Dr. Bbosa Science

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Current I

This is the flow of charge

I = $\frac{\delta Q}{\delta t}$

The S.I. unit of current is Ampere (A)

An Ampere is the current flowing in a circuit when a charge of one coulomb passes any point through the circuit in one second.

Mechanism of conduction in metals (heat effect on metals)

Conduction in metals is due to free electrons. Free electrons have thermal energy, and move randomly through metal from one atom to another. When a battery is connected across the ends of the metal, an electric field is set up. The electrons are accelerated by the field; they gain velocity and kinetic energy. When they collide with an atom vibrating about its fixed mean position (called a lattice site) they give up some of their energy to it. Kinetic energy lost is transferred into heat energy within the metal and causes the temperature of the metal to increase.

Although the movement of electron is erratic, on average the electrons drifts in a direction of a field with mean average speed depending on the strength of the field. It is this electron which constitutes an electric current.

The electrical resistance is caused by the obstruction due to atoms to electron movements.

Derivation of I = nAve and J = nve

Consider a section of a metallic conductor in which a current is flowing.

Let I = current through the conductor (A)

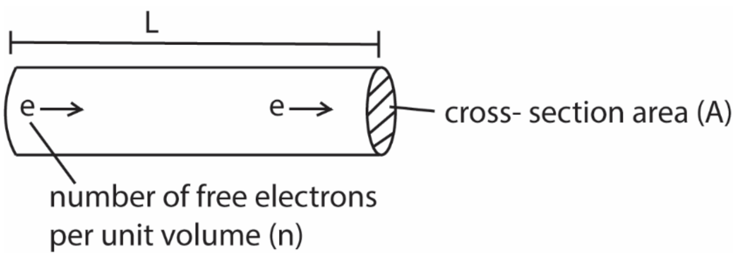
L = length of the conductor (m)

A = cross-section area (m^2)

n = number of free electrons per unit volume (n)

e = charge on each electron (C)

v = average (drift) velocity of the electron (ms^{-1})



It follows that

Volume of the section = LA

Number of free electrons in the section = nLA

Total quantity of charge which is free to move = nLAe

Time taken for an electron to travel through the section = $\frac{L}{v}$

Rate of flow of charge = $\frac{\mathrm{nLAe}}{L/v}$ = nLAve

Hence current, I = nLAve

Current density, = $\frac{I}{A}$ = $\frac{\mathrm{nLAve}}{A}$ = nve

Potential difference (P.d.)

This is work done in transferring a charge of 1 coulomb from one point to another in a circuit. Whenever current flows from one point to another, it does so because the electrical potential at two points are different. If two points are at the same potential, no current can flow between them.

Resistance (R)

This is the opposition to the flow of current in the material.

Resistance R =$\frac{V}{I}$. The S.I. units of resistance is ohms (Ω)

An ohm is a resistance of a conductor through which a current of one Ampere is flowing when the potential difference across is one volt. i.e. 1Ω = $1VA^{-1}$.

Resistivity (ρ, rho)

Resistivity is electrical resistance across opposite faces of a cube of 1m long

$R \propto \frac{l}{A}$ or $R = \rho\frac{l}{A}$ and $\rho = R\frac{A}{l}\ \Omega m$

Conductivity (λ)

It is the reciprocal of resistivity

$\lambda = \frac{1}{\rho}\ \Omega^{-1}m^{-1}$

Some conductors have resistances which depend on current flowing through them; but in most cases the resistance of many conductors (metals) depend only on their physical circumstances e.g. temperature. This was discovered by Ohm and such conductors are called ohmic conductors and obey Ohm's law.

Coulomb

Is the quantity of charge passing in one second through a given cross-section area of a conductor when current flowing is one Ampere.

Electromotive force (e.m.f)

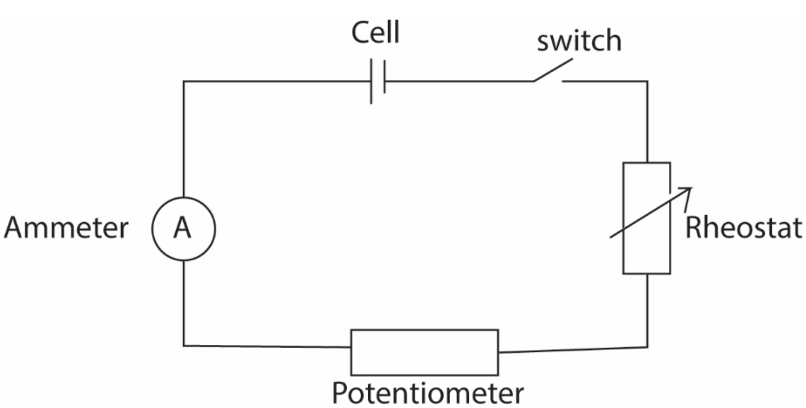
This is the work done in transferring one coulomb of charge around a circuit in which a battery is connected. E.m.f is denoted by E.

Ohm's law

States that the current flowing through a conductor is directly proportional to the P.d. across it provided that there is no change in physical conditions such as temperature of the conductor.

i.e. I = RV

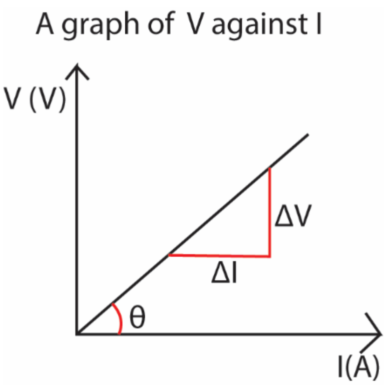
Experiment to determine Ohm's law



The switch is closed the current (I) from ammeter and P.d. (V) from the potentiometer are recorded.

The rheostat is adjusted to obtain several values of I and V

The graph of V against I is plotted.



A straight line graph through the origin implies that V is directly proportional to I

From the graph R = $\frac{\Delta V}{\Delta I}$ = tan θ

Factors affecting resistance

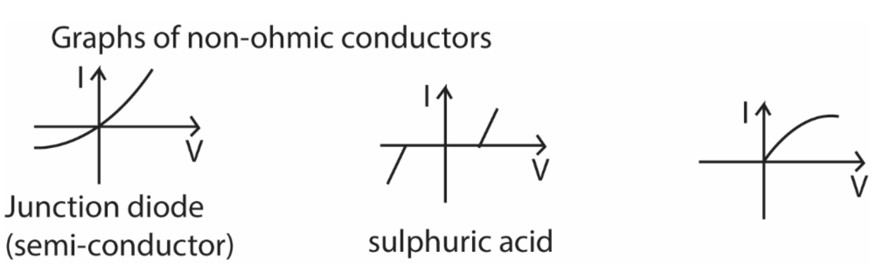
1. Length: Resistance increases with the length of a conductor; electron make more frequent collision with atoms when the length of the conductor increases. This reduces drift velocity of free electron hence resistance increases.
2. Cross sectional area (A): Resistance reduces with increase in cross sectional area. When the cross-sectional area increases, the number of electrons that drift along the conductor increases. This implies that there is an increase in the number of electrons per second that pass a given point, thus an increase in current and consequently a decrease in resistance.
3. Temperature: Resistance increases with temperature increase. Increase in temperature causes atoms to vibrate with greater amplitude and frequency about their mean positions. The velocity of electrons also increases making more collision between electrons and atoms. This increase in collision between electron and atoms reduces the drift velocity of electrons and increases resistance.

Ohmic conductors

These are conductors that obey ohm's law.

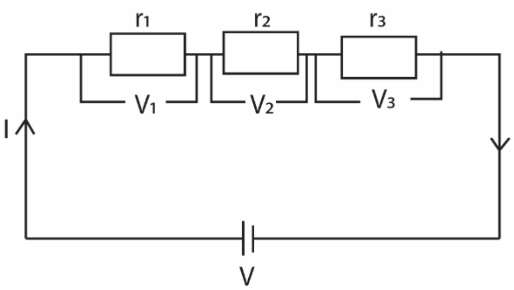
Non-ohmic conductors

Are conductors that do not obey ohm's law; examples



Resistance in combination

(i) Series combination



For series arrangement, the same current I flows through the resistors but each resistor has its own P.d. across

For r_1, $V_1 = r_1I$ ………………. (i)

For r_2, $V_2 = r_2I$ ………………. (ii)

For r_3, $V_3 = r_3I$ ………………. (iii)

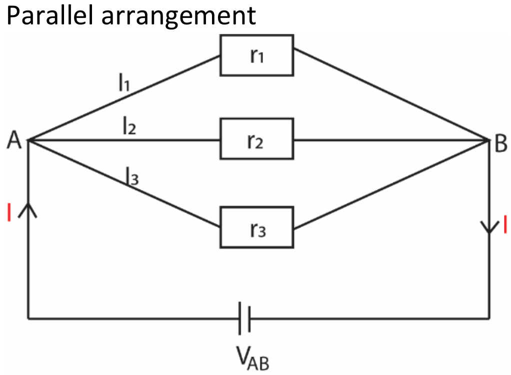
$V = V_1 + V_2 + V_3$

$= r_1I + r_2I + r_3I$

$= I(r_1 + r_2 + r_3)$

Resultant resistance, $R = \frac{V}{I} = (r_1 + r_2 + r_3)$

(ii) Parallel arrangement



The p.d. across each resistor is the same = V_{AB}

Total sum of current $I = I_1 + I_2 + I_3$

$$= \frac{V_{AB}}{r_1} + \frac{V_{AB}}{r_2} + \frac{V_{AB}}{r_3}$$

$$= V_{AB}\left[\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_2}\right]$$

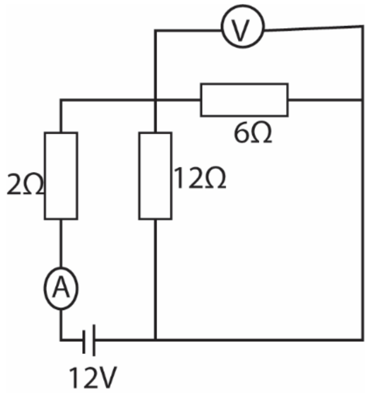
$$\frac{1}{R} = \frac{I}{V_{AB}} = \left[\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_2}\right]$$

Example 1

(i) Define a volt

(ii) Derive the formula of resultant resistance for 3 combined resistors in parallel.

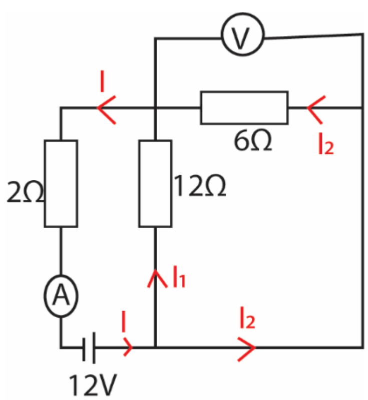
(iii) In the circuit, the battery has negligible internal resistance.



Find ammeter and voltmeter reading

Solution

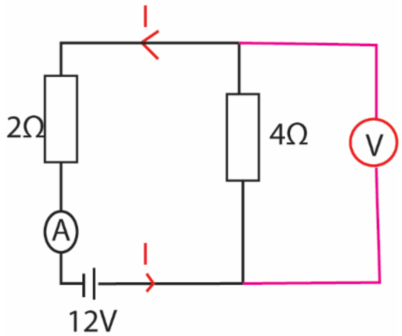
(iii)



6Ω and 12 Ω are in parallel, let their effective resistance be R_1

$\frac{1}{R_1} = \frac{1}{6} + \frac{1}{12}$ then $R_1 = 4\Omega$

Equivalent circuit



2Ω and 4Ω resistors are in series

The effective resistance R = 2 + 4 = 6Ω

From the circuit formula I = $\frac{e.m.f}{total\ resistance} = \frac{12}{6} = 2A$

From ohm's law, V = IR = 2 x 4 = 8Ω

Example 2

The resistance of a nichrome element of an electric wire is 50.9Ω at20.0^{0}C when operating a 240V supply the current flowing through it is 4.17A. Calculate the stead temperature reached by the electric fire if the temperature coefficient of resistance of nichrome is 1.7 x $10^{-4}K^{-1}$.

Solution

R_{20} = 50.9Ω, α = 1.7 x $10^{-4}K^{-1}$, R_θ = ? V = 240V, I = 4.17A

From Ohm's law

$R_\theta = \frac{V}{I} = \frac{240}{4.17} = 57.55\Omega$

Using $R_\theta = R_0(1 + \alpha\theta)$

$R_{20} = R_0(1 + 20\alpha) = 50.9$ …………….(i)

$R_\theta = R_0(1 + \alpha\theta) = 57.55$ ………….. (ii)

From equation (i) and (ii) and substituting for α

θ = 791.1^{0}C

Example 3

A nichrome wire of length 1m and uniform diameter 0.722mm at 25^{0}C is made into a coil. The coil is immersed in 200cm^3 at the same temperature and a current 5.0A is passed through the coil for 8 minutes when the water starts to oil at 100^{0}C. Find

(i) The resistance of the coil
(ii) The electrical energy expended assuming all of it goes to heating the water
(iii) The mean temperature coefficient of resistance of nichrome between 0^0 and 100^{0}C. (Resistivity of nichrome at 25^{0}C =1.2 x 10^{-6})

Solution

(i) L = 1.0m, d = 0.722mm = 7.2 x 10^{-4}m

A = $\pi \frac{d^2}{4} = \pi \frac{[7.2 \ x\ 10^{-4}]^2}{4}$ = 4.07 x 10^{-7}m^2

Using $R_{25} = \rho_{25} . \frac{L}{A}$

$$R_{25} = 1.2\ x\ 10^{-6}\ x\ \frac{1}{4.07 \text{ x } 10-7} = 2.948\Omega$$

(ii) Assuming no heat losses

Electric energy dissipated = heat gained by water = mcθ

= (200 x 10^{-3})(4200)(100-75) = 63000J

Electric energy dissipated = I^2R_{mean} xt

I = 5.0A, t = 8minutes = 8 x 60 = 480s

$R_{mean} = \frac{R_{25} + R_{100}}{2}$

5^2 x $R_{mean} x$ 4800 = 6300

$R_{mean} = 5.2\Omega$

$5.2 = \frac{R_{25} + R_{100}}{2}$

$R_{100} = 7.552\Omega$

Using $R_\theta = R_0(1 + \alpha\theta)$

$R_{20} = R_0(1 + 25\alpha) = 2.948$ ……………(i)

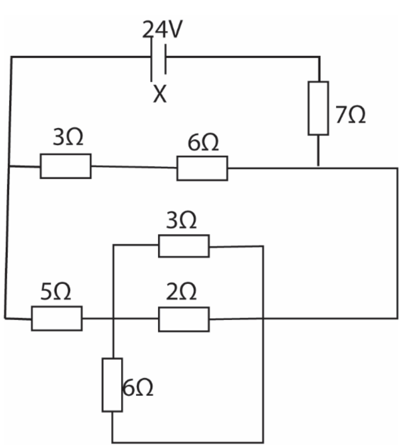
$R_\theta = R_0(1 + 100\alpha) = 7.552$ ………….. (ii)

Divide eqn (ii) by (i)

$\frac{1+100\alpha}{1+25\alpha} = \frac{7.552}{2.948}$

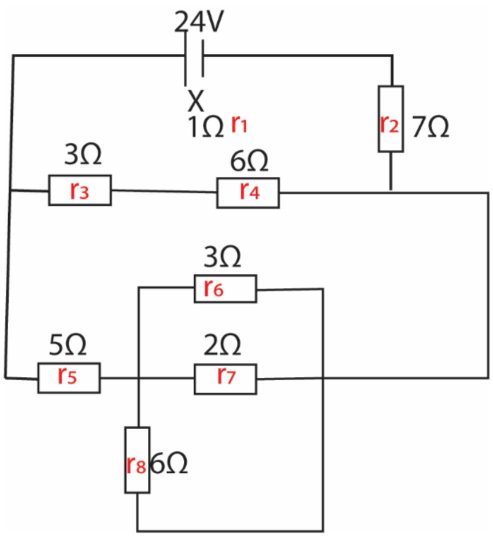
α = 0.0434

Example 4



In the figure X is an accumulator of the e.m.f 24V and having internal resistance of 1Ω. Find the effective resistance of the circuit.

Solution



r6, r7, and r8 are in parallel; let their effective resistance be R_1

$$\frac{1}{R_1} = \frac{1}{3} + \frac{1}{2} + \frac{1}{6}$$

$$R_1 = 1\Omega$$

R_1 and r_5 are in series; their effective resistance $R_2 = 5 + 1 = 6\Omega$

r_3 and r_4 are in series, their effective resistance $R_3 = 3 + 6 = 9\Omega$

R_2 and R_3 are in parallel, let their effective resistance be R_4

$$\frac{1}{R_4} = \frac{1}{6} + \frac{1}{9} = \frac{5}{18}$$

$R_4 = 3.6\Omega$

R_4, r_1 and r_2 are in series their effective resistance, R =3.6 + 1+ 7 = 11.6Ω

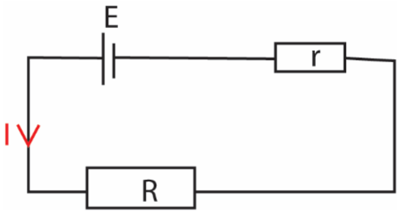
Therefore, effective resistance of the circuit = 11.6Ω

Example 5

A battery of unknown e.m.f and internal resistance is connected in series with a load resistance of R ohms. If a very high resistance voltmeter connected across the load reads 3.2V and the power dissipated in the battery is 0.032W and the efficiency of the circuit is 80%. Find

(i) The current flowing
(ii) The internal resistance of the battery
(iii) The e.m.f of the battery

Solution



Total resistance = R + r

From circular formula I = $\frac{E}{R+r}$ ……………………… (i)

Power dissipated in the battery = I^2r = 0.032w..(ii)

From Ohm's law

Terminal p.d. = IR = 3.2V ………………………………… (iii)

Efficiency = $\frac{power\ out\ put}{power\ in\ put}\ x100$

Power generated = EI = $I^2(R+r)$

Power output = I^2R

- ⇨ $\frac{I^2R}{I^2(R+r)} = \frac{80}{100}$
- ⇨ 100R = 80(R + r)
- ⇨ R = 4r

From equation (3)

I x 4r = 3.2 ……………………………………………………(iv)

Divide eqn. (ii) by (iv)

$\frac{I^2r}{4Ir} = \frac{0.032}{0.32}$

I = 0.04A

From equation (ii) r = $\frac{0.032}{I^2}$

r = 20Ω

but R = 4r = 4 x 20 = 80Ω

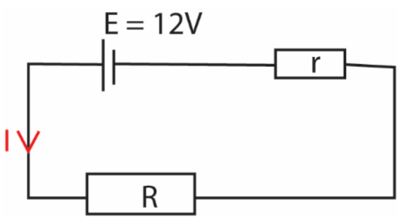
E = I(R + r) = 0.04(80 + 20) = 4V

Example 6

A battery of e.m.f 12V and having unknown internal resistance is connected in series with load R. if a very high resistance voltmeter connected across R reads11.4V and the power dissipated in thee battery is 0.653W. Find

(i) Current flowing
(ii) Internal resistance of the battery
(iii) The value R
(iv) The efficiency of the circuit.

Solution



NB= p.d. across the internal resistance is the lost volts

Ir = E – IR

From Ohm's law

V= IR = 11.4V ……………………………………………………….………(i)

Lost volts = e.m.f of battery – terminal p.d

Ir = 12 – 11.4 = 0.6V ……………………………………………….…(ii)

Power dissipated in the battery = I^2r = 0.653W ………..(iii)

(i) Divide eqn. (iii) by (ii)

$\frac{I^2r}{Ir} = \frac{0.653}{0.6}$

I = 1.088A

(ii) From eqn. (ii)

r $= \frac{0.653}{I^2} = \frac{0.653}{[1.088^2]}$ = 0.55Ω

(iii) From eqn. (i)
V = IR
R = $\frac{11.4}{1.088}$ = 10.48Ω

(iv) Efficiency = $\frac{power\ output}{power\ in\ put}\ x\ 100\%$
$= \frac{I^2R}{EI} = \frac{1.088^2\ x\ 10.48}{12\ x\ 1.088}$ x 100 = 95%

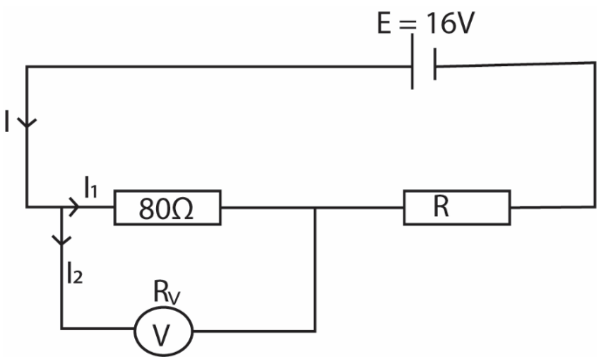
Example 7

A battery of e.m.f 16V and having negligible internal resistance is connected in series with 80Ω and RΩ. When a voltmeter is connected across an 80Ω resistor, it reads 11.28 V while it reads 2.83V when connected across R. find

(i) The value R.

