MATHEMATICS SCENARIO BASED ITEMS

Topic 1: NUMERICAL CONCEPTS

Sub-topic 1.1: Indices

Scenario 1

A rapidly growing tech startup in Kampala,"ByteHub Uganda," is projecting its user base growth for investors. Their analytics team has modeled the growth using the formula $U(t) = 500 \times 2^{0.5t}$, where U is the number of users and t is the time in months. The board of directors needs to make critical decisions about server capacity and funding rounds based on these projections. They need to know the expected user base 6 months from now and understand how long it will take to reach a milestone of 10,000 users to secure the next phase of venture capital funding. Accurately applying the laws of indices is essential for these calculations, as even a small miscalculation could lead to either costly over-provisioning of resources or catastrophic system failure due to underestimation.

Task:

a)Calculate the projected number of users after 6 months.

b)Determine how many months it will take for the user base to reach 10,000.

c)State what the base and the exponent represent in the context of this model.

Scenario 2

The National Agricultural Research Organisation(NARO) is studying the propagation of a beneficial soil bacterium in a new fertilizer. Under ideal conditions in the lab, a single bacterium divides into two every 30 minutes. The population growth is modeled by $P(t) = P_0 \text{ X } 2^{2t}$, where P_0 is the initial population and t is the time in hours. Researchers need to predict the population size after 4 hours starting from an initial colony of 100 bacteria to ensure there are enough bacteria to effectively treat one hectare of farmland. This helps in determining the correct and cost-effective amount of fertilizer to produce for Ugandan farmers, directly impacting crop yields and food security in the region.

Task:

a)Calculate the population of bacteria after 4 hours.

b)If the culture medium can only support 1,000,000 bacteria, how long will it take to reach this capacity from the start?

c)Explain why the exponent in the model is $2t$ and not just t .

Scenario 3

A financial advisor at Stanbic Bank Uganda is explaining the power of compound interest to a young client who wants to save for university. A savings account offers an annual interest rate of 8%, compounded annually. The future value of the investment is given by the formula $A = P(1 + r)^n$, where P is the principal, r is the interest rate, and n is the number of years. The client, who has UGX 500,000 to invest, wants to know how much the investment will be worth in 5 years and how long it will take for her money to double, enabling her to make an informed decision about her financial future and educational goals.

Task:

a)Calculate the future value of the UGX 500,000 investment after 5 years.

b)Determine the number of years it will take for the initial investment to double.

c)State one way the client could decrease the time it takes for her investment to double.

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Sub-topic 1.2: Logarithms

Scenario 4

Acoustic engineers are assessing noise pollution at a new factory site in Namanve Industrial Park.They measure the sound intensity at the perimeter wall to be 5 x 10^{-4} Wm^{-2}. Ugandan law mandates that industrial noise must not exceed 75 dB at residential boundaries, calculated using $P = 10\log_{10}(\frac{I}{L})$, with $L = 10^{-12}$ Wm^{-2}. The factory managers must determine if they are compliant or if they need to invest in expensive sound-dampening measures to avoid fines and legal action from the nearby community, which has already raised concerns about the potential impact on their quality of life.

Task:

a) Calculate the sound level in decibels for the measured intensity.

b) Determine the maximum permissible sound intensity I that corresponds to the 75 dB legal limit.

c) Explain why using a logarithmic scale is more practical than a linear scale for measuring sound intensity in this context.

Scenario 5

A data scientist at a telecommunications company is analyzing the server load for their new mobile money platform. The load on the servers follows the model $L(t) = k\log_2(t + 1)$, where t is the number of concurrent users. During a peak hour, with 1024 users, the load is measured at 50 units. The operations team needs to predict the server load when 4096 users are active simultaneously to decide if the current server infrastructure can handle the anticipated growth during the festive season or if a costly upgrade is necessary to prevent system failure.

Task:

a)Use the given data to find the value of the constant k .

b)Calculate the predicted server load for 4096 concurrent users.

c)State what the value '1' in the term (t + 1) likely represents in this model.

Scenario 6

Public health officials are tracking the spread of an infectious disease in a rural Ugandan district.The effective reproduction number R_t is a crucial metric. If R_t is 2, it means each infected person spreads the disease to 2 others. The growth rate can be analyzed using logarithms. If the number of cases increases from 10 to 160 over a period, officials need to calculate the average R_t to understand the speed of transmission and decide whether to implement lockdowns, which have significant economic and social consequences for the community.

Task:

a)The number of cases follows $C = C_0 X (R_t)^n$, where n is the number of transmission cycles. If cases grow from 10 to 160, and n=3 , calculate R_t.

b)Express the number of transmission cycles n in terms of C , C_0 , and R_t using logarithms.

c)If R_t is found to be 1.5, is the outbreak growing or shrinking? Explain.

Sub-topic 1.3: Surds

Scenario 7

An architect is designing a uniquely shaped national monument for a Kampala roundabout.The design features a large equilateral triangle with a side length of $8\sqrt{3}$ meters. To source the correct amount of a special, expensive cladding material for the perimeter, the contractor needs the exact perimeter length. Providing a decimal approximation could lead to a costly over-ordering or under-ordering of material, so the exact value in surd form is essential for precise costing and procurement before construction begins.

Task:

a)Calculate the exact perimeter of the equilateral triangle in surd form.

b)Find the area of the triangle, also in surd form.
c)Provide a rational approximation of the perimeter to 2 decimal places for the logistics team, who require numerical values for transport planning.

Scenario 8

A land surveyor is demarcating a rectangular plot of land for a new school in Wakiso District. The plot is $(5\sqrt{2} + 10)$ meters long and $(5\sqrt{2} - 10)$ meters wide. The school administration needs to know the exact area of the land for legal documentation and to plan the placement of buildings and playgrounds. Using surds ensures the legal documents are mathematically precise, avoiding future boundary disputes with neighboring landowners.

Task:

a)Show that the area of the plot is a rational number.
b)Calculate the exact area.
c)If a fence is to be built around the plot, calculate the exact length of the diagonal to determine the amount of fencing material needed for the corners.

Scenario 9

A civil engineer is inspecting a collapsed section of a road embankment.The cross-section of the stable embankment is a right-angled triangle with a vertical height of $4\sqrt{5}$ meters and a base of $2\sqrt{5}$meters. To design a reinforcing support, the engineer needs the exact length of the sloping side (the hypotenuse). Using surds provides the precision required for structural calculations, ensuring the support is designed to the correct specifications to prevent further collapses and ensure public safety.

Task:

a)Calculate the exact length of the hypotenuse of the triangular cross-section.
b)Find the exact perimeter of this triangular cross-section.
c)Rationalize the denominator of the expression for the hypotenuse if it were written as a fraction.

Topic 2: EQUATIONS AND INEQUALITIES

Sub-topic 2.1: Linear and Simultaneous Equations

Scenario 10

A manager at a Nakawa market cooperative needs to track the week's sales of maize and beans.The cooperative sold a total of 300 kg of both crops. The total revenue was UGX 1,200,000. If maize is sold at UGX 3,000 per kg and beans at UGX 5,000 per kg, the manager must determine the exact breakdown to calculate profits, pay the farmers, and plan next week's stock. An error in this calculation could lead to financial losses or disputes with the suppliers.

Task:

a)Formulate a pair of simultaneous equations to represent this situation.
b)Solve the equations to find the kilograms of maize and beans sold.
c)If the price of beans increases by 10%, how would this affect the solution if the total revenue and quantity remained the same?

Scenario 11

A delivery company in Kampala uses a fleet of motorcycles(bodas) and vans. On a particular day, the total number of vehicles used was 25. The total number of wheels counted (excluding spares) was 70. A logistics planner needs to know how many of each vehicle type were operational that day to analyze fuel consumption patterns and plan for maintenance schedules, which are different for the two vehicle types.

Task:

a)Formulate a pair of simultaneous equations to represent this situation. (Assume a motorcycle has 2 wheels and a van has 4).
b)Solve the equations to find the number of motorcycles and vans.

c)If three vans were undergoing repair that day, what would the new total number of wheels have been?

Scenario 12

A chemist at a Jinja water treatment plant is preparing a disinfectant solution.She needs to mix two solutions: one with a 30% chlorine concentration and another with a 60% chlorine concentration, to obtain 10 liters of a 45% chlorine solution. The exact volumes of each starting solution are critical. Too much chlorine wastes resources and poses an environmental hazard, while too little fails to purify the water, risking public health.

Task:

a)Formulate a pair of simultaneous equations to represent this situation.

b)Solve the equations to find the required volume of each solution.

c)If she only had 8 liters of the 60% solution, what concentration would the resulting mixture have if she used all of it and made up the rest with the 30% solution to get 10 liters?

Sub-topic 2.2: Quadratic Equations

Scenario 13

A local football club is analyzing the trajectory of a penalty kick.The height h of the ball in meters is modeled by the equation $h = 2 + 15t - 5t^2$, where t is the time in seconds. The coach wants to know if the ball will be high enough to clear a 10-meter-high defensive wall at the time $t = 1.5$ seconds, and also the total time the ball spends in the air to coach players on timing their runs.

Task:

a)Calculate the height of the ball at $t = 1.5$ seconds.

b)Determine the total time the ball remains in the air

c)What is the maximum height reached by the ball?

Scenario 14

A farmer in Masaka wants to fence a rectangular vegetable garden against a wall.She has 40 meters of fencing material for the other three sides. To maximize the growing area, which is crucial for her income, she needs to find the dimensions that will give the largest possible area. This is a classic optimization problem that can be solved by forming and analyzing a quadratic equation.

Task:

a)If the side perpendicular to the wall is x meters, show that the area A is given by $A = x(40 - 2x)$

b)Rewrite this expression in the standard quadratic form.

c)Find the dimensions that maximize the area and state this maximum area.

Scenario 15

The profit P in thousands of Ugandan Shillings for a small business selling handmade crafts is modeled by the equation $P = -2n^2 + 40n - 72$, where n is the number of items sold in hundreds. The business owner needs to know the "break-even" points (where profit is zero) to set realistic sales targets and to determine the number of items that must be sold to start making a profit.

Task:

a)Find the number of items sold when the profit is zero (solve for n when $P=0$).

b)How many items must be sold to maximize profit?

c)What is the maximum profit achievable?

Sub-topic 2.3: Quadratic Inequalities

Scenario 16

A manufacturing plant in Kasese produces solar panel components.The daily profit P in millions of Ugandan Shillings is modeled by the inequality $P > -x^2 + 14x - 33$, where x is the number of workers per shift. The plant manager needs to determine the range of workers per shift that will ensure the company remains profitable ($P > 0$) to optimize labor costs while maintaining production efficiency and avoiding operational losses.

Task:

a)Solve the quadratic inequality $-x^2 + 14x - 33 > 0$ to find the range of x .

b)If the plant can only accommodate a maximum of 12 workers per shift, does this fall within the profitable range?

c)Interpret what happens to the profit if the number of workers is 3.

Scenario 17

An architect is designing an arched window for a new community center in Gulu.The arch is modeled by the equation $y = -\frac{1}{4}x^2 + 3x$, where y is the height in meters and x is the horizontal distance from the left side. For the window to be functional, its height must be greater than 5 meters. The architect needs to find the horizontal distance x where the arch meets this height requirement to ensure the design complies with both aesthetic and practical specifications.

Task:

a)Set up the quadratic inequality that represents the condition $y > 5$.

b)Solve the inequality to find the range of x where the arch height exceeds 5 meters.

c)What is the maximum height of the arch, and at what x value does it occur?

Scenario 18

A ball is thrown vertically upward from a building in Kampala.Its height h in meters above the ground at time t seconds is given by $h = 40 + 15t - 5t^2$. Safety regulations require that during a fireworks display, the ball must be at least 30 meters above the ground to be visible and safe. The event planner needs to determine the time interval during which the ball meets this safety requirement.

Task:

a)Set up the quadratic inequality for $h \geq 30$.

b)Solve the inequality to find the time interval t for which the ball is at least 30 meters high.

c)For how many seconds is the ball above 40 meters?

Sub-topic 2.4: Polynomials

Scenario 19

An engineer is testing the stress tolerance of a new composite beam.The beam's deflection D in millimeters under a load is modeled by the polynomial $D(x) = x^3 - 6x^2 + 11x - 6$, where x is the load in tons. The beam fails if deflection exceeds 10mm. The engineer needs to find the load values where deflection is zero to understand the beam's fundamental behavior before testing its limits.

Task:

a)Show that $x = 1$ is a root of the polynomial $D(x)$.

b)Factorize the polynomial $D(x)$ completely.

c)Based on the roots, between what load values does the deflection change direction?

Scenario 20

A financial analyst at a Ugandan bank is modeling an investment's growth over time. The value V of the investment after t years is given by the polynomial $V(t) = t^3 - 9t^2 + 24t - 16$ (in millions of UGX). The client wants to know when the investment will return to its initial value (i.e., when $V(t) = 0$) to assess the break-even point and make decisions about early withdrawal.

Task:

a)Use the Factor Theorem to show that $t = 4$ is a root of $V(t)$.

b)Factorize $V(t)$ completely.

c)What are all the times when the investment returns to its initial value?

Scenario 21

A chemist is studying the rate of a catalytic reaction.The reaction rate R at temperature T (in °C) is given by $R(T) = T^3 - 12T^2 + 41T - 42$. The reaction becomes inefficient when the rate is zero. The lab needs to identify the critical temperatures where this occurs to avoid operating under those conditions and to optimize the reaction for industrial-scale chemical production.

Task:

a)Verify that $T = 2$ is a root of the polynomial $R(T)$.

b)Factorize $R(T)$ completely to find all temperatures where the reaction rate is zero.

c)If the optimal operating temperature is between 5°C and 10°C, is the reaction rate positive in this range?

Topic 3: COORDINATE GEOMETRY 1

Sub-topic 3.1: Straight Lines

Scenario 22

The Kampala Capital City Authority(KCCA) is planning a new public bus route to connect the bustling commercial hub of Nakasero Market with the rapidly growing residential area of Ntinda. City planners are using coordinate geometry to design the most efficient and straightest possible route on their city grid map. They have placed Nakasero Market at coordinates (2, 5) and a key transfer point at (8, 17) on their grid, where each unit represents 500 meters. The primary challenge is to determine the exact equation of the bus route line to calculate its length, project fuel costs, and identify where it will intersect with other major routes for scheduling purposes. Furthermore, they need to ensure the route maintains a gradient that is feasible for the city's buses, especially during the rainy season when steep inclines can become hazardous. Accurate calculation is essential for budgeting and for creating an effective public transport system that reduces traffic congestion in the city center.

Task:

a)Calculate the gradient of the straight line representing the proposed bus route.

b)Determine the equation of the line in the form $y = mx + c$.

c)The existing Busuba Road follows the line $y = 3x + 1$. Determine the coordinates of the point where the new bus route will intersect Busuba Road.

Scenario 23

A large-scale agricultural project in the Teso sub-region is dividing a vast,rectangular piece of land for maize, sorghum, and soybean cultivation. The land is mapped on a coordinate grid with corners at A(1, 2), B(7, 5), C(10, 11), and D(4, 8). The project manager needs to verify that the land is indeed rectangular to ensure the irrigation plans and plot allocations are accurate. This involves checking if the angles at the vertices are right angles. Additionally, they need to find the coordinates of the central point where a shared water reservoir will be located, ensuring it is equidistant from all four corners for efficient water distribution. An error in these geometric calculations could lead to unequal water access for different crop sections, potentially jeopardizing the yield of the entire project, which is critical for regional food security.

Task:

a)Show that the quadrilateral ABCD is a rectangle by calculating the gradients of its sides.

b)Calculate the coordinates of the midpoint of diagonal AC.

c)Verify that the midpoint of diagonal BD is the same as that of AC, confirming it is the center.

Scenario 24

The Ministry of Education is rolling out a new program to deliver digital learning tablets to remote rural schools.A drone delivery service is trialed to fly from a distribution warehouse at point W(0, 10) to a school at point S(12, 2) on a perfectly straight path. Air traffic control needs the equation of this flight path for monitoring. Partway through the flight, a storm cell is detected along the line $y = \frac{1}{2}x + 6$. It is critical to determine if the drone's path will intersect this hazardous weather system, necessitating a course change. The drone operator must calculate the shortest distance from the warehouse to the storm line to assess the initial risk and the point of potential intersection to plan an alternative route, ensuring the safe and timely delivery of the educational resources.

Task:

a)Find the equation of the straight-line flight path from W to S.

b)Calculate the point of intersection between the drone's path and the storm line.

c)If the storm is active within 2 km of its line, will the warehouse be at risk based on the shortest distance?distance

Topic 4: PARTIAL FRACTIONS

Sub-topic 4.1: Linear Factor

Scenario 25

A team of civil engineers in Uganda is working on a complex structural analysis for a new bridge over the Nile River.The stress distribution on a critical beam is modeled by a complex rational function: $\frac{5x+11}{x^2+3x+2}$. To integrate this function and calculate the total load-bearing capacity of the beam over a specific interval, they must first decompose it into simpler partial fractions. This process is crucial because the integral of the complex function is not readily apparent, but the integrals of the resulting simpler fractions are standard and easy to compute. The safety of the entire bridge structure depends on accurately determining the total stress, and any miscalculation in this decomposition could lead to an underestimation of the load, potentially resulting in catastrophic structural failure. The team must carefully factor the denominator and solve for the constants in the numerators to proceed with their integration and subsequent safety verification.

Task:

a)Factorize the denominator of the expression $\frac{5x+11}{x^2+3x+2}$.

b)Express the function in the form of partial fractions: $\frac{A}{(X+1)} + \frac{B}{(X+2)}$.

c)Find the values of the constants A and B.

Sub-topic 4.2: Quadratic Factor

Scenario 26

In the development of a new mobile signal processing algorithm for a Ugandan tech startup,engineers encounter a transfer function that describes the filter's behavior: $\frac{3x^2+4x+5}{(x+1)(x^2+1)}$. To analyze the filter's response to different frequencies, they need to perform an inverse Laplace transform, which requires decomposing this complex function into partial fractions. The denominator includes an irreducible quadratic factor (x^2+1) that cannot be broken down into real linear factors. This presents a specific challenge, as the numerator over the quadratic factor must be of the form $Bx + C$. Correct decomposition is vital for the subsequent transformation into the time domain, which will determine the filter's effectiveness in clearing up signal noise in Uganda's often congested mobile networks.

Task:

a)Set up the correct form for the partial fraction decomposition of $\frac{3x^2+4x+5}{(x+1)(x^2+1)}$

b)Find the values of the constants A, B, and C.

c)Once decomposed, the term $\frac{2x+3}{x^2+1}$ can be split. Show how this would be done.

Sub-topic 4.3: Repeated Factor
Scenario 27
A financial modeling team at the Bank of Uganda is analyzing the long-term growth of a national investment fund.The rate of growth is given by a complex rational function: $\frac{x^2+x+1}{(x-1)3}$. To project the total value of the fund over a 20-year period, they need to integrate this growth rate function. The presence of a repeated linear factor $(x-1)^3$ in the denominator necessitates a specific approach to partial fractions. The decomposition must account for each power of the repeated factor, leading to three separate terms. An accurate decomposition is critical here, as even a small error in the constants could lead to a multi-billion shilling miscalculation in the projected value of the national fund, affecting future economic policy and public sector budgeting.
Task:
a)Write the correct form for the partial fraction decomposition of $\frac{x^2+x+1}{(x-1)3}$.
b)Find the values of the constants A, B, and C.
c)If the denominator were $(x-1)^2(x+2)$, how would the form of the partial fractions change? Write the new form.

Topic 5: TRIGONOMETRY
Sub-topic 5.1: Trigonometrical Ratios
Scenario 28
A team of archaeologists is excavating a newly discovered ancient pyramid-like structure in the Karamoja region. To document the site accurately, they need to calculate the height of the structure without climbing it, to avoid causing damage. They set up a surveyor's theodolite 50 meters from the base of the structure and measure the angle of elevation to the top as 35 degrees. The instrument itself is 1.5 meters tall. Using the tangent ratio, which relates the opposite side (the height of the structure above the instrument) to the adjacent side (the distance from the base), they can calculate the total height. This non-invasive method is crucial for preserving the integrity of the historical site while still gathering essential data for their research and for reporting to the Uganda Museum.
Task:
a)Sketch a right-angled triangle representing this situation.
b)Calculate the height of the structure above the theodolite's sightline.
c)Calculate the total height of the ancient structure from the ground to its peak.

Sub-topic 5.2: Graphs of sinθ, cosθ, tanθ
Scenario 29
An electrical engineer at Umeme,Uganda's main electricity distributor, is analyzing the voltage in a national grid circuit. The voltage fluctuates sinusoidally with time according to the function $V(t) = 240\sin(100\pi t)$ volts, where t is in seconds. To ensure the grid's stability and protect connected appliances, the engineer must visualize this sine wave. She needs to understand key features of the graph: its amplitude, which indicates the peak voltage; its period, which determines the frequency of the alternating current; and its phase, which is crucial for synchronizing multiple power sources. Understanding this graph is fundamental to diagnosing power quality issues, such as sags or surges, that can affect industries and homes across the country.
Task:
a)State the amplitude and period of the voltage function $V(t)$.
b)Calculate the frequency of the alternating current in Hertz.
c)Determine the voltage at time $t = 0.01$ seconds.

Sub-topic 5.3: Compound Angle Formulae

Scenario 30

A pilot flying a small charter plane from Kajjansi Airfield to Kidepo National Park needs to calculate the most fuel-efficient heading.The plane needs to fly due north, but a strong wind is blowing from the direction N30°E at 80 km/h. The plane's airspeed (speed relative to the air) is 300 km/h. To find the correct compass heading and the resulting ground speed, the pilot must resolve the velocities into components and use the sine and cosine rules, which are derived from compound angle concepts. The ground speed is the vector sum of the airspeed and wind velocity. An error in this calculation could cause the plane to drift significantly off course, wasting fuel and potentially leading to a dangerous situation over a remote national park.

