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ARCH 1107
YEAR 1 SEM 1
TEST TWO

Question 1

let the matrix $\begin{pmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{pmatrix}$ be A

for skew-symmetrical matrix; $A^T = -A$

$$A^T = \begin{pmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{pmatrix}$$

$$-(A) = -\begin{pmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{pmatrix} = -\begin{pmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{pmatrix}$$

$\therefore a = -2$

$b = 0$ since diagonal elements $= 0$

$c = -3$

Question 2;

let the matrix be ~~B~~ A

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$$

If det $= 0$ then matrix has no inverse

If det $\neq 0$ then matrix has an inverse.

$$\det(A) = 2\begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} - (-1)\begin{vmatrix} -5 & 1 \\ -3 & 3 \end{vmatrix} + 3\begin{vmatrix} -5 & 3 \\ -3 & 2 \end{vmatrix}$$

$$\det(A) = 2\left[(3\times3)-(2\times1)\right] - (-1)\left[(-5\times3)-(-3\times1)\right] + 3\left[(-5\times2)-(-3\times2)\right]$$

$$\det(A) = 2\left[9-2\right] + 1\left[-15+3\right] + 3\left[-10+9\right]$$

$$\det(A) = 14 - 12 - 3$$

$\underline{\det(A) = -1}$ hence the inverse exists.

$$\operatorname{adj}(A) = (\text{cofactors})^T$$

$$= \begin{bmatrix} +[(3\times3)-(2\times1)] & -[(-5\times3)-(1\times-3)] & +[(-5\times2)-(3\times-3)] \\ -[(-1\times3)-(3\times2)] & +[(2\times3)-(3\times-3)] & -[(2\times2)-(-1\times-3)] \\ +[(-1\times1)-(3\times3)] & -[(2\times1)-(-5\times3)] & +[(2\times3)-(-1\times-5)] \end{bmatrix}^T$$

$$C = \begin{bmatrix} 7 & 12 & -1 \\ 9 & 15 & -1 \\ -10 & -17 & 1 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ -1 & -1 & 1 \end{bmatrix}$$

$$\operatorname{adj}(A) = \begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ -1 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ -1 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix}$$

Question 3;

$$A = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$$

Find AB

$$AB = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$$

$$AB = \begin{bmatrix} [(2\times1)+(2\times2)+(-4\times0)] & [(2\times-1)+(2\times3)+(-4\times1)] & [(2\times0)+(2\times4)+(2\times4 \\ [(-4\times1)+(2\times2)+(-4\times0)] & [(-4\times-1)+(2\times3)+(-4\times1)] & [(-4\times0)+(2\times4)+(-4\times2 \\ [(2\times1)+(-1\times2)+(5\times0)] & [(2\times-1)+(-1\times3)+(5\times1)] & [(2\times0)+(-1\times4)+(5\times2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2+4+0 & -2+6-4 & 0+8-8 \\ -4+4+0 & 4+6-4 & 0+8-8 \\ 2-2+0 & -2-3+5 & 0-4+10 \end{bmatrix}$$

$$\therefore AB = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

Question 4

let matrix $\begin{pmatrix} 1 & 0 & -1 \\ 8 & 8 & 10 \\ 8 & 8 & 8 \end{pmatrix}$ be A

$R_2' = R_2 - 8R_1$

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 8 & 18 \\ 8 & 8 & 8 \end{pmatrix}$$

$R_3' = R_3 - 8R_1$

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 8 & 18 \\ 0 & 8 & 16 \end{pmatrix}$$

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$R_3'' = R_3' - R_2'$

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 8 & 18 \\ 0 & 0 & -2 \end{pmatrix}$$

$\therefore$ Since there are 3 non-zero rows, the rank is 3.

$\therefore$ The rank of matrix A is 3

Question 5;

$$\lim_{x \to 2}(x^2-1)$$

x^2-1 is a polynomial and $x=2$

$$x^2-1 = 2^2-1$$
$$= 4-1$$
$$= 3$$

Hence $\lim_{x \to 2}(x^2-1) = 3$

Question 6;

$$\lim_{x \to 2} \frac{x^2+4x-12}{x^2-2x}$$

$$\lim_{x \to 2} \frac{x^2+4x-12}{x^2-2x} = \lim_{x \to 2} \frac{(x+6)\cancel{(x-2)}}{x\cancel{(x-2)}}$$

$$= \lim_{x \to 2} \frac{x+6}{x}$$

when $x=2$; $\frac{x+6}{x} = \frac{2+6}{2} = \frac{8}{2}$

$$= 4$$

$\therefore \lim_{x \to 2} \frac{x^2+4x-12}{x^2-2x} = 4$

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Question 7

$$f(x) = x^2 + 2x$$

$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

$$f'(x) = \lim_{h \to 0} \left[\frac{((x+h)^2 + 2(x+h)) - (x^2 + 2x)}{h} \right]$$