(ii) The resistance of the voltmeter

Solution



Case 1: let R1 be effective resistance of 80Ω and R_v

$$\frac{1}{R_1} = \frac{1}{80} + \frac{1}{R_v}$$

$R_1 = \frac{80R_V}{80+R_v}$ ……………………………………………….(i)

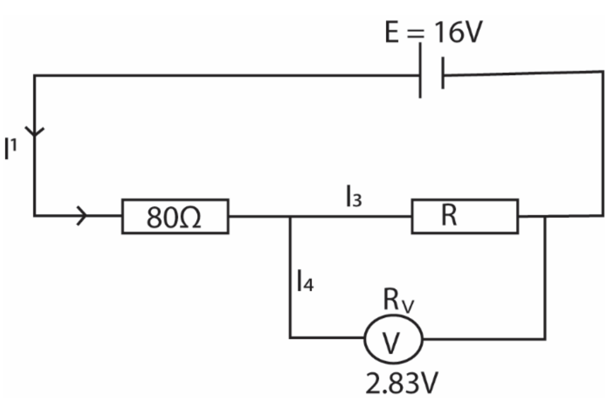
IR_1 = 11.28………………………………………………….(ii)

$\left[\frac{80R_V}{80+R_v}\right] I$ = 11.28 ………………………………………. (iii)

IR = 4.72 ……………………………………………………. (iv)

Case II

Let effective resistance between Rv and R be R_2



$\frac{1}{R_2} = \frac{1}{R} + \frac{1}{R_v}$

$R_2 = \frac{R.R_V}{R+ R_v}$

$\left[\frac{R.R_V}{R+ R_v}\right] I^1$ = 2.83 ....(v)

$80I^1$ = 13.17 .... (vi)

$I^1 = 0.165A$

$\frac{R.R_V}{R+ R_v} = \frac{2.83}{I^1} = \frac{2.83}{0.165} = 17.152$

$RR_v = 17.152R + 17.152R_v$ ....(vii)

Eqn. (iii) ÷ Eqn. (iv)

$\frac{80R_v}{R[80+R_v]} = \frac{11.28}{4.72}$ = 2.39..(viii)

$80R_v$ = 2.39R(80 + R_v) .... (ix)

$80R_v$ = 191.186R + 2.390RR_v .... (x)

Substituting RRv from equation (vii) into equation (x)

$80R_v$ = 191.186R + 2.390R($17.152R + 17.152R_v$)

$80R_v$ = 232.179R + 40.993R_v

$R_v = \frac{232.179R}{(80-40.993)} = 5.952R$

Substituting R_v = $5.952R$in equation (vii)

$5.952R^2 = 17.152R + 17.152 \ x \ 5.952R$

R = $\frac{17.152+102.089}{5.952} = 20.03\Omega$

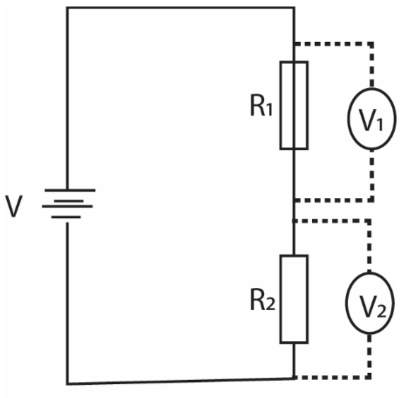
$R_v = 5.952 \ x \ 20 = 119.04\Omega$

Example 8

(a) What is meant by e.m.f and internal resistance of a battery?
(b) A d.c source of e.m.f 16V and negligible internal resistance is connected in series with two resistors of 400Ω and RΩ respectively. When voltmeter is connected across 400Ω resistor it reads4.0V while it reads 6.0V when connected across the resistor of RΩ. Find
    (i) The resistance of the voltmeter (= 400Ω)
    (ii) The value R. (600Ω)

Potential divider

When resistors are arranged in series, the form a potential divider.



Consider two resistor of resistance R_1 and R_2 connected in series with a d.c supply of V volts as shown and having negligible internal resistance.

Total resistance R = $R_1 + R_2$ ………………………………….(i)

From circuit formula;

Current flowing, = $\frac{e.m.f}{total\ resistance}$

I = $\frac{V}{R_1 + R_2}$ …………………………………………… (ii)

From Ohm's Law

$V_1 = IR_1$ ……………………………………………………… (iii)

Putting (ii) in (iii)

$V_1 = \left[\frac{R_1}{R_1 + R_2}\right] V$ ………………………………………………(iv)

$V_2 = IR_2$ ……………………………………………………… (v)

Put (ii) in (v)

$V_2 = \left[\frac{R_2}{R_1 + R_2}\right] V$ ………………………………………………(vi)

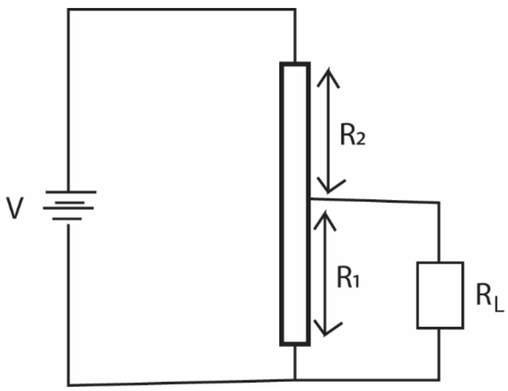
Dividing (iv) : (vi)

$V_1 : V_2 = R_1 : R_2$

For any resistor arranged in series, the p.d. across R1

$V_1 = \frac{R_1}{Total\ resistance}\ x\ supply\ voltage$

Note: in potential dividers, the load is connected across a section of a divider whose p.d. is equal to the operating voltage of the load. The load may be a cooker, electric bulb, etc. the load resistance, say R1, will now be considered to be in parallel to the resistance of the lower section of the divider. Let the resistance of the lower section be R_1 and that of the upper section be R_2.



Now R_1 is in parallel with R_L

P.d. across R_1 = p.d. across the load

Let R be effective resistance of R_1 and R_L

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_L} = \frac{R_1 + R_L}{R_1 R_L}$

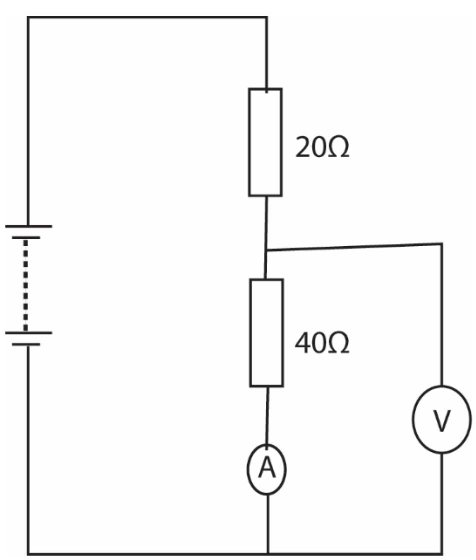
R = $\frac{R_1 R_L}{R_1 + R_L}$

Since R is in series with R_2

Total resistance = R + R_2.

Example 9

The figure shows a potential divider, V, is a very high resistance voltmeter. A is an accurate ammeter



(i) Find the ammeter and voltmeter readings

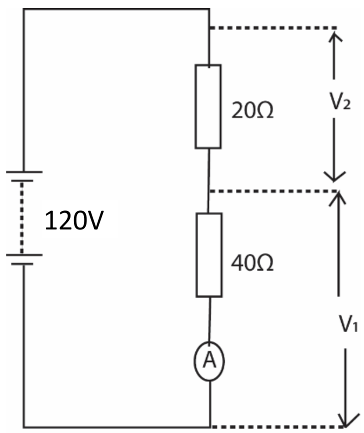
(ii) If the voltmeter above was replaced by another voltmeter of resistance 120, what will be the new reading?

(iii) Find the percentage change in the ammeter reading

(iv) If the voltmeter is replaced by a C.RO, what will be its reading

Solution

(i)

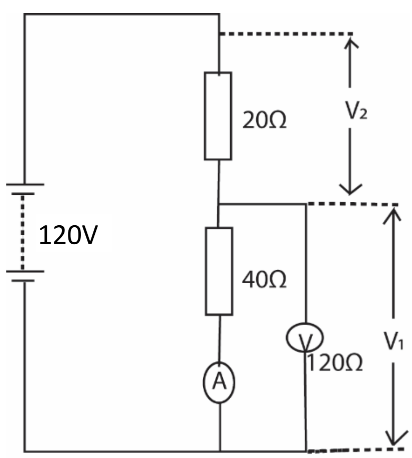


Total resistance R = 20 + 40 = 60Ω

From the circuit formula I = $\frac{e.m.f}{total\ resistance} = \frac{120}{60} = 2A$

From Ohm's law V = IR = 2 x 40 = 80V

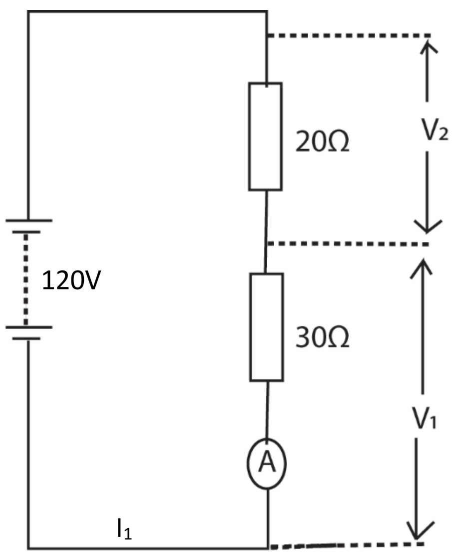
(ii)



Now 40Ω and 120Ω are in parallel, their effective resistance R_1

$\frac{1}{R_1} = \frac{1}{40} + \frac{1}{120}$; R_1 = 30Ω

Equivalent circuit



Total resistance = 20 + 30 = 50Ω

Current I = $\frac{120}{50}$ = 2.4A

From Ohm's law, V = IR_1 = 2.4 x 30 = 72V

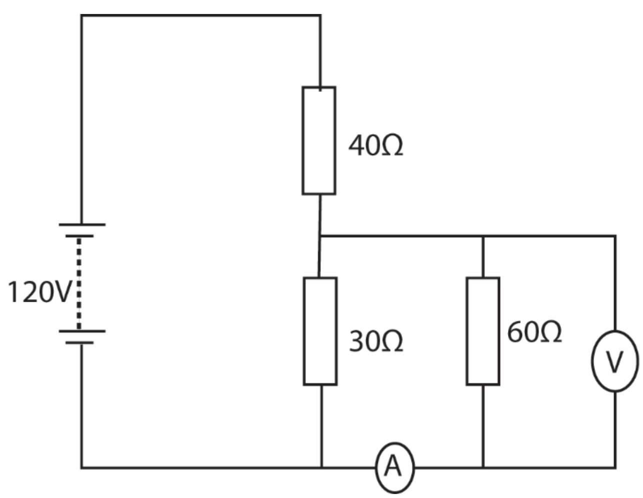
From Ohm's law, I_1 = $\frac{V}{R} = \frac{72}{40} = 1.8A$

(iii) Percentage change in current = $\frac{I - I_1}{I} = \frac{2-1.8}{2}\ x\ 100\% = 10\%$

(iv) A C.R.O is considered as an ideal voltmeter of infinite resistance which makes it to be very accurate, hence, its reading would be the same as in (i) above i.e. 80V

Example 10

The figure shows a potential divider. V is a very sensitive voltmeter.



(i) Find the ammeter and voltmeter readings

(ii) If the voltmeter above was replaced by another voltmeter having a resistance of 120Ω, what is the new reading of voltmeter?

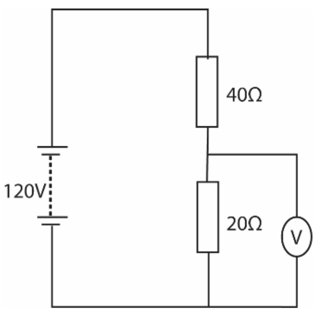
(iii) Find the percentage change in ammeter reading

Solution

(i) 30Ω and 60Ω are in parallel. Let their effective resistance be R

$\frac{1}{R} = \frac{1}{30} + \frac{1}{60} = \frac{3}{60}$; R = 20Ω

Equivalent circuit



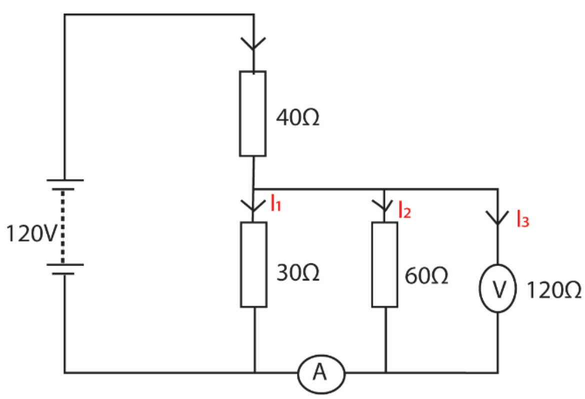
Total resistance = 40 + 20 = 60Ω

Current I = $\frac{120}{60} = 2A$

From Ohm's law V = IR = 2 x 20 = 40V

From Ohm's law ammeter reading = $\frac{V}{R} = \frac{40}{60} = \frac{2}{3}A$

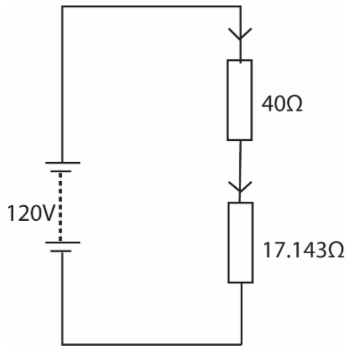
(ii)



30Ω, 60Ω and 120Ω are in parallel, their effective resistance R is given by

$\frac{1}{R} = \frac{1}{30} + \frac{1}{60} + \frac{1}{120}$; R = 17.143Ω

Effective circuit



40Ω and 17.143Ω are in series

Total resistance = 40 + 17.143 = 57.143Ω

From the circuit formula, I = $\frac{e.m.f}{total\ resisitance} = \frac{120}{57.143} = 2.10A$

From Ohm's law, V = IR = 2.1 x 17.143 = 36V

Voltmeter reading = 36V

From Ohm's law I = $\frac{V}{R}$

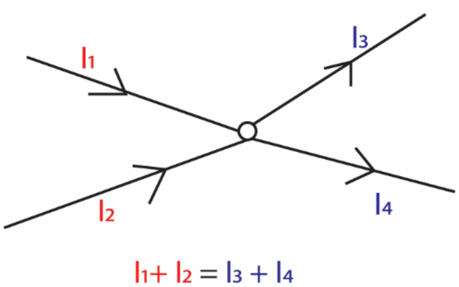
Ammeter reading = $I_2 + I_3 = \frac{36}{60} + \frac{36}{120} = 0.9A$

Percentage change in ammeter reading = $\frac{2-0.9}{2}\ x\ 100$ = 55%

Kirchhoff's laws (1824-87)

First law or law of conservation of current a junction

States that the sum of currents entering a junction is equal to the sum of currents leaving the junction



Or the algebraic sum of all currents at a junction is zero. i.e. $I_1 + I_2 - I_3 - I_4 = 0$

Here a current, I, is reckoned positive if it floes towards the point and negative if it flows away from it.

Second law or closed loop equation

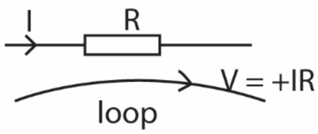
In any closed loop, the algebraic sum of all the potential drops is equal to the algebraic sum of all the e.m.fs.

$\sum V = \sum E$ Where V is the p.d and E is the e.m.f.

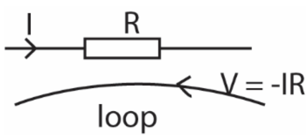
Sign allocation to p.ds in circuit

When allocating signs to the p.d across a resistor, the direction of current flowing through a resistor is considered in relation to the direction of the loop.

The p.d across a resistor is considered be positive if the current is in the same direction as that of the loop.

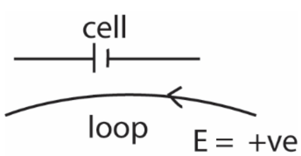


And the p.d is considered to be negative if the current is in an opposite direction to that of the loop.

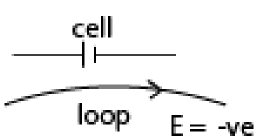


Sign allocation to e.m.fs

When allocating signs to e.m.fs of cells, the polarity of the cell is considered in relation to the direction of the loop. The e.m.f of a cell is considered to be positive if the loop moves from negative terminal of the cell towards the positive terminal.



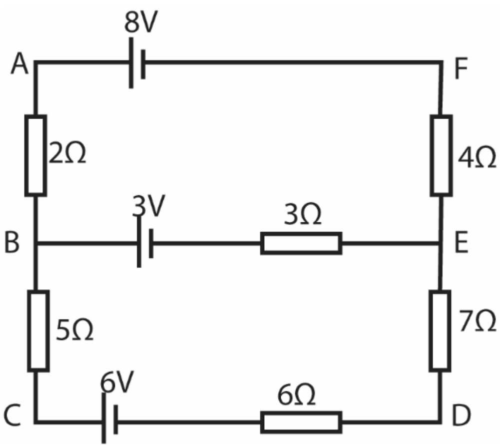
And it is considered to be negative if the loop moves from positive terminal towards the negative terminal



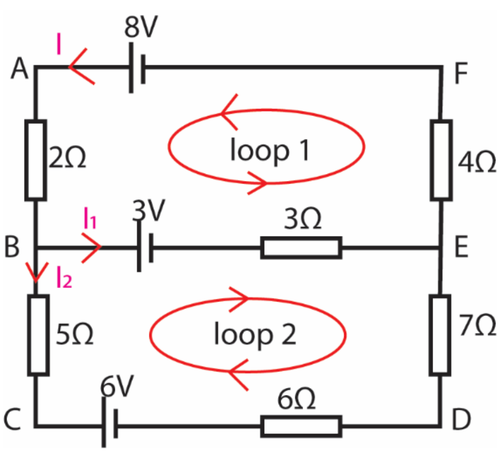
After application of Kirchhoff's law for electric circuit networks independent simultaneously equations involving both the known and unknowns will be obtained and these can now be solved using the available appropriate methods of techniques.

Example 11

In the figure, find the current flowing through the 2Ω, 3Ω and 6Ω resistor.



