**Surds, indices and logarithms**

1. Nambi designed a mat and recorded its width as $25^{\frac{w}{2}}$, the length as $5^{w+1}$ and covered an area of 125 square units. She tasked you to find the value of w, and also determine for which value of w will the length be 3

2. The number of goals scored by Arsenal and Manchester city during the English Premier league are represented by x and y respectively are related by two equations: $1 + \log_2 (2x+y) = \frac{\log_3 12}{\log_3 2}$. Following the analysis, Arsenal won the league with points given by the expression 17(K - c) where $K = \sqrt{28+10\sqrt{x}}$. Determine the number of goals scored by each team and the number of points Arsenal had at the end of the season.

3. Your scout patrol at Kaazi Camp finds a locked treasure box with a 4-digit passcode. To open it, you must solve two mathematical statements written on the lid.

   **Statement One:** $2^{2x} + 2 = 3(2^x)$

   **Statement Two:** $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$

   **Statement Three:** $\frac{x-y}{x+y} = \frac{\log_b c-\log_b a}{\log_b c+\log_b a}$

   **Rules:**

   I. Solving statement one yields two values of x where the smaller of the two is the first digit and the bigger is the 2nd digit.

   II. Expressing statement two in the form $a - b\sqrt{c}$ yields the remaining digits such that a and b are the 3rd and 4th digits respectively.

   III. If $\log_a n=x$ and $\log_{bc} n=y$, prove the statement three to claim the prize.

   **Task:** As the patrol leader, help your unit to find the 4-digit passcode and thus claim the prize in locked in the treasure box.

4. A youth farming group uses a solar-powered computer to track their stored data. The data stored in gigabytes after t weeks is modeled by the function $P(t) = 2^{0.5t+3}$. A system report reveals the log entry: $\log P(t) = \log 8 + \log 10^{\sqrt{t}}$. A student named Henry found this expression in the system files: $\sqrt{35+8\sqrt{6}}$, and he had to express it as a summation of two positive square roots so as to get the product of the numbers in the square root which complete the passcode to their joint account. Determine, to the nearest years (1 year has

52 weeks), the exact values of t, and the final digit(s) of the joint account passcode.

5. Josephine installs a solar panel in a rural village. Its power output P (in watts) after t hours of sunlight is modeled by: $P = k \cdot 2^{0.5t}$ (where k is a constant dependent on the panel type. After 4 hours of sunlight, the panel generates 40 watts. Determine:

   a. How many hours will produce 160 watts?

   b. How much power will be produced by the panel after 1 hour, write your answer on the form $\sqrt{a}$

6. A biologist models the population sizes, x and y, of two interacting bacterial species X and Y using logarithmic equations.

   The individual growth profile of species X is controlled by the equation:

   $$\log_4 (6-x) = \log_2 x$$

   The ecological interaction between species X and Y follows the simultaneous conditions:

   $$\log_{10} x - \log_{10} y = \log_{10} 2.5$$

   $$\log_{10} x + \log_{10} y = 1$$

   Help the biologist to determine valid biological population value for x and thus verify if this value remains consistent even in the ecological interaction model.

7. As an engineering intern at Kiira Motors Corporation, you are analyzing the dimensions of a triangular battery mounting bracket for a new electric vehicle. The chief engineer provided you with two preliminary side lengths of the bracket: $\sqrt{75}$ cm and $\sqrt{108}$ cm. Express the total length of the two sides in the form $b\sqrt{a}$, and determine the exact length of the third side given that total length of the lining around the entire bracket is exactly 25 cm.

## Simultaneous equations

1. Three schools A, B and C participated in a football tournament where a unique awarding system was used. School A obtained 3 wins, 2 draws and 6 losses, B had 4 wins, 7 draws and 3 losses while C got 2 wins, 2 draws and 10 losses. The total points scored by A, B and C at the end of the tournament were 42, 44 and 26 respectively. Determine the number of points awarded

for a win, a draw and a loss.

2. Three school clubs: Drama, Science and Maths raised a total of Shs. 600,000 for a joint event. The treasurer recorded these details: The Science club raised Shs. 50,000 less than the Drama club. The product of the amount raised by the drama and the math club is 'two hundred fifty' squared, times the total amount contributed by the three clubs. No club contributed more than twice any other club. The Drama, Science and Maths teachers contributed $\frac{2}{5}$, $\frac{3}{4}$ and $\frac{1}{3}$ respectively of what their individual clubs raised. Help the club patron to determine the amount of money contributed by each teacher.

