1. An online banking platform uses two distinct machine learning models to scan transactions flag risks: Model X (flags suspicious velocity) and Model Y (flags irregular geo-location), which run in parallel on the same dataset. The likelihood that Model X and Model Y flag a transaction are 0.22 and 0.15 respectively. The likelihood that at least one of the models flags a transaction is 0.31.
    a) Find the probability that a transaction is flagged by both Model X and Model Y simultaneously. Verify if these two evaluation streams independent.
    b) Given that Model Y has already flagged a transaction, what is the probability that Model X will also flag it?
2. An automotive assembly plant depends on the on-time delivery of semiconductor microchips and lithium batteries. Logistics bottlenecks are unpredictable. The probability that the chip delivery is delayed is 0.45. The probability that the battery delivery is delayed is 0.35. The probability that only the microchip delivery is delayed and the battery arrives exactly on time) is 0.25.
    i. Calculate the probability that both shipments experience a delay at the same time.
    ii. Find the probability that the assembly plant experiences absolutely no disruptions (both deliveries arrive safely on time).
    iii. Given that the chip shipment arrives successfully on time, what is the chance that the battery shipment is delayed?
3. A cloud-based gaming server experiences performance bottlenecks caused by three separate, unrestricted infrastructure strains: High Player Concurrency (A), Network Packet Routes Dropping (B) and Database Query Lag (C). A diagnostic log reveals the following data: $P(A) = 0.50$, $P(A \cap B) = 0.18$, $P(B) = 0.40$, $P(A \cap C) = 0.15$, $P(C) = 0.35$ and $P(B \cap C) = 0.12$. The probability that all three infrastructure strains occur at the exact same peak minute is 0.05.
    i. Find the probability that a server experiences at least one of these three strains during a peak minute.
    ii. Calculate the probability that a server experiences only Network Packet Drops while the other two systems run perfectly.
    iii. Given that the server is experiencing Database Query Lag, what is the probability that it is free from both high player concurrency and packet drops?
4. A tech company is launching a major software update. The chance that a user experiences a user-interface (UI) glitch is 70%. The chance that they experience a database connection error is 0.2. Given that a user encounters a database connection error, the chance that they also experience a UI glitch is 0.1.
    a. Find the probability that a user experiences either a UI glitch, a database error or both.
    b. Find the probability that a user experiences a UI glitch but does not experience a database error.
5. A streaming platform analyses consumer viewing habits regarding movie content and live sports. The probability that a randomly chosen subscriber watches live sports is $\frac{1}{8}$. One in every ten subscribers watches both movies and live sports. Given that a subscriber watches movies, the chance that they also watch live sports is $\frac{1}{3}$.
    a) Determine the probability that a subscriber watches movies.
    b) Determine the probability that a subscriber watches movies, live sports, or both.
    c) Given that a subscriber does not watch live sports, determine the probability that they watch movies.
6. A busy retail agent in Wandegeya tracks whether daily customers make transactions using MTN Mobile Money or Airtel Money. There is a 60% chance that a customer uses MTN MoMo. If a

customer uses Airtel Money, there is a $\frac{5}{7}$ chance that they do *not* use MTN MoMo. The probability that a customer uses either MTN or Airtel (or both) is 0.85. The remaining 0.15 of customers use cash or banking apps.

a. Find the chance that a customer uses:
   i. MTN only
   ii. Both MTN and Airtel for their transaction.
   iii. Airtel only.
b. Find the likelihood that a customer uses MTN MoMo, given that they have already used Airtel Money.

7. A weekend sports and entertainment analyst in Kampala evaluates two events: the Uganda Cranes winning their upcoming fixture and a local artist selling out Lugogo Cricket Oval. The Cranes winning is twice as likely to happen as the concert selling out. The probability of the Uganda Cranes winning 0.5.

i. Prove that if the Cranes winning is independent of the concert's selling out, they're not happening are also independent.
ii. Find the probability that either the Uganda Cranes win or the concert sells out.
iii. The chance that both events happen, given that the Uganda Cranes have won their match.
iv. The chance that the Cranes do not win or the Concert does not sell out.

