

P425/2
PRINCIPAL
MATHEMATICS
Paper 2
2026
3 hours



UGANDA NATIONAL EXAMINATIONS BOARD
Uganda Advanced Certificate of Education

PRINCIPAL MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES

*This examination paper consists of **two** sections; **A** and **B**. It has **five** examination items.*

*Section **A** consists of **one compulsory** item.*

*Section **B** has two parts, **I** and **II** each with two optional items. Respond to only **two** items from Section **B**, choosing **one** item from **each** of the parts, **I** and **II**.*

*Respond to **three** items in all.*

*Any additional item(s) responded to will **not** be scored.*

Begin each item on a fresh page.

***All** necessary working **must** be shown clearly.*

Graph paper is provided.

Where necessary, use $g = 9.8\text{ms}^{-2}$.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A

The item in this section is compulsory.

Item 1

A District Headquarter does not have sufficient water supply and the District Council has resolved to obtain additional water from one of the rivers that is exactly 20m wide. The District Council has tasked the District Water Engineer to install a pump for pumping water from the river to the district headquarters. To do this, the engineer has modeled the river's depth $H(x)$ in metres using the function;

$H(x) = 12 - 0.12(x-10)^2$; $0 \leq x \leq 20$, where x is the distance from the river bank.

For the pump to work efficiently, it requires a minimum cross-sectional area of the river to be 145m^2 , and according to the engineer the model conforms to that area.

The engineer has also informed the council that when he takes measurements at exactly 5m intervals from the river bank and uses the **approximate** method, he can easily prove that his model satisfies the condition of the minimum cross-sectional area required to install the pump.

He can also prove that his model satisfies the condition of the minimum cross-sectional area required to install the pump using the **exact** method.

According to the engineer, the base of the water pump requires a depth $H(x) = 8\text{m}$ and the engineer has assured the council that using a distance of 4m from the river bank as an initial approximation, he can easily determine the least distance and hence the suitable position with the depth required for a suitable installation of the pump with a tolerance error not exceeding 0.1m, for the pump to be stable.

However, the district council needs proof that using the two methods, the model yields the minimum area required, to inform decision about installation of the pump. The council also needs to know exactly how far from the bank, the pump shall be installed.

Task:

- (a) Provide the proof needed by the council.
- (b) Investigate and Inform the council, the exact position from the river bank, to install the pump.

SECTION B

Answer only **two** items from this section, choosing **one** item from each of the parts I and II.

PART I.

Item 2

A teachers' association of a certain district conducted a physics examination. The association used a sample of 85 students to guide in the analysis of the students' performance.

The distribution of the scores is shown in the table below.

Scores (%)	No. of students
5- < 30	17
30- < 40	21
40- < 50	30
50- < 60	14
60- < 80	3

Basing on the association's standards, the performance is considered good when both the average mark and dominant mark are at least 45%, and the examination is considered reliable when the standard deviation is at most 10%.

The association would like to know if the performance was good or not and also establish the reliability of the examination.

In the whole district, 500 students sat for the physics examination and their scores were approximately normally distributed.

The officer in charge of physics is challenged with providing the analysis of all the results that include determining the:

- (i) Cut off marks for a distinction and a fail basing on the condition that the top 15% of the students score a distinction and bottom 20% fail.
- (ii) Number of students who passed the examination and those who got a distinction.
- (iii) Range within which the average mark of all the students who sat the examination lies basing on 95% degree of confidence.

Task:

- (a) Based on calculated results, report about;
 - (i) The performance of students in physics.
 - (ii) The reliability of the examination.
- (b) Address the challenges of the officer in charge of physics.

Item 3

A District Education Officer (DEO) is monitoring the utilisation of sciences laboratories in his district. On visiting one of the schools that has both O and A-Level students, he has found out that the probability a student utilised a Physics, Chemistry and Biology laboratory is 0.3, 0.5 and 0.2 respectively.

The DEO has also discovered that the probability that a student is in A-Level given that he/she utilised a Physics, Chemistry and Biology laboratory is 0.65, 0.85 and 0.5 respectively.

The DEO wants to make a recommendation to the district council for construction of new laboratories if the proportion of students who utilised the Chemistry laboratory given that they are A-Level students exceeds 0.5 and the expected number of laboratories utilised is 2.

Furthermore, the DEO has sampled 10 students from the school and would wish to establish the proportion of at least half of the number of students who utilised the Chemistry laboratory. If the proportion exceeds 70% then more equipment will be supplied.