Solution



At junction B;

$I = I_1 + I_2$ ……………………………….. (1)

Considering loop 1 (ABEFA)

$2I + 3I_1 + 4I = 8 + -3$

$6I + 3I_1 = 5$ …………………………… (2)

Considering loop 2 (BEDCB)

$3I_1 + -7I_2 - 6I_2 - 5I_2 = -3 + 6 =$

$3I_1 - 18I_2 = 3$ ………..(3)

Divide equation3 by 3

$I_1 - 6I_2 = 1$ ……………(4)

Substituting eqn. 1 in eqn.2

$6(I_1 + I_2) + 3I_1 = 5$

$9I_1 + 6I_2 = 5$ …………… (5)

Eqn. (4) and eqn. (5)

10I1 = 6

$I_1 = \frac{6}{10} - 0.6A$

From eqn. (4)

$I_2 = \frac{I_1 - 1}{6} = \frac{0.6-1}{6} = -0.067A$

$I = I_1 + I_2 = 0.6 + -0.067 = 0.533A$

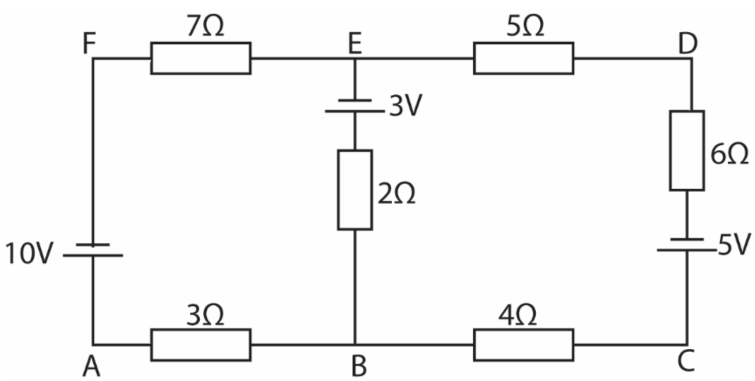
∴ Currents are

$I_1 = 0.6A$

$I_2 = 0.067A$ (in a direction opposite to that assumed in the diagram)

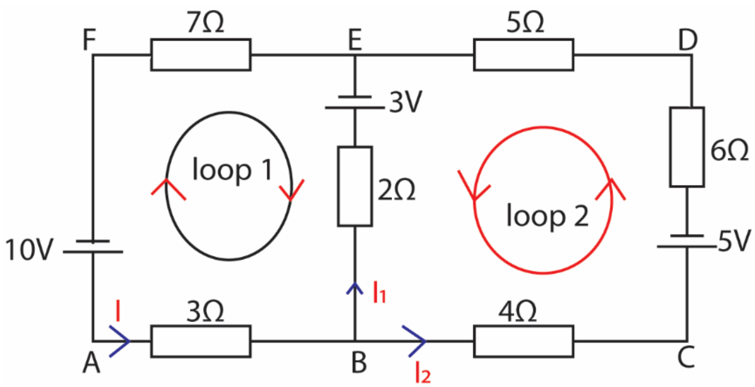
$I_3 = 0.533A$

Example 12



(a) In the figure, find the current flowing through the 3Ω, 2Ω and 4Ω resistors.
(b) If a vey high resistance voltmeter is connected across BD, what will be its readings?

Solution



At junction B;

$I = I_1 + I_2$ ……………………………….(i)

Considering loop 1 (ABEFA)

$3I + 2I_1 + 7I = 10 + -3$ …………….. (ii)

$10I + 2I_1 = 7$ ………………………… (iii)

Considering loop 2 (BEDCB)

$2I_1 - 5I_2 - 6I_2 - 4I_2 = -3 + 5$

$2I_1 - 15I_2 = 2$…………………. (iv)

Substitute eqn. (i) into eqn. (iii)

$10(I_1 + I_2) + 2I_1 = 7$

$12I_1 + 10I_2 = 7$ ………………………….(v)

Multiply 6 into eqn. (iv)

$6 \times [2I_1 - 15I_2 = 2]$

$12I_1 - 90I_2 = 12$ ……………………….(6)

Eqn. (v) – eqn. (6)

$100I_2 = -5$

$I_2 = -0.05A$

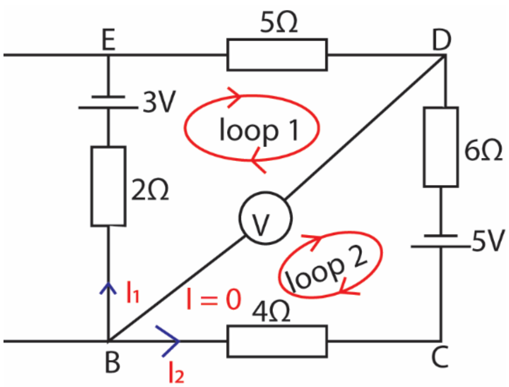
From eqn. (iv)

$$I_1 = \frac{2+15(-0.05)}{2} = 0.624A$$

From eqn. (i)

$I = I_1 + I_2 = 0.625 + -0.05 = 0.575A$

∴ currents are 0.575A, 0.625A, and 0.05A (in direction opposite to that assumed in the diagram)



Considering loop 1(BEDB)

Let V be p.d across BD

$2I_1 - 5I_2 - V = -3$

$V = 3 + 2I_1 - 5I_2 = 3 + (2 \times 0.625) - (10 \times - 0.05) = 4.5V$

Alternatively

Considering loop 2 (BDCB)

$V - 6I_2 - 4I_2 = 5$

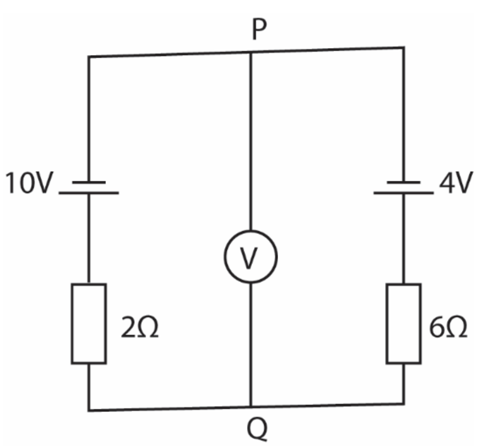
V = 5 + 10I2

= 5 + 10 x - 0.05

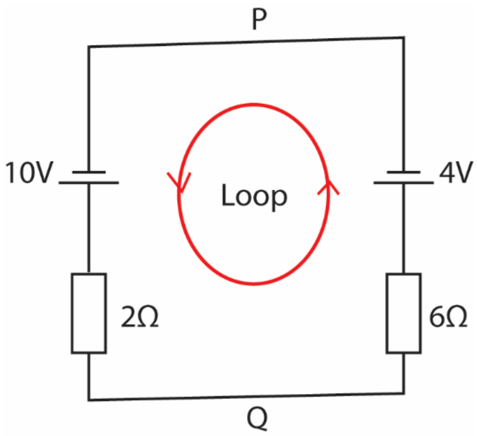
= 4.5V

Example 13

In the diagram, V is a very high resistance voltmeter. Find its reading



Solution



Considering the loop $\sum V = \sum E$

2I + 4I = 10 -4 = 6

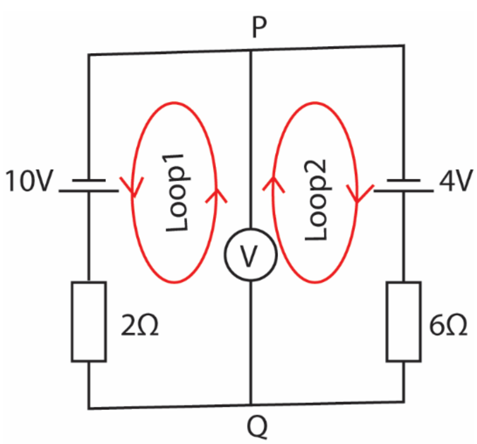
I = 1A

Alternatively, net p.d = 10 -4 = 6

Total resistance = 4+2 = 6Ω

From circuit formula

Current, I = $\frac{net\ p.d}{total\ resistance} = \frac{6}{6}$ = 1A



Considering loop 1

V + 2I = 10

V = 10 -2I = 10 – (2 x 1) = 8V

Alternatively

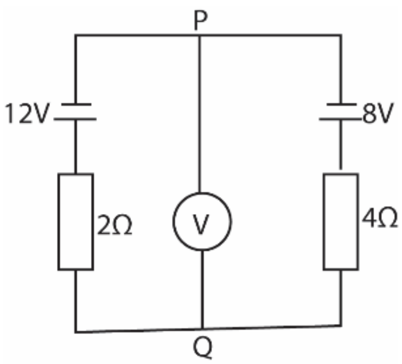
Considering loop 2

V – 4I = 4

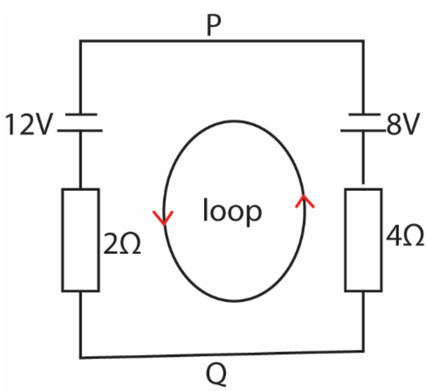
V = 4 + 41 = 4 + 4 x 1 = 8V

Example 14

In the diagram, V is a high resistance voltmeter. Find its reading.



Solution



Considering the loop $\sum V = \sum E$

2I + 4I = 12 -8 = 4

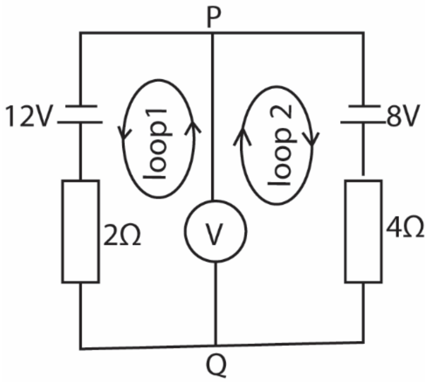
I = 0.667A

Alternatively, net p.d = 12 - 8 = 4

Total resistance = 4+2 = 6Ω

From circuit formula

Current, I = $\frac{net\ p.d}{total\ resistance} = \frac{4}{6}$ = 0.66A



Considering loop 1

V + 2I = 12

V = 12 -2I = 12 – (2 x 0.667) = 10.67V

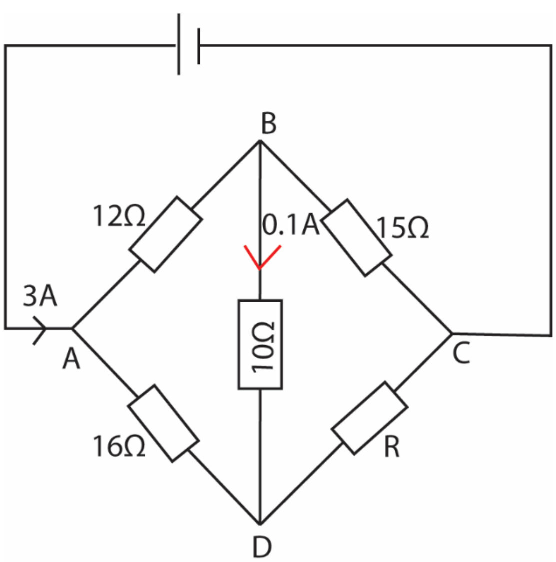
Alternatively

Considering loop 2

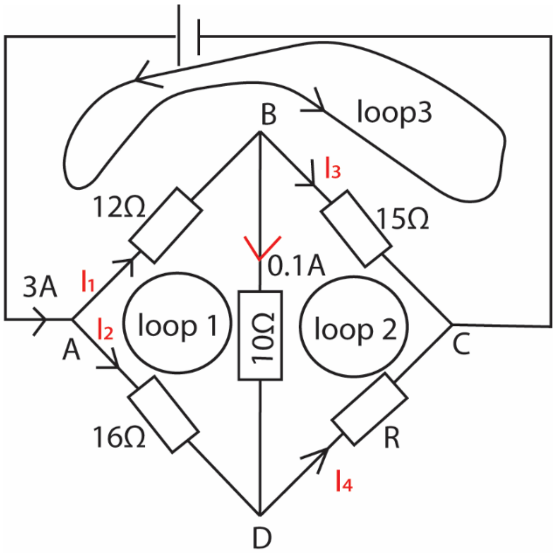
V – 4I = 8

V = 8 + 4 x 0.667 = 10.67V

Example 16



Solution



At junction A;

$I_1 + I_2 = 3$ …………………………….. (i)

At junction B

$I1 = I2 + 0.1$ ……………………….. (ii)

At junction D

$I_2 + 0.1 = I_4$ ……………………….. (iii)

Considering loop 1 (ABDA)

$12I_1 + 10 \times 0.1 - 16I_2 = 0$

$12I_1 - 16I_2 = -1$ …………………… (4)

Considering loop 2 (BCDB)

$15I_3 - RI_4 = 1$ …………………(5)

Considering loop 3

$12I_1 + 15I_3 = x$ ……………. (6)

From eqn. (1)

$I_2 = 3 - I_1$ ……………………(7)

Put (7) into (4)

$12I_1 - 16(3 - I_1) = -1$

$28I_1 = 47$

$I_1 = 1.679A$

$I_2 = 3 - I_1$

$I_2 = 3 = 1.679 = 1.321A$

From equation (2)

$I_3 = I_1 - 0.1 = 1.679 - 1 = 1.579A$

From eqn. (3)

$I_4 = I_2 + 0.1 = 1.321 + 0.1 = 1.421A$

(ii) From eqn. (5)

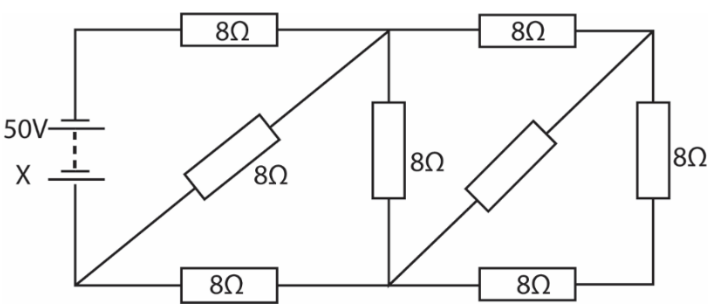
$RI_4 = 15I_3 - (10 \times 0.1)$

$R = \frac{(15 \ x \ 1.579) - 1}{1.421} = 15.964\Omega$

For cell X,

$X = 12I_1 + 15I_3 = (12 \times 1.679) + (15 \times 1.579) = 43.833V$

Example 17

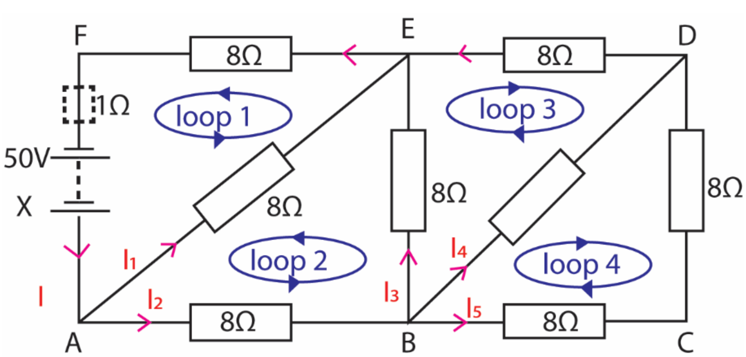


In the figure A is a battery of e.m.f 50V and having internal resistance of 1Ω. Find

(i) The effective resistance of the circuit

(ii) The power in the battery.

Solution



At junction A

$I = I_1 + I_2$ ....(i)

At junction B

$I_2 = I_3 + I_4 + I_5$ ....(ii)

Considering loop 1 (AEFA)

$8I_1 + 8I + I= 50$ .... (iii)

$8I_1+ 9I = 50$ .... (iv)

Considering loop 2 (ABEA)

$8I_2 + 8I_3 – 8I_1 = 0$ .... (v)

Considering loop 3 (BEDB)

$8I_3- 8(I_4 + I_5) -8I_4= 0$

$8I_3 – 16I_4 – 8I_5 = 0$ .... (vi)

Considering loop 4 (BDCB)

$8I_4 -8I_5 -8I_5 = 0$

$8I_4 – 16I_5 = 0$ ....(vii)

From eqn. (vii)

$8I_4 = 16I_5$

$I_4 = 2I_5$ .... (viii)

From eqn.(vi) and (viii)

$8I_3 – 16(2I_5) – 8I_5 = 0$

$I_3 = 5I_5$ .... (ix)

From eqn. (2)

$I_2 = 5I_5 + 2I_5 + I_5 = 8I_5$ .... (x)

From eqn. (v)

$8(8I_5) + 8(5I_5) – 8I_1 = 0$

$I_1 = 13I_5$ .... (xi)

From eqn. (i)

$I = 13I_5 + 8I_5 = 21I_5$

From eqn. (iv)

$8(13I_5) + 9(21I_5) = 50$

$I_5 = \frac{50}{293}$

$I = 21 \, x \, \frac{50}{293} = \frac{E}{R} = \frac{50}{R}$

R = 13.952Ω

Alternatively

Taking path BCD

Effective resistance = 8 + 8 = 16Ω

16Ω and 8Ω are in parallel

$\frac{1}{R} = \frac{1}{16} + \frac{1}{8}$; R = $\frac{16}{3}\Omega$

Now $\frac{16}{3}\Omega$ is in series with 8Ω

Effective resistance = $\frac{16}{3}\Omega$ + 8 = $\frac{40}{3}\Omega$

$\frac{40}{3}\Omega$ I s in parallel with 8Ω along BE

$\frac{1}{R} = \frac{1}{8} + \frac{3}{40}$; R = 5Ω

5Ω and 8Ω (AB) are in series and their effective resistance = 5 + 8 = 13Ω

13Ω and 8Ω (AE) are in parallel

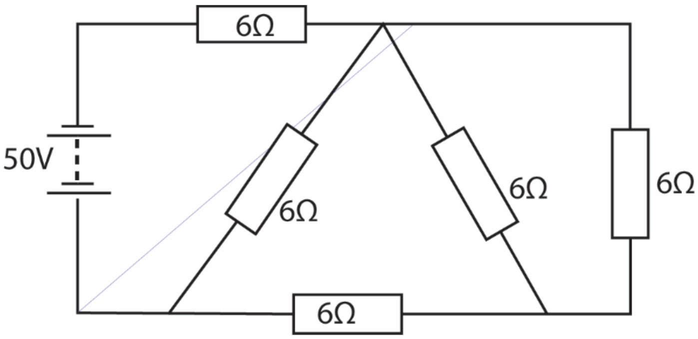
$\frac{1}{R} = \frac{1}{13} + \frac{1}{8}$; R = $\frac{104}{21}\Omega$

Now $\frac{104}{21}\Omega$, 8Ω (EF), 1Ω are in series

Total resistance = $\frac{104}{21}\Omega$ + 8Ω + 1Ω = 13.952Ω

(ii) power dissipated = $I^2r = (21 \text{ x } \frac{50}{293})^2 \text{ x } 1$ = 12.84W

Example 18

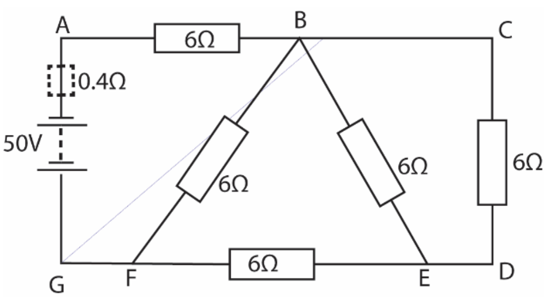


The figure shows a network of resistors connected to a battery of e.m.f 50V and internal resistance 0.4Ω. find

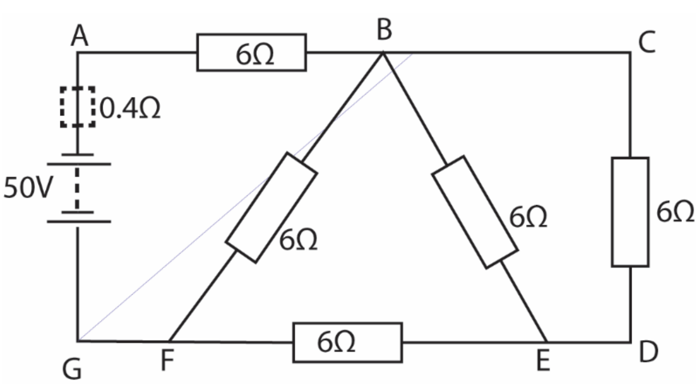
(i) Effective resistance in the circuit

(ii) Power dissipated in the battery

Solution



Solution



R_1 is the effective resistor for Resistors EB is in parallel with resistor DC

$\frac{1}{R_1} = \frac{1}{6} + \frac{1}{6}; R_1 = 3\Omega$

R_1 is in series with resistor FE

$R_2 = 3 + 6 = 9\Omega$

R_2 is parallel with resistor FB

$\frac{1}{R_3} = \frac{1}{9} + \frac{1}{6}$; $R_3 = 3.6\Omega$

R_3 is in series with resistors BA and AG

Effective resistor R = 3.6 + 6 + 0.4 = 10Ω

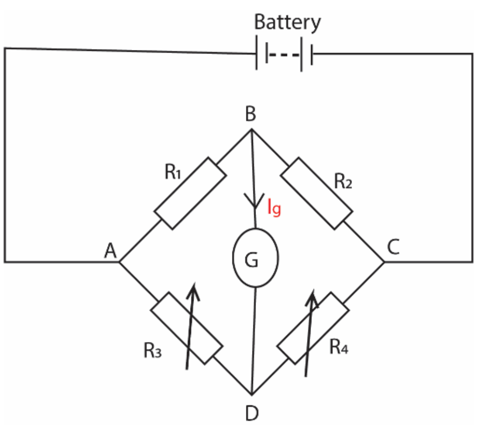
From V = IR

$I = \frac{V}{R} = \frac{50}{10} = 5A$

(ii) power = $I^2r = 5^2 \times 0.4 = 10W$

The Wheatstone bridge (1843) circuit network

It an arrangement of four resistors; the unknown resistor, the standard resistor, and the variable resistors. It is used in comparison of resistances. The circuit is arranged as shown below:



R1 – unknown

R2 – standard resistor

R3 and R4 are variable resistor

G –center zero galvanometer

Ig- current flowing through the galvanometer.

