WAVES

A wave is a periodic disturbance that transfers energy from one point to another in a medium without causing any permanent displacement of the medium itself.

CATEGORIES (CLASSIFICATION) OF WAVES

Waves are categorized or classified into two categories namely;

(i) Mechanical waves.

(ii) Electromagnetic waves.

MECHANICAL WAVES (MATERIAL WAVES)

These are waves that require a material medium for their propagation and therefore cannot travel in a vacuum.

Examples include; sound waves, water waves, waves in vibrating strings.

These waves cannot travel in a vacuum. They are produced by a disturbance of particles in a material medium and propagated by vibration of these particles about their fixed points.

ELECTROMAGNETIC WAVES

These are waves that do not require a material medium for their transmission (propagation) and can travel in a vacuum.

Examples include; light waves, ultraviolet light, infrared radiation, gamma rays, x-rays.

These waves travel at the speed of light i.e $3\times10^{8}\,ms^{-1}$. For this reason, they travel faster than mechanical waves.

Electromagnetic waves consist of disturbances in form of varying electric and magnetic fields which are perpendicular.

GENERATION AND PROPAGATION OF MECHANICAL WAVES

Mechanical waves are generated by a disturbance at one point in a material medium called the source of the wave. The disturbance causes the particles at the source to vibrate about their mean positions.

These vibrating particles collide with the neighboring particles causing these neighboring particles to also vibrate in turn.

In this way, the disturbance is transmitted from the source to the surrounding regions without the particles of the transmitting medium travelling between the regions.

The disturbance travelling outwards constitutes the wave.

N.B:

Because vibrating particles possess kinetic energy, the wave is able to transmit energy from one point in the transmitting medium to another without particles of the transmitting medium travelling between the points.

DIFFERENCES BETWEEN MECHANICAL AND ELECTROMAGNETIC WAVES

MECHANICAL WAVES	ELECTROMAGNETIC WAVES
• Require a material medium for propagation so can't travel in a vacuum	• Don't require a material medium for their propagation so can travel in a vacuum
• Travel at lower speed	• Travel at higher speeds
• Have lower frequency	• Have higher frequency
• Have longer wavelength	• Have shorter wavelength
• Propagated by oscillating particles of a material medium	• Propagated by varying electric and magnetic fields

TYPES (KINDS) OF WAVES

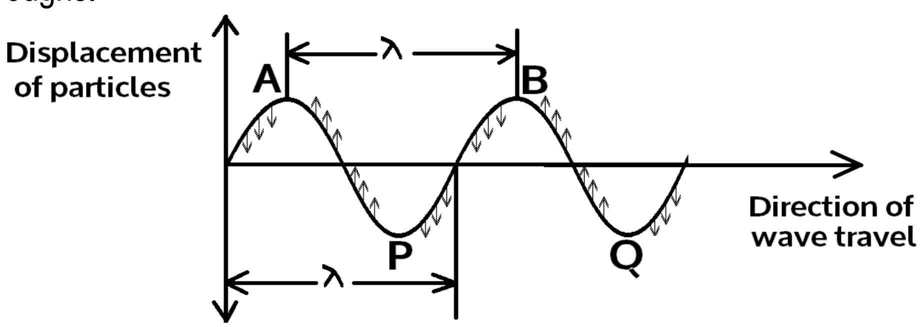
There are two types of waves, namely;

(i) Transverse waves

(ii) Longitudinal waves

TRANSVERSE WAVES

These are waves in which the direction of vibration of the particles of the medium is perpendicular to the direction of travel of the wave. These waves consist of crests and troughs.



- A and B are crests
- P and Q are troughs

A crest is a point of maximum positive displacement of a particle from the equilibrium position along a transverse wave profile.

At a region of a crest, a vibrating particle is above the equilibrium position and is at the maximum displacement from the rest position.

A trough is a point of maximum negative displacement of a particle from the equilibrium position along a transverse wave profile.

In the region of a trough, a vibrating particle is below the equilibrium position and is also at the maximum displacement from the rest position but in the negative direction of the displacement.

In a transverse wave, the distance between any two successive crests or troughs is called the **wavelength.**

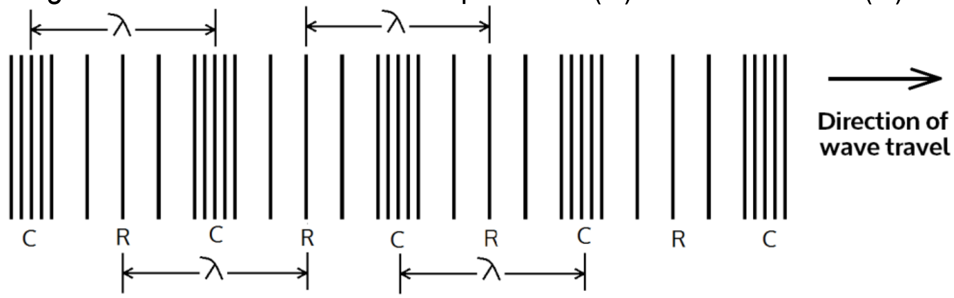
Examples of transverse waves include; -water waves, electromagnetic waves, etc.

LONGITUDINAL WAVES

These are waves in which the direction of vibration of the particles is parallel to the direction of propagation of the wave.

In these waves, the particles vibrate in the same direction as the direction of wave propagation or travel.

Longitudinal waves consist of compressions(C) and rarefactions(R) as shown below.



A compression is a region of maximum particle density along a longitudinal wave profile.

A rarefaction is a region of minimum particle density along a longitudinal wave profile.

Examples of longitudinal waves

- Sound waves
- Waves along the length of spring

DIFFERENCES BETWEEN TRANSVERSE AND LONGITUDINAL WAVES

TRANSVERSE WAVES	LONGITUDINAL WAVES
• Direction of vibration of the particles is perpendicular to the direction of propagation of the wave.	• Direction of vibration of particles is parallel to the direction of propagation of the wave.
• There is no variation in particle density along the wave profile.	• There is variation in particle density along the wave profile giving rise to compressions and rarefaction
• Consist of crests and troughs	• Consist of compressions and rarefactions
• Undergo polarization	• Cannot undergo polarization
• A phase change of π radians occurs when the wave is reflected from a fixed surface.	• No phase change occurs when the wave is reflected.

DIFFERENCES BETWEEN SOUND WAVES AND LIGHT WAVES

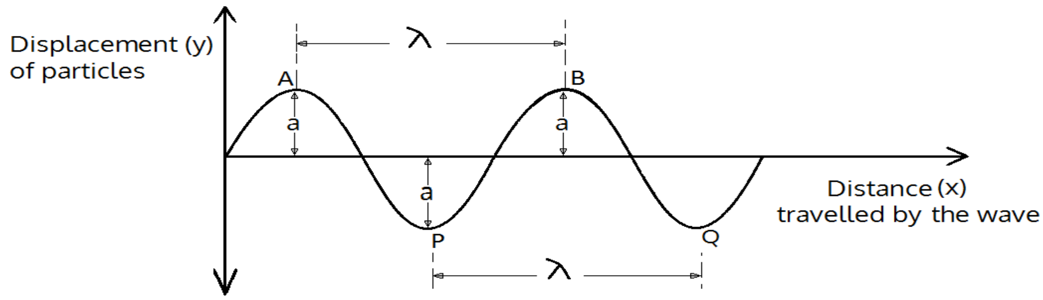
SOUND WAVES	LIGHT WAVES
• They are longitudinal in nature	• They are transverse in nature
• They are mechanical waves	• They are electromagnetic waves
• They cannot be polarized	• They can be polarized
• They travel at a lower speed	• They travel at a higher speed
• They have lower frequency and longer wavelength	• They have a higher frequency and short wavelength

• They cannot travel in a vacuum	• They can travel in a vacuum

GRAPHICAL REPRESENTATION OF WAVES

All waves can be represented graphically using sine curves/waves. These curves are obtained by plotting a graph of displacement(y) of a particle from the equilibrium position against the distance(x) of the particle from the source or against time of travel of the wave.
Note: x can also be considered as distance travelled by the wave from the source.

The graphs have the following forms.



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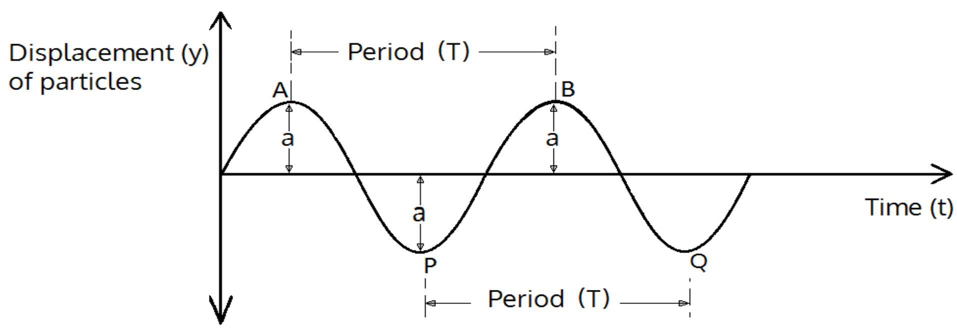


TERMS USED

1. **The rest position/equilibrium position/mean position:**
   This line represents the equilibrium of the particles.
   It is defined as the line/plane along which the particles of the transmitting medium rest when not displaced.
   It can also be defined as the line/plane along which all particles have zero displacement.

2. **Displacement**
   This is the distance travelled by a particle in a transmitting medium from the equilibrium position. The S.I unit of displacement is metres (m).

3. **Amplitude (a)**
   This is the maximum displacement of a particle in a transmitting medium from its equilibrium position. The S.I unit of amplitude is the metre (m).

4. **Wavelength, (λ)**

This is the distance between any two successive particles along the wave profile which are in phase.

Two particles are said to be in phase if they have equal displacement from the equilibrium position and are travelling in the same direction i.e if they are at the same step of vibration.

As applied to **transverse waves**, the wavelength is the distance between any two successive crests or troughs.

As applied to **longitudinal waves**, the wavelength is the distance between any two successive compressions or rarefactions.

The S.I unit of wavelength is metres (m).

The wavelength is equivalent to the distance travelled by a wave when it performs one cycle(oscillation).

5. **Cycle/oscillation**

This is a complete to and fro motion of a particle in a medium.

6. **Frequency**

This is the number of complete oscillations or cycles performed by a wave per second.

Whenever a wave performs a cycle, it travels forward by one wavelength.

The number of cycles performed by a wave in one second is therefore equal to the number of wavelengths travelled by the wave in one second.

The frequency can also be defined as the number of complete wavelengths travelled by the wave in one second.

The S.I unit of frequency is hertz (Hz).

The hertz(Hz) is the frequency of a wave which makes one cycle in one second.

7. **Period (T)**

It is the time taken by a wave to perform one complete cycle or oscillation.

The S.I unit of period is second (s)

8. **Phase**

Particles which are in phase are those which are in the same step of vibration with one another.

Particles which are in anti-phase are those which are in opposite steps of vibration with one another.

9. **Velocity/speed of a wave, V**

This is the distance moved by the wave in its direction in one second.

The S.I unit of velocity is ms^{-1}

10. A pulse

Is a single disturbance that transfers energy from one point to another in a medium without causing any permanent disturbance of the medium

11. A wave profile

Is a side view of the wave showing its pattern and features including crests and troughs or compressions and rarefactions, amplitudes, wavelengths among others.

WAVEFRONTS AND RAYS

A wave front is a section or line taken through an advancing wave along which all particles are in the same phase.

There are two types of wave fronts, namely;

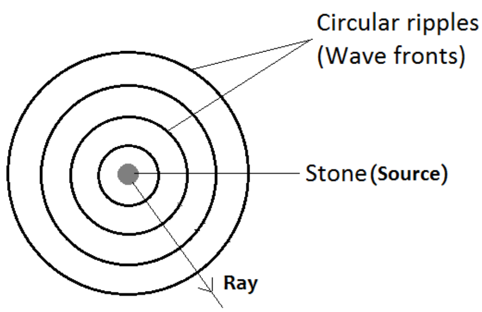
(i) Circular wave front

(ii) Plane wave front

CIRCULAR WAVE FRONTS

Circular wave fronts are wave fronts that form concentric circles having the centre at the source of the wave.

An example of circular wave fronts can be obtained when a still water surface is disturbed by a round object.

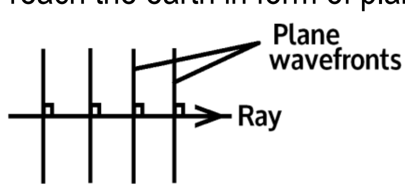


PLANE WAVE FRONTS

These are wave fronts that are parallel to each other.

An example of such wave fronts can be obtained if a still water surface is disturbed using a plane object (e.g a stick, ruler).

Light from the sun is thought to reach the earth in form of plane wave fronts.



NOTE

Successive wave fronts are supposed to be equidistant from each other.
The distance between any two successive wave fronts is equal to **wavelength.**

RAYS

A ray is a line perpendicular to the wave fronts with an arrow that shows the direction of travel of the wave.

RELATIONSHIP BETWEEN FREQUENCY (f) AND PERIOD (T)

Consider a wave that performs n cycles in t seconds.

Period (T) is given by;

$$T = \frac{Time\,(t)\ taken\ to\ perform\ \ n\ cycles}{Number\ of\ cycles} = \frac{t}{n} ---------------(i)$$

Frequency (f) is given by ;

$$f = \frac{Number\ of\ cycles\ (n)}{time\ taken\,(t)} = \frac{n}{t} -----------------------(ii)$$

From (ii) $n = ft$

Substituting for n in (i) gives

$$T = \frac{t}{ft}$$

$$\Rightarrow T = \frac{1}{f}$$

$$OR \quad f = \frac{1}{T}$$

THE GENERAL WAVE EQUATION

The general wave equation relates the velocity of the wave to its frequency and wavelength.
The equation is $v = \lambda f$
Where f is frequency, v is velocity, λ is wavelength.

Derivation

$\text{The distance travelled by a wave in one cycle} = \lambda$

$\Rightarrow \text{In } n \text{ complete cycles, the wave travels a distance} = n\lambda$

$$Velocity = \frac{\text{distance travelled in } n \text{ cycles}}{\text{time taken}}$$

$$\therefore v = \frac{n\lambda}{t} ------------------(i)$$

$$But\ f = \frac{\text{Number of cycles}(n)}{\text{time taken}(t)}$$

$$\therefore f = \frac{n}{t} -------------------(ii)$$

$From\ (ii)\ n = ft$

$\text{Substituting for } n \text{ in } (i) \text{ gives;}$

$$v = \frac{(ft)\lambda}{t}$$

$$v = \lambda f$$

Alternatively;

By definition ,

$$Speed / velocity = \frac{distance}{time}$$

$\text{In 1 cycle, distance} = \text{wavelength}(\lambda) \text{ and time} = \text{period } (T)$

$$\Rightarrow v = \frac{\lambda}{T} = \left(\frac{1}{T}\right)\lambda$$

$$But\ f = \frac{1}{T}$$

$$\therefore v = \lambda f$$

WAVE MOTION AS OSCILLATORY MOTION

Wave motion is a form of oscillatory motion.
Oscillatory motion is a periodic motion in which a body moves to and fro in the same path.
Periodic motion is motion that repeats itself at equal time intervals. These time intervals are referred to as period.
Bodies in oscillatory motion perform oscillations.

OSCILLATION

An oscillation is a complete to and fro motion of a particle about an equilibrium position.
The time taken for a particle to perform one oscillation is called a **period**.
The maximum displacement of a particle from the equilibrium position is called **amplitude**.

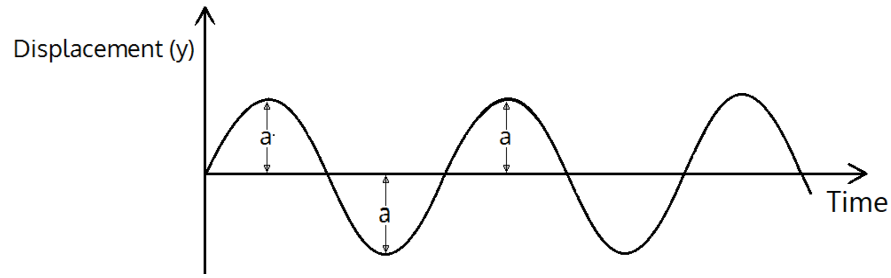
TYPES OF OSCILLATION

There are three types of oscillations namely,

(i) Free oscillation
(ii) Damped oscillations
(iii) Forced oscillation

1. FREE OSCILLATIONS

These are oscillations of a system that occur in the absence of dissipative forces such that the energy and amplitude of the oscillating system remain constant and the system remains in oscillation indefinitely.



Dissipative forces are forces that resist motion of a body and cause a body to lose energy.
The body loses energy because it has to do work to overcome the dissipative forces.
Examples of dissipative forces include; friction, viscosity, air resistance.
An example of a free oscillation is the oscillation of a pendulum in a vacuum.

2. DAMPED OSCILLATIONS

These are oscillations of a system that occur in the presence of dissipative forces so that the system progressively loses energy to the surrounding and the amplitude of oscillation decreases progressively until the system comes to rest.
In damped oscillations, the system experiences resisting or dissipative forces and does work against these dissipative forces at the expense of its energy. Therefore the system loses energy progressively resulting into the amplitude of oscillation reducing and the system eventually comes to rest.

DIFFERENCES BETWEEN FREE AND DAMPED OSCILLATIONS

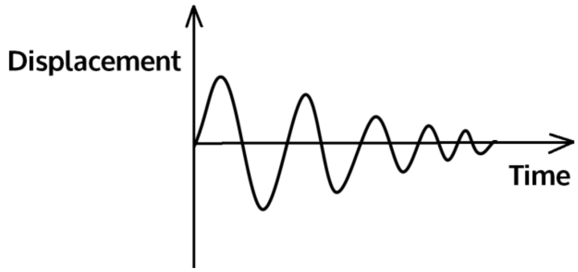
FREE OSCILLATIONS	DAMPED OSCILLATIONS
• Oscillations occur in absence of dissipative forces	• Oscillations occur in presence of dissipative forces
• Amplitude remains constant	• Amplitude decreases with time
• Wave energy remains constant	• Wave energy decreases with time
• The system oscillates indefinitely/forever	• The system eventually comes to rest

TYPES OF DAMPED OSCILLATIONS

There are three types damped oscillations
(i) Under damped oscillations
(ii) Critically damped oscillations
(iii) Over damped oscillations

(a) UNDER DAMPED OSCILLATIONS

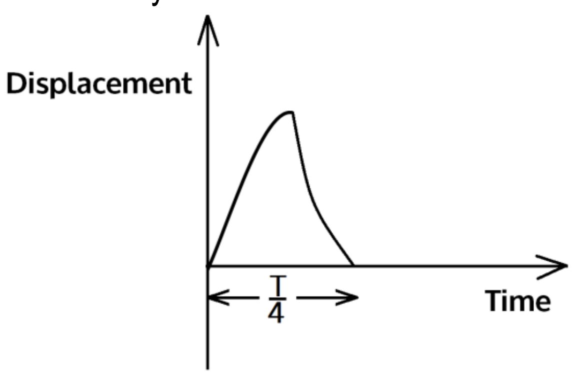
These are oscillations in which a system experiences low resistive forces so that energy is lost gradually causing the amplitude of oscillation to reduce until the system comes to rest.
An example of an under damped oscillation would be a pendulum oscillating in air, and a horizontal spring oscillating over a surface of little roughness.



(b) CRITICALLY DAMPED OSCILLATIONS

These are oscillations in which when a system is displaced, it does not oscillate beyond the equilibrium position but it returns to rest or equilibrium position in the shortest time possible.

The magnitude of the dissipative forces is such that they bring the system to rest immediately.



APPLICATIONS OF CRITICALLY DAMPED OSCILLATIONS

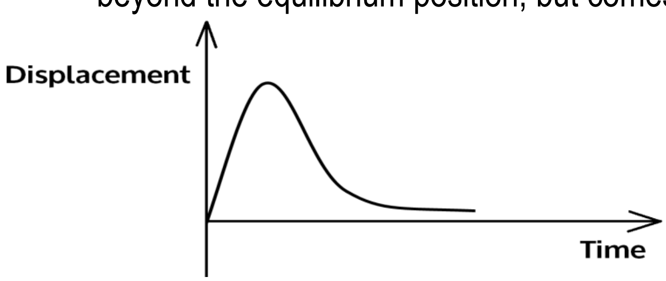
(i) Office doors are critically damped so that they come to rest in the shortest possible time after displacement.

(ii) Critical damping is also applied in car shock absorbers to prevent the body of a car swinging about e.g after hitting a hump.

(iii) Critically damped oscillations are also applied in moving coil instruments so that when in use, the pointer is brought to rest immediately without swinging about to allow the reading to be taken. The motion of such a pointer is said to be ‘’dead beat’’.

(c) OVER DAMPED OSCILLATIONS (HEAVILY DAMPED OSCILLATIONS)

These are oscillations in which when a system is displaced, it does not oscillate beyond the equilibrium position, but comes to rest after a very long period of time**.**



In this case, the system returns to rest very slowly without oscillating past the equilibrium position.

EXAMPLES OF OVER DAMPED OSCILLATIONS

- A horizontal spring oscillating over a very rough horizontal surface
- A metal object attached to a vertical spring and made to vibrate in a viscous liquid.

3. FORCED OSCILLATIONS

A system experiencing damping eventually comes to rest as a result of loss of energy due to dissipative forces.
In order to keep a system experiencing a degree of damping in continuous oscillatory motion, the system must be subjected to an external periodic force which replaces the energy lost as a result of damping.
The system in this case will continue oscillating for a required period of time because it is forced to do so by the external periodic force. This system is said to be performing forced oscillation.

Definition: **Forced oscillations** are oscillations in which a system experiencing some degree of damping is set into continuous oscillatory motion by subjecting it to an external periodic force.

The external periodic force replaces any energy lost due to damping.
The frequency of this external periodic force is the **forcing frequency** while the frequency of the oscillating system experiencing no damping is referred to as the **natural frequency** of the system.
When the forcing frequency becomes equal to the natural frequency of the system, **resonance** occurs.
At resonance, the system absorbs maximum energy from the forcing system and oscillates with maximum constant amplitude at its resonant frequency.
All the energy lost due to damping at this point will be replaced by the forcing system. In fact, the energy absorbed by the system from the eternal periodic force equals the energy lost.
The net energy absorbed by the system is therefore zero and so the system's energy remains constant.
The amplitude of the oscillations will also remain constant since the amplitude of oscillation is proportional to the energy of the system.

Examples of forced oscillations

- Oscillations of objects due to an earth quake.
- A child being swung in air on a swing by an elder or her colleagues.
- Alternating current.
- An oscillating bridge due to soldiers matching on it.

RESONANCE

This is the oscillation (vibration) of a system at its natural frequency as a result of impulses received from a nearby system vibrating at the same frequency.
Every system has a natural frequency of oscillation and if it receives impulses from a nearby system vibrating at the same frequency, resonance will occur.

Resonant frequency is the frequency of a vibrating system receiving impulses from a nearby vibrating system at which maximum energy is being absorbed by the receiving system.

PRACTICAL EXAMPLES OF RESONANCE

(i) Glasses in concert halls may break suddenly when music having the same frequency as the natural frequency of glass is played.
The impulses received by the glass rom the sound waves having the same frequency as the natural frequency of glass cause resonance to occur.
The glass absorbs maximum energy and breaks as it attempts to vibrate with maximum amplitude.

(ii) A bridge may break when soldiers match on it at the same frequency as its own natural frequency.
When this occurs, resonance occurs and the bridge absorbs maximum energy fromthe matching soldiers.
As it attempts to vibrate with maximum amplitude, it may break. For this reason, soldiers are required to break their march when crossing a bridge.

Question:
Explain how the amplitude of a forced oscillation builds up to a constant value.
Consider an oscillating system which is subjected to an external force of variable frequency.
When the system is subjected to a periodic force, the system absorbs energy from the external periodic force.
When the frequency of the external periodic force increases, the amount of energy absorbed by the system also increases causing the amplitude of the oscillation to increase since the amplitude is directly proportional to the energy of the system.
The increase in amplitude of the oscillations increases the magnitude of dissipative force and so the amount of energy lost by the system also increases due to the dissipative forces.
A point is reached when the rate at which energy is absorbed by the system equals the rate at which energy is lost from the system.
The system will have no net gain or loss of energy and so the energy of the system will remain constant causing the amplitude to remain constant. This occurs when the frequency of the external periodic force equals the natural frequency of the oscillating system.
Resonance occurs causing the amplitude to remain maximum and constant since the energy of the system is constant.

HARZARDS OF RESONANCE

- Collapse of structures due earth quakes
- Collapse of bridges due to passing of matching soldiers
- Collapse of window glasses of buildings due vibrations of a nearby passing Aeroplan

APPLICATIONS OF RESONANCE

- Determining the speed or velocity of sound in air using the resonance tube
- Determining the frequency of a note
- Tuning a musical instrument to a desired note

PROGRESSIVE WAVES

A progressive wave is a wave in which the wave profile travels outwards with the wave speed while transferring energy from the source.
Progressive waves consist of disturbances moving from the source to the surrounding region leading to the transfer of energy from the source to the surrounding region.
In a progressive wave, the particles vibrate with constant amplitude but the phase of vibration varies from point to point along the wave profile.
The phase of vibration can be represented in form of phase angle and phase differences.

CHARACTERISTICS OF PROGRESSIVE WAVES

(i) The wave profile moves along with the wave speed.
(ii) The amplitude of the wave remains constant.
(iii) The phase of vibration of the particles varies from point to point along the wave profile.
(iv) They transfer energy as the wave travels.

PHASE ANGLE AND PHASE DIFFERENCE

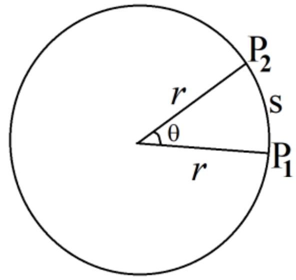
Phase angle

This is the angular displacement of a particle in a wave.
The phase angle is measured in radians and it is represented by the symbol ϕ.

Phase difference

This is the difference in the phase angles of two particles in a wave oscillation or in two different waves.
The phase difference is also measured in radians and may be represented by $\Delta\phi$.
Consider two particles A and B with phase angles ϕ_1 and ϕ_2 respectively. The phase difference between A and B can be given by $\Delta\phi = \phi_2 - \phi_1$.
Two particles are said to be in phase if their phase angles are equal or if the phase difference between them is zero.

RELATION BETWEEN DEGREES AND RADIANS



Consider a circle of radius r along which a particle moves through an arc length s form P_1 to P_2 through an angle θ.

$$s = \frac{\theta}{360^{\circ}} \times 2\pi r$$

But $\frac{s}{r} = sin\theta$ or $tan\theta$

For small angles in radians; $sin\theta \approx tan\theta \approx \theta$

$$\therefore s = r\theta$$

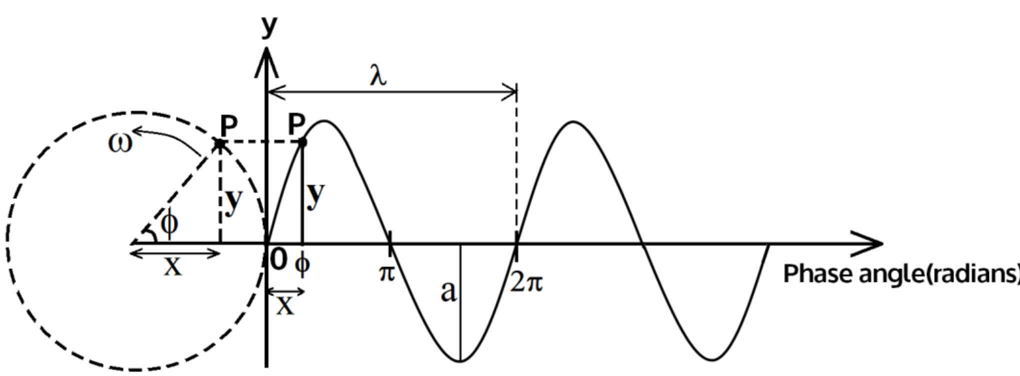
$$\therefore r\theta = \frac{\theta}{360^{\circ}} \times 2\pi r$$

$$1 = \frac{2\pi}{360^{\circ}}$$

$$\therefore 360^{\circ} = 2\pi\, radians, \qquad 180^{\circ} = \pi\, rads, \qquad 90^{\circ} = \frac{\pi}{2} rads$$

THE EXPRESSION OF PHASE ANGLE OF A PARTICLE

The displacement y of a particle in a wave may be plotted against the phase angle.



Distance travelled by a wave is equivalent to angular displacement of a particle

$$x \equiv \Phi \ldots\ldots\ldots\ldots (i)$$

$$\lambda \equiv 2\pi \ldots\ldots\ldots\ldots. (ii)$$

(i) divided by (ii) gives; $\frac{x}{\lambda} = \frac{\Phi}{2\pi} \implies \Phi = \frac{2\pi}{\lambda}x$ or $\frac{x}{\lambda}2\pi$

The above expression gives the phase angle of any particle at a distance x from the source of the wave.

$\Phi = \mathrm{k}x$ where $k = \frac{2\pi}{\lambda}$ is called the wave number in *radians per metre*

EXPRESSION FOR THE PHASE DIFFERENCE

Consider two particles at distances x_1 and x_2 respectively from the source having phase angles ϕ_1 and ϕ_2 respectively.

$\phi_1 = \frac{2\pi}{\lambda} x_1$ and $\phi_2 = \frac{2\pi}{\lambda} x_2$

Phase difference, $\Delta\phi = \phi_2 - \phi_1$

$\Delta\phi = \frac{2\pi}{\lambda} x_2 - \frac{2\pi}{\lambda} x_1 = \frac{2\pi}{\lambda}(x_2 - x_1)$

$(x_2 - x_1)$ is the distance between particles.

Let $(x_2 - x_1) = x$ (distance of particles apart)

$Phase\ difference,\ \Delta\Phi = \frac{2\pi}{\lambda} x$ or $\Delta\Phi = \frac{2\pi}{\lambda}\Delta x$

Where x is distance between the particles.

In the above expression, the unit of x and λ must be the same.

EXAMPLE

1. A wave travelling at $3.4 \times 10^{-1} kms^{-1}$ has a frequency of $17Hz$. Determine the;
   (ii) Phase angle of the particle $10m$ from the source.
   (iii) Phase difference between two particles in a wave which are respectively at a distance $5m$ and $10m$ from the source.

   Solution

   (i) $\Phi = \frac{2\pi}{\lambda} x$

$$\lambda = \frac{v}{f} = \frac{340}{17} = 20m$$

$$\Phi = \frac{2\pi}{20} \times 10 = \pi\ radians$$

   (ii) $\Delta\Phi = \frac{2\pi}{\lambda}(x_2 - x_1) = \frac{2\pi}{20}(10 - 5) = \frac{2\pi}{20} \times 5 = \frac{10\pi}{20} = 0.5\pi\ radians$

2. Two waves of frequencies 256Hz and 280Hz respectively travel with a speed of $340ms^{-1}$ through a medium. Find the phase difference at a point 2.0m from where the waves were initially in phase.

   Solution

   $\Delta\Phi = \Phi_2 - \Phi_1$

$$\Phi_1 = \frac{2\pi}{\lambda_1} x_1, \Phi_2 = \frac{2\pi}{\lambda_1} x_2 \quad and\ x_1 = x_2 = 2m$$

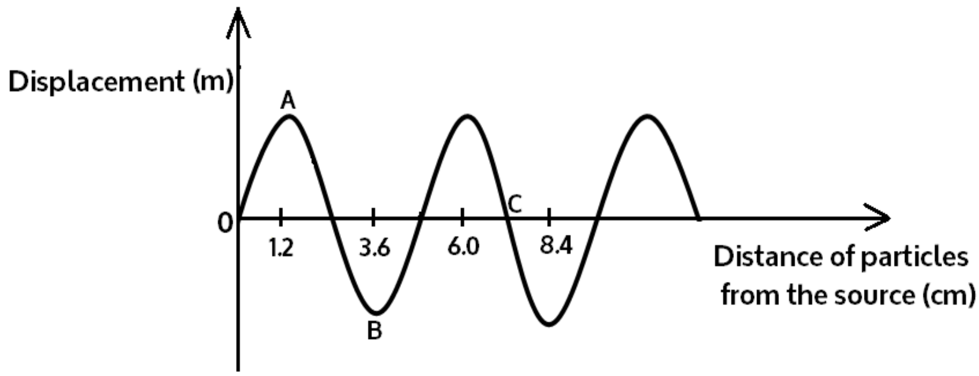
$$But \lambda_1 = \frac{v}{f_1} = \frac{340}{256} = 1.3281m$$

$$\Rightarrow \Phi_1 = \frac{2\pi}{\lambda_1} x_1 = \frac{2\pi \times 2}{1.3281} = 9.462\ radians$$

$$Also\lambda_2 = \frac{v}{f_2} = \frac{340}{280} = 1.2143m$$
$$\Rightarrow \Phi_2 = \frac{2\pi}{\lambda_2}x_2 = \frac{2\pi \times 2}{1.2143} = 10.349\, radians$$
$$\therefore \Delta\Phi = \Phi_2 - \Phi_1 = 10.349 - 9.462 = 0.887\, radians$$

Exercise

1. The graph below shows the displacement of particles in a wave profile.



Determine the;

(i) Phase angle of particle A

(ii) Phase difference between B and C.

2. What is the phase difference between two waves of wavelength 12cm when one leads the other by;

(i) 6cm?

(ii) 9cm?

(iii) 12cm?

THE PROGRESSIVE WAVE EQUATION

The progressive wave equation gives the displacement, y of a particle in a progressive wave at any time, t.

The equation is obtained from the sine curve/wave equation $y = a\sin\omega t$;

where $\omega t = \theta$ =angular displacement,

ω is the angular velocity

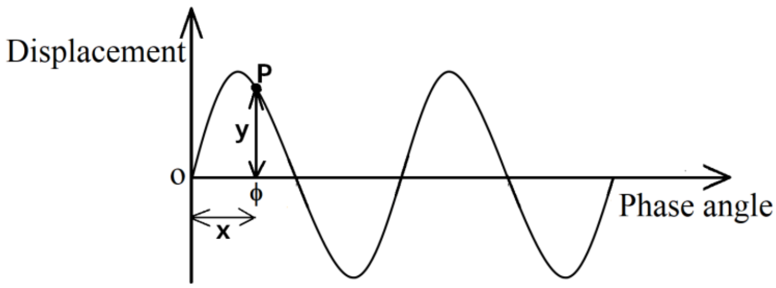
and a is the amplitude of the wave

Angular velocity is the rate of change of angular displacement.

$$\omega = \frac{\theta}{t} \Rightarrow \theta = \omega t$$

The S.I unit of ω is $rad\ s^{-1}$.

Consider the wave below.



(a) A particle such as **P** lags behind a particle at the origin by a phase angle of ϕ radians.
The particle **P** in other words must travel an angle of ϕ radians in order to be in phase with the particle at the origin.
To account for such a particle, a phase angle ϕ is subtracted from ωt in the general sine wave equation.
The displacement y of this particle at any time t is therefore given by the equation
$y = a\sin(\omega t - \phi)$−−−−−−−−−−−−−(1)
This is the general progressive wave equation. It gives the displacement y of a particle at a time t whose phase angle is ϕ radians.
The above equation represents a progressive wave travelling in the positive $x-$direction (from left to right from the source).

(b) For a wave travelling in the opposite direction (negative $x-$direction) or towards the left from the source, the sign of ϕ is changed to positive and the equation becomes
$y = a\sin(\omega t + \phi)$.
This equation represents a wave travelling in the negative $x-$direction.
The angular velocity, ω of the particle can be represented in terms of the period from the expression, $\omega = \frac{2\pi}{T}$.

Alternatively

ω may be expressed in terms of frequency, f where f is the frequency.

$\omega = 2\pi f$ since $f = \frac{1}{T}$

Also $\Phi = \frac{2\pi}{\lambda}x$

Substituting for ω and Φ in **equation (1)**

$$y = a\ sin(\omega t - \Phi) = a\ sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$$

$y = a\ sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right) = \text{a sin}\left(\frac{2\pi}{T}t - kx\right)$ where $k = \frac{2\pi}{\lambda}$ in rads per metre and is called wave number.

$$y = asin2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) ---------------(2)$$

Where T is the period of the wave, λ is wavelength, x is distance of the particle from the source.

Alternatively

Substituting for $\omega = 2\pi f$ in **equation (1)**

$$y = asin(\omega t - \phi) = asin\left(2\pi ft - \frac{2\pi}{\lambda}x\right)$$

$$y = asin2\pi\left(ft - \frac{x}{\lambda}\right) ---------------(3)$$

Using $v = \lambda f \Rightarrow f = \frac{v}{\lambda}$

Substituting for f equation (3)

$$y = asin2\pi\left(\frac{vt}{\lambda} - \frac{x}{\lambda}\right) = asin2\pi\left(\frac{vt - x}{\lambda}\right)$$

$$y = asin\frac{2\pi}{\lambda}(vt - x) ---------------(4)$$

Any of the above equations can be used to represent the progressive wave travelling in the positive $x-$ direction.