Task:

Assist the DEO to;

- (a) Make an informed recommendation to the district council about construction of new laboratories.
- (b) Decide on the supply of equipment.

PART II.

Item 4

A construction company has been contracted to renovate a building in town.

To lift heavy crates to the service deck, the company's technical team has set up a pulley system, shown in **figure.1**, consisting of a light in-extensible cable attached to a ceiling beam.

The cable loops under a smooth movable pulley which holds a counter weight of 50kg and then runs over a smooth fixed pulley mounted on a service deck. A crate of 20kg to be moved to the deck is attached to the other end of the cable.

The cable sections remain in the vertical plane during operation.

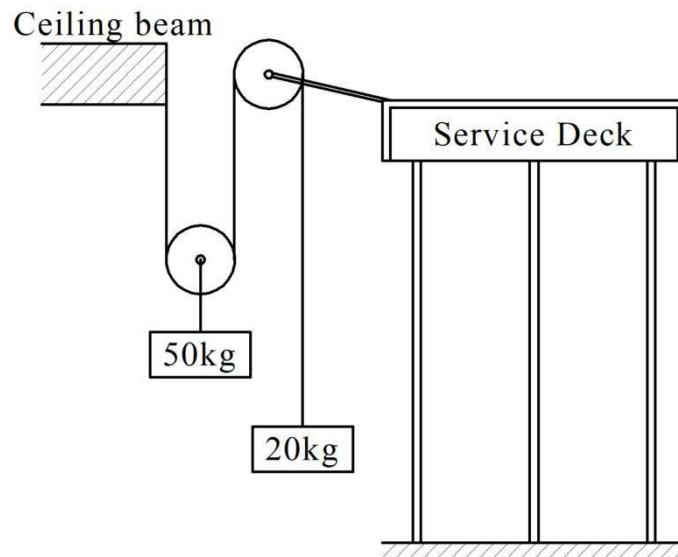


Fig.1

The system is currently held at rest but the engineer is about to release it.

For the safety of passersby, the engineer has set a safe ground zone of 5m from the service desk in case of any falling debris.

However, according to the engineering team the system must meet the following requirements before a system run is made:

- i) For safety and stability, the acceleration should not exceed 2ms^{-2} .
- ii) The cable is designed to handle a maximum tension of 300N as a safety rating.
- iii) If the system is released from rest, the team needs to know how far the movable pulley will travel after 5 seconds.

Immediately after setting the system, their tool box slides off the service deck horizontally at a speed of 2ms^{-1} , falling from a height of 20m above the ground thus creating fear among the engineering team about the safety of the passersby near the site.

Task:

- (a) Investigate whether the system will meet the requirements needed before the system run is made and provide an informed decision.
- (b) Assess the safety of the passersby near the site.

Item 5

During a leavers' party, members of the organising committee want to move a box of decorations and a container of soda along two levels of a storeyed building using a pulley system.

The box is of mass 300kg and rests on a rough horizontal level with coefficient of friction being 0.3 while the container is 500kg and sits on a rough plane inclined at 30° to the horizontal with coefficient of friction 0.1 as shown in **figure.2**.

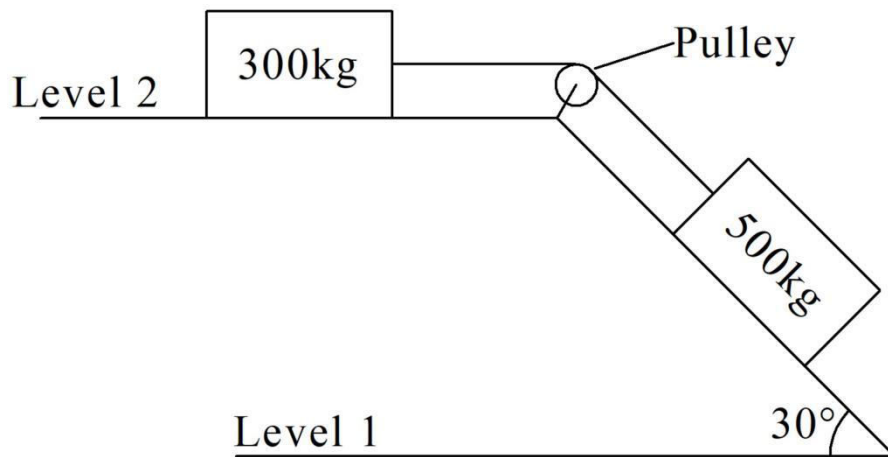
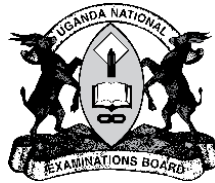


Fig.2

If the container is to be successfully lowered to level 1 without breaking the bottles of soda, its speed after 3s should not exceed 5ms^{-1} and the magnitude of the reaction at the pulley should be at most 700N.

