MAGNETISM NOTES

1.0 MAGNETISM

Is a group of phenomena or concepts associated with the field of force that exists around a magnet or a current carrying conductor.

A magnetic field

A magnetic field is a region of space in which any one of the following occurs:

(i) A magnetic dipole (a small bar magnet) experiences a force.

(ii) A current carrying conductor experiences a force, or a moving charge experiences a force.

(iii) An e.m.f. is, induced across a moving conductor.

Definition:

A magnetic field is thus defined in short as, a field of force that exists around a magnet or a current carrying conductor.
The magnetic properties of a magnetic body appear to originate at certain regions in the magnet called "poles". In a bar magnet these are near the ends of the magnet.

Experiments show that:

(i) Magnetic poles are of two types, north poles and south poles.

(ii) Like poles repel each other while the unlike poles attract each other.

(iii) Magnetic poles always seem to occur in equal and opposite pairs and

(iv) When no other magnet is near a freely suspended bar magnet, it rests in such a position that its magnetic axis is approximately parallel to the Earth's magnetic North – South axis.

Magnetic field lines.

The direction of a magnetic field at a point – is taken to be the direction of force on a north pole of magnetic dipole there under the influence of the field at that point.
The path that such a pole would follow is called ***a magnetic field line*** or (line of force)

Definition:

A magnetic field line – is the path or direction followed by the North Pole of a magnetic needle or magnetic dipole or a very small bar magnet when placed at that point in an electric field.
A magnetic field can therefore be represented by magnetic field lines so that,

(i) The line (or the tangent to it if it is a curved path) gives the direction of the magnetic field at that point.

(ii) The number of lines per unit cross sectional area is an indication of the strength of the field. i.e. the strength of the magnetic field is proportional to the density of the field lines.

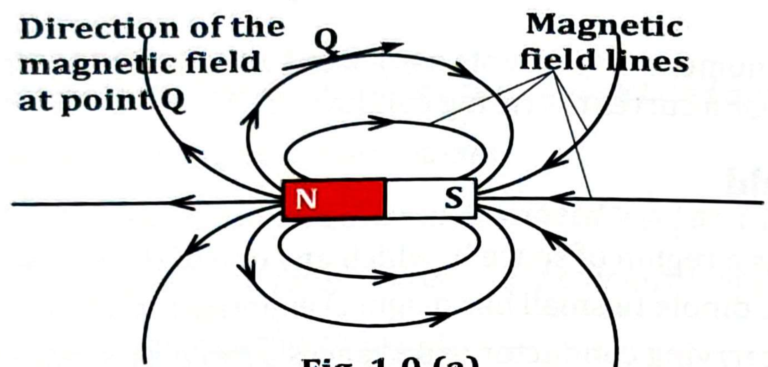


Fig. 1.0 (a)

The direction of the magnetic field is always directed away from the North pole of a magnet towards the South Pole.

For the case of a compass needle placed in the magnetic field of a bar magnet, the needle will be tangential to the magnetic field line at that point. The north pole of the needle points from the N-pole of the bar magnet to the South Pole.

Representations of magnetic field patterns

1. Around a magnet.

(a) Bar magnet.

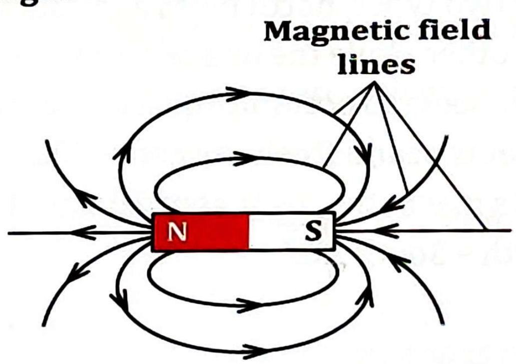


Fig. 1.0 (b)

The magnetic field around a bar magnet is non – uniform i.e. varies in strength and direction from point to point.

(b) U-shaped (Horseshoe) magnet.

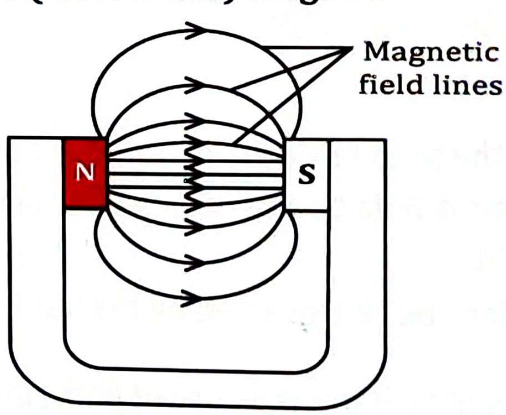
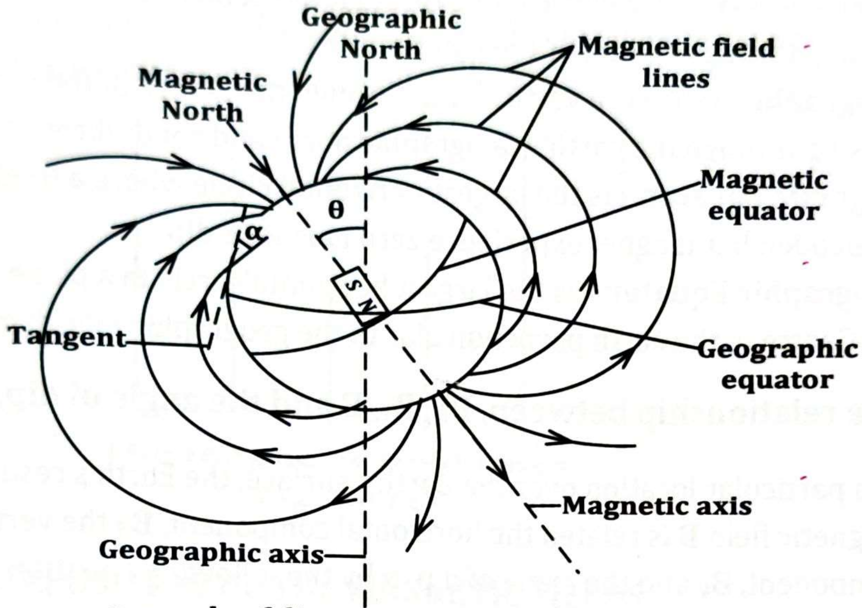


Fig. 1.0 (c)

2. The Earth's magnetic field

The following features represent the Earth's magnetic field pattern:



α = angle of dip
θ = angle of declination (magnetic variance)

Fig. 1.0 (d)

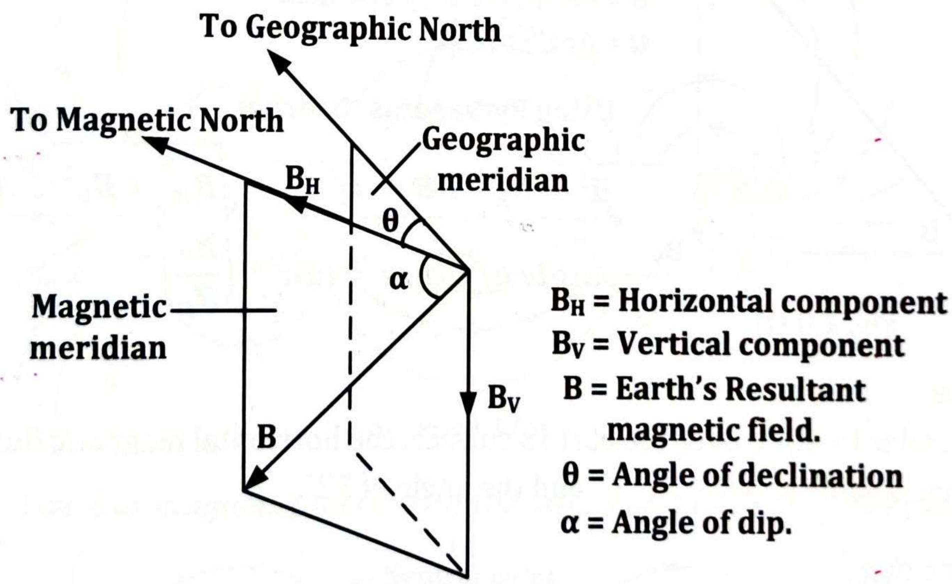


Fig. 1.0 (e)

Definitions associated with the Earth's Magnetic field

(i) **Magnetic Meridian:** - is a vertical plane passing through the Earth's Magnetic North and South poles.

(ii) **Geographic Meridian:** - is a vertical plane passing through the Earth's Geographic North and South directions.

(iii) **Angle of declination or Magnetic variance:** - is the angle between the Earth's magnetic meridian and the geographic meridian.

(iv) **Angle of dip**: - is the angle between the Earth's resultant magnetic field

and the horizontal component of the earth's magnetic field (Horizontal).
Or the angle between the horizontal and the axis through the poles of a freely suspended bar magnet when it sets.

(v) **Magnetic Axis** – is a vertical line through the Centre of the earth and passing through the earth's magnetic poles.

(vi) **Geographic Axis** – is a vertical line through the Centre of the earth and passing through the earth's Geographic north and south directions.

(vii) **Magnetic Equator** – is the largest horizontal circle where a freely suspended bar magnet experience zero magnetic dip.

(viii) **Geographic Equator** – is the largest horizontal circle in a plane through the Centre of the earth perpendicular to the geographic meridian.

The relationship between, B_H, B_V, B and the angle of dip, α

At a particular location over the Earth's surface, the Earth's resultant magnetic field **B** is related the horizontal component, $\mathbf{B_H}$ the vertical component, $\mathbf{B_V}$ and the angle of dip, **α** by the following equation:

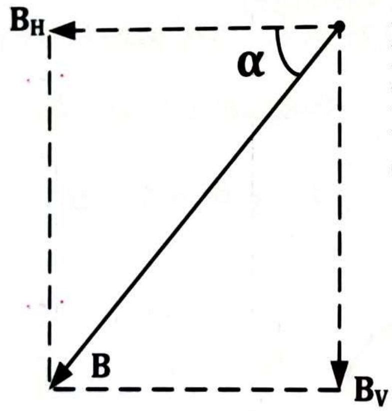


Fig. 1.0 (f)

$\mathbf{B_H}$ = Horizontal component
$\mathbf{B_V}$ = Vertical component
B = Resultant magnetic field
α = Angle of dip.

Using Pythagoras theorem,

$$B^2 = B_H{}^2 + B_V{}^2 \Rightarrow B = \sqrt{B_H{}^2 + B_V{}^2} \quad \text{....}(i)$$

$$Angle\ of\ dip, \alpha = tan^{-1}\left(\frac{B_V}{B_H}\right) \quad \text{...............}(ii)$$

Example:

At a particular location over the earth's surface, the horizontal magnetic flux density has a value of 3.0×10^{-4} T and the angle of 52°.

Determine the;

(i) Earth's resultant magnetic field.
(ii) Vertical component of the earth's magnetic field.

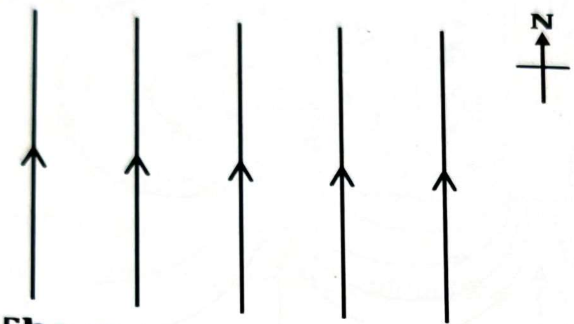
Solution:

(i) $\cos\alpha = \frac{B_H}{B} \Rightarrow The\ resutant\ field, \mathbf{B} = \frac{B_H}{\cos\alpha} = \frac{3.0\times10^{-4}}{\cos 52°} = \mathbf{4.87} \times 10^{-4}\,T$

(ii) $\tan\alpha = \frac{B_V}{B_H} \Rightarrow \mathbf{B_V} = B_H \tan\alpha = 3.0\times10^{-4}\tan 52° = \mathbf{3.84} \times 10^{-4}\,T$

The Earth's local magnetic field

This is a representation of the earth's magnetic field pattern as observed at a particular location over the earth's surface.
It's denoted or represented by a uniform parallel beam of the magnetic field directed towards the earth's geographic North.



The magnetic field lines
Fig. 1.0 (g)

THE SUPERPOSITION OF THE MAGNETIC FIELDS

(a) ***Two Bar magnets placed with the adjacent poles being un-like poles.***

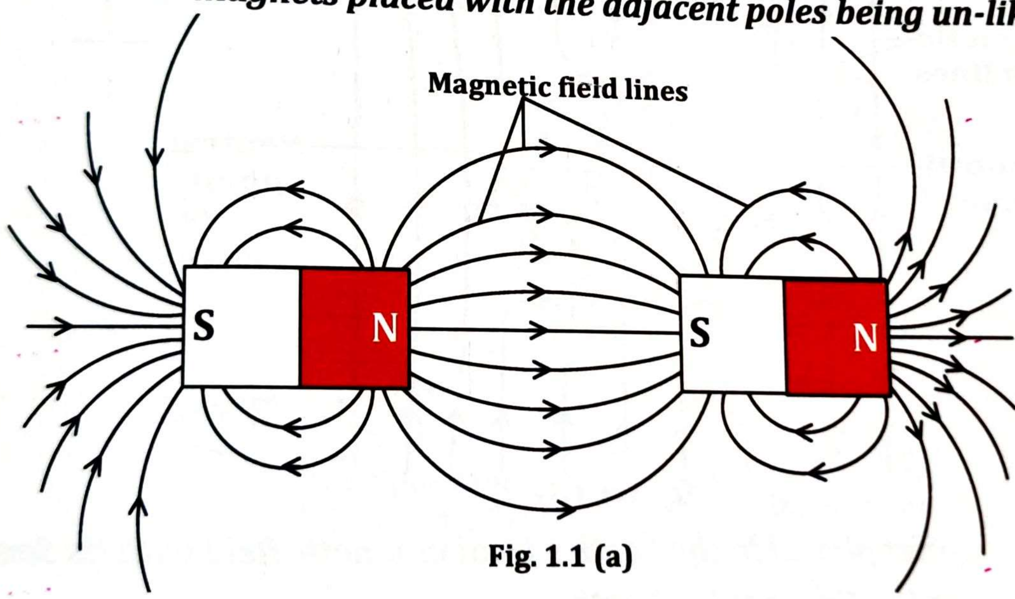


Fig. 1.1 (a)

(b) ***Two Bar magnets placed with the adjacent poles being like poles***

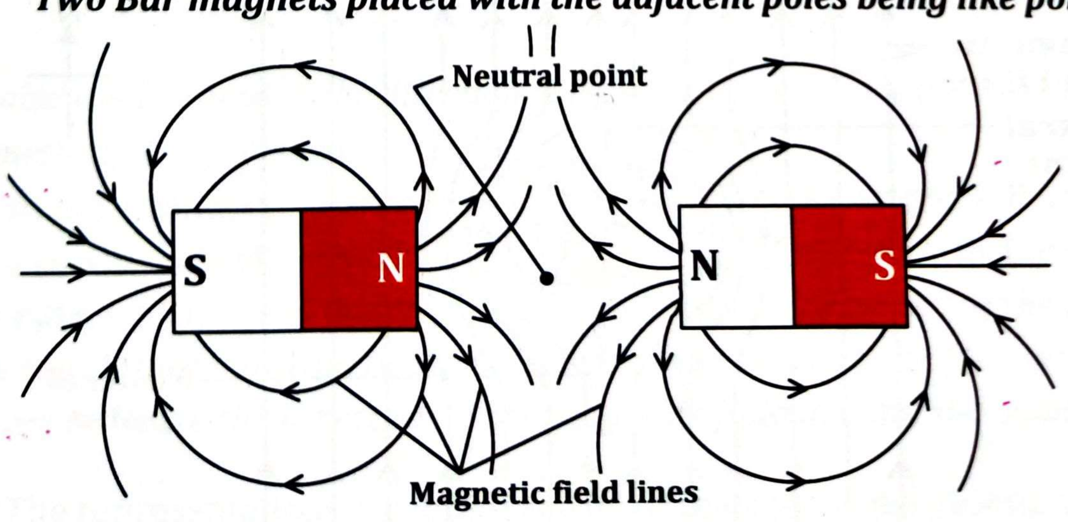


Fig. 1.1 (b)

(c) Two Bar magnets placed parallel and near each other with adjacent poles being unlike poles

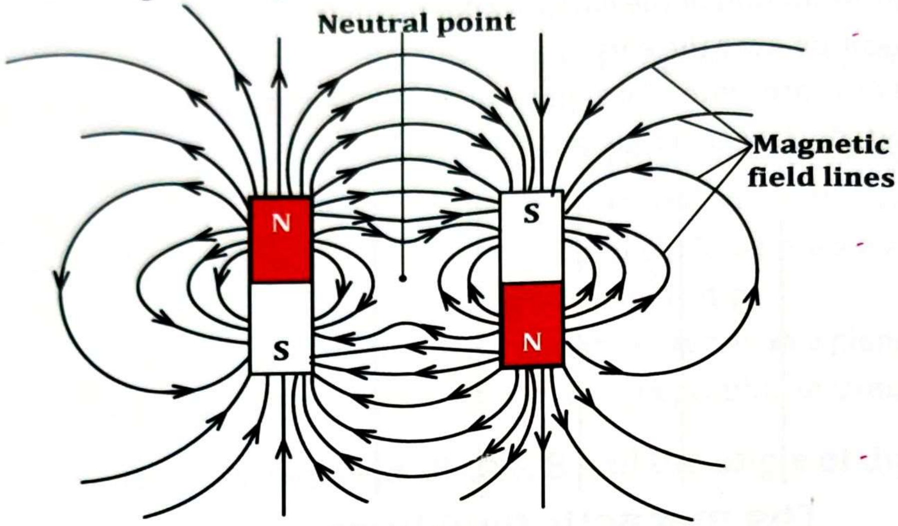


Fig. 1.1 (c)

(d) ***A bar magnet placed in the Earth's local magnetic field, with its North Pole facing the Geographic North***

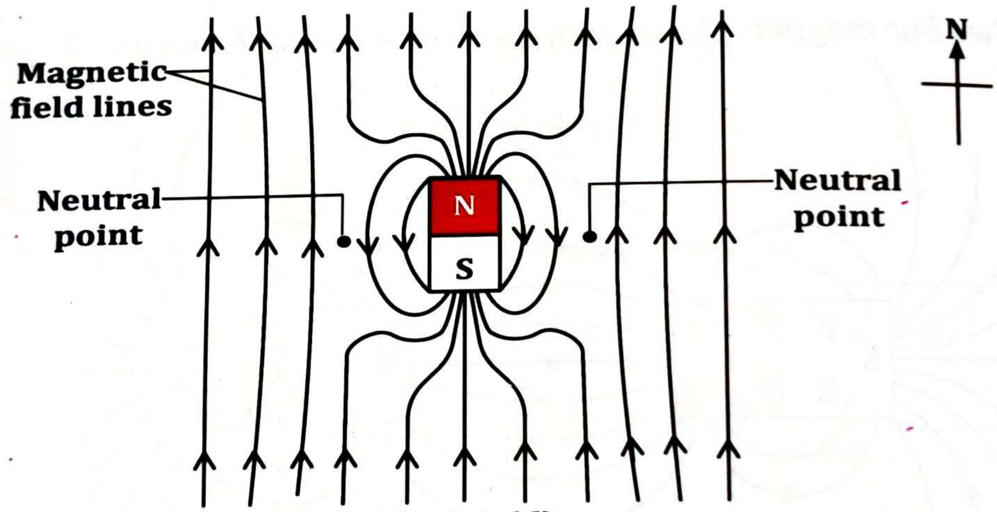


Fig. 1.1 (d)

(e) ***A bar magnet placed in the Earth's local magnetic field with its South Pole facing the Geographic North***

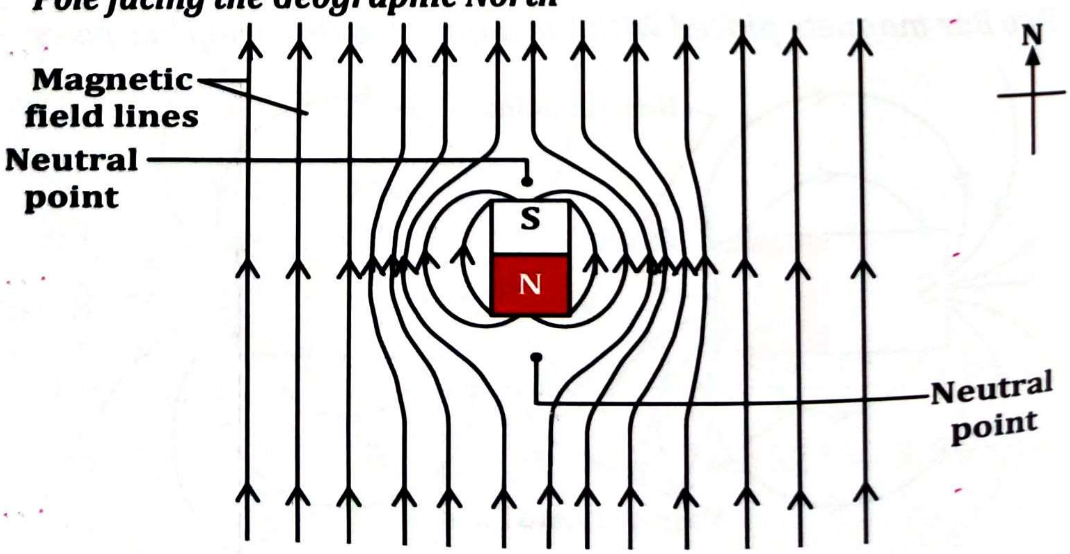


Fig. 1.1 (e)

A neutral point – *is a region of space in a magnetic field where two magnetic fields have equal magnitudes but opposite in their directions i.e. the resultant magnetic field is zero.*
At such a point, the resultant force on a magnetic dipole or on a small freely suspended bar magnet is zero.

1.2 THE RIGHT HAND GRIP RULE

*It is the rule used to predict the direction of a magnetic field when a current is passed through a conductor or a wire in such a way that, if the right hand is gripped, the **Thumb** represents the **direction** of the **current** provided. While the **four gripped, fingers** will provide the **direction of the magnetic field** around the current carrying conductor wire.*

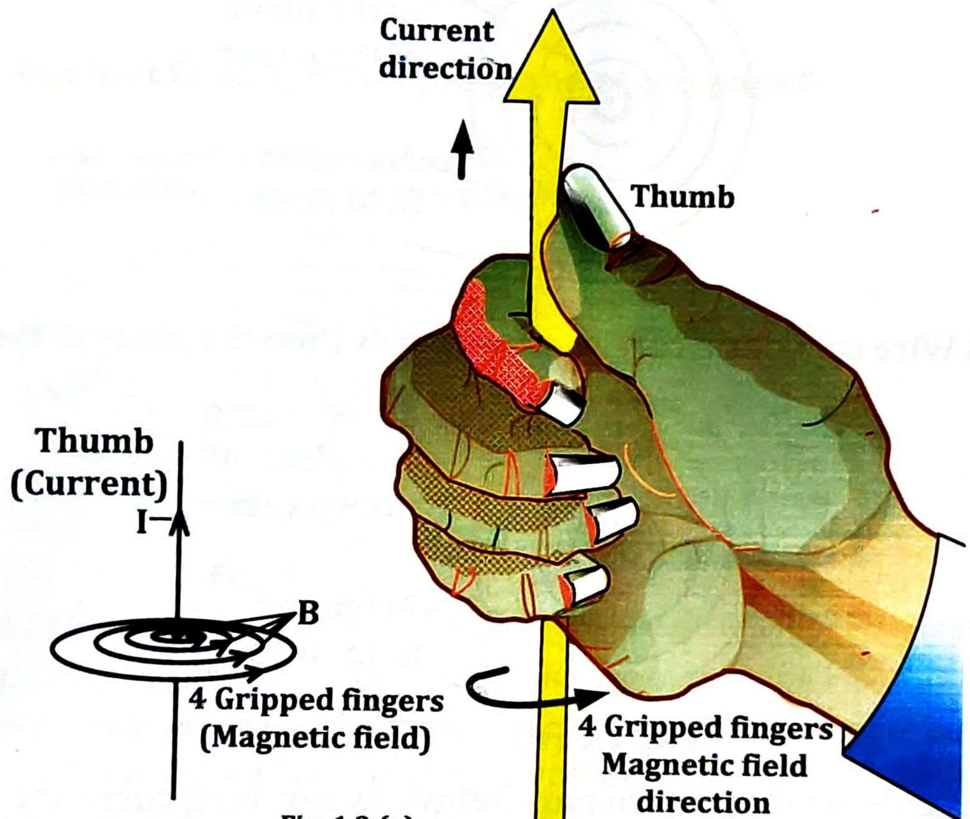


Fig. 1.2 (a)

It is also used to predict the direction of flow of a current if the direction of the Magnetic field is provided.
Any straight conductor carrying a current, experiences a magnetic field around it. The direction of a magnetic field around the conductor is given by the **right hand grip rule**, which states that ***imagine a conductor to be griped in the right hand with the thumb pointing in the direction of the current, the four gripped fingers indicate the direction of the magnetic field around the conductor***.

NB: The representations are interchanged or reversed when dealing with ***a coil or solenoid, carrying a current.***

Magnetic field patterns due to current – carrying conductors

1. A Straight wire carrying a current vertically upwards

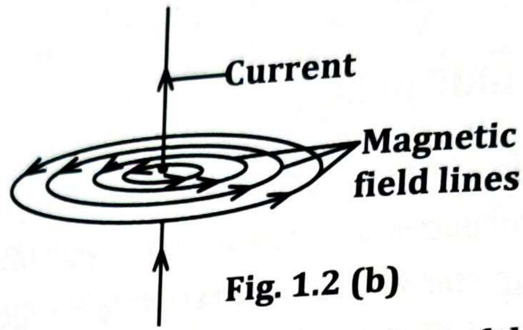


Fig. 1.2 (b)

(a) A Wire carrying a current upwards (out of the plane of the paper)

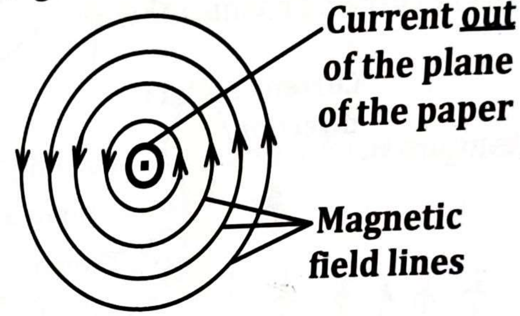


Fig. 1.2 (c)

(b) A Wire carrying a current downwards (into the plane of the paper)

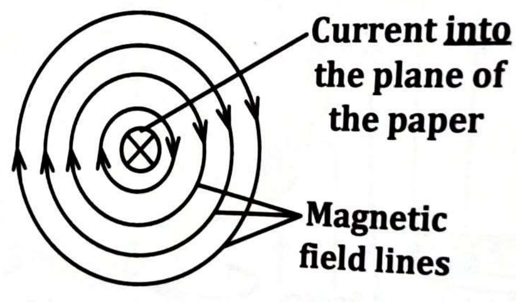


Fig. 1.2 (d)

2. Magnetic fields due to two parallel wires carrying currents

(a) In the opposite directions

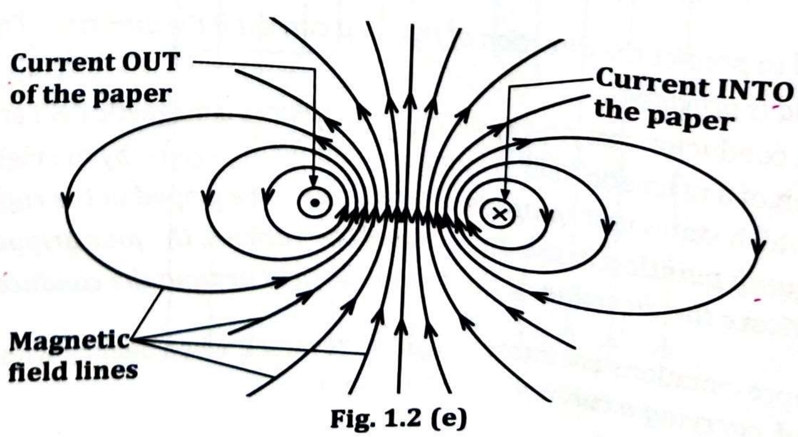


Fig. 1.2 (e)

(b) In the same direction

(i) Current flowing OUT of the plane of the paper

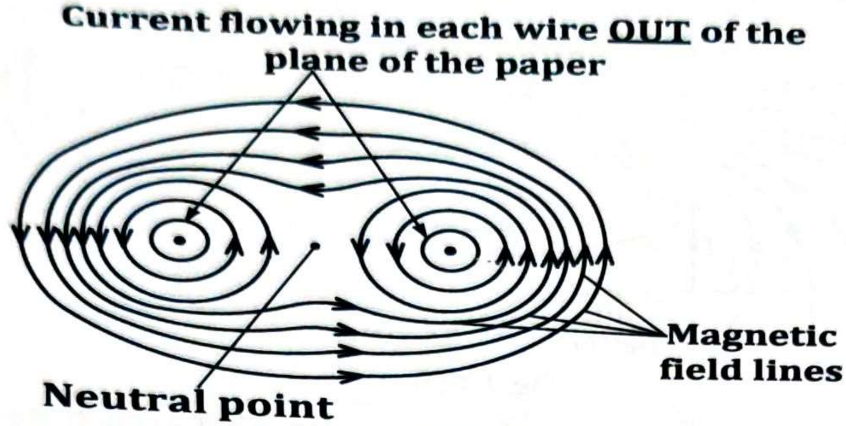


Fig. 1.2 (f)

(ii) Current flowing INTO the plane of the paper.

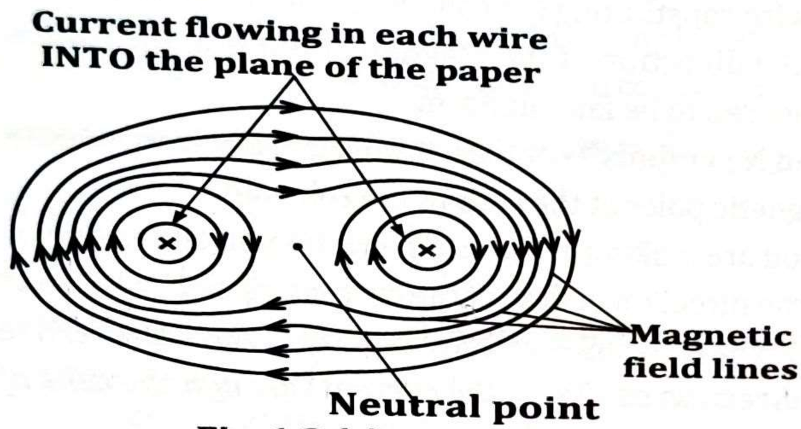


Fig. 1.2 (g)

NB: *In each of the cases in (i) and (ii) above, there are more magnetic field lines at the extreme outer positions of each wire than at the inner sections. This implies the existence of a stronger magnetic field at the outer positions and a weaker resultant field in the region between the wires. A resultant magnetic force always acts towards the centre of the two wires (weaker field).*

3. Magnetic field pattern due to a plane circular coil of N – turns

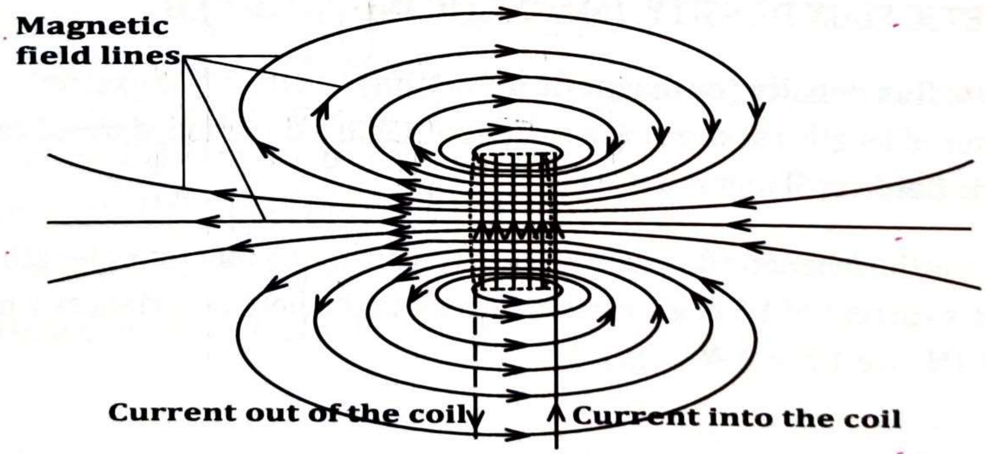


Fig. 1.2 (h)

4. **Magnetic field pattern around a Solenoid (a long coil)**

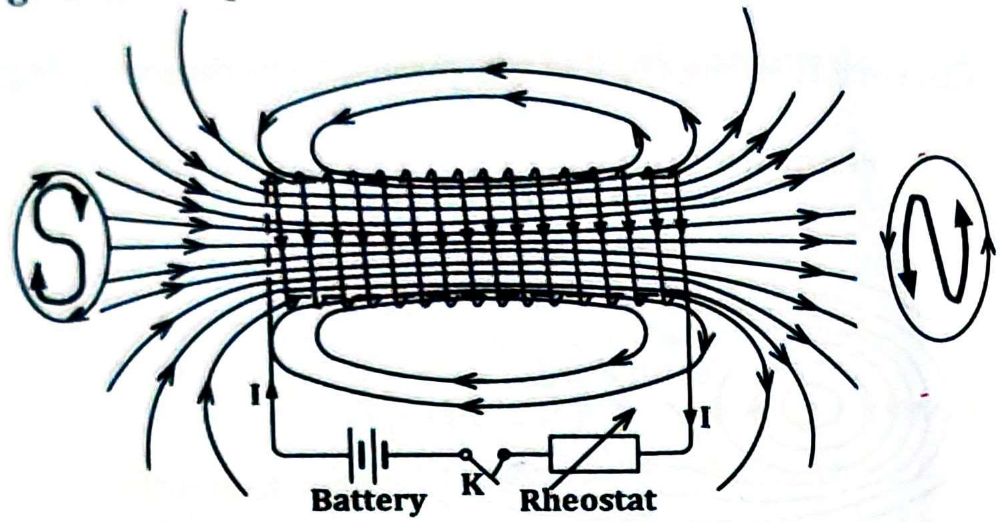


Fig. 1.2 (*i*)

NB: When direct current is passed through a solenoid or a coil in the direction indicated, in the figure 1.2 (*i*) the **four gripped fingers of** the **right hand** are used to indicate the **direction of** the several **turns of the current** carrying wire constituting the coil. While on the other hand, the **thumb** indicates the **direction of the magnetic field B**, at the **centre of such a coil**, considered to be fairy uniform.

The "S and N symbols" – at the extreme ends, help to identify the polarity of the magnetic poles at the ends of the solenoid.

Assume you are looking at the cross – section on either side of the solenoid; consider the direction of flow of the current, as clockwise or anti-clockwise. The "letter **S** or **N**" whose end arrows agrees with the direction of current i.e. ***same direction as that of the current through the coils of solenoid***, is the **correct pole** at that end of the solenoid.

Example, from the diagram in the figure 1.2 (*i*), on the **left hand side** of the solenoid, **current direction is clockwise**, so **"letter S"** agrees with the current direction, hence the **South Pole, S.**

Likewise on the **right hand side of the solenoid,** current flows in the **anti-clockwise direction**, and **"letter, N"** agrees with **the current direction**, hence the **North pole, N,** is obtained at the right hand side of the solenoid.

1.3 MAGNETIC FLUX DENSITY, (MAGNETIC INDUCTION) B

Magnetic flux density (or magnetic induction) – Is the force exerted on a conductor of length **1m** carrying a current of **1A** in a direction **normal to the** magnetic **field.** Its SI unit is a tesla (**T**).

A tesla – is the magnetic flux density across which, a conductor of length **1m** and carrying a current of **1A** in a direction normal to the field experiences a magnetic force of **1N**. $i.e\ 1\,T = 1\,N\,(A\,m)^{-1}$

Derivation for the force exerted on a current carrying wire or (conductor)

Suppose a straight wire of length, **L**, having **N** free electrons each of charge **e**, has a current, **I**, flowing through it and is placed perpendicularly across a uniform magnetic field of flux density **B**, the electrons attain an average drift velocity, **v**.

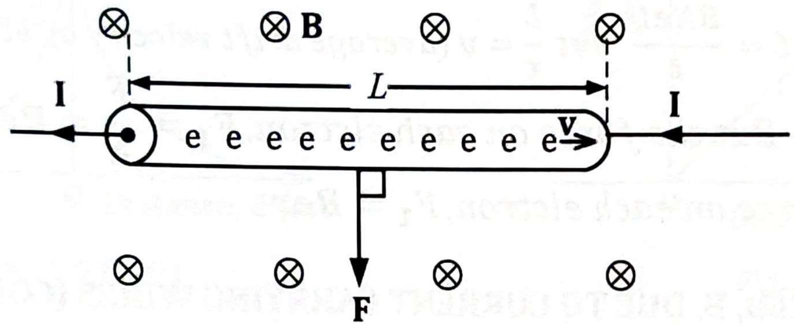


Fig. 1.3 (a)(i)

The magnetic force acting on each electron is given by;

Force on each electron, $F_1 = Bev$, and the total force on N-free electrons,

$F = NF_1 = NBev$ But Ne = Q (total charge on the conductor)

$\therefore F = BQv$, $But\ Q = It \Rightarrow F = BIt\ v$, where, $v = \frac{L}{t}$

$\therefore \quad F = BIt \times \frac{L}{t}$

Hence, $\boldsymbol{F = BIL}$

NB, suppose the conductor is at an angle θ to the magnetic field, B

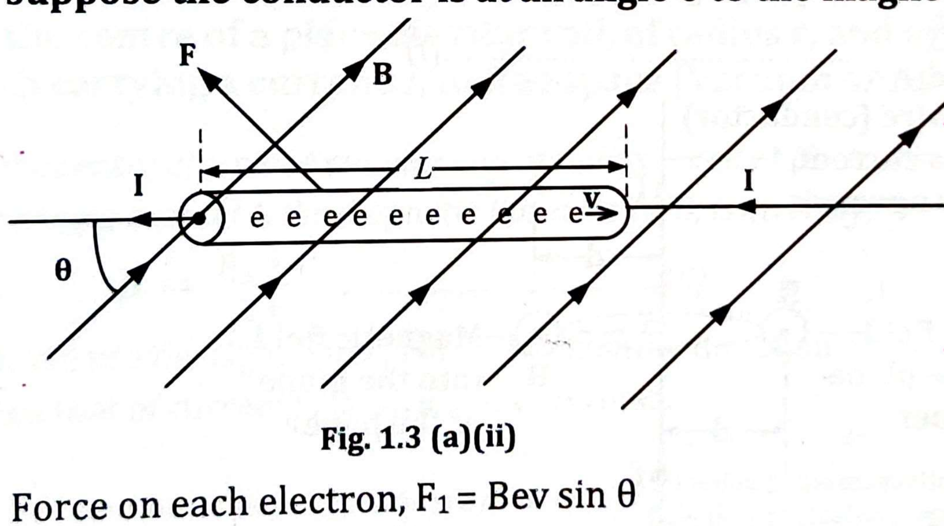


Fig. 1.3 (a)(ii)

Force on each electron, $F_1 = Bev \sin\theta$

Total force on N-free electrons,

$F = NF_1 = NBev \sin\theta \quad but,\ Q = Ne$

$\therefore F = BQv \sin\theta,\ But\ also\ Q = It$

$\Rightarrow F = BItv \sin\theta$, But, $v = \frac{L}{t}$

$\therefore F = BIt \times \frac{L}{t} \times \sin\theta$

Hence, $\boldsymbol{F = BIL \sin\theta}$

Derivation for the force exerted on an electron moving at a speed v, in a magnetic field, B, within a current - carrying wire (conductor)

Total force on a conductor of length, L, carrying a current I, across a magnetic field of flux density B, is $F = BIL$ but $current\ flowing\ I = \frac{Q}{t}$

$\therefore F = BIL = B\left(\frac{Q}{t}\right)L$ $where\ Q = Ne,\ N = Total\ no.of\ free\ electrons$

$\therefore F = B\left(\frac{Ne}{t}\right)L = \frac{BNeL}{t}$ $but\ \frac{L}{t} = v\ (average\ drift\ velocity\ of\ electrons)$

$\therefore F = BNev$ $But\ the\ force\ on\ each\ elctron, F_1 = \frac{F}{N} = Bev$

$Hence, the\ force\ on\ each\ elctron, F_1 = Bev$

1.4 MAGNETIC FIELD, B, DUE TO CURRENT CARRYING WIRES (CONDUCTORS)

NB, *The derivations for the expressions for the magnetic fields due to current carrying conductors involves the use of* ***The Biot - Savat law*** *which is out of the context of our A' level physics syllabus as of now. However, learners are expected to memorize the expressions, and make use of them in various applications of the syllabus.*

1. A straight wire carrying a current, I, in free space (Vacuum or Air)

*At a perpendicular distance, **d**, from the straight wire carrying a current **I**, the magnetic flux density, **B**, is given by,*

$$B = \frac{\mu_o I}{2\pi d} \quad \text{...(i)}$$

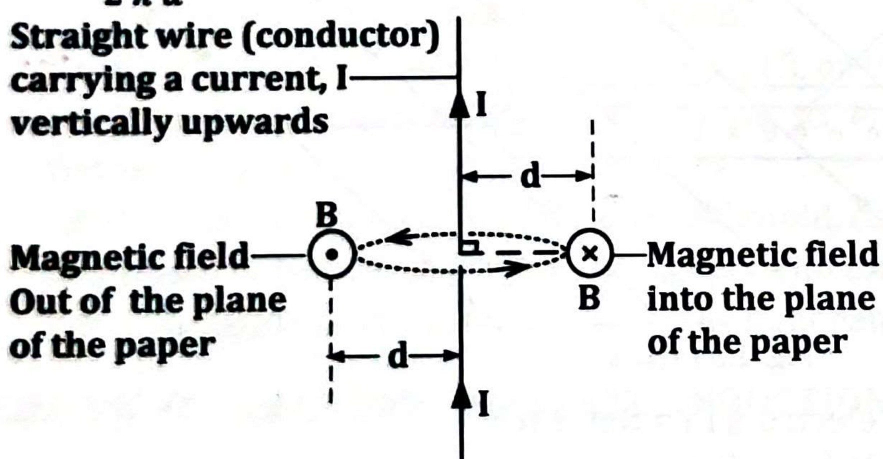


Fig. 1.4 (a)

NB: If the wire is surrounded by ***any other medium*** of permeability μ, where

$\mu = \mu_o\mu_r$ ***where,*** μ_r = ***Relative permeability of the media.***

The magnetic flux density, equals, $B = \frac{\mu I}{2\pi d} = \frac{\mu_o\mu_r I}{2\pi d}$

Magnetic flux density, B, due to a current carrying wire varies inversely with distance, d, as shown on the graph below.

A graph of magnetic flux density with distance, and magnetic flux density against reciprocal of distance from the wire.

(i) Variation of B with d

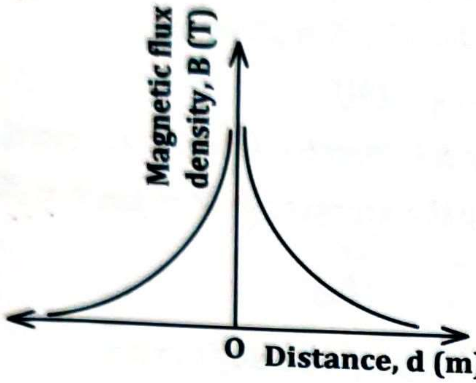


Fig. 1.4 (b)(i)

(ii) Variation of B with $\frac{1}{d}$

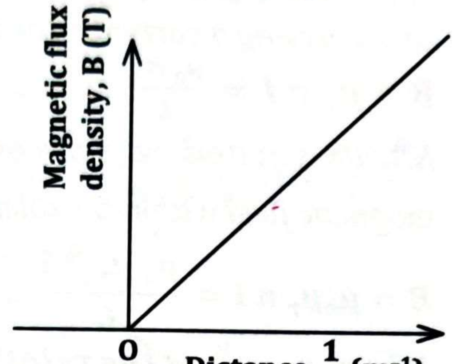


Fig. 1.4 (b)(ii)

NB: Graph (i) above shows that, as distance tends to zero, i.e. very close to the wire, the magnetic flux density becomes infinitely large and as the distance tends to infinity, the magnetic flux density tends to zero. While for graph (ii) **B** increases with increase in the value, $\frac{1}{d}$.

Ampere's law:

States that *– the magnetic field due to a straight conductor is proportional to the magnetic field produced and inversely is proportional to the perpendicular distance from the wire. i.e.* $B \propto \frac{I}{d}$

2. At the centre of a plane circular coil, of radius r, and of N – turns each carrying a current, I, in free space (Vacuum or Air)

At the centre of a plane circular coil of radius r, and of N – turns of wire each carrying a current I, the magnetic flux density, B, is given by,

$$B = \frac{\mu_o N I}{2r} \quad \text{.....................................(ii)}$$

NB: We use the right hand grip rule to identify direction of the magnetic field when that of current is known and vice versa.

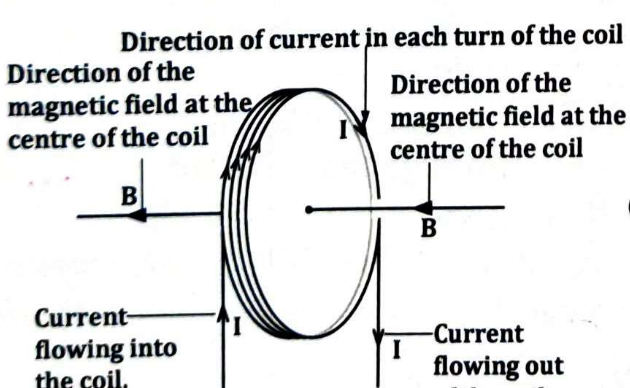


Fig. 1.4 (c)(i)

OR

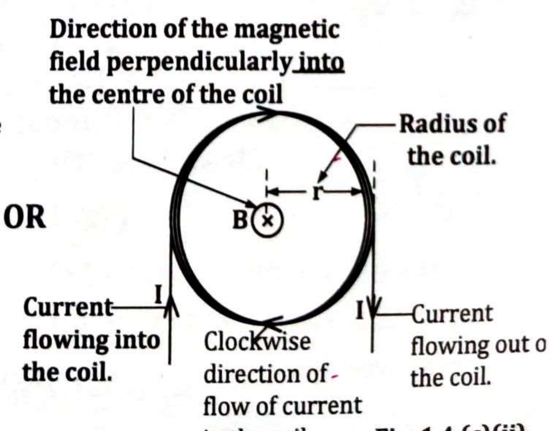


Fig. 1.4 (c)(ii)

3. **At the centre of a long coil, (Solenoid) of Length, L and N – turns or of (n – turns per metre), each carrying a current, I, in free space (Vacuum or Air)**

At the centre and along the axis of a long solenoid of n – turns per metre of wire each carrying a current I, the magnetic flux density, B, is given by,

$B = \mu_o n I = \frac{\mu_o N I}{L}$ ....(iii)

NB: *If a soft iron metal piece of permeability* μ *is inserted into the solenoid, the magnetic field within the solenoid is given by the expression,* $B = \mu n I = \frac{\mu N I}{L}$

$B = \mu_o \mu_r n I = \frac{\mu_o \mu_r N I}{L}$ ... ... (iv)

Where, $\mu_r = \frac{\mu}{\mu_o}$ ***is the relative permeability of the soft iron.***

Relative permeability, μ_r *– of a material is a measure of the conductivity of the medium for magnetic field lines.*

- *The greater the permeability of a material, the greater is its conductivity for the magnetic field lines and vice versa. Since the magnetic flux density (B) is a measure of the magnetic field lines passing normally per unit area of the material, thus it is also a measure of the magnetic permeability of the material.*
- *Suppose the magnetic flux density in a vacuum or air is B_o, If tis air is replaced by a material and the magnetic flux density in the material is B, then the ratio,* $\frac{B}{B_o} = \mu_r$ *is called the relative permeability of the material.*
- *For a vacuum / air,* $\mu_r = 1$, *since* $\frac{B_o}{B_o} = 1$ *implying that relative permeability of a material is just a number or numerical value without units.*
- *It is also observed that magnetic forces between two objects is much weaker than the electrostatic force between the same objects.*

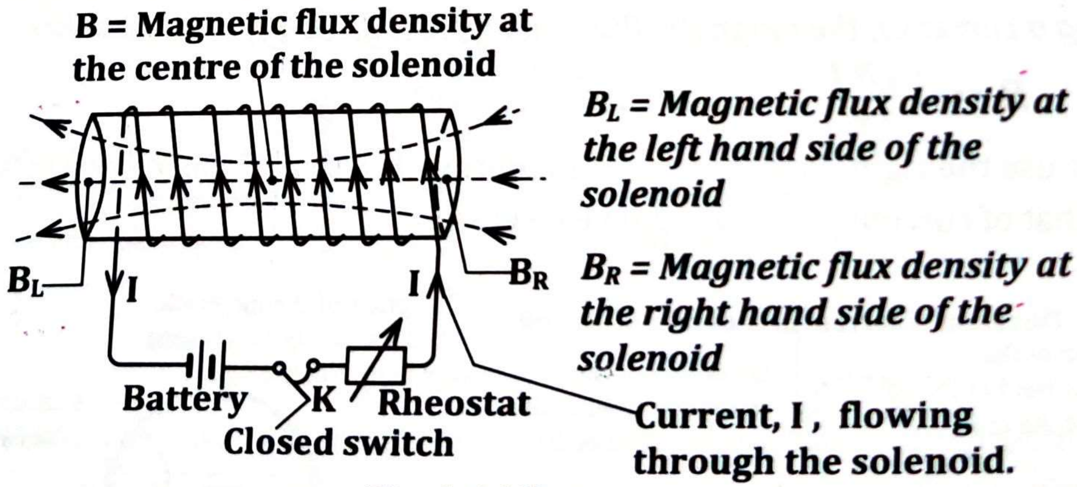


Fig. 1.4 (d)

At the ends, L and R, along the axis of a long solenoid of n – turns per metre of wire each carrying a current I, the magnetic flux density, B_L and B_R, is the same and given by,

$$B_L = \frac{1}{2}(\mu_o n I) = \frac{\mu_o N I}{2L}$$

$$B_L = B_R = \frac{1}{2}(\mu_o n I) = \frac{\mu_o N I}{2L} \quad ....(v)$$

A graph of magnetic flux density, B, with distance, x, from the centre of a long solenoid, of n – turns per metre carrying a current I

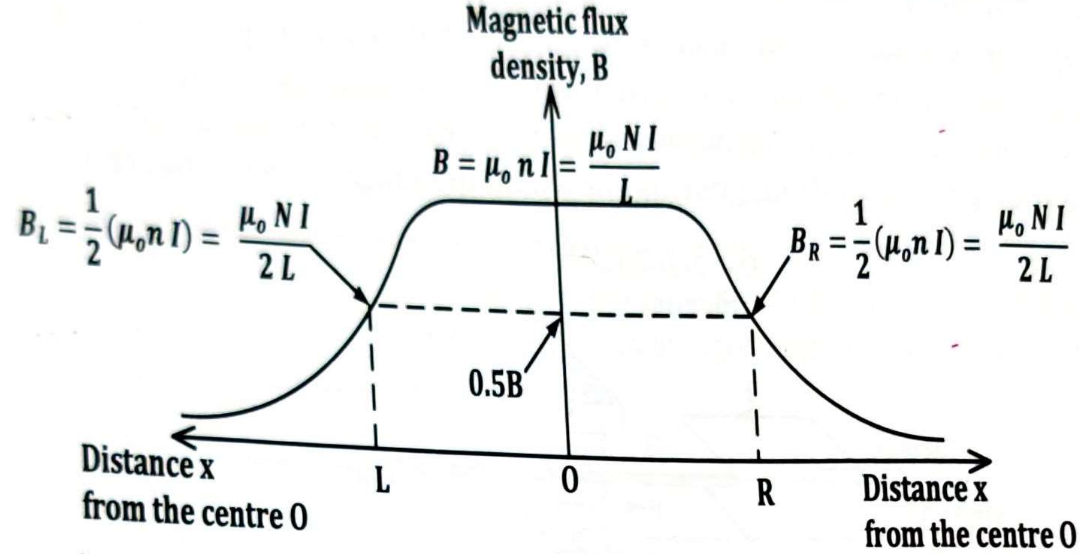


Fig. 1.4 (e)

NB, *The magnetic flux density at the centre of a solenoid is approximately uniform, and is maximum at a value* $\boldsymbol{B = \mu_o n I}$ *and this value progressively reduces as distance, x, from the centre O increases.*

At the extreme positions L and R of the solenoid, the value of the magnetic flux density reduces to half the value at centre O, on each side, hence, $\boldsymbol{B_L = B_R = \frac{1}{2}(\mu_o n I)}$

1.5 FLEMING'S LEFT HAND RULE: (OFTEN CALLED THE MOTOR RULE)

States that – *Whenever a current carrying conductor is placed perpendicularly across a magnetic field, it will experience a magnetic force* $F = BIL$*, that acts in the direction of the* ***Thumb*** *of the left hand, while the* ***first*** *and second. Fingers, placed perpendicular to each other and whose plane is normal to the thumb, represent Magnetic* ***Field, B*** *and current* ***I*** *respectively, as shown in the figure 1.5 (a)*

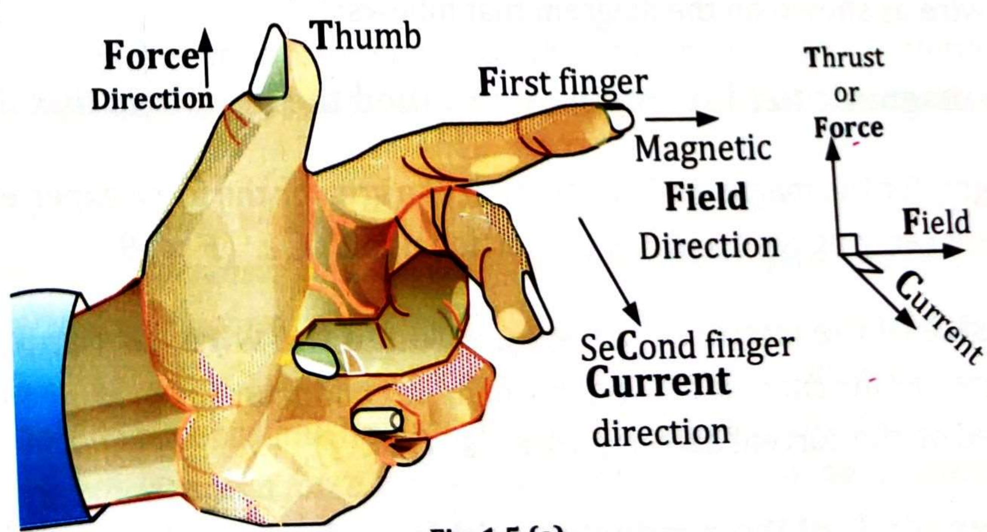


Fig. 1.5 (a)

Force on a straight conductor currying current in a magnetic field

When a conductor carrying current is placed in an external magnetic field (produced by a permanent magnet for example), it experiences a force that will move it, if it is free to do so. This can be demonstrated using the apparatus shown below. A metal rod is placed across metal rails that lie between the poles of a horse shoe (U – shaped) magnet, as shown on the diagram in the figure 1.5(b)

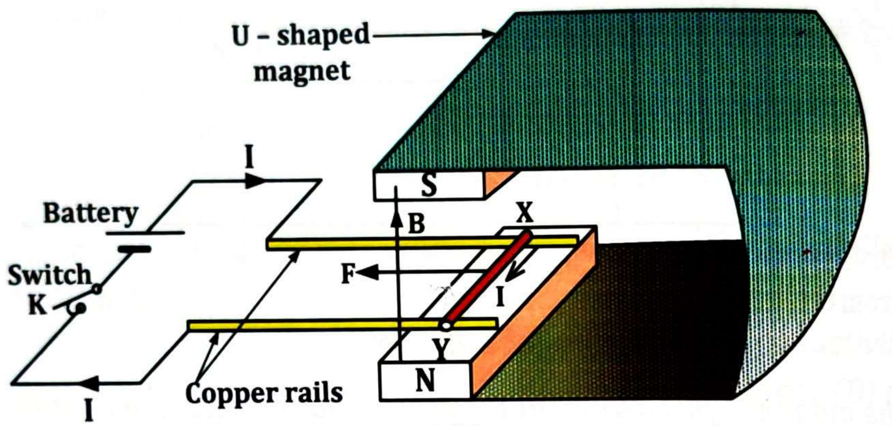


Fig. 1.5 (b)

When the switch K is closed, a current **I** is passed through the rod, XY in the direction, X towards Y. By Flemings left hand rule, a magnetic force F = BIL acts on the rod XY, acting from right to left, hence causing it to roll from the right hand side along the raids towards the left hand side direction as shown. The direction of force and hence the movement of the metal rod was predicted by Flemings left hand rule.

Factors affecting the size of the force on a conductor

Simple experiments show that, the magnitude of the force (**F**) on a wire carrying a current in a magnetic field depends on **four** basic factors as follows and the magnitude of this force can be investigated by measuring the angle **θ**, of swing of the wire as shown on the diagram that follows:

1. **The magnetic field strength often called the magnetic flux density,** B

   The greater the magnetic field strength, the greater the force experienced by a current carrying wire and **F** is proportional to **B**. $i.e.\ (\boldsymbol{F} \propto \boldsymbol{B})$

2. **The size of the current I, flowing through the wire (or conductor)**
   The greater the current flowing through the conductor the greater the force exerted on the current carrying wire $i.e.\ (\boldsymbol{F} \propto \boldsymbol{I})$

3. **The length, L, of the conductor, within and across the magnetic field**
   The greater the length, **L** is the greater the force $i.e.\ (\boldsymbol{F} \propto \boldsymbol{L})$.

The angle θ of inclination of the conductor to the direction of the magnetic field, B

Only the ***component of the magnetic field perpendicular to the current*** or ***vice versa***, produces the magnetic force on the conductor, but NOT the component parallel to the current or magnetic field. It can be shown that *i.e.* ($F \propto \mathbf{B} \sin \theta$) Or ($F \propto \mathbf{I} \sin \theta$) as shown on the diagram below. A conductor parallel to direction of the magnetic field experiences **no magnetic force.**

The converse is thus true, i.e. **maximum force** is experienced by the conductor when, $\theta = \pi/2$ *or* **90°** (i.e. The conductor is at right angles to the magnetic field) and becomes **zero** when, $\theta = 0°$

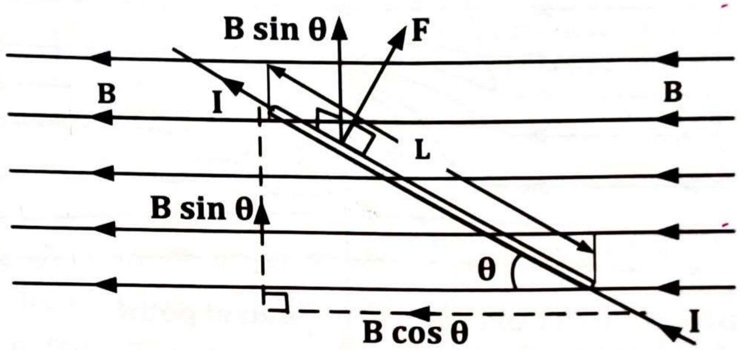


Fig. 1.5 (c)

Combining all the four factors given above, together.

$F \propto BIL \sin \theta$

or $F = kBIL \sin \theta$ where, $k = constant, and\ experiments\ show\ that, k = 1$

$\therefore \boldsymbol{F = BIL \sin \theta}$

Explanations

- *The magnetic force experienced by an electron within the conductor is given by* $\boldsymbol{F = Bev}$ *where* $\boldsymbol{v}$ *is the average drift velocity of the electrons. Each electron interacts with one magnetic field line.*
- *When the magnetic field strength is increased, the number of electrons interacting with the field increases so the total force on the conductor as a whole therefore increases.*
- *From I = nevA, I ∝ n, thus when the current flowing in the conductor increases, the number of electrons increases, and since each experiences a force, F = Bev, so the total force on the conductor as a whole therefore increases.*
- *Increasing the length, L of the conductor within the magnetic field, implies increasing the number of conducting electrons in the region of the magnetic field and since each experiences a force, F = Bev, so the total force on the conductor as a whole therefore increases.*
- *When the conductor is parallel to the magnetic field Fleming's left hand rule does not hold so electrons do not experience a magnetic force. When the angle of tilt θ is*

increased, the component of length L across the field increases, since L∝F, the force on the conductor increases reaching a maximum value when θ = 90° since the $\sin 90° = 1$, *from the expression,* $F = BIL\sin\theta \Rightarrow F \propto \sin\theta$ when all the *other factors are kept constant*

Why a conductor carrying a current across a magnetic field experiences a magnetic force

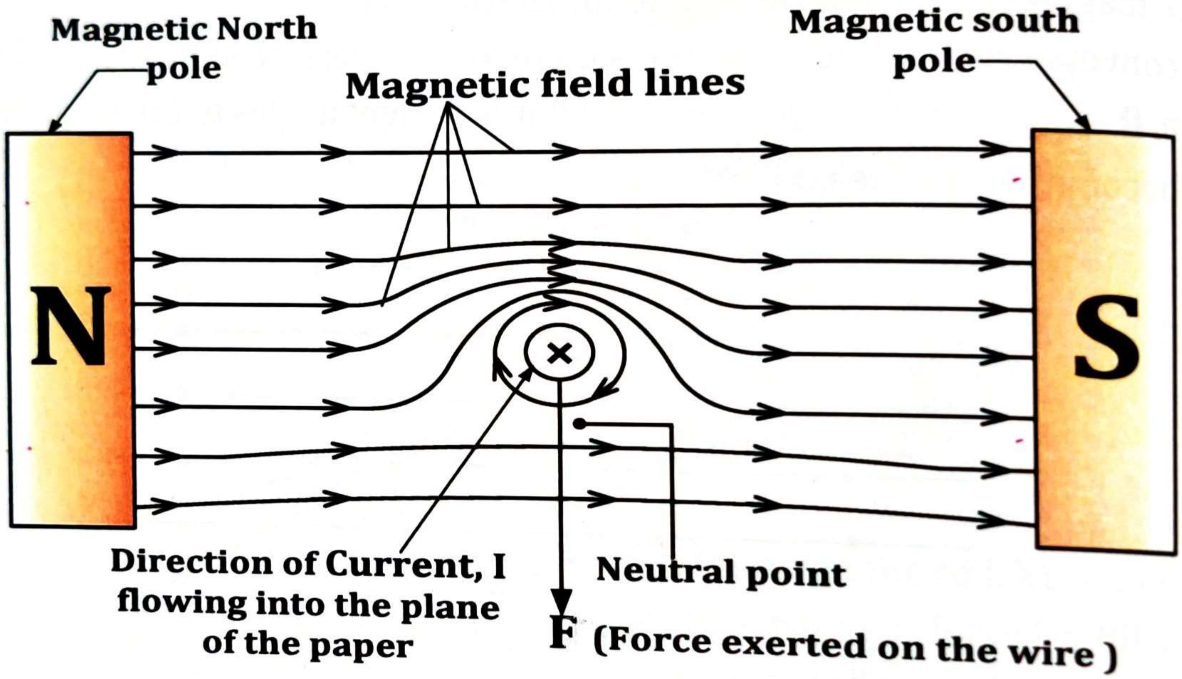


Fig. 1.5 (d)

- *The wire carrying a current, produces a magnetic field around itself in a direction given by the right hand grip rule.*
- *The magnetic field of the wire interacts with the external magnetic field, B, provided by the pole pieces of the magnet.*
- *The two magnetic fields interact and reinforce each other above the wire Creating a region of stronger resultant magnetic field above the wire.*
- *On the other hand, a weaker resultant magnetic field is created below the Wire and a neutral point is also created below the wire where two magnetic fields are equal in magnitude but opposite in directions.*
- *The wire then experiences a resultant magnetic force F = BIL acting vertically downwards Form the region of stronger magnetic field towards the region of weaker magnetic field, as shown on the diagram above.*
- *When the wire is light and free to move, this may cause motion of the wire along the direction of the force.*

Experiment to investigate some of the factors affecting the size of force exerted on a wire PQ carrying a current across a magnetic field.

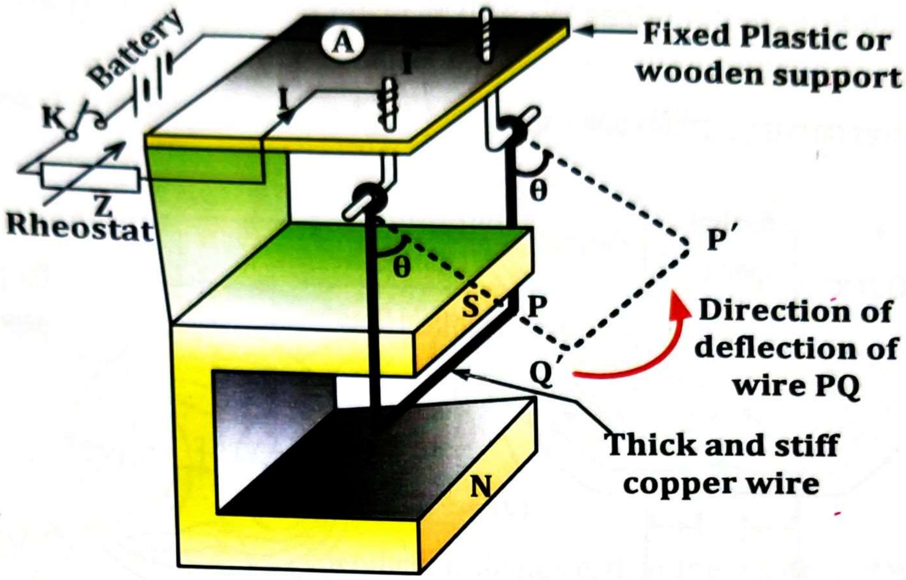


Fig. 1.5 (e)

- Using the set up shown in the figure 1.5 (e), using length PQ = L, switch K is closed, and wire PQ kicks off to the right, and the maximum angle θ_0 of deflection is noted. The switch is then opened and the wire PQ left to settle down without movement.
- Current **I** is increased using a rheostat Z, then switch K is closed, the new deflection θ_1 is noted. It is observed that, $\theta_1 > \theta_0$. ***This implies the magnetic force on wire PQ increases with increase in current flowing through the wire.***
- Bar magnets of the same size are then added on each pole of the **"C"** shaped magnet, and the experiment is repeated and the new deflection, θ_2. It is observed that, $\theta_2 > \theta_0$. ***This implies the magnetic force on wire PQ increases with increase in the magnetic field strength, B, across the wire.***
- The **length, L** of wire PQ, is **increased** and the experiment is repeated and the new deflection, θ_3 is noted. It is observed that, $\theta_3 > \theta_0$. ***This implies the magnetic force on wire PQ increases with increase in length L of the wire, across the field.***
- The orientation, β of the wire PQ to the magnetic field, is reduced where $\beta < 90°$ and the experiment is repeated. The new deflection θ_4 is noted. It is observed that, $\theta_4 < \theta_0$. ***This implies magnetic force on wire PQ decreases with decrease in angle β of inclination of the wire to the magnetic field.***

1.6 MAGNETIC FORCES EXPERIENCED BY CURRENT CARRYING WIRES

When a current currying wire is placed across a magnetic field provided by another current carrying wire, the two magnetic fields interact creating regions of reinforced magnetic fields and reduced or cancelled out magnetic fields. This causes the wires to experience magnetic forces as demonstrated by the following set ups.

1. **Two straight parallel wires W_1 and W_2 carrying currents I_1 and I_2 in the same direction**

(i) Two wires carrying currents upwards

(ii) Two wires carrying currents perpendicularly into the plane of the paper.

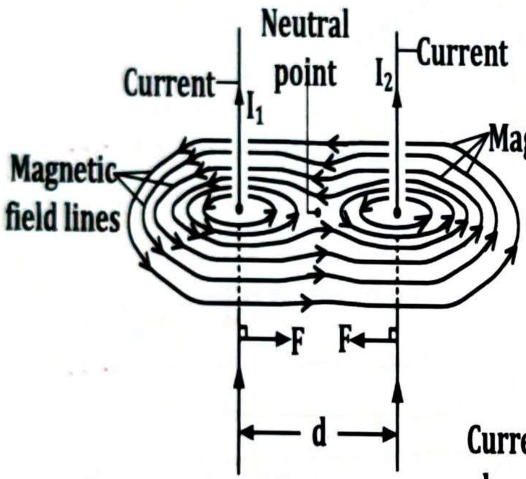


Fig. 1.6 (a)(i)

OR

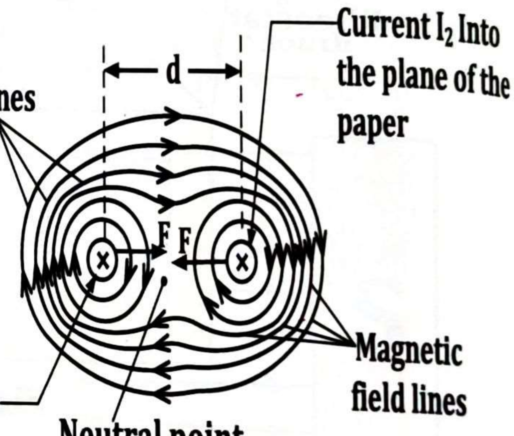


Fig. 1.6 (a)(ii)

Magnetic flux density at a perpendicular distance, d, at the position of wire 2 due to wire W_1 is given by $\boldsymbol{B_1} = \frac{\mu_0 I_1}{2\pi d}$ ……………………… (i)

Similarly, magnetic flux density at a perpendicular distance, d, at the position of wire 1 due to wire W_2 is given by $\boldsymbol{B_2} = \frac{\mu_0 I_2}{2\pi d}$ ……………… (ii)

Thus, by Fleming's left hand rule, a magnetic force,

$F_1 = B_2 I_1 L$ *and substituting for* B_2 *from equation (ii) we obtain;*

$$F_1 = B_2 I_1 L = \frac{\mu_0 I_2}{2\pi d} I_1 L \text{ where L is the length of each wire.}$$

$$\therefore F_1 = \left(\frac{\mu_0 I_2 I_1 L}{2\pi d}\right) \text{acting towards } W_2 \text{ ……………… (iii)}$$

Likewise, by Fleming's left hand rule, a magnetic force, on wire W_2,

$F_2 = B_1 I_2 L$ *and substituting for* B_1 *from equation (i) we obtain;*

$$F_2 = B_1 I_2 L = \frac{\mu_0 I_1}{2\pi d} I_2 L \text{ where L is the length of each wire.}$$

$$\therefore F_2 = \left(\frac{\mu_0 I_1 I_2 L}{2\pi d}\right) \text{acting towards } W_1 \text{ ……………… (iv)}$$

NB: *From equations (iii) and (iv), the two wires have the same magnitude of force*, i.e. $|F_1| = |F_2| = \boldsymbol{F} = \left(\frac{\mu_0 I_1 I_2 L}{2\pi d}\right)$ and is an attractive force pulling the two wires together, towards each other.

2. **Two straight parallel wires W_1 and W_2 carrying currents I_1 and I_2 in opposite directions.**

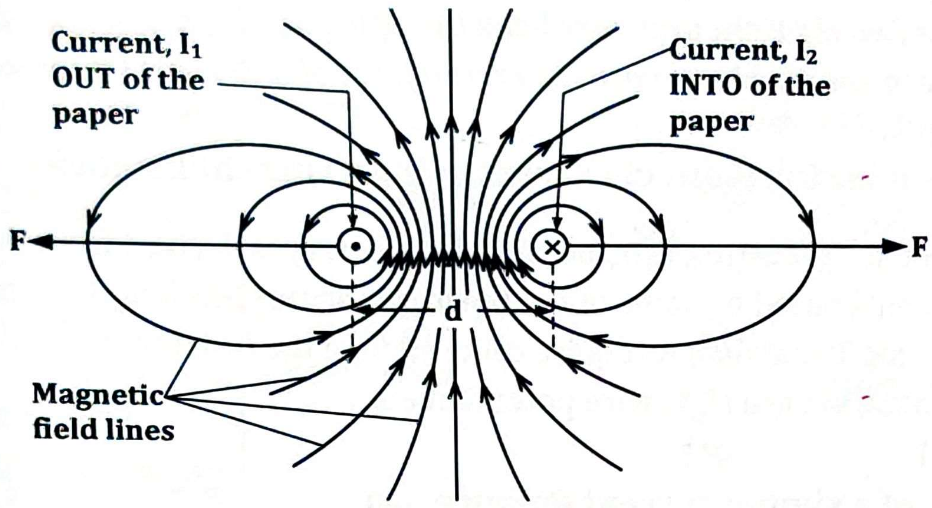


Fig. 1.6 (b)

Magnetic flux density at a perpendicular distance, **d**, at the position of wire W_2 due to wire $\mathbf{W_1}$ is given by $\boldsymbol{B_1} = \frac{\mu_0 I_1}{2\pi d}$ ……………………………… ***(i)***

Similarly, magnetic flux density at a perpendicular distance, d, at the position of wire 1 due to wire W_2 is given by $\boldsymbol{B_2} = \frac{\mu_0 I_2}{2\pi d}$ ……………………………… ***(ii)***

Thus, by Fleming's left hand rule, a magnetic force,

$F_1 = B_2 I_1 L$ *acting to the left of* W_1 *& sub. for* B_2 *from (ii), we obtain;*

$F_1 = B_2 I_1 L = \frac{\mu_0 I_2}{2\pi d} I_1 L$ *where L is the length of each wire.*

$\therefore F_1 = \left(\frac{\mu_0 I_2 I_1 L}{2\pi d}\right)$ *acting away from* W_2 *i.e. to the left of* W_1 ……… *(iii)*

Likewise, by Fleming's left hand rule, a magnetic force, on wire W_2,

$F_2 = B_1 I_2 L$ *and substituting for* B_1 *from equation (i), we obtain;*

$F_2 = B_1 I_2 L = \frac{\mu_0 I_1}{2\pi d} I_2 L$ *where L is the length of each wire.*

$\therefore F_2 = \left(\frac{\mu_0 I_1 I_2 L}{2\pi d}\right)$ *acting to the Right of* W_2 *i.e away from* W_2 …… *(iv)*

NB: From equations (iii) and (iv), the two wires have the same magnitude of force,

i.e. $|F_1| = |F_2| = F = \left(\frac{\mu_0 I_1 I_2 L}{2\pi d}\right)$

and is a repulsive force pushing wires apart or away from each other.

3. The definition of an ampere.

From the expression, $\boldsymbol{F} = \left(\frac{\mu_0 I_1 I_2 L}{2\pi d}\right)$, when the two parallel wires above, each of length, ***one metre***, placed ***one metre apart*** in ***free space***, and if each is to carry a current of ***one ampere***, then substituting, $\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}, I_1 = 1\ A$, $I_2 = 1\ A, d = 1\ m$ ***and*** $L = 1\ m$ then, $F = \left(\frac{4\pi\times10^{-7}\times 1.0\times 1.0\times1.0}{2\pi\times1.0}\right) = 2.0\times10^{-7} N$,

then an ampere is defined as follows,

Definition

An ampere *– is the* ***steady, (direct or constant) current*** *which when flowing in each of the* ***two straight and parallel*** *wires of negligible cross sectional area, placed one metre apart in a vacuum, exert a force of* $2.0 \times 10^{-7} N$ *per metre of each other's length.*

4. Absolute measurement of current using a current balance

The method is called **absolute**, because its accuracy is derived from the measurements based on some of the **basic quantities** like length, **L** , mass **M** and probably time **T** that do not require calibration of the instrument used in the measurement, using a slide wire potentiometer.

Method I

Diagram: of a simple current Balance.

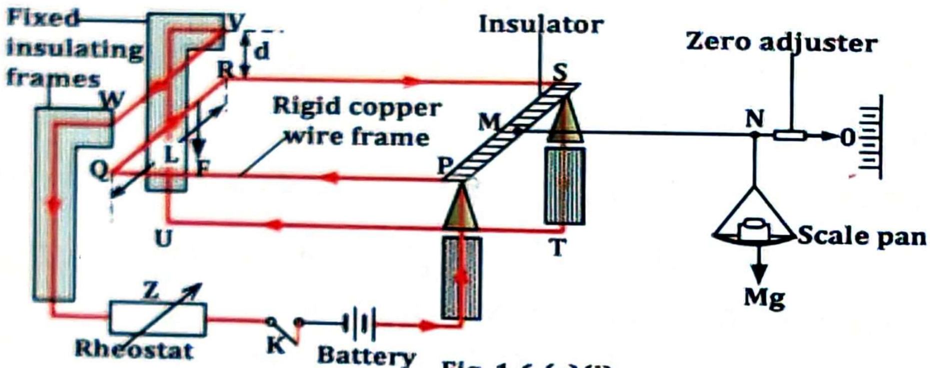


Fig. 1.6 (c)(i)

The mode of operation:

- When switch K is open, the rigid rectangular copper wire frame **PQRS** is made horizontal by the help of the zero adjuster.
- Switch K is then closed and a suitable current **I** is made to flow through wires **QR** and **VW** in opposite directions and produce a downward deflection on QR
- Small weights of are carefully added into the scale pan, until the wire frame **PQRS** balances horizontally about pivots **P** and **S.**
- The total weight, **Mg** in scale pan is then noted.
- The length, **L** of wire **QR** is then measured using *a metre rule* and recorded down.
- The distance, **d**, between the two wires **QR** and **VW** when the frame is horizontal is measured using *a travelling microscope* and recorded down.
- Taking moments of forces, F, acting on QR and that of the weight of the components of the scale pan, about a common axis, **PS**

$$i.e.\ F \times RS = Mg \times MN \text{ where, } F = \frac{\mu_0 I^2 L}{2\pi d}$$

- Now, since practically the distance, **RS = MN**, and **QR = *L***
- The value of the current, **I** is then calculated from the expression,

$$I = \sqrt{\frac{2\pi d\, mg}{\mu_0 L}} \quad \text{where, } \mu_0 = 4\pi \times 10^{-7} Hm^{-1}$$

Method II

Diagram: of a simple current Balance.

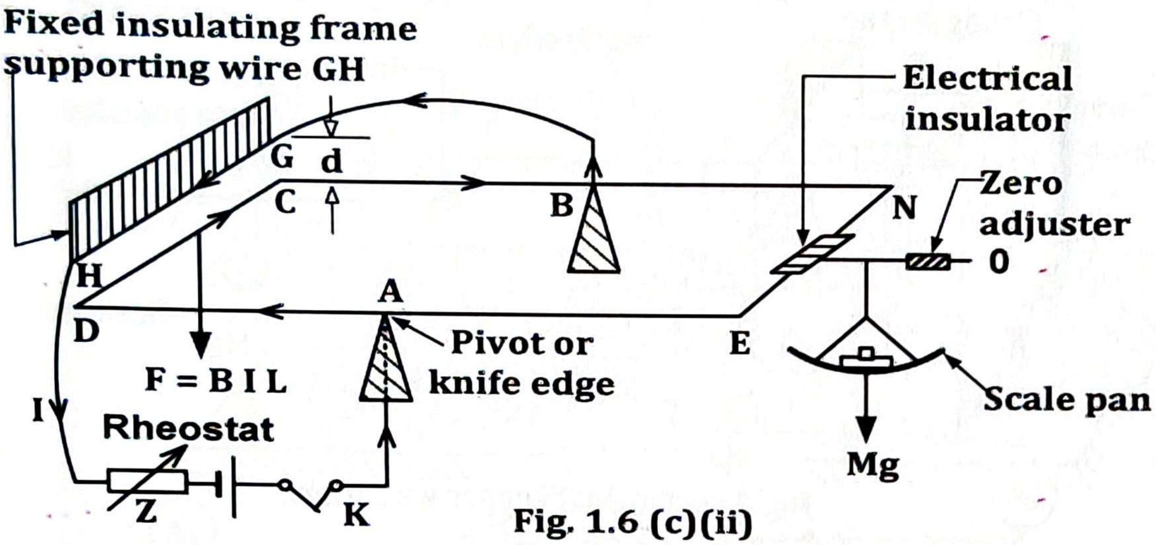


Fig. 1.6 (c)(ii)

- The apparatus is set up as shown on the diagram in figure 1.6(c)
- DCNE is a conducting copper wire frame such that **AD = AE.**
- When switch K is open, i.e. with no current flowing in the circuit, the zero screw (adjuster) is adjusted until the rigid, rectangular copper wire frame CDEN balances horizontally.
- The switch, K, is then closed and the current flows through the two parallel wires **CD** and **GH** in ***opposite directions***, causing wire CD to be repelled vertically downwards.
- Small masses are carefully added into the scale pan until the horizontal wire frame CDEN balances horizontally again.
- The total weight Mg, in the scale pan is noted, and recorded down.
- The length **L** of wire CD is measured using a metre rule and recorded down.
- The distance, **d**, of separation of the wires CD and GH is also measured using a travelling microscope and recorded down.
- Now since practically the distance, **AE = AD**, and **CD =** ***L***
- The value of the current I is calculated from the expression,
  - $I = \sqrt{\frac{2\pi\, mgd}{\mu_0\, L}}$ where, $\mu_0 = 4\pi \times 10^{-7} Hm^{-1}$

NB: Other versions of current balances do exist but use the same concept for their operations.

This knowledge can be used to determine the size of the magnetic flux density **B**, provided by a given source of magnetic field. For example, the magnetic flux density, B, between the pole pieces of a U – shaped magnet can be determined, the magnetic flux density at the centre of a plane circular coil and at the centre of a solenoid can similarly be determined.

5. Magnetic flux density between the pole pieces of a U – shaped magnet

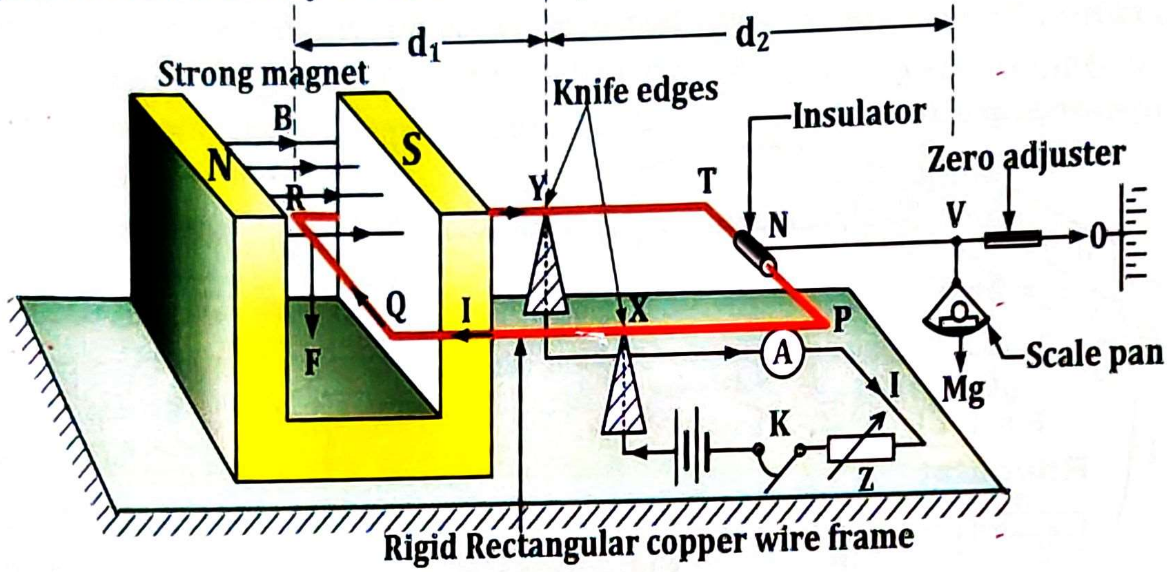


Fig. 1.6 (d)

- The experiment is set up as shown on the diagram in figure 1.6 (d)
- With switch **K** open, the rigid rectangular copper wire frame **PQRT** is made horizontal using the zero adjuster.
- Using a suitable setting of the rheostat **Z**, switch **K** is then closed, such that a current **I** flows through the wire QR normal to the magnetic field, B, in such a direction as to cause wire QR to move vertically downwards. The setting of Z may be altered until wire QR registers a reasonable and measurable downward force, **F = BIL**.
- Small weights are added into the scale pan until the wire frame PQRT balances horizontally.
- The ammeter reading **I** is then noted.
- The total weight, **Mg** in the scale pan is also noted.
- The length **L** = QR of the part of the wire between the pole pieces of the magnet, is measured using a metre rule and recorded down.
- Taking the distances RY = YT = NV as known values provided on the instrument or that can be measured, the magnetic flux density, **B** is calculated from,
- $BIL \times RY = Mg \times (YT + NV)$ *where* $RY = d_1$ *and* $(YT + NV) = d_2$
- $\Rightarrow B = \left(\frac{Mg \times d_2}{d_1 I L}\right)$ *assuming* $d_1 = d_2$, *then*, $B = \left(\frac{Mg}{IL}\right)$

NB: In order to investigate effect of length, L on the force F exerted on the wire carrying a current in a magnetic field, a number of equally strong U – shaped magnets are placed in series so that the varying length, of the wire is between the adjacent pole pieces of the magnets.

6. Measurement of magnetic flux density at the centre of a plane circular coil carrying a current

Diagram

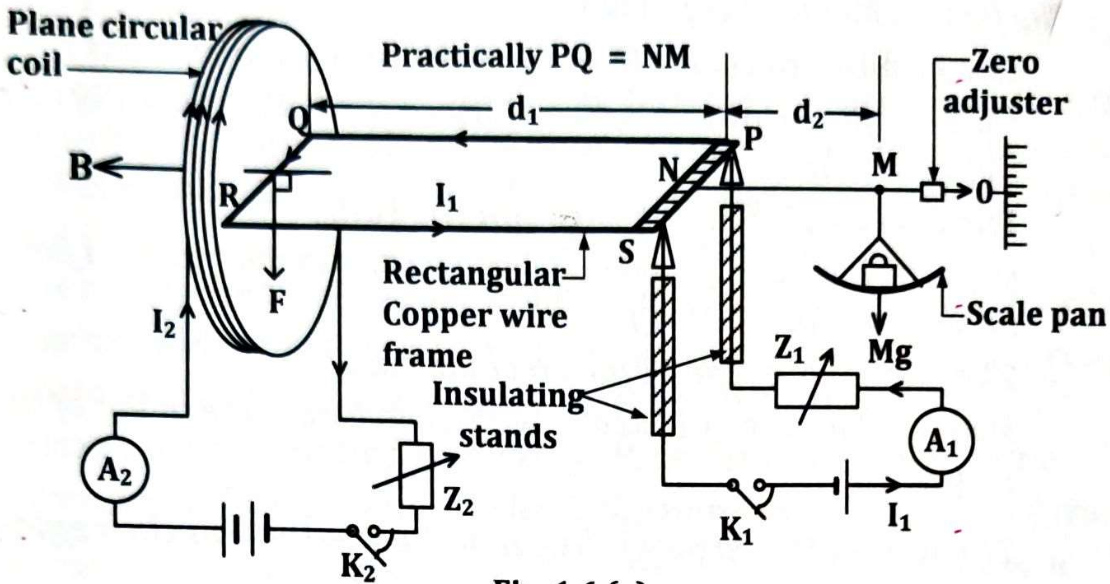


Fig. 1.6 (e)

- The experiment is set up as shown on the diagram in the figure 1.6 (e)
- With switches K_1 and K_2 both open, the rigid rectangular copper wire frame PQRS is made horizontal using the zero adjuster.
- Using suitable settings of the rheostats Z_1 and Z_2, switches K_1 and K_2 are then closed, such that a current I_1 flows through the wire QR normal to the magnetic field, **B**, in such a direction as to cause wire QR to move vertically downwards.
- The setting of Z_1 may be altered until wire QR registers a reasonable downward force.
- Small weights are then added into the scale pan until, the wire frame PQRS again balances horizontally.
- The ammeter reading I_1 is then noted.
- The total weight **Mg** in the scale pan is also noted.
- The length **L** = QR of the part of the wire, at the centre of the coil is measured using a metre rule and recorded down.
- Taking the distances **PQ = NM** as known values provided on the instrument or that are measured using a metre rule, the magnetic flux density, **B** at the centre of the plane circular coil is calculated from,
- $BI_1L \times PQ = Mg \times NM$ *where* $PQ = d_1$ *and* $NM = d_2$

$$\Rightarrow B = \left(\frac{Mg \times d_2}{d_1 I_1 L}\right)$$

NB: The same principles are used for measuring the magnetic flux density at the centre of a long solenoid.

7. **Factors affecting the size of the magnetic flux density at the centre of a plane circular coil**

The factors include the following:

(i) ***The number of turns, N, of the coil carrying a current I.***
Magnetic flux density increases with increase in the number of turns of the coil. i.e. $B \propto N$

(ii) ***The current, I, flowing through the coil.***
Magnetic flux density increases with increase in the current, I flowing through the coil. i.e. $B \propto I$

(iii) ***The radius r, of the plane circular coil.***
Magnetic flux density increases with a decrease in the radius of the coil. i.e. $B \propto \frac{1}{r}$

(iv) ***The magnetic permeability, μ of the medium in the region of the coil.***
Magnetic flux density increases with increase in the strength of the magnetic permeability threading the plane of the coil. i.e. $B \propto \mu$

8. **Variation of the factors affecting Magnetic flux density at the Centre of a plane circular coil.**

(a) ***Variation of Magnetic flux density at the centre of a plane circular coil, with the number of turns, N of the coil.***

- A plane circular coil of known number of turns N is connected to a d.c source via a rheostat Z_2 and an ammeter, A_2.
- When switches K_1 and K_2 are both open, the plane of the rigid rectangular copper wire frame **PQRS** is made horizontal using a zero adjuster.

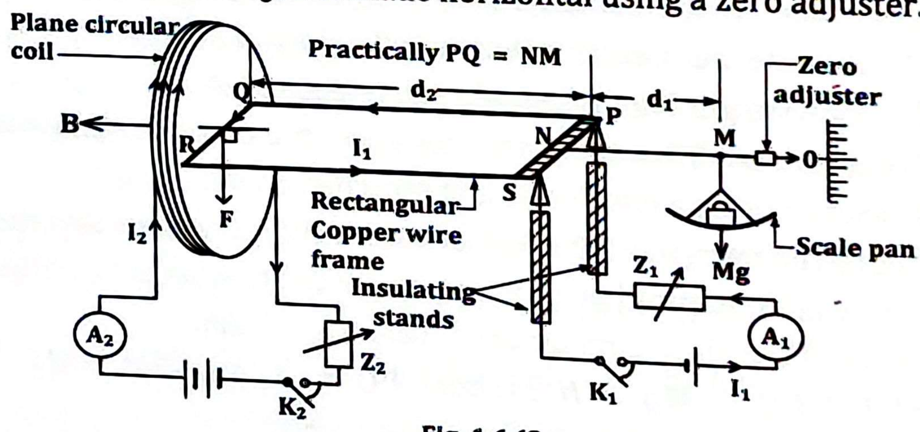


Fig. 1.6 (f)

- Switches K_1 and K_2 are then closed and current I_1 through wire QR is noted on ammeter A_1. Current through the coil may also adjusted using rheostat Z_2 until a reasonable downward deflection on wire QR is registered.
- The ammeter reading I_2 is noted.
- Small weights are added into the scale pan until the wire frame **PQRS** balances horizontally.

- The length **L** of wire QR is measured using a metre rule and the total weight **Mg** in the scale pan is noted. i.e. $B\, I_1 L\, d_2 = Mg\, d_1$, where, $B = \frac{\mu_0 N\, I_2}{2\,r}$

$$B\, I_1 L\, d_2 = Mg\; d_1 \; since, B = \frac{\mu_0 N\, I_2}{2\,r} \Rightarrow N = \frac{2r\; d_1\, Mg}{\mu_0 I_1\, I_2 L\, d_2} \Rightarrow N \propto\; Mg$$

- Keeping values of radius, r, of the coil, currents I_1 and I_2 all constant, the experiment is **repeated** using different increasing number of turns, **N** of different **samples of the coil** of the same material of the wire each time and in each case, the total weight **Mg** in the scale pan is noted.
- The results are tabulated in a suitable table including values of N and Mg.
- A graph of N against Mg is then plotted and gives a straight line through the origin. i.e. $\Rightarrow N \propto\; Mg$ *and since,* $B \propto\; Mg \;\Rightarrow B \propto\; N$

Hence, the magnetic flux density, B, at the centre of a plane circular coil varies directly with the number of turns, N, of the coil.

(i) Variation of number of turns N, with weight, Mg

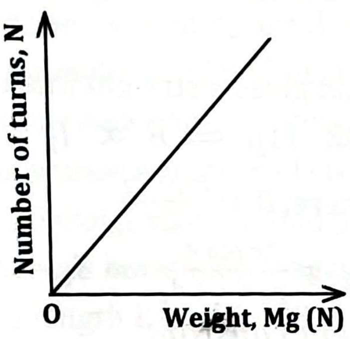


Fig. 1.6 (g)(i)

$\Rightarrow$

(ii) Variation of Magnetic flux density, B with number of turns N

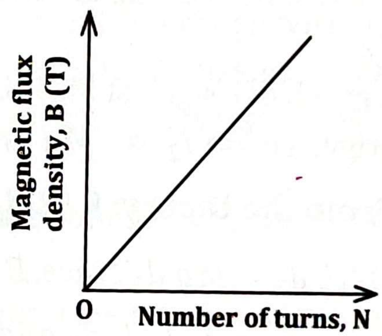


Fig. 1.6 (g)(ii)

$\Rightarrow$ N $\propto$ mg but, assuming, $d_2 = d_1$, $Mg = F = BI_1L \Rightarrow$ mg $\propto$ B

$\therefore B \propto N$ i.e. magnetic flux density varies directly with the number of turns of the coil.

(b) ***Variation of Magnetic flux density at the centre of a plane circular coil, with the Current, I, flowing through the coil.***

- A plane circular coil of known number of turns N is connected to a d.c source via a rheostat Z_2 and an ammeter, A_2 as shown in the figure 1.6 (h)

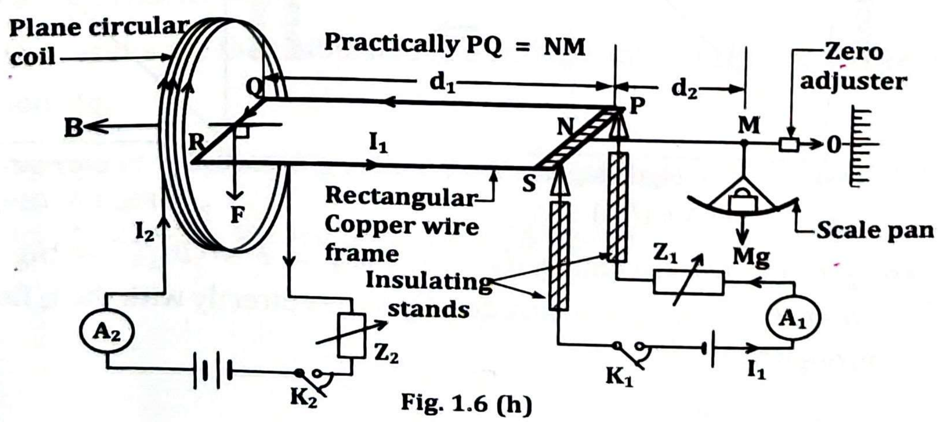


Fig. 1.6 (h)

- When switches K_1 and K_2 are both open, the plane of the rigid rectangular copper wire frame PQRS is made horizontal using the zero adjuster.
- Switches K_1 and K_2 are then closed and current I_1 through wire QR is noted on ammeter A_1.
- The current, I_2 through the coil is also adjusted using rheostat Z_2 until a reasonable downward deflection on wire QR is registered.
- The ammeter reading I_2 is noted.
- Small weights are then added into the scale pan until the wire frame **PQRS** again balances horizontally.
- The length **L** of wire QR is measured, using a metre rule and the total weight **Mg** in the scale pan is noted.
- Keeping values of currents I_1, r, d_1, d_2, L and N constant, the experiment is **repeated,** using different increasing values of current, I_2 flowing through the coil each time and in each case, adjusted using rheostat, Z_2, the total weight Mg in the scale pan is noted each time.
- The results are then tabulated in a suitable table including values of I_2 and Mg.
- A graph of I_2 against Mg is then plotted and gives a straight line through the origin. i.e. $\Rightarrow I_2 \propto Mg$ *and since,* $B \propto Mg \Rightarrow B \propto I_2$

From the theory: $B\, I_1 L\, d_2 = Mg\, d_1$, where, $B = \frac{\mu_0 N I_2}{2r}$

$B\, I_1 L\, d_2 = Mg\, d_1$ *since,* $B = \frac{\mu_0 N I_2}{2r} \Rightarrow I_2 = \frac{2r\, Mg\, d_1}{\mu_0 N I_1 L\, d_2} \Rightarrow I_2 \propto Mg$

Since, μ_0, r, N, I_1, L, d_1 *and* d_2 *are all kept constant.*

Since, $I_2 \propto Mg$ *and* $B \propto Mg \Rightarrow B \propto I_2$

Hence, the magnetic flux density, B, at the centre of a plane circular coil varies directly with the number of turns, N, of the coil

(i) Variation of current, I_2 flowing in the coil with weight, Mg.

