UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

P425/1 PRINCIPAL MATHEMATICS

SCORING GUIDE
FOR THE SAMPLE PAPER

 Turn Over

In response to the tasks, the candidate is expected to: correctly identify the relevant concepts to manipulate in response to the task(s) in the scenario, select and apply (a)correct method(s) to correctly manipulate the concepts identified, and correctly interpret accurate outputs from correctly identified and manipulated concepts to justify/inform decision.

SECTION A

ITEM 1

TASK(a)

Proving that points/trading centres $A(2,3), B(-2,1), and\ \ C(4,7)\ form\ a\ triangle$

For line/road AB,

$$m_1 = \frac{1-3}{-2-2} = \frac{1}{2}$$

For line/road AC

$$m_2 = \frac{7-3}{4-2} = 2$$

$$tan\ \theta = \left|\frac{\{m_2 - m_1\}}{\{1 + m_1 m_2\}}\right|$$

$$tan\ \theta = \left|\frac{\{2 - \frac{1}{2}\}}{\left\{1 + 2 \times \frac{1}{2}\right\}}\right|$$

$$\theta = tan^{-1}\left(\frac{3}{4}\right)$$

$$\theta = 36.87^o$$

Since the lines AB and AC meet at an angle of 36.87^o, then trading centres A, B and C for a triangle, and it is possible to construct the triangular road network.

TASK(b)

$$Circle\ Through\ Points\ A, B, C$$

$$General\ equation: x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\{For\ A(2,3):\}\ 4 + 9 + 4g + 6f + c = 0$$

$$4g + 6f + c = -13\,\{---(i)\}$$

$$\{For\ B(-2,1):\}\ 4 + 1 - 4g + 2f + c = 0$$

$$-4g + 2f + c = -5\,\{---(ii)\}$$

$$\{For\ C(4,7):\}\ 16 + 49 + 8g + 14f + c = 0$$

$$8g + 14f + c = -65\,\{---(iii)\}$$

$$\{(iii) - (ii):\}\ 12g + 12f = -60$$

$$g + f = -5\,\{---(iv)\}$$

$$\{(iv) - (ii):\}\ 2g + f = -2$$

$$3 + f = -5$$

$$\Rightarrow f = -8$$

$$g = 3$$

$$\{Substitute\ in\ (ii):\}\ 4(3) + 6(-8) + c = -13$$

$$12 - 48 + c = -13$$

$$c = 23$$

$$\boldsymbol{Equation\ of\ the\ circular\ road: x^2 + y^2 + 6x - 16y + 23 = 0}$$

Equations to model the triangular road network:

For road $\overline{AB}$, $m_1 = \frac{1}{2}, \Rightarrow \frac{y-3}{x-2} = \frac{1}{2}$ Therefore, $y = \frac{1}{2}x + 2$

For road $\overline{AC}$, $m_2 = 2, \Rightarrow \frac{y-3}{x-2} = 2$ Therefore, $y = 2x - 1$

For road $\overline{BC}$, $m_3 = \frac{7-1}{4-(-2)} = 1, \Rightarrow \frac{y-7}{x-4} = 1$ Therefore, $y = x + 3$

Task(c)

$$Lengths\ of\ Roads$$

$$For\ the\ circular\ road, \qquad r = \sqrt{\{50\}}$$

$$Length\ of\ the\ road = circumference = 2\pi\sqrt{\{50\}} = 44.4288$$

$$\{For\ the\ triangular\ road:\}$$

$$|AB| = \sqrt{\{20\}} = 2\sqrt{\{5\}}$$

$$|AC| = \sqrt{\{20\}} = 2\sqrt{\{5\}}$$

$$|BC| = \sqrt{\{72\}} = 6\sqrt{\{2\}}$$

$$Total\ length\ of\ the\ road = \sqrt{\{20\}} + \sqrt{\{20\}} + \sqrt{\{72\}} = 17.4296$$

$$\boldsymbol{The\ triangular\ road\ network\ should\ be\ constructed\ since\ it\ has\ shorter\ length}$$

$$\boldsymbol{Control\ tower\ for\ traffic\ lights\ should\ be\ installed\ at\ (-3, 8)}$$

