 5.

- The narrower the gap, the more the waves spread out.
- The longer the wavelength, the more the waves spread out.

To explain why the waves are diffracted on passing through the gap, consider each point on a wave front as a secondary emitter of 'wavelets'. The wavelets from the points along a wave front travel only in the direction in which the wave is travelling, not in the reverse direction, and they combine to form a new wave front spreading beyond the gap.

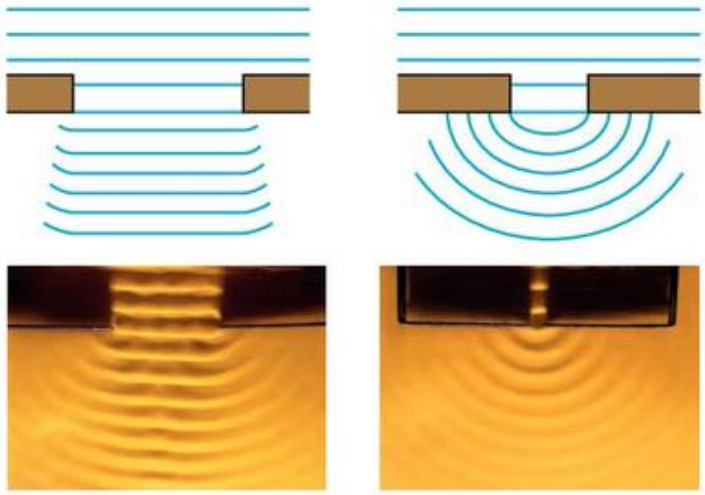


Fig 5 The effect of the gap width

QUESTIONS

1 Copy and complete Figure 6, by showing the wave front after it has been reflected from the straight reflector. Also, show the direction of the reflected wave front.

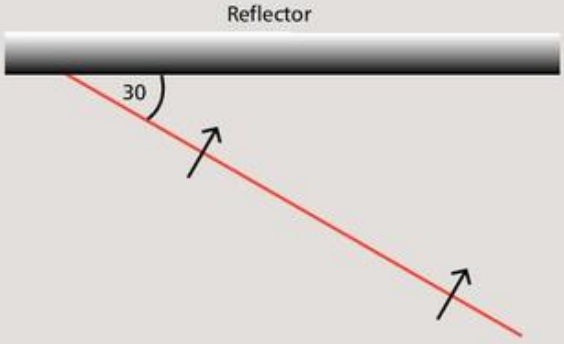


Fig 6

2 A circular wave spreads out from a point P on a water surface, which is 0.50 m from a flat reflecting wall. The wave travels at a speed of 0.20 m s^{-1}. Sketch the arrangement and show the position of the wavefront:

a 2.5 s,

b 4.0 s after the wave front was produced at P.

3 Copy and complete Figure 7, by showing the wave fronts after they have passed across the boundary and have been refracted. Also, show the direction of the refracted waves.

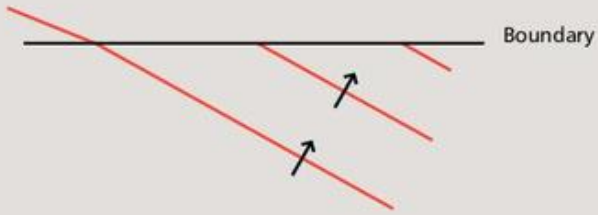


Fig 7

4 Water waves are diffracted on passing through a gap. How is the amount of diffraction changed as a result of:

a widening the gap without changing the wavelength,

b increasing the wavelength of the water waves without changing the gap width,

c increasing the wavelength of the water waves and reducing the gap width,

d widening the gap and increasing the wavelength of the waves?

8.4 Wave properties 2

On these pages you will learn to:

- Explain the principle of superposition in simple applications
- Use the principle of superposition in simple applications, including interference and stationary waves
- Describe and explain experiments that demonstrate two-source interference using water ripples and microwaves

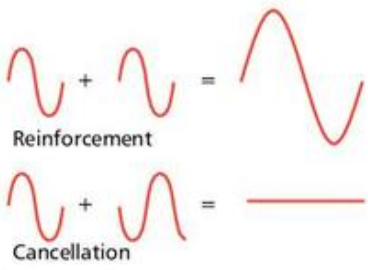


Fig 1 *Superposition*

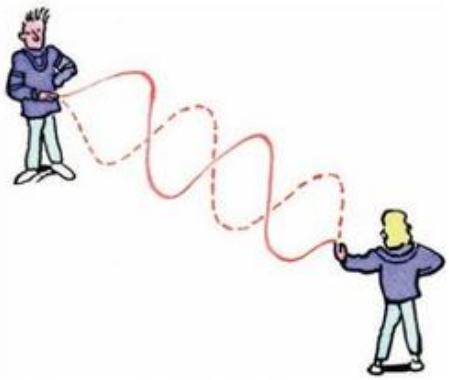


Fig 2 *Making stationary waves*

Fig 3 *Interference of water waves*

The Principle of Superposition

When waves meet, they pass through each other. At the point where they meet, they combine for an instant before they move apart. This combining effect is known as **superposition**. Imagine a boat hit by two wave crests at the same time from different directions. Anyone on the boat would know it had been hit by a **supercrest**, the combined effect of two wave crests.

The Principle of Superposition states that when two waves meet, the total displacement at a point is equal to the sum of the individual displacements at that point.

- Where a crest meets a crest, a **supercrest** is created; the two waves reinforce each other.
- Where a trough meets a trough, a **supertrough** is created; the two waves reinforce each other.
- Where a crest meets a trough, the resultant displacement is **zero**; the two waves cancel each other out.

Further examples of superposition

1 Stationary waves on a rope

Stationary waves are formed on a rope if two people send waves continuously along a rope from either end, as shown in Figure 2. The two sets of waves are referred to as **progressive waves**, to distinguish them from stationary waves. They combine at fixed points along the rope to form points of no displacement, or **nodes**, along the rope. At each node, the two sets of waves are always 180° out of phase so they cancel each other out. Stationary waves are described in more detail in topic 8.5.

2 Water waves in a ripple tank

A vibrating dipper on a water surface sends out circular waves. Figure 3 shows a snapshot of two sets of circular waves produced in this way in a ripple tank. The waves pass through each other continuously:

- Points of **cancellation** are created where a crest from one dipper meets a trough from the other dipper. These points of cancellation are seen as gaps in the wave fronts.
- Points of **reinforcement** are created where a crest from one dipper meets a crest from the other dipper, or where a trough from one dipper meets a trough from the other dipper.

As the waves are continuously passing through each other at constant frequency and at a constant phase difference, cancellation and reinforcement occur at fixed positions. This effect is known as **interference**. The two dippers are said to be **coherent** emitters of waves, because they vibrate with a constant phase difference. If the phase difference changed at random, the points of cancellation and reinforcement would move about at random and no interference pattern would be seen. Interference of light is described in more detail in topic 9.1.

Note The points of cancellation and reinforcement would be further apart if the wavelength were increased or the dippers were closer together.

Tests using microwaves

A microwave transmitter and receiver can be used to demonstrate reflection, refraction, diffraction, interference and polarisation of microwaves. The transmitter produces microwaves of wavelength 3.0 cm. The receiver can be connected to a suitable meter, which gives a measure of the intensity of the microwaves at the receiver.

- Place the receiver in the path of the microwave beam from the transmitter. Move the receiver gradually away from the transmitter and note that the receiver signal decreases with distance from the transmitter. This shows that the microwaves become weaker as they travel away from the receiver.
- Place a metal plate between the transmitter and the receiver to show that microwaves cannot pass through metal.
- Use two metal plates to make a narrow slit and show that the receiver detects microwaves that have been diffracted as they pass through the slit. Show that if the slit is made wider, less diffraction occurs.
- Use a narrow metal plate with the two plates from above to make a pair of slits, as in Figure 4. Direct the transmitter at the slits and use the receiver to find points of cancellation and reinforcement where the microwaves from the two slits overlap.

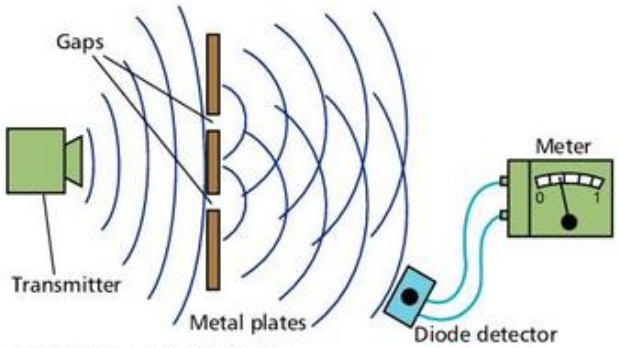


Fig 4 Interference of microwaves

QUESTIONS

1 Figure 5 shows two wave pulses on a rope travelling towards each other. Sketch a snapshot of the rope:

Fig 5

a when the two waves are passing through each other,

b when the two waves have passed through each other.

2 How would you expect the interference pattern in Figure 3 to change if:

a the two dippers are moved further apart,

b the frequency of the waves produced by the dippers is reduced?

3 Microwaves from a transmitter are directed at a narrow slit between two metal plates. A receiver is placed in the path of the diffracted microwaves, as shown in Figure 6.

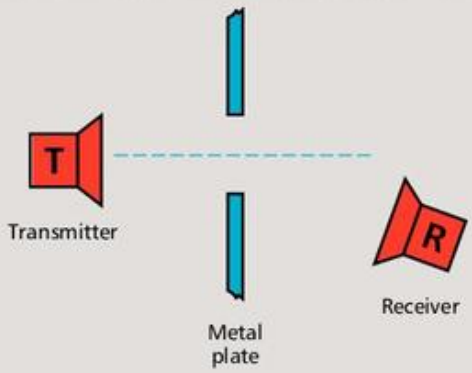


Fig 6

How would you expect the receiver signal to change if:

a the receiver is moved directly away from the slit,

b the slit is then made narrower?

4 Microwaves, from a transmitter, are directed at two parallel slits in a metal plate. A receiver is placed on the other side of the metal plate. When the receiver is moved a short distance along a line AB parallel to the plate, the receiver signal decreases then increases again.

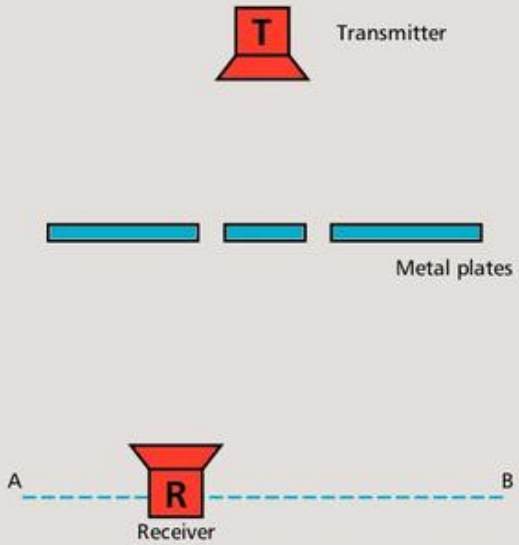


Fig 7

a Explain why the signal decreased when it was moved from A along the line AB, and then increased.

b Explain why the signal increased as it moved towards B.

8.5 Stationary and progressive waves

On these pages you will learn to:

- Compare progressive and stationary waves in terms of particle vibrations
- Explain the formation of a stationary wave on a string and identify nodes and antinodes
- Describe and explain experiments that demonstrate stationary waves using microwaves and stretched strings

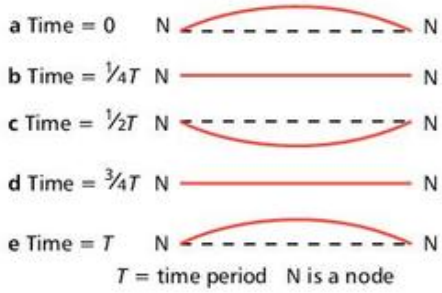


Fig 1 Fundamental vibrations

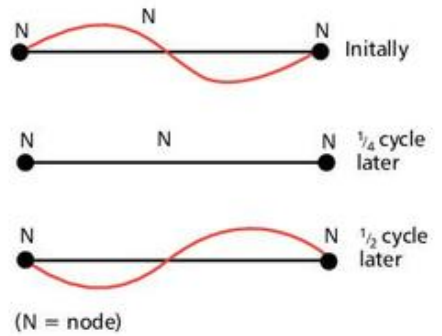


Fig 2 A stationary wave of two loops

Stationary waves do not transfer energy

The amplitude of vibration is zero at the nodes, so there is no energy at the nodes. The amplitude of vibration is a maximum at the antinodes, so there is maximum energy at the antinodes. Because the nodes and antinodes are at fixed positions, no energy is transferred in a stationary wave pattern.

Formation of stationary waves

When a guitar string is plucked, the sound produced depends on the way in which the string vibrates. If the string is plucked gently at its centre, a stationary wave of constant frequency is set up on the string. The sound produced therefore has a constant frequency. If the guitar string is plucked harshly, the string vibrates in a more complicated way and the note produced contains other frequencies as well as the frequency produced when it is plucked gently.

As explained in topic 8.4, a stationary wave is formed when two progressive waves pass through each other. This can be achieved on a string in tension by:

- Sending progressive waves along the string from either end.
- Fixing one end of the string and sending progressive waves along it from the other end. The waves reflect at the fixed end and pass through progressive waves moving towards the fixed end.
- Fixing both ends and making the middle part vibrate, so progressive waves travel towards each end, reflect at the ends, and then pass through each other.

The simplest stationary wave pattern on a string is shown in Figure 1. This is the **fundamental** mode of vibration of the string. It consists of a single loop that has a **node** (i.e. a point of no displacement) at either end. The string vibrates with maximum amplitude mid-way between the nodes. This position is referred to as an **antinode**. In effect, the string vibrates from side-to-side repeatedly. For this pattern to occur, the distance between the nodes at either end (i.e. the length of the string) must be equal to one half-wavelength of the waves on the string:

$$\text{Distance between adjacent nodes} = \tfrac{1}{2}\lambda$$

If the frequency of the waves sent along the rope from either end is raised steadily, the pattern in Figure 1 disappears: a new pattern is observed with two equal loops along the rope. This pattern (Figure 2) has a node at the centre as well as at either end. It is formed when the frequency is twice as high as in Figure 1, corresponding to half the previous wavelength. Because the distance from one node to the next is equal to half a wavelength, the length of the rope is therefore equal to one full wavelength.

Explanation of stationary waves

Consider a snapshot of two progressive waves passing through each other:

- When they are in phase, they reinforce each other to produce a large wave (Figure 3a).
- A quarter of a cycle later, the two waves have each moved one quarter of a wavelength in opposite directions. They are now in **antiphase**, so they cancel each other (Figure 3b).
- After a further quarter cycle, the two waves are back in phase. The resultant is again a large wave as in Figure 3a, except reversed.

The points where there is no displacement (i.e. the nodes) are fixed in position throughout. Between these points, the stationary wave oscillates between the nodes.

In general, in any stationary wave pattern:

The amplitude of a vibrating particle in a stationary wave pattern varies with position from:

- zero at a node,
- to maximum amplitude at an antinode.

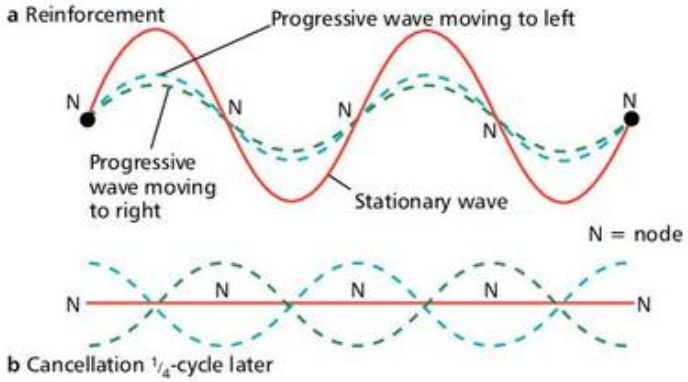


Fig 3 Explaining stationary waves

The phase difference between two vibrating particles:

- is zero if the two particles are between adjacent nodes or separated by an even number of nodes,
- is 180° if the two particles are separated by an odd number of nodes.

Table 1 Comparison between stationary waves and progressive waves in terms of particle vibrations

	Stationary waves	Progressive waves
Frequency	All particles, except at the nodes, vibrate at the the same frequency.	All particles vibrate at same frequency.
Amplitude	The amplitude varies from zero at the nodes to a maximum at the antinodes.	The amplitude is the same for all particles.
Phase difference between two particles	$m\pi$, where m is the number of nodes between the two particles.	$2\pi x/\lambda$, where x = distance apart and λ is the wavelength.

More examples of stationary waves

Sound in a pipe

Sound resonates at certain frequencies in an air-filled tube or pipe. In a pipe closed at one end, these resonant frequencies occur when there is an antinode at the open end and a node at the other end. (See topic 8.7.)

Using microwaves

Microwaves from a transmitter are directed normally at a metal plate, which reflects the microwaves back towards the transmitter. When a detector is moved along the line between the transmitter and the metal plate, the detector signal is found to be zero at equally spaced positions along the line. The reflected waves and the waves from the transmitter form a stationary wave pattern. The positions where no signal is detected are where nodes occur. They are spaced at intervals of one half of a wavelength.

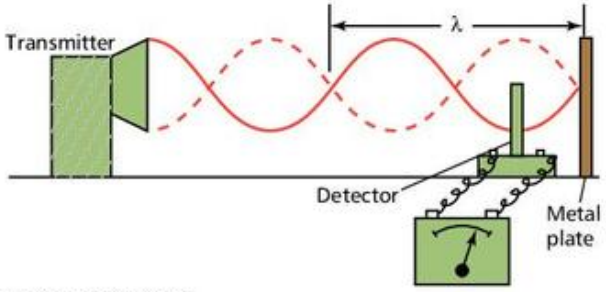


Fig 4 Using microwaves

QUESTIONS

1 a Sketch the stationary wave pattern seen on a rope, when there is a node at either end and an antinode at its centre.

b If the rope in part **a** is 4.0 m in length, calculate the wavelength of the waves on the rope for the pattern you have drawn.

2 The stationary wave pattern shown in Figure 5 is set up on a rope of length 3.0 m.

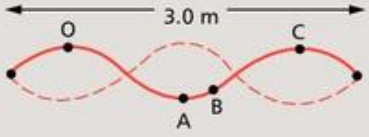


Fig 5

a Calculate the wavelength of these waves.

b State the phase difference between the particle vibrating at O and the particle vibrating at:

i A

ii B

iii C

3 State two differences between a stationary wave and a progressive wave in terms of the vibrations of the particles.

4 Microwaves from a transmitter are reflected back to the transmitter using a flat metal plate. A detector is moved along the line between the transmitter and the metal plate. The detector signal is zero at positions 15 mm apart.

a Explain why the signal is zero at certain positions.

b Calculate the wavelength of the microwaves.

8.6 More about stationary waves on strings

On these pages you will learn to:

- Explain the stationary wave patterns formed on a string
- Know how the frequencies of the stationary wave patterns on a string relate to each other

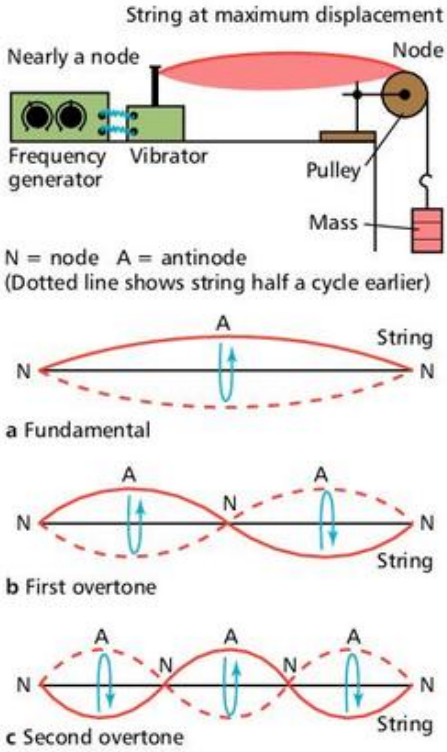


Fig 1 *Stationary waves on a string*

Stationary waves on a vibrating string

A controlled arrangement for producing stationary waves is shown in Figure 1. A string or wire is tied at one end to a mechanical vibrator, connected to a frequency generator. The other end of the string passes over a pulley and supports a mass, which keeps the tension in the string constant. As the frequency of the generator is increased from a very low value, different stationary wave patterns are seen on the string. In every case, the length of string between the pulley and the vibrator has a node at either end.

- The **fundamental pattern** of vibration is seen at the lowest possible frequency. This has an antinode at the middle, as well as a node at either end. Because the length L of the vibrating section of the string is between adjacent nodes, the wavelength of the waves that form this pattern, the fundamental wavelength λ_0, is equal to $2L$. Therefore, the fundamental frequency:

$$f_0 = \frac{v}{2L}$$

where v is the speed of the progressive waves on the wire.

- The next stationary wave pattern, the **first overtone**, is where there is a node at the middle so the string is in two loops. The wavelength of the waves that form this pattern λ_1 is L, because each loop has a length of half a wavelength. Therefore, the frequency of the first overtone vibrations:

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{L} = 2f_0$$

- The next stationary wave pattern, the **second overtone**, is where there are nodes at a distance of $\frac{1}{3}L$ from either end and an antinode at the middle. The wavelength of the waves that form this pattern λ_2 is $\frac{2}{3}L$, because each loop has a length of half a wavelength. Therefore, the frequency of the second overtone vibrations:

$$f_2 = \frac{v}{\lambda_2} = \frac{3v}{2L} = 3f_0$$

In general, stationary wave patterns occur at frequencies f_0, $2f_0$, $3f_0$, $4f_0$, etc., where f_0 is the frequency of the fundamental vibrations. This is the case in any vibrating linear system that has a node at either end.

Explanation of the stationary wave patterns on a vibrating string

In the arrangement shown in Figure 1, consider what happens to a wave sent out by the vibrator. The crest reverses its phase when it reflects at the fixed end, and travels back along the string as a trough. When it reaches the vibrator, it reflects and reverses phase again, travelling away from the vibrator once more as a crest. If this crest is reinforced by a crest created by the vibrator, the amplitude of the wave is increased. This is how a stationary wave is formed. The key condition is that the time taken for a wave to travel along the string and back should be equal to the time taken for a whole number of cycles of the vibrator:

- The time taken for a wave to travel along the string and back:

$$t = \frac{2L}{v}$$

where v is the speed of the waves on the string.

- The time taken for the vibrator to pass through a whole number of cycles:

$$t = \frac{m}{f}$$

where f is the frequency of the vibrator and m is a whole number (i.e. $m = 1$ or 2 or 3, etc.).

Therefore the key condition may be expressed as $\frac{2L}{v} = \frac{m}{f}$

Rearranging this equation gives:

$$f = \frac{mv}{2L} = mf_0 \text{ and } \lambda = \frac{v}{f} = \frac{2L}{m}$$

In other words:

- Stationary waves are formed at frequencies f_0, $2f_0$, $3f_0$, etc.
- The length of the vibrating section of the string

$L = \frac{m\lambda}{2}$ = whole number of half wavelengths

Making music

A guitar produces sound when its strings vibrate as a result of being plucked. In an electronic guitar, the vibrations of the string are detected by a microphone which produces electrical waves. These are amplified and converted back to sound waves in a loudspeaker. In an acoustic guitar, the string vibrations make the guitar surfaces vibrate and send out sound waves.

When a stretched string or wire vibrates, its **pattern of vibration** is a mix of its fundamental mode of vibration and the overtones. The sound produced is the same mix of frequencies which change with time as the pattern of vibration changes. A **spectrum analyser** can be used to show how the intensity of a sound varies with frequency and with time. Combined with a **sound synthesiser**, the original sound can be altered by amplifying or suppressing different frequency ranges.

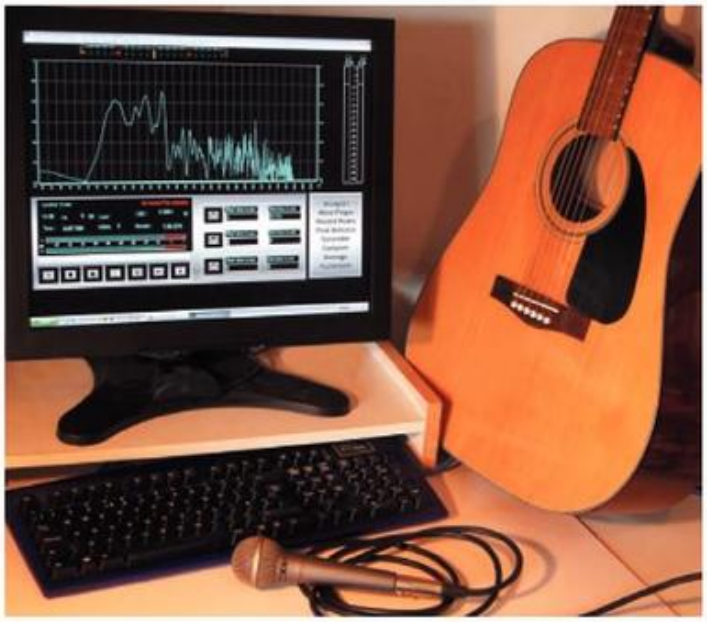


Fig 2 A sound synthesiser

The **pitch** of a note produced by a stretched string can be altered by changing the **tension** of the string, or by altering its **length**. The pitch is raised by raising the tension or shortening the length. Lowering the tension or increasing the length lowers the pitch. By changing the length or altering the tension, a vibrating string or wire can be tuned to the same pitch as a vibrating tuning fork. However, the sound from a vibrating string includes the overtone frequencies as well as the fundamental frequency, whereas a tuning fork vibrates only at its fundamental frequency. The wire is tuned when its fundamental frequency is the same as the tuning fork frequency. A simple visual check is to balance a small piece of paper on the wire at its centre, and place the base of the vibrating tuning fork on one end of the wire. If the wire is tuned to the tuning fork, it will vibrate in its fundamental mode and the piece of paper will fall off.

QUESTIONS

1 A stretched wire of length 0.80 m vibrates in its fundamental mode at a frequency of 256 Hz. Calculate:

a the wavelength of the progressive waves on the wire,

b the speed of the progressive waves on the wire.

2 The fundamental frequency of vibration of a stretched wire is inversely proportional to the length of the wire. For the wire in question **1**, calculate the length of the wire needed to produce a frequency of:

a 512 Hz

b 384 Hz

Assume that the tension of the wire is not changed.

3 Describe how you would expect the note from a vibrating guitar string to change if the string is:

a shortened,

b tightened.

4 The speed, v, of the progressive waves on a stretched wire varies with the tension T in the wire, in accordance with the equation $v = \left(\frac{T}{\mu}\right)^{1/2}$, where μ (Greek *mu*) is the mass per unit length of the wire. Use this formula to explain why a nylon wire and a steel wire, of the same length, diameter and tension, produce notes of different pitch. State, with a reason, which wire would produce the higher pitch.

8.7 Stationary waves in pipes

On these pages you will learn to:

- Explain the formation of a stationary wave in a pipe closed at one end
- Explain the formation of a stationary wave in a pipe open at both ends
- Identify nodes and antinodes
- Determine the wavelength of sound using stationary waves in a pipe

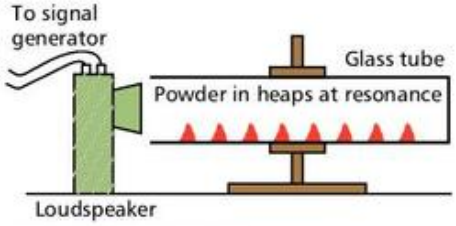
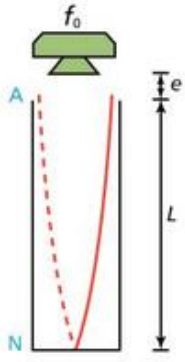


Fig 1 Acoustic resonance



a Fundamental

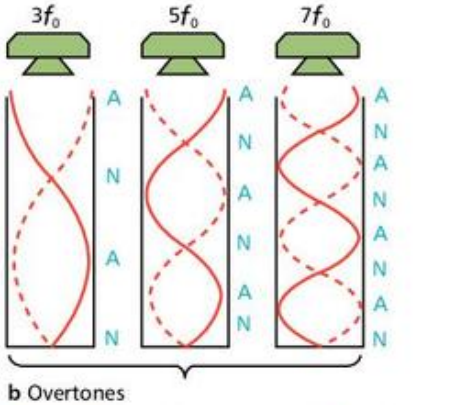


Fig 2 Resonances in a pipe closed at one end

Stationary waves in a pipe closed at one end

If a small loudspeaker connected to a signal generator is placed near the open end of a pipe, the pipe resonates with sound at certain frequencies as the signal frequency is changed. Each **resonance** is due to stationary sound waves in the pipe. Sound waves from the transmitter at the open end are reflected at the other end of the pipe. The reflected waves, and the waves directly from the transmitter, pass along the pipe in opposite directions and therefore form a stationary wave pattern at certain frequencies. Figure 1 shows how this effect can be seen, as well as heard. The powder forms small heaps at the nodes, where there are no vibrations. Antinodes occur mid-way between the nodes and just beyond the open end. For greatest effect, the position of the loudspeaker needs to be at the antinode just beyond the open end.

- **At the fundamental frequency**, the lowest frequency at which the pipe resonates with sound, there is a node at the closed end and an antinode at the open end with no nodes or antinodes between. As shown in Figure 2, the pipe length $L = \frac{1}{4}\lambda_0$ (ignoring the small end-correction, e, at the open end). Therefore,
  the fundamental wavelength $\lambda_0 = 4L$,
  the fundamental frequency, $f_0 = \frac{v}{4L}$, where v is the speed of sound in the pipe.
- **Overtones** occur at frequencies $3f_0$, $5f_0$, $7f_0$, etc. In each case, there is a node at the closed end and an antinode at the open end, with one or more equally spaced nodes or antinodes between. As shown in Figure 2:

1 At the **first overtone**, the pipe length $L = \frac{3}{4}\lambda_1$ (ignoring the small end-correction at the open end). Therefore, the first overtone wavelength $\lambda_1 = \frac{4L}{3}$, where L is the pipe length, and

$$\text{the first overtone frequency, } f_1 = \frac{3v}{4L} = 3f_0$$

2 At the **second overtone**, the pipe length $L = \frac{5}{4}\lambda_2$ (ignoring the small end-correction).

Therefore, the second overtone wavelength $\lambda_2 = \frac{4L}{5}$, where L is the pipe length,

and the second overtone frequency, $f_2 = \frac{5v}{4L} = 5f_0$

Further resonances occur at $7f_0$, $9f_0$, etc., corresponding to an odd number of quarter wavelengths equal to the pipe length.

To determine the wavelength of sound in a pipe closed at one end, a vertical glass tube is used as the pipe, as shown in Figure 3. The length of the air column in the pipe is varied by using the valves to alter the level of water in the pipe. A small loudspeaker (or a tuning fork) is used to direct sound waves of constant frequency into the open end of the pipe.

- The water level in the pipe is gradually lowered from near the top of the pipe until the pipe resonates at its shortest length. The length, L_0, of the air column at this resonance is then measured. This length plus the small end-correction, e, is equal to one-quarter of a wavelength.
- The water level is then lowered further until the pipe resonates with sound at its first overtone. The length, L_1, of the air column at this resonance is then measured. This length plus the small end-correction is equal to three-quarters of a wavelength.

The difference between the two length measurements, $L_1 - L_0$, is equal to half the wavelength of the sound waves in the pipe. Hence the wavelength λ $(= 2(L_1 - L_0))$ can be calculated.

Stationary waves in a pipe open at both ends

Sound waves travelling along the pipe partially reflect at the open end because the speed changes at the exit. An antinode is formed at either end. Therefore the pipe resonates with sound at any frequency corresponding to the pipe length L equal to a whole number of half wavelengths (the distance between two adjacent antinodes). Figure 4 shows the stationary wave patterns in this situation.

- **At the fundamental frequency,** there is an antinode at either end of the pipe with a node mid-way. As shown in Figure 4, the pipe length $L = \frac{1}{2}\lambda_0$ (ignoring the small end-corrections at each end).
  Therefore, the fundamental wavelength $\lambda_0 = 2L$, where L is the pipe length,

  $$\text{the fundamental frequency, } f_0 = \frac{v}{2L},$$

  where v is the speed of sound in the pipe.
- **Overtones** occur at frequencies $2f_0$, $3f_0$, $4f_0$, etc. In each case, there is an antinode at either end, with one or more equally spaced nodes and antinodes between, as shown in Figure 4,

1 At the **first overtone**, the wavelength λ_1 = the pipe length L (ignoring the small end-corrections).

Therefore, the first overtone frequency, $f_1 = \frac{v}{L} = 2f_0$

2 At the **second overtone**, the wavelength λ_2 is such that the pipe length $L = \frac{3}{2}\lambda_2$ (ignoring the small end-corrections).

Therefore, the second overtone wavelength $\lambda_2 = \frac{2L}{3}$, where L is the pipe length,

the second overtone frequency, $f_2 = \frac{3v}{2L} = 3f_0$

Further resonances occur at $4f_0$, $5f_0$, etc., corresponding to a whole number of half-wavelengths equal to the pipe length.

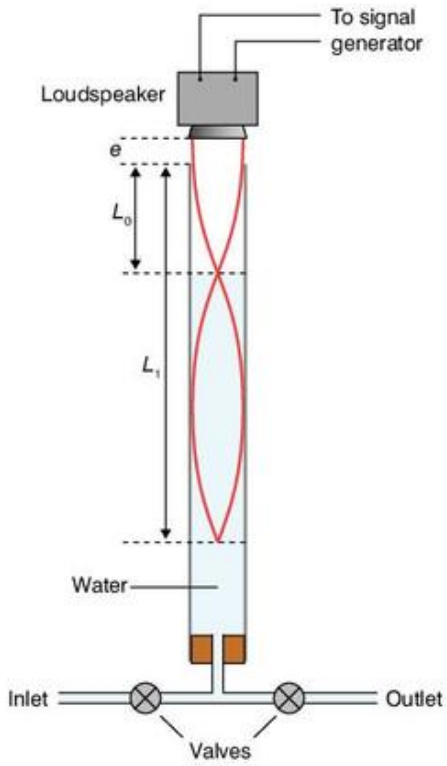


Fig 3 Using stationary waves to measure the wavelength of sound

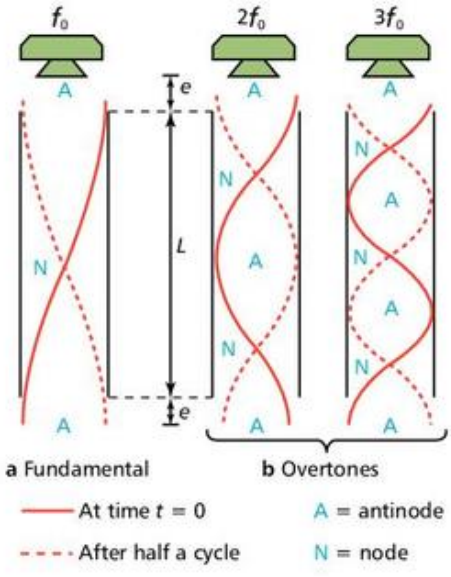


Fig 4 Stationary waves in an open pipe

QUESTIONS

1 A pipe, of length 0.60 m, is closed at one end and open at the other end. The speed of sound in the pipe is 340 m s^{-1}. Estimate:
  a its fundamental wavelength and frequency,
  b the frequency of its first overtone.

2 A pipe of length 1.50 m, closed at one end and open at the other end, resonates at a frequency of 170 Hz. The speed of sound in the pipe is 340 m s^{-1}. Calculate:
  a the wavelength of the sound waves in the pipe,
  b the fundamental frequency of this pipe.

3 A pipe, of length 2.40 m, is open at both ends. The speed of sound in the pipe is 340 m s^{-1}. Calculate:
  a its fundamental frequency,
  b its first overtone frequency.

4 A wind organ has pipes open at both ends, which are of lengths from 0.25 m to 2.50 m. Calculate the range of fundamental frequencies from these pipes. The speed of sound in the pipes is 340 m s^{-1}.

8.8 The Doppler effect

On these pages you will learn to:

- Show an understanding of the Doppler effect and recall that the change of frequency is called a Doppler shift
- Appreciate that Doppler shift is observed with all waves, including sound and light
- Use the expression $f_0 = \frac{f_s v}{v \pm v_s}$ for a source of sound waves moving relative to a stationary observer

When a vehicle sounding its horn speeds past, the note heard by a bystander changes pitch. When approaching, the pitch is higher; when moving away, the pitch is lower. This is an example of the Doppler effect: when there is relative motion between a source of sound and an observer, the observed frequency differs from the emitted frequency.

The Doppler effect occurs with any form of wave motion, provided there is relative motion between the observer and the source of the waves along the line between them. To understand the cause of the Doppler effect, consider a source of waves moving at constant speed along a straight line, as in Figure 1, emitting waves of a constant frequency f_s.

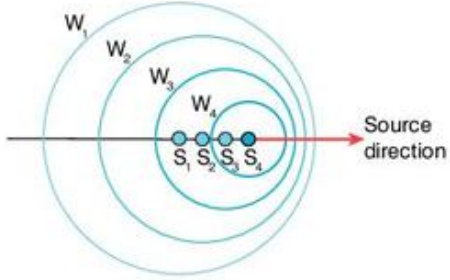


Fig 1 The Doppler effect

In Figure 1, the source has moved from position S_1 to S_4, where it emits wavefront W_4. It emitted wavefront W_1 when it was at S_1, wavefront W_2 when it was at S_2 and wavefront W_3 when it was at S_3. Because the source is moving in one direction, the wavefronts ahead of the source are bunched together and the wavefronts behind it are further apart.

More generally, if the wave speed is v and the speed of the source is v_s, then in one second the source emits f_s wavefronts and moves through a distance v_s, the leading wavefront having moved through a distance v.

Ahead of the source, the waves are bunched up because the source is moving in the same direction as the waves. So the distance from the leading wavefront to the source is equal to $v - v_s$ after the source has emitted f_s wavefronts. As this distance is equal to f_s wavelengths, the wavelength λ of the waves moving in the same direction as the source is given by:

$$\lambda = \frac{v - v_s}{f_s}$$

A stationary observer ahead of the source would therefore detect waves of frequency f_0, where:

$$f_0 = \frac{\text{wave speed}}{\text{wavelength}} = \frac{v}{\left(\frac{v - v_s}{f_s}\right)} = \frac{v f_s}{(v - v_s)}$$

Note that the observed frequency f_0 is greater than the source frequency f_s because:

$$\frac{v}{v - v_s} > 1$$

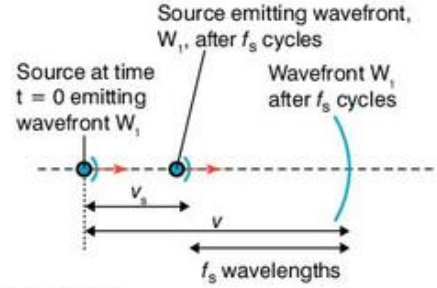


Fig 2 Ahead of the source

Behind the source, the waves are stretched out because the waves are moving in the opposite direction to the source direction. So the distance from the leading wavefront to the source is equal to $v + v_s$ after the source has emitted f_s wavefronts. As this distance is equal to f_s wavelengths, the wavelength λ of the waves moving in the opposite direction to the source is given by:

$$\lambda = \frac{v + v_s}{f_s}$$

A stationary observer behind the source would therefore detect waves of frequency f_0 where:

$$f_0 = \frac{\text{wave speed}}{\text{wavelength}} = \frac{v}{\left(\frac{v + v_s}{f_s}\right)} = \frac{v f_s}{(v + v_s)}$$

The equation shows that the observed frequency f_0 is less than the source frequency f_s because:

$$\frac{v}{(v + v_s)} < 1$$

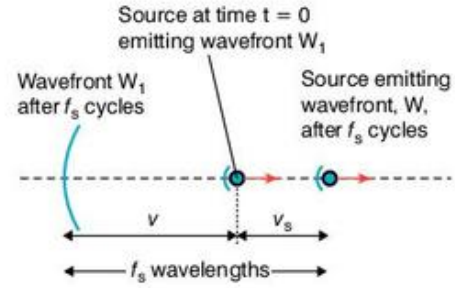


Fig 3 Behind the source

Notes

1 The above equations may be summarised as a single equation:

$$f_0 = \frac{vf_s}{(v \pm v_s)}$$

where the − sign applies ahead of the source and the + sign applies behind it.

2 The change of frequency:

$$\Delta f = f_0 - f_s = \frac{vf_s}{(v \pm v_s)} - f_s$$

$$= \frac{vf_s}{(v \pm v_s)} - \frac{(v \pm v_s)f_s}{(v \pm v_s)} = \frac{(v_s f_s)}{(v \pm v_s)}$$

where the − sign applies when the observer is ahead of the source (i.e. the source is moving towards the observer) and the + sign applies when the observer is behind the source (i.e. the source is moving away from the observer).

3 For electromagnetic waves, where the speed of the source is much less than the speed of the waves, it can be shown that the change of frequency (or **Doppler shift**)

$$\Delta f = \frac{v_s f_s}{c},$$

where c is the speed of electromagnetic waves.

Worked example

A train sounds its horn as it travels at a speed of $30\,m\,s^{-1}$ towards a level crossing. The sound is emitted from the horn at a frequency of 840 Hz. Calculate:

a the wavelength of the sound waves travelling towards the level crossing

b the observed frequency of the sound waves at the level crossing.

The speed of sound in air = $340\,m\,s^{-1}$.

a $\lambda = \frac{v - v_s}{f_s} = \frac{340 - 30}{840} = 0.37\,m$

b $f_0 = \frac{v}{\lambda} = \frac{340}{0.37} = 920\,Hz$

Speed measurement using the Doppler effect

A radar speed camera emits pulses of microwave radiation and detects any pulses reflected from a vehicle back to the camera. If the vehicle is moving towards or away from the camera, the frequency of such a reflected pulse differs from the emitted pulse. The difference in frequency is proportional to the speed of the vehicle and so can be used to measure the vehicle speed.

The same principle is used in:

- the Doppler anemometer, which uses pulses of laser light to measure wind speed by detecting pulses reflected by small particles in the wind
- Doppler echocardiography, which uses ultrasound pulses to create an image of the heart showing the speed and direction of blood flow inside it.

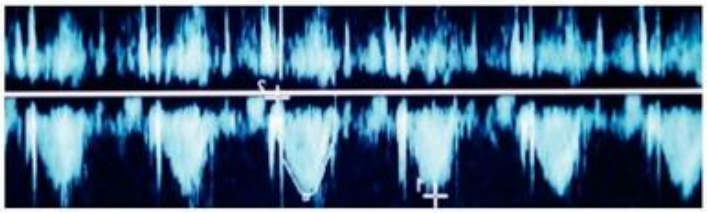


Fig 4 An echocardiogram

QUESTIONS

The speed of sound in air = $340\,m\,s^{-1}$.

The speed of electromagnetic waves in a vacuum = $3.0 \times 10^8\,m\,s^{-1}$.

1 A car on a motorway sounds its horn as it approaches an overhead bridge at a speed of $28\,m\,s^{-1}$. The horn emits sound waves at a frequency of 1100 Hz.
   a Calculate the frequency of the sound heard by an observer on the bridge as the car approaches.
   b State and explain what the frequency of the sound heard by the observer would be when the car was directly below the bridge.

2 A train is travelling at a speed of $40\,m\,s^{-1}$ on a straight section of track between two overhead bridges X and Y when it sounds its horn. An observer on bridge X hears the sound from the horn at a lower frequency than does an observer on bridge Y.
   a State whether the train is moving towards X or towards Y and explain why.
   b The observer at X hears sound of frequency 950 Hz from the horn. Calculate the frequency of the sound emitted by the horn.

3 The Sun has an equatorial diameter of 1.4×10^9 m and spins at its equator at a steady rate of one revolution every 25 days.
   a Calculate the speed of rotation of an atom at the Sun's equator.
   b Calculate the change in frequency observed in light of frequency 5.0×10^{14} Hz emitted by atoms on the Sun's equator when they are moving away from the Earth at the speed calculated in **a**.

4 The Doppler anemometer uses pulses of laser light to measure wind speed by detecting pulses reflected by small particles in the wind. Explain why **a** ultrasound and **b** microwaves would be unsuitable for use instead of laser pulses.

8 End of chapter questions

1 a i Explain the difference between longitudinal waves and transverse waves.
ii State one example of a longitudinal wave.
iii State one example of a transverse wave.

b With the aid of a diagram, explain what is meant by:
i diffraction of a wave,
ii refraction of a wave.

2 a Explain what is meant by a 'polarised' transverse wave.

b Explain why sound waves cannot be polarised, whereas light waves can.

3 An oscilloscope is used to display the signal from a microphone when sound waves of constant frequency are directed at the microphone (see graph below).

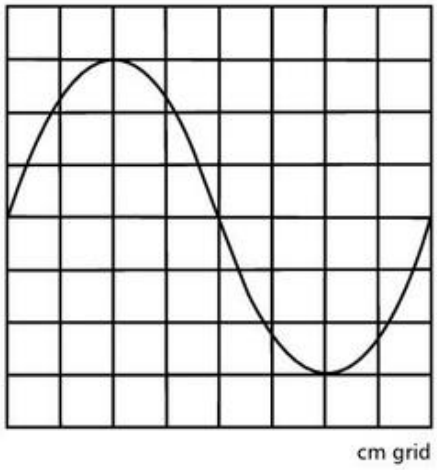


Fig 3.1

a The oscilloscope time base is set at $0.5\,\text{ms}\,\text{cm}^{-1}$. Calculate the frequency of the sound waves.

b Sketch the display on the oscilloscope you would observe, if the frequency of the sound waves was doubled with no change of loudness.

4 a Explain what is meant by the 'superposition' of waves.

b Two small loudspeakers, 1.2 m apart, are connected to a signal generator, as shown opposite. The signal generator is adjusted to produce an alternating voltage of constant amplitude, at a frequency of 1.0 kHz.
i An observer moves along the line XY at a perpendicular distance of 2.0 m from the loudspeakers, as shown. The intensity of the sound rises and falls as she moves along the line. Explain this effect.
ii In part **i**, the observer notes that successive minima are further apart along the line XY when the test is repeated at a lower frequency. Explain this observation.

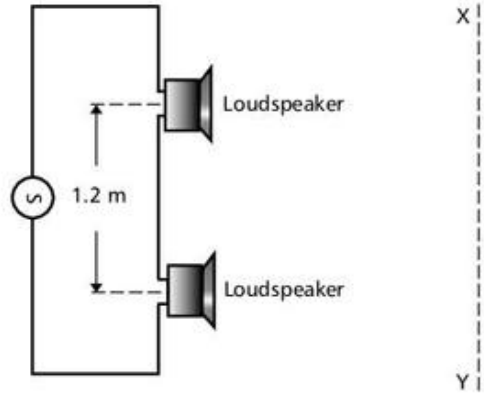


Fig 4.1

5 Microwaves from a transmitter were directed at two narrow gaps between three metal plates. A detector was placed on the other side of the plates at P, as shown below.

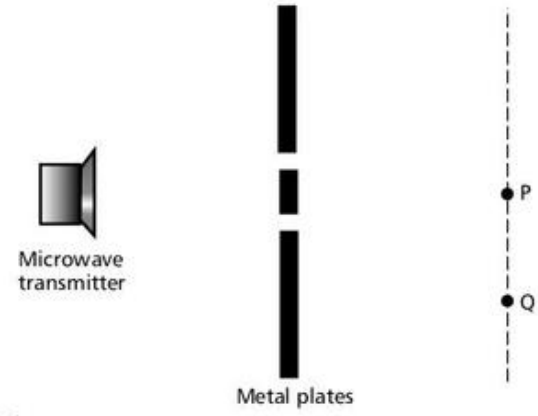


Fig 5.1

a The detector was moved along a line parallel to the plates to Q. The signal from the detector decreased from a maximum at P to a minimum mid-way between P and Q, then to a maximum at Q.
i Explain why there was a maximum at P and at Q.
ii Explain why there was a minimum mid-way between P and Q.

b Explain how the detector signal would change when the detector is mid-way between P and Q and a metal plate is placed over one of the gaps.

6 A stationary wave pattern of a wire, of length 0.60 m, vibrating at a frequency of 300 Hz is shown below.

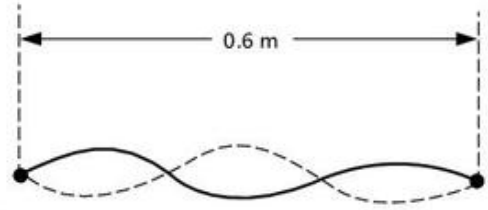


Fig 6.1

a In terms of amplitude and phase difference, compare the motion of the wire 0.10 m from the left-hand end with:
- **i** its motion at the mid-point,
- **ii** 0.10 m from the other end.

b **i** Calculate the wavelength of the waves on the wire.
- **ii** The frequency of vibrations is increased to 400 Hz, and a different stationary wave pattern is produced. Sketch the pattern you would expect to observe at 400 Hz.

7 Sound from a small loudspeaker connected to a signal generator was directed into a vertical glass tube containing some water, as shown below. The length of the air column in the tube could be varied by altering the level of water in the tube.

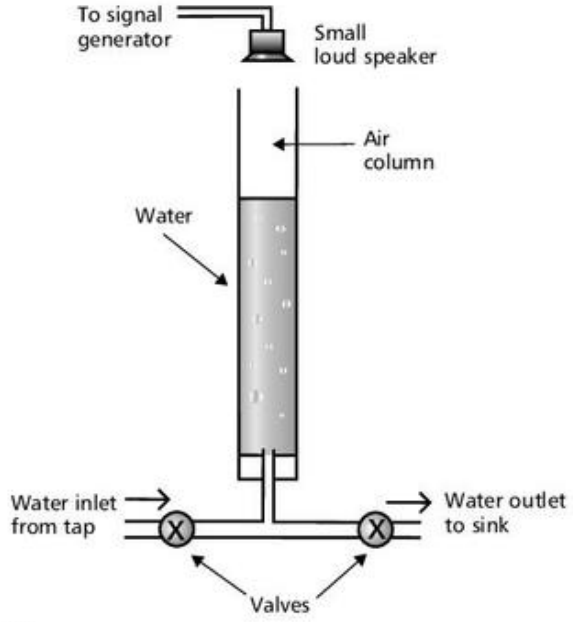


Fig 7.1

a The length of the air column was gradually increased from a few centimetres, by lowering the level of water in the tube slowly. The tube resonated with sound at certain positions of the water level. Explain why this effect happened only at certain lengths of the air column.

b The tube resonated with sound of frequency 300 Hz when the length of the air column was 270 mm and 820 mm:
- **i** Diagram **a** shows how the amplitude of the sound waves in the tube varied with position along the tube when the air column length was 270 mm. Using a copy of diagram **b**, sketch the corresponding pattern when the air column length was 820 mm.
- **ii** Show that the wavelength of the sound in the tube was 1.10 m, and hence calculate the speed of sound in the tube.

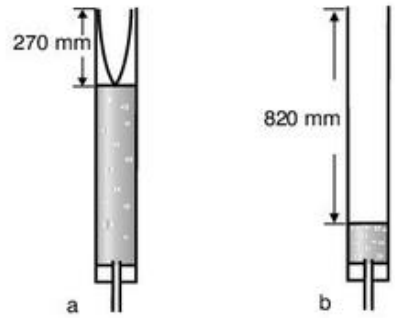


Fig 7.2

8 An open-ended pipe, of length 0.80 m, resonates with sound at a frequency of 200 Hz. The speed of sound in the pipe is 330 m s^{-1}.

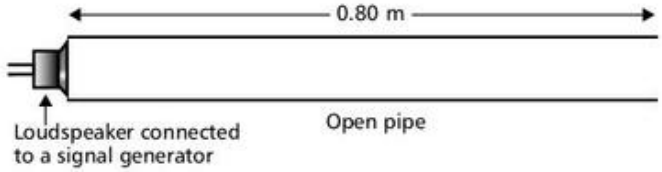


Fig 8.1

a **i** Calculate the wavelength of sound waves of frequency 200 Hz in the pipe.
- **ii** Sketch the stationary wave pattern in the pipe, when it resonates at 200 Hz.

b **i** Calculate the next highest frequency at which the pipe would resonate if the frequency was gradually increased.
- **ii** Explain why the pipe resonates at certain frequencies only.

9 **a** State what is meant by a *progressive wave*. *(2 marks)*

b The variation with distance x along a progressive wave of a quantity y, at a particular time, is shown in Figure 9.1.

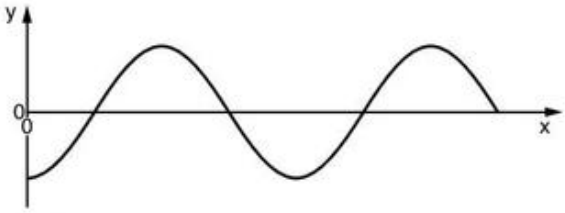


Fig 9.1

i State what the quantity y could represent. *(1 mark)*

ii Distinguish between the quantity y for
- **1** a transverse wave, *(1 mark)*
- **2** a longitudinal wave. *(1 mark)*

Cambridge International AS and A Level Physics 9702
Paper 21 Q5a, b November 2009

9 Optics

9.1 Interference of light

On these pages you will learn to:

- Explain what is meant by interference
- Describe and explain the double slits experiment using light to demonstrate two-source interference
- Recall and solve problems using the equation $\lambda = ax/D$ for double-slit interference using light

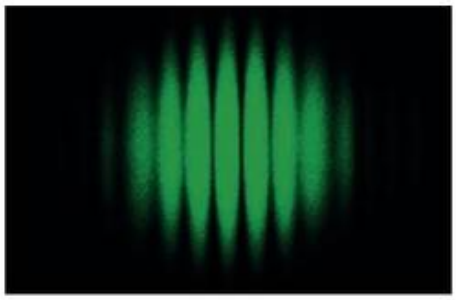


Fig 1 *Double-slit interference pattern as seen on a screen*

The wave nature of light

Young's double slits experiment

The wave nature of light was first suggested by Christian Huygens in the seventeenth century, but it was rejected at the time in favour of Sir Isaac Newton's corpuscular theory of light. Newton considered that light was composed of tiny particles he referred to as corpuscles and he was able to explain reflection and refraction using his theory. Huygens was also able to explain reflection and refraction using his wave theory. However, the two theories differed about whether or not light in a transparent substance travels faster (as predicted by Newton's theory) or slower (as predicted by Huygens' theory) than in air. Because of Newton's much stronger scientific reputation, Newton's theory of light remained unchallenged for over a century, until 1803, when Thomas Young at the Royal Institution first demonstrated interference of light. Even so, Newton's theory of light was not rejected in favour of Huygens' wave theory until several decades later when the speed of light in water was measured and found to be slower than in air.

An arrangement like the one used by Thomas Young to observe interference is shown in Figure 2. Young would have used a candle instead of a light bulb to illuminate a narrow single slit. A pair of narrow slits, referred to as the 'double slits', is illuminated by light from the single slit. Alternate bright and dark fringes can be seen using a microscope, or on a white screen placed where the diffracted light from the double slits overlaps. The fringes are evenly spaced and parallel to the double slits.

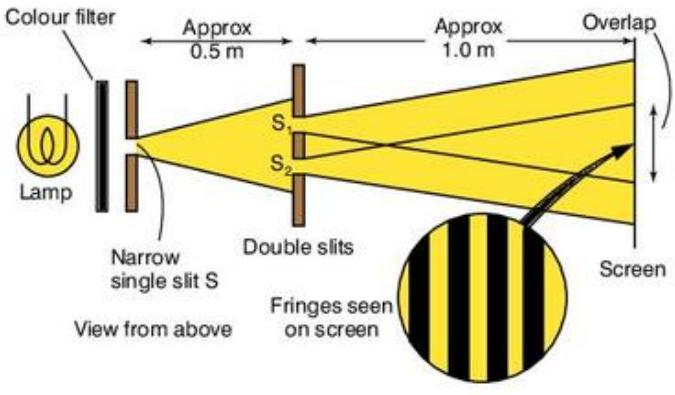


Fig 2 *Young's double slits experiment*

The fringes are formed due to **interference of light** from the two slits:

- Where a **bright fringe** is formed, the light from one slit reinforces the light from the other slit. In other words, the light waves from each slit arrive **in phase** with each other.
- Where a **dark fringe** is formed, the light from one slit cancels the light from the other slit. In other words, the light waves from the two slits arrive **180° out of phase**.

The distance from the centre of a bright fringe to the centre of the next bright fringe is called the **fringe separation,** x. This depends on the slit spacing a and the distance D from the slits to the screen, in accordance with the equation:

$$\textbf{Fringe separation, } x = \frac{\lambda D}{a},$$

where λ is the wavelength of light.

The equation shows that the fringes become more widely spaced if:

- the distance D from the slits to the screen is increased,
- the wavelength λ of the light used is increased,
- the slit spacing, a, is reduced.

Note The slit spacing is the distance between the **centres** of the slits.

The theory of the double slits equation

Consider the two slits S_1 and S_2 shown in Figure 3. At a point P on the screen where the fringes are observed, light emitted from S_1 arrives later than light from S_2 emitted at the same time. This is because the distance S_1P is greater than the distance S_2P. The difference between distances S_1P and S_2P is referred to as the **path difference**.

- **For reinforcement at P**, the path difference $S_1P - S_2P = m\lambda$, where $m = 0, 1, 2$, etc.

  Therefore, light emitted simultaneously from S_1 and S_2 arrives in phase at P, if reinforcement occurs at P.
- **For cancellation at P**, the path difference $S_1P - S_2P = (m + \frac{1}{2})\lambda$, where $m = 0, 1, 2$, etc.

  Therefore, light emitted simultaneously from S_1 and S_2 arrives at P out of phase by 180°, if cancellation occurs at P.

In Figure 3, a point Q along line S_1P has been marked such that $QP = S_2P$. Therefore, the path difference $S_1P - S_2P$ is represented by the distance S_1Q.

Because triangles S_1S_2Q and MOP are very nearly similar in shape, where M is the mid-point between the two slits and O is the mid-point of the central bright fringe of the pattern, then:

$$\frac{S_1Q}{S_1S_2} = \frac{OP}{OM}$$

If P is the mth bright fringe from the centre (where $m = 0$, 1, 2, etc.), then $S_1Q = m\lambda$ and $OP = mx$, where x is the distance between centres of adjacent bright fringes.

Also, OM = distance D and S_1S_2 = slit spacing a.

Therefore, $$\frac{m\lambda}{a} = \frac{mx}{D}$$

Rearranging this equation gives: $$\boldsymbol{\lambda = \frac{ax}{D}}$$

By measuring the slit spacing, a, the fringe separation, x, and the slit–screen distance D, the wavelength λ of the light used can be calculated. The formula is valid only if the fringe separation, a, is much less than the distance D from the slits to the screen. This condition is to ensure the triangles S_1S_2Q and MOP are very nearly similar in shape.

Note Light wavelengths are usually expressed in **nanometres (nm)**, where $1\,\text{nm} = 10^{-9}\,\text{m}$.

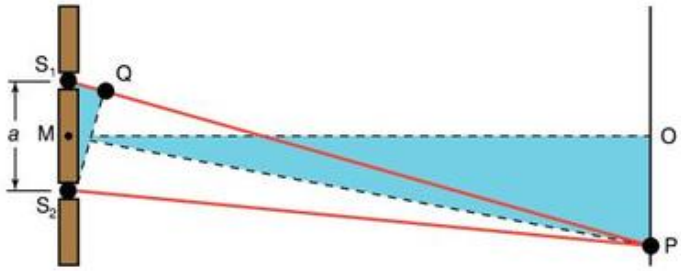


Fig 3 The theory of the double slits experiment

QUESTIONS

1 In a double slits experiment using red light, a fringe pattern is observed on a screen at a fixed distance from the double slits. How would the fringe pattern change if:

a the screen was moved closer to the slits,

b one of the double slits was blocked completely?

2 The following measurements were made in a double slits experiment:

Slit spacing, $a = 0.4\,\text{mm}$; fringe separation, $x = 1.1\,\text{mm}$; slit–screen distance, $D = 0.80\,\text{m}$.

Calculate the wavelength of light used.

3 In question **2**, the double slits were replaced by a pair of slits with a slit spacing of 0.5 mm. Calculate the fringe separation for the same slit–screen distance and wavelength.

4 The following measurements were made in a double slits experiment:

Slit spacing, $a = 0.4\,\text{mm}$; fringe separation, $x = 1.1\,\text{mm}$; wavelength of light used, $\lambda = 590\,\text{nm}$.

Calculate the distance from the slits to the screen.

9.2 More about interference

On these pages you will learn to:

- Explain the terms monochromatic light and coherence
- Show an understanding of the conditions required to observe two-source interference
- Describe two-source interference fringes using monochromatic light and white light

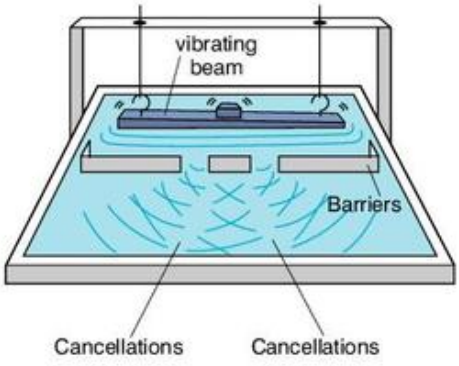


Fig 1 *Interference of water waves*

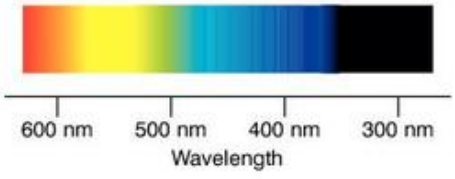


Fig 2 *Wavelength and colour*

Coherence

The double slits are described as **coherent sources,** because they emit light waves with a constant phase difference. This is because each wave crest, or wave trough, from the single slit always passes through one of the double slits a fixed time after it passes through the other slit. The double slits therefore emit wave fronts with a constant phase difference.

The arrangement is like the ripple tank demonstration in Figure 1. Straight waves from the beam vibrating on the water surface diffract after passing through the two gaps in the barrier, and produce an interference pattern where the diffracted waves overlap. If one gap is closer to the beam than the other, each wave front from the beam passes through the nearer gap first. However, the time interval between the same wave front passing through the two gaps is always the same, so the waves emerge from the gaps with a constant phase difference.

Light from two separate light bulbs could not form an interference pattern, because the two light sources emit light waves at random. The points of cancellation and reinforcement would change at random, so no interference pattern is possible.

Wavelength and colour

In the double slits experiment, the fringe separation depends on the colour of light used. White light is composed of a continuous spectrum of colours, corresponding to a continuous range of wavelengths from about 350 nm for violet light to about 650 nm for red light. Each colour of light has its own wavelength, as shown in Figure 2.

The fringe patterns shown in Figure 3 show that the fringe separation is greater for red light than for blue light. This is because red light has a longer wavelength than blue light. The fringe spacing, x, depends on the wavelength, λ, of the light according to the formula: $\frac{x}{D} = \frac{\lambda}{a}$ as explained in topic 9.1.

Rearranging this formula gives $x = \frac{\lambda D}{a}$. Thus the longer the wavelength of the light used, the greater the fringe separation is.

Light sources

- **Vapour lamps and discharge tubes** produce light with a dominant colour. For example, the sodium vapour lamp produces a yellow/orange glow which is due to light of wavelength 590 nm. Other wavelengths of light are also emitted from a sodium vapour lamp, but the colour due to light of wavelength 590 nm is much more intense than any other colour. A sodium vapour lamp is, in effect, a **monochromatic** light source because its spectrum is dominated by light of a certain colour.
- **Light from a filament lamp or from the Sun** is composed of the colours of the spectrum, and therefore covers a continuous range of wavelengths from about 350 nm to about 650 nm.

Fig 3 *The double slits fringe pattern*

- **Light from a laser** is of a specific wavelength, and therefore laser light is highly monochromatic. For example, a helium–neon laser produces red light of wavelength 635 nm only. Because a laser beam is almost perfectly parallel and monochromatic, a convex lens can focus it to a very fine spot. The beam power is then concentrated in a very small area. This is why a laser beam is very dangerous if it enters the eye. The **eye lens** would focus the beam on a tiny spot on the **retina** and the intense concentration of light at that spot would destroy the retina.

Always wear safety goggles in the presence of a laser beam. Never look along a laser beam, even after reflection.

Observing Young's fringes

Provided the light source is **not** a laser, a microscope may be used to observe the fringe pattern and to measure the fringe spacing. If so, the plane of viewing of the fringe pattern must be located to measure the distance D from the slits to the fringes accurately. Figure 4 shows how this can be done.

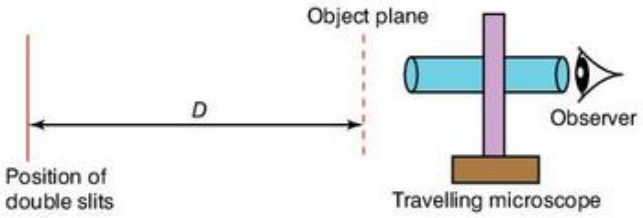


Fig 4 Measuring the fringes using a microscope

The contrast between the bright and dark fringes can be improved by narrowing the single slit (see topic 9.1 Figure 2). If this slit is too wide, each part of it produces a fringe pattern which is displaced slightly from the pattern due to adjacent parts of the single slit. As a result, the dark fringes of the double slit pattern become narrower than the bright fringes, and contrast is lost between the dark and the bright fringes.

If a laser is used as the light source, a screen **must** be used on which to observe the fringe pattern produced by a laser. Never look along a laser beam, even after reflection. Safety goggles must always be worn in the presence of a laser beam.

White light fringes

Figure 3 shows the fringe patterns observed with blue light and with red light. As explained above, the blue light fringes are closer together than the red light fringes. The fringe pattern produced by white light is shown in Figure 5. Each component colour of white light produces its own fringe pattern, each pattern centred on the screen at the same position. As a result:

- The central fringe is **white**, because every colour contributes at the centre of the pattern.
- The inner fringes are tinged with **blue on the inner side and red on the outer side**. This is because the red fringes are more spaced out than the blue fringes, and the two fringe patterns do not overlap exactly.
- The outer fringes merge into an indistinct **background of white light**. This is because, where the fringes merge, different colours reinforce and therefore overlap.

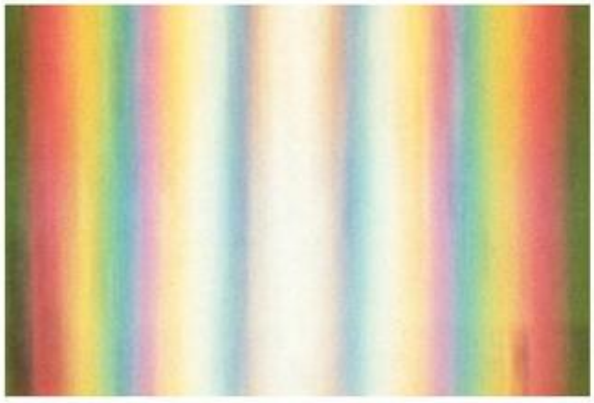


Fig 5 White light fringes

QUESTIONS

1 a Sketch an arrangement that may be used to observe the fringe pattern observed when light from a narrow slit, illuminated by a sodium vapour lamp, is passed through a pair of double slits.

b Describe the fringe pattern you would expect to observe in part **a**.

2 In question **1**, describe how the fringe pattern would change if:

a one of the double slits is blocked,

b the narrow single slit is replaced by a wider slit.

3 Double slit interference fringes are observed using light of wavelength 590 nm and a pair of double slits of slit spacing 0.50 mm. The fringes are observed on a screen at a distance of 0.90 m from the double slits. Calculate the fringe separation of these fringes.

4 Describe and explain the fringe pattern that would be observed in question **3**, if the light source were replaced by a white light source.

9.3 The diffraction grating

On these pages you will learn to:

- Explain how a diffraction grating works
- Recall and solve problems using the formula $d \sin\theta = n\lambda$
- Describe the use of a diffraction grating to determine the wavelength of light

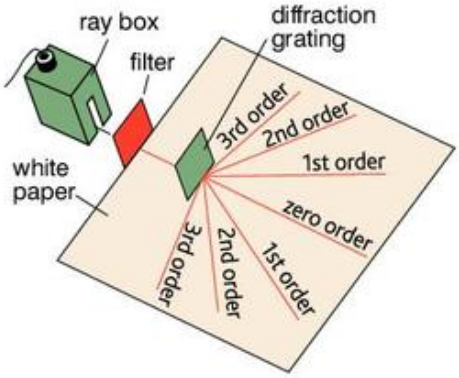


Fig 1 *The diffraction grating*

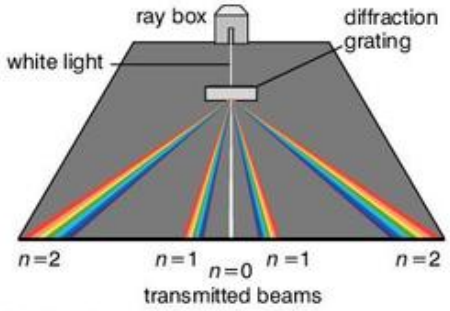


Fig 2 *Using white light*

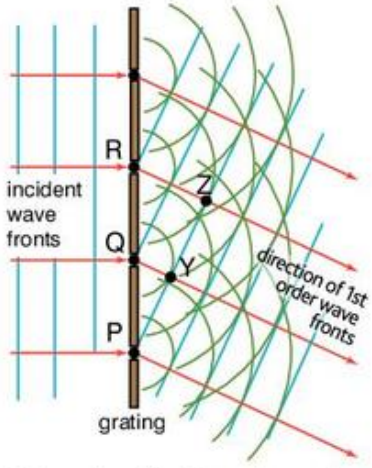


Fig 3 *Formation of the first order wavefront*

Investigating the diffraction grating

A **diffraction grating** consists of a transparent plate with many closely-spaced parallel slits ruled on it. When a parallel beam of monochromatic light is incident normally on a diffraction grating, as shown in Figure 1, light is transmitted by the grating in certain directions only. This is because:

- the light passing through each slit is diffracted
- the diffracted light waves from adjacent slits reinforce each other in certain directions only, including the incident light direction, and cancel out in all other directions.

The central beam, referred to as the 'zero order beam', is in the same direction as the incident beam. The other transmitted beams are numbered outwards from the zero order beam and are referred to as 'first order', 'second order', etc.

The **angle of diffraction**, θ, between each transmitted beam and the central beam increases if:

- light of a longer wavelength is used (e.g. by replacing a blue filter with a red filter)
- a grating with closer slits is used.

Note Figure 2 shows what happens if white light instead of monochromatic light is directed at the grating. The central transmitted beam is white and each non-central transmitted beam is spread out as a spectrum of colour from violet to red. This is because white light consists of a continuous range of wavelengths from about 350 nm (violet) to about 650 nm (red). Therefore, in each order, red light is diffracted more than violet light as it has a longer wavelength than violet light.

The diffraction grating equation

The wavelength of the light in each transmitted beam can be calculated by measuring the angle of diffraction, θ, of the beam and using the **diffraction grating equation**

$$d \sin \theta = n\lambda$$

where n is the order number of the beam and d is the slit spacing of the grating.

To prove the diffraction grating equation, consider a magnified view of part of a diffraction grating, as shown in Figure 3. Each slit diffracts the light waves that pass through it. As each diffracted wavefront emerges from a slit, it reinforces a wavefront from each of the adjacent slits. For example, in Figure 3, the wavefront emerging at P reinforces the wavefront emitted from Q one cycle earlier which reinforces the wavefront emitted from R one cycle earlier, etc. The effect is to form a new wavefront PYZ which travels in a certain direction and contributes to the first order diffracted beam.

Figure 4 shows the formation of a wavefront of the nth order beam. The wavefront emerging from slit P reinforces a wavefront emitted n cycles earlier by the adjacent slit Q. This earlier wavefront therefore must have travelled a distance of n wavelengths from the slit. Thus the perpendicular distance QY from the slit to the wavefront is equal to $n\lambda$ where λ is the wavelength of the light waves.

Since the angle of diffraction of the beam, θ, is equal to the angle between the wavefront and the plane of the slits, it follows that $\sin\theta$ = QY/QP where QP is the grating spacing (i.e. the centre-to-centre distance d between adjacent slits).

Substituting d for QP and $n\lambda$ for QY therefore gives $\sin\theta = n\lambda/d$. Rearranging this equation gives the diffraction grating equation above.

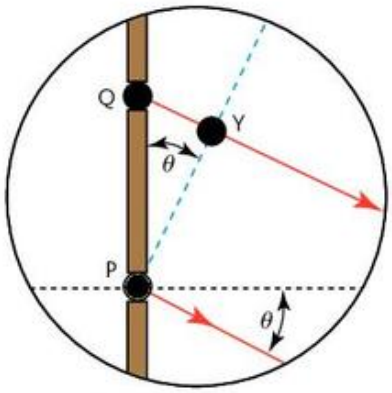


Fig 4 The nth order wavefront

Notes

1 The number of slits per metre on the grating, $N = 1/d$ where d is the grating spacing.
2 For a given order and wavelength, the smaller the value of d, the greater the angle of diffraction is. In other words, the larger the number of slits per metre is, the bigger the angle of diffraction is.
3 Fractions of a degree are usually expressed either as a decimal or in minutes (abbreviated ′) where $1° = 60′$.
4 To find the maximum number of orders produced, substitute $\theta = 90°$ ($\sin 90° = 1$) into the diffraction grating equation and calculate n using $n = d/\lambda$.

The maximum number of orders is given by the value of d/λ rounded down to the nearest whole number.

Worked example

For a diffraction grating with 600 slits per millimetre, calculate the maximum order number for light of wavelength 580 nm.

Solution

$$d = 1/N = 1/600\,\text{mm} = 1.67 \times 10^{-3}\,\text{mm}$$

$$d/\lambda = \frac{1.67 \times 10^{-3}\,\text{mm}}{580\,\text{nm}} = \frac{1.67 \times 10^{-6}\,\text{m}}{580 \times 10^{-9}\,\text{m}} = 2.88$$

Therefore the maximum order number = 2 (as 2.88 rounded down to the nearest whole number = 2).

Note There would be a first order beam and a second order beam either side of the zero order beam.

Diffraction gratings in action

We can use a diffraction grating in a **spectrometer** to study the spectrum of light from any light source and to measure light wavelengths very accurately.

- A filament lamp produces a continuous spectrum of colour from deep violet at wavelengths of about 350 nm to deep red at about 650 nm, as shown in topic 9.2 Figure 2.
- A glowing gas in a vapour lamp or a discharge tube produces a spectrum consisting of narrow vertical lines of different colours as shown in Figure 5. The wavelengths of the lines are characteristic of the chemical elements that produce the light.

Fig 5 A line spectrum

A spectrometer is designed to measure angles to within 1 arc minute which is a sixtieth of a degree. The angle of diffraction of a diffracted beam can be measured very accurately. Using light of a known wavelength, the grating spacing of a diffraction grating can therefore be measured very accurately with a spectrometer. The spectrometer and the grating can then be used to measure light of any wavelength.

QUESTIONS

1 A laser beam of wavelength 630 nm is directed normally at a diffraction grating which has 300 lines per millimetre. Calculate:
  a the angle of diffraction of each of the first two orders
  b the number of diffracted orders produced.

2 Light directed normally at a diffraction grating contains wavelengths of 580 nm and 586 nm only. The grating has 600 lines per mm.
  a How many diffracted orders are observed in the transmitted light?
  b For the highest order, calculate the angle between the two diffracted beams.

3 Light of wavelength 430 nm is directed normally at a diffraction grating. The first order transmitted beams are at 28° to the zero order beam. Calculate:
  a the number of slits per millimetre on the grating
  b the angle of diffraction for each of the other diffracted orders of the transmitted light.

4 A diffraction grating is used in a spectrometer to view light of wavelength 430 nm. The first order diffracted beam is observed at an angle of 14° 55′ to the zero order beam.
  a Calculate the number of lines per mm on the grating.
  b i Determine the order number of the highest order beam.
  ii Calculate the angle of diffraction of this order.
  (Reminder: 1 degree = 60 minutes of arc)

REVISION

9.4 Reflection and refraction of light

On these pages you will learn to:

- Explain reflection and refraction of light using wave theory
- Recall that the refractive index of a substance is the ratio of the speed of light in a vacuum to the speed of light in the substance
- Recall and use Snell's law and the equation $n_1 \sin i = n_2 \sin r$

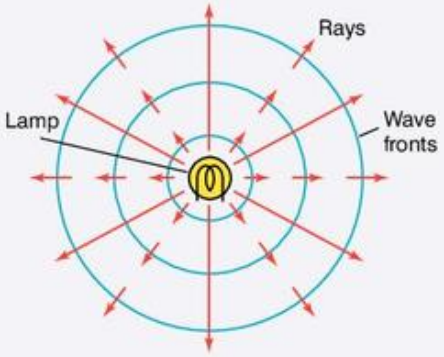


Fig 1 *Rays and waves*

The **wave theory of light** can be used to explain reflection and refraction of light. However, when we consider the effect of lenses or mirrors on the path of light, we usually prefer to draw diagrams using light rays. Light rays represent the direction of travel of wave fronts.

Reflection of light at a plane mirror

The laws of reflection

A **plane mirror** has a very smooth flat surface which reflects light waves without distorting or scattering the waves. Figure 2 shows the **incident ray**, representing the direction of the incident wave fronts, is at the same angle to the normal as the **reflected ray**, which represents the direction of the reflected wave fronts.

The two laws of reflection are:

1 The angle between the incident ray and the normal = the angle between the reflected ray and the normal.
2 The incident ray, the reflected ray and the normal are all in the same plane.

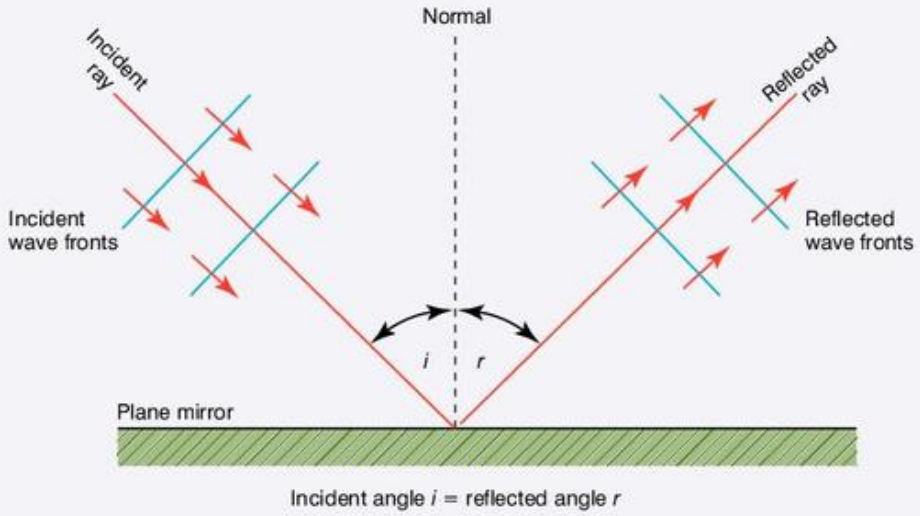


Fig 2 *Reflection by a plane mirror*

Refraction of light

Refraction is the change of direction that occurs when light passes non-normally across a boundary between two transparent substances. Figure 3 shows the change of direction of a light ray when it enters, and when it leaves, a rectangular glass block in air. The light ray bends:

- **towards** the normal when it passes from air into glass,
- **away from** the normal when it passes from glass into air.

No refraction takes place if the incident light ray is along the normal.

At a boundary between two transparent substances, the light ray bends:

- **towards** the normal if it passes into a **more** refractive substance,
- **away from** the normal if it passes into a **less** refractive substance.

Figure 3 shows a light ray directed into a rectangular glass block.

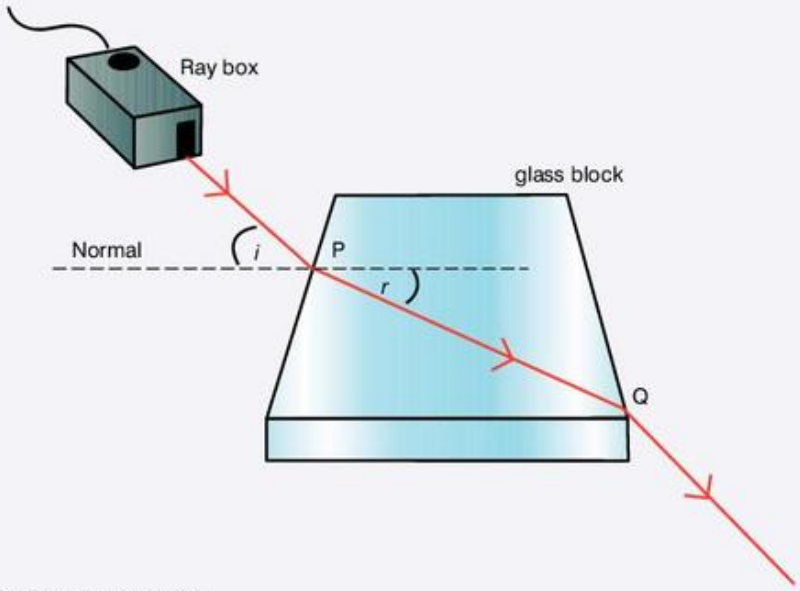


Fig 3 *Investigating refraction*

The **angle of incidence**, ***i***, is the angle between the incident light ray and the normal at the point of incidence.

The angle of refraction, ***r***, is the angle between the normal at P and the refracted ray in the glass.

Measurements of the angles of incidence and refraction for different incident rays show that the ratio of $\frac{\sin i}{\sin r}$ is the same for each light ray.

This is known as **Snell's law**.

The ratio is referred to as the **refractive index, *n***, of glass.

For a light ray travelling from air into a transparent substance:

$$\textbf{Refractive index of the substance, } n = \frac{\sin i}{\sin r}$$

Explaining refraction

Refraction occurs because the **speed of the light waves** is different in each substance. The amount of refraction that takes place depends on the speed of the waves in each substance.

Consider a wave front of a wave when it passes across a straight boundary, as shown in Figure 4.

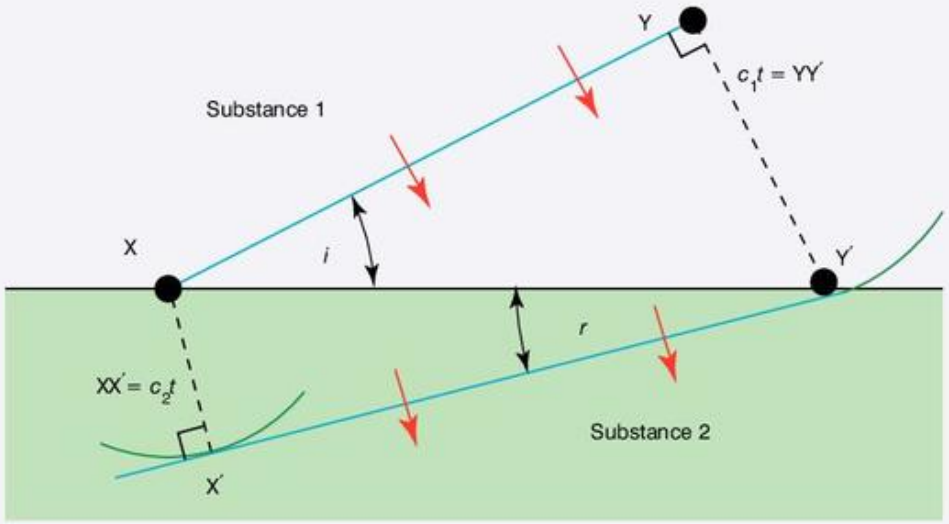


Fig 4 *Explaining refraction*

Suppose the wave front moves from XY to X′Y′ in time t, crossing the boundary between X and Y′. In this time, the wave front moves:

- a distance c_2t at speed c_2 in the second substance from X to X′, and
- a distance c_1t at speed c_1 in the first substance from Y to Y′.

Considering triangle XYY′, $YY' = c_1t = XY' \sin i$

Considering triangle XX′Y′, $XX' = c_2t = XY' \sin r$

Combining these two equations gives:

$$\frac{\sin i}{\sin r} = \frac{c_1}{c_2}$$

This is in agreement with Snell's law. If substance 1 is a vacuum, the **refractive index, *n***, of the second substance is therefore the ratio of the speed of light in a vacuum to the speed of light in the (second) substance.

Multiplying both sides of this equation by c_0, the **speed of light in a vacuum**, therefore gives:

$$\frac{c_0}{c_1}\sin i = \frac{c_0}{c_2}\sin r$$

Substituting n_1 for $\frac{c_0}{c_1}$, where n_1 is the refractive index of substance 1

and n_2 for $\frac{c_0}{c_2}$, where n_2 is the refractive index of substance 2 gives:

$$n_1 \sin i = n_2 \sin r$$

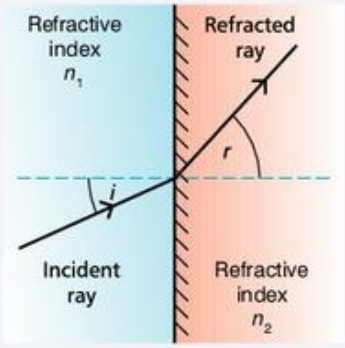


Fig 5 *The 'n sin i' rule*

Worked example

A light ray crosses the boundary between water, of refractive index 1.30, and glass, of refractive index 1.50, at an angle of incidence of 40°. Calculate the angle of refraction of this light ray.

Solution

$n_1 = 1.30$, $n_2 = 1.50$, $i = 40°$

Using $n_1 \sin i = n_2 \sin r$ gives $1.30 \sin 40 = 1.50 \sin r$

$$\therefore \quad \sin r = \frac{1.30 \sin 40}{1.5} = 0.56$$

$$r = 34°$$

Note The speed of light in air at atmospheric pressure is 99.97% of the speed of light in a vacuum. Therefore, the **refractive index of air** is 1.0003. For most purposes, the refractive index of air may be assumed to be 1.

Worked example

Water waves at a speed of $0.020\,\text{m s}^{-1}$ cross a straight boundary and enter shallow water, where the speed of the waves is $0.015\,\text{m s}^{-1}$. Calculate the angle of refraction of the refracted waves when the angle of incidence is 30°.

Solution

Using $\frac{\sin i}{\sin r} = \frac{c_1}{c_2}$ with $c_1 = 0.020\,\text{m s}^{-1}$, $c_2 = 0.015\,\text{m s}^{-1}$ and $i = 30°$

$$\frac{\sin 30}{\sin r} = \frac{0.020}{0.0150}$$

$$\therefore \quad \sin r = \frac{0.015 \times \sin 30}{0.020} = 0.375$$

$$r = 22°$$

QUESTIONS

1 Water waves of frequency 4.0 Hz, travelling at a speed of $0.16\,\text{m s}^{-1}$, travel across a boundary from deep to shallow water where the speed is $0.12\,\text{m s}^{-1}$.

a Calculate the wavelength of these waves:
i in the deep water,
ii in the shallow water.

b The incident wave fronts cross the boundary at an angle of 25° to the boundary. Calculate the angle of the refracted wave fronts to the boundary.

2 a The speed of light in a vacuum is $3.00 \times 10^8\,\text{m s}^{-1}$. Calculate the speed of light in:
i glass of refractive index 1.52,
ii water (of refractive index 1.33).

b A light ray passes across a plane boundary from water into glass at an angle of incidence of 55°. Use the refractive index values from part **a** to show that the angle of refraction of this light ray is 46°.

3 A light ray is directed from air into a glass block of refractive index 1.50. Calculate:

a the angle of refraction at the point of incidence, if the angle of incidence is:
i 30° **ii** 60°

b the angle of incidence, if the angle of refraction at the point of incidence is:
i 35° **ii** 40°

4 A light ray enters an equilateral glass prism of refractive index 1.55 at the mid-point of one side of the prism at an angle of incidence of 35°:

a Sketch this arrangement and show that the angle of refraction of the light ray in the glass is 22°.

b i Show that the angle of incidence where the light ray leaves the glass prism is 38°.

ii Calculate the angle of refraction of the light ray where it leaves the prism.

9.5 Total internal reflection

On these pages you will learn to:

- Explain what is meant by total internal reflection
- Recall the conditions for total internal reflection
- Recall and use the equation $n = 1/\sin c$
- Describe applications of optical fibres, including the endoscope and in communications

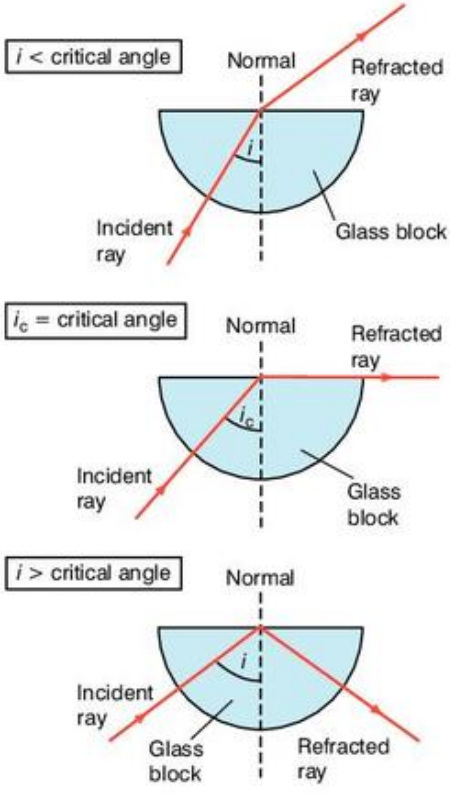


Fig 1 Total internal reflection

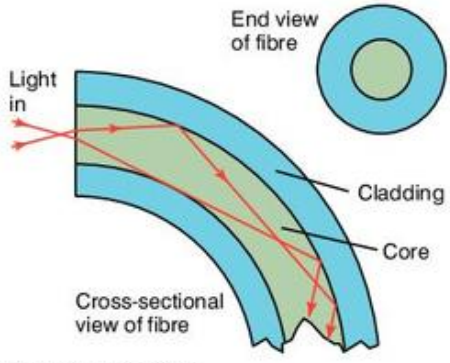


Fig 2 An optical fibre

Investigating total internal reflection

When a light ray travels from glass into air, it refracts away from the normal. If the angle of incidence is increased to a certain value known as the **critical angle,** the light ray refracts along the boundary. Figure 1 shows this effect. If the angle of incidence is increased further, the light ray undergoes **total internal reflection** at the boundary, the same as if the boundary was replaced by a plane mirror.

In general, total internal reflection can only take place if:

- the incident substance has a **larger refractive index** than the other substance,
- the angle of incidence **exceeds the critical angle**.

At the critical angle i_c, the angle of refraction is 90° because the light ray emerges along the boundary. Therefore, $n_1 \sin i_c = n_2 \sin 90$ where n_1 is the refractive index of the incident substance and n_2 is the refractive index of the other substance. Since $\sin 90 = 1$, then:

$$\sin i_c = \frac{n_2}{n_1}$$

Prove for yourself that the critical angle for the boundary between glass of refractive index 1.5 and air of refractive index 1 is 42°. This means that if the angle of incidence of a light ray in the glass is greater than 42°, the light ray undergoes total internal reflection back into the glass at the boundary.

Optical fibres

Optical fibres are used in medical **endoscopes** to see inside the body, and in communications to carry light signals. Figure 2 shows the path of a light ray along the core of an optical fibre. The light ray is totally internally reflected each time it reaches the core boundary.

- **A communications optical fibre** allows pulses of light that enter at one end from a transmitter to reach a receiver at the other end. Such fibres need to be highly transparent, to minimise absorption of light. Each fibre consists of a core surrounded by a layer of cladding of lower refractive index.
  1. Total internal reflection takes place at the core–cladding boundary. At any point where two fibres are in direct contact, light would cross from one fibre to the other if there was no cladding. Such crossover would mean that the signals would not be secure, as they would reach the wrong destination.
  2. The core must be very narrow to prevent **multi-path dispersion**. This occurs in a wide core, because light travelling along the axis of the core

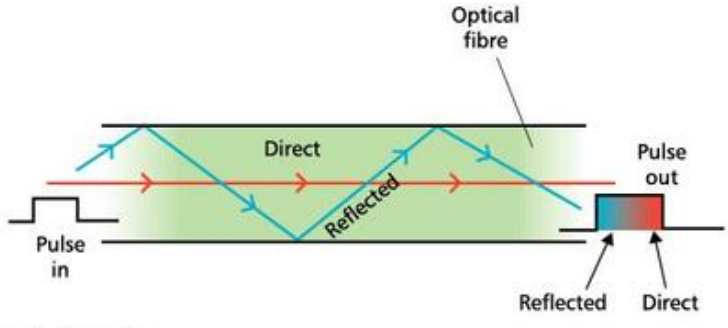
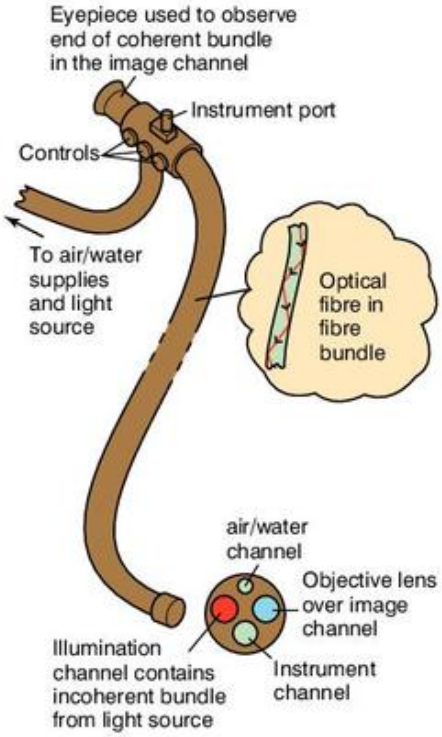


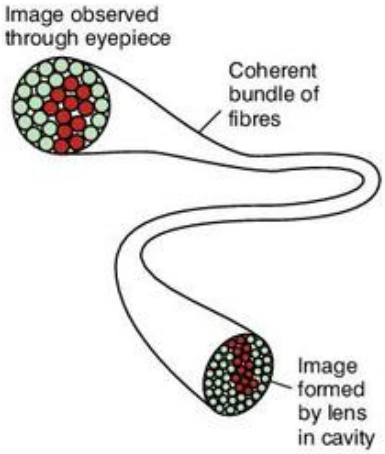
Fig 3 Multi-path dispersion

travels a shorter distance per metre of fibre than light that repeatedly undergoes total internal reflection. A pulse of light sent along a wide core would spread out and be longer than it ought to be. If it were too long, it would merge with the next pulse.

- **The medical endoscope** contains two bundles of fibres. The endoscope is inserted into a body cavity, which is then illuminated using light sent through one of the fibre bundles. A lens over the end of the other fibre bundle is used to form an image of the body cavity on the end of the fibre bundle. The light that forms this image travels along the fibres to the other end of the fibre bundle, where the image can be observed. This fibre bundle needs to be a **coherent** bundle, which means that the fibre ends at each end are in the same relative positions.



a Endoscope design



b A coherent bundle

Fig 4 *The endoscope*

QUESTIONS

1 a State two conditions for a light ray to undergo total internal reflection at a boundary between two transparent substances.

b Calculate the critical angle for:
i glass of refractive index 1.52 and air,
ii water (of refractive index 1.33) and air.

2 a Show that the critical angle at a boundary between glass of refractive index 1.52 and water (of refractive index 1.33) is 61°.

b Figure 6 shows the path of a light ray in water (of refractive index 1.33) directed at an angle of incidence of 40° at a thick glass plate of refractive index 1.52. Calculate:
i the angle of refraction of the light ray at P,
ii the angle of incidence of the light ray at Q.

c Sketch the path of the light ray beyond Q.

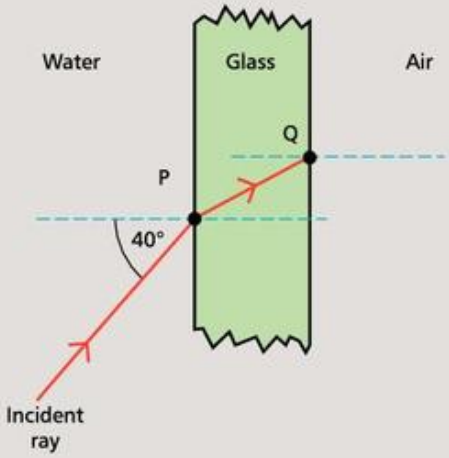


Fig 6

3 A window pane, made of glass of refractive index 1.55, is covered on one side only with a transparent film of refractive index 1.40.

a Calculate the critical angle of the film–glass boundary.

b A light ray in air is directed at the film at an angle of incidence of 45°, as shown in Figure 5. Calculate:
i the angle of refraction in the film,
ii the angle of refraction of the ray where it leaves the pane.

4 a In a medical endoscope, the fibre bundle used to view the image is coherent.
i What is meant by a 'coherent' fibre bundle?
ii Explain why this fibre bundle needs to be coherent.

b i Why is an optical fibre used in communication composed of a core surrounded by a layer of cladding of lower refractive index?
ii Why is it necessary for the core of an optical communications fibre to be narrow?

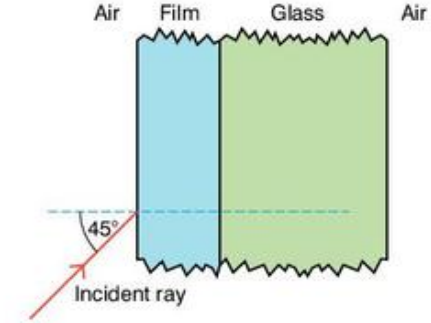


Fig 5

9 End of chapter questions

1 a Choose from the list below to complete each of the following statements about how the frequency, speed and wavelength of light change when it passes from air into glass:

decreases increases stays the same

When light passes from air into glass:

i its speed _____

ii its frequency _____

iii its wavelength _____

b Light of wavelength 600 nm, travelling in air at a speed of $3.0 \times 10^8\,m\,s^{-1}$, enters a liquid of refractive index 1.4. Calculate its speed, frequency and wavelength in the liquid.

2 An interference pattern of bright and dark fringes is observed when two closely spaced slits are illuminated by a parallel beam of monochromatic light, as shown below:

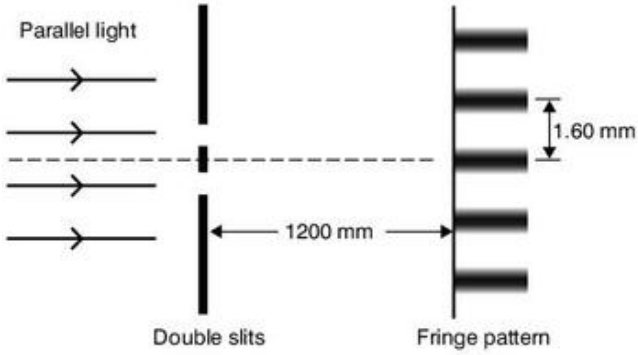


Fig 2.1

a Explain the formation of:

i a bright fringe,

ii a dark fringe.

b The two slits act as coherent emitters of light:

i Explain what is meant by this statement.

ii Explain why the fringe pattern could not be observed from two closely spaced light sources, such as two nearby filaments.

c The two slits were at a distance of 0.4 mm apart. The spacing between two adjacent bright fringes was 1.60 mm, when the fringes were observed at a distance of 1200 mm from the slits. Calculate the wavelength of the monochromatic light.

3 a State what is meant by the *diffraction* of a wave. *(2 marks)*

b Plane wavefronts are incident on a slit, as shown in Figure 3.1. *(2 marks)*

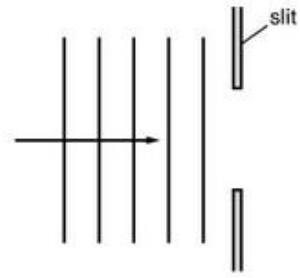


Fig 3.1

Complete Figure 3.1 to show four wavefronts that have emerged from the slit. *(2 marks)*

c Monochromatic light is incident normally on a diffraction grating having 650 lines per millimetre, as shown in Figure 3.2.

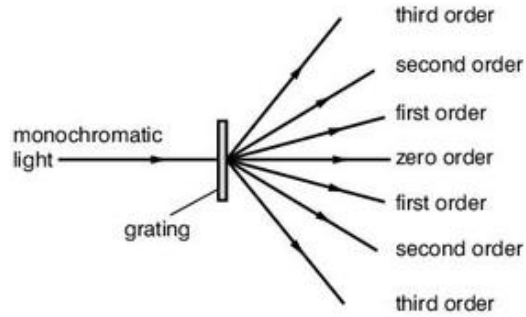


Fig 3.2

An image (the zero order) is observed for light that has an angle of diffraction equal to zero.

For incident light of wavelength 590 nm, determine the number of orders of diffracted light that can be observed on each side of the zero order. *(3 marks)*

d The images in Figure 3.2 are viewed, starting with the zero order and then with increasing order number.

State how the appearance of the images changes as the order number increases. *(1 mark)*

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4 The diagram shows a beam of red light directed from air into a rectangular glass block:

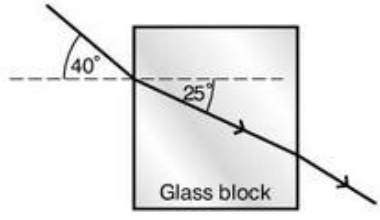


Fig 4.1

a The angle of refraction of the light ray at the point where the light ray entered the glass block was 25° when the angle of incidence was 40°.

i Calculate the refractive index of the glass for red light.

ii Calculate the angle of refraction if the angle of incidence had been 60°.

b White light consists of all the colours of the spectrum. The refractive index of glass is greater the smaller the wavelength of the incident light. Sketch what you would have observed if the beam of red light above had been replaced by a beam of white light at the same angle of incidence.

5 a i Explain, with the aid of a diagram, what is meant by 'total internal reflection'.

ii State two essential conditions for the total internal reflection of a light ray at a boundary between two transparent substances.

b The diagram shows a beam of yellow light directed normally into a semi-circular glass block through its curved surface. The refractive index of this glass block for yellow light is 1.5.

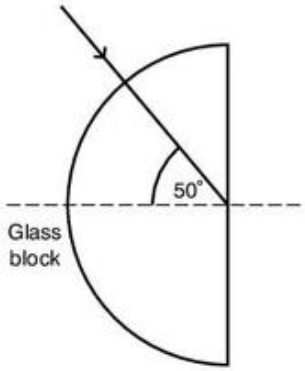


Fig 5.1

i Calculate the critical angle of yellow light in this glass block.

ii The light ray is incident on the flat surface at an angle of 50°. Sketch its path into and out of the glass block.

6 The refractive index of a transparent liquid can be measured by using two semi-circular glass blocks of known refractive index, pressed together with a film of the liquid in between, as shown below.

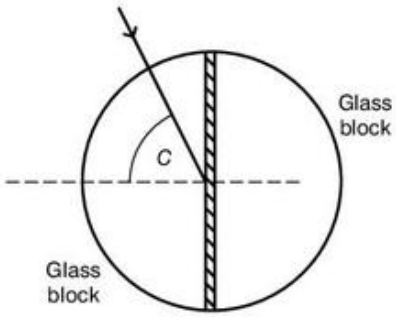


Fig 6.1

A narrow light ray is directed at the centre of the blocks through the curved face of one block. The angle of incidence, *i*, at the glass–liquid interface is increased until the light ray is totally internally reflected. At this position, the angle of incidence is equal to the critical angle, *c*, of the glass–liquid boundary.

a The method was tested using glass blocks of refractive index 1.48 and water (which has a refractive index of 1.33). Calculate the critical angle for the boundary between these two substances.

b Using a different liquid with the same glass blocks, the critical angle was 62°. Calculate the refractive index of the liquid.

7 Diagram **a** shows an optical fibre, which has a core of refractive index 1.49, surrounded by transparent cladding. The critical angle of the core–cladding boundary is 65°.

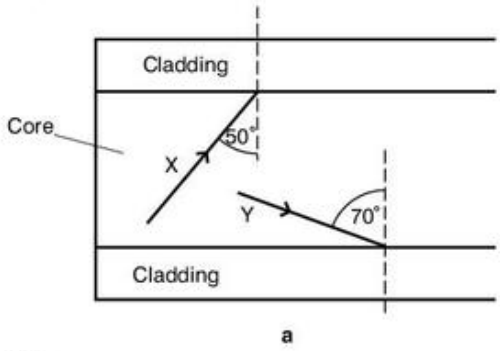


Fig 7.1

a i A light ray, X, in the core is incident on the boundary at an angle of incidence of 50°. Sketch the path of this light ray after the boundary.

ii A second light ray, Y, in the core is incident on the boundary at an angle of incidence of 70°. Sketch the path of this light ray after the boundary.

b A pulse of light enters the fibre at its flat end and travels a distance of 1000 m to the far end of the fibre.

i Calculate the least time taken for this pulse of light to travel along this length of fibre.

ii Show that the maximum time taken for this pulse of light to travel along this length of fibre is about 10% longer than the least time.

iii Diagram **b** shows how the intensity of the pulse of light varies with time, for the point where it enters the fibre. Sketch the corresponding intensity-time graph for the pulse when it reaches the far end of the fibre.

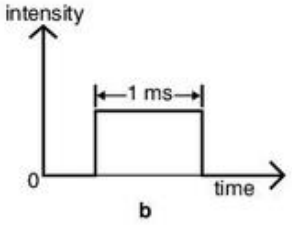


Fig 7.2

10 Radioactivity

10.1 The discovery of the nucleus

On these pages you will learn to:

- Describe the basic principles of Rutherford's α-particle scattering experiment
- Describe what Rutherford discovered about the scattering of the alpha particles
- State and explain the key conclusions that Rutherford came to about the structure of the atom

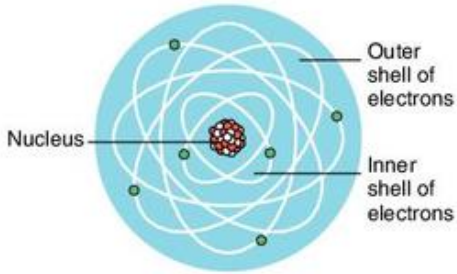


Fig 1 The structure of an atom

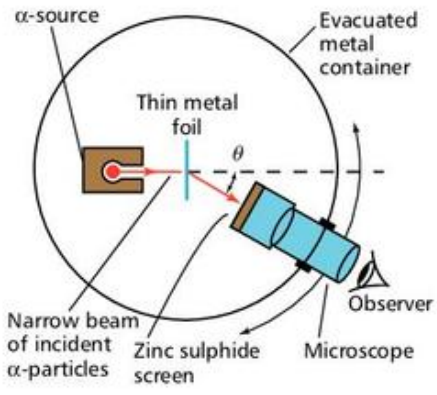


Fig 2 Rutherford's α-scattering apparatus

Rutherford's alpha scattering experiment

The nucleus was discovered by Ernest Rutherford in 1914. He knew from the work of J.J. Thomson that every atom contains one or more electrons. Thomson had shown that the electron is a negatively charged particle inside every atom but no one knew, until Rutherford's discovery, how the positive charge in the atom was distributed.

Rutherford knew that the atoms of certain elements were unstable and emitted radiation. It had been shown that there were three types of such radiation, referred to as **alpha radiation** (symbol α), **beta radiation** (symbol β) and gamma radiation (symbol γ). Rutherford knew that α-radiation consisted of fast-moving positively charged particles. He used this type of radiation to probe the atom. He reckoned that a beam of the particles directed at a thin metal foil might be scattered slightly by the atoms of the foil if the positive charge was spread out throughout each atom. He was astonished when he discovered that some of the particles bounced back from the foil – in his own words 'as incredible as if you fired a 15-inch naval shell at tissue paper and it came back'.

Rutherford's alpha scattering experiment in more detail

Rutherford used a narrow beam of alpha particles, all of the same kinetic energy, in an evacuated container to probe the structure of the atom. The diagram shows an outline of the arrangement he used. A thin metal foil was placed in the path of the beam. Alpha particles scattered by the metal foil were detected by a detector which could be moved round at a constant distance from the point of impact of the beam on the metal foil. See Figure 2.

Rutherford used a microscope to observe the pinpoints of light emitted by alpha particles hitting a fluorescent screen. He measured the number of alpha particles reaching the detector per minute for different angles of deflection from zero to almost 180°. His measurements showed that:

1 most alpha particles pass straight through the foil with little or no deflection,
2 a small percentage of alpha particles deflect through angles of more than 90°.

Imagine throwing tennis balls at a row of vertical posts separated by wide gaps. Most of the balls would pass between the posts and therefore would not be deflected much. However, some would rebound as a result of hitting a post. Rutherford realised the alpha scattering measurements could be explained in a similar way by assuming every atom has a 'hard centre' much smaller than the atom. His interpretation of each result was that:

1 most of the atom's mass is concentrated in a small region, the **nucleus**, at the centre of the atom,
2 the nucleus is positively charged because it repels alpha particles (which carry positive charge) that approach it too closely.

Figure 3 shows the paths of some alpha particles which pass near a fixed nucleus. The closer an alpha track deflection passes to a nucleus, the greater its deflection is because the alpha particle and the nucleus repel each other as they carry the same type of charge. Using **Coulomb's law of force** (i.e. the law of force between

charged objects) and Newton's laws of motion, Rutherford used his nuclear model to explain the exact pattern of the results. By testing foils of different metal elements, he also showed that the magnitude of the charge of a nucleus is $+Ze$, where e is the charge of the electron and Z is the **atomic number** of the element.

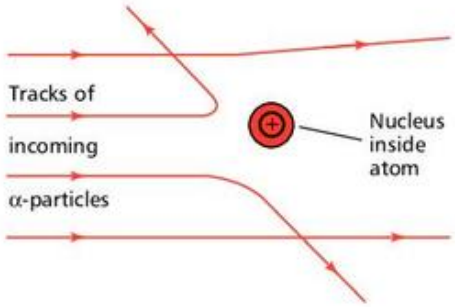


Fig 3 α-scattering paths

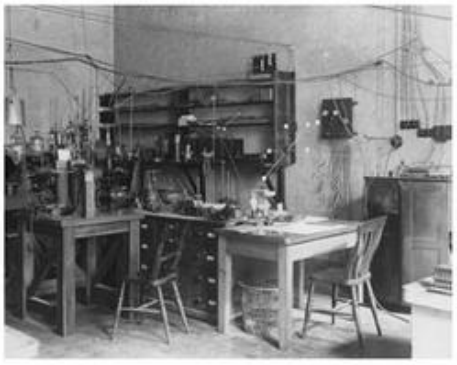


Fig 4 Rutherford's Cavendish laboratory, 1920s

Notes

1 The alpha particles must have the same speed otherwise slow alpha particles would be deflected more than faster alpha particles on the same initial path.
2 The tube must be evacuated or the alpha particles would be stopped by air molecules.
3 The activity of the source (i.e. the number of alpha particles emitted each second) decreases because more and more nuclei become stable and the number of radioactive nuclei in the source decreases. The source of the alpha particles must therefore have a long half-life, otherwise later readings would be lower than earlier readings due to radioactive decay of the source nuclei.

Ernest Rutherford 1871–1937

Ernest Rutherford arrived in Britain from New Zealand in 1895. By the age of 28, he was a professor. He made important discoveries about radioactivity and was awarded the Nobel Prize for chemistry in 1908 for his investigations into the disintegration of radioactive substances. He worked in the universities of Montreal, Manchester and Cambridge. He put forward the nuclear model of the atom and proved it experimentally using α-scattering experiments. He was knighted in 1914 and made Lord Rutherford of Nelson in 1931. His co-worker Otto Hahn described him as a 'very jolly man'. In 1915, he expressed the hope that 'no one discovers how to release the intrinsic energy of radium until man has learned to live at peace with his neighbour'. After his death in 1937, his ashes were placed close to Newton's tomb in Westminster Abbey.

QUESTIONS

1 **a** In the Rutherford α-particle scattering experiment, most of the alpha particles passed straight through the metal foil. What did Rutherford deduce about the atom from this discovery?

b A small fraction of the alpha particles were deflected through large angles. What did Rutherford deduce about the atom from this discovery?

2 In Rutherford's α-particle scattering experiment, why was it essential that:

a the apparatus was in an evacuated chamber,

b the foil was very thin,

c the α-particles in the beam all had the same speed,

d the beam was narrow?

3 An alpha particle collided with a nucleus and was deflected by it, as shown in Figure 5.

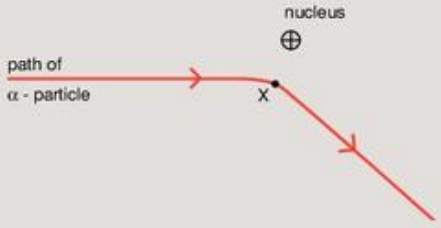


Fig 5

a Copy the diagram and show on it the direction of the force on the alpha particle when it was at the position marked X.

b Describe how:

i the kinetic energy of the alpha particle, and

ii the potential energy of the alpha particle changed during this interaction.

4 Explain why the radioactive source in Rutherford's alpha scattering experiment needed to have:

a a very long half-life,

b to emit alpha particles with the same kinetic energy.

10.2 Inside the atom

On these pages you will learn to:

- Describe a simple model for the nuclear atom in terms of protons, neutrons and orbital electrons
- Distinguish between nucleon number and proton number
- Represent different nuclides in terms of mass number and proton number using the accepted notation
- State what is meant by the term isotope and explain why an element can exist in various isotopic forms

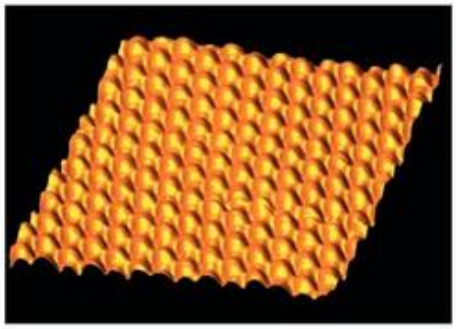


Fig 1 Atoms seen using a Scanning Tunnelling Microscope (STM)

The structure of the atom

Atoms are so small, less than a millionth of a millimetre in diameter, that we can only see images of them using the latest electron microscopes. We can't see inside them yet we know, from Rutherford's alpha scattering investigations, that every atom contains a positively charged nucleus composed of:

- **protons** and **neutrons**,
- **electrons** that surround the nucleus.

Each electron has a negative charge and is held in the atom by the electrostatic force of attraction between it and the nucleus because the nucleus is positively charged. Rutherford's investigations showed that the nucleus contains most of the mass of the atom and its diameter is of the order of 0.00001 times the diameter of a typical atom.

Table 1 shows the charge and the mass of the proton, the neutron and the electron in SI units (i.e. coulombs for charge and kilograms for mass) and relative to the charge and mass of the proton. Notice that:

1 the electron has a much smaller mass than the proton and the neutron,

2 the proton and the neutron have almost equal mass,

3 the electron has equal and opposite charge to the proton. The neutron is uncharged.

Table 1 *Inside the atom*

	Charge		Mass	
	/C	/charge of the proton	/kg	/mass of the proton
proton	$+1.60 \times 10^{-19}$	1	1.67×10^{-27}	1
neutron	0	0	1.67×10^{-27}	1
electron	-1.60×10^{-19}	−1	9.11×10^{-31}	0.0005

Isotopes

Every atom of a given element has the same number of protons as any other atom of the same element. The proton number is usually called the **atomic number** (symbol Z) of the element. For example,

- $Z = 6$ for carbon because every carbon atom has 6 protons in its nucleus,
- $Z = 92$ for uranium because every uranium atom has 92 protons in its nucleus.

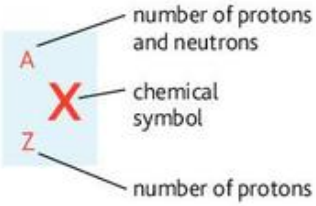
The atoms of an element can have different numbers of neutrons. Atoms of the same element with different numbers of neutrons are called **isotopes**. For example, the most abundant isotope of natural uranium contains 146 neutrons and the next most abundant contains 143 neutrons.

Isotopes are atoms with the same number of protons and different numbers of neutrons.

A proton or a neutron in the nucleus is referred to as a **nucleon**. The total number of protons and neutrons in an atom is called the **nucleon number** (symbol A) or sometimes the **mass number** of the atom. This is because it is almost numerically equal to the mass of the atom in relative units (where the mass of a proton or neutron is approximately 1).

The isotopes of an element are labelled according to their atomic number Z, their mass number A and the chemical symbol of the element. Figure 2 shows how we do this. Notice that:

- Z is at the bottom left of the element symbol and gives the number of protons in the nucleus,
- A is at the top left of the element symbol and gives the number of protons and neutrons in the nucleus.
- The number of neutrons in the nucleus $= A - Z$



Example: the symbol for the uranium isotope with 92 protons and 146 neutrons is

$^{238}_{92}U$ (or sometimes U-238)

Fig 2 *Isotope notation*

Each type of nucleus is called a **nuclide** and is labelled using the isotope notation. For example, a nuclide of the carbon isotope $^{12}_{6}C$ has two fewer neutrons and two fewer protons than a nuclide of the oxygen isotope $^{16}_{8}O$.

About neutrons

From his α-scattering results, Rutherford deduced that the charge of a nucleus is $+Ze$, where Z is the atomic number of the element and the mass of the nucleus is $A\ m_u$, where A is the mass number of the nucleus. The $^{1}_{1}H$ nucleus is the smallest known nucleus, and physicists concluded that it is a single particle, which became known as the proton. They knew that Z and A are integers and that for all nuclei larger than the $^{1}_{1}H$ nucleus, A is greater than Z. So they put forward the hypothesis that the difference between A and Z could be due to the existence of an uncharged particle (which therefore did not contribute to Z) with about the same mass as the proton (as the difference is always an integer). They called this neutral particle the neutron, even though there was no direct evidence for it at the time. Such evidence was eventually found by Sir James Chadwick, who was one of Rutherford's former students.

QUESTIONS

You will need to use data from Table 1 to answer some of the questions below.

1 State the number of protons and the number of neutrons in a nucleus of:

a $^{12}_{6}C$, **b** $^{16}_{8}O$, **c** $^{235}_{92}U$, **d** $^{24}_{11}Na$, **e** $^{63}_{29}Cu$

2 Name the particle in an atom which:

a has zero charge,

b has the largest charge per unit mass,

c when removed leaves a different isotope of the element.

3 **a** A $^{63}_{29}Cu$ atom loses two electrons. For the ion formed:

i calculate its charge in C,

ii state the number of nucleons it contains.

b An ion has a mass of 2.67×10^{-26} kg and a negative charge of 3.2×10^{-19} C.

The ion has eight protons in its nucleus. How many neutrons and how many electrons does it have?

4 A block of wax contains hydrogen and carbon atoms. When a beam of neutrons is directed at a block of wax, the neutrons knock protons out of the block. Explain why protons are knocked out of the block rather than carbon nuclei.

10.3 The properties of α, β and γ radiation

On these pages you will learn to:

- Show an understanding of the nature and properties of α-, β- and γ-radiations

Fig 1 *Marie Curie (1867–1934)*

Marie Curie established the nature of radioactive materials. She showed how radioactive compounds could be separated and identified. She and her husband Pierre won the 1903 Nobel Prize for their discovery of two new elements, polonium and radium. After Pierre's death in 1906 she continued her painstaking research and was awarded a second Nobel Prize in 1911 – an unprecedented honour.

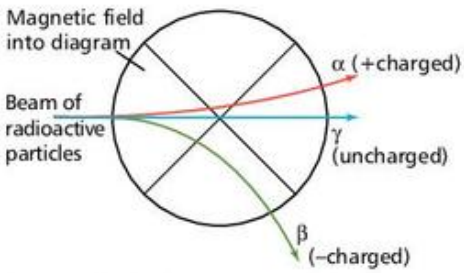


Fig 2 *Deflection by a magnetic field*

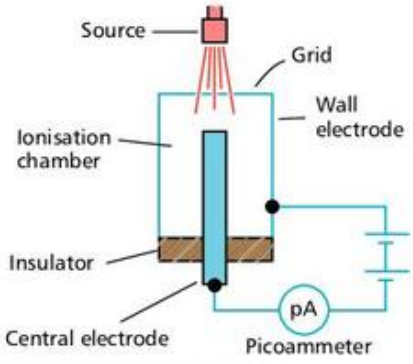


Fig 3 *Investigating ionisation*

The discovery of radioactivity

In 1896, Henri Becquerel was investigating materials that glow when placed in an X-ray beam. He wanted to find out if strong sunlight could make uranium salts glow. He prepared a sample and placed it in a drawer on a wrapped photographic plate, ready to test the salts on the next sunny day. When he developed the plate, he was amazed to see the image of a key. He had put the key on the plate in the drawer and then put the uranium salts on top of the key. He realised that uranium salts emit radiation which can penetrate paper and blacken a photographic film. The uranium salts were described as being **radioactive**. The task of investigating radioactivity was passed on by Becquerel to one of his students, Marie Curie. Within a few years, Marie Curie discovered other elements which are radioactive. One of these elements, radium, was found to be over a million times more radioactive than uranium.

Rutherford's investigations into radioactivity

Rutherford wanted to find out what the radiation emitted by radioactive substances was and what caused it. He found that the radiation:

- Ionised air, making it conduct electricity. He made a detector which could measure the radiation from its ionising effect.
- Was of two types. One type which he called **alpha** (α) radiation was easily absorbed. The other type which he called **beta** (β) radiation was more penetrating. A third type of radiation, called **gamma** (γ) radiation, even more penetrating than β radiation, was discovered a year later.

Further tests showed that a magnetic field deflects α and β radiation in opposite directions and has no effect on γ radiation. From the deflection direction, it was concluded that α radiation consists of positively charged particles and β radiation consists of negatively charged particles. γ radiation was later shown to consist of high-energy photons.

1 Ionisation

The ionising effect of each type of radiation can be investigated using an ionisation chamber and a picoammeter, as shown in Figure 3. The chamber contains air at atmospheric pressure. Ions created in the chamber are attracted to the oppositely charged electrode where they are discharged. Electrons pass through the picoammeter as a result of ionisation in the chamber. The current is proportional to the number of ions per second created in the chamber.

- α radiation causes strong ionisation. However, if the source is moved away from the top of the chamber, ionisation ceases beyond a certain distance. This is because α radiation has a range in air of no more than a few centimetres.
- β radiation has a much weaker ionising effect than air. Its range in air varies up to a metre or more. A β-particle, therefore, produces fewer ions per millimetre along its path than an α-particle does.
- γ radiation has a much weaker ionising effect than either α or β radiation. This is because photons carry no charge so they have less effect than α- or β-particles do.

2 Cloud chamber observations

A cloud chamber contains air saturated with a vapour at a very low temperature. Due to ionisation of the air, an α- or a β-particle passing through the cloud chamber leaves a visible track of minute condensed vapour droplets. This is

because the air space is supersaturated. When an ionising particle passes through the supersaturated vapour, the ions produced trigger the formation of droplets.

- α-particles produce straight tracks that radiate from the source and are easily visible. The tracks from a given isotope are all approximately the same length, indicating that the α-particles have the same range.
- β-particles produce wispy tracks that are easily deflected as a result of collisions with air molecules. The tracks are not as easy to see as α-particle tracks because β-particles are less ionising than α-particles.

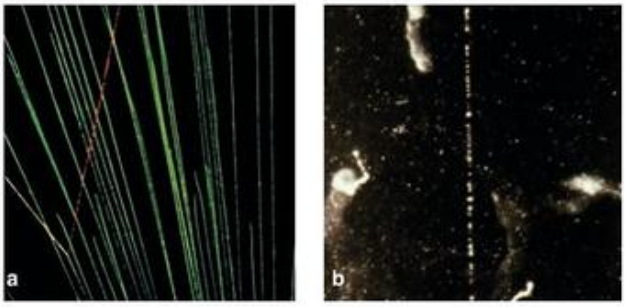


Fig 4 *Cloud chamber photographs* ***a*** *α-particle tracks* ***b*** *β-particle tracks*

3 Absorption tests

Figure 5 shows how a Geiger tube and a counter may be used to investigate absorption by different materials. Each particle of radiation that enters the tube is registered by the counter as a single count or 'click'. The clicks occur randomly which indicates that radioactive emission is a **random** process. The number of counts in a given time is measured and used to work out the **count rate** which is the number of counts divided by the time taken. Before the source is tested, the count rate due to **background radioactivity** must be measured. This is the count rate without the source present.

- The count rate is then measured with the source at a fixed distance from the tube without any absorber present. The background count rate is then subtracted from the count rate with the source present to give the **corrected (i.e. true) count rate** from the source.
- The count rate is then measured with the absorber in a fixed position between the source and the tube. The corrected count rates with and without the absorber present can then be compared.

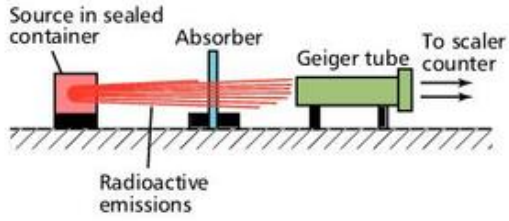


Fig 5 *Investigating absorption*

By using absorbers of different thicknesses of the same material, the effect of the absorber thickness can be investigated. Figure 6 shows a typical set of measurements for the absorption of β radiation by aluminium.

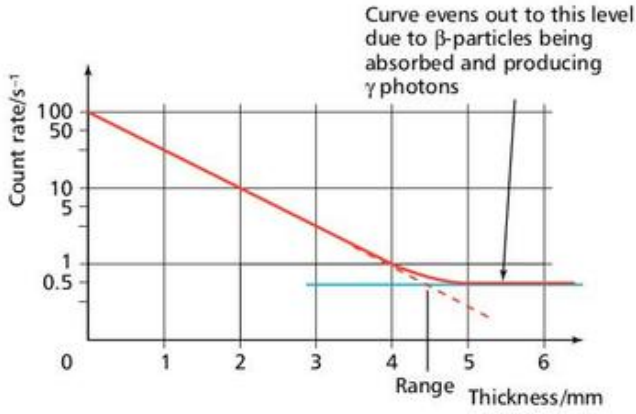


Fig 6 *Count rate v. absorber thickness*

QUESTIONS

1 A beam of radiation from a radioactive substance passes through paper and is then stopped by an aluminium plate of thickness 5 mm.

a What type of particles are in this beam?

b Describe a further test you could do to check your answer in **a**.

2 **a** What type of radioactivity was responsible for the image of the key seen by Becquerel in the effect described on p.148?

b Explain why an image of the key was produced on the photographic plate.

3 **a** Which type of radiation from a radioactive source is:
i least ionising, **ii** most ionising?

b When an α-emitting source above an ionisation chamber grid was moved gradually away from the grid, the ionisation current suddenly dropped to zero. Explain why the current suddenly dropped to zero.

4 In an absorption test as shown in Figure 5 using a β-emitting source and a Geiger counter, a count rate of 8.2 counts per second was obtained without the absorber present and a count rate of 3.7 counts per second was obtained with the absorber present. The background count rate was 0.4 counts per second. What percentage of the β-particles hitting the absorber:

a pass through it,

b are stopped by the absorber?

10.4 More about α, β and γ radiation

On these pages you will learn to:

- State that (electron) antineutrinos and (electron) neutrinos are produced during β^- and β^+ decay
- Use equations to represent the nuclear change when:
  - an α-particle is emitted
  - a β^- particle is emitted
  - a β^+ particle is emitted
- Recognise that a γ-photon is emitted by nuclei that have excess energy after emitting an α-particle or a β^- particle or a β^+ particle

The range of α, β and γ radiation in air

The arrangement in topic 10.3 Figure 5 without the absorbers may be used to investigate the range of each type of radiation in air. The corrected count rate is measured for different distances between the source and the tube, starting with the source close to the tube.

- α radiation has a range of several centimetres in air. The count rate decreases sharply once the tube is beyond the range of the α-particles. This can be seen in topic 10.3 Figure 4 as the α-particle tracks are the same length indicating that the particles from the source have the same range and, therefore, the same initial kinetic energy. The range differs from one source to another indicating that the initial kinetic energy differs from one source to another.
- β radiation has a range in air of up to a metre or so. The count rate gradually decreases with increasing distance until it is the same as the background count rate at a distance of about 1 metre. The reason for the gradual decrease of count rate as the distance increases is that the β-particles from any given source have a range of initial kinetic energies up to a maximum. Faster β-particles travel further in air than slower β-particles as they have greater initial kinetic energy.
- γ radiation has an unlimited range in air. The count rate gradually decreases with increasing distance because the radiation spreads out in all directions so the proportion of the γ-photons from the source entering the tube decreases (Figure 2).

The nature of α, β and γ radiation

Alpha radiation consists of positively charged particles. Each α-particle is composed of two protons and two neutrons, the same as the nucleus of a helium atom.

Some years before his discovery that every atom contains a nucleus, Rutherford discovered that neutralised α-particles are the same as helium atoms. After he established the nuclear model of the atom, it was realised that the nucleus of the hydrogen atom, the lightest known atom, was a single positively charged particle which became known as the **proton**. Rutherford realised that other nuclei contain protons and he predicted the existence of neutral particles of similar mass, **neutrons**, in the nucleus. For example, the helium nucleus carries twice the charge of the hydrogen nucleus and therefore contains two protons. However, its mass is four times the mass of the hydrogen nucleus so Rutherford predicted that it contained two neutrons as well as two protons.

β-radiation is emitted by a nucleus with either too many neutrons or too many protons.

A nucleus with too many neutrons emits a negative β-particle (symbol β^-) as a result of one of its neutrons changing into a proton. In addition, an antineutrino (symbol $\bar{\nu}$) is emitted. The β^- particles were shown to be fast-moving electrons by deflecting them in electric and magnetic fields in order to measure their specific charge. This was found to be the same as the specific charge of the electron.

A nucleus with too many protons emits a positive β-particle (symbol β^+), also called a positron, as a result of one of its protons changing into a neutron. In addition, a neutrino (symbol ν)is emitted. See Figure 1 for more about the discovery of the positron.

Particles and antiparticles

For every known type of particle, there is a corresponding antiparticle with equal and opposite charge. The theory of antiparticles was predicted in 1928 by English physicist Paul Dirac. In his theory, he also predicted:

- **pair production**, whereby a photon of sufficient energy could produce a particle and its corresponding antiparticle
- **annihilation**, when a particle and its corresponding antiparticle collide, annihilate each other and produce photons.

Antiparticles were discovered in 1932 by American physicist Carl Anderson, who used a cloud chamber and a camera to photograph the trails produced by cosmic rays. He found trails that were curved by a magnetic field in the opposite direction to the trails produced by β^- particles, so they must have been produced by positively charge particles. He measured the specific charge of these particles and found it was the same as that of the β^- particles. So he concluded that each such positively charged particle was a **positron**, the antiparticle of the electron.

Fig 1 The discovery of the positron

An elusive particle

When the energy spectrum of beta particles was first measured, scientists were puzzled when they discovered the kinetic energy of β-particles varied up to a maximum even though the nucleus always lost a certain amount of energy in the process of emitting a β-particle. Either energy was not conserved in the change or some of it was carried away by unknown uncharged particles and antiparticles, which they called **neutrinos** and **antineutrinos**. This latter hypothesis was eventually shown to be correct when antineutrinos from a nuclear reactor were detected as a result of their interaction with cadmium nuclei in water. Now we know that billions of these elusive particles from the Sun sweep though our bodies every second without interacting.

γ radiation consists of photons with a wavelength of the order of a fraction of a nanometre or less. This discovery was made by using a crystal to diffract a beam of γ radiation in a similar way to the diffraction of light by a diffraction grating.

The equations for radioactive change

A nuclide ${}^{A}_{Z}\mathrm{X}$ contains Z protons and $A - Z$ neutrons.

- its charge $= +Ze$, where e is the magnitude of the charge of an electron
- its mass in **atomic mass units, u**, $= A$ approximately.

Note The mass of a proton is 1.00728 u and the mass of a neutron is 1.00866 u, where 1 u $= \frac{1}{12}$ of the mass of a ${}^{12}_{6}\mathrm{C}$ atom.

1 α-emission

An α-particle is represented by the symbol ${}^{4}_{2}\alpha$ as its charge $= +2e$ so $Z = 2$ and it consists of 2 neutrons and 2 protons so $A = 4$.

When a nucleus ${}^{A}_{Z}\mathrm{X}$ emits an α-particle, it loses two protons and two neutrons. Therefore, its proton number (Z) is reduced by 2 and its mass number (A) is reduced by 4.

$$ {}^{A}_{Z}\mathrm{X} \rightarrow {}^{4}_{2}\alpha + {}^{A-4}_{Z-2}\mathrm{Y} $$

2 β-emission

β^- and β^+ particles are represented in equations by the symbols ${}^{0}_{-1}\beta$ and ${}^{0}_{+1}\beta$, respectively, as their mass is much less than 1 u and their charge is $-e$ and $+e$, respectively. Antineutrinos and neutrinos are uncharged and have very little mass compared with β-particles.

When a nucleus ${}^{A}_{Z}\mathrm{X}$ emits a β^- particle (i.e. an electron), a neutron in the nucleus changes into a proton and an antineutrino, $\bar{\nu}$, is also emitted. Therefore, the proton number of the nucleus increases by 1 and the mass number is unchanged.

$$ {}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z+1}\mathrm{Y} + {}^{0}_{-1}\beta + \bar{\nu} $$

When a nucleus ${}^{A}_{Z}\mathrm{X}$ emits a β^+ particle (i.e. a positron), a proton in the nucleus changes into a neutron and a neutrino (ν) is also emitted. Therefore, the proton number of the nucleus decreases by 1 and the mass number is unchanged.

$$ {}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z-1}\mathrm{Y} + {}^{0}_{+1}\beta + \nu $$

3 γ-emission

A γ-photon is emitted if a nucleus has excess energy after it has emitted an α- or a β-particle.

No change occurs in the number of protons or neutrons of a nucleus when it emits a γ-photon.

Note The number of γ-photons per second entering the tube when it is at distance r from the source is proportional to $\frac{1}{r^2}$.

This is because all the photons emitted from the source are spread over a total area of $4\pi r^2$ (the surface area of a sphere of radius r) at the tube. If the tube area facing the radiation is A, the proportion of photons entering the tube from the source is $\frac{A}{4\pi r^2}$. So, the number of photons per second entering the tube is proportional to $\frac{1}{r^2}$ (Figure 2).

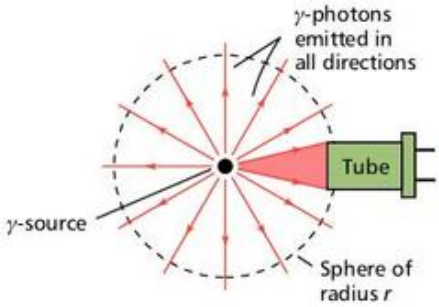


Fig 2 γ radiation from a point source

QUESTIONS

1 Copy and complete each of the following equations representing α-emission.

a ${}^{238}_{92}\mathrm{U} \rightarrow {}_{90}\mathrm{Th} +$ **b** ${}_{90}\mathrm{Th} \rightarrow {}^{224}_{88}\mathrm{Ra} +$

2 Copy and complete each of the following equations representing β-emission.

a ${}^{64}_{29}\mathrm{Cu} \rightarrow {}_{30}\mathrm{Zn} + {}^{0}\beta$ **b** ${}_{15}\mathrm{P} \rightarrow {}^{32}\mathrm{S} + {}^{0}_{-1}\beta$

3 The bismuth isotope ${}^{213}_{83}\mathrm{Bi}$ decays by emitting a β-particle to form an unstable isotope of polonium (Po) which then decays by emitting an α-particle to form an unstable isotope of lead (Pb). This isotope then decays by emitting a β-particle to form a stable isotope of bismuth.

a Write down the symbol for each of the three product nuclides in this sequence.

b Write down the number of protons and the number of neutrons in a nucleus of:
i the bismuth isotope ${}^{213}_{83}\mathrm{Bi}$,
ii the stable bismuth isotope.

4 A point source of γ radiation is placed 200 mm from the end of a Geiger tube. The corrected count rate was measured at 12.7 counts per second. Calculate:

a the corrected count rate if the source was moved to a distance of 400 mm from the tube,

b the distance between the source and the tube for a corrected count rate of 20 counts per second.

EXTENSION

10.5 The dangers of radioactivity

On these pages you will learn to:

- Explain what is meant by background radioactivity
- Recall different types of ionising radiation
- Recognise the hazards of ionising radiation and how it may be monitored
- Describe procedures used to ensure safe use of radioactive materials

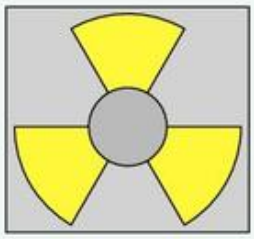


Fig 1 A radioactive warning sign

Table 1 Sources of background radioactivity

Source	Typical annual dose*
Natural radioactivity in the air	800
Ground and buildings	380
Food and drink	370
Cosmic rays	310
Nuclear weapons testing	10
Air travel	8
Nuclear power	3

* in microsieverts, a measure of the effect of radioactivity on cells.

The hazards of ionising radiation

Ionising radiation is hazardous because it damages living cells. **Ionising radiation** is any form of radiation that creates ions in substances it passes through. Such radiation includes **X-rays**, protons and neutrons as well as α, β and γ radiation. Ionising radiation affects living cells because:

- it can destroy cell membranes which causes cells to die, or
- it can damage vital molecules such as DNA directly or indirectly by creating 'free radical' ions which react with vital molecules. Normal cell division is affected and nuclei become damaged. Damaged DNA can cause cells to divide and grow uncontrollably, causing a tumour which may be cancerous. Damaged DNA in a sex cell (i.e. an egg or a sperm) can cause a mutation which might be passed on to future generations.

As a result of exposure to ionising radiation, living cells die or grow uncontrollably or mutate, affecting the health of the affected person (somatic effects) and possibly affecting future generations (genetic effects). High doses of ionising radiation kill living cells. Cell mutation and cancerous growth occur at low doses as well as at high doses. There is no evidence of the existence of a threshold level of ionising radiation below which living cells would not be damaged.

Background radioactivity occurs naturally due to cosmic radiation and from radioactive materials in rocks, soil and in the air. Everyone is exposed to background radioactivity which does vary with location due to local geological features. In addition, radon gas which is radioactive can accumulate in poorly ventilated areas of buildings in such locations. Table 1 shows the many sources of background radioactivity that occur in most populated locations.

Radiation monitoring

The Geiger tube

The Geiger tube is a sealed metal tube that contains argon gas at low pressure. The thin mica window at the end of the tube allows α- and β-particles to enter the tube. γ-photons can enter the tube through the tube wall as well. A metal rod down the middle of the tube is at a positive potential as shown in Figure 2. The tube wall is connected to the negative terminal of the power supply and is earthed.

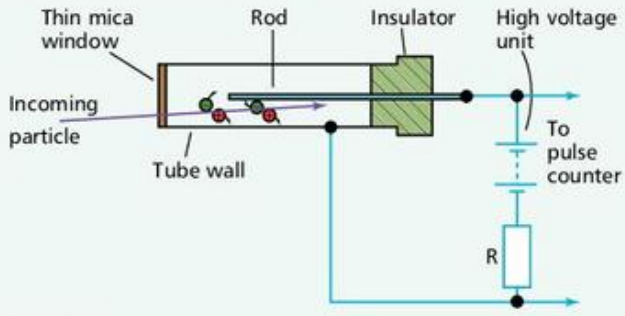


Fig 2 A Geiger tube

When a particle of ionising radiation enters the tube, the particle ionises the gas atoms along its track. The negative ions are attracted to the rod and the positive ions to the wall. The ions accelerate and collide with other gas atoms, producing more ions which produce further ions in the same way. Within a very short time, many ions are created and discharged at the electrodes. A pulse of charge passes round the circuit through resistor *R*, causing a voltage pulse across *R* which is recorded as a single count by the pulse counter.

The **dead time** of the tube, the time taken to regain its non-conducting state after an ionising particle enters it, is typically of the order of 0.2 ms. Another particle that enters the tube in this time will not cause a voltage pulse. Therefore, the count rate should be no greater than about $5000\,s^{-1}$ $(= \frac{1}{0.2\,ms})$.

The film badge

Anyone using equipment that produces ionising radiation must wear a film badge to monitor his or her exposure to ionising radiation. The badge contains a strip of photographic film in a light-proof wrapper. Different areas of the wrapper are covered by absorbers of different materials and different thicknesses. When the film is developed, the amount of exposure to each form of ionising radiation can be estimated from the blackening of the film. If the badge is overexposed, the wearer is not allowed to continue working with the equipment.

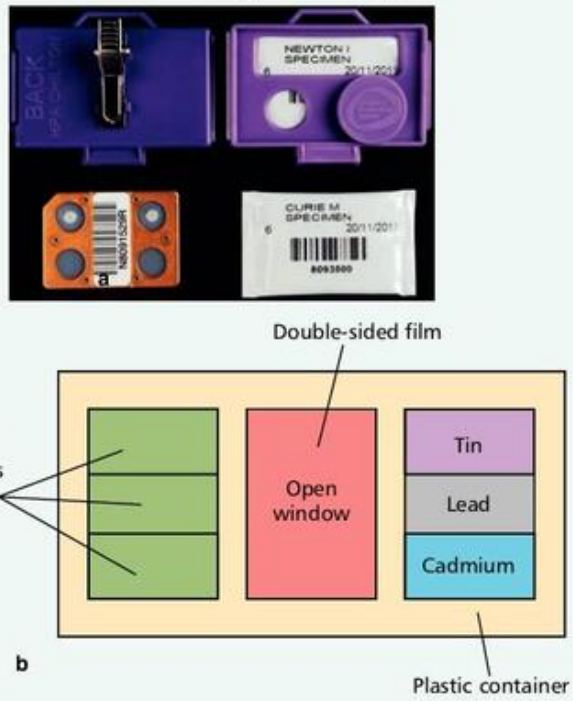


Fig 3 ***a*** *A film badge* ***b*** *Inside a film badge*

The spark counter

This device consists of a thin wire stretched about 1 mm above a flat metal plate which is earthed. The wire is in series with a resistor and is at a positive potential of between 3 kV and 4 kV, just less than that required to create sparks between it and the plate. When an α-particle passes between the wire and the plate, its ionising effect creates a spark. The resulting flow of charge through the resistor produces a detectable voltage pulse which can be counted electronically. The spark counter cannot detect β or γ radiation.

Safe use of radioactive materials

Because radioactive materials produce ionising radiation, they must be stored and used with care. In addition, disposal of a radioactive substance must be done in accordance with specific regulations. Only approved institutions are allowed to use radioactive materials. Approval is subject to regular checks and approved institutions are categorised according to purpose.

1 **Storage of radioactive materials** should be in lead-lined containers. Most radioactive sources produce γ radiation as well as α or β radiation so the lead lining of a container must be thick enough to absorb all the γ radiation from the sources in the container. In addition, regulations require that the containers are under 'lock and key' and a record of the sources is kept.