Task:

a)Represent this situation with a vector diagram.

b)By resolving vectors, show that the eastward component of the plane's velocity must cancel the eastward component of the wind.

c)Calculate the heading (angle from north) the pilot must fly and the resulting ground speed.

Sub-topic 5.4: Compound Angle Formulae and Derived Identities

Scenario 31

A team of telecommunications engineers is designing a new cellular tower on a hill overlooking Kampala.The signal strength between the new tower and an existing tower is modeled by a complex waveform that depends on the precise calculation of signals arriving at different phases. The engineers encounter an expression for the combined signal: $\sin(45^\circ + 30^\circ)$. To simplify their analysis and input this into their signal simulation software, they must expand this using the compound angle formula. The accuracy of this expansion is critical, as it affects the predicted coverage area of the new tower. An error could lead to gaps in network coverage for thousands of users or an overestimation of the service area, resulting in costly post-installation adjustments. Using the exact values derived from the compound angle formulae, rather than decimal approximations, ensures the mathematical model of the signal propagation remains precise.

Task:

a)Use the compound angle formula for sine to expand $\sin(75^\circ)$.

b)Using known exact values for $\sin(45^\circ)$, $\cos(45^\circ)$, $\sin(30^\circ)$, and $\cos(30^\circ)$, calculate the exact value of $\sin(75^\circ)$.

c)Hence, prove that $\sin(75^\circ) + \sin(15^\circ) = \sqrt{\frac{3}{4}}$.

Scenario 32

In an advanced physics experiment at Makerere University,students are studying the interference pattern of two coherent light waves. The intensity I of the resulting wave at a point is given by the equation $I = 4I_0 \cos^2\left(\frac{\Delta\theta}{2}\right)$, where $\Delta\theta$ is the phase difference. To analyze how the intensity changes with phase difference, they need to express $\cos^2\left(\frac{\Delta\theta}{2}\right)$ in a simpler form. This requires the use of the double-angle identity derived from compound angle formulae, specifically $\cos(2A) = 2\cos^2(A) - 1$. Understanding this identity allows them to rewrite the intensity formula as $I = 2I_0(1 + \cos(\Delta\theta))$, which is much easier to graph and interpret. This interpretation is fundamental to understanding phenomena like diffraction gratings and thin-film interference, which have applications in laser technology and optical sensors.

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Task:

a)Starting from the double-angle identity $\cos(2A) = 2\cos^2(A) - 1$, show that $\cos^2(A) = \frac{1+\cos 2A}{2}$.

b)Hence, express the intensity $I = 4I_0 \cos^2 (\frac{\Delta\theta}{2})$ in the form $I = kI_0(1 + \cos(\Delta\theta))$, stating the value of k .

Scenario 33

A structural engineer is modeling the vibration of a suspension bridge cable during a storm.The vertical displacement, y , of the cable is found to be a combination of two harmonic motions described by $y = 3\sin(x) + 4\cos(x)$, where x is related to time and position. To analyze the amplitude of the total vibration—a critical factor for assessing structural fatigue—the engineer must combine these two trigonometric terms into a single sine function. This involves using the identity $R\sin(x + \alpha)$, which is derived from the compound angle formula. Finding the amplitude R and the phase shift \alpha is essential for determining the maximum stress on the cable and ensuring the bridge's design can withstand extreme weather conditions, a vital safety consideration for infrastructure in Uganda.

Task:

a)Express $3\sin(x) + 4\cos(x)$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

b)Calculate the amplitude R of the resulting vibration.

c)Calculate the phase angle α to the nearest degree.

Sub-topic 5.5: Solution of Triangles

Scenario 34

A team of geographers from Makerere University is conducting a survey to map a remote,triangular-shaped section of the Rwenzori Mountains. They need to calculate the exact area of this region for a conservation study. Due to the rugged terrain, they can only directly measure two sides and the angle between them. The sides measured are 8 km and 11 km, with an included angle of 60°. Using the formula for the area of a triangle, Area= $\frac{1}{2}ab\sin C$, they can determine the precise area without having to traverse the dangerous third side. This calculation is crucial for accurately reporting the size of the conservation zone to the Uganda Wildlife Authority and for planning future research expeditions in the area.

Task:

a)State the formula for the area of a triangle given two sides and the included angle.

b)Calculate the area of the triangular region using the given measurements.

c)If the included angle were 30° instead of 60°, how would the area change? Calculate the new area.

Scenario 35

A disaster response team from the Office of the Prime Minister needs to airlift supplies to a village cut off by floods.The village is located between two relief camps. From Camp A, the village is 15 km away on a bearing of 050°. From Camp B, the village is 12 km away on a bearing of 110°. The team at Camp A needs to know the direct distance between Camp A and Camp B to coordinate the logistics and share fuel resources. Using the cosine rule, they can calculate this distance accurately, ensuring efficient coordination of the emergency response and avoiding any delays in delivering critical supplies like food, medicine, and tents to the affected population.

Task:

a)Sketch a diagram representing the positions of Camp A, Camp B, and the village, labeling all known sides and angles.

b)Calculate the angle at the village between the lines to Camp A and Camp B.

c)Apply the cosine rule to find the direct distance between Camp A and Camp B.

Scenario 36

Engineers are assessing the stability of a historic monument,the Kasubi Tombs, which has a triangular facade. They need to find all the angles of this triangular structure to analyze stress distribution. They measure the three sides of the triangle to be 25 meters, 32 meters, and 40 meters. However, they cannot safely measure the angles directly with traditional tools. By applying the cosine rule, they can calculate each angle accurately. This information is vital for creating a precise digital model of the structure, which will be used to plan restoration work and ensure the long-term preservation of this UNESCO World Heritage Site for future generations.

Task:

a)State the cosine rule formula used to find an angle when three sides are known.

b)Calculate the largest angle in the triangular facade.

c)Using the fact that angles in a triangle sum to 180°, find the remaining two angles.

Topic 6: DESCRIPTIVE STATISTICS

Sub-topic 6.1: Measures of Dispersion

Scenario 37

The Ministry of Health is analyzing the consistency of medication delivery times from a central warehouse to 10 regional hospitals.Timely delivery is critical for patient care. The delivery times (in hours) for the last shipment were: 24, 26, 28, 22, 35, 20, 27, 25, 29, 24. The Minister is concerned about the reliability of the service. While the average delivery time might seem acceptable, a high dispersion would indicate unpredictable service, which is dangerous for hospitals relying on essential drugs. The logistics team must calculate the range, variance, and standard deviation of these times to identify if the delivery system needs a complete overhaul or just minor adjustments. A high standard deviation would confirm that some hospitals face significant delays, risking drug stock-outs and impacting healthcare quality across the country.

Task:

a)Calculate the range of the delivery times.

b)Calculate the standard deviation of the delivery times.

c)If the standard deviation is high, what does this indicate about the reliability of the delivery service?

Scenario 38

The Uganda National Examinations Board(UNEB) is reviewing the performance of two secondary schools in the 2024 Advanced Level Mathematics examinations. School A has a mean grade of B, and School B also has a mean grade of B. However, UNEB suspects that School B's performance is highly inconsistent, with many students failing and a few scoring very highly, possibly due to a focus on only the top students. To investigate this fairly, they analyze the scores. School A's scores: 70, 75, 72, 68, 75. School B's scores: 90, 40, 85, 35, 80. Calculating the variance and standard deviation for both sets will reveal the consistency of teaching and learning. This analysis helps UNEB identify schools that need systemic support rather than just those with low average grades.

Task:

a)Calculate the mean score for both School A and School B.

b)Calculate the variance for School B's scores.

c)Based on your calculations, which school has more consistent performance? Explain your answer.

Scenario 39

An agronomist at the National Agricultural Research Organisation(NARO) is testing two new varieties of maize (Variety A and Variety B) for drought resistance. After a controlled drought period, the yield (in kg per hectare) from several test plots is recorded. The mean yield for both varieties is similar, around 1500 kg/ha. However, the agronomist needs to recommend the variety that provides a more reliable and consistent yield to farmers, as stability is crucial for food security and income. A variety with a lower standard deviation is less risky for a subsistence farmer. The yield data for Variety A is tightly clustered, while for Variety B it is spread out. Calculating the interquartile range (IQR) and standard deviation will provide a clear picture of consistency, guiding the final recommendation to the Ministry of Agriculture.

Task:

a)Explain why the mean yield alone is not sufficient for making a recommendation to farmers.

b)Describe what a larger standard deviation indicates about the yield of a maize variety.

c)If you were to advise a subsistence farmer, which statistical measure (mean, median, standard deviation) would be most critical for their decision? Justify your answer.

Sub-topic 6.2: Measures of Relative Positions

Scenario 40

The Uganda Ministry of Public Service is conducting a nationwide salary review for civil servants. They have collected the annual salary data for all Grade 5 officers, which follows a normal distribution with a mean of UGX 18,000,000 and a standard deviation of UGX 2,500,000. A teachers' union is advocating for a salary increase, arguing that their members are falling behind. To make a data-driven case, they need to determine the relative standing of a teacher earning UGX 21,000,000. By calculating the z-score, they can express this salary in terms of standard deviations from the mean, showing whether it is average, above average, or exceptional compared to their peers. This objective measure is more powerful in negotiations than simply stating the salary figure, as it contextualizes the income within the entire distribution.

Task:

a)Calculate the z-score for a Grade 5 officer with an annual salary of UGX 21,000,000.

b)Interpret what this z-score means in the context of the salary distribution.

c)If another officer has a z-score of -1.2, what is their approximate annual salary?

Scenario 41

A large university in Uganda,such as Makerere, uses standardized test scores for postgraduate admissions. The scores for the Graduate Entrance Exam (GEE) are normally distributed with a mean of 60 and a standard deviation of 8. The admissions committee needs to set a fair cutoff score to select the top 15% of applicants for a competitive scholarship program. They cannot simply use a raw score; they must find the score that corresponds to the 85th percentile (since the top 15% means the score is higher than 85% of applicants). Using percentile ranks and z-tables, they can determine the exact cutoff score, ensuring a transparent and statistically sound selection process for awarding these valuable scholarships.

Task:

a)Explain what the 85th percentile represents in this context.

b)Find the z-score that corresponds to the 85th percentile (you may use a z-table or the known approximation).

c)Calculate the minimum GEE score required to be in the top 15% of applicants.

Scenario 42

A national hospital is monitoring the recovery times for a specific surgical procedure.The recovery time is normally distributed with a mean of 14 days and a standard deviation of 3 days. Hospital management wants to identify patients with exceptionally slow recovery times who might need additional medical support or have underlying complications. They decide to flag any patient whose recovery time is above the 95th percentile for a special review by a senior medical team. To implement this protocol, they need to calculate the recovery time that corresponds to the 95th percentile. This use of percentiles helps in allocating limited medical resources efficiently and proactively managing patient care.

Task:

a)What is the z-score corresponding to the 95th percentile?

b)Calculate the recovery time (in days) that corresponds to the 95th percentile.

c)If a patient has a recovery time with a z-score of -2, what is their recovery time, and how would you interpret this?

Topic 7: SCATTER DIAGRAMS AND CORRELATIONS

Sub-topic 7.1: Scatter Diagram

Scenario 43

The Kampala City Council Authority (KCCA) is investigating the relationship between daily hours of sunshine and the sales of bottled water in the city center. They suspect that hotter, sunnier days lead to higher water sales. A data analyst collects data over 15 days, recording the average daily sunshine hours and the number of cases of water sold by a major vendor. Before calculating any complex statistics, the analyst must create a scatter diagram. This visual tool will provide an immediate, intuitive understanding of the potential relationship. Plotting sunshine hours on the x-axis and water sales on the y-axis will reveal if the points generally form an upward trend, a downward trend, or no pattern at all. This first step is crucial for deciding whether to proceed with a formal correlation analysis, which could influence how vendors manage their inventory and staffing based on weather forecasts.

Task:

a)State which variable should be the independent variable (plotted on the x-axis) and which should be the dependent variable (plotted on the y-axis).

b)Describe what the analyst would expect to see on the scatter diagram if there is a positive correlation between sunshine hours and water sales.

c)What might a scatter diagram that shows no correlation look like?

Sub-topic 7.2: Correlation

Scenario 44

The management of a large matatu(public transport) company in Uganda wants to understand the factors affecting their daily fuel costs. They hypothesize that the number of kilometers driven by their fleet is a major driver of cost. After collecting data for a month, they plot a scatter diagram which shows a strong linear pattern. To quantify the strength and direction of this relationship, they calculate the Pearson's product-moment correlation coefficient (r). This numerical value, which will fall between -1 and +1, will provide an objective measure. A value close to +1 would confirm a strong positive correlation, allowing management to create more accurate budgets and identify routes that are less fuel-efficient. Understanding this correlation is a key step towards implementing a cost-saving strategy for the entire company.

Task:

a)State the possible range of values for the Pearson's correlation coefficient, r .

b)If the calculated value of r is 0.88, how would you interpret the strength and direction of the relationship?

c)Why is it important to obtain a scatter diagram before calculating the correlation coefficient?

Scenario 45

A real estate agency in Entebbe is building a model to advise clients on property pricing. They want to investigate the claim that there is a relationship between the size of a house (in square meters) and its selling price. After gathering data from recent sales, they create a scatter diagram. The diagram suggests a relationship, but it is not perfectly linear. To confirm the strength of this monotonic relationship (whether one variable tends to increase as the other increases, but not necessarily at a constant rate), they decide to calculate Spearman's rank correlation coefficient. This method is more appropriate than Pearson's if the relationship is non-linear or the data is based on ranks. The result will help the agency set more realistic and data-driven asking prices for their clients' properties.

Task:

a)Differentiate between Pearson's and Spearman's correlation coefficients.

b)When is it more appropriate to use Spearman's rank correlation coefficient?

c)If Spearman's coefficient is calculated to be -0.95, what does this indicate about the relationship between house size and price?

Topic 8: DYNAMICS 1

Sub-topic 8.1: Resultant and Components of Forces

Scenario 46

A team of engineers is designing a new cable-stayed footbridge over a stream in a Kampala park.A key cable is attached to a central pylon and exerts a force of 1000 N at an angle of 30° to the horizontal. To ensure the bridge's stability, the engineers must calculate the horizontal and vertical components of this force. The horizontal component will affect the lateral stability of the pylon, while the vertical component supports the weight of the bridge deck. Accurate calculation of these components is crucial for selecting appropriately strong materials for the pylon and the deck supports, ensuring the safety of hundreds of daily pedestrians.

Task:

a)Calculate the horizontal component of the force in the cable.

b)Calculate the vertical component of the force in the cable.

c)If the angle were increased to 45°, what would happen to the magnitude of the horizontal component? Explain your reasoning.

Scenario 47

During the setup for the annual Kampala City Festival,workers need to position a heavy stage lighting rig weighing 1500 N. Two ropes are attached to the rig. One rope is pulled with a force of 800 N due East. The other rope is pulled with a force of 600 N due North. The site manager needs to know the magnitude and direction of the resultant force acting on the rig to ensure the ropes and anchoring points are strong enough to handle the total force without snapping, which could cause serious injury and damage to the equipment.

Task:

a)Sketch a vector diagram representing the two forces.

b)Calculate the magnitude of the resultant force.

c)Calculate the direction of the resultant force as a bearing.

Scenario 48

A boat is being pulled by two tugboats in Port Bell on Lake Victoria.Tugboat A exerts a force of 4000 N in a direction N30°E. Tugboat B exerts a force of 3000 N in a direction N60°E. The harbour master needs to find the resultant force on the boat to predict its path and avoid collisions with other vessels in the busy port. This requires resolving both forces into their North and East components before summing them to find the overall resultant force and its direction.

Task:

a)Resolve the 4000 N force into its North and East components.

b)Resolve the 3000 N force into its North and East components.

c)Hence, find the magnitude and direction (as a bearing) of the resultant force acting on the boat.

Sub-topic 8.2: Friction

Scenario 49

A factory owner in Namanve Industrial Park is designing a conveyor belt system to move heavy crates.A crate weighing 200 N is placed on the belt, which is inclined at an angle of 15° to the horizontal. The coefficient of friction between the crate and the belt is 0.4. The engineer needs to determine whether the crate will slide down the belt when the conveyor is stopped for loading, or if friction is sufficient to hold it in place. This analysis is critical for designing a safe system that prevents crates from sliding back and causing accidents or damage to goods.

Task:

a)Calculate the component of the crate's weight acting parallel to the inclined belt.

b)Calculate the maximum frictional force available to prevent sliding.

c)Based on your calculations, state whether the crate will remain at rest or slide down the incline.

Scenario 50

A driver is traveling along a wet road in Jinja when they need to stop suddenly.The car has a mass of 1200 kg and the coefficient of friction between the tires and the wet road is 0.3. The driver needs to know the minimum stopping distance when braking from 60 kmh^{-1}. Understanding the role of friction in deceleration is essential for promoting safe driving speeds, especially in adverse weather conditions, and is a key part of driver education programs run by the Uganda Police Force.

Task:

a)Calculate the maximum frictional force that can act to decelerate the car.

b)Calculate the deceleration of the car due to this frictional force.

c)Using equations of motion, calculate the minimum stopping distance from 60 km/h.

Scenario 51

A construction worker in Entebbe is pushing a 50 kg toolbox across a concrete floor. The worker applies a force of 200 N at an angle of 25° downwards from the horizontal. The coefficient of friction between the toolbox and the floor is 0.5. The foreman needs to know if this force is sufficient to move the toolbox, or if the worker needs assistance. This ensures efficient workflow and prevents worker injury from straining to move a stuck object.

Task:

a)Calculate the normal reaction force between the toolbox and the floor. (Remember the applied force has a downward component).

b)Calculate the maximum static frictional force.

c)Calculate the horizontal component of the applied force and determine if the toolbox will move.

Sub-topic 8.3: Connected Particles

Scenario 52

In a warehouse in Kampala's Industrial Park,a worker needs to lift a heavy crate of mass 80 kg using a rope thrown over a fixed pulley. The worker, who has a mass of 70 kg, pulls downwards on the other end of the rope. The system is modeled as two particles connected by a light, inextensible rope over a smooth pulley. The warehouse manager needs to calculate the acceleration of the crate and the tension in the rope to ensure that the rope's breaking strain is not exceeded and that the crate can be lifted safely without the worker losing control. This analysis is crucial for workplace safety and for selecting the appropriate equipment for the job.

Task:

a)Sketch the system and label the forces acting on the crate and the worker.

b)Apply Newton's Second Law to both the crate and the worker to form two simultaneous equations.

c)Solve the equations to find the acceleration of the system and the tension in the rope.

Scenario 53

A farmer in Masaka is using a pulley system to draw water from a well.A bucket of water with a total mass of 15 kg is connected by a rope to a counterweight of mass 10 kg. The rope passes over a fixed, smooth pulley. When released, the system begins to move. The farmer needs to know how long it will take for the bucket to be lifted 8 meters from rest. This information helps the farmer estimate the time needed to fetch water, which is crucial for daily planning, especially during the dry season when water is scarce and multiple trips are necessary.

Task:

a)Calculate the acceleration of the system when it is released.

b)Using the equations of motion, calculate the time taken for the bucket to rise 8 meters from rest.

c)What would be the effect on the acceleration if the pulley was not smooth? Explain.

Scenario 54

During a physics demonstration at a school in Gulu,a teacher sets up a system with two masses on a smooth, horizontal table. Mass A (4 kg) is connected by a light, inextensible string to Mass B (6 kg), which hangs vertically over the edge of the table. The string passes over a smooth pulley at the table's edge. The teacher releases the system from rest and asks the students to predict the motion. The students must calculate the acceleration of the system and the tension in the string to understand the principles of connected particles and Newton's Laws of Motion, which are fundamental concepts in mechanics.

Task:

a)Draw a diagram showing all the forces acting on both masses.

b)Write down the equation of motion for each mass.

c)Solve the equations to find the acceleration of the system and the tension in the string.

Topic 9: PROBABILITY THEORY

Sub-topic 9.1: Probability Theorems

Scenario 55

A mobile money agent in Kampala has two separate lines for customers:one for deposits and one for withdrawals. The probability that a customer arrives for a deposit in any given minute is 0.6, and the probability that a customer arrives for a withdrawal is 0.4. These events are independent. The agent needs to calculate the probability that in a given minute, at least one customer arrives (either for deposit or withdrawal) to determine whether to keep both lines open during slow periods or consolidate them to reduce operational costs while maintaining service quality.

Task:

a)Calculate the probability that at least one customer arrives in a given minute.

b)What is the probability that both types of customers arrive in the same minute?

c)If the events were mutually exclusive, how would your calculation in part (a) change?

Scenario 56

A hospital in Jinja is studying patient flow through its outpatient department.The probability that a patient needs to see a doctor is 0.7, and the probability that a patient needs laboratory tests is 0.5. The probability that a patient needs both is 0.3. The hospital administrator wants to know the probability that a randomly selected patient needs either a doctor or laboratory tests to better allocate staff resources and reduce patient waiting times, which is a key performance indicator for the hospital's quality of care.

Task:

a)Represent this situation using a Venn diagram.

b)Calculate the probability that a patient needs either a doctor or laboratory tests.

c)Calculate the probability that a patient needs only laboratory tests.

Scenario 57

A quality control inspector at a beverage factory in Entebbe is testing bottles from two production lines.Line A produces 60% of the bottles, with a defect rate of 2%. Line B produces 40% of the bottles, with a defect rate of 4%. The inspector randomly selects a defective bottle. The production manager needs to know the probability that this defective bottle came from Line B to identify which production line requires maintenance and quality improvement measures, potentially saving the company significant costs in wasted materials and reputation damage.

Task:

a)Draw a tree diagram to represent this situation.

b)Calculate the overall probability of selecting a defective bottle.

c)Use Bayes' Theorem to find the probability that a defective bottle came from Line B.