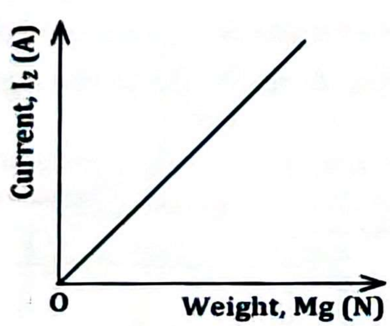


Fig. 1.6 (*i*)(i)

⇒

(ii) Variation of Magnetic flux density, B with I_2 flowing in the coil.

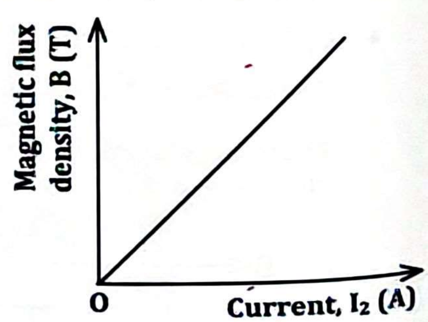


Fig. 1.6 (*i*)(ii)

$\Rightarrow I_2 \propto$ mg but, assuming, $d_2 = d_1$, $Mg = F = BI_1L \Rightarrow$ mg $\propto B$
$\therefore B \propto I_2$ i.e. magnetic flux density varies directly with the I_2 flowing through the coil.

(c) ***Variation of Magnetic flux density, B, at the centre of a plane circular coil, with the radius, r , of the coil***

- A plane circular coil of known number of turns **N**, is connected to a d.c source via a rheostat Z_2 and an ammeter, A_2 as shown in figure 1.6 (j)

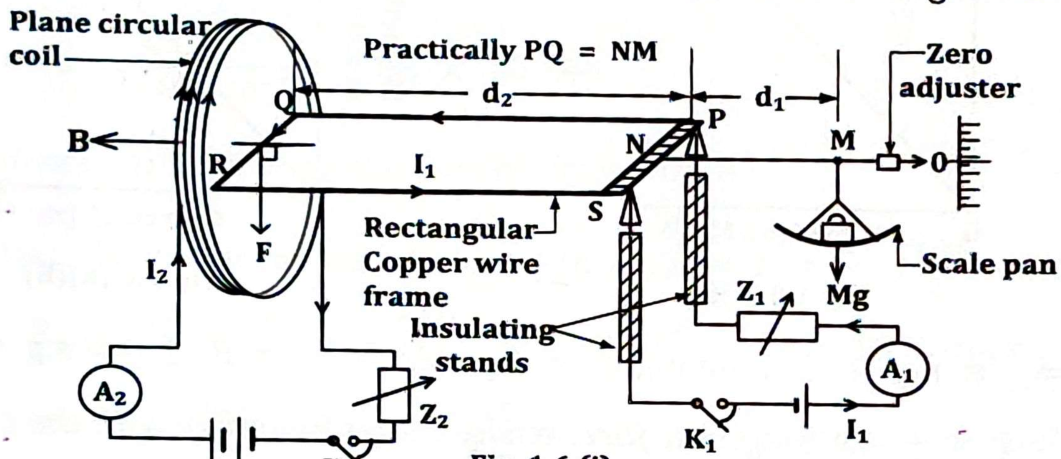


Fig. 1.6 (j)

- When switches K_1 and K_2 are both open, the plane of the rigid rectangular copper wire frame **PQRS** is made horizontal using the zero adjuster.
- Switches K_1 and K_2 are then closed and current I_1 through wire QR is noted on ammeter A_1. Current through the coil is also adjusted using rheostat Z_2 until a reasonable downward deflection on wire QR is registered.
- The ammeter reading I_2 is noted.
- Small weights are added into the scale pan until the wire frame **PQRS** balances horizontally again.
- The length **L** of wire QR is measured, using a metre rule and the total weight **Mg** in the scale pan is noted. i.e $B\,I_1 L\, d_2 = Mg\, d_1$, where, $B = \frac{\mu_0 N I_2}{2r}$
- $B\,I_1 L\, d_2 = Mg\, d_1$ *since*, $B = \frac{\mu_0 N I_2}{2r} \Rightarrow \frac{1}{r} = \frac{2r\, Mg\, d_1}{\mu_0 N I_2 I_1 L\, d_2} \Rightarrow \frac{1}{r} \propto Mg$
- Keeping values of I_1, I_2, d_1, d_2, L and N all constant, the experiment is **repeated,** using different samples of the coils of **the same wire** but of different **increasing radii, r,** of the coil each time and in each case, the sample coil is connected to the above circuit in figure 1.6(J).
- The total weight Mg in the scale pan is noted when the wire frame PQRS balances horizontally.
- The results are then tabulated in a suitable table including values of, **r,** $\frac{1}{r}$ and Mg.
- A graph of $\frac{1}{r}$ against Mg is then plotted and gives a straight line through the origin. i.e. $\Rightarrow \frac{1}{r} \propto Mg$ *and since*, $B \propto Mg \Rightarrow B \propto \frac{1}{r}$

(i) Variation of $\frac{1}{r}$ with weight, Mg.

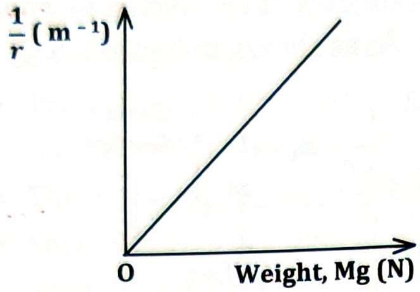


Fig. 1.6 (k)(i)

⇒

(ii) Variation of Magnetic flux density, B with $\frac{1}{r}$

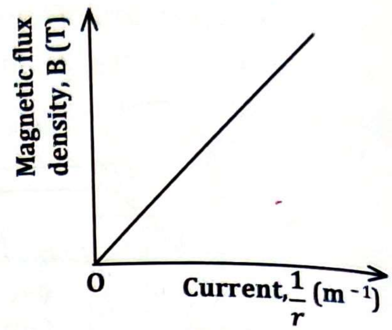


Fig. 1.6 (k)(ii)

$\Rightarrow \frac{1}{r} \propto$ mg but, assuming, $d_2 = d_1$, $Mg = F = BI_1L \Rightarrow mg \propto \frac{1}{r}$

$\therefore B \propto \frac{1}{r}$ i.e. ***magnetic flux density varies inversely with the radius,*** r of the coil.

1.7 EXAMPLES & EXERCISES ON CURRENT CARRYING CONDUCTORS

1. (a) Write down the expressions for the magnetic flux density due to each one of the following current – carrying conductors placed in air.
   - *(i) At a perpendicular distance,* ***d****, due to a straight wire carrying a current I, in air.*
   - *(ii) A plane circular coil of N – turns and of radius R, each carrying a current I.*
   - *(iii) A long solenoid, of n – turns per metre carrying a current I.*

   (b) Two straight and parallel wires W_1 and W_2 each of length 0.25 m are carrying currents of 2A and 5A respectively in the same direction and are separated by a distance of 10.0 cm in air. Determine the;

   - *(i) Position from W_1 for which the resultant magnetic flux density is zero.*
   - *(ii) Magnetic flux density mid-way between the wires.*
   - *(iii) Magnetic force exerted by W_1 on W_2*

Solutions:

(a) (i) $B = \frac{\mu_0 I}{2\pi d}$ (ii) $B = \frac{\mu_0 NI}{2R}$ (iii) $B = \mu_0 nI$

(b) (i)

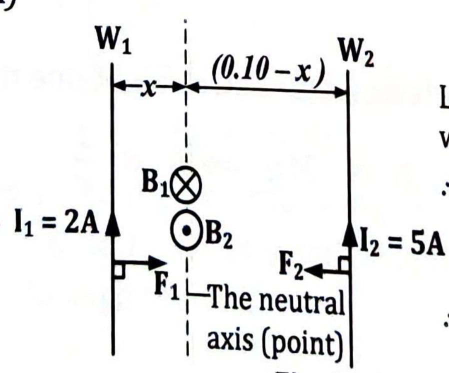


Fig. 1.7 (a)

Let the neutral axis be a distance, x from wire W_1,

$\therefore B_1 = \frac{\mu_0 I_1}{2\pi x}$ *into the paper while*

$B_2 = \frac{\mu_0 I_2}{2\pi(0.10-x)}$ *out of the paper*

$\therefore B = (B_1 - B_2) = \frac{\mu_0 I_1}{2\pi x} - \frac{\mu_0 I_2}{2\pi(0.10-x)}$

$$B = \frac{\mu_0}{2\pi}\left(\frac{I_1}{x} - \frac{I_2}{(0.10-x)}\right) = \frac{4\pi\times10^{-7}}{2\pi}\left[\frac{2}{x} - \frac{5}{(0.10-x)}\right] \text{ but } B = 0$$

$$\frac{4\pi\times10^{-7}}{2\pi}\left(\frac{2}{x} - \frac{5}{(0.10-x)}\right) = 0 \Rightarrow \frac{2\times10^{-7}}{x}\left[\frac{2(0.10-x)-5x}{(0.10-x)}\right] = 0$$

$$\Rightarrow x = \frac{4.0\times10^{-8}}{14.0\times10^{-7}} = 2.86\times10^{-2}m$$

Hence, the neutral point is 2.86 cm from wire W_1

(ii) Mid – way between the wires, $x = (d-x) \Rightarrow x = \frac{d}{2} = \frac{0.10}{2} = 0.05m$

$$\therefore B_1 = \frac{\mu_0 I_1}{2\pi x} = \frac{4\pi\times10^{-7}\times2.0}{2\pi\times(0.05)} = 8.0\times10^{-6}T,\ \textit{into the paper}$$

and $\therefore B_2 = \frac{\mu_0 I_2}{2\pi x} = \frac{4\pi\times10^{-7}\times5.0}{2\pi\times(0.05)} = 2.0\times10^{-5}T, \textit{out of the paper}$

Hence, the magnetic flux density mid-way the wires W_1 and W_2 is

$\therefore B = (B_2 - B_1) = (2.0\times10^{-5} - 8.0\times10^{-6}) = 1.20\times10^{-5}T$ *acting perpendicularly out of the plane of the paper.*

(iii) Let $F_1 = B_2 I_1 L = \frac{\mu_0 I_1 I_2 L}{2\pi d} = \frac{4\pi\times10^{-7}\times2.0\times5.0\times0.25}{2\pi\times(0.10)}$

$\therefore F_1 = 5.0\times10^{-6}N$ *acting from W_1 towards W_2.*

The figure 1.7 (b) shows two wires **AB** and **CD** each of length 5.0 cm and each carrying of 10.0 A in the directions shown. A long conductor carrying a current of 15.0A is placed parallel to the wire **CD**, 2.0 cm below it.

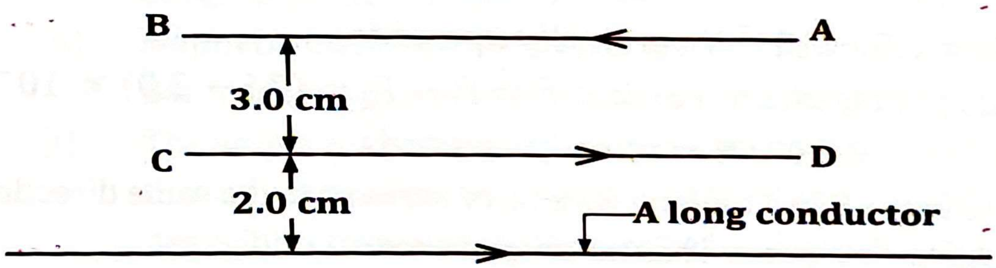


Fig. 1.7 (b)(i)

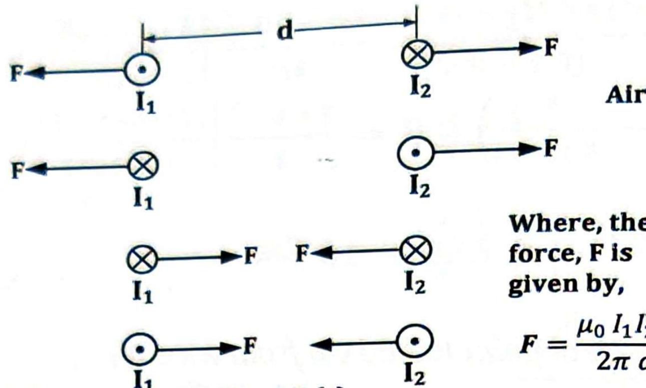
(i) Calculate the net force on the long wire.

(ii) Sketch the magnetic field pattern between the long wire and wire **CD** after removing wire **AB**. Use the field pattern to define a neutral point.

Solution:

NB: Two parallel wires carrying currents in opposite directions exert repulsive forces on each other, but of equal magnitudes.

Two parallel wires carrying currents in same directions exert attractive forces on each other, but of equal magnitudes as summarized by the diagram in figure 1.7 (c)



Where, the force, F is given by,

$$F = \frac{\mu_0 I_1 I_2 L}{2\pi d}$$

Fig. 1.7 (c)

(i)

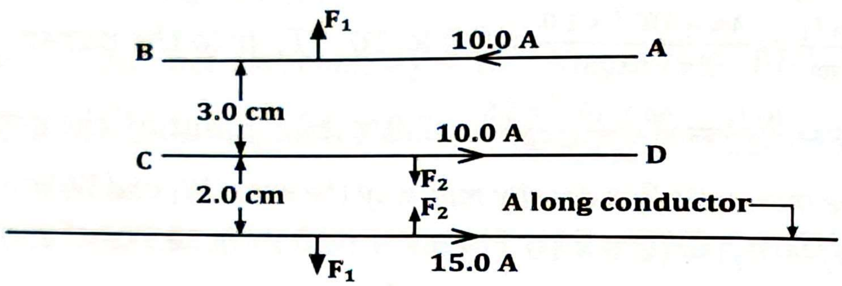


Fig. 1.7 (b)(ii)

Let F_1 be the force on the long wire due to current flowing in the wire AB
Let F_2 be the force on the long wire due to current flowing in the wire CD

$$F_1 = \frac{\mu_0 I_1 I_3 L}{2\pi d} = \frac{4\pi \times 10^{-7} \times 10.0 \times 15.0 \times 0.05}{2\pi \times (0.05)}$$

$\therefore F_1 = 3.0 \times 10^{-5}\, N$ *vertically downwards.*

$$F_2 = \frac{\mu_0 I_2 I_3 L}{2\pi d} = \frac{4\pi \times 10^{-7} \times 10.0 \times 15.0 \times 0.05}{2\pi \times (0.02)}$$

$\therefore F_1 = 7.5 \times 10^{-5}\, N$ *vertically upwards*

Thus the resultant force, $\therefore F = F_1 - F_2 = (7.5 - 3.0) \times 10^{-5}$

$\therefore F = 4.5 \times 10^{-5}\, N$ *vertically upwards*

(ii) Since wire CD and the long wire carry currents in the same direction, the magnetic field pattern has the shape below.

NB: Since the long wire carries a much bigger current than wire CD, the long wire produces more magnetic field lines than wire CD, and the neutral point lies closer to wire CD than the long wire.

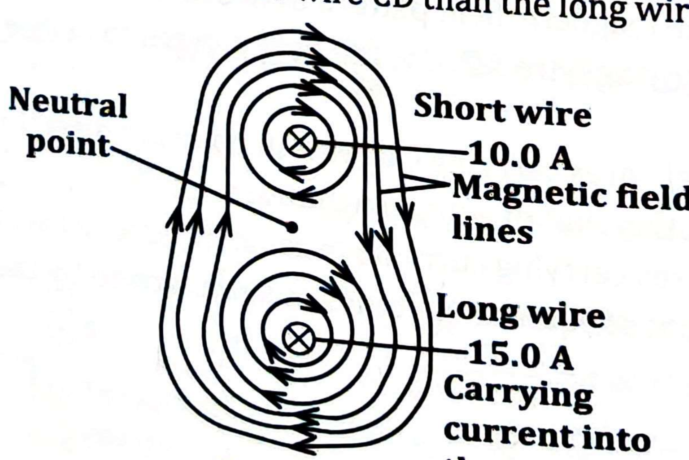


Fig. 1.7 (d)

The neutral point – is the region between the two wires where two magnetic fields are equal in magnitudes but opposite in directions and the resultant magnetic field is zero.

3. (a) Define the following terms:

(i) Magnetic variance.

(ii) Magnetic meridian

(b) (i) Define an ampere.

(ii) Three parallel wires P, Q and R each of length 0.8 m carrying currents of 3.0 A, 6.0A and 8.0 A respectively, are arranged at the corners of a triangle PQR of sides 5.0 cm, 12.0 cm and 13.0 cm as shown in the figure 1.7 (e)

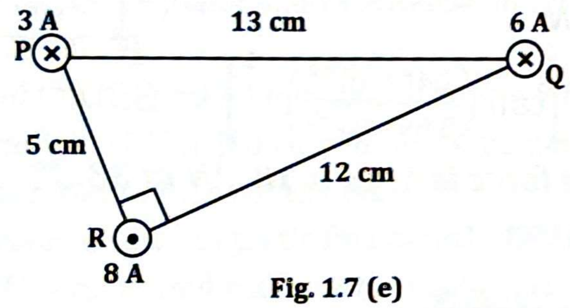


Fig. 1.7 (e)

Determine the resultant force experienced by wire Q.

(c) Describe an experiment to measure current accurately.

Solutions

(a) (i) ***Magnetic variance*** – is the angle between the Earth's Magnetic and Geographic meridians.

(ii) ***Magnetic meridian*** – is a vertical plane in which a freely suspended Bar magnet sets and contains the earth's magnetic poles.

(b) (i) The **ampere** – is the steady or constant current which when flowing through each of the two straight, parallel and infinitely long wires of negligible cross-sectional area separated by a distance of 1m apart in a vacuum exert a force of $2 \times 10^{-7} Nm^{-1}$ on each wire.

(ii) Let F_P and F_R be the magnetic forces on the charge at Q due to charges at points P and R respectively.

NB: The triangle of charges is a right angled Δ.

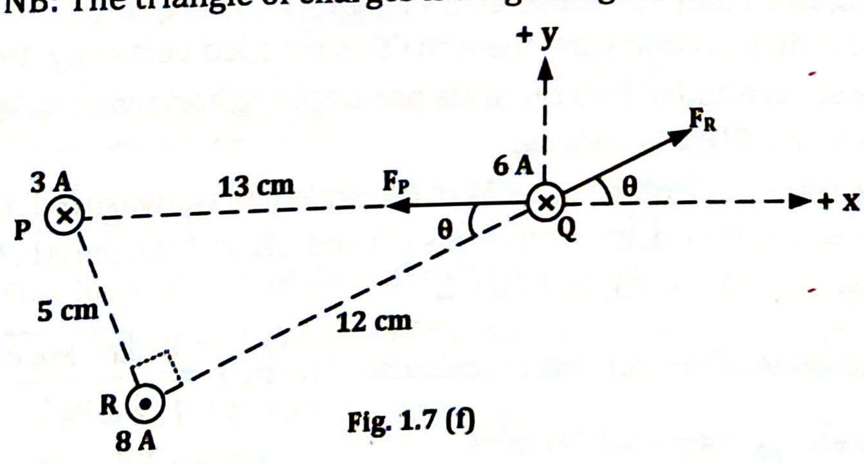


Fig. 1.7 (f)

$\sin\theta = \frac{5}{13}$ or $\cos\theta = \frac{12}{13}$ or $\tan\theta = \frac{5}{12} \Rightarrow \theta = 22.6°$

$F_P = BI_qL_q = \left(\frac{\mu_0 I_P}{2\pi a}\right) I_q L_q = \frac{4\pi\times10^{-7}\times3\times6\times0.8}{2\pi\times0.13} = 2.22\times10^{-5}N$

$F_R = BI_qL_q = \left(\frac{\mu_0 I_R}{2\pi a}\right) I_q L_q = \frac{4\pi\times10^{-7}\times6\times8\times0.8}{2\pi\times0.12} = 6.40\times10^{-5}N$

$\uparrow F_y = F_R\sin\theta = 6.40\times10^{-5}\times\frac{5}{13} = 2.46\times10^{-5}N$

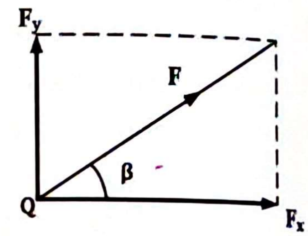
$\rightarrow F_x = F_R\cos\theta - F_p = \left(6.40\times10^{-5}\times\frac{12}{13}\right) - 2.22\times10^{-5}$

$\therefore F_x = \mathbf{3.69\times10^{-5}N}$

Resultant force, $F = \sqrt{F_x^2 + F_y^2}$

$\Rightarrow F = \sqrt{(3.69\times10^{-5})^2 + (2.46\times10^{-5})^2}$

$\therefore\ \mathbf{F = 4.43\times10^{-5}N}$



In the direction $\beta = \left[\tan\left(\frac{2.46\times10^{-5}}{3.69\times10^{-5}}\right)^{-1}\right] = 33.7°$

Hence, the resultant force is $4.43\times10^{-5}N$ at $33.7°$ to the $+X-$ direction.

i) **Diagram of A current Balance.**

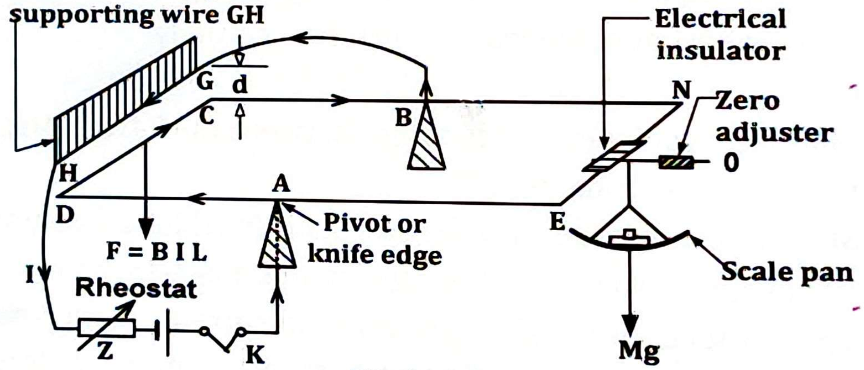


Fig. 1.7 (g)

- The apparatus is set up as shown up as shown on the diagram in the figure 1.7 (g).
- DCNE is a conducting wire frame such that **AD = AE.** With no current flowing, i.e. when switch K is open, the zero screw (adjuster) is adjusted until the frame CDEN balances horizontally.
- The switch is closed and the arm CD is repelled vertically downwards.
- Masses are added into the scale pan until the horizontal balance position of the frame CDEF is restored.
- The total value of the mass M in the scale pan is measured, together with the separation, **d**, between arms CD and GH and the length, **L** of wire CD.
- Now since **AE = AD**, and CD = L
- The value of the current I is calculated from, $I = \sqrt{\frac{2\pi Mg\,d}{\mu_0 L}}$

Where, $\mu_0 = 4\pi\times10^{-7}\ H\ m^{-1}$

(a) Two parallel wires P and Q each of length 0.20 m carry currents of 10A and 1A respectively in opposite directions as shown on the diagram in the figure 1.7 (h)(i)

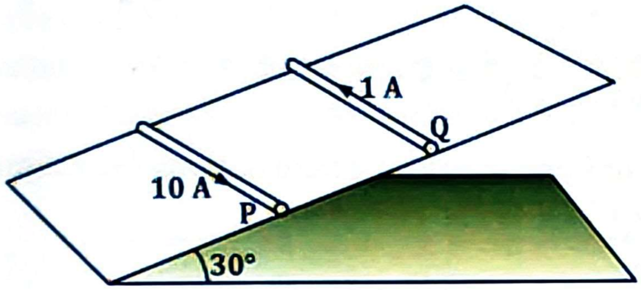


Fig. 1.7 (h)(i)

The distance between the wires is 0.04 m. if both wires remain stationary and the angle the plane makes with the horizontal is 30°, Calculate the weight of wire Q.

(b) A wire of thickness 0.34 mm and of length 7.85 m is wound into a circular coil of radius 0.05 m. If a current of 2A passes through the coil, find the;

(i) number of turns of the oil.

(ii) value of the magnetic flux density at the centre of the coil.

(c) A coil of 50 turns and radius 4 cm is placed with its plane in the earth's magnetic meridian. A compass needle is placed at the centre of the coil. When a current of 0.1 A passes through the coil, the compass needle deflects through 40°. When the current is reversed, the needle deflects through 43° in the opposite direction.

(i) Calculate the horizontal component of the earth's magnetic flux density.

(ii) Calculate the earth's resultant magnetic flux density at the location where the angle of dip is 15°.

Solutions

(a)

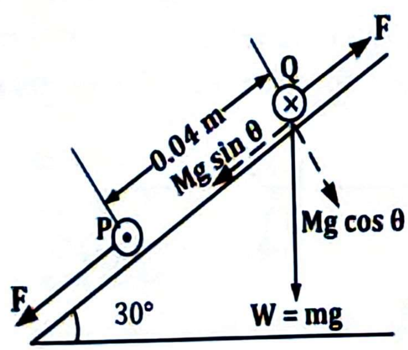


Fig. 1.7 (h)(ii)

Assuming wire P is fixed, The, repulsive magnetic force F = BIL just prevents the weight component Mg sin θ from moving it downwards. i.e.

$$W \sin\theta = \frac{\mu_0 I_P I_Q L}{2\pi d}$$

$$\therefore W = \frac{\mu_0 I_P I_Q L}{2\pi d \sin\theta}$$

$$\therefore W = \frac{4\pi\times10^{-7}\times10\times1\times0.20}{2\pi\times0.04}$$

$$\therefore W = 1.0\times10^{-5}\ N$$

(b) (i) One loop or turn of the coil = circumference of a circle = 2πR

$i.e.\ (2\pi R)N = L$ (Length of the wire) $\therefore N = \frac{L}{(2\pi R)} = \frac{7.85}{2\pi\times0.04}$

$\Rightarrow N = 31.2\ turns$

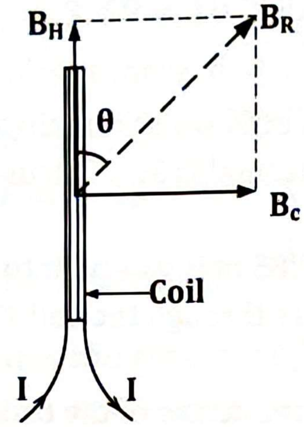
$\therefore$ **N ≅ 31 turns.**

(ii) Assuming the turns of the coil touch each other, it implies 31 turns × the thickness, d, of each wire = total length, l, of the coil.

$\therefore N\,d = l$ and using $B = \mu_0 n I = \frac{\mu_0 N I}{l} = \frac{\mu_0 N I}{N d} = \frac{\mu_0 I}{d}$

$$\therefore B = \frac{\mu_0 I}{d} = \frac{4\pi\times10^{-7}\times 2.0}{0.34\times10^{-3}} = 7.39\times10^{-3}\ \mathrm{T}$$

(c) Horizontal component of Earth's magnetic field is along the magnetic meridian.



$\theta_1 = 40°, \theta_2 = 43° \therefore \theta = \left(\frac{\theta_1+\theta_2}{2}\right) = \left(\frac{40°+43°}{2}\right) = 41.5°$

Thus, $\tan 41.5° = \frac{B_c}{B_H} = \frac{\mu_0 NI}{2r B_H} \Rightarrow B_H = \frac{\mu_0 NI}{2r \tan 41.5°}$

$$B_H = \frac{4\pi\times10^{-7}\times 50\times 0.1}{2\times 0.04\times \tan 41.5°} = 8.88\times10^{-5}T$$

$\therefore B_H = 8.88\times10^{-5}\ T$

Fig. 1.7 (i)

(b) At the location where angle of dip, α = 15° ⟹ $\cos 15° = \frac{B_H}{B}$

$$\therefore B = \frac{B_H}{\cos 15°} = \frac{8.88\times10^{-5}}{\cos 15°} = 9.19\times10^{-5}\ T$$

5. (a) (i) Define the ***Magnetic flux density.***

(ii) Write does the expression for the force on a conductor of length **L** carrying a current **I** when it is inclined at an angle **θ** to a uniform magnetic field of flux density **B.** Use the expression, above to derive an expression for the force experienced by one electron of charge, e, and moving with an average drift velocity, v.

(b) The figure 1.6 (u) represents a simple current balance. When switch, K is open the force required to balance the magnet is 0.20 N. When switch, K is closed and a current of 0.50A flows, a force of 0.22 N is required for balance.

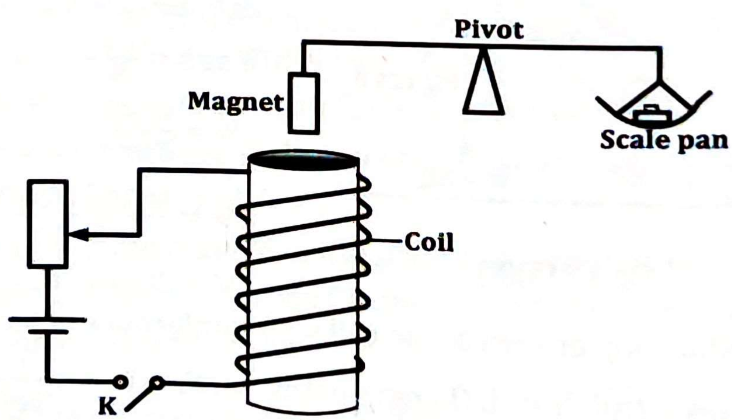


Fig. 1.7 (j)

(i) Determine the polarity at the end of the magnet closest to the coil.

(ii) Calculate the weight required for the balance when a current 2A flows through the coil.

(c) In the diagram in figure 1.7 (k)(i), a copper rod PQ of length 12.0 cm is guided between a pair of parallel vertical metal rails with perfect electrical contact. PQ is directly above wire WX fixed on a flat horizontal table and parallel to PQ.

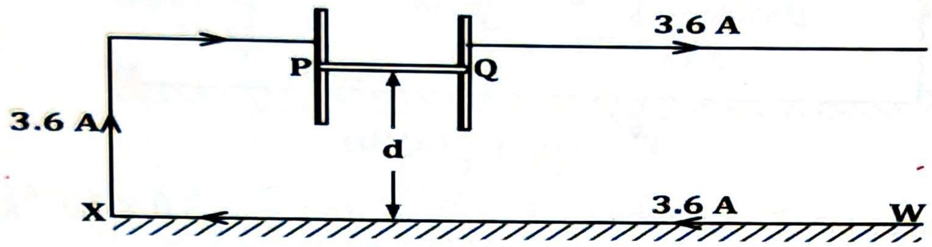


Fig. 1.7 (k)(i)

Given that wire PQ has a mass per cm of 3.0 mg cm^{-1} and carries a current of 3.6 A.

Determine the distance, d, between the wires PQ and WX for which PQ remains stationary at equilibrium.

Solution

(a) (i) **Magnetic flux density** – is the force exerted of a 1 m long conductor carrying a current of 1 A in a direction normal to the field.

(ii) **F = BIL sin θ**

Current flowing $I = \frac{Ne}{t}$

$\therefore F = BNe\frac{l}{t}\sin\theta$

$\therefore F = B\,Nev\sin\theta$ but $\frac{l}{t} = v$

Force on one electron $F_1 = \frac{F}{N}$

$F_1 = Bev\sin\theta$

(b) (i) When switch K is closed, a current flows through the solenoid, in a clockwise direction while observing at the top.
This makes the top end of the coil a **South pole**, but since there is an increase in the size of the force required to establish horizontal equilibrium, it implies the lower end of the magnet a **north pole** being attracted by the south pole of the top side of the solenoid.

(ii) When K is open, weight of the magnet = weight in the pan = 0.20 N
When K is closed, a current flows through the coil and a magnetic force created is proportional to square of current flowing i.e. $F \propto I^2$

$\Rightarrow F = kI^2 \;\therefore\; 0.20 + kI_1^{\;2} = 0.22$ ………… (i) when, $I_1 = 0.50$ A

$\Rightarrow k = 0.08\ NA^{-2}$ thus when the current becomes, $I_2 = 2.0$ A

$\Rightarrow 0.20 + kI_2^{\;2} = F'$ ……………………… (ii)

$\therefore F' = 0.20 + 0.08 \times 2.0^2 = 0.20 + 0.32 = 0.52\ N$

(c) The current flowing in the two wires WX and PQ in opposite directions causes each wire to experience a repulsive magnetic force. This force acts against the weight of wire PQ. When the two forces equalize, the wire PQ stops rising or falling, thus horizontal equilibrium of PQ is established.

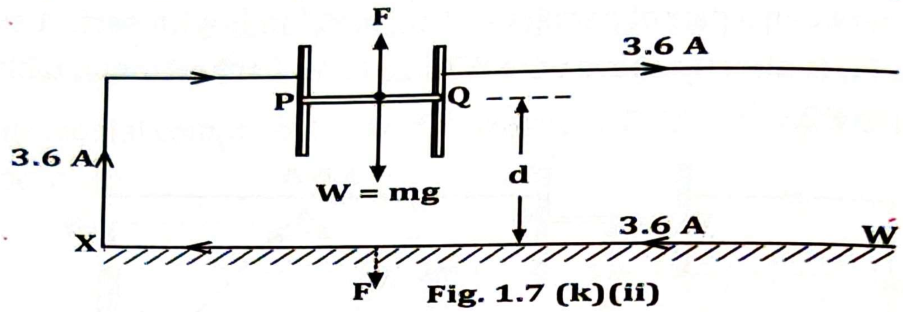


Fig. 1.7 (k)(ii)

At equilibrium, $F = mg$, where, $F = \frac{\mu_0 I^2 L}{2\pi d}$ and $\frac{m}{L} = 3.0 \times 10^{-6} kgcm^{-1}$

$\therefore$ mass of PQ, $m = 3.0 \times 10^{-6} kgcm^{-1} \times 12\ cm = 3.6 \times 10^{-5} kg$

Thus at equilibrium, $\frac{\mu_0 I^2 L}{2\pi d} = mg \Rightarrow d = \frac{\mu_0 I^2 L}{2\pi mg}$

$$d = \frac{\mu_0 I^2 L}{2\pi mg} = \frac{4\pi \times 10^{-7} \times (3.6)^2 \times 0.12}{2\pi \times 3.6\times10^{-5} \times 9.81} = 8.81 \times 10^{-4} m$$

$\therefore$ $\mathbf{d = 8.81 \times 10^{-4} m}$

6. (a) The figure 1.7 (l)(i) shows two parallel wires **P** and **Q** of infinite length Carrying currents of 30 A and 2 A respectively and are separated by a distance of 10.0 cm apart.

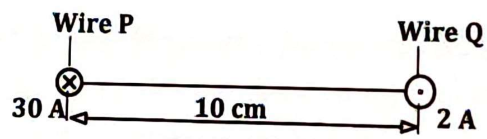


Fig. 1.7 (l)(i)

(i) Determine the resultant magnetic field midway between the wires.

(ii) At what distance from wire q is the resultant magnetic flux density zero?

(b) The figure 1.7 (m)(i) shows a beam of electrons directed into a region of uniform magnetic field of flux density 0.80 T, perpendicularly out of the plane. The electrons enter the magnetic field with a speed of $2.0 \times 10^6 ms^{-1}$ at 45°.Where necessary, use $\left(\frac{e}{m} = 1.76 \times 10^{11}\ C\ kg^{-1}\right)$

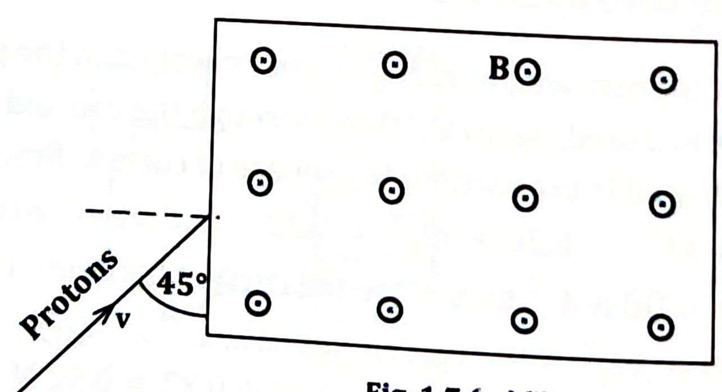


Fig. 1.7 (m)(i)

(i) Explain the motion of the electrons while inside and out of the magnetic field.

(ii) Calculate the radius of the circular path described.

Solutions

(a) (i) Let B_P and B_Q be magnetic flux densities due to wires **P** and **Q** respectively.

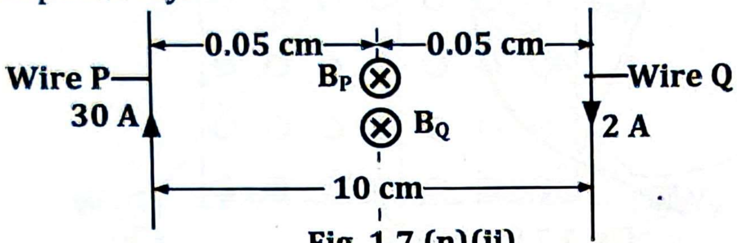


Fig. 1.7 (n)(ii)

$$B_P = \frac{\mu_0 I_P}{2\pi x} \text{ and } B_Q = \frac{\mu_0 I_Q}{2\pi x}$$

Mid-way between the wires, d = 0.05 m and $\mathbf{B} = B_P + B_Q$

$$B = \frac{\mu_0}{2\pi x}(I_P + I_Q) = \frac{4\pi \times 10^{-7}}{2\pi \times 0.05}(30 + 2) = 8.96 \times 10^3 \text{ T}$$

(ii) Let the neutral point be a distance, y, from wire Q to the right.

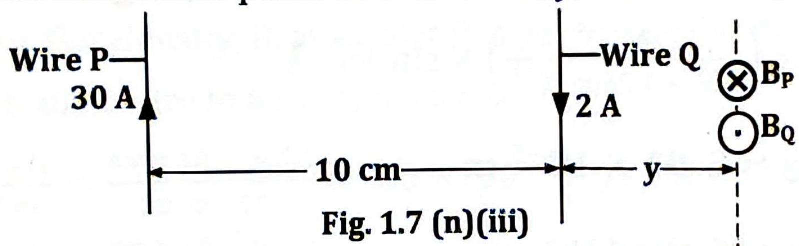


Fig. 1.7 (n)(iii)

$$B_P = \frac{\mu_0 I_P}{2\pi(0.10 + y)} \text{ and } B_q = \frac{\mu_0 I_q}{2\pi y} \text{ since B = 0}$$

$$\Rightarrow B_P - B_q = 0 \text{ or } B_P = B_q$$

$$\therefore \frac{\mu_0 I_P}{2\pi(0.10 + y)} = \frac{\mu_0 I_q}{2\pi y} \Rightarrow \frac{10}{(0.10 + y)} = \frac{2}{y}$$

$$5y = (0.10 + y) \Rightarrow y = 2.5 \times 10^{-2} m$$

(c) (i) When electrons enter the region of uniform magnetic field, by Fleming's left hand rule the electrons (charged particles) experience a magnetic force, $F = Bev$, and as a result of changing velocity, the electrons experience an acceleration that acts towards the centre of a circular path, as provided by the centripetal force, $F = \frac{mv^2}{r}$.

Where, m is the mass of an electron and v is the velocity of the electron within the magnetic field, of flux density, B.

When the electrons get out of the region of uniform magnetic field, the magnetic force ceases to exist and so they continue to move in a straight line along the tangent to the final path of the charged particle while in the region of uniform magnetic field as shown on the diagram in figure 1.6 (x)(ii)

(ii) **Method I**

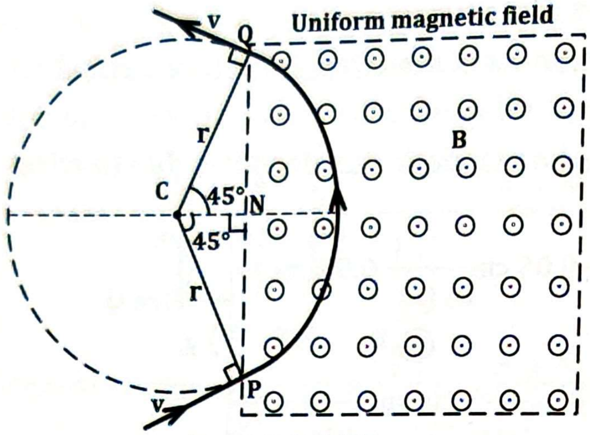


Fig. 1.7 (m)(ii)

In the region of magnetic field, $Bev = \frac{m v^2}{r} \Rightarrow r = \frac{m v}{B e}$ …………… (i)

$PN = r \sin 45°$ *and also* $NQ = r \sin 45° \Rightarrow PQ = 2r \sin 45°$ …. (ii)

∴ substituting (i) into (ii)

$$\Rightarrow PQ = 2r \sin 45° = 2\left(\frac{m v}{B e}\right) \sin 45° = 2\left(\frac{v}{B \times \frac{e}{m}}\right) \sin 45°$$

$$\therefore PQ = 2 \times \left(\frac{2.0 \times 10^6}{0.80 \times 1.76 \times 10^{11}}\right) \times \sin 45°$$

$$Hence, \boldsymbol{PQ = 2.01 \times 10^{-5}\, m}$$

Alternative Method II

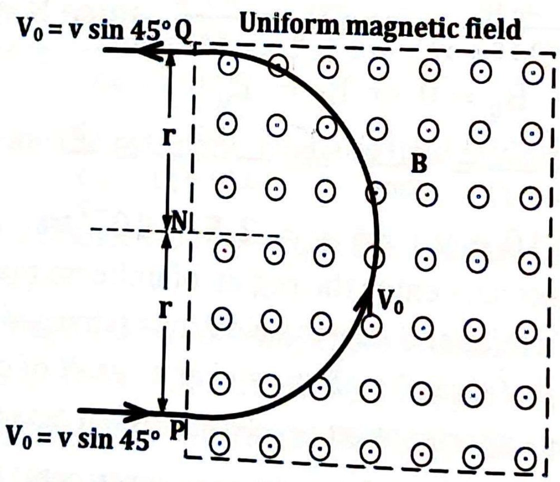


Fig. 1.7 (m)(iii)

Suppose, V_0 is the velocity of the electron normal to the magnetic field at P, Since the electron emerges out of the uniform magnetic field, and is not moving towards P again, it's assume to emerge normal to the magnetic field again at Q, a distance 2r away from P.

$$\Rightarrow BeV_0 = \frac{mV_0^2}{r} \quad but, V_0 = v \sin 45° \Rightarrow Be(v \sin 45°) = \frac{m(v \sin 45°)^2}{r}$$

$$\therefore r = \frac{m v \sin 45°}{B e} = \frac{v \sin 45°}{B\left(\frac{e}{m}\right)} = \left(\frac{2.0 \times 10^6}{0.80 \times 1.76 \times 10^{11}}\right) \times \sin 45° = 1.00 \times 10^{-5}\, m$$

$$\therefore \boldsymbol{PQ} = 2r = 2 \times 1.00 \times 10^{-5} = \boldsymbol{2.00 \times 10^{-5} m}$$

7. Figure 1.7 (n)(i) shows three identical straight and parallel wires W_1, W_2, and W_3 of infinite length arranged along the x – axis and carrying currents of 5A, 2A and 5A respectively as shown.

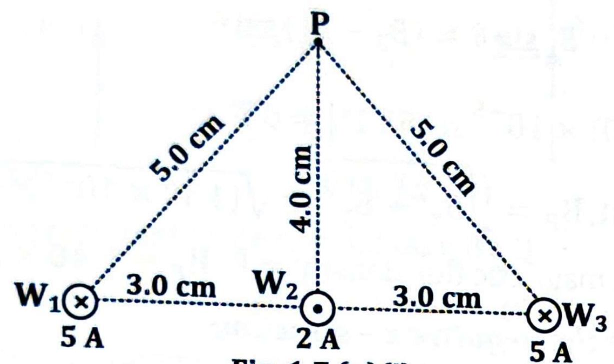


Fig. 1.7 (n)(i)

(i) Determine the resultant magnetic flux density at point **P**.

(ii) A fourth wire of mass 4 mg and of length 0.250 m carrying a current of 4.5A into the plane of the paper is placed at point **P**,
Deduce the acceleration of this wire from the result of (i) above.

Solution

(i) Magnetic flux density, B, at a point D, a perpendicular distance d, from a given straight wire in air is given by, $B = \frac{\mu_0 I}{2\pi d}$

$$B_1 = \frac{\mu_0 I_1}{2\pi d} = \frac{4\pi \times 10^{-7} \times 5.0}{2\pi \times (0.05)} = 2.0 \times 10^{-5} T, \textit{from P towards } W_3$$

$$B_2 = \frac{\mu_0 I_2}{2\pi h} = \frac{4\pi \times 10^{-7} \times 2.0}{2\pi \times (0.04)} = 1.0 \times 10^{-5} T, \textit{to the left of } P$$

$$B_3 = \frac{\mu_0 I_3}{2\pi d} = \frac{4\pi \times 10^{-7} \times 5.0}{2\pi \times (0.05)} = 2.0 \times 10^{-5} T, \textit{away from P along } W_1P$$

Let angle, $PW_1W_2 = \textit{angle } PW_3W_2 = \theta$ where, $\tan\theta = \frac{4}{3}$

$\Rightarrow \theta = 53.1°$ Now resolving magnetic flux density in two perpendicular directions.

NB: The direction of the magnetic flux density due to each current carrying wire at point P is obtained by drawing a tangent the path or loop of the magnetic field line at that point.

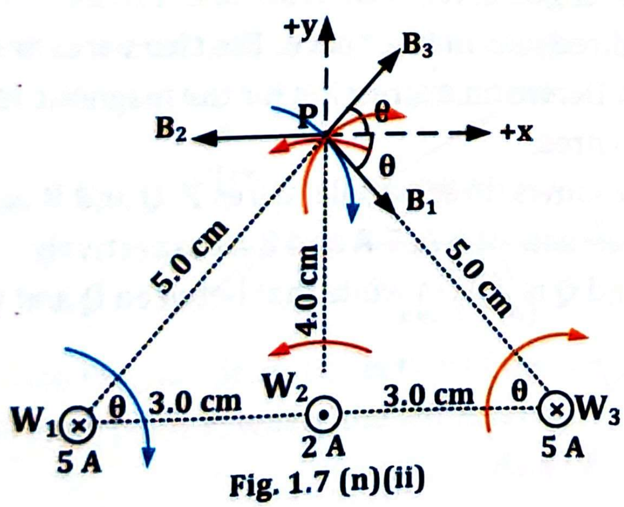


Fig. 1.7 (n)(ii)

$$\sum B_X = B_1 \cos\theta + B_3 \cos\theta - B_2 = [(B_1 + B_3)\cos\theta - B_2]$$

$$B_X = \left[(2.0 + 2.0) \times 10^{-5} \cos 53.1° - 1.0 \times 10^{-5}\right] = 1.40 \times 10^{-5}\, T$$

$$\sum B_y = B_3 \sin\theta - B_1 \sin\theta = (B_3 - B_1)\sin\theta$$

$$B_y = \left[(2.0 - 2.0) \times 10^{-5} \sin 53.1°\right] = 0\, T$$

∴ The resultant, $B_P = \sqrt{B_x{}^2 + B_y{}^2} = \sqrt{(1.40 \times 10^{-5})^2 + 0^2}$

∴ The resultant magnetic flux density at P, $\mathbf{B_P = 1.40 \times 10^{-5}\, T}$

Acting towards the negative x – direction.

(ii) *From Newton's 2^{nd} law, $F = ma$, where $F = BIL$*

$$\Rightarrow a = \frac{F}{m} = \frac{BIL}{m} = \frac{1.40 \times 10^{-5} \times 4.5 \times 0.250}{4.0 \times 10^{-6}} = 3.94\, ms^{-2}$$

∴ **Acceleration of of the wire = $3.94\, ms^{-2}$ *vertically upwards.***

Exercises

1. A long straight wire carries a current of 50.0 A. An electron, of charge 1.6×10^{-19} C travelling at $1.0 \times 10^{7}\, ms^{-1}$ is 5.0 cm from the wire at that instant. Determine the force (both in magnitude and direction) acting on the electron,
   (i) If the electron's velocity is directed towards the wire. **Ans: [3.20×10^{-16}]** parallel to the wire in the same direction with the current.
   (ii) If the electron's velocity is in the same direction as that of the current. **Ans: [3.20×10^{-16}]** Perpendicular to the wire away from the wire.
2. Two long parallel wires X and Y each carries a current of 10 A and are 5 cm apart. Calculate the;
   (i) Magnetic flux density at the position of wire Y due to the current in X. **Ans:[4.00×10^{-5} T]**
   (ii) Force per metre on wire Y. **Ans:[$4.00 \times 10^{-4}\, Nm^{-1}$]**
3. (a) Two parallel wires each of length, **L**, carry currents of the same magnitude, **I**, in opposite directions in free space. The two wires are separated by a distance, **d**. Derive an expression for the magnetic force exerted on any one of the wires.
   (b) The diagram figure 1.7 (a) shows three parallel wires **P**, **Q** and **R** each of length 0.500 m carrying currents of **6 A**, **5 A** and **2 A** respectively. The distance between **P** and **Q** is 2.0 cm while that between **Q** and **R** is 3.0 cm.

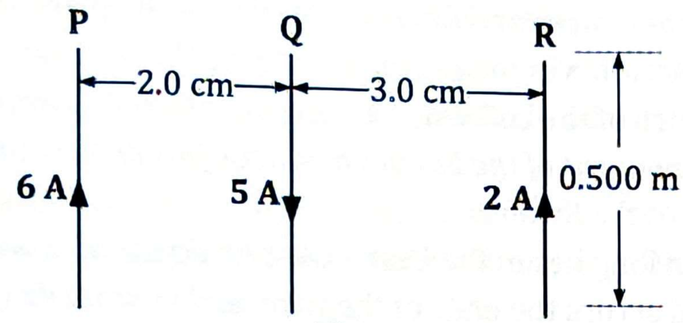


Fig. 1.7 (o)

Calculate the resultant force exerted on wire Q.

Ans:[F = 1.167×10^{-4} N (to the right)

4. Describe how a simple current balance is used to investigate the effect of the number of turns of a coil, on the magnetic flux density at the centre of the same plane circular coil.

5. The figure 1.7 (p) shows four straight infinitely long parallel wires A, B, C, and D carrying currents of 10 A, 4.5 A, 5 A, and 1 A respectively. Wires A, B, and C are fixed wires in a horizontal plane, while D is free to move and is suspended in free space. The separations of the wires are as indicated on the diagram.

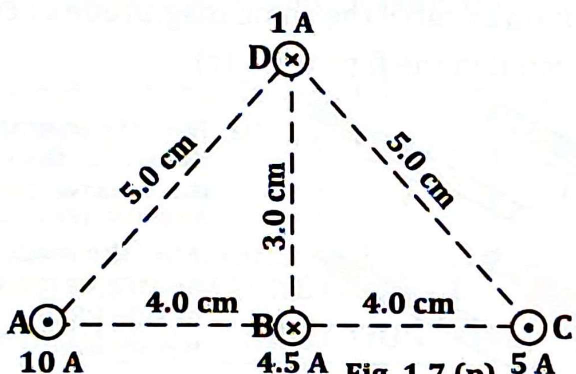


Fig. 1.7 (p)

Determine the resultant force per metre on wire D.

Ans:[F = 2.41×10^{-5} Nm^{-1}, Direction, $\beta = 48.4°$ to the +X - direction]

6. Two long insulated wires lie in the same horizontal plane. A current of 20.0 A flows towards the north inside wire A while a current of 10.0 A flows towards the east in wire B as shown in the figure 1.7 (q)

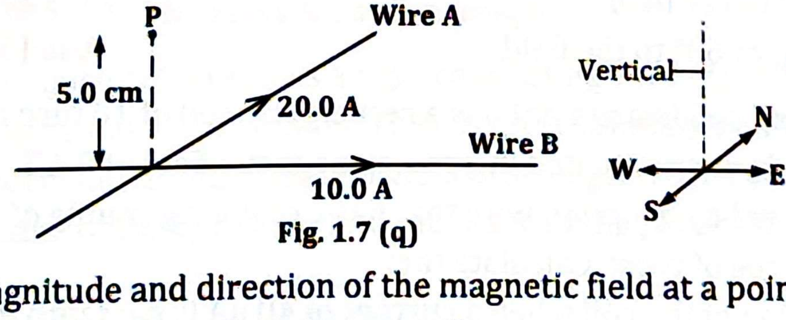


Fig. 1.7 (q)

What is the magnitude and direction of the magnetic field at a point that is 5.00 cm above the point P, where the wires cross?

Ans:[8.94×10^{-5} T, in direction, S26.6°E]

7. A circular coil of **four** turns and diameter 11 cm has its plane vertical and parallel to the magnetic meridian of the Earth. Determine the resultant magnetic flux density at the centre of the coil when a current of 0.35 A flows through it. (*Take the horizontal component of the Earth's magnetic flux density to be* 1.6×10^{-5} T)

Ans: [1.12×10^{3} N]

8. A metal wire 10 m long lies in the East – west direction on a wooden table. What p.d. has to applied across the ends of the wire, and in what direction, in order to just make the wire rise from the surface? Assume electrical connections to the wire make no appreciable restraint. (Density of the metal $= 1.0 \times 10^{4}$ kg m^{-3}, resistivity of the metal $= 2.0 \times 10^{-8}\,\Omega$ m, Horizontal component of the Earth's magnetic field $= 1.8 \times 10^{-5}$ T, Earth's gravitational field strength, $g = 9.81$ N kg^{-1})

Ans: [1.09×10^{3} V]

9. A long solenoid with 3000 turns per metre carries a current of 4.0A. A horizontal wire X 4.0 cm long is in the middle of the solenoid perpendicular to its axis and also carrying a current of 4.0A.
   (i) Determine the force experienced by wire X. **Ans:[2.41×10^{-3} N]**
   (ii) Suggest a design for a current balance based on this principle.

10. Four long and parallel wires are arranged at the corners of a square of side 0.10 m. All the four wires carry the same magnitude of current I = 10.0 A in the directions indicated in the figure 1.7 (r)

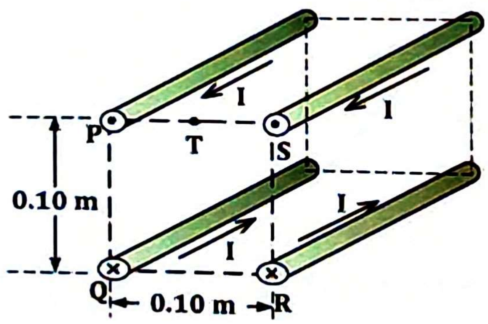


Fig. 1.7 (r)

(i) Find the magnetic flux density at the centre of the square.
Ans: [80 μT to the right.]
(ii) Find the magnetic flux density at the mid point, T of side PS of the square.
Ans: [0.11 mT to the right.]

11. A rectangular coil of 20 turns and dimensions 4 cm by 2 cm is suspended with its plane and longer side vertical in a horizontal field of 2.0×10^{-2} T. If a current of 2A flows in the coil, calculate the torque or moment of the couple initially on the coil when its plane is,
   (i) Parallel to the field. **Ans:[6.40×10^{-4} Nm]**
   (ii) Inclined at 60° to the field. **Ans:[3.20×10^{-4} Nm]**

12. A moving coil instrument has a rectangular coil of 10 turns and dimensions of 5 cm by 2 cm situated in a radial magnetic field of 0.4 T. The coil is suspended by a torsion wire that has a restoring couple of 2.0×10^{-6} Nm per degree of twist. Calculate the;
   (i) Deflection of the coil when a current of 40 μA is pass into it. **Ans:[0.08°]**
   (ii) Sensitivity of the instrument. **Ans:[2.0° mA]**

13. A flat circular coil of wire of 20 turns and of radius 10.0 cm is placed with its plane vertical and at 45° to the magnetic meridian. Assuming that horizontal component of the earth's magnetic flu density = 2.0×10^{-8} T, $\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$.
Calculate the current in the coil, if a compass needle, free to move in a horizontal plane, points in the East – West direction, when placed at the centre of the coil. **Ans:[0.23 A]**

14. A rectangular loop of wire PQRS, carrying a current I_1= 2.0 mA, is next to a very long wire XY carrying a current I_2 = 8.0 A as shown on the diagram in the figure 1.7 (s)

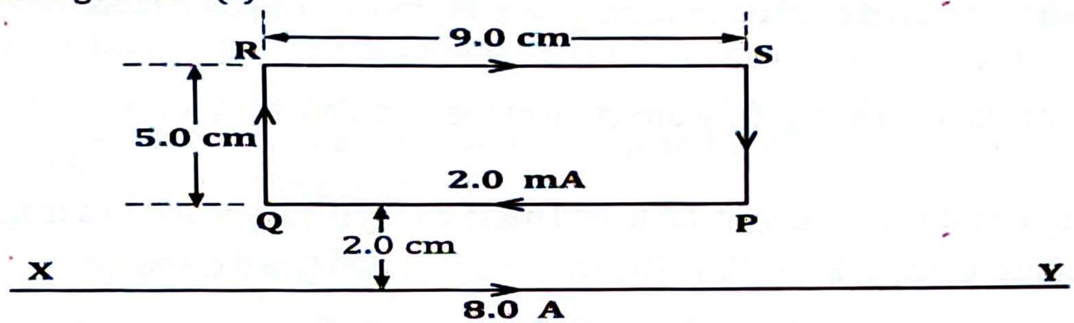


Fig. 1.7 (s)

(a) What is the magnitude and direction of the magnetic force on each of the four sides of the rectangular loop of wire, due to the long wire's magnetic field?

(b) Calculate the net magnetic force on the rectangular loop due to the long wire's magnetic field. [**Hint:** *The long wire does **not** produce a uniform magnetic field.*]

(a) Answers

SIDE	CURRENT DIRECTION	MAGNETIC FIELD DIRECTION	DIRECTION OF THE FORCE
RS	Right	Out of the page	Attracted to the long wire.
PQ	Left	Out of the page	Repelled by the long wire.
QR	Up	Out of the page	To the Right.
SP	Down	Out of the page	To the left.

(b) Ans:[1.00×10^{-8} N] away from the long wire.

15. The diagram in figure 1.7 (t) shows a rigid conducting wire PQ connected to a 6.0 V battery through a 6.0 V, 3.0 W lamp. The circuit is standing on the top pan of a balance. A uniform horizontal magnetic field of strength 50 mT acts at right angles to the plane of the wire loop into the plane of the paper. The balance reads 153.86 g.

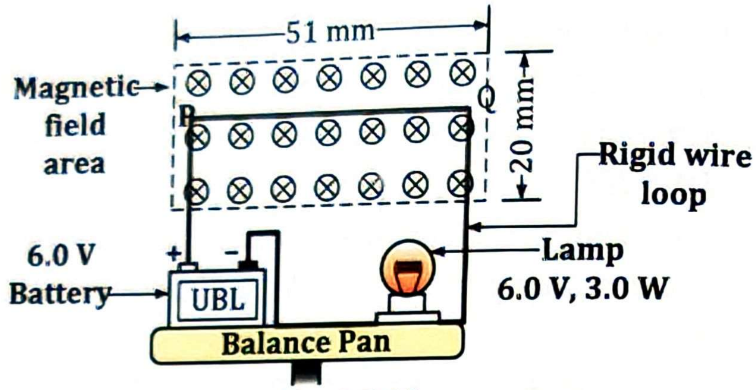


Fig. 1.7 (t)

Calculate the;

(a) Force exerted on the conducting wire PQ by the magnetic field.

Ans: [1.28×10^{-3} N]

New balance reading if the direction of the magnetic field is reversed.

Ans: [1.51 N]

16. A straight stiff wire of length 1.0 m and mass 25.0 g is suspended in a magnetic field of flux density, B = 0.75 T. The wire is connected to a d.c. source.

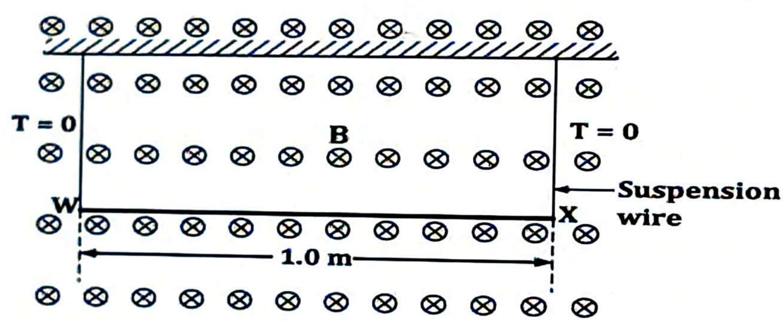


Fig. 1.7 (u)

Determine the magnitude and direction of the current that must flow in the suspension wire so that the tension in the supporting wires is zero.

Ans: [0.327 A]

17. A light power line 125 m long is held horizontally between two pylons and carry a current of 2500 A towards the south. The Earth's magnetic field at that location is 0.52 mT towards the north at an angle of dip of 62°. Determine the magnetic force experienced by the wire.

Ans: [0.140 N due East]

18. A square loop of wire of side 0.60 m carries a current of 9.0 A as shown in the Figure 1.7 (v). When there is no applied magnetic field, the plane of the loop is horizontal and non – conducting nonmagnetic spring of force constant, $k = 550\ Nm^{-1}$ is un-stretched. A Horizontal magnetic field of magnitude 1.3 T is now applied.

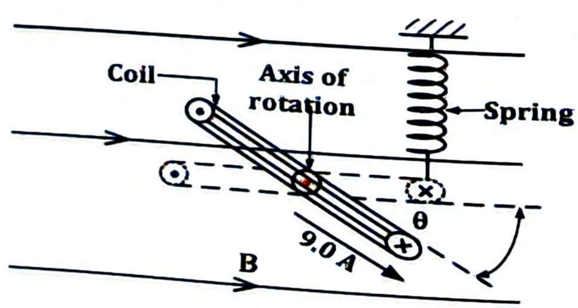


Fig. 1.7 (v)

At what angle θ is wire loop's new equilibrium position? Assuming the spring remains vertical because θ is small.
Ans: [4.9°]

19. An electron revolves in a circular orbit of radius 2.0×10^{-10} m at a frequency of 6.8×10^{15} Hz. Calculate the magnetic flux density at the centre of the coil.
Ans: [2.43×10^{5} T]

20. A wire of length 7.85 m is wound into a circular coil of radius 0.05 m. If a current of 2 A passes through the coil, find the magnetic flux density at the centre of the coil.
Ans: [6.28×10^{-4} T]

21. A current of 3.25 A flow through a long solenoid f 400 turns and length 40.0 cm. Determine the magnitude of the force exerted on a particle of charge 15.0μC moving at 1.0×10^{3} ms^{-1} through the centre of the solenoid at an angle of 11.5° relative to the axis of the solenoid.
Ans: [1.22×10^{-5} N]

22. Figure 1.7 (w) shows two parallel conductors A and B, each carrying a current of 2.0 A into the plane of the paper.

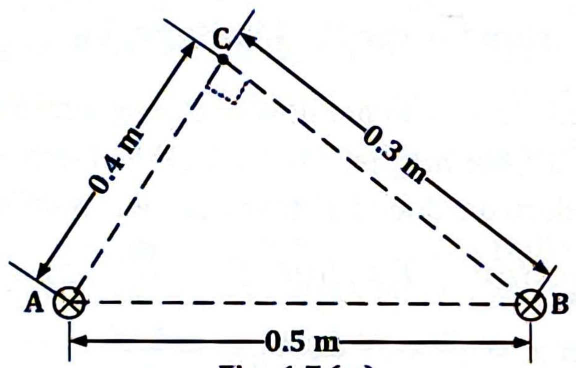


Fig. 1.7 (w)

(i) Find the resultant magnetic flux density at point, C.
Ans: [1.66×10^{-6} T at an angle β = 0.04° below +x – direction]

(ii) Draw the magnetic field pattern due to currents through A and B.
Ans: [See the magnetic field pattern on Fig. 1.2 (g)]

1.8 THE HALL EFFECT (Named after Edwin Herbert hall 1855 – 1938)

Whenever a current carrying conductor is placed or positioned a cross a magnetic field, the majority charge carriers get urged by the field towards one side of the conductor in a direction predicted by Fleming's left hand rule. As a result a large p.d. is set across the sides of the conductor. This p.d. is called the Hall p.d.

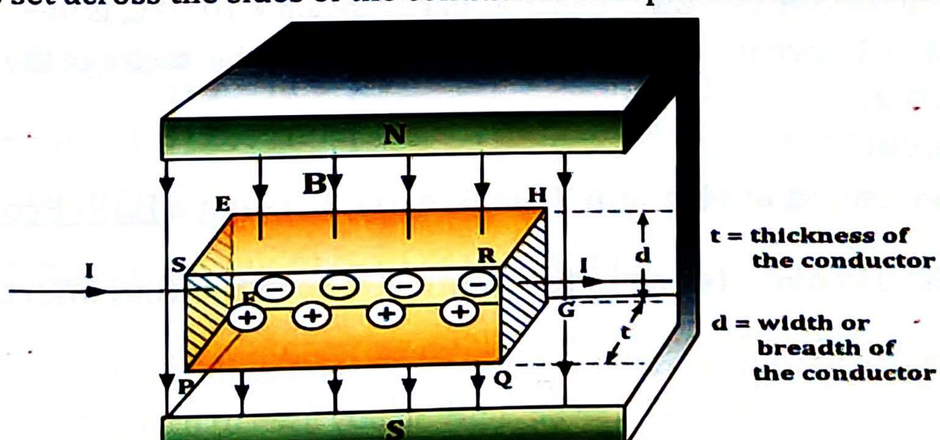


Fig. 1.8 (a)

- When current I and a magnetic field of flux density B are applied to The conducting slab at right angles to each other as shown in the diagram in figure 1.8 (a), the magnetic field urges the majority charge carriers (electrons) towards face EFGH, according to Fleming's left hand rule, leaving an equal magnitude of positive charge on the opposite face PQRS.
- A magnetic force F = Bev, acts on conduction electrons as they drift along the conductor and eventually drift off in the direction from face PQRS to face FGHE.
- As the process continues, face PQRS acquires excess positive charge while face EFGH acquires excess negative charge.
- When maximum separation of charge has occurred so that no more net flow of charge occurs across oppositely charged faces, a large voltage or a p.d develops across the faces PQRS and EFGH.
- This voltage is called a **hall voltage,** V_H and the effect is called the hall effect.

Derivation of the expression for the Hall Voltage, V_H

From the above setup, when there is no net flow of charge across opposite charged faces PQRS and EFGH, the magnetic force on the electrons, equals the electric force on the same electron, due to a strong electric field intensity across the charged faces. i.e. $Bev = Ee$, but $E = \frac{V_H}{d}$, and $v = \frac{I}{nAe}$

$$Bv = E \Rightarrow \frac{BI}{nAe} = \frac{V_H}{d} \quad but\ A = (d \times t) \Rightarrow V_H = \frac{BI\cancel{d}}{n(\cancel{d} \times t)\,e}$$

$$\therefore V_H = \frac{BI}{net}$$ *is the expression for the Hall voltage.*

Example

(a) Describe an experiment to measure magnetic flux density between the pole pieces of a U – shaped magnet.

(b) An electric current of 1.0 A passed through the smallest face of a rectangular metal slab of thickness 1mm, placed with its largest face normal to a uniform magnetic field of 0.80T, causes a p.d of 0.48 mV to be set up across the slab when electron, of charge 1.6×10^{-19} C; is un-deflected. Determine the number of electrons per cubic metre of the conductor.

Solution:

(a) **Measurement of Magnetic Flux Density, B, using a <u>Hall- Probe</u>**

The Hall Probe – Is one of the applications of the Hall effect.

This device has a small wafer of Germanium semiconductor mounted along a narrow handle, so that it can be conveniently used to probe the magnetic field being examined.

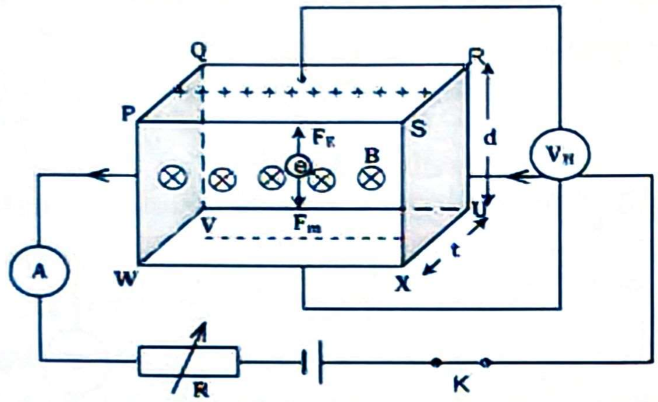


Fig. 1.8 (b)

- A small current I, measured by a suitable ammeter, **A or mill-ammeter, mA** is passed between two opposite faces of the semi-conductor by closing switch **K** and adjusting the rheostat **R** to a suitable value of the current **I.**
- The semiconductor being placed with the largest face **PSXW** perpendicular to a uniform magnetic field of flux density **B** has its electrons experiencing a magnetic force **F_m = Bev** acting down towards the lower face **XUVW** and positive charges left on the upper face **PQRS.**
- The opposite charges continue moving to the opposite faces until no more charges can move across the faces.
- A maximum steady p.d. V_H called the **Hall voltage** is then set up across the opposite charged faces PQRS and UVWX. This p.d. called the **Hall voltage** is then measured using a high impedance voltmeter, **V.**
- At the same time when the p.d. V_H registered by the voltmeter, V is steady and maximum, the ammeter reading, I is noted.
- Using a known value, **"net"** provided by the manufacturer of the semi-conductor wafer, the magnetic flux density, B, is calculated from the expression, $V_H = \frac{BI}{net}$

  From which, **B** = $\frac{V_H(net)}{I}$ the magnetic flux density **B** is determined when $n, e, t, I,$ *and* V_H *are all known values.*

(b) Using the equation, $V_H = \frac{BI}{net} \Rightarrow n = \frac{BI}{V_H e t}$

$$\Rightarrow n = \frac{0.8 \times 1.0}{(1.6 \times 10^{-19} \times 1.0 \times 10^{-3} \times 0.48 \times 10^{-3})}$$

$\therefore n = 1.042 \times 10^{25}$ per m^3 Is the number of charge carriers per unit volume.

ELECTROMAGNETIC BLOOD FLOW METER

This is yet another interesting application of the Hall effect similar to experiment of J J Thompson's set up used in the determination of the charge – to – mass ratio of an electron, where an electron is accelerated into the velocity selector region. The electric and magnetic fields in the velocity selector are adjusted until the electron passes through the region un-deflected.

Diagram

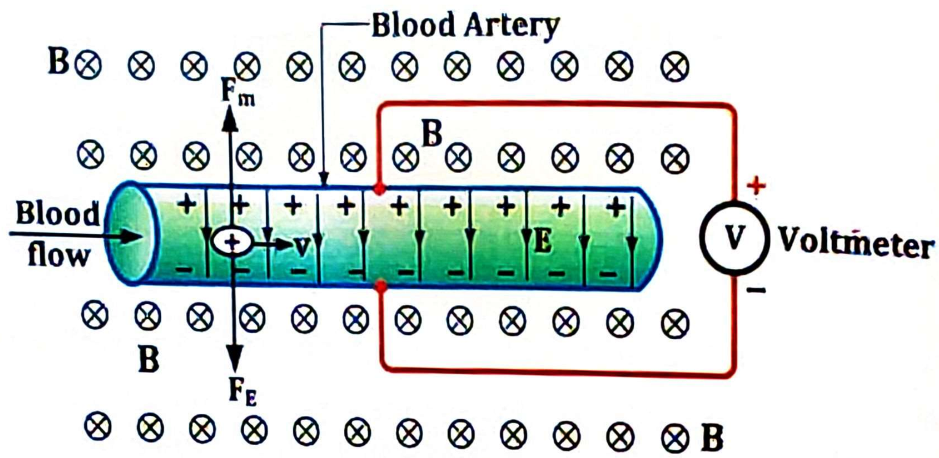


Fig. 1.8 (c)

- **Electromagnetic flowmeter** measures ***the speed of blood flow*** through a major artery during **cardio-vascular surgery.**
- **Blood** contains ions; the motion of the ions can be affected by a magnetic field.
- In an electromagnetic flow meter, a magnetic field is applied perpendicular the flow direction of blood in the artery.
- The magnetic force exerted by the field on the positive ions is towards one side of the artery according to Fleming's left hand rule, while the negative ions move in the opposite direction of the artery.
- This separation of the charge with the positive charge on one side and negative charge on the opposite side creates a large p.d. across the artery, resulting in the generation of a strong electric field, E.
- Electric force is then exerted on the moving ions in the opposite direction to that of the magnetic force.
- When the magnetic force on the ions = electric force on the same ions, i.e.

  $$F_m = F_E \Rightarrow Bqv = Eq \Rightarrow v = \frac{E}{B} \quad \dots\dots\dots (i)$$

  where v = average speed of the ion, and that of the blood flow.
- A voltmeter attached to the opposite sides of the artery is used to measure the potential difference, V, and knowing the magnetic field strength, B, and the electric field intensity in the region, the speed of the blood flow, v, is calculated from, $\therefore v = \frac{V_H}{Bd} \quad \dots\dots\dots\dots\dots\dots (ii)$ the ions in the blood are primarily sodium, Na^+ ions.

1.9 Exercises

1. The diagram in figure 1.9 (a) shows a rectangular piece of semiconductor with leads attached to metal end faces.

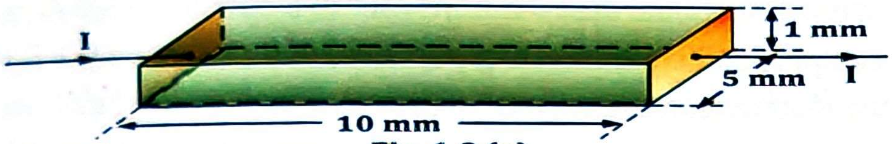


Fig. 1.9 (a)

(i) If the resistance of the specimen is approximately 100 Ω, show that the resistivity of the material of the material is about 0.050 Ω m.

(ii) Suppose a uniform magnetic field of flux density 0.5 T is applied vertically and perpendicularly to the largest area, determine the hall voltage **V_H** if a current of 40 mA, flows across the opposite smaller faces.

2. The current in a strip of copper is given by $I = nevA$ where, A is the cross sectional area, of the strip and n is the number of free electrons per unit volume. If d is the thickness of the strip and b is the breadth;
   (i) Express ev *in terms of* I, a, b *and* d.
   (ii) Show that the hall voltage, $V_H = \frac{BI}{ned}$, *when a field B is applied.*
   (iii) Calculate the hall voltage, given that $B = 1.0\,T, I = 6.0\,A, n = 7.5 \times 10^{28}$, $d = 1\,mm$ *and* $e = 1.6 \times 10^{-19} C$. **Ans:[$5.00 \times 10^{-7}$ V]**

3. The concentration of free electrons in silver is 5.85×10^{28} per m³. A strip of silver of thickness 0.050 mm and width 20.0 mm is placed in a magnetic field of 0.80 T. A current of 10.0 A is sent down along the strip as shown on the figure 1.9 (b).

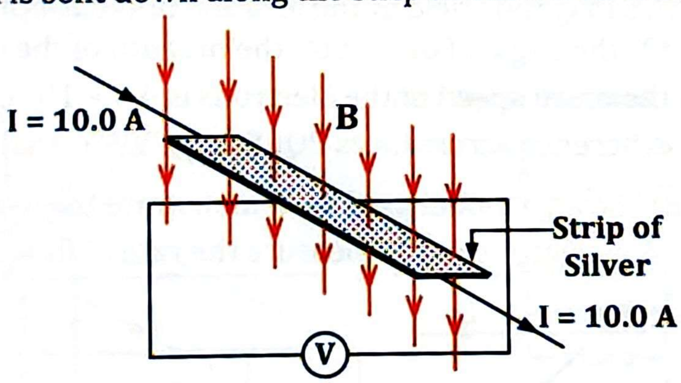


Fig. 1.9 (b)

Determine the,

   (i) Drift velocity of the electrons.
   (ii) Hall voltage measured by the moving coil meter.
   (iii) Side of the voltmeter that is at a higher potential.

4. An electromagnetic flowmeter is used is to be used to measure the blood speed. A magnetic field of 0.115 T is applied across an artery of inner diameter 3.80 mm. The Hall voltage is measured to be 88.0 μV. What is the average speed of the blood flowing in the artery? **Ans:[20.14 cm s^{-1}]**

5. (a) A circular coil of 10 turns and radius 5.0 cm caries a current of 1.0 A. Find the magnetic flux density at the centre of the coil. **Ans:[1.26×10^{-4}T]**

(b) A copper wire of cross sectional area 1.5 mm^2 carries a current of 5.0 A. The wire is placed perpendicular to a magnetic field of flux density 0.2 T. If the density of the electrons in the wire is 10^{29} m^{-3}. Calculate the force on each electron. **Ans:[6.67×10^{-24} N]**

6. The diagram in the figure 1.9 (c) shows a cuboid of a conductor of length **L**, breadth, **b** and thickness, **t**, placed with its largest face PQVW perpendicular to the horizontal component of the Earth's magnetic field of flux density B_H. A current, I is passed through it as shown in figure 1.9 (c)

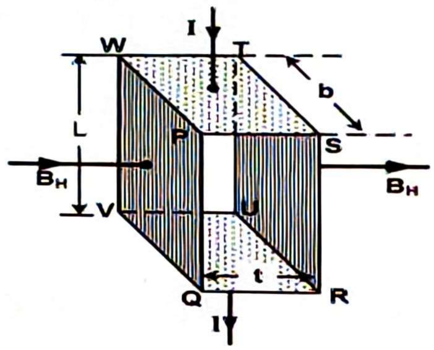


Fig. 1.9 (c)

(i) Account for the occurrence of a large potential difference across faces PQRS and UVWT and derive an expression for this voltage in terms of B_H, b and the average velocity of the charge carriers, v. **Ans:** $\left[V_H = \frac{B_H I}{net}\right]$

(ii) If the Earth's magnetic field at the location of the conductor is 2.0×10^{-4} T, the angle of dip is 60°, the breadth of the conductor is 5 cm and the mean speed of the electrons is 4.0×10^{-2} ms^{-1}. Calculate the potential difference across faces PQRS and UVWT. **Ans:** [$V_H = 3.46 \times 10^{-7}$ V]

7. The figure 1.9 (d) shows a model used to demonstrate the working principle of an electromagnetic flow meter used to measure the rate of flow of blood through an artery.

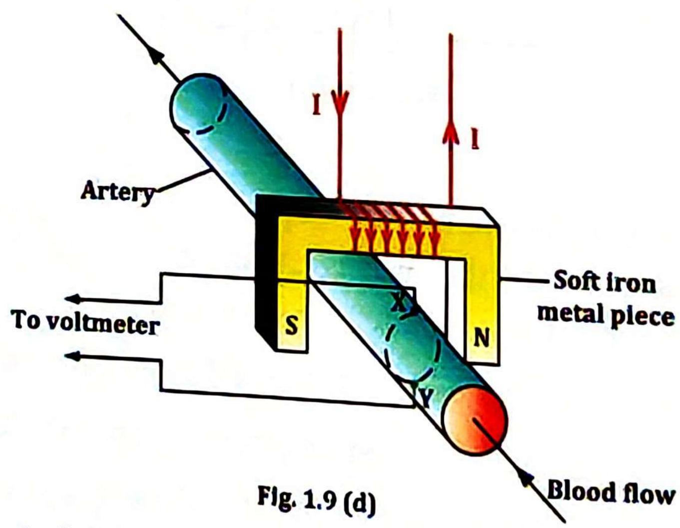


Fig. 1.9 (d)

When a magnetic field of 2.0 T is produced by the electromagnet, a potential difference (p.d.) of 600 μV is developed between the two electrodes, X and Y. The cross-sectional area of the artery is 1.5×10^{-6} m^2 and the separation of the electrodes is 1.4×10^{-3} m

Write down an expression for the force on an ion in the blood which is moving at right angles to the field. Define your symbols used. Which electrode is positive? An ion has a charge of 1.6×10^{-19} C. Determine the force on the ion due to the electric field between X and Y. **Ans: [6.86×10^{-20} N]**

Given that the p.d. of 600 μV is developed when the electric and magnetic forces on an ion are equal and opposite, calculate the;

(i) Speed of the blood through the artery. **Ans: [0.214 m s^{-1}]**

(ii) Volume of blood flowing through each section of the artery per second. **Ans: [$3.21 \times 10^{-7} m^3$]**

Magnetic Torque on a coil carrying a current in a magnetic field

Definition Magnetic Torque is the product of the magnitude of one of the forces constituting a couple and the distance between the lines of actions of the forces.

SI unit: is newton metre (Nm)

Derivation for the Torque experienced by a rectangular coil.

The Plane of the coil making an angle θ with the magnetic field, B

Consider a rectangular coil of wire of N – turns each carrying a current **I** in an external uniform magnetic filed of flux density **B**, with the plane of the coil inclined ***at an angle θ*** to the magnetic field as shown in the figure 2.0 (a).

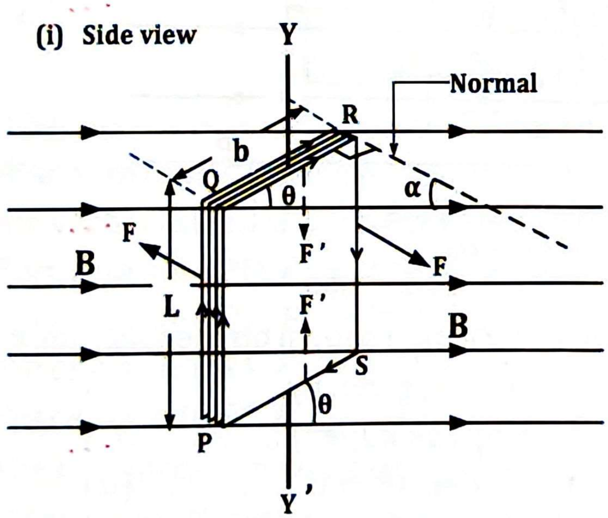


Fig. 2.0 (a)(i)

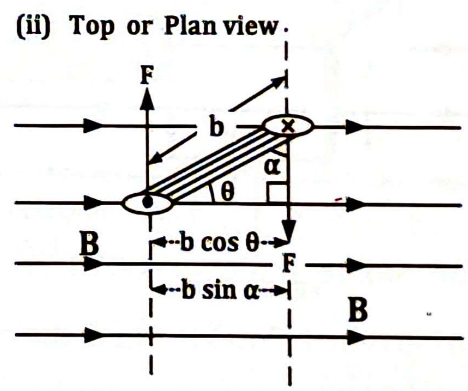


Fig. 2.0 (a)(ii)

When current, **I**, flows through the coil, in the direction shown on the diagram, each side of the conductor experiences magnetic force, F, given by.

Force on side PQ, $F = NBIL(\text{into the plane of the paper}) \ldots\ldots\ldots (i)$

Force on side RS, $F' = NBI\, b \sin\theta\ (\text{vertically downwards}) \ldots\ldots\ldots (ii)$

Force on side RS, $F = NBIL(\text{Out of the plane of the paper}) \ldots\ldots\ldots (iii)$

Force on side SP, $F' = NBI\, b \sin\theta\ (\text{vertically upwards}) \ldots\ldots\ldots (iv)$

The two ***forces on sides QR and SP*** *are equal in magnitude but are in opposite* directions and so they ***cancel out*** due to the rigidity of the coil.

Side **PQ** experiences force ***NBIL*** perpendicularly ***into*** the page, while **QR** experiences force ***NBIL*** perpendicularly ***out*** of the page.

The two forces ***constitute a couple whose turning moment or torque***

$T = F \times b \cos\theta$ or $T = F \times b \sin\alpha$

$T = NBIL \times b\cos\theta = NBIL\, b\cos\theta$ or $T = NBIL \times b\sin\alpha$ but $(L \times b) = A$

$\therefore \boldsymbol{T = NBIA\cos\theta}$ or $\boldsymbol{T = NABI\sin\alpha}$ *is the* ***Torque on the coil.***

Torque – is the product of the magnitude of one of the forces constituting a couple and the distance between the lines of the actions of the forces.

SI unit – is newton metre (Nm)

(b) The Plane of the coil being parallel to the magnetic field, B, direction.

Consider a rectangular coil of wire of N – turns each carrying a current **I** in an external uniform magnetic filed of flux density **B**, with the ***plane of the coil parallel*** to the ***magnetic field*** as shown in the figure 2.0 (b)

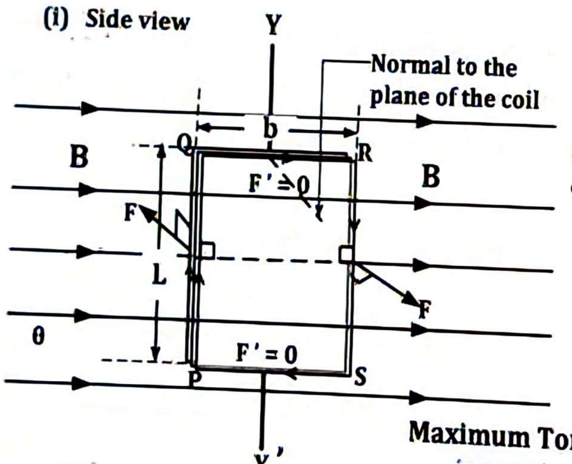
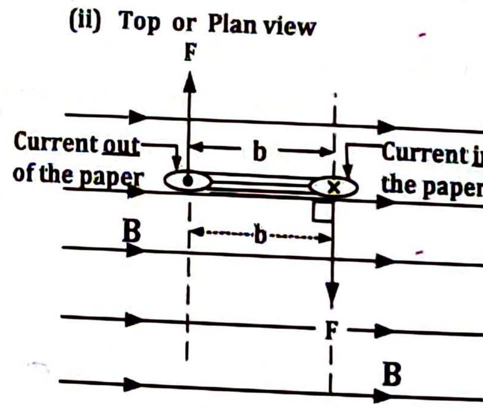


Fig. 2.0 (b)(i)



Maximum Torque, T_{max} = BINA
is experienced by the coil.

Fig. 2.0 (b)(ii)

When current, **I**, flows through the coil, in the direction shown on the diagram, each side of the conductor experiences magnetic force, F, given by.

Force on side PQ, $F = NBIL$ *(into the plane of the paper)* ……………(i)

Force on side QR, $F' = NBI\, b \sin 0° = 0$, *since* $\sin 0° = 0$ ……………(ii)

Force on side RS, $F = NBIL$ *(Out of the plane of the paper)* ……………(iii)

Force on side SP, $F' = NBI\, b\sin 0° = 0$, *since,* $\sin 0° = 0$ ……………(iv)

The two ***forces on sides QR and SP*** are *each* ***zero***, *since* ***current flows*** *in each in the* ***same direction*** *as that of the* ***magnetic field*** *thus Fleming's left hand rule doesn't apply* on the sides QR and SP of the rigid rectangular coil.

Side **PQ** experiences force ***NBIL*** perpendicularly ***into*** the page, while **QR** experiences force ***NBIL*** perpendicularly ***out*** of the page.

The two forces ***constitute a couple whose moment or torque***

$T = F \times b$ or $T = NBIL \times b = NBI(L \times b)$, but $(L \times b) = A$

$\therefore T = NBIA$ or $T = NABI$ *is the* ***Torque on the coil.***

NB: This gives the **maximum torque** the same coil can experience under the conditions given or indicated.

Angle of Rotation of the coil placed <u>Parallel to the magnetic Field</u>

When the current passed through the coil causes the coil to experience a deflection torque, **T = BINA,** the coil rotates through an angle **θ'**, until stopped by the restoring torque, **T'**, provided by a pair of hair springs, where, **T'= k θ'**

At equilibrium, **T= T'**

c) The Plane rectangular coil being placed in a radial magnetic field, B

As opposed to a uniform magnetic field, B, in a radial magnetic field, the plane of the coil is parallel the magnetic field at all positions of the coil in the magnetic field, hence maximum torque is experienced by the coil each time a current flows in the coil.

Top or Plan view of the Coil

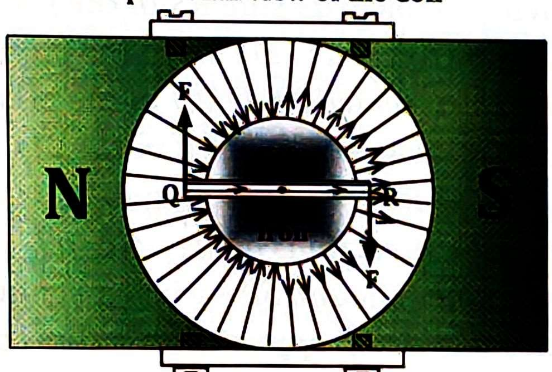


Fig. 2.0 (c)

When current, **I,** flows through the coil, in the direction shown on the diagram, only the vertical sides of the conductor experiences magnetic force, **F,** given by.

$Force\ on\ side\ PQ, F = NBIL(perpendicularly\ INTO\ the\ plane\ of\ the\ coil)$

$Force\ on\ side\ QR, F' = NBI\ b\ sin\ 0° = 0\ , since\ \sin 0° = 0$

$Force\ on\ side\ RS, F = NBIL(perpendicularly\ OUT\ of\ the\ plane\ of\ the\ coil)$

$Force\ on\ side\ SP, F' = NBI\ b\ sin\ 0° = 0\ , since, \sin 0° = 0$

The two ***forces on sides QR and SP*** are *each* <u>**zero**</u>, *since* ***current flows*** *in each side in the* ***same direction as*** *that of the* ***magnetic field,*** *thus* ***Fleming's left hand rule doesn't apply*** on the sides QR and SP of the rigid rectangular coil.

Side **PQ** experiences force $NBIL$ perpendicularly ***into*** the page, while **QR** experiences force $NBIL$ perpendicularly ***out*** of the page.

The two forces ***constitute a couple whose turning moment or torque***

$T = F \times b$ *or* $T = NBIL \times b = NBI(L \times b)$, *but* $(L \times b) = A$

$\therefore T = NBIA$ or $T = NABI$ *is the* ***Torque on the coil.***

NB, The ***torque here is maximum*** for ***all positions*** of the coil in the magnetic field.

(d) **The Plane of the coil being perpendicular to the magnetic field, B.**

Consider a rectangular coil of wire of N – turns each carrying a current **I** in an external uniform magnetic filed of flux density **B**, with the ***plane of the coil perpendicular to the magnetic field*** as shown below.

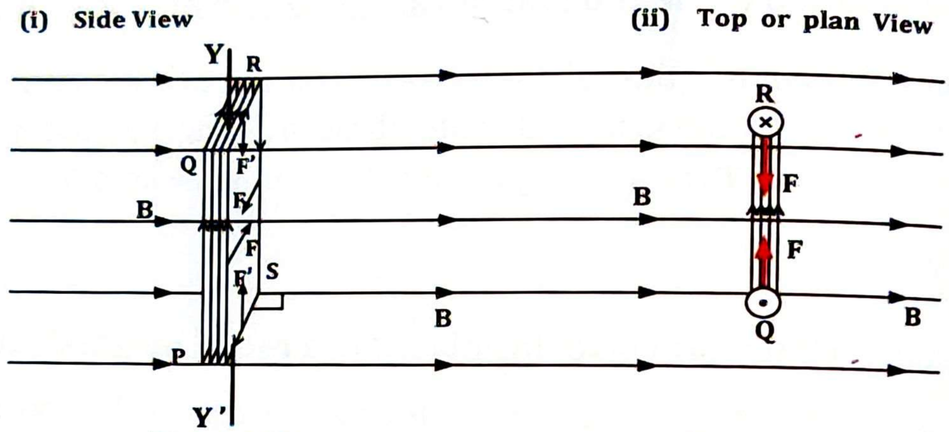


Fig. 2.0 (d)(i) Fig. 2.0 (d)(ii)

When current, **I**, flows through the coil, in the direction shown on the diagram, ALL the sides of the coil experience magnetic forces, **F**, given by.

$Force\ on\ side\ PQ, F = NBIL$ (towards side RS in the plane of the coil)

$Force\ on\ side\ QR, F' = NBIb$ (towards side SP in the plane of the coil)

$Force\ on\ side\ RS, F = NBIL$ (towards side PQ in the plane of the coil)

$Force\ on\ side\ SP, F' = NBIb$ (towards side QR in the plane of the coil)

The two ***forces on sides PQ and RS*** each of magnitude $F = NBIL$ ***cancel out*** each other since they act in ***opposite directions*** *of the rigid coil.*

Similarly, the two ***forces on sides QR and SP*** each of magnitude $F' = NBIb$ also ***cancel out*** each other since they act in ***opposite directions*** *of the rigid coil.*

Since none of the pairs of forces acting on the coil, **constitutes a couple, there is *NO*** *turning moment of a couple or* **torque** *generated and so* **the torque is zero.**

$T = 0$

NB: The ***torque here is minimum*** i.e. <u>***ZERO***</u> ***torque when the*** plane of the coil is normal to the magnetic field direction.

ELECTROMAGNETIC MOMENT (m) OF A CURRENT CARRYING COIL

- It has been found convenient to define the quantity known as ***electromagnetic moment, m*** (sometimes called ***magnetic moment***) of a current – carrying coil as that property which determines;
- *The magnitude of the magnetic torque that acts on the coil when it is at a given angle to the given magnetic field.*
- *The angle at which the coil ultimately comes to rest in the magnetic field.*

Definition:

Electromagnetic (magnetic) moment – *is <u>numerically</u> the magnetic torque acting on a coil whose plane is parallel to a uniform magnetic field of flux density one tesla.*

Electromagnetic moment – *is a vector quantity whose* ***magnitude*** *is,* $m = IAN$ *and whose* ***direction is along the normal to plane of the coil*** *such that it agrees with the direction of advance of Maxwell's Right Handed Advancing Screw Rule. Or it is provided by the direction of the magnetic field at the centre of the coil in the Right Hand Grip Rule in relation to the direction of flow of current in the coil.*

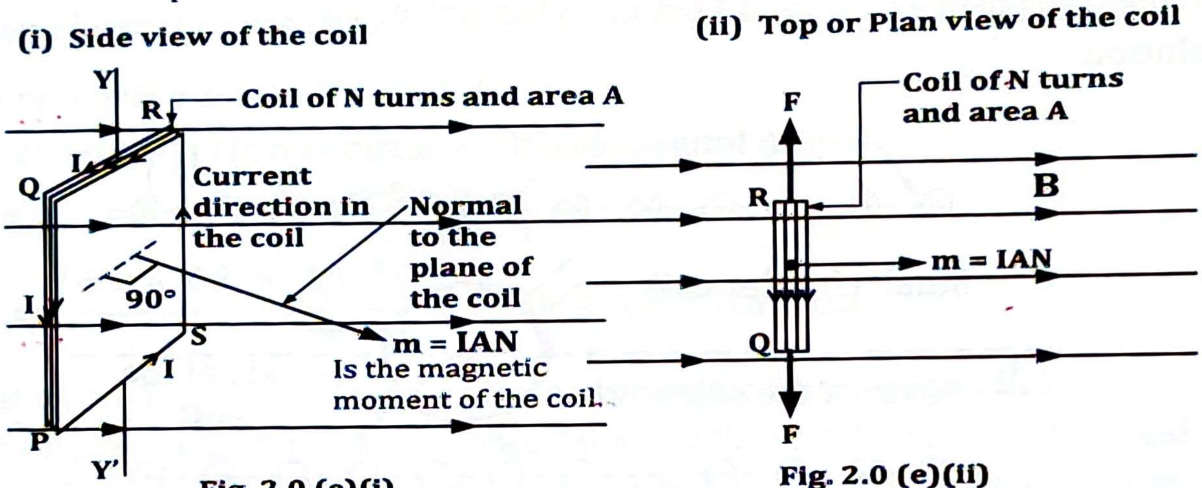


Fig. 2.0 (e)(i) Fig. 2.0 (e)(ii)

From the equation for magnetic torque, $T = BANI\cos\theta$ *or* $T = BANI\sin\alpha$
Where, θ = Angle between the plane of the coil and the magnetic field.
and, α = Angle that the normal to the plane of the coil makes with the field(B),
Thus, $T = B\,m\cos\theta$ *or* $T = B\,m\sin\alpha$

The SI unit of the magnetic moment (m) is ampere metre squared (**Am²**)
NB: - The magnetic torque on the current – carrying coil acts so as to align the electromagnetic moment, **m**, with the direction of the magnetic field, **B**.