Task:

Determine whether the container will be delivered safely when the system is released from rest.



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SCORING GUIDE FOR THE SAMPLE PAPER

In response to the tasks, the candidate is expected to: correctly identify the relevant concepts to manipulate in response to the task(s) in the scenario, select and apply (a) correct method(s) to correctly manipulate the concepts identified, and correctly interpret accurate outputs from correctly identified and manipulated concepts to justify/inform decision.

SECTION A

ITEM 1

Task(a)

Trapezium Rule Approximation

x		
0	0	
5		9
10		12
15		9
20	0	
SUM	0	30

$$\int_{\{0\}}^{\{20\}} [12 - 0.12(x - 10)^2] dx \approx \frac{5}{2} [0 + 2(30)]$$

$$\approx 150m^2$$

Exact Integration

$$\int_{\{0\}}^{\{20\}} (2.4x - 0.12x^2) dx = [1.2x^2 - 0.04x^3]_{\{0\}}^{\{20\}}$$

$$= (1.2(20)^2 - 0.04(20)^3) - 0$$

$$= 160 m^2$$

Conclusion: The cross-sectional area of the river satisfies the requirement for the pump to be installed since both $150m^2$ and $160 m^2$ are greater than the required minimum cross-sectional area of $145m^2$.

Task(b)*Newton – Raphson Method*

$$2.4x - 0.12x^2 = 8$$

$$f(x) = 0.12x^2 - 2.4x + 8$$

$$f'(x) = 0.24x - 2.4$$

$$x_{\{n+1\}} = x_n - \frac{\{0.12x_n^2 - 2.4x_n + 8\}}{\{0.24x_n - 2.4\}}$$

Iterations with $x_0 = 4, e = 0.1$

$$x_1 = 4 - \frac{\{0.12(4)^2 - 2.4(4) + 8\}}{\{0.24(4) - 2.4\}} = 4.2222$$

$$x_2 = 4.2222 - \frac{\{0.12(4.2222)^2 - 2.4(4.2222) + 8\}}{\{0.24(4.2222) - 2.4\}} = 4.2265$$

$$e = |4.2265 - 4.2222| = 0.0043 < 0.1$$

*Conclusion: The suitable position to install the pump from the river bank is 4.2265m.***SECTION B****PART I****ITEM 2****Task(a)(i)**

C.B	x	f	fx	fx ²	L	fd
5-30	17.5	17	297.5	5206.25	25	0.68
30-40	35	21	735	25725	10	2.1
40-50	45	30	1350	60750	10	3.0
50-60	55	14	770	42350	10	1.4
60-80	70	3	210	14700	20	0.15
		Σf=85	Σfx=3362.5	Σfx ² =148731.25		

Mean:

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{3362.5}{85} = 39.5588$$

Mode:

$$\text{Mode} = 40 + \left[\frac{\{30 - 21\}}{\{(30 - 21) + (30 - 14)\}} \right] \times 10 = 43.6$$

The performance is not good since the average and dominant mark are less than 45%.

Task(a)(ii)

Standard Deviation:

$$\begin{aligned}\sigma &= \sqrt{\left\{ \left(\frac{85}{85-1} \right) \left(\frac{\{148731.25\}}{\{85\}} - \left(\frac{\{3362.5\}}{\{85\}} \right)^2 \right) \right\}} \\ &= \sqrt{\left\{ \frac{85}{84} (1749.779 - 1565.99) \right\}} \\ &= 13.6777\end{aligned}$$

The examination is not reliable since $\sigma = 13.6777 > 10\%$ which is required.