For a wave travelling in the opposite direction, the sign in the bracket is changed to a (+).

NOTE: In a progressive wave equation, the units of y are the same as those of the amplitude while the units of x must be the same as those of wavelength and may differ from those of y.

VELOCITY OF A PARTICLE IN A PROGRESSIVE WAVE

As shown above, the displacement $\boldsymbol{y}$ of the particle in the positive x direction can be given

$$y = asin(\omega t - \phi)$$

But $v = \frac{dy}{dt} = \frac{d}{dt}(asin(\omega t - \phi)) = a\omega cos(\omega t - \phi)$

Since velocity v is the rate of change of displacement, then;

velocity, $v = a\omega\cos(\omega t - \phi)$

The above equation gives the velocity of a particle in a progressive wave travelling in the positive direction at any time, t.

MAXIMUM VELOCITY OF A PARTICLE

The velocity of a particle varies with time because $cos(\omega t - \phi)$ also varies with time.

For the velocity to be maximum, $\cos(\omega t - \phi)$ must therefore be maximum.

From trigonometry, the maximum value of ***cosine*** of angles is 1. Therefore,

$$[cos(\omega t - \phi)]_{\max} = 1$$

$$v_{\max} = \omega a[cos(\omega t - \phi)]_{\max}$$

$$v_{\max} = \omega\ a$$

The above expression gives the maximum velocity that can be attained by any particle in a progressive wave.

Since $\omega = \frac{2\pi}{T}$

$$\Rightarrow v_{max} = \frac{2\pi}{T}a$$

Also, since $\frac{1}{T} = f$

$$v_{max} = 2\pi fa$$

EXAMPLES

1. A wave is represented by $y = 0.20\sin 0.4\pi(60t - x)$, where all distances are in cm and time in seconds. Find the;
   (i) Amplitude
   (ii) Wavelength
   (iii) Frequency
   (iv) Speed.

Solution

(i) Given

$$y = 0.20sin0.4\pi(60t - x)$$

Comparing with

$$y = asin\frac{2\pi}{\lambda}(vt - x)$$

$$Amplitude\ \ A = 0.20cm$$

(ii) From $y = 0.20 sin 0.4\pi(60t - x)$

$y = 0.20 sin\left(24\pi t - 0.4\pi x\right)$

$\frac{2\pi x}{\lambda} = 0.4\pi x \Rightarrow \frac{2\pi}{\lambda} = 0.4\pi$

$\Rightarrow \lambda = 5cm$

(iii) From $\omega = 2\pi f$

But $\omega = 24\pi$

$\Rightarrow 2\pi f = 24\pi$

$f = 12Hz$

(iv) From $v = f\lambda = 12 \times 5 = 60cms^{-1} = 0.6ms^{-1}$

2. Given that a wave is represented by $y = asin\left(2000\pi t - \frac{\pi}{17}x\right)$ where t is in seconds, and y in cm. Find the velocity of the wave.

Solution

$y = asin\left(2000\pi t - \frac{\pi}{17}x\right)$ compared to $y = a\ sin\left(\omega t - \frac{2\pi}{\lambda}x\right)$

$$From \omega = 2\pi f = 2000\pi$$

$$f = \frac{2000\pi}{2\pi} = 1000Hz$$

$$From k = \frac{2\pi}{\lambda} = \frac{\pi}{17}$$

$$\Rightarrow \lambda = 2 \times 17 = 34cm$$

$$From v = f\lambda$$

$v = 1000 \times 34 = 34000cms^{-1}$ or $v = 1000 \times 0.34 = 340ms^{-1}$

$\therefore v = 340ms^{-1}$.

TRIAL QUESTIONS

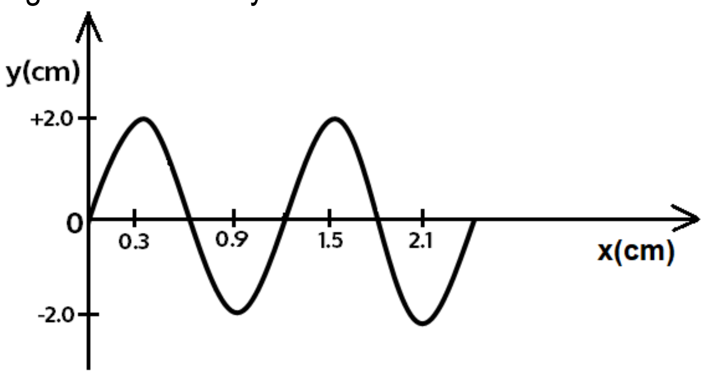
1. The equation $y = 2sin(200t - 50x)$ represents a progressive wave travelling in the $x-$direction, where y is the displacement of the particle in ***metres*** and x is the distance of the particle from the source in ***cm***.
   (a) Deduce the direction in which the wave is travelling.
   (b) Determine the;

(i) amplitude of the wave ($a = 2m$)
(ii) frequency of the wave ($f = 31.83Hz$)
(iii) wavelength of the wave ($\lambda = 0.00126m$)
(iv) velocity of the wave ($v = 0.04ms^{-1}$)
(v) phase difference between two particles in the wave which are 2cm apart. ($99.7\ rads$)

(c) What is the maximum velocity attained by the particle in this wave? ($400ms^{-1}$)

2. The displacement y of a wave travelling in the x direction is given at any time t by $y = asin2\pi\left(\frac{t}{0.5} - \frac{x}{2.0}\right)$ ***metres***.
Find the speed of the wave.

3. The figure below shows a wave travelling in the positive x direction away from the origin with a velocity of $9ms^{-1}$



(a) What is the period of the wave.

(b) Show that the displacement equation for the wave $y = 2sin\frac{5}{3}\pi(9t - x)$.

4. The displacement of a particle in a progressive wave is $y = 2sin2\pi(0.25x - 100t)$, where x and y are in cm and t is in seconds. Calculate the;
(a) wavelength,
(b) velocity of the wave.

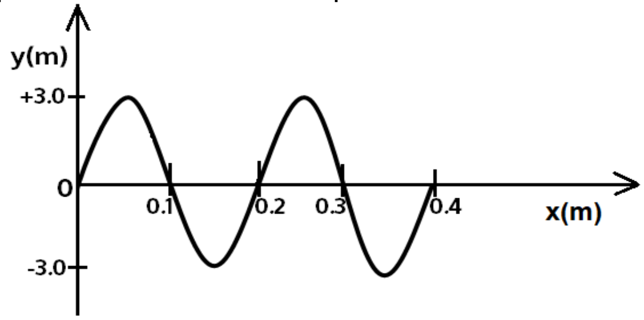
5. A plane progressive wave is represented by the equation

$$y = 0.2sin\left(200\pi t - \frac{20\pi x}{17}\right)$$

where y is in *mm, t* is in *seconds* and x is in *m.*
Find the;
(i) speed of the wave

(ii) phase difference between a point *0.25m* and another point *1.00m* from the origin.

6. The figure below shows the wave profile of a progressive wave travelling in the positive x-direction at a speed of $15ms^{-1}$.



Show that the equation of the wave is given by $y = 3.0\,sin150\pi\left(t - \frac{x}{15}\right)$ metres.

7. When a plane wave traverses a medium, the displacement of the particles is given by $y = 0.01sin2\pi(2t - 0.01x)$, where y and x are in metres and t in seconds. Calculate the;
(i) frequency of the wave
(ii) wave velocity
(iii) phase difference at a given instant of time, between two particles 50m apart.

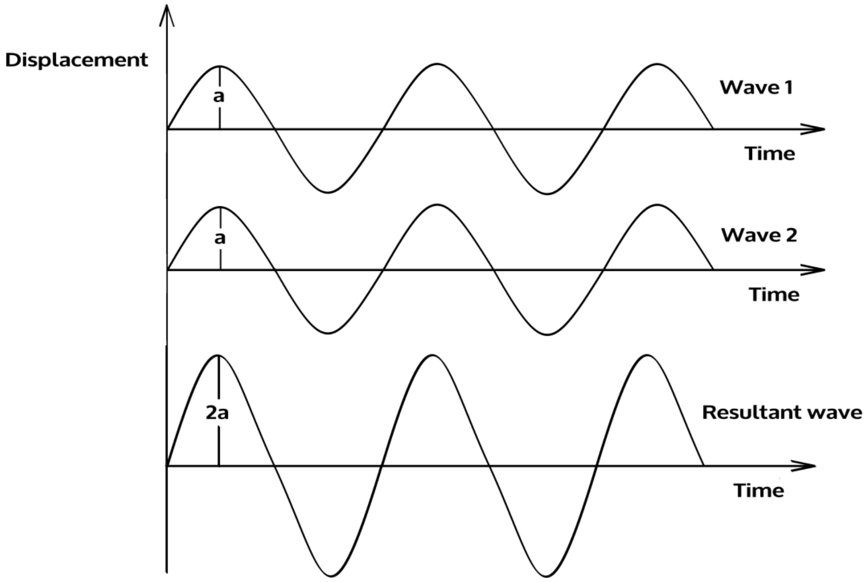
8. The displacement of a particle on a progressive wave is $y = asin2\pi\left(\frac{t}{10} - \frac{x}{2}\right)$ metres at a time,t. Find the;
(i) velocity of the wave,
(ii) period of the wave.

9. The equation $y = 10sin(20\pi\ x - 150\pi\ t)$ shows a progressive wave travelling in the $x-$ direction, where y is in mm and x is in m.
(a) Determine the amplitude and velocity of the wave.
(b) Deduce the direction in which the wave travels.

THE SUPERPOSITION PRINCIPLE

The superposition principle states that when two waves travelling in the same medium meet, the resultant displacement at any point is the vector sum of the displacements due to each wave at that point.
Waves, unlike particles, do not travel past each other or collide. When they meet, they practically ‘’add up’’ or superpose/superimpose.
The resultant displacement at any point in the medium will then be equal to the vector sum of their individual displacements at a point.

Consider the wave below.



When waves 1 and 2 each having amplitude a meet, they super impose giving a resultant wave whose displacement and amplitude are the sum of the displacements and amplitudes of the super imposed waves.

APPLICATIONS OF SUPERPOSITION PRINCIPLE

The superposition principle is used to explain the formation of stationary waves and beats.

STATIONARY (STANDING) WAVES

A standing (or stationary) wave is a wave formed when two progressive waves of equal frequency, wavelength, speed and amplitude travelling in opposite directions meet/superpose.
In such waves, the wave profile doesn't travel along the medium and the amplitude of vibration of the particles varies from point to point along the wave profile.
Stationary waves consist of regions in which particles vibrate with maximum amplitude called **antinodes** alternating with regions in which the particles are permanently at rest called **nodes.**
An antinode is a point on a stationary wave profile in which the particles vibrate with maximum amplitude.
A node is a point on a stationary wave profile in which the particles are permanently at rest or have minimum displacement.

CHARACTERISTICS OF STATIONARY WAVES

(i) The wave profile does not travel along the medium.
(ii) The amplitude of vibration of the particles varies from point to point along the wave profile.
(iii) The phase of vibration remains constant for all particles on the wave profile.
(iv) Stationary waves don't transmit energy from one point to another along the medium.
(v) They consist of nodes and antinodes alternating with each other along the wave profile.

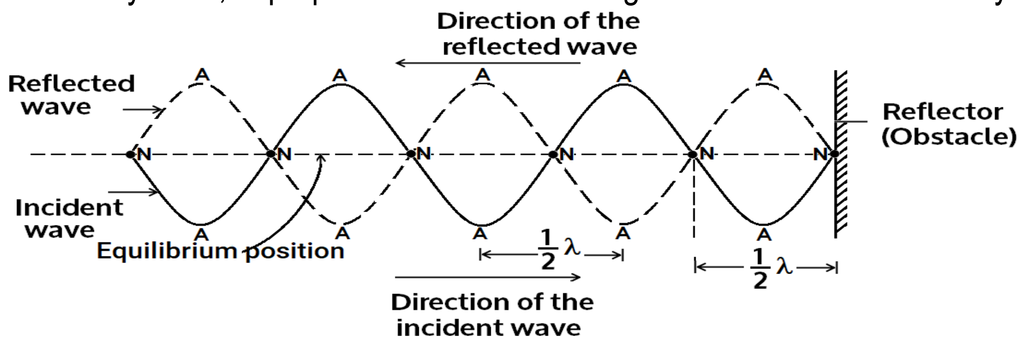
FORMATION OF STATIONARY WAVES

Stationary waves are formed when two progressive waves having the same frequency, amplitude and speed meet when travelling in opposite directions.

Such waves result when a progressive wave travels towards a barrier and is reflected so that it travels back in the opposite direction.

The incident and reflected waves have the same frequency and amplitude and travel in opposite directions at the same speed.

When they meet, superposition occurs resulting into formation of stationary waves.



The stationary wave formed consists of alternating regions of maximum displacement called antinodes and regions of minimum displacement called nodes.

The distance of separation of successive nodes and antinodes is constant and equal to half the wavelength of the super imposing wave, i.e $Distance\ between\ two\ successive\ nodes = \frac{\lambda}{2}$

Similarly;

$$Distance\ between\ two\ successive\ antinodes = \frac{\lambda}{2},$$

$$Distance\ between\ a\ node\ and\ an\ antinode = \frac{1}{2}\left(\frac{\lambda}{2}\right) = \frac{\lambda}{4}$$

Where λ is the wavelength of the super imposing wave.

CONDITIONS FOR THE FORMATION OF STATIONARY WAVES

Stationary waves are formed by superposition of two progressive waves which must;

(i) Have the same frequency.

(ii) Have equal amplitudes.

(iii) Be travelling at the same speed.

(iv) Be travelling in opposite directions.

EXPLANATION OF THE FORMATION OF STATIONARY WAVES USING THE SUPERPOSITION PRINCIPLE

Stationary waves are formed when two waves having the same frequency, equal amplitudes and same wavelength meet when travelling in opposite directions.

When they meet, the resultant displacement is equal to the vector sum of the displacements due to each wave.

At some point, the waves meet when in phase resulting into a region of maximum displacement called an **antinode.** At other points, the waves meet when completely out of

phase (in anti-phase) resulting into a region of minimum or zero displacement called a **node.**
The **nodes** and **antinodes** alternate with each other and make up (constitute) the stationary wave.

DIFFERENCES BETWEEN STATIONARY WAVES AND PROGRESSIVE WAVES

STATIONARY WAVES	PROGRESSIVE WAVES
• The wave profile doesn't travel along the medium	• The wave profile moves along with the wave speed
• There is no transfer energy	• There is energy transfer from point to another along the medium
• The amplitude of vibration of the particles varies from point to point	• The amplitude of vibration of particles is constant
• The phase of vibration of particles is constant between nodes	• The phase of vibration of particles varies from point to point along the wave profile
• They consist of nodes and antinodes	• They consist of crests and troughs/compressions and rarefactions
• Distance between two successive nodes or antinodes is half of the wavelength	• Distance between two successive crests or troughs or between two successive compressions or rarefactions is equal to wavelength

THE STATIONARY WAVE EQUATION
The stationary wave equation gives the displacement y of particles in a stationary wave at any time, t.
To derive the stationary wave equation, we use the fact that a stationary wave is formed when two progressive waves having the same frequency and wavelength and equal amplitude meet when travelling in opposite directions.
If $y_1 = asin(\omega t - kx)$ is the equation of one of the progressive waves, the equation of the other progressive which can give a stationary wave when super imposed with the above wave would be $y_2 = asin(\omega t + kx)$ where wave number $k = \frac{2\pi}{\lambda}$.
By the superposition principle, the displacement, y of the stationary wave formed is equal to the vector sum of the displacement due to the above wave, i.e;

$$y = y_1 + y_2$$

$$y = asin(\omega t - kx) + asin(\omega t + kx)$$

$y = a[sin(\omega t - kx) + sin(\omega t + kx)]$. By factor formula, $sin\ P + sin\ Q = 2sin\ \frac{P+Q}{2} cos\ \frac{P-Q}{2}$

$$y = a\left[2sin\left(\frac{\omega t - kx + \omega t + kx}{2}\right)cos\left(\frac{\omega t - kx - \omega t - kx}{2}\right)\right]$$

$$y = 2asin\omega t\ cos(-kx)$$

$$But\ cos(-kx) = coskx$$

$$\Rightarrow y = 2asin\omega t\ coskx$$

$$\therefore y = 2acoskx\ sin\omega t$$

Let $A = 2acoskx$

$$Wherek = \frac{2\pi}{\lambda}$$

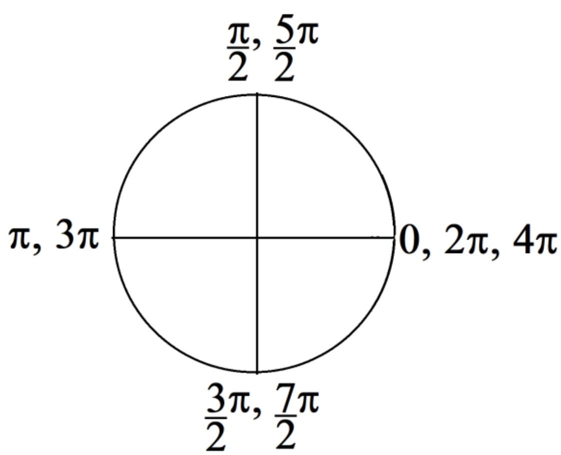
$$\Rightarrow A = 2acos\frac{2\pi}{\lambda}x ---------------------(*)$$

$Where\ \ A\ \ is\ the\ amplitude\ of\ the\ resulting\ stationary\ wave$

$$\therefore y = Asin\omega t ------------------------ (**)$$

Equation (*) represents the amplitude of the stationary wave which depends on the distance,x of a particle from the source and varies from particle to particle along the stationary wave profile.
Thus, the amplitude of vibration of particles in a stationary wave varies from point to point.
In general, any wave in which the amplitude of vibration of the particles varies from point to point along the wave profile is a **stationary wave.**
Equation (**) is the equation of the resulting stationary wave.

NOTE: The angles $0°,\ 90°,\ 180°,\ 270°,\ 360°,\ 450°,\ 540°,\ 630°,\ 720°,\ $ etcin radians are respectively $0, \frac{\pi}{2}\ ,\ \pi,\ \frac{3\pi}{2}\ , 2\pi,\ \frac{5\pi}{2}\ , 3\pi,\ \frac{7\pi}{2}\ , 4\pi$ radians, etc.



DERIVATION OF THE EQUATION FOR THE DISTANCE BETWEEN SUCCESSIVE NODES OR ANTINODES

Distance between two successive nodes

By definition, at any node, the amplitude of vibration of the particle is zero.

i.e at the node, amplitude $A = 0$.

But $A = 2acos\frac{2\pi}{\lambda}x$ where x is the distance of the particle from the source of the wave.

Thus at the anode, $2acos\frac{2\pi}{\lambda}x = 0 \Rightarrow cos\frac{2\pi}{\lambda}x = 0$

Any cosine function is zero (0) when the angles are $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, etc$.

Thus for a node; $\frac{2\pi}{\lambda}x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} ---$

For first node

$$\frac{2\pi}{\lambda}x_1 = \frac{\pi}{2}$$

$$x_1 = \frac{\lambda}{4}$$

For second node

$$\frac{2\pi}{\lambda}x_2 = \frac{3\pi}{2}$$

$$x_2 = \frac{3\lambda}{4}$$

For the third node

$$\frac{2\pi}{\lambda}x_3 = \frac{5\pi}{2}$$

$$x_3 = \frac{5\lambda}{4}$$

Distance between two successive nodes, $x = x_2 - x_1$

$$x = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{2\lambda}{4}$$

$$x = \frac{\lambda}{2}$$

Therefore the distance between two successive nodes on a stationary wave is $\frac{\lambda}{2}$.

Distance between two successive antinodes

By definition, at any antinode, the amplitude of vibration of the particle is maximum.

i.e at the antinode, $amplitude = \pm 2a$.

But $A = \pm 2acos\frac{2\pi}{\lambda}x$ where x is the distance of the particle from the source of the wave.

Thus at the antinode, $A = \pm 2a$

$$\pm 2acos\frac{2\pi}{\lambda}x = \pm 2a \Rightarrow cos\frac{2\pi}{\lambda}x = \pm 1$$

Any *cosine* function is ± 1 when the angles are $0, \pi, 2\pi, 3\pi, etc$.

Thus, for an antinode; $\frac{2\pi}{\lambda}x = 0, \pi, 2\pi, 3\pi ---$

For first antinode

$$\frac{2\pi}{\lambda}x_1 = 0$$

$$x_1 = 0$$

For second antinode

$$\frac{2\pi}{\lambda}x_2 = \pi$$

$$x_2 = \frac{\lambda}{2}$$

For the third antinode

$$\frac{2\pi}{\lambda}x_3 = 2\pi$$

$$x_3 = \lambda$$

Distance between two successive antinodes, $x = x_2 - x_1$

$$x = \frac{\lambda}{2} - 0 = \frac{\lambda}{2}$$

$$x = \frac{\lambda}{2}$$

Therefore, the distance between two successive antinodes on a stationary wave is $\frac{\lambda}{2}$.

EXAMPLES

1. A plane progressive wave is given by $y = asin\left(100\pi t - \frac{10}{9}\pi x\right)$ where x and y are in mm and t is in *seconds.*
   (i) Write the equation of the progressive wave which would give rise to a stationary wave if superposed with the one above.
   (ii) Find the equation of the stationary wave and hence determine the amplitude of vibration.
   (iii) Determine the frequency and velocity of the stationary wave.

Solution

(i) $y = asin\left(100\pi t + \frac{10}{9}\pi x\right)$

(i) From the principle of superposition, $y = y_1 + y_2$

$$y = asin\left(100\pi t - \frac{10}{9}\pi x\right) + asin\left(100\pi t + \frac{10}{9}\pi x\right)$$

$$y = a\left[sin\left(100\pi t - \frac{10}{9}\pi x\right) + sin\left(100\pi t + \frac{10}{9}\pi x\right)\right]$$

$$y = a\left[2sin100\pi t\ cos\left(-\frac{10}{9}\pi x\right)\right]$$

$$But\ \ cos\left(-\frac{10}{9}\pi x\right) = cos\left(\frac{10}{9}\pi x\right)$$

$$y = 2acos\frac{10}{9}\pi x\ sin100\pi t$$

$Let A = 2a\cos\frac{10}{9}\pi x$ and is the varying amplitude

$$\Rightarrow y = Asin100\pi t$$

(ii) Consider $y = Asin100\pi t$

$But \ \ generally \ \ y = Asin\omega t$

$Comparing \ \ the\ equations$

$\Rightarrow \omega = 100\pi$

$But\ also \ \ \omega = 2\pi f$

$\Rightarrow 2\pi f = 100\pi$

$\therefore f = 50Hz$

$$Also \quad \frac{2\pi}{\lambda}x = \frac{10}{9}\pi x$$

$\Rightarrow \lambda = 1.8mm$

$From \ \ v = f\lambda = 50\times1.8\times10^{-3}$

$\therefore v = 9\times10^{-2}\,ms^{-1}$

EXERCISE

1. The equation $y = 5sin(50\pi t - 200\pi x)$ shows a progressive wave travelling in the $x -$ direction. Where y is in m and x in cm.
   (a) Write an equation for the wave which when super imposed with the above wave will form a stationary wave.
   (b) Derive an expression for the wave formed when the above two waves are super imposed and hence deduce that the wave is stationary.
   (c) Determine the;
      (i) Distance between any two successive nodes of the above wave.
      (ii) Velocity of the wave in (b) above

2. A progressive wave represented by $y = asin2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ is reflected back along the same path. Show how the overlap of the two waves may give rise to a stationary wave.

WAVES IN STRINGS AND PIPES

Strings produce transverse stationary waves and pipes produce longitudinal stationary waves. The nature of the stationary wave produced by the string or pipe depends on the length of the string or pipe.
Stationary waves in pipes are produced by vibrating air columns.

1. STATIONARY TRANSVERSE WAVES (PRODUCED BY VIBRATING STRINGS/WIRES)

When a stretched string tightly fixed at both ends is plucked, transverse waves travel towards both ends. Since the ends are fixed, the waves are reflected back.
Interference between the incident and reflected waves occurs and a stationary wave pattern is formed with **nodes** at both ends.
The simplest mode of vibration is obtained when the string is plucked midway. The note given out in this case is called the **fundamental note** and its frequency is called the **fundamental frequency.**
A note is a sound of regular frequency.
In a string instrument, the string vibrates in several modes when plucked but where and how hard it is plucked determines the quality or timbre of the note produced.

VELOCITY OF TRANSVERSE STATIONARY WAVES IN A VIBRATING STRING

The speed of stationary waves produced in stretched vibrating strings is independent of the amplitude and frequency of the wave.
It depends on the **tension**, τ in the string and **mass per unit length**, μ of the string/wire.
Using dimensional analysis, the velocity of the wave can be obtained as follow.
From the factors above,

$$v = k\tau^x \mu^y - - - - - - - - - - - - - (1) \quad where\ k\ is\ a\ \text{dimensionless}\ constant.$$

Applying dimensions

$$[v] = [\tau]^x [\mu]^y - - - - - - - - - - - - (2)$$

$$But \left.\begin{aligned} [v] &= LT^{-1} \\ [\tau] &= MLT^{-2} \\ [\mu] &= ML^{-1} \end{aligned}\right\} - - - - - - - - - (3)$$

Substituting (3) into (2) gives

$$LT^{-1} = \left(MLT^{-2}\right)^x \left(ML^{-1}\right)^y = M^x.L^x.T^{-2x}.M^y.L^{-y}$$

$$M^0LT^{-1} = M^{(x+y)}L^{(x-y)}T^{-2x}$$

Comparing LHS and RHS

$$M: 0 = x + y - - - - - - - - - - - - - (i)$$

$$L: 1 = x - y - - - - - - - - - - - - - (ii)$$

$T: -1 = -2x - - - - - - - - - - - - - (iii)$

$From\ \ (iii)\ x = \frac{1}{2}$

$From\ \ (ii)\ y = x - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$

$Or\ From\ (i) y = -x\ \ \therefore y = -\frac{1}{2}$

$Substituting\ \ for\ \ x\ \ and\ \ y\ \ into\ \ (1)\ \ gives$

$v = k\tau^{\frac{1}{2}}\mu^{-\frac{1}{2}} = k\left(\frac{\tau}{\mu}\right)^{\frac{1}{2}}$

$v = k\sqrt{\frac{\tau}{\mu}}$

$From\ \ experiments, \qquad k = 1$

$Thus\ \ v = \sqrt{\frac{\tau}{\mu}}$

FACTORS AFFECTING THE FREQUENCY OF WAVES IN VIBRATING STRINGS

The frequency of the waves produced by the vibrating strings depends on the following factors.

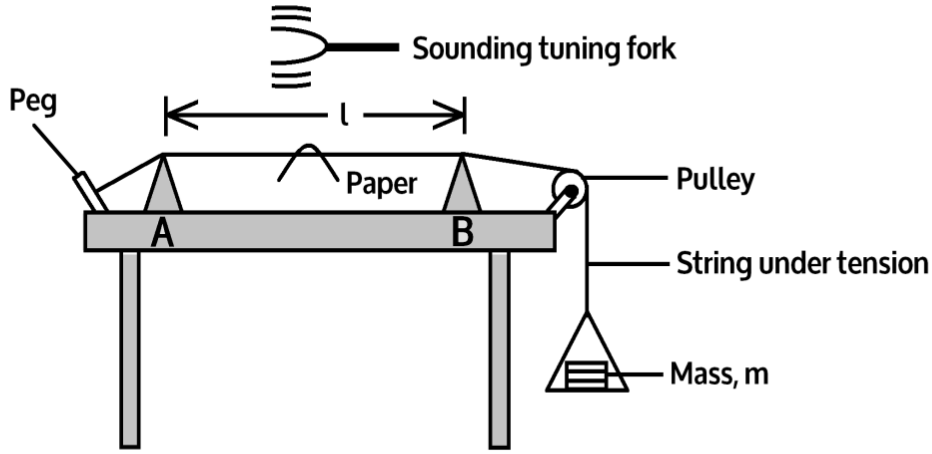
(i) Length of the string/wire. Short strings produce high pitched notes and vice versa.
(ii) Tension in the string/wire. Tight strings produce high pitched notes and vice versa.
(iii) Mass per unit length (thickness) of the string. Strings with smaller mass per unit length (i.e thin strings/wires) produce high pitched notes and vice versa.
(iv) Nature of the string/wire.

Note:

(i) Pitch is the highness or lowness of a sound note. Pitch depends on frequency. The higher the frequency, the higher the pitch and vice versa.
(ii) A set up called a sonometer or monochord was designed to verify or investigate the above factors.

AN EXPERIMENT TO INVESTIGATE THE FACTORS AFFECTING FREQUENCY OF NOTES PRODUCED BY STRETCHED STRINGS

1. LENGTH (keeping τ constant)



The string is fixed at one end of a sonometer table/box and then passed over two bridges A and B and over a smooth pulley. A constant mass tied on the other end as shown in the diagram above, so that the tension in the string is constant.

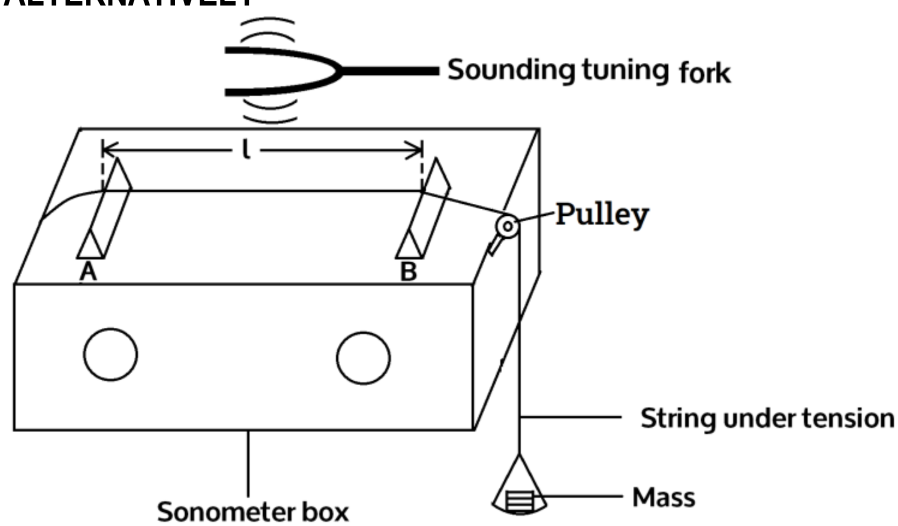
A tuning fork is sounded and placed near the stretched string on which a light paper is placed.

The bridge B is moved along the table until a point where the paper is thrown off the string. The length, l is measured and the frequency, f of the tuning fork is recorded.

The experiment is repeated with other tuning forks and length, l and the corresponding frequencies, f recorded in a table including values of $\frac{1}{l}$.

A graph of f against $\frac{1}{l}$ is plotted and is a straight line through the origin. This implies that frequency is proportional to $\frac{1}{l}$.

ALTERNATIVELY



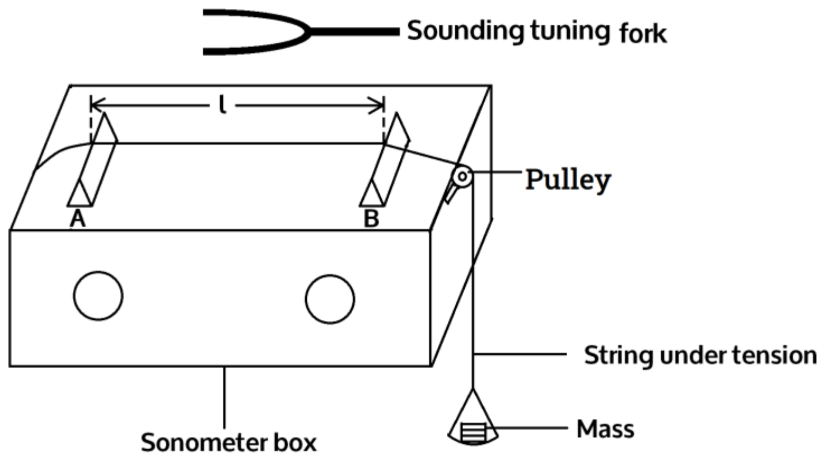
A string or wire fixed at one end on a wooden box is passed over two bridges A and B and over a pulley.
On the other end of the string is hung a mass, m which is kept constant so that the tension in the string is constant.
The string is plucked in the middle and a sounding tuning fork is brought near it.
The length of the wire or string is varied by moving bridge B towards A until a loud sound is heard. The length, l of the string is measured and recorded together with the frequency, f of the tuning fork.
The procedure is repeated with other tuning forks and the results are tabulated including values of $\frac{1}{l}$.

A graph of f against $\frac{1}{l}$ is plotted and is a straight line passing the origin.

This implies that for a fixed tension T and a particular wire, frequency, f is proportional to $\frac{1}{l}$.

Question
Describe with the aid of a diagram, an experiment to investigate the variation of frequency of a stretched string with length.

2. TENSION (keeping length l constant)

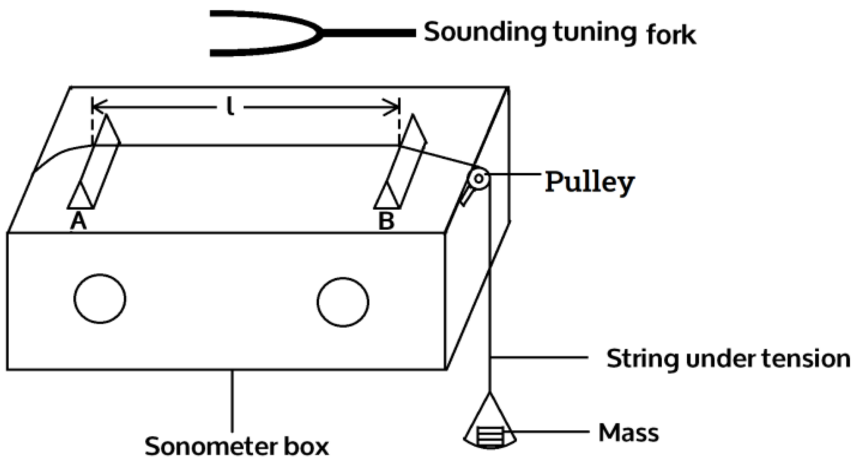


A string/wire fixed at one end of a wooden box (sonometer box) is passed over two bridges A and B and over a pulley.
On the other end of the string is hung a mass m.
Keeping distance, l between the bridges A and B constant, the string is plucked in the middle and the sounding tuning fork is brought near it.
The mass on the pan is gradually increased by small amounts to increase the tension in the string until a loud sound is heard. The total mass, m of the scale pan and its contents and the frequency, f of the tuning fork are recorded.
The procedure is repeated with different tuning forks of different frequencies and a table of results is filled which has columns of; f, m, and f^2.
A graph of f^2 against m is plotted and is a straight line through the origin; hence $f^2 \propto m$.

Since tension $T = mg$ where g is acceleration due to gravity, it implies that $f^2 \propto T$ or $f \propto \sqrt{T}$.

ALTERNATIVELY;
After recording m and f, the procedure is repeated with different tuning forks of different frequencies and the results are recorded in a table including values of $\sqrt{T}$, where $T = mg$ and g is acceleration due to gravity, and T is the tension.
A graph of f against $\sqrt{T}$ is plotted and is a straight line through the origin. This implies that $f \alpha \sqrt{T}$.

3. MASS PER UNIT LENGTH (Keeping the length and tension constant**)**



Two bridges A and B are placed on a sonometer box at a constant distance l apart.
Strings or wires of the same material of known density ρ but different thickness are cut to the same length.
Using a micrometer screw gauge, the diameter average d of one of the wires is obtained.
The wire is then set on the sonometer box as above with a constant value of the mass, m.
The wire is plucked in the middle and different tuning forks are sounded and brought near the wire, one at a time. The one which produces a loud sound with the wire is selected and its frequency f recorded together with the average diameter d of the wire.
The procedure is repeated with other wires and the results are recorded in a table which has d, f, and $\frac{1}{d}$.
A graph of **f**against $\boldsymbol{\frac{1}{d}}$ is plotted and is a straight line through the origin. Hence $f \propto \frac{1}{d}$.
So, f is inversely proportional to the thickness of the wires.
Since $\mu = \frac{Mass\ of\ the\ wire}{Length\ of\ the\ wire}$

$\mu = \frac{Volume \times \rho}{L}$, where L is the whole length of the wire.

$$\mu = \frac{AL\rho}{L}$$

μ =Aρ *,where A is the cross sectional area of the wire.*
But $A = \frac{\pi d^2}{4}$

So, $\mu = \frac{\pi d^2 \rho}{4}$ thus $\mu \propto d^2$ and $\sqrt{\mu} \propto d$.

This implies that $\frac{1}{\sqrt{\mu}} \propto \frac{1}{d}$.