Comparison of resistance

In comparison of resistance, the unknown resistor is connected on one side of the bridge with a standard resistor on the opposite side as shown in the circuit.

Procedure

The variable resistor R3 and R4 are varied until the galvanometer indicates zero deflection.

As the resistors R3 and R4 are varied, the potentials at B and D change.

The galvanometer reads zero when V_D = V_B

When the potential at B (V_B) is greater than the potential at D (V_D), then I_g flows from B to D, giving a deflection to one side of the galvanometer. When (V_D) > (V_B) current flows from D to B and the galvanometer deflects to another side.

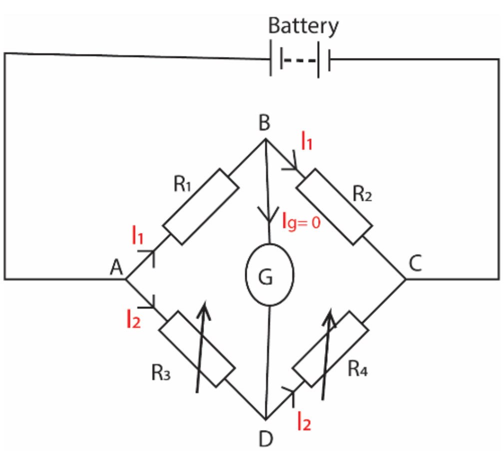
Conditions for balance

These can be derived in two ways

(i) By considering the physical principles involved
(ii) By application of Kirchhoff's laws

Considering the physical principle

Consider a Wheatstone bridge below



At balance, $V_B + V_D$

⇨ $V_{AB} + V_{AD}$
$I_1R_1 = I_2R_3$ ………………………(i)

Similarly

VBC = VCD

$I_1R_2 = I_2R_4$ ……………………………..(ii)

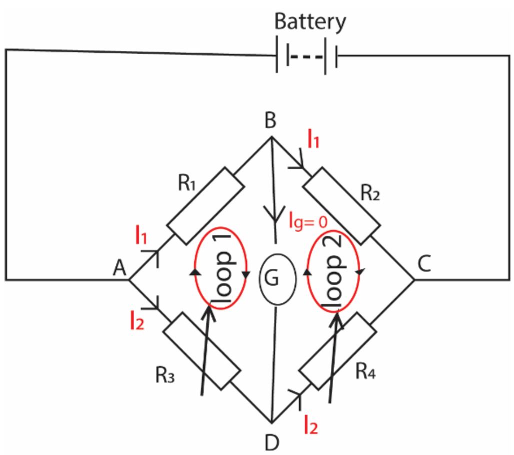
Divide eqn. (i) by eqn. (ii)

$\frac{I_1R_1}{I_1R_2} = \frac{I_2R_3}{I_2R_4}$

Or at balancing point

$\frac{R_1}{R_2} = \frac{R_3}{R_4}$

(ii) Applying Kirchhoff's laws



Considering loop 1 (ABDA)

$I_1R_1 + 0 \times R_g - I_2R_3 = 0$

$I_1R_1 = I_2R_2$ ……………. (i)

Considering loop 2 (BCDB)

$I_1R_3 + 0 \times R_g - I_2R_4 = 0$

$I_1R_3 = I_2R_4$ ……………. (ii)

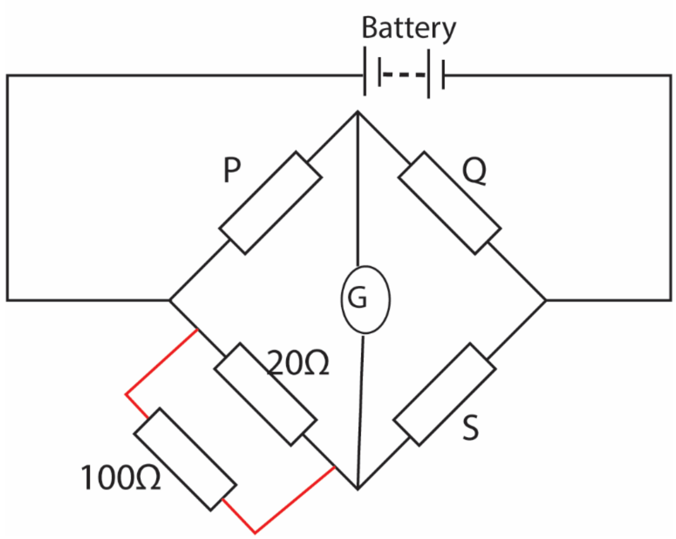
Divide (i) by (ii)

$\frac{R_1}{R_2} = \frac{R_3}{R_4}$

The resistance of unknown resistor, $R_1 = \frac{R_3}{R_4} x R_2$

Example 19

(a) Draw a Wheatstone bridge circuit network and use to derive the balancing condition.
(b) Explain how the network in (a) can be used to determine resistance of unknown resistor.
(c)



In the figure, the 20Ω resistor is shunted with a 100Ω resistor for the bridge to balance. When P and Q are interchanged, the shunt resistance changes to 50Ω for the bridge to balance again. Find the value of S and true ratio P:Q.

Solution

Effective resistance of 20 and 100

$\frac{1}{R} = \frac{1}{20} + \frac{1}{100}$; R = $\frac{50}{3}$

At balance

$\frac{P}{Q} = \frac{R}{S}$

⇨ $\frac{P}{Q} = \frac{50}{3S}$ ……………….(i)

Effective resistance, R1, for 20Ω and 50Ω

$\frac{1}{R_1} = \frac{1}{20} + \frac{1}{50}$; $R_1 = \frac{100}{7}\Omega$

At balance

$\frac{P}{Q} = \frac{S}{R_1}$

$\frac{P}{Q} = \frac{7S}{100}$ ……………….(ii)

Eqn. (i) and eqn. (ii)

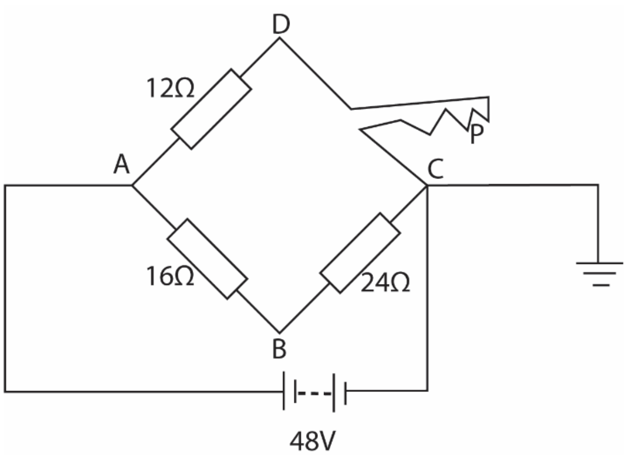
$\frac{7S}{100} = \frac{50}{3S}$; S = 15.43Ω

From eqn. (ii)

$\frac{P}{Q} = \frac{7S}{100} = \frac{7 \ x \ 17.43}{100}$ = 1.08

Example 20

The figure below shows unbalanced Wheatstone bridge circuit network. P is a coil of resistance 16.5Ω at 00C.



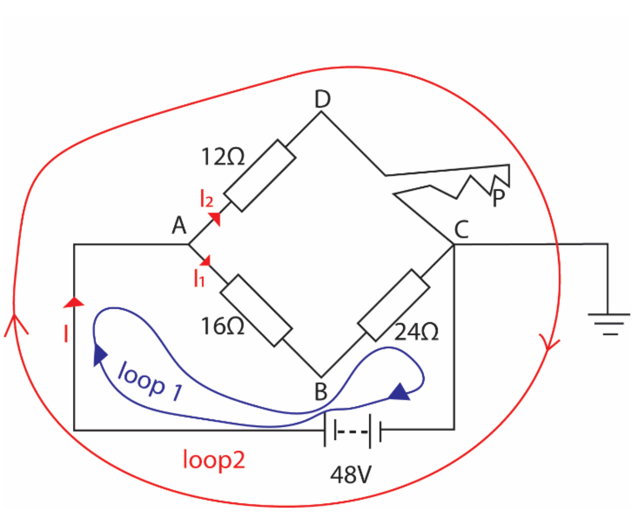
Find

(i) The potential at B and D

(ii) If the galvanometer is connected across BD, in which direction would the current flow

(iii) If the temperature coefficient of resistance of the coil is 2.0 x 10^3K^{-1}, to what temperature must the coil be raised in order to balance?

Solution



Considering loop $16I_1 + 24I_1 = 48$

$40I_1 = 48$

$I_1 = 1.2A$

Considering loop 2

$12I_2 + 16.5I_2 = 48$

$I_2 = 1.684A$

$V_{CB} = V_B - V_C = IR$ (since current flows from region of high potential to that of low potential)

But $V_C = 0$ (since potential due to the earth is zero)

$V_B = IR = 1.684 \times 16.5 = 27.79V$

Potential at B, $V_B = 27.79V$

$V_{CD} = V_D - V_C = IR$

But $V_C = 0$

$V_D = IR = 1.2 \times 24 = 28.8V$

Potential at D = 28.8V

(ii) The current would flow from D to B since D is at a higher potential that at B.

(iii) Let the temperature be θ

At balance;

$\frac{R_0}{12} = \frac{24}{16}$; $R\theta = 18\Omega$

Using $R_\theta = R_0(1 + \alpha\theta)$

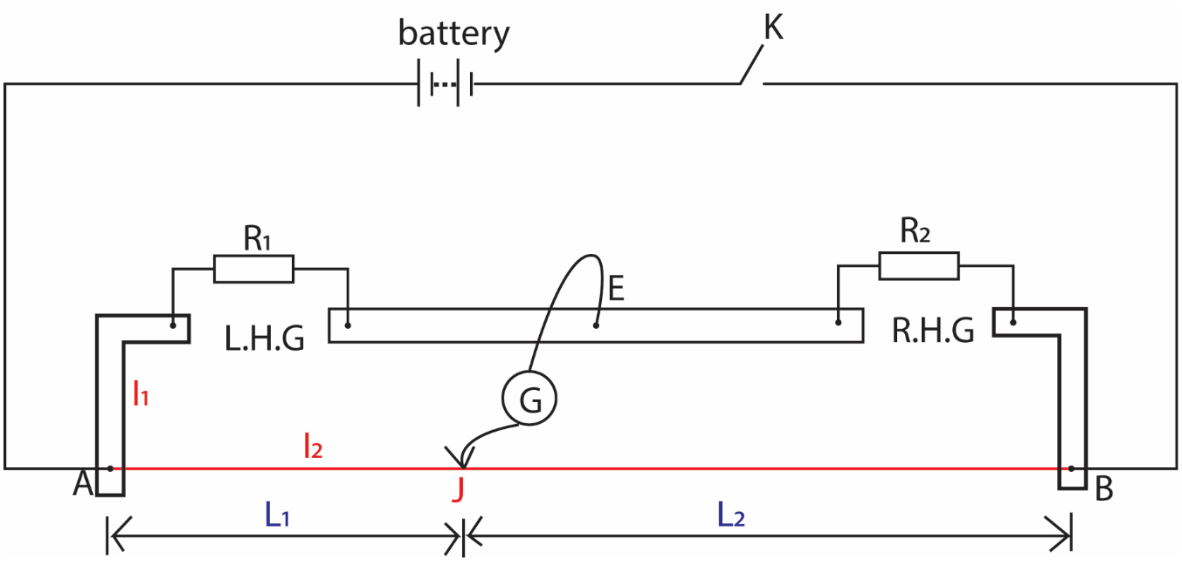
$18 = 16.5(1 + 2.0 \times 10^{-3}\theta)$

$\theta = 45.5^0$

The coil must be raised to 45.5^0 to balance

The meter bridge

A meter bridge consist of a uniform slide wire 1m long (or mounted on a meter rule). The wire is connected in series with accumulator (driver cell) which maintains a steady current through the wire. A meter bridge is used in measurement of resistances, resistivity and temperature coefficient of resistance. The circuit is arranged as shown below.



Procedure

The unknown resistor is connected to L.H.G of the meter bridge. Switch K is closed and Jockey is placed at different places on the slide wire until the galvanometer indicates zero deflection. The meter bridge is said to be balanced. The balancing lengths L_1 and L_2 are noted and recorded.

At balancing point

$$\frac{R_1}{R_{AJ}} = \frac{R_2}{R_{JB}}$$

But $R_{AJ} = KI_1$ and $R_{JB} = KI_2$, where K is resistance per cm of the wire

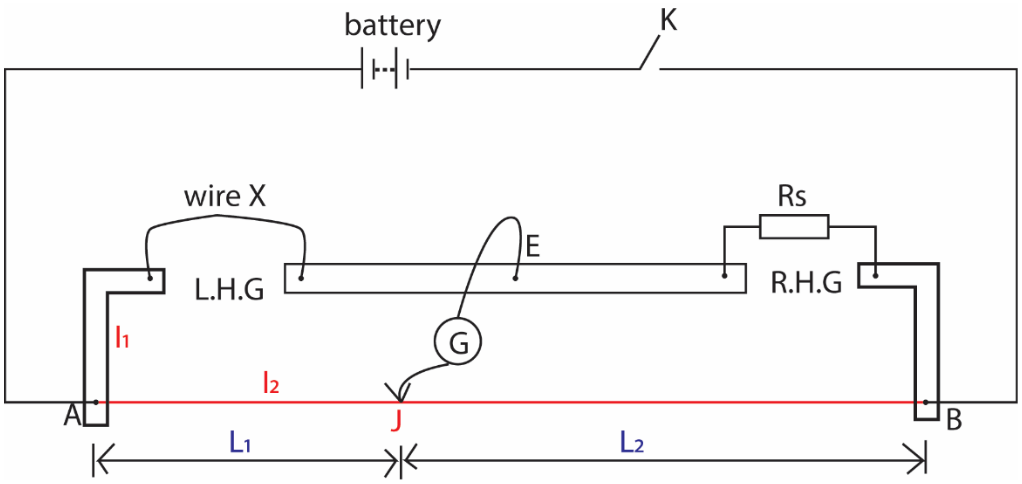
$$\frac{R_1}{KI_1} = \frac{R_2}{KI_2}$$

Thus, $\frac{R_1}{R_2} = \frac{L_1}{L_2}$

Example 21

Draw a circuit diagram of a meter bridge and use it to derive the balancing conditions.

Experiment to determine the resistivity of a material of a wire



Procedure

(i) The mean diameter (d) of wire X is measured using a micrometer screw gauge, and cross section area, A = $\frac{\pi d^2}{4}$ of the wire is calculated

(ii) Starting with a suitable measured length of X in meters, the balancing point is determined using a Jockey.

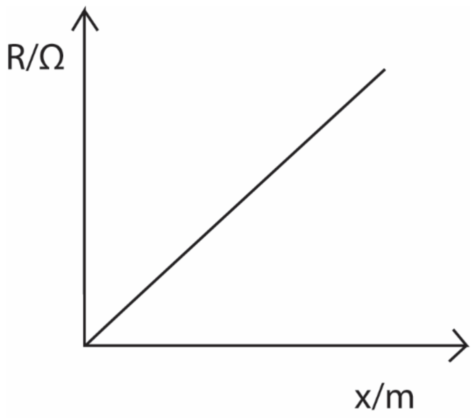
(iii) The balancing length L_1 and L_2 are noted.

(iv) The resistance of specimen wire X is calculated from
R = $R_s\ x\ \frac{L_1}{L_2}$

(v) The experiment is repeated for different lengths of X and corresponding balance lengths L_1 and L_2 are obtained.
Table of results

Length of wire X	L_1 (m)	L_2(m)	R = $R_s\ x\ \frac{L_1}{L_2}$

A graph of R of R against X is plotted. This is a straight line within limits of experimental errors passing through the origin



The slope of the graph is determined.

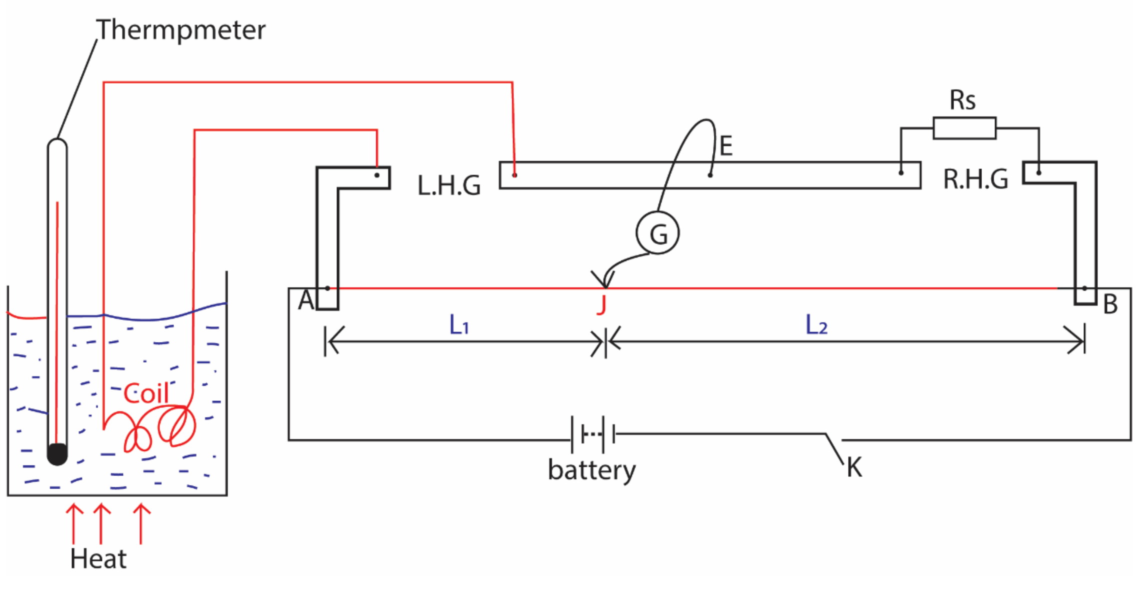
The resistivity of the wire can be obtained from ρ = slope x A

Theory of the experiment

From $\frac{\rho X}{A}$. Ths is in the form y = mx; gradient = ρ

Experimental determination of temperature coefficient of resistance.

A specimen wire in form of a coil immersed in a water bath where temperature θ is varied and measured by a thermometer. The end of the coil are connected to L.H.G of the meter bridge with a standard resistor in R.H.G. the circuit is as below:



Procedure

(i) Starting with suitable temperature, stop heating and switch K is closed.

(ii) Balancing point is determined

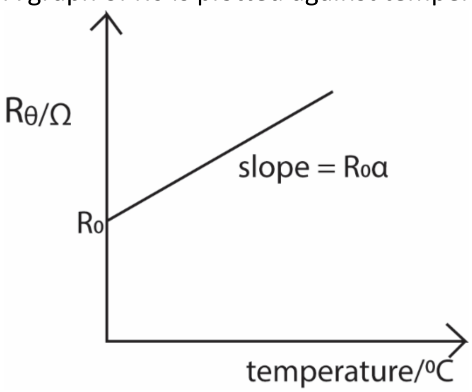
(iii) The balance lengths L_1 and L_2 are obtained

(iv) The resistance R_θ of the coil at temperature θ is calculate. $R_\theta = R_0 \frac{L_1}{L_2}$

(v) The experiment is repeated for different values of θ and corresponding values of L_1 and L_2 are tabulated.