Sub-topic 9.2: Applications of Probability in Real Life

Scenario 58

An insurance company in Kampala is developing a new life insurance product.Based on national health statistics, the probability that a 40-year-old person lives to age 70 is 0.85. The company needs to calculate the probability that out of 5 randomly selected 40-year-old policyholders, at least 4 will live to age 70. This calculation is crucial for pricing the insurance product correctly and ensuring the company remains financially solvent while offering competitive rates to customers in Uganda's growing insurance market.

Task:

a)State whether this is a binomial probability situation and justify your answer.

b)Calculate the probability that exactly 4 out of 5 policyholders live to age 70.

c)Calculate the probability that at least 4 out of 5 policyholders live to age 70.

Scenario 59

A public health researcher is studying the effectiveness of a new malaria prevention program in a rural Ugandan district.Before the program, the prevalence of malaria in children under 5 was 30%. After implementing the program, the researcher randomly selects 20 children and finds that 4 have malaria. The researcher needs to determine the probability of observing 4 or fewer cases if the prevalence rate is still 30% to assess whether the program has been statistically significantly effective, which could influence future public health policy and funding decisions.

Task:

a)Calculate the probability of finding exactly 4 malaria cases in the sample of 20 children.

b)Calculate the probability of finding 4 or fewer malaria cases in the sample.

c)Based on your result, what might you conclude about the effectiveness of the program?

Scenario 60

A telecommunications company is planning the capacity for its new data center in Wakiso.The company knows that on average, each customer connects to the service for 2 hours per day. The company needs to calculate the probability that more than 1000 customers are simultaneously connected during peak hours to ensure the system can handle the load without crashing. This probability analysis helps determine the optimal server capacity, balancing infrastructure costs against the risk of service interruption in Uganda's rapidly expanding digital economy.

Task:

a)What probability distribution would be appropriate for modeling this situation?

b)State the parameter(s) of this distribution for this scenario.

c)Explain how the company would use this probability distribution in its capacity planning.

Topic 10: DIFFERENTIATION 1

Sub-topic 10.1: Gradient of a Curve

Scenario 61

An environmental scientist is studying the pollution level in Lake Victoria.The concentration of a pollutant, C in mg/liter, at a point x meters from a discharge point is given by the function $C(x) = x^3 - 6x^2 + 9x + 2$. The scientist needs to find the point where the pollution concentration is increasing most rapidly to prioritize cleanup efforts. This requires finding the gradient function (derivative) of $C(x)$ and then analyzing its behavior. Identifying this critical point will help environmental agencies in Uganda and Tanzania allocate limited resources effectively to protect the lake's ecosystem and the livelihoods of millions who depend on it.

Task:

a)Find the derivative (gradient function) of $C(x) = x^3 - 6x^2 + 9x + 2$.

b)Calculate the gradient of the curve at $x = 1$ meter and interpret what this value means.

c)Find the coordinates of the point where the gradient of the concentration curve is zero.

Scenario 62

A sports physiologist at the Uganda Olympic Committee is analyzing the performance of a 400m runner.The distance, s meters, covered by the runner after t seconds is modeled by the function $s(t) = 0.2t^3 + 4t^2 + 5t$. The coach wants to know the runner's instantaneous velocity at exactly $t = 5$ seconds to assess their mid-race performance and compare it with their starting and finishing speeds. The instantaneous velocity is given by the derivative of the displacement function with respect to time. This analysis helps in tailoring training programs to improve an athlete's performance in specific phases of a race.

Task:

a)State what the derivative represents in this context.

b)Find the function for the runner's instantaneous velocity.

c)Calculate the runner's velocity at $t = 5$ seconds.

Scenario 63

A mechanical engineer is testing the efficiency of a new engine design.The efficiency, E, as a function of the engine speed, R in revolutions per minute (RPM), is given by $E(R) = -0.001R^3 + 0.15R^2 + 20R$. The engineer needs to find the engine speed at which the efficiency is maximized. This requires finding the derivative of the efficiency function and solving for where the gradient is zero. Determining this optimal RPM is crucial for programming the engine's control unit to operate at its most efficient point, saving fuel and reducing emissions for vehicles in Uganda's transport sector.

Task:

a)Find the derivative $E'(R)$.

b)Set $E'(R) = 0$ to find the critical points.

c)Which of these critical points represents a realistic operating RPM for a maximum? Justify your answer.

Sub-topic 10.2: Gradient Functions

Scenario 64

An urban planner is designing a new road over a hilly terrain in Kabale District.The elevation of the road surface, y meters, at a horizontal distance x meters from the start, is modeled by the function $y = 0.01x^3 - 0.15x^2 + 2x$. The planner needs to ensure that the road's gradient never exceeds 8% for safety reasons. The gradient of the road at any point is given by the derivative $\frac{dy}{dx}$. By analyzing this gradient function, the planner can identify the steepest sections of the road and modify the design if necessary before construction begins.

Task:

a)Find the gradient function $\frac{dy}{dx}$ for the road's elevation.

b)Calculate the gradient at $x = 50$ meters.

c)Find the maximum gradient of the road and the point where it occurs.

Scenario 65

An economist at the Bank of Uganda is modeling the country's GDP growth.The GDP, G in trillion UGX, is projected to follow the model $G(t) = 120 + 2t + 0.1t^2 - 0.005t^3$, where t is the number of years from now. The economist is interested in the rate of economic growth, which is the derivative of the GDP function. Analyzing this gradient function will help predict when the growth rate will start to slow down, informing long-term fiscal and monetary policy decisions for the nation's economic stability.

Task:

a)Find the gradient function $G'(t)$, which represents the GDP growth rate.

b)Calculate the projected growth rate 5 years from now.

c)Determine the year when the GDP growth rate is expected to be at its maximum.

Scenario 66

A civil engineer is inspecting a slightly bent metal beam in a building structure in Kampala.The deflection of the beam from its original straight position is given by the function $y = 0.0001x(20 - x)^2$, where x is the distance from one end. To assess the severity of the bend, the engineer needs to find the points where the beam is steepest (maximum gradient) and the points where it is flat (zero gradient). This analysis is critical for deciding whether the beam needs reinforcement or replacement to ensure the structural integrity of the building.

Task:

a)Expand the function for y and then find its derivative $\frac{dy}{dx}$.

b)Find the values of x where the gradient of the beam is zero.

c)Evaluate the gradient at the midpoint, $x = 10$.

Sub-topic 10.3: Composite Functions

Scenario 67

A meteorologist at the Uganda National Meteorological Authority is modeling how temperature changes with altitude in the Rwenzori Mountains.The temperature T in °C depends on the atmospheric pressure P in kPa, given by $T(P) = 20 - 5P$. Furthermore, the pressure P decreases with altitude h in meters according to $P(h) = 100 - 0.01h$. The meteorologist needs to find a single function that directly gives the temperature as a function of altitude for hikers and aviation services. This requires forming the composite function $T(P(h))$, which will allow for direct prediction of temperature at any given altitude, crucial for flight planning and mountaineering safety.

Task:

a)Form the composite function $T(h)$ by substituting $P(h)$ into $T(P)$.

b)Simplify the composite function $T(h)$.

c)Using the composite function, calculate the temperature at an altitude of 1500 meters.

Scenario 68

An environmental scientist is studying the spread of an invasive plant species in Lake Victoria.The area covered by the plant, A in square kilometers, depends on the nutrient concentration N in the water, given by $A(N) = 2N^2$. The nutrient concentration N itself increases over time t in weeks due to agricultural runoff, following $N(t) = 5 + 0.5t$. The scientist wants to predict the area covered by the plant after 10 weeks to assess the urgency of intervention measures. This requires differentiating the composite function $A(N(t))$ with respect to time to find the rate at which the plant is spreading.

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Task:

a)Form the composite function A(t) that gives area directly as a function of time.

b)Use the chain rule to find the derivative $\frac{dA}{dt}$.

c)Calculate the rate at which the plant is spreading after 10 weeks.

Scenario 69

An automotive engineer at Kiira Motors is testing the fuel efficiency of a new electric vehicle prototype.The vehicle's range R in kilometers is a function of its battery charge level C in kWh, given by $R(C) = 8C - 0.1C^2$. During a test drive, the charge level decreases over time t in hours according to $C(t) = 60 - 10t$. The engineer needs to find how quickly the vehicle's predicted range is decreasing after 2 hours of driving to validate the battery management system's accuracy. This requires applying the chain rule to the composite function $R(C(t))$.

Task:

a)State the chain rule for differentiation.

b)Use the chain rule to find an expression for $\frac{dR}{dt}$.

c)Calculate the rate of change of the vehicle's range with respect to time after 2 hours.

Sub-topic 10.4: Implicit Functions

Scenario 70

A civil engineer is designing a curved arch for a bridge in Jinja.The shape of the arch is defined by the equation $x^2 + 4y^2 = 100$, where x and y are in meters. To determine the steepness of the arch at specific points for structural analysis and material stress calculations, the engineer needs to find the gradient $\frac{dy}{dx}$ without explicitly solving for y . Using implicit differentiation is the most efficient method, as solving for y would result in two functions (top and bottom halves of the arch).

Task:

a)Differentiate both sides of the equation $x^2 + 4y^2 = 100$ with respect to x .

b)Solve the resulting equation for $\frac{dy}{dx}$.

c)Find the gradient of the arch at the point where $x = 6$ meters and $y > 0$.

Scenario 71

An economist at Makerere University is analyzing the relationship between a country's inflation rate x and its unemployment rate y , modeled by the equation $xy + x + y = 10$. This relationship is known as a Phillips curve variant. The economist wants to find the rate at which unemployment changes with respect to inflation ($\frac{dy}{dx}$) when the inflation rate is 2%. This analysis helps in understanding the trade-off between these two critical economic indicators and in formulating national economic policy.

Task:

a)Differentiate the equation $xy + x + y = 10$ implicitly with respect to x .

b)Solve for $\frac{dy}{dx}$ in terms of x and y .

c)Given that when $x = 2$, $y = 2$, calculate the value of $\frac{dy}{dx}$.

Scenario 72

A physicist at the International University of East Africa is studying the path of a charged particle in a magnetic field.The particle's path is described by the equation $\sin(xy) = x + y$. To analyze the particle's velocity components, the physicist needs to find the relationship between $\frac{dy}{dx}$ and the variables x and y at any point on its trajectory. This requires implicit differentiation of the transcendental equation, which cannot be easily solved for y .

Task:

a)Differentiate both sides of $\sin(xy) = x + y$ with respect to x .

b)Apply the chain rule carefully to differentiate $\sin(xy)$.

c)Solve the resulting equation for $\frac{dy}{dx}$.

Sub-topic 10.5: Products and Quotients of Functions

Scenario 73

A pharmaceutical company in Kampala is modeling the concentration of a new malaria drug in a patient's bloodstream over time.The concentration $C(t)$ in mg/L is given by the function $C(t) = 5t\, e^{-0.2t}$, where t is time in hours. This function is a product of a polynomial term $(5t)$ and an exponential decay term $(e^{-0.2t})$. Medical researchers need to find the rate of change of the drug concentration, $C'(t)$, to determine the time when the concentration is at its peak, which is crucial for establishing the optimal dosage interval for maximum therapeutic effect.

Task:

a)Identify the two functions $u(t)$ and $v(t)$ whose product gives $C(t)$.

b)Apply the product rule to find the derivative $C'(t)$.

c)Set $C'(t) = 0$ to find the time t when the drug concentration is at its maximum.

Scenario 74

An environmental engineer is studying the rate of cooling of a geothermal spring in Fort Portal after being disturbed.The temperature T in °C at time t hours is modeled by $T(t) = 80/t^2 + 4$. This function is a quotient, with a constant numerator and a polynomial denominator. The engineer needs to find how quickly the temperature is decreasing at $t = 2$ hours to understand the recovery rate of the spring's ecosystem. This requires applying the quotient rule for differentiation.

Task:

a)State the quotient rule for differentiation.

b)Use the quotient rule to find $T'(t)$.

c)Calculate $T'(2)$ and interpret what this value means in the context of the problem.

Scenario 75

An economist at the Bank of Uganda is analyzing the country's debt-to-GDP ratio.The ratio $R(t)$ is modeled as $R(t) = 0.5t^2 /(1 + 0.1t)$, where t is the number of years from the present. This is a quotient of two functions of t . To forecast future fiscal stability, the economist needs to find the rate of change of this ratio, $R'(t)$. This will help predict whether the debt burden is growing at a sustainable rate or if policy interventions are needed.

Task:

a)Identify the functions $u(t)$ and $v(t)$ for the numerator and denominator.

b)Apply the quotient rule to find an expression for $R'(t)$.

c)Calculate $R'(5)$ and state whether the debt-to-GDP ratio is increasing or decreasing after 5 years.

Sub-topic 10.6: Applications of Differentiation 1

Scenario 76

A local artisan in Mpigi makes and sells wooden sculptures.Her profit P in Ugandan Shillings from selling x sculptures is given by $P(x) = -2x^3 + 30x^2 - 96x$. To maximize her earnings, she needs to determine the number of sculptures she should produce and sell each month. This involves finding the turning points of the profit function by setting its derivative to zero and then using the second derivative to distinguish between maximum and minimum profit points.

Task:

a)Find the first derivative $P'(x)$.

b)Find the critical points by solving $P'(x) = 0$.

c)Use the second derivative test to determine which critical point gives a maximum profit.

Scenario 77

A water tank at a school in a rural area is being filled.The volume V of water in the tank in liters at time t minutes is given by $V(t) = 0.1t^3 - 2t^2 + 12t$ for $0 \leq t \leq 10$. The school custodian needs to find the time when the water is flowing into the tank at the fastest rate. The rate of water flow is the derivative of the volume, $V'(t)$. The maximum flow rate corresponds to the maximum of this derivative function.

Task:

a)Find the flow rate function $V'(t)$.

b)Find the critical points of the flow rate function $V'(t)$.

c)Determine the time t during the filling process when the flow rate is greatest.

Scenario 78

A drone is being tested for delivering medical supplies.Its height h in meters at time t seconds is given by $h(t) = t^3 - 9t^2 + 24t$. The engineering team needs to analyze the drone's vertical motion, specifically finding when it is ascending and when it is descending. They also need to find its velocity and acceleration at specific times to ensure safe and stable flight. The velocity is the first derivative of height, and acceleration is the second derivative.

Task:

a)Find the velocity function $v(t) = h'(t)$.

b)Find the acceleration function $a(t) = h''(t)$.

c)Determine the time intervals during the first 6 seconds when the drone is ascending (i.e., $v(t) > 0$).

Topic 11: INTEGRATION 1

Sub-topic 11.1: Indefinite Integral

Scenario 79

An environmental scientist is studying the rate of absorption of a new organic fertilizer in the soil.The rate of absorption, $R(t)$ in grams per day, is given by the function $R(t) = 6t^2 - 4t + 5$, where t is the time in days. To determine the total amount of fertilizer absorbed by the soil after a certain period, the scientist needs to find the integral of this rate function. This indefinite integral will provide a general formula for the total absorption, which is crucial for calculating the optimal quantity of fertilizer to use in Ugandan agricultural projects to maximize crop yield while minimizing cost and environmental impact.

Task:

a)Find the indefinite integral $\int 6t^2 - 4t + 5 \, dt$.

b)Explain what the constant of integration represents in this context.

c)If it is known that 10 grams were absorbed by day 1, find the particular solution for the total absorption.

Scenario 80

A mechanical engineer at a manufacturing plant in Namanve is analyzing the rate of production of a machine.The machine's production rate, $P'(t)$ in units per hour, is modeled by $P'(t) = 3e^{0.1t} + 4$. The production manager needs a function that predicts the total number of units produced after t hours to plan inventory and shipments. Finding the indefinite integral of the production rate function will give the general form of the total production function, which is essential for efficient supply chain management in Uganda's growing manufacturing sector.

Task:

a)Find the indefinite integral $\int (3e^{0.1t} + 4) dt$.

b)If the machine had produced 15 units by the start of the measurement (t=0), determine the constant of integration.
c)Write the specific total production function $P(t)$.
Scenario 81
An economist at Makerere University is modeling the marginal cost of producing a new solar lamp.The marginal cost function is $C'(x) = 0.04x + 20$, where x is the number of units produced. To find the total cost function, which is essential for pricing and profitability analysis, the economist needs to integrate the marginal cost function. This will help Ugandan solar companies determine the most cost-effective production levels to make clean energy affordable.
Task:
a)Find the indefinite integral $\int(0.04x + 20)dx$.
b)If the fixed costs (cost when x=0) are 500,000 UGX, find the constant of integration.
c)Write the specific total cost function $C(x)$.

Sub-topic 11.2: Definite Integral
Scenario 82
A civil engineer is calculating the amount of material needed to build a curved road embankment.The cross-sectional area of the embankment can be modeled by the function
$A(x) = x^2 + 2x + 1$ (in square meters) between $x = 0$ and $x = 5$ meters. The total volume of material required for a 100-meter long embankment is found by integrating this area function over the given interval and then multiplying by the length. Accurate calculation is crucial for budgeting and procuring the correct amount of soil and aggregate, preventing costly project delays in Uganda's infrastructure development.
Task:
a)Set up the definite integral to find the average cross-sectional area from x=0 to x=5 .
b)Calculate the definite integral $\int_0^5 x^2 + 2x + 1\, dx$.
c)Find the total volume of material needed for the 100-meter long embankment.
Scenario 83
A public health researcher is analyzing the total number of malaria cases reported in a district during a rainy season. The rate of new cases per week is given by $R(t) = 50 + 10t - t^2$, where t is time in weeks for $0 \le t \le 10$. The researcher needs to find the total number of cases over the entire 10-week period to assess the severity of the outbreak and allocate medical resources effectively. This requires calculating the definite integral of the rate function over the given time interval.
Task:
a)Set up the definite integral for the total number of cases from week 0 to week 10.
b)Calculate $\int_0^{10} 50 + 10t - t^2\, dt$.
c)Interpret what the result represents in the context of the malaria outbreak.
Scenario 84
A water resources engineer is studying the total water flow in a river during the rainy season.The flow rate, $F(t)$ in cubic meters per second, is given by $F(t) =(100 + 20\sin\frac{\pi t}{6})$, where t is the time in months $(0 \le t \le 6)$. The engineer needs to calculate the total volume of water that passed a monitoring station during this 6-month period to manage reservoir levels and plan for irrigation needs in the dry season. This involves finding the definite integral of the flow rate function.
Task:
a)Set up the definite integral for the total water volume from t=0 to t=6.

b)Calculate $\int_0^6 100 + 20\sin\frac{\pi t}{6}\,dt$ dt .

c)If the reservoir capacity is 16,000,000 m³, what percentage of the reservoir did this flow represent?

Sub-topic 11.3: Applications of Integration (Area)

Scenario 85

The Kampala City Council Authority(KCCA) is planning to create a new public park in a previously unused plot of land. The boundaries of the park are defined by two curves: the path of a small stream, modeled by $y = x^2 + 1$, and a walking path, modeled by $y = x + 3$, where x and y are in meters. The area between these two curves, from their intersection points, will be developed into a green space. The parks department needs to calculate the exact area of this land to budget for sod, plants, and irrigation systems. This requires using integration to find the area between two curves, a fundamental application that ensures efficient use of public funds and optimal urban planning.

Task:

a)Find the points of intersection between the curves $y = x^2 + 1$ and $y = x + 3$.

b)Set up the definite integral to find the area enclosed between these two curves.

c)Calculate this area.

Scenario 86

An architect is designing a modern building with a unique curved wall.The wall's profile follows the curve $y = \sqrt{x}$ from $x = 1$ to $x = 4$, and the ground level is at $y = 0$. To estimate the cost of the special glass needed for this wall, the architect must calculate the area of the wall's surface. In the initial design phase, this can be approximated by finding the area under the curve $y = \sqrt{x}$ between the specified bounds. Accurate area calculation is crucial for material procurement and cost estimation in Uganda's growing construction industry.

Task:

a)Sketch the curve $y = \sqrt{x}$ from $x = 1$ to $x = 4$.

b)Set up the definite integral to find the area under the curve between these x-values.

c)Calculate this area.

Scenario 87

An agricultural researcher is studying the yield of a new variety of maize.The yield distribution across a test field is modeled by the probability density function $f(x) = -\frac{1}{50}(x - 10)^2 + 2$ for $0 \le x \le 20$, where x represents the distance in meters from a central irrigation point and $f(x)$ represents the yield in kgm^{-2}. The researcher needs to find the total yield from a 1-meter wide strip running from $x = 5$ to $x = 15$ to understand how yield changes with distance from water. This involves finding the area under the yield curve, which represents the total yield.

Task:

a)Set up the definite integral for the total yield from $x = 5$ to $x = 15$.

b)Calculate $\int_5^{15} -\frac{1}{50}(x - 10)2 + 2\,dx$.

c)Interpret the result in the context of the maize yield

Sub-topic 11.4: Mean Value of a Function

Scenario 88

An energy company is analyzing the power output of a small solar farm in Mbarara over a 12-hour period(from 6:00 to 18:00). The power output $P(t)$ in kilowatts is modeled by the function $P(t) = 100\sin(\frac{\pi t}{12}) + 50$, where t is the time in hours after 6:00. To design an effective battery storage system and plan for grid integration, engineers need to know the average power output over this period. This requires calculating the mean value of the power function, which provides a crucial benchmark for energy production planning in Uganda's renewable energy sector.

Task:

a)State the formula for the mean value of a function $f(x)$ over the interval $[a, b]$.

b)Set up the calculation for the mean power output from $t=0$ to $t=12$.

c)Calculate the mean value of the power function.

Scenario 89

A water treatment plant in Gulu is monitoring the concentration of a chemical additive in the water supply throughout the day.The concentration $C(t)$ in parts per million varies according to $C(t) = 0.1t^2 - 2.4t + 15$ for $0 \leq t \leq 12$ hours. Health regulations require that the average concentration over any 12-hour period must not exceed 10 ppm. The plant manager needs to calculate the mean concentration to ensure compliance with safety standards and protect public health.