So, f $\propto \frac{1}{\sqrt{\mu}}$

ALTERNATIVELY;
After recording f and d, the procedure is repeated with other wires and the results are recorded in a table including valuesd^2, $\mu = \frac{M}{L}$,$\sqrt{\mu}$, $\frac{1}{\sqrt{\mu}}$, where$M$ is the mass of wire and L is the whole length of the wire, hence $\mu = \frac{Volume \times \rho}{L} = \frac{AL\rho}{L} = A\rho = \frac{\pi d^2 \rho}{4}$

d(m)	f(Hz)	$d^2(m^2)$	$\mu(kgm^{-1})$	$\sqrt{\mu}\left(kg^{\frac{1}{2}}m^{-\frac{1}{2}}\right)$	$\frac{1}{\sqrt{\mu}}\left(kg^{-\frac{1}{2}}m^{\frac{1}{2}}\right)$

A graph of f against $\frac{1}{\sqrt{\mu}}$ is plotted and is a straight line through the origin. This implies that $f\alpha\frac{1}{\sqrt{\mu}}$.

Note: $\mu = \frac{M}{L} = \frac{volume \times density}{length} = \frac{(A \times L)\rho}{L} = A\rho = \frac{\pi d^2 \rho}{4}$.

The above results 1, 2, and 3 are called **laws of vibrations of stretched strings or wires.**
They can be combined as $f = \frac{k}{l}\sqrt{\frac{\tau}{\mu}}$ where k is a constant.

Experiments and theory show that $k = \frac{1}{2}$.

Thus $f = \frac{1}{2l}\sqrt{\frac{\tau}{\mu}}$

LAWS OF VIBRATION OF STRINGED INSTRUMENTS

(i) The frequency of vibration of a stretched string is inversely proportional to the length of the wire provided tension and its mass per metre are kept constant.

i.e $f \ \alpha \ \frac{1}{l}$ where T and μ are constant.

(ii) The frequency of vibration of a stretched string is directly proportional to the square root of the tension in the string provided the length of the string and its mass per metre are kept constant.

i.e $f \ \alpha \ \sqrt{T}$ where l and μ are constant.

(iii) The frequency of vibration of a stretched string is inversely proportional to the square root of its mass per metre of the string provided its length and tension are kept constant.

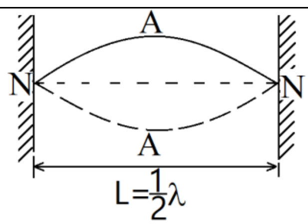
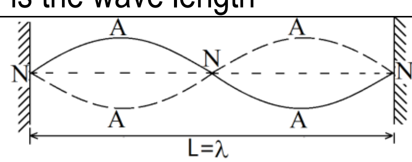
i.e $f \ \alpha \ \frac{1}{\sqrt{\mu}}$ where l and T are constant.

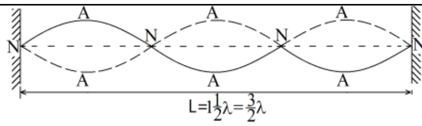
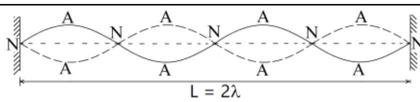
MODES OF VIBRATION OF A STRETCHED STRING

When a stretched string fixed at both of its ends is plucked at a point, a transverse wave is set up which travels towards either ends of the string.

Since the ends are fixed, the waves are reflected and when they meet, a stationary wave is formed with the fixed ends being **nodes(N)**

The strings vibrate under different modes as discussed below.

Mode of vibration and how it is produced	Diagram	Formula for the frequency
1. 1st mode or simplest mode or fundamental mode or 1st harmonic. This is produced by plucking the stretched string in the middle	 L is length of the string and λ is the wave length	$\lambda = 2L$ $V = \lambda f \Rightarrow f = \frac{V}{\lambda}$ $\boldsymbol{f_o = \frac{V}{2L}}$ This is the fundamental frequency
2. Second mode or second harmonic. This is produced by plucking the stretched string $\frac{1}{4}$ way from one fixed end		$\lambda = L$ $V = \lambda f \Rightarrow f = \frac{V}{\lambda} \therefore f_1 = \frac{V}{L}$ From $f_o = \frac{V}{2L} \Rightarrow f_o = \frac{1}{2}\left(\frac{V}{L}\right)$ $f_o = \frac{1}{2} f_1$ and $\therefore$ $\boldsymbol{f_1 = 2f_o}$ This is the 2nd harmonic

3. Third mode or 3rd harmonic. This is also produced by plucking the middle of the stretched string.		$\lambda = \frac{2}{3}L$ $f = \frac{V}{\lambda} \therefore f_2 = V \div \frac{2L}{3}$ $f_2 = V \times \frac{3}{2L} \Longrightarrow f_2 = \frac{3V}{2L}$ $\boldsymbol{f_2 = 3f_o}$ This is the 3rd harmonic or the 1st main overtone.
4. Fourth mode or 4th harmonic. This is produced by plucking the stretched string $\frac{1}{2}\left(\frac{1}{4}\right)$ way i.e. $\left(\frac{1}{8}\right)^{th}$ way of the string.		$\lambda = \frac{1}{2}L$ $f = \frac{V}{\lambda} \therefore f_3 = V \div \frac{L}{2}$ $f_3 = V \times \frac{2}{L} \Longrightarrow f_3 = \frac{2V}{L}$ $\boldsymbol{f_3 = 4f_o}$ This is the 4th harmonic
5. Fifth mode or fifth harmonic. This is produced by plucking the stretched string in the middle.		$\lambda = \frac{2}{5}L$ $f = \frac{V}{\lambda} \therefore f_4 = V \div \frac{2L}{5}$ $f_4 = V \times \frac{5}{2L} \Longrightarrow f_4 = \frac{5V}{2L}$ $\boldsymbol{f_4 = 5f_o}$ This is the 5th harmonic or the 2nd main overtone.
6. Sixth mode or 6th harmonic produced by plucking the string $\frac{1}{2}\left(\frac{1}{8}\right)$ way i.e. $\left(\frac{1}{16}\right)^{th}$ way from one fixed end.		$\boldsymbol{f_5 = 6f_o}$ This is the 6th harmonic
7. Seventh mode of vibration or 7th harmonic produced by plucking the middle.		$\boldsymbol{f_6 = 7f_o}$ This is the 7th harmonic or the 3rd main overtone.

Definitions

1. **Fundamental frequency** is the lowest predominant frequency produced by an instrument.

2. **The fundamental note** is the note of the lowest frequency that can be produced by a given length of a string under a particular value of tension and mass per unit lengthOr is a note of the lowest predominant frequency produced by an instrument.
3. **A harmonic** is a note whose frequency is an integral multiple of the fundamental frequency.
4. **An overtone** is a note of frequency higher than the fundamental frequency.
   OR it is a higher integral multiple of the fundamental frequency which is produced together with the fundamental note on which other frequencies depend.

Note:
A stretched string can vibrate in more than one mode i.e performing one loop or more. When it is vibrating with only one loop, the note produced is called the **main** or **fundamental note.**

In general, the frequency of the n[th] harmonic is given by $f_n = nf_0 \; , n = 1,2,3,4 ---$

Wheref_0 is the fundamental frequency.
Therefore, the possible frequencies when the string is plucked and it forms stationary waves are; $f_0, 2f_0, 3f_0, 4f_0, 5f_0, ---$
The notes corresponding to the above frequencies are called harmonics. This is one of the frequencies that can be produced by a particular instrument (string or pipe).
From the cases above, the frequencies of the various harmonics are integral multiples of the fundamental frequency, f_0.

NOTE:
1. All the higher odd harmonics are called **<u>main overtones</u>** i.e. $3f_o,\ 5f_o,\ 7f_o,\ 9f_o,$ $11f_o$ etc
2. All the main overtones are produced when the string is plucked in the middle of its length dividing it into equal parts and are odd harmonics.
3. **In general,** the **n[th] main overtone** of a plucked string is the **(2n+1)[th] harmonic** i.e
   $$f_n = (2n+1)f_0 \; , n = 1,2,3,4 -----$$
4. Notes of frequencies $3f_0, 5f_0, 7f_0, ----$ are all produced by plucking in the middle. They can be produced together with the main (fundamental) note i.e they may be accompanying overtones. **<u>Other overtones</u>** are produced plucking the stretched string $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, --- way$ and they include $2f_0, 4f_0, 6f_0, ----$ respectively.
5. Where the string is plucked becomes the antinode(A) . Suppose the string is plucked $\frac{1}{3}$ way, it will not be possible to produce a mode of vibration.

GENERAL EXPRESSION FOR FREQUENCY OF THE nTHHARMONIC

The wavelength of an nth harmonic is generally given as $\lambda_n = \frac{2l}{n}$; $n = 1, 2, 3, 4$ ...

The frequency, f of an nth harmonic is generally given as $f_n = nf_0$, $n = 1,2,3,4 ---$

$$From\ \ v = f_n \lambda_n$$

$$f_n = \frac{v}{\lambda_n} = \frac{v}{\left(\frac{2l}{n}\right)} = \left(\frac{n}{2l}\right)v$$

$$For\ \ a\ \ stretched\ \ string,\ v = \sqrt{\frac{T}{\mu}}$$

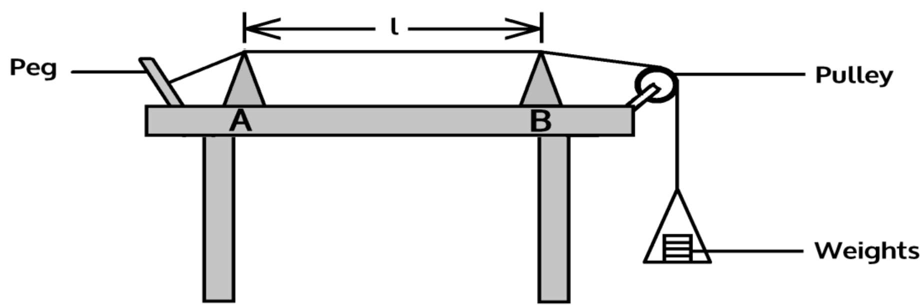
$$\Rightarrow f_n = \left(\frac{n}{2l}\right)\sqrt{\frac{T}{\mu}}\ \ = n\left(\frac{1}{2l}\sqrt{\frac{T}{\mu}}\right) = nf_0$$

$$\therefore f_n = nf_0$$

$$where\ \ n = 1, 2, 3, 4 ----$$

Where T is tension in the string, μ is the mass per unit length of the string.

AN EXPERIMENT TO SHOW THAT A STRETCHED STRING VIBRATES IN MORE THAN ONE MODE



A wire is stretched on two bridges A and B as shown above.

The stretched wire AB is plucked in the middle. The tuning forks of different frequencies are struck and brought, one at a time, near the wire.

The one which produces a loud sound with it is identified.

The procedure is repeated when the wire is plucked $\frac{1}{4}$ and $\frac{1}{8}$ way from B.

It is found that the frequency of the tuning forks resonating with the wire in each of the cases is different.

Hence a wire under tension resonates with more than one frequency or mode.

ALTERNATIVELY

A string is stretched on two bridges as shown above.

The string is plucked in the middle and tuning forks are sounded and brought near it, one at a time.

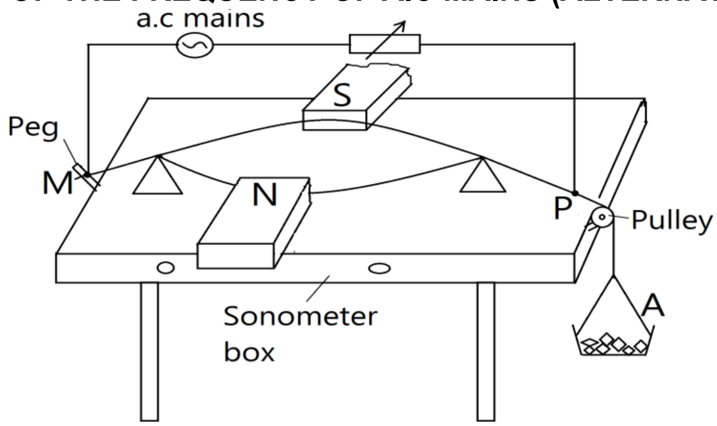
The fork which makes a loud sound with the string is selected and its frequency, f is recorded.

Another tuning fork whose frequency is equal to an odd integral multiple of frequency, f is selected.
The fork is sounded and brought near the string after plucking it in the middle. It is observed that the string again makes a loud sound i.e the note of the fork is resonating with the note from the string.

Questions

1. Describe an experiment to show that a stretched string vibrates in more than one mode.
2. Describe an experiment to determine the frequency of a vibration of a stretched string.

MEASUREMENT OF THE FREQUENCY OF A.C MAINS (ALTERNATING CURRENT)



The a.c whose frequency f is to be measured is passed through a conducting sonometer wire MP of known diameter d and whose material has known density ρ.
The poles N, S of a powerful magnet are placed on either sideof the wire so that its magnetic field is perpendicular to the wire.
The wire is subjected to a transverse force due to the magnetic effect on the current carrying conductor.
The magnitude of the force varies at the rate of the frequency f of the a.c.
The tension in the wire is adjusted by varying the weights on the scale pan A until the wire is seen vibrating through a large amplitude. In this case, the wire is resonating to the applied force.
The length L of the wire between the bridges is now measured, the tension $T = mg$ is found and the mass per unit length $\mu = \pi r^2 \rho$ of the wire is also found where m is the total mass of A and its contents, g is the acceleration due to gravity and $r = \frac{d}{2}$ is the radius of the wire.

The frequency of the a.c is calculated from $f = f_o = \frac{1}{2L} V = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

SUMMARY OF THE FORMULAE

1. $V = \sqrt{\frac{\boldsymbol{T}}{\boldsymbol{\mu}}} = \sqrt{\frac{\boldsymbol{TL}}{\boldsymbol{M}}}$is the velocity of a transverse stationary wave in a vibrating string

2. **(a)** For fundamental frequency of vibration of a string
(i) $\lambda = 2L$

(ii) $f_o = \frac{V}{\lambda} = \frac{V}{2L} = \frac{1}{2L}\sqrt{\frac{TL}{M}}$

(b) $f_n = nf_o$ for n[th] harmonic

$$f_n = \frac{n}{2L}\sqrt{\frac{TL}{M}}$$

EXAMPLES

1. A wire of length $400mm$ and mass $1.2g$ is under tension of $120N$. Find the fundamental frequency.

Solution

$Given\ l = 40cm = 0.4m, m = 1.2g, T = 120N$

$$f_0 = \frac{v}{2l} \quad But\ v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Tl}{m}}$$

$$f_0 = \frac{1}{2l}\sqrt{\frac{T\,l}{m}} = \frac{1}{2\times 0.4}\sqrt{\frac{120\times 0.4}{1.2\times 10^{-3}}}$$

$$f_0 = 250Hz$$

2. The mass of a vibrating length of a wire is $1.20g$ and it is found out that a note of frequency $512Hz$ is produced when the wire is sounding its second overtone. If the tension in the wire is $100N$, calculate the vibrating length of the wire.

Solution

$Given\ l = ??\ , m = 1.2g, T = 100N,\ f = 512Hz$

$For\ second\ overtone,\ f = 5f_0$

$$But\ \ f_0 = \frac{v}{2l}$$

$$Also\ \ v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Tl}{m}}$$

$$f = \frac{5v}{2l} = \frac{5}{2l}\sqrt{\frac{Tl}{m}} = \frac{5}{2}\sqrt{\frac{Tl}{ml^2}} = \frac{5}{2}\sqrt{\frac{T}{ml}}$$

$Squaring\ \ both\ \ sides$

$$f^2 = \frac{25}{4}\left(\frac{T}{ml}\right)$$

$$l = \frac{25T}{4mf^2} = \frac{25\times 100}{4\times 1.2\times 10^{-3}\times 512^2}$$

$$l = 1.99m$$

3. A string of length $50cm$ and mass $5.0g$ is stretched between two points. If the tension in the string is $100N$, find the frequency of the second harmonic.

Solution

$Given\ \ l = 50cm$, $\mathrm{m} = 5g$, $\mathrm{T} = 100\mathrm{N}$

$For\ second\ harmonic, f = 2f_0$

$$But f_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2l}\sqrt{\frac{T\,l}{m}}$$

$$f = 2\left(\frac{1}{2l}\sqrt{\frac{T\,l}{m}}\right) = \frac{1}{l}\sqrt{\frac{T\,l}{m}}$$

$$f = \frac{1}{0.5}\sqrt{\frac{100\times 0.5}{5\times 10^{-3}}}$$

$$f = 200Hz$$

4. A sonometer wire of length $76cm$ is maintained under tension of $40N$ and an alternating current passes through the wire. A horse shoe magnet is placed with its poles either side of the wire at its midpoint and the resulting forces set the wire in resonant vibration. If the density of the material of the wire is $8800kgm^{-3}$ and the diameter of the wire is $1mm$, what is the frequency of the alternating current?

Solution

$$At\ \ resonance\ ,\ f_{a.c} = f_{wire}$$

$$For\ \ 1^{st}\ \ resonance\ \ f_{wire} = \left(\frac{1}{2l}\right)\sqrt{\frac{\tau}{\mu}}$$

$$\Rightarrow f_{a.c} = \left(\frac{1}{2l}\right)\sqrt{\frac{\tau}{\mu}}$$

$$\mu = \frac{m}{l} = \frac{volume\times density}{length} = \frac{Al\times\rho}{l} = A\rho = \pi r^2\rho$$

$$But\ \ \rho = 8800kgm^{-3},\ r = 0.5mm = 5\times 10^{-4}m,\ l = 0.76m,\ \tau = 40N$$

$$\Rightarrow f_{a.c} = \left(\frac{1}{2l}\right)\sqrt{\frac{\tau}{\pi r^2\rho}} = \left(\frac{1}{2\times 0.76}\right)\sqrt{\frac{40}{\pi\left(5\times 10^{-4}\right)^2\times 8800}}$$

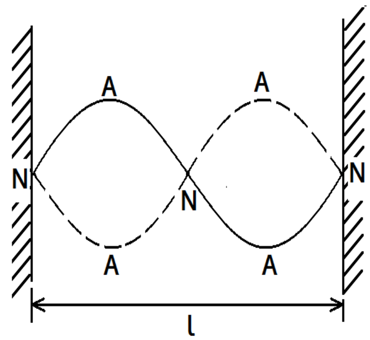
$$\therefore\ f_{a.c} = 50Hz$$

5. A string of length $0.5m$ and mass $5g$ is stretched between two fixed points. If the tension in the string is $100N$, find the frequency of the second harmonic.

Solution

Given $l = 0.5m, m = 5g, \tau = 100N$

For 2nd harmonic



From the diagram $\lambda = l$

But $v = f\lambda \Rightarrow f = \frac{v}{\lambda}$

$\Rightarrow f_1 = \frac{v}{l}$

For a stretched string $v = \sqrt{\frac{\tau}{\mu}}$ But $\mu = \frac{m}{l}$

$$\Rightarrow v = \sqrt{\frac{\tau}{\left(\frac{m}{l}\right)}} = \sqrt{\frac{\tau\, l}{m}}$$

$$f_1 = \frac{v}{l} = \frac{1}{l}\sqrt{\frac{\tau\, l}{m}}$$

$$f_1 = \frac{1}{0.5}\sqrt{\frac{100 \times 0.5}{5 \times 10^{-3}}} = \frac{100}{0.5}$$

$$f_1 = 200Hz$$

6. A uniform wire of length $1.00m$ and mass $2 \times 10^{-2}kg$ is stretched between two fixed points. The tension in the wire is $200N$. The wire is plucked in the middle and released. Calculate the;

(i) Speed of the transverse wave.

(ii) Frequency of the fundamental note.

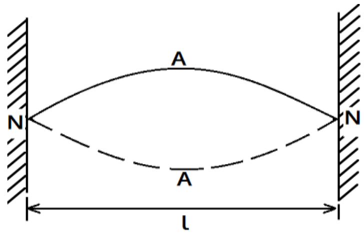
Solution

(i) For a stretched string, $v = \sqrt{\frac{\tau}{\mu}}$ But $\mu = \frac{m}{l}$

$$\Rightarrow v = \sqrt{\frac{\tau}{\left(\frac{m}{l}\right)}} = \sqrt{\frac{\tau\, l}{m}} = \sqrt{\frac{200 \times 1}{2 \times 10^{-2}}}$$

$$v = 100ms^{-1}$$

(ii)



$$l = \frac{\lambda}{2} \Leftrightarrow \lambda = 2l$$

From $v = f\lambda \Rightarrow f = \frac{v}{\lambda}$

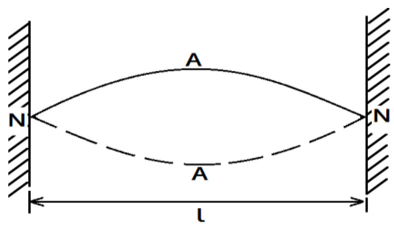
$$\Rightarrow f_0 = \frac{v}{2l} = \frac{100}{2 \times 1}$$

$$f_0 = 50Hz$$

7. The wire of a guitar of length 50cm and mass per metre $1.5 \times 10^{-3} kgm^{-1}$ is under tension of 173.4N. If it is plucked at its midpoint, find the;
   (i) Wavelength,
   (ii) Frequency of the fundamental note.

Solution

(i)



$$l = \frac{\lambda}{2} \Rightarrow \lambda = 2l$$

$$\lambda = 2 \times 0.5$$

$$\lambda = 1.0m$$

(ii) For a stretched string,

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{173.4}{1.5 \times 10^{-3}}} = 340ms^{-1}$$

From $v = f\lambda$

$$f_0 = \frac{v}{\lambda} = \frac{340}{1.0}$$

$$f_0 = 340Hz$$

8. A certain string has a linear mass gradient of $0.25kgm^{-1}$ and is stretched with a tension of 25N. One end is given a sinusoidal motion with a frequency of 5Hz and amplitude of 0.01m. At time $t = 0$, the free end has zero displacement and the original wave is moving in the positive x direction.
   (i) Find the wave speed, angular frequency, period, wavelength, and wave number.
   (ii) write down the wave function describing the motion of the wave.
   (iii) Find the position of a particle at a point $x = 0.5m$ at a time $t = 0.1s$.

Solution

(i) Given $a = 0.01m,\ f = 5Hz,\ T = 25N,\ \mu = 0.25kgm^{-1}$

$$From\ v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{25}{0.25}} = \sqrt{100} = 10ms^{-1}$$

$Angular\ \ frequency\ \ \omega = 2\pi f\ = 2\pi \times 5 = 10\pi\ \ rads^{-1}$

$Period\ \ T = \frac{1}{f} = \frac{1}{5} = 0.2s$

$From\ \ v = f\lambda$

$\Rightarrow wavelength\ \ \lambda = \frac{v}{f} = \frac{10}{5} = 2m$

$wave\ number\ \ k = \frac{2\pi}{\lambda} = \frac{2\pi}{2} = \pi$

(ii) $The\ wave\ equation\ is\ of\ the\ form\ y = asin(\omega t - kx)$

$\Rightarrow y = 0.01sin(10\pi t - \pi x)$

$\therefore y = 0.01sin\,\pi(10t - x)$

(iii) Given $x = 0.5m,\ t = 0.1s,\ y = ??$

$From y = 0.01\sin\pi(10t - x)$

$$\Rightarrow y = 0.01sin\pi(10 \times 0.1 - 0.5)$$

$\therefore y = 0.01m$ from the equilibrium position

TRIAL QUESTIONS

1. A uniform wire of length $0.8m$ and mass $2 \times 10^{-2}kg$ is stretched between two fixed points so that the tension in the wire is 200N. If the wire is plucked in the middle, calculate the;
   (i) Speed of the transverse wave produced.
   (ii) Frequency of the fundamental note.
2. A wire of length $50cm$ and density $8gcm^{-3}$ is stretched between two points. The wire vibrates at a fundamental frequency of 1.5Hz. Calculate the;
   (i) Velocity of the wave along the wire
   (ii) Tension per unit area of cross section of the wire.
3. A wire of length $0.5m$ and mass per unit length $1 \times 10^{-3}kgm^{-1}$ is stretched by a load of 4kg. If it is plucked at its mid point, find the;
   (i) Wavelength of the note.
   (ii) Frequency of the note produced.
4. A wire of length 100cm is under tension of 2N and produces a note of 100 cycles per second when plucked in the middle. Calculate the frequency of the first overtone.

2. STATIONARY LONGITUDINAL WAVES IN PIPES

(a) CLOSED PIPES

Consider a pipe open at one end and closed at the other end. When air is blown into it at the open end, a sound wave travels to the closed end and is reflected back.

The resulting wave is formed by interference between the two waves. The closed end is the node since the layers of air in contact with it are permanently at rest.

At the open end, air is free to vibrate and so vibrates with maximum amplitude resulting into an antinode.

The simplest stationary wave in the closed pipe therefore is one with half a loop.

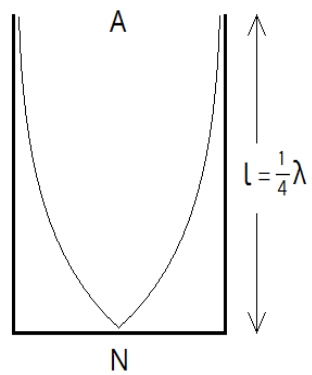
So, the boundary condition is a node at the closed end and an open anti-node at the open end.

POSSIBLE MODES

(i) Fundamental mode

In this first mode, air vibrates at the fundamental frequency.

This is the minimum frequency that can be obtained in a closed pipe. It is also called the 1st harmonic.



From the diagram

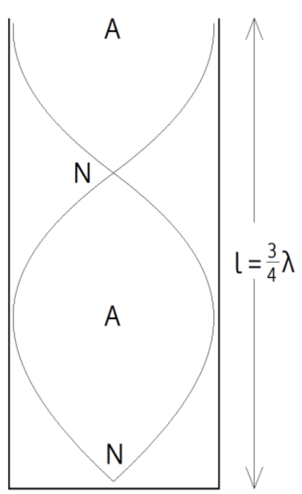
$$l = \frac{\lambda}{4} \Rightarrow \lambda = 4l$$

From $v = f\lambda$

$$f_0 = \frac{v}{\lambda}$$

$f_0 = \frac{v}{4l}$ This frequency f_o is the fundamental frequency

(ii) 1st overtone (3rd harmonic)



From the diagram, $l = \frac{3}{4}\lambda \Rightarrow \lambda = \frac{4}{3}l$

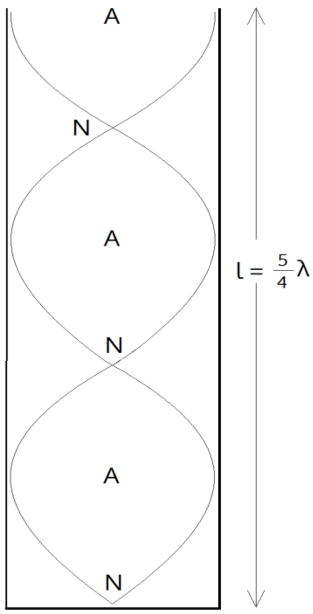
From $v = f\lambda \Rightarrow f = \frac{v}{\lambda}$

$$f_1 = \frac{v}{\left(\frac{4l}{3}\right)} = \frac{3v}{4l} = 3\left(\frac{v}{4l}\right) \text{ But } f_0 = \frac{v}{4l}$$

$$\Rightarrow f_1 = 3f_0$$

The frequency f_1 is said to be the 3rd harmonic because it is obtained by multiplying thefundamental frequency by 3.

(iii) 2nd overtone (5th harmonic)



From the diagram $l = \frac{5}{4}\lambda \Rightarrow \lambda = \frac{4l}{5}$

$v = f\lambda \Rightarrow f = \frac{v}{\lambda}$

$f_2 = \frac{v}{\left(\frac{4l}{5}\right)} = 5\left(\frac{v}{4l}\right)$

But $f_0 = \frac{v}{4l}$

$\therefore f_2 = 5f_0$

f_2 is called the 5th harmonic because it is obtained by multiplying the fundamental frequency by 5.

In general, the **nth overtone** of a closed pipe is the **(2n+1)thharmonic** i.e

$f_n = (2n+1)f_0,\ n = 1,2,3,--$

For an mthmode of vibration of air in a closed pipe, $\lambda_m = \frac{4L}{(2m-1)}$; $m = 1, 2, 3, \ldots$ and

$$(2m-1)f_o = f;\ m = 1, 2, 3, \ldots$$

Conclusion: Closed pipes produce only odd harmonics i.e $f = nf_0$ where n is an odd number.

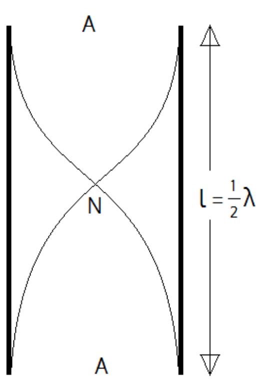
(b) OPEN PIPES

An open pipe is one which is open at both ends.

In open pipes, antinodes occur at both ends whose air is free to move.

The mode of vibration with the lowest frequency has two antinodes and one node.

(i) Fundamental mode (1st harmonic)



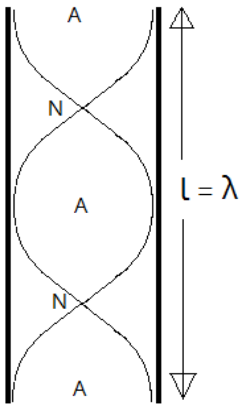
From the diagram, $l = \frac{\lambda}{2} \Rightarrow \lambda = 2l$

From $v = f\lambda \Rightarrow f = \frac{v}{\lambda}$

$f_0 = \frac{v}{2l}$

This frequency is the fundamental frequency (1st harmonic) which is similar to that of strings.

(ii) 1st overtone or 2nd harmonic



From the diagram, $l = \lambda$

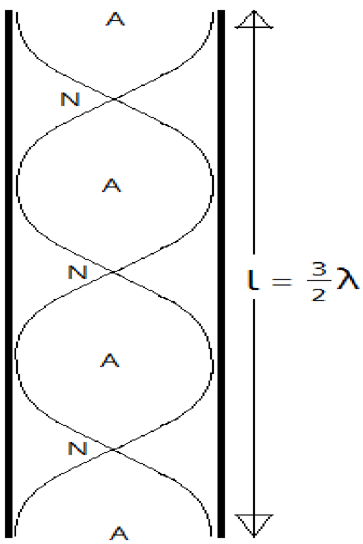
From $v = f\lambda$

$f_1 = \frac{v}{l} = 2\left(\frac{v}{2l}\right)$,But $f_0 = \frac{v}{2l}$

$f_1 = 2f_0$

f_1 is the second harmonic because it is obtained by multiplying the fundamental frequency by 2.

(iii) 2nd overtone or 3rd harmonic



From the diagram, $l = \frac{3}{2}\lambda \Rightarrow \lambda = \frac{2l}{3}$

From $v = f\lambda$

$f_2 = \frac{v}{\lambda} = \frac{v}{\left(\frac{2l}{3}\right)} = 3\left(\frac{v}{2l}\right)$

But $f_0 = \frac{v}{2l}$

$\therefore f_2 = 3f_0$

f_2 is the third harmonic because it is obtained by multiplying the fundamental frequency by 3.

The harmonics produced by an ope similar to those of vibrating strings.

Conclusion

Generally, the open pipes produce both odd and even harmonics.

Note:

1. The fundamental frequency of an open pipe is twice that of the closed pipe.
2. Since the open pipe overtones have different frequencies from those of the closed pipe, the quality of the same note is different when sounded on a closed pipe and on an open pipe.

ADVANTAGE OF OPEN PIPES OVER CLOSED PIPES

Open pipes produce notes of better quality than closed pipes of the same length. This is because all harmonics (both odd and even) can be produced in an open pipe while only odd harmonics can be produced in closed pipes.

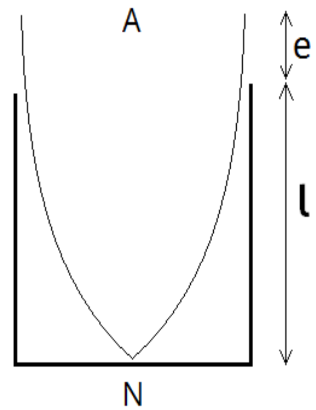
Question: Explain why open pipes are preferred to closed pipes as musical instruments.

Solution:
Closed pipes produce only odd harmonics while open pipes produce both odd and even harmonics. A note with many overtones is of better quality than the same note with fewer overtones. Thus a note played on an open pipe is of better quality than the same note played on a closed pipe.

END CORRECTION

The air at the open end of the pipe is free to move and hence the vibrations of air at this end of a sounding pipe extend a little into the air outside the pipe.
The antinode of the stationary wave due to any note is at a distance, e from the open end of the pipe. This distance, e is known as the **end correction**.
End correction is a small length added to the length of the pipe due to extensions of vibrating air beyond the open end of the pipe.

(a) For a closed pipe

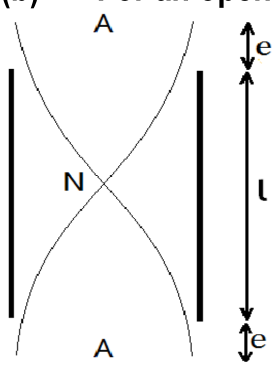


From the diagram above, $l + e = \frac{\lambda}{4} \Rightarrow \lambda = 4(l + e)$

Fundamental frequency, f_0 is obtained from; $f_0 = \frac{v}{\lambda} = \frac{v}{4(l + e)}$

Where; v is velocity of sound in air
l is length of air column or pipe
e is end correction of the pipe.

(b) For an open pipe



$$l + 2e = \frac{\lambda}{2}$$
$$\lambda = 2(l + 2e)$$

The fundamental frequency f_0 can be obtained as;

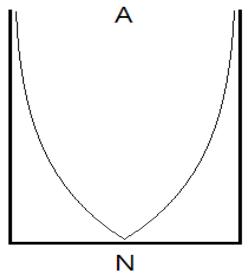
$$f_0 = \frac{v}{\lambda} = \frac{v}{2(l + 2e)}$$

NOTE:
The mathematical theory of the end correction shows that $e = 0.58r \approx 0.6r$.
Where r is radius of the pipe.
Hence the wider the pipe, the bigger the end correction.
Identical pipes have the same end correction value.

VARIATION OF PRESSURE OF VIBRATING AIR IN A PIPE

The disturbance set up at one place causes gas or air particles in that area to move away. As they move, they form regions of high concentration of gas particles hence high pressure, and regions of low concentration thus low pressure. High pressure is again formed in the regions which originally had low pressure and vice versa.
This continues repeatedly in all directions, thus forming a progressive wave, and energy is thus transferred from points of disturbance to others.

MOTION OF AIR IN A CLOSED TUBE



Air at **A** vibrates with maximum amplitude. The amplitude of vibration decreases as the end **N** is approached. Air at **N** is stationary.

A is the antinode and **N** is the node.

RESONANCE

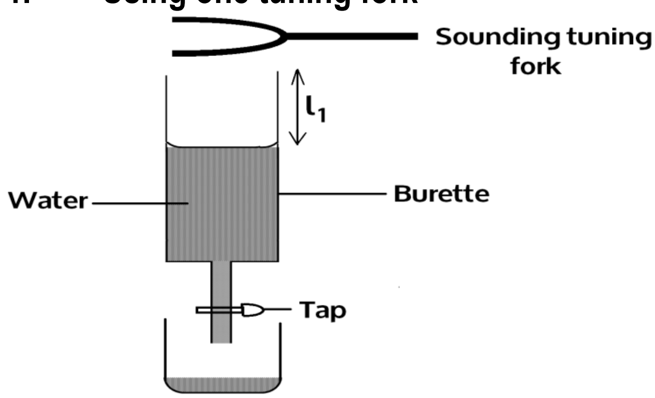
Resonance is a condition where a system is set into vibration/oscillation at its natural frequency due to impulses received from a nearby system oscillating at the same frequency.
Resonance is used in tuning musical instruments, measuring frequencies and measuring speed of sound. However it can cause collapse of big structures like buildings.

RESONANCE IN A TUBE OR PIPE

When a sounding tuning fork is held just above the mouth of a pipe, air inside the pipes is set into vibrations by the periodic force exerted on it by the prongs of the fork.
For other tuning forks, the vibrations are feable/weak and the intensity of the sound heard is low since they are forced vibrations.
When the tuning fork is of the same frequency as the fundamental frequency (or other allowed frequencies) of the pipe, the air inside it is set into resonance and the vibration is large resulting into a loud sound.

MEASUREMENT OF VELOCITY OF SOUND USING A RESONANCE TUBE AND DETERMINING OF THE END CORRECTION

1. Using one tuning fork



A burette is filled with water and a sounding tuning fork is held above it.
The tap is opened and water allowed to drain out slowly until a loud sound is heard.
The tap is immediately closed and the length, l_1 of the air column in the burette is measured.

The fork is again sounded and held over the mouth of the tube. Water is drained out further until another loud sound is heard.
The tap is closed and the length l_2 of the air column measured and recorded together with the frequency *f* of the tuning fork.
The velocity v of sound is then calculated from $v = 2f(l_2 - l_1)$.