2 **When using radioactive materials**, established rules and regulations must be followed. No source should be allowed to come into contact with the skin.
   - Solid sources should be transferred using handling tools such as tongs or a glove-box or using robots. The handling tools ensure the material is as far from the user as practicable so the intensity of the γ radiation from the source at the user is as low as possible and the user is beyond the range of α or β radiation from the source.
   - Liquid and gas sources and solids in powder form should be in sealed containers. This is to ensure radioactive gas cannot be breathed in and radioactive liquid cannot be splashed on the skin or drunk.

3 **Disposal** requires long-term storage until the radioactive material is no longer radioactive. The key aim is to ensure that people and the environment are not put at risk. As explained in the next section, radioactive half-lives differ according to the isotope. The half-life of a radioactive isotope is the time it takes for the activity of the isotope to decrease to half. For example, an isotope with a half-life of 5 years would decay to 6.25% of its initial activity after 20 years.

QUESTIONS

1 a What is meant by ionisation?

b Explain why a source of α radiation is not as dangerous as a source of β radiation provided the sources are outside the body.

2 a Discuss the reasons why ionising radiation is hazardous to a person exposed to the radiation.

b i What is the purpose of a film badge worn by a radiation worker?

ii With the aid of a diagram, describe what is in a film badge and how the film badge is tested.

3 Explain why a radioactive source should be:

i kept in a lead-lined storage box when not in use,

ii transferred using a pair of tongs with long handles.

4 Discuss the precautions you would take when carrying out an experiment using a source of γ radiation.

10.6 Fundamental particles

On these pages you will learn to:

- Appreciate that protons and neutrons are not fundamental particles since they contain quarks
- Describe a simple quark model of hadrons in terms of up, down and strange quarks and their respective antiquarks
- Describe protons and neutrons in terms of a simple quark model
- Appreciate that there is a weak interaction between quarks, giving rise to β decay
- Describe β^- and β^+ decay in terms of a simple quark model
- Appreciate that electrons and neutrinos are leptons

The fundamental forces of nature

In your physics course, you will meet four fundamental forces:

- the **electromagnetic force**, which acts between charged particles. This force is due to the charged particles creating and exchanging 'virtual photons', and its range is unlimited.
- the **strong nuclear force**, which holds the nucleus together and acts between neutrons and protons. Its range is of the order of 2–3 femtometres.
- the **weak nuclear force**, which causes a neutron to change into a proton or a proton to change into a neutron. Its range is of the order of a fraction of a femtometre.
- the **gravitational force**, which acts between any two matter particles. Its range is unlimited.

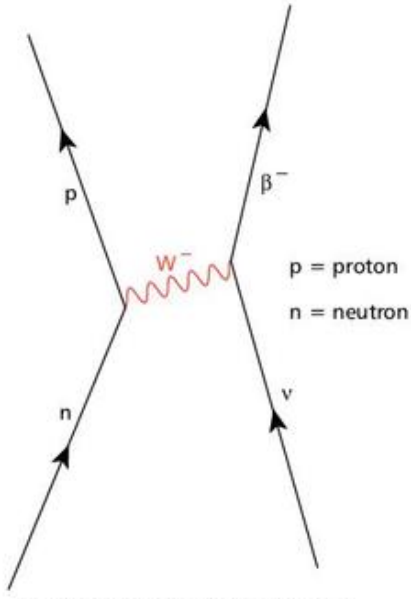
Physicists now know that at very high energies, the electromagnetic force and the weak nuclear force are unified as the 'electroweak' force. At even higher energies, the electroweak force and the strong nuclear force may possibly be unified. But the nature of gravity, the commonest force because we experience it all the time, may be beyond unification!

Forces and interactions

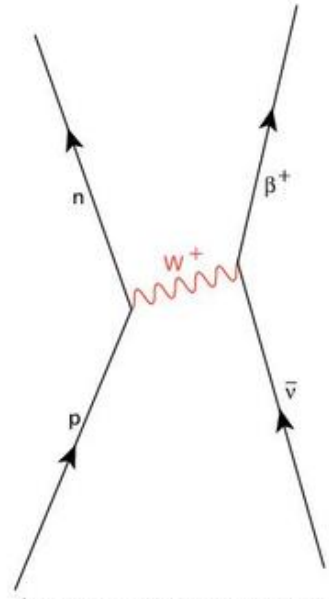
A stable isotope has nuclei that do not disintegrate, so there must be a force holding them together. We call this force the **strong nuclear force** or the 'strong interaction', because it overcomes the electrostatic force of repulsion between the protons in the nucleus and keeps the protons and neutrons together, except in unstable nuclei. Its range is no more than about 3–4 femtometres (fm), where $1\,\text{fm} = 10^{-15}\,\text{m}$. In comparison, the electrostatic force between two charged particles has an infinite range.

The strong nuclear force holds the neutrons and protons in a nucleus together. However, it doesn't cause a neutron to change into a proton in β^- decay or a proton to change into a neutron in β^+ decay. These changes cannot be due to the electromagnetic force, as the neutron is uncharged. There must be a different force at work in the nucleus causing these changes. It must be weaker than the strong nuclear force, otherwise it would affect stable nuclei; hence we refer to it as the **weak nuclear force** or the 'weak interaction'. When a nucleus emits a beta particle, it also emits a neutrino or an antineutrino. These particles hardly interact with other particles but they sometimes do. For example:

- a neutrino can interact with a neutron and make it change into a proton. A β^- particle is created and emitted at the same time. The interaction is due to the exchange of a particle called a **W^- boson**, which is created in the neutron and absorbed by the neutrino, as shown in Figure 1a.
- an antineutrino can interact with a proton and make it change into a neutron. A β^+ particle is created and emitted at the same time. The interaction is due to the exchange of a particle called a **W^+ boson**, which is created in the proton and absorbed by the antineutrino, as shown in Figure 1b.



a *A neutron–neutrino interaction*



b *A proton–antineutrino interaction*

Fig 1 *Weak interactions*

Collisions and colliders

Cosmic rays are high-energy particles that travel through space from the stars, including the Sun. When a cosmic particle enters the Earth's atmosphere, it interacts with the nuclei of atoms in the atmosphere and creates new short-lived particles and antiparticles. When they were first discovered, most physicists thought cosmic rays were from terrestial radioactive substances. This theory was disproved when physicist and amateur balloonist Victor Hess found that the ionising effect of the rays was significantly greater at 5000 m than at the ground.

Further investigations showed that most cosmic rays are fast-moving protons or small nuclei. They collide with the nuclei of gas atoms in the atmosphere, making them unstable and creating showers of particles and antiparticles that can be detected at ground level. By using cloud chambers and other detectors, new types of short-lived particles and antiparticles were discovered, including:

- the **muon** or 'heavy electron' (symbol μ), a negatively charged particle with a rest mass over 200 times the rest mass of the electron. Muons and antimuons decay into electrons and antineutrinos or positrons and neutrinos, respectively.
- the **pion** or 'π meson', a particle that can be positively charged (π^+), negatively charged (π^-) or neutral (π^0) and has a rest mass greater than a muon but less than a proton. Charged π mesons decay into muons and antineutrinos or antimuons and neutrinos.
- the **kaon** or 'K meson', which also can be positively charged (K^+), negatively charged (K^-) or neutral (K^0) and has a rest mass greater than a pion but still less than a proton. Like π mesons, kaons are produced through the strong interaction when protons moving at high speed crash into nuclei. Because they are produced in pairs through the strong interaction and decay through the weak interaction, they are called **strange** particles.

More strange and non-strange particles were discovered using high-energy accelerators such as:

- the Stanford Linear Accelerator in California. It accelerates electrons over a distance of 3 km through a potential difference (p.d.) of 50 000 million volts. The energy of an electron accelerated through this p.d. is 50 000 MeV (= 50 GeV). When the electrons collide with a target, they can create lots of particle–antiparticle pairs.
- the Large Hadron Collider at CERN near Geneva designed to accelerate charged particles to energies of more than 7000 GeV. Unlike a linear accelerator, this accelerator is a 28 km diameter ring constructed in a circular tunnel below the ground. It is used by physicists from many countries to find out more about the fundamental nature of matter and radiation.

Every strange particle is assigned a **strangeness** number, which is 1 or −1. The strangeness of the K^- meson is defined as −1. Because strangeness is conserved in strong interactions (but not in weak interactions), the strangeness of all other strange particles and antiparticles can be determined. For example, when a proton crashes into nucleus, suppose a particle X and a K^- meson are created. The strangeness of X must be +1, as the total strangeness after the event must be zero, the same as before the event.

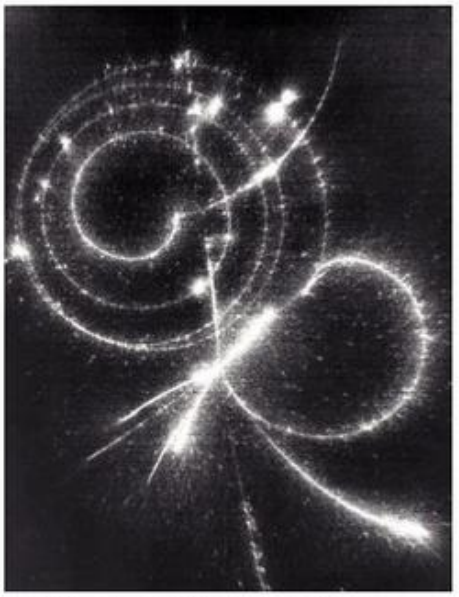


Fig 2 Creation and decay of a π meson

Fig 3 The Large Hadron Collider

Hadrons and leptons

Protons, neutrons, π mesons and K mesons and their antiparticles are examples of particles and antiparticles that we call **hadrons**, because they experience the strong interaction. Electrons, muons and neutrinos and their antiparticles are examples of particles and antiparticles that we call **leptons**, because they experience the weak interaction.

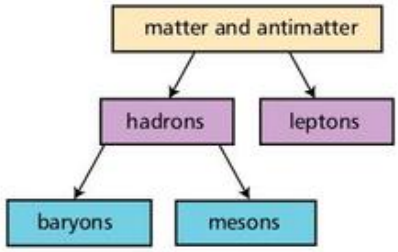


Fig 4 Hadrons and leptons

All leptons are fundamental particles in that they are elementary. Except for the proton, which is stable, hadrons decay in one or more stages into leptons and/or protons. Hadrons that include a proton in their decay products are referred to as **baryons**. Thus, a hadron is either a baryon or a meson, according to whether or not its decay includes a proton.

- Leptons do not interact through the strong interaction. They interact through the weak interaction and, if charged, through the electromagnetic interaction.
- Hadrons interact through the strong interaction and, if charged, through the electromagnetic interaction. Apart from the proton, which is stable, hadrons decay through the weak interaction.

Quarks

The properties of hadrons – such as charge, strangeness and rest mass – can be explained by assuming they are composed of smaller particles known as **quarks** and **antiquarks**. Three different types of quarks and their corresponding antiquarks are necessary in the simple **quark model**. These three types are referred to as up, down and strange quarks and are denoted by the symbols u, d and s, respectively. The equivalent antiquarks are denoted by the symbols $\bar{u}$, $\bar{d}$ and $\bar{d}$. The properties of these three quarks are shown in Table 1.

Evidence for quarks

Experimental evidence for the three-quark model was first obtained using the Stanford Linear Accelerator to make electrons collide at high speed with protons. The results showed that the electrons were scattered by **three** scattering centres in each proton.

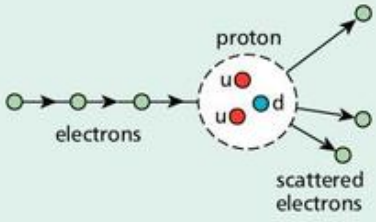


Fig 5 Evidence for quarks

Table 1 Quark properties

	Quarks			Antiquarks		
	up	down	strange	up	down	strange
	u	d	s	$\bar{u}$	$\bar{d}$	$\bar{s}$
Charge Q	$+\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$
Strangeness S	0	0	−1	0	0	+1

The rules for combining quarks and antiquarks to form baryons and mesons are astonishingly simple.

Mesons: each meson is composed of a quark and an antiquark (e.g. a π^- meson is the $\bar{u}d$ combination).

Baryons: each baryon is composed of three quarks (e.g. the proton is the uud combination).

Antibaryons: each antibaryon is composed of three antiquarks (e.g. the antiproton is the $\bar{u}\bar{u}\bar{d}$ combination).

Quark combinations

Mesons are hadrons that each consist of a quark and an antiquark. Figure 6 shows all nine different quark–antiquark combinations and the meson each combination forms in each case. Notice that:

- a π^0 meson can be any quark-corresponding antiquark combination
- each pair of charged mesons is a particle–different antiparticle pair
- there are two uncharged K mesons, the K^0 meson and the $\overline{K^0}$ meson.

The antiparticle of any meson is a quark–antiquark pair and is therefore a meson.

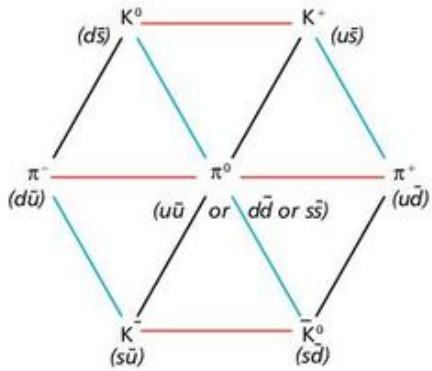


Fig 6 Quark combinations for the mesons

Baryons and antibaryons are hadrons that consist of three quarks or three antiquarks:

- a proton is the uud combination
- a neutron is the udd combination.

The proton is the only stable baryon. A free neutron decays into a proton.

Quarks and beta decay

In β^- decay, a neutron in a neutron-rich nucleus changes into a proton and an electron and an antineutrino are released. In quark terms, a down quark changes to an up quark. In the process, a W^- boson is created and emitted from the down quark. The W^- boson then decays into a β^- particle and an antineutrino. A simple diagram for this change is shown in Figure 7(a).

In β^+ decay, a proton in a proton-rich nucleus changes into a neutron and a positron and an neutrino are released. In quark terms, an up quark changes to a down quark. In the process, a W^+ boson is created and emitted from the up quark. The W^+ boson then decays into a β^+ particle and a neutrino. A simple diagram for this change is shown in Figure 7(b).

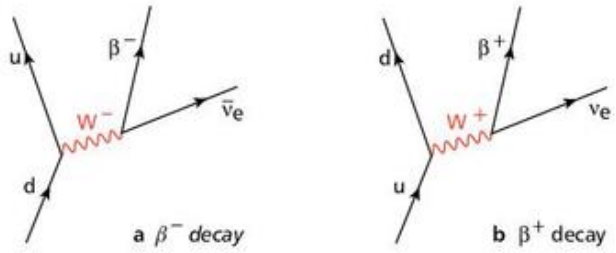


Fig 7 Quark changes in beta decay

QUESTIONS

1 **a** State two differences between a W boson and a virtual photon.

b State the approximate range of a W boson and estimate its lifetime, given it cannot travel faster than the speed of light, which is $3.0 \times 10^8\,m\,s^{-1}$.

2 **a** State the difference between a lepton and a hadron in terms of their interactions.

b Give an example of **i** a baryon, **ii** a non-strange meson, **iii** a strange meson.

3 **a** State the quark composition of **i** a proton, **ii** a neutron.

b With the aid of a diagram, explain in terms of quarks the changes that take place when a proton changes into a neutron in a nucleus.

4 The omega minus (Ω^-) consists of three strange quarks.

a Determine **i** its charge relative to the proton and **ii** its strangeness.

b It decays into a π^- meson and a baryon X, which contains two strange quarks and another quark.

i State the quark composition of the π^- meson.

ii Determine the quark composition of X. Give a reason for your answer.

10 End of chapter questions

$c = 3.00 \times 10^8 \text{m s}^{-1}$, $e = 1.60 \times 10^{-19}\text{C}$,
$h = 6.63 \times 10^{-34}\text{ Js}$

1 A beam of α-particles was directed at normal incidence towards a thin metal foil. A detector was used to measure the number of α-particles per second scattered by different angles.

a Explain why:
- **i** most α-particles passed through the metal foil with little or no deflection,
- **ii** some α-particles were scattered through very large angles.

b The figure below shows the path of two α-particles moving at the same speed in the same direction as they approached a nucleus of an atom. Copy the diagram and complete the paths of these particles.

Fig 1.1

2 An α-particle collided head-on with a nucleus, as shown below.

Fig 2.1

a Describe and explain how the kinetic energy of the α-particle changed as it approached then moved away from the nucleus.

b i Discuss the factors that determine how closely the α-particle can approach the nucleus.

ii Explain why an α-particle with more kinetic energy on such a track might not rebound from the nucleus.

3 In a nuclear reaction, a neutron collided with a nucleus of the lithium isotope ${}^{6}_{3}\text{Li}$. A tritium nucleus ${}^{3}_{1}\text{H}$ and another nucleus X was formed as a result.

Write down an equation that represents this reaction and identify the nucleus X.

4 A narrow beam of ionising radiation from a radioactive substance was directed at a detector at a distance of 30 cm from the source. The detector reading was unchanged when an aluminium plate 5 mm thick was placed between the source and the detector.

a What type of radiation was emitted by this source?

b With the source at a constant distance from the detector, discuss the effect on the detector reading of:
- **i** placing a second identical aluminium plate in the path of the beam,
- **ii** placing a lead plate of thickness 10 mm in the path of the beam.

5 a A Geiger tube in a room recorded a background count in 5 minutes of 130 counts. When a point source of γ radiation was placed at a distance of 0.25 m from the Geiger tube, the Geiger counter recorded 2450 counts in exactly 5 minutes. Calculate:
- **i** the count rate in counts per second due to the source at this distance,
- **ii** the number of counts in 5 minutes with the source at 0.50 m from the tube.

b When an experiment is carried out using a radioactive source in a school laboratory, explain why:
- **i** the source should be kept in a lead-lined container when it is not in use,
- **ii** the user should keep as far from the source as possible and keep the time of use as short as possible.

6 a A ${}^{238}_{92}\text{U}$ nucleus emits an α-particle. The nucleus then emits a β^--particle. Calculate the number of protons and neutrons in the nucleus formed immediately after the emission of:
- **i** the α-particle,
- **ii** the β^--particle.

b A copper disc is placed in the core of a nuclear reactor where the copper nuclei are bombarded by neutrons. As a result, the sample becomes radioactive and emits β^--particles.

Copy and complete the following equations representing these changes:

$${}^{63}_{29}\text{Cu} + {}^{1}_{0}\text{n} \rightarrow \text{X}$$

$$\text{X} \rightarrow \text{Zn} + {}^{0}_{-1}\beta$$

7 A radioactive nucleus decays with the emission of a beta particle and a gamma-ray photon.

a Describe the changes that occur in the proton number and the nucleon number of the nucleus.

b Comment on the relative penetrating powers of the two types of ionising radiation.

c Gamma rays from a point source are travelling towards a detector. The distance from the source to the detector is changed from 1.0 m to 4.0 m. Calculate:

$$\frac{\text{intensity of radiation at 4.0 m}}{\text{intensity of radiation at 1.0 m}}$$

8 An α-particle A approaches and passes by a stationary gold nucleus N. The path is illustrated in Figure 8.1.

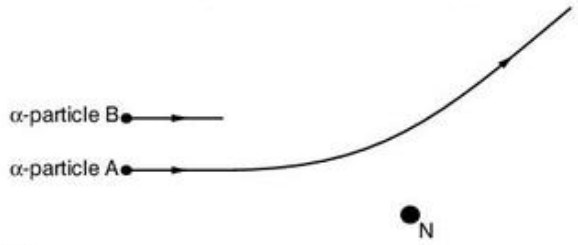


Fig 8.1

a On Figure 8.1, mark the angle of deviation *D* of this α-particle as a result of passing the nucleus N. *(1 mark)*

b A second α-particle B has the same initial direction and energy as α-particle A.

On Figure 8.1, complete the path of α-particle B as it approaches and passes by the nucleus N. *(2 marks)*

c State what can be inferred about atoms from the observation that very few α-particles experience large deviations. *(2 marks)*

d The nucleus N could be one of several different isotopes of gold.

Suggest, with an explanation, whether different isotopes of gold would give rise to different deviations of a particular α-particle. *(2 marks)*

Cambridge International AS and A Level Physics 9702
Paper 21 Q7 November 2009

9 One of the isotopes of uranium is uranium-238 ($^{238}_{92}$U).

a State what is meant by *isotopes*. *(2 marks)*

b For a nucleus of uranium-238, state

i the number of protons, *(1 mark)*

ii the number of neutrons. *(1 mark)*

c A uranium-238 nucleus has a radius of 8.9×10^{-15} m.

Calculate, for a uranium-238 nucleus,

i its mass, in kg, *(2 marks)*

ii its mean density, in kgm^{-3}. *(2 marks)*

d The density of a lump of uranium is 1.9×10^{4} kg m^{-3}.

Using your answer to **c ii**, suggest what can be inferred about the structure of the atom. *(2 marks)*

Adapted from Cambridge International AS and A Level Physics 9702
Paper 21 Q7 June 2010

10 a Uranium (U) has at least fourteen isotopes.

Explain what is meant by *isotopes*. *(2 marks)*

b One possible nuclear reaction involving uranium is

$$^{235}_{92}\mathbf{U} + {}^{1}_{0}\mathbf{n} \rightarrow {}^{141}_{56}\mathbf{Ba} + {}^{92}_{Z}\mathbf{Kr} + x\ {}^{1}_{0}\mathbf{n} + \mathbf{energy.}$$

For this reaction, determine the value of

1. Z, *(1 mark)*

2. x. *(1 mark)*

Cambridge International AS and A Level Physics 9702
Paper 21 Q7 a and b(ii) November 2010

Key concepts: Electricity, waves and radioactivity

In studying electricity in Chapters 6 and 7, you will have gained a clear understanding of how the 'conduction electron' model is used to explain why metals conduct and insulators do not and why the resistivity of an intrinsic semiconductor decreases as its temperature is increased. Electrons are negatively charged particles, and their movement round an electric circuit causes energy to transfer from the battery or power supply to the components in the circuit. Conservation of charge and of energy are important concepts in electric circuit theory because these two principles underpin Kirchoff's laws and other circuit rules. In order to apply these rules to electric circuits, the characteristics of the components in a circuit need to be known before the currents and potential differences within the circuit can be determined. Such component characteristics include knowledge of its resistance and its variation, if any, with temperature and whether or not the component is an ohmic conductor. Internal resistance is an important characteristic of a battery or power source, as internal resistance transfers energy by heating from the power source to the surroundings and is therefore a source of inefficiency in the transfer of energy in an electric circuit.

You should also have developed your mathematical skills in studying electricity as a result of using circuit rules to calculate potential differences, currents and power supplied to the components in a circuit. Conservation of energy applied to a circuit means that the power supplied by a power source less the power dissipated to its internal resistance should be equal to the sum of the powers supplied to all the components.

Electricity is a practical subject, and you should now know how to measure currents and potential differences accurately and to use variable resistors and potential dividers to control currents and potential differences. The correct use of a potential divider in a sensor circuit requires a very clear grasp of the concepts of potential difference and resistance in order to predict how a sensor works.

Waves transfer energy and information. After studying chapters 8 and 9, you should understand the main differences between longitudinal waves and transverse waves and the difference between mechanical waves and electromagnetic waves. You should also know what is meant by wave characteristics such as wavelength, frequency and phase difference. You should also be able to describe wave properties such as reflection, refraction, diffraction and interference and be able to explain two-source interference. You should know how a diffraction grating works and how it is used to measure light wavelengths. Also, you should be able to describe stationary wave patterns and be able to explain their formation. A clear grasp of the principle of superposition is important in gaining a good understanding of interference and also of the formation of stationary waves.

The final chapter on radioactivity develops previous knowledge from earlier chapters about the structure of the atom and about conservation of charge. In studying the nuclear model of the atom, the role of experimentation should be recognised as being crucial to our understanding of matter. Rutherford used his experimental skills to obtain measurements of the deflection of alpha particles and then successfully applied his theoretical knowledge of mechanics and electric fields to his nuclear model of the atom. Using the nuclear model of the atom, you should have a clear understanding of what is meant by an isotope and how equations based on conservation principles are used to represent nuclear changes. The nuclear model of the atom enables us to understand the nature and properties of alpha, beta and gamma radiation as well as gaining knowledge

and awareness of the practical methods and devices used in determining such properties. In such practical work you should have become aware of the hazards of ionising radiation and of the importance of adhering to vital safety rules when radioactive materials are used.

The topic on fundamental particles provides awareness of how experimental discoveries have led to our present knowledge of the structure of matter, including matter and antimatter, the quark model and the weak nuclear force. Conservation rules on nuclear equations developed earlier provide the basic principles that underpin such knowledge.

The following table provides an overview of the topics in chapters 6 to 10, showing where key concepts have been developed.

Chapters 6–10: Key concepts

	Topic		Key concepts	
6.1–6.2	conductors and insulators	electrons as charge carriers	models	drift velocity
		calculations	use of maths	problem-solving
	electric fields	nature of an electric field	forces and fields	action at a distance
6.3–6.5	potential difference and resistance	potential difference	energy and matter	energy transfer per unit charge
		I–V characteristics		
		resistance and resistivity		practical skills using an ammeter and a voltmeter and graph work
		I–V graphs		
		calculations involving resistance and resistivity		problem-solving
6.6	electric field strength	lines of force	forces and fields	force per unit charge
		uniform and non-uniform fields		
7.1–7.2	Kirchhoff's laws	currents at a junction	models	conservation of charge and of energy applied to electric circuits
		p.d. round a circuit	energy and matter	
		resistor rules		
		circuit calculations	use of maths	problem-solving
7.3–7.4	e.m.f. and internal resistance	measurement of e.m.f. and internal resistance	testing	practical skills involving electrical measurements and graph work
		conservation of energy	energy and matter	
		maximum power		
		circuit calculations	use of maths	application of Kirchoff's laws
7.5	the potential divider	theory and use of the potential divider	testing	practical skills involving measurements of p.d.
		sensor circuits	models	predicting output changes in sensor circuits
8.1–8.3	wave properties	longitudinal and transverse waves	energy and waves	amplitude, phase difference
		polarisation of transverse waves		
		reflection, refraction, diffraction, interference	testing	practical skills involved in measuring amplitudes and frequencies
8.4–8.7	stationary waves	stationary wave formation, nodes	energy and waves	principle of superposition
8.8	Doppler effect	Doppler shift	use of maths	problem-solving
		calculations using Doppler effect equations		

continued on p166

Chapters 6–10: Key concepts *(continued)*

	Topic		Key concepts	
9.1–9.2	interference of light	theory of two-slit interference	models	principle of superposition
			energy and waves	path difference
				coherence
		calculations involving use of the two-slit interference equation	use of maths	problem-solving
9.3	diffraction grating	theory of the diffraction grating	energy and waves	Superposition of diffracted wavefronts
		observation and measurement of spectra	testing	practical skills
		calculation of wavelength	use of maths	problem-solving
10.1–10.2	structure of the atom	Rutherford's alpha scattering experiment	models	nuclear model of the atom
		isotopes		
		calculation of nuclear size and nuclear density	use of maths	problem-solving
10.3–10.5	α, β and γ radiation	properties	testing	investigation of range, absorption, ionisation
		ionisation		
		background radioactivity		
		nuclear equations		conservation rules
10.6	fundamental particles	matter and antimatter	energy and matter	
		strong and weak nuclear forces	forces and fields	
		classification of particles		
		quarks	models	the quark model

Question

This question is about an investigation into the heating effect of an electric current and the use of physics equations to predict and test the results. It tests how we use models to make predictions about a physical system and how we then test these predictions with an experimental investigation. It also tests practical skills, when electrical and temperature measurements are used to plot a graph. Accuracy in making temperature measurements is often more difficult to achieve than in electrical or mechanical measurements such as distance or time intervals and simple practical techniques such as stirring the water being heated are important.

Note the use of physics theory and mathematics to predict the relationship between the current I *and the temperature change,* ΔT*, of the water being heated. In this case, the predicted relationship is of the form* $\Delta T = kI^2$*. So by plotting a graph of* ΔT *against* I^2*, we can tell if the relationship is valid from whether or not the graph is linear (i.e. a straight-line graph). If the graph is linear, we can also determine the constant* k*, as it is equal to the gradient of the graph.*

This investigation involves a predicted relationship of the same form as that in the Key concepts 'projectile motion' question on page 88. In both situations, although the measurements give a straight-line graph, the spacing between the plotted points increases along the line. This uneven spacing causes unnecessary difficulty in judging the best-fit line. In practice, the values of I *(or* x *in the previous question) should be selected to give approximately even intervals between successive values of* I^2 *rather than between successive values of* I *(or* x^2 *in the previous question).*

Before answering the question, students should be given the opportunity to do this or a similar investigation.

1 A student carried out an investigation to find out how the heating effect of an electric current changes when the current is increased. A 12 V heater in series with an ammeter, a variable resistor, a switch and a 12 V battery was used to heat some water in an insulated beaker. A thermometer was used to measure the temperature change of the water. The test was repeated several times with a different current each time. In each test, the same amount of water was heated for 600 seconds.

a i Sketch the circuit diagram.

ii In each test, the current was kept constant. Describe how this was achieved using the circuit you sketched. *(2 marks)*

b i Outline a test you would carry out to ensure the beaker insulation was effective.

ii Why was it important to heat the same volume of water for the same length of time in each test? *(4 marks)*

c The table below shows the results of these tests.

current *I* / A	1.2	2.0	3.1	3.7	4.5
temperature increase ΔT / °C	3.0	6.5	16.5	23.5	35.0
I^2 / A^2					

The student predicted that the temperature rise ΔT should be directly proportional to the square of the current.

i Give a justification for this prediction.

ii Copy the table and complete the third row.

iii Plot a suitable graph to find out if the prediction is valid. *(9 marks)*

d The student noticed in each test the resistance of the variable resistor had to be decreased to keep the current constant during the time the water was heated. Give an explanation of this observation. *(2 marks)*

11 Motion in a circle

11.1 Uniform circular motion

On these pages you will learn to:

- Explain what is meant by angular displacement and express angular displacement in radians
- Explain what is meant by angular speed and angular velocity
- Use the concept of angular speed to solve problems
- Recall and use $v = r\omega$ to solve problems

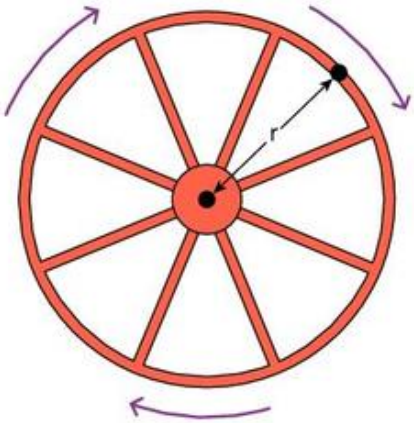


Fig 1 *In uniform circular motion*

Fig 2 *The London Eye*

In a cycle race, the cyclists pedal furiously at top speed. The speed of the perimeter of each wheel is the same as the cyclist's speed, provided the wheels do not slip on the ground. If the cyclist's speed is constant, the wheels must turn at a steady rate. An object rotating at a steady rate is said to be in **uniform circular motion**.

Consider a point on the perimeter of a wheel of radius r rotating at a steady speed.

- The circumference of the wheel $= 2\pi r$
- The frequency of rotation $f = \frac{1}{T}$, where T is the time for one rotation.

The speed of a point on the perimeter $= \frac{\text{circumference}}{\text{time for 1 rotation}} = \frac{2\pi r}{T} = 2\pi rf$

Worked example

A cyclist is travelling at a speed of $13\,\text{m s}^{-1}$ on a bicycle which has wheels of radius 390 mm. Calculate:

a the time for one rotation of the wheel

b i the frequency of rotation of the wheel.

ii the number of rotations of the wheel in 1 minute.

Solution

a Rearranging speed $v = \frac{2\pi r}{T}$ gives the time for 1 rotation, $T = \frac{2\pi r}{v}$

Therefore, $T = \frac{2\pi \times 0.39\,\text{m}}{13\,\text{m s}^{-1}} = 0.19\,\text{s}$

b i Frequency $f = \frac{1}{T} = \frac{1}{0.19\,\text{s}} = 5.3\,\text{Hz}$

ii Number of rotations in 1 minute $= 60 \times 5.3 = 318$

Angular displacement

In the UK, the London Eye is a very popular tourist attraction. The wheel has a diameter of 130 m and takes passengers high above the surrounding buildings, giving a glorious view on a clear day. Each full rotation of the wheel takes 30 minutes. Each capsule therefore takes its passengers through an angle of 0.2° each second or $= \frac{0.2\pi}{180}$ radians each second as $360° = 2\pi$ radians.

For any object in uniform circular motion, the object turns through an angle of $\frac{2\pi}{T}$ radians each second, where T is the time taken for 1 complete rotation.

In other words, the angular displacement of the object each second is $\frac{2\pi}{T}$.

Therefore, the **angular displacement** of the object in time t is given by

$$\theta \text{ (in radians)} = \frac{2\pi t}{T} = 2\pi ft$$

Angular velocity

The angular velocity of an object moving in uniform circular motion is defined as its angular displacement per second.

The unit of angular velocity is the radian per second (rad s^{-1}).

Using the previous equation $\theta = \frac{2\pi t}{T}$ therefore gives

$$\text{angular velocity, } \omega = \frac{\text{angular displacement, } \theta}{\text{time taken, } t} = \frac{2\pi}{T}$$

The object travels once round the circle in its **time period**. For a circle of radius r, its circumference is equal to $2\pi r$. Hence its speed v is given by

$$v = \frac{\text{circumference of the circle}}{\text{time period}} = \frac{2\pi r}{T}$$

Substituting $\omega = \frac{2\pi}{T}$ gives:

$$\boldsymbol{v = \omega r}$$

Figure 3 shows the angular displacement of the object in time t which is less than the time period T.

1 The angular displacement of the object, $\theta = \omega t$.
2 The object travels along an arc of the circle of length $s = vt = \frac{2\pi rt}{T} = r\theta$

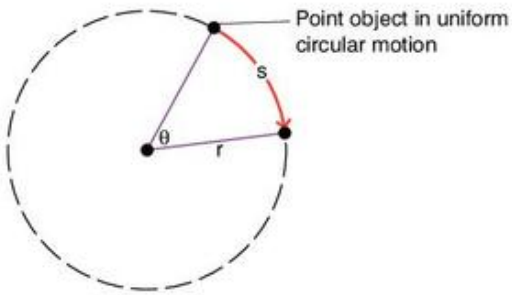


Fig 3 Arcs and angles

Worked example

A cyclist travels at a speed of 12 m s^{-1} on a bicycle which has wheels of radius 0.40 m. Calculate:
a the frequency of rotation of each wheel,
b the angular velocity of each wheel,
c the angle the wheel turns through in 0.10 s:
i in radians, **ii** in degrees.

Solution

a Circumference of wheel $= 2\pi r = 2\pi \times 0.4 = 2.5$ m

Time for 1 wheel rotation, $T = \frac{\text{circumference}}{\text{speed}} = \frac{2.5}{12} = 0.21$ s

Frequency $= \frac{1}{T} = \frac{1}{0.21} = 4.8$ Hz

b Angular velocity $= \frac{2\pi}{T} = 30$ rad s^{-1}

c i Angle the wheel turns through in 0.10 s, $\theta = \frac{2\pi t}{T} = \frac{2\pi \times 0.10}{0.21}$
$= 3.0$ radians

ii $\theta = 3.0 \times \frac{360}{2\pi} = 172°$

One radian is defined as the angle subtended by a circular arc of length equal to the radius of the circle. It is based on a scale of 2π radians being equal to 360°. This is because the formula for the circumference of a circle $2\pi r$ (where r is the circle radius) then leads to the following useful formula for the length, s, of an arc of a circle:

$$s = r\theta$$

where θ is the angle in radians subtended by the arc at the centre of the circle. See Figure 3.

QUESTIONS

1 Calculate the angular displacement in radians of the tip of the minute hand of a clock in:
a 1 second,
b 1 minute,
c 1 hour.

2 An electric motor turns at a frequency of 50 Hz. Calculate:
a its time period,
b the angle it turns through in radians in:
i 1 ms,
ii 1 second.

3 The Earth takes exactly 24 hours for 1 full rotation. Calculate:
a the speed of rotation of a point on the equator,
b the angle the Earth turns through in 1 second in:
i degrees, **ii** radians.

The radius of the Earth = 6400 km.

4 A satellite in a circular orbit of radius 8000 km takes 120 minutes per orbit. Calculate:
a its speed,
b its angular displacement in 1.0 s in:
i degrees, **ii** radians.

11.2 Centripetal acceleration

On these pages you will learn to:

- Describe uniform circular motion
- Understand centripetal acceleration in the case of uniform circular motion
- Recall and use centripetal acceleration equations $a = r\omega^2$ and $a = v^2/r$
- Recall and use centripetal force equations $F = mr\omega^2$ and $F = mv^2/r$

The **velocity** of an object moving round a circle at constant speed continually changes direction. Because its velocity changes, the object therefore accelerates. If this seems odd because the speed is constant, remember that acceleration is change of velocity per second. Passengers on the London Eye might not notice they are being accelerated but if the wheel was made to rotate much faster, they undoubtedly would notice.

The velocity of an object in uniform circular motion at any point is along the tangent to the circle at that point. The direction of the velocity changes continuously as the object moves round on its circular path. The change in the direction of the velocity is towards the centre of the circle. So its acceleration is towards the centre of the circle and is referred to as **centripetal acceleration**. Centripetal means 'towards the centre of the circle'.

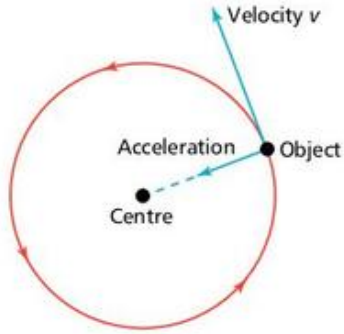


Fig 1 Centripetal acceleration

For an object moving at constant speed v in a circle of radius r, it can be shown that

$$\textbf{its centripetal acceleration, } a = \frac{v^2}{r}$$

Proof of this equation is not required for the specification. However, a proof is given to provide a better understanding of the idea of centripetal acceleration.

Proof of $a = \frac{v^2}{r}$

- Consider an object in uniform circular motion at speed v moving in a short time interval δt from position A to position B along its path. Therefore the distance AB along the circle, $\delta s = v\,\delta t$. Figure 2 shows the idea.
- The line from the object to the centre of the circle at C turns through angle θ when the object moves from A to B. The velocity direction of the object turns through the same angle θ, as shown in Figure 2.
- The change of velocity, δv = velocity at B − velocity at A, is shown in the velocity vector triangle in Figure 2.
- The triangles ABC and the velocity vector triangle have the same shape because they both have two sides of equal length with the same angle θ between the two sides.

Provided θ is small, then $\dfrac{\delta v}{v} = \dfrac{\delta s}{r}$

Because $\delta s = v\,\delta t$, then $\dfrac{\delta v}{v} = \dfrac{v\,\delta t}{r}$

Therefore, acceleration, $a = \dfrac{\text{change of velocity } \delta v}{\text{time taken } \delta t}$

$= \dfrac{v^2}{r}$ towards the centre.

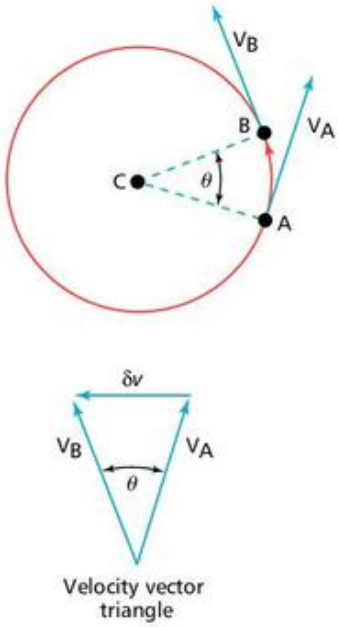


Fig 2 Proving $a = \frac{v^2}{r}$

Note Since $v = \omega r$, then $a = \dfrac{v^2}{r} = \dfrac{(\omega r)^2}{r} = \omega^2 r$

Centripetal force

To make an object move round on a circular path, it must be acted on by a resultant force which changes its direction of motion. Figure 3 shows a 'hammer' being whirled round in a circle. The tension in the cable pulls on the ball and changes its direction continuously. When the cable is released, the ball flies off at a tangent.

The resultant force on an object moving round a circle at constant speed is referred to as the **centripetal force** because it acts towards the centre of the circle.

- For an object whirling round on the end of a string, the tension in the string is the centripetal force.
- For a satellite moving round the Earth, the force of gravity between the satellite and the Earth is the centripetal force. See Figure 4.
- For a planet moving round the Sun, the force of gravity between the planet and the Sun is the centripetal force.
- For a capsule on the London Eye, the centripetal force is the resultant of the support force on the capsule and the force of gravity on it.
- In Chapter 18, you will meet the use of a magnetic field to bend a beam of charged particles (e.g. electrons) in a circular path. The magnetic force on the moving charged particles is the centripetal force.

Fig 3 Centripetal force in action

Any object that moves in circular motion is acted on by a resultant force which always acts towards the centre of the circle. The resultant force is the centripetal force and therefore causes a centripetal acceleration.

Notes

1 If the object is acted on by a single force only (e.g. a satellite in orbit round the Earth), that force is the centripetal force and causes the centripetal acceleration.

2 The centripetal force is at right angles to the direction of the object's velocity. Therefore, no work is done by the centripetal force on the object because there is no displacement in the direction of the force. The kinetic energy of the object is therefore constant so its speed is unchanged.

Equation for centripetal force

For an object moving at constant speed v along a circular path of radius r, its centripetal acceleration $a = \frac{v^2}{r}$ $(= \omega^2 r)$

Therefore, applying Newton's second law for constant mass in the form '$F = ma$' gives

$$\text{centripetal force } F = \frac{mv^2}{r} = m\omega^2 r$$

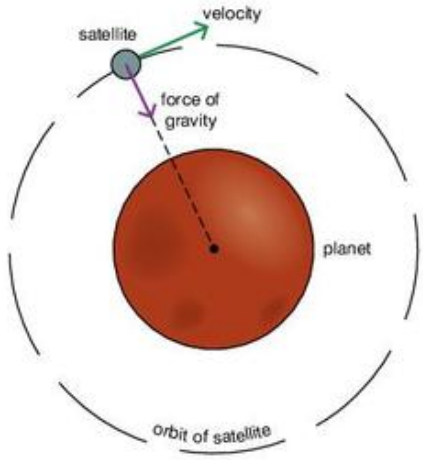


Fig 4 A satellite in uniform circular motion

QUESTIONS

1 The wheel of the London Eye has a diameter of 130 m and takes 30 minutes for a full rotation. Calculate:

a the speed of a capsule,

b **i** the centripetal acceleration of a capsule,
ii the centripetal force on a person of mass 65 kg in a capsule.

2 An object of mass 0.15 kg moves round a circular path of radius 0.42 m at a steady rate once every 5.0 seconds. Calculate:

a the speed and acceleration of the object,

b the centripetal force on the object.

3 **a** The Earth moves round the Sun on a circular orbit of radius 1.5×10^{11} m, taking $365\frac{1}{4}$ days for each complete orbit. Calculate:
i the speed, **ii** the centripetal acceleration of the Earth on its orbit round the Sun.

b A satellite is in orbit just above the surface of a spherical planet which has the same radius as the Earth and the same acceleration of free fall at its surface. Calculate:
i the speed,
ii the time for 1 complete orbit of this satellite.

The radius of the Earth = 6 400 km
Acceleration of free fall = 9.8 m s^{-2}

4 A hammer thrower whirls a 2.0 kg hammer on the end of a rope in a circle of radius 0.8 m. The hammer took 0.6 s to make one full rotation just before it was released. Calculate:

a the speed of the hammer just before it was released,

b its centripetal acceleration,

c the centripetal force on the hammer just before it was released.

11.3 On the road

On these pages you will learn to:

- Recall the equations for angular speed, centripetal acceleration and centripetal force
- Solve problems on uniform circular motion in the context of road vehicles and aircraft

Even on a very short journey, the effects of circular motion can be important. For example, a vehicle that turns a corner too fast could skid or topple over. A vehicle that goes over a curved bridge too fast might even lose contact briefly with the road surface. To make any object move on a circular path, the object must be acted on by a resultant force which is always towards the centre of curvature of its path.

Examples

1 Over the top of a hill

Consider a vehicle of mass m moving at speed v along a road that passes over the top of a hill or over the top of a curved bridge.

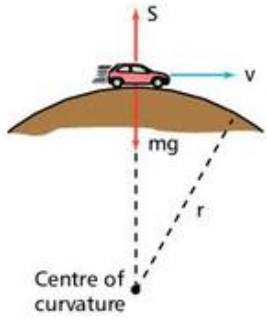


Fig 1 Over the top

At the top of the hill, the support force S from the road on the vehicle is directly upwards in the opposite direction to its weight, mg. The resultant force on the vehicle is the difference between the weight and the support force. This difference acts towards the centre of curvature of the hill as the centripetal force. In other words,

$$mg - S = \frac{mv^2}{r}, \text{ where } r \text{ is the radius of curvature of the hill}$$

The vehicle would lose contact with the road if its speed is equal to or greater than a certain speed, v_0. If this happens, then $S = 0$ so $mg = \frac{mv_0^2}{r}$

Therefore, the vehicle speed should not exceed v_0, where $v_0^2 = gr$, otherwise the vehicle will lose contact with the road surface at the top of the hill. Prove for yourself that a vehicle that travels over a curved bridge of radius of curvature 5 m would lose contact with the road surface if its speed exceeded 7 m s^{-1}.

2 On a roundabout

Consider a vehicle of mass m moving at speed v in a circle of radius r as it moves round a roundabout on a level road. The centripetal force is provided by the force of friction between the vehicle's tyres and the road surface. In other words,

$$\text{force of friction } F = \frac{mv^2}{r}$$

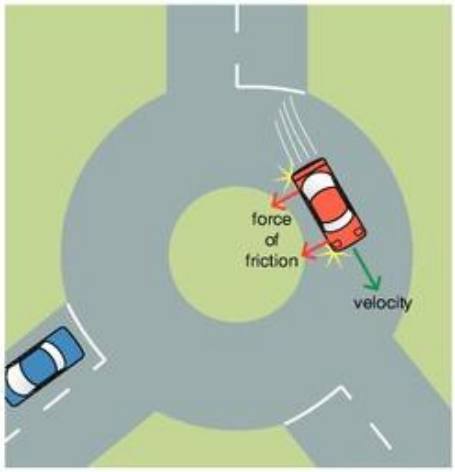


Fig 2 On a roundabout

For no skidding or slipping, the force of friction between the tyres and the road surface must be less than a limiting value F_0 which is proportional to the vehicle's weight.

Therefore, for no slipping, the speed of the vehicle must be less than a certain value v_0 which is given by the equation,

$$\text{limiting force of friction } F_0 = \frac{mv_0^2}{r}$$

3 On a banked track

A race track is often banked where it curves. Motorway slip roads in cities often bend in a tight curve. Such a road is usually banked to enable vehicles to drive round without any sideways friction on the tyres. Rail tracks on curves are usually banked to enable trains to move round the curve without slowing down too much.

- Without any banking, the centripetal force is provided only by sideways friction between the vehicle wheels and the road surface. As explained in the previous example, the vehicle on a bend slips outwards if its speed is too high.

- On a banked track, the speed can be higher. To understand why, consider Figure 3 which represents the front-view of a racing car of mass m on a banked track, where θ = the angle of the track to the horizontal. For there to be no sideways friction on the tyres due to the road, the horizontal component of the support forces N_1 and N_2 must act as the centripetal force.

Resolving these forces into horizontal components ($= (N_1 + N_2)\sin\theta$) and vertical components ($= (N_1 + N_2)\cos\theta$),

- because $(N_1 + N_2)\sin\theta$ acts as the centripetal force, then $(N_1 + N_2)\sin\theta = \frac{mv^2}{r}$
- because $(N_1 + N_2)\cos\theta$ balances the weight (mg), then $(N_1 + N_2)\cos\theta = mg$.

Therefore $$\tan\theta = \frac{(N_1 + N_2)\sin\theta}{(N_1 + N_2)\cos\theta} = \frac{mv^2}{mgr}$$

Simplifying this equation gives the condition for no sideways friction; $\tan\theta = \frac{v^2}{gr}$

In other words, there is no sideways friction if the speed v is such that $\boldsymbol{v^2 = gr\tan\theta}$.

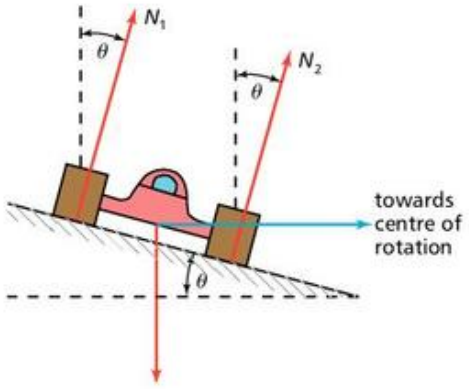


Fig 3 A racing car taking a bend

Notes

1 An aircraft banks when it turns in a circle. The horizontal component of the lift force acts as the centripetal force. Figure 4 shows the idea. The radius of the circle, r, depends on the speed and the angle of banking in accordance with the equation $\boldsymbol{v^2 = gr\tan\theta}$.

2 The **conical pendulum** in Figure 5 shows an object in uniform circular motion due to the horizontal component of a force, in this case the tension in the supporting thread. Resolving the thread tension T into horizontal and vertical components gives:

- vertical component $T\cos\theta = mg$
- horizontal component $T\sin\theta = mv^2/r$ where the circle radius $r = L\sin\theta$

Hence $$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{mv^2/r}{mg} = \frac{v^2}{gr}$$

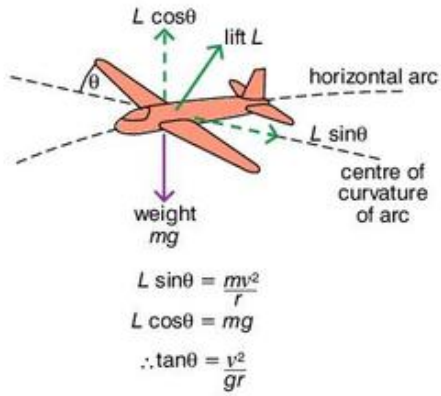


Fig 4 An aircraft circling

QUESTIONS

$g = 9.8\,\text{m s}^{-2}$

1 A vehicle of mass 1 200 kg passes over a bridge of radius of curvature 15 m at a speed of $10\,\text{m s}^{-1}$. Calculate:
   a the centripetal acceleration of the vehicle on the bridge,
   b the support force on the vehicle when it is at the top.

2 The maximum speed for no skidding of a vehicle of mass 750 kg on a roundabout of radius 20 m is $9.0\,\text{m s}^{-1}$. Calculate, for this speed:
   a the centripetal acceleration,
   b the centripetal force on the vehicle.

3 Explain why a circular athletics track that is banked is suitable for sprinters but not for marathon runners.

4 At a racing car circuit, the track is banked at an angle of 25° to the horizontal on a bend which has a radius of curvature of 350 m.
   a Calculate the speed of a vehicle on the bend if there is to be no sideways friction on its tyres.
   b Discuss and explain what could happen to a vehicle that took the bend too fast.

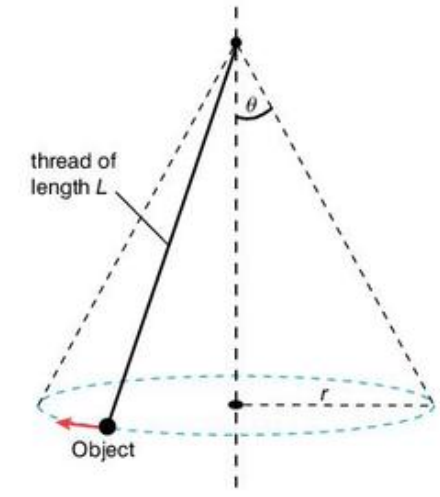


Fig 5 The conical pendulum

11.4 At the fairground

On these pages you will learn to:

- Recall the equations for angular speed, centripetal acceleration and centripetal force
- Solve problems on uniform circular motion in the fairground rides

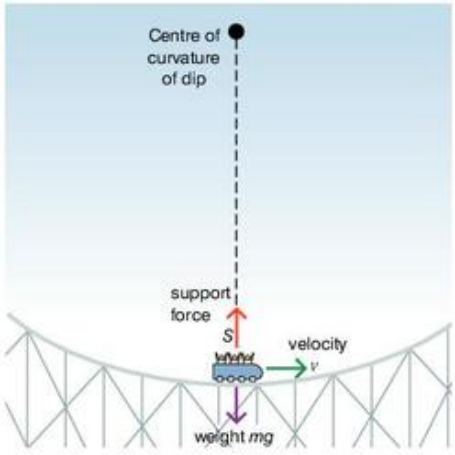


Fig 1 In a dip

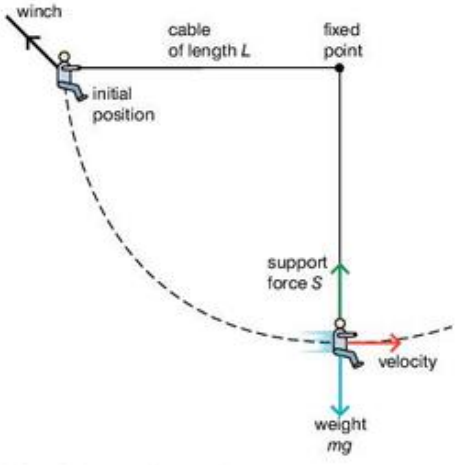


Fig 2 The very long swing

Many of the rides at a fairground take people round in circles. Some examples are analysed below. It is worth remembering that centripetal acceleration values of more than 2–3 g can be dangerous to the average person.

Examples

1 The Big Dipper

A ride that takes you at high speed through a big dip pushes you into your seat as you pass through the dip. The difference between the support force on you (acting upwards) and your weight acts as the centripetal force.

At the bottom of the dip, the support force S on you is vertically upwards, as shown in Figure 1.

Therefore, for a speed v at the bottom of a dip of radius of curvature r,

$$S - mg = \frac{mv^2}{r}$$

So the support force $S = mg + \frac{mv^2}{r}$

The extra force you experience due to your motion is therefore $\frac{mv^2}{r}$.

2 The very long swing

Consider a person of mass m on a very long swing of length L, released from height h above the equilibrium position. The maximum speed is when the swing passes through the lowest point. This can be worked out by equating the loss of potential energy to the gain of kinetic energy.

$$\tfrac{1}{2}mv^2 = mgh$$

where v is its speed as it passes through the lowest point.

$$\text{Therefore } v^2 = 2gh$$

The person on the swing is on a circular path of radius L. At the lowest point, the support force S on the person due to the rope is in the opposite direction to the person's weight, mg. The difference, $S - mg$, acts towards the centre of the circular path and provides the centripetal force. Therefore

$$S - mg = \frac{mv^2}{L}$$

Because $v^2 = 2gh$, $S - mg = \frac{2mgh}{L}$

In other words, $\frac{2mgh}{L}$ represents the extra support force the person experiences due to circular motion. Prove for yourself that for $h = L$ (i.e. a 90° swing), the extra support force is equal to twice the person's weight.

3 The Big Wheel

This ride takes its passengers round in a vertical circle on the inside of the circumference of a very large wheel. The wheel turns fast enough to stop the passengers falling out as they pass through the highest position.

At maximum height, the reaction R from the wheel on each person acts downwards. Therefore, the resultant force at this position $= mg + R$. This reaction force and the weight provide the centripetal force. Therefore, at the highest position when the wheel speed is v,

$$mg + R = \frac{mv^2}{r} \qquad \text{where } r \text{ is the radius of the wheel}$$

$$\therefore \quad R = \frac{mv^2}{r} - mg$$

At a certain speed v_0 such that $v_0^2 = gr$, then $R = 0$ so there would be no force on the person due to the wheel.

A person in a Big Wheel with capsules (e.g. the London Eye) would be unsupported at speed v_0. Such a wheel must turn slower otherwise passengers would lose contact with the capsule floor.

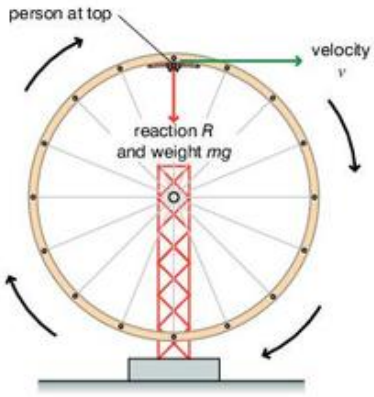


Fig 3 The Big Wheel

QUESTIONS

$g = 9.8\,\text{m s}^{-2}$

1 A train on a fairground ride is initially stationary before it descends through a height of 45 m into a dip which has a radius of curvature of 78 m, as shown in Figure 4.

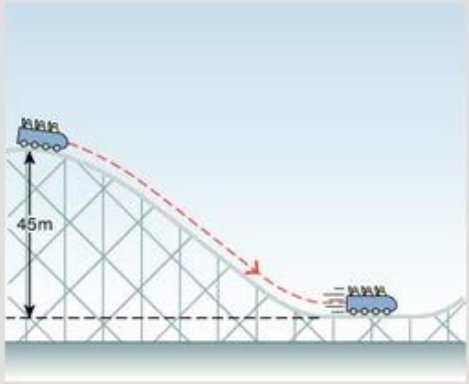


Fig 4

a Calculate the speed of the train at the bottom of the dip, assuming air resistance and friction are negligible.

b Calculate:
i the centripetal acceleration of the train at the bottom of the dip,
ii the extra support force on a person of weight 600 N in the train.

2 A very long swing at a fairground is 32 m in length. A person of mass 69 kg on the swing descends from a position when the swing is horizontal. Calculate:

a the speed of the person at the lowest point,

b the centripetal acceleration at the lowest point,

c the support force on the person at the lowest point.

3 The Big Wheel at a fairground has a radius of 12.0 m and rotates in a vertical plane once every 6.0 seconds. Calculate:

a the speed of rotation of the perimeter of the wheel,

b the centripetal acceleration of a person on the perimeter,

c the support force on a person of mass 72 kg at the highest point.

4 The wheel of the London Eye has a diameter of 130 m and takes 30 minutes to complete one revolution. Calculate the change, due to rotation of the wheel, of the support force on a person of weight 500 N in a capsule at the top of the wheel.

CHAPTER SUMMARY

1 For an object of mass m in uniform circular motion on a circle of radius r:

- its **speed** $v = \frac{2\pi r}{T}$, where T is the time for one rotation,
- its **frequency of rotation** $f = \frac{1}{T}$,
- its **angular displacement**, in radians, in time $t = 2\pi ft = \frac{2\pi t}{T}$,
- its **centripetal acceleration**, $a = \frac{v^2}{r}$ towards the centre of the circle,
- the **centripetal force** on the object $= \frac{mv^2}{r}$

2 Examples

- Car of mass m going over the top of a hill; support force $S = mg - \frac{mv^2}{r}$
- Object of mass m going through the bottom of a dip; support force $S = mg + \frac{mv^2}{r}$
- Person on the inside of a vertical fairground wheel at the top; reaction force $R = \frac{mv^2}{r} - mg$
- Banked track at angle θ to horizontal; $v^2 = gr\tan\theta$ for no sideways friction

11 End of chapter questions

$g = 9.8\ \text{m s}^{-2}$

1 An object moves on a circular path at constant speed. Explain:

a why the object's velocity continually changes even though its speed is constant,

b the object accelerates even though its speed is constant.

2 The Moon moves round the Earth once every $27\frac{1}{3}$ days on a circular orbit of radius 380 000 km. Calculate:

a the speed of the Moon,

b its centripetal acceleration.

3 A pulley wheel of diameter 24 mm fitted to an electric motor in a machine rotates at a frequency of 30 Hz when the machine is in normal operation. A belt fitted to the wheel is used to drive a drum in the machine, as shown below.

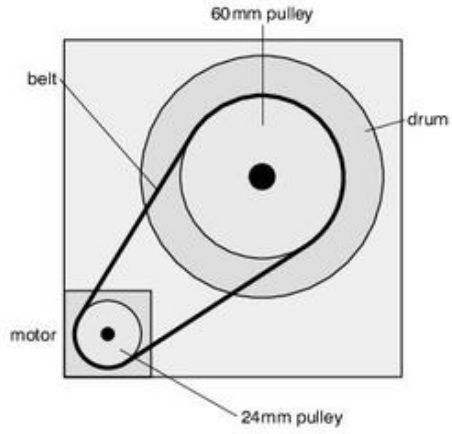


Fig 3.1

a Calculate:

i the speed of the belt on the wheel,

ii the centripetal acceleration of the belt attached to the wheel as it moves round the pulley wheel.

b The belt drives the drum via a second pulley wheel of diameter 60 mm attached to the drum axle. Calculate:

i the frequency of rotation of the second pulley wheel,

ii the centripetal acceleration of the belt as it passes round the second pulley wheel.

4 A cyclist travels at a speed of $15\ \text{m s}^{-1}$ on a bicycle fitted with wheels of diameter 850 mm.

a Calculate:

i the frequency of rotation of each wheel,

ii the centripetal acceleration of the tyre on each wheel.

b The rear wheel of the bicycle is driven by a chain which passes round a gear wheel of diameter 55 mm attached to the axle of the rear wheel.

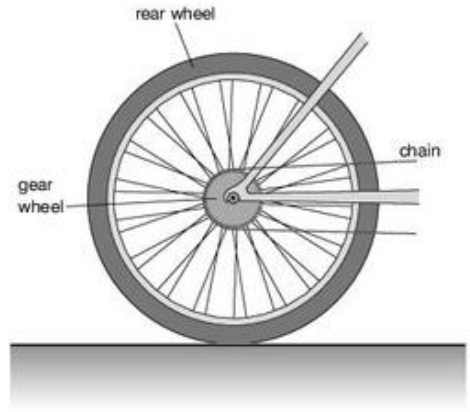


Fig 4.1

i Calculate the centripetal acceleration of the chain as it passes round the gear wheel.

ii Explain why the chain will come off the gear wheel if its speed is too great.

5 **a** Explain why a car on a roundabout will slide to the outside of the roundabout if it travels too fast on the roundabout.

b **i** Calculate the centripetal acceleration of a vehicle travelling round a roundabout of diameter 40 m when the speed of the vehicle is $8\ \text{m s}^{-1}$.

ii For no slippage on the roundabout, the frictional force on the vehicle tyres must be less than 0.6 × the vehicle weight. Calculate the maximum speed of the vehicle on the roundabout for no slippage.

6 On a fairground ride, a train of mass 1500 kg moving at a speed of $1.5\ \text{m s}^{-1}$ descends from the highest point of the track through a height of 42 m to the bottom of a dip and then passes over a 'hill' which is 15 m higher than the bottom of the dip.

Fig 6.1

a i Calculate the speed of the train at the bottom of the dip.

ii Show that the speed of the train as it passes over the top of the hill is 23 m s^{-1}. (Assume air resistance and friction are negligible.)

b i The dip has a radius of curvature of 65 m. Show that a person of mass 80 kg in the train experiences an extra support force of 1020 N on passing through the bottom of the dip.

ii The passengers in the train momentarily leave their seats when the train passes over the hill. Show that the radius of curvature of the hill is 54 m.

7 a Explain why a train can travel round a horizontal curve at a higher speed if the track is banked rather than flat.

b Discuss why the train would leave the track if it travelled round the curve too fast.

8 The figure below shows a cross-section of an automatic brake fitted to a rotating shaft. The brake pads are held on the shaft by springs.

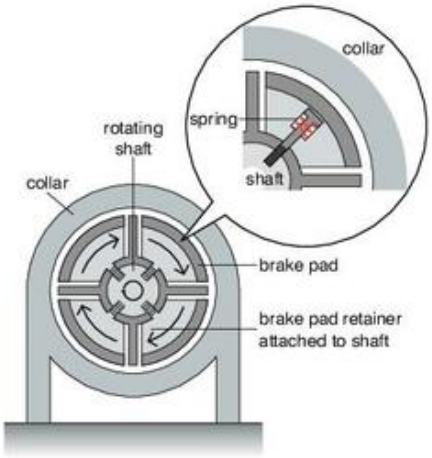


Fig 8.1

a Explain why the brake pads press against the inner surface of the stationary collar if the shaft rotates too fast.

b Each brake pad and its retainer has a mass of 0.30 kg and its centre of mass is 60 mm from the centre of the shaft. The tension in the spring attached to each pad is 250 N. Calculate the maximum frequency of rotation of the shaft for no braking.

9 a Define the *radian*. *(2 marks)*

b A stone of weight 3.0 N is fixed, using glue, to one end P of a rigid rod CP, as shown in Figure 9.1.

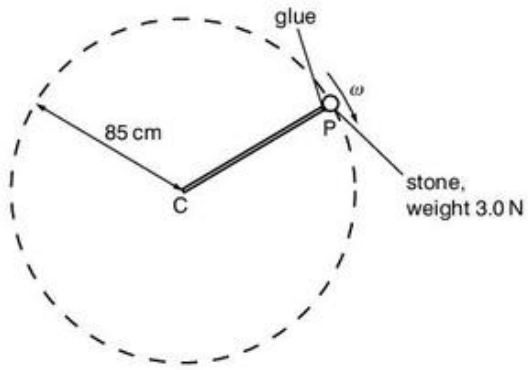


Fig 9.1

The rod is rotated about end C so that the stone moves in a vertical circle of radius 85 cm.
The angular speed ω of the rod and stone is gradually increased from zero until the glue snaps. The glue fixing the stone snaps when the tension in it is 18 N.
For the position of the stone at which the glue snaps,

i on a copy of the diagram, on the dotted circle of Figure 9.1, mark with the letter S the position of the stone, *(1 mark)*

ii calculate the angular speed ω of the stone, in rad s^{-1} *(4 marks)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 41 Q1 June 2010

12 Oscillations

12.1 Measuring oscillations

On these pages you will learn to:

- Describe simple examples of free oscillations and describe the motion of an oscillating object
- Explain what is meant by the amplitude, the period, the frequency and the angular frequency for an oscillating object
- Explain what is meant by phase difference between two objects oscillating at the same frequency
- Relate the frequency and angular frequency of an oscillating object to its time period

There are many examples of oscillations in everyday life. A car that travels over a bump bounces up and down for a short time afterwards. Every microcomputer has an electronic oscillator to drive its internal clock. A child on a swing moves forwards then backwards repeatedly. In this simple example, one full cycle of motion is from maximum height at one side to maximum height on the other side then back again. The lowest point is referred to as the **equilibrium** position as it is where the child eventually comes to a standstill. The child in motion is said to **oscillate** about equilibrium.

Further examples of oscillating motion include:

- an object on a spring moving up and down repeatedly,
- a pendulum moving to-and-fro repeatedly,
- a ball bearing rolling from side-to-side,
- a small boat rocking from side to side.

Displacement v. time for an oscillating object

An oscillating object moves repeatedly one way then in the opposite direction through its equilibrium position. The **displacement** of the object (i.e. distance and direction) from equilibrium continually changes during the motion. In one full cycle after passing through equilibrium, the displacement of the object:

- increases as it moves away from equilibrium, then
- decreases as it returns to equilibrium, then
- reverses and increases as it moves away from equilibrium in the opposite direction, then
- decreases as it returns to equilibrium.

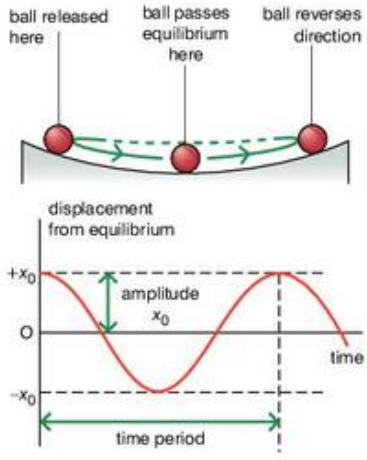


Fig 1 Oscillating motion

The **amplitude** of the oscillations is the maximum displacement of the oscillating object from equilibrium. If the amplitude is constant and no frictional forces are present, the oscillations are described as **free** oscillations. See topic 12.5.

The **time period** of the oscillating motion is the time for one complete cycle of oscillations. One full cycle after passing through any position, the object passes through that same position in the same direction.

The **frequency** of oscillations is the number of cycles per second made by an oscillating object.

The unit of frequency is the hertz (Hz) which is 1 cycle per second.

For oscillations of frequency f, the time period $T_P = \frac{1}{f}$

The **angular frequency** ω of the oscillating motion is defined as $\frac{2\pi}{T_P}$ $(= 2\pi f)$.

The unit of ω is the radian per second (rad s^{-1}).

Phase difference

Imagine two children on adjacent identical swings. The time period, T_P of the oscillating motion is the same, as the swings are identical. If one child reaches maximum displacement on one side a certain time, Δt, later than the other child, they oscillate out of phase. Their **phase difference** stays the same as they oscillate, always corresponding to a fraction of a cycle equal to $\frac{\Delta t}{T_P}$. For example, if the time period is 2.0 s and one child reaches maximum displacement on one side 0.5 seconds later than the other child, the later child will always be a quarter of a cycle $\left(= \frac{0.5\,\text{s}}{2.0\,\text{s}}\right)$ behind the other child. Their phase difference, in radians, is therefore $0.5\pi \left(= 2\pi\frac{\Delta t}{T_P}\right)$.

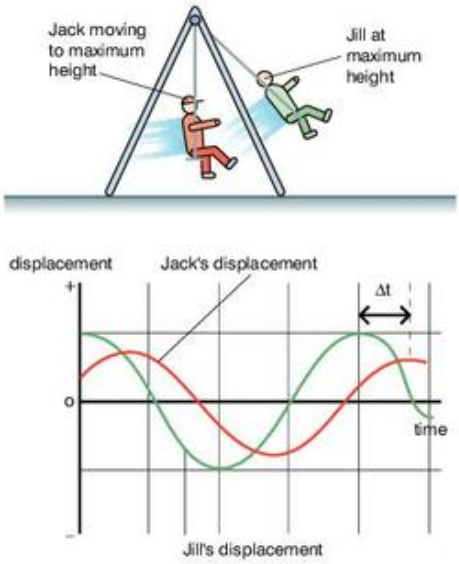


Fig 2 Phase difference

In general, for two objects oscillating at the same frequency,

$$\textbf{their phase difference, in radians, } = 2\pi\frac{\Delta t}{T_P}$$

where Δt is the time between successive instants when the two objects are at maximum displacement in the same direction.

Notes

1 2π radians $= 360°$ so the phase difference in degrees is $360\,\frac{\Delta t}{T_P}$.

2 The two objects oscillate in phase if $\Delta t = T_P$. The phase difference of 2π is therefore equivalent to zero.

3 Table of phase differences

Δt	0	$0.25T_P$	$0.50T_P$	$0.75T_P$	T_P
Phase difference in radians	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
in degrees	0	90	180	270	360

QUESTIONS

1 Describe how the velocity of a bungee jumper changes from the moment he jumps off the starting platform to the moment he next returns to the platform.

2 **a** What is meant by free oscillations?

b A metre rule is clamped to a table so that part of its length projects at right angles from the edge of the table, as shown in Figure 3. A 100 g mass is attached to the free end of the rule. When the free end of the rule is depressed downwards then released, the mass oscillates. Describe how you would find out if the oscillations of the mass are free oscillations.

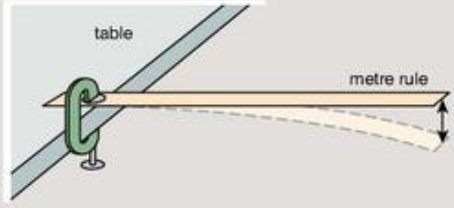


Fig 3

3 An object suspended from the lower end of a vertical spring is displaced downwards from equilibrium by a distance of 20 mm then released. It takes 9.6 s to undergo 20 complete cycles of oscillation. Calculate:

a its time period,

b its frequency of oscillation.

4 Two identical pendulums X and Y each consist of a small metal sphere attached to a thread of a certain length. Each pendulum makes 20 complete cycles of oscillation in 16 s. State the phase difference, in radians, between the motion of X and that of Y if:

a X passes through equilibrium 0.2 s after Y passes through equilibrium in the same direction,

b X reaches maximum displacement at the same time as Y reaches maximum displacement in the opposite direction.

12.2 The principles of simple harmonic motion

On these pages you will learn to:

- Draw and describe graphs to show the changes in displacement, velocity and acceleration during simple harmonic motion
- Define simple harmonic motion

An oscillating object speeds up as it returns to equilibrium and it slows down as it moves away from equilibrium. Figure 1 shows one way to record the displacement of an oscillating pendulum.

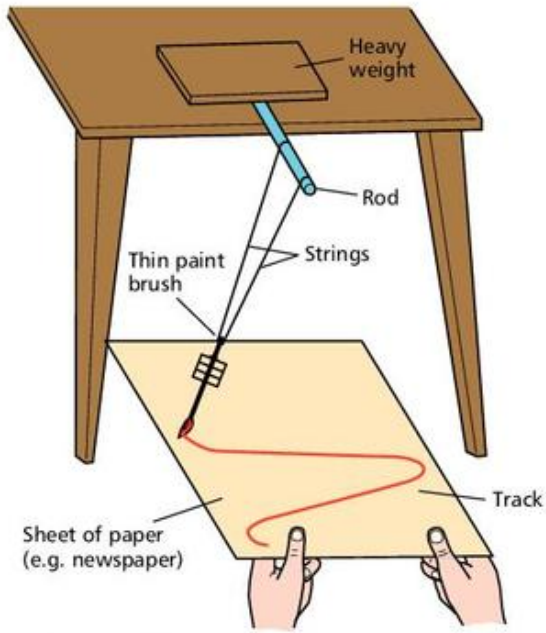


Fig 1 *Investigating oscillations*

The variation of displacement with time is shown in Figure 2 (i). Provided friction is negligible, the amplitude of the oscillations is constant.

The variation of velocity with time is given by the gradient of the displacement v. time graph, as shown by Figure 2 (ii).

- The velocity is greatest where the gradient of the displacement v. time graph is greatest (i.e. at zero displacement when the object passes through equilibrium).
- The velocity is zero where the gradient of the displacement v. time graph is zero (i.e. at maximum displacement).

The variation of acceleration with time is given by the gradient of the velocity v. time graph, as shown by Figure 2 (iii).

- The acceleration is greatest where the gradient of the velocity v. time graph is greatest. This is when the velocity is zero and occurs at maximum displacement.
- The acceleration is zero where the gradient of the velocity v. time graph is zero. This is when the displacement is zero.

By comparing Figure 2 (i) and (iii) directly, it can be seen that

the acceleration is always in the opposite direction to the displacement.

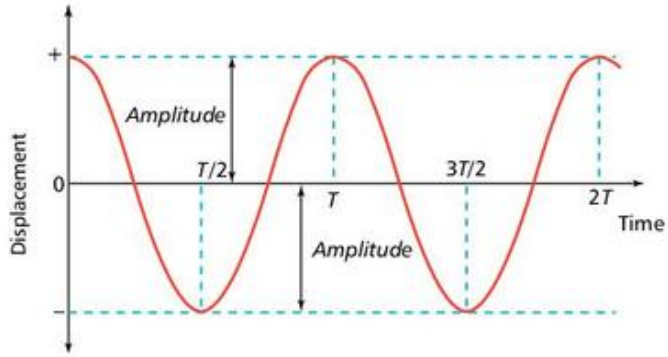


Fig 2 (i) *Displacement v. time*

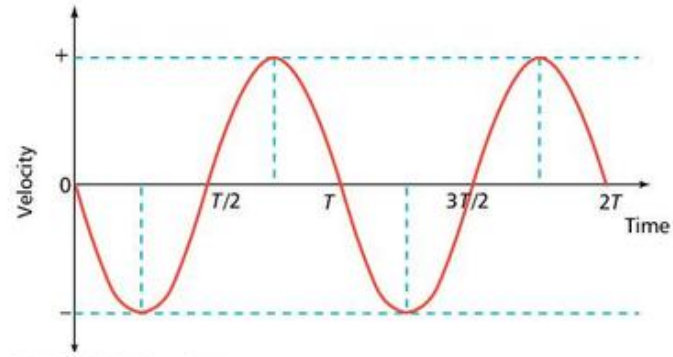


Fig 2 (ii) *Velocity v. time*

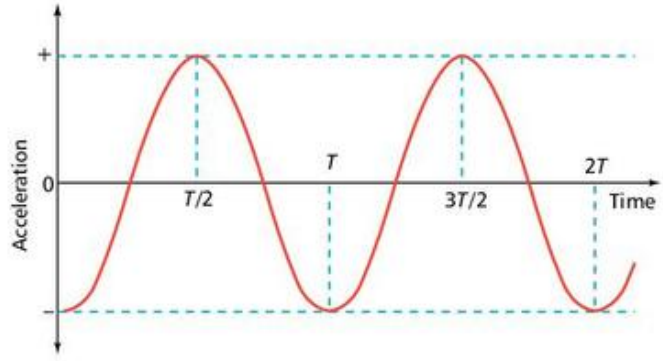


Fig 2 (iii) *Acceleration v. time*

In other words, if one direction is referred to as the positive direction and the other as the negative direction, the acceleration direction is always the opposite sign to the displacement direction.

Simple harmonic motion is defined as oscillating motion in which the acceleration is

1 proportional to the displacement,
2 always in the opposite direction to the displacement.

Acceleration, a = − constant × displacement x

The constant depends on the time period T_P of the oscillations. The shorter the time period, the faster the oscillations which means the larger the acceleration at any given displacement. So the constant is greater the shorter the time period. As shown in topic 12.3, the constant in this equation is ω^2, where the angular frequency, $\omega = \frac{2\pi}{T_P} (= 2\pi f)$.

Therefore the defining equation for simple harmonic motion is

acceleration, $a = -\omega^2 x$, where x = displacement

ω = angular frequency $= \frac{2\pi}{T_P}$

This equation may also be written as

$a = -(2\pi f)^2 x$, where f = frequency

NOTES

1 The time period is independent of the amplitude of the oscillations.
2 Maximum displacement $x_{MAX} = \pm x_0$, where x_0 is the amplitude of the oscillations. Therefore,
- when $x_{MAX} = +x_0$, the acceleration $a = -(2\pi f)^2 x_0$, and
- when $x_{MAX} = -x_0$, the acceleration $a = +(2\pi f)^2 x_0$.

QUESTIONS

1 A small object attached to the end of a vertical spring (Figure 3) oscillates with an amplitude of 25 mm and a time period of 2.0 s. The object passes through equilibrium moving upwards at time $t = 0$. What is the displacement and direction of motion of the object:
a $\frac{1}{4}$ cycle later, **b** $\frac{1}{2}$ cycle later,
c $\frac{3}{4}$ cycle later, **d** 1 cycle later?

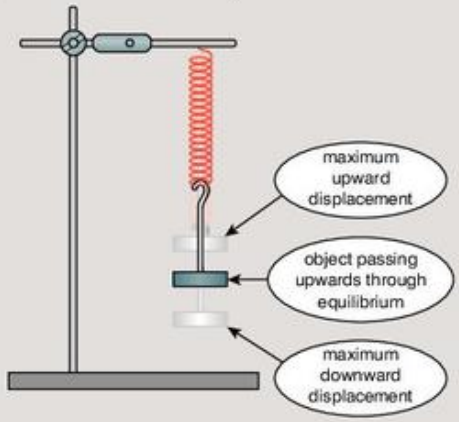


Fig 3

2 For the oscillations in **1**, calculate:
a the frequency,
b the acceleration of the object when its displacement is:
i +25 mm, **ii** 0, **iii** −25 mm.

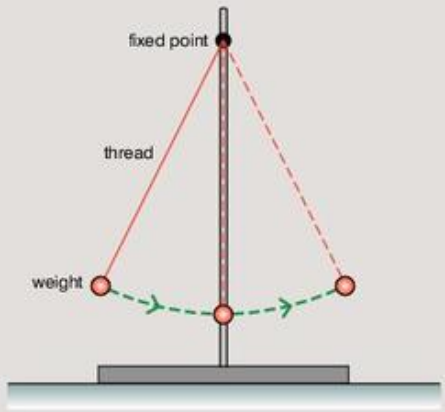


Fig 4

3 A simple pendulum consists of a small weight on the end of a thread. The weight is displaced from equilibrium and released. It oscillates with an amplitude of 32 mm, taking 20 s to execute 10 oscillations. Calculate:
a its frequency,
b its initial acceleration.

4 For the oscillations in **3**, the object is released at time $t = 0$. State the displacement and calculate the acceleration when:
a $t = 1.0$ s,
b $t = 1.5$ s.

12.3 More about sine waves

On these pages you will learn to:

- Recall and use the equation $a = -\omega^2 x$ to solve simple harmonic motion problems
- Recall and use $x = x_0 \sin\omega t$ as a solution to the equation $a = -\omega^2 x$

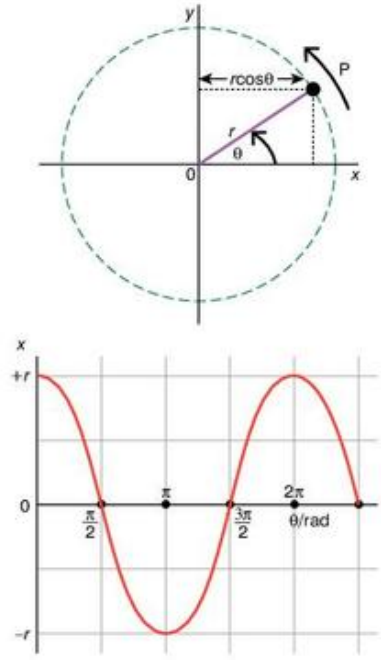


Fig 1 Circles and waves

Circles and waves

Consider a small object P in uniform circular motion, as shown in Figure 1. Measured from the centre of the circle at O, the coordinates of P are therefore $x = r\cos\theta$ and $y = r\sin\theta$, where θ is the angle between the x-axis and the radial line OP. Figure 1 shows how the x-coordinate changes as angle θ changes. The curve is a cosine wave. It has the same shape as the simple harmonic motion curves in topic 12.2 Figure 2.

Oscillating shadows

To see the link between simple harmonic motion and sine curves, consider the motion of the ball and the pendulum bob in Figure 2. A projector is used to cast a shadow of the ball (P) in uniform circular motion on to a screen alongside the shadow of the bob (Q) of a simple pendulum oscillating above the circle. The two shadows keep up with each other exactly when their time periods are matched.

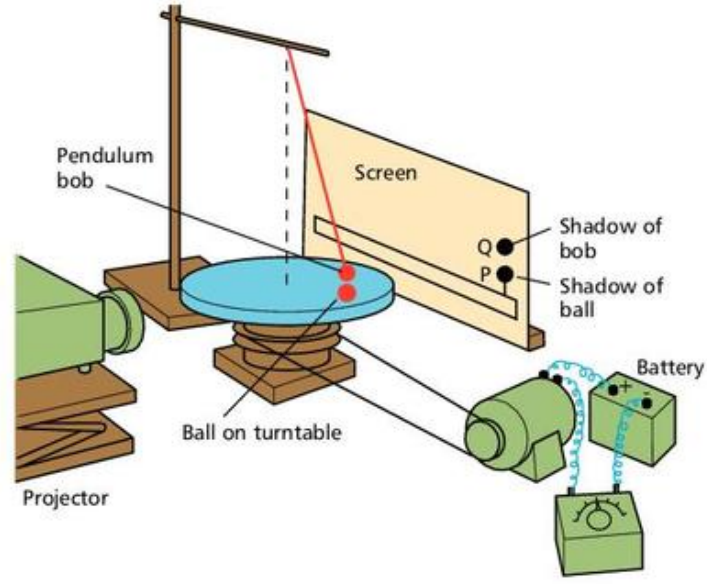


Fig 2 Comparing simple harmonic motion with circular motion

In other words, P and Q at any instant have the same horizontal position and the same horizontal motion. The acceleration of Q is therefore the same as the acceleration of P.

- Because the ball is in uniform circular motion, its acceleration $a = -\frac{v^2}{r}$ where the minus sign indicates its direction towards O. Since speed $v = 2\pi rf$ (see topic 11.1), then $a = -(2\pi f)^2 r$.
- The component of acceleration of the ball parallel to the screen, $a_x = a\cos\theta$, $\therefore$ the acceleration of P, $a_x = -(2\pi f)^2 r\cos\theta = -(2\pi f)^2 x$

Because the bob's motion is the same as the motion of the ball's shadow,

the acceleration of the bob, $a_x = -(2\pi f)^2 x$

This is the defining equation for simple harmonic motion and it shows why the constant of proportionality is $(2\pi f)^2$ or ω^2 where $\omega = 2\pi f$.

Notes

1 The bob oscillates along the x-axis between $x = -r$ and $x = +r$. Its amplitude of oscillation, $x_0 = r$.

2 The displacement of the bob from equilibrium is therefore given by $x = x_0 \cos\theta$, where θ is the angle the ball moves through from its position when $x = x_0$. At time t after the ball passes through this position,

$$\theta \text{ (in radians)} = \frac{2\pi t}{T_P} = 2\pi f t = \omega t$$

Therefore $x = x_0 \cos(2\pi f t) = x_0 \cos \omega t$

Sine wave solutions

For any object oscillating at frequency f in simple harmonic motion, its acceleration a at displacement x is given by

$$a = -(2\pi f)^2 x = -\omega^2 x$$

The variation of displacement with time depends on the initial displacement and the initial velocity (i.e. the displacement and velocity at time $t = 0$). For example:

1 If $x = 0$ when $t = 0$ and the object is moving to maximum displacement $+x_0$, then its displacement at time t is given by $x = x_0 \sin \omega t$. This is how the displacement of the bob's shadow Q varies with time if $t = 0$ is when the ball is nearest the screen.

2 If $x = +x_0$ when $t = 0$ and the object has zero velocity at that instant, then its displacement at time t later is given by $x = x_0 \cos \omega t$. This is how the displacement of Q varies with time if $t = 0$ had been at the instant when Q was at maximum positive displacement.

Notes

1 The shape of the curves in Figure 3, described as sinusoidal, is the same as the shape of the simple harmonic motion curves in Figure 2 in topic 12.2.

2 The general solution is $x = x_0 \sin(\omega t + \varphi)$, where φ is the phase difference between the instants when $t = 0$ and when $x = 0$. The general solution is not required in this physics specification.

3 The time period T_P does not depend on the amplitude of the oscillating motion. For example, the time period of an object oscillating on a spring is the same, regardless of whether the amplitude is large or small.

4 The symbol A is sometimes used for amplitude instead of x_0.

Worked example

A small object on a spring oscillates with a time period of 0.48 s and an amplitude of 15 mm. Its displacement, x, from equilibrium at time t is given by $x = x_0 \cos \omega t$. Calculate:

a its frequency,
b its displacement and acceleration at $t = 0.20$ s.

Solution

a $f = \frac{1}{T_P} = \frac{1}{0.48} = 2.08\,\text{Hz}$

b $x_0 = 15\,\text{mm} = 0.015\,\text{m}$,
$\omega = 2\pi f = 2\pi \times 2.08 = 13.1\,\text{rad s}^{-1}$
$x = 0.015 \cos \omega t = 0.015$
$\cos(13.1 \times 0.20) = -1.30 \times 10^{-2}\,\text{m}$
$a = -\omega^2 x = -(13.1^2 \times -1.30 \times 10^{-2}) = 2.23\,\text{m s}^{-2}$

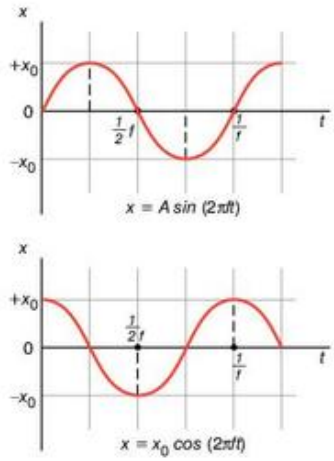


Fig 3 *Graphical solutions*

QUESTIONS

1 An object oscillates in simple harmonic motion with a time period of 3.0 s and an amplitude of 58 mm. Calculate:

a its frequency,

b its maximum acceleration.

2 The displacement of an object oscillating in simple harmonic motion varies with time in accordance with the equation $x/\text{mm} = 12 \cos 10t$, where t is the time in seconds after the object's displacement was at its maximum positive value.

a Determine:
i the amplitude, **ii** the time period.

b Calculate the displacement of the object at $t = 0.1$ s.

3 An object on a spring oscillates with a time period of 0.48 s and a maximum acceleration of $9.8\,\text{m s}^{-2}$. Calculate:

a its frequency,

b its amplitude.

4 An object oscillates in simple harmonic motion with an amplitude of 12 mm and a time period of 0.27 s. Calculate:

a its frequency,

b its displacement and its direction of motion:
i 0.10 s,
ii 0.20 s after its displacement was +12 mm.

12.4 Applications of simple harmonic motion

On these pages you will learn to:

- Describe how to measure the period of the oscillations of a loaded spring and a simple pendulum
- Use the equations for the period of the oscillations of a loaded spring and a simple pendulum to solve problems

For any oscillating object, the resultant force acting on the object acts towards the equilibrium position. The resultant force is described as a restoring force as it always acts towards equilibrium. Provided the restoring force is proportional to the displacement from equilibrium, the acceleration is proportional to the displacement and always acts towards equilibrium. Therefore, the object oscillates with simple harmonic motion.

Investigating an oscillating system

Use two stretched springs and a trolley, as shown in Figure 1. When the trolley is displaced then released, it oscillates backwards and forwards.

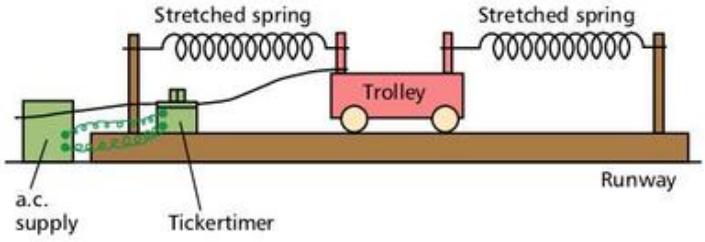


Fig 1 *Investigating oscillations*

- The first half-cycle of the trolley's motion can be recorded using a length of ticker tape attached at one end to the trolley. When the trolley is released, the tape is pulled through a ticker timer that prints dots on the tape at a rate of 50 dots per second.
  A graph of displacement v. time for the first half-cycle can be drawn using the tape, as shown in Figure 2. The graph can be used to measure the time period which can be checked as explained on the next page if the trolley mass m and the combined spring constant k are known.
- A motion sensor linked to a computer can be used to record the oscillating motion of the trolley. (See p.25.)

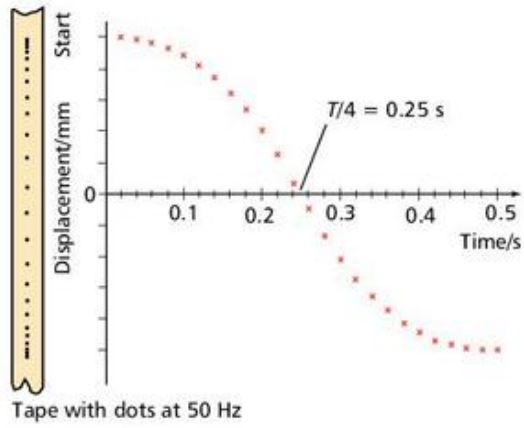


Fig 2 *Displacement–time curve from a tickertape*

What determines the frequency of oscillation of a loaded spring?

In the above investigation, the frequency of oscillation of the trolley can be changed by loading the trolley with extra mass or by replacing the springs with springs of different stiffness. The frequency is reduced by:

1. Adding extra mass. This is because the extra mass increases the inertia of the system. At a given displacement, the trolley would therefore be slower than if the extra mass had not been added. Each cycle of oscillation would therefore take longer.
2. Using weaker springs. The restoring force on the trolley at any given displacement would be less, so the trolley's acceleration and speed at any given displacement would be less than it would have been if the weaker springs had not been added. Each cycle of oscillation would therefore take longer.

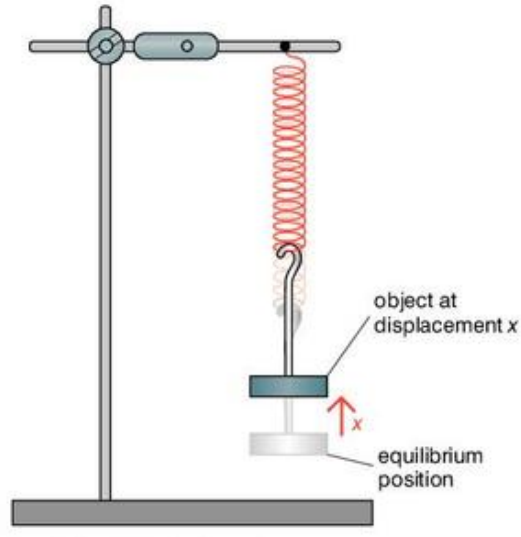


Fig 3 *The oscillations of a loaded spring*

To see exactly how the mass and the spring constant affect the frequency, consider a small object of mass m attached to a spring.

- Assuming the spring obeys **Hooke's law**, the tension T in the spring is in proportion to its extension e from its unstretched length. This relationship can be expressed by means of the equation $T = ke$, where k is the spring constant.
- When the object is oscillating and is at displacement x from its equilibrium position, the change of tension in the spring provides the restoring force on the object. Using the equation $T = ke$, the change of tension ΔT from equilibrium is therefore given by $\Delta T = -kx$, where the minus sign represents the fact that the change of tension always tries to restore the object to its equilibrium position. Hence, the restoring force on the object $= -kx$
- Therefore, the acceleration,

$$a = \frac{\text{restoring force}}{\text{mass}} = -\frac{kx}{m} = -\omega^2 x$$

where $\omega^2 = \dfrac{k}{m}$

The object therefore oscillates in simple harmonic motion with a time period

$$T_P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

Notes

1 The formula for the time period shows that T_P is increased by increasing m (i.e. using a larger mass) or by reducing k (i.e. using a weaker spring).

2 The time period does not depend on g. A mass–spring system on the Moon would have the same time period as it would on Earth.

3 The tension T in the spring varies from $mg + kx_0$ to $mg - kx_0$, where x_0 = amplitude.

Maximum tension is when the spring is stretched as much as possible (i.e. $x = -x_0$)

Minimum tension is when the spring is stretched as little as possible (i.e. $x = +x_0$).

Worked example

$g = 9.8\,\text{m s}^{-2}$

A spring of natural length 300 mm hangs vertically with its upper end attached to a fixed point. When a small object of mass 0.20 kg is suspended from the lower end of the spring in equilibrium, the spring is stretched to a length of 379 mm. Calculate

a i the extension of the spring at equilibrium, **ii** the spring constant,

b the time period of oscillations that the mass on the spring would have if the mass was to be displaced downwards slightly then released.

Solution

a i Extension of spring at equilibrium,
$e_0 = 79\,\text{mm} = 0.0790\,\text{m}$

ii Spring constant $k = mg/e_0 = 0.20 \times 9.8/0.079$
$= 24.8\,\text{N m}^{-1}$

b $T_P = 2\pi\sqrt{\dfrac{0.20}{24.8}} = 0.56\,\text{s}$

Investigating the oscillations of a loaded spring

To verify the formula $T_P = 2\pi\sqrt{\dfrac{m}{k}}$ for a loaded spring,

- measure the time period for at least six different known masses,
- time 20 complete cycles of oscillation three times for each mass to give an average value of the time period for each mass.

A graph of T_P^2 on the vertical axis against m on the horizontal axis should give a straight line through the origin. This is because $T_P^2 = \dfrac{4\pi^2}{k}m$ which means that a graph of T_P^2 against m should be a straight line through the origin with a gradient equal to $\dfrac{4\pi^2}{k}$.

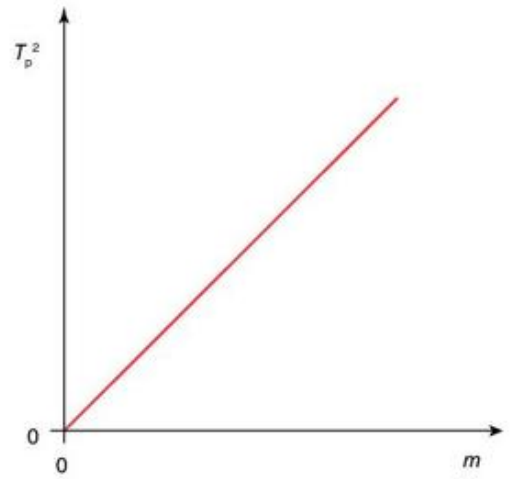


Fig 4 *T_P^2 against m for a loaded spring*

EXTENSION

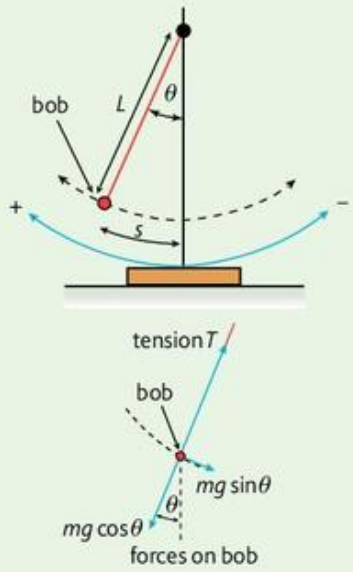


Fig 5 The simple pendulum

The theory of the simple pendulum

Consider a simple pendulum consisting of a bob of mass m attached to a thread of length L, as shown in Figure 5. If the bob is displaced from equilibrium then released, it oscillates about the lowest point. At displacement x, from the lowest point, when the thread is at angle θ to the vertical, the weight, mg, has components

- $mg\cos\theta$ perpendicular to the path of the bob, and
- $mg\sin\theta$ along the path towards the equilibrium position.

The restoring force $F = -mg\sin\theta$, so the acceleration, $a = \frac{F}{m} = -\frac{mg\sin\theta}{m}$

$$= -g\sin\theta$$

Provided θ does not exceed about 10°, then $\sin\theta = \frac{x}{L}$,

therefore the acceleration $a = -\frac{g}{L}x = -\omega^2 x$ where $\omega^2 = \frac{g}{L}$

The object therefore oscillates in simple harmonic motion with a time period

$$T_P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

Notes

1 The length of the pendulum is the distance from the point of support to the centre of the bob.
2 As the bob passes through equilibrium, the tension T acts directly upwards. At this instant, the difference between the tension in the thread and the weight of the bob is equal to the centripetal force on the bob. In other words, $T - mg = mv^2/L$, where v is the speed as it passes through equilibrium.

Investigating the simple pendulum

To verify the formula $T_P = 2\pi\sqrt{\frac{L}{g}}$ for a simple pendulum:

- Measure the time period for at least six different measured lengths.
- Time 20 complete cycles of oscillation three times for each length to give an average value of the time period for each length. Ensure the angle between the thread and the vertical does not exceed 10°.
- Measure the length from the point of suspension of the thread to the centre of the spherical bob.

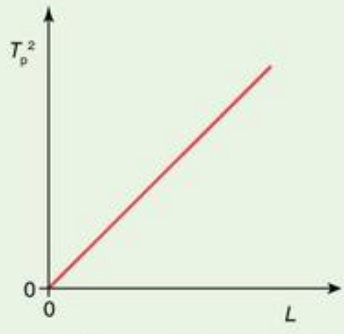


Fig 6 T_p^2 against L for a simple pendulum

A graph of T_p^2 on the vertical axis against L on the horizontal axis should give a straight line through the origin. This is because $T_P^2 = \frac{4\pi^2}{g}L$ which means that a graph of T_p^2 against L should be a straight line through the origin with a gradient $\frac{4\pi^2}{g}$. Therefore g can be calculated from $g = \frac{4\pi^2}{\text{gradient}}$

QUESTIONS

$g = 9.8\,\text{m s}^{-2}$

1 An object was suspended from the end of a vertical spring and set into oscillating motion along a vertical line. The amplitude of its oscillations was 20 mm and it took 6.5 s to perform 20 oscillations. Calculate:

a **i** its time period,
ii its frequency,

b its acceleration when its displacement was:
i 0 mm, **ii** 10 mm, **iii** 20 mm.

2 In the arrangement described in **1**, the object was replaced by an object of different mass.

When the second object was oscillating vertically, its acceleration, a, at displacement x was given by $a = -360x$.

a Calculate:
i the frequency,
ii the time period of the oscillations.

b By comparing the frequency of oscillation of the second object with that of the first object, discuss whether or not the mass of the second object is greater than or less than the mass of the first object.

3 The upper end of a vertical spring of natural length 250 mm is attached to a fixed point. When a small object of mass 0.15 kg is attached to the lower end of the spring, the spring stretches to an equilibrium length of 320 mm.

a Calculate:
i the extension of the spring at equilibrium,
ii the spring constant.

b When the object oscillates vertically, its acceleration, a, at displacement x is given by $a = -\left(\frac{k}{m}\right)x$, where m is the mass of the object and k is the spring constant of the spring.
i Show that the object oscillates at a frequency of 1.9 Hz.
ii Calculate the time period of the oscillations.

4 An object of mass 0.50 kg is attached to the lower end of a vertical spring which has a spring constant of $25\,\text{N m}^{-1}$. The mass is displaced downwards from equilibrium by a distance of 50 mm then released. Calculate:

a **i** the force on the object at a displacement of 50 mm,
ii the acceleration of the object at the instant it was released.

b **i** Show that the acceleration, a, at displacement x is given by $a = -50x$.
ii Calculate the frequency of the oscillations and the displacement of the mass 0.50 s after it was released.

5 Calculate the time period of a simple pendulum

a of length **i** 1.0 m, **ii** 0.25 m

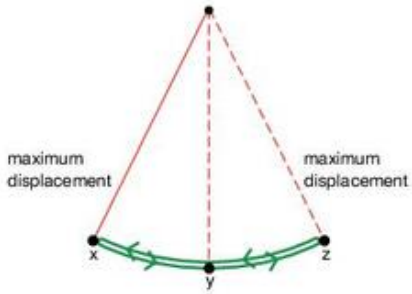
b Calculate the time period of a simple pendulum of length 1.0 m if it was on the surface of the Moon where $g = 1.6\,\text{m s}^{-2}$.

6 A simple pendulum and a mass suspended on a vertical spring have equal time periods on the Earth. Discuss whether or not they would have the same time periods on the surface of the Moon where $g = 1.6\,\text{m s}^{-2}$.

12.5 Energy and simple harmonic motion

On these pages you will learn to:

- Describe the interchange between kinetic and potential energy during simple harmonic motion
- Recognise and use the equations $v = v_0 \cos \omega t$ and $v = \pm \omega\sqrt{(x_0^2 - x^2)}$ for free oscillations
- Explain what is meant by damped oscillations and give practical examples of different degrees of damping including critical damping



Position	E_p	E_k
x	E_{TOTAL}	O
y	O	E_{TOTAL}
z	E_{TOTAL}	O

Fig 1 *The energy changes of a simple pendulum*

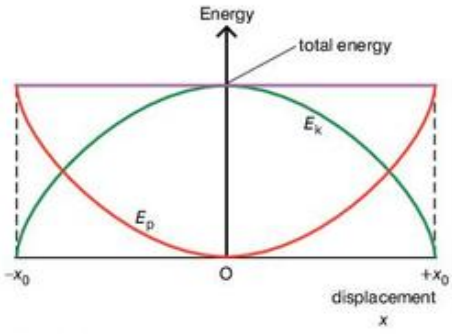


Fig 2 *Energy curves*

Free oscillations

A freely oscillating object oscillates with a constant amplitude because there is no friction acting on it. The only forces acting on it combine to provide the restoring force. If friction was present, the amplitude of oscillations would gradually decrease and the oscillations would eventually cease.

Observe the oscillations of a simple pendulum over many cycles and you should find that the decrease of amplitude from one cycle to the next is scarcely measurable. Nevertheless, over many cycles the amplitude does decrease noticeably. So, friction is present, even if its effect is insignificant over a single cycle.

Consider the example of a mass oscillating on a spring. The energy of the system changes from kinetic energy to potential energy and back again every half-cycle after passing though equilibrium. Provided friction is absent, the total energy of the system is constant and is equal to its maximum potential energy.

1 The potential energy changes with displacement x, in accordance with the equation below. See topic 5.4 if necessary.

$$E_p = \tfrac{1}{2}kx^2$$

2 The total energy, E_{TOTAL}, of the system is, therefore, $\tfrac{1}{2}kx_0^2$, where x_0 is the amplitude of the oscillations.

3 The kinetic energy of the oscillating mass, $E_k = \tfrac{1}{2}k(x_0^2 - x^2)$ $(= E_{TOTAL} - E_p)$

The SHM speed equation

Using $E_k = \tfrac{1}{2}mv^2$ gives $\tfrac{1}{2}mv^2 = \tfrac{1}{2}k(x_0^2 - x^2)$ where v is the speed of the object at displacement x.

As $\omega^2 = \dfrac{k}{m}$, the above equation can be written as $v^2 = \omega^2(x_0^2 - x^2)$

Hence $$v = \pm\omega\sqrt{(x_0^2 - x^2)} \qquad \text{where } \omega = 2\pi f$$

Note that making $x = 0$ in this equation gives the maximum speed $v_{max} = 2\pi f x_0$

Energy v. displacement graphs

- The potential energy curve is parabolic in shape, given by $E_p = \tfrac{1}{2}kx^2$
- The kinetic energy curve is an inverted parabola, given by $E_k = \tfrac{1}{2}k(x_0^2 - x^2)$.
- The sum of the kinetic energy and the potential energy is always equal to $\tfrac{1}{2}kx_0^2$ which is the potential energy at maximum displacement. This is the same as the kinetic energy at zero displacement. So the two curves add together to give a flat line for the total energy.

Damped oscillations

The oscillations of a simple pendulum gradually die away because air resistance gradually reduces the total energy of the system. In any oscillating system where friction or air resistance is present, the amplitude decreases. The forces causing the amplitude to decrease are described as **dissipative forces** because they dissipate the energy of the system to the surroundings as thermal energy. The motion is said to be **damped** if dissipative forces are present. Figure 3 shows how the displacement of a lightly-damped oscillating system decreases with time. Note that:

- The amplitude gradually decreases, reducing by the same fraction each cycle.

- The time period is independent of the amplitude so each cycle takes the same length of time as the oscillations die away.

The greater the damping, the faster the amplitude decreases. For example, the amplitude of a mass on a spring oscillating in air decreases gradually because the damping is very light. However, the same mass oscillating on the same spring in oil would be subjected to much greater dissipative forces. The amplitude of its oscillations would decrease much more rapidly than in air.

Where the damping is strong enough, an object displaced from equilibrium and released returns to equilibrium without oscillating. The damping is said to be **critical** if the object returns to equilibrium in the shortest possible time without oscillating. If the damping is greater than critical, the time taken to return to equilibrium is greater.

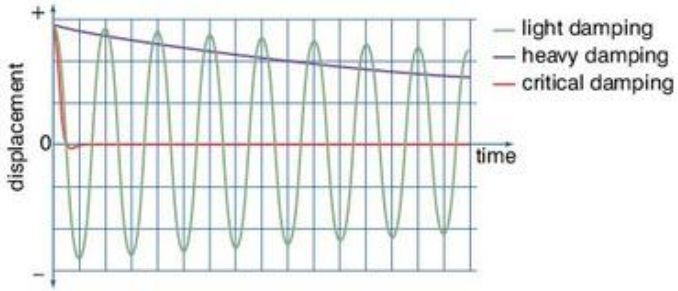


Fig 3 Damping

Practical examples

1 **The suspension system of a car** consists of a coiled spring near each wheel between the wheel axle and the car chassis. When the wheel is jolted, for example on a bumpy road, the spring smoothes out the force of the jolts. An oil damper fitted with each spring prevents the chassis from bouncing up and down too much.

Without oil dampers, the occupants of the car would continue to be thrown up and down until the oscillations died away. The flow of oil through valves in the piston of each damper provides a frictional force which damps the oscillating motion of the chassis. The dampers are designed to ensure the chassis returns to its 'equilibrium' position in the shortest possible time after each jolt with little or no oscillations. The system is therefore at or near critical damping.

2 **Swing doors** need to be damped to prevent them swinging back on people. Imagine walking through a swing door without dampers; if you walk too slowly through it, it might bounce back and hit you as you are walking away from it. A damper fitted to the door slows its motion and stops it from swinging back. The damper needs to provide heavier than critical damping so the door closes slowly.

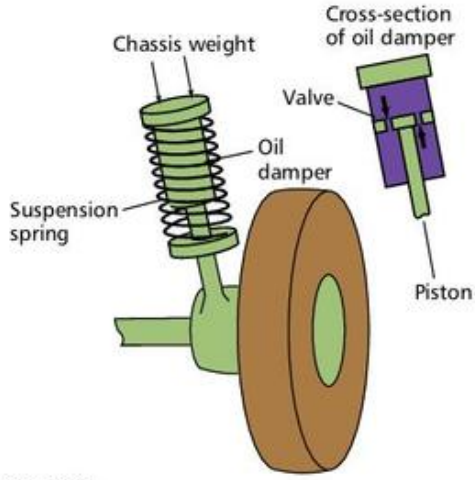


Fig 4 Car suspension

QUESTIONS

1 a Describe the energy changes of a simple pendulum oscillating in air during one cycle of oscillation after it passes through equilibrium.

b Sketch graphs on the same axes to show how the potential energy and the kinetic energy of a freely oscillating object vary with its displacement from equilibrium.

2 A glider of mass 0.45 kg on a frictionless air track is attached to two stretched springs at either end, as shown in Figure 5. A force of 3.0 N is needed to displace the glider from equilibrium and hold it at a displacement of 50 mm. The glider is then released and it oscillates freely on the air track. Calculate:

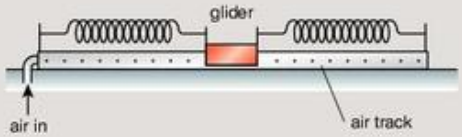


Fig 5

a the spring constant k for the system,

b i the initial potential energy of the system when the glider is held at a displacement of 50 mm,

ii the maximum kinetic energy of the glider.

3 a In each of the following examples, describe the energy changes after the instant of release:

i A child on a swing displaced from equilibrium then released.

ii Water in a U-shaped tube displaced from equilibrium then released.

b Discuss how effective a car suspension damper would be if the oil in the damper was replaced by oil that was much more viscous.

4 The amplitude of an oscillating mass on a spring decreases by 4% each cycle from an initial amplitude of 100 mm. Calculate the amplitude after:

a 5 cycles of oscillation,

b 20 cycles of oscillation.

12.6 Forced oscillations and resonance

On these pages you will learn to:

- Describe practical examples of forced oscillations and resonance
- Draw and describe graphs to show how the amplitude of a forced oscillation changes with frequency near to the natural frequency of the system
- Show an understanding of the factors that determine the frequency response and sharpness of the resonance
- Describe examples of circumstances in which resonance should be avoided

Imagine pushing someone on a swing at regular intervals. If each push is timed suitably, the swing goes higher and higher. The pushes are a simple example of a **periodic force** which is a force applied at regular intervals.

- When the system oscillates without a periodic force being applied to it, its frequency is referred to as its **natural frequency**.
- When a periodic force is applied to an oscillating system, the response depends on the frequency of the periodic force. The system undergoes **forced oscillations** when a periodic force is applied to it.

Investigating forced oscillations

Figure 1 shows how a periodic force can be applied to an oscillating system consisting of a mass attached to two stretched springs.

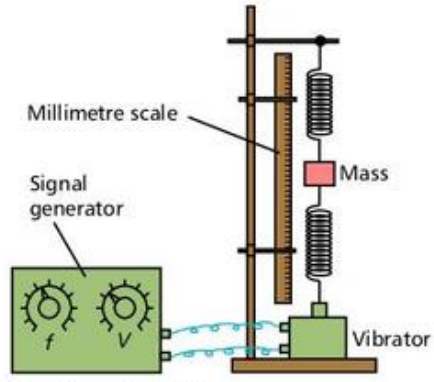


Fig 1 Forced oscillations

The bottom end of the lower spring is attached to a mechanical oscillator which is connected to a signal generator. The top end of the upper spring is fixed. The mechanical oscillator pulls repeatedly on the lower spring at a frequency that can be adjusted by adjusting the signal generator. The frequency of the oscillator is referred to as the **applied frequency**. The response of the system is measured from the amplitude of oscillations of the mass.

The variation of the response with the applied frequency is shown in Figure 2.

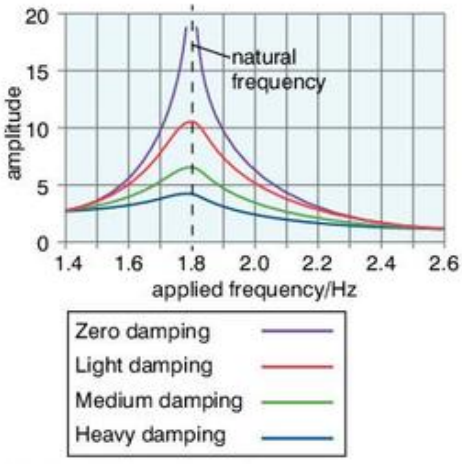


Fig 2 Resonance curves

For each curve, the frequency of the periodic force at the peak amplitude is **the resonant frequency**. As the damping becomes less and less:

- the frequency at which resonance occurs tends towards the natural frequency (which is at 1.80 Hz in the diagram above),
- the curve becomes sharper in that the system becomes more responsive to change of frequency near the peak.

Consider the effect of increasing the applied frequency from zero:

1 As the applied frequency approaches the natural frequency of the mass–spring system:
   - the amplitude of oscillations of the object increases more and more,
   - the phase difference between the displacement and the periodic force increases from zero to $\frac{1}{2}\pi$ at the natural frequency.

2 When the applied frequency is equal to the natural frequency of the mass–spring system:
   - the amplitude of oscillations becomes very large. The lighter the damping in the system, the larger the amplitude becomes. The system is said to be in resonance when the applied frequency is equal to the natural frequency.
   - the phase difference between the displacement and the periodic force is $\frac{1}{2}\pi$ at resonance. The periodic force is then exactly in phase with the velocity of the oscillating object.

3 As the applied frequency becomes increasingly larger than the natural frequency of the mass–spring system:
   - the amplitude of oscillations decreases more and more,
   - the phase difference between the displacement and the periodic force increases from $\frac{1}{2}\pi$ until the displacement is π radians out of phase with the periodic force.

The amplitude of oscillations is greatest when the applied frequency is equal to the natural frequency, provided the damping is light.

For an oscillating system with little or no damping, at resonance,

the applied frequency of the periodic force = the natural frequency of the system

1 At resonance, the periodic force acts on the system at the same point in each cycle, causing the amplitude to increase to a maximum value limited only by damping. At maximum amplitude, energy supplied by the periodic force is lost at the same rate because of the effects of damping. So the system oscillates with a constant amplitude at resonance.
2 The applied frequency at resonance, the resonant frequency, is equal to the natural frequency only when there is little or no damping. Resonance occurs at a lower frequency than the natural frequency if the damping is not light. The lighter the damping, the closer the resonant frequency is to the natural frequency.

More examples of resonance

Barton's pendulums

Figure 3 shows five simple pendulums P, Q, R, S and T, of different lengths hanging from a supporting thread which is stretched between two fixed points. A single 'driver' pendulum, D, of the same length as one of the other pendulums is also tied to the thread.

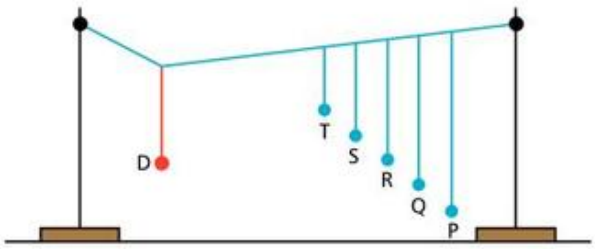


Fig 3 Barton's pendulums

The driver pendulum, D, is displaced and released so it oscillates in a plane perpendicular to the plane of the pendulums at rest. The effect of the oscillating motion of D is transmitted along the support thread, subjecting each of the other pendulums to forced oscillations. Pendulum R responds much more than any other pendulum. This is because it has the same length and, therefore, the same time period as D. So its natural frequency is the same as D's natural frequency. Therefore, R oscillates in resonance with D because it is subjected to forced oscillations of the same frequency as its own natural frequency of oscillation. The response of each of the other pendulums depends on how close its length is to the length of D.

Bridge oscillations

A bridge span can oscillate due to its 'springiness' and its mass. If a bridge span is not fitted with 'dampers', it can be made to oscillate at resonance if subjected to a suitable periodic force.

1 A cross-wind can cause a periodic force on a bridge span because of eddy currents created by the wind along the bridge span. If the wind speed is such that the frequency of the periodic force is equal to the natural frequency of the bridge span, resonance can occur in the absence of damping. The dramatic collapse of the Tacoma Narrows Bridge in the United States in 1940 was due to such resonance.

Fig 4 The collapse of the Tacoma Narrows Bridge

2 A steady trail of people in step with each other walking across a footbridge can cause resonant oscillations of the bridge span if there is insufficient damping. Soldiers marching in columns are taught to break step when crossing a footbridge to avoid causing resonance. Shortly after it was opened, the Millennium Bridge in London had to be closed and fitted with a suitable damping system because it swayed in resonance when people first walked across it.

Fig 5 The Millennium Bridge, London

QUESTIONS

1 A mass suspended on a vertical spring is made to resonate by applying a periodic force of frequency, f_0.

a Explain what is meant by resonance.

b Explain why the frequency of the periodic force needs to be f_0 to cause resonance.

2 With reference to the mass–spring system shown in Figure 1, state and explain what the effect would be on the resonant frequency of:

a increasing the mass,

b replacing the springs with stiffer springs.

3 The panel of a certain washing machine vibrates loudly when the drum rotates at a certain frequency. Explain why this happens only when the drum rotates at this frequency.

4 A vehicle has a suspension system that is lightly damped. When it is driven without passengers at a certain speed over speed bumps spaced 15 m apart, the vehicle chassis bounces up and down violently.

a Explain why this effect happens.

b Discuss whether or not the effect would happen if:

i the vehicle had been travelling at a different speed,

ii the vehicle had been carrying passengers.

CHAPTER SUMMARY

Measurements

1 For an oscillating object:

- its **amplitude** is its maximum displacement from equilibrium
- its **time period** is the time taken for one complete cycle of oscillations
- its **frequency** is the number of cycles of oscillation per unit time.

2 For two objects oscillating with the same time period T_P,

$$\textbf{their phase difference, in radians, } = \frac{2\pi\Delta t}{T_P}$$

where Δt is the time between successive instants when the two objects are at maximum displacement in the same direction.

Simple harmonic motion

Definition:

An object oscillates in simple harmonic motion if its acceleration is:

1 proportional to the displacement of the object from equilibrium
2 always directed towards equilibrium

Equation:

Acceleration, $a = -\omega^2 x$, where x = displacement from equilibrium, and ω = angular frequency of oscillations $= \frac{2\pi}{T_P}$

Solutions:

1 $x = x_0 \sin \omega t$ if $x = 0$ and the object is moving in the + direction at time $t = 0$,
2 $x = x_0 \cos \omega t$ and the object is at maximum displacement, $+x_0$, at time $t = 0$,
3 Speed at displacement x, $v = \omega\sqrt{(x_0^2 - x^2)}$

Applications

1 Time period of an oscillating mass m on the end of a vertical spring $= 2\pi\sqrt{\frac{m}{k}}$, where k is the spring constant.
2 Time period of a simple pendulum of length $L = 2\pi\sqrt{\frac{L}{g}}$

Resonance condition

Frequency of the periodic force = natural frequency of the system, provided the damping is negligible.

12 End of chapter questions

1 In a test of an undamped suspension spring of a car, the chassis above the wheel is pushed down by 80 mm then released. It undergoes 5 complete cycles of oscillation in 7.0 seconds.

a Calculate:
i the frequency of oscillation,
ii the acceleration of the chassis when its displacement is +80 mm.

b Discuss the effect on the oscillating motion if the test had been carried out with the damper fitted.

2 **a** Describe the energy changes of an oscillating simple pendulum that is lightly damped.

b A simple pendulum is displaced 50 mm from equilibrium then released.
It takes 35 s to complete 20 oscillations. Calculate:
i the frequency of oscillation of the pendulum,
ii its initial acceleration.

3 Two identical simple pendulums X and Y with a time period of 2.0 s oscillate with an amplitude of 60 mm. When X is at maximum displacement, Y is at zero displacement and moving towards X.

a State the phase difference between X and Y.

b Sketch graphs to show how the displacement:
i of X, and
ii of Y changes during the next 4.5 seconds.

4 The diagram below shows a vehicle accelerometer that consists of a steel ring attached to springs on a horizontal slider.

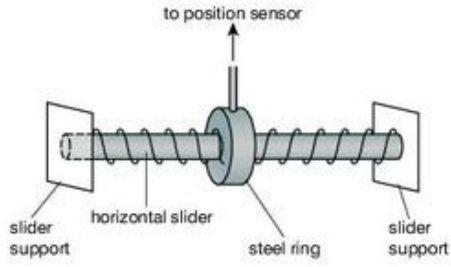


Fig 4.1

a **i** Explain why the ring is displaced from equilibrium when the vehicle accelerates.
ii Describe how the displacement of the ring from equilibrium depends on the magnitude of the acceleration.

b Describe the motion of the ring when the vehicle brakes sharply and stops.

c What would be the effect on the accelerometer if:
i the spring was replaced by a weaker spring,
ii the ring was replaced by a ring with more mass?

5 An object of mass 0.15 kg is suspended from the lower end of a spring, as shown below.

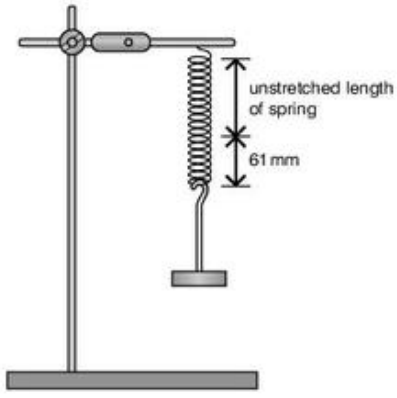


Fig 5.1

a When the object is in equilibrium, the spring is extended by 61 mm from its unstretched length. Calculate the spring constant of the spring.

b When the object is displaced vertically downwards and then released, it oscillates in simple harmonic motion. Its acceleration, a, at displacement x is given by $a = -\left(\frac{k}{m}\right)x$, where m is the mass of the object and k is the spring constant of the spring. Calculate:
i the frequency,
ii the time period of these oscillations.

6 The pendulum bob of a simple pendulum is displaced from equilibrium by a distance of 36 mm with the thread taut then released. It performs 20 oscillations in 70 s. Its displacement at time t after being released may be represented by the equation $x = x_0 \cos(2\pi ft)$.

a What do x_0 and f represent in this equation?

b State the values of x_0 and f.

c Calculate the displacement of the object 1.0 s after it was released.

d Calculate the maximum acceleration of the pendulum bob.

7 When a new car was tested, one of the metal panels of the car body vibrated loudly when the engine ran at a certain speed. When the engine speed was increased or decreased, the vibrations stopped.

a Explain why the panel vibrated loudly at a certain engine speed.

b A design engineer suggested the panel could be replaced by a thicker panel to eliminate the vibrations. Discuss whether or not this is a reasonable suggestion.

8 A floor-mounted diesel engine is used to drive an alternating current generator in a power station. To prevent vibrations from the engine shaking the building, the engine was mounted on a spring suspension system fitted with dampers, as shown below.

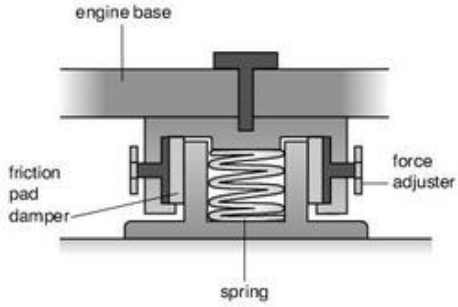


Fig 8.1

a **i** Describe how the vibrations of the engine are absorbed in this system.

ii Explain why the friction pads become warm when the engine is running.

b At a certain engine speed, vibrations reached certain parts of the building until the force of the friction pads in the suspension system was increased. Explain why this stopped the vibrations.

9 A vertical peg is attached to the edge of a horizontal disc of radius r, as shown in Figure 9.1.

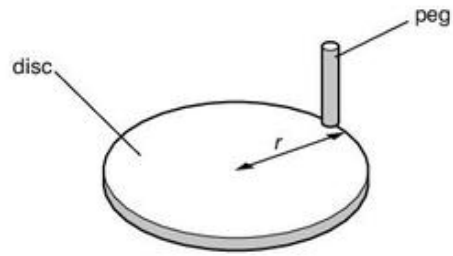


Fig 9.1

The disc rotates at constant angular speed ω. A horizontal beam of parallel light produces a shadow of the peg on a screen, as shown in Figure 9.2.

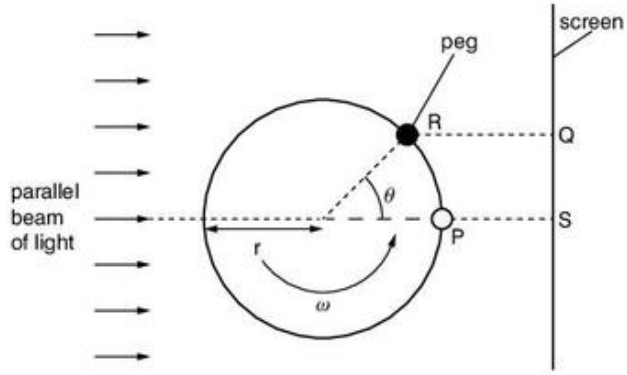


Fig 9.2 (plan view)

At time zero, the peg is at P, producing a shadow on the screen at S.

At time t, the disc has rotated through angle θ. The peg is now at R, producing a shadow at Q.

a Determine,

i in terms of ω and t, the angle θ, *(1 mark)*

ii in terms of ω, t and r, the distance SQ. *(1 mark)*

b Use your answer to **a ii** to show that the shadow on the screen performs simple harmonic motion. *(2 marks)*

c The disc has radius r of 12 cm and is rotating with angular speed ω of 4.7 rad s^{-1}.

Determine, for the shadow on the screen,

i the frequency of oscillation, in Hz *(2 marks)*

ii its maximum speed, in cm s^{-1}. *(2 marks)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 4 Q4 June 2009

10 A student sets up the apparatus illustrated in Figure 10.1 in order to investigate the oscillations of a metal cube suspended on a spring.

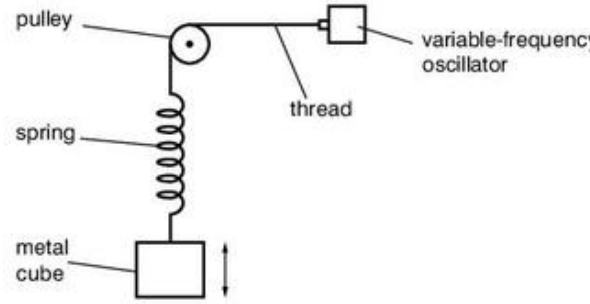


Fig 10.1

The amplitude of the vibrations produced by the oscillator is constant.

The variation with frequency of the amplitude of the oscillations of the metal cube is shown in Figure 10.2.

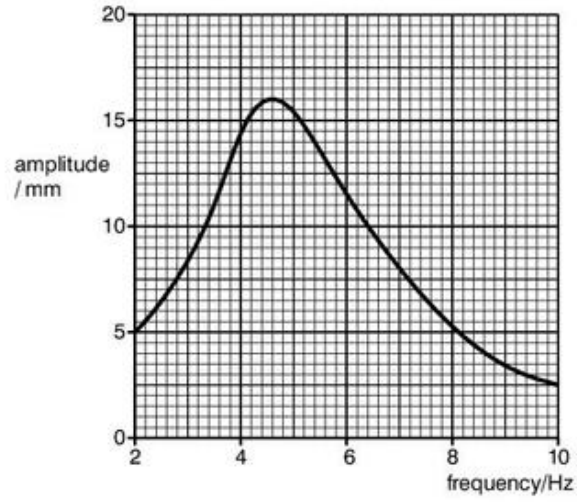


Fig 10.2

a i State the phenomenon illustrated in Figure 10.2. *(1 mark)*

ii For the maximum amplitude of vibration, state the magnitudes of the amplitude, in mm, and the frequency, in Hz. *(1 mark)*

b The oscillations of the metal cube of mass 150 g may be assumed to be simple harmonic.

Use your answers in **a ii** to determine, for the metal cube,

i its maximum acceleration, in $m\,s^{-2}$ *(3 marks)*

ii the maximum resultant force on the cube in N. *(2 marks)*

c Some very light feathers are attached to the top surface of the cube so that the feathers extend outwards, beyond the vertical sides of the cube.

The investigation is now repeated.

On a copy of Figure 10.2, draw a line to show the new variation with frequency of the amplitude of vibration for frequencies between 2 Hz and 10 Hz. *(2 marks)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 41 Q3 November 2010

13 Gravitational fields

13.1 Gravitational field strength

On these pages you will learn to:

- Recognise a gravitational field as an example of a field of force
- Define gravitational field strength, g, as force per unit mass
- Use the equation $g = F/m$
- Recognise that on and near the surface of the Earth g is approximately constant and is called the acceleration of free fall

Fig 1 Earth in space

'What goes up must come down', or must it? Throw a ball into the air and it returns to you because of the Earth's gravity. The force of gravity on the ball pulls it back to Earth. The force of attraction between the ball and the Earth is an example of gravitational attraction which exists between any two masses. It isn't obvious that there is a force of attraction between you and any object near you, but it is true. Any two masses exert a gravitational pull on one another. But the force is usually too weak to notice unless at least one of the masses is very large.

The mass of an object creates a force field around itself. Any other mass placed in the field is attracted towards the object. The second mass also has a force field around itself and this pulls on the first object with an equal force in the opposite direction. The force field around a mass is called a **gravitational field**.

If a small mass is placed close to a massive body, the small mass and the body attract each other with equal and opposite forces. However, this force is too small to move the massive body noticeably. The small mass, assuming it is free to move, is pulled by the force towards the massive body. The path which the smaller mass would follow is called a **field line** or, sometimes, **a line of force**. Figure 2 shows the field lines near a planet. The lines are directed to the centre of the planet as a small object released near the planet would fall towards its centre.

The strength of a gravitational field, g, is the force per unit mass on a small test mass placed in the field.

The test mass needs to be small, otherwise it might pull so much on the other object that it changes its position and alters the field. In general, the force on a small mass in a gravitational field varies from one position to another. If a small test mass, m, is at a certain position in a gravitational field where it is acted on by a gravitational force F, the gravitational field strength at that position is given by

$$g = \frac{F}{m}$$

The unit of gravitational field strength is the newton per kilogram ($N\,kg^{-1}$). For example, the gravitational field strength of the Earth at the surface of the Earth is $9.8\,N\,kg^{-1}$.

Free fall in a gravitational field

The weight of an object is the force of gravity on it. If an object of mass m is in a gravitational field, the gravitational force on the object $F = mg$, where g is the gravitational field strength at the object's position. If the object is not acted on by any other force, it accelerates with an acceleration given by

$$\text{acceleration } a = \frac{\text{force}}{\text{mass}} = \frac{mg}{m} = g.$$

The object therefore falls freely with acceleration, g. Thus, g is also described as the acceleration of a freely falling object. See topic 1.5 for more about the measurement of g.

An object that falls freely is unsupported. Although the object in this situation is commonly described as being weightless, it is better to describe it as 'unsupported' as it is acted on by the force of gravity alone.

Field patterns

1 **A radial field** is where the field lines are like the spokes of a wheel, always directed to the centre. Figure 2 shows an example of a radial field. The force of gravity on a small mass near a much larger spherical mass is always directed to the centre of the larger mass. For example, the force on a small object near a spherical planet always acts towards the centre of the planet, regardless of the position of the object. The magnitude of g in a radial field decreases with increased distance from the massive body.

2 **A uniform field** is where the gravitational field strength is the same in magnitude and direction throughout the field. The field lines are therefore parallel to one another.

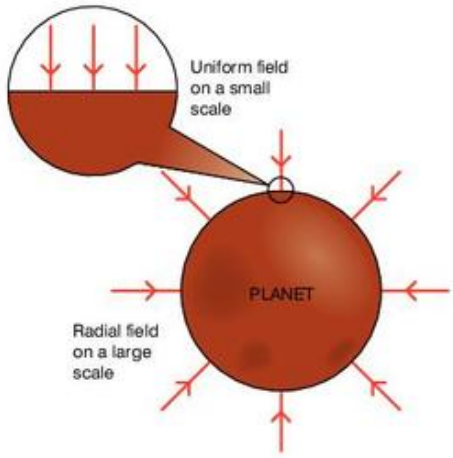


Fig 2 *Field patterns*

Is the Earth's gravitational field uniform or radial? The force of gravity due to the Earth on a small mass decreases with distance from the Earth so the gravitational field strength of the Earth falls with increasing distance from the Earth. The field is therefore radial. However, over small distances which are much less than the Earth's radius, the change of gravitational field strength is insignificant. In other words, over such small distances, the acceleration of free fall, g, (i.e. gravitational field strength) is constant. For example, the measured value of g has the same magnitude ($= 9.8\,\text{N kg}^{-1}$) and direction (downwards) 100 m above the Earth as it has on the surface. In theory, g is smaller higher up, but the difference is too small to be noticeable – provided we don't go too high! Only over distances which are small compared with the Earth's radius can the Earth's field be considered uniform.

QUESTIONS

1 **a** What is meant by a field line or a line of force of a gravitational field?

b With the aid of a diagram in each case, explain what is meant by:
i a radial field, **ii** a uniform field.

2 **a** Calculate the gravitational force on:
i an object of mass 3.5 kg in a gravitational field at a position where $g = 9.5\,\text{N kg}^{-1}$,
ii an object of mass 100 kg in a gravitational field at a position where $g = 1.6\,\text{N kg}^{-1}$.

b Calculate the gravitational field strength at a position in a gravitational field where:
i an object of mass 2.5 kg experiences a force of 40 N,
ii an object of mass 18 kg experiences a force of 72 N.

3 Show that the acceleration of an object falling freely in a gravitational field is equal to g, where g is the gravitational field strength at that position.

4 Figure 3 represents a small part of the Earth's surface. Sketch the lines of force near this part of the Earth's surface:

a if the density of the Earth in this part is uniform,

b if there is a large mass of dense matter under this part of the surface.

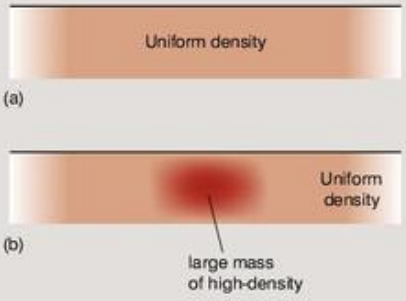


Fig 3

*

13.2 Gravitational potential

On these pages you will learn to:

- Define gravitational potential at a point as the work done in bringing unit mass from infinity to the point
- Explain what is meant by an equipotential
- Solve problems using the equation $\Delta\phi = \Delta W/m$

Fig 1 Into space

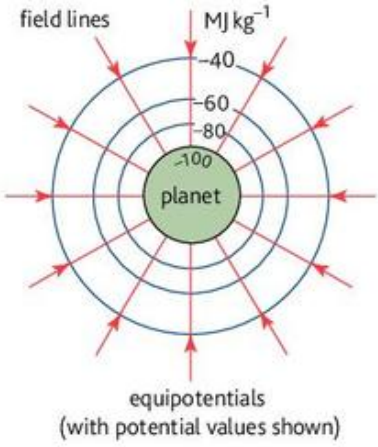


Fig 2 Equipotentials near a planet

Working in a gravitational field

Gravitational potential energy is the energy of an object due to its position in a gravitational field. The position for zero gravitational potential energy is at infinity – in other words, the object would be so far away that the gravitational force on it is negligible. A rocket climbing out of a planet's gravitational field needs to increase its gravitational potential energy to zero to escape completely. At the surface, its gravitational potential energy was negative so it needs to do work to escape from the field completely.

The gravitational potential at a point is defined as the work done per unit mass to move a small object from infinity to that point. In this book, the symbol for gravitational potential is ϕ (pronounced 'phi'). Note that ϕ is a scalar quantity.

For a small object of mass m at a position where the gravitational potential is ϕ, the work W that must be done on the object to enable it to escape completely is given by $W = m\phi$. Rearranging this gives

$$\phi = \frac{W}{m}$$

Note that the unit of gravitational potential is J kg^{-1}.

Suppose a rocket has a 'payload' mass of 1000 kg and the gravitational potential at the surface of the planet is -100 MJ kg^{-1}. Assume the fuel is used quickly to boost the rocket to high speed. For the rocket to escape completely, the gravitational potential energy of the 1000 kg payload must increase from -100×1000 MJ to zero. So the work done on the payload must be at least 100 000 MJ to escape. If the rocket payload is only given 40 000 MJ of kinetic energy from the fuel, then it can only increase its gravitational potential energy by 40 000 MJ. So it can only reach a position in the field where the gravitational potential is -60 MJ kg^{-1}.

Note

In general, if a small object of mass m is moved from gravitational potential ϕ_1 to gravitational potential ϕ_2, its change of gravitational potential energy $\Delta E_P = m(\phi_2 - \phi_1) = m\Delta\phi$ where $\Delta\phi = (\phi_2 - \phi_1)$.

As the work done ΔW to move it from ϕ_1 to ϕ_2 is equal to the change of its gravitational potential energy, then $\Delta W = m\Delta\phi$

Remember that $\Delta E_P = mg\Delta h$ can only be applied for values of Δh which are very small compared with the Earth's radius. $\Delta E_P = m\Delta\phi$ can always be applied.

Equipotentials

Equipotentials are lines of constant potential. Hillwalkers ought to know all about equipotentials since a map contour line is a line of constant potential. A contour line joins points of equal height above sea level. So a hillwalker following a contour line has constant potential energy. Sensible hillwalkers take great care where the contour lines are very close to one another. One slip and their gravitational potential energy might fall dramatically!

The equipotentials near a spherical planet are circles as shown in Figure 2. At increasing distance from the surface, the gravitational field becomes weaker so

the gain of gravitational potential energy per metre of height gain becomes less. In other words, away from the surface, the equipotentials for equal increases of potential are spaced further apart.

However, near the surface over a small region, the equipotentials are horizontal (i.e. parallel to the ground) as shown in Figure 3. This is because the gravitational field over a small region is uniform. A 1 kg mass raised from the surface of the Earth by 1 m gains 9.8 J of gravitational potential energy; if it is raised another 1 m, it gains another 9.8 J. So its gravitational potential energy rises by 9.8 J for every metre of height it gains above the surface.

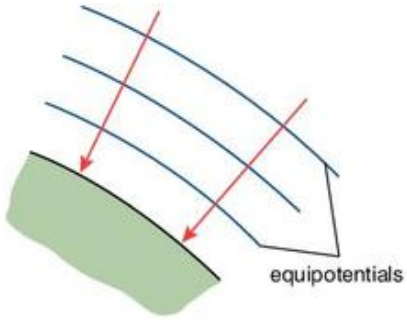


Fig 3 Equipotentials near a surface

QUESTIONS

$g = 9.8\,\text{N kg}^{-1}$

1 **a** Calculate the gain of gravitational potential energy of an object of mass 12 kg when its centre of mass is raised through a height of 2.0 m.

b Show that the gravitational potential difference between the Earth's surface and a point 2.0 m above the surface is $19.6\,\text{J kg}^{-1}$.

2 A rocket of mass 35 kg launched from the Earth's surface gains 70 MJ of gravitational potential energy when it reaches its maximum height.

a Calculate the gravitational potential difference between the Earth's surface and the highest point reached by the rocket.

b The gravitational potential of the Earth's gravitational field at the surface of the Earth is $-63\,\text{MJ kg}^{-1}$. Calculate: **i** the gravitational potential at the highest point reached by the rocket, **ii** the work that would need to have been done by the rocket to escape from the Earth's gravitational field.

3 Figure 4 shows the equipotentials near a non-spherical object.

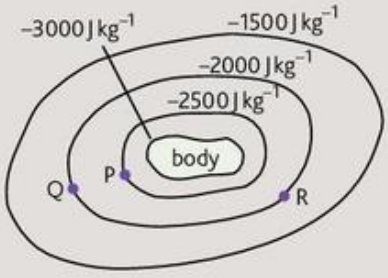


Fig 4

a Calculate the gravitational potential energy of a 0.1 kg object at **i** P, **ii** Q, **iii** R

b How much work must be done on the object to move it from **i** P to Q, **ii** Q to R?

4 Figure 5 shows equipotentials at a spacing of 1.0 km near a planet. The point labelled X is on the $-500\,\text{kJ kg}^{-1}$ equipotential.

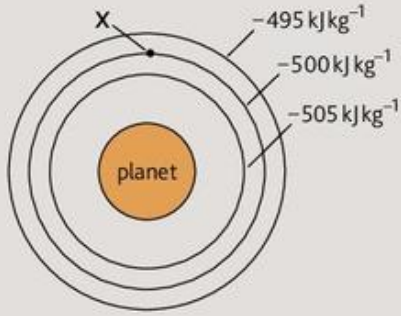


Fig 5

a Show that the work done to move a 1 kg object from X to a position 10 m higher is 50 J.

b Hence calculate the gravitational field strength at X.

c Calculate the work that would need to be done to remove an object of mass 50 kg from X to infinity.

13.3 Newton's law of gravitation

On these pages you will learn to:

- Recall and use Newton's law of gravitation in the form $F = Gm_1m_2/r^2$
- Understand that, for a point outside a uniform sphere, the mass of the sphere may be considered to be a point mass at its centre

We owe our understanding of gravitation to Isaac Newton. 'The notion of gravity was occasioned by the fall of an apple!', said Newton when asked what made him develop the idea of gravity. Newton's theory of gravitation was an enormous leap forward because it explains events from the 'down-to-earth' falling apple to the motion of the planets. Like any good theory, it can be used to make predictions; for example, the return of a comet and its exact path can be calculated using the law of gravitation.

Newton realised that gravity is universal. Any two masses exert a force of attraction on each other. He knew about the careful measurements of planetary motion made by astronomers such as Johannes Kepler. Forty or more years before Newton established the theory of gravitation, Kepler had shown that the motion of the planets was governed by a set of laws. Kepler had measured the motion of each planet and had shown that each planet orbits the Sun. The measurements that he made for each planet were its time period T (i.e. the time for one complete orbit of the Sun) and the average radius r of its orbit. He showed that the value of $\frac{r^3}{T^2}$ was the same for all the planets. This is known as Kepler's third law.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn
Average radius r of orbit/10^{10} m	6	11	15	23	78	143
Time T for one orbit/10^7 s	0.8	1.95	3.2	5.9	37.4	93.0
$r^3/T^2/10^{16}$ m^{3}s^{-2}	337	350	330	349	340	338

Fig 1 *Kepler's third law*

To explain Kepler's third law, Newton started by assuming that the planets and the Sun were point masses. A scale model of the Solar System with the Sun represented by a marble would put the Earth about a metre away, represented by a grain of sand! Newton assumed that the force of gravitation between a planet and the Sun varied inversely with the square of their distance apart. In other words, if the force is F at distance d apart, then

- at distance $2d$ apart, the force is $\frac{F}{4}$,
- at distance $3d$ apart, the force is $\frac{F}{9}$,
- at distance $4d$ apart, the force is $\frac{F}{16}$.

