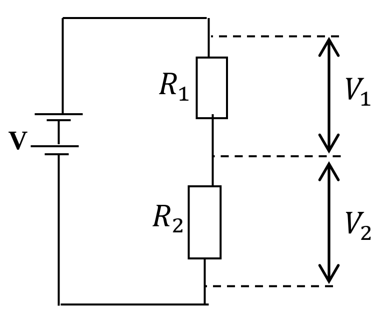
$E = I(R_2 + r)$
$E = 0.3(1 + r)$
$E = 0.3 + 0.3r \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(ii)$

Solving equations (i) and (ii) simultaneously, we get;
$\underline{E = 1.5V, \text{and } r = 4\Omega}$

Example:6
A d.c source of e.m.f 16V and negligible internal resistance is connected in series with two resistors of 400Ω and RΩ respectively. When the voltmeter is connected across the 400Ω resistor, it reads 4.0V, while it reads 6.0V when connected across the RΩ resistor. Find the;
(i). Resistance of the voltmeter [$R_v = 400\Omega$].
(ii). Value of R [$R = 600\Omega$].

POTENTIAL DIVIDERS
When resistors are arranged in series, they form what is called a potential divider.

Consider two resistors R_1 and R_2 connected in series with a d.c supply of V volts and negligible internal resistance.



Total resistance: $R = R_1 + R_2 \ldots\ldots\ldots$ (i)

From the circuit formula;
Current flowing, $I = \frac{\text{E.m.f}}{\text{Total resistance}}$
$I = \frac{V}{R_1+R_2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (ii)

From Ohm's law;
$V_1 = IR_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (iii)
$V_2 = IR_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (iv)

Put Eqn.(ii) into Eqn.(iii) and Eqn.(iv)
$V_1 = \left(\frac{V}{R_1+R_2}\right)R_1.$

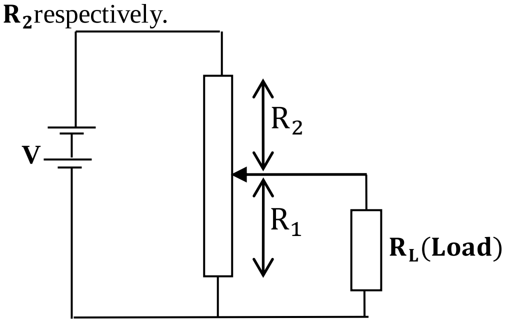
$V_1 = \left(\frac{R_1}{R_1+R_2}\right)V \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (v)

$V_2 = \left(\frac{R_2}{R_1+R_2}\right)V \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (vi)

Thus $\mathbf{V_1 : V_2 = R_1 : R_2}$

Therefore, for any resistors arranged in series, the p.d V_i across R_i is given by:
$$\mathbf{V_i = \left(\frac{R_i}{TotalResistance}\right) \times Supplyvoltage}$$

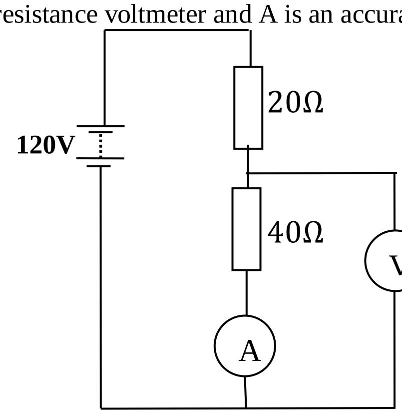
Note:
In potential dividers, the load is connected across a section of the divider whose P.d is equal to the operating voltage of the load. The load may be a cooker, an electric bulb, Radio or Tv sets, e.t.c. The load resistance, $\mathbf{R_L}$, will now be considered to be in parallel with the resistance of the lower section of the divider.
Let the resistance of the lower and upper sections be $\mathbf{R_1}$and $\mathbf{R_2}$respectively.