Theory of the experiment

1st loudsound (1st resonance)

$$l_1 + e = \frac{\lambda}{4}$$

$$\lambda = 4(l_1 + e) \text{ ------------ } (1)$$

2nd loud sound (2nd resonance)

$$l_2 + e = \frac{3}{4}\lambda$$

$$3\lambda = 4(l_2 + e) \text{ ---------- } (2)$$

Equation (2) – Equation (1)

$$3\lambda - \lambda = 4(l_2 + e) - 4(l_1 + e)$$

$$2\lambda = 4(l_2 - l_1)$$

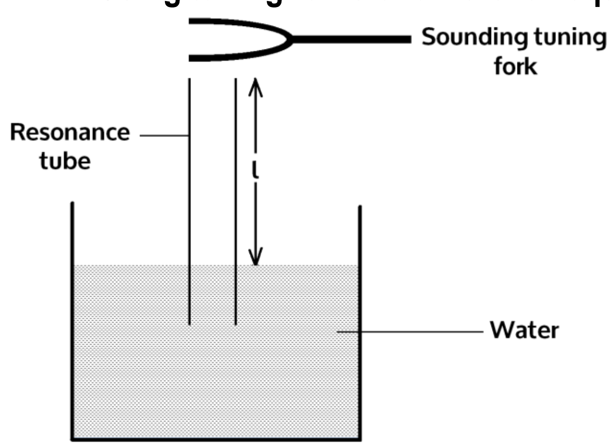
$$\lambda = 2(l_2 - l_1)$$

Using $v = f\lambda$

$$v = 2f(l_2 - l_1)$$

NOTE: After finding $\lambda = 2(l_2 - l_1)$,the end correction e can be obtained by using equation 1 or 2.

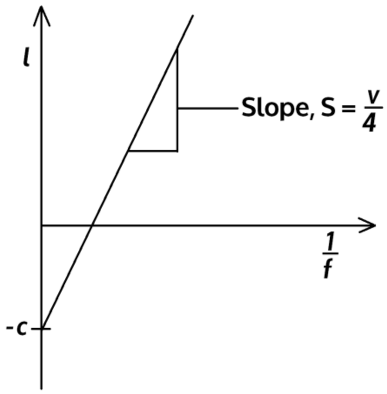
2. Using tuning forks of different frequencies



A resonance tube is placed to stand in a tall jar of water.
Starting with a very short length of the air column, a vibrating tuning fork is held over the mouth of the resonance tube. The tube is then raised slowly until a point where a loud sound is heard.
The length, l of the air column in the tube is measured and recorded together with the frequency f of the tuning fork.

The experiment is repeated with other tuning forks and the results recorded in a suitable table including values of $\frac{1}{f}$

A graph of $l$against $\frac{1}{f}$ is plotted and the slope, S calculated.



The velocity v of sound is now calculated from $V = 4S$.

Note: The end correction is $e = -c$

Theory of the experiment

Consider the first point of resonance

$l + e = \frac{\lambda}{4} \Rightarrow \lambda = 4(l + e)$

$From \ v = f\lambda$

$v = f \times 4(l + e) = 4f(l + e)$

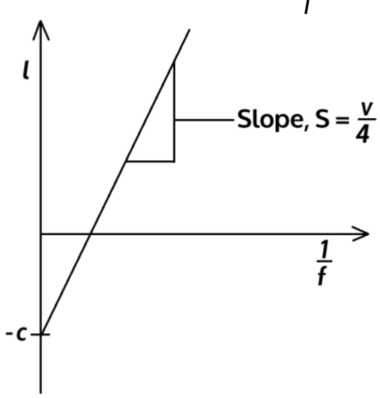
$l + e = \frac{v}{4f} \Rightarrow l = \frac{v}{4f} - e$

$l = \left(\frac{v}{4}\right)\frac{1}{f} - e$

Comparing with the general equation of the line $y = mx + c$

$$s = \frac{V}{4} \Longrightarrow V = 4s \ and - e = -c$$

A graph of l against $\frac{1}{f}$ gives a straight line with an intercept on the l –axis (y –intercept)



End correction, $e = -c$

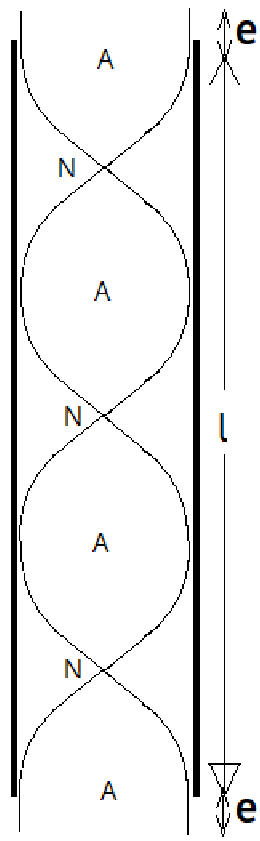
From the graph above, slope, $S = \frac{v}{4} \Rightarrow v = 4S$

Thus the velocity v of sound can be obtained from the equation above.

EXAMPLES

1. A vibrating tuning fork of frequency $330Hz$ is held over a long glass tube open at both ends. If the velocity of sound is $320ms^{-1}$ and the end correction is $0.01m$, determine the length of the tube if the third harmonic of a sound note is set up.

Solution



For 3rd harmonic, $l + 2e = \frac{3}{2}\lambda$

$$\lambda = \frac{2}{3}(l + 2e)$$

$$f = \frac{v}{\lambda} = \frac{v}{\left(\frac{2}{3}(l + 2e)\right)} = \frac{3v}{2(l + 2e)}$$

$But\ v = 320ms^{-1}, e = 0.01m, f = 330Hz$

$$330 = \frac{3 \times 320}{2(l + 2 \times 0.01)}$$

$$l = 1.43m$$

OR For 1st harmonic (fundamental frequency)

$$l + 2e = \frac{\lambda}{2} \Longrightarrow \lambda = 2(l + 2e)$$

$$f = \frac{v}{\lambda} \Longrightarrow f_o = \frac{V}{2(l + 2e)}$$

For 3rd Harmonic $f = 3f_o$

$$\Longrightarrow f = \frac{3V}{2(l + 2e)}$$

$$\therefore 330 = \frac{3 \times 320}{2(l + 0.02)}$$

$$l = 1.43m$$

2. A cylindrical pipe of length $29cm$ is closed at one end. The air in the pipe resonates with a tuning fork of frequency $860Hz$ sounded near the open end of the tube.

Determine the mode of vibration if the velocity of sound in air is $340\ ms^{-1}$. (Neglect the end correction)

Solution

For a closed pipe, $f_0 = \frac{v}{4(l+e)}$

A closed pipe produces only odd harmonics given by;

$(2m-1)f_0 = f; m = 1, 2, 3, \ldots$ where m is the mode of vibration

$$(2m-1)\frac{V}{4(l+e)} = f$$

$But\ l + e \approx l$

$$(2m-1)\frac{V}{4l} = f$$

$$(2m-1)\frac{340}{4 \times 0.29} = 860$$

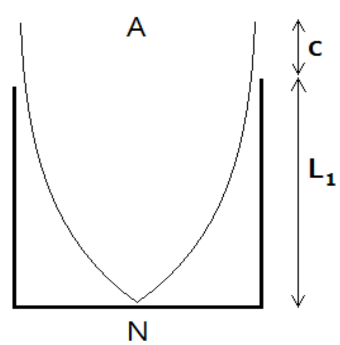
$m = 1.967 \approx 2$

Thus, the vibration is the 2nd mode or 1st overtone or 3rd harmonic since $f = (2 \times 2 - 1)f_0 = 3f_0$

3. In an experiment using a resonance tube, the first two successive positions of resonance were obtained when the lengths of the air column were 15.4cm and 48.6cm respectively. If the velocity of sound in air at that time was $34000 cms^{-1}$,
   (a) Calculate the frequency of the source used and the end correction.
   (b) Find the length when the next resonance occurs, when the air column is further increased in length.

Solution

(a)

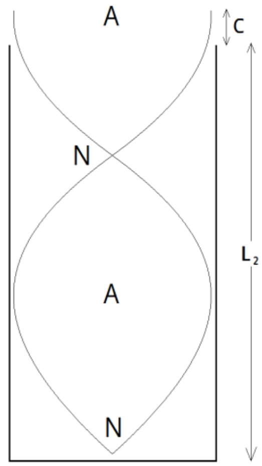


$Given\ l_1 = 15.4cm,\ v = 34000cms^{-1}$

$From\ l_1 + c = \frac{\lambda}{4} \Rightarrow \lambda = 4(l_1 + c)$

$\lambda = 4(15.4 + c)$

$\lambda = 61.6 + 4c$ ------------- (1)



$$l_2 = 48.5cm$$
$$l_2 + c = \frac{3}{4}\lambda$$
$$3\lambda = 4(l_2 + c)$$
$$3\lambda = 194.4 + 4c \quad ------------(2)$$
$$Eqn(2) - Eqn(1)$$
$$3\lambda - \lambda = 194.4 + 4c - (61.6 + 4c)$$
$$2\lambda = 194.4 - 61.6$$
$$2\lambda = 132.8$$
$$\lambda = 66.4cm$$

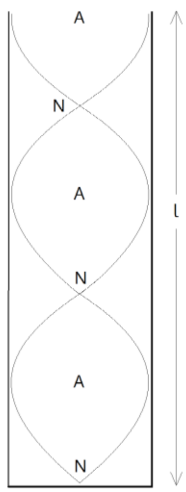
From $v = f\lambda \Rightarrow f = \frac{v}{\lambda}$

$$f = \frac{34000\ cms^{-1}}{66.4\ cm}$$
$$f = 512.05Hz$$

From $eqn(1)$

$$\lambda = 61.4 + 4c$$
$$4c = 66.4 - 61.6$$
$$4c = 4.8$$
$$c = 1.2cm$$

(b) For the third harmonic



From the diagram,

$$l_3 + c = \frac{5}{4}\lambda \Rightarrow l_3 = \frac{5}{4}\lambda - c$$

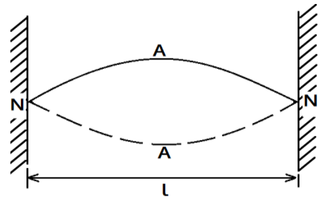
$$l_3 = \frac{5}{4} \times 66.4 - 1.2$$
$$l_3 = 81.8cm$$

4. A stretched wire of length 0.75m, radius 1.36mm and density $1380kgm^{-3}$ is clamped at both ends and plucked in the middle. The fundamental note produced by the wire has the same frequency as the first overtone in a pipe of length $0.15m$ closed at one end.

(a) Sketch the standing wave pattern in the wire.
(b) Calculate the tension in the wire. ($velocity\ of\ sound\ in\ air = 330ms^{-1}$)

Solution

(a)



(b) **For the wire**

$l = 0.75m, r = 1.36mm, \rho = 1380kgm^{-3}$

$Volume,\ V = \pi r^2 l$

$But\ \ mass\ , m = \rho V = \rho\pi r^2 l$

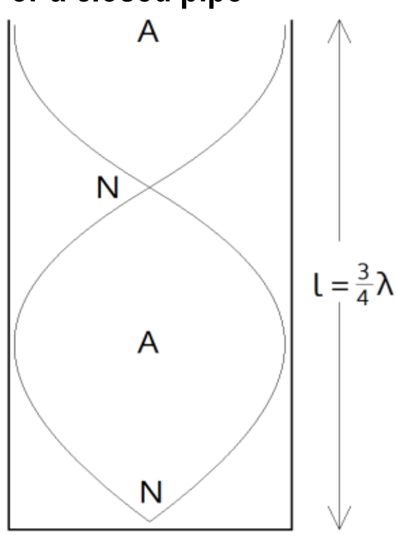
$$Mass\ per\ unit\ length\ , \mu = \frac{m}{l} = \frac{\rho\pi r^2 l}{l} = \rho\pi r^2$$

$$\text{Velocity; } V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho\pi r^2}} ----------------(1)$$

$$f_0 = \frac{V}{2L}$$

$$f_0 = \frac{1}{2l}\sqrt{\frac{T}{\rho\pi r^2}} --------------------(2)$$

For a closed pipe



From the diagram,

$$l = \frac{3}{4}\lambda \Rightarrow \lambda = \frac{4}{3}l$$

From $v = f\lambda \Rightarrow f = \frac{v}{\lambda}$

$$f_1 = \frac{v}{\left(\frac{4l}{3}\right)} = \frac{3v}{4l}$$

But $v = 330ms^{-1}, l = 0.15m$

$$f_1 = \frac{3\times 330}{4\times 0.15} = 1650Hz$$

OR

$f_1 = 3f_o$ where $f_o = \frac{V}{4L}$

$$f_1 = \frac{3V}{4L}$$

$$f_1 = \frac{3\times 330}{4\times 0.15}$$

$$f_1 = 1650Hz$$

But $f_0 = f_1$

$$\frac{1}{2l}\sqrt{\frac{T}{\rho\pi r^2}} = 1650$$

Squaring both sides gives

$$\frac{1}{4l^2}\frac{T}{\rho\pi r^2} = (1650)^2$$

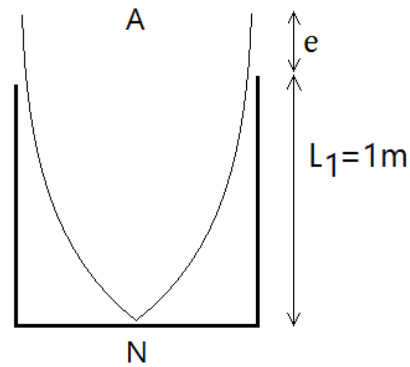
$$T = 4l^2\rho\pi r^2(1650)^2$$

$$T = 4\pi(0.75)^2(1380)(1.36\times10^{-3})^2(1650)^2$$

$$T = 49119.867\ N$$

5. A tube of length 1m has its lowest resonant frequency at 86.2Hz. With a tube of identical dimensions but open at both ends, the first resonance occurs at 171Hz. Calculate the;
   (a) Speed of sound.
   (b) End correction.

Solution



$$l_1 + e = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4(l_1 + e)$$

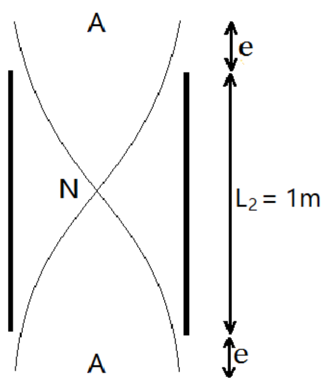
$$\lambda_1 = 4(1 + e)$$

$$\lambda_1 = 4 + 4e - - - - - - - - - - - - - - (1)$$

From $V = f\lambda$; $V = f_1\lambda_1$ But $f_1 = 86.2Hz$

$$V = 86.2(4 + 4e)$$

$$\frac{V}{86.2} - 4 = 4e \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$



$$l_2 + 2e = \frac{\lambda_2}{2} \Rightarrow \lambda_2 = 2(l_2 + 2e)$$

$$\lambda_2 = 2(1 + 2e)$$

$$\lambda_2 = 2 + 4e - - - - - - - - - - - - - (3)$$

$$Also\ \ v = f_2\lambda_2 \text{But} f_2 = 171Hz$$

$$v = 171(2 + 4e)$$

$$\frac{V}{171} - 2 = 4e \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Equating the equations (2) and (4)

$$\frac{V}{86.2} - 4 = \frac{V}{171} - 2$$

$$\frac{V}{86.2} - \frac{V}{171} = 4 - 2$$

$$84.8V = 29480.4$$

$$V = 347.6\,ms^{-1}$$

(b) Substituting in (2)

$$\frac{V}{86.2} - 4 = 4e$$

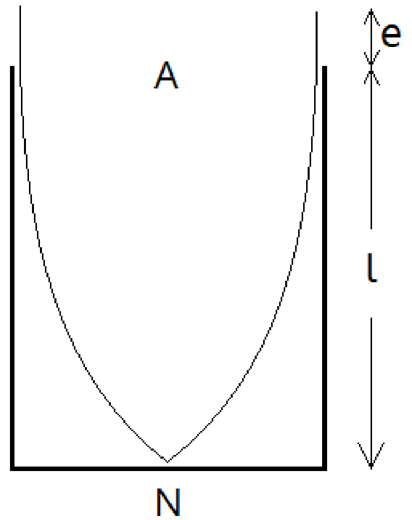
$$\frac{347.6}{86.2} - 4 = 4e$$

$$0.0325 = 4e$$

$$e = 0.00813m$$

6. A closed pipe of length $68cm$ is blown at its open end so that it produces the first harmonic.
   (a) Sketch the wave pattern in the pipe.
   (b) Determine the frequency of the note produced.
   (c) What is the radius of the pipe? (Given the end correction of the pipe is 0.02m and the velocity of sound in air is 340 ms^{-1})

Solution: (a)



(b) Given $l = 68cm = 0.68m, e = 0.02m$

$l + e = \frac{\lambda}{4} \Rightarrow \lambda = 4(l + e)$

$\lambda = 4(0.68 + 0.02) = 2.8m$

From $v = f\lambda \Rightarrow f = \frac{v}{\lambda}$

$f = \frac{v}{\lambda} = \frac{340}{2.8}$

$f = 121.4Hz$

(c) From $e = 0.6r$

$0.02 = 0.6r$

$r = \frac{0.02}{0.6}$

$r = 0.033m$

TRIAL QUESTIONS

1. A cylindrical pipe of length $28cm$ closed at one end is found to be at resonance when a tuning fork of frequency $64Hz$ is sounded near its open end. Determine the mode of vibration of air in the pipe and hence deduce the value of end correction. $(v = 340ms^{-1})$.

   Hint: For an m^th^ mode; $\lambda_m = \frac{4L}{2m-1}$ where $\frac{V}{\lambda_m} = 64 \Longrightarrow \lambda_m = 5.3125m$.

   $\therefore m = 0.6 \approx 1$

   $(2m - 1)f_o = \frac{340}{4(L+e)} = 64$, hence find e.

2. A cylindrical pipe of length $30cm$ is closed at one end. The air in the pipe resonates with a tuning fork of frequency off $825Hz$ sounded near the open end of the pipe. Determine the mode of vibration of air assuming there is no end correction. (Take $v = 330ms^{-1}, Answer; m = 2$)

3. A uniform glass tube open at the top is held vertically with its lower open end dipped into water. The tube resonates to a vibrating tuning fork of frequency $384Hz$ when the air column above water is $18cm$ and again when it is $59.4cm$. Determine the;
   (i) Speed of sound in the air column
   (ii) End correction.

4. A uniform tube $80cm$ long is filled with water and a small loud speaker connected to a signal generator held over the open end of the tube. With the signal generator set at $600Hz$, the water level in the tube is lowered until resonance is first obtained

when the length of the air column is $13cm$. If the third resonance is obtained when the air column is $69.8cm$ long. Calculate the;

(i) Velocity of sound in air

(ii) Fundamental frequency for the tube if it were open at both ends.

5. A uniform tube $50cm$ long is filled with water and vibrating tuning fork of frequency $512Hz$ is sounded and held above it. When the level of water is gradually lowered, the air column resonates with the tuning fork when its length is $12cm$ and again when it is $43.3cm$. Estimate the lowest frequency to which the air in the tube could resonate if the tube was empty.

6. Determine the frequency of the first overtone of a sound note set up in a pipe of length $0.5m$ if it is;

   (i) Open at both ends

   (ii) Closed at one end. (Take the velocity of sound in air to be $330ms^{-1}$ and neglect the end correction).

7. A steel wire of length $40.0cm$ and diameter $0.025cm$ vibrates in unison with a tube, open at both ends, of effective length $60.0cm$. Find the tension in the wire when both the wire and the tube are sounding their fundamental note. (Assume velocity of sound in air is $340ms^{-1}$ and density of steel is $7800kgm^{-3}$).

8. A steel wire of length $40.0cm$ and diameter $0.025cm$ vibrates in unison with a tube, closed at one end and of length $56.4cm$ and end error $3.6cm$. When the tube is sounding its fundamental note, the temperature is $27°$C. Find the tension in the wire. (Assume velocity of sound in air at $0°$C is $331ms^{-1}$ and density of steel is $7800kgm^{-3}$). $\left[\text{Hint}: \left(\frac{v_1}{v_2}\right)^2 = \frac{T_1}{T_2}\right]$

9. The results below were obtained in an experiment to determine the velocity of sound in air using the resonance tube.

l(cm)	f (Hz)
16.0	512.0
17.5	480.0
20.3	426.6
22.0	384.0
26.5	320.0

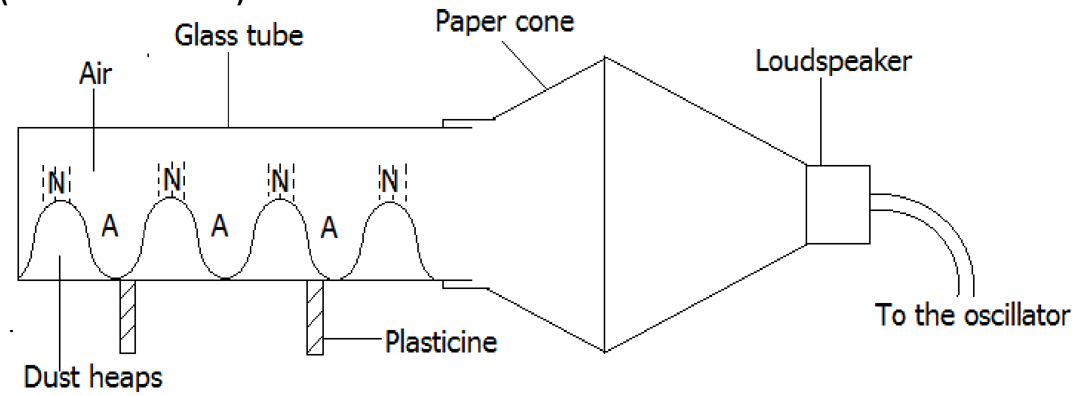
   Plot an appropriate graph and use it to determine the;

   (a) Velocity of sound in air

   (b) End correction

10. A wire of length *0.5m* and diameter *0.34mm* is made of a material of density $7.8\times10^3 kgm^{-3}$ and having a tension of *81N* produces a fundamental note when plucked in the middle. Find the length of the open pipe whose first overtone produces resonance with the wire, assuming end corrections are negligible.

MEASUREMENT OF VELOCITY OF SOUND IN AIR BY DUST TUBE METHOD (KUNDT'S TUBE)



A glass tube closed at one end is placed on plasticine on a table and made to lie horizontally.

Chalk dust or lycopodium powder is sprinkled evenly on the inside of the tube.

A paper cone attached to the loud speaker is fitted over the open end of the tube.The loud speaker is connected to a sensitive oscillator of variable frequency.

The oscillator is turned on and its frequency is slowly increased until the dust finally settles in regularly spaced heaps along the tube.

The peaks at the heaps are nodes of the stationary wave formed along the tube.

The average distance, l between two successive heaps is measured and the frequency, f of the oscillator is noted.

The velocity, v of sound in air is then calculated from the expression,

$v = \lambda f = 2fl \; where \; \lambda = 2l.$

Note: Heaps are formed at the points where there are no vibrations (the nodes)

Source of error: Measuring distance l from outside of the tube may not be accurate hence errors.

Revision Question

Describe an experiment to determine the speed of sound in air using Kundt's dust tube method.

EXAMPLES

1. In the dust tube experiment, the vibrating air due to a nearby loudspeaker causes 7 consecutive heaps of powder to occupy a total distance of $0.6m$ when the air in the tube is set into resonance at a frequency, f. Determine the value of the frequency, f. [Take $v = 340ms^{-1}$]

   Solution

   $Distance \; covered \; by \, 7 \; heaps = 3\lambda$

   $0.6 = 3\lambda$

   $\lambda = 0.20m$

   $From \;\; v = f\lambda$

   $f = \frac{v}{\lambda} = \frac{340}{0.20}$

   $f = 1700Hz$

2. In an experiment to determine the velocity of sound in air in a tube, chalk dust settled in heaps and the distance between three successive heaps is 1.328m, calculate the speed of sound in air if the oscillator frequency is 252Hz.

Solution

$From \quad v = f\lambda$

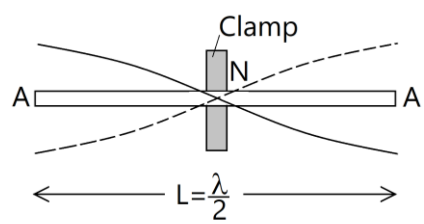
$But \ \lambda = 1.328m, \ f = 252Hz$

$v = 252 \times 1.328$

$v = 334.6ms^{-1}$

WAVES IN A ROD

Consider a rod AA of length L clamped tightly at its midpoint N.



If the rod is stroked along its length by a rosined cloth (a cloth powdered with rosin), a stationary longitudinal wave is set up in the rod due to reflection; and a high-pitched note is heard.

Since the midpoint of the rod is fixed, then it is the node, N of the stationary wave; and since the ends of the rod are free to vibrate, then they are the anti-nodes A.

Thus, the length L of the rod is equal to half of the wavelength $\frac{\lambda}{2}$ of the wave in the rod. $L = \frac{\lambda}{2} \Longrightarrow \lambda = 2L$

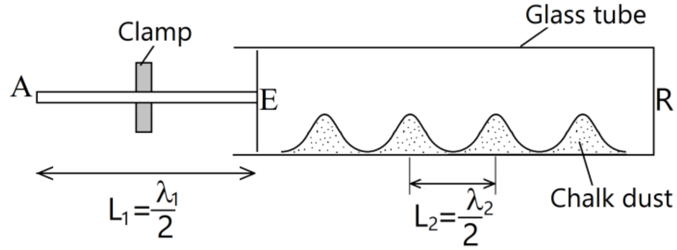
The velocity of the sound wave in the rod $V_r = \lambda f = 2Lf$ where f is the frequency of the note from the rod.

MEASUREMENT OF THE SPEED OF SOUND IN A ROD USING KUNDT'S DUST TUBE

Some chalk dust is evenly sprinkled along the interior of a glass tube closed at one end.

A rod AE of length L_1 is clamped at its midpoint with one end E projecting into the tube.

A disc E which just clears or touches the sides of the tube is connected to end E of the rod.



The rod is stroked at A by a rosined cloth in the direction EA such that the rod vibrates longitudinally and a high-pitched note is heard.
The rod is stroked until chalk dust in the tube settles in heaps.
The average distance L_2 between two consecutive heaps is measured and recorded.
The velocity of sound in the rod is obtained from $V_r = \frac{L_1 V_a}{L_2}$ where V_a is the velocity of sound in air which is in the tube.

Note: A rosined cloth is a cloth powdered or smeared with rosin to increase its grip on the rod as it is slid/stroked over the rod.

THEORY/EXPLANATION
The end E of the vibrating rod AE acts as a vibrating source of a sound wave of the same frequency as the wave in the rod.
A sound wave thus travels through air in the glass tube and is reflected at the closed end R.
From $V = \lambda f$;
The velocity V_r of the wave of frequency f_r in the rod is:

$$V_r = 2L_1 f_r \dots\dots\dots\dots\dots\dots\dots\dots (i)$$

The velocity of sound waves in air in the tube is

$$V_a = 2L_2 f_a \dots\dots\dots\dots\dots\dots\dots\dots (ii)$$

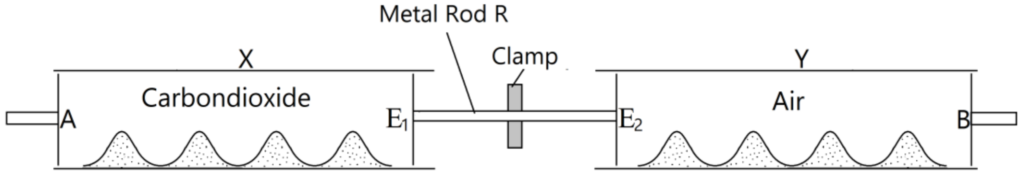
But $f_a = f_r \implies \frac{V_r}{V_a} = \frac{L_1}{L_2} \therefore V_r = \frac{L_1}{L_2} V_a$ where V_a is known

Question
Describe an experiment to compare the velocity of sound in a rod and in air.

Note: To determine the velocity V_g of sound in a gas using Kundt's tube, the air in the tube is replaced by the gas after finding the velocity V_r of sound in the rod. When the tube is filled with the gas, the rod is stroked until the chalk dust settles in heaps. The average distance L_3 between two successive heaps is measured. From $V_r = 2L_1 f_r$ and $V_g = 2L_3 f_g$ where $f_r = f_g, \frac{V_g}{V_r} = \frac{L_3}{L_1}$. So, knowing V_r enables V_g to be obtained.

Assignment: Describe clearly the procedure for obtaining the velocity V_g of sound in a gas using Kundt's dust tube.

COMPARISON OF THE SPEEDS OF SOUND IN GASES USING KUNDT'S DUST TUBE



Lycopodium powder or chalk dust is evenly sprinkled along the interiors of open tubes X and Y into which the ends of a metal rod R project.
Discs E_1 and E_2 which just touch or clear the sides of the tubes are attached to the ends of the metal rod R.
The middle of the metal rod R is clamped.
Two gases; carbon-dioxide and air are contained or put in the tubes X and Y respectively, and movable discs A and B which also just touch or clear thesides of the tubes are used to close the tubes.
The metal rod R is stroked while adjusting the positions of the movable discs A and B in turn until the powder or dust in each tube settles in heaps at various nodes.
The average distances d_x and d_y between successive nodes in tubes X and Y respectively are measured.
The frequency f of the sound wave in X and Y is the same.
$f = \frac{V}{\lambda} \therefore \frac{V_c}{\lambda_c} = \frac{V_a}{\lambda_a}$ where V_c and λ_c are velocity and wavelength respectively in carbon-dioxide and V_a and λ_a are velocity and wavelength respectively in air.

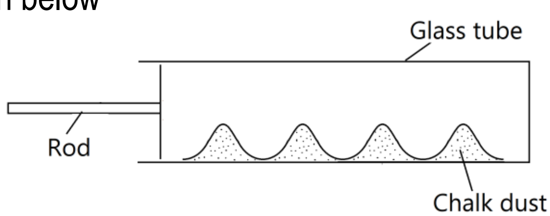
$$\therefore \frac{V_c}{V_a} = \frac{\lambda_c}{\lambda_a}$$

But $\lambda_c = 2d_x$ and $\lambda_a = 2d_y \Longrightarrow \frac{V_c}{V_a} = \frac{d_x}{d_y}$

Hence V_c and V_a compared
If V_a is known, then V_c can be calculated.

Example

in an experiment to determine the speed of sound in air contained in a tube, chalk dust settled in heaps as shown below



If the frequency of the vibrating rod is 220Hz and the distance between their successive heaps is 1.50m, calculate the speed of sound in air.

Solution

$$\lambda = 1.50m, V = \lambda f = 1.5 \times 220 = 330\ ms^{-1}$$

TRANSMISSION OF ENERGY BY A WAVE

In all travelling (progressive) waves, energy propagates through the medium in the direction of travel of the wave.
Each particle of the medium has energy of vibration and passes it on to the succeeding particles.
In oscillations or vibrations where there is no damping, the energy of the vibrating particles changes from kinetic to potential energy and back, but the total energy, E remains constant.

$Energy\ , E =$ Maximum kinentic energy $(K.E_{max})$

$E = \frac{1}{2}mv_{max}^2$ where v_{max} is the maximum velocity of a particle.

From the wave equation, $y = a\sin(\omega t - kx)$

$$v = \frac{dy}{dt} = a\omega\ cos\ (\omega t - kx)$$

$v_{max} = a\omega$ or $A\omega$ where **a** or **A** is amplitude.

$$\therefore E = \frac{1}{2}m(\omega A)^2 = \frac{1}{2}m\omega^2 A^2$$

$$But\ \ \omega = 2\pi\ f$$

$$E = \frac{1}{2}m(2\pi\ f)^2 A^2$$

$$E = 2m\pi^2 f^2 A^2$$

$Where\ m\ is\ mass\ of\ the\ vibrating\ particle\ and\ f\ is\ frequency\ of\ vibration.$

For N vibrating particles in the medium, $E = 2(Nm)\pi^2 f^2 A^2$, where (Nm) is the total mass of the vibrating layer of the medium.

ENERGY PER UNIT VOLUME

As a wave passes through a medium, the energy per unit volume of the medium is the energy of the particle times the number of particles per unit volume.

$$\boldsymbol{\frac{E}{Volume} = \frac{2(Nm)\pi^2 f^2 A^2}{V} = 2\left(\frac{Nm}{V}\right)\pi^2 f^2 A^2} = 2m\pi^2 f^2 A^2 \times \frac{N}{V}$$

$\boldsymbol{But \frac{Nm}{V} = density, \rho\ \ of\ the\ \ medium}$, where *V* is the volume of the medium

$$\frac{E}{Volume} = 2\rho\pi^2 f^2 A^2$$

INTENSITY (I)

This is the rate of energy transfer through a medium by a wave per square metre of the area perpendicular to the direction of propagation of a wave.

The S.I unit of intensity is Wm^{-2} or $Js^{-1}m^{-2}$

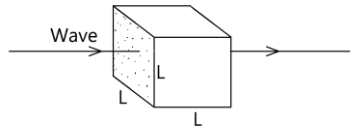
$$I = \frac{Rate\ \ of\ \ \ energy\ \ transfer}{Area} = \frac{Power}{Area}$$

$$I = \frac{P}{Area} = \frac{E}{t \times Area}$$

Where P is the power of the wave or the rate of energy transfer by the wave, E is the energy transferred by the wave, t is the time for energy transfer.

$$I = \frac{2Nm\pi^2 f^2 A^2}{t \times Area}$$

Consider a cube of the medium of side L and t is the time of travel of the wave through the cube across opposite faces.



$t = \frac{l}{v}$ and area $= l^2$ where v is velocity of the wave

$$\Rightarrow I = \frac{2Nm\pi^2 f^2 A^2}{\left(\frac{l}{v}\right) \times l^2} = \frac{2Nm\pi^2 f^2 A^2 v}{l \times l^2} = \frac{2Nm\pi^2 f^2 A^2 v}{l^3}$$

$$But\ \frac{Nm}{l^3} = \rho$$

$$\therefore I = 2\rho\pi^2 f^2 A^2 v - - - - - - - - - - - - - - \otimes$$

But $\omega = 2\pi f \Longrightarrow \omega^2 = 4\pi^2 f^2$

$$\frac{\omega^2}{4\pi^2} = f^2$$

$$\therefore I = 2\rho\pi^2 \frac{\omega^2}{4\pi^2} A^2 v$$

$$\therefore I = \frac{1}{2}\rho\omega^2 A^2 v - - - - - - - - - - - - - - - \otimes\otimes$$

Conclusions:

- The intensity, I of a wave is directly proportional to the square of the amplitude and square of the frequency.So, intensity I of a sound wave depends on its amplitude and its frequency.
- The greater the intensity of sound, the louder the sound since loudness also depends on amplitude of sound.

VARIATION OF THE INTENSITY OF SOUND WITH DISTANCE FROM THE SOURCE

The intensity of sound decreases as the distance from the source increases following the inverse square law. This is because;

- As sound travels further away from the source, sound energy is lost to the transmitting medium due to air resistance.
- As sound travels further away from the source, its energy becomes spread over a wider area.

A sound wave travelling outwards from the source has its energy spreading over the surface of a sphere centered at the source.
The intensity at any point which is at a distance, r from the source can be given by;

$$I = \frac{Power(P)}{Area(A)} \quad where\ A\ is\ area\ of\ a\ sphere$$

$$I = \frac{P}{4\pi r^2} = \left(\frac{P}{4\pi}\right)\frac{1}{r^2}$$

$$But\ \frac{P}{4\pi} = k\ (constant)$$

$$\Rightarrow I = k\left(\frac{1}{r^2}\right)$$

$$\therefore I\alpha\frac{1}{r^2}$$

From the equation above, it can be observed that the intensity of sound is inversely proportional to the square of the distance from the source. This explains why the loudness of sound decreases with increasing distance from the source.

Question

Explain why the amplitudeor loudness of a sound wave decreases as the distance from the source increases.

Solution

As the wave progresses, some energy is absorbed by the transmitting medium. Also as the wave spreads out, the energy is spread out over a wider area.

Thus, as the distance r from the source increases, there is a decrease in intensityI of the wave (sound) received by the observer, since from the inverse square law, $I\alpha\frac{1}{r^2}$.But

$I\ \alpha\ A^2$ where A is the amplitude

Thus, $A^2\alpha\frac{1}{r^2} \Rightarrow A\alpha\frac{1}{r}$

Therefore, the amplitude(A) is inversely proportional to the distance from the source. Thus, increasing the distance leads to a reduction in amplitude of vibration, and thus a reduction in loudness.

Example

The velocity of sound in air at STP is 340ms^{-1}. A source of sound of frequency 300Hz radiates energy uniformly in all directions at a rate of 10W. Find the;

(i) Intensity of sound at distance of 20m from the source.

(ii) Amplitude of the sound at this distance given that density of air at STP is 1.29kgm^{-3}

Solution

(i) Given $v = 330ms^{-1},\ f = 300Hz, P = 10W,\ radius, r = 20m$

$$From\ \ I = \frac{P}{A} \quad But\ \ A = 4\pi r^2$$

$$\Rightarrow I = \frac{P}{4\pi r^2}$$

$$\therefore I = \frac{10}{4\pi \times 20^2}$$

$$I = 2\times 10^{-3}Wm^{-2}$$

(ii) $From\quad I = 2\rho\pi^2 f^2 A^2 v$

$$A = \sqrt{\frac{I}{2\rho\pi^2 f^2 v}} = \sqrt{\frac{2\times10^{-3}}{2\times1.29\times\pi^2\times300^2\times340}}$$

$$A = 1.60\times10^{-6}\,m$$

Revision question

Show that intensity, I of a wave is directly proportional to the squares of amplitude and frequency of the wave.

SOUND

Sound is a form of energy produced by vibration of particles of a medium and its sensation is detected by the ear.

Sound is a longitudinal wave. It therefore consists of compressions and rarefactions.

Sound waves are generated when particles of a medium are set into oscillation by a vibrating object.

The vibrating object superposes an oscillation of the particles of the transmitting medium along the direction of the wave.

Sound exhibits all properties of waves except polarization which is not exhibited by longitudinal waves.

Sound is a mechanical wave and therefore requires a material medium for its propagation.

Thus, it cannot travel through a vacuum.

HOW SOUND TRAVELS IN AIR

When a body is set into vibration, it sets nearby molecules of air into vibration about their mean positions. These molecules in turn also set the neighbouring molecules into vibrations and so on. This results into regions of high particle density called compressions and those of low particle density called rarefactions in air. Hence sound is able to travel from one point to another in air.

WHY SOUND PROPAGATION IS CONSIDERED AN ADIABATIC PROCESS

Sound propagation (travel) in air is by compressions and rarefactions. In a compression, the temperature of air rises unless heat is withdrawn while in a rarefaction, there is decrease in temperature.

The compressions and rarefactions occur so fast that heat does not enter or leave the gas, hence the process is adiabatic.

WHY THE MOON IS OFTEN REFERRED TO AS A SILENT SATELLITE

The escape velocity of the moon is very small and the gas molecules easily escape, leaving the moon with no atmosphere (no air). There is thus no medium through which sound can propagate, making the moon a silent satellite.

PROPERTIES OF SOUND

Sound exhibits properties of waves including;

(i) Reflection
(ii) Refraction
(iii) Interference
(iv) Diffraction

REFLECTION OF SOUND WAVES

Sound waves can be reflected by obstacles obeying the laws of reflection.

A reflected sound from an obstacle is known as **an echo**.

The time that elapses between the original sound and the echo determines whether the observer will be able to distinguish the original sound and the echo. This time depends on;

(i) Speed of sound in that medium
(ii) Distance travelled by the echo from the reflector or obstacle.

If the time is less than 0.1 seconds, a human ear cannot distinguish between the original sound and the echo.

If the time is above 0.1 seconds, the original sound heard is prolonged to the listener. This effect is called **reverberation.**

Definition

Reverberation is the production of a prolonged sound when an echo merges with the original sound.

OR

Reverberation is a phenomenon which occurs when the original sound appears prolonged to the observer or listener as a result of reflection.

EXPLANATION OF REVERBERATION

When sound is reflected from a hard surface close to the observer, an echo follows the original incident sound so closely that the listener cannot distinguish between the original sound and the echo.

The observer however gets the impression of a prolonged original sound and this effect is known as **reverberation.**

The time taken for the intensity of the sound to die out completely is called **reverberation time.**

WHY ECHOES ARE NOT HEARD IN A SMALL ROOM

For an echo to be heard, the time lag between the original sound and the echo should be at least 0.1 seconds. In a small room, this time is less than 0.1 seconds, hence an echo cannot be heard.

Advantage of reverberation

It improves the audibility of sound when one makes a speech in a large hall.

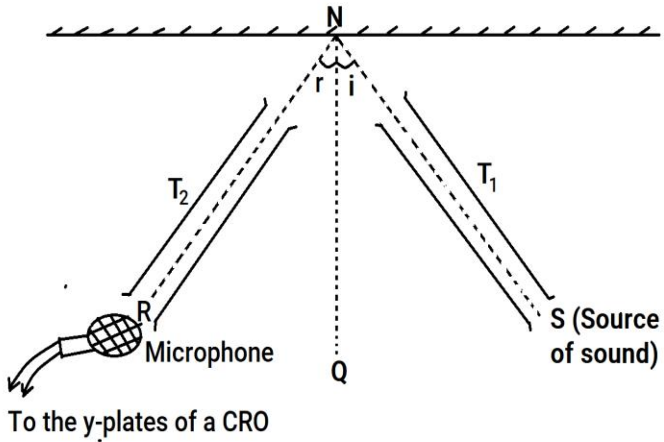
Disadvantage of reverberation

Excess reverberation makes a speech or music indistinct and confused in a small hall.

HOW TO PREVENT THE NEGATIVE EFFECTS OF REVERBERATION

By using soft clothing, cushions on the walls and soft human body, original sound is absorbed to reduce on the echoes that would merge with the original sound.

AN EXPERIMENT TO DEMONSTRATE REFLECTION OF SOUND AND TO SHOW THAT SOUND OBEYS THE LAWS OF REFLECTION



A line AB is drawn horizontally on a sheet of paper and a normal line NQ is drawn at N on AB.
Another line NS is drawn at a measured angle i to the normal NQ.
A hard cardboard is placed on the paper with its plane surface vertical and along AB.
A hollow open tube T_1 is placed lying along NS.
A source of sound is sounded and placed at S.
Another hollow open tube T_2 is placed on the opposite side of NQ facing point N.
A microphone is connected to the y-plates of a Cathode Ray Oscilloscope whose time base is switched off and it is placed at the mouth of tube T_2 furthest from point N.
A vertical trace/line is observed on the screen of a CRO.
While facing point N, the tube T_2 is slowly rotated together with the microphone about point N and away or towards NQ until the length of the vertical trace is maximum.
This shows that sound from the source under goes reflection at the cardboard at point N and is being received by the microphone.
The position R of the tube T_2 is now noted and T_2 is removed.
A line through the central axis of T_2 while at R is drawn to point N. The angle r which this line makes with the normal NQ is measured and is the angle of reflection.
It is found that $r = i$ hence obeying the laws of reflection.

REFRACTION OF SOUND WAVES

Sound waves can be refracted.
The refraction of sound occurs when a sound wave travels from;

(i) Medium of low density to a medium of higher density or vice versa.
(ii) Region of one density to a region of a different density within the same medium. For example, when the wave travels in air from a hotter region to a cooler region. Since sound travels faster in air at higher temperature, there is a change in velocity of sound when it travels from hotter region to a cooler region hence the sound is refracted.

The refraction of sound can be used to explain the following observations.

(i) Why sound is more audible at night than during the day.

(ii) Why sound is more audible when wind blows towards the observer in the same direction as the sound wave fronts than when it blows in the opposite direction of the sound wave fronts.

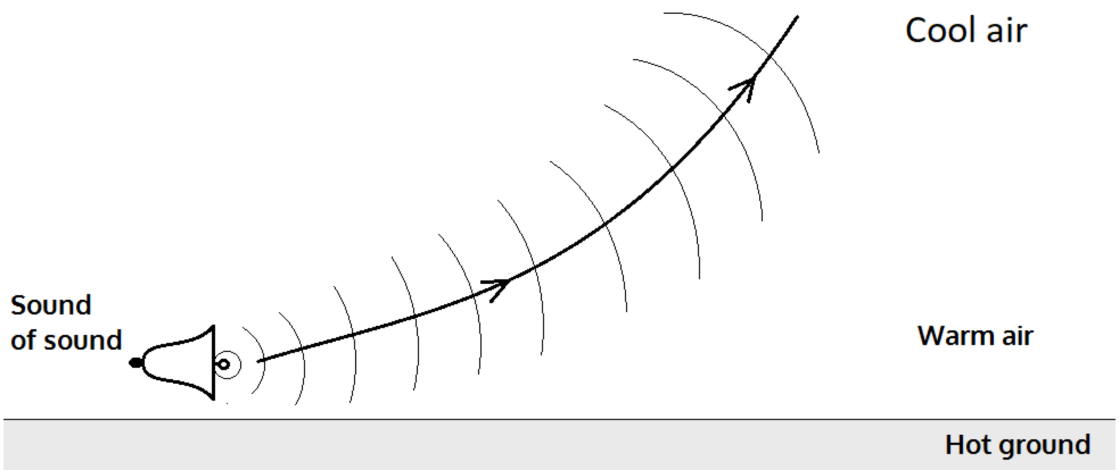
WHY SOUND IS MORE AUDIBLE AT NIGHT THAN DURING DAY

This is explained in terms of refraction of sound as it travels through layers of air.

During the day, the layers of air close to the ground are hotter and less dense than the layers further above the ground.

Since sound travels faster in air at higher temperature,then as sound waves travel from the lower less dense layers of air to the upper denser layers, it becomes progressively refracted upwards towards the normals and away from the observer on the ground.

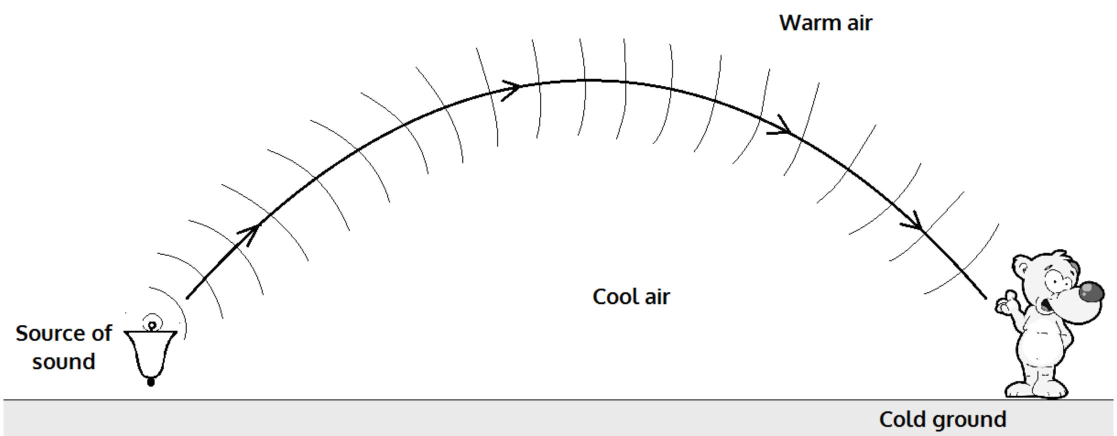
The intensity of sound detected by the observer therefore diminishes, causing the sound to be less audible.



At night, the layers of air close to the ground are cooler and denser than the layers above.

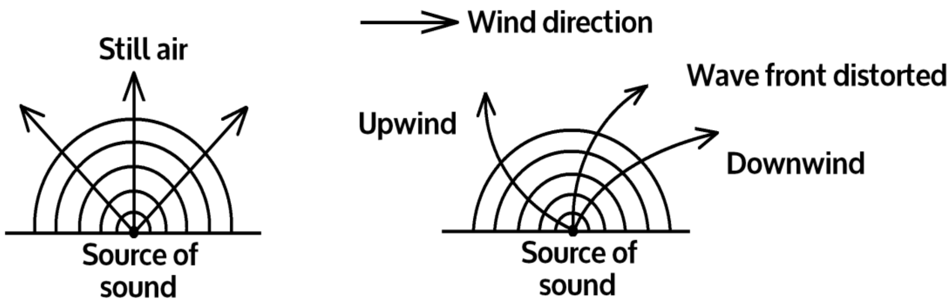
As the sound wave travels from the lower denser layers to the upper less dense layers, it is progressively refracted away from the normals towards the observer.

The observer therefore receives sound of increased intensity which is more audible.



EFFECT OF WIND DIRECTION ON AUDIBILITY OF SOUND

If there is wind in the air carrying sound waves, the effective velocity relative to the ground is equal to the vector sum of the velocity of the wind and sound. Since wind velocity usually increases with increase in height, wave fronts far above the source get more distorted than those below as in the diagram below leading to greater audibility downwind compared to upwind.



Revision question

Explain why sound is more audible when wind blows towards the observer than when blowing away from the observer.

MUSICAL NOTES

A musical note or tone is a sound of regular frequency.
Music is a combination of soundnotes of regular frequencies.
Noise on the other hand isa combination of sound notes with irregular frequencies.

CHARACTERISTICS (PROPERTIES) OF MUSICAL NOTES/SOUND

Musical sounds have three major characteristics i.e;

- Pitch
- Loudness(intensity)
- Quality(timbre)

1. PITCH

This is the characteristic of sound which enables one to distinguish between a high note from a low note. i.e Pitch is the highness or lowness of sound.
Pitch depends on the frequency of the sound thus the higher the frequency, the higher the pitch and the lower the frequency, the lower the pitch.
E.g pitch enables one to distinguish between sound produced by a woman from that produced by a man.

2. LOUDNESS

Loudness of sound is the amount of sound energy entering the ear per second.
The greater the amount of energy entering the ear, the louder the sound.
The loudness of sound depends on the amplitude of sound and its intensity.
The greater the amplitude, the greater the intensity of sound and thus the louder the sound.

3. QUALITY

This is the characteristic of a sound note that distinguishes it from another note of the same pitch and loudness.
The quality of a note depends on the intensity of the harmonics (overtones) present in a note.
Notes that contain many harmonics are usually of better quality than those with fewer harmonics.

SPEED OF SOUND IN DIFFERENT MEDIA

The speed of a mechanical wave in a medium depends on elasticity and density of the medium i.e

$$\text{Speed} = \left(\frac{\text{elastic modulus of the medium}}{\text{density of the medium}}\right)^{\frac{1}{2}}$$

(a) In a solid of density ρ and young's modulus, E, the speed of sound $v = \left(\frac{E}{\rho}\right)^{\frac{1}{2}}$.

(i) For a steel rod, $\rho = 7.8\times10^3 kgm^{-3}$ and $E = 2.0\times10^{11} Pa$

$$v = \left(\frac{E}{\rho}\right)^{\frac{1}{2}} = \left(\frac{2.0\times10^{11}}{7.8\times10^3}\right)^{\frac{1}{2}} = 5.1\times10^3 ms^{-1}$$

(ii) For an iron rod, $\rho = 7.7\times10^3 kgm^{-3}$ and $E = 2\times10^{11} Pa$

$$v = \left(\frac{E}{\rho}\right)^{\frac{1}{2}} = \left(\frac{2\times10^{11}}{7.7\times10^3}\right)^{\frac{1}{2}} = 5096.5 ms^{-1}$$

(b) In gases, volume may change and therefore the bulk modulus K becomes the relevant elastic modulus under the conditions in which the sound wave travels.
$K = \gamma P$, where P is pressure of the gas and γ is the ratio of its principal heat capacities.

$$\text{Speed of sound in gases},\ v = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

$$\text{At STP}\ \rho = 1.3 kgm^{-3},\ \gamma = 1.4\ \ P = 1.035\times10^5 Pa$$

$$\Rightarrow v = \sqrt{\frac{1.4\times1.035\times10^5}{1.3}} = 334 ms^{-1}$$

(c) Effect of pressure, temperature and humidity in gases
For 1 mole of an ideal gas having volume, V and pressure, P at temperature, T,

$$PV = RT$$

$$V = \frac{RT}{P}$$

$$But\ \ \rho = \frac{m}{V} = \frac{m}{\left(\frac{RT}{P}\right)} = \frac{mP}{RT}$$

$$v = \sqrt{\frac{\gamma P}{\left(\frac{mP}{RT}\right)}} = \sqrt{\frac{\gamma RT}{m}} = \left(\sqrt{\frac{\gamma R}{m}}\right)T^{\frac{1}{2}}$$

$$v\ \alpha\ T^{\frac{1}{2}}$$

$\boldsymbol{OR}$

$PV = RT$ and $\rho = \frac{M}{V}$

$$\therefore V = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma\left(\frac{RT}{V}\right)}{\left(\frac{M}{V}\right)}}$$

$$= \left(\sqrt{\frac{\gamma R}{M}}\right)T^{\frac{1}{2}}$$

$$\Rightarrow V \propto T^{\frac{1}{2}}$$

- The speed of sound in gases is therefore independent of pressure. Since R has the same value for all gases and γ and m are constant for a particular gas, then $v\ \alpha\ T^{\frac{1}{2}}$
- The speed of sound in gases is directly proportional to the square root of absolute temperature of the gas (provided γ is independent of the temperature).
- For moist air, ρ is less than that of dry air and γ is slightly greater. The net result is that the speed of sound increases with increase in humidity.

DIFFERENCES BETWEEN SOUND AND LIGHT WAVES

Sound	Light
Mechanical wave	Electromagnetic wave
Longitudinal wave	Transverse wave
Doesn't undergo polarization	Undergoes polarization
Requires a material medium for propagation(transmission)	Can be propagated in a vacuum
Has low speed	Has higher speed

Question

The speed of sound in air is given by $v = \left(\frac{\gamma P}{\rho}\right)^{\frac{1}{2}}$ where P is pressure, ρ is density and γ is the ratio $c_p : c_v$. Use this expression to show and explain the effect of temperature on the speed of sound in air.

Solution

Let m be mass of air, V be volume of air and n be number of moles of air at an absolute temperature, T.

$$From\ \ v = \left(\frac{\gamma P}{\rho}\right)^{\frac{1}{2}}$$

$$But\ \rho = \frac{m}{V}$$

$$v = \left(\frac{\gamma P}{\left(m/V\right)}\right)^{\frac{1}{2}} = \left(\frac{\gamma PV}{m}\right)^{\frac{1}{2}}$$

$$But\ \ PV = nRT$$

$$Thus\ \ v = \left(\frac{\gamma nRT}{m}\right)^{\frac{1}{2}} = \left(\frac{\gamma nR}{m}\right)^{\frac{1}{2}} T^{\frac{1}{2}}$$

$$Let\ \left(\frac{\gamma nR}{m}\right)^{\frac{1}{2}} = constant\ , k$$

$$v = kT^{\frac{1}{2}}$$

$$v \,\alpha\, T^{\frac{1}{2}} - - - - - - - - - - - - - - - - (\otimes)$$

Thus, from equation $(\otimes)$ above, the velocity of sound in air is directly proportional to the square root of the absolute temperature.
This implies that when temperature of the air molecules increases, their vibrations also increase and thus allowing easy and fast transmission of sound in air.

ULTRASONICS

Ultrasonic sound is sound of frequency at least 20kHz which is above the maximum audible frequency of a human ear.
A human ear is sensitive to frequency of up to approximately 18kHz and the sound of higher frequency cannot be detected by a human ear.
Ultrasonics therefore deals with sound beyond the sensitivity of the human ear.

APPLICATIONS OF ULTRASONIC SOUND

(a) In medicine

- It is used in medical ultra sound imaging to image soft tissues that cannot be easily seen with x-rays or which will be damaged by x-rays e.g unborn babies.
- They are used in selective destruction of tumors.
- It is used in physiotherapy.
- It can be used together with Doppler effect to measure the speed of movement of blood in different parts of the body and monitoring heartbeat.

(b) Non-medical applications

- It is used as a cleansing agent
- It is used in SONAR (Sound Navigation And Ranging) to detect under- water objects such submarines, fish, etc.

- It is applied in echo sounding to measure the depth of seas and under-water substances.

BEATS

Beats refers to the periodic rise and fall in the intensity of sound heard when two sound notes of equal amplitude and nearly equal frequency are sounded together.

CONDITIONS FOR FORMATION OF BEATS

(i) Two notes having the same amplitude must be sounded together.

(ii) The two notes must have nearly equal frequency.

The formation of beats may be explained using the superposition principle.

EXPLANATION OF FORMATION OF BEATS USING SUPERPOSITION PRINCIPLE

Beats are formed when two sound notes of equal amplitude and nearly equal frequency are sounded together.

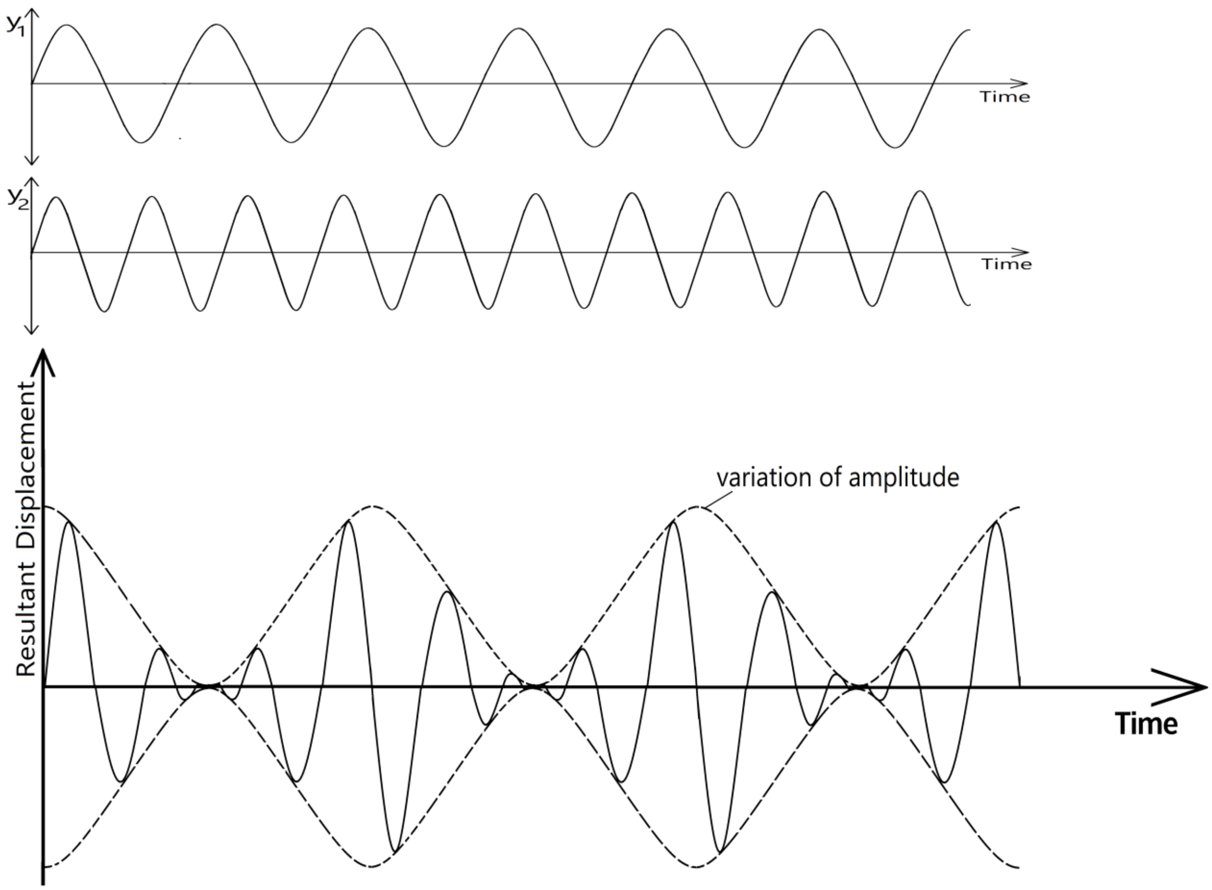
When the notes reach the observer's ear, superposition occurs and the resultant displacement will be equal to the vector sum of the displacement due to each wave.

At a certain time, the two waves reach the observer's ear when in phase resulting into maximum displacement and sound of high intensity.

Subsequently, the sound waves reach the observer's ear when completely out of phase resulting into minimum displacement and sound of low intensity. This repeats periodically and the periodic rises and fall in sound intensity are what is referred to as **beats.**

GRAPHICAL ILLUSTRATION OF BEAT FORMATION

Consider a layer of air at some distance away from two vibrating tuning forks producing two pure notes of equal amplitude and nearly equal frequency, say 48Hz and 56Hz respectively. The layer of air will vibrate due to each of the forks. The varying displacements y_1 and y_2 due to each fork are illustrated below and the resultant varying displacements of air due to the superposition principle is drawn which shows the beats formed.



Experimental evidence shows that in the interval of time when a beat is formed following the previous one (beat period), the note of higher frequency performs one cycle more than the note of lower frequency.

BEAT PERIOD

This is the interval of time between any two successive beats or loud sounds.
It is also defined as the time taken for the formation of a beat following the previous one.
It is denoted by T and measured in seconds.

BEAT FREQUENCY

Thisis the number of beats formed in one second.
It is commonly represented by f_{beat} and it is measured in hertz (Hz).
It is measured by counting the number of beats of loud sound or high intensity sound, n produced after a time, t seconds.
The beat frequency is then calculated from;

$$f_{beat} = \frac{n}{t}.$$

Also $f_b = \frac{1}{T}$

DERIVATION OF THE EQUATION OF BEAT FREQUENCY

Suppose the time between two successive loud sounds heard when two sound notes P and Q of equal amplitude and of nearly equal frequencies f_1 and f_2 respectively is T (beat period), where $f_2 > f_1$.

From $f = \frac{n}{t}$ where n is number of cycles in time t.
$f_1 = \frac{n_1}{T}$and$f_2 = \frac{n_2}{T}$
$n_1 = f_1 T$and$n_2 = f_2 T$
Where $n_1$is the number of cycles made by sound note P in time $t = T$ and n_2 is the number of cycles made by sound note Q in the same time $t = T$. The time T is the beat period not the period for P or Q.
$In\ this\ time\ T, since\ f_2 > f_1\ and\ Q\ makes\ 1\ \ cycle$more than P, then:

$$Number\ of\ cycles\ \ made\ by\ Q - Number\ of\ cycles\ made\ by\ P = 1$$

$$n_2 - n_1 = 1$$

$$(f_2 T - f_1 T) = 1$$

$$\Rightarrow (f_2 - f_1)T = 1$$

$$(f_2 - f_1) = \frac{1}{T}$$

$$But\ f_b = \frac{1}{T}$$

$$\therefore f_b = f_2 - f_1$$

$$Hence\ beat\ frequency\ , f_b = f_2 - f_1$$

In summary: $f_b = f_2 - f_1$ when $f_2 > f_1$ Or $f_b = f_1 - f_2$ when $f_1 > f_2$

BEAT WAVELENGTH

Thisis the distance between two successive locations of minimum or maximum intensities formed when two sound notes of equal amplitude and nearly equal frequency are sounded together.
$\lambda_{beat} = \frac{V}{f_{beat}}$where v is speed of sound.

VARIATION OF INTENSITY WITH BEAT FREQUENCY

Consider two displacements at a single point, one of frequency f_1 and another of frequency f_2.
$y_1 = asin2\pi f_1 t$ and $y_2 = asin2\pi f_2 t$. Where f_1 and f_2 are nearly equal frequencies.
Using the superposition principle

$$y = y_1 + y_2$$

$$y = asin2\pi f_1 t + asin2\pi f_2 t$$

$$y = a\left(2sin\left(\frac{2\pi f_1 t + 2\pi f_2 t}{2}\right)cos\left(\frac{2\pi f_1 t - 2\pi f_2 t}{2}\right)\right)$$

$$y = a\left(2sin\left(\frac{2\pi(f_1 + f_2)t}{2}\right)cos\left(\frac{2\pi(f_1 - f_2)t}{2}\right)\right)$$

$$y = 2asin(f_1 + f_2)\pi t\ \ cos(f_1 - f_2)\pi t$$

$$y = 2acos(f_1 - f_2)\pi t\ \ sin(f_1 + f_2)\pi t$$

$$Let\ A = 2acos(f_1 - f_2)\pi t$$

$Where\ A\ is\ variable\ amplitude\ of\ the\ resultant\ wave$

$y = Asin(f_1 + f_2)\pi t$

$Let\ A = 2acos(f_1 - f_2)\pi t$

$Where\ A\ is\ variable\ amplitude\ of\ the\ resultant\ wave$

$y = Asin(f_1 + f_2)\pi t$

Intensity, I of the resultant wave is directly proportional to the square of the amplitude, *A*i.e

$I\ \alpha\ A^2$

$I\ \alpha\ (2acos(f_1 - f_2)\pi\, t)^2$

$I\ \alpha\ 4a^2cos^2(f_1 - f_2)\pi\, t$

In summary, the intensity of the wave varies with the beat frequency, $f_1 - f_2$.

EXAMPLES

1. Two tuning forks X and Y sounding together produce sound waves that make 5020 and 5000 cycles respectively in 10 seconds. Determine the frequency of beats produced when these forks are sounding together and deduce the beat period.

   Solution

   $$f_1 = \frac{n_1}{t} = \frac{5020}{10} = 502Hz$$

   $$f_2 = \frac{n_2}{t} = \frac{5000}{10} = 500Hz$$

   Since $f_1 > f_2 \Rightarrow f_{beat} = f_1 - f_2 = 502 - 500 = 2Hz$

   $$T = \frac{1}{f_{beat}} = \frac{1}{2} = 0.5s$$

2. Two whistles are sounded simultaneously and the wavelengths of the sounds emitted are 5.5m and 6.0m respectively. Find the beat frequency if the speed of sound in air is 330ms^{-1} .

   Solution

   $For\ wave\ 1$

   $\lambda_1 = 5.5m,\ v = 330ms^{-1},\ f_1 = ??$

   $$f_1 = \frac{v}{\lambda_1} = \frac{330}{5.5} = 60Hz$$

   $For\ wave\ 2$

   $\lambda_2 = 6.0m,\ v = 330ms^{-1},\ f_2 = ??$

   $$f_2 = \frac{v}{\lambda_2} = \frac{330}{6.0} = 55Hz$$

   Since $f_1 > f_2 \Rightarrow f_{beat} = f_1 - f_2 = 60 - 55 = 5Hz$

   $f_{beat} = 5.0Hz$

3. Two sources of sound are vibrating simultaneously with frequencies of *2000Hz* and *2040Hz* respectively. If the speed of sound in air is 340ms^{-1}.

(a) How many beats are formed?
(b) What is the distance between the successive locations of maximum intensity?

Solution

(a) Given $f_1 = 2000Hz \ and \ f_2 = 2040Hz$

$$f_{beat} = f_2 - f_1 \text{Since } f_2 > f_1$$

$$f_{beat} = 2040 - 2000 = 40Hz$$

$$Number \ of \ beats = f_{beat}$$

$$\therefore Number \ of \ beats = 40 beats \ per \ second$$

(b) $Distance \ between \ two \ successive \ points \ of \ maximum \ intensity = beat \ wavelength$

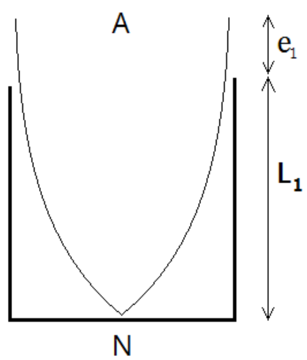
$$v = f\lambda \Rightarrow \lambda = \frac{v}{f} = \frac{340}{40}$$

$$\lambda = 8.5m$$

4. When two stopped pipes each of length $62cm$ with end corrections of $1.2cm$ and $1.8cm$ respectively are sounding their fundamental notes, beats are formed. If the velocity of sound in air is $340ms^{-1}$, find the beat period.

Solution

For 1st pipe



Given $l_1 = 62cm = 0.62m, e_1 = 1.2cm = 0.012m, v = 340ms^{-1}$

From the diagram

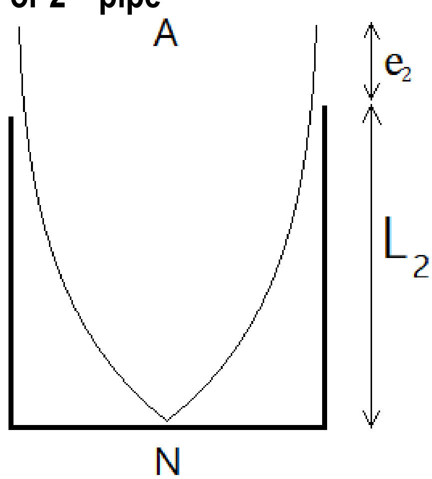
$$l_1 + e_1 = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4(l_1 + e_1)$$

$$\lambda_1 = 4(0.62 + 0.012) = 2.528m$$

From $f_1 = \frac{v}{\lambda_1} = \frac{340}{2.528}$

$$f_1 = 134.49Hz$$

For 2nd pipe



Given $l_2 = 62cm = 0.62m, e_2 = 1.8cm = 0.018m$

$$l_2 + e_2 = \frac{\lambda_2}{4} \Rightarrow \lambda_2 = 4(l_2 + e_2)$$

$$\lambda_2 = 4(0.62 + 0.018) = 2.552m$$

From $f_2 = \frac{v}{\lambda_2} = \frac{340}{2.552}$

$$f_2 = 133.23Hz$$

Beat frequency, $f_b = |f_1 - f_2|$

$$f_b = |134.49 - 133.23| = 1.26Hz$$

Beat period $T = \frac{1}{f_b} = \frac{1}{1.26}$

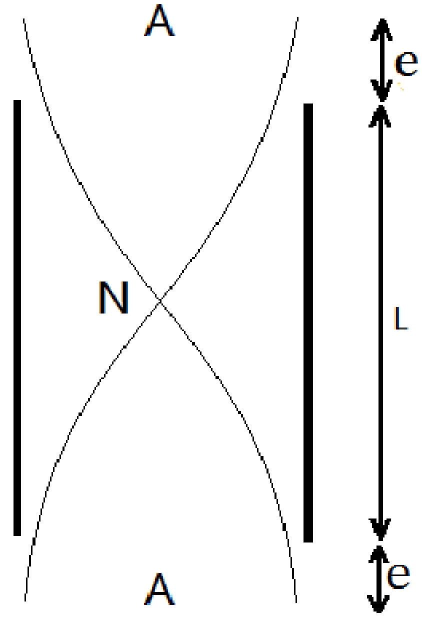
$$T = 0.79s$$

5. Two open pipes of length $92cm$ and $93cm$ are found to give beat frequency of $3Hz$ when each is sounding in its fundamental mode. If the end corrections are $1.5cm$ and $1.8cm$ respectively. Calculate the;

(a) velocity of sound in air

(b) frequency of each note

Solution



Given $l_1 = 92cm = 0.92m, e_1 = 1.5cm = 0.015m,$

$l_2 = 93cm = 0.93m, e_2 = 1.8cm = 0.018m$

For 1st pipe

$$l_1 + 2e_1 = \frac{\lambda_1}{2}$$

$$\lambda_1 = 2(l_1 + 2e_1) = 2(0.92 + 2 \times 0.015)$$

$$\lambda_1 = 1.9m$$

For 2nd pipe

$$l_2 + 2e_2 = \frac{\lambda_2}{2} \Rightarrow \lambda_2 = 2(l_2 + 2e_2) = 2(0.93 + 2 \times 0.018)$$

$$\lambda_2 = 1.932m$$

$$From\ \ v = f\lambda$$

$$v = 1.9f_1 --------------------(1)$$

$$v = 1.932f_2 -------------------(2)$$

$But\ \ f_1 - f_2 = 3$ since $f_1 > f_2$ $\ i.e\ f = \frac{v}{2l}$; $For\ l_2 > l_1$ So $f_1 > f_2$.

$$\Rightarrow f_1 = 3 + f_2 -----------------(3)$$

$Substituting\ (3)\ into\ (1)\ gives$

$v = 1.9(3 + f_2) ---------------- (4)$

$Eqn(2) = Eqn(4)$

$1.932 f_2 = 1.9(3 + f_2) = 5.7 + 1.9 f_2$

$0.032 f_2 = 5.7$

$f_2 = 178.125Hz$

$Hence\ \ v = 1.932 \times 178.125 = 344.14ms^{-1}$

$\Rightarrow f_1 = 3 + 178.125$

$f_1 = 181.125Hz$

6. Two closed pipes of length $50cm$ and $51cm$ respectively are sounding their fundamental notes together. Neglecting end corrections, determine the frequency of the beats produced. ($v = 340ms^{-1}$).

Solution: $L_1 = 50cm$

$$L_1 = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4 \times 50 = 200cm = 2m$$

$$f_1 = \frac{V}{\lambda_1} = \frac{340}{2} = 170Hz$$

$L_2 = 51cm$

$L_2 = \frac{\lambda_2}{4}$

$\Rightarrow \lambda_2 = 4 \times 51$
$= 204cm$
$= 2.04m$

$f_2 = \frac{V}{\lambda_2} = \frac{340}{2.04}$
$= 166.67Hz$

$$f_b = f_1 - f_2 = 170 - 166.67$$
$$= 3.33Hz$$

USES OF BEATS

1. **Beats are used to measure the unknown frequency of a sound note**.

Method 1:Loading the tuning fork of known frequency with plasticine

A tuning fork with a note of known frequency, f is sounded together with a note of unknown frequency, f_u The number of beats heard in time, t are counted. This can be done about three times and the average number of beats heard in time t is calculated, as n.

The beat frequency, f_b is calculated from, $f_b = \frac{n}{t}$.

Thus from $f_b = f - f_u$ OR $f_b = f_u - f$,

$\Rightarrow f_u = f - f_b$ OR $f_u = f + f_b$

To find out which value of f_u is true, **one of the prongs of the tuning fork with known frequency is loaded** with plasticine to slightly decrease its frequency f .

The experiment is repeated and the new beat frequency f_b' is determined.