ITEM 2:

Task(a)

$$let\ x\ be\ the\ number\ of\ sacks\ of\ maize.$$

$$23x - x^2 \geq 120$$

$$x^2 - 23x + 120 \leq 0$$

$$(x - 15)(x - 8) \leq 0$$

Critical values are; $x = 8$, $x = 15$

	$x < 8$	$8 < x < 15$	$x > 15$
$(x - 15)(x - 8)$	$+$	$-$	$+$

$$Range:\ 8 \leq x \leq 15$$

The number of sacks of maize given to each farmer should be $\boldsymbol{8 \leq x \leq 15}$

TASK(b)

$$Combinations$$

^nC_r

$$n = 7, \quad for\, r \leq 4$$

$$No\ of\ ways = {}^{7}C_{4} + {}^{7}C_{3} + {}^{7}C_{2} + {}^{7}C_{1} + {}^{7}C_{0}$$

$$= 35 + 35 + 21 + 7 + 1$$

$$= 99\, ways$$

$$(3 + 2x)^8 = 3^8 + 8(2x)(3)^7 + \frac{\{8 \times 7\}(2x)^2(3)^6}{2!} + \frac{\{8 \times 7 \times 6\}(2x)^3(3)^5}{3!} + \cdots$$

$$= 6561 + 21870x + 29160x^2 + 19440x^3 + \cdots$$

Number of ways to reward prizes to the helpers is as follows:

4th position, the prize amount = $6561

3rd position, the prize amount = $21870

2nd position, the prize amount = $29160

1st position, the prize amount = $19440

Task(c)

$$Arithmetic\ Progression\ \&\ Time$$

$$Let\ the\ number\ of\ visitors\ be\ n$$

$$Therefore\ \ n = 45$$

$$The\ intervals\ in\ which\ the\ visitors\ arrive\ form\ an\ AP\ whose$$

$$first\ term\ a = 0\ and\ common\ difference\ d = 5$$

$$T_n = a + (n-1)d$$

$$Total\ time\ taken\ for\ last\ guest\ to\ arrive\ T_{\{45\}} = 0 + 44 \times 5$$

$$T_{45} = 220 \text{ minutes}$$

$$T_{45} = 3\ hours\text{: 40 minutes}$$

$$Time\ when\ the\ Chief\ \ guest(last\ guest)arrives = 8:00 + 3:40$$

$$= 11:40 \text{ AM}$$

$$\boldsymbol{Time\ for\ official\ opening\ of\ the\ festival = 11{:}40\ +\ 0{:}10}$$

$$\boldsymbol{= 11{:}50\ \text{AM}}$$

ITEM 3:

Task(a)

$$Yield\ Calculation$$

$$(Ax\ +\ By)^3 = A^3x^3 + 3A^2x^2By + 3AxB^2y^2 + B^3y^3$$

$$x\ = 6,\ y = 1$$

$$(6A\ +\ B)^3 = 216A^3 + 108A^2B + 18AB^2 + B^3$$

1^{st} season, $1728\ = 216A^3$

$$A\ =\ 2$$

Last season, $216 = B^3$

$$B\ =\ 6$$

$$2^{nd} season, yield = 108(2)^2 \times 6 = 2592kg$$

$$3^{rd} season, yield = 18(2)(6)^2 = 1296kg$$

Therefore, $\boldsymbol{2^{nd} season}$ gives the highest yield when the farmer uses the fertilizer mixture.

