CONDITIONAL PROBABILITY

Summary:

1. *For two events* ***A*** *and* ***B:***

(i) $P(A/B)$ *is known as the probability of* ***A*** *given that* ***B*** *has already occurred.*

(ii) $P(A/B) = \frac{P(A \cap B)}{p(B)}$

(iii) $P(B/A) = \frac{P(A \cap B)}{p(A)}$

(iv) *The equations in* ***(ii)*** *and* ***(iii)*** *above can be written as follows****:***

$$P(A \cap B) = P(A/B) \cdot P(B) \qquad \textbf{or} \qquad P(A \cap B) = P(B/A) \cdot P(A)$$

2. *If the events are mutually exclusive, then* $P(A/B) = 0$ *and* $P(B/A) = 0$

since $P(A \cap B) = 0$.

3. *If the events are independent, then* $P(A/B) = P(A)$ *and* $P(B/A) = P(B)$

since $P(A \cap B) = P(A) \cdot P(B)$.

EXAMPLES:

1. *Events* ***A*** *and* ***B*** *are a such that* $P(A) = \frac{1}{3}$, $P(B/A) = \frac{1}{4}$ *and* $P(\overline{B}/\overline{A}) = \frac{4}{5}$.

*Find****:***

(i) $P(A \cap B)$

(ii) $P(\overline{B}/A)$

(iii) $P(B)$

(iv) $P(A \cup B)$

2. *Independent events* A *and* B *are such that* $P(A/B) = \frac{3}{4}$ *and* $P(B) = P(\overline{A} \cap \overline{B})$.

Find:

(i) $P(\overline{B})$

(ii) $P(A \cap \overline{B})$

3. *Events* A *and* B *are such that* $P(A) = 0{\cdot}5$, $P(B) = 0{\cdot}4$ *and* $P(A/B) = 0 \cdot 3$.

(i) *State with a reason whether the events are independent.*

(ii) *Find* ***P(A or B*** *but not both* A *and* ***B)***

4. *Given that* $P(A) = \frac{1}{4}$, $P(A/B) = \frac{1}{2}$ *and* $P(B/A) = \frac{2}{3}$.

(a) *State with reasons whether* A *and* B *are:*

(i) *independent events*

(ii) *mutually exclusive events.*

(b) *Find:*

(i) $P(A \cap B)$

(ii) $P(A \cup B)$

(iii) $P(A/\overline{B})$

5. *Show that for any two events* M *and* N*:*

(i) $P(M \cup N) = P(M) + P(N) - P(M \cap N)$

(ii) $P(M/N) + P(\overline{M}/N) = 1$.

6. *The chance that it will be windy today is* $\frac{1}{3}$*. If it is windy, the chance that Tom plays tennis is* $\frac{4}{5}$*. If it is not windy, his chance of playing is* $\frac{2}{5}$*. Find the chance that:*

(i) *Tom plays tennis.*

(ii) *it was not windy given that he does not play tennis.*

7. *A good football striker is nursing an injury in his leg. The probability that his team will win the next match when he is playing is* $\frac{4}{5}$*, otherwise it is* $\frac{2}{3}$*. The probability that he will have recovered by the time of the match is* $\frac{1}{4}$*. Find the probability that his team will win the match.*

8. *On a given day, it is twice as likely to be fine as not fine. If it is a fine day, the probability that Bob goes to play football is* ***0·9*** *and that of Tom is* ***0·75.*** *If it is not a fine day, Bob's probability is* ***0·5*** *and that of Tom is* ***0·25.***

(i) *Find the probability that both go to play.*

(ii) *If they both go to play, find the probability that it is a fine day.*

9. *The table below shows the likely hood of where* ***P*** *and* ***Q*** *spend their leisure time.*

	P	***Q***
Goes to dance	$\frac{9}{20}$	$\frac{1}{2}$
Visits a neighbor	$\frac{1}{4}$	$\frac{1}{3}$
Stays at home	$\frac{3}{10}$	$\frac{1}{6}$

(i) *Find the probability that both go out.*

(ii) *If they both go out, find the probability that both went to dance.*

10. ***Otim's*** *chances of passing* ***Arabic, French*** *and* ***History*** *are* ***0·6, 0·75*** *and* ***0·8*** *respectively.*

(i) *Find the chance that he passes only one subject.*

(ii) *If it is known that he passed at least two subjects, find the probability that he failed French.*

11. *At the bus park,* ***40%*** *of the buses are Isuzu make,* ***25%*** *are Styer type and the rest are of Tata make. Of the Isuzu make,* ***30%*** *have radios while for Styer and Tata types* ***20%*** *and* ***10%*** *have radios respectively. If a bus is selected at random from the park, find the probability that:*

(i) *it has a radio.*

(ii) *a Styer type is selected given that it has a radio.*

12. *The chance of Tom going to work by bus, by train or on foot is* ***0·3, 0·45*** *and* ***0·25*** *respectively. If he uses these methods of travel, the chance that he will be late is* ***0·4, 0·65*** *and* ***0·8*** *respectively. Find the chance that he will:*

(i) *be late on a particular day*

(ii) *use a bus given that he is not late*

(iii) *not go on foot given that he is late*

13. *On a certain day, a petrol station that deals in selling grade* ***A*** *and grade* ***B*** *petrol serves three times as many men as women.* ***70%*** *of the men and* ***40%*** *of the women buy grade* ***A*** *petrol. Of the men buying grade* ***A*** *petrol* ***80%*** *pay by cheque and of the men buying grade* ***B*** *petrol,* ***60%*** *pay by cheque. Of the women buying grade* ***A*** *petrol half pay by cheque and of the women buying grade* ***B*** *petrol,* ***40%*** *pay by cheque. Find the probability that a customer selected at random:*