Length of wire X	L_1 (m)	L_2(m)	$R_\theta = R_0 \; x \; \frac{L_1}{L_2}$

A graph of Rθ is plotted against temperature. A straight line is obtained.



The slope and intercept of the graph are determined. The temperature coefficient of resistance (T.C.R) can be obtained by the intercept

$\alpha = \frac{slope}{intercept}$

Theory of the experiment

From $R_\theta = R_0(1 + \alpha\theta) = R_0 + R_0\alpha\theta$

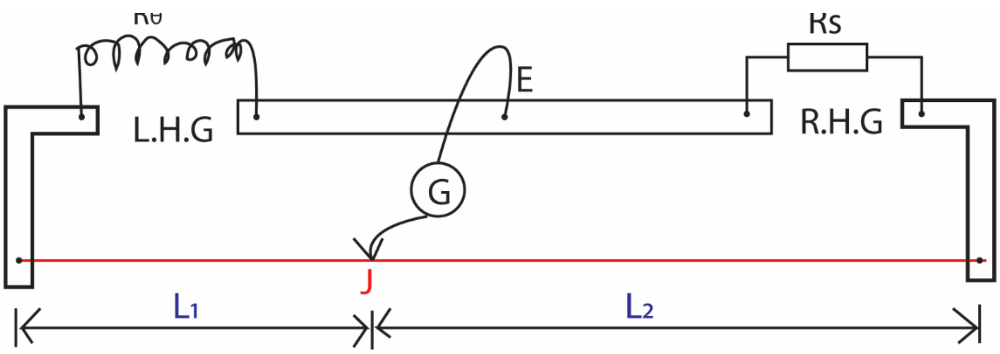
The intercept is = R0

Slope = $R_0\alpha$

Example 22

A specimen wire in the form of a coil was connected in the L.H.G of a meter bridge with a standard resistor in the R.H.G. when the temperature of the coil reached 45^0C, a balance length of 45.2 was obtained and when the temperature was raised to 80^0C a balance length of 51.2cm was obtained. Find the temperature coefficient of resistance of the coil.

Solution



Let R_θ be the resistance at θ^0C

At balance

$\frac{R\theta}{l} = \frac{R_S}{(100-l)}$

$\frac{R_{45}}{45.2} = \frac{R_S}{(54.8)}$

$R_{45} = 45.2\,x\,\frac{R_S}{54.8}$ ………………………………(i)

$\frac{R_{80}}{51.8} = \frac{R_S}{(48.2)}$

$R_{80} = 51.8\,x\,\frac{R_S}{48.2}$ ………………………………(ii)

Using $R_\theta = R_0(1 + \alpha\theta)$

$R_{45} = R_0(1 + 45\alpha)$ …………………………(iii)

$R_{80} = R_0(1 + 80\alpha)$ ……………………….. (iv)

Eqn. (iv) ÷ eqn.(iii)

$\frac{R_{80}}{R_{45}} = \frac{(1+80\alpha)}{(1+45\alpha)}$ …………………………………. (v)

Eqn. (ii) ÷ eqn.(i)

$\frac{R_{80}}{R_{45}} = \frac{51.8R_S}{48.2}\,x\,\frac{54.8}{45.2R_2}$ ……………………… (vi)

Equating (v) and (vi)

$$\frac{(1+80\alpha)}{(1+45\alpha)} = \frac{51.8R_S}{48.2} x \frac{54.8}{45.2R_2}$$

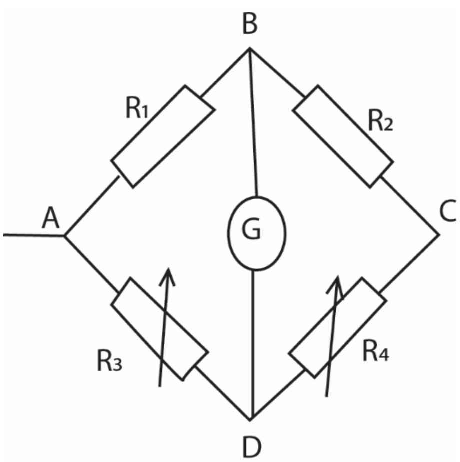
$\alpha = 1.4182 \times 10^{-3}K^{-1}$

Example 23

Explain why a Wheatstone bridge/meter bridge is unsuitable for comparison of very high or low resistance.

Solutions

Consider a Wheatstone bridge circuit network below



If R_1 is much larger than R_2, the current would pass through R_2 and the galvanometer will deflect on one side for all adjustment on R_3 and R_4.

In case of Meter Bridge the current would pass through the slide wire only and the galvanometer would deflect on one side for all positions of the jockey along the slide wire and hence a balance point cannot be obtained.

Hence a Wheatstone bridge becomes unsuitable for comparison of resistances which differ very much in magnitude.

Very low resistances

Where the resistances to be compared are very low, the resistance of the connecting wires become substantial and the balancing point $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ becomes inaccurate.

For accuracy, the balancing condition becomes $\frac{R_1+r_1}{R_2+r_2} = \frac{R_3+r_3}{R_4+r_4}$, where r_1, r_2, r_3 and r_4 are the resistances of the connecting wires

In the case of a meter bridge, the balancing length will be very low that the resistances of end connection become substantial. Thus end correction needs to be added to improve the accuracy.

$$\frac{R_1}{R_2} = \frac{L_1 + e}{L_2 + e}$$

An end correction is the length of the slide wire which has the same resistance at the contact at zero end of the wire.

Measurements of current ant voltage

Current is measured by milliammeter (in milliamperes) or by ammeters in amperes. The milliammeter and ammeters are low resisitance instruments to allow much current through them.

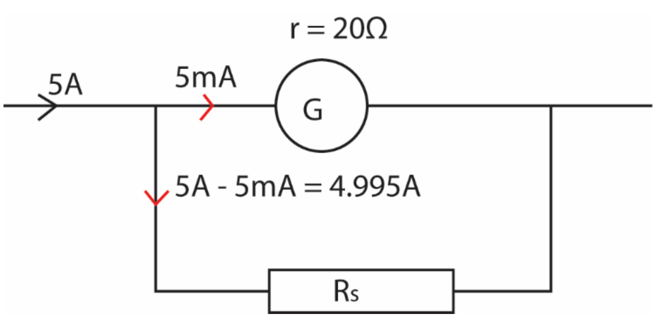
Potential differences are measured by voltmeters in volts. Voltmeters are high resistance instruments not to allow current through them

Conversion of a milliammeter into an ammeter.

Moving coil instruments give full scale deflections (f.s.d) for currents smaller than those generally encountered in the laboratory.

A moving coil ammeter can be converted into an ammeter in order to read big currents by connecting low resistance (S) called a **shunt** across its terminals. The shunt diverts most of the current to be measured away from the coils (not to damage it)

Suppose the coil of the meter has a resisitance r = 20Ω and full scale deflection I_s of 5 millamperes. And we wish to shunt it so as to give f.s.d of 5 amperes.



Then the current through the shut = $(I - I_s) = (5 - 0.005) = 4.995A$

The potential difference across the shunt is the same as that across the coil =IR

$$= 20 \times 0.005 = 4.995 \times Rs$$

$$Rs = 0.02002\Omega$$

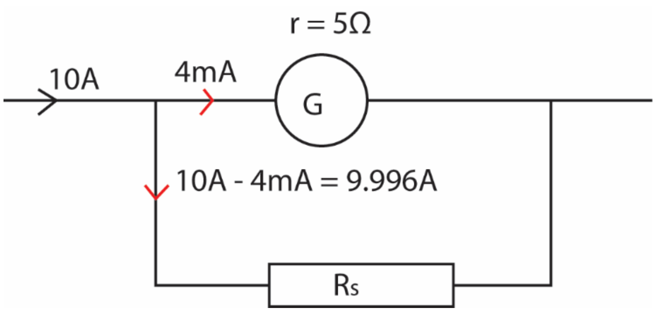
Therefore the resistsnce of the shunt = 0.02002Ω

Example 23

A moving coil galvanometer gives a full scale deflection of 4mA and has a resisitance of 5Ω. How can such an instrument be conveted into an ammeter that give f.s.d of 10A

Solution

Let the required resistance be Rs.



Current through the shunt resistor = 10A – 4mA = 10 – 0.004 =9.996A

The potential difference across the shunt is the same as that across the coil =IR

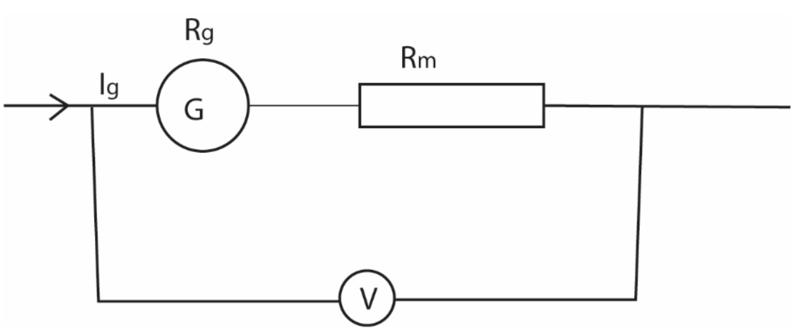
$$= 5 \times 0.004 = 9.996 \times Rs$$

$$Rs = 0.002\Omega$$

Conversion of a milliammeter into a volitmeter.

Moving coil instrument are more sensitive and accurate than other form of voltmeters.

To convert a milliammeter, a large resistance called a multiplier is connected in series to draw extra p.d. to it.



From the diagram

V = (p.d. across the galvanometer) + (p.d across the resistor)

$= I_gR_g + I_gR_m$

Example 24

A moving coil galvanometer gives af.s.dof 6mA and has a resistance of 4Ω. How can the instrument be converted into

(i) Ammeter giving a f.s.d of 15A
(ii) Voltimeter reading up to 20Ω

Solution

(i) A shunt of resistance R_s is used
Current through the shunt = 15 – 0.006 = 14.994A
p.d across the galvanometer = p.d across the shunt = IR
$0.006 \times 4 = 14.994 \times R_s$
Rs = 0.0016Ω

(ii) A multiplier resistor of resistance Rm used
voltage across the galvanometer = IR = 0.006 x 4 = 0.024
voltage across the multiplier resistor = 20 – 0.024 = 19.976V
Rm = $\frac{19{,}976}{0.006}$ = 3329Ω

Example 25

A moving coil galvanometer of resistance 5Ω and current sensitivity of 2div/mA gives a full scale deflection of 16div. how can such an instrument be converted into

(i) Ammeter reading up to 20A
(ii) A voltmeter in which each division represents 2V.

Solution

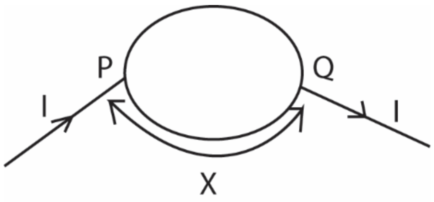
Full scale deflection galvanometer reading = $\frac{16div\ x\ 1}{2}\ 8mA$

(i) A shunt of resistance R_s is used
Current through the shunt = 20 – 0.008 = 19.992A
p.d across the galvanometer = p.d across the shunt = IR
$0.008 \times 5 = 19.992 \times R_s$
Rs = 0.002Ω

(iii) A multiplier resistor of resistance Rm used
voltage across the galvanometer = IR = 0.008 x 5 = 0.04V
Total voltage to be read = 2V x 16 = 32V
voltage across the multiplier resistor = 32 – 0.04 = 31.96V
Rm = $\frac{31.96}{0.008}$ = 3995Ω

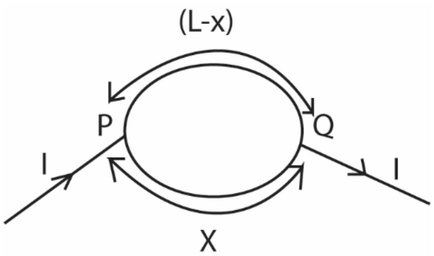
Example 25

A wire of diameter, d, length, L and resistivity, ρ, is made in form of a circular loop. A current enters and leaves the loop at points P and Q respectively.



Show that the resistance R of the wire is given by R =$\frac{4\rho x(L-x)}{\pi d^2 l}$

Solution



Let R_1 and R_2 be the resistance of wire of length x and (L-x)

$R_1 = \frac{\rho x}{A}$ ……………………………(i)

$R_2 = \frac{\rho(L-x)}{A}$ ……………………………(ii)

Where A is the cross section area = $\frac{\pi d^2}{4}$ …………………… (ii)

Effective resistance, R, for the loop

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{A}{\rho x} + \frac{A}{\rho(L-x)}$$

Substituting for A

R = $\frac{4\rho x(L-x)}{\pi d^2 l}$

Example 26

(i) Define the term resistivity and temperature coefficient
(ii) Explain why metals have positive temperature coefficient of resistance.
(iii) Explain why semi-conductors have negative temperature coefficient of resistance.
(iv) Give two substance with negative coefficient of temperature.

Solution

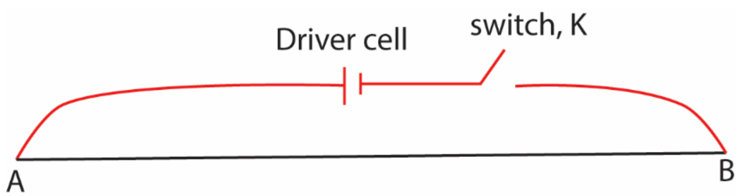
(i) Resistivity is the resistance across opposite faces of a cube of material of side 1m
(ii) Metals have positive temperature coefficient of resistance because their resistance increases with temperature. Increasing temperature increases the speed and kinetic energy of electrons and the amplitude of vibration of atoms. The vibration of atoms hinder the movement of electrons increasing resistance with temperature.

(iii) Semi-conductors have negative temperature coefficient of resistance because increase in temperature breaks the strong forces of attraction between outer electrons and the nuclear attraction. This sets electrons free to conduct heat and electricity.

Potentiometer

It consist of a uniform slide wire which can be of any length (but usually 1m) connected in series with an accumulator or driver cell which maintains a steady current through the wire.

The driver cell supplies a uniform p.d across the wire and hence the p.d across any length of the wire is directly proportional to the length of the section.



Principle of a slide wire potentiometer

Potentiometers are used to measure resistance, current and voltages. In all cases the unknown p.d is balanced against the p.d across a given section of the slide wire when the galvanometer shows no deflections.

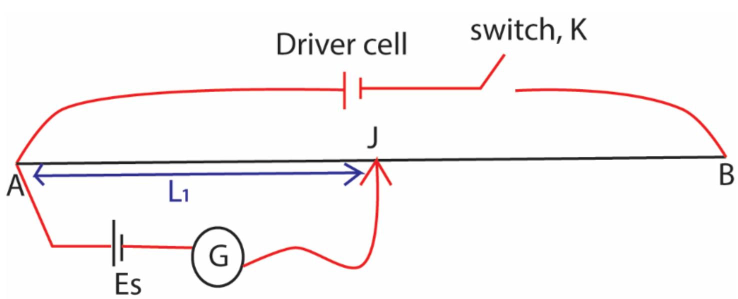
The unknown p.d could be the p.d across a resistor, terminal p.d of a cell on either an open circuit or closed circuit or e.m.f of thermocouples.

N.B. In order to balance, the positive terminal of the unknown p.d is connected to the positive terminal of the driver cell.

Standardization or calibration of the slide wire

Standardization is a process of calibrating a slide wire so that its p.d per cm is known.

A standard cell is balanced against the slide wire; this is done using the circuit below.



Procedure

K is closed and the jockey is placed at different places along the slide wire until the galvanometer shows zero deflection. The balance point L_1 is obtained

At balance Es = kL_1 ……………………….(i)

Where k is the p.d per centimeter of the slide wire

From equation (i)

k $= \frac{E_S}{L_1}$

note that: if the resistance of the wire is known, then the steady current flowing in the slide wire can be calculated given the e.m.f of the standard cell.

At balance

Let I be the steady current flowing

$E_s = V_{AJ} = IR_{AJ}$

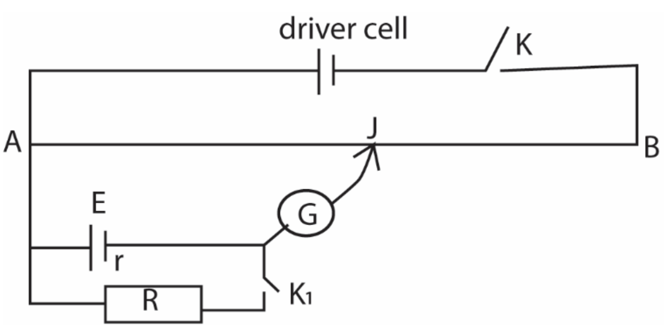
Let r be the resistance per cm, then

$R_{AJ} = rL_1$

$E_s = IrL_1$

Hence I can be obtained from I = $\frac{E_S}{rL_1}$

Consider the following cases



When K is closed and K_1 open, the balance length is L_0 and when K and K_1 are closed, the balance length changes to L_1.

Procedure

Case 1 K closed, K1 open

E = $V_{AJ} = kL_0$ ………………………………….(i)

When K and K_1 are closed

$IR = kL_1$ …………………………………………. (ii)

From circuit formula, I = $\frac{E}{R+r}$ ……………(iii)

Substitute eqn, (iii) in eqn.(ii)

$\frac{ER}{R+r}$ = Kl

Divide (i) by (iv)

$\frac{E(R+r)}{ER} = \frac{L_0}{L_1}$

$1 + \frac{r}{R} = \frac{L_0}{L_1}$

$\frac{r}{R} = \left(\frac{L_0}{L_1} - 1\right)$

$r = R\left(\frac{L_0}{L_1} - 1\right)$

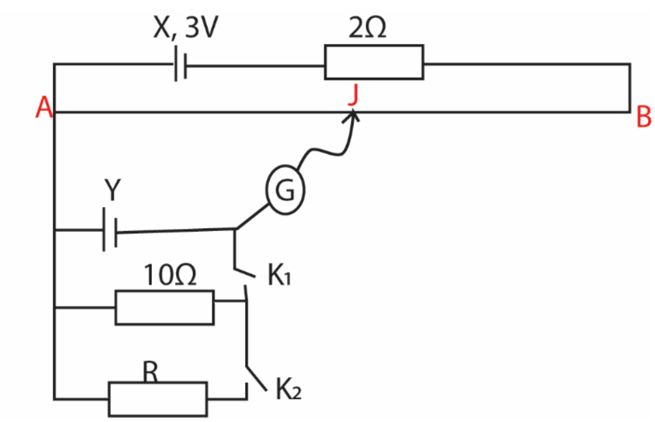
or

$\frac{1}{R} = \left(\frac{L_0}{r}\right)\frac{1}{L_1} - \frac{1}{r}$

A graph of $\frac{1}{R}$ against $\frac{1}{L_1}$ gives a straight line with the slope $\frac{L_0}{r}$ and intercept = $\frac{-1}{r}$

Example 27

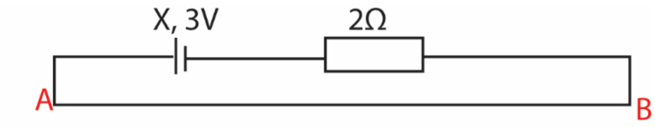
In the figure, X is a cell of e.m.f 3V and having negligible internal resistance. AB is a uniform slide wire 1.2m long and having a resistance of 4Ω. Y is a cell of unknown e.m.f and internal resistance.