Task(b)

$$Logarithm\ \&\ Expiry\ Date$$

$$\log P = \log P_0 - C\log(t-4)$$

$$\log P = \log P_0 - \log(t-4)^C$$

$$\log P = \log \frac{P_0}{(t-4)^C}$$

$$P\ =\ \frac{P_0}{(t-4)^C}$$

$$t == ((\frac{P_0}{P})^{1/c}) + 4$$

$$C = 0.2, P_0 = 95 \; and \; P = 47.5$$

$$t = ((\frac{95}{47.5})^{1/0.2}) + 4$$

$$t = 36 \; months = 3 years$$

Manufacture date: 1st September, 2026

Expiry date:01st September, 2029

Task(c)

$$z^3 - 5z^2 + 17z - 13 = 0$$

$$Let \; P(z) = z^3 - 5z^2 + 17z - 13$$

$$When \; z = 1, \quad P(1) = 0, \quad \Rightarrow z - 1 \; is \; a \; factor$$

$$Using \; long \; division;$$

$$Z^2 - 4Z + 13$$

$$Z - 1 \quad \overline{)(z^3 - 5z^2 + 17z - 13)}$$

$$-(Z^3 - Z^2)$$

$$-4Z^2 + 17Z - 13$$

$$-(-4Z^2 + 4Z)$$

$$13Z - 13$$

$$-(13Z - 13)$$

$$0$$

$$Therefore \quad (z-1)(z^2 - 4z + 13) = 0$$

$$\Rightarrow (z^2 - 4z + 13) = 0$$

$$Z = \frac{4 \pm \sqrt{((4^2) - 4 \times 1 \times 13)}}{2 \times 1}$$

$$Z = \frac{4 \pm \sqrt{-36}}{2}$$

$$z = 2 \pm 3\mathrm{i}$$

$$Therefore \quad z = 1, \qquad 2 \pm 3\mathrm{i}$$

$$\boldsymbol{The\ coordinates\ of\ the\ sales\ outlets\ are}\ (\mathbf{1},\mathbf{0}), (\mathbf{2},\mathbf{3}), (\mathbf{2},-\mathbf{3})$$

ITEM 4:

Task(a)(i)

1. $Fencing\ \&\ Optimization$

$Let\ x\ and\ y\ be\ the\ length\ and\ width\ of\ the\ Rectangular\ piece\ of\ land\ to\ be\ fenced$

$$Area\ of\ the\ land, \quad xy = 39200$$

$$Perimeter, \quad P = 2y + x - 12$$

$$From\ xy = 39200,\ y = \frac{39200}{x}$$

$$P = \frac{78400}{x} + x - 12$$

$$\frac{dP}{dx} = \frac{-78400}{x^2} + 1$$

$$For\ minimun\ length, \frac{dP}{dx} = 0$$

$$\Rightarrow \frac{-78400}{x^2} + 1 = 0$$

$$Therefore, x = 280m$$

$$y = \frac{39200}{x} = \frac{39200}{280} = 140\text{ m}$$

$$The\ required\ lenght\ of\ the\ fence = 2y + x - 12 = 280 + 280 - 12$$

$$\boldsymbol{The\ required\ lenght\ of\ the\ fence = 548m}$$

Task(a)(ii)

$$\frac{dL}{dt} = 12 - 0.1t$$

$$\int_0^R dL = \int_0^{12} (12 - 0.1t)\,\text{dt}$$

$$L = |12t - \frac{0.1}{2}t^2|_0^{12}$$

$$L = 144 - 7.2$$

$$L = 136.8m$$

$$Time\ taken = \frac{548}{136.8}$$

$$Time\ taken = 4.0058\ days \approx 5 \text{ days}$$

It will take five days to fence the land.

Task(b)(i)