Task:

a)Set up the definite integral to find the mean value of $C(t)$ from $t=0$ to $t=12$.

b)Calculate the mean concentration over this period.

c)Determine if the plant is operating within the regulatory limit.

Scenario 90

A traffic engineer is studying vehicle flow rate on Entebbe Road during morning rush hour(7:00 to 9:00 AM). The flow rate $F(t)$ in vehicles per minute is given by $F(t) = -5(t - 1)^2 + 80$, where t is measured in hours after 7:00 AM. To plan for road widening and traffic management systems, the engineer needs to find the average flow rate during this 2-hour period. This mean value will help determine the necessary capacity for future infrastructure projects in Uganda's rapidly growing urban centers.

Task:

a)Determine the interval for t corresponding to 7:00-9:00 AM.

b)Calculate the mean value of $F(t)$ over this interval.

c)If the road's current capacity is 70 vehicles per minute, is the average flow exceeding capacity?

Topic 12: PERMUTATIONS AND COMBINATIONS

Sub-topic 12.1: Permutations and Combinations

Scenario 91

The Uganda National Examinations Board(UNEB) is designing the seating arrangement for a national scholarship examination to be held at a major venue in Kampala. There are 20 distinct scholarship candidates from different regions of the country, and they must be seated in a row of 20 specially assigned seats. To ensure the integrity of the examination process and prevent any possibility of collaboration or cheating, UNEB officials need to calculate the total number of possible distinct seating arrangements for these candidates. This calculation involves understanding the concept of permutations, where the order of arrangement matters significantly. The officials must consider that each candidate is unique and changing the position of any two candidates creates a completely different seating arrangement. This thorough planning is essential for maintaining the credibility of the national scholarship program and ensuring fair conditions for all exceptional students competing for these prestigious awards.

Task:

a)Calculate the total number of possible distinct seating arrangements for the 20 candidates.

b)If two particular candidates from the same school must not sit next to each other, how would this restriction affect the calculation?

c)Explain why this is a permutation problem rather than a combination problem.

Scenario 92

The Ministry of Health is forming specialized rapid response teams to address disease outbreaks across Uganda's diverse regions.From a pool of 15 highly qualified medical professionals with different specializations (epidemiologists, virologists, public health experts, and clinicians), the ministry needs to select 5 members for each regional team. The selection committee must calculate how many different teams can be formed, considering that each professional brings unique expertise and the composition of the team affects its effectiveness in responding to specific health emergencies. This scenario requires understanding combinations rather than permutations, as the order of selection doesn't matter - only which professionals are included in the team. Accurate calculation is crucial for ensuring that all possible team configurations are considered when deploying resources to handle potential outbreaks in different parts of the country, from the border regions to remote rural areas.

Task:

a)Calculate the number of different teams of 5 that can be selected from 15 professionals.

b)If the team must include at least 2 epidemiologists from the 4 available, how would this constraint change the calculation?

c)Explain why this is a combination problem rather than a permutation problem.

Scenario 93

A major telecommunications company in Uganda is implementing enhanced security protocols for its mobile money platform.The new system requires users to create a 6-character PIN where the first 3 characters must be distinct letters from the English alphabet and the last 3 characters must be distinct digits from 0-9. The security team needs to calculate the total number of possible unique PIN combinations to assess the system's vulnerability to brute-force attacks. This complex scenario involves both permutations of letters and permutations of digits, requiring the application of the fundamental counting principle. The calculation must account for the case sensitivity of letters and the requirement that all characters within each section be distinct. This security analysis is critical for protecting millions of Ugandan mobile money users from potential fraud and ensuring the integrity of one of the country's most important financial technologies.

Task:

a)Calculate the number of ways to choose and arrange 3 distinct letters from the 26-letter alphabet.

b)Calculate the number of ways to choose and arrange 3 distinct digits from 0-9.

c)Apply the fundamental counting principle to find the total number of possible 6-character PINs.

Topic 13: SERIES

Sub-topic 13.1: Arithmetic Progression (A.P.)

Scenario 94

The Ministry of Works and Transport is planning a massive infrastructure project to install solar-powered streetlights along a newly constructed highway stretching 100 kilometers from Kampala to Masaka.The project design specifies that streetlights must be placed at regular intervals, with the first light at the 2-kilometer mark and the final light exactly at the 98-kilometer mark. If the total number of streetlights to be installed is 50, the project engineers need to determine the exact interval between consecutive streetlights and the precise position of each light along the highway. This problem represents a classic arithmetic progression where the positions of the streetlights form a sequence with a constant difference. Accurate calculation is essential for budgeting the exact number of materials needed, planning the construction schedule, and ensuring uniform illumination along the entire highway for driver safety, especially during nighttime travel when the road is used by both commercial trucks and private vehicles.

Task:

a)Identify the first term and the number of terms in this arithmetic progression.

b)Calculate the common difference between the positions of consecutive streetlights.
c)Find the position of the 25th streetlight along the highway.

Sub-topic 13.2: Geometric Progression (G.P.)
Scenario 95
A virologist at the Uganda Virus Research Institute is studying the transmission pattern of a new viral strain detected in the country.In a controlled observation, a single infected individual transmits the virus to 3 new people in the first transmission cycle. Each of these newly infected individuals then transmits the virus to 3 more people in the next cycle, and this pattern continues consistently. The public health team needs to project the total number of infected people after 6 complete transmission cycles to prepare adequate medical facilities, isolation centers, and preventive measures. This scenario perfectly illustrates a geometric progression where each term is obtained by multiplying the previous term by a constant ratio. Understanding this exponential growth pattern is crucial for implementing timely interventions to prevent a potential epidemic that could overwhelm Uganda's healthcare system, particularly in rural areas with limited medical resources.
Task:
a)Identify the first term and common ratio of this geometric progression.
b)Calculate the number of newly infected people in the 5th transmission cycle.
c)Calculate the total number of infected people after 6 transmission cycles (including the initial case).

Sub-topic 13.3: Proof by Induction
Scenario 96
A computer science researcher at Makerere University is developing a new algorithm for optimizing data storage in Uganda's growing digital infrastructure.During the algorithm analysis, the researcher encounters a mathematical pattern where the sum of the first n odd positive integers appears to equal n^2 for all natural numbers n. Before implementing this finding into the algorithm's core logic, which will be used by various government agencies and private companies, the researcher must provide a rigorous mathematical proof that this relationship holds true for all possible cases. Proof by mathematical induction provides the perfect method for this verification, establishing the truth for the base case and then demonstrating that if the statement holds for an arbitrary case k, it must also hold for the next case k+1. This thorough mathematical validation is essential before deploying the algorithm in critical systems where data integrity and storage efficiency are paramount for Uganda's digital transformation journey.
Task:
a)Verify the statement that the sum of the first n odd numbers equals n^2 for the base case n=1.
b)Form the inductive hypothesis by assuming the statement is true for n=k.
c)Using the inductive hypothesis, prove that the statement must also be true for n=k+1.

Sub-topic 13.4: Binomial Expansion
Scenario 97
An agricultural engineer is developing a predictive model for crop yield under varying weather conditions for Uganda's National Agricultural Research Organization.The model incorporates a complex probability component expressed as $(1 + 0.05)^8$, which represents the cumulative effect of small daily growth variations over an 8-day critical growth period. Calculating this value directly would be computationally intensive, but using the binomial theorem provides an efficient approximation method. The engineer needs to expand this expression to estimate the expected yield multiplier accurately, which will help farmers make informed decisions about planting schedules and resource allocation. This application of binomial expansion is particularly valuable for subsistence farmers who rely on precise yield predictions

for food security and income stability in a country where agriculture employs over 70% of the workforce and contributes significantly to the national economy.

Task:

a)Write out the first four terms of the binomial expansion of $(1 + 0.05)^8$.

b)Calculate the approximate value using these first four terms.

c)Estimate the percentage error of your approximation compared to the exact value.

Scenario 98

A financial analyst at the Bank of Uganda is modeling compound interest scenarios for the country's emerging small and medium enterprise sector.The analyst encounters expressions of the form $(a + b)^n$ where a represents the principal amount, b represents the interest rate, and n represents the number of compounding periods. Using the binomial theorem, the analyst can expand these expressions to understand how different components contribute to the final amount. This analysis helps in developing financial products tailored to Ugandan businesses, particularly those in rural areas where access to capital remains a significant challenge for economic development and poverty reduction initiatives supported by both government and international development partners.

Task:

a)Expand $(1 + x)^5$ using the binomial theorem.

b)Use your expansion to approximate $(1.02)^5$.

c)If a small business borrows 1,000,000 UGX at 2% monthly interest for 5 months, use your approximation to estimate the total amount owed.

Scenario 99

A statistics officer at the Uganda Bureau of Statistics is working on demographic projections for the country's rapidly growing population.The officer needs to calculate probabilities involving binomial distributions where expressions like $(p + q)^n$ frequently appear, with p representing the probability of an event and q representing the probability of its complement. The binomial theorem provides a systematic way to expand such expressions and extract specific terms corresponding to particular demographic scenarios. This mathematical tool is indispensable for accurate population forecasting, which informs national planning in critical areas such as healthcare, education, housing, and infrastructure development across Uganda's diverse regions from the densely populated Kampala area to the more sparsely populated Karamoja region.

Task:

a)Expand $(x + y)^4$ completely using the binomial theorem.

b)Find the coefficient of the x^3y term in the expansion of $(2x + y)^5$.

c)In a population study, if the probability of a household having internet access is 0.3, what is the probability that exactly 3 out of 5 randomly selected households have internet access?

Topic 14: RANDOM VARIABLES

Sub-topic 14.1: Discrete Random Variables

Scenario 100

The Uganda National Meteorological Authority is analyzing the number of days with significant rainfall during the crucial planting season in the Mbale region,known for its high agricultural output. They define a "significant rainfall day" as one with more than 20mm of precipitation. Historical data suggests that in the 90-day planting season, the number of such days, which we can call the random variable X , varies between 20 and 40. The authority needs to model X as a discrete random variable to calculate the probability of different rainfall scenarios. This model is vital for advising farmers on optimal planting times, crop selection, and irrigation planning. Accurate predictions can significantly impact food security and the economic stability of thousands of farming households in this fertile region.

Task:

a)Explain why the number of significant rainfall days, X, is considered a discrete random variable.

b)If the probability distribution is given by $P(X = x) = kx$ for $x = 20, 21, ..., 40$, find the value of the constant k.

c)Calculate the expected number of significant rainfall days during the planting season.

Scenario 101

A mobile network operator is analyzing the number of daily customer service calls received at its Kampala call center.The random variable Y represents the number of calls related to mobile money transaction issues. Based on past data, the probability distribution of Y is partially known. The company wants to fully define this distribution to staff the call center appropriately, ensuring short wait times and high customer satisfaction. Understaffing leads to customer frustration, while overstaffing increases operational costs unnecessarily. This analysis is crucial for maintaining competitiveness in Uganda's rapidly growing telecommunications sector.

Task:

The probability distribution of Y is given by the table below. Find the missing probability a.

Y	0	1	2	3	4
P(Y)	0.1	0.3	a	0.2	0.1

a) Find the value of a.

b)Calculate the expected number of daily mobile money related calls, $E(Y)$.

c)Calculate the variance of Y, $Var(Y)$.

Scenario 102

A public health official is studying the number of new malaria cases reported per week at a specific clinic in a high-risk district.The random variable Z follows a distribution where its probabilities are proportional to the number of cases. Understanding the behavior of Z is essential for resource allocation, such as stocking antimalarial drugs, diagnostic kits, and scheduling medical personnel. Efficient management directly impacts the clinic's ability to effectively combat malaria, a major public health challenge in Uganda.

Task:

Suppose the probability mass function of Z is $P(Z = z) = c.z$ for $z = 1, 2, 3, 4$, and 0 otherwise.

a)Find the normalization constant c.

b)Find the probability that there are more than 2 new cases in a week, $P(Z > 2)$.

c)Calculate the expected value $E(Z)$.

Sub-topic 14.2: Continuous Random Variables

Scenario 103

A team of water quality engineers is monitoring the concentration of a specific mineral in the water supply of a newly developed urban area in Wakiso District.The concentration C (in milligrams per liter) is modeled as a continuous random variable with a probability density function (pdf) that is constant between 0.5 mg/L and 2.5 mg/L. Concentrations outside this range are considered abnormal. The team needs to determine the probability that a randomly collected water sample falls within the national safety standard of 1.0 to 2.0 mg/L. This analysis is critical for ensuring the health of the community and complying with Uganda's national water quality regulations.

Task:

a)If C is uniformly distributed between 0.5 and 2.5, write down its probability density function, $f(c)$

b)Calculate the probability that a water sample has a mineral concentration between 1.0 and 2.0 mg/L, $P(1.0 \leq C \leq 2.0)$.
c)Find the expected (mean) mineral concentration.

Scenario 104

An electrical engineer at the Uganda Electricity Transmission Company Ltd(UETCL) is analyzing the daily peak load (maximum power demand) on a particular substation serving a growing industrial park. The peak load L (in Megawatts) can be modeled by a continuous random variable with the probability density function $f(l) = k\,l\,(10 - l)$ for $0 \leq l \leq 10$, and 0 elsewhere. Understanding this distribution is vital for planning grid upgrades, preventing blackouts, and ensuring a reliable power supply that supports industrial growth and job creation in the region.

Task:

a)Find the value of k that makes $f(l)$ a valid probability density function.
b)Calculate the probability that the daily peak load exceeds 8 MW, $P(L > 8)$.
c)Determine the median peak load.

Scenario 105

A biologist studying the growth patterns of a particular tree species in the Mabira Forest Reserve measures the height of fully matured trees.The height H (in meters) is modeled as a continuous random variable with the cumulative distribution function (CDF) $F(h) = 1 - e^{-0.2h}$ for $h \geq 0$. This information helps in understanding the forest's ecology, estimating carbon sequestration, and developing sustainable forestry management practices that balance conservation with the economic needs of local communities.

Task:

a)Use the CDF to find the probability that a randomly selected tree is taller than 10 meters, $P(H > 10)$.
b)Find the probability density function (pdf) $f(h)$ of the tree height.
c)Calculate the probability that a tree's height is between 5 and 15 meters.

Topic 15: PROBABILITY DISTRIBUTIONS

Sub-topic 15.1: Binomial Distribution

Scenario 106

A quality control inspector at a bottled water plant in Mbarara is responsible for ensuring the purity of the final product.The production line has a known historical defect rate where 2% of bottles fail a stringent purity test. Each day, the inspector randomly selects and tests 50 bottles from the production line. The management needs to know the probability of finding a certain number of defective bottles in this daily sample to assess the consistency of their production process and maintain their certification with the Uganda National Bureau of Standards (UNBS). This scenario perfectly models a binomial distribution, where each bottle test is an independent Bernoulli trial with two outcomes: pass or fail (defective).

Task:

a)State the two parameters, n and p , for the binomial distribution in this context.
b)Calculate the probability that the inspector finds exactly one defective bottle in the daily sample of 50.
c)Calculate the probability that the inspector finds at least two defective bottles.

Scenario 107

A vaccination team is conducting a door-to-door measles immunization campaign in a rural village in Nakaseke District.Based on previous campaigns, they estimate that the probability of a household agreeing to vaccinate their eligible children is 0.75. The team plans to visit 20 households in a day. The district health officer needs to predict the number of households that will likely consent to the vaccination to plan for the required number of vaccine doses and logistical support. This allows for efficient resource allocation, minimizing waste of precious vaccines and ensuring the campaign's success in protecting children from preventable diseases.

Task:

a)Define a suitable random variable X for this scenario and state its distribution.

b)Calculate the expected number (mean) of households that will consent to vaccination.

c)Find the probability that more than 15 households will consent.

Scenario 108

A mathematics tutor in Kampala is preparing students for a national exam.She knows that a student has an 80% chance of correctly solving a particular type of problem. In a practice test containing 10 of these problems, she wants to analyze a student's performance. Understanding the binomial distribution allows her to distinguish between random variation in scores and a genuine change in a student's understanding, enabling her to provide targeted help and improve learning outcomes.

Task:

a)What is the probability that a student solves exactly 8 out of the 10 problems correctly?

b)What is the probability that a student solves at least 8 problems correctly?

c)Calculate the variance of the number of correctly solved problems.

Sub-topic 15.2: Uniform Rectangular Distribution

Scenario 109

A city bus on a specific route in Jinja is scheduled to arrive at a particular stop every 30 minutes.However, due to unpredictable traffic conditions, a passenger arriving randomly at the stop experiences a waiting time that is uniformly distributed between 0 and 30 minutes. The city's transport authority wants to understand the passenger experience to assess the need for more frequent services or better schedule adherence. Analyzing this uniform distribution of waiting times provides concrete data on passenger wait times, which is a key metric for public satisfaction and the efficiency of the urban transport system.

Task:

a)Write down the probability density function (pdf) for the waiting time T.

b)Calculate the probability that a passenger waits for less than 5 minutes.

c)Find the average (expected) waiting time for a passenger.

Scenario 110

A soil scientist is analyzing the pH level of soil samples from a large,uniformly managed agricultural field in the Kigezi highlands. Preliminary analysis suggests that the pH levels in the field are uniformly distributed between 5.5 and 7.0. This information is critical for determining the correct amount of lime needed to neutralize the soil acidity for optimal crop growth, directly impacting the yield and profitability for the local farming cooperative.

Task:

Let the random variable X represent the soil pH level.

a)State the probability density function $f(x)$ for X.

b)The ideal pH for the intended crop is between 6.0 and 6.5. What is the probability that a randomly selected soil sample is in this ideal range?

c)Calculate the standard deviation of the pH level.

Scenario 111

An IT manager at a large organization in Entebbe is analyzing the time it takes for the central server to respond to a user request(server latency). During a stable operational period, the latency is found to be uniformly distributed between 50 milliseconds and 150 milliseconds. Understanding this distribution is essential for setting performance benchmarks, identifying potential system slowdowns, and ensuring a smooth user experience for hundreds of employees who rely on the system for their daily tasks.

Task:
Let L be the server latency in milliseconds.
a)Sketch the probability density function (pdf) for L .
b)The service level agreement (SLA) states that latency should not exceed 100 ms. What is the probability that a random request violates this SLA?
c)Find the median server latency.

Sub-topic 15.3: Normal Distribution

Scenario 112

The Uganda National Examinations Board(UNEB) is analyzing the scores of the recent Uganda Advanced Certificate of Education (UACE) Mathematics examination. The scores are found to be normally distributed with a mean of 65 and a standard deviation of 12. UNEB needs to determine the percentage of candidates who scored above 80 to award distinctions, and the score that represents the top 10% of candidates for scholarship considerations. This use of the normal distribution allows for fair and standardized comparison of candidate performance across different years and examination cycles.

Task:
a)Calculate the proportion (probability) of candidates who scored more than 80.
b)Find the minimum score required to be in the top 10% of candidates.
c)What percentage of candidates scored between 50 and 75?

Scenario 113

A manufacturer of medical syringes in Kampala must ensure that the volume of each syringe is highly consistent.The filling machine produces syringes with volumes that are normally distributed with a mean of 5.0 ml and a standard deviation of 0.05 ml. Regulatory standards require that 99% of syringes must contain between 4.9 ml and 5.1 ml. The quality assurance team needs to verify if the manufacturing process meets this strict requirement, as under-dosing or over-dosing can have serious implications for patient care and drug efficacy in Ugandan hospitals and clinics.

Task:
a)Calculate the probability that a randomly selected syringe has a volume between 4.9 ml and 5.1 ml.
b)Does the process meet the regulatory standard of 99%? Justify your answer.
c)To what value would the standard deviation need to be reduced to ensure that 99.9% of syringes fall within the 4.9 ml to 5.1 ml range?

Scenario 114

An anthropologist is studying the heights of adult males in a specific ethnic group in the Karamoja region.The heights are normally distributed with a mean of 172 cm and a variance of 49 cm^2. This research contributes to understanding human physical variation and is also useful for designing ergonomic tools, furniture, and infrastructure that are better suited to the local population, thereby improving comfort and productivity.

Task:
a)Find the probability that a randomly selected adult male from this group is taller than 180 cm.
b)Find the height that is exceeded by 75% of the population.
c)If four men are selected at random, what is the probability that their average height is less than 170 cm?

Sub-topic 15.4: Normal Approximation to the Binomial Distribution

Scenario 115

A large regional hospital in Mbale is analyzing its outpatient department's patient satisfaction survey.Historically, from extensive data, they know that 70% of patients rate the service as "satisfactory." In the latest survey, they collected responses from 200 randomly selected patients. The hospital administration wants to know the probability that more than 150 of these patients reported being satisfied. Calculating this directly using the binomial distribution would be computationally intensive due to the large sample size (n=200). The statistician on staff suggests using the normal distribution as an approximation to the binomial distribution to simplify the calculation while maintaining acceptable accuracy, allowing management to quickly gauge recent performance trends.

Task:

a)State the conditions for using a normal approximation for a binomial distribution. Check if they are satisfied in this case.

b)Calculate the mean (μ) and standard deviation (σ) of the approximating normal distribution.

c)Using the normal approximation with continuity correction, find the probability that more than 150 patients reported satisfaction.

Scenario 116

A nationwide telecommunications company in Uganda is auditing the success rate of its mobile money cash-out transactions.The company knows that the probability of any single transaction failing due to network or system error is 5%. On a typical day, a single agent performs 500 such transactions. The risk management department needs to estimate the probability that the agent experiences more than 30 failed transactions in a day. This helps in identifying agents or regions that might need technical support or system upgrades. Using the normal approximation to the binomial distribution makes this estimation feasible and efficient for monitoring the performance of thousands of agents across the country in near real-time.

Task:

a)Justify why the normal approximation is suitable for this problem.

b)Find the mean and standard deviation for the normal approximation.

c)Use the normal approximation with continuity correction to estimate the probability of having more than 30 failed transactions.