Now R_1 is in parallel with and R_L.
P.d across R_1= P.d across R_L.
Let R_{eff} be the effective resistance of R_1 and R_L.
$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_L}$$

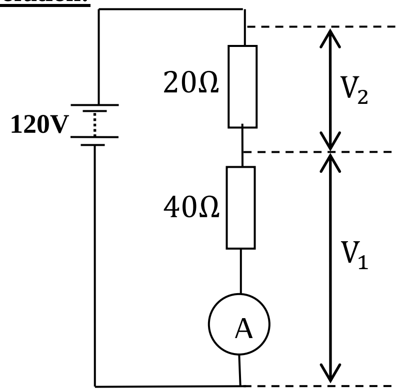
Now R_{eff} is in series withR_2. Thus;
$\mathbf{Total\ resisrance\ ; R = R_{eff} + R_2}$

Example: 1
The figure below shows a potential divider. V is a very high resistance voltmeter and A is an accurate ammeter.



(i) Find the ammeter and voltmeter readings.
(ii) If the voltmeter was replaced by another voltmeter of resistance120Ω., what will be its new reading?
(iii) Find the percentage change in the ammeter reading.
(iv) If the voltmeter above is replaced by a C.R.O, what will be its new reading?

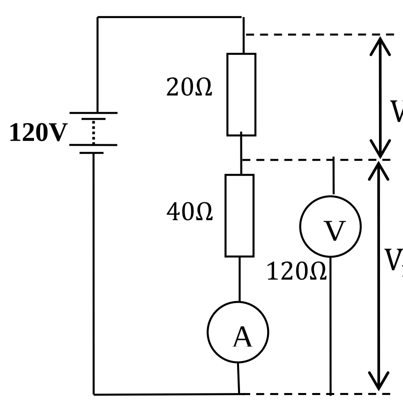
Solution:



(i)
Total resistance; $R = 20 + 40 = 60\Omega$
From the circuit formula;
$$I = \frac{\text{E.m.f}}{\text{Total resistance}}$$
$$I = \frac{120}{60} = 2A$$
Thus the ammeter reading was 2 amperes.

(ii)
From Ohm's law;
$V = IR$
$V = 2(40)$
$V = 80V.$

(iii)



Now the 40Ω and 120Ω are in parallel.
Let R be the effective resistance in parallel.
$$\frac{1}{R} = \frac{1}{40} + \frac{1}{120}$$
$R = 30\Omega$

Now R and the 20Ω are in series. Thus;
Total resistance; $R = 20 + 30 = 50\Omega$
From the circuit formula;
$$I = \frac{\text{E.m.f}}{\text{Total resistance}}$$
$$I = \frac{120}{50} = 2.4A$$
Thus the ammeter reading will be 2.4 amperes.

From Ohm's law;
$V = IR_1$
$V = 2.4(30)$
$V = 72V.$
Thus the new voltmeter reading will be 72Volts.

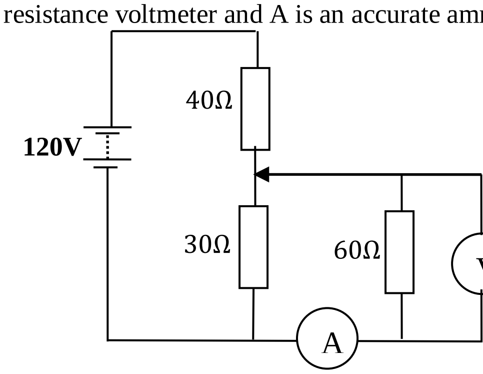
From Ohm's law;
$V = I_1R$
$120 = I_1(40)$
$I_1 = 1.8A.$

Percentage change in current will be;

$$\%\ change; = \frac{Change\ in\ current}{Original\ value} \times 100\%$$
$$\%\ change; = \left(\frac{I - I_1}{I}\right) \times 100\%$$
$$\%\ change; = \left(\frac{2 - 1.8}{2}\right) \times 100\%$$
$\%\ change; = 10\%$

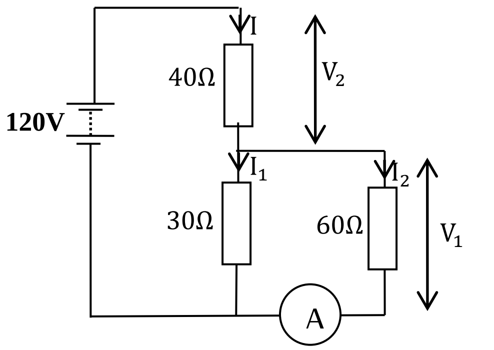
(v) A C.R.O is considered as an ideal voltmeter of infinite resistance, which makes it very accurate. Hence its reading would be the same as in (i) above. i.e. **80V**.