$$y = 12 - 3x^2$$

$$\frac{dy}{dx} = -6x$$

$$\frac{d^2y}{dx^2} = -6 \Rightarrow a\ maxima$$

$$For\ maximum, -6x = 0$$

$$\Rightarrow x = 0$$

$$y = 12 - 3(0^2)$$

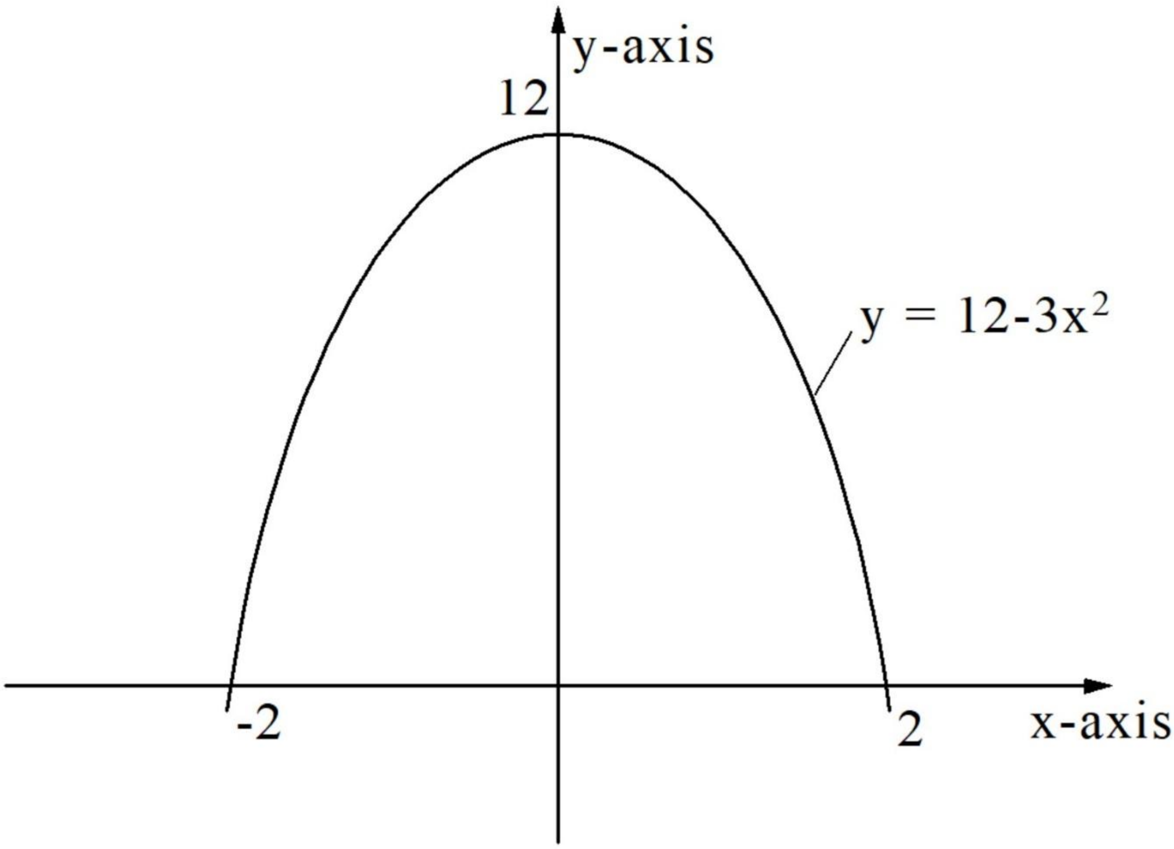
$$y = 12$$

$$Therefore, (12{,}0)\ is\ a\ maximum\ point$$

$$From\ y = 12 - 3x^2,$$

$$when\ y = 0, x = \pm 2$$

$$\Rightarrow (2{,}0)\ and\ (-2{,}0)\ are\ intercepts$$



Task(b)(ii)

$$Volume\ of\ the\ Ant-hill = \int_0^{12} \pi x^2 dy$$

$$From\ y = 12 - 3x^2, \quad x^2 = \frac{12-y}{3}$$

$$Volume\ of\ the\ Ant-hill = \int_0^{12} \pi(\frac{12-y}{3})dy$$

$$Volume\ of\ the\ Ant-hill = \int_0^{12} (\frac{12\pi}{3} - \frac{\pi y}{3})dy$$