Using this **inverse-square law of force**, Newton was able to prove that $\frac{r^3}{T^2}$ was the same for all the planets. The actual proof is outlined in topic 13.5. Newton then went on to use the inverse-square law of force to explain and make predictions for many other events involving gravity.

Newton's law of gravitation assumes that the gravitational force between any two point objects is

- always an attractive force,
- proportional to the mass of each object,
- proportional to $\frac{1}{r^2}$, where r is their distance apart.

These three requirements can be summarised as

$$\text{gravitational force } F = -\frac{Gm_1m_2}{r^2}$$

where m_1 and m_2 = masses of the two objects. The minus sign in the equation is because the force is always an attractive force.

The constant of proportionality, G, in the above equation, is called the **universal constant of gravitation**. The unit of G can be worked out from $F = -\frac{Gm_1m_2}{r^2}$; rearranged, the equation gives $G = -\frac{Fr^2}{m_1m_2}$. So G can be given units of $\text{N m}^2\text{kg}^{-2}$.

The value of G is $6.67 \times 10^{-11}\,\text{N m}^2\text{kg}^{-2}$.

Work out for yourself the gravitational force between two point masses, each of mass 10 kg at 0.1 m apart. The values of m_1, m_2 and r must be put into the equation in units of kilograms and metres. The force of gravitational attraction works out at 6.7×10^{-7} N which is far too small to notice except with extremely sensitive equipment. Only if one of the masses is very large does the force become noticeable, unless special techniques are used, as described later.

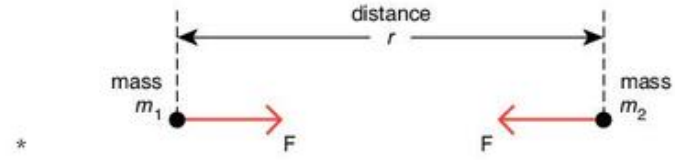


Fig 2 *Newton's law of gravitation*

Worked example

The distance from the centre of the Sun to the centre of the Earth is 1.5×10^{11} m. The mass of the Sun is 2.0×10^{30} kg and the mass of the Earth is 6.0×10^{24} kg.

a The Earth has a diameter of 1.3×10^{7} m. The Sun has a diameter of about 1.4×10^{9} m. Why is it reasonable to consider the Sun and the Earth at a distance of 1.5×10^{11} m apart as point masses on this distance scale?

b Calculate the force of gravitational attraction between the Sun and the Earth.

$G = 6.67 \times 10^{-11}\,\text{N m}^2\text{kg}^{-2}$.

Solution

a On a scale model where the centre of the Sun was 1 m away from the centre of the Earth, the Sun would be a sphere of diameter about 1 cm and the Earth would be a sphere of diameter about 0.1 mm, no larger than a dot. The distance from the Earth to any part of the Sun is, therefore, the same to within 1%.

b $$F = -\frac{6.67 \times 10^{-11} \times 2.0 \times 10^{30} \times 6.0 \times 10^{24}}{(1.5 \times 10^{11})^2}$$
$$= -3.6 \times 10^{22}\,\text{N}$$

Cavendish's measurement of G

The first accurate measurement of G was made by Henry Cavendish in 1798. He devised a torsion balance made of two small lead balls at either end of a rod. The rod was suspended horizontally by a torsion wire, as in Figure 3. The wire was calibrated by measuring the couple required to twist it per degree. Then, with the rod at rest in equilibrium, two massive lead balls were brought near the torsion balance to make the wire twist. By measuring the angle which it twisted through, the force of attraction between each massive lead ball and the small ball nearest to it was calculated. The distance between the centres of the small and large masses was also measured. Then G was calculated using the equation for the law of gravitation.

Fig 3 *Cavendish's measurement of G*

QUESTIONS

$G = 6.67 \times 10^{-11}\,\text{N m}^2\text{kg}^{-2}$

1 a Calculate the force of gravitational attraction between two 'point objects' of masses 60 kg and 80 kg at a distance of 0.5 m apart.

b Calculate the distance between two identical point objects, each of mass 0.20 kg, that exert a force of 9.0×10^{-8} N on each other.

2 a Calculate the force of gravitational attraction between the Earth and an object of mass 80 kg on the surface of the Earth where $g = 9.8\,\text{N kg}^{-1}$.

b Use the result of your calculation in **a** to estimate the mass of the Earth. Assume that the mass of the Earth is concentrated at its centre.

The radius of the Earth = 6.4×10^{6} m.

3 The Sun exerted a force of 6.0 N on a 1000 kg comet when it was at a distance of 1.5×10^{11} m from the Sun. Calculate the force due to the Sun on the comet when it was at a distance of:

a 0.5×10^{11} m from the Sun,

b 7.5×10^{11} m from the Sun.

4 A space rocket of mass 1500 kg travelled from the Earth to the Moon, a distance of 3.8×10^{8} m.

a When the space rocket was mid-way between the Earth and the Moon, calculate the force of gravitational attraction on it:
i due to the Earth
ii due to the Moon.

b Calculate the magnitude and direction of the force of gravity of the Earth and the Moon on the space rocket when it was mid-way between the Earth and the Moon.

The mass of the Earth = 6.0×10^{24} kg; the mass of the Moon = 7.4×10^{22} kg

13.4 Planetary fields

On these pages you will learn to:

- Derive the equation $g = \frac{GM}{r^2}$ for the gravitational field strength of a point mass
- Recall and solve problems using the above equation
- Solve problems using the equation $\phi = -\frac{GM}{r}$ for the potential in the field of a point mass

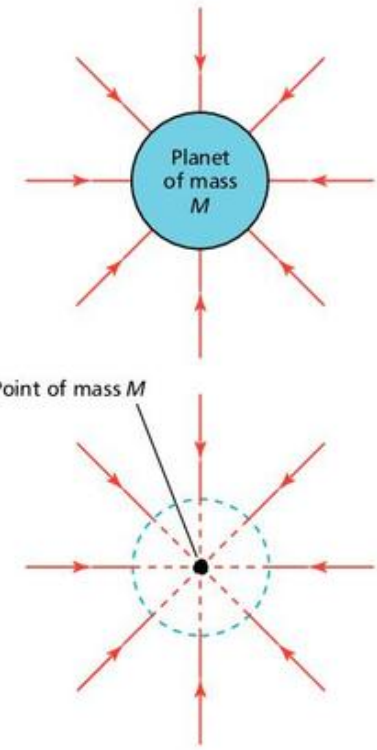


Fig 1 *Comparing fields*

Gravitational field strength near a spherical planet

The law of gravitation can be used to determine the gravitational field strength at any point in the field of a planet or any other spherical mass. Newton showed that the field of a spherical mass is the same as if the mass were concentrated at its centre. The field lines of a spherical mass are always directed towards the centre, so the field pattern is just the same as for a point mass, as shown in Figure 1.

- For a point mass M, the force of attraction on a 'test mass' m (where $m \ll M$) at distance r from M is given by Newton's law of gravitation $F = \frac{GMm}{r^2}$
  Therefore, the gravitational field strength at distance r is given by $g = \frac{F}{m} = \frac{GM}{r^2}$
- For a spherical mass M of radius R, the force of attraction on a 'test mass' m at distance r **from the centre of M** is the same as if mass M was concentrated at its centre. Therefore,
  the force of attraction between m and M, $F = \frac{GMm}{r^2}$
  Therefore, the gravitational field strength at distance r is given by $g = \frac{F}{m} = \frac{GM}{r^2}$,
  provided distance r is greater than or equal to the radius R of the sphere.

  Gravitational field strength, $g = \frac{GM}{r^2}$

  at distance r from a point object or the centre of a sphere of mass M

Note The gravitational field strength at the surface of a sphere of radius R and mass M is given by, $g_S = \frac{GM}{R^2}$ as the distance from the centre $r = R$ at the surface.

Worked example

$G = 6.67 \times 10^{-11}\,\text{N m}^2\text{kg}^{-2}$

The gravitational field strength at the surface of the Earth is $9.8\,\text{N kg}^{-1}$. Calculate:

a the mass of the Earth,
b the gravitational field strength of the Earth at a height of 1000 km above the surface.

The radius of the Earth = 6400 km.

Solution

a Rearranging $g_S = \frac{GM}{r^2}$ gives $M = \frac{g_S R^2}{G} = \frac{9.8 \times (6400 \times 10^3)^2}{6.67 \times 10^{-11}} = 6.0 \times 10^{24}\,\text{kg}$

b At height $h = 1000\,\text{km}$, $r = R + h = 7400\,\text{km}$

$$\therefore g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{(7400 \times 10^3)^2} = 7.3\,\text{N kg}^{-1}$$

The variation of *g* with distance from the centre of a spherical planet (or star)

1 At and beyond the surface of a spherical planet of mass M and radius R,

$$g = \frac{GM}{r^2},$$

where r is the distance from the centre of the sphere

Because the surface gravitational field strength, $g_S = \frac{GM}{r^2}$, then $GM = g_S R^2$

Therefore, $\boldsymbol{g = g_S \frac{R^2}{r^2}}$

The equation shows how g changes with increase of distance r.

- At distance $r = 2R$, $g = \frac{g_S R^2}{(2R)^2} = \frac{g_S}{4}$
- At distance $r = 3R$, $g = \frac{g_S R^2}{(3R)^2} = \frac{g_S}{9}$
- At distance $r = 4R$, $g = \frac{g_S R^2}{(4R)^2} = \frac{g_S}{16}$

Figure 2 shows how g varies with distance r. The shape of the curve beyond $r = R$ is an 'inverse-square law' curve.

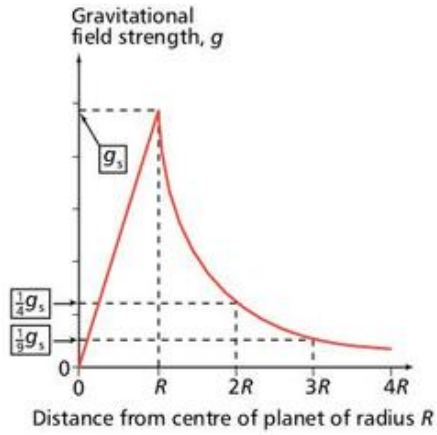


Fig 2 *Gravitational field strength*

2 Inside the planet

From the equation $g = \frac{GM}{r^2}$, you might think that g inside the Earth becomes ever larger and larger as r becomes smaller and smaller. However, inside the planet, only the mass in the sphere of radius r contributes to g. The rest of the mass outside r up to the surface gives no resultant force. So, as r becomes smaller, the mass M which contributes to g becomes smaller too. At the centre, the mass that contributes to g is zero. So g is zero at the centre. Figure 2 also shows how g varies with distance from the centre inside the planet.

Gravitational potential near a spherical planet

At or beyond the surface of a spherical planet, the gravitational potential ϕ at distance r from the centre of the planet of mass M is given by

$$\boldsymbol{\phi = -\frac{GM}{r}}$$

Applying this formula to the surface of the Earth with $M = 6.0 \times 10^{24}$ kg and $r = 6.4 \times 10^7$ m gives a value of $-63\,\text{MJ kg}^{-1}$. This means that 63 MJ of work needs to be done to remove a 1 kg mass from the surface of the Earth to infinity.

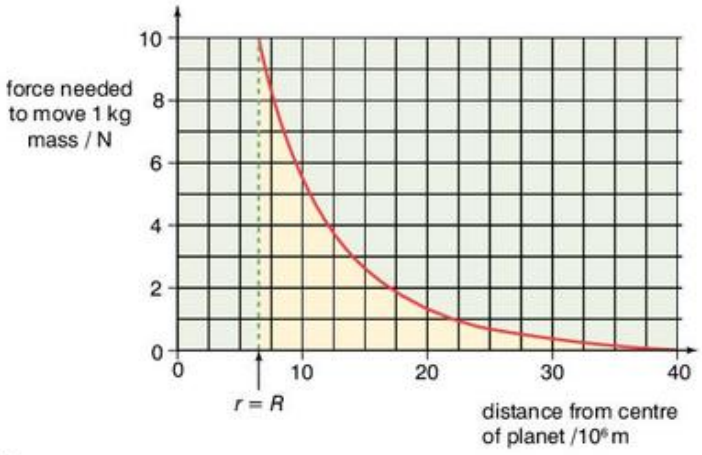


Fig 3 *Work done*

Figure 3 shows how the force of gravity on a 1 kg mass varies with distance r from the centre of the Earth. As explained previously, the mathematical equation for this curve is $g = g_S R^2/r^2$. The area under the curve represents the work done to move the 1 kg mass from infinity to the surface.

- Each grid square in Figure 3 represents a 1 N force acting for a distance of 2.5×10^6 m and therefore represents 2.5 MJ ($= 1\,\text{N} \times 2.5 \times 10^6\,\text{m}$) of work done.

- The work done to move the 1 kg mass from infinity to the surface can therefore be estimated by counting the number of grid squares under the curve and multiplying this number by 2.5 MJ.

Counting part-filled squares that are half-filled or more as wholly-filled squares and neglecting part-filled squares that are less than half-filled, show for yourself that this method gives an estimate for the work done which is very close to the value of 63 MJ determined on p.203.

In effect, the area method used above is an application of 'work done = force × distance moved' with a variable force $F = GMm/r^2$ and $m = 1\,\text{kg}$.

Consider one small step in moving the 1 kg mass from infinity to the surface. Suppose the distance from the centre of the Earth changes from r_1 to r_2 in making this small step of distance Δr.

The work done ΔW to make this small step is given by

$$\Delta W = F\Delta r = \frac{GMm\Delta r}{r^2} \qquad \text{where } \Delta r = r_2 - r_1$$

In Figure 3, the work done ΔW in making each small step $\Delta r = 2.5 \times 10^6\,\text{m}$ is represented by each column of grid squares under the curve. Thus the total work done to move from infinity to the surface is given by the total area under the curve.

Proof of the formula for gravitational potential $\phi = -GM/r$

Although the 'proof' below is not required in the specification, it is provided to give some further insight into the use of maths in physics.

The change of potential $\Delta\phi$ for the small step (from $r_1 = r$ to $r_2 = r - \Delta r$) $= \phi_2 - \phi_1$, where ϕ_2 is the potential at r_2 and ϕ_1 is the potential at r_1.

$$\Delta\phi = \frac{\Delta W}{m} = \frac{F\Delta r}{m} = -\frac{GM\Delta r}{r^2}$$

where the minus sign indicates a decrease in r.

As $$\frac{1}{r_1} - \frac{1}{r_2} = \frac{r_2 - r_1}{r_1 r_2} = \frac{(r - \Delta r) - r}{r(r - \Delta r)} = -\frac{\Delta r}{r^2} \qquad \text{provided } \Delta r \ll r$$

then $$\Delta\phi = \phi_2 - \phi_1 = GM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Hence $$\phi_2 = -\frac{GM}{r_2} \qquad \text{and} \qquad \phi_1 = -\frac{GM}{r_1}$$

Therefore, in general, the potential ϕ at distance r from the centre of a planet is given by $\phi = -\dfrac{GM}{r}$

The variation of gravitational potential with distance from the centre of a spherical planet

The gravitational potential ϕ near a spherical planet is inversely proportional to the distance r from the centre of the planet, as given by the equation $\phi = -\dfrac{GM}{r}$ where M is the mass of the planet. Figure 4 shows how the gravitational potential of the Earth varies with distance. Note the potential curve is a $1/r$ curve not an inverse square (i.e. $1/r^2$) curve like the field strength curve in Figure 3. So the potential:

- at distance $2R$ from the centre is 0.50 × the potential at distance R from the centre,
- at distance $3R$ from the centre is 0.33 × the potential at distance R from the centre,

- at distance $4R$ from the centre is $0.25 \times$ the potential at distance R from the centre, etc.

The gradient of the potential curve at any point is equal to $-g$, where g is the gravitational field strength at the point.

Consider moving a small mass m a distance Δr away from the planet.

The work done on $m = m\Delta\phi$ where $\Delta\phi$ is the change of potential.

The force applied to m = work done ÷ distance moved in the direction of the force

$$= m\frac{\Delta\phi}{\Delta r}$$

As the gravitational force F is equal and opposite to the applied force, $F = -m\frac{\Delta\phi}{\Delta r}$

Hence $$g = \frac{F}{m} = -\frac{\Delta\phi}{\Delta r} = -\text{gradient of the potential curve.}$$

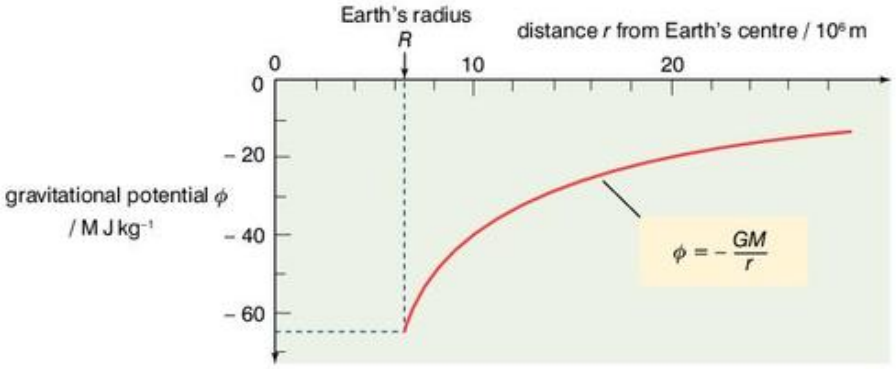


Fig 4 *Gravitational potential near the Earth*

QUESTIONS

$G = 6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$

1 The Moon has a radius of 1740 km and its surface gravitational field strength is $1.62\,\text{N kg}^{-1}$ to 3 significant figures.

a Calculate the mass of the Moon.

b The Moon's gravitational pull on the Earth causes the ocean tides. Show that the gravitational pull of the Moon on the Earth's oceans is approximately 3 millionths of the gravitational pull of the Earth on its oceans. The distance from the Earth to the Moon = 3.8×10^8 m.

2 The Sun has a mass of 2.0×10^{30} kg and a mean radius of 1.4×10^9 m. Calculate:

a its gravitational field strength at:

i its surface,

ii the Earth's orbit which is at a distance of 1.5×10^{11} m from the Sun.

b The Earth has a mass of 6.0×10^{24} kg. Show that the gravitational field strength of the Earth is equal and opposite to the gravitational field strength of the Sun at a distance of 260000 km from the centre of the Earth.

3 The tip of the tallest mountain on the Earth, Mount Everest, is 9 km above sea level. The mean radius of the Earth to the nearest kilometre is 6378 km.

a Calculate the difference between the gravitational field strength of the Earth at sea level and the top of Mount Everest.

b Discuss if it is reasonable to assume that the Earth's gravitational field is uniform between the surface and a height of 10 km above the surface.

c Calculate the gain of potential energy of a mountaineer of mass 80 kg who travels to the top of the mountain from sea level.

4 Use the data in Q1 to calculate the gravitational potential at the surface of the Moon and hence calculate the work done to launch a 500 kg rocket from the surface so it escapes from the Moon's gravitational field.

13.5 Satellite motion

On these pages you will learn to:

- Analyse circular orbits in inverse square law fields by relating the gravitational force to the centripetal acceleration it causes
- Use the equation $r^3/T^2 = GM/4\pi^2$ for circular orbits
- Explain what is meant by a geostationary orbit

Fig 1 Space station in orbit

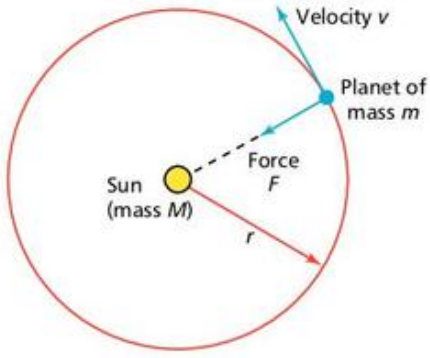


Fig 2 Explanation of planetary motion

On any clear night you ought to be able to see satellites passing overhead in the night sky. Although they are pinpoints of light, they are noticeable because they move steadily through the constellations. Websites supply information to enable you to identify some of them from their directions. However, **satellite motion** is not confined to artificial satellites orbiting the Earth. Any small mass which orbits a larger mass is a satellite. The Moon is the Earth's only natural satellite. Mars has two moons, *Phobos* and *Deimos*. Jupiter has more than 60 satellites including the four innermost satellites, *Io*, *Callisto*, *Ganymede* and *Europa*, first observed by Galileo four centuries ago.

From Kepler to Newton

Newton knew that the time period, T, of a planet orbiting the Sun depends on the mean radius, r, of the orbit in accordance with Kepler's third law,

$$\frac{r^3}{T^2} = \text{constant.}$$

He realised that the force of gravitational attraction between each planet and the Sun is the centripetal force that keeps the planet on its orbit. By assuming the gravitational force is given by $\frac{GMm}{r^2}$, the gravitational field strength, $\frac{GM}{r^2}$, is therefore equal to the centripetal acceleration $\frac{v^2}{r}$, where M is the mass of the Sun, m is the mass of the planet, r is the radius of the orbit and v is the speed of the planet.

Therefore, the speed of the planet is given by $v^2 = \frac{GM}{r}$

Because speed, $v = \frac{\text{circumference of the orbit}}{\text{time period}} = \frac{2\pi r}{T}$

then $\frac{(2\pi r)^2}{T^2} = \frac{GM}{r}$

Rearranging this equation gives $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$

As $\frac{GM}{4\pi^2}$ is the same for all the planets, then $\frac{r^3}{T^2}$ is the same for all the planets.

So, by assuming the force of attraction F varies with distance according to the inverse-square law (i.e. $F \propto \frac{1}{r^2}$), Newton was able to prove Kepler's third law.

Newton's theory not only explains Kepler's laws, but it also allows the mass M to be calculated if the value of G is known. The Earth orbits the Sun once per year on a circular orbit of radius 1.5×10^{11} m so you can prove for yourself that the value of r^3/T^2 for any planet is $3.4 \times 10^{18}\,\text{m}^3\,\text{s}^{-2}$. Given $G = 6.67 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$, show that the mass of the Sun is 2.0×10^{30} kg.

Geostationary satellites

A geostationary satellite orbits the Earth directly above the equator and has a time period of exactly 24 hours. It therefore remains in a fixed position above the equator because it has exactly the same time period as the Earth's rotation.

The radius of orbit of a geostationary satellite can be calculated as follows using the equation $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$.

$$T = 24 \text{ hours} = 24 \times 3600\,\text{s} = 86400\,\text{s}$$

$$r^3 = \frac{GM}{4\pi^2}T^2 = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times (86400)^2}{4\pi^2} = 7.6 \times 10^{22}\,\text{m}^3$$

$$\therefore \quad r = 4.23 \times 10^7\,\text{m} = 42300\,\text{km}$$

The radius of the Earth is 6400 km. Therefore, the height of a geostationary satellite above the Earth is 36000 km (= 42300 − 6400 km to 2 significant figures).

CHAPTER SUMMARY

Definitions

Gravitational field strength, g, is the force per unit mass on a small test mass placed in the field.

Gravitational potential, ϕ, at a point, is the work done per unit mass to move a small object from infinity to that point.

A line of force or **a field line** is the line followed by a small mass acted on by no other forces than the force of gravity.

A uniform field exists in a region where g is the same in magnitude and direction everywhere in the region.

Equations

1 $g = \frac{F}{m}$, where F is the gravitational force on a small mass m.

$\phi = \frac{W}{m}$ where W is the work done to move a small mass m from infinity.

2 Newton's law of gravitation; the gravitational force F between two point masses m_1 and m_2 at distance r apart is given by $F = \frac{Gm_1m_2}{r^2}$

3 At distance r from a point mass M,

$$g = \frac{GM}{r^2} \qquad \phi = -\frac{GM}{r}$$

4 At or beyond the surface of a sphere of mass M,

$$g = \frac{GM}{r^2}$$

where r is the distance to the centre.

$$\phi = -\frac{GM}{r}$$

5 At the surface of a sphere of mass M and radius R,

$$g_S = \frac{GM}{R^2}$$

6 For a satellite in a circular orbit, its centripetal acceleration $\frac{v^2}{r} = g$

QUESTIONS

$G = 6.67 \times 10^{-11}\,\text{N m}^2\text{kg}^{-2}$

The radius of the Earth = 6400 km, $g = 9.8\,\text{N kg}^{-1}$ at the Earth's surface.

1 a Two satellites X and Y are seen from the ground crossing the night sky at the same time. Satellite X crosses the sky faster than Y. State with a reason which satellite is higher.

b Explain why satellite TV dishes must be aligned carefully so they always point to the same position above the equator.

2 A space probe moving at a speed of $3.2\,\text{km s}^{-1}$ is in a circular orbit about a planet of mass M. The time period of the satellite is 110 minutes. Calculate:

a the radius of the orbit,

b the centripetal acceleration of the satellite,

c the mass of the planet.

3 a A satellite moves at speed v in a circular orbit of radius r.

i Write down an expression for the centripetal acceleration of the satellite.

ii Show that the speed of the satellite is given by the equation $v^2 = gr$, where g is the gravitational field strength at the orbit.

b A satellite orbits the Earth in a circular orbit at a height of 100 km. Calculate:

i the gravitational field strength of the Earth at this distance,

ii the speed of the satellite,

iii the time period of the satellite.

4 a Show that the speed, v, of a satellite in a circular orbit of radius r about a planet of mass M is given by the equation $v^2 = \frac{GM}{r}$.

b A weather satellite is in a polar orbit at a height of 1600 km.

i Show that its speed is $7.1\,\text{km s}^{-1}$.

ii Calculate its time period.

iii Explain why such a satellite can survey global weather patterns every day.

13 End of chapter questions

$G = 6.67 \times 10^{-11}\,\text{N m}^2\text{kg}^{-2}$

The Earth; radius = 6400 km, mass = 6.0×10^{24} kg, $g = 9.8\,\text{N kg}^{-1}$ at the surface.

1 a Sketch the pattern of lines of force of the gravitational field surrounding a uniform sphere.

b On your diagram:

i mark two points X and Y where the gravitational field strength has the same magnitude but is in opposite directions,

ii mark a point Z where the gravitational field strength is 0.25 times the gravitational field strength at X and in the same direction.

2 Jupiter's mass is 318 times the mass of the Earth.

a Calculate the gravitational force between the Earth and Jupiter when Jupiter is 6.3×10^{11} m from the Earth.

b Calculate the gravitational field strength of Jupiter at a distance of 6.3×10^{11} m from its centre.

3 a Sketch a graph to show how the gravitational field strength of the Earth varies with height from the surface to a height of 13 000 km above its surface.

b Calculate the gravitational field strength of the Earth at a distance of 5000 km above the surface.

4 a The Moon is 380 000 km from the Earth. Calculate the gravitational field strength of the Earth at the Moon.

b Use your answer to **a** to calculate:

i the centripetal acceleration of the Moon,

ii the speed of the Moon,

iii the time period of the Moon.

5 The mean radius of Jupiter's orbit round the Sun is 7.8×10^{11} m. Jupiter has a mass of 1.9×10^{27} kg. The Sun has a mass of 2.0×10^{30} kg.

a Calculate the distance from Jupiter to the point along the line between the Sun and Jupiter at which the gravitational field strength of Jupiter is equal and opposite to the gravitational field strength of the Sun.

b The asteroid belt consists of minor planets and smaller bodies in orbits round the Sun between the Sun and Jupiter. Discuss the view that the asteroids could be the remains of a larger body which was pulled apart by the gravitational force of Jupiter and of the Sun.

6 a Calculate the gravitational force of the Earth on an astronaut who weighs 690 N on the Earth's surface when the astronaut is in a spacecraft orbiting the Earth at a height of 100 km.

b Discuss whether or not an astronaut in an orbiting spacecraft is weightless.

7 A spy satellite orbits the Earth in a polar orbit at a height of 300 km.

a Calculate the gravitational field strength of the Earth at this height.

b i Show that the satellite has a speed of $7.7\,\text{km s}^{-1}$.

ii Calculate the time period of the satellite at this height.

8 a i What is meant by a geostationary orbit?

ii A communications satellite is in a geostationary orbit directly above the equator. Explain why it is advantageous for such a satellite to be in a geostationary orbit.

b A communications network company proposes to place 12 satellites in the same orbit, equally spaced along the orbit. The orbit is to be 500 km above the Earth.

i Calculate the gravitational field strength of the Earth at this height.

ii Show that the time period of a satellite in this orbit is 1 hour and 35 minutes.

iii Explain why a mobile phone at ground level using this system would need to switch to a different satellite every 8 minutes.

9 a Define *gravitational field strength*. *(1 mark)*

b A spherical planet has diameter 1.2×10^4 km. The gravitational field strength at the surface of the planet is $8.6\,\text{N kg}^{-1}$.

The planet may be assumed to be isolated in space and to have its mass concentrated at its centre.

Calculate the mass of the planet in kg. *(3 marks)*

c The gravitational potential at a point X above the surface of the planet in **b** is $-5.3 \times 10^7\,\text{J kg}^{-1}$.

For point Y above the surface of the planet, the gravitational potential is $-6.8 \times 10^7\,\text{J kg}^{-1}$.

i State, with a reason, whether point X or point Y is nearer to the planet. *(2 marks)*

ii A rock falls radially from rest towards the planet from one point to the other.

Calculate the final speed of the rock, in m s^{-1}. *(2 marks)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 4 Q1 June 2009

10 a State Newton's law of gravitation. *(2 marks)*

b The Earth may be considered to be a uniform sphere of radius R equal to 6.4×10^6 m.

A satellite is in a geostationary orbit.

i Describe what is meant by a *geostationary orbit*. *(3 marks)*

ii Show that the radius x of the geostationary orbit is given by the expression

$$gR^2 = x^3\omega^2$$

where g is the acceleration of free fall at the Earth's surface and ω is the angular speed of the satellite about the centre of the Earth. *(3 marks)*

iii Determine the radius x of the geostationary orbit in m. *(3 marks)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 41 Q1 November 2009

11 a Define *gravitational field strength*. *(1 mark)*

b An isolated star has radius R. The mass of the star may be considered to be a point mass at the centre of the star.

The gravitational field strength at the surface of the star is g_s.

On Figure 11.1, sketch a graph to show the variation of the gravitational field strength of the star with distance from its centre. You should consider distances in the range R to $4R$.

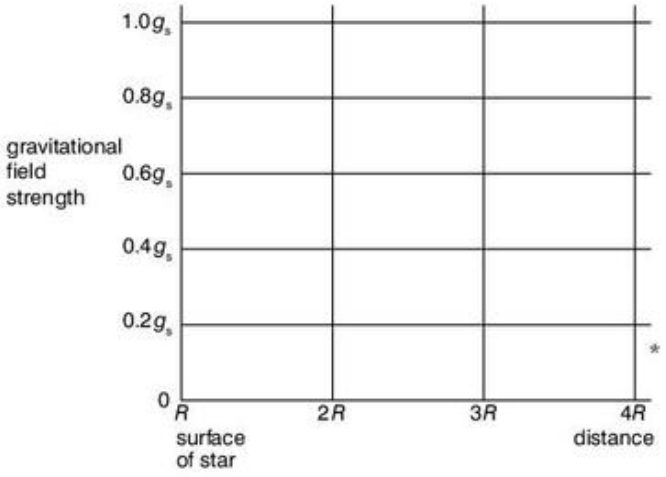


Fig 11.1

(2 marks)

c The Earth and the Moon may be considered to be spheres that are isolated in space with their masses concentrated at their centres.

The masses of the Earth and the Moon are 6.00×10^{24} kg and 7.40×10^{22} kg respectively.

The radius of the Earth is R_E and the separation of the centres of the Earth and the Moon is 60 R_E, as illustrated in Figure 11.2.

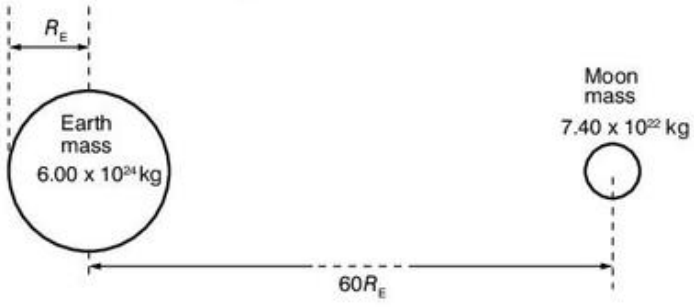


Fig 11.2 *(not to scale)*

i Explain why there is a point between the Earth and the Moon at which the gravitational field strength is zero. *(2 marks)*

ii Determine the distance, in terms of R_E, from the centre of the Earth at which the gravitational field strength is zero. *(3 marks)*

iii On the axes of Figure 11.3, sketch a graph to show the variation of the gravitational field strength with position between the surface of the Earth and the surface of the Moon.

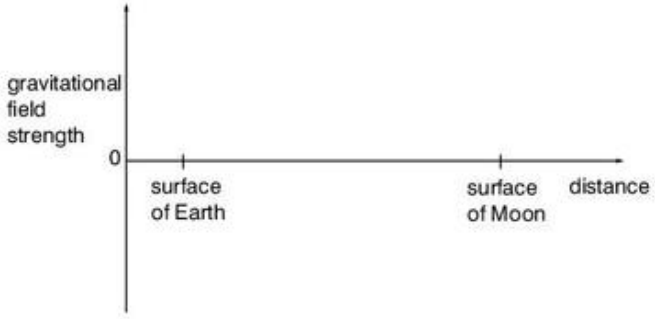


Fig 11.3

(3 marks)

Cambridge International AS and A Level Physics 9702 Paper 41 Q1 November 2010

14 Electric fields

14.1 Coulomb's law

On these pages you will learn to:

- Recall and use Coulomb's law in the form $F = Q_1Q_2/4\pi\varepsilon_0 r^2$ for the force between two point charges in free space or air
- Understand that, for any point outside a spherical conductor, the charge on the sphere may be considered to act as a point charge at its centre

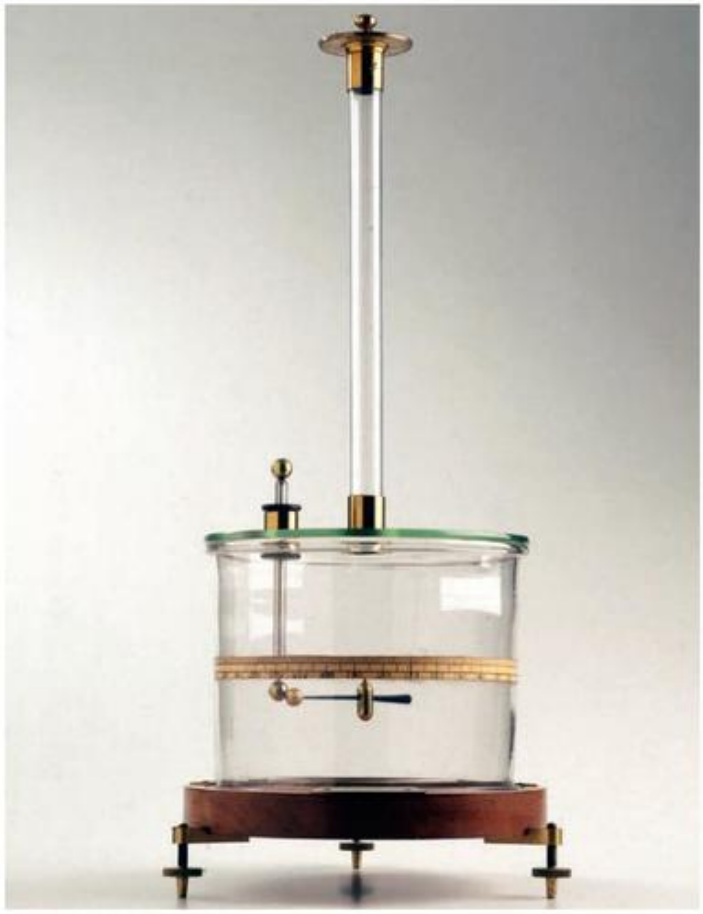


Fig 1 *Coulomb's torsion balance*

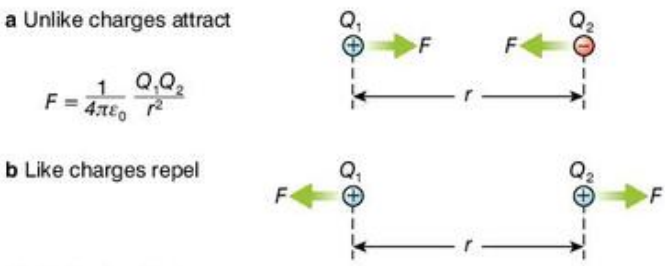


Fig 2 *Coulomb's law*

Like charges repel and unlike charges attract. The force between two charged objects depends on how close they are to each other. The exact link was first established by Charles Coulomb in France in 1784. He devised a very sensitive torsion balance to measure the force between charged pith balls. Figure 1 shows the arrangement. A needle with a ball made of pith (a substance obtained from plants) at one end and a counterweight at the other end was suspended horizontally by a vertical wire. Another pith ball on the end of a thin vertical rod could be placed in contact with the first ball.

The pith balls were small enough to be considered as point objects. The ball on the rod was charged and then placed in contact with the other ball on the needle. The contact between them charged the second ball which was then repelled by the ball fixed on the rod. This caused the wire to twist until the electrical repulsion was balanced by the twist built up in the wire. By turning the torsion head at the top of the wire, the distance between the two balls could be set at any required value. The amount of turning needed to achieve that distance gave the force. Some of Coulomb's many measurements are below.

Table 1 *Some of Coulomb's results*

Distance, *r*	36	18	8.5
Force, *F*	36	144	567

Measurements for both variables were actually made in degrees, so the above values are in relative units. Can you make out a pattern for these measurements? Halving the distance from 36 to 18 makes the force increase by a factor of 4. Halving the distance from 18 to 8.5 (near enough 9) increases the force again by a factor of about 4. The measurements fit the link that the force F is proportional to $\frac{1}{r^2}$. All the other measurements made by Coulomb fitted the same link.

Because the force is also proportional to the charge on each ball, Coulomb deduced the following equation, known as Coulomb's law, for the force F between two 'point charges' Q_1 and Q_2

$$F = k\frac{Q_1Q_2}{r^2}$$

where r is the distance between the charges.

The constant of proportionality, k, can be shown to be equal to $\frac{1}{4\pi\varepsilon_0}$, where ε_0 is the absolute permittivity of free space. Coulomb's law is therefore written as

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$$

where r = distance between two point charges Q_1 and Q_2
A method of measuring ε_0 is outlined in topic 15.2. The accepted value of ε_0 is $8.85 \times 10^{-12}\,\text{F m}^{-1}$ so

$$\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9\,\text{m F}^{-1}$$

Worked example

$e = 1.6 \times 10^{-19}\,\text{C}$, $\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9\,\text{m F}^{-1}$

Calculate the force between a proton and an electron at a separation of $3.0 \times 10^{-10}\,\text{m}$.

Solution

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2} = \frac{9.0 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(3.0 \times 10^{-10})^2}$$

$$= 2.6 \times 10^{-9}\,\text{N}$$

Note on $k = \frac{1}{4\pi\varepsilon_0}$

If Coulomb's law in the form $F = k\frac{Q_1Q_2}{r^2}$ is applied to the force on a 'test' charge q at distance r from a point charge Q, the force on the test charge $F = \frac{kQq}{r^2}$ so the electric field strength at distance r, $E = \frac{F}{q} = \frac{kQ}{r^2}$.

By introducing $\frac{1}{4\pi\varepsilon_0}$ as k, the equation $E = \frac{kQ}{r^2}$ may be written as $\frac{Q}{4\pi r^2} = \varepsilon_0 E$ or $\frac{Q}{A} = \varepsilon_0 E$. In this form, E is the electric field strength at the surface of a sphere of radius r and surface area $A(= 4\pi r^2)$ which has a charge Q evenly distributed on its surface. The equation $\frac{Q}{A} = \varepsilon_0 E$ gives the surface charge density $\frac{Q}{A}$ needed to produce an electric field of strength E at the surface. Thus ε_0 represents the charge per unit area on a surface in a vacuum that produces an electric field of strength 1 volt per metre above the surface. The equation applies to any arrangement, for example, as in Figure 3. The explanation here is not part of this specification and is provided to give a better understanding of Coulomb's law.

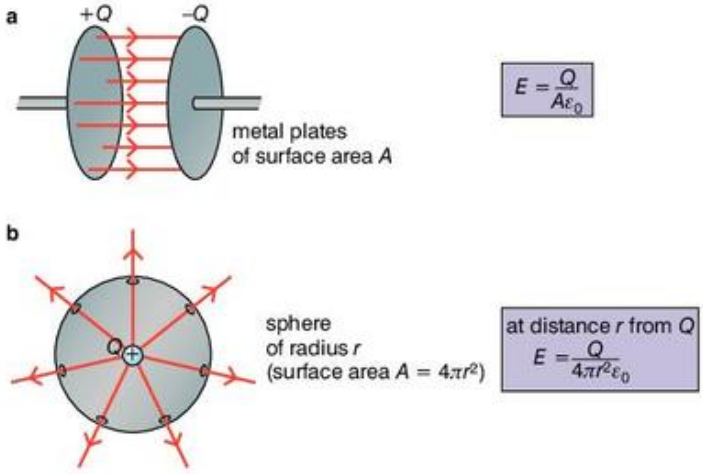


Fig 3 *Comparison of surface electric field strengths*

QUESTIONS

$\varepsilon_0 = 8.85 \times 10^{-12}\,\text{F m}^{-1}$, $\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9\,\text{m F}^{-1}$, $e = 1.6 \times 10^{-19}\,\text{C}$

1 Calculate the force between an electron and:
- **a** a proton at a distance of $2.5 \times 10^{-9}\,\text{m}$,
- **b** a nucleus of a nitrogen atom (charge $+7e$) at a distance of $2.5 \times 10^{-9}\,\text{m}$.

2 **a** Two point charges $Q_1 = +6.3\,\text{nC}$ and $Q_2 = -2.7\,\text{nC}$ exert a force of $3.2 \times 10^{-5}\,\text{N}$ on each other when they are at a certain distance, d, apart. Calculate:
- **i** the distance, d, between the two charges,
- **ii** the force between the two charges if they are moved to distance $3d$ apart.

b A charge of $+4.0\,\text{nC}$ is added to each charge in Question **2a**. Calculate the force between Q_1 and Q_2 when they are at separation d.

3 A $+30\,\text{nC}$ point charge is at a fixed distance of 6.2 mm from a point charge Q. The charges attract each other with a force of $4.3 \times 10^{-2}\,\text{N}$.
- **a** Calculate the magnitude of charge Q and state whether Q is a positive or a negative charge.
- **b** The two charges are moved 2.5 mm further apart. Calculate the force between them in this new position.

4 Two point objects, X and Y, carry equal and opposite amounts of charge at a fixed separation of $3.6 \times 10^{-2}\,\text{m}$. The two objects exert a force on each other of $5.1 \times 10^{-5}\,\text{N}$.
- **a** Calculate the magnitude, Q, of each charge and state whether the charges attract or repel each other.
- **b** The charge of each object is increased by adding a positive charge of $+2Q$ to each object. Calculate the separation at which the two objects would exert a force of $5.1 \times 10^{-5}\,\text{N}$ on each other and state whether the objects attract or repel each other.

14.2 Electric potential

On these pages you will learn to:

- Define electric potential at a point in terms of the work done in bringing unit positive charge from infinity to the point
- Recall the definition of electric field strength and the use of the equation $F = QE$
- State that the electric field strength of the field at a point is equal to the negative of potential gradient at that point

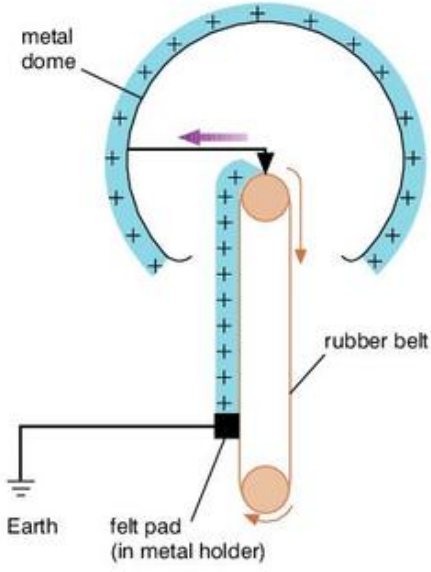


Fig 1 The Van de Graaff generator

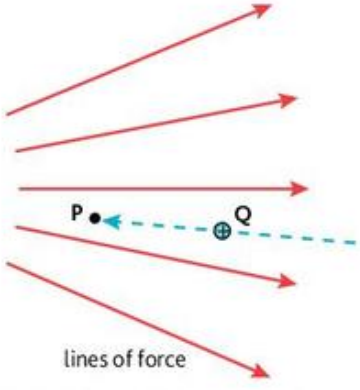


Fig 2 Moving charge Q to position P

The Van de Graaff generator

A Van de Graaff generator can easily produce sparks in air several centimetres in length. Figure 1 shows how a Van de Graaff generator works. Charge created when the rubber belt rubs against a pad is carried by the belt up to the metal dome of the generator. As charge gathers on the dome, the potential difference between the dome and Earth increases until sparking occurs.

A spark suddenly transfers energy from the dome. Work must be done to charge the dome because a force is needed to move the charge on the belt up to the dome. So the electric potential energy of the dome increases as it charges up. Some or all of this energy is transferred from the dome when a spark is created.

In general, work must be done to move a charged object A towards another object B that has the same type of charge. Their electric potential energy increases as A moves towards B.

The electric potential energy of A increases from zero if it is moved from infinity towards B. The electric field of B causes a force of repulsion to act on A and this force must be overcome to move A closer to B.

The **electric potential** at a certain position in any electric field is defined as the work done per unit positive charge on a 'positive test charge' (i.e. a small positively charged object) when it is moved from infinity to that position. By definition, the position of zero potential energy is infinity. Thus the electric potential at a certain position is the potential energy per unit positive charge of a 'positive test charge' at that position P.

The unit of electric potential is the volt (V), equal to $1\,\mathrm{J\,C^{-1}}$.

Consider a positive test charge Q placed at a position in an electric field where its electric potential energy is E_P. The electric potential V at this position is given by

$$V = \frac{E_P}{Q}$$

Note that rearranging this equation gives $E_P = QV$

Note If a test charge $+Q$ is moved in an electric field from one position where the electric potential is V_1 to another where the electric potential is V_2, the work done ΔW on it is given by

$$\Delta W = Q(V_2 - V_1)$$

Potential gradients

Equipotentials are lines of constant potential. A test charge moving along an equipotential has constant potential energy. No work is done by the electric field on the test charge because the force due to the field is at right angles to the equipotential. In other words, the lines of force of the electric field cross the equipotential lines at right angles.

The equipotentials for an electric field are like equipotentials for a gravitational field; both are lines of constant potential energy for the appropriate test object, in one case a test charge and in the other case a test mass.

Figure 3 shows the equipotentials of the electric field due to two positively charged objects.

Suppose a +2.0μC test charge is moved from X to Y.

The potential at X, V_X, is +1000 V so the test charge at X has potential energy equal to $+2.0 \times 10^{-3}$ J

$(= QV_X = +2.0 \times 10^{-6}\text{C} \times +1000\text{V})$

The potential at Y, V_Y, is +400 V so the test charge at Y has potential energy equal to $+8.0 \times 10^{-4}$ J

$(= QV_Y = +2.0 \times 10^{-6}\text{C} \times +400\text{V})$

Therefore, moving the test charge from X to Y lowers its potential energy by 1.2×10^{-3} J.

Note that if the test charge is moved from Y to Z along the +400 V equipotential, its potential energy remains constant at $+8.0 \times 10^{-4}$ J.

The **potential gradient** at any position in an electric field is the change of potential per unit change of distance in a given direction.

1 If the field is non-uniform as in Figure 3, the potential gradient varies according to position and direction. The closer the equipotentials are, the greater the potential gradient. The potential gradient is at right angles to the equipotentials.
2 If the field is uniform, such as the field between the two oppositely charged parallel plates shown in Figure 4, the equipotentials **between the plates** are equally-spaced lines parallel to the plates. Figure 4 also shows how the potential relative to the negative plate changes with perpendicular distance x from the negative plate.

The graph shows that the potential relative to the negative plate is proportional to distance x. In other words, the potential gradient is constant (such that the potential increases in the opposite direction to the electric field) and equal to V/d.

The electric field strength E between the plates is equal to V/d and is directed from the + to the − plate. In other words,

the electric field strength is equal to the negative of the potential gradient.

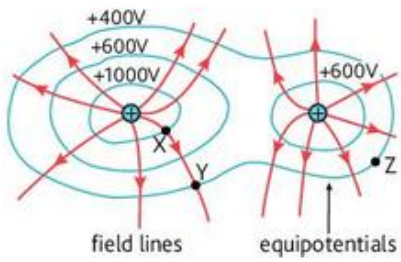
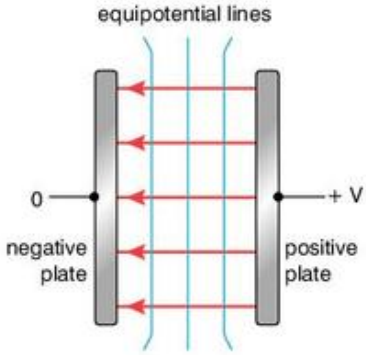


Fig 3 Equipotentials



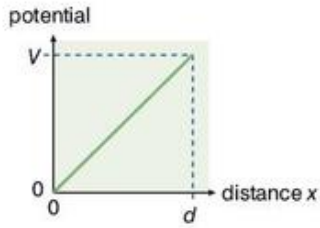


Fig 4 A uniform potential gradient

QUESTIONS

$e = 1.6 \times 10^{-19}$ C

1 An electron in a beam is accelerated from a potential of −50 V to a potential of +450 V. Calculate:
   a the potential energy of the electron at:
      i −50 V,
      ii +450 V,
   b the change of potential energy of the electron.

2 In Figure 3, a test charge q is moved from X to Z. Calculate the change of potential energy of the test charge:
   a if $q = +3.0\mu$C,
   b if $q = -2.0\mu$C.

3 An oil droplet carrying a charge of $+2e$ is in air between two parallel metal plates separated by a distance of 20 mm. The p.d. between the plates is 5.0 V.
   a Calculate:
      i the potential gradient between the two plates,
      ii the force on the droplet.
   b Calculate the change of electrical potential energy of the oil droplet if it moves from the midpoint of the plates to the negative plate.

4 **a** Define electric potential and state its unit.
   b Two parallel horizontal metal plates are placed one above the other at a separation of 20 mm. A potential difference of +60 V is applied between the plates with the top plate positive.
      i Calculate the electric field strength between the plates.
      ii Sketch a graph to show how the electric potential V between the plates varies with height h above the lower plate.

14.3 Electric field strength of point charges

On these pages you will learn to:

- Recall the definition of electric field strength and use $F = QE$
- Recall and use $E = Q/4\pi\varepsilon_0 r^2$ for the field strength of a point charge in free space or air
- Use the equation $V = Q/4\pi\varepsilon_0 r$ for the potential in the field of a point charge

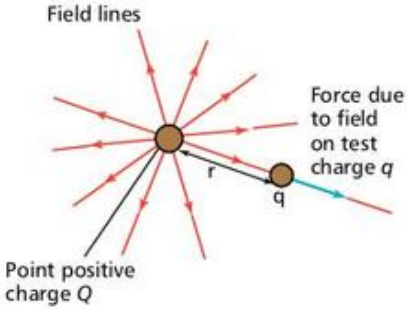


Fig 1 Force near a point charge Q

A point charge is a convenient expression for a charged object in a situation where distances under consideration are much greater than the size of the object. The same idea applies to a distant star which is considered as a point object because its diameter is much smaller than the distance to it from the Earth.
A 'test' charge in an electric field is a point charge that does not alter the electric field in which it is placed. This would happen if an object with a sufficiently large charge is placed in an electric field and it causes a change in the distribution of charge that creates the field.

Consider the electric field due to a point charge $+Q$, as shown in Figure 1.
The field lines radiate from the point charge because a test charge $+q$ in the field would experience a force directly away from Q wherever it was placed. Coulomb's law gives the force F on the test charge q as

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$$

Therefore, as electric field strength $E = \frac{F}{q}$ by definition, the electric field strength at distance r from Q is given by

$$\boldsymbol{E = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

Note that if Q is negative, the above formula gives a negative value of E corresponding to the field lines pointing inwards towards Q.

Worked example

$\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9\,\text{m F}^{-1}$, $e = 1.6 \times 10^{-19}\,\text{C}$

Calculate the electric field strength due to a nucleus of charge $+82e$ at a distance of 0.35 nm.

Solution

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{9.0 \times 10^9 \times (+82 \times 1.6 \times 10^{-19})}{(0.35 \times 10^{-9})^2} = 9.6 \times 10^{11}\,\text{V m}^{-1}$$

Electric field strength as a vector

If a test charge is in an electric field due to several point charges, each charge exerts a force on the test charge. The resultant force F on the test charge gives the resultant electric field strength at the position of the test charge. Consider the following situations:

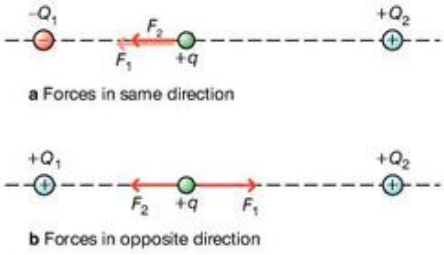


Fig 2 Forces along the same line

- **Forces in the same direction**: Figure 2**a** shows a test charge $+q$ on the line between a negative point charge Q_1 and a positive point charge Q_2. The test charge experiences a force $F_1 = q E_1$ where E_1 is the electric field strength due to Q_1 and a force $F_2 = q E_2$ where E_2 is the electric field strength due to Q_2. The two forces act in the same direction because Q_1 attracts q and Q_2 repels q. So the resultant force $F = F_1 + F_2 = q E_1 + q E_2$. Therefore, the resultant electric field strength $E = \frac{F}{q} = \frac{qE_1 + qE_2}{q} = E_1 + E_2$
- **Forces in opposite directions**: Figure 2**b** shows a test charge $+q$ on the line between two positive point charges Q_1 and Q_2. The test charge experiences a force $F_1 = q E_1$ where E_1 is the electric field strength due to Q_1 and a force $F_2 = q E_2$ where E_2 is the electric field strength due to Q_2. The two forces act in

opposite directions because Q_1 repels q and Q_2 repels q. Assuming F_1 is greater than F_2, the resultant force $F = F_1 - F_2 = qE_1 - qE_2$.

Therefore, the resultant electric field strength $E = \frac{F}{q} = \frac{qE_1 - qE_2}{q} = E_1 - E_2$

- **Forces at angle θ to each other**: Figure 3 shows a test charge $+q$ on perpendicular lines from two positive point charges Q_1 and Q_2. The test charge experiences a force $F_1 = qE_1$ where E_1 is the electric field strength due to Q_1 and a force $F_2 = qE_2$ where E_2 is the electric field strength due to Q_2. The two forces are at angle θ to each other, F_1 along the line between q and Q_1 and F_2 along the line between q and Q_2. The magnitude of the resultant force F is given by the formula $F^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$ as explained in topic 1. Therefore, the resultant electric field strength $E = F/q$.

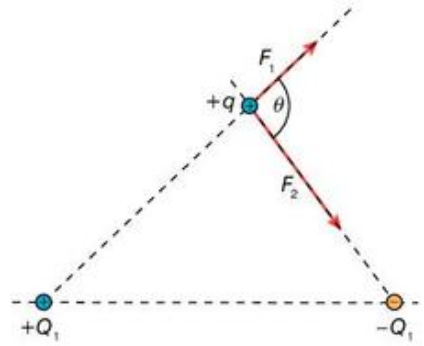


Fig 3 Forces at angle θ

In general, the resultant electric field strength is the vector sum of the individual electric field strengths.

Worked example

$$\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9\,\text{m F}^{-1}$$

A $+65\,\mu$C point charge Q_1 is at a distance of 50 mm from a $+38\,\mu$C charge Q_2.

A $+12$ pC charge q is placed at M, midway between Q_1 and Q_2.

Calculate:
a the resultant electric field strength at M,
b the magnitude and direction of the force on q.

Solution

a The electric field strength due to Q_1 at M, $E_1 = \frac{Q_1}{4\pi\varepsilon_0 r_1^2}$ where $r_1 = 25$ mm

Therefore, $E_1 = 9.0 \times 10^9 \times \frac{65 \times 10^{-6}}{(25 \times 10^{-3})^2} = 9.4 \times 10^8\,\text{V m}^{-1}$ away from Q_1

The electric field strength due to Q_2 at M, $E_2 = \frac{Q_2}{4\pi\varepsilon_0 r_2^2}$, where $r_2 = 25$ mm

Therefore, $E_2 = 9.0 \times 10^9 \times \frac{38 \times 10^{-6}}{(25 \times 10^{-3})^2} = 5.5 \times 10^8\,\text{V m}^{-1}$ away from Q_2

As E_1 and E_2 are in opposite directions, the resultant electric field strength $E = E_1 - E_2 = 9.4 \times 10^8 - 5.5 \times 10^8 = 3.9 \times 10^8\,\text{V m}^{-1}$ away from Q_1 towards Q_2

b The resultant force on $q = qE = +12 \times 10^{-12} \times 3.9 \times 10^8 = 4.7 \times 10^{-3}$ N

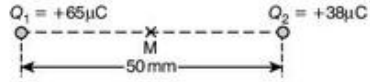


Fig 4

Electric potential near a point charge

Because Coulomb's law $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$ and Newton's law $F = G\frac{m_1m_2}{r^2}$ are both inverse-square relationships, the forces vary with distance in the same way. Therefore the formula for the electrical potential near a point charge Q is of the same form as the formula for the gravitational potential near a point mass (or spherical mass) i.e. $\phi = \frac{-GM}{r}$ which was derived in topic 13.4.

Using this analogy and replacing M by Q and G by $\frac{1}{4\pi\varepsilon_0}$ in the gravitational potential formula above therefore gives the following formula for the electric potential V at distance r from a point charge Q

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

The equation shows that the electric potential V is inversely proportional to the distance r. The curve is not an 'inverse-square law' curve as V is proportional to $1/r$. See Figure 6.

NOTE

A test charge inside the sphere would experience zero force due to the charge on the sphere, because the electric field due to the charge on the sphere is in all directions from any point inside the sphere.

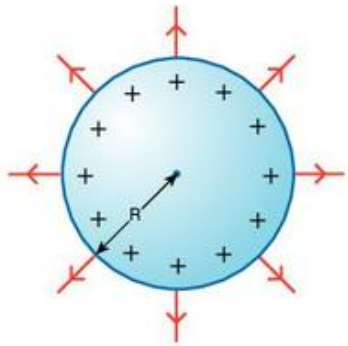


Fig 5 The electric field near a charged hollow metal sphere

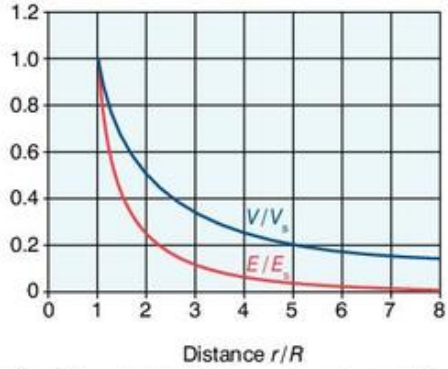


Fig 6 The electric field strength and potential near a charged sphere

NOTES

1 Gravitational potential is always negative as the force is always attractive, whereas electric potential in the electric field near a point charge Q can be positive or negative according to whether Q is a positive or a negative charge.

2 The potential energy of two point charges q and Q at distance d apart

$$= qV = \frac{qQ}{4\pi\varepsilon_0 d}$$

More about radial fields

The electric field surrounding a charged metal sphere is radial, like the field due to a point charge. The charge on the metal surface is evenly distributed on its surface. The field has the same strength as if all the charge was concentrated at the centre of the sphere.

For a sphere of radius R with a charge Q,

- Outside the sphere at distance r from the centre of the sphere,
  the electric field strength $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ and the electric potential $V = \frac{Q}{4\pi\varepsilon_0 r}$
- At the surface of the sphere,
  the electric field strength $E_s = \frac{Q}{4\pi\varepsilon_0 R^2}$ and the electric potential $V_s = \frac{Q}{4\pi\varepsilon_0 R}$
- If the sphere is hollow, there is no electric field inside it. Therefore the electric field strength is zero and the electric potential is V_s.

Figure 6 shows how the electric field strength and potential vary with distance from the centre of the sphere.

Outside the sphere,

- the field strength curve is an 'inverse-square law' curve as E is proportional to $\frac{1}{r^2}$.
- the potential curve is an 'inverse' curve as V is proportional to $\frac{1}{r}$.

QUESTIONS

$\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9\,\text{m F}^{-1}$

1 a Calculate the electric field strength at a distance of 3.2 mm from a +6.0 nC point charge.

b Calculate the distance from the point charge in **a** at which the electric field strength is $5.4 \times 10^5\,\text{V m}^{-1}$.

2 A + 25 μC point charge Q_1 is at a distance of 60 mm from a +100 μC charge Q_2.

Fig 7

a A +15 pC charge q is placed at M, 25 mm from Q_1 and 35 mm from Q_2. Calculate:
i the resultant electric field strength at M,
ii the magnitude and direction of the force on q.

b Show that the electric field strength due to Q_1 and Q_2 is zero at the point which is 20 mm from Q_1 and 40 mm from Q_2.

3 A +15 μC point charge Q_1 is at a distance of 20 mm from a +10 μC charge Q_2.

a Calculate the resultant electric field strength:
i at M, the midpoint between the two charges,
ii at the point P along the line between Q_1 and Q_2 which is 25 mm from Q_1 and 45 mm from Q_2.
iii at a point S which is 20 mm from Q_1 and 20 mm from Q_2.

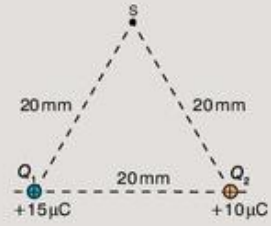


Fig 8

b i Explain why there is a point along the line between the two charges at which the electric field strength is zero.
ii Calculate the distance from this point to Q_1 and to Q_2.

4 A +15 μC point charge Q_1 is at a distance of 30 mm from a −30 μC charge Q_2.

a Calculate the electric potential at the midpoint of the two charges.

b i Show that the electric potential is zero at a point between the two charges which is 10 mm from Q_1 and 20 mm from Q_2.
ii Calculate the electric field strength at this position and state its direction.

14.4 Comparison between electric and gravitational fields

The similarities and differences between the two types of fields are listed in the table below. In the mid-nineteenth century, James Maxwell showed that electric and magnetic forces are different manifestations of the electromagnetic force. Towards the end of the twentieth century, physicists proved that the electromagnetic force and the nuclear force responsible for radioactive decay are different manifestations of a more fundamental force, the electroweak force. At the present time, the force of gravity remains outside this theoretical framework, despite repeated attempts to establish a unified framework. The fundamental nature of the force of gravity remains mysterious even though we use it in everyday situations more than any other force.

Table 1

	Gravitational force	**Electric force**
Similarities		
Line of force or a field line	Path of a free 'test' mass in the field	Path of a free 'test' charge in the field
Inverse-square law of force	Newton's law of gravitation $F = \frac{Gm_1m_2}{r^2}$	Coulomb's law of force $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$
Field strength	force per unit mass, $g = F/m$	force per unit + charge, $E = F/q$
Unit of field strength	Nkg^{-1} or ms^{-2}	NC^{-1} or Vm^{-1}
Potential	Gravitational potential energy per unit mass	Electric potential energy per unit + charge
Unit of potential	Jkg^{-1}	V (= JC^{-1})
Potential energy between two point masses or charges	$E_p = -\frac{Gm_1m_2}{r}$	$E_p = -\frac{Q_1Q_2}{4\pi E_0 r}$
Uniform fields	g is the same everywhere, field lines parallel	E is the same everywhere, field lines are parallel
Radial fields	due to a point mass or a uniform spherical mass M, $g = \frac{GM}{r^2}$ $\phi = -\frac{Gm}{r}$	due to a point charge or a charged metal sphere of charge Q, $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ $V = \frac{Q}{4\pi\varepsilon_0 r}$
Differences		
Action at a distance	between any two masses	between any two charged objects
Force	attracts only	unlike charges attract, like charges repel
Constant of proportionality in force law	G	$\frac{1}{4\pi\varepsilon_0}$

On these pages you will learn to:

- Recognise the analogy between certain qualitative and quantitative aspects of electric fields and gravitational fields

CHAPTER SUMMARY

A **line of force** or a **field line** of an electric field is a line which a free positive 'test' charge would follow in the field.

The electric field strength, E, at a point in the field is defined as the force per unit charge on a positive test charge placed at that point.

Electric potential, V, at a point is defined as the work done per unit charge to move a positive test charge from infinity to that point.

A uniform electric field is one where the electric field strength has the same magnitude and direction everywhere between the plates.

Electric field strength $E = \frac{F}{Q}$

Coulomb's law of force

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$$

Uniform electric field $E = \frac{V}{d}$

where V is the p.d. between two oppositely charged parallel plates at separation d.

Radial electric field

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \qquad V = \frac{Q}{4\pi\varepsilon_0}$$

where r is the distance from a point charge Q or from the centre of a metal sphere carrying a charge Q.

Electric potential

$$V = \frac{E_p}{Q}$$

14 End of chapter questions

$\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9\,\text{m F}^{-1}$, $e = 1.6 \times 10^{-19}\,\text{C}$,
$g = 9.8\,\text{N kg}^{-1}$, $G = 6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$

1 **a** Explain what is meant by a uniform electric field.

b A beam of electrons is directed horizontally into a uniform electric field which acts vertically downwards, as shown in the diagram below.

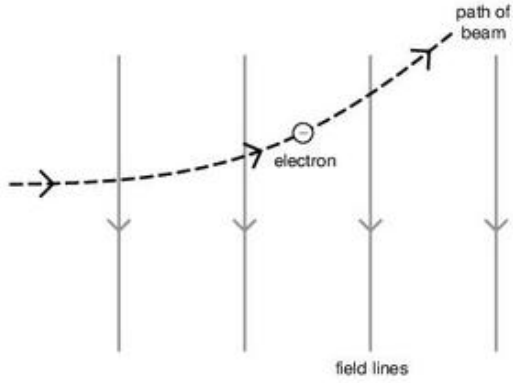


Fig 1.1

i Explain why the beam follows a path which curves upwards.

ii Each electron in the beam experiences a constant force of 1.8×10^{-13} N. Calculate the strength of the electric field.

2 A small metal sphere on an insulating rod is placed on a top pan balance, as shown in the diagram below. An identical metal sphere carrying a charge of $2Q$ on an insulating rod is brought into contact with the sphere on the balance so each sphere acquires the same charge.

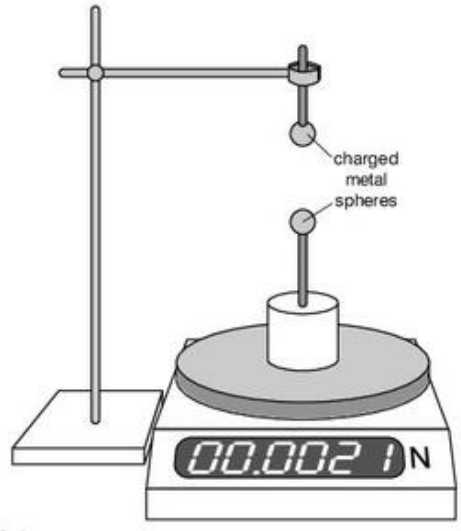


Fig 2.1

a When the two spheres are at a separation of 68 mm, the top pan balance reading increases by 2.1×10^{-3} N. Calculate the charge, Q, on each sphere.

b Explain why the electric field strength is zero at the midpoint between the two charges.

3 A +36 nC point charge X is at a distance of 75 mm from a −9 nC point charge Y.

a **i** Sketch the pattern of the electric field lines between the point charges.

ii Calculate the electric field strength at the midpoint between X and Y.

b At a certain position along the line XY, the resultant electric field strength is zero.

i Explain why this position is more than 75 mm further from X than it is from Y.

ii Calculate the distance from this position to X and to Y.

4 **a** Explain why an air molecule can be ionised by an electric field that is sufficiently strong.

b The spherical dome of a Van de Graaff machine has a diameter of 0.30 m. Sparks are produced from the dome if the electric field strength at the surface reaches $40\,\text{kV m}^{-1}$. Calculate:

i the maximum charge that can be stored on the dome,

ii the electric field strength at a distance of 1.0 m from the surface of the dome when the surface electric field strength is $40\,\text{kV m}^{-1}$.

5 Two identical small conducting spheres are supported at either end of an insulating fibre which is attached at its midpoint to a fixed support. When the two spheres are in contact with each other, they are charged by contact with a charged rod. The two spheres repel each other as shown in the diagram opposite.

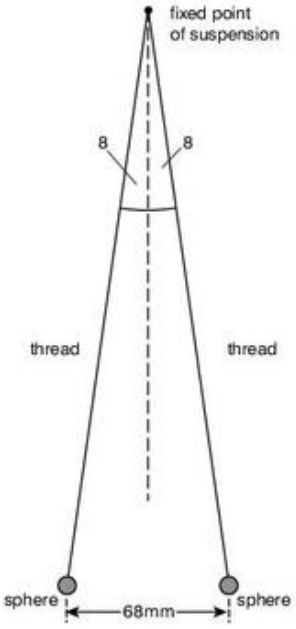


Fig 5.1

a In equilibrium, each section of the thread makes an angle of 8° to the vertical. Show that the electrostatic force of repulsion on each sphere $F = mg \tan 8°$, where m is the mass of each sphere.

b The mass of each sphere is 3.4×10^{-3}kg. Their separation at equilibrium is 68 mm. Calculate the charge on each sphere.

6 a State two similarities and two differences between an electric field and a gravitational field.

b Show the electrostatic force of repulsion between two protons is $1.2 \times 10^{36} \times$ the gravitational force between them at the same distance.

mass of a proton = 1.67×10^{-27}kg

7 a Define ***electric potential*** at a point. *(2 marks)*

b An α-particle is emitted from a radioactive source with kinetic energy of 4.8 MeV.

The α-particle travels in a vacuum directly towards a gold ($^{197}_{79}$Au) nucleus, as illustrated in Figure 7.1.

Fig 7.1

The α-particle and the gold nucleus may be considered to be point charges in an isolated system.

i Explain why, as the α-particle approaches the gold nucleus, it comes to rest. *(2 marks)*

ii For the closest approach of the α-particle to the gold nucleus determine

1 their separation, in m *(3 marks)*

2 the magnitude of the force, in N, on the α-particle. *(2 marks)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 41 Q5 November 2009

15 Capacitors

15.1 Capacitance

On these pages you will learn to:

- Define capacitance and the farad, as applied to both isolated conductors and to parallel plate capacitors
- State and explain how the p.d. across a capacitor changes when the charging current is constant
- Recall and use $C = Q/V$

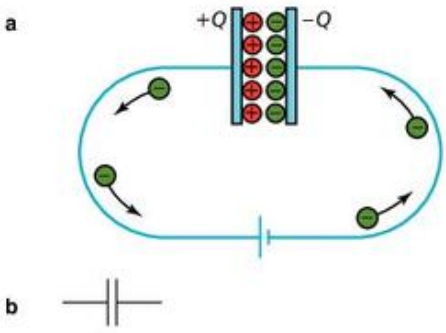


Fig 1 ***a*** *Storing charge*
b *Capacitor symbol*

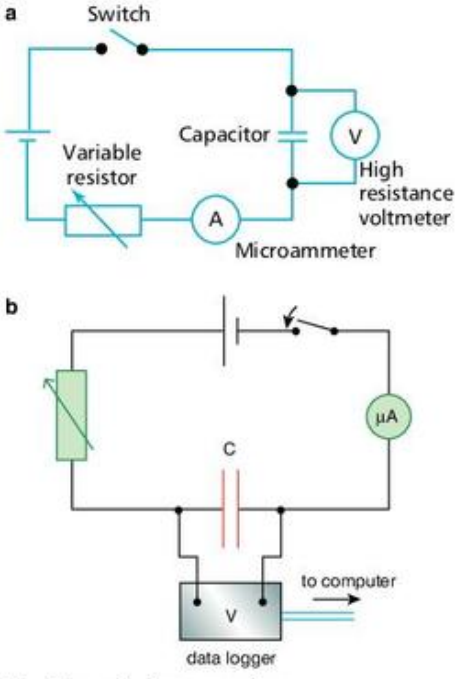


Fig 2 *Investigating capacitors*
a *Circuit diagram* ***b*** *Using a data logger*

A capacitor is a device designed to store charge. Two parallel metal plates placed near each other form a capacitor. When the plates are connected to a battery, electrons from the negative terminal of the battery flow onto one of the plates. An equal number of electrons leave the other plate to return to the battery via its positive terminal. So each plate gains an equal and opposite charge.

A capacitor consists of two conductors insulated from each other. An isolated conductor also acts as a capacitor as it can store charge because is is not 'earthed'. The isolated conductor and the Earth (which acts as the second conductor) are insulated from each other. The symbol for a capacitor is shown in Figure 1b.
As explained above, when a capacitor is connected to a battery, one of the two conductors gains electrons from the battery and the other conductor loses electrons to the battery. When we say that the charge stored by the capacitor is Q, we mean that one conductor has charge $+Q$ and the other conductor has charge $-Q$.

Charging a capacitor at constant current

Figure 2 shows how this can be achieved using a variable resistor, a switch, a microammeter and a cell in series with the capacitor. When the switch is closed, the variable resistor is continually adjusted to keep the microammeter reading constant. At any given time, t, after the switch is closed, the charge Q on the capacitor can be calculated using the equation $Q = It$, where I is the current.

A high-resistance voltmeter connected in parallel with the capacitor enables the capacitor p.d. to be measured. To investigate how the capacitor p.d. changes with time for constant current, use the variable resistor to keep the current constant and either

- use a stopwatch and measure the voltmeter reading at measured times, or
- use a datalogger as shown in Figure 2**b**.

Typical readings for a current of 15 μA are shown in the following table. The charge Q has been calculated using $Q = It$.

Table 1 *Current = 15 mA*

Time, *t*/s	0	20	40	60	80	100
p.d., *V*/volts	0	0.29	0.62	0.90	1.22	1.50
charge, *Q*/μC	0	300	600	900	1200	1500

The graph of charge stored Q against p.d. V for these measurements is shown in Figure 3. The measurements define a straight line passing through the origin. Therefore, the charge stored Q is proportional to the p.d. V. In other words, the charge stored per volt is constant.

The capacitance C of a capacitor is defined as the charge stored per unit p.d.

The unit of capacitance is the farad (F), equal to 1 coulomb per volt. Note that $1\,\mu F = 10^{-6}\,F$.

For a capacitor which stores charge Q at p.d. V, its capacitance can be calculated using the equation

$$C = \frac{Q}{V}$$

Note Rearranging this equation gives $Q = CV$ or $V = \frac{Q}{C}$.

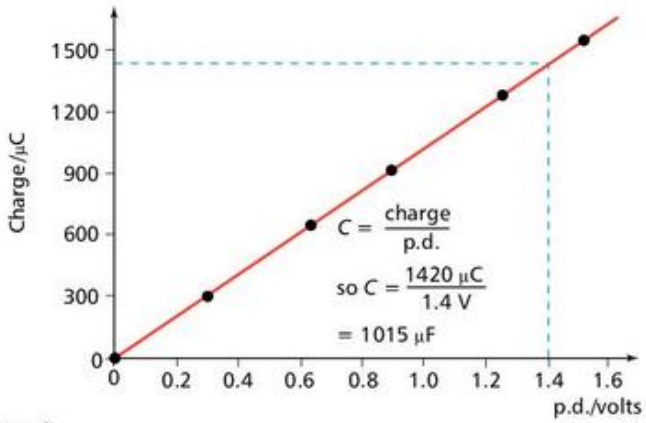


Fig 3 Graph of results

Worked example

A capacitor which is initially uncharged is charged at a constant current of 16μA for 40s. At the end of this time, the capacitor p.d. increases to 5.8V. Calculate:

a the charge stored by the capacitor,
b the capacitance of the capacitor.

Solution

a $Q = It = 16 \times 10^{-6} \times 40 = 6.4 \times 10^{-4}$C,

b $C = \frac{Q}{V} = \frac{6.4 \times 10^{-4}}{5.8\text{V}} = 1.1 \times 10^{-4}$F $(= 110\mu\text{F})$

QUESTIONS

1 Complete the following table

	(a)	(b)	(c)	(d)	(e)	(f)
charge/μC	60	330		6.30	52	
p.d./V	12		9.0	4.5		50
capacitance/μF		150	1100		4.7	68

2 A 22μF capacitor is charged by means of a constant current of 2.5μA to a p.d. of 12.0V. Calculate:
a the charge stored on the capacitor at 12.0V,
b the time taken.

3 A capacitor is charged by means of a constant current of 0.5μA to a p.d. of 5.0V in 55s. Calculate:
a the charge stored, **b** the capacitance of the capacitor.

4 A capacitor is charged by means of a constant current of 24μA to a p.d. of 4.2V in 38s. The capacitor is then charged from 4.2V by means of a current of 14μA in 50s. Calculate:
a the charge stored at a p.d. of 4.2V,
b the capacitance of the capacitor,
c the extra charge stored at a current of 14μA,
d the p.d. after the extra charge was stored.

EXTENSION

Practical capacitors

Most practical capacitors are designed using two strips of aluminium foil as plates separated by a thin layer of **dielectric** which is an insulating material that increases the charge that can be stored for a given p.d. Capacitances of less than about 10μF contain dielectrics such as mica or polyester.

Figure 4 shows the construction of such a capacitor in which the foil and the dielectric are rolled up into a convenient size. See topic 15.2 for more about dielectrics.

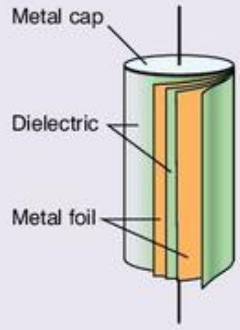


Fig 4 A practical capacitor

Electrolytic capacitors contain paper soaked with an electrolyte such as aluminium borate between the metal strips. When used in a d.c. circuit, a thin oxide layer which is a dielectric forms on the positive plate. As explained in topic 15.3, because the dielectric layer is very thin, the capacitance can be much greater than that of non-electrolytic capacitors. Electrolytic capacitors must be connected with the correct polarity otherwise the dielectric is damaged.

15.2 Capacitors in series and parallel

On these pages you will learn to:

- Derive formulae for the total capacitance of capacitors in series and in parallel
- Solve problems using the capacitance formulae for capacitors in series and in parallel

Capacitors in parallel

For two or more capacitors of capacitances C_1, C_2, C_3 … in parallel,

the combined capacitance $C = C_1 + C_2 + C_3 + \ldots$

Consider two capacitors C_1 and C_2 in parallel, as shown in Figure 1. Components in parallel have the same p.d. so the charge stored by each capacitor, for p.d. V, is

- $Q_1 = C_1 V$ for capacitor C_1, and
- $Q_2 = C_2 V$ for capacitor C_2.

Therefore, the total charge stored $\quad Q = Q_1 + Q_2 = C_1V + C_2V$

So the combined capacitance $\quad C = \dfrac{Q}{V} = \dfrac{C_1V + C_2V}{V} = C_1 + C_2$

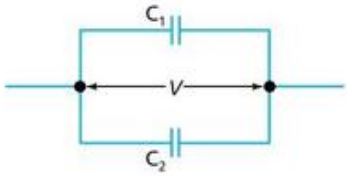


Fig 1 Capacitors in parallel

Note

For two capacitors in parallel, the ratio of the charge stored by each capacitor,

$$\frac{Q_1}{Q_2} = \frac{C_1V}{C_2V} = \frac{C_1}{C_2}$$

So the ratio of the charge stored by each capacitor is equal to the ratio of the capacitances $\dfrac{C_1}{C_2}$.

Worked example

A 3.0 V battery is connected to a 4.0 μF capacitor and a 2.0 μF capacitor in parallel with each other. Calculate:

a the combined capacitance of the two capacitors,
b i the charge stored on each capacitor,
ii the total charge stored.

Solution

a $C = C_1 + C_2 = 4.0 + 2.0 = 6.0\,\mu F$

b i For the 4.0 μF capacitor, $Q_1 = C_1V = 4.0\,\mu F \times 3.0\,V = 12.0\,\mu C$
For the 2.0 μF capacitor, $Q_2 = C_2V = 2.0\,\mu F \times 3.0\,V = 6.0\,\mu C$

ii The total charge stored $Q = CV = 6.0\,\mu F \times 3.0\,V = 18\,\mu C$.

Note The total charge stored, Q, could be calculated using $Q = Q_1 + Q_2$

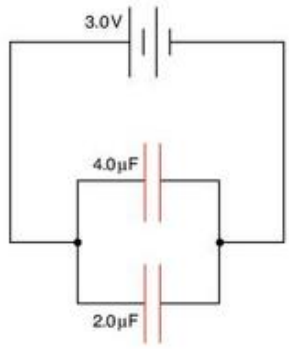


Fig 2

Capacitors in series

Components in series store the same charge. Figure 3 shows why this is so for two capacitors. The battery is connected to the left-hand plate of C_1 and to the right-hand plate of C_2. Electrons transfer from the negative terminal of the battery to the left-hand plate of C_1 and from the right-hand plate of C_2 to the battery. Transfer of electrons takes place to the left-hand plate of C_2 from the right-hand plate of C_1 but there is no transfer of electrons to either of these plates from the battery. Each plate stores the same magnitude of charge Q so the total charge stored is Q.

For two or more capacitors of capacitances C_1, C_2, C_3 … in series,

the combined capacitance $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \ldots$

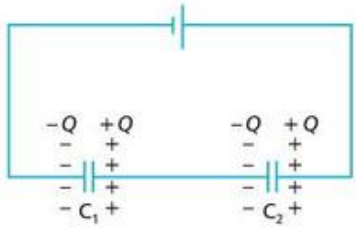


Fig 3 Capacitors in series

Consider two capacitors C_1 and C_2 in series, as shown in Figure 3. Because they store the same amount of charge Q, the p.d. across each capacitor is

- $V_1 = \frac{Q}{C_1}$ for capacitor C_1, and
- $V_2 = \frac{Q}{C_2}$ for capacitor C_2.

The capacitors are in series so the total p.d. $V = V_1 + V_2$

$$\therefore \quad V = \frac{Q}{C_1} + \frac{Q}{C_2}$$

Because the combined capacitance $C = \frac{Q}{V}$, then

$$\frac{1}{C} = \frac{V}{Q} = \frac{\left(\frac{Q}{C_1} + \frac{Q}{C_2}\right)}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

Worked example

A 3.0 V battery, a 4.0 μF capacitor C_1 and a 2.0 μF capacitor C_2 are connected in series with each other. Calculate:

a the combined capacitance of the two capacitors,

b **i** the charge stored on each capacitor,

ii the p.d. across each capacitor.

Solution

a $\frac{1}{C} = \frac{1}{4.0} + \frac{1}{2.0} = 0.25 + 0.50 = 0.75\,\mu F^{-1}$

Therefore $C = \frac{1}{0.75} = 1.33\,\mu F$

b **i** $Q = CV = 3.0\,V \times 1.33\,\mu F = 4.0\,\mu C$

ii For C_1, $V_1 = \frac{Q}{C_1} = \frac{4.0\,\mu C}{4.0\,\mu F} = 1.0\,V$. For C_2, $V_2 = \frac{Q}{C_2} = \frac{4.0\,\mu C}{2.0\,\mu F} = 2.0\,V$

Note The p.d. across C_2, V_2, could have been calculated using $V = V_1 + V_2$.

Combinations with capacitors in series and parallel

Figure 5 shows two arrangements where 3 capacitors are connected to a battery.

- In **a**, two capacitors C_1 and C_2 in parallel are connected in series to a third capacitor C_3 and the battery.
- In **b**, the two capacitors C_1 and C_2 in series are connected in parallel with the third capacitor C_3 and the battery.

The steps below show how to work out the charge and p.d. of each capacitor, given the capacitance values and the battery p.d. V_0.

1 Calculate the total capacitance C by using the appropriate combination rule to find the combined capacitance C' of C_1 and C_2 then use the other combination rule to work out the combined capacitance of C_3 and C'.
2 Work out the total charge stored Q using $Q = CV$.
3 For **a**, charge Q is stored by C_3 and the same amount of charge is shared between C_1 and C_2 in proportion to their capacitances. After working out the charge stored by each capacitor, use $V = \frac{Q}{C}$ to work out the p.d. across each capacitor.
For **b**, the battery p.d. V_0 is across C_3 and is shared between C_1 and C_2 in inverse proportion to their capacitances. After working out the p.d. across each capacitor, use $Q = CV$ to work out the charge stored by each capacitor.

Notes

For two capacitors in series,

1 the ratio of the p.ds,

$$\frac{V_1}{V_2} = \frac{Q}{C_1} \div \frac{Q}{C_2} = \frac{C_2}{C_1}$$

So the ratio of the p.ds across the two capacitors is equal to the inverse ratio of the capacitances.

2 $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$

Making C the subject of this expression gives

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{\text{product of the capacitances}}{\text{sum of the capacitances}}$$

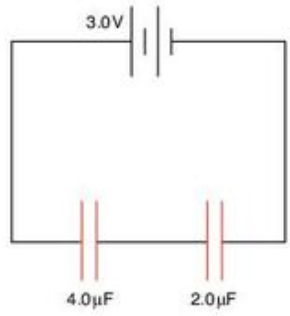


Fig 4

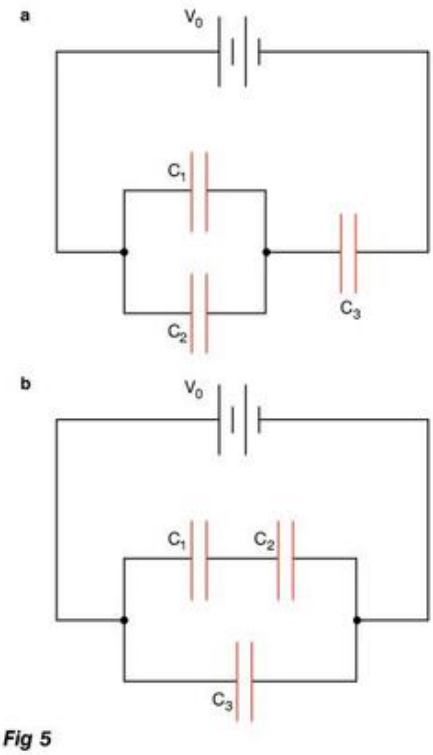


Fig 5

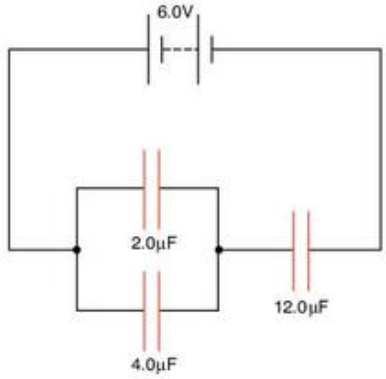


Fig 6

Worked example 1

A 2.0μF capacitor and a 4.0μF capacitor are in parallel with each other. The parallel combination is in series with a 12.0μF capacitor and a 6.0V battery, as shown in Figure 6. Calculate:

a the total capacitance of the combination,
b the charge stored by the combination,
c the charge and p.d. for each capacitor.

Solution

a $C' = 2.0 + 4.0 = 6.0\,\mu F$

$\therefore$ the total capacitance C is given by $\frac{1}{C} = \frac{1}{6.0} + \frac{1}{12.0} = 0.25\,\mu F^{-1}$

$\therefore C = \frac{1}{0.25} = 4.0\,\mu F$

b The charge stored $Q = CV = 4.0 \times 6.0 = 24\,\mu C$

c C_3; charge stored $= Q = 24\,\mu C$. $\therefore$ p.d. across $C_3 = \frac{Q}{C_3} = \frac{24\,\mu C}{12\,\mu F} = 2.0\,V$

C_1 and C_2; $\frac{C_1}{C_2} = \frac{2\,\mu F}{4\,\mu F} = 0.5$, then $\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = 0.5$

As $Q = Q_1 + Q_2 = 24\,\mu C$, then $Q_1 = 8\,\mu C$ and $Q_2 = 16\,\mu C$

$\therefore \quad V_1 = \frac{Q_1}{C_1} = \frac{8\,\mu C}{2\,\mu F} = 4\,V.$

$V_2 = \frac{Q_2}{C_2} = \frac{16\,\mu C}{4\,\mu F} = 4\,V.$

Note $V_1 = V_2 = 6.0\,V - V_3$

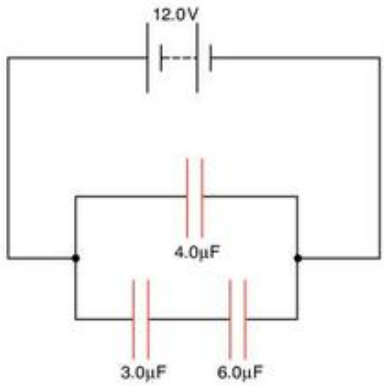


Fig 7

Worked example 2

A 3.0μF capacitor and a 6.0μF capacitor are in series with each other. The series combination is in parallel with a 4.0μF capacitor and a 12.0V battery, as shown in Figure 7. Calculate:

a i the total capacitance of the combination,
ii the charge stored by the combination,
b the charge and p.d. for each capacitor.

Solution

a i $\frac{1}{C'} = \frac{1}{3.0} + \frac{1}{6.0} = 0.5\,\mu F^{-1}$, $\therefore C' = \frac{1}{0.5} = 2.0\,\mu F$

$\therefore$ the total capacitance $C = C' + C_3 = 2.0 + 4.0 = 6.0\,\mu F$

ii The charge stored $Q = CV = 6.0 \times 12.0 = 72\,\mu C$

b C_3; p.d. V_3 = battery p.d. = 12V

charge stored $Q_3 = C_3 V_3 = 4.0\,\mu F \times 12.0\,V = 48\,\mu C.$

C_1 and C_2; charge stored by the combination $= Q - Q_3 = 72 - 48 = 24\,\mu C$

They store the same amount of charge as they are in series with each other.

$\therefore Q_1 = Q_2 = 24\,\mu C$

$\therefore V_1 = \frac{Q_1}{C_1} = \frac{24\,\mu C}{3.0\,\mu F} = 8.0\,V.$

$V_2 = \frac{Q_2}{C_2} = \frac{24\,\mu C}{6.0\,\mu F} = 4.0\,V.$

Note $V_1 + V_2 = V_3 = 12\,V$

EXTENSION

More about dielectrics

When a dielectric is placed between oppositely charged metal plates, the electrons in each dielectric molecule are pulled a little towards the positive plate. Each molecule of the dielectric becomes **polarised**, as shown in Figure 8. The surface of the dielectric facing the positive plate gains a layer of negative charge due to polarisation and the surface facing the negative plate gains positive charge. The p.d. between the plates is equal to the battery p.d. and is constant. As a result,

- the positive charge on the dielectric surface facing the negative plate attracts electrons from the battery onto the negative plate
- the negative charge on the dielectric surface facing the positive plate pushes electrons back to the battery from the negative plate.

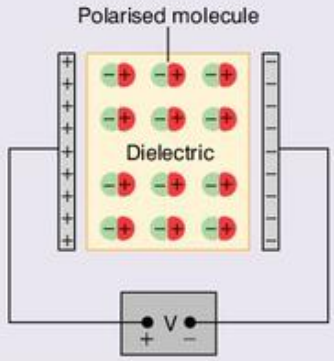


Fig 8 Dielectrics

So the dielectric increases the charge stored on the plates without changing the plate p.d. In other words, the dielectric increases the capacitance of the parallel plates.

The relative permittivity ε_r or dielectric constant, may be defined as

$$\varepsilon_r = \frac{C}{C_0}$$

where C is the capacitance with a dielectric completely filling the space between the plates and C_0 is the capacitance with empty space between the plates.

Typical values for ε_r are 7 for mica, 2.3 for polythene, 2.7 for paper and 81 for water.

The capacitance C of a pair of parallel metal plates depends on:

- the perpendicular distance d between the plates,
- the surface area A of the plates,
- the presence of a dielectric between the plates.

For a constant p.d. V between the plates, it can be shown using Coulomb's law, as outlined in topic 14.1, that in the absence of a dielectric between the plates, the electric field strength $E = \frac{Q}{A\varepsilon_0}$, where Q is the magnitude of the charge on each plate Q.

Since $E = \frac{V}{d}$, rearranging $\frac{V}{d} = \frac{Q}{A\varepsilon_0}$ gives the following formula for the capacitance C_0 of the 'empty' parallel plates:

$$C_0 = \frac{A\varepsilon_0}{d}$$

If the space between the plates is completed filled with a dielectric of relative permittivity ε_r, the capacitance $C = \varepsilon_r C_0 = \frac{A\varepsilon_r\varepsilon_0}{d} = \frac{A\varepsilon}{d}$ where the permittivity of the dielectric $\varepsilon = \varepsilon_r\varepsilon_0$.

Note ε_0 may be determined by measuring the capacitance C_0 of 'empty' parallel plates of known area A and known spacing d. then using the equation for C_0 above to calculate ε_0.

QUESTIONS

1 A 3.0 V battery is connected to a 2.0 μF capacitor in parallel with a 3.0 μF capacitor. Sketch the circuit diagram and calculate:

a the combined capacitance of the two capacitors,

b the charge stored and the p.d. across each capacitor.

2 a A 4.5 V battery is connected to a 6.0 μF capacitor in series with a 4.0 μF capacitor. Calculate:

i the combined capacitance of the two capacitors,

ii the charge stored and the p.d. across each capacitor.

b A 10 μF capacitor is connected in parallel with the 6.0 μF capacitor.

i Sketch the new circuit diagram and calculate the total capacitance of the three capacitors.

ii Calculate the charge and p.d. across each capacitor.

3 A student is given 3 capacitors of values 10 μF, 22 μF and 47 μF. Sketch the 8 different possible combinations using all three capacitors and calculate the capacitance of each combination.

4 A 4.0 μF capacitor in series with a 10.0 μF capacitor are connected to a 6.0 V battery. A 2.0 μF capacitor is then connected to the battery in parallel with the two capacitors in series.

a Sketch the circuit diagram for this arrangement and calculate its total capacitance.

b Calculate the charge and p.d. for each capacitor in the arrangement.

15.3 Energy stored in a charged capacitor

On these pages you will learn to:

- Derive from the area under a potential–charge graph the equations $E = \frac{1}{2}QV = \frac{1}{2}CV^2$ for the energy stored in a capacitor
- Solve problems using the above equations

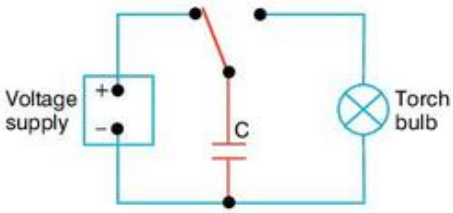


Fig 1 Releasing stored energy

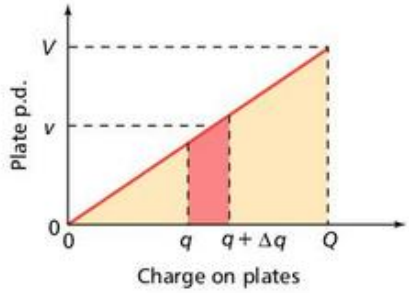


Fig 2 Energy stored in a capacitor

When a capacitor is charged, energy is stored in it. A charged capacitor discharged across a torch bulb will release its energy in a brief flash of light from the bulb, as long as the capacitor has been charged initially to the operating p.d. of the bulb. Charge flow is rapid enough to give a large enough current to light the bulb, but only for a brief time. The bulb could be replaced by a miniature electric motor which would spin briefly when the capacitor is discharged through it.

How much energy is stored in a charged capacitor? The charge is forced onto the plates by the battery. In the charging process, the p.d. across the plates increases in proportion to the charge stored, as shown in Figure 2.

Consider one step in the process of charging a capacitor of capacitance C when the charge on the plates increases by a small amount Δq from q to $q + \Delta q$. The work done ΔW to force the extra charge Δq on to the plates is given by $\Delta W = v\Delta q$, where v is the average p.d. during this step. $v\Delta q$ is represented in Figure 2 by the area of the vertical strip of width Δq and height v under the line. Therefore, the area of this strip represents the work done ΔW in this small step.

Now consider all the small steps from zero p.d. to the final p.d. V. The total work done W is obtained by adding up the work done for each small step. In other words, W is represented by the total area under the line from zero p.d. to p.d. V. As this area is a triangle of height V and base length Q ($= CV$), the total work done W = triangle area = $\frac{1}{2} \times$ height $\times$ base = $\frac{1}{2}VQ$

Because the energy stored in the capacitor, or **capacitor energy**, is equal to the work done on it to charge it, the energy stored = $\frac{1}{2}VQ$.

Energy stored by the capacitor, $W = \frac{1}{2}QV$

Note Using $Q = CV$, the above equation may be written as $W = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$

Measuring the energy stored in a charged capacitor

A joulemeter is used to measure the energy transfer from a charged capacitor to a light bulb when the capacitor discharges. The capacitor p.d. V is measured and the joulemeter reading recorded before the discharge starts. When the capacitor has discharged, the joulemeter reading is recorded again. The difference of the two joulemeter readings is the energy transferred from the capacitor during the discharge process. This is the total energy stored in the capacitor before it discharged. This can be compared with the calculation of the energy stored using $W = \frac{1}{2}CV^2$.

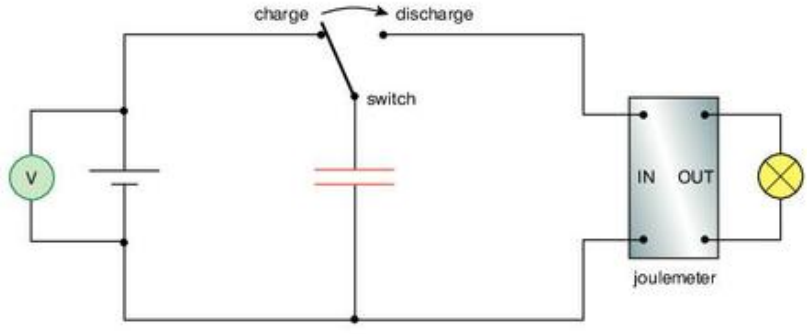


Fig 3 Measuring energy stored

The energy stored in a thundercloud

Imagine a thundercloud and the Earth below like a pair of charged parallel plates. Because the thundercloud is charged, a strong electric field exists between the thundercloud and the ground. The potential difference between the thundercloud and the ground, $V = Ed$, where E is the electric field strength and d is the height of the thundercloud above the ground.

- For a thundercloud carrying a constant charge Q, the energy stored $W = \frac{1}{2}QV = \frac{1}{2}QEd$.
- If the thundercloud is forced by winds to rise to a new height d', the energy stored $W' = \frac{1}{2}QEd'$.
- As the electric field strength is unchanged (since it depends on the charge per unit area), therefore, the increase in the energy stored $= W' - W = \frac{1}{2}QEd' - \frac{1}{2}QEd = \frac{1}{2}QE\,\Delta d$, where $\Delta d = d' - d$.

This increase in the energy stored is because work is done by the force of the wind to overcome the electrical attraction between the thundercloud and the ground and make the charged thundercloud move away from the ground.

The insulating property of air breaks down if it is subjected to an electric field strength more than about $300\,\text{kV m}^{-1}$. Prove for yourself that, for every metre rise of the thundercloud carrying a maximum charge of 20 C, the energy stored would increase by 3 MJ. At a height of 500 m, the energy stored would be 1500 MJ.

QUESTIONS

1 Calculate the charge and energy stored in a 10 μF capacitor charged to a p.d. of:

a 3.0 V,

b 6.0 V.

2 A 50000 μF capacitor is charged from a 9 V battery then discharged through a light bulb in a flash of light lasting 0.2 s. Calculate:

a the charge and energy stored in the capacitor before discharge,

b the average power supplied to the light bulb.

3 A 2.2 μF capacitor is connected in series with a 10 μF capacitor and a 3.0 V battery. Calculate the charge and energy stored in each capacitor.

4 In Figure 4, a 4.7 μF capacitor is charged from a 12.0 V battery by connecting the switch to X. The switch is then reconnected to Y to charge a 2.2 μF capacitor from the first capacitor.

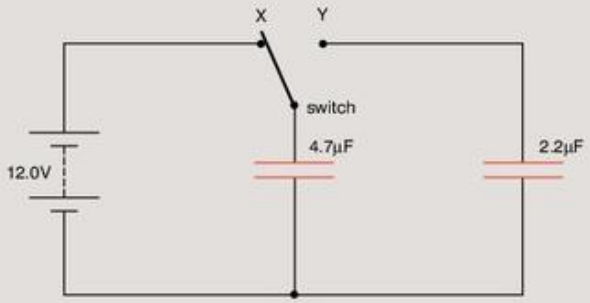


Fig 4

Calculate:

a the initial charge and energy stored in the 4.7 μF capacitor,

b the combined capacitance of the two capacitors,

c the final p.d. across the two capacitors,

d the final energy stored in each capacitor. Account for the loss of energy stored.

EXTENSION

15.4 Capacitor discharge

On these pages you will learn to:

- Draw a graph to show how the p.d. across a capacitor decreases as the capacitor discharges through a fixed resistor
- Recall and use the equation $V = V_0 e^{-t/RC}$
- Explain what is meant by the time constant of a circuit that has a capacitor in series with a resistor
- Recall and use the time constant formula $\tau = RC$
- Show an understanding of the function of capacitors in simple circuits

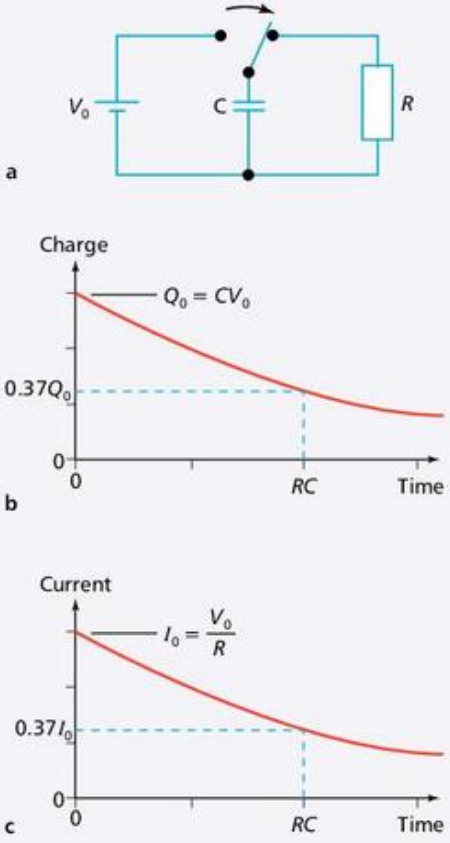


Fig 1 *Capacitor discharge*

When a capacitor discharges through a fixed resistor, the discharge current decreases gradually to zero. Figure 1 shows a circuit in which a capacitor is discharged through a resistor when the switch is changed over. The reason why the current decreases gradually is that the p.d. across the capacitor decreases as it loses charge. Because the resistor is connected directly to the capacitor, the resistor current (= p.d./resistance) decreases as the p.d. decreases.

The situation is not unlike water emptying through a pipe at the bottom of the container. When the container is full, the flow rate out of the pipe is high because the water pressure at the pipe is high. As the container empties, the water level falls so the water pressure at the pipe falls and the flow rate drops.

The graphs in Figure 1 show how the current and the charge decrease with time. Both curves have the same shape as both the current and charge (and p.d.) decrease **exponentially**. This means that any of these quantities decreases by the same factor in equal intervals of time. For example, for initial charge Q_0, if the charge is $0.9Q_0$ after a certain time t_1, the charge will be

- $0.9 \times 0.9Q_0$ after time $2t_1$,
- $0.9 \times 0.9 \times 0.9Q_0$ after time $3t_1$ …
- $0.9^n Q_0$ after time $n\,t_1$

About exponential decrease

To understand why the decrease is exponential, consider one small step in the discharge process of a capacitor C through a resistor R when the charge decreases from Q to $Q - \Delta Q$ in time Δt.

At this stage, the current $I = \dfrac{\text{p.d. across the plates, } V}{\text{resistance, } R} = \dfrac{Q}{CR}$ as $V = \dfrac{Q}{C}$

Note that the current is proportional to the charge which is proportional to the p.d. So the curves all have the same shape.

The decrease of charge $\Delta Q = -I\,\Delta t$ ($-$ as Q decreases)

Therefore, $\Delta Q = -\dfrac{Q}{CR}\Delta t$, which gives

$$\frac{\Delta Q}{Q} = -\frac{\Delta t}{CR}$$

The equation tells us that the fractional drop of charge $\dfrac{\Delta Q}{Q}$ is the same in any short interval of time Δt during the discharge process. For example, suppose $\Delta t = 10\,\text{s}$ and $CR = 100$. Then $\dfrac{\Delta Q}{Q} = -0.1 \left(= -\dfrac{\Delta t}{CR}\right)$. So the charge decreases to 0.9 of its value at the start of the 10 s interval. So, if the initial charge is Q_0, then the charge still on the plates will be

- $0.9Q_0$ after 10 s,
- $0.9 \times 0.9Q_0$ after a further 10 s,
- $0.9 \times 0.9 \times 0.9Q_0$ after a further 10 s, etc.

In theory, the charge on the plates never becomes zero.

Exponential changes occur whenever the rate of change of a quantity is proportional to the quantity itself. Rearranging the equation $\frac{\Delta Q}{Q} = -\frac{\Delta t}{CR}$ gives

$$\frac{\Delta Q}{\Delta t} = -\frac{Q}{CR}$$

For very short intervals of time (i.e. $\Delta t \to 0$), $\frac{\Delta Q}{\Delta t}$ represents the rate of change of charge and is written $\frac{dQ}{dt}$.

Therefore, $$\frac{dQ}{dt} = -\frac{Q}{CR}$$

The graphical solution to this equation is shown in Figure 1**b**.
The mathematical solution is

$Q = Q_0 e^{-t/RC}$ where Q_0 = initial charge, and
e = the exponential function (sometimes written 'exp')

The quantity RC is called the **time constant**, τ, for the circuit. At time τ after the start of the discharge, the charge falls to 0.37 ($= e^{-1}$) of its initial value.

Time constant $= RC$ where R = circuit resistance, C = capacitance

The unit of RC is the second. This is because 1 ohm = 1 volt/ampere and 1 farad = 1 coulomb/volt so the unit of RC = volt/ampere × coulomb/volt = coulomb/ampere = the second.

Notes

1 The current, the p.d. and the charge are all proportional to one another. All three quantities decrease exponentially in capacitor discharge in accordance with the equation $x = x_0 e^{-t/CR}$, where x represents either the current or the charge or the p.d.

2 The inverse function of e^x is $\ln x$, where ln is the natural logarithm. To calculate t, given x, x_0, R and C, use of the inverse function of e^x gives $\ln x = \ln x_0 - (t/RC)$.

3 The function $e^z = 1 + z + \frac{z^2}{2 \times 1} + \frac{z^3}{3 \times 2 \times 1} + \ldots$, etc. It can be shown mathematically that the rate of change of this function with respect to z is the same function. This is why the function appears whenever the rate of change of a quantity is proportional to the quantity itself. Note that $z = 1$ gives $e = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24}$ etc. = 2.718. You can check this on your calculator by keying in 'e^x' then pressing 1 to give 2.718. Keying in −1 instead of 1 gives 0.37 for e^{-1}.

Investigating capacitor discharge

Figure 2 shows how to measure the p.d. across a capacitor as it discharges through a fixed resistor. An oscilloscope is used as it has a very high resistance so the discharge current from the capacitor passes only through the fixed resistor.

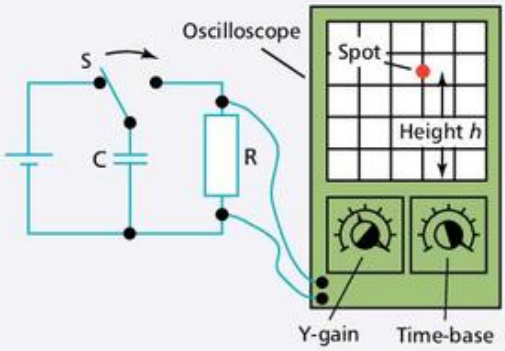


***Fig 2** Measuring capacitor discharge*

Worked example

A 2200 µF capacitor is charged to a p.d. of 9.0 V then discharged through a 100 kΩ resistor using a circuit as shown in Figure 1a.

a Calculate:
- **i** the initial charge stored by the capacitor,
- **ii** the time constant of the circuit.

b Calculate the p.d. after:
- **i** a time equal to the time constant, **ii** 300 s.

Solution

a i $Q_0 = CV_0$
$= 2200 \times 10^{-6}\,\text{F} \times 9.0\,\text{V}$
$= 2.0 \times 10^{-2}\,\text{C}$,

ii Time constant $= RC$
$= 100 \times 10^3\,\Omega \times 2.2 \times 10^{-3}\,\text{F}$
$= 220\,\text{s}$

b i When $t = RC$, $V = V_0 e^{-1}$
$= 0.37 \times 9.0 = 3.33\,\text{V}$

ii When $t = 300\,\text{s}$,
$\frac{t}{RC} = \frac{300}{220} = 1.36$
$\therefore\ V = V_0 e^{-t/RC} = 9.0 e^{-1.36}$
$= 2.3\,\text{V}$

See Mathematical skills for more about exponential decrease.

The oscilloscope is used to measure the capacitor p.d. at regular intervals. A data logger or a digital voltmeter could be used instead of the oscilloscope.

The measurements may be used to plot a graph of voltage against time. The time taken for the voltage to decrease to 37% (= 1/e) of the initial value can be measured from the graph and compared with the calculated value of RC.

The significance of the time constant

The time constant RC is the time taken, in seconds, for the capacitor to discharge to 37% of its initial charge. Given values of R and C, the time constant can be quickly calculated and used as an approximate measure of how quickly the capacitor discharges. In the worked example above, the time constant of 220 s gives a 'rule of thumb' estimate of the time taken to discharge significantly but not completely. Also, $5RC$ gives a 'rule of thumb' estimate for the time taken to discharge by over 99%. Prove for yourself that $t = 5RC$ gives a value which is less than 1% of the initial value.

Applications of capacitor discharge

1 **Any electronic timing circuit or time-delay circuit** makes use of capacitor discharge through a fixed resistor. Figure 3 shows an alarm circuit where the alarm rings if the input voltage to the electronic circuit drops below a certain value after the switch is reset. The time delay between resetting the switch and the alarm ringing can be increased by increasing the resistance R or the capacitance C. Such a change to the circuit would make the discharge of C through R slower so increasing the time for the capacitor voltage to decrease sufficiently to make the alarm ring.

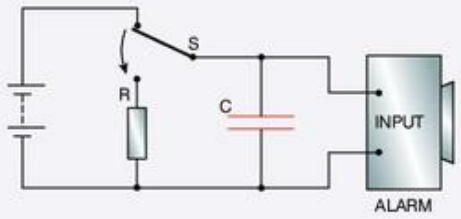


Fig 3 *A time-delayed alarm circuit*

2 **Capacitor smoothing** is used in applications where sudden voltage variations or 'glitches' can have undesirable effects. For example, mains appliances being switched on or off in a building could affect computers connected to the mains supply in the building. A large capacitor in a computer supplies current if the mains supply is interrupted so the computer circuits continue to function normally.

QUESTIONS

1 A 50 μF capacitor is charged by connecting it to a 6.0 V battery then discharged through a 100 kΩ resistor.

a Calculate:

i the charge stored in the capacitor immediately after it has been charged,

ii the time constant of the circuit.

b i Estimate how long the capacitor would take to discharge to about 2 V.

ii Estimate the resistance of the resistor that you would use in place of the 100 kΩ resistor if the discharge is to be 99% completed within about 5 s.

2 A 68 μF capacitor is charged to a p.d. of 9.0 V then discharged through a 20 kΩ resistor.

a Calculate:

i the charge stored by the capacitor at a p.d. of 9.0 V,

ii the initial discharge current.

b Calculate the p.d. and the discharge current 5.0 s after the discharge started.

3 A 2.2 μF capacitor is charged to a p.d. of 6.0 V and then discharged through a 100 kΩ resistor. Calculate:

a the charge and energy stored in this capacitor at 6.0 V,

b the p.d. across the capacitor 0.5 s after the discharge started,

c the energy stored at this time.

4 A 4.7 μF capacitor is charged to a p.d. of 12.0 V and then discharged through a 220 kΩ resistor. Calculate:

a the energy stored in this capacitor at 12.0 V,

b the time taken for the p.d. to fall from 12.0 V to 3.0 V,

c the energy lost by the capacitor in this time.

CHAPTER SUMMARY

The **capacitance** of a capacitor is defined as the charge stored per unit p.d.

The **unit of capacitance** is the farad (F), equal to 1 coulomb per volt. Note that $1\mu F = 10^{-6}$ F.

Capacitor equation $C = \dfrac{Q}{V}$

Capacitor combination rules

1 Capacitors in parallel; combined capacitance
$C = C_1 + C_2 + C_3 + \ldots$

2 Capacitors in series; combined capacitance

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

Energy stored by the capacitor, $W = \frac{1}{2}QV = \frac{1}{2}CV^2$

Capacitor discharge

1 Time constant, $\tau = RC$

2 Exponential decrease equation for current or charge or p.d.; $x = x_0\, e^{-t/RC}$

15 End of chapter questions

1 a Define the capacitance of a capacitor.

b i With the aid of a circuit diagram, describe how a capacitor may be charged using a battery and a variable resistor to keep the charging current constant.

ii The diagram below shows how the p.d. changes with time when a capacitor is charged at a constant current of 16 μA. Calculate the charge stored after 20 s and calculate the capacitance of the capacitor.

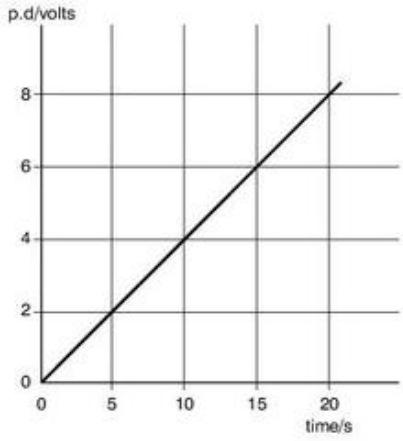


Fig 1.1

2 a A 2.0 μF capacitor is connected in series with a 4.0 μF capacitor and a 6.0 V battery.

i Sketch the circuit diagram and calculate the combined capacitance of the two capacitors.

ii Calculate the charge, p.d. and energy stored for the 2.0 μF capacitor.

b A capacitor *C* is connected in parallel with the 4.0 μF capacitor in the circuit in **a**. The p.d. across the 2.0 μF capacitor changes to 5.0 V. Sketch the new circuit diagram and calculate the capacitance of *C*.

3 A 4.0 μF capacitor is charged by connecting it to a 9.0 V battery, as shown below. The capacitor is then disconnected from the battery and connected to an uncharged 6.0 μF capacitor.

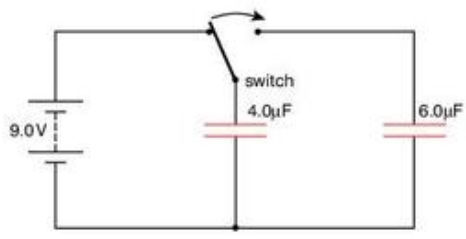


Fig 3.1

a Calculate the charge and energy stored in the 4.0 μF capacitor when the p.d. across its terminals was 9.0 V.

b The p.d. across the two capacitors becomes the same when they are connected together.

i Calculate the combined capacitance of the two capacitors.

i Show that the p.d. across the two capacitors is 3.6 V after they have been connected together.

iii Calculate the total energy stored by the two capacitors after they have been connected together and explain why it differs from the initial energy stored.

4 The flashlight circuit of a camera includes two 47 000 μF capacitors in parallel with each other. The capacitors are charged using a 4.5 V battery. A flash of light is emitted from the flashbulb when the charged capacitors are connected across the flashbulb.

a i Calculate the energy stored in the capacitor when the p.d. across its terminals is 4.5 V.

ii The duration of the discharge is approximately 50 ms. Estimate the power of the flashlight, stating any assumptions made in your estimate.

b Explain why less energy would be stored in the capacitors if they were in series with each other.

5 A capacitor *C* of capacitance 470 μF is connected to a 12 V battery then discharged through a 100 kΩ resistor *R*.

a Calculate the charge and energy stored in *C* when the p.d. across its terminals is 12 V.

b i Calculate the time constant of the discharge circuit.

ii Show that the p.d. across the capacitor decreases to 3.3 V in 60 s.

iii Calculate the energy transferred from the capacitor to the resistor when the p.d. decreases from 12 V to 3.3 V.

6 A capacitor circuit consists of 4 identical 10 mF capacitors connected in parallel to a 12 V voltage supply unit.

a Calculate the total charge and energy stored.

b The circuit is used as a 'back-up' supply in a data capture device in case the p.d. from the voltage supply unit is interrupted. The device switches off if the voltage supplied to it falls below 10 V. In normal operation, the voltage supply unit supplies a current of 80 mA at 12 V.

i Show that the circuit resistance in normal operation is approximately 150 Ω.

ii If the voltage supply unit is switched off, estimate how much longer the device would continue to operate for.

7 A solid metal sphere, of radius r, is insulated from its surroundings. The sphere has charge $+Q$.

This charge is on the surface of the sphere but it may be considered to be a point charge at its centre, as illustrated in Figure 7.1.

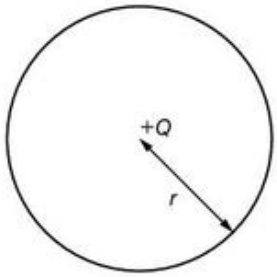


Fig 7.1

a **i** Define *capacitance*. *(1 mark)*

ii Show that the capacitance C of the sphere is given by the expression

$$C = 4\pi\varepsilon_0 r.$$

b The sphere has radius 36 cm.

Determine, for this sphere,

i the capacitance, in F. *(1 mark)*

ii the charge in C required to raise the potential of the sphere from zero to 7.0×10^5 V. *(1 mark)*

c Suggest why your calculations in **b** for the metal sphere would not apply to a plastic sphere. *(3 marks)*

d A spark suddenly connects the metal sphere in **b** to the Earth, causing the potential of the sphere to be reduced from 7.0×10^5 V to 2.5×10^5 V.

Calculate the energy dissipated in the spark, in J. *(3 marks)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 4 Q5 June 2009

16 Electronics

16.1 Electronic sensors

On these pages you will learn to:

- Show an understanding of the change in resistance with light intensity of a light-dependent resistor (LDR)
- Sketch the temperature characteristic of a negative temperature coefficient thermistor
- Show an understanding of the action of a piezo-electric transducer and its application in a simple microphone
- Describe the structure of a metal-wire strain gauge and explain how it works

When you approach an automated door, an **electronic sensor** detects your presence and causes the door to open. Sensors are used in any electronic system that responds to an external change. A list of such electronic systems would be very long and very diverse and would include everyday items such as smoke alarms and touch screens and much more sophisticated devices such as remote-controlled weather stations and robotic machines in vehicle assembly plants.

A sensor supplies a signal to a **processing unit**, which can operate an **output device** such as an indicator lamp or a relay. The processing unit may receive signals from more than one sensor and it is programmed to respond according to the signals it receives.

An electronic sensor contains a **sensing device**, which responds to an external change by producing:

- a change of p.d. because it is in a **potential divider** and its resistance changes (as in a **thermistor** or a **light-dependent resistor** or a **strain gauge** – see below), or
- a p.d. directly as in a **piezo-electric transducer** as explained below.

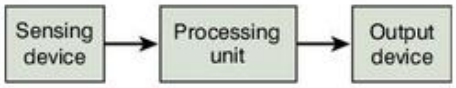


Fig 1 A sensor-operated electronic system

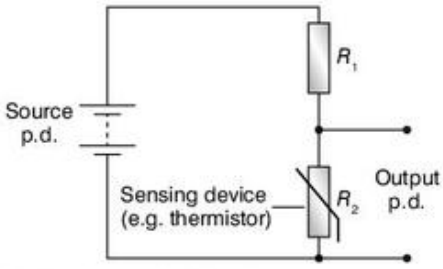


Fig 2 Using a potential divider

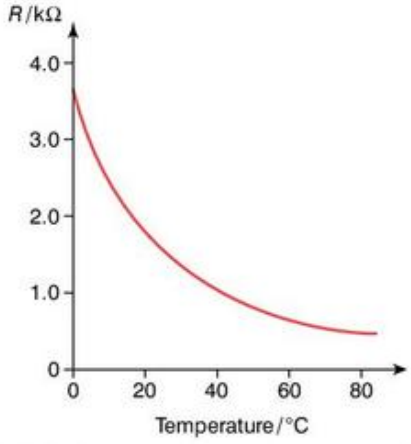


Fig 3 Resistance variation with temperature for an n.t.c. thermistor

Using a potential divider

A potential divider consists of two resistors in series, of resistances R_1 and R_2 connected to a source of fixed potential difference V_S as shown in Figure 2.

Because the current I through the resistors is equal to the source p.d. divided by the total resistance of the two resistors, $I = \frac{V_S}{R_1 + R_2}$.

The p.d. across each resistor is equal to its resistance × the current. Therefore,

the p.d. across $R_1 = IR_1 = \frac{R_1}{R_1 + R_2} \times V_S$, and

the p.d. across $R_2 = IR_2 = \frac{R_2}{R_1 + R_2} \times V_S$.

One of the resistors is a sensing device such as a thermistor. The other resistor is a fixed resistance. When the resistance of the sensing device changes, the p.d. across one resistor decreases and the p.d. across the other one increases. If R_1 and R_2 are the resistances of the fixed resistor and the sensing device respectively, as shown in Figure 2, when the resistance of the sensing device increases, the current decreases so the p.d. across the fixed resistor decreases. Therefore, the p.d. across the sensing device increases.

Note In Figure 2, the p.d. across the sensing device provides the output p.d. from the potential divider. If the resistors were interchanged in the circuit with the output p.d. provided by the p.d. across the fixed resistor, the output p.d. would increase when the resistance of the sensing device decreases.

a A temperature sensor uses a thermistor as the sensing device, as shown in Figure 2.

A thermistor is a resistor whose resistance changes significantly with change of temperature. See topic 6.5 for information about thermistors. Thermistors that have a resistance that falls with increase of temperature are said to have a

negative temperature coefficient and are referred to as n.t.c. thermistors. Figure 3 shows how the resistance of an n.t.c. thermistor decreases as the temperature increases. Notice that:

- the variation of resistance with temperature is non-linear
- the rate of change of resistance with temperature decreases as the temperature increases.

Assuming the thermistor in Figure 2 is an n.t.c. thermistor, when its temperature increases, its resistance decreases. As a result, the current in the circuit increases so the p.d. across the fixed resistance (= current × resistance of the fixed resistor) must therefore increase. The p.d. across the thermistor therefore decreases because it is equal to the source p.d. minus the p.d. across the fixed resistor. Since the output p.d. in Figure 2 is across the thermistor, the output p.d. therefore decreases.

Notice that if the output p.d. had been across the fixed resistor, the output p.d. would have increased.

b A light sensor uses a **light-dependent resistor (LDR)** as the sensing device. The resistance of an LDR decreases non-linearly when the intensity of light incident on it increases. Typically, the resistance of an LDR decreases from about 1 MΩ in darkness to about 100 Ω in sunlight.

Figure 4 shows an LDR in a potential divider.

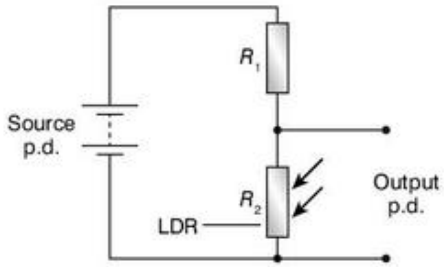
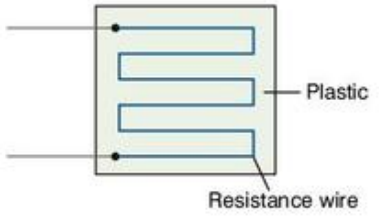


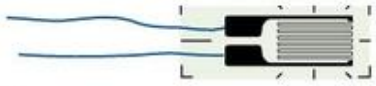
Fig 4 Using an LDR in a potential divider

- When the light intensity is increased, the resistance of the LDR decreases so the current in the circuit increases. As a result, the p.d. across the fixed resistor increases. This means that the p.d. across the LDR decreases and so the output p.d. decreases.
- When the light intensity is decreased, the current in the circuit decreases so the p.d. across the fixed resistor decreases. Hence the p.d. across the LDR increases so the output p.d. increases.

c A strain gauge in a potential divider can be used to measure or monitor the strain of a beam or the movement of cracks or other faults in structures such as buildings and bridges.



a Construction



b

Fig 5 The strain gauge

A strain gauge consists of a length of very thin wire sealed in a small plastic rectangle. The rectangle is fixed securely across the fault so that it is stretched if the fault widens. This would make the wires in the strain gauge longer and slightly thinner so increasing the resistance of the strain gauge. The resistance R of the wire in the strain gauge is given by the equation $R = \frac{\rho L}{A}$ where L is the length of the wire, A is its area of cross-section and ρ is its resistivity. See topic 6.4 if necessary.

Suppose the wire is stretched so its length increases from L to $L + \Delta L$, causing its resistance to increase from R to $R + \Delta R$. Therefore $R + \Delta R = \frac{\rho(L + \Delta L)}{A}$, assuming the area of cross-section does not change significantly.

Hence the increase of its resistance $\Delta R = \frac{\rho \Delta L}{A}$, which means that its increase of resistance is proportional to its change of length ΔL.

As the strain on wires in the strain gauge is defined as $\frac{\text{change of length } \Delta L}{\text{initial length } L}$, it follows that the increase of resistance of the strain gauge is directly proportional to the strain.

The strain gauge could be connected as the sensing device in a circuit such as Figure 2. The output p.d. could be recorded using a data logger. If the output p.d.

is found to change over time, the strain gauge must have been gradually stretched more if its resistance increased or stretched less if its resistance decreased.

Using a piezo-electric transducer

A **transducer** is any device that is designed to convert energy from one form to another. A piezo-electric transducer generates a p.d. when it is squeezed. The piezo-electric material contains positive and negative ions, which are held together by the electrostatic forces they exert on each other. When pressure is applied to opposite surfaces of the material, the centres of charge of the positive and negative ions are displaced slightly in opposite directions, causing a potential difference between the two surfaces. The effect is known as the **piezo-electric effect** and is displayed by crystals such as quartz.

To detect and measure the p.d., the opposite surfaces to which the pressure is applied must be coated with metal so an electrical connection can be made to each surface.

- **The greater the pressure is, the larger the p.d. that is generated**. A piezo-electric transducer can therefore be used directly as a pressure sensor or a force sensor. Piezo-electric ceramics are used in pressure sensors, force sensors, microphones and gas lighters. A piezo-electric rod, about 1 mm in diameter, of length 10 mm squeezed by a force of 10 N between its ends, can generate a p.d. in excess of 1000 V.

Variations in the applied pressure cause corresponding variations in the p.d. generated by the transducer. The effect is so sensitive that sound waves directed at the crystal cause it to generate an alternating p.d. This happens because the compressions and rarefactions of the sound waves cause pressure variations in the substance in which the sound waves are travelling. So a piezo-electric transducer, connected to an **amplifier** if necessary, can be used as a **microphone**.

The piezo-electric effect is reversible in that the application of a p.d. across a piezo-electric material causes the distance across the material to increase or decrease according to the polarity of the p.d. This effect is used in the ultrasonic transducer described in topic 24.1.

QUESTIONS

1 a State the physical property that causes change of resistance of:
i a thermistor, **ii** a light-dependent resistor, **iii** a strain gauge.

b In Figure 4, the output p.d. will change if there is a change in the incident light intensity on the LDR. State and explain the change in the output p.d. if the light intensity decreases.

2 A potential divider consists of a thermistor and a 1.0 kΩ resistor in series with each other connected to a 6.0 V battery, as shown in Figure 2. Assume the variation of the thermistor resistance with temperature is as shown in Figure 3.

a Calculate the output p.d. of the potential divider when the temperature of the thermistor is 40 °C.

b Determine the temperature of the thermistor when the output p.d. is 4.0 V.

3 A 150 Ω strain gauge and a 100 Ω resistor are connected in series to a 3.0 V battery.

a Calculate the p.d. across the fixed resistor.

b Calculate the change of p.d. across the fixed resistor when the strain gauge is subjected to a 2% strain.

4 a i State what is meant by the piezo-electric effect.
ii Describe how a piezo-electric microphone works.

16.2 Operational amplifiers

On these pages you will learn to:

- Recall the main properties of the ideal operational amplifier (op-amp)
- Describe and explain the use of an operational amplifier as a comparator
- Show an understanding of the effects of negative feedback on the gain of an operational amplifier
- Recall the circuit diagrams for both the inverting and the non-inverting amplifier for single signal input
- Show an understanding of virtual earth and derive an expression for the gain of inverting amplifiers
- Recall and use expressions for the voltage gain of inverting and of non-inverting amplifiers

Operational amplifier characteristics

As outlined earlier, the output p.d. from an electronic sensor is supplied to a processing unit which operates an output device. The processing unit may be designed to compare the output p.d. from the sensor with another p.d. or it may need to amplify or invert the output p.d. from the electronic sensor.

An **amplifier** is a circuit designed to produce an output p.d. that is proportional to the input p.d. supplied to it. The **voltage gain** of an amplifier is the ratio of the output p.d. to the input p.d. In effect, the amplifier multiplies the input p.d. by a constant (i.e. the p.d. gain). An amplifier that is used for a specific operation is referred to as an **operational amplifier**. Different types of operational amplifiers (op-amps) include:

- comparators that compare two different p.d.s,
- inverting amplifiers that invert a p.d. and multiply it by a constant,
- non-inverting amplifiers that multiply a p.d. by a positive constant.

An operational amplifier is an integrated circuit that includes transistors, resistors and capacitors formed on a small silicon chip. It has two input terminals and a single output terminal and needs a power supply with a positive, a zero and a negative line (e.g. +15 V, 0 V and −15 V for a 741 op-amp). The p.d.s applied to the input terminals and the output p.d. are measured relative to the 0 V (i.e. earthed) line of the power supply. The power supply and its terminals are not normally shown on circuit diagrams.

The input terminals are referred to as the inverting input, P, and the non-inverting input, Q. The output p.d. has:

- the opposite polarity to the p.d. applied at P if Q is earthed,
- the same polarity as the p.d. applied at Q if P is earthed.

For p.d.s V^- and V^+ applied to inputs P and Q respectively,

the output p.d. $\quad V_{OUT} = A_0(V^+ - V^-)$,

where A_0 is referred to as the **open-loop gain** of the op-amp and is its gain with no additional components added. Figure 2 shows the open-loop characteristics as a graph of V_{OUT} against $(V^+ - V^-)$.

Notes

1 The open-loop gain of an op-amp for direct p.d.s is typically of the order of 10^5. This is represented on the graph by the sloped section. An ideal op-amp would have an infinite open-loop gain.

2 The output p.d. cannot exceed the $\pm$15 V limits determined by the power supply. Therefore, on open-loop, $(V^+ - V^-)$ should not exceed about $\pm$150 μV ($= \pm 15\,V/10^5$) if the output p.d. is not to reach its limits. If it does, the output is said to be saturated.

3 An ideal op-amp has an infinite input impedance (i.e. input resistance) and zero output impedance (i.e. output resistance). A 741 op-amp has an input resistance of the order of 1 MΩ; provided its output does not saturate, the input current is therefore of the order of 10^{-10} A ($= 150\,\mu V/1\,M\Omega$).
In addition, an ideal op-amp would have:
- an **infinite bandwidth**, which means it would amplify all alternating p.d.s by the same factor regardless of the frequency of the alternating p.d.,
- an **infinite slew rate**, which means that there is no delay between a change to the input p.d. to the op-amp and the corresponding change to its output p.d.

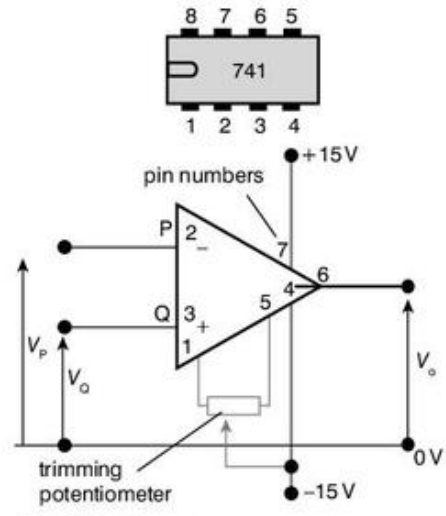


Fig 1 The 741 op-amp

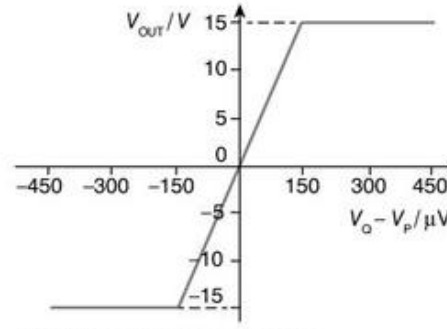


Fig 2 Open-loop characteristics

4 Provided the output p.d. is not saturated, the p.d. at P will differ from the p.d. at Q by less than 150 μV so P and Q are virtually at the same p.d. If either P or Q is earthed, the terminal that is not earthed is said to be at **virtual earth**, provided the output p.d. is not saturated.

The comparator

The open-loop op-amp is a comparator as it can be used directly to compare a p.d. applied to P with a p.d. applied to Q. Assuming the difference in the input p.d.s exceeds 150 μV, the output is either at positive saturation (i.e. the maximum positive output p.d. e.g. + 15 V) or negative saturation (i.e. the maximum negative output p.d. e.g. −15 V). In other words, if:

- the output p.d. V_{OUT} is at positive saturation, then $V^+ > V^-$,
- the output p.d. V_{OUT} is at negative saturation, then $V^- > V^+$,

The comparator may be used to provide a switched response to a gradual change of input p.d.

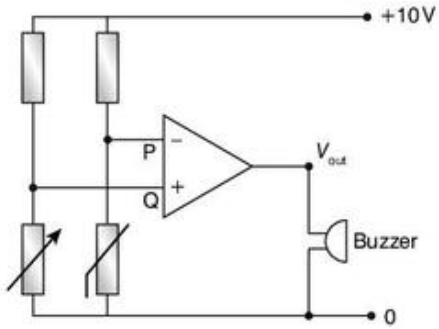


Fig 3 A temperature-operated buzzer

Figure 3 shows a temperature-operated buzzer in which the buzzer sounds if the temperature of the thermistor increases above a pre-set level. This happens because the output p.d. switches from negative to positive saturation when the p.d. at P falls below the p.d. at Q as the thermistor becomes warmer.

The p.d. at Q can be changed by changing the resistance of the variable resistor.

This alters the pre-set level at which the output p.d. switches.

- If the variable resistor is reduced, the p.d. at Q is reduced. Therefore, the thermistor needs to reach a higher temperature before the p.d. at P becomes lower than the p.d. at Q to make the output p.d. switch.
- If the variable resistor is increased, the p.d. at Q is increased. Therefore, the thermistor does not need to reach such a high temperature to make the p.d. at P lower than the p.d. at Q. So the output p.d. switches at a lower temperature.

Therefore, by setting the variable resistor to a suitable resistance, the buzzer switches on when the thermistor temperature increases to the desired temperature.

Negative feedback

Feedback is the process of taking some of the output signal from an amplifier and adding it to the input signal. The effect is to increase or decrease the output signal according to whether the feedback is negative (which reduces the output) or positive which increases the output. Figure 4 shows the basic idea applied to an amplifier.

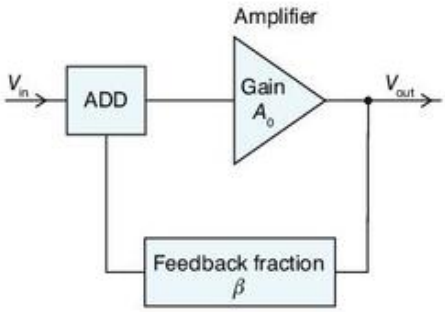


Fig 4 Feedback

In Figure 4, an input p.d. V_{in} is supplied to the amplifier and the output p.d. is V_{out}.

Without feedback, $V_{out} = A_0 V_{in}$, where A_0 is the open-loop gain of the amplifier.

With feedback, a fraction β of the output p.d. is added to the input p.d.

Therefore the total input p.d. to the amplifier with feedback $= V_{in} + \beta V_{out}$.

Hence the output p.d. $V_{out} = A_0(V_{in} + \beta V_{out})$.

Rearranging this equation gives:

the overall voltage gain $$\frac{V_{out}}{V_{in}} = \frac{A_0}{1 - \beta A_0}$$

The feedback fraction β needs to be positive so that the denominator $(1 - \beta A_0)$ in the above equation is less than 1. This will give an overall voltage gain greater than 1, as required for an amplifier.

Feedback at a chosen fraction can be supplied using selected resistors without consideration of the open-loop gain of the op-amp. This is because the above equation can be written as $\frac{V_{out}}{V_{in}} = \frac{1}{-\beta}$ provided $\beta A_0 >> 1$. As the value of A_0 is typically about 10^5, if $\beta >> 10^{-5}$, the above condition is met and overall voltage gain is independent of A_0. The open-loop gain of the op-amp does not therefore need to be considered provided $\beta >> 10^{-5}$.

The condition $\beta A_0 >> 1$ means that the gain is reduced considerably from its open-loop value by having feedback. However, as explained later in this topic, this has several benefits which are:

- a wider range for the input p.d. V_{in} without the output p.d. 'saturating' at the power supply limits,
- an increased frequency range as the frequency range is reducing when the gain is increased,
- less distortion of the output waveform compared with the input waveform,
- greater stability of the output voltage.

The inverting amplifier

To make an op-amp circuit that can amplify an input p.d. greater than 150μV without causing saturation, the voltage gain must be reduced. As explained above, this can be achieved by 'feeding back' a proportion of the output p.d. to the inverting input of the op-amp. Such feedback is negative feedback because it reduces the voltage gain. Figure 5 shows how negative feedback can be achieved in an op-amp circuit.

- The two resistors form a potential divider between the output terminal of the op-amp and the input terminal.
- The potential divider is used to provide a fixed proportion of the p.d. between the output and the input to the inverting input of the op-amp. With the non-inverting input earthed, it can be shown that the voltage gain is given by:

$$\textbf{Voltage gain} \qquad \frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = -\frac{\mathbf{R_f}}{\mathbf{R_{in}}}$$

where R_f is the feedback resistor and R_{in} is the input resistor.

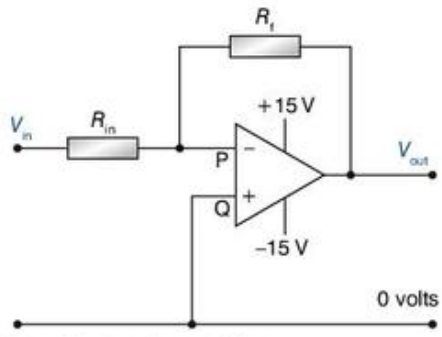


Fig 5 The inverting amplifier

To prove the voltage gain $= -R_f/R_{in}$, consider the inverter amplifier circuit in Figure 5. All p.d.s are measured relative to the zero volt line. For an input p.d. V_{in}, current I passes between the voltage source at the input and the output of the op-amp through the two resistors. Assuming the output p.d. is not saturated, no current enters the op-amp at P because the input resistance of the op-amp is very high and P is at virtual earth (i.e. $V^- \approx 0$). Therefore, as $V^- \approx 0$,

- $V_{in} - V^- = IR_{in}$ gives $V_{in} = IR_{in}$
- $V^- - V_{out} = IR_f$ gives $V_{out} = -IR_f$

Hence the voltage gain $= \frac{V_{out}}{V_{in}} = -\frac{IR_f}{IR_{in}} = -\frac{R_f}{R_{in}}$

Notes

1 The voltage gain of the inverter amplifier depends only on the resistances R_{in} and R_f and is independent of the open-loop voltage gain, A_0.
2 The above formula applies to input p.d.s that are direct voltages and alternating voltages up to a certain frequency as explained at the end of this section.
3 The inverting amplifier gives an output p.d. which is inverted compared with the input p.d. In other words, when the input p.d. is positive, the output p.d. will be negative and when the input p.d. is negative the output p.d. will be positive.

EXTENSION

The summing amplifier

Figure 6 shows an inverting amplifier with three input terminals and three input resistors. Because P is at virtual earth, the current through each input resistor is equal to the corresponding input voltage ÷ input resistance.

Provided the output voltage does not saturate, the current I through the feedback resistor is equal to the sum of the currents through the feedback resistors. Hence

$$I = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

Therefore the output p.d. $V_{out} = -IR_f = -\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)R_f$

Choosing input resistances of equal resistance R therefore gives

$$V_{out} = -(V_1 + V_2 + V_3) \times \frac{R_f}{R}$$

Therefore, **the output voltage = – the sum of the input voltages** $\times \frac{R_f}{R}$

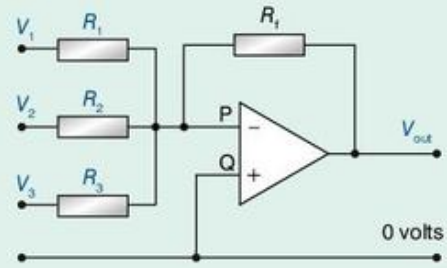


Fig 6 *The summing amplifier*

The non-inverting amplifier

A non-inverting amplifier amplifies an input p.d. greater than 150 μV without inverting the input p.d. Figure 7 shows how this is achieved.

- The two resistors form a potential divider between the output terminal of the op-amp and 0 volts.
- This potential divider is used to provide a fixed proportion of the output p.d. to the inverting input of the op-amp. With the non-inverting input earthed, it can be shown that the voltage gain is given by:

Voltage gain $\frac{V_{out}}{V_{in}} = 1 + \frac{R_f}{R_{in}}$

where R_f is the feedback resistor and R_{in} is the input resistor.

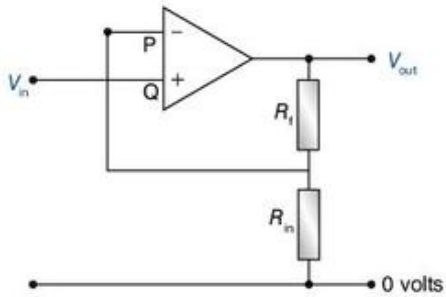


Fig 7 *The non-inverting amplifier*

To prove the voltage gain = $1 + \boldsymbol{R_f/R_{in}}$, consider the non-inverter amplifier circuit in Figure 7.