Example:2
The figure below shows a potential divider. V is a very high resistance voltmeter and A is an accurate ammeter.



(vi) Find the ammeter and voltmeter readings.
(vii) If the voltmeter was replaced by another voltmeter of resistance120Ω., what will be its new reading?
(viii) Find the percentage change in the ammeter reading.

Solution.
(i)



The 30Ω and 60Ω are in parallel.
Let R be the effective resistance in parallel.

$\frac{1}{R} = \frac{1}{30} + \frac{1}{60}$

<u>R = 20Ω</u>

Total resistance; $R = 20 + 40 = 60\Omega$

From the circuit formula;

$I = \frac{E.m.f}{Total\ resistance}$

$I = \frac{120}{60} = 2A$

<u>Thus the ammeter reading was 2 amperes</u>.

From Ohm's law;

V = IR

V = 2(20)

<u>V = 40V.</u>

<u>Thus the voltmeter reading was 40 volts.</u>

From Ohm's law;

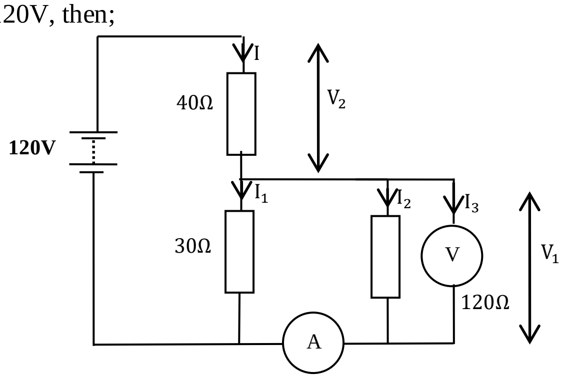
$V = I_2R$

$40 = I_2(60)$

$I_2 = 0.667A$.

$I_1 = (I - I_2) = (2 - 0.667) = 1.333A$

(ii)
If the voltmeter is replaced by another voltmeter of resistance 120V, then;



The 30Ω, 60Ω and the 120Ω are in parallel.
Let R be the effective resistance in parallel.

$\frac{1}{R} = \frac{1}{30} + \frac{1}{60} + \frac{1}{120}$

<u>R = 17.143Ω</u>

Total resistance; $R = 17.143 + 40 = 57.143\Omega$

From the circuit formula;

$I = \frac{E.m.f}{Total\ resistance}$

$I = \frac{120}{57.143} = 2.10A$

<u>Thus the ammeter reading will be 2.10 amperes</u>.

From Ohm's law;

$V_1 = IR$

$V_1 = 2.10(17.143)$

$V_1 = 36V$

<u>Thus the voltmeter reading will be 36 volts.</u>

From Ohm's law;

$V = IR$

$I = \frac{V}{R}$

$I_1 = \frac{V}{R_1} = \frac{36}{30} = 1.2A$

$I_2 = \frac{V}{R_2} = \frac{36}{60} = 0.6A$

$I_3 = \frac{V}{R_1} = \frac{36}{120} = 0.3A$

Ammeter reading $= I_2 + I_3$

Ammeter reading $= 0.6 + 0.3$

<u>Ammeter reading = 0.9A</u>

(iii)

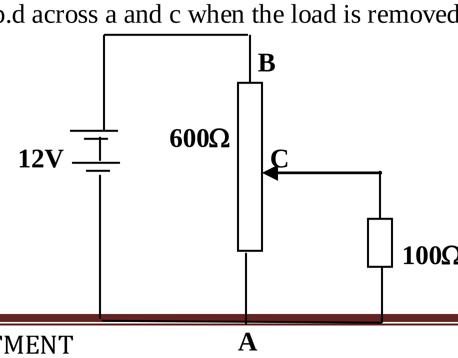
$\%\ change; = \frac{Change\ in\ current}{Original\ value} \times 100\%$

$\%\ change; = \left(\frac{I - I_1}{I}\right) \times 100\%$

$\%\ change; = \left(\frac{2 - 0.9}{2}\right) \times 100\%$

<u>$\%\ change; = 55\%$</u>

3. a 12V battery is connected across a potential divider of resistance 600Ω as shown below. If the load of 100Ω is connected across the terminals a and c when the slider is half way up the divider, find:
(i) p.d across the load
(ii) p.d across a and c when the load is removed.