Restoring torque on a current – carrying coil

When a coil with its plane placed at an angle θ to the magnetic field direction, it experiences a ***deflection torque*** $T = BANI\cos\theta$ ……………………(*i*)
that twists the suspension wire through an **angle β** as it turns.
The suspension wire in turn, then provides ***a restoring torque,***

$T' = k\beta$, *where k is a proportionality constant* ……………… (*ii*)

k = *torsion suspension constant of the torsion wire* ($Nm\,rad^{-1}$)

When the coil stops turning or rotating in a magnetic field, the deflection torque equals the restoring torque. i.e. from (i) and (ii) above,

$BANI\cos\theta = k\beta$ *where θ is expressed in degrees while β is in* $Nm\,rad^{-1}$

β = *is the angle turned by the coil due to the deflection torque.*

2.1 Examples & Exercises on magnetic torque on a coil

1. A small circular coil of 10 turns and mean radius 2.5 cm is mounted at the centre of a long solenoid of 750 turns per metre with its axis at right angles to the axis of the solenoid. If a current in the solenoid is 2.0 A. Calculate the initial torque on the circular coil when a current of 1.0 A flows through it.

Solution:

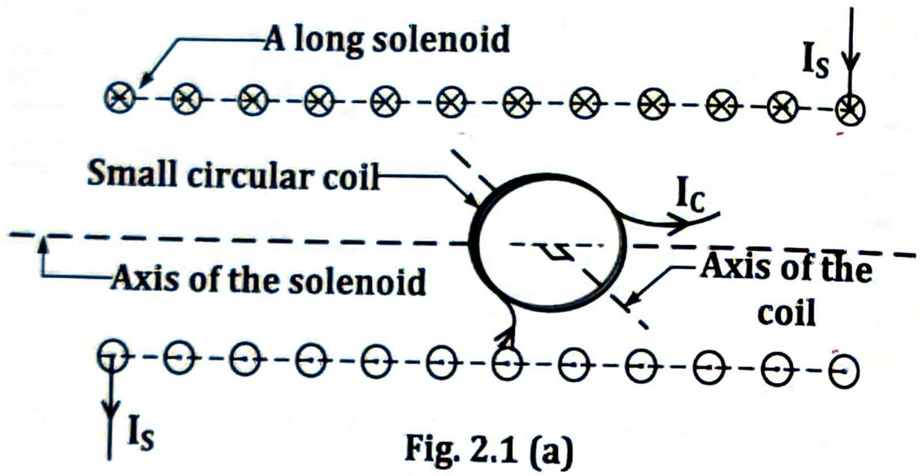


Fig. 2.1 (a)

The plane of the coil is parallel to the magnetic field at the centre of the solenoid, given by, $B = \mu_0 nI = 4\pi \times 10^{-7} \times 750 \times 2.0 = 1.88 \times 10^{-3}\,T$

The initial magnetic torque experienced by the circular coil is given by,

$T = BANI \cos 0° = BANI \sin 90° = BANI$

$T = 1.88 \times 10^{-3} \times \pi(0.025)^2 \times 10 \times 1.0$

$\therefore \boldsymbol{T = 1.88 \times 10^{-5}\,Nm}$

2. A small rectangular coil of 10 turns and dimensions 4 cm × 2 cm is suspended inside a long solenoid of 1000 turns per metre so that its plane lies along the axis of the solenoid as shown in the figure 2.1 (b) below. The coil is connected in series with the solenoid. When a current of 2.0 A is passed through the solenoid, the coil deflects through 30°. Calculate the torsion constant of the suspension.

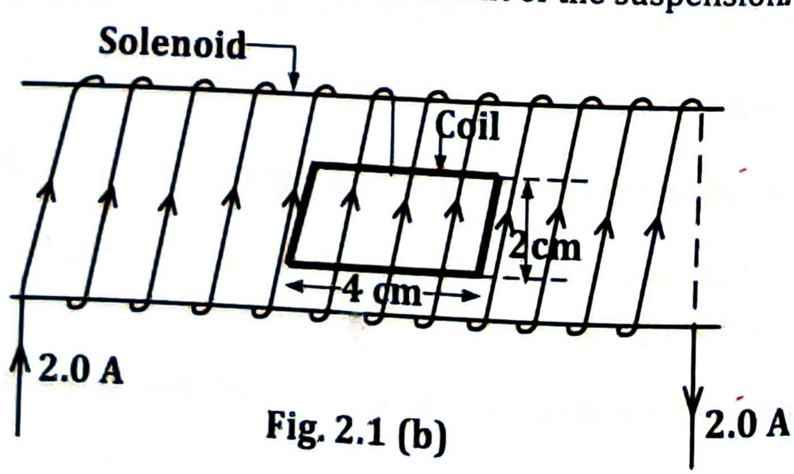


Fig. 2.1 (b)

Solution:

The plane of the rectangular coil is parallel to the magnetic field at the centre of the solenoid, i.e, $B = \mu_0 nI = 4\pi \times 10^{-7} \times 1000 \times 2.0 = 2.51 \times 10^{-3}\,T$

The initial magnetic torque experienced by the circular coil is given by,
$T = BANI\cos 0° = BANI\sin 90° = BANI$ since $\theta = 0°$ and $\alpha = 90°$
$T = 2.51 \times 10^{-3} \times (0.02 \times 0.04) \times 10 \times 2.0$
$\therefore \boldsymbol{T = 4.02 \times 10^{-5}\ Nm}$

When the coil turns through angle, $\beta = 30° = \left(\frac{\pi}{180°} \times 30°\right)$ *radians,*
The restoring torque $\boldsymbol{T' = k\beta}$
But the ***deflection torque = restoring torque***
$\therefore \boldsymbol{T = T' \Rightarrow 4.02 \times 10^{-5} = k \times \frac{\pi}{6}}$
$\therefore \boldsymbol{k = \left(\frac{6 \times 4.02 \times 10^{-5}}{\pi}\right) = 7.68 \times 10^{-5}\ Nm\,rad^{-1}}$

3. A flat circular coil X of 30 turns and mean diameter 30 cm is fixed in a vertical plane and carries a current of 3.0 A. Another coil Y of 2 cm × 2 cm and having 200 turns is suspended in a vertical plane, at the centre of the circular coil. Initially the planes of the two coils coincide. Determine the torque on coil Y when a current of 2.0 A is passed through it.

Solution:

Coil X provided a uniform magnetic field to coil Y, where $B_X = \frac{\mu_0 N_X I_X}{2R}$

$$B_X = \frac{4\pi \times 10^{-7} \times 30 \times 3.0}{2 \times 0.15} = 3.77 \times 10^{-4}\ T$$

But the planes of the two coils, coincide ⟹ the magnetic field B_X is normal to the plane of coil Y, ⟹ $\theta = 90°$ *and* $\beta = 0°$
$T_Y = B_X A_Y N_Y I_Y \cos 90° = B_X A_Y N_Y I_Y \sin 0° = 0$ $B'se\ \cos 90° = \sin 0° = 0$
$T_Y = 3.77 \times 10^{-4} \times (0.02 \times 0.02) \times 200 \times 2.0\cos 90°$, *but* $\cos 90° = 0$
$\therefore \boldsymbol{T_Y = 0\ Nm}$ ***i.e. the coil Y does not experience a turning effect.***

4. A rectangular coil of sides 2 cm by 4 cm having 10 turns and carrying a current of 2.0 A is freely pivoted at the centre of a large plane circular coil of 200 turns and of radius 8 cm, carrying a current of 5.0 A. If the planes of the coils are initially at an angle of 30° to each other. Determine the torsion constant of the suspension wire supporting the rectangular coil, when it has turned through 45°.

Solution

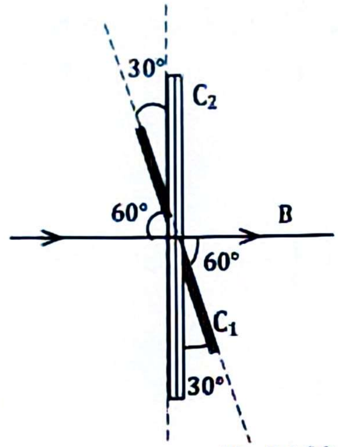


Fig. 2.1 (c)

C_1 area $A_1 = 4 \times 2 = 8\ cm^2 = 8.0 \times 10^{-4} m^2$,
$N_1 = 10$ turns
C_2 area $A_2 = \pi r^2 = 3.14 \times (0.08)^2 = 2.01 \times 10^{-2}$ m^2, $N_2 = 200$ turns
$I_2 = 5.0\ A$, $\theta_1 = 30°$, $\theta_2 = 60°$, $\beta = 30° = \frac{\pi}{6}$ $radian$

$$B_2 = \frac{\mu_o N I}{2r} = \frac{4\pi \times 10^{-7} \times 200 \times 5}{2 \times 0.08} = 7.85 \times 10^{-3}\ T$$

$$T = BANI \cos\theta = k\beta$$

$$k = \frac{6 \times 7.85 \times 10^{-3} \times 8.0 \times 10^{-4} \times 10 \times 2 \cos 60°}{\pi}$$

$$k = 1.20 \times 10^{-4}\ Nm\ rad^{-1}$$

Exercises

1. A flat coil of 50 turns and of mean diameter 40 cm is in a fixed vertical plane and has a current of 5 A flowing through it. A small coil, 1.0 cm square and having 120 turns, is suspended at the centre of a circular coil in a vertical plane at an angle of 30° to that of the larger coil. Calculate the magnetic torque experienced by the small coil when it carries a current of 2 mA. **Ans: [$9.42 \times 10^{-9}\ Nm$]**

2. A fixed vertical circular coil has a diameter of 15.0 cm and 120 turns. At the centre of the coil is a small coil of radius 2.0 cm and 100 turns. Pivoted through the centre so that it can rotate about a horizontal axis which lies along the diameter of the larger coil. A rider of mas 0.05 g must be moved 13.0 cm from the axis of the small coil along an arm fixed to the small coil to keep the plane of the latter horizontal when the same current is passed through both coils. Determine the value of this current.
Ans: [$0.734\ A$]

3. A coil of radius 7.5 cm and 500 turns is suspended vertically with the plane of the coil in the east – west direction. If the horizontal component at the centre of the coil is 1.8×10^{-5} T, what current must be passed through the coil to just neutralize this field? Explain why there are two possible answers. **Ans: [$4.30 \times 10^{-3} A$]**

4. At a distance of 5.0 cm from a vertical wire carrying a current in air, the resultant magnetic field is zero as a result of the Earth's horizontal component of magnetic field of 1.8×10^{-5} T. Calculate the current flowing through the wire. **Ans: [4.5 A]**

5. A circular coil of 100 turns and mean radius 10.0 cm is set up with its plane vertical and at right angles to the magnetic meridian. A short magnetic needle suspended at its centre makes 8 oscillations per minute when slightly deflected. How many oscillations per minute will the needle make when a current of 0.5 A flows in the coil. If the horizontal component of the Earth's magnetic field is
2.0×10^{-5} T. Explain clearly why there are two possible answers.
Ans: [7.88 and 8.27 vibrations per minute]

2.2 THE MOVING COIL INSTRUMENTS

1. The moving coil galvanometer

This is one of the instruments that uses the principle of the deflection magnetic torque for it operation.

The structure of a moving coil galvanometer

- *A moving coil galvanometer has a rectangular coil of fine insulated copper wire wound on an aluminium frame (to provide electromagnetic damping, due to eddy currents).*
- *The coil is mounted over a soft iron cylinder, (to boost or enhance the magnetic field in the air gaps) and held between the concave pole pieces of a strong permanent magnet (to provide a radial magnetic field to the coil, and to ensure maximum deflection torque or turning moment).*
- *The coil is freely suspended by a torsion wire over jewelled bearing, (to minimise friction at the contacts and support points for free movement of the coil).*
- *The suspension torsion wire is also supported by a pair of oppositely coiled hair springs (to provide the restoring torque to the coil and also act as input and outlet terminals to current in the coil)*
- *Attached to the suspension system is an aluminium pointer that moves over the scale when current passes through the coil, and shows a deflection on the linear scale that is proportional to input current flowing through the coil.*

The Diagram

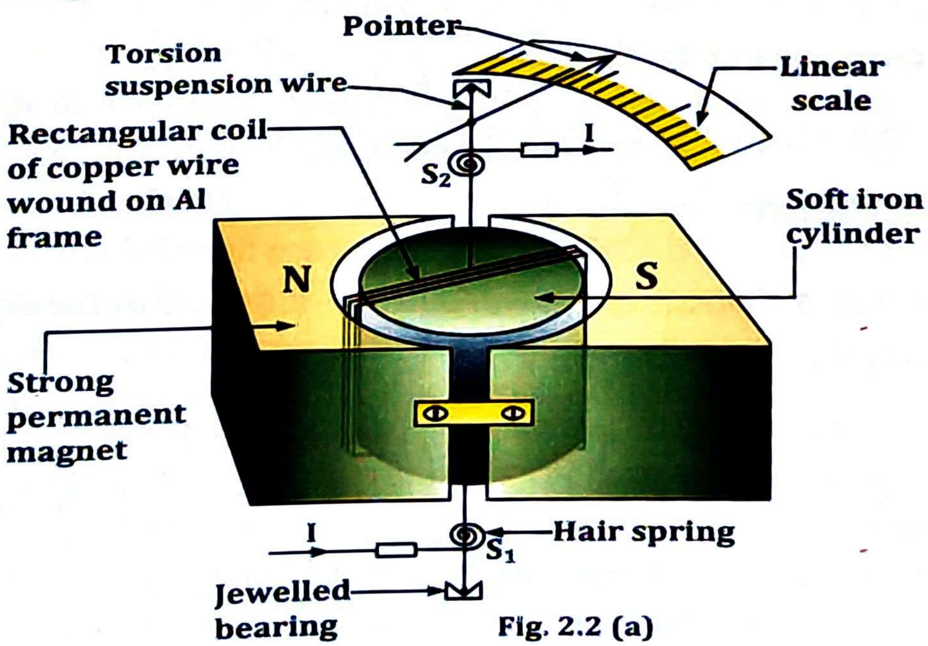


Fig. 2.2 (a)

Mode of operation (Working principle) of a moving coil galvanometer

Current, I, is passed through the coil placed in a radial magnetic field via the hair springs S_1 and S_2 and causes the coil to experience a couple of forces causing a deflection torque, $T = BANI$ ……………………….. (i) which causes the coil to rotate together with the pointer that moves over the scale through an angle θ. The rotation of the coil and pointer due to the deflection torque is eventually stopped by the restoring torque, $T' = k\,\beta$ ……………. (ii) provided by the pair of hairsprings S_1 and S_2.

At equilibrium, from (i) and (ii) deflection torque equals restoring torque

i.e. $T = T' \Rightarrow BANI = k\,\beta$ ……………………… (iii)

Hence; $I = \left(\frac{k}{BAN}\right)\beta$ ……………………………… (iv)

Where, k = The torsion or sensitivity constant of the galvanometer.
A = Area of the plane of the coil.
B = Magnetic field strength in the air gap.
N = Number of turns of the coil would on aluminium metal frame.
β = the angle turned by the coil and the pointer over the scale.

From equation (i) above, k, A, B, and N are all constant for a given meter so, ***current I is directly proportional to the deflection β***, **i.e. I ∝ β.**
Hence, the ***instrument has a linear scale***.

SENSITIVITY OF A MOVING COIL GALVANOMETER

This is a measure of the extent to which the instrument responds to the passage of current of varying magnitudes through it. There are two types of sensitivity namely:

(i) Current sensitivity, $S_I = \frac{\beta}{I} = \left(\frac{BAN}{k}\right)$

(ii) Voltage sensitivity, $S_V = \frac{\beta}{V} = \frac{\beta}{IR} = \left(\frac{BAN}{kR}\right)$

(a) Current sensitivity, $\frac{\beta}{I} = \left(\frac{BAN}{k}\right)$

The current sensitivity, S_I, of a galvanometer – is defined as the deflection per unit current. i.e. $S_I = \frac{\beta}{I}$

Units: ***Current Sensitivity*** – is usually expressed in mm per microampere ($mm\ \mu A^{-1}$) owing to the small values of currents involved, and usually read on a linear scale of the instrument graduated in millimetres (mm).

However other units can be used depending on both the calibration and the adaptation of the instrument to the given value of current its meant to measure. **E.g** It can be expressed in mm per mA *i.e.* ($mm\ mA^{-1}$)

(b) **Voltage sensitivity, $S_V = \frac{\beta}{V} = \frac{\beta}{IR} = \left(\frac{BAN}{kR}\right)$**

The voltage sensitivity, Sv, of a galvanometer *– is defined as the deflection per unit voltage. i.e.* $S_V = \frac{\beta}{V}$

Where β is the deflection produced on the graduated scale, and V is the potential difference (p.d.) across the galvanometer

Units: ***Voltage Sensitivity*** – is usually expressed in mm per microvolt ($mm\ \mu V^{-1}$) owing to the small values of voltages involved, across the instrument and usually read on a linear scale of the instrument graduated in millimetres (mm).

However, other units can be used depending on both the calibration and the adaptation of the instrument to the given value of the voltage it is meant to measure.

E.g. It can be expressed in millimetres per millivolt $i.e. (mm\ mV^{-1})$

(c) Factors affecting the sensitivity of a moving coil meter

From the expressions of current sensitivity $S_I = \frac{\beta}{I}$ *and* $S_V = \frac{\beta}{V}$ voltage sensitivity, current and voltage sensitivities are higher when the values of B, A and N must be large and the value of k must be small as follows:

(i) ***A large magnetic flux density, B*** – is achieved using a narrow air gap and using a strong permanent magnet (typically B = 0.4 T), which also overshadows the external influence of stray magnetic fields like the Earth's magnetic field (i.e. $B \approx 4.0 \times 10^{-3}$ T).

(ii) ***A large number of turns, N of the coil*** – makes the instrument more sensitive, however this number, N should not compromise the size of the magnetic air gap. i.e. N – should make the coil still fit in the air gap.

(iii) ***A large area A of the plane of the coil*** – leads to increase in the sensitivity of the instrument. However a compromise with the other factors must be resolved. A coil with an extremely large area, may swing about its equilibrium deflected position for a longer time before a reading can be taken.

(iv) ***A small value of the sensitivity constant, k*** – leads to increase in the sensitivity of the instrument. Thus, a suspension wire of low rigidity value or of low torsion constant, is required. However too weak a wire will again increase the time the swing about its equilibrium deflected position for a longer time before a reading can be taken.

(v) ***A small value of resistance, R*** *– of the coil especially for the sensitivity of the voltmeter. A small resistance of the instrument allows a large current to flow through the coil hence setting up a large p.d. across the instrument.*

Conversions of a moving coil galvanometer to other meters

When choosing a galvanometer, the resistance of the rest of the circuit has to be considered. Thus moving-coil galvanometers can be ***adapted*** for use ***as ammeters*** and ***voltmeters*** by the ***addition*** of ***shunts*** and ***multipliers*** respectively. ***Moving coil galvanometers can*** also be modified to ***measure alternating currents*** and ***voltages*** if an appropriate value of ***a rectifier is placed in series with the coil.***

(i) Conversion of a galvanometer into an ammeter

In order to convert an ordinary moving coil galvanometer into an ammeter, a very ***low value resistance*** *called a* ***shunt*** *resistor is connected* ***across the*** *coil of the instrument. The value or choice of the resistor depends on the maximum deflections of both the galvanometer and the ammeter and on the resistance of the coil of the instrument, so that the p.d. across the coil equals the p.d. across the shunt as shown on the diagram in figure 2.2 (b)*

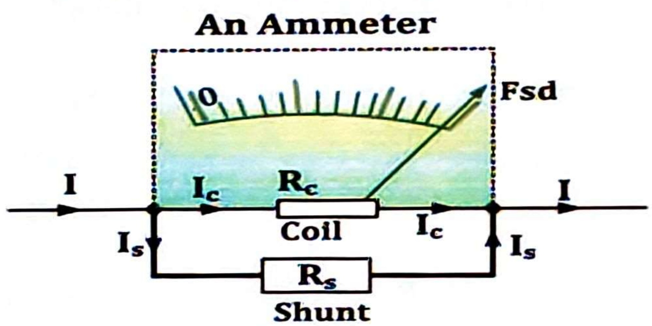


Fig. 2.2 (b)

If **I** is the total current to be measured, **Ic** is the current through the coil (fsd) of the instrument, then the rest ($I_S = I - I_C$) is the current that passes through the shunt resistor, **Rs**.

$$The\ p.d.\ across\ the\ coil = The\ p.d.\ across\ the\ shunt$$

$$i.e.\ I_C \times R_C = I_S \times R_s\ from\ which, R_s\ is\ calculated\ using,$$

$$\therefore\ R_s = \left(\frac{I_C \times R_C}{I_S}\right)\ is\ the\ value\ of\ the\ shunt\ to\ be\ used.$$

(ii) Conversion of a galvanometer into a voltmeter.

In order to convert an ordinary moving coil galvanometer into a voltmeter, a very ***High value resistance*** *called a* ***Multiplier*** *resistor is connected* ***in series with the coil*** *of the instrument. The value or choice of the resistor depends on the maximum deflections of both the galvanometer and the voltmeter and on the resistance of the coil of the instrument, so that the p.d. across the coil plus the p.d. across the multiplier equals the p.d. across the entire voltmeter as shown on the diagram in the figure 2.2 (c)*

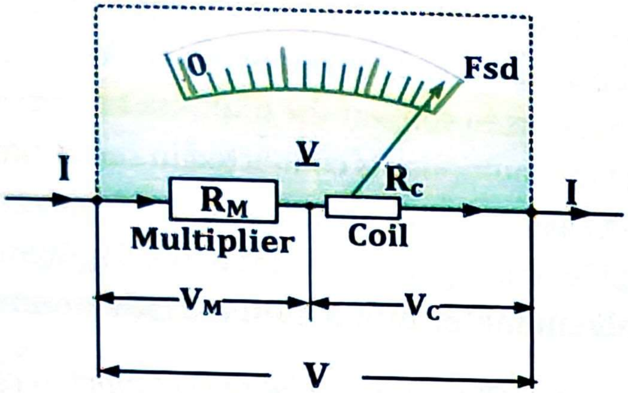


Fig. 2.2 (c)

If **V** is the total p.d. to be measured, (f s d), V_C is the p.d. across the coil and V_M is the p.d. across the multiplier resistor, R_M. The same current, **I** flows through the two resistors, R_c and R_M.

The p.d. across the instrument = The p.d. across the coil plus the p.d. across the Multiplier. i.e. $V = V_c + V_M$

i.e. $V = I_C(R_C + R_M)$ *from which*, R_M *is calculated.*

$\therefore\ \boldsymbol{R_M} = \left[\left(\frac{V}{I_c}\right) - \boldsymbol{R_c}\right]$ *is the value of the Multiplier to be used.*

(iii) A multi-meter that measures both current and voltage

In order to convert an ordinary moving coil galvanometer into both a voltmeter, and an Ammeter, a very ***High value resistance*** *called a* ***Multiplier*** *resistance is connected* ***in series with the coil*** *of the instrument and a* ***low value resistance*** *called a* ***Shunt*** *is connected* ***across the coil, then appropriate switches*** *are incorporated accordingly. The value or choice of a given resistor depends on the maximum deflections of both the galvanometer and the voltmeter or ammeter and on the resistance of the coil of the instrument, so that the p.d. across the coil plus the p.d. across the multiplier equals the p.d. across the entire voltmeter or the p.d. across the coil equals the p.d across the shunt as shown on the diagram below.*

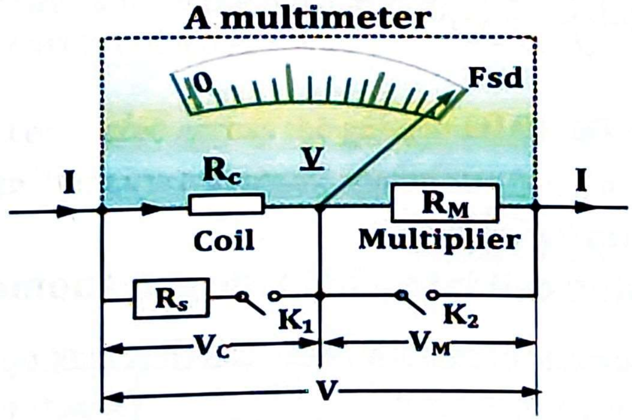


Fig. 2.2 (d)

In order for the above multi-meter to be used as an ammeter, a suitable value of a shunt is connected across the coil and switches K_1 and K_2 are closed, the value of the desired resistance R_S is then calculated.
On the other hand, in order to convert the multi-meter into a voltmeter, an appropriate value of the multiplier is connected in series with the coil, and with both switches K_1 and K_2 open, the value of the desired resistance R_m is calculated.

(iv) Conversion of a galvanometer into a Ballistic galvanometer

*A **Ballistic galvanometer (B.G)** is a special type of moving coil galvanometer used to **measure charge** by measuring an "electric blow" due to current flow into the coil. If a galvanometer is to be used ballistically, two major conditions need to be fulfilled.*

- *All the charge being measured should be delivered to the instrument before Its coil has moved appreciably. This motion of the coil, is as a result of the momentary current that flows.*
- *The damping of the motion of the coil should be as small as possible, by winding the coil on a non – conducting frame so that the only electromagnetic damping present should be that due to induced current which flows through both the coil and the external circuit. This current however, should be kept minimal by including a large resistance in the circuit so as to reduce the associated damping.*
- *When charge passes through the instrument, its coil is deflected through a maximum angle θ_m and then oscillates with a decreasing amplitude about its initial (zero) position.*
- *The maximum angular deflection, θ_m is $\propto$ the charge, Q, delivered.*

  i.e. $\theta_m \propto Q \Rightarrow \boldsymbol{\theta_m = kQ}$

  *where, **k** is a constant of proportionality known as the charge sensitivity of the instrument expressed in radians per coulomb* $\mathbf{rad.\,C^{-1}}$

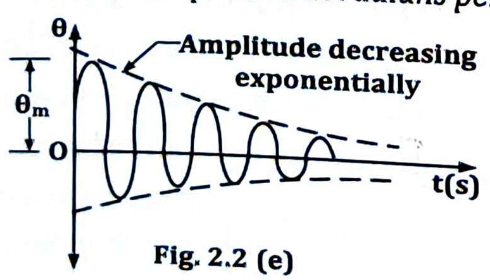


Fig. 2.2 (e)

NB: *Damping in a ballistic galvanometer is very small as evidenced by the **decaying amplitude.***

Converting a moving coil into a ballistic galvanometer involves,

(a) ***Increasing the number of turns N*** of the coil thus making the coil heavier to increase on the inertia of the coil.

(b) ***Replacing the aluminum metal frame*** on which the coil is wound on with an ***Insulating frame***, to minimize electromagnetic damping, due to eddy currents.

(c) ***Replacing a rigid suspension torsion wire*** with a fine but ***flexible wire of low rigidity modulus*** or of a low torsion constant to increase on the period of oscillation of the coil.

(d) ***Replacing the aluminium pointer*** with a ***beam of light that acts*** as a ***pointer of negligible inertia***, producing a spot of light on the scale which gives an instant response.

Examples

1. If a coil of a moving coil galvanometer having 10 turns and resistance of 4.0 Ω is removed and replaced by a second coil of 100 turns and of resistance 160 Ω. Calculate the factor by which the;

(i) Current sensitivity changes and

(ii) Voltage sensitivity changes, assuming that all the other features remain unchanged in both cases.

Solution

(i) Current sensitivity,

$\frac{\beta}{I} = \left(\frac{BAN}{k}\right)$ *For coil*, 1, $\Longrightarrow N_1 = 10$ *turns*, $R_1 = 4.0\ \Omega$

and For coil, 2, $\Longrightarrow N_2 = 100$ *turns*, $R_2 = 160\ \Omega$

$\therefore$ ***Sensitivity of coil 2: coil 1*** $\frac{(S_I)_2}{(S_I)_1} = \left(\frac{BAN_2}{k}\right) \div \left(\frac{BAN_1}{k}\right) = \frac{N_2}{N_1}$

$$\therefore \frac{(S_I)_2}{(S_I)_1} = \frac{N_2}{N_1} \Longrightarrow \frac{(S_I)_2}{(S_I)_1} = \frac{100}{10} = \mathbf{10}$$

Thus the current sensitivity changes by factor **10**

(ii) Similarly, voltage sensitivity of coil 2 to that of coil 1

i.e. S_V ***of coil 2:*** S_V ***of coil 1*** $\frac{(S_V)_2}{(S_V)_1} = \left(\frac{BAN_2}{R_2k}\right) \div \left(\frac{BAN_1}{R_1k}\right) = \frac{N_2}{R_2} \times \frac{R_1}{N_1}$

$$\therefore \frac{(S_V)_2}{(S_V)_1} = \left(\frac{BAN_2}{R_2k}\right) \div \left(\frac{BAN_1}{R_1k}\right) = \frac{100}{160} \times \frac{4}{10} = \frac{1}{4}$$

Thus the voltage sensitivity changes by factor $\frac{1}{4}$ ***or*** **0.25**

2. A galvanometer having a coil of 2.0 Ω has a full scale deflection of 2.0 mA. Determine the value of the most appropriate resistance needed to enable it measure a current of 5.0 A.

Solution

A shunt is connected across the galvanometer and its value **Rs** is obtained.

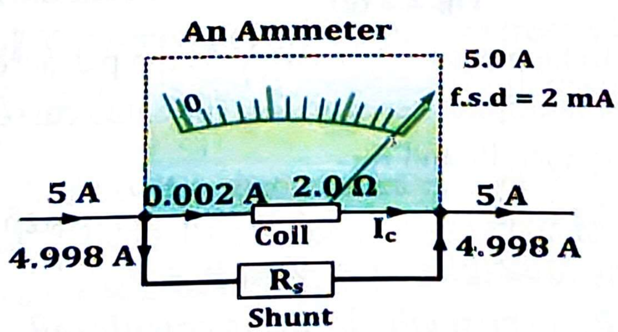


Fig. 2.2 (f)

$\text{The p.d. across the coil} = \text{The p.d. across the shunt}$

$i.e.\ I_c \times R_c = I_s \times R_s$ *from which*, R_s *is calculated using,*

$\therefore R_s = \left(\frac{I_c \times R_c}{I_s}\right)$ *is the value of the shunt to be used.*

$\therefore R_s = \left(\frac{0.002 \times 2.0}{4.998}\right) = \mathbf{8.00 \times 10^{-4}\ \Omega}$

3. A moving coil galvanometer has a resistance of 25 Ω and gives a full scale division when carrying a current 4.4 μA.

(a) What current will give a full scale deflection when the galvanometer is shunted by a 0.10 Ω resistance?

(b) How can the above instrument be converted to a voltmeter having a range of (0 – 3.00 V)

Solution

(a) $R_c = 25\ \Omega \quad I_c = 4.4 \times 10^{-6} A$

$\text{The p.d. across the coil} = \text{The p.d. across the shunt}$

$i.e.\ I_c \times R_c = I_s \times R_s$ *from which*, I_s *is calculated using,*

$\therefore I_s = \left(\frac{I_c \times R_c}{R_S}\right)$ *is the value of the shunted current.*

$\therefore I_s = \left(\frac{4.4 \times 10^{-6} \times 25}{0.10}\right) = 1.10 \times 10^{-3} A$

$\therefore \textbf{Total current}, I = (I_c + I_s) = (4.4 \times 10^{-6} + 1.10 \times 10^{-3})$

$\therefore \textbf{Total current}, I = \mathbf{1.1044 \times 10^{-3}} A$

(b) $R_c = 25\ \Omega,\ I_c = 4.4 \times 10^{-6} A$

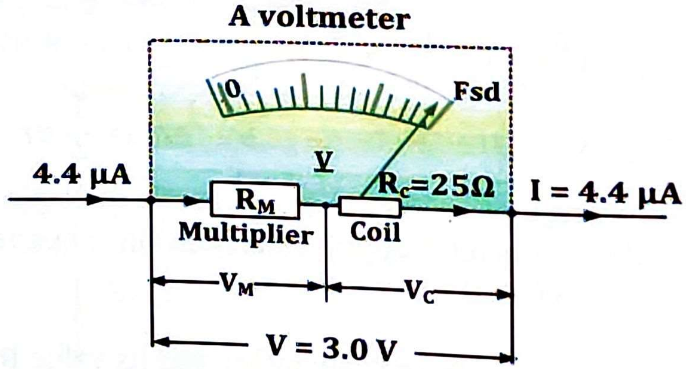


Fig. 2.2 (g)

If **V** is the total p.d. to be measured, (f s d), V_C is the p.d. across the coil and V_M is the p.d. across the multiplier resistor, R_M. The same current, **I,** flows through the two resistors, R_c and R_M.

The p.d. across the instrument = The p.d. across the coil plus the p.d. across the Multiplier. i.e. $V = V_c + V_M$

$i.e.\ V = I_c(R_c + R_M)$ *from which*, R_M *is calculated.*

$\therefore R_M = \left[\left(\frac{V}{I_c}\right) - R_c\right]$ *is the value of the Multiplier to be used.*

$\therefore R_M = \left[\left(\frac{3.00}{4.4 \times 10^{-6}}\right) - 25\right] = 68.18\ M\Omega$

$\therefore R_M = 6.818 \times 10^7\ \Omega$ is the multiplier connected in series with the meter.

2.3 Exercises

1. A moving coil galvanometer has a resistance of 2.0 Ω and gives a full scale division when carrying a current 5.0 mA.

(i) What current will give a full scale deflection when the galvanometer is shunted by a 2.0 mΩ resistance? **Ans: [5.01 A]**

(ii) How can the above instrument be converted to a voltmeter to measure up to a voltage of 15.00V. **Ans: [Use of multiplier of 2,998 Ω]**

2. The diagram below is of a multi-meter, having a coil of 0.2 Ω and full scale deflection of 10 mA. Two resistances R_1 and R_2 are connected to the circuit via switches K_1 and K_2 as shown in the figure 2.3 (a)

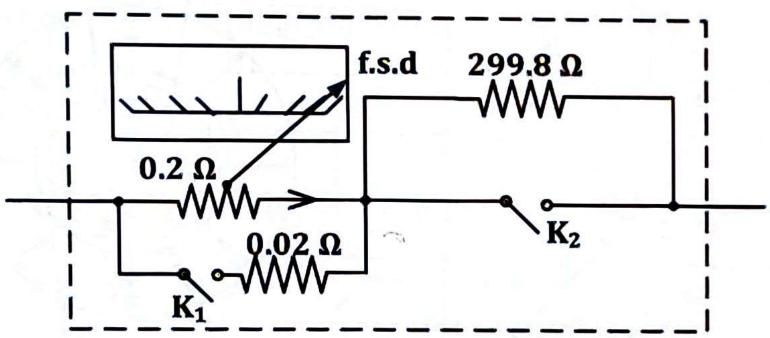


Fig. 2.3 (a)

Determine the reading of the instrument when switch

(i) K_1 is closed and switch K_2 is also closed. **Ans: [1.0 A]**

(ii) K_1 is open and switch K_2 is also open. **Ans: [3.0 V]**

3. A moving coil galvanometer meant to measure large current of 10.00 A has a current of 100μA passed through the galvanometer to give a full scale deflection.

Determine the size of the most appropriate resistance suitable for the task. **Ans: [1.0 mΩ]**

4. A rectangular coil 10 cm × 2.0 cm consisting of 100 turns is suspended vertically from the middle of the short side in a radial magnetic field of flux density, 0.02 T and supplied with a current from a 25 V d.c. supply. If the resistance of the coil is 100 Ω, calculate the deflecting torque on the suspension. **Ans: [1.0×10^{-3} Nm]**

5. A voltmeter has a switch that enables voltages to be measured with a maximum of 25.0V or 10.0 V. For a range of voltages up to 25.0 V, the switch connects to a resistor of 9850 Ω in series with a galvanometer. For the range

of voltages to 10.0 V, the switch connects a resistor of magnitude 3850 Ω in series with the galvanometer. Find the;

(i) Coil resistance of the galvanometer and **Ans: [150 Ω]**

(ii) The galvanometer current that causes a full-scale deflection. **Ans: [2.50 mA]**

The Moving Coil Loud Speaker

The moving coil loud speaker is a device that converts electrical energy into sound energy. It uses the principles of positioning of a current carrying coil of an insulated metal wire placed in a strong radial magnetic field, together with Fleming's left hand rule for its operation.

Diagram / Structure:

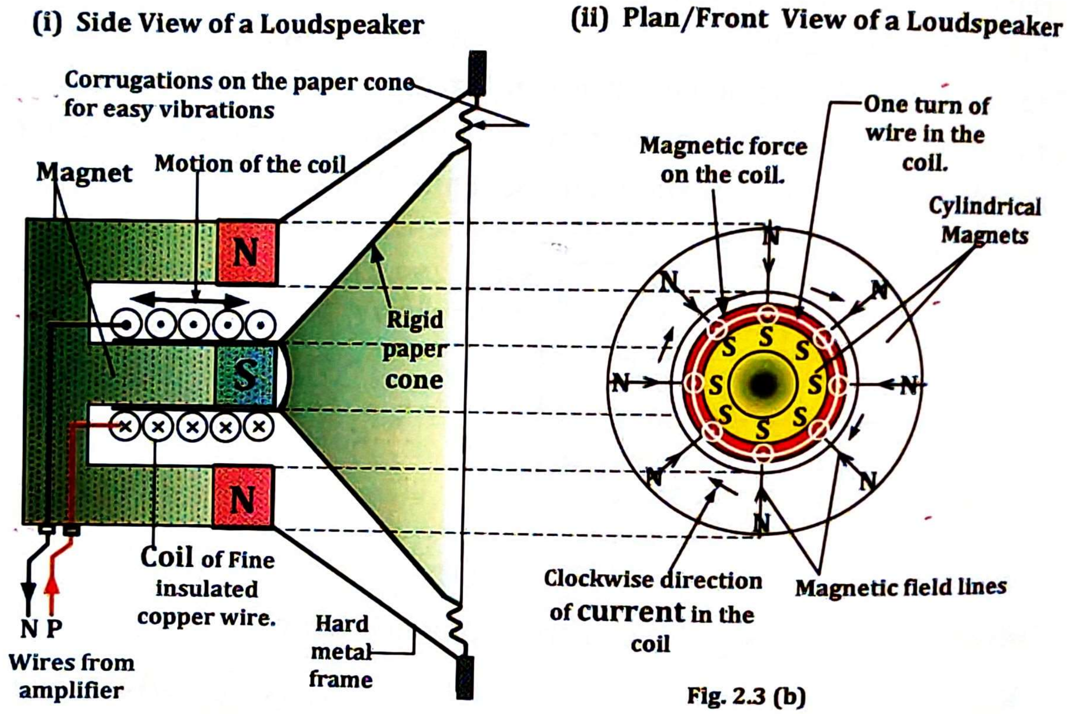


Fig. 2.3 (b)

Mode of operation of a moving coil loud speaker: (How it works)

- A varying Current from the amplifier as provided by a given source (*e.g signal vibrator, microphone, radio or record player*), is fed into the coil via the leads (terminals) of the loud speaker.
- The coil is wound on a cylindrical insulating paper that leaves a small air gap between the coil and the cylindrical South pole of a permanent magnet, that provides a radial magnetic field from the north pole radially into the south pole.
- The ***coil*** that is attached to the rigid paper cone, perpendicular to the magnetic field, experiences; a magnetic force, F, (By Fleming's Left Hand Rule) that moves the coil outward and hence pushing a large mass of air attached to the cone out wards. i.e. ($F = BIL$, *where L is total the length of the wire in the coil*)
- When the current in the coil, say drops to zero, the springy corrugations on the paper cone, pull back the coil to its equilibrium central position.

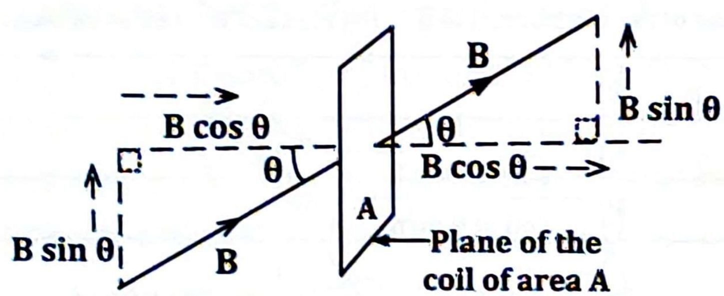
- The process repeats itself by moving the coil in and out of the cylindrical magnet, depending on the direction of flow of current through the coil, and its magnitude.
- As the current changes magnitude and direction, the coil vibrates back and forth setting the neighbouring air molecules adjacent to the cone into vibration at the same frequency as of the source.
- The air movement to and from causes the vibration of the eardrum of the listener at the same frequency, hence, producing sound in the ear.

NB, *Even if the coil is not a straight wire, the magnetic field direction is such that the magnetic force on every part of the coil is in the same direction (in or out)*

Magnetic Flux (ϕ)

Definition: Magnetic flux - is the product of the magnitude of magnetic flux density and the area it links perpendicularly i.e. $\phi = B \times A = BA$

Suppose the magnetic field lines are incident onto the plane of the coil at an angle θ as shown on the diagram below.



Magnetic flux , $\phi = BA\cos\theta$

Fig. 2.3 (c)

SI unit is weber (Wb)

Definition

The ***weber*** is the magnetic flux that passes normally through an area of 1m^2 when the magnetic flux density linking the coil is 1T.

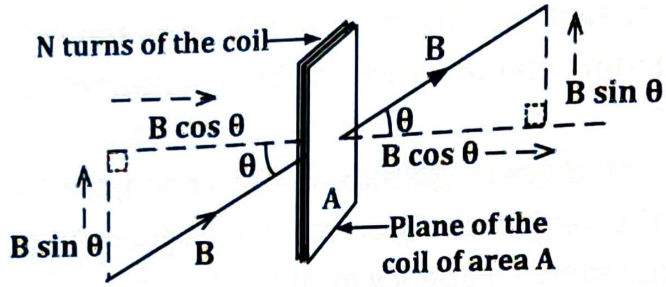
Magnetic Flux Linkage $N\phi$ = NAB or NAB cos θ

Definition

Magnetic flux linkage is the product of total magnetic flux density and the total area it links perpendicularly.

Suppose a coil has N turns each of area A being threaded normally by a magnetic field of flux density, B, tesla. **Magnetic Flux Linkage $N\phi$ = NAB**

If the plane of N turns of the coil make an angle θ with the magnetic field, as shown on the diagram below, then



Magnetic flux linkage, $N\phi = NBA\cos\theta$

Fig. 2.3 (d)

Magnetic Flux Linkage $N\phi = NAB\cos\theta$

Magnetic Flux – charge relationship $\left[Q = \frac{NAB\,(1-\cos\theta)}{R}\right]$

Suppose a coil of wire of N – turns each having a plane of area, A, has its free ends joined together to form a continuous loop of wire via a calibrated Ballistic galvanometer, **BG.** Let the plane of the coil initially be normal to a uniform magnetic field of flux density, **B** and after a short while, the coil if flipped about an axis parallel to the plane of the coil, through an angle θ as shown below.

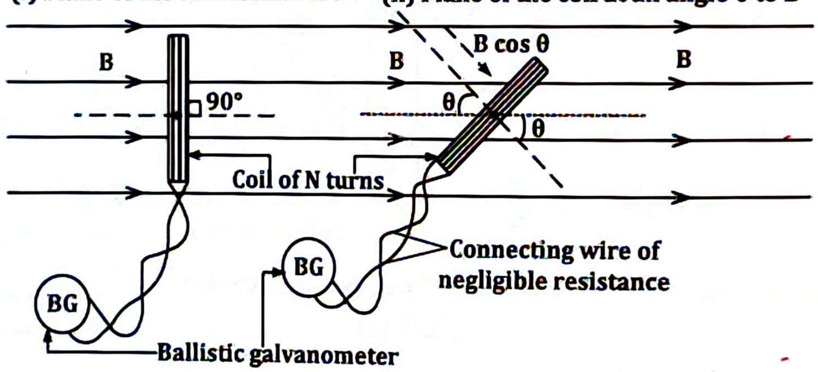


Fig. 2.3 (e)

- Whenever the magnetic flux linking the plane of the coil changes, wth time, an e.m.f. gets induced in the coil in acordance with the laws of electrmagnetic induction.
- When the coil is flipped about an axis through its centre parallel to its plane,the e.m.f. induced $\boldsymbol{E} = -\frac{d(N\emptyset)}{dt}$ *where* $E = IR$

$\therefore IR = -\frac{d(N\emptyset)}{dt} \Rightarrow I = -\frac{N\,d(\emptyset)}{R\,dt}$ *but current,* $I = \frac{dQ}{dt}$

$\therefore \frac{dQ}{dt} = -\frac{N\,d(\emptyset)}{R\,dt} \Rightarrow \int_0^Q dQ = -\frac{N}{R}\int_{\emptyset_1}^{\emptyset_2} d\emptyset$

$\therefore Q = -\frac{N}{R}(\emptyset_2 - \emptyset_1)$ *but* $\emptyset_2 = BA\cos\theta$ *and* $\emptyset_1 = BA$

$\Rightarrow$ ***Induced charge,*** $\boldsymbol{Q = -\frac{NBA}{R}(\cos\theta - 1)} \ldots\ldots\ldots\ldots (i)$

<u>Special Cases</u>

(i) When the ***coil is turned through 90°*** from initial position of maximum threading of the plane of the coil, θ = 90° and from (i) above, it implies

$$\textit{Induced charge}, Q = \frac{NBA}{R} \quad since \ \cos 90° = 0 \ldots\ldots\ldots (ii)$$

(ii) When the ***coil is turned through 180°*** from initial position of maximum threading of the plane of the coil, θ = 180° and from (i) above, it implies

$$\textit{Induced charge}, Q = \frac{2\,NBA}{R} \quad since \ \cos 180° = -1 \ldots\ldots\ldots (iii)$$

(iii) When the ***coil is turned through 60°*** from initial position of maximum threading of the plane of the coil, θ = 60° and from (i) above, it implies

$$\textit{Induced charge}, Q = \frac{NBA}{2\,R} \quad since \ \cos 60° = \frac{1}{2} \ldots\ldots\ldots (iv)$$

(iv) When the ***coil is turned through 120°*** from initial position of maximum threading of the plane of the coil, θ = 120° and from (i) above, it implies

$$\textit{Induced charge}, Q = \frac{3NBA}{2\,R} \quad since \ \cos 120° = -\frac{1}{2} \ldots\ldots (v)$$

(v) Generall when the ***coil is turned through any angle θ,*** from initial position of maximum threading of the plane of the coil, to the final position when the plane of the coil makes an angle, θ from (i) above, it implies $\textit{Induced charge}, Q = \frac{NBA}{R}(1 - \cos\theta) \ldots\ldots\ldots (vi)$

(vi) When the ends of the coil are connected to a calibrated Ballistic galvonometer, where the total resistance of the coil, the galvanometer and the circuit equals **R , a deflection, β on the B.G is proportional to the charge, Q set in motion.**

$$\textit{Induced charge}, Q = \frac{NBA}{R}(1 - \cos\theta) = k\beta \ldots\ldots\ldots (vii)$$

Applications of the Magnetic Flux – Charge relationship

The search coil

is a small flat coil of fine insulated wire made of a large number of turns. Those used in schools usually have between 500 to 2000 turns withan average diameter of 0.5 cm.

It is mounted on an insulating handle with the leads enclosed inside the insulating handle.

1. Measurement of Magnetic flux density B, due to a straight wire carrying a current in air

Whenever a straight wire is connected to a d.c source and a circuit in switched on, a rapidly changing current flowing in the wire during the closing or opening up of the switch causes an e.m.f. to be induced in a closed coil of wire smartly placed in the region of changing magnetic field.

The charge set in motion in the closed loop of the coil is directly proportional to the deflection it causes on a B.G. connected in series with the small plane circular coil (searh coil).

Procedure

- A search coil of known geometry (i.e. Known number of turns N, and known are of the plane, A) is connected in series with a calibrated ballistic galvanometer of known sensitivity constant, k.
- The experiment is then set up as shown on the diagram in the figure 2.3 (f)

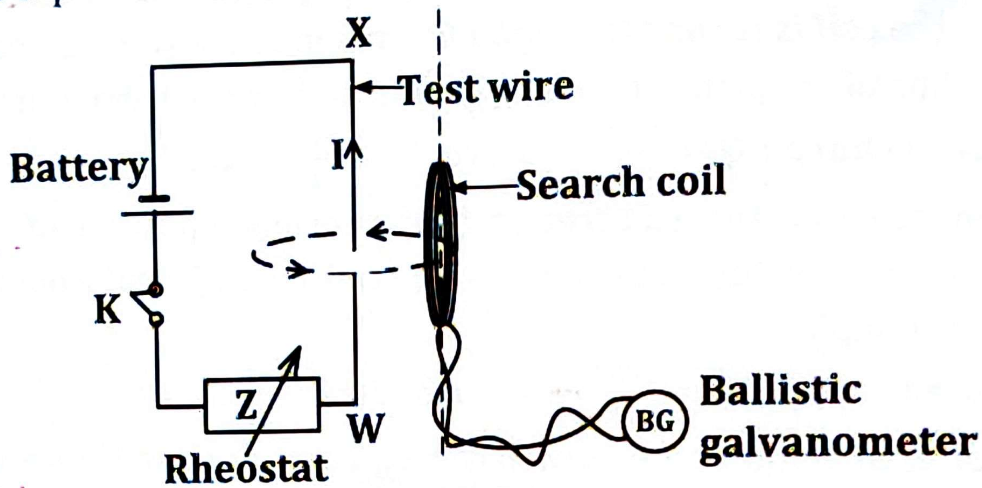


Fig. 2.3 (f)

- The seach coil connected in series with the B.G. is placed with its plane parallel to the test wire WX.
- Using a suitable setting of the rheostat Z, switch K is closed and the maximum deflection, θ_1 on the scale of the B.G. is noted.
- After a short while when the B.G. registers a steady zero deflection, while using the same setting of rheostat, Z, the switch K is opened.
- The maximum deflection θ_2 on the B.G. scale in the opposite direction is noted.
- The average deflection, $\theta = \left(\frac{\theta_1 + \theta_2}{2}\right)$ *is then calculated.*
- The magnetic flux density, at a specified perpendicular distance, d, from the straight wire, WX carrying a current, I, is then calculated from the expression, $B = \frac{Rk\theta}{NA}$ ……………………… (i)

Where, R = Total resistance of the search coil + Ballistic galvanometer.

N = Number of turns of the search coil.

A = Area of the plane of the search coil.

K = Sensitivity constant of the B.G. expressed in (C rad^{-1})

NB: In case the ***B.G. does not have a known value of, k,*** a standard capacitor of known capacitance, C_S is charged by connecting it across a known p.d. V_S as shown on the diagram in the figure 2.3 (g), charge it, by closing switch K_1 while switch K_2 is left open.

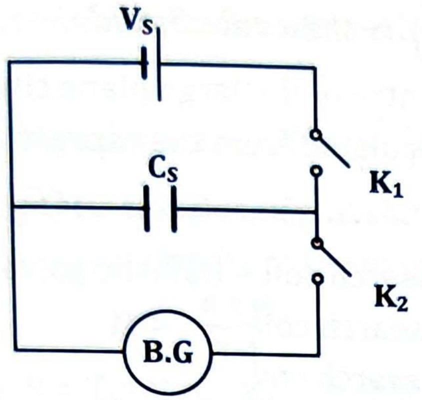


Fig. 2.3 (g)

After a short while, when the capacitor has charged fully, switch K_1 is opened and K_2 is then closed. The capacitor then discharges through the B,G. and the maximum deflection, θ_m is noted, from which the sensitivity constant, **k**, is determined from, $k = \frac{C_S V_S}{\theta_m}$ …………………. *(ii)*

Substituting (ii) into (i), the magnetic flux density at a distance, d, from a straight wire, carrying a current, I, is calculated from:

$$B = \left(\frac{R\, C_S\, V_S}{N\, A}\right)\left(\frac{\theta}{\theta_m}\right) \quad \ldots\ldots\ldots\ldots (iii)$$

1. Measurement of Magnetic flux density B, at the centre of a plane circular coil of N_1 turns, area A_1, carrying a current, I_1, in air

Procedure

- A search coil of known geometry (i.e. Known number of turns **N**, and known are of the plane, **A**) is connected in series with a calibrated ballistic galvanometer of known sensitivity constant, **k**.
- The experiment is then set up as shown on the diagram in the figure 2.3 (h)

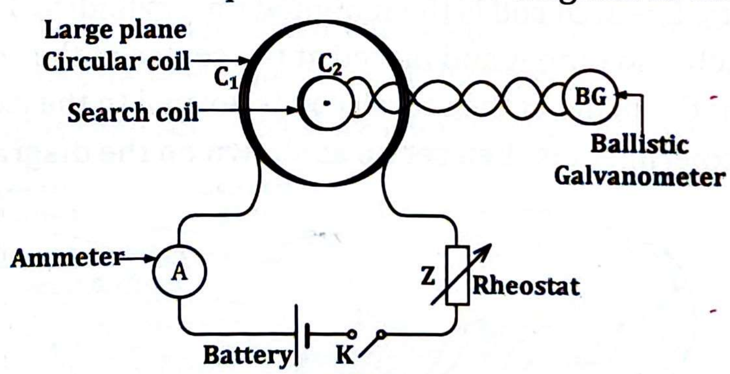


Fig. 2.3 (h)

- The seach coil C_2 connected in series with the B.G. is placed at the centre of the large plane circular coil C_1 with their planes coinciding (parallel).
- Using a suitable setting of the rheostat Z, switch K is closed and the maximum deflection, θ_1 on the scale of the B.G. is noted.
- After a short while when the B.G. registers a steady zero deflection, while using the same setting of rheostat, Z, the switch K is opened.
- The maximum deflection θ_2 on the B.G. scale in the opposite direction is noted.

- The average deflection, $\theta = \left(\frac{\theta_1 + \theta_2}{2}\right)$ *is then calculated.*
- The magnetic flux density, at the centre of the large plane circular coil C_1, and carrying a current, I, is then calculated from the expression,

$$B = \frac{R k \theta}{N A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Where, R = Total resistance of the search coil + Ballistic galvanometer.
N = Number of turns of the search coil.
A = Area of the plane of the search coil.
K = Sensitivity constant of the B.G. expressed in ($C\ rad^{-1}$)

NB: In case the ***B.G. does not have a known value of, k***, a standard capacitor of known capacitance, C_S is charged by connecting it across a known p.d. V_S, and then discharge it through the B.G, with the maximum deflection, θ_m noted. Then from, $C_S V_S = k\,\theta_m \Rightarrow k = \frac{C_S V_S}{\theta_m} \ldots\ldots\ldots (ii)$

Then substituting (ii) into (i), the magnetic flux denssity B at the centre of a large plane circular coil C_1 is calculated from equation (iii) below.

$$B = \left(\frac{R C_S V_S}{N A}\right)\left(\frac{\theta}{\theta_m}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

2. Measurement of Magnetic flux density B, at the centre of a long Solenoid of n turns per metre, carrying a current, I_S in air

Procedure

- A search coil of known geometry (i.e. Known number of turns **N**, and known are of the plane, **A**) is connected in series with a calibrated ballistic galvanometer (B.G) of known sensitivity constant, **k**.
- The small search coil is the mounted on a cylindrical manila tube and carefully slid onto it and placed at the centre of the solenoid along its axis, so that the plane of the search coil is normal to the axis of the solenoid.
- The experiment is then set up as shown on the diagram in the figure 2.3(i).

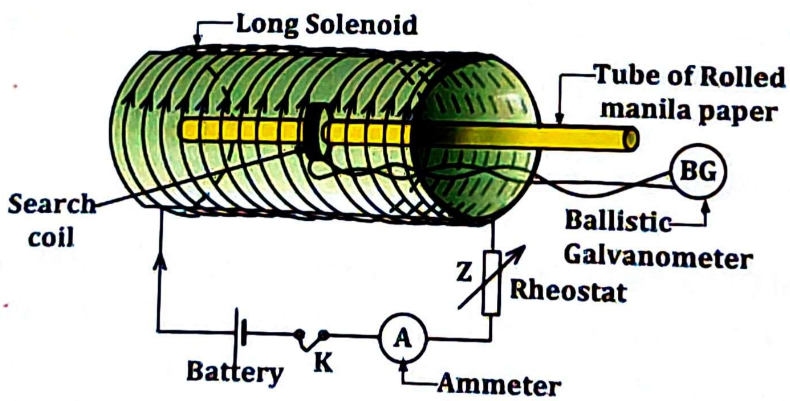


Fig. 2.3 (i)

- Using a suitable setting of the rheostat Z, switch K is closed and the maximum deflection, θ_1 on the scale of the B.G. is noted.
- After a short while, i.e. when the B.G. registers a steady zero deflection, while using the same setting of rheostat, Z, the switch K is opened.

- The maximum deflection θ_2 on the B.G. scale in the opposite direction is noted.
- The average deflection, $\theta = \left(\frac{\theta_1 + \theta_2}{2}\right)$ *is then calculated.*
- The magnetic flux density, at the centre of the solenoid, carrying a current, I, is then calculated from the expression,

$$B = \frac{R k \theta}{N A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Where, R = Total resistance of the search coil + Ballistic galvanometer.
N = Number of turns of the search coil.
A = Area of the plane of the search coil.
K = Sensitivity constant of the B.G. expressed in (C rad^{-1})

NB: In case the ***B.G. does not have a known value of, k***, a standard capacitor of known capacitance, **C_S** is charged by connecting it across a known p.d. **V_S**, and then discharge it through the B.G, with the maximum deflection, **θ_m** noted. Then from, $C_S V_S = k\,\theta_m \Rightarrow k = \frac{C_S V_S}{\theta_m}$ ………… (ii)

Then substituting (ii) into (i), the magnetic flux denssity B at the centre of a solenoid is calculated from equation (iii) below.

$$B = \left(\frac{R C_S V_S}{N A}\right)\left(\frac{\theta}{\theta_m}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

3. Magnetic flux density between the pole pieces of a U – shaped magnet

- A search coil of known geometry (i.e. known number of turns **N**, and known are of the plane, **A**) is connected in series with a calibrated ballistic galvanometer (B.G) of known sensitivity constant, **k**.
- The small search coil is then placed with its plane initially, normal to the magnetic field lines between the poles of the magnet.
- The experiment is then set up as shown on the diagram in the figure 2.3 (j)

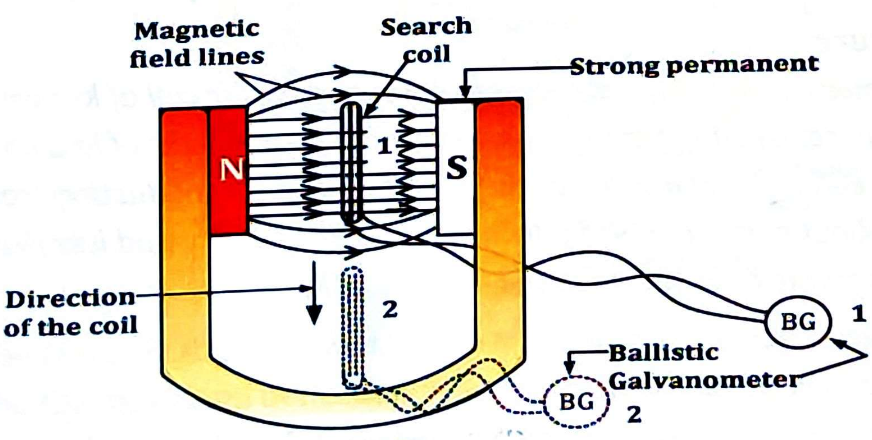


Fig. 2.3 (j)

- Starting with the plane of the coil normal to the magnetic field between the pole pieces of the magnet, the coil is *smartly* pulled ***completely out*** of the magnetic field (i.e. without changing the orientation of the plane of the search coil) and the maximum deflection, θ_1 on the scale of the B.G. is noted.

- After a short while, i.e. when the B.G. registers a steady zero deflection, the experiment can be repeated.
- The new maximum deflection θ_2 on the B.G. scale then is noted.
- The average deflection, $\theta = \left(\frac{\theta_1 + \theta_2}{2}\right)$ *is then calculated.*
- The magnetic flux density, between the poles of the magnet, is then calculated from the expression,

$$B = \frac{R\,k\,\theta}{N\,A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (i)$$

Where, R = Total resistance of the search coil + Ballistic galvanometer.

N = Number of turns of the search coil.

A = Area of the plane of the search coil.

K = Sensitivity constant of the B.G. expressed in (C rad^{-1})

NB: In case the ***B.G. does not have a known value of, k***, a standard capacitor of known capacitance, C_S is charged by connecting it across a known p.d. V_S, and then discharge it through the B.G, with the maximum deflection, θ_m noted. Then from, $C_S V_S = k\,\theta_m \Rightarrow k = \frac{C_S V_S}{\theta_m}$ ……… (ii)

Then substituting (ii) into (i), the magnetic flux denssity B at the centre of a solenoid is calculated from equation (iii) below.

$$B = \left(\frac{R\,C_S\,V_S}{N\,A}\right)\left(\frac{\theta}{\theta_m}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (iii)$$

4. Measurement of the Earth's magnetic field using an Earth Inductor

*An Earth inductor is a device that can be used to measure both **the horizontal component** and the vertical coponent of the Earth's magnetic together with the angle of dip at any paticular location over the Earth's surface.*

The Structure

The instrument usually consists of a small plane circular coil of known geometry connected in series with a callibrated ballistic galvanometer of known constant. The search coil is freely pivoted at the centre of a non – conducting frame F_1 and rotatable about pivots P_1 and P_2 drilled onto a more bulky and less flexible non - conducting frame F_2 as shown in the figure 2.3 (k)

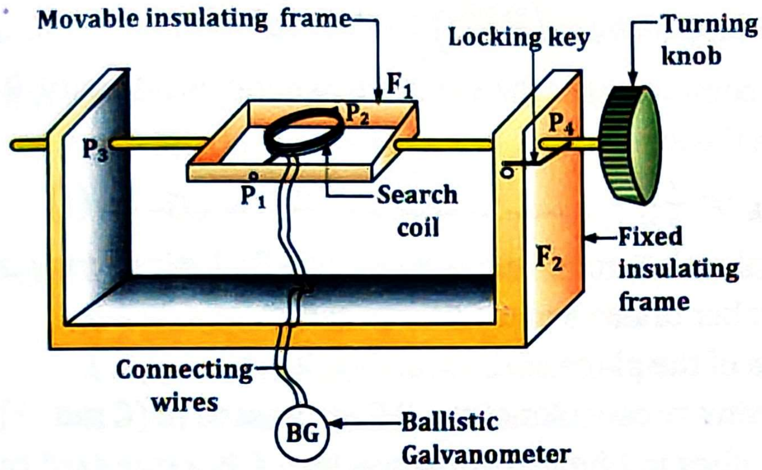


Fig. 2.3 (k)

The instrument can be adapted to measure

(i) *The Horizontal component B_H of the Earth's magnetic field.*

(ii) *The Vertical component B_V of the Earth's magnetic field.*

(iii) *The Angle of dip, α.*

(a) Measurement of the Horizontal component B_h of the Earth's magnetic field.

- A search coil of known geometry (i.e. known number of turns, N and area A) is connected in series with a ballistic galvanometer (B.G) of known sensitivity, k, and its plane is made to coincide with that of the movable frame, F_1.
- With the help of a compass needle or a freely suspended bar magnet that has set, the planes of the search coil and F_1 are placed perpendicular to the magnetic meridian and vertical so as to be threaded normally by the horizontal component B_h as shown in the figure 2.3 (l).

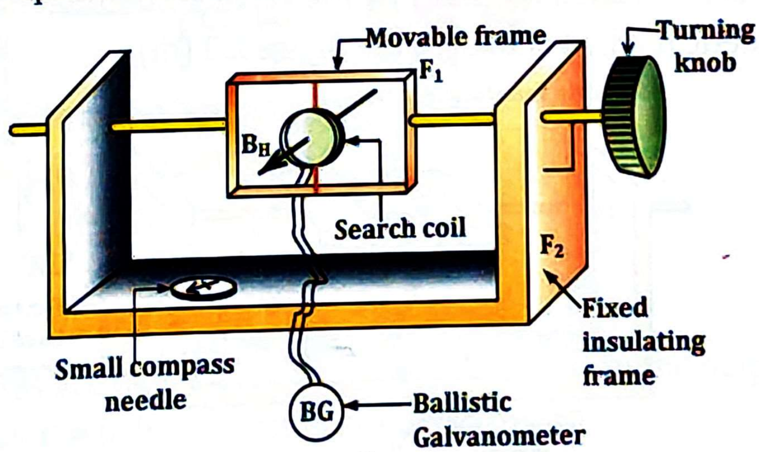


Fig. 2.3 (l)

- The frame F_1 together with the search coil, are quickly turned through 180° and the maximum deflection θ_1 on the scale of the BG is noted.
- The coil is taken back to its original position and left for a short while for the pointer of the BG to return to the zero position.
- The coil is again turned through 180° in the opposite direction, and the maximum deflection θ_2 in the opposite side of the scale of the BG is noted.

- The average deflection, $\theta_h = \left(\frac{\theta_1 + \theta_2}{2}\right)$ *is then calculated.*
- The horizontal component of the Earth's magnetic flux density, B_H, is then calculated from the expression,

$$B_h = \frac{R\,k\,\theta_h}{2\,N\,A} \text{ .... } (i)$$

Where, R = Total resistance of the search coil + Ballistic galvanometer.
N = Number of turns of the search coil.
A = Area of the plane of the search coil.
K = Sensitivity constant of the B.G. expressed in ($C\,rad^{-1}$)

NB: In case the ***B.G. does not have a known value of, k***, a standard capacitor of known capacitance, C_S is charged by connecting it across a known p.d. V_S, and then discharge it through the B.G, with the maximum deflection, θ_m noted. Then from, $C_S V_S = k\,\theta_S \Rightarrow k = \frac{C_S V_S}{\theta_S}$ .... (ii)

Then substituting (ii) into (i), the magnetic flux denssity B at the centre of a solenoid is calculated from equation (iii) below.

$$B_h = \left(\frac{R\,C_S\,V_S}{2\,N\,A}\right)\left(\frac{\theta_h}{\theta_m}\right) \text{ .... } (iii)$$

(b) Measurement of the Vertical component B_V of the Earth's magnetic field.

- A search coil of known geometry (i.e. known number of turns, N and area A) is connected in series with a ballistic galvanometer (B.G) of known sensitivity, k, and its plane is made to coincide with that of the movable frame, F_1.
- The planes of the search coil and frame F_1 are placed horizontally so that they are threaded normally by the vertical component B_V of the Earth's magnetic field, as shown in the figure 2.3 (m)

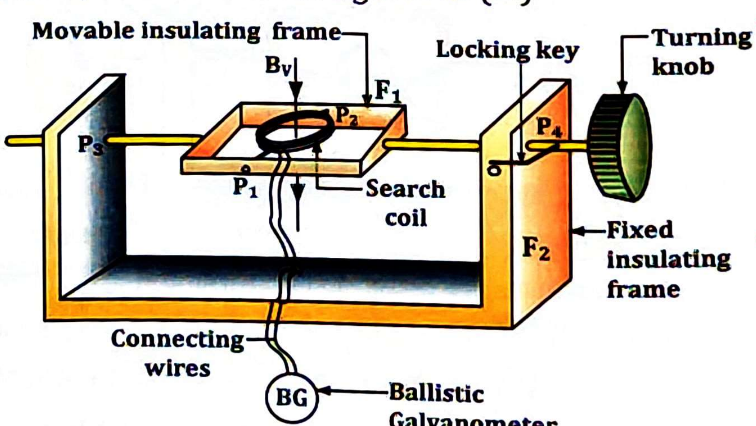


Fig. 2.3 (m)

- The frame F_1 together with the search coil, are quickly turned through 180° and the maximum deflection θ_1 on the scale of the BG is noted.
- The coil is taken back to its original position and left for a short while, for the pointer of the BG to return to the zero position.
- The coil is again turned through 180° in the opposite direction, and the maximum deflection θ_2 in the opposite side of the scale of the BG is noted.

- The average deflection, $\theta_V = \left(\frac{\theta_1 + \theta_2}{2}\right)$ *is then calculated.*
- The vertical component of the Earth's magnetic flux density, Bv, is then calculated from the expression,

$$B_V = \frac{R\,k\,\theta_V}{2\,N\,A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Where, R = Total resistance of the search coil + Ballistic galvanometer.
N = Number of turns of the search coil.
A = Area of the plane of the search coil.
K = Sensitivity constant of the B.G. expressed in ($C\ rad^{-1}$)

NB: In case the ***B.G. does not have a known value of, k***, a standard capacitor of known capacitance, C_S is charged by connecting it across a known p.d. V_S, and then discharge it through the B.G, with the maximum deflection, θ_m noted. Then from, $C_S V_S = k\,\theta_S \Rightarrow k = \frac{C_S V_S}{\theta_S}$ ……………… (ii)

Then substituting (ii) into (i), the magnetic flux denssity B at the centre of a solenoid is calculated from equation (iii) below.

$$B_V = \left(\frac{R\,C_S\,V_S}{2\,N\,A}\right)\left(\frac{\theta_V}{\theta_m}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

(c) Measurement of the angle of dip of the Earth's magnetic field

- A search coil of known geometry is connected in series with a ballistic galvanometer (BG) and its plane is made to coincide with that of the movable frame, F_1.

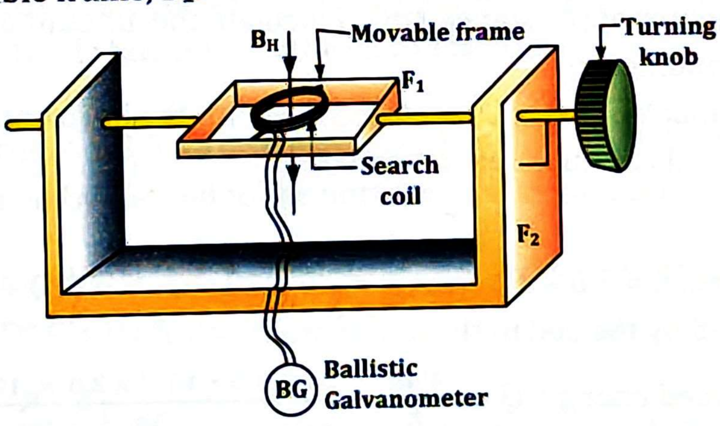


Fig. 2.3 (n)

- The two planes are then aligned and perpendicular to the magnetic meridian with the help of a compass needle.
- The movable frame is then used to rotate the search coil through 180° about a horizontal axis.
- The maximum deflection θ_h on the scale of the BG is noted.
- The planes of the search coil and the movable frame are now made horizontal so as to be threaded by the Earth's vertical magnetic field, B_V.
- The movable frame is then used to rotate the search coil through 180° about a horizontal axis.
- The maximum deflection θ_V on the scale of the BG is noted.

- The angle of dip α, is then calculated from the expression

$$\tan\alpha = \frac{B_v}{B_h} = \frac{\theta_v}{\theta_h} \qquad \Rightarrow \alpha = \tan^{-1}\left(\frac{\theta_v}{\theta_h}\right)$$

Examples

1. A plane circular coil of 20 turns each of mean radius 5.0 cm is placed on a flat horizontal surface in the Earth's magnetic field of flux density 55μT at an angle of dip of 60°. Determine the magnetic flux linkage of the coil.

Solution:

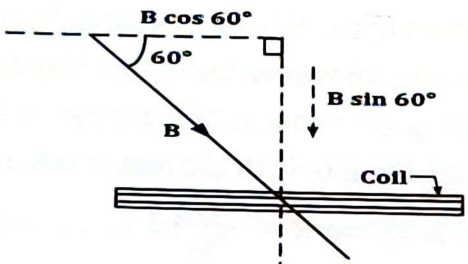


Fig. 2.3 (o)

N = 20 turns, r = 5.0 cm, B = 55 μT = 55×10^{-6} T, angle of dip α = 60°

$\Phi = NAB \sin 60°$

$= 20 \times \pi (0.05)^2 \times 55 \times 10^{-6} \times \sin 60°$

$= \mathbf{7.48 \times 10^{-6}\,Wb}$

2. A plane circular coil of 20 turns and area 3.0×10^{-7} m² is placed in a horizontal magnetic field of flux density 0.02 T so that the magnetic flux is normal to all the turns. The coil has a resistance of 10 Ω and is connected to a ballistic galvanometer of resistance 40Ω . Calculate the amount of charge sent through the galvanometer;
   (i) When the plane of the coil is rotated at 90° to the horizontal.
   (ii) When the coil is completely reversed.

Solution:

N = 20 turns, A = 3.0×10^{-7} m², B = 2.0×10^{-2} T, R = (10 + 40) = 50 Ω

(i) Angle turned by the coil to the horizontal, θ = 90°

Using, $Induced\ charge, Q = \frac{NAB}{R} = \frac{20 \times 3.0 \times 10^{-7} \times 2.0 \times 10^{-2}}{50}$

$\therefore Q = 2.40 \times 10^{-9}\ C\ or\ 2.4\ nC$

(ii) The coil being reversed ⟹ *Turning it through* 180°

$\therefore Induced\ charge, Q = \frac{2NAB}{R} = \frac{2 \times 20 \times 3.0 \times 10^{-7} \times 2.0 \times 10^{-2}}{50}$

$\therefore Q = 4.80 \times 10^{-9}\ C\ or\ 4.8\ nC$

3. A Circular coil of 50 turns and radius 0.5 m is placed with its plane perpendicular to the earth's magnetic meridian. It is connected to a ballistic galvanometer of sensitivity 5.7 10³ rad. C⁻¹ and circuit resistance of 100 Ω. When the coil is rotated through 180° about a horizontal axis, the galvanometer deflects through 0.8 rads.

Calculate the,

(i) Horizontal component of the earth's magnetic field.

(ii) P.d across the solenoid of 2000 turns and resistance 5 Ω that produces the same magnetic flux density as that calculated in (i) above.

Solution:

(i) N = 50 turns, A = 7.85 × 10^{-1} m^2, B_H = ? R = 100 Ω, k = 5.7 × 10^3 rad C^{-1}

Angle turned by the coil to the horizontal, θ = 180° and $\beta = 0.8\ rad$

Using, $Induced\ charge, Q = \frac{2NAB}{R} = k\beta$

$$\frac{2 \times 50 \times 7.85 \times 10^{-1} \times B}{100} = 0.8 \times 5.7 \times 10^3$$

$\therefore \boldsymbol{B_h = 5.81 \times 10^3\ T}$

(ii) Using $B = \mu_0 nI,\ where\ I = \frac{V}{R} = \frac{V}{5}, n = 2000\ and$

$B = B_h = 5.81 \times 10^3\ T$

$$\Rightarrow \mu_0 n \times \frac{V}{R} = B_h \Rightarrow V = \frac{R \times B_h}{\mu_0\ n} = \frac{5 \times 5.81 \times 10^3}{4\pi \times 10^{-7} \times 2000}$$

$\therefore \boldsymbol{P.d, V = 1.16 \times 10^7\ V}$

4. A search coil has 40 turns of wire and cross sectional area 5 cm^2. The coil is connected to a ballistic galvanometer and then placed with its plane perpendicular to a uniform magnetic field of flux density B. When the coil is smartly withdrawn from the field, the galvanometer gives a deflection of 240 divisions. When a capacitor of 4 µF is charged to 20 V and then discharged through the circuit, the galvanometer deflection is 180 divisions. Find the value of B, if the total resistance of the circuit is 20 Ω.

Solution

N = 40 turns, A = 5.0 × 10^{-4} m^2, B_H = ? R = 20 Ω, k = ? $C_S = 4$ µF, $V_S = 20\ V$

Angle turned by the coil to the horizontal, θ ≅ 90° and

Deflection on the B.G $\boldsymbol{\beta_1} = 240\ divisions$ and when the Capacitor is discharged through the BG, $\boldsymbol{\beta_2} = 180\ divisions$

$$\boldsymbol{B} = \frac{R\,k\,\beta_1}{N\,A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Where, R = Total resistance of the search coil + Ballistic galvanometer.

N = Number of turns of the search coil.

A = Area of the plane of the search coil.

k = Sensitivity constant of the B.G. expressed in (C rad^{-1})

NB: In case the ***B.G. does not have a known value of,* k**, a standard capacitor of known capacitance, **C_S** is charged by connecting it across a known p.d. **V_S**, and then discharge it through the B.G, with the maximum deflection, θ_m noted. Then from, $\boldsymbol{C_S\,V_S = k\,\beta_2} \Rightarrow \boldsymbol{k} = \frac{C_S\,V_S}{\beta_2} \ldots\ldots\ldots (ii)$

Then substituting (ii) into (i), the magnetic flux denssity B at the centre of a solenoid is calculated from equation (iii) below.

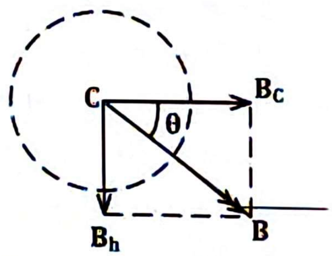
$$B = \left(\frac{R\,C_S\,V_S}{N\,A}\right)\left(\frac{\beta_1}{\beta_2}\right) \ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

$$B = \frac{20 \times 4 \times 10^{-6} \times 20 \times 240}{40 \times 5.0 \times 10^{-4} \times 180} = 0.107\ T$$

$$\therefore B_h = 1.07 \times 10^{-1}\ T$$

2.4 THE TANGENT GALVANOMETER (T.G)

This is an instrument used for measuring a horizontal magnetic field placed perpendicular to the Earth's horizontal component of the magnetic field, $\mathbf{B_h}$. It can be used for measuring the horizontal component of the Earth's magnetic field, $\mathbf{B_h}$ when a current is passed through the coil of the tangent galvanometer of known geometry, (i.e. known number of turns, N and known radius r, or area A of the plane of the coil). Since the two magnetic fields are perpendicular (Tangential) to each other, the compass needle (Two pointed end aluminium pointers) respond to both fields and provide the resultant of these fields.



$B^2 = (B_h)^2 + (B_C)^2$

$B = \sqrt{[(B_h)^2 + (B_C)^2]}$

Is the resultant magnetic field

Fig. 2.4 (a)

The Structure of a Tangent Galvanometer:

The tangent galvanometer consists of a **deflection magnetometer** mounted at the centre and along the vertical diameter of a large coil of known geometry as shown on the diagram in the figure 2.4 (b)

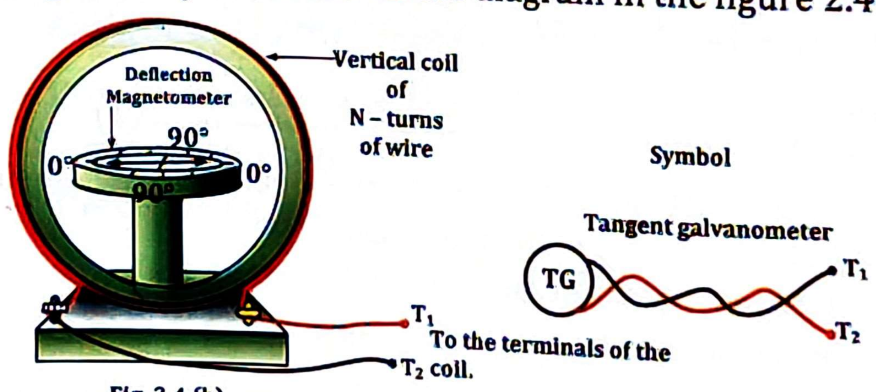


Fig. 2.4 (b)

The Deflection Magnetometer

Consists of a freely suspended or pivoted ***small bar magnet*** or magnetic ***compass needle*** on ***a vertical axis*** inside an Aluminium frame having a

circular scale, over which the double pointed ***aluminium pointers*** slide over or move adjacent to.

The scale is graduated in degrees ranging from 0° to 90° at the centre and similar graduations on the opposite end of the magnetometer.

The Deflection Magnetometer

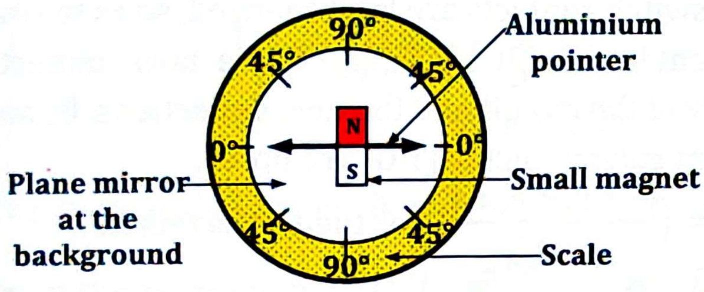


Fig. 2.4 (c)

The pointer can rotate over the circular scale. The deflection magnetometer is used to compare two magnetic flux densities, one of which being the horizontal component of the Earth's magnetic flux density, B_h.

The two fields B_h and any other horizontal flux density B are arranged at right angles to each other. The compass needle then sets itself, at an angle say θ to the original direction when it was in the field B_h alone.

The needle finally points in the direction of the resultant of B_h and B.

The angles of deflection θ_1 and θ_2 of the needle on the scale are measured and the average deflection, $\theta = \left(\frac{\theta_1 + \theta_2}{2}\right)$ is then determined, from which

The ratio, $\frac{B}{B_h} = \tan\theta$ *is obtained and used to determine,* B *or* B_h

(a) Measurement of Horizontal component of Earth's magnetic Flux density B_h, Using a Tangent galvanometer (TG).

- A coil of known geometry (*i.e. known number of turns, N and known radius, r*), containing a deflection magnetometer is placed with its plane in a magnetic meridian i.e. facing the Earth's magnetic North – South poles.

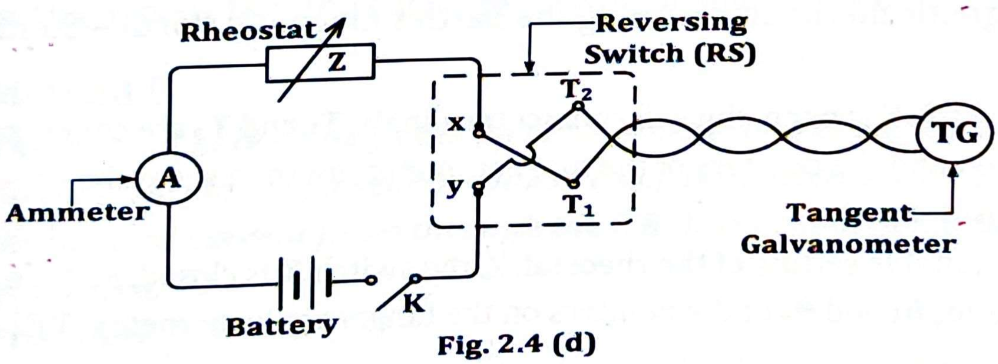


Fig. 2.4 (d)

- When switch K is open, the reversing switch contacts, T_1 and T_2 are connected to contacts **x** and **y** respectively of the circuit and the pointers of the magnetometer are then set at the 0° – 0° scale positions.

- Using a suitable setting of the rheostat, Z, the switch K is closed, and the deflections, θ_1 and θ_2 of the pointers on the tangent galvanometer (T.G) are noted.
- The steady current reading, **I** of the ammeter is also noted.
- Keeping the switch K, closed, and using the same setting of the rheostat, Z as in the first case, the reversing switch contacts are interchanged, so as to reverse the direction of flow of current in the coil i.e. **T₁** and **T₂** are now connected to contacts **y** and **x** respectively of the circuit and the new deflections, θ_3 and θ_4 of the pointers on the tangent galvanometer (T.G) are noted.
- The average deflection, $\theta = \left(\frac{\theta_1 + \theta_2 + \theta_3 + \theta_4}{4}\right)$ is then determined.

From, $\tan\theta = \frac{B_C}{B_h} \Rightarrow B_h = \frac{B_C}{\tan\theta}$

$\therefore \boldsymbol{B_h} = \frac{\mu_0 N I}{2 r \tan\theta}$ ***is then calculated using known values of:***

N = Number of turns of the coil.
r = Radius of each of the turns of the coil
$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1} (Permeability of free space) and
I = the current flowing through the coil.

NB, The use of the reversing switch (RS) is to improve on the accuracy of the results by enabling a second set of deflections to be obtained by reversing the direction of flow of current in the coil, hence reversing the deflections of the pointers as well.
The experiment however can be carried out using only one set of values in the absence of the reversing switch as may be observed in the next experiment.

(b) Measurement of the magnetic flux density, B_C at the centre of a coil, using a Tangent galvanometer (TG).

- A coil containing a deflection magnetometer is placed with its plane in the magnetic meridian i.e. facing the Earth's magnetic North – South poles.
- When switch K is open, the coil contact terminals, **T_1** and **T_2** are connected to the circuit and the pointers of the magnetometer are then set at the 0° – 0° scale positions.
- Using a suitable setting of the rheostat, Z, the switch **K** is closed, and the deflections, θ_1 and θ_2 of the pointers on the tangent galvanometer (T.G) are noted.

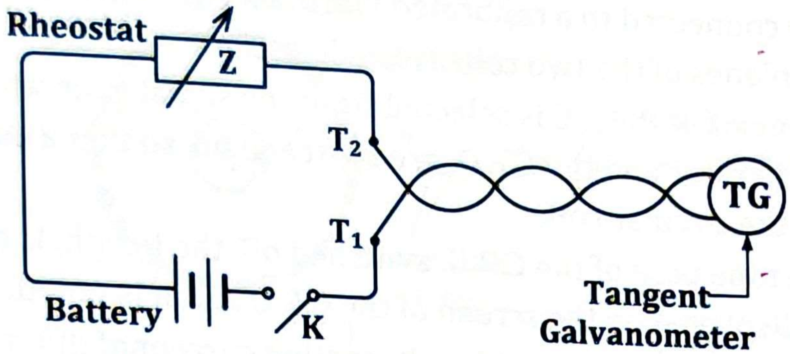


Fig. 2.4 (e)

- The average deflection, $\theta = \left(\frac{\theta_1 + \theta_2}{2}\right)$ is then determined.

  From the expression, $\tan\theta = \frac{B_c}{B_h} \Rightarrow B_c = B_h \tan\theta$

  $\therefore B_c = B_h \tan\theta$, **the magnetic flux density at the centre of the coil, *is then calculated using known values of the horizontal component of the Earth's magnetic field, B_h at that location.***

NB, In order to determine any other uniform horizontal magnetic field, B, the plane of the coil is placed in the magnetic meridian, the pointers of the magnetometer set at the 0° - 0° positions, and the test uniform magnetic field is arranged so as to be perpendicular to the plane of the coil. The deflections θ_1 and θ_2 are then noted and the average, deflection $\theta = \left(\frac{\theta_1 + \theta_2}{2}\right)$ is then determined.

From the expression, $\tan\theta = \frac{B}{B_h} \Rightarrow B = B_h \tan\theta$

$\therefore \boldsymbol{B = B_h \tan\theta}$, the magnetic flux density at the centre of the coil, *is then calculated using known values of the horizontal component of the Earth's magnetic field, B_H at that location.*

Variation of Magnetic Flux Density at the centre of a circular coil with current flowing through it using a search coil.

a) Method I

The method here employs the fact that whenever a changing magnetic flux threads the plane of a search coil, an e.m.f. is induced across the coil and the induced current flows through the C.R.O. is proportional to the e.m.f. generated.

Procedure:

- A search coil of known geometry is connected to Y – plates of a cathode ray oscilloscope whose time base is switched off.
- The search coil is then placed at the centre of a large circular coil whose free

ends are connected to a calibrated vibrating signal generator, in such a way that the planes of the two coils coincide as shown on the diagram.

- A frequency, f of the A.C is selected from the signal generator, then both the signal generator and the C.R.O. are switched on, so that a current passes through the circular coil.
- With the time base of the C.R.O. switched off, the length, L, of the vertical line trace is displayed on the screen of the C.R.O and it is noted.
- The experiment is repeated for alternating current of different rms values from a signal generator, each is passed through the circular coil,
- The length, L of the vertical line trace displayed on the screen of the C.R.O in each case is noted.

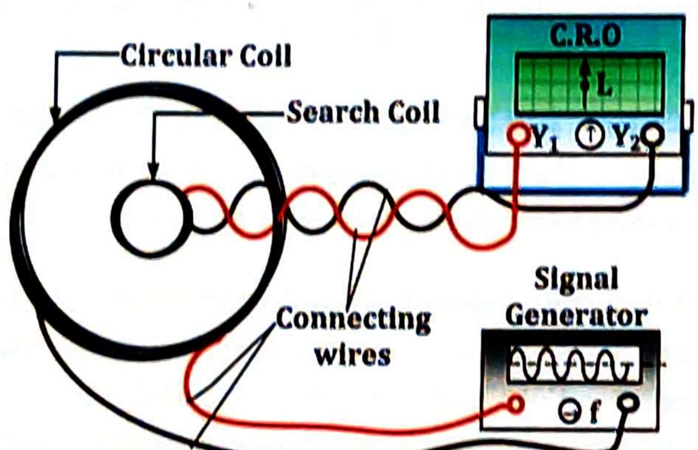


Fig. 2.4 (f)

- The results are then tabulated in a suitable table, including values of I_{rms} and L.
- A graph of L against, I_{rms} is then plotted and gives a straight line through the origin. This implies, $L \propto I_{rms}$ *but since,* $L \propto B$
- Hence, magnetic flux density at the centre of the coil, $B \propto I_{rms}$ is directly proportional to the current flowing through it.

(b) Method II

The method here also employs the fact that, whenever a changing magnetic flux threads the plane of a search coil, an e.m.f. is induced across the coil and the induced current flows through the ballistic galvanometer that is proportional to the e.m.f. generated.

Procedure

- A search coil of known geometry is connected in series with a ballistic galvanometer.
- The search coil is then placed at the centre of a large circular coil whose free ends are connected to a calibrated ballistic galvanometer, in such a way that the planes of the two coils coincide as shown on the diagram.

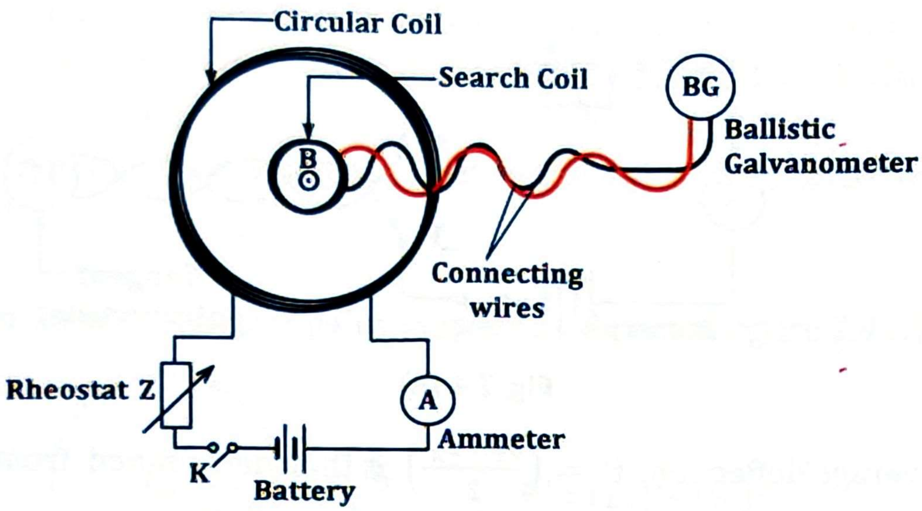


Fig. 2.4 (g)

- Switch K, is closed and current flowing through the circular coil is adjusted to a suitable value using rheostat Z.
- The maximum deflection θ_1 on the scale of the BG is noted, together with current **I** on the ammeter.
- The switch K is opened and the maximum deflection θ_2 of the BG in the opposite direction is noted
- The average deflection θ is noted, where, $\theta = \left(\frac{\theta_1 + \theta_2}{2}\right)$
- The experiment is repeated a number of times using different settings of Z, each time average deflection θ and the corresponding current **I** are noted.
- The results are then tabulated, in a suitable table of results including values of, $\theta_1, \theta_2, \theta$ and **I**.
- A graph of θ against **I** is then plotted and gives a straight line through the origin. $\Rightarrow \theta \propto I$
- *Since*, $B \propto \theta$ *and* $\theta \propto I \Rightarrow B \propto I$
  Hence, Magnetic flux density varies directly with current through the coil.

(c) Method III (using a tangent galvanometer)

- A coil containing a deflection magnetometer has its plane made to lie in the magnetic meridian i.e. facing the Earth's magnetic North – South poles.
- When switch K is open, the coil contact terminals, T_1 and T_2 are connected to the circuit and the pointers of the magnetometer are then set at the 0° – 0° positions.
- Switch K is then closed and using a suitable setting of the rheostat, Z, a suitable current, I, as measured on the ammeter A, is passed through the coil and deflections, θ_1 and θ_2 of the pointers on the tangent galvanometer (T.G) are noted., then switch K is opened

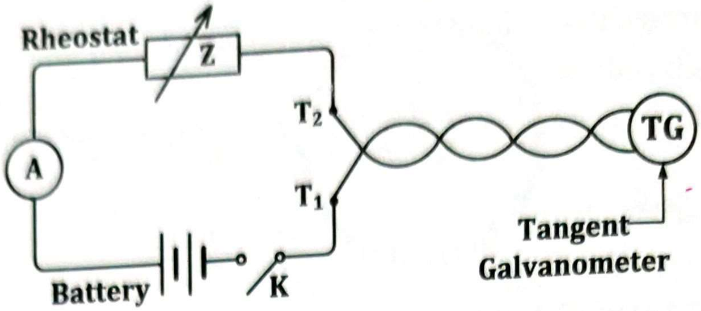


Fig. 2.4 (h)

- The average deflection, $\theta = \left(\frac{\theta_1 + \theta_2}{2}\right)$ is then determined, from which $\tan\theta$ is also determined From the expression, $\tan\theta = \frac{B_C}{B_h} \Rightarrow B_c = B_h \tan\theta$
- The experiment is repeated using different settings of Z and different currents flowing through the coil. In each case, the average deflection, θ on the TG is obtained.
- The results are tabulated in a suitable table, including values of, **θ_1, θ_2, θ, tan θ** and **I**.
- A graph of, **tan θ** against **I** is then plotted and gives a straight line through the origin. $\Rightarrow \tan\theta \propto I$
- *Since*, $\tan\theta \propto B_C$ *and* $\tan\theta \propto I \Rightarrow B_C \propto I$
- *Hence, Magnetic flux density varies directly with current through the coil.*

2.5 Examples & Exercises

1. A large plane circular coil of mean radius 5 cm and having 20 turns connected to a d.c. source, has a small compass needle suspended at its centre along its vertical diameter and lying along the horizontal diameter of the coil. When a current of 2A is passed through the coil, the needle deflects through 41°. When the current in the coil is reversed, it deflects through 39°. Find the horizontal component of the Earth's magnetic field.

   Solution:

   r = 5.0cm = 0.05 m, N = 20 turns, I = 2.0 A, $\theta_1 = 41°$, $\theta_2 = 39°$

   $\therefore$ *Average deflection*, $\theta = \frac{\theta_1 + \theta_2}{2} = 40°$

   Using tangent galvanometer, $\tan\theta = \frac{B_C}{B_h} = \frac{\mu_o N I}{2r B_h}$

   $$B_h = \frac{\mu_o N I}{2r \tan\theta} = \frac{4\pi \times 10^{-7} \times 20 \times 2.0}{2 \times 0.05 \times \tan 40°}$$

   $\therefore$ $\mathbf{B_h = 5.99 \times 10^{-4} T}$

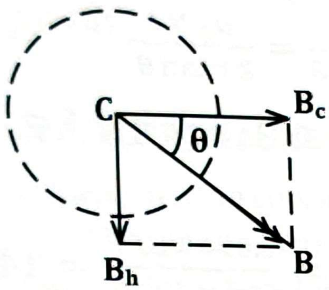
2. A circular coil of 10 turns and diameter 12.0 cm caries a current I. The coil is placed with its plane in the magnetic meridian. A small compass needle placed at the centre of the coil makes 30 oscillations per minute about a vertical axis.

When the current is switched off, it makes 15 oscillations per minute. If the horizontal component of the Earth's magnetic field is 2.0×10^{-5} T. Calculate the magnitude of current, I.

[Assume the square of the frequency f oscillation is proportional to the magnetic flux density].

Solution:

Using the set up of the tangent galvanometer as shown in figure 2.4 (i) below,



$B^2 = (B_h)^2 + (B_c)^2$

$B = \sqrt{[(B_h)^2 + (B_c)^2]}$

Is the resultant magnetic field

Fig. 2.4 (i)

$$\mathbf{N = 10,\ d = 12.0\ cm} \Rightarrow r = 6.0\ cm,\ B = kf_1^{\ 2} = \left(\frac{30}{60}\right)^2 = 0.50^2$$

$$B_h = kf_2^{\ 2} = \left(\frac{15}{60}\right)^2 = 0.25^2 \Rightarrow \frac{B}{B_h} = \left(\frac{0.50}{0.25}\right)^2 = 4$$

$$\therefore B = 4B_h \Rightarrow B_C = \sqrt{B^2 - B_h^{\ 2}} = \sqrt{(4B_h)^2 - B_h^{\ 2}} = B_h\sqrt{3}$$

$$But, B_c = \frac{\mu_0 N\,I}{2\,r} = B_h\sqrt{3} \Rightarrow I = \frac{2rB_h\sqrt{3}}{\mu_0 N} = \frac{2 \times 0.06 \times 2.0 \times 10^{-5} \times \sqrt{3}}{4\pi \times 10^{-7} \times 10}$$

$$\therefore \boldsymbol{I = 3.31 \times 10^{-1}}\ A$$

3. A capacitor of capacitance 2000 μF is fully charged to 10 V. When the capacitor is discharged through a ballistic galvanometer, the galvanometer gives a maximum deflection of 20 divisions. A coil of 25 turns, each of radius 10 cm is placed with its plane perpendicular to a uniform magnetic field. The coil is connected in series with the ballistic galvanometer. When the coil is rotated through 180°, the galvanometer gives a throw of 15 divisions. Calculate the magnetic flux density, if the total resistance in the circuit is 3 Ω.

