UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

S475/1 SUBSIDIARY MATHEMATICS

SCORING GUIDE

FOR THE SAMPLE PAPER

In response to the tasks, the candidate is expected to: correctly identify the relevant concepts to manipulate in response to the task(s) in the scenario, select and apply (a)correct method(s) to correctly manipulate the concepts identified, and correctly interpret accurate outputs from correctly identified and manipulated concepts to justify/inform decision.

$$SECTION\ A$$

ITEM 1

Task

$$Points: B(-70, 30), \quad A(90, 30), \quad S(0, 0)$$

$$Let\ \beta\ be\ the\ angle\ between\ \overrightarrow{SB}\ and\ \overrightarrow{SA}.$$

$$\overrightarrow{SA} = \begin{pmatrix} 90 \\ 30 \end{pmatrix}$$

$$\overrightarrow{SB} = \begin{pmatrix} -70 \\ 60 \end{pmatrix}$$

$$\overrightarrow{SA} \cdot \overrightarrow{SB} = \begin{pmatrix} 90 \\ 30 \end{pmatrix} \cdot \begin{pmatrix} -70 \\ 60 \end{pmatrix}$$

$$= -6300 + 1800$$

$$= -4500$$

$$-4500 = (\sqrt{\{90^2 + 30^2\}} \cdot \sqrt{\{(-70)^2 + 60^2\}})cos\beta$$

$$-4500 = (\sqrt{\{9000\}} \cdot \sqrt{\{8500\}})cos\beta$$

$$-4500 = 8746.4278\, cos\beta$$

$$cos\ \beta = -0.5145$$

$$\beta = 120.96°$$

$$\approx 121.0^o$$

$$Therefore\ a\ third\ speaker\ is\ needed\ since\ the\ angle\ between\ the\ sound\ projections\ \overrightarrow{SA}\ and\ \overrightarrow{SB}\ is\ 121.0^o$$

$$which\ is\ more\ than\ 90^o$$

$$2sin^2\theta + cos\theta = 1$$

$$Using\ \{sin^2\theta = 1 - cos^2\theta\}\ gives\ \ 2(1 - cos^2\theta) + cos\theta = 1$$

$$2 - 2cos^2\theta + cos\theta = 1$$

$$2cos^2\theta - cos\theta - 1 = 0$$

$$Let\ a = 2, \quad b = -1, \quad c = -1.$$

$$Using\ the\ quadratic\ formula:$$

$$cos\ \theta = \frac{\left\{-b \pm \sqrt{\{b^2 - 4ac\}}\right\}}{\{2a\}}$$

$$cos\ \theta = \frac{\left\{-(-1) \pm \sqrt{\{(-1)^2 - 4 \times 2 \times (-1)\}}\right\}}{\{2 \times 2\}}$$

$$\cos\theta = \frac{\left\{1 \pm \sqrt{\{9\}}\right\}}{\{4\}}$$

$$= \frac{\{1 \pm 3\}}{\{4\}}$$

$$Either\ \ cos\ \theta = 1\ or\ cos\ \theta = -\frac{1}{2}$$

$$\theta = 0° \quad or \quad \theta = 120^o$$

$$For\ quality\ sound, the\ microphone\ should\ be\ projected\ at\ 0°\ to\ the\ horizontal.$$