(i) *buys grade* ***A*** *petrol.*

(ii) *pays by cheque.*

(iii) *pays by cheque given that is a woman.*

(iv) *is a man given that he pays by cheque for grade* ***A*** *petrol.*

EER:

1. *Given that* $P(A) = \frac{2}{3}$, $P(B) = \frac{5}{12}$, *and* $P(A/B) = \frac{2}{5}$. *Find the probability that:*

(i) *at least one event occurs*

(ii) *only one event occurs*

(iii) *neither events occur.*

[Ans: (i) $\frac{11}{12}$ ***(ii)*** $\frac{3}{4}$ ***(iii)*** $\frac{1}{12}$ ***]***

2. *Events* ***A*** *and* ***B*** *are a such that* $P(A) = 0 \cdot 6$, $P(B) = 0 \cdot 5$ *and* $P(A/B) = 0 \cdot 4$.

Find:

(i) *the probability that neither events occur*

(ii) $P(A/A \cup B)$

[Ans: (i) $0 \cdot 1$ ***(ii)*** $\frac{2}{3}$ ***]***

3. *Events* ***A*** *and* ***B*** *are a such that* $P(B) = \frac{3}{8}$, $P(A \cap B) = \frac{1}{4}$ *and* $P(B/A) = \frac{2}{5}$.

(a) *State with reasons whether* ***A*** *and* ***B*** *are:*

(i) *independent events*

(ii) *mutually exclusive events.*

(b) *Find:*

(i) $P(A)$

(ii) $P(A/B)$

(iii) $P(A/\overline{B})$

[Ans: b(i) $\frac{5}{8}$ ***(ii)*** $\frac{2}{3}$ ***(iii)*** $\frac{3}{5}$ ***]***

4. *Events* **A** *and* **B** *are such that* $6P(A) = 5P(B) = 15P(A \cap B) = 30P(\overline{A} \cap \overline{B})$, *find:*

(i) $P(\overline{A} \cup \overline{B})$

(ii) $P(\overline{A}/\overline{B})$

(iii) **P(A or B** *but not* **A** *and* **B)**

[**Ans: (i)** $\frac{4}{5}$ **(ii)** $\frac{1}{4}$ **(iii)** $\frac{7}{10}$]

5. *Given that* $P(A) = 0 \cdot 6$, $P(A/B) = 0 \cdot 4$ *and* $P(A \cup B) = 0 \cdot 9$, *find:*

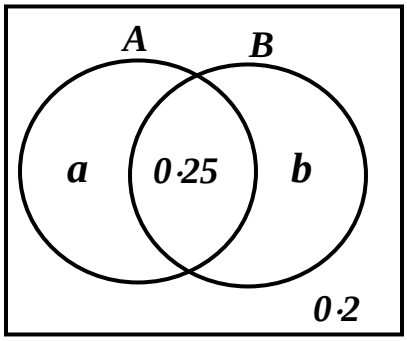
(i) **P(B)**

(ii) $P(B/A)$

(iii) $P(B/\overline{A})$

[**Ans: (i)** $\frac{1}{2}$ **(ii)** $\frac{2}{3}$ **(iii)** $\frac{3}{4}$]

6. *The Venn diagram below shows events* **A** *and* **B.**



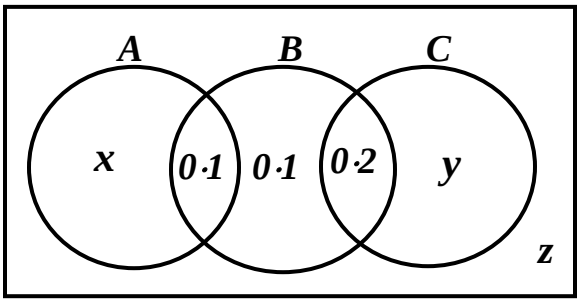
Given that $P(\overline{A}/\overline{B}) = \frac{1}{3}$, *find:*

(i) *the values of* **a** *and* **b.**

(ii) $P(\overline{B}/A)$

[**Ans: (i)** $0 \cdot 4$, $0 \cdot 15$ **(ii)** $\frac{8}{13}$]

7. *The Venn diagram below shows events* ***A, B*** *and* ***C.***



Given that events ***A*** *and* ***B*** *are independent and that* $P(B/C) = \frac{5}{11}$, *find:*

(i) *the values of* ***x, y*** *and* ***z***

(ii) $P(A \cup C/B)$

[Ans: (i) 0·15, 0·24, 0·21 (ii) 0·75]

8. *Events* ***A*** *and* ***B*** *are a such that* $4P(A) = 3P(B)$, $P(A \cup B) = \frac{7}{10}$ *and* $P(A/B) = \frac{7}{12}$.