If the beat frequency decreases i.e if $f_b' \langle f_b$ then $f_b = f - f_u \Rightarrow f_u = f - f_b$

If the beat frequency increases i.e if $f_b' \rangle f_b$ then $f_b = f_u - f \Rightarrow f_u = f + f_b$

Method II: Loading the tuning fork of unknown frequency with plasticine after sounding it together with another tuning fork of known frequency

A tuning fork of known frequency f is sounded together with another tuning fork of unknown frequency f_u

The number of beats heard in time t are counted. This can be done about three times and the average number of beats heard in this time is calculated, recorded as n

The beat frequency, f_b is calculated from, $f_b = \frac{n}{t}$.

Thus from $f_b = f - f_u \quad OR \quad f_b = f_u - f$,

$\Rightarrow f_u = f - f_b \quad OR \quad f_u = f + f_b$

To find out which value of f_u is true, **one of the prongs of the tuning fork with unknown frequency is loaded** with plasticine to slightly decrease its frequency f_u.

The experiment is repeated and the new beat frequency f_b' is determined.

If the beat frequency decreases i.e if $f_b' \langle f_b$ then $f_b = f_u - f \Rightarrow f_u = f + f_b$

If the beat frequency increases i.e if $f_b' \rangle f_b$ then $f_b = f - f_u \Rightarrow f_u = f - f_b$

2. Beats are also used for tuning a musical instrument to a given desirable note.

A tuning fork of known frequency is sounded together with an instrument such as a stringed instrument which is needed to be tuned.

Beats are heard.

The frequency of the instrument is adjusted for example by adjusting the tension or length until no beats are heard.

This now means that the frequency of the instrument is equal to the frequency of the fork and the instrument is said to have been tuned.

QUESTIONS

1. Two tuning forks X and Y are sounded together to produce beats of frequency 8Hz. Fork X has a known frequency of 512Hz. When Y is loaded with a small piece of plasticine, beats of frequency 2Hz are heard when the two forks are sounded together. Calculate the frequency of Y when unloaded.

   Solution

$Given\ f_X = 512Hz\ \ f_y = ??\ \ f_b = 8Hz$

$f_b = f_X - f_Y \quad OR \ \ f_b = f_Y - f_X$

$\Rightarrow f_Y = f_X - f_b \quad OR \ \ f_Y = f_X + f_b$

$since\ the\ fork\ with\ unknown\ frequency\ was\ loaded\ and\ f_b^{'} \langle f_b$

$\Rightarrow f_Y = f_X + f_b$

$f_Y = 512 + 8$

$f_Y = 520Hz$

2. A tuning fork of unknown frequency and a standard fork of frequency 440Hz are sounded simultaneously and beats of frequency 4Hz are heard. What deductions can you make regarding the unknown frequency of the first fork? A small piece of plasticine is attached to the prongs of the first fork and both forks are sounded again. It is found that the beat frequency is now 3Hz. What further deductions can you make?

Solution

$\boldsymbol{In\ the\ first\ case, the}\ unknown\ \boldsymbol{frequency\ \ of\ the}\ first\ \boldsymbol{fork\ is}$

$$= \mathbf{440 \pm 4}\boldsymbol{Hz}$$

$\boldsymbol{i.ef = 444Hz\text{or}f = 436Hz}$

$\boldsymbol{In\ the\ second\ \ case, since\ the\ beat\ frequency\ \ decreases}$**, then the unknown frequency of the first fork must have originally been 444Hz.**

3. A tuning fork X of known frequency 540Hz is sounded together with another tuning fork Y of unknown frequency. The frequency of beats obtained is 10Hz. When X is loaded with plasticine and the two forks sounded together, the frequency of the beats obtained is 12Hz. Determine the frequency of Y.

Solution

$Given\ f_X = 540Hz,\ \ f_y = ??\ \ f_b = 10Hz$

$f_b = f_Y - f_X\ \ OR\ f_b = f_X - f_Y$So, $f_Y = f_X + f_b\ \ OR\ \ f_Y = f_X - f_b$

$Since\ the\ fork\ X\ with\ known\ frequency\ was\ loaded\ and\ f_b^{'} \rangle f_b$

$\Rightarrow f_Y = f_X + f_b$

$f_Y = 540 + 10$

$f_Y = 550Hz$

4. A guitar is sounded together with a tuning fork of frequency 500Hz to produce 12 beats heard in 4 seconds. If it is known that the note produced by the guitar has a lower frequency than the tuning fork, determine the frequency of the note produced by the guitar.

Solution

$Let\ \ f_t = frequency\ of\ tuning\ fork\ and\ \ f_g = frequency\ of\ guitar\ note$

$Given\ \ f_t = 500Hz$

$f_{beat} = \frac{n}{t} = \frac{12}{4} = 3Hz$

$f_{beat} = f_t - f_g \Rightarrow f_g = f_t - f_{beat}$

$f_g = 500 - 3 = 497Hz$

5. When determining the frequency of a note produced by a guitar, the guitar is sounded together with a tuning fork of frequency 612Hz and 10 beats are heard after 5 seconds. When the turning fork is loaded with plasticine and the guitar and tuning fork sounded again, the same number of beats is heard in 2 seconds. Determine the frequency of the note produced by the guitar.

Solution

$Let\ \ f_t = frequency\ of\ tuning\ fork\ and\ \ f_g = frequency\ of\ guitar\ note$

$Given\ \ f_t = 612Hz$

$f_{beat} = \frac{n}{t} = \frac{10}{5} = 2Hz\ \ and\ \ f_{beat}^{'} = \frac{n}{t} = \frac{10}{2} = 5Hz$

$since\ the\ fork\ with\ known\ frequency\ was\ loaded\ and\ f_b^{'} \rangle f_b$

$\Rightarrow f_g = f_t + f_{beat}$

$f_g = 612 + 2$

$f_g = 614Hz$

6. When a tuning fork of frequency 512Hz is sounded together with a sonometer wire emitting its fundamental frequency, 3 beats are heard every second. When the sonometer wire is tightened slightly, the beat frequency increases to 4Hz. If the length of the sonometer wire is 25cm and the mass per unit length of the material of the wire is $9\times10^{-3}kgm^{-1}$, calculate the tension in the sonometer wire.

Solution

$Let\ \ f_t\ and\ f_s\ be\ the\ respective\ frequencies\ of\ the\ tuning\ fork\ and\ sonometer\ wire\ respectively$

$f_b = f_t - f_s\ \ OR\ \ f_b = f_s - f_t$

$On\ increasing\ the\ tension\ in\ the\ wire,\ f_s\ increases\ to\ f_s^{'} \rangle f_s\ and\ f_b\ increases$

$f_s = f_t + f_b$

$\Rightarrow f_s = 512 + 3 = 515Hz$

$f_s = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \Rightarrow 515 = \frac{1}{2\times0.25}\sqrt{\frac{T}{9\times10^{-3}}} \Rightarrow (257.5)^2 = \frac{T}{9\times10^{-3}}$

$T = 596.8N$

TRIAL QUESTIONS

1. A guitar and a tuning fork are sounded together to produce beats of frequency 4Hz. When the tuning fork is loaded with plasticine to lower its frequency, the new beat frequency becomes 2Hz. If the frequency of the fork when unloaded is 600Hz, determine the frequency of a note produced by the guitar.
[Ans: 596Hz]
2. A tuning fork of 480 Hz is sounded together with a musical note of unknown frequency and beats of frequency 2Hz are produced. When the tuning fork is loaded with plasticine and again sounded with a note, the new beat frequency is 5Hz. Determine the frequency of the note. **[Ans: 482Hz]**
3. A piano is being played together with a guitar, with the guitar producing a note of frequency 740Hz. In the same situation, a keen observer hears 60 beats in 15 seconds. The guitar string is then loosened to reduce its frequency and the observer hears 90 beats in the same time. Determine the frequency of the note produced by the piano.**[Ans: 744Hz]**
4. The wire of a sonometer of mass per unit length $10^{-3}kgm^{-1}$ is stretched on the two bridges by a load of *40N*. The wire is plucked at the central point so that it executes its fundamental vibration and at the same time a tuning fork of *264Hz* is sounded and beats are heard and found to have a frequency of *3Hz*. If the load is slightly increased, the beat frequency is lowered. Calculate the separation of the standing wave. (Hint: $f_u = f_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}\ and\ f_b = f - f_u\ \ or\ \ f_u - f$, but when f_u is increased by increasing the load, $f_b' < f_b;\ So,\ f_b = f - f_u$ implying that $f_u = f - f_b = 264 - 3 = 261Hz$ then find the value of $l.$)

DOPPLER EFFECT

When either a source of a wave (like sound) or an observer is in motion, there is an apparent change in the frequency of the wave or of the sound compared to that observed when there is no relative motion between the source and the observer e.g the pitch of the note of sound from the siren appears to reduce suddenly after it has just bypassed a stationary observer. This effect is called Doppler Effect.

Definition:

Doppler Effect is the apparent change in the frequency of a wave when there is relative motion between the source and the observer.

Doppler effect occurs in electromagnetic waves and sound waves.

RELATIVE VELOCITY

This is the velocity of body A as if body B was at rest although body B may be moving.

(a) If a body A is moving with a velocity v_A in the same direction as body B moving with velocity v_B.

$\longrightarrow v_B \quad \longrightarrow v_A$

Then the velocity of A relative to B (or the relative velocity of A with respect to B) is given by

$v_{AB} = {}_A v_B = v_A - v_B$

(b) If they are moving in opposite directions,

$\longrightarrow v_B \quad v_A \longleftarrow$

Then $v_{AB} = {}_A v_B = v_A - -v_B = v_A + v_B$

CALCULATION OF APPARENT FREQUENCY

Let v = velocity of the wave in air

u_s = Velocity of the source of the wave

u_o = Velocity of the observer

f = frequency of the wave from the source

v_{ws} = Relative velocity of the wave with respect to the source (or velocity of wave relative to the source)

v_{wo} = Relative velocity of the wave with respect to the observer (or velocity of wave relative to the observer)

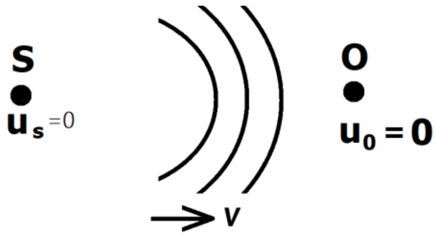
f' = Apparent frequency

λ' = Apparent wavelength

Note: The direction of the sound or the wave considered is that towards the observer in all cases.

Case 1: ***Stationary source, S and stationary observer, O.***
Since the source and observer are stationary, there is no apparent frequency heard. What is heard (received) is the actual frequency, f.

Proof:



$$Apparent\ wave\ length = \frac{Velocity\ of\ the\ wave\ relative\ to\ the\ source}{Actual\ frequency}$$

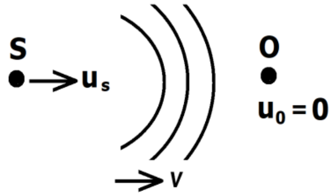
$\lambda' = \frac{V_{ws}}{f} = \frac{V-0}{f} = \frac{V}{f}$, *hence equal to the actual wave length*, λ.

$$Apparent\ frequency, f' = \frac{Velocity\ of\ the\ wave\ relative\ to\ the\ observer}{Apparent\ wavelength, \lambda'}$$

$$f' = \frac{V_{wo}}{\lambda'}$$

$$f' = \frac{V - O}{(\frac{V}{f})} = f = actual\ frequency.$$

Case 2: ***A source, S moving towards a stationary observer, O.***



$$Apparent\ wavelength, \lambda' = \frac{velocity\ of\ wave\ relative\ to\ source}{actual\ frequency} = \frac{v_{ws}}{f}$$

$$\lambda' = \frac{v - u_s}{f} ---------------- (1)$$

$$Apparent frequency, f' = \frac{velocity\ of\ wave\ relative\ to\ observer}{\lambda'} = \frac{v_{wo}}{\lambda'}$$

$$f' = \frac{v - 0}{\lambda'} = \frac{v}{\lambda'} ---------------- (2)$$

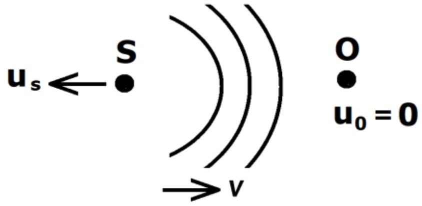
Substituting (1) into (2)

$$f' = \frac{v}{\left(\frac{v - u_s}{f}\right)}$$

$$f' = \left(\frac{v}{v - u_s}\right) f$$

Thus the frequency appears to increase when the source is moving towards a stationary observer.

Case 3: ***Asource Smovingawayfromastationaryobserver O.***



$$Apparent\ wavelength,\ \lambda' = \frac{velocity\ of\ \ wave\ relative\ to\ source}{actual\ \ frequency} = \frac{v_{ws}}{f}$$

$$\lambda' = \frac{v - -u_s}{f} = \frac{v + u_s}{f} - - - - - - - - - - - - - - - - (1)$$

$$Apparent frequency,\ f' = \frac{velocity\ of\ \ wave\ relative\ to\ observer}{\lambda'} = \frac{v_{wo}}{\lambda'}$$

$$f' = \frac{v - 0}{\lambda'} = \frac{v}{\lambda'} - - - - - - - - - - - - - - - - (2)$$

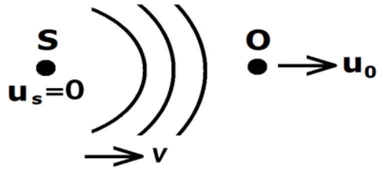
$Substituting\ \ (1)\ \ \text{int}\ o\ \ (2)$

$$f' = \frac{v}{\left(\dfrac{v + u_s}{f}\right)}$$

$$f' = \left(\frac{v}{v + u_s}\right) f$$

Thus the frequency of a wave appears to decrease when the source is moving away from a stationary observer.

Case 4: ***A stationary source S and an observer O moving away from it.***



$$Apparent\ wavelength, \lambda' = \frac{velocity\ of\ wave\ relative\ to\ source}{actual\ frequency} = \frac{v_{ws}}{f}$$

$$\lambda' = \frac{v-0}{f} = \frac{v}{f} ----------------(1)$$

$$Apparent frequency,\ f' = \frac{velocity\ of\ wave\ relative\ to\ observer}{\lambda'} = \frac{v_{wo}}{\lambda'}$$

$$f' = \frac{v-u_o}{\lambda'} ----------------(2)$$

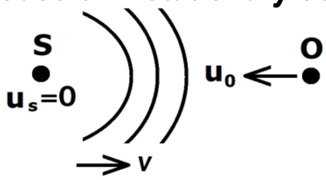
$Substituting$ (1) into (2)

$$f' = \frac{v-u_o}{\left(\frac{v}{f}\right)}$$

$$f' = \left(\frac{v-u_o}{v}\right)f$$

Thus the frequency of the wave appears to decrease as the observer moves away from a stationary source.

Case 5: ***A stationary source S and an observer O moving towards it.***



$$Apparent\ wavelength,\ \lambda' = \frac{velocity\ of\ wave\ relative\ to\ source}{actual\ frequency} = \frac{v_{ws}}{f}$$

$$\lambda' = \frac{v-0}{f} = \frac{v}{f} ----------------(1)$$

$$Apparent\ frequency,\ f' = \frac{velocity\ of\ wave\ relative\ to\ observer}{\lambda'} = \frac{v_{wo}}{\lambda'}$$

$$f' = \frac{v--u_o}{\lambda'} = \frac{v+u_o}{\lambda'} ----------------(2)$$

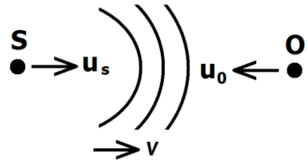
Substituting (1) *into* (2),

$$f' = \frac{v+u_o}{\left(\frac{v}{f}\right)}$$

$$f' = \left(\frac{v+u_o}{v}\right)f$$

Thus the frequency of the wave appears to increase as the observer moves towards a stationary source.

Case 6: ***A source S of waves and an observer O both moving towards each other.***



$$Apparent\ wavelength,\ \lambda' = \frac{velocity\ of\ wave\ relative\ to\ source}{actual\ frequency} = \frac{v_{ws}}{f}$$

$$\lambda' = \frac{v - u_s}{f} ---------------(1)$$

$$Apparent\ frequency,\ f' = \frac{velocity\ of\ wave\ relative\ to\ observer}{\lambda'} = \frac{v_{wo}}{\lambda'}$$

$$f' = \frac{v - -u_o}{\lambda'} = \frac{v + u_o}{\lambda'} ---------------(2)$$

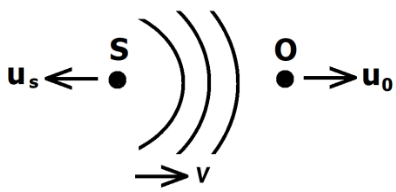
Substituting (1) *into* (2),

$$f' = \frac{v + u_o}{\left(\frac{v - u_s}{f}\right)}$$

$$f' = \left(\frac{v + u_o}{v - u_s}\right) f$$

Thus, the frequency of the wave appears to increase as both the observer and the source move towards each other.

Case 7: ***source S of waves and an observer O both moving away from each other.***



$$Apparent\ wavelength,\ \lambda' = \frac{velocity\ of\ wave\ relative\ to\ source}{actual\ frequency} = \frac{v_{ws}}{f}$$

$$\lambda' = \frac{v - -u_s}{f} = \frac{v + u_s}{f} ---------------(1)$$

$$Apparent\ frequency,\ f' = \frac{velocity\ of\ wave\ relative\ to\ observer}{\lambda'} = \frac{v_{wo}}{\lambda'}$$

$$f' = \frac{v - u_o}{\lambda'} ---------------(2)$$

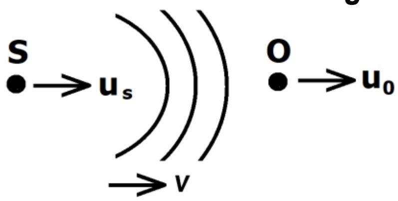
Substituting (1) *into* (2),

$$f' = \frac{v - u_o}{\left(\frac{v + u_s}{f}\right)}$$

$$f' = \left(\frac{v - u_o}{v + u_s}\right) f$$

Thus the frequency of the wave appears to decrease as the source and the observer move away from each other.

Case 8: ***Source S moving towards an observer O moving away from it.***



$$Apparent\ wavelength, \lambda' = \frac{velocity\ of\ wave\ relative\ to\ source}{actual\ frequency} = \frac{v_{ws}}{f}$$

$$\lambda' = \frac{v-u_s}{f} = \frac{v-u_s}{f} ----------------(1)$$

$$Apparent\ frequency,\ f' = \frac{velocity\ of\ wave\ relative\ to\ observer}{\lambda'} = \frac{v_{wo}}{\lambda'}$$

$$f' = \frac{v-u_o}{\lambda'} ----------------(2)$$

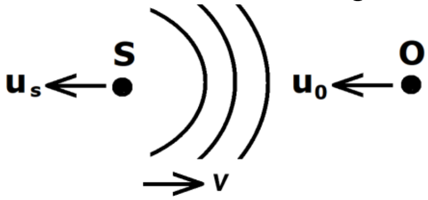
Substituting (1) *into* (2),

$$f' = \frac{v-u_o}{\left(\frac{v-u_s}{f}\right)}$$

$$f' = \left(\frac{v-u_o}{v-u_s}\right)f$$

Thus the frequency of the wave appears to increase if $u_o < u_s$ or it appears to decrease if $u_s < u_o$.

Case 9: *Source S moving away from an observer O who is moving towards it*.



$$\text{Apparent wavelength, } \lambda' = \frac{\text{velocity of wave relative to source}}{\text{actual frequency}} = \frac{v_{ws}}{f}$$

$$\lambda' = \frac{v - -u_s}{f} = \frac{v + u_s}{f} \text{----------------} (1)$$

$$\text{Apparent frequency, } f' = \frac{\text{velocity of wave relative to observer}}{\lambda'} = \frac{v_{wo}}{\lambda'}$$

$$f' = \frac{v - -u_o}{\lambda'} = \frac{v + u_o}{\lambda'} \text{----------------} (2)$$

Substituting (1) *into* (2),

$$f' = \frac{v + u_o}{\left(\frac{v + u_s}{f}\right)}$$

$$f' = \left(\frac{v + u_o}{v + u_s}\right) f$$

Thus, the frequency of the wave appears to decrease if $u_0 < u_s$ or it appears to increase if $u_0 > u_s$

SUMMARY

$$\text{The apparent frequency of the wave, } f' = \left(\frac{v \pm u_o}{v \pm u_s}\right) f$$

NOTE

1. In calculations, consider the observer and the source independently (separately) when using the above formula.
2. If the observer is moving towards the source, f' will increase, so make the numerator big i.e $v + u_o$ and vice versa, i.e. when the observer is moving away from the source, make the numerator $v - u_o$
3. If the source is moving towards the observer, f' will increase, so make the denominator small i.e $v - u_s$ and vice versa i.e when the source is moving away from the observer, make the denominator $v + u_s$.
4. For a stationary observer: $u_o = 0ms^{-1}$ and for a stationary source $u_s = 0ms^{-1}$

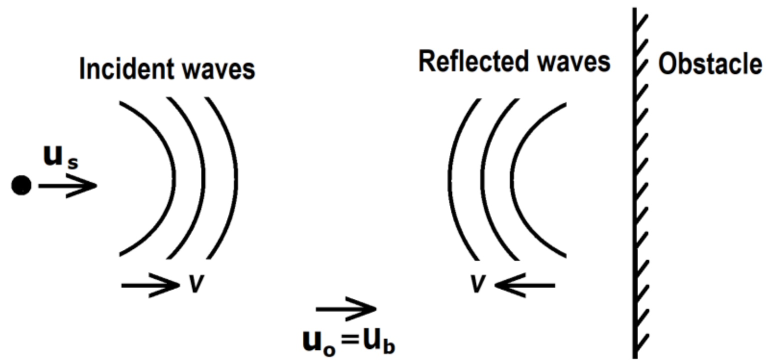
CASES INVOLVING A BAT OR A SIREN VEHICLE WITH AN OBSERVER SEATED INSIDE MOVING TOWARDS AN OBSTACLE SUCH AS A WALL OR CLIFF

(a) A bat:

This moving bat is the source of incident waves and it is the observer/ receiver of the reflected waves.

Let the velocity of the bat be u_b

$\Rightarrow u_b = u_s = u_o$



$$The\ velocity\ of\ the\ incident\ wave\ relative\ to\ the\ source;\ v_{ws} = v - u_s ----------(1)$$

$$Apparent\ wavelength, \lambda' = \frac{v_{ws}}{f} = \frac{v - u_s}{f} ------(2)$$

$$The\ velocity\ of\ the\ reflected\ wave\ relative\ to\ the\ observer;\ v_{wo} = v - -u_o = v + u_o --------(3)$$

$$Apparent\ frequency\ of\ the\ wave;\ f' = \frac{v_{wo}}{\lambda'} = \frac{v + u_o}{\lambda'} ----(4)$$

$Substitutig\ (2)\ into\ (4)$

$$f' = \frac{v + u_o}{\left(\frac{v - u_s}{f}\right)} = \left(\frac{v + u_o}{v - u_s}\right) f$$

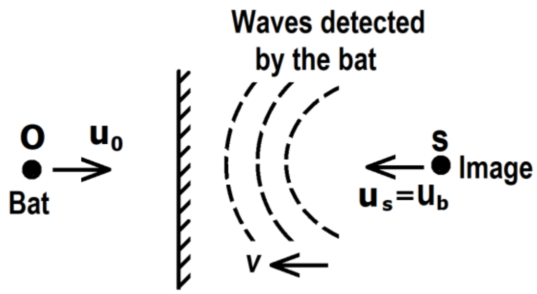
$$f' = \left(\frac{v + u_b}{v - u_b}\right) f$$

ALTERNATIVELY

Consider an image of the bat approaching the actual bat from behind the wall(obstacle) towards the object.

So the bat is the moving observer/receiver of the waves from behind the wall. Its image is the source.

Let velocity of the bat be u_b



$$Apparent\ wavelength,\ \lambda' = \frac{velocity\ of\ wave\ relative\ to\ source}{actual\ frequency} = \frac{v_{ws}}{f}$$

$$\lambda' = \frac{v - u_s}{f} ----------------(1)$$

$$Apparent\ frequency,\ f' = \frac{velocity\ of\ wave\ relative\ to\ observer}{\lambda'} = \frac{v_{wo}}{\lambda'}$$

$$f' = \frac{v - -u_o}{\lambda'} = \frac{v + u_o}{\lambda'} ----------------(2)$$

Substituting (1) into (2)

$$f' = \frac{v + u_o}{\left(\frac{v - u_s}{f}\right)} = \left(\frac{v + u_o}{v - u_s}\right) f$$

$$f' = \left(\frac{v + u_b}{v - u_b}\right) f$$

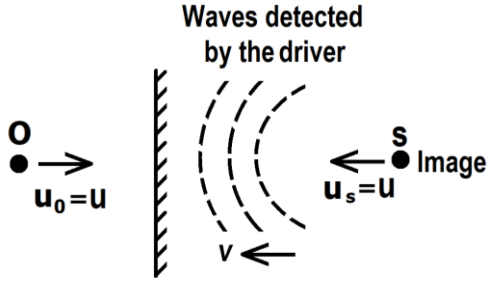
EXAMPLES

1. A bat flying at $1.0ms^{-1}$ emits waves of frequency 78kHz as it flies away from a tall building. Calculate the apparent frequency of the reflected waves from the wall that are received by the bat. (Velocity of sound in air is 320 ms^{-1}).
   Hint: $f' = \left(\frac{v - u_b}{v + u_b}\right) f$

2. UNEB 2011 No. 3(d) *"Copy that question and work it out."*

(b) (i) A moving siren Car or vehicle towards an obstacle when it has an observer in it

Let velocity of the car be u



$$Apparent\ wavelength, \lambda^{'} = \frac{velocity\ of\ wave\ relative\ to\ source}{actual\ frequency} = \frac{v_{ws}}{f}$$

$$\lambda^{'} = \frac{v - u_s}{f} ------------------(1)$$

$$Apparent\ frequency, f^{'} = \frac{velocity\ of\ wave\ relative\ to\ observer}{\lambda^{'}} = \frac{v_{wo}}{\lambda^{'}}$$

$$f^{'} = \frac{v - -u_o}{\lambda^{'}} = \frac{v + u_o}{\lambda^{'}} ------------------(2)$$

Substituting (1) *into* (2)

$$f^{'} = \frac{v + u_o}{\left(\frac{v - u_s}{f}\right)} = \left(\frac{v + u_o}{v - u_s}\right) f$$

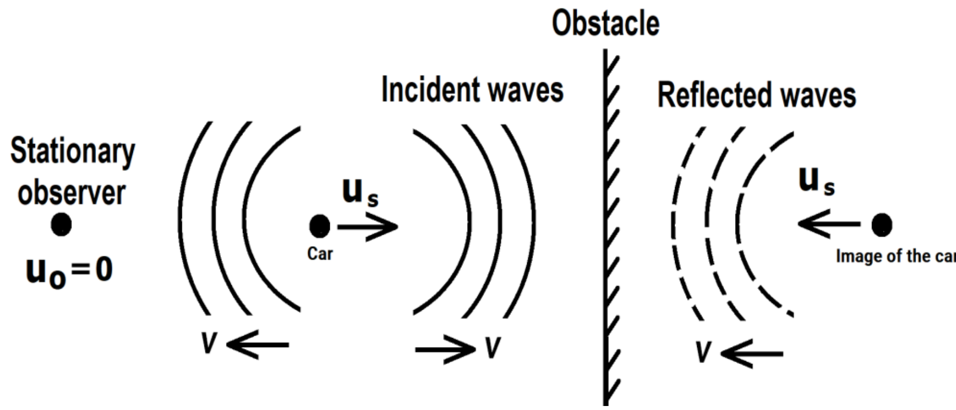
$$f^{'} = \left(\frac{v + u}{v - u}\right) f$$

NB. The beat frequency heard by the observer in the siren vehicle is $f_b = f' - f$ since $f' > f$.

(b)(ii). A siren vehicle sounding its siren moves away from an obstacle with a speed u. Derive an expression for the apparent frequency f' heard by the driver if the speed of sound waves produced by the siren is V and f is the actual frequency.

(c) The beats heard by an observer when a siren car recedes him or approaches him and it is moving towards an obstacle:

(i) A stationary observer and a receding siren vehicle which is approaching an obstacle.



The observer hears beats due to waves from the receding siren and the echoes he receives.

The reflected waves (echoes) are treated as though they originate from an approaching image of the siren car.

Let f' and λ' be the apparent frequency and apparent wavelength of waves received directly from the receding car/vehicle respectively and f'' and λ'' be the apparent frequency and apparent wavelength of the echoes respectively.

Part I: Receding car/vehicle

$$\textit{Apparent wavelength},\ \lambda' = \frac{v_{ws}}{f} = \frac{v--u_s}{f} = \frac{v+u_s}{f}$$

$$\textit{Apparent frequency}, f' = \frac{v_{wo}}{\lambda'} = \frac{v-0}{\lambda'} = \frac{v}{\lambda'}$$

$$\Rightarrow f' = \frac{v}{\left(\frac{v+u_s}{f}\right)} = \left(\frac{v}{v+u_s}\right)f -----(1)$$

Part II: Reflected waves received by the observer

$$\textit{Apparent wavelength},\ \lambda'' = \frac{v_{ws}}{f} = \frac{v-u_s}{f}$$

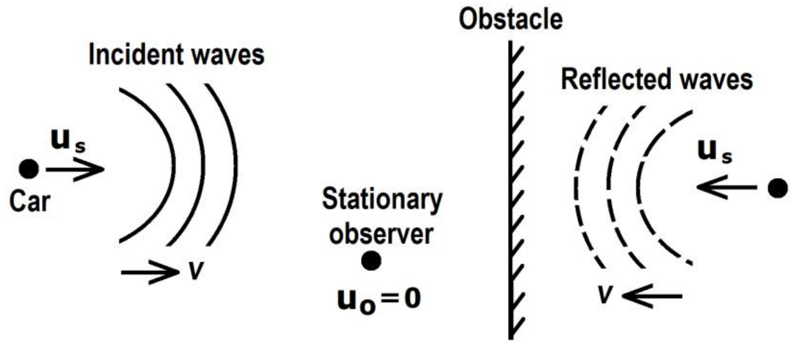
$$\textit{Apparentfrequency},\ f'' = \frac{v_{wo}}{\lambda''} = \frac{v-0}{\lambda''} = \frac{v}{\lambda''}$$

$$\Rightarrow f'' = \frac{v}{\left(\frac{v-u_s}{f}\right)} = \left(\frac{v}{v-u_s}\right)f -----(2)$$

Since $f'' \rangle f'$ from the above expressions, then $f_b = f'' - f'$

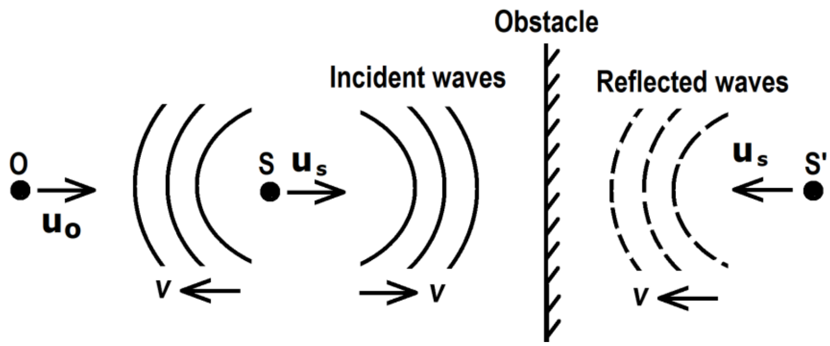
Where f_b is the beat frequency and $u_s = u = velocity\ of\ the\ vehicle$

(ii) A siren car moving towards an obstacle and approaching a stationary observer infront of the obstacle.



If the car moves with a velocity u, derive the expression for the beat frequency heard by the observer. (**Assignment**)

(iii) An observer running with a speed u_o towards a receding sounding siren car moving with a speed u_s towards an obstacle.



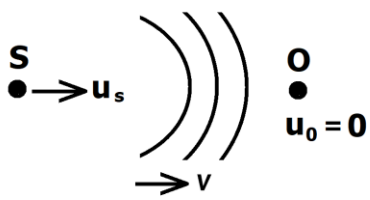
If v is the speed of the sound waves and f is the frequency of the sound, derive an expression for the beat frequency f_b received by the observer. (**Assignment**)

EXAMPLES

1. A police car moving at speed of 80 ms^{-1} sounds a siren at a frequency of $1000Hz$ as it approaches a stationary observer. What is the apparent frequency of the siren as heard by the observer if the speed of sound in air is $340\ ms^{-1}$?

Solution

Given $v = 340ms^{-1},\ f = 1000Hz,\ u_s = 80ms^{-1}$



Let v be the velocity of the wave and f be the frequency of the wave

Velocity of the wave relative to the source is given by;

$v_{ws} = v - u_s$

$$Apparent\ \ wavelength,\ \lambda' = \frac{v_{ws}}{f} = \frac{v - u_s}{f} --------(1)$$

Velocity of the wave relative to the observer is

$v_{wo} = v$ *since the observer is stationary*

$$Apparent\ \ frequency\ f' = \frac{v_{wo}}{\lambda'}$$

$$f' = \frac{v}{\lambda'}................(2)$$

Substituting (1) *in* (2),

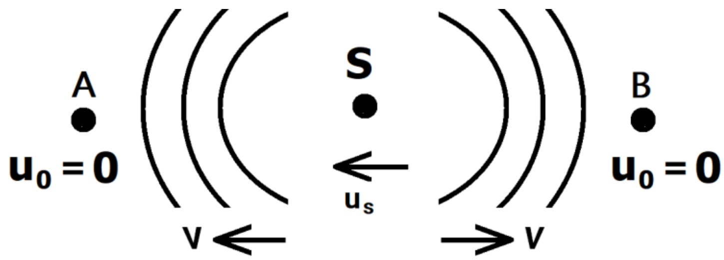
$$f' = \frac{v}{\left(\frac{v - u_s}{f}\right)} = \left(\frac{v}{v - u_s}\right)f$$

$$f' = \left(\frac{340}{340 - 80}\right)1000$$

$$f' = 1307.69Hz$$

2. A car sounds its horn as it travels at a speed of $15ms^{-1}$ along a straight road between two stationary observers A and B. Observer A hears a frequency of $538Hz$ while observer B hears a lower frequency. Calculate the frequency heard by B assuming that the speed of sound in air is $340ms^{-1}$.

Solution



Given $f'_A = 538Hz,\ u_s = 15ms^{-1},\ u_A = 0,\ u_B = 0,\ v = 340ms^{-1}$

For observer A

$$v_{ws} = v - u_s$$

$$\lambda'_A = \frac{v_{ws}}{f} = \frac{v - u_s}{f}$$

$$Apparent\ frequency, f'_A = \frac{v_{wo}}{\lambda'_A} = \frac{v - 0}{\lambda'_A} = \frac{v}{\lambda'_A}$$

$$\therefore f'_A = \frac{v}{\left(\frac{v - u_s}{f}\right)} = \left(\frac{v}{v - u_s}\right) f$$

$$f = \left(\frac{v - u_s}{v}\right) f'_A = \left(\frac{340 - 15}{340}\right) 538$$

$$f = 514.26Hz = actual\ frequency$$

For observer B

$$v_{ws} = v + u_s$$

$$\lambda'_B = \frac{v_{ws}}{f} = \frac{v + u_s}{f}$$

$$Apparent\ frequency, f'_B = \frac{v_{wo}}{\lambda'_B} = \frac{v - 0}{\lambda'_B} = \frac{v}{\lambda'_B}$$

$$f'_B = \frac{v}{\left(\frac{v + u_s}{f}\right)} = \left(\frac{v}{v + u_s}\right) f$$

$$f'_B = \left(\frac{340}{340 + 15}\right) 514.26$$

$f_B' = 492.53Hz$

3. A train approaching a railway crossing at $20ms^{-1}$ sounds a whistle of frequency $440Hz$ when $1.0km$ from the crossing. If there is no wind and the speed of sound in air is $320ms^{-1}$, what frequency is heard by a stationary observer at the crossing?