$$\boldsymbol{ITEM\ 2}:$$

$$\boldsymbol{TASK\ (a)}$$

$$T(x) = 200000 - 1500x^2$$

$$When\ T = 0:$$

$$0 = 200000 - 1500x^2$$

$$1500x^2 = 200000$$

$$x^2 = \frac{200000}{1500} = 133.33$$

$$x = \pm 11.547$$

$$Taking\ the\ positive\ value \therefore x = 11.5$$

$$x\ intercepts: (-11.5, 0), (11.5, 0).$$

$$At\ x = 0:$$

$$T(0) = 200000 - 1500(0)^2 = 200000$$

$$y\ intercept\ : (0, 200000).$$

$$\{Determining\ the\ nature\ of\ the\ Turning\ Point\}$$

$$T(x) = 200000 - 1500x^2$$

$$\frac{dT}{dx} = -3000x$$

$$At\ the\ turning\ point, \frac{dT}{dx} = 0$$

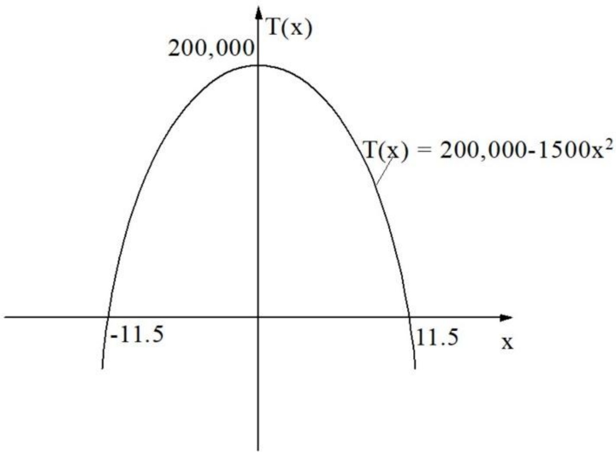
$$-3000x = 0$$

$$implies\ x = 0$$

$$T(0) = 200000 - 1500(0)^2 = 200000$$

$$Turning\ point: (0, 200000).$$

$$\frac{d^2T}{dx^2} = -3000 < 0, it\ implies\ a\ maximum\ turning\ point.$$



$Sketch\ of\ parabola\ with\ vertex\ at\ (0, 200000), x - intercepts\ at\ x = \pm 11.5.$

$\boldsymbol{Conclusion: Initially, the\ forest\ had\ 200,000\ trees.}$

$\boldsymbol{If\ no\ intervention, the\ forest\ will\ take\ 11.5\ years\ for\ complete\ destruction.}$

$$\boldsymbol{TASK\ (b)}$$

$Let\ A\ represent\ the\ part\ of\ the\ forest\ burnt, and\ t\ represent\ time\ taken\ for\ the\ fire\ to\ consume\ the\ fores$

$$\frac{dA}{dt} \propto A$$

$$implies\ \frac{dA}{dt} = kA$$

$$but\ k = 0.1, \qquad \frac{dA}{dt} = 0.1A$$

$$Separating\ variables:$$

$$\int \frac{1}{A} dA = \int 0.1dt$$

$$ln(A) = 0.1t + C$$

$$At\ t = 0, \qquad A = 100:$$

$$Thus:$$

$$ln(100) = 0.1(0) + C$$

$$C = ln(100)$$

$$ln(A) = 0.1t + ln(100)$$

$$After\ fire\ fighters\ intervention, t = 5,$$

$$ln(A) = 0.1(5) + ln(100)$$

$$ln\ A - ln(100) = 0.1(5)$$

$$\ln(\frac{A}{100}) = 0.5$$

$$(\frac{A}{100}) = e^{0.5}$$

$$A = 100e^{\{0.5\}}$$

$$= 100 \times 1.6487$$

$$= 164.87$$

$$\therefore \mathbf{164.87\ hectares\ burnt\ after\ the\ fire\ fighters'intervention}.$$

$$\boldsymbol{SECTION\ B}$$

$$\boldsymbol{PART\ 1}$$

$$\boldsymbol{ITEM\ 3}$$

$$\boldsymbol{Task\ (a)(i)}$$

$$Day\ 1 = 6\ trays$$

$$Day\ 2 = 9\ trays$$

$$Day\ 3 = 12\ trays$$

$$Pattern: 6, 9, 12, \ldots$$

$$This\ is\ an\ Arithmetic\ Progression\ with\ a = 6, \quad d = 3.$$

$$Sum\ of\ trays\ of\ bread\ produced\ in\ the\ first\ 10\ days:$$

$$S_{\{10\}} = \frac{10}{2}[2(6) + (10-1)3]$$

$$S_{\{10\}} = 5[12 + 27]$$

$$= 5 \times 39$$

$$= 195$$

$$Following\ the\ pattern, the\ manager\ will\ produce\ only\ 195\ trays\ in\ 10\ days, which\ is$$

$$less\ than\ 690\ trays\ required.$$

$$Conclusion: Therefore\ the\ manager\ will\ not\ be\ able\ to\ fulfill\ the\ contract.$$