Solution:

$C = 2000\ \mu F = 2.0 \times 10^{-3}\ F$, $\theta_m = 20$ div. $N = 25$ turns, $r = 0.10$ m, $\theta_S = 15$ div.

$R = 3\ \Omega$, $V_S = 10$ V

$$\text{Using } \boldsymbol{B} = \frac{R\,k\,\theta_S}{2\,N\,A}\ \ where\ k = \frac{C\,V_S}{\theta_m}$$

$$\Rightarrow B = \frac{R\,C\,V_S\,\theta_S}{2\,N\,A\,\theta_m} = \frac{3 \times 2.0 \times 10^{-3} \times 10 \times 15}{2 \times 25 \times \pi \times (0.10)^2 \times 20} = 0.0286\,T$$

$$\therefore \boldsymbol{B = 2.86 \times 10^{-2}\ T}$$

4. A circular coil of 5 turns of mean diameter 10.0 cm is mounted with its plane vertical and along the magnetic meridian. A small compass needle is mounted on a vertical axis at the centre of the coil. When a current of 0.50 A is passed through

the coil, the compass needle deflects through 61°. When the current in the coil is reversed, the compass needle deflects through 59°. Calculate the horizontal component of the earth's magnetic field intensity.

Solution

$N = 5\ turns, d = 10.0\ cm \Rightarrow r = 0.05\ m, I = 0.50\ A,$

$\theta_1 = 61°, \theta_2 = 59° \Rightarrow \theta = \left(\frac{61° + 59°}{2}\right) = 60°$

Now using, $\tan\theta = \frac{B_C}{B_h} \Rightarrow B_h = \frac{B_C}{\tan\theta} = \frac{\mu_0 N I}{2r\tan\theta}$

$$B_h = \frac{\mu_0 N I}{2r\tan\theta} = \frac{4\pi \times 10^{-7} \times 5 \times 0.50}{2 \times 0.05 \times \tan 60°} = 1.814 \times 10^{-5}\ T$$

$\therefore B = 1.814 \times 10^{-5}\ T$

NB, Magnetic field intensity, $h = \frac{B}{\mu_0} = \frac{1.814 \times 10^{-5}}{4\pi \times 10^{-7}} = \mathbf{14.44}\ A\,m^{-1}$

$\therefore$ **Magnetic field intensity,** $h = \mathbf{14.44}\ \mathbf{Am^{-1}}$

2.5 Exercises

1. What current must be passed through a flat circular coil of 10 turns and radius 5.0 cm in order to produce a magnetic flux density of 2.0×10^{-4} T at its centre? ($\mu_0 = 4\pi \times 10^{-7}\ H\,m^{-1}$) **Ans: (1.59 A)**
2. A coil of 50 turns and area 10^{-2} m² is placed with its plane normal to the magnetic field between the pole pieces of a powerful magnet. The coil has a resistance of 5 Ω and is connected to a ballistic galvanometer of 35 Ω. When the coil is removed completely from the field, a deflection of 120 divisions is registered on the galvanometer. Calculate the magnetic flux density if the sensitivity of the galvanometer is 15 divisions per micro coulomb. **Ans: [6.40×10^{-4}T]**
3. A steady current of 2 A flows in a long air cored solenoid. A small narrow coil of 50 turns and area 10^{-3} m² is placed in the middle of the solenoid and coaxial with it and connected to a ballistic galvanometer. When the current in the solenoid is reversed, a charge of 10 μC circulates in the galvanometer. If the total resistance of the galvanometer and the coil is 5 Ω. Calculate the;
   (i) Magnetic flux density at the middle (centre) of the solenoid. **Ans: [10^{-3}T]**
   (ii) Inductance of the solenoid if it has 200 turns and an area of 1.2×10^{-2} m². **Ans: [6.0×10^{-5}H]**

2.6 ELECTROMAGNETS & some of their applications

An electromagnet consists of a coil of wire wound on a soft magnetic material such as soft iron. When a current is passed through the coil, the magnetic core becomes magnetized and temporarily becomes a magnet. When the circuit is broken, the magnetic core loses its magnetism.

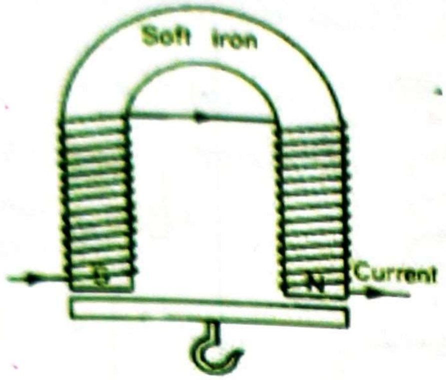


Fig. 2.6 (a)

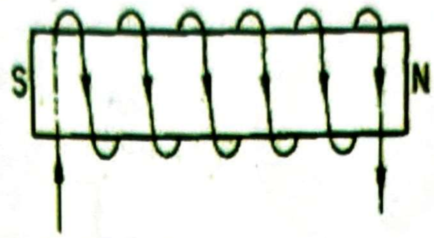


Fig. 2.6 (b)

ACTION OF AN ELECTROMAGNET

When currents pass thru the coil, a strong but temporary magnetism is induced on the soft magnetic material. The magnetism is lost when current is switched off and again produced when current is switched on, repeating the process a number of times depending on its designed purpose.

FACTORS AFFECTING THE STRENGTH OF ELECTROMAGNETS

- Current in the coil
- Number of turns in the coil
- Nature of the magnetic material i.e. soft iron.

APPLICATIONS OF ELECTROMAGNETS

Some of the applications of electromagnets include some of the following examples:

- Solenoid switch in car doors.
- An Electric Bell operation.
- Electromagnetic brakes.
- Car ignition induction coils.
- Ac. Transformers.
- Magnetic relay switches.
- Moving coil loud speakers.
- Telephone receiver.
- Electromagnetic metal separators
- Operation of electric guitars.
- Induction ammeters.

The structure and mode of operation of an Electric Bell

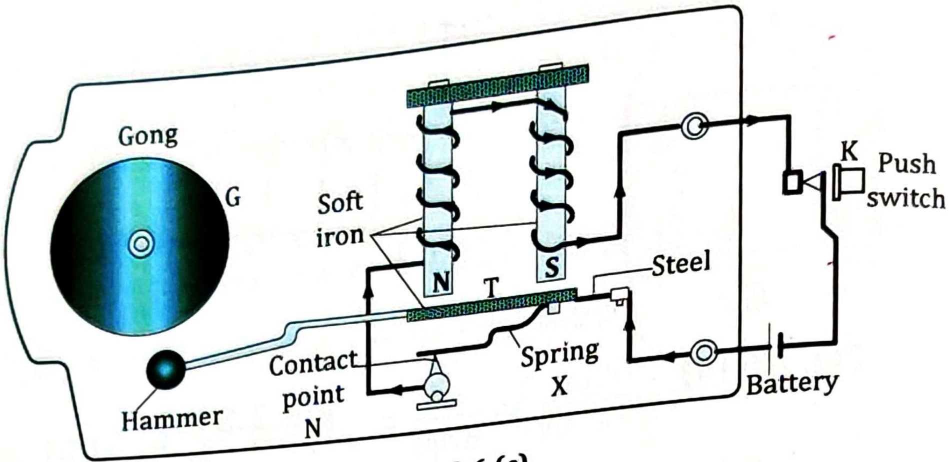


Fig. 2.6 (c)

- When switch K is pushed in (closed), a current I flows through the circuit causing the parallel soft iron metal pieces to get magnetized as shown on the diagram in the figure 2.6 (c)
- A soft iron metal piece T, then gets attracted by the electromagnet and gets pulled towards the magnet.
- The hammer then hits or strikes the gong G, and hence sound is produced.
- At the same time the contact point N, attached to spring X, loses contact with the spring, thus cutting off current in the circuit.
- The soft iron parallel metal pieces then lose their magnetism, thereby forcing the steel bar and the spring X to pull back the metal T and reconnect the circuit.
- So long as K is still on (depressed), the process repeats itself.

2.7 ELECTROMAGNETIC INDUCTION

Definition

Electromagnetic induction – is the production or generation of an induced e.m.f in a coil, conductor whenever there is relative change of magnetic flux linked with it.(i.e. coil, or conductor).

This e.m.f. can be produced in any one of the following ways

(i) A closed loop of wire or coil has its plane changed within the magnetic field or removed completely out of the region of the magnetic field.

(ii) A straight conductor is made to move across a magnetic field.

(iii) A changing current is passed through a coil, or the coil is moved in a magnetic field.

(iv) A changing current is passed through a primary coil placed co-axially with a secondary coil, the magnetic flux linking the secondary coil causes an e.m.f. to be induced in the secondary coil.

(v) A metal disc is span or rotated in a magnetic field with its plane perpendicular to the magnetic field.

(vi) A coil of metal e.g. copper wire is rotated in a magnetic field.

LAWS OF ELECTROMAGNETIC INDUCTION

There are two laws namely,

1. **Faraday's law**
2. **Lenz's law.**

FARADAY'S LAW

States that – The ***magnitude*** of the e.m.f. induced in a coil or across the ends of a conductor is directly proportional the rate of change magnetic flux linkage or to the rate of cutting of the magnetic flux.

$$i.e.\ |E| \propto \frac{d(N\Phi)}{dt} \Rightarrow |E| = k\frac{d(N\Phi)}{dt} \quad experiments\ show\ that, k = 1$$

$$|E| = \frac{d(N\Phi)}{dt} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

LENZ'S LAW

States that – The ***direction*** *of the induced e.m.f.* in a coil or closed circuit acts in such a way ***as to oppose the change of the magnetic flux that causing it.***

Or it can also be stated as – the ***direction*** of the induced e.m.f. is such that the induced current that it causes to flow in a closed circuit, opposes the change of flux, which is producing it.

NB: the two laws above when combined can be expressed as Newman's Equation.

$$\text{i.e. } \boldsymbol{E = -\frac{d(N\Phi)}{dt}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii) \quad \textbf{i.e. Newman's equation.}$$

The minus sign (–) in equation (ii) above accounts for Lenz's law.
For example, if the magnetic flux linking the plane of a coil in a closed circuit, is due to an approaching North pole of a bar magnet. The induced e.m.f. causes an induced current to flow in such a way as to make ***the near end of the coil*** being approached by the ***North pole*** of the bar magnet, also a ***North pole,*** so as to oppose the incoming north pole.

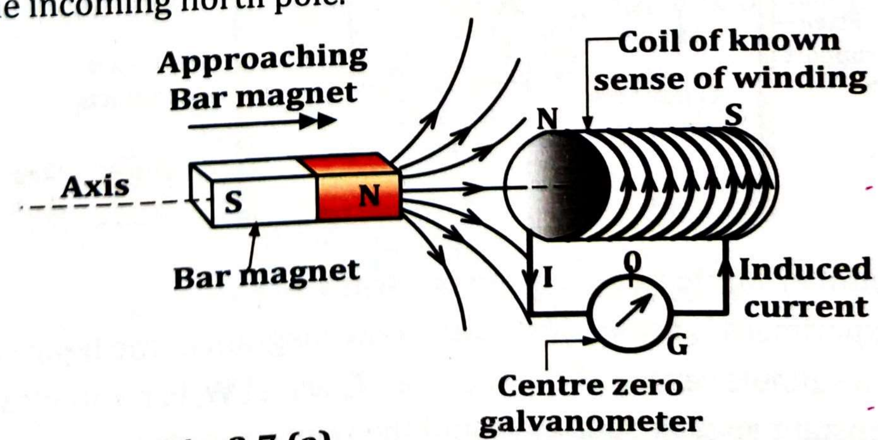


Fig. 2.7 (a)

Experimental verification of the laws of electromagnetic induction

Experiments to verify Faraday's Law

(a) Method I (Movement of a metal rod across a magnetic field)

- The apparatus is set up as shown in the figure 2.7 (b), where PQ is a copper metal frame that is free to rotate round the cylindrical magnet. X, Y are metal slip contacts connecting the centre zero galvanometer G, via rheostat Z to the metal frame PQ.
- When the copper rod is rotated say clockwise, it cuts the magnetic flux lines, and by Flemings Right Hand Rule, an e.m.f. is induced across the ends P and Q of the copper rod.
- Since the circuit is closed by the slip contacts X and Y via the centre zero Galvanometer, G, an induced current, I, flows in the circuit in a clockwise direction, causing G to deflect to the left (direction of flow of current).
- The galvanometer G, then gives a deflection, θ, that is proportional to current, I, which also depends of the e.m.f. induced across PQ.
- The induced e.m.f. on the other hand depends on the rate of cutting of the magnetic flux linked with the copper rod, PQ, which also depends on the rate of rotation of the wheel, W. $i.e. \theta \propto I, and\ I \propto E,\ E \propto f, while\ f \propto \frac{d\Phi}{dt}$
- Thus, when $\theta \propto f, and\ \theta \propto I \Rightarrow I \propto f, Hence,\ E \propto \frac{d\Phi}{dt}$ this then leads to the verification of Faraday's law.

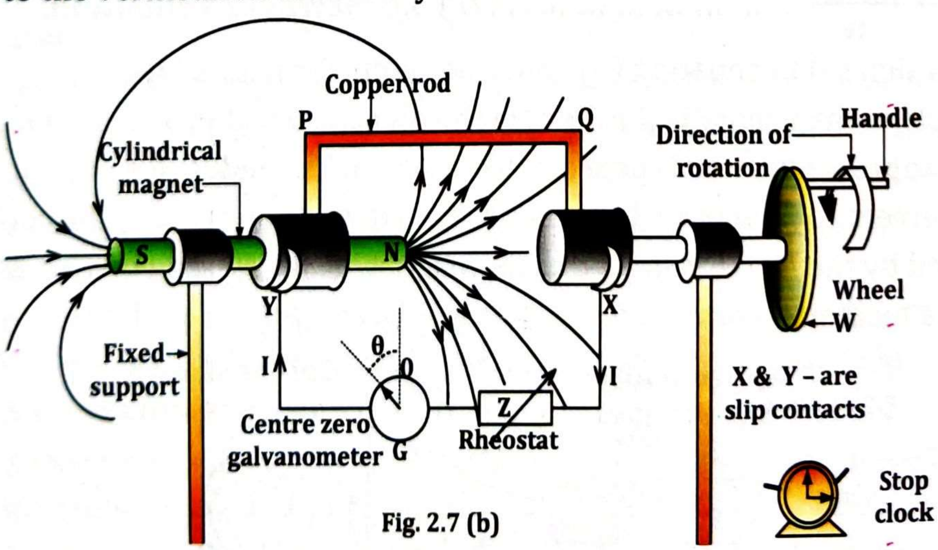


Fig. 2.7 (b)

Procedure/ Mode of operation (How it works)

- The experiment is set up as shown on the diagram in the figure 2.7 (b)
- Using a suitable setting of the rheostat, **Z**, wheel **W**, is rotated say clockwise at a constant angular speed, **w**, until the centre zero galvanometer G registers a steady reading, **θ** on its scale.

- The number of revolutions, **n**, is noted and the time, **t**, taken for these, **n** revolutions is also, noted on a stop clock or stop watch.
- The frequency, $f = \frac{n}{t}$ is calculated for a given speed of rotation of the wheel, **W**.
- The experiment is repeated, for several other different steady speeds rotation of the wheel, **W**, and in each case, the frequencies **f**, together with their corresponding deflections **θ** are recorded in a suitable table of results.
- A graph of **θ** against **f** is then plotted and a straight line through the origin is obtained. This shows that $\theta \propto f$

  $but\ \theta \propto I, while, I \propto (emf, E)\ and\ so\ \frac{d\Phi}{dt} \propto f.$

- Hence , $E \propto \frac{d\Phi}{dt}$ where $\frac{d\Phi}{dt}$ is the rate of change of magnetic flux linkage, thus Faraday's law is verified.

(b) **Method II (The Magnet – Coil experiment)**
Verification of Faraday's law

- *The principle behind this method is that, when a changing magnetic flux threads a plane of coil say normally, an e.m.f. gets induced in the coil.*
- *The rate at which the magnetic flux linking the coil changes, depends on the velocity of the bar magnet towards or away from the coil.*
- *When the circuit connected to the coil is closed or completed via a centre zero galvanometer, G, the induced e.m.f. causes an induced current to flow in the circuit in such a direction as to produce a magnetic pole at the end of the coil near the adjacent end of the magnet, which opposes the magnetic flux that has caused it.*
- *The flow of the current through the circuit causes the centre zero galvanometer, G, to give a deflection, θ, that is proportional to the current I, flowing which in turn is proportional to the induced e.m.f.* $\theta \propto I, while, I \propto (emf, E)$ *and so* $v \propto \frac{d\Phi}{dt}$.
- *Thus since,* $\theta \propto I$ *and* $I \propto E$ *while* $v \propto \frac{d\Phi}{dt}$ *but* $v \propto \theta \Rightarrow E \propto \frac{d\Phi}{dt}$

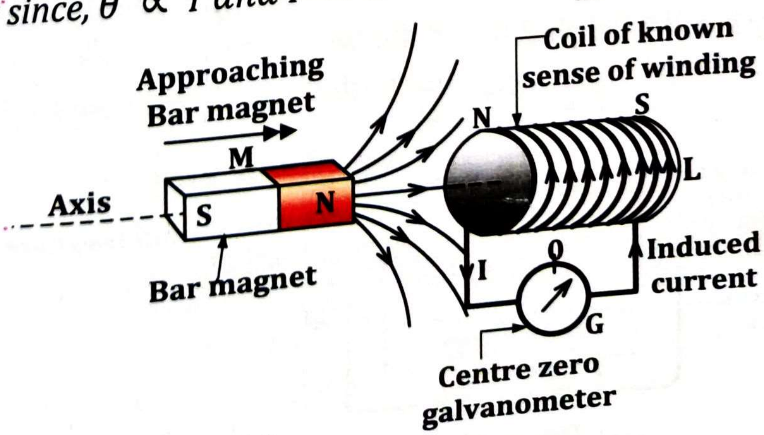


Fig. 2.7 (c)

Procedure/ Mode of operation (How it works)or Description of the experiment

- The experiment is set up as shown on the diagram in the figure 2:7 (c)
- With the magnet **M**, stationary relative to the coil **L**, the centre zero galvanometer G, shows no deflection.
- The magnet is now moved towards coil **L**, with a fairly low speed, v_1 and the corresponding deflection θ_1 of G is noted.
- M is pulled back to its original position and the pointer of G allowed to settle.
- M is now moved towards **L** at a faster speed, v_2, and the deflection θ_2 is noted.
- θ_2 is then found to be greater than θ_1 (i.e. $\theta_2 > \theta_1$) but the speed of motion of the magnet is proportional to the rate of change of magnetic flux linking the coil.
- The deflection θ is also proportional to the current I, flowing in the circuit, which in turn is proportional to the e.m.f. E, induced in the coil. *i. e.* $\theta \propto I$ *and* $I \propto E$
- Hence, $E \propto \frac{d\Phi}{dt}$ *where* $\frac{d\Phi}{dt}$ *is the rate of change of magnetic flux linkage.*

  Thus, Faraday's law is verified.

(c) Method III (Coil – Coil experiment or Transformer effect)

Verification of Faraday's law

- *The principle behind the operation of the system is that, when a rapidly changing current supplied by the signal generator is passed through the solenoid, an e.m.f. called back e.m.f. is induced in it, due to changing magnetic flux threading its turns.*
- *This magnetic flux from the primary also links the nearby secondary coil and the rate of change of the magnetic flux linkage is proportional to the e.m.f. induced in the secondary coil.*
- *An induced current, flows in the secondary circuit connected to the Y – plates of a C.R.O. whose time – base is switched off and produces a line trace on the screen, whose length L, is proportional to the root mean square value of the a.c. input to the signal generator.*
- *Since* $f \propto \frac{d\Phi}{dt}$ *and* $E \propto I$ *where* $I \propto L \Rightarrow E \propto \frac{d\Phi}{dt}$

  Hence, Faraday's law is verified.

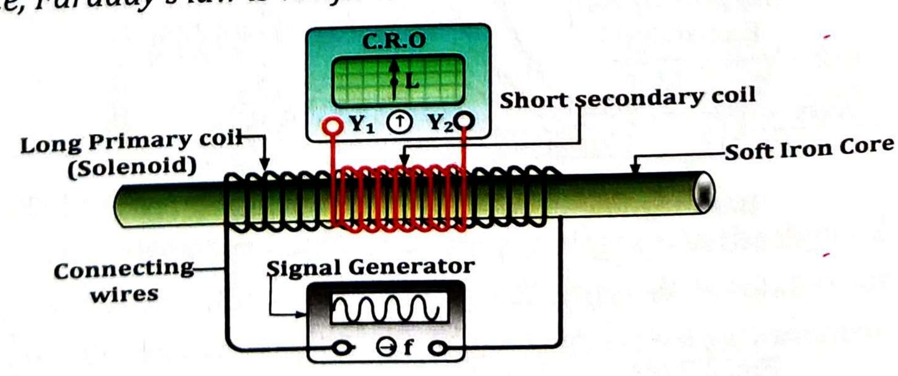


Fig. 2.7 (d)

Procedure/ Mode of operation (How it works) or Description of the Experiment

- A signal generator is connected to the primary coil (solenoid), i.e. insulated copper wire wound on a soft iron core.
- A short secondary coil wound tightly on and in the middle portion of the primary coil, is connected to the **Y**-plates of the **C.R.O** whose ***time base is switched off.***
- The generator is set at a particular frequency, **f** and switched on.
- The length, **L**, of the vertical line trace on the C.R.O screen is then noted.
- The experiment is repeated with other different settings of frequency, **f**, and the corresponding values of length **L**, of the line trace are recorded in a table.
- A graph of ***L*** against **f** is plotted and gives a straight line through the origin.
- This implies that, $L \propto f$, *but,* $L \propto E$ (induced in the coil) while $f \propto \frac{d\Phi}{dt}$

$\therefore \Rightarrow E \propto \frac{d\Phi}{dt}$ ***Hence, Faraday's law is verified.***

2. Experiment to verify Lenz's Law (*Discovered by Emil Lenz the Russian Physicist in 1834*)

Procedure:

- A centre zero galvanometer **G**, a switch **K**, resistor **R** and source of e.m.f., **B**, are connected in series to ascertain the direction of deflection of the galvanometer, G for a given direction of flow of current as shown in the figure 2.7 (e)

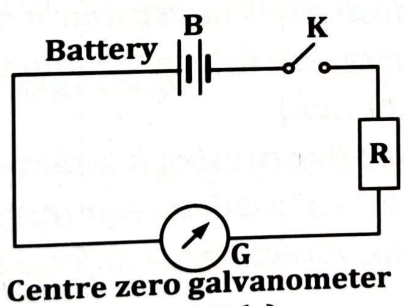


Fig. 2.7 (e)

- Switch K is closed and the direction of deflection of the centre zero galvanometer is noted.
- The source of e.m.f., a switch K and a resistor R are then replaced by a coil of known sense of winding as shown in the figure 2.7 (f)

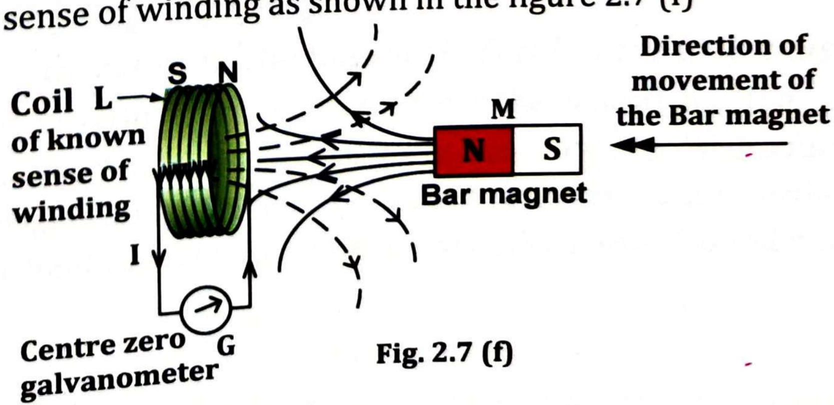


Fig. 2.7 (f)

- A bar magnet with its **North pole** closer to the coil is moved ***towards*** the coil, and the centre zero galvanometer **G** is seen to deflect in *one direction* ***(to the right).***
- When the magnet is withdrawn and pulled ***away from*** the coil, the galvanometer G deflects in the *opposite direction* ***(to the left).***
- When the bar magnet is moved towards the coil, the increasing magnetic flux threading and linking the plane of the coil causes an e.m.f. to be induced in the coil.
- An induced current I flows in such a direction to make the end of the coil nearer the approaching magnet, a north pole also opposing the approaching north pole of the bar magnet.
- When the magnet recedes (is withdrawing), the reducing magnetic flux threading the coil, induces an induced e.m.f. which acts in such a direction as to cause an induced current I to flow in the closed circuit in such a way as to make the end of the coil a south pole attracting the receding north pole.
- ***Thus, in every action of the magnet the coil responds in a way as crate activity that opposes the action that caused it, hence Lenz's law.***

NB, Whenever an induced current flows in a loop of wire (coil), it always flows in such a direction as to counteract the change of flux that is producing it.(Due to the external field)

(i) If the ***external magnetic flux causing it*** is ***increasing***, it acts in such a direction as to create a counter magnetic field in the ***opposite direction*** to *counteract the increasing magnetic flux.)*

(ii) If however, the ***external magnetic flux causing it*** is ***decreasing, or reducing***, it acts in such a direction as ***to enhance or increase*** *the decaying external magnetic field* in ***the same direction*** to *reinforce the decreasing magnetic flux.*)

These two can be illustrated by diagrams in figures (i) and (ii) below, where the ***direction of the loop*** depicts the direction of ***induced current*** in the coil or loop of wire due to the induced e.m.f.

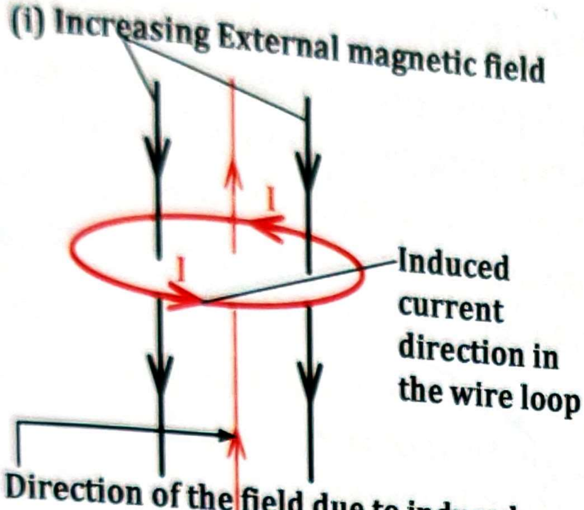


Fig. 2.7 (g)(i)

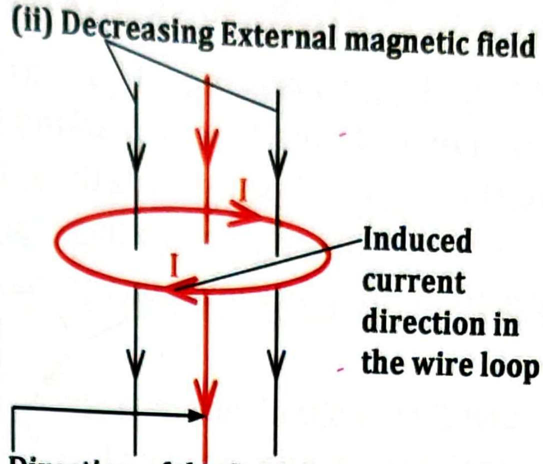


Fig. 2.7 (g)(ii)

Anti – clockwise current loop

When an ***increasing*** ***magnetic flux*** *links* the plane of the ***wire loop***, an e.m.f. is induced in the loop in such a way that the induced current that it causes to flow generates a *magnetic flux that tends to oppose (i.e.* ***reduces****) the increasing external flux*. The net magnetic field in the loop of wire is the resultant of the two magnetic fields, but since the external field is larger than the induced field, there would be a net reduction of the field in the loop. From $F = BIL \Rightarrow I = \frac{F}{BL} \Rightarrow I \propto \frac{1}{B}$

$\Rightarrow$ ***A reduction in the magnetic field causes induced current to increase***

With increase in the magnetic flux linkage to the loop of wire, hence increase in the deflection of the galvanometer when connected in series with the loop or coil.

Clockwise current loop

When a ***decreasing or reducing*** ***magnetic flux*** *links* the plane of the ***wire loop***, or coil, an e.m.f. is induced in the loop in such a way that the induced current that it causes to flow generates a *magnetic flux that tends to* ***increase*** *(i.e.* ***enhance****) the decreasing or decaying external magnetic flux* i.e. it compensates for the reducing flux. The net magnetic field in the loop of wire is the resultant of the two magnetic fields, thus it tends to temporally increase but cannot replace completely the lost external field, and this however then tends to reduce induced current in the loop.

From $F = BIL \Rightarrow I = \frac{F}{BL} \Rightarrow I \propto \frac{1}{B}$

$\Rightarrow$ *A temporary increase in the magnetic field in the loop causes induced current* to reduce, hence a decrease in the deflection of the galvanometer if connected in series with the coil or wire loop. With a further decrease in the magnetic flux linkage, the induced current reduces further, as the induced magnetic field cannot sustain the decaying magnetic flux indefinitely.

CURRENT BRAKING DUE TO LENZ'S LAW

A fairly light ***metal ring*** falling vertically towards ***a solenoid carrying*** a large ***current***, will experience a ***braking force***, that slows down its speed of fall. As the ring is falling towards the solenoid, the magnetic field strength and flux linked with the ring will be increasing causing a large induced current to flow round the loop as shown on the diagram in the figure 2.7 (h)

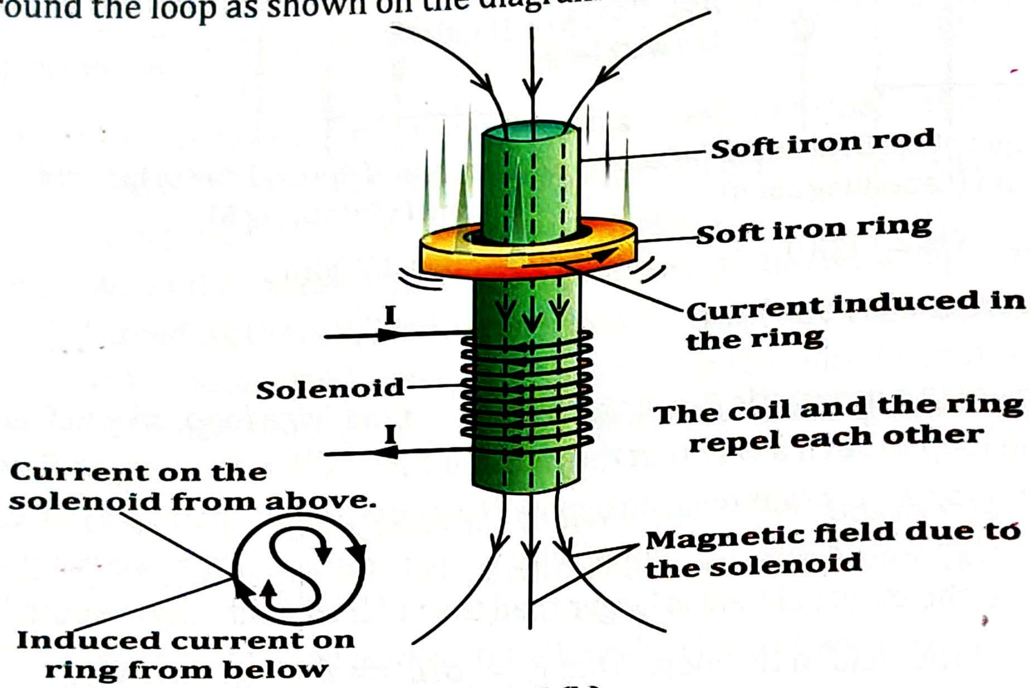


Fig. 2.7 (h)

Lenz's law shows that, the current induced in the ring circulates in such a direction that allows the magnetic field of the current to compensate for the change in the magnetic flux.

The ring acts like a small coil, magnetized with the opposite pole facing downwards and so it is repelled by a like (similar) pole at the top of the solenoid due to the current flowing through the solenoid. This is demonstrated on the diagram by the South Pole with arrows at end of letter "S" surrounded by a loop with arrows pointing in the same direction as that on the letter, "S".

NB: If the same experiment was carried out using a **superconducting ring**, it is possible for the ring to **float** or **hover** just above the solenoid. This is because the currents induced in super conductors are always large enough to completely cancel out the magnetic flux crossing their surface. (This is known as **Meissner effect)**. The repulsive force on the ring then equals the weight of the ring.

In ordinary metal rings, the induced e.m.f. s. are generally small, the eddy currents induced in them along low resistance paths may be very large and act in such a way as to oppose the motion that created or produced them. They can ∴ act as very efficient braking mechanism as the kinetic energy is dissipated as heat.

The only disadvantage or draw back here is that as the motion slows down, the eddy currents become smaller, and the braking effect is reduced, hence automobiles use this alongside other braking mechanisms like hydraulic brakes.

Lenz's law and the conservation of Energy (Explanation)

In an experiment to verify Lenz's law, a bar magnet in mechanically pushed towards the plane of the coil in a closed circuit.
An e.m.f. is induced in the coil that causes a current induced in the circuit to flow in such a way as to create a similar like pole (N – pole) at the adjacent near end of the coil. The like poles then repel each other, hence work has to be done in order to overcome the repulsion of the like poles.
The mechanical energy then is converted to electrical energy in the circuit with some of it being expended in form of heat at the coil and the connecting wires, while the rest of the energy does mechanical work to cause a deflection torque on the coil, observed as a deflection of the pointer of the galvanometer.

$$\Rightarrow Mechanical\ energy \longrightarrow Electrical\ energy \longrightarrow (Heat\ + Mechanical\ energy)$$

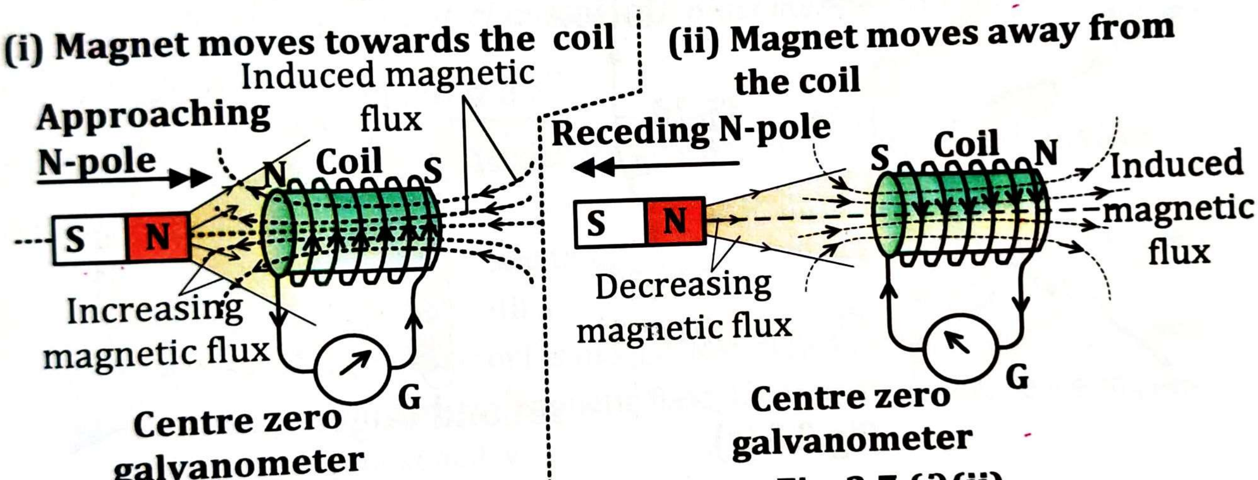


Fig. 2.7 (*i*)(i) Fig. 2.7 (*i*)(ii)

When the magnet is pulled away from the coil, the reducing magnetic flux linked with the coil, induces an e.m.f. in the coil that causes an induced current to flow in such a direction as to make the adjacent end of the coil, unlike pole ***(South Pole) attracting back the receding North Pole***.
Work has to be done again to push the bar magnet backwards *against the magnetic attraction* by the South Pole on the coil. Hence, electrical energy causes mechanical energy to expend in moving the magnet against the force of attraction.
Thus, there is a continuous interchange of energy between mechanical and electrical energy in the system.
Hence, whenever a magnet is moved along the axis of a coil in a closed circuit, work is always done against the force of either repulsion or attraction.
Lenz's law is therefore the law of conservation of energy that applies to electromagnetic induction.

2.8 FLEMING'S RIGHT HAND (DYNAMO) RULE

The direction of an induced current in a conductor can always be found using Lenz's law, however if the current is being induced due to the motion of a <u>*straight*</u>

conductor, *it is more convenient to use Fleming's Right Hand Rule commonly referred to as the Dynamo rule.*

Fleming's Right Hand Rule – states that, if the first finger, the second finger and the thumb of the right hand are held comfortably at right angles to each other. The **F**irst finger pointing in the direction of the magnetic **F**ield and the thu**M**b pointing in the direction of **M**otion, then the se**C**ond finger points in the direction of the induced current.

F = Field (for the First finger)
C = Current induced in the conductor (for the seCond finger)
M = Motion of the conductor across the magnetic field (for the thuMb)

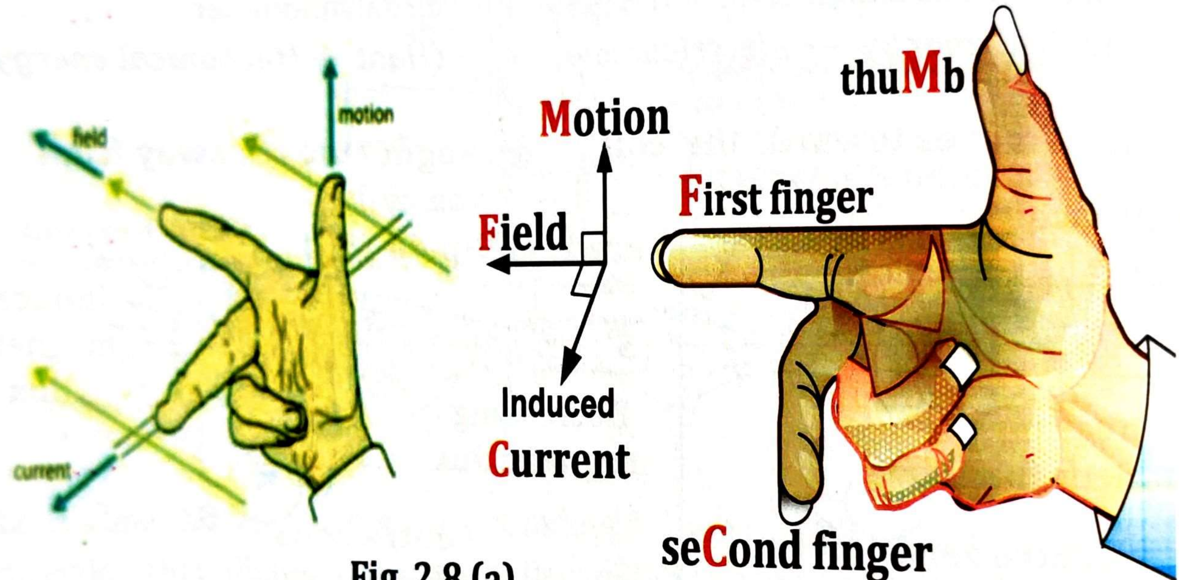


Fig. 2.8 (a)

METHODS OF GENERATING AN INDUCED E.M.F.

E.m.f produced when a straight metal rod is moved across a magnetic field

(a) Method I

- Consider a straight conductor of length, L, moved with a velocity v, across a uniform magnetic field of flux density B, tesla, as the conductor cuts the magnetic flux, electrons move to one end of the conductor leaving an equal magnitude of positive charge.
- A potential difference is then set up across the opposite charged faces of the conductor, this p.d is called an induced e.m.f. E, whose magnitude is given by $E = BLv$ and its direction is in such a way as to oppose the action that caused it.
- The proof or derivation comes from the diagram in the figure 2.8 (b)

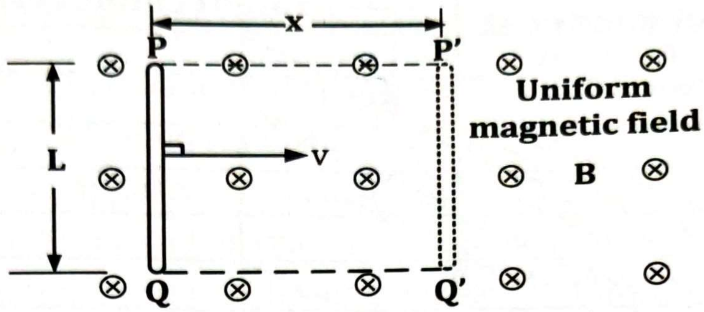


Fig. 2.8 (b)

- Suppose the metal conductor moves from PQ to P'Q', a displacement x, the area swept out by the rod $A = L \times x$
- Magnetic flux cut, $\Phi = BA = BL\,x$
- The magnitude of induced e.m.f. , $|E| = \frac{d\Phi}{dt} = \frac{d\,(BLx)}{dt} = BL\left(\frac{dx}{dt}\right) = BLv$

  Where $\frac{dx}{dt} = v\ (rate\ of\ change\ of\ displacement\ with\ time)$
- Thus induced e.m.f., $\boldsymbol{E = BLv}$

(b) Method II (Induced e.m.f. due to force on moving electrons inside a wire)

- Consider a conductor of length L, being moved across a uniform magnetic field of flux density, B, with a velocity, v.
- When the wire (conductor) is moved vertically downwards perpendicularly across a magnetic field, B, electron of charge **– e** moves down at the same speed, v.
- At the same time there is an equivalent upward movement of positive charge in the conventional direction of current I. Applying Fleming's left hand rule, (or the Hall effect), a magnetic force, F = Bev acts on the electron from side **W** towards **X**.
- If the wire is not in a closed circuit, electrons pile up on side **X** of the conductor leaving an equivalent quantity of positive charge at the opposite end **W** of the wire.
- After some time, the negative charge at **X,** will ***oppose*** any further movement of electrons along the wire WX and so the drifting of charges stops.
- The opposite charges of the same magnitude, sets up a large p.d. ***E***, known as ***induced e.m.f.*** across the metal conductor (wire), PQ, when the electric force on the stationary electron equals the magnetic force on the same electron.

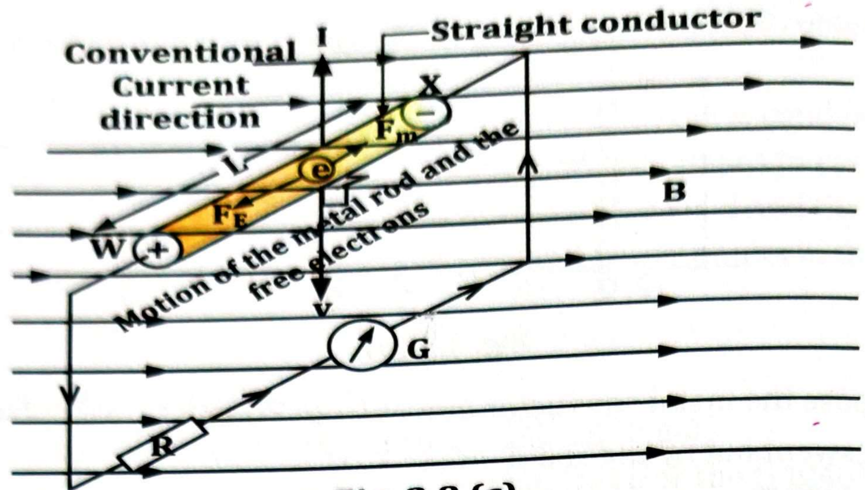


Fig. 2.8 (c)

i.e. $F_E = F_m \Rightarrow Bev = E_0 e$

where $E_0 = \frac{E}{L}$ is the electric field intensity in XW

$Bv = E_0 = \frac{E}{L} \Rightarrow E = BLv$

Thus induced electromotive force across, wire WX, $E = BLv$

NB:

- *The ends WX of the moving wire acts like a generator, with W acting as the positive terminal while X is the negative terminal.*
- *When the ends of the conductor are connected via a resistor **R** and a centre zero galvanometer, **G**, or ammeter, an induced current, **i**, flows from W via G to X then back to W, in the direction indicated causing, G, to deflect to the right.*
- *When Fleming's right hand rule is applied to the motion of the wire across a magnetic field, the polarity of the ends of the wire (e.m.f.) induced and the direction of flow of the induced current i, agrees with the above conventions, and the induced e.m.f. is given by, E = BLv.*

(c) Method III (Induced e.m.f. due to motion of a wire metal rod across a magnetic field, B.
(Using the conservation of energy and Lenz's law)

Consider a metal rod of length L, placed across a magnetic field of flux density B and moved perpendicularly across the field with a velocity, v.

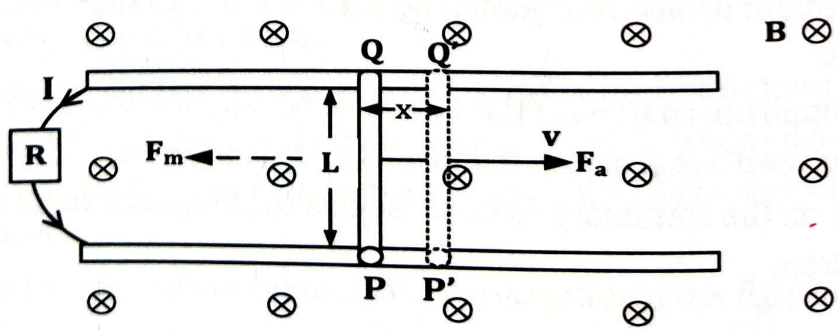


Fig. 2.8 (d)

- Work done by an external force, F_a, against magnetic force, F_m, i.e. $W = F_a \times x$

- Mechanical power applied to move the metal rod PQ $P_a = F_a \times \frac{dx}{dt} = F \times v$
- By Flemings' Right hand rule an induced current I flows from ***P towards Q***
- By Flemings' Left hand rule, a magnetic force F_m = BIL acts in such a direction as to oppose, the applied force causing motion of the rod PQ.
- As the speed of the rod increases, the induced e.m.f and consequently Induced current increases, leading increase in F_m until it equals F_a. The rod then attains a constant speed v. i.e. $F_a = F_m = BIL$, and mechanical power is converted into electrical power.
- $i.e.\ P_a = P_m \Rightarrow F_a \times v = E \times I$, But $F_a = F_m = BIL$
  $\therefore BILv = EI$
  Hence, *induced e.m.f.* $\mathbf{E = BLv}$ *with end* **P** *at a* ***higher positive potential*** *while Q is at a lower negative potential*

NB, ***When the circuit has a resistance, R, induced e.m.f. E = IR where I is the induced current that flows round the closed part of the circuit.***

2.9 Examples & Exercises on Electromagnetic induction

1. The diagrams (a) to (c) in figure 2.9 (a) show a metal rod PQ of length **8.0 cm** being moved in the plane of the paper at **3.0 m s⁻¹** through a magnetic field of flux density
$\mathbf{4.0 \times 10^{-2}}$ ***T*** Which is directed into the paper.

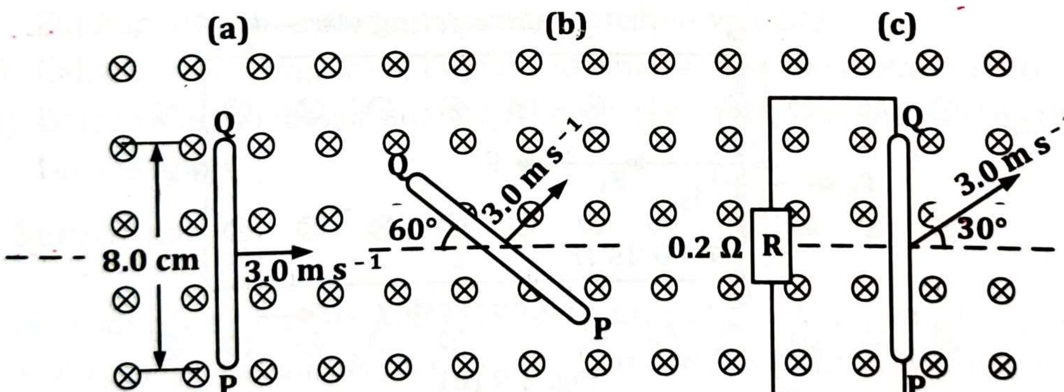


Fig. 2.9 (a)

(i) Find the magnitude of the e.m.f. induced in the rod in each case.
(ii) When a resistor, **R**, of **0.2 Ω** is connected across the rod in (c) how much current flows through it.

Solution

(a) Induced e.m.f., $E = BLv = 4.0 \times 10^{-2} \times 0.08 \times 3.0$
$\therefore \boldsymbol{E = 9.60 \times 10^{-3}\ V}$

(b) Since, the magnetic field, B, the motion of the rod PQ and the position of the metal rod PQ itself are all perpendicular to each other, Fleming's right hand rule holds, so 60° has no effect on the e.m.f. induced on the metal rod.
Induced e.m.f. , $E = BLv = 4.0 \times 10^{-2} \times 0.08 \times 3.0$
$\therefore \boldsymbol{E = 9.60 \times 10^{-3}\ V}$

(c) (i) Since, the magnetic field, B, the motion of the rod PQ and the position of the metal rod PQ itself are NOT perpendicular to each other, Fleming's right hand rule does not hold, so 30° has an effect on the e.m.f. induced on the metal rod.

Component of velocity normal to the rod PQ, $v_h = v\cos 30° = 3.0\cos 30°$

Induced e.m.f., $E = BLv\cos 30° = 4.0 \times 10^{-2} \times 0.08 \times 3.0\cos 30°$

$\therefore \mathbf{E = 8.31 \times 10^{-3}\ V}$

(ii) Using $E = BLv\cos 30°$ but $E = IR \Rightarrow IR = BLv\cos 30°$

$$\therefore I = \frac{BLv}{R} = \frac{4.0 \times 10^{-2} \times 0.08 \times 3.0\cos 30°}{0.2} = 4.16 \times 10^{-2} A$$

$\therefore$ ***Induced current,*** $\mathbf{I = 4.16 \times 10^{-2} A}$

2. A straight wire of length 20.0 cm and resistance 0.25 Ω lies at right angles to a magnetic field of flux density 0.40 T. The wire moves when a p.d. of 2.0 V is applied across its ends.

Calculate the;

(i) Initial force exerted on the wire.

(ii) Force on the wire when it moves at a speed of 15 m s⁻¹.

(iii) Maximum speed attained by the wire.

Solution

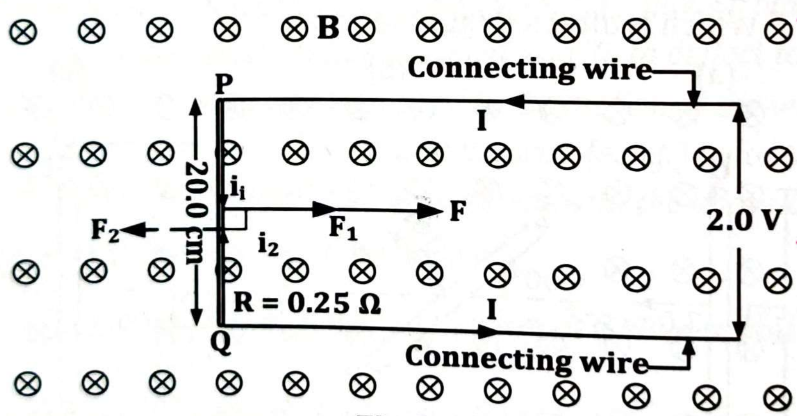


Fig. 2.9 (b)

(i) From Fleming's left hand rule, $F_1 = Bi_1L$, where, $i_1 = \frac{V}{R} \Rightarrow F = B\left(\frac{V}{R}\right)L$

$\Rightarrow F_1 = 0.40 \times \left(\frac{2.0}{0.25}\right) \times 0.20 = 0.064\ N$

$\therefore$ *The initial force,* $\mathbf{F_1 = 0.064\ N}$

(ii) Electrical power = Mechanical power.

$V i_1 = F_2 v$, where, $V = i_1 R \Rightarrow i_1^2 R = F_2 v$

$$\therefore F_2 = \frac{i_1^2 R}{v} = \frac{\left(\frac{2.0}{0.25}\right)^2 \times 0.25}{15} = \mathbf{1.07\ N}$$

(iii) When the rod attains constant velocity, v, mechanical force = magnetic force

$\Rightarrow F_1 = F_3 \Rightarrow F_1 = B(i_1 - i_2)L$

$0.064 = 0.40\left[\left(\frac{2.0}{0.25}\right) - i_2\right] \times 0.20$

$\therefore i_2 = \mathbf{7.2\ A}$

$\therefore BLv = i_2R \Rightarrow v = \frac{i_2\,R}{B\,L} = \frac{7.2 \times 0.25}{0.40 \times 0.20}$ *where i_2 is the induced current*

$\therefore v = \mathbf{22.5\ ms^{-1}}$ ***is the constant velocity attained by the rod.***

When the wire begins to move, an e.m.f. is induced across it, given by the expression, $E = BLv$ but another force $F_2 = Bi_2L$ *acts against the motion.*

3. In the figure 2.9 (c), a conducting rod PQ of length 2.0 cm rests on a smooth conducting frame to form a complete circuit of resistance 4.0 Ω. When a force, F, is applied, the rod moves at a constant velocity of 6.0m s^{-1} perpendicular to a uniform magnetic field of flux density 1.5 T.

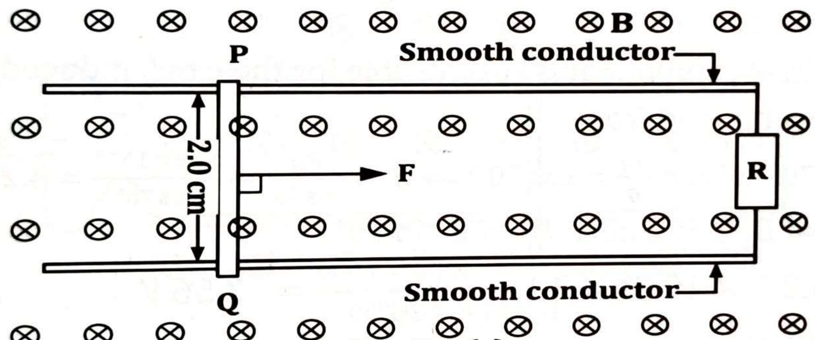


Fig. 2.9 (c)

(i) Explain why the rod moves with a constant velocity.
(ii) Calculate the magnitude of induced e.m.f. across the metal rod PQ.
(iii) Determine the magnitude of the force, F, and the current flowing through the resistor R.

Solution

(i)

- As soon as the metal rod is PQ ***accelerates*** to the right along the rails, the ***rod cuts the magnetic flux*** lines and ***an e.m.f. is induced*** *across the ends P and Q of the rod so that end* ***P*** *is positive while end* ***Q*** *is negative* ***(By Fleming's Right Hand Rule)****.*
- When the rod gains speed, the ***rate of cutting*** of the magnetic flux liked with it ***increases***, and ***a larger e.m.f.*** *is induced across rod PQ.*
- Since the ***circuit is closed*** from right hand side of the rails by the resistor R, an ***induced current, I, flows*** *clockwise i.e. from Q towards P* (By Fleming's Right Hand Rule), setting up ***a magnetic force F = BIL acting to the left*** against the applied force F, on the metal rod.(***By Fleming's left hand rule***)
- This force starts to reduce the size of acceleration of the metal rod, since the increase in induced e.m.f., increases the induced current and hence ***increases the magnetic force*** acting to the left until it equals the applied force, F.

- Hence the resultant force on the metal rod PQ becomes zero, thus the rod attains ***a constant or uniform terminal velocity, v.***

(ii) Induced e.m.f., $E = BLv = 1.5 \times 2.0 \times 10^{-2} \times 6.0 = 0.18\,V$

$\therefore \boldsymbol{E = 1.80 \times 10^{-1}\,V}$

(iii) When the rod PQ attains a constant speed, applied force = magnetic force.

$BIL = F \text{ where } IR = BLv \Rightarrow I = \frac{BLv}{R} \Rightarrow F = \frac{(B\,L)^2\,v}{R}$

$\therefore \boldsymbol{F} = \frac{(1.5 \times 0.02)^2 \times 6.0}{4} = \boldsymbol{1.35 \times 10^{-3} N}$

Using, $E = IR \Rightarrow \boldsymbol{I} = \frac{BLv}{R} = \frac{1.80 \times 10^{-1}}{4} = \boldsymbol{0.045\,A} \text{ or } \boldsymbol{4.50 \times 10^{-2} A}$

4. An aeroplane of wing span 30 m flies horizontally at a speed of 1000 km h^{-1} What is the p.d across the tips of the wings, if the horizontal component of the Earth's magnetic is 1.46 × 10^{-4}T? [*Angle of dip, at the place is 70°*]

Solution:

It's the vertical component is responsible for the e.m.f. induced, E, across the wing tips is obtained from;

$B_V = B \sin 70° \text{ where}, \frac{B_H}{B} = \cos 70° \Rightarrow B = \frac{B_H}{\cos 70°} = \frac{1.46 \times 10^{-4}}{\cos 70°} = 4.27 \times 10^{-4}\,T$

$Using, E = B_V\,L\,v \text{ where } B_V = B \sin 70°$

$\Rightarrow E = 4.27 \times 10^{-4} \times 30 \times \frac{1000 \times 1000}{60 \times 60} = 3.56\,V$

$\therefore \boldsymbol{E = 3.56\,V}$

5. An aircraft moving horizontally over the Earth's surface from East to West at a velocity of 900 km h^{-1} generates an e.m.f. of 50 mV across the tips of its wings. If the magnetic flux density at that location is 2.0 × 10^{-5}T downwards and the angle of dip at that location is 60°.

(i) Determine the length of the wings (i.e. wingspan) of the aircraft.

(ii) Which wing is at a positive potential?

Solution:

(i) $v = 900$ km h$^{-1} \Rightarrow v = \frac{900 \times 1000}{60 \times 60} = 250\,ms^{-1}$, e.m.f. E = 50 mV = 50 ×10^{-3}V

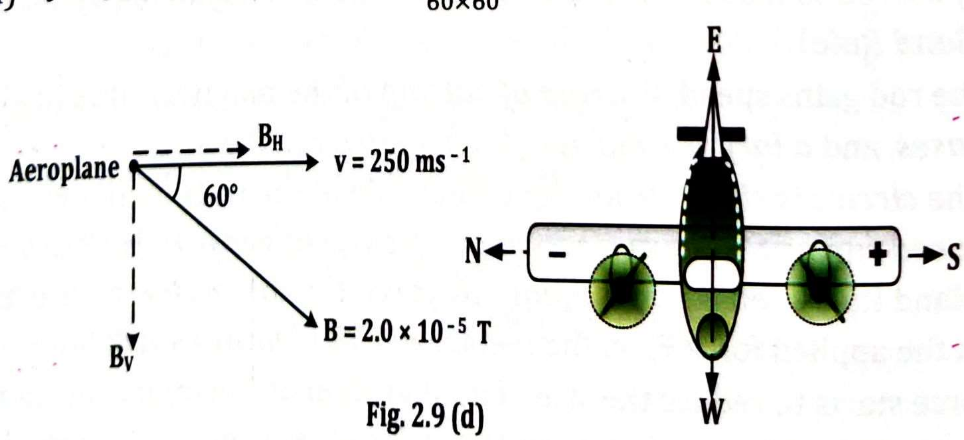


Fig. 2.9 (d)

$B_V = B \sin 60° = 2.0 \times 10^{-5} \sin 60° = 1.732 \times 10^{-5}\,T$

$Using\ E = B_V\,L\,v \text{ where } B_V = B \sin 60°$

$$\Rightarrow 50 \times 10^{-3} = 1.732 \times 10^{-5} \times L \times 250$$

$$\therefore L = \frac{50 \times 10^{-3}}{1.732 \times 10^{-5} \times 250}$$

$$\Rightarrow \mathbf{L = 11.55\ m}$$

(ii) Using *Fleming's* **Right hand** *rule*, and applying it to the motion of the aeroplane, with the **F**irst finger (**Field**) pointing downwards, the thu**M**b (**M**otion) pointing to the west, then the se**C**ond finger (E.m.f. or Induced Current) will automatically point to the south signifying the direction of Positive (+) charge. As seen on the diagram above.

6. In the figure 2.9 (e) below, **X** and **Y** are smooth conducting metal rails connected to a source of e.m.f, $\mathbf{E_0}$. **CD** is a metal rod of length **L metres** placed horizontally on **X** and **Y** perpendicular to a uniform magnetic field of flux density, **B.**

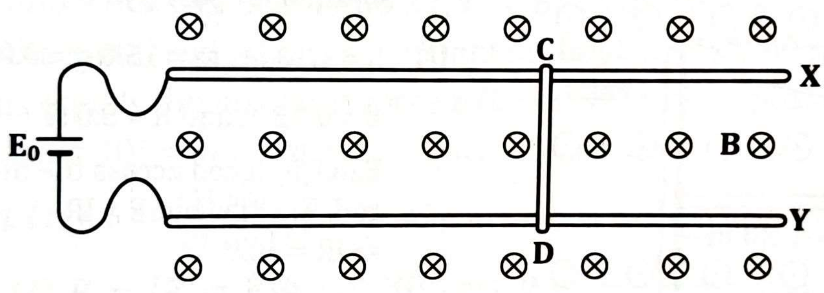


Fig. 2.9 (e)(i)

(i) Copy the diagram and indicate the force, F, acting on the rod CD.
(ii) Using the principle of conservation of energy, show that F = BIL, where I is the current supplied by the source.

Solution:

(i) When rod CD is across a magnetic field, B, the current I supplied by the source (Battery), it experiences a magnetic force, F = BIL as predicted by Fleming's left hand rule, to the right, as shown on thefigure 2.9 (e)(ii)

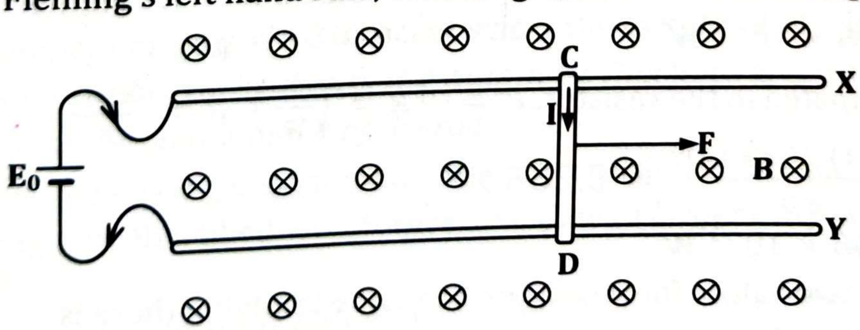


Fig. 2.9 (*e*)(ii)

(ii) When rod CD moves to the right, and cuts the magnetic flux, an e.m.f. E = BLv is induced across the rod CD, an since an induced current flows anti clockwise againtst the current supplied by the battery. Suppose the net current is I, electrical power expended in the circuit, is EI. Assuming the terminal velocity attained by the rod CD is v, then mechanical power, is Fv.

Thus the mechanical power = Electrical power

$\Rightarrow F\,v = E\,I \quad but\ E = BLv \quad \Rightarrow F\,v = BLv\,I$

$\therefore \boldsymbol{F = BIL}$

7. A 15.0 g conducting rod of length 1.3 m is free to slide downwards between two frictionless vertical rails. The rails are connected to an 8.0 Ω resistor, and the entire apparatus is placed in a 0.45 T uniform magnetic field. Assuming the rails have negligible resistance, determine the;

(i) Terminal velocity of the metal rod.

(ii) Change in the gravitational potential energy per second when the rod attains a terminal velocity.

(iii) Power dissipated in the resistor.

Solutions:

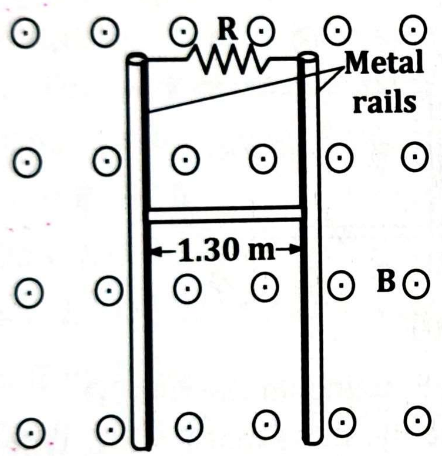


Fig. 2.9 (f)

Solution for part (i):

(i) L = 1.30 m , m = 15.0 g = 0.0150 kg

B = 0.45 T, and R = 8.0 Ω

E.m.f. induced across the metal rod, E = BLv but E = IR

⇒ IR = BLv

$\therefore I = \dfrac{BLv}{R}$ .... (i)

At terminal velocity, BIL = mg

$\therefore v = \dfrac{mgR}{(BL)^2}$ .... (ii)

$$\Rightarrow v = \frac{15.0\times10^{-3} \times 9.81 \times 8.0}{(0.45\times1.30)^2} = 3.44\ ms^{-1}$$

(ii) Gravitational power, $P = F \times v = mg \times v$

$\Rightarrow P = 15.0 \times 10^{-3} \times 9.81 \times 3.44 = 0.506\ W$

$\therefore \boldsymbol{P = 5.06 \times 10^{-1}\ W}$

(iii) Power dissipated in the resistor, $P = I^2R = \left(\frac{BLv}{R}\right)^2 R = \frac{(BLv)^2}{R}$

$$P = \frac{(0.45\times1.30\times3.44)^2}{8.0} = 0.506\ W$$

$\therefore \boldsymbol{P = 5.06 \times 10^{-1}\ W}$

Hence, the two values for power are the same implying there is ***conservation of energy*** in the system.

8. A metal rod XY of mass 0.2 kg, length 0.8 m and negligible resistance rolls down PP' and QQ' inclined at 30° to the horizontal . The rails lie in a uniform vertical magnetic field of fluc density 0.4 T. The ends PQ of the rails are connected to a resistance of 5 Ω as shown in the figure 2.9 (g)

(i) Calculate the constant speed the rod attains.

(ii) Explain the occurance of the constant speed.

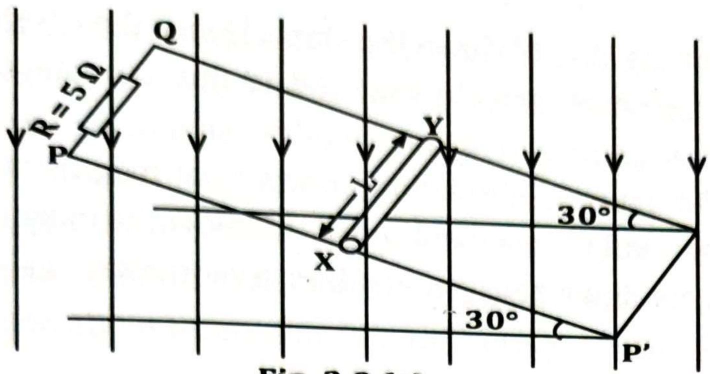


Fig. 2.9 (g)

Solution:

(i) The component of the vertical magnetic field, $B\cos 30°$ is normal to the plane of metal rails and $B\sin 30°$ down the plane of the rails.
The weight component, $mg\cos 30°$ of metal rod down the rails accelerates the rod down the rails.
E.m.f. induced across metal rod XY, $E = BLv\cos 30°$ ……………. (i)
When the metal rod attains a constant speed, v, weight component down the rails equals the magnetic force $BIL\cos 30°$ *up the plane.*
$i.e. BIL\cos 30° = mg\sin 30°$ …………………………………… (ii)

$$\text{From } (ii), I = \frac{mg\sin 30°}{BL\cos 30°} \quad \ldots\ldots\ldots (iii)$$

$$\text{From } (i)\ E = IR = BLv\cos 30° \Rightarrow v = \frac{IR}{BL\cos 30°} \quad \ldots\ldots (iv)$$

$$\text{Substituting (iii) into (iv)} \Longrightarrow v = \frac{mgR\sin 30°}{(BL\cos 30°)^2}$$

$$v = \frac{0.2 \times 9.81 \times 5\ \times \sin 30°}{(0.4 \times 0.8\ \times \cos 30°)^2} = 6.387 \times 10^3 m\,s^{-1}$$

$\boldsymbol{v = 6.39 \times 10^3 m\,s^{-1}}$ ***down the rails.***

(ii) As soon as the metal rod is released from up the inclined plane, the weight

- Component, $\boldsymbol{mg\sin 30°}$ ***accelerates*** the metal rod XY, down the slope, and as it does so, the ***rod cuts the magnetic flux*** lines and ***an e.m.f. is induced*** *across the ends X and Y of the rod.*
- When the rod gains momentum down the slope, the ***rate of cutting*** of the magnetic flux liked with it ***increases***, and *a larger e.m.f. is induced across rod XY.*
- Since the ***circuit is closed*** from above the rails by the resistor R, an ***induced current, I, flows*** *anti-clockwise i.e. From X towards Y* (By Fleming's Right Hand Rule), setting up ***a magnetic force F = BIL cos 30°*** (B cos 30° is component of B, normal to the plane of the rails or the slope) ***acting upwards the slope*** against the downward motion of the metal rod.
- This force starts to reduce the size of acceleration of the metal rod down the slope, since the increase in induced e.m.f. increases the induced current and hence increases the magnetic force up the inclined plane until it equals the

weight component mg sin 30° down the slope. Hence the resultant force on the metal rod XY becomes zero, thus the rod attains a constant or uniform terminal velocity, v.

9. (a) The diagram in the figure 2.9 (h) below shows a bar magnet attached to a spring whose other end is on a fixed support. Below the magnet is a circuit containing a coil, of known sense of winding, a centre zero galvanometer G and a switch K.

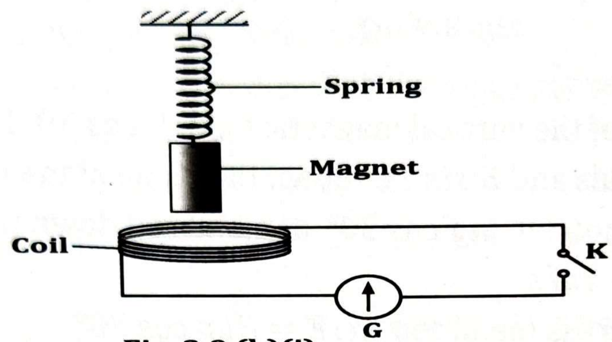


Fig. 2.9 (h)(i)

The magnet is slightly pulled down vertically, and then released to oscillate, the switch K is later closed. State and explain what is observed when the switch is,

(i) Open
(ii) Closed.

Solution:

(i) When switch **K is open**, the magnet ***takes a longer time*** to stop oscillating. The changing magnetic flux linking the coil from the magnet, induces an e.m.f. in the coil, but since the circuit is open, **no induced current flows** in such a way as to make the coil a magnet that creates an opposite pole to that of the approaching magnet (By Lenz's law). Thus, no opposing magnetic flux is created as to damp the motion of the magnet and hence **no deflection** on the centre zero galvanometer is noted.

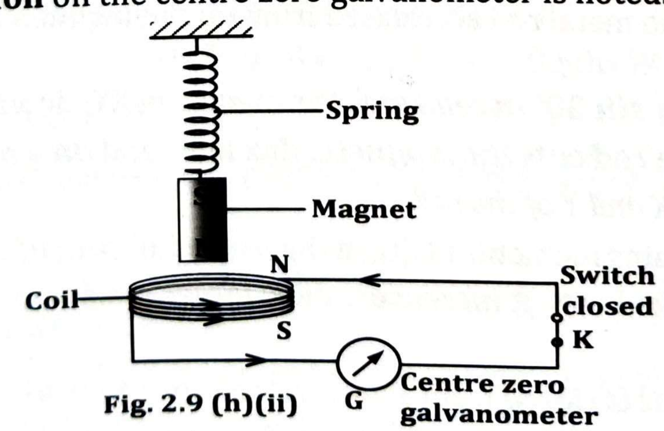


Fig. 2.9 (h)(ii)

(ii) When switch **K is closed**, the magnet takes ***a shorter time*** to stop oscillating.
The changing magnetic flux linking the coil from the moving bar magnet, induces an e.m.f. in the coil but since the circuit is closed and complete, an **induced current flows** in such a way as to create a magnetic pole whose magnetic flux **opposes the flux due to the approaching or receeding**

pole of the bar magnet at the top of the coil. E.g. when a north pole approaches the coil, a north pole is also induced at the top of the coil and when the north pole of the magnet is receeding, a south pole trying to attract the magnet is induced at the top of the coil. Thus opposing magnetic flux due to the oscillating magnet is induce at the top of the coil; hence, it damps the motion of the magnet while the **galvanometer oscillates** back and forth about its equilibrium the zero position.

(b) The diagram in Figure 2.9(i) shows a metal rod PQ of length 0.200m, mass 40 mg and resistance 6.0 Ω made to slide down thick frictionless rails inclined at 10° to the horizontal and connected at the top with a wire of negligible resistance, in a region of uniform downward vertical magnetic field of flux density B = 0.80 T.

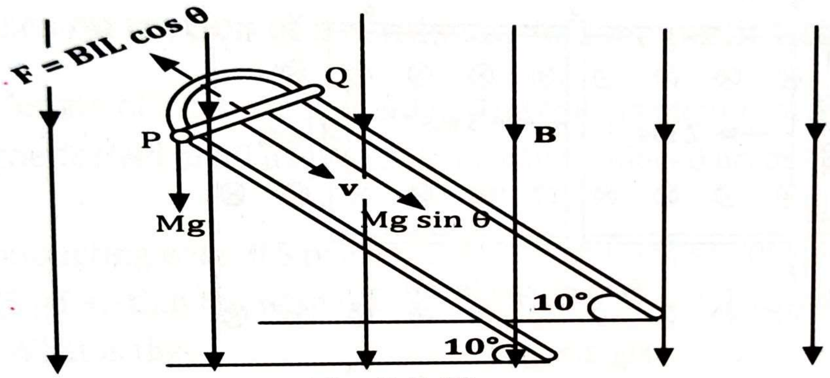


Fig. 2.9 (i)

(i) Explain what happens when the rod is released from the top.
(ii) Calculate the terminal velocity of the metal rod.

Solution

(i) θ = 10° , length of rod PQ = L = 0.200 m , R = 6.0 Ω, B = 0.50 T
When the rod PQ is released, it will ***accelerate*** down the plane and cuts across a uniform magnetic, field, B cos θ, normally.
An ***e.m.f. is induced*** across it, and an ***induced current I*** ,flows in the direction ***P towards Q*** (By Fleming's Right Hand Rule).
A ***magnetic force F' = BIL cos θ*** (By Fleming's Left Hand Rule), then acts in the opposite direction i.e. (Upwards),to the weight component Mg sin θ
As the rod PQ gains speed down the plane the *rate of cutting magnetic flux increases*, a larger e.m.f. gets induced across it and the ***induced current I, increases***.
The magnetic force, F = BIL cos θ increases until its value equals the weight component down, i.e. Mg sin θ i.e. ***BIL cos θ = Mg sin θ***
Thus the rod PQ, then attains a constant ***terminal velocity***.

(ii) $E = BLv\cos 10°$ but E = IR

$IR = BLv\cos 10° \Rightarrow v = \frac{IR}{Bl\cos 10°}$ but at terminal velocity,

$Mg\sin 10° = BIL\cos 10° \therefore\Rightarrow BIL = Mg\tan 10°$

$$\therefore I = \frac{Mg\ \tan 10°}{BL}$$

$$v = \frac{MgR\ \tan 10°}{(BL)^2 \cos 10°} = \frac{40 \times 10^{-3} \times 9.81 \times 6 \times \tan 10°}{(0.50 \times 0.200)^2 \cos 10°}$$

$\therefore$ $\boldsymbol{v = 42.15\ ms^{-1}}$ down the slope.

Two metal rods moved across a uniform magnetic field

Consider two metal rods **PQ** and **ST** each of length 0.5 m are moving at 2.0 m s^{-1} and 5.0 m s^{-1} have resistance of 1 Ω and 2 Ω respectively. The smooth parallel conductors are joined at the end with a resistor of 4 Ω and a uniform downward magnetic field of flux density 0.85 T threads the plane of the rails normally as shown on the diagram in the figure 2.9 (j)(i)

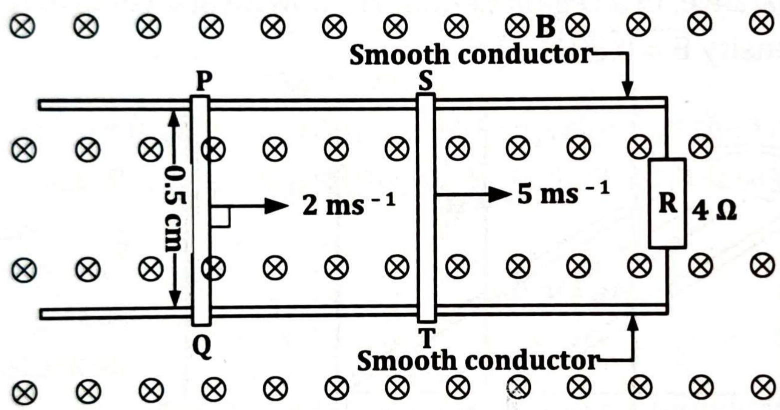


Fig. 2.9 (j)(i)

Determine the;

(i) E.m.f induced in each rod.

(ii) Current passing through the resistor

(iii) Power dissipated in the 4Ω resistor labelled R.

Solution

(i) E.m.f. induced across rod PQ, $E_1 = BLv_1 = 0.85 \times 0.5 \times 10^{-2} \times 2.0$

$\therefore\ \boldsymbol{E_1 = 8.5\ mV\ \ or\ \ 8.5 \times 10^{-3}\ V}$

E.m.f. induced across rod ST, $E_2 = BLv_2 = 0.85 \times 0.5 \times 10^{-2} \times 5.0$

$\therefore\ \boldsymbol{E_1 = 21.25\ mV\ \ or\ \ 2.125 \times 10^{-2}\ V}$

(ii) Considering the two metal rods PQ and ST as sources of e.m.f., E_1, and E_2 of internal resistances, r_1 and r_1 respectively as shown in figure 2.9 (j)(ii).

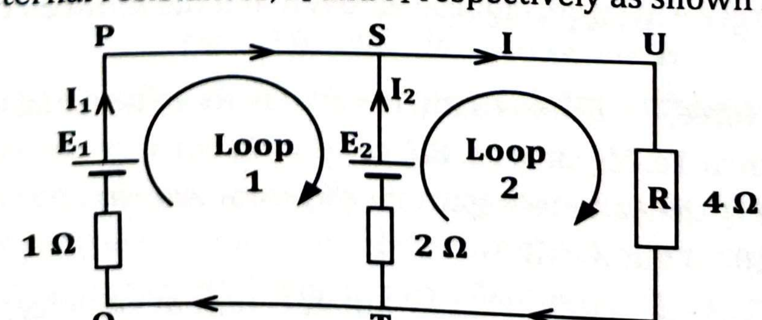


Fig. 2.9 (j)(ii)

Using Kirchhoff's laws; at junction, S, $I_1 + I_2 = I$ ………………… (i)

Considering loop 1, $E_1 - E_2 = I_1 - 2I_2$ ……………………………… (ii)

Considering loop 2, $E_2 = 4I + 2I_2$ ………………………………… (iii)

Equation (ii) + (iii) $\Rightarrow E_1 = 4I + I_1$ ……………………………… (iv)

Substituting for I_2 from (i) into (iii)

$\Rightarrow E_2 = 4I + 2(I - I_1) \Rightarrow \boldsymbol{E_2 = 6I - 2I_1}$ ………………………… (v)

Equation [(iv)×2] + (v) $\Rightarrow 2E_1 + E_2 = (8I + 2I_1) + (6I - 2I_1)$.. (iv)

$$\Rightarrow I = \frac{2E_1 + E_2}{14} = \frac{(2\times21.25 + 8.5)\times10^{-3}}{14} = 3.64 \times 10^{-3} A$$

$\therefore$ ***Current through*** 4Ω ***resistor,*** $\boldsymbol{I = 3.64 \times 10^{-3} A}$

(iii) Electrical power dissipated in a resistor R, $P = I^2R$

$\Rightarrow P = I^2R = (3.64 \times 10^{-3})^2 \times 4.0 = 5.30 \times 10^{-5}\ W$

$\therefore$ Power dissipated in the $4\ \Omega$ ***resistor,*** $\boldsymbol{P = 5.30 \times 10^{-5}\ W}$

Exercises on motion of a conductor in a magnetic field

1. The length of a train axle is 3.0 m. The train is moving at 40 km h $^{-1}$ at 90° to a magnetic field of 50 μT. What is the e.m.f. induced across the axle?

   Ans: [1.67 mV]

2. A conducting wire, 0.5 m long is moving at 40 m s $^{-1}$ through a magnetic field of 25 μT so that the wire is at 90° to the field lines. The induced current is 250 μA. What is the;
   (i) Force required to maintain the constant speed? **Ans: [3.125 × 10 $^{-9}$ N]**
   (ii) Work done per second by this force? **Ans: [6.25 × 10 $^{-8}$ W]**
   (iii) Resistance of the wire? **Ans: [1.0 Ω]**

3. An orbiting shuttle is travelling at 3100 m s $^{-1}$. It extends a 20 km long conducting tether, so that the line of the tether is perpendicular to the Earth's surface. A current of 5 A is measured in the tether and the power generated is 15 kW. Determine the average magnetic field strength along the length of the tether. **Ans: [4.84 × 10 $^{-5}$ T]**

4. A metal aircraft with a wing span of 40 m flies with a ground speed of 1000 km h $^{-1}$ in the direction due east at a constant altitude in a region of the northern hemisphere where the horizontal component of the Earth's magnetic field is
   1.6 × 10 $^{-5}$ T and the angle of dip is 71.6°. Find the potential difference in volts that exists between the wing tips and state with reason which wing is at a higher potential. **Ans: [5.34 × 10 $^{-1}$ T]**

5. A Boeing with a wingspan of 64.4 m moves over the earth's surface horizontally at 910 km h $^{-1}$ in the Earth's magnetic field of 100 μT at a location whose angle of dip is 30°. Determine the magnitude of the p.d. generated across the tips of the wings. **Ans: [0.814 V]**

6. The diagram in figure 2.9 (k) shows an aeroplane of wing span 20 m moving horizontally to the north at a speed of 100 m s^{-1} in the Earth's magnetic field of flux density 5.0×10^{-5} T at an angle of dip of 71°.

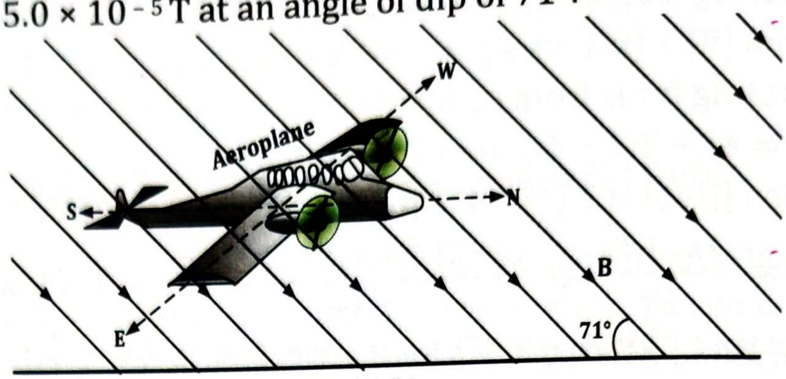


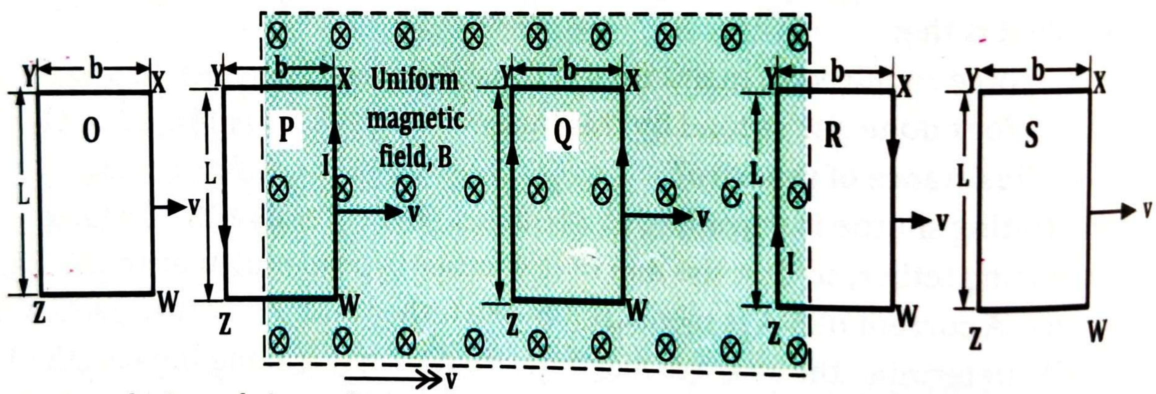
Fig. 2.9 (k)

(i) Calculate the magnitude of the e.m.f. induced across the tips of the wings. **Ans: [9.46×10^{-2} T]**

(ii) Suppose the aircraft dives vertically downwards, what is the size of e.m.f. generated across the wing tips. **Ans: [3.26×10^{-2} T]**

3.0 E.M.F. Induced in a rectangular loop of wire moved in a magnetic field

Consider a rectangular loop of a wire of length, L and width (breadth) b, of total resistance, R, placed in a uniform magnetic field of flux density, B, perpendicular into the plane of the wire loop, as shown on figure 3.0



Rectangular Loop of wire moving into, through and out of the uniform magnetic field, of flux density, B.

Fig. 3.0 (a)

When the loop is moved at a constant velocity, **v**, from the position **O** outside the influence of a uniform magnetic field **B**, through position **P** partly in the field, to position **Q** completely inside the field, to position **R** partly in the field and finally to position **S** completely outside the magnetic field, the following observations are made and explained.

At position **O** of the wire loop, all portions of the wire are outside the field. Despite the motion of the **loop WXYZ** to the right, ***no magnetic flux is cut*** and so there is **no e.m.f.** induced in the loop. i.e. **E = 0**

At position **P** of the wire loop, side **YZ** of the wire is outside the field. Despite the motion of the loop WXYZ to the right, no magnetic flux is cut by YZ and so there is no e.m.f. induced across YZ. However, side WX is moving inside the field and is cutting the magnetic flux. E.m.f, **E = BLv** is induced across WX. It thus drives induced current **I**, anti-clockwise. Generally there is increase in the magnetic flux linkage to the wire loop.

At position **Q** of the wire loop, all portions of the wire are inside the field. Despite the motion of the loop WXYZ to the right, ***there is no change of the magnetic flux cut*** and so there is **no e.m.f.** induced in the loop. i.e. **E = 0.**

Alternatively, both sides WX and YZ cut the magnetic flux in the same sense, so e.m.f. is induced across each portion of the loop in opposite directions, so as to drive the same current I in the loop in opposite directions, so that the net current, $I_{net} = (I - I) = 0$, hence the net e.m.f., **E = 0**

At position **R** of the wire loop, side **WX** of the wire has moved outside the field. Despite the motion of the loop WXYZ to the right, no magnetic flux is cut by WX and so there is no e.m.f. induced across WX. However, side YZ is still moving within the field and is cutting the magnetic flux. E.m.f, **E = BLv** is induced across YZ, and thus drives induced current I, clockwise in the loop. Generally, there is a reducing magnetic flux linked with the wire loop WXYZ as it gets out of the magnetic filed.

At position **S** of the wire loop, all portions of the wire are outside the field, again. Despite the motion of the loop WXYZ to the right, ***no magnetic flux is cut*** by any section of the loop and so there is **no e.m.f.** induced in the loop. i.e. **E = 0**

Notes

(i) Sides **XY** and **WZ** ***do not cut the magnetic flux***, at all the various positions of the wire loop, since they move parallel to the direction of the magnetic field lines. Hence, ***no, e.m.f.*** is induced ***across*** the ends of sides XY and WZ of the continuous wire loop WXYZ.

(ii) At positions **O**, **Q** and **S**, the magnetic flux is not changing, even though the loop WXYZ of the wire is moving. In each case, a small displacement of the loop, causes no magnetic flux change. i.e. The magnetic flux is zero at positions O and S, and nonzero but constant at position Q. Thus, for these three positions O, Q and S of the loop, the induced e.m.f. is zero and so is the induced current. i.e. I = 0.

(iii) There was a change of magnetic flux, from position **Q** of the loop to position **R**, or from position **O** to position **P**, since **portion of the area** threaded by the magnetic flux changed, with time. $\Delta\phi = \phi_2 - \phi_1 = B(A_2 - A_1)$

Thus induced e.m.f., $E = -\frac{\Delta\Phi}{\Delta t} = -B\frac{(A_2 - A_1)}{\Delta t}$

(iv) **A graph of current, I, flowing in the loops against time has the shape shown on the sketch in figure 3.0 (b) below.**

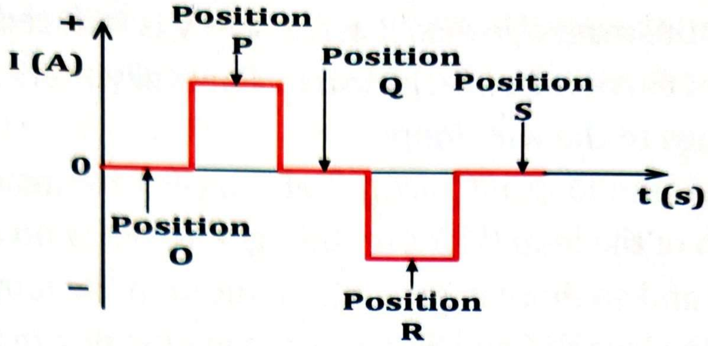


Fig. 3.0 (b)

3.1 Examples & Exercises on motion of wire loops in a magnetic field

1. A single rectangular loop of wire with dimensions 35 cm by 75 cm is arranged such that part of it is inside a region of uniform magnetic field of flux density 0.45 T and part of it is outside the field. The longer side of the loop is parallel to the magnetic field lines, while shorter side is normal to the magnetic field lines. The total resistance of the loop is 0.23Ω. Calculate the force required to pull the loop from the field at a constant velocity of 3.4 m s^{-1} perpendicular to the field.

Solution

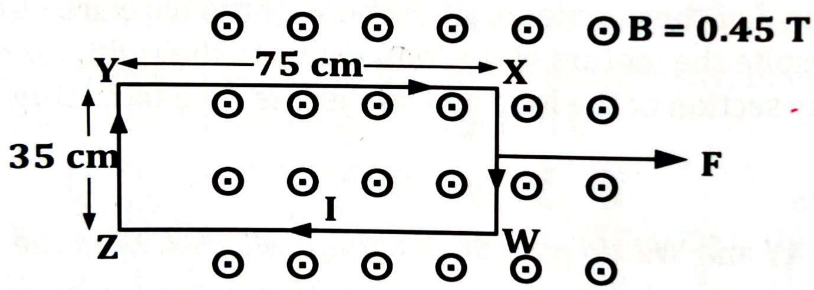


Fig. 3.1 (a)

When the loop is moved to the right, side WX cuts the magnetic flux, causing an e.m.f. to be induced in the rod, whose magnitude is given by $E = BLv$

Where, $E = 0.45 \times 0.35 \times 3.4 = \mathbf{5.355 \times 10^{-1}\ V}$

When the loop is at a constant velocity, $\Rightarrow \boldsymbol{F\,v = E\,I}$ ***where*** $\boldsymbol{I = \frac{E}{R}}$

$$\therefore F = \frac{E^2}{vR} = \frac{(5.355 \times 10^{-1})^2}{3.4 \times 0.23} = 3.667 \times 10^{-1}\ N$$

$\therefore \boldsymbol{F = 0.367\ N}$ ***to the right.***

2. In the figure 3.1 (b), a horizontal square frame ABCD, of side **d**, moves with a velocity, v, parallel to sides AB and DC from a magnetic field free region to a region of uniform magnetic field, of flux density B_0. The boundaries of the field are parallel to the sides BC and AD of the frame, and the field is directed normal into the plane of the paper.
Write down expressions for, the electromotive force induced in the frame, when;

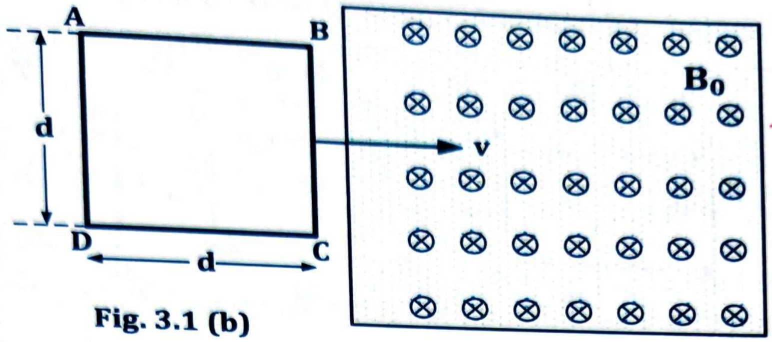


Fig. 3.1 (b)

(i) Side BC has entered the field while AD has not.

(ii) The whole frame is entirely within the magnetic field region.

(iii) Side BC has left the field but AD has not.

(iv) For each position in (i) to (iii) above, derive an expression for the magnitude and direction of the current in the frame and the resultant force acting on the frame due to the current. The total resistance of the wire frame is R, and its self-inductance may be neglected.

Solutions

(i) $\boldsymbol{E = B_0 v\, d}$ Only side BC is cutting the flux, so e.m.f. is induced in it.

(ii) $\boldsymbol{E = 0}$, Both side BC and AD are cutting the magnetic flux, in the same sense, so e.m.f.s, are induced in it in opposite directions and so they cancel out leaving the frame a net e.m.f. of **zero**.

(iii) $\boldsymbol{E = B_0 v\, d}$ Only side AD is cutting the flux, so e.m.f. is induced in it.

(iv) For case (i) By Fleming's right hand rule, induced current flows from, C towards B and flows round the loop ***anti-clockwise.*** E.m.f. induced across BC, $E = B_0 v\, d,\ but\ E = IR \Rightarrow I = \frac{E}{R} = \frac{B_0 v\, d}{R}$

For case (ii) By Fleming's right hand rule, induced current flows from, C towards B on side BC, while for side AD, induced current of the same magnitude flows from, side, D towards A on side AD and the net current in the metal frame becomes zero, i.e. I = 0

For case (iii) By Fleming's right hand rule, induced current flows from, D towards A and flows round the metal frame, ***clockwise.*** E.m.f. induced across AD, $E = B_0 v\, d,\ but\ E = IR \Rightarrow I = \frac{E}{R} = \left(\frac{B_0 v\, d}{R}\right)$

In each case when e.m.f. is induced in the metal loop, since the loop moves with a constant velocity, v, mechanical power, P = F × v and this compensates for the electrical power dissipated in the resistance R of the metal frame, P = I^2 R. Thus, $Fv = I^2 R \Rightarrow \boldsymbol{F} = \left(\frac{B_0 v\, d}{R}\right)^2 \times \frac{R}{v}$

Thus, $\boldsymbol{F} = \frac{v\,(B_0\, d)^2}{R}$

3. In the figure 3.1 (c)(i), PQRS is a rectangular metal wire loop of dimensions, PQ = 0.15 m by QR = 0.20 m, placed with its plane perpendicular to a uniform magnetic field, B = 0.50 T occupying a total area of 0.50 m by 0.80 m. Sides PQ = P'Q' = P''Q'' = 0.15 m, while sides QR = Q'R' = Q''R'' =

0.20 m. Similarly distances RQ' = R'Q'' = 0.10 m.