3. A Ugandan agribusiness enterprise in Mbarara packages three types of organic fertilizers: Super-Gro, Yield-Max, and Soil-Fix. A production manager tracks the weekly raw material allocations based on three strict factory constraints. Packaging one sack of Super-Gro, two sacks of Yield-Max, and one sack of Soil-Fix consumes a combined total of 2,400 kg of raw phosphate. Processing two sacks of Super-Gro, one sack of Yield-Max, and three sacks of Soil-Fix requires a total of 3,900 factory minutes. Transporting three sacks of Super-Gro, four sacks of Yield-Max, and two sacks of Soil-Fix utilizes a total distribution volume of 5,100 cubic units. Determine exact number of sacks of each fertilizer that must be packaged to fully utilize the allocated resources.

4. A community group in Gulu is managing the costs for drilling three boreholes (A, B, and C). The total cost was UGX 25,000,000. The cost of Borehole B was UGX 1,000,000 less than Borehole A. The combined cost of Boreholes A and C was three times the cost of Borehole B.

   Determine the cost of drilling all the bore holes if drilling each meter costs Sh. 250,000.

5.

**Geometry 1**

1. On a construction site, a ladder was inclined on a vertical wall such that it passes through points A( - 2, 0) and B(0, 4) on the ground and top of the wall respectively. The horizontal ground from the ladder meets the vertical wall at the origin, O. Determine the centroid, orthocentre and the circumcentre of the resulting triangle.

2. During the 130years anniversary celebrations at Mengo senior School, the first lady's tent was set up in the triangular shape with vertices (3, 2), (1, 4) and (5, 4). The tent was surrounded by a circular bulletproof glass which passed through its vertices and the first lady's seat was placed at the center

of this circular fence. A security van had to be parked as close to the first lady's seat as possible, either on the straight stretch before the tent defined by the equation $2x = 12 + 5y$, or at the goal post positioned at (7,8). Determine which position was the van parked.

## Trigonometry

1. A security drone has arms given by the parametric equations $x = 3\cos 2A$ and $y=8\sin A\cos A$ where A is a reflex angle of projection. The maximum height H in meters reached by the drone is given by the equation: $H = x + y$. Show that $16x^2 + 9y^2 = 144$ and find the maximum height reached by the drone with the corresponding angle of projection.

2. In a mathematics test, learners were required to verify whether it is true that $\cos x$ can be written as $\frac{\cos x}{\sin^2 x} - \cos x\cot^2 x$. Two learners approached you after the test to confirm if this was true. Help the learners to verify if statement is true.

3. An information security analyst uses two trigonometric models to design a secure alarm system, an Encryption key generator modelled by a periodic function: $\cos 2x = \sin 3x$ (where x is an acute angle) and an Alarm audio wave modelled by: $y = \sin(\theta) + 2\cos\ \ (\theta)$. Determine the possible value(s) of x that satisfy the encryption key and the values of θ within the interval $|\theta| \le \pi$ where audio wave crosses the horizontal axis.

4. A robot arm moves such that the position of its end is described by: $x = a\sin\theta$, $y = b\cot\theta$ where θ is the angle of rotation and a, b are constants. Show that $x = \pm\frac{ab}{\sqrt{b^2+y^2}}$. Given that $a = \frac{5}{3}$, $b = 6$ and $xy = 8$, determine the smallest reflex value of θ.

## Roots of quadratic equations

1. In a mathematics contest, the learners were required to prove that $c(a-b)^3 - a(c-b)^3$ if one of the roots of the equation $ax^2 + bx + c = 0$ is a square of the other.

2. 2 and 3 are factors of 6, hence 6 can be written as 2×6, express $x^3 - 3x^2 - 10x + 24$ as a product.

3. Determine the possible values of p if the equation $x^2 - 8x = 8px - 64$ has repeated roots.

4. Mrs. Nabukalu, a commercial maize farmer in Masaka, models her seasonal crop yield, Y (in bags per acre), based on the amount of fertilizer applied, x (in kg per acre), using the quadratic function: $y = -0.5x^2 + 20x + 50$. Fertilizer

costs UGX 1,500 per kg and she wants to optimize her application rates to avoid wasteful spending. She intends to find the exact amount of fertilizer that maximizes her production, while also determining the operational range of fertilizer she can purchase to safely maintain a strict baseline yield of at least 200 bags per acre.

a. Help her to calculate the maximum possible yield she should expect from her land and the exact mass of fertilizer she will apply.

b. Determine the precise range of fertilizer amounts she should buy.

