

UGANDA ADVANCED CERTIFICATE OF EDUCATION

Pure Mathematics: Scenario-Based Examination

Instructions to Candidates: This question requires the application of roots of quadratic equations, simultaneous equations, and partial fractions. All mathematical derivations must be shown clearly.

Scenario: Structural Stress Analysis

A structural engineer is analyzing the stress distribution $S(x)$ across a reinforced beam. The stress at a distance x is modeled by the rational function:

$$S(x) = \frac{7x^2 - 11x + 13}{(x - 1)(x^2 - px + q)}$$

The quadratic component $f(x) = x^2 - px + q$ has two distinct real roots, α and β . The following design constraints have been established:

- The sum of the roots ($\alpha + \beta$) exceeds their product ($\alpha\beta$) by exactly 1.
- The sum of the reciprocals of the squares of the roots is $\frac{5}{4}$; that is, $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{5}{4}$.
- To resolve the stress into component vectors, $S(x)$ must be expressed in partial fractions:

$$S(x) \equiv \frac{A}{x - 1} + \frac{Bx + C}{x^2 - px + q}$$

(a)

(i) Express the structural constraints in terms of p and q to form a system of simultaneous equations. **(04 marks)**

(ii) Show that p satisfies the quartic equation $5p^4 - 10p^3 - p^2 + 12p - 4 = 0$. **(04 marks)**

(iii) Given that $p = 2$ is one solution, determine the valid pair of values for (p, q) such that the roots α and β are distinct and real, and $q > 0$. **(03 marks)**

(b)

(i) Find the exact roots α and β using the values of p and q from part (a). **(03 marks)**

(ii) Construct a new quadratic equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. **(05 marks)**

(c)

By substituting the confirmed values of p and q into $S(x)$, resolve the expression into partial fractions to find the values of the constants A , B , and C . **(07 marks)**