

UNEB UACE PURE MATHS 2018

$$P(X=x) = \begin{cases} kx, & x = 1, 2, 3, 4, 5. \\ 0, & \text{Otherwise,} \end{cases}$$

where k is a constant.

Determine; the value of k .

- $P(2 < X < 5)$
- Expectation, $E(X)$
- Variance, $\text{Var}(X)$.

10. A particle of mass 3 kg is acted upon by a force $F = 6i - 36t^2j + 54tk$ Newtons at time t . At time $t = 0$, the particle is at the point with a position vector $i - 5j - k$ and its velocity is $3i + 3jm/s$. Determine the

- position vector of the particle at time $t = 1$ second.
- distance of the particle from the origin at time $t = 1$ second.

11. A student used the trapezium rule with five sub-intervals to estimate $\int_2^3 \frac{x}{(x^2-3)} dx$ correct to three decimal places.

Determine;

- the value the student obtained.
- the actual value of the integral
- the error the student made in the estimate
 - how the student can reduce the error.

12. The times taken for 55 students to have their lunch to the nearest minute are given in the table below.

Time (minutes)	3 - 4	5-9	10-19	20-29	30-44
Number of students	2	7	16	21	9

- Calculate the mean time for the students to have lunch.
- Draw a histogram for the given data
 - Use your histogram to estimate the modal time for the students to have lunch.

A non-uniform rod AB of mass 10kg has its centre of gravity at a distance $\frac{1}{4}AB$ from B . The rod is smoothly hinged at A . It is maintained in equilibrium at 60° above the horizontal by a light inextensible string tied at B and at a right angle to AB . Calculate the magnitude and direction of the reaction at A .

14. By plotting graphs of $y = x$ and $y = 4 \sin x$ on the same axes, show that the root of the equation $x - 4 \sin x = 0$ lies between 2 and 3.

Hence use Newton Raphson's method to find the root of the equation correct to 3 decimal places.

15. The number of cows owned by residents in a village is assumed to be normally distributed. 15% of the residents have less than 60 cows. 5% of the residents have over 90 cows.

- Determine the values of the mean and standard deviation of the cows.
- If there are 200 residents, find how many have more than 80 cows.

16. At 12 noon a ship A is moving with constant velocity of 20.4 kmh^{-1} in the direction $N\theta^\circ E$, where $\tan \theta = \frac{1}{5}$.

A second ship B is 15km due north of A . Ship B is moving with constant velocity of 5 kmh^{-1} in the direction $S\theta^\circ W$, where $\tan \theta = \frac{3}{4}$. If the shortest distance between the ships is 4.2km, find the time to the nearest minute when the distance between the ships is shortest.

END

PAPER 1 SECTION A

1. In triangle ABC , $a = 7 \text{ cm}$, $b = 4 \text{ cm}$ and $c = 5 \text{ cm}$. Find the value of:

- $\cos A$
- $\sin A$

$$\frac{x+4}{8} = \frac{y-2}{2} = \frac{z+1}{-4}$$

2. Determine the angle between the line $4x + 3y - 3z + 1 = 0$ and the plane

3. Find $\int x^2 e^x dx$

4. Express the function $y = x^2 + 12x + 32$, in the form $a(x+b)^2 + c$. Hence find the minimum value of the function $f(x)$.

5. A point P moves such that its distances from two points $A(-2, 0)$ and $B(8, 6)$ are in the ratio $AP : PB = 3 : 2$. Show that the locus of P is a circle.

6. Determine the equation of the tangent to the curve $y^3 + y^2 - x^4 = 1$ at the point $(1, 1)$.

7. Show that $2 \log 4 + \frac{1}{2} \log 25 - \log 20 = 2 \log 2$

8. The region bounded by the curve $y = x^2 - 2x$ and the x -axis from $x = 0$ to $x = 2$, is rotated about the x -axis. Calculate the volume of the solid formed.

SECTION B

Answer any five questions from this section.