$$\boldsymbol{Task}\ (\boldsymbol{a})(\boldsymbol{ii})$$

$$Given\ a = 6, \quad n = 10, \quad S_{\{10\}} = 690, \quad find\ d:$$

$$690 = \frac{10}{2}[2(6) + (10 - 1)d]$$

$$690 = 5(12 + 9d)$$

$$138 = 12 + 9d$$

$$9d = 126$$

$$d = 14$$

$$The\ manager\ needs\ to\ increase\ production\ by\ 14\ trays\ every\ subsequent\ day\ to\ fulfill\ the\ contract.$$

$$\boldsymbol{Task}\ (\boldsymbol{b})$$

$$Finding\ different\ combinations\ of\ the\ 6\ types\ taking\ 3\ at\ a\ time:$$

$$^{6}C_{3} = \frac{\{6!\}}{\{3!(6-3)!\}}$$

$$= \frac{\{6 \times 5 \times 4\}}{\{3 \times 2 \times 1\}}$$

$$= 20$$

$$Therefore\ the\ shop\ keeper\ has\ 20\ possible\ selections\ of\ the\ bread\ type.$$

***ITEM* 4**

Task

$$Given: n = 10, \quad r = 2, \quad a = 1000$$

$$Using\ sum\ of\ geometric\ series:$$

$$S_n = a\frac{\{r^n - 1\}}{\{r - 1\}}$$

$$S_{\{10\}} = 1000\frac{\{2^{\{10\}} - 1\}}{\{2 - 1\}}$$

$$= 1000(1024 - 1)$$

$$= 1{,}023{,}000$$

$$Using\ the\ well-wishers\ formula:$$

$$A_n = 5000 \times 3^n$$

$$But\ A_n = \frac{1}{2}S_{\{10\}}$$

$$\Rightarrow \frac{1}{2}S_{\{10\}} = 5000 \times 3^n$$

$$\frac{1}{2}(1{,}023{,}000) = [5000 \times 3^n]$$

$$511{,}500 = [5000 \times 3^n]$$

$$\frac{511{,}500}{5000} = 3^n$$

$$102.3 = 3^n$$

$$n = \frac{log102.3}{log3}$$

$$n = 4.212$$

Since n = 4.212, they need to save for 5 months if they are to follow the well-wisher's suggestion.

Therefore, they will have their charity in July since they would have started saving in February.

$$\boldsymbol{Number\ of\ Committees\ with\ at\ most\ 2\ men}$$

$Given\ 5\ women\ (W) and\ 3\ men\ (M)$. At most 2 men $means\ 0, 1, or\ 2\ men.$

No. of committees = ${}^5C_5 \times {}^3C_1 + {}^5C_4 \times {}^3C_2$

$$No. of\ committees = 3 + 5 \times 3$$

$$= 3 + 15$$

$$= 18$$

$There\ are\ 18\ ways\ of\ selecting\ the\ organizing\ committee.$

PART II

ITEM 5

Task (a)

$$f(t) = \begin{cases} K(2t - t^2), & 0 \le t \le 2 \\ 0, & otherwise \end{cases}$$

$$To\ find\ K, \quad \int_{\{0\}}^{\{2\}} f(t)\,dt = 1:$$

$$K \int_{\{0\}}^{\{2\}} (2t - t^2)\,dt = 1$$

$$K\left[t^2 - \frac{t^3}{3}\right]_0^2 = 1$$

$$K[(4 - \frac{8}{3}) - 0] = 1$$

$$K\left(\frac{4}{3}\right) = 1$$

$$\Rightarrow K = \frac{3}{4}$$

$$Thus, f(t) = \begin{cases} \frac{3}{4}(2t - t^2), & 0 \le t \le 2 \\ 0, & otherwise \end{cases}$$

$$P(1 \le T \le 2) = \int_{\{1\}}^{\{2\}} \frac{3}{4}(2t - t^2)\,dt$$

$$= \frac{3}{4}\left[t^2 - \frac{t^3}{3}\right]_1^2$$

$$= \frac{3}{4}[(4 - \frac{8}{3}) - (1 - \frac{1}{3})]$$

$$\frac{3}{4}[(\frac{4}{3}) - (\frac{2}{3})]$$

$$= \frac{1}{2}$$

There is a 50% likelihood that a randomly selected patient arrived between 9:00AM and 10:00 AM.