Find:

(i) $P(A \cap B)$ ***(ii)*** $P(A \cup \overline{B})$ ***(iii)*** $P(A/\overline{B})$

[Ans: (i) $\frac{7}{20}$ ***(ii)*** $\frac{3}{4}$ ***(iii)*** $\frac{1}{4}$ ***]***

9. *The chances of a day being* ***snowy, rainy or foggy*** *are* ***0·3, 0·45*** *and* ***0·25*** *respectively. The corresponding probabilities of having car accidents are* ***0·4, 0·6*** *and* ***0·8.*** *If there was no accident, find the probability that it was foggy.*

[Ans: 0·1220]

10. *Events* ***A*** *and* ***B*** *are such that* $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{5}$ *and* $P(A/B) = 0$,

(i) *State with a reason the relationship between* ***A*** *and* ***B.***

(ii) *find* $P(A \cup B)$ ***[Ans: (ii)*** $\frac{7}{10}$ ***]***

11. *In a factory biscuits may be packed by* ***P, Q*** *or* ***R.*** ***P*** *packs* ***55%, Q*** *packs* ***30%*** *and the rest is for* ***R.*** *The corresponding probabilities of breaking some biscuits are* ***0·7, 0·2*** *and* ***0·1.*** *Find the probability that a packet with broken biscuits was packed by* ***P.***

[Ans: 0·8370]

12. *The chance that Joan goes to work by bus is* ***40%*** *and the rest is for train.* *If she goes by bus,* *the chance of being late for work is* ***20%,*** *otherwise it is* ***25%.*** *Find the chance that she will:*

(i) *be late on a particular day.*

(ii) *go by train given that she is on time*

[Ans: (i) 0·23 (ii) $\frac{45}{77}$ ***]***

13. *In a city,* ***65%*** *are conservatives,* ***20%*** *are liberals and* ***15%*** *are independents. In an election,* ***82%*** *of conservatives voted,* ***45%*** *of liberals voted and* ***40%*** *of independents voted. Find the probability that a person picked at random from the city:*

(i) *voted*

(ii) *is liberal given that did not vote.*

[Ans: (i) 0·683 (ii) 0·3470]

14. *Exhaustive events* ***A*** *and* ***B*** *are such that* ***P(A) = 0·75*** *and* ***P(B) = 0·4.*** *Find:*

(i) $P(A/B)$ ***(ii)*** $P(B/A)$ ***(iii)*** $P(\overline{A}/B)$

[Ans: (i) $\frac{3}{8}$ ***(ii)*** $\frac{1}{5}$ ***(iii)*** $\frac{5}{8}$ ***]***

15. *64% of the students take science subjects and* ***36%*** *take arts subjects.*

The chance of passing is $\frac{3}{4}$ *for science students and* $\frac{5}{6}$ *for arts students.*

Find the probability that a student chosen at random will fail.

[Ans: 0·22]

16. *A shop stocks* ***40%, 24%*** *and the rest of its spare parts from countries* ***A, B*** *and* ***C*** *respectively.* ***30%*** *of the stock from country* ***A*** *is defective and the respective percentages for* ***B*** *and* ***C*** *are* ***60%*** *and* ***45%.*** *If a spare part is drawn at random from the stock, find the probability that:*

(i) *It is defective.*

(ii) *It is from country* ***B*** *given that it is defective.*

[Ans: (i) 0·426 (ii) 0·33802]

17. *At a certain party,* ***25%*** *of the guests are women. Nile beer and Bell beer are the only drinks available for the guests.* ***40%*** *of the women and* ***70%*** *of the men take Nile beer. Of the men taking Nile beer,* ***80%*** *got drunk and of the men taking Bell beer,* ***60%*** *got drunk. Of the women taking Nile beer,* ***50%*** *got drunk and of the women taking Bell beer,* ***40%*** *got drunk. Find the probability that a randomly selected guest:*

(i) *takes Nile beer.*

(ii) *got drunk.*

(iii) *got drunk given that is a woman.*

(iv) *is a man given that he got drunk for taking Nile beer.*

[Ans: (i) $\frac{5}{8}$ ***(ii)*** $\frac{133}{200}$ ***(iii)*** $\frac{11}{25}$ ***(iv)*** $\frac{42}{47}$ ***]***

18. *Events* ***A*** *and* ***B*** *are a such that* $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{8}$ *and* $P(A/B) = \frac{7}{12}$.

(a) *State with reasons whether* ***A*** *and* ***B*** *are:*

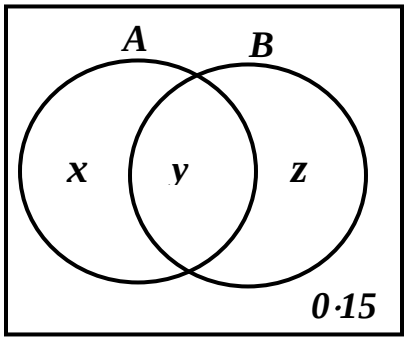
(i) *independent events*

(ii) *mutually exclusive events.*

(b) *Find:*

(i) $P(A \cap B)$ ***(ii)*** $P(B/\overline{A})$ ***[Ans: b(i)*** $\frac{7}{32}$ ***(ii)*** $\frac{5}{16}$ ***]***

19. *The Venn diagram below shows events* ***A*** *and* ***B.***



Given that $P(A) = 0{\cdot}6$ *and* $P(\overline{A}/B) = \frac{5}{7}$, *find:*

(i) *the values of* ***x, y*** *and* ***z.***

(ii) $P(B/\overline{A})$ ***[Ans: (i) 0·5, 0·1, 0·25 (ii)*** $\frac{5}{8}$ ***]***

20. *The probability of a bus leaving on time is* ***0·8.*** *If it leaves on time, the chance that it arrives on time is* ***0·7,*** *otherwise it is* ***0·4.*** *Find the chance that the bus:*

