UGANDA ADVANCED CERTIFICATE OF EDUCATION

MOCK EXAMINATION

MATHEMATICS

PAPER 1 (PURE MATHEMATICS) — SCENARIO FORMAT

TIME: 2½ HOURS

INSTRUCTIONS TO CANDIDATES:

- This paper consists of two sections: **Section A** and **Section B**.
- **Section A is compulsory.** It contains **one (1) complex scenario** on Geometry.
- **Section B** consists of two parts:
  - **Part (i): Algebra Scenarios** (Contains two scenarios; answer only **one**).
  - **Part (ii): Calculus Scenarios** (Contains two scenarios; answer only **one**).
- In total, candidates must solve **three (3) full scenarios**: the compulsory scenario in Section A, one from Section B Part (i), and one from Section B Part (ii).
- Clearly show all structural formulations, coordinate models, and mathematical reasoning.

SECTION A (COMPULSORY)

ANSWER THE SINGLE SCENARIO IN THIS SECTION.

Scenario 1: The Memorial Park Infrastructure Project (Geometry 1 & 2)

The Situation:
A local council has allocated a plot of land to build a community structural memorial park dedicated to historical local leaders, including the late matriarch **Mwatumu Kantono Katooko**. A civil engineer maps out the layout on a Cartesian grid system where 1 unit represents 10 meters.

Two central pedestrian walkways cross through the park. On the grid map, the path for Walkway Alpha is represented by the linear equation $2x + 3y - 7 = 0$, while the path for Walkway Beta is modeled by $3x - y - 5 = 0$. The council intends to construct a large, uniform circular water fountain at the exact point where these two walkways intersect. To ensure the fountain fits within the landscape design, its outer perimeter concrete foundation ring must pass directly through a designated botanical garden corner post situated at the coordinates $P(2, 6)$.

Furthermore, for security and crowd control during public visits, two straight, protective iron railings running perfectly parallel to the horizontal x-axis of the grid map must be installed. These railings must act as tangents that just touch the northernmost and southernmost boundary edges of the circular fountain.

The Task:
As the structural land surveyor, write an engineering report to the construction committee to guide the site workers:

1. Formulate and solve the simultaneous linear models to determine the exact grid coordinates of the fountain's central axis.
2. Calculate the precise radius length of the water fountain foundation and derive the standard expanded Cartesian equation of the circle ($x^2 + y^2 + 2gx + 2fy + c = 0$) to map its perimeter.
3. Determine the mathematical equations representing the exact placement lines for the two horizontal protective safety railings.

SECTION B

ANSWER ONE SCENARIO FROM PART (I) AND ONE SCENARIO FROM PART (II).

PART (I): ALGEBRA SCENARIOS

Scenario 2: Cooperative Farm Resource Management (Algebra 1 & 2)

The Situation:

An agricultural cooperative runs three mixed-farming blocks to supply food to a trading center. The operational costs and resource allocations rely on managing three primary inputs: specialized nitrogen fertilizer (x), organic compost (y), and irrigation water pumping hours (z).

The data tracking system records the resource dynamics across three distinct processing cycles, leading to the following weekly structural balance equations:

$$3x + 2y - z = 4$$

$$2x - y + 3z = 9$$

$$x + 3y - 2z = -1$$

Separately, the cooperative's research team analyzes the seasonal crop yield variability. They determine that the optimal efficiency thresholds are governed by a quadratic function $3x^2 - 6x + 2 = 0$, whose mathematical roots are denoted by α and β. To calibrate a new automated macro-nutrient dispenser, the engineers require a new mathematical boundary function whose root properties are precisely defined as $(2\alpha + 1/\beta)$ and $(2\beta + 1/\alpha)$.

The Task:

As the technical data analyst for the agricultural cooperative:

1. Apply systematic elimination or matrix techniques to solve the simultaneous system of resource equations, determining the precise operational levels of x, y, and z.
2. Without solving the quadratic yield function directly, determine the values of $(\alpha + \beta)$ and $\alpha\beta$.
3. Construct the new calibrated quadratic function equation for the automated dispenser using the transformed root parameters provided.