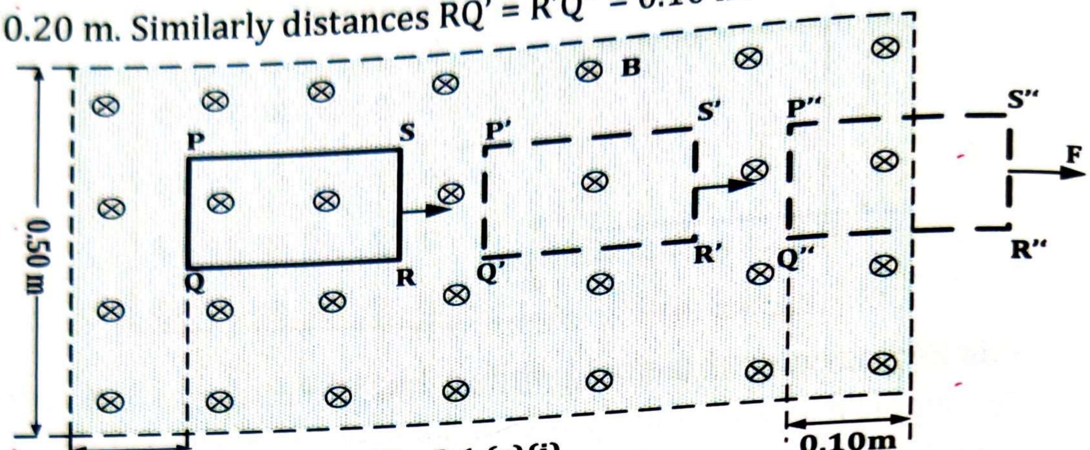


Fig. 3.1 (c)(i)

(i) Find the change of magnetic flux linked with the loop as it is moved from position PQRS to P'Q'R'S'.

(ii) Explain whether e.m.f. is induced in the loop or not when it moved from position PQRS to P'Q'R'S' in 0.5 seconds.

(iii) Determine the magnitude of the e.m.f. induced in the loop as it is moved from position P'Q'R'S' to P''Q''R''S'' in a time of 0.25 seconds.

Solution

(i) Area of the plane, PQRS, A = (L × W) = 0.20 × 0.15 = 0.03 m^2

Magnetic flux B = 0.50 T

$\therefore \Phi_1 = BA = 0.50 \times 0.03 = 0.015$ Wb

When the coil is at P'Q'R'S'

$\Phi_2 = BA = 0.50 \times 0.03 = 0.015$ Wb

Change of magnetic flux, $\Delta\Phi = \Phi_1 - \Phi_2$

$$\therefore \Delta\Phi = \Phi_1 - \Phi_2 = 0.03 - 0.30 = 0\,Wb$$

(ii) Induced E.m.f., $E = \frac{\Delta\Phi}{\Delta t} = \frac{0}{0.5} = 0\,V$

Since the change of magnetic flux = 0

No e.m.f. is induced in the loop when it moves from PQRS to P'Q'R'S'

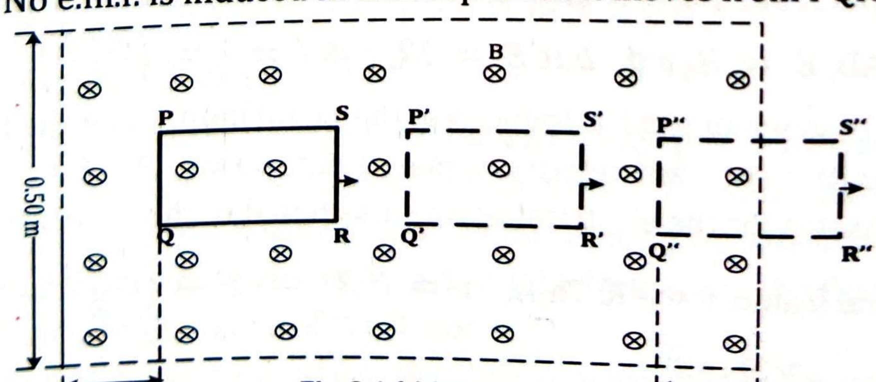


Fig. 3.1 (c)(ii)

(iii) For the loop at P'Q'R'S' area of the loop threaded by magnetic field

$A_1 = 0.20 \times 0.15 = 0.03\ m^2$

$\therefore \Phi_1 = BA = 0.50 \times 0.03 = 0.015\,Wb$

For the loop at P'Q'R'S' area of the loop threaded by magnetic field
$A_3 = 0.10 \times 0.15 = 0.015\ m^2$

$\therefore \Phi_3 = BA = 0.50 \times 0.015 = 0.0075\ Wb$

$\therefore$Induced E.m.f., $|E| = \frac{\Delta\Phi}{\Delta t} = \frac{\Phi_1 - \Phi_3}{0.25} = \frac{0.015-0.0075}{0.25} = 0.03\ V$

Alternatively

When the loop is displaced to the right by a distance 0.10 m in a time of 0.25 s$\Rightarrow$ $velocity, of\ the\ loop, v = \frac{\Delta d}{\Delta t} = \frac{0.10}{0.25} = 0.40\ ms^{-1}$

$\therefore E = BLv = 0.50 \times 0.015 \times 0.40 = 0.030$ V

Hence, induced e.m.f, $\mathbf{E = 0.030\ V}$

Exercises

1. The figure 3.1 (d) shows a uniform magnetic field of 2.0 T, normal into the plane of the paper. Loop Y is a rigid rectangular loop of wire of dimensions, 50 mm × 100 mm and resistance 0.5 Ω. It is pulled through the magnetic field at a constant velocity of 2.0 cm s^{-1} as shown.

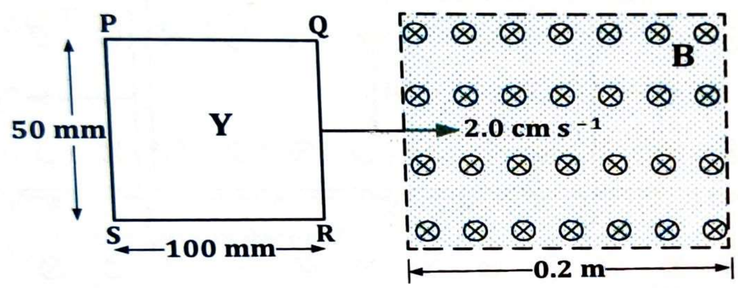


Fig. 3.1 (d)

It's leading edge enters the field at a time, t = 0 s. Determine the value in each case and sketch graphs giving explanations, of how the following vary with time from; t = 0 to t = 20s

(i) The magnetic flux linking the loop. **Ans: [0.01 Wb]**

(ii) The induced e.m.f. around the loop. **Ans: [0.5 mV]**

(iii) The current in the loop. Neglect any effect of self – inductance. **Ans: [4.0 mA]**

2. A closed wire loop in form of a square of side 4.0 cm is mounted with its plane horizontal. The loop has a resistance of $2.0 \times 10^{-3}\ \Omega$, and negligible inductance. The loop is situated in a magnetic field of flux density 0.70 T directed vertically downwards. When the field is switched off, it decreases to zero at a uniform rate in 0.80 s. What is the value of the;

(i) Current induced in the loop? **Ans: [0.7 A]**

(ii) Energy dissipated in the loop during the change in the magnetic field? **Ans: [7.84×10^{-4} J]**

(iii) Show on the diagram, the direction of the induced current. Justify your statement.

3. A rectangular loop of wire of dimensions 8.0 cm by 5.0 cm having a total resistance of 0.04 Ω is placed with its plane normal to a uniform magnetic field of

flux density 6.0×10^{-3} T directed into the plane of the wire frame, and one of the sides of the rectangular frame being of the field. Determine the velocity at which the frame must be moved in order to generate an e.m.f. of 1.5 mV in it, if;

(i) Moved in a direction parallel to the 8.0 cm side. **Ans: [5.00 m s^{-1}]**

(ii) Moved in a direction parallel to the 5.0 cm side. **Ans: [3.125 m s^{-1}]**

(iii) Sketch the position of part of the coil in the field in each case the magnitude and indicate the direction of the current flowing in it.

Ans: [Anti-clockwise; 3.75×10^{-2} A]

4. In the figure 3.1 (e), two conducting rod PQ and XY each of length 0.500 m rests on a pair of smooth conducting metal rails having a resistor of resistance, R = 2.0 Ω, between them as shown. Rod PQ has a resistance of 0.1 Ω while rod XY has negligible resistance. When forces , F_1 and F_2 are respectively applied, to the rods PQ and Y each attains a constant velocity, perpendicular to a uniform magnetic field of flux density 8.0×10^{-4} T.

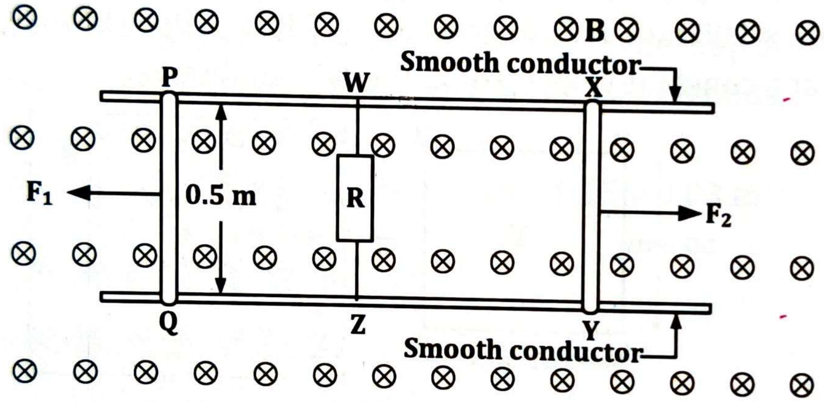


Fig. 3.1 (e)

Determine the;

(i) Constant velocity attained by rod PQ if a p.d of 1.50 mV is generated across it. **Ans: [5.00 m s^{-1}]**

(ii) Constant velocity attained by rod XY if a p.d of 2.00 mV is generated across it. **Ans: [3.75 m s^{-1}]**

(iii) Current flowing through rod PQ. **Ans: [35.0 mA, Anti-clockwise]**

(iv) Current flowing through rod XY. **Ans: [36.0 mA, Clockwise]**

(v) Power dissipated in the resistor, R. **Ans: [2.0×10^{-6} W]**

(vi) Determine the magnitudes of the forces, F_1 and F_2.

Ans: [1.40×10^{-5} N; 1.44×10^{-5} N]

5. A closed loop in form of a square of side 4.0 cm is placed with its plane perpendicular to a uniform magnetic field, which is increasing at a rate of 0.3 T s^{-1}. The loop has negligible inductance, and resistance of 2.0×10^{-3} Ω. Calculate the current induced in the loop, and explain with the aid of a clear diagram the relation between the direction of the induced current and the direction of the magnetic field. **Ans: [0.24 A]**

6. A solid loop of wire of side 2.3 cm and electrical resistance 79 Ω is near a long straight wire that carries a current of 6.8 A in the direction indicated. The long wire and the loop both lie in the plane of the paper page.

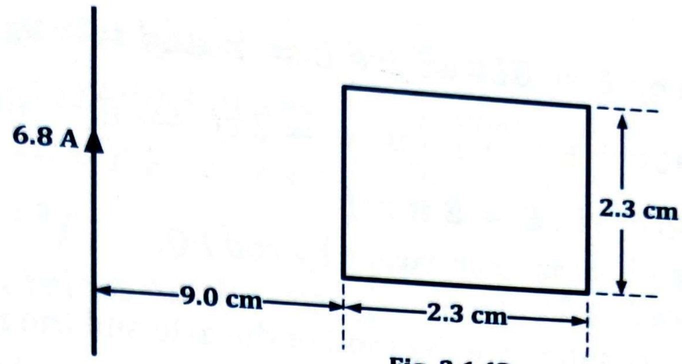


Fig. 3.1 (f)

If the loop is moving to the right at a constant speed of 45 cm s $^{-1}$.
Determine the;

(i) Induced e.m.f. in the loop. **Ans: [3.18 × 10 $^{-8}$ V]**

(ii) Magnitude and direction of induced current. **Ans: [4.03 × 10 $^{-10}$ A; clockwise]**

(iii) Size of the force acting on the loop. **Ans: [2.85 × 10 $^{-17}$ N; to the left]**

(iv) Electrical power dissipated in the loop and show that it is equal to the rate at which an external force, pulling the loop does work in keeping the speed of the loop constant. **Ans: [1.28 × 10 $^{-17}$ W]**

3.2 E.M.F. PRODUCED BY FARADAY'S DISC (HOMOPOLAR GENERATOR)

(a) Method I

- A Homopolar/disc generator consists of a copper disc, which is rotated between the pole pieces of a strong magnet or within a uniform magnetic field of flux density B.
- As the disc is rotated in a clock-wise direction at a constant angular velocity ω, assuming the magnetic field B, is uniform over the radius, PQ of the disc.
- The velocity at the axle P, is zero, while that at the rim Q is maximum and is, $v = r\omega$.
- Thus the average velocity of the one radius metal rod PQ,

$$v_a = \frac{1}{2}(0 + r\omega) = \frac{r\omega}{2}$$

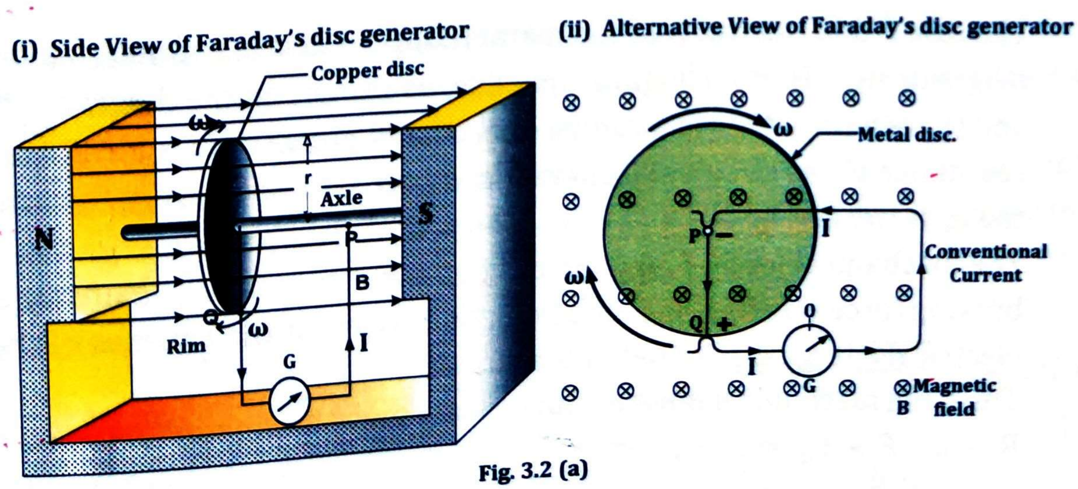


Fig. 3.2 (a)

- The radius PQ = r, cut the magnetic flux continuously, causing an induced e.m.f., E, given by, $E = BLv$ where, $L = r$ and $v = v_a = \frac{r\omega}{2}$.

- Thus, $E = BLv = Br\left(\frac{r\omega}{2}\right)$ but, $\omega = 2\pi f \Rightarrow E = Br\left(\frac{2\pi r f}{2}\right)$
- Hence, induced e.m.f. , $\boldsymbol{E = B\pi r^2 f}$

  where, $\boldsymbol{A = \pi r^2}$ *is the area swept by rod PQ.*

NB: (i) Suppose r_1 and r_2 are the radii of the axle and the rim respectively, P and Q are the contacts at the axle and the rim of the disc respectively and B is the magnetic flux density of the field perpendicular to the plane of the copper disc.

(ii) The disc is treated like a wheel with many spokes each of radius r. In one revolution of the disc, all radii cut across the magnetic flux between P and Q. the flux is given by:

$$\boldsymbol{\phi = BA = B \times \pi(r_2{}^2 - r_1{}^2)}$$

If the disc makes f, revolutions in one second, then the rate of cutting of the magnetic flux $\boldsymbol{\frac{d\Phi}{dt} = B\pi(r_2{}^2 - r_1{}^2)f}$

(b) Alternative Method II

By Faraday's law, the magnitude of e.m.f. induced in a conductor is directly proportional to the rate of cutting of the magnetic flux linkage. i.e.

$$E = -\frac{d\Phi}{dt} \quad \text{But } \mathbf{\Phi = B\,A} \Rightarrow E = -\frac{d\Phi}{dt} = -\frac{d(BA)}{dt}$$

$$\Rightarrow E = -\frac{Bd(A)}{dt} \quad but\ A = \pi r^2$$

$$|E| = \frac{Bd(\pi r^2)}{dt} = B\pi r^2 f \quad since\ the\ rate\ of\ change\ of\ area\ with\ time = \pi r^2 f$$

Thus induced e.m.f. $|E| = B\pi r^2 f$ or $|E| = BA\,f$

(c) Alternative method III (Derivation using the conservation of energy)

The axle **P** and the rim **Q** of the metal (copper) disc is rotated in a uniform magnetic field **B**, at an angular speed ω, as shown on the diagram. When P and Q are connected via a centre – zero galvanometer G., so that the total resistance of the circuit is R, an induced current I flows from the axle P towards the rim Q.

The mechanical force, F, applied to keep the disc rotating against the braking force due the magnetic field, does work at the expense of the electrical energy dissipated in the circuit.

Thus, the mechanical power = Electrical power dissipated in the resistance, R *i.e.* $F \times v_a = E \times I$ *where*

$v_a = \frac{r\omega}{2}$ *is the average velocity between P and Q,*

ω *is the angular speed and* $F = B\,I\,L$

where $L = r$ *(radius of the disc)and* $\omega = 2\pi f$

$\therefore\ B\,I\,r \times \frac{r\omega}{2} = E\,I \Rightarrow B I r \times \frac{2\pi f r}{2} = E\,I$

$\therefore\ \mathbf{E = B\pi\, r^2 f}$

3.3 Examples & Exercises on the Disc generator/Dynamo

1. A metal spoke in a wheel is 80.0 cm long. If the wheel makes 30 revolutions per minute in a plane perpendicular to the Earth's magnetic field where the flux density is 5.0×10^{-5} T. Find the p.d generated between the rim and the axle.

Solution

L = 0.80 m, $frequency, f = \frac{No.\ of\ revolutions}{Time\ in\ seconds} = \frac{30}{60} = 0.5\ Hz$, B = 5.0×10^{-5} T.

Using, $E = B\,\pi\, r^2 f = 5.0 \times 10^{-5} \times \pi \times (0.800)^2 \times 0.5$

$\therefore\ \boldsymbol{E = 5.03 \times 10^{-5}\ V}$

2. A copper disc having its central and outer parts of the axle of radii of 82.5 cm and 80.0 cm respectively has its plane vertical in the Earth's magnetic meridian. The region has a magnetic field of flux density 2.0×10^{-4} T where the angle of dip is 62°. The disc is span about its axle at 360 revolutions per minute. Determine the e.m.f. induced between its axle and the rim.

Solution

$r_1 = 0.825\ m,\ r_2 = 0.800\ m, frequency, f = \frac{No.\ of\ revolutions}{Time\ in\ seconds} = \frac{360}{60}$

$\therefore\ f = 6.0\ Hz,\qquad B_h = 2.0 \times 10^{-4} \sin 62° = 1.77 \times 10^{-4}\ T,$

Using, $E = B_h\,(\pi\, r_1{}^2 - \pi\, r_1{}^2) f = B_h\,\pi(r_1{}^2 - r_1{}^2) f$

$E = 1.77 \times 10^{-4} \times \pi \times [(0.825)^2 - (0.800)^2] \times 6.0$

$\therefore\ \boldsymbol{E = 1.36 \times 10^{-4}\ V}$

3. A copper disc of radius 10.0 cm is situated in a uniform magnetic field of flux density 1.0×10^{-5} T with its plane perpendicular to the field. The disc is rotated about an axis through its centre parallel to the field at 3.0×10^3 revolutions per minute.

(i) Calculate the e.m.f. between the rim and the centre of the disc.

(ii) Draw a circle to illustrate the disc. Show the direction of rotation as clockwise and consider the field directed into the plane of the diagram.

(iii) Explaining how you obtain your result, state the direction of the current flowing in a stationary wire whose ends touch the rim and the centre of the disc.

Solution

(i) r = 0.10 m, $frequency, f = \frac{No.\ of\ revolutions}{Time\ in\ seconds} = \frac{3.0 \times 10^3}{60} = 50\ Hz,$

B = 1.0×10^{-5} T. radius, r = 10 m = 0.100 m

Using, $E = B\,\pi\, r^2 f = 1.0 \times 10^{-5} \times \pi \times (0.100)^2 \times 50$

$\therefore\ \boldsymbol{E = 1.57 \times 10^{-5}\ V}$

(ii) When considering the direction of rotation of the disc, treat a line drawn from the centre, **X** to the rim, **Y** as ***one straight conductor***, then take a tangent to the disc at the rim where the line that you have drawn ends.
Apply Fleming's right hand rule accordingly, with the thu**M**b pointing in the direction of **Motion**, i.e. clockwise, the **F**irst finger pointing in the direction of the magnetic **Field**. Then the convenient direction of the se**C**ond finger perpendicular to the first finger, indicates the direction of the induced **Current** (Where the second finger is pointing, shows the **positive terminal** of our induced e.m.f. or position of **higher potential**)

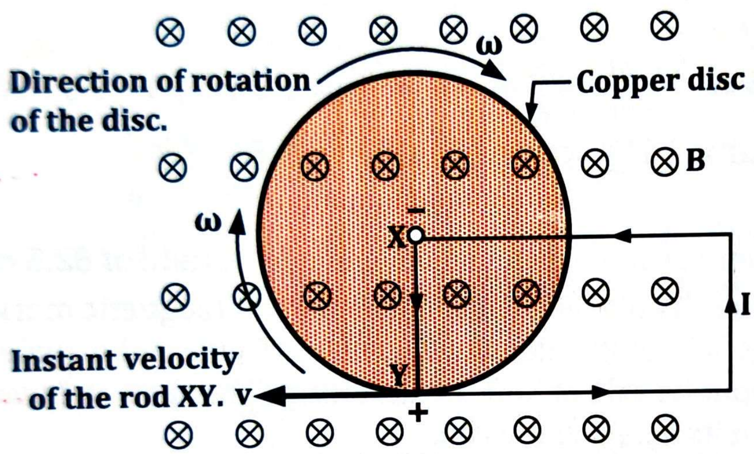


Fig. 3.3 (a)

(iii) As stated in (ii) above using Fleming's right hand rule to conductor XY. When the magnetic flux density is perpendicularly into the page, and motion from right to left. Point your ***first finger*** of the right hand ***towards the page*** and the ***thumb pointing to the left***, the direction of the ***second finger***, is pointing ***towards you*** i.e. pointing from **X** towards **Y**, with the tip of the finger at Y (Positive) while the beginning part of the second finger is at X (Negative). Hence, the induced current, I, then flows round the wire in an anti-clockwise direction as indicated on the diagram above.

4. A circular aluminium disc, of radius 30 cm is mounted inside a long solenoid of 2.0×10^3 turns per metre carrying a current of 20.0 A such that its axis coincides with that of the solenoid. If the disc is rotated about its axis at 40 revolutions per minute. What will be the e.m.f. induced?

Solution:

r = 0.30 m, $frequency, f = \frac{No.\ of\ revolutions}{Time\ in\ seconds} = \frac{2.0\times10^3}{60} = 33.3\ Hz,$

Magnetic flux density at the centre of the solenoid, is given by;

$B = \mu_0 nI = 4\pi \times 10^{-7} \times 2.0 \times 10^3 \times 20.0 = 5.03 \times 10^{-2}\ T$

Using, $E = B\,\pi\, r^2 f = 5.03 \times 10^{-2} \times \pi \times (0.300)^2 \times 33.3$

$\therefore\ \boldsymbol{E = 4.74 \times 10^{-1}\ V}$ is the e.m.f. induced between the centre and rim of the aluminium disc.

Exercises

1. A vertical metal disc of radius 0.2 m is rotated at 5 rev. s^{-1} about its centre in a horizontal uniform magnetic field of 0.1 T which is normal to the plane of the disc.
   (i) Find the e.m.f. produced between the centre and top of the disc.
   Ans: [$6.28 \times 10^{-2} V$]
   (ii) Explain the formula used and show the direction of the induced e.m.f. in a sketch.
   (iii) What is the e.m.f. between the ends of the diameter of the disc. **Ans: [0 V]**

2. (i) A circular metal disc of radius R, rotates in an anticlockwise direction at an angular speed, ω, in a uniform magnetic field of flux density, B directed into the paper as shown in the figure 3.3 (b) with A and C being contact points.

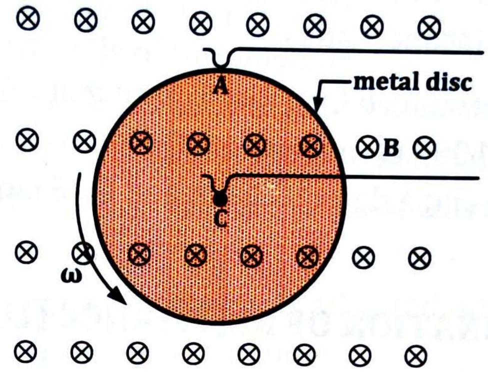


Fig. 3.3 (b)

   Derive an expression for the e.m.f. induced between A and C.
   (ii) A copper disc of radius 10 cm is placed is placed in a uniform magnetic field of flux density, 0.02 T, with its plane perpendicular to the field. If the disc is rotated about an axis parallel to the field through its centre at 3000 revolutions per minute. Calculate the e.m.f. that is generated between its rim and the centre. **Ans: [3.14×10^{-2} V]**
   (iii) Suppose the same disc in (ii) is rotated with its plane parallel to the field about an axis perpendicular to the field through its centre at 3000 revolutions per minute. Calculate the e.m.f. that is generated between its rim and the centre. **Ans: [0 V]**

3. A circular and uniform metal disc of having a plane of area, $3.0 \times 10^{-4} m^2$ is rotated at an angular speed, ω about an axis through its centre, parallel to a uniform magnetic field of flux density 5.0×10^{-4} T, in the direction, left to right. An e.m.f. of 1.5 mV is then generated between the centre and the rim of the disc. Determine the;
   (i) Value of the angular speed, ω. **Ans: [6.28×10^{4} rad. s^{-1}]**
   (ii) Number of revolutions per second made by the disc.
   Ans: [1.0×10^{4}Hz]
   (iii) If the disc is rotating in an anti-clockwise direction, which part of the disc is at a higher potential?
   Ans: [Axle or centre is at a higher, (i.e. + ve) potential]

4. A bicycle wheel with metal spokes each of length 31.0 cm and attached to the axle of radius 33.0 cm and has its plane vertical in the Earth's magnetic meridian. The region has a magnetic field of flux density 8.0×10^{-3} T where the angle of dip is 70°. The wheel is rotated about its axle at 3000 revolutions per minute. Determine the e.m.f. induced between its axle and the rim.

Ans: [1.61×10^{-2} V]

5. A Westland Lynx helicopter has a rotor with four blades each 6.4 m long and hovers in an area where the vertical component of the Earth's magnetic field is 4.0×10^{-5} T. If, as the rotor rotates, the tips of the rotor blade move with a speed of 200 m s^{-1}, calculate the induced e.m.f. between the
   (i) Tip of one blade and the axle. **Ans: [25.6 mV]**
   (ii) Tips of two diametrically opposite blades. **Ans: [0 V]**
   (iii) Tips of two adjacent blades. **Ans: [0 V]**

6. Calculate the e.m.f. produced by a disc rotating at 20 revolutions per second inside a solenoid of 1000 turns and length 0.2 m carrying a current of 2.0 A. The radii of the disc and axle are 2.0 cm and 0.25 cm respectively.

Ans: [124.4 μV]

3.4 ABSOLUTE DETERMINATION OF RESISTANCE (LORENTZ DISC EXPT.)

The method is known as ***absolute method***, because its accuracy is derived from the measurements made basing on the physical or fundamental quantities of, length (L), mass (M) and time (T) that are standard and need no calibration using a potentiometer.

When the metal disc at the centre of the solenoid is rotated, when current is flowing through the solenoid, the disc cuts the magnetic flux causing an e.m.f. to be induced between the centre and the rim of the disc. The direction of rotation of the disc is carefully chosen so that the p.d. set between the axle and the rim should drive the induced current I_2 in the opposite direction through the resistor R, opposing current I_1 flowing through the solenoid as provided by the battery. When the two currents pass through the centre zero galvanometer, G, in opposite directions, and the value of one of them adjusted steadily until G shows no deflection. This implies induced e.m.f. $E = B\pi r^2 f$, across R, equals the p.d, $V=IR$ across the resistor R.

Procedure

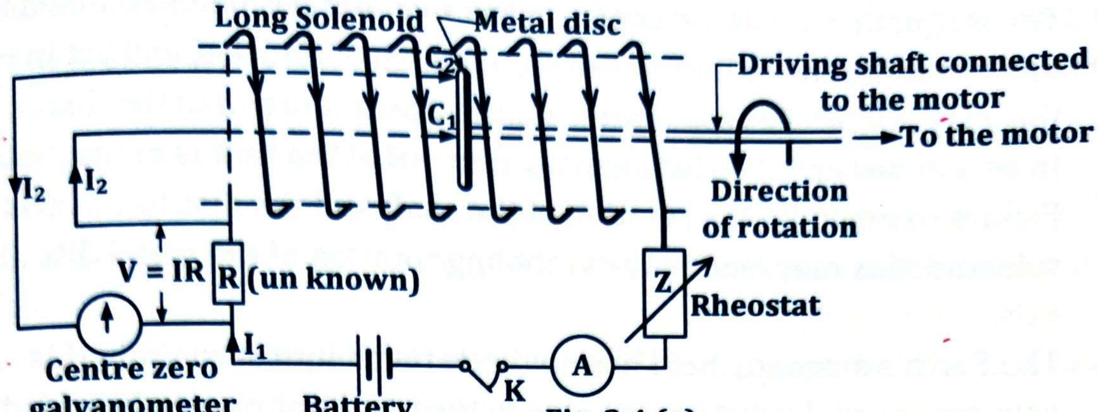


Fig. 3.4 (a)

- The experiment is set up as shown on the diagram in the figure 3.4 (a)
- The metal (copper) disc of a known measured radius, r, is placed at the centre of a solenoid of, n, turns per metre, with the plane of the disc perpendicular to the axis of the solenoid.
- Switch, K is closed and the rheostat, Z, is adjusted to a suitable value, then the copper disc is rotated via the shaft connected to an electric motor, in an appropriate direction to reduce the value of current flowing through, G, to zero.
- The speed of rotation of the motor, is adjusted until the centre – zero galvanometer, G, shows no deflection.
- The number of revolutions per second, f, made by the metal disc is noted from the revolution meter attached to the motor.
- Using p.d. across, R, equals the induced e.m.f. $IR = B\pi r^2 f$ *where* $B = \mu_0 nI \Rightarrow IR = \mu_0 nI\pi r^2 f$

  Hence, the resistance, R, is calculated from, $\boldsymbol{R = \mu_o n\pi r^2 f}$.

  Alternatively, $f = \frac{\omega}{2\pi} \Rightarrow R = \frac{\mu_o n r^2 \omega}{2} = \frac{\mu_o N r^2 \omega}{2L}$ *where,*

  $n = \frac{N}{L}$ is the number of turns per metre of the solenoid.

  Thus, the absolute resistance, R, is determined from any of the equations above.

  Angular velocity $\omega = \frac{2\pi}{T}$ can also be used, where T is the period of rotation of the motor measured using a stroboscope.

Limitations of the Lorentz disc. Experiment or possible sources of errors:

The Lorentz disc experiment described is not all, that free from some shortfalls due to the following reasons.

(i) The e.m.f. s generated between the axle and the rim are usually very small and therefore are generally difficult to measure.

(ii) The friction generated at the contacts C_1 and C_2 may generate thermo-electric e.m.f. s in the system that may not be negligible when compared to the induced e.m.f produced.

(iii) The magnetic field at the centre of the solenoid has been assumed to be uniform over the entire cross sectional area of the disc, and yet in reality this may not be the case right from the axis to the rim of the disc.

(iv) In accuracies involved in the measurement of the radius of the metal disc.

(v) Failure to maintain the rotation of the shaft and the disc to the axis of the solenoid, this may lead to the wobbling rotation of the metal disc about its axis.

(vi) The Earth's magnetic field has not been taken into account in this experiment, and yet it contributes to the resultant magnetic field at the centre of the solenoid.

3.5 Exercises on the disc generator/dynamo

1. A copper disc of radius 10.0 cm is situated in a uniform magnetic field provided at the centre of a long current carrying solenoid having 2500 turns per metre with its plane perpendicular to the field. The disc is rotated about an axis through its centre parallel to the field and to the axis of the solenoid. A resistor of resistance R = 49.3 Ω is connected to the circuit and the disc is rotated at an appropriate direction until the centre – zero galvanometer connected across the resistor, R, shows no deflection. Determine the number of revolutions per second required to achieve no deflection in the galvanometer. **Ans: [50 Hz]**

2. The figure 3.5 (a) shows a uniform copper disc of radius 8.0 cm placed inside a solenoid of length 10.8 cm and radius 12.5 cm made up of 235.62 m of fine insulated copper wire. While observing from the left hand side of the coil, the disc is rotated clockwise at a frequency of 60 Hz so that zero net current flows through the galvanometer, G.

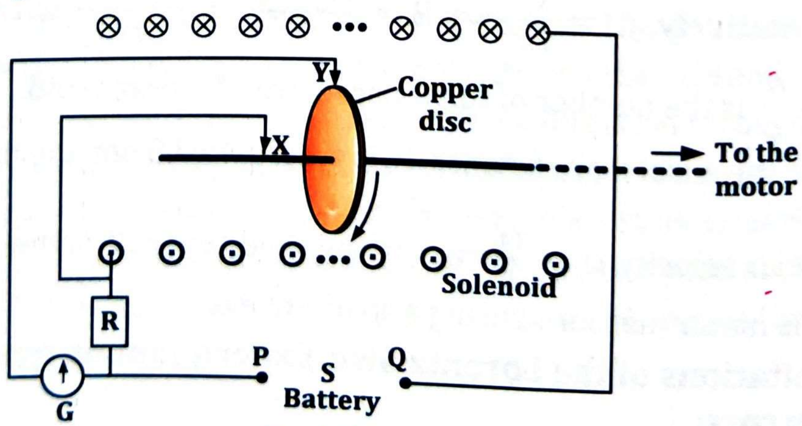


Fig. 3.5 (a)

(i) Redraw the diagram and indicate the polarity of the battery, S, and the disc.

(ii) Determine the number of turns of the solenoid. **Ans: [300 turns]**

(iii) What is the e.m.f. induced between the axis and the rim of the disc, when a current of 5.0 A flows through the solenoid? **Ans: [17.5 mV]**

(iv) Determine the value of R. **Ans: [3.5 m Ω]**

3. A solenoid having 250 turns per metre and carrying a current of 0.050A lies with its axis in the east – west direction. Well inside and at the centre of the solenoid is a small compass needle pointing 37° west of north.
   (i) Calculate the Earth's horizontal magnetic field component, B_H
   Ans: [2.08×10^{-5} T]
   (ii) Calculate the resultant magnetic field at the centre of the solenoid.
   Ans: [2.61×10^{-5} T]
   (iii) If a uniform copper disc of radius 0.05 m replaces the compass needle, sketch the position of the coil in the magnetic field and use it, to determine the e.m.f. induced between the axis and the rim of the disc, if its rotated at 3000 revolutions per minute. **Ans: [6.17×10^{-6} V]**
   (iv) Determine the resistance of the resistor connected up in the circuit in (iii) above for which the centre zero galvanometer G connected across enables G to show a zero deflection. **Ans: [1.23×10^{-4} Ω]**

4. A horizontal metal disc of radius 10.0 cm is rotated at 1800 revolutions per minute about a vertical axis through its centre, in the region where the Earth's magnetic induction is 5.4×10^{-5} T and the angle of dip is 70°. A sensitive galvanometer having a resistance of 0.15 Ω has its terminals connected to the axle and rim of the disc respectively. Assuming there is no friction at the positions of contact and the disc has negligible resistance, determine the;
   (i) E.m.f. induced in the coil. **Ans: [4.78×10^{-5} V]**
   (ii) Current that would flow through the galvanometer. **Ans: [3.19×10^{-4} A]**
   (iii) Power required to maintain the rotation of the disc.
   Ans: [1.52×10^{-5} W]

3.6 E.M.F. PRODUCED DUE TO CHANGING MAGNETIC FLUX LINKAGE

Whenever the magnetic flux linked with the plane of a coil or any metal conductor changes in ***whatever way*** or for ***whatever reason the flux changes***, an e.m.f. gets induced in the associated coil or conductor, in accordance with the laws of electromagnetic induction.

i.e. From Faraday's law, the magnitude of the e.m.f. induced, $|E| = \frac{d\Phi}{dt} = \frac{\Delta\Phi}{\Delta t}$

Some of the instances associated with the changes in the magnetic flux are,

- A conducting loop might be moving in a region where the magnetic field is not constant.
- A conducting loop rotating in a magnetic field.
- A conducting loop is changing size or shape.
- A conductor is placed in a region where the magnetic field is changing due to changes in the current flowing through a coil of wire.

Consider a plane circular coil of N – turns with its ends joined together to form a continuous and complete loop placed with its plane initially perpendicular to a uniform magnetic field of flux density B. The coil is turned through an angle θ smartly so that its plane is finally at an angle θ to the initial position, as shown on the diagram below.

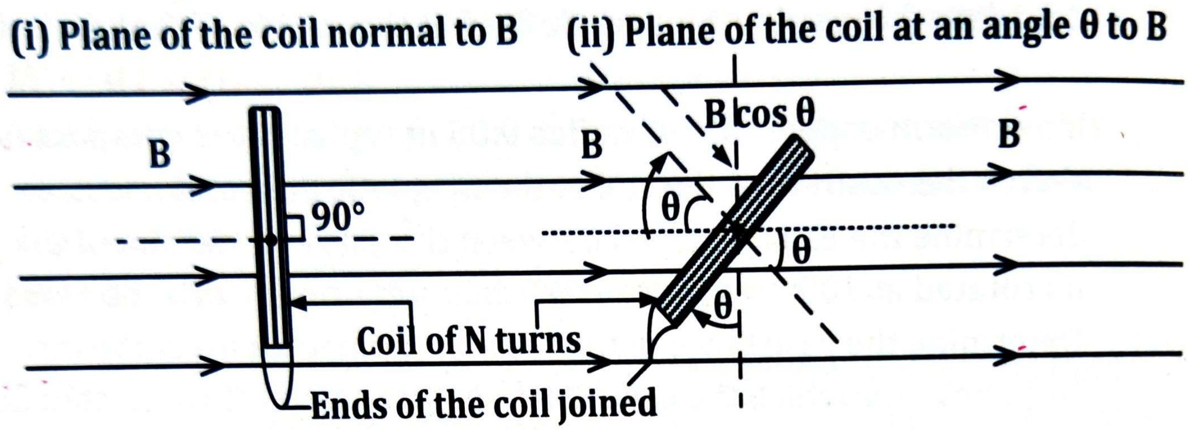


Fig. 3.6 (a)

- The magnetic flux threading a plane of the coil of one turn, $\phi = BA$,
- The magnetic flux linking N – turns of the coil, is given by, $N\phi_1 = NBA$
- When the coil is flipped about an axis through its centre parallel to its plane, the new magnetic flux linked with the coil, $N\phi_2 = NBA\cos\theta$
- The e.m.f. induced $E = -\left(\frac{final\ flux\ linkage - initial\ flux\ linkage}{time\ interval}\right)$

$$\therefore E = -\frac{d(N\phi_2 - N\phi_1)}{dt} = -\frac{d(NBA\cos\theta - NBA)}{dt} = \frac{NBA(1-\cos\theta)}{\Delta t}$$

$$\Rightarrow \textit{Induced e.m.f}, E = \frac{NBA}{\Delta t}(1-\cos\theta) \ldots\ldots\ldots\ldots\ldots (i)$$

Special Cases

When the ***coil is turned through 90°*** from initial position of maximum threading of the plane of the coil, θ = 90° and from (i) above, it implies

$$\textbf{Induced } e.m.f., E = \frac{NBA}{\Delta t} \quad since\ \cos 90° = 0 \ldots\ldots\ldots (ii)$$

When the ***coil is turned through 180°*** from initial position of maximum threading of the plane of the coil, θ = 180° and from (i) above, it implies

$$\textbf{Induced } e.m.f., E = \frac{2\,NBA}{\Delta t} \quad since\ \cos 180° = -1 \ldots (iii)$$

When the ***coil is turned through 60°*** from initial position of maximum threading of the plane of the coil, θ = 60° and from (i) above, it implies

$$\textbf{Induced } e.m.f., E = \frac{NBA}{2\,\Delta t} \quad since\ \cos 60° = \frac{1}{2} \ldots\ldots\ldots (iv)$$

When the ***coil is turned through 120°*** from initial position of maximum threading of the plane of the coil, θ = 120° and from (i) above, it implies

$$\textbf{Induced } e.m.f., E = \frac{3NBA}{2\,\Delta t} \quad since\ \cos 120° = -\frac{1}{2} \ldots\ldots (v)$$

Generally when the ***coil is turned through any angle θ,*** from initial position of maximum threading of the plane of the coil, to the final position when the plane of the coil makes an angle, θ from (i) above, it implies

Induced $e.m.f., E = \frac{NBA}{\Delta t}(1 - \cos\theta)$ ……………….(*vi*)

Examples

1. A plane circulr coil of 20 turns and of mean radius 5.0 cm is placed with its plane perpendicular to a uniform magnetic field of flux density 0.08 T. The coil is smartly pulled completely out of the field in 0.25 s, while maintaining its direction of orientation to the field. Determine the magnitude of e.m.f. nduced in the coil.

   Solution:

   $N = 20\ turns, r = 0.05\ m, A = \pi r^2 = \pi \times (0.05)^2 = 7.85 \times 10^{-3} m^2$

   $$E = -\frac{d\,N\Phi}{d\,t} = \frac{NBA(1-0)}{\Delta\, t}, \Rightarrow E = \frac{20 \times 0.08 \times \pi \times (0.05)^2}{0.25}$$

   $\therefore \boldsymbol{E = 5.03 \times 10^{-2}\ V}$

2. A circular coil of 100 turns and cross sectional area 0.2 m² is plcedd with its plane perpendicular to a horizontal magnetic field of flux density 1.0 × 10 ⁻² T. The coil is rotated about a vertical axis so that it turns through 60° in 2 s. Calculate the,

   (i) Initial flux magnetic linkage through the coil.

   (ii) E.m.f. induced in the coil.

   Solution

   (i) $N = 100\ turns, A = \pi r^2 = 0.2\ m^2, B = 1.0 \times 10^{-2}\ T, t = 2.0\ s$

   $Initial\ magnetic\ flux, linkage, N\Phi_1 = \text{NBA} = 100 \times 1.0 \times 10^{-2} \times (0.2)$

   $\therefore\ Initial\ magnetic\ flux, linkage, N\Phi_1 = 0.20\ Wb$

   (ii) $$E = -\frac{d\,N\Phi}{d\,t} = \frac{NBA(1 - \cos 60°)}{\Delta\, t}, \Rightarrow E = \frac{0.20(1-0.50)}{2.0}$$

   $\therefore \boldsymbol{E = 5.00 \times 10^{-2}\ V}$

3. A coil of 100 turns and cross sectional area 2.0 × 10 ⁻³ m² is placed in a magnetic field of 8.0 × 10 ⁻³ T so that the flux threas all the turns normally. Calculate the average e.m.f. induced in the coil if the field is reversed in 1/50 s.

   Solution

   $N = 100\ turns, A = \pi r^2 = 2.0 \times 10^{-3}\ m^2, B = 8.0 \times 10^{-3}\ T, t = 0.02\ s$

   $Initial\ flux, linkage, N\Phi_1 = \text{NBA} = 100 \times 8.0 \times 10^{-3} \times 2.0 \times 10^{-3}$

   $\therefore\ Initial\ magnetic\ flux, linkage, N\Phi_1 = 1.60 \times 10^{-3}\ Wb$

   $Final\ flux, linkage, -N\Phi_2 = -\text{NBA} = -100 \times 8.0 \times 10^{-3} \times 2.0 \times 10^{-3}$

   $\therefore\ Initial\ magnetic\ flux, linkage, N\Phi_2 = -1.60 \times 10^{-3}\ Wb$

   The e.m.f. induced $\boldsymbol{E} = -\left(\frac{final\ flux\ linkage - initial\ flux\ linkage}{time\ interval}\right)$

   $$\therefore E = -\frac{d(N\Phi_2 - N\Phi_1)}{dt} = -\frac{d[N\Phi_1 - (-N\Phi_2)]}{dt} = \frac{NBA[1-(-1)]}{\Delta\, t} = \frac{2NBA}{\Delta\, t}$$

$\therefore E = \left(\frac{2 \times 1.60 \times 10^{-3}}{0.02}\right) = 1.60 \times 10^{-1}\ V$

NB: ***Reversing magnetic field in the coil***, is equivalent to ***reversing the direction of flow of current*** in the coil and is also equivalent to ***flipping the coil through two right angles (i.e. 180°)*** when the magnetic field is steady or uniform.

4. A coil of 40 turns of wire and of radius 3.0 cm is placed between the poles of an electromagnet. The magnetic field increases from 0 to 0.75 T at a constant rate in a time interval of 225 s. What is the magnitude of the e.m.f. induced in the coil if the magnetic field is;
   (i) Perpendicular to the plane of the coil?
   (ii) Making an angle of 30° with the plane of the coil?

Solution

(i) $N = 40\ turns, A = \pi \times (0.03)^2 = 2.83 \times 10^{-3}\ m^2, t = 225\ s, B = 0.75\ T$

Using, E.m.f. induced $E = -\left(\frac{final\ flux\ linkage - initial\ flux\ linkage}{time\ interval}\right)$

$\therefore E = -\frac{d(N\phi_2 - N\phi_1)}{dt} = -\frac{40 \times 2.83 \times 10^{-3}(0.75-0)}{225}$

$\therefore |E| = 3.77 \times 10^{-4}\ V$

NB: When the magnetic field increases at a constant rate, the magnetic flux is also changing at constant rate and so the induced e.m.f. in the coil is constant.

If the rate of change of the field were not constant, then the 0.377mV would be the average e.m.f. during the time interval. The instanteneous e.m.f. would sometimes be higher or lower than the average value.

(ii) When the field is at 30° to the plane of the coil or (90°– 0°) = 60° to the normal to the plane, B cos 60° is the component normal to the field.

Thus from, $\therefore E = -\frac{d(N\phi_2 - N\phi_1)}{dt} = -NA\frac{(B\cos 60° - 0)}{\Delta t}$

$\therefore E = -\frac{40 \times 2.83 \times 10^{-3}(0.75\cos 60° - 0)}{225}$

$\therefore |E| = 1.89 \times 10^{-4}\ V$

5. A circular conducting coil with a radius of 3.4 cm is placed in a uniform magnetic field of 0.880 T with the plane of the coil perpendicular to the magnetic field. The coil is rotated through 180° about a vertical axis through its centre in 0.222 s.
   (i) What is the average induced e.m.f. in the coil during the rotation?
   (ii) If the coil is made up of copper wire of diameter 0.90 mm, what is the average current that flows through the coil during the rotation? *(Electrical resistivity of copper = 1.67 × 10 -⁸ Ω m)*

Solution

(i) Using, E.m.f. induced $E = -\left(\frac{final\ flux\ linkage - initial\ flux\ linkage}{time\ interval}\right)$

$\therefore E = -\frac{d(N\Phi_2 - N\Phi_1)}{dt} = -\frac{NBA(\cos 180° - 1)}{\Delta t}$

$\therefore Induced\ e.m.f., E = \frac{2\,NBA}{\Delta t}$ $since\ \cos 180° = -1$

$\therefore E = -\frac{2 \times 1 \times 0.880 \times \pi \times (3.4 \times 10^{-2})^2}{0.222}$

$\therefore |E| = 2.88 \times 10^{-2}\ V$

(ii) Diameter of the copper wire, d = 0.90 mm, resistivity, $\rho = 1.67 \times 10^{-8}\ \Omega\, m$

Using, $Resistance, R = \rho\frac{L}{A} = \rho\frac{2\pi r}{\left(\frac{\pi d^2}{4}\right)} = \frac{8\rho r}{d^2} = \frac{8 \times 1.67 \times 10^{-8} \times 3.4 \times 10^{-2}}{(0.90 \times 10^{-3})^2}$

$\therefore Resistance, R = 5.61 \times 10^{-3}\Omega$

$Now, induced\ e.m.f. E = IR \Rightarrow induced\ current, I = \frac{E}{R}$

$\therefore I = \frac{2.88 \times 10^{-2}}{5.61 \times 10^{-3}} = 5.13\ A$

$\therefore I = 5.13\ A$

6. A window frame of a house standing in the East – west direction is made up of 200 turns of fine copper wire joined at the free ends to make a complete and continueous loop. The dimensions of the window are 1.2 m by 0.8 m. The window is carefully opened about the hinges through an angle of 150° in 1.5 s. If the Earth's magnetic field at the location is 8.0 × 10 -4 T and the angle of dip is 60°, determine the;
   (i) Horizontal component of the Earth's magnetic field at that location.
   (ii) E.m.f. induced in the window frame.

Solution

(i) Horizontal component, $B_h = B\cos 60° = 8.0 \times 10^{-4} \times \cos 60°$

$\therefore B_h = 4.0 \times 10^{-4}\ T$

(ii) Using, E.m.f. induced $E = -\left(\frac{final\ flux\ linkage - initial\ flux\ linkage}{time\ interval}\right)$

$\therefore E = -\frac{d(N\Phi_2 - N\Phi_1)}{dt} = -\frac{(NBA\cos 150° - NBA)}{\Delta t} = -\frac{NBA(\cos 150° - 1)}{\Delta t}$

$\therefore Induced\ e.m.f., E = \frac{NBA \times (1.886)}{\Delta t}$ $since\ \cos 150° = -0.866$

$\therefore E = \frac{200 \times 4.0\times10^{-4} \times (1.2 \times 0.8) \times (1 + \cos 150°)}{1.5}$

$\therefore |E| = 9.55 \times 10^{-2}\ V$

NB, When the window is completely shut (Closed), its plane is threaded normally by the horizontal component of the Earth's magnetic field. When is being opened it "cuts" the magnetic flux threading its plane and when opened through 150° the flux begins to reverse direction in the coil. This change of the magnetic flux linked with the plane of the window (coil) in the given time interval causes an e.m.f. to be induced in this window frame or coil.

3.7 Exercises on magnetic flux change with time

1. The magnetic flux passing through a coil of 80 turns is reduced quickly but steadily from 2.0 mWb to 0.5 mWb in a time interval of 4.0 s. Determine the magnitude of the e.m.f. induced in the coil. **Ans: [3.00×10^{-2} V]**
2. A flat circular coil of 100 turns and mean radius 5.0 cm lying on a horizontal surface is turned over in 0.20 s. Calculate the mean e.m.f. induced in the coil, if the vertical component of the Earth's magnetic flux density is 4.0×10^{-5} T. **Ans: [3.14×10^{-1} V]**
3. A flat circular coil of area 4.5 cm^2 having 200 turns of wire of total resistance 20 Ω lies with plane perpendicular to a uniform magnetic field B = 0.60 T. If the coil is turned through 90° in 0.50 s, what is the average induced e.m.f. and current in the coil, if the external circuit resistance is zero. **Ans: [1.08×10^{-1} V, 5.4 mA]**
4. The figure 3.7 (a) shows a horse track at the beginning of a horse – race with a horizontal wire of length 20 m being raised vertically through a height of 3.0 m in 0.20 s.

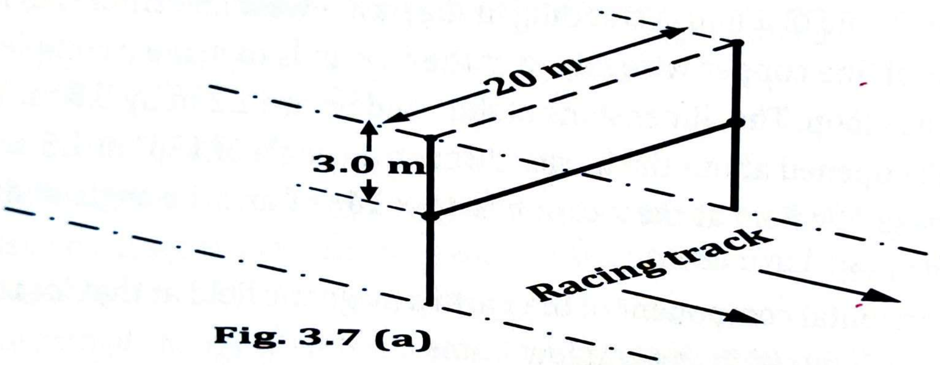


Fig. 3.7 (a)

Given that the horizontal component of the Earth's magnetic field strength is 2.0×10^{-5} T. What is the average e.m.f. induced across the ends of the wire? **Ans: [6.0 mV]**

5. A metal window frame 1.3 m high and 0.7 m wide is pivoted about a vertical edge and faces due south in a region where the horizontal and vertical components of the earth's magnetic field are 20 μT and 50 μT respectively. The window is opened through an angle of 90° in a time of 0.80 s.
   (i) What is the change of magnetic flux associated with the opening of the window? **Ans: [1.82×10^{-5} Wb]**
   (ii) Calculate the average e.m.f. induced at the window. **Ans: [22.75 μV]**
   (iii) State and explain the effect on the induced e.m.f. on converting the window to a sliding mechanism for opening. **Ans: [No change of flux, so e.m.f.=0V]**
6. A flat circular coil of 120 turns, each of area 0.070 m^2, is placed with its axis normal to a uniform magnetic field of 80 mT. The coil is rotated about its vertical diameter through 70° in 4.0 s. Determine the,
   (i) Change of the magnetic flux linked with the coil. **Ans: [5.82×10^{-1} Wb]**
   (ii) Average e.m.f. induced in the coil. **Ans: [1.45×10^{-1} V]**

7. A coil of 100 turns and radius 10.0 cm is mounted so that the axis of the coil can be oriented in any horizontal direction. Initially the axis is oriented so that magnetic flux form the Earth's magnetic field is maximized. If the coil's axis is rotated through 90° in 0.08 s, an e.m.f. of 0.687 mV is induced in the coil.
   (i) Determine the magnitude of the horizontal component of the Earth's magnetic field at this location. **Ans: [1.75×10^{-5} T]**
   (ii) If the same coil is transferred to a position where the Earth's resultant magnetic flux density is 1.60×10^{-4} T and the angle of dip is 60°, what would the e.m.f. induced in the coil be, if the time of rotation of the coil is maintained as 0.08s ? **Ans: [5.44×10^{-3} V]**

8. A flat circular coil of 150 turns, each of area 0.090 m², is placed with its plane making an angle of 30° to a uniform magnetic field of 800 μT. The coil is rotated about its horizontal diameter until its plane makes an angle of 120° to the magnetic field in 2.5 s. Determine the,
   (iii) Change of the magnetic flux linked with the coil. **Ans: [3.95×10^{-3} Wb]**
   (iv) Average e.m.f. induced in the coil. **Ans: [1.58×10^{-3} V]**

9. The magnetic field between the poles of an electromagnet is 2.6 T. A coil of wire is placed in this region so that the field is parallel to the axis of the coil. The coil has electrical resistance of 25 Ω, radius 1.8 cm, and length 12.0 cm. When the current supply to the electromagnet is shut off, the total charge that flows through the coil is 9.0 mC. Determine the number of turns of the coil. **Ans: [1.76×10^{8}]**

10. A bar magnet is initially far from a closed loop of wire. The magnet is moved at a constant speed along the axis of the loop as shown on the diagram in figure 3.7(b)

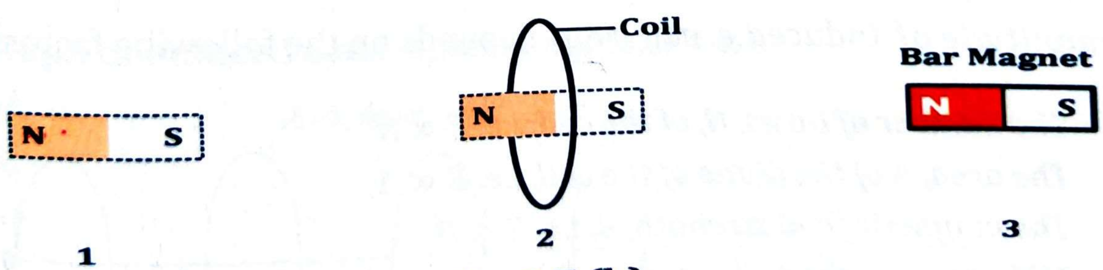


Fig. 3.7 (b)

It moves towards the loop, proceeds to pass through it, and then continues until it is far away on the right hand side of the loop.
   (i) Sketch a qualitative graph of the current in the loop as a function of the position of the bar magnet. Take the current to be positive anti – clockwise as viewed from the left.
   (ii) Explain the shape of the graph.

3.8 E.M.F. PRODUCED DUE TO ROTATION OF A COIL IN A MAGNETIC FIELD

Consider a rectangular coil, **abcd** of **N** – turns being rotated about an axis through the centre of its shorter sides **ad** and **bc** and perpendicular to a uniform magneic field of flux density, **B**, at a constant angular speed ω.

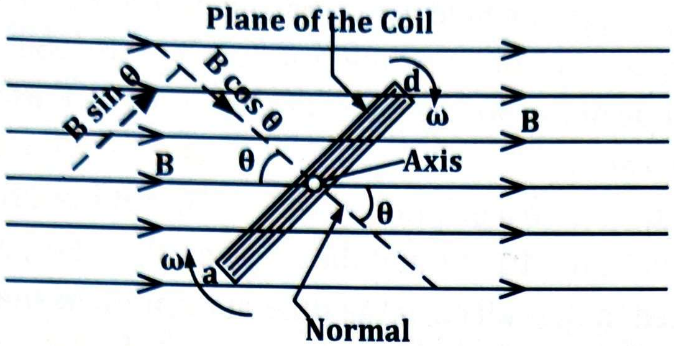


Fig. 3.8 (a)

Initially at t = 0 s the plane of the coil is Normal to the magnetic field.
Magnetic flux linking the coil after a time t, $\Phi = BAN\cos\theta$ $where\ \theta = \omega t$
E.m.f. induced in the coil because of its rotation,

$$E = -\frac{d(BAN\cos\theta)}{dt} = -NAB\frac{d(\cos\omega t)}{dt}$$

$$\therefore E = \boldsymbol{NAB\omega\ sin\ \omega t}$$

$\therefore$ $\boldsymbol{E = E_0\ sin\ \omega t}$ $where\ E_0 = NAB\omega\ is\ the\ peak\ value\ of\ induced\ e.m.f.$
Where, E is expressed in volts (V),
N = the number of turns of the coil,
A = the area of the plane of the coil, expressed in (m^2)
B = the magnetic field strength (Flux density) in the region of the coil, expressed in tesla (T)
ω = the angular speed of the coil expressed in radians per second ($rad.s^{-1}$)

NB: $E_0 = NAB\omega$, *is the* ***Peak*** *or maximum value or amplitude of induced e.m.f. that* depends on the following factors:

(i) ***The number of turns, N, of the coil, i.e. E ∝ N***
(ii) ***The area, A of the plane of the coil, i.e. E ∝ A***
(iii) ***The magnetic field strength, B, i.e. E ∝ B***
(iv) ***The angular speed, ω = 2π f, i.e. E ∝ ω or E ∝ f***
(v) ***The position or angle θ of inclination of the plane of the coil to the magnetic field, as a function of time, e.g. E = E_{max} = E_0 = NABω***
When θ = 90° i.e. when the plane of the coil has turned through 90° from its initial position when the plane of the coil is normal to the field.
Hence, when the plane of the coil is normal to the magnetic field, the magnetic flux ϕ= NAB (a maximum), but the rate of cutting of the magnetic flux is zero, implying the induced e.m.f. E = 0.
However, when the magnetic flux linking the plane of the coil is zero (i.e. the plane of the coil is parallel to the filed), the rate of cutting of the magnetic flux is zero. i.e. the induced e.m.f. is maximum.

Position of the coil in a magnetic field with time and the associated graphs.

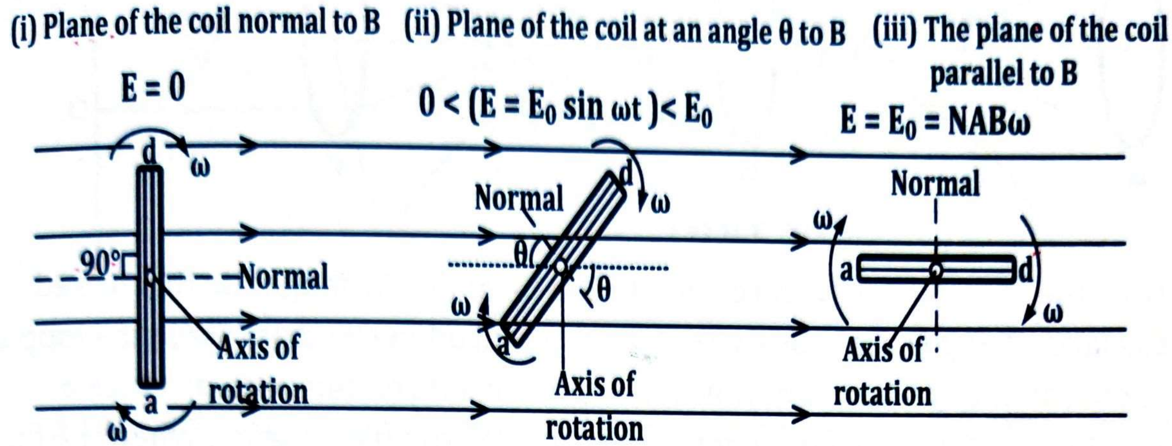


Fig. 3.8 (b)(i) Fig. 3.8 (b)(ii) Fig. 3.8 (b)(iii)

From the diagrams (i) to (iii) above, induced e.m.f. E = 0, when the plane of the rotating coil is normal to the magnetic field. The induced e.m.f. begins to increase when the rate of cutting of the magnetic flux increases, but when the magnitude of the flux linked with the plane of the coil reduces to zero.

Graphs of induced e.m.f. and magnetic flux linkage with time.

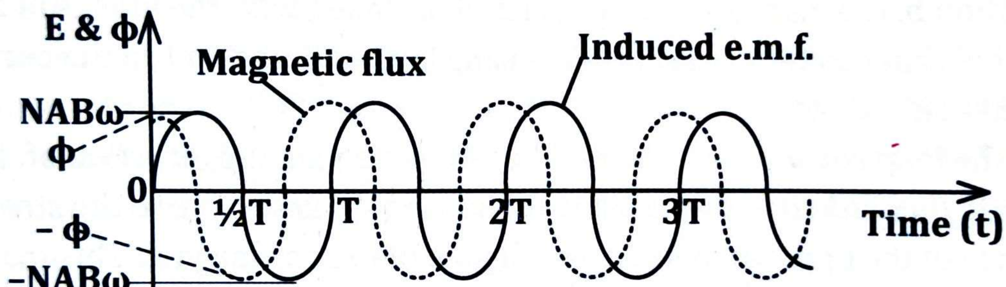


Fig. 3.8 (c)

A Graph of induced e.m.f. against angle, θ = ωt.

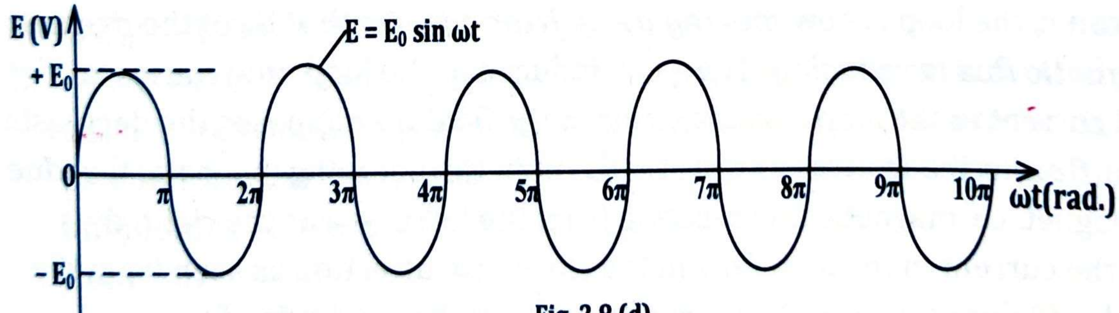


Fig. 3.8 (d)

Movement of a coil of wire towards a bar magnet or vice – versa

Consider a circular loop of wire moving towards a bar magnet at a constant velocity, v. the loop passes around the magnet and continues away from it to the side as shown in the figure 3.8 (e)

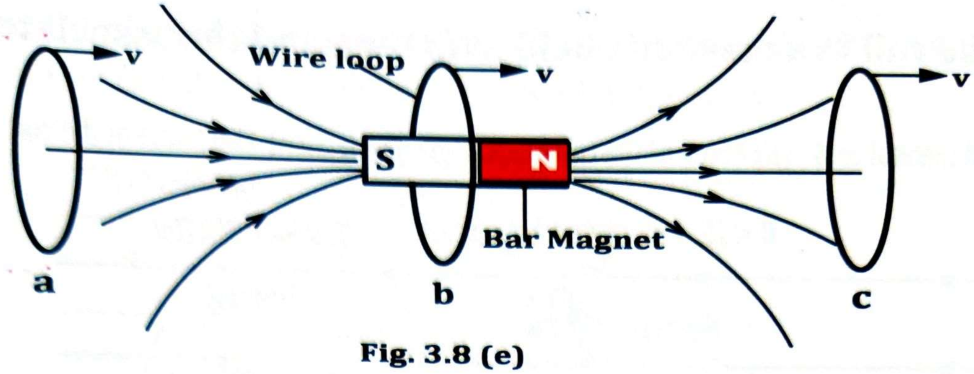


Fig. 3.8 (e)

Each time the loop moves with respect to the magnet, the magnetic flux linked with the loop changes causing an e.m.f. to be induced in the loop. Since the loop is continuous, an induced current flows in it in such a direction as to produce a magnetic flux that ***opposes*** or *enhances* that due to the bar magnet, when the flux is ***increasing*** or *decreasing* respectively.

At position **a**, the *loop is moving towards the magnet*, so there is *increasing magnetic flux linking the loop*. To oppose the increase, the current induced in the loop makes the magnetic field lines act towards the left to *oppose the increasing flux* due to the bar magnet. Hence, induced current flows *anti clockwise* as observed from the left hand side.

At position **b**, the magnetic flux associated or linked with the loop is in the same direction as in position **a**, but is decreasing in magnitude and so the current induced in it reduces.

When the loop goes beyond the middle of the magnet, the direction of the magnetic ***flux linked*** with the plane of the loop ***reverses*** due to the stronger influence of the opposite pole i.e. North pole of the bar magnet. This magnetic flux ***begins to grow or increase*** as the coil *approaches the other pole i.e. the North Pole,* reaching its maximum value as the loop just reaches the north pole of the magnet. Thus from the middle of the magnet ***the current flowing in the loop also reverses direction.***

At positon c, the loop is now ***moving away from the North Pole*** of the magnet, the ***magnetic flux is reducing.*** The e.m.f. induced in the loop now causes an induced current in the loop to create a magnetic field that opposes the decreasing magnetic flux i.e. it enhances or tries to maintain the decaying magnetic flux due to the magnet. i.e. magnetic flux is acting from the left towards the right, thus making the current in the loop flow in the clockwise direction as seen from the left hand side or anti-clockwise as seen from the right hand side of the loop.

As the loop moves very far from the magnet the influence of the magnetic field on it seizes to exist and e.m.f. and current in the loop both become zero.

If the clockwise direction of flow of current through the loop is taken as positive, a graph of induced current against time for motion of the loop with respect to the stationary bar magnet has the shape shown in figure 3.8 (f)

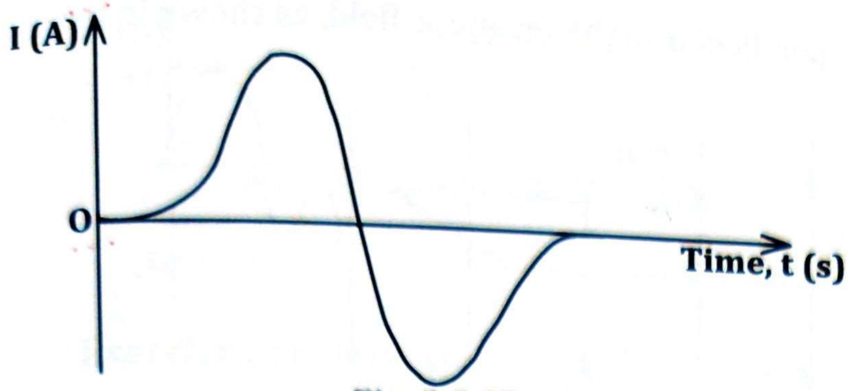


Fig. 3.8 (f)

9 EXAMPLES & EXERCISES ON MOTION OF A COIL IN A MAGNETIC FIELD:

1. A coil of 500 turns and area 80 cm² is rotated at 1200 revolutions per minute about an axis perpendicular to its plane and magnetic flux density 0.25 T Calculate the e.m.f. induced in the coil.

   Solution:

   $Given\ information: N = 500\ turns, A = 80 \times 10^{-4}\ m^2, B = 0.25\ T$

   $E = NAB\omega \sin \omega t\ but\ when, \omega t = 90°\ E = E_0 = BAN\omega, and\ \omega = 2\pi f$

   $\therefore E_0 = 500 \times 80 \times 10^{-4} \times 0.25 \times (2 \times \pi \times 20) = 125.66\ V$

   $\therefore \boldsymbol{E_0 = 1.26 \times 10^2\ V}$

2. A rectangular coil of 50 turns is 15.0 cm wide and 30.0 cm long. If it is rotated at a constant rate of 2000 revolutions per minute about an axis parallel to its long side and at right angles to a uniform magnetic field of flux density 0.04 T. Determine the peak value of the e.m.f. induced in the coil.

   Solution:

   $Given\ information: N = 50\ turns, A = (15.0 \times 30.0) \times 10^{-4}\ m^2, B = 0.04\ T$

   $E = NAB\omega \sin \omega t\ but\ when, \omega t = 90°\ E = E_0 = BAN\omega, and\ \omega = 2\pi f$

   $$\therefore E_0 = 50 \times 4.50 \times 10^{-2} \times 0.04 \times \left(2 \times \pi \times \frac{2000}{60}\right) = 18.85\ V$$

   $\therefore \boldsymbol{E_0 = 1.89 \times 10^1\ V}$

3. A flat circular coil with 2000 turns, each of radius 50 cm, is rotated at a uniform rate of 600 revolutions per minute about its diameter at right angles to a uniform magnetic field of flux density 5.0 × 10⁻⁴ T. Calculate the amplitude of the amplitude of the induced e.m.f.

   Solution:

   $Given\ that: N = 2000\ turns, A = \pi(0.50)^2 = 7.85 \times 10^{-1}\ m^2, B = 5.0 \times 10^{-4}\ T$

   $E = NAB\omega \sin \omega t\ but\ when, \omega t = 90°\ E = E_0 = BAN\omega, and\ \omega = 2\pi f$

   $$\therefore E_0 = 2000 \times 7.85 \times 10^{-1} \times 5.0 \times 10^{-4} \times \left(2 \times \pi \times \frac{2000}{60}\right) = 164.4\ V$$

   $\therefore \boldsymbol{E_0 = 1.64 \times 10^2\ V}$

4. A coil of 100 turns and cross sectional area 2.0×10^{-2} m² lies in a magnetic field of flux density 3.0×10^{-3} T and rotates uniformly at 100 revolutions per

second about an axis perpendicular to the magnetic field, as shown in the figure 3.9 (a)

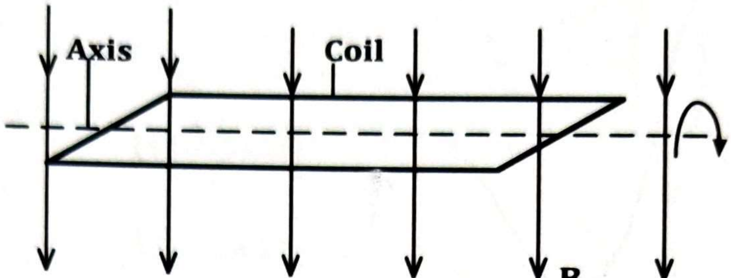


Fig. 3.9 (a)

Calculate the;

(i) E.m.f. induced when the plane of the oil makes an angle of 60° with B.

(ii) Amplitude of the induced e.m.f.

Solution:

(i) *Given that*: $N = 100\ turns, A = 2.0 \times 10^{-2}\, m^2, B = 3.0 \times 10^{-3}\, T$

$E = NAB\omega \sin \omega t$ *but where*, $\omega t = (90° - 60°)$ *and* $\omega = 2\pi f$

$\therefore E = 100 \times 2.0 \times 10^{-2} \times 3.0 \times 10^{-3} \times (2 \times \pi \times 100) \sin 30° = 1.885\ V$

$\therefore \boldsymbol{E = 1.89\ V}$

(ii) Amplitude is the peak value of the induced e.m.f. E_0 where,

$E_0 = NAB\omega$, *and* $\omega = 2\pi f$

$\therefore E_0 = 100 \times 2.0 \times 10^{-2} \times 3.0 \times 10^{-3} \times (2 \times \pi \times 100) = 3.77\ V$

$\therefore \boldsymbol{E_0 = 3.77\ V}$ *is the amplitude of the induced e.m.f.*

5. A current $I = 10 \cos 120\, \pi t$ is passed through a solenoid of 1000 turns per metre. A small circular coil of 500 turns of radius 3.5 cm has an axis through its centre being parallel to that of the solenoid.

(i) Determine the peak value of the e.m.f. induced in the circular coil when the current is flowing in the solenoid.

(ii) Sketch a graph of induced e.m.f. with time,

Solution

(i) $E = -\frac{d\, N\phi}{dt} = -\frac{d}{dt}(BAN) = -\mu_0 nAN \frac{d}{dt}(I)$

$E = -\mu_0 nAN \frac{d}{dt}(10 \cos 120\, \pi t) = \mu_0 nAN \times 10 \times 120\pi \sin 120\pi t$

$\therefore \boldsymbol{E = 1200\, \pi\, \mu_0 nAN \sin 120\pi t}$ ***is the e.m.f. induced in the coil***

The amplitude, $E = E_0$, *where*, $E_0 = 1200\pi\, \mu_0 nAN$

$\Rightarrow E_0 = \mu_0 nAN \times 10 \times 120\pi$

$\therefore\ E_0 = 4\pi \times 10^{-7} \times 1000 \times [\pi \times (0.035)^2] \times 500 \times 1200\pi$

$\therefore\ \boldsymbol{E_0 = 9.12\ V}$

(ii)

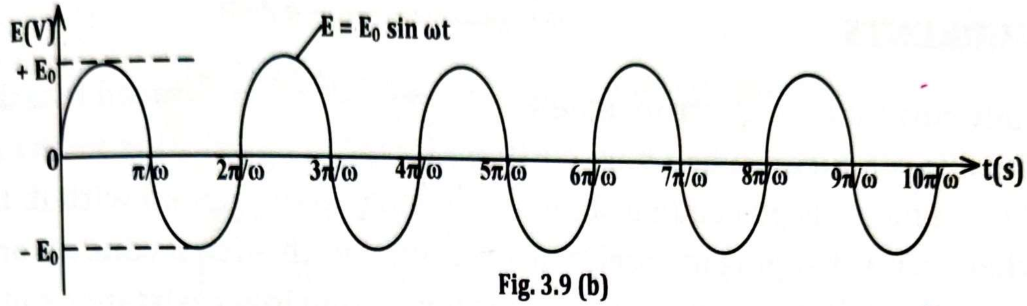


Fig. 3.9 (b)

Exercises on rotation of a coil in a magnetic field

1. A rectangular coil of 200 turns and of dimensions 5 cm by 4 cm is placed with its plane normal to a uniform magnetic field of flux density 0.008 T is rotated about an axis through the centre of the shorter sides perpendicular to the magnetic field at 420 revolutions per minute. Determine the peak value of the e.m.f. induced in the coil. **Ans: [1.41×10^{-1} V]**
2. A coil of 300 turns and with area of 0.05 m² is rotated 20 times per second in a magnetic field of flux density 0.2 T. Calculate the;
   (i) Maximum e.m.f. induced in the coil. **Ans: [3.77×10^{2} V]**
   (ii) Torque required to maintain this rate of rotation if the current in the coil is 0.8 A when the e.m.f. generated is a maximum. **Ans: [2.4 Nm]**
3. A current $I = 10 \cos 120\pi t$ is passed through a solenoid of 1000 turns per metre. A small circular coil of 500 turns of radius 3.5 cm has an axis through its centre being parallel to that of the solenoid. Determine the torque on the coil.
4. A rectangular coil of 30.0 cm long and 20.0 cm wide has 25 turns. It rotates at a uniform rate of 3000 revolutions per minute about an axis parallel to its longer side and at right angles to a uniform magnetic field of flux density 5.00×10^{-2} T Find the;
   (i) Frequency of rotation of the coil. **Ans: [50 Hz]**
   (ii) Peak value of the e.m.f. induced in the coil. **Ans: [23.6 V]**
   (iii) Describe with the aid of diagrams how you arrange for the rotating coil to supply an external circuit with direct current and also alternating current separately.
5. The e.m.f. generated by a simple-coil a.c. generator may be represented by the equation $E = E_0 \sin \omega t$.
   (i) State the meanings of each symbol employed in the above equation, and give the associated units used in each symbol.
   (ii) Draw diagrams showing the relative positions of the coil and the magnetic field – when t = 0 , and – when $E = E_0$
   (iii) Discuss the factors that in practice determine the maximum current, which may be generated by such a generator.
   (iv) Deduce a formula for the torque on the coil at the moment when the maximum current is flowing assuming the coil is rectangular and state the units of any new symbols that you may employ.

4.0 EDDY CURRENTS

- Eddy currents are electromagnetic currents that are induced in a thick metallic conductor when it is "cutting across" magnetic flux lines or when the conductor is placed in a changing magnetic flux linked with it. I.e. whenever a changing magnetic flux is linked with such a conductor, eddy currents are induced in it and always flow along low resistance paths.
- Eddy currents always flow in such a direction to oppose the changes causing them to be produced.
- Eddy currents are also due to (or result from) the induced e.m.f s caused by changing magnetic flux or when a conductor is moved across a uniform magnetic field.
- Eddy currents always follow low resistance paths in metals and may be large even if the associated e.m.f s induced in the conductor may be small and they usually cause considerable heating and magnetic effects.

Explanation about the production of Eddy currents in a metal

- When **a thick metal block or plate *is moved across a magnetic field***, or is placed in a changing magnetic field, ***a magnetic force*** acts on the ***conduction electrons.***
- **This force**, displaces the electrons in ***a particular direction***, in accordance to **Fleming's right hand rule** or cause the displacement of opposite charges, with the electrons pushed to one direction while the positive charges left behind.
- The positive charges begin to attract the electrons from another direction causing movement of the electrons towards the positive charge.
- This ***movement of electrons lead to a current loop called eddy current*** that always flows along low resistance paths within the thick metal block or plate.
- These eddy currents also flow in such a direction as to oppose the action that caused them. (Lenz's law)

The Heating Effect of Eddy currents

Experiments to demonstrate the heating effect of eddy currents:

(a) Induction furnace.

Diagram

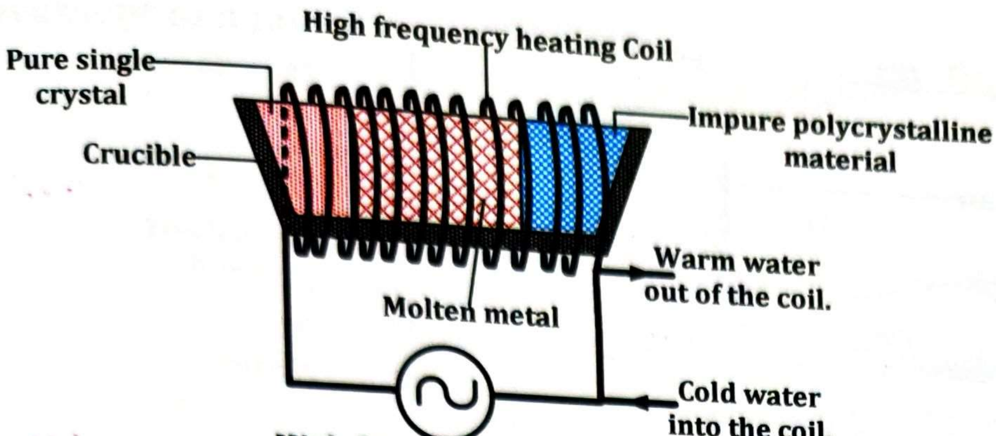


Fig. 4.0 (a)

Mode of operation

- The metal pieces to be melted are fed into the crucible surrounded by a water-cooled high frequency a.c. coil as shown on the diagram.
- When the system is switched on, the rapidly changing magnetic flux caused by high frequency alternating current flowing in the coil, induces large eddy currents in the conducting parts of the material, causing them to melt off.
- Insulators and any non-conducting materials (i.e. impurities) are not affected by the eddy currents.
- When the crucible is moved very slowly in through the coil, the impurities tend to collect in the molten zone, which moves to one end of the crucible.
- After cooling, this end is removed leaving, a very pure single crystal sample.
- This is a typical example employed in zone – refining of metals and semi – conductors by eddy current heating.

(b) The Induction cooking stove

In induction stove has a structure resembling an ordinary hot plate cooking stove. However induction stove has an electromagnet just generates an oscillating magnetic field just beneath the cooking surface as opposed to a heating element or coil as is the case of a hot plate.

Secondly the cooking surface of an induction stove does not feel hot when touched except for the heat conducted to it by the metal pan heated by eddy currents.

Diagram

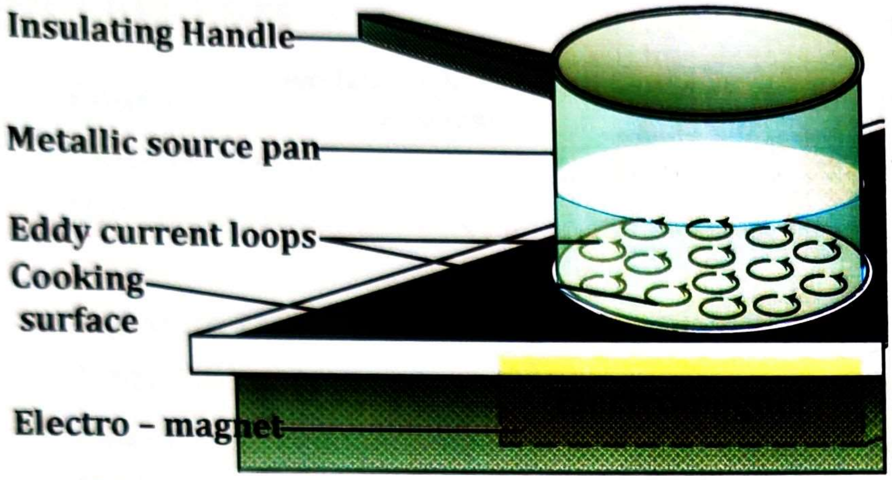


Fig. 4.0 (b)

Mode of operation

- An electro-magnet placed underneath the cooking surface is switched on, creating a high alternating magnetic field.
- When a metallic pan (e.g. Aluminium pan) containing food to be cooked is placed on the cooking surface of the stove, the induced e.m.f. causes induced eddy currents to be spontaneously created and flow in small loops in relation to Lenz's law.
- The resistance of the metal pan causes the eddy currents to dissipate energy in the pan in the I^2R – mechanism, leading to the generation of heat in the source pan, used to cook the food in it.

The magnetic Effect of Eddy currents

- When a thick metal block is pulled across a uniform magnetic field, the magnetic flux linked with it keeps on changing, and an e.m.f is generated across the ends of the conductor and this set up induced current loops within the material of the conductor, by Fleming's Right hand Rule as shown on the sketch diagram below.
- Conversely when the induced current, I, begins to flow in the thick conductor, a magnetic force, F = BIL, will be generated acting in such a direction as to oppose to the mechanical force causing motion of the conductor across the magnetic field (By Fleming's Left Hand Rule), this force in turn will oppose motion, and gradually reduces the velocity of the conductor (thus acting as brakes to motion).
- The conductor's motion then becomes damped, it slows down and eventually stops.
- As soon as the conductor stops, eddy currents also seize to exist and the opposing magnetic force disappears.

Movement of a metal conductor across a magnetic field

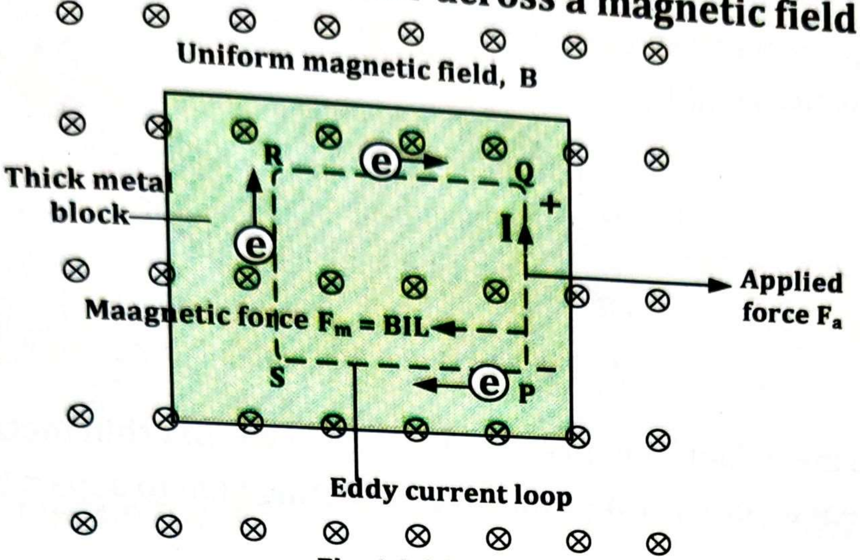


Fig. 4.0 (c)

Consider side **PQ** of the current loop **PQRS**, from Fleming's Right hand rule, the magnetic Field **B** is ***into*** the plane of the paper, while the applied Force, $\mathbf{F_a}$ acts to the Right. The induced current **I**, will act or flow vertically upwards, because of motion of electrons in the opposite direction (Clockwise direction) as shown in the figure 4.0 (c).

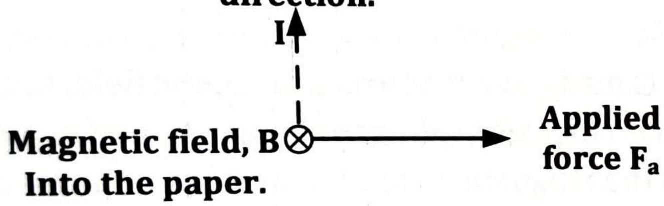


Fig. 4.0 (d)(i)

Considering the same side **PQ** of the current loop, and the magnetic field perpendicular and **into** the plane of the paper, a magnetic force $\mathbf{F_m}$, acts in the opposite direction, thus opposing the original motion causing damping of the motion of the metal block.

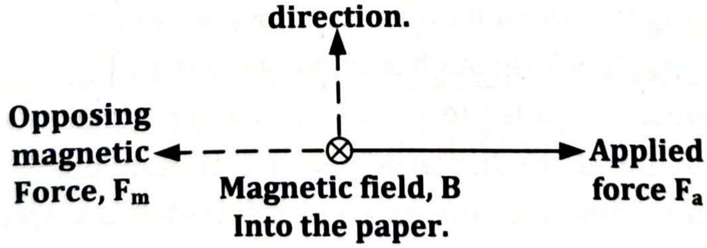


Fig. 4.0 (d)(ii)

Minimising / Reducing the effect of induced Eddy Currents

The effect above due to Eddy currents can be minimised by Laminating the metal block with strips of the metal glued together with an insulator, thus breaking down the original large induced current loop to negligible loops.

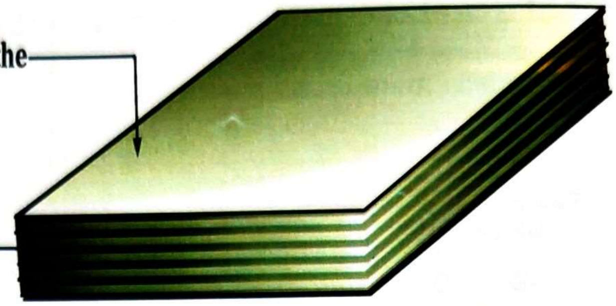


Fig. 4.0 (e)

Laminating the metal block involves slicing it into tiny and thin metal sheets and then joining them together using adhesive insulating glue to separate the metal sheets.

This brakes the originally large and energetic eddy current loops to tiny loops, this increases the resistance of the metal and eventually reducing the net or resultant eddy currents flowing in the metal block.

Experiments to demonstrate the damping effects of Eddy currents

(a) Motion of a solid and a slotted metal block across a magnetic field

(i) When a metal conductor is made to cut across a magnetic field, the magnetic flux linked with it changes leading to the creation of eddy currents that get induced in it and the magnetic field acts on these induced currents in such a direction as to create a magnetic force to oppose the motion of the conductor.

Procedure

- In figures 4.0 (f) (i) & (ii), a slotted metal piece and a non-slotted metal Piece respectively, are set into free swinging motion at the same time between the pole pieces of similar strong permanent magnets.
- The insulating string supporting the metal block is set to swing by displacing each metal block through a small angle θ and its left to swing freely in the region between the pole pieces of the strong magnet.
- The metal blocks **A** and **B** are displaced either simultaneously or one at a time, and at the same time the stopwatch or stop clock is started.
- The time it takes the respective blocks, t_1 and t_2 to stop swinging are noted on each of the stopwatches or stop clocks.
- It is observed that the time taken, t_1, taken by the slotted metal piece, **A** to stop swinging is much longer than that of the solid or the un–slotted metal piece, **B**. i.e. $t_1 > t_2$.
- Implying large eddy currents are induced in the solid metal block **B**, caused a magnetic force that acts in a direction that opposes motion of the solid metal block each time.

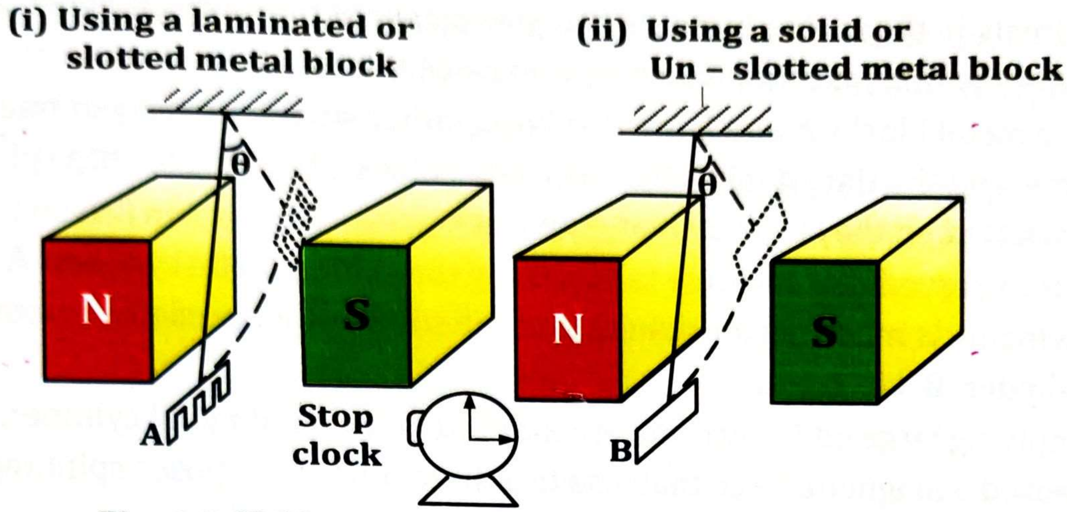


Fig. 4.0 (f)(i) Fig. 4.0 (f)(ii)

- While the slotted metal block has negligible induced eddy currents loops in it that create a high resistance to current flow therefore minimizing the magnetic force opposing the applied force causing the swinging.
- This greatly minimizes the damping thus having almost no resistive effect on its motion.

(ii) **Spinning of a metal cylinder between the pole pieces of a strong magnet**

An experiment having almost a similar principle to that in (i) above can also be adopted to demonstrate the damping effect of eddy currents, as shown on the diagram in the figure 4.0 (g) (i) & (ii)

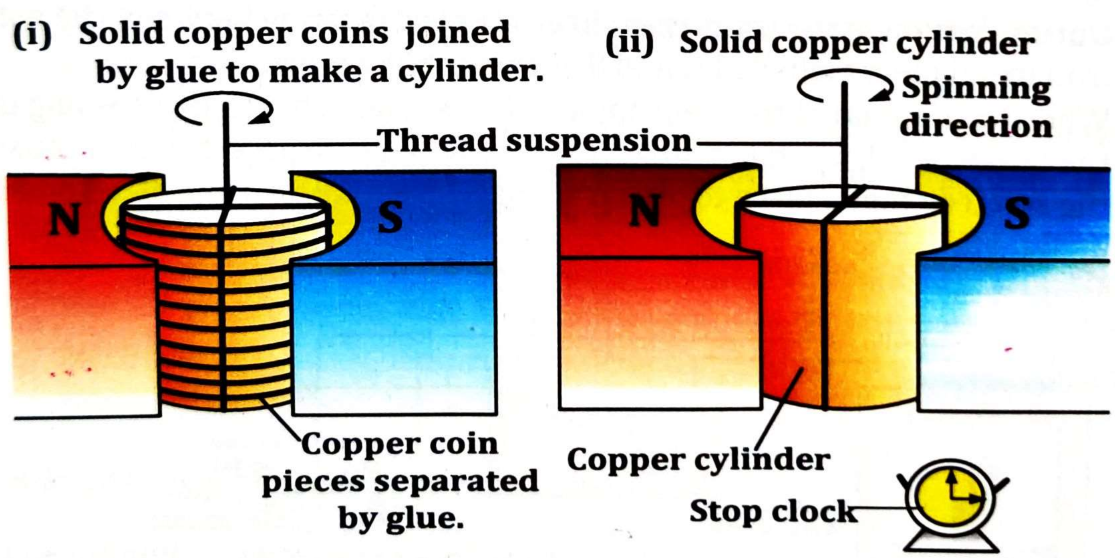


Fig. 4.0 (g)(i) Fig. 4.0 (g)(ii)

Procedure

- In figures (i) & (ii) above, a slotted metal piece (copper coins) and a non-slotted metal Piece (copper cylinder) respectively, are set into free rotation motion about a vertical axis of the insulating thread in each case at the same time between the pole pieces of strong permanent magnets.
- The insulating string supporting the metal block is spun clockwise to rotate

it freely in the region between the pole pieces of the strong magnet, and at the same time, the stopwatch or stop clock is started.

- The metal blocks **A** and **B** are displaced either simultaneously or one at a time, and the time it takes the respective blocks, t_1 and t_2 to stop spinning are noted on the stopwatch or stop clock.
- It is observed that the time taken, t_1, by the slotted metal cylinder, **A** to stop swinging is much bigger than that of the solid or the un–slotted metal cylinder, **B**. i.e. $t_1 > t_2$.
- Implying large eddy currents are induced in the solid metal cylinder, **B**, caused a magnetic force that acts in a direction that opposes spinning motion of the solid metal cylinder.
- While the slotted metal pieces has negligible induced eddy currents loops in it that create a high resistance to current flow therefore minimizing the magnetic force opposing the applied spinning force causing the rotation.
- This greatly minimizes the damping thus having almost no resistive effect on its spinning motion.

(b) Electromagnetic brake in Auto-mobiles e.g. Trains

Large auto-mobiles like trains and Lifting cranes, employ electromagnetic brakes in addition to Hydraulic brakes in bringing motion of the system to a stop, with the help of electromagnets.

Mode of operation

- The system is set up as shown on the diagram in the figure 4.0 (f)
- During motion of the train, the wheel W keeps rotating between the soft iron metal pieces labelled N and S of an electromagnet.
- When the Captain of the train applies the hydraulic brake by pressing down the brake pedal, a current flows from the battery through the coil, causing the soft iron metal piece to get magnetized creating a strong electromagnet as shown.

Diagram:

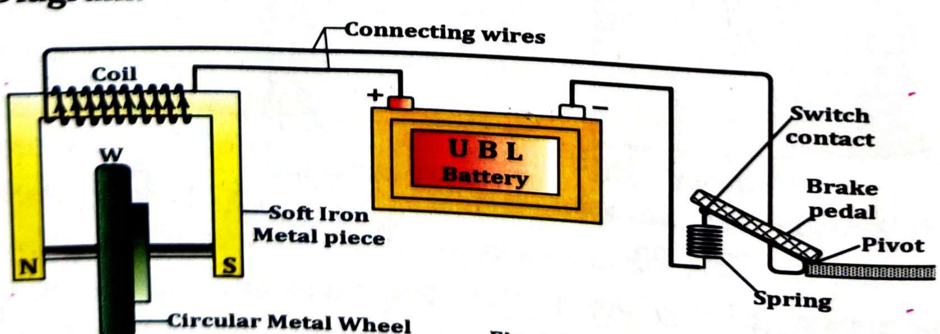


Fig. 4.0 (f)

- A strong magnetic field is created between the pole pieces from North Pole to the South Pole.
- The wheel, **W**, begins to cut the magnetic flux lines, causing eddy currents to quickly get induced in the wheel W, and create an electro-magnetic force that acts in an opposite direction to oppose the direction of motion of the wheel **W**.