$$Volume\ of\ the\ Ant-hill = |(4\pi y - \frac{\pi y^2}{6})|_0^{12}$$

$$Volume\ of\ the\ Ant-hill = (4\pi \times 12) - \frac{144\pi}{6}$$

$$Volume\ of\ the\ Ant-hill = 48\pi - 24\pi$$

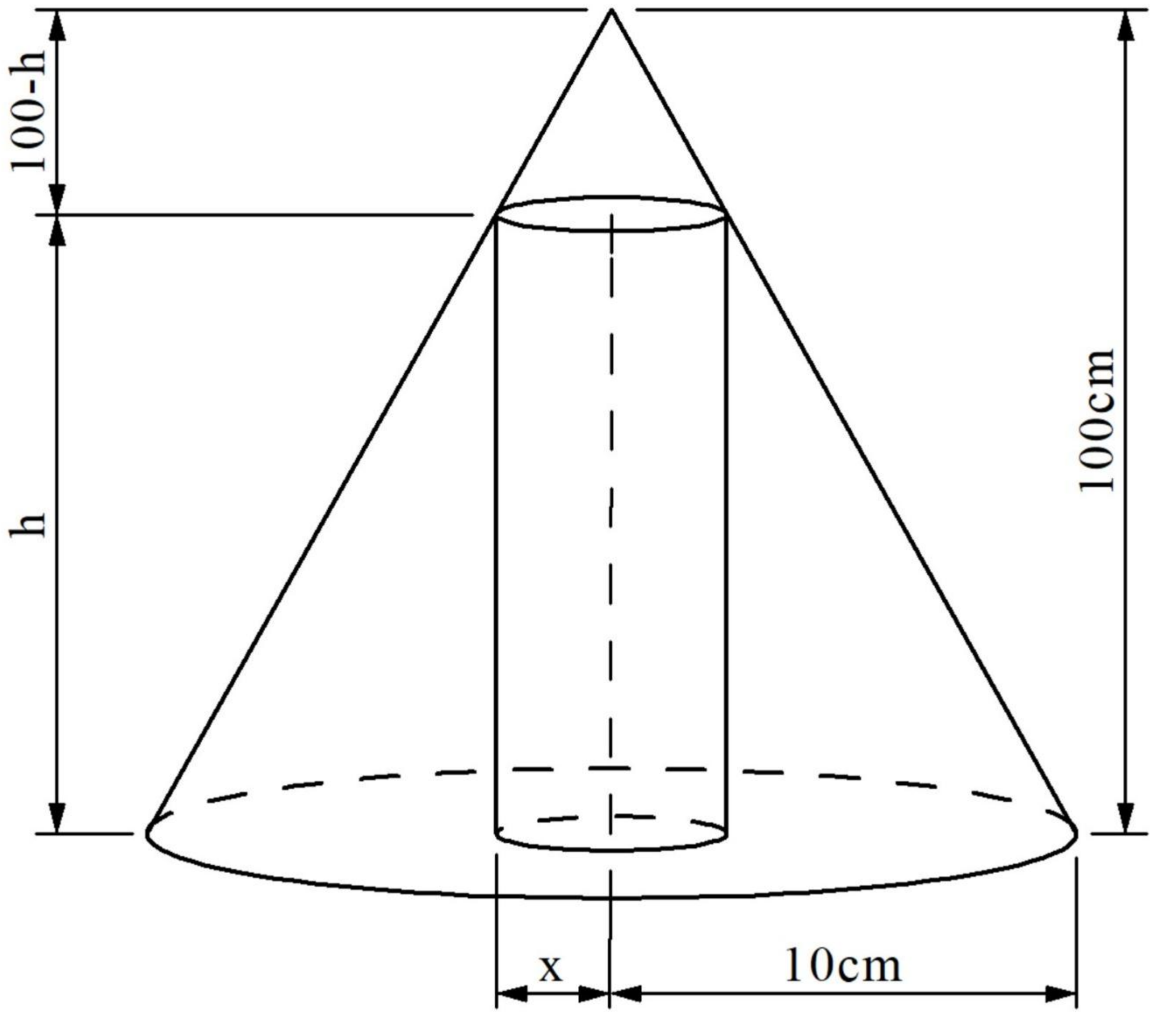
$$Volume\ of\ the\ Ant-hill = 24\pi$$

$$Volume\ of\ the\ Ant-hill = 75.40m^3$$

The soil obtained by clearing the Ant-hill will not be enough to fill the depression since it is <200m^3.