5. A school in Mbarara wants to create a rectangular vegetable garden. They have 80 meters of fencing available. Determine the possible range of values for the length of the garden such that the area of the garden exceeds 300 square meters.

**POLYNOMIALS**

1. An automotive engineering team at Kiira Motors Corporation in Jinja is modelling the fuel-injection profile for an eco-friendly engine. The fluid velocity profile is governed by a cubic polynomial function, $P(x) = x^3 + ax^2 + bx + c$, where a, b, and c are design constants. For the engine to run without stalling, the fuel-injection system must experience absolute zero pressure drops (a factor state where the net remainder is zero) at exactly three test calibrations: $x = -1$, $x = 2$, and $x = 4$. Determine the values of the design constraints a, b and c, and verify their correctness by using the given factors to form the polynomial function.

2. A software engineer at a logistics hub in Malaba is programming an automated tracking system that uses a cubic polynomial function, f(x), to generate security verification codes. During a system calibration check, the engineer observes that when the polynomial configuration path is divided by an indexing factor of $x + 1$, it leaves a data remainder of 7. Similarly, when the same polynomial configuration is divided by an indexing factor of $x - 3$, it leaves a data remainder of -8. To optimize the system's security architecture, the engineer needs to determine the resulting algebraic remainder when this polynomial function is processed through a combined dual-channel filter modelled by the product factor $(x + 1)(x - 4)$. Help the engineer to determine the remainder upon dividing by the combined dual-channel and the values of

x for which $f(x) = 0$ given that the quotient is $(2x-1)$ upon dividing by the combined dual-channel.

3. An environmental engineer at the National Environment Management Authority (NEMA) uses a polynomial function, $f(x) = 2x^3 - kx^2 + px - 12$, to model the waste decay rate in a landfill outside Kampala. The engineer knows that the chemical breakdown process stabilizes perfectly without any baseline remnants when the treatment path follows the exact linear dimensions of $(x - 2)$ and $(x + 3)$. The operations director needs to identify the hidden system variables, k and p, to forecast long-term landfill safety. Help the engineer to determine the values of k and p for which $(x - 2)$ and $(x + 3)$ exactly divide $f(x)$

4. A logistics company in Malaba optimizes its cross-border warehouse storage distribution using an algebraic capacity function modelled by a symmetric polynomial: $V = x^3 - 2x^2 - 5x + k$. The data operations manager notes that the warehouse operates at absolute perfect capacity efficiency whenever the delivery schedules, represented by the linear variables $(x + 2)$ and $(x - 3)$, perfectly divide the capacity function without any leftover tracking errors. Prove that both $(x + 2)$ and $(x - 3)$ are simultaneously valid factors of V by finding a single unique value for the capacity constant k that satisfies both constraints. Hence deduce the other hidden linear factor of the warehouse system.

5. A structural engineer working with UNRA is designing a reinforced concrete support beam for a new flyover bridge in Kampala. The bending moment of the beam along its length x is modelled by the quartic polynomial expression $P = 2x^4 - 9x^3 + ax^2 + bx - 12$. To prevent structural cracks under maximum load, the beam design requires that the clearance expression $(x-2)^2$ must be a perfect repeated factor of $P(x)$. If this condition is met, the beam will seamlessly bear the traffic weight without generating any internal tension remnants. Determine the exact values of the reinforcement parameters a and b. Hence, find the remaining two safety coordinates where the bending moment is zero.

6. A telecommunications technician in Kampala is configuring a network signal booster. The signal strength over a distance x is represented by the polynomial equation $S(x) = 3x^3 + 7x^2 + mx + n$. To prevent dropped calls, the signal profile must be perfectly divisible by the square of a specific frequency configuration path, modelled by $(x+2)^2$, leaving a net data remainder of 0. Deduce the precise values of the tuning variables m and n that ensure

perfect network signal division.