For an input p.d. V_{in}, current I passes between the output of the op-amp and the zero volt line through the two resistors. Assuming the output voltage is not saturated, no current enters the op-amp at P because the input resistance of the op-amp is very high and P is virtually at the same p.d. as Q (i.e. $V^- \approx V_{in}$). Therefore:

- $V_{out} - V^- = IR_f$ gives $V_{out} - V_{in} = IR_f$,
- $V^- - 0 = IR_{in}$ gives $V_{in} = IR_{in}$.

Hence $V_{out} - V_{in} = V_{in}R_f/R_{in}$

Rearranging this equation gives voltage gain $\frac{V_{out}}{V_{in}} = 1 + \frac{R_f}{R_{in}}$

The frequency response of an operational amplifier

For an alternating voltage as the input voltage, the voltage gain is independent of frequency up to a certain frequency and decreases at higher frequencies, as shown in Figure 8.

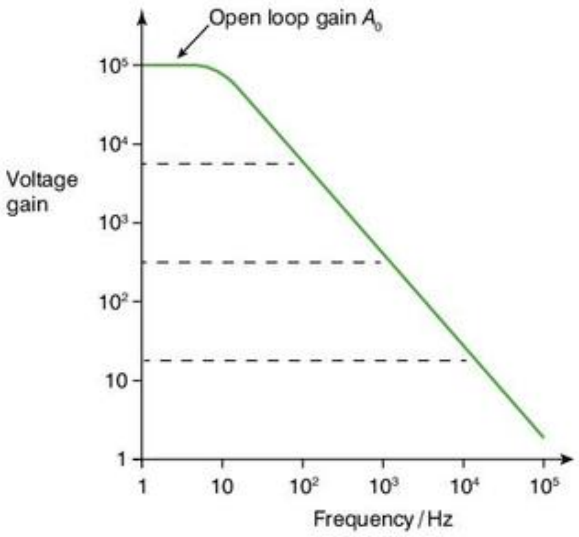


Fig 8 *Frequency response*

The **bandwidth** is the frequency range over which the voltage gain is constant.

Figure 8 shows that the bandwidth decreases as the voltage gain increases. For example, the bandwidth at a voltage gain of 10 is approximately 10^5 Hz whereas at a voltage gain of 10^4, the bandwidth is approximately 100 Hz.

EXTENSION

The voltage follower

The voltage follower shown in Figure 9 gives an output voltage equal to the input voltage. Because the output terminal is connected to the inverting input terminal P, the feedback factor $\beta = -1$. Therefore, as the open loop gain A_0 is very high, the voltage gain $\frac{V_{out}}{V_{in}} = \frac{A_0}{1 - \beta A_0} \cong 1$.

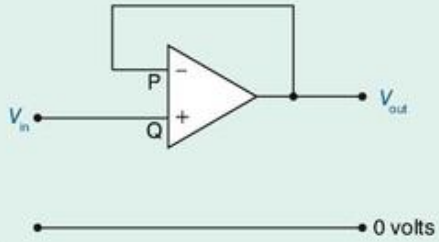


Fig 9 *The voltage follower*

The voltage follower has a very high input resistance. With a voltmeter at its output, it can therefore be used to measure voltages, drawing negligible current from the circuit under test.

QUESTIONS

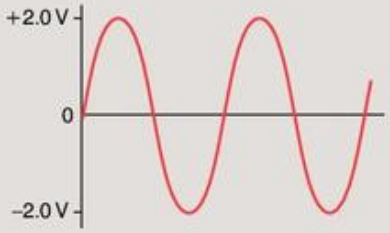
1 a When the buzzer in Figure 3 is off, state whether the p.d. at P is greater than or smaller than the p.d. at Q.

b When the temperature of the thermistor in Figure 3 changes, the buzzer switches on at a certain temperature. State and explain whether this temperature is higher or lower than the initial temperature of the thermistor.

2 a Draw the circuit diagram for an inverting amplifier.

b If the feedback resistance of an inverter amplifier is 1.0 MΩ, calculate the input resistance that will give a voltage gain of −4.

3 A sinusoidal alternating voltage with a peak voltage of 2.0 V, as shown below, is applied to the input of a non-inverting operational amplifier which has a voltage gain of × 5.



a Draw graphs on the same axes to show how the input p.d. and the output p.d. vary with time.

b If the output p.d. saturates at ± 15 V, draw a graph on the same axes to show how the output p.d. varies with time if the peak voltage of the input p.d. is increased to 4.0 V.

4 a Explain what is meant by negative feedback in an amplifier.

b Describe and explain how the output p.d. in question 3b would change if the feedback resistor was removed.

16.3 Output devices

On these pages you will learn to:

- Show an understanding that an output device may be required to monitor the output of an op-amp circuit
- Describe the use of relays in electronic circuits
- Describe the use of light-emitting diodes (LEDs) to indicate the state of the output of electronic circuits
- Recognise the need for calibration where digital or analogue meters are used as output devices

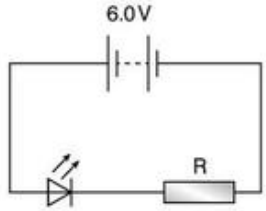


Fig 1 Forward biased LED

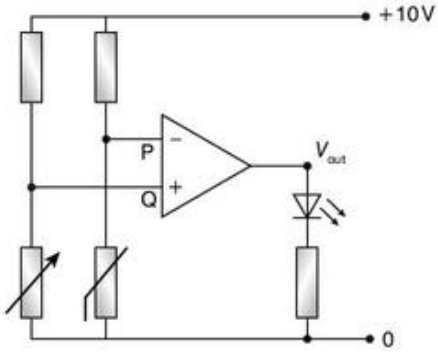


Fig 2 Using an LED as an output indicator

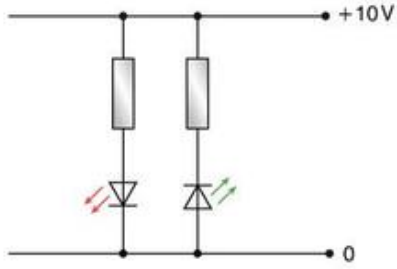


Fig 3 Using two LEDS

Light emitting diodes (LEDs)

A diode conducts current in one direction only. This happens when one of its two terminals, referred to as its anode, is positive and the other, its cathode, is negative. When a diode conducts, it is said to be **forward-biased**. If the p.d. across it is reversed so its anode is negative and its cathode is positive, it does not conduct and is said to be **reverse-biased**.

A light-emitting diode emits light when it is 'forward-biased' and a current passes through it in its forward direction. Hence it may be used to indicate the output state of an op-amp comparator or a digital circuit. Unlike a filament lamp, an LED emits light of a characteristic colour which depends on the type of LED. Also, it consumes less power than a filament lamp and it is more robust.

The electrical characteristics of an LED are similar to that of a semiconductor diode (as described in topic 6.5) in that:

- An LED conducts in one direction only provided it is forward-biased and the p.d. across it is greater than a certain non-zero value. This value, its **forward voltage**, is greater than the value of about 0.6 V for an ordinary diode and depends on the type of LED.
- An LED has a **maximum forward current** which, if exceeded, overheats and destroys the LED.
- An LED does not conduct in its reverse direction unless the applied p.d. exceeds a certain value, referred to as the **reverse breakdown voltage**. Provided the applied p.d. is less than this value, the resistance of an LED in the reverse direction is very large and the current through the LED is negligible.

To limit the current through an LED to no more than the maximum forward current, a resistor needs to be connected in series with the LED. The resistance of the resistor is chosen to limit the current to less than the maximum forward current of the LED. The resistance can be calculated if the LED characteristics are known and the p.d. to be applied to the resistor and LED is known. For example, in Figure 1, the LED is in series with a 6.0 V battery. If the maximum forward current through the LED is 5 mA and its forward voltage is 1.8 V, the p.d. across the resistor R is 4.2 V (= 6.0 − 1.8 V). Therefore, to limit the current to no more than 5 mA, the resistance of the resistor must be at least 840 Ω (= 4.2 V ÷ 0.005 A). See topic 7.4 for similar diode calculations.

An LED used as an indicator in an op-amp circuit needs to be in series with a limiting resistor. In the comparator circuit shown in Figure 2, the LED and the resistor are connected between the output terminal of the op-amp and the zero volt line of the op-amp power supply.

As explained on p.238, the output p.d. of the op-amp is either at positive saturation or negative saturation depending on whether or not the p.d. at the non-inverting input is greater than or less than the p.d. at the inverting input.

- If the LED is on, the output p.d. of the op-amp must be positive. Therefore, the p.d. at the non-inverting input (Q) must be greater than the p.d. at the inverting input (P).
- If the LED is off, the output p.d. of the op-amp must be negative. Therefore, the p.d. at the non-inverting input (Q) must be less than the p.d. at the inverting input (P).

In Figure 2, the p.d. at P and the p.d. at Q are provided by separate potential dividers.

- The p.d. at P depends on the temperature of the thermistor.
- The p.d. at Q can be adjusted by altering the resistance of the variable resistor.

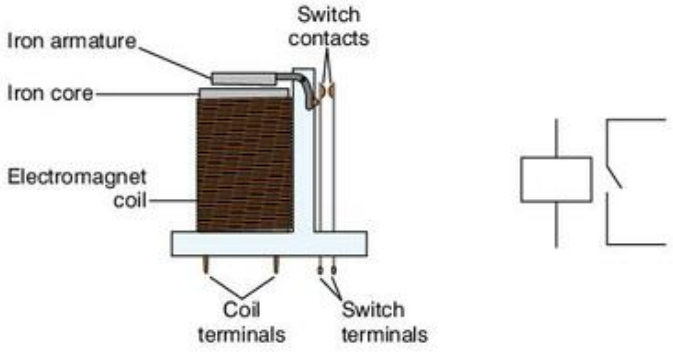
Suppose the LED is initially **off** which means that the p.d. at P is more than the p.d. at Q initially. If the temperature of the thermistor is increased continually, the thermistor resistance decreases and the p.d. at P decreases. When the p.d. at P becomes less than the p.d. at Q, the LED switches **on**.

Notes

1 The variable resistor can be adjusted so that the LED switches on at a different temperature. If the resistance of the variable resistor had been greater initially, the p.d. at Q would have been greater. The p.d. at P would not have had to decrease as much to switch the LED on so the temperature of the thermistor would not have had to increase as much.
2 Reversing the LED in Figure 2 would result in the LED being **on** initially and switching **off** at a certain temperature as the thermistor temperature is increased.
3 Two LEDS of different colours could be used in opposite directions in parallel, as in Figure 3, to show if the output p.d. is positive or negative. This would be useful if the op-amp also switches a relay on or off.

Relays

Relays are used in control circuits to switch machines on or off. Figure 4 shows the construction of a relay.



a *Construction* **b** *Circuit symbol*

Fig 4 *A relay*

- When a current passes through the coil of the electromagnet, the iron armature is attracted onto the iron core of the electromagnet.
- The armature turns about a pivot and closes the gap between the switch contacts.

In this way, a small current can be used to switch on a much greater current. For example, when the ignition switch of a car is turned on, a small current passes through the electromagnet coil so the relay switch closes. This allows a much greater current to pass through the starter motor.

The output current from an op-amp cannot be greater than about 25 mA. For this reason, an op-amp cannot be used directly to control 'high-current' devices such as heating appliances and high-power electric motors.

To control any device that needs a current of more than 25 mA, the op-amp is used to operate a relay, which is used to control the device. For example, Figure 5 shows a light-operated op-amp used to control an electric motor via a relay. Notice that the sensing device (the LDR) in this circuit is connected between the +10 V line and the inverting input P in this example.

- **When the LDR is in daylight**, the motor is off because the LDR resistance is low enough to cause a p.d. at the inverting input P of the op-amp which is greater than the p.d. at the non-inverting input Q. Therefore V_{out} is negative and the diode D_1 ensures the relay operates only when V_{out} is positive.
- **When the LDR is in darkness**, the motor is on because the LDR resistance is high enough to reduce the p.d. at the inverting input P of the op-amp to below the p.d. at the non-inverting input Q. Therefore V_{out} is positive so the relay operates.

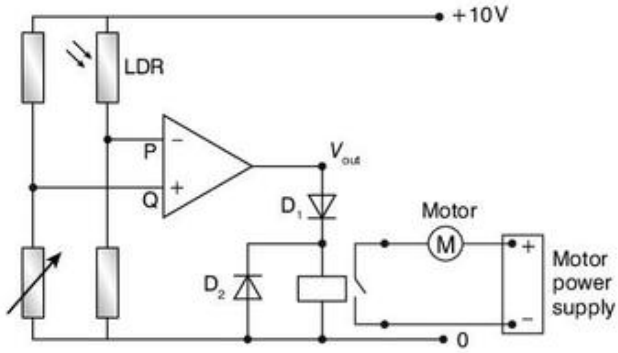


Fig 5 *Using a relay*

The level of the light intensity at which the motor switches on when the light intensity decreases can be set by adjusting the variable resistor. Note that the diode D_2 is necessary to short-circuit the back e.m.f. generated by the collapsing magnetic field in the relay when the relay is switched off. If diode D_2 was not present, this back e.m.f. would destroy the op-amp.

Making measurements

A sensing device that depends on its change of resistance can be used in a circuit to measure or monitor its characteristic physical property X (i.e. the physical property that changes its resistance). For example:

- a thermistor can be used to measure temperature,
- an LDR can be used to measure light intensity,
- a strain gauge can be used to monitor strain.

The device could be connected directly to a resistance meter or it could be in a circuit such as a potential divider with an analogue or digital voltmeter across one of its resistors. In

general, the p.d. or resistance will not vary linearly with X. The meter reading would need to be measured for a range of known values of X, its characteristic physical property, and the results plotted to give a calibration curve. The value of X for any reading on the meter could then be read from the calibration curve, as shown in Figure 6.

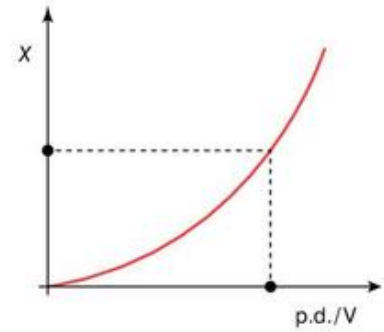


Fig 6 A calibration curve

A sensing device may be connected in an op-amp circuit. A digital or analogue voltmeter connected to the output of the op-amp can then be used to monitor the physical property that the sensing device responds to. For example, if the sensing device is a thermistor connected as shown in Figure 7, the reading on the voltmeter will change when the temperature of the thermistor changes (provided the output voltage of the op-amp does not saturate).

Notice that the op-amp is an inverting op-amp in which the input resistance is $R_{th} + R_1$ where R_{th} is the resistance of the thermistor and R_1 is the resistance of the resistor in series with the thermistor.

Therefore, the output p.d. from the op-amp,

$$V_{out} = \frac{R_f}{R_{th} + R_1} V_{in}.$$

The input p.d. V_{in} is a constant negative voltage so the output p.d. is always positive. However, the formula above shows that the voltage gain of the op-amp depends on the thermistor resistance R_{th} as well as resistances R_f and R_1. Therefore, when the temperature of the thermistor increases, the output voltage increases because the thermistor resistance decreases and the voltage gain becomes larger in accordance with the above equation. The resistances of R_f and R_1 need to be chosen to ensure the highest possible temperature to be monitored does not make the output p.d. exceed the maximum reading of the voltmeter.

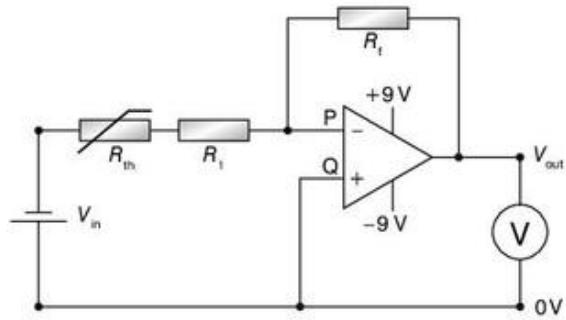


Fig 7 Using a thermistor to monitor temperature changes

QUESTIONS

1 a Explain what is meant by
- **i** the forward voltage,
- **ii** the reverse breakdown voltage of a diode.

b An LED with a forward voltage of 2.2 V is connected in series with a resistor and a 3.0 V battery of negligible internal resistance, as in Figure 1. Calculate the minimum resistance of the resistor if the current through the LED is not to exceed 2.0 mA.

2 In Figure 2, the variable resistor is then adjusted so its resistance decreases and the LED switches off. The LED then switches back on again because the thermistor temperature changes. State whether this change of temperature is an increase or a decrease and explain why it caused the LED to switch back on again.

3 a In Figure 5, state the function of each of the two diodes.

b i When the motor in Figure 5 is on, what change made to the variable resistor would switch the motor off?

ii What change made to the LDR would cause the motor to switch back on again? Explain your answer.

4 The thermistor in Figure 7 has a resistance of 50 kΩ at 20 °C and a resistance of 10 kΩ at 80 °C. The feedback resistor has a resistance of 120 kΩ, the input resistance has a resistance of 10 kΩ and the input p.d. V_{in} is 2.0 V.

a i Show that the output p.d. of the op-amp at 20 °C is 4.0 V.

ii Calculate the output p.d. at 80 °C.

b Explain why the output p.d. would not be exactly midway between the output p.d. at 20 °C and 80 °C when the thermistor temperature is 60 °C.

EXTENSION

16.4 Semiconductor devices

On these pages you will learn to:

- Explain what an intrinsic semiconductor is and what p-type and n-type semiconductors are
- Describe the structure and operation of a p–n junction diode and an npn transistor
- State the function of the following logic gates: OR, AND, NOR, NAND, NOT, EOR, EXOR, EXNOR
- Describe the function of a half-adder, a full-adder, an SR flip flop as a latch and a series of T-type flip flop as a binary counter

Intrinsic and extrinsic semiconductors

Semiconductors are used to make a wide range of electronic components including ordinary diodes, LEDs, transistors and integrated circuits. Apart from LEDs, most semiconductor devices are made from silicon, which is an abundant chemical element.

Silicon is a Group 4 chemical element and has four electrons in the outer shell of each atom. Each atom forms a covalent or 'shared' bond with each of four other atoms to form the structure of the solid. At absolute zero, the outer electrons are localised in the covalent bonds so there are no conduction electrons. Above absolute zero, some of the outer shell electrons break free from the atoms to become conduction electrons. The electron vacancies in the covalent bonds are referred to as **holes**.

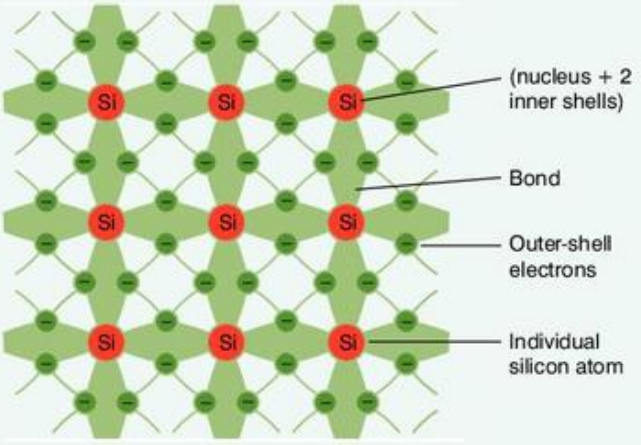


Fig 1 Silicon atoms in the solid state

Semiconductors are characterised by resistivity that decreases with increase of temperature. This happens because the vibrations of the atoms cause electrons in the outer shell of each atom to break free to become conduction electrons. An increase of temperature makes the atoms vibrate more, so more outer shell electrons break free. The resistivity of the semiconductor therefore decreases because it contains more conduction electrons per unit volume. In other words, the population density of conduction electrons is increased when the temperature is increased, so the resistivity of the material decreases.

Pure silicon is an example of an **intrinsic** semiconductor. The electrical properties of an intrinsic semiconductor can be changed by doping it with a tiny fraction of Group 3 or Group 5 atoms so it becomes an **extrinsic** or 'doped' semiconductor.

n-type extrinsic semiconductor is produced using Group 5 atoms which each have five electrons in their outer shells. Since only four electrons from the outer shell of each such atom are needed for the covalent bonds the atom forms with its neighbours, the surplus electron contributes to the conduction electrons in the material. As conduction in the material is due to electrons which are negative **charge carriers**, the material is referred to as an n-type semiconductor. It has more conduction electrons per unit volume compared with an intrinsic semiconductor of the same material at the same temperature.

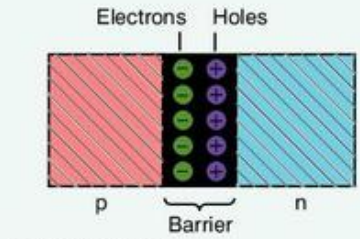
p-type extrinsic semiconductor is produced using Group 3 atoms which each have three electrons in their outer shells. Since four electrons from the outer shell of each such atom are needed for the covalent bonds the atom forms with its neighbours, there is a vacancy or 'hole' in the covalent bonds of each donor atom. The hole may be filled by a conduction electron or by an electron from a neighbouring atom. As a result, holes move about from atom to atom inside the material as if they were positive charge carriers. The material is therefore referred to as a **p-type semiconductor**. It has fewer conduction electrons per unit volume compared with an intrinsic semiconductor of the same material at the same temperature.

The p–n junction diode

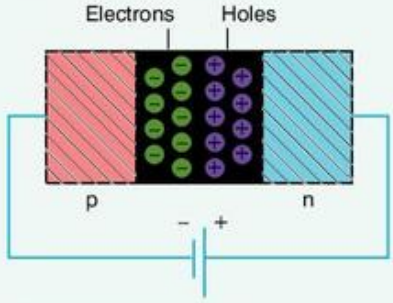
A **p–n junction diode** consists of p-type semiconductor material in contact with n-type semiconductor material. At the interface or 'junction', electrons from the n-type material diffuse into the p-type material and fill the electron vacancies in the p-type material. As a result, in an 'unbiased' or isolated p–n diode, a p.d. develops across the junction between the p-type and the n-type material, which prevents further diffusion.

- The narrow layer of p-type material next to the interface becomes negative as it gains electrons.
- The narrow layer of n-type material next to the interface becomes positive as it loses electrons.

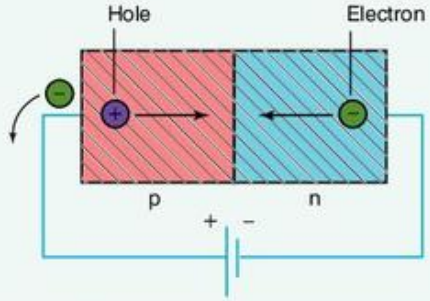
The layers are referred to as **depletion layers** because they contain fewer of the majority charge carriers than the rest of the material. If a p.d. is then applied across the p–n diode terminals such that:



a A p–n junction



b Reinforcing the barrier



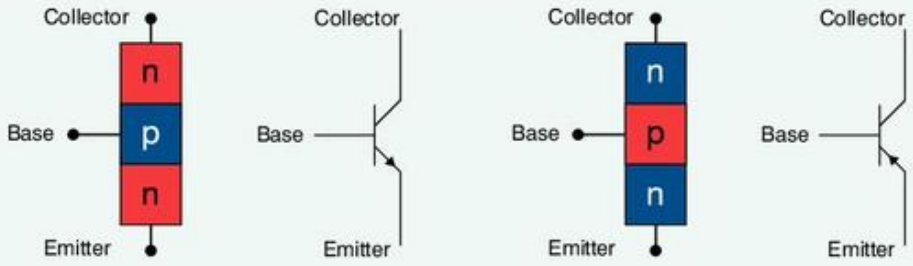
c Removing the barrier

Fig 2 *The p–n junction diode*

1 **The p-type material is made more negative and the n-type material more positive**, the depletion layer is strengthened and the junction p.d. is increased. No conduction electrons can cross the junction, which means no current can pass through the diode. The diode is then said to be **reverse-biased**.
2 **The p-type material is made positive and the n-type material negative**, the junction p.d. is reduced or eliminated and the depletion layer is weakened or removed. Conduction electrons can now cross the junction from the n-type to the p-type material which means a current now passes through the diode. The diode is then said to be **forward-biased**.

Notes

1 **Diodes** are used in rectifier circuits. See topic 19.3. The current–p.d. characteristics of a silicon p–n junction diode is shown in topic 6.5 Figure 3.
2 **LEDs** emit light from the p–n interface when they conduct. The range of wavelengths emitted by an LED depends on the semiconductor materials used to make the LED.
3 A **junction transistor** is basically two p–n junctions in opposite directions such that a piece of n-type material is between two pieces of p-type material (pnp) or a piece of p-type is between two pieces of n-type material (npn). The symbol for each type of transistor is shown in Figure 3. In each type of transistor, the current between the collector and the emitter is controlled by a much smaller current between the base and the emitter. A transistor can therefore be used as a current-operated switch or a current amplifier.
4 A **limiting resistor** should always be in series with a diode or the base and collector or emitter of a transistor. If too much current passes through a semiconductor device, excessive resistance heating of the semiconducting material occurs, destroying the diode. See topic 7.4.



a *An npn transistor* ***b*** *An pnp transistor*

Fig 3 *Transistors*

Logic gates

Electronic logic

Decisions are made by each of us all the time. Each decision involves making a choice of some sort. A logical decision is one with an outcome that depends on the 'input' conditions. For example, suppose you are about to go out and you are thinking about whether or not you should lock up your home when you go. The table below shows the possible 'input' conditions and the outcome or 'output' for each input condition. Each input and output 'state' is entered as 0 for 'No' and 1 for 'Yes' in the table. Such a table is referred to as a truth table.

Inputs		Output
Am I the only person in at the moment?	**Will I be out for more than a few minutes?**	**Should I lock up?**
0	0	0
0	1	0
1	0	0
1	1	1

1 = Yes 0 = No

A logic gate is a digital circuit designed to make a decision. Any logic gate has one or more input terminals and an output terminal. The voltage at the output terminal (i.e. the output state) depends on the voltage at the input terminals (i.e. the input states) according to the type of gate. The output state for each combination of input states can be displayed using a truth table. We don't need to consider how a logic gate works, only what it does. Figure 4 shows the circuit symbol and the truth table of some common logic gates.

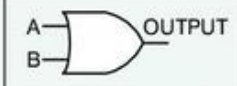
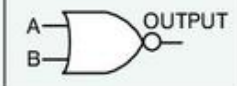
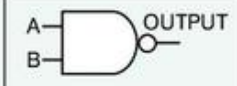
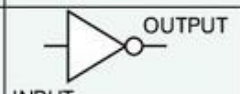
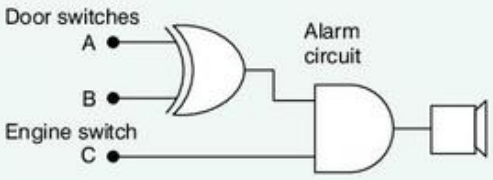
Gate	Symbol	Function (High voltage = 1, Low voltage = 0)	Truth Table INPUTS A	B	OUTPUT
OR		OUTPUT = 1 if A OR B = 1	0	0	0
			0	1	1
			1	0	1
			1	1	1
AND		OUTPUT = 1 if A AND B = 1	0	0	0
			0	1	0
			1	0	0
			1	1	1
NOR		OUTPUT = 0 if A OR B = 1	0	0	1
			0	1	0
			1	0	0
			1	1	0
NAND		OUTPUT = 0 if A AND B = 1	0	0	1
			0	1	1
			1	0	1
			1	1	0
NOT		OUTPUT = 1 if INPUT = 0 OUTPUT = 0 if INPUT = 1	0		1
			1		0
EXOR (exclusive OR)		OUTPUT = 1 if A unequal to B	0	0	0
			1	0	1
			0	1	1
			1	1	0
EXNOR (exclusive NOR)		OUTPUT = 1 if A equal to B	0	0	1
			1	0	0
			0	1	0
			1	1	1

Fig 4 *Logic gates*

An example of a logic gate in use is a door alarm for the 2-door car that sounds only when one of the two doors is opened and the engine is running. Figure 5 shows the truth table and a suitable logic gate combination for this purpose. The truth table can be summed up by the condition

the output = 1 if input A OR input B = 1 AND input C = 1



a *Using an OR and an AND gate*

INPUT			OUTPUT
A	B	C	
0	0	0	0
1	0	0	0
0	1	0	0
1	1	0	0
0	0	1	0
1	0	1	1
0	1	1	1
1	1	1	1

b *Truth table*

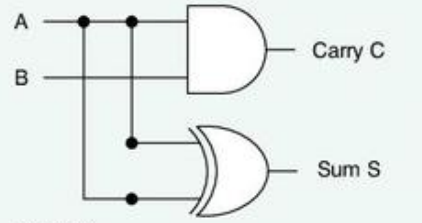
Fig 5 *A logic gate combination*

Note
NOR gates only (or NAND gates only) can be used to construct other logic gates. For example:

- A NOT gate can be constructed by joining the two inputs of a NOR gate
- An OR gate can be constructed by connecting the output of a NOR gate to the input of a NOT gate (which can be constructed from a NOR gate as explained above).

Adders

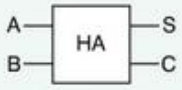
A **half-adder** is an electronic circuit that adds two **bits** to give a sum bit and a carry bit. For example, adding bits 1 and 1 gives 10 (i.e. a carry of 1 and a sum of 0). Figure 6 shows how a half-adder can be constructed from an EXOR gate and an AND gate.



a Circuit

A	B	carry	sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

b Truth table



C Symbol

Fig 6 *The half-adder*

A **full-adder** adds three bits to give sum and carry bits. One of the three input bits is usually a carry bit from another full-adder. The circuit for a full-adder is shown in Figure 7.

To understand how the full adder works, consider its truth table shown below. The three inputs, A, B and C_0 are shown in blue; the outputs from each half-adder and the OR gate are in the white columns. The final sum and carry outputs are in red. These are both 1 (or 0) only when all three inputs (shown in blue) are 1 (or 0); otherwise, the final carry is 1 and the final sum is 0 corresponding to 2 inputs only equal to 1.

Truth table for a full-adder

Half-adder 1				Half-adder 2				OR		
A	B	C_1	S_1	C_0	S_1	C_2	S_2	C_1	C_2	C
0	0	0	0	0	0	0	0	0	0	0
0	1	0	1	0	1	0	1	0	0	0
1	0	0	1	0	1	0	1	0	0	0
1	1	1	0	0	0	0	0	1	0	1
0	0	0	0	1	0	0	1	0	0	0
0	1	0	1	1	1	1	0	0	1	1
1	0	0	1	1	1	1	0	0	1	1
1	1	1	0	1	0	0	1	1	0	1

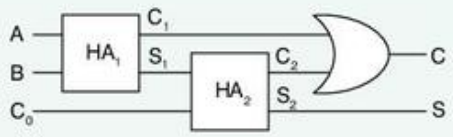


Fig 7 *A full-adder*

Flip-flops

A flip-flop is a memory circuit, also called a 'latch', because it stores a data bit. Two NAND gates or two NOR gates may be used to construct a flip-flop, as shown in Figure 8.

Inputs S and R are each connected to both inputs of NAND gates which therefore acts as a NOT gate. The truth table below gives the output state for different combinations of inputs S and R.

Table for Figure 8

State	S	R	Q	$\bar{Q}$
Indeterminate	0	0	1	0
			0	1
Set	1	0	1	0
Reset	0	1	0	1
Not used	1	1	1	1

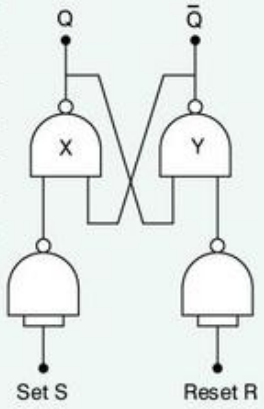


Fig 8 *A flip-flop*

When S = 0, R = 0 the outputs can be in either one of two stable states.

Step 1 Making R = 1 when S = 0 resets output Q to 0 and $\bar{Q}$ to 1. Changing R back to 0 does not alter the state of the outputs.

Step 2 Applying a data bit (either 0 or 1) to S sets Q to the same state as the data bit. Removing the data bit does not alter the state of the outputs.

Therefore, the data bit is stored at Q until the circuit is reset again.

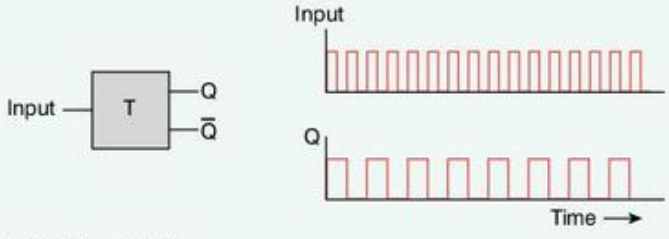


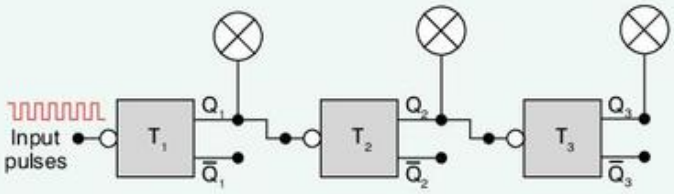
Fig 9 *A T-type flip-flop*

Counters

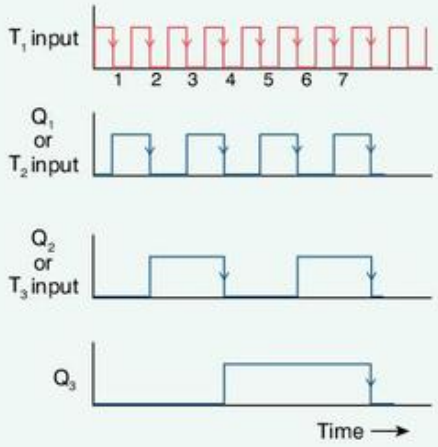
A triggered bistable or **T-type flip-flop** is designed to give an output pulse at Q (i.e. 0 to 1 to 0) for every two pulses at its input, as shown in Figure 9. In other words, the output

pulse frequency is half the input pulse frequency. Each input pulse is said to 'toggle' or change the state of the output at Q.

- Each time the input changes from 0 to 1, output Q changes from 0 to 1 or 1 to 0.
- When the input changes from 1 to 0, output Q is unchanged.
- Therefore one input pulse (0 to 1 to 0) changes Q and a second input pulse changes Q back. So two input pulses give one output pulse.



a *Circuit*



b *Input and output states*

Pulse number	Q_3	Q_2	Q_1
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

c *Binary counting*

Fig 10 *A 3-bit binary counter*

Note

The T-type flip-flop shown changes the output state when the input changes from 0 to 1 so it is called a *'rising edge'* flip-flop. T-type flip-flops that change their output state when the input changes from 1 to 0 are called *'falling edge'* flip-flops. In circuit diagrams, a falling edge flip-flop is indicated by a '0' at its input, as shown in the flip-flops used in Figure 10.

A **binary counter** is designed to count pulses and display the number counted as a binary number. It can be constructed using falling edge T-type flip-flops in series with the Q output of each one connected to the input of the next one. An indicator is connected to the Q output. Figure 10 shows a 3-bit binary counter. The table in Figure 10 shows the state of its outputs after n pulses where $n = 0$ to 7. If the counter had a fourth flip-flop with Q_3 as its input, it would be a 4-bit binary counter; the state of the outputs after 8 pulses would be $Q_4Q_3Q_2Q_1 = 1\,0\,0\,0$.

QUESTIONS

1 a Explain what is meant by an intrinsic semiconductor.

b State two differences between a p-type semiconductor and an n-type semiconductor.

2 a Explain why a p–n junction diode conducts electricity in one direction only.

b Explain why an LED emits light when it conducts.

3 Draw and complete the truth table for each of the logic gate combinations in Figure 11.

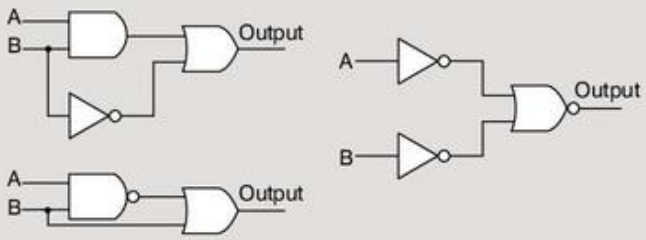


Fig 11

4 a With the aid of a diagram, explain the operation of a half-adder circuit.

b Describe the function of a full-adder.

16 End of chapter questions

1 a By reference to an amplifier, explain what is meant by *negative feedback*. *(2 marks)*

b An amplifier circuit incorporating an ideal operational amplifier (op-amp) is shown in Figure 1.1.

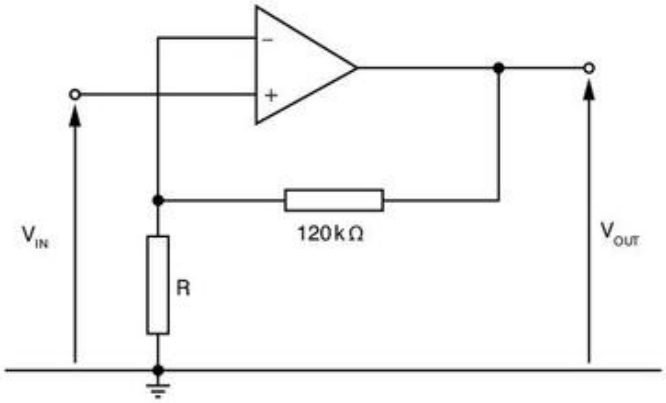


Fig 1.1

The supply for the op-amp is ± 9.0 V.

The amplifier circuit is to have a gain of 25.

Calculate the resistance of resistor R in Ω. *(2 marks)*

c State the value of the output voltage V_{OUT} of the amplifier in **b** for input voltages V_{IN} of

i − 0.08 V, *(1 mark)*

ii +0.4 V. *(1 mark)*

Adapted from Cambridge International AS and A Level Physics 9702
Paper 04 Q10 June 2009

2 The circuit of Figure 2.1 may be used to indicate temperature change.

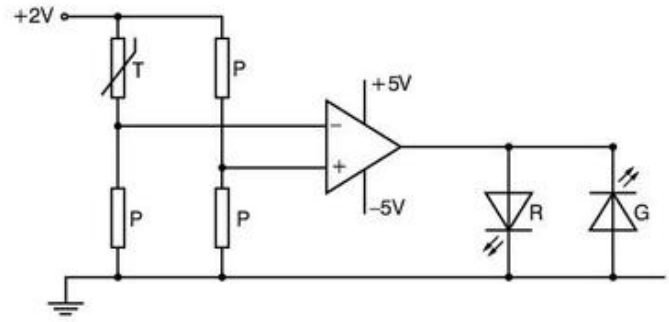


Fig 2.1

The resistance of the thermistor T at 16 °C is 2100 Ω and at 18 °C, the resistance is 1900 Ω. Each resistor P has a resistance of 2000 Ω.

Determine the change in the states of the light-emitting diodes R and G as the temperature of the thermistor changes from 16 °C to 18 °C. *(4 marks)*

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17 Communications

17.1 Communication systems

On these pages you will learn to:

- Explain the term modulation and distinguish between amplitude modulation (AM) and frequency modulation (FM)
- Recognise that a carrier wave, amplitude modulated by a single audio frequency, is equivalent to the carrier wave frequency together with two sideband frequencies
- Explain what is meant by bandwidth
- Recall the frequencies and wavelengths used in different channels of communication
- Describe the relative advantages of AM and FM transmissions

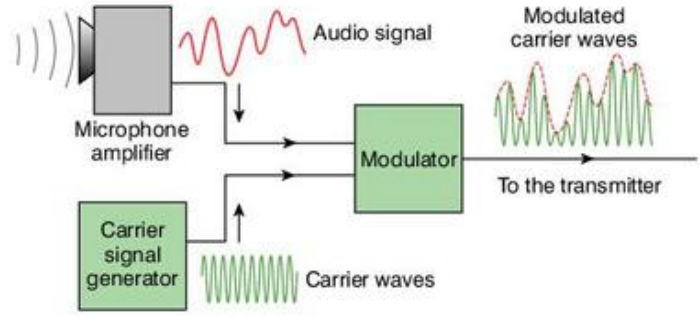


Fig 1 Amplitude modulation

Carrier waves and modulation

A communication system has a source, a receiver and a means of sending information. When you listen directly to someone, their vocal cords vibrate and produce sound waves that are 'modulated' (i.e. varied by the vocal cords) so they carry information. The sound waves travel through the air and are detected by your ears.

When you listen to a radio programme, your radio is detecting radio waves of a particular frequency transmitted by a radio station. In the studio at the radio station, sound waves are converted to electrical waves, referred to as **audio waves**, using a microphone. These waves are then used to modulate the radio waves sent out by the radio station. The radio waves from the transmitter are called **carrier waves**. They have much higher frequencies than the upper frequency limit of audio waves which is about 15 kHz. Radio stations send out modulated carrier waves rather than 'audio-frequency' radio waves because such 'audio-frequency' radio waves would:

- require very long aerials,
- have a very short range compared with carrier waves of the same intensity,
- be impossible to select from the waves sent out by other radio stations.

Audio waves are an example of an **information signal**. Further examples of information signals include TV signals, mobile phone signals and data transfer to and from computers. Information signals can be carried by electromagnetic waves or by electrical waves in the form of alternating voltages or by light waves. The signal, which could be either analogue or digital is used as described below to modulate the carrier wave.

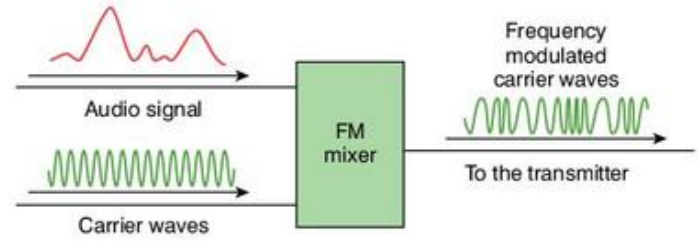


Fig 2 Frequency modulation

An **analogue signal** can have any value from zero to an upper limit. It can be carried by a wave by using the signal to vary the **amplitude** or the **frequency** of the carrier wave:

- **Amplitude modulation (AM)** The amplitude of a carrier wave of constant frequency is modulated by the amplitude of the signal to be carried. Figure 1 outlines how an analogue signal is produced using amplitude modulation.
- **Frequency modulation (FM)** The frequency of a carrier wave is modulated by the amplitude of the signal to be carried. Figure 2 outlines how an analogue signal is produced using frequency modulation.

A **digital signal** is a signal which has only two values. It can be carried by a wave by using the signal to switch the amplitude or the frequency between two different levels. See topic 17.2.

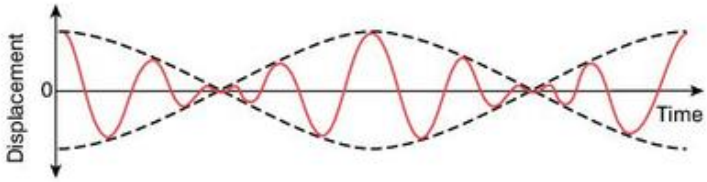
Note

Amplitude modulation and frequency modulation are terms usually applied only to analogue signals and not to digital signals.

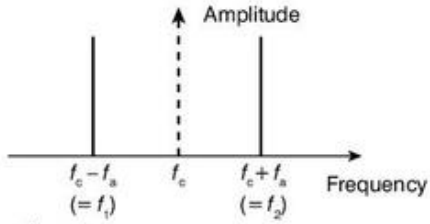
Bandwidth

Consider a carrier wave of frequency f_c modulated by an audio signal of constant frequency f_a. The carrier wave is a wave of constant frequency. Before it is used to carry a signal, its frequency spectrum is a single line at frequency f_c. To modulate it, the signal to be carried is added to the carrier wave. The frequency spectrum of the modulated carrier wave consists of the same line as before and two lines at either side at frequencies $f_1 = f_c - f_a$ and $f_2 = f_c + f_a$, as shown in Figure 3.

The reason why two side frequencies are produced is because any waveform can be resolved into sinusoidal components (or can be synthesized from sinusoidal components). The modulated carrier wave can be resolved into a sine wave of frequency $f_c - f_a$, a sine wave of frequency f_c, and a sine wave of frequency $f_c + f_a$.



a *Displacement v time*



b *The frequency spectrum*

Fig 3 *A modulated carrier wave*

Side bands

If the signal frequency varies between zero and an upper limit f_a, the frequency spectrum consists of a line corresponding to the carrier frequency, f_c, and two side bands of width f_a either side of the carrier frequency.

- The range of frequencies is referred to as the **bandwidth**. This is equal to the difference between the upper and lower sideband frequencies and is therefore equal to $2f_a$.
- The relative amplitude of the sidebands and the carrier depends on the relative amplitudes of the audio and the carrier waves. No sidebands are produced if there is no audio signal.

Notes

1 A radio or TV channel is a band of the frequency spectrum allocated to carry the channel signal. The width of the allocated band is the channel bandwidth. The signal bandwidth is restricted to fit the channel bandwidth. See Figure 4.

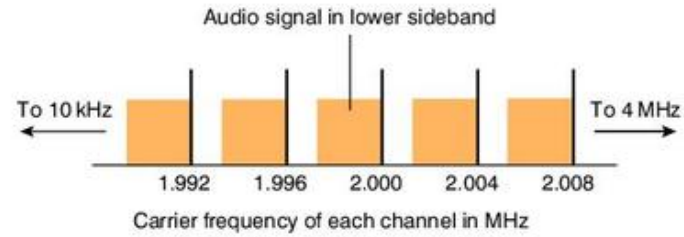


Fig 4 *Frequency channels*

2 The bandwidth of an FM signal is much greater than that of an AM signal.

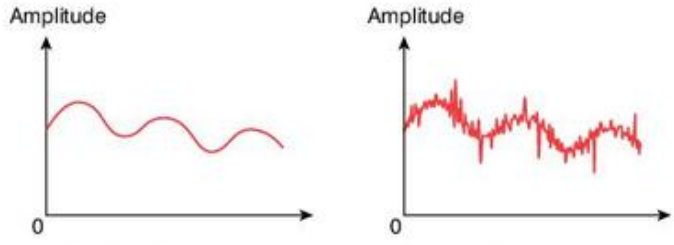
- For AM signals, the channel bandwidth is restricted to 9 kHz to prevent frequency overlap from nearby stations. As a result, the upper frequency limit of the audio waves carried by AM signals is restricted before modulation to 4.5 kHz.
- FM signals have further upper and lower sidebands at multiples of the audio frequencies (whereas AM signals only have one upper and one lower sideband). Due to these extra side bands, FM bandwidths are about 200 kHz compared with 9 kHz for AM bandwidths. The much wider FM bandwidth means that audio waves of frequencies up to 15 kHz can be carried by FM signals.

The relative advantages of FM and AM transmission

Quality

The quality of a signal is affected by **noise** which is unwanted variations in the signal amplitude that are superimposed on the transmitted signal. Noise has various causes, including the random motion of conduction electrons in resistors in electrical circuits and in radio and TV communications by atmospheric effects. A signal is difficult to detect if too much noise is present. Figure 5 shows the waveform of an analogue signal which is difficult to discern because so much noise is present.

FM radio transmission is less affected by noise, signal distortion and other unwanted amplitude variations; for example due to atmospheric effects. This is because amplitude variations do not affect the frequency modulation of the signal. As a result, received FM signals are of higher quality than received AM signals. In addition, because FM signals carry a higher range of audio frequencies than AM signals, they reproduce higher-quality sounds (e.g. for music programmes) than AM signals.



a *Signal without noise* **b** *Signal with noise*

Fig 5 *A noisy signal*

Number of channels

FM radio stations use carrier frequencies that are much higher than those used by AM stations. This is because FM bandwidths are much greater than AM bandwidths. A single FM channel with a bandwidth of 200 kHz using carrier waves of frequencies 100 kHz would use up the entire frequency spectrum up to 200 kHz. By allocating FM stations to the VHF waveband which has a frequency range from 30 to 300 MHz, FM stations can operate at different carrier frequencies (e.g. 98.2 MHz, 98.6 MHz) without their bandwidths overlapping.

Note

AM transmitters and receivers are simpler and cheaper than FM transmitters and receivers.

Range

An AM transmitter has a much greater range than an FM transmitter broadcasting at the same transmitter power. A typical FM station has a range of about 30 km by 'line-of-sight'. This is because the **attenuation** (i.e. reduction of intensity) of an electromagnetic wave travelling through air increases with increase of frequency. In other words, high frequency radio waves are attenuated more than low-frequency radio waves. So the signal from a single transmitter can reach a much larger area if it is an AM signal rather than an FM signal. To reach the same area, many FM transmitters would be needed to replace an AM transmitter. For this reason, national and international radio stations use AM transmission and local radio stations use FM transmission. Table 1 shows the wavebands that are used by each type of radio stations.

Table 1

Waveband	Frequency range	Wavelength range	Uses
Low frequency (LF) / long wave (LW)	<300 kHz	>1 km	International long-wave AM radio
Medium frequency (MF) / medium wave (MW)	300 kHz to 3 MHz	100 m to 1 km	AM medium-wave radio
High frequency (HF)	3 to 30 MHz	10 to 100 m	AM short-wave radio
Very high frequency (VHF)	30 to 300 MHz	1 to 10 m	local FM radio
Ultra high frequency (UHF)	300 to 3000 MHz	0.1 to 1 m	TV broadcasting, mobile phones, person-to-person radios
Microwave	>3000 MHz (= 3 GHz)	0.1 mm to 0.1 m	Satellite links, satellite TV

QUESTIONS

1 List the following communication systems in order of increasing frequency:
FM radio International radio National radio Satellite TV

2 With the aid of diagrams, explain the difference between an analogue signal that is amplitude-modulated and an analogue signal that is frequency-modulated.

3 A radio transmitter broadcasts a frequency-modulated signal of bandwidth 250 kHz at a carrier frequency of 101 MHz. Explain what is meant by the bandwidth of a signal and calculate the upper and lower limits of the allocated frequency channel, assuming only the lower side band is used.

4 FM radio stations have a bandwidth of 250 kHz and broadcast in the VHF waveband. Give a reason why FM radio is not allocated to the HF band or to the UHF band.

17.2 Digital transmission

On these pages you will learn to:

- Recall the advantages of the transmission of data in digital form, compared to the transmission of data in analogue form
- Explain the need for and use of analogue-to-digital conversion (ADC) and digital-to-analogue conversion (DAC)
- Show an understanding of the effect of the sampling rate and the number of bits in each sample on the reproduction of an input signal

Notes about binary numbers

The two digits of the binary system of counting are 0 and 1. Any binary number can be expressed as a series of these two digits. Each digit or 'bit' in the series represents the presence or absence of a different power of two in ascending order from right to left. For example, the binary number **101** represents $\mathbf{1} \times 2^2 + \mathbf{0} \times 2^1 + \mathbf{1} \times 2^0$ which is 5 in the denary (or base 10) number system.

Table 1

0	**0000**	8	**1000**
1	**0001**	9	**1001**
2	**0010**	10	**1010**
3	**0011**	11	**1011**
4	**0100**	12	**1100**
5	**0101**	13	**1101**
6	**0110**	14	**1110**
7	**0111**	15	**1111**

Bits and bytes

A **digital signal** is a signal that has two states only, which it switches between. Digital radio and TV signals are pulses of electromagnetic waves that switch between two states. The two states may be two different amplitudes or two different frequencies. Electronic signals are pulses defined by two voltage values which are usually referred to as 'high' or 'low' (or numerically as 1 or 0).

An **analogue signal** is a signal that can vary continuously in amplitude. A microphone produces an alternating voltage when sound waves are incident on it. The alternating voltage is an analogue signal which has an amplitude that depends on the amplitude of the sound waves. The lower limit of the signal is zero and the upper limit would depend on the design of the microphone.

Data storage and transmission

The 0s and 1s of a digital signal are referred to as **bits**. A series of bits is referred to as a **byte**. A code must be used when digital pulses are used to send information. For example, the ASCII code (American Standard Code for Information Interchange) is used to represent every letter, number and symbol on a computer keyboard by a unique 8-bit byte. When you press a particular key on a computer keyboard, a unique stream of electronic pulses is sent from the keyboard to the microprocessor in the computer.

Table 1 shows the value of the sixteen 4-bit bytes.

The advantages of digital transmission

Elimination of noise

In the process of transmission, electrical signals along wires and cables become weaker due to resistance and signals carried by electromagnetic waves become weaker due to absorption and spreading. Because the noise on a signal increases and the signal becomes weaker as it travels, the **signal-to-noise ratio** decreases the further the signal travels. If the ratio decreases too much, the signal is masked by the noise and cannot be detected. Noise cannot be eliminated from analogue systems as amplifiers used to boost the amplitude of a signal would also boost the noise amplitude.

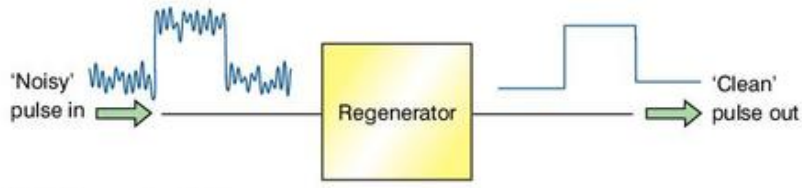


Fig 1 *Cleaning a noisy pulse*

Noise can be eliminated in digital systems by using **regenerator amplifiers**, which are designed to produce a 'clean' output pulse from each noisy input pulse, provided the noise does not mask the signal. With regenerator amplifiers at regular intervals along a transmission link, a digital signal transmitted without noise would be received at the other end of the link without noise. One of the key advantages of digital transmission over analogue transmission for long-distance links is the elimination of noise which means received signals are of the same quality as transmitted signals.

Extra information

Because digital signals can be compressed by using pulses of shorter duration, more bits per second and therefore more information can be sent using digital transmission. In addition to sending extra data bytes, the extra bits can be used:

- To code the data bytes so each byte goes to a particular destination. For example, a 16-bit byte might consist of an 8-bit data byte and an 8-bit address byte.
- To check and correct errors in transmission. For example, an extra bit may be used to check that the 'parity' of a data byte has not changed in transmission. The parity is set at either 0 or 1 according to whether or not there are an odd or an even number of 1s in the data byte when it is transmitted. If one of the data bits changes its state in transmission, the parity check will detect the change.

For the above reasons, digital transmission is more reliable and more cost-effective than analogue transmission.

Converter circuits

Analogue signals can be converted into digital signals for transmission using an analogue-to-digital converter (ADC) and then converted back on reception using a digital-to-analogue converter (DAC). In this way, audio and video signals can be transmitted digitally, giving programmes of higher technical quality as well as more channels.

Conversion of an analogue signal to a digital signal

Figure 2 outlines how an analogue signal is converted into a digital signal.

- The analogue signal is 'sampled' at fixed intervals to produce a 'pulse amplitude modulated' (PAM) signal. This can be achieved by supplying the analogue signal to an op-amp which is switched on for a brief interval. The output voltage of the op-amp is equal to the sample voltage.
- The sample is then converted to a binary number on a scale where all the bits equal to 1 represents the maximum amplitude that can be sampled. To do this, each sample voltage of the PAM signal is converted to a binary number with a certain number of bits (e.g. an 8-bit byte or 8-bit 'word') on a scale where the maximum amplitude is represented with 1 for every bit (e.g. 1111 1111 for an 8-bit word). This is achieved using an ADC circuit which has a single input and a suitable number of parallel outputs (e.g. 8 for conversion of an 8-bit byte).
- Each byte on the parallel outputs of the ADC circuit is then converted using a parallel-to-serial converter into a sequence of bits at a single output terminal. The bytes representing successive samples are then transmitted as a stream of pulses, as outlined in Figure 2.

Table 2 shows all the 4-bit binary numbers that result for an analogue signal that can vary between 0 and 15 V. Each row shows the minimum voltage needed to produce the corresponding digital signal. For example, an analogue voltage of 6.8 V would produce 0110 as the digital signal.

Table 2

Analogue voltage/V	Digital signal	Analogue voltage/V	Digital signal
0	**0000**	8	**1000**
1	**0001**	9	**1001**
2	**0010**	10	**1010**
3	**0011**	11	**1011**
4	**0100**	12	**1100**
5	**0101**	13	**1101**
6	**0110**	14	**1110**
7	**0111**	15	**1111**

Notes

1 In the example above, each bit has 2 states (0 or 1) and there are 4 bits in each digital sample. The analogue signal can therefore be converted into 2^4 (= 16) different digital samples as shown in Table 2.

2 The least significant bit of a byte (i.e. the bit at the right hand end) in the above table changes when the analogue voltage changes by 1 volt or more.

Sampling frequency

If the sampling frequency is too low, the analogue signal cannot be recovered from the digital signal. In order to recreate the analogue signal, it can be shown theoretically that **the frequency of the sampling process must be at least twice the highest frequency of the analogue signal.**

For example, as the sampling frequency of a phone signal is 8 kHz, phone signals are restricted to audio frequencies of no more than 3.4 kHz.

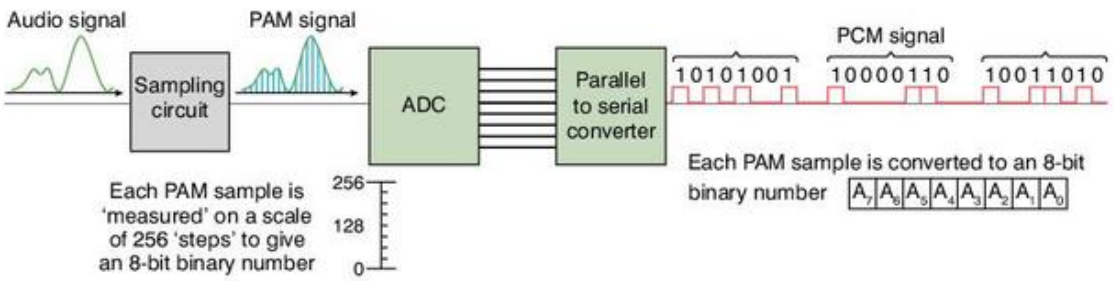


Fig 2 *Analogue-to-digital conversion*

Note

The number of bits transmitted per second is the sample frequency × the number of bits per sample. In addition to the data byte representing the signal sample, further bits are added to signal the arrival of a byte at the receiver and to check the byte has not been corrupted in transmission. Thus the bit rate for a sample frequency of 8 kHz would be 128 kilobits per second for 16-bit words, equal to 1 bit transmitted every 7.8 µs.

Conversion of a digital signal to an analogue signal

To convert a digital signal to an analogue signal, the digital signal must be decoded at the receiver. A digital signal can be converted to an analogue signal by converting each byte into an analogue voltage on a scale where the maximum analogue voltage allowed is produced when all the bits are equal to 1.

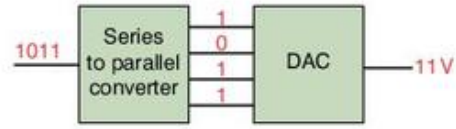
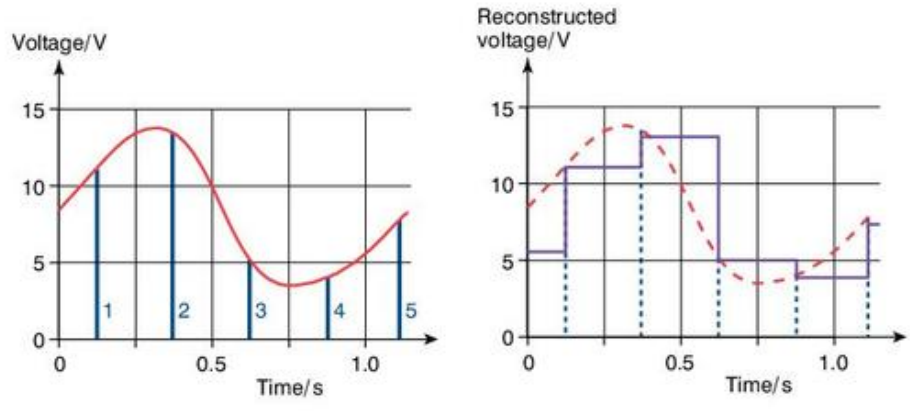


Fig 3 *Digital-to-analogue conversion*

Figure 3 shows the result of converting a digital signal into an analogue signal on a scale where 1011 produces an analogue voltage of 11 V.

Figure 4 shows the result of converting 4-bit byte digital signals produced by sampling an analogue signal back into the analogue signal. The recreated analogue signal is not smooth like the original signal. This is because the digital signal has only 4 bits and so can only create 2^4 different samples. In other words, the voltage of the recreated analogue signal can only have 16 different values. Using an ADC converter that produces signals with more bits would result in a recreated signal with smaller steps. For example, an ADC converter that produces 8-bit digital signals would give 2^8 or 256 different values of the digital signal and therefore a much smoother recreated signal.



Sample	1	2	3	4	5
Voltage/V	11.0	13.5	5.2	4.0	7.8
Digital value	1011	1101	0101	0100	0111
Reconstructed voltage/V	11.0	13.0	5.0	4.0	7.0

Fig 4 *Reconstructing an analogue signal*

QUESTIONS

1 State the difference between an analogue signal and a digital signal and give one example of each.

2 **a** What is meant by noise in a communication system and why can it be problematic?

b State the function of a regenerator amplifier in a digital system.

3 An audio signal with a maximum amplitude of 6.4 V is converted into a digital signal. Each sample of the audio signal is converted into an 8-bit byte.

a What byte is produced when the voltage of the audio signal is **i** 6.4 V, **ii** 3.0 V?

b What is the least change of the audio voltage that would produce a different byte?

4 An analogue signal is converted to a digital signal which is then used to reconstruct the analogue signal.

a Explain why a digital signal with 8-bits per byte would give a better reconstructed signal than signals with 4-bits per byte.

b When the analogue signal is sampled and converted to a digital signal, the sampling frequency must be at least twice the maximum signal frequency. State and explain the result of using a lesser sampling frequency.

17.3 Communication links

On these pages you will learn to:

- Recognise different channels of communication, including wire-pairs, coaxial cables, radio and microwave links and optic fibres
- Discuss the relative advantages and disadvantages of different channels of communication in terms of available bandwidth, noise, cross linking, security, signal attenuation, repeaters and regeneration
- Recall the relative merits of both geostationary and polar orbiting satellites for communicating information
- Understand and use signal attenuation expressed in dB and dB per unit length
- Recall and use the decibel equation $\text{dB} = 10\log_{10} P_1/P_2$ for the ratio of two powers

Types of channels

Information may be transmitted along different types of channels including:

- twisted **wire pairs** and coaxial cables, as shown below, carrying electrical signals
- radio and microwave links, including satellite links
- optical fibres carrying light signals.

Wire pairs are used to carry low-frequency signals over short distances. Attenuation over longer distances occurs because:

- the heating effect of the current in the wires causes energy to be transferred as heat to the surroundings
- the wires act as aerials and radiate electromagnetic waves which carry energy away from the wires. Twisting the wires together reduces this effect but cannot eliminate it.

The wires may pick up external **interference** due to electromagnetic waves from other sources and, where several twisted wire pairs are near each other, signals in one pair may be picked up by the other pairs.

A **coaxial cable** consists of an inner conductor in the form of a thick copper wire separated by polythene insulation from an outer 'copper braid' conductor, as shown in Figure 1. The outer conductor is covered in a layer of plastic insulation.

Attenuation of signals is much less along a coaxial cable than along a twisted wire pair.

A typical coaxial cable has a bandwidth of about 60 MHz. Unlike a twisted wire pair, a coaxial cable of the same length can therefore carry high-frequency signals up to 60 MHz in frequency without significant attenuation. For this reason, a coaxial cable can carry much more information that a wire pair. For example:

- a coaxial cable is capable of carrying 100 million bits per second over a distance of 1 km
- a twisted wire pair can transfer no more than about 10 million bits per second over a distance of 100 m.

NOTE

When you make a telephone call to someone in a distant country, you may think the person you are speaking to is almost next door because the call is so clear. Such a call passes through different channels of communication including twisted wire pairs that connect phones to local telephone exchanges, coaxial cables or optical fibres that connect the circuits in the local exchange to other exchanges and microwave transmitters that send and receive signals to and from satellites. Each type of channel used at each stage provides the most appropriate link for the stage, according to location, distance and bandwidth. For example, satellite links for long-distance communications are much more cost-effective and reliable than cables.

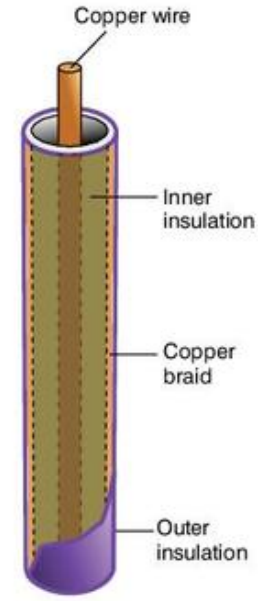


Fig 1 A coaxial cable

Radio waves

Radio waves and microwaves are used for radio and TV broadcasting, satellite communications and for two-way communications including mobile phone communications. Radio waves are electromagnetic waves so they travel at the speed of light. Electromagnetic waves radiate from a transmitter aerial when an alternating voltage of frequency greater than about 30 kHz is applied to the aerial.

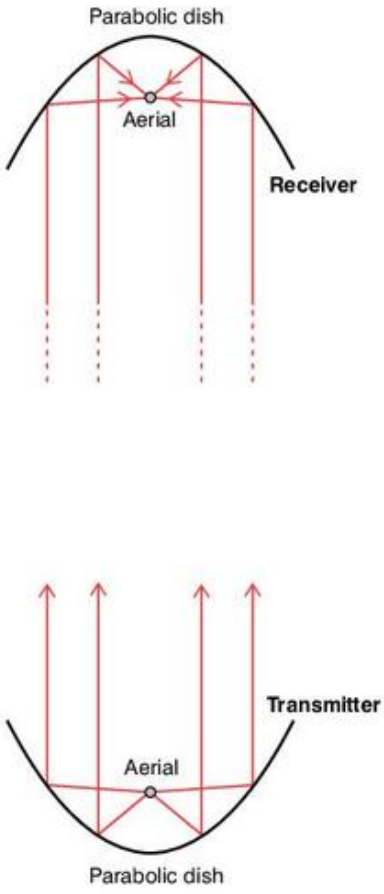


Fig 2 A microwave link

The frequency range covered by radio waves and microwaves is divided into wavebands for different users, as shown in Table 1 on p.253.

Transmitter aerials used for local, national and international broadcasting stations need to be designed so they transmit radio waves in all directions. The further a receiver aerial is from the transmitter aerial, the weaker the signal it detects. This is because the waves spread out as they travel away from the transmitter and they lose some energy due to atmospheric absorption as they travel through the atmosphere.

Radio waves are affected by the Earth's atmosphere and surface according to frequency.

- **Surface waves** – low frequency radio waves (below 3 MHz) stay near the surface as they spread out from the transmitter and they travel further over the sea than the land. Their range over land is about 1000 km. They are used for long-distance broadcasts and maritime communications.
- **Sky waves** – medium and high frequency waves reflect from the ionosphere in the upper atmosphere back to the ground. The ionosphere reflects radio waves of frequencies up to 30 MHz. Above this frequency, radio waves pass through the ionosphere. Therefore MF and HF radio signals can travel round the Earth as a result of being repeatedly reflected by the ionosphere and the ground surface.
- **Space waves** – radio waves of frequencies greater than 30 MHz (VHF, UHF and microwaves) are used for line-of-sight communications and satellite links. VHF and UHF radio waves have a lower range than radio waves of lower frequencies transmitted at the same power so they are used for local transmissions.

Microwave links

Microwaves are electromagnetic waves in the frequency range from 3 GHz to 30 GHz. They have a bandwidth of the order of GHz so they can carry more information than radio waves can. They are used for satellite communications and for point-to-point communications between line-of-sight transmitters and receivers. For long-distance ground communications, many **repeater** stations are needed with line-of-sight adjacent transmitter/receiver towers.

Microwaves in air have wavelengths of about 10 cm or less. The transmitter aerial or 'element' is fixed at the 'focal point' of a parabolic reflecting dish. As a result, the waves from the transmitter element are reflected by the dish to form a parallel beam. When the beam is directed at a receiver consisting of a similar reflector with a receiving aerial at its focus, they are focused by the receiver reflector onto the receiver aerial. In this way, signals can be carried by the microwave beam along the line-of-sight between the transmitter and the receiver.

The parabolic shape of the reflectors ensures the beam is:

- parallel so as much power as possible is in the beam at the receiver
- focused onto the reflector aerial to ensure as much power as possible is received from the beam.

Compared with radio waves, microwave beams:

- spread out much less than longer wavelength electromagnetic waves so they weaken less,
- have a much greater bandwidth so they can carry many more channels,
- pass straight through the atmosphere, unlike radio waves at frequencies below 30 MHz, so can be used for satellite links.

Satellite communication signals such as phone signals are carried by microwave beams between geostationary satellites and ground-based receivers. The carrier frequency of the link between the ground and the satellite (i.e. the 'uplink') is different to that of the link between the satellite and the ground (i.e. the 'downlink'). This is so that the transmitted signal does not swamp the reception of the signal received at the satellite.

NOTE

The transmitter aerial is not a point object so the waves diverge slightly when they reflect from the transmitter dish. Also, diffraction occurs when the waves are reflected by each dish. To minimise diffraction, the diameter of the transmitter dish must be much larger than the wavelength of the waves.

Long-distance communications using satellite links are more reliable than using radio waves because long wave radio waves are poorly received in hilly areas and medium and short wave radio waves rely on reflection by the ionosphere which varies in reflectivity during the day. Satellite TV broadcasting is from geostationary satellites that each transmit a microwave beam to dish receivers on a wide area of the Earth's surface.

Satellites are placed in orbits that depend on the purpose of the satellite:

1 **Geostationary satellites** orbit the Earth directly above the equator with a period of 24 hours in the same direction of rotation as the Earth. Such a satellite remains directly over the same point on the equator. The orbital radius for such an orbit is 42 000 km. Geostationary satellites are used for international phone links and satellite TV broadcasts. A single geostationary satellite can be used for TV broadcasting or for continuous communication between any two points on the Earth in the 'line of sight' of the satellite. Several satellites sending signals between each other and the ground are needed for global communications. However, because the transit time of phone signals that travel long-distances between geostationary satellites is unacceptably long, such signals are carried on the surface by optical fibre links as well.

2 **Polar satellites** pass directly over both of the Earth's poles with a time period of about 90 minutes.
For such satellites:
- their orbits must be much lower than geostationary orbits,
- successive passes across the equator in the same direction are further west each time; for a 90 minute orbit, each such pass is 22.5° further west (= 360 × 1.5 hours/24 hours).

Polar satellites are used for many purposes including surveillance, monitoring and weather forecasting. Because a polar satellite is in a much lower orbit than a geostationary orbit, its signals cover a much smaller area of the surface. Continuous communication outside the area covered by a single satellite is possible if there are several equally spaced satellites linked by microwave beams along the orbit.

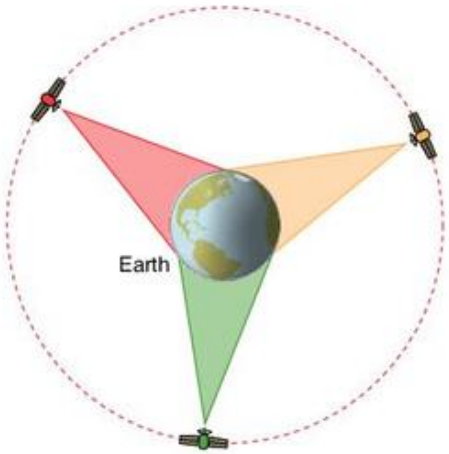


Fig 3 Geostationary satellites

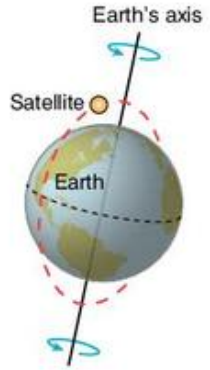


Fig 4 A polar satellite

Optical fibres

As explained in topic 9.5, optical fibres are used in communications to transmit infrared or visible light pulses coded as digital signals. Infrared light of wavelength about 1200 nm (i.e. frequency of the order of 250 THz) is usually used to carry optical signals in optic fibres.

Noise can be removed from such signals using regenerator circuits at intervals. The fibres used are **step-index fibres**, which means each fibre consists of a core of transparent material surrounded by a cladding of transparent material of lower refractive index. Light rays travelling along a fibre that reach the core–cladding boundary are totally internally reflected at the boundary, provided the fibre does not bend too much. Figure 5 shows how a light ray travels along a step-index fibre as a result of successive reflections at the core–cladding boundary.

Light pulses from a laser source can be sent along an optical fibre at a rate of 10^8 pulses per second. The speed of light in a fibre depends on the light frequency; light from an LED has a range of frequencies so a pulse of light from an LED becomes longer as it travels along a fibre because its different frequencies travel at different speeds. This doesn't happen with a laser light because it has a single frequency.

The duration of a single pulse or 'bit' can in theory be as short as 10^{-14} s, corresponding to the sampling frequency needed for light at frequencies of about 10^{14} Hz. This would allow bits from hundreds of thousands of different signals to be transmitted between successive bits from any one signal. However, at present, laser pulses of duration less than about 10^{-10} s cannot be generated.

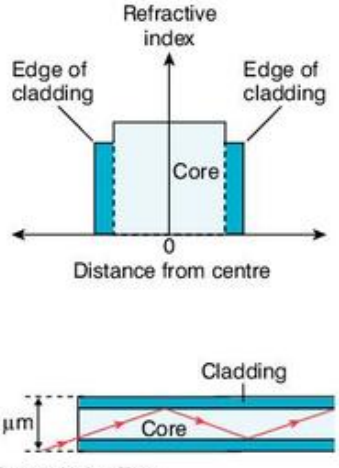


Fig 5 A step-index fibre

Notes

- A light pulse also spreads out because of **multi-path dispersion**, also referred to as modal dispersion. This happens because a light ray that is repeatedly reflected at the core–cladding boundary travels further than a light ray that travels straight along the fibre. The leading edge of the pulse is defined by the light rays that travel along the shortest path while the trailing edge is defined by the light rays that travel along the longest path. The longer the length of the fibre, the greater the difference between the shortest and longest path and therefore the longer the pulse. The effect is reduced considerably by using **mono-mode fibre**. Such a fibre has a very thin core of about 5 µm in diameter. Because the core is so thin, modal dispersion is effectively eliminated because light travelling along the core effectively travels along the shortest path.
- A laser pulse sent along a step-index fibre can become distorted due to absorption and scattering by the fibre material as well as loss through the core–cladding boundary if the fibre is bent too much. All these effects reduce the amplitude of the pulse.

The advantages of using optical fibres for transmission compared with other forms of transmission include:

- a greater transmission capacity in terms of bits per second,
- a wider bandwidth (i.e. range of frequencies that can be transmitted),
- less signal power loss,
- cheaper, lighter and smaller than other cables,
- high security because cross-talk is negligible and they do not radiate electromagnetic waves,
- no interference due to external electromagnetic waves.

Signal power

Signals become attenuated (i.e. weaker) the further they travel. Light is partly absorbed or scattered as it passes through glass. Radio waves and microwaves spread out and are partly absorbed as they travel through the atmosphere. Electrical signals along a wire become weaker due to the electrical resistance of the wire. **Repeaters** and **regenerators** are used to strengthen signals.

- Repeaters are amplifiers used at intervals to amplify the signal. They amplify noise as well as signals.
- Regenerators are used in digital circuits to regenerate digital signals by removing noise and restoring the signal to its original amplitude.

The effect of a channel or an amplifier on the power of the signal can be expressed in **bels** where the number of bels, n, is defined by the equation

$$\textbf{power } P_1 = 10^n \times \textbf{power } P_2$$

where P_1 and P_2 for an amplifier are the output power and input power respectively and vice versa for a channel. The distinction above between an amplifier and a channel ensures n always has a positive value.

Rearranging the above equation gives:

$$n = \log_{10}\left(\frac{P_1}{P_2}\right)$$

Attenuation and amplification values are usually expressed in decibels (dB) where 1 bel = 10 dB.

Hence the above equation gives

$$\textbf{the number of decibels} = 10\log_{10}\left(\frac{P_1}{P_2}\right)$$

For a signal that is amplified from power P_{in} to power P_{out}

the amplification of the signal, measured in decibels (dB)

$$= 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

For example, an amplifier with 30 dB of amplification gives an output signal that is $10^3 \times$ more powerful than the input signal.

For a signal that is attenuated from power P_{in} on transmission to power P_{out} on reception,

the attenuation of the signal, measured in decibels (dB)

$$= 10\log_{10}\left(\frac{P_{in}}{P_{out}}\right)$$

For example, if the attenuation of a signal by a channel is 50 dB, the input signal is $10^5 \times$ more powerful that the output signal. In other words, the output signal power is $10^5 \times$ less powerful than the input signal.

Where a signal passes through a series of amplifiers and channels, the overall effect on the signal is given by subtracting the decibel changes produced by the channels from the decibel changes produced by the amplifiers. If the overall effect is positive, the signal is stronger; if negative, the signal is weaker.

For example, if a signal carried by a channel is transmitted at power P_0 then received at power $0.001P_0$ and then amplified to power $0.01P_0$,

- the attenuation due to the channel $= 10\log_{10}(P_0/0.001P_0) = 30$ dB
- the amplification due to the amplifier $= 10\log_{10}(0.01P_0/0.001P_0) = 10$ dB

The signal after amplification is therefore 20 dB less powerful than when it was transmitted.

Note

- A 3 dB change is a change by a factor of 2.
- A $10n$ dB change is a change by a factor of 10^n.

Maximum length of a channel

The attenuation of a channel is directly proportional to the distance travelled by the signal (i.e. the length L of the channel). For example, the attenuation of a signal that passes through two identical channels will be twice the attenuation produced by each channel.

$$\text{The attenuation per unit length of a channel} = \frac{\text{attenuation, in dB,}}{\text{length } L}$$

The maximum length of a channel is determined by the signal-to-noise ratio of the signal. The further a signal travels, the more it is attenuated. If the signal-to-noise ratio becomes too low, the signal cannot be detected because there is too much noise present.

In general, to detect a signal, the signal power needs to be 100 times greater than the power of the noise in the channel. Using the decibel formula $10\log_{10}\left(\frac{P_1}{P_2}\right)$ therefore gives 20 dB for the minimum acceptable signal-to-noise ratio.

Worked example

A cable has an attenuation per unit length of $3\,\text{dB km}^{-1}$. In use, the background noise power in it is 4×10^{-13} W. Calculate the maximum length of this cable for an input signal power of 4×10^{-8} W. The minimum acceptable signal-to-noise ratio is 20 dB.

Solution

The output signal must be 20 dB more powerful than the noise signal.

Therefore, the minimum value of the output power of the signal, P_{out} is given by

$$20 = 10\log_{10}\left(\frac{P_{out}}{4 \times 10^{-13}}\right)$$

Hence $P_{out} = 100 \times 4 \times 10^{-13}\,\text{W} = 4 \times 10^{-11}\,\text{W}$

As the cable is supplied with a signal of power 4×10^{-8} W, the attenuation of the signal should not exceed a value in decibels given by $10\log_{10}\left(\frac{4 \times 10^{-8}}{4 \times 10^{-11}}\right)$ which equals 30 dB.

Because the attenuation due to the cable is 3 dB for each kilometre of length, the cable should be no longer than 10 km ($= 30\,\text{dB} \div 3\,\text{dB km}^{-1}$).

Table 1 *Summary of the advantages and disadvantages of each type of channel*

Type of channel	Twisted wire pair	TV coaxial cable	Radio/microwave link	Optic fibre
Typical attenuation	7 dB at 10 MHz per 100 m	11 dB at 100 MHz per 100 m	90 dB at 1 GHz per km	0.2 dB for infrared, per km
Bandwidth	500 kHz	50 MHz	FM 200 kHz Microwaves 1 GHz	
Noise	electrical interference from electric motors etc, cross-over from other wires	electrical interference from electric motors etc	electrical storms, cosmic sources	no electrical interference, noise is removed by regenerators
Security	← low, unless coded digital signals used →			high
Cross-over	causes noise	insignificant	if atmospheric conditions extend local signals	insignificant
Cost / convenience	low / high	medium / medium	high / low	low / high
Repeater/ regenerator spacing	short distances	up to 2–3 km	line of sight for microwaves	up to 100 km or more

EXTENSION

Mobile phone communications

In the early telephone network, each call went through the network on a fixed route via manually operated telephone switchboards in local telephone exchanges and, where necessary, via trunk cables to 'trunk exchanges'. In comparison, modern digital networks can switch a signal automatically between different routes to the same destination.

A **mobile phone network** consists of a network of local **cells**, each with a local **base station** with its own transmitter/receiver mast that sends and receives signals from users in the cell area. Each user has a handset that is a radio transmitter and receiver.

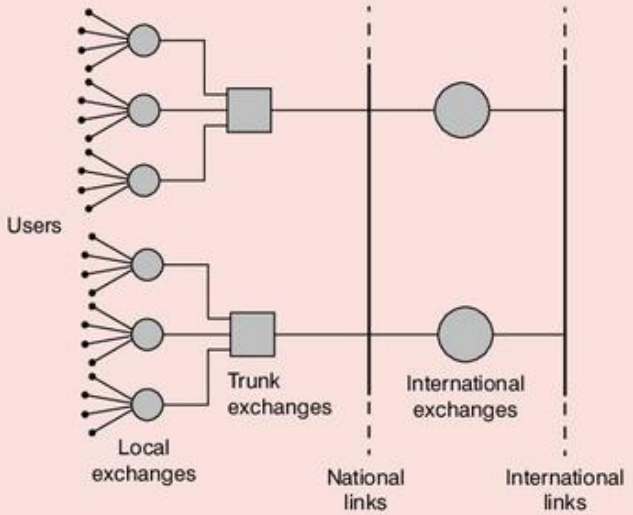


Fig 6 *The Public Switched Telephone Network*

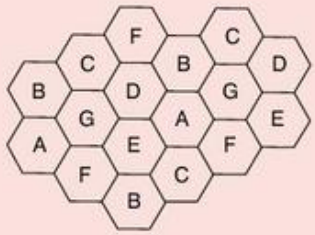


Fig 7 Mobile phone cells

The base stations are linked via cables to a **cellular exchange** which is connected to the **public switched telephone network** (PSTN) so that each user stays connected if the user moves into an adjacent cell. The power transmitted from the base station (of the order of a few watts) is just sufficient to reach users near the edge of a cell.

When a mobile phone is switched on, it detects the control signal from the nearest base station and transmits a signal in return to register its location. When a call is made from the mobile phone, the base station allocates to it a time slot in an uplink frequency channel (from the handset to the base station) and a time slot in a downlink frequency channel (from the base station to the handset). Figure 9 shows how the signals to and from the handset aerial are processed.

- The signal from the handset microphone is sampled and converted to a digital signal which is used to modulate the 'uplink' carrier wave.
- The signal from the base station is 'demodulated' (i.e. separated from the 'downlink' carrier wave) and converted to an analogue audio signal which is supplied to the loudspeaker.

During a call, the allocated frequency channels may be changed by the control signal thus reducing fading and interference. If the signal strength from a mobile phone falls on account of the user moving out of a cell, the base station is programmed to handover the call to a different frequency channel of the adjacent base station which receives the strongest signal from the mobile phone. The handover takes place within a fraction of a second and is not noticed by the user.

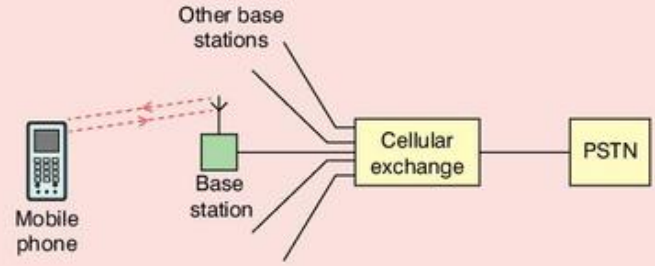


Fig 8 A mobile phone network

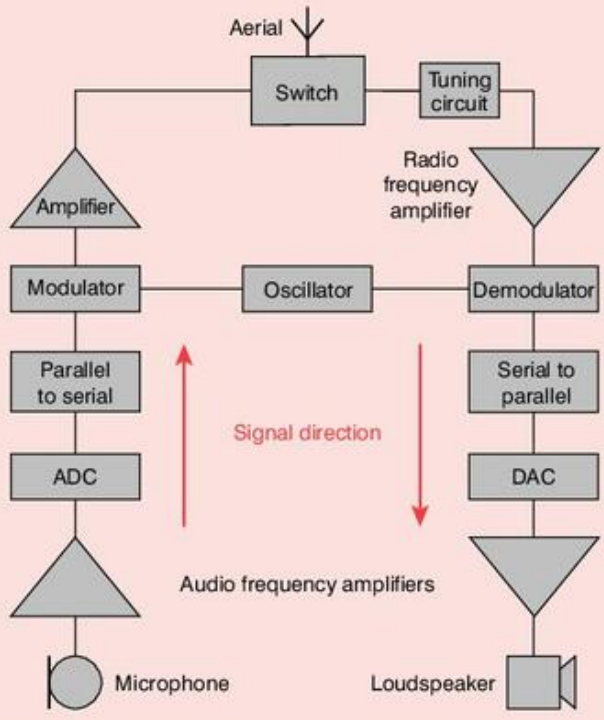


Fig 9 Inside a mobile phone

Note

The power transmitted by a mobile phone is little more than 1 W and base stations transmit less than 10 W of power. The scientific evidence of possible adverse health effects of radiation from mobile phones has led some governments to recommend caution, particularly for young people, until more is known.

QUESTIONS

1 State one advantage and one disadvantage of a fibre optic link in comparison with a microwave link between a transmitter and a receiver 30 km away.

2 Explain what is meant by a geostationary satellite and describe how such satellites can be used for global communications.

3 **a** Estimate the attenuation and power of a 100 MHz, 10 mW electrical signal transmitted along a coaxial cable to a receiver at a distance of **i** 100 m, **ii** 500 m from the transmitter.

b Explain why a signal transmitted along a coaxial cable cannot be detected if the cable is too long.

4 **a** Explain why different frequency channels are allocated to adjacent mobile phone cells.

b Describe how a mobile phone frequency channel can carry up to eight different calls.

17 End of chapter questions

1 a State and explain two advantages of the transmission of information in digital, rather than analogue, form. *(4 marks)*

b Convert

i the decimal number 13 to a four-bit digital number, *(1 mark)*

ii the digital number 0101 to a decimal number. *(1 mark)*

c An analogue signal is to be transmitted digitally. A block diagram for part of the transmission system is shown in Figure 1.1.

Fig 1.1

i Complete Figure 1.1 by labelling block X and block Y. *(2 marks)*

ii State the purpose of the parallel-to-serial converter. *(2 marks)*

d The original analogue signal is shown in Figure 1.2. The recovered signal after transmission is shown in Figure 1.3.

Suggest and explain two ways in which the reproduction of the input signal may be improved. *(4 marks)*

Cambridge International AS and A Level Physics 9702
Paper 41 Q12 November 2009

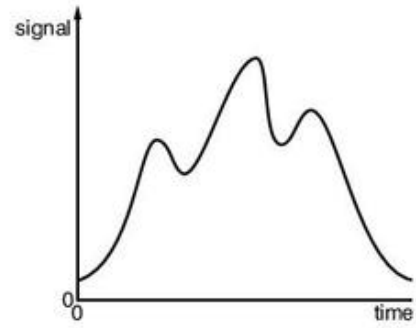


Fig 1.2

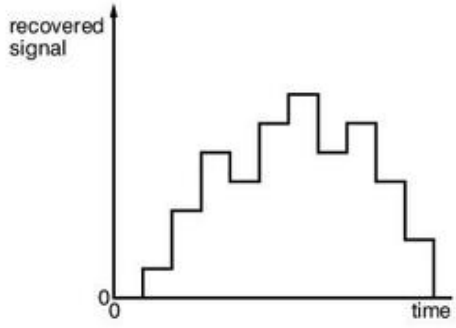


Fig 1.3

2 A ground station on Earth transmits a signal of frequency 14 GHz and power 18 kW towards a communications satellite orbiting the Earth, as illustrated in Figure 2.1.

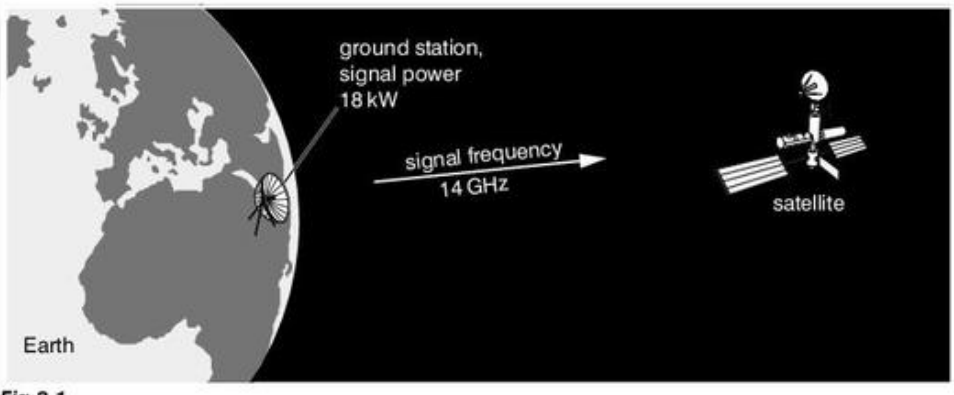


Fig 2.1

The loss in signal power between the ground station and the satellite is 190 dB.

a Calculate the power, in W, of the signal received by the satellite. *(3 marks)*

b The signal received by the satellite is amplified and transmitted back to Earth.

i Suggest a frequency, in GHz, for the signal that is sent back to Earth. *(1 mark)*

ii Give a reason for your answer in **i**. *(1 mark)*

Adapted from Cambridge International AS and A Level Physics 9702
Paper 41 Q12 June 2010

18 Magnetic fields

18.1 Magnetic field patterns

On these pages you will learn to:

- Recognise that a magnetic field is an example of a field of force produced either by current-carrying conductors or by permanent magnets
- Explain what is meant by a magnetic field line
- Sketch magnetic field patterns due to a bar magnet, a long straight wire, a flat circular coil and a long solenoid

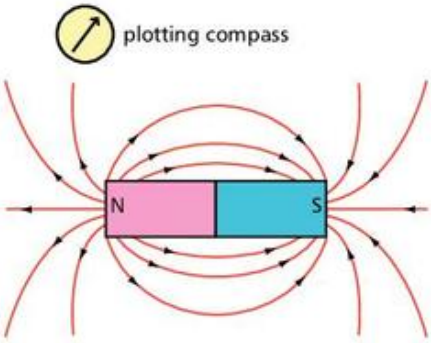


Fig 1 The magnetic field near a bar magnet

Lines of force

- A **magnetic field** is a force field surrounding a magnet or current-carrying wire which acts on any other magnet or current-carrying wire placed in the field.
- The magnetic field of a bar magnet is strongest near its ends, which are referred to as '**poles**'. A bar magnet free to turn horizontally about its centre aligns itself with one end pointing North and the other pointing South. This occurs because the Earth's magnetic field attracts one end and repels the other end. The poles are referred to as **north-seeking** and **south-seeking**, according to the direction in which each pole points.
- A line of force of a magnetic field is a line along which a 'free' north pole would move in the field. Lines of force are often referred to as '**magnetic field lines**'. Note that the lines of force of a permanent magnet loop round from the north pole to the south pole of the magnet. A plotting compass points in the direction of a line of force.

The force between two magnets

Two bar magnets placed end-to-end attract or repel, depending on whether the nearest poles are:

- **like polarity**, in which case they repel, or
- **unlike polarity**, in which case they attract.

Electromagnetism

A magnetic field is created round a wire whenever **a current passes** along the wire. The pattern of the magnetic field lines for a long straight wire, a solenoid and a flat coil are shown opposite. Note that the lines of force are **complete loops**. Also, the **direction of the lines of force** depends on the direction of current. If the current is reversed, the direction of the lines of force is reversed.

- **For a long straight wire**, the lines of force are circles centred on the wire in a plane perpendicular to the wire. The direction of the lines of force depends on the current direction and can be worked out using the 'corkscrew rule' as shown in Figure 2.
- **For a solenoid**, the lines of force pass through the solenoid along its axis and loop round outside the solenoid. The direction of the lines of force depends on the current direction and can be worked out using the solenoid rule, as shown in Figure 3.

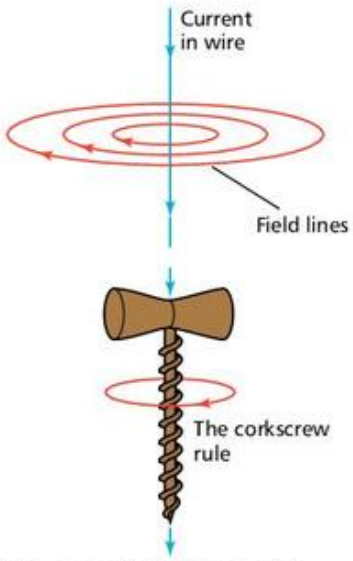


Fig 2 The magnetic field near a wire

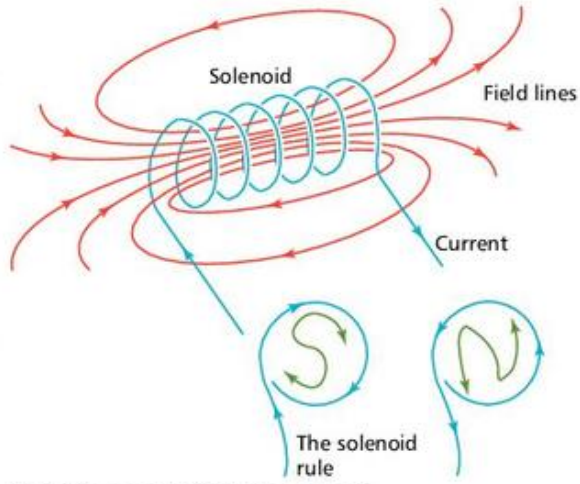


Fig 3 The magnetic field of a solenoid

- **For a flat coil**, the lines of force are lines that pass through the coil and loop round outside the coil. The direction of the lines of force can be worked out from the current direction using the solenoid rule, as explained on the previous page.

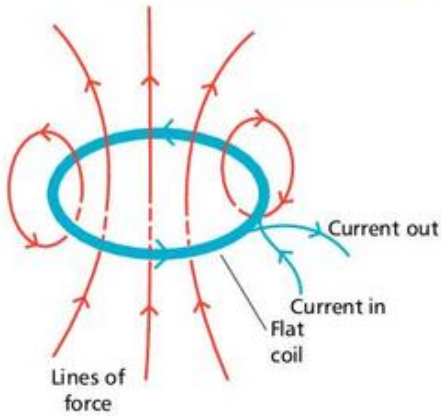


Fig 4 The magnetic field of a flat coil

QUESTIONS

1 A plotting compass is placed at the intersection of two perpendicular lines drawn on a sheet of paper. The plotting compass points North along one of the lines. When a bar magnet is placed along the other line near the plotting compass, as shown in Figure 5, the plotting compass points north-east.

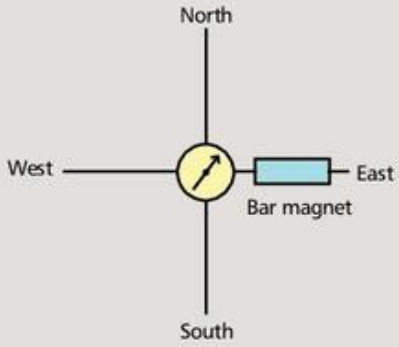


Fig 5

a What is the polarity of the pole of the magnet nearest the plotting compass?

b If the magnet is turned round, what direction will the plotting compass then point to?

2 An underground cable is aligned horizontally in an east-west direction. The cable carries a direct current from west to east, as shown in Figure 6.

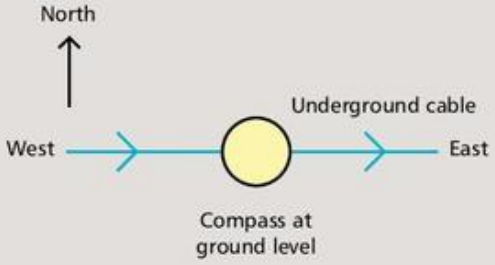


Fig 6

a What would be the direction of a magnetic compass directly above the wire, assuming the magnetic field due to the cable is much stronger than the Earth's magnetic field?

b How would the direction of the compass change if the current in the cable is gradually reduced to zero?

3 A plotting compass is placed at point P at the end of a solenoid. When a direct current is passed through the solenoid, the compass points into the solenoid, as shown in Figure 7.

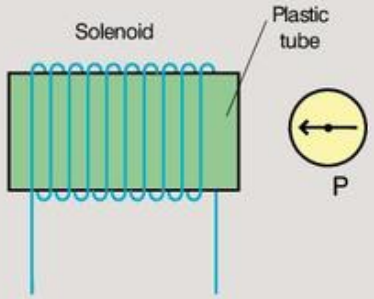


Fig 7

a State the direction of the current round the solenoid, as seen by an observer looking directly at the end of the solenoid at P.

b How would the direction of the plotting compass change if the current in the solenoid was reversed?

4 A student makes a model ammeter using a pair of flat coils in series with each other and a plotting compass, as shown in Figure 8.

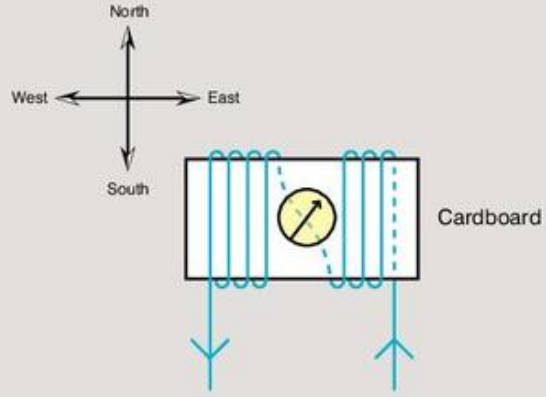


Fig 8

a Explain why the compass needle deflects when a direct current is passed through the coils.

b Explain why the compass needle cannot deflect more than 90° no matter how much current is passed through the coils.

18.2 The motor effect

On these pages you will learn to:

- Recognise that a force acts on a current-carrying conductor placed in a magnetic field at a non-zero angle to the field lines
- Describe the operation of a simple electric motor

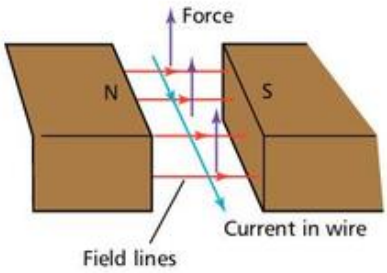


Fig 1 The motor effect

NOTE

Notice that the force is always perpendicular to the field lines, unlike electric forces on point charges (and gravitational forces on small objects) which are always parallel to the field lines.

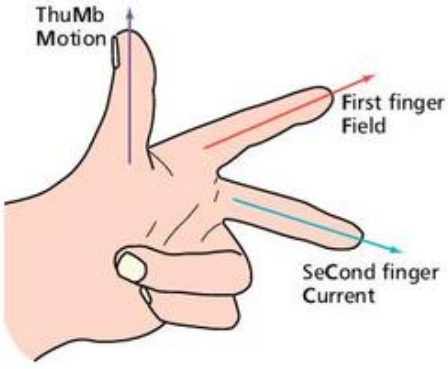


Fig 3 Fleming's left-hand rule

The force on a current-carrying wire in a magnetic field

A current-carrying wire placed at a non-zero angle to the lines of force of an external magnetic field experiences a force due to the field. This effect is known as the **motor effect.** The force is perpendicular to the wire and to the lines of force.

The motor effect can be tested using the simple arrangement shown in Figure 1. The wire is placed between opposite poles of a U-shaped magnet so it is at right angles to the lines of force of the magnetic field. When a current is passed through the wire, the section of the wire in the magnetic field experiences a force that pushes it out of the field. The combined magnetic field due to the wire and the magnet is stronger on one side of the wire than on the opposite side. The wire is pushed in the direction where the combined field is weakest, as shown in Figure 2.

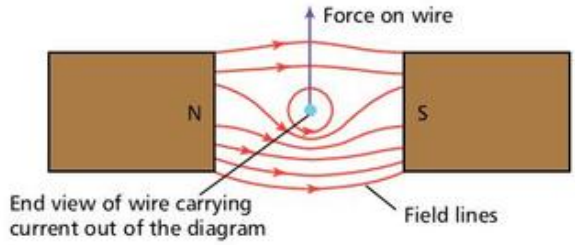


Fig 2 A field pattern

Force factors

The magnitude of the force depends on the **current**, the strength of the **magnetic field**, the **length** of the wire and on the **angle** between the lines of force of the field and the current direction.

The force is:

- greatest when the wire is at **right angles** to the magnetic field,
- zero when the wire is **parallel** to the magnetic field.

The direction of the force is **perpendicular** to the direction of the field and to the direction of the current, as indicated by Fleming's left-hand rule, shown in Figure 3. If the current is reversed, or if the magnetic field is reversed, the direction of the force is reversed.

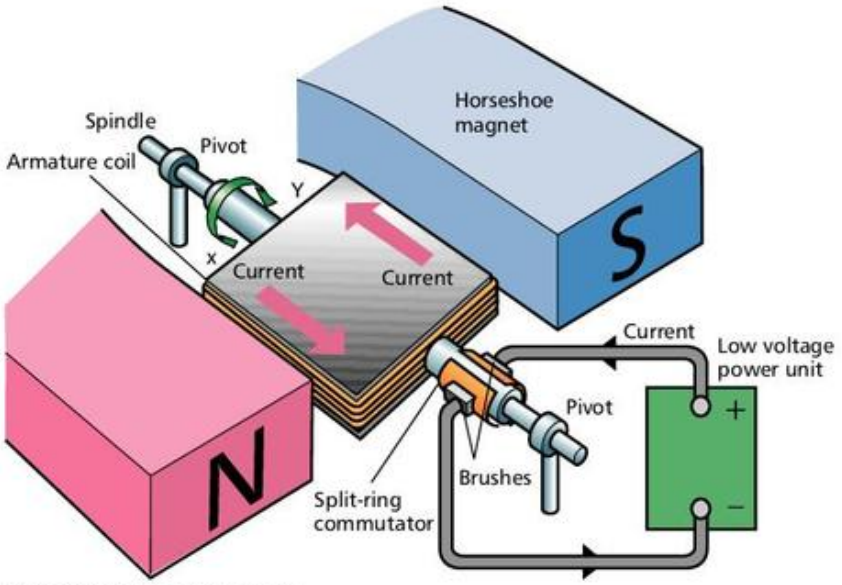
The electric motor

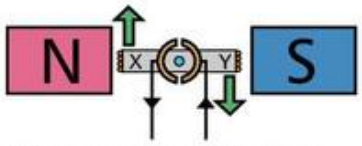
The simple electric motor consists of a coil of insulated wire, the **armature**, which spins between the poles of a U-shaped magnet. When a direct current passes round the coil:

- The wires at opposite edges of the coil are acted on by forces in opposite directions.
- The force on each edge makes the coil **spin** about its axis.

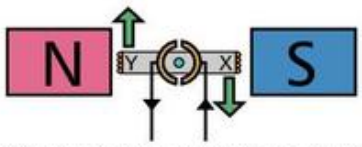
Current is supplied to the coil via a **split-ring commutator**. The direction of the current round the coil is reversed by the split-ring commutator each time the coil rotates through half a turn. This ensures the current along an edge changes direction when it moves from one pole face to the other. The result is that the force on each edge continues to turn the coil in the same direction (Figure 4).

The **direction of rotation** of the motor is reversed by either reversing the current **or** the direction of the magnetic field. If both the current and the magnetic field are reversed, the direction of rotation is unchanged.





Initially, current is up side X and down side Y. Therefore the coil turns clockwise



After half a turn, current is up side Y and down side X. Therefore the coil continues to turn clockwise

Fig 4 The simple electric motor

The **a.c. motor** has an electromagnet instead of a permanent magnet connected to the same a.c. supply as the armature. The direction of rotation does not change when the current reverses because the magnetic field of the electromagnet reverses as well.

The **speed of rotation** depends on the **current**, the **strength** of the magnet and the **number of turns** of the coil. The strength of the magnetic field is increased if an armature with an iron core is used.

A practical electric motor

A practical electric motor has an armature with several **evenly spaced coils** wound on it. Each coil is connected to its own section of the commutator. The result is that each coil in sequence experiences a turning effect when it is connected to the voltage supply, so the motor runs smoothly.

Fig 5 A practical electric motor

QUESTIONS

1 A fixed vertical wire is in a horizontal magnetic field. State the direction of the force on the wire if:

a the current is upwards and the magnetic field lines are from East to West,

b the current is upwards and the magnetic field lines are from South to North,

c the current is downwards and the magnetic field lines are from East to West.

2 A fixed horizontal wire lies along a line from East to West in a magnetic field. State the direction of the magnetic field lines if the current in the wire is from East to West and the force on it is:

a vertically up,

b horizontal and due North.

3 A rectangular coil carries a current in a magnetic field:

a Explain why the coil experiences a turning effect when the plane of the coil is parallel to the magnetic field lines, as in Figure 6a.

b Explain why the coil experiences no turning effect when the plane of the coil is perpendicular to the magnetic field lines, as in Figure 6b.

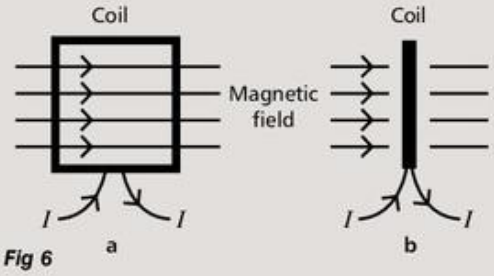


Fig 6

4 a What is the function of the split-ring commutator in a d.c. electric motor?

b A simple electric motor containing a permanent magnet is connected to a battery and a variable resistor. What would be the effect on the motor of:

i increasing the current,

ii reversing the current,

iii using an a.c. supply instead of the battery?

18.3 Magnetic flux density

On these pages you will learn to:

- Define magnetic flux density and the tesla
- Describe how the force on a current-carrying conductor can be used to measure the flux density of a magnetic field using a current balance
- Recall and solve problems using the equation $F = BIl\sin\theta$, with directions as interpreted by Fleming's left-hand rule
- Use appropriate formulas to calculate the magnetic flux density near a straight wire, at the centre of a coil and along the axis of a solenoid
- Recognise that the magnetic flux density in a solenoid is increased if a ferrous material is placed in the solenoid
- Explain the forces between current-carrying conductors and predict the direction of the forces

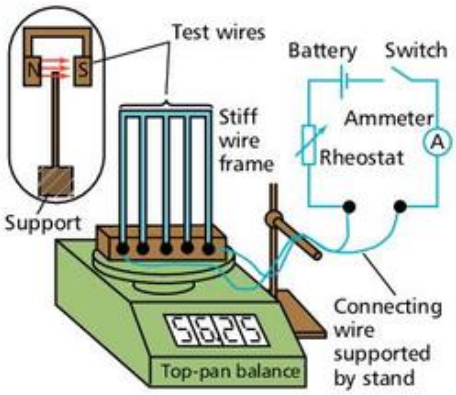


Fig 1 *Measuring the force on a current-carrying wire in a magnetic field*

Investigating the force on a current-carrying wire in a magnetic field

The magnitude of the force on a current-carrying wire in a magnetic field can be investigated using the arrangement shown in Figure 1. The stiff wire frame is connected in series with a switch, an ammeter, a variable resistor and a battery. When the switch is closed, the magnet exerts a force on the wire which can be measured from the change of the top-pan balance reading.

- To test the variation of force with the current through the wire, the variable resistor is adjusted to change the current. Before switching the current on, the top-pan balance reading should be noted. The top-pan balance reading is then measured for different measured values of the current. The length of the test wire in the field is kept the same. The force due to the magnetic field is worked out from the change of the top-pan balance reading. If this is in grams, the reading must be converted to kilograms then multiplied by g ($= 9.8\,\text{m s}^{-2}$) to give the force. For example, if the change of the top-pan balance reading is 20.5 g, the force due to the magnetic field is 0.20 N ($= 20.5 \times 10^{-3}\,\text{kg} \times 9.8\,\text{m s}^{-2}$).

  A graph of a typical set of results is shown in Figure 2.
- To test the variation of force with the length of the wire, the current is kept the same throughout by using the variable resistor as necessary. The length of the wire in the field is changed by reconnecting the wires to the frame. For each length, the reading of the top-pan balance is noted and the force due to the magnetic field is calculated. A graph of a typical set of results is shown in Figure 3.
- To test the variation of the force with the angle between the wire and the magnetic field lines, the magnet can be turned gradually. This test shows that the force is a maximum when the wire is perpendicular to the magnetic field lines.

The tests above show that the force F on the wire is proportional to:

- the current I,
- the length l of the wire.

Magnetic flux density

The magnetic flux density, B, is defined as the force per unit current per unit length on a current-carrying wire placed perpendicular to the field lines.

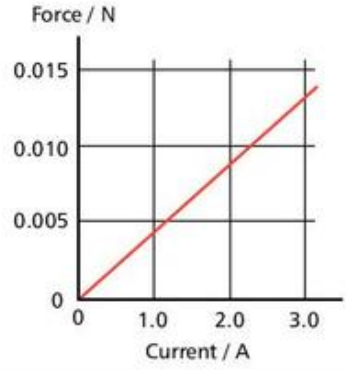


Fig 2 *Force v. current*

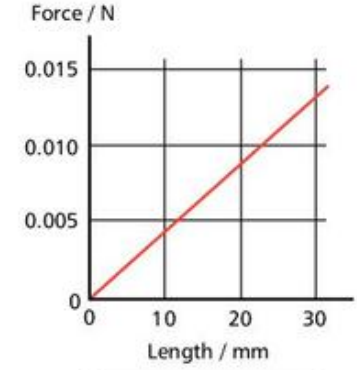


Fig 3 *Force v. length*

The unit of B is the **tesla (T)**, equal to $1\,\text{N A}^{-1}\text{m}^{-1}$. The magnetic flux density is sometimes also referred to as the **magnetic field strength**.

For a wire of length l at right angles to a uniform magnetic field of flux density B, the force on the wire when current I passes through it is given by:

$$F = BIl$$

Worked example

A horizontal wire of length 0.050 m is in a uniform magnetic field directed vertically upwards. The wire lies along a line from North to South. When a current of 4.0 A is passed along the wire, a force of 5.6×10^{-2} N is exerted on the wire, as shown in Figure 4.

a Calculate the magnetic flux density of the magnetic field.
b State the direction of the force on the wire if the current in the wire was from North to South.

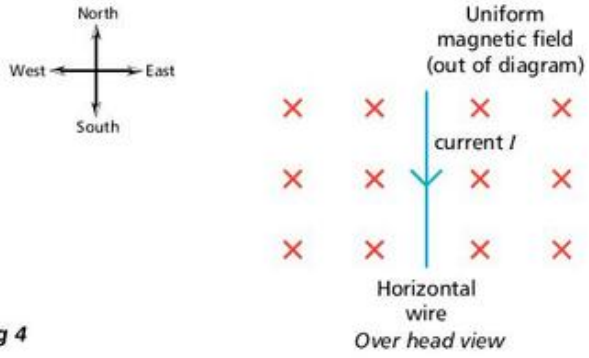


Fig 4

Solution

a $B = \dfrac{F}{Il} = \dfrac{5.6 \times 10^{-2}}{4.0 \times 0.050} = 0.28\,\text{T}$

b Using Fleming's left-hand rule (Figure 3, topic 18.2), gives due West for the direction of the force.

For a straight wire that is not at right angles to the magnetic field lines, the force on the wire due to the field is determined using the component of the magnetic field at right angles to the wire.

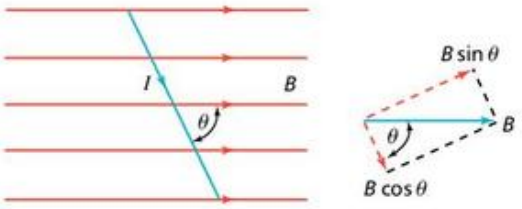


Fig 5 $F = BIl\sin\theta$

For angle θ between the wire and the field lines,

- the component of B perpendicular to the wire $= B\sin\theta$,
- the component of B parallel to the wire $= B\cos\theta$.

Because no force acts on the wire due to the parallel component,

1 the magnitude of the force on the wire $F = (B\sin\theta)Il$
$= BIl\sin\theta$

2 the direction of the force on the wire is given by Fleming's left hand rule, where the field direction is the direction of the perpendicular component of B at the wire.

For a wire of length l carrying a current I in a uniform magnetic field B at angle θ to the field lines,

the force on the wire, $\boldsymbol{F = BIl\sin\theta}$

Worked example

A straight horizontal wire XY of length 5.0 m is in a uniform horizontal magnetic field of magnetic flux density 120 mT. The wire is at an angle of 30° to the field lines, as shown in Figure 6. When the wire conducts a current of 14 A from X to Y, calculate the magnitude of the force on the wire and state its direction.

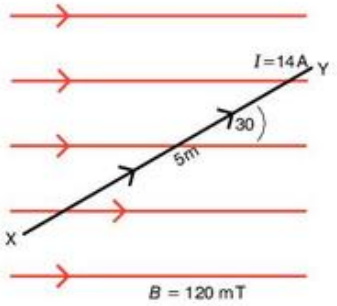


Fig 6

Solution

$\boldsymbol{F = BIl\sin\theta} = 0.12\,\text{T} \times 14\,\text{A} \times 5.0\,\text{m} \times \sin 30 = 4.2\,\text{N}$

The force on the wire is vertically downwards.

The couple on a coil in a magnetic field

Consider a rectangular current-carrying coil in a uniform horizontal magnetic field, as shown in Figure 7. The coil has n turns of wire and can rotate about a vertical axis.

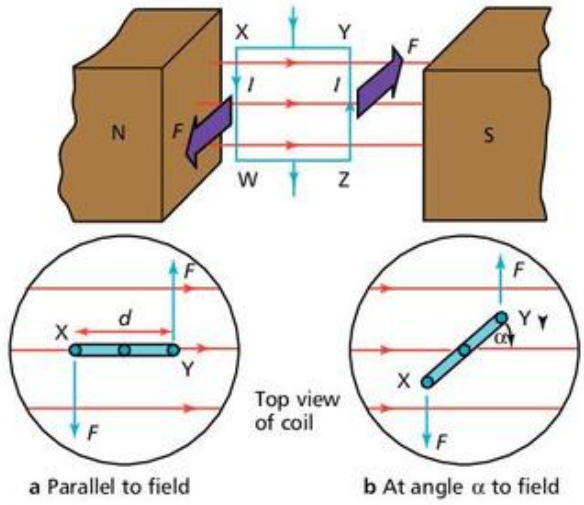


Fig 7 Couple on a coil

- The long sides of the coil are vertical and of length l. Each side therefore experiences a horizontal force $F = BIln$ in opposite directions at right angles to the field lines.
- The pair of forces acting on the long sides form a couple as the forces are not directed along the same line. The torque of the couple $= Fd$, where d is the perpendicular distance between the line of action of the forces on each side. If the plane of the coil is at angle α to the field lines, then $d = w\cos\alpha$ where w is the width of the coil.
- Therefore, the torque $= Fw\cos\alpha = BIlnw\cos\alpha$ $= BIAn\cos\alpha$, where $A = lw =$ the coil area.
  If $\alpha = 0$ (i.e. the coil is parallel to the field) the torque $= BIAn$ since $\cos 0 = 1$.
  If $\alpha = 90°$ (i.e. the coil is perpendicular to the field) the torque $= 0$ since $\cos 90° = 0$.

In an electric motor, the coil is wound on an iron core. The field is much stronger as a result so the torque on the motor is much stronger. Also, the presence of the core makes the field radial so the coil is in the plane of the field (i.e. $\alpha = 0$) for most of the time. As a result, the torque is steady and the motor runs more smoothly.

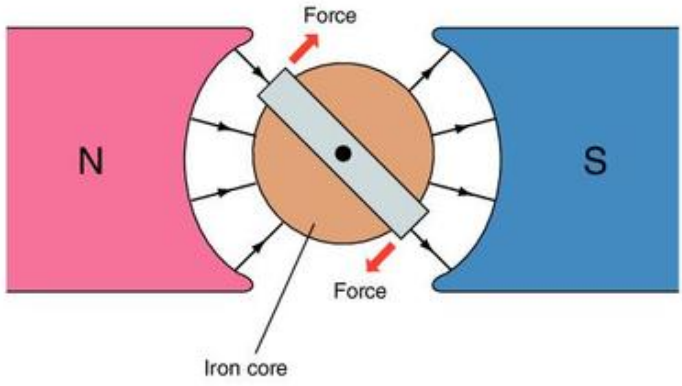


Fig 8 In an electric motor

Magnetic flux density due to current-carrying conductors

In a solenoid, experiments show that the magnetic flux density B inside a solenoid does not vary with position and is proportional to

- the current I, and
- the number of turns per metre, n, on the solenoid.

Therefore, inside a solenoid,

$$B = \mu_0 nI$$

where the constant of proportionality μ_0 is referred to as the **absolute permeability of free space**.

The unit of μ_0 may be written as the tesla metre per ampere (corresponding to the above equation) or the henry per metre ($\mathrm{H\,m^{-1}}$). As explained opposite, the value of μ_0 is $4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$.

Notes

1 The henry, H, is the unit of inductance which is a property of a coil to do with the induced voltage in it caused by a changing current.

2 Figure 9 shows how the magnetic flux density in a solenoid varies along its length. The formula above gives B at any position inside the solenoid away from its ends. At each end, the magnetic flux density is half the value given by the above formula.

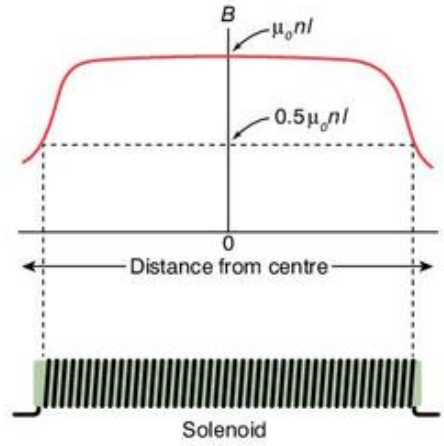
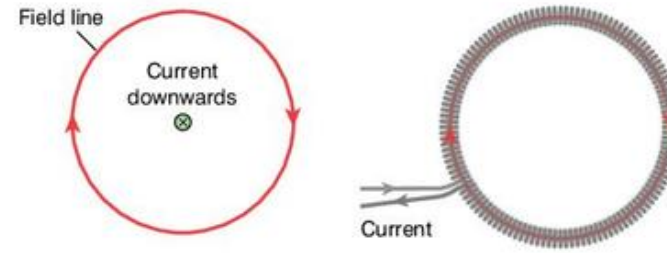


Fig 9 B in a solenoid

3 For a solenoid of length L with N turns, the number of turns per metre, $n = N/L$. Hence the magnetic flux density at its centre, $B = \dfrac{\mu_0 NI}{L}$

4 The magnetic flux density of a solenoid is increased considerably if a ferrous material such as iron or steel is placed inside the solenoid. The **relative permeability** μ_r of such a material is defined as the ratio of magnetic flux density with and without the solenoid core filled with the material. For example, if a solenoid with an iron core produces a magnetic flux density which is 2000 times greater than without the core (for the same current), the relative permeability of the material is 2000. Hence the magnetic flux density with the material present is given by the equation $B = \mu_0\mu_r nI$.

Near a long straight wire, the magnetic field lines are circles centred on the wire, as in Figure 10a.



a Near a long straight wire *b Inside a solenoid*

Fig 10 Circular field lines

Circular field lines would also be produced if the solenoid in Figure 9 was bent round as shown in Figure 10b. Its length would then be equal to the circumference of the circle which is $2\pi r$ where r is the circle radius. So its magnetic flux density would be $B = \frac{\mu_0 NI}{2\pi r}$. As all N coils contribute to B, each turn must contribute $\frac{\mu_0 I}{2\pi r}$ to B. Applying this idea to Figure 10a where only one wire contributes, the magnetic flux density at perpendicular distance r from the wire is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

At the centre of a flat circular coil of radius r with N turns, the field lines are perpendicular to the plane of the coil. Away from the centre of the coil, the field lines form non-circular loops. The magnetic flux density B is given by the equation

$$B = \frac{\mu_0 NI}{2r}$$

The definition of the ampere

Two parallel current-carrying wires exert equal and opposite forces on each other because the current in each wire creates a magnetic field that causes a force on the other wire.

As shown in Figure 11, the wires

- repel when the currents are in opposite directions, and
- attract when the currents are in the same direction.

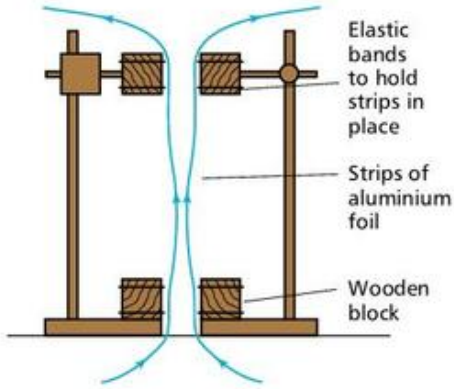


Fig 11 The force between two current-carrying conductors

Using the formulae $F = BIl$ and $B = \frac{\mu_0 I}{2\pi r}$, the force per unit length on one conductor (X) due to the magnetic field of the other conductor (Y), $\frac{F}{l} = \frac{\mu_0 I_x I_y}{2\pi r}$ where r is the perpendicular distance between the two wires and I_x and I_y are the currents in X and Y respectively.

The **ampere** is defined from this effect as the current in two parallel conductors that causes a force of 2.0×10^{-7} newtons per metre length on each wire when the wires (of negligible cross-section) are one metre apart in a vacuum.

Using the above formula with $I_x = I_y = 1\,\text{A}$, $r = 1.0\,\text{m}$ and $F/l = 2.0 \times 10^{-7}$ newtons per metre therefore gives $\mu_0 = 4\pi \times 10^{-7}\,\text{H m}^{-1}$.

QUESTIONS

1 a Use Figure 2 to work out the magnetic flux density of the magnet that was used in the test.

b Calculate the magnitude of the force on a straight horizontal wire of length 0.10 m carrying a current of 4.0 A in a uniform horizontal magnetic field of flux density 55 mT when the angle between the wire and the field lines is:
i 0, **ii** 30°, **iii** 90°.

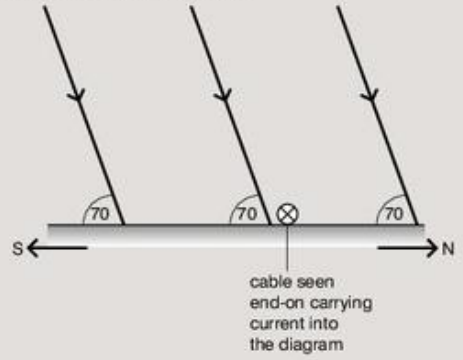
2 Table 1 relates the force on a current-carrying wire, at right angles to the lines of force of a magnetic field, to the magnetic flux density and the current. Complete the table by working out the missing data in each column.

Table 1

	a	b	c	d
B/T	0.20 T vertically down	0.20 T vertically down	?	0.1 T horizontal due?
I/A	3.0 A horizontal due North	?	3.0 A horizontal due North	2.0 A vertically up
l/m	0.040 m	0.040 m	0.040 m	0.040 m
F/N	?	0.036 N horizontal due South	0.024 N horizontal due West	? horizontal due East

3 At a certain location on the Earth's surface, the magnetic flux density B of the Earth is 70 μT in a direction due North at 70° to the horizontal. A horizontal cable of length 52 m aligned from East to West carries a current of 28 A.

Calculate the magnitude of the magnetic force on the cable and state its direction.



4 A rectangular coil of width 60 mm and of length 80 mm has 50 turns. The coil is placed horizontally in a uniform horizontal magnetic field of flux density 85 mT with its shorter side parallel to the field lines. A current of 8.0 A was passed through the coil. Sketch the arrangement and determine the force on each side of the coil.

18.4 Moving charges in a magnetic field

On these pages you will learn to:

- Recognise that a charged particle moving at a non-zero angle to the lines of a magnetic field experiences a force
- Predict the direction of the force on a charge moving in a magnetic field
- Recall and solve problems using $F = BQv\sin\theta$
- Derive and use the equation $V_H = BI/ntq$ for the Hall voltage

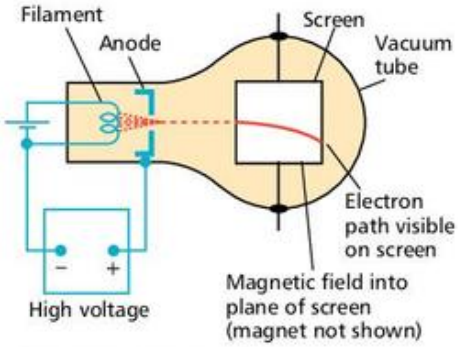


Fig 1 An electron deflection tube

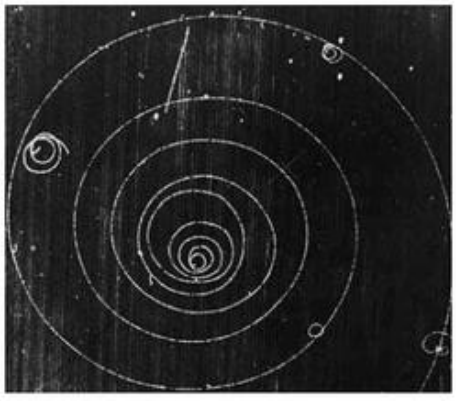


Fig 3 Charged particles in a magnetic field. The large spiral is due to a charged particle created by a collision (not shown) between a fast-moving incoming particle and the nucleus of an atom. The charged particle is forced onto a curved path by the magnetic field. It spirals inwards because kinetic energy and momentum decreases gradually to the atoms it passes through so it is deflected more and more by the field.

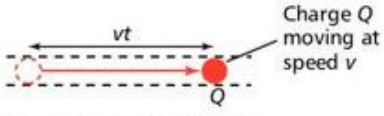


Fig 4 Force on a moving charge

Electron beams

Figure 1 shows a vacuum tube designed to show the effect of a magnetic field on an electron beam. The production of the electron beam is explained in topic 22.1. The path of the beam can be seen where it passes over the fluorescent screen in the tube. The beam is deflected downwards when a magnetic field is directed into the plane of the screen. Each electron in the beam experiences a force due to the magnetic field. The beam follows a circular path because the direction of the force on each electron is perpendicular to the direction of motion of the electron (and to the field direction). The direction of the force on an electron in the beam can also be worked out using Fleming's left hand rule, provided we remember the convention that the current direction is opposite to the direction in which the electrons move.

The reason why a current-carrying wire in a magnetic field experiences a force is that the electrons moving along the wire are pushed to one side by the field. If the electrons in Figure 2 had been confined to a wire, the whole wire would have been pushed downwards.

Fig 2 Electrons in a magnetic field

Force on a moving charge in a magnetic field

A beam of charged particles crossing a vacuum tube is an electric current across the tube. Suppose each charged particle has a charge Q and moves at speed v. In a time interval t, each particle travels a distance vt. Its passage is equivalent to current $I = \frac{Q}{t}$ along a wire of length $l = vt$. See Figure 4.

If the particles pass through a uniform magnetic field in a direction at right angles to the field lines, each particle experiences a force F due to the field. If the particles were confined to a wire, the force would be given by $F = BIl$.

For moving charges, the same equation applies where $I = \frac{Q}{t}$ and $l = vt$.

Therefore, for a charged particle moving across a uniform magnetic field in a direction at right angles to the field, $F = BIl = B \times \frac{Q}{t} \times vt = BQv$.

More generally, if the direction of motion of a charged particle in a magnetic field is at angle θ to the lines of the field, then $B\sin\theta$ is used in the equation for F. This is because $B\sin\theta$ is the component of the magnetic field perpendicular to the direction of motion of the charged particle.

For a particle of charge Q moving through a uniform magnetic field at speed v in a direction at angle θ to the field, the force on the particle is given by

$$F = BQv\sin\theta$$

Notes

1 If the velocity of the charged particle is at right angles to the direction of the magnetic field, $\theta = 90°$ so the equation becomes $F = BQv$ because $\sin 90° = 1$.

2 If the velocity of the charged particle is parallel to the direction of the magnetic field, $\theta = 0$ so $F = 0$ because $\sin 0 = 0$.

The Hall probe

Hall probes are used to measure magnetic field strength and also as magnetic field sensors.

A Hall probe contains a slice of semiconducting material. Figure 5 shows the slice in a magnetic field with the field lines perpendicular to the flat side of the slice. A constant current passes through the slice as shown. The **charge carriers** (which are electrons in an n-type semiconductor) are initially deflected by the field. As a result, a potential difference is created between the top and bottom edges of the slice. This effect is known as the **Hall Effect** after its discoverer.

The p.d., referred to as the Hall voltage, is proportional to the magnetic flux density, provided the current is constant. This is because each charge carrier passing through the slice is subjected to a magnetic force $F_{mag} = BQv$, where v is the speed of the charge carrier. Once the Hall voltage has been created, the magnetic deflection of a charge carrier entering the slice is opposed by the force on it due to the electric field created by the Hall voltage so each charge carrier passes through undeflected. The electric field force $F_{elec} = \frac{QV_H}{d}$, where V_H represents the Hall voltage and d is the distance between the top and bottom edges of the slice. Therefore, $\frac{QV_H}{d} = BQv$ gives $V_H = Bvd$.

For constant current I passing through a slice of cross-sectional area A, $I = nAvq$, where n is the number of charge carriers per unit volume and v is their drift velocity. See topic 6.2.

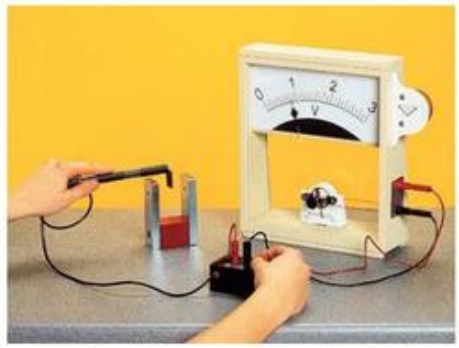
Combining this equation with the equation $V_H = Bvd$ gives:

$$V_H = Bvd = \frac{B(nAvq)d}{nAq} = \frac{BId}{nAq}$$

As $A = d \times t$, where t is the thickness of the slice:

$$V_H = \frac{BId}{nAq} = \frac{BId}{n(d \times t)q}$$

Therefore $$\boldsymbol{V_H = \frac{BI}{ntq}}$$



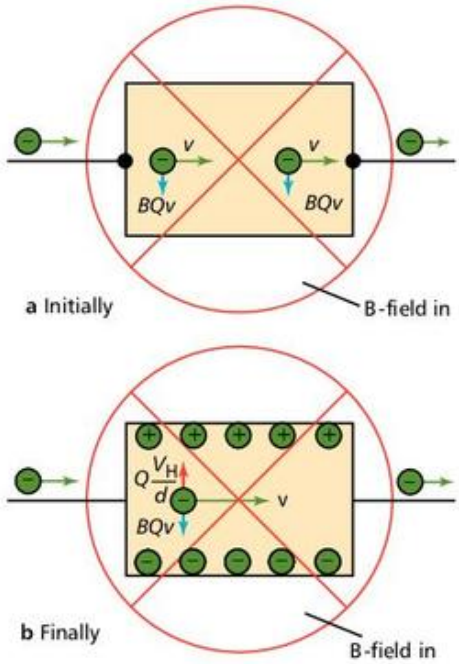


Fig 5 The Hall Effect

QUESTIONS

$e = 1.6 \times 10^{-19}$ C

1 In Figure 2, how would the force on the electrons in the magnetic field differ if:
- **i** the magnetic field was reversed in direction,
- **ii** the magnetic field was reduced in strength,
- **iii** the speed of the electrons was increased?

2 Calculate the force on an electron that enters a uniform magnetic field of flux density 150 mT at a velocity of 8.0×10^6 m s^{-1} at an angle of:
- **i** 90°, **ii** 30° to the field.

3 A beam of protons moving at constant speed is directed into a uniform magnetic field in the same direction as the field.
- **a** Explain why the beam is not deflected by the field.
- **b** Describe and explain how the path of the beam in the field would have differed if the beam had been directed into the field at a slight angle to the field lines.

4 In a Hall probe, electrons passing through the semiconductor slice experience a force due to a magnetic field.
- **a** Explain why a potential difference is created across the slice as a result of the application of the magnetic field.
- **b** When the magnetic flux density is 90 mT, each electron moving through the slice experiences a force of 6.4×10^{-20} N due to the magnetic field. Calculate:
  - **i** the mean speed of the electrons passing through the slice,
  - **ii** the force on each electron if the magnetic flux density is increased to 120 mT.

18.5 Charged particles in circular orbits

On these pages you will learn to:

- Describe and analyse the deflection of beams of charged particles by uniform electric and uniform magnetic fields
- Explain how electric and magnetic fields can be used in velocity selection
- Recognise that a uniform magnetic field causes the circular orbit of a charged particle in devices such as a cyclotron, a synchrotron and a mass spectrometer

Magnetic fields are used to control beams of charged particles in many devices, from television tubes to high energy accelerators. The force of the magnetic field on a moving charged particle is at right angles to the direction of motion of the particle.

- No work is done by the magnetic field on the particle as the force acts at right angles to the velocity of the particle. Its direction of motion is changed by the force but not its speed. The kinetic energy of the particle is unchanged by the magnetic field.
- In accordance with Fleming's left hand rule, the magnetic force is perpendicular to the velocity at any point along the path. The force therefore acts towards the centre of curvature of the circular path.
- The particle moves on a circular path. The force causes a centripetal acceleration because it is perpendicular to the velocity. Figure 1 shows the deflection of a beam of electrons in a uniform magnetic field. The path is a complete circle because the magnetic field is uniform and the particle remains in the field.

The radius, r, of the circular orbit in Figure 1 depends on the speed v of the particles and the magnetic flux density B.

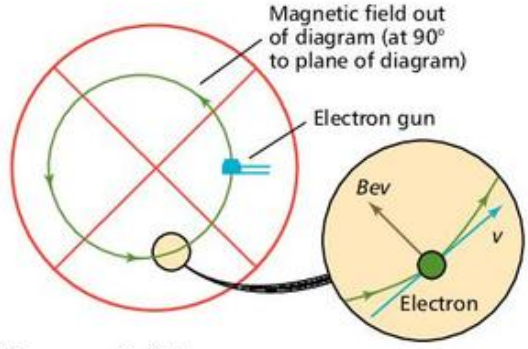


Fig 1 *A circular orbit in a magnetic field*

At any point on the orbit, the particle is acted on by a magnetic force $F = BQv$ and it experiences a centripetal acceleration, $a = \frac{v^2}{r}$ towards the centre of the circle.

Applying Newton's second law in the form $F = ma$ gives

$$BQv = \frac{mv^2}{r}$$

Rearranging this equation gives

$$r = \frac{mv}{BQ}$$

The equation for r shows that

1. r decreases if B is increased,
2. r increases if v is increased,
3. particles in a beam with different values of specific charge, Q/m, are separated by a magnetic field.

Applications

The following applications make use of the essential principle that charged particles move on circular paths when in a magnetic field and moving at right angles to the field lines.

1 Electrons moving in a circle

The fine beam tube shown in Figure 2 contains hydrogen gas at low pressure. When a beam of electrons from the electron gun in the tube passes through the gas, the atoms along the beam path emit light due to collisions with electrons. So the path of the beam is seen as a fine trace of light in the tube. A pair of coils, placed either side of the tube, is supplied with a direct current to produce a uniform magnetic field through the tube.

Provided the initial direction of the beam is at right angles to the magnetic field lines, the beam curves round in a circle.

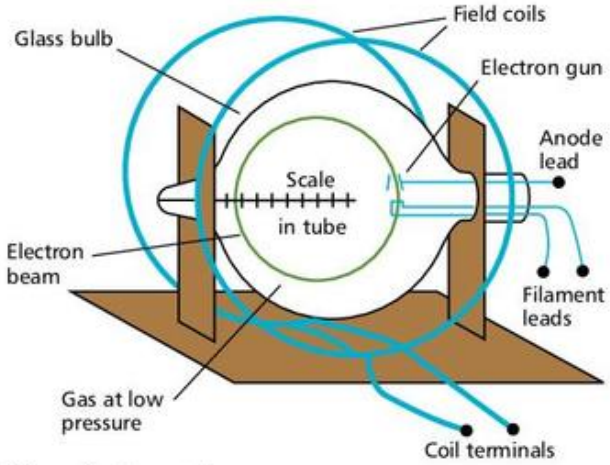


Fig 2 Using a fine beam tube

As explained opposite, the radius of the circle $r = \frac{mv}{Be}$ where B is the magnetic flux density and v is the speed of the electrons.

The equation shows that the radius of the circle can be reduced by:

- increasing the magnetic flux density, or
- reducing the anode voltage of the electron gun since this reduces the speed of the electrons.

As explained in topic 22.2, the fine beam tube may be used to measure e/m, the specific charge of the electron. This was first determined by J. J. Thomson in 1895. Its value is $1.76 \times 10^{11}\,\mathrm{C\,kg^{-1}}$.

Before Thomson made this measurement, the hydrogen ion was known to have the largest specific charge of any charged particle. Thomson showed that the electron's specific charge is 1860 times larger than that of the hydrogen ion. However, Thomson could not conclude that the electron has a much smaller mass than the hydrogen ion as the charge of the electron was not known at that time. The charge of the electron was measured by R. Millikan in 1915.

2 The cyclotron

The cyclotron was invented in 1930 by E.O. Lawrence. It consists of 2 hollow D-shaped electrodes (referred to as 'dees') in a vacuum chamber. A high-frequency alternating voltage is applied between the dees. A beam of charged particles is directed into one of the dees near the centre of the cyclotron. A uniform magnetic field is applied perpendicular to the plane of the dees.

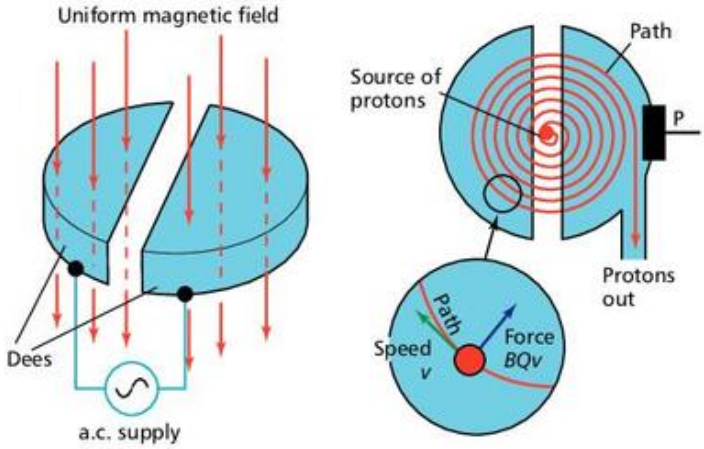


Fig 3 The cyclotron

The charged particles are forced on a circular path by the magnetic field, causing the particles to emerge from the dee they were directed into. The particles emerging from the dee when the alternating voltage reverses are accelerated into the other dee where they are forced on a circular path by the magnetic field. When they emerge from this dee, the alternating voltage reverses again and accelerates the particles into the first dee where the process is repeated. This occurs because the time taken, T, by a particle to move round its semi-circular path in the dee $= \frac{\pi r}{v}$, where r is the radius of the path and v is the particle speed. As explained previously, $r = \frac{mv}{BQ}$ so $T = \frac{m\pi}{BQ}$. For an alternating voltage of frequency f, the time for one half cycle $= \frac{1}{2f}$. Therefore, $\frac{1}{2f} = \frac{m\pi}{BQ}$ so $f = \frac{BQ}{2\pi m}$. The equation shows that the frequency is independent of the radius and the speed so the charged particles cross between the dees each time the voltage reverses.

The particles gain speed each time they are accelerated from one dee to the other. The radius of the circular path, therefore, is larger each time a particle travels into and out of a dee. The particles emerge from the cyclotron when the radius of orbit is equal to the dee radius R. As $v = \frac{BQr}{m}$ the speed of the particles on exit from the cyclotron $= \frac{BQR}{m}$.

The **synchrotron** accelerates charged particles to much higher energies than a cyclotron. The magnetic field is increased to keep the particles in an orbit of constant radius as they are boosted to higher and higher speeds.

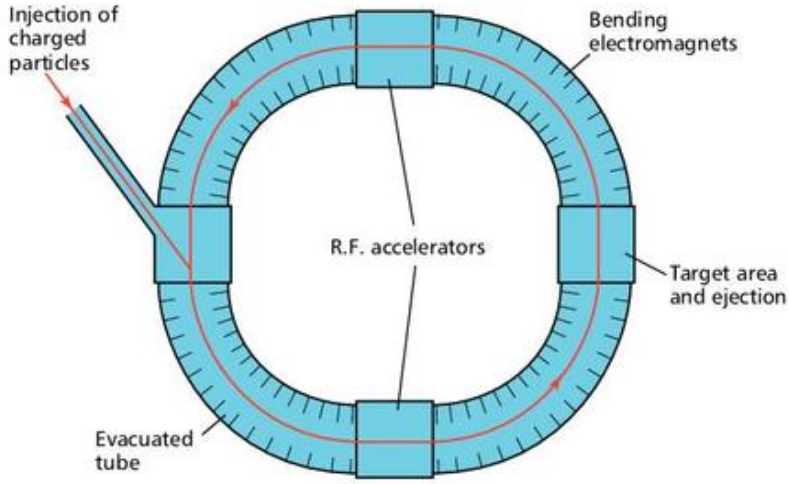


Fig 4 *The synchrotron*

As the radius of orbit is constant, semi-circular dees are not necessary and the particles move round a ring-shaped tube in a magnetic field created by a large number of electromagnets positioned round the ring. A high-frequency alternating voltage, applied between electrodes positioned in the ring, is used to accelerate the charged particles in the ring to high energies. In operation, the particles are injected into the ring and are boosted in 'bursts' to high energies each time the magnetic field is increased.

EXTENSION

The Van Allen belts

Charged particles from space are trapped in belts above the Earth's atmosphere by the Earth's magnetic field. These belts were predicted by Van Allen and were discovered by Geiger counters aboard the US Explorer 1 satellite in 1958. The charged particles in the belts spiral around the field lines, bouncing back near the poles. The inner belt consists of protons and other charged particles with energies of 10 to 100 MeV that stream from the Sun in the 'solar wind' or as a result of cosmic rays (i.e. high energy particles from space) colliding with atoms in the upper atmosphere. The outer belt stretches out to about 20000 km above the Earth and consists mostly of electrons with energies of 10 to 100 keV from the Sun.