Scenario 117

A large-scale bean seed producer in Masaka supplies seeds to farmers across East Africa.The producer claims that their germination rate is 90%. A quality control inspector from the Uganda National Bureau of Standards (UNBS) visits the facility and randomly tests a batch of 1000 seeds. The inspector will flag the batch for further review if the number of seeds that germinate is below a certain threshold, suggesting the true germination rate might be lower than advertised. To set this threshold appropriately, the UNBS needs to know the distribution of the number of germinated seeds. Using the normal approximation to the binomial distribution allows them to easily calculate probabilities and set fair, statistically sound quality control limits that protect farmers without unfairly penalizing the producer.

Task:

a)Calculate the mean and standard deviation for the number of germinated seeds in the batch of 1000.

b)Using the normal approximation, find the probability that fewer than 875 seeds germinate.

c)The producer wants to be 95% confident that a batch will not be flagged. What is the minimum number of seeds (in a test of 1000) that must germinate to meet this standard? (Hint: Find the value k such that $P(X \geq k) = 0.95$).

Topic 16: ERROR ANALYSIS

Sub-topic 16.1: Errors

Scenario 118

A team of civil engineers is conducting a land survey for a new road project connecting two towns in the Busoga region. They need to calculate the total area of a rectangular plot of land. The length of the plot is measured as 250.5 meters, with a possible error of ±0.2 meters. The width is measured as 180.3 meters, with a possible error of ±0.1 meters. The procurement department needs to know the maximum possible error in the calculated area to budget for extra materials (like asphalt and gravel) that might be needed due to measurement uncertainties. This analysis of error propagation ensures the project stays within budget and is completed on schedule, even if the initial measurements were at the extreme ends of their error ranges.

Task:

a)Calculate the nominal area of the plot (using the measured length and width).

b)Estimate the maximum possible absolute error in the area calculation.

c)Calculate the relative error and the percentage error in the area.

Sub-topic 16.2: Propagation of Errors

Scenario 119

A pharmacist at Mulago National Referral Hospital is preparing an intravenous(IV) saline solution. The concentration C of the solution depends on the mass of salt m and the volume of water V, given by $C = \frac{m}{V}$. The mass is measured as 9.0 grams with an absolute error of ±0.1 grams. The volume is measured as 1.0 liter with an absolute error of ±0.02 liters. An incorrect concentration can be dangerous for patients. The hospital's quality control requires an understanding of how the errors in mass and volume measurement propagate to create an error in the final concentration, ensuring patient safety and adherence to strict medical standards.

Task:

a)Calculate the nominal concentration of the saline solution.

b)Using the formula for error propagation in a quotient, estimate the absolute error in the concentration, ΔC .

c)Find the percentage error in the concentration.

Sub-topic 16.3: Errors in Functions

Scenario 120

A physicist at Makerere University is determining the period T of a simple pendulum using the formula $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the string. In an experiment, the length is measured as L = 1.00 meter with a possible error of $\Delta L = \mp 0.005$ meters. The value of g (acceleration due to gravity) is taken as 9.81 m/s² and is assumed to be exact for this purpose. The student needs to find out how the error in measuring the length affects the calculated value of the period. This understanding is crucial for evaluating the precision of the experimental results and for reporting them with appropriate significant figures and confidence intervals in their final research paper.

Task:

a)Calculate the nominal value of the period T .

b)Use differentials (or error propagation for a function of one variable) to estimate the absolute error in the period, ΔT .

c)Calculate the percentage error in the period.

Topic 17: VECTORS

Sub-topic 17.1: Vectors in Three Dimensions

Scenario 121

A team of geologists from the Directorate of Geological Survey and Mines is conducting a subsurface mineral exploration in the Kasese region.They are using 3D seismic imaging technology to map a potential mineral vein. The coordinates of three key sensor points relative to a central base camp are given as: Sensor A (500, 200, -50), Sensor B (300, -100, -80), and Sensor C (-100, 400, -30), where the units are in meters and the z-coordinate represents depth (negative for below surface). The team needs to calculate the vectors between these sensors and their magnitudes to understand the spatial geometry of the mineral vein and plan their drilling operations accurately. This 3D vector analysis is crucial for minimizing drilling costs and maximizing the yield of valuable minerals for Uganda's growing mining sector.

Task:

a)Find the vector AB from Sensor A to Sensor B.

b)Calculate the magnitude (length) of vector AB , which represents the straight-line distance between the two sensors.

c)Find the position vector of the midpoint between Sensor A and Sensor C.

Scenario 122

An air traffic controller at Entebbe International Airport is tracking the position of an ascending aircraft shortly after takeoff. The control tower is at the origin (0, 0, 0). At a specific moment, the aircraft is at position A (2000, 1500, 500) meters, and one minute later, it is at position B (5000, 3000, 1500) meters. The controller needs to determine the displacement vector of the aircraft during that minute, the distance it traveled, and its average velocity vector. This information is vital for maintaining safe separation between aircraft and ensuring efficient management of Ugandan airspace.

Task:

a)Find the displacement vector AB of the aircraft.

b)Calculate the distance the aircraft traveled in that minute (the magnitude of its displacement).

c)If the time taken was exactly 60 seconds, what is the average velocity vector of the aircraft? (Velocity = Displacement / Time)

Scenario 123

A civil engineer is designing the support structure for a new communication tower in Kampala.Three support cables are attached to the tower at a point T(0, 0, 50) meters and anchored to the ground at points A(20, 0, 0), B(0, 20, 0), and C(-15, -15, 0). The engineer needs to find the force vectors along each cable, assuming they are perfectly straight and under tension. Understanding these 3D vectors is essential for calculating the stresses on the tower and ensuring its stability during strong winds, which is a critical safety consideration for tall structures in Uganda.

Task:

a)Find the vector representing cable TA.

b)Calculate the magnitude of the vector for cable TB.

c)Find a unit vector in the direction of cable TC.

Sub-topic 17.2: Lines in Two and Three Dimensions

Scenario 124

A search and rescue team is operating in the mountainous Rwenzori region.The path of a missing hiker's last known route is modeled as a straight line in 3D space. The hiker started at point P(1, 2, 0) and was moving in the direction of vector d = (2, -1, 3). The rescue team, located at base camp R(5, 0, 6), needs to determine the equation of the hiker's path and find the closest point on this path to their base camp to plan the most efficient interception route. This application of 3D line equations can significantly reduce search time in difficult terrain, potentially saving lives.

Task:

a)Find the vector equation of the line representing the hiker's path.

b)Write the parametric equations of the line.

c)Calculate the shortest distance from the base camp R(5, 0, 6) to the hiker's path.

Scenario 125

An architect is designing a modern building with a prominent straight steel beam that runs diagonally through a large atrium.In the building's 3D coordinate system, the beam passes through points M(2, 1, 4) and N(5, 7, -2). The structural engineer needs the equations of the line along this beam to interface with other structural elements and to calculate loads and stresses accurately. This precise mathematical description is crucial for the structural integrity of the building, which is set to become a new landmark in Kampala's skyline.

Task:

a)Find the direction vector of the line through points M and N.

b)Write down the Cartesian (symmetric) equations of the line.

c)Determine if the point Q(8, 13, -8) lies on this beam.

Scenario 126

A drone is programmed to fly on a straight-line patrol path over a wildlife reserve to monitor for poaching activity. The path is defined by the line $r = (1, 0, 2) + \lambda (3, 1, -1)$, where the coordinates are in kilometers from a central ranger station. The rangers receive a signal from a poaching incident at point S(10, 3, -1). They need to determine if the incident is on the drone's patrol path and, if not, how far it is from the path to decide if they should redirect the drone or send a ground team. This efficient use of resources is key to protecting Uganda's valuable wildlife.

Task:

a)Write the parametric equations for the drone's path.

b)Determine whether the point S(10, 3, -1) lies on the drone's path.

c)Find the perpendicular distance from point S to the drone's path.

Sub-topic 17.3: Planes

Scenario 127

A geotechnical engineer is analyzing a large,flat rock slab on a hillside in Kabale that is at risk of sliding. The slab is modeled as a plane in 3D space that passes through three points measured by GPS: U(1, 1, 2), V(3, 4, 1), and W(2, 3, 5), with coordinates in meters. The engineer needs to find the equation of this plane to calculate its orientation and the stress forces acting upon it. This analysis is critical for assessing landslide risk and designing preventive measures to protect the nearby community and infrastructure.

Task:

a)Find two vectors that lie in the plane, for example, UV and UW.

b)Calculate the normal vector to the plane using the cross product.

c)Hence, find the Cartesian equation of the plane.

Scenario 128

An interior designer is planning the layout for a large exhibition hall in the new Uganda National Museum. One of the main walls is a vast, flat surface. In the building's 3D coordinate system, the wall (a plane) has a normal vector $n = (2, -1, 3)$ and contains the point (0, 5, 0). The designer needs to install a large, heavy display screen that must be mounted perpendicular to this wall. Knowing the exact equation of the plane is necessary to design the mounting brackets and ensure the screen is perfectly aligned, enhancing the visitor experience.

Task:

a)Write down the equation of the plane representing the wall.

b)A mounting point is at P(2, 4, 1). Determine if this point lies on the plane.

c)Find the distance from the origin (0, 0, 0) to the plane.

Scenario 129

An aviation engineer is simulating the flight path of an aircraft approaching Entebbe International Airport for landing.The ideal landing approach is defined by a specific glide path, which can be represented as a line in space. This glide path must lie within a "safe approach" plane, defined by the equation $2x - y + 3z = 6$. The engineer needs to verify that a proposed flight path, given by the line $r = (0, 6, 0) + \mu(1, 2, 0)$, lies entirely within this safe approach plane to ensure the aircraft meets all safety regulations during its final descent.

Task:

a)Show that the direction vector of the line is parallel to the plane. (Hint: It should be perpendicular to the plane's normal vector).

b)Show that a point on the line satisfies the plane's equation.

c)Based on (a) and (b), conclude whether the entire line lies in the plane.

Topic 18: DIFFERENTIATION 2

Sub-topic 18.1: Trigonometric Functions

Scenario 130

A marine engineer is designing a floating pontoon system for a new ferry terminal on Lake Victoria.The pontoon rises and falls with the water waves, and its vertical displacement h (in meters) from its average position is modeled by the function $h(t) = 2\sin(0.5t) + 0.5\cos(0.5t)$, where t is time in seconds. To design a safe and stable gangway that connects the pontoon to the shore, the engineer needs to analyze the pontoon's velocity and acceleration. This involves differentiating the displacement function, which is a combination of trigonometric functions. Understanding these rates of change is crucial for ensuring the gangway can accommodate the pontoon's movement without breaking or becoming unsafe for passengers, especially during stormy weather on the lake.

Task:

a)Find the velocity function $v(t)$ by differentiating $h(t)$ with respect to time.

b)Find the acceleration function $a(t)$ by differentiating $v(t)$.

c)Calculate the initial velocity and acceleration (at $t = 0$).

Scenario 131

An energy company is modeling the alternating current(AC) generated by a new hydroelectric turbine on the Nile. The voltage V (in volts) varies with time t (in seconds) according to the function $V(t) = 340\sin(120\pi t)$. To understand how the voltage changes at any given instant—which is vital for synchronizing the power grid and protecting electrical equipment from sudden surges—engineers need to find the rate of change of voltage. This requires differentiating the sine function, a fundamental concept in electrical engineering for analyzing AC circuits that power homes and industries across Uganda.

Task:

a)Find the derivative $\frac{dV}{dt}$, which represents the instantaneous rate of change of voltage.

b)Calculate the rate of change of voltage at time $t = 0.01$ seconds.

c)Find the maximum rate at which the voltage can change.

Scenario 132

An architect is designing a spectacular arched entrance for a new cultural center in Gulu.The arch is shaped like an inverted cosine curve. The height y (in meters) of the arch at a horizontal distance x (in meters) from its center is given by $y = 5\cos(0.2x)$ for $-15 \leq x \leq 15$. To ensure the arch is structurally sound and to plan the installation of decorative elements, the architect needs to know the steepness (gradient) of the arch at various points. This requires differentiating the cosine function, allowing the architect to identify where the curve is steepest and where it is flat, which influences both the design and the construction methodology.

Task:

a)Find the gradient function $\frac{dy}{dx}$.

b)Calculate the gradient of the arch at a point 5 meters from the center ($x = 5$).

c)Find the points where the arch is perfectly horizontal (i.e., where the gradient is zero).

Sub-topic 18.2: Exponential, Logarithmic and Inverse Trigonometric Functions

Scenario 133

A epidemiologist at the Ministry of Health is modeling the early growth of a disease outbreak in a densely populated area of Kampala.In the initial phase, the number of infected people N can be modeled by the exponential function $N(t) = 100e^{0.2t}$, where t is the time in days. To allocate medical resources effectively and plan containment strategies, health officials need to predict not just the number of cases, but also the daily growth rate of new infections. This requires differentiating the exponential function to understand how quickly the outbreak is accelerating, which is critical for implementing timely public health interventions.

Task:

a)Find the derivative $\frac{dN}{dt}$, which represents the daily growth rate of infections.

b)Calculate the number of new infections per day at $t = 5$ days.

c)Explain what the value 0.2 in the exponent represents in the context of the outbreak.

Scenario 134

A financial analyst at the Bank of Uganda is studying the impact of inflation on the purchasing power of the Ugandan Shilling.The real value V of a certain amount of money after t years of constant inflation is modeled by $V(t) = V_0e^{-0.05t}$, where V_0 is the initial value. To advise the public and policymakers on the erosion of savings over time, the analyst needs to find the instantaneous rate at which purchasing power is decreasing. This involves differentiating the exponential decay function, providing a clear measure of how inflation affects the economy and individual savings.

Task:

a)Find the derivative $\frac{dV}{dt}$.

b)Calculate the rate of decrease in purchasing power after 2 years if the initial value V_0 is 1,000,000 UGX.

c)Interpret the meaning of the negative sign in your answer for part (b).

Scenario 135

An acoustic engineer is calibrating the sound system for the National Theatre to ensure the sound intensity levels are perfect for both quiet dialogues and loud musical performances.The perceived loudness L (in decibels) is related to the sound intensity I by the logarithmic function $L(I) = 10\log(I)$, where I is measured in watts per square meter. To understand how small changes in amplifier power (which affects intensity) will impact the perceived volume, the engineer needs to differentiate this logarithmic function. This ensures that fine-tuning the equipment leads to the desired auditory experience for the audience.

Task:

a)Differentiate $L(I) = 10\log(I)$ with respect to I

b)Calculate the rate of change of loudness with respect to intensity when $I = 10^{-6} W/m^2$ (a typical conversation level).

c)What does the result from part (b) tell you about how sensitive the perceived loudness is to changes in intensity at that level?

Sub-topic 18.3: Maclaurin's Series

Scenario 136

A computer graphics engineer at a Ugandan animation studio is developing a 3D modeling software for local architects.The software needs to render complex curved surfaces, but the rendering engine can only process polynomial functions efficiently. The engineer encounters the function $f(x) = \sin(x)$, which is essential for creating smooth, organic shapes. To approximate this function within the software, they decide to use a Maclaurin series expansion. This allows them to replace the computationally expensive sine function with a polynomial that is accurate for small values of x (like those used in detailed architectural elements), significantly speeding up the rendering process without sacrificing visual quality for Uganda's growing digital design industry.

Task:

a)Write down the first four non-zero terms of the Maclaurin series for $f(x) = \sin(x)$.

b)Use your series to approximate $\sin(0.1)$ radians.

c)Estimate the percentage error of your approximation compared to the calculator value of $\sin(0.1)$.

Scenario 137

A financial quantitative analyst("quant") at a Kampala investment firm is modeling the growth of a complex financial derivative. The value of the derivative depends on the function $g(x) = e^X$, but the firm's risk assessment model requires a polynomial form to run thousands of rapid simulations. The analyst uses the Maclaurin series expansion for the exponential function to create a simplified, yet accurate, model for small fluctuations in the market variable x . This enables the firm to quickly assess potential risks and returns on investments, contributing to more stable and informed financial decision-making in Uganda's emerging capital markets.

Task:

a)Find the Maclaurin series for $g(x) = e^x$ up to the term in x^4 .

b)Use this series to approximate the value of $e^{0.2}$.

c)If the analyst used only the first three terms of the series, what would be the absolute error in the approximation for $e^{0.2}$?

Scenario 138

A civil engineer is working on the structural analysis of a newly designed,gently curved bridge in Jinja. The curve of the main support cable is described by the function $h(x) = \ln(1 + x)$, where x is the horizontal distance from the bridge's center. To calculate the bending moments and stresses in the cable using standard engineering software that prefers polynomial inputs, the engineer approximates the logarithmic function using its Maclaurin series. This approximation is valid for the small values of x relevant to the bridge's curvature and ensures the structural calculations are both accurate and feasible, guaranteeing the safety and longevity of the infrastructure.

Task:

a)Derive the Maclaurin series for $h(x) = \ln(1 + x)$ up to the term in x^4 .

b)State the interval of values for x for which this series expansion is valid.

c)Use the series to approximate $\ln(1.05)$.

Sub-topic 18.4: Further Curve Sketching

Scenario 139

An environmental scientist is modeling the concentration C of a pollutant in a stream near an industrial area in Namanve over time. The concentration is given by the function $C(t) = \frac{t}{t^2+1}$ for $t \geq 0$, where t is time in days. To communicate the findings effectively to policymakers and the public, the scientist needs to sketch a clear and accurate graph of this function. This involves finding intercepts, asymptotes, critical

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points (maximum concentration), and intervals of increase and decrease. A well-drawn graph can powerfully illustrate how the pollutant concentration peaks and then gradually dissipates, informing decisions on environmental regulation and clean-up efforts.

Task:

a)Find the y-intercept and any horizontal asymptotes of the function $C(t)$.

b)Find the first derivative $C'(t)$ and use it to determine the time t at which the pollutant concentration is at its maximum.

c)Sketch the graph of $C(t)$ for $t \geq 0$, clearly labeling the maximum point and the horizontal asymptote.

Scenario 140

An economist at Makerere University is studying a model for the cost of producing a new agricultural tool.The average cost per tool, $A(x)$ in Ugandan Shillings, when x tools are produced is given by $A(x) = \frac{x^2-5x+20}{x}$, for $x > 0$. To advise local manufacturers on the most efficient scale of production, the economist needs to sketch the graph of this average cost function. The sketch will visually reveal the production level that minimizes the average cost, which is crucial for enhancing the profitability and sustainability of small-scale manufacturing in Uganda.

Task:

a)Simplify the function $A(x)$ and find the equation of any oblique asymptote.

b)Find the first derivative $A'(x)$ and determine the number of tools x that minimizes the average cost.

c)Sketch the graph of $A(x)$, showing the asymptote and the minimum point.

Scenario 141

A physicist is analyzing the energy distribution E(\lambda) of radiation emitted from a prototype solar panel developed at a Ugandan tech hub. The function is given by $E(\lambda) = \frac{5\lambda^4}{e^{\lambda}-1}$ for $\lambda > 0$, where λ is the wavelength. This complex function has a distinctive peak, and sketching its curve is essential for understanding at which wavelength the panel emits the most energy. This information helps engineers optimize the panel's material composition to capture the most sunlight, boosting the efficiency of solar energy—a key renewable resource for Uganda.

Task:

a)State what happens to $E(\lambda)$ as $\lambda \longrightarrow 0^+$ and as $\lambda \longrightarrow \infty$ (find the horizontal asymptote).

b)The derivative of this function is complex, but it is known that the graph has a single turning point. Based on the function's behavior, sketch a plausible graph of $E(\lambda)$ for $\lambda > 0$, showing its behavior near zero and infinity and indicating the single maximum point.

c)What feature of the graph is most important for the solar panel engineers?

Topic 19: INTEGRATION 2

Sub-topic 19.1: Function and its Derivative (Change of Variables)

Scenario 142

A water engineer at the National Water and Sewerage Corporation is modeling the rate at which a new reservoir in Kiruhura District is being filled.The rate of water flow into the reservoir is given by the function $R(t) = 20t(t^2 + 4)^2$ cubic meters per hour, where t is the time in hours. To calculate the total volume of water accumulated in the reservoir after the first 3 hours of operation, the engineer needs to integrate this rate function. The presence of the composite function $(t^2 + 4)^2$ suggests that the method of integration by substitution (change of variables) is the most efficient approach. Accurately determining this volume is crucial for managing water release schedules and ensuring a stable supply for irrigation and domestic use in the surrounding communities.

Task:

a)Identify a suitable substitution u for the integral $\int 20t(t^2 + 4)^2\, dt$.

b)Using this substitution, find the indefinite integral.
c)Hence, calculate the total volume of water that has flowed into the reservoir in the first 3 hours.

Scenario 143

A biologist at Mbarara University is studying the growth rate of a bacterial culture used in a biogas production experiment. The growth rate is modeled by the function $G(t) = \frac{3t^2}{\sqrt{t^3+9}}$ bacteria per minute. To predict the total increase in the bacterial population between $t = 0$ and $t = 2$ minutes, the biologist must integrate this growth rate function. The structure of the function, with t^2 in the numerator and a square root of a cubic function in the denominator, makes integration by substitution the ideal method. This calculation helps in optimizing the biogas production process, a key renewable energy technology for rural Uganda.

Task:

a)For the integral $\int \frac{3t^2}{\sqrt{t^3+9}}\,dt$, state an appropriate substitution for u.
b)Rewrite the integral in terms of u and du, and then solve it.
c)Evaluate the definite integral from $t = 0$ to $t = 2$ to find the total increase in the bacterial population over that time.

Scenario 144

An economist is analyzing the marginal revenue for a locally manufactured product in Kampala.The marginal revenue function is given by $MR(x) = x\sqrt{2x^2+5}$, where x is the number of units sold in hundreds. To find the total revenue function, the economist needs to integrate the marginal revenue. The composite function under the square root indicates that a substitution method will simplify the integration process. This analysis is vital for the company to understand its revenue structure and make informed production and pricing decisions in a competitive market.

