P425/1

ARUA SS MATH

PURE MATHEMATICS

Paper 1

3 Hours

UGANDA ADVANCED CERTIFICATE OF EDUCATION

SENIOR FIVE END OF TERM TWO ASSESSMENT

PURE MATHEMATICS

TIME: 3 HOURS

Instructions

- This paper consists of two sections A and B
- Section A has five compulsory items
- Section B has two parts (part I and part II). Choose two items from each part in section B

SECTION A

ITEM 1

Mr James a prominent trader in **Arua**, wants to invest in a new venture. He has narrowed his option down to two.

- **Venture A:** exporting vanilla beans

  He's to rely on key logarithmic identity that $a^{\log b} = b^{\log a}\ ; a, b > 0$

  He prospects that the time, **T (in weeks)** required to achieve his profit target, using this venture, satisfies the equation; $3^{\log T} + 3\ X\ T^{\log 3} = