



PURE MATHEMATICS SEMINAR

TROGONOMETRY

Pythagoras theorem

1. Solve the following equations, giving the values of θ from 0° to 360° inclusive.

a. $3 - 3\cos\theta = 2\sin^2\theta$

b. $\operatorname{cosec}^2\theta = 3\tan\theta - 1$

c. $2\cot^2\theta + 8 = 7\operatorname{cosec}\theta$

2. Prove the following identities

i. $\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$

ii. $\sec^2\theta - \operatorname{cosec}^2\theta = \tan^2\theta - \cot^2\theta$

iii. $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

iv. $\sec^2\theta - \operatorname{cosec}^2\theta = \frac{\sin^2\theta - \cos^2\theta}{\cos^4\theta\sin^4\theta}$

v. $\frac{1 - \cos^2\theta}{\sec^2\theta - 1} = 1 - \sin^2\theta$

3. Eliminate θ from the following equations

i. $x = a\cos\theta, y = b\sin\theta$

ii. $x = a\tan\theta, y = b\cos\theta$

iii. $x = a\sec\theta, y = b + d\cos\theta$

iv. $x = \cos\theta, y = \operatorname{cosec}\theta - \cot\theta$

The formula for $\sin(A \pm B)$, $\cos(A \pm B)$

1. Given that $\sin A = \frac{4}{5}$ and $\cos B = \frac{12}{13}$, where A is obtuse and B is acute, find the values of;

a. $\sin(A + B)$

b. $\cos(A + B)$

c. $\cot(A + B)$

d. $\sin(A - B)$

2. If $\tan(A + B) = \frac{1}{7}$ and $\tan A = 3$, find the value of $\tan B$

3. If $\sin(x - \alpha) = \cos(x - \alpha)$, prove that $\tan x$

4. Prove the following

a. $\sin(A + B) + \sin(A - B) = 2 \sin A \sin B$

b. $\cos(A + B) - \cos(A - B) = -2 \sin A \sin B$

c. $\tan A + \tan B = \frac{\sin(A+B)}{\cos A \cos B}$

5. Prove that

$$\tan(A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan A \tan C - \tan B \tan C}$$

Hence prove that if A, B, C are angles of a triangle, then

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$

The double angle formula

1. Prove that $\sin 3A = 3 \sin A - 4 \sin^3 A$
2. Prove that $\cos 3A = 4 \cos^3 A - 3 \cos A$
3. Solve the following equations for the values of θ from 0° to 360°
 - a. $3 \cos 2\theta - \sin \theta + 2 = 0$
 - b. $\sin 2\theta \cos \theta + \sin^2 \theta$
 - c. $3 \cot 2\theta + \cot \theta = 1$

4. Eliminate θ from the following equations

a. $x = \cos \theta, y = \cos 2\theta$

b. $x = 2 \sin \theta, y = 3 \cos 2\theta$

c. $x = \tan \theta, y = \tan 2\theta$

d. $x = 2 \sec \theta, y = \cos 2\theta$

5. Prove that

$$a. \frac{\cos 2\theta}{\cos A + \sin A} = \cos A - \sin A$$

$$b. \frac{\cos A}{\sin B} - \frac{\sin A}{\cos B} = \frac{2 \cos(A+B)}{\sin 2B}$$

$$c. \cot A - \tan A = 2 \cot 2A$$

$$d. \frac{\sin 2A}{1 + \cos 2A} = \tan A = \frac{1 - \cos 2A}{\sin 2A}$$

Section D: The t-Formulae

Solve the following equations for $0^\circ \leq \theta \leq 360^\circ$

a. $2 \cos \theta + 3 \sin \theta = -2$

b. $7 \cos \theta + \sin \theta - 5 = 0$

c. $3 \cos \theta - 4 \sin \theta + 1 = 0$

d. $3 \cos \theta + 4 \sin \theta = 2$

Section E: the form $a \cos \theta + b \sin \theta$

1. Show that $\sqrt{3} \cos \theta - \sin \theta$ maybe written as $2 \cos(\theta + 30^\circ)$ or $2 \sin(60^\circ - \theta)$. find the maximum and minimum values of the expression, state the values of θ between 0° and 360° for which they occur.
2. Show that $3 \cos \theta + 2 \sin \theta$ maybe written in the form $R \cos(\theta - \alpha)$; where α is acute. Find the values of R and α

GENERAL QUESTIONS

1. Solve the following equations for the values of x from 0° to 360° .

a. $\cos x + \cos 5x = 0$

b. $\sin 3x - \sin x = 0$

c. $\sin(x + 10^\circ) + \sin x = 0$

d. $\cos(2x + 10^\circ) + \cos(2x - 10^\circ) = 0$

2. Prove that $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$

3. Prove that $\frac{\sin B - \sin C}{\sin B + \sin C} = \cot \frac{B+C}{2} \tan \frac{B-C}{2}$

4. If $2A + B = 45$, show that $\tan B = \frac{1 - 2 \tan A - \tan^2 A}{1 + 2 \tan A - \tan^2 A}$

5. If A , B , and C are angles of a triangle, prove that

$$\cos A + \cos B + \cos C = 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$