When both K_1 and K_2 are open, G shows no deflection when AJ = 86.5cm. When both K_1 and K_2 are closed, G shows no deflection when AJ = 83.3cm. Find

(i) The e.m.f of cell Y
(ii) The internal resistance of cell Y
(iii) The value of R

Solution



Resistance AB, R_{AB} = 4

Total resistance = 4 + 2 = 6Ω

From circuit formula I = $\frac{e.m.f}{total\ resistance} = \frac{3}{6} = 0.5A$

V_{AB} = IR_{AB} = 0.5 x 4 = 2V

p.d. per cm = $\frac{V_{AB}}{L} = \frac{2}{120} = \frac{1}{60}Vcm^{-1}$

(i) With K_1 and K_2 open,

At balance; E = V_{AJ} = R_{AJ} = kL = $\frac{1}{60}\ x\ 90 = 1.5V$

e.m.f of cell, E = 1.5V

Alternatively

E = V_{AJ} = IR_{AJ}

R_{AJ} = resistance per cm x L = $\frac{4}{120}\ x\ 90 = \ 1.5V$

(ii) At balance IR = kL ……………………….(i)

From the circuit formula I = $\frac{E}{R+r}$

⇨ $\frac{ER}{R+r}$ = kL

$\frac{1.5\ x\ 10}{10+r} = \frac{1}{60}\ x\ 86.5$

$r\ = 0.4\Omega$

(iii) Let R_1 be the effective resistance between R and 10Ω

$R_1\ = \frac{10R}{10+R}$

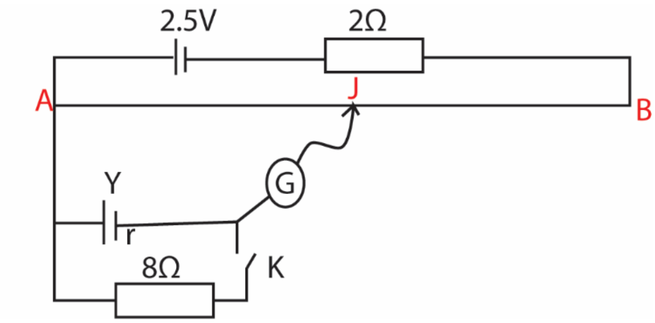
$IR_1\ = \frac{E}{R_1+0.4} = \frac{1}{60}\ x\ 84.3$

$R_1\ = 6\Omega\ = \frac{10R}{10+R}$

R = 15Ω

Example 27

In the figure AB is a uniform wire 1m long and having a resistance of 4Ω. Y is a cell of unknown e,m.f and internal resistance.

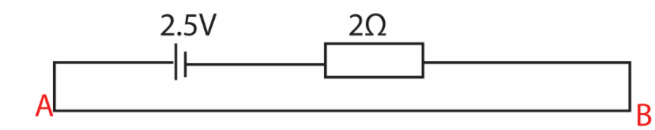


When K is open, G shows no deflection when AJ = 90.0cm. When K is closed, G shows no deflection when AJ = 87.8cm. Find

(i) E.m.f of the cell Y

(ii) Internal resistance of Y.

Solution



R_{AB} = 4Ω

Total resistance = (4 + 2) = 6Ω

Total current I = $\frac{e.m.f}{total\ resstance} = \frac{2.5}{6}A$

V_{AB} = IR_{AB} = $\frac{2.5}{6}\ x\ 4 = \frac{5}{2}V$

p.d. per cm k = $\frac{5}{2} \div 100 = \frac{1}{60}Vcm^{-1}$

when K is open, L = 90.0cm

At balance , E = V_{AJ} = Kl = $\frac{1}{60}\ x\ 90 = 1.5V$

Alternatively

E = V_{AJ} = IR_{AJ}

Resistance per cm = $\frac{R_{AB}}{L}$ = $\frac{4}{100}\ 0.04\Omega cm^{-1}$

R_{AJ} =0.04 x 90 = 3.6Ω

E = $\frac{2.5}{6}\ x\ 3.6 = 1.5V$(which is e.m.f of cell Y)

(ii) when K is closed L = 87.8cm

From the formula, I = I = $\frac{E}{R+r}$

I = $\frac{ER}{R+r}$ = kL

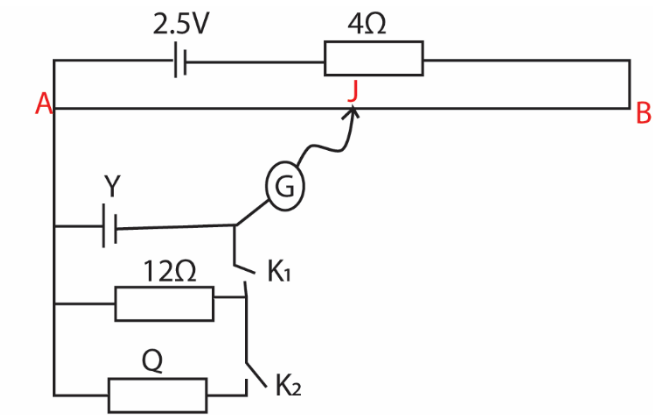
R = 8Ω, E = 1.5V

⇨ $\frac{1.5 \, x \, 8}{8+r} = \frac{1}{60} \, x \, 87.8$

⇨ $r = 0.2\Omega$

Example 27

In the figure Ab is a uniform slide wire of 1.0m long and having a resistance of 4Ω. Y is a cell of unknown e.m.f and internal resistance.



When K_1 and K_2 are open G shows no deflection when AJ = 96cm. when K_2 is closed and K_1 is left open, G shows no deflection when AJ = 92.9cm. When K_1 and K_2 are closed, G shows no deflection when AJ = 91.4cm. Find

(i) The e.m.f of cell Y
(ii) Internal resistance of Y
(iii) The value of Q

Solution

Resistance AB, R_{AB} = 4

Total resistance = 4 + 4 = 8Ω

From circuit formula I = $\frac{e.m.f}{total\ resistance} = \frac{2.53}{8} = 0.0.3125A$

V_{AB} = IR_{AB} = 0.3125 x 4 = 1.25V

p.d. per cm = $\frac{V_{AB}}{L} = \frac{1.25}{100} = \frac{1}{80} Vcm^{-1}$

(i) With K_1 and K_2 open,

At balance; E = V_{AJ} = R_{AJ} = kL = $\frac{1}{80}\ x\ 96 = 1.2V$

e.m.f of cell, E = 1.2V

Alternatively

E = V_{AJ} = IR_{AJ}

R_{AJ} = resistance per cm x L = $\frac{1.25}{100}\ x\ 96 =\ 1.2V$

(ii) At balance IR = kL ……………………..(i)

From the circuit formula I = $\frac{E}{R+r}$

⇨ $\frac{ER}{R+r}$ = kL

$\frac{1.2\ x\ 12}{12+r}$ = $\frac{1}{80}\ x\ 92.9$

$r\ = 0.4\Omega$

(iii) Let R_1 be the effective resistance between Q and 12Ω

$\frac{1}{R_1} = \frac{1}{Q} + \frac{1}{12}$ …………………………………. (i)

At balance IR_1 = kL

$\frac{ER_1}{R_1+r}$ = kL

$\frac{1.2R_1}{R_1+0.4}$ = $\frac{1}{80} = 91.4 = 1.1425$

$R_1\ = 7.9$

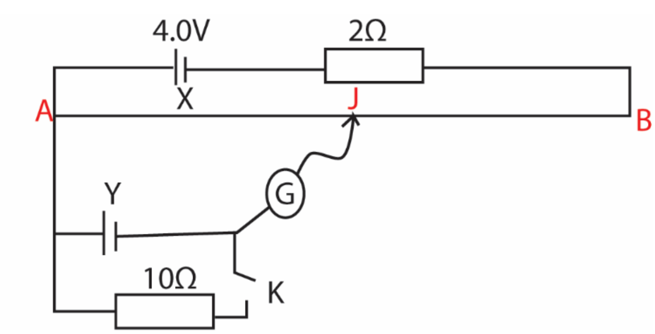
Substituting R1 in equation (i)

$\frac{1}{7.9} = \frac{1}{Q} + \frac{1}{12}$

Q = 24Ω

Example 28

In the figure AB is a uniform slide wire of 1m long and having a resistance of 4Ω.



X is an accumulator of e.m.f 4.0Ω and having negligible internal resistance.

When K is open, G shows no deflection when AJ = 56.25cm. When K is closed, G shows no deflection when AJ = 51.5cm. Find

(i) E.m.f of the cell Y
(ii) Internal resistance of Y

Solution

R_{AB} = 4Ω

Total resistance = (4 + 2) = 6Ω

Total current I = $\frac{e.m.f}{total\ resstance} = \frac{4.0}{6}A$

V_{AB} = IR_{AB} = $\frac{4.0}{6}\ x\ 4 = \frac{8}{3}V$

p.d. per cm k = $\frac{V_{AB}}{L}$ = $\frac{\frac{8}{3}}{100} = \frac{2}{75}Vcm^{-1}$

(i) When K is open, L = 56.25cm

At balance, E = V_{AJ} = Kl = $\frac{2}{75}\ x\ 56.25 = 1.5V$ (e.m.f of Y)

Alternatively

E = V_{AJ} = IR_{AJ}

Resistance per cm = $\frac{R_{AB}}{L}$ = $\frac{4}{100}\ 0.04\Omega cm^{-1}$

R_{AJ} =0.04 x 56.25 = 2.25Ω

E =IR_{AJ}= $\frac{4}{6}\ x\ 2.25 = 1.5V$(which is e.m.f of cell Y)

(ii) When K is closed L = 51.5cm

From the formula, I = I = $\frac{E}{R+r}$

I = $\frac{ER}{R+r}$ = kL

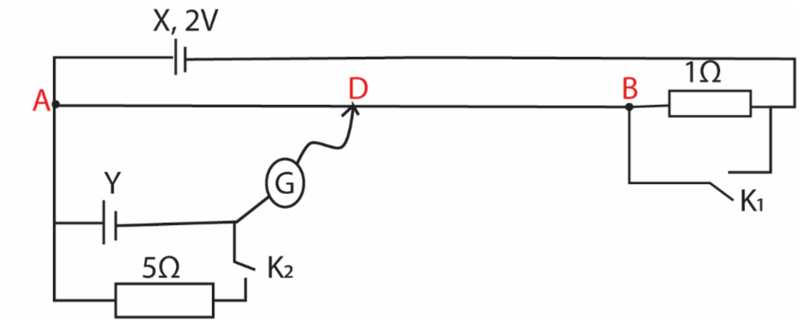
R = 10Ω, E = 1.5V

⇨ $\frac{1.5\ x\ 10}{10+r}$ = $\frac{2}{75}\ x\ 51.1$
⇨ $r = 1\Omega$

∴ internal resistance of a cell = 1Ω

Example 29

In the figure, cell X is of e.m.f 2.0V and negligible internal resistance, AB is a uniform slide wire of length 100cm and resistance 5Ω



When both K_1 and K_2 is open, the balance length AD = 90cm. When K_2 is closed, the balance length change to 75cm. Find

(i) E.m.f of the cell Y
(ii) Internal resistance of Y
(iii) The balance length if both K_1 and K_2 are closed

Solution

K1 is open

R_{AB} = 5Ω

Total resistance = (5 + 1) = 6Ω

Total current I = $\frac{e.m.f}{total\ resstance} = \frac{2.0}{6} = \frac{1}{3}\ A$

V_{AB} = IR_{AB} = $\frac{1}{3}\ x\ 5 = \frac{5}{3}V$

p.d. per cm k = $\frac{V_{AB}}{L}$ = $\frac{\frac{5}{3}}{100} = \frac{5}{300}Vcm^{-1}$

(i) When K_1 is closed

Total resistance = 5Ω

Total current = $\frac{2}{5}A$

V_{AB} = IR_{AB} = $\frac{2}{5}\ x\ 5 =\ 2V$

p.d. per cm k = $\frac{V_{AB}}{L}$ = $\frac{2}{100} = \frac{1}{50}Vcm^{-1}$

kl = Ey = $\frac{5}{300}\ x$ 90 = 1.5V

∴ e.m.f of a cell = 1.5V

(ii) When K is open L = 75cm

From the formula, I = I = $\frac{E}{R+r}$

I = $\frac{ER}{R+r}$ = kL

R = 5Ω, E = 1.5V

⇨ $\frac{1.5 \, x \, 5}{5+r} = \frac{5}{300} \, x \, 75$

⇨ $r = 1\Omega$

∴ internal resistance of a cell = 1Ω

(iii) Balance length if both K_1 and K_2 are closed

kl = IR = $\frac{ER}{R+r}$

$\frac{1}{50}L = \frac{1.5 \, x \, 5}{5+1}$

L = 62.6cm

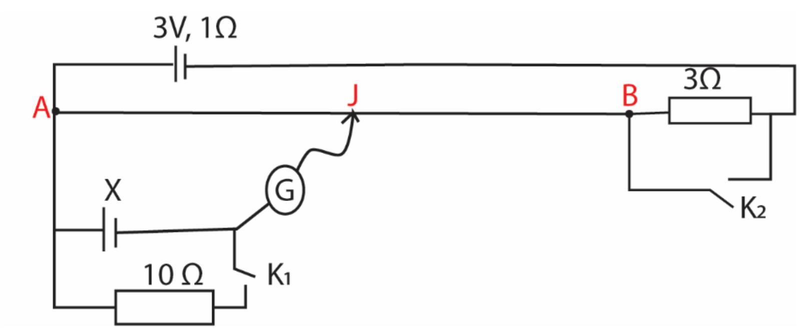
NB when K1 is closed, current passes through the switch alone where there is less resistance.

When K1 is closed, current passes through 1Ω resistor

Hence the different values of resistance.

Example 30

In the figure below when K_1 is open and K_2 closed, G shows no deflection for AJ = 83.3cm.



When K_1 is closed and K_2 open, G shows no deflection when AJ = 67.5cm

(a) Find the resistivity of the potentiometer wire 1m long with a diameter 0.5mm and having a resistance of 5Ω

(b) Calculate

(i) e.m.f of cell X

(ii) internal resistance of X

(iii) the balance length when both switches K_1 and K_2 are closed

(iv) the balance length when both K_1 and K_2 are open.

Solution

(a) ρ = $\frac{RA}{L}$ = $\frac{R\pi d^2}{4L}$ = $\frac{5\,x\,\pi\,x\,(0.5\,x10^{-3})^2}{4\,x\,1}$ = 9.81 x 10^{-7}Ωm

(b) when K2 is open

R_{AB} = 5Ω

Total resistance = (5 +3 + 1) = 9Ω

Total current I = $\frac{e.m.f}{total\ resstance} = \frac{3}{9} = \frac{1}{3}\ A$

V_{AB} = IR_{AB} = $\frac{1}{3}\ x\ 5 = \frac{5}{3}V$

p.d. per cm k = $\frac{V_{AB}}{L}$ = $\frac{\frac{5}{3}}{100} = \frac{1}{60}Vcm^{-1}$

(i) When K_2 is closed

Total resistance = 5 + 1 = 6Ω

Total current = $\frac{3}{6} = \frac{1}{2}A$

Resistance per centimeter =$\frac{5}{100}$

p.d. per cm I$R_{/cm}$ k = $\frac{5}{100}\ x\ \frac{1}{2} = \frac{1}{40}Vcm^{-1}$

(i) Ex = k l = $\frac{1}{40}\ x$ 83.3 = 2.1V

∴ e.m.f of a cell = 2.1V

(ii) When K_1 is closed and K_2 is open L = 67.5cm

From the formula, I = I = $\frac{E}{R+r}$

I = $\frac{ER}{R+r}$ = kL

R = 10Ω, E = 2.1V

⇨ $\frac{2.1\ x\ 10}{10+r}$ = $\frac{1}{60}\ x\ 67.5$

⇨ $r\ = 8.67\Omega$

(iii) Balance length if both K_1 and K_2 are closed

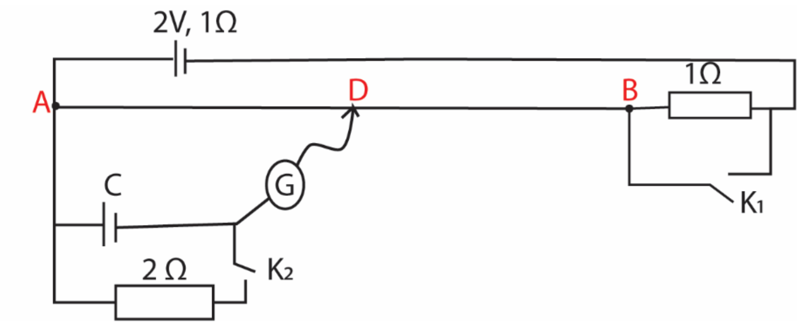
kl = IR = $\frac{ER}{R+r}$

$\frac{1}{60}L$ =$\frac{2.1\ x\ 9}{9\ +8.67}$

L = 64.18cm

Example 31

The figure below shows a slide wire AB of length 1m and resistance 0.04Ωcm^{-1}



When K_1 and K_2 are open, galvanometer shows no deflection when AD = 90.0cm. When K_1 is open and K_2 is closed, AD changes to 60.0cm. Find

(i) E.m.f cell C
(ii) Internal resistance of cell C
(iii) The value of Ad when K_1 and K_2 are closed

Solution

K1 is open

R_{AB} = 0.04 x 100 = 4Ω

Total resistance = (4 +1 + 1) = 6Ω

Total current I = $\frac{e.m.f}{total\ resstance} = \frac{2.0}{6} = \frac{1}{3}\ A$

$V_{AB} = IR_{AB} = \frac{1}{3}\ x\ 4 = \frac{4}{3}V$

p.d. per cm k = $\frac{V_{AB}}{L} = \frac{\frac{4}{3}}{100} = \frac{5}{300} = \frac{1}{75}Vcm^{-1}$

(i) When K_1 is closed, K_2 is open

Total resistance = 4 + 1 = 5Ω

Total current = $\frac{2}{5}A$

$V_{AB} = IR_{AB} = \frac{2}{5}\ x\ 4 = 1.6V$

p.d. per cm k = $\frac{V_{AB}}{L} = \frac{1.6}{100} = \frac{2}{125}Vcm^{-1}$

kL = Ey = $\frac{1}{75}\ x\ 90$ = 1.5V

∴ e.m.f of a cell = 1.5V

(iv) When K_1 is open and K_2 closed L = 60cm

From the formula, I = I = $\frac{E}{R+r}$

I = $\frac{ER}{R+r}$ = kL

R = 2Ω, E = 1.5V

⇨ $\frac{1.5\ x\ 2}{2+r} = \frac{1}{75}\ x\ 60$

⇨ $r\ = 1\Omega$

∴ internal resistance of a cell = 1.75Ω

(v) Balance length if both K_1 and K_2 are closed

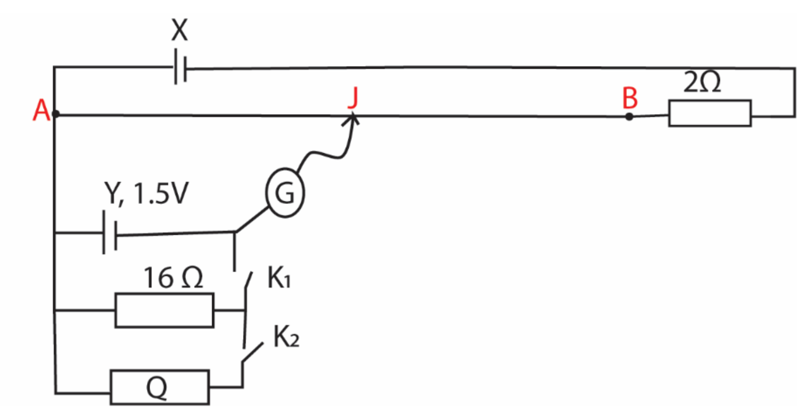
kI = IR = $\frac{ER}{R+r}$

$\frac{2}{125}L = \frac{1.5\ x\ 2}{2+1.75}$

L = 50.0cm

Example 32

In the figure, AB is a uniform slide wire 1.2m long and having resistance 4Ω. Y is a cell of e.m.f 1.5V and having unknown internal resistance.