- This then damps the motion of the wheel, W, and retards the motion of the train as a whole body. When the train stops eddy currents disappear and no electromagnetic damping exists.

(c) Electromagnetic damping in Moving coil instruments

The coil of a moving coil galvanometer is wound on an aluminium metal former or frame to aid in the creation of eddy currents whenever the coil rotates in a magnetic field.

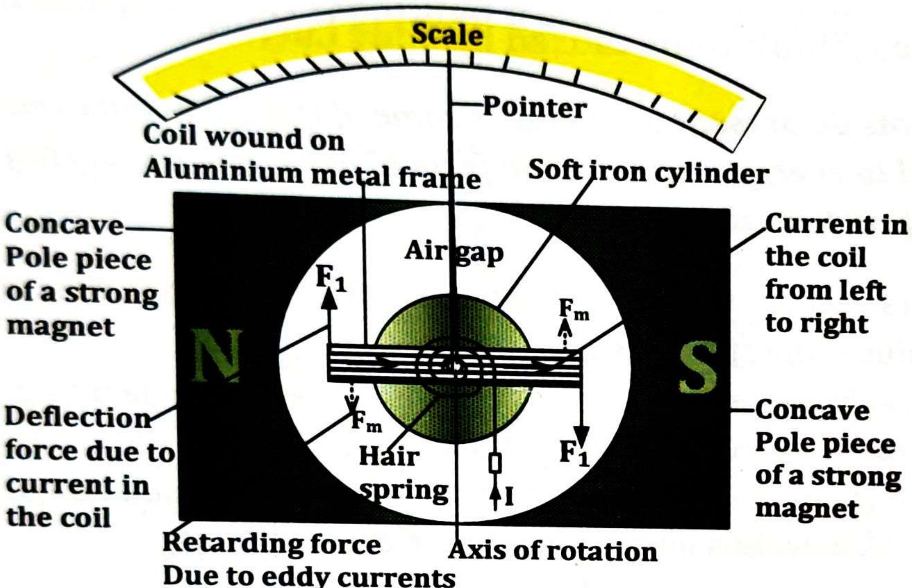


Fig. 4.0 (g)

- When a current, **I**, is passed through the coil of **N** – turns, wound on an aluminium frame, whose plane of area, **A** is parallel to a radial magnetic field, of flux density, **B**, the coil experiences a maximum deflection torque, **T = BINA,** causing it to rotate clock wise.
- As the coil rotates it moves together with an aluminium frame on which its wound, eddy currents are induced in the aluminium metal frame in the opposite direction to the applied current.
- By Fleming's left hand rule, a magnetic force, F_m then acts on the aluminium metal frame, exerting this force in the opposite direction to the deflecting force and as a result, the motion of the coil gets critically damped, and this prevents oscillation of the coil and the pointer s it moves over the scale.

Advantages of Eddy Currents/Their industrial uses

The following are a summary of some of the advantages or applications of Eddy Currents

(i) Electromagnetic damping in moving coil instruments e.g. moving coil galvanometers, voltmeters, ammeters e.t.c.
(ii) Electromagnetic Brakes in Large Automobiles, such as lifting cranes.
(iii) Detection of cracks in metals.
(iv) Sorting metallic objects from solid waste, such as in zone refining.
(v) Heating in Induction Furnaces.

(vi) Used in telephone booths that use coins for their operation.
(vii) Used to control opening and closing of electric doors that use "smart cards" for their operation, when swiped over special electronic devices.
(viii) Used in automated vending machines using coins.
(ix) Used in the damping mechanism of the arms of very sensitive weighing scales.
(x) Used in the operation of speedometers of motorbikes and vehicles.
(xi) Used in Ground Fault Interrupter (GFI) used in electric outlets like Bathrooms.

Disadvantages /limitations caused by Eddy Currents

Eddy currents *do present problems in some of the following areas and usually lead to energy dissipation in form of heat, hence lowering the efficiency of such machines.*

(i) Transformers – where they are produced in the soft iron core, thus producing heat that reduces the efficiency of the transformer.
(ii) The Ballistic galvanometer – where they cause undesirable damping.
(iii) Electric motors – where energy is dissipated as heat in the I^2R mechanism in the armature windings. This would cause damage to the insulation of the windings these may then burn up because of excess heating.

Other applications of Electromagnetic induction and Eddy currents

1. Separating metals from non – metals used in Car scrap yards

(a) Magnetic materials separated from non – magnetic materials.

Diagram

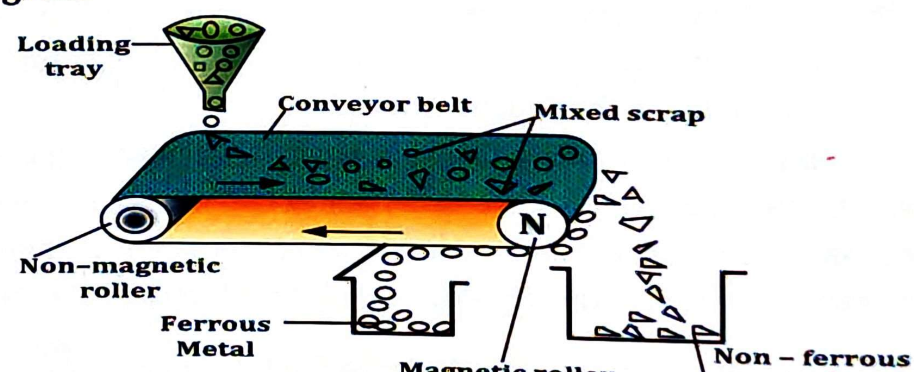


Fig. 4.0 (h)

- Electromagnetic metal separator, used in car scrap yards is one the most important industrial applications of electromagnetic induction.
- Crushed pieces of say a scrap car sent for recycling, are passed along a conveyor belt passed over a d.c. electromagnet at the end of the belt inform of a roller.
- The ferrous (magnetic) metal scrap pieces get attracted on to the belt as it passes over the final roller and are scooped off from underneath the belt onto a special container containing the ferrous metal.

- On the other hand, the remainder of all the non – ferrous (Non – magnetic) metal scrap pieces such as rubber, plastics, and a large amount of aluminium fly over under the influence of gravity into the hopper meant to collect the non –ferrous scrap.
- Aluminium and other metals can further be separated from the other solid waste by passing them over another conveyor belt having an electromagnet at the base s shown on the diagram in the figure 4.0 (f)

(b) Metallic materials separated from non – metallic materials Diagram

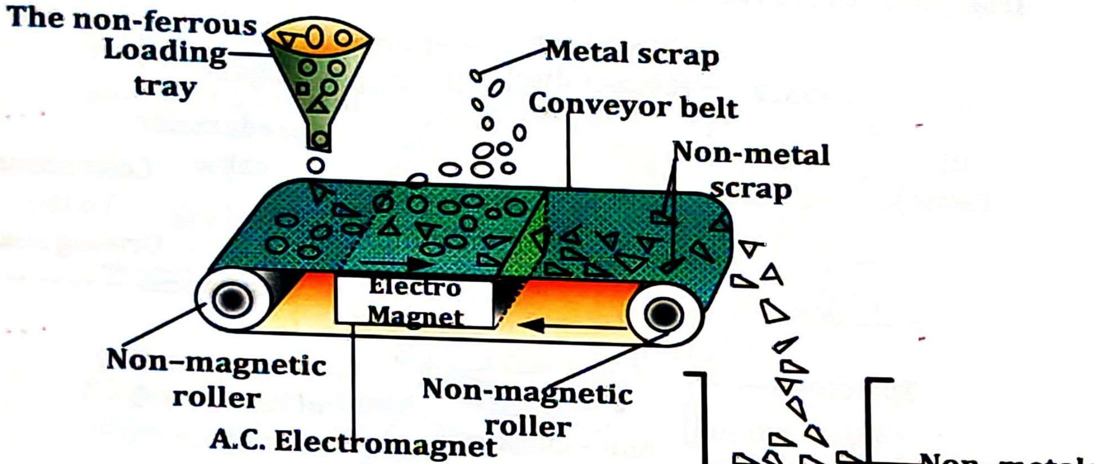


Fig. 4.0 (*i*)

- From the process in (a) above, the Non-ferrous scrap, can further be separated into metallic scrap and non-metal scrap when it is loaded into a conveyor belt having a strong electromagnet underneath it.
- When metallic scrap passes over the high frequency a.c. electro-magnet, the rapidly changing magnetic flux threads the metallic pieces, inducing eddy currents into each piece. E.g in the aluminium metal scrap.
- The eddy currents get induced in such a direction that a magnetic repulsive force, is exerted on each of the metal pieces and it throws these metal pieces off the belt in one specific direction while, the other non-metal continue along the belt into a hopper at the end of the conveyor belt.

2. Operation of a Mechanical speedometer of a vehicle or Motorcycle

The car speedometer is a device or gauge that helps the driver to know the speed at which his/her car is moving and the mileage covered in relation to the whole journey, by displaying this information on the dial or scale on the dash board. There are two types of speedometers namely;

(i) **The electronic or digital speedometers** – that uses electronic motion-sensors to pass on the speed transmission from the driveshaft of the car to the dial (scale) equipped with LED displays, where some other navigation information can be displayed like; Navigation range, Blue –tooth audio information display etc.

(ii) **The Analog or Mechanical speedometers** – that pass on the speed transmission from the driveshaft of the car to the dial (scale) via a long twisted steel cable called the speedometer cable.

Some automobiles have a combination of the Digital and the Analog speed display dials that are greatly synchronized.

The diagram in the figure 4.0 (j) shows a simplified model of the Analog or Mechanical car speedometer, which operates on the principle of eddy currents.

Diagram: of A Mechanical (Analog) speedometer

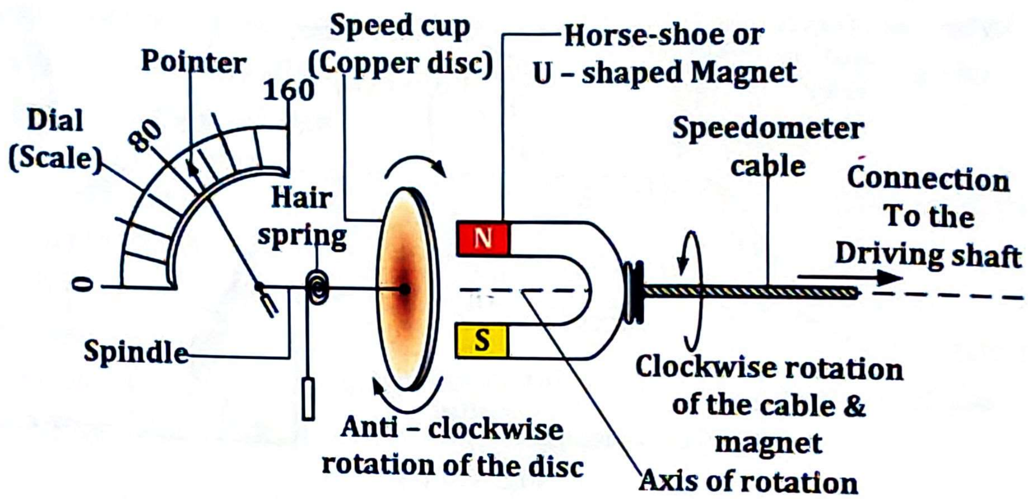


Fig. 4.0 (j)

Mode of Operation (How it works)

- When the driver presses the foot on the accelerator paddle to increase the speed of the car, the speed of rotation of the drive shaft increases. As a result, the speed of the drive cable connected to it also increases in the same proportion, causing the U – shaped magnet to rotate at the same speed as that of the shaft and the car.
- As the magnet spins about the axis at the speed of the shaft, a changing magnetic flux links the freely pivoted copper disc that is not attached to the magnet in any way but attached to a hairspring. This changing magnetic field due to rotation of the magnet, causes eddy currents to quickly get induced in the speed cup or metal (copper) disc and create a counter magnet field that opposes what led to their creation.
- The copper disc as a result begins to spin about the same axis but in the opposite direction (Anti-clockwise) but in a controlled manner that is proportional to speed of the vehicle due to the help of the control hairspring.
- The hairspring restrains the speed cup (copper disc) of excessive movements and allows a controlled synchronized rotation of the spindle attached to the pointer that moves over the dial (scale). The hairspring also brings the pointer back to zero position when the vehicle stops.
- As the speed cup (copper disc) rotates, the spindle attached to it and having a pointer at its end also rotates at the same controlled speed of the disc.

- The pointer then moves over the dial (scale) and shows the speed of the disc which is directly proportional to the speed of the vehicle.

NB: In ***a bicycle speedometer***, a small bar magnet is attached to the spokes of the wheel, while a coil is fixed to the frame of the bicycle, so that the north pole of the magnet moves past the coil once in every revolution of the wheel.

As the magnet moves past the coil, a pulse of current is induced in the coil. The in-built computer then measures the time between the current pulses displayed on a calibrated screen and computes the bicycle's speed.

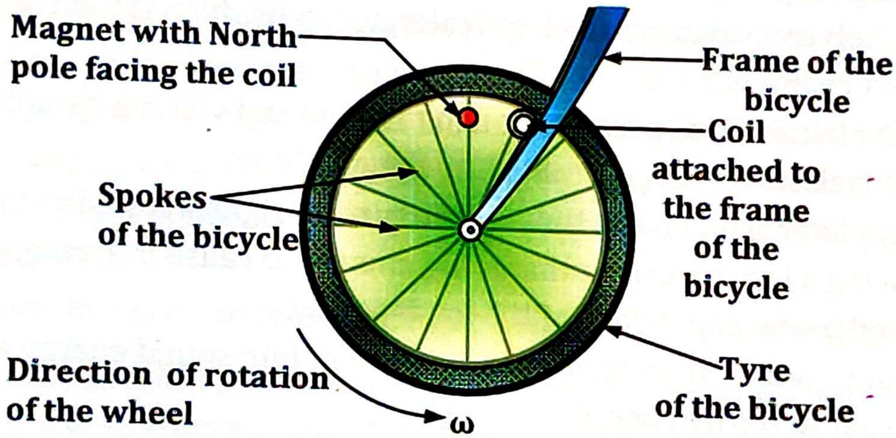


Fig. 4.0 (k)

3. The Electric Guitar

An electric guitar is one of the instruments that has revolutionized the entertainment industry.

The strings of an electric guitar are made of ferromagnetic metals that vibrate back and forth transmitting energy into the hollow wooden body of the guitar making it (and the air inside) resonate and eventually amplifying the sound.

An electric guitar converts electrical energy into sound energy and does this following about six stages or involving six different parts, as summarized on the diagram below, with each wire having the same type of setup adjacent to it.

Diagram showing the essential parts of An Electric Guitar

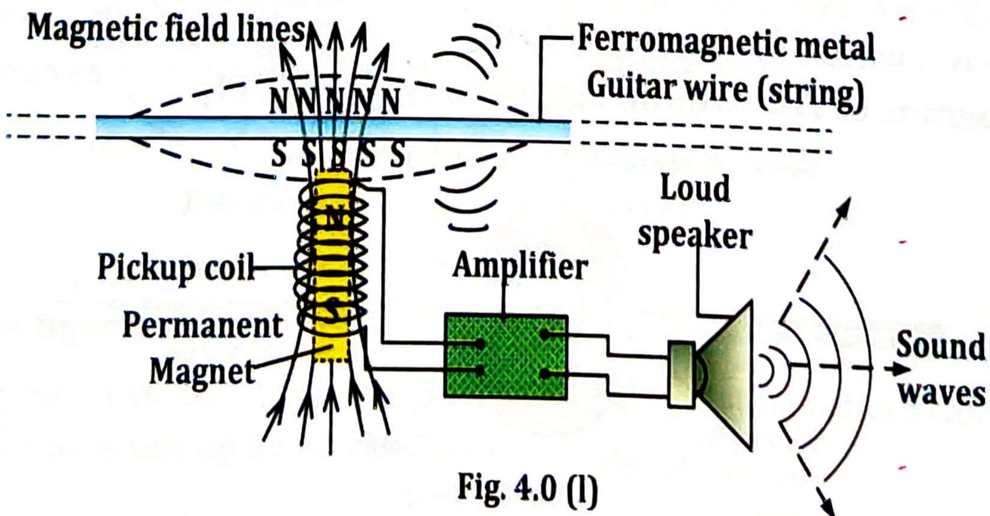


Fig. 4.0 (l)

Mode of Operation (How it works)

- A ferromagnetic string (wire) placed near the pickups (magnet + coil) causes the wire to get induced magnetism with the side of the wire near the magnet, picking up opposite pole to that of the magnet. i.e. part of the wire near the North Pole of the magnet becomes South Pole while the extreme upper side attains the same pole as of the permanent magnet (i.e. South Pole).
- When a guitar string is plucked, the string movements creates a region of changing magnetic field of its own and, induces tinny electric currents in the pickup coil, creating an electrical signal in it which is then passed on to the next chamber.
- The electrical signal passes through the tone and volume circuits to the output jack and through a cable into the amplifier.
- The amplifier then boosts the electric current signals from the pickups producing a larger current that is big enough to cause movement of the coil attached to the paper cone of the loudspeaker.
- The loud speaker then turns electric current into sound energy as received by the ear of the listener or observer.

4. The Ground Fault Interrupter (GFI)

A ground fault interrupter (GFI) is a device commonly used in a.c. electric outlets such as bathrooms, laundry points, kitchens and other places where the risk of electric shock is high.

It consists of a ferromagnetic ring having a coil wrapped at one of its positions and two electric wires, the live wire and the neutral wire running through the middle of the ferromagnetic ring, so that their distances from the ring are balanced.

The two wires carry alternating current of high frequency say from 50 Hz up to 120 Hz in two opposite directions each time. They set up magnetic fields on the opposite sides of the soft iron ring acting in two opposite directions as to cancel out each other each time the same magnitude of current flows in the two wires in opposite directions.

Diagram: of The Ground Fault Interrupter (GFI)

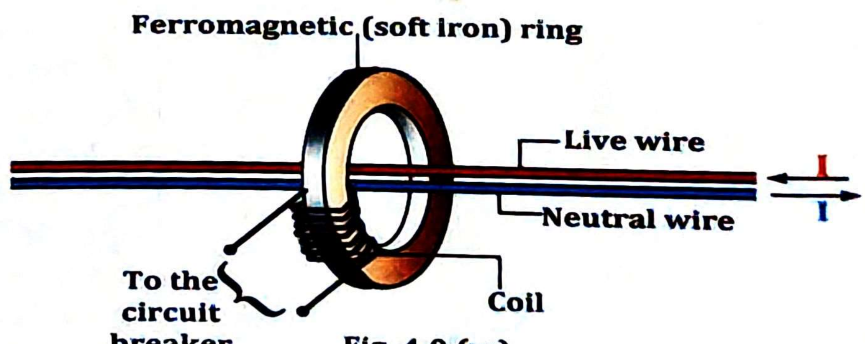


Fig. 4.0 (m)

Mode of Operation (How it works)

- Using the set shown above, if a person having wet hands accidentally comes into contact with the electric circuit, the person provides a very low resistance path to the current making the current to flow through the person to the ground, instead of flowing through the return neutral wire.
- This causes an imbalance in the current through the two wires, i.e. the two currents through the live and neutral wires are now different.
- The magnetic fields linking the iron ring by the two wires are now different and hence do not cancel out to deny the ring an induced e.m.f. A net or resultant magnetic flux links the ring at the frequency of the source, causing an induced e.m.f. to be generated across the coil.
- The induced e.m.f. causes a small induced current to flow to the circuit breaker and trips the circuit breaker, hence it disconnects the current flow to the point of the accident where the person made contact with the live circuit wire.

5. **The moving coil microphone (or The telephone mouth piece)**

A moving coil microphone works less in the reverse way a moving coil loud speaker works where sound energy is converted into electrical energy then finally back to amplified sound.

There are a number of different types of microphones, the common of which include,

(i) **The moving coil microphone** This is a versatile type and one of the most common type that is ideal for general-purpose use and quite resilient to rough handling. It works on the principle that, when a coil of wire moves in a magnetic field, a current is induced in it that is proportional to the rate of cutting of the magnetic flux.

The Diagram of a moving coil microphone

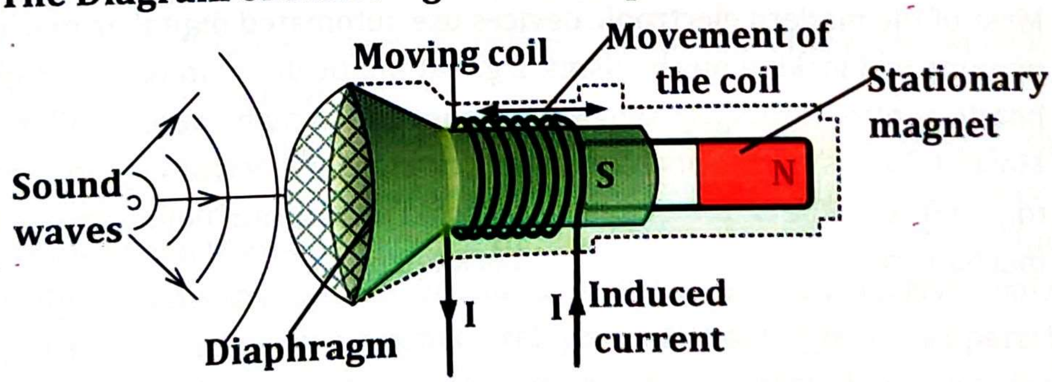


Fig. 4.0 (n)

Mode of Operation (How it works)

- Using the set shown above, a thin metal diaphragm attached to a coil is hit by sound waves setting it into vibration.

- The coil together with the diaphragm as a response to the sound waves, causes the coil to move back and forth (backwards and forwards) past the permanent magnet.
- The magnetic flux linked with the vibrating coil changes, causing an induced e.m.f. in the coil resulting into an induced electrical current that flows in the coil and is then channeled via the microphone cable to the amplifier.
- The amplifier then magnifies the electrical output and feeds into the loud speaker where the electrical energy is converted back into sound output at the receiver's ear.

Other types of microphones include

(ii) **The moving ribbon microphone**
Is the type that uses a thin corrugated and light aluminium foil instead of a moving coil for its operation. When sound waves strike the aluminium foil, it moves in a magnetic field provided by a permanent magnet and current is induced in it that is proportional to the speed of movement of the foil. The current is fed into the amplifier and finally to the output loud speaker.

(iii) **The condenser (Capacitor type) microphone**
Has one of the plates of a parallel plate capacitor, being very thin and light to act as a diaphragm. When sound waves hit the thin disc (diaphragm), the diaphragm vibrates back and forth the second metal plate. The variation of distance, between the two plates of the capacitor causes the variation of capacitance of the capacitor, and when a bias p.d. is connected across the plates of the capacitor, there will be changes in voltage across the capacitor with changes in distance between the plates resulting to changes in the output via the amplifier.

6. The Solenoid Car Door Lock

Most of the modern electronic devices use automated digital or mechanical opening and locking mechanisms. E.g. Automatic doors in Banks, High-class hotels, central locking system in car doors and in many automobiles. Most of these systems operate on the principle of electromagnetic induction together with the action of a relay switch. The diagram below shows a simple mechanism on which the Solenoid Lock operates.

The Solenoid Lock

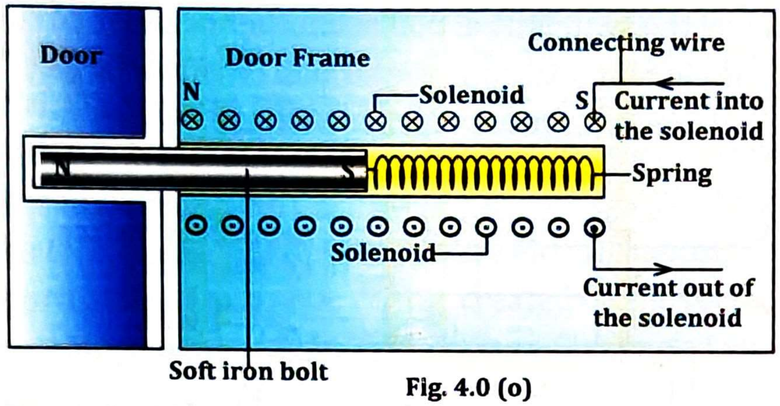


Fig. 4.0 (o)

Mode of Operation (How it works)

- Using the set shown above, a current from the system is fed into the solenoid.
- The current flowing through the solenoid creates a magnetic field in and around the solenoid that magnetizes the soft iron bolt, in the same sense as the solenoid.
- The magnetic pole at the end of the iron bolt is opposite to that at the near end of the solenoid. Consequently, the iron bolt gets drawn into the solenoid with the magnetic force of attraction between un-like poles of two bar magnets and compresses the spring against the force of the spring, which then stores energy as elastic potential energy.
- This now allows the iron bolt to be sucked into the solenoid axis, thus enabling the door to open.
- When the current is switched off or is cut off by any means, the solenoid loses its magnetism and the iron bold also loses its magnetic attractive force by the solenoid, consequently, the stored elastic potential energy in the spring forces the iron bolt out of the solenoid and the door locks again.

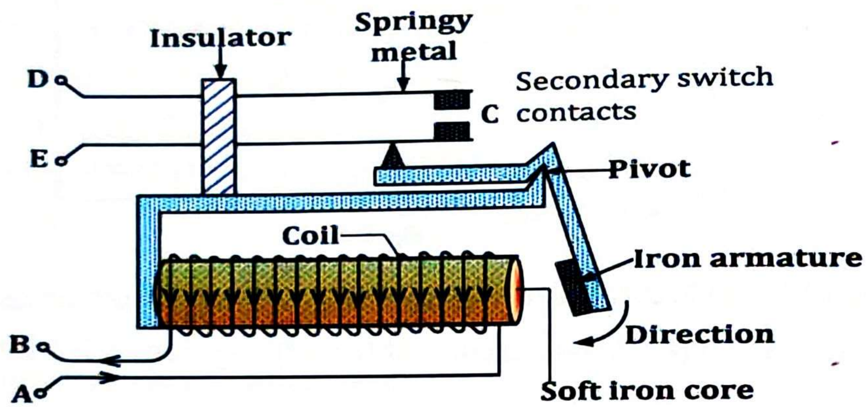
NB:

- The central locking system of car doors operates on the solenoid lock mechanism via a lever system.
- Some central locking system in vehicles are operated depending on the speed of the car like in the case of the operation of a car speedometer. When a vehicle attains a certain threshold speed from take-off, the induced eddy currents in a smooth circular copper disc, energized a given solenoid coil, that switches on a magnetic relay switch that triggers the operation of the solenoid lock designed in the reverse sense to that above and closes simultaneously all the doors of the car centrally. When the speed of the car falls below the critical speed e.g. below 10 km hr^{-1} the induced current falls below the value required to energize the solenoid to operate the relay that switches on the solenoid lock.

Electromagnetic Relay Switch

This is a switch operated by an electro-magnet. It is useful if we want one circuit to control the switching action of another or other circuits, more especially if the current operating the second circuit is larger.

Electromagnetic Relay Switch



A, B, C, D and E – are electrical contacts

Fig. 4.0 (p)

When contacts **A** and **B** are connected to the primary circuit, and its switch is closed, current flows through the coil causing the soft iron core to get magnetized and attracts the **L** – shaped soft iron armature. This rocks the pivot, causing the left end of the **L** – shaped iron armature to move upwards about the pivot.

The springy metal strip attached to contact **E**, is pushed upwards towards that connected to **D** and the contacts at **C** are made or connected.

This acts as the second switch to the secondary circuit, i.e. the relay switch is energized, triggering a fairly large current to flow in the secondary circuit.

Magnetic relays are commonly used in,

- Car ignition circuits involving the starter motor.
- Automatic electronic operated doors.
- Operation of fog lamps of a car, wind screen wipers, heated rear windscreens of a car, etc.
- Controlling the switching actions of security lights in homes and streets.
- The operation of electric bells.
- Switching power on the national grid.
- The operation push switch buttons in a lift relay circuit.
- The operation of auxiliary secondary transformers in power stations.

The advantage of the relay switch is that, operator or person controlling the system, is electrically isolated or detached from the potentially dangerous large current circuit.

NB, Minor modifications in the relay circuit above can be made, that would also allow the current in the main secondary circuit to be switched off rather than to be switched on, when a current flows through the coil.

A DIRECT CURRENT (D.C) ELECTRIC MOTOR

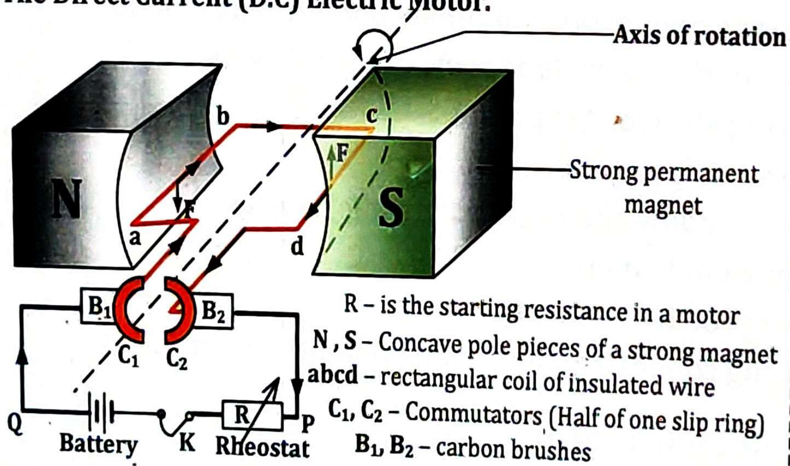
An electric motor is a device that converts electrical energy to mechanical energy. It converts electrical energy to the rotational energy that can be used to do work for example in *electric blenders, motors of grinding mills, electric shaving machines, rotating wheels in radio cassettes, C.D disc players, motors in lifting cranes and most of the moving parts in machines* are triggered by electric motors.
It works on the principle of magnetic torque created on the rectangular coil placed across a strong magnetic field in relation to Fleming's Left Hand Rule.

The structure:

The d.c. motor – consists of ***a rectangular coil*** **abcd** of fine insulated copper wire that rotates freely about an axis at the centre of the coil and between the concave pole pieces of a strong permanent magnet.
The free ends of the coil are each, connected to the flat circular halves of a split brass ring, known as ***commutators***, **C_1** and **C_2**.
Placed in contact with and pressing on the commutators by springy metal strips are the small metal blocks called the **carbon brushes, B_1** and **B_2** that link the ends of the coil via the ***commutators*** to the source of e.m.f. or Battery, and the switch, **K** and in series with a starting variable resistor, **R.**

Diagram/Structure:

The Direct Current (D.C) Electric Motor.



R – is the starting resistance in a motor
N , S – Concave pole pieces of a strong magnet
abcd – rectangular coil of insulated wire
C_1, C_2 – Commutators (Half of one slip ring)
B_1, B_2 – carbon brushes

Fig. 4.1 (a)(i)

A simplified version of A d.c. motor

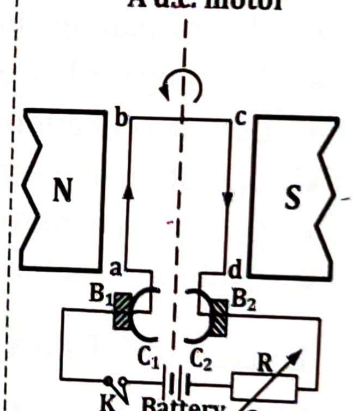


Fig. 4.1 (a)(ii)

The mode of operation of the d.c. motor (How it works)

- The switch, K is closed, and a current, **I**, flows from the battery via the carbon brushes **B**, to the commutators, **C** and to side **ab** of the coil, **abcd,** that experiences a downward force **F**, (By Fleming's Left Hand Rule), while side, **cd** experiences an equal magnitude of the upward force, **F.**
- The two forces that are equal in magnitude constitute a turning moment or torque on the coil, causing the coil to rotate about the axis through the mid-point of sides **ad** and **bc** anti-clockwise as shown on the diagram.
- When the coil is vertical, the split – ring commutators C_1 and C_2 lose the

contacts with Carbon brushes B_1 and B_2 and current stops flowing in the coil shortly.

NB: The commutators – help the swap the current – direction every half turn. It also stops the wires being tangled.

- However, the coil gets carried over past this vertical position by its original momentum.
- After the vertical position, contact is re-established but interchanged, *(i.e. C_1 makes contact with B_2, while C_2 makes contact with B_1)*, and the current reverses direction of flow in the coil but the coil continues to rotate in the original direction.
- The process then repeats itself for all the subsequent cycles or rotations of the coil, thus enabling the coil to pick up speed and do mechanical work its intended for.

NB: ***Factors affecting the speed of rotation of the armature of the motor are:***

(i) The current I flowing through the coils of the motor. $i.e.$ $\textbf{speed} \propto \boldsymbol{I}$

(ii) The magnetic field strength, B. $i.e.$ $\textbf{speed} \propto \boldsymbol{B}$

(iii) The number of turns N of the coil used on armature, $i.e.$ $\textbf{speed} \propto \boldsymbol{N}$

Applications of Direct Currents:

Direct current has some applications that may not be applicable to alternating currents, and these include some of the following:

- Electro-plating during electrolysis process.
- Charging car batteries and any other batteries.
- Used in operations of radios and other electronic devices.
- Operation of thermocouple meters.
- Operation of moving coil instruments.
- Calibration of moving coil instruments using potentiometers.

Back e.m.f. In an electric motor

Back e.m.f. in an electric motor – is the e.m.f. induced in the coil of the motor because of the changes in the magnetic flux linked with the coil due to rotation of the coil in a magnetic field and thereby cutting the magnetic flux.

The **back e.m.f., E_b, opposes** the source of **e.m.f. E**, that caused it to be produced. Since the circuit is closed, the induced e.m.f. called back e.m.f. E_b opposes the current supplied by the source, E, through the coil, because the back e.m.f. produces an induced current flowing in a direction to oppose the current supplied by the battery.

NB: The back e.m.f. in a coil acts like a small cell connected in series with the coil, but in opposite direction to the main cell (source of e.m.f.) that is driving the current in the whole circuit, as shown in the figure 4.1 (b)

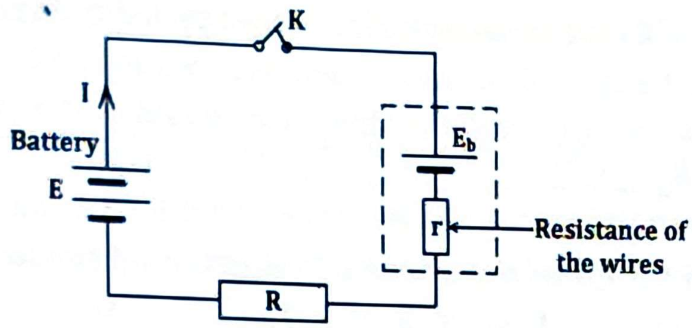


Fig. 4.1 (b)

Suppose, E is the e.m.f. of the source (Battery), r, the armature resistance, R is any external resistance, e.g. starting resistance, of a motor and E_b is the back e.m.f. generated in the motor when its coil starts rotating in a magnetic field.

Since the back e.m.f. depends on the rate of cutting of the magnetic flux, it implies, the ***back e.m.f. increases with increase in the speed of rotation of the coil***, and consequently the current flowing through the armature windings becomes small.

When switch K is just closed, the current I supplied by the battery, E, is at a maximum value, $I_{max} = \left(\frac{E}{R+r}\right)$ and the back e.m.f. E_b is zero since the coil is not rotating.

However, the back e.m.f. induced in the coil opposes the current flow as soon as the switch is closed, and the current slowly reduces in the circuit until it reaches its minimum value, as the back e.m.f. increases to a given maximum value as shown in the graph in the figure 4.1 (c)

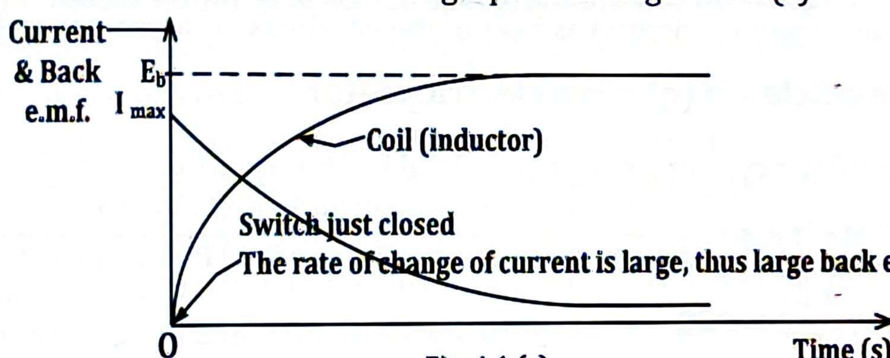


Fig. 4.1 (c)

NB: The rotating part of the motor is called the **armature** (or the **rotor**) of the motor.

In practice, the armature consists of several equally spaced coils wound on a soft iron core and connected to a commutator that has a corresponding number of sections.

- The several coils provide the motor with a constant torque so that its motion is not jerky but instead smooth.
- The concave pole pieces of the magnet together with the soft iron core provide a radial magnetic field and also to make the torque constant.
- The laminated core with the thin iron sheets separated with insulating varnish minimizes the heating up of the core caused by eddy currents.

- Suppose V and E_b are the magnitudes of the applied voltage and back e.m.f. that opposes the applied voltage respectively, while I and R_a is the total armature resistance, then all the above are related by the equation;

$$V - E_b = I\,R_a \quad \ldots\ldots\ldots\ldots\ldots (i)$$

Where

E_b is proportional to the speed of rotation of the armature.

i.e. $E_b \propto \omega \Rightarrow E_b = k_0\omega \ \ or\ \ E_b = k_1 f \ \ldots\ldots\ldots (ii)$

where, $\omega = 2\pi f$ *is the angular speed, and* $f =$ *frequency of rotation.*

Practically, $E_b = |NAB\omega \sin\omega t|$ *and for maximum e.m.f., $\sin\omega t = 1$*

$\Rightarrow NAB = k_0$ *(a constant) when N, A and B are all kept constant.*

- When a motor is loaded, its speed of rotation falls or reduces to some new steady value say ω_2. This is because a loaded motor has to exert a torque in order to perform mechanical work, and this causes its speed of rotation to reduce and this therefore requires a larger current to be drawn from the source, and more power is also supplied to the motor.
- Multiplying equation (i) by I, and rearranging gives
- $IV = IE_b + I^2 R_a \quad \ldots\ldots\ldots\ldots\ldots (iii)$

Where, $IV =$ *power supplied to the motor (Power input)*

$IE_b =$ *mechanical power output fron the motor (Useful Power)*

Or the rate at which the motor is performing mechanical work.

and $I^2 R_a =$ *rate at which energy is dissipated as heat in the coils.*

Or the power dissipated as heat in the windings (Wasted power)

The efficiency (η) of an electric motor

The efficiency, η of a motor is defined by the equation;

$$\boldsymbol{Efficiency, \eta = \frac{Mechanical\ power\ obtained\ (Output)}{Power\ supplied\ by\ the\ source\ (Power\ input)} \times 100}$$

$$\therefore \eta = \frac{I\,E_b}{I\,V} \times 100$$

$$\therefore \eta = \frac{E_b}{V} \times 100$$

NB: *The efficiency is usually high, when the coil resistance is small. As a consequence, practical motors have coil resistances of less than 1.0 Ω.*

The use of the starting Resistance, R , in an electric motor

When the motor is just switched on, the back e.m.f. is initially zero, i.e. $E_b = 0$, the whole supply voltage, V, would be across the coil of the motor. This would set up a very large current across the coil, since the resistance of the armature (coil windings), r, is quite negligible the current supplied by the source could easily burn off the coil. Because $E_b = 0$, and R is missing,

$\therefore\ I = \frac{V}{r}$, $when, r \to 0 , I \to \infty$ $(current\ tends\ to\ \infty), since\ E_b = 0$

So, in order to limit the current, I, flowing through the coils at the start, a variable starting resistance R, (rheostat) is incorporated in series with the coils.

Thus at the start of the motor, the armature current, I_a is given by the equation;

$I_a = \frac{V - 0}{R + r}$ $where\ E_b = 0\ and\ resistance, R, has\ a\ fairly\ large\ value.$

$\Rightarrow$ *At the start of the motor,* $I_a < I$ *(when R is shorted out), This then ensures the current flowing through the coil is small enough not to destroy or to burn off the coils or windings of the motor.* $I_a = \frac{V - E_b}{R + r}$ By the time the armature or rotor of the motor is running at its full operating speed, the rheostat (Starting resistance) has its value, gradually reduced to zero. At such a time when the motor has full operating speed, the back e.m.f. would be having its value very close to that of the operating supply voltage but slightly less than the supply p.d., V. This then limits the current I_a flowing through the coils, to a small value.

NB: Basing *on the above facts,*

(i) A d.c. ***motor running*** at its full speed, ***should not be stopped abruptly*** when it is still connected to the supply. This is because the back e.m.f. abruptly drops to zero, making a large current to flow through the coil instantly and thus burning off the coils of the motor.

(ii) Industrial motors are usually fitted with ***either*** *two switches*, a starting switch that places a large value resistance in series with the armature coils and a running switch that shorts out the resistor when the motor has gained nearly its maximum speed, **or** *an automated variable resistance* (Rheostat) that gradually goes off with increase in speed of the motor.

(iii) The armature ***current should never drop to zero, to maintain*** the deflection ***torque*** on the coil necessary to enable the motor to do ***mechanical work***, relevant for its mode of operation.

(iv) Whenever a motor is loaded, the angular speed ω reduces, thereby reducing the back e.m.f. of the motor, but the turning moment (or Torque, T) of the motor increases, i.e. larger power is required to turn the motor, thus, the current flowing through the coil then increases. Hence, a large current is drawn from the source, this may burn off the coil of the motor and it increases the power consumption.

(v) The back e.m.f. E_b in the armature of the motor is directly proportional to the rate of cutting of the magnetic flux, and therefore to the angular speed, ω of the motor. *i.e* $E_b = k\omega$, *where k is a constant of proportionality*

(vi) From $V - E_b = rI_a \Rightarrow I_aV - I_aE_b = r\,(I_a)^2$ *The rate at which the motor works in order to overcome the back e.m.f. is* I_aE_b *and this is equal to rate at which the motor is performing mechanical work given by,* $(T + T_0)\omega$*, where* T_0 *is a constant torque used to overcome friction within the motor. Thus* $I_aE_b = (T + T_0)\omega$*, where,* $T = BINA$ *is the maximum torque on the coil. If the frictional forces in the motor are negligible, then,* $I_aE_b = T\omega$ *and from the above equations,* $V - E_b = rI_a \Rightarrow, V - k\omega = rI_a$ *but* $I_a = \frac{(T+T_0)}{k}$

Thus back e.m.f. can be expressed as, $E_b = \left\{V - \left[\frac{(T+T_0)}{k}\right]r\right\} \cong \left\{V - \left[\frac{T}{k}\right]r\right\}$

Series wound and shunt wound motors

In large electric motors, the magnetic field is usually provided by an electromagnets as may be opposed to permanent magnets. The coils that provide the magnetic fields that thread the plane of the armature are known as ***field coils.***

(a) Series wound motors

These are motors where the field coils are connected in series with the armature coil windings.

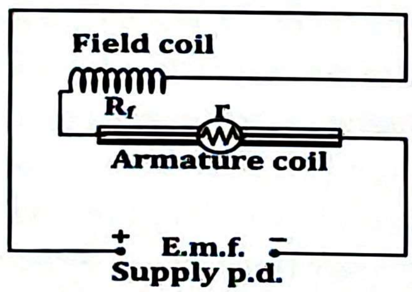


Fig. 4.1 (d)

The characteristics that these type of motors have are,

(i) The same current flows from the source to both the armature and the field coils.

(ii) The field coils are usually few and made up of thick copper wires of low resistance.

(iii) Different p.ds are set up across the armature and field coils hence they experience different powers.

(iv) They have a large starting torques at low speeds, since both the current and the magnetic field due to the series field coils are large.

(v) They are high speed motors; however their speeds vary considerably with the load input. i.e. They are very sensitive when loaded. Implying there is great variation in speeds with different loads.

(vi) They are suitable for use in electric fans, lifting cranes, power units of electric locomotives (trains), winches and for high-speed motors used in grinding wheels, blenders where speed control is not very important.

(b) Shunt – wound motors

These are motors where the ***field coils*** are connected in ***parallel*** with the ***armature*** coils (windings).

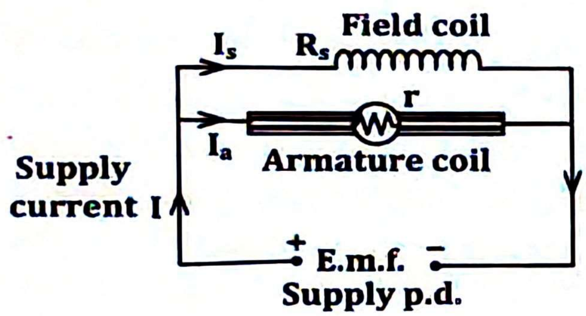


Fig. 4.1 (e)

The characteristics these type of motors have are;

(i) The armature and field coils have the same p.d. across them.

(ii) The field coils are usually many and of fine copper wire so as the limit the current flowing through them, so as to allow a larger current to flow through the shunt.

(iii) Different currents I_a and I_s flow through the armature and field coils respectively.

(iv) They have a lower starting torque at low speeds when compared to the series wound motors.

(v) There is very small variation in the speed of the motor when the load on it is varied and the speeds can easily be controlled.

(vi) A shunt wound motor keeps nearly a steady speed, because if the load is increased, the speed of the motor falls a little, the back e.m.f. also falls in equivalent proportion to the speed, and the current rises, thus enabling the motor to develop more power to overcome the increased load. This is not the case with a series wound motor.

(vii) Shunt wound motors are commonly used in practical applications where speed regulation is very important such as in record players, C.D players, compact cassette radio players, curving tool machines say in wood workshops.

Factors affecting the efficiency of an electric motor

Practical electric motors may not attain an efficiency of 100% due to some of the following factors.

(i) **Eddy currents Power loss**

The changing magnetic flux in the core causes eddy currents to be induced in it and they cause the heating up of the core of the motor in the I^2R – mechanism leading reduction of efficiency of the motor.

Remedy: This defect is minimized by using a laminated core e.g. a Laminated soft iron core which breaks down originally large eddy current loops into tiny loops rendering eddy currents negligible.

(ii) **Resistance of the armature windings**

The insulated wires used for making the armature coils have some resistance that cause heat dissipation in the motor windings in the I^2R – mechanism thus leading to the reduction of efficiency of the motor.

Remedy: This defect is minimized by using insulated ***thick copper wires of low resistance*** for making the armature coil windings.

(iii) **Hysteresis power loss**

The constantly changing magnetic flux in the core of the rotor creates internal friction in the core as magnetic dipoles keep changing directions with that of the magnetic field. This causes increase in the internal energy of the atoms leading energy dissipation in form of heat in the core of the motor.

Remedy: This defect is minimized by using magnetic materials of low hysteresis loss for making the core of the motor such as soft iron, perm alloy and mumetal.

4.2 EXAMPLES & EXERCISES ON EDDY CURRENTS & MOTORS

1. The diagram in the figure 4.2 (a) shows an arrangement by which a laboratory balance is critically damped. The aluminium beam supporting the pan moves in magnetic field of two powerful magnets.

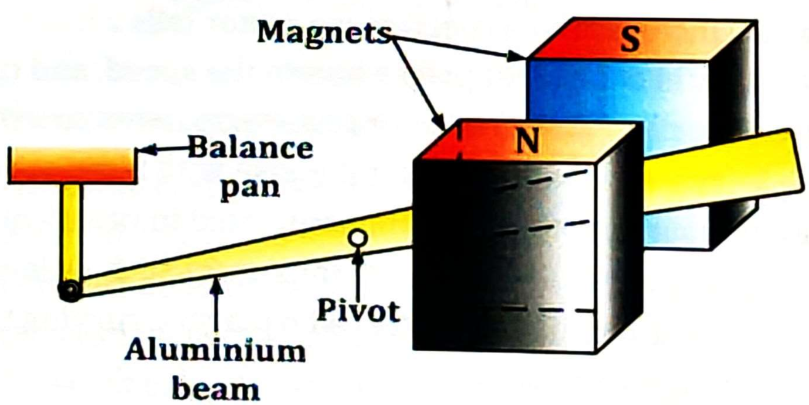


Fig. 4.2 (a)

(i) Explain how damping is caused.

(ii) What change would occur in the performance of the balance if the magnets were replaced with much weaker ones?

Solution

(i) When the balance pan is loaded, the aluminium metal beam moves in and out of the magnetic field between the two pole pieces of the magnet. As the beam cuts the magnetic flux, induced e.m.f in the beam causes eddy currents to flow in a direction as the oppose the changes in magnetic flux, thereby creating a magnetic force that opposes the force causing motion of the beam across the magnetic field. This makes the beam to come to rest much sooner than it would in the absence of the magnetic field, i.e. ***the motion of the metal beam is damped.***

Alternatively

The damping of the motion of the beam can be explained in terms of ***energy conservation.***

When an object is placed into the balance pan, the potential energy of the object is converted into the kinetic energy of the aluminium metal beam.

As the metal beam moves in and out of the magnetic field, eddy currents get induced into the beam, causing energy dissipation in the beam in form of heat, in the I^2R – mechanism.

The kinetic energy of the beam then reduces causing it to come to rest much sooner than it would otherwise have done in the absence of the magnetic field.

Effect of speed of the metal beam in the magnetic field on the damping force.

If the aluminium metal beam is moving faster, between the pole pieces of the magnet, the rate of cutting of the magnetic flux would increase. By Faraday's law, states that, the magnitude of the induced e.m.f. in the beam is proportional the rate of change of the magnetic flux linked with it. This also leads to the increase on the eddy currents induced in the metal beam. Since, the damping force, depends on the eddy currents, this would cause the damping force to increase, or to become larger.

Hence, the bigger the speed of the metal beam in the magnetic field, the larger the damping effect is experiences.

(ii) From Faraday's law; the magnitude of the induced e.m.f. in the beam is proportional the rate of change of the magnetic flux linked with it.

Symbolically; $|E| = \frac{d\Phi}{dt} = \frac{d(BA)}{dt} \Rightarrow E \propto B$, it implies, induced e.m.f. increases with increase in the magnetic field strength and vice versa.

Thus, when a much weaker magnet is used, the induced e.m.f. in the metal beam is small and so are the induced eddy currents.

Since the damping force is proportional the size of eddy currents, the motion of the beam would be lightly damped, thus making the balance to take a much longer time to come to rest at the zero balance position.

2. A small bar magnet is attached to a spring as shown in the figure 4.2 (b)

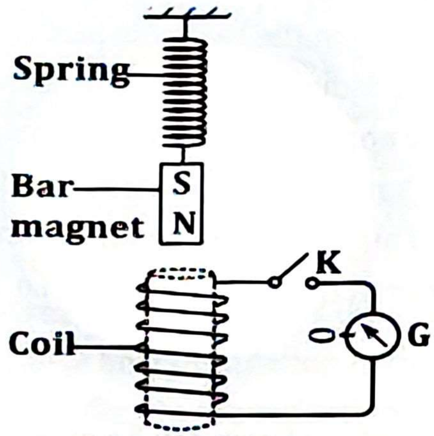


Fig. 4.2 (b)

Switch K is closed and the magnet is displaced downwards slightly and released to oscillate vertically.

Explain;

(i) The observations made.

(ii) Why the magnet takes long to come to rest when switch, K is opened.

Solution

(i) When switch, K, is closed, the circuit is completed, and when the magnet is displaced and allowed to oscillate vertically, the galvanometer pointer starts to deflect back and forth in opposite directions about the zero position, however, but with a higher degree of damping of the oscillations.

As the North Pole of the magnet approaches the coil, the magnetic flux linkage to the coil increases, causing an e.m.f. to be induced in the coil. Since the circuit is closed, an induced current flows in the direction of induced e.m.f. i.e. the top part of the coil becomes a north pole so as to oppose the increasing flux. This causes the pointer to deflect to the right.

As the North Pole of the magnet withdraws (recedes), the reducing magnetic flux linked with the coil causes an e.m.f. to be induced in it in such a way as to make the top end of the coil a south pole. Thus, this makes the galvanometer to deflect in the opposite direction (to the left) because of induced current flowing in the new direction of the induced e.m.f.

(ii) When the switch K is opened, there is no closed circuit, and so there is no flow of induced current. The coil does not produce the opposing magnetic field and flux that oppose the movement of the bar magnet, hence there is no damping to the motion of the magnet, causing the oscillations to take a much longer time to die out or to stop.

3. (i) Define the term back e.m.f. in an electric motor.

(ii) A d.c. motor having a coil of resistance 2 Ω is connected a cross a 240V steady voltage supply. When the coil rotates at 120 revolutions per minute, it draws a current of 5.0A from the supply. Find the back e.m.f. and the frequency of rotation of the coil when the current of 8.0 A is drawn from the supply.

Solution

(i) **Back e.m.f.** – Is the *induced p.d or e.m.f.* set up across the armature or coil of

the motor when it's rotated in a magnetic field and acts in such a direction as to oppose the applied voltage across the motor windings.

(ii) $V = 240\ V$ $r = 2\ \Omega$ $I_1 = 5.0A$ $f = \frac{120}{60} = 2Hz$

Using, $V - E_b = I\,r, E_b = V - I\,r$ $\Rightarrow E_{b1} = 240 - 5.0 \times 2 = 230\ V$

But, $E_b = k\omega = k'f$ $E_{b1} = k'f_1 \Rightarrow k' = \frac{230}{2} = 115\ V$

$\Rightarrow E_{b2} = 240 - 8.0 \times 2 = 224\ V,$ *is the back e.m.f. when I = 8.0 A*

$\therefore \mathbf{f_2} = \frac{E_{b2}}{k'} = \frac{224}{115} = \mathbf{1.95\ Hz}$

4. John is using a cordless electric weed trimmer with a d.c. motor to cut the long weeds in his backyard. The trimmer generates a back e.m.f. of 18.0 V when it is connected to an e.m.f. of 24.0 V d.c. The total electrical resistance of the motor is 8.0 Ω.

(i) How much current flows through the motor when its running steadily?

(ii) Suppose the string of the trimmer suddenly is wrapped around a pole in the ground causing the spinning of the rotor of the motor to stop. What current now flows through the motor and what advice would you give to John?

Solution

(i) Using, $V - E_b = I_1 R \Rightarrow I_1 = \frac{V - E_b}{R}$

$\Rightarrow I_1 = \frac{24.0 - 18.0}{8.0} = 0.75\ A$

$\therefore I_1 = 0.75\ A$ *is the current that flows through the motor.*

(ii) When the motor suddenly stops rotating, the back e.m.f. becomes zero, i.e. $E_b = 0$, from $V - E_b = I_1 R \Rightarrow I_1 = \frac{V - 0}{R}$

$\Rightarrow I_1 = \frac{24.0 - 0}{8.0} = 3.00\ A$

$\therefore I_1 = 3.00\ A$ *is the current that flows through the motor.*

This is a very ***large current!!*** and if left to flow through the motor for a fairly longer time it ***burns off the coil*** .

Thus, John is advised to ***quickly turn off the power supply*** to the motor.

5. An electric lawn – mower has an armature of 0.5 Ω. When running freely and connected to a 240 V supply, the motor takes 3.0 A, but when the mower is on full load when cutting through long grass, the current rises to 50 A. Calculate the;

(i) Back e.m.f. on each case.

(ii) Electrical efficiency in the second case.

Solution

(i) For the first case, with no load, From $V - E_{b1} = I_1R \Rightarrow E_{b1} = V - I_1R$

$\Rightarrow E_{b1} = 240 - 3.0 \times 0.5 = 238.5$ V

$\therefore E_{b1} = 238.5\ V$

When a load is applied to the mower, a current $I_2 = 50$ A is drawn from it,

$\Rightarrow E_{b2} = V - I_2R$

$\Rightarrow E_{b2} = 240 - 50 \times 0.5 = 215.0$ V

(ii) $$Efficiency, \eta = \frac{\textit{Mechanical power obtained (Output)}}{\textit{Power supplied by the source (Power input)}} \times 100$$

$$\therefore \eta = \frac{I\,E_b}{I\,V} \times 100$$

$$\therefore \eta = \frac{215}{240} \times 100$$

$$\therefore \boldsymbol{\eta = 89.6\ \%}$$

6. A shunt – wound motor is connected to a 240 V d.c. supply. With no load exerted on the motor, it rotates at 15 Hz and the armature current is 2.5 A. When the motor is loaded its rate of rotation falls to 5 Hz and the armature current rises to 25 A. What is the resistance of the armature?

Solution

Back e.m.f. is proportional to the frequency of rotation of the armature.

i.e. $E_b = kf$ *where k is a constant of proportionality*

Using, $V - E_b = I_aR_a \Rightarrow$ *when current is* 2.5 *A*

$\Rightarrow 240 - 15k = 2.5R_a$ ……………………………… (i)

When the motor is loaded the current $I_2 = 25$ *A*

$\Rightarrow 240 - 5k = 25R_a$ ……………………………… (ii)

Equation (ii) × 3 – Equation (i)

$\Rightarrow 720 - 240 = 75R_a - 2.5R_a$ ……………………… (iii)

$\therefore 480 = 72.5R_a$

$\therefore \boldsymbol{R_a = 6.62\ \Omega}$

7. (a) Give the importance of back e.m.f. in a motor.

(b) A shunt wound d.c electric motor takes a current of 10 A from a 200 volts mains. The shunt field coils have a resistance of 40 Ω, and the armature has a resistance of 0.5 Ω. Find the back e.m.f and the electrical energy converted into mechanical work per second.

Solution

(a) **Importance of back e.m.f. in a motor**

- *It provides the necessary mechanical power of the motor required to do work.*
- *It minimizes heat generated in a coil, by allowing only a small current to flow through the armature winding when the motor is working, since too much heat may burn the coil of the motor windings.*

(b) Since a shunt, is always connected in parallel to the armature coil, they have the same p.d. E = 200 V of the supply battery.

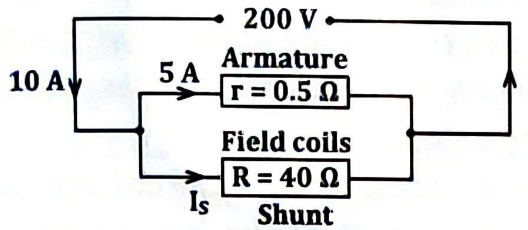


Fig. 4.2 (c)

Current in the shunt field coils, $I_S = \frac{E}{R_s} = \frac{200}{40} = 5.0\ A$

Armature current, $I_a = (10 - 5) = 5.0\ A.$

P.d across the armature

$V_a = I_a R_a = 5.0 \times 0.5 = 2.5\ V$

$\therefore$ *Back e.m.f,* $E_b = (E - V_a) = (200 - 2.5) = \mathbf{197.5\ V}$

Energy per second in the armature,

$P = [IE - (I_a^2 R_a + I_s^2 R)]$

$= \{(10 \times 200) - [(5.0^2 \times 0.5) + (5.0^2 \times 40)]\}$

$\therefore$ *Power expended,* $P = \mathbf{987.5\ watts.}$

Alternatively

Since mechanical power, $\boldsymbol{P_m} = \boldsymbol{I_a E_b} = (5.0 \times 197.5) = \mathbf{987.5\ W}$

8. (a) What are ***eddy currents?***

(b) The figure 4.2 (d) shows a flattened aluminium bottle top supported on light cotton threads at the top of a solenoid made of copper wire and connected to a d.c. source of e.m.f.

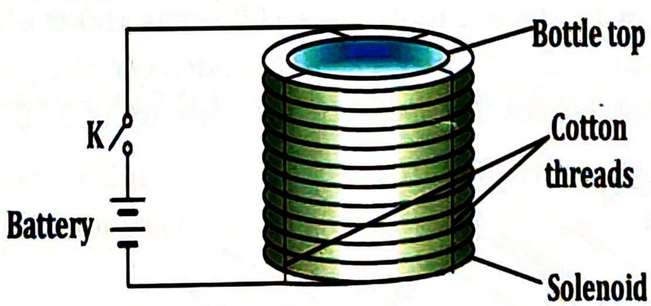


Fig. 4.2 (d)

When Switch K is just closed momentarily then shortly opened, the bottle top immediately jumps up and falls down again, on the threads.

Explain these observations.

Solution

(a) **Eddy currents** – are currents induced in a thick metal conductor whenever the magnetic flux linked with it changes.
Eddy currents flow in such a direction as to oppose the changes that caused them.

(b) When switch, K, is just closed, a rapidly increasing current flows through the solenoid in a clockwise direction.
A rapidly changing magnetic flux is created inside and at around the solenoid, which in turn thread the bottle top. This induced e.m.f. in the bottle top and eddy currents get induced in the bottle top in such a way that the lower surface of the bottle top has the same polarity as the top part of the coil (i.e. South pole). The two like poles (south poles) then repel each other. This causes the bottle top to jump off.
When the switch is opened, current is cut off from the coil and no Eddy currents are induced in the bottle top, so it falls back down to the threads on top of the coil due to the influence only its own weight.

Exercises on Electric motors

1. A small electric motor with permanent magnets to produce the magnetic field is connected to a 12 V supply of negligible internal resistance. With no load the motor rotates with a frequency of 10 Hz and the armature current is 2 A. If the armature resistance is 1.5 Ω. Calculate the rate of rotation when a load that causes a current of 5 A to flow in the armature.

Ans: [5.0 Hz]

2. A motor having an armature resistance of 4.0 Ω is connected to a 240 V supply. When the motor is on a light load, the motor speed is 200 revolutions per minute, while the armature current is 5.0 A. When the motor is put on full load, the armature current increases to 20.0 A. Determine the;
(i) Back e.m.f. in each case. **Ans: [220 V and 160 V respectively].**
(ii) Speed of the motor when under full load. **Ans: [145 rev. min^{-1}]**

3. The armature coil of an electric motor has 100 turns and is of length 0.12 m as shown in the figure 4.2 (e). The coil rotates between the pole pieces of a U – shaped magnet, where the magnetic flux density is 0.18 T.

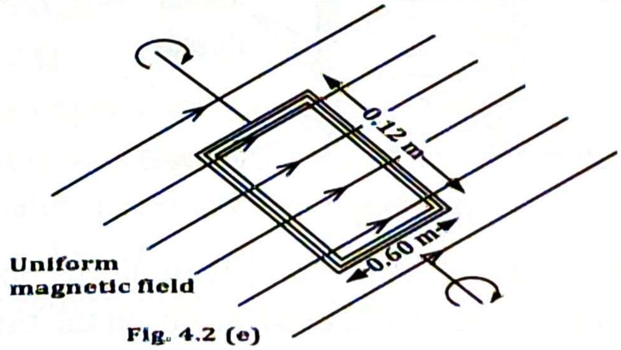


Fig. 4.2 (e)

(a) (i) Calculate the force acting on each side of the coil when the current flowing through the coil is 0.8 A.

(ii) Discuss how the force on each side of the coil changes during one complete rotation of the coil.

(b) Explain why the force acting on each side of the coil has its maximum turning effect when the plane of the coil is parallel to the lines of force of the magnetic field.

(c) If the coil is rotated at 1200 revolutions per minute, what is the maximum value of the e.m.f. generated in it?

(d) What factors determine the value of the e.m.f. generated in the coil.

4. A d.c. motor is connected to a 240 V supply. When no load is connected to the motor and is rotating at 3000 revolutions per minute, an armature of resistance 0.2 Ω draws a current of 5.0 A is drawn from the supply. When the load is connected. When a load is connected to the armature a current of 7.5 A is drawn from the mains supply. Determine the back e.m.f. of the loaded motor and its speed of rotation. **Ans: [238.5V & 2994 rev. min^{-1} respectively].**

5. A d.c. motor has coils with a resistance of 16.0 Ω and is connected to an e.m.f. of 120 V. When the motor is operated at full speed, the back e.m.f. is 72 V. What is

(i) The current in the motor at the start of its rotation. **Ans: [7.5 A]**

(ii) The current when the motor is operated at its full speed.

Ans: [3.0 A]

(iii) If the motor is drawing a current of 4.0 A and its not operated at full

(iv) speed, what is the back e.m.f. at that speed and time? **Ans: [56 V]**

.3 ELECTRIC GENERATORS

A generator – is an electrical device that converts mechanical energy into electrical energy. It uses the principle of rotation of a coil of metal wire in a magnetic field, and tapping the output of the induced e.m.f. across the open ends of the coil.

There are essentially two types of generators namely:

(i) The Direct Current (D.C) generators (Dynamos)
(ii) The Alternating Current (A.C) generators (Alternators)

The Direct Current (D.C) generator (Dynamo)

The structure of a simple d.c. generator

The d.c. generator – consists of ***a rectangular coil* abcd** of fine insulated copper wire that rotates freely about an axis at the centre of the coil and between the concave pole pieces of a strong permanent magnet.
The free ends of the coil are each, connected to the flat circular halves of a split brass ring, known as ***commutators*, C_1 and C_2.**
Placed in contact with and pressing on the commutators on springy metal strips are the small blocks called the carbon brushes, that link the ends of the coil via the ***commutators*** to the output terminals, **P** and **Q** across which is connected the load, **R.**

The Direct Current (D.C) Generator (Dynamo).

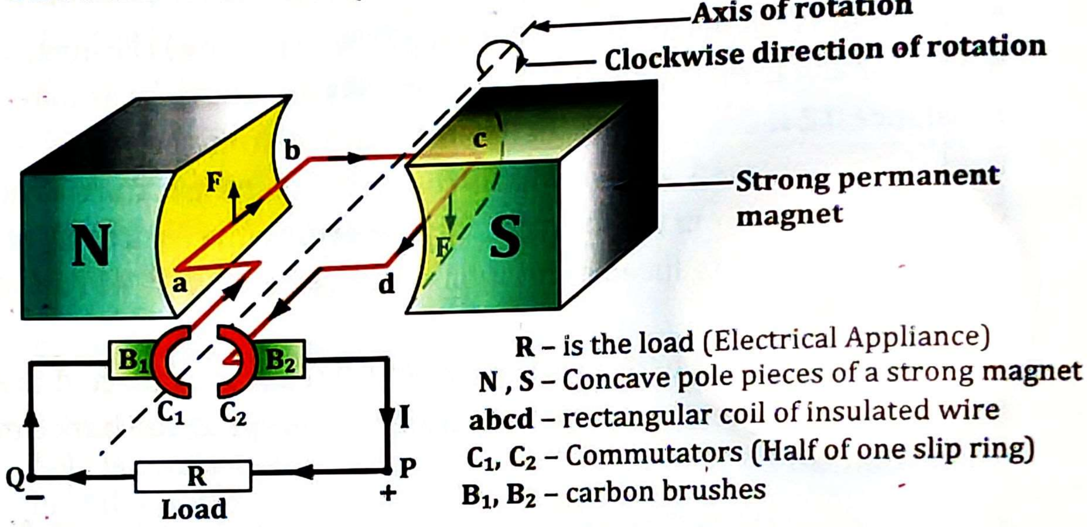


Fig. 4.2 (a)

The mode of operation (How it works)

- Using the set up shown on the diagram, the coil **abcd** is mechanically rotated in the clockwise direction by pulling side **ab** upwards and pushing the opposite side, **cd** downwards, it cuts the magnetic flux lines causing an e.m.f. to be induced in the coil.
- An induced current **I** (By Flemings's Right Hand Rule), flows through the circuit in a clockwise direction. So that current flows out of the carbon brush **B_2** to terminal **P**, and then across the load **R** in the direction **PQ**.
- When the coil has turned through 90° (i.e. when it reaches its vertical position), the rate of cutting of the magnetic flux reduces to zero, hence the induced e.m.f. reduces to zero and so does the induced current flowing in the circuit.
- The commutators **C_1** and **C_2** now lose contact with the carbon brushes **B_1** and **B_2** respectively.
- When the inertia and the momentum of the coil carries it past the vertical position, contacts between the commutators and the carbon brushes are re-established but interchanged i.e. **C_1** and **C_2** make contact with the carbon brushes **B_2** and **B_1** respectively.
- The rate of cutting the magnetic flux by the coil now begins to increase past the vertical position however the direction of cutting of the flux by sides ab

and cd reverses, hence, the induced e.m.f. and current, I, both reverse directions.

- After passing past the vertical position up to half a revolution, the current reverses direction of flow in the circuit i.e. flowing from **d** to **c** on the left hand side of the coil and flows from **b** to **a** on the right hand side of the same coil.
- However, ***the direction of flow of the current across the load, R, remains*** the same as in the first case, i.e. flowing from **P** towards **Q**, hence the name ***direct current*** generator.
- The direction of rotation of the coil also is *maintained* ***clock-wise for one complete revolution of the coil*** and the process ***repeats itself*** for all the subsequent revolutions of the coil enabling the motor to attain a steady speed and converts electrical energy to mechanical energy, thus enabling it to do mechanical work.
- A graph of induced e.m.f. (or induced current) with time for a number of revolutions of the coil has the shape shown below, together with the associated positions of the coil in the magnetic field with respect to time.

A graph of induced e.m.f. against time (with corresponding positions of the rotating coil in the magnetic field)

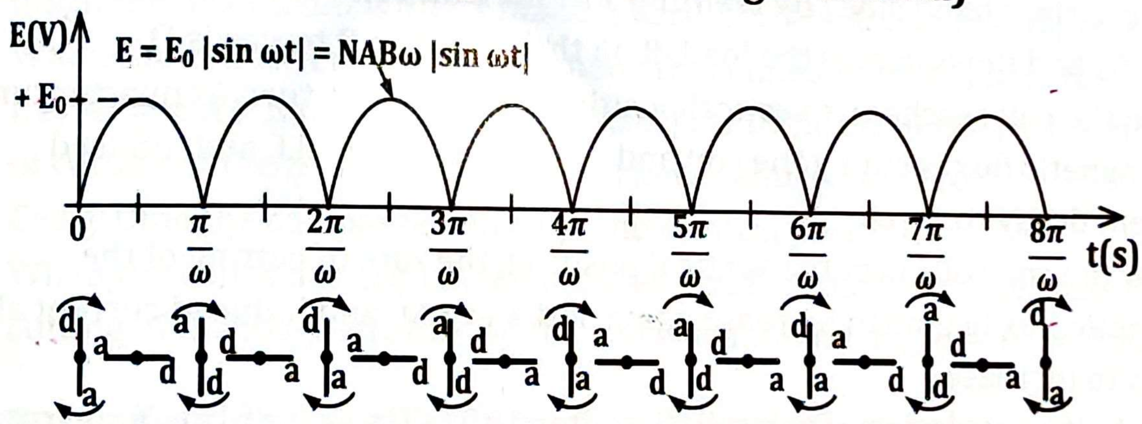


Fig. 4.2 (b)

The A.C. generator (Alternator)

The structure of a simple a.c. generator

- **An a.c. generator** – consists of ***a rectangular coil*** **abcd,** of fine insulated copper wire that rotates freely about an axis at the centre of the coil, and between the concave pole pieces of a strong permanent magnet.
- The free ends of the coil are each, connected to flat circular brass rings, known as ***Slip rings,*** S_1 and S_2.
- Placed in contact with the slip rings and pressing on them, by springy metal strips, are the small metal conducting blocks called the ***carbon brushes,*** that link the ends of the coil via the slip rings to the output terminals, **P** and **Q** across which is connected the load, **R.**

The A.C. Generator (The Alternator)

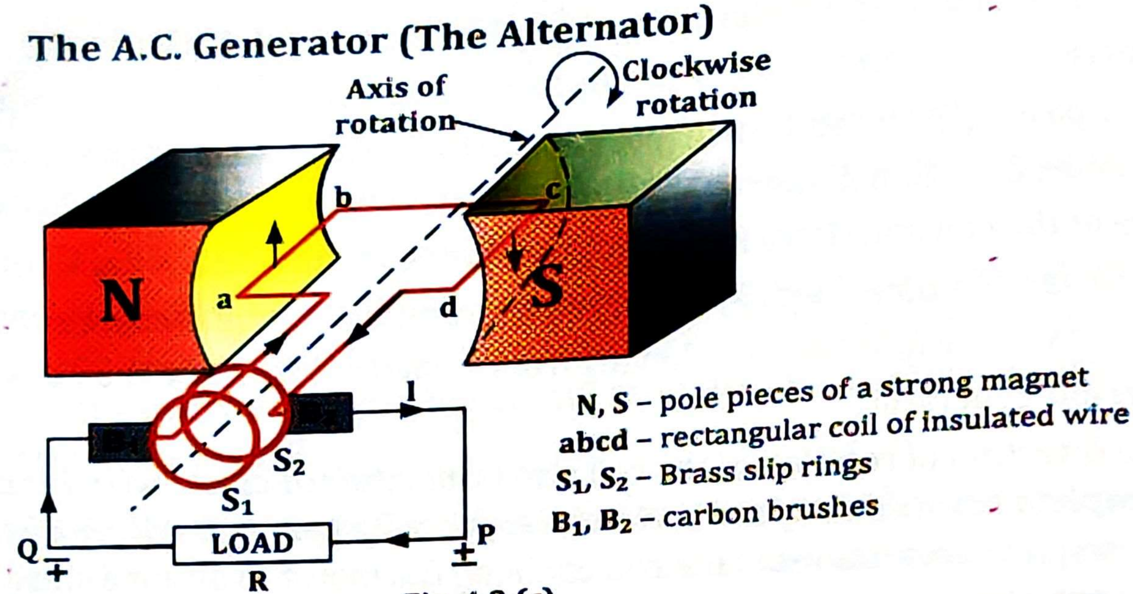


Fig. 4.2 (c)

The mode of operation (How it works)

- When the coil is mechanically rotated in a clockwise direction as shown by the directions of the arrows, its sides **ab** and **cd** cut the magnetic flux causing an e.m.f. to be induced across the ends of the coil.
- An induced current, **I**, flows from position **a** to **b** on the left hand side and **c** to **d** on the right hand side (***By Fleming's Right Hand Rule***) and out of the coil from B_2 and flows across the load, **R**, in the direction **P** towards **Q**.
- When the coil reaches its vertical position (i.e. after 90° turn or quarter turn), no magnetic flux is cut by the coil and so the induced e.m.f. and induced current decay to zero.
- When the coil rolls past the vertical position, the rate of cutting of the magnetic flux begins to increase again and an e.m.f. and induced current also begin to increase.
- After half a revolution of the coil, (i.e. after 180° turn or half-turn), the current ***reverses direction*** in the coil i.e. flowing from **d** to **c** on the left and **b** to **a** on the right hand side of the coil **abcd**.
- The current also reverses direction through the load i.e. flowing from terminal **Q** towards terminal **P**.
- The process ***repeats itself periodically*** at the frequency of the source, making a current to keep on alternating in the load hence a.c. output voltage is generated across the load terminals having an output shown below together with the positions of the coil in the magnetic field with time, **t**.

A graph of induced e.m.f. against time (with corresponding positions of the rotating coil in the magnetic field)

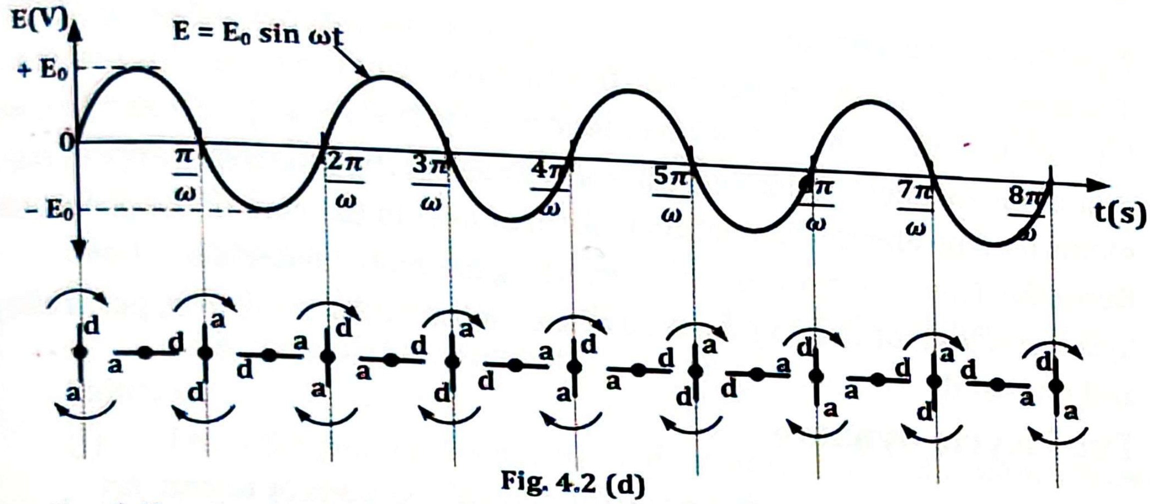


Fig. 4.2 (d)

Note the following,

- Most practical generators use ***field coils*** to provide the magnetic field as opposed to the use of permanent magnets, with the shunt wound generators producing a fairly steady current output compared to a series wound generator.
- When, $t = \frac{n\pi}{\omega}$ *where* $n = 0, 1, 2, 3, \ ...,$ the coil is horizontal, and the magnetic flux linked with its plane is zero. i.e. $\Phi = 0$ *but* $\boldsymbol{E = E_0 = NAB\omega}$.
- When the coil is vertical, the magnetic flux linked with its plane is maximum, *i.e.* $\boldsymbol{\Phi = NAB}$ *but* $E = 0$ *thus no e.m.f. is induced in the coil,* since the rate of cutting of the magnetic flux would have reduced to zero.
- Every time the coil passes through the vertical position
- When the coil is horizontal, the induced e.m.f. is maximum, because the rate of cutting of the magnetic flux by the coil is maximum, $E_{max} = E_0 = NAB\omega$

Factors affecting the efficiency of an electric Generator

Just like electric motors, practical electric generators are not 100% efficient due to some of the following factors:

(i) Eddy currents Power loss

The changing magnetic flux in the core causes eddy currents to be induced in it and they cause the heating up of the core of the generator in the I^2R – mechanism leading reduction of efficiency of the generators.

Remedy: This defect is minimized by using a laminated core e.g. a Laminated soft iron core which breaks down originally large eddy current loops into tiny loops rendering eddy currents negligible.

(ii) Resistance of the armature windings

The insulated wires used for making the armature coils have some resistance that cause heat dissipation in the generators windings in the I^2R – mechanism thus leading to the reduction of efficiency of the generators.

Remedy: This defect is minimized by using insulated ***thick copper wires of low resistance*** for making the armature coil windings.

(iii) Hysteresis power loss

The constantly changing magnetic flux in the core of the generator creates internal friction in the core as magnetic dipoles keep changing directions with that of the magnetic field. This causes increase in the internal energy of the atoms leading energy dissipation in form of heat in the core of the generator.

Remedy: This defect is minimized by using magnetic materials of low hysteresis loss for making the core of the motor such as soft iron, perm alloy and mumetal.

The bicycle dynamo

This is a device that uses the same principle of a generator to convert mechanical energy to electrical energy and finally light energy from the headlamp of the bicycle. However it slightly differs from the other in that instead of rotating a rectangular coil of wire in a stationary magnetic field, the magnet is rotated inside the magnetic field of stationary coils.

Its mode of operation is dependent on motion of the bicycle, since its driving wheel leans against the bicycle tyre.

Diagram:

The Bicycle Dynamo

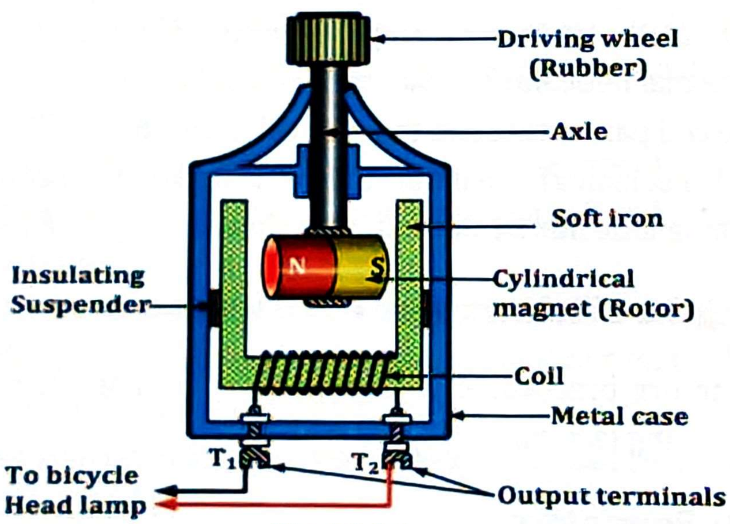


Fig. 4.2 (e)

Mode of operation (How it works)

- The gear lever of the dynamo is turned on, so that the driving wheel makes contact with the tyre by leaning on the tyre of the bicycle.
- When the bicycle is pedaled, the tyre rotates the driving wheel of the dynamo. As a result the cylindrical magnet connected to the driving wheel via the shaft axle, rotates.
- A changing magnetic field and flux (at a rate proportional to speed of the wheel or bicycle), links the soft iron and the coil of the dynamo and this causes an e.m.f. to be induced across the terminals of the dynamo.
- When the circuit is completed via the bulb of the head lamp, an induced current flows through the connecting wires and lights up the lamp.

Examples on generators

1. The output voltage E, in volts of a simple a.c. generator varies with time in seconds according to the equation, $E = 300 \sin 314\, t$.
Calculate the;
   (i) Frequency of the output voltage generated.
   (ii) Minimum time for the output of the generator to rise to 150 V in each cycle of the operation.

Solution:

(i) From the generation equation of a.c. output, $\boldsymbol{E = E_0 \sin \omega t}$, relate it to the given equation, $E = 300 \sin 314\, t$

$\Rightarrow \boldsymbol{\omega = 314 = 2\pi f}$

$\Rightarrow \boldsymbol{frequency, f = \frac{314}{2\times\pi} = 49.97 \cong 50\ Hz}$

(ii) $Using,\ E = 300 \sin 314\, t$

$\Rightarrow 150 = 300 \sin 314\, t$

$\Rightarrow \sin 314\, t = \frac{150}{300}$

$314\, t = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\therefore \boldsymbol{Time, t = 1.67 \times 10^{-3}\ s}$

2. A simple generator has a 300 turn rectangular coil of dimensions 20 mm by 35 mm the coil rotates in a uniform magnetic field of flux density 0.25 T. How many revolutions per second must the coil make in order to produce a peak output of 12.0 V?

Solution:

$E_{max} = NAB\omega = 2\pi fNAB$

$E_{max} = 2\pi fNAB \Rightarrow f = \frac{E_{max}}{2\pi NAB} = \frac{12.0}{2\pi\times300\times(20\times35)\times10^{-6}\times0.25}$

$\therefore \boldsymbol{The\ frequency, f = 36.4\ Hz}$

3. A generator having 100 turns is rotated in a uniform magnetic field of flux density 0.2 T at 20 revolutions per minute. Given that, the coil has an area of 2.5 cm^2.
   (i) Determine the maximum e.m.f. obtained from the device.
   (ii) Sketch using the same axes, graphs of induced e.m.f. with time and the magnetic flux linked with the coil with time for at least two revolutions.

Solution

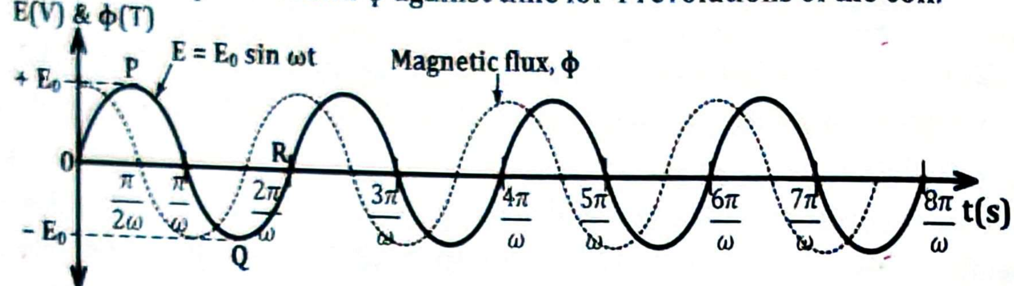
(i) $E_{max} = NAB\omega = 2\pi fNAB$

$E_{max} = 2\pi fNAB \Rightarrow E_{max} = 2\pi \times \frac{20}{60} \times 100 \times 2.5 \times 10^{-4} \times 0.2$

$\therefore \boldsymbol{The\ maximum\ e.m.f,\ E_{max} = 1.05 \times 10^{-2}\ V}$

(ii)

Graphs of E and ϕ against time for 4 revolutions of the coil.



At positions O and R – The coil is vertical & Perpendicular to the magnetic field
At positions P and Q – The coil is horizontal & Parallel to the magnetic field

Fig. 4.2 (f)

4. A bicycle generator (Dynamo) is made up of a circular coil of radius 1.8 cm and has 150 turns. The magnetic field in the region of the coil is 0.20 T. When the generator supplies an e.m.f. of amplitude 4.2 V to the light bulb, the bulb consumes an average power of 6.0 W and a maximum instantaneous power of 12.0 W.

What is the;

(i) Rotational speed of the armature of the generator in revolutions per minute (rpm)?

(ii) Average torque and maximum instantaneous torque that must be applied by the bicycle tire to the generator, assuming the generator is an ideal type?

(iii) If the radius of the tire is 32 cm and the radius of the shaft of the generator where it contacts the tire is 1.0 cm, at what linear speed must the bicycle move to supply an e.m.f. of amplitude 4.2 V?

Solution

(i) $E.m.f.$ *is a function of time, i.e.* $E = NAB\omega|\sin\omega t|$ and this e.m.f. has a maximum value when, $|\sin\omega t| = 1$ thus the amplitude is, $NAB\omega$

$E_{max} = NAB\omega = 2\pi fNAB$, *where* $N = 150, A = \pi r^2, \omega = 2\pi f$

$$f = \frac{E_{max}}{2\pi NAB} = \frac{4.2}{2\pi \times 150 \times \pi(0.018)^2 \times 0.20} = 21.89\ Hz\ (or\ rev.\ s^{-1})$$

But $1\ second = \frac{1}{60}\ minute$

$\therefore f = 21.89 \times 60 = 1313\ rev.\ min^{-1}$

(ii) Using maximum power, $P_{max} = E_{max}I_{max} \Rightarrow I_{max} = \frac{P_{max}}{E_{max}} = \frac{12.0}{4.2}$

$\Rightarrow I_{max} = 2.86\ A$, *now using magnetic torque,* $\tau = NABI\sin\alpha$

where, α = *angle between the normal to the plane of the coil & field, B*

At the position where the e.m.f. is maximum, $|\sin\alpha| = 1$

Then, , $\tau_{max} = NABI_{max} = 150 \times \pi(0.018)^2 \times 0.20 \times 2.86$

$\therefore \tau_{max} = \mathbf{8.73 \times 10^{-2}\ Nm}$

Alternatively

Assuming that, the generator is an ideal type, the torque applied to the crank, must do work at the same rate as that the electrical energy is generated. i.e. $P = \frac{W}{\Delta t}$ and since for a small angular displacement, $\Delta\theta$, the work done by a couple of forces, $W = \text{torque} \times \Delta\theta = \tau\Delta\theta$

$$\therefore P = \frac{W}{\Delta t} = \frac{\tau\Delta\theta}{\Delta t} = \tau\omega$$

$$The\ average\ torque, \tau_{ave} = \frac{P_{ave}}{\omega} = \frac{6.0}{2\pi f} = \frac{6.0}{2\pi \times 21.89} = \mathbf{4.36 \times 10^{-2}\ Nm}$$

$$The\ maximum\ torque, \tau_{max} = \frac{P_{max}}{\omega} = \frac{12.0}{2\pi f} = \frac{12.0}{2\pi \times 21.89} = \mathbf{8.72 \times 10^{-2}\ Nm}$$

(iii) The tangential speed of the tire where it touches the generator shaft is the same, since the shaft rolls without slipping on the tire. Since the generator (Dynamo) is almost outside the edge of the tire, the tangential speed at the outer radius of the tire is approximately the same.

The tangential speed of the generator shaft is $v_{tan} = r\omega = 2\pi fr$

$$v_{tan} = r\omega = 2\pi fr = 2\pi \times 21.89 \times 0.010 = 1.375\ ms^{-1}$$

$$\therefore v_{tan} = 1.375\ ms^{-1}$$

4.3 SELF INDUCTION

Self-induction *- is the production or appearance of an induced e.m.f in a coil when a changing current flows in the same coil.*

Explanation:

When a changing current flows in a coil, this result of changing magnetic flux linking the same coil, this produces an induced electric field that gives rise to an induced e.m.f. in the coil.

By Lenz's law, the e.m.f induced in a given coil opposes the change of magnetic flux that has caused it (if changing current flows in the coil, the induced current opposes the current that caused it), this type of induced e.m.f within the same coil, is called ***Back e.m.f*** because of its tendency to oppose external e.m.f.

NB: When a coil, solenoid, toroid or other circuit element is used in a circuit primarily for its self – inductance effects, it's often referred as an inductor. Self – inductance denoted by "**L**" is often shortened to inductance.

Self-inductance (L)

The measure of the ability of a coil to give rise to ***back e.m.f.*** is called the ***self-inductance L*** of the coil and is defined by the equation below.

$$\mathbf{L} = -\frac{\mathbf{E}}{\left(\frac{dI}{dt}\right)}, where,\ L = self\ inductance\ of\ a\ coil\ in\ henries\ (H)$$

$E = the\ back\ e.m.f\ induced\ in\ the\ coil\ in\ volts\ (V)$

$\left(\frac{dI}{dt}\right)$ = *the rate of change of current in the coil.*

Definition of self - inductance

Self-Inductance, L – *is the ratio of the back e.m.f induced in the coil, to the rate of change of current flowing in the same coil.*

SI unit: of inductance is ***a henry (H)***

Definition of: (henry H)

A henry – *is the inductance of a coil (or circuit) in which an e.m.f. of one volt is induced, in it when the current flowing in it changes at a rate of one ampere per second. i.e.* $1\ H = 1\ V\,s\,A^{-1}$.

Alternatively

Self-inductance **L**, can be obtained from the fact that, Magnetic flux linkage through the coil is directly proportional to the current I flowing through the coil. i.e. $N\phi \propto I$, this $\Longrightarrow N\phi = LI$, where L = a constant.

Therefore $\Longrightarrow L = \left(\frac{N\Phi}{I}\right)$

Self-inductance, L – is the magnetic flux linkage per unit current (or per ampere)

NB: ***The back e.m.f.*** in a coil acts like a small cell connected in series with the coil, but in ***opposite direction*** to the main cell (source of e.m.f.) that is driving the current in the circuit, ***when the current is increasing***, and acts in the opposite sense when the current is reducing or decreasing.

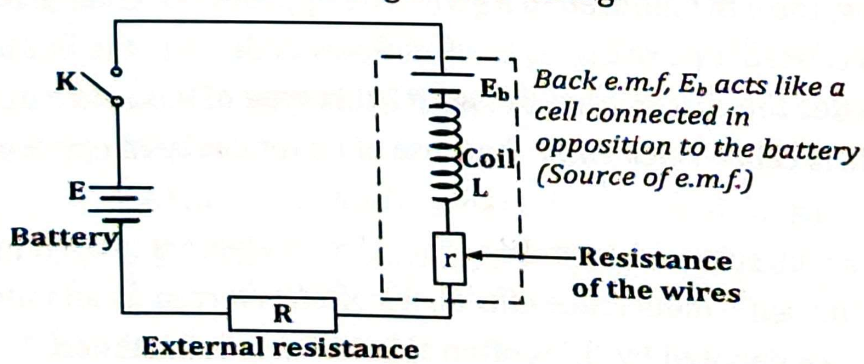


Fig. 4.3 (a)

When switch **K** is closed, the current, I supplied by the battery, E steadily increases from zero, until it stabilises at some maximum value, $I_{max} = \frac{E}{R+r}$

However, the coil opposes the increase in current as soon as the switch is closed, and thus the current grows slowly in the circuit until it reaches its maximum value, as shown in the graph in the figure 4.3 (b)

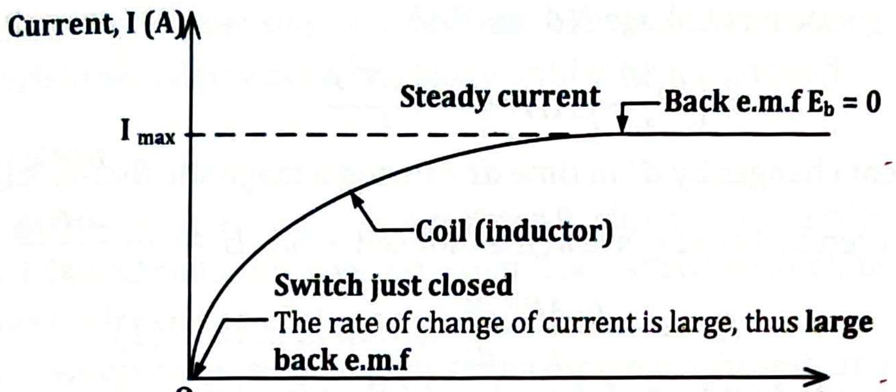
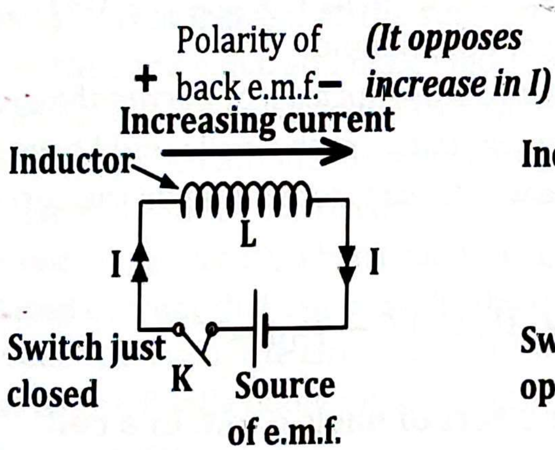


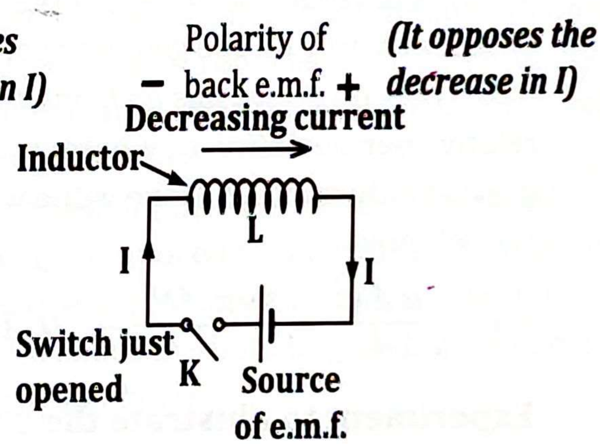
Fig. 4.3 (b)

NB: Inductors in a circuit behave like current ***stabilizers.*** i.e. it tries to maintain the status quo. When the current is constant, there is no induced e.m.f. to the extent that we can ignore the resistance, r, of its windings. The inductor acts like short circuit. When the current is changing, the induced e.m.f. is ∝ this rate of change of current. Consider two inductors (a) and (b) shown in the figure below.



(a)

Fig. 4.3 (c)(i)



(b)

Fig. 4.3 (c)(ii)

NB: The current through each of the inductors in (a) and (b) above flows to the right i.e. clockwise.