Solution

$$\boldsymbol{v = 320ms^{-1}},\ \boldsymbol{f = 440Hz}, \qquad \boldsymbol{u_s = 20ms^{-1}}$$

For source moving towards a stationary observer,

$$Apparent\ \ frequency, f' = \left(\frac{v}{v - u_s}\right) f$$

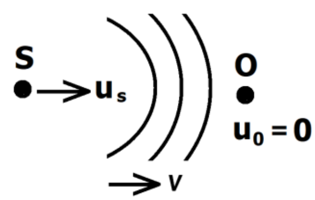
$$f' = \left(\frac{320}{320 - 20}\right) 440$$

$$f' = 469.33Hz$$

4. A car sounding a horn producing a note of $500Hz$, approaches and passes a stationary observer, O at a steady speed of $20ms^{-1}$. Find the change in pitch of the note heard by O. (Take velocity of sound $= 340ms^{-1}$)

Solution

For a source moving toward a stationary observer:

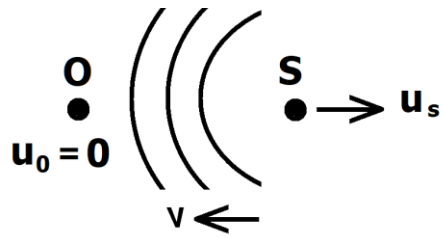


$$Apparent\ wavelength, \lambda' = \frac{v - u_s}{f}$$

$$Apparent\ \ frequency\ \ f' = \frac{v}{\lambda'} = \frac{v}{\left(\frac{v - u_s}{f}\right)} = \left(\frac{v}{v - u_s}\right) f$$

$$f' = \left(\frac{340}{340 - 20}\right) 500 = 531.25Hz$$

For a source moving away from *a stationary oberver*,



$$Apparent\ wavelength, \lambda'' = \frac{v + u_s}{f}$$

$$Apparent\ frequency, f'' = \frac{v}{\lambda''} = \frac{v}{\left(\frac{v+u_s}{f}\right)} = \left(\frac{v}{v + u_s}\right)f$$

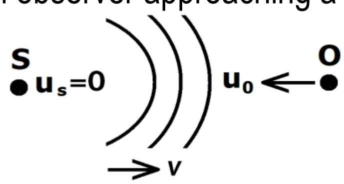
$$f'' = \left(\frac{340}{340 + 20}\right)500 = 472.22Hz$$

$$Change\ in\ pitch\ \Delta f = f' - f'' = 531.25 - 472.22 = 59.03Hz$$

5. A bell at a rail road crossing emits a sound whose frequency is *400Hz*. What is the apparent frequency heard by a passenger on a train that is;
   (a) Approaching the crossing at $30ms^{-1}$?
   (b) Moving away from the crossing at $30ms^{-1}$?
   (Speed of sound in still air = $330ms^{-1}$)

Solution

(a) For an observer approaching a stationary source.



$Velocity\ of\ the\ wave\ relative\ to\ the\ source\ is\ given\ by;$

$v_{ws} = v ---------- (1)\ since\ the\ source\ is\ stationary$

$Apparent\ wavelength\ \lambda' = \frac{v_{ws}}{f} = \frac{v}{f} -------- (2)$

$Apparent\ frequency,\ f' = \frac{v_{wo}}{\lambda'}$

$But\ v_{wo} = v - -u_o = v + u_o$

$\Rightarrow f' = \frac{v + u_o}{\lambda'} ------------ (3)$

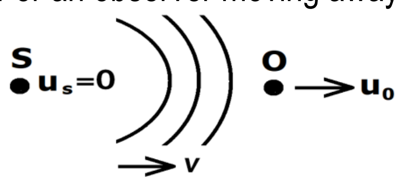
$Substituting\ (2)\ into\ (3),$

$$f' = \frac{v + u_o}{\left(\frac{v}{f}\right)} = \left(\frac{v + u_o}{v}\right) f$$

$$f' = \left(\frac{330 + 30}{330}\right) 400$$

$$f' = 436 Hz$$

(b) For an observer moving away from a stationary source



$Velocity\ of\ the\ wave\ relative\ to\ the\ source\ is\ given\ by;$

$v_{ws} = v ---------- (1)\ since\ the\ source\ is\ stationary$

$Apparent\ wavelength,\ \lambda' = \frac{v_{ws}}{f} = \frac{v}{f} -------- (2)$

$Apparent\ frequency,\ f' = \frac{v_{wo}}{\lambda'}$

$But\ v_{wo} = v - u_o$

$\Rightarrow f' = \frac{v - u_o}{\lambda'} ------------ (3)$

$Substituting\ (2)\ into\ (3)$

$$f' = \frac{v - u_o}{\left(\frac{v}{f}\right)} = \left(\frac{v - u_o}{v}\right) f$$

$$f' = \left(\frac{330 - 30}{330}\right) 400 = 364 Hz$$

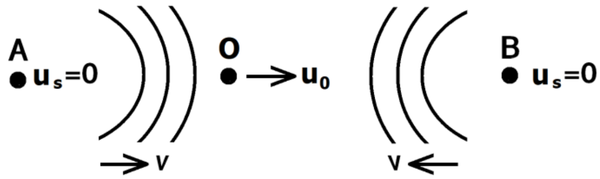
6. An observer moving at a velocity, u_o along a straight line joining two stationary identical sources hears beats. If the frequency of the sources is f and the velocity of the sound in air at that time is v.

(a) Derive the expression for the beat frequency, f_b.

(b) Find the frequency of the sources given that beat frequency $f_b = 5s^{-1}, v = 320ms^{-1}$ *and* $u_o = 1.75ms^{-1}$.

Solution

(a)



$For\ sound\ from\ A$

$v_{ws} = v$ since the source is stationary

$$Apparent\ wavelength,\ \lambda' = \frac{v_{ws}}{f} = \frac{v}{f}$$

$$Apparent\ frequency, f'_A = \frac{v_{wo}}{\lambda'} = \frac{v - u_o}{\left(\frac{v}{f}\right)} = \left(\frac{v - u_o}{v}\right) f$$

$For\ sound\ from\ B$

$v_{ws} = v$since the source is stationary

$$Apparent\ wavelength, \lambda' = \frac{v_{ws}}{f} = \frac{v}{f}$$

$$Apparent\ \ frequency, f'_B = \frac{v_{wo}}{\lambda'} = \frac{v + u_o}{\left(\frac{v}{f}\right)} = \left(\frac{v + u_o}{v}\right) f$$

$Beat\ \ frequency\ \ heard\ \ by\ \ the\ \ observer, f_b = f'_B - f'_A$ since $f'_B > f'_A$

$$f_b = \left(\frac{v + u_o}{v}\right) f - \left(\frac{v - u_o}{v}\right) f = \left(\frac{v + u_o}{v} - \frac{v - u_o}{v}\right) f$$

$$\Rightarrow f_b = \frac{2u_o f}{v}$$

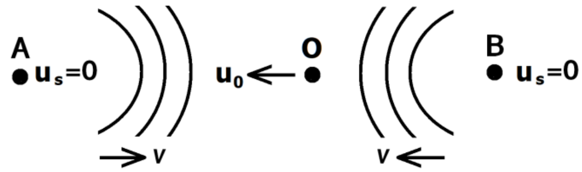
(b) $f_b = \dfrac{2u_o f}{v}$

$$f = \frac{vf_b}{2u_o} = \frac{320 \times 5}{2 \times 1.75} = 457.14Hz$$

7. An observer moving between two identical stationary sources of sound along a straight line hears beats at a rate of $5s^{-1}$. If the frequencies of the sources are

$600Hz$ and the velocity of sound in air is $330ms^{-1}$, calculate the velocity at which the observer is moving.

Solution



Apparent frequency of sound from A

$$f_A' = \left(\frac{v - -u_o}{v}\right)f = \left(\frac{v+u_o}{v}\right)f$$

Apparent frequency of sound from B

$$f_B' = \left(\frac{v-u_o}{v}\right)f$$

Beat frequency $f_b = f_A' - f_B'$

$$f_b = \left(\frac{v+u_o}{v}\right)f - \left(\frac{v-u_o}{v}\right)f = \left(\frac{v+u_o}{v} - \frac{v-u_o}{v}\right)f$$

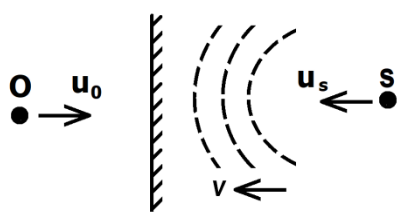
$$f_b = \frac{2u_o f}{v}$$

$$5 = \frac{2u_o \times 600}{330}$$

$$u_o = 1.38ms^{-1}$$

8. A train approaching a tunnel in a cliff face at $95kmhr^{-1}$ is sounding a whistle of frequency $100Hz$. Find the apparent frequency of the echo from the cliff face heard by thedriver.(Take velocity of sound in air $= 330ms^{-1}$)

Solution



The image of the train acts as a source of waves received by the driver after reflection

$$\Rightarrow u_s = u_o = u$$

$$Apparent\ wavelength, \lambda' = \frac{v - u_s}{f}$$

$$Apparent\ \ frequency, f' = \frac{v+u_o}{\lambda'} = \frac{v+u_o}{\left(\frac{v-u_s}{f}\right)} = \left(\frac{v+u}{v-u}\right)f$$

$$\Rightarrow u = \frac{95 \times 1000}{60 \times 60} = 26.39ms^{-1}$$

$$f' = \left(\frac{330 + 26.39}{330 - 26.39}\right) \times 100$$

$$f' = 117.38Hz$$

Revision questions

1. A motorcyclist and a police car are approaching each other. The motorcycle is moving at $10ms^{-1}$ and the police car at $20ms^{-1}$. The police siren is sounded at $480Hz$. Calculate the frequency of the note heard by the cyclist after the police car passes by. (speed of sound in air is$340ms^{-1}$)
2. An engine travelling at a constant speed towards a tunnel emits a short burst of sound of frequency 400Hz which is reflected from the tunnel entrance. The engine driver hears an echo of frequency 500Hz two seconds after the sound is emitted. Assuming the speed of sound is $340ms^{-1}$, calculate the speed of the engine.
3. A train moving with a uniform velocity sounds a horn as it approaches a stationary observer.
   (a) Derive the expression for the apparent frequency of the sound detected by the observer.
   (b) If the frequency of the sound detected by the observer after the train passes is 1.5 times lower than the frequency detected in (a) above, find the speed of the train given that the speed of sound in air is $340ms^{-1}$.
4. A car travelling at $20ms^{-1}$ has a siren that produces a sound of $500Hz$. Determine the difference between the frequency of sound heard by an observer at the roadside as the car approaches and as it recedes the observer. (speed of sound in air is$330ms^{-1}$).
5. Calculate the frequency of beats heard by a stationary observer when a source of sound of frequency $80Hz$ is receding her with a speed of $5.0ms^{-1}$ towards a vertical wall. (speed of sound in air is$340ms^{-1}$)
6. An observer moving between two identical stationary sources of sound, along the line joining them, hears beats at the rate of $3.0ms^{-1}$. At what velocity is the observer moving if the frequencies of the sources are $480Hz$ and the velocity of sound when the observation was made was $340ms^{-1}$?
7. An observer moving at a speed of $10ms^{-1}$ between two stationary sources of sound A and B hears beats at $5s^{-1}$. If the frequency of waves produced by source A is $515Hz$ and the observer is moving towards A, find the frequency of sound produced by B. (speed of sound in air is$340ms^{-1}$)
8. A driver of a car speeding at $18ms^{-1}$ receives a note of frequency $714Hz$ from a hoofer of a factory behind the car. Find the true frequency of the note. (speed of sound in air is$330ms^{-1}$)
9. An observer travelling with a constant velocity of $20ms^{-1}$ passes close to a stationary source of sound and notices that there is a change of frequency of $50Hz$

as she passes the source. Find the frequency of the source. (speed of sound in air is$340ms^{-1}$)

10. Calculate the frequency of beats heard by a stationary observer when a source of sound of frequency $120Hz$ is receding with speed of $8ms^{-1}$ towards a vertical wall. (speed of sound in air is$340ms^{-1}$)
11. A police car travelling at $108kmhr^{-1}$ is chasing a lorry which is travelling at $72kmhr^{-1}$. Both are about to pass a stationary bystander and the police car siren emits a sound of frequency $400Hz$. Calculate the apparent frequency of a note from the siren as observed by the lorry driver. (speed of sound in air is$340ms^{-1}$)
12. Derive an expression for the apparent frequency, f_0 of sound waves reaching a stationary observer when the source is producing sound of frequency, f_s and the source moves towards the observer at a speed, u_s. Assume the velocity of sound in air is v.
13. A whistle giving out $500Hz$ moves away from a stationary observer in a direction towards a perpendicular flat wall with a velocity of $1.5ms^{-1}$. How many beats per second will be heard by the observer? (Take speed of sound in air as$340ms^{-1}$ and assume no wind)
14. A person carrying a whistle sounding at 600 cycles per second moves towards a wall with a velocity of $4ms^{-1}$. If the wall is smooth and perpendicular to the direction of motion of the whistle, calculate the beats heard by the person. (Velocity of sound in air is$320ms^{-1}$)
15. Calculate the frequency of beats heard by a stationary observer when a source of sound of frequency 80Hz is receding with a speed of 5.0ms^{-1} towards a vertical wall. (speed of sound in air=340ms^{-1})
16. An observer moving on a straight road towards a tall wall is approached at the back by a police car moving at $30ms^{-1}$ and sounding a siren of frequency $620Hz$. He then hears beats at a rate of $6s^{-1}$. Find the speed of the observer.(speed of sound in air is 340ms^{-1})
17. (UNEB 2020)
    A car, p, moving at a speed of $108kmhr^{-1}$ towards a stationary observer and another observer in car, Q moving in the opposite direction with the same speed as P, sounds a horn of frequency 256Hz. Find the frequency of sound heard by the:
    (i) Stationary observer (03marks)
    (ii) Observer in car, Q. (03 marks)
    (Speed of sound in air is 340ms^{-1})

APPLICATIONS OF DOPPLER EFFECT

In light, it is used in:

1. Determination of direction of a star.
2. Determination of speed of motion of a star.
3. Measurement of high (plasma) temperatures.
4. Measurement of the speed of rotation of the sun

Other applications include:

5. In police radar speed traps and in tracking satellites

Note that the electromagnetic spectrum is an arrangement of the electromagnetic waves in order of increasing wave length or decreasing frequency.

V I B G Y O R

Gamma rays	X-rays	Ultra violet								Infra-red	Micro waves	T.V waves	Radio waves

Visible light

→ Increasing wave length or decreasing frequency.

So, as an example, the wave length of red light (R) is longer or greater than the wave length of blue light (B). In comparison to blue and red lights, red shift means increase in wave length of light while blue shift means a decrease in wave length of light.

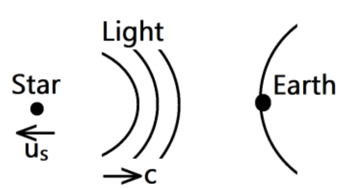
DETERMINATION OF DIRECTION OF MOTION OF A STAR AND ITS SPEED

Revision Question

(a) Explain how to measure the speed of a star that is moving away from the earth.
(b) Explain how to measure the speed of a star that is moving towards the earth.
(c) Explain how to determine the direction of a moving star and hence to determine/measure its speed.

Solution

(a) A star moving away from an observer on the earth with a velocity u_s produces light of actual wavelength λ but an apparent wavelength λ' is received by the observer on the earth.



$\lambda' = \frac{c--u_s}{f}$ where c is speed of light

$$f = \frac{c}{\lambda} \therefore \lambda' = \frac{c + u_s}{\left(\frac{c}{\lambda}\right)} = \left(\frac{c + u_s}{c}\right)\lambda = \left(1 + \frac{u_s}{c}\right)\lambda$$

The wavelength shift, $\Delta\lambda = \lambda' - \lambda = \left(1 + \frac{u_s}{c}\right)\lambda - \lambda = +\frac{u_s\lambda}{c} = \text{Red shift}$ since $\lambda' > \lambda$, which implies a reduction in frequency received from the moving star.
This **confirms** that the star is moving away from the earth.
The actual wavelength λ of light and the Doppler shift or wavelength shift $\Delta\lambda$ are measured in the laboratory using a diffraction grating and the speed u_s of the star can be calculated from $u_s = \frac{c\Delta\lambda}{\lambda}$.

OR; The photograph of a spectrum of a star (moving star) is taken and its spectral lines of wavelength λ' are observed.
Another photograph of a spark spectrum of an element known to be present in the star (stationary star) is taken and the same spectral lines of actual wavelength λ are observed. The spectral lines are then compared.
It will be observed that the spectral lines of wavelength λ' are displaced towards the red end $(\lambda' > \lambda)$which implies a reduction in frequency received from the moving star, **confirming** that the star is receding the earth.
The Doppler shift $\Delta\lambda = \lambda' - \lambda$ is obtained.
The velocity of the star is $u_s = \frac{c\Delta\lambda}{\lambda}$, where c is the speed of light and λ is the wavelength of light which is measured in the laboratory using a diffraction grating.

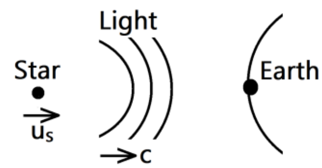
i.e $\lambda' = \frac{c - -u_s}{f}$ where c is speed of light.

$$f = \frac{c}{\lambda} \qquad \therefore \lambda' = \frac{c + u_s}{\left(\frac{c}{\lambda}\right)} = \left(\frac{c + u_s}{c}\right)\lambda = \left(1 + \frac{u_s}{c}\right)\lambda$$

The wavelength shift, $\Delta\lambda = \lambda' - \lambda = \left(1 + \frac{u_s}{c}\right)\lambda - \lambda = \frac{u_s\lambda}{c}$

$$\therefore u_s = \frac{c\Delta\lambda}{\lambda}$$

(b) A star moving towards an observer on the earth with a velocity u_s produces light of actual wavelength λ but an apparent wavelength λ' is received by the observer on the earth.



$\lambda' = \frac{c - u_s}{f}$ where c is speed of light

$$f = \frac{c}{\lambda} \therefore \lambda' = \frac{c - u_s}{\left(\frac{c}{\lambda}\right)} = \left(\frac{c - u_s}{c}\right)\lambda = \left(1 - \frac{u_s}{c}\right)\lambda$$

The doppler shift or wavelength shift,

$\Delta\lambda = \lambda' - \lambda = \left(1 - \frac{u_s}{c}\right)\lambda - \lambda = -\frac{u_s\lambda}{c} = \text{Blue shift}$ since $\lambda' < \lambda$, which implies an increase in frequency received from the moving star.
This **confirms** that the star is moving towards the earth.

$$|\Delta\lambda| = \frac{u_s\lambda}{c}$$

$$u_s = \frac{c|\Delta\lambda|}{\lambda}$$

The actual wavelength λ of light and the Doppler shift or wavelength shift $\Delta\lambda$ are measured in the laboratory using a diffraction grating and the speed u_s of the star can be calculated from $u_s = \frac{c\Delta\lambda}{\lambda}$.

OR; The photograph of a spectrum of a star (moving star) is taken and its spectral lines of wavelength λ' are observed.
Another photograph of a spark spectrum of an element known to be present in the star (stationary star) is taken and the same spectral lines of the actual wavelength λ are observed. The spectral lines are then compared.
It will be observed that the spectral lines of wave length λ' are displaced towards the blue end (i.e. $\lambda' < \lambda$) which implies an increase in frequency received from the moving star, confirming that the star is moving towards the earth.
The doppler shift $|\Delta\lambda| = |\lambda' - \lambda|$.
The velocity of the star, $u_s = \frac{c|\Delta\lambda|}{\lambda}$ where c is the speed of light and λ is the wavelength of light measured in the laboratory using a diffraction grating.

(c) The photograph of the spectrum of the star (moving star) is taken and its spectral lines of apparent wavelength $\lambda'$are observed.
Another photograph of a spark spectrum of an element known to be present in the star (stationary star) is taken and the same spectral lines of actual wavelength λ are observed.
The spectral lines are then compared.
If the spectral lines of apparent wave length λ' are displaced towards the red (red shift; $\lambda' > \lambda$); **then the star is receding (moving away from) the earth** since shifting towards red implies an increase in wavelength hence a reduction in frequency of light received from the moving star.
If the spectral lines of apparent wave length λ' are displaced towards the blue or violet (blue shift; $\lambda' < \lambda$); **then the star is approaching the earth**.
The doppler shift $|\Delta\lambda| = |\lambda' - \lambda|$ in either case is obtained and the speed of the star is given by
$u_s = \frac{c|\Delta\lambda|}{\lambda}$ where c is the speed of light and λ is the actual wavelength of light which is measured in the laboratory using a diffraction grating.

EXAMPLE

A certain spectral line of hydrogen has a frequency of $4.5657 \times 10^{14} Hz$ as measured in a laboratory on earth. The same line in the light reaching the earth from a star has frequency $4.56711 \times 10^{14} Hz$. Calculate the speed of the star relative to the earth (given the speed of light is $3.0 \times 10^{8} ms^{-1}$)

Solution

$Using \quad v = \lambda f \quad But \quad v = c$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^{8}}{4.5657 \times 10^{14}}$$

$$\lambda = 6.5707 \times 10^{-7} m$$

Similarly

$$\lambda^{'} = \frac{c}{f^{'}} = \frac{3 \times 10^{8}}{4.56711 \times 10^{14}}$$

$$\lambda^{'} = 6.5687 \times 10^{-7} m$$

So, since $\lambda' < \lambda$, blue shift occurs.

Doppler shift, $\Delta\lambda = \lambda' - \lambda$

$$\Delta\lambda = 6.5687 \times 10^{-7} - 6.5707 \times 10^{-7}$$

$$\Delta\lambda = -1.9946 \times 10^{-10} m.$$

Since $\Delta\lambda$ is negative, the star is moving towards the earth with a speed u_s given by

$$u_s = \frac{c|\Delta\lambda|}{\lambda} = \frac{3 \times 10^{8} \times 1.9946 \times 10^{-10}}{6.5707 \times 10^{-7}} = 9.1068 \times 10^{4}\ ms^{-1}.$$

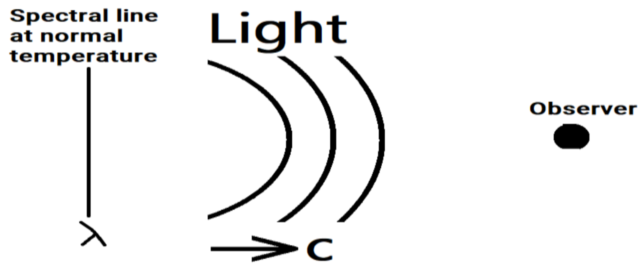
MEASUREMENT OF PLASMA TEMPERATURES (VERY HIGH TEMPERATURES SUCH AS THE TEMPERATURE OF THE PLASMA STATE OF MATTER)

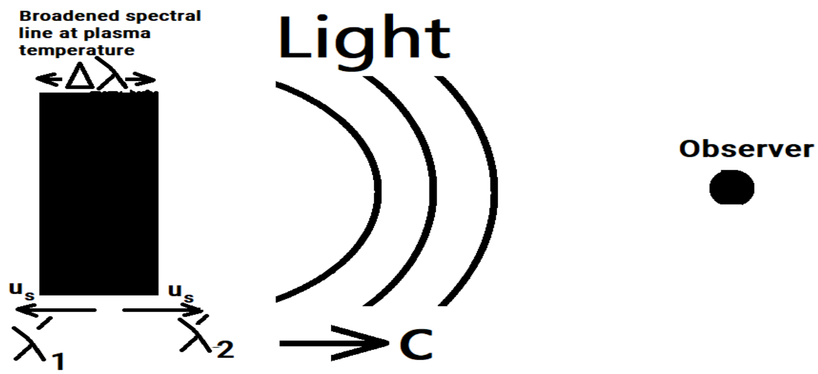
In thermal nuclear fusion experiments, the temperatures are of the order of millions of degrees Celsius. At very high temperatures ($\approx \times 10^{6}$°C) molecules of the glowing gas (source of light) are moving away and towards the observer with very high speeds of mean value $\boldsymbol{u_s}$.

Due to Doppler effect, the wavelength λ of a particular spectral line is apparently changed and the spectral line **appears broadened**.

One edge (end) of the broadened spectral line is far from the observer and it corresponds to an apparently increased wavelength,$\boldsymbol{\lambda_1'}$due to molecules moving directly away from the observer and the other edge near the observer corresponding to an apparently decreased wavelength,$\boldsymbol{\lambda_2'}$due to molecules moving directly towards the observer.

The line thus appears broadened by an amount$\Delta\lambda$.





If the mean velocity of the molecules is u_s, then;

$\boldsymbol{\lambda_1'} = \frac{c--u_s}{f}$, $f = \frac{c}{\lambda}$ and $\boldsymbol{\lambda_2'} = \frac{c-u_s}{f}$. So;

$$\boldsymbol{\lambda_1'} = \left(\frac{c+u_s}{c}\right)\lambda \text{ and} \boldsymbol{\lambda_2'} = \left(\frac{c-u_s}{c}\right)\lambda$$

$$The\ width\ of\ the\ line; \boldsymbol{\lambda_1'} - \boldsymbol{\lambda_2'} = \left(\frac{c+u_s}{c} - \frac{c-u_s}{c}\right)\lambda = \frac{2u_s}{c}\lambda = \Delta\lambda$$

$$\therefore u_s = \frac{c\Delta\lambda}{2\lambda} = \left(\frac{c|\lambda_1 - \lambda_2|}{2\lambda}\right)$$

Where λ is the actual wavelength of the light obtained in the laboratory and c is the speed of light.

The broadening (width) $\Delta\lambda$ is measured by a diffraction grating and then, u_s can be calculated.

The plasma temperature, $T\ in\ kelvins$ can be calculated from the kinetic theory of gases from which we know that $\boldsymbol{\frac{1}{2}Mu_s^2 = \frac{3}{2}RT}$, where **T** is absolute temperature, **R** is molar gas constant and **M** is molar mass of a gas.

$\therefore T = \frac{Mu_s^2}{3R}$.

If u_s is required and T is known, then $\boldsymbol{u_s^2 = \frac{3RT}{M}}$.

POLICE RADAR SPEED TRAPS

A transmitter carried by a police officer sends out radar waves or micro waves towards a speeding car. The waves are reflected back and the time taken is used to measure the speed of the car which is recorded by a meter calibrated in $kmhr^{-1}$ or ms^{-1}.

Application in radar speed traps

Microwaves or radar waves of frequency f from a stationary radar set are directed towards a motor car or air craft moving with speed v.

Microwaves reflected from the car are detected at the radar set. The reflected signal superimposes with the transmitted signal to obtain beats since they have slightly different frequencies.

The beat frequency Δf which is equal to the difference in frequency of the received and transmitted signals is determined or computed by the radar set or speed gun computer.

The speed v of the car is given by $v = \frac{c\Delta f}{2f}$ where c is the speed of the electromagnetic wave $3.0 \times 10^8 ms^{-1}$.

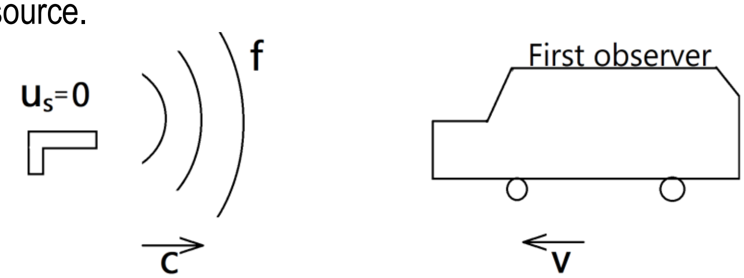
PROOF:

Prove that the velocity v of a moving car is $v = \frac{c\Delta f}{2f}$ or $\frac{cf_b}{2f}$ where c is velocity of microwaves or radar waves of frequency f transmitted from a stationary radar set (speed gun) and reflected from an incoming car resulting into beats of frequency Δf or f_b.

Solution

Let f be the actual frequency of the radar waves or microwaves of speed c produced and transmitted from a stationary radar set or speed gun as the source.

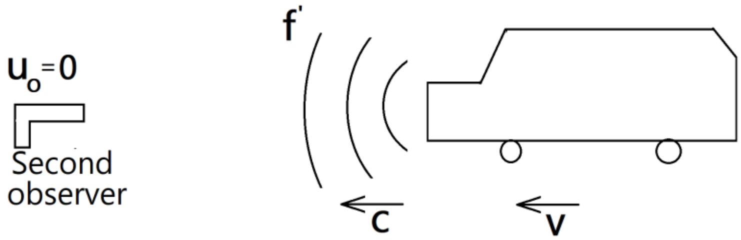
Considering the driver or the car as the first observer or observer 1 moving at a speed v towards the radar set, let f' be the apparent frequency of the waves received by the car from a stationary source.



Apparent wavelength, $\lambda' = \frac{c}{f}$

$$f' = \frac{c - -v}{\lambda'} = \left(\frac{c + v}{c}\right) f \ldots\ldots\ldots\ldots (i)$$

Let f'' be the apparent frequency of the reflected waves from the moving car as the moving source received by the stationary radar set (speed gun) acting as the second observer or observer 2.



$$\lambda'' = \frac{c-v}{f'}$$

$$f'' = \frac{c-0}{\lambda'} = \left(\frac{c}{c-v}\right)f' \ldots\ldots\ldots\ldots (ii)$$

Substituting f' from (i) into (ii)

$$f'' = \left(\frac{c}{c-v}\right)\left(\frac{c+v}{c}\right)f$$

$$f'' = \left(\frac{c+v}{c-v}\right)f$$

The transmitted waves of frequency f combine with the reflected waves of frequency f'' to produce beats of beat frequency Δf or f_b which is computed by the speed gun computer.

$$\Delta f = f'' - f \text{ since} f'' > f$$

$$\Delta f = \left(\frac{c+v}{c-v}\right)f - f$$

$$\Delta f = \left[\frac{c+v}{c-v} - 1\right]f$$

$$\Delta f = \left[\frac{c+v-c+v}{c-v}\right]f$$

$$\Delta f = \frac{2vf}{c-v}$$

But since $c \gg v; c - v \approx c \therefore \Delta f = \frac{2vf}{c}$

$$v = \frac{c\Delta f}{2f}$$

Assignment:

Show that the same equation $v = \frac{c\Delta f}{2f}$ is obtained for a car moving away from (receding) the stationary radar set at a speed v.

Hint: You will get $f' = \left(\frac{c-v}{c}\right)f$, $f'' = \left(\frac{c}{c+v}\right)f' = \left(\frac{c-v}{c+v}\right)f$,

$$\Delta f = f - f'' \text{ since } f > f''$$

$$\Delta f = \frac{2vf}{c+v} \text{ But since } c \gg v; c + v \approx c$$

$$\therefore \Delta f = \frac{2vf}{c} \Longrightarrow v = \frac{c\Delta f}{2f}$$

GENERAL PROPERTIES OF WAVES

All waves show the following general properties or characteristics

(i) Interference
(ii) Diffraction
(iii) Reflection
(iv) Refraction

In addition, transverse (not longitudinal) waves show the property of polarization.

Longitudinal waves do not undergo polarization.

Any wave capable of undergoing polarization is transverse in nature.

1. INTERFERENCE

This is the superposition of two coherent waves resulting into alternate regions of maximum and minimum intensity.

In light, the regions of maximum intensity are called bright fringes and those of minimum intensity are called dark fringes.

Coherent sources are sources that produce waves of the same frequency, equal (or nearly equal) amplitude and which have a constant phase difference between them.

Interference occurs only when the waves undergoing superposition are from coherent sources.

Coherent waves are waves having the same frequency, equal amplitude and a constant phase difference between them.

CONDITIONS FOR INTERFERENCE

For interference of waves to occur;

(i) The waves must have equal frequency
(ii) The waves must have equal or nearly equal amplitudes
(iii) The waves must have a constant phase difference between them.

Such waves are produced by coherent sources.

TYPES OF INTERFERENCE

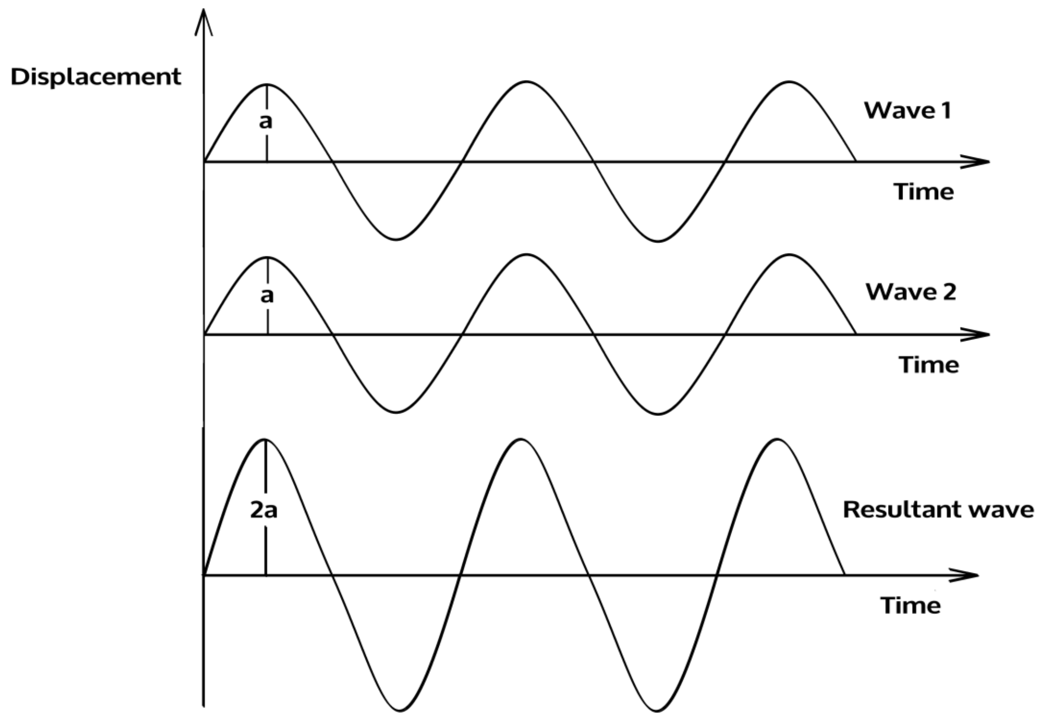
(i) Constructive interference
(ii) Destructive interference

CONSTRUCTIVE INTERFERENCE

This is a form of interference that occurs when coherent waves meet when in phase such that reinforcement occurs resulting into a region of maximum intensity.

By the superposition principle, the two waves reinforce each other resulting into a wave of maximum amplitude and intensity.

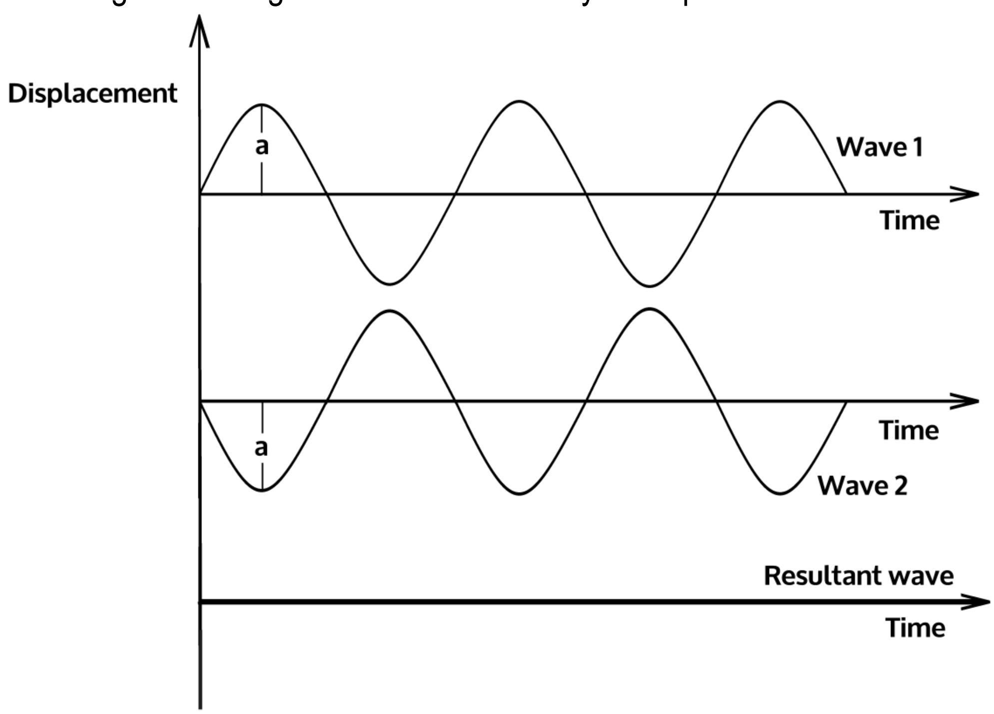
In transverse waves, constructive interference occurs when crests of coherent waves overlap and also troughs of the same waves overlap resulting into increased amplitude or intensity.



If the two superimposing waves are light waves, and constructive interference occurs, a bright fringe or light is observed.
If they are sound waves, a loud sound is heard.

DESTRUCTIVE INTERFERENCE

This is a form of interference that occurs when coherent waves meet when completely out of phase so that cancellation occurs resulting into a region of minimum intensity.
In transverse waves, destructive interference occurs when crests of coherent waves overlap with troughs resulting into minimum intensity or amplitude.



With light waves, destructive interference results into a dark band or fringe while with sound, near silence results.

Note

Normally, the regions of maximum intensity alternate with those of minimum intensity.
In light, this results into an alternating pattern of bright and dark bands.
In sound, an alternation of loud sound and near silence is heard along the line of superposition.
The alternating maximum and minimum intensity regions form what is referred to as an interference pattern.

GEOMETRICAL PATH LENGTH (x):

This is the distance travelled by a wave.