Scenario 3: Academic Development Workshop Planning (Progressions & Investment)

The Situation:
An educational institution is organizing a multi-phase professional development workshop series to improve classroom competencies under the New Lower Secondary Curriculum (NLSC). The planning committee designs a rolling recruitment model where the number of teachers trained increases over sequential sessions.

The number of teachers attending follows a strict arithmetic progression (A.P.). During the 3rd workshop session, exactly 11 teachers are trained, and by the 7th session, the attendance rises to 27 teachers. The regional education department will grant the institution a premium center-of-excellence status as soon as the cumulative number of teachers trained across all consecutive sessions strictly exceeds 500.

To sustainably fund this ongoing teacher professional development, the school board sets up a structural development asset fund. They invest a fixed principal sum of Shs 800,000 at the start of each fiscal year into an account that yields a guaranteed compound interest rate of 6% per annum.

The Task:
As the educational curriculum developer and planner, evaluate the project's growth and financial sustainability:

1. Determine the baseline number of teachers trained in the 1st session and the constant growth rate of attendees per session.
2. Calculate the minimum number of workshop sessions that must be executed for the school to hit its target and unlock the premium center-of-excellence status.
3. Compute the total accumulated financial value of the structural development asset fund at the end of 8 years to ensure long-term workshop funding.

<u>PART (II): CALCULUS SCENARIOS</u>

Scenario 4: Industrial Metalwork Design & Optimization (Calculus 1)

The Situation:

A vocational skill-acquisition center secures a contract to fabricate open-topped, rectangular sheet-metal storage bins to distribute to local health centers for medical waste management. The material must be utilized with maximum efficiency to conserve resources. Each bin must be constructed with a perfectly square base of side length x ***cm*** and a vertical height of h ***cm***.

The sheet metal sheets provided for each bin have a strict, constant total surface area of exactly ***108 cm^2***. The fabrication instructor wants the students to understand how structural shape dimensions directly dictate internal carrying capacity. They must first prove the mathematical relationship from first principles using a base quadratic function $f(x) = 2x^2 - 5x + 3$ to demonstrate how rates of surface area accumulation scale.

The Task:

As the industrial design instructor, generate the mathematical proof and optimization guidelines for the students:

1. Using the fundamental definition of a derivative from first principles, find the gradient function of $f(x) = 2x^2 - 5x + 3$.
2. Show structurally that since the total surface area is limited to ***108 cm^2***, the total internal holding volume V of the fabricated medical bin can be modeled by the single-variable function:

$$V = 27x - x^3/4$$

3. Apply differential calculus to determine the exact base dimension x and height h that will maximize the volume of the storage bin, and state this maximum capacity.

Scenario 5: Hydro-Engineering & Infrastructure Planning (Calculus 2)

The Situation:

A district civil engineering department is designing a local water supply and drainage channel network near a rural trading center. The cross-sectional profile of a natural hill ridge valley is modeled mathematically by the parabolic function curve $y = 6x - x^2$. To manage overflow during heavy downpours, a straight drainage pipeline must be constructed along the valley floor, following a path modeled by the linear equation $y = x$.

To clear the land and lay the structural concrete foundations for the drainage pipeline safely, engineers must first establish the exact geometric boundaries where the pipeline cuts through the hill ridge. They must then map out a cross-sectional safety zone. The local planning authority requires the exact measurement of the total surface area enclosed between the parabolic hill ridge and the straight pipeline path to allocate the correct environmental compensation funds to the surrounding community.

The Task:

As the leading hydraulic structural engineer on this project:

1. Determine the precise coordinate points on the grid map where the drainage pipeline intersects the parabolic hill ridge.
2. Sketch the enclosed cross-sectional profile of the valley safety zone, showing the boundaries clearly.
3. Set up a structural definite integral expression and evaluate it to compute the exact regional area enclosed between the curve $y = 6x - x^2$ and the line $y = x$.