In **(a),** the current is ***increasing*** and thus, induced e.m.f. in the inductor ***"tries"*** to ***prevent the increase.***

In **(b),** the current is ***decreasing*** and thus, induced e.m.f. in the inductor ***"tries"*** to ***prevent the decrease.*** i.e. ***It tries to enhance the decaying current.***

(a) **Self-Inductance, L, of a long coil (Solenoid)**

Consider a long, air-cored solenoid of length l, cross-sectional area A, having N turns and carrying current I. The flux density B is almost constant over A and neglecting the ends, is given by: $B = \mu_0 n I$, where $n = \frac{N}{l}$,

thus $B = \frac{\mu_0 N I}{l}$

The magnetic flux linkage $N\phi = BAN$

$$= \left(\frac{\mu_0 NI}{l}\right) AN = \frac{\mu_0 AN^2 I}{l}$$

If current changes by dI in time dt causing a magnetic flux-linkage change, $d(N\phi)$ then by Faraday's law, the induced e.m.f, $E = -\frac{d(N\phi)}{dt}$

$$E = -\frac{\mu_0 AN^2}{l}\frac{dI}{dt} \dots\dots\dots\dots\dots\dots (i)$$

If, **L**, is the ***inductance of the solenoid***, then from the defining equation, we get,

$$E = -L\frac{dI}{dt} \dots\dots\dots\dots\dots\dots\dots\dots (ii)$$

Comparing these two equations, (i) *and* (ii) above, it follows that:

$$L = \frac{\mu_0 AN^2}{l} \quad or \quad L = \mu_0 n^2 A\, l \dots\dots\dots\dots (iii)$$

In terms of volume, $V = Al$, of the coil; $L = \mu_0 n^2 V \dots\dots\dots (iv)$
L, depends only on the geometry of the solenoid.
NB: If the coil above has an Iron-core along its axis, of ***permeability,*** μ, or relative permeability μ_r, where, $\mu = \mu_0\mu_r$, such a coil would have a much greater inductance, but the value would vary depending on the current in the solenoid.

$$\therefore L = \frac{\mu AN^2}{l} = \frac{\mu_0\mu_r AN^2}{l} = \mu_0\mu_r n^2 A\, l = \mu_0\mu_r n^2 V$$

Experiment to illustrate the effect of back-e.m.f. in a coil

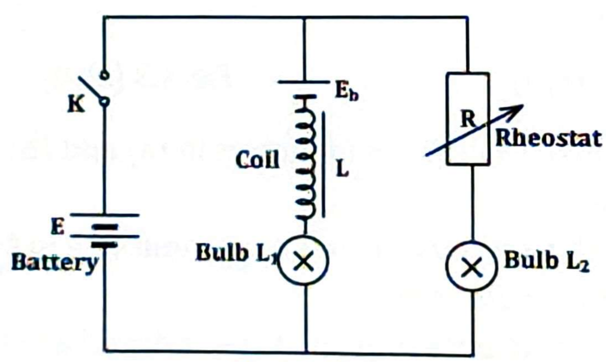


Fig. 4.3 (d)

- Initially, switch K is closed and the current is allowed to flow in the system until bulb L_1 attains full brightness. Rheostat R, is then adjusted until the brightness of the two bulbs L_1 and L_2 becomes the same.
- The switch K, is then opened.
- Using the set up shown above, switch K, is closed, bulb L_2 is seen to attain full brightness almost instantly, while bulb L_1 has its brightness gradually increasing from zero to full brightness after a reasonable length of time.

- When the switch is just switched off, bulb L_2 goes off almost instantly, while L_1 has its brightness decreasing gradually until it finally goes off with time.

Explanation

- Bulb L_2 connected in series with a resistor R, glows on instantly when **K** is just switched on and suddenly goes off when K is just switched off because it has negligible back e.m.f. to changing current.
- Bulb L_1 however takes time to glow to maximum brightness, and to go off, when the switch K is respectively switched on and off. This is because an inductor has a high back e.m.f. to changing current.
- Thus when the switch K, is just switched on, the **increasing current** through the coil **causes changing magnetic flux** in the coil, **creating an induced e.m.f.** called **back e.m.f.** in the coil which in turn causes an induced current to flow in such a **direction as to oppose** the current supplied by the source. This then tends to suppress the supplied current, thus little current flows through bulb L_1, at that instant. As the supplied current attains fairly steady or constant value, the back e.m.f. and hence induced current reduces until it eventually becomes zero when supplied current becomes constant at its maximum value. The bulb L_1 then attains maximum brightness.
- Similarly, when the switch is just switched off, there is rapid decrease of current in the circuit, a back e.m.f. is again induced in the coil creating to an induced current that opposes the decreasing current through it. The net current flowing in the circuit therefore keeps the bulb L_1 glowing for some time before finally going off. (*See the graph below*).

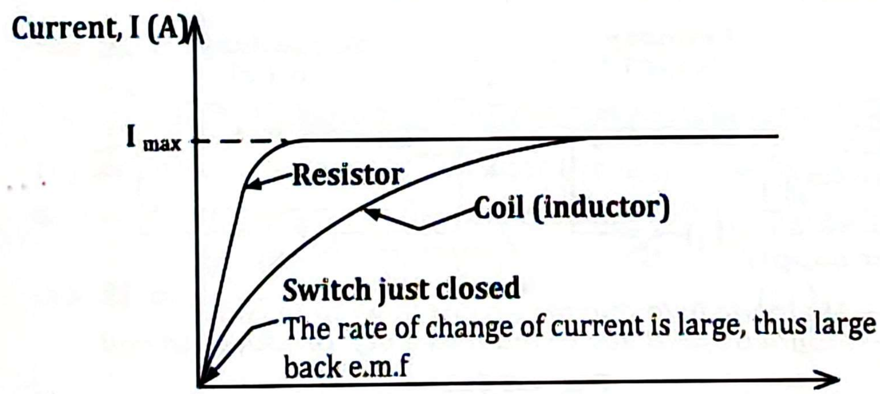


Fig. 4.3 (e)

NB: Resistance, $R = \frac{V}{I}$ resistance of the coil (dissipative opposition to the flow of current)

Inductance, $L = -\frac{E}{\left(dI/_{dt}\right)}$

Energy stored in an inductor

Whenever a switch is ***closed or opened in a circuit*** containing a coil, work δW is done in driving a charge q, throughout the circuit against back e.m.f. given by:

$\delta W = -V\ \delta q$ where, $\delta q = I\ \delta t$ and $V = -E$ the back e.m.f.

$\delta W = -E\ (I\ \delta t)$ where, $E = -L\ \frac{dI}{dt}$

$\delta W = L\frac{dI}{dt}(I\ \delta t)$

For the current to build up from I = 0, to a maximum steady value, $I = I_0$, the total work done,

$$W = \int_{I=0}^{I_0} dW$$

$$W = L\int_0^{I_0} I\ dI$$

$$\therefore \boldsymbol{W = \frac{1}{2}LI_0^{\ 2}}$$

Thus, this also equals the ***energy stored in an inductor.***

This expression is likened to the energy stored in a capacitor, i.e. $E = \frac{1}{2}CV^2$

4.4 MUTUAL INDUCTION (THE TRANSFORMER EFFECT)

Definition

Mutual induction *- is the process of generating of an* ***induced e.m.f*** *in the neighbouring* ***secondary coil*** *caused by changing magnetic flux linking it from the primary coil because of the passage of* ***changing current*** *through the* ***primary coil.***
It can also be defined as – a process of generating an induced e.m.f. in nearby secondary coil placed coaxially with a primary coil, when a changing current is passed through the primary coil and the coils are magnetically linked.

The transformer effect (Mutual induction)

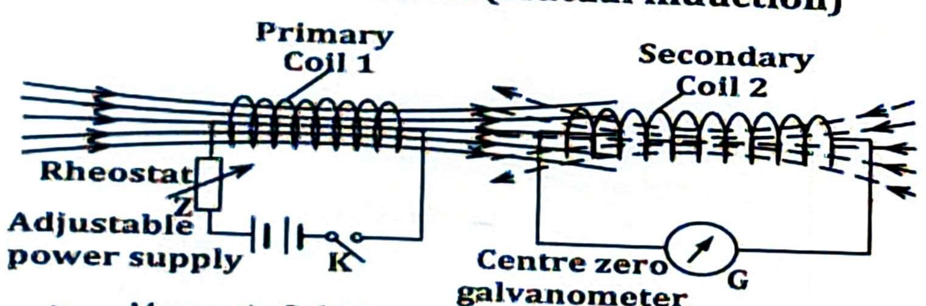


Fig. 4.4 (a)

- When switch K is closed and the rheostat, **Z**, is adjusted, or when the d.c. source is replaced by an a.c. source, a changing current **I_1** flows through the primary **coil 1** instantaneously in ***a clockwise direction,*** a changing magnetic flux **ϕ_1** is generated in and around coil 1, and magnetically links up the turns of **coil 2**.
- An induced e.m.f. is generated in **coil 2**, this then causes an induced current, **I_2** to flow in the ***secondary circuit*** in such a way as to make the end of **coil 2** next to **coil 1** have the ***same polarity*** as that of the adjacent end of ***coil 1***. i.e. (North pole)

- An induced current I_2 then flows in the **secondary coil 2** in such a direction as to oppose that in coil A, ***i.e. anti-clockwise***, this causes the centre zero galvanometer to deflect to the right.
- A magnetic flux linkage, between the two coils $N_2\phi_{21}$ has then been created, in the nearby coil 2, where $N_2\phi_{21}$ is proportional to the current flowing in coil 1. $i.e.\ N_2\phi_{21} \propto I_1$ where

  ϕ_{21} *stands for* ***total flux*** *through* **coil 2 due to** *the field produced by* **coil 1**
- The constant of proportionality, is called the mutual inductance, **M**, thus $N_2\phi_{21} = M I_1$

Mutual inductance, M

Definition

Mutual inductance, M – Is the ratio of the electromotive force induced in a neighbouring secondary coil or circuit due to the corresponding change of current flowing in the primary coil or circuit magnetically liked with it.

Mutual inductance, M – Is also the electromotive force induced in a neighbouring secondary coil, magnetically liked with the primary when the current flowing in the primary coil or circuit is changing at a rate of one ampere per second($1\ A\ s^{-1}$)

Alternatively:

Mutual inductance, M can be **defined** as the proportionality between the e.m.f. generated in **coil 2** due to the change of current flowing in coil 1, which produced it.

$$i.e.\ N_2\phi_{21} = M I_1$$

$$\Rightarrow M = \frac{N_2\,\phi_{21}}{I_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

NB: The mutual inductance, M, is found to be the same for the two coils, regardless of whether we consider the magnetic flux linkage through coil 2 due to changing current flowing through coil 1 or vice versa.

$$\Rightarrow M = \frac{N_2\,\phi_{21}}{I_1} = \frac{N_1\,\phi_{12}}{I_2} \ldots\ldots\ldots\ldots\ldots (ii)$$

The mutual inductance, M, depends on the following factors:

- *The shape and size of the two coils (i.e. cross – sectional area A)* $i.e.\ M \propto A$
- *The magnetic field strength,* ***B****, generated by the changing current in coil 1.*
- *The number of turns of each of the two coils.* $i.e.\ M \propto N$
- *The distance of separation between the two coils.* $i.e.\ M \propto d$
- *The nature of the magnetic core that links the two coils.* $i.e.\ M \propto \mu$
- *The size of the changing current flowing in the primary coil.* $i.e.\ M \propto \frac{1}{I}$

The size of ***mutual inductance, M,*** that links one coil to another depends very much on the relative positioning of the two coils. If one coil is positioned very close to the other coil, that their physical **distance apart is small,** then nearly all of the magnetic flux generated by the first coil will interact with the coil turns of the second coil inducing a relatively **large e.m.f** and therefore producing a large mutual inductance value, **M.**
Likewise, if the two coils are farther apart from each other or at different angles of inclinations of their axes, the amount of induced magnetic flux from the first coil into the second coil, will be weaker producing a much smaller induced e.m.f in the second coil and therefore a much smaller mutual inductance value. Thus, the effect of mutual inductance is very much dependent upon the relative positions or spacing, **d** of the two coils and this is demonstrated in the figure below.

Mutual Inductance, M, between two Coils

Mutually coupled coils.

(i) Two coils arranged in series (ii) Two coils arranged in parallel (Adjacent coils)

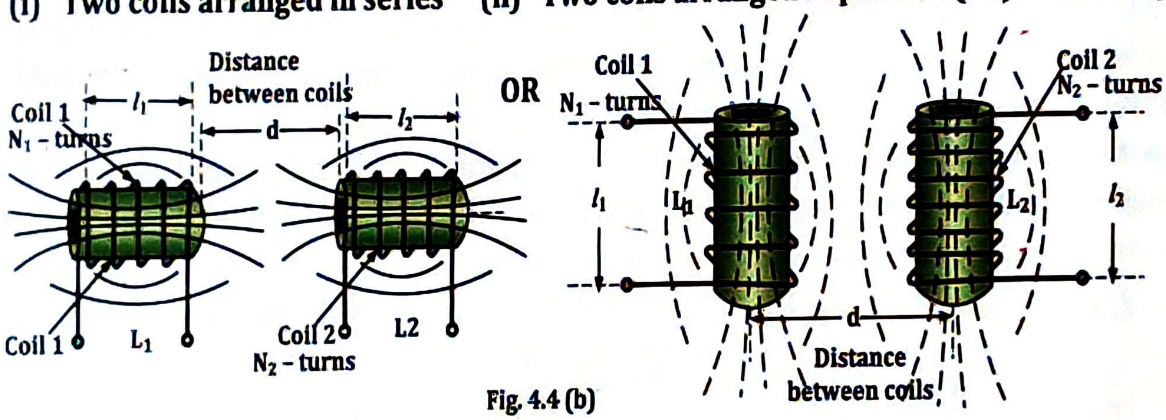


Fig. 4.4 (b)

The **mutual inductance, M** that exists between the two coils can be greatly <u>**increased**</u> by positioning them on a **common soft iron core** or **by increasing** the **number of turns of any one of the coils** as would be found in an a.c. transformer.
If the *two coils are tightly wound one on top of the other over a common soft iron core unity coupling* is said to exist between them as any losses due to the leakage of the magnetic flux will be extremely small or negligible if any.
Then assuming a perfect magnetic flux linkage between the two coils, the mutual inductance, **M,** that exists between the two coils can be given as.

$$M = \frac{\mu_0 \mu_r N_1 N_2 A}{l}$$

Where,

$\mu = \mu_0 \mu_r$ *is permeability of magnetic core.*

μ_0 *is the permeability of free space* $(4\pi \times 10^{-7} F^{-1} m^{-1})$

μ_r *is the relative permeability of the soft iron core*

N *is in the number of coil turns*

A *is in the cross − sectional area in* m^2

l *is the length of coil in metres*

Mutual Induction between two coils linked magnetically:

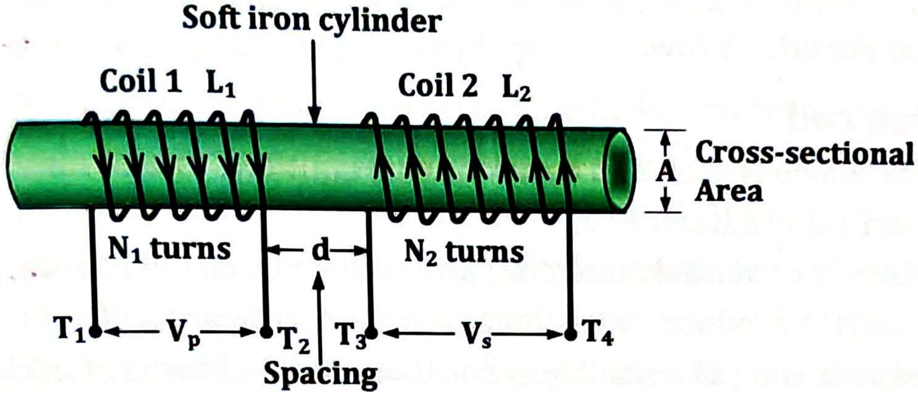


Fig. 4.4 (c)

Here, the current I_1 flowing in coil one, L_1 sets up a magnetic field around itself with some of these magnetic field lines passing through coil two, L_2 giving us mutual inductance M_{12}. Coil **one** has a current of I_1 and N_1 turns while, coil **two** has N_2 turns. Therefore, the mutual inductance, M_{12} of ***coil two*** that exists ***with respect to coil one*** depends on their position with respect to each other and is given as:

$$M_{12} = \frac{N_2\Phi_{12}}{l_1} = \frac{N_2N_1\mu_0\mu_r A}{l_1}$$

Likewise, the magnetic flux linking coil one, Φ_{21} when a current, I_2 flows around coil two, L_2 is exactly the same as the magnetic flux linking coil two, Φ_{12} when the same current flows around coil one L_1 above, then the mutual inductance of coil one with respect of coil two is defined as M_{21}. This mutual inductance is true irrespective of the *size, number of turns, relative position or orientation of the two coils.* Because of this, we can write the mutual inductance, **M**, between the two coils as: $M_{12} = M_{21} = M$

I hope that, we remember that the self-inductance of each individual coil is given

as; $L_1 = \frac{\mu_0\mu_r N_1^{\ 2} A}{l}$ *and* $L_2 = \frac{\mu_0\mu_r N_2^{\ 2} A}{l}$

Mutual inductance, M, that exists between the two coils, can be expressed in terms of the self-inductance of each coil.

Then, by cross-multiplying the two equations above, the mutual inductance, **M,** is obtained and given by the expression below.

$i.e.\ M^2 = L_1L_2 \Rightarrow$ ***The mutual inductance,*** $M = \sqrt{L_1L_2}$ *(henries)*

This gives us a final and more common expression for the mutual inductance between two coils. However, the above equation assumes zero magnetic flux leakage between the two coils. i.e. **ALL** the ***magnetic flux*** from ***one coil links*** the second adjacent or ***neighbouring coil.***

Applications of mutual induction

Mutual induction is applicable in some of the following:

1. *An induction coil.*
2. *An a.c. transformer.*
3. *Verification of Faraday's law.*

1. An induction coil

This is a device, which is used as a basis of many ignition systems for vehicles, motorcycles and many automobiles.

It uses the principle of mutual induction, where the mutual inductance between two coils to produce a high voltage in the secondary coil.

A primary coil consisting of a small number (hundreds) of turns of thick insulated copper wire is wound round a bundle of soft iron rods which are insulated from each other using adhesive non-conducting glue. (Laminated iron strips).

A secondary coil of insulated copper wire consisting more number of turns than the primary coil, (thousands of turns) is tightly wound on top of the primary coil with fewer turns as shown on the diagram below.

A capacitor connected across the make and break switch contacts helps to eliminate sparking at the contacts and to make the magnetic field to die away much faster than it would in the absence of the capacitor, hence making the rate of generation of induced secondary e.m.f. much faster.

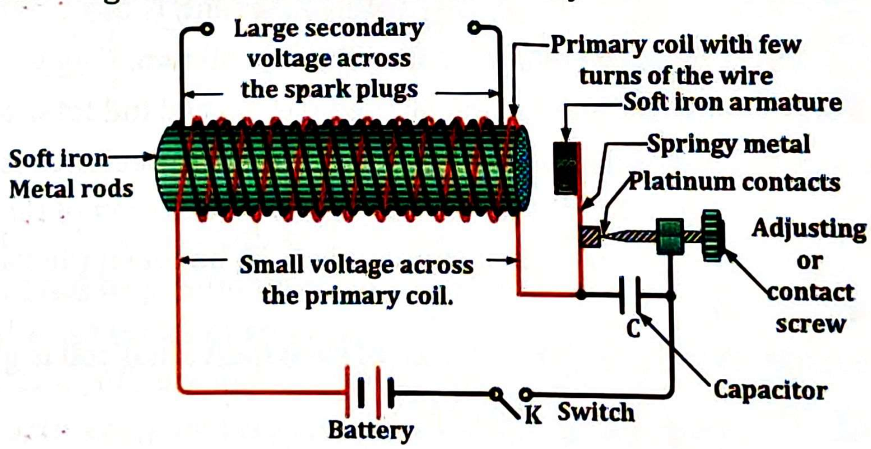


Fig. 4.4 (d)

The mode of operation (How it works)

- Using the set up shown, switch K, is closed and causes a current to flow in the primary circuit, and magnetizes the soft iron core, which attracts the soft iron armature piece causing the circuit to break up at the platinum contacts.
- The magnetic field at the soft iron rods dies off (it gets demagnetized), the springy metal piece pulls back the soft iron metal piece which then causes the platinum contacts to be re-joined (made) and the primary circuit is completed again causing a current to flow in the primary coil and the process then repeats itself.

- Since the above process occurs very rapidly (i.e. several times per second), the rapidly changing magnetic field linking the primary and secondary coils each time, produces a large back e.m.f. in the primary and consequently very high or large voltage across the secondary coil maintained across a narrow air gap at the spark plugs.
- A capacitor, C, connected across the make and break platinum contacts, reduces the sparking at the contacts and also causes the magnetic field to die away much more rapidly than it would if the capacitor were absent.
- As a result, the induced voltage in the secondary coil is much greater when the circuit is broken than when it is made.
- The secondary current therefore pulsates, but it is always in the same direction.
- Sparks of several centi-metres in length may be obtained through air at Atmospheric pressure in small coils as those used in ignition circuits of automobiles and in larger coils as of those used originally to power X-ray tubes.

NB: When the voltages produced by the induction coil vary with time, the graphs shown in the figure below are obtained. Note the rapid decrease in the primary voltage at the break and the corresponding high value of the e.m.f. induced in the secondary.

Graphs of Primary and Secondary voltages across the coils against time.

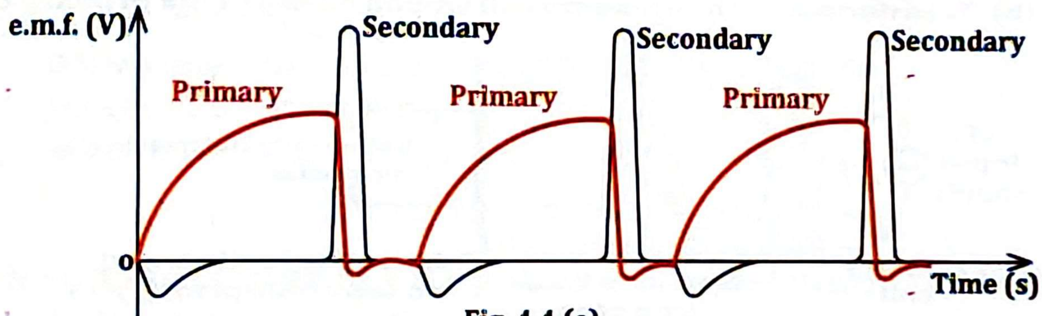


Fig. 4.4 (e)

4.5 THE A.C. TRANSFORMER

- Transformers are devices that make use of the principle of ***mutual induction*** to **"step up"** or **"step down"** a.c. voltages.
- Transformers consist of essentially two coils namely the primary coil (connected across a varying source of e.m.f.) and the secondary coil (where the output load or devices are connected.
- The two coils need not to have physical contact between them, but they are magnetically by proxy.
- In order to enhance proper magnetic flux linkage, the two insulated coils usually of copper wire, are either wound tightly on opposite limbs of a magnetically soft-core material such as; mumetal or soft iron or the

secondary coil is wound on top of the primary coil, which is in contact with the magnetic core.

- Transformers aid in the high-voltage electric power transmission on the National grid, which makes long-distance transmission economically practical.
- In home wiring applications, a transformer will step down the voltage in an electric circuit from say, 240 volts to around 6, 9,12 volts etc. for use in the device, such as electric doorbells, thermostats and low-voltage lighting systems, etc.

Diagram/Structure:

THE A.C. TRANSFORMER

(a) With the primary and secondary coils linked via the magnetic core.

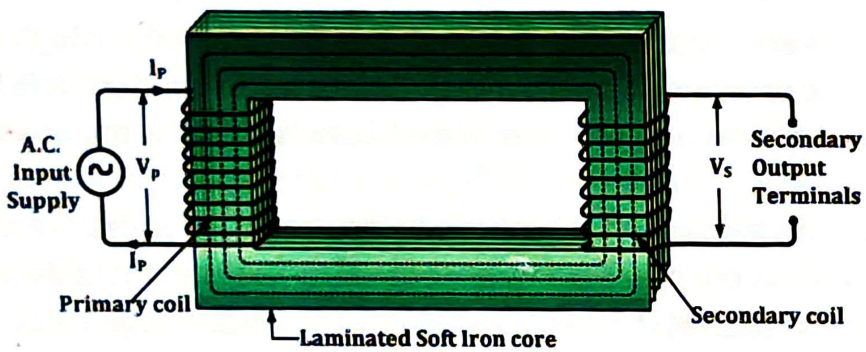


Fig. 4.5 (a)

The alternative design of an a.c. Transformer:

(b) Transformer with secondary coil wound on top of the primary coil.

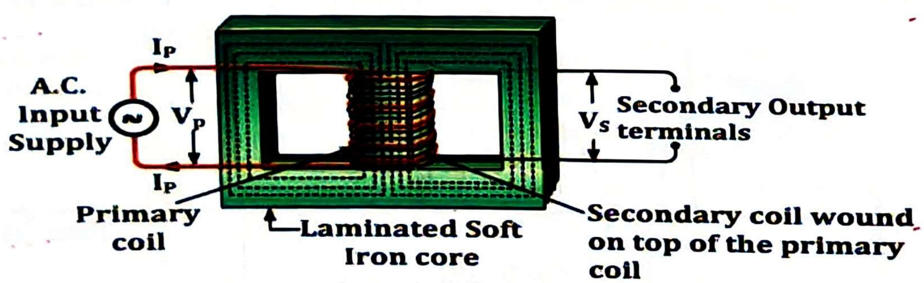


Fig. 4.5 (b)

The mode of operation of an a.c. Transformer

- An alternating voltage connected to the primary coil produces an alternating current in it.
- This sets up an ***alternating or changing magnetic flux*** in the core that links up with the secondary coil and thus *induces an alternating* e.m.f, V_s in the secondary coil.
- The changing magnetic flux linking the secondary coil causes an e.m.f to be induced in the secondary coil given by;

$$V_s = -N_s\frac{d\Phi}{dt}, where\ \Phi = B_pA \ldots\ldots\ldots\ldots (i)$$

- The changing current in the primary coil, causes a changing magnetic flux

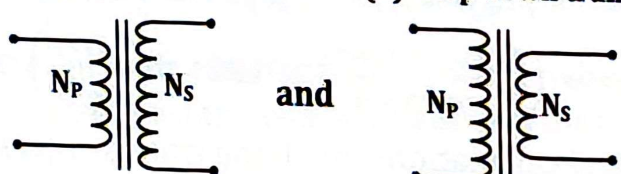
to link the primary coil itself due to ***self-induction***, in the primary coil a back e.m.f, E_b is induced in the same coil, given by; $E_b = -N_p \frac{d\Phi}{dt}$.
However, if the primary coil has negligible resistance, the applied voltage V_p is equal to the back e.m.f. i.e. $E_b = -V_p$

$$\therefore V_p = N_p \frac{d\Phi}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

- Thus, $\frac{E.m.f.\ induced\ in\ the\ Secondary}{Voltage\ applied\ to\ the\ primary} = \frac{V_s}{V_p} = -\frac{N_s}{N_p} = t\ (turns\ ratio) \ldots (iii)$
- The negative sign shows that the primary and secondary voltages are usually 180° out of phase i.e. **Anti-phase.** i.e. If say half cycle of the ***primary input*** voltage is ***a crest***, then corresponding half cycle of the ***secondary output*** would be ***a trough*** and ***vice versa.***
- When $N_s > N_p \Rightarrow V_s > V_p$ and the transformer is called ***a step - up Transformer*** *and* ***steps the voltage up to a higher value.***
- When, on the other hand, $N_s < N_p \Rightarrow V_s < V_p$, and the transformer is called ***a step down transformer*** and ***steps the voltage down to a lower value.***

Types and Symbols used for A.C. Transformers:

(a) Step up transformer **(b) Step down transformer**



Where number of turns in the primary coil is less than those of secondary coil. i.e. $N_P < N_S$

Where number of turns in the primary is greater than those of secondary coil. i.e. $N_P > N_S$

Fig. 4.5 (c)

Why High voltage is used in EHT power transmission on the National grid instead of High Current

- The cables that transmit power, have resistance and therefore power is lost due to production of heat in the cables as current flows through them.
- Power loss is given by, $P = I^2R$. To reduce power loss, current supplied must be small.
- Now power, $P = IV \Rightarrow I = \frac{P}{V}$, hence for current to be small, voltage must be high.

Alternatively

- Transmission at high voltage ***reduces power loss.***
- Power supplied $P = IV \quad \Rightarrow \quad I = \frac{P}{V}$
- Hence, when voltage, V is high, the current, I is small
- Thus the power loss $P_0 = I^2R$ is minimal, hence conserving power.

- Therefore, when the transmission current, I is small, the power loss, P_0 along the transmission power lines or cables is small.

Efficiency of an a.c. transformer

- An ideal transformer is one considered to be100percentage efficient, with no energy or power losses in it.
- In such a transformer, the power output in the secondary circuit is equal to the power input (or Power generated) in the primary circuit
  $i.e. \textit{Power output} = \textit{power input (power generated)}$
- Thus when there no power or energy losses in a transformer, $I_sV_s = I_PV_P$
- However if Power output is not equal to the power input (power generated), then the transformer is not 100% efficient due to a number of factors as seen shortly.
- Hence, the Power output, $I_sV_s = (\textit{proper fraction})\ of\ I_PV_P$ and thus the transformer is less than 100% efficient.

Definition

Efficiency η - is the ratio of the power output to the power input of the transformer.

$$Efficiency\ \eta = \frac{Power\ Output}{Power\ Input} = \frac{I_SV_S}{I_PV_P}$$

$$\%\ Efficiency = \left(\frac{Power\ output}{Power\ input}\right) \times 100 = \left(\frac{I_sV_s}{I_pV_p}\right) \times 100$$

NB: Numerical calculations involving transformers always involve the r.m.s. values of current and voltages i.e. I_{rms} and V_{rms}

Power and Energy Losses in a Transformer

In Practical transformers, power output (I_sV_s) is always less than the power input (I_PV_P), due to some of the following factors:

(i) Eddy currents Power loss:

The changing magnetic flux in the core causes eddy currents to be induced in it and they cause the heating up of the core in the I^2R – mechanism leading reduction of efficiency.

Remedy: This defect is minimized by using a Laminated core e.g. Laminated soft iron core that breaks down eddy current loops rendering eddy currents negligible.

(ii) Resistance of the windings:

The coils used for making the primary and second coils have some resistance that cause heat dissipation in the transformer in the I^2R – mechanism leading to loss of efficiency.

Remedy: This defect is minimized by using thick copper wires of low resistance for making the primary and secondary coil windings.

(iii) **Hysteresis power loss**

The constantly changing current in the primary circuit causing changing magnetic flux in the core creates internal friction in the core as magnetic dipoles keep changing direction with that of the magnetic field. This causes increase in the internal energy of the atoms leading energy dissipation in form of heat in the core.

Remedy: This defect is minimized by using magnetic materials of low hysteresis loss for making the core such as soft iron, perm alloy and mumetal.

(iv) **Magnetic flux leakage**

Sometimes not all the magnetic flux generated in the primary circuit may link up with the secondary circuit, some may be lost on its way. This reduces the efficiency of the transformer.

Remedy: This defect is minimized by using well designed core of the transformer with no air gaps between the laminations using "E – Shaped" interleaved magnetic laminations of the core.

Alternatively this defect is also minimized by winding the secondary coil on top of the primary coil all of which using insulated copper wires.

Why much current is drawn from the source when a load is connected to the secondary coil of a transformer

NB: Whenever, a load (Resistance) is connected across the secondary coil of a transformer, a current begins to flow in the secondary circuit and windings. The current flow is in such a direction as to reduce or oppose the magnetic flux in the core linking the secondary coil, since V_S and V_P are antiphase (i.e. 180° out of phase).

The reduction in the magnetic flux linkage, causes a reduction in the back e.m.f. in the primary coil, hence the current drawn from the supply increases, in order to compensate for the drop or reduction in the value of back e.m.f.

Transmission of power on the national grid at high voltage & at smaller currents

- The national grid – is a network of transmission lines which connect power stations to the electricity consumers, via a number of step up and step down transformers, pylons and wire lines.
- Since power is a product of voltage and current, a given amount of power can be transmitted and delivered either at high voltage and low current through the grid, or at low voltage and high current.

- However, since the transmission cables themselves have some electrical resistance, R, some energy will be dissipated and lost along these power lines in the I^2Rt – *mechanism* in form of heat energy, where, **t**, is the time lapse from the source to the consumer along the wires.
- The amount of energy that is wasted is thus proportional to the square of the current in the transmission cables as current passes through them.
- Thus, the most effective way to transmit power is at high voltage with low current to minimize power waste along the power lines.
- Now power $P = IV \Rightarrow I = \frac{P}{V}$, hence for ***current to be small***, voltage must be high.
- Another advantage of high voltage transmission is that, lower current requires thinner wires that become cost effective as opposed to thicker cables that would carry larger current and yet they would be very costly to install on top of larger dissipation and wastage of electrical energy or power on the way.
- The excess weight of the cables also could lead to easy snapping of the cables under the influence of ***their own weight*** or probably ***large sagging*** and may even pull down the pylons with their own weight.
- The other disadvantage is the cost of building tall pylons to suspend the high voltage cables above the ground (or deep below the ground in high protective casings) so as to protect against human life.
- In order to compromise the thickness and weight of the cables, thinner, light and cheaper multi - twisted strands of aluminium wires with a steel strand at the centre of the aluminium cables.

Alternatively

- Transmission at high voltage rather than at high current, ***reduces power Losses inn the I^2R mechanism.***
- Power supplied, P = I V $\Rightarrow$ $I = \frac{P}{V}$
- Hence when the voltage, V is high, then the current, I, will be smaller.
- Thus the power loss $P_0 = I^2R$ will be minimized since, current, I is low.
- Therefore, when current, I, flowing in the transmission cables is small, the power loss along these transmission cables, P_0 is also small.

Diagram representing of the power transmission on the National Grid

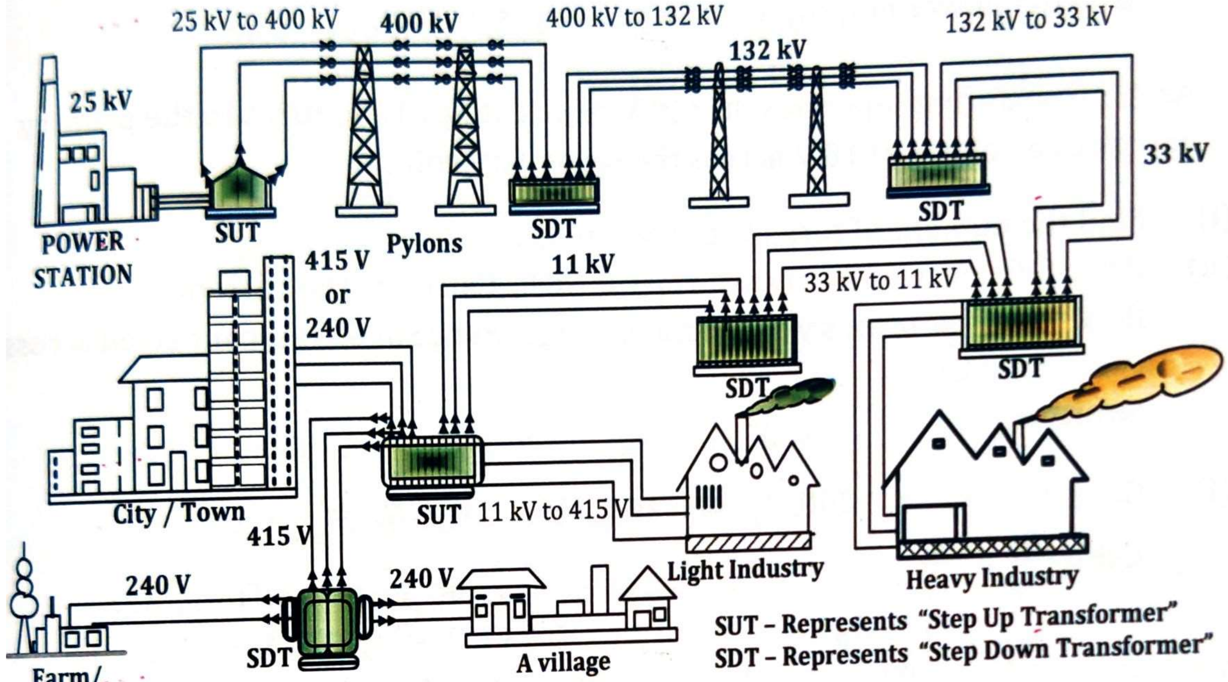


Fig. 4.5 (d)

EXAMPLES & EXERCISES ON SELF/MUTUAL INDUCTION & TRANSFORMERS

. An a.c transformer of input voltage of 100 V is applied to the primary circuit having 500 turns and 2400 turns in the secondary. With no load in the secondary, a negligible current flows in the primary circuit, but when a resistive load is connected to the secondary, a current of 2.0 A flows in the primary circuit. Calculate the;

(i) Voltage in the secondary
(ii) Current flowing in the secondary
(iii) State any assumptions that you have made.

Solution

Assuming there are no energy losses in the transformer.

(i) From, $\frac{N_s}{N_p} = \frac{V_s}{V_p}$ given that, $N_s = 2400,\ N_p = 500\ turns$

$V_s = \frac{N_s}{N_p} \times V_p = \frac{2400}{500} \times 100$ where, $V_p = 100\,V, V_s = ?$

$\therefore \boldsymbol{V_s = 480\ V}$

(ii) From $\frac{N_s}{N_p} = \frac{I_p}{I_s}$

$I_s = \frac{N_p \times I_p}{N_s}$

$I_s = \frac{500}{2400} \times 2.0$

$\therefore \boldsymbol{I_s = 0.417\ A}$

(iii) The assumptions employed here are:
The transformer is an ideal transformer, where all the power input is equal to all the power output. Hence no energy is wasted.

2. An A.C transformer operates on 240 V mains. It has 1200 turns in the primary and gives a voltage of 18 V across the secondary coil.

(i) Find the number of turns in the secondary.
(ii) If the Efficiency of the transformer is 90%. Calculate the current flowing in the primary coil, if a load of resistance of 50 Ω is connected across the secondary.

Solution

(i) Given that $N_p = 1200,\ V_p = 240\,V, V_s = 18\,V, N_s = ?$

Using $\frac{N_s}{N_p} = \frac{V_s}{V_p}$

$$N_S = \frac{V_S \times N_p}{V_p}$$

$$= \frac{18 \times 1200}{240}$$

$$\therefore \boldsymbol{N_S = 90\ turns}$$

(ii) Given $V_s = 18\,V\ while\ , I_s = ?\ and\ R = 50\,\Omega$
From V = IR

$I_S = \frac{Vs}{R} = \frac{18}{50} = \frac{9}{25} A = 0.36\,A\ \ if\ transformer\ is\ ideal.$

If the transformer is 90 % efficient

$\Rightarrow only\ 90\%\ of\ the\ Power\ input\ is\ delivered\ to\ the\ secondary\ circuit.$

$90\%\ of\ V_pI_p = V_sI_s \Rightarrow \frac{90}{100} \times V_pI_p = V_sI_s$

$$\therefore, I_p = \frac{100}{90} \times \frac{V_S \times \left(\frac{V_S}{R}\right)}{V_p} = \frac{100}{90} \times \frac{18 \times \left(\frac{18}{50}\right)}{240} = 0.03\,A$$

$\therefore Current\ in\ the\ primary, \boldsymbol{I_p = 0.03\,A}$

3. (a) A transformer connected to an a.c supply of peak voltage 240 V, is to supply a peak voltage of 12 V to a mini lighting system of resistance 5Ω. Calculate the;

(i) ratio of the number of primary to secondary turns.
(ii) r.m.s current supplied by the secondary circuit. (2 marks)
(iii) average power delivered to the lighting system. (2 marks)

Solution

(i) When using the formulas generated above, the expressions correspond to ***root mean square values*** of ***current*** I_p or I_s and ***root mean square values*** of ***voltages,*** V_p or V_s

Using $\frac{N_p}{N_s} = \frac{V_p}{V_s}$ where $V_{rms} = \frac{V_o}{\sqrt{2}} \Rightarrow V_p = \frac{V_{op}}{\sqrt{2}}$ and $V_s = \frac{V_{os}}{\sqrt{2}}$

$$\frac{N_p}{N_s} = \frac{\frac{V_{op}}{\sqrt{2}}}{\frac{V_{os}}{\sqrt{2}}} = \frac{V_{op}}{V_{os}} = \frac{240}{12} = 20$$

Thus the ratio of primary to secondary turns, $N_p : N_s = 20 : 1$

(ii) Using, $I_{rms} = \frac{V_{os}}{R\sqrt{2}} = \frac{12}{50 \times \sqrt{2}} = 0.170\ A$

$\therefore \boldsymbol{I_{rms} = 1.70 \times 10^{-1}\ A}$

(iii) The average power dissipation , $P = I_{rms}{}^2 R = (0.170)^2 \times 50$

$\therefore \boldsymbol{P_{average} = 1.445\ W}$

4. The figure 4.6 (a) shows two electric bulbs A and B having the same power rating connected across d.c. source via a switch K. bulb A is connected in series with a pure inductor of self inductance L, while bulb B is connected in series with a resistor of resistance R.

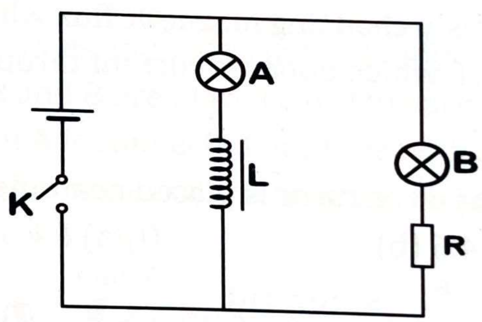


Fig. 4.6 (a)

Explain the following observations:

(i) When switch **K** is closed, initially, bulb **A** lights dimly than bulb B. After a short time, **A** lights brighter than **B.**

(ii) When the d.c. source is replaced by an a.c. source and switch **K** is closed, Bulb A lights dimly while bulb B lights brightly.

Solution

(i) When switch **K** is closed, initially, bulb **A** lights much more dimly than bulb **B.** A changing or increasing current produces a changing or an increasing magnetic flux, linking the coil, **L**, thus it induces, ***a back e.m.f.*** in the coil, **L,** which then ***opposes the applied voltage*** and hence an ***opposite current*** flows through the coil and bulb, **A**, in the opposite direction.

Little (i.e. the **net**) current flows through bulb **A,** making it light dimly compared to bulb **B.**

When the ***current supplied*** by the battery ***becomes steady***, after a short while, ***no changing magnetic flux*** is induced in the coil, hence the ***back e.m.f. becomes zero, enabling a large applied current to flow through bulb A***.

Bulb **A** lights brighter than bulb **B** afterwards, because the current through it becomes steady, thus the back e.m.f. in the coil, L, becomes zero and hence the net current through **A** increases, making bulb **A** lights brighter than **B**, connected in series with a resistor R, that opposes current flow through **B** continuously.

When **K** is opened, both bulbs **A** and **B** flash for a short time and stop lighting, with bulb **A**, dims down progressively for a slightly longer time than bulb, **B**. This is because, a ***back e.m.f.*** is induced in such a direction as to support of the decaying or dying current, through bulb, **A**, and it drives induced current in same direction as of the battery, hence the bulb, **A**, has decaying light.

When, applied current reduces to zero, the Back e.m.f. and induced current also progressively become zero and the bulb **A** goes off.

(ii) Replacing a d.c. source with an a.c. source and closing switch K, causes a continuously Bulb **A** continuously lights dimmer than **B**. A changing current creates a continuously changing magnetic flux which then continuously induces a back e.m.f. which opposes current through bulb **A**, hence it continuously dims.

5. A circular coil of wire of three turns is placed near one open end of the solenoid as shown in the figure 4.6 (b).

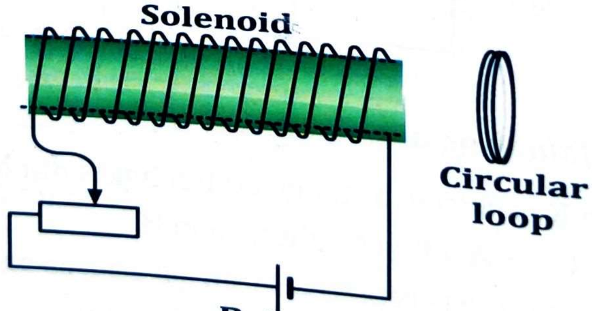


Fig. 4.6 (b)

When the current in the solenoid is 550 mA, the magnetic flux linking the circular loops of the coil is 2.7×10^{-5} Wb. When the current in the solenoid changes at 6.0 A s^{-1}, the induced current in the circular loop is 0.36 mA. Determine the;

(i) mutual inductance of the solenoid and the circular coil.

(ii) resistance of the circular loop.

Solution

Let subscript 1 represent the solenoid and 2 the coil

(i) From $E = -M\frac{dI_1}{dt}$ *and* $E = -N_2\frac{d\Phi_{21}}{dt} \Rightarrow MI_1 = N_2\Phi$

$\therefore M = \left(\frac{N_2\Phi}{I_1}\right) = \frac{3 \times 2.7 \times 10^{-5}}{0.550}$

$\therefore \mathbf{M = 1.47 \times 10^{-4}\ H}$ - is the mutual inductance of the solenoid and the circular coil.

(ii) A changing current in the solenoid (1) induces an e.m.f. $E = -M\frac{dI_1}{dt}$ in the loops of the coil, whose magnitude, is given by $|E| = M\frac{dI_1}{dt}$ *but* $E = IR$

$\Rightarrow IR = M\frac{dI_1}{dt}$ $\therefore$ **Resistance,** $\mathbf{R} = \left[\mathbf{M} \times \frac{\left(\frac{dI_1}{dt}\right)}{I}\right] = \left[\frac{1.47 \times 10^{-4} \times 6.0}{3.60 \times 10^{-4}}\right]$

$\therefore$ **Resistance of the coil,** $\mathbf{R = 2.45\ \Omega}$

Note

- The ***mutual inductance,*** *determines the* ***size of the e.m.f. induced in the coil*** for *a given rate at which current is changing in the solenoid.*
- The size of the current that flows in the coil depends on the electrical resistance of the coil.
- The coil with a higher resistance would have the same size of e.m.f. induced in it, but with a small or low current flowing in it and vice versa.

6. (a) (i) Define a henry as applied to mutual induction?

(ii) Two coils **A** and **B** are placed close to each other with their planes parallel. Coil **A** is connected in series with a cell and switch K. Coil **B** is connected in series with a centre zero galvanometer, G, as shown in the figure 4.6 (c)(i)

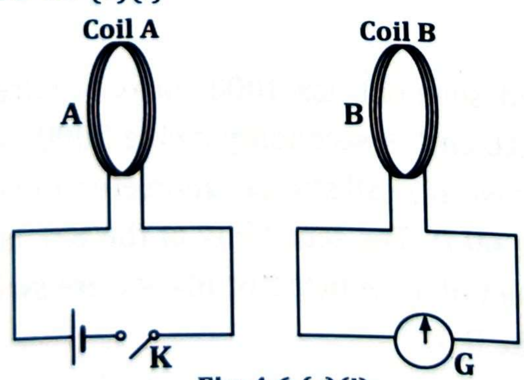


Fig. 4.6 (c)(i)

Explain what happens when switch K is momentarily closed, and later opened.

Solution

(i) A henry is a mutual inductance due to magnetic flux linkage of two coils when an e.m.f. of 1V is induced in a neighbouring secondary coil when the current flowing in the first primary coil is changing at the rate of one ampere per second ($1\ A\ s^{-1}$).

(ii)

- When switch **K** is closed, a rapidly increasing current flowing though coil A, creates a changing magnetic flux that links up with coil B.
- This causes an e.m.f. to be induced in coil B, and since the circuit is closed an induced current flows in coil B in such a direction as to create a magnetic flux in coil B that opposes that linking it from coil A. As a result, the pointer of the galvanometer G deflects in one direction.
- When the current becomes steady, the pointer goes back to its original zero position.

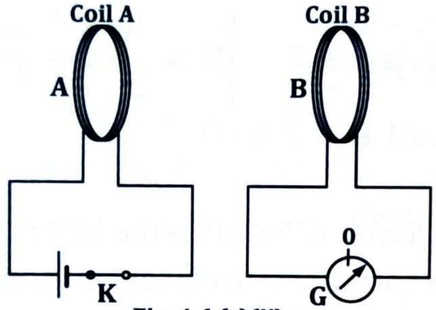


Fig. 4.6 (c)(ii)

- When switch K is opened, a rapidly decreasing current flowing though coil A, creates a reducing magnetic flux that links up with coil B.
- This causes an e.m.f. to be induced in coil B, and since the circuit is closed, an induced current flows in coil B in such a direction as to create a magnetic flux in coil B that enhances the decaying magnetic flux linking it from coil A.
- As a result, the pointer of the galvanometer, G deflects in the opposite direction to that in the first case when K was closed.

Exercises

1. A long air – cored solenoid has 1000 turns of wire per metre and cross – sectional area of 8.0 cm^2. A secondary coil of 2000 turns, is wound around its centre and connected to a ballistic galvanometer, the total resistance of the coil and the B.G. being 60 Ω. The sensitivity of the B.G. is 2.0 divisions per micro-coulomb. If a current of 4.0 A in the primary were switched off, what would be the deflection of the B.G?
[Permeability of free space = $4\pi \times 10^{-7}$ H m^{-1}] **Ans: [26.8 divisions]**

2. A transformer connected to an a.c supply of peak voltage 240V, is to supply a peak voltage of 9V to a mini lighting system of resistance 5 Ω.

Calculate the,

(i) The ratio of the primary to secondary turns. (1 mark)
(ii) The r.m.s current supplied by the secondary circuit. (2 marks)
(iii) The average power delivered to the lighting system. (2 marks)

(iv) Explain why the voltage of the electricity generated at Owen Falls dam has to be stepped up to about 132kV for transmission to very far places like Parts of Western Uganda and then later stepped down for the general use. (3 marks)

(v) Give any two power losses in an a.c transformer and state how they are minimized. (3 marks)

(vi) What is meant by the National Grid?

(vii) Explain why the voltage of the electricity generated at Owen Falls dam has to be stepped up to about 132kV for transmission to very far places like Parts of Western Uganda and then later stepped down for the general use. (3 marks)

(viii) Give any two power losses in an a.c transformer and state how they are minimized. (3 marks)

=END=

4.7 ALTERNATING CURRENTS

Alternating current – is the current that reverses its direction of flow periodically in one second or with time. The polarity of the voltage is also reverses periodically with time in an alternating current source.

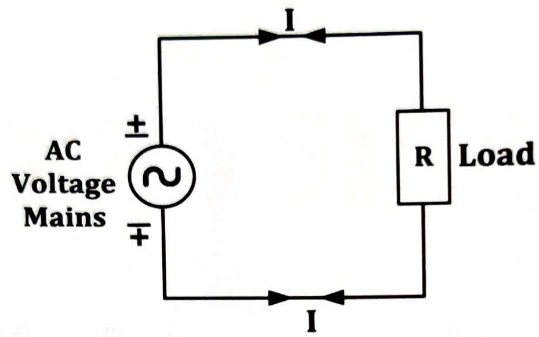
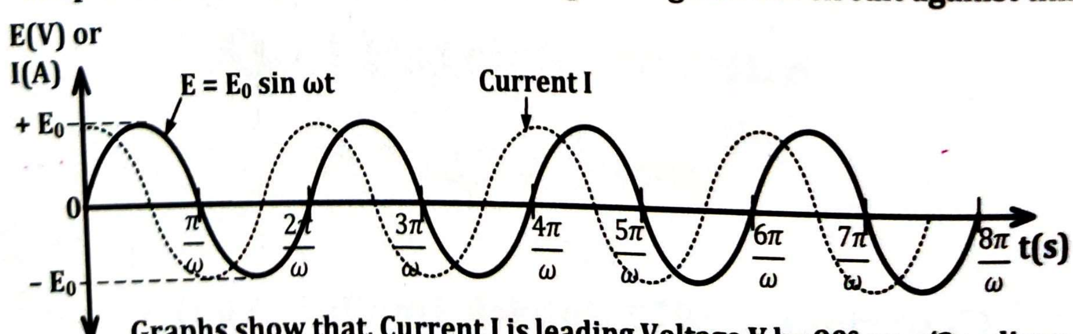


Fig. 4.7 (a)

The number of times this current changes its direction in one second is defined as the **frequency** of alternating current.
In this current, the charged particles move from zero to maximum value in one direction and fall to zero and goes for another cycle in the opposite direction with the same values (in negative direction).
The values processed by alternating current in both directions are equal at all instances.
Generally A.C. waveform is represented by a sinusoidal wave form and given by equation, $y = a \sin 2\pi f t$
When one waveform having the same frequency and wavelength is compared to another having the same reference point/position or when a position of one point on the wave form is compared to another point on the same waveform, one may lead or lag behind the other by a phase angle ϕ. Such a relation may be represented by the general equation, $y = a \sin(2\pi f t \pm \phi)$
where ϕ is the **phase angle** between the points or waves.

Graphs of e.m.f. E, or Current, I flowing through an a.c. circuit against time



Graphs show that, Current I is leading Voltage V by 90° or π/2 radians

Fig. 4.7 (b)

When a source of e.m.f. has its polarity charging with time, it is known as an alternating e.m.f. e.g. that produced by an A.C generator.

Thus, the current that such an e.m.f. source causes to flow through a circuit also changes its direction repeatedly with time and is called an <u>alternating current</u> (A.C).

Definition:

An alternating current or voltage is one whose polarity varies periodically with time, both in magnitude and direction.

Waveforms that represent alternating currents include the following:

(a) *Sinusoidal a.c. waveform*

Graphs of e.m.f. E, or Current, I flowing through an a.c. circuit with time, t.

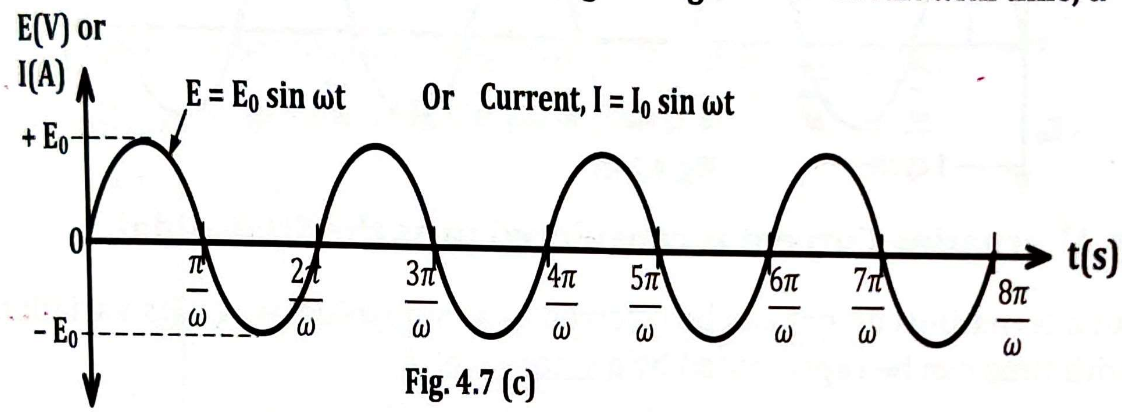


Fig. 4.7 (c)

(b) *Square – wave A.C. wave form*

A Graph of e.m.f. E, or Current, I flowing through an a.c. circuit with time, t.

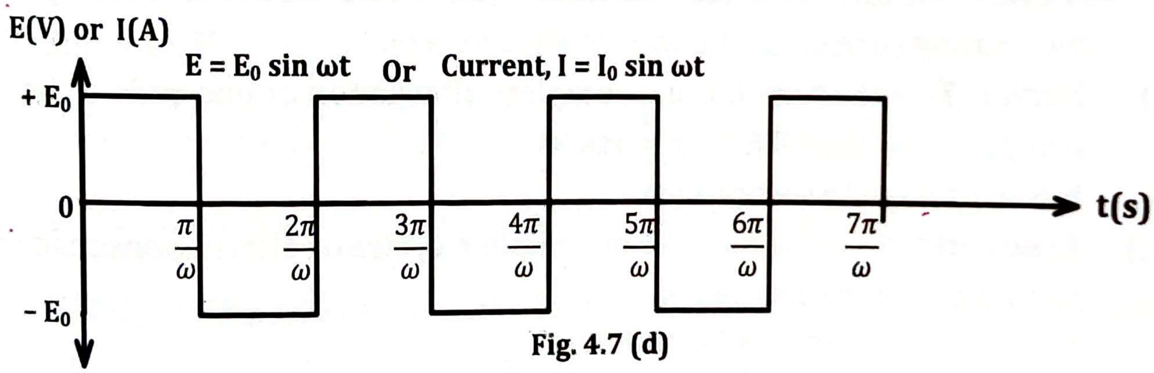


Fig. 4.7 (d)

(c) *Saw-tooth A.C. waveform*

A Graph of e.m.f. E, or Current, I flowing through an a.c. circuit with time, t.

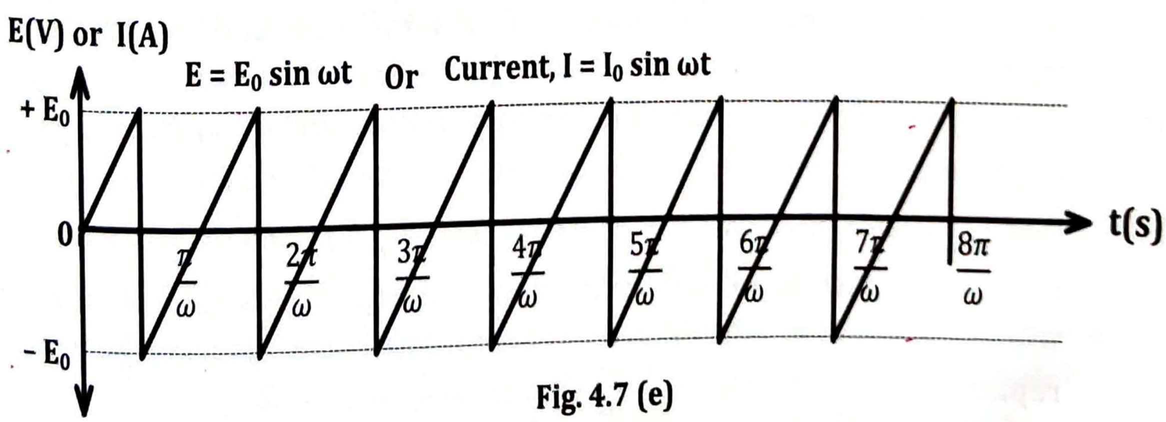


Fig. 4.7 (e)

Other waveforms are considered as a combination of many waveforms of either one of the above or a combination of more than one type of the above waveforms.
The simplest and the most commonly applicable and important waveform is a sinusoidal waveform.

A Graph of e.m.f. E, or Current, I flowing through an a.c. circuit with time, t.

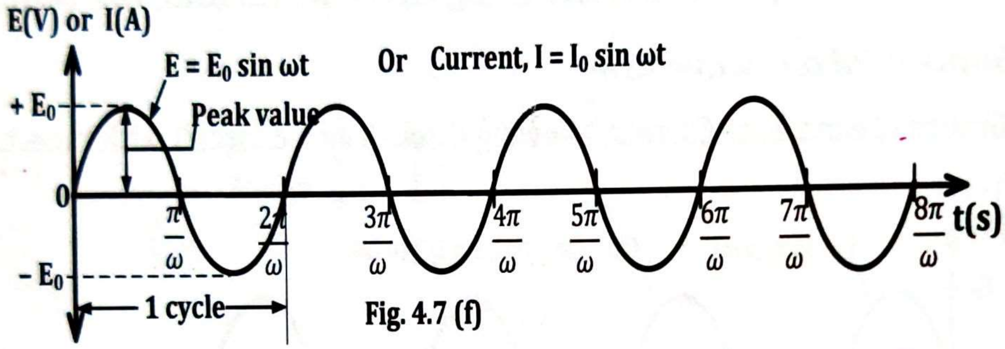


Fig. 4.7 (f)

Why Alternating Current is considered to as the Sinusoidal.

An alternating current can be referred to as sinusoidal when its variations with time can be represented by a sine wave.

Definitions of the terms used in alternating current:

(i) ***A cycle:*** - is one complete alternation (**back and forth** movement) of the alternating current particles in the waveform.

(ii) ***Period T***- is the time for one complete alternation or one cycle to be completed by the vibrating particle.
It is expressed in seconds (s).

(iii) ***Frequency f*** - is the number of complete cycles or alternations made by the wave particle in one second.
S.I Unit of frequency is **hertz** (Hz).

(iv) ***Amplitude or Peak value of a.c. voltage or current, V_0 or I_0*** **– is the maximum value** of the alternating e.m.f./voltage or current respectively taken over one complete cycle. i.e. it is numerically equal to the distance between the average value of a given a.c. wave form and its crest or trough. It represents the amplitude.

(v) ***Average or Mean value of alternating current or voltage.***
Is the average of all the instantaneous values of an alternating current or voltage taken over one complete cycle. i. e. $\sum_1^n I_n = \left(\frac{I_1+I_2+I_3+...+I_n}{n}\right)$
However, if current or voltage is taken as a continuous variable function f(t) which is a function of time over one complete cycle, It's average value is represented as; $\langle f(t)\rangle_T = \frac{1}{T}\int_0^T f(t)dt$ $where, T = \frac{2\pi}{\omega}$

Examples

(a) Suppose, $I = I_0 \sin \omega t \Rightarrow \langle I(t)\rangle_T = \frac{1}{T}\int_0^T I(t)dt$ *where*, $T = \frac{2\pi}{\omega}$

$$\Rightarrow \langle I(t)\rangle_T = I_0\left(\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \sin \omega t\, dt\right) = \frac{I_0\omega}{2\pi}\left[\left(-\frac{1}{\omega}\cos \omega t\right)\right]_0^{\frac{2\pi}{\omega}}$$

$$= \frac{I_0\omega}{2\pi}\left(-\frac{1}{\omega}\cos 2\pi - \frac{-1}{\omega}\cos 0\right) = \frac{I_0\omega}{2\pi}\left(-\frac{1}{\omega}\times 1 + \frac{1}{\omega}\times 1\right) = 0$$

$\therefore$ $\boldsymbol{\langle I(t)\rangle_T = \langle I_0 \sin \omega t\rangle_T = 0}$ Similarly, $\boldsymbol{\langle V(t)\rangle_T = \langle V_0 \sin \omega t\rangle_T = 0}$

This is equivalent to finding the area under the graph of $I_0 \sin \omega t$ against time, t, as a function of the angles in radians over one cycle.

Similarly, $\boldsymbol{\langle I_0 \cos \omega t\rangle_T = \langle V_0 \cos \omega t\rangle_T = 0}$

This is illustrated by the graphs in figure 4.7 (g)(i).

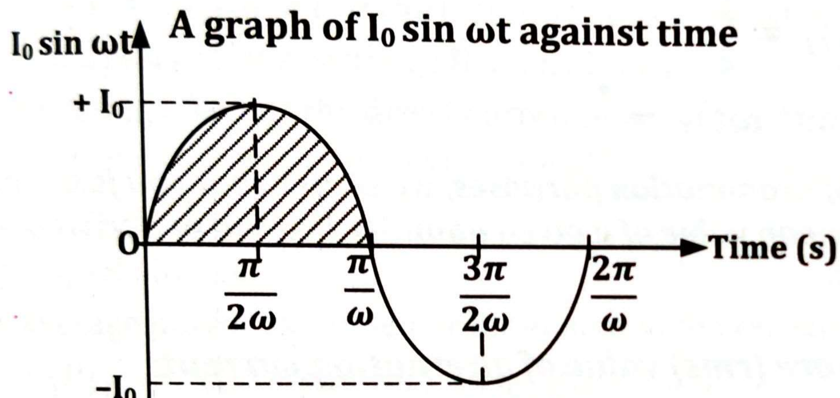


Fig. 4.7 (g)(i)

(b) Suppose, $V = V_0 \cos \omega t \Rightarrow \langle V(t)\rangle_T = \frac{1}{T}\int_0^T V(t)dt$ *where*, $T = \frac{2\pi}{\omega}$

$$\Rightarrow \langle V(t)\rangle_T = V_0\left(\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \cos \omega t\, dt\right) = \frac{I_0\omega}{2\pi}\left[\left(\frac{1}{\omega}\sin \omega t\right)\right]_0^{\frac{2\pi}{\omega}}$$

$$= \frac{I_0\omega}{2\pi}\left(\frac{1}{\omega}\sin 2\pi - \frac{1}{\omega}\sin 0\right) = \frac{I_0\omega}{2\pi}\left(\frac{1}{\omega}\times 0 - \frac{1}{\omega}\times 0\right) = 0$$

$\therefore$ $\boldsymbol{\langle I(t)\rangle_T = \langle V_0 \cos \omega t\rangle_T = 0}$ Similarly, $\boldsymbol{\langle V(t)\rangle_T = \langle V_0 \cos \omega t\rangle_T = 0}$

This is equivalent to finding the area under the graph of $V_0 \cos \omega t$ against time, t as a function of the angles in radians over one cycle.

Similarly, $\boldsymbol{\langle I_0 \cos \omega t\rangle_T = \langle I_0 \cos \omega t\rangle_T = 0}$

This is illustrated by the graphs in figure 4.7 (g)(ii) below.

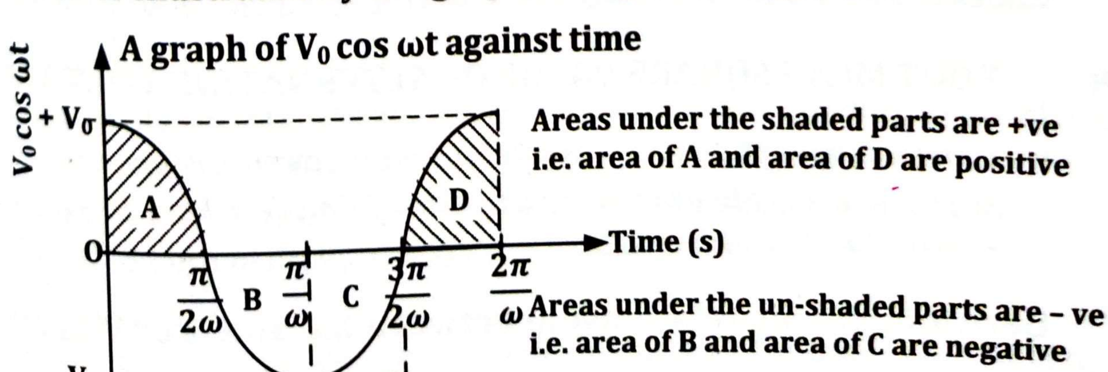


Fig. 4.7 (g)(ii)

Total area = (A + B + C + D) = 0

(c) Suppose, $I^2 = I_0{}^2 \sin^2 \omega t \Rightarrow \langle I^2(t)\rangle_T = \frac{1}{T}\int_0^T I^2(t)dt$ *where*, $T = \frac{2\pi}{\omega}$

From the trigonometric identity, $\cos 2\omega t = 1 - 2\sin^2 \omega t$

$\Rightarrow \sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t)$

$\therefore \langle \sin^2 \omega t\rangle_T = \langle \frac{1}{2}(1 - \cos 2\omega t)\rangle_T = \frac{1}{2} - \frac{1}{2}\langle \cos 2\omega t\rangle_T$ where,

$$\Rightarrow \langle \cos 2\omega t\rangle_T = \left(\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \cos \omega t\, dt\right) = \frac{\omega}{2\pi}\left[\left(-\frac{1}{\omega}\sin \omega t\right)\right]_0^{\frac{2\pi}{\omega}}$$

$$= \frac{\omega}{2\pi}\left(-\frac{1}{\omega}\sin 2\pi - \frac{-1}{\omega}\sin 0\right) = \frac{\omega}{2\pi}\left(-\frac{1}{\omega}\times 0 + \frac{1}{\omega}\times 0\right) = 0$$

$$\langle \sin^2 \omega t\rangle_T = \langle \frac{1}{2}(1 - \cos 2\omega t)\rangle_T = \frac{1}{2} - \frac{1}{2}\langle \cos 2\omega t\rangle_T = \left(\frac{1}{2} - 0\right) = \frac{1}{2}$$

$$\therefore \langle \sin^2 \omega t\rangle_T = \frac{1}{2}$$

Similarly, $\langle \cos^2 \omega t\rangle_T = \frac{1}{2}$

NB: In case of examination purposes, we prefer that you just state, final value of the mean value of a given quantity instead of deriving it from 1st principles.

(vi) ***Root mean square (rms) value of alternating current:***

Definition

The root mean square value – of an alternating current (also called ***effective value***) – is the steady or direct current that converts electrical energy to other forms of energy in a given resistance at the same rate as the alternating current.

Alternatively:

The root mean square value of an A.C – is the value of steady current which dissipates heat in a given resistor at the same rate as the a.c under the same conditions.

Note that,

The Room Mean Square Value (rms) – is the square root of the mean of square values of the current or voltage taken over one whole cycle of the a.c. input.

4.8 ROOT MEAN SQUARE VALUE OF ALTERNATING VOLTAGE, (V_{RMS})

- *Is the steady voltage which applied across a given resistance, would cause electrical energy to be converted to other forms (e.g. heat energy) at the same rate as the alternating voltage.*

Derivation of the relationship between I_{rms} and I_0 or V_{rms} and V_0

Consider two resistors of the same resistance, R, connected respectively to an a.c. source and a d.c. source. The current in the d.c. circuit is then adjusted until it *produces the same desired output (e.g. light or heating effect) in the same value of the resistance, R, as in the case alternating current, then, this value of direct current is termed as the root mean square value (I_{rms}) of the alternating current.*

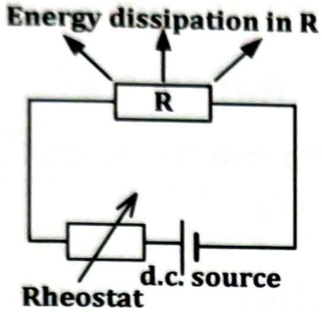


Fig. 4.8 (a)(i)

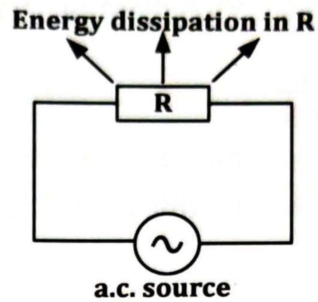


Fig. 4.8 (a)(ii)

Suppose $\boldsymbol{I = I_0 \sin \omega t}$ is the instantaneous alternating current flowing through a resistor of resistance **R** at any time for a time, **t** and produced the same heating effect as the direct current, $\boldsymbol{I_d}$

Instantaneous, alternating power expended, in the resistance, R

$$P = I^2R = (I_0 \sin \omega t)^2 R$$

$$P = I_0^{\,2} R \sin^2 \omega t$$

The average power expended or dissipated in the resistor over one complete cycle, $\langle P \rangle_T = \langle I_0^{\,2} R \sin^2 \omega t \rangle = I_0^{\,2} R \langle \sin^2 \omega t \rangle$

But, $\langle \sin^2 \omega t \rangle_T = \frac{1}{2}$

$$\therefore \langle P \rangle_T = \frac{1}{2}\left(I_0^{\,2} R\right) = \frac{I_0^{\,2} R}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

When the same output, due to the d.c. is achieved in the resistance, R, then the d.c. power dissipated in the same resistance is given by

$$\therefore P_d = I_d^{\,2} R \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Since the same power output is achieved in both circuits having the same value of the resistance, R, the two power outputs are now equal.

$\Rightarrow \boldsymbol{P_d = \langle P \rangle_T}$ *and this implies,*

$$\therefore I_d^{\,2} R = \frac{I_0^{\,2} R}{2} \Rightarrow \sqrt{I_d^{\,2}} = \sqrt{\frac{I_0^{\,2}}{2}} \quad \text{But, } \sqrt{I_d^{\,2}} = I_{rms}$$

$$\therefore I_{rms} = \frac{I_0}{\sqrt{2}} = 0.7071\, I_0$$

Alternatively

Since alternating current and voltage in a resistor in a resistor, R, are in phase

Given that current, $\boldsymbol{I = I_0 \sin \omega t}$ and voltage, $\boldsymbol{V = V_0 \sin \omega t}$

Instantaneous, alternating power expended, in the resistance, R, in 1 cycle

$$P = IV = (I_0 \sin \omega t)(V_0 \sin \omega t)$$

$$P = I_0 V_0 \sin^2 \omega t$$

The average power expended or dissipated in the resistor over one complete cycle, $\langle P \rangle_T = \langle I_0 V_0 \sin^2 \omega t \rangle = I_0 V_0 \langle \sin^2 \omega t \rangle$ But, $\langle \sin^2 \omega t \rangle_T = \frac{1}{2}$

$$\therefore \langle P\rangle_T = \frac{1}{2}(I_0V_0) = \frac{I_0V_0}{2} \text{ but}, V_0 = I_0R$$

$$\therefore \langle P\rangle_T = \frac{I_0{}^2R}{2} \text{ or using, } I_0 = \frac{V_0}{R}$$

$$\therefore \langle P\rangle_T = \frac{V_0{}^2}{2R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

When the power output, due to the d.c. is achieved across the resistance, R, then the d.c. power expended in the same resistance is given by

$$\therefore P_d = \frac{V_d{}^2}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (iv)$$

*Since the same power output is achieved in both circuits having the same value of the resistance, **R**, the two power outputs are now equal.*

$\Rightarrow \boldsymbol{P_d = \langle P\rangle_T}$ *and this implies,*

$$\therefore \frac{V_d{}^2}{R} = \frac{V_0{}^2}{2R} \Rightarrow \sqrt{V_d{}^2} = \sqrt{\frac{V_0{}^2}{2}} \quad \text{But, } \sqrt{V_d{}^2} = V_{rms}$$

$$\therefore V_{rms} = \frac{V_0}{\sqrt{2}} = 0.7071\, V_0$$

Graphs of $\sin\omega t$, $\sin^2 \omega t$ and $\langle \sin^2 \omega t\rangle$

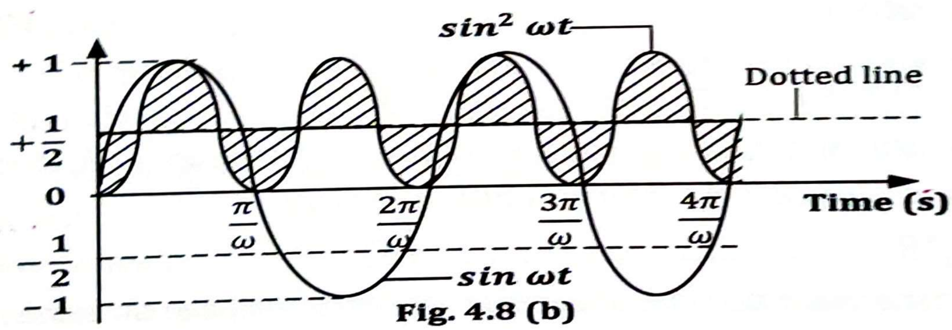


Fig. 4.8 (b)

Notes (i) From the graphs in figure 4.8 (b), it can be seen that, all the values of $\sin^2 \omega t$ *are all positive and lie between* 0 *and* 1.

(ii) The shaded areas above and below the dotted line are equal since a sine curve (wave) is symmetrical about that line, thus the mean value of $\sin^2 \omega t$ ***is therefore half. i.e.*** $\langle \sin^2 \omega t\rangle_T = \frac{1}{2}$

(iii) This implies therefore, $\therefore I_{rms} = \frac{I_0}{\sqrt{2}} = \mathbf{0.7071}\, I_0$

(iv) Root mean square values – can be treated like direct current (d.c) values for finding the average power dissipated in a resistor. i.e.

$$\therefore \langle P\rangle_T = I_{rms}V_{rms} = I_{rms}{}^2R = \frac{1}{2}(I_0V_0) = \frac{V_0{}^2}{2R} = \frac{I_0{}^2R}{2}$$

(v) The numerical values of alternating currents and voltages used and often quoted are root mean square values, unless specifically quoted as instantaneous or peak values, since they represent a steady values of these currents and voltages. For example voltages used in domestic consumption of electricity supply. i.e. V = 240 volts represents the root mean square voltage.

$\Rightarrow \boldsymbol{V_{rms} = 240\ V}$ *thus,* $\boldsymbol{Peak\ voltage\ V_0 = V_{rms}\sqrt{2}}$

$\therefore \boldsymbol{V_0 = 240\sqrt{2} = 339\ V}$

Examples

1. An alternating current, $I = 2.0\ sin\ 100\pi t$ is passed through an electric kettle having a coil of resistance 5.0 Ω.
Determine the;
(i) Peak value of the current
(ii) Root mean square value of the current.
(iii) Frequency of the alternating current.
(iv) The power rating of the kettle.
(v) The maximum power expended in the kettle.

Solutions

(i) Compare $I = 2.0\ sin\ 100\pi t$ with the general equation of alternating current, $I = I_0 \sin \omega t = I_0 \sin 2\pi f\ t$

$\Rightarrow \boldsymbol{Peak\ value, I_0 = 2.0\ A}$

(ii) Using, $I_{rms} = \frac{I_0}{\sqrt{2}} = 0.7071\ I_0 = 0.7071 \times 2.0$

$\Rightarrow \boldsymbol{Root\ mean\ square\ value, I_{rms} = 1.4142\ A}$

(iii) Comparing, $2\pi f$ *to* $100\pi \Rightarrow 2f = 100$

$\therefore \boldsymbol{Frequency, f = 50\ Hz}$

(iv) Average power expended in the kettle, $\langle \boldsymbol{P} \rangle_{\boldsymbol{T}} = \frac{\boldsymbol{I_0{}^2 R}}{\boldsymbol{2}}$

$\therefore \langle \boldsymbol{P} \rangle_{\boldsymbol{T}} = \frac{(2.0)^2 \times 5.0}{2} = \boldsymbol{10.0\ W}$

(v) Maximum power, $P_{max} = I_0{}^2 R, since\ maximum\ of\ \sin^2 \omega t = 1$

$P_{max} = I_0{}^2 R = (2.0)^2 \times 5.0$

$\therefore \boldsymbol{P_{max} = 20.0\ W}$

2. An electric bulb having a rating of 100 W is connected to an a.c voltage of 120 V (rms). Determine the;
(a) Resistance of the electric light bulb filament at normal operating temperature.
(b) Root mean square value and peak current through the filament.
(c) Comment on the size of the power of the filament lamp when cold and the switch is just closed in relation to the filament's average working power of 100 W.

Solutions

(a) Since the average power rating of the lamp =100 W, and when connected to the d.c. supply, it would dissipate the same power output.

i.e. $\therefore P_d = \frac{V_d^{\,2}}{R} \Rightarrow Reistaance, R = \frac{V_d^{\,2}}{P_d} \quad \therefore R = \frac{(120)^2}{100}$

$\therefore R = 144\ \Omega$

(b) Using, average power, $\langle P \rangle = I_{rms}V_{rms} \Rightarrow I_{rms} = \frac{\langle P \rangle}{V_{rms}}$

(c) $\therefore I_{rms} = \frac{\langle P \rangle}{V_{rms}} = \frac{100}{120} = 0.833\ A$

Thus the peak value or the amplitude of the a.c., $I_0 = I_{rms}\sqrt{2}$

$\therefore$ *Peak current,* $I_0 = 0.833\sqrt{2} = \mathbf{1.18\ A}$

(d) From the expression for power, $P = \frac{V_{rms}^{\,2}}{R} \Rightarrow P \propto \frac{1}{R}$ *when the p.d.* Is kept constant.

For metals, the resistance increases with increase in temperature of the metal wire. Thus when the filament is cold when the switch is just closed, its resistance is smaller and the average power dissipated in the filament would be larger, than when the filament heats up, since both are connected across the same p.d.

4.9 MEASUREMENT OF ALTERNATING CURRENT

Unlike direct currents (d.c), whose deflection of the meter's pointer depend on the direction of flow of current through the instrument, the deflection of the pointers of a.c. meters must not depend on the direction of the current through the instrument but on either the repulsion or attraction of magnetic iron rods or on the heating effect caused by the current, on a resistance wire used in the set up.

This explains why a moving coil galvanometer pointer just vibrates about the zero position when an alternating current is passed through it, and so cannot be used for the measurement of alternating currents.

A.C. METERS (Alternating Current Measuring Instruments)

Instruments used for a.c. measurements include the following:

1. Hot – wire ammeter
2. Thermocouple meter
3. Moving iron-meter
   (a) Attraction type
   (b) Repulsion type
4. Rectifier meter

1. The hot – wire ammeter

The structure / Labelled diagram of the hot wire ammeter:

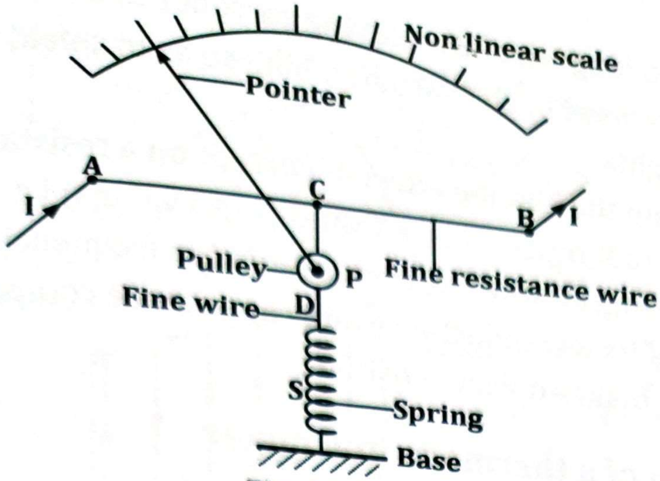


Fig. 4.9 (a)

- Current, **I** to be measured is passed through the fine resistance wire **AB**.
- The wire gets heated up, it expands and sags.
- The sag is then picked up by the second fine wire **CD** that passes round a grooved pulley **P** and attached to the pointer.
- The pulley rotates and causes the pointer to rotate as it moves over the scale, Until it is stopped by the controlling torque provided by a pair of hair springs wen the pointer is deflected through angle θ.
- The deflection, **θ** is proportional to the sag, and is therefore proportional to the square of the average or mean current. i.e. $\theta \propto \langle I^2 \rangle$
- Hence, the instrument has a non-linear or square scale.

NB: For convenience of taking readings on the scale, is calibrated to read I_{rms} values where, $I_{rms} \propto sag$. The graduations on the scale are crowded together for small values r.m.s values of current but nearly equally spaced for large values of current.

2. The thermocouple – meter

The structure / Labelled diagram of the thermocouple meter:

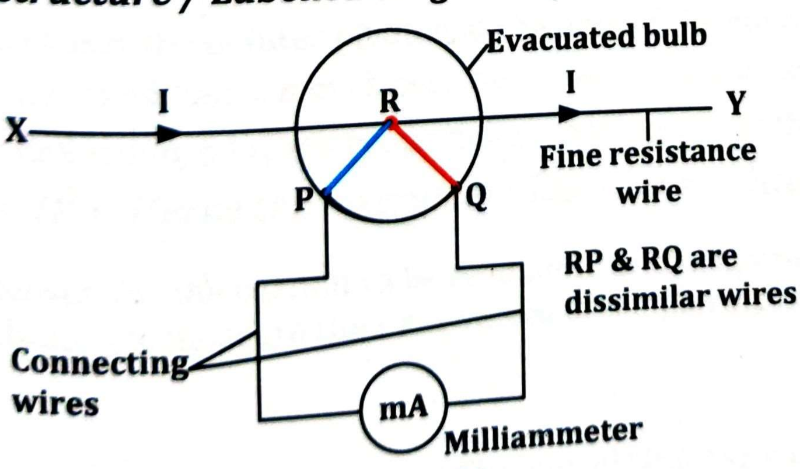


Fig. 4.9 (b)

- P and Q are wires made up of two ***different metals*** joined together at **R** so as

to form a hot junction a long wire **XY**, carrying current **I** to be measured.
The arrangement is enclosed in an evacuated bulb so as to shield off the hot Junction, **R**, from draughts.
The instrument relies on the heating effect of current on a resistance wire XY. It therefore, measures root mean square values (r.m.s values) i.e. (d.c) and can also be used for measuring alternating currents of high frequencies of ***several maga-hertz*** because of its low inductance and capacitance compared with other meters, this thus makes it very sensitive.

The mode of Operation of a thermocouple meter

Current, **I** being measured is passed through the fine ***resistance wire XY*** and warms it up.
Contact **R** at the centre of the bulb and shielded off from draughts acts as the ***hot junction*** with junctions **P** and **Q** made up of two ***dissimilar wires*** acting as ***cold junctions***.
A temperature gradient is then set up between **R**, and the other two junctions **P** and **Q** thus causing a thermo-electric e.m.f. to be produced that in turn causes a thermo-electric current, (***the root mean square current, I_{rms}***) to flow through the **mA** or the **μA** connected in series with the set up already calibrated to measure direct current.
A current through the meter causes a deflection, **θ** which is proportional to the the root mean square current (I_{rms})
The thermocouple meter has ***high sensitivity*** because of its low inductance and capacitance.

The moving Iron – ammeter (Attraction type

The structure/diagram of an attraction type of moving iron ammeter.

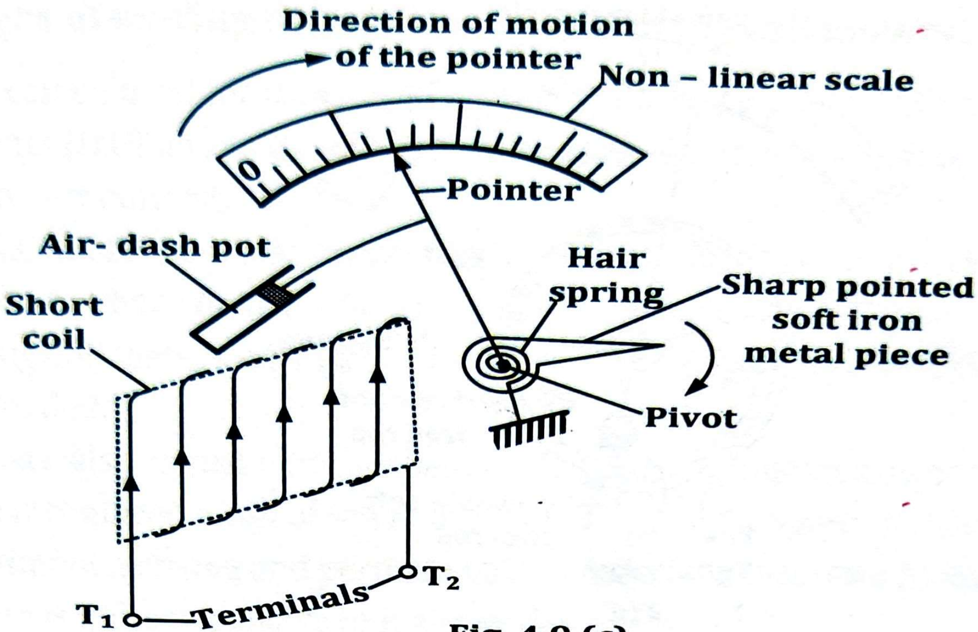


Fig. 4.9 (c)

- The instrument consists of a specially shaped piece of soft iron metal with a pointer attached to it and pivoted near the open end of a short coil (solenoid) whose terminals T_1 and T_2 are connected to an a.c. source.
- At the base of a specially designed soft iron metal piece is attached a pair of hairsprings that provide the restoring couple of forces.
- The air dash pot provides the necessary damping so that the movement of the pointer is not erratic, but smooth.

The mode of operation of the thermocouple meter.

- Current, **I** is fed into the coil via terminals T_1 and T_2, creating a magnetic field at the centre of the coil with the ends of the coils being the North and South poles depending on the direction of flow of current in the coil.
- The sharp pointed soft iron metal piece pivoted along the axis of the coil, just beyond the open end of the coil then gets attracted towards the coil, with a magnetic force that is directly proportional to the square of the current, I flowing through the coil.
- The sharpest point of the Iron piece has the greatest attraction for magnetic field at the open end of the coil.
- This causes the pointer pivoted at the base of the metal piece to rotate about the pivot and hence moves over the scale through an angle of deflection, θ.
- The deflection, θ is then proportional to the mean of the square current i.e. $\theta \propto \langle I^2 \rangle$. *Hence the instrument has* ***a non – linear scale***.

NB, However, for the current to be read directly in amperes, the scale has to be calibrated to measure the r.m.s values of the current.

4. **The moving Iron Ammeter. (The repulsion type).**

The structure / Labelled diagram of a moving Iron meter

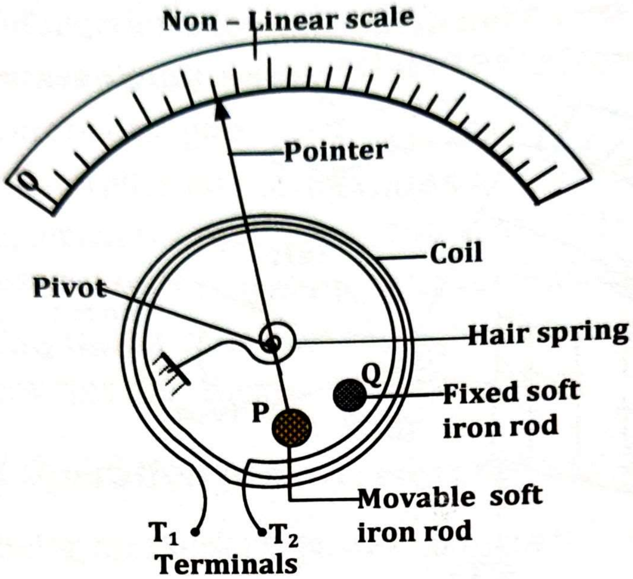


Fig. 4.9 (d)

- The instrument consists of two iron rods **P** and **Q** surrounded by a coil of insulated copper wire carrying a current to be measured.
- The coil and one of the soft iron rods, **Q** are fixed to the frame of the meter, while the movable soft iron rod **P**, is attached to a pointer pivoted at the centre, along the axis of the coil.
- The other rod **P** is attached to an axle which also carries a pointer whose motion alongside a non-linear scale is controlled by a pair of hair springs provide the opposing restoring torque and control the rotation of the pointer.
- An air – dashpot attached to the pointer provides an additional air resistance to the motion of the pointer, hence damping its motion.

The mode of operation of a repulsion type of moving iron meter.

- Current, **I** is fed into the coil via terminals T_1 and T_2, creating a magnetic field at the centre of the coil.
- The two soft iron rods **P** and **Q** get *magnetized in the same sense* irrespective of the direction of flow of current in the coil and begin to *repel each other with an average force* which is proportional to the square of the current flowing through the coil.
- The fixed soft iron rod **Q** repels rod **P**, causing it to rotate about the pivot and moves over the scale through an angle **θ**, until it is stopped by the restoring torque due to the couple provided by a pair of hair springs.
- The deflection, **θ** produced is proportional to the average of the square current.

  i.e. $\theta \propto \langle I^2 \rangle$. *Hence the instrument has a non – linear scale.*

Advantages of moving iron meters over moving coil meters

(i) They can be used for measuring both alternating currents (A.C) and direct currents (D.C) unlike the moving coil meters that can be used for measuring only direct currents.

(ii) The A.C. meters are also more durable than the moving coil meters, because they do not have delicate coils that can easily be blown off like those of moving coil meters when a large current is passed through them i.e. when overloaded.

(iii) They are also robust and cheaper to construct or manufacture and purchase, Since it required a coil of fewer number of turns compared to moving coil instrument a strong and permanent U – shaped magnet used in moving coil meters is quite expensive to make or design.

(iv) The A.C. meters can be adapted to measure large currents and voltages even at ***high frequencies*** for the case of a.c. by connecting appropriate shunts and multipliers respectively to the meter.

(v) Less friction error – this is very small in moving iron instruments because their torque to weight ratio is high since their current carrying part stationary and the moving parts are lighter in weight.

Disadvantage of moving iron – meters.

(i) Like most A.C meters, moving iron meters have uneven or non – uniform scales. Thus each current being measured has to be read directly from the instrument itself, making it less accurate.

(ii) Errors introduced because of hysteresis frequency and stray magnetic fields compromise the accuracy of the instrument.

(iii) Waveform error. This is because the deflection torque is not directly proportional to the square of current for all the current values as is the case with moving coil meters, this causes the waveform errors.

(iv) Since moving iron instruments measure both A.C. and D.C. the calibrations of the A.C. and D.C are different because of the effect of inductance of the meter and eddy currents in the meter which is used on A.C.

Differences between moving iron meters and moving coil meters.

(i) The moving iron instrument (MII) is used for measuring for measuring both A.C. and D.C. whereas the moving coil instrument (MCI) is used for measuring only D.C.

(ii) The moving iron instrument (MII) has a non – uniform scale whereas the moving coil instrument (MCI) has a uniform scale.

(iii) The moving iron instrument (MII) is less accurate compared to the moving coil instrument (MCI).

(iv) The moving iron instrument (MII) can be used as an ammeter, voltmeter and a wattmeter whereas the moving coil instrument (MCI) can be used as an ammeter, a voltmeter, galvanometer and ohmmeter.

(v) The moving iron instrument (MII) is free from hysteresis loss whereas the moving coil instrument (MCI) present hysteresis loss as a possible energy loss.

(vi) In moving iron instrument (MII) gravity or the spring provides the controlling torque whereas in the moving coil instrument (MCI) only te spring provides the controlling torque to the instrument.

(vii) The moving iron instrument (MII) the deflection of the pointer is proportional to the square of the current whereas in the moving coil instrument (MCI) is proportional to the current.

(viii) The moving iron instrument (MII) consume more power than the moving coil instrument (MCI) that consume less power.

(ix) The moving iron instrument (MII) uses the soft iron pieces as a moving element whereas the moving coil instrument (MCI) uses the coil as the moving element.

(x) The working principle of the (MII) is also dependent on air friction whereas the working principle of a moving coil instrument (MCI) is dependent on eddy currents damping.

The Rectifier Meter

A Rectifier – is a device used for converting alternating current to direct current. It has a low resistance to the flow of current in one direction and a high resistance to the flow of current in the reverse (or opposite) direction.

Thus, when a rectifier is connected to an a.c. supply, it allows pulses of varying but direct current to pass it.

A moving coil meter is connected in series with a rectifier and a source of A.C as shown in the figure 4.9 (e).

The structure / Labelled diagram of the Rectifier meter

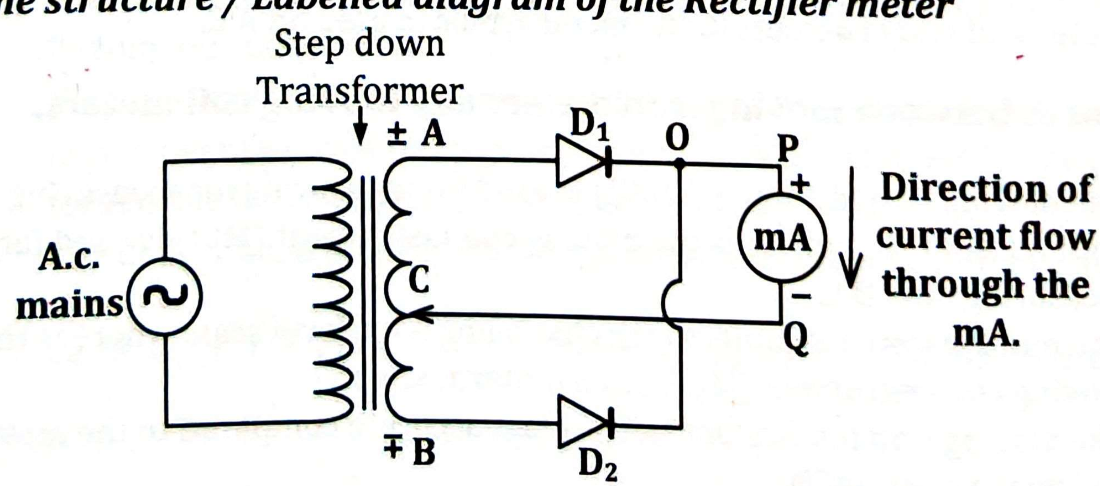


Fig. 4.9 (e)

- When terminal **A** of the a.c. power input is positive with respect to **B**, the diode **(rectifier) D_1** conducts current while **D_2** does not.
- Current flows from A through diode D_1, to P through the milliammeter (***mA***) in the direction PQ to C and back to A.
- Current flows from A through **D_1**, to P through resistor the ***mA*** to Q in the direction PQ causing a deflection on the scale of the mA then a current pulse flows to C and back to A.
- The first half of the a.c input cycle is then rectified.
- During the change of current polarity when terminal **A** is negative with respect to terminal positive terminal **B**, current drops to zero in the mA.
- When **B** is positive with respect to **A**, diode **D_2** conducts current while D_1 does not. Current flows from B through D_2, to O to P , through the mA then to Q in the direction PQ causing a deflection on the scale of the mA due to a current pulse in it and flows to C and back to B.
- Hence, the second half cycle of the a.c input is now also rectified.
- The process repeats itself several times per second for all the subsequent cycles producing direct current flow through the mA of output shown below.

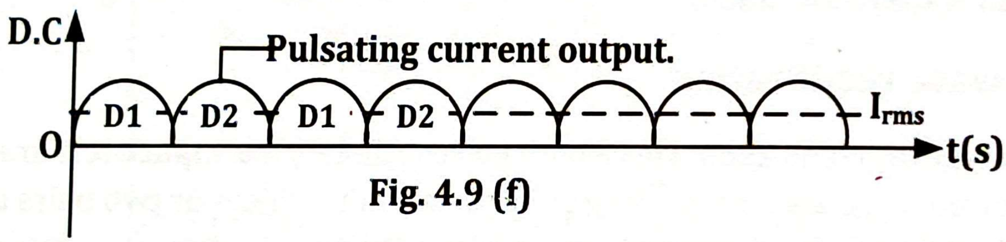


Fig. 4.9 (f)

The output of the A.C input signal pulsates as it passes through the moving coil meter, which however measures the average value of these pulses known as root mean square current (I_{rms}).

NB Rectifier instruments based on the moving coil meter, are more sensitive than A.C meters and are used in multi-meters that measure both a.c and d.c and have their corresponding ranges marked accordingly.

The scale of a rectifier meter is calibrated to read r.m.s values of currents and p.d values directly with sinusoidal waveforms.