OPTICAL PATH LENGTH (OR OPTICAL PATH);nx:

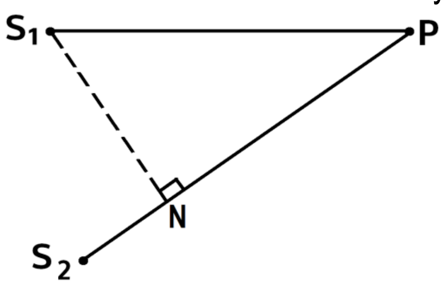
This is the product of the geometricalpath length of a wave and the refractive index of the medium in which the wave is moving.

PATH DIFFERENCE

This is the difference in the optical paths travelled by two wave trains before they superpose or meet or cross each other.

OR

It is the difference between the distances travelled by two wave fronts from their secondary sources to the point of overlap.
Consider S_1 and S_2 as secondary sources producing wave trains/fronts that meet at P.



$$Path\ \ difference = S_2N = S_2P - S_1P$$

SIGNIFICANCE OF PATH DIFFERENCE

Path difference can be used to predict whether constructive or destructive interference occurs at a point of overlap.
Constructive interference occurs when the path difference is a whole number multiple of a full wavelength. i.e $path\ \ difference = n\lambda\ where\ n = 0,1,2,3, etc$

For destructive interference, the path difference must be an odd number multiple of half wavelength, i.e

$Path\ \ difference = (2n+1)\frac{\lambda}{2}\ \ where\ \ n = 0,1,2,3,$ etc or $(2n-1)\frac{\lambda}{2}$, where n=1,2,3,….

The above are the conditions for constructive and destructive interference respectively.

EXPLANATION OF INTERFERENCE USING SUPERPOSITION PRINCIPLE

It occurs when two waves of equal frequency, equal or nearly equal amplitude having a constant phase difference between them meet.

When they meet, superposition occurs such that the resultant displacement at any point is the vector sum of the displacement due to each wave at that point.

At certain points, the two waves meet when the path difference between them is a whole number multiple of a full wavelength. At these points, the waves will be in phase and the resultant displacement will be maximum resulting into a region of maximum amplitude and intensity. Constructive interference is said to occur at such points.

At other points, the waves meet with a path difference between them being an odd number multiple of half wavelength. The waves at these points meet when completely out of phase resulting into cancellation and a region of minimum amplitude and intensity. Destructive interference is then said to occur.

The regions of constructive interference alternate with those of destructive interference resulting into an alternating pattern of regions of maximum and minimum intensity referred to as an interference pattern.

DIFFERENCES BETWEEN CONSTRUCTIVE AND DESTRUCTIVE INTERFERNCE

Constructive interference	Destructive interference
Occurs when waves from coherent sources meet when in phase.	Occurs when waves from coherent sources meet when completely out of phase.
Reinforcement occurs	Cancellation occurs
Results into a region of maximum intensity	Results into a region of minimum intensity
Occurs when the path difference is a whole number multiple of the full wave length of the coherent waves.	Occurs when the path difference is an odd number multiple of half the wave length.

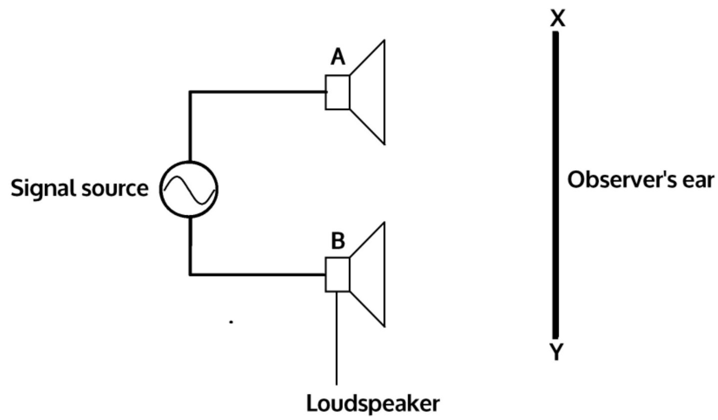
INTERFERENCE IN SOUND

Interference occurs in sound when sound waves from two coherent sources meet and superpose.

This can occur, for example, when two loud speakers A and B are connected to the same signal sources.

The loud speakers produce sound waves of the same frequency and amplitude since they are connected to the same source and therefore act as coherent sources.

The sound waves superimpose producing an interference pattern audible to an observer moving along a line parallel to the line joining the loudspeakers i.e XY.



Observation

The observer on the walkway hears alternate loud sound and near silence as he moves along XY.

The loud sound and near silence occur at equal distances from each other.

Explanation

The loudspeakers act as coherent sources producing sound waves of equal frequency and amplitude and having a constant phase difference between them.

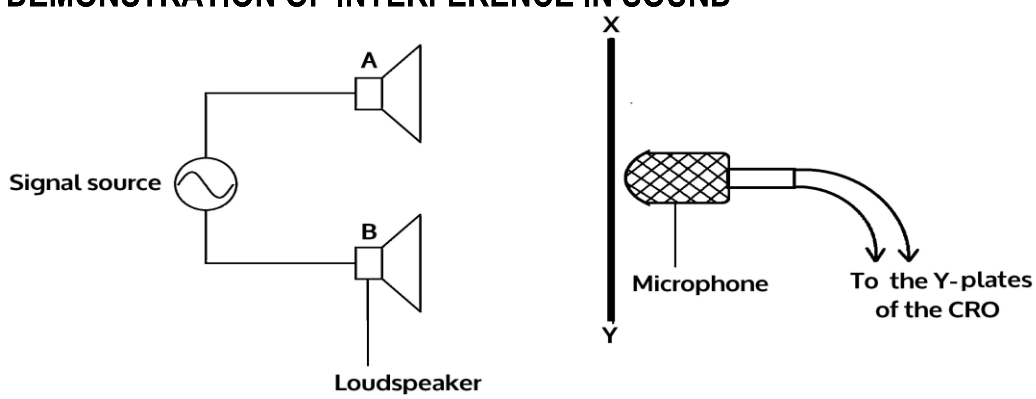
When the sound waves meet, superposition occurs resulting into interference.

At points where the path difference is a whole number multiple of a full wavelength of the sound, constructive interference occurs resulting into a loud sound being heard.

At other points, the path difference will be an odd multiple of a half wavelength resulting into destructive interference, producing near silence due to cancellation of displacements due to sound waves.

The alternate loud sounds and near silence form an interference pattern.

DEMONSTRATION OF INTERFERENCE IN SOUND



The apparatus is set up as shown above.

Two loudspeakers are connected to the same signal source.

The microphone is connected to Y-plates of the CRO with the time base switched off.

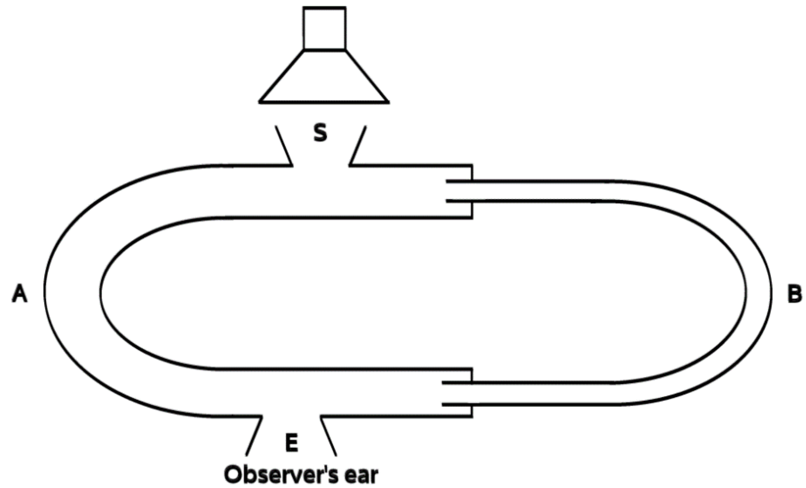
The microphone is moved along XY parallel to the line joining loud speakers.

The vertical trace on the screen of the CRO is seen to increase to maximum length and then reduce to a minimum length at equal distance intervals.

Where the length of trace is maximum, constructive interference is occurring whereas where it is minimum, destructive interference is occurring.

The alternate maximum and minimum length of the vertical trace indicates an interference pattern.

ALTERNATIVELY

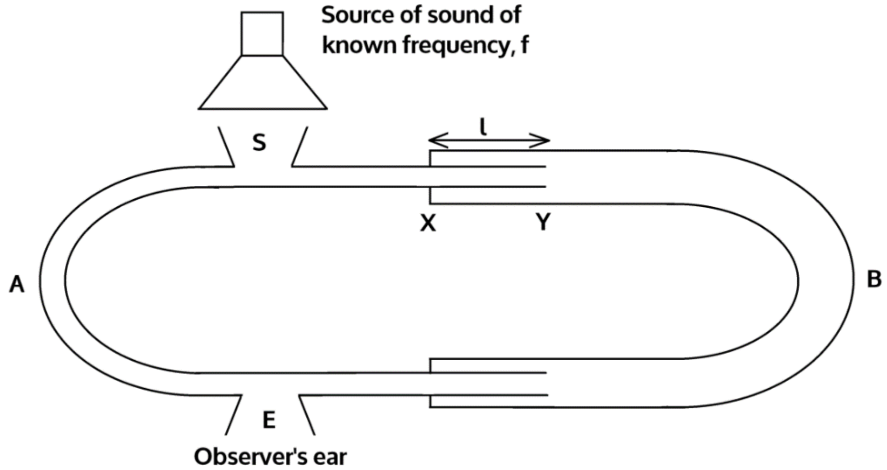


The apparatus is made as shown above.
Tube A is made fixed while tube B is free to move in and out of tube A.
A source of sound is placed at S and the sound is detected at E by an observer.
The tube B is slowly pulled outwards while listening at E.
It is observed that the sound observed at E alternately increases to a maximum intensity and decreases to a minimum intensity at equal intervals of length of the tube B moved outwards.
The alternate maximum and minimum intensity demonstrates interference.

THEORY OF THE EXPERIMENT

The sound produced at S travels to different parts along the tubes A and B.
Sound waves travelling along the two tubes meet at E where they superpose.
Constructive or destructive interference will occur at E depending on the path difference between the two. i.e P.D = SBE – SAE.
If a path difference is a whole number multiple of a full wave length, constructive interference will occur resulting into sound of maximum intensity.
On the other hand, when the path difference is an odd multiple of half wavelength, destructive interference occurs resulting into sound of minimum intensity.
A path difference may be varied by pulling tube B outwards or by adjusting the position of tube B.

MEASUREMENT OF VELOCITY OF SOUND IN AIR BY INTERFENCE METHOD

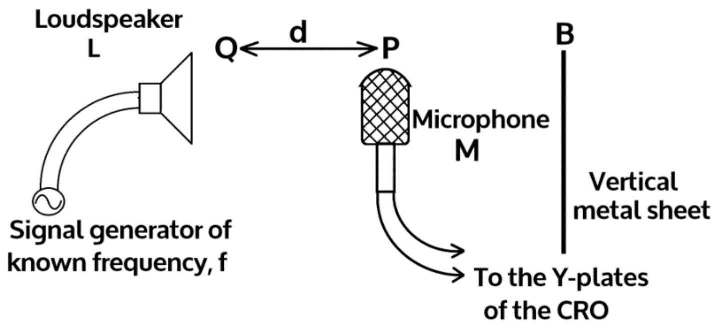


The apparatus is set up as shown above.
Tube A is fixed while tube B is free to move.
A source producing sound of known frequency, f is placed at S and then detected at E.
Tube B is adjusted so that a loud sound is detected at E.
The position X of the end of the tube B is noted.
Tube B is then pulled slowly outwards until another loud sound is detected at E. The new position Y of the end of the tube B is noted.

The distance l between X and Y is measured and recorded and it is $l = \frac{\lambda}{2}$. Thus $\lambda = 2l$, where λ is the wave length of sound.

The velocity of sound in air can then be determined from the expression, $v = f\lambda = 2fl$.

MEASUREMENT OF VELOCITY OF SOUND IN AIR BY STATIONARY WAVE METHOD



A loud speaker L, microphone, M and a vertical metal sheet B are arranged as shown above.
The loud speaker is connected to a signal generator of known frequency, f.
The microphone is connected is connected to Y–plates of a CRO with time base switched off.
The microphone is moved slowly away from B towards L until the length of the vertical trace on the screen of the CRO is maximum. The position P of the microphone is noted and the distance BP measured.

The microphone is further moved away from B towards L until the length of the vertical trace again becomes maximum. The new position Q of the microphone is noted and the distance BQ measured and recorded.

The distance between P and Q is calculated and recorded as d, i.e $d = BQ - BP = \frac{\lambda}{2}$.

$\lambda = 2d$

$v = f\lambda = f \times 2d$

$v = 2fd$

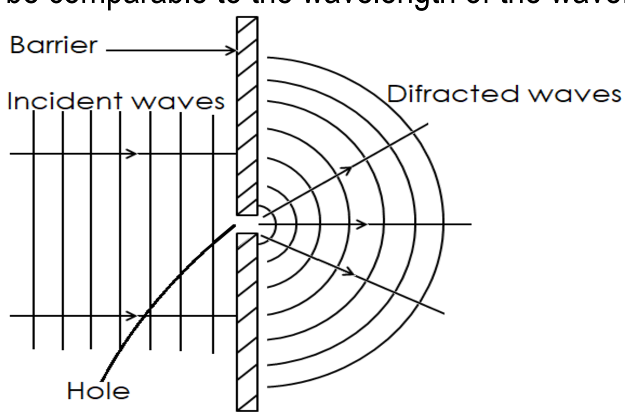
The velocity of sound is determined from the above equation.

2. DIFFRACTION

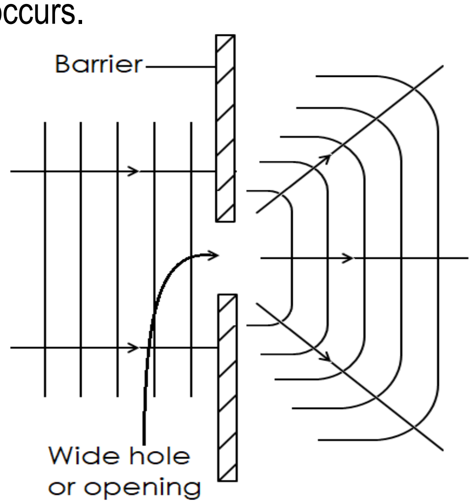
This is the spreading of a wave beyond its geometrical shadow resulting into interference.
Diffraction occurs when a wave approaches an obstacle with a narrow aperture (gap) in it.
If the dimensions of the aperture (gap) are comparable with the wave length of the wave, the wave spreads outwards at the aperture.
The wave then travels into regions where we would have instead expected its shadow to be.

CONDITIONS FOR DIFFRACTION

For diffraction to occur, the diameter or width the aperture must be of the order of the wavelength of the wave. This means that the size of the aperture must be small enough to be comparable to the wavelength of the wave.



If the aperture is much wider than the wavelength then little or no spreading of the wave occurs.



FACTORS AFFECTING THE EXTENT OF DIFFRACTION

The extent of diffraction depends on two factors namely;

(i) Dimensions (size) of the aperture (gap).
Narrow apertures give more diffraction. The wider the aperture, the less the extent of diffraction.

(ii) The wavelength of the wave.
Waves of short wavelength do not undergo diffraction so easily. The longer the wavelength, the easier it is for the wave to be diffracted and hence the greater the extent of diffraction. This explains why it is easier to hear sound around corners than it is to see light around corners.

Sound waves have long wavelength and undergo diffraction when they reach barriers with apertures e.g the space between doors and walls. This causes the wave to spread in order to reach individuals behind the corner.

On the other hand, light has very short wavelength and usually doesn't undergo diffraction on reaching such apertures because the diameters of the apertures are wider hence not of the order of the wavelength of the light.

Question: Write down the applications of diffraction of waves.

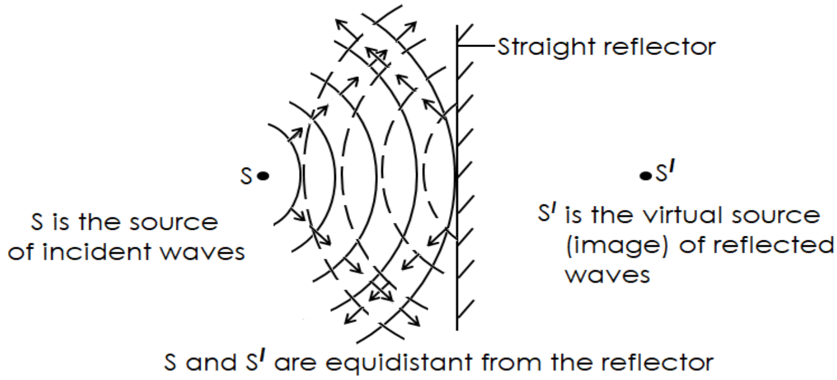
3. REFLECTION

All waves can be reflected and the laws of reflection are obeyed.
Reflection occurs when a wave reaches a barrier which acts as a reflecting surface.
The reflecting surfaces may be plane or curved and the incident wave fronts may be circular or straight/plane.

(a) REFLECTION AT PLANE SURFACES

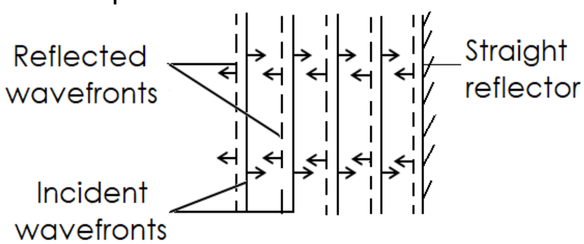
At planes surfaces, a plane incident wave front gives rise to a plane reflected wave front while a circular incident wave front is also reflected as a circular reflected wave front.
It is important to note that after reflection, the wavelength and speed of the wave front do not change.
The reflected wave front has an apparent source behind the reflecting surface whose distance from the reflecting surface is equal to the distance of the source of the incident wave front from the reflecting surface.

(i) Incident circular waves

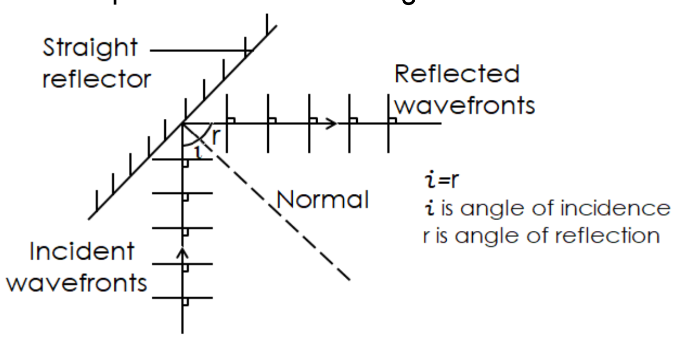


(ii) Incident straight(plane) wave

⊗ Incident plane waves normal to the reflectors



⊗ Incident plane waves at an angle of incidence



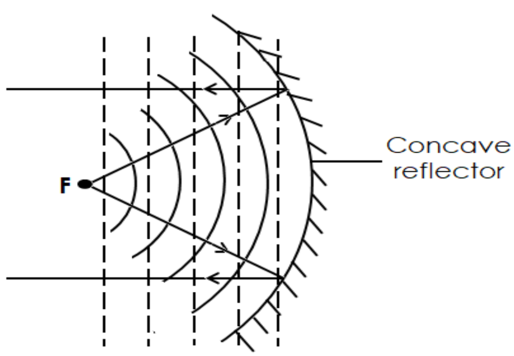
(b) REFLECTION AT CURVED SURFACES

Curved surfaces may be concave or convex.

REFLECTION OF WATER WAVES BY A CONCAVE REFLECTOR

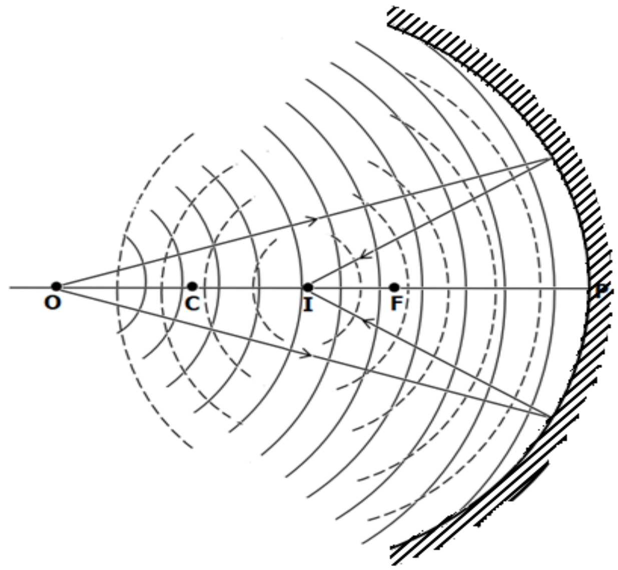
(i) Incident circular waves whose source(S) is at the principal focus(F).

Here, the reflected wave fronts are plane waves.



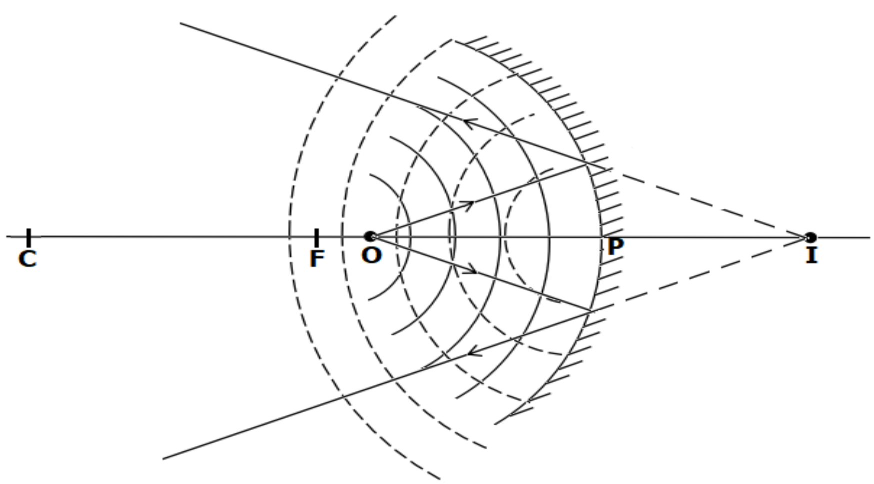
(ii) Incident circular waves whose source is at a point (O) beyond the centre of curvature(C).

In this case, the reflected waves are also circular converging to a point (I) between C and F and then diverging beyond

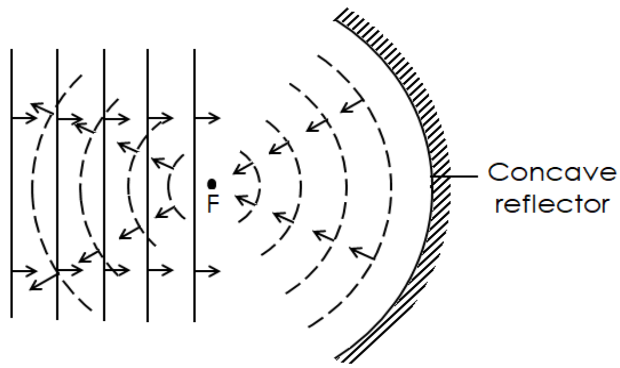


(iii) Incident circular waves whose source is at point (O) between F and P of the concave reflector.

In this case, the reflected waves are circular, diverging from a virtual source(I) behind the reflector.

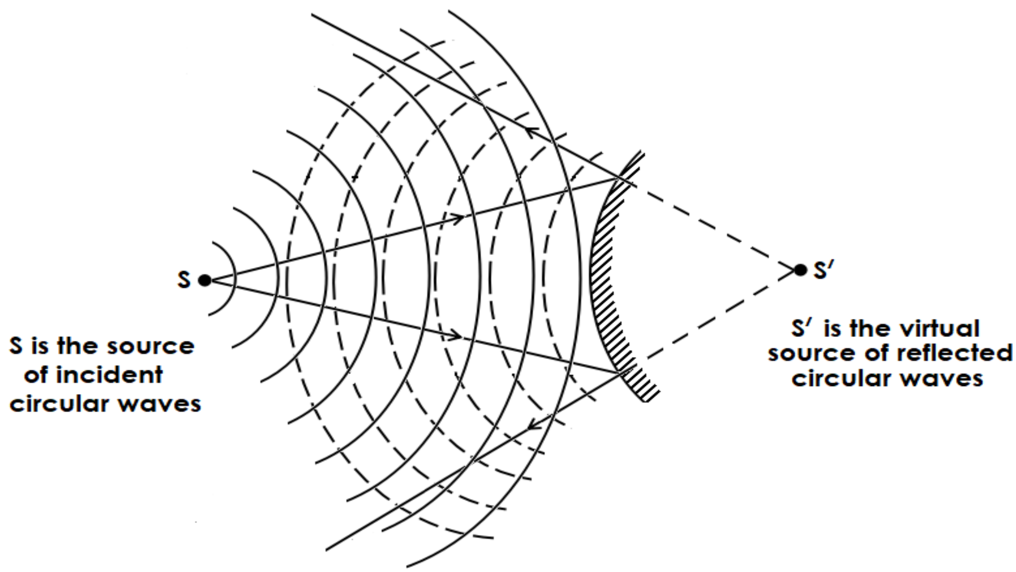


(iv) Incident plane or straight waves

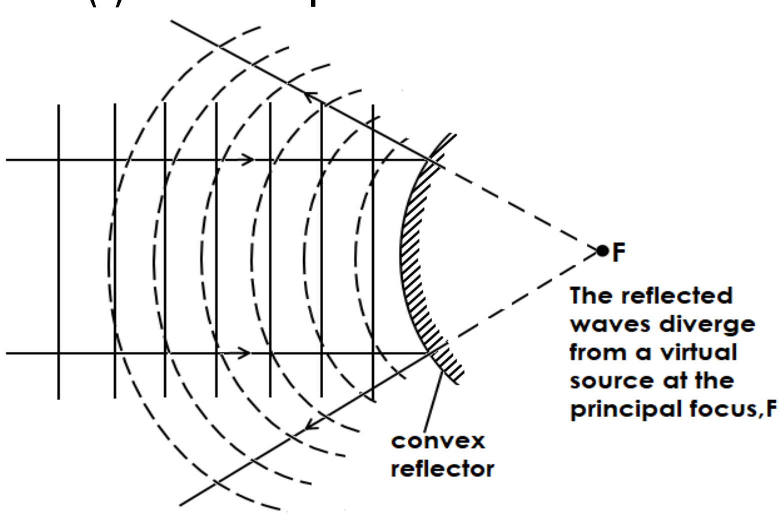


REFLECTION OF WATER WAVES FROM A CONVEX REFLECTOR

(i) Incident circular waves on a convex reflector



(ii) Incident plane waves on a convex reflector



4. REFRACTION

All waves can be refracted when they move from one medium to another of different densities.

Refraction is the change in the speed and direction of a wave when it travels from one medium to another of different density.

Refraction of wave fronts also obeys the laws of refraction.

How refraction of waves occurs

When a wave travels from a less dense medium to a denser medium, the ray which is perpendicular to the wave fronts bends towards the normal. So, the refracted wave fronts perpendicular to the refracted ray become closer together hence both the wave length and speed reduce.

On the other hand, when a wave travels from a denser medium to a less dense medium, the ray which is perpendicular to the wave fronts bends away from the normal. So, the refracted wavefronts perpendicular to the refracted ray become farther apart hence both the wave length and speed increase.

So, when a wave front travels from one medium to another of a different density;

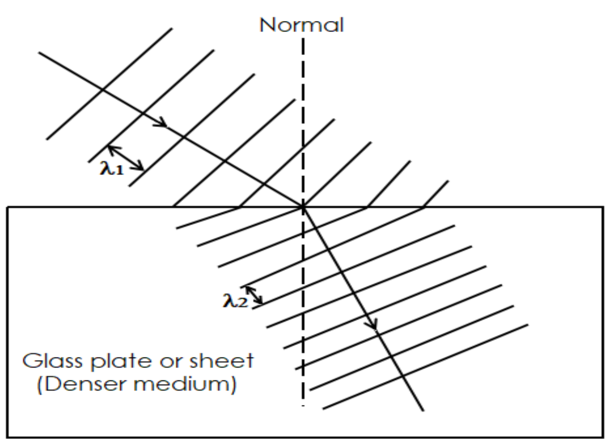
(i) The wave length of the wave changes.

The wave length increases when the wave travels from a denser medium to a less dense medium while it decreases when the wave travels into a denser medium from a less dense medium.

(ii) The frequency of the wave remains unchanged.

(iii) The speed and direction of the wave changes i.e the wave is refracted. When the wave travels from a less dense medium to a denser medium, its speed decreases as a result of the decrease in its wavelength while when it travels from a denser medium to a less dense medium, the velocity increases as a result of increase in the wavelength.

Consider a wave travelling from a less dense mediumof refractive index, n_1 to a denser mediumof refractive index,n_2 $(n_1 < n_2)$.



If the velocities of the wave in the two media are v_1 and v_2 respectively while wavelengths are λ_1 and λ_2 respectively,then$\lambda_2 < \lambda_1$ and $v_2 < v_1$.

REFRACTION OF WATER WAVES

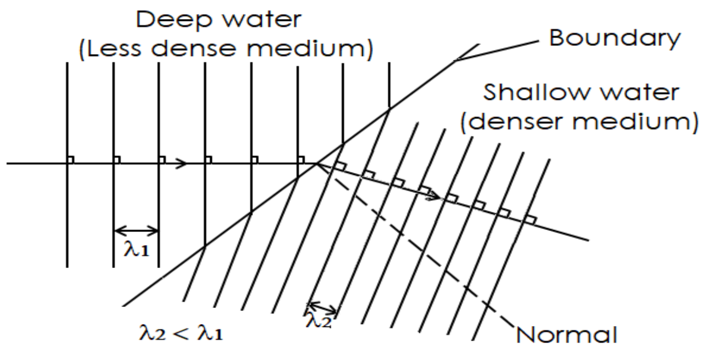
In the ripple tank, the shallow water is the denser medium and the deep water is the less dense medium.

The shallow water (denser medium) in the ripple tank is made by placing a plate/sheet of glass in water on one side of the tray.

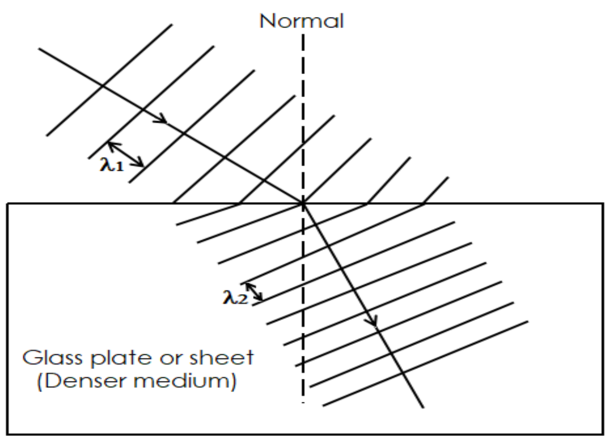
Note: Refraction of water waves in the ripple tank results into change of speed and hence wavelength changes but frequency remains the same.

(a) Refraction of waves from deep water(less dense medium) to shallow water(denser medium)

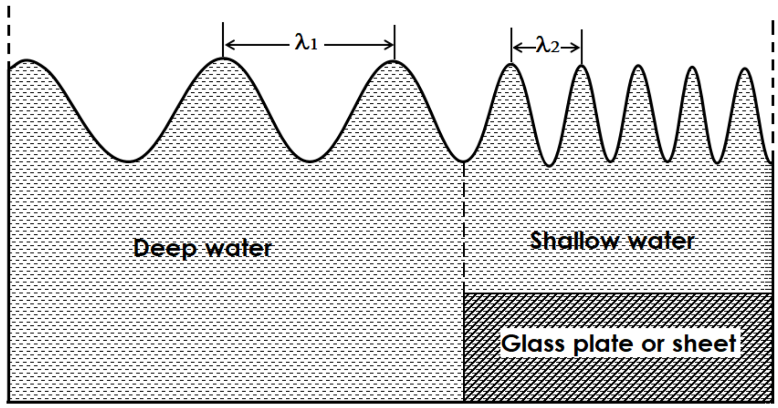
(i)



(ii)



(iii) Cross sectional view



$\lambda_1 > \lambda_2$

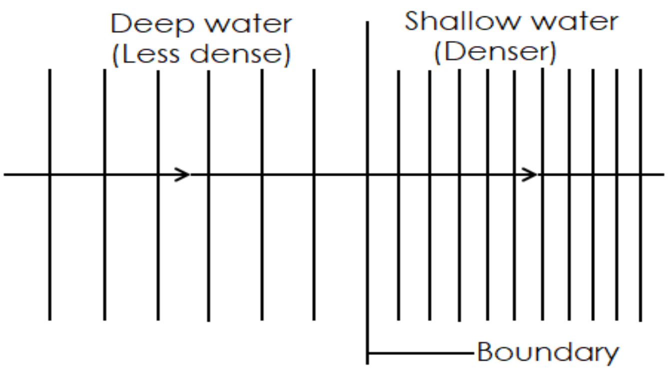
When water waves move from deep water(less dense medium) to shallow water(denser medium), the speed reduces and therefore wavelength reduces but frequency remains the same. i.e

$v_1 = \lambda_1 f$ and $v_2 = \lambda_2 f$

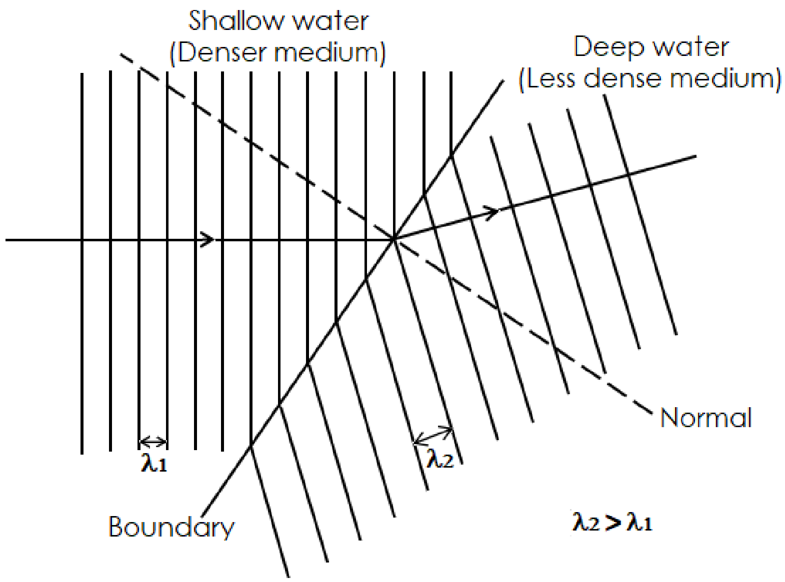
$$\frac{v_1}{v_2} = \frac{\lambda_1 f}{\lambda_2 f}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

NOTE: If the incident wave strikes the boundary normally, there will be no change in direction and frequency but the speed and wavelength will reduce when moving from less dense to a denser medium and vice versa.



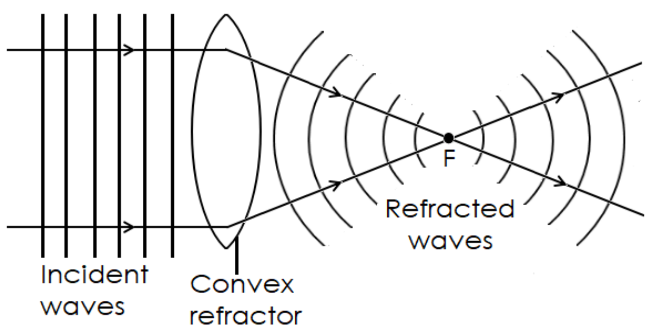
(b) Refraction of waves from shallow water(denser medium) to deep water(less dense medium)



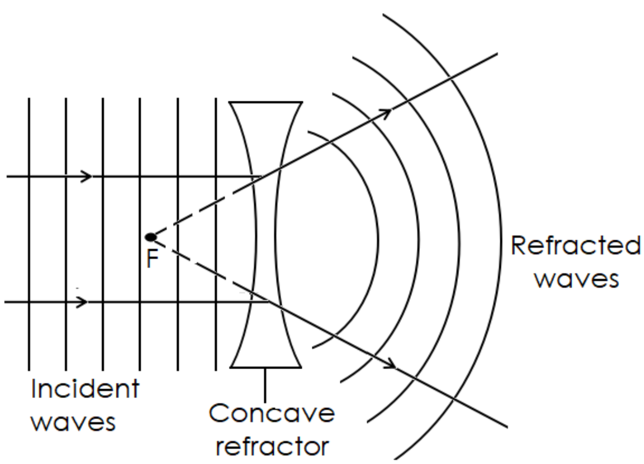
NOTE: When waves move from shallow water(denser) to deep water(less dense), the speed and wavelength increase but frequency remains the same.

REFRACTION OF STRAIGHT WAVES AT CURVED SURFACES BY:

(a) A converging refractor (convex refractor/lens)



(b) A diverging refractor (concave refractor/lens)



REFRACTION OF PLANE/STRAIGHT WAVES IN A PRISM