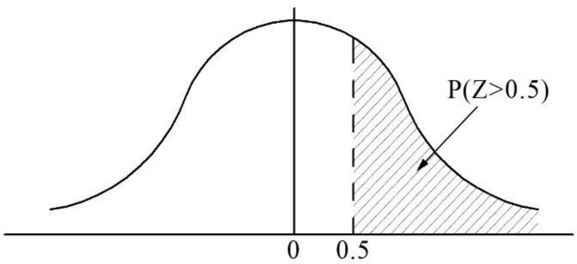
$$\boldsymbol{Task\ (b)}$$

$$Let\ X\ representNo.of\ patients\ arriving, X\ \sim N(\mu, \sigma^2)\ with\ \mu = 60, \qquad \sigma^2 = 100.$$

$$P(X > 65) = P\left(Z > \frac{\{65 - 60\}}{10}\right)$$

$$= P(Z > 0.5)$$

$$= 1 - \emptyset(0.5)$$



$$= 1 - 0.6915 = 0.3085$$

$The\ probability\ that\ the\ number\ of\ patients\ arriving\ on\ a\ given\ day\ exceeds$ 65 $available\ testing\ kits$ is 0.3085.

$$\boldsymbol{Task\ (c)}$$

$$Let\ M\ represent\ No.of\ days\ with\ shortage\ of\ kits. M\ \sim B(n,p)\ with\ n = 5, \quad p = 0.3085.$$

$$P(M \geq 2) = 1 - P(M < 2)$$

$$= 1 - [P(M = 0) + P(M = 1)]$$

$$= 1 - {}^5C_1(0.3085)^1(0.6915)^5 + {}^5C_0(0.3085)^0(0.6915)^5$$

$$= 1 - [0.1527 + 0.3627]$$

$$= 1 - 0.5158$$

$$= 0.4842$$

$$\boldsymbol{Since\ P(M \geq 2) = 0.4842\ is > 0.45, then\ more\ kits\ have\ to\ be\ availed\ by\ the\ officer}.$$

ITEM 6

(i)

Data for tests P and Q:

P	P^2	Q	Q^2
70	4900	68	4624
60	3600	58	3364
65	4225	65	4225
45	2025	50	2500
55	3025	50	2500
50	2500	48	2304
63	3969	62	3844
52	2704	54	2916
$\sum P = 460$	$\sum P^2 = 26948$	$\sum Q = 455$	$\sum Q^2 = 26277$

$$\bar{P} = \frac{460}{8} = 57.5, \qquad \bar{Q} = \frac{455}{8} = 56.875$$

Standard deviation for P:

$$\sigma_P = \sqrt{\left\{\frac{\sum P^2}{n} - (\bar{P})^2\right\}} = \sqrt{\left\{\frac{26948}{8} - (57.5)^2\right\}} = \sqrt{\{3368.5 - 3306.25\}} = \sqrt{\{62.25\}} = 7.8899$$

Standard deviation for Q:

$$\sigma_Q = \sqrt{\left\{\frac{\sum Q^2}{n} - (\bar{Q})^2\right\}} = \sqrt{\left\{\frac{26277}{8} - (56.875)^2\right\}} = \sqrt{\{3284.625 - 3234.7656\}} = \sqrt{\{49.8594\}}$$

$$= 7.0611$$

Students' performance was more consistent in Test Q than test P because the standard deviation for Q is less than that of P.

(ii)

Spearman Rank Correlation

P	R_P	Q	R_Q	$d = (R_P - R_Q)$	d^2
70	1	68	1	0	0
60	4	58	4	0	0
65	2	65	2	0	0
45	8	50	6.5	1.5	2.25
55	5	50	6.5	-1.5	2.25
50	7	48	8	-1	1
63	3	62	3	0	0
52	6	54	5	1	1
					$\sum d^2 = 6.5$

$$\rho = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6 \times 6.5}{8(64 - 1)}$$

$$= 1 - \frac{39}{504}$$

$$= 1 - 0.07738$$

$$= 0.9226$$

There is a high positive correlation between test P and test Q.

(iii)

$$P(bove\ 52\ in\ test\ Q) = \frac{5}{8} = \ 0.625$$

$$\therefore \boldsymbol{62.5\%\ of\ the\ learners\ scored\ above\ 52.}$$