Task (b)

Normal Distribution & Probability

Given:

$$\{x\} = 39.5588, \quad \sigma = 13.6777, \quad n = 500$$

Let $X \sim$ marks scored, $D \sim$ distinction mark and $F \sim$ fail mark

$$P(X > D) = 0.15$$

$$P\left(Z > \frac{D - 39.5588}{13.6777}\right) = 0.15$$

$$\frac{D - 39.5588}{13.6777} = 1.036$$

$$D = 53.7289$$

The cut-off for a distinction is 53.7%(1dp)

Fail Mark:

$$P(X < F) = 0.2$$

$$\Rightarrow P\left(Z < \frac{F - 39.5588}{13.6777}\right) = 0.2$$

$$\left(\frac{F - 39.5588}{13.6777}\right) = -0.842$$

$$F = 28.0422$$

The cut-off for a fail is 28.0%(1dp)

Number of Students that:

$$**Passed: (1 - 0.2) \times 500 = 400**$$

$$**Got a Distinction: 0.15 \times 500 = 75**$$

Confidence Interval (95%):

$$\alpha/2 = 0.475, \quad Z_{0.475} = 1.96$$

$$\mu = \bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$= 39.5588 \pm \frac{1.96 \times 13.6777}{\sqrt{500}}$$

$$\mu = [38.3599, \quad 40.7577]$$

The range within which the average mark of all the students that sat the examination lies basing on 95% degree of confidence is $38.3599 \leq \mu \leq 40.7577$.

ITEM 3

Probability Distribution

Given:

$$P(P) = 0.3, \quad P(\bar{P}) = 0.7, \quad P(A/P) = 0.65,$$

$$P(C) = 0.5, \quad P(\bar{C}) = 0.5, \quad P(A/C) = 0.85,$$

$$P(B) = 0.2, \quad P(\bar{B}) = 0.8, \quad P(A/B) = 0.5$$

Total Probability of A:

$$P(A) = P(P)P(A/P) + P(C)P(A/C) + P(B)P(A/B)$$

$$= 0.3(0.65) + 0.85(0.5) + 0.2(0.5)$$

$$= 0.72$$

Bayes' Theorem:

$$\begin{aligned}
 P(A/C) &= \frac{P(C \cap A)}{P(A)} \\
 &= \frac{\{0.5 \times 0.85\}}{\{0.72\}} \\
 &= 0.5903
 \end{aligned}$$

Probability Distribution of X:

X ~ number of laboratories used

$$P(X = 0) = 0.7 \times 0.5 \times 0.8 = 0.28$$

$$P(X = 1) = (0.3 \times 0.5 \times 0.8) + (0.5 \times 0.7 \times 0.8) + (0.2 \times 0.7 \times 0.5) = 0.47$$

$$P(X = 2) = (0.3 \times 0.5 \times 0.8) + (0.7 \times 0.5 \times 0.2) + (0.3 \times 0.5 \times 0.2) = 0.22$$

$$P(X = 3) = 0.3 \times 0.5 \times 0.2 = 0.03$$

x	0	1	2	3
$P(X = x)$	0.28	0.47	0.22	0.03
$xP(X = x)$	0	0.47	0.44	0.09

$$E(X) = 0.47 + 0.44 + 0.09 = 1.00$$

There will be no construction of new laboratories since the two conditions were not met.

Task(b)

Binomial Probability

Let Y = number of students who used the chemistry laboratory.

$$n = 10, \quad p = 0.5, \quad q = 0.5$$

$$P(Y \geq 5) = P(Y = 5) + P(Y = 6) + P(Y = 7) + P(Y = 8) + P(Y = 9) + P(Y = 10)$$

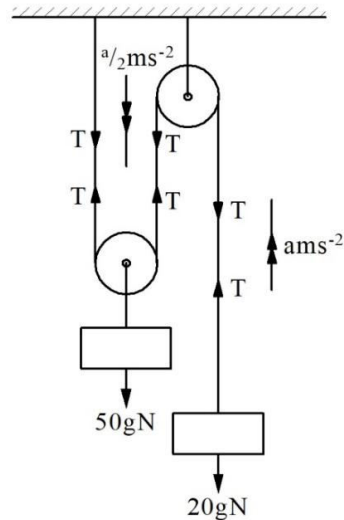
$$= 0.2461 + 0.2051 + 0.1172 + 0.0439 + 0.0098 + 0.0010$$

$$= 0.6231$$

SECTION B: PART II

ITEM 4

Pulley System – Mechanics



$$T - 196 = 20a \quad \text{--- (1)}$$

$$490 - 2T = 50\left(\frac{a}{2}\right)$$

$$245 - T = 12.5a \quad \text{--- (2)}$$

Adding (1) and (2) gives:

$$49 = 32.5a$$

$$a = 1.5077 \text{ m/s}^2$$

Since $a < 2 \text{ m/s}^2$, the system is safe and stable.