Task:

a)For the integral $\int x\sqrt{2x^2+5}\,dx$, choose a substitution u that will simplify the expression.
b)Find the indefinite integral using this substitution.
c)If the total revenue is zero when no units are sold, find the constant of integration and write the specific total revenue function.

Sub-topic 19.2: Exponential and Logarithmic Functions

Scenario 145

A public health official is tracking the rate of administration of a new vaccine during a mass vaccination campaign in the Lango sub-region.The rate at which vaccines are administered is modeled by $A(t) = 500e^{0.1t}$ vaccines per day, where t is time in days. To evaluate the campaign's success and plan for future initiatives, the official needs to calculate the total number of vaccines administered from day 5 to day 10 of the campaign. Integrating this exponential function will provide the total, demonstrating the power of exponential growth in successful public health interventions.

Task:

a)Find the indefinite integral $\int 500e^{0.1t}\,dt$.
b)Calculate the total number of vaccines administered between day 5 and day 10.
c)What does the value 0.1 in the exponent represent in the context of the campaign?

Scenario 146

A chemical engineer at a Jinja-based sugar factory is studying the rate of a catalytic reaction used in processing molasses.The reaction rate $R(t)$ is given by $R(t) = \frac{10}{t+1}$ grams per second, for $t \geq 0$. To determine the total amount of product formed in the first 9 seconds of the reaction, the engineer must integrate this function. The integral of $\frac{1}{t+1}$ is a natural logarithm, making this a key application of integrating functions that yield logarithmic results, which is essential for optimizing industrial processes.

Task:

a)Find the indefinite integral $\int \frac{10}{t+1}\, dt$.

b)Calculate the total product formed from $t = 0$ to $t = 9$ seconds.

c)Explain why the result is a logarithmic function.

Scenario 147

A financial analyst is modeling the depreciation of a fleet of new motorcycles(boda bodas) for a Kampala transport cooperative. The value $V(t)$ of a motorcycle decreases at a rate proportional to its current value, leading to a differential equation $\frac{dV}{dt} = -kV$. The solution to this equation is an exponential decay function. To find the average value of a motorcycle over its first 3 years of service, the analyst needs to integrate this exponential decay function and divide by the time interval. This calculation is critical for the cooperative's accounting, insurance, and long-term financial planning.

Task:

Suppose the value of a motorcycle is $V(t) = 5{,}000{,}000e^{-0.2t}$ UGX, where t is in years.

a)Find the average value of the motorcycle over the first 3 years.

(Recall: Average value $= \frac{1}{b-1}\int f(x)\, dx$)

b)Interpret what the value 0.2 in the exponent means for the depreciation.

c)After how many years will the motorcycle's value be half of its original value?

Sub-topic 19.3: Trigonometric Functions

Scenario 148

A power engineer at the Uganda Electricity Generation Company Ltd(UEGCL) is analyzing the alternating current (AC) power output from the Isimba Hydropower Plant. The instantaneous power delivered to a resistive load is given by $P(t) = V_0I_0\sin^2(\omega t)$, where V_0 and I_0 are peak voltage and current, and \omega is the angular frequency. To calculate the average power over one complete cycle a crucial value for billing and grid stability—the engineer must integrate this trigonometric function. Using the trigonometric identity $\sin^2(\theta) = \frac{1-\cos 2\theta}{2}$ simplifies the integration process, demonstrating a direct application of integrating even powers of sine.

Task:

a)Using the identity, rewrite $P(t) = 1000\sin^2(100\pi)$ in an integrable form. (Assume $V_0I_0 = 1000$ Watts).

b)Find the average power over one period, $T = \frac{2\pi}{100\pi} = 0.02$ seconds. (Average $= \frac{1}{T}\int_0^T P(t)dt$)

c)State the final average power in Watts.

Scenario 149

A civil engineer is designing a parabolic arch for a bridge over the River Nile in Pakwach. For a specific stress analysis, they need to calculate the integral of an odd power of cosine over a symmetric interval. The force distribution along a cross-section of the arch is modeled by

$F(\theta) = \cos^3(\theta)$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. The total load is found by integrating this function. The engineer simplifies the integral by factoring and using a Pythagorean identity, a common technique for integrating odd powers of trigonometric functions, ensuring the bridge design can handle the calculated stress.

Task:

a)Express $\cos^3(\theta)$ as $\cos(\theta)(1 - \sin^2(\theta))$.

b)Use the substitution $u = \sin(\theta)$ to find the indefinite integral \int \cos^3(\theta) \, d\theta .

c)Evaluate the definite integral $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos^3 \theta \, d\theta$. Explain the result based on the properties of the function.

Scenario 150

A signal processing engineer at a Ugandan telecommunications company is working on a noise cancellation algorithm. The algorithm requires calculating the integral of the product of two different trigonometric functions, which appears when analyzing signal interference. The specific integral encountered is $\int \sin(3t)\cos(2t)\,dt$. Instead of using a complicated substitution, the engineer employs the Factor Formulae (product-to-sum identities) to rewrite the product as a sum of simpler sine functions, which are straightforward to integrate. This efficient method helps in processing signals faster, improving call quality for millions of users.

Task:

a)Use the product-to-sum identity: $\sin(A)\cos(B) = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$ to rewrite the integrand $\sin(3t)\cos(2t)$.

b)Integrate the resulting expression with respect to t .

c)Hence, find $\int_0^{\frac{\pi}{2}} \sin(3t)\cos(2t)\,dt$.

Sub-topic 19.4: Partial Fractions

Scenario 151

A chemical engineer at a Tororo-based fertilizer plant is modeling the concentration of a catalyst during a batch reaction process.The rate of change of concentration is given by the rational function $\frac{dC}{dt} = \frac{5}{(t+1)(t+3)}$. To find the concentration C as a function of time t , the engineer must integrate this expression. The presence of distinct linear factors in the denominator makes the method of partial fractions the most efficient technique. Accurately modeling the catalyst concentration is essential for controlling the reaction rate and ensuring the final fertilizer product meets quality standards.

Task:

a)Decompose $\frac{5}{(t+1)(t+3)}$ into partial fractions.

b)Hence, find the indefinite integral $\int \frac{5}{(t+1)(t+3)}\,dt$.

c)If the initial concentration at $t=0$ is 0, find the constant of integration and write the specific solution for $C(t)$.

Scenario 152

An environmental scientist is studying the decay of a pollutant in Lake Victoria. The decay rate is modeled by the function $R(x) = \frac{x+7}{x^2+x-2}$, where x is time in weeks. To determine the total amount of pollutant that has decayed over a certain period, the scientist needs to integrate this function. The first step is to factor the denominator and apply partial fractions to decompose the complex rational function into simpler terms that can be easily integrated, providing vital data for environmental protection efforts.

Task:

a)Factor the denominator $x^2 + x - 2$.

b)Express $\frac{x+7}{x^2+x-2}$ in partial fractions.

c)Find the indefinite integral $\int \frac{x+7}{x^2+x-2}\,dx$.

Scenario 153

An economist is analyzing the cumulative investment flow $I(t)$ into Uganda's renewable energy sector. The rate of investment is given by the function $I'(t) = \frac{2}{t(t+2)}$ (in billions of UGX per year). To project the total investment over a 5-year plan, the economist must integrate this rate function from t=1 to t=5 . Using partial fractions to handle the integral of this rational function allows for a clear and accurate projection, which is crucial for government policy and attracting further international investment.

Task:

a)Decompose $\frac{2}{t(t+2)}$ into partial fractions.

b)Find the indefinite integral $\int \frac{2}{t(t+2)} dx$.

c)Calculate the total investment from year 1 to year 5.

Sub-topic 19.5: Integration by Parts

Scenario 154

A mechanical engineer at Kiira Motors is analyzing the total energy dissipated by the brakes of a new electric bus model during a specific deceleration test in Kampala.The power dissipation as a function of time is given by $P(t) = 5te^{-0.2t}$ (in kilowatts). The total energy dissipated is the integral of power over time. The function is a product of a polynomial (t) and an exponential function ($e^{-0.2t}$) , making integration by parts the ideal method. Accurately calculating this energy is crucial for designing the brake cooling system and ensuring the vehicle's safety and reliability on Uganda's demanding urban routes.

Task:

a)For the integral $\int 5te^{-0.2t}$ dt, identify the functions u and dv for the integration by parts formula

b)Apply integration by parts to find the indefinite integral.

c)Calculate the total energy dissipated (in kilowatt-seconds) from t = 0 to t = 10 seconds.

Scenario 155

A software developer at a tech hub in Kampala is writing an algorithm to calculate the area under a curve that represents signal processing data.The function to be integrated is $f(x) = x\ln(x)$ for $x \geq 1$. This function is the product of an algebraic term (x) and a logarithmic term (ln(x)) . The developer recognizes that integration by parts is necessary, choosing u =ln(x) to simplify the integral, as its derivative $\frac{1}{x}$ is easier to manage. This calculation is a key step in optimizing the signal processing algorithm for use in mobile applications across Uganda.

Task:

a)Apply the integration by parts formula to find $\int x\ln(x)dx$

b)Use the result from part (a) to evaluate $\int_1^e x\ln(x)dx$.

c)Confirm your result by differentiating your answer from part (a).

Scenario 156

An audio engineer is designing a sound filter for a new community radio station in a rural area.The filter's impulse response involves integrating a function of the form $\int \frac{t^2}{\cos t} dt$. This requires applying the integration by parts method twice in succession due to the t^2 term. Mastering this technique allows the engineer to precisely model the filter's behavior, ensuring clear audio transmission for educational and informational programs that are vital for rural development.

Task:

a)To find $\int \frac{t^2}{\cos t} dt$, apply integration by parts once, using $u = t^2$ and $dv = \cos(t)\, dt$.

b)The resulting integral will still contain a 't' term. Apply integration by parts a second time to complete the solution.

c)Write down the final expression for the indefinite integral $\int \frac{t^2}{\cos t} dt$.

Topic 20: DYNAMICS 2
Sub-topic 20.1: Resultant Velocity
Scenario 157

A ferry captain is navigating a boat across Lake Victoria from Entebbe to Lukaya.The boat's engine can propel it at 15 km/h due north relative to the water. However, a strong current is flowing at 5 km/h from the west (i.e., due east). The captain needs to determine the boat's actual speed and direction over the lakebed (its resultant velocity) to accurately estimate the time of arrival and set the correct course to reach the intended destination. Miscalculation could lead to the boat drifting significantly off course, wasting fuel and time. This vector addition problem is fundamental to navigation on all of Uganda's major water bodies.

Task:

a)Represent the boat's velocity and the current's velocity as vectors.

b)Calculate the magnitude of the resultant velocity (the boat's actual speed).

c)Calculate the true bearing of the boat's path across the lake.

Scenario 158

An aircraft pilot needs to fly from Gulu Airport to Kidepo Valley National Park on a bearing of 060°.The aircraft's airspeed (speed relative to the air) is 200 km/h. A wind is blowing from the north at 30 km/h. To follow the desired ground track, the pilot must point the aircraft's nose (its heading) into the wind. The navigator must calculate this heading and the resulting ground speed to file an accurate flight plan and ensure sufficient fuel for the journey over the remote Karamoja region.

Task:

a)Resolve the wind velocity into components acting along and perpendicular to the desired track (060°).

b)Determine the aircraft's required heading to compensate for the wind drift.

c)Calculate the aircraft's ground speed along the desired track.

Scenario 159

A search and rescue team is operating a drone to locate a missing person in the Rwenzori Mountains.The drone can fly at 10 m/s relative to the air. A steady wind is blowing from the southwest at $4\sqrt{2}$ m/s (which gives components of 4 m/s from the west and 4 m/s from the south). The operator needs to direct the drone on a straight-line path with a bearing of 045° (northeast) relative to the ground. To achieve this, the drone must be pointed in a different direction to counteract the wind. Calculating the correct resultant velocity and heading is critical for efficiently covering the search area.

Task:

a)Find the resultant velocity vector that the drone must have to track 045° over the ground.

b)Calculate the heading (direction) the drone must be pointed to achieve this resultant velocity.

c)What will the drone's ground speed be?

Sub-topic 20.2: Relative Motion
Scenario 160

Two boats,A and B, are on Lake Albert. Boat A is located 2 km due east of a lighthouse and is moving due north at 20 km/h. Boat B is located 3 km due north of the same lighthouse and is moving due east at 15 km/h. The coast guard needs to determine if the boats are on a collision course and, if not, find the distance of closest approach between them to assess any risk. This problem involves analyzing the relative motion of one boat with respect to the other.

Task:

a)Find the initial position vector of Boat B relative to Boat A.

b)Find the velocity vector of Boat B relative to Boat A.

c)Calculate the shortest distance between the two boats during their motion.

Scenario 161

Two public buses,the "Nalubaale Express" and the "Speke Coach," are traveling on two straight, perpendicular roads that intersect at a junction in Kampala. The Nalubaale Express is approaching the junction from the west at a constant speed of 60 km/h and is 2 km away. The Speke Coach is approaching from the south at 80 km/h and is 1.5 km away. A traffic control AI needs to calculate the time before they are closest to each other and that minimum distance to evaluate the risk of a collision and manage traffic signals if necessary.

Task:

a)Set up equations for the position of each bus as a function of time, t hours.

b)Find an expression for the distance between the two buses at time t.

c)Find the time t at which this distance is a minimum and calculate the minimum distance.

Scenario 162

During military training exercises in Nakasongola,two armored vehicles are maneuvering. Vehicle P is moving with a velocity of $(3i + 4j)$ m/s, and Vehicle Q has a velocity of $(5i - 2j)$ m/s. The commander in Vehicle Q needs to know the velocity of Vehicle P as seen from his own vehicle. This relative velocity is crucial for tactical positioning, targeting, and avoiding friendly fire incidents during the simulated combat scenario.

Task:

a)State the formula for the velocity of P relative to Q.

b)Calculate the velocity of Vehicle P relative to Vehicle Q.

c)What is the speed of Vehicle P as observed by the commander in Vehicle Q?

Sub-topic 20.3: Projectiles

Scenario 163

The Uganda Wildlife Authority (UWA) is designing a new system to safely tranquilize aggressive elephants from a distance. The tranquilizer dart is fired from a high-pressure gun at an initial speed of 80 m/s. To ensure the dart reaches an elephant typically 150 meters away, the ranger needs to know the correct launch angle. Ignoring air resistance, the ranger must calculate the required angle so that the dart's horizontal range is exactly 150 meters. This ensures the humane and effective immobilization of the animal for relocation or medical treatment.

Task:

a)State the formula for the horizontal range R of a projectile launched with speed u at an angle θ to the horizontal.

b)Calculate the two possible launch angles that will give a range of 150 m with an initial speed of 80 m/s.

c)Which of the two angles would be more practical for this application and why?

Scenario 164

During a cultural festival in Jinja,a cannon is used to launch a payload of confetti over the crowd. The confetti capsule is launched from ground level with an initial velocity of 25 m/s at an angle of 60° to the horizontal. The event organizers need to know the maximum height reached by the capsule and the total time it remains in the air to coordinate the explosion for maximum visual effect and ensure it happens safely above the spectators.

Task:

a)Calculate the maximum height reached by the projectile.

b)Calculate the total time of flight.

c)Determine the horizontal distance from the launch point where the confetti capsule should be set to explode (at the peak of its trajectory).

Scenario 165

An engineer is testing the water jets of a new fountain for the Kampala City Square.One jet is designed to launch water at 10 m/s from a nozzle inclined at 30° to the horizontal. The water is meant to land in a catchment pool that starts 5 meters away from the nozzle. The engineer needs to verify if the water will clear the edge of the pool and determine the maximum height of the fountain to ensure it meets the aesthetic design specifications.

Task:

a)Find the time taken for the water to reach the point 5 meters horizontally from the nozzle.

b)Calculate the height of the water at this horizontal distance. Will it have already landed, or will it clear a 0.5-meter high pool edge?

c)Calculate the maximum height of this water jet.

Topic 21: TRAPEZIUM RULE

Sub-topic 21.1: Estimating an Integral

Scenario 166

A hydrologist at the Ministry of Water and Environment is analyzing the cross-sectional area of the River Nile at a point near Jinja to estimate the flow rate.The width of the river at this point is 20 meters. Depth measurements are taken at 4-meter intervals across the river, yielding the following depths (in meters): 0, 1.2, 2.5, 3.1, 2.8, 1.9, 0. The hydrologist needs to estimate the cross-sectional area of the river. Since the shape is irregular, the Trapezium Rule provides a suitable numerical method for this estimation, which is crucial for calculating the volume of water flowing downstream to the power plant.

Task:

a)Sketch the cross-section with the given data points.

b)Use the Trapezium Rule with 6 strips (7 ordinates) to estimate the cross-sectional area.

c)State one way to improve the accuracy of this estimate.

Scenario 167

An environmental scientist is studying the rate of carbon dioxide absorption by a forest plantation in the Bugoma Forest area.The rate of absorption, $R(t)$ in tons per day, was recorded at the start of each month for a year. The values are complex and do not fit a simple function. To find the total amount of CO_2 absorbed over the year, the scientist needs to integrate the rate function. Using the available discrete data points, the Trapezium Rule offers a practical way to approximate this definite integral and assess the forest's carbon sequestration potential, a key metric for climate change mitigation efforts in Uganda.

Task:

Suppose the following simplified data represents the rate at the start of each month for 6 months: $R = \{2, 5, 7, 6, 4, 3\}$ tons/day.

a)State the width h of each strip if the data covers a 6-month period.

b)Apply the Trapezium Rule to estimate the total CO_2 absorbed over the 6 months.

c)What is the unit of your final answer, and what does it represent?

Scenario 168

An electrical engineer at Umeme Ltd.is analyzing the power consumption of a small factory in Namanve over a 24-hour period. The power $P(t)$ in kilowatts was logged every 4 hours. The values are: $P = \{50, 150, 200, 180, 120, 80, 60\}$. The total energy consumed is the integral of power over time. Since the function is not known, the engineer uses the Trapezium Rule to estimate the total energy usage in kilowatt-hours (kWh). This helps the factory manager understand daily energy costs and identify peak usage periods.

Task:

a)List the 7 ordinates y_0 to y_6 from the data.

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b)Apply the Trapezium Rule formula to estimate the integral $\int_0^{24} P(t)\,dt$.
c)State the estimated total energy consumption in kWh.

Sub-topic 21.2: Percentage Error
Scenario 169
A civil engineer needs to calculate the area of a land plot bounded by a curved road and a straight boundary.The perpendicular distances from the straight boundary to the curve are known at 5-meter intervals. The exact area can be found by integrating the function $f(x) = 10 + \sqrt{x}$ from $x=0$ to $x=20$. To check the reliability of a field measurement done using the Trapezium Rule with 4 strips, the engineer calculates the percentage error between the trapezium estimate and the exact integral value. This validates the field method for rapid land area assessment in Uganda's ongoing road expansion projects.
Task:
a)Calculate the exact value of $\int_0^{20} 10 + \sqrt{x}\ dx$.
b)Use the Trapezium Rule with 4 strips (h=5) to estimate the integral.
c)Calculate the percentage error of the trapezium estimate compared to the exact value.
Scenario 170
A physics student at Makerere University is using a sensor to measure the force exerted on a moving object over time.The data is collected at discrete intervals, and the student uses the Trapezium Rule to estimate the total impulse (the integral of force with respect to time). To understand the accuracy of this method for their experiment, they test it on a function with a known integral: $\int_0^{6} x^2\,dx$. Comparing the trapezium estimate with the exact value allows the student to report a percentage error, which is essential for evaluating the experimental setup's precision.
Task:
a)Find the exact value of $\int_0^{6} x^2\,dx$
b)Estimate the integral using the Trapezium Rule with 3 strips (h=2).
c)Calculate the percentage error of this estimate.
Scenario 171
A financial analyst is modeling the total revenue over a quarter (90 days) where the daily revenue rate r(t) is given by the function $r(t) = 1000e^{0.001t}$. The exact total revenue is $\int_0^{90} r(t)dt$. The analyst wants to use the Trapezium Rule for a quick approximation but needs to quantify the potential error if they use only 9 strips (10 data points, one every 10 days). Calculating the percentage error helps decide if this level of approximation is acceptable for the preliminary report to the board of directors.
Task:
a)Calculate the exact value of $\int_0^{90} r(t)dt$.
b)Estimate the integral using the Trapezium Rule with 9 strips (h=10).
c)Find the percentage error in the trapezium estimate.

Topic 22: SAMPLING DISTRIBUTION
Sub-topic 22.1: Distribution of Sampling Mean
Scenario 172
The Uganda Bureau of Statistics(UBOS) is conducting a study on household electricity consumption in Kampala. From historical data, they know that the monthly consumption for a household is normally distributed with a mean (μ) of 250 kWh and a standard deviation (σ) of 50 kWh. UBOS plans to take a random sample of 100 households to estimate the average consumption. The statisticians need to determine the probability that the sample mean will be greater than 260 kWh. This involves

understanding the distribution of the sample mean, which is crucial for planning energy distribution and identifying potential shifts in consumption patterns that could strain the national grid.

Task:

a)State the mean and standard deviation (standard error) of the sampling distribution of the sample mean for a sample size of $n = 100$.

b)Calculate the z-score for a sample mean of $x = 260$ kWh.

c)Find the probability that the sample mean exceeds 260 kWh.

Scenario 173

A large maize mill in Mbale receives shipments of maize from local farmers. The weight of a fully loaded truck is known to be normally distributed with a mean of 10,000 kg and a standard deviation of 400 kg. The quality control manager randomly selects 25 trucks and weighs them to check if the average load is consistent with the expected mean. The manager needs to know the probability that the average weight of this sample is less than 9,900 kg, which would indicate a potential issue with the loading process or the scales.

