That's an excellent question! 👍 This is actually the point where many students get confused.

The expression

$$a\cos\theta+b\sin\theta$$

did not come from nowhere. It comes from the **question itself**.

For example, an exam question might say:

Express

$$3\cos\theta+4\sin\theta$$

in the form $R\cos(\theta-\alpha)$.

Here,

- $a=3$
- $b=4$

So we write the general form as

$$a\cos\theta+b\sin\theta.$$

The letters a and b are just placeholders for **any numbers**.

Think of it like algebra

Suppose I teach you how to factor

$$ax+ay.$$

Later you see

$$
3x+3y.
$$

Here,

- $(a=3)$

Or you see

$$
7x+7y.
$$

Here,

- $(a=7)$

The formula uses letters so it works for **every** question.

The same idea applies here.

Example 1

Question:

$$
5\cos\theta+12\sin\theta.
$$

Compare with

$$
a\cos\theta+b\sin\theta.
$$

So,

[
a=5,\qquad b=12.
]

Example 2

Question:

[
8\cos x-6\sin x.
]

Compare with

[
a\cos x+b\sin x.
]

Therefore,

[
a=8,\qquad b=-6.
]

Notice that **(b) can be negative**.

But why do mathematicians write it this way?

Because they want **one method** that works for every expression.

Instead of proving separately for

- (3\cos\theta+4\sin\theta),
- (5\cos\theta+12\sin\theta),
- (8\cos\theta-6\sin\theta),

they prove it once for the general expression

$$
\boxed{a\cos\theta+b\sin\theta.}
$$

Then you simply replace a and b with the numbers in your question.

Where does $R\cos(\theta-\alpha)$ come from?

This is the clever part.

Mathematicians **assume** that the expression can be rewritten in the form

$$
R\cos(\theta-\alpha).
$$

Why?

Because it is much easier to work with a **single cosine** than with two separate terms.

Now expand it using the cosine subtraction identity:

$$
\cos(\theta-\alpha)=\cos\theta\cos\alpha+\sin\theta\sin\alpha.
$$

So,

$$
\mathbf{R\cos(\theta-\alpha)}
$$

$$
R\cos\theta\cos\alpha
+
R\sin\theta\sin\alpha.
$$

Rearrange it as

$$
(R\cos\alpha)\cos\theta
$$

$$
+
(R\sin\alpha)\sin\theta.
$$

Now compare this with

$$
a\cos\theta+b\sin\theta.
$$

Since the coefficients of the same trig functions must be equal, we get

$$
R\cos\alpha=a,
$$

and

$$
R\sin\alpha=b.
$$

These two equations lead to the familiar formulas:

$$
R=\sqrt{a^2+b^2},
$$

and

$$
\tan\alpha=\frac{b}{a}.
$$

The big picture

The expression

$$
\boxed{a\cos\theta+b\sin\theta}
$$

is simply a **general template**. It represents **any** expression that is made up of:

- a multiple of $\cos\theta$, and
- a multiple of $\sin\theta$.

The numbers a and b come from the specific question you're solving.

My question for you

Would you like me to **derive the entire formula** from scratch—starting with

$$
R\cos(\theta-\alpha)
$$

and ending with

$$
R=\sqrt{a^2+b^2}
\quad\text{and}\quad
\tan\alpha=\frac{b}{a},
$$

showing **every algebraic step**? That's the proof many UACE teachers expect students to understand.