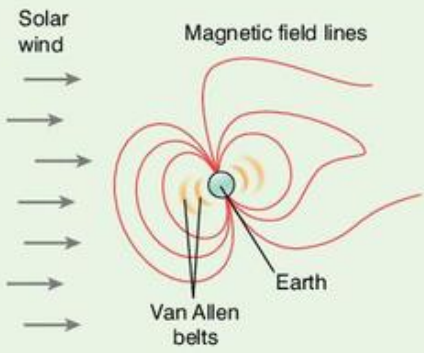


Fig 5 *The Van Allen belts*

3 The mass spectrometer

The mass spectrometer is used to analyse the type of atoms present in a sample. The atoms of the sample are ionised and directed in a narrow beam at constant velocity into a uniform magnetic field. Each ion is deflected in a semi-circle by the magnetic field onto a detector, as shown in Figure 6. The radius of curvature of the path of each ion depends on the specific charge $\frac{Q}{m}$ of the ion in accordance with the equation $r = \frac{mv}{BQ}$. Each type of ion is deflected by a different amount onto the detector. The detector is linked to a computer which is programmed to show the relative abundance of each type of ion in the sample.

The ions in the beam enter the magnetic field at the same speed. When they are produced from the sample, they have a continuous range of speeds. Before they enter the magnetic field, they are formed into a beam and directed through a velocity selector, as shown in Figure 6. The velocity selector consists of a magnet and a pair of parallel plates at spacing d and voltage V_p due to a high voltage supply. The magnet and the plates are aligned so each ion passing through the velocity selector is acted on by an electric field force, $F_{elec} = \frac{QV_P}{d}$, in the opposite direction to the magnetic field force $F_{mag} = B_S Qv$ where B_S is the magnetic flux density of the magnet in the velocity selector. Ions moving at a certain speed such that $B_S Qv = \frac{QV_P}{d}$ experience equal and opposite forces and so pass through undeflected. All other ions are deflected and do not pass through the collimator slit. So the beam emerging from the collimator consists of different types of ions, all with the same speed $v = \frac{V_P}{B_S d}$.

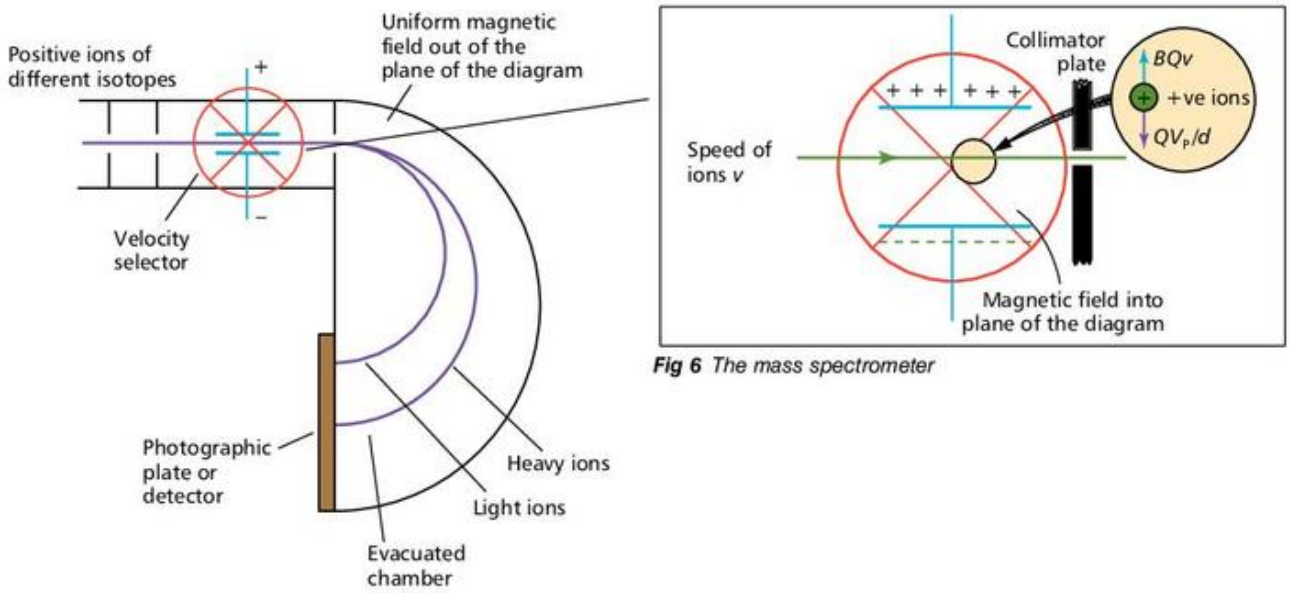


Fig 6 The mass spectrometer

QUESTIONS

$e = 1.6 \times 10^{-19}$ C, e/m for the electron $= 1.76 \times 10^{11}$ C kg^{-1}

1 A beam of electrons at a speed of 3.2×10^7 m s^{-1} is directed into a uniform magnetic field of flux density 8.5 mT in a direction perpendicular to the field lines. The electrons move on a circular orbit in the field.

a i Explain why the electrons move on a circular orbit.
ii Calculate the radius of the orbit.

b The flux density is adjusted until the radius of orbit is 65 mm. Calculate the flux density for this new radius.

2 In a fine beam tube, electrons were accelerated from rest through a certain p.d. before being directed at a speed of 2.9×10^7 m s^{-1} in a narrow beam into a uniform magnetic field.

a The beam followed a circular path of radius 35 mm in the magnetic field. Calculate the flux density of the magnetic field.

b The speed of the electrons in the beam was halved as a result of reducing the anode voltage. Calculate the new radius of curvature of the beam in the field.

3 The first cyclotron, used to accelerate protons, was 0.28 m in diameter and was in a magnetic field of flux density 1.1 T.

a Show that protons emerged from this cyclotron at a maximum speed of 1.5×10^7 m s^{-1}.

b Calculate the maximum kinetic energy, in MeV, of a proton from this accelerator.

The mass of a proton $= 1.67 \times 10^{-27}$ kg.
1 MeV $= 1.6 \times 10^{-13}$ J

4 In a mass spectrometer, a beam of different ions moving at a speed of 7.6×10^4 m s^{-1} was directed into a uniform magnetic field of flux density 680 mT, as shown in Figure 6.

a An ion was deflected through 180° to a position on the detector which was 28 mm from where it entered the field. Calculate the specific charge of the ion.

b A different type of ion was deflected onto the same position on the detector when the magnetic flux density was changed to 400 mT. Calculate the specific charge of this ion.

CHAPTER SUMMARY

1 a $F = BIl \sin\theta$ gives the force F on a current-carrying wire of length l in a uniform magnetic field B at angle θ to the field lines, where I is the current.

b The direction of the force is given by Fleming's left hand rule where the field direction is the direction of the field component perpendicular to the wire.

2 a $F = BQv \sin\theta$ gives the force F on a particle of charge Q moving through a uniform magnetic field B at speed v in a direction at angle θ to the field.

b If the velocity of the charged particle is perpendicular to the field, $F = BQv$.

c The direction of the force is given by Fleming's left hand rule, provided the current is in the direction that positive charge would flow in.

3 $BQv = \frac{mv^2}{r}$ gives the radius of the orbit of a charge moving in a direction at right angles to the lines of a magnetic field.

18 End of chapter questions

$e = 1.6 \times 10^{-19}$C, e/m for the electron $= 1.76 \times 10^{11}$C kg^{-1}
The mass of a proton is 1.67×10^{-27}kg

1 The diagram shows a plotting compass mid-way between a bar magnet and one end of a solenoid. The solenoid is connected in series with a battery, a variable resistor and a switch.

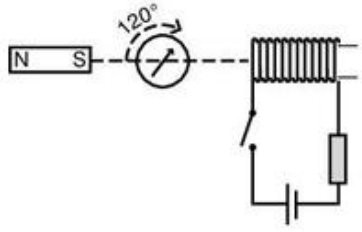


Fig 1.1

a When the switch is open, the plotting compass points directly towards the bar magnet. When the switch is closed, the needle of the plotting compass turns through 120°. Explain why the needle turns when the switch is closed.

b With the switch closed, the variable resistor is adjusted, making the needle turn back by 30°.
i What must have been the effect of the adjustment of the variable resistor on the solenoid current?
ii What would be the effect on the direction of the compass needle if the magnet was to be moved away from the plotting compass?

2 A U-shaped magnet is placed on the pan of a top-pan balance, as shown below. A straight wire is placed horizontally between the poles of the magnet, which are of length 32 mm. The wire is connected to a variable resistor, an ammeter, a switch and a battery.

Fig 2.1

a When the switch is closed, the reading of the top-pan balance changes by 0.028 N when the ammeter reads 3.8 A. Calculate the magnetic flux density between the poles of the magnet.

b Calculate the force on the wire if the current is increased to 7.0 A.

3 The Earth's magnetic field at a certain location has a downward vertical component of 58 μT, and a horizontal component of 18 μT in a direction due North. A horizontal cable of length 50 m lies along a line from North to South. The cable carries a direct current of 26 A from South to North.

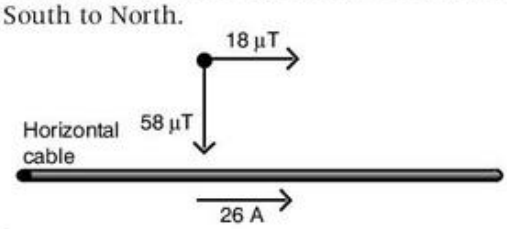


Fig 3.1

a **i** Show that the magnitude of the force on the cable due to the Earth's magnetic field is 7.5×10^{-2}N.
ii State the direction of the force on the cable.

b Without further calculation, explain why the force on the cable for the same current would have been different, had the cable been aligned along a line from East to West instead of from North to South.

4 The armature coil of an electric motor has 100 turns and is of length 0.12 m as shown below. The coil spins between the poles of a U-shaped magnet, where the magnetic flux density is 0.18 T.

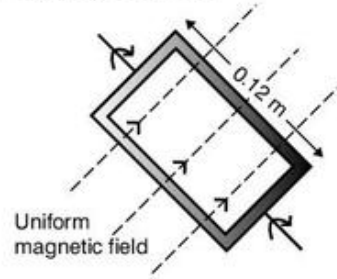


Fig 4.1

a **i** Calculate the force on each side of the coil when the current through it is 0.8 A.
ii Discuss how the force on each side of the coil changes during one complete rotation of the coil.

b Explain why the force acting on each side of the coil has its maximum turning effect when the plane of the coil is parallel to the lines of force of the magnetic field.

5 **a** The magnetic flux density between the poles of a U-shaped magnet was determined by measuring the force on a wire of length 32 mm positioned at right angles to the field lines. When a current of 4.5 A was passed along the wire, the force was 0.013 N. Calculate the magnetic flux density at the wire.

b Calculate the angle the wire would need to be moved through to reduce the force on the wire by 5% when the current is 4.5 A.

6 The Earth's magnetic field at a certain location has a flux density of 0.070 mT in a direction due North at an angle of 70° to the surface, as shown in the diagram below. A straight wire of length 35 mm carrying a current of 8.2 A upwards is placed vertically in the field.

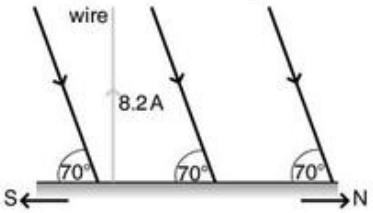


Fig 6.1

a Show that the wire experiences a force of 6.9×10^{-6} N.

b Determine the direction of the force.

7 A flat rectangular coil of length 65 mm and width 42 mm consists of 20 turns of insulated wire. The coil is placed in a horizontal uniform magnetic field of flux density 95 mT with its plane parallel to the field and with its shorter sides horizontal.

a Sketch this arrangement and explain why the coil experiences a torque when it carries a direct current.

b The current in the coil is adjusted to 4.2 A. Calculate the force on each of the longer sides due to the magnetic field and hence show that the torque on the coil is 0.022 N m.

8 In a d.c. electric motor, the coil consists of 60 turns of insulated wire wound on a rectangular frame of length 25 mm and width 20 mm. The coil spins in a magnetic field of magnetic flux density 110 mT.

a Calculate the maximum torque on the coil due to the magnetic field when a current of 0.80 A passes through the coil.

b A 1.0 N weight is attached to a thread wrapped round a pulley of diameter 5 mm fitted to the motor spindle. Discuss if this weight could be raised by the motor when the coil current is 0.80 A.

9 A beam of electrons in a uniform magnetic field of magnetic flux density 3.6 mT travelled on a circular path of radius 55 mm.

a Explain why the electrons travelled at constant speed in the field.

b Calculate the speed of the electrons.

10 A beam of electrons moving at a speed of 2.7×10^7 m s^{-1} is directed horizontally into a uniform magnetic field of flux density 8.6 mT which is directed vertically upwards.

a i Explain why the beam moves on a circular orbit in the field.

ii Calculate the radius of curvature of the beam in the field.

b The magnetic flux density is reduced steadily to zero then reversed and increased steadily to 8.6 mT in the opposite direction. Describe and explain the effect on the beam of these changes.

11 A cyclotron has a diameter 0.80 m. Protons are accelerated by the cyclotron to a maximum kinetic energy of 4.2 MeV.

a Show that the maximum speed of the protons is 2.8×10^7 m s^{-1}.

b i Calculate the magnetic flux density of the magnet used in this cyclotron.

ii Show that the frequency of the alternating voltage applied to the cyclotron was 11 MHz.

12 In a mass spectrometer, a beam of different ions travelling at the same speed is directed into a uniform magnetic field at right angles to the field.

a Explain why the magnetic field separates the ions according to their specific charge $\frac{Q}{m}$.

b i In a test of a mass spectrometer, protons at a speed of 4.8×10^6 m s^{-1} were directed into the field. The protons moved in the field in a semi-circular orbit of radius 60 mm. Calculate the magnetic flux density of the magnetic field.

ii Calculate the specific charge of an ion that moved on an orbit of radius 420 mm in the same flux density.

13 a Define the *tesla*. *(3 marks)*

b A large horseshoe magnet produces a uniform magnetic field of flux density B between its poles. Outside the region of the poles, the flux density is zero.

The magnet is placed on a top-pan balance and a stiff wire XY is situated between its poles, as shown in Figure 13.1.

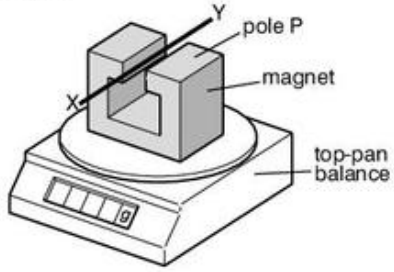


Fig 13.1

The wire XY is horizontal and normal to the magnetic field. The length of wire between the poles is 4.4 cm.

A direct current of magnitude 2.6 A is passed through the wire in the direction from X to Y.

The reading on the top-pan balance increases by 2.3 g.

i State and explain the polarity of the pole P of the magnet. *(3 marks)*

ii Calculate the flux density between the poles in T. *(3 marks)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 04 Q6 June 2009

19 Electromagnetic induction

19.1 Generating electricity

On these pages you will learn to:

- Describe experiments on electromagnetic induction to show:
  - that a changing magnetic flux can induce an e.m.f. in a circuit
  - that the direction of the induced e.m.f. opposes the change producing it
  - the factors affecting the magnitude of the induced e.m.f.
- Explain in terms of electrons why an e.m.f. is induced in a conductor when it cuts across the lines of a magnetic field

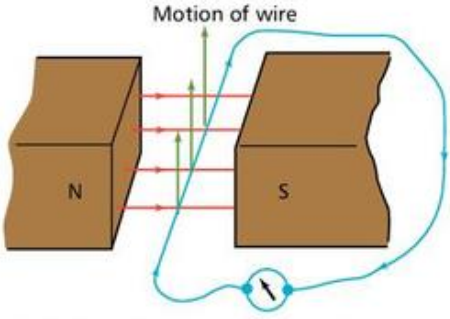


Fig 1 Generating an electric current

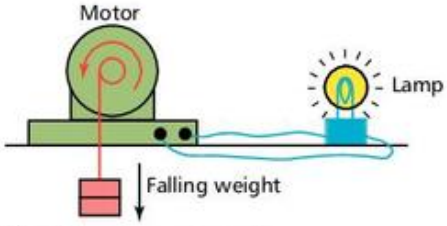


Fig 2 A motor as a generator

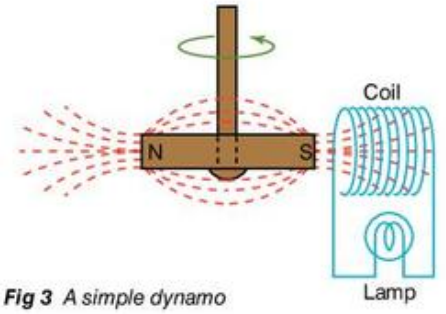


Fig 3 A simple dynamo

To generate electricity, all you need is a magnet and some wire which is connected to a sensitive meter, as shown in Figure 1. When the magnet is moved near the wire, a small current passes through the meter. This happens because an e.m.f. is **induced** in the wire. This effect, known as **electromagnetic induction**, occurs whenever a wire cuts across the lines of a magnetic field. If the wire is part of a complete circuit, the induced e.m.f. forces electrons round the circuit. The induced e.m.f. can be increased by:

- moving the wire faster,
- using a stronger magnet,
- making the wire into a coil and pushing the magnet in or out of the coil, as in Figure 3 below.

No e.m.f. is induced in the wire if the wire moves parallel to the magnetic field lines as it moves through the field. The wire must cut across the lines of the magnetic field for an e.m.f. to be induced in the wire.

Other methods of generating induced e.m.fs include:

1 **Using an electric motor in reverse**, as in Figure 2. The falling weight makes the motor coil turn between the poles of the magnet in the motor. The e.m.f. induced in the coil forces a current round the circuit and so causes the lamp to light. The faster the coil turns, the brighter the lamp is.
2 **Using a cycle dynamo**, as in Figure 3. When the magnet in the dynamo spins, an e.m.f. is induced in the coil. If the coil is connected to a lamp, the lamp lights because the e.m.f. forces a current round the circuit.

In both examples above, an e.m.f. is induced because there is relative motion between the coil and the magnet. In the electric motor in reverse, the coil spins and the magnet is fixed. In the dynamo, the magnet spins and the coil is fixed.

Energy changes

When a magnet is moved relative to a conductor (e.g. a wire or a coil), an e.m.f. is induced in the circuit. If the conductor is part of a complete circuit which has no other sources of e.m.f., a current passes round the circuit just as if the circuit included a battery. However, unlike the e.m.f. of a battery which is constant, the induced e.m.f. becomes zero when the relative motion between the magnet and the wires ceases.

An electric current transfers energy from the source of the e.m.f. in a circuit to the other components in the circuit. For example, when a dynamo is used to light a lamp, energy is transferred from the dynamo to the lamp. The current through the dynamo coil causes a reaction force on the coil due to the magnet. Work must therefore be done to keep the magnet spinning. The energy transferred from the coil to the lamp is equal to the work done on the coil to keep it spinning.

The rate of transfer of energy from the source of e.m.f. to the other components of the circuit is equal to the product of the induced e.m.f. and the current. This is because:

- the induced e.m.f. is the energy transferred from the source per unit charge that passes through the source,
- the current is the charge flow per second.

So the induced e.m.f. × the current = energy transferred per unit charge from the source × the charge flow per second = energy transferred per second from the source.

Michael Faraday 1791–1861

Electromagnetic induction was discovered by Michael Faraday in 1831 at the Royal Institution, London. Faraday knew that a current passing along a wire produced a magnetic field near the wire and he wanted to know if a magnet could be used to produce a current. Using a magnetic compass near a loop of wire as a detector of current, he showed that the compass deflected whenever the magnet was moved in or out of the wire. He used the term **'electromotive force' (e.m.f.)** to describe the voltage induced in a wire. When he demonstrated his discoveries to an invited audience at the Royal Institution, he was asked the question 'What use is electricity, Mr Faraday?'. He replied with another question 'What use is a new baby?'. No one can tell what can grow from a new discovery.

Fig 4 Apparatus for electromagnetic induction

Understanding electromagnetic induction

When a beam of electrons is directed across a magnetic field, each electron experiences a force at right angles to its direction of motion and to the field direction. A metal rod is like a tube containing lots of free electrons. If the rod is moved across a magnetic field, as shown in Figure 5, the magnetic field forces the free electrons in the rod to one end away from the other end. So, one end of the rod becomes negative and the other end positive. In this way, an e.m.f. is induced in the rod. The same effect happens if the magnetic field is moved and the rod is stationary. As long as there is relative motion between the rod and the magnetic field, an e.m.f. is induced in the rod. If the relative motion ceases, the induced e.m.f. becomes zero because the magnetic field no longer exerts a force on the electrons in the rod. Note that when the rod is part of a complete circuit, the electrons are forced round the circuit. In other words, the induced e.m.f. drives a current round the circuit.

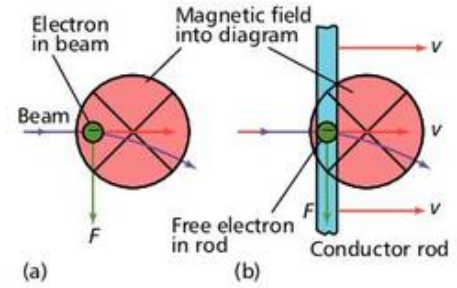


Fig 5 Deflection of electrons in a magnetic field

The dynamo rule

In Figure 5, the magnetic field is into the plane of the diagram and the motion of the conductor is towards the right. The electrons in the rod are forced downwards. The direction of the induced current can also be worked out using Fleming's right-hand rule, also referred to as the dynamo rule, as shown in Figure 6. The direction of the induced current is, in accordance with the current convention, opposite to the direction of the flow of electrons in the conductor.

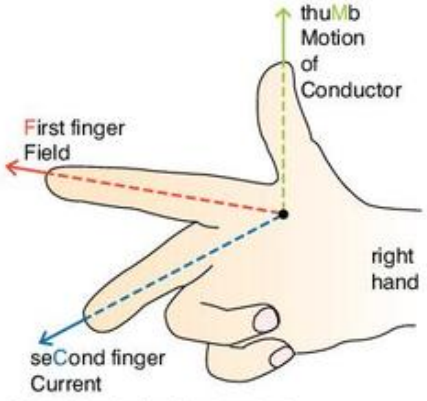


Fig 6 Fleming's right-hand rule

QUESTIONS

1 A coil of wire is connected to a sensitive meter.

- **a** Explain why the meter shows a brief reading when a magnet is pushed into the coil.
- **b** State two ways in which the meter reading could be made larger.

2 An electric motor consists of a coil of wire between the poles of a magnet. The motor is connected to a lamp. A thread wrapped round the motor spindle is used to support a weight, as shown in Figure 2.

- **a** Explain why the lamp lights when the weight descends.
- **b** What difference would have been made if the magnet had been much stronger?

3 **a** Explain why a lamp connected to a dynamo lights when the dynamo turns.

- **b** Why is the dynamo easier to turn when the lamp is disconnected?

4 A horizontal rod aligned along a line from East to West is dropped through a horizontal magnetic field which is directed from South to North.

- **a** **i** What is the direction of the velocity of the rod?
  ii Determine which end of the rod is positive. Explain your answer.
- **b** Explain why no e.m.f. is induced in the rod if it is aligned from north to south then dropped in the field.

19.2 The laws of electromagnetic induction

On these pages you will learn to:

- Define magnetic flux and the weber
- Recall and use $\Phi = BA$
- Define magnetic flux linkage
- Recall Faraday's law of electromagnetic induction and Lenz's law
- Use the above laws to explain and solve problems
- Explain Lenz's law using the principle of conservation of energy

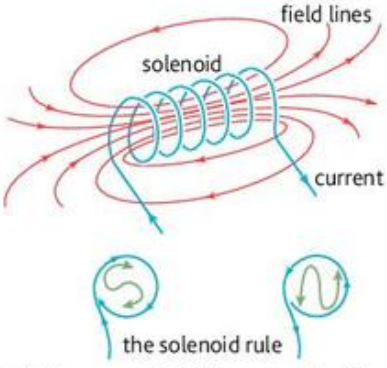


Fig 1 The magnetic field near a solenoid

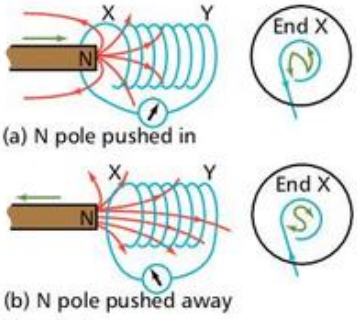


Fig 2 Lenz's Law

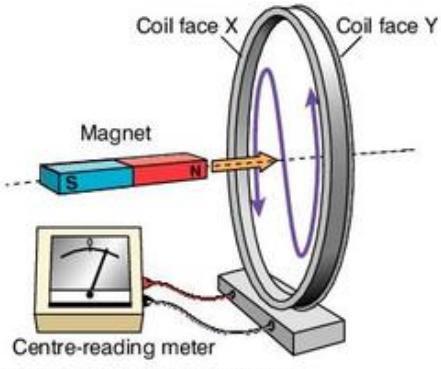


Fig 3 Induced polarity of a coil

Coils, currents and fields

A magnetic field is produced in and around a coil when an electromagnet is connected to a battery and a current is passed through it. A magnetic compass near the electromagnet is deflected when current passes through the coil. For a long coil or solenoid, the pattern of the magnetic field lines is like the pattern for a bar magnet – except the magnetic field lines near a bar magnet loop round from the north pole to the south pole of the magnet. Figure 1 shows the magnetic field pattern of a current-carrying solenoid. The field lines pass through the solenoid and loop round outside the solenoid from one end (the north pole) to the other end (the south pole). If each end in turn is viewed from outside the solenoid:

- current passes a**N**ticlockwise (or cou**N**terclockwise) round the '**N**orth pole' end,
- current passes clockwise round the 'south pole' end.

Lenz's law

When a bar magnet is pushed into a coil connected to a meter, the meter deflects. If the bar magnet is pulled out of the coil, the meter deflects in the opposite direction. What determines the direction of the induced current? Consider the North pole of a bar magnet approaching end X of a coil, as shown in Figure 2.

The induced current passing round the circuit creates a magnetic field due to the coil. The coil field must act against the incoming North pole, otherwise it would pull the N-pole in faster, making the induced current bigger, pulling the N-pole in even faster still, etc. Clearly, conservation of energy forbids this creation of kinetic and electrical energy from nowhere. So, the induced current creates a magnetic field in the coil which opposes the incoming N-pole. The induced polarity of end X must therefore be a N-pole so as to repel the incoming N-pole. Therefore, the current must go round end X of the coil in an anticlockwise direction, as shown.

If the magnet is removed from inside the coil, the induced current passes round end X of the coil in a clockwise direction. This corresponds to an induced S-pole at end X which, therefore, opposes the magnet moving away.

Lenz's law states that the direction of the induced current is always such as to oppose the change that causes the current.

The explanation of Lenz's law is that energy is never created or destroyed. The induced current could never be in a direction to help the change that causes it; that would mean producing electrical energy from nowhere, which is forbidden!

An induced current is generated if the coil is moved instead of the magnet and the magnet is at rest. In general, whenever the magnet and the coil move relative to each other, an induced current is generated and Lenz's law can be used to work out the direction of the current.

Some further examples of the applications of Lenz's law are given below.

1 **A bar magnet moves towards and through a flat coil**, as shown in Figure 3.
   - When the bar magnet approaches face X of the coil with its N-pole leading, the induced polarity of face X of the coil must be a N-pole in order to oppose the movement of the bar magnet. So the induced current in the coil is anticlockwise as seen in face X.
   - After passing through the coil, the bar magnet moves away from face Y of the coil with its S-pole trailing. The induced polarity of face Y of the coil

must be a N-pole in order to oppose the S-pole of the bar magnet moving away. So the induced current in the coil is anticlockwise as seen in face Y and therefore clockwise as seen in face X, opposite in direction to when the magnet was approaching the coil.

2 **A flat coil or a solenoid (P) carrying a direct current moves towards another flat coil (Q)**, as shown in Figure 4. The direction of the current in P determines its magnetic polarity.
When P and Q are approaching each other, the induced polarity in Q is the same as the 'leading' polarity of P (and therefore opposite to the 'trailing' polarity of P). The same ideas apply if Q is replaced by a large solenoid.

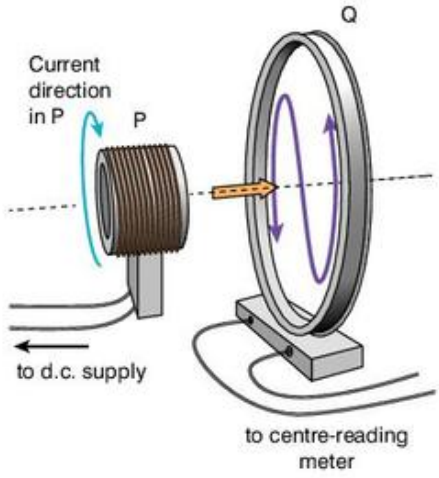


Fig 4 Induced polarity due to another coil

Faraday's law of electromagnetic induction

Consider a conductor of length ℓ, which is part of a complete circuit cutting through the lines of a magnetic field of flux density B.

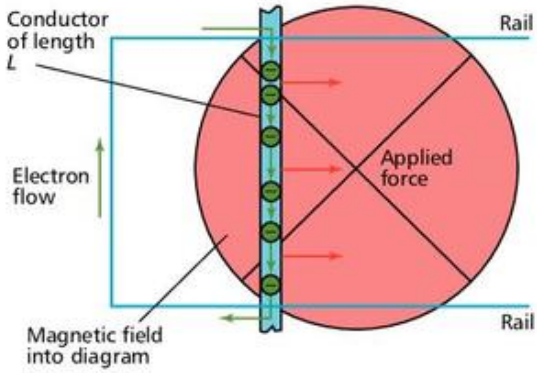


Fig 5 Induced e.m.f. in a conductor

An e.m.f., E is induced in the conductor and an induced current I passes round the circuit.

The conductor experiences a force $F = BI\ell$ due to carrying a current in a magnetic field. The force opposes the motion of the conductor and so an equal and opposite force must be applied to the conductor to keep it moving in the field. If the conductor moves a distance Δs in time Δt,

- the work done W by the applied force is given by $W = F\,\Delta s = BI\ell\,\Delta s$.
- the charge transfer along the conductor in this time $Q = I\Delta t$.

Therefore, the induced e.m.f. $$E = \frac{W}{Q} = \frac{BI\ell\,\Delta s}{I\,\Delta t} = \frac{B\ell\,\Delta s}{\Delta t}$$

As $\ell\Delta s = \Delta A$ 'swept out' in time Δt,

$$\mathrm{E} = \frac{B\Delta A}{\Delta t} = \frac{\Delta BA}{\Delta t} = \frac{\boldsymbol{\Phi}}{\Delta t}$$

where $\boldsymbol{\Phi} = BA$ = magnetic flux.

The product of the magnetic flux density, B, and the area, A, is called the **magnetic flux**. The concept of magnetic flux is very useful for calculating induced e.m.fs. The example of the conductor cutting across the field lines shows that the induced e.m.f. is equal to the magnetic flux swept out by the conductor each second. Michael Faraday was the first person to show how induced e.m.fs could be calculated from magnetic flux changes.

- **Magnetic flux, $\boldsymbol{\Phi} = BA$.**
- **Flux linkage through a coil of N turns $= N\boldsymbol{\Phi} = NBA$** where B is the magnetic flux density perpendicular to area A.
- The unit of magnetic flux is the **weber** (Wb), equal to $1\,\mathrm{T\,m^2}$.

Fig 6 An electric car

Regenerative braking

A battery-powered electric vehicle contains an **alternator** that can be used as an electric motor or as a generator. When the alternator is used as an electric motor, it is driven by the batteries. When the brakes are applied, the alternator is used to generate electricity which is used to recharge the battery. Some of the kinetic energy is transferred to electrical energy in the battery. The induced current through the alternator coil creates a magnetic field that acts against the magnetic field of the alternator. So, the alternator experiences a braking force which helps to slow the vehicle down.

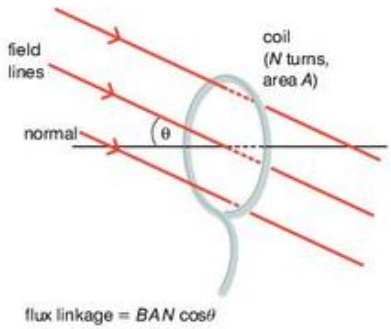


Fig 7 Flux linkage

NOTES

1 The minus sign represents the fact that the induced e.m.f. acts in such a direction as to oppose the change that causes it.
2 The unit of flux change per second, the weber per second, is the same as the volt. Therefore, the weber is equal to 1 volt second.

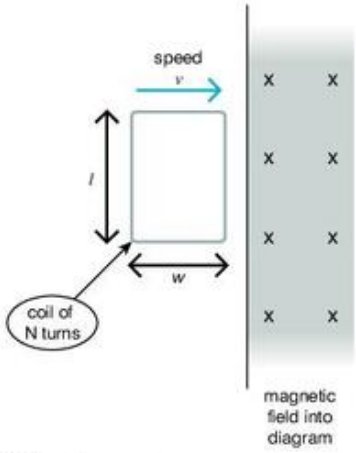


Fig 8 Flux changes

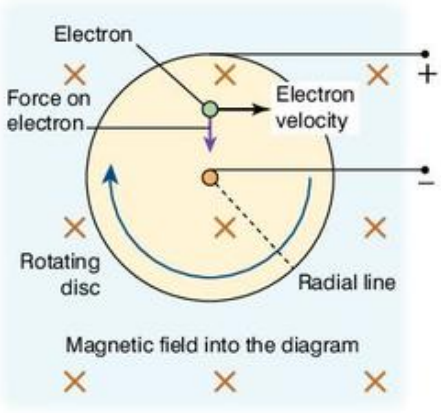


Fig 9 The rotating disc

Note that magnetic flux density B (in teslas) is the flux per unit area passing through an area at right angles to the area (i.e. normally). Therefore 1 tesla = 1 weber per square metre.

In general, when the magnetic field is at angle θ to the normal at the coil face, as shown in Figure 7, the flux linkage through the coil $N\Phi = BAN\cos\theta$

1 When the magnetic field is along the normal (i.e. perpendicular) to the coil face, the flux linkage = BAN.
2 When the coil is turned through 180°, the flux linkage = $-BAN$.
3 When the magnetic field is parallel to the coil area, the flux linkage = 0 as no field lines pass through the coil area.

Faraday's law of electromagnetic induction states that the induced e.m.f. in a circuit is proportional to the rate of change of flux linkage through the circuit.

$$\textbf{Induced e.m.f. } E = -N\frac{\Delta\Phi}{\Delta t}$$

where $N\frac{\Delta\Phi}{\Delta t}$ is the change of flux linkage per second.

Whenever the flux linkage through a circuit changes, an e.m.f. is induced in the circuit. The flux can be due to a permanent magnet or due to a current-carrying wire.

- If the flux is due to a permanent magnet, motion of the magnet relative to the circuit is necessary to cause an induced e.m.f. This is how an e.m.f. is generated in an a.c. generator or a dynamo.
- If the flux is due to a current-carrying wire, changing the current in the wire causes an induced e.m.f. in the circuit. This is how an e.m.f. is generated in a transformer or an induction coil.

Examples

1 A moving conductor in a magnetic field

An e.m.f. is induced in the conductor provided the conductor cuts across the lines of the magnetic field. The direction of motion of the conductor in Figure 5 is at right angles to the field lines.

As explained on the previous page, the magnitude of the induced e.m.f. $E = \frac{Bl\,\Delta s}{\Delta t}$, where l is the length of the conductor and Δs is the distance it moves in time Δt. Note that the change of flux in this time, $\Delta\Phi = Bl\,\Delta s$ so the change of flux per second, $\frac{\Delta\Phi}{\Delta t}$, is equal to the magnitude of the induced e.m.f.

Because the speed of the conductor, $v = \frac{\Delta s}{\Delta t}$, the induced e.m.f. $E = \frac{Bl\,\Delta s}{\Delta t} = Blv$.

$$\textbf{Induced e.m.f. } E = Blv$$

2 A rectangular coil moving into a uniform magnetic field

Consider a rectangular coil of N turns, length l and width w moving into a uniform magnetic field of flux density B at constant speed v, as shown in Figure 8. Suppose the coil enters the field at time $t = 0$.

- The time taken by the coil to enter the field completely $= \frac{\text{coil width}}{\text{speed}} = \frac{w}{v}$.
  During this time, the flux linkage $N\Phi$ increases steadily from 0 to $BNlw$. Therefore, the induced e.m.f. = change of flux linkage per second, $\frac{N\Delta\Phi}{\Delta t} = \frac{BNlw}{w/v} = BlvN$
- When the coil is completely in the field, the flux linkage through it ($= BNlw$) does not change so the induced e.m.f. is zero.

3 A spinning disc in a uniform magnetic field

When the disc spins at constant frequency, a constant e.m.f. is induced between the centre of the disc and its edge. In Figure 9, the conduction electrons in the rotating disc are forced to the centre by the magnetic field so the induced e.m.f. is negative at the centre and positive at the edge. Reversing the field or rotation direction reverses the induced e.m.f.
In one revolution, each radial line on the disc sweeps out an area $A = \pi r^2$ where r is the disc radius. Therefore, the magnetic flux swept out:

- in one revolution is $B \times \pi r^2 \ (= B \times A)$
- in one second is $B \times \pi r^2 \times f$ where f is the frequency of rotation of the disc.

Therefore, the induced emf E = change of magnetic flux per second = $B \times \pi r^2 \times f$

4 A fixed coil in a changing magnetic field

Figure 10 shows a small coil on the axis of a current-carrying solenoid. The magnetic field of the solenoid passes through the small coil. If the current in the solenoid changes, an e.m.f. is induced in the small coil. This is because the magnetic field through the coil changes so the flux linkage through it changes, causing an induced e.m.f. Because the induced e.m.f. is proportional to the rate of change of flux linkage through the coil and the flux linkage is proportional to the current I in the solenoid, the magnitude of the induced e.m.f is, therefore, proportional to the rate of change of current in the solenoid.

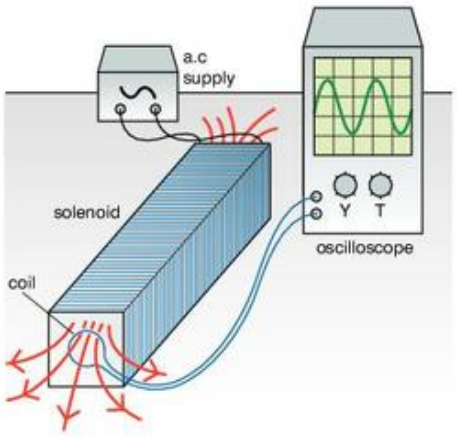


Fig 10 A changing magnetic field

QUESTIONS

1 A uniform magnetic field of flux density 72 mT is confined to a region of width 60 mm, as shown in Figure 11. A rectangular coil of length 50 mm and width 20 mm has 15 turns. The coil is moved into the magnetic field at a speed of 10 mm s^{-1} with its longer edge parallel to the edge of the magnetic field.

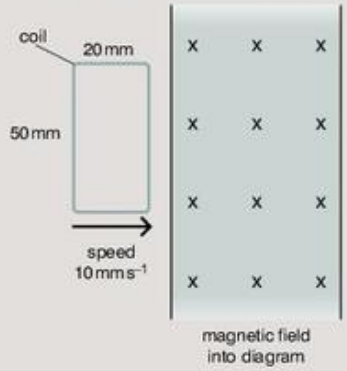


Fig 11

a Calculate:
- **i** the flux linkage through the coil when it is completely in the field,
- **ii** the time taken for the flux linkage to increase from zero to its maximum value,
- **iii** the induced e.m.f. in the coil as it enters the field.

b i Sketch a graph to show how the flux linkage through the coil changes with time from the instant the coil enters the field to when it leaves the field completely.
ii Sketch a graph to show how the induced e.m.f. in the coil varies with time.

2 A rectangular coil of length 40 mm and width 25 mm has 20 turns. The coil is in a uniform magnetic field of flux density 68 mT.
a Calculate the flux linkage through the coil when the coil is at right angles to the field lines.
b The coil is removed from the field in 60 ms. Calculate the mean value of the induced e.m.f.

3 A circular coil of diameter 24 mm has 40 turns. The coil is placed in a uniform magnetic field of flux density 85 mT with its plane perpendicular to the field lines.
a Calculate:
- **i** the area of the coil in m^2,
- **ii** the flux linkage through the coil.

b The coil was reversed in a time of 95 ms. Calculate:
- **i** the change of flux linkage through the coil,
- **ii** the magnitude of the induced e.m.f.

4 A small circular coil of diameter 15 mm and 25 turns is placed in a fixed position on the axis of a solenoid, as shown in Figure 10. The magnetic flux density of the solenoid at this position varies with current according to the equation $B = kI$, where $k = 1.2 \times 10^{-3}\,\text{T A}^{-1}$.
a Calculate the flux linkage through the coil when the current in the solenoid is 1.5 A.
b The current in the solenoid was reduced from 1.5 A to zero in 0.20 s. Calculate the magnitude of the induced e.m.f. in the small coil.

19.3 Alternating current

On these pages you will learn to:

- Describe and explain the a.c. generator as a simple application of electromagnetic induction
- Explain what is meant by a back e.m.f.
- Explain what is meant by the rms value and the peak value of an alternating current and know how they relate to each other
- Deduce that the mean power in a resistive load is half the maximum power for a sinusoidal alternating current
- Represent a sinusoidally alternating current or voltage by an equation of the form $x = x_0 \sin \omega t$
- Describe rectifier circuits with and without a smoothing capacitor

Alternating current generators are used in power stations and in mobile and emergency generators. In this section, the simple a.c. generator is considered as an application of electromagnetic induction.

The simple a.c. generator

The simple a.c. generator consists of a rectangular coil that spins in a uniform magnetic field, as shown in Figure 1. When the coil spins at a steady rate, the flux linkage changes continuously. At an instant when the normal to the plane of the coil is at angle θ to the field lines, the flux linkage through the coil, $N\Phi = BAN\cos\theta$, where B is the magnetic flux density, A is the coil area and N is the number of turns on the coil.

For a coil spinning at a steady frequency, f, $\theta = 2\pi ft$ at time t after $\theta = 0$. So the flux linkage $N\Phi$ ($= BAN\cos 2\pi ft$) changes with time as shown in Figure 2.

- The gradient of the graph is the change of flux linkage per second, $N\frac{\Delta\Phi}{\Delta t}$, so it represents the induced e.m.f. It can be shown mathematically that the induced e.m.f. alternates in accordance with the equation $E = E_0 \sin 2\pi ft$, where f is the frequency of rotation of the coil and E_0 is the peak e.m.f. Substituting the angular frequency ω for $2\pi f$ therefore gives $\boldsymbol{E = E_0 \sin \omega t}$. Figure 3 shows how the induced e.m.f., E, varies with time t.
- The induced e.m.f is zero when the sides of the coil move parallel to the field lines. At this position, the rate of change of flux is zero and the sides of the coil do not cut the field lines.
- The induced e.m.f. is a maximum when the sides of the coil cut at right angles across the field lines. At this position, the e.m.f. induced in each wire of each side $= Blv$, where v is the speed of each wire and l is its length. So, for N turns and two sides, the induced e.m.f. at this position $E_0 = 2NBlv$. The equation shows that the peak e.m.f. can be increased by increasing the speed (i.e. the frequency of rotation) or using a stronger magnet or a bigger coil or a coil with more turns.

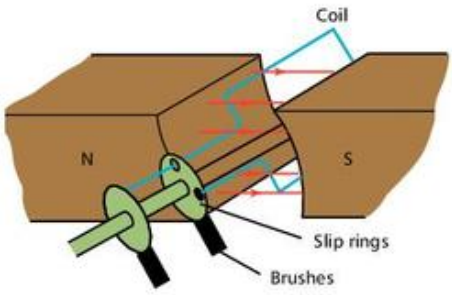


Fig 1 The a.c. generator

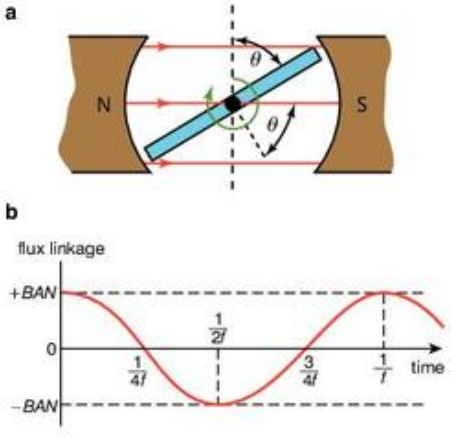


Fig 2 Flux linkage in a spinning coil

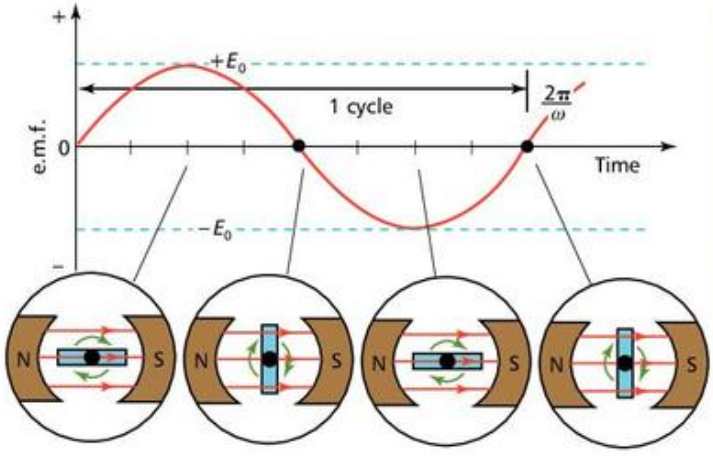


Fig 3 E.m.f. v. time for an a.c. generator

An a.c. generator in a power station has 3 coils at 120° to each other. Each coil produces an alternating voltage 120° out of phase with the voltage from the other coils.

Note A d.c. generator can be made by replacing the two slip rings of the a.c. generator with a split-ring, as shown in Figure 4. The e.m.f. does not reverse its polarity because the connections between the split-ring and the brushes reverse every half-cycle.

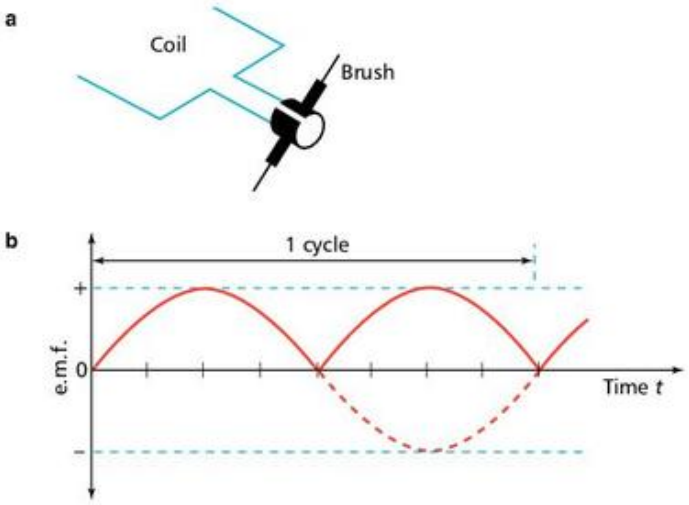


Fig 4 The d.c. generator **a** *The split-ring commutator,* **b** *e.m.f. v. time*

Back e.m.f.

An e.m.f. is induced in the spinning coil of an electric motor because the flux linkage through the coil changes. The induced e.m.f. E is referred to as a **back e.m.f.** because it acts against the p.d. V applied to the motor in accordance with Lenz's law. At any instant, $V - E = IR$, where I is the current through the motor coil and R is the circuit resistance.

Because the induced e.m.f. is proportional to the speed of rotation of the motor, the current changes as the motor speed changes.

- At low speed, the current is high because the induced e.m.f. is small.
- At high speed, the current is low because the induced e.m.f. is high.

Note that multiplying the equation $V - E = IR$ by I throughout gives $IV - IE = I^2R$.

Rearranging this equation gives

electrical power supplied by the source	=	electrical power transferred to mechanical power	+	electrical power wasted due to the circuit resistance
IV		IE		I^2R

The above power equation shows that electrical power supplied to the motor that is not used as mechanical power is wasted due to the resistance heating effect of the current.

- When the motor spins without driving a **load**, it spins at high speed so the current is very small. Its speed is limited by friction in the bearings and air resistance. It uses little or no power.
- When the motor is used to drive a load, its speed is less than when it is 'off-load' so the current is much larger. The power it uses from the voltage source that is not transferred as mechanical power to the load is wasted due to the resistance heating effect of the current.

Rectifier circuits

Diodes are used to 'rectify' or convert alternating current to direct current.

Half-wave rectification is produced using a single diode, as shown in Figure 5. The resistor is needed to limit the current otherwise the diode would be destroyed by overheating. The diode conducts every other half-cycle when the a.c. supply across it is in the forward direction. The current is negligible when the supply is in the reverse direction.

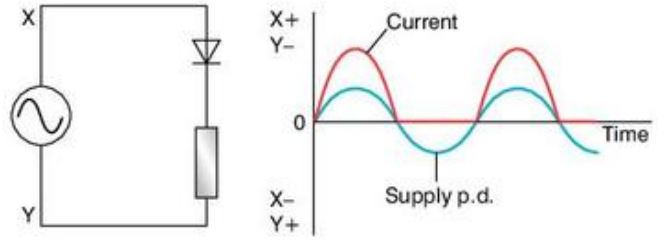


Fig 5 Half-wave rectification

Full-wave rectification is produced using four diodes in a **bridge rectifier** circuit as shown in Figure 6. Diodes D_1 and D_3 conduct when X is positive (and Y is negative); diodes D_4 and D_2 conduct when Y is positive (and X is negative). As a result, the direction of the current through R is the same in both half-cycles. The current waveform in Figure 6 shown in purple is the full-wave rectified wave form.

The waveform can be made smoother by connecting a capacitor across the output terminals of the bridge rectifier. The capacitor charges up from the bridge rectifier as the output p.d. increases then discharges through the resistor as the output p.d. decreases. In this way, the current through the resistor is smoothed out, as shown in red in Figure 6. Note that the capacitance C of the capacitor should be sufficiently large in relation to the load resistance R so that the time constant RC is much greater than the time period of the a.c. supply.

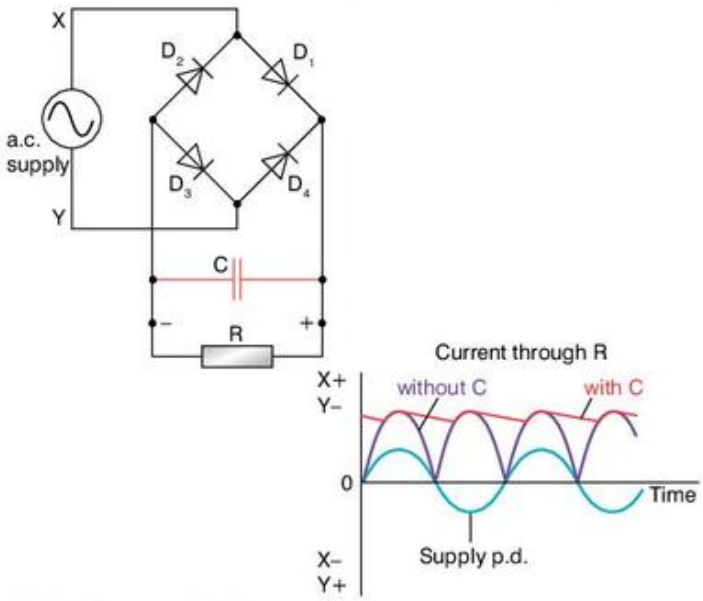


Fig 6 Full-wave rectification

The heating effect of an alternating current

Imagine an electric heater supplied with alternating current at a very low frequency. The heater would heat up and then cool down repeatedly as the current changed.

As explained in topic 7.2, the heating effect of an electric current varies according to the square of the current. This is because the electrical power P supplied to the heater for a current I is given by

$P = IV = I^2R$, **where R is the resistance of the heater element.**

Figure 7 shows how the power ($= I^2R$) varies with time.

- At the peak current I_0 in either direction, maximum power is supplied equal to I_0^2R.
- At zero current, zero power is supplied.

For a sinusoidal current, the **mean power** over a full cycle is half the peak (i.e. maximum) power. This can be seen from the symmetrical shape of the power curve in Figure 7 about the mean power. The mean power is therefore $\frac{1}{2}I_0^2R$.

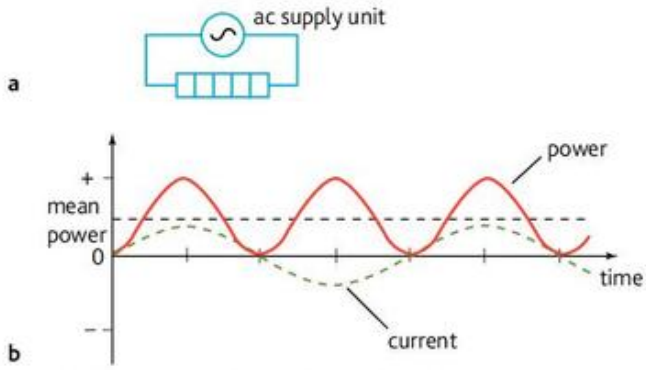


Fig 7 Variation of power with time for an alternating current

The direct current that would give the same power as the mean power is called the **root mean square value** of the alternating current, I_{rms}.

The root mean square value of an alternating current is the value of direct current that would give the same heating effect as the alternating current in the same resistor.

Therefore, $I_{rms}^2R = 0.5I_0^2R$

Cancelling R from this equation and rearranging the equation gives $I_{rms}^2 = 0.5I_0^2$

Therefore $I_{rms} = \frac{1}{\sqrt{2}}I_0$

Also, the root mean square value of an alternating p.d. is given by $V_{rms} = \frac{1}{\sqrt{2}}V_0$

The root mean square value of an alternating current or p.d. $= \frac{1}{\sqrt{2}} \times$ the peak value

For example, if the peak voltage of an a.c. supply is 50 V, the rms value will be 35 V.

Note For any resistor of known resistance in an alternating circuit, if we know the rms p.d. or current for the resistor, we can calculate the mean power supplied to it using the rms values.

$$P = I_{rms}^2R = V_{rms}^2/R = I_{rms}V_{rms}$$

For example, if an alternating current of rms value 4 A is passed through a 5 Ω resistor, the mean power supplied to the resistor $= 4^2 \times 5 = 80$ W.

QUESTIONS

1 a An a.c. generator produces an alternating e.m.f. with a peak value of 8.0 V and a frequency of 20 Hz. Sketch a graph to show how the e.m.f. varies with time.

b The frequency of rotation of the a.c. generator in **a** is increased to 30 Hz. On the same axes, sketch a graph to show how the e.m.f. varies with time at 30 Hz.

2 A rectangular coil of N turns and area A spins at a constant frequency f in a uniform magnetic field of flux density B. Complete the table to show how the flux linkage and induced e.m.f. relate to the orientation of the coil during one cycle of rotation.

Time	Orientation of coil	Flux linkage	Induced e.m.f.
0	Parallel to field	?	$+E_0$
0.25/f	Perpendicular to field	$+BAN$	?
0.50/f	Parallel to field	?	?
0.75/f	Perpendicular to field	?	?

3 The coil of an a.c. generator has 80 turns, a length of 65 mm and a width of 38 mm. It spins in a uniform magnetic field of flux density 130 mT at a constant frequency of 50 Hz.

a Calculate the maximum flux linkage through the coil.

b i Show each side of the coil moves at a speed of 6.0 m s^{-1}.

ii Show that the peak voltage is 8.1 V.

4 An electric motor is to be used to move a variable load. The motor is connected in series with a battery and an ammeter.

a Explain why the motor current is very small when the load is zero.

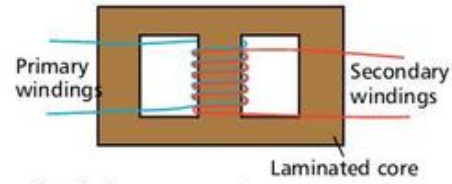
b Explain why the motor current increases when the load is increased.

19.4 The transformer

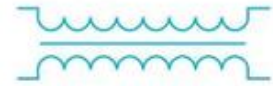
A transformer changes an alternating p.d. to a different peak value. Any transformer consists of two coils: the primary coil and the secondary coil. The two coils have the same iron core. When the primary coil is connected to a source of alternating p.d., an alternating magnetic field is produced in the core. The field passes through the secondary coil. So, an alternating e.m.f. is induced in the secondary coil by the changing magnetic field. The symbol for the transformer is shown in Figure 1.

On these pages you will learn to:

- Describe and explain the transformer as a simple application of electromagnetic induction
- Recall and use the transformer rule $V_S/V_P = N_S/N_P$
- Calculate the efficiency of a transformer from the primary and secondary current and p.d.
- Explain the sources of energy loss in a practical transformer
- Explain why transmission of electrical power is more efficient at high voltage than at low voltage



a Practical arrangement



b Transformer symbol

Fig 1 *The transformer*

The transformer rule

A transformer is designed so that all the magnetic flux produced by the primary coil passes through the secondary coil.

Let Φ = the flux in the core passing through each turn at an instant when an alternating p.d. V_P is applied to the primary coil.

- The flux linkage in the secondary coil = $N_S\phi$, where N_S is the number of turns on the secondary coil. From Faraday's law, the induced e.m.f. in the secondary coil, $V_S = N_S\dfrac{\Delta\Phi}{\Delta t}$
- The flux linkage in the primary coil = $N_P\Phi$, where N_P is the number of turns on the primary coil. From Faraday's law, the induced e.m.f. in the primary coil = $N_P\dfrac{\Delta\Phi}{\Delta t}$. The induced e.m.f. in the primary coil opposes the p.d. applied to the primary coil, V_P.

  Assuming the resistance of the primary coil is negligible, the applied p.d. $V_P = N_P\dfrac{\Delta\Phi}{\Delta t}$

  Dividing the equation for V_S by the equation for V_P gives $\dfrac{V_S}{V_P} = N_S\dfrac{\Delta\Phi}{\Delta t}\Big/N_P\dfrac{\Delta\Phi}{\Delta t}$

  Cancelling $\dfrac{\Delta\Phi}{\Delta t}$ from this equation gives the **transformer rule**

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

- **A step-up transformer** has more turns on the secondary coil than on the primary coil. So the secondary voltage is stepped up compared with the primary voltage (i.e. $N_S > N_P$ so $V_S > V_P$)
- **A step-down transformer** has fewer turns on the secondary coil than on the primary coil. So the secondary voltage is stepped down compared with the primary voltage (i.e. $N_S < N_P$ so $V_S < V_P$).

Transformer efficiency

$$\text{The efficiency of a transformer} = \frac{\text{power delivered by the secondary coil}}{\text{power supplied to the primary coil}} = \frac{I_S V_S}{I_P V_P}\ (\times 100\%)$$

When a device (e.g. a lamp) is connected to the secondary coil, because the efficiency of a transformer is almost equal to 100%,

$$\text{the electrical power supplied to the primary coil} = \text{the electrical power supplied by the secondary}$$

Therefore, the current ratio $\dfrac{I_S}{I_P} = \dfrac{V_P}{V_S} = \dfrac{N_P}{N_S}$

Transformer materials

Transformers are almost 100% efficient because they are designed with:

- Low-resistance windings to reduce power wasted due to the heating effect of the current.
- A laminated core, which consists of layers of iron separated by layers of insulator. Induced currents in the core itself, referred to as **eddy currents,** are reduced in this way so the magnetic flux is as high as possible. Also, the heating effect of the induced currents in the core is reduced.
- The core consists of '**soft iron**', which is easily magnetised and demagnetised. This reduces power wasted through repeated magnetisation and demagnetisation of the core.

- In a step-up transformer, the voltage is stepped up and the current is stepped down.
- In a step-down transformer, the voltage is stepped down and the current is stepped up.

The grid system

Electricity from power stations in most countries is fed into a national grid system which supplies electricity to most parts of the country. The grid is a network of cables, either underground or on pylons. Each power station generates alternating current in 3 phases (see topic 19.3) at a precise frequency of 50 Hz (or 60 Hz in some countries) at about 25 kV.

Step-up transformers at the power station increase the alternating voltage to 400 kV or more for long-distance transmission via the grid system. Step-down transformers operate in stages, as shown in Figure 2. Factories are supplied with all three phases at either 33 kV or 11 kV. Homes are supplied via a local transformer sub-station with single-phase a.c. at 220–240 V in most countries except in America (110–130 V), Japan (100 V) and a few other countries.

Transmission of electrical power over long distances is much more efficient at high voltage than at low voltage. The reason is that the current needed to deliver a certain amount of power is reduced if the voltage is increased. So power wasted due to the heating effect of the current through the cables is reduced. To deliver power P at voltage V, the current required $I = \frac{P}{V}$.

If the resistance of the cables is R, the power wasted through heating the cables is $I^2R = \frac{P^2R}{V^2}$. Therefore, the higher the voltage, the smaller the ratio of the wasted power to the power transmitted.

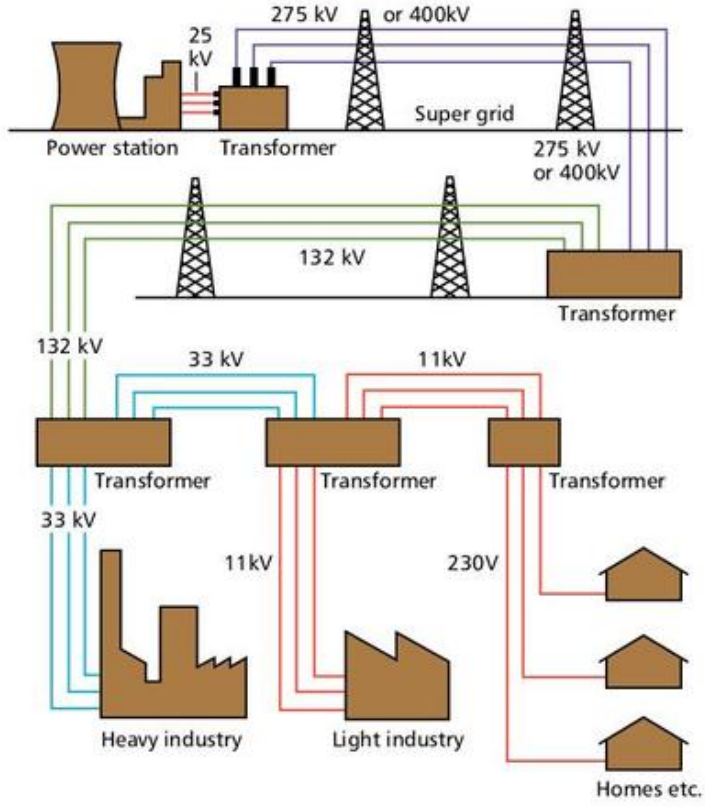


Fig 2 *The grid system*

For example, for transmission of 1 MW of power through cables of resistance 500 Ω at 25 kV, the current necessary would be 40 A (= 1 MW/25 kV) so the power wasted would be 0.8 MW (= $I^2R = 40^2 \times 500$ W). Prove for yourself that at 400 kV, the power wasted would be about 3 kW.

Voltage adaptors

Portable electronic devices such as laptop computers contain batteries that need to be recharged using a voltage adaptor. Figure 3 shows the circuit of such an adaptor. It supplies a constant low voltage when it is plugged into a mains socket.

- The transformer steps down the alternating p.d. from the mains to an alternating p.d. with a much smaller peak p.d. The turns ratio of the transformer is chosen to give a suitable p.d. from the transformer.
- The bridge rectifier converts the alternating p.d. from the transformer to a full-wave direct p.d.
- The capacitor across the bridge rectifier smooths out the full-wave p.d. to give a constant output p.d, as explained in topic 19.3. The greater the capacitance of C, the smaller the variation in the output p.d. as current is drawn from the voltage adaptor.

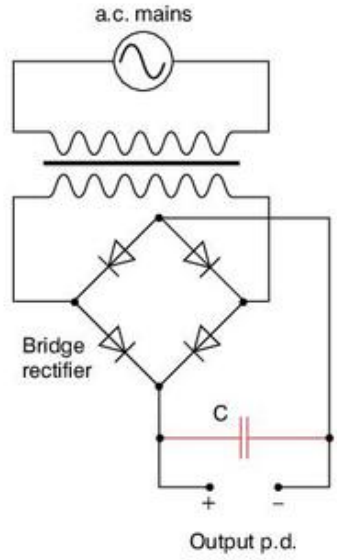


Fig 3 A voltage adaptor

QUESTIONS

1 a Explain why an alternating e.m.f. is induced in the secondary coil of a transformer when the primary coil is connected to an alternating voltage supply.

b In terms of electrical power, explain why the current through the primary coil of a transformer increases when a device is connected to the secondary coil.

2 a Explain why a transformer is designed so that as much of the magnetic flux produced by the primary coil of a transformer as possible passes through the secondary coil.

b Explain why a transformer works using alternating current but not using direct current.

3 A transformer has a primary coil with 120 turns and a secondary coil with 2400 turns.

a Calculate the primary voltage needed for a secondary voltage of 230 V.

b A 230 V, 60 W lamp is connected to its secondary coil. Calculate the current through:
i the secondary coil,
ii the primary coil. State any assumptions made in this calculation.

4 a Explain why transmission of electrical power over a long distance is more efficient at high voltage than at low voltage.

b A power cable of resistance 200 Ω is to be used to deliver 2.0 MW of electrical power at 120 kV from a power station to an industrial estate. Calculate:
i the current through the cable,
ii the power wasted in the cable.

CHAPTER SUMMARY

Magnetic flux, $\Phi = BA$

Flux linkage through a coil of N turns = $N\Phi = NBA$ where B is the magnetic flux density perpendicular to area A.

Lenz's law states that the direction of the induced current is always such as to oppose the change that causes the current.

Faraday's law of electromagnetic induction states that the induced e.m.f. in a circuit is proportional to the rate of change of flux linkage through the circuit.

Equation for Faraday's law; $E = -N\frac{\Delta\Phi}{\Delta t}$, where $N\frac{\Delta\Phi}{\Delta t}$ is the change in flux linkage per second.

For a moving conductor in a uniform magnetic field, the induced e.m.f. = $B\ell v$

For a changing magnetic field in a fixed coil, induced e.m.f. = $NA\frac{\Delta B}{\Delta t}$

Units

The unit of magnetic flux density B is the **tesla** (T).

The unit of magnetic flux and of flux linkage is the **weber** (Wb), equal to 1 Tm^2 or 1 V s.

The unit of rate of change of flux (or rate of change of flux linkage) is the weber per second (Wb s^{-1}), equal to 1 V.

19 End of chapter questions

1 In a ribbon microphone, a metal ribbon vibrates between the poles of a magnet when sound waves reach the microphone.

a Explain why an e.m.f. is induced across the ends of the ribbon when it vibrates.

b How would the e.m.f. be affected if:
i a stronger magnet was used,
ii a ribbon of greater mass was used?

2 a A bar magnet was positioned near a coil connected to a centre-reading meter. When the bar magnet was pushed into the coil, the meter pointer deflected briefly to the right.
i Explain why the pointer deflected briefly.
ii State and explain what is observed when the magnet is withdrawn from the coil.

b In **a**, the flux density of the magnet was 25 mT and the area of the 30 turn coil was $4.0 \times 10^{-4}\,m^2$. The magnet was pushed into the coil in 0.20 s. Calculate:
i the flux linkage through the coil,
ii the mean induced e.m.f.

3 A U-shaped magnet was placed at the centre of a horizontal stretched steel wire such that the magnetic field was vertical. When the wire was plucked at its centre, an alternating e.m.f. was induced between the ends of the wire.

a Explain why an alternating e.m.f. was induced between the ends of the wire.

b The length of wire between the poles was 28 mm and the magnetic flux density of the magnet was 78 mT. The peak voltage produced was 3.4 mV. Calculate the maximum speed of the wire between the poles.

4 a A straight conducting rod PQ of length 0.10 m moves through a uniform magnetic field at a speed of $20\,mm\,s^{-1}$. The induced e.m.f. was 0.60 mV when the conductor was moving perpendicular to the field. Calculate:
i the flux swept out in 5.0 s,
ii the magnetic flux density.

b In **a**, the field was vertically downwards and the rod was horizontal, as shown in the diagram opposite.
i State the polarity of each end of the rod.
ii If the rod was part of a complete circuit, state and explain the direction in which the induced current would pass through it.

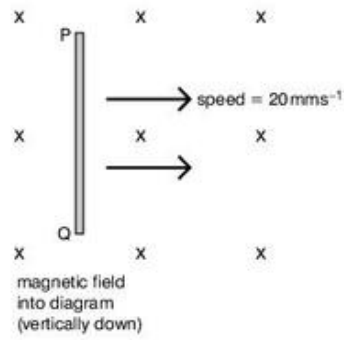


Fig 4.1

5 A rectangular coil of length 50 mm and width 20 mm has 25 turns. The diagram below shows the coil just before it was moved at a constant speed of $5\,mm\,s^{-1}$ into a uniform magnetic field of flux density 86 mT. The field lines are perpendicular to the plane of the coil.

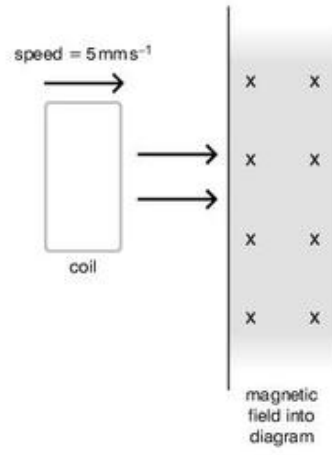


Fig 5.1

a Show that the coil takes 4.0 s to enter the field and calculate the flux linkage through the coil when it is completely in the field.

b Sketch a graph to show how:
i the flux linkage through the coil changes with time,
ii the induced e.m.f. varies with time t from $t = 0$ when the coil enters the field to $t = 5.0$ s.

6 An a.c. generator is used to provide electricity for a lighting circuit.

a Explain why the generator is easier to turn when the lamps are switched off.

b The generator coil has 120 turns on a rectangular coil of length 40 mm and width 30 mm. The coil spins in a magnetic field of flux density 220 mT.

i When the generator spins at a frequency of 20 Hz, show that the peak voltage is 4.0 V.

ii Calculate the peak voltage when the frequency is 25 Hz.

7 A bar magnet was held vertically in a horizontal coil connected to a data recorder, as shown in the diagram below. When the magnet was released, the data recorder was used to measure the voltage induced in the coil every 5 ms. The diagram shows how the voltage changed with time.

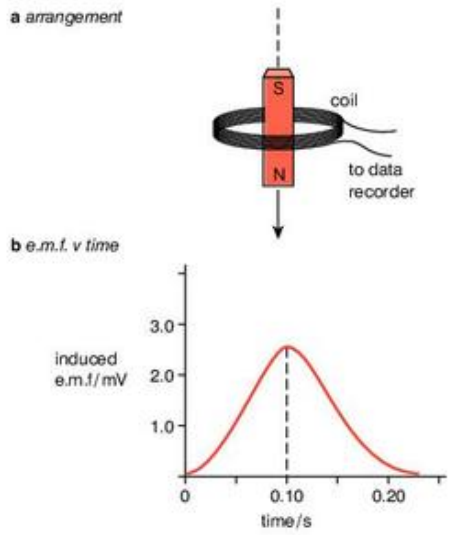


Fig 7.1

a i Without calculations, sketch a graph to show how the flux linkage through the coil changed with time.

ii Use your graph to explain the shape of the voltage v. time graph.

b Sketch the voltage v. time graph that would have been produced if the bar magnet had been released from a position above the coil so that it dropped through the coil.

8 A transformer is used to step down an alternating voltage of 230 V to 12 V. The transformer has a primary coil with 1000 turns.

a Calculate the number of turns on the secondary coil.

b The transformer is used to supply power to a 12 V, 60 W lamp. Calculate the current in:

i the secondary coil,

ii the primary coil when the lamp is on.

c The lamp is connected to the transformer by means of a cable of resistance 0.4 Ω.

i Estimate the power wasted due to the heating effect of the current in the cable.

ii Discuss whether or not it would be better to replace the lamp and transformer with a 230 V, 60 W lamp connected to the mains using the same cable.

9 You are provided with a coil of wire, a bar magnet and a sensitive ammeter.

Outline an experiment to verify Lenz's law. *(6 marks)*

Cambridge International AS and A Level Physics 9702
Paper 04 Q7 June 2009

10 A student is asked to design a circuit by which a direct voltage of peak value 9.0 V is obtained from a 240 V alternating supply.

The student uses a transformer that may be considered to be ideal and a bridge rectifier incorporating four ideal diodes.

The partially completed circuit diagram is shown in Figure 10.1.

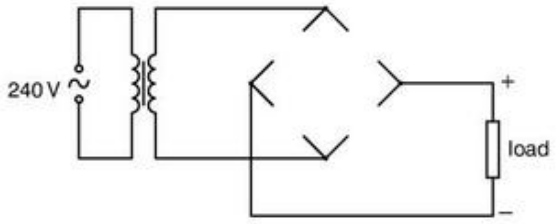


Fig 10.1

a On a copy of Figure 10.1, draw symbols for the four diodes so as to produce the polarity across the load as shown on the diagram. *(2 marks)*

b Calculate the ratio

$$\frac{\text{number of turns on the secondary coil}}{\text{number of turns on the primary coil}}.$$ *(3 marks)*

Adapted from Cambridge International AS and A Level Physics 9702
Paper 41 Q6 June 2010

Key concepts: Fields

In studying chapters 11 and 12, you will have gained a clear understanding of the force conditions necessary for an object to undergo uniform circular motion and simple harmonic motion. You should also appreciate that in uniform circular motion, because the velocity direction is always perpendicular to the resultant force, no work is done on the object and hence its kinetic energy and speed is constant. This consideration also is important where a satellite is in uniform circular motion about a spherical planet. In considering simple harmonic motion, you should appreciate that the energy of an oscillating system is constant provided that dissipative forces such as air resistance are negligible. In addition, you will have developed your mathematical skills to a level where you can apply non-linear equations including sinusoidal equations and graphs to calculate the acceleration and the time period of the motion and the speed and position at any time. In addition, by investigating oscillating systems such as a loaded spring, you will have developed your practical and analytical skills by plotting straight-line graphs to test related non-linear theoretical equations.

In studying Chapters 13, 14 and 18, you should have gained awareness of important field concepts such as lines of force, uniform fields and field strength. Beyond these concepts, electric and gravitational fields share common features such as an inverse square law of force, similar equations for field strength and potential in radial fields and similar trajectories for objects moving across a uniform field. Magnetic fields differ from electric and gravitational fields in respect of these and other features. For example, charged objects moving across a uniform magnetic field follow a circular path because the magnetic force is perpendicular to the direction of the object's velocity (as well as being perpendicular to the field direction). The force on a moving charged object in a magnetic field is the reason why a current-carrying conductor in a magnetic field experiences a force when it is not aligned along the field lines, and it provides the basis of explanations of the motor effect and the Hall effect. Knowledge and understanding gained in Chapter 18 ought to have given you a sound basis for the study of electromagnetic induction in Chapter 19. Conservation of energy is also important in understanding and applying Lenz's law and Faraday's law to important applications such as the a.c. generator and the transformer. In all these chapters, you will have developed your mathematical skills by applying them to further non-linear equations and related graphs.

Your practical skills as well as your knowledge of electricity should have developed considerably in your studies of chapters 15, 16 and 17. Practical investigations into capacitors in Chapter 15 involve timing measurements as well as measurements of currents and potential differences. By making such measurements with and without a data logger, you should appreciate the benefits of using a data logger in these experiments. In addition, your mathematical knowledge and skills will have been enhanced by carrying out calculations and plotting graphs involving exponential decrease equations and graphs. In Chapter 16, you should have gained a good understanding of how electronic sensors are designed and used to switch light-emitting diodes (LEDs) and relays on and off. The study of operational amplifiers will have allowed you to develop a sound grasp of concepts such as feedback, voltage gain and bandwidth. From Chapter 17, you will have gained a good understanding of analogue and digital communications and of techniques such as modulation, analogue to digital conversion and digital to analogue conversion. You will also have gained knowledge of the different channels that are used for communications and the effect such channels have on the attenuation of signals.

The following table provides an overview of the topics in chapters 11 to 19, showing where key concepts have been developed.

Chapters 11–19 Key concepts

Topic			Key concepts	
11.1–11.4	uniform motion in a circle	centripetal force and centripetal acceleration	forces and fields	vectors
		calculations involving angular speed, centripetal acceleration and Newton's laws of motion	use of maths	problem-solving
12.1–12.4	principles and applications of simple harmonic motion	conditions for shm	forces and fields	restoring force
		calculations involving shm equations and sinusoidal functions	use of maths	problem-solving
		simple pendulum to measure *g*	testing	practical skills including analysis and evaluation
		oscillations of a loaded spring		
12.5–12.6	energy and simple harmonic motion	kinetic and potential energy of an object	energy and matter	conservation of energy
		damping, forced oscillations and resonance		work done by dissipative forces
		SHM calculations of kinetic and potential energies	use of maths	problem-solving
13.1–13.4	gravitational forces and fields	Newton's law of gravitation	forces and fields	inverse-square law
		gravitational field strength, uniform and radial fields		field strength
		gravitational potential	energy and matter	conservation of energy
		calculations involving formulae for gravitational field strength and potential	use of maths	problem-solving
13.5	satellite motion	Newton's law of gravitation applied to uniform circular motion leading to speed and time period equations	forces and fields	use of inverse-square law
		calculations involving above equations	use of maths	problem-solving
14.1–14.3	electric fields	electric charge, line of force, Coulomb's law, electric field strength, uniform and radial fields	forces and fields	inverse-square law for radial fields, uniform fields
		electric potential, equipotentials	energy and matter	conservation of energy
		calculations involving formulae for electric field strength and potential	use of maths	problem-solving
14.4	comparison of electric and gravitational fields	field strength and potential	forces and fields energy and matter	inverse and inverse-square equations and graphs
15.1–15.4	capacitors	capacitance	forces and fields	electric charge
		capacitor combination rules		
		energy stored in a capacitor	energy and matter	conservation of energy
		capacitor charging and discharging	testing	practical skills including analysis
		calculations and graphs involving capacitor formulae, including capacitor discharge	use of maths	problem-solving

continued on p296

Chapters 11–19 Key concepts *(continued)*

	Topic		Key concepts	
16.1–16.4.	electronic systems	electronic sensors, operational amplifiers, output devices	testing	practical skills
				predicting
		calculations of voltage gain and interpretation of graphs	use of maths	problem-solving
17.1–17.3	communications	digital and analogue signals	energy and waves	carrier waves
		modulation, bandwidth		noise
		communication channels		
		use of the decibel equations for attenuation and amplification	use of maths	problem-solving
18.1–18.4	magnetic fields	magnetic field lines, magnetic flux density, the motor effect, magnetic field patterns	forces and fields	force perpendicular to conductor and to the field lines
		calculations and graphs involving magnetic field formula	use of maths	problem-solving
	moving charges in a magnetic field	force on a moving charge in a magnetic field, Hall effect		
		use of a Hall probe		
18.5	charged particles in circular orbits	circular motion of charged particles in a magnetic field	forces and fields	force perpendicular to velocity
		calculations involving charged particles in circular orbits and use of the Hall voltage equation	testing	practical skills; measurement of magnetic flux density
			use of maths	problem-solving
19.1–19.4	electromagnetic induction	origin of induced e.m.f., magnetic flux	forces and fields	
		Lenz's law and Faraday's law	energy and matter	conservation of energy
		calculations involving magnetic flux, Faraday's law, the a.c. generator and the transformer	use of maths	problem-solving
				transformer efficiency

Question

This question is about the oscillations of a loaded spring. It tests how we use models to make predictions about a physical system and how we then test these predictions with an experimental investigation. It also tests practical skills involving timing the oscillations and measuring the extension of a loaded spring. Systematic errors can arise when making extension measurements, for example as a result of not using a plane mirror correctly when reading a pointer against a millimetre scale and also when timing oscillations, for example by miscounting the number of oscillations. Random errors can be minimised when timing oscillations by making repeat measurements of at least 20 oscillations.

As in the previous key concepts questions on pages 88 and 167, physics theory and mathematics are used to predict the relationship between the static extension $\mathbf{e}$ *of the spring and the time period* $\mathbf{T}$ *of the oscillations. In this case, the predicted relationship is of the form* $\mathbf{T^2 = ke}$. *(or* $\mathbf{T = k^{½}e^{½}}$*). So by plotting a graph of* $\mathbf{T^2}$ *against* $\mathbf{e}$ *(or* $\mathbf{T}$ *against* $\mathbf{e^{½}}$*), we can tell if the relationship is valid from whether or not the graph is linear (i.e. a straight-line graph) and passes through the origin. If the graph is linear, we can also determine the constant* $\mathbf{k}$*, as it is equal to the gradient of the graph. Also, the value of* $\mathbf{g}$ *can then be found from the result for* $\mathbf{k}$.

In some investigations, the relationship between two variables may not be easy to predict from the relevant theory or the theory might not give a straight-line graph. In these cases, assuming the relationship is of the form $y = kx^n$*, a graph of* $\log y$ *against* $\log x$ *may be plotted; a straight-line graph would confirm that the relationship is of the form* $y = kx^n$ *and the power* n *could then be found, as it is equal to the gradient of the graph.*

Before answering the question, students should be given the opportunity to do this or a similar investigation.

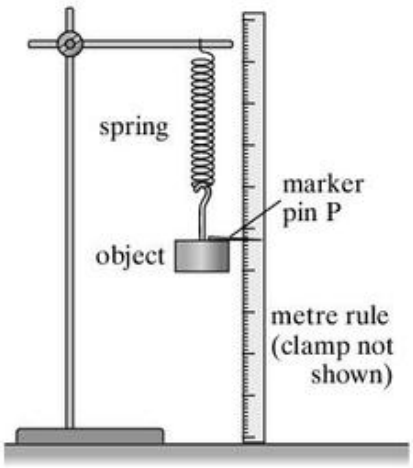


Fig 1

1 In an investigation, a small object was suspended from the lower end of a vertical steel spring, which was fixed at its upper end, as shown in Figure 1.

A horizontal marker pin P was attached to the lower end of the spring. The vertical position x of the pin was measured against the millimetre scale of a metre rule clamped vertically in a fixed position. The measurement was made three times without, then with, the small object suspended from the spring.

a The readings obtained are shown in Table 1.

Table 1

	x/mm			mean x/mm	extension e/mm
without the object on the spring	2	2	2	2.0	0
with the object on the spring	71	72	73		

ii Copy and complete Table 1 by calculating the mean vertical position of P and the extension of the spring when the object was placed on it.

iii The readings were taken to a precision of 0.5 mm using a millimetre ruler. Estimate the percentage 'uncertainty' in the extension. *(2 marks)*

b The time period, T, of small vertical oscillations of the object on the spring was also measured by timing 20 oscillations three times. The timing readings for 20 oscillations were 10.98 s, 11.11 s and 10.97 s.

i Calculate the time period T.

ii Use the readings to estimate the percentage 'uncertainty' in T. *(2 marks)*

c **i** Give an expression for the extension e of the spring in terms of the mass m of the object and the spring constant k of the spring.

ii Hence show that $T = 2\pi\sqrt{\frac{e}{g}}$ *(3 marks)*

d The experiment was repeated with objects of different mass suspended from the spring. The measurements obtained are given in Table 2.

Table 2

object	e/mm	T/s
1	70	0.551
2	139	0.761
3	205	0.923
4	271	1.062
5	341	1.187
6	409	1.291

Plot a suitable graph using the above measurements to confirm the equation and to determine g. *(9 marks)*

e Discuss the accuracy of your determination of g. *(4 marks)*

20 Thermal physics

20.1 Internal energy and temperature

On these pages you will learn to:

- Explain what is meant by the internal energy of a system in terms of the energy of the molecules of a system
- Recall and use the first law of thermodynamics $\Delta U = Q + W$, expressed in terms of the increase in internal energy, the heating of the system and the work done on the system
- Explain using a simple kinetic model for matter the structure of solids, liquids and gases

Fig 1 *Infrared image showing heat transfer from a house*

When you are outdoors in a cool climate in winter, you need to wrap up well otherwise heat transfer from your body to the surroundings takes place and you lose energy. In hot weather, if you are in a very hot room, you gain energy from the room due to heat transfer.

Energy transfer between two objects takes place if:

- One object exerts a force on the other one and makes it move. In other words, one object does work on the other (see topic 2.4 for more about work).
- One object is hotter than the other so heat transfer takes place by means of conduction, convection or radiation. In other words, heat transfer is energy transfer due to a temperature difference.

Internal energy

The brake pads of a moving vehicle become hot if the brakes are applied for long enough. The work done by the frictional force between the brake pads and the wheel heats the brake pads. The brake pads gain energy from the kinetic energy of the vehicle. The temperature of the brake pads increases as a result and the internal energy of each brake pad increases.

As explained below, the internal energy of an object is the energy of its molecules due to their individual movements. The internal energy of an object due to its temperature is sometimes referred to as **thermal energy**. However, some of the internal energy of an object might be due to other causes. For example, an iron bar that is magnetised has more internal energy than if it is unmagnetised because of the magnetic interaction between its atoms.

The internal energy of an object changes as a result of:

- heat transfer or energy transfer by radiation to or from the object, or
- work done on or by the object, including work done by electricity.

If Q represents the heat transfer to an object (including energy transfer by radiation) and W represents the work done on it (leaving out work done to make an object move faster or to raise it), then

the object's change of internal energy, $\Delta U = Q + W$

The equation follows from the principle of conservation of energy. The above equation is known as the **first law of thermodynamics**.

If the internal energy of an object is constant, either:

- there is no heat transfer or energy transfer due to radiation and no work is done, or
- heat transfer, energy transfer due to radiation and work done 'balance' each other out.

For example, the internal energy of a lamp filament increases when the lamp is switched on because work is done by the electricity supply pushing electrons through the filament. The filament becomes hot as a result. When it reaches its operating temperature, heat transfer to the surroundings takes place and it radiates light. Work done by the electricity supply pushing electrons through the filament is balanced by heat transfer and light radiated from the filament.

About molecules

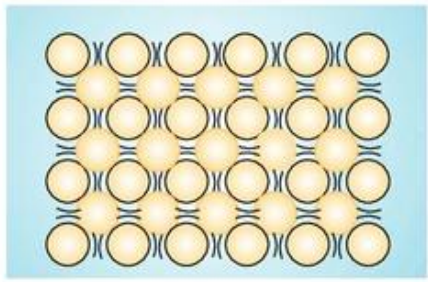
A molecule is the smallest particle of a pure substance that is characteristic of the substance. For example, a water molecule consists of two hydrogen atoms joined to an oxygen atom.

An atom is the smallest particle of an element that is characteristic of the element. For example, a hydrogen atom consists of a proton and an electron.

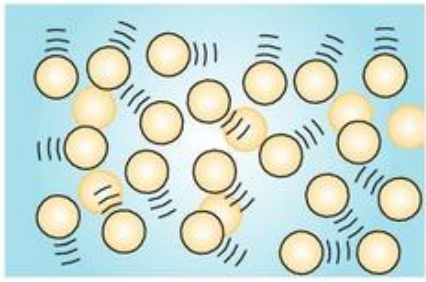
- In a solid, the atoms and molecules are held to each other by forces due to the electrical charges of the protons and electrons in the atoms. The molecules in a solid vibrate randomly about fixed positions. The higher the temperature of the solid, the more the molecules vibrate. The energy supplied to raise the temperature of a solid increases the kinetic energy of the molecules. If the temperature is raised sufficiently, the solid melts. This happens because its molecules vibrate so much that they break free from each other and the substance loses its shape. The energy supplied to melt a solid raises the potential energy of the molecules because they break free from each other.
- In a liquid, the molecules move about at random in contact with each other. The forces between the molecules are not strong enough to hold the molecules in fixed positions. The higher the temperature of a liquid, the faster its molecules move. The energy supplied to a liquid to raise its temperature increases the kinetic energy of the liquid molecules. Heating the liquid more and more causes it to vaporise. The molecules have sufficient kinetic energy to break free and move away from each other.
- In a gas or vapour, the molecules also move about randomly but much further apart on average than in a liquid. Heating a gas or a vapour makes the molecules speed up and so gain kinetic energy.

The internal energy of an object is the sum of the random distribution of the kinetic and potential energies of its molecules.

Increasing the internal energy of a substance increases the kinetic and/or potential energy associated with the random motion and positions of its molecules. The kinetic and potential energy of each individual molecule differs between molecules and changes at random.

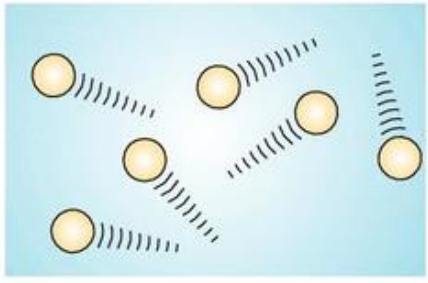


A solid is made up of particles arranged in a regular 3-dimensional structure. There are strong forces of attraction between the particles. Although the particles can vibrate, they cannot move out of their positions in the structure.



In a liquid the particles are free to move around. A liquid therefore flows easily and has no fixed shape. There are still forces of attraction between the particles.

When a liquid is heated, some of the particles gain enough energy to break away from the other particles. The particles which escape from the body of the liquid become a gas.



In a gas, the particles are far apart. There are almost no forces of attraction between them. The particles move about at high speed. Because the particles are so far apart, a gas occupies a very much larger volume than the same mass of liquid.

The molecules collide with the container. These collisions are responsible for the pressure which a gas exerts on its container.

Fig 2 *Particles in a solid, a liquid and a gas*

QUESTIONS

1 Describe the energy transfers that occur when a low voltage heater connected to a battery is used to heat some water in a beaker.

2 **a** Explain why an electric motor becomes warm when it is used.

b A battery is connected to an electric motor which is used to raise a weight at a steady speed. When in operation, the electric motor is at a constant temperature which is above the temperature of its surroundings. Describe the energy transfers that take place.

3 **a** State what is meant by internal energy.

b Describe a situation in which the internal energy of an object is constant even though work is done on the object.

4 **a** State one difference between the motion of the molecules in a solid and the molecules in a liquid.

b Describe how the motion of the molecules in a solid changes when the solid is heated.

20.2 Temperature scales

On these pages you will learn to:

- Know what is meant by a fixed point of temperature
- Relate the Celsius scale of temperature to the thermodynamic (Kelvin) scale
- Recognise the gas thermometer is the standard against which other thermometers are calibrated

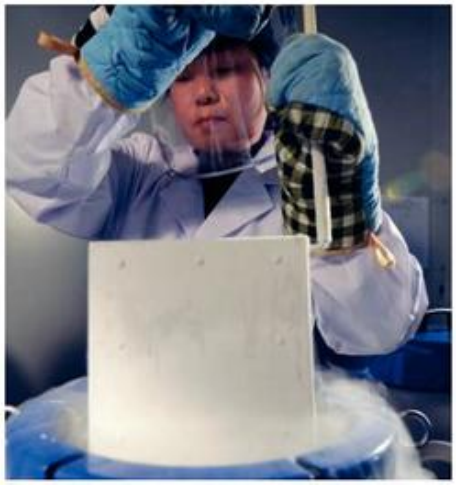


Fig 1 A low temperature research laboratory

The coldest places in the world

You don't need to travel to the South Pole to find the coldest places in the world. Go to the nearest university physics department that has a low temperature research laboratory. Substances have very strange properties at very low temperatures. For example, metals cooled to a few degrees within absolute zero become superconductors which means they have zero electrical resistance.

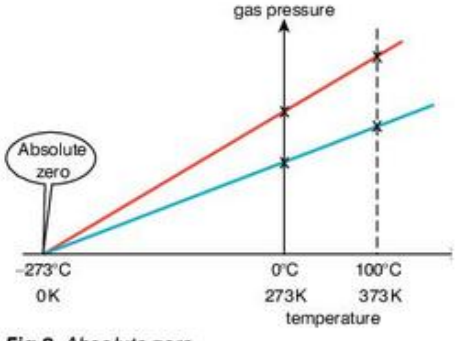


Fig 2 Absolute zero

The temperature of an object is a measure of the degree of hotness of the object. The hotter an object is, the more internal energy it has. Place your hand in cold water and it loses internal energy due to heat transfer. Place it in warm water and it gains internal energy due to heat transfer. If the water is at the same temperature as your hand, no overall heat transfer takes place. Your hand is in **thermal equilibrium** with the water. No overall heat transfer takes place between two objects at the same temperature.

The 'baby in the bath' rule

Before dipping the baby in the bath water, the parent tests the water by putting a hand (not the baby's hand!) in the water. If the baby is at the same temperature as the parent and the parent's hand is at the same temperature as the bath water, the baby will be at the same temperature as the bath water.

Practical temperature scales

A temperature scale is defined in terms of **fixed points** which are standard degrees of hotness which can be accurately reproduced.

- **The Celsius scale of temperature**, in °C, is defined in terms of:
  1. ice point, 0 °C, which is the temperature of pure melting ice,
  2. steam point, 100 °C, which is the temperature of steam at standard atmospheric pressure.
- **The thermodynamic (Kelvin) scale of temperature**, in kelvins (K) is defined in terms of:
  1. absolute zero, 0 K, which is the lowest possible temperature,
  2. the triple point of water, 273.16 K, which is the temperature at which ice, water and water vapour are in thermal equilibrium.

Because ice point on the absolute scale is 273.15 K and steam point is 100 K higher,

temperature in °C = absolute temperature in kelvins − 273.15

About absolute zero

The thermodynamic scale of temperature, also referred to as the absolute or kelvin scale, is based on absolute zero, the lowest possible temperature. No object can have a temperature below absolute zero. **An object at absolute zero has minimum internal energy**, regardless of the substances the object consists of.

As explained in topic 21.1, the pressure of a fixed mass of gas in a sealed container of fixed volume decreases as the gas temperature is reduced. If the pressure measured at ice point and at steam point is plotted on a graph as shown in Figure 2, the line between the two points always cuts the temperature axis at −273 °C, regardless of which gas is used or how much gas is used.

The thermodynamic scale of temperature starts at absolute zero. Its unit, the kelvin, is defined so that a temperature change of 1 K is the same as a temperature change of 1 °C. The kelvin scale depends on a fundamental feature of nature, namely the lowest possible temperature. In comparison, the Celsius scale depends on the properties of a substance, water, chosen for convenience rather than for any fundamental reason.

Thermometer	Thermometric property	Advantages	Disadvantages
Liquid-in-glass	length of the liquid thread	easy to use, portable	fragile, limited range
Thermocouple	e.m.f. E across the junction of two different metal wires in contact with each other	fast response, wide range, remote readings	millivoltmeter needed
Resistance	resistance R of a thermistor (or a metal wire)	wide range, accurate, measures small changes accurately	slow response
Gas	pressure p of an ideal gas at constant volume	wide range, accurate, standard thermometer	bulky, slow response

Thermometers

The **thermometric property** of a thermometer is a physical property that varies smoothly with change of temperature and can therefore be used to measure temperature. Four different thermometers are compared in the table above and in Figure 3.

All thermometers are calibrated in terms of the temperature measured by a gas thermometer. This is a thermometer consisting of dry gas in a sealed container. The pressure of the gas is proportional to the absolute temperature of the gas.
In other words, equal increases of temperature cause equal increases of gas pressure.

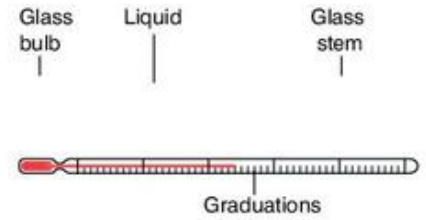
By measuring the gas pressure, p_{Tr}, at the triple point of water (= 273.16 K by definition) and an unknown temperature T/K, the unknown temperature, in kelvins, can be calculated using

$$\frac{T}{273.16} = \frac{p}{p_{Tr}}$$

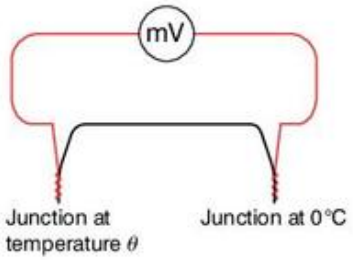
where p is the gas pressure at the unknown temperature.

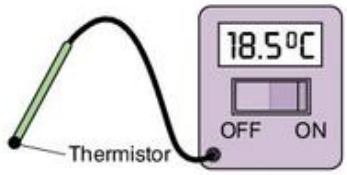
To determine the temperature on the Celsius scale or the thermodynamic (i.e. Kelvin) scale using any other thermometer, it must be calibrated against a gas thermometer (or indirectly against a thermometer previously calibrated against a gas thermometer). This involves measuring the thermometric property at different temperatures, T, measured using the gas thermometer (or indirectly as above). A graph or 'calibration curve' can then be plotted of temperature T on the y-axis against the thermometric property on the x-axis.

Note If the thermometric property is X_0 at ice-point, X_{100} at steam point and X at an unknown temperature θ, the unknown temperature can be calculated on a centigrade scale using the equation $\theta = \dfrac{X - X_0}{X_{100} - X_0} \times 100$. This gives the values that would be obtained by dividing the interval between X_0 and X_{100} into 100 equal degrees. If a thermometer is graduated with such a scale, a calibration curve of the 'gas thermometer' temperature T against θ would need to be used to determine the temperature T from a thermometer reading, θ.

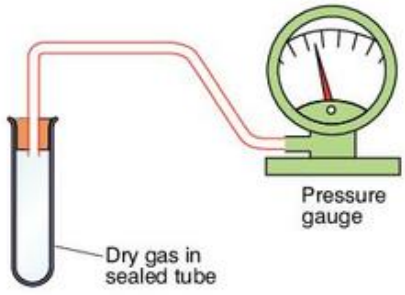


a *A liquid-in-glass thermometer*





c *A resistance thermometer*



d *A gas thermometer*

Fig 3 *Different types of thermometers*

QUESTIONS

1 **a** Define the thermodynamic scale of temperature and state its unit.

b State each of the following temperatures to the nearest degree on the thermodynamic scale:
i the temperature of pure melting ice,
ii 20 °C, **iii** −196 °C.

2 The pressure of a constant-volume gas thermometer was 100 kPa at a temperature of 273 K.
a Calculate the temperature of the gas when its pressure was 120 kPa.
b Calculate the pressure of the gas at 100 °C.

3 Explain why the 50 °C mark on the stem of a liquid-in-glass thermometer is not exactly midway between the 0 and 100 °C marks.

4 Explain why a gas thermometer is used to calibrate other types of thermometers.

20.3 Specific heat capacity

On these pages you will learn to:

- Relate a rise in temperature of a body to an increase in its internal energy
- Define and use the concept of specific latent heat, and identify the main principles of its determination by electrical methods

Heating and cooling

Sunbathers on hot sandy beaches dive into the sea to cool off. Sand heats up much more readily than water does. Even when the sand is almost too hot to walk barefoot across, the sea water is refreshingly cool. The temperature rise of an object when it is heated depends on:

- the mass of the object,
- the amount of energy supplied to it,
- the substance or substances from which the object is made.

The specific heat capacity, c, of a substance is the energy needed to raise the temperature of unit mass of the substance by 1 K without change of state.

The unit of c is $J\,kg^{-1}\,K^{-1}$.

Specific heat capacities of some common substances are shown in Table 1.

Table 1 *Some specific heat capacities*

Substance	Specific heat capacity /J kg^{-1}K^{-1}
Aluminium	900
Concrete	850
Copper	390
Iron	490
Lead	130
Oil	2100
Water	4200

To raise the temperature of mass m of a substance from temperature T_1 to temperature T_2,

$$\text{the energy needed } \Delta Q = mc\,(T_2 - T_1)$$

For example, to calculate the heat that must be supplied to raise the temperature of 5.0 kg of water from 20 °C to 100 °C, using the above formula gives $\Delta Q = 5.0 \times 4200 \times (100 - 20) = 1.7 \times 10^6$ J.

The heat capacity, C, of an object is the heat supplied to raise the temperature of the object by 1 K.

Therefore, for an object of mass m made of a single substance of specific heat capacity c, its heat capacity $C = mc$. For example, the heat capacity of 5.0 kg of water is $21\,000\,J\,K^{-1} = 5.0\,kg \times 4200\,J\,kg^{-1}\,K^{-1}$.

The inversion tube experiment

In this experiment, the gravitational potential energy of an object falling in a tube is converted into internal energy when it hits the bottom of a tube. Figure 1 shows the idea. The object is a collection of tiny lead spheres.

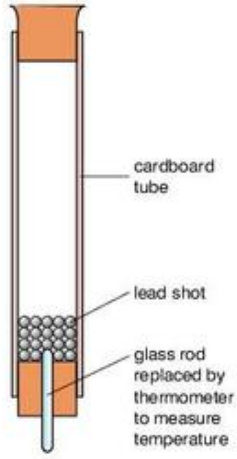


Fig 1 *The inversion tube experiment*

The tube is inverted each time the spheres hit the bottom of the tube. The temperature of the lead shot is measured initially and after a certain number of inversions.

Let m represent the mass of the lead shot.

For a tube of length L, the loss of gravitational potential energy for each inversion $= mgL$

Therefore, for n inversions, the loss of gravitational potential energy $= mgLn$

The gain of internal energy of the lead shot $= mc\,\Delta T$, where c is the specific heat capacity of lead and ΔT is the temperature rise of the lead shot.

Assuming all the gravitational potential energy lost is transferred to internal energy of the lead shot,

$$mc\Delta T = mgLn$$

$$\therefore \quad c = \frac{gLn}{\Delta T}$$

The experiment can therefore be used to measure the specific heat capacity of lead with no other measurements than the length of the tube, the temperature rise of the lead and the number of inversions.

Specific heat capacity measurements using electrical methods

1 Measurement of the specific heat capacity of a metal

A block of the metal of known mass m in an insulated container is used. A 12 V electrical heater is inserted into a hole drilled in the metal and used to heat the metal by supplying a measured amount of electrical energy. A thermometer inserted into a second hole drilled in the metal is used to measure the temperature rise ΔT (= its final temperature − its initial temperature).

The electrical energy supplied = heater current, I × heater p.d., V × heating time, t

$\therefore$ assuming no heat loss to the surroundings, $mc\Delta T = IVt$

$$\therefore \quad c = \frac{IVt}{m\Delta T}$$

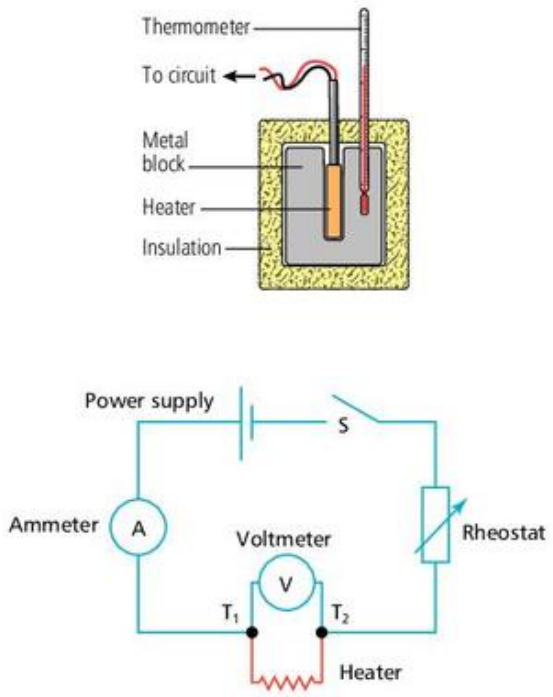


Fig 2 *Measuring c*

2 Measurement of the specific heat capacity of a liquid

A known mass of the liquid is used in an insulated calorimeter of known mass and known specific heat capacity. A 12 V electrical heater is placed in the liquid and used to heat it directly. A thermometer inserted into the liquid is used to measure the temperature rise, ΔT. Assuming no heat loss to the surroundings:

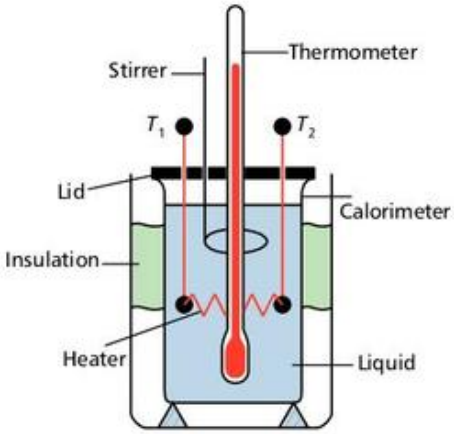


Fig 3 *Measurement of the specific heat capacity of a liquid*

- The electrical energy supplied = current I × voltage V × heating time t.
- The energy needed to heat the liquid =
  mass of liquid (m_l) × specific heat capacity of liquid (c_l) × temperature rise ΔT.
- The energy needed to heat the calorimeter =
  mass of calorimeter (m_{cal}) × specific heat capacity of calorimeter (c_{cal}) × temperature rise ΔT

$$\therefore\ IVt = m_l c_l \Delta T + m_{cal} c_{cal} \Delta T.$$

Hence, c_l can be calculated from this equation as all the other quantities are known.

EXTENSION

Method of mixtures experiments can be carried out as follows to determine specific heat capacities. No electrical heating is required. An insulated calorimeter as shown in Figure 3 is used without the heater. The masses of the calorimeter (m_c) and of the liquid (m_l) in it must be measured.

To determine the specific heat capacity, *c*, of a small object of known mass m, the object is firstly suspended on a thread in a beaker of boiling water to raise its initial temperature T_1 to 100 °C. The initial temperature T_2 of the liquid (e.g. water or oil) in the calorimeter is measured then the object is quickly transferred from the boiling water into the liquid in the calorimeter. As a result, the liquid becomes warmer and it is then stirred until its temperature reaches a steady value, T_3.

Assuming no heat transfer to or from the surroundings:

- the heat energy transferred from the object (to the liquid and calorimeter) = $mc(T_1 - T_3)$ where $T = 100\,^{\circ}\text{C}$,
- the heat energy gained by the calorimeter = $m_c c_0 (T_3 - T_2)$ where c_0 is the specific heat capacity of the calorimeter,
- the heat energy gained by the liquid = $m_l c_l (T_3 - T_2)$ where c_l is the specific heat capacity of the liquid.

Therefore, because no heat energy is transferred to or from the surroundings, the heat energy transferred from the object = the heat energy gained by the object and the calorimeter.

$$mc(T_1 - T_3) = m_l c_l (T_3 - T_2) + m_c c_0 (T_3 - T_2)$$

Hence c can be calculated by substituting the measured values of all the other quantities in the above equation.

To determine the specific heat capacity of a liquid, the method above can be used with the liquid in the calorimeter and with an object of known specific heat capacity.

Note To increase the accuracy of the experiments above:

- the initial temperature of the calorimeter and the liquid should be at room temperature so the temperature difference between T_1 (= 100 °C) and T_2 is as large as possible, and
- the masses should be chosen so the final temperature T_3 is approximately midway between T_1 and T_2.

Worked example

An insulated copper calorimeter of mass 0.127 kg contains 0.170 kg of water at an initial temperature of 20.0 °C. A stone of mass 0.050 kg suspended on a thread is transferred from boiling water at 100 °C to the water, causing the temperature of the water to increase to 31.5 °C. Calculate:

a the heat energy gained by the water and the calorimeter,
b the specific heat capacity of the stone.

specific heat capacities: copper 390 J kg^{-1} K^{-1}, water 4200 J kg^{-1} K^{-1}.

Solution

a Heat energy gained by calorimeter = $0.127 \times 390 \times (31.5 - 20.0) = 570\,\text{J}$
Heat energy gained by the water = $0.170 \times 4200 \times (31.5 - 20.0) = 8210\,\text{J}$
Heat energy gained by calorimeter and water = 8780 J
b Heat energy lost by the stone = $0.050 \times c \times (100 - 31.5) = 8780\,\text{J}$

$$\text{Therefore } c = \frac{8780}{0.050 \times (100 - 31.5)} = 2560\,\text{J kg}^{-1}\,\text{K}^{-1}$$

Continuous flow heating

In an electric shower, water passes steadily through copper coils heated by an electrical heater. The water is hotter at the outlet than at the inlet. This is an example of continuous flow heating. For mass m of fluid passing through the heater in time t at a steady flow rate, assuming no heat loss.

$$\textbf{the electrical energy supplied per second } IV = \frac{mc\Delta T}{t}$$

where ΔT is the temperature rise of the water.

QUESTIONS

Use the data in Table 1 for the following calculations.

1 Calculate:
- **a** the energy needed to heat an aluminium pan of mass 0.30 kg from 15 °C to 100 °C,
- **b** the energy needed to heat 1.5 kg of water from 15 °C to 100 °C.

2 **a** Calculate the time taken to heat the water and pan in Question **1** from 15 °C to 100 °C using a 2.0 kW electric hot plate.
- **b** Calculate the energy needed to raise the temperature of 80 kg of water in an insulated copper tank of mass 20 kg from 20 °C to 50 °C.

3 In an inversion tube experiment, 0.50 kg of lead shot at an initial temperature of 18 °C was inverted fifty times in a tube of length 1.30 m. The final temperature of the lead shot was 23 °C. Calculate:
- **a** the total gravitational potential energy released by the lead,
- **b** the specific heat capacity of lead.

4 An electric shower is capable of heating water from 10 °C to 40 °C when the flow rate is 0.025 kg s^{-1}. Calculate the minimum power of the heater. The specific heat capacity of water = 4200 J kg^{-1} K^{-1}.

20.4 Change of state

On these pages you will learn to:

- Explain in terms of molecules why melting and boiling take place without a change in temperature
- Define specific latent heat capacity and carry out calculations involving specific latent heat values
- Identify the main principles of electrical methods to determine specific latent heat values
- Explain why the specific latent heat of vaporisation is higher than the specific latent heat of fusion for the same substance
- Explain why a cooling effect accompanies evaporation

When a solid is heated and heated, its temperature increases until it melts. If it is a pure substance, it melts at a well-defined temperature, its **melting point**. Once all the solid has melted, continued heating causes the temperature of the liquid to increase until the liquid boils. This occurs at a certain temperature, the **boiling point**. The substance turns to a vapour as it boils away.

The three physical states of a substance, solid, liquid and vapour, have different physical properties. For example:

- The density of a gas is much less than the density of the same substance in the liquid or the solid state. This is because the molecules of a liquid and of a solid are packed together in contact with each other. In contrast, the molecules of a gas are, on average, separated from each other by relatively large distances.
- Liquids and gases can flow but solids cannot. This is because the atoms in a solid are locked together by strong force bonds which the atoms are unable to break free from. In a liquid or a gas, the molecules are not locked together because they have too much kinetic energy and the force bonds are not strong enough to keep the molecules fixed to each other.

Latent heat

When a solid or a liquid is heated so its temperature increases, its molecules gain kinetic energy. In a solid, the atoms vibrate more about their mean positions. In a liquid, the molecules move about faster, still keeping in contact with each other but free to move about.

1 **When a solid is heated at its melting point**, its atoms vibrate so much that they break free from each other. The solid therefore becomes a liquid due to energy being supplied at the melting point. The energy needed to melt a solid at its melting point is referred to as **latent heat of fusion**. Fusion is used for the melting of a solid because the solid 'fuses' into a liquid as it melts. Latent heat is released when a liquid solidifies. This happens because the liquid molecules slow down as the liquid is cooled; at the melting point, the molecules move slowly enough for the force bonds to lock the molecules together.

2 **When a liquid is heated at its boiling point**, the molecules gain enough kinetic energy to overcome the bonds that hold them close together. The molecules therefore break away from each other to form bubbles of vapour in the liquid. The energy needed to vaporise a liquid is referred to as **latent heat of vaporisation**.

Latent heat is released when a vapour condenses. This happens because the vapour molecules slow down as the vapour is cooled; the molecules move slowly enough for the force bonds to pull the molecules together to form liquid.

Some solids vaporise directly when heated. This process is known as **sublimation**.

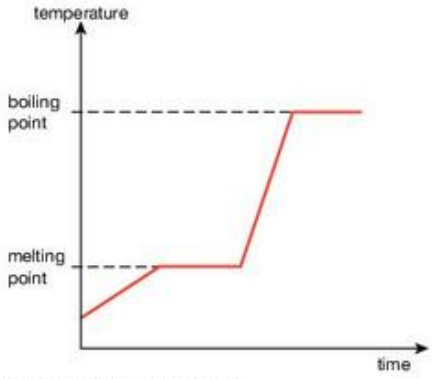


Fig 1 Melting and boiling

'Latent' means 'hidden'; latent heat supplied to melt a solid or boil a liquid may be thought of as hidden because no temperature change takes place even though the substance is being heated. The energy supplied as latent heat increases the internal potential energy of the molecules.

Specific latent heat

In general, much more energy is needed to vaporise a substance than to melt it. This is because the molecules need to move completely away from each other when vaporisation occurs (unlike when melting occurs). For example, 2.2 MJ is needed to vaporise 1 kg of water at its boiling point. In comparison, 0.34 MJ is needed to melt 1 kg of ice at its melting point.

The energy needed to change the state of 1 kg of a substance at its melting point (or its boiling point) is referred to as its specific latent heat of fusion (or vaporisation).

- The specific latent heat of fusion, L_f, of a substance is the energy needed to change the state of unit mass of the substance from solid to liquid without change of temperature.
- The specific latent heat of vaporisation, L_v, of a substance is the energy needed to change the state of unit mass of the substance from liquid to vapour without change of temperature.

Therefore, the heat energy Q needed to change the state of mass m of a substance from solid to liquid or liquid to vapour without change of temperature is given by

$$Q = mL$$

where L is the specific latent heat of fusion or vaporisation as appropriate.

The unit of specific latent heat is $\mathrm{J\,kg^{-1}}$.

Note Where a substance changes its state and changes its temperature, to calculate the heat energy transferred,

- use $Q = mL$ to calculate the heat energy transferred when its state changes
- use $Q = mc(T_2 - T_1)$ when its temperature changes.

Worked example

Calculate the energy needed to melt 5.0 kg of ice at 0 °C and heat the melted ice to 50 °C.

Specific latent heat of fusion of ice = $3.36 \times 10^5\,\mathrm{J\,kg^{-1}}$

Specific heat capacity of water = $4200\,\mathrm{J\,kg^{-1}\,K^{-1}}$

Solution

To melt 5.0 kg of ice, energy needed $Q_1 = mL = 5.0 \times 3.36 \times 10^5 = 1.68 \times 10^6\,\mathrm{J}$

To heat 5.0 kg of melted ice (i.e. water) from 0 °C to 50 °C, energy needed

$$Q_2 = mc(T_2 - T_1) = 5.0 \times 4200 \times (50 - 0) = 1.05 \times 10^6\,\mathrm{J}$$

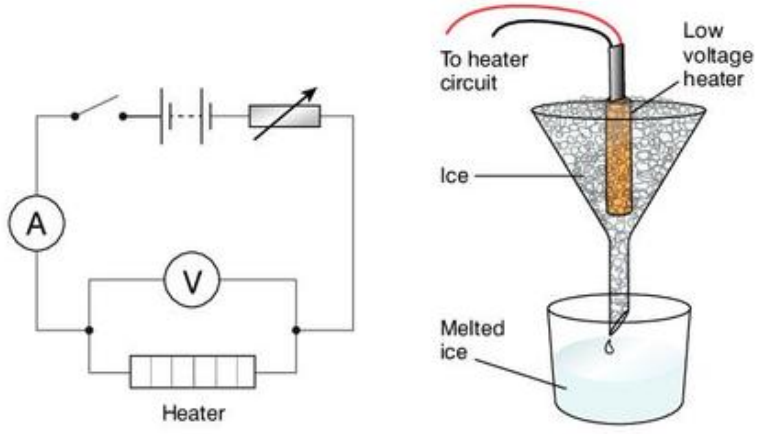
Therefore, the total energy needed $= Q_1 + Q_2 = 2.73 \times 10^6\,\mathrm{J}$

Measurement of the specific latent heat of fusion of ice

In this experiment, a low voltage heater is to be used to melt crushed ice in a funnel. The melted ice is collected using a beaker under the funnel, as shown in Figure 3.

To take account of heat transfer from the surroundings, the mass of ice melted in a certain time must be measured with the heater off then with it on. The difference in the two measurements gives the mass of ice melted due to the heater only.

1 With the heater off, water from the funnel is collected in the beaker for a measured time (e.g. 10 minutes). The mass of the beaker and water, m_1, is then measured. The beaker is then emptied for the next stage.
2 With the heater on, the procedure is repeated for exactly the same time. The ammeter and voltmeter readings are recorded with the heater switched on. After the heater is switched off, the mass of the beaker and the water, m_2, is measured once more.



a *Heater circuit* **b** *Collecting melting ice*

Fig 3 *Measuring the specific latent heat of fusion of ice*

Cooling by evaporation

If you have an injection, the doctor or nurse might 'numb' your skin by dabbing it with a liquid that easily evaporates. As the liquid evaporates, your skin becomes too cold to feel any pain.

Evaporation occurs at the open surface of a liquid. Relatively weak attractive forces exist between the particles in the liquid. Cooling by evaporation occurs because the faster particles in the liquid near the surface have enough kinetic energy to break away from the other particles and escape from the liquid. After they leave, the liquid is cooler because the average kinetic energy of the particles left in the liquid has decreased.

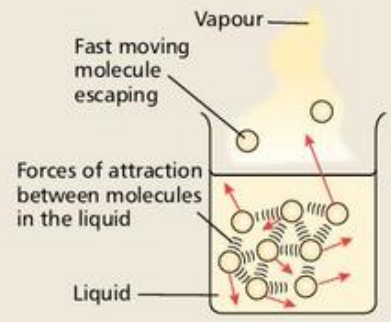


Fig 2 *Cooling by evaporation*

To calculate the specific latent heat of fusion of ice, note that:

- the mass of ice melted due to the heater, $m = m_2 - m_1$
- the energy supplied E to the heater = current × p.d. × time
- the specific latent heat of fusion of ice, $l = \frac{E}{m} = \frac{E}{m_2 - m_1}$

EXTENSION

Note The method of mixtures described in topic 20.3 can also be used to measure the specific latent heat of fusion of ice. Pieces of ice dried with a tissue are melted one by one in warm water in the calorimeter. To compensate for heat loss to the surroundings ice is added until the temperature of the water is as much below room temperature as it was above room temperature before any ice was added.

The mass of the calorimeter, m_c, the initial mass of water, m_1, and the final mass of water, m_2, must be measured. The mass of ice, m, melted is therefore $(m_2 - m_1)$.

The initial temperature, T_1, and the final temperature, T_2, of the water must also be measured.

- The energy gained by the ice in melting and warming to temperature $T_2 = mL + mc_w(T_2 - 0)$, where L is the specific latent heat of fusion of ice and c_w is the specific heat capacity of water.
- The energy lost by the calorimeter and initial mass of water in it $= m_1c_w(T_1 - T_2) + m_cc_c(T_1 - T_2)$ where c_c is the specific heat capacity of the calorimeter.

Assuming no heat loss to the surroundings occurs, equating the heat loss to the heat gain enables L to be calculated:

$$mL + mc_w(T_2 - 0) = m_1c_w(T_1 - T_2) + m_cc_c(T_1 - T_2)$$

Measurement of the specific latent heat of vaporisation of a liquid

In this experiment, a low voltage heater is used to heat the liquid in the inner flask to its boiling point. The heater circuit is the same as that shown in Figure 3. When the liquid is boiling, vapour from the liquid leaves the inner flask, condenses in the condenser and drips into the flask underneath the condenser.

The ammeter and voltmeter readings are recorded at the normal current rating of the heater and the mass of liquid collected in a certain time is measured. The procedure is repeated with the liquid boiling at a reduced heater power.

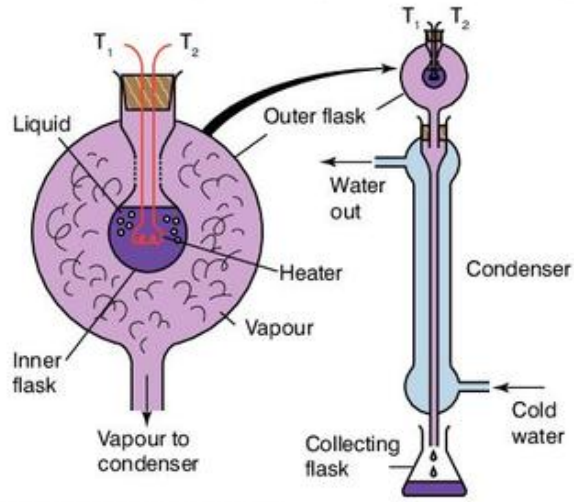


Fig 4 Measuring the specific latent heat of steam

In each collecting time, the energy supplied = current × p.d. × time.

Let E_1 and E_2 represent the energy supplied during each 'collecting' time, and let m_1 and m_2 represent the mass of liquid collected each time,

Therefore, $E_1 = m_1L + Q$ and $E_2 = m_2L + Q$ where L is the specific latent heat of the liquid and Q is the heat loss to the surroundings during the collecting time.

Hence $$E_1 - E_2 = m_1L - m_2L \quad \text{so} \quad L = \frac{E_1 - E_2}{m_1 - m_2}$$

Note The rate of heat loss to the surroundings is the same at normal power as at reduced power because the liquid is at the same temperature (i.e. its boiling point) in both situations. Since the collecting time is the same, the heat loss Q is therefore the same.

Temperature v. time graphs

If a pure solid is heated to its melting point and beyond, its temperature v. time graph will be as shown in Figure 5.

Assuming no heat loss occurs during heating,

- before the solid melts, the energy supplied each second $= mc_S\left(\frac{\Delta T}{\Delta t}\right)_S$ where $\left(\frac{\Delta T}{\Delta t}\right)_S$ is the rise of temperature per second of the solid and c_S is the specific heat capacity of the solid.
- at the melting point, the energy supplied = energy supplied per second × time taken to melt,
- after the solid melts, the energy supplied each second $= mc_L\left(\frac{\Delta T}{\Delta t}\right)_L$ where $\left(\frac{\Delta T}{\Delta t}\right)_L$ is the rise of temperature per second of the liquid and c_L is the specific heat capacity of the liquid.

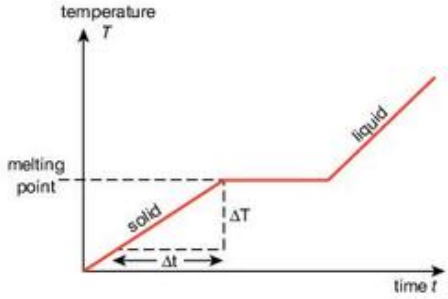


Fig 5 Temperature v. time for a solid being heated

QUESTIONS

1 **a** Explain why energy is needed to melt a solid.

b Explain why the internal energy of the water in a beaker must be reduced to freeze the water.

2 Calculate the mass of water boiled away in a 3 kW electric kettle in 2 minutes, given that 2.2 MJ of energy must be supplied to boil away 1 kg of water at atmospheric pressure.

3 A plastic beaker containing 0.080 kg of water at 15 °C was placed in a refrigerator and cooled to 0 °C in 1200 s.

a Calculate how much energy each second was removed from the water in this process. The specific heat capacity of water = 4200 J kg^{-1} K^{-1}.

b Calculate how long the refrigerator would take to freeze the water in **a**, given that 340 kJ of energy must be removed from 1.0 kg of water at 0 °C to freeze it.

4 The temperature v. time graph shown in Figure 6 was obtained by heating 0.12 kg of a substance in an insulated container. The specific heat capacity of the substance in the solid state is 1200 J kg^{-1} K^{-1}. Calculate:

a the energy per second supplied to the substance in the solid state if its temperature increased from 60 °C to its melting point at 78 °C in 120 s,

b the energy needed to melt the solid if it took 300 s to melt with energy supplied at the same rate as in **a**.

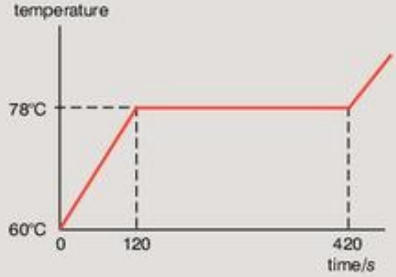


Fig 6

EXTENSION

20.5 Heat transfer

On these pages you will learn to:

- Explain what is meant by thermal conduction, convection and radiation and give examples of heat transfer by each method
- Explain why metals are better conductors of heat than non-metals
- Recognise the factors that determine the rate of heat transfer along a uniform conductor
- Use the equation $Q/t = kA(T_1 - T_2)/L$ to solve simple problems on thermal conduction
- Define thermal resistance and use it to solve simple problems
- Use Stefan's law to solve simple problems on thermal radiation

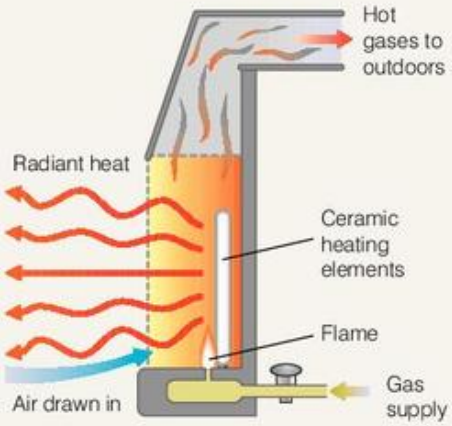


Fig 1 A gas heater

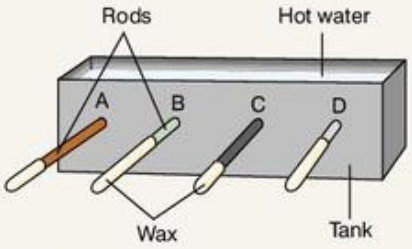


Fig 2 Comparing heat conductors

Heat is energy transfer due to a temperature difference. The three methods by which heat transfer occurs are thermal conduction, convection and radiation. Evaporation from the surface of a liquid also causes transfer of energy from the liquid. This effect causes a liquid to cool because the faster molecules leave it. Conduction takes place in solids, liquids and gases. Convection takes place in liquids and gases only. For example, heat from a hot radiator reaches other parts of a room due to thermal conduction through the radiator panel, which heats the air and causes convection currents and also radiates infrared radiation throughout the room.

Convection

When a fluid is heated it becomes less dense and it rises because it is less dense than the cold fluid. In a closed space, the fluid circulates as the hot fluid rises and cold fluid is drawn in to the source of heating where it heats up and rises. This process is known as **convection**. Some examples are listed below.

1 A hot air balloon rises because a gas burner heats the air in the balloon and makes it less dense than the surrounding air. Without repeated bursts of heating, the hot gas in the balloon would cool and the balloon would sink as the air in it becomes more dense.

2 Ocean currents such as the Gulf Stream are convection currents of warm water that flow at or near the ocean surface from hot to colder regions.

3 Hot gases from the flames of a gas fire rise, drawing air into the gas fire. The products of combustion include carbon monoxide, which is lethal if allowed to build up. For this reason, gas fires must be well-ventilated so that the products of combustion escape into the atmosphere and fresh air is drawn into the room where the heater is.

Thermal conduction

Some materials conduct heat much more effectively than others. Figure 2 shows how rods of different materials, which are otherwise identical, can be compared in terms of thermal conductivity. Each rod protrudes from a tank with its protruding length coated evenly with a thin wax layer. After the tank is filled with hot water, each rod conducts heat along its length, causing its wax layer to melt progressively along its length. The length of the wax layer remaining after a certain time indicates how effectively each rod conducts heat. The thermal conductivities of the rods can thus be compared.

Metals are better conductors of heat than non-metals because of the presence of conduction electrons in metals. When a metal is heated, the electrons gain energy and move about faster, transferring energy to atoms and electrons elsewhere.

Non-metals do not contain conduction electrons and therefore do not conduct as well as metals. Thermal conduction in a non-metal takes place as a result of vibrations of the atoms spreading throughout the non-metal. Heating a non-metal makes the atoms at the point of heating vibrate more and these vibrations cause atoms to vibrate in other parts of the non-metal. This process does occur in a metal but much less than energy transfer due to conduction electrons.

Consider the heat flow along a conductor of uniform cross-sectional area A and of length L which has one end at fixed temperature T_1 and the other end at a lower fixed temperature T_2. Provided the conductor is in an insulated jacket so there is no heat loss from its sides, the temperature falls uniformly along its length from

T_1 to T_2, as shown in Figure 3. This shows that the temperature gradient along the bar, the temperature drop per unit length, is constant.

The heat flow per second conducted along the bar, Q/t:

- is proportional to the area of cross section, A
- is proportional (and in the opposite direction) to the temperature gradient $\frac{T_1 - T_2}{L}$
- depends on the thermal conductivity of the material.

Hence the heat flow per second, Q/t, conducted along the bar is given by the equation

$$\frac{Q}{t} = -\frac{kA(T_1 - T_2)}{L}$$

where k is the **coefficient of thermal conductivity** of the material. The unit of k is $\mathrm{W\,m^{-1}\,K^{-1}}$. Table 1 gives values of k for different materials. The minus sign in the equation represents the fact that the heat flow direction is in the opposite direction to the temperature gradient.

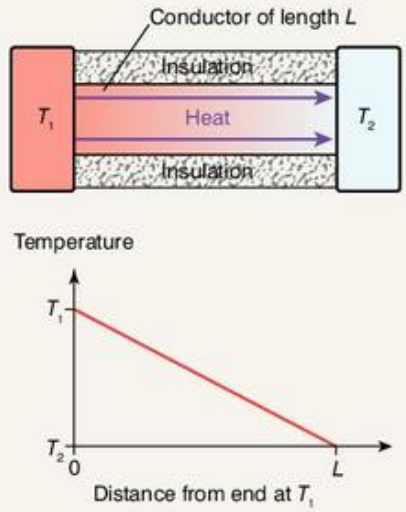


Fig 3 *Temperature v. position along an insulated thermal conductor*

Measurement of the thermal conductivity of copper

One end of an insulated copper bar is heated using steam, as shown in Figure 4. The heat conducted along the bar is removed by water flow through the copper cooling coils wrapped round the cold end of the bar. Steady heat flow along the bar is attained when the temperature readings of the thermometers becomes steady.

1. The temperature gradient is measured using the thermometers T_1 and T_2 at measured distance L apart.
2. The diameter d of the bar is measured and used to calculate the area of cross-section using $A = \pi d^2/4$,
3. The rate of flow of heat along the bar Q/t is determined by measuring the water flow rate through the cooling coils and the outflow and inflow temperatures T_3 and T_4.

If m is the mass of water flowing through the coils in time t, the heat flow Q/t = the energy removed per second by the water = $\frac{mc(T_3 - T_4)}{t}$, where c is the specific heat capacity of water.

The thermal conductivity k can then be calculated using the equation

$$\frac{Q}{t} = \frac{kA(T_1 - T_2)}{L}$$

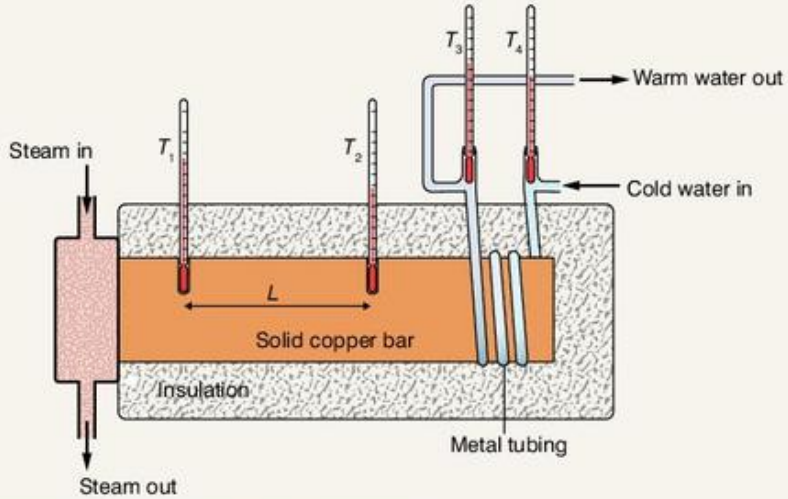


Fig 4 *Measuring the thermal conductivity of copper*

Worked example

Calculate the heat transfer per second through a brick wall of thickness 90 mm, height 2.50 m and length 3.60 m for a temperature difference of 20 K.

Solution

$$\frac{Q}{t} = \frac{kA(T_1 - T_2)}{L}$$

$$= \frac{0.40 \times (2.50 \times 3.60) \times 20}{90 \times 10^{-3}}$$

$$= 800\,\mathrm{W}$$

Table 1 *Thermal conductivity values*

Material	Thermal conductivity $k/\mathrm{W\,m^{-1}\,K^{-1}}$
aluminium	210
brick	0.40
cardboard	0.036
copper	390
glass	0.72
felt	0.21
plaster	0.29
steel	450

Worked example

Calculate the heat transfer per second across a brick wall of thickness 90 mm, height 2.50 m and length 3.60 m covered with a 32 mm layer of plaster board when the temperature difference across the wall is 20 K.

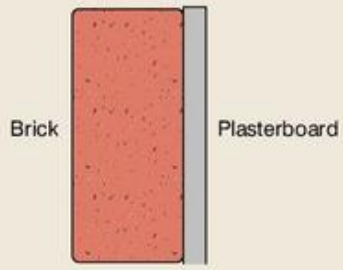


Fig 5

Thermal conductivity of plaster $= 0.29\,\text{W m}^{-1}\text{K}^{-1}$

Solution

Thermal resistance of the brick wall

$$= \frac{L}{kA} = \frac{90 \times 10^{-3}}{0.40 \times 2.50 \times 3.60}$$

$$= 2.50 \times 10^{-2}\,\text{KW}^{-1}$$

Thermal resistance of the plaster

$$= \frac{L}{kA} = \frac{32 \times 10^{-3}}{0.29 \times 2.50 \times 3.60}$$

$$= 1.19 \times 10^{-2}\,\text{KW}^{-1}$$

Total thermal resistance

$$= 2.50 \times 10^{-2} + 1.19 \times 10^{-2}$$

$$= 3.7 \times 10^{-2}\,\text{KW}^{-1}$$

to 2 significant figures.

Heat flow/second,

$$= \frac{Q}{t} = \frac{\text{temperature difference}}{\text{total thermal resistance}}$$

$$= \frac{20}{3.7 \times 10^{-2}} = 540\,\text{W}$$

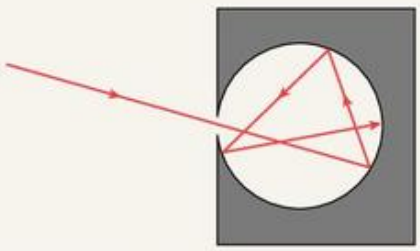


Fig 6 A black body

Composite conductors

The analogy between heat flow and charge flow is useful here. If the above equation is compared with $I = \frac{V}{R}$,

- electric current I is equivalent to $\frac{Q}{t}$,
- potential difference V is equivalent to temperature difference $T_1 - T_2$
- electrical resistance R is equivalent to **thermal resistance** $\frac{L}{kA}$

The concept of thermal resistance is useful in thermal conduction calculations where more than one material is present. Where two or more thermal conductors are:

1 **In series**, their total thermal resistance is the sum of the individual thermal resistances. Therefore, heat flow per second through them = temperature difference across them ÷ total thermal resistance.
2 **In parallel**, the total heat flow per second through each material = the sum of the individual heat flow per second through each material (which can be calculated from the temperature difference ÷ the thermal resistance).

Radiation

Every object emits electromagnetic radiation due to its temperature. This radiation is known as **thermal radiation** and consists mostly of infrared radiation although it would include visible radiation if the temperature is sufficiently high.

Thermal radiation is absorbed most effectively by matt black surfaces and least effectively by shiny silvered surfaces. **A black body** is a body that absorbs all the radiation incident on its surface. For example, a small hole in the surface of a hollow object would act as a black body as any radiation entering the hole would be completely absorbed by the surface of the cavity. See Figure 6. The Sun and the stars may be considered as black bodies since any radiation incident on them would be absorbed.

A surface that is a good absorber of thermal radiation is also a good emitter. The radiation from a black body is referred to as **black body radiation**. The energy per second radiated from a surface depends on:

- the area, A, of the surface; the greater the area, the more energy radiated each second
- the surface temperature, T; the hotter an object's surface is, the more energy it radiates per second
- the nature of the surface; the surface of a black body radiates more energy per second than any other surface of the same area and at the same temperature.

Stefan's law of radiation states that the energy per second, P, radiated from a surface of area A at absolute temperature T is given by the equation

$$P = e\sigma AT^4$$

where σ is a constant known as the **Stefan constant**, and e, the emissivity of the surface, is the ratio of the energy per second radiated from the surface to energy per second radiated by a black body surface of equal area at the same temperature. The value of σ is $5.67 \times 10^{-8}\,\text{W m}^{-2}\text{K}^{-4}$.

Note For a black body, $e = 1$ therefore $P = \sigma AT^4$ for a black body.

The greenhouse effect

Many scientists now believe that the increased burning of fossil fuels is causing global warming due to the increasing amounts of carbon dioxide in the Earth's atmosphere. The effect is called the greenhouse effect because the Earth's atmosphere acts like a greenhouse made of glass. In a greenhouse:

- short wavelength infrared radiation (and light) from the Sun can pass through the glass to warm the objects inside the greenhouse
- infrared radiation from these warm objects is trapped inside by the glass because the objects emit longer wavelength infrared radiation that can't pass through the glass.

So the greenhouse stays warm. But the Earth may become too warm to avoid disastrous consequences such as rising sea levels and more 'extreme' weather events unless the use of fossil fuels is reduced. Alternative technology such as solar panels, biofuel and wind farms could reduce fossil fuel dependency in hot coastal regions.

QUESTIONS

$\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$

1 Figure 7 shows a cross-section of a vacuum flask designed to keep a liquid hot.

Fig 7

a Why is the glass vessel in the flask silvered?

b Why is there a vacuum between the inner and outer surfaces of the vessel?

c Why is a lid essential?

d What would be a suitable material to surround the vessel? Give a reason for your choice.

2 Estimate the heat radiated per second from a hot plate of diameter 0.15 m which is at a surface temperature of 800 K.

3 Calculate the heat flow per second per square metre through:

a a single-brick wall of thickness 150 mm when the temperature on one side is 25 °C and the temperature on the other side is 10 °C,

b a double-brick wall with a 50 mm air gap between.

thermal conductivity of air $= 0.024\,\mathrm{W\,m^{-1}\,K^{-1}}$.

4 An aluminium pan of diameter 110 mm was used to boil water on a gas cooker. In two minutes, 0.14 kg of water was boiled away.

a Calculate the energy per second needed to boil water away at this rate.

b The thickness of the base of the pan was 3.2 mm. Estimate the temperature difference between the underside and the top of the base of the pan.

The specific latent heat of steam is $2.3\,\mathrm{MJ\,kg^{-1}}$ and the thermal conductivity of aluminium is $210\,\mathrm{W\,m^{-1}\,K^{-1}}$.

Worked example

Calculate the energy per second radiated from a glowing fuse wire of length 20 mm and diameter 0.3 mm at a surface temperature of 1600 K, given its surface emissivity is 0.2.

Solution

Surface area of wire, $A = 2\pi rL$
where $r = 0.15\,\mathrm{mm} = 1.5 \times 10^{-4}\,\mathrm{m}$
and $L = 20\,\mathrm{mm} = 0.020\,\mathrm{m}$

Hence

$$A = 2\pi \times 1.5 \times 10^{-4} \times 0.020$$
$$= 1.89 \times 10^{-5}\,\mathrm{m^2}$$

Using Stefan's law, energy per second radiated from the surface

$$= e\sigma AT^4$$
$$= 0.2 \times 5.67 \times 10^{-8} \times 1.89 \times 10^{-5}\,\mathrm{m^2} \times (1600)^4$$
$$= 1.40\,\mathrm{J\,s^{-1}}$$

CHAPTER SUMMARY

The internal energy of an object is the sum of the random distribution of the kinetic and potential energies of its molecules.

Absolute zero is the temperature at which an object has minimum internal energy.

Temperature in °C = absolute temperature in kelvins − 273.15.

Specific heat capacity, *c*, of a substance is the energy needed to raise the temperature of 1 kg of the substance by 1 K without change of state.

Energy needed to raise the temperature of mass m of a substance from T_1 to T_2 = $mc\,(T_2 - T_1)$, where c is the specific heat capacity of the substance.

Energy to melt (or boil) mass m of a substance = mL, where L is the specific latent heat of fusion (or vapourisation) of the substance.

20 End of chapter questions

$g = 9.8\,\text{m s}^{-2}$

1 **a** In terms of molecules, explain why:
- **i** a solid has its own shape,
- **ii** a liquid and a gas can flow,
- **iii** a gas is much less dense than a solid or a liquid.

b Describe the effect on the molecules of a solid of:
- **i** supplying energy to it to raise its temperature,
- **ii** supplying energy to it to melt it.

2 When a certain object is heated, its internal energy increases. Explain, in molecular terms:

a what is meant by internal energy,

b why increasing the internal energy of the object differs from increasing its kinetic energy.

3 A vehicle of mass 1200 kg is fitted with brake pads of total mass 23 kg. When the car is travelling at a speed of $30\,\text{m s}^{-1}$, the brakes are applied and the car stops in a distance of 110 m. The temperature of the brake pads increases by 7.5 K during this time.

a Calculate:
- **i** the initial kinetic energy of the car,
- **ii** the gain of internal energy of the brake pads, assuming no heat transfer from the brake pads occurs.

b Show that the frictional force on the car due to the brakes was at least 3.9 kN.

The specific heat capacity of the brake pad material $= 2500\,\text{J kg}^{-1}\,\text{K}^{-1}$.

4 **a** An electric motor in prolonged use becomes hotter than its surroundings.
- **i** State two reasons why some of the electrical energy transferred to the motor is wasted as heat.
- **ii** Describe the energy transfers in the motor when it is in prolonged use at a constant temperature which is higher than the temperature of its surroundings.

b A 250 W electric winch raises a load of weight 110 N through a height of 4.0 m in 20 s.
- **i** Calculate the efficiency of the winch.
- **ii** The temperature of a pulley wheel in the winch increases by 2.0 K when the load in (i) is raised. The pulley wheel is made of steel and has a mass of 0.095 kg. Calculate the energy needed to raise the temperature of the wheel by 2.0 K and determine the increase of efficiency of the winch if friction at the pulley could be eliminated.

Specific heat capacity of steel $= 460\,\text{J kg}^{-1}\,\text{K}^{-1}$

5 An insulated metal cylinder of mass 1.4 kg was heated by a 24 W electric heater placed in a slot in the cylinder, as shown in Figure 2 on p.303. The temperature of the metal was measured using a thermometer placed in a different slot in the metal. The measurements were used to plot a graph of the temperature v. time, as shown in the diagram below.

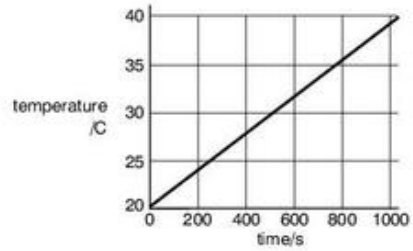


Fig 5.1

a Determine the temperature rise per second of the cylinder.

b Calculate the specific heat capacity of the metal.