(i) *arrives on time*

(ii) *leaves on time given that it arrives late*

[Ans: (i) 0·64 (ii) $\frac{2}{3}$ ***]***

21. *5% of the population is infected with a certain disease. A test for such a disease has a probability of* ***0·8*** *of detecting the disease when it is present and its probability of detecting the disease falsely when it is not present is* ***0·1.*** *If a person gives a positive result when tested, find the probability that this person actually has the disease.*

[Ans: 0·2963]

22. *In a class,* ***25%*** *of the students are girls and the rest are boys.* ***80%*** *of the boys and* ***90%*** *of the girls are right-handed, while the rest are left handed. The chance that a right-handed student breaks a conical flask in any practical session is* ***0·3*** *and the corresponding chance for a left-handed student is* ***0·4.*** *Use a probability tree diagram to find the probability that a student chosen at random from the class:*

(i) *is left-handed and does not break the flask.*

(ii) *is left-handed.*

(iii) *is a girl given that is left-handed.*

[Ans: (i) 0·105 (ii) 0·175 (iii) 0·1429]

23. *The probability that an aircraft departs on time is* ***0·83*** *and the probability that it arrives on time is* ***0·92.*** *The probability that it departs on time and arrives on time is* ***0·78.*** *Find the probability that the aircraft:*

(i) *arrives on time given that it departs on time*

(ii) *departs on time given that it arrives on time*

[Ans: (i) 0·9398 (ii) 0·8478]

24. *The probability of a man sleeping late is* ***0·6.*** *If he sleeps late, the chance of waking up early is* ***0·2,*** *otherwise it is* ***0·75.*** *Find the probability that he:*

(i) *sleeps early and wakes up late*

(ii) *wakes up early*

(iii) *sleeps late given that he wakes up late*

[Ans: (i) 0·1 (ii) 0·42 (iii) $\frac{24}{29}$ ***]***

25. *The probability that a blue-eyed person is left-handed is* $\frac{1}{7}$. *The probability that a left-handed person is blue-eyed is* $\frac{1}{3}$. *The probability that a person has neither of these attributes is* $\frac{4}{5}$. *Find the probability that a person has both attributes.*

[Ans: $\frac{1}{45}$ ***]***

26. *In the market,* ***30%*** *of the fish sold is from lake* ***X, 25%*** *from* ***Y*** *and the rest from* ***Z.*** *Each lake had an estimated ratio of poisoned fish of* ***4%, 6%*** *and* ***5%*** *respectively. If a fish is drawn at random, find the probability that:*

(i) *It is poisoned.*

(ii) *It is from lake* ***Y*** *given that it is poisoned.*

[Ans: (i) 0·0495 (ii) 0·3030]

27. *Events* ***A*** *and* ***B*** *are such that* $P(A \text{ or } B \text{ but not both } A \text{ and } B) = \frac{7}{18}$, $P(A) = \frac{5}{9}$ *and* $P(B) = \frac{1}{2}$. *Find:* ***(i)*** $P(A/B)$ ***(ii)*** $P(B/\overline{A})$ ***[Ans: (i)*** $\frac{2}{3}$ ***(ii)*** $\frac{3}{8}$ ***]***

28. *Events* A *and* B *are such that* $P(A \cup B) = \frac{9}{10}$, $P(A/B) = \frac{1}{3}$ *and* $P(B/A) = \frac{2}{5}$.

Find:

(i) $P(A)$ **(ii)** $P(\overline{A}/\overline{B})$ **(iii)** $P(A/A \cup B)$

(iv) $P(A \cap B/A)$ **(v)** $P(A \cap B/A \cup B)$

[Ans: (i) $\frac{1}{2}$ **(ii)** $\frac{1}{4}$ **(iii)** $\frac{5}{9}$ **(iv)** $\frac{2}{5}$ **(v)** $\frac{2}{9}$ **]**

29. **30** *in every* **48** *people on the street are employed. The chance that the person is a university graduate given that is employed is* **0·6.** *Find the:*

(i) *probability that a person picked at random from the street is a university graduate and employed.*

(ii) *number of people that are non graduates and are employed from a group of* **120** *people.*

[Ans: (i) 0·375 (ii) 30]

30. *Events* A *and* B *are independent and the events* A *and* C *are mutually exclusive. Given that* $P(A) = 0{\cdot}4$, $P(B) = 0{\cdot}2$, $P(C) = 0{\cdot}3$ *and* $P(C \cap B) = 0{\cdot}1$,

(a) *draw a Venn diagram to illustrate the events* A, B *and* C *and the probabilities for each region*

(b) *find:*

(i) $P(A \cup B)$ **(ii)** $P(C/B)$ **(iii)** $P(B/A \cup C)$

(iv) *the probability that only one of the events* B, C *will occur*

[Ans: (i) 0·52 (ii) 0·5 (iii) 0·2571 (iv) 0·22]

31. *In a group of* ***20*** *adults,* ***4*** *of the* ***7*** *women and* ***2*** *of the* ***13*** *men wear glasses.*

Find the probability that a randomly selected person will be:

(i) *a woman or someone who wears glasses*

(ii) *a man given that the person does not wear glasses* ***[Ans: (i)*** $\frac{9}{20}$ ***(ii)*** $\frac{11}{14}$***]***

32. *From a medical study of* ***400*** *patients, it was found that* ***250*** *were smokers,* ***75*** *died from lung cancer and of these* ***50*** *were smokers. Find the probability of:*