8. A coffee processing factory in Masaka uses two independent grading systems, Sensor P and Sensor Q, to detect export-quality coffee beans. The probability that both independent sensors flag a coffee batch as premium grade together is 0.125. The probability that either sensor or both sensors flag the batch is 0.625. Find the individual probabilities of each independent grading system successfully identifying premium coffee.

9. The constitutional rules at a secondary school in Gayaza state that a student cannot hold more than one major post. A student is randomly selected from the student council to assess leadership distribution. The probability of selecting a School Prefect is 0.4 and the probability of selecting a Sports Captain is 0.5. Find the probability that the selected student is:

a. Either a School Prefect or a Sports Captain.
b. A School Prefect but not the Sports Captain.
c. Neither of the two student leaders.

10. A commercial poultry farmer in Wakiso district manages risks on his farm. He monitors two completely independent operational threats: a Newcastle disease outbreak and a commercial feed shortage. The probability of a Newcastle disease outbreak is 0.45. The combined probability that the farm experiences either a disease outbreak or a feed shortage (or both) is 0.8.

a. Calculate the individual probability of a commercial feed shortage.
b. Calculate the probability that the farm does not get the disease outbreak or experiences a feed shortage.

11. An innovation hub in Kampala tracks the progress of local mobile app start-ups. They look at two milestones: securing seed funding and expanding service to the Mbarara regional market. The probability of a start-up expanding to Mbarara is $\frac{1}{3}$. The probability of securing funding and expanding to Mbarara is $\frac{1}{12}$. Given that a startup secures seed funding, the probability of expanding to Mbarara is $\frac{1}{3}$.

a. Calculate the probability of a start-up securing funding
b. Calculate the likelihood of securing the seed funding and expanding to Mbarara.

c. What is the chance of securing the seed funding or not expanding to Mbarara?
d. State with mathematical reasons whether securing funding and regional expansion are independent or mutually exclusive.

12. A clinic screens patients for two related health indicators: high cholesterol and high blood pressure. One in every five randomly selected patients has high cholesterol and one in every four randomly selected patients has high blood pressure. The likelihood that a patient has high cholesterol given that a patient also has high blood 40%.
a. Find the probability that a patient has both high cholesterol and high blood pressure.
b. Find the probability that a patient has high cholesterol or high blood pressure.

13. An automated assembly line uses two completely independent laser sensors, Sensor X and Sensor Y, to detect product flaws. The probability that Sensor X fails to detect a flaw is 0.2 short of that of Sensor Y failing to detect a flaw. There is a 65% chance that at least one of the sensors fails to detect a flaw. Find the baseline failure value of sensor X.

14. An enterprise data centre relies on a Primary Firewall and a Secondary Intrusion Detection System to protect its main server. Historically, the chance that a sophisticated cyber-attack breaches the Primary Firewall is 8%. If the Primary Firewall is breached, the panic-mode routing changes the network structure. The likelihood that the Secondary System fails *given* that the Primary Firewall has already been breached is 15%.
Conversely, if the Primary Firewall successfully deflects the attack, the Secondary System operates at its baseline vulnerability, with a failure probability of 0.02.
At the same time, the server might experience a completely separate Internal System Crash. However, a crash and an external breach cannot physically happen at the exact same millisecond due to system locking protocols.
i. Determine the absolute probability that the Secondary Intrusion Detection System fails during an attack.
ii. Verify whether the events "Primary Firewall breached" and "Secondary System fails" are independent.
iii. What is the exact mathematical relationship between the internal system crash and an external breach?

