P425/1
PRINCIPAL MATHEMATICS
Paper 1
2026

UGANDA NATIONAL EXAMINATIONS BOARD
Uganda Advanced Certificate of Education

PRINCIPAL MATHEMATICS — PAPER 1
3 HOURS

INSTRUCTIONS TO CANDIDATES:

- This paper consists of three sections: A, B, and C.
- Candidates must observe the following choices:
  - From Section A (Algebra), attempt one item.
  - From Section B (Geometry), attempt one item.
  - From Section C (Calculus), attempt two items, selecting exactly one item from each designated sub-part.
- A total of four items must be answered.
- All items carry equal marks.
- Graph paper and relevant mathematical tables are provided where necessary.
- Silent, non-programmable scientific calculators may be used.
- Clear, logically step-by-step mathematical working must be shown. Statistical or final answers without structural proof may not award marks.

SECTION A: ALGEBRA

(Attempt exactly one item from this section)

ITEM 1

A national agricultural cooperative monitors the seasonal output of maize across three regional zones in Uganda: Albertine, Central, and Eastern. During three consecutive agricultural quarters, the logistical transport yields satisfy a system of linear equations mapping total output capacity:

$x + y + z = 12$

$2x + y + 3z = 25$

$x + 3y + 2z = 23$

Where x, y, and z represent the metric tonnage produced by the Albertine, Central, and Eastern regions respectively.

(a) Formulate the system of equations into a matrix formulation of the form AX = B.
(b) Determine the specific inverse of the coefficient matrix A^{-1}.
(c) Compute the exact metric tonnage distribution for each of the three agricultural zones using your matrix inverse solution.

ITEM 2

An engineering firm designs an array of corporate telecommunication signals. The mechanical stress threshold S(t) over time depends heavily on finding the complex root profiles of the specialized baseline characteristic function:
$Z^3 - 4Z^2 + 6Z - 4 = 0$

(a) Given that Z = 1 + i is a confirmed complex root of the characteristic polynomial, determine all remaining roots over the complex plane C.
(b) Plot these roots clearly on an Argand diagram.
(c) Find the modulus and principal argument of each root, expressing them in standard polar form $r(\cos\theta + i\sin\theta)$.

SECTION B: GEOMETRY

(Attempt exactly one item from this section)

ITEM 3

A logistics company maps drone flight paths between a high-altitude loading station and a delivery point. The flight paths follow straight-line vector trajectories in a Cartesian coordinate space. Line L_1 passes through the point A(1, -2, 5) and is parallel to the direction vector $d_1 = 2i + j - k$. Line L_2 passes through the point B(3, 1, 0) and is parallel to the direction vector $d_2 = i - j + 2k$.

(a) Write down vector parametric equations for both flight lines L_1 and L_2.
(b) Show conclusively whether or not the two flight lines intersect in space.
(c) Calculate the shortest distance between the two lines.

ITEM 4

A construction layout requires mapping out a precise parabolic drainage channel on a building plot. The edge boundary line of the drainage zone satisfies the general conic locus equation:
$y^2 - 4y - 8x + 28 = 0$

(a) Transform the equation into the standard vertex form $(y - k)^2 = 4p(x - h)$.
(b) State the exact coordinates of the vertex, the focus, and the equation of the directrix line.
(c) Sketch the conic boundary section on standard coordinate axes, marking the focal point and axis of symmetry clearly.

SECTION C: CALCULUS

(Attempt exactly two items, selecting one from Part I and one from Part II)

PART I: DIFFERENTIAL CALCULUS

ITEM 5

An industrial manufacturing facility fabricates closed cylindrical aluminum canisters designed to store liquid fertilizer. The corporate engineering specification mandates that each canister must hold a strict constant internal volume of exactly $V = 54\pi \text{ cm}^3$.

(a) Show that the total surface area A of the canister as a function of its base radius r is given by: $A = 2\pi r^2 + 108\pi/r$
(b) Using differential calculus, determine the exact dimensions (radius r and height h) that minimize the total surface area of aluminum required per canister.
(c) Verify that your calculated dimensions represent a true minimum surface area configuration.

ITEM 6

A chemical reaction model maps the concentration profile of a compound dissolving into a solute mixture over time. The concentration curve follows the specific parameterized implicitly defined function:
$x^3 + y^3 - 3axy = 0$

(a) Find an explicit expression for the first derivative dy/dx in terms of x, y, and the constant parameter a.
(b) Determine the exact coordinates of any points on the curve where the tangent line is perfectly horizontal (parallel to the x-axis).
(c) Find the equation of the normal line to the curve at the specific coordinate boundary point where $x = 3a/2$ and $y = 3a/2$.

PART II: INTEGRAL CALCULUS

ITEM 7

A civil engineering design team is calculating the surface volume of a concrete architectural arch structure. The cross-sectional boundary profile of the arch is bounded by the curves $y = x^2$ and $y = 2x - x^2$.

(a) Find the precise coordinate points of intersection where these two geometric boundary lines meet.
(b) Sketch the enclosed regional area mapped between the intersecting curves.
(c) Calculate the exact volume of the solid generated when this enclosed region is rotated through 360° (one full revolution) about the horizontal x-axis line.

ITEM 8

An electronics testing lab tracks a fluctuating power surge running through an automated grid node. The mathematical model for the electrical energy dissipation curve over a tracking interval requires solving the definite integral:

I = ∫[0 to π/2] (x^2 sin x) dx

(a) Evaluate the absolute value of the integral I using the method of integration by parts.

(b) Approximate the exact same integral using the Trapezoidal Rule with 4 equal intervals (sub-intervals) to 3 decimal places.

(c) Compute the percentage error introduced by the trapezoidal approximation compared to your exact analytical evaluation in part (a).