5.0 RECTIFICATION OF ALTERNATING CURRENT

Definition

Rectification – is a process of converted alternating current (A.C) into direct current (D.C) using **a rectifier** (or Diode).

Rectifiers

A rectifier – is a devide that allows current to flow through it and the circuit to which its connected in only one direction, known as the ***forward bias*** direction and blocks it for the flow of current in the opposite direction called the ***reverse bias*** direction.

Symbols of the rectifier

Figure 5.0 (a) shows rectifiers in two symbols connected in the forward and reverse bias settings.

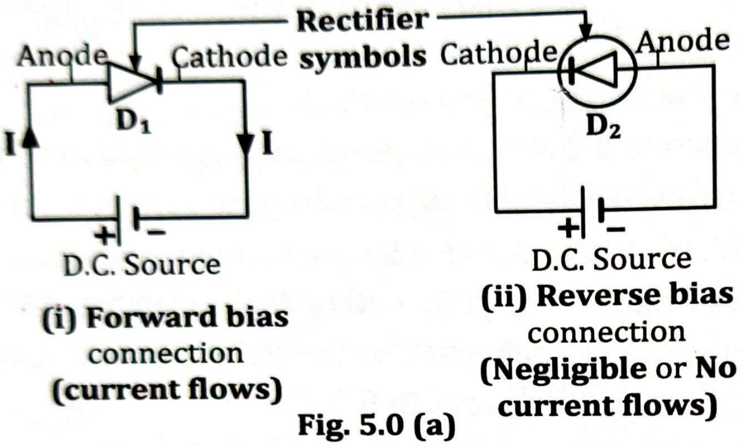


Fig. 5.0 (a)

There are two types of rectification namely

1. Full wave rectification
2. Half wave rectification.

Full wave rectification

Is the type of rectification where both halves of every a.c. input cycle are converted to d.c. each time, using either a pair of rectifiers or two pairs of rectifiers (4 rectifiers) arranged in a bridge network.

(a) The Full wave bridge Rectifier

This involves the setup of four rectifiers or diodes in a bridge network as shown in figure 5.0 (b).

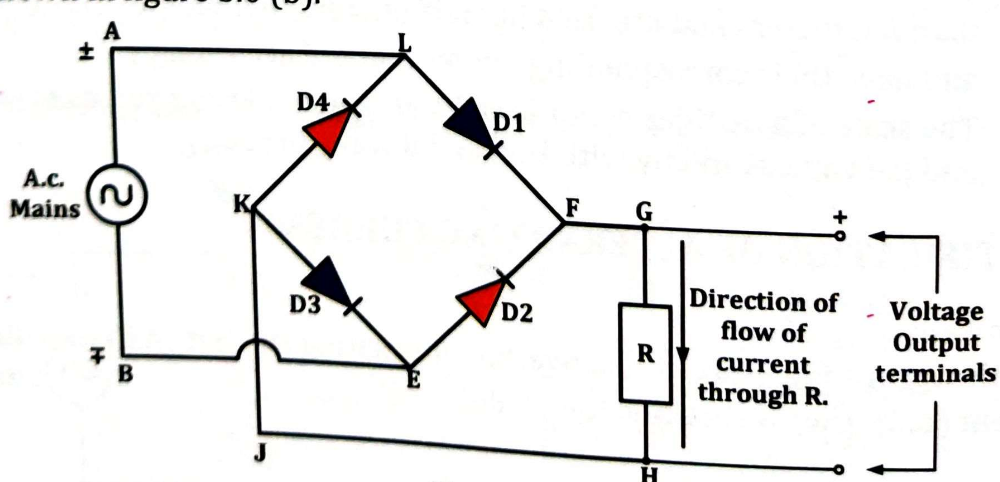


Fig. 5.0 (b)

- Using the set up shown in fig. 5.0 (b), when terminal **A** of the a.c input is positive with respect to B, diodes (rectifiers) **D₁ and D₃** conduct current since they are forward biased while D_2 and D_4 do not conduct, because they are reverse biased.

- Current I, flows from A, to L, through D_1, to F to G and passes through resistor R in the direction GH then from H to J, to K, through D_3 to E to B and back to A. The first half of the a.c input cycle is then rectified.
- When terminal B is positive with respect to A , rectifiers D_2 and D_4 conducts current while D_1 and D_3 do not conduct (i.e. they are reverse biased). Current flows from the source (B) through D_2 to F, then to H through resistor R in the same direction as before, then from H to J to K through D_4 to L to A and back to B.
- Thus the second half cycle of the a.c input is also now rectified.
- The process repeats itself several times per second for all the subsequent cycles producing direct current flow through the load resistor R but with pulsating output shown in figure 5.0(c) (ii) in comparison with the un rectified a.c. voltage input in figure 5.0 (c)(i).

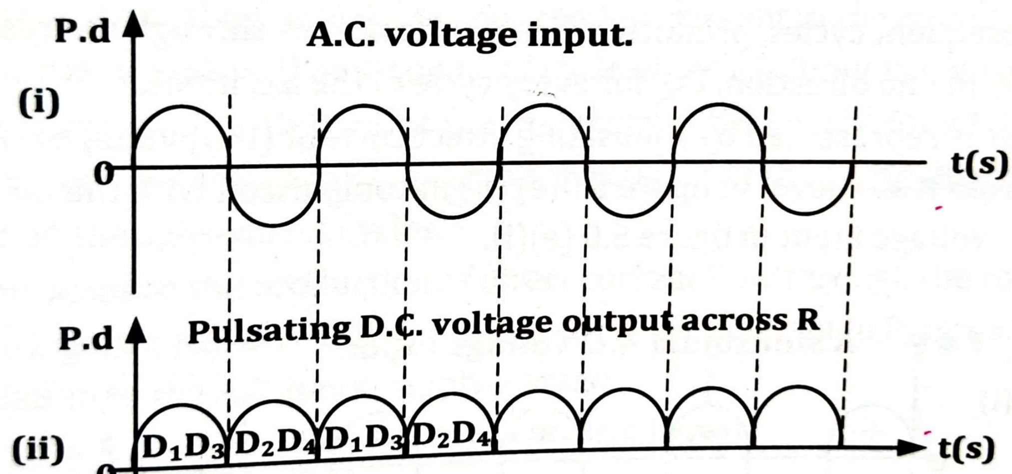


Fig. 5.0 (c)

NB: In the pulsating d.c. output D_1, D_3 and D_2,D_4 on the diagram shows the forward biased (conducting) pair of rectifiers (Diodes) at that moment.

(b) Full wave rectification using two Rectifiers (Diodes)

Diagram of the two diode full wave rectifier

The set up in figure 5.0 (d) uses a centre – tap step down transformer

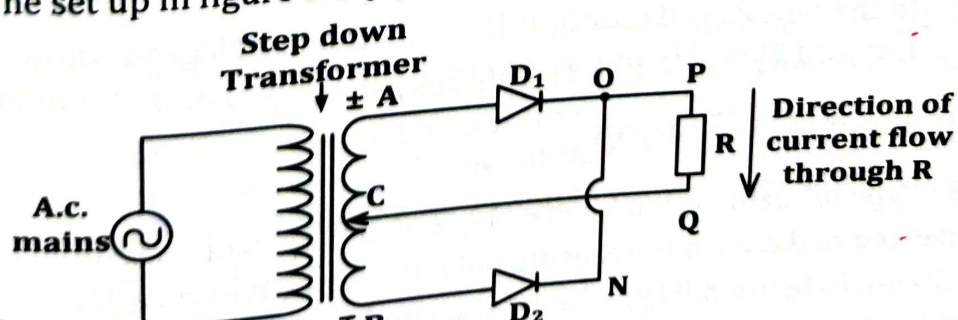


Fig. 5.0 (d)

- When terminal **A** of the a.c. power input is **positive** with respect to **B** and C, the diode (rectifier) D_1 conducts current (i.e. its forward biased) while rectifier D_2 does not (i.e. it is reverse biased).
- Current flows from **A** through diode D_1, to **P** the flows through the resistor **R** (*where it sets up a p.d. across*) with the current flowing in the direction **PQ** to **C** and back to **A**, as it completes the circuit.
- The first half-cycle of a complete a.c input cycle is then rectified.
- During the second half cycle of the a.c. input, (*i.e. during change over of current from terminal **A** to terminal **B***), i.e. when **B** is positive with respect to both **A** and **C**, diode **(rectifier)** D_2 conducts current (i.e. its forward biased) while rectifier D_1 does not (i.e. it is reverse biased).
- Current flows from **B** through diode D_2 to N to O, to P and flows through the resistor **R** again in the **same direction** **PQ** as before, then to C and back to B, hence completing the circuit.
- The process then repeats itself several times per second for all the subsequent cycles, producing direct current flow through the resistor **R** only in one direction, **PQ**, for every cycle of the a.c. input.
- This is represented by a pulsating direct current (D.C) voltage output across R as shown in figure 5.0(e) (ii) in comparison with the un rectified a.c. voltage input in figure 5.0 (e)(i).

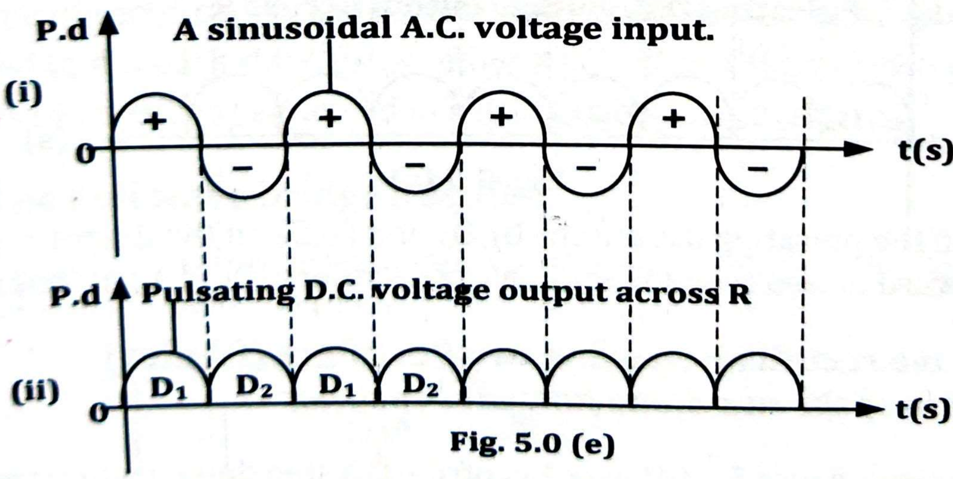


Fig. 5.0 (e)

NB: In the pulsating d.c. output D_1 and D_2 on the diagram shows the forward biased (conducting) rectifier (Diode) at that moment.

Half wave rectification

Is the type of rectification where only one half of every a.c. input cycle is converted to d.c. each time, using only one of the two rectifiers arranged in as circuit, in figure 5.0 (g) while the second half is each time lost (Un rectified).

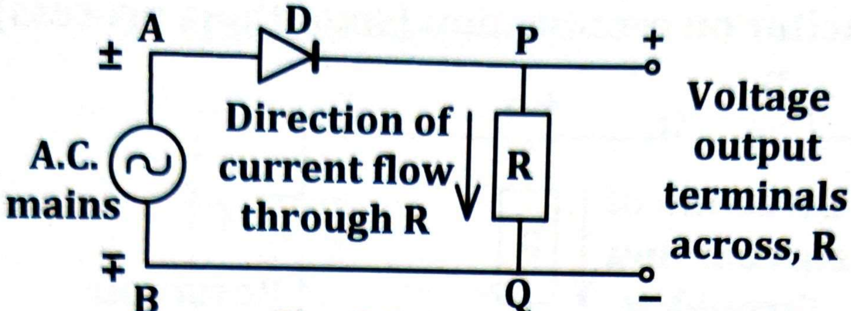


Fig. 5.0 (g)

- During the first half of the a.c. input, when terminal **A** of the a.c. input with respect to **B**, the diode (rectifier) **D** conducts current through the resistor **R** in the direction P towards Q. i.e. D is forward biased.
- The first half cycle of the a.c input is then rectified, while the second half is lost (un –rectified).
- During the change of current from terminal **A** to terminal **B**, current drops to zero in the circuit and no current flows through R.
- During the second half cycle of the a.c. input, when **B** is positive with respect to A, the diode D does not conduct current in the circuit, since it is reverse biased, so that second half cycle of the a.c. input is not rectified.
- The process repeats itself at the frequency of the source for all the subsequent cycles allowing current through R in the **same direction PQ** for all the subsequent cycles.
- This leads to the production of direct current flow through the resistor and setting up a p.d. across R as illustrated on the diagram in figure 5.0 (h) in relation to the A.C. input supply voltage.

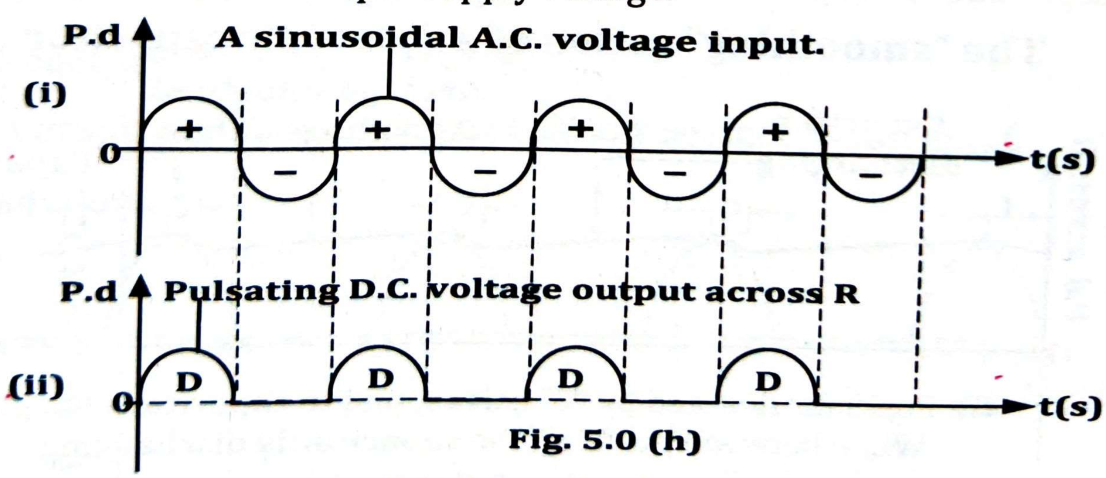


Fig. 5.0 (h)

NB: In the pulsating d.c. output, the diode (rectifier) **D** on the fig. 5.0 (h) shows the forward biased connection (i.e. when D is conducting) while the flattened portion corresponds to the reverse bias connection for the (Diode) at that moment (i.e. when the diode does not conduct current, for example when B is positive while A is negative)

(c) Action of a Capacitor on rectification (smoothing process)

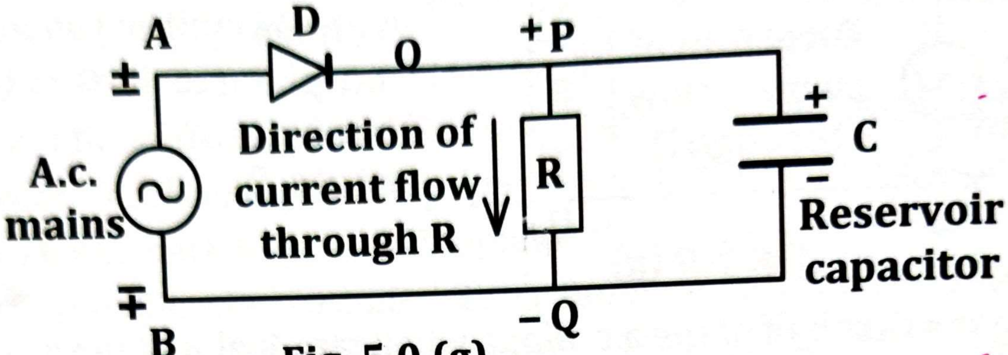
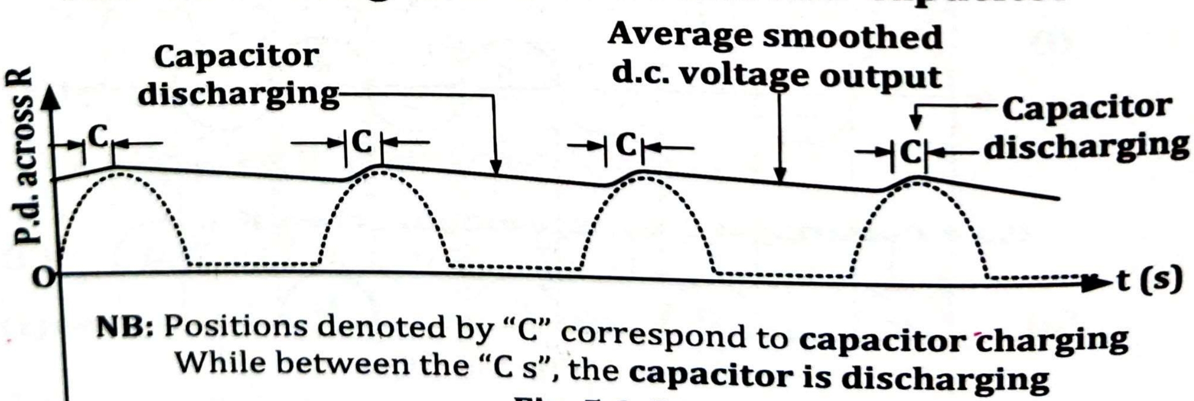


Fig. 5.0 (g)

- During the first half of the a.c. input, when terminal **A** of the a.c is positive with respect to **B**, the diode (rectifier) D conducts current through the resistor R in the direction P to Q.
- At the same time the capacitor C, charges as indicated on the diagram.
- The first half of the a.c input cycle is then rectified.
- During the change of current from terminal A to terminal B, current drops to zero in the circuit and the capacitor quickly discharges through R, to maintain the current flow in the circuit.
- During the second half cycle of the a.c. input, when B is positive with respect to A, the diode D_1 does not conduct current in the circuit, so that half of the a.c. input is not rectified
- The process repeats itself several times per second for all the subsequent cycles producing direct current flow through the resistor and a smoothed output due to the presence of the capacitor with the output shown below

The "smoothing" action of a reservoir capacitor



NB: Positions denoted by "C" correspond to **capacitor charging**
While between the "C s", the **capacitor is discharging**

Fig. 5.0 (h)

5.1 A.C THROUGH A PURE RESISTOR

Suppose a sinusoidal voltage $V = V_0 Sin\ \omega t$ is connected across a resistor of resistance, R.

A.C. Through a resistor, R

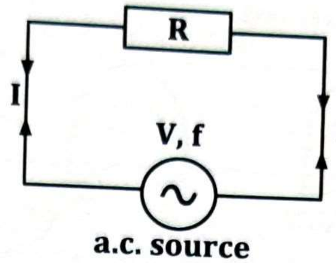


Fig. 5.1 (a)

A sinusoidal instantaneous current, I, flows through the resistor, where, $I = \frac{V}{R}$ $I = \frac{Vo}{R}\, sin\omega t$ and can be written as, $I = I_0 \sin \omega t$

Where, $I_0 = \frac{Vo}{R} \Rightarrow Resistance\ of\ the\ reistor, \boldsymbol{R} = \frac{\boldsymbol{Vo}}{\boldsymbol{I_0}} = \frac{\boldsymbol{Vrms}}{\boldsymbol{I_{rms}}}$

Since $Vo = Vrms\sqrt{2}\ and\ Io = Irms\sqrt{2} \Rightarrow Io \propto Irms\ and\ Vo \propto Vrms$

NB: For a resistor, the voltage across it, is $V = Vo \sin \omega t$ ....(i)
While, the current flowing through it, is given by, $I = Io \sin \omega t$ ....(ii)
$Where, Vo = \ peak\ voltage\ and\ Io = \ peak\ value\ of\ current.$

From equations (i) and (ii) ***Current I and Voltage V are in Phase*** *in a resistor.*

NB: In all a.c. circuits, currents and voltages vary in both magnitude and direction as they pass through a given device and so are treated as **vector quantities.**

From the equation, $\boldsymbol{R} = \frac{\boldsymbol{Vo}}{\boldsymbol{I_0}} = \frac{\boldsymbol{Vrms}}{\boldsymbol{I_{rms}}}$ the resistance of a resistor is independent of the frequency of the supply source.

A graph of resistance, R of a resistor against frequency, f.

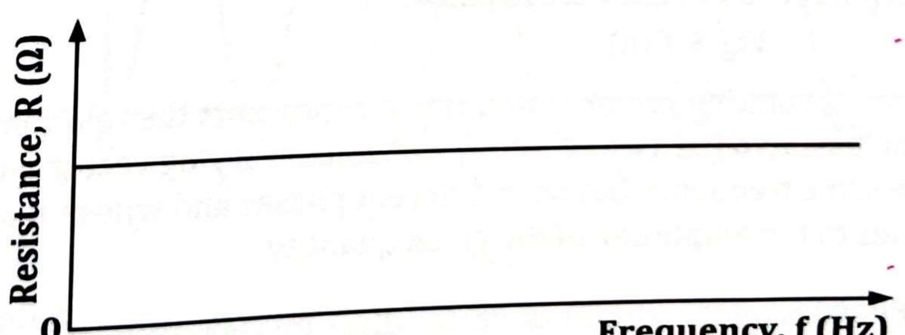
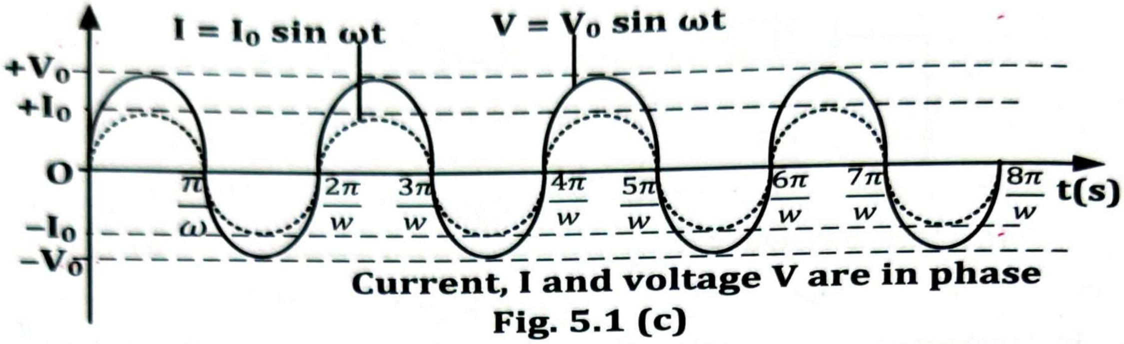


Fig. 5.1 (b)

Graphs of Current I and voltage, V against time for a resistor.



Current, I and voltage V are in phase

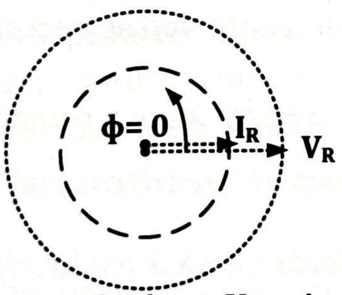
Fig. 5.1 (c)

Phasor or vector diagrams

- A sinusoidal alternating quantity can be represented by a rotating vector in the anticlockwise direction, known as **a phasor diagram.**
- Perpendicular to the time axis, shows the locus of the vector and for more than one vector, the phase difference or phase angle ϕ between any two vectors must be shown.

  NB. From (i) and (ii) above, voltage $V = Vo \sin \omega t$ across a resistor and current $I = Io \sin \omega t$ across the same resistor, are in a phase as shown in figure 5.1(b). This can be represented by the phasor or vector diagram shown in the figure 5.1 (c).

Phasor diagram for a resistor



Current, **I** and voltage, **V** are **in phase.**

Fig. 5.1 (d)

A phasor – is a rotating vector diagram that represents the relationship between at least two (i.e. two or more) sinusoidal varying vector quantities having the same frequency but with different phases and whose magnitude corresponds to the amplitude of the given quantity.

NB: The **Phasors** on the Phasor or vector diagram represent, either ***peak values*** or ***r.m.s values*** of a given vector quantity.

Power Absorbed in a pure resistor

The instantaneous power absorbed in a resistor

$P = IV, but\ also\ V = IR, Implying\ P = I^2R = (I_0 \sin \omega t) \times (V_0 \sin \omega t)$

$P = I_0V_0\ sin^2\ \omega t\ \ or\ P = {I_0}^2R\ sin^2\ \omega t$

Average power absorbed over one cycle is written as,

$$\langle P\rangle_T = \langle I_0^2 R \sin^2 \omega t\rangle = I_0^2 R \langle \sin^2 \omega t\rangle$$

But, $\langle \sin^2 \omega t\rangle_T = \frac{1}{2}$ obtained from; $\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t)$

Where, the $\langle \cos 2\omega t\rangle_T = 0$, *which is the same as area under the curve*

Thus, average power expended in a resistor over one cycle, is given by any one of the following expressions or equations

$$\langle P\rangle_T = \frac{1}{2} I_0^2 R = \frac{V_0^2}{2R} = \frac{I_0 V_0}{2} = I_{rms}^2 R = \frac{V_{rms}^2}{R} = I_{rms} V_{rms}$$

Note that the average of a cosine curve or wave over 1 cycle = 0

Cosine wave over 1 complete cycle

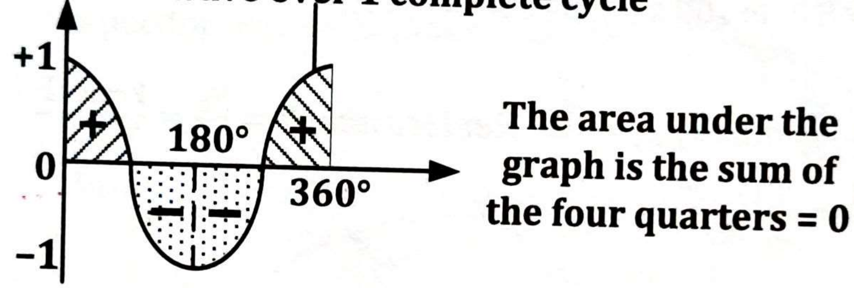


Fig. 5.1 (e)

A graph of power absorbed in a resistor as a function of time.

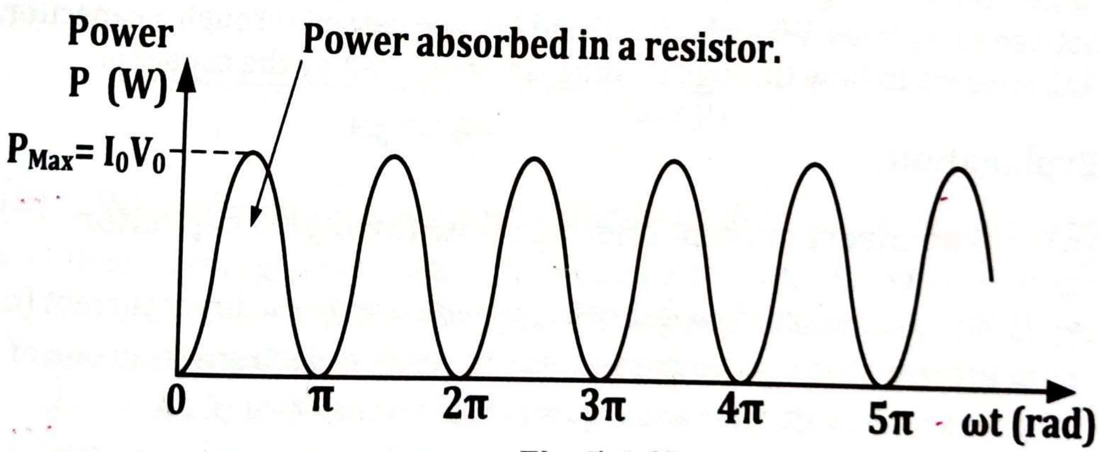


Fig. 5.1 (f)

Examples:

1. An alternating current, is represented by, $I = 5.0 \sin 200\,\pi t$ where I is in amperes while, t is the time in seconds. What is the:

   (i) Peak value of current.
   (ii) root mean square value of the current.
   (iii) current flowing, 1/1200 s after the current changes direction.
   (iv) power dissipated in the non – inductive resistance of 4 Ω.

Solution

(i) Compare $I = 5.0 \sin 200\,\pi t$ to $I = I_0 \sin 2\pi f t$

$\Rightarrow$ *Peak value*, $\boldsymbol{I_0 = 5.0\ A}$

(ii) $\boldsymbol{I_{rms}} = \frac{I_0}{\sqrt{2}} = \frac{5.0}{\sqrt{2}} = \boldsymbol{3.54\ A}$

(iii) $I = 5.0 \sin 200\,\pi t = 5.0 \sin\left(200\,\pi \times \frac{1}{1200}\right) = 5.0 \sin\left(\frac{\pi}{6}\right)$

$\boldsymbol{I} = 5.0 \sin 30° = \boldsymbol{2.5\ A}$

(iv) *Power*, $\boldsymbol{P} = \frac{1}{2} \times (5.0)^2 \times 4.0 = \boldsymbol{50\ W}$

2. Calculate the root mean square voltage across a resistance of 100 Ω having a current, $I = 8.0 \sin 100\,\pi t$ *flows through it.*

Solution:

Using, $\boldsymbol{I_{rms}} = \frac{I_0}{\sqrt{2}}$ *and* $\boldsymbol{V_{rms}} = \frac{V_0}{\sqrt{2}}$ ***Resistance***, $\boldsymbol{R} = \frac{V_0}{I_0} = \frac{V_{rms}\sqrt{2}}{I_0}$

$$\boldsymbol{V_{rms}} = \frac{I_0 \times R}{\sqrt{2}} = \frac{8.0 \times 100}{\sqrt{2}} = \boldsymbol{565.69\ V}$$

5.2 A.C. THROUGH A PURE CAPACITOR

A pure capacitor is one of infinite dielectric resistance, thus direct current cannot flow through a capacitor because of the insulating medium placed between its plates. When both A.C and D.C are passed through a capacitor, only A.C. appears to flow through it while D.C is blocked by the capacitor.

Explanations

(a) Why direct current does not flow through a capacitor

- When an uncharged capacitor is connected across a direct current (d.c.) source, the battery provides energy to remove electrons from one of the plates of the capacitor and deposit them on the other plate.
- An instantaneous current I then begins to flow through part of the circuit and since the quantity of charge on each plate initially is low, the rate of charge transfer (current) is high, but lasts only for a short period of time.
- The momentary flow of current, is as a result of electrons being drawn from plate **A** by the positive terminal of the battery, whilst at the same time, the electrons are being deposited to plate B by the action of the negative terminal of the battery. [***see fig. 5.2 (a)***]
- As the plates acquire more charge of opposite signs, but of equal magnitudes, the rate of charge transfer reduces as the p.d. across the capacitor builds up.

- When the capacitor is fully charged, the rate of charge transfer becomes zero. i.e. the current stops flowing in the circuit stops, when the p.d. across the plates of the capacitor equals the supply voltage or e.m.f.

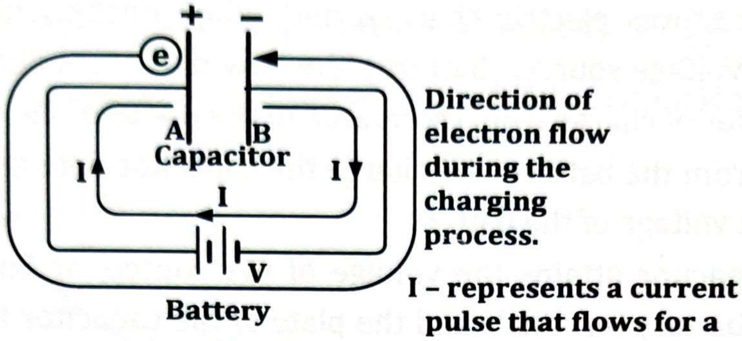


Fig. 5.2 (a)

A sketch of a graph of current against time during the charging of a capacitor using a d.c. source of e.m.f.

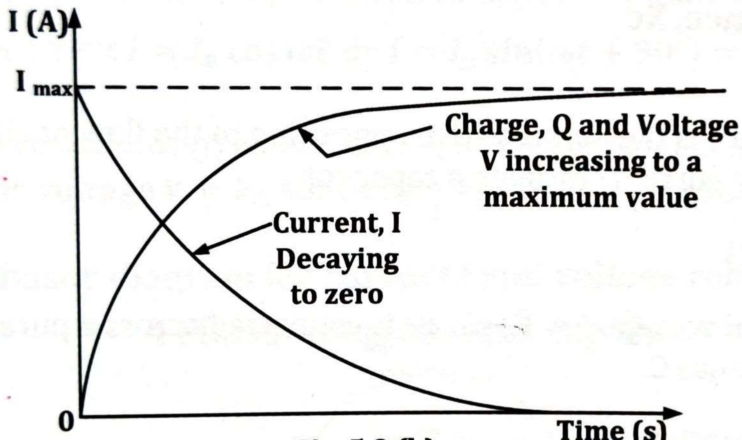


Fig. 5.2 (b)

(b) Why alternating current appears to flow through a capacitor:

- When un charged capacitor is connected across an a.c. source, during the first quarter cycle of the a.c. input, **the capacitor charges** until the p.d between its places equals to the supply p.d. energy is drawn from the source and stored in the electric field of the capacitor.
- In the second quarter cycle, the supply voltage decreases and the p.d across the plates of the capacitor drives charge in the opposite direction thus, **the capacitor discharges**, through the source and the energy that it had stored in the first quarter cycle is returned to the source.
- In the third quarter cycle, **the capacitor again charges** as in the first quarter cycle but in the opposite direction and stores energy in its electrostatic field between the plates of the capacitor.
- In the 4th quarter (last quarter) of the a.c. input cycle, when p.d across the capacitor is maximum current becomes zero and so **the capacitor discharges** in an attempt to maintain flow of charge through circuit and so it returns all the energy it had stored back to the source.

Hence, when a capacitor is connected across an a.c. source, the capacitor charges and discharges continuously, each time sending a pulse of current through part of the circuit between its plates. Since the frequency of a.c. is usually high, a continuous current appears to flow through the circuit.

(c) Why direct current does not flow through a capacitor:

- The capacitor stores electric charges, and When uncharged capacitor is connected to voltage source, (battery), the flow of current almost instantly enables transfer of charge from the source to the plates of the capacitor.
- The current from the battery will charge the capacitor with zero voltage to the maximum voltage of the battery.
- When the capacitor attains the voltage of the source, or battery the p.d between the battery terminals and the plate of the capacitor becomes zero and hence current stops flowing.
- When the instantaneous current drops to zero, it stays this way indefinitely, creating an impression that direct current flowing in the capacitor is zero.

Capacitive reactance, Xc

Definition

Capacitive reactance – is the non-resistive opposition to the flow of alternating current (or changing current) through a capacitor.

SI unit is an ohm (Ω)

Suppose a sinusoidal voltage $V = V_0 \sin \omega t$ is connected across a pure capacitor of capacitance C.

A.C. Through a Capacitor, C

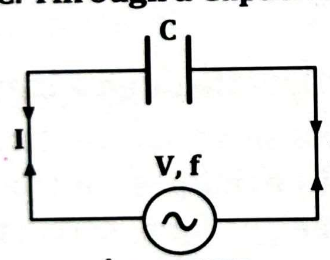


Fig. 5.2 (c)

The instantaneous charge Q, at any time, t, stored in the capacitor plates is given by, $Q = CV, Q = CV_0 \sin \omega t$

The instantaneous current $I = rate\ of\ change\ of\ charge$

$I = rate\ of\ change\ of\ charge\ with\ time$

$$i.e\quad I = \frac{dQ}{dt} \quad \Rightarrow I = \frac{d}{dt}(CV_0 \sin \omega t) = \omega CVo \cos \omega t$$

$$I = \omega CV_0 \cos\omega t \Rightarrow I = I_0 \cos \omega t$$

$Where, I_0 = \omega CV_0 = Peak\ value\ of\ the\ current$

Reactance of the capacitor, $X_C = \frac{V_0}{I_0} = \frac{V_0}{\omega C\, Vo} = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

Since $V_0 = V_{rms}\sqrt{2}$ *and* $I_0 = I_{rms}\sqrt{2} \Rightarrow X_C = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}} = \frac{1}{2\pi f C}$

SI unit of X_c is an ohm (Ω)

From the expression for capacitive reactance, $X_C = \frac{1}{2\pi f C} \Rightarrow X_C \propto \frac{1}{f}$

A graph of reactance against frequency for a capacitor across, a.c. circuit.

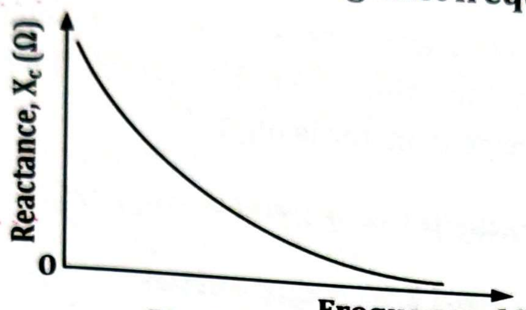


Fig. 5.2 (d)

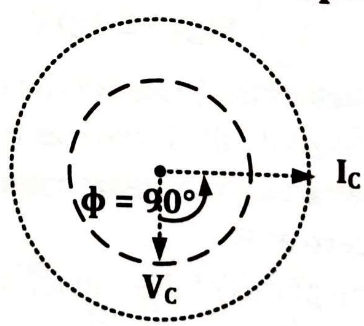
The phase relationship between voltage, $V = V_0 \sin \omega t$ … … … … … … … … (i)

and current $I = I_0 \cos \omega t$ *or* $I = I_0 \sin(\omega t + 90°) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$. … (ii)

From equations (i) and (ii) above, it is observed that the current, $I = I_0 \cos \omega t$ **leads voltage** $V = V_0 \sin \omega t$ by $\frac{\pi}{2}$ *radians or by* 90°

A phasor diagram for current I and voltage relations in a capacitor.

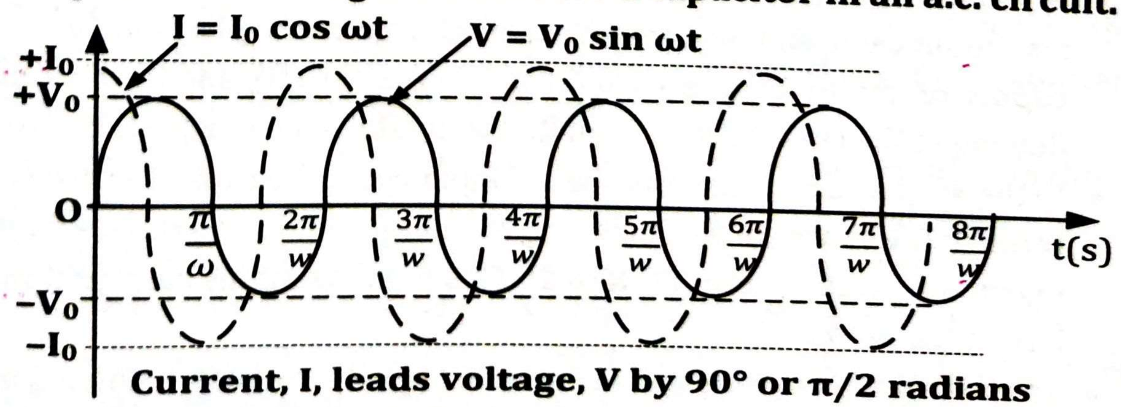
A Phasor diagram for a capacitor



Current, I, leads voltage, V are **in a capacitor by 90°**

Fig. 5.2 (e)

Graphs of I and V against time for a capacitor in an a.c. circuit.



Current, I, leads voltage, V by 90° or π/2 radians

Fig. 5.2 (f)

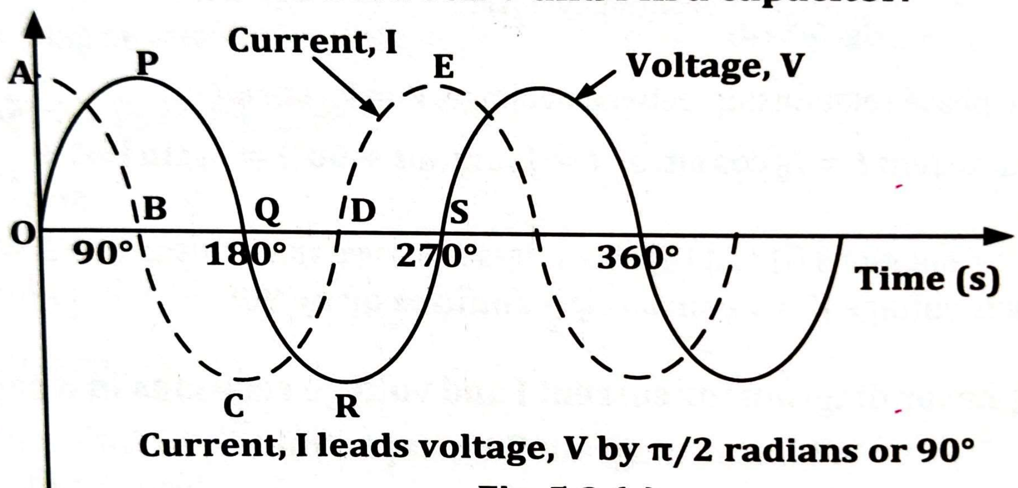
NB: Since charge induced on the plates of the capacitor during the charging and discharging process due to a.c. is proportional to the p.d. generated across the plates, a graph of charge against time has the same shape and phase with respect to current as that for voltage in figure 5.2 (f).

Explanation why Current I, leads voltage, V in a capacitor

- When uncharged capacitor is connected across an a.c. source, the charge on the plates of the capacitor is initially zero, thus the p.d. between the plates is also zero. However the rate of charge transfer is high,

i.e. $\frac{dQ}{dt} = I\ (Maximum\ current\ flows\ in\ part\ of\ the\ circuit)$

Phase difference between V and I in a capacitor.



Current, I leads voltage, V by π/2 radians or 90°

Fig. 5.2 (g)

- As the charge builds in each of the plates of the capacitor, the p.d between the plates of the capacitor gradually increases from zero at **O** to the maximum value at **P**. The rate of charge transfer gradually reduces from maximum value at **A** to zero at **B**.
- When the p.d between the plates of the capacitor is maximum, the e.m.f. of the source would have dropped to zero. The capacitor then starts to discharge (from **P** to **Q**) thus driving current through the source of e.m.f. in the opposite direction (i.e. **B** to **C**).
- When the capacitor has fully discharged, the e.m.f. of the source is at its maximum value and begins to transfer **charge** to the plates ***of the capacitor***, but in the **opposite direction** (i.e. **Q** to **R**), and the current flowing in the circuit drops from its maximum value to zero (i.e. **C** to **D**)
- In the 4th (i.e. last) quarter of the a.c. input cycle, the capacitor discharges through the source of e.m.f. causing its charge and p.d. to reduce from maximum value to zero (i.e. **R** to **S**). This then maintains current flow in the circuit (i.e. **D** to **E**)
- Thus as shown on the graph in figure 5.2 (g), the current, I, flowing in the circuit leads voltage, V by $\frac{\pi}{2}$ *radians.*

Power absorbed by a pure capacitor

For a pure capacitor, $I = I_0 \cos\omega t$ and $V = V_0 \sin\omega t$

Instantaneous power $P = IV = I_0V_0 \sin\omega t \cos\omega t$

$P = \frac{1}{2}I_0V_0(2\sin\omega t\cos\omega t) = \frac{1}{2}I_0V_0\ \sin 2\omega t = \frac{1}{2}I_0V_0\ \sin 2\pi f t$

$\langle P\rangle_T = \frac{1}{2}I_0V_0\langle \sin 2\omega t\rangle_T$

∴ The Average power over one cycle is zero.

Since, $\langle \sin 2\omega t\rangle_T = 0$

∴ $\langle P\rangle_T = 0$

NB: (i) The frequency of power in a capacitor is twice the frequency of the applied voltage.

(ii) The average power expended in a capacitor over one cycle is zero ie $\langle P\rangle_T = 0$
Hence, a capacitor is considered to be a wattles component in an a.c. circuit.

A graph of power absorbed in a capacitor as a function of time

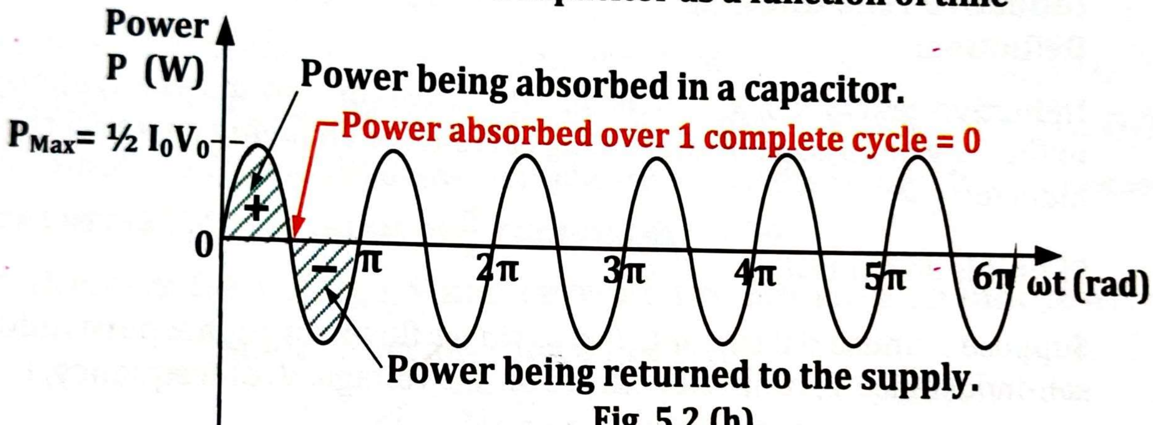


Fig. 5.2 (h)

Explanations as to why NO Power is absorbed by a capacitor

- When a capacitor is connected across an a.c. source, during the first quarter cycle of the a.c. input, the capacitor charges in one direction until the potential difference (p.d) across its plates is equal to the supply p.d. Therefore, energy is drawn from the source and stored in the electric field of the capacitor.
- In the second quarter cycle, the supply voltage decreases and the p.d. across the plates of the capacitor drives charge in the opposite direction, thus the capacitor discharges, and the energy that it had stored in the first quarter of the input cycle is returned to the source.
- In the third quarter of the input cycle, the capacitor again charges in the opposite direction and stores energy in its electrostatic field in the plates of the capacitor. Energy is again drawn from the source of e.m.f.
- In the 4th quarter (last quarter) of a.c. input cycle, when p.d. across the capacitor plates is maximum, the current flowing in the circuit reduces zero and so, the capacitor discharges through the battery in an attempt to maintain

flow of charge through circuit and so it returns all the energy it had stored back to the generator (source of e.m.f)

- Hence, for each complete cycle of the a.c. input, the net energy and average power stored and expended in the capacitor is zero.

5.3 A.C. THROUGH A PURE INDUCTOR

A pure inductor is a coil of wire having negligible resistance, (or a non-dissipative component), but has appreciable self-inductance, L, and reactance, X_C.
A non – inductive coil on the other hand is one with no opposition to the passage of changing current through it.

However, in daily practice, inductive coils have both reactance and resistance, and the total opposition that the two parts exert towards the passage of a.c. through it is called impedance, denoted by, $Z = \sqrt{X^2 + R^2}$

Inductive reactance, X_L
Definition:

Inductive reactance, X_L – is the non-resistive (or non-dissipative) opposition to the flow or passage of alternating current (or changing current) through an inductor.

SI unit is an ohm (Ω)

Suppose a sinusoidal current, $I = I_0 \sin \omega t$ flows through a pure inductor of self-inductance, L, connected across an a.c. voltage, V, of frequency, f.

A.C. Through an Inductor, L

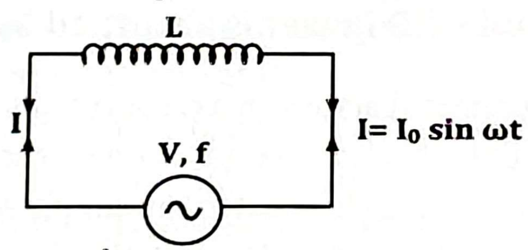


Fig. 5.3 (a)

The instantaneous back e.m.f., E_b at any time, t, induced across the inductor is given by $E_b = -L\frac{dI}{dt}$ *and always opposes the applied voltage.*
Where, $I = I_0 \sin \omega t$

$$i.e \quad E_b = -L\frac{dI}{dt} = -L\frac{d}{dt}(I_0 \sin \omega t) = -\omega L I_0 \cos \omega t$$

$But, E_b = -V$, since back e. m. f. opposes the applied voltage.

$$-V = -\omega L I_0 \cos\omega t \text{ and } V = I_0 \omega L \cos \omega t \Rightarrow V = V_0 \cos \omega t$$

Where, $V_0 = I_0 \omega L$ *is the Peak value of the applied voltage*

Reactance of the Inductor, $X_L = \frac{V_0}{I_0} = \frac{I_0 \omega L}{I_0} = \omega L = 2\pi f L$

Since $V_0 = V_{rms}\sqrt{2}$ and $I_0 = I_{rms}\sqrt{2} \Rightarrow X_L = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}} = 2\pi f L$

SI unit of X_L is an ohm (Ω)

From the expression for Inductive reactance, $X_L = 2\pi f L \Rightarrow X_L \propto f$

∴ Inductive reactance, X_L **is directly proportional to frequency, f.**

A graph of reactance against frequency for an inductor in a.c. circuit.

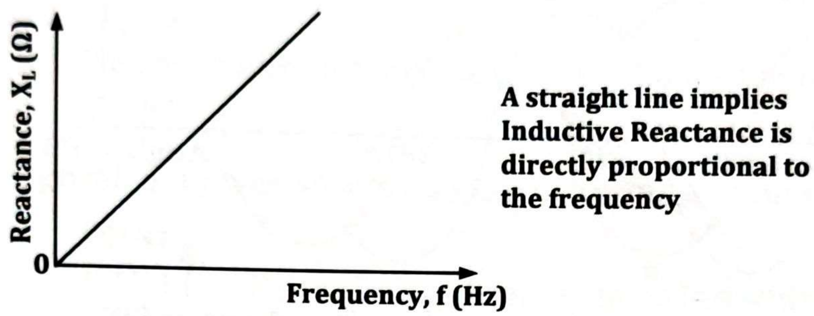


Fig. 5.3 (b)

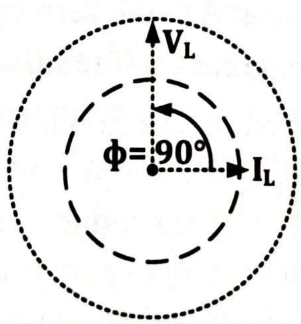
The phase relationship between current, $I = I_0 \sin \omega t$ ………………………. **(i)**

and voltage, $V = V_0 \cos \omega t$ or $V = V_0 \sin(\omega t + 90°) = V_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ … **(ii)**

From equations (i) and (ii) above, it is observed that the voltage, $V = V_0 \cos \omega t$ **leads Current,** $I = I_0 \sin \omega t$ by $\frac{\pi}{2}$ $radians$ or by $90°$

A phasor diagram for voltage, V and current, I relations in an inductor.

A Phasor diagram for an inductor



Voltage, **V** leads current, **I** are in **an Inductor by 90°**

Fig. 5.3 (c)

Graphs of V and I against time for an inductor in an a.c. circuit.

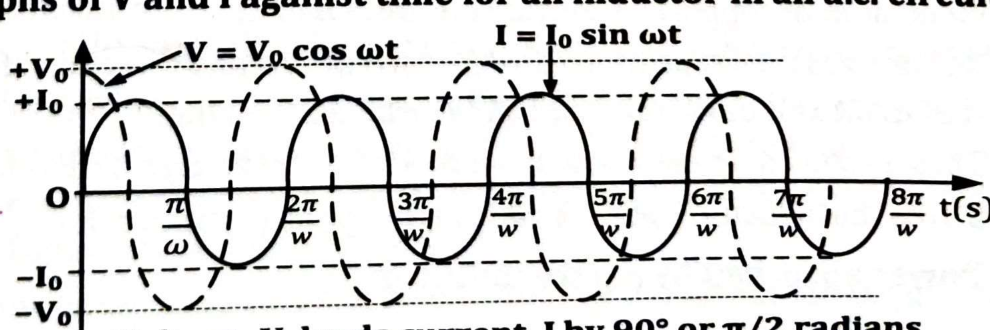
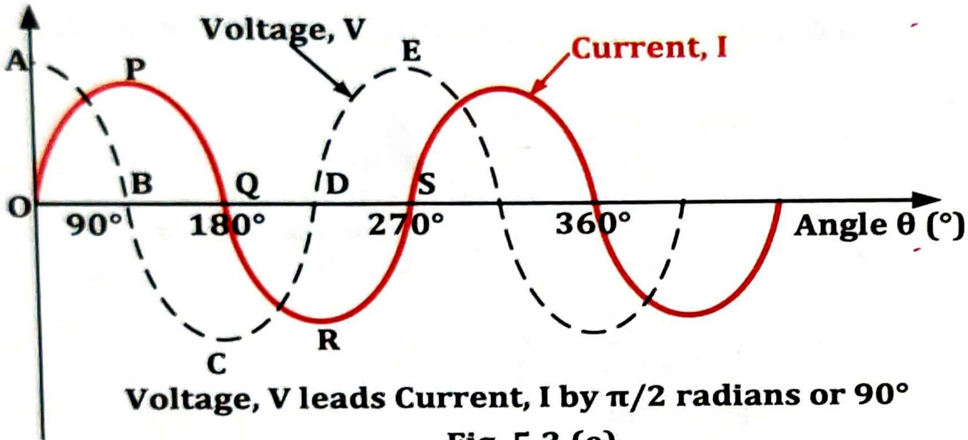


Fig. 5.3 (d)

Explanation why voltage V, leads current, I in an inductor

- When an inductor is connected across an a.c. source and the circuit is completed, (i.e. switch is closed), the rate of current flow in the circuit is very high, this causes a large back e.m.f. to be induced in the inductor.
  i.e. $E = -L\frac{dI}{dt} = V\ (Maximum\ p.d\ across\ the\ inductor, when\ I = 0)$

Phase difference between V and I in an inductor.



Voltage, V leads Current, I by π/2 radians or 90°

Fig. 5.3 (e)

This causes energy to be stored in the magnetic field of the inductor seen from the graph in figure 5.3 (e) where voltage V is maximum initially at **A** when the current **I** is zero at **O**.

- As the current flow in the coil (inductor) builds up gradually to a maximum value (i.e. **O** to **P**), the rate of current flow, $i.e.\left(\frac{dI}{dt} \propto E_b\right)$ in the inductor reduces gradually to zero and back e.m.f. induced in the coil also gradually reduces from maximum value at **A** to the zero value at **B**. this shows that the voltage **V** is leading the current **I** by $\frac{\pi}{2}\ radians.$
- When the rate of flow of current in the circuit becomes zero at **P**, the back e.m.f. also becomes zero at **B**. When the current flowing in the circuit begins reducing, (i.e. from **P** to **Q**), the inductor liberates the energy originally stored in its magnetic field to enhance the decaying magnetic field in the coil and to maintain decaying current flowing in the circuit in the same direction as before. The back e.m.f. begins to increase again in the opposite direction causing energy to be stored in the magnetic field of the inductor in the opposite direction (i.e. from **B** to **C**)
- The process then repeats itself when current reverses direction of flow in the circuit as it did in the first half cycle of the a.c. input.
- Thus, as shown on the graph in figure 5.3 (e), the voltage, V, developed across the inductor, leads the current, **I**, flowing in the circuit by $\frac{\pi}{2}\ radians.$

Power absorbed by a pure inductor

For a pure inductor, $V = V_0 \cos\omega t$ and $I = I_0 \sin\omega t$

Instantaneous power, $P = IV = I_0V_0 \sin\omega t \cos\omega t$

$$P = \frac{1}{2}I_0V_0(2\sin\omega t\cos\omega t) = \frac{1}{2}I_0V_0\ \sin 2\omega t = \frac{1}{2}I_0V_0\ \sin 2\pi f t$$

$$\langle P\rangle_T = \frac{1}{2}I_0V_0\langle\ \sin 2\omega t\rangle_T$$

∴ The Average power over one cycle is zero.

Since, $\langle\ \sin 2\omega t\rangle_T = 0$

$\therefore \langle P\rangle_T = 0$

NB: (i) The frequency of power in an inductor is twice the frequency of the applied voltage.

(ii) The average power expended in an inductor over one cycle is zero ie $\langle P\rangle_T = 0$
Hence, an inductor is considered to be a wattles component in an a.c. circuit.

A graph of power absorbed in an Inductor as a function of time.

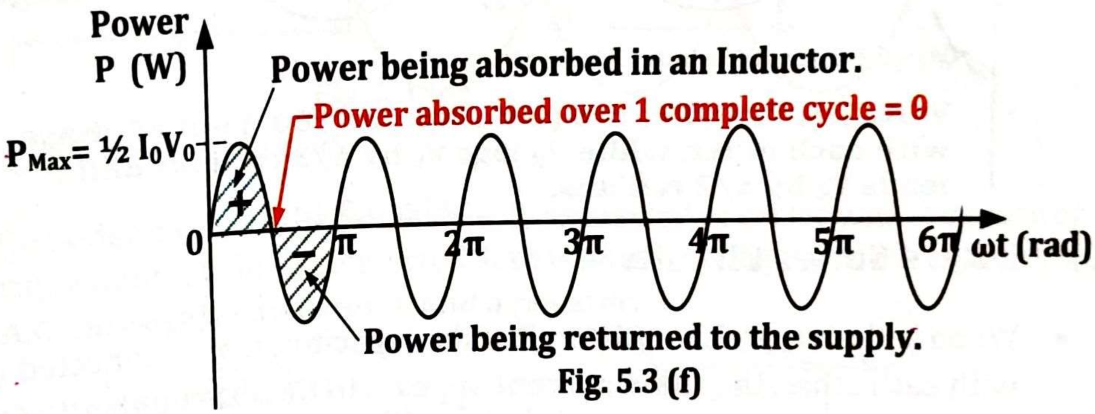


Fig. 5.3 (f)

Explanations as to why <u>NO Power</u> is absorbed by an inductor

- During the first quarter of the a.c. input cycle, the increasing current flowing through the inductor, causes a changing magnetic flux to link the plane of the coil (inductor), a large back e.m.f. is then induced in the inductor, causing energy to be stored in the magnetic field of the inductor.
- During the second quarter of the a.c. input cycle, the current flowing through the inductor, and the magnetic flux linking the plane of the coil (inductor), gradually reduce to zero. The energy originally stored in the magnetic field of the inductor is then liberated and given back to the source to maintain the decaying current in the circuit and magnetic flux linking the inductor.
- During the third quarter of the a.c. input cycle, the current flowing through the inductor from the source again increases, but in the opposite direction, causing an increase in the magnetic flux linkage to the coil (or inductor), a large back e.m.f. to be induced in the inductor, and energy is stored in the magnetic field of the inductor.
- During the last quarter of the a.c. input cycle, current and the magnetic flux again decay to zero in the inductor, causing it to release an equivalent amount of energy it had gained during the third quarter back to the source.

- Thus, after one complete cycle of every a.c. input, the inductor gains energy twice and also loses an equal amount of energy twice thereby remaining with no energy and power at the end of every cycle. Hence, it is considered as a **"wattles component."**

Graphs of voltages across a capacitor, an inductor and a resistor against time.

Phase differences of p.ds across the components in the circuit.

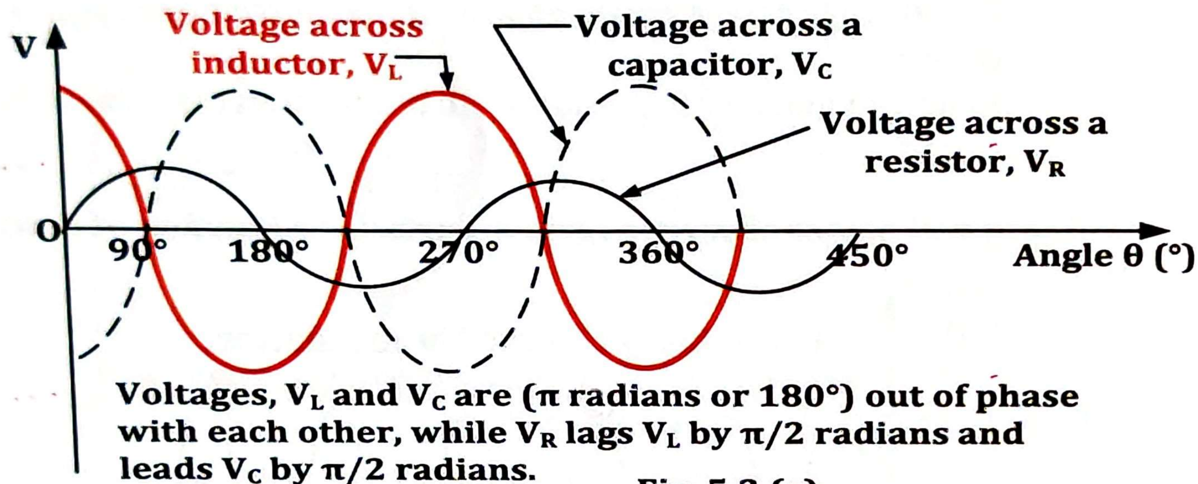


Fig. 5.3 (g)

5.4 L R C – Series Circuits

- When an inductor, **L**, a resistor, **R** and a capacitor, **C** are connected in series with each other, the **same current** appears to **flow through all** of them, and so it is regarded as the common vector to all the components.
- Thus, when the vector or phasor diagrams are drawn to represent the phase relationships of these components, the phase relationship between current and voltages are put into consideration.
- For a series combination of the vectors, current which is the common vector is the current and is permanent and constitutes the reference vector that lies in the positive **X – direction.**
- For a parallel combination of the vectors, voltage across all the components is the common vector is the current and is permanent and constitutes the reference vector that lies in the positive **X – direction.**
- The numerical values of **V** and **I** used in the calculations are considered mainly as root mean square (**r.m.s**) values, unless specifically stated, as peak values or as instantaneous value of current or voltage involved.
- The phase relationships between current and voltage are summarized by an acronym **"CIVIL"** as follows:
  CIV ⇒ For a capacitor, Current **I**, **leads** Voltage **V**, by **π/2** radians or **90°**
  VIL ⇒ For an inductor, Voltage **V**, **leads** Current **I**, by **π/2** radians or **90°**

NB: In a resistor Current I and Voltage V are in phase.

Thus the phase relationship between V and I for a resistor is represented as shown in figure 5.4 (a)

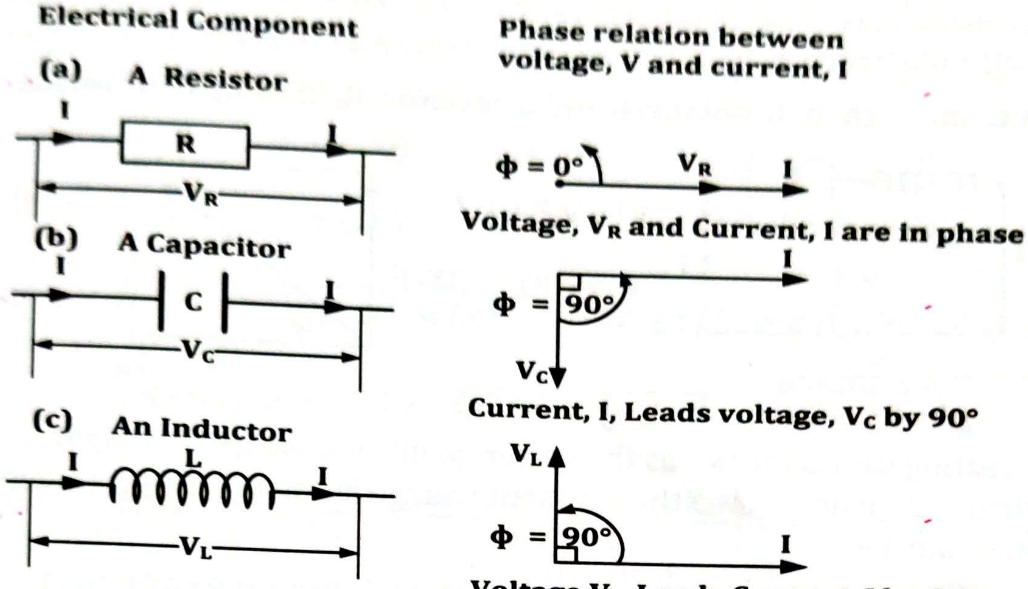
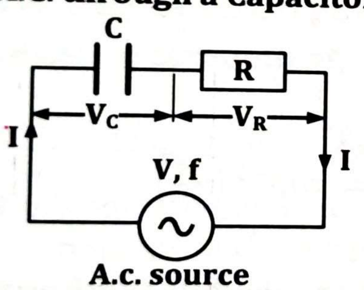


Fig. 5.4 (a)

(a) C R – Series Circuit

Consider a resistor of resistance, R connected in series with a capacitor of capacitance, C, both connected across an a.c. voltage, V at a frequency f.

A.C. through a Capacitor, C and a resistor, R



$V^2 = V_R^2 + V_C^2$

$V^2 = (IR)^2 + (IX_C)^2$

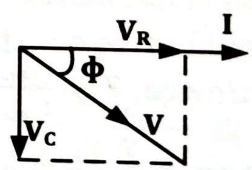


Fig. 5.4 (b)

Treating the quantities as the vector quantities, we use Pythagoras theorem, to determine the resultant voltage, following the vector diagram.

$$V^2 = V_C^2 + V_R^2$$
$$= (IX_C)^2 + (IR)^2$$
$$= I^2(X_C^2 + R^2)$$
$$V = I\sqrt{X_C^2 + R^2} \quad where, X_C = \frac{1}{2\pi f C}$$

$$Impedance, Z = \frac{V_{rms}}{I_{rms}} = \sqrt{X_C^2 + R^2} \quad while, \tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \left(\frac{X_C}{R}\right)$$

$$\therefore The\ phase\ angle, \phi = \tan^{-1}\left(\frac{X_C}{R}\right) \ for\ which\ V\ is\ ahead\ of\ I$$

Impedance, Z

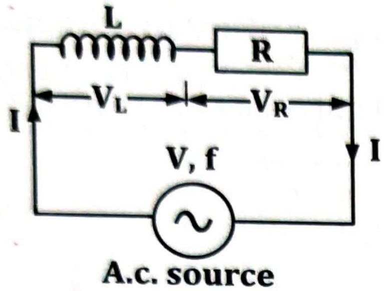
Is defined as the ***total opposition*** offered ***by a reactive*** and ***resistive circuit*** that involves a capacitor or an inductor with any other component say a resistor to the passage of alternating current (a.c.) through it.

SI unit – is an **ohm (Ω)**

(b) L R – Series Circuit

Consider a resistor of resistance, **R** connected in series with an inductor of self-inductance, **L**, both connected across an a.c. voltage, **V** at a frequency **f**.

A.C. through an inductor, L and a resistor, R, arranged in series



$$V^2 = V_R^2 + V_L^2$$

$$V^2 = (IR)^2 + (IX_L)^2$$

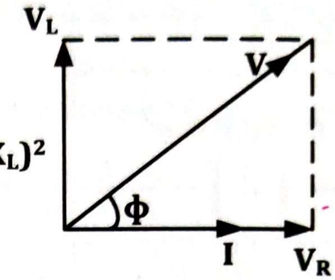


Fig. 5.4 (c)

Treating the quantities as the vector quantities, we use Pythagoras theorem, to determine the resultant voltage following the given vector diagram. i.e.

$$V^2 = V_L^2 + V_R^2$$

$$= (IX_L)^2 + (IR)^2$$

$$= I^2(X_L^2 + R^2)$$

$$V = I\sqrt{X_L^2 + R^2} \quad where, X_L = 2\pi f L$$

$$\therefore \boldsymbol{Impedance, Z = \frac{V_{rms}}{I_{rms}} = \sqrt{X_L^2 + R^2}}$$

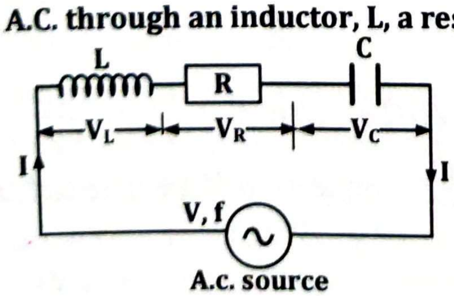
$$while,\ \boldsymbol{\tan\phi = \frac{V_L}{V_R} = \frac{I\,X_L}{I\,R} = \left(\frac{X_L}{R}\right) \Rightarrow \phi = \tan^{-1}\left(\frac{X_L}{R}\right)}$$

$$\therefore \boldsymbol{The\ phase\ angle, \phi = \tan^{-1}\left(\frac{X_L}{R}\right)\ for\ which\ V\ is\ ahead\ of\ I}$$

(c) L R C – Series Circuit

Consider a resistor of resistance, **R** connected in **series** with an inductor of self-inductance, **L**, and a capacitor of capacitance, **C**, all of which are connected across an a.c. voltage, **V** at a frequency **f**.

A.C. through an inductor, L, a resistor, R, & a capacitor, C all arranged in series



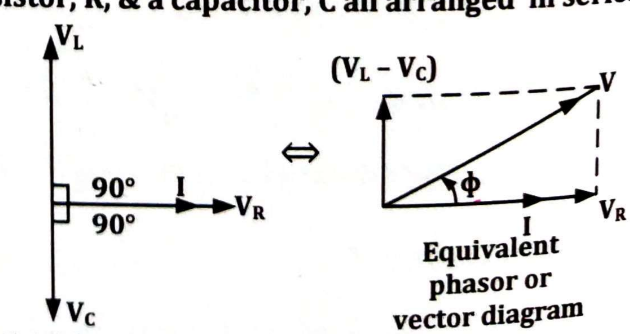


Fig. 5.4 (c)

Treating the components as the vector quantities, we use Pythagoras theorem, to determine the resultant voltage following the given vector diagram. i.e.

$$V^2 = (V_L - V_C)^2 + V_R^2$$

$$V^2 = (IX_L - IX_C)^2 + (IR)^2$$

$$V^2 = I^2(X_L - X_C)^2 + (IR)^2$$

$$V = I\sqrt{(X_L - X_C)^2 + R^2} \ \ where, X_L = 2\pi fL \ and \ X_C = \frac{1}{2\pi fC}$$

$$\therefore \ \textbf{Impedance}, Z = \frac{V_{rms}}{I_{rms}} = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$while, \tan\phi = \frac{(V_L - V_C)}{V_R} = \frac{I\,(X_L - X_C)}{I\,R} = \left[\frac{(X_L - X_C)}{R}\right] \Rightarrow \phi = \tan^{-1}\left[\frac{(X_L - X_C)}{R}\right]$$

$\therefore$ ***The phase angle,*** $\phi = \tan^{-1}\left[\frac{(X_L - X_C)}{R}\right]$ *for which* ***V is ahead of I.***

Alternatively, $\cos\phi = \frac{V_R}{V} = \frac{(I_{rms})\,R}{(I_{rms})\,Z} = \frac{R}{Z} \Rightarrow \phi = \cos^{-1}\left(\frac{R}{Z}\right)$

The Power Factor, (cos ϕ) in A.C. circuit

The **true, or actual power** consumed in an a.c. circuit is only by **the resistive component**, i.e. in **R** and any resistive part of the inductor if any.

$$\therefore P_t = (I_{rms})^2 R = (I_{rms})(V_{rms})\cos\phi = \frac{(V_{rms})^2}{R}\cos^2\phi \ldots\ldots (i)$$

The apparent power, consumed in the circuit is as though all the components of the circuit would consume power, thus the apparent power $P_a = (I_{rms})(V_{rms}) \ldots\ldots\ldots\ldots (ii)$ from equations (i) and (ii)

$$Power\ factor, \frac{P_t}{P_a} = \frac{(I_{rms})(V_{rms})\cos\phi}{(I_{rms})(V_{rms})} = \cos\phi$$

Thus, power factor, $= \cos\phi$

Where the phase angle **ϕ** is the angle by which the applied p.d. leads the current, I, flowing in the circuit.

(d) L R C – Resonance Series Circuit

Assuming the capacitor in the circuit in figure 5.4 (d) is a variable air capacitor, then its capacitive reactance can be varied by varying the capacitance **C** of the capacitor until its value equals the inductive reactance, hence ***resonance occurs.***

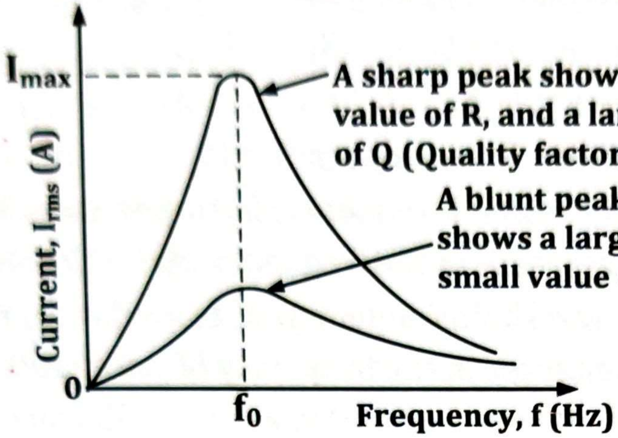


Fig. 5.4 (e)

The Quality factor (Q – factor)

The peak values or the root mean square values of voltages across a resistor, a capacitor and an inductor, are vector quantities denoted by the following respective expressions;
$V_R = I\,R$, $V_C = IX_C$ and $V_L = IX_L$ and if X_L and X_C are much larger than, R, then these values of V_L and V_C are much larger than the peak value of the applied p.d, $V = V_R$.

Thus, the **Q – factor** (or **Quality factor**) of the circuit is defined by the expression, $Q = \frac{V_L}{V_0} = \frac{V_C}{V_0} = \frac{V_L}{V_R} = \frac{X_L}{R}$ since $Q \propto \frac{1}{R}$ it can be shown that a circuit that has a high Q – factor gives rise to a sharp resonance.

Graphs of reactance, resistance and impedance against frequency

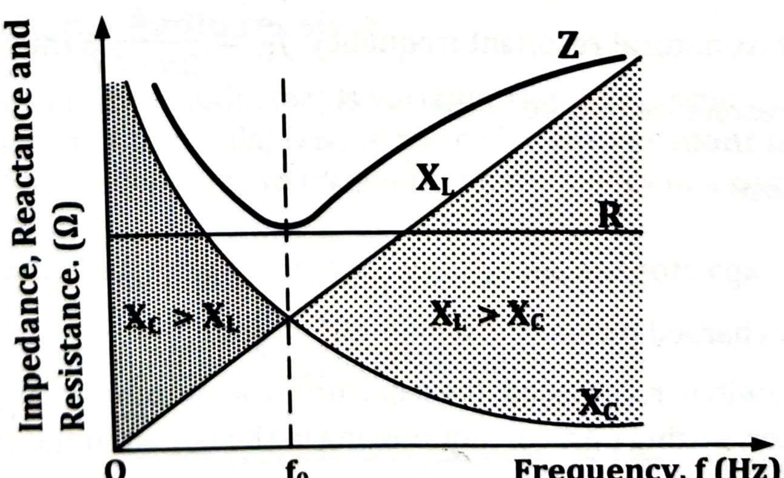


Fig. 5.4 (f)

From the graph in the figure 5.4 (f), **at resonance**, it is observed that:

- Resistance, **R**, is independent of the frequency, **f**.
- Capacitive reactance, X_C is inversely proportional to the frequency, **f**.
- Inductive reactance, X_L is directly proportional to the frequency, **f**.
- Impedance, **Z**, has its lowest value being equal to **R**. i.e. at f_0, Z = R.

- Below the threshold frequency, f_0, $X_C > X_L$.
- Above the threshold frequency, f_0, $X_L > X_C$
- At the threshold frequency, f_0, $X_C = X_L$
- At the threshold frequency, f_0, impedance, **Z = R**
- Below the value of the threshold frequency, **f_0, Impedance, Z** of the circuit decreases with increase in the frequency of the circuit.
- Above the value of the threshold frequency, **f_0, Impedance, Z** of the circuit increases with increase in the frequency of the circuit.

5.5 Oscillating Electrical Circuits

The oscillations of the current within an electrical circuit are of fundamental importance in the generation of waveforms of a variety of shapes for radios, T.Vs , oscilloscopes, signal generators etc.
One of the simplest circuits for producing these oscillations is a charged capacitor and a pure inductor connected via a switch as shown in the figure 5.5 (a)

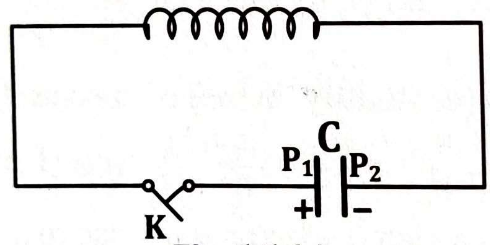


Fig. 5.5 (a)

When a charged capacitor is connected across a pure inductor of negligible resistance, an alternating current flows through the circuit causing it to oscillate at its natural resonant frequency, $f_0 = \frac{1}{2\pi\sqrt{LC}}$ thus causing electrical resonance to take place.

Explanations

- A charged capacitor has energy stored in the electric field between its oppositely charged plates. $i.e.\ E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$
- When the switch, K, is closed, the capacitor C starts to discharge through the inductor, setting up a current flowing in the circuit in the clockwise direction.
- An increasing current flowing in the coil (inductor) sets up a changing magnetic flux in the inductor, which in turn causes a back e.m.f. to be induced in the inductor hence, energy gets stored in the magnetic field of the inductor. The induced back e.m.f. eventually opposes the current flow through the inductor, thus making the capacitor to discharge slowly.

- Below the threshold frequency, f_0, $X_C > X_L$.
- Above the threshold frequency, f_0, $X_L > X_C$
- At the threshold frequency, f_0, $X_C = X_L$
- At the threshold frequency, f_0, impedance, $Z = R$
- Below the value of the threshold frequency, f_0, **Impedance, Z** of the circuit decreases with increase in the frequency of the circuit.
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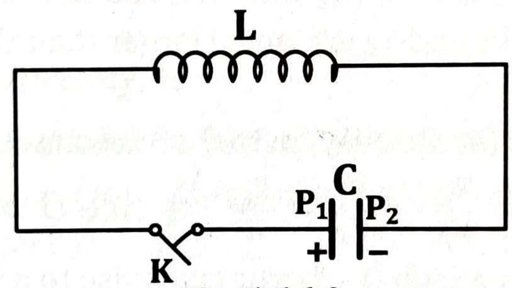

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- When the capacitor is completely discharged, all the electrical energy originally stored in the capacitor has now been transferred to the magnetic field in the inductor, **L**.
- At the instant the capacitor energy is completely depleted, maximum energy is in the inductor, **L** and the inductor in turn begins to liberate the energy originally stored in its magnetic field. The rapidly reducing magnetic field liberated in the inductor causes an e.m.f. to be induced in the inductor L, that acts in such a way as to drive a current in the circuit in the same direction as before i.e. in the clockwise direction, in an attempt to maintain the decaying magnetic field and current in the circuit, until all the energy stored in the inductor is depleted.
- The flow of current in the circuit in the clockwise direction, caused by the liberated energy in the inductor causes the capacitor to charge again, in the opposite sense to that in the first case, i.e. plate P_2 becomes positive while plate P_1 becomes negative. Thus energy gets stored in the electric field of the capacitor.
- The process then repeats itself, setting up an alternating current flowing in the circuit, due to exchange of energy in the electrical circuit from electrical or electrostatic potential energy to magnetic energy in the circuit.

$i.e. E = \frac{1}{2}\ QV\ \textbf{or}\ \frac{1}{2}\ CV^2\ in\ the\ capacitor\ to\ \frac{1}{2}\ L\ I^2\ in\ the\ inductor$

NB: The amplitudes of oscillations decrease with time since some energy is lost as other forms for example in the inductor and connecting wires as heat, due to flow of current through them.

The simple Radio receiver

If a capacitor (or inductor) is variable, then the circuit may be tuned to resonate at a particular frequency, called the **resonant frequency, f_0** of the circuit corresponding to transmission frequency of a particular radio or T.V. station.

Resonance circuit for tuning a radio receiver.

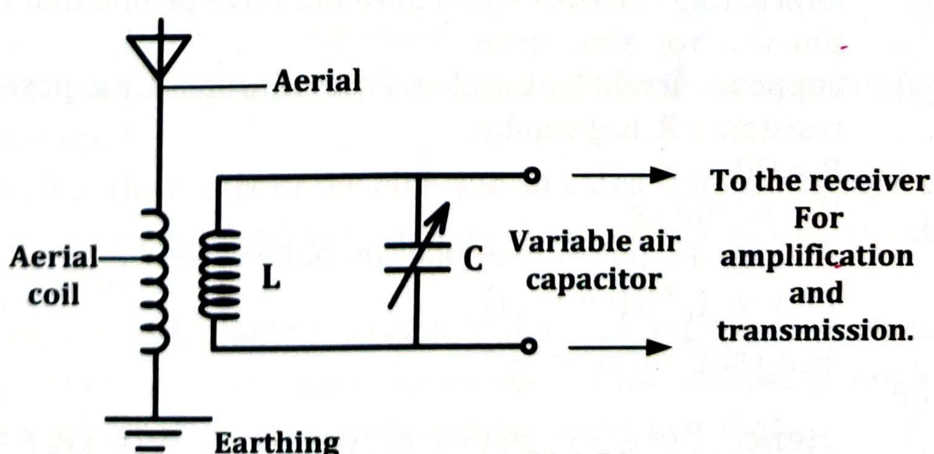

Fig. 5.5 (b)

Mode of operation: (How it works)

- Radio waves from the different transmitting radio or T.V. stations induce e.m.f.s of different frequencies at the aerial coil, which in turn induce currents of the same frequencies in the inductor, **L**, by mutual induction and connected in series with the variable air capacitor, **C**.
- By altering or tuning the variable air capacitor, C, the circuit is tuned to resonate with the frequency of the desired signal.
- At a particular frequency, it responds and stores a large amount of energy that passes on to and fro between the electric field and the magnetic fields of the inductor.
- The currents due to unwanted signals are negligibly small in comparison to the desired values. At resonance, the impedance whose value Z = R, is very small in comparison to X_L and X_C hence making the Q – factor in the circuit very high, thus making the circuit **highly selective.**

5.6 Worked out Examples & Exercises on Alternating Currents

1. (a) (i) Why is alternating current considered as sinusoidal? (1 mark)

(ii) Derive an expression for the average power dissipated in a resistor when a sinusoidal current is passed through it. (3 marks)

(b) (i) Sketch using the same axes graphs of current and voltage against time for a capacitor connected across a p.d. $V = V_0 \sin(2\pi f t)$(3 mks)

(ii) Explain why current leads voltage in the capacitor. (2 marks)

(c) (i) Describe the structure and mode of operation of a thermocouple meter. (5 marks)

(ii) State two advantages of a thermocouple meter over a moving coil ammeter. (2 marks)

(d) (i) Define the term impedance. (1 mark)

(ii) A pure inductor of self-inductance 5.0 mH is connected in series with a resistor of resistance 2.0 Ω and both are across a 240V a.c mains of frequency 50 Hz. Determine the impedance of the circuit. (3 marks)

Solution

(a) (i) Alternating current – has a periodic wave profile that resembles a sine wave or sine curve.

(ii) Suppose current $I = I_o \sin \omega t$, flows through R, a.c. power in the resistance R, is given by;

$P = I^2R$

$P_a = I_o^2 \sin^2 \omega t$

Average a.c. power over one complete cycle

$\langle P_a \rangle = I_o^2 R \langle \sin^2 \omega t \rangle$

But $\langle \sin^2 \omega t \rangle_T = \frac{1}{2}$

Hence, the average power, over 1 cycle, $\therefore \langle P_a \rangle = \frac{1}{2} I_o^2 R$

(b) (i) Graphs of Current and Voltage against time for a capacitor.

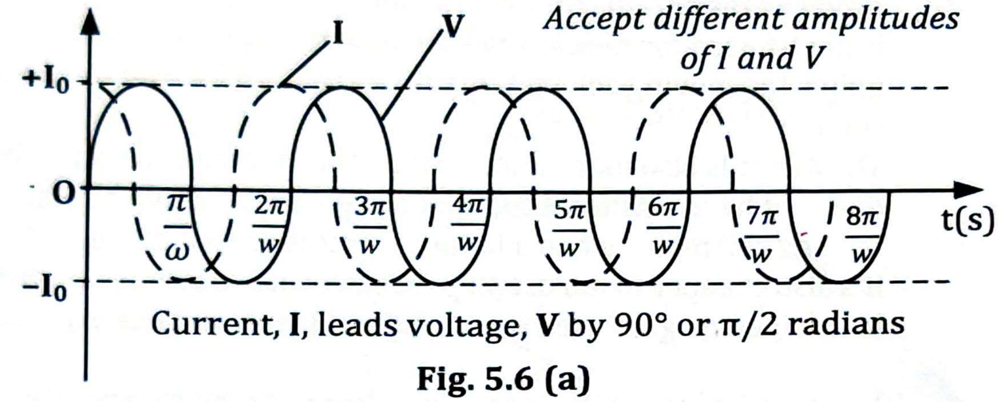

Fig. 5.6 (a)

(ii) The capacitor stores electric charges.
When uncharged capacitor is connected to voltage source, the flow of current almost instantly enables transfer of charge from the source to the plates of the capacitor.
The current from the battery will charge the capacitor with zero voltage to the maximum voltage of the battery.
When the capacitor attains the voltage of the source, or battery the p.d between the battery and the plate of the capacitor becomes zero and hence current stops flowing.
Thus, current occurs first and ***leads the voltage*** in a capacitor by $^{\pi}/_{2}$ radians.

(c) (i) The structure and mode of operation of a thermocouple meter.

The Thermo-couple meter.

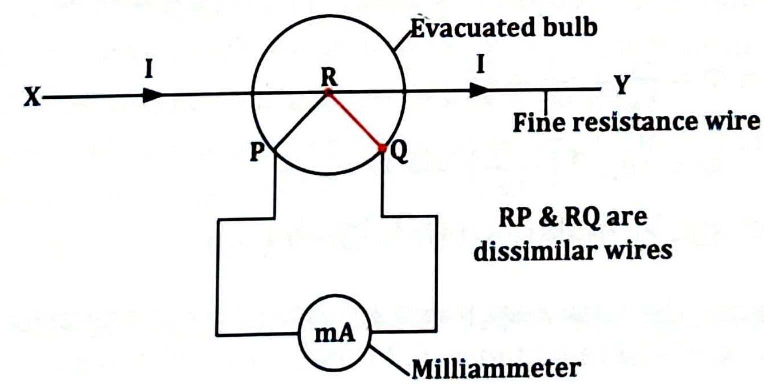

Fig.5.6 (b)

- Current **I** being measured is passed through the fine resistance wire XY and warms it up.
- Contact R at the centre of the bulb and shielded from draughts acts as the hot junction with junctions P and Q of the two dissimilar wires acting as cold junctions.
- A temperature gradient between R, P and Q causes a thermo-electric e.m.f. to be produced and a thermo electric current I_{rms} to flows through the **mA** or the **μA** already calibrated to measure direct current.
- A current through the meter causes a deflection **θ** which is proportional to the $\mathbf{I_{rms}}$

(ii) **Advantages *of a thermo-couple meter over a moving coil meter***
It can be used for measuring both alternating current and direct current unlike the moving coil meter that can be used for measuring only direct current.
The meter is also more durable than the moving coil meter, because it does not have a delicate coil that can easily be blown off like that of a moving coil meter when a large current is passed through it.
It's also cheaper to make and purchase, since a strong and permanent. U – shaped magnet used in moving coil meter is quite expensive to make.

(d) (i) **Impedance** – is the ***total opposition*** offered by a reactive circuit containing a capacitor and/or an inductor to the passage of changing current or a.c through it.

(ii) $L = 5.0\,mH = 5.0 \times 10^{-3} H$, R = 2.0 Ω

$V_{rms} = 240\,V,\ f = 50H_z$

$V^2 = V_L^2 + V_R^2$

$= (IX_L)^2 + (IR)^2$

$= I^2(X_L^2 + R^2)$

$V = I\sqrt{X_L^2 + R^2}$

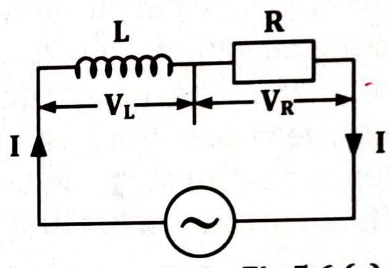

Fig.5.6 (c)

$\therefore$ Impedance, $Z = \frac{V_{rms}}{I_{rms}} = \sqrt{X_L^2 + R^2}$

Where $X_L = 2\pi fL = 2\pi \times 50 \times 5.0 \times 10^{-3} = \mathbf{1.571\ \Omega}$

$\therefore \quad Z = \sqrt{(1.571)^2 + (2.0)^2} \quad = \mathbf{2.543\ \Omega}$

$\tan\Phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}$

$\therefore \quad \Phi = \tan^{-1}\left(\frac{1.571}{2.0}\right) \Rightarrow \Phi = 38.1°$

i.e. Voltage, **V**, leads Current, **I**, by 38.1°.

2. (a) (i) Define the term ***root mean square*** value of alternating current.
(ii) a coil of self inductance, 0.2 H is connected across an a.c. source of voltage, $V = 4.0 \sin 100\,\pi t$ volts. Determine the peak voltage, frequency of the source and the steady current flowing in the circuit.

(b) (i) Describe the structure and mode of operation of a repulsion type of moving iron ammeter.
(ii) State the advantages of a moving iron ammeter over a moving coil ammeter.

Solution

(a) (i) **Root mean square value** – is ***the steady current*** that dissipates (heat) or energy in a given resistor at the ***same rate*** as ***the*** alternating current.

(ii) From, $V = 4.0 \sin 100\pi t$ and L = 0.2 H, $X_L = \frac{V_0}{I_{rms}\sqrt{2}} = 2\pi f L$

$f = 50\,Hz$, Peak voltage, $V_0 = 4.0\,V$

$$\therefore I_{rms} = \frac{V_0}{2\pi f L\sqrt{2}} = \frac{4.0}{2\pi \times 50 \times 0.2\sqrt{2}} = 4.50 \times 10^{-2}\,A$$

(b) (i) ***The repulsion type of moving iron ammeter.***

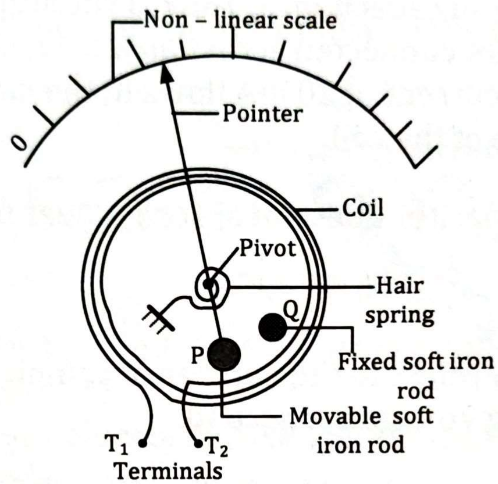

Fig.5.6 (d)

- Current, **I** is fed into the coil via terminal T_1 and T_2, creating a magnetic field at the centre of the coil.
- The two soft iron rods **P** and **Q** get *magnetized in the same sense* and begin to *repel each other with an average force* which is proportional to the square of the current flowing through the coil.
- The fixed rod Q ***repels*** rod P, causing rod P, to rotate about the pivot and the pointer moves over the scale through an angle θ, until its stopped by the restoring couple due to a pair of hair springs.
- The deflection, θ produced on the scale is proportional to the average of the square current. i.e. $\theta \propto \langle I^2 \rangle$
- *Hence, the innstrument has a non – linear scale.*

(ii) ***Advantages of a moving iron meter over a moving coil meter***

- Moving iron ammeter can be used for measuring both alternating current and direct current unlike the moving coil meters that can be used for measuring only direct current.
- The moving iron meter is also more durable than the moving coil meter, because it does not have a delicate coil that can easily be blown off or burn off like that of a moving coil meter, that easily burns up or blows when a large current is passed through it. i.e. moving iron meters can be used to measure large currents unlike moving coil meters.
- Moving iron meter is also much cheaper to make or purchase, since it has a strong and permanent U – shaped magnet used in moving coil meter is quite expensive to make.

(a) (i) Define the term frequency of alternating current.
(ii) Why is alternating current commonly referred to as sinusoidal?

(b) Describe how full wave rectification of a.c. is achieved using only two rectifiers.

(c) (i) Define the terms, *resistance, reactance and impedance* of an a.c. circuit.
(ii) A capacitor of capacitance, 16μF, a resistor of 300 Ω and a pure inductor, are connected across an a.c. source of 20 V and frequency, 50 Hz. If a current of 20 mA flows in the circuit, determine the self inductance of the coil.

(d) Explain why a capacitor does not absorb power from an a.c. circuit.

Solution

(a) (i) ***Frequency*** refers to the number of complete cycles made by an alternating Voltage per second.

(ii) ***Root mean square value*** is the value of the steady voltage which dissipates heat energy in a given resistor at the same rate as the alternating voltage.
(ii) It's variation with time is represented by a sine curve.

(b) ***The full wave Rectifier meter***

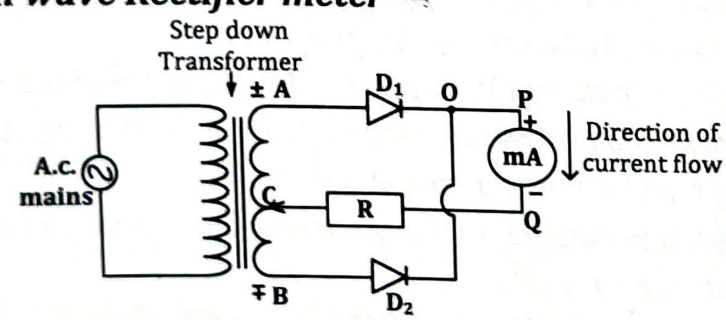

Fig.5.6 (e)

- During the first half of the a.c. input when terminal **A** of the a.c is positive with respect to **B**, diode (rectifier) **D_1** conducts current while D_2 does not.
- Current flows from **A** through diode D_1, to **P** through the milli ammeter in the direction **PQ** to **C** and back to **A**.
- Current flows from **A** through **D_1**, to **P** through resistor the mA to **Q** in the direction **PQ** causing a deflection on the scale of the mA then a current pulse flows to **C** and back to A.
- The first half of the a.c input cycle is then rectified.
- During the change of current from terminal **A** to terminal **B**, current drops to zero in the mA
- When **B** is positive with respect to **A**, diode **D_2** conducts current while **D_1** does not. Current flows from B through **D_2**, to **O** to **P**, through the mA then

to Q in the direction PQ causing a deflection on the scale of the mA due to a current pulse in it and flows to C and back to B.

- Hence, the second half cycle of the a.c. input is now also rectified.
- In both half cycles, current flows through R in the same direction **Q** to **C**.
- The pointer of the mill-ammeter connected in series with **R** deflects as the deflection is proportional to the average of the square of the current.
- The process repeats itself several times per second for all the subsequent cycles producing direct current flow through the mA of output shown in figure 5.6 (f)

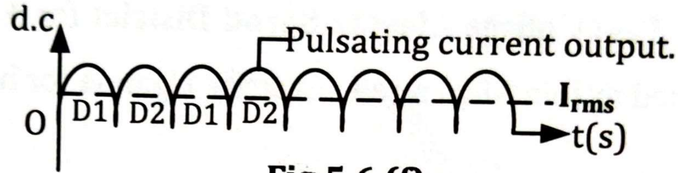

Fig.5.6 (f)

(ii) *It can measure both A.C and D.C unlike moving coil meters that can only measure D.C. and not A.C.*
It can measure large values of current unlike moving coil meters whose coil burns up when overloaded.
It's relatively cheaper to construct compared to the moving coil meter.
It's also more durable i.e. long lasting since its coil is not as delicate as that of a moving coil meter that easily burns up when overloaded.

(c) (i) ***Resistance*** is the dissipative opposition to the flow of current in a conductor.
Reactance is the non-resistive (or non – dissipative) opposition to the flow of current through a capacitor and an inductor.
Impedance is the resultant opposition to the flow of current through both reactive and resistive components connected together.

(ii) Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$\frac{V}{I} = Z = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$\frac{20}{40\times10^{-3}} = \sqrt{300^2 + \left(2\pi\times50\times L - \frac{1}{2\pi\times50\times16\times10^{-6}}\right)^2}$$

$$\therefore L = 1.906H$$

(d) *During the first quarter cycle, the capacitor charges and energy is drawn from the source and stored in the electric field of the capacitor when applied voltage increases to a maximum value.*
During the second quarter cycle, applied voltage decreases, the capacitor discharges and energy is returned to the source.
During the third quarter cycle, the capacitor charges in the opposite direction as applied voltage increases, again energy is stored in the electric field of the capacitor.
In the last quarter cycle, applied voltage decreases, the capacitor discharges and energy returns to the source.
Thus in one cycle, there is a no net energy in the capacitor, hence a capacitor is a wattles component.