ITEM 5:

Task(a)



$$By\ similar\ triangles: \frac{100}{100 - h} = \frac{10}{x}$$

$$100x = 1000 - 10h$$

$$\Rightarrow h = 100 - 10x$$

$$Volume: V = \pi r^2 h = \pi x^2(100 - 10x)$$

$$V = \pi(100x^2 - 10x^3)$$

$$\frac{dV}{dx} = \pi(200x - 30x^2)$$

$$For\frac{max}{min}: \pi(200x - 30x^2) = 0$$

$$(20 - 3x) = 0$$

$$\Rightarrow x = 20/3 = 6⅔\ cm$$

$$V_{max} = \pi\left(\frac{20}{3}\right)^2\left(100 - 10 \times \frac{20}{3}\right)$$

$$V_{max} = \frac{400\pi}{9}\left(\frac{300 - 200}{3}\right)$$

$$V_{max} = \frac{4000\pi}{27}$$

$$V_{max} = 465.4211cm^3$$

The volume of the biggest cylinder Jalia can mould from her conical shaped piece is $\mathbf{465.4211cm^3}$

Task(b)

$$b)\ Error\ in\ r = 10 - 9.8 = 0.2$$

$$\%\ error = \frac{0.2}{10} \times 100 = 2\%$$

$$\delta r = \frac{2r}{100} = 0.02r$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\delta V = \frac{dV}{dr}.\delta r$$

$$\delta V = 4\pi r^2(0.02r)$$

$$\frac{\delta V}{V} \times 100 = \frac{4\pi r^2(0.02r)}{\frac{4}{3}\pi r^3} \times 100$$

$$= 6\%$$

Margaret's mould will be approved for the competition since the error in the volume of her mould is 6% and therefore does not exceed the 8% error required for approval.

Task(c)

Moses – Bottle Problem

$$\frac{dN}{dt} \propto (100 - N)^{\frac{1}{2}}$$

$$\Rightarrow \frac{dN}{dt} = k(100 - N)^{\frac{1}{2}}$$

$$\frac{dN}{(100 - N)^{\frac{1}{2}}} = k\,dt$$

$$\int (100 - N)^{-\frac{1}{2}} dN = \int k\ dt$$

$$-2(100 - N)^{\frac{1}{2}} = kt + C$$

$$When\ t = 0, N = 0: \ -2\sqrt{100} = C \Rightarrow C = -20$$

$$When\ t = 1, N = 19: \ -2\sqrt{81} = k - 20 \Rightarrow k = 2$$

$$-2\sqrt{100 - N} = 2t - 20$$

$$-2\sqrt{100 - 100} = 2t - 20$$

t= $10\ minutes$

Conclusion: Moses is likely to win the competition since the time he takes to pack the bottles during practice is less than the time the win in that category took last year.

Alternative

Let x represent number of bottles remaining at time t.

$$\frac{dx}{dt} \propto \sqrt{x}$$

$$\Rightarrow \frac{dx}{dt} = k\sqrt{x}$$

$$\frac{dx}{\sqrt{x}} = k\,dt$$

$$\int x^{-\frac{1}{2}}\,dx = \int k\,dt$$

$$2x^{\frac{1}{2}} = kt + C$$

$$At\ t = 0, x = 100:\ 2\sqrt{100} = C \Rightarrow C = 20$$

$$At\ t = 1, x = 81:\ 2\sqrt{81} = k + 20 \Rightarrow k = -2$$

$$2\sqrt{x} = -2t + 20$$

$$When\ x = 0: 0 = -2t + 20 \Rightarrow t = 10\ minutes$$

Conclusion: Moses is likely to win the competition since the time he takes to pack the bottles during practice is less than the time the win in that category took last year.