6 A 3.0 kW electric kettle took 260 s to heat 1.8 kg of water from 18 °C to 100 °C.

a Calculate:
- **i** the electrical energy supplied to the kettle in this time,
- **ii** the internal energy gained by the water in the kettle.

b The mass of the kettle was 1.2 kg and it was made from aluminium.
- **i** Calculate the internal energy gained by the kettle.
- **ii** Account for the difference between the electrical energy supplied to the kettle and the internal energy gained by the water and the kettle.

specific heat capacity of water $= 4200\,\text{J kg}^{-1}\,\text{K}^{-1}$,
specific heat capacity of aluminium $= 900\,\text{J kg}^{-1}\,\text{K}^{-1}$

7 A shower fitted with an electric heater raised the temperature of water passing through the shower from 12 °C to 46 °C when the flow rate was $0.050\,\text{kg s}^{-1}$.

a Calculate the power of the electric heater, assuming there is no heat loss to the surroundings.

b The shower is fitted with a thermostat that restricts the temperature of the outflowing water to 50 °C. Calculate the flow rate that would give this outflow temperature when the temperature of the inflow is 12 °C.

Specific heat capacity of water $= 4200\,\text{J kg}^{-1}\,\text{K}^{-1}$

8 When a liquid is boiling, thermal energy must be supplied in order to maintain a constant temperature.

a State two processes for which thermal energy is required during boiling.

b A student carries out an experiment to determine the specific latent heat of vaporisation of a liquid.

Some liquid in a beaker is heated electrically as shown in Figure 8.1.

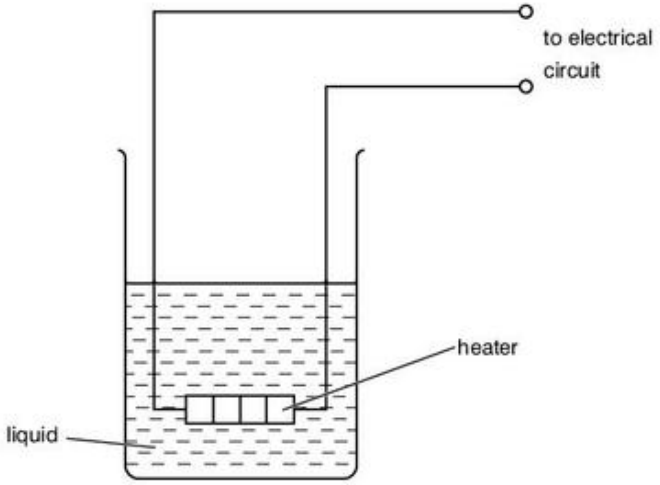


Fig 8.1

Energy is supplied at a constant rate to the heater. When the liquid is boiling at a constant rate, the mass of liquid evaporated in 5.0 minutes is measured.

The power of the heater is then changed and the procedure is repeated.

Data for the two power ratings are given in Figure 8.2.

power of heater/W	mass evaporated in 5.0 minutes/g
50.0	6.5
70.0	13.6

Fig 8.2

i Suggest

1 how it may be checked that the liquid is boiling at a constant rate, *(1 mark)*

2 why the rate of evaporation is determined for two different power ratings. *(1 mark)*

ii Calculate the specific latent heat, in $J\,g^{-1}$, of vaporisation of the liquid. *(3 marks)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 04 Q3 June 2009

9 a A student states, quite wrongly, that temperature measures the amount of thermal energy (heat) in a body.

State and explain two observations that show why this statement is incorrect. *(4 marks)*

b A thermometer and an electrical heater are inserted into holes in an aluminium block of mass 960 g, as shown in Figure 9.1.

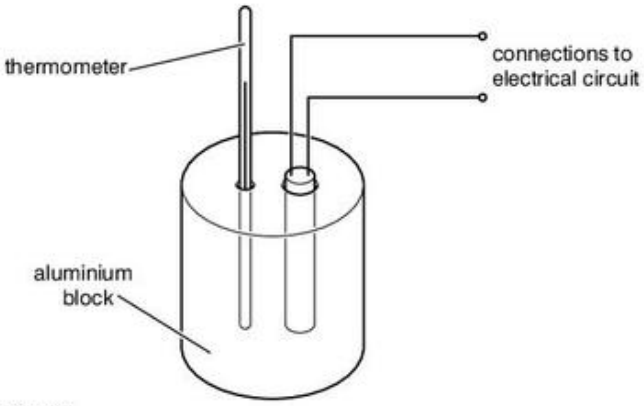


Fig 9.1

The power rating of the heater is 54 W.

The heater is switched on and readings of the temperature of the block are taken at regular time intervals. When the block reaches a constant temperature, the heater is switched off and then further temperature readings are taken. The variation with time t of the temperature θ of the block is shown in Figure 9.2.

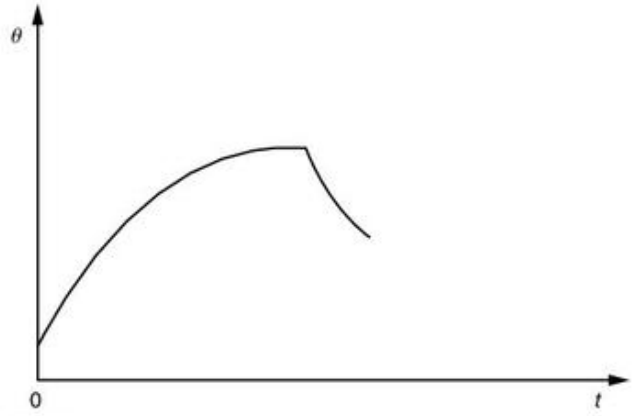


Fig 9.2

i Suggest why the rate of rise of temperature of the block decreases to zero. *(2 marks)*

ii After the heater has been switched off, the maximum rate of fall of temperature is 3.7 K per minute.

Estimate the specific heat capacity, in $J\,kg^{-1}\,K^{-1}$, of aluminium. *(3 marks)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 41 Q3 November 2009

21 Gases

21.1 Experiments on gases

On these pages you will learn to:

- Know what is meant by an isothermal change
- Recall that temperature in kelvins = temperature in °C + 273(.15)
- Recognise that for a fixed mass of gas at pressure p, volume V and temperature T in kelvins:
- pV = constant at constant temperature T
- V/T = constant at constant pressure
- p/T = constant at constant volume

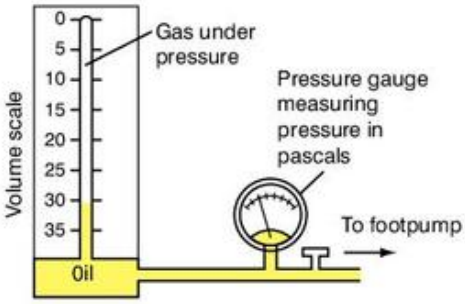
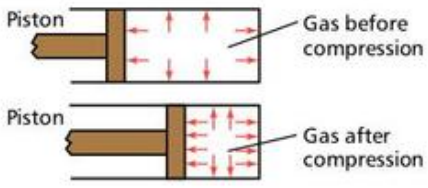


Fig 1 Testing Boyle's law



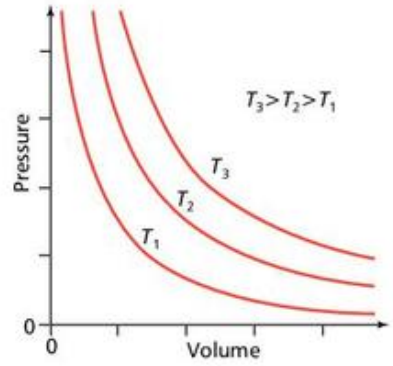


Fig 2 Boyle's law

The experimental gas laws

When you use a cycle pump to inflate a tyre, you raise the air pressure in the tyre because the pump pushes air through a valve into the tyre. The valve lets the air in but does not allow it out. The tyre is a buffer between the wheel frame and the ground. If the tyre pressure is too low, the wheel frame will rub on the ground when cycling.

The pressure of a gas is the force per unit area that the gas exerts on a surface. Pressure is measured in pascals (Pa), where $1\,\text{Pa} = 1\,\text{N m}^{-2}$. The pressure of a gas depends on its temperature, the volume of the gas container and on the mass of gas in the container.

1 Boyle's law

The apparatus shown in Figure 1 may be used to investigate how the pressure of a fixed mass of gas depends on its volume when the temperature remains the same. Measurements using this apparatus show that the gas pressure times its volume is constant for a fixed mass of gas at constant temperature. This is known as Boyle's law, after Robert Boyle who first discovered the law in 1662. Note that a change at constant temperature is called an **isothermal change**.

Boyle's law states that for a fixed mass of gas at constant temperature,

$$pV = \textbf{constant},$$

where p = gas pressure, and V = gas volume.

The measurements plotted as a graph of pressure v. $\frac{1}{\text{volume}}$ give a straight line through the origin. This is because Boyle's law may be written as $p = \text{constant} \times \frac{1}{V}$ which represents the equation $y = mx$ for a straight-line graph through the origin if p is plotted on the y-axis and $\frac{1}{V}$ on the x-axis.

Notes

1 A graph of pressure v. volume is a curve which tends towards each axis, as shown in Figure 2.
2 An ideal gas is a gas that obeys Boyle's law.

2 Charles' law

The volume of a fixed mass of gas at constant pressure increases with temperature. Plotting the measurements of the volume of the gas at 0 °C and 100 °C on a graph leads to the idea of absolute zero. No matter how much gas is used, provided the gas is an ideal gas, its volume would be zero at absolute zero which is −273.15 °C.

Figure 3 shows how the volume of a fixed mass of gas at constant pressure varies with absolute temperature T in kelvins. The graph is a straight line. The relationship, known as **Charles' law**, between the gas volume V and the temperature T in kelvins can therefore be written as

$$\frac{V}{T} = \textbf{constant}.$$

3 Pressure law

Figure 4 shows how the pressure of a fixed mass of gas at constant volume can be measured at different temperatures. If the measurements are plotted on a graph of pressure against temperature in kelvins, they give a straight line through the origin. The relationship between pressure p and temperature T, in kelvins, can therefore be written

$$\frac{p}{T} = \text{constant}$$

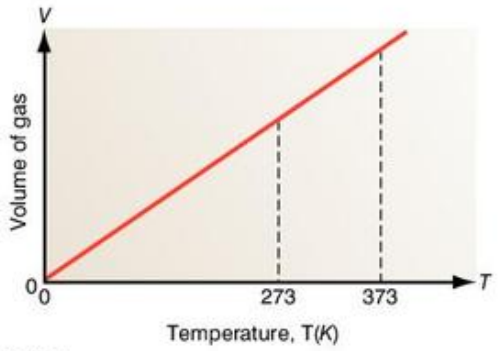


Fig 3 Charles' law

EXTENSION

Measurement of the density of a powder using Boyle's law

A hand pump of volume V_P is used to compress the air in a sealed flask of known volume V_F. A pressure gauge is used to measure the pressure of the air in the flask before compression (p_0) and after compression. See Figure 5.

1 Without any powder in the flask; applying Boyle's law gives $p_1V_F = p_0\,(V_F + V_P)$, where p_1 is the measured pressure after compression.

$\therefore\ p_0V_F = p_1V_F - p_0V_P$

2 With a measured mass of powder in the flask; applying Boyle's law gives $p_2\,(V_F - v) = p_0\,(V_F - v + V_P)$, where p_2 is the measured pressure after compression and v is the powder volume.

$\therefore\ p_0V_F = p_2V_F - p_0V_P - (p_2 - p_0)v$

Subtracting the two equations gives

$0 = p_1V_F - p_2V_F + (p_2 - p_0)v$

$\therefore\ (p_2 - p_0)v = p_2V_F - p_1V_F$

$$v = \frac{(p_2 - p_1)}{(p_2 - p_0)}V_F$$

Hence, the volume and the density of the powder can be determined.

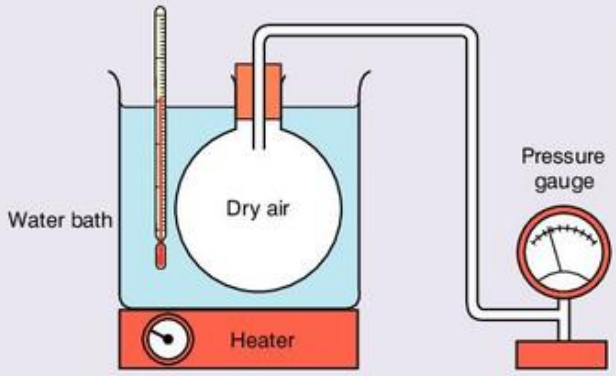


Fig 4 The pressure law

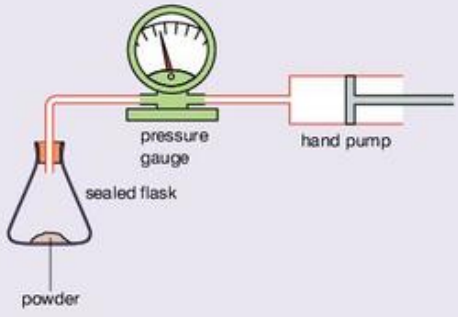


Fig 5 Measuring the volume of a powder

QUESTIONS

1 A hand pump of volume $2.0 \times 10^{-4}\,m^3$ is used to force air through a valve into a container of volume $8.0 \times 10^{-4}\,m^3$ which contains air at an initial pressure of 101 kPa. Calculate the pressure of the air in the container after one stroke of the pump, assuming the temperature is unchanged.

2 A sealed can contains air at a pressure of 101 kPa at 100 °C. The can is then cooled to a temperature of 20 °C. Calculate the pressure of the air in the can assuming the volume of the can is unchanged.

3 The volume of a fixed mass of gas at 15 °C was $0.085\,m^3$. The gas was then heated to 55 °C without change of pressure. Calculate the new volume of this gas.

4 In an experiment to measure the density of a powder, a hand pump was used to raise the pressure of the air in a flask of volume $1.20 \times 10^{-4}\,m^3$ first without, and then with the powder in the flask.

1 Without the powder in the flask, the pressure increased from 110 kPa to 135 kPa.
2 With 0.038 kg of powder in the flask, the pressure increased from 110 kPa to 141 kPa.

a Show that the volume of air in the hand pump initially was $2.7 \times 10^{-5}\,m^3$.

b Calculate the volume and the density of the powder.

21.2 The ideal gas law

On these pages you will learn to:

- Know that the Avogadro constant N_A is the number of atoms in 0.012 kg of carbon-12
- Use molar quantities where one mole of any substance is the amount containing a number of particles equal to the Avogadro constant N_A
- Recall and solve problems using the ideal gas equation $pV = nRT$, where n = number of moles
- Recognise the experimental basis of the ideal gas equation
- Infer from a Brownian motion experiment the evidence for the movement of molecules

Molecules in a gas

The molecules of a gas move at random with different speeds. When a molecule collides with another molecule or with a solid surface, it bounces off without loss of speed. The pressure of a gas on a surface is due to the gas molecules hitting the surface. Each impact causes a tiny force on the surface. Because there are a very large number of impacts each second, the overall result is that the gas exerts a measurable pressure on the surface.

Molecules are too small to see individually. The effect of individual molecules in a gas can be seen if smoke particles are observed using a microscope. If a beam of light is directed through the smoke, the smoke particles are seen as tiny specks of light wriggling about unpredictably. This type of motion is called **Brownian motion** after Robert Brown who first observed it in 1827 with pollen grains in water. The motion of each particle is due to it being bombarded unevenly and at random by individual molecules. The particle is therefore subjected to a force due to the impacts which changes its magnitude and direction at random.

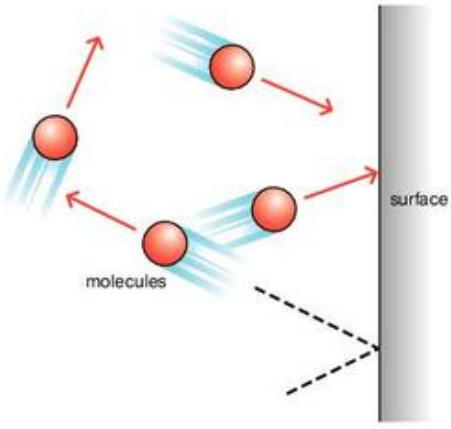


Fig 1 Molecules in motion

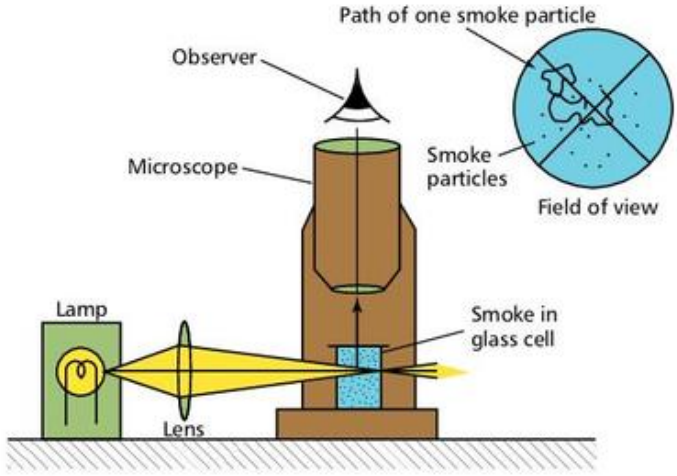


Fig 2 Brownian motion

The Avogadro number

The density of oxygen gas is 16 times that of hydrogen gas at the same temperature. Therefore, the mass of a certain volume of oxygen is 16 times that of the mass of the same volume of hydrogen at the same temperature. When such measurements were first made in the nineteenth century, Amadeo Avogadro put forward the hypothesis that equal volumes of gases at the same temperature and pressure contain an equal number of molecules.

How many molecules are in a certain amount of gas? Avogadro thought of the idea of counting atoms and molecules in terms of the number of atoms in 1 gram of hydrogen. Now we use 12 grams of the carbon isotope ${}^{12}_{6}C$ as the standard amount as hydrogen contains a small proportion of the isotope of hydrogen ${}^{2}_{1}H$ which cannot easily be removed.

The Avogadro constant, N_A, is defined as the number of atoms in exactly 12 grams of the carbon isotope ${}^{12}_{6}C$.

The value of N_A (to 4 significant figures) is 6.023×10^{23}. Therefore the mass of an atom of ${}^{12}_{6}C$ is 1.993×10^{-23} grams (= 12 grams/6.02×10^{23}).

One atomic mass unit (u) is $\frac{1}{12}$th of the mass of a $^{12}_{6}C$ atom.

As the mass of a carbon atom is 1.993×10^{-26} kg, then $1\,u = 1.661 \times 10^{-27}$ kg.

Notes

1 The masses in atomic mass units of some different atoms (to 1 u): hydrogen, H = 1 u, carbon, C = 12 u, nitrogen, N = 14 u, oxygen, O = 16 u, copper, Cu = 64 u

2 The masses in atomic mass units of some different molecules (to 1 u): water H_2O = 18 u, carbon monoxide CO = 28 u, carbon dioxide CO_2 = 44 u, oxygen O_2 = 32 u

Molar mass

- **One mole** of a substance consisting of identical particles is defined as the quantity of substance that contains N_A particles. The number of moles in a certain quantity of a substance is its **molarity**. The unit of molarity is the mol.
- **The molar mass** of a substance is the mass of 1 mole of the substance. The unit of molar mass is kg mol^{-1}. For example, the molar mass of oxygen gas is 0.032 kg mol^{-1}. So 0.032 kg of oxygen gas contains N_A oxygen molecules.

Therefore,

1 the number of moles in mass m of a substance $= \frac{m}{M}$, where M is the molar mass of the substance,

2 the number of molecules in mass m of a substance $= N_A\frac{m}{M}$.

For example, because the molar mass of carbon dioxide is 44 grams:

2 moles of carbon dioxide has a mass of 88 grams and contains $2N_A$ molecules,

10 moles of carbon dioxide has a mass of 440 grams and contains $10N_A$ molecules,

n moles of carbon dioxide has a mass of $44n$ grams and contains nN_A molecules.

The ideal gas equation

An ideal gas is a gas that obeys Boyle's law. The three experimental gas laws can be combined to give the equation

$$\frac{pV}{T} = \textbf{constant, for a fixed mass of ideal gas,}$$

where p is the pressure, V is the volume and T is the **absolute** temperature. This equation takes in all situations where the pressure, volume and temperature of a fixed mass of gas change.

As explained above, equal volumes of ideal gases at the same temperature and pressure contain equal numbers of moles. Further measurements show that one mole of any ideal gas at 273 K and a pressure of 101 kPa has a volume of 0.0224 m^3.

Therefore, for 1 mole of any ideal gas, the value of $\frac{pV}{T}$ for 1 mole is equal to $8.31\,J\,mol^{-1}\,K^{-1}\left(= \frac{pV}{T} = \frac{101 \times 10^3\,Pa \times 0.0224\,m^3}{273\,K}\right)$. This value is known as the **molar gas constant, R.**

Hence, the combined gas law may be written as

$$pV_m = RT$$

where V_m = volume of 1 mole of ideal gas at pressure p and temperature T.

NOTE

The unit of R is the joule per mol per kelvin (J mol^{-1} K^{-1}) which is the same as the unit of $\frac{\text{pressure} \times \text{volume}}{\text{temperature}}$.

This is because the unit of pressure (the pascal = 1 N m^{-2}) × the unit of volume (m^3) is the joule (= 1 N m).

Therefore, for n moles of ideal gas,

$$pV = nRT$$

where V = volume of the gas at pressure p and temperature T.

This equation is known as the **ideal gas equation**.

Notes

1 T is the temperature in kelvins.

2 The equation $pV = nRT$ can be written as $pV = NkT$ where N is the number of molecules in the gas and $k = \frac{R}{N_A}$.

k is referred to as the Boltzmann constant. See topic 21.4.

3 A graph of pV against temperature T is a straight line through absolute zero and has a gradient equal to nR.

4 The density of an ideal gas of molar mass M, $\rho = \frac{\text{mass}}{\text{volume}} = \frac{nM}{V} = \frac{pM}{RT}$.

Therefore, for an ideal gas at constant pressure, its density ρ is inversely proportional to its temperature T (as $\rho = \frac{pM}{RT} = \frac{\text{constant}}{T}$ for constant pressure).

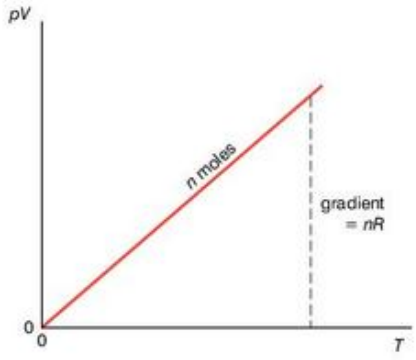


Fig 3 A graph of pV against T for an ideal gas

Worked example

$R = 8.31\,\text{J mol}^{-1}\,\text{K}^{-1}$

Calculate the number of moles of air in a balloon when the air pressure in the balloon is 170 kPa, the volume of the balloon is $8.4 \times 10^{-4}\,\text{m}^3$ and the temperature of the air in the balloon is 17 °C.

Solution

$T = 273 + 17 = 290\,\text{K}$

Using $pV = nRT$ gives $n = \frac{pV}{RT} = \frac{170 \times 10^3 \times 8.4 \times 10^{-4}}{8.31 \times 290} = 5.9 \times 10^{-2}$ moles

QUESTIONS

$N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$,
$R = 8.31\,\text{J mol}^{-1}\,\text{K}^{-1}$

1 A gas cylinder has a volume of $0.024\,\text{m}^3$ and is fitted with a valve designed to release the gas if the pressure of the gas reaches 125 kPa. Calculate:

a the maximum number of moles of gas that can be contained by this cylinder at 50 °C,

b the pressure in the cylinder of this amount of gas at 10 °C.

2 In an electrolysis experiment, $2.2 \times 10^{-5}\,\text{m}^3$ of a gas is collected at a pressure of 103 kPa and a temperature of 20 °C. Calculate:

a the number of moles of gas present,

b the volume of this gas at 0 °C and 101 kPa.

3 **a** Sketch a graph to show how the pressure of 2 moles of gas varies with temperature when the gas is heated from 20 °C to 100 °C in a sealed container of volume $0.050\,\text{m}^3$.

b The molar mass of the gas in **a** is 0.032 kg. Calculate the density of the gas.

4 The molar mass of air is 0.029 kg.

a Calculate the density of air at 20 °C and a pressure of 101 kPa.

b Calculate the number of molecules in $0.001\,\text{m}^3$ of air at 20 °C and a pressure of 101 kPa.

21.3 The kinetic theory of gases

The gas laws can be explained by assuming a gas consists of point molecules moving about at random, continually colliding with the container walls. Each impact causes a force on the container. The force of many impacts is the cause of the pressure of the gas on the container walls.

Explanation of Boyle's law: the pressure of a gas at constant temperature is increased by reducing its volume because the gas molecules travel less distance between impacts at the walls due to the reduced volume. Hence there are more impacts per second so the pressure is greater.

Explanation of the pressure law: the pressure of a gas at constant volume is increased by raising its temperature. The average speed of the molecules is increased by raising the gas temperature so the impacts of the molecules on the container walls are harder and more frequent. Hence the pressure is raised as a result.

On these pages you will learn to:

- Explain how molecular movement causes the pressure exerted by a gas
- State the basic assumptions of the kinetic theory of gases and derive the relationship $pV = \frac{1}{3}Nm\langle c^2\rangle$ where N = number of molecules
- Solve problems using the above equation and the equation $p = \frac{1}{3}\rho c^2_{rms}$, where ρ is the density of the gas

Molecular speeds

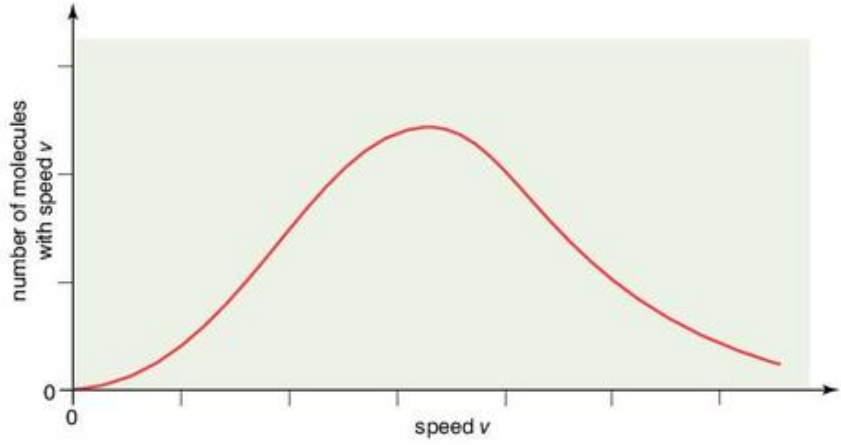


Fig 1 Distribution of molecular speeds

The molecules in an ideal gas have a continuous spread of speeds, as shown in Figure 1. The speed of an individual molecule changes when it collides with another gas molecule but the distribution stays the same, provided the temperature does not change.

The **root mean square speed** of the molecules, $c_{rms} = \frac{[(c_1^2 + c_2^2 + \ldots + c_N^2)]^{1/2}}{N}$

where c_1, c_2, c_3,... c_N represent the speeds of the individual molecules and N is the number of molecules in the gas.

Notes

1 The root mean square (rms) speed of the molecules of a gas is not the same as the mean speed which is the sum of the speeds divided by the number of molecules.
2 The symbols $\langle c^2\rangle$ and $\overline{c^2}$ are sometimes used for the mean square speed, c^2_{rms}.

If the temperature of a gas is raised, its molecules move faster on average. The rms speed of the molecules increases. The distribution curve becomes flatter and broader as there are molecules at higher speeds. See Figure 2.

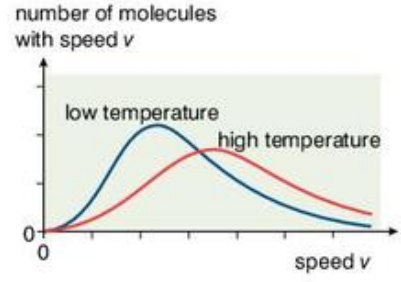


Fig 2 The effect of temperature on the distribution of speeds

The kinetic theory equation

For an ideal gas consisting of N identical molecules, each of mass m, in a container of volume V, the pressure p of the gas is given by

$$pV = \frac{1}{3}Nmc^2_{rms}$$

where c_{rms} is the root mean square speed of the gas molecules.

We need to apply the laws of mechanics and statistics to the molecular model of a gas to derive the kinetic theory equation. In doing so, certain assumptions must be made about the molecules in a gas:

1 The molecules are point molecules. The volume of each molecule is negligible compared with the volume of the gas.
2 They do not attract each other. If they did, the effect would be to reduce the force of their impacts on the container surface.
3 They move about in continual random motion.
4 The collisions they undergo with each other and with the container surface are elastic collisions (i.e. there is no overall loss of kinetic energy in a collision).
5 Each collision with the container surface is of much shorter duration than the time between impacts.

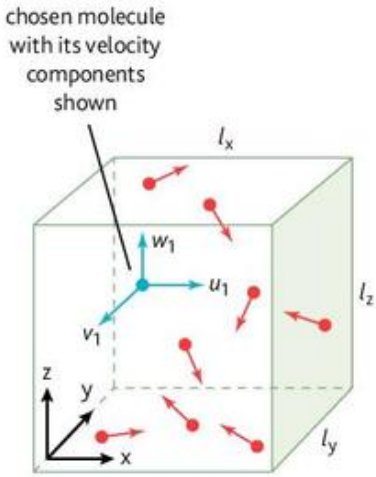


Fig 3 *Molecules in a box*

Part 1

Consider one molecule of mass m in a rectangular box of dimensions l_x, l_y and l_z as shown in Figure 3. Let u_1, v_1 and w_1 represent its velocity components in the x, y, and z directions respectively.

Note that the speed, c_1, of the molecule is given by the following rule for adding perpendicular components (in this case the three velocity components u_1, v_1 and w_1),

$$c_1^2 = u_1^2 + v_1^2 + w_1^2,$$

We will need to use this rule in Part 2.

Each impact of the molecule with the shaded face in Figure 3 reverses the x-component of velocity thus changing the x-component of its momentum from $+mu_1$ to $-mu_1$.

Therefore, the change of its momentum due to the impact = final momentum − initial momentum

$$= (-mu_1) - (mu_1) = -2mu_1$$

The time, t, between successive impacts on this face is given by

$$t = \frac{\text{the total distance to the opposite face and back}}{x\text{-component of velocity}} = \frac{2l_x}{u_1}$$

Using Newton's second law therefore gives:

$$\text{the force on the molecule} = \frac{\text{change of momentum}}{\text{time taken}} = \frac{-2mu_1}{(2l_x/u_1)} = \frac{-mu_1^2}{l_x}$$

Since the force F_1 of the impact on the surface is equal and opposite to the force on the molecule in accordance with Newton's third law, then $F_1 = \frac{+mu_1^2}{l_x}$

As pressure = force/area, the pressure p_1 of the molecule on the surface is given by

$$p_1 = \frac{\text{force}}{\text{area of the shaded face } (l_y \times l_z)} = \frac{mu_1^2}{l_x \times l_y \times l_z} = \frac{mu_1^2}{V}$$

where V = the volume of the box = $l_x \times l_y \times l_z$

Part 2

For N molecules in the box moving at different velocities, the total pressure p is the sum of the individual pressures $p_1, p_2, p_3, \ldots p_N$ where each subscript refers to each molecule.

$$\text{Hence } p = \frac{mu_1^2}{V} + \frac{mu_2^2}{V} + \frac{mu_3^2}{V} + \ldots + \frac{mu_N^2}{V} = \frac{m}{V}(u_1^2 + u_2^2 + u_3^2 + \ldots + u_N^2)$$

$$= \frac{Nm\overline{u^2}}{V}$$

where $\overline{u^2} = (u_1^2 + u_2^2 + u_3^2 + \ldots + u_N^2)/N$

As the motion of the molecules is random, there is no preferred direction of motion. The equation above could equally well have been derived in terms of the y-components of velocity $v_1, v_2, v_3 \ldots v_N$ or the z-components of velocity $w_1, w_2, w_3 \ldots w_N$.

i.e. $$p = \frac{Nm\overline{v^2}}{V} \quad \text{where } \overline{v^2} = (v_1^2 + v_2^2 + v_3^2 + \ldots + v_N^2)/N$$

$$p = \frac{Nm\overline{w^2}}{V} \quad \text{where } \overline{w^2} = (w_1^2 + w_2^2 + w_3^2 + \ldots + w_N^2)/N$$

Therefore

$$p = \frac{Nm}{3V}(\overline{u^2} + \overline{v^2} + \overline{w^2})$$

The note below shows that, because the motion of the molecules is random, **the root mean square speed** of the gas molecules is given by the equation:

$$c_{rms}^2 = \overline{u^2} + \overline{v^2} + \overline{w^2}$$

Hence $$p = \frac{Nmc_{rms}^2}{3V} \quad \text{or} \quad pV = \frac{1}{3}Nmc_{rms}^2$$

Note

As explained in Part 1, the speed c of each molecule is related to its velocity components according to equations of the form:

$$c_1^2 = u_1^2 + v_1^2 + w_1^2, \quad c_2^2 = u_2^2 + v_2^2 + w_2^2, \quad c_3^2 = u_3^2 + v_3^2 + w_3^2, \ldots$$
$$\ldots c_N^2 = u_N^2 + v_N^2 + w_N^2$$

The root mean square speed of the molecules, c_{rms} is defined by:

$$c_{rms}^2 = (c_1^2 + c_2^2 + c_3^2 + \ldots + c_N^2)/N$$

$$= (u_1^2 + v_1^2 + w_1^2 + u_2^2 + v_2^2 + w_2^2 + u_3^2 + v_3^2 + w_3^2 + \ldots + u_N^2 + v_N^2 + w_N^2)/N$$

$$= \overline{u^2} + \overline{v^2} + \overline{w^2}$$

Pressure and density

In the kinetic theory equation $pV = \frac{1}{3}Nmc_{rms}^2$, Nm is the total mass of gas present because N is the number of molecules and m is the mass of each molecule.

Therefore the density of the gas, $\rho = \dfrac{\text{mass}}{\text{volume}} = \dfrac{Nm}{V}$

Rearranging the kinetic theory equation therefore gives $p = \frac{1}{3} \times \frac{Nm}{V} \times c_{rms}^2$

$$= \frac{1}{3}\rho c_{rms}^2$$

$$\boldsymbol{p = \frac{1}{3}\rho c_{rms}^2}$$

QUESTIONS

$N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$,
$R = 8.31\,\text{J mol}^{-1}\,\text{K}^{-1}$

1 Explain in molecular terms why the pressure of a gas in a sealed container increases when its temperature is raised.

2 Calculate the rms speed of the molecules of a gas when its pressure is 1.10×10^5 Pa and its density is $1.20\,\text{kg m}^{-3}$.

3 A sealed flask of volume $1.50 \times 10^{-4}\,\text{m}^3$ contains oxygen gas at a pressure of 1.2×10^5 Pa at a temperature of 15 °C. Calculate:

a the number of moles of oxygen in the flask,

b the rms speed of the oxygen molecules.

molar mass of oxygen = 0.032 kg

4 An ideal gas of molar mass 0.028 kg is in a container of volume $0.037\,\text{m}^3$ at a pressure of 100 kPa and a temperature of 300 K. Calculate

a the number of moles,

b the mass of gas present,

c the rms speed of the molecules of the gas.

21.4 Thermodynamics of ideal gases

On these pages you will learn to:

- Recall that the Boltzmann constant $k = R/N_A$ and express the ideal gas equation as $pV = NkT$
- Deduce that the average translational kinetic energy of a molecule is proportional to T using the equations $pV = \frac{1}{3}Nmc^2$ and $pV = NkT$
- Use the first law of thermodynamics, expressed in terms of the increase in internal energy, the heating of the system and the work done on the system ($\Delta V = Q + W$), to solve problems

Molecules and kinetic energy

The mean **kinetic energy of a molecule of an ideal gas**

$$= \frac{\text{total kinetic energy of all the molecules}}{\text{total number of molecules}}$$

$$= (\tfrac{1}{2}mc_1^2 + \tfrac{1}{2}mc_2^2 + \tfrac{1}{2}mc_3^2 + \ldots + \tfrac{1}{2}mc_N^2)/N$$

$$= \tfrac{1}{2}m(c_1^2 + c_2^2 + c_3^2 + \ldots + c_N^2)/N = \tfrac{1}{2}mc_{rms}^2$$

The higher the temperature of a gas, the greater is the mean kinetic energy of a molecule of the gas.

For an ideal gas, by assuming the mean kinetic energy of a molecule $\frac{1}{2}mc_{rms}^2 = \frac{3}{2}kT$, where $k = R/N_A$, then $3kT = mc_{rms}^2$.

Substituting $3kT$ for mc_{rms}^2 in the kinetic theory equation $pV = \frac{1}{3}Nmc_{rms}^2$ therefore gives $pV = \frac{1}{3}N \times 3kT = NkT$

As $Nk = NR/N_A = nR$, we then obtain the ideal gas equation $pV = nRT$.

We have derived the ideal gas equation (which is an experimental law) from the kinetic theory equation by assuming that the mean kinetic energy of an ideal gas molecule $= \frac{3}{2}kT$. Therefore we can say for an ideal gas at absolute temperature T, **the mean kinetic energy of a molecule of the gas $= \frac{3}{2}kT$, where $k = R/N_A$.**

The constant k is referred to as the Boltzmann constant.

Its value ($= R/N_A$) is $1.38 \times 10^{-23}\,\text{J K}^{-1}$

Note

- The total kinetic energy of 1 mole of an ideal gas $= N_A \times \frac{3}{2}kT = \frac{3}{2}RT$ (as $k = R/N_A$),
- The total kinetic energy of n moles of an ideal gas $= n \times \frac{3}{2}RT = \frac{3}{2}nRT$

NOTE

The idea that the mean kinetic energy of a particle is proportional to absolute temperature can be applied in many other situations, for example to explain why conduction electrons can't escape from a metal at room temperature. See topic 22.6.

The total kinetic energy of n moles of an ideal gas $= \frac{3}{2}nRT$

Worked example

$N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$, $k = 1.38 \times 10^{-23}\,\text{J K}^{-1}$.

Calculate the rms speed of oxygen molecules at 0 °C.

The molar mass of oxygen $= 0.032\,\text{kg mol}^{-1}$

Solution $T = 273\,\text{K}$

The mass of an oxygen molecule, $m = \dfrac{0.032}{6.02 \times 10^{23}}$

$$= 5.3 \times 10^{-26}\,\text{kg}$$

Rearranging $\frac{1}{2}mc_{rms}^2 = \frac{3}{2}kT$ gives

$$\therefore \quad c_{rms}^2 = \frac{3kT}{m} = \frac{3 \times 1.38 \times 10^{-23} \times 273}{5.3 \times 10^{-26}}$$

$$= 2.13 \times 10^5\,\text{m}^2\,\text{s}^{-2}$$

$\therefore$ root mean square speed, $c_{rms} = (2.13 \times 10^5)^{1/2} = 460\,\text{m s}^{-1}$

Using the first law of thermodynamics

As explained in topic 20.1, the first law of thermodynamics tells us that the change of internal energy of a system is equal to the sum of the work done on or by it and the heat transfer to or from it.

The internal energy U of an ideal gas is due to the kinetic energy of its molecules. No internal forces of attraction exist between 'ideal gas' molecules so an ideal gas does not possess molecular potential energy. As explained above, for n moles of an ideal gas at temperature T, the total kinetic energy $= \frac{3}{2}nRT$. Therefore, its internal energy $= \frac{3}{2}nRT$.

To change the temperature of an ideal gas, its internal energy must be changed. To do this, work must be done on or by the gas or heat must be transferred to or from it.

Work W is done on a gas when its volume is reduced (i.e. it is compressed). At constant volume, no work is done because the forces due to pressure in the gas do not move the container walls.

When a gas is compressed, work is done on it by the applied forces which push the pressure forces back.

When a gas expands, the forces due to its pressure push back the container walls. So the gas does work when it expands. As explained in topic 5.2, when a gas at constant pressure p expands, the work done W by the gas is given by the equation $W = p\Delta V$, where ΔV is the change of volume.

- **If the pressure of the gas remains constant**, the work done can be calculated directly using the above equation. Also, since the volume V is related to the temperature T according to the ideal gas equation $pV = nRT$ where n is the number of moles, then $p\Delta V = nR\Delta T$.
  Therefore, the work done $= p\Delta V = nR\Delta T$.

- **If the pressure changes**, the work done can be estimated from a graph of pressure against volume as shown in Figure 1. The work done is given by the area under the curve. This is because the work done in each small change of volume ΔV is represented by the area of the corresponding strip under the curve; so the total work done when the volume changes from V_1 to V_2 is represented by the total area under the curve from V_1 to V_2.

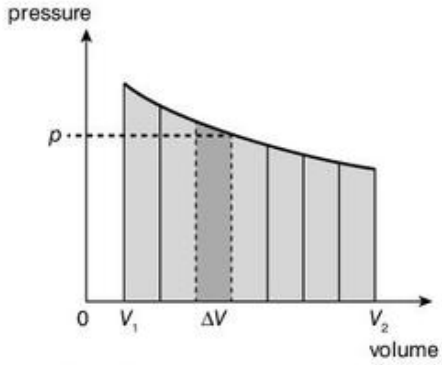


Fig 1 Estimating work done

Heat transfer Q to or from a gas may or may not cause a change of its temperature, depending on whether or not work is done on or by the gas. For example, if 1000 J of work is done on a gas by compressing it and 1200 J of heat energy is transferred from it by cooling it at the same time, its internal energy will decrease by 200 J and its temperature will decrease.

QUESTIONS

$N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$, $R = 8.31\,\text{J K}^{-1}$, $k = 1.38 \times 10^{-23}\,\text{J K}^{-1}$.

1 The molar mass of oxygen is 0.032 kg. A cylinder of volume 0.025 m^3 contains oxygen gas at a pressure of 120 kPa and a temperature of 373 K. Calculate **a** the number of moles of oxygen in the cylinder, **b** the mean kinetic energy of an oxygen molecule in the cylinder, **c** the total kinetic energy of all the gas molecules in the container.

2 Calculate **a** the mean kinetic energy of a hydrogen molecule at 0 °C, **b** the root mean square speed of a hydrogen molecule at 0 °C. The molar mass of hydrogen gas = 0.002 kg.

3 Air consists mostly of nitrogen and oxygen in proportions 1 : 4 by mass.

a Explain why the mean kinetic energy of a nitrogen molecule in air is the same as that of an oxygen molecule in the same sample of air.

b Show that the rms speed of a nitrogen molecule in air is 1.07 × that of an oxygen molecule in the same sample of air.

molar mass nitrogen = 0.028 kg; oxygen = 0.032 kg

4 An ideal gas in a cylinder at an initial temperature of 290 K is compressed and cooled from a volume of 0.025 m^3 to a volume of 0.019 m^3 at a constant pressure of 1.45×10^5 Pa. Calculate:

a **i** the number of moles of the gas,
ii the temperature of the gas after the change,

b the work done on the gas,

c the decrease of internal energy of the gas,

d the heat transferred from the gas.

CHAPTER SUMMARY

The ideal gas law $pV = nRT$, where n is the number of moles of gas, T is the absolute temperature and R is the molar gas constant.

The Avogadro constant, N_A is defined as the number of atoms in 12 grams of the carbon isotope $^{12}_{6}C$

One mole of a substance consisting of identical particles is the quantity of substance that contains N_A particles of the substance.

The molar mass of a substance is the mass of one mole of that substance.
The kinetic theory of gases equation $pV = \frac{1}{3}Nmc_{rms}^2$, where c_{rms} is the root mean square speed of the gas molecules

The mean kinetic energy of a molecule in a gas at absolute temperature $T = \frac{3}{2}kT$, where k is the Boltzmann constant ($= R/N_A$)

EXTENSION

Molar heat capacities

The molar heat capacity at constant volume C_v is defined as the heat transfer required to raise the temperature of 1 mole by 1 K at constant volume.

If 1 mole of gas is heated at constant volume so its temperature rises by ΔT, the heat transfer must equal $C_v\Delta T$. Since $\Delta V = 0$, the work done is zero. Therefore, the rise of internal energy = $C_v\Delta T$.

The molar heat capacity at constant pressure C_p is defined as the heat transfer required to raise the temperature of 1 mole by 1 K at constant pressure.

If 1 mole of gas is heated at constant pressure p so its temperature rises by ΔT and its volume increases by ΔV, the heat transfer must equal $C_p\Delta T$. This is greater than the heat transfer at constant volume as work is done on the gas and its internal energy is increased.

Therefore, as explained above, if 1 mole of gas is heated at constant pressure:

- heat transfer = $C_p\Delta T$
- its rise of internal energy = $C_v\Delta T$
- work done = $R\Delta T$.

Since the heat transfer at constant pressure = the rise of internal energy + the work done, then $C_p\Delta T = C_v\Delta T + R\Delta T$.

Hence $C_p = C_v + R$ or $\boldsymbol{C_p - C_v = R}$.

21 End of chapter questions

$N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$, $R = 8.3\,\text{J mol}^{-1}\text{K}^{-1}$

1 a In molecular terms, explain why the pressure of a gas increases:
- **i** if the temperature of the gas is raised at constant volume,
- **ii** if the amount of gas is increased at constant volume.

b Sketch a graph to show how the pressure of a gas in a sealed cylinder increases with the absolute temperature of the gas if the cylinder contains:
- **i** 1 mole of gas,
- **ii** 2 moles of gas.

2 a Assuming that air at atmospheric pressure consists of 80% nitrogen and 20% oxygen, show that the molar mass of air is 0.029 kg.

molar mass of nitrogen = 0.028 kg,
molar mass of oxygen = 0.032 kg.

b A rectangular room has a length of 5.0 m, a width of 4.0 m and a height of 2.5 m. The air in the room has a pressure of 100 kPa and a temperature of 290 K. Calculate:
- **i** the number of moles of air in the room,
- **ii** the number of gas molecules in the room.

3 A vehicle air bag inflates rapidly when an impact causes the production and release of a large quantity of nitrogen in a chemical reaction. In a test of an air bag, the bag inflates to a volume of 1.2 m^3 and a pressure of 103 kPa at a final temperature of 280 K. Calculate:

a the number of moles of gas in the bag,

b the initial pressure of the gas if it was released from a container of volume $5.6 \times 10^{-4}\,m^3$ at the same temperature.

4 A hot air balloon rises from the ground because it is partly filled with helium which is less dense than air.

a A certain hot air balloon contains 370 m^3 of helium gas at a temperature of 300 K. Calculate:
- **i** the number of moles of helium gas in the balloon when the pressure of the gas is 110 kPa,
- **ii** the density of the gas.

b The helium gas in the balloon fills most of the balloon from the top down. Heating the gas causes it to expand and displace some of the air in the lower part of the balloon.
- **i** Explain why heating the gas causes its density to decrease.
- **ii** Explain why the balloon ascends when the helium gas is heated.

The molar mass of helium = 4.0×10^{-3} kg

5 A hollow steel cylinder fitted with a pressure gauge is used to store nitrogen gas. The cylinder has an internal volume of $5.0 \times 10^{-3}\,m^3$.

The molar mass of nitrogen = 0.028 kg

a i Calculate the mass of nitrogen gas that must be stored in the cylinder at 300 K to give a gas pressure of 150 kPa.
- **ii** Describe how the mass of gas remaining in the cylinder could be estimated from the pressure reading of the pressure gauge.

b The cylinder is fitted with a valve that releases gas from the cylinder if the gas pressure exceeds 160 kPa.

Calculate the maximum mass of nitrogen that could be stored in the cylinder at 350 K.

6 Argon is an inert gas. The gas is composed of single atoms that do not combine with each other. A certain light bulb contains $8.0 \times 10^{-5}\,m^3$ of argon gas. When the light bulb was at a temperature of 300 K, the pressure of the gas in the light bulb was 15 kPa.

a i Calculate the number of moles of argon gas in the light bulb.
- **ii** Calculate the number of argon atoms in the light bulb.
- **iii** The mean kinetic energy of an argon atom at 300 K is 6.2×10^{-21} J. Calculate the speed of an argon atom which has 6.2×10^{-21} J of kinetic energy.

b When the light bulb was switched on, the temperature of the gas in the bulb increased to 350 K. Calculate:
- **i** the pressure in the light bulb at this temperature,
- **ii** the increase of kinetic energy of the gas molecules in the light bulb.

The molar mass of argon = 0.040 kg.

7 Helium gas released with oil from an oil well was collected and stored in a sealed underground cavern of volume 26000 m^3 at a temperature of 280 K. As a result of storing the helium gas, the pressure of the gas in the cavern increased from 100 kPa to 125 kPa.

a Calculate:
- **i** the mass of helium gas stored in the cavern,
- **ii** the mass of a helium molecule.

b The mean kinetic energy of a helium gas molecule at 280 K is 5.8×10^{-21} J. Calculate:
- **i** the speed of a helium gas molecule which has 5.8×10^{-21} J of kinetic energy,
- **ii** the kinetic energy of all the helium gas molecules in the cavern at 280 K.

The molar mass of helium = 0.004 kg

8 In a vehicle braking system, pressure applied by the driver to the footbrake is transmitted from the master cylinder to the brake pads via sealed pipes filled with brake fluid.

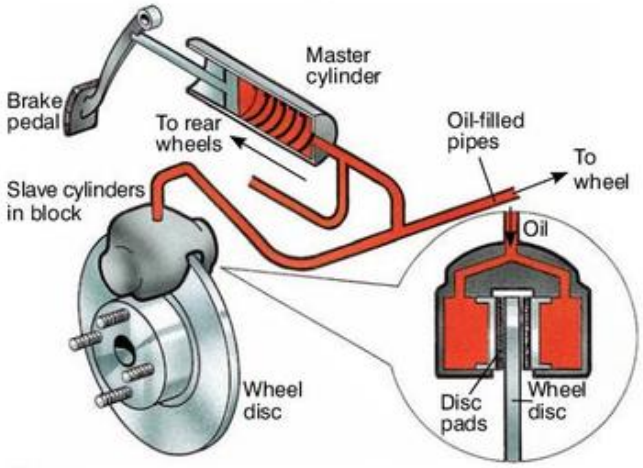


Fig 8.1

a Explain why a bubble of air in the brake fluid makes the brakes less effective than they should be.

b A vehicle brake system is filled with a volume of $1.2 \times 10^{-4}\,m^3$ of brake fluid. When a force of 20 N is applied to the piston in the master cylinder, the pressure in the brake system increases from 105 kPa to 155 kPa.

i Calculate the area of cross-section of the master cylinder.

ii Air leaks into the system when the brake fluid is replaced. When a force of 20 N is applied to the piston in the master cylinder, it travels 5 mm more than in **a** and the pressure in the system increases from 105 kPa to 145 kPa. Show that the volume of the air bubble before the brakes were applied was $7.3 \times 10^{-6}\,m^3$.

iii The temperature of the brake fluid was 290 K. Calculate the mass of air in the bubble.

The molar mass of air = 0.029 kg.

9 a Some gas, initially at a temperature of 27.2 °C, is heated so that its temperature rises to 38.8 °C. Calculate, in kelvin, to an appropriate number of decimal places,

i the initial temperature of the gas, *(2 marks)*

ii the rise in temperature. *(1 mark)*

b The pressure p of an ideal gas is given by the expression

$$p = \frac{1}{3}\rho\langle c^2\rangle$$

where ρ is the density of the gas.

i State the meaning of the symbol $\langle c^2\rangle$. *(1 mark)*

ii Use the expression to show that the mean kinetic energy $\langle E_K\rangle$ of the atoms of an ideal gas is given by the expression

$$\langle E_K\rangle = \frac{3}{2}kT$$

Explain any symbols that you use. *(4 marks)*

c Helium-4 may be assumed to behave as an ideal gas.

A cylinder has a constant volume of $7.8 \times 10^3\,cm^3$ and contains helium-4 gas at a pressure of 2.1×10^7 Pa and at a temperature of 290 K.

Calculate, for the helium gas,

i the amount of gas, in mol *(2 marks)*

ii the mean kinetic energy of the atoms, in J *(2 marks)*

iii the total internal energy, in J. *(3 marks)*

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10 a i State the basic assumption of the kinetic theory of gases that leads to the conclusion that the potential energy between the atoms of an ideal gas is zero. *(1 mark)*

ii State what is meant by the *internal energy* of a substance. *(2 marks)*

iii Explain why an increase in internal energy of an ideal gas is directly related to a rise in temperature of the gas. *(2 marks)*

b A fixed mass of an ideal gas undergoes a cycle PQRP of changes as shown in Figure 10.1.

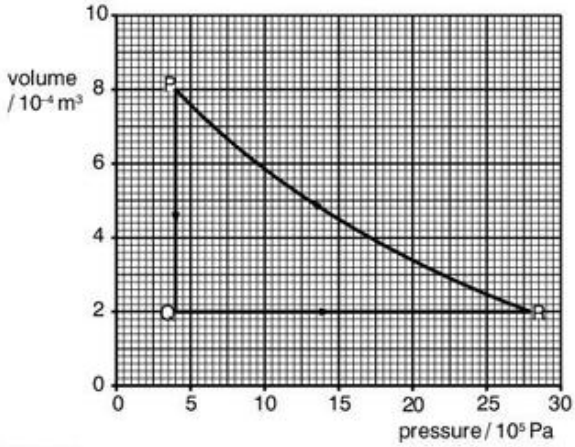


Fig 10.1

i State the change in internal energy, in J, of the gas during one complete cycle PQRP. *(1 mark)*

ii Calculate the work done, in J, on the gas during the change from P to Q. *(2 marks)*

iii Some energy changes during the cycle PQRP are shown in Figure 10.2.

change	work done on gas / J	heating supplied to gas / J	increase in internal energy / J
P → Q	.....	−600	.....
Q → R	0	+720	.....
R → P	.....	+480	.....

Fig 10.2

Copy and complete Figure 10.2 to show all of the energy changes. *(3 marks)*

Adapted from Cambridge International AS and A Level Physics 9702
Paper 41 Q2 November 2010

22 Electrons

22.1 The discovery of the electron

On these pages you will learn to:

- Explain what is meant by thermionic emission
- Recognise how thermionic emission is used to produce an electron beam
- Explain why an electron beam is deflected in a uniform electric field and a uniform magnetic field
- Calculate the force on an electron in an electric field and in a magnetic field

Cathode rays

Discharge tubes used in colourful advertising displays were first developed by William Crookes in the 1870s. He discovered that gases at sufficiently low pressure in such tubes conduct electricity and emit light of a characteristic colour. For example, neon gas in such a tube emits red light. Investigations into the cause of the glow showed that:

- The glowing gas near the anode, the positive column, was easily distorted when a magnet was brought near the tube. This observation showed that charged particles move through the gas when it conducts electricity.
- Radiation from the direction of the cathode forced a 'paddle wheel' placed in the tube to rotate. The radiation was referred to as 'cathode rays'. Placing a magnet near the tube stopped the paddle wheel rotating by deflecting the cathode rays away from it.

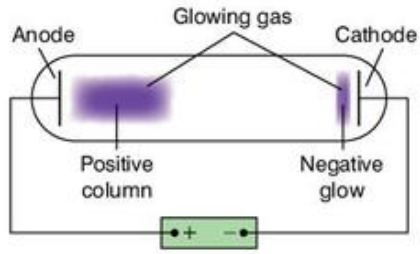


Fig 1 A discharge tube

Further tests showed the cathode rays to be negatively charged particles. Exactly what cathode rays are puzzled scientists for over 20 years until Joseph J. Thomson carried out a series of experiments and proved that the negative particles are the same, regardless of which gas is used in the tube. The cathode ray particles were referred to as **electrons**.

The principle of thermionic emission

Thermionic emission is a much simpler way of producing an electron beam than using a discharge tube. When a metal is heated, some of the electrons that move about freely inside the metal (referred to as 'free' or 'conduction' electrons) gain sufficient kinetic energy to leave the metal at its surface. In practice, the metal is a wire filament which is heated by passing an electric current through it. The filament or 'cathode' is at one end of an evacuated glass tube which has a metal plate or 'anode' at the other end, as shown in Figure 2.

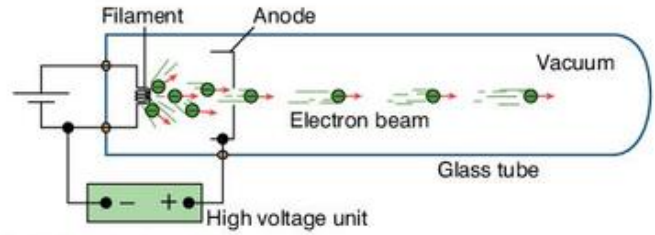


Fig 2 Thermionic emission

The electrons emitted from the filament are attracted to the anode by connecting a high voltage power supply between the anode and the cathode, with the anode positive relative to the filament. Because there are no gas molecules in the tube to stop the electrons, the electrons are accelerated to the anode where they pass through a small hole to form a narrow beam.

The speed of each electron passing through the hole can be worked out by equating:

- The work done by the p.d. V between the anode and the cathode on each electron $= eV$, where e is the charge of the electron.
- The kinetic energy of each electron passing through the hole $= \frac{1}{2}mv^2$, where v is the speed of each electron at this position.
- Since the work done on each electron increases its kinetic energy from a negligible value at the cathode, then the speed, v, of each electron on leaving the anode is given by

$$\frac{1}{2}mv^2 = eV$$

Notes

1 The speed of each electron in the beam from the anode is given by the above equation. This is because the electric field between the anode and the cathode does not act on the electrons once they have passed through the hole in the anode. So their kinetic energy does not change after leaving the anode.

2 The above equation assumes that:
- each electron starts from the filament with negligible kinetic energy in comparison with the work done on it by the accelerating p.d. V,

- the speed of the electrons in the beam, v, is much less than the speed of light in free space, c, so the non-relativistic formula for kinetic energy ($\frac{1}{2}mv^2$) applies.

Deflection of an electron beam

An electron beam can be deflected by an electric field or a magnetic field. By adjusting the strength of the field in each case, the extent of the deflection can be controlled.

Using a uniform electric field

Figure 3 shows a glass vacuum tube in which an electron beam is produced so that it can be deflected by an electric field or a magnetic field or both.

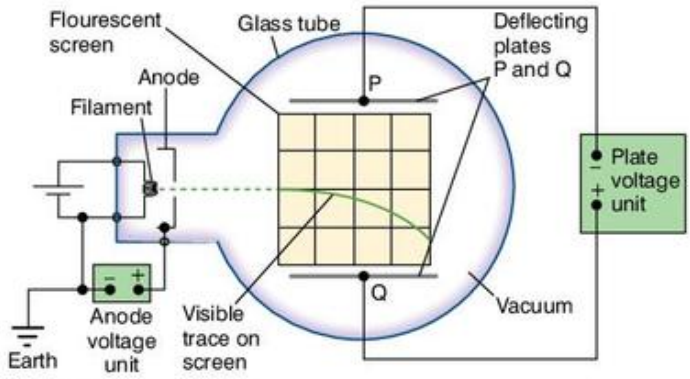


Fig 3 Deflecting electrons

The electric field used to deflect the beam is produced in Figure 3 by applying a constant p.d. between the metal deflecting plates P and Q. The beam enters the field at right angles to the field lines and is deflected towards the positive plate.

The force on each electron in the field is constant in magnitude and direction because the field is uniform so the beam curves in a parabolic path (just as a projectile projected horizontally does). The screen in the tube enables the path of the beam to be observed and measured.

If the p.d. between plates P and Q is V_P, the force F on each electron is given by the equation

$$F = eE = \frac{eV_P}{d}$$

where d is the perpendicular distance between plates P and Q, and $E(= V_P/d)$ is the electric field strength between the plates.

Notes

1 The acceleration of each electron towards the positive plate, $a = F/m = eV_P/md$, where m is the mass of the electron.

2 The time taken, t, by each electron to cross the field = L/v, where L is the length of each plate and v is the initial speed of the electron on entry to the field. This is the horizontal component of the electron's velocity in the field as it has no horizontal acceleration.

3 The deflection, y, of the electron on leaving the field is given by the equation $y = \frac{1}{2}at^2$. Using the above equations, it can be shown that y is directly proportional to the plate p.d. V_P.

Using a uniform magnetic field

A uniform magnetic field can be applied to the electron beam by placing a pair of large circular coils in series coaxially either side of the tube such that the plane of each coil is parallel to the screen, as shown in Figure 4. The magnetic flux density can be varied by varying the current in the coils.

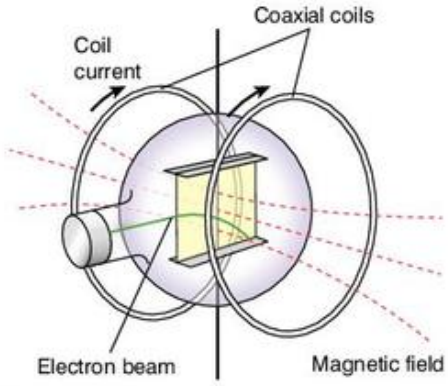


Fig 4 Using coaxial coils

Each electron in the beam experiences a force due to the magnetic field at right angles to the direction of motion of the electrons and to the direction of the magnetic field. As explained in topic 18.5:

- the force F on each electron in the field = Bev, where B = the magnetic flux density of the magnetic field and v = the speed of the electron
- the electron moves on a circular path of radius $r = \frac{mv}{Be}$.

QUESTIONS

$e = 1.60 \times 10^{-19}$C,
rest mass of an electron = 9.11×10^{-31}kg.

1 Calculate the speed of an electron after it has been accelerated in a vacuum tube from rest through a p.d. of 4000 V. Give your answer to 3 significant figures.

2 **a** In Figure 3, if the top plate is at a negative potential relative to the lower plate, state the direction of **i** the electric field between the plates, **ii** the force on an electron in the field, **iii** the acceleration of an electron in the field.

b An electron beam is directed horizontally into a uniform electric field which acts vertically downwards. Explain why the electron beam curves upwards with increasing speed.

3 An electron beam is directed horizontally into a uniform magnetic field that is perpendicular to the initial direction of the beam. Explain why the beam curves round in a vertical circle at constant speed.

4 A beam of electrons moving at a speed of 2.85×10^7m s^{-1} passes through perpendicular electric and magnetic fields without being deflected. The magnetic field has a flux density 3.20×10^{-3}T. Calculate the strength of the electric field.

22.2 The measurement of the specific charge of the electron, e/m

On these pages you will learn to:

- Explain the use of perpendicular electric and magnetic fields to determine the speed of the electrons in an electron beam
- Explain the main principles of one method for the determination of the specific charge e/m_e for electrons
- Solve problems on the deflection of electrons moving in uniform electric and magnetic fields

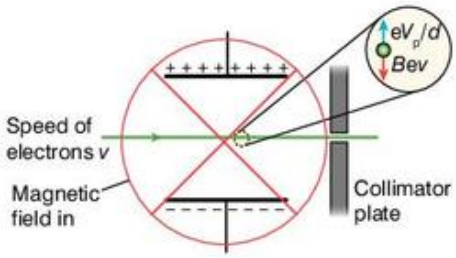


Fig 1 Balanced fields

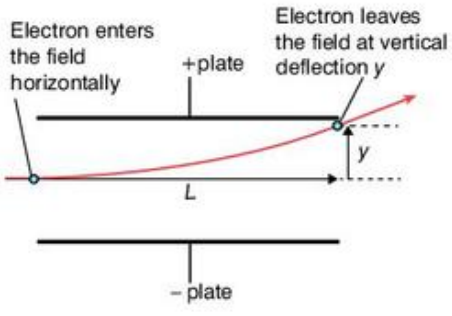


Fig 2 Deflection by a uniform electric field

Balanced fields

After the discovery of cathode rays, J. J. Thomson in 1895 measured the specific charge, e/m, of the electron (i.e. its charge e divided by its mass m, sometimes referred to as its charge/mass ratio). Using a deflection tube, he measured the speed of the electrons by passing them through balanced electric and magnetic fields, adjusted so the electron beam was undeflected, as shown in Figure 1.

The electric force on each electron is therefore equal and opposite to the magnetic force on it. The two fields may be said to be 'balanced' because they produce zero resultant force on each electron. See topic 18.5.

- The electric force $= eV_P/d$, where V_P is the p.d. between the plates and d is the plate spacing.
- The magnetic force $= Bev$, where B is the magnetic flux density and v is the speed of the electrons.

Therefore $Bev = eV_P/d$

Rearranging this equation gives $v = \dfrac{V_P}{Bd}$

Hence the speed of the electrons could then be calculated by measuring the deflecting p.d. V_P, the plate spacing d and the magnetic flux density B and using the equation $v = V_P/Bd$.

With one of the fields switched off, the deflection of the beam due to the field still on was then measured and used with the known value of the initial speed to determine e/m.

Using the electric field only, as explained in topic 22.1, with electrons of known speed v:

- the time taken, t, for the electrons to travel the length L of the deflecting plates $= L/v$
- the acceleration, a, towards the positive plate $= 2y/t^2$ (from $y = \frac{1}{2}at^2$).

Since the acceleration $a = eV_P/md$, rearranging this equation gives $e/m = ad/V_P$.

Worked example

Electrons moving at a constant speed are directed horizontally into a uniform electric field due to two parallel plates of length 60.0 mm and spaced 50.0 mm apart, which have a p.d. of 3000 V between them. The electrons are deflected by 22.0 mm as a result. When a uniform magnetic field of flux density 2.05 mT is applied at right angles to the beam and to the electric field, the beam is undeflected. Calculate the specific charge e/m of the electron.

Solution

$$\text{The speed of the beam, } v = \frac{V_P}{Bd} = \frac{3000}{2.05 \times 10^{-3} \times 50 \times 10^{-3}} = 2.93 \times 10^7 \text{ m s}^{-1}$$

$$\text{time taken, } t = \frac{L}{v} = \frac{60.0 \times 10^{-3}}{2.93 \times 10^7} = 2.05 \times 10^{-9} \text{ s}$$

$$\text{acceleration, } a = \frac{2y}{t^2} = \frac{2 \times 22.0 \times 10^{-3}}{(2.05 \times 10^{-9})^2} = 1.05 \times 10^{16} \text{ m s}^{-2}$$

$$\frac{e}{m} = \frac{ad}{V_P} = \frac{1.05 \times 10^{16} \times 50 \times 10^{-3}}{3000}$$
$$= 1.75 \times 10^{11}\,\text{C kg}^{-1}$$

Measurement of *e*/*m* using the fine beam tube

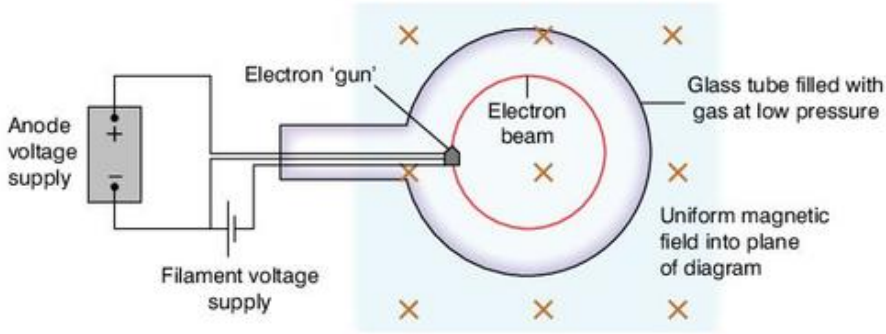


Fig 3 *The fine beam tube*

The 'fine beam' tube described in topic 18.5 may be used to measure e/m. With a sufficiently strong magnetic field, provided the initial direction of the beam is at right angles to the magnetic field lines, the beam path can be seen as a complete circle, enabling its diameter and its radius to be measured.

With the anode voltage V_A constant, the coil current is adjusted in steps so the beam diameter can be measured directly from a scale inside the tube. In this way, the magnetic flux density B is measured for different values of the beam radius, r.

As explained in topic 22.1, the speed of the electrons depends on the anode voltage, V_A, in accordance with the equation $\frac{1}{2}mv^2 = eV_A$.

Combining this with the equation $r = mv/Be$ rearranged as $v = Ber/m$ gives:

$$\tfrac{1}{2}m(Ber/m)^2 = eV_A$$

Making e/m the subject of this equation therefore gives:

$$\boldsymbol{e/m = \frac{2V_A}{B^2r^2}}$$

Notes

1 Rearranging the above equation gives $\boldsymbol{r = k/B}$ where the constant $k = (2mV_A/e)^{1/2}$.
Using the measurements of r and B to plot a graph of r against $1/B$ gives a straight line through the origin with a gradient equal to k. Therefore, e/m can be determined if the value of k is measured from the graph and used in the equation $e/m = 2V_A/k^2$.

2 B can be measured using a Hall probe without the tube present. V_A is measured directly using a suitable voltmeter connected across the high voltage supply unit.

Worked example

Electrons emitted from the heated filament are accelerated through a p.d. of 490 V to form a beam which is then directed into a uniform magnetic field of magnetic flux density 3.4 mT. The magnetic field deflects the beam into a circle of diameter 44 mm. Use these data to calculate the specific charge of the electron.

Solution

beam radius $r = 22$ mm

$$\frac{e}{m} = \frac{2V_A}{B^2r^2} = \frac{2 \times 490}{(3.4 \times 10^{-3} \times 22 \times 10^{-3})^2} = 1.75 \times 10^{11}\,\text{C kg}^{-1}$$

QUESTIONS

1 Electrons moving at a constant speed are directed horizontally into a uniform electric field due to two parallel plates of length 85.0 mm and spaced 40.0 mm apart which have a p.d. of 4000 V between them. When a uniform magnetic field of flux density 2.95 mT is applied at right angles to the beam and to the electric field, the beam is undeflected. Calculate the speed of the electrons.

2 In the arrangement in Q1, when the magnetic field is switched off, the beam is deflected vertically by the electric field by 55 mm where it leaves the field. Show that the each electron takes 2.51 ns to pass through the field and calculate the specific charge e/m of the electron.

3 Electrons moving at a speed of $1.35 \times 10^7\,\text{m s}^{-1}$ are directed into a uniform magnetic field of flux density 3.08×10^{-3} T in such a direction as to form a circle of diameter 50 mm. Calculate the specific charge, e/m, of the electron.

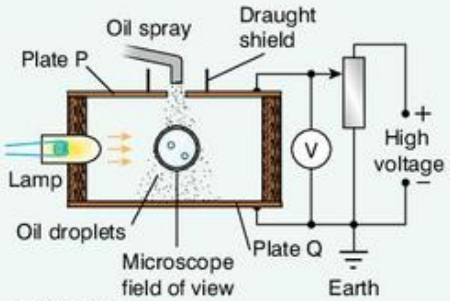
4 Electrons emitted from the heated filament are accelerated through a p.d. of 550 V to form a beam which is then directed into a uniform magnetic field of magnetic flux density 2.80 mT. The magnetic field deflects the beam into a circle of radius 28 mm. Use these data to calculate the specific charge of the electron.

EXTENSION

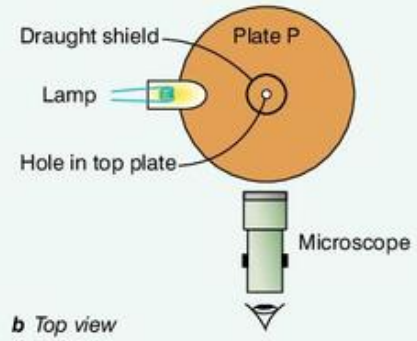
22.3 The determination of the charge of the electron, *e*, by Millikan's method

On these pages you will learn to:

- Describe the principles of Millikan's method to determine the charge *e* of the electron
- Recognise the significance of Millikan's experiment
- Solve problems relating to a charged oil drop in a uniform vertical electric field



a Side view



b Top view

Fig 1** Millikan's method for finding **e

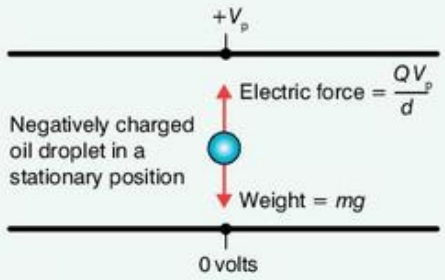


***Fig 2** A stationary oil droplet*

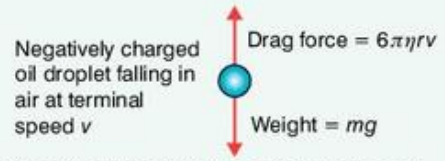


***Fig 3** An oil droplet falling at terminal speed*

Measurement of the charge of a charged oil droplet of known mass

The accurate measurement of the charge *e* of the electron was first successfully made by R. A. Millikan in 1915. He discovered he could control the motion of charged oil droplets from an oil spray using the electric field between oppositely charged parallel metal plates. He found that if the plates were horizontal so the field was vertical, he could slow down or stop or reverse the descent of an oil droplet falling between the plates. Using a microscope as shown in Figure 1, he was able to observe individual charged oil droplets.

He was able to make any charged droplet stay in a stationary position by adjusting the p.d. between the plates until the electric force on the droplet acting vertically upwards was equal and opposite to the weight of the droplet. Figure 2 shows the forces on such a charged droplet.

For a droplet of charge Q and mass m:

- the electric force on the droplet $= \frac{QV_P}{d}$ where V_P is the plate p.d. and d is the perpendicular distance between the plates,
- the weight of the droplet $= mg$.

Therefore, when the droplet is stationary, $\frac{QV_P}{d} = mg$

Hence the charge on the droplet, $Q = \frac{mgd}{V_P}$.

If the mass of the droplet is known, the charge Q on the droplet can therefore be calculated.

Note

The upthrust on the droplet due to buoyancy is equal to about 0.1% of its weight and therefore may be considered to be negligible. Knowledge of the upthrust is not required in this specification.

The significance of Millikan's results

As explained below, Millikan was able to measure the mass of each droplet accurately by measuring its terminal speed with the electric field switched off. He was then able to calculate its charge as explained above. After making many measurements on many charged oil droplets, he found that the charge Q was always a whole number $\times$ 1.6×10^{-19}C. In other words, he showed that electric charge is **quantised** in whole number multiples of 1.6×10^{-19}C. He therefore concluded that the charge of the electron is 1.6×10^{-19}C and that the whole number n corresponds to how many electrons on the droplet are responsible for its charge.

Measurement of the mass of an oil droplet

When no electric field is present, an oil droplet falls at its terminal speed according to its weight. The greater the weight of an oil droplet, the greater is its terminal speed. Millikan realised the drag force on the droplet due to air resistance is equal and opposite to the weight of the droplet when it falls at terminal speed. The resultant force on it is zero and hence its acceleration is zero. By measuring the terminal speed of the droplet, he was able to calculate the mass of the droplet as follows:

Figure 3 shows the forces acting on the droplet as it falls at its terminal speed. Assuming the droplet is a sphere of radius r,

- volume $= \frac{4}{3}\pi r^3$ therefore mass m = volume $(\frac{4}{3}\pi r^3) \times$ density $(\rho) = \frac{4}{3}\pi r^3\rho$. Therefore, weight $= mg = \frac{4}{3}\pi r^3\rho\, g$
- the drag force, $F_D = 6\pi\eta r v$ according to Stokes' law, where η is the viscosity of air. Note the drag force is always in the opposite direction to its velocity. Hence $\frac{4}{3}\pi r^3\rho\, g = 6\pi\eta r v$

Rearranging this equation gives $r^2 = \dfrac{9\eta v}{2\rho g}$

The equation above shows the radius r of the droplet and hence its mass can be calculated if its terminal speed v is measured and the values of the oil density ρ and the viscosity of air η are known.

Worked example

A charged oil droplet was held stationary between oppositely charged metal plates 5.0 mm apart when the p.d. between the plates was 490 V. When the p.d. was switched off, the droplet fell at a terminal speed of $1.3 \times 10^{-4}\,\text{m s}^{-1}$. Calculate:

a the mass of the droplet,
b the charge of the droplet.
density of the oil $= 960\,\text{kg m}^{-3}$, viscosity of air $= 1.8 \times 10^{-5}\,\text{N s m}^{-2}$

Solution

a $r^2 = \dfrac{9\eta v}{2\rho g} = \dfrac{9 \times 1.8 \times 10^{-5} \times 1.3 \times 10^{-4}}{2 \times 960 \times 9.81} = 1.12 \times 10^{-12}\,\text{m}^2$

hence $r = 1.06 \times 10^{-6}\,\text{m}$

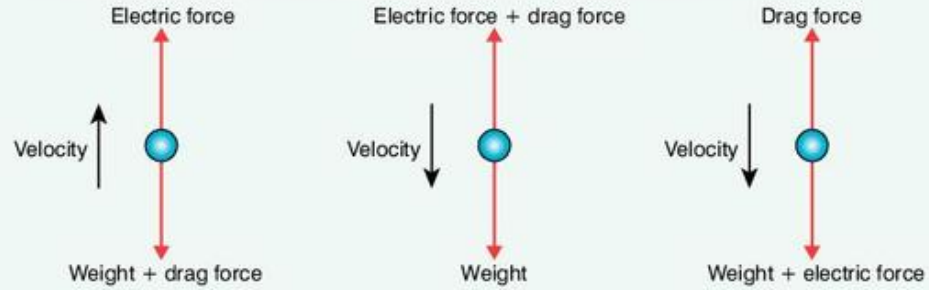
$m = \frac{4}{3}\pi r^3\rho = \frac{4}{3}\pi(1.06 \times 10^{-6})^3 \times 960 = 4.75 \times 10^{-15}\,\text{kg}$

b $Q = \dfrac{mgd}{V_P} = \dfrac{4.75 \times 10^{-15} \times 9.81 \times 5.0 \times 10^{-3}}{490} = 4.8 \times 10^{-19}\,\text{C}$

Notes

1 When the droplet is held stationary by the electric field and the field is switched off, the droplet falls and reaches its terminal speed in a fraction of a second. Its initial acceleration is g as it is due to gravity only as the drag force is initially zero. As the droplet gains speed, the drag force increases so the resultant force (and hence the acceleration) decreases to zero.

2 When the electric field is on and the droplet is moving vertically, it moves at a constant speed that depends on its weight, the drag force and the electric force. Figure 4 shows the forces acting on a droplet that is made to move up or down by changing the electric force.
- In **a**, the electric force is greater than the droplet weight so the droplet moves upwards.
- In **b**, the electric force is less than the weight so the droplet falls.
- In **c**, the electric force is downwards so the droplet falls.



a *Electric force acting upwards:* droplet moving up at constant speed
b *Electric force acting upwards:* droplet moving down at constant speed
c *Electric force acting downwards:* droplet moving down at constant speed

Fig 4 *Using the electric field*

QUESTIONS

$g = 9.81\,\text{m s}^{-2}$, $e = 1.6 \times 10^{-19}\,\text{C}$

1 A charged oil droplet of mass $2.60 \times 10^{-15}\,\text{kg}$ is held stationary by the electric field between two parallel horizontal metal plates which are 6.00 mm apart. The top plate is at a positive potential of 320 V relative to the lower plate which is earthed.
a Calculate the charge on the droplet and state its sign.
b State how many electrons are responsible for the droplet's charge.

2 A charged oil droplet of radius $1.10 \times 10^{-6}\,\text{m}$ is held stationary in a uniform electric field of strength $8.20 \times 10^4\,\text{V m}^{-1}$ which acts vertically upwards.
a Show that the mass of droplet is $5.35 \times 10^{-15}\,\text{kg}$.
b i Calculate the charge on the droplet and state its sign.
ii State how many electrons are responsible for the droplet's charge.
density of oil $= 960\,\text{kg m}^{-3}$

3 A charged oil droplet was held stationary between oppositely charged metal plates 5.00 mm apart when the p.d. between the plates was 610 V. When the p.d. was switched off, the droplet fell at a terminal speed of $1.15 \times 10^{-4}\,\text{m s}^{-1}$. Calculate:
a the mass of the droplet,
b the charge of the droplet.
density of the oil $= 960\,\text{kg m}^{-3}$, viscosity of air $= 1.80 \times 10^{-5}\,\text{N s m}^{-2}$

4 a State the general result obtained by Millikan for the charge on a charged oil droplet and explain its significance.
b A charged oil droplet is held stationary in a uniform electric field which is then switched off. Describe and explain the subsequent motion of the droplet.

22.4 Electromagnetic waves

On these pages you will learn to:

- Recognise the nature of an electromagnetic wave
- Recognise the main parts of the electromagnetic spectrum and recall the main properties of electromagnetic waves in each part of the spectrum, including their effect on living cells

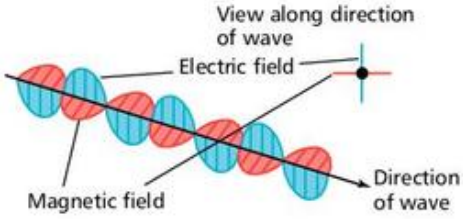


Fig 1 Electromagnetic waves

The nature of electromagnetic waves

Electromagnetic waves were predicted by James Clerk Maxwell in 1862. Maxwell knew that a magnetic field is created round a wire when an electric current passes along the wire. He knew about Michael Faraday's discovery that a changing magnetic field in a coil of wire induces a voltage in the coil. He wondered if the two effects could be linked, and he used his mathematical skills to discover the link. In effect, Maxwell showed that the changing magnetic field created by an alternating current in a wire creates an alternating electric field, which creates an alternating magnetic field further away, which creates an alternating electric field, and so on.

Maxwell showed that the result is an **electromagnetic wave** consisting of an alternating magnetic field in phase with an alternating electric field. He worked out that the speed of an electromagnetic wave should be $300\,000\,km\,s^{-1}$, the same as the speed of light in a vacuum!

Maxwell realized, from his theory, that light must be an electromagnetic wave and that electromagnetic waves must exist beyond the visible spectrum. He knew that infrared and ultraviolet radiation are outside the visible spectrum, and he correctly predicted electromagnetic waves beyond these two invisible forms of radiation.

An electromagnetic wave consists of an **alternating magnetic field** (the magnetic wave) and an **alternating electric field** (the electric wave), in which the magnetic wave and the electric wave:

- are at right angles to each other,
- vibrate in phase with each other,
- both vibrate at right angles to the direction of propagation of the electromagnetic wave.

Beyond the visible spectrum

Radio waves and microwaves

- Radio waves were first discovered by Heinrich Hertz in 1887. Hertz showed that radio waves are produced when high voltage sparks jump across a gap. He used a wire loop with a small gap in it as a detector. When radio waves pass through the loop, tiny sparks are induced across the detector gap.
- Radio waves and microwaves are used to carry information from aerials supplied with high-frequency alternating current. The carrier waves are modulated by the **information signal** which is used to vary the frequency or the amplitude of the alternating current. See topic 17.1.
- Microwaves can be directed in a beam from a transmitter dish aerial. **Diffraction** by the dish is limited because the wavelength is much less than the dish diameter. Therefore, the beam can be directed and detected by a receiver dish along the line of sight of the beam. Microwaves travel through the atmosphere without significant absorption and, therefore, can be used for satellite communications.
- Microwaves are used for heating. Food in a microwave oven becomes hot throughout because the microwaves penetrate into the food and make the water molecules in the food vibrate.

Infrared radiation

- Every object emits infrared radiation. The higher the temperature of the source, the greater the amount of infrared radiation it emits.
- Infrared radiation can be detected by producing a visible spectrum using white light and placing a blackened thermometer bulb just beyond the red part of the visible spectrum. The thermometer shows a rise of temperature because it absorbs infrared radiation.
- Infrared radiation is used for imaging, for heating and drying, and for communications. Infrared beams are used to carry data in digital form along optical fibres.

Ultraviolet radiation

- Ultraviolet radiation can be detected by placing a strip of fluorescent paper just beyond the violet part of the visible spectrum. The paper glows due to absorbing ultraviolet light. Ultraviolet marker pens contain ink that is not visible except when it absorbs ultraviolet radiation and glows.
- Ultraviolet radiation is harmful to the skin and to the eyes. Protective goggles should be worn by anyone using a sunbed. The ozone layer of the Earth's atmosphere filters out most, but not all, of the ultraviolet radiation from the Sun. The thinning of the ozone layer is a cause for concern and is the reason why people should wear hats and suncream when outdoors in summer.

X-rays and gamma rays

- X-rays and gamma rays cover the same range of wavelengths. They differ in that gamma rays are produced when an atomic nucleus loses energy, whereas X-rays are emitted when electrons in a beam are suddenly stopped or when an inner shell electron in an atom moves to a shell closer to the nucleus.
- X-rays and gamma rays pass through human tissue, but are stopped by bone. When an X-ray photograph is taken, the relevant part of the patient is exposed to X-rays from an X-ray tube. A photographic film, in a light-proof wrapper, is placed in the path of the X-rays that pass through the patient.
  A 'negative' image of the bones is formed on a photographic film.

Electromagnetic waves and living cells

Radio waves and microwaves penetrate tissues and are absorbed by the water content of living cells, causing internal heating which may damage or even destroy the cell.

Infrared radiation is absorbed by skin cells, causing internal heating which may damage or destroy the cells.

Ultraviolet radiation damages the cells of the retina of the eye. Special protective goggles must be worn by users of sunbeds. UV radiation also damages cells below the skin, because it can penetrate more than infrared radiation. Users of sunbeds must **not** exceed recommended exposure times or severe burns will result.

X-rays and gamma radiation create ions in substances which they pass through. Living cells are damaged by ionising radiation. High doses kill cells and low doses can cause cell mutation and cancerous growth. There is no evidence for a lower limit below which living cells would not be damaged. The maximum permissible radiation dose is a legal limit decided on the basis of acceptable risk.

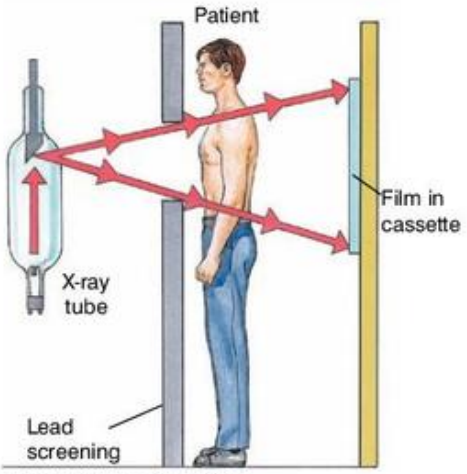


Fig 2 Taking a chest X-ray

QUESTIONS

1 a Why must sunbed users wear protective goggles?

b Why is the thinning of the Earth's ozone layer a matter of great concern?

2 a Explain why an infrared TV camera can be used to observe the movement of animals at night.

b Explain why a hot object wrapped in silver foil cools more slowly than a similar unwrapped object.

3 a State two types of electromagnetic wave used for:
i communications,
ii heating.

b Give two reasons why microwaves, rather than radio waves, are used for satellite communications.

4 a State the difference between X-rays and gamma rays.

b State one property of X-rays and gamma rays not shared with other forms of electromagnetic radiation.

22.5 Photoelectricity

On these pages you will learn to:

- Describe the photoelectric effect and its principal features
- Recall the significance of threshold frequency and explain why it cannot be explained using wave theory
- State Einstein's photon theory of light and use it to explain the photoelectric effect
- Recall and use the equation $E = hf$
- Explain photoelectric phenomena in terms of photon energy and work function energy

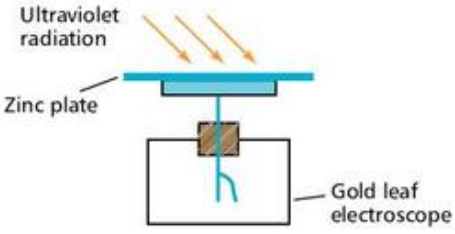


Fig 1 *Demonstrating photoelectricity*

The discovery of photoelectricity

A metal contains conduction electrons which move about freely inside the metal. These electrons collide with each other and with the positive ions of the metal. Heinrich Hertz discovered how to produce and detect radio waves (see topic 22.4). He found that the sparks produced in his spark gap detector, when radio waves were being transmitted, were stronger when ultraviolet radiation was directed at the spark gap. Further investigations of the effect of electromagnetic radiation on metals showed that electrons are emitted from the surface of a metal when electromagnetic radiation above a certain frequency is directed at the metal. This effect is known as the **photoelectric effect**.

Demonstration of the photoelectric effect

Ultraviolet radiation from a UV lamp is directed at the surface of a zinc plate placed on the cap of a gold leaf electroscope, as shown in Figure 1. This device is a very sensitive detector of charge. When it is charged, the thin gold leaf of the electroscope rises; it is repelled from the metal 'stem', because they are both charged with a like charge.

- If the electroscope is **charged negatively**, the leaf rises and stays in position. However, if ultraviolet light is directed at the zinc plate, the leaf gradually falls. The leaf falls because conduction electrons at the zinc surface leave the zinc surface when ultraviolet light is directed at it. The emitted electrons are referred to as **photoelectrons**.
- If the electroscope is **charged positively**, the leaf rises and stays in position, regardless of whether or not ultraviolet light is directed at the zinc plate. This is because the conduction electrons in the zinc plate are held on the zinc plate, as the plate is charged positive and electrons carry negative charge.

Puzzling problems

The following observations were made about photoelectricity after Hertz's discovery. These observations were a major problem, because they could not be explained using the wave theory of electromagnetic radiation.

- Photoelectric emission of electrons from a metal surface does not take place if the frequency of the incident electromagnetic radiation is below a certain value, known as the **threshold frequency**. This minimum frequency depends on the type of metal.
  Note The **wavelength** of the incident light must be less than a **maximum** value equal to the speed of light/the threshold frequency.
- The number of electrons emitted per second is proportional to the **intensity** of the incident radiation, provided the frequency is greater than the threshold frequency. If the frequency of the incident radiation is less than the threshold frequency, no photoelectric emission from that metal surface can take place, no matter how intense the incident radiation is.
- Photoelectric emission occurs without delay, as soon as the incident radiation is directed at the surface, provided the frequency of the radiation exceeds the threshold frequency. No matter how weak the intensity of the incident radiation is, electrons are emitted as soon as the source of radiation is switched on.

The wave theory of light cannot explain either the existence of a threshold frequency, or why photoelectric emission occurs without delay. According to wave theory, each conduction electron at the surface of a metal should gain some energy from the incoming waves, regardless of how many waves arrive each second. Wave theory therefore predicted that:

- Emission should take place with waves of any frequency.
- Emission would take longer using low intensity waves than using high intensity waves.

The discovery of radio waves and X-rays confirmed the prediction by Maxwell (topic 22.4) that electromagnetic radiation exists beyond the known spectrum from ultraviolet to infrared radiation. Using Maxwell's theory of electromagnetic waves, physicists were very successful in predicting the properties of electromagnetic waves until the discovery of the photoelectric effect.

Einstein's explanation of photoelectricity

The photon theory of light was put forward by Einstein in 1905 to explain photoelectricity. Einstein assumed that light is composed of wave packets, or **photons**, each of energy equal to hf, where f is the frequency of the light and h is the Planck constant. The accepted value for h is 6.63×10^{-34} J s:

$$\textbf{Energy of a photon} = hf$$

For electromagnetic waves of wavelength λ, the energy of each photon $E = hf = \frac{hc}{\lambda}$

where c is the speed of the electromagnetic waves.

To explain photoelectricity, Einstein said that:

- When light is incident on a metal surface, an electron at the surface absorbs a **single** photon from the incident light and therefore gains energy equal to hf, where hf is the energy of a light photon.
- An electron can leave the metal surface, if the energy gained from a single photon exceeds **the work function,** ϕ, of the metal. This is the minimum energy needed by an electron to escape from the metal surface.

Hence the maximum kinetic energy of an emitted electron:

$$E_{K(max)} = hf - \phi$$

Emission can take place from a surface at zero potential, provided $E_{Kmax} > 0$, i.e. $hf > \phi$. Thus the threshold frequency of the metal:

$$f_{min} = \frac{\phi}{h}$$

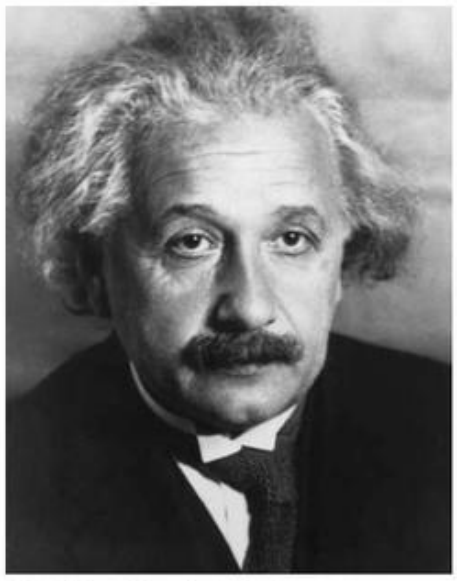


Fig 2 Albert Einstein 1879–1955

In 1905 Einstein published three papers, each of which changed existing ideas. His paper on Brownian motion showed that atoms must exist, and he established the photon theory of light in the second paper. Perhaps he is best remembered for his theories of relativity which he stated in the third paper.

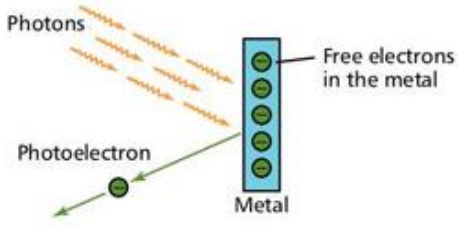


Fig 3 Explaining photoelectricity

QUESTIONS

$h = 6.63 \times 10^{-34}$ J s, $c = 3.00 \times 10^{8}$ m s^{-1}

1 a What is meant by 'photoelectric emission' from a metal surface?

b Explain why photoelectric emission from a metal surface only takes place if the frequency of the incident radiation is greater than a certain value.

2 a Calculate the frequency and energy of a photon of wavelength:
i 450 nm **ii** 1500 nm

b A metal surface at zero potential emits electrons from its surface if light of wavelength 450 nm is directed at it, but not if light of wavelength 650 nm is used. Explain why photoelectric emission happens with light of wavelength 450 nm but not with light of wavelength 650 nm.

3 The work function of a certain metal plate is 1.1×10^{-19} J. Calculate:

a the threshold frequency of incident radiation,

b the maximum kinetic energy of photoelectrons emitted from this plate when light of wavelength 520 nm is directed at the metal surface.

4 Light of wavelength 635 nm is directed at a metal plate at zero potential. Electrons are emitted from the plate with a maximum kinetic energy of 1.5×10^{-19} J. Calculate:

a the energy of a photon of this wavelength,

b the work function of the metal,

c the threshold frequency of electromagnetic radiation incident on this metal.

22.6 More about photoelectricity

On these pages you will learn to:

- Explain why the maximum photoelectric energy is independent of intensity, whereas the photoelectric current is proportional to intensity
- Recall, use and explain the significance of $hf = \Phi + \frac{1}{2}mv_{max}^2$

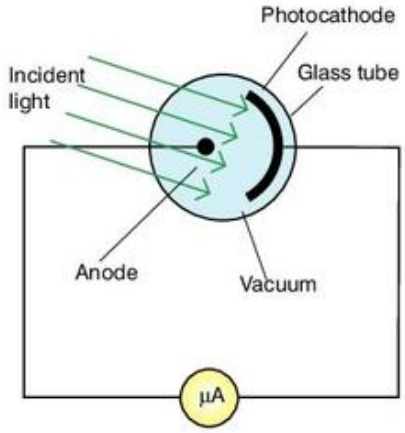


Fig 1 Using a vacuum photocell

The electron volt

The electron volt (eV) is a unit of energy equal to the work done when an electron is moved through a p.d. of 1 V.

For a charge q moved through a p.d., V:

$$\text{Work done} = qV$$

Therefore, the work done when an electron moves through a potential difference of 1 V is equal to **1.6×10^{-19} J** ($= 1.6 \times 10^{-19}$ C $\times$ 1 V). This amount of energy is defined as **1 electron volt.**

Examples

The work done on:

- an electron when it moves through a potential difference of 1000 V = 1000 eV,
- an ion of charge +2e when it moves through a potential difference of 10 V = 20 eV.

More about conduction electrons

The **average kinetic energy** of a conduction electron in a metal depends on the **temperature** of the metal. As the conduction electrons move about at random in the metal, they can be likened to the molecules of a gas. The average kinetic energy of a gas molecule is proportional to the absolute temperature of the gas. It can be shown that the average kinetic energy of an conduction electron in a metal at 300 K is therefore about 6×10^{-21} J.

- The **work function of a metal** is the **minimum** energy needed by a conduction electron to escape from the metal surface when the metal is at zero potential. The work function of a metal is of the order of 10^{-19} J, which is about 20 times greater than the average kinetic energy of a conduction electron in a metal at 300 K. In other words, a conduction electron in a metal at about 20 °C does not have sufficient kinetic energy to leave the metal.
- When a conduction electron **absorbs a photon**, its kinetic energy increases by an amount equal to the energy of the photon. Provided the energy of the photon exceeds the work function of the metal, the conduction electron can leave the metal. If the electron does not leave the metal, it collides repeatedly with other electrons and positive ions, and it quickly loses its extra kinetic energy.

Photoelectricity investigations

The vacuum photocell

A **vacuum photocell** is a glass tube that contains a metal plate, referred to as the **photocathode**, and a smaller metal electrode referred to as the **anode**. Figure 1 shows a vacuum photocell in a circuit. When light of frequency greater than the threshold frequency for the metal is directed at the photocathode, electrons emitted from the cathode transfer to the anode. The micro ammeter in the circuit can be used to measure the photoelectric current, which is proportional to the number of electrons per second that transfer from the cathode to the anode.

- For a photoelectric current I, the **number of photoelectrons per second** that transfer from the cathode to the anode is I/e, where e is the charge of the electron.
- The photoelectric current is proportional to the **intensity** of the light incident on the cathode. This is because the intensity of the incident light is a measure of the **energy per second** carried by the incident light, which is proportional to the number of photons per second incident on the cathode. Because each photoelectron must have absorbed one photon to escape from the metal surface, the number of photoelectrons emitted per second (i.e. the photoelectric current) is therefore proportional to the intensity of the incident light.
- The intensity of the incident light does **not** affect the maximum kinetic energy of a photoelectron. No matter how intense the incident light is, the energy gained by a photoelectron is due to the absorption of one photon only. Therefore, the maximum kinetic energy of a photoelectron is still given by $E_{Kmax} = hf - \phi$, as explained in topic 22.5.

Measurement of the work function of a metal

Photoelectric emission can be stopped by making the photocathode sufficiently positive. Figure 2 shows how a potential divider can be used to make the photocathode increasingly positive, relative to the anode. As the potential difference is increased from zero, the micro-ammeter reading decreases to zero. This happens because each photoelectron leaving the metal surface needs to

do extra work, as the plate is at a positive potential. The kinetic energy of a photoelectron is therefore reduced, because each electron has to do work to overcome the attraction of the plate. When the micro ammeter reading is zero, photoelectric emission stops; this is because the kinetic energy of a photoelectron is reduced to zero before the electron can escape from the attraction of the plate.

- As explained earlier, the work done by a charged particle when it moves through a potential difference V is qV, where q is the charge of the particle. Therefore, to escape from a metal plate at positive potential V, the **extra work** needed to be done by a photoelectron is $\boldsymbol{eV}$, where e is the charge of the electron.
- At zero potential, the **maximum kinetic energy** of an emitted photoelectron is $\boldsymbol{hf - \phi}$, where f is the frequency of the incident radiation and ϕ is the work function of the metal.

Therefore, photoelectric emission is stopped when the potential V is such that:

$$eV_S = hf - \phi$$

where V_S is the potential needed to stop emission.

In other words, the maximum kinetic energy of a photoelectron from a surface at zero potential:

$$E_{K(max)} = eV_S$$

By measuring V_S for different frequencies f, $E_{K(max)}$ can be calculated for each frequency; then a graph of $E_{K(max)}$ against frequency f can be plotted. Because $E_{K(max)} = hf - \phi$, the graph is a straight line with:

- a gradient h,
- a y-intercept equal to $-\phi$, and
- an x-intercept equal to the threshold frequency, $f_{min} = \frac{\phi}{h}$
- The above equation can be written as $hf = \phi + \frac{1}{2}mv^2_{max}$ where v_{max} is the maximum speed of the emitted photoelectrons and $\frac{1}{2}mv^2_{max}$ is the maximum kinetic energy $E_{K(max)}$.

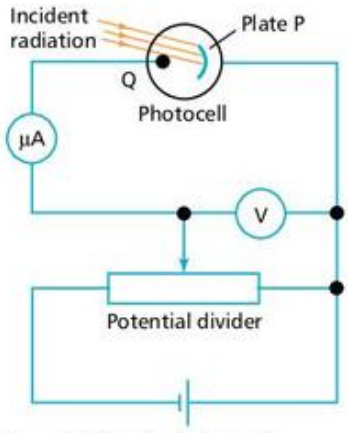


Fig 2 Investigating photoelectricity

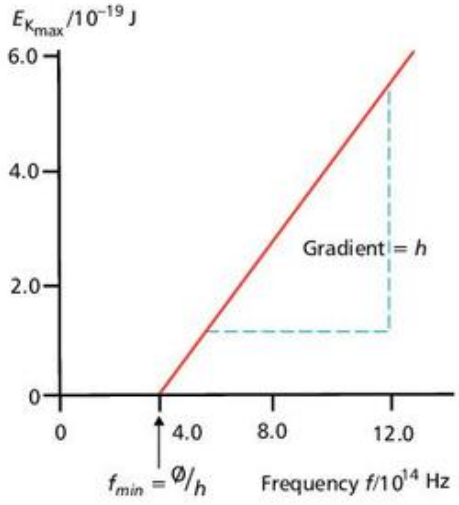


Fig 3 A graph of $E_{K(max)}$ against frequency

QUESTIONS

$h = 6.63 \times 10^{-34}$ J s, $c = 3.00 \times 10^8$ m s^{-1}, $e = 1.6 \times 10^{-19}$ C

1 A vacuum photocell is connected to a microammeter. Explain the following observations:

a When the cathode was illuminated with blue light of low intensity, the microammeter showed a non-zero reading.

b When the cathode was illuminated with an intense red light, the microammeter reading was zero.

2 A vacuum photocell is connected to a micro ammeter. When light is directed at the photocell, the micro ammeter reads 0.25 μA.

a Calculate the number of photoelectrons emitted per second by the photocathode of the photocell.

b Explain why the micro ammeter reading is doubled if the intensity of the incident light is doubled.

3 A narrow beam of light of wavelength 590 nm and of power 0.5 mW is directed at the photocathode of a vacuum photocell, which is connected to a micro-ammeter that reads 0.4 μA. Calculate:

a the energy of a single light photon of this wavelength,

b the number of photons per second incident on the photocathode,

c the number of electrons emitted per second from the photocathode.

4 a Use Figure 3 to estimate:

i the threshold frequency,

ii the work function of the photocathode that gave the results used to plot the graph.

b A metal surface has a work function of 1.9×10^{-19} J. Light of wavelength 435 nm is directed at the metal surface. Calculate the maximum kinetic energy of the photoelectrons emitted from this metal surface.

22.7 Energy levels and spectra

On these pages you will learn to:

- Recognise the existence of discrete electron energy levels in isolated atoms (e.g. atomic hydrogen) and deduce how this leads to spectral lines
- Distinguish between emission and absorption line spectra
- Recall and solve problems using the relation $hf = E_1 - E_2$

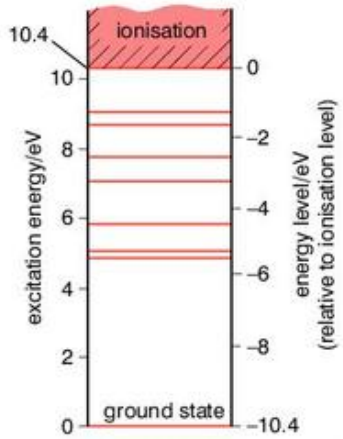


Fig 1 *The energy levels of the mercury atom*

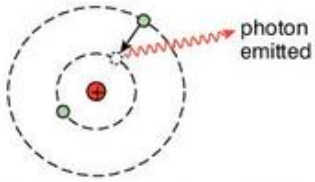


Fig 2 *De-excitation by photon emission*

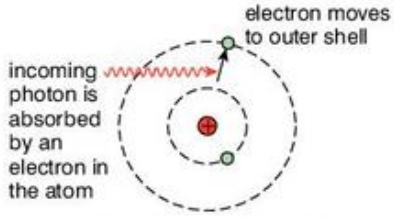


Fig 3 *Excitation by photon absorption*

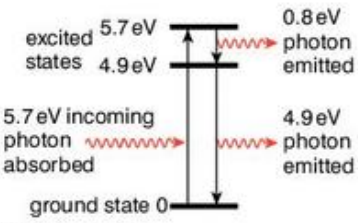


Fig 4 *Fluorescence*

Electrons in atoms

The electrons in an atom are trapped by the electrostatic force of attraction of the nucleus. They move about the nucleus in allowed orbits or 'shells' surrounding the nucleus. The energy of an electron in a shell is constant. An electron in a shell near the nucleus has less energy than an electron in a shell further away from the nucleus. Each shell can only hold a certain number of electrons. For example, the innermost shell (that is, the shell nearest to the nucleus) can only hold two electrons and the next nearest shell can only hold eight electrons.

Each type of atom has a certain number of electrons. For example, a helium atom has two electrons and a lithium atom has three electrons. Thus, in its lowest energy state:

- a helium atom has both electrons in the innermost shell,
- a lithium atom has two electrons in the innermost shell and one in the next shell.

The lowest energy state of an atom is called its **ground state**. When an atom in the ground state absorbs energy, one of its electrons moves to a shell at higher energy, so the atom is now in an **excited state**. We can use the excitation energy measurements to construct an energy level diagram for the atom, as shown in Figure 1. This shows the allowed energy values of the atom. Each allowed energy corresponds to a certain electron configuration in the atom. Note that the ionisation level may be considered as the 'zero' reference level for energy, instead of the ground state level.

De-excitation

Did you know that gases at low pressure emit light when they are made to conduct electricity? For example, a neon tube emits red-orange light when it conducts. The gas-filled tube used to measure excitation energies emits light when excitation occurs. This happens because the atoms absorb energy as a result of excitation by collision but they do not retain the absorbed energy permanently.

The electron configuration in an excited atom is unstable because an electron that moves to an outer shell leaves a vacancy in the shell it moves from. Sooner or later, the vacancy is filled by an electron from an outer shell transferring to it. When this happens, the electron emits a photon of energy equal to the energy lost by the electron. The atom therefore moves to a lower energy level. See Figure 2.

The energy of the photon is equal to the energy lost by the electron and therefore by the atom. For example, when a mercury atom at an excitation energy level of 4.9 eV de-excites to the ground state, it emits a photon of energy 4.9 eV.

An atom in an excited state may de-excite to the ground state indirectly. For example, a mercury atom at an excitation energy of 5.7 eV may de-excite:

- to the 4.9 eV level, emitting a 0.8 eV photon in the process, then
- to the ground state from the 4.9 eV level, emitting a 4.9 eV photon in this process.

In general, when an electron moves from energy level E_1 to a lower energy level E_2, **the energy of the emitted photon $hf = E_1 - E_2$**

Excitation using photons

An electron in an atom can absorb a photon and move to an outer shell where a vacancy exists – but only if the energy of the photon is exactly equal to the gain in the electron's energy (see Figure 3). In other words, the photon energy must be exactly equal to the difference between the final and initial energy levels of the atom. If the photon's energy is smaller or larger than the difference between the two energy levels, it will not be absorbed by the electron.

Fluorescence

An atom in an excited state can de-excite directly or indirectly to the ground state, regardless of how the excitation took place. Therefore, an atom can absorb photons of certain energies and then emit photons of the same or lesser energies. For example, a mercury atom in the ground state could be excited to its 5.7 eV energy level by absorbing a photon of energy 5.7 eV; the mercury could then de-excite to its 4.9 eV energy level by emitting a photon of energy 0.8 eV; then de-excite to the ground state by emitting a photon of energy 4.9 eV.

Figure 4 represents these changes on an energy level diagram.

This overall process explains why certain substances **fluoresce** or glow with visible light when they absorb ultraviolet radiation. Atoms in the substance absorb ultraviolet photons and become excited. When the atoms de-excite, they emit visible photons. When the source of ultraviolet radiation is removed, the substance stops glowing.

Line spectra

A rainbow is a natural display of the colours of the spectrum of sunlight. Raindrops split sunlight into a continuous spectrum of colours. Figure 5 shows the continuous spectrum produced by passing a beam of white light from a filament lamp through a glass prism. The wavelength of the light photons that produce the spectrum increases across the spectrum from deep violet at less than 400 nm to deep red at about 650 nm.

If we use a tube of glowing gas as the light source instead of a filament lamp, we see a spectrum of discrete lines of different colours, as shown in Figure 6. This type of spectrum is referred to as a **line emission spectrum**.

If the spectrum of light from a filament lamp is observed after passing it through a glowing gas, we see dark vertical lines in the continuous spectrum, as shown in Figure 7. The lines are due to absorption of photons of certain energies by gas atoms. This type of spectrum is referred to as a **line absorption spectrum**.

The wavelengths of the lines of a line spectrum of an element are characteristic of the atoms of that element. By measuring the wavelengths of a line spectrum, we can therefore identify the element that produced the light. No other element produces the same pattern of light wavelengths. This is because the energy levels of each type of atom are unique to that atom. So the photons emitted are characteristic of the atom.

- Each line in a line spectrum is due to light of a certain colour and therefore a certain wavelength.
- The photons that produce each line all have the same energy, which is different to the photons that produce any other line.
- Each photon is emitted when an atom de-excites due to one of its electrons moving to an inner shell.
- As explained above, if the electron moves from energy level E_1 to a lower energy level E_2, the energy of the emitted photon $hf = E_1 - E_2$.

For each wavelength λ, we can calculate the energy of a photon of that wavelength as its frequency $f = c/\lambda$, where c is the speed of light. Given the energy level diagram for the atom, we can therefore identify on the diagram the transition that causes a photon of that wavelength to be emitted.

Worked example $c = 3.0 \times 10^8\,\text{m s}^{-1}$, $e = 1.6 \times 10^{-19}\,\text{C}$, $h = 6.63 \times 10^{-34}\,\text{J s}$

A mercury atom de-excites from its 4.9 eV energy level to the ground state. Calculate the wavelength of the photon released.

Solution

$E_1 - E_2 = 4.9 - 0 = 4.9\,\text{eV} = 4.9 \times 1.6 \times 10^{-19}\,\text{J} = 7.84 \times 10^{-19}\,\text{J}$

Therefore, $f = \dfrac{E_1 - E_2}{h} = \dfrac{7.84 \times 10^{-19}}{6.63 \times 10^{-34}} = 1.12 \times 10^{15}\,\text{Hz}$

$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^8}{1.12 \times 10^{15}} = 2.52 \times 10^{-7}\,\text{m}$$

Fig 5 Observing a continuous spectrum

Fig 6 A line emission spectrum

Fig 7 A line absorption spectrum

QUESTIONS

$e = 1.6 \times 10^{-19}\,\text{C}$

1 Figure 1 shows some of the energy levels of the mercury atom.

a Estimate the energy needed to excite the atom from the ground state to the highest excitation level shown in the diagram.

b Mercury atoms in an excited state at 5.7 eV can de-excite directly or indirectly to the ground state. Show that the photons released could have six different energies.

2 a In terms of electrons, state two differences between excitation and de-excitation.

b A certain type of atom has excitation energies of 1.8 eV and 4.6 eV.

i Sketch an energy level diagram for the atom using these energy values.

ii Calculate the possible photon energies from the atom when it de-excites from the 4.6 eV level, indicating on your diagram, by a downward arrow, the energy change responsible for each photon energy.

3 Explain why the line spectrum of an element is unique to that element and can be used to identify it.

22.8 Energy bands

On these pages you will learn to:

- Explain what is meant by energy bands in solids and the terms valence band, conduction band and forbidden band (band gap)
- Use simple band theory to explain the temperature dependence of the resistance of metals and of intrinsic semiconductors
- Use simple band theory to explain the dependence on light intensity of the resistance of an LDR

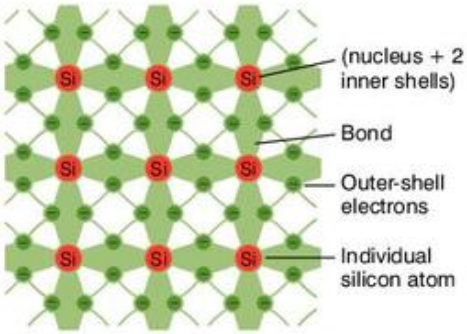


Fig 1 *Silicon atoms in the solid state*

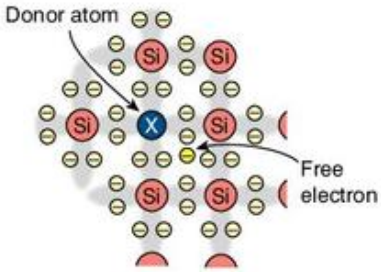


Fig 2 *n-type semiconductor*

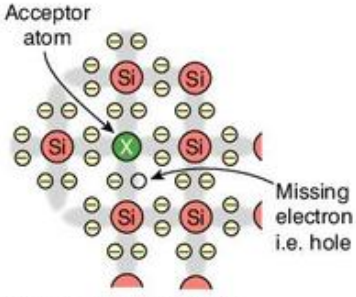


Fig 3 *p-type semiconductor*

Semiconductors

Most semiconductor devices are made from silicon, which is an abundant chemical element.

Silicon is a Group 4 chemical element and has four electrons in the outer shell of each atom. Each atom forms a covalent or 'shared' bond with each of four other atoms to form the structure of the solid. At absolute zero, the outer electrons are localised in the covalent bonds, so there are no conduction electrons. Above absolute zero, some of the outer shell electrons break free from the atoms to become conduction electrons. The electron vacancies in the covalent bonds are referred to as **holes**.

Semiconductors are characterised by a resistivity that decreases with an increase in temperature. This happens because the vibrations of the atoms cause electrons in the outer shell of each atom to break free to become conduction electrons. An increase in temperature makes the atoms vibrate more, so more outer shell electrons break free. The resistivity of the semiconductor therefore decreases because it contains more conduction electrons per unit volume. In other words, the population density of conduction electrons is increased when the temperature is increased, so the resistivity of the material decreases.

Pure silicon is an example of an **intrinsic semiconductor**. The electrical properties of an intrinsic semiconductor can be changed by doping it with a tiny fraction of Group 3 or Group 5 atoms so that it becomes an **extrinsic semiconductor**, or 'doped' semiconductor.

- **An n-type extrinsic semiconductor** is produced using Group 5 atoms, which each have five electrons in their outer shells. As only four electrons from the outer shell of each such atom are needed for the covalent bonds the atom forms with its neighbours, the surplus electron contributes to the conduction electrons in the material. As conduction in the material is due to electrons that are negative **charge carriers**, the material is referred to as an **n-type semiconductor**. It has more conduction electrons per unit volume compared with an intrinsic semiconductor of the same material at the same temperature. The added atoms are called **donor atoms** because they donate free electrons to the material, as shown in Figure 2. When a voltage is applied across n-type material, the free electrons in the material move towards the positive end of the material. So *n*-type semiconductors conduct due to the presence of free electrons as negative charge carriers in the material.
- **A p-type extrinsic semiconductor** is produced using Group 3 atoms, which each have three electrons in their outer shells. As four electrons from the outer shell of each such atom are needed for the covalent bonds the atom forms with its neighbours, there is a vacancy or 'hole' in the covalent bonds of each donor atom. The hole may be filled by a conduction electron or by an electron from a neighbouring atom. As a result, holes move about from atom to atom inside the material as if they are positive charge carriers. The material is therefore referred to as a **p-type semiconductor**.
  The added atoms are called **acceptor atoms** because they accept any free electrons to fill the holes in the incomplete bonds. When a voltage is applied across *p*-type material, incomplete bonds are filled by electrons jumping across from other bonds, as shown in Figure 3. When an electron jumps from one bond to fill an incomplete bond, it creates a hole in the bond it left. In this way, the holes in the material move towards the negative end of the material. So p-type semiconductors conduct due to the presence of holes as positive charge carriers in the material.

Energy bands

The electrons in the shells of an isolated atom have well-defined energies, depending on which shell each electron is in. However, atoms in a solid vibrate about their mean positions, causing the energy levels to spread out into **energy bands**, as shown in Figure 4. Electrons fill the bands from the lowest levels up.

Metals

The highest energy band containing electrons is only partly filled. So electrons in this band can easily move to unfilled levels further up the band. These electrons, referred to as conduction electrons, have broken free from individual atoms and are able to move through the material. So this band is called the **conduction band**. The **valence band** is immediately below the conduction band. Electrons in the valence band are firmly attached to individual atoms.

Insulators

Each energy band containing electrons is filled completely in an insulator. The highest filled band contains the outer-shell electrons, which are firmly held to atoms. This band, the valence band, is separated by an **energy gap** from the next band up, the conduction band, which is empty. The gap is too large for electrons from the valence band to jump across. Hence there can be no conduction.

Intrinsic semiconductors

At absolute zero, the band picture is like an insulator but the energy gap is not so large. So at temperatures above absolute zero, valence electrons can gain enough energy to jump across the gap to become conduction electrons. The higher the temperature, the greater the number of electrons that jump across the gap. An electron jumping across the gap corresponds to the electron breaking away from an individual atom to become a conduction electron.

A light-dependent resistor (LDR) is made of a semiconductor material. In the dark the resistance of the LDR can be several megaohms (MΩ), but in strong sunlight its resistance can fall to a few hundred ohms. Photons with enough energy are absorbed by the semiconductor material, enabling electrons in the valence band to jump into the conduction band. These electrons are then free to form part of an electric current.

Extrinsic semiconductors

- *n*-type semiconductors have added energy levels due to the donor atoms. The donor levels are in the energy gap between the valence and conduction bands just below the conduction band. Surplus electrons from the donor atoms occupy these levels and can easily jump into the conduction band to become conduction electrons.
- *p*-type semiconductors have added energy levels due to the acceptor atoms. The acceptor levels are in the energy gap between the valence and conduction bands just above the valence band. So electrons from the valence band can jump into the acceptor levels, leaving holes in the valence band.

For both types of extrinsic semiconductor, conduction due to electrons jumping from the valence band to the conduction band is insignificant unless the material is at much higher temperatures than room temperature.

Increased light intensity also causes the resistance of a semiconductor to decrease. When light photons are incident on a semiconductor, more charge carriers are created when electrons gain energy by absorbing photons. An electron that absorbs a photon of sufficient energy jumps to the conduction band or to an acceptor level as shown in Figures 5 and 6.

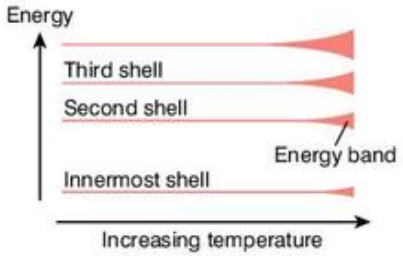
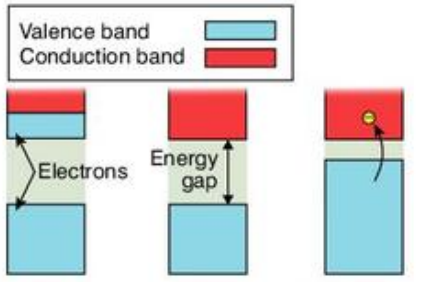


Fig 4 Energy bands for a solid



a Metal *b Insulator* *c Semiconductor*

Fig 5 Different types of solid

QUESTIONS

1 **a** State the type of charge carrier responsible for conduction in **i** a metal and **ii** a p-type semiconductor.

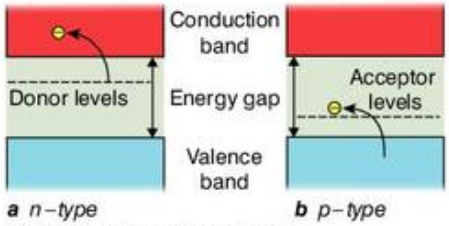
b Explain why the resistivity of a semiconductor decreases when its temperature is raised.

2 **a** State what is meant by an extrinsic semiconductor.

b Explain how an n-type semiconductor differs from a p-type semiconductor.

3 Use band theory to describe how a p-type semiconductor conducts when a potential difference is applied across it.

4 An n-type semiconductor slice contains 6.0×10^{21} charge carriers per cubic metre. The slice has a cross-sectional area of 8.2 mm^2. Calculate the drift velocity of the charge carriers when the current is 0.055 A.



a n–type *b p–type*

Fig 6 Extrinsic semiconductors

22.9 Wave–particle duality

On these pages you will learn to:

- Show an understanding that the photoelectric effect provides evidence for a particulate nature of electromagnetic radiation while phenomena such as interference and diffraction provide evidence for a wave nature
- Describe and interpret qualitatively the evidence provided by electron diffraction for the wave nature of particles
- Recall and use the relation for the de Broglie wavelength $\lambda = h/p$

The dual nature of light

Light is part of the electromagnetic spectrum of waves. The theory of electromagnetic waves predicted the existence of electromagnetic waves beyond the visible spectrum. The subsequent discovery of X-rays and radio waves confirmed these predictions and seemed to show that the nature of light had been settled. Many scientists in the late nineteenth century reckoned that all aspects of physics could be explained using Newton's laws of motion, and the theory of electromagnetic waves. They thought that the few minor problem areas, such as photoelectricity, would be explained sooner or later using Newton's laws of motion and Maxwell's theory of electromagnetic waves. However, photoelectricity was not explained until Einstein put forward the radical theory that light consists of photons, which are 'particle-like' packets of electromagnetic waves. Light has a dual nature, in that it can behave as a wave or as a particle, according to circumstances.

- The **wave-like nature** is observed when **diffraction** of light takes place. This happens, for example, when light passes through a narrow slit. The light emerging from the slit spreads out in the same way as water waves spread out after passing through a gap. The narrower the gap or the longer the wavelength, the greater the amount of diffraction.

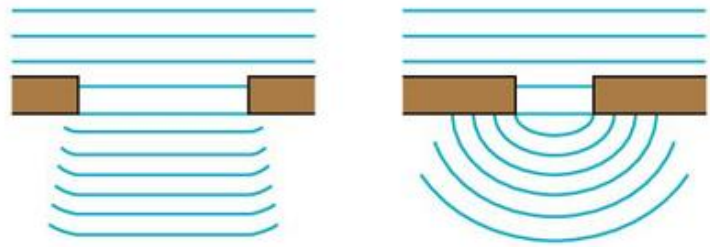


Fig 1 *Diffraction*

- The **particle-like nature** is observed, for example, in the photoelectric effect. When light is directed at a metal surface and an electron at the surface absorbs a photon of frequency f, the kinetic energy of the electron is increased from a negligible value by hf. The electron can escape if the energy it gains from a photon exceeds the work function of the metal.

Matter waves

If light has a dual wave–particle nature, perhaps particles of matter also have a dual wave–particle nature. Electrons in a beam can be deflected by a magnetic field. This is evidence that electrons have a particle-like nature. The idea that matter particles also have a wave-like nature was first considered by de Broglie in 1923. By extending the ideas of duality from photons to matter particles, de Broglie put forward the hypothesis that:

- matter particles have a dual wave–particle nature
- the wave-like behaviour of a matter particle is characterised by a wavelength, its **de Broglie wavelength**, λ, which is related to the momentum, p, of the particle by means of the equation

$$\lambda = \frac{h}{p}$$

Since the momentum of a particle is defined as its mass × its velocity, according to de Broglie's hypothesis, a particle of mass m moving at velocity v has a de Broglie wavelength given by

$$\lambda = \frac{h}{mv}$$

Note that the de Broglie wavelength of a particle can be altered by changing the velocity of the particle.

Evidence for de Broglie's hypothesis

The wave-like nature of electrons was discovered when, three years after de Broglie put forward his hypothesis, it was demonstrated that a beam of electrons can be diffracted. Figure 2 shows in outline how this is done. After this discovery, further experimental evidence, using other types of particles, confirmed the correctness of de Broglie's theory.

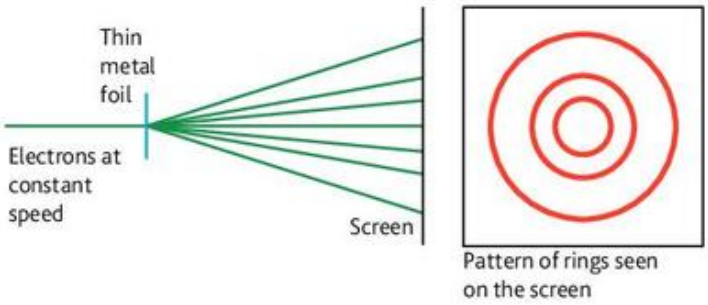


Fig 2 Diffraction of electrons

- A narrow beam of electrons in a vacuum tube is directed at a thin metal foil. A metal is composed of many tiny crystalline regions. Each region or 'grain' consists of positive ions arranged in fixed positions in rows in a regular pattern. The rows of atoms cause the electrons in the beam to be diffracted, just as a beam of light is diffracted by the slits.
- The electrons in the beam pass though the metal foil and are diffracted in certain directions only, as shown in Figure 2. They form a pattern of rings on a fluorescent screen at the end of the tube. Each ring is due to electrons diffracted by the same amount from grains of different orientations, at the same angle to the incident beam.
- The beam of electrons is produced by attracting electrons from a heated filament wire to a positively charged metal plate, which has a small hole at its centre. Electrons that pass through the hole form the beam. The speed of these electrons can be increased by increasing the potential difference between the filament and the metal plate. This makes the diffraction rings smaller, because the increase of speed makes the de Broglie wavelength smaller. So less diffraction occurs and the rings become smaller.

Energy levels explained

An electron in an atom has a fixed amount of energy that depends on the shell it occupies. Its de Broglie wavelength has to fit the shape and size of the shell. This is why its energy depends on the shell it occupies.

For example, an electron in a spherical shell moves round the nucleus in a circular orbit. The circumference of its orbit must be equal to a whole number of de Broglie wavelengths (circumference = $n\lambda$, where n = 1 or 2 or 3, etc.). You don't need to know this for your exam but this condition can be used to derive the energy level formula for the hydrogen atom – and it gives you a deeper insight into quantum physics.

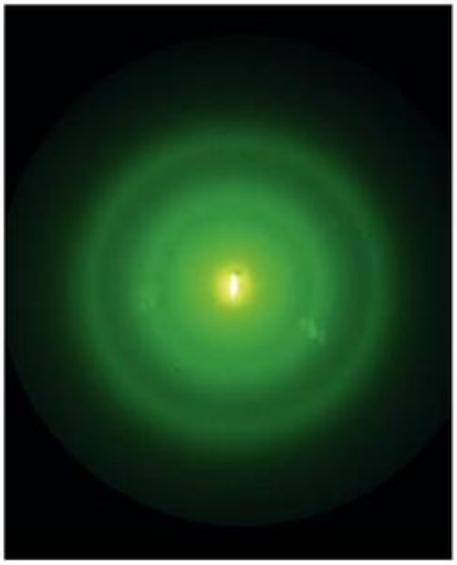


Fig 1 Electron diffraction

QUESTIONS

$h = 6.63 \times 10^{-34}$ J s, the mass of an electron = 9.11×10^{-31} kg, the mass of a proton = 1.67×10^{-27} kg

1 With the aid of an example in each case, explain what is meant by the dual wave–particle nature of:
a light, **b** matter particles.

2 State whether each of the following experiments demonstrates the wave nature or the particle nature of matter or of light: (a) photoelectricity, (b) electron diffraction.

3 Calculate the de Broglie wavelength of:
a an electron moving at a speed of 2.0×10^{7} m s^{-1}
b a proton moving at the same speed.

4 Calculate the momentum and speed of:
a an electron that has a de Broglie wavelength of 500 nm,
b a proton that has the same de Broglie wavelength.

22 End of chapter questions

1 The diagram below shows an electron gun in an evacuated tube. Electrons emitted by *thermionic emission* from the metal filament are attracted to the metal anode which is at a fixed potential, V, relative to the filament. Some of the electrons pass though a small hole in the anode to form a beam which is directed into a uniform magnetic field.

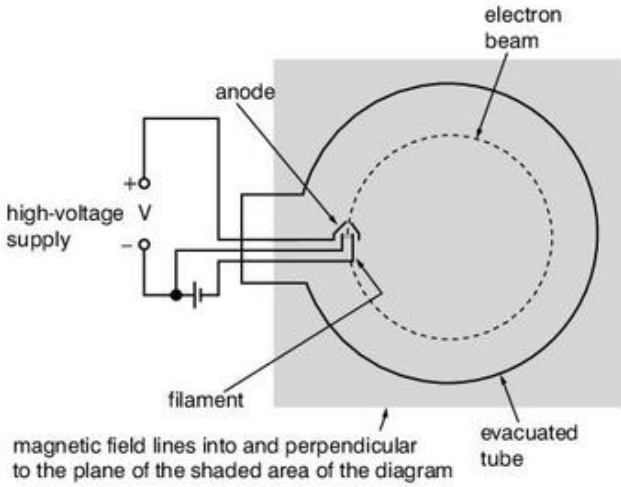


Fig 1.1

a i Explain what is meant by thermionic emission.

ii Show that the speed, v, of the electrons in the beam is given by

$$v = \sqrt{\frac{2eV}{m}}$$

where m is the mass of the electron and e is the charge of the electron.

b The beam of electrons travels through the field in a circular path at constant speed.

i Explain why the electrons travel at constant speed in the magnetic field.

ii Show that the radius, r, of the circular path of the beam in the field is given by

$$r = \sqrt{\frac{2mV}{B^2e}}$$

where B is the magnetic flux density and V is the p.d. between the anode and the filament.

iii The arrangement described above was used to measure the specific charge of the electron, e/m. Use the following data to calculate e/m.

$B = 3.1\,\text{mT}$

$r = 25\,\text{mm}$

$V = 530\,\text{V}$

2 A narrow beam of electrons, all with the same kinetic energy, is directed between two horizontal deflecting plates, X and Y, in a vacuum tube, as shown.

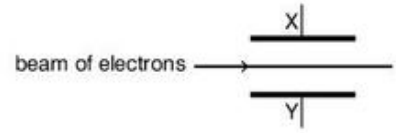


Fig 2.1

a State and explain the effect on the electron beam of applying a constant p.d. between X and Y, with X negative relative to Y.

b With a constant pd, V, between X and Y, a uniform magnetic field is applied perpendicular to the plane of the diagram between the plates. The magnetic flux density is adjusted to a certain value B_0, so that the beam is undeflected.

i Explain why the beam is undeflected at this value of the magnetic flux density.

ii Show that the speed, v, of the electrons in the beam is given by

$$v = \frac{V}{B_0 d},$$

where d is the perpendicular distance between plates X and Y.

c Electrons are accelerated from rest through a p.d. of 3800 V to a speed of $3.7 \times 10^7\,\text{m s}^{-1}$. Use this information to calculate the specific charge e/m of the electron.

3 The diagram below shows a charged oil droplet between two oppositely-charged horizontal parallel plates X and Y which are 6.0 mm apart.

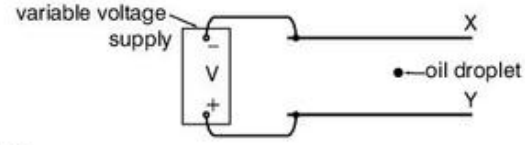


Fig 3.1

a When the potential difference between the two plates is zero, the droplet falls vertically at a steady speed of $7.8 \times 10^{-5}\,\text{m s}^{-1}$.

density of oil droplet $= 960\,\text{kg m}^{-3}$

viscosity of air $= 1.8 \times 10^{-5}\,\text{N s m}^{-2}$

i Explain why the droplet falls at a steady speed.

ii Show that the radius of the droplet is $8.2 \times 10^{-7}\,\text{m}$.

iii Show that the mass of the droplet is $2.2 \times 10^{-15}\,\text{kg}$.

b The potential difference between X and Y is adjusted until the droplet becomes stationary.

i Explain why the droplet becomes stationary.

ii The droplet is stationary when the potential difference is 410 V. Show that the charge of the droplet is $3.2 \times 10^{-19}\,\text{C}$.

iii Discuss the significance of this result and the results of similar tests on other charged droplets.

4 Negatively-charged particles are moving through a vacuum in a parallel beam. The particles have speed v.

The particles enter a region of uniform magnetic field of flux density 930 μT. Initially, the particles are travelling at right-angles to the magnetic field. The path of a single particle is shown in Figure 4.1.

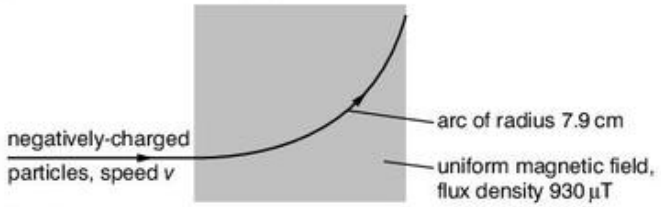


Fig 4.1

The negatively-charged particles follow a curved path of radius 7.9 cm in the magnetic field. A uniform electric field is then applied in the same region as the magnetic field. For an electric field strength of 12 kV m^{-1}, the particles are undeviated as they pass through the region of the fields.

a On a copy of Figure 4.1, mark with an arrow the direction of the electric field. *(1 mark)*

b Calculate, for the negatively-charged particles,

i the speed v, in m s^{-1} *(3 marks)*

ii the ratio $\frac{\text{charge}}{\text{mass}}$ in C kg^{-1} *(3 marks)*

Adapted from Cambridge International AS and A Level Physics 9702 Paper 41 Q7 June 2010

5 **a** Explain why, for the photoelectric effect, the existence of a threshold frequency and a very short emission time provide evidence for the particulate nature of electromagnetic radiation, as opposed to a wave theory. *(4 marks)*

b State and explain two relations in which the Planck constant h is the constant of proportionality. *(6 marks)*

Cambridge International AS and A Level Physics 9702 Paper 04 Q8 June 2009

6 Electrons can escape from a certain metal surface when it is illuminated by blue light, but not when it is illuminated by red light:

a Explain why blue light causes emission of electrons from this metal, whereas red light does not.

b What difference would be made to the emission of electrons from this metal surface if:

i the intensity of the blue light were increased,

ii the surface were at a positive potential?

7 Light of wavelength 600 nm was used to illuminate a metal surface. The maximum kinetic energy of an electron emitted as a result was 1.3×10^{-19} J. Calculate:

a the energy of a photon of this wavelength,

b the work function of the metal surface.

8 **a** Explain the meaning of the term 'work function' of a metal surface.

b **i** Calculate the wavelength of a photon of energy 2.0×10^{-19} J.

ii Explain why photons of wavelength longer than the value calculated in part (i) could not cause photoelectric emission from a metal surface which has a work function of 2.0×10^{-19} J.

9 A vacuum photocell connected to a microammeter is shown below.

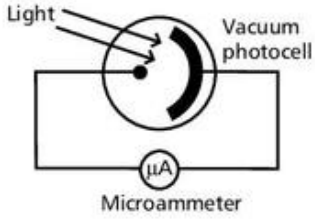


Fig 9.1

a Explain why:

i the microammeter registers a current when light of a certain wavelength is directed at the photocell,

ii the current increases when the light intensity is increased.

b When light of a longer wavelength is directed at the photocell, no current is registered:

i Explain why there is no current in this situation.

ii Explain why making the light more intense would not cause a current with light of this longer wavelength.

10 The spectrum of light from a sodium lamp has two prominent closely-spaced lines at wavelengths of 589.0 nm and 589.6 nm. These lines are due to electron transitions from two closely-spaced energy levels X and Y to the same lower energy level Z, as shown below (not to scale).

a Copy the diagram and show the electron transition that causes the emission of photons of wavelength **i** 589.0 nm, **ii** 589.6 nm

b Calculate the energy difference between the two closely-spaced energy levels.

11 State whether each of the following experiments or observations using light demonstrates the wave-like or the particle-like nature of light:

i When light passes through a narrow gap, it diffracts.

ii When light is directed at a metal surface, electrons are emitted from the surface only if the light wavelength is less than a certain value.

12 **a** An electron in a beam has a speed of 1.5×10^{7} m s^{-1}. Calculate:

i the kinetic energy,

ii the momentum and de Broglie wavelength of this electron.

b Calculate the wavelength of a photon with the same energy as the electron in part **a**.

23 Nuclear physics

23.1 Radioactive decay

On these pages you will learn to:

- Show an appreciation from the fluctuations in count rate that radioactive decay has a spontaneous and random nature
- Define the terms activity and decay constant and recall and solve problems using $A = \lambda N$

Half-life

When a nucleus of a radioactive isotope emits an α- or a β-particle, it becomes a nucleus of a different isotope because its proton number changes. The number of nuclei of the initial radioactive isotope therefore decreases. The mass of the initial isotope decreases gradually as the number of nuclei of the isotope decreases. Figure 1 shows how the mass decreases with time. The curve is referred to as a decay curve. The mass of the isotope decreases at a slower and slower rate. Measurements show that the mass decreases exponentially which means that the mass drops by a constant factor (e.g. ×0.8) in equal intervals of time. For example, if the initial mass of the radioactive isotope is 100 g and the mass decreases by a factor of ×0.8 every 1000 seconds, then:

- after 1000 s, the mass remaining = 80 g (= 0.8 × 100 g),
- after 2000 s, the mass remaining = 64 g (= 0.8 × 0.8 × 100 g),
- after 3000 s, the mass remaining = 51 g (= 0.8 × 0.8 × 0.8 × 100 g).

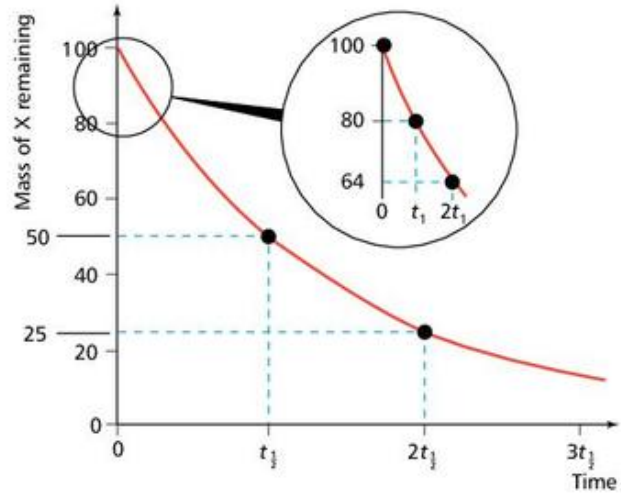


Fig 1 *A radioactive decay curve*

A convenient measure for the rate of decrease is the time taken for a decrease by half. This is the half-life of the process.

The half-life, $t_{\frac{1}{2}}$, of a radioactive isotope is the time taken for the mass of the isotope to decrease to half the initial mass.

This is the same as the time taken for the number of nuclei of the isotope to decrease to half the initial number.

Consider a sample of a radioactive isotope X which initially contains 100 g of the isotope.

- After 1 half-life, the mass of X remaining = $0.5 \times 100 = 50$ grams
- After 2 half-lives from the start, the mass of X remaining = $0.5^2 \times 100 = 25$ grams
- After 3 half-lives from the start, the mass of X remaining = $0.5^3 \times 100 = 12.5$ grams
- After n half-lives from the start, the mass of X remaining = $0.5^n m_0$, where m_0 = the initial mass

The mass of X decreases exponentially. This is because radioactive decay is a **random** process. We know this because the fluctuations in the count rate of a Geiger counter are random variations. So the number of nuclei that decay in a certain time is in proportion to the number of nuclei of X remaining.

Randomness

To understand the idea of randomness, consider a game of dice starting with 1000 dice, each representing a nucleus of X. The throw of a dice is a random process in which each face has a 1 in 6 chance of being uppermost.

After each throw, suppose all the dice showing '1' uppermost are removed. For example, after the first throw 167 dice (= 1000/6) would be removed and 833 (= 1000 − 167) would remain. The table below shows how many dice remain after each throw.

Throw	Number of dice		
	initially	removed	remaining
1st	1000	167	833
2nd	833	139	694
3rd	694	116	578
4th	578	96	482
5th	482	80	402

The analysis shows that 4 throws are needed to reduce the number of dice remaining to less than half the initial number. Prove for yourself that a further 4 throws would reduce the number of dice remaining to 25% of the initial number. Figure 2 shows how the number of dice remaining decreases with time. The curve has the same shape as Figure 1. The half-life of the process is 3.8 'throws'.

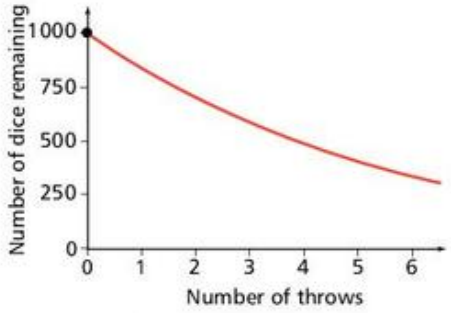


Fig 2 Exponential decrease

Activity

The activity A of a radioactive isotope is the number of nuclei of the isotope that disintegrate per second. In other words, it is the rate of change of the number of nuclei of the isotope. The unit of activity is the **becquerel (Bq)**, where 1 Bq = 1 disintegration per second.

The activity of a radioactive isotope is proportional to the mass of the isotope. Because the mass of a radioactive isotope decreases with time due to radioactive decay, the activity decreases with time. Figure 3 shows an experiment in which the activity of a radioactive isotope of protoactinium $^{234}_{91}$Pa is measured and recorded using a Geiger tube and a counter. This isotope is a β-emitter produced by the decay of the radioactive isotope of thorium $^{234}_{90}$Th. In this experiment, an organic solvent in a sealed bottle is used to separate protoactinium from thorium to enable the activity of the protoactinium to be monitored.

Before the experiment is carried out, the background count rate is measured without the bottle present. The bottle is then shaken to mix the aqueous and solvent layers and then placed near the end of the Geiger tube. The layers are allowed to separate as shown in Figure 3. The protoactinium is collected by the solvent and the thorium by the aqueous layer. The Geiger tube detects β-particles emitted by the decay of the protoactinium nuclei in the solvent layer.

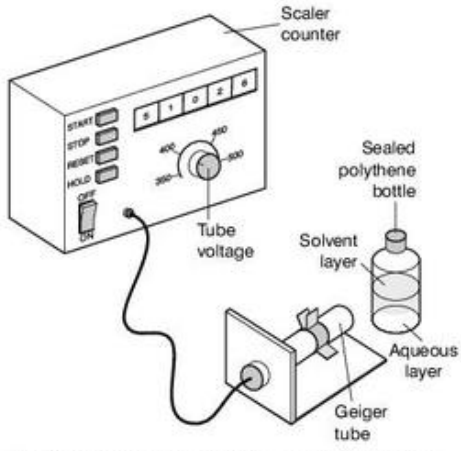


Fig 3 Measuring the activity of protoactinium

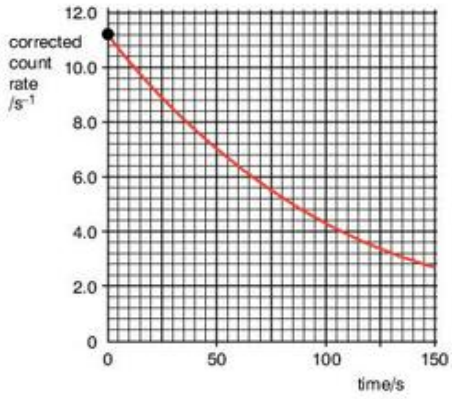


Fig 4 A radioactive decay curve

Reminder about molar mass and the Avogadro constant

As explained in topic 21.2 for an element with a mass number A,

- its molar mass is the mass of 1 mole,
- one mole of the element contains N_A atoms where N_A is the Avogadro constant,
- mass m of the element contains $\left(\frac{m}{M}\right) N_A$ atoms.