15. An AI-driven hospital intake system categorizes arriving emergency patients into exactly three structural urgency tiers: **Immediate** Tier ($\boldsymbol{T_1}$), **Urgent** Tier ($\boldsymbol{T_2}$) or **Standard** Tier ($\boldsymbol{T_3}$). Every incoming patient must be assigned to exactly one tier. The administration establishes that the likelihood of admitting standard tier patients is twice that of admitting immediate tier patients, and the likelihood of admitting urgent tier patients is three times that of admitting immediate tier patients. Patients receive an intensive care unit (ICU) bed allocation within 2 hours according to their tiers. Immediate, Urgent and Standard tier patients have 80%, 30% and 5% chances of being allocated ICU beds respectively.
a. Explain why the events $\boldsymbol{T_1}$, $\boldsymbol{T_2}$ and $\boldsymbol{T_3}$ are considered both mutually exclusive and exhaustive. Find the baseline probability for each tier.
b. Calculate the total probability that any randomly arriving patient will receive an ICU bed allocation.
c. If a patient ends up needing an ICU bed allocation within 2 hours, what is the probability they were originally misclassified into the lowest priority tier, $\boldsymbol{T_3}$?

16. A Mars transport rover uses two separate communications antennae: a High-Gain Antenna and a Low-Gain Antenna. Due to atmospheric dust interference, the operational status of one antenna affects the environmental load on the other. The probability that the High-Gain Antenna operates

perfectly throughout the landing phase is 0.92. If the High-Gain Antenna fails, the power surge increases the strain on the Low-Gain backup, resulting in a conditional failure probability of 0.25. If the High-Gain Antenna operates perfectly, the Low-Gain Antenna functions with a normal operational success probability of 0.95.

i. Find the probability that the Low-Gain Antenna fails during the landing phase.
ii. Calculate the probability that the rover completely loses all communications during landing.
iii. Given that the Low-Gain Antenna successfully operationalizes during landing, find the probability that the High-Gain Antenna has failed.

17. A prominent wholesale trader in Owino Market buys produce from a cooperative union in Kapchorwa. She tracks the quality of her weekly shipments of Matooke and Irish Potatoes. The probability that a Matooke shipment arrives in perfect condition is $\frac{4}{7}$. The probability that the Matooke arrives in perfect condition but the Irish potatoes are damaged or delayed $\frac{1}{3}$. If the Irish potatoes arrived in perfect condition, the chance that the Matooke also arrives in perfect condition is $\frac{5}{14}$.

Calculate:

a. The probability that a shipment of Irish potatoes arrives in perfect condition.
b. The probability that both food shipments arrive damaged or delayed.

18. An S.5 student living in Mukono commutes to a school in Kampala city centre daily. They analyze two events affecting their evening travel: encountering a major traffic jam on Jinja Road and experiencing an evening downpour. There is a 20% chance of encountering a heavy traffic jam and a 50% chance of an evening downpour. Help the student to find the probability that it rains but they do not get stuck in a traffic jam, under two different assumptions:

a. The traffic jams and the rain showers occur entirely independently of each other.
b. The traffic jams and rain are mutually exclusive (e.g. traffic control measures ensure gridlocks never happen when it rains).

19. The school ICT club carries out a survey among S.5 students to understand their favourite communication platforms. The survey reveals that the probability of a student using only Whats App but not X is three times that of a student using Whats App. The probability of using X but not Whats App is twice that of using Whats App. The probability of a student using neither platform is equal to the chance of using Whats App. Overall, the probability that a randomly chosen student uses Whats App is $\frac{4}{7}$. Find the probability that a student uses both X and Whats App actively.

20. At a career guidance day at a Ugandan secondary school, Senior 5 students are surveyed about their subject combination interests. Two popular optional target areas are Medicine & Science and Nursing & Allied Health. The probability that a randomly chosen S.5 student wants to pursue Medicine & Science is 0.7. The probability that a student is interested in both Medicine and Nursing is 0.45. The probability that a student is interested in neither of these fields is 0.18. Find:

a. The probability that a student is not interested in Nursing & Allied Health.
b. The probability that a student is interested in either Medicine or Nursing, but not both fields.