When both K1 and K2 are open G shows no deflection when AJ = 90.0cm; when K1 is closed and K2 open, G shows no deflection when AJ = 88.3cm. When Both K1 and K2 are closed, G shows no deflection when AJ = 87.3. Find

(i) The current flowing through the slide wire
(ii) The e.m.f of cell X
(iii) Internal resistance of Y
(iv) The value Q

K1 is open

R_{AB} = 4Ω

Total resistance = (4 + 2) = 6Ω

Total current I = $\frac{e.m.f}{total\ resstance} = \frac{E_x}{6}$ ....(i)

(i) When K_1 and K2 are open, L= 90
At balance
$E_Y = V_{AJ} = IR_{AJ}$ = 1.5V .... (ii)
Resistance per cm = $\frac{4}{1.2\ x\ 100} = \frac{1}{30}\Omega cm^{-1}$
RAJ = $\frac{1}{30}\ x\ 90 = 3\Omega$
From eqn. (ii)
I x 3 = 1.5
I = 0.5A

(ii) From eqn.
Ex = IR = 6 x 0.5 = 3V

∴ e.m.f of cell X = 3V

(iii) When Ki is closed, K2 is open, L = 88.3cm
At balance, IR = $V_{AJ} = IR_{AJ}$
From circuit formula
I = $\frac{E}{R+r}$
Then
$\frac{ER}{R+r}$ = IR_{AJ}; but $R_{AJ} = \frac{1}{30}\ x\ 88.3$
$\frac{1.5\ x\ 16}{16+r} = 0.5\ x\ \frac{1}{30}\ x\ 88.3$
$r = 0.3\Omega$

(iv) When K1 and K2 are closed, L= 87.3cm, let the effective resistance R_1

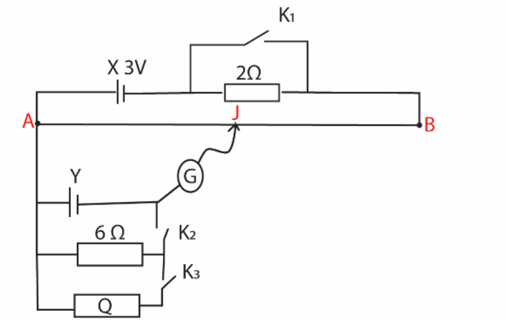
From circuit formula
I = $\frac{E}{R+r}$
Then
$\frac{ER}{R+r}$ = IR_{AJ}; but $R_{AJ} = \frac{1}{30}\ x\ 87.3$
$\frac{1.5\ x\ R_1}{R_1+0.3} = 0.5\ x\ \frac{1}{30}\ x\ 87.3$
$R_1 = 9.7\Omega$

But
$\frac{1}{R_1} = \frac{1}{16} + \frac{1}{Q}$

$\frac{1}{9.7} = \frac{1}{16} + \frac{1}{Q}$
Q = 24.63Ω

Example 32

In the figure X is accumulator of e.m.f 3V and having negligible internal resistance. AB is a uniform slide wire 1.2m long and having a resistance 4Ω. Y is a cell of unknown e.m.f and internal resistance



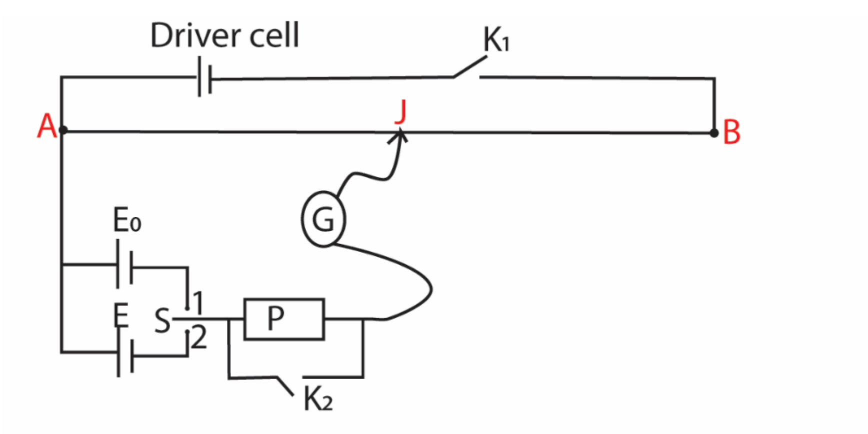
When K_1, K_2 and K_3 are open the balance length, AJ = 90cm
When K_1, K_2 are closed and K_3 is open the balance length, AJ = 43.9cm
When K_2, K_3 are closed and K_1 are open the balance length, AJ = 58.1cm

Find

(i) The e.m.f of cell Y (ans. 1.5V)
(ii) Internal resistance of a cell Y (ans. 0.2Ω)
(iii) Value Q (ans. 12Ω)
(iv) Balancing length when K1, K2 and K3 are all closed (ans.38.71cm)

Comparison of e.m.f

The e.m.f of two cells can be compared using circuit below:



P= protective resistor

E_0= standard cell

E = cell of unknown e.m.f

G center zero galvanometer

Procedure

(a) The standard cell is first balanced against the slide wire. This is done as follows:

- K_1 if closed and S is put in position 1 with K_2 open, the jockey is placed at different places along the slide wire until the galvanometer indicates zero deflection.
- An approximate balance point is obtained, K_2 is closed to obtain the actual balance length L_0.
  $E_0 = kL_0$ …………………………..(i) (where k is the p.d per cm of the slide wire.

(b) S is placed in position 2 and the new balance length L is obtained

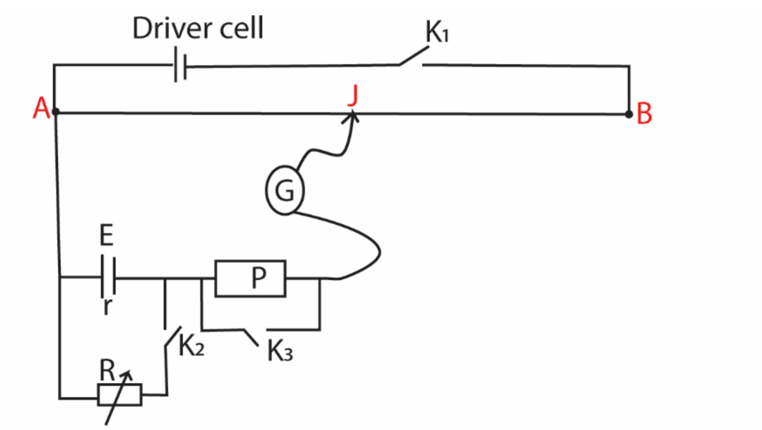
$E = kL$ ………………………… (ii)

Dividing (ii) by (i)

$$\frac{E}{E_0} = \frac{L}{L_o}$$

Or $E = \frac{L}{L_o}\, x\, E_0$

Experiment to determine internal resistance of a cell



P = protective resistor

R= variable resistor

G = center zero galvanometer

E = dry cell whose internal resistance is required

J = jockey

Procedure

(a) The dry cell is first balanced against the slide wire as follows:
K_1 is closed while K_2 and K_3 are left open, the jockey is placed in different places on the slide wire until the galvanometer shows no deflection. K_3 in the closed and the actual balance length L_o is noted.
At balance
$E = kL_0$ ……………………………(i) where k is the p.d per cm

(b) The variable resistor is adjusted to a suitable value and the p.d across it is balanced against the slide wire as follows
With K_1 and K2 closed, K_3 open, an approximate balance length is obtained. K_3 is then closed and actual balance length, L is obtained.
At balance
IR = kL ……………………………..(ii) where I is current flowing through the resistor

The experiment is repeated for different values of R and corresponding lengths, L are recorded.
The results tabulated including values of $\frac{1}{R}$ and $\frac{1}{L}$
A plot of $\frac{1}{R}$ against $\frac{1}{L}$ give a straight line with intercept = $\frac{-1}{r}$

Theory of the experiment

E = kL0 …………………………………………………..(i)

IR = kL ………………………………………………… (ii)

From the formula

I = $\frac{E}{R+r}$

⇨ $\frac{ER}{R+r} = kL$……………………………..(iii)

Divide (i) by (iii)

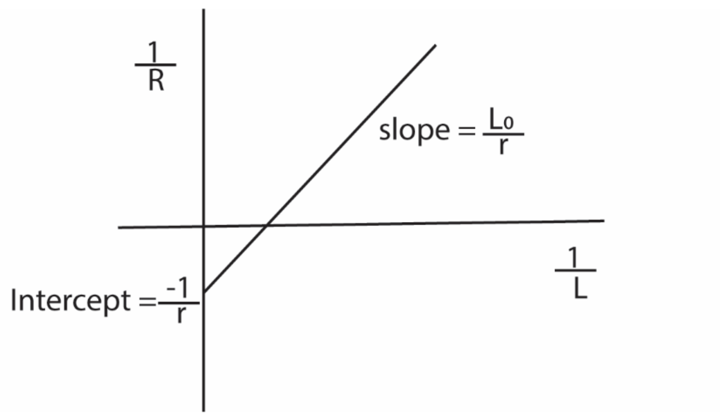
$\frac{(R+r)E}{ER} = \frac{L_0}{L}$

$\frac{R+r}{R} = \frac{L_0}{L}$

$1 + \frac{r}{R} = \frac{L_0}{L}$

$\frac{1}{R} = \left(\frac{L_0}{r}\right)\frac{1}{L} - \frac{1}{r}$

A graph of $\frac{1}{R}$ $against$ $\frac{1}{L}$



Example 33

Outline the principles of a slide potentiometer

Solution

A potentiometer consists of a uniform slide wire which can be any length (about 1m long) connected with a driver cell which maintains a steady current through the wire. Since the wire is uniform, has a constant resistance per cm, driver cell supplies a constant p.d per cm of the wire or uniform p.d so that p.d across any length, L. of the wire is directly proportional to the length.

i.e. $V \propto L$ or V = kL

the unknown p.d is balanced against the slide wire by placing the jockey along the slide wire until the galvanometer shows no deflection.

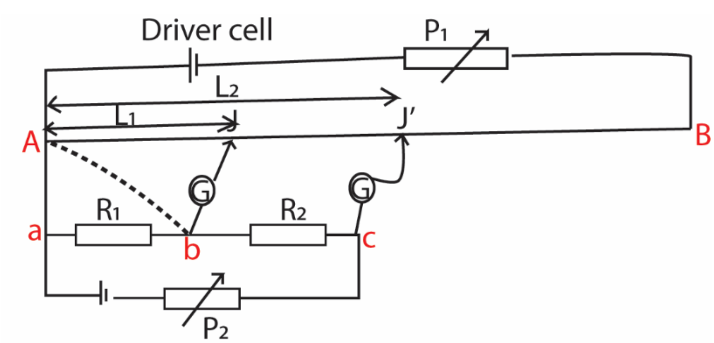
In order to balance, the unknown p.d must be connected in the opposition with the driver cell.

Example 34

With the aid of a circuit diagram, describe how you would determine the internal resistance of a dry cell using a potentiometer.

Comparison of resistance using a potentiometer

The resistance can be compared using the circuit below



G = center zero galvanometer

P_1 and P_2 = rheostat

R_1 = standard resistor

R_2= unknown resistor

Procedure

With connections at a and b (shown by full lines,) the p.d across R1 is balanced against the slide wire. The jockey is placed at different points along the slide wire until the galvanometer shows no deflection. The balance length L_1 is noted.

At balance

$IR_1 = kL_1$ …………………… (i)

Where k is the p.d per cm of the slide wire and I is the current flowing through the resistance.

The connection at a and b are now replaced by those at b and c (shown by dotted line). The p.d across R_2 is now balanced against the slide wire. The balance length $L_2$is noted

At balance

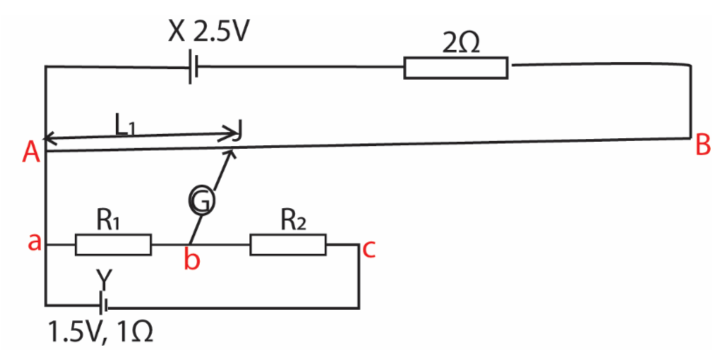
$IR_2 = kL_2$ ……………………………………….(ii)

Divide eqn. (i) by eqn. (ii)

$$\frac{R_1}{R_2} = \frac{L_1}{L_2}$$

Example 35

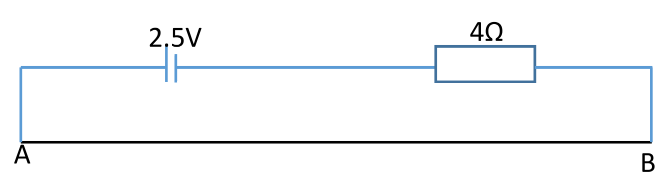
In the figure X is an accumulator of e.m.f 2.5V and having negligible internal resistance. Y is a cell of e.m.f 1.5V having internal resistance 1Ω.



AB is a uniform wire 1m long and having a resistance of 40Ω. With the connection at a and b a balance length of 39.8cm was obtained. When the connection at a and b were replaced by those at a and c, a balance length of 82.4cm was obtained. Find

(i) Current flowing through slide wire
(ii) The resistance R_1 and R_2.

Solution



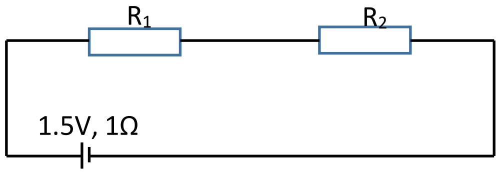
Total resistance = 4 + 2 = 6Ω

Steady current flowing

I = $\frac{e.m.f}{total\ resistance} = \frac{2.5}{6} = \frac{5}{12}A$

Resistance per cm = $\frac{4}{100} = 0.04\Omega cm^{-1}$

p.d per cm, k = IR/cm = $\frac{5}{12}\ x\ \frac{4}{100} = \frac{1}{60}Vcm^{-1}$



Total resistance = $R_1 + R_2 + 1$

Current flowing, I = $\frac{1.5}{R_1 + R_2 + 1}$ ……………………………..(i)

with connections at a and b

At balance

$IR_1 = kL_1$; L_1 = 39.8cm

$IR_1 = \frac{1}{60}\ x\ 39.8 = 0.6633$ ……………………………(ii)

With connections at a and c

$I(R_1 + R_2) = kl_2 = \frac{1}{60}\ x\ 82.4 = 1.3733$ ………… (iii)

From eqn. (i)

$I(R_1 + R_2 + 1) = 1.5$ ……………………………………… (iv)

Substituting eqn. (iii) into eqn. (iv)

$I(R_1 + R_2) + I = 1.5$

1.3733 + I + 1.5

I = 0.1267A

From eqn. (ii)

$R_1 = \frac{1.6633}{0.1267}$ = 5.24Ω

From eqn. (iii)

$R_1 + R_2 = \frac{1.3733}{0.1267}$ = 10.84Ω

R_2 = 10.84 – 5.24 = 5.60Ω

Possible problems of the experiment

If balance point cannot be obtained, then either the p.d. per cm is too small or too big. The variable resistor P1 is adjusted until measurable balance length is obtained
Or
The p.d across the resistances may be too small or too big. The variable resistor P2 is adjusted until balance point is restored. (The variable resistors P_1 and P_2 are adjusted until balance point is obtained.

Note

P_1 adjust the current through the slide wire or current delivered by the driver cell and consequently this changes p.d per cm of the slide wire and directly affects the balance point.

P_2 adjusts the current through the resistors and hence the p.d across them.

If the resistances to be compared are very small, then the balance length L1 and L2 will be very small. An end error correction needs to be added for accurate results.

$\frac{R_1}{R_2} = \frac{L_1}{L_2}$ becomes inaccurate

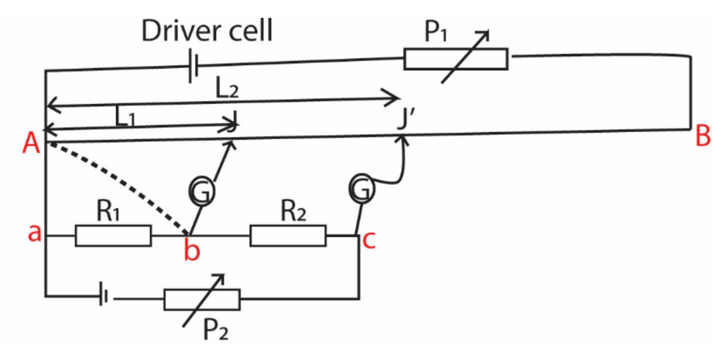
For accuracy, $\frac{R_1}{R_2} = \frac{L_1+e}{L_2+e}$

Note that

An end correction is the length of the slide wire which has the same resistance as the contact at the zero end or extreme end of the wire.

How to obtain end correction

- The p.d across R1 is balanced against the slide wire using the circuit below



- The balance length L1 is measured and recorded. Let e be end correction
  At balance
  $IR_1 = k(L_1 + e)$ ……………………………………….(i)
- The p.d across R_2 is now balanced against the slide wire and its balance say L_2 is noted
  At balance
  $IR_2 = k(L_2 + e)$ …………………………………. (ii)

- The p.d across both R_1 and R_2 is now balanced against the slide (with connection at a and c) and the balance length L_3 is noted.
  $I(R_1 + R_2) = k(L_3 + e)$ …………………………… (iii)
  The end correction of the wire is obtained from
  $(e = L_3-(L_1 – L_2))$

  Theory
  Add (i) and (ii)
  $I(R_1+ R_2) =k(L_1 + L_2 + 2e)$ …………………… (iv)
  Eqn. (iii) and (iv)
  $L_1+ L_2 + 2e = L_3 + e$
  $L_1+ L_2 + e = L_3$
  $e = L_3-(L_1 – L_2)$

  Note that:
- If the e.m.f out of the dry cell is greater than that of the driver cell, the balance point will not be obtained.
- Similarly, if the positive terminal of the driver cell is connected to the negative terminal of the dry cell the balance point cannot be obtained since current flows in one direction.

Example 36

When resistors 4Ω and 8Ω are connected respectively in the LHG and RHG of meter bridge, balance point is obtained at a point a distance 32.0cm from left hand of the bridge wire. When the resistors are interchanged a balance point of 68.0cm from the left is obtained.

The resistance of the slide wire is uniform equal to 5Ω. Calculate the end error.

Solution

If e_1 is the end error on the LHS and e_2 is the end error of RHS

From,

$\frac{R_1}{L_1+e}=\frac{R_2}{L_2+e}$

$\frac{4}{32+e_1} = \frac{8}{68+e_2}$

$2e_1 - e_2 = 4$ ……………………………………(i)

$on\ interchanging\ the\ resistors,$

$\frac{4}{68+e_1} = \frac{8}{32+e_2}$

$2e_2 - e_1 = 4$ ……………………………………(ii)

$solving\ simultaneous\ equation\ for\ eqn. (i) and\ (ii)$

$e_1 = e_2$ = 4cm

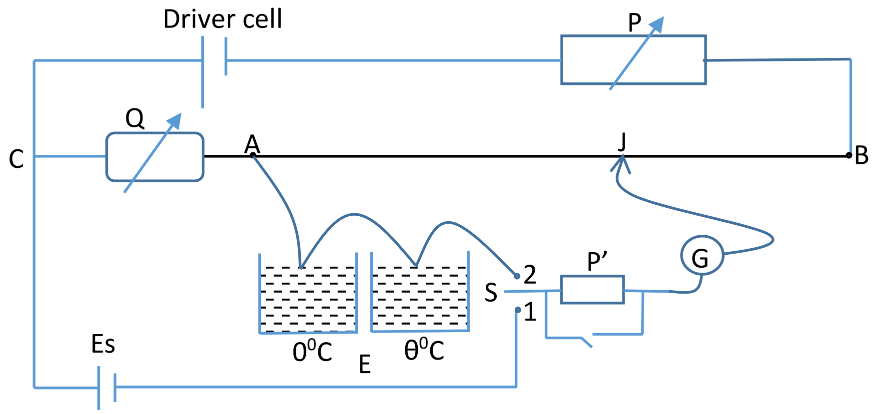
$$resisitance\ per\ cm = \frac{5}{100} = 0.05\Omega cm^{-1}$$

$$resistance\ e_1 = e_2 = 4\ x\ 0.05 = 0.2\Omega$$

Measurement of thermoelectric e.m.f

Thermoelectric e.m.fs are of order 10^{-3} volts or 10^{-4}volts. In the measurements of thermoelectric e.m.f, the uniform slide wire is connected in series with very high variable resistance and the driver cell so that the current flowing through the wire is very small and consequently the p.d across the wire will be of the same order as the thermoelectric e.m.f to be measured.