Effective resistance
$R = \frac{300 \times 100}{300 + 100} + 300 = 375\Omega$
Current supplied by the battery, I; From Ohm's law
$V = IR$
$12 = 375I$
$I = 0.032A$
This the current through parallel combination of resistors

P.d across parallel combination of resistors, $V = 0.032 \times 75 = 2.4V$

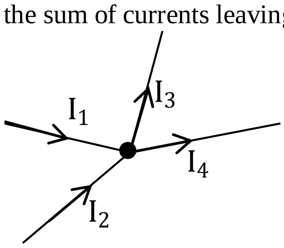
Hence the p.d across the load is 2.4V.

(ii) When the load is removed; Effective resistance = 600Ω.
From Ohm's law;
$V = IR$
$12 = 600I$
$I = 0.02A$
Hence p.d across AC is given by;
$$V_{AC} = 0.02 \times 300 = 6V$$

3. The circuit below, the battery has negligible internal resistance. Find the ammeter and voltmeter reading.

KIRCHHOFF'S LAWS

These are two laws:
1st law: This law is also referred to as the law of conservation of current at a junction.
It is derived from the fact that charges do not accumulate at a point.
It states that the sum of currents entering a junction is equal to the sum of currents leaving the junction.



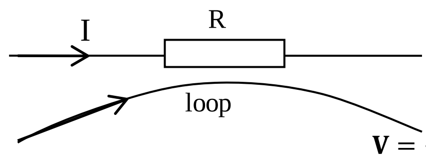
From Kirchhoff's 1st law;
$\mathbf{I_1 + I_2 = I_3 + I_4}$

OR:
It states that the algebraic sum of all currents at a junction is zero.
Here all currents entering a junction are given a positive sign and those leaving the junction are given a negative sign.
That is;
$$\sum \mathbf{I = 0}$$
$I_1 + I_2 + (-I_3) + (-I_4) = 0$
$I_1 + I_2 - I_3 - I_4 = 0$

2nd law: This law is sometimes referred to as the closed loop equation.
It states that in any closed loop, the algebraic sum of all potential drops is equal to the algebraic sum of all the E.m.fs.
That is;
$$\sum \mathbf{P.d} = \sum \mathbf{E.m.f}$$

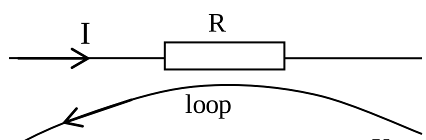
Note: A loop is a closed path.

Sign allocation to P.ds
When allocating signs to P.ds across a resistor in a closed loop, the direction of current flowing through the resistor is considered in relation to the direction of the loop.
The p.d across the resistor is considered positive if the current is in the same direction as that of the loop.



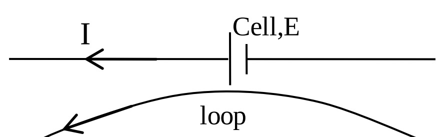
$\mathbf{V = +IR}$

The p.d across the resistor is considered negative if the current is in the opposite direction to that of the loop.



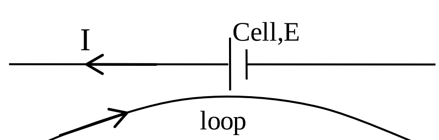
$\mathbf{V = -IR}$

Sign allocation to E.m.fs
When allocating signs to E.m.f a cell in a closed loop, the polarity of the cell is considered in relation to the direction of the loop.
The E.m.f of the cell is considered positive if the loop moves from negative terminal to the positive terminal.



$\mathbf{E = +ve}$

The E.m.f of the cell is considered negative, if the loop moves from positive terminal towards the negative terminal.



$\mathbf{E = -ve}$

Remarks:
In some circuit networks, the direction of current may not be given. In such cases, a student is free to choose his own convenient direction. But after the calculation, a positive value of current will imply that the actual current flows in the