(i) *dying from lung cancer given that the person smokes*

(ii) *dying from lung cancer given that the person does not smoke*

[Ans: (i) $\frac{1}{5}$ ***(ii)*** $\frac{1}{6}$ ***]***

33. *The table below shows information about* ***200*** *adults and their marital status:*

	Married	***Single***	***Total***
Male	***56***	***40***	***96***
Female	***72***	***32***	***104***
Total	***128***	***72***	***200***

(a) *Find the probability that a randomly selected person will be:*

(i) *a female given that the person is married*

(ii) *single given that the person is a male*

(iii) *married given that the person is a female*

(b) *Given that* ***75%*** *of the married own cars and* ***60%*** *is for the single adults, find the probability that a randomly selected person:*

(i) *owns a car*

(ii) *is married given that the person owns a car*

[Ans: (a) (i) $\frac{9}{16}$ ***(ii)*** $\frac{5}{12}$ ***(iii)*** $\frac{9}{13}$ ***(b) (i)*** $\frac{87}{125}$ ***(ii)*** $\frac{20}{29}$***]***

34. *Events* ***A*** *and* ***B*** *are such that* $\mathbf{P}(A/B) = \frac{1}{2}$, $P(B/A) = \frac{3}{4}$, $P(B) = \frac{4}{5}$,

$P(C) = \frac{2}{3}$ *and* $P(C/A \cap B) = \frac{1}{3}$. *Find:*

(i) P(A n B) ***(ii) P(A)*** ***(iii) P(A n B n C)*** ***(iv)*** $P(A \cap B/C)$

[Ans: (i) $\frac{2}{5}$ ***(ii)*** $\frac{8}{15}$ ***(iii)*** $\frac{2}{15}$ ***(iv)*** $\frac{1}{5}$ ***]***

35. *The table below shows the likely hood of where* ***A*** *and* ***B*** *spend their leisure time.*

	A	***B***
Goes to dance	$\frac{1}{2}$	$\frac{2}{3}$
Visits a neighbor	$\frac{1}{3}$	$\frac{1}{6}$
Stays at home	$\frac{1}{6}$	$\frac{1}{6}$

(i) *Find the probability that both go out.*

(ii) *If they both go out, find the probability that both went to dance.*

[Ans: (i) $\frac{25}{36}$ ***(ii)*** $\frac{12}{25}$ ***]***

36. Ben's *chances of passing* ***Arabic, French*** *and* ***History*** *are* ***0·7, 0·8*** *and* ***0·6*** *respectively. Find the probability that he failed:*

(i) *only one subject*

(ii) *history given that at least two subjects were passed*

[Ans: (i) 0·452 (ii) 0·496]

37. *In a certain country**, 60%** of the cars are privately owned. Of the privately owned cars **70%** are small**,** whereas of the cars which are not privately owned **40%** are small. Independent of ownership **20%** of the small cars and **30%** of the large ones are new. If a car is chosen at random**,** calculate the probability that the car is**:***

(i) *small*

(ii) *privately owned given that it is large*

(iii) *privately owned, large and old*

(iv) *large, given that it is privately owned and old*

[Ans: (i) 0·58 (ii) 0·4286 (iii) 0·126 (iv) 0·2727]

38. *Two teams **P** and **Q** play a foot ball match against each other and the number of goals each scores in a match are distributed as follows.*

No of Goals	***Probability***	
	P	***Q***
0	***0·35***	***0·2***
1	***0·25***	***0·5***
2	***0·4***	***0·3***

Calculate the probability of:

(i) P *winning*

(ii) Q *winning*

(iii) *both teams scoring two goals, given that they will make a draw.*

(b) *If a tournament of three matches is played, calculate the probability of **P** winning the tournament.*

[Ans: (i) 0·33 (ii) 0·355 (iii) 0·38095 (iv) 0·35306]

CONDITIONAL PROBABILITIES AND INDEPENDENT PRACTICE

1. *A box contains **5** red and **3** blue pens. Two pens are drawn in succession at random from the box with replacement. Find the probability of picking:*

(i) *pens of the same colour*

(ii) *exactly one red pen*

(iii) *at least one red pen*

(iv) *at most one blue pen*

2. *A box contains **8** red and **3** blue pens. Two pens are drawn in succession at random from the box without replacement. Find the probability of picking:*

(i) *pens of different colours*

(ii) *exactly one red pen*

(iii) *at least one red pen*

(iv) *at most one blue pen*

3. *Bag **P** contains **4** red and **3** white beads, while bag **Q** contains **3** red and **2** white beads. A bag is selected at random and two beads are drawn from it without replacement. Find the probability that:*

(i) *they are of different colours*

(ii) *bag **Q** is selected given that the beads drawn are of the same colour .*

4. *Three bags contain coloured balls as follows**:***

*Bag **X** has **4** red and **3** white balls*

*Bag **Y** has **3** red and **4** white balls.*

*Bag **Z** has **5** red and **2** white balls.*

*A bag is selected at random and two balls are drawn from it without replacement. Find the probability that**:***

(i) *at least one red ball is drawn.*

(ii) *the second ball drawn is white*

(iii) *the balls drawn are of different colours*

(iv) *bag **Y** is selected given that the balls drawn are of the same colour*

5. *Box **X** contains **4 red** and **3 green** sweets. Box **Y** contains **5 red** and **6 green** sweets. Box **X** is twice as likely to be picked as box **Y.** If a box is chosen at random and two sweets are removed from it, one at a time without replacement, Find the probability that**:***

(i) *the sweets removed are of the same colour*

(ii) *box **Y** is selected given that the sweets removed are of the same colour.*

