



**UGANDA NATIONAL EXAMINATIONS BOARD**  
**Uganda Advanced Certificate of Education**

**P425/2 PRINCIPAL  
MATHEMATICS**

**SCORING GUIDE  
FOR THE SAMPLE PAPER**

***In response to the tasks, the candidate is expected to: correctly identify the relevant concepts to manipulate in response to the task(s) in the scenario, select and apply (a) correct method(s) to correctly manipulate the concepts identified, and correctly interpret accurate outputs from correctly identified and manipulated concepts to justify/inform decision.***

## SECTION A

### ITEM 1

#### Task(a)

*Trapezium Rule Approximation*

$x$		
0	0	
5		9
10		12
15		9
20	0	
SUM	0	30

$$\int_{\{0\}}^{\{20\}} [12 - 0.12(x - 10)^2] dx \approx \frac{5}{2} [0 + 2(30)]$$

$$\approx 150m^2$$

*Exact Integration*

$$\int_{\{0\}}^{\{20\}} (2.4x - 0.12x^2) dx = [1.2x^2 - 0.04x^3]_{\{0\}}^{\{20\}}$$

$$= (1.2(20)^2 - 0.04(20)^3) - 0$$

$$= 160 m^2$$

***Conclusion:*** The cross-sectional area of the river satisfies the requirement for the pump to be installed since both  $150m^2$  and  $160 m^2$  are greater than the required minimum cross-sectional area of  $145m^2$ .

### Task(b)

*Newton – Raphson Method*

$$2.4x - 0.12x^2 = 8$$

$$f(x) = 0.12x^2 - 2.4x + 8$$

$$f'(x) = 0.24x - 2.4$$

$$x_{\{n+1\}} = x_n - \frac{\{0.12x_n^2 - 2.4x_n + 8\}}{\{0.24x_n - 2.4\}}$$

*Iterations with  $x_0 = 4, e = 0.1$*

$$x_1 = 4 - \frac{\{0.12(4)^2 - 2.4(4) + 8\}}{\{0.24(4) - 2.4\}} = 4.2222$$

$$x_2 = 4.2222 - \frac{\{0.12(4.2222)^2 - 2.4(4.2222) + 8\}}{\{0.24(4.2222) - 2.4\}} = 4.2265$$

$$e = |4.2265 - 4.2222| = 0.0043 < 0.1$$

*Conclusion: The suitable position to install the pump from the river bank is 4.2265m.*

## SECTION B

### PART I

#### ITEM 2

##### Task(a)(i)

C.B	x	f	fx	fx <sup>2</sup>	L	fd
5-30	17.5	17	297.5	5206.25	25	0.68
30-40	35	21	735	25725	10	2.1
40-50	45	30	1350	60750	10	3.0
50-60	55	14	770	42350	10	1.4
60-80	70	3	210	14700	20	0.15
		$\sum f=85$	$\sum fx=3362.5$	$\sum fx^2=148731.25$		

Mean:

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{3362.5}{85} = 39.5588$$

Mode:

$$\text{Mode} = 40 + \left[ \frac{\{30 - 21\}}{\{(30 - 21) + (30 - 14)\}} \right] \times 10 = 43.6$$

**The performance is not good since the average and dominant mark are less than 45%.**

**Task(a)(ii)**

Standard Deviation:

$$\begin{aligned} \sigma &= \sqrt{\left\{ \left( \frac{85}{85-1} \right) \left( \frac{\{148731.25\}}{\{85\}} - \left( \frac{\{3362.5\}}{\{85\}} \right)^2 \right) \right\}} \\ &= \sqrt{\left\{ \frac{85}{84} (1749.779 - 1565.99) \right\}} \\ &= 13.6777 \end{aligned}$$

**The examination is not reliable since  $\sigma = 13.6777 > 10\%$  which is required.**

**Task (b)**

Normal Distribution & Probability

Given:

$$\{x\} = 39.5588, \quad \sigma = 13.6777, \quad n = 500$$

Let  $X \sim$  marks scored,  $D \sim$  distinction mark and  $F \sim$  fail mark

$$P(X > D) = 0.15$$

$$P\left(Z > \frac{D - 39.5588}{13.6777}\right) = 0.15$$

$$\frac{D - 39.5588}{13.6777} = 1.036$$

$$D = 53.7289$$

**The cut-off for a distinction is 53.7% (1dp)**

Fail Mark:

$$P(X < F) = 0.2$$

$$\Rightarrow P\left(Z < \frac{F - 39.5588}{13.6777}\right) = 0.2$$

$$\left(\frac{F - 39.5588}{13.6777}\right) = -0.842$$

$$F = 28.0422$$

**The cut-off for a fail is 28.0% (1dp)**

***Number of Students that:***

***Passed:  $(1 - 0.2) \times 500 = 400$***

***Got a Distinction:  $0.15 \times 500 = 75$***

*Confidence Interval (95%):*

$$\alpha/2 = 0.475, \quad Z_{0.475} = 1.96$$

$$\mu = \bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$= 39.5588 \pm \frac{1.96 \times 13.6777}{\sqrt{500}}$$

$$\mu = [38.3599, \quad 40.7577]$$

**The range within which the average mark of all the students that sat the examination lies basing on 95% degree of confidence is  $38.3599 \leq \mu \leq 40.7577$ .**

### **ITEM 3**

#### ***Probability Distribution***

*Given:*

$$P(P) = 0.3, \quad P(\bar{P}) = 0.7, \quad P(A/P) = 0.65,$$

$$P(C) = 0.5, \quad P(\bar{C}) = 0.5, \quad P(A/C) = 0.85,$$

$$P(B) = 0.2, \quad P(\bar{B}) = 0.8, \quad P(A/B) = 0.5$$

*Total Probability of A:*

$$P(A) = P(P)P(A/P) + P(C)P(A/C) + P(B)P(A/B)$$

$$= 0.3(0.65) + 0.85(0.5) + 0.2(0.5)$$

$$= 0.72$$

*Bayes' Theorem:*

$$\begin{aligned}
 P(A/C) &= \frac{P(C \cap A)}{P(A)} \\
 &= \frac{\{0.5 \times 0.85\}}{\{0.72\}} \\
 &= 0.5903
 \end{aligned}$$

*Probability Distribution of X:*

*X ~ number of laboratories used*

$$P(X = 0) = 0.7 \times 0.5 \times 0.8 = 0.28$$

$$P(X = 1) = (0.3 \times 0.5 \times 0.8) + (0.5 \times 0.7 \times 0.8) + (0.2 \times 0.7 \times 0.5) = 0.47$$

$$P(X = 2) = (0.3 \times 0.5 \times 0.8) + (0.7 \times 0.5 \times 0.2) + (0.3 \times 0.5 \times 0.2) = 0.22$$

$$P(X = 3) = 0.3 \times 0.5 \times 0.2 = 0.03$$

<i>x</i>	0	1	2	3
$P(X = x)$	0.28	0.47	0.22	0.03
$xP(X = x)$	0	0.47	0.44	0.09

$$E(X) = 0.47 + 0.44 + 0.09 = 1.00$$

There will be no construction of new laboratories since the two conditions were not met.

### **Task(b)**

*Binomial Probability*

*Let Y = number of students who used the chemistry laboratory.*

$$n = 10, \quad p = 0.5, \quad q = 0.5$$

$$P(Y \geq 5) = P(Y = 5) + P(Y = 6) + P(Y = 7) + P(Y = 8) + P(Y = 9) + P(Y = 10)$$

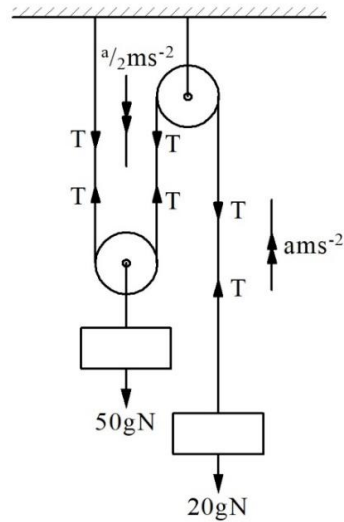
$$= 0.2461 + 0.2051 + 0.1172 + 0.0439 + 0.0098 + 0.0010$$

$$= 0.6231$$

## **SECTION B: PART II**

### **ITEM 4**

*Pulley System – Mechanics*



$$T - 196 = 20a \quad \text{--- (1)}$$

$$490 - 2T = 50\left(\frac{a}{2}\right)$$

$$245 - T = 12.5a \quad \text{--- (2)}$$

Adding (1) and (2) gives:

$$49 = 32.5a$$

$$a = 1.5077 \text{ m/s}^2$$

**Since  $a < 2 \text{ m/s}^2$ , the system is safe and stable.**

$$T = 12.5 \times 1.5077 + 245$$

$$= 226.1538 \text{ N}$$

**The cable will brake since  $T > 100 \text{ N}$**

$$s = ut + \frac{1}{2}at^2$$

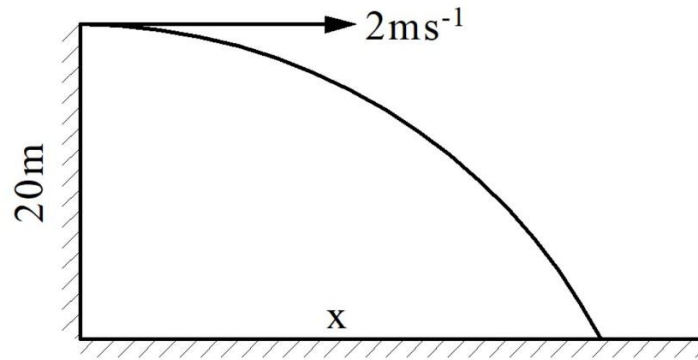
$$= 0 + \frac{1}{2}\left(\frac{1.5077}{2}\right)(5^2)$$

$$= 9.4231 \text{ m}$$

**The movable pulley will travel 9.4231m in 5s if the system is released from rest.**

**Task(b)**

**Projectile Motion**



$$y = \frac{1}{2}gt^2$$

$$20 = \frac{1}{2} \times 9.8t^2$$

$$t = \sqrt{\left\{\frac{20}{4.9}\right\}}$$

$$= 2.0203s$$

$$x = ut$$

$$= 2 \times 2.0203$$

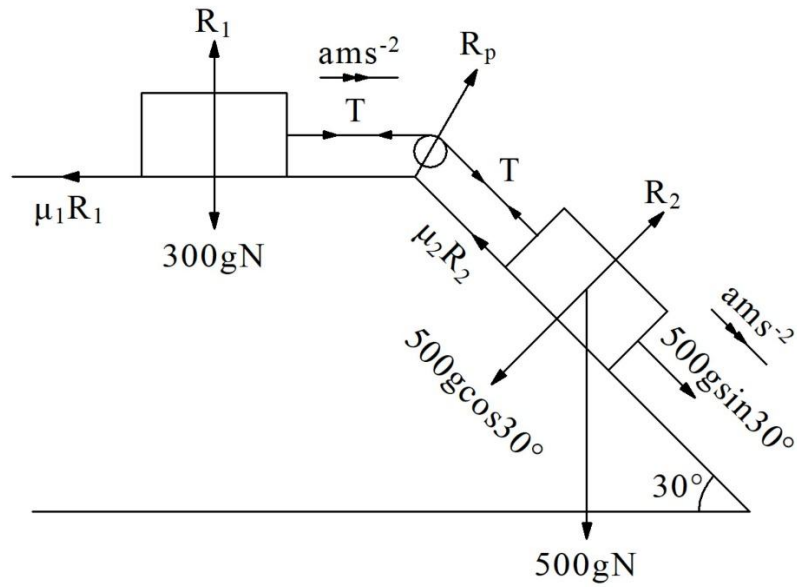
$$= 4.0406m$$

**The passers-by are safe since the toll box falls in a distance of 4.0406m of the service deck which is less than the 5m safe zone set by the engineer.**

## **ITEM 5**

### **Task**

#### *Inclined Plane – Mechanics*



$$2450 - T - 424.3524 = 500a \quad \text{--- (1)}$$

$$T - 882 = 300a \quad \text{--- (2)}$$

Adding (1) and (2):

$$1143.6476 = 800a$$

$$\Rightarrow a = 1.4296 \text{ m/s}^2$$

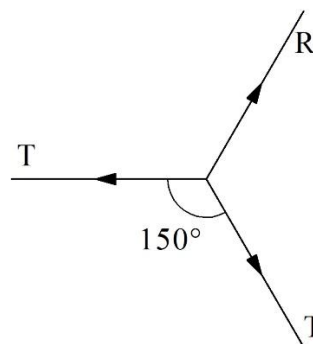
Velocity after 3s:

$$V = u + at = 0 + 1.4296 \times 3 = 4.2887 \text{ m/s}$$

Tension:

$$T = 300(1.4296) + 882 = 1310.88 \text{ N}$$

Resultant Tension in Rope:



$$R^2 = T^2 + T^2 + 2T^2 \cos \theta$$

$$R = \sqrt{\{(1310.88)^2 \times 2 + 2(1310.88) \cos 150^\circ\}}$$

$$R = 678.5614 \text{ N}$$

***Since  $V < 5\text{m/s}$  and  $R < 700\text{N}$ , the container will be delivered safely.***