7. A civil engineer designing the slope profile of a new highway flyover in Entebbe maps the structural load using a cubic polynomial curve, $P(x) = x^3 - 4x^2 + ax + b$. Initial safety checks reveal that dividing this structural curve by a drainage clearing width of $(x - 1)$ leaves a structural stress remainder of 3. A secondary check reveals that dividing the exact same curve by an alternative clearing width of $(x + 2)$ leaves a stress remainder of -24. The construction team cannot clear the ground without knowing the exact shape of the flyover slope. Find the values of a and b, and write down the fully completed polynomial expression for P(x). Also, determine the stress remainder if the team decides to use a clearing width of $(x - 3)$.

8. A cyber-security firm in Kampala is auditing a data encryption matrix governed by a high-degree polynomial function, f(x). The audit log shows that when the encryption routine is divided by individual security keys $(x - 1)$, $(x - 2)$, and $(x + 3)$, it generates data leak remainders of 5, 11 and -9 respectively. The network security team needs to deploy an emergency patch filter modelled by the cubic divisor expression $(x - 1)(x - 2)(x + 3)$. To safely configure the patch, they must first identify the unique quadratic remainder function, R(x), that will be left behind by this operation. As the lead cryptographic analyst, solve the system to determine the quadratic remainder, R(x).

9. An aerospace engineering research group at Makerere University is testing a structural flight path modelled by the quartic polynomial equation $P(x) = ax^4 + bx^3 - 7x^2 + 12x - 4$. For the flight path to maintain maximum aerodynamic stability, the polynomial must be perfectly divisible by a safety clearance factor modelled by the perfect square trinomial $(x-1)^2$. If any algebraic remainder occurs, the vehicle will experience dangerous turbulence. The chief engineer requires the exact values of the design parameters a and b to eliminate any remainder from the system. Help the chief engineer to calculate the exact values of the design parameters a and b and Confirm your results by rewriting the complete polynomial and using long division or factorization to show that the net remainder is identically zero.

10. An agricultural processing plant in Kasese uses an automated sorting machine whose speed control is calibrated by a polynomial function $Q(x) = 3x^3 - 5x^2 + qx + 8$. The operations manual states that the machine registers an identical operational stress value when calibrated at two different node settings. Specifically, dividing Q(x) by the low-frequency

setting $(x - k)$ yields the exact same numerical remainder as dividing it by the high-frequency setting $(x + k)$, where k is a non-zero operational constant. Furthermore, a secondary mechanical check shows that dividing $Q(x)$ by $(x - 2k)$ produces a peak load remainder of 72. As the industrial systems technician, determine the exact values of the operational constant k and the tracking coefficient q, assuming k must be a positive integer.

**11.** A telecommunications engineer at a base station in Gulu is troubleshooting a signal attenuation curve modelled by the polynomial function $f(x) = 2x^3 - 3x^2 - 4x + 12$. The automated system runs a diagnostic routine and outputs a clean, simplified signal quotient of $Q(x) = 2x^2 + x - 2$, but leaves a constant background noise remainder of 8. The engineer needs to identify the exact linear hardware filter expression, $d(x)$, that was used as the divisor in this operation to update the system log. As a junior network analyst, identify the linear hardware filter that was used.

**12.** A financial data analyst for an agribusiness firm in Jinja uses a revenue projection polynomial model given by $P(x) = x^4 - 2x^3 - 7x^2 + kx + 14$, where k is an unrecorded tracking coefficient. A software summary reports that dividing $P(x)$ by a specific quadratic investment index, $d(x)$, results in a precise quotient of $Q(x) = x^2 - 4x + 2$ and leaves a linear correction remainder of $R(x) = 4x + 10$. The analyst must recover both the missing tracking coefficient k and the structural quadratic divisor expression $d(x)$ to complete the financial audit. As the lead data auditor, determine the missing tracking coefficient k and the quadratic divisor, $d(x)$.

**13.** An industrial automation plant in Tororo programs a robotic packaging arm using a structural path function $h(x) = 3x^4 + 11x^3 + ax^2 - 14x + b$. Due to a system glitch, the coefficients a and b were corrupted in the main file. However, the backup drive successfully preserved the operational quotient of $(3x^2 + 2x - 5)$ and a linear safety remainder of $(x + 15)$. The plant supervisor needs to find the exact quadratic divisor component that represents the mechanical physical limits of the robotic arm. As the control systems technician, determine the missing coefficients and quadratic divisor used.