6. *Box **P** contains **4** red and **3** blue pens, box **Q** contains **3** red and **4** blue pens, while box **R** contains **5** red and **2** blue pens. Box **P** is twice as likely to be picked as box **Q,** also it is thrice as likely to pick box **P** as box **R.** If a box is selected at random and two pens are picked from it without replacement, find the probability of picking**:***

(i) *pens of different colours*

(ii) *box **Q** given that the pens drawn are of the same colour*

Hint:

A: B *= 2:1 =* $(2\times3):(1\times3) =$ ***6 : 3*** ***A: B*** *= 3:1 =* $(3\times2):(1\times2) =$ ***6 : 2***

If ***A:B:C =6:3:2,*** *then* $P(A) = \frac{6}{11}$, $P(B) = \frac{3}{11}$ *and* $P(C) = \frac{2}{11}$

7. *A box contains* ***2*** *green and* ***5*** *blue balls. A second box contains* ***5*** *green and* ***3*** *blue balls. One ball is drawn at random from the second box and placed into the first box. Find the probability that the ball now drawn at random from the first box is green*

8. *Box* ***P*** *contains* ***3*** *white and* ***4*** *blue balls while box* ***Q*** *contains* ***5*** *white and* ***3*** *blue balls. A ball is drawn at random from* ***P*** *and put into* ***Q,*** *and then a ball is taken from* ***Q*** *and put into* ***P.*** *Find the probability that****:***

(i) *each box remains as it was initially*

(ii) *box* ***P*** *now contains* ***2*** *white and* ***5*** *blue balls*

(iii) *the ball drawn now from box* ***P*** *is white****.***

9. *A box contains* ***3 red*** *and* ***5 white*** *balls. When a red ball is picked from the box****,*** *it is returned otherwise it is not returned****.*** *If two balls are picked in succession at random from the box****,*** *find the probability of picking at least one red ball.*

10. *A box contains* ***2 red*** *and* ***3 white*** *beads. When a bead is drawn from the box****,*** *it is returned together with the bead of the other colour****.*** *If two such random draws are made****,*** *find the probability that****:***

(i) *the beads drawn are of the same colour*

(ii) *the box now contains* ***3 red*** *and* ***4 white*** *beads.*

11. *In a group of* ***12*** *international judges there are* ***3*** *from Africa,* ***4*** *from Asia and* ***5*** *from Europe. Three judges are picked at random from the group without replacement to preside over an international criminal case. Find the probability of picking:*

(i) *a judge from each continent*

(ii) *two judges from Asia*

(iii) *three judges from the same continent*

12. *A committee of* ***5*** *members is chosen at random from a group of* ***6*** *boys and* ***8*** *girls. Find the probability that the committee contains:*

(i) *exactly* ***2*** *boys.*

(ii) *more boys than girls.*

13. *A bookshelf has* ***5*** *History,* ***4*** *French and* ***3*** *Biology books. Four books are drawn at random from the shelf without replacement. Find the probability that of the drawn books, two are of History, one is of French and the other is of Biology.*

EER:

1. *A box contains **3** red, **4** green, and **5** blue beads. Two beads are selected at random from the box without replacement . Find the probability that**:***

(i) *the beads are of the same colour*

(ii) *at least one red bead is selected*

$$[\textbf{Ans: (i) } \frac{19}{66} \quad \textbf{(ii) } \frac{5}{11} \ ]$$

2. *A box contains two types of balls **red** and **black**. When a ball is picked from the box, the probability that it is red is $\frac{7}{12}$. Two balls are selected at random from the box without replacement. Find the probability that**:***

(i) *the second ball is black*

(ii) *the first ball is red given that the second one is black*

$$[\textbf{Ans: (i) } \frac{5}{12} \quad \textbf{(ii) } \frac{7}{11} \ ]$$

3. *A bag contains **5** white, **3** red and **2** green counters. **3** counters are drawn without replacement. Find the probability that there:*

(i) *is no green counter*

(ii) *are **2** white counters and a green counter*

$$[\textbf{Ans: (i) } \frac{7}{15} \quad \textbf{(ii) } \frac{1}{6} \ ]$$

4. *A committee of five members is chosen at random from a group of **8** men and **4** women. Find the probability that the committee contains exactly three men.*

$$[\textbf{Ans: } \frac{14}{33} \ ]$$

5. *A box contains **2** green and **5** blue balls. A second box contains **6** green and **4** blue balls. One ball is drawn at random from the second box and placed into the first box. Find the probability that the ball drawn now at random from the first box is green*

$$[\textbf{Ans: } \frac{27}{40} \ ]$$

6. *Bag **X** contains **5** blue and **4** red beads, bag **Y** contains **7** blue and **5** red beads, while bag **Z** contains **3** blue and **5** red beads. A bag is selected at random and a bead is drawn from it. Find the probability of picking:*

(i) *a red bead.*

(ii) *a bead from bag **Y,** given that it is blue.*

$$[\textbf{Ans: (i) } \frac{107}{216} \quad \textbf{(ii) } \frac{42}{109} \ ]$$

7. *Box **P** contains **5** white and **3** blue balls, while box **Q** contains **3** white and **5** blue balls. A ball is drawn at random from **P** and put into **Q,** and then a ball is taken from **Q** and put into **P.** Find the probability that:*

(i) *each box remains as it was initially*

(ii) *each box now contains **4** white and **4** blue balls*

(iii) *the ball drawn now from box **P** is white.*

$$[\textbf{Ans: (i) } \frac{19}{36} \quad \textbf{(ii) } \frac{25}{72} \quad \textbf{(iii) } \frac{43}{72} \ ]$$