$$T = 12.5 \times 1.5077 + 245$$

$$= 226.1538 \text{ N}$$

The cable will brake since $T > 100 \text{ N}$

$$s = ut + \frac{1}{2}at^2$$

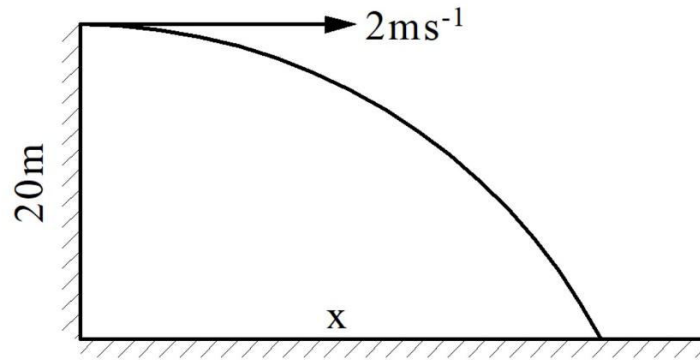
$$= 0 + \frac{1}{2}\left(\frac{1.5077}{2}\right)(5^2)$$

$$= 9.4231 \text{ m}$$

The movable pulley will travel 9.4231m in 5s if the system is released from rest.

Task(b)

Projectile Motion



$$y = \frac{1}{2}gt^2$$

$$20 = \frac{1}{2} \times 9.8t^2$$

$$t = \sqrt{\left\{\frac{20}{4.9}\right\}}$$

$$= 2.0203s$$

$$x = ut$$

$$= 2 \times 2.0203$$

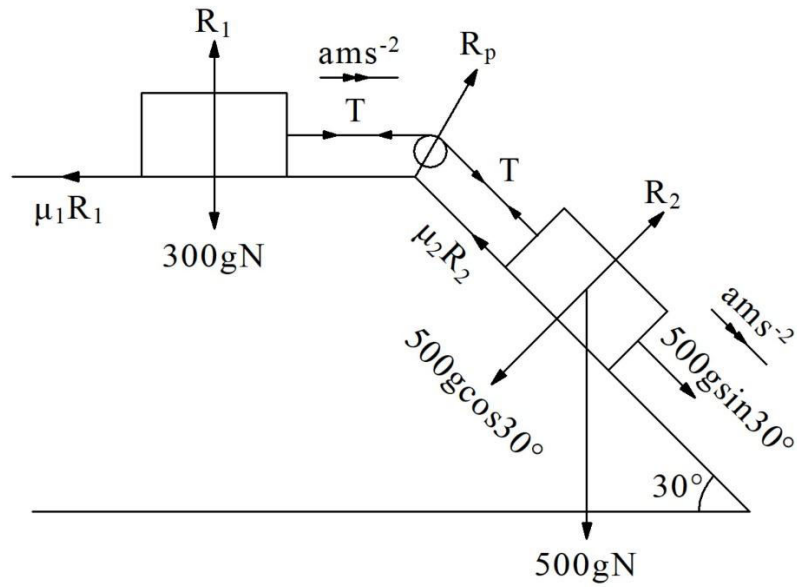
$$= 4.0406m$$

The passers-by are safe since the toll box falls in a distance of 4.0406m of the service deck which is less than the 5m safe zone set by the engineer.

ITEM 5

Task

Inclined Plane – Mechanics



$$2450 - T - 424.3524 = 500a \quad \text{--- (1)}$$

$$T - 882 = 300a \quad \text{--- (2)}$$

Adding (1) and (2):

$$1143.6476 = 800a$$

$$\Rightarrow a = 1.4296 \text{ m/s}^2$$

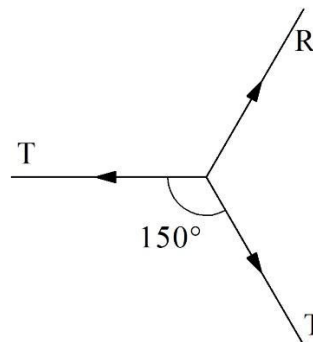
Velocity after 3s:

$$V = u + at = 0 + 1.4296 \times 3 = 4.2887 \text{ m/s}$$

Tension:

$$T = 300(1.4296) + 882 = 1310.88 \text{ N}$$

Resultant Tension in Rope:



$$R^2 = T^2 + T^2 + 2T^2 \cos \theta$$

$$R = \sqrt{\{(1310.88)^2 \times 2 + 2(1310.88) \cos 150^\circ\}}$$

$$R = 678.5614 \text{ N}$$

Since $V < 5\text{m/s}$ and $R < 700\text{N}$, the container will be delivered safely.

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