

**TIME: 2 HOURS 30 MINUTES**

**Instructions**

- ✓ Section A is compulsory, attempt one item in section B part I and one item in section B part II
- ✓ All presentation should be shown clearly
- ✓ Acceleration due to gravity ( $g=9.8\text{ms}^{-2}$ )
- ✓ Silent non-programmable calculators and a table with a list of formulae may be used

**SECTION A: Numerical Methods**

**Compulsory item**

**ITEM ONE**

An architect is designing a compound for a businessman. The businessman has a GPS (Global Positioning System) that shows that the area is bounded by the line  $y=x$  and the curve  $y=\cos x$  along the distances from  $x=0.5\text{km}$  to  $x=1.5\text{km}$ . The businessman is encountered with the problem of a camera to be set at the point of intersection between the line and the curve along the x-axis but would like to know the exact value of this point of intersection using a suitable method so that the error is less than 0.05.

The businessman is also encountered with knowing the approximate area covered by the curve and the line and also to check whether there is an error less than 0.03 between the approximate and the exact values so that, in case the error is minimal, he can use either the exact value or the approximate value.

The architect wants the help of a student to highlight the problem of the businessman.

**Task**

As a student of Mathematics, help the architect to give a response to the businessman that has interpretation, methodology, and judgment.

**SECTION B**

**PART I: Probability and Statistics**

(Attempt one item)

**ITEM 2**

The board of governors of a school have a problem of analysis of data represented by the performance of their students in the mock examination. The marks and their respective numbers of students in interval form are as below:

Marks	0-<15	15-<30	30-<35	35-<40	40-<55	55-<65	65-<75
No of students	6	8	15	20	10	7	4

The board of governors requested the director of studies to analyse the data using graphical and calculation approaches for them to know the median mark, average mark, and standard deviation of the data.

When this data was inserted in the computer, it produced approximately normal data. The board wanted to know the 80% confidence interval around the mean mark to be between 15 and 65 and wanted this mean mark as a representative of the probability of success for a sample of 10 students to know the probability of those who were representing at most 3 of the above students, less than not exceeding 0.3.

The director of studies consulted the mathematics students to analyze the above for the board of governors.

**Task**

As a mathematics student consulted by the Director of Studies, analyze this information for the Board of Governors.

### ITEM 3

An entrepreneur has an A-level student, he consults over analysis of his two companies: one in Arua (A) that produces sweets in the ratios of 4 red and 3 green sweets, and another in Bushenyi (B) in ratios of 7 red and 4 green in a thousand. A buyer's selection of A is twice as likely as that of B. If two sweets are randomly picked one at a time without replacement, the entrepreneur wants the student to analyze the probability that both sweets are of the same colour and that they are from A given they are of the same colour.

He would also want the student to prepare a probability distribution of green sweets in his companies, of which this distribution he wants to know its mean and standard deviation.

There are another 15 companies that follow the same trend as in A and B of the entrepreneur, and he would wish to know the probability that at least 13 of them have a probability of success as represented by company A.

*As a MATC student, help the entrepreneur to interpret, show Task him the methodology and make the judgements for him.*

### PART II: Mechanics

(Attempt one item)

### ITEM 4

A transportation and logistics company is developing an integrated system to move goods from a lakeside dock to an automated warehouse. The project involves two key stages: launching a ferry boat and testing a prototype warehouse transport system. The following analyses are required to ensure operational efficiency and safety.

A ferry boat, used to transport goods on a lake, has a mass of 500kg. It is initially stationary on a ramp of rollers that make an angle of  $15^\circ$  with the horizontal. The coefficient of kinetic friction between the boat and the rollers is 0.002.

Two dockworkers pull the boat up the ramp to launch it into the water using two ropes. The ropes are parallel to the ramp. Worker A pulls with a force of 580N, and Worker B pulls with a force of FN. The boat moves up the ramp with a constant acceleration of  $0.5\text{ms}^{-2}$

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Once in the water, the boat is untied. A single tugboat's cable is attached to the front of the ferry and exerts a force of 2500N at an angle of  $10^\circ$  upwards from the horizontal. Assume the water's resistance is negligible. It is known that the ferry moves smoothly in water if its acceleration lies in the range of  $2\text{ms}^{-2}$  to  $5\text{ms}^{-2}$ .

Simultaneously, the same company is designing a prototype for an automated warehouse system to ensure standards for a multi-million-dollar setup. A simulation of forces and motions must be done to ensure efficiency and prevent damage.

A 4kg transport cart (mass A) on a smooth floor is connected to a 1kg mobile sorting robot (pulley Q) via a cable over a fixed smooth pulley. The sorting robot itself carries a second mechanism: A light cable over its body with a 2kg package (mass B) and a 1kg package (mass C).

The lead engineer demands the following data to plan control software, stability requirements, and select cables with adequate strength:

- 1) The acceleration of A and the packages (B and C) so as to plan the control software and stability requirements
- 2) The tensions in the main cable and secondary cable so as to select the cables with adequate strength

The tensions in the main cable and secondary cable.

Task:

- a) determine the value of F
- b) Explain whether the tugboat cable will provide the smooth movement of the Ferry
- c) if you are part of the team designing the prototype, provide a complete analysis of the system to answer the Engineer's questions

#### ITEM 5

An engineer wants to construct a rectangular roof design ABCD of mass 20kg with dimensions  $AB = 12\text{m}$  and  $AD = 5\text{m}$  that corresponds to axes x and y respectively, with forces of 5N, 6N, 3N, 2N, and 13N acting along sides AB, BC, CD, DA, and AC respectively. The businessman would wish to know the single force that represents the five forces and its direction and would wish it does not exceed 17N.

The engineer is worried that the roof can fall to the ground, which is at a height of 8 m from ground level, and does not want the design to have a final velocity greater than  $13\text{ms}^{-1}$  on falling on the ground. It moves with a force of 20N on a rough horizontal plane with the coefficient of friction as  $1/5$  as

it comes to rest. The wall is 50m from the point of dropping and he would wish to know the safety of the wall.

**Task**

As a mathematics student, explain and justify the problem to the engineer.

**END**