8. *Bag* ***X*** *contains* ***5*** *red and* ***4*** *white beads, while bag* ***Y*** *contains* ***7*** *red and* ***5*** *white beads. A bag is selected at random and a bead is picked from it. Find the probability of picking:*

(i) *a red bead.*

(ii) *a bead from bag* ***Y,*** *given that it is white.*

[Ans: (i) $\frac{41}{72}$ ***(ii)*** $\frac{15}{31}$ ***]***

9. *Bag* ***X*** *contains* ***5*** *red and* ***4*** *white beads, bag* ***Y*** *contains* ***7*** *red and* ***5*** *white beads, while bag* ***Z*** *contains* ***3*** *red and* ***5*** *white beads. A bag is selected at random and two beads are picked from it without replacement. Find the probability that:*

(i) *they are of different colours*

(ii) *bag* ***Y*** *is selected given that the beads drawn are of the same colour*

[Ans: (i) 0·54052 (ii) 0·34075]

10. *Boxes* ***A, B*** *and* ***C*** *contain coloured pens as follows:*

Boxes	***A***	***B***	***C***
Red Pens	***4***	***3***	***5***
Blue pens	***3***	***4***	***2***

Box ***A*** *is twice as likely to be picked as box* ***B,*** *while* ***A*** *and* ***C*** *are equally likely to be picked. If a box is chosen at random and two pens are drawn from it without replacement, find the probability of picking:*

(i) *pens of different colours*

(ii) *box* ***B*** *given that the pens drawn are of the same colour*

[Ans: (i) $\frac{8}{15}$ ***(ii)*** $\frac{9}{49}$ ***]***

11. *A box contains* ***20*** *good and* ***4*** *bad oranges. If* ***5*** *oranges are picked at random without replacement, determine the probability that* ***4*** *are good and the other is bad.*

[Ans: 0·45596]

12. *Three balls are drawn at random one after the other without replacement from a bag containing* ***4*** *white,* ***8*** *blue ,* ***5*** *red and* ***3*** *pink balls. Find the probability that the first ball is blue, the second red or blue and the third is white.*

[Ans: $\frac{16}{285}$ ***]***

13. *A box contains* ***5*** *red and* ***4*** *blue pens. Three pens are drawn in succession at random from it without replacement. Find the probability of picking:*

(i) *pens of the same colour*

(ii) *exactly* ***3*** *red pens*

(iii) ***2*** *red pens and* ***1*** *blue pen*

(iv) *more than* ***1*** *blue pen*

(v) *at most* ***2*** *blue pens*

(vi) *the first and last with the same colour*

[Ans: (i) $\frac{1}{6}$ ***(ii)*** $\frac{5}{42}$ ***(iii)*** $\frac{10}{21}$ ***(iv)*** $\frac{17}{42}$ ***(v)*** $\frac{20}{21}$ ***(vi)*** $\frac{4}{9}$ ***]***

14. *A box contains* ***5*** *red and* ***3*** *blue balls. Two balls are drawn in succession at random from it without replacement. Find the probability that:*

(i) *they are of the same colour*

(ii) *the second ball drawn is red.*

[Ans: (i) $\frac{13}{28}$ ***(ii)*** $\frac{5}{8}$ ***]***

15. *A Box contains* ***4*** *red and* ***3*** *blue balls, while a second box contains* ***3*** *red and* ***4*** *blue balls. A ball is drawn at random from the first box and put into the second box, and then a ball is taken from the second box and put into the first box. Find the probability that the ball drawn now from the first box is red.*

$$[\text{Ans: } \frac{31}{56} \quad]$$

16. *Bag* ***P*** *contains* ***3*** *red and* ***2*** *white beads, while bag* ***Q*** *contains* ***2*** *red and* ***3*** *white beads. A bag is selected at random and two beads are drawn from it without replacement. Find the probability that they are of different colours.*

$$[\text{Ans: } \frac{3}{5} \quad]$$

17. *Box* ***P*** *contains* ***6*** *red and* ***4*** *white cards, while box* ***Q*** *contains* ***2*** *red and* ***3*** *white cards. A box is selected at random and a card is picked from it. Find the probability of picking:*

(i) *a white card.*

(ii) *a card from box* ***Q,*** *given that it is white.*

$$[\text{Ans: (i) } \frac{1}{2} \quad \text{(ii) } \frac{2}{5} \quad]$$

18. *A bag contains* ***3*** *white and* ***5*** *black balls. If two balls are picked in succession at random from it, find the probability that the second ball picked is white if picking is done:*

(i) *with replacement.*

(ii) *without replacement.*

$$[\text{Ans: (i) } \frac{3}{8} \quad \text{(ii) } \frac{3}{8} \ ]$$

19. *A box contains* ***4*** *red and* ***6*** *blue beads. Four beads are drawn at random from it with replacement. Find the probability that****:***

(i) *they are of the same colour*

(ii) *two of the drawn beads are of the same colour*

[Ans: (i) $\frac{97}{625}$ ***(ii)*** $\frac{216}{625}$ ***]***

20. *A box contains* ***1 red*** *bead and* ***1 white*** *bead. When a bead is drawn from the box****,*** *it is returned together with the bead of the other colour****.*** *If three such random draws are made****,*** *find the probability that****:***