Task:

a)Describe the distribution of the sample mean weight for samples of size $n = 25$.

b)Calculate the standard error of the mean.

c)Find the probability that the sample mean weight is less than 9,900 kg.

Scenario 174

A mobile money company analyzes the average transaction value. The transaction values are heavily right-skewed with a mean of UGX 50,000 and a standard deviation of UGX 20,000. Due to the Central Limit Theorem, the company knows that the distribution of the sample mean for large samples will be approximately normal. For their annual audit, they take a random sample of 64 transactions. The auditors need to know the probability that the average transaction value in this sample falls between UGX 48,000 and UGX 52,000 to assess the accuracy of their financial reporting.

Task:

a)State the mean and standard error of the sampling distribution for $n = 64$.

b)Calculate the z-scores for sample means of UGX 48,000 and UGX 52,000.

c)Find the probability that the sample mean lies between UGX 48,000 and UGX 52,000.

Sub-topic 22.2: Point Estimation

Scenario 175

A pharmaceutical company in Entebbe is testing the purity of a new batch of antimalarial tablets.The active ingredient content per tablet should be 500 mg. The quality control team takes a random sample of 30 tablets from the production line and measures the active ingredient. The sample mean is calculated to be 498 mg. The team uses this single value, the point estimate, as the best guess for the true population mean of the entire batch. This point estimate is critical for deciding whether to release the batch for distribution or to reject it, ensuring that patients receive the correct dosage.

Task:

a)What is the point estimate for the population mean active ingredient content?

b)Is this point estimate a parameter or a statistic?

c)State one limitation of relying solely on a point estimate.

Scenario 176

The Ministry of Agriculture wants to estimate the average yield of maize(in tons per hectare) for smallholder farmers in the Lango sub-region this season. It is impractical to measure every farm, so they commission a survey. A random sample of 50 farms is selected, and the sample mean yield is calculated to be 2.5 tons per hectare. This value serves as the point estimate for the true average yield across the entire region. This estimate is vital for forecasting national food security and planning for potential imports or exports.

Task:

a)Identify the parameter being estimated and the point estimator used.

b)The sample standard deviation is found to be 0.4 tons/hectare. What is the point estimate for the population variance?

c)Why is it important that the sample is random?

Scenario 177

A university administrator wants to estimate the proportion of students who are satisfied with the new online learning platform.A random sample of 200 students is surveyed, and 150 of them express satisfaction. The sample proportion $p = \frac{150}{200} = 0.75$ is used as a point estimate for the true population proportion p of all students who are satisfied. This information helps the administration decide whether to continue investing in the platform.

Task:

a)What is the point estimate for the population proportion p ?

b)Calculate the standard error of this point estimate.

c)If another sample of 200 students were taken, would you expect the point estimate to be exactly 0.75 again? Explain.

Sub-topic 22.3: Interval Estimation

Scenario 178

Following the maize yield survey in Lango sub-region(Scenario 176), the Ministry of Agriculture realizes that a single point estimate of 2.5 tons/hectare does not convey the uncertainty in the estimate. They decide to construct a 95% confidence interval for the true average yield. With a sample mean of 2.5, a sample standard deviation of 0.4, and a sample size of 50, they can calculate an interval that they are 95% confident contains the true population mean. This interval provides a range of plausible values, which is more informative for policy-making than a single number.

Task:

a)Calculate the standard error of the mean.

b)Find the critical z-value for a 95% confidence level.

c)Construct a 95% confidence interval for the true average maize yield.

Scenario 179

A public health researcher wants to estimate the average blood pressure of adults in a specific district.From a random sample of 40 adults, the sample mean is 128 mmHg and the sample standard deviation is 15 mmHg. Since the population standard deviation is unknown and the sample size is less than 60, the researcher uses the t-distribution to construct a 99% confidence interval. This interval will provide a range that is very likely to contain the true district-wide average blood pressure, helping to identify potential public health risks.

Task:

a)State the degrees of freedom for the t-distribution in this case.

b)Find the critical t-value for a 99% confidence interval.

c)Construct the 99% confidence interval for the population mean blood pressure.

Scenario 180

The university administrator from Scenario 177,who found a sample proportion of 0.75, now wants to construct a 90% confidence interval for the true proportion of satisfied students. This interval will show a range of values that is likely to contain the **actual** satisfaction rate for the entire student population, giving a better sense of the estimate's precision than the point estimate alone.

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Task:
a)Calculate the standard error for the sample proportion.
b)Find the critical z-value for a 90% confidence level.
c)Construct a 90% confidence interval for the true population proportion p .

Topic 23: ITERATIVE METHODS
Sub-topic 23.1: Interpolation and Extrapolation
Scenario 181
A hydrologist at the Ministry of Water and Environment is analyzing water level data from Lake Victoria. The water level L (in meters above sea level) was recorded at specific times. At 6:00 AM, the level was 1133.50 m; at 10:00 AM, it was 1133.65 m; and at 2:00 PM, it was 1133.55 m. The hydrologist needs to estimate the water level at 12:00 PM (interpolation) to fill a data gap and predict the level at 6:00 PM (extrapolation) for dam management purposes. Using linear interpolation between the closest known points provides a simple yet effective method for these estimations, which are critical for managing water release schedules at the Owen Falls Dam.
Task:
a)Using the data points for 10:00 AM (1133.65 m) and 2:00 PM (1133.55 m), estimate the water level at 12:00 PM via linear interpolation.
b)Estimate the water level at 6:00 PM by extrapolating from the 2:00 PM data point, assuming the rate of change between 10:00 AM and 2:00 PM continues.
c)State one risk associated with using extrapolation for prediction.
Scenario 182
An economist at the Bank of Uganda is studying the relationship between the interest rate(%) and the volume of loans (UGX billions) issued by commercial banks. Data is available for interest rates of 5%, 10%, and 15%, with corresponding loan volumes of 500, 400, and 200 billion UGX respectively. The economist needs to estimate the loan volume that would correspond to an interest rate of 12%, a value not directly observed in the data. This process of interpolation helps the central bank understand the potential impact of its monetary policy decisions on private sector credit.
Task:
a)Identify the two data points that should be used to interpolate the loan volume at 12%.
b)Use linear interpolation between these points to estimate the loan volume at a 12% interest rate.
c)If the interest rate were to rise to 18%, extrapolate to estimate the potential loan volume.
Scenario 183
An agricultural researcher is measuring the growth of a bean plant over time.The height of the plant was recorded on day 5 (10 cm), day 10 (16 cm), and day 15 (19 cm). The researcher needs to estimate the plant's height on day 12, a day for which data is missing, to complete a growth chart. This is a classic case for interpolation. Additionally, predicting the height on day 20 via extrapolation can help forecast the plant's maturity, although this is less reliable.
Task:
a)Estimate the height of the bean plant on day 12 using linear interpolation.
b)Estimate the height on day 20 using extrapolation from the last two data points.
c)Why is the estimate for day 20 likely to be less accurate than the estimate for day 12?

Sub-topic 23.2: Location of Roots
Scenario 184
A civil engineer is designing a new water pipeline that must follow a specific gradient.The elevation of the pipeline is modeled by the function $f(x) = x^3 - 3x^2 - 4x + 12$, where x is the horizontal distance in

kilometers. The pipeline must be laid where the elevation is exactly 10 meters above the baseline (i.e., where $f(x) = 10$). The engineer needs to find the horizontal distance x where this occurs. Rearranging gives $g(x) = x^3 - 3x^2 - 4x + 2 = 0$. The first step is to locate the interval where the root lies by evaluating $g(x)$ for different values of x and finding a sign change, which indicates a root lies between them.

Task:

a)Evaluate $g(x)$ for $x = 0$ and $x = 2$.

b)Based on your results, does a root exist between $x = 0$ and $x = 2$? Justify your answer.

c)Evaluate $g(3)$ and state another interval that contains a root.

Scenario 185

A financial analyst is trying to determine the internal rate of return(IRR) for a potential investment in a solar farm. The net present value (NPV) is a function of the discount rate r , given by $NPV(r) = -100 + \frac{40}{1+r} + \frac{60}{(1+r)^2} + \frac{50}{(1+r)^3}$. The IRR is the value of r that makes $NPV(r) = 0$. The analyst needs to find this root. By evaluating the function for different values of r , they can locate an interval where the sign of $NPV(r)$ changes, confirming the existence of a root within that interval.

Task:

a)Calculate $NPV(r)$ for $r = 0.0$ (0%).

b)Calculate $NPV(r)$ for $r = 0.1$ (10%).

c)Based on your calculations, confirm that a root (the IRR) lies between $r= 0.0$ and $r= 0.1$.

Scenario 186

An environmental scientist is modeling the concentration of a pollutant in a lake over time. The concentration $C(t)$ is given by $C(t) = e^{-0.2t} - 0.5t + 1$. The scientist wants to find the time t when the concentration drops to zero (i.e., find the root of $C(t) = 0$). This time represents when the lake is expected to be free of the pollutant. The first step is to locate the interval where this root lies by testing values of t and looking for a sign change in $C(t)$.

Task:

a)Evaluate $C(t)$ for $t = 0$ and $t = 2$.

b)Is there a root between $t=0$ and $t=2$? Explain.

c)Evaluate $C(t)$ for $t=3$ to further narrow down the interval containing the root.

Sub-topic 23.3: Newton Raphson Method

Scenario 187

A structural engineer needs to find the critical load factor \lambda for a bridge design, which is a root of the equation $x^3 + 2x - 5 = 0$. An initial rough estimate suggests x is close to 1.3. The Newton-Raphson method provides a fast, iterative way to find an accurate solution. This precise calculation is essential for ensuring the bridge can withstand expected loads without costly over-engineering, a key consideration for infrastructure projects in Uganda.

Task:

Let $f(x) = x^3 + 2x - 5$.

a)Find the derivative $f'(x)$.

b)Using $x_0 = 1.3$ as the initial guess, perform one iteration of the Newton-Raphson method to find x_1 .

c)State the formula for the Newton-Raphson method.

Scenario 188

A computer scientist is developing a graphics algorithm that requires calculating the inverse of a number using only addition and multiplication(i.e., without division). This can be done by finding the root of $f(x) = a - \frac{1}{x}$, which simplifies to $f(x) = ax - 1 = 0$, where a is the number. For $a = 5$, the root is $x = 0.2$ The scientist uses the Newton-Raphson method with an initial guess of $x_0 = 0.5$ to quickly converge to

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the solution, optimizing the algorithm's performance for real-time rendering in Ugandan-made mobile applications.

Task:

For $f(x) = 5x - 1$

a)Find the derivative $f'(x)$.

b)Perform two iterations of the Newton-Raphson method starting from $x_0 = 0.5$.

c)How does the speed of this convergence demonstrate the strength of the Newton-Raphson method?

Scenario 189

An electrical engineer is analyzing a circuit where the voltage V satisfies the equation $V + 2\ln(V) - 3 = 0$. An approximate solution is needed for simulation software. The engineer knows that V is positive and roughly around 2. The Newton-Raphson method is an efficient way to find a precise solution for this transcendental equation, which cannot be solved algebraically. This allows for accurate modeling of the circuit's behavior.

Task:

Let $f(V) = V + 2\ln(V) - 3$.

a)Find the derivative $f'(V)$.

b)Using $V_0 = 2$ as the initial guess, perform one iteration of the Newton-Raphson method to find V_1 .

c)Why is the Newton-Raphson method particularly suitable for this problem compared to simpler methods like bisection?

Topic 24: COORDINATE GEOMETRY 2

Sub-topic 24.1: Locus

Scenario 190

A city planner in Kampala is designing a new park.A key feature is a fountain that will be placed such that its distance from two intersecting footpaths (which act as the x-axis and y-axis) is always equal. The planner needs to find the equation of the path that the fountain's center must follow. This path is a locus defined by the condition that its perpendicular distances to the two axes are equal. Understanding this locus helps in precisely positioning the fountain and other symmetrical elements within the park's design, ensuring aesthetic balance.

Task:

a)If a point $P(x, y)$ is equidistant from the x-axis and the y-axis, write an equation relating x and y . (Hint: Distance from the x-axis is $|y|$, from the y-axis is $|x|$).

b)Simplify this equation to find the locus of all such points P .

c)Sketch the locus on a coordinate plane.

Scenario 191

An architect is designing a modern building with a curved glass facade.The curve is defined by a locus: every point on the curve is twice as far from the point (3, 0) as it is from the line $x = -3$. This specific geometric property will create a unique parabolic reflection of sunlight. The architect needs the equation of this locus to create precise digital models and construction plans for the builders.

Task:

a)Let $P(x, y)$ be a point on the locus. Write an expression for its distance from the point (3, 0).

b)Write an expression for the perpendicular distance from $P(x, y)$ to the line $x = -3$.

c)Use the given condition (distance from point is twice the distance from line) to derive the equation of the locus.

Scenario 192

A telecommunications company is installing a new radio tower.The signal strength is strongest for receivers that are exactly 3 km closer to this new tower (located at (0, 4)) than to an existing tower (located at (0, -4)). The engineers need to map the area of optimal signal strength. The boundary of this area is a locus defined by the constant difference in distances to two fixed points, which forms a hyperbola. This analysis is crucial for network planning and customer communication.

Task:

a)Let $P(x, y)$ be a point on the locus. Write an equation stating that the distance from P to (0, 4) is 3 km less than the distance from P to (0, -4).

b)Simplify this equation to find the standard form of the hyperbola.

c)What are the coordinates of the foci of this hyperbola?

Sub-topic 24.2: The Circle

Scenario 193

A civil engineer is designing a circular roundabout at a major intersection in Entebbe.The center of the roundabout is planned at coordinates (5, 2) on the city grid, and it must have a radius of 30 meters to accommodate the expected traffic flow. The engineer needs the equation of this circle to interface with road design software and to calculate the exact coordinates for the curb and landscaping elements. This precise mathematical definition ensures the roundabout is built correctly the first time, saving time and resources.

Task:

a)Write down the standard equation of a circle with center (h, k) and radius r.

b)State the equation of the roundabout with center (5, 2) and radius 30.

c)A lamppost is planned at point (20, 2). Determine if this lamppost lies on the circumference of the roundabout, inside it, or outside it.

Scenario 194

A land surveyor is mapping a plot of land bounded by three straight fences.The owner wants to build a circular water reservoir that touches all three fences. The equations of the fences are $y = 0$, $x = 0$, and $y = 8 - 2x$. The surveyor realizes that the center of the required circle must be equidistant from all three lines. This center is the incenter of the triangle formed by the fences, and the distance to any line is the radius. Finding this circle is essential for planning the construction.

Task:

a)The center (h, k) of the circle is equidistant from the lines $y=0$, $x=0$, and $y=8-2x$. Write an equation relating h and k using the distance from (h, k) to $y=0$ and $x=0$.

b)Write another equation using the perpendicular distance from (h, k) to the line $y = 8 - 2x$ (or $2x + y - 8 = 0$).

c)Solve these equations to find the coordinates of the center (h, k).

Scenario 195

An archaeologist discovers the remnants of a ancient circular stone structure.Through excavation, three points on the original circumference are found: A(1, 1), B(2, 4), and C(5, 3). To reconstruct the full plan of the structure, the archaeologist needs to find the equation of the circle that passes through these three points. This involves finding the center and radius from the general equation of a circle.

Task:

a)Substitute each point A, B, and C into the general equation of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ to form three equations.

b)Solve these simultaneous equations to find the values of g, f, and c.

c)Hence, state the center and radius of the ancient structure.

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Sub-topic 24.3: Parabola

Scenario 196

An engineer is designing a solar cooker for a rural community in Uganda.The cooker uses a parabolic reflector to focus sunlight onto a single point (the focus) where the cooking pot is placed. The reflector's cross-section is a parabola with its vertex at the origin (0, 0) and its axis along the x-axis. If the focus is placed 0.5 meters from the vertex, the engineer needs the equation of the parabola to manufacture the reflector with the correct curvature for maximum efficiency.

Task:

a)For a parabola with vertex at (0,0) and focus at (a, 0), what is its standard equation?

b)Given the focus is at (0.5, 0), state the value of a and write the equation of the parabola.

c)How wide is the reflector (the length of the latus rectum) to ensure the pot holder is the correct size?

Scenario 197

A suspension bridge is being planned over a river.The main cable hangs in the shape of a parabola. The towers on either side are 100 meters high and 200 meters apart. The lowest point of the cable is 20 meters above the water level, midway between the towers. Civil engineers need the equation of this parabola to calculate the length of the cable and the forces acting on the towers, which is critical for the structural design and safety of the bridge.

Task:

a)Set up a coordinate system with the origin at the lowest point of the cable. The parabola opens upwards. If the towers are 100 m apart horizontally from the origin, and the cable is 80 m higher at the towers, what is a point on the parabola?

b)Using the standard form $x^2 = 4ay$, find the value of a .

c)Write the equation of the parabola.

Scenario 198

A satellite dish is being installed to receive educational broadcasts in a remote school.The dish has a parabolic shape to reflect signals to a receiver at the focus. The dish is 2 meters wide and 0.5 meters deep. The installer needs to know how far from the vertex (the deepest point) to place the receiver. This requires finding the focus of the parabola that models the dish's cross-section.

Task:

a)Model the dish's cross-section as a parabola with its vertex at (0,0) and opening upwards. If the dish is 2 m wide and 0.5 m deep, what are the coordinates of the rim points?

b)Using the standard form $x^2 = 4ay$, substitute the coordinates of a rim point to find a .

c)How far from the vertex should the receiver (the focus) be placed?

Sub-topic 24.4: Ellipse

Scenario 199

An architect is designing a"whispering gallery" for a new national museum. The room has an elliptical shape. When a person stands at one focus and whispers, the sound is reflected by the walls and can be clearly heard at the other focus. The room is to be 20 meters long (the major axis) and 12 meters wide (the minor axis). The architect needs to find the precise location of the two foci to design the seating and acoustic panels correctly.

Task:

a)For an ellipse centered at the origin with a horizontal major axis, what is its standard equation in terms of a and b , where $2a$ is the major axis and $2b$ is the minor axis?

b)Given $2a = 20$ and $2b = 12$, find a and b .

c)Calculate the distance c from the center to each focus using the relationship $c^2 = a^2 - b^2$.

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Scenario 200

A gardener is planning an elaborate elliptical flower bed for a city park.The bed will be marked out with string tied between two stakes (the foci). The total length of the string is 10 meters. The gardener wants the ellipse to be 6 meters wide at its narrowest point (the minor axis). To set this up, the gardener needs to calculate how far apart to place the two stakes and the length of the semi-major axis.

Task:

a)The total length of the string is $2a$. Given this is 10 m, find a.

b)The minor axis is $2b = 6$ m. Find b.

c)Use the relationship $c^2 = a^2 - b^2$ to find c, the distance from the center to each focus. How far apart should the stakes be placed?

Scenario 201

An astronomer is tracking the orbit of a Ugandan satellite around the Earth.The orbit is elliptical. At its closest point (perigee), the satellite is 500 km from the Earth's center, and at its farthest point (apogee), it is 700 km away. The Earth's center is at one focus of the ellipse. The astronomer needs the equation of the elliptical orbit to predict the satellite's position at any given time for communication and data collection purposes.

Task:

a)In an elliptical orbit, the perigee distance is $a - c$ and the apogee distance is $a + c$. Given these are 500 km and 700 km respectively, form two equations and solve for a and c.

b)Find b, the semi-minor axis, using $b^2 = a^2 - c^2$.

c)Write the equation of the satellite's orbit, assuming the major axis is horizontal and the center of the ellipse is at the origin.

Topic 25: COMPLEX NUMBERS

Sub-topic 25.1: Imaginary Numbers

Scenario 202

An electrical engineering student at Kyambogo University is analyzing an AC circuit that has both resistance and inductance. The impedance Z of such a circuit is a complex number, given by $Z = R + j\omega L$, where R is the resistance, ω is the angular frequency, L is the inductance, and j is the imaginary unit (where $j^2 = -1$). The student needs to calculate the impedance for a circuit where $R = 4\omega$, $\omega L = 3\omega$. Understanding how to represent and manipulate this complex number is fundamental to predicting the circuit's behavior, such as the phase difference between voltage and current, which is crucial for designing efficient electrical systems across Uganda.

Task:

a)Write the impedance Z in the form $a + bj$.

b)Calculate the magnitude (modulus) of the impedance, $|Z|$.

c)Explain why the impedance cannot be represented by a single real number in this AC circuit.

Scenario 203

A control systems engineer is designing a stability analysis for an automated irrigation system.The system's transfer function has a pole at $s = -2 + 5j$. The engineer needs to understand the nature of this complex root. If the real part is negative, the system is stable; if positive, it is unstable. The imaginary part relates to the oscillation frequency of the system's response. Identifying and interpreting these complex roots is essential for ensuring the irrigation system responds predictably and without destructive oscillations to commands from the central control unit.

Task:

a)Plot the complex number $-2 + 5j$ on an Argand diagram.

b)State the real part and the imaginary part of this number.

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c)Based on the real part, would you predict the system to be stable or unstable?

Scenario 204

A mathematician is exploring the fundamental theorem of algebra,which states that a polynomial of degree n has exactly n roots in the complex number system. They consider the simple quadratic equation $x^2 + 1 = 0$. This equation has no solutions in the real number system because no real number squared gives -1. The introduction of the imaginary unit j (where $j^2 = -1$) is necessary to solve such equations, forming the foundation for complex numbers which are indispensable in advanced mathematics, engineering, and physics.

Task:

a)Solve the quadratic equation $x^2 + 1 = 0$ for x .

b)Write the two solutions in the form $a + bj$.

c)Verify your solutions by substituting them back into the original equation.

Sub-topic 25.2: Algebra of Complex Numbers

Scenario 205

A signal processing engineer at a telecommunications company in Kampala is combining two signal waves.The first wave is represented by the complex number $z_1 = 3 + 4j$ and the second by $z_2 = 1 - 2j$. The combined signal is the sum of these two complex numbers. The engineer needs to find this resultant signal to analyze its amplitude and phase. Correctly adding complex numbers is a routine but critical operation in signal processing for mobile networks, ensuring clear communication for millions of users.