9. The position vectors of the vertices of a triangle are \mathbf{a} , \mathbf{r} and \mathbf{s} , where \mathbf{O} is the origin. Show that its area (A) is given by

$$4A^2 = |\mathbf{r}|^2 |\mathbf{s}|^2 - (\mathbf{r} \cdot \mathbf{s})^2 \quad \text{Hence, find the area of a triangle when } \mathbf{r} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ and } \mathbf{s} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

10. Express $5 + 12i$ in polar form. Hence, evaluate $\sqrt[3]{(5 + 12i)}$, giving your answers in the form $a + ib$ where a and b are corrected to two decimal places.

11. a) Differentiate $\frac{x^5}{\sqrt{(1-2x^2)}}$ with respect to x .

b) The period, T of a swing of a simple pendulum of length, l is given by the equation

$$T^2 = \frac{4\pi^2 l}{g}$$

where g is the acceleration due to gravity. An error of 2% is made in measuring the length, l . Determine the resulting percentage error in the period, T .

$$\tan 4\theta = \frac{4t(1-t^2)}{t^4 - 6t^2 + 1}, \text{ where } t = \tan \theta.$$

12. a) Show that

b) Solve the equation

$$\sin x + \sin 5x = \sin 2x + \sin 4x \text{ for } 0^\circ < x < 90^\circ$$

UNEB UACE APPLIED MATHS 2018

13. a) The first three terms of a Geometric Progression (G.P) are 4,8 and 16. Determine the sum of the first ten terms of the G.P
 b) An arithmetic Progression (A.P) has a common difference of 3. A Geometric Progression (G.P) has a common ratio of 2. A sequence is formed by subtracting the terms of the A.P from the corresponding terms of the G.P. The third term of the sequence is 4. The sixth term of the sequence is 79. Find the first term of the
 i) A.P
 ii) G.P

14. Evaluate:

a) $\int_0^{\frac{\pi}{2}} \sin 5x \cos 3x \, dx$

b) $\int_0^{\sqrt{5}} \frac{dx}{9+4x^2}$

15. The line $y = mx + c$ is a tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

- a) Obtain an expression for c in terms of a, b, and m
 b) Calculate the gradients of the tangents to the ellipse through the

$$\left(\sqrt{(a^2 + b^2)}, 0\right)$$

16. The rate at which the temperature of a body falls is proportional to the difference between the temperature of the body and that of its surrounding. The temperature of the body is initially 60°C. After 15minutes the temperature of the body is 50°C. The temperature of the surrounding is 10°C.
 a) Form a differential equation for the temperature of the body.
 b) Determine the time it takes for the temperature of the body to reach 30°C.

SECTION A

1. A stone is thrown vertically upwards with a velocity of 21ms⁻¹
 Calculate the:
 a) maximum height attained by the stone.
 b) time the stone takes to reach the maximum height.

2. Two events A and B are such that

$$P(A/B) = 2/5, P(B) = 1/4 \text{ and } P(A) = 1/5$$

Find:

- a) P(A#B)
 b) P(A#B)

3. The table below shows how T varies with S

T	-2.9	-0.1	2.9	3.1
S	30	20	12	9

Use linear interpolation/ extrapolation to estimate the value of

- a) T when S = 26
 b) S when T =3.4

4. A particle of mass 15kg is pulled up a smooth slope by a light inextensible string parallel to the slope. The slope is 10.5m long and inclined at $\sin^{-1}(\frac{4}{7})$ to the horizontal. The acceleration of the particle is 0.98ms⁻².
 Determine the:
 a) tension in the string
 b) work done against gravity when the particle reaches the end of the slope.

5. The price index of an article in 2000 based on 1998 was 130. The price index for the article in 2005 based on 2000 was 80.
 Calculate the:

- a) price index of the article in 2005 based on 1998.
 b) price of the article in 1998 if the price of the article was 45,000 in 2005.

6. Two numbers A and B have maximum possible errors e_a and e_b respectively.

- a) Write an expression for the maximum possible error in their sum
 b) If A = 2.03 and B = 1.547, find the maximum possible error in A+B.