(a) *the second and third beads drawn are of the same colour*

(b) *one of the drawn beads is white*

If the event in ***(a)*** *above is* ***X*** *and the event in* ***(b)*** *is* ***Y,*** *find****:***

(i) $P(X/Y)$

(ii) $P(Y/X)$

[Ans: (a) $\frac{5}{12}$ ***(b)*** $\frac{11}{24}$ ***(i)*** $\frac{4}{11}$ ***(ii)*** $\frac{2}{5}$ ***]***

21. *There are* ***3*** *black and* ***2*** *white balls in each of two bags. A ball is taken from the first bag and put in the second, then a ball is taken from the second to the first, find the probability that each bag remains as it was initially*

[Ans: $\frac{3}{5}$ ***]***

22. *A bag contains* ***30*** *white,* ***20*** *blue and* ***20*** *red balls. Three balls are drawn in succession at random without replacement. Find the probability that the first ball and third ball is white.*

[Ans: $\frac{29}{161}$ ***]***

23. *Box* ***P*** *contains* ***3*** *red and* ***4*** *blue balls, while box* ***Q*** *contains* ***2*** *red and* ***5*** *blue balls. A ball is drawn at random from* ***P*** *and put into* ***Q.*** *Find the probability that the ball drawn now from box* ***Q*** *is red.*

[Ans: $\frac{17}{56}$ ***]***

24. *Box* ***P*** *contains* ***5*** *red and* ***3*** *blue balls, while box* ***Q*** *contains* ***4*** *red and* ***4*** *blue balls. Two balls are drawn in succession at random from* ***P*** *and put into* ***Q.***

Find the probability that:

(i) *the second box now contains* ***5*** *red and* ***5*** *blue balls*

(ii) *the ball drawn now from box* ***Q*** *is red.*

[Ans: (i) $\frac{15}{28}$ ***(ii)*** $\frac{21}{40}$ ***]***

25. *A box contains* ***4*** *red and* ***5*** *blue pens. Three pens are picked at random from it without replacement . Find the probability of picking:*

(i) *pens of the same colour*

(ii) *exactly one red pen*

(iii) *at least two red pens*

(iv) *at most two red pens*

[Ans: (i) $\frac{1}{6}$ ***(ii)*** $\frac{10}{21}$ ***(iii)*** $\frac{17}{42}$ ***(iv)*** $\frac{20}{21}$ ***]***

26. *Box* ***Q*** *contains* ***4*** *red and* ***3*** *white beads, while box* ***R*** *contains* ***3*** *red and* ***2*** *white beads. A thrown biased coin is twice as likely to show heads as tails. Box* ***Q*** *is selected when a head shows up otherwise it is box* ***R.*** *Two beads are then drawn at random from that box without replacement. Find the probability that:*

(i) *the beads drawn are of different colours.*

(ii) *box* ***R*** *is selected given that the beads drawn are of the same colour.*

$$\textbf{[Ans: (i) } \frac{61}{105} \quad \textbf{(ii) } \frac{7}{22} \textbf{]}$$

27. *A box contains* ***1 red*** *bead and* ***2 white*** *beads. When a bead is drawn from the box, it is returned together with the bead of the other colour. If two such random draws are made, find the probability that:*

(i) *the beads drawn are of the same colour*

(ii) *the box now contains* ***2*** *red and* ***3*** *white beads.*

$$\textbf{[Ans: (i) } \frac{5}{12} \quad \textbf{(ii) } \frac{7}{12} \textbf{]}$$

28. *A committee of* ***5*** *members is chosen at random from a group of* ***6*** *boys and* ***7*** *girls. Find the probability that the committee contains:*

(i) *exactly* ***2*** *boys.*

(ii) *more boys than girls.*

(iii) *members of the same gender*

$$\textbf{[Ans: (i) } \frac{175}{429} \quad \textbf{(ii) } \frac{59}{143} \quad \textbf{(iii) } \frac{3}{143} \textbf{]}$$

29. *A bag contains* ***6*** *white and* ***4*** *black balls. Three balls are drawn in succession at random without replacement . Find the probability of picking:*

(i) *only white balls*

(ii) *two black balls*

(iii) *a white ball first then the last two constitute a white and black ball.*

[Ans: (i) $\frac{1}{6}$ ***(ii)*** $\frac{3}{10}$ ***(iii)*** $\frac{1}{3}$ ***]***

30. *A box contains* ***3*** *red,* ***2*** *green, and* ***5*** *blue beads. Two beads are selected at random from the box without replacement . Find the probability that:*

(i) *the beads are of the same colour*

(ii) *at least one red bead is selected* ***[Ans: (i)*** $\frac{14}{45}$ ***(ii)*** $\frac{8}{15}$ ***]***

31. *A box contains* ***5*** *red,* ***3*** *green, and* ***2*** *blue beads. Four beads are drawn in succession at random from it without replacement . Find the probability that at least one bead of each colour is drawn* ***[Ans:*** $\frac{1}{2}$ ***]***

32. *A box contains* ***4*** *red and* ***6*** *blue pens. Four pens are drawn in succession at random from it without replacement . Find the probability that:*

(i) *all the pens drawn are blue*

(ii) *at least one pen of each colour is drawn*

(iii) *at least two blue pens are drawn*

(iv) *at least two blue pens are drawn, given that at least one pen of each colour is drawn*

[Ans: (i) $\frac{1}{14}$ ***(ii)*** $\frac{97}{105}$ ***(iii)*** $\frac{37}{42}$ ***(iv)*** $\frac{85}{97}$ ***]***