Task:

a)Find the sum $z_1 + z_2$.

b)Find the product $z_1 \times z_2$.

c)Find the quotient of z_1 & z_2.

Scenario 206

A physicist is modeling the interference pattern of two light waves.The waves are represented by complex numbers $A = 2 + j$ and $B = 1 + 3j$. The intensity of the interference is related to the square of the magnitude of the sum of these complex numbers. The physicist needs to calculate $|A + B|^2$ to predict the brightness of the interference fringes in an experiment. This application is key to understanding wave optics, which has applications in laser technology and precision measurement instruments.

Task:

a)Calculate $A + B$.

b)Find the magnitude $|A + B|$.

c)Hence, find $|A + B|^2$.

Scenario 207

An economist is using a quadratic formula to model economic growth,which yields complex solutions. The solutions are $z = 2 + 3j$ and its complex conjugate. The economist needs to verify that these are indeed roots of the quadratic equation $z^2 - 4z + 13 = 0$. Understanding how complex roots work in pairs (conjugates) and how to substitute them into equations is vital for accurate economic forecasting models that can handle oscillatory behavior.

Task:

a)State the complex conjugate of $z = 2 + 3j$.

b)Substitute $z = 2 + 3j$ into the expression $z^2 - 4z + 13$ and show that the result is zero.

c)Explain why complex roots of real-coefficient polynomials always come in conjugate pairs.

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Sub-topic 25.3: Argand Diagram and Polar Form

Scenario 208

An avionics engineer is plotting the position of an aircraft on a radar screen.The aircraft's position relative to the control tower is given by the complex number $z = 3 + 4j$, where the real axis represents East-West and the imaginary axis represents North-South (units in km). The engineer needs to convert this Cartesian coordinate into polar form (modulus and argument) to determine the actual distance and bearing of the aircraft from the tower, which is the standard format used for air traffic control communications.

Task:

a)Plot the point $z = 3 + 4j$ on an Argand diagram.

b)Calculate the modulus $r = |z|$, which represents the distance from the origin.

c)Calculate the argument $\theta = \arg(z)$, which represents the bearing from due east.

Scenario 209

An electrical engineer is analyzing the phase relationship between voltage and current in a capacitive circuit. The complex impedance is $Z = 5 - 5j$ ohms. To find the phase angle π by which the voltage lags the current, the engineer needs to find the argument of this complex number. The polar form $Z = r(\cos\theta + j\sin\theta)$ makes it easy to extract this phase information, which is crucial for designing power factor correction circuits to improve the efficiency of Uganda's electrical grid.

Task:

a)Find the modulus r of $Z = 5 - 5j$.

b)Find the argument θ of Z . (Ensure you identify the correct quadrant).

c)Write Z in polar form.

Scenario 210

A mathematician is solving the cubic equation $z^3 = 8$. They know one real root is $z = 2$, but the fundamental theorem of algebra states there should be three roots in total. To find the other two complex roots, they decide to write the number 8 in polar form and then use De Moivre's Theorem. This approach elegantly reveals all roots, demonstrating the power of the polar form for solving polynomial equations.

Task:

a)Write the real number 8 in polar form.

b)Use the formula for the n th roots of a complex number to find all three cube roots of 8.

c)Represent all three roots on a single Argand diagram.

Sub-topic 25.4: Locus

Scenario 211

A telecommunications engineer is designing a cellular network for a hilly region in Kabale.A signal booster must be placed such that its distance from two existing towers (represented by complex numbers $z_1 = -3 + 0j$ and $z_2 = 3 + 0j$) is equal. This ensures balanced signal distribution. The set of all possible points for the booster forms a locus. The engineer recognizes this as the perpendicular bisector of the line segment joining the two towers. Finding the equation of this line is crucial for selecting optimal booster locations on the map.

Task:

a)The condition is $|z - z_1| = |z - z_2|$. Substitute $z_1 = -3$ and $z_2 = 3$.

b)Let $z = x + yj$. Substitute into the equation from (a) and simplify to find the Cartesian equation of the locus.

c)Describe the geometric shape of this locus.

Scenario 212

A satellite operator needs to define a"keep-out zone" around a satellite located at position $z_0 = 1 + 2j$ in the complex plane (representing coordinates in space). The zone is defined as all points within a distance of 4 units from the satellite to avoid collisions with space debris. This region is a circular disk. The operator needs the inequality that describes this locus to program into the satellite's collision avoidance system.

Task:

a)Write the inequality that represents all points z whose distance from $z_0 = 1 + 2j$ is less than or equal to 4.

b)Let $z = x + yj$. Substitute into the inequality and express it in Cartesian form.

c)What is the center and radius of the circle that forms the boundary of this locus?

Scenario 213

A robotics engineer is programming an autonomous vehicle to follow a specific path in a warehouse.The path is defined by the locus of points z such that the argument of $(z - (1 + j))$ is $\frac{\pi}{4}$ radians. This means the vehicle must move along a straight line emanating from the point 1+j at a fixed angle. The engineer needs the Cartesian equation of this line to code the vehicle's navigation system.

Task:

a)The condition is $\arg(z - (1+j)) = \frac{\pi}{4}$. Let $z = x + yj$.

b)Find an expression for $\arg(x - 1 + (y-1)j)$.

c)Using the fact that $\tan(\arg(u)) = \frac{Im}{Re}$, find the Cartesian equation of this locus.

Sub-topic 25.5: De Moivre's Theorem

Scenario 214

An electrical engineer is analyzing a three-phase power system,a common method for electrical power generation, transmission, and distribution used in Uganda's national grid. The three phase voltages are separated by 120°. Using De Moivre's Theorem, the engineer can represent these voltages as complex numbers: $V, V\omega, V\omega^2$, where ω is the complex cube root of unity. This representation simplifies the analysis of power flow and balance in the system, which is essential for maintaining a stable and efficient grid.

Task:

a)Find the complex cube roots of unity, i.e., solve $z^3 = 1$. (Hint: Write 1 in polar form).

b)Show that $1 + \omega + \omega^2 = 0$.

Scenario 215

A computer graphics programmer needs to compute $(1 + j)^{10}$ to rotate an object in a 2D simulation for an educational app developed in Kampala. Calculating this directly would be very inefficient. Instead, the programmer converts 1+j to polar form and applies De Moivre's Theorem, which states $[r(\cos\theta + j\sin\theta)]^n = r^n(\cos n\theta + j\sin n\theta)$. This allows for a quick and precise calculation of the high power, enabling smooth animations.

Task:

a)Convert $z = 1 + j$ to polar form.

b)Use De Moivre's Theorem to find z^{10}.

c)Convert your answer back to Cartesian (a+bj) form.

Scenario 216

A mathematician is exploring patterns in trigonometric identities.They suspect that $\cos(3\theta)$ can be expressed in terms of powers of $\cos\theta$. Using De Moivre's Theorem and the binomial expansion, they can derive a triple-angle formula. This process demonstrates the deep connection between complex numbers and trigonometry, with applications in signal processing and physics.

Task:

a)State De Moivre's Theorem.

b)Write $(\cos\theta + j\sin\theta)^3$ in two ways: using De Moivre's Theorem and using the binomial expansion.

c)By equating the real parts of both expressions, derive the identity for $\cos(3\theta)$.

Topic 26: DIFFERENTIAL EQUATIONS

Sub-topic 26.1: Differential Equations

Scenario 217

A biomedical engineer at Mulago Hospital is modeling the concentration C(t) of a drug in a patient's bloodstream. The rate at which the drug is eliminated is proportional to its current concentration. This leads to the differential equation $\frac{dC}{dt} = -kC$, where $k > 0$ is the elimination constant. Solving this equation allows doctors to predict how long the drug remains at therapeutic levels, which is critical for determining correct dosing intervals for antibiotics and other essential medicines.

Task:

a)Classify the differential equation $\frac{dC}{dt} = -kC$. (Is it ordinary/partial? What order? Linear/Non-linear?).

b)Solve this differential equation by separating the variables to find the general solution for C(t)

c)If the initial concentration at t=0 is C_0 , find the particular solution.

Scenario 218

An environmental scientist is studying the population P(t) of a protected bird species in the Mabira Forest Reserve. The population grows at a rate proportional to the current population (exponential growth), but also faces a limitation due to carrying capacity K of the forest (logistic growth). A simple logistic model is given by $\frac{dP}{dt} = rP(1 - \frac{P}{K})$. Solving this differential equation helps conservationists predict the long-term population and assess the success of their protection efforts.

Task:

a)The equation $\frac{dP}{dt} = rP(1 - \frac{P}{K})$ is a first-order differential equation. Is it separable?

b)By separating variables, show that the equation can be written as: $\int \frac{dP}{1-\frac{P}{K}} = \int r\, dt$.

c) (Optional/Challenge) The solution involves partial fractions. Set up the partial fraction decomposition for $\frac{1}{1-\frac{P}{K}}$.

Scenario 219

A mechanical engineer is analyzing the displacement x(t) of a shock absorber on a newly assembled bodaboda (motorcycle). The system is modeled by the differential equation $\frac{d^{2x}}{dt^2} + 3\frac{dx}{dt} + 2x = 0$, which represents damped harmonic motion. Solving this second-order differential equation reveals whether the shock absorber will smoothly return to equilibrium or oscillate violently, directly impacting rider safety and comfort on Uganda's often bumpy roads.

Task:

a)What is the order of the differential equation $\frac{d^{2x}}{dt^2} + 3\frac{dx}{dt} + 2x = 0$?

b)To solve it, we assume a solution of the form $x = e^{mt}$. Substitute this into the equation to find the auxiliary (characteristic) equation.

c)Solve the auxiliary equation for m .

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Sub-topic 26.2: Solving First Order Differential Equations

Scenario 220

A chemical engineer at a Tororo-based fertilizer plant is studying the mixing process in a large tank.A salt solution with a concentration of 2 kgL^{-1} enters the tank at 5 $Lmin^{-1}$. The well-stirred mixture leaves the tank at the same rate. The tank initially contains 100 L of pure water. The engineer needs to find the amount of salt A(t) in the tank at any time t . This leads to a first-order linear differential equation of the form $\frac{dA}{dt}$ = Rate in - Rate out. Solving this equation is essential for quality control and ensuring the final product has the correct concentration.

Task:

a)The rate of salt entering is 2 kgL^{-1} x 5$Lmin^{-1}$ = 10 $kgmin^{-1}$. The rate leaving is $\frac{A(t)}{100} X\,5$. Form the differential equation for $\frac{dA}{dt}$

b)Write the equation in the standard linear form $\frac{dA}{dt}$ + P(t)A = Q(t) .

Scenario 221

An ecologist is modeling the population N(t) of an invasive plant species in Queen Elizabeth National Park. The growth rate is proportional to the population, but the park authorities are implementing a constant removal strategy, harvesting h plants per unit time. This situation is modeled by the differential equation $\frac{dN}{dt}$ = kN - h , where k is the growth constant. Solving this equation helps predict if the removal strategy will eventually eradicate the invasive species or if more aggressive measures are needed.

Task:

a)The equation $\frac{dN}{dt}$ - kN = -h is a first-order linear differential equation. Find the integrating factor.

b)Use the integrating factor to find the general solution for N(t) .

Scenario 222

A food scientist at a dairy processing plant in Kampala is modeling the temperature T(t) of a milk pasteurization vat as it cools down. According to Newton's Law of Cooling, the rate of change of temperature is proportional to the difference between the vat's temperature and the ambient room temperature T_a . This gives the equation $\frac{dT}{dt}$ = -k(T - T_a) . Solving this separable differential equation allows the scientist to determine how long the milk must cool before it can be safely packaged, ensuring product quality and safety.

Task:

a)Show that the equation $\frac{dT}{dt} = -k(T - T_a)$ is separable.

b)Separate the variables and integrate both sides to find the general solution for T(t) .

c)If the milk starts at $T(0) = 90^C$, the room is $T_a = 25^C$, and k=0.1 , find the particular solution.

Sub-topic 26.3: Application of Differential Equations

Scenario 223

An epidemiologist at the Uganda Virus Research Institute is modeling the early spread of an infectious disease in a densely populated area of Kampala using a simple SIR model.In the initial phase, the rate of new infections $\frac{dI}{dt}$ is proportional to the number of infected people I and the number of susceptible people S . Assuming S is approximately constant initially, this leads to $\frac{dI}{dt} = \varphi S_1$, which has an exponential solution. Solving this equation helps predict the initial growth rate of the outbreak, which is critical for allocating medical resources and planning containment strategies.

Task:

a)Solve the differential equation $\frac{dI}{dt}$ = kI (where $k = \varphi S$) to find I(t) .

b)If the number of infected people doubles every 5 days, find the value of the constant k .

c)If 100 people are initially infected, how many will be infected after 15 days?

Scenario 224

An economist at the Bank of Uganda is modeling the price P(t) of a staple food commodity. They propose that the rate of change of price is proportional to the difference between the demand D and supply S , where both demand and supply are linear functions of price:

D = a - bP and S = -c + dP . This leads to the differential equation

$\frac{dP}{dt}$ = k[(a - bP) - (-c + dP)] = k(a + c) - k(b+d)P . Solving this equation helps predict price stability and informs market intervention policies to control inflation.

Task:

a)Show that the differential equation can be written in the linear form $\frac{dP}{dt}$+ k(b+d)P = k(a+c) .

b)What is the equilibrium price P_e (the price where $\frac{dP}{dt}$= 0)?

c)Describe the significance of this equilibrium price for the Ugandan market.

Scenario 225

A mechanical engineer is designing the suspension system for a new bus model intended for Ugandan roads.The vertical displacement y(t) of the bus after hitting a pothole is modeled by the second-order differential equation $m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + ky = 0$, where m is mass, c is the damping coefficient, and k is the spring constant. Solving this equation reveals whether the suspension will provide a comfortable ride (underdamped, with oscillations) or a stiff, jarring one (overdamped).

Task:

a)For a simplified case with no damping (c=0), the equation becomes $\frac{d^2y}{dt^2} + \phi^2 y = 0$, where $\phi^2 = km^{-1}$. What is the general solution to this equation?

b)This solution describes simple harmonic motion. What physical quantity does ϕ represent?

c)How would the introduction of damping (c > 0) affect the motion of the bus?

Topic 27: FLOW CHARTS

Sub-topic 27.1: Algorithms and Presenting Them on Flow Charts

Scenario 226

A software developer at a fintech startup in Kampala is designing the core logic for a mobile money transfer approval system. The algorithm must check if a user has sufficient balance, if the recipient's number is valid, and if the transaction amount is within the daily limit. The developer needs to represent this decision-making process clearly for the rest of the team and for client presentations. A flowchart is the perfect tool to visualize the algorithm's logical flow, using standardized symbols for start/end, processes, decisions, and input/output.

Task:

a)List the key steps (inputs, decisions, outputs) involved in the mobile money transfer process described.

b)Draw a flowchart that represents this algorithm. Use appropriate symbols for Start/End, Process, Decision, and Input/Output.

c)Why is a flowchart preferable to a paragraph of text for explaining this algorithm to non-programmers?

Scenario 227

A mathematics teacher at a secondary school in Gulu wants to create a clear guide for students on how to find the roots of a quadratic equation $ax^2 + bx + c = 0$. The process involves calculating the discriminant $D = b^2 - 4ac$ and then following different paths based on whether $D > 0$, $D = 0$, or $D < 0$. The teacher decides to present this algorithm as a flowchart to help students visualize the logical steps and understand the different cases for the roots (real and distinct, real and equal, or complex).

Task:

a)Write down the step-by-step algorithm for solving a quadratic equation.

b)Represent this algorithm using a flowchart.

c)In the flowchart, how many decision diamonds are needed?

Scenario 228

A logistics manager for a delivery company in Jinja needs to optimize the package sorting process at their warehouse.The algorithm for sorting involves checking the package's weight, dimensions, and destination zone to assign it to the correct delivery truck. The manager sketches a flowchart to standardize the procedure, ensuring all workers follow the same efficient process. This reduces errors and speeds up operations, which is crucial for meeting delivery targets in Uganda's growing e-commerce sector.

Task:

a)Identify the inputs and outputs for the package sorting algorithm.

b)Identify the key decisions that determine which truck a package is assigned to.

c)Draw a simple flowchart for this sorting process, including at least two decision points.

Sub-topic 27.2: Performing Dry Runs

Scenario 229

A programmer is testing a simple algorithm designed to calculate the factorial of a non-negative integer n, where factorial $n! = n \times (n-1) \times \ldots \times_1$, and $0! = 1$. The algorithm uses a loop. Before writing the actual code, the programmer performs a dry run (a paper test) with a small input value like n=4 to verify the logic. This involves manually stepping through each instruction, tracking the values of variables, and checking if the final output is correct (which should be 24). This process helps catch logical errors early, saving valuable debugging time later.

Task:

The algorithm is:

1. Start
2. Read n
3. Set f = 1 , i = 1
4. While $i \leq n$:
    f = f x i
    i = i + 1
5. Print f
6. End

a)Perform a dry run of this algorithm for n=4 . Create a table to track the values of n, f, and i after each step.

b)What is the final output?

c)What is the purpose of initializing f to 1?

Scenario 230

A student is learning about an algorithm that finds the largest number in a list. The algorithm iterates through the list, comparing each element to the current maximum. The student performs a dry run with the sample list [7, 2, 9, 4] to understand how the algorithm works. Manually tracing the values helps the student grasp the concept of iteration and comparison, which are fundamental building blocks in computer science and data analysis.

The algorithm is:

1. Start with a list of numbers.
2. Set max = the first number in the list.
3. For each subsequent number in the list:
   If the number > max , then set max = number
4. Output max .

Task:

a)Perform a dry run with the list [7, 2, 9, 4] . Track the value of max after each comparison.

b)What is the final output of the algorithm?

c)Why is it important to initialize max to the first element of the list?

Scenario 231

A business analyst has designed a flowchart for a loan approval process at a microfinance institution.The process checks an applicant's credit score and income level. Before implementing this process, the analyst performs a dry run with test cases to ensure it correctly approves qualified applicants and rejects unqualified ones. This validation step is crucial to prevent financial losses for the institution and to ensure fair access to credit for small business owners in Uganda.

Consider this simplified loan criteria from a flowchart:

Start
Input Credit Score, Annual Income
If Credit Score > 650 AND Annual Income > 5,000,000 UGX then
 Output "Approved"
Else
 Output "Denied"
 End

Task:

a)Perform a dry run for an applicant with a Credit Score of 700 and an Annual Income of 6,000,000 UGX. What is the output?

b)Perform a dry run for an applicant with a Credit Score of 600 and an Annual Income of 10,000,000 UGX. What is the output?

c)What is the advantage of testing an algorithm with multiple dry runs using different inputs?

Scenario 232

A software developer at a Ugandan hospital is designing a triage algorithm to prioritize patient care in the emergency department. The flowchart checks vital signs like heart rate, blood pressure, and consciousness level to categorize patients as "Critical," "Urgent," or "Non-Urgent." Before implementing this system in a live environment, the developer must perform exhaustive dry runs with various patient profiles to ensure the logic is sound and no critical cases are misclassified. A single error in the flowchart could have serious consequences for patient outcomes.

Task:

Consider a simplified triage flowchart:

1. Start
2. Input Heart Rate (HR), Systolic BP, Conscious (Y/N)
3. If Conscious = 'N' OR Systolic BP < 90, output "CRITICAL"
4. Else, if HR > 130 OR HR < 50, output "URGENT"
5. Else, output "NON-URGENT"
6. End

Task:

a)Perform a dry run for an unconscious patient with HR=80, BP=120. What is the output?

b)Perform a dry run for a conscious patient with HR=140, BP=100. What is the output?

Scenario 233

An agricultural engineer is designing a smart irrigation algorithm for a large-scale maize farm in the Teso sub-region. The system uses soil moisture sensors and weather forecast data to decide whether to activate the sprinklers. The algorithm's flowchart is complex, involving multiple decision points. The engineer performs a dry run for a scenario where the soil moisture is low but the weather forecast predicts heavy rain within the hour. This tests whether the algorithm correctly avoids unnecessary irrigation, saving water and energy costs for the farm.

Task:

A simplified version of the algorithm is:

1. Start
2. Input Soil_Moisture, Rain_Forecast (mm/h)
3. If Soil_Moisture < 30 AND Rain_Forecast < 5 then
   · Output "IRRIGATE"
4. Else
   · Output "DO NOT IRRIGATE" 5. End

Task:

a)Perform a dry run for Soil_Moisture=25%, Rain_Forecast=10 mm/h. What is the output?

b)Perform a dry run for Soil_Moisture=35%, Rain_Forecast=2 mm/h. What is the output?

Scenario 234

A financial technology company in Kampala is automating its"Know Your Customer" (KYC) verification process. The algorithm checks a user's submitted documents and flags accounts for manual review if certain risk factors are present, such as mismatched information or a high-risk country of origin. A business analyst performs a dry run with a test case where a user's national ID name doesn't match their mobile money registration name. This ensures the algorithm correctly flags the account for further investigation, helping to prevent fraud and comply with Ugandan financial regulations.

Task:

A simplified KYC flowchart:

1. Start
2. Input ID_Name, Registered_Name, Country
3. If ID_Name ≠ Registered_Name then
   · Output "FLAG FOR REVIEW"
4. Else, if Country is in [High-Risk List] then
   · Output "FLAG FOR REVIEW"
5. Else
   · Output "APPROVED" 6. End

Task:

a)Perform a dry run for a user where ID_Name="Kato John", Registered_Name="John Kato", Country="Uganda". What is the output?

b)Perform a dry run for a user where ID_Name="Jane Auma", Registered_Name="Jane Auma", Country=[A high-risk country]. What is the output?

c)In a dry run, what is the purpose of tracking the path taken through the flowchart's decision diamonds?

END