The circuit is arranged as shown below:



G – Center zero galvanometer

P' - protective resistor

Es- standard cell

P and Q variable resistors

S – Switch

E - Thermocouple

Procedure

- The variable resistor P and Q are first adjusted so that the current flowing through the slide wire is very small and consequently the p.d. across AB is of the same order as thermoelectric e.m.f
- S is put in position 1. With K open, the jockey is placed at different places of the slide wire until the galvanometer shows no deflection. P and Q are adjusted until a measurable/reasonable approximate balance length is obtained. K is then closed and actual balance length L_s obtained

  At balance
  $E_s = V_{CJ} = I(Q+ rL_s)$………………………………………. (i)

Where r is the resistance per cm of the slide wire

- The e.m.f of the thermocouple is now balanced against the slide when S is in position 2; the balance length L is obtained.

  At balance

  E = V_{AJ} = Irl ……………………………………………(ii)

  From (i) I = $\frac{E_s}{Q+rL_s}$ ……………………………… (iii)

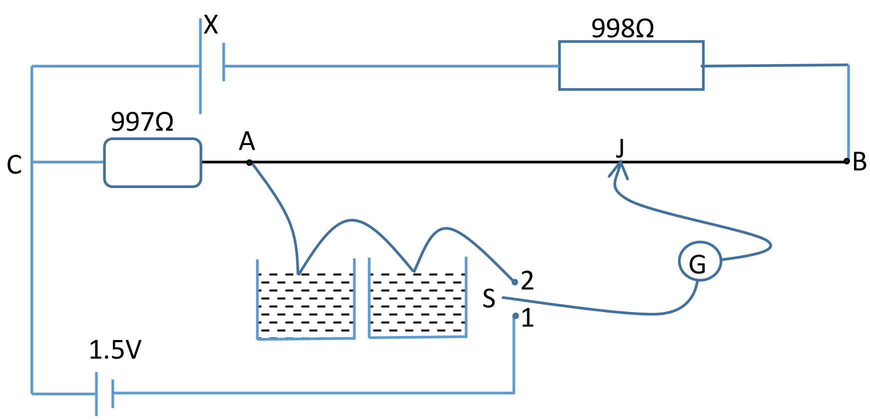
  Put (iii) in (ii)

  E = $\frac{E_s rL}{Q+rL_s}$

  Thus, the e.m.f of the thermocouple can be obtained fromE_s, r, L, Q and L_s which are known.

Example 37

In the figure AB is a uniform slide wire 1m long and having a resistance of 5Ω. X is a cell of unknown e.m.f and having negligible internal resistance.



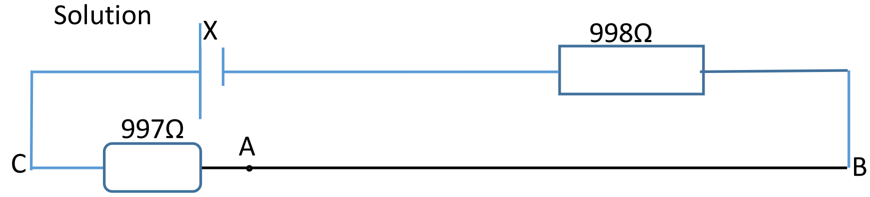
When S is put in position 1, G shows no deflection when AJ = 60.0cm. When S is put in position 2 a balance length of 40.0cmis obtained. Find

(i) The current flowing in the slide wire

(ii) The e.m.f of cell X

(iii) The e.m.f of the thermocouple

Solution



Resistance AB = 5Ω

Total resistance = 997 + 998 + 5 = 2000Ω

Steady current flowing

I = $\frac{e.m.f}{total\ resistance}$ = $\frac{E_x}{2000}$ ………………. (i)

At balance

$E_s = V_{CJ}$ = I(997 + R_{AJ})= 1.5

$\frac{resistance}{cm} = \frac{5}{100} = 0.05\Omega cm^{-1}$

L = 60cm

R_{AJ} = rL = 0.05 x 60 = 3Ω

I(997 + 3) = 1.5

I = $\frac{1.5}{1000} = 1.5\ x\ 10^{-3}A$

Current flowing I = 1.5 x 10^{-3}A

From (i) Ex = 2000I = 2000 x 1.5 x 10^{-3} = 3V

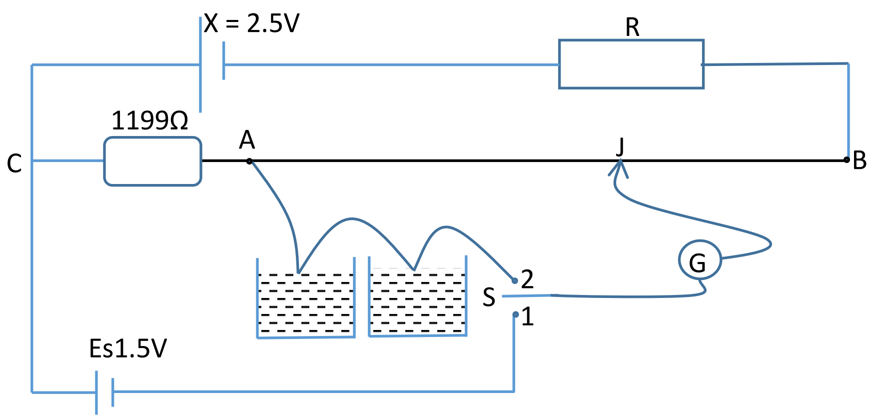
At balance e.m.f of thermocouple E = V_{AJ} = IR_{AJ}

R_{AJ} = rL= 0.05 x 40 = 2Ω

E = 1.5 x 10^{-3} x 2 = 3.0mV

Example 38

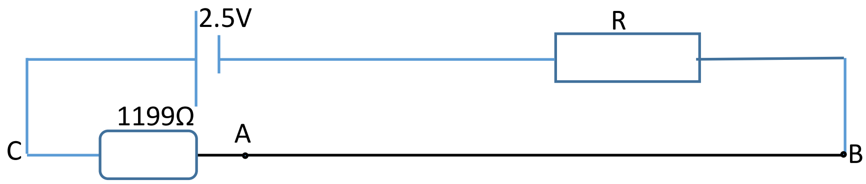
In the figure AB is a uniform slide wire 1m long and having a resistance of 4Ω. X is a cell of e.m.f 2.5V and having negligible internal resistance.



When S is put in position 1, G shows no deflection when AJ = 25.0cm. When S is put in position2, the balance length changes to 40cm. Find

(i) The current flowing
(ii) The resistance R
(iii) The e.m.f of thermocouple

Solution



Resistance of AB = 4Ω

Total resistance = (1199 + 4 + R)Ω

Steady current flowing

I = $\frac{e.m.f}{total\ resistance}$ = $\frac{2.5}{(1199+4+R)}$ ………………… (i)

At balance, Es = I(1199 + R_{AJ}) = 1.5

Resistance /cm = $\frac{4}{100} = 0.04cm^{-1}$

L = 25cm

R_{AJ} = rL = 0.04 x 25Ω = 1Ω

I = $\frac{1.5}{1200}$ = 1.25x 10^{-3}A

Current flowing = 1.25 x 10^{-3}A

(ii) From eqn. (i)

I = $\frac{2.5}{(1199+4+R)}$ = 1.25 x 10^{-3}A

R = 797Ω

(iii) At balance

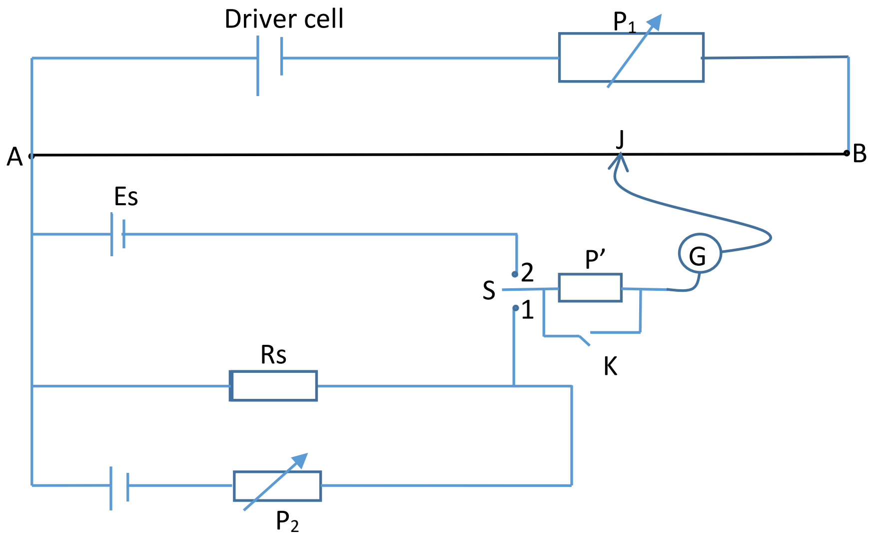
E = VAJ = IRAJ

R_{AJ} = rL = 0.04 x 40 = 1.6Ω

E = 1.24 x 10^{-3} x 1.6 = 2mV

Measurement of current

The current flowing through a resistor can be determined by measuring the potential difference which it sets across a standard resistor. The circuit is arranged as shown below:



Procedure

(i) The slide wire is first calibrated so that p.d per cm is known. This is done as follows:

- With S in position 1, and K open, the standard cell is balanced against the slide wire.
- The jockey is placed at different points along the slide wire until the galvanometer shows no deflection. K is then closed and actual balance length, Ls is measured and recorded.

At balance

Es = kLs where k is the p.d per cm

k = $\frac{E_S}{L_S}$ ………………………………….. (i)

(ii) The p.d across a standard resistor is now balanced against the slide wire as follows

- With S in position 2 and K open, the rheostats are adjusted the experiment repeated until a measurable approximate balance length obtained, K is then closed, the actual balance length say L is measured and recorded

At balance

I_aR_s = kL ……………………….(ii)

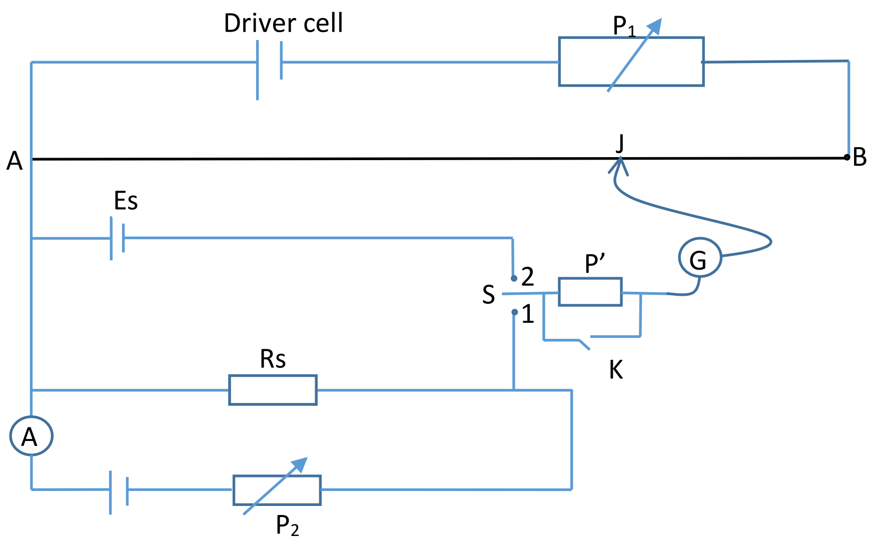
Where I_a is the actual current flowing in the resistors

$I_a = \frac{E_S L}{R_S L_S}$

Thus, the current flowing in the resistors can be obtained from values of Es, Ls, Rs and L.

Calibration of an ammeter

The circuit is arranged as shown below



P_1 and P_2 = rheostat

Rs = Standard resistor

G = Center zero galvanometer

Es – Standard cell

AB – Uniform slide wire

A – Ammeter to be calibrated

Procedure

- The slide wire is first calibrated so that its p.d per cm is known. This is done as follows: with S put in position 1, and K open, the standard cell is balanced against the slide wire until the galvanometer shows no deflection. K is then closed and the actual balance Ls is measured and recorded.
  At balance
  Es = kLs ………………………… (i) where k is the p.d. per cm
- The p.d across a standard resistor is now balanced against the slide wire. This is done as follows: with S put in position 2 and K open, the rheostats are adjusted until the ammeter indicates a suitable reading I and an approximate balance length is obtained. K is then closed and the actual balance length L is measured.
  At balance
  I_aR_s =kL ……………………..(ii) where I_a is the current flowing through the resistors.

$$I_a = \frac{E_s L}{R_s L_s}$$

- The experiment is repeated for different ammeter readings I and corresponding balance length, L are measured and recorded. The results are tabulated including values of I_a.

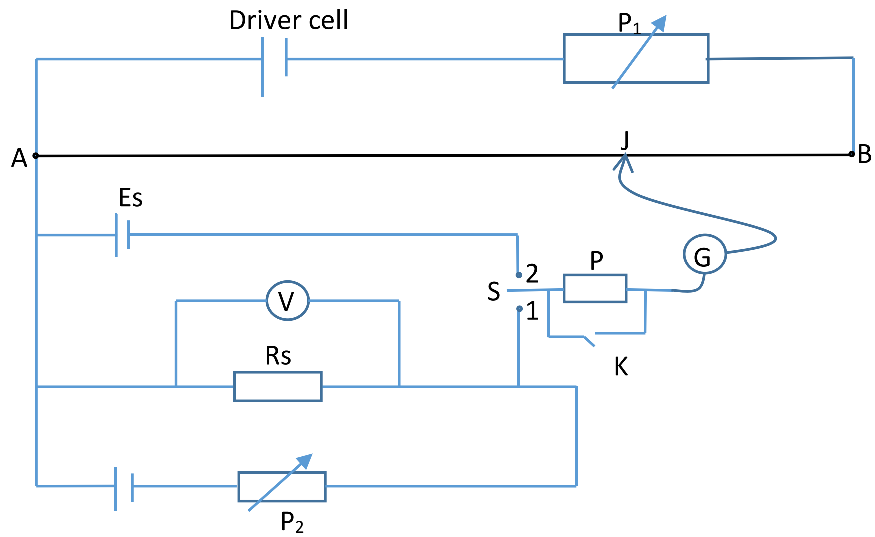
Table of results

I(A)	L	I_a

A graph of I_a against I is plotted. This gives a calibration curve for the ammeter.

Calibration of a voltmeter

The circuit is arranged as shown below



P_1 and P_2 = rheostat

Rs = Standard resistor

G = Center zero galvanometer

Es – Standard cell

AB – Uniform slide wire

V – Voltmeter to be calibrated

J – sliding contact (jockey)

Procedure

The standard cell is first balanced against the slide wire. This is done as follows:-

With S placed in position 1 and K open, the jockey is tapped at different places of the slide wire until the galvanometer shows no deflection. This is an approximate point.

K is closed and actual balance length Ls is measured.

At balance

$E_s = kL_s$ ………………….(i) where k is the p.d per cm.

The p.d across the standard resistor is now balanced against the slide wire as follows
With S placed in position 2 and K open, the rheostat P_2 is first adjusted until the voltmeter indicates suitable reading.

The experiment is repeated and an approximate balance length is obtained. K is closed and actual balance length L is measured.

If Va is the actual p.d. across, then

At balance

$V_a = kL$ ……………. (ii) but $k = \frac{E_S}{L_S}$

$V_a = \frac{E_S L}{L_S}$ …………………..(iii)

- The experiment is repeated for different voltmeter readings and the corresponding balance lengths, L are obtained.
- The results are tabulated including values of V_a.

Table of results

V(V)	L(cm)	$V_a = \frac{E_S L}{L_S}$

A graph of V against V is plotted. This gives a calibration curve for the voltmeter.

Advantage of potentiometer

- At balance it does not draw current from the circuit and hence the resistance of the connecting wires are irrelevant in the calculations.
- It is more accurate compared to a voltmeter since at balances and does not draw current.
- It can measure a wide range of voltages only limited by the e.m.f of driver cell

Disadvantages of a potentiometer

- It does not give direct readings
- It requires a skilled person to operate
- It is bulky

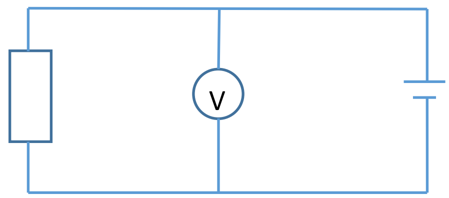
Example 39

(a) Define current density and the ohm and state their units.

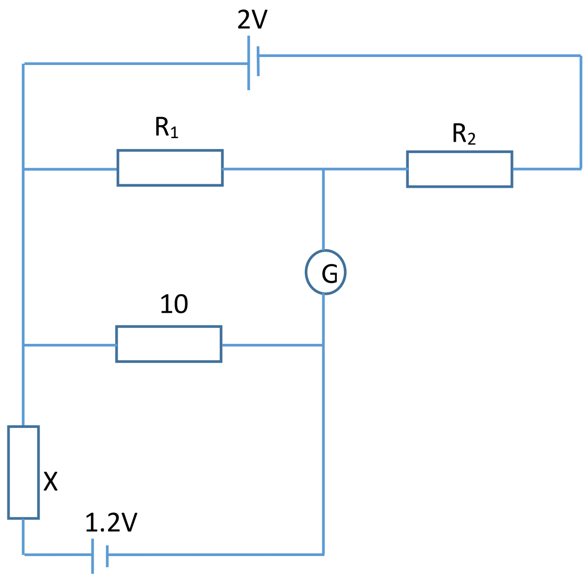
(b) (i) Sketch the current versus voltage characteristic for a gas discharge tube.

(ii) Explain the main features of the graph in (b)(i).

(c) The figure shows a cell of e.m.f, E and internal resistance, r, connected to a voltmeter, V and variable resistance, R. explain how the value of V varies with R.



(d) In figure R1 and R2 are resistor of 10Ω and 90Ω respectively. If the cells have negligible internal resistances, find the value of X at which G shows no deflection



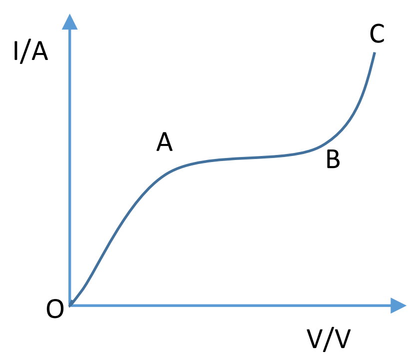
(e) Describe how the internal resistance of a cell can be measured using a slide wire potentiometer

Solution

(a) Current density is the amount of current flowing through a conductor having a cross sectional area of $1m^2$.

Ohm is the resistance of a conductor that has a p.d of 1V across it when carrying a current of 1A.

(b) (i)



(ii) Along OA the p.d is low so the electrons and the positive ions have low velocities. Consequently the current flowing is low. The current flowing is also proportional to the applied p.d so ohms law is obeyed

Along AB, all ions produced by the ionizing agent are collected by the electrode. So a constant current called saturated current flows

Also in this region the different velocity is sufficient to prevent appreciable recombination of positive ions and electrons.

Along BC the p.d is high enough to enable the electrons to have sufficient K.e for knocking electrons out of the electrons they collide with. At higher p.d even the electrons produced by collision acquire enough K.e to enable them to cause ionization when they collide with neutral atoms. So there is uncontrollable growth of current and the gas is said to be broken down

(c) As R increases, I decreases, since r is constant it implies that the quantity Ir decrease. But because E is constant it follows that V = E – Ir will increase.

When R decreases, I increases, V decreases

(d) Suppose current flowing in driver circuit is I

$I = \frac{2}{(10+90)} = 0.02A$

p.d across $R_1 = IR_1 = 0.02 \times 10 = 0.2V$

p.d across 10Ω =0.2V

but also p.d across 10Ω resistor = $0.2 = \frac{10}{(x+10)}\, x\ 1.2$; x = 50Ω

Thank you

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