$$f'(x) = \lim_{h \to 0} \left[\frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h - \cancel{x^2} - \cancel{2x}}{h} \right]$$

$$f'(x) = \lim_{h \to 0} \left[\frac{2xh + h^2 + 2h}{h} \right]$$

$$f'(x) = \lim_{h \to 0} (2x + h + 2)$$

hence $h = 0$

$$\therefore f'(x) = 2x + 2$$

Question 8

a) $f(x) = \dfrac{x-2}{x^4+1}$

using quotient rule,

$$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$$

let $u = x-2$, $v = x^4+1$

$$\left(\frac{u}{v}\right)' = \frac{[(1)(x^4+1)] - [(4x^3)(x-2)]}{[x^4+1]^2}$$

$$\left(\frac{u}{v}\right)' = \frac{(x^4+1) - (4x^4 - 8x^3)}{[x^4+1]^2}$$

$$\left(\frac{u}{v}\right)' = \frac{x^4 + 1 - 4x^4 + 8x^3}{(x^4+1)^2}$$

$$\left(\frac{u}{v}\right)' = \frac{8x^3 - 3x^4 + 1}{(x^4+1)^2}$$

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$$\therefore f(x) = \frac{(-3x^4+8x^3+1)}{(x^4+1)^2}$$

b) $f(x) = \sqrt{1-x^2}$

$f(x) = (1-x^2)^{1/2}$

~~Let~~ Using chain rule;

let $u = (1-x^2)^{1/2}$ $v = (1-x^2)$

~~&~~ $f'(x) = \frac{du}{dx} \cdot v'$

$$f'(x) = \frac{1}{2} \cdot (1-x^2)^{-1/2} \cdot (-2x)$$

$$f'(x) = (1-x^2)^{-1/2} \cdot (-x)$$

$$f'(x) = \frac{1}{(1-x^2)^{1/2}} \cdot (-x)$$

$$\therefore f'(x) = \frac{-x}{\sqrt{(1-x^2)}}$$

Question 9

Maximum and minimum values of $f(x) = x^2 - 6x + 5$ on the interval $(1, 4)$

$f(x) = x^2 - 6x + 5$

$f'(x) = 2x - 6$

for $f'(x) = 0$

$2x - 6 = 0$

$2x = 6$

$x = 3$

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at point (1, 4)

when $x = 1$

$f(x) = x^2 - 6x + 5$

$= (1)^2 - 6(1) + 5$

$= 1 - 6 + 5$

$= 0$

when $x = 3$

$f(x) = (3)^2 - 6(3) + 5$

$= 9 - 18 + 5$

$= -4$

when $x = 4$

$f(x) = 4^2 - 6(4) + 5$

$= 16 - 24 + 5$

$= -3$

x	fx
1	0
3	-4
4	-3

$\therefore$ hence maximum value is 0 ~~and~~;

minimum value is -4

Question 10

$f(x) = x^4 - 4x^2$

$f'(x) = 4x^3 - 8x$

$f''(x) = 12x^2 - 8$

Critical points;

set; $f'(x) = 4x^3 - 8x$

$f'(x) = 4x(x^2 - 8)$

when $f'(x) = 0$

$4x(x^2 - 8) = 0$

$4x = 0$ | $x^2 - 8 = 0$ | $x = \pm\sqrt{2}$

$x = 0$ | $\sqrt{x^2} = \sqrt{8}$

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critical points are; $x = 0$, $x = \sqrt{2}$, $x = -\sqrt{2}$

using $f''x$ on critical points.

when $x = 0$

$f''x = 12x^2 - 8$

$= 12(0)^2 - 8$

$f''x = -8 < 0$ (local maximum)

when $x = \sqrt{2}$

$f''(x) = 12(\sqrt{2})^2 - 8$

$f''x = (12 \times 2) - 8$

$f''x = 24 - 8$

$f''(x) = 16 > 0$ (local minimum)

when $x = -\sqrt{2}$

$f''(x) = 12(-\sqrt{2})^2 - 8$

$= (12 \times 2) - 8$

$= 24 - 8$

$= 16 > 0$ (local minimum)

hence when $x = 0$

local maximum is $(0, 0)$

when $x = \sqrt{2}$

$f(x) = x^4 - 4x^2$

$= (\sqrt{2})^4 - 4(\sqrt{2})^2$

$= -4$

$(\sqrt{2}, -4)$

when $x = -\sqrt{2}$

$f(x) = (-\sqrt{2})^4 - 4(-\sqrt{2})^2$

$= -4$

$(-\sqrt{2}, -4)$

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Hence the local ~~maxima~~ is $(0,0)$ and local minima is $(\pm\sqrt{2}, -4)$.