The counter is used to measure the number of counts every 10 seconds. The count rate is the number of counts in each ten-second interval divided by 10 s. The background count rate is subtracted to give the corrected count rate. Since the activity is proportional to the corrected count rate, a graph of the corrected count rate against time, as in Figure 4, shows how the activity of the protoactinium decreases with time.

Activity and power

For a radioactive source of activity A that emits particles (or photons) of the same energy E, the energy per second released by radioactive decay in the source by the radiation is the product of its activity and the energy of each particle. In other words, the power of the source $= AE$.

The energy transfer per second from a radioactive source $= AE$

If the source is in a sealed container and emits only α-particles which are all absorbed by the container, the container gains thermal energy from the absorbed radiation equal to the energy transferred from the source. For example, for a source that has an activity of 30 MBq and emits particles of energy 2.5 MeV, the energy transfer per second from the source is given by

$$30 \times 10^6\,\text{Bq} \times 2.5\,\text{MeV} = 7.5 \times 10^7\,\text{MeV s}^{-1} = 1.2 \times 10^{-5}\,\text{J s}^{-1}.$$

QUESTIONS

$N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$, $1\,\text{MeV} = 1.6 \times 10^{-13}\,\text{J}$

1 Figure 4 shows how the activity of protoactinium $^{234}_{91}\text{Pa}$ decreases with time.

a Use the graph to work out the half-life of this isotope,

b If the initial mass of the isotope was 48 g, calculate the mass of the isotope remaining after three half lives.

2 A freshly prepared sample of a radioactive isotope X contains 1.8×10^{15} atoms of the isotope. The half-life of the isotope is 8.0 hours. Calculate:

a the number of atoms of this isotope remaining after:
i 8 hours,
ii 24 hours.

b the number of atoms of X that would have decayed after:
i 8 hours,
ii 24 hours.

c the energy transfer from the sample in 24 hours if the isotope emits α-particles of energy 5 MeV.

3 $^{131}_{53}\text{I}$ is a radioactive isotope of iodine which has a half-life of 8.0 days.

A sample of this isotope has an initial activity of 38 kBq. Calculate the activity of this sample:

a 8.0 days later,

b 32 days later.

4 $^{137}_{55}\text{Cs}$ is a radioactive isotope of caesium which has a half-life of 35 years.

A sample of this isotope has a mass of 1.0×10^{-3} kg.

a Calculate the number of atoms in 1.0×10^{-3} kg of this isotope.

b Calculate the number of atoms of the isotope remaining in the sample after 70 years.

23.2 The theory of radioactive decay

On these pages you will learn to:

- Solve problems using the relationship $x = x_0e^{-\lambda t}$, where x could represent activity, number of undecayed nuclei or count rate
- Sketch a graph to show the exponential nature of radioactive decay
- Define half-life and solve related problems using the equations above and the relationship $t_{\frac{1}{2}} = 0.693/\lambda$

The random nature of radioactive decay

An unstable nucleus becomes stable by emitting an α- or a β-particle or a γ-photon. This is an unpredictable event. Every nucleus of a radioactive isotope has an equal probability of becoming stable in any given time interval. Therefore, for a large number of nuclei of a radioactive isotope, the number of nuclei that disintegrate in a certain time interval depends only on the total number of nuclei present. The same idea was considered in the dice experiment. The greater the number of dice used, the more likely it is that 1 in every 6 dice show a particular number on the upper face.

Consider a sample of a radioactive isotope X that initially contains N_0 nuclei of the isotope.

Let N represent the number of nuclei of X remaining at time t after the start.

Suppose in time Δt, the number of nuclei that disintegrate is ΔN.

Because radioactive disintegration is a random process, ΔN is proportional to:

1 N, the number of nuclei of X remaining at time t,
2 the duration of the time interval Δt.

Therefore, $\Delta N = -\lambda N \Delta t$, where λ is a constant referred to as the **decay constant**. The minus sign is necessary because ΔN is a decrease.

So, the rate of disintegration, $\dfrac{\Delta N}{\Delta t} = -\lambda N$

For a given radioactive isotope, its activity is the rate of disintegration $\dfrac{\Delta N}{\Delta t}$

Therefore, the activity A of N atoms of a radioactive isotope is given by

$$A = \lambda N$$

The solution of the equation $\dfrac{\Delta N}{\Delta t} = -\lambda N$ is $N = N_0e^{-\lambda t}$

where e^x is the exponential function.

Figure 1 shows that a graph of N against t gives a decay curve. The number of nuclei N decreases exponentially with time. In other words,

- in one half-life, the remaining number of nuclei $N_1 = 0.5N_0$
- in two half-lives, the remaining number of nuclei $N_2 = 0.25N_0$
- in n half-lives, the remaining number of nuclei $N = 0.5^nN_0$

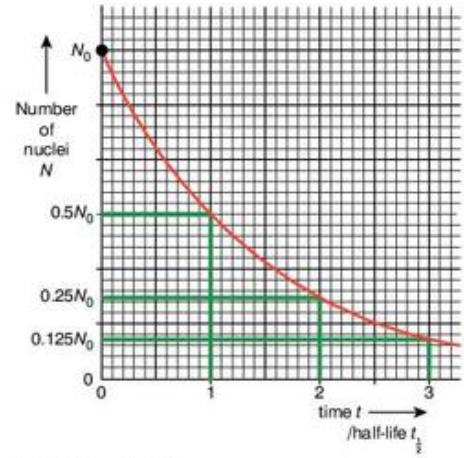


Fig 1 $N = N_0e^{-\lambda t}$

The graph of the number of nuclei N against time t as represented by the equation $N = N_0e^{-\lambda t}$ is shown in Figure 1 above. It is a curve with exactly the same shape as Figure 1 of topic 23.1.

The mass, m, of a radioactive isotope decreases from initial mass, m_0, in accordance with the equation $m = m_0e^{-\lambda t}$ because the mass, m, is proportional to the number of nuclei, N, of the isotope.

The activity, A, of a sample of N nuclei of an isotope decays in accordance with the equation

$$A = A_0e^{-\lambda t}$$

where A_0 is the initial activity (or the activity at $t = 0$).

This is because the activity, A = the magnitude of the number of disintegrations per second $= \lambda N$. Hence, $A = \lambda N_0e^{-\lambda t} = A_0e^{-\lambda t}$ where $A_0 = \lambda N_0$.

The count rate, C, due to a sample of a radioactive isotope at a fixed distance from a Geiger tube is proportional to the activity of the source. Therefore, the count rate decreases with time in accordance with the equation $\boldsymbol{C = C_0 e^{-\lambda t}}$, where C_0 is the count rate at time $t = 0$.

The above equations for the number of nuclei, N, the activity, A, and the count rate, C, are all of the same general form, namely $\boldsymbol{x = x_0 e^{-\lambda t}}$, where x represents N or A or C and x_0 represents the initial value.

1 The exponential function appears in any situation where the rate of change of a quantity is in proportion to the quantity itself. This is because the rate of change of each term in the function sequence is equal to the previous term in the sequence.

2 The exponential function, $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ (See Mathematical skills)

Differentiating e^x with respect to x gives e^x $\left(\text{i.e. } \frac{d(e^x)}{dx} = e^x\right)$ because differentiating each term in the expression for e^x gives the previous term.
The exponential function is indicated on a calculator as 'exp' or 'e^x' or 'inv ln'. (See Mathematical skills)

3 The natural logarithm function, $\ln x$, is the inverse exponential function. In other words, if $y = e^x$, then $\ln y = x$. Therefore, $N = N_0 e^{-\lambda t}$ may be written $\ln N = \ln N_0 - \lambda t$.
The graph of $\ln N$ against t is therefore a straight line with
- a gradient $= -\lambda$, and
- a y-intercept $= \ln N_0$.

4 The exponential decrease formula is also used in the theory of capacitor discharge. (See topic 15.4)

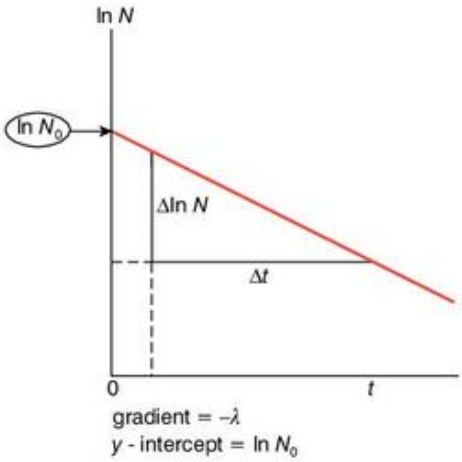


Fig 2 ln N versus t

Worked example

A sample of a radioactive isotope initially contains 1.2×10^{20} atoms of the isotope. The decay constant for the isotope is $3.6 \times 10^{-3}\,s^{-1}$. Calculate:

a the number of atoms of the isotope remaining after 1000 s,
b the activity of the sample after 1000 s.

Solution

a $N_0 = 1.2 \times 10^{20}$, $\lambda = 3.6 \times 10^{-3}\,s^{-1}$, $t = 1000\,s$,
$\lambda t = 3.6 \times 10^{-3}\,s^{-1} \times 1000\,s = 3.6$
$\therefore N = N_0 e^{-\lambda t} = 1.2 \times 10^{20} e^{-3.6} = 1.2 \times 10^{20} \times 2.7 \times 10^{-2} = 3.2 \times 10^{18}$

b Activity, $A = \lambda N = 3.6 \times 10^{-3} \times 3.2 \times 10^{18} = 1.2 \times 10^{16}$ Bq

The decay constant

The decay constant λ is the probability of an individual nucleus decaying per second. If there are 10 000 nuclei present and 300 decay in 20 seconds, the decay constant is $0.0015\,s^{-1}$ $\left(= \frac{\left(\frac{300}{10\,000}\right)}{20\,s}\right)$.

In general, if the change of the number of nuclei ΔN in time Δt is given by $\Delta N = -\lambda N \Delta t$, then, the probability of decay, $\frac{\Delta N}{N} = \lambda \Delta t$ (the minus sign is not needed here as reference is made to decay).

So, the probability per unit time $= \frac{\frac{\Delta N}{N}}{\Delta t} = \lambda$

As explained on p.348, the **half-life**, $t_{½}$, of a radioactive isotope is the time taken for half the initial number of nuclei to decay. The longer the half-life, the smaller the decay constant because the probability of decay per second is smaller.

The half-life $t_{½}$ is related to the decay constant λ according to the equation

$$t_{½} = \frac{\ln 2}{\lambda}$$

As $\ln 2 = 0.693$, this equation may be written as $t_{½} = \frac{0.693}{\lambda}$

Proof of $t_{½} = \frac{\ln 2}{\lambda}$

The proof of this equation is not part of this specification. It is provided below to help you develop a better understanding of the topic.

Let the number of nuclei $N = N_0$ at time $t = 0$, so at time $t = t_{½}$, $N = 0.5\,N_0$

Inserting $t = t_{½}$, $N = 0.5\,N_0$ into $N = N_0 e^{-\lambda t}$ gives $0.5\,N_0 = N_0 e^{-\lambda t_{½}}$

Cancelling N_0 and taking the natural logarithm (ln) of each side gives $\ln 0.5 = -\lambda t_{½}$

Because $\ln 0.5 = -\ln 2$, then $\ln 2 = \lambda t_{½}$.

Rearranging this equation gives $t_{½} = \frac{\ln 2}{\lambda}$

Note To calculate N at time t, given values of N_0 and $t_{½}$,

- **either** calculate λ using $\lambda = \frac{\ln 2}{t_{½}}$ then use the equation $N = N_0 e^{-\lambda t}$,
- **or** calculate the number of half-lives, n, using $n = \frac{t}{t_{½}}$ then use $N = 0.5^n N_0$

QUESTIONS

$N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$, $1\,\text{MeV} = 1.6 \times 10^{-13}\,\text{J}$

1 $^{131}_{53}\text{I}$ is a radioactive isotope of iodine which has a half-life of 8.0 days. A fresh sample of this isotope contains 4.2×10^{16} atoms of isotope. Calculate:

a the decay constant of this isotope,

b the number of atoms of this isotope remaining after 24 hours.

2 A radioactive isotope has a half-life of 35 years. A fresh sample of this isotope has an activity of 25 kBq. Calculate:

a the decay constant in s^{-1},

b the activity of the sample after 10 years.

3 **a** Calculate the number of atoms present in 1.0 kg of $^{226}_{88}\text{Ra}$.

b The isotope $^{226}_{88}\text{Ra}$ has a half-life of 1620 years. For an initial mass of 1.0 kg of this isotope, calculate:

i the mass of this isotope remaining after 1000 years,

ii how many atoms of the isotope will remain after 1000 years.

4 A fresh sample of a radioactive isotope has an initial activity of 40 kBq. After 48 hours, its activity has decreased to 32 kBq. Calculate:

a the decay constant of this isotope,

b its half-life.

23.3 Radioactive isotopes in use

On these pages you will learn to:

- Show an understanding of the uses of radioactive isotopes in terms of the properties of the isotopes used and the radiation emitted
- Use the equations in topic 23.2 to solve problems in connection with the use of radioactive isotopes

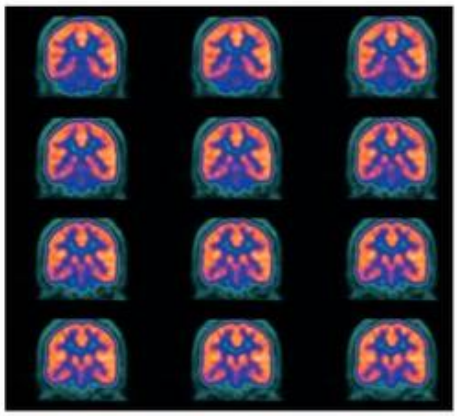


Fig 1 PET (positron emission tomography) scan of the brain – one of the many uses of radioactive tracers

Radioactive isotopes are used for many purposes. The choice of an isotope for a particular purpose depends on its half-life and on the type of radiation it emits. For some uses, the choice also depends on how the isotope is obtained and on whether or not it produces a stable decay product. The following examples are intended to provide a wider awareness of important uses of radioactive substances and to set contexts in which knowledge and understanding of radioactivity is developed further. Read again topics 10.3 and 10.4 about α, β and γ radiation if necessary.

Radioactive tracers

A radioactive tracer is used to follow the path of a substance through a system, as illustrated below.

- **Environmental uses**; for example, the detection of leaks in underground pipes that carry water or oil. Such a leak can be detected by injecting a radioactive tracer into the flow. Geiger tubes on the surface above the pipeline can then be used to detect leakage. The radioactive isotope used should have a half-life short enough so it decays quickly after use and long enough so the test can be completed before its activity becomes too low. In addition, it should be a β-emitter as α-radiation would be absorbed by the pipes and γ radiation would pass through the pipes without absorption.
- **Geological uses**; for example, to improve the recovery of oil from an underground reservoir. Water containing a radioactive tracer is injected into an oil reservoir at high pressure, forcing some of the oil out. Detectors at the production wells monitor breakthrough of the radioactive isotope. The results are used to build up a model of the reservoir to improve and control recovery. Because the time from injection to breakthrough can be many months, the tracer must have a suitably long half-life. A suitable tracer is 'tritiated' water ^3H_2O, a β-emitter with a half-life of 12 years.
- **Medical uses**; for example, to monitor the uptake of iodine by the thyroid gland. The thyroid gland absorbs iodine to maintain its function of producing a hormone. The rate of uptake is measured by giving the patient a solution containing sodium iodide which includes a small quantity of the radioactive isotope of iodine, $^{131}_{53}I$, which is a β-emitter with a half-life of 8 days. The activity of the patient's thyroid and the activity of an identical sample prepared at the same time are measured 24 hours later. The percentage uptake by the patient is then calculated from (the corrected count rate of the thyroid/the corrected count rate of the identical solution) × 100%. A normal thyroid has a percentage uptake of 20–50% after 24 hours.
- **Agricultural research**; for example, to investigate the uptake of fertilisers by plants. This can be done by using a fertiliser which contains the radioactive isotope of phosphorus, $^{32}_{15}P$, which is a β-emitter with a half-life of 14 days. By measuring the radioactivity of the leaves, the amount of fertiliser reaching them can be determined.

Radioactive dating

- **Carbon dating**; living plants and trees contain a small percentage of the radioactive isotope of carbon, $^{14}_{6}C$, which is formed in the atmosphere as a result of cosmic rays knocking out neutrons from nuclei. These neutrons then collide with nitrogen nuclei to form carbon-14 nuclei.

$$^{1}_{0}n + {}^{14}_{7}N \rightarrow {}^{14}_{6}C + {}^{1}_{1}p$$

Carbon dioxide from the atmosphere is taken up by living plants as a result of photosynthesis. So a small percentage of the carbon content of any plant is carbon-14. This isotope has a half-life of 5570 years so there is negligible decay during the life-time of a plant. Once a tree has died, no further carbon is taken in so the proportion of carbon-14 in the dead tree decreases as the carbon-14 nuclei decay. Because activity is proportional to the number of atoms still to decay, measuring the activity of the dead sample enables its age to be calculated, provided the activity of the same mass of living wood is known.

Worked example

A certain sample of dead wood is found to have an activity of 0.28 Bq. An equal mass of living wood is found to have an activity of 1.3 Bq. Calclulate the age of the sample.

The half-life of carbon-14 is 5570 years.

Solution

The half-life, $t_{\frac{1}{2}}$, in seconds $= 5570 \times 365 \times 24 \times 3600\,\text{s} = 1.76 \times 10^{11}\,\text{s}$

$\therefore$ the decay constant of carbon-14, $\lambda = \dfrac{0.693}{t_{\frac{1}{2}}} = \dfrac{0.693}{1.76 \times 10^{11}} = 3.95 \times 10^{-12}\,\text{s}^{-1}$

Using activity $A = A_0 e^{-\lambda t}$ where $A = 0.28$ Bq and $A_0 = 1.30$ Bq gives

$$0.28 = 1.3e^{-\lambda t} \quad \text{so} \quad e^{-\lambda t} = \left(\frac{0.28}{1.30}\right) = 0.215$$

$$\therefore \quad \lambda t = 1.535$$

$$t = \frac{1.535}{\lambda} = \frac{1.535}{3.95 \times 10^{-12}\,\text{s}} = 3.88 \times 10^{11}\,\text{s} = 12300 \text{ years}$$

Note A useful check is to estimate the number of half-lives needed for the activity to decrease from 1.30 Bq to 0.28 Bq. You should find that just over 2 half-lives are needed, corresponding to about 11 000 years.

- **Argon dating**; ancient rocks contain trapped argon gas as a result of the decay of the radioactive isotope of potassium, ${}^{40}_{19}\text{K}$ into the argon isotope ${}^{40}_{18}\text{Ar}$. This happens when its nucleus captures an inner shell electron. As a result, a proton in the nucleus changes into a neutron and a neutrino is emitted. The equation for the change is

$${}^{40}_{19}\text{K} + {}^{0}_{-1}\text{e} \rightarrow {}^{40}_{18}\text{Ar} + \nu$$

The potassium isotope ${}^{40}_{19}\text{K}$ also decays by β-emission to form the calcium isotope ${}^{40}_{20}\text{Ca}$. This process is 8 times more probable than electron capture.

$${}^{40}_{19}\text{K} \rightarrow {}^{0}_{-1}\beta + {}^{40}_{20}\text{Ca} + \bar{\nu}$$

The effective half-life of the decay of ${}^{40}_{19}\text{K}$ is 1250 million years. The age of the rock (i.e. the time from when it solidified) can be calculated by measuring the proportion of argon-40 to potassium-40. For every n potassium-40 atoms now present, if there is 1 argon-40 atom present, there must have originally been $n + 9$ potassium atoms. (i.e. 1 that decayed into argon-40 + 8 that decayed into calcium-40 + n remaining). The radioactive decay equation $N = N_0 e^{-\lambda t}$ can then be used to find the age of the sample. For example, suppose for every 4 potassium-40 atoms now present, a certain rock now has 1 argon-40 atom. Therefore, $N = 4$ and $N_0 = 13$. Substituting these values into the equation $N = N_0 e^{-\lambda t}$ gives $4 = 13e^{-\lambda t}$.

Therefore, $e^{-\lambda t} = \dfrac{4}{13} = 0.308$ which gives $t = \dfrac{-\ln 0.308}{\lambda}$. Substituting $\dfrac{0.693}{t_{\frac{1}{2}}}$ for λ into this equation gives $t = \left(\dfrac{-\ln 0.308}{0.693}\right) t_{\frac{1}{2}} = 1.70\, t_{\frac{1}{2}}$. The age of the sample is therefore 2120 million years.

Note A useful check is to estimate the number of half-lives needed for N to decrease from 13 to 4. You should find that between 1 and 2 half-lives are needed, corresponding to an age of between 1250 and 2500 million years.

Fig 2 *Pistons (and piston rings) fit into this engine block*

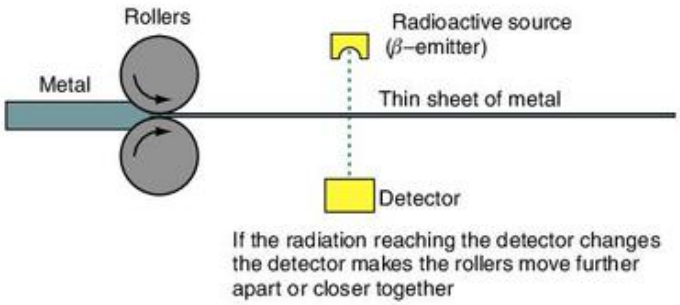


Fig 3 *The manufacture of metal foil*

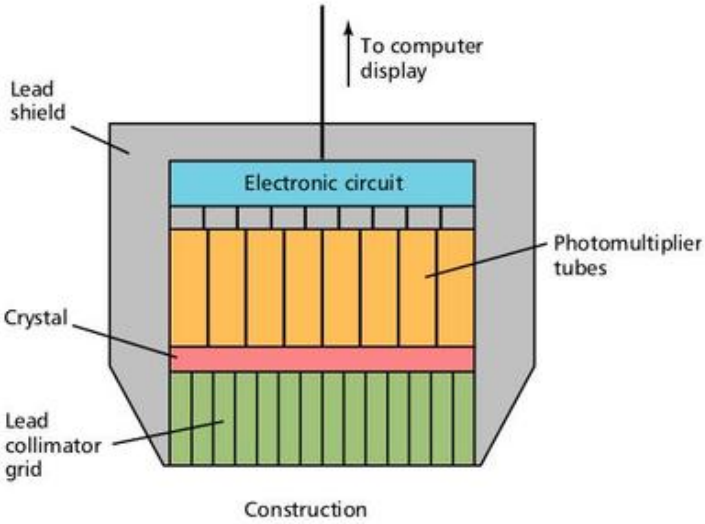


Fig 4 *The gamma camera*

Industrial uses

The examples below are just two of a wide range of applications of radioactivity in industry.

- **Engine wear**
  The rate of wear of a piston ring in an engine can be measured by fitting a ring that is radioactive. As the ring slides along the piston compartment, radioactive atoms transfer from the ring to the engine oil. By measuring the radioactivity of the oil, the mass of radioactive metal transferred from the ring can be determined and the rate of wear calculated. A metal ring can be made radioactive by exposing it to neutron radiation in a nuclear reactor. Each nucleus that absorbs a neutron becomes unstable and disintegrates by β-emission.
- **Thickness monitoring**
  Metal foil is manufactured by using rollers to squeeze plate metal on a continuous production line, as shown in Figure 3. A detector measures the amount of radiation passing through the foil. If the foil is too thick, the detector reading drops. A signal from the detector is fed back to the control system to make the rollers move closer together and so make the foil thinner. The source used is a β-emitter with a long half-life. α radiation would be absorbed completely by the foil and γ radiation would pass straight through without absorption.
- **Power for remote devices** such as satellites and weather sensors can be obtained using a radioactive isotope in a thermally insulated sealed container which absorbs all the radiation emitted by the isotope.
  A thermocouple attached to the container produces electricity as a result of the container becoming warm through absorbing radiation. For mass, m, of the isotope, its activity, $A = \lambda N$ where N is the number of radioactive atoms present in mass m. If each disintegration of a nucleus releases energy E, the energy transfer per second from the source $= \lambda NE$. The source needs to have a reasonably long half-life so it does not need to be replaced frequently but a very long half-life would require too much mass to generate the necessary power.

Medical and health uses

As explained earlier, radioactive isotopes are used as medical tracers. Further uses of radioactive isotopes in medicine include the gamma camera used to form images of joints and organs, the PET scanner and gamma therapy to destroy cancerous tissues.

- **The gamma camera** is designed to detect γ radiation from sites inside the body where a γ-emitting isotope is located. For example, bone deposits can be located using a phosphate tracer containing the radioactive isotope of technetium, $^{99}_{52}$Te which is a γ-emitter that has a half-life of 6 hours.
  The γ-photons from inside the body are absorbed by a lead collimator grid unless they travel parallel to narrow

channels through the collimator. Each γ-photon that passes through the grid strikes a large sodium iodide crystal, causing a flash of light which is detected by a photomultiplier tube in an array of tubes. The tubes are connected to a computer which displays an image of the γ-emitting sources in the body.

- **Gamma therapy** is used to destroy tumours inside the body. A narrow beam of γ radiation from the radioactive isotope of cobalt, $^{60}_{27}$Co, is directed at the tumour from different directions by moving the source or by moving the patient. This movement is necessary to ensure healthy tissue in the path of the beam is exposed much less than the target tissue. The cobalt-60 source has a half-life of 5.3 years and emits γ-photons of energies 1.17 MeV and 1.33 MeV. The source is enclosed in a thick lead container. When the source is to be used, it is rotated to the inner end of an exit channel so that a beam of γ radiation emerges after passing along the exit channel. When the source is not in use, it is rotated away from the inner end of the exit channel so no γ radiation can emerge from the container.
- **Food preservation** can be achieved by irradiating food with γ radiation. About 20% of the world's food is lost through spoilage. The major cause is bacteria, moulds and yeast which grow on food. Some bacteria produce toxic waste products that cause food poisoning. Irradiation of food with γ radiation kills 99% of the disease-carrying organisms in the food, such as *Salmonella* which infects poultry and *Clostridium*, the cause of botulism. The treatment is not suitable for all foods. Red meat turns brown and develops an unpleasant taste, eggs develop a smell and tomatoes go soft.

QUESTIONS

$N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$, $1\,\text{MeV} = 1.6 \times 10^{-13}\,\text{J}$

1 a Explain why living wood is slightly radioactive.

b A sample of ancient wood of mass 0.5 g is found to have an activity of 0.11 Bq. A sample of living wood of the same mass has an activity of 0.13 Bq. Calculate the age of the sample of wood.

The half-life of radioactive carbon $^{14}_{6}$C is 5570 years.

2 The radioactive isotope of iodine, $^{131}_{53}$I, is used for medical diagnosis of the kidneys. The isotope has a half-life of 8 days. A sample of the isotope is given to a patient in a glass of water. The passage of the isotope through each kidney is then monitored using two detectors outside the body. The isotope is required to have an activity of 800 kBq at the time it is given to the patient.

a Calculate:

i the activity of the sample 24 hours after it was given to the patient,

ii the activity of the sample when it was prepared 24 hours earlier,

iii the mass of $^{131}_{53}$I in the sample when it was prepared.

b The reading from the detector near one of the patient's kidneys rises then falls. The reading from the other detector which is near the other kidney rises and does not fall. Discuss the conclusions that can be drawn from these observations.

3 a i In the manufacture of metal foil, describe how the thickness of the foil is monitored using a radioactive source and a detector.

ii Explain why the source needs to be a β-emitter, not an α-emitter or a γ-emitter.

b i Explain why a cobalt-60 source used for γ-therapy is enclosed in a thick lead lined container.

ii Explain why a beam of γ radiation used to destroy a tumour inside a patient is directed at the tumour from different directions during treatment.

4 a A cardiac pacemaker is a device used to ensure that a faulty heart beats at a suitable rate. The required electrical energy in one type of pacemaker is obtained from the energy released by a radioactive isotope. The radiation is absorbed inside the pacemaker. As a result, the absorbing material gains thermal energy and heats a thermocouple attached to the absorbing material. The voltage from the thermocouple provides the source of electrical energy for the pacemaker.

i Discuss whether the radioactive source should be an α-emitter or a β-emitter or a γ-emitter.

ii The radioactive source needs to have a reasonably long half-life, otherwise it would need to be replaced frequently. Discuss the disadvantages of using a radioactive source with a very long half-life.

b The energy source for a remote weather station is the radioactive isotope of strontium, $^{90}_{38}$Sr which has a half-life of 28 years. It emits β-particles of energy 0.40 MeV. For a mass of 10 g of this isotope, calculate:

i its activity,

ii the energy released per second.

23.4 Energy from the nucleus

On these pages you will learn to:

- Show an appreciation of the association between energy and mass as represented by $E = mc^2$ and recall and use this relationship
- Show an understanding of the energy released in radioactive decay
- Solve problems relating the energy released in radioactive decay to the change of total mass

Energy and mass

In 1905, Einstein published the theory of special relativity in which he showed that moving clocks run more slowly than stationary clocks, fast-moving objects appear shorter than when stationary, the mass of a moving object changes with its speed and no material object can travel as fast as light. He also showed that the mass of an object increases (or decreases) when it gains (or loses) energy, E, in accordance with the equation

$$E = mc^2$$

where m is the change of its mass and c is the speed of light in free space which is $3.0 \times 10^8\,\text{m s}^{-1}$.

For example:

- A sealed torch that radiates 10 W of light for 10 h (= 36000 s) would lose 0.36 MJ of energy (= 10 W × 36000 s). Its mass would therefore decrease by 4.0×10^{-12} kg (= $0.36\,\text{MJ}/c^2$), an insignificant amount compared with the mass of the torch.
- A mass of a 1000 kg car that speeds up from a standstill to $30\,\text{m s}^{-1}$ would gain 450 kJ of kinetic energy so its mass when moving at $30\,\text{m s}^{-1}$ would be 5.0×10^{-12} kg (= $450\,\text{kJ}/c^2$)more than when it is rest.
- An unstable nucleus that releases a 5 MeV γ photon would lose 8.0×10^{-13} J of energy. Its mass would therefore decrease by 8.9×10^{-30} kg (= $8.0 \times 10^{-13}\,\text{J}/c^2$) which is not an insignificant amount compared with the mass of a nucleus.

The equation applies to all energy changes of any object. These two examples show that such changes are important in nuclear reactions but are not usually significant otherwise. A century after Einstein published his theory, the reason why the mass of an object changes when energy is transferred to or from it is still not clearly understood. However, it is known for every type of particle, there is a corresponding antiparticle with the same mass and opposite charge (if charged). We also know that:

- When a particle and its corresponding antiparticle meet, they **annihilate** each other and two gamma (γ) photons are produced, each of energy mc^2 where m is the mass of the particle or antiparticle.
- A single γ photon of energy in excess of $2mc^2$ can produce a particle and an antiparticle, each of mass m, in a process known as pair production.

Energy changes in reactions

Reactions on a nuclear or sub-nuclear scale do involve significant changes of mass. For example, in radioactive decay, if we know the exact rest mass of each particle involved, we can calculate the energy released (Q) from the difference Δm in the total mass before and after the reaction. In general, for a spontaneous reaction in which no energy is supplied

the energy released $\quad Q = \Delta mc^2$

In any change where energy is released such as radioactive decay, the total mass after the change is always less than the total mass before the change. This is because, in the change, some of the mass is converted to energy which is released.

1 In α decay, the nucleus recoils when the α-particle is emitted so the energy released is shared between the α-particle and the nucleus. Applying conservation of momentum to the recoil, you should be able to show that the energy released is shared between the α-particle and the nucleus in inverse proportion to their masses.
2 In β decay, the energy released is shared in variable proportions between the β-particle and the neutrino or antineutrino released in the decay. When the β-particle has maximum kinetic energy, the neutrino or antineutrino has negligible kinetic energy in comparison. The maximum kinetic energy of the β-particle is very slightly less than the energy released in the decay because of recoil of the nucleus.

NOTE

1 To calculate the energy corresponding to a mass difference of 1 atomic mass unit ($1\,\text{u} = 1.6605 \times 10^{-27}$ kg), using $E = mc^2$ gives
$E = 1.6605 \times 10^{-27}\,\text{kg} \times (2.9979 \times 10^8\,\text{m s}^{-1})^2$
$= 1.4923 \times 10^{-10}\,\text{J} = 931.5\,\text{MeV}$ using exact values.
2 When calculating Q in beta decay, assume the mass of the neutrino is negligible.
3 If the mass of each atom is given instead of the mass of its nucleus, calculate the mass of each nucleus by subtracting the mass of the electrons ($= Z_{m_e}$) in the atom from the mass of each atom.

Worked example

The polonium isotope $^{210}_{84}$Po emits α-particles and decays to form the stable isotope of lead $^{206}_{82}$Pb. Write down an equation to represent this process and calculate the energy released when a $^{210}_{84}$Po nucleus emits an α-particle.

mass of $^{210}_{84}$Po nucleus = 209.93667 u

mass of $^{206}_{82}$Pb nucleus = 205.92936 u

mass of α-particle = 4.00150 u

1 u is equivalent to 931.5 MeV

Solution

$$^{210}_{84}\text{Po} \longrightarrow {}^{4}_{2}\alpha + {}^{206}_{82}\text{Pb} \ (+ \text{ energy released } Q)$$

mass difference = total initial mass − total final mass

= 209.93667 − (205.92936 +4.00150)

= 5.81×10^{-3} u

energy released Q = mass difference in u × 931.5

= 5.41 MeV

More about the strong nuclear force

The fact that most nuclei are stable tells us there must be an attractive force, the **strong nuclear force**, between any two protons or neutrons in the nucleus.

- The strength of the strong nuclear force can be estimated by working out the force of repulsion between two protons at a separation of 1 fm (= 10^{-15} m), the approximate size of the nucleus. The strong nuclear force must be greater in magnitude that this force of repulsion. Prove for yourself, using Coulomb's law of force (see topic 14.1), that the force of repulsion between two protons at a separation of 10^{-15} m is of the order of 200 N. So the strong nuclear force is at least 200 N.

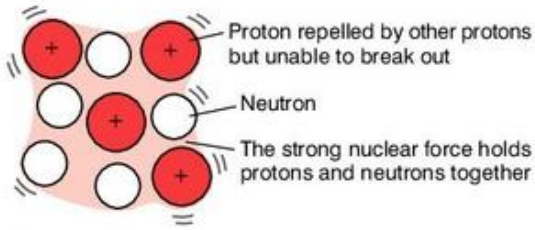


Fig 1 *The strong nuclear force*

- The range of the strong nuclear force is no more than about 2 to 3 × 10^{-15} m. The diameter of a nucleus can be measured from high-energy electron scattering experiments. The results show that nucleons are evenly spaced at about 10^{-15} m in the nucleus and therefore the strong nuclear force acts only between nearest neighbour nucleons.
- The energy needed to pull a nucleon out of the nucleus is of the order of millions of electron volts (MeV). This can be deduced because the strong nuclear force is at least about 200 N and it acts over a distance of about 2 to 3 × 10^{-15} m. The work done by the strong nuclear force over this distance is therefore about 5 × 10^{-13} J (= 200 N × 2.5 × 10^{-15} m) which is about 3 MeV as 1 MeV = 1.6 × 10^{-13} J.
- The strong nuclear force between two nucleons must become repulsive at separations of about 0.5 fm or less, otherwise nucleons would pull each other closer and closer together and be much smaller than it is.

QUESTIONS

Magnitude of the charge of the electron = 1.60 × 10^{-19} C, rest mass of an electron = 9.11 × 10^{-31} kg, 1 u = 931.5 MeV, g = 9.81 m s^{-2}

1 Calculate the increase of mass of:

a a 10 kg object when it is raised through a height of 2.0 m,

b an electron when it is accelerated from rest through a p.d. of: **i** 5000 V, **ii** 5 MV.

2 The bismuth isotope $^{212}_{83}$Bi emits α-particles and decays to form the stable isotope of thallium $^{208}_{81}$Tl.

a Write down an equation to represent this process and calculate the energy released.

mass of $^{212}_{83}$Bi nucleus = 211.94562 u,

mass of $^{208}_{81}$Tl nucleus = 207.93746 u,

mass of α-particle = 4.00150 u

b Explain without calculation why the thallium nucleus in the above decay gains a small proportion of the energy released.

3 The strontium isotope $^{90}_{38}$Sr emits β$^-$ particles and decays to form the stable isotope of yttrium $^{90}_{39}$Y.

a Write down an equation to represent this process and calculate the energy released.

mass of $^{90}_{38}$Sr nucleus = 89.88640 u,

mass of $^{90}_{39}$Y nucleus = 89.88525 u,

mass of β$^-$ particle = 0.00055 u

b Explain without calculation why the kinetic energy of the β-particle released when the strontium nucleus decays varies from zero up to a maximum.

4 The sodium isotope $^{25}_{11}$Na emits β$^-$ particles and decays to form the stable isotope of magnesium $^{25}_{12}$Mg.

Calculate the Q-value of this decay.

mass of $^{25}_{11}$Na nucleus = 24.98931 u,

mass of $^{25}_{12}$Mg nucleus = 24.98528 u

23.5 Binding energy

On these pages you will learn to:

- Define and understand the terms mass defect and binding energy
- Calculate the binding energy per nucleon of a nuclide
- Sketch the variation of binding energy per nucleon with nucleon number
- Explain the relevance of binding energy per nucleon to nuclear fusion and to nuclear fission

Binding energy and mass defect

Suppose all the nucleons in a nucleus were separated from one another, removing each one from the nucleus in turn. Work must be done to overcome the strong nuclear force and separate each nucleon from the others. The potential energy of each nucleon is therefore increased when it is removed from the nucleus.

The binding energy of the nucleus is the work that must be done to separate a nucleus into its constituent neutrons and protons.

When a nucleus forms from separate neutrons and protons, energy is released as the strong nuclear force does work pulling the nucleons together. The energy released is equal to the binding energy of the nucleus. Because energy is released when a nucleus forms from separate neutrons and protons, the mass of a nucleus is less than the mass of the separated nucleons.

The mass defect Δm of a nucleus is defined as the difference between the mass of the separated nucleons and the combined mass of the nucleus.

- Calculation of the mass defect of a nucleus of known mass; a nucleus of an isotope ${}^{A}_{Z}\mathrm{X}$ is composed of Z protons and $(A - Z)$ neutrons. Therefore, for a nucleus ${}^{A}_{Z}\mathrm{X}$ of mass M_N, **its mass defect** $\Delta m = Zm_p + (A - Z)m_n - M_N$
- Calculation of the binding energy of a nucleus; the mass defect Δm is due to energy released when the nucleus formed from separate neutrons and protons. The energy released in this process is equal to the binding energy of the nucleus. Therefore, **the binding energy of a nucleus** $= c^2(\Delta m)$

Worked example

The mass of a nucleus of the bismuth isotope ${}^{212}_{83}\mathrm{Bi}$ is 211.80012 u. Calculate the binding energy of this nucleus in MeV.

The mass of a proton, $m_p = 1.00728$ u; the mass of a neutron, $m_n = 1.00866$ u

1 u is equivalent to = 931.5 MeV

Solution

Mass defect $\Delta m = 83m_p + (212 - 83)m_n - M_N = 1.92126$ u

$\therefore$ binding energy = 1.92126 u × 931.5 MeV/u = 1790 MeV

Note

1. The mass of an atom of an isotope ${}^{A}_{Z}\mathrm{X}$ is measured using a mass spectrometer. The mass of a nucleus can then be calculated by subtracting the mass of Z electrons from the atomic mass.
2. The atomic mass unit, 1 u = 1.661×10^{-27} kg. This is defined as $\frac{1}{12}$th of the mass of an atom of the carbon isotope ${}^{12}_{6}\mathrm{C}$.
3. The energy corresponding to a mass of 1 u = 931.5 MeV. See topic 23.4 if necessary.

Nuclear stability

The binding energy of each nuclide is different. The binding energy per nucleon of a nucleus is the work done to remove a nucleon from a nucleus; it is therefore a measure of the stability of a nucleus. For example, the binding energy per nucleon of the ${}^{212}_{83}\mathrm{Bi}$ nucleus is 8.4 MeV per nucleon (= 1790 MeV/212 nucleons).

If the binding energy per nucleon of two different nuclides are compared, the nucleus with more binding energy per nucleon is the more stable of the two nuclei. Figure 1 shows a graph of the binding energy per nucleon v. mass number A for all the known nuclides. This graph is a curve which has a maximum value of 8.7 MeV per nucleon between $A = 50$ and $A = 60$. Nuclei with mass numbers in this range are the most stable nuclei. As explained below, energy is released in:

- **nuclear fission**, the process in which a large unstable nucleus splits into two fragments which are more stable than the original nucleus. The binding energy per nucleon increases in this process, as shown in Figure 1.
- **nuclear fusion**, the process of making small nuclei fuse together to form a larger nucleus. The product nucleus has more binding energy per nucleon than the smaller nuclei. So the binding energy per nucleon also increases in this process, provided the nucleon number of the product nucleus is no greater than about 50.

Note The change of binding energy per nucleon is about 0.5 MeV in a fission reaction and can be more than 20 times as much in a fusion reaction.

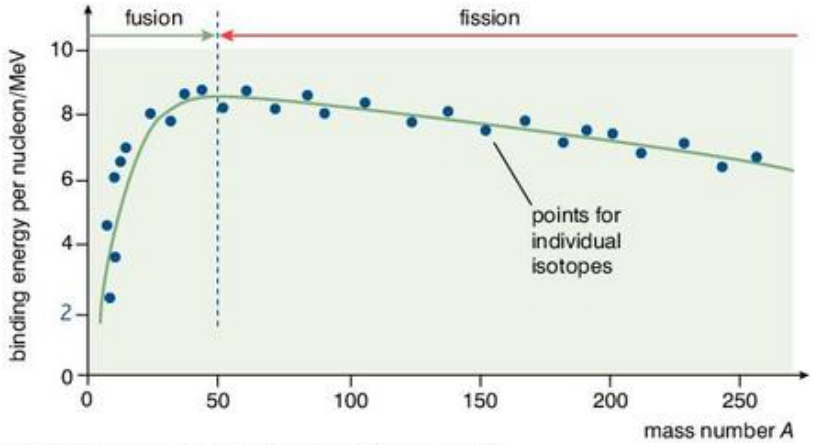


Fig 1 *Binding energy per nucleon for all known nuclides*

QUESTIONS

mass of a proton, $m_p = 1.00728$ u; mass of a neutron, $m_n = 1.00866$ u,
1 u is equivalent to = 931.5 MeV

1 **a** Explain what is meant by the binding energy of a nucleus.

b Sketch a curve to show how the binding energy per nucleon of a nucleus varies with its mass number A, showing the approximate scale on each axis.

2 Calculate the binding energy per nucleon, in MeV per nucleon, of:

a a $^{12}_{6}$C nucleus (mass = 12 u by definition),

b a $^{56}_{26}$Fe nucleus (mass = 55.92067 u)

3 **a** Calculate the binding energy per nucleon in MeV per nucleon, of
i an α-particle **ii** a $^{3}_{2}$He nucleus.
mass of an α-particle = 4.00150 u; mass of a $^{3}_{2}$He nucleus = 3.01493 u

b Use the results of your calculations in (a) to explain why an α-particle rather than a $^{3}_{2}$He nucleus is emitted by a large unstable nucleus.

4 Calculate the binding energy per nucleon, in MeV, of the $^{2}_{1}$H nucleus.
mass of $^{2}_{1}$H nucleus = 2.01355 u

23.6 Fission and fusion

On these pages you will learn to:

- Explain what is meant by nuclear fission and by a chain reaction
- Calculate the energy released in a fission or fusion event from the masses of the nuclei and other particles involved
- Explain what is meant by nuclear fusion and explain in terms of forces the necessary condition to fuse two nuclei

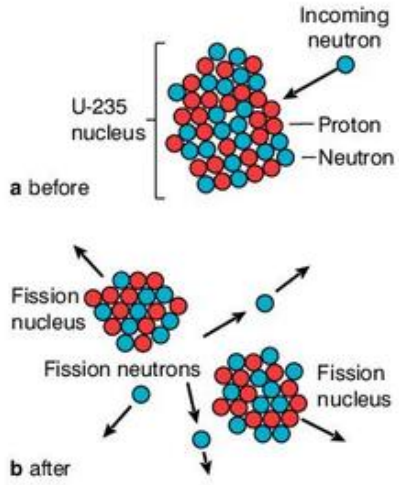


Fig 1 Induced fission

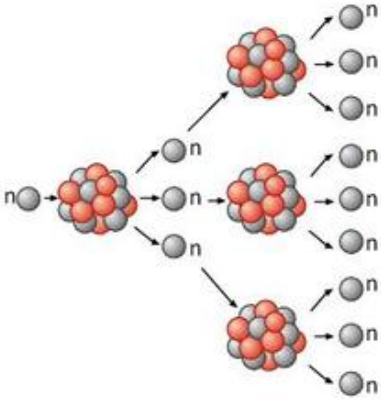


Fig 2 A chain reaction in a nuclear reactor

NOTE

The fragment nuclei have excess neutrons compared with stable nuclei of the same proton number. They become stable by emitting β^- particles. See topic 10.4.

Induced fission

Fission of a nucleus occurs when a nucleus splits into two approximately equal fragments. This happens when the uranium isotope $^{235}_{92}U$ is bombarded with neutrons, a discovery made by Hahn and Strassmann in 1938. The process is known as induced fission. The plutonium isotope, $^{239}_{94}Pu$, is the only other isotope that is fissionable. This isotope is an artificial isotope formed by bombarding nuclei of the uranium isotope $^{238}_{92}U$ with neutrons.

Hahn and Strassman knew that bombarding different elements with neutrons produces radioactive isotopes. Uranium is the heaviest of all the naturally occurring elements; scientists thought that neutron bombardment could turn uranium nuclei into even heavier nuclei. Hahn and Strassmann undertook the difficult work of analysing chemically the products of uranium after neutron bombardment to try to discover any new elements heavier than uranium. Instead, they discovered that many lighter elements such as barium were present after bombardment, even though the uranium was pure before. The conclusion could only be that uranium nuclei were split into two approximately equal fragment nuclei as a result of neutron bombardment.

Further investigations showed that each fission event releases energy and two or three neutrons.

- Fission neutrons, the neutrons released in a fission event, are each capable of causing a further fission event as a result of a collision with another $^{235}_{92}U$ nucleus. A **chain reaction** is therefore possible in which fission neutrons produce further fission events which release fission neutrons and cause further fission events and so on. If each fission event releases two neutrons on average, after n 'generations' of fission events, the number of fission neutrons would be 2^n. Prove for yourself that fission of 6×10^{23} $^{235}_{92}U$ nuclei (i.e. 235 g of the isotope) would happen in 79 generations. As explained below, each fission event releases about 200 MeV of energy. Because each event takes no more than a fraction of a second, a huge amount of energy is released in a very short time. Using the above figures, complete fission of 235 g of $^{235}_{92}U$ would release about 10^{13} J ($= 6 \times 10^{23} \times 200$ MeV). This is about a million times more than the energy released as a result of burning a similar mass of fossil fuel.
- Energy is released when a fission event occurs because the fragments repel each other (as they are both positively charged) with sufficient force to overcome the strong nuclear force trying to hold them together. The fragment nuclei and the fission neutrons therefore gain kinetic energy. The two fragment nuclei are smaller and therefore more tightly bound than the original $^{235}_{92}U$ nucleus. In other words, they have more binding energy so they are more stable than the original nucleus. The energy released is equal to the change of binding energy. The binding energy of each nucleon increases from about 7.5 MeV to about 8.5 MeV as a result of the fission event. As there are about 240 nucleons in the original nucleus, the energy released in a fission event is of the order of 200 MeV ($= 240 \times$ about 1 MeV).
- Many fission products are possible when a fission event occurs. For example, the equation below shows a fission event in which a $^{235}_{92}U$ nucleus is split into a barium $^{144}_{56}Ba$ nucleus and a krypton $^{90}_{36}Kr$ nucleus and two neutrons are released.

$$^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{144}_{56}Ba + {}^{90}_{36}Kr + 2\,{}^{1}_{0}n + \text{energy released, } Q$$

- The energy released, Q, can be calculated using $E = mc^2$ in the form $Q = c^2(\Delta m)$, where Δm is the difference between the total mass before and after the event.
- In the above equation, the mass difference

$$\Delta m = M_{\text{U-235}} - M_{\text{Ba-144}} - M_{\text{Kr-90}} - m_{\text{n}}$$

where M represents the appropriate nuclear mass and m_{n} is the mass of the neutron.

Nuclear fusion

Fusion takes place when two nuclei combine to form a bigger nucleus. The binding energy curve (topic 23.5) shows that if two light nuclei are combined, the individual nucleons become more tightly bound together. The binding energy per nucleon of the product nucleus is greater than of the initial nuclei. In other words, the nucleons become even more trapped in the nucleus when fusion occurs. As a result, energy is released equal to the increase of binding energy.

Nuclear fusion can only take place if the two nuclei that are to be combined collide at high speed. This is necessary to overcome the electrostatic repulsion between the two nuclei so they can become close enough to interact through the strong nuclear force. Some examples of nuclear fusion reactions are shown in Figure 4 and described below.

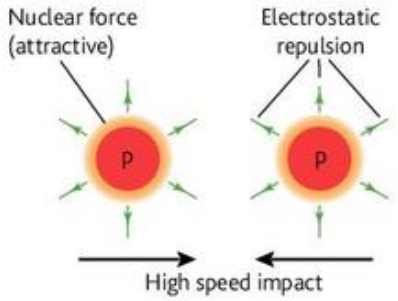


Fig 3 Fusion of two protons

1 The fusion of two protons produces a nucleus of deuterium (the hydrogen isotope ${}^{2}_{1}\text{H}$), a β^+ particle and a neutrino

$${}^{1}_{1}\text{p} + {}^{1}_{1}\text{p} \longrightarrow {}^{2}_{1}\text{H} + {}^{0}_{+1}\beta + \nu$$

2 The fusion of a proton and a deuterium nucleus ${}^{2}_{1}\text{H}$ produces a nucleus of the helium isotope ${}^{3}_{2}\text{He}$ and 5.5 MeV of energy.

$${}^{2}_{1}\text{H} + {}^{1}_{1}\text{p} \longrightarrow {}^{3}_{2}\text{He}$$

3 The fusion of two nuclei of helium isotope, ${}^{3}_{2}\text{He}$, produces a nucleus of the helium isotope ${}^{4}_{2}\text{He}$, two protons and 12.9 MeV of energy.

$${}^{3}_{2}\text{He} + {}^{3}_{2}\text{He} \longrightarrow {}^{4}_{2}\text{He} + 2{}^{1}_{1}\text{p}$$

In each case, the energy released in the reaction may calculated using $E = mc^2$ in the form $Q = c^2(\Delta m)$, where Δm is the difference between the total mass before and after the event.

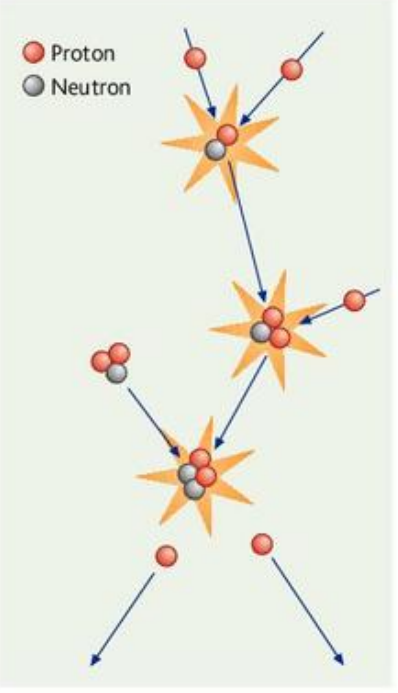


Fig 4 Fusion reactions inside the Sun

Solar energy is produced as a result of fusion reactions inside the Sun. The temperature at the centre of the Sun is thought to be 10^8 K or more. At such temperatures, atoms are stripped of their electrons. Matter in this state is referred to as 'plasma'. The nuclei of the plasma move at very high speeds because of the enormous temperature. When two nuclei collide, they fuse together because they overcome the electrostatic repulsion due to their charge and approach each other closely enough to interact through the strong nuclear force. Protons (i.e. hydrogen nuclei) inside the Sun's core fuse together in stages (corresponding to equations 1, 2 and 3 above) to form helium ${}^{4}_{2}\text{He}$ nuclei. For each helium nucleus formed, 25 MeV of energy is released. This corresponds to 7 MeV per proton, considerably more than the energy released per nucleon in a fission event.

Fusion power

Fusion reactors are still at the prototype stage even though scientific teams in several countries have been working on fusion research for more than 50 years. Prototype fusion reactors such as JET, the Joint European Torus, in the United Kingdom have produced large amounts of power but only for short periods of time. JET produces less power than it uses but the less powerful International Thermonuclear Experimental Reactor (ITER) due to start up in 2016 is designed to produce several times more power than it uses.

Fig 5 The JET fusion reactor

Energy is released in JET by fusing nuclei of deuterium ${}^{2}_{1}\mathrm{H}$ and tritium ${}^{3}_{1}\mathrm{H}$ to produce nuclei of the helium isotope ${}^{4}_{2}\mathrm{He}$ and neutrons, as below.

$$ {}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \longrightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} + 17.6\,\mathrm{MeV} $$

The neutrons are absorbed by a 'blanket' of lithium surrounding the reactor vessel. The reaction between the neutrons and the lithium nuclei, as shown below, produces tritium which is then used in the main reaction. Deuterium occurs naturally in water as it forms 0.01% of naturally occurring hydrogen.

$$ {}^{6}_{3}\mathrm{Li} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{4}_{2}\mathrm{He} + {}^{3}_{1}\mathrm{H} + 4.8\,\mathrm{MeV} $$

The plasma is contained in a doughnut-shaped steel container and is heated by passing a very large current through it. A magnetic field is used to confine the plasma so it does not touch the sides of its steel container, otherwise it would lose its energy. In theory, the energy released per second should be more than is needed to heat the plasma so the reactor ought to give a continuous output of power. However, at the present time, power can only be produced for a short time as the plasma becomes unstable at such high temperatures.

Chapter summary

The **half-life**, $t_{\frac{1}{2}}$, of a radioactive isotope is the time taken for the mass of the isotope to decrease to half the initial mass. This is the same as the time taken for the number of nuclei of the isotope to decrease to half the initial number.

The decay constant, λ, is the probability of an individual nucleus decaying per second.

The **activity**, A, of a radioactive isotope is the number of nuclei of the isotope that disintegrate per second. The unit of activity is the becquerel (Bq), equal to one disintegration per second.

Binding energy of a nucleus is the work that must be done to separate a nucleus into its constituent neutrons and protons.

Binding energy = mass defect $\times c^2$

Binding energy/nucleon is greatest for nuclei of mass number 57.

Fission is the splitting of a ${}^{235}_{92}\mathrm{U}$ nucleus or a ${}^{239}_{94}\mathrm{Pu}$ nucleus into two approximately equal fragments. Induced fission is fission caused by an incoming neutron colliding with a ${}^{235}_{92}\mathrm{U}$ nucleus or a ${}^{239}_{94}\mathrm{Pu}$ nucleus.

Fusion is the fusing together of light nuclei to form a heavier nucleus.

Equations

1 $A = \lambda N$

2 $N = N_0 e^{-\lambda t}$, $A = A_0 e^{-\lambda t}$, $C = C_0 e^{-\lambda t}$

3 Binding energy = mass defect $\times c^2$

QUESTIONS

1 u is equivalent to 931.5 MeV

1 a Explain why the protons in a nucleus do not leave the nucleus even though they repel each other.

b Explain why the mass of a nucleus is less than the mass of the separated protons and neutrons from which the nucleus is composed.

2 a What is meant by nuclear fission?

b i The incomplete equation below represents a reaction that takes place when a neutron collides with a nucleus of the uranium isotope ${}^{235}_{92}\mathrm{U}$. Determine the values of a and b in this equation.

$$ {}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{136}_{a}\mathrm{Xe} + {}^{b}_{36}\mathrm{Kr} + 2{}^{1}_{0}\mathrm{n} + \text{energy released, } Q $$

ii Calculate the energy, in MeV, released in this fission reaction.
masses:
${}^{235}_{92}\mathrm{U}$ nucleus 234.993 u, ${}^{136}_{a}\mathrm{Xe}$ nucleus 135.877 u,
${}^{b}_{36}\mathrm{Kr}$ nucleus 97.886 u, neutron 1.00866 u

3 a What is meant by nuclear fusion?

b Hydrogen nuclei fuse together to form helium nuclei in the Sun. Two stages in this process are represented by the following equations:

$$ {}^{1}_{1}\mathrm{p} + {}^{1}_{1}\mathrm{p} \longrightarrow {}^{2}_{1}\mathrm{H} + {}^{0}_{+1}\beta $$

$$ {}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{p} \longrightarrow {}^{3}_{2}\mathrm{He} $$

i Describe the reactions that these equations represent.

ii Calculate the energy released in each reaction.
masses: β-particle 0.00055 u, proton 1.00728 u,
${}^{2}_{1}\mathrm{H}$ nucleus 2.01355 u, ${}^{3}_{2}\mathrm{He}$ nucleus 3.01493 u

4 a Explain why light nuclei do not fuse when they collide unless they are moving at a sufficiently high speed.

b Calculate the energy released in the following fusion reaction:

$$ {}^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \longrightarrow {}^{4}_{2}\mathrm{He} + 2{}^{1}_{1}\mathrm{p} $$

masses: proton 1.00728 u, ${}^{3}_{2}\mathrm{He}$ nucleus 3.01493 u, α-particle 4.00150

c Show that about 25 MeV of energy is released when a ${}^{4}_{2}\mathrm{He}$ nucleus is formed from 4 protons.

23 End of chapter questions

1 u is equivalent to 931.5 MeV

1 a A radioactive isotope of polonium, $^{210}_{84}$Po, has a half-life of 140 days. Calculate:
- **i** the decay constant in s^{-1},
- **ii** the mass of a sample of this isotope which has an activity of 1.0×10^{12} Bq.

b The isotope emits α-particles and forms a stable isotope of lead. Each disintegration in the isotope releases 5.3 MeV of energy. The isotope is to be used to supply power for a remote weather station for at least 1 year.
- **i** Show that 0.72 g of this isotope will release 100 W of power.
- **ii** Calculate the power supplied after 1 year by an initial mass of 0.72 g of this isotope.

2 a Calculate the number of atoms present in 1.0 kg of $^{226}_{88}$Ra.

b The radioactive isotope $^{226}_{88}$Ra has a half-life of 1620 years. Calculate the activity of 1 milligram of this isotope.

3 a Radioactive disintegration is a random process yet it is possible to calculate reasonably accurately the number of atoms in a radioactive source of known activity and half-life. Explain why.

b A radioactive isotope X with a half-life of 44 hours disintegrates to form a stable product. A pure sample of X is prepared with an activity of 60 kBq. Calculate:
- **i** the activity of the sample after 24 hours,
- **ii** the time taken for the activity of the sample to decrease to 10% of its initial activity.

4 The radioactive isotope of cobalt, $^{60}_{27}$Co, has a half-life of 5.3 years. It emits γ-photons of energy 1.3 MeV.

a Calculate:
- **i** the activity of a sample of mass of 10 g of this isotope,
- **ii** the energy transfer per second from this sample.

b The sample is in a thick lead container of mass 920 kg. Calculate the temperature rise of the container in 24 hours, assuming no energy transfer occurs from the container.

Specific heat capacity of lead = 130 J kg^{-1} K^{-1}.

c $^{60}_{27}$Co is used for gamma therapy to destroy tumours.
- **i** What property of γ radiation is made use of in this application?
- **ii** What precautions are taken to ensure the patient is not exposed to the beam unnecessarily?

5 a Explain what is meant by the binding energy of a nucleus.

b Calculate the binding energy per nucleon, in MeV, of:
- **i** an α-particle,
- **ii** a deuterium $^{2}_{1}$H nucleus.

Masses: neutron = 1.00866 u, proton = 1.00728 u, $^{2}_{1}$H nucleus = 2.01355 u, α-particle = 4.00150 u

c Discuss why an α-particle does not break up into two $^{2}_{1}$H nuclei.

6 a In a nuclear reaction, a neutron collided with a nucleus of the lithium isotope $^{6}_{3}$Li. A tritium nucleus $^{3}_{1}$H and another nucleus X was formed as a result.

Write down an equation that represents this reaction and identify the nucleus X.

b The mass loss in the above reaction was 0.00514 u.
- **i** Calculate the energy released, in J, in this reaction.
- **ii** Calculate the mass of X, given the masses of the other nuclei are as follows: $^{3}_{1}$H 3.0155 u, $^{6}_{3}$Li 6.01348 u, neutron 1.00866 u

7 a i State two properties of the strong nuclear force.

ii Explain why energy is released when a $^{235}_{92}$U nucleus undergoes induced fission.

b i Copy and complete the induced fission equation below:

$$^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{140}_{?}\text{Xe} + {}^{93}_{38}\text{Sr} + ? \, {}^{1}_{0}\text{n}$$

ii Calculate the energy released, in J, in the above reaction.

Masses: $^{235}_{92}$U 235.0439 u, $^{1}_{0}$n 1.00866 u, $^{140}_{?}$Xe 139.9216 u, $^{93}_{38}$Sr 92.9140 u

8 a State one difference and one similarity between a fusion reaction and a fission reaction.

b i Explain why fusion in a plasma containing light nuclei only takes place if the temperature of the plasma is of the order of 10^8 K.

ii Two deuterium ($^{2}_{1}$H) nuclei fuse together to form a tritium ($^{3}_{1}$H) nucleus and a proton.

Write down the equation which represents this reaction and use the information below to calculate the energy released, in MeV, in this reaction.

Masses $^{2}_{1}$H 2.0136 u, $^{3}_{1}$H 3.0155 u, proton 1.0073 u

9 a Explain why the spent fuel rods from a nuclear reactor are more radioactive after removal from the reactor than they were before they were used in the reactor.

b The radioactive isotope, $^{90}_{38}$Sr, is a β$^-$-emitter which has a half-life of 28 years. It is produced as a fission product in a fuel rod in a nuclear reactor. If it escapes

into the environment, it can be absorbed by the body in place of calcium. Calculate:

i the activity of 1 mg of this isotope,

ii the activity in 100 years of a sample of this isotope that has a mass of 1 mg at the present time.

10 The controlled reaction between deuterium ($^{2}_{1}H$) and tritium ($^{3}_{1}H$) has involved ongoing research for many years. The reaction may be summarised as

$$^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n + Q$$

where $Q = 17.7$ MeV.

Binding energies per nucleon are shown in Figure 10.1.

	binding energy per nucleon/MeV
$^{2}_{1}H$	1.12
$^{1}_{0}n$	–
$^{4}_{2}He$	7.07

Fig 10.1

a Suggest why binding energy per nucleon for the neutron is not quoted. *(1 mark)*

b Calculate the mass defect, in kg, of a helium $^{4}_{2}He$ nucleus. *(3 marks)*

c **i** State the name of the type of reaction illustrated by this nuclear equation. *(1 mark)*

ii Determine the binding energy per nucleon, in MeV, of tritium ($^{3}_{1}H$). *(3 marks)*

Cambridge International AS and A Level Physics 9702
Paper 41 Q8 November 2009

11 In some power stations, nuclear fission is used as a source of energy.

a State what is meant by *nuclear fission*. *(2 marks)*

b The nuclear fission reaction produces neutrons. In the power station, the neutrons may be absorbed by rods made of boron-10.

Copy and complete the nuclear equation for the absorption of a single neutron by a boron-10 nucleus with the emission of an α-particle.

$$^{10}_{5}B + \ldots\ldots\ldots\ldots \rightarrow {}^{\ldots\ldots}_{3}Li + \ldots\ldots\ldots\ldots \quad \textit{(3 marks)}$$

c Suggest why, when neutrons are absorbed in the boron rods, the rods become hot as a result of this nuclear reaction. *(3 marks)*

Adapted from Cambridge International AS and A Level Physics 9702
Paper 41 Q8 November 2010

24 Medical imaging

24.1 Ultrasonic imaging

On these pages you will learn to:

- Explain the principles of the generation and detection of ultrasonic waves using piezo-electric transducers
- Explain the main principles behind the use of ultrasound to obtain diagnostic information about internal structures
- Define specific acoustic impedance and explain the importance in relation to the intensity reflection coefficient at a boundary
- Recall and solve problems by using the equation $I = I_0 e^{-\mu x}$ for the attenuation of ultrasound in matter

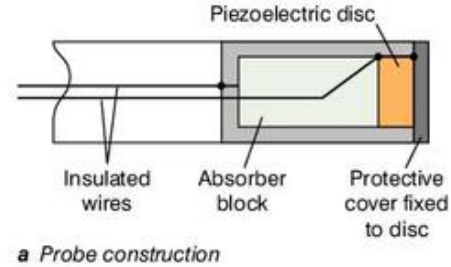
Producing ultrasonic waves

Ultrasonics are sound waves at frequencies of more than about 18 kHz above the range of the human ear. Unlike X-rays, ultrasonic radiation is non-ionising radiation and therefore does not damage living tissue. For medical imaging, ultrasonics at frequencies between about 1 and 10 MHz are used as diffraction would be significant at lower frequencies and reduced intensity due to absorption would be significant at higher frequencies.

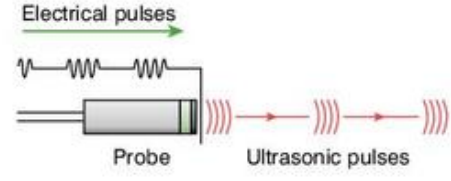
An **ultrasonic probe** used in ultrasonic scanning is a hand-held device placed in contact with the body surface to direct pulses of ultrasound into the body. Each emitted pulse is partially reflected by internal boundaries in the body. The reflected pulses are then detected by the probe before the next pulse is emitted.

Figure 1 shows the construction of an ultrasonic probe.

- The probe contains a piezo-electric **transducer** in the shape of a disc. Piezo-electricity is a property of certain solids whereby a p.d. applied between opposite faces causes a change of distance between the two faces. When an alternating p.d. is applied between the faces of the disc, the disc vibrates due to its changing thickness.
- By applying an alternating p.d. of frequency equal to the resonant frequency of vibration of the disc, the disc vibrates at resonance and creates ultrasonic waves in the surrounding medium. The thickness of the disc determines its resonant frequency.
- An absorber pad of 'backing material' behind the disc prevents ultrasonic waves created at the two surfaces of the disc from cancelling each other out. The pad also damps the vibrations of the disc rapidly after each pulse is emitted.



a Probe construction



b Ultrasonic pulses

***Fig 1** An ultrasonic probe*

More about piezo-electricity

A **transducer** is any device that is designed to convert energy from one form to another. A piezo-electric transducer generates a p.d. when it is squeezed. The piezo-electric material contains positive and negative ions, which are held together by the electrostatic forces they exert on each other. The centre of the negative charge of each molecule is in the same position as the centre of positive charge. When pressure is applied to opposite surfaces of the material, the centres of charge of the positive and negative ions are displaced slightly in opposite directions, causing a potential difference between the two surfaces. The effect is known as the **piezo-electric effect** and is displayed by crystals such as quartz.

To apply a p.d. across a quartz crystal, opposite surfaces to which the pressure is applied must be coated with metal so an electrical connection can be made to each surface.

The piezo-electric effect is reversible in that the application of a p.d. across a piezo-electric material causes the distance across the material to increase or decrease according to the polarity of the p.d.

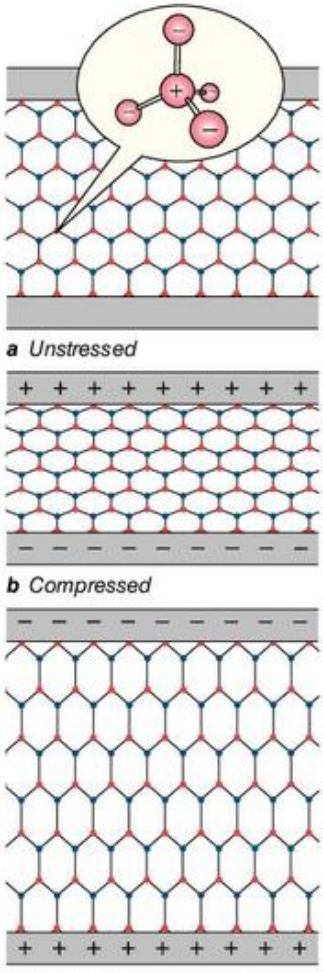


Fig 2 *Piezo-electricity*

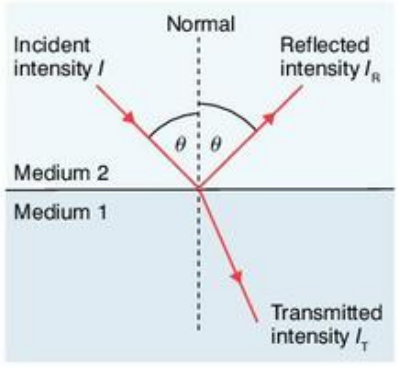


Fig 3 *Reflection and transmission at a boundary*

Absorption and reflection of ultrasonic waves

Ultrasonic waves can be reflected and refracted just like sound waves. When ultrasonic waves reach a boundary between two substances, some of the wave energy is reflected and some is transmitted as shown in Figure 3.

Considering the energy reaching the boundary in 1 second, as energy cannot be created or destroyed, the sum of the reflected energy and the transmitted energy is equal to the incident energy. Hence the incident intensity I = the reflected intensity I_R + the transmitted intensity I_T.

The fraction of the incident energy that is reflected or transmitted depends on:

- the angle of incidence θ of the incident waves,
- the densities ρ_1 and ρ_2 of the two substances,
- the wave speeds c_1 and c_2 of ultrasonic waves in the two substances.

The **acoustic impedance** of a substance, Z is defined by the equation $\boldsymbol{Z = \rho c}$

The unit of Z is given by the product of the unit of density (i.e. $kg\,m^{-3}$) and the unit of speed (i.e. $m\,s^{-1}$). Hence the unit of Z is $kg\,m^{-2}s^{-1}$.

Specific values for different substances are given in Table 1. Notice the very small value for air and the large values for quartz and bone compared with the other substances listed in the table.

Table 1

Substance	Acoustic impedance $Z/kg\,m^{-2}s^{-1}$
air	430
blood	1.59×10^6
bone	6.80×10^6
fat	1.38×10^6
muscle	1.70×10^6
quartz	1.52×10^7
soft tissue	1.63×10^6
water	1.50×10^6

When ultrasonic waves are incident on a boundary between two substances with acoustic impedances Z_1 and Z_2, the ratio of the reflected intensity to the incident intensity I_R/I, the reflection coefficient, is given by the equation:

$$\frac{I_R}{I} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

- If Z_1 and Z_2 are almost equal, the ratio is close to zero and the reflected intensity is very small compared with the incident intensity. In other words, most of the wave energy is transmitted.
- If Z_1 and Z_2 are very different, the ratio is close to 1 so most of the wave energy is reflected. The transmitted intensity is very small compared with the incident intensity. In other words, most of the wave energy is reflected.

The intensity of the reflected pulses from the ultrasonic probe when they return to it depends on:

1 the ratio of the reflected intensity to the incident intensity at each boundary, and
2 the absorption of the ultrasonic waves by each substance they pass through.

When ultrasound is directed from a probe into the body, any air trapped between the probe and the body will cause most of the ultrasound energy to be reflected from the body. This is because the acoustic impedances of air and soft tissue are very different so the ratio I_R/I is close to 1. To eliminate such trapped air, a **coupling medium** such as a gel is applied between the probe and the body surface. Such substances have similar acoustic impedances to soft tissue so the ratio I_R/I is close to zero. In other words, most of the wave energy is transmitted into the body.

Reflection at tissue boundaries is significant and can't be avoided. For example, the ratio I_R/I for a boundary between soft tissue and fat is 6.9×10^{-3} (using values from Table 1).

- The reflected pulses are therefore much weaker than the pulses leaving the probe.
- Also, the further a boundary is from the probe, the further the pulses reflected from that boundary travel and the weaker the reflected pulses from it will be due to absorption (see below).

Absorption of ultrasonic waves depends on the substances the waves pass through and the distance travelled through each substance. The energy absorbed by the substance causes the temperature of the substance to increase.

When a parallel beam of ultrasonic waves travels through a substance, the intensity of the waves decreases exponentially with distance.

The intensity I of the waves after travelling through distance x of a substance is given by the equation

$$I = I_0 e^{-\mu x}$$

where I_0 is the incident intensity and μ is the absorption coefficient of the substance. The unit of μ is m^{-1}.

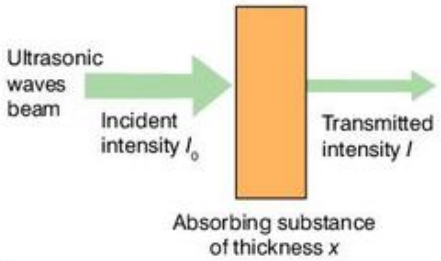


Fig 4 Absorption

Note

1 The greater the absorption coefficient μ, the greater the energy absorbed from the waves over any given distance.
2 μ depends on the frequency of the ultrasonic waves.
3 Table 2 gives some values of μ for different substances.

Table 2

Substance	Absorption coefficient μ/m^{-1}
air	120
bone	130
muscle	23
water	0.02

Ultrasonic scans

An **ultrasonic scanner** consists of an ultrasonic probe connected to a control unit and a visual display unit. In the simplest scan system, referred to as the **A-scan system**, a pulse generator is used to supply electrical pulses to the probe and to trigger the oscilloscope time base each time a pulse is generated. Figure 5 shows an A-scan of an eye.

- In each scan, a pulse is generated by the probe and, before the next pulse is generated, the probe detects reflected pulses from the boundaries in the path of the pulse. The reflected pulses or 'echoes' detected by the probe are amplified and displayed on the oscilloscope.
- Each time the time base is triggered, the oscilloscope beam sweeps across the screen from left to right. The time base of the oscilloscope is adjusted to display on the screen all the reflected pulses for each transmitted pulse. As a result, each pulse on the screen will be very narrow as the duration of each pulse is much shorter than the time for each 'sweep' of the oscilloscope screen.

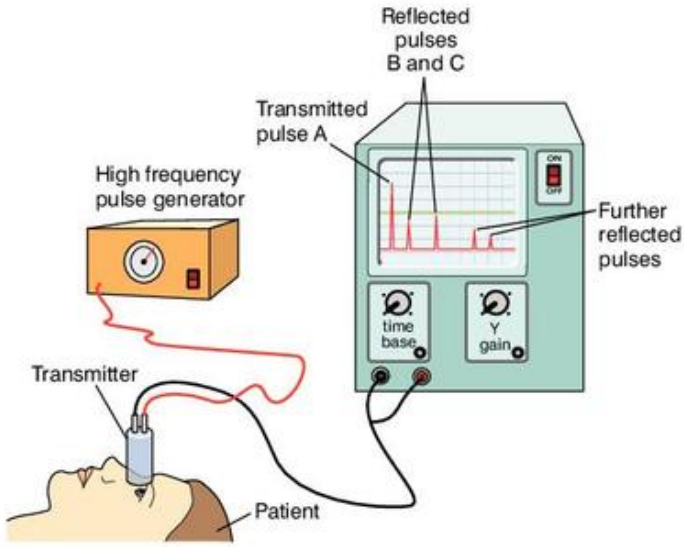


Fig 5 The A-scan system

1 The position of each reflected pulse on the screen depends on the transit time of the ultrasonic pulse (i.e. the time taken by the ultrasonic pulse to travel from the probe to the internal boundary that reflected the pulse and back).
2 The further a reflected pulse on the screen appears from the transmitted pulse:
   - the longer the transit time of the pulse in the body and the further away the boundary is from the probe,
   - the smaller the pulse height will be because the ultrasonic pulse is partially reflected by the boundaries it passes through and the substances it passes through absorb some of its energy.
3 The transit time is proportional to the distance from the probe to the boundary. Therefore, the greater the distance from the probe to the boundary, the further the pulse appears across the screen. The oscilloscope can be used to measure the transit time, t, of a pulse. The distance travelled by the pulse $s = vt$, where v is the speed of ultrasonic waves in the body. Therefore, the distance from the probe to the internal boundary causing the pulse $= \frac{1}{2}vt$. Using this equation, the screen could be calibrated in terms of distance from the probe so the distance between a boundary and any other boundary or the probe can be measured directly.

In the **B-scan system**, the probe has a number of ultrasonic transducers side by side, each one sending out ultrasonic pulses in a slightly different direction to the others. The signals from the transducers due to the reflected pulses are processed by a computer such that each reflected pulse is displayed as a bright spot on the screen in the correct direction and at the correct distance from the probe. As the probe is moved over the body surface, the bright spots on the screen build up a two-dimensional image of the reflecting boundaries scanned. The image may be enhanced and stored electronically.

Comparison with X-rays

- B-scans are used for pre-natal scans (i.e. to observe unborn babies in the womb) rather than X-ray CT scans. This is because ultrasonic waves are non-ionising and, at the intensities used in scanning, do not damage human tissue.
- Ultrasound reflects at bone/tissue boundaries as well as at internal boundaries between soft substances such as fat, muscle and tissue because the acoustic impedance of such substances differs. Therefore, ultrasonic images show such 'soft' boundaries whereas X-ray images do not (because X-rays are not reflected at such boundaries).

Fig 6 A B-scan image of an unborn baby

QUESTIONS

Use the data in Table 1 where necessary.

1 An ultrasonic probe generates ultrasonic waves at a frequency of 2.5 MHz. The speed of ultrasound in air = $350\,m\,s^{-1}$ and in soft tissue = $1550\,m\,s^{-1}$.

a Calculate the wavelength of the ultrasonic waves from this probe **i** in air, **ii** in soft tissue.

b Explain why ultrasonic waves of much lower frequency are unsuitable for medical imaging.

2 a i With the aid of a diagram, describe the construction of an ultrasonic probe and how it produces ultrasonic waves.

ii Explain the function of the backing block in an ultrasonic probe.

b In the A-scan arrangement shown in Figure 5, the furthest boundary from the probe is the retina.

i Explain the presence of each pulse on the screen in terms of the cross-section of the patient's eye.

ii Calculate the distance between the boundary responsible for pulse B and the retina in Figure 5 if the distance from the probe to the furthest boundary is 24mm.

3 a Use the data in Table 1 to calculate the reflection coefficient of the boundary between **i** air and skin, **ii** water and skin. Assume skin is soft tissue.

b Use the results of your calculation to explain why a gel must be applied between an ultrasonic probe and the skin when the probe is used.

c A body organ has a density of $1040\,kg\,m^{-3}$ and the speed of sound through it is $1580\,m\,s^{-1}$.

i Calculate the acoustic impedance of the organ tissue.

ii Use the data in Table 1 to calculate the reflection coefficient of the boundary between the organ and the surrounding soft tissue.

4 a State the main differences between an A-scan and a B-scan.

b Ultrasonic waves and X-rays are both used for medical imaging. Explain why an ultrasonic scan rather than an X-ray scan is used for scanning a baby in the womb.

24.2 X-rays

On these pages you will learn to:

- Explain the principles of the production of X-rays by electron bombardment of a metal target
- Describe the main features of a modern X-ray tube, including control of the intensity and hardness of the X-ray beam
- Show an understanding of the use of X-rays in imaging internal body structures, including a simple analysis of the causes of sharpness and contrast in X-ray imaging
- Recall and solve problems by using the equation $I = I_0 e^{-\mu x}$ for the attenuation of X-rays in matter
- Show an understanding of the purpose and principles of computed tomography (CT scanning)
- Show an understanding of how the image of an 8-voxel cube can be developed using CT scanning

The production and properties of X-rays

X-ray imaging in medicine is an example of a diagnostic technique that is non-invasive. X-rays are electromagnetic waves of wavelength of the order of 0.1 nm or less. Figure 1 shows how a diagnostic X-ray tube works. The current through the filament wire heats the wire, which causes electrons to be emitted from the wire. These electrons are attracted from the filament or 'cathode' to the **anode** when the anode is positive relative to the filament, typically 20–100 kV for X-ray imaging. The electrons are stopped when they collide with the anode and they emit X-rays in the process.

For an anode potential V, the maximum energy of an X-ray photon $= eV$ so the maximum frequency $f_{max} = eV/h$
Hence the minimum wavelength λ_{min} is given by:

$$\lambda_{min} = \frac{hc}{eV}$$

Note

X-ray tubes used for therapy to destroy tumours are designed differently because they need to produce photons at higher energies. Such tubes operate at voltages from 250 kV to an upper limit (due to insulation breakdown) of 300 kV. X-ray photons from such tubes can destroy tumours no deeper than 5 cm beneath the skin. For deeper tumours, gamma photons with energies of the order of 1 MeV from radioactive isotopes are used.

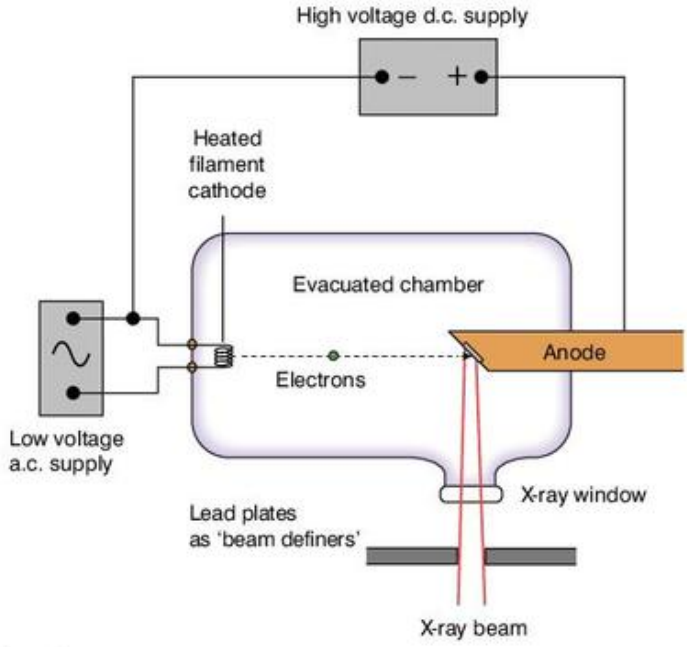


Fig 1 *The X-ray tube*

The spectrum of photon energies from an X-ray tube

An X-ray tube produces a continuous spectrum of photon energies up to the maximum value of eV, where V is the maximum tube voltage, as shown in Figure 2. Raising the tube voltage increases the intensity at all photon energies up to the maximum photon energy as well as increasing the maximum photon energy.

In addition, intensity 'spikes' are produced, which are characteristic of the atoms of the anode and do not change position when the tube voltage is altered. However, if the tube voltage is reduced sufficiently, each spike will disappear when the maximum photon energy is less than the energy of the photons at the spike.

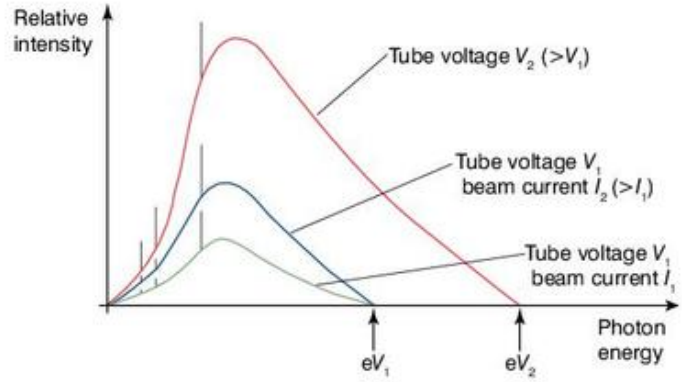


Fig 2 *The energy spectrum of an X-ray tube*

The spikes are caused by the excitation of atoms in the anode when electrons from the filament collide with them. As a result, electrons in the atoms move temporarily from the innermost shells of the atom to higher energy levels. When these electrons return to their original levels, they emit X-ray photons at energies which are characteristic of the anode atoms. These emitted X-rays form patterns of line spectra, each pattern corresponding to electrons returning from the outer energy levels to a particular electron shell. The pattern for electrons returning to:

- the innermost energy shell ($n = 1$) is referred to as the K-series,
- the second energy shell ($n = 2$) is referred to as the L-series,
- the third shell ($n = 3$) is referred to as the M-series.

Notes

1 The energy of an X-ray photon is often expressed in electronvolts (eV) where $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$.

2 The power supplied to an X-ray tube = IV, where I is the beam current. The % efficiency of an X-ray tube is the percentage of the power supplied emitted as X-radiation. A typical X-ray tube has an efficiency of about 1%. The wasted energy is dissipated as heat at the anode.

X-ray imaging

When an X-ray picture is made, X-rays from the X-ray tube are directed for a specified time at the relevant area of the body with a film cassette on the other side of the body. Bones, teeth and other dense matter in the path of the X-rays absorb X-rays much more than muscle and body tissue does. When the film is developed, the areas of the film exposed to X-rays are darker than the unexposed areas so a negative image of the bones, teeth, etc. is formed on the developed film.

For any given application, the following factors must be taken into account before the X-ray tube is used.

1 The penetrating power or 'hardness' of the X-ray beam is increased by increasing the tube voltage. The higher the energy of the X-ray photon, the further it can travel through matter. The X-rays used to give an image of the bones of a broken arm do not need to penetrate as far as the X-rays used to give an image of an organ in the body. Increasing the tube voltage increases the maximum energy of the photons emitted by the tube so the beam is more penetrating. Low energy photons are easily absorbed. An **aluminium filter** placed between the X-ray tube and the patient is used to absorb such photons

which would otherwise be absorbed by the body and cause unnecessary exposure of the body to X-rays.

2 **The intensity of the X-ray beam** is the radiation energy per second passing through unit area at right angles to the area. The darkening of an X-ray film depends on the intensity of the X-radiation as well as on the duration of exposure. The greater the intensity or the longer the duration of exposure, the darker the exposed parts of the film will be. An organ that moves would need a shorter duration of exposure and therefore greater intensity than a bone in the arm which can be held still. The intensity depends on the number of electrons per second reaching the anode and therefore on the tube current (since the tube current is a measure of the number of electrons per second reaching the anode). The tube current is controlled by the current through the filament wire. If the filament current is increased, the intensity of the X-ray beam is increased because:
   - the filament becomes hotter and emits more electrons per second,
   - more electrons per second hit the anode so more X-ray photons are released each second.
3 **The width of the beam** is set using 'beam definer' lead plates to ensure only the part of the patient to be X-rayed is exposed to X-rays. Lead plates surrounding the X-ray tube are used to prevent people other than the patient being exposed to X-rays (see Figure 3).

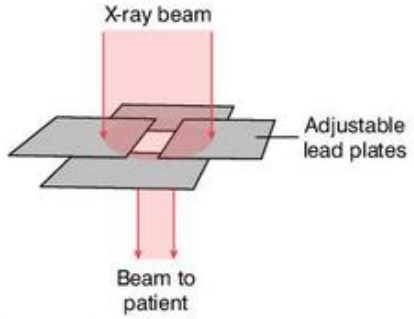


Fig 3 *Beam definers*

Image quality

Image sharpness

The sharpness of an X-ray image is determined by how clearly the edges of structures in the image are defined. A sharp image is one in which such edges can clearly be seen.

To form a sharp image on the film, the X-rays need to originate from a small area of the anode and X-rays scattered by body organs and tissues need to be stopped from reaching the film.

If the area of the anode is too large, the images will be blurred at the edges by large partial shadows as shown in Figure 4. However if the area is too small, the intense concentration of electrons in this area of the target area will damage the anode. To prevent overheating of the anode, it usually consists of a tungsten metal block set in a copper cylinder which is kept cool by pumping water or oil through it. Tungsten is chosen as it has a high melting point and copper is chosen as it is an excellent conductor of heat.

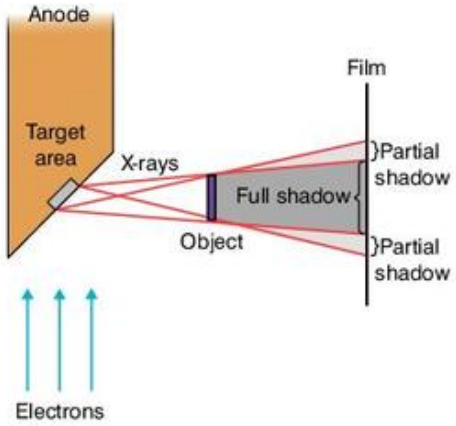


Fig 4 *Sharpness*

- X-ray photons may be scattered by atoms in the body tissues which they pass through. Some scattered X-ray photons may be scattered into the shadow areas on the film of bones or body organs. This would lessen the contrast between the images on the film of bones and body organs and the surrounding tissues. To eliminate scattered X-rays, a lead grid is placed between the patient and the film, as shown in Figure 5. Lead is used because it is a very effective absorber of X-rays.

The grid holes are aligned with the direction in which the unscattered X-rays are travelling so unscattered X-rays that enter the holes of the grid pass straight through it. However, scattered X-rays are absorbed by the grid because they travel mostly through lead after reaching the grid.

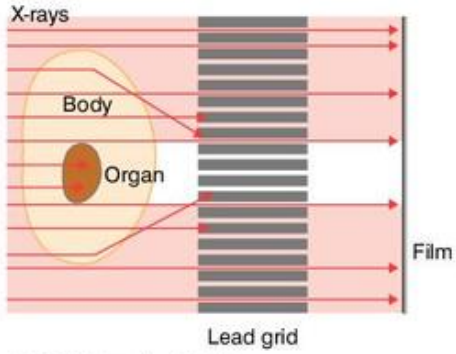


Fig 5 *Using a lead grid*

Contrast

An X-ray image with good contrast has areas where the film is very dark due to exposure to X-rays and other areas that are hardly darkened by X-rays. Bones and teeth are good absorbers of X-rays so they give images that stand out in good contrast with the surrounding tissue.

Body organs such as the stomach are not as effective at absorbing X-rays as bones and teeth. In order to obtain good X-ray images of an organ, a **contrast medium** is used. For example, a patient about to undergo a stomach X-ray is given a drink containing barium sulphate. Because barium is a good absorber of X-rays, X-rays that would otherwise pass through the stomach are absorbed so the contrast between the image of the stomach and its surroundings is vastly improved. A contrast medium is also used to obtain X-ray images of blood vessels where the contrast medium is injected into the bloodstream.

Contrast is lessened if:

- **The duration of exposure is too long**. The light areas and the dark areas of the film both become darker but the increase of darkness is greater in the light areas of the film. So the difference in darkness between the light and dark areas is reduced.
- **The X-rays are too penetrating**. Increasing the energy of the photons by increasing the tube voltage would increase their penetrating power. So more X-rays would pass through the organ and reach the shadow area of the film.
- **Too much scattering of X-ray photons** occurs when they pass through the tissue surrounding the organ.

Contrast can be improved if the film in its cassette is covered with a sheet of fluorescent substance. X-rays directed at a fluorescent substance cause the atoms of the substance to emit light photons. Each X-ray photon might cause many light photons to be emitted, darkening the film more in the areas which are exposed to X-rays. In addition to improving the contrast, the exposure of the patient to X-rays can be reduced as the film is more sensitive to darkening.

Absorption of X-rays by matter

When a beam of X-rays spreads out in a vacuum from a source, the intensity decreases as the distance from the source increases. If the beam spreads out in all directions, the intensity at distance r from the source is proportional to $1/r^2$. This is the same rule as for gamma photons from a point source. See topic 10.4 for an explanation.

When a beam of X-rays passes through matter, some X-ray photons are absorbed and some are scattered. The transmitted beam is said to be **attenuated** because it is less intense than the incident beam.

For a parallel beam of intensity I_0 directed at normal incidence at an absorber of thickness x, the transmitted intensity I is given by:

$$I = I_0 e^{-\mu x}$$

where μ is the **attenuation** (or absorption) **coefficient** of the absorber.

The variation of intensity with thickness is shown in Figure 7. The curve decreases exponentially with increase of thickness.

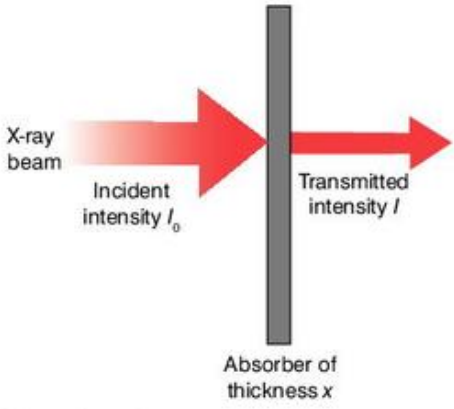


Fig 6 Attenuation

The half thickness, $X_{½}$, of an absorber is the thickness required to reduce the intensity of the beam to half its initial value.

For thickness $x = X_{½}$, $I = \frac{1}{2}I_0$.

Substituting these values into the above equation gives
$\frac{1}{2}I_0 = I_0 e^{-\mu X_{½}}$

Cancelling I_0, rearranging and taking natural logs of both sides of this equation gives:

$$\mu X_{½} = \ln 2$$

Notes

1 The unit of μ is m^{-1} (or cm^{-1} or mm^{-1}).
2 μ is proportional to the density of the absorber. Bones absorb X-rays more than body tissue does because bones are more dense.
3 Absorption of X-rays due to knocking electrons out of atoms is proportional to Z^3, where Z is the atomic number of the atom. This is why a suitable element with a high value of Z (e.g. barium) is used as a contrast medium when an X-ray picture is made of a soft organ such as the stomach.

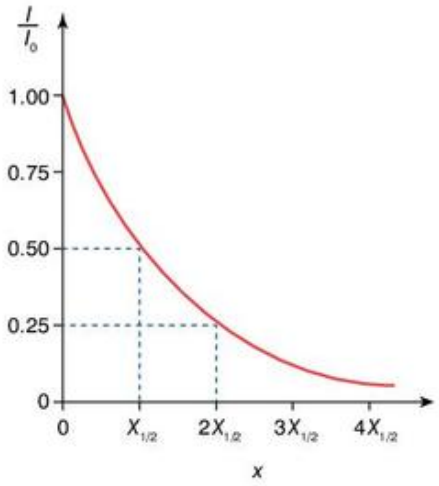


Fig 7 *Transmitted intensity against thickness*

Computer tomography (CT)

In a CT scan, a narrow collimated beam of X-rays is directed through the patient at an array of detectors. The X-ray tube and the detectors are rotated in small steps around an axis along the length of the patient. The detectors are connected to a computer and the signals from the **detectors** are processed to form an image of the cross-section of the body which is exposed to the beam.

- The tube and the detectors are mounted in a gantry which is rotated through 360° about the patient lying in a stationary position on a flat couch between the tube and the detector.
- Each detector receives X-rays that have travelled along a straight path between the tube and the detector.

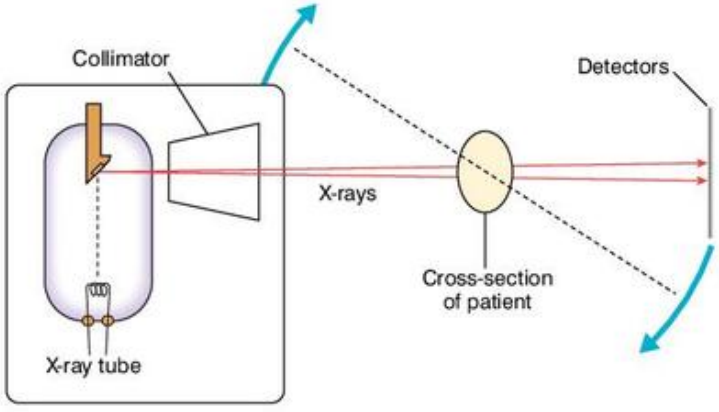


Fig 8 *CT scanning*

The intensity detected by each detector depends on the absorption of X-rays along the path from the tube to the detector. This absorption depends on the different densities and thicknesses of the tissue along the path.

By considering the body divided into small volume elements, referred to as 'voxels', each voxel would contribute a certain 'pixel' value to the intensity reduction along any given path.

- Each detector records the sum of the pixel values along each path,
- Each voxel contributes its pixel value to the detector reading (i.e. the detected intensity) many times as the tube and detectors are rotated round the patient.

The signals from the detectors can be processed by a computer to work out each pixel value for each voxel and hence display a 2-D 'mosaic' image of the cross-section. The contrast and sharpness of the pixels can be adjusted to give the best possible image. In addition, the image can be stored and transmitted electronically.

Note

A complete scan of a single cross-section takes about 10 s. By taking further scans in parallel planes, the computer can produce an image of a cross-section through the patient in any plane, not just in the scanning planes.

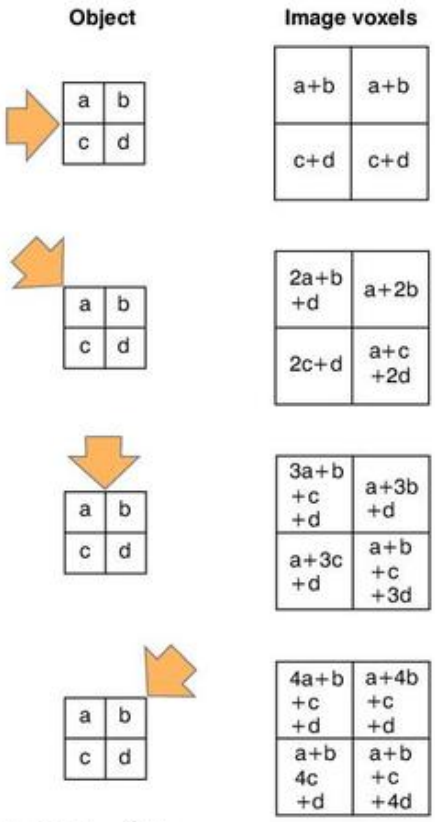


Fig 9 *A simplified scan*

A model scan

Figure 9 shows a simplified version of the process where four voxels are scanned from four different directions which are horizontal, diagonal, vertical and the opposite diagonal.

The pixel value of each voxel is worked out by reconstructing an image consisting of four image voxels and assigning the detector readings to the image voxels according to the path the X-rays pass through in each direction. In this 2 × 2 example:

- Each image voxel receives a contribution four times (once for each direction) from its corresponding object voxel and once from each of the other three object voxels. The total value of all the image voxels after the complete scan is therefore seven times the sum of the object voxel values.
- The sum of the object voxel values is therefore the total value of all the image voxels ÷ 7. This sum is effectively a background value for each of the four image voxels.

Subtracting the background value from the final value of each image voxel leaves a value equal to three times the contribution from the corresponding object voxel. The value of each object voxel can then be calculated.

Suppose the image voxel values shown in Figure 9 after a complete scan are 22, 16, 25 and 28.

- Their total value = 22 + 16 + 25 + 28 = 91
- The sum of the object voxel values = 91 ÷ 7 = 13
- Subtracting this background value from the values of the image voxels gives 9, 3, 12 and 15.
- Dividing these 'corrected' values by 3 gives 3, 1, 4 and 5 for the object voxel values.

QUESTIONS

1 An X-ray tube operates at an anode potential of 50 kV.

a i Show that the minimum wavelength of the X-rays from this tube is 0.025 nm.

ii When the beam current at 50 kV is 0.2 mA, the tube operates at an efficiency of 1.5%. Calculate the radiation energy per second produced by the tube.

b State and explain what change in the X-ray beam occurs if:

i the tube current is increased,

ii the anode voltage is increased.

2 a What is meant by **i** the sharpness, **ii** the contrast of an X-ray image.

b State the function of a scattering grid and explain why it is necessary when an X-ray image is obtained.

3 a A metal plate of thickness 1.3 mm placed in the path of a collimated X-ray beam reduces the beam intensity by 70%. Show that the absorption coefficient of the metal plate is 9.3×10^{-2} m^{-1}.

b Explain why a contrast medium is used when an X-ray picture of the stomach is made.

4 a With the aid of a diagram, outline the principle of operation of a CT scanner.

b In the simplified model in Figure 9, the values of the image voxels after a complete scan were 21, 15, 21 and 27. Calculate **i** the background value of the image voxels, **ii** the pixel value of each object voxel.

24.3 Magnetic resonance imaging (MRI)

On these pages you will learn to:

- Explain the main principles behind the use of nuclear magnetic resonance imaging (NMRI) to obtain diagnostic information about internal structures
- Show an understanding of the function of the non-uniform magnetic field, superimposed on the large constant magnetic field, in diagnosis using NMRI

Nuclear magnetic resonance

The magnetic resonance (MR) scanner is used in hospitals to scan the hydrogen content in the body. Hydrogen atoms are present in the body mostly in water molecules but also in many other molecules in the body. As explained later, the type of tissue which the molecules are in affects the way hydrogen nuclei in the molecules respond to changing magnetic fields which is why MR scans can distinguish different types of tissue.

A hydrogen nucleus (and certain other nuclei, e.g. carbon, phosphorus) is like a tiny bar magnet as it possesses intrinsic magnetism, technically referred to as a magnetic moment, which makes it turn in a magnetic field. This intrinsic magnetism is due to the motion of the nucleons in the nucleus, which also makes the nucleus spin.

In a strong magnetic field:

- A bar magnet can be either lined up in the direction of the magnetic field or in a higher energy state in the opposite direction to the magnetic field in an unstable state.
- A nucleus which has a magnetic moment acts like a spinning top in a magnetic field as its spin axis precesses about the direction of the external magnetic field just as the axis of a tilted spinning top precesses about a vertical line. The frequency of precession is called the **Larmor frequency** and it depends on the magnetic flux density and the type of nucleus. The Larmor frequency is proportional to the magnetic flux density. For example, a hydrogen nucleus in a magnetic field of flux density 1 T precesses at a frequency of 43 MHz.
- Relative to the magnetic field, the nucleus has just two energy levels corresponding to the two different alignments relative to the magnetic field as shown in Figure 1.

A hydrogen nucleus in a **strong** magnetic field can be excited into the higher energy level using radio waves of the same frequency as the frequency of precession of the nucleus. This process of excitation is known as **nuclear magnetic resonance**. As the frequency of precession is proportional to the magnetic flux density of the external magnetic field, it can be changed by changing the magnetic flux density of the external field. Nuclear magnetic resonance can therefore occur by adjusting the magnetic flux density until the precession frequency is equal to the frequency of the radio waves.

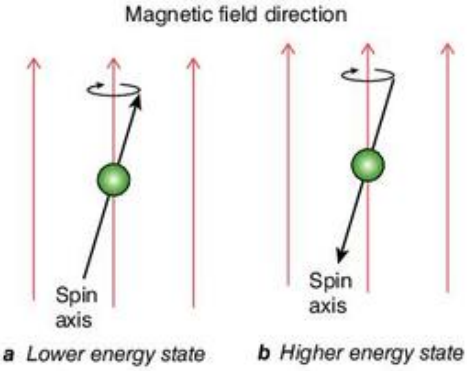


Fig 1 Precession

In a sample of hydrogen nuclei in a magnetic field, some nuclei will be in the higher energy level due to the thermal energy of the atoms. At room temperature, there are a slightly higher proportion of nuclei in the lower energy state.

- When a pulse of radio waves of the same frequency as the precession frequency is applied to the sample, some of the nuclei in the lower energy state each absorb a radio-wave photon and flip into the higher energy level.
- When the radio pulse ends, some of the excited nuclei flip back to the lower energy state, each emitting a radio wave photon. In an MR scanner, the detection of the emitted photons is used by the scanning software to determine the location of the nucleus that emitted it.

In the MR scanner

The key design feature of an MR scanner is that it applies a magnetic field of specific flux density to the patient at a precise and well-defined location that changes systematically with time so as to scan a cross-section of the patient. At this specific flux density, the magnetic field causes excitation of the hydrogen nuclei at a definite location. As the nuclei de-excite before the next pulse is emitted, the signal received by the detector must therefore have come from that location. The intensity of the signal depends on the number of nuclei excited at that location. The detector signal is processed by a computer to construct an image of the number of hydrogen atoms per unit volume in the patient.

The **magnetic field** is produced by a large superconducting electromagnet which generates a large constant magnetic field and by coils that provide magnetic fields with gradients that ripple through the patient in perpendicular directions.

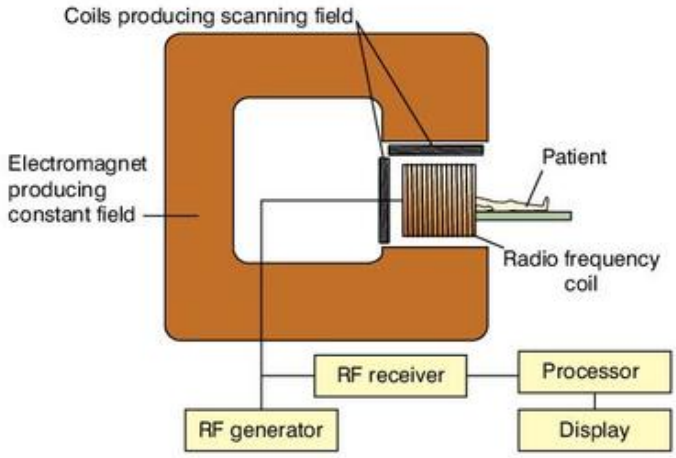


Fig 2 An MR scanner system

The combined effect is to scan the patient with a magnetic field of the required flux density. In addition a radio frequency coil supplied with a high-frequency alternating current is used to excite the nuclei.

In the scan, the location of the excited nuclei changes along a 'raster' of successive straight lines in a single plane through the patient, like the lines that build up a visual image on a TV screen. The detector, pulse transmitter and magnetic field coils that create the scanning raster are connected to a computer which is programmed to create a visual image of a cross-section of the patient. By scanning adjacent cross-sections systematically, a 3-dimensional image of any scanned part of the patient can be displayed.

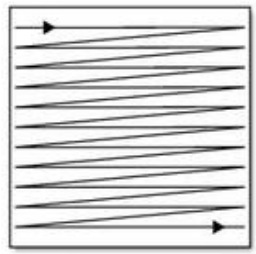


Fig 3 A raster of lines

Tissue discrimination is possible because the rate of decay of the detected signal after each pulse, the relaxation time, depends on the type of molecules surrounding the water molecules.

The magnetic fields of surrounding magnetic nuclei delay the de-excitation of excited nuclei to different extents, according to their relative arrangement. For example, grey matter in the brain has a relaxation time of 0.37 s compared with 0.30 s for white matter. In addition, the strength and length of the pulse affects the proportion of excited to unexcited nuclei in different types of tissue and therefore contributes to the relaxation time of the signal. The relaxation time is therefore used to identify tissue types.

In addition MRI scans do not expose patients to ionising radiation as X-ray scans do and seem to cause little or no physiological harm. Brain research and medical diagnosis have moved forward very dramatically as a result of MRI scanners which, although extremely expensive, are now in operation at major hospitals in many countries. Examples of brain research include monitoring the sites in the brain of increased blood flow due to increase of brain activity when undertaking specific physical or mental activities.

Examples of MRI applications in medicine generally involve distinguishing between different types of soft tissue including imaging:

- arteries to detect narrowing or potential ruptures
- brain tissue to detect scars, tumours and other damage
- spinal discs to assess damage such as rupture or movement out of place
- hip and knee joints to detect early osteoarthritis (by early detection of a chemical substance known to be associated with osteoarthritis).

***Table 1** Comparison of MRI, ultrasound and X-rays*

	MRI	Ultrasound	X-rays
Comparative advantages	• non-ionising radiation • distinguishes between cancerous and normal tissue as well as between different types of tissue	• non-ionising radiation • distinguishes between different types of soft tissue	• detects bone fractures • image enhancement through use of contrast media • good resolution, • real-time images possible
Comparative disadvantages	• expensive, • superconducting magnet required,	• poor resolution	• ionising radiation is dangerous
Preferred uses	• brain scans • whole body scans	• pregnancy scans • tumour scans	• bone fracture images • soft organ images with use of contrast media

QUESTIONS

1 Give two reasons why MR scanners can locate hydrogen nuclei in the body.

2 a What physical quantity is changed systematically when an MR scan is carried out?

b What physical property of hydrogen nuclei is made use of in an MR scan?

3 a What physical quantity is detected in an MR scan?

b Describe the changes that take place to the hydrogen nuclei in the body when they absorb radio wave photons in an MR scan.

4 a What characteristic of the detected signals in an MR scan is used to discriminate between different types of tissue?

b State and explain **one** advantage of an MR scan in comparison with an X-ray scan.

24 End of chapter questions

1 Outline briefly the main principles of the use of magnetic resonance to obtain diagnostic information about internal body structures. *(6 marks)*

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2 a i State what is meant by the *acoustic impedance* of a medium. *(1 mark)*

ii Data for some media are given below.

medium	speed of ultrasound /m s^{-1}	acoustic impedance /kg m^{-2} s^{-1}
air	330	4.3×10^{2}
gel	1.5×10^{6}	1.53×10^{6}
soft tissue	1.6×10^{6}	1.63×10^{6}
bone	7.0×10^{6}	6.80×10^{6}

Fig 2.1

Use data from Figure 2.1 to calculate a value for the density of bone in kg m^{-3}. *(1 mark)*

b A parallel beam of ultrasound has intensity I. It is incident at right-angles to a boundary between two media, as shown in Figure 2.2.

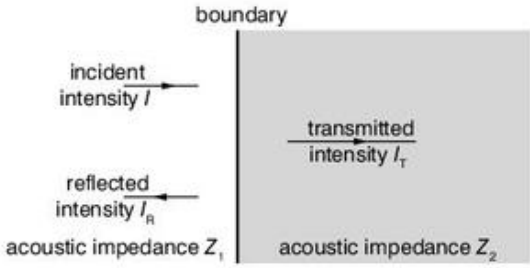


Fig 2.2

The media have acoustic impedances of Z_1 and Z_2. The transmitted intensity of the ultrasound beam is I_T and the reflected intensity is I_R.

i State the relation between I, I_T and I_R. *(1 mark)*

ii The reflection coefficient a is given by the expression

$$a = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}.$$

Use data from Figure 2.1 to determine the reflection coefficient a for a boundary between

1. gel and soft tissue, *(2 marks)*

2. air and soft tissue. *(1 mark)*

c By reference to your answers in **b ii**, explain the use of a gel on the surface of skin during ultrasound diagnosis. *(3 marks)*

Adapted from Cambridge International AS and A Level Physics 9702
Paper 41 Q10 November 2010

Key concepts: Thermal and nuclear physics

In studying Chapter 20, you will have gained a sound understanding of internal energy and absolute temperature and a good grasp of the practical skills involved in measuring specific heat capacities and specific latent heats. In addition, by considering conservation of energy, you will have learned how to analyse problems involving specific heat and latent heat and to carry out the necessary calculations in order to solve such problems.

After studying Chapter 21 on gases, you should appreciate the experimental basis of the ideal gas equation and how the kinetic theory of gases provides a full explanation of the properties of an ideal gas. The random motion of the molecules of an ideal gas is a key concept here and you should recognise that the observation of Brownian motion provides direct evidence of this random motion. In studying gases, you will have developed your mathematical skills in carrying out calculations using the ideal gas equation and the kinetic theory of gases equation. In addition, you should be able to combine these two equations to show that the mean kinetic energy of the molecules of an ideal gas is proportional to the absolute temperature of the gas.

Chapter 22 provides insight into the methods used to determine the properties of electrons – in particular their specific charge. Such methods require the use of electric and magnetic fields to produce and control electron beams. You should also be aware that the results of investigations in photoelectricity could not be explained using the wave theory of light and consequently led to the establishment by Einstein of the photon theory of light. The success of the photon theory should be evident to you from your studies of energy levels and spectra as well as from the successful explanation of photoelectricity. In contrast to the properties described above, where experiments and investigations preceded important theoretical work, the wave nature of matter was predicted by de Broglie and only accepted when experimental evidence of electron diffraction was discovered. Throughout Chapter 22, in addition to furthering your knowledge and understanding of the structure and properties of matter, you should have further developed your problem-solving skills as well as your analytical and mathematical skills.

Your studies of nuclear physics in Chapter 23 build on previous studies on radioactivity in Chapter 10. After studying Chapter 23, you should be aware that radioactive decay is a random process that gives a full explanation of the exponential decrease in the activity of a radioactive isotope. In addition, you should be able to appreciate how the concept of binding energy and mass defect enables us to explain why energy is released in nuclear changes including fission and fusion. In studying radioactive decay, you will have further developed your mathematical skills in solving problems and carrying out calculations involving exponential decreases and nuclear reactions.

The final chapter, Chapter 24 on medical imaging, provides awareness of the important role of physics in medicine. The three topics, ultrasonic imaging, X-ray imaging and magnetic resonance imaging are not the only imaging techniques used, but they do involve techniques applicable to different situations.

The following table provides an overview of the topics in chapters 20 to 24, showing where key concepts have been developed.

Chapters 20–24: Key concepts

	Topic		Key concepts	
20.1–20.2	internal energy and temperature	first law of thermodynamics	energy and matter	conservation of energy
		absolute temperature		internal energy
20.3–20.4	specific heat capacity and specific latent heat	measuring specific heat capacity and specific latent heat	testing	practical skills
		heat capacity, latent heat	energy and matter	conservation of energy
		calculations involving specific heat capacity and specific latent heat	use of maths	problem-solving,
21.1–21.2	ideal gases	testing the gas laws	testing	practical skills
		Brownian motion molar mass	energy and matter	random motion molecular theory
		calculations involving the ideal gas equation	use of maths	problem-solving
21.3–21.4	kinetic theory of gases	derivation of the kinetic theory of gases equation	models	kinetic theory of gases model
		random motion, root mean square speed, internal energy of an ideal gas	energy and matter	random motion
		calculations involving the kinetic theory of gases equation and the ideal gas equation	use of maths	problem-solving
22.1–22.3	properties of electrons	thermionic emission measurement of e/m, velocity selector	energy and matter	resultant forces
		calculations on deflection of electron beams in electric and magnetic fields	use of maths	problem-solving
22.5–22.7	electrons and photons	electromagnetic waves, photoelectricity, threshold frequency, work function energy levels and spectra	energy, matter and waves	nature of light
		the photon model used to explain photoelectricity, Einstein's photoelectric equation	models	photon model
		energy bands		model of energy levels and energy bands
		calculations on photoelectricity and energy levels	use of maths	problem-solving,
22.8	band theory	conduction and valence electrons	models	band theory
		intrinsic and extrinsic semiconductors		
22.9	wave particle duality	wave properties of electrons, electron diffraction	energy, matter and waves	dual nature of matter
		de Broglie's hypothesis and equation		
		calculations involving de Broglie's equation	use of maths	problem-solving

continued on p382

Chapters 20–24: Key concepts *(continued)*

	Topic		Key concepts	
23.1–23.3	radioactive decay	activity, half-life	energy, matter and waves	
		random nature of radioactive decay	models	randomness
		theory of radioactive decay		exponential decrease
		calculations using the radioactive decay equation	use of maths	problem-solving
23.4–23.6	energy from the nucleus	binding energy, mass defect	energy and matter	conservation of energy
		binding energy curve		
		fission and fusion	models	chain reaction
		induced fission		
		calculations of binding energy per nucleon and of energy released in nuclear reactions	use of maths	problem-solving
24.1–24.3	medical imaging	ultrasonic imaging	energy, matter and waves	use of waves to probe matter
		X-ray imaging in medicine		
		MR scanning		ionising and non-ionising radiation
		CT scans	models	3D images
		calculations involving acoustic impedance, reflection coefficients, absorption using $I = I_0 e^{-\mu x}$ and minimum photon wavelength from an X-ray tube	use of maths	problem-solving

Question

The first part of the question is about the nuclear model of the atom and why scientists predicted that the nucleus is composed of neutrons as well as protons before there was any direct evidence of the existence of neutrons. It also gives some insight into how such direct evidence was obtained using alpha radiation. The question also tests the application of the principles of conservation of energy and of momentum to the emission of α-particles from an α-emitting isotope to explain why the α-particles are emitted with well-defined energy.

This well-defined energy is the reason why α-particles from an α-emitting isotope have a well-defined range in air, and the final part of the question also tests the relationship between the range $\boldsymbol{R}$ *in air of α-particles and their initial kinetic energy* $\boldsymbol{E}$ *using experimental data from several α-emitting isotopes. A graph of* $\log \boldsymbol{R}$ *against* $\log \boldsymbol{E}$ *is plotted on the assumption that the relationship is of the form* $\boldsymbol{R} = \boldsymbol{kE}^{\mathrm{n}}$*, where* n *is an unknown constant. A straight-line graph would confirm that the relationship is of this form, enabling the power* n *to be found as it is equal to the gradient of the graph. Knowledge of this value of* n *provided physicists with a further challenge beyond the question to develop a theoretical model that 'confirms' the value of* n *found using the graphical method. The question thus shows how we can use data from experimental investigations to find relationships that can then be investigated by devising theoretical models that explain the relationships. In addition, knowing the value of* n *enables* $\boldsymbol{E}$ *to be calculated for other α-emitting isotopes by measuring the range* $\boldsymbol{R}$ *and using the now-known relationship between* $\boldsymbol{E}$ *and* $\boldsymbol{R}$*.*

Before answering the question, students should be given the opportunity to do this or a similar investigation.

1 The neutron–proton model of the nucleus was first put forward by Rutherford to explain the general composition of the nucleus. The existence of the neutron was not proved experimentally until some years later.

a Rutherford knew from his alpha-scattering experiments that the charge of a nucleus is equal to $+Ze$, where Z is the atomic number of the nucleus and e is the magnitude of the charge of the electron. He also knew that the mass of a nucleus is approximately equal to $A\,m_u$, where A is the mass number of the nucleus and m_u is the mass of a hydrogen nucleus.

i By reference to the isotopes ${}^{1}_{1}\mathrm{H}$ and ${}^{4}_{2}\mathrm{He}$, give **two** reasons why Rutherford's neutron–proton model was considered more than an untested hypothesis when it was first put forward. *(2 marks)*

ii Explain why Rutherford concluded that the hydrogen ${}^{1}_{1}\mathrm{H}$ nucleus is a single proton. *(1 mark)*

b Evidence of the existence of neutrons was found when it was discovered that α-particles knocked uncharged particles out of beryllium foil. Investigations into the nature of the uncharged radiation showed it consisted of particles each uncharged and of about the same mass as a proton.

i Complete the nuclear reaction below to show the result of an inelastic collision between an α-particle and a beryllium ${}^{9}_{4}\mathrm{Be}$ nucleus.

$${}^{4}_{2}\alpha + {}^{9}_{4}\mathrm{Be} \longrightarrow {}^{\ }_{6}\mathrm{C} + {}^{1}_{0}\mathrm{n}$$ *(1 mark)*

ii Explain why the above change only takes place if the α-particle has sufficient kinetic energy. *(3 marks)*

c The α-particles from any α-emitting isotope have the same initial kinetic energy and a well-defined range in air at atmospheric pressure.

i The isotope ${}^{210}_{84}\mathrm{Po}$ emits α-particles with initial kinetic energy of 5.3 MeV. Show that the nucleus recoils with an initial kinetic energy of 0.1 MeV and hence calculate the total energy released by the nucleus when it emits an α-particle. *(4 marks)*

ii The table below shows the range R in air and the initial kinetic energy E of α-particles from several α-emitting isotopes

***R*/mm**	39	48	53	57	66	78
***E*/MeV**	5.3	6.0	6.5	6.8	7.4	8.3

Plot a suitable graph to find out if the relationship between R and E is of the form $R = kE^n$, where k and n are constants, and determine a value for n. Explain your choice of graph. *(9 marks)*

25 Physics of the eye and the ear

EXTENSION

25.1 The eye as an optical refracting system

On these pages you will learn to:

- Describe the structure and characteristics of the eye as a refracting system
- State the function and location of rods and cones on the retina
- Sketch graphs to show the sensitivity of rods and the three types of cones to light of different wavelengths
- State and explain what is meant by persistence of vision

The eye is an optical instrument that can focus automatically on objects over a wide range of distances, adjust automatically to a wide range of light intensities and is sensitive to a continuous range of electromagnetic waves from less than 400 nm to about 650 nm in wavelength.

Figure 1 shows the internal structure of the eye. The main optical parts of the eye and their principal features are described below:

- The **cornea** is a protective transparent layer at the front of the eye. It has a fixed convex curvature and therefore acts as a 'fixed focus' lens.
- The **eye lens** is flexible and attached to the ciliary muscles. These muscles change the thickness of the eye lens which alters its optical power. This enables it to form an image on the retina of the eye of any object within a range of distances.
- The **ciliary muscle fibres** lie along concentric circles round the rim of the eye lens.
- To view a near object, the eye muscles must become taut so that the muscle fibres shorten and make the eye lens thicker and more powerful.
- To view a distant object, the eye muscles must relax so that the muscle fibres lengthen, allowing the eye lens to become thin and less powerful.

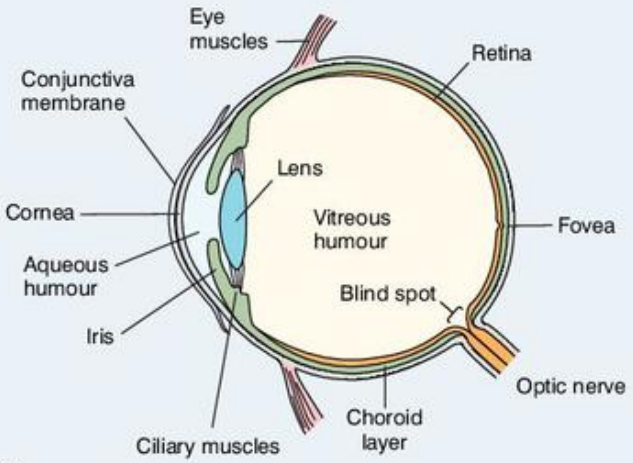
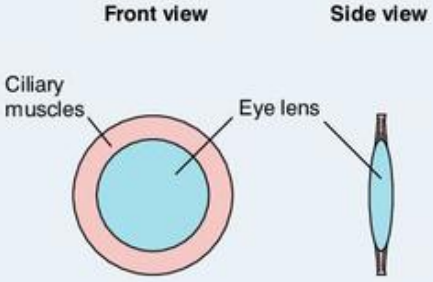
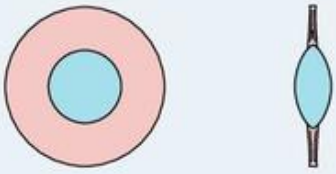


Fig 1 *The eye*



a *Ciliary muscle relaxed = thin eye lens*



b *Ciliary muscle taut = thick eye lens*

Fig 2 *The eye lens*

The **iris** controls the amount of light entering the eye. It consists of concentric and radial muscle fibres.

- In bright light, the concentric fibres contract and the radial fibres relax so the iris expands, making the eye pupil narrower so less light passes through it.
- In dim light, the concentric fibres relax and the radial fibres contract so the iris contracts, dilating (i.e. widening) the eye pupil so more light passes into the eye.

The range of the eye pupil's diameter is typically from less than 1 mm up to 10 mm. The area of the pupil determines the amount of light entering the eye which therefore increases by a factor of 100 when the diameter of the eye pupil increases from 1 mm to 10 mm.

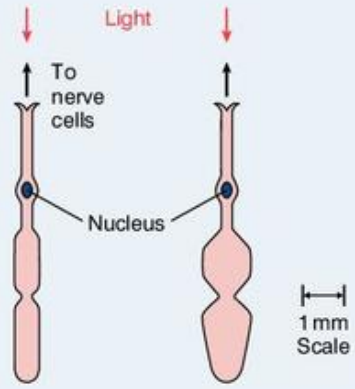
The sensitivity of the eye

The **retina** is a layer of light-sensitive cells at the back of the eye. There are two types of retinal cells, **rods** and **cones**. The retinal cells are most dense at the **fovea** which is the region of the retina near the principal axis of the eye lens. The fovea consists mostly of cones whereas rods predominate near the periphery of the retina.

Rods are sensitive to low levels of light intensity but cannot distinguish between colours. Because rods predominate at the periphery of the retina, dim objects viewed in dark conditions can often be seen at the edge of your field of view (but not at the centre) but you cannot tell their colour.

Rods contain rhodopsin, also known as visual purple, which consists of complex molecules that can be split in two by light photons. This causes a change in the cell potential which helps to 'trigger' the nerve fibre to which the cell is connected. Up to ten photons need to be absorbed to trigger a rod. Several rods connected to the same nerve fibre need to be triggered to send an electrical impulse to the brain. The rhodopsin molecules regenerate slowly. In very bright light, the molecules are unable to regenerate so the rods have reduced sensitivity. Adaptation to dark conditions takes over 30 minutes as the rhodopsin molecules slowly re-form.

Cones are of three types, each sensitive to a different range of wavelengths. These ranges correspond broadly to red or green or blue light, as shown in Figure 3. Cones do not respond to very low levels of light intensity and automatically become less sensitive at very high intensities. The after-image seen after a very bright image disappears is because the cones take up to a second or so to regain normal sensitivity. This is known as persistence of vision. If the original image is coloured, the after-image will be the complementary colour. This is because the type of coloured cones stimulated by the original image does not respond effectively until they regain their normal sensitivity. During this time, the other cones operate normally and hence form a temporary after-image in the complementary colour. Television viewers rely on persistence of vision as the TV picture is 'renewed' at least 25 times every second ; because of persistence of vision, the viewer sees a continuous sequence of 'frames'. If the frame rate was significantly lower, the pictures would flicker noticeably.



a A rod cell *b A cone cell*

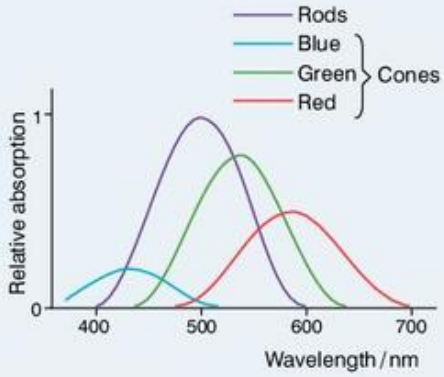


Fig 3 *Rods and cones*

QUESTIONS

1 Describe and explain the adjustment that takes place within the eye when it changes from viewing a nearby object to a distant object.

2 State two ways in which the eye adapts when the incident light becomes:
   a very intense
   b very dim.

3 Sketch graphs on the same axes to show the response of rods and the three different types of cones to light of different wavelengths from about 350 nm to 700 nm. Label each graph with the type of retinal cell.

4 Explain why objects in dim light at the edge of the field of view are colourless.

Fig 4 *Stare at the picture for a few seconds. Then close and cover your eyes to see the after-image.*

REVISION

25.2 Lenses

On these pages you will learn to:

- Define the focal length of a converging lens and a diverging lens
- Use ray diagrams to explain the formation of images by a thin lens
- Use the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ in calculations

Converging and diverging lenses

Lenses are used in optical devices such as the camera, the telescope and the eye. A lens works by changing the direction of light at each of its two surfaces. Figure 1 shows the effect of a converging lens and of a diverging lens on a beam of parallel light rays.

- A **converging lens** makes parallel rays converge to a focus. The point where the parallel rays are focused to is called the principal focus or the focal point of the lens.
- A **diverging lens** makes parallel rays diverge (i.e. spread out). The point where the rays appear to come from is the principal focus or focal point of this type of lens.
- In both cases, the distance from the lens to the principal focus is the focal length of the lens.

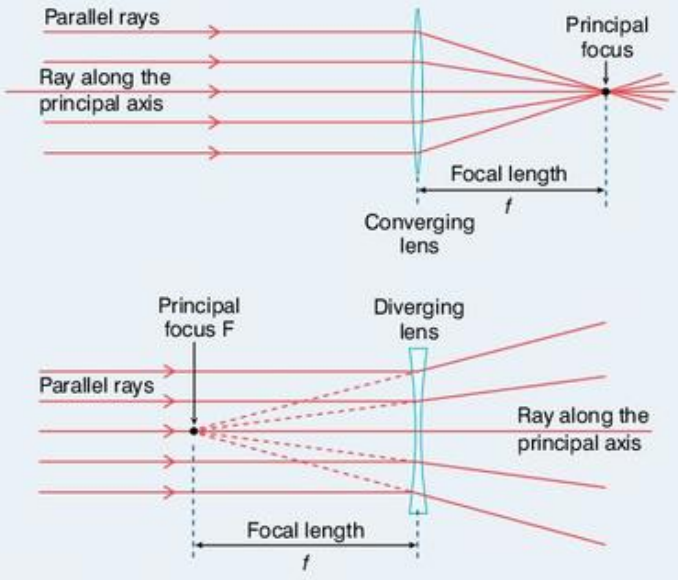


Fig 1 Focal length

NOTE

The principal axis of a lens is the straight line that passes normally through both surfaces at their centres. The plane on each side of the lens perpendicular to the principal axis containing the principal focus is called the focal plane.

Investigating the converging lens

The arrangement in Figure 2 can be used to investigate the image formed by a converging lens. Light rays from the illuminated crosswires are refracted by the lens to form an image of the crosswires.

1 With the object at different distances beyond the principal focus of the lens, the position of the screen is adjusted until a clear image of the object is seen on the screen. The image is described as a **real image** because it is formed on the screen where the light rays meet. If the object is moved nearer the lens towards its principal focus, the screen must be moved further from the lens to see a clear image. The nearer the object is to the lens, the larger the image is.

2 With the object nearer to the lens than the principal focus, a magnified image is formed. The lens acts as a magnifying glass. But the image can only be seen when you look into the lens from the other side to the object. The image is called a **virtual image** because it is formed where the light rays appear to come from.

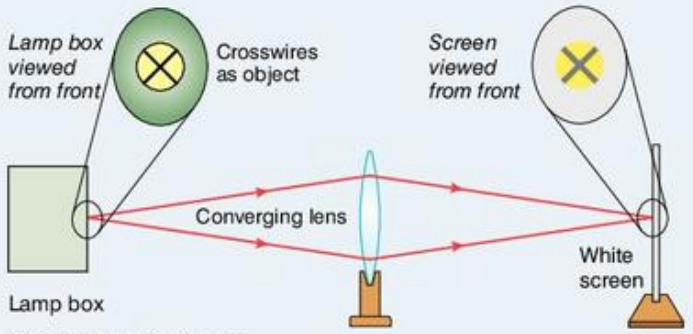


Fig 2 Investigating images

Ray diagrams

The position and nature of the image formed by a lens depends on the focal length of the lens and the distance from the object to the lens. If we know the focal length, f, and the object distance, u, we can find the position and nature of the image by drawing a ray diagram to scale in which the lens is assumed to be thin so it can be represented by a single line at which refraction takes place. The principal focus F is marked on the principal axis at the same distance from the lens on each side of the lens. The object is represented by an 'upright' arrow as shown in Figure 3 opposite.

Note that the 'horizontal' scale of the diagram must be chosen to enable you to fit the object, the image and the lens on the diagram.

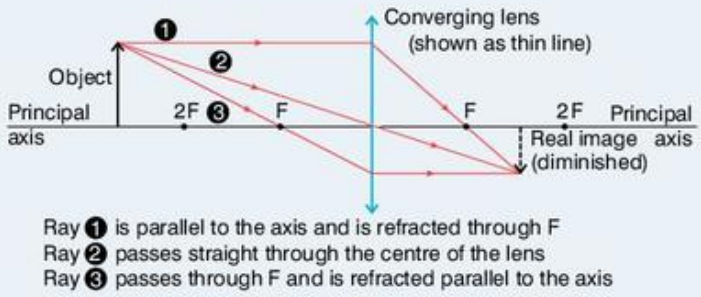


Fig 3 Formation of a real image by a converging lens

1. Formation of a real image by a converging lens

To form a real image, the object must be beyond the principal focus F of the lens. The image is formed on the other side of the lens to the object.

To locate the tip of the image, three key 'construction' rays from the tip of the object are drawn, as described below, through the lens. The tip of the image is formed where these three rays meet. The image is real and inverted.

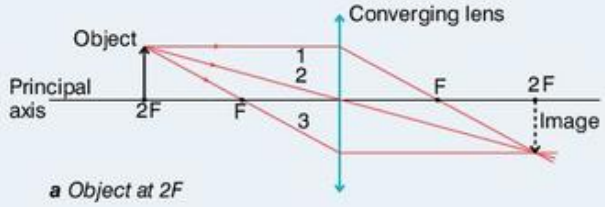
1 Ray 1 is drawn parallel to the lens axis before the lens so it is refracted by the lens through F.
2 Ray 2 is drawn through the lens at its centre without change of direction. This is because the lens is thin and its surfaces are parallel to each other at the axis.
3 Ray 3 is drawn through F before the lens so it is refracted by the lens parallel to the axis.

In Figure 3, the image is smaller than the object. This is because the object is beyond 2F.

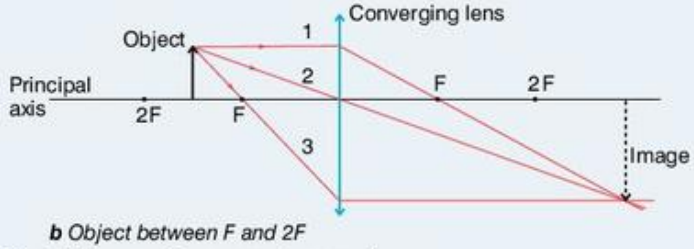
Figure 4a and b show respectively ray diagrams for the object at 2F and between F and 2F. The results for Figures 3 and 4 are described in Table 1.

Notice that the image is:

- diminished in size when the object is beyond 2F as in Fig 3
- the same size as the object when the object is at 2F as in Fig 4a
- magnified when the object is between F and 2F as in Fig 4b.



a Object at 2F



b Object between F and 2F

Fig 4 Using ray diagrams to locate an image

2. Formation of a virtual image by a converging lens

The object must be between the lens and its principal focus, as shown in Figure 5. The image is formed on the same side of the lens as the object.

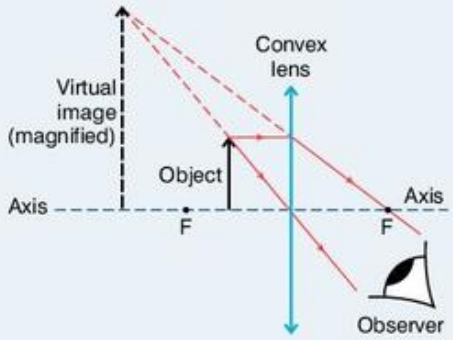


Fig 5 Formation of a virtual image by a converging lens

The diagram shows that the image is virtual, upright and larger than the object. The image is on the same side of the lens as the object and can only be seen by looking at it through the lens. This is how a magnifying glass works.

If the object is placed in the focal plane, light rays from any point on the object are refracted by the lens to form a parallel beam. A viewer looking at the object through the lens would therefore see a virtual image of the object at infinity.

Table 1 Image formation by a converging lens

Object position	Image position	nature of image	magnified or diminished	upright or inverted	application
>2F	Between F and 2F	real	diminished	inverted	camera
2F	2F	real	same size	inverted	inverter
Between F and 2F	>2F	real	magnified	inverted	projector
<F	same side as object	virtual	magnified	upright	magnifying lens

3. Formation of a virtual image by a diverging lens

The lens makes light rays from a point object diverge. The image is formed on the same side of the lens as the object, as shown in Figure 6. To locate the tip of the image, three key 'construction' rays from the tip of the object are drawn, as described below, through the lens. The tip of the image is formed where these three rays appear to come from. The image is always virtual, upright and smaller than the object.

1 Ray 1 is drawn parallel to the lens axis before the lens so it is refracted as if it passed through F before the lens.
2 Ray 2 is drawn through the lens at its centre without change of direction. This is because the lens is thin and its surfaces are parallel to each other at the axis.

The image position can be checked by drawing a third ray (shown between rays 1 and 2) from the tip of the object towards F on the other side of the lens so it is refracted by the lens parallel to the axis.

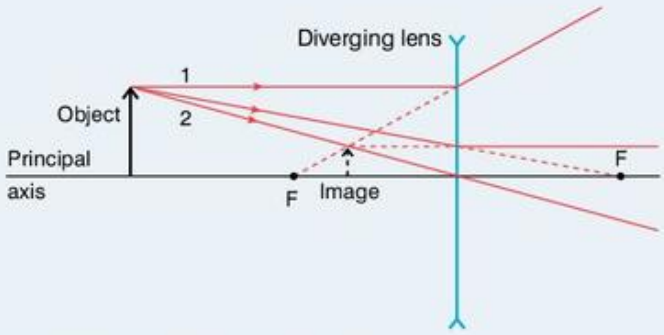


Fig 6 Ray diagram for a diverging lens

Note

The linear magnification, m, of the image = height of the image/height of the object. It can be shown that this ratio is equal to $\frac{\text{the image distance } v}{\text{the object distance } u}$.

The image is said to be magnified if the image height is greater than the object height and diminished if it is smaller.

The lens formula

For an object on the principal axis of a thin lens of focal length f at distance u from the lens, the distance from the image to the lens, v, is given by

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Notes

1 Proof of the lens formula is not required in the Extension specification.

2 When numerical values are substituted into the formula, the sign convention 'real is positive; virtual is negative' is used for the object and image distances. The focal length, f, of a converging lens is always assigned a positive value. A diverging lens is always assigned a negative value.

Worked example

An object is placed on the principal axis of a convex lens of focal length 150 mm at a distance of 200 mm from the centre of the lens. Calculate the image distance and state whether the image is real or virtual.

Solution

$f = +0.150\,\text{m}$, $u = +0.200\,\text{m}$,

Using the lens formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives

$$\frac{1}{0.200} + \frac{1}{v} = \frac{1}{0.150}$$

Hence

$$\frac{1}{v} = \frac{1}{0.150} - \frac{1}{0.200}$$

$$= 6.67 - 5.00 = 1.67$$

Therefore $v = +0.600\,\text{m}$

The image is real (because v is positive).

QUESTIONS

1 a i Copy and complete the ray diagram in Figure 7 to show how a converging lens in a camera forms an image of an object.

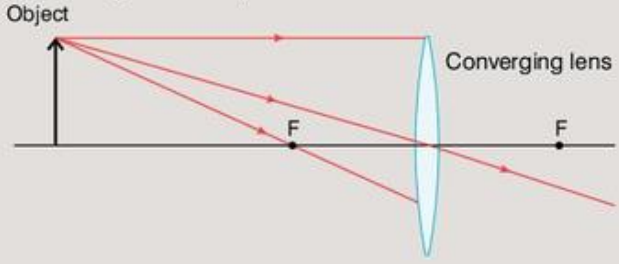


Fig 7

ii State whether the image in Figure 7 is real or virtual, magnified or diminished, upright or inverted.

b i Draw a ray diagram to show how a converging lens is used as a magnifying glass.

ii State whether the image in your diagram is real or virtual, magnified or diminished, upright or inverted.

2 An object is placed on the principal axis of a thin converging lens at a distance of 400 mm from the centre of the lens. The lens has a focal length of 150 mm.

a Draw a ray diagram to determine the distance from the image to the lens.

b State whether the image: **i** is real or virtual, **ii** upright or inverted.

c Use the lens formula to check the accuracy of your ray diagram.

3 An object is placed on the principal axis of a thin converging lens at a distance of 100 mm from the centre of the lens. The lens has a focal length of 150 mm.

a Draw a ray diagram to determine the distance from the image to the lens.

b State whether the image **i** is real or virtual, **ii** upright or inverted.

c Use the lens formula to check the accuracy of your ray diagram.

4 An object of height 10 mm is placed on the principal axis of a diverging lens of focal length 0.200 m.

Calculate the image distance and the height of the image for an object distance of **a** 0.150 m, **b** 0.250 m.

EXTENSION

25.3 Defects of vision

On these pages you will learn to:

- Define the lens power of a lens and state its unit
- Describe, with the aid of diagrams, the causes of myopia, hypermetropia and astigmatism and how to correct these sight defects
- Use the lens formula to calculate the focal length of the correcting lenses for myopia and hypermetropia

Lens power

The power of a lens is defined as

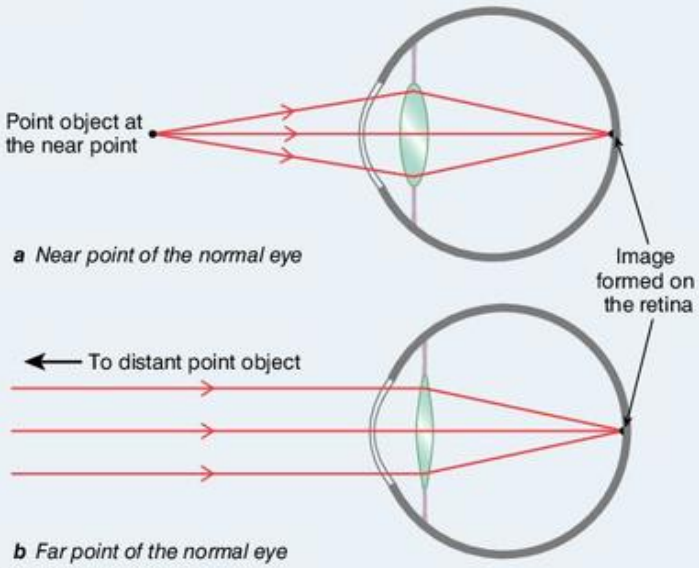
$$\frac{1}{\text{its focal length in metres}}$$

The unit of power is the dioptre (D). For example, for:

- a converging lens with a focal length of 0.20 m (i.e. $f = +0.20$ m), its lens power = +5.0 D
- a diverging lens with a focal length of 0.25 m (i.e. $f = -0.25$ m), its lens power = −4.0 D.

The eye is capable of focusing objects at different distances. This process is called **accommodation** and is achieved by automatic adjustment of the thickness of the eye lens. A normal eye can see objects in focus in the range from infinity to 25 cm. In other words, the normal eye has a near point of 25 cm and a far point at infinity. A normal eye is that of a 40-year-old person with normal vision. Young people have a much wider range but the range decreases gradually with age.

For near objects, the eye lens must be thicker and hence more powerful than for distant objects. Figure 1a shows the light rays from a nearby object brought to a focus on the retina.



a *Near point of the normal eye*

b *Far point of the normal eye*

Fig 1 *The normal eye*

For distant objects, the eye lens must be thinner and hence less powerful than for near objects. Figure 1b shows the light rays from a distant object brought to a focus on the retina.

Note
The depth of focus of the image is the range of the image distance which gives a sharp image on the retina.

Myopia and hypermetropia

Myopia or short-sight occurs when an eye cannot focus on distant objects. The uncorrected far point of the defective eye is nearer than infinity. This is because the eye muscles cannot make the eye lens thin enough to focus an image on the retina of an object at infinity. The eye can focus nearby objects hence the defect is referred to as 'short-sight'.

The cause of myopia is that light, after passing through the eye lens, converges in front of the retina, as shown in Figure 2. This happens if the eye lens cannot become thin enough to focus light onto the retina or if the eyeball is too long.

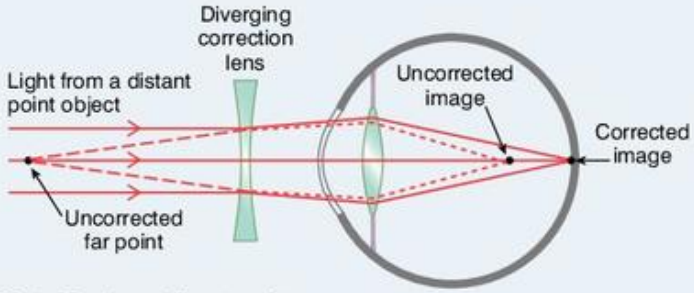


Fig 2 *Myopia and its correction*

To correct myopia using a lens, a diverging lens of a suitable focal length must be placed in front of the eye as shown in Figure 2. The correcting lens makes parallel rays from a distant object diverge so they appear to come from the uncorrected far point. Therefore, the correcting lens for myopia must:

- be a diverging lens,
- have a focal length equal to the distance from the eye to the uncorrected far point.
- form a virtual image of the distant point object at the uncorrected far point.

Note
The correcting lens effectively moves the far point of the uncorrected eye to infinity. It also 'moves' the near point away as shown in Figure 3. The correcting lens makes the image of an object placed at the least distance of distinct vision (i.e. 'new' near point) appear at the 'unaided' near point. Therefore, the image is virtual and nearer the lens than the object, as shown.

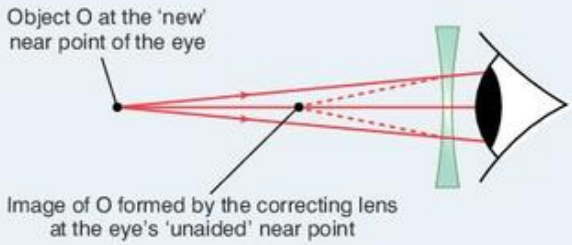


Fig 3 Effect on the near point

In Figure 3, if the object is at distance u and the image is at distance v, applying the lens formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$. Since f is negative (as the correcting lens is a diverging lens), $1/v$ is more negative than $-1/u$ so the distance to the 'unaided' near point (i.e. image distance v) is smaller than the 'new' near point distance (i.e. object distance u).

Worked example

A short-sighted eye has a far point of 5.00 m and a near point of 0.25 m.

a State the type of lens needed to correct this defect and calculate the power of the correcting lens.

b Calculate the distance from the lens to the near point of the eye with the correcting lens in front of the eye.

Solution

a A diverging lens is needed. Its focal length is −5.0 m (= the distance to its far point).
Therefore the power of the correcting lens = $1/f = -0.20$ D.

b Let u = least distance of distinct vision to an object with the correcting lens in place.
The image of this object is a virtual image formed at 0.25 m from the eye. Hence $v = -0.25$ m

Using $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ with $v = -0.25$ m and $f = -5.0$ m gives

$$\frac{1}{u} = \frac{1}{f} - \frac{1}{v} = \frac{1}{-5.0} - \frac{1}{-0.25}$$

$$= -0.20 + 4.00$$

$$= +3.80\,\text{m}^{-1}$$

Hence $u = +1/3.80 = 0.26(3)$ m.

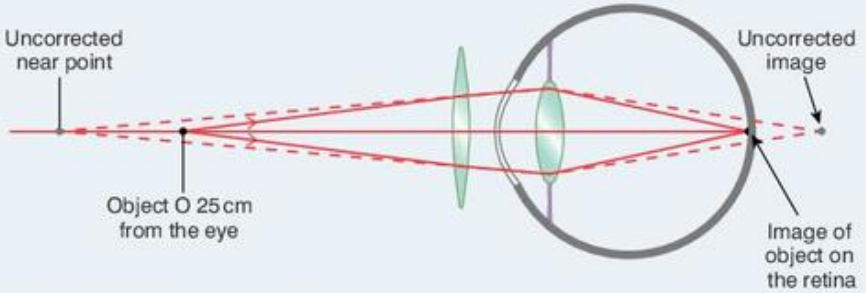


Fig 5 Correction of hypermetropia

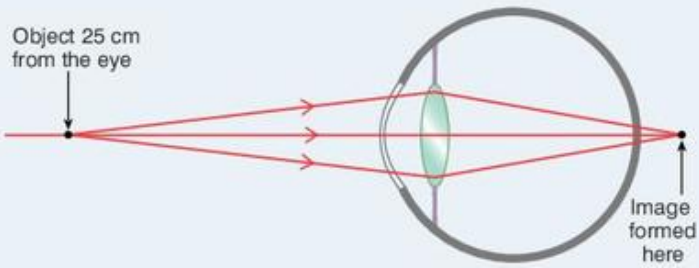


Fig 4 Hypermetropia

Hypermetropia or long-sight occurs when an eye cannot focus on nearby objects. The uncorrected near point of the defective eye is further away than 25 cm. This is because the eye muscles cannot make the eye lens thick enough to focus an image on the retina of an object 25 cm away. The eye can focus distant objects hence the defect is referred to as 'long-sight'.

The cause of hypermetropia is that light, after passing through the eye lens, does not converge enough to form an image on the retina, as shown in Figure 4. This happens if the eye lens cannot become thick enough to focus light onto the retina or if the eyeball is too short.

To correct hypermetropia using a lens, a converging lens of a suitable focal length must be placed in front of the eye as shown in Figure 5. The correcting lens makes the rays from an object 25 cm away diverge less so they appear to come from the uncorrected near point. Therefore, the correcting lens for hypermetropia must:

- be a converging lens,
- have a focal length which makes an object placed 25 cm from the eye appear as if it is at the uncorrected near point.

Figure 5 shows that: the correcting lens forms a virtual image of the point object at the uncorrected near point,

Note

The correcting lens effectively moves the near point of the uncorrected eye nearer the lens to 25 cm away (the near point of a normal eye). It also 'moves' the far point from infinity nearer to the eye as shown in Figure 6. The correcting lens makes the image of an object at the 'new' far point appear to be at infinity. Therefore, the object must be at the focal point of the converging lens for the rays to become parallel after the lens to make the image at infinity.

EXTENSION

Cataracts

A cataract is cloudiness in the eye lens which causes loss of clear vision and which may progress to cause blindness. To restore clear vision, the eye lens is removed and replaced by a plastic lens which may be a fixed focus lens or a multi-focal lens that can be used for distance vision and for reading.

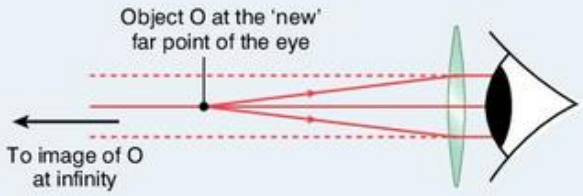


Fig 6 Effect on the far point

Therefore, the new far point is at distance f from the lens, where f is the focal length of the lens.

Worked example

A long-sighted eye has a far point at infinity and a near point which is 40 cm from the eye.

a State the type of lens needed to correct this defect and calculate the power of the correcting lens.

b Calculate the distance from the lens to the far point of the eye with the correcting lens in front of the eye.

Solution

a A converging lens is needed.

For an object at 25 cm from the eye, the object distance $u = +25\,\text{cm} = +0.25\,\text{m}$. The lens must form a virtual image of the object at the uncorrected near point of the eye.

Hence the image distance $v = -40\,\text{cm}$

Using $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ with $u = +0.25\,\text{m}$ and $v = -0.40\,\text{m}$ gives

$$\frac{1}{f} = \frac{1}{+0.25} + \frac{1}{-0.40} = 4.00 - 2.50 = 1.50$$

Hence the power of the correcting lens $= 1/f = +1.50\,\text{D}$

b The distance from the lens to the 'new' far point $= f = 1/1.50 = 0.67\,\text{m}$

Astigmatism

Astigmatism is a sight defect in which objects are seen to be sharper in focus in one direction than in other directions. It is most noticeable when observing parallel lines in perpendicular directions, as shown in Figure 7. If one set of lines stands out more than the other set at a certain orientation (e.g. vertical), the eye is astigmatic.

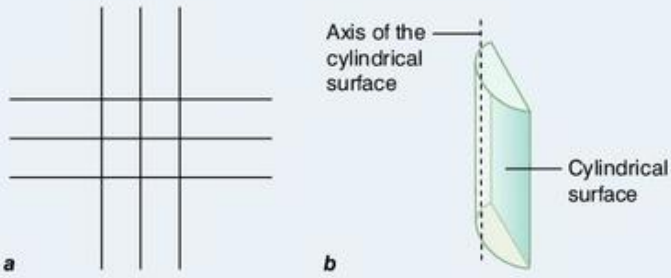


Fig 7 **a** *An astigmatism test* **b** *A cylindrical lens*

The cause of astigmatism is uneven curvature of the cornea. If the curvature is different in different directions, straight lines at different orientations cannot be focused on the retina at the same time. The more prominent line(s) will be in focus on the retina; the other lines will be out of focus.

The correction of astigmatism requires a lens with a cylindrical-shaped surface orientated so that it compensates for the uneven curvature of the cornea. Figure 7b shows the shape of a cylindrical lens. An optician's prescription for astigmatism will state:

- the curvature of the cylindrical-shaped surface,
- the orientation of the axis of the cylindrical surface.

QUESTIONS

1 a State what is meant by myopia.

b i With the aid of a diagram, describe how myopia is corrected.

ii A short-sighted eye has a far point 8.0 m away. Calculate the power of the correcting lens to give a corrected far point at infinity.

2 a State what is meant by hypermetropia.

b i With the aid of a diagram, describe how hypermetropia is corrected.

ii A long-sighted eye has a near point at 0.50 m away. Calculate the power of the correcting lens to give a corrected near point at 25 cm from the eye.

3 a A short-sighted eye has a far point at 5.0 m away and a near point 25 cm away. Calculate:

i the power of the correcting lens needed to give a corrected far point at infinity,

ii the distance from the lens to the corrected near point when the lens is in front of the eye.

b A long-sighted eye has a near point at 0.80 m away and a far point at infinity. Calculate:

i the power of the correcting lens needed to give a corrected near point at 25 cm from the eye,

ii the distance from the lens to the corrected far point when the lens is in front of the eye.

4 a State what is meant by astigmatism and explain why it occurs.

b Describe how an astigmatic eye is corrected using a suitable lens, stating what information needs to be supplied to a lens maker to ensure the lens is suitable.

EXTENSION

25.4 The structure of the ear

On these pages you will learn to:

- Explain what is meant by the loudness and the pitch of a sound
- Describe the structure of the ear and the function of the outer ear, the middle ear and the inner ear
- Explain why the pressure on the oval window is much greater than the pressure on the eardrum

Sound waves

Sound waves are longitudinal waves that pass through any solid, liquid or gas. Consider how they are produced in air using a loudspeaker connected to a signal generator, as shown in Figure 1.

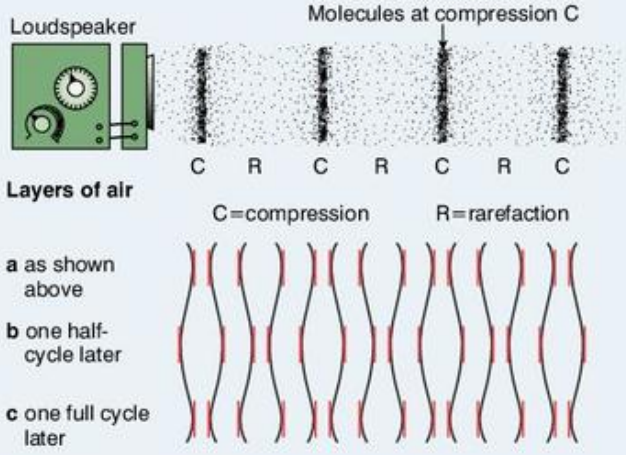


Fig 1 Creating sound waves

The alternating current supplied by the signal generator makes the loudspeaker diaphragm vibrate.

- Each time the diaphragm moves forward, it compresses the surrounding air slightly, pushing the surrounding air molecules into molecules further away which push into other molecules further away again so the compression travels through the air away from the loudspeaker.
- When the diaphragm moves backwards, it expands the air slightly so creating a low-density region, referred to as a rarefaction, which nearby air molecules move into so vacating space which molecules further away move into.

In this way, the vibrations of the diaphragm create sound waves consisting of alternate compressions and rarefactions that travel through the air away from the loudspeaker. Any vibrating object has this effect on a solid, liquid or gas it is in contact with.

The loudness of a sound depends on the amplitude of the sound waves. The bigger the amplitude is, the louder the sound will be.

The pitch of a sound increases if the frequency of the sound waves is increased.

Notes and noise

A note of sound consists of sound waves that move very smoothly and rhythmically, which makes them easy to listen to. In contrast, noise generally consists of sound waves that change abruptly and randomly. In effect, noise is unwanted sound.

Inside the ear

The normal human ear can hear sound waves that can range in amplitude by a factor of about 10^{12} and in frequency from about 20 to 18000 Hz. It is largely maintenance-free and it filters out unwanted body noises such as from the blood system and the digestive system.

Figure 2 shows a cross-section of the human ear.

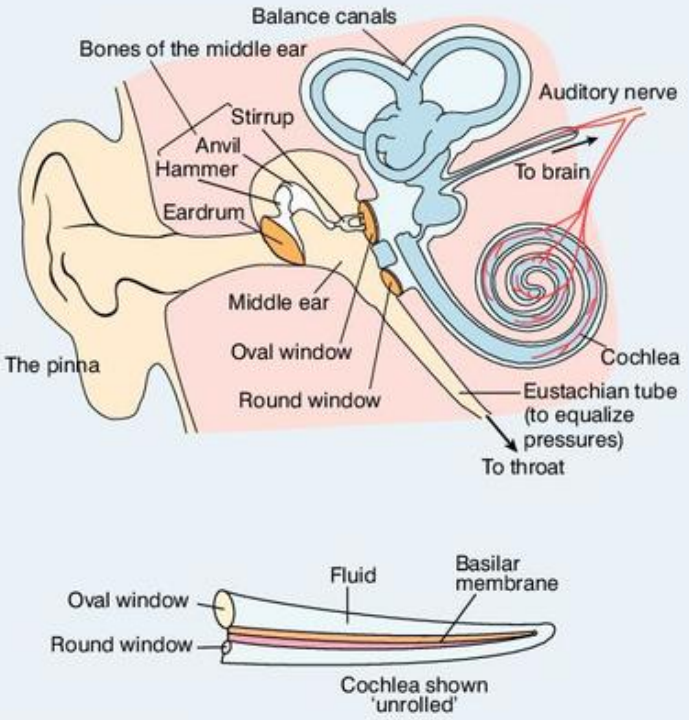


Fig 2 The human ear

The **outer ear** consists of the pinna and the ear canal. The pinna reflects sound waves into the ear canal. The pressure waves travel along the ear canal to the tympanic membrane (i.e. eardrum), which vibrates as a result. Sounds of frequency about 3000 Hz corresponding to wavelengths of about 0.03 m resonate in the ear canal, which is why the ear is most sensitive around this frequency.

The **middle ear** is a cavity containing the three bones of the middle ear, named the hammer, the anvil and the stirrup due to their shape, which transmit the vibrations of the tympanic membrane to the oval window of the **inner ear**. Pressure differences between the middle ear and outside are equalised by air flow through the Eustachian tube, which connects the middle ear to the pharynx.

The bones of the middle ear act as a lever system to amplify the force of the vibrations by about 50%, giving a $\times 1.5$ increase in force. They also:

- filter out noise generated in the body,
- protect the ear from excessive vibrations by switching to a less efficient mode of vibration at high sound levels.

In a quiet environment, the muscles that hold the bones of the middle ear together contract to pull the bones tightly against each other, giving maximum effectiveness as transmitters of sound energy. If the ear is subjected to excessive noise, the muscles become slack and allow the bones to vibrate differently so they are less efficient as transmitters of sound energy. Temporary deafness can occur on leaving a noisy environment until the muscles become taut again. Hearing damage occurs if the exposure to excessive noise is prolonged.

The **inner ear** is a fluid-filled enclosed cavity which receives the sound vibrations from the middle ear at its oval window. The vibrations are transmitted through the fluid of the cochlea as pressure waves, making the basilar membrane vibrate, and then onto the round window where they leave the inner ear. Tiny sensitive 'hair cells' on the basilar membrane send nerve impulses to the brain as a result of being stimulated by the vibrations of the membrane. Different frequencies cause the hair cells along the basilar membrane to vibrate at different positions, each pattern of vibration causing a pattern of nerve impulses which the brain learns to interpret.

The oval window has an area of about $3\,\text{mm}^2$ which is about 15 times less than the area of the ear drum. The pressure on the oval window is over 20 times greater than the pressure on the ear drum because:

1 the force on the oval window is about 1.5 times the force on the eardrum,
2 the area of the oval window is about 15 times smaller than the area of the ear drum.

Therefore

$$\frac{\text{pressure on oval window}}{\text{pressure on ear drum}} = \frac{\text{force on oval window}}{\text{area of oval window}} \div \frac{\text{force on ear drum}}{\text{area of ear drum}}$$

$$= \frac{\text{force on oval window}}{\text{force on ear drum}} \div \frac{\text{area of ear drum}}{\text{area of oval window}}$$

$$= 1.5 \times 15 \approx 20$$

QUESTIONS

1 Outline the process by which vibrations of the ear drum due to sound waves reach the inner ear.

2 **a** Explain how the ear automatically responds to excessive loudness.

b Explain why temporary deafness can occur after leaving a very noisy environment.

3 An ear detects sound waves of constant frequency, which causes the pressure on the eardrum to vary with an amplitude of $2.0 \times 10^{-5}\,\text{Pa}$. The eardrum has an area of $50\,\text{mm}^2$. The bones of the middle ear amplify the force from the eardrum by 50%. Calculate:

a the maximum force on the eardrum,

b the maximum pressure on the oval window, assuming its area is $3.0\,\text{mm}^2$.

4 State the function of the following parts of the inner ear:

a the oval window,

b the basilar membrane,

c the round window.

EXTENSION

25.5 Sound measurements

On these pages you will learn to:

- Define the intensity and the intensity level of a sound
- Carry out calculations using the equation
  intensity level/dB $= 10 \log (I/I_0)$
- Define the threshold of hearing and the decibel scale

Fig 1 *The decibel scale*

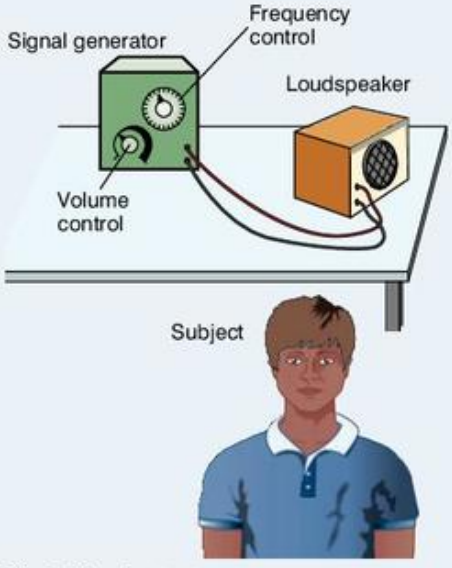


Fig 2 *A hearing test*

The decibel scale

The intensity of a sound wave (and of any type of wave) is defined as the energy per second per unit area incident normally on a surface. The unit of intensity is $J\,s^{-1}\,m^{-2}$ or $W\,m^{-2}$.

For example, if an eardrum that has an area of $3.0\,mm^2$ ($= 3.0 \times 10^{-6}\,m^2$) receives sound energy at normal incidence at a rate of $3.0 \times 10^{-12}\,J\,s^{-1}$, the intensity of the sound waves at the eardrum is $1.0 \times 10^{-6}\,J\,s^{-1}\,m^{-2}$ ($= 3.0 \times 10^{-12}\,J\,s^{-1} / 3.0 \times 10^{-6}\,m^2 = 1.0\,\mu W\,m^{-2}$). In case you think that this value of intensity would be too small to hear, it is in fact about the same as the sound intensity you experience when people are talking to you.

The least intensity the human ear can hear at a frequency of 1000 Hz is $1.0 \times 10^{-12}\,W\,m^{-2}$ ($= 1$ picowatt/m^2). This is referred to as the threshold intensity of hearing, I_0. At the other extreme, sound intensities over about $1\,W\,m^{-2}$ are painful to the ear and can cause permanent deafness.

The intensity level in decibels (dB) of a sound of intensity I is defined as

$$\textbf{Intensity level in decibels} = 10 \log \left(\frac{I}{I_0} \right)$$

where I_0 is the threshold intensity of the human ear.

Figure 1 shows the intensity level in decibels of a range of different sounds. Intensity level in decibels is a more convenient measure than intensity because it covers the very large range of intensities which the ear responds to. In addition, as explained below, it matches the response of the human ear in terms of changes of loudness. Notice the upper limit of about 140 dB above which the listener would experience pain and the eardrum may rupture.

Worked example

$I_0 = 1.0 \times 10^{-12}\,W\,m^{-2}$

Calculate the intensity level in decibels of a sound of intensity $2.0 \times 10^{-6}\,W\,m^{-2}$.

Solution

Intensity level in dB $= 10 \log (I/I_0) = 10 \times \log (2.0 \times 10^{-6} / 1.0 \times 10^{-12})$

$= 10 \times 6.3 = 63\,dB.$

The response of the ear to changes of sound intensity is directly proportional to the percentage change of intensity. Suppose a hearing test is conducted using sound of constant frequency from a loudspeaker connected to a signal generator, as shown in Figure 2. If the sound intensity is increased in steps by a certain percentage each step, the person who is the subject of the test will detect equal responses (i.e. equal increases of loudness).

The frequency control on the signal generator should be set at a constant value and the loudness control adjusted so the sound intensity can just be heard by the subject. A sound level meter at a fixed position near the subject's ear could be used to measure the intensity level of the sound in decibels.

Increasing the 'volume' control of the signal generator would increase the sound intensity at the subject and the reading of the sound level meter. Equal increases in the decibel reading of the meter would be due to equal percentage increases in the sound intensity. For example, successive increases of 10 decibels would be due to successive '×10' increases in the sound intensity. Such increases would give equal increases of loudness as judged by the subject.

For example, if the initial intensity is I_0 (the threshold of hearing) and the intensity is increased by $\times 10$ in each step (i.e. increased by 1000% in each step), equal increases of loudness will be heard when the intensity is increased from I_0 to $10I_0$, then from $10I_0$ to 10^2I_0 then from 10^2I_0 to 10^3I_0, etc.

After n steps, the intensity $I = 10^nI_0$ and the subject would have experienced n successive equal increases of loudness. Using base 10 **logarithms** gives $n = \log(I/I_0)$. This tells us how many '$\times 10$' increases of loudness would be needed to increase the intensity of a sound from I_0 to I. The response of the ear is therefore logarithmic. Because the decibel scale is also logarithmic, it is therefore a useful scale to use as a measure of the response of the human ear to changes of sound intensity.

Using the decibel formula

1 **To calculate the intensity I of a sound**, given the intensity level in decibels, it is necessary to rearrange the formula: Intensity level (dB) $= 10\log(I/I_0)$ to make I/I_0 the subject.
Step 1 $\log(I/I_0) =$ Intensity level dB/10
Step 2 $I/I_0 =$ inv log (Intensity level dB/10)

(Note inv log $n = 10^n$)

Worked example

$I_0 = 1.0 \times 10^{-12}\,\mathrm{W\,m^{-2}}$

Calculate the intensity of a sound that has an intensity level of 94 dB.

Solution
Substituting 94 dB into the intensity level equation gives

$$94 = 10\log(I/I_0).$$
$$\log(I/I_0) = 9.4$$
$$I/I_0 = \text{inv log } 9.4 = 2.5 \times 10^9$$
$$I = 2.5 \times 10^9 I_0 = 2.5 \times 10^9 \times 1.0 \times 10^{-12}$$
$$= 2.5 \times 10^{-3}\,\mathrm{W\,m^{-2}}$$

(**Note** inv log $9.4 = 10^{9.4} = 10^{0.4} \times 10^9 = 2.5 \times 10^9$)

2 **To calculate the intensity level of two sounds**, given the intensity level of each sound in decibels, it is necessary to:
- calculate the intensity of each sound as explained in **1** above,
- add the two intensities to give the total intensity, then
- use the intensity level formula again to calculate the intensity level of the total intensity.

Worked example

A lathe in a workshop exposes its operator to sound at an intensity level of 80 dB when it is in operation. The operator standing at the lathe also experiences sound at an intensity level of 78 dB from a drill when that is in operation with the lathe switched off. Calculate the intensity level experienced by the lathe operator at the lathe when both machines are switched on.

Solution
Intensity level due to the lathe = 80 dB.

$\therefore$ intensity due to lathe, I_1, is given by $80 = 10\log(I_1/I_0)$

Hence $I_1/I_0 =$ inverse log (80/10)
$= 10^8$ so $I_1 = 1.0 \times 10^8 I_0$

Intensity level due to the drill = 78 dB

$\therefore$ intensity due to drill, I_2, is given by $78 = 10\log(I_2/I_0)$

Hence $I_2/I_0 =$ inverse log (78/10) $= 10^{7.8}$ so $I_2 = 6.3 \times 10^7 I_0$

Intensity due to both machines $I = I_1 + I_2$
$= 1.0 \times 10^8 I_0 + 6.3 \times 10^7 I_0$
$= 1.63 \times 10^8 I_0$

$\therefore$ Intensity level due to both machines $= 10\log(I/I_0)$
$= 10\log(1.63 \times 10^8)$
$= 82$ dB

QUESTIONS

$I_0 = 1.0 \times 10^{-12}\,\mathrm{W\,m^{-2}}$

1 An eardrum that has an area of 3.0 mm^2 ($= 3.0 \times 10^{-6}$ m^2) receives sound energy at normal incidence at a rate of $6.6 \times 10^{-8}\,\mathrm{J\,s^{-1}}$. Calculate:
a the intensity of the sound waves at the eardrum,
b the intensity level of sound of this intensity.

2 A certain aircraft taking off creates sound at an intensity level of 123 dB near a building when it passes over the building. Calculate the intensity of the sound waves from the aircraft near the building.

3 A signal generator is connected to a loudspeaker which is near a decibel meter. When the loudness of the sound from the loudspeaker is increased at constant frequency, the meter reading increases from 60 to 65 dB.
a What would be the increase of the reading of the meter if the loudness of the sound were to be increased again by the same amount?
b Calculate the intensity of a 65 dB sound.

4 A toll booth operator experiences an intensity level of 102 dB when a lorry is next to the booth. Calculate:
a the intensity of a 102 dB sound,
b the intensity level in decibels due to two such lorries either side of the toll booth.

EXTENSION

25.6 Frequency response

On these pages you will learn to:

- Draw 'equal loudness' frequency response curves for the human ear and explain how they may be used to determine hearing loss
- Explain what is meant by the dBA scale

Frequency response of the normal ear

The normal human ear can detect sounds of frequencies from about 20 Hz to 18000 Hz. Figure 1 shows equal loudness curves for the human ear. The lowest curve shows how the least detectable intensity level varies with frequency. This and the other curves show that the ear is less and less responsive at frequencies approaching the lower and upper limits of about 20 Hz and 18000 Hz respectively.

The lowest curve shows the variation of the threshold of hearing with frequency. Notice that the zero level for the decibel scale is at 1000 Hz. This is because I_0 in the intensity level formula $10\log(I/I_0)$ is defined as the least detectable intensity at 1000 Hz.

All the curves are equal loudness curves. For example, the middle curve (labelled '40 phon') shows that a 100 dB sound at 10000 Hz has the same loudness as a 40 dB sound at 1000 Hz.

Note The phon is not required in this specification and is included here to develop a clear grasp of the meaning of loudness. The loudness of a sound in phons is defined as the intensity level in decibels of a sound at 1000 Hz that has the same loudness as the sound. Thus a 100 dB sound at 10000 Hz has a loudness of 40 phons.

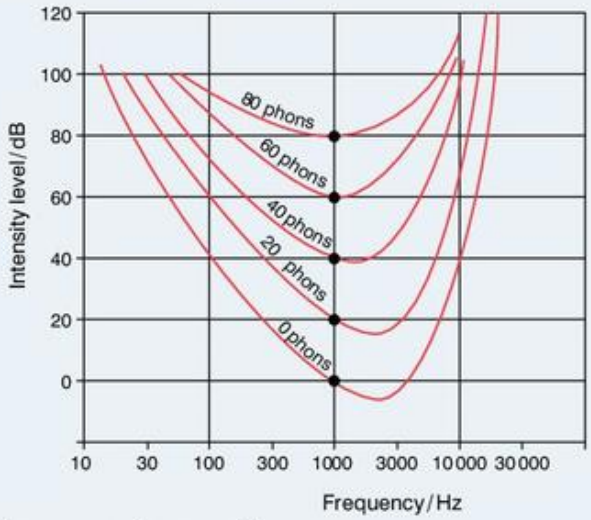


Fig 1 *Equal loudness curves for a normal human ear*

To obtain equal loudness curves:

- The frequency of the signal generator is first set to 1000 Hz then the volume control is adjusted until the sound level meter indicates a certain intensity level in decibels which is then noted as the loudness.
- The frequency is then set to a different value (e.g. 2000 Hz) and the volume control readjusted to make the loudness as judged by the subject the same as at 1000 Hz. The intensity level of the sound in then noted from the sound level meter.
- The procedure is then repeated for other frequencies to cover the whole frequency range from about 100 Hz to the upper frequency limit of the subject.

To obtain further loudness curves, the procedure outlined above is repeated for different initial intensity levels at 1000 Hz.

The dBA scale

Sound level meters are usually calibrated on an adjusted decibel scale, the dBA scale, which matches the ear's response at different frequencies. This is particularly important where people are subjected to sounds dominated by low or high sound frequencies as the normal ear is less sensitive at these frequencies than it is in the middle part of its frequency range. For example, using Figure 1:

- a 100 dB sound at 10 kHz would give a reading of 60 dBA on the dBA scale corresponding to the normal ear's response. The reading of 60 dBA means the intensity level of the sound is 60 dB above the threshold of hearing at 10 kHz which is 40 dB.
- a 60 dB sound at 100 Hz would give a reading of 20 dBA on the dBA scale corresponding to the normal ear's response. The reading of 20 dBA means the intensity level of the sound is 20 dB above the threshold of hearing at 10 kHz which is 40 dB.

Defects of hearing

Deterioration of hearing occurs due to age and due to exposure to excessive noise. Figure 2 shows the effect of age on the threshold of hearing.

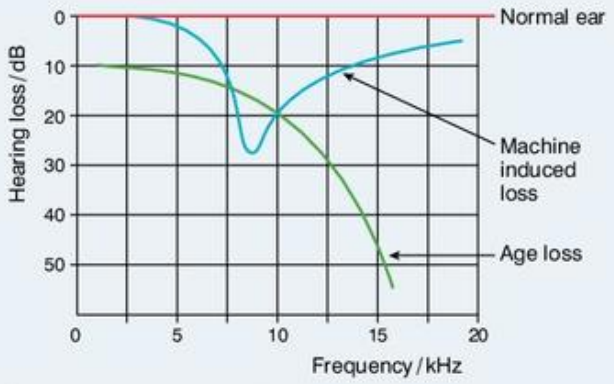


Fig 2 The effect of hearing defects

- Age-related deterioration occurs at all frequencies and is greater the higher the frequency.
- Excessive and prolonged exposure to noise in a narrow frequency range (e.g. machine noise) can cause deterioration in that frequency range only.

Hearing loss can be tested by obtaining equal loudness curves as explained above and comparing the results with the corresponding curves for normal hearing. The hearing loss at any given frequency may be expressed as the difference between the threshold intensity level for normal hearing and for the person being tested. For example, a person with a threshold intensity level of 65 dB at 10 kHz in comparison with 40 dB for a person with normal hearing has a hearing loss of 25 dB at that frequency.

The results for different frequencies can be presented on a grid of hearing loss against frequency as shown in Figure 3. Some results for a person with machine-related hearing loss are shown. Further measurements would need to be made to find the frequency at which the hearing loss is a maximum and the extent of the hearing loss over 10 kHz.

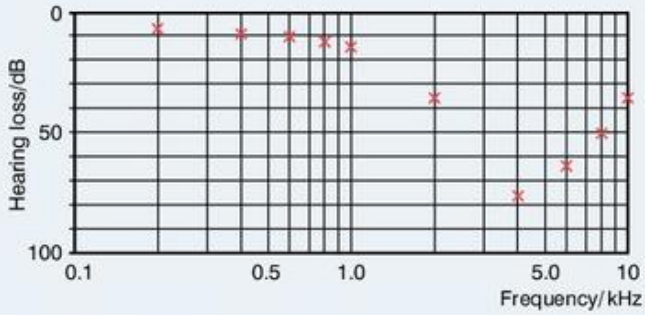


Fig 3 Hearing loss results

QUESTIONS

1 Describe the frequency response of a person with normal hearing from:

a 1000 to 5000 Hz,

b 10 000 Hz upwards.

2 A sound level meter has a dB scale and a second meter has a dBA scale.

a What is meant by a dBA scale?

b If the meters are used to read the sound level of a sound of constant intensity level 60 dB, use Figure 1 to determine the dBA reading and the difference between the readings of the two meters if the frequency of the sound is:
i 100 Hz, **ii** 1000 Hz,
iii 10 000 Hz.

3 Describe how you would carry out a test to measure the hearing loss of a person at a particular frequency. State the equipment you would use and the measurements you would make to determine the hearing loss in decibels.

4 **a** State two types of hearing loss.

b Describe one similarity and one difference between the two types of hearing loss stated in **a**.

Glossary

A-scan system ultrasound scan in which pulses from an ultrasonic transducer are detected by the probe after being reflected by internal boundaries and then displayed on an oscilloscope

absolute temperature, T, in kelvins = temperature in °C + 273(.15)

absolute zero is the temperature at which an object has minimum internal energy

acceleration change of velocity per unit time

acceleration of free fall acceleration of an object acted on only by the force of gravity

acceptor atoms Group 3 atoms added to a pure semiconductor to make it into an extrinsic p-type semiconductor

acoustic impedance the product of the density of a substance and the speed of the ultrasonic or sound waves through it

activity, A, of a radioactive isotope is the number of nuclei of the isotope that disintegrate per second. The unit of activity is the becquerel (Bq), equal to 1 disintegration per second

air resistance the force of the air opposing the motion of an object moving through the air (see drag force)

alpha radiation consists of particles that are each composed of two protons and two neutrons. An alpha (α) particle is emitted by a heavy unstable nucleus which is then less unstable as a result. Alpha radiation is easily absorbed by paper, has a range in air of no more than a few centimetres and is more ionising than beta (β) or gamma (γ) radiation

aluminium filter aluminium plate used in X-radiography to absorb 'soft' (i.e. low energy) X-rays

amplifier a circuit designed to produce an output pd that is proportional to the input pd supplied to it

amplitude maximum displacement of a vibrating particle; for a transverse wave, it is the distance from the middle to the peak of the wave

amplitude modulation modulation of the amplitude of a carrier wave of constant frequency by the amplitude of the signal

analogue signal signal that can have any value from zero to an upper limit (eg the varying pd from a microphone)

angular displacement, in radians, in time $t = 2\pi ft = \frac{2\pi t}{T}$ for an object in uniform circular motion

angular frequency $2\pi \times$ frequency of oscillating motion. The unit of ω is the radian per second (rad s^{-1})

annihilation process whereby a particle and its corresponding antiparticle collide, annihilate each other and produce photons

anode positive terminal of an electrical device

antineutrino an uncharged antiparticle with a very low rest mass compared with an electron and which is emitted from a nucleus when it emits a β^- particle

antinode fixed point in a stationary wave pattern where the amplitude is a maximum

antiquark the antiparticle of a quark

atomic mass unit, u, (correctly referred to as the unified atomic mass constant) is equal to 1.66×10^{-27} kg. It is defined as $\frac{1}{12}$ of the mass of an atom of the carbon isotope $^{12}_{6}C$

atomic number number of protons in the nucleus of the atom. It is also the order number of the element in the Periodic Table

attenuation reduction of intensity of a beam due to absorption or scattering

attenuation coefficient measure of the reduction of intensity per unit distance of a beam travelling through a substance and is equal to ln2/half thickness of the substance

audio wave electrical waves produced from sound waves for example by a microphone

Avogadro constant, N_A, is defined as the number of atoms in 12 grams of the carbon isotope $^{12}_{6}C$

background radioactivity is radioactivity due to radioactive substances which may be in the ground or in building materials or elsewhere in the environment. Background radioactivity is also caused by cosmic radiation

band theory a theory using the concept of energy bands to explain why metals and semiconductors conduct and why insulators do not conduct

bandwidth range of frequencies of a signal equal to the difference between the upper and lower sideband frequencies

baryons matter particles and antiparticles that each consist of three quarks or three antiquarks

base station transmitter/receiver mast that sends and receives signals from users in the surrounding area

base units the five units that define the SI system

beta minus (β^-) particle an electron created and emitted by an unstable neutron-rich nucleus when a neutron in the nucleus changes into a proton – an antineutrino is created and emitted at the same time

beta plus (β^+) particle a positron created and emitted by an unstable proton-rich nucleus when a proton in the nucleus changes into a neutron – a neutrino is created and emitted at the same time

beta radiation consists of beta-particles (β) which are electrons emitted by unstable nuclei with too many neutrons compared to protons. Beta radiation has a range in air of about

a metre and is less ionising than alpha (α) radiation and more ionising than gamma (γ) radiation

binding energy of a nucleus:
- the work that must be done to separate a nucleus into its constituent neutrons and protons,
- binding energy = mass defect $\times c^2$,
- binding energy/nucleon is greatest for nuclei of mass number 57

biofuel fuel obtained from biomass; see biomass

biomass biological material from living or recently living organisms used as fuel (eg animal waste or woodchip)

bit binary digit , 0 or 1

bonds forces that hold atoms or molecules together

brittle snaps without stretching or bending when subject to stress

Brownian motion is the random and unpredictable motion of a particle such as a smoke particle and is caused by molecules of the surrounding substance colliding at random with the particle

byte a series of bits (eg 0101 is a 4-bit byte)

capacitance of a capacitor is defined as the charge stored per unit p.d. The unit of capacitance is the farad (F), equal to 1 coulomb per volt

For a capacitor of capacitance C at p.d. V, the charge stored, $Q = CV$

capacitor combination rules
1 **Capacitors in parallel**; combined capacitance $C = C_1 + C_2 + C_3 + \ldots$
2 **Capacitors in series**; combined capacitance is given by $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$

capacitor energy; energy stored by the capacitor, $W = \frac{1}{2}QV = \frac{1}{2}CV^2$

capacitor discharge through a fixed resistor R;
1 Time constant $= RC$
2 Exponential decrease equation for current or charge or p.d.; $x = x_0e^{-t/RC}$

carrier wave electromagnetic waves used to carry a signal

cell area surrounding a mobile phone base station

cellular exchange exchange that controls and transfers the signals to and from local base stations and is linked to the public phone network

centre of gravity point where the weight of a body may be considered to act

centripetal acceleration:
1 For an object moving at speed v in uniform circular motion, its centripetal acceleration $a = \frac{v^2}{r}$ towards the centre of the circle
2 For a satellite in a circular orbit, its centripetal acceleration $\frac{v^2}{r} = g$

centripetal force is the resultant force on an object that moves along a circular path. For an object of mass m moving at speed v along a circular path of radius r, the centripetal force $= \frac{mv^2}{r}$

chain reaction A series of reactions in which each reaction causes a further reaction. In a nuclear reactor, each fission event is due to a neutron colliding with a $^{235}_{92}U$ nucleus which splits and releases two or three further neutrons which can go on to produce further fission. A steady chain reaction occurs when one fission neutron on average from each fission event produces a further fission event

charge carriers charged particles that move through a substance when a p.d. is applied across it

ciliary muscle fibres muscle fibres that lie along concentric circles round the rim of the eye lens

coaxial cable cable with an inner copper wire surrounded by insulation and copper braid covered by insulation

coherence two sources of waves are coherent if they emit waves with a constant phase difference

collisions An **elastic** collision is one in which the total kinetic energy after the collision is the same as before the collision. A **totally inelastic** collision is where the colliding objects stick together

conduction band the energy band occupied by conduction electrons

cone three types of colour-sensitive retinal cells , each sensitive to a different range of wavelengths corresponding broadly to red, green or blue light

contrast medium substance passed through a body organ or blood vessel to enhance the X-ray image of the body part by increasing the absorption of X-rays

converging lens a lens shaped so that it makes parallel rays of light converge to a fixed point, referred to as the focal point

cooling by evaporation the decrease in the temperature of a liquid due to evaporation from its open surface

cornea protective transparent layer at the front of the eye. It has a fixed convex curvature and therefore acts as a 'fixed focus' lens

Coulomb's law of force between two point charges states that the force is proportional to the product of the charges and inversely proportional to the square of the distance between the charges. For two point charges Q_1 and Q_2 at distance apart r, the force F between the two charges is given by the equation $F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$, where ε_0 is the absolute permittivity of free space

couple pair of equal and opposite forces acting on a body but not along the same line

coupling medium gel applied between the body surface and the surface of an ultrasonic probe to exclude air at the interface to ensure the ultrasonic waves are not reflected at the body surface

critical mass This is the minimum mass of the fissile isotope (e.g. the uranium isotope $^{235}_{92}U$) in a nuclear reactor necessary to produce a chain reaction. If the mass of the fissile isotope in the reactor is less than the critical mass, a chain reaction does not occur because too many fission neutrons escape from the reactor or are absorbed without fission

cycle interval for a vibrating particle (or a wave) from a certain displacement and velocity to the next time the particle (or the next particle) has the same displacement and velocity

damped oscillations of an oscillating system are due to the presence of resistive forces due to friction and drag. For a lightly damped system, the amplitude of oscillations decreases gradually. For a heavily damped system displaced from equilibrium then released, the system slowly returns to equilibrium without oscillating. For a critically damped system, the system returns to equilibrium in the least possible time without oscillating

de Broglie wavelength A particle of matter has a wave-like nature because it can behave as a wave. For example, electrons directed at a thin crystal are diffracted by the crystal. The de Broglie wavelength, λ, of a matter particle depends on its momentum, p, in accordance with de Broglie's equation:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

decay constant, λ is the probability of an individual nucleus decaying per second

density of a substance mass per unit volume of the substance

depletion layer narrow layer of the semiconducting material either side of a p-n junction

depth of focus the range of the image distance for the eye which gives a sharp image on the retina

diffraction is the spreading of waves when they pass through a gap or round an obstacle. X-ray diffraction is used to determine the structure of crystals, metals and long molecules. Electron diffraction and neutron diffraction are also used to probe the structure of materials. High-energy electron scattering is used to determine the diameter of the nucleus

diffraction grating plate with many close equally-spaced parallel slits that diffracts light at normal incidence into a direction that depends on its wavelength

digital signal a signal which has only two values

diode an electrical device that conducts in one direction only

dispersion splitting of a beam of white light by a glass prism into the colours of a spectrum

displacement distance in a given direction

diverging lens a lens shaped so that it makes parallel rays of light diverge from the focal point

donor atoms Group 5 atoms added to a pure semiconductor to make it into an extrinsic n-type semiconductor

Doppler effect the effect of relative motion between a source of waves and an observer on the observed frequency, causing it to differ from the emitted frequency

Doppler shift the difference between the observed frequency and the emitted frequency of the waves from a source due to relative motion between the source and the observer

drag force the force of fluid resistance on an object moving through the fluid

ductile stretches easily without breaking

efficiency $= \dfrac{\text{useful energy transferred}}{\text{total energy supplied}}$

effort the force applied to a machine to make it move

elastic limit point beyond which a wire is permanently stretched

electric potential (at a point) work done in bringing unit positive charge from infinity to the point

electrolysis process of electrical conduction in a solution or molten compound due to ions moving to the oppositely charged electrode

electrolyte a solution or molten compound that conducts electricity

electromotive force (e.m.f.) the amount of electrical energy per unit charge produced inside a source of electrical energy

electron fundamental particle with a fixed negative charge equal and opposite to that of a proton and a mass approximately 1/1840th of the mass of a proton

electron volt amount of energy equal to 1.6×10^{-19} J, defined as the work done when an electron is moved through a p.d. of 1 V

electronic sensor an electronic circuit that responds to an external change

endoscope optical fibre device used to see inside cavities

energy bands atoms in a solid vibrate about their mean positions causing their energy levels to spread out into energy bands

energy gap the energy gap between the bottom of the conduction band and the top of the valence band

equilibrium state of an object when at rest or in uniform motion

equipotential a line or a surface of constant gravitational potential

error bar representation of an uncertainty on a graph

error of measurement uncertainty of a measurement

evaporation the process by which a liquid turns to vapour below its boiling point

excited state an energy state of an atom with more energy than the lowest energy state

explosion In an explosion where two objects fly apart, the two objects carry away equal and opposite momentum

exponential decrease Exponential change happens when the rate of change of a quantity is proportional to the quantity

itself. For an exponential decrease of a quantity x, $\frac{dx}{dt} = -\lambda x$, where λ is referred to as the decay constant

The solution of this equation is $x = x_0 e^{-\lambda t}$, where x_0 is a constant

extrinsic semiconductor a semiconductor containing added substances consisting of either Group 3 atoms or Group 5 atoms, which change its electrical properties

eye lens flexible lens that forms an image on the retina of the eye of any object within a range of distances

Faraday's law of electromagnetic induction states that the induced e.m.f. in a circuit is proportional to the rate of change of magnetic flux linkage through the circuit

For a changing magnetic field in a fixed coil, induced e.m.f. $= -NA\frac{\Delta B}{\Delta t}$

fission is the splitting of a $^{235}_{92}U$ nucleus or a $^{239}_{94}Pu$ nucleus into two approximately equal fragments. Induced fission is fission caused by an incoming neutron colliding with a $^{235}_{92}U$ nucleus or a $^{239}_{94}Pu$ nucleus

fluoresce light emitted from a substance as a result of high energy radiation or particles being directed at the substance

force = rate of change of momentum

$$= \frac{\text{change of momentum}}{\text{time taken}}$$

(=mass × acceleration for fixed mass)

forward biased the direction of a diode in a circuit in order for the diode to conduct

forward voltage the potential difference necessary across a diode to make it conduct in its forward direction

fovea the region of the retina near the principal axis of the eye lens where the retinal cells are most dense. The fovea consists mostly of cones whereas rods predominate near the periphery of the retina

free and forced oscillations

- **free oscillations** are oscillations where there is no damping and no periodic force acting on the system so the amplitude of the oscillations is constant
- **forced oscillations** are oscillations of a system that is subjected to an external periodic force

frequency the number of cycles of a wave that pass a point per second

frequency modulation modulation of the frequency of a carrier wave by the amplitude of the signal

fusion is the fusing together of light nuclei to form a heavier nucleus

geothermal energy energy transferred towards the Earth's surface from underground rocks heated by radioative substances deep within the Earth

gravitational field strength, g, is the force per unit mass on a small test mass placed in the field

- $g = \frac{F}{m}$, where F is the gravitational force on a small mass m
- At distance r from a point mass M, $g = \frac{GM}{r^2}$
- At or beyond the surface of a sphere of mass M, $g = \frac{GM}{r^2}$ where r is the distance to the centre
- At the surface of a sphere of mass M and radius R, $g_S = \frac{GM}{r^2}$

gravitational potential (at a point) the work done per unit mass to move a small object from infinity to the point

gravitational potential energy energy due to position in a gravitational field ; and is equal to the work done to move a small object from infinity to a point in the field

ground heat heat flow from underground

ground state lowest energy state of an atom

hadrons matter particles and antiparticles that can interact through the strong interaction. Hadrons are subdivided into baryons (which consist of three quarks or three antiquarks) and mesons (which consist of a quark and an antiquark). Protons and neutrons are baryon hadrons

half-life, $t_{1/2}$, of a radioactive isotope is the time taken for the mass of the isotope to decrease to half the initial mass. This is the same as the time taken for the number of nuclei of the isotope to decrease to half the initial number

Hall effect the effect of causing a potential difference between the opposite sides of a conductor or semiconductor when a magnetic field is used to deflect charge carriers passing though the material

Hall voltage the potential difference between the opposite sides of a conductor or semiconductor when a magnetic field is used to deflect charge carriers passing though the material

hole the charge carrier in a p-type semiconductor

Hooke's law the extension of a spring is proportional to the force needed to extend it

hypermetropia hypermetropia or long-sight occurs when an eye cannot focus on nearby objects. The uncorrected near point of the defective eye is further away than 25 cm

ideal gas law, $pV = nRT$, where n is the number of moles of gas, T is the absolute temperature and R is the molar gas constant

information signal a signal (which may be analogue or digital) used to transfer information

inner ear a fluid-filled enclosed cavity which receives the sound vibrations from the middle ear at its oval window and transmits them through the fluid of the cochlea as pressure waves, making the basilar membrane and its hair cells vibrate

intensity of radiation at a surface is the radiation energy per second per unit area at normal incidence to the surface. The unit of intensity is $J\,s^{-1}\,m^{-2}$ or $W\,m^{-2}$

interference formation of points of cancellation and reinforcement where two coherent waves pass through each other

internal energy of an object is the sum of the random distribution of the kinetic and potential energies of its molecules

internal resistance resistance inside a source of electrical energy

intrinsic semiconductor a pure semiconductor containing no other substances. Silicon is the most common intrinsic semiconductor

inverse-square laws
- **force**; Newton's law of gravitation and Coulomb's law of force between electric charges are described as inverse-square laws because the force between two point objects (masses in the case of gravitation and charge in the case of charges) is inversely proportional to the square of the distance between the two objects. Because the two laws above are inverse-square laws, the field strength due to a point mass or a point charge varies with distance according to the inverse of the square of the distance to the point object
- **intensity**; The intensity of γ radiation from a point source varies with the inverse of the square of the distance from the source. The same rule applies to radiation from any point source that spreads out equally in all directions and is not absorbed

ion a charged atom

ionising radiation produces ions in substances it passes through. It destroys cell membranes and damages vital molecules such as DNA directly or indirectly by creating 'free radical' ions which react with vital molecules

iris concentric and radial muscle fibres that control the amount of light passing through the pupil of the eye

isotope atoms which have the same number of protons in each nucleus but different numbers of neutrons

junction transistor an electronic device consisting of a layer of p-type semiconductor between two layers of n-type semiconductor (npn) or a layer of n-type semiconductor between two layers of p-type semiconductor

kaon a meson that contains a strange quark and an antiquark or a strange antiquark and a quark

kinetic energy is the energy of a moving object due to its motion. For an object of mass m moving at speed v, its kinetic energy $E_K = \frac{1}{2}mv^2$, provided $v \ll c$ (the speed of light in free space)

kinetic energy of a molecule of an ideal gas The mean kinetic energy of a molecule of an ideal gas $= \frac{3}{2}kT$,

where the Boltzmann constant $k = \dfrac{R}{N_A}$

kinetic theory of a gas
- Assumptions; a gas consists of identical point molecules which do not attract one another. The molecules are in continual random motion colliding elastically with each other and with the container
- It can be shown that the pressure p of N molecules of such a gas in a container of volume V is given by the equation $pV = \frac{1}{3}Nm\langle c^2\rangle$, where m is the mass of each molecule and $\langle c^2\rangle$ is the mean square speed of the gas molecules
- Assuming that the mean kinetic energy of a gas molecule $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$, where $k = \dfrac{R}{N_A}$, it can be shown from $pV = \frac{1}{3}N\,m\langle c^2\rangle$ that $pV = nRT$ which is the ideal gas law

Kirchhoff's first law at a junction, the total current in = the total current out

Kirchhoff's second law the sum of the e.m.fs round a complete loop in a circuit = the sum of the p.ds round the loop

Larmor frequency frequency of precession of a nucleus in a magnetic field

latent heat of fusion (energy to melt a solid) is used to break the bonds that lock the molecules of the solid into fixed positions

latent heat of vaporisation (energy to boil a liquid) is used to break the bonds that prevent molecules moving away from each other

Lenz's law states that the direction of the induced current is always such as to oppose the change that causes the current

lepton matter particles and antiparticles that can interact through the weak interaction. They cannot interact through the strong interaction. Leptons are thought to be elementary. Electrons, positrons, neutrinos and antineutrinos are examples of leptons

light-dependent resistor (LDR) resistor which is designed to have a resistance that changes with change of intensity

light-emitting diode (LED) a diode that emits light when it conducts

line of force or a field line of a gravitational field (or electrical field) A line followed by a small mass (or small charge) acted on by no other forces than the force of the field.
Line of force or a field line of magnetic field is the line along which a free north pole would move

linear two quantities are said to have a linear relationship if the change of one quantity is proportional to the change of the other

load the force to be overcome by a machine when it shifts or raises an object

logarithmic scale This is a scale such that equal intervals correspond to a change by a constant factor or multiple (e.g. ×10)

logarithms For a number $n = b^p$ where b is the base number, then $p = \log_b n$
- $\log(nm) = \log n + \log m$ and $\log(n/m) = \log n - \log m$
- $\log(m^p) = p\log m$
- natural logs; for $n = e^p$, then $\ln n = p$
- base 10 logs; for $n = 10^p$, then $\log_{10} n = p$

log graphs
1. For $y = kx^n$, $\log_{10} y = \log_{10} k + n\log_{10} x$; the graph of $\log_{10} y$ (on the vertical axis) against $\log_{10} x$ is therefore a straight line of gradient n with an intercept equal to $\log k$
2. For $x = x_0 e^{-\lambda t}$, $\ln x = \ln x_0 - \lambda t$; the graph of $\ln x$ (on the vertical axis) against t is a straight line with a gradient equal to $-\lambda$ and a y-intercept equal to $\ln x_0$

longitudinal waves waves with a direction of vibration parallel to the direction of travel of the waves

magnetic field region near a magnet or a current-carrying wire in which another magnet or current-carrying wire experiences a force

magnetic flux, $\Phi = BA$ for a uniform magnetic field that is perpendicular to an area A

magnetic flux density is defined as the force per unit length per unit current on a current-carrying conductor at right angles to the field lines. The unit of magnetic flux density B is the tesla (T). B is sometimes referred to as the magnetic field strength

magnetic flux linkage through a coil of N turns $= N\Phi = NBA$ where B is the magnetic flux density perpendicular to area A. The unit of magnetic flux and of flux linkage is the **weber** (Wb), equal to $1\,\text{T m}^2$ or $1\,\text{V s}$

magnetic force

- $F = BIl \sin\theta$ gives the force F on a current-carrying wire of length l in a uniform magnetic field B at angle θ to the field lines, where I is the current. The direction of the force is given by Fleming's left hand rule where the field direction is the direction of the field component perpendicular to the wire
- $F = BQv \sin\theta$ gives the force F on a particle of charge Q moving through a uniform magnetic field B at speed v in a direction at angle θ to the field. If the velocity of the charged particle is perpendicular to the field, $F = BQv$. The direction of the force is given by Fleming's left hand rule, provided the current is in the direction that positive charge would flow in
- $BQv = \frac{mv^2}{r}$ gives the radius of the orbit of a charge moving in a direction at right angles to the lines of a magnetic field

mass defect the difference between the mass of the separated nucleons (ie. protons and neutrons from which the nucleus is composed) and the nucleus

mass measure of the inertia or resistance to change of motion of an object

matter waves the wave-like behaviour of particles of matter

mean kinetic energy of a molecule in a gas at absolute temperature $T = \frac{3}{2}kT$, where k is the Boltzmann constant $\left(= \frac{R}{N_A}\right)$

meson matter particles and antiparticles that each consist of a quark and an antiquark

microphone an electrical device that converts sound waves into electrical waves

middle ear a cavity containing the three bones of the middle ear, named the hammer, the anvil and the stirrup due to their shape, which transmit the vibrations of the tympanic membrane to the oval window of the inner ear

mobile phone network network of local mobile phone cells and cellular exchanges linked to the public telephone network

mole One mole of a substance consisting of identical particles is the quantity of substance that contains N_A particles of the substance. The **molar mass** of a substance is the mass of one mole

molecule the smallest particle in a substance that can be identified as belonging to the substance

moment of a force about a point force × perpendicular distance from line of action of force to the point

momentum is defined as mass × velocity. The unit of momentum is kg m s^{-1}

mono-mode fibre optical fibre with a very narrow core in order to reduce multi-path dispersion

motive force the force that drives a vehicle

multi-path dispersion the lengthening of a light pulse as it travels along an optical fibre due to rays that repeatedly undergo total internal reflection having to travel a longer distance than rays that undergo less total internal reflection

muon (symbol μ) a negatively charged particle with a rest mass over 200 times the rest mass of the electron. Muons and antimuons decay into electrons and antineutrinos or positrons and neutrinos, respectively

myopia myopia or short-sight occurs when an eye cannot focus on distant objects. The uncorrected far point of the defective eye is nearer than infinity

n-type semiconductor an extrinsic semiconductor that contains Group 5 atoms as well as silicon atoms. The Group 5 atoms provides extra charge carriers in the form of conduction electrons

neutrino an uncharged particle with a very low rest mass compared with an electron and which is emitted from a nucleus when it emits a β^+ particle

neutron a particle with no charge and a mass approximately the same as a proton. One or more neutrons are in every atomic nucleus except that of the smallest hydrogen nucleus

Newton's law of gravitation; the gravitational force F between two point masses m_1 and m_2 at distance r apart is given by $F = Gm_1m_2/r^2$

Newton's laws of motion

- **first law** An object continues at rest or in uniform motion unless it is acted on by a resultant force
- **second law** The rate of change of momentum of an object is proportional to the resultant force on it
- **third law** When two objects interact, they exert equal and opposite forces on one another

Newton's second law may be written as $F = \frac{\Delta p}{\Delta t}$, where p is the momentum ($= mv$) of the object and F is the force in newtons. For constant mass, $\Delta p = m\Delta v$ so $F = \frac{m\Delta v}{\Delta t} = ma$

node fixed point of no displacement

noise unwanted variations in signal amplitude superimposed on the transmitted signal

nuclear fission splitting of certain large nuclei such as a $^{235}_{92}\text{U}$ nucleus or a $^{239}_{94}\text{Pu}$ nucleus into two approximately equal fragments. Induced fission is fission caused by an incoming neutron colliding with a $^{235}_{92}\text{U}$ nucleus or a $^{239}_{94}\text{Pu}$ nucleus

nuclear fusion fusing together of light nuclei to form a heavier nucleus

nuclear magnetic resonance excitation of a (hydrogen) nucleus in a strong magnetic field using radio waves of the same frequency as the frequency of precession (the Larmor frequency) of the nucleus

nucleon a neutron or a proton in the nucleus

nucleon number the number of neutrons and protons in a nucleus

nuclide a nucleus of a certain isotope

nuclide of an isotope ${}^{A}_{Z}\mathbf{X}$ is a nucleus composed of Z protons and (A − Z) neutrons, where Z is the proton number (and also the atomic number of element X) and A is the number of protons and neutrons in a nucleus

Ohm's law the p.d. across a metallic conductor is proportional to the current through it provided the physical conditions do not change

operational amplifier (op-amp) an amplifier that is used for a specific operation such as comparison of two pds or multiplication of a pd by a constant

optical fibre a thin flexible transparent fibre used to carry light pulses from one end to the other

outer ear the pinna and the ear canal. The pinna reflects sound waves into the ear canal

output device any electrical device such as an indicator lamp or a relay connected to the output of a processing unit

p-type semiconductor an extrinsic semiconductor that contains Group 3 atoms as well as silicon atoms. The Group 3 atoms provide extra charge carriers in the form of 'holes'

p–n junction diode a diode consisting of p-type semiconductor in contact with n-type semiconductor

pair production process whereby a photon of sufficient energy produces a particle and its corresponding antiparticle

particle-like nature properties that are characteristic of particles such as momentum or deflection by electric or magnetic fields

path difference the difference in distances from two coherent sources to an interference fringe

period of a wave time for one complete cycle of a wave to pass a point

phase difference the fraction of a cycle between the vibrations of two vibrating particles, measured either in radians or degrees

phase difference, in radians, $= \dfrac{2\pi\Delta t}{T_{\mathrm{P}}}$, for two objects oscillating with the same time period, T_{P}, where Δt is the time between successive instants when the two objects are at maximum displacement in the same direction

photoconduction electrical conduction due to light

photoelectricity emission of electrons from a metal surface when the surface is illuminated by light of frequency greater than a minimum value, known as the threshold frequency

photon electromagnetic radiation consists of photons. Each photon is a wave packet of electromagnetic radiation. The energy of a photon, $E_{\mathrm{ph}} = hf$, where f is the frequency of the radiation and h is the Planck constant

piezo-electric effect property of certain solids whereby a pd applied between opposite faces causes a change of distance between the two faces and where applied stress causes a potential difference

piezo-electric transducer the component in an ultrasonic probe that produces ultrasonic waves when an alternating pd is applied to it

pion a meson that contains an up or down quark and an up or down antiquark

plastic deformation deformation of a solid beyond its elastic limit

polarised waves transverse waves that vibrate in one plane only

positron the antimatter counterpart of the electron. It is positively charged and its rest mass is the same as that of the electron

potential divider two or more resistors in series connected to a source of fixed potential difference .The source pd is divided between the resistors as they are in series with each other

potential energy the energy of an object due to its position

potential gradient change of potential per unit change of distance in a given direction

power = rate of transfer of energy $= \dfrac{\text{energy transferred}}{\text{time taken}}$

pressure force per unit area acting on a surface perpendicular to the surface

principle of conservation of energy This states that in any change, the total amount of energy after the change is always equal to the total amount of energy before the change

principle of conservation of momentum This states that when two or more bodies interact, the total momentum is unchanged, provided no external forces act on the bodies

probable error estimate of the uncertainty of a measurement

processing unit an electronic circuit designed to give an output that depends on the input signals supplied to it

projectile a projected object in motion acted on only by the force of gravity

proton a particle with a fixed positive charge of $+1.60 \times 10^{-19}$ C and a mass of 1.663×10^{-27} kg . One or more protons are in every atomic nucleus

public switched telephone network (PSTN) network of telephone exchanges used to transmit phone signals

quark protons and neutrons and all other hadrons consist of quarks and/or antiquarks. There are six types of quarks (up, down, strange, charm, top, bottom). This syllabus only requires knowledge of the up, down and strange quarks and their antiquarks; see quark model, baryon and meson

quark model (or standard model) a quark can join with an antiquark to form a meson or with two other quarks to form a baryon. An antiquark can join with two other antiquarks to form an antibaryon

radian measure of an angle defined such that 2π radians = 360°

random error error of measurement with no obvious cause

real image image formed by light rays from each point of an object being brought to a focus at corresponding image points

refraction change of direction of a wave when it crosses a boundary where its speed changes

regenerator circuit used to regenerate digital signals by removing noise and restoring the signal to its original amplitude

renewable energy This is energy from a source that is continually renewed. Examples include hydroelectricity, tidal power, geothermal power, solar power, wave power and wind power

repeater repeaters are amplifiers used at intervals to amplify a signal. They amplify noise as well as the signal

resistance $\frac{\text{p.d.}}{\text{current}}$

resistivity resistance per unit length × area of cross-section

resonance large-amplitude oscillations that occur in a lightly-damped system when the frequency of the applied force is equal to the natural frequency of oscillation of the system

retina layer of light-sensitive cells at the back of the eye. There are two types of retinal cells, rods and cones

reverse biased the direction of a diode in a circuit in order for the diode not to conduct

rod retinal cells sensitive to low levels of light intensity but unable to distinguish between colours. They predominate at the periphery of the retina

Rutherford's α-particle scattering experiment This experiment demonstrated that every atom contains a positively charged nucleus which is much smaller than the atom and where all the positive charge and most of the mass of the atom is located

satellite motion A satellite is a small object in orbit round a larger object. For a satellite moving at speed v in a circular orbit of radius r round a planet, its centripetal acceleration, $\frac{v^2}{r} = g$. Substituting $v = \frac{2\pi r}{T}$, where T is its time period, and $g = \frac{GM}{r^2}$, where M is the mass of the planet, it can be shown that $T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$. A satellite in a geostationary orbit is always directly above the same point on the equator. This is because it is in a circular orbit in the same plane as the equator and it has a time period of exactly 24 hours

scalar a physical quantity with magnitude only

semiconductor a substance in which the number of charge carriers increases when its temperature is raised

sensing device device that responds to an external change by producing a potential difference or a change of potential difference

SI system the scientific system of units

signal-to-noise ratio ratio of signal power to the power of the noise in the channel, usually expressed in decibels

simple harmonic motion An object oscillates in simple harmonic motion if its acceleration is proportional to the displacement of the object from equilibrium and is always directed towards equilibrium

- The acceleration, a, of an object oscillating in simple harmonic motion is given by the equation $a = -(2\pi f)^2 x$, where x = displacement from equilibrium, and f = frequency of oscillations
- The solution of this equation depends on the initial conditions. If $x = 0$ and the object is moving in the + direction at time $t = 0$, then $x = A \sin(2\pi f t)$. If the object is at maximum displacement, $+A$, at time $t = 0$, then $x = A \cos(2\pi f t)$

specific heat capacity, c, of a substance is the energy needed to raise the temperature of 1 kg of the substance by 1 K without change of state. The energy needed to raise the temperature of mass m of a substance from T_1 to $T_2 = mc(T_2 - T_1)$, where c is the specific heat capacity of the substance

spectrometer optical instrument used to measure the wavelengths of the lines in a spectrum

speed change of distance per unit time

stationary waves wave pattern with nodes and antinodes formed when two or more progressive waves of the same frequency and amplitude pass through each other

step-index fibre optical fibre with a core of constant refractive index surrounded by cladding of lower refractive index

strain extension per unit length of a solid when deformed

strain gauge a thin wire sealed in a plastic rectangle such that the resistance of the wire changes when the base is stretched

strangeness a quantum number assigned to a particle or an antiparticle on the basis that the total strangeness is conserved in a strong interaction. The strangeness of the K^- meson is defined as −1

stress force per unit area of cross-section in a solid perpendicular to the cross-section

strong interaction/strong nuclear force the force in a nucleus responsible for holding the protons and neutrons in the nucleus together and for interactions between hadrons

superposition the effect of two waves adding together when they meet

systematic error error of measurement with a known cause

T-type flip-flop a digital circuit designed to give an output pulse for every two input pulses supplied to it

terminal speed the maximum speed reached by an object when the drag force on it is equal and opposite to the force causing the motion of the object

thermistor resistor which is designed to have a resistance that changes with temperature

threshold frequency minimum frequency of light that can cause photoelectric emission

time period (or period) is the time taken for one complete cycle of oscillations

torque of a couple force × perpendicular distance between the lines of action of the forces

total internal reflection a light ray travelling in a substance is totally internally reflected at a boundary with a substance of lower refractive index, if the angle of incidence is greater than a certain value known as the critical angle

transducer any device designed to convert energy from one form to another . A piezo-electric transducer generates a pd when it is squeezed

transformer A transformer converts the amplitude of an alternating p.d. to a different value. It consists of two insulated coils, the primary coil and the secondary coil, wound round a soft iron laminated core

- The transformer rule states that the ratio of the secondary voltage to the primary voltage is equal to the ratio of the number of secondary turns to the number of primary turns
- For a transformer that is 100% efficient, the output power (= secondary voltage × secondary current) = the input power (= primary voltage × primary current)

transverse waves waves with a direction of vibration perpendicular to the direction of travel of the waves

ultrasonic probe hand-held device used in medicine to direct ultrasonic pulses into the body

ultrasonics sound waves at frequencies above the range of the human ear, which is about 18 kHz

uniform field A region where the field strength is the same in magnitude and direction at every point in the field

- The electric field between two oppositely charged parallel plates is uniform. The electric field strength $E = \frac{V}{d}$, where V is the p.d. between the plates and d is the perpendicular distance between the plates
- The gravitational field of the Earth is uniform over a region which is small compared to the scale of the Earth
- The magnetic field inside a solenoid carrying a constant current is uniform

valence band the energy band occupied by valence electrons below the conduction band

vector a physical quantity with magnitude and direction

velocity change of displacement per unit time

virtual earth the virtually zero potential of one of the two input terminals of an operational amplifier when the other input terminal is at zero potential and the output is not saturated

virtual image image formed where light rays from each point of an object appear to diverge from

voltage gain the ratio of the output pd from an amplifier to its input pd

W-boson a charged particle that is created in a nucleus when it undergoes beta decay. The W^- boson then decays into an electron and an antineutrino. The W^+ boson then decays into a positron and a neutrino

wave-like nature properties that are characteristic of waves such as interference or diffraction

wave particle duality

- Matter particles have a wave-like nature as well as a particle-like nature. For example, electrons directed at a thin crystal are diffracted by the crystal. This is wave-like behaviour in contrast with the particle-like behaviour of electrons in a beam which is deflected by a magnetic field. See de Broglie wavelength
- Photons have a particle-like nature, as shown in the photoelectric effect, as well as a wave-like nature as shown in diffraction experiments

wavelength distance between two adjacent wave peaks

weak nuclear force/weak interaction the force in a nucleus responsible for beta decay and for interactions between leptons

weight the force of gravity acting on an object

wire pair two wires used to carry a signal twisted together to reduce energy loss by electromagnetic radiation

work force × distance moved in the direction of the force

work done Work is energy transferred when a force moves its point of application in the direction of the force. The work done W by a force F when its point of application moves through displacement s at angle θ to the direction of the force is given by $\boldsymbol{W = Fs\cos\theta}$

work function of a metal minimum amount of energy needed by an electron to escape from a metal surface

X-rays Electromagnetic radiation of wavelength less than about 1 nm. X-rays are emitted from an X-ray tube as a result of fast-moving electrons from a heated filament (as the cathode) being stopped on impact with the metal anode. X-rays are ionising and they penetrate matter. Thick lead plates are needed to absorb a beam of X-rays.

Answers to in-topic questions

Chapter 1

1.1

1 **a** 63 km **b** 18°
2 **a** 40 m s^{-1} East, 69 m s^{-1} North **b** 21 km
3 6.1 kN vertically up, 2.2 kN horizontal
4 **a** **i** 10.4 km **ii** 6.0 km
b **i** 20.4 km **ii** 10.0 km
c 22.7 km
5 **a** 3.7 N at 33° to 3.1 N
b 17.1 N at 21° to 16 N
c 1.4 N at 45° to 3 N and to 1 N
6 **a** 14.0 N **b** 6.0 N **c** 10.8 N at 22° to the 10 N force
d 13.5 N at 10° to the 10 N force

1.2

1 **a** 80 km h^{-1} **b** 22 m s^{-1}
2 **a** 2.5×10^4 km h^{-1} **b** 7.0×10^3 m s^{-1}
3 **a** 45 000 m **b** 28.3 m s^{-1}
4 **b** **i** 4.0 km
ii 30 m s^{-1} then, 25 m s^{-1} in the opposite direction

1.3

1 **a** 1.5 m s^{-2} **b** −2.5 m s^{-2}
2 **a** 0.45 m s^{-2} **b** 7.9 m s^{-1}
3 **b** 0.60 m s^{-2}, 0, −0.40 m s^{-2}

1.4

1 **a** 2.0 m s^{-2} **b** 221 m
2 **a** 43 s **b** −0.93 m s^{-2}
3 **a** **i** 0.2 m s^{-2} **ii** 90 m
b **i** −0.75 m s^{-2} **ii** 8.0 s **d** 3.0 m s^{-1}
4 **a** 5.0 m s^{-2} **b** 7.5 m **c** 18 m **d** 6.4 m s^{-1}

1.5

1 **a** 4.0 m **b** 8.8 m s^{-1}
2 **a** 3.2 s **b** 31 m s^{-1}
3 **a** **i** 3.9 s **ii** 38 m s^{-1}
4 **a** 1.6 m s^{-2} **b** 3.6 m s^{-1} **c** 0.64 m

1.6

1 **a** **i** 83 s **ii** 127 s
2 **a** 600 s
3 **b** **i** 5250 m **ii** 9750 m
4 **a** **i** 0.61 s **ii** 5.9 m s^{-1} **iii** 0.43 s **iv** 4.2 m s^{-1}

1.7

1 **a** **i** 52 s **ii** 0.49 m s^{-2} **b** **i** 406 m **ii** −1.04 m s^{-2}
2 **a** 15 m **b** −0.13 m s^{-2}
c 0.67 m s^{-1} downwards, 13.4 m from the start
3 **a** **i** 80 m **ii** 8.0 m s^{-1} **b** **i** 65 s **ii** −0.12 m s^{-2}
4 **a** **i** 180 m s^{-1} **ii** 2.7 km
b 4.4 km
c 290 m s^{-1}

1.8

1 **a** 32 m s^{-1} **b** 2.8 s **c** 39 m
2 **a** 3.0 s **b** 49 m **c** 34 m s^{-1} (at 62° to the horizontal)
3 **a** 0.20 s **b** 11.7 m s^{-1}
4 **a** 354 m **b** **i** 1020 m **ii** 1020 m **c** 146 m s^{-1}

1.9

1 **a** 470 mm **b** 3.0 m s^{-2}
c 2.7 m s^{-1} (at 79° to the vertical)
2 **a** 25.8 m **b** 2.3 s **c** 4.6 s **d** 179 m
3 **a** 3.5 m s^{-1}, 3.0 m s^{-1} **b** **i** 150 m **ii** 20 m
4 **a** 21 m s^{-1}, 10 m s^{-1} **b** 8.7 m **c** 27 m s^{-1}

Chapter 2

2.1

1 **a** 0.24 m s^{-2} **b** 190 N **c** 0.024
2 **a** −2.4 m s^{-2} **b** 12 000 N
3 **a** 360 N **b** 23 s
4 **a** -1.3×10^5 m s^{-2} **b** 260 N

2.2

1 **a** 5400 N **b** 7700 N
2 **a** 60 N **b** 270 N
3 **a** 11.8 kN **b** 11.8 kN **c** 12.2 kN **d** 12.2 kN
4 **a** 1.0 m s^{-2} **b** 12.5 N

2.3

1 **a** **i** 0.04 m s^{-1} **ii** 1.5 N
3 **a** 0.14 m s^{-2} **b** 520 m

2.4

1 **a** 200 J **b** 4.5 J
2 **a** 48 J **b** 24 J **c** 0
3 **a** 1000 J **b** 600 J **c** 400 J
4 **a** 2.4 N **b** 0.12 J

2.5

1 **a** 9.0 J **b** 9.0 J **c** 1.8 m
2 **a** **i** 2.9 J **ii** 2.4 J **b** 0.5 J
3 **a** **i** 16 kJ **ii** 5.8 kJ **b** **i** 10.2 kJ **ii** 20 N
4 **a** 590 kJ **b** 2.4 kJ **c** 470 kJ **d** 122 kJ **e** 1.6 kN

2.6

1 **a** 1.13 kJ **b** 62.5 J s^{-1}
2 500 MW
3 **a** 156 MJ **b** 140 MJ **c** 12 MW
4 122 m

2.7

1 **a** 450 J **b** 1800 J s^{-1}
2 **a** 480 J **b** 50 J **c** 10%
3 **a** 570 MJ s^{-1} **b** 6.2×10^5 kg
4 **a** 600 s **b** 3.7 MJ **c** 8%

2.8

1 1.0×10^7 m^2
2 4 MW
3 125 m^3 s^{-1}
4 **a** 6.3×10^{11} kg **b** 430 MW

Chapter 3

3.1
1 **a** 7.3 N **b** 7.3 N at 31.5° to vertical
2 **b** **i** 2.7 N **ii** 4.7 N
3 **a** 139 N **b** 95 N
4 **a** 73° **b** 7.0 N

3.2
1 300 N
2 **b** 6.2 N
3 27 cm
4 6.75 N

3.3
1 0.51 N at 100 mm mark, 0.69 N at 800 mm mark
2 **a** 122 N at 1.0 m end and 108 N from the other end, both vertically upwards
b 122 N at 1.0 m end and 108 N at the other end, both vertically downwards
3 620 kN, 640 kN
4 **a** 100 N, 50 N **b** 150 N

3.4
2 89 N
3 **a** 48°
b Yes, they will raise the overall centre of mass so it will topple on a less steep slope

3.5
1 **a** 50 N **b** 250 N
2 **a** 1800 N m **b** 1800 N
3 6 kN
4 10.8 kN

3.6
1 **a** 15 N **b** 3.0 N **c** 10.8 N
2 7 N
3 **a** 6.8 N **b** 52°
4 18.0 N
5 **a** 17 kN **b** 17 kN
6 **a** 6.2 N **b** 11.2 N
7 50 mm away from pivot
8 **a** 6.8 N **b** 9.8 N
9 **a** 2200 N **b** 3100 N
10 **b** 950 N at X , 750 N at Y
11 **a** 2820 kN **b** 1660 kN and 1540 kN
12 **a** 8.0 N and 16 N **b** 38 N and 76 N
13 **b** 11 kN, 11 kN
14 **b** 28.4 N

Chapter 4

4.1
1 **a** **i** 1.2×10^{-18} kg m s^{-1} **ii** 0.050 kg m s^{-1} iii 14 kg m s^{-1}
b **i** 6.0 kg **ii** 20 m s^{-1}
2 **a** 3.6×10^{5} kg m s^{-1} **b** 60 s
3 **a** 5.4×10^{6} kg m s^{-1} **b** 45 s
4 **b** **i** 9.0×10^{3} kg m s^{-1} **ii** -8.4×10^{3} kg m s^{-1} **iii** 1.0 m s^{-1}

4.2
1 **a** 1600 kg m s^{-1} **b** 3200 N
2 **a** 3000 kg m s^{-1} **b** 7.5 kN
3 **a** 4.2×10^{-23} kg m s^{-1} **b** 1.9×10^{-13} N
4 **a** 2.1×10^{-23} kg m s^{-1} **b** 9.5×10^{-14} N

4.3
1 0.72 m s^{-1},
2 0.70 m s^{-1} in the same direction
3 0.050 m s^{-1} in the direction the 1.0 kg trolley was moving in
4 0.63 m s^{-1} in the opposite direction to its initial direction.

4.4
2 **a** 9.0 m s^{-1} in the same direction **b** 24 kJ
3 **a** 1.1 m s^{-1} in the reverse direction **b** 20 J
4 **a** **i** 1.0 m s^{-1}
b **i** 0.9 m s^{-1} **ii** 0.9 m s^{-1}

4.5
1 0.35 m s^{-1}
2 **a** 0.25 m s^{-1}; the mass of A and X was greater than the mass of B, so B moved away faster.
3 **a** **i** 0.10 m s^{-1} **ii** 15 mJ **b** 0.19 m s^{-1}
4 **a** 8.9 m s^{-1} **b** **i** 1.1 J **ii** 79 J

Chapter 5

5.1
1 **a** 8.0×10^{-4} m^3 **b** 3.1×10^{3} kg m^{-3}
2 **a** 6.3 kg **b** 2.0×10^{-3} m^3 **c** 3.1×10^{3} kg m^{-3}
3 **a** 9.6×10^{-6} m^3 **b** 7.5×10^{-2} kg
4 **a** **i** 0.29 kg **ii** 0.12 kg **b** 2.3×10^{3} kg m^{-3}

5.2
1 1.2 kPa
2 120 kN
3 13 kN
4 **a** 10.3 m
b **i** 1.47 kPa **ii** This gas pressure is 1.5% ((= 1.47 kPa/101 kPa) × 100%) above atmospheric pressure so it is not normal.

5.3
1 **a** 0.40 m **b** 12.5 N
2 **a** 20 N **b** 100 mm **c** 200 N m^{-1}
3 **a** 40 N **b** 200 mm
4 **a** 12.3 N m^{-1} **b** 8.8×10^{-2} J **c** 2.2 N

5.4
1 1.0×10^{9} Pa
2 1.3×10^{11} Pa
3 **a** 9.4×10^{8} Pa **b** 1.2×10^{-2} m
4 **a** No, the UTS for glass is greater than for copper
b No, the initial gradient for steel is greater than for glass
c Yes, the copper curve extends more than the glass curve.

5.5
1 **a** 3.3×10^{6} Pa **b** 2.8×10^{-4} m **c** 0.21 J
2 **a** 2.3 mm **b** 1.7×10^{-2} J
3 **a** 470 kN **b** 47 J
4 **a** 10.5 J **b** 3.5 J

Chapter 6

6.2
1 a i 3.5 C **ii** 210 C **b i** 3.0 A **ii** 0.15 A
2 a 3.8×10^{14} **b** 1.9×10^{21}
3 a 800 C **b i** 1600 s **ii** 8000 s
4 1.0 mm s^{-1}

6.3
1 a 29 kJ **b** 720 J
2 a 2 A **b** 22 kJ
3 a i 48 kJ **ii** 3.5 A **b** 5 A
4 a 12 kJ **b** 4.5 W **c** 2700 s

6.4
1 a 6.0 Ω, 9.9 V, 0.125 mA, 160 Ω, 2.5 mA **b** 7.5 Ω
2 31 Ω
3 0.11 mΩ
4 a 1.8×10^{-6} Ω m **b** 33 mm

6.5
1 a 0.25 A, 12 Ω
b The filament would become brighter and hotter until it melts and breaks as a result.
2 a 0.03 mA **b** 0.38 mA
4 a 30.4 Ω **b** 46 °C

6.6
1 a 1.4×10^{-3} N **b** 4.0×10^{4} V m^{-1}
2 a i negative **ii** 1.3×10^{-7} C
b i 7.3×10^{-3} N **ii** towards the metal surface
3 a i 9.0×10^{4} V m^{-1} **ii** 7.2×10^{-14} N **b** 80 mm
4 a 2.9 kV **b i** 5.6×10^{-15} N

Chapter 7

7.1
1 a 1.0 A, 4.0 A **b** 5.0 A **c** 30 W
2 b i 2.0 V **ii** 0.20 A
3 b i 4.0 V **ii** 2.0 V **iii** 10 Ω
4 a 3.6 V **b** 30 Ω

7.2
1 a 16 Ω **b** 3.0 Ω **c** 4 Ω
2 a 2 Ω **b** 6 Ω **c** 1.0 A **d** 4.0 W
3 a 3.6 Ω **b** 0.83 A
c 2 Ω: 0.5 W; 4 Ω: 1.0 W; 9 Ω: 1.0 W, **d** 2.5 W
4 a 14.4 W **b** 2.4 Ω

7.3
1 a 6.0 Ω **b** 2.0 A **c** 3.0 V **d** 9.0 V
2 a 0.5 A **b** 1.25 V **c** 0.63 W **d** 0.13 W
3 a 2.0 Ω **b** 1.5 V
4 12 V, 2 Ω

7.4
1 a 12.0 Ω **b** 0.25 A
c 4 Ω: 0.25 A, 1.0 V; 24 Ω: 0.08 A, 2.0 V; 12 Ω: 0.17 A, 2.0 V
2 a i 20.0 Ω **ii** 1.05 A **iii** 1.05 A, 15.8 V
b i 2.49 A **ii** 3.17 A
3 a i 2.0 W **ii** 2.0 W **b i** 2.0 W **ii** 8.0 W
4 a Q: 0.6 V, 0.06 mA; P: 2.4 V, 0.48 mA
b P: 0.6 V, 0.12 mA: Q: 2.4 V, 0.24 mA

7.5
1 a 1 kΩ: 0.75 V; 5.0 kΩ: 3.75 V
b 1 kΩ: 1.3 V; 5.0 kΩ: 3.2 V
3 a i 0.5 A **ii** 8.0 Ω: 4.0 V; 4.0 Ω: 2.0 V
b i 3.0 V **ii** 4.0 V
4 a i 2.8 V **ii** 6.4 kΩ

Chapter 8

8.2
1 a 0.10 m **b** 1.9×10^{-2} m
2 a 10 GHz **b** 5.0×10^{14} Hz
3 1.0 V, 1.0 kHz
4 a i amplitude = 9 mm **ii** 180° **iii** 270°
b +9 mm

8.5
1 b 8.0 m
2 a 2.0 m **b i** 180° **ii** 225° **iii** 0
4 b 30 mm

8.6
1 a 1.6 m **b** 410 m s^{-1}
2 a 0.4 m **b** 0.53 m

8.7
1 a 2.40 m, 140 Hz **b** 425 Hz
2 a 2.0 m **b** 57 Hz
3 a 71 Hz **b** 142 Hz
4 68–680 Hz

8.8
1 a 1200 Hz
b When the car is directly under the bridge, its direction of motion is perpendicular to the straight line between an overhead observer and the car. So at that instant, the car is neither approaching or moving away from the bridge. So the observed frequency is equal to the emitted frequency, which is 1100 Hz.
2 a towards Y, because Y observes a higher frequency than the frequency emitted by the horn
b 1060 Hz
3 a 2040 m s^{-1} **b** 3.4 GHz
4 a The reflected pulses would be too weak to be detected, as they would diffract too much on reflection. Also, the variable speed of the air would affect the speed of the waves.
b The small particles in the air would be very poor reflectors of microwaves because they are so small compared with the wavelength of the microwaves.

Chapter 9

9.1
2 550 nm
3 0.9 mm
4 0.75 m

9.2
3 1.1 mm

9.3
1 a 10.9°, 22.2° **b** 5
2 a 2 **b** 0.58° (= 35′)

3 **a** 1090 **b** 69.9°
4 **a** $599\,\text{mm}^{-1}$ **b** **i** 3 **ii** 50° 40′

9.4

1 **a** **i** 4.0 cm **ii** 3.0 cm **b** 18°
2 **a** **i** $1.97 \times 10^{8}\,\text{m s}^{-1}$ **ii** $2.26 \times 10^{8}\,\text{m s}^{-1}$
3 **a** **i** 19° **ii** 35° **b** **i** 59° **ii** 75°
4 **b** **ii** 74°

9.5

1 **b** **i** 41° **ii** 49°
2 **b** **i** 34° **ii** 34°
3 **a** 65° **b** **i** 30° **ii** 45°

Chapter 10

10.1

4 **a** The activity of the source decreases because more and more nuclei become stable and the number of radioactive nuclei in the source decreases. If the source did not have a long half-life, later readings would be significantly lower than earlier readings due to the decrease of the source activity.
b Alpha particles with different kinetic energies approaching a nucleus along the same path would be deflected by different amounts so the reading at each angle of deflection would not be due to alpha particles of the same kinetic energy.

10.2

1 **a** 6p, 6n **b** 8p, 8n **c** 92p, 143n
d 11p, 13n **e** 29p, 34n
2 **a** neutron **b** electron **c** neutron
3 **a** **i** $+3.2 \times 10^{-19}\,\text{C}$ **ii** 63 **b** 8 neutrons and 10 electrons
4 Most of the hydrogen atoms are ${}^{1}_{1}\text{H}$ atoms and therefore have a single proton as a nucleus. Less energy is needed for a neutron to knock a ${}^{1}_{1}\text{H}$ nucleus (i.e. a proton) out compared with knocking a carbon nucleus out. This is because a neutron hitting a carbon nucleus is likely to recoil and retain some kinetic energy, whereas a neutron hitting a ${}^{1}_{1}\text{H}$ nucleus is likely to be stopped and use most of its kinetic energy to eject the ${}^{1}_{1}\text{H}$ nucleus.

10.3

4 **a** 42% **b** 58%

10.4

1 **a** ${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\alpha$ **b** ${}^{228}_{90}\text{Th} \rightarrow {}^{224}_{88}\text{Ra} + {}^{4}_{2}\alpha$
2 **a** ${}^{64}_{29}\text{Cu} \rightarrow {}^{64}_{30}\text{Zn} + {}^{0}_{-1}\beta\,(+\bar{\nu})$
b ${}^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + {}^{0}_{-1}\beta\,(+\bar{\nu})$
3 **a** ${}^{213}_{84}\text{Po}$, ${}^{209}_{82}\text{Pb}$, ${}^{209}_{83}\text{Bi}$
b **i** 83 p + 130 n **ii** 83 p + 126 n
4 **a** 3.2 counts per second **b** 160 mm

10.6

1 **a** A W boson is charged; a photon is uncharged. A W boson has a very short range; a photon has an unlimited range.
b lifetime ≅ range / speed of light $= 3 \times 10^{-15}\,\text{m} / 3 \times 10^{8}\,\text{s} = 10^{-23}\,\text{s}$
2 **a** Leptons interact through the weak interaction; hadrons interact through the strong interaction.
b **i** a proton or a neutron **ii** a π meson
iii **a K meson**
3 **a** **i** uud **ii** udd
b One of the up quarks in the proton changes into a down quark and emits a W^{+} boson, which then decays into a β^{+} particle and a neutrino.
4 **a** **i** −1 **ii** −3
b $\pi^{-} = \bar{\text{u}}\text{d}$, X = ssu. X must contain a u quark rather than a d quark to ensure conservation of charge.

Chapter 11

11.1

1 **a** $1.75 \times 10^{-3}\,\text{rad}$ **b** 0.105 rad **c** 6.28 rad
2 **a** 20 ms **b** **i** 0.31 rad **ii** 310 rad
3 **a** $470\,\text{m s}^{-1}$ **b** **i** 0.0042° **ii** $7.3 \times 10^{-5}\,\text{rad}$
4 **a** $7.0\,\text{km s}^{-1}$ **b** **i** 0.050° **ii** $8.7 \times 10^{-4}\,\text{rad}$

11.2

1 **a** $0.23\,\text{m s}^{-1}$ **b** **i** $7.9 \times 10^{-4}\,\text{m s}^{-2}$ **ii** $5.1 \times 10^{-2}\,\text{N}$
2 **a** $0.53\,\text{m s}^{-1}$, $0.66\,\text{m s}^{-2}$ **b** $9.9 \times 10^{-2}\,\text{N}$
3 **a** **i** $3.0 \times 10^{4}\,\text{m s}^{-1}$ **ii** $5.9 \times 10^{-3}\,\text{m s}^{-2}$
b **i** $7.9 \times 10^{3}\,\text{m s}^{-1}$ **ii** $5.1 \times 10^{3}\,\text{s}$
4 **a** $8.4\,\text{m s}^{-1}$ **b** $88\,\text{m s}^{-2}$ **c** 175 N

11.3

1 **a** $6.7\,\text{m s}^{-2}$ **b** 3.8 kN
2 **a** $4.1\,\text{m s}^{-2}$ **b** 3.0 kN
4 **a** $40\,\text{m s}^{-1}$

11.4

1 **a** $30\,\text{m s}^{-1}$ **b** **i** $11.3\,\text{m s}^{-2}$ **ii** 690 N
2 **a** $25\,\text{m s}^{-1}$ **b** $20\,\text{m s}^{-2}$ **c** 2000 N
3 **a** $13\,\text{m s}^{-1}$ **b** $13\,\text{m s}^{-2}$ **c** 240 N
4 −0.04 N

Chapter 12

12.1

3 **a** 0.48 s **b** 2.1 Hz
4 **a** $\frac{\pi}{2}$ radians **b** π radians

12.2

1 **a** +25 mm, changing direction from up to down
b 0, moving down
c −25 mm, changing direction from down to up
d 0, moving up
2 **a** 0.5 Hz **b** **i** $-0.25\,\text{m s}^{-2}$ **ii** 0 **iii** $0.25\,\text{m s}^{-2}$
3 **a** 0.5 Hz **b** $-0.32\,\text{m s}^{-2}$
4 **a** −32 mm, $0.32\,\text{m s}^{-2}$ **b** 0, 0

12.3

1 **a** 0.33 Hz **b** $0.25\,\text{m s}^{-2}$
2 **a** **i** 12 mm **ii** 0.63 s **b** 6.5 mm
3 **a** 2.1 Hz **b** 0.057 m
4 **a** 3.7 Hz
b **i** −8.2 mm towards maximum negative displacement
ii −0.7 mm towards maximum positive displacement.

12.4

1 **a** **i** 0.33 s **ii** 3.1 Hz
b **i** 0 **ii** $-3.7\,\text{m s}^{-2}$ **iii** $-7.5\,\text{m s}^{-2}$

2 a i 3.0 Hz **ii** 0.33 s **b** $f_2 < f_1 \therefore m_2 > m_1$
3 a i 70 mm **ii** 21 N m^{-1} **b ii** 0.53 s
4 a i 1.25 N **ii** 2.5 m s^{-2} **b ii** 1.1 Hz, +46 mm
5 a i 2.0 s **ii** 1.0 s **b** 5.0 s
6 The mass–spring system would have the same time period on the Moon as it has on the Earth. The simple pendulum would have larger time period on the Moon than it has on the Earth.

12.5

2 a 60 N m^{-1} **b i** 7.5×10^{-2} J **ii** 7.5×10^{-2} J
4 a 82 mm **b** 44 mm

Chapter 13

13.1

2 a i 33 N **ii** 160 N **b i** 16 N kg^{-1} **ii** 4.0 N kg^{-1}

13.2

1 a 235 J
2 a 2.0 MJ kg^{-1} **b i** −61 MJ kg^{-1} **ii** 2.1×10^9 J
3 a i −250 J **ii** −200 J **iii** −200 J **b i** 50 J **ii** 0
4 b 5 N kg^{-1} **c** 25 MJ

13.3

1 a 1.3×10^{-6} N **b** 5.4 mm
2 a 780 N **b** 6.0×10^{24} kg
3 a 54 N **b** 0.24 N
4 a i 16.6 N **ii** 0.2 N
b 16.4 N towards the centre of the Earth

13.4

1 a 7.35×10^{22} kg
2 a i 68 N kg^{-1} **ii** 5.9×10^{-3} N kg^{-1}
3 a 0.028 N kg^{-1} **c** 7.1×10^6 J
4 −2.8 MJ kg^{-1}, 1410 MJ

13.5

2 a 3.4×10^6 m **b** 3.0 N kg^{-1} **c** 5.2×10^{23} kg
3 b i 9.5 N kg^{-1} **ii** 7.9 km s^{-1} **iii** 5200 s
4 b ii 7100 s

Chapter 14

14.1

1 a 3.7×10^{-11} N **b** 2.6×10^{-10} N
2 a i 69 mm **ii** 3.6×10^{-6} N **b** 2.5×10^{-5} N repulsion
3 a 6.1 nC, negative **b** 2.2×10^{-2} N
4 a 2.7 nC, attract **b** 6.2×10^{-2} m, repel

14.2

1 a i -8.0×10^{-18} J **ii** $+7.2 \times 10^{-17}$ J **b** $+8.0 \times 10^{-17}$ J
2 a -1.8×10^{-3} J **b** $+1.2 \times 10^{-3}$ J
3 a i 250 V m^{-1} **ii** 8.0×10^{-17} N (towards the negative plate)
b -8.0×10^{-19} J
4 b i 3000 V m^{-1}

14.3

1 a 5.3×10^6 V m^{-1} **b** 10 mm
2 a i 3.7×10^8 V m^{-1} **ii** 5.6×10^{-3} N towards Q_1
3 a i 4.5×10^8 V m^{-1} towards Q_2
ii 2.6×10^8 V m^{-1} away from Q_1
iii 4.90×10^8 V m^{-1} at 83.4° to the line between the two charges
b ii 11 mm from Q_1, 9 mm from Q_2
4 a -9.0×10^6 V
b ii 2.0×10^9 V m^{-1} directly towards Q_2

Chapter 15

15.1

1 a 5.0 μF **b** 2.2 V **c** 9.9 mC
d 1.4 μF **e** 11 V **f** 3.4 mC
2 a 264 μC **b** 106 s
3 a 27.5 μC **b** 5.5 μF
4 a 910 μC **b** 220 μF **c** 700 μC **d** 7.4 V

15.2

1 a 5.0 μF
b 2.0 μF; 6.0 μC, 3.0 V; 3.0 μF; 9.0 μC, 3.0 V
2 a i 2.4 μF
ii 6.0 μF; 11.0 μC, 1.8 V, 4.0 μF; 11.0 μC, 2.7 V
b i 3.2 μF
ii 4 μF; 14.4 μC, 3.6 V, 10 μF; 9.0 μC, 0.90 V, 6.0 μF; 5.4 μC, 0.90 V
3 1. All in series 6.0 μF 2. All in parallel 79 μF
3. Two in series with the third in parallel, 25 μF, 30 μF, 54 μF
4. Two in parallel in series with the third, 9 μF, 16 μF, 19 μF
4 a 4.9 μF
b 2 μF; 12 μC, 6.0 V, 4 μF; 17 μC, 4.3 V, 10 μF; 17 μC, 1.7 V

15.3

1 a 30 μC, 45 μJ **b** 60 μC, 180 μJ
2 a 0.45 C, 2.0 J **b** 10 W
3 2.2 μF; 5.4 μC, 6.6 μJ, 10 μF; 5.4 μC, 1.5 μJ
4 a 56 μC, 340 μJ **b** 6.9 μF
c 8.2 V **d** 4.7 μF; 160 μJ, 2.2 μF; 73 μJ

15.4

1 a i 300 μC **ii** 5.0 s **b i** 5 s approx **ii** 20 kΩ
2 a i 0.61 mC **ii** 0.45 mA **b** 0.23 V, 11 μA
3 a 13 μC, 40 μJ **b** 0.62 V **c** 0.42 μJ
4 a 0.34 mJ **b** 1.4 s **c** 0.32 mJ

Chapter 16

16.1

1 a i temperature **ii** light intensity **iii** length of wire
b The output p.d. will increase because the LDR resistance increases so the current decreases and the p.d. across the fixed resistor therefore decreases. Hence the p.d. across the LDR (and therefore across the output) increases.
2 a 3.0 V **b** 18°C
3 a 1.2 V **b** -1.4×10^{-2} V
4 a The piezo-electric effect is the generation of a p.d. across opposite surfaces of certain solid materials when pressure is applied to the two surfaces. The effect is reversible in that application of a p.d. between the two opposite surfaces causes the surfaces to move towards or away from each other.
b The microphone contains a flat piezo-electric disc with electrical connections to each surface. The compressions and rarefactions of a sound wave alternately compress and expand the piezo-electric crystal, generating electrical signals.

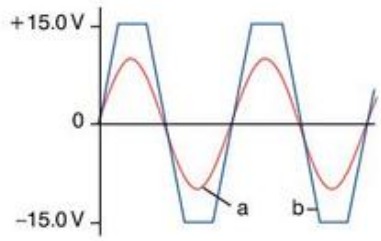
16.2

1 a The p.d. at P is higher than that at Q.

b Higher; the temperature must be increased so the resistance of the thermistor decreases and the p.d. at P decreases until it is below the p.d. at Q.

2 a See topic 16.2 Figure 5. **b** 250 kΩ

3 a and **b**



4 a Negative feedback is where a fraction of the output p.d. is added to the input p.d. to reduce the output p.d.

b The waveform would become a square wave with positive saturation throughout each positive half-cycle of the input p.d. and negative saturation throughout each negative half-cycle.

16.3

1 a i The minimum p.d. at which the LED conducts in its forward direction

ii The p.d. across the LED in its reverse direction which would cause it to breakdown and conduct

b 400 Ω

2 The temperature increases. This causes the thermistor resistance to fall so the p.d. at P falls below that at Q. Therefore the output p.d. from the op-amp becomes positive and the LED switches on.

3 a D_1 prevents the relay operating when the output p.d. is negative. D_2 prevents the op-amp being damaged when the motor is switched off because it short-circuits the back e.m.f. induced in the coil when the relay is switched off.

b i The resistance of the variable resistor needs to be reduced.

ii The LDR needs to be illuminated less so that its resistance increases sufficiently to make the p.d. at P less than the p.d. at Q and thus switch the relay on.

4 a ii 12.0 V

b The output p.d. of the thermistor varies non-linearly with the temperature of the thermistor. This is because the resistance of the thermistor varies non-linearly with temperature and the output p.d. is inversely proportional to the total input resistance. Therefore the output p.d. will not be exactly midway between the output p.d. at 20 °C and 80 °C when the thermistor temperature is exactly midway between 20 °C and 80 °C (i.e. 50 °C).

16.4

1 a A semiconductor is a material with resistivity that decreases as the temperature of the semiconductor is increased. An intrinsic semiconductor is a pure semiconductor with no other substances added.

b The charge carriers in p-type semiconductors are holes (which are electron vacancies in the atomic bonds) whereas the charge carriers in n-type semiconductors are electrons. An n-type semiconductor is an intrinsic semiconductor to which a small proportion of Group 5 atoms has been added in place of the intrinsic semiconductor atoms whereas a p-type semiconductor contains a small proportion of Group 3 atoms not Group 5 atoms.

2 a In an isolated p–n diode, some electrons from the n-type material move into the p-type material and some holes from the p-type material move into the n-type material. As a result, a barrier p.d. develops across the junction as the p-type becomes negative and the n-type positive. This p.d. stops further movement of electrons and holes across the p–n junction. When the diode is in a circuit, conduction through it can only occur by making the p-type positive and the n-type negative.

b When a p–n diode conducts, electrons and holes move in opposite directions across the junction. If an electron and a hole meet, the electron falls into the hole and releases energy in the form of a photon.

3

INPUTS		OUTPUT		
A	B	top left	top right	bottom left
0	0	1	0	1
1	0	1	0	1
0	1	0	0	1
1	1	1	1	1

4 a See topic 16.4 Figure 6. The SUM output from the EXOR gate is 1 when the inputs differ. The CARRY output from the AND gate is 1 when the inputs are both 1. Hence adding 0 and 1 gives 01 and adding 1 and 1 gives 10.

b A full-adder adds two two-bit binary numbers and a 'carry' bit to give sum and carry outputs. If all three inputs are 1, the output is 11; if two inputs are 1 and one is 0, the output is 10; if one input is 1 and two inputs are 0, the output is 01.

Chapter 17

17.1

1 International radio, national radio, FM radio, satellite TV

2 Amplitude modulation is where the amplitude of a carrier wave of constant frequency is modulated by the amplitude of the signal to be carried (see topic 17.1 Figure 1). Frequency modulation is where the frequency of a carrier wave is modulated by the amplitude of the signal to be carried (see topic 17.1 Figure 2).

3 The bandwidth of a signal is the range of frequencies in the signal: 100.75 to 101.00 MHz.

4 An FM station in the HF waveband would cover too much of the frequency range of the waveband. This would reduce the number of channels in the waveband. In the UHF waveband, local radio stations would need to use more powerful transmitters to cover the same area. This is because the carrier waves would be at higher frequency and would be attenuated more than in the VHF waveband.

17.2

1 An analogue signal can take any value between two limits. A digital signal only has two values. The alternating voltage from a microphone is an example of an analogue signal. A mobile phone signal or a digital TV signal is an example of a digital signal.

2 a Random variations referred to as 'noise' occur in communication systems. The signal cannot be detected if noise is excessive. Noise can be eliminated from digital signals but not from analogue signals.
b A regenerator amplifier is a digital circuit that removes the noise from a digital signal.
3 a i 1111 1111 **ii** 0111 0111 **b** 0.025 V
4 a An 8-bit signal has 256 equally-spaced values from zero to its maximum value where all the bits are 1. A 4-bit signal only has 16 equally-space values. The reconstructed analogue signal from 4-bit bytes would have much larger step changes than if 8-bit bytes were used.
b The digital signal would not reconstruct the original analogue signal. The higher frequencies of the analogue signal would be missing.

17.3

1 Advantages: more secure, no interference, noise eliminated. Disadvantages: regenerator circuits would be needed at intervals; an optical cable would need to be laid down over 30 km.
2 A geostationary satellite is a satellite in an orbit which is directly above the equator moving in a direction which keeps it directly above a fixed point on the equator. Signals are carried to the satellite by microwave beams from transmitter/receiver stations on the ground. The satellite sends the signals at a different carrier frequency to other ground stations directly or via other geostationary satellites.
3 a i 11 dB, 0.79 mW **ii** 55 dB, 32 μW
b The attenuation of a signal increases the further it travels. If the cable is too long, the signal will be attenuated too much and will not be detected. This is because it will be indistinguishable from the noise.
4 a If two adjacent cells sent out signals of the same frequency, the two signals would interfere with each other in the overlap area. The user would be unable to detect either signal properly.
b Signal samples from the eight different users in the frequency channel are sent in successive time slots. Each user's signal is sampled every eighth time slot.

Chapter 18

18.1

1 a S **b** NW
2 a S **b** sudden switch to N
3 a clockwise **b** reversed
4 a field due to coil along axis
b 90° if coil field >> Earth's field

18.2

1 a S **b** W **c** N
2 a N → S **b** vertical up

18.3

1 a 0.14 T **b i** 0 **ii** 11 mN **iii** 22 mN
2 a 2.4×10^{-2} N West
b 4.5 A West
c 0.20 T vertically down
d South, 8.0×10^{-3} N
3 0.10 N due South at 20° below the horizontal
4 Long sides; 2.7 N on each side perpendicular to the plane of the coil and in opposite directions, Short sides; 0

18.4

2 i 1.9×10^{-13} N **ii** 9.6×10^{-14} N
4 b i 4.4 m s^{-1} **ii** 8.5×10^{-20} N

18.5

1 a ii 21 mm **b** 2.8 mT
2 a 4.7 mT **b** 17.5 mm
3 b 1.1 MeV
4 a 8.0×10^{6} C kg^{-1} **b** 1.4×10^{7} C kg^{-1}

Chapter 19

19.1

4 a i Vertically downwards **ii** The eastern end

19.2

1 a i 1.1 mWb **ii** 2.0 s **iii** 0.54 mV
2 a 1.4 mWb **b** 23 mV
3 a i 4.5×10^{-4} m^2 **ii** 1.5(4) mWb
b i 3.1 mWb **ii** 32 mV
4 a 8.0 μWb **b** 40 μV

19.3

2 flux linkage 0, 0, $-BAN$; induced e.m.f. 0, $-E_0$, 0
3 a 26 mWb

19.4

3 a 11.5 V **b i** 0.26 A **ii** 5.2 A
4 b i 17 A (16.7 A to 3 sig. fig.) **ii** 56 kW

Chapter 20

20.2

1 b i 273 K **ii** 293 K **iii** 77 K
2 a 328 K **b** 137 kPa

20.3

1 a 23 kJ **b** 536 kJ
2 a 270 s **b** 10.3 MJ
3 a 320 J **b** 130 J kg^{-1} K^{-1}
4 3.2 kW

20.4

2 0.16 kg
3 a 4.2 J s^{-1} **b** 6500 s
4 a 22 J s^{-1} **b** 6.5 kJ

20.5

1 a The vessel is silvered to reduce heat loss by radiation from its outer surface to outer container.
b Heat transfer across the gap due to conduction and convection is eliminated because there is no material in the gap.
c The lid prevents heat loss due to evaporation from the surface of the liquid in the flask.
d Felt or polystyrene could be used as such material is a good thermal insulator and so reduces heat loss due to thermal conduction from the flask to the outer container.
2 410 J s^{-1}
3 a 40 J s^{-1} **b** 5.3 J s^{-1}
4 a 2.7 kJ s^{-1} **b** 4.3 K

Chapter 21

21.1

1 126 kPa
2 79 kPa
3 0.097 m^3
4 **b** $2.33 \times 10^{-5} m^3$, 1600 kg m^{-3}

21.2

1 **a** 1.1 moles **b** 109 kPa,
2 **a** 9.3×10^{-4} moles **b** $2.1 \times 10^{-5} m^3$
3 **b** 1.3 kg m^{-3},
4 **a** 1.2 kg m^{-3} **b** 2.5×10^{22}

21.3

1 Increasing the temperature of the gas causes the root mean square speed of the molecules to increase. Therefore the molecules have more momentum on average so their change of momentum on impact is greater. Also, because the container volume is constant, the molecules move faster on average so they collide with the container walls more frequently. Therefore, the mean force exerted by each molecule is greater.
2 524 $m s^{-1}$
3 **a** 7.52×10^{-3} **b** 474 $m s^{-1}$
4 **a** 1.48 moles **b** 4.2×10^{-2} kg **c** $5.2 \times 10^2 m s^{-1}$

21.4

1 **a** 0.97 moles **b** 7.7×10^{-21} J **c** 4.5 kJ
2 **a** 5.7×10^{-21} J **b** 1.8×10^3 $m s^{-1}$
3 **a** The mean kinetic energy of a molecule of an ideal gas depends only on the temperature of the gas. The gas temperature is the same throughout the gas so the mean kinetic energy of any gas molecule is the same as that of any other molecule.
b In any sample of air, the mean kinetic energy of a nitrogen molecule is equal to that of an oxygen molecule in the same sample. The kinetic energy of a gas molecule is equal to $\frac{1}{2} m c_{rms}^2$ where m is its mass and c_{rms}^2 is its mean square speed. Therefore $\frac{1}{2} m_n (c_{rms})_n^2 = \frac{1}{2} m_o (c_{rms})_o^2$ where the subscripts n and o are used respectively to denote the masses and mean square speeds of a nitrogen molecule and an oxygen molecule.
Rearranging this equation gives

$$\frac{\text{the mean square speed of a nitrogen molecule, } (c_{rms})_n^2}{\text{the mean square speed of an oxygen molecule, } (c_{rms})_o^2} = \frac{\text{mass of an oxygen molecule, } m_o}{\text{mass of a nitrogen molecule, } m_n}$$

The ratio of molecular masses m_o/m_n = the ratio of the molar masses $= \frac{0.032}{0.028} = 1.14$
Therefore the above ratio of mean square speeds = 1.14
Hence the rms speed of a nitrogen molecule = $1.14^{0.5}$ = 1.07 × the rms speed of an oxygen molecule.
4 **a** **i** 1.50 **ii** 220 K **b** 870 J **c** 1310 J **d** 2180 J

Chapter 22

22.1

1 $3.75 \times 10^7 m s^{-1}$
2 **a** **i** vertically upwards, **ii** and **iii** vertically downwards
b The force on the electron is constant and vertically upwards so its acceleration is constant and vertically upwards. Therefore the vertical component of its velocity increases steadily with time. As the horizontal component of its velocity is constant, its vertical displacement increases at a faster rate than its horizontal displacement so its path curves upwards.
3 The force on an electron in the beam is perpendicular to the magnetic field direction and to the electron's velocity direction. Therefore the magnetic force does no work on the electrons so the speed of each electron in the beam is constant. The magnetic force is therefore constant in magnitude and always perpendicular to the direction of motion of the electron. Therefore the force causes each electron to move round on a circular path with a centripetal acceleration of constant magnitude. The path of the beam is in a vertical plane as the magnetic field is horizontal.
4 9.12×10^4 V m^{-1}

22.2

1 $3.39 \times 10^7 m s^{-1}$
2 1.75×10^{11} C kg^{-1}
3 1.75×10^{11} C kg^{-1}
4 1.79×10^{11} C kg^{-1}

22.3

1 **a** 4.78×10^{-19} C, negative **b** 3
2 **b** **i** 6.40×10^{-19} C, positive **ii** 4
3 **a** 3.96×10^{-15} kg **b** 3.18×10^{-19} C
4 **a** The charge on a droplet is equal to a whole number × e where e, the magnitude of the charge of the electron = 1.6×10^{-19} C. The significance is that charge is quantised and the quantum of charge is e.
b The droplet falls vertically with an acceleration, initially equal to g, that decreases to zero as its speed becomes constant. The droplet is acted on by the force of gravity which is downwards and constant and a drag force which is upwards and which increases with speed. Therefore, as the droplet falls and gains speed, the drag force increases so the resultant force on it decreases and its acceleration decreases. The acceleration becomes zero and the speed becomes constant when the drag force is equal and opposite to the force of gravity on the droplet.

22.5

2 **a** **i** 6.7×10^{14} Hz, 4.4×10^{-19} J
ii 2.0×10^{14} Hz, 1.3×10^{-19} J
3 **a** 1.7×10^{14} Hz **b** 2.7×10^{-19} J
4 **a** 3.1×10^{-19} J **b** 1.6×10^{-19} J **c** 2.5×10^{14} Hz

22.6

2 **a** 1.6×10^{12}
3 **a** 3.4×10^{-19} J
b 1.5×10^{15}
c 2.5×10^{12}
4 **a** **i** 4.0×10^{14} Hz **ii** 2.7×10^{-19} J,
b 2.7×10^{-19} J

22.7

1 **a** 9.0 eV
b There are three energy levels below the 5.7 eV level. There are three possible transitions to the ground state, two to the first excited state and one to the second excited state making six transitions in total.

2 a 1. The energy of an electron in an atom increases in an excitation and decreases in a de-excitation.
2. Excitation can occur through photon absorption or electron collision. De-excitation only occurs through photon emission.
b i The diagram should show the ground state and two excited levels at 1.8 eV and 4.6 eV
ii photon energies 1.8 eV, 2.8 eV, 4.6 eV

3 The energy levels of each type of atom are unique to that atom. So the photons emitted are characteristic of that atom.

22.8

1 a i electrons **ii** holes
b Increasing the temperature makes the atoms vibrate more so more electrons break free from the atoms to become conduction electrons. So the number of charge carriers per unit volume increases and hence the resistivity decreases.

2 a A semiconductor, which has a small percentage of Group 3 or Group 5 atoms added to provide the majority of its charge carriers.
b An n-type semiconductor has Group 5 atoms added to provide electrons as charge carriers. A p-type semiconductor has Group 3 atoms added to provide holes as charge carriers.

3 The Group 3 atoms in a p-type semiconductor have unfilled energy levels just above the valence band. Some of the electrons from the valence band have sufficient energy to fill the Group 3 levels, leaving holes as charge carriers in the valence band. When a potential difference is applied, the holes migrate towards the negative terminal as electrons in the valence band or the group 3 levels fill the holes, leaving holes elsewhere.

4 $7.0\,m\,s^{-1}$

22.9

3 a $3.6 \times 10^{-11}\,m$ **b** $1.9 \times 10^{-14}\,m$

4 a $1.3 \times 10^{-27}\,kg\,m\,s^{-1}$, $1.5 \times 10^{3}\,m\,s^{-1}$
b $1.3 \times 10^{-27}\,kg\,m\,s^{-1}$, $0.78\,m\,s^{-1}$

Chapter 23

23.1

1 a 73 s **b** 6 g

2 a i 9.0×10^{14} **ii** $2.2(5) \times 10^{14}$
b i 9.0×10^{14} **ii** 15.8×10^{14} **c** $1.3 \times 10^{3}\,J$

3 a 19 kBq **b** 2.4 kBq

4 a 4.4×10^{21} **b** 1.1×10^{21} atoms

23.2

1 a $1.0 \times 10^{-6}\,s^{-1}$ **b** 3.9×10^{16}

2 a $6.3 \times 10^{-10}\,s^{-1}$ **b** 20.5 kBq

3 a 2.7×10^{24} **b i** 0.65 kg **ii** 1.7×10^{24}

4 a $1.3 \times 10^{-6}\,s^{-1}$ **b** 149 hours

23.3

1 b 1340 years

2 a i 730 kBq **ii** 870 kBq **iii** $1.9 \times 10^{-13}\,kg$

4 b i $5.2 \times 10^{13}\,Bq$ **ii** $3.3\,J\,s^{-1}$

23.4

1 a $2.18 \times 10^{-15}\,kg$
b i $8.89 \times 10^{-33}\,kg$ **ii** $8.89 \times 10^{-30}\,kg$

2 6.2 MeV

3 0.56 MeV

4 3.2 MeV

23.5

2 a 7.4 MeV **b** 8.8 MeV

3 a i 7.1 MeV **ii** 2.6 MeV

4 1.1 MeV

23.6

2 b i $a = 56, b = 98$ **ii** 206 MeV

3 b ii 0.43 MeV, 5.5 MeV

4 b 12.9 MeV

Chapter 24

24.1

1 a i 0.14 mm **ii** 0.62 mm
b At this frequency, their wavelength in the body is significant compared with the transducer and with body structures so diffraction is significant. As a result, they spread out and weaken too much so they do not form distinct images.

2 a i See topic 24.1 Figure 1 and related notes
ii The backing block prevents ultrasonic waves created at the two surfaces of the disc from cancelling each other out. The pad also damps the vibrations of the disc rapidly after each pulse is emitted.
b i A is due to reflection at the cornea; B is due to reflection at the front surface of the eye lens; C is due to reflection at the back surface of the eye lens; the furthest pulse (at the right side of the screen) is due to reflection at the retina.
ii about 13 mm

3 a i 0.999 **ii** 1.73×10^{-3}
b Gel and skin have similar acoustic impedances so a gel–skin interface hardly reflects any ultrasonic waves from the probe. If the gel was not used, trapped air between the probe and the body would reflect most of the ultrasonic waves because air and skin have very different acoustic impedances.
c i $1.64 \times 10^{6}\,kg\,m^{-2}\,s^{-1}$ **ii** 1.6×10^{-5}

4 a A B-scan gives a 2-dimensional image whereas an A-scan only gives information about the distances from the probe to reflecting boundaries in one direction. A B-scan requires a multi-transducer probe whereas an A-scan uses a probe with a single transducer.
b Ultrasonic waves are non-ionising unlike X-rays so they would not harm the baby whereas X-rays might.

24.2

1 a ii 0.15 W
b i The intensity increases because more electrons strike the anode each second so more X-ray photons are produced.
ii The beam becomes more penetrating because the maximum energy of the X-ray photons in the beam is increased. This happens because the electrons strike the anode with more kinetic energy when the tube voltage is increased.

2 a i Sharpness is to do with how clearly the edges of structures in an X-ray image can be seen.
ii Contrast is to do with the relative darkening of an X-ray film in different areas of the film. Poor contrast

occurs if the lightest areas are not much lighter than the darkest areas.

b A scattering grid placed between the patient and the film prevents X-rays scattered in the body of the patient from reaching the film. Without the grid, such X-rays would darken the shadow areas of the film which would reduce the contrast and sharpness of the image.

3 b A contrast medium is necessary when an X-ray image of a soft body organ such as the stomach is being created. If the organ is filled with the contrast medium, it absorbs X-rays more effectively. This improves the contrast of the image so the edges and structures of the image can be seen more clearly.

4 a See topic 24.2 Figure 8 for a suitable diagram. The X-ray tube produces a narrow beam which is detected by electronic detectors on the other side of the patient. The tube and the detectors are on a gantry which is rotated in steps about the patient so that the beam always passes through the same cross-section of the patient. The detector signals are recorded at each step and used to construct a cross-sectional image of the patient.

b i 12 **ii** 3, 1, 3, 5

24.3

1 Hydrogen nuclei absorb energy from the radio waves produced by MR scanners and emit them so the presence of the nuclei can be detected. The body contains many hydrogen nuclei in water molecules and other molecules.

2 a The magnetic flux density of the magnetic field produced by the scanning coil

b The spin of the nuclei make them precess at a certain frequency when they are in a strong magnetic field.

3 a The intensity of the radio waves emitted from the hydrogen nuclei after they are excited by the scanning field.

b A hydrogen nucleus precesses at a higher energy level when it absorbs a radio frequency photon of suitable energy. When it de-excites, it returns to the lower energy level and emits a radio wave photon.

4 a The relaxation time is used to discriminate between different tissue types. This is the time the nucleus takes to de-excite after it has been excited.

b Radio waves are non-ionising and therefore harmless, unlike X-rays. An MRI scan uses radio waves and therefore does not expose the patient to ionising radiation as an X-ray scan would.

Chapter 25

25.1

1 The eye lens becomes thinner because the radial ciliary muscles relax and lengthen.

2 a The iris contracts so the eye pupil becomes narrower and less light enters the eye. The retinal cells become less sensitive.

b The iris dilates and the eye pupil widens allowing more light to enter the eye. The rods in the retina become more sensitive.

3 See topic 25.1. Figure 3.

4 There are more rods than cones at the edge of the field of view and more cones than rods near the centre. Unlike cones, rods are not colour-sensitive. In dim light, the rods become more sensitive and the cones switch off. Therefore objects near the edge of the field of view viewed in dim light are seen in black and white.

25.2

1 a i The top ray should refract at the lens and pass through F; the bottom ray should refract at the lens and then become parallel to the principal axis; the image should be formed at 1.4(3)*f* on the right-hand side of the lens.

ii real, inverted, diminished

b i See topic 25.2 Figure 5. **ii** virtual, magnified, upright

2 a and **c** $v = +0.240\,\text{m}$ **b i** real **ii** inverted

3 a and **c** $v = -0.300\,\text{m}$ **b i** virtual **ii** upright

4 a $v = -0.086\,\text{m}$, image height = 5.7 mm

b $v = -0.111\,\text{m}$, image height = 4.4 mm

25.3

1 a Myopia or short-sightedness is where distant objects cannot be seen clearly but nearby objects can.

b i See topic 25.3 Figure 2. **ii** $-0.125\,\text{D}$

2 a Hypermetropia or long-sightedness is where nearby objects cannot be seen clearly but distant objects can.

b i See topic 25.3 Figure 5. **ii** $+2.0\,\text{D}$

3 a i $-0.20\,\text{D}$ **ii** 0.26(3) m

b i $+2.75\,\text{D}$ **ii** 0.36(3) m

4 a Astigmatism is a sight defect in which objects are seen to be sharper in focus in one direction than in other directions. It is due to uneven curvature of the cornea.

b It is corrected by using a lens (in front of the eye) with a cylindrical-shaped surface orientated so that it compensates for the uneven curvature of the cornea. The curvature of the cylindrical-shaped surface of the lens and the orientation of the axis of the cylindrical surface need to be supplied.

25.4

1 The sound waves incident on the tympanic membrane (or ear drum) make it vibrate. The vibrations are transmitted through the middle ear via the three bones of the middle ear: the hammer, which is in contact at one end with the tympanic membrane, the anvil and the stirrup. The stirrup is in contact with the oval window and makes it vibrate.

2 a If the ear is subjected to excessive noise, the muscles that hold the bones of the middle ear become slack, allowing the bones to vibrate differently so they are less efficient as transmitters of sound energy.

b Temporary deafness can occur on leaving a noisy environment until the muscles holding the bones of the middle ear together become taut again.

3 a $1.0 \times 10^{-9}\,\text{N}$ **b** $5.0 \times 10^{-4}\,\text{Pa}$

4 a The oval window transmits vibrations from the middle ear into the inner ear.

b The vibrations from the oval window make the basilar membrane vibrate. Tiny sensitive 'hair cells' on the basilar membrane send nerve impulses to the brain as a result of being stimulated by the vibrations of the membrane.

c The vibrations from the oval window pass through fluid in the inner ear and leave through the round window.

25.5

1 a $2.2 \times 10^{-2}\,\text{W m}^{-2}$ **b** 103 dB

2 $2.0\,\text{W m}^{-2}$

3 **a** 70 dB **b** $3.2 \times 10^{-6}\,\text{W m}^{-2}$
4 **a** $1.6 \times 10^{-2}\,\text{W m}^{-2}$ **b** 105 dB

25.6

1 **a** The ear becomes more sensitive to different frequency sounds as the frequency increases from 1000 Hz to about 3000 Hz then it becomes less sensitive.
b The ear becomes increasingly less sensitive as the frequency is increased beyond 1000 Hz and cannot detect sound at frequencies above 18 kHz.
2 **a** The dBA scale is a dB scale that is adjusted to match the normal ear's response at different frequencies.
b **i** 20 dBA, 40 dB **ii** 60 dBA, 0 dB
iii 15 dBA, approx 45 dB
3 Using a signal generator connected to a loudspeaker and a sound level meter, adjust the frequency of the signal generator to the required frequency and adjust the amplitude of the signal so the sound can be heard. Place the sound level meter near the person under test and adjust the meter to read on the dBA scale. Reduce the signal amplitude until the sound can no longer be heard. The reading on the meter gives the hearing loss of the person in dB at that frequency.
4 **a** Deterioration in hearing can occur due to age and due to prolonged exposure to excessive noise.
b Age-related deterioration occurs at all frequencies and is greater the higher the frequency. Excessive and prolonged exposure to noise in a narrow frequency range (e.g. machine noise) can cause deterioration in that frequency range only.

Chapter 26

26.1

1 **a** **i** 0.500 m **ii** 320 cm **iii** 95.6 m
b **i** 450 g **ii** 1.997 kg **iii** 5.4×10^{7} g
c **i** $2.0 \times 10^{-3}\,\text{m}^2$ **ii** $5.5 \times 10^{-5}\,\text{m}^2$
iii $5 \times 10^{-6}\,\text{m}^2$
2 **a** **i** 1.5×10^{11} m **ii** 3.15×10^{7} s
iii 6.3×10^{-7} m **iv** 2.57×10^{-8} kg
v 1.50×10^{5} mm **vi** 1.245×10^{-6} m
b **i** 35 km **ii** 650 nm **iii** 3.4×10^{3} kg
iv 870 MW (= 0.87 GW)
3 **a** **i** $20\,\text{m s}^{-1}$ **ii** $20\,\text{m s}^{-1}$ **iii** $1.5 \times 10^{8}\,\text{m s}^{-1}$
iv $3.0 \times 10^{4}\,\text{m s}^{-1}$
b **i** $6.0 \times 10^{3}\,\Omega$ **ii** $5.0\,\Omega$ **iii** $1.7 \times 10^{6}\,\Omega$
iv $4.9 \times 10^{8}\,\Omega$ **v** $3.0\,\Omega$
4 **a** **i** 301 **ii** 2.8×10^{9} **iii** 1.9×10^{-23}
iv 1.2×10^{-3} **v** 2.0×10^{4} **vi** 7.9×10^{-2}
b **i** 1.6×10^{-3} **ii** 5.8×10^{-6} iii 1.7 **iv** 3.1×10^{-2}

26.2

1 **a** 1.57 m
b **i** 1.57 m **ii** 1.05 m **iii** 0.26 m **iv** 0.47 m
c **i** 35 km **ii** 31°
2 **a** The angle subtended in degrees should be equal to 0.115 × the diameter in mm. **b** **ii** 0.5°
3 **a** **i** 0.035 **ii** 0.140
b **i** 72 mm **ii** 72 mm **iii** 148 mm
c **i** $B = 74°, C = 46°$ **ii** $B = 78°, C = 32°$
iii $B = 45°, C = 15°$
4 **a** **i** 3.9 N, 4.6 N, **ii** 3.4 N, 9.4 N **iii** 4.8 N , 5.7 N
b 4.8 N at 21° to the 3.5 N

26.3

1 **a** **i** 0.2 **ii** 0.1, 0.25 **iii** < 0.1
b **i** $t = \frac{v - u}{a}$ **ii** $t = \sqrt{\frac{2s}{a}}$
iii $t = \frac{y}{k} + t_0$ **iv** $t = \frac{mv}{F}$
c **i** 2 **ii** -1 **iii** 4.25 **iv** $\frac{1}{3}$
2 **a** **i** $x = \frac{y^2}{4}$ **ii** $x = \frac{1}{4y^2}$
iii $x = \frac{1}{y^3}$ **iv** $x = \sqrt{\frac{k}{y}}$
b **i** 0.25 **ii** ± 2.8 **iii** ± 0.5 **iv** 4
3 **a** **i** $3.1 \times 10^{-7}\,\text{m}^2$ **ii** 6.2×10^{-3} m
iii 2.5 s **iv** $17\,\text{m s}^{-1}$
b **i** $\text{kg m}^2\text{s}^{-2}$ **ii** kg **iii** s
4 **a** **i** ½ or -3 **ii** 1.44 or 5.56 **iii** 1 or -1.67
b **i** $t = -\frac{20}{6}$ s or 2 s **ii** $R = 1.0\,\Omega$ or $4.0\,\Omega$

26.4

1 **a** **i** 3 **ii** -3 **iii** 1
b **i** -4 **ii** 8 **iii** 2
c **i** -1 **ii** 5 **iii** 5
d **i** -1.5 **ii** 3 **iii** 2
2 **a** **i** $y = 2x - 8$ **ii** -8
b **i** $3\,\text{m s}^{-2}$ **ii** $5\,\text{m s}^{-1}$
3 **a** (1, 4) **b** $y = 4x$
4 **a** $x = 2, y = 0$
b $x = 3, y = 5$
c $x = 2, y = 0$

26.5

1 **b** **i** gradient **ii** area under line
2 **b** Resistance $= \frac{1}{\text{gradient}}$
3 **b** The power is constant and is represented by the gradient of the line.
4 **b** **i** **1** acceleration **2** distance fallen **ii** velocity

26.6

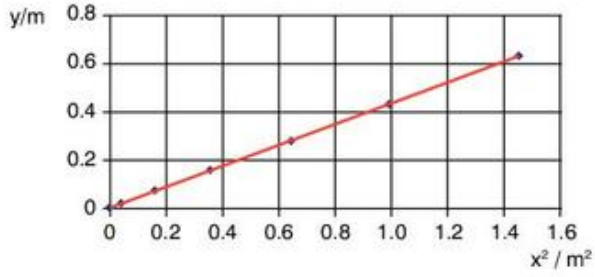
1 **a** **i** 0.477 **ii** 1.176 **b** **i** 1.653 **ii** 0.699
2 **a** **i** 10.8 dB **ii** 7.0 dB **b** 17.8 dB
3 **a** $n = 5, k = 3$ **b** $n = 3, k = \frac{1}{2}$ **c** $n = 2, k = 1$
4 **a** **i** 1.10 **ii** 2.71 **b** **i** 3.81 **ii** 1.61

26.7

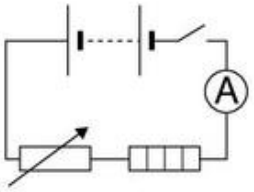
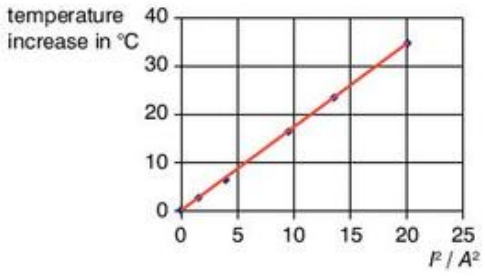
1 **a** **i** 2, 3 **ii** 12, 0.2 **iii** 4, 0.02
b **i** 0.23 s **ii** 3.5 s **iii** 35 s
2 **a** 11.3 kBq **b** 9.0 kBq **c** 0.38 kBq
3 **a** i 2.20 s **ii** 1.52 s **b** **i** 4.83 V **ii** 1.24 V
4 **a** **i** 0.14 s **ii** 82 **b** $a = 6.9, b = -5$

Answers to key concept questions

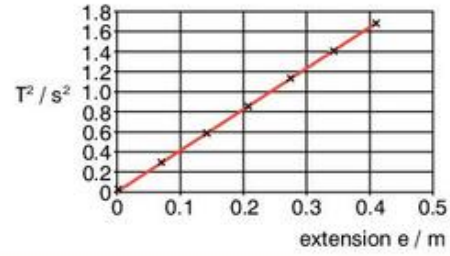
Answers to questions for Key concepts: Forces (pages 86–89)

	Answer	Marks	Comments
1 a i	to ensure its speed of projection was the same every time the test was carried out	**1**	
ii	It might dislodge the bar on impact.	**1**	
	It might bend the tube as it rolled down inside it.	**1**	
b i	mean value of y / mm: 20.3, 71.7, 159, 278.3, 433, 637.7	**2**	2 marks for all correct
		2	1 mark for 3 correct
	x^2 / m^2: 0, 0.0420, 0.162, 0.355, 0.645, 0.990, 1.452		as above
ii	$t = x/U$ (from $x = Ut$ rearranged)	**1**	
	substituting in $y = \frac{1}{2}gt^2$ gives $y = \frac{1}{2}g(x/U)^2 = (\frac{1}{2}g/U^2)x^2 = kx^2$		
	where $k = \frac{1}{2}g/U^2$	**1**	
iii	graph:  		Three sets of measurements are given of the vertical distance fallen for different horizontal distances. In part b, the hypothesis is given that vertical distance is directly proportional to the horizontal distance squared. The mean value of each vertical distance has to be calculated then used to plot a graph of the vertical distance against the horizontal distance squared in order to test the hypothesis.
	labelled correctly	**1**	
	correct units shown on axes	**1**	
	points plotted correctly	**1**	
	best-fit line drawn	**1**	
iv	k = the gradient of the straight line	**1**	Accept answer for U in the range 3.1–3.5 m s^{-1}.
	$= 0.61/1.4 = 0.44\,\text{m}^{-1}$	**1**	
	so $U = \sqrt{\frac{g}{2k}} = \sqrt{\frac{9.8}{2 \times 0.44}} = 3.3\,\text{m s}^{-1}$	**1**	
c	The y values for each distance measurement have a range of more than 2 mm. (For example, for $x = 1.205$ m, the range of y is 12 mm.)	**1**	Part c asks about the precision and accuracy of the measurements. You need to realise here that the uncertainty of each mean value is not given by the precision of a mm scale, but by estimating the spread of each set of measurements (e.g. half the range of the vertical distance measurements).
	So, the y values have a measurement error of more than 2 mm.	**1**	
	For the x-values, although the readings using a mm rule can be made to within 1 mm, the exact position of impact on the bar is uncertain and introduces a further measurement error equal to the diameter of the bar.	**1** **1**	
	Total mark	**20**	

Answers to questions for Key concepts: Electricity, waves and radioactivity (pages 164–167)

	Answer	Marks	Comments
1 a i	four out of five components shown correctly	**1**	The question is about an investigation into the heating effects of an electric current. Part a tests your knowledge of electric circuits.
	for all components shown correctly	**1**	
			
ii	by adjusting the variable resistor if the current (or the ammeter reading) changed to keep the ammeter reading the same in each test	**1** **1**	Correct reference to ammeter and variable resistor needed for both marks.
b i	(Use the thermometer to) measure the water temperature at intervals after the current is switched off.	**1**	
	The insulation is effective if the water temperature does not decrease (or change) over at least 600 s.	**1**	Reference to time scale of at least 600 s necessary for the second mark.
ii	The temperature rise depends on the heating time and the volume (or mass of water).	**1**	
	If either of these quantities differs in a test, the temperature rise will be unreliable.	**1**	
c i	the energy supplied to the heater in the time $t = I^2Rt$	**1**	Here you have to use your knowledge of electrical power and resistance to justify a given prediction and then use your data analysis skills to plot a graph and use it to test the prediction.
	The temperature rise is proportional to the energy supplied.	**1**	
ii	I^2/A^2: 1.44, 4.00, 9.61, 13.7, 20.3	**1**	
iii	graph		
			
	for correctly labelled axes	**1**	
	for suitable scales	**1**	
	for correctly plotted points	**1**	
	for best-fit line	**1**	
	The graph is a straight line through the origin, so the temperature increase is directly proportional to the current squared, as predicted.	**1** **1**	
d	The resistance of the heater element increased as the water temperature increased.	**1**	You are expected to use your knowledge and understanding about the effect of temperature on the resistance of a metal to explain the observation.
	The battery provided a constant e.m.f.	**1**	
	So the resistance of the variable resistor had to be reduced to keep the total circuit resistance constant and hence keep the current constant.	**1**	
	Total mark	**18**	

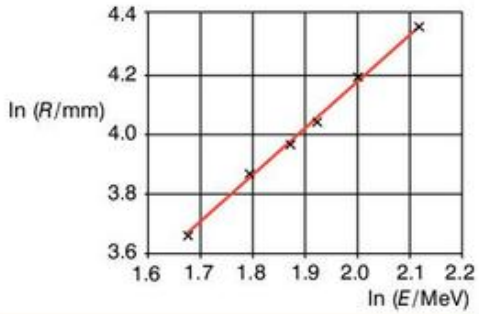
Answers to questions for Key concepts: Fields (pages 294–297)

	Answers	Marks	Comments
1 a i	With the object on the spring: mean value of $x = 72\,\text{mm}$ $e = 70\,\text{mm}$	**1**	
ii	1.4%	**1**	Each reading was $\pm 0.5\,\text{mm}$. As the extension was the subtraction of two readings, the absolute uncertainties are added to give an absolute uncertainty of $\pm 1.0\,\text{mm}$ and a percentage uncertainty of $\frac{1}{70} \times 100 = \pm 1.4\%$.
b i	0.551 s	**1**	$T_{av} = 11.02\,\text{s}$
ii	0.6%	**1**	The absolute uncertainty can be taken as half the range of the values, so the uncertainty in T_{av} is: $(11.11 - 10.97)/2 = \pm 0.07\,\text{s}$
c i	$mg = ke$, therefore $e = mg/k$	**1**	
ii	Using the above equation gives $\frac{m}{k} = \frac{e}{g}$ Substituting this expression for $\frac{m}{k}$ into the mass–spring time period equation $T = 2\pi\sqrt{\frac{m}{k}}$ gives the required equation.	**1** **1**	
d	Plot either T^2 against e or T against $\sqrt{e}$: correct labels and units suitable scales all points plotted correctly best-fit line According to the equation, the line should pass through the origin and the gradient is equal to $4\pi^2/g$ for the T^2 against e line *or* $2\pi/\sqrt{g}$ for the T against $\sqrt{e}$ line. To determine g, the gradient of the line should be measured. Gradient given the correct unit (s^2m^{-1} or s^2mm^{-1}). A large triangle used correctly to determine the gradient and used with the appropriate gradient formula above to find g.	**4** **1** **1** **1** **1** **1**	Draw a triangle or use points that cover over half of the line you have drawn. It improves accuracy. Make sure the points used are on your line and not just two of the plotted points. For example, the graph for T^2 against e is shown below. Its gradient $= 4.074\,\text{s}^2\text{m}^{-1}$. Hence $g = 4\pi^2/4.074$ $= 9.69\,\text{ms}^{-2}$ 
e	The line is straight through the origin. The uncertainty in the gradient can be estimated by considering the uncertainty in each measurement and put error bars on each plotted point, as explained on p.xxi. Draw best-fit lines through the error bars with maximum and minimum gradients, and then give an estimate of the uncertainty in the gradient value.		An alternative method is to estimate the uncertainty in the y-coordinate of the point where the extension is 0.400 m, which is between $1.62\,\text{s}^2$ and $1.65\,\text{s}^2$, giving a gradient of between $1.62\,\text{s}^2/0.400\,\text{m}$ ($= 4.05\,\text{s}^2\text{m}^{-1}$) and $1.65\,\text{s}^2/0.400\,\text{m}$ ($= 4.13\,\text{s}^2\text{m}^{-1}$) or $4.09 \pm 0.04\,\text{s}^2\text{m}^{-1}$. Prove for yourself that these estimates give $g = 9.65 \pm 0.09\,\text{ms}^{-2}$.
	Total mark	**20**	

Answers to questions for Key concepts: Thermal and nuclear physics (pages 380–383)

	Answers	Marks	Comments
1 a i	The $^{4}_{2}$He nucleus has twice the charge of a $^{1}_{1}$H nucleus or proton but it cannot be just two protons because its mass is four times greater than the mass of proton.	1	
	• The extra mass of 2 units cannot be due to a single particle as mass numbers are integer units.	1	
	Alternative:		
	Rutherford used the neutron–proton model to explain why the mass number of any nucleus heavier than the $^{1}_{1}$H nucleus is greater than its atomic number.		
	The extra mass must be due to 'unit mass' particles, as mass numbers are integer units.		
ii	The $^{1}_{1}$H nucleus is the smallest known nucleus and has the least charge / is a single particle / is a proton.	1	
b i	$^{9}_{4}$Be, $^{12}_{6}$C	1	
b ii	any 3 from:	3	You may recognise that low-energy alpha particles are scattered elastically.
	The α-particle has to have sufficient kinetic energy:		
	• to overcome the electrostatic repulsion of the nucleus		
	• to reach the nucleus closely enough		
	• to experience the strong nuclear force		
	• which pulls the α-particle into the nucleus		
	• as the strong nuclear force is stronger than the electrostatic force at close range.		
c i	$E_K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$, where p = momentum = mv		Don't forget that the recoil nucleus has a mass of $206m_u$ because the a-particle has left the Po-210 nucleus.
	From conservation of momentum, the momentum of the emitted α-particle $p_\alpha = -p_{nuc}$, where p_{nuc} is the momentum of the recoil nucleus.	1	
	Therefore $\frac{E_k \text{ for the recoil nucleus}}{E_k \text{ for the } \alpha \text{ particle}} = \frac{\left(\frac{p^2}{2m}\right)_{nuc}}{\left(\frac{p^2}{2m}\right)_{\alpha}} = \frac{m_\alpha}{m_{nuc}} = \frac{4}{206}$	1	
	E_K for the recoil nucleus $= \frac{4}{206} \times 5.3\,\text{MeV} = 0.1\,\text{MeV}$	1	
	So the total energy released = 5.3 + 0.1 = 5.4 MeV.	1	
c ii	If $R = kE^n$, a graph of ln$R$ on the y-axis against ln$E$ on the x-axis should give a straight line with a gradient equal to n.		This is an excellent example of the use of a log–log graph to determine the numerical power of an equation of the form $y = kx^n$. If this method does not generate a straight-line graph, the equation cannot be of the form above.
		1	

***R*/mm**	39	48	53	57	66	78
***E*/MeV**	5.3	6.0	6.5	6.8	7.4	8.3
ln*R*	3.66	3.87	3.97	4.04	4.19	4.36
ln*E*	1.67	1.79	1.87	1.92	2.00	2.12

Correct values to 2 or more significant figures for:		
• ln$R$	1	
• ln$E$	1	
suitable scales	1	
correctly labelled scales	1	
all points plotted correctly	1	
best-fit line	1	
correct calculation of gradient	1	
e.g. gradient $= \frac{4.34 - 3.72}{2.10 - 1.70} = 1.55$		
$n = 1.55$		
n in the range = 1.5 to 1.6	1	
Total mark	**20**	

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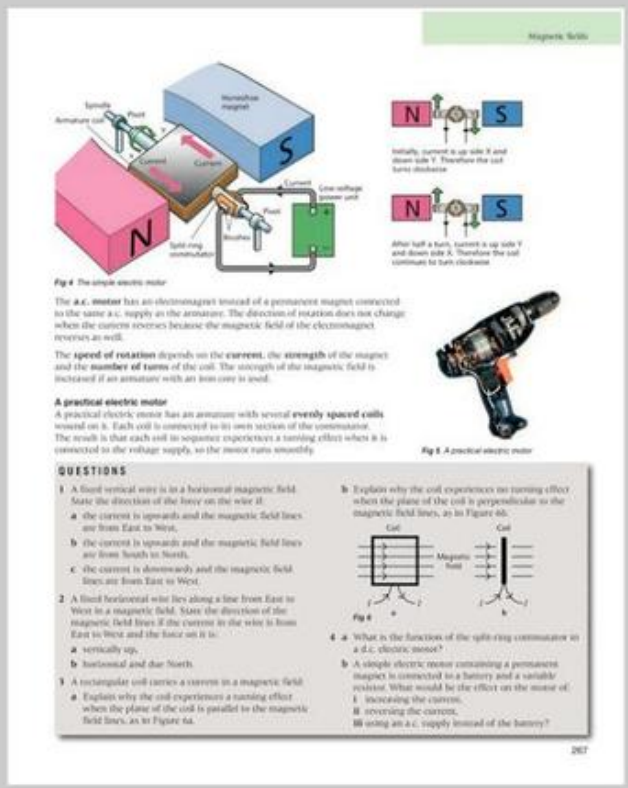

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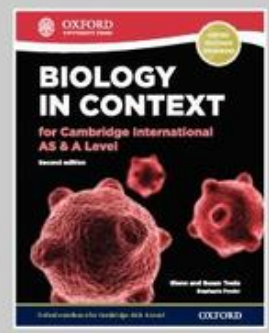

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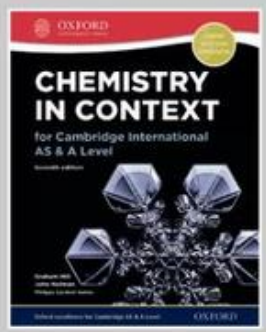

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email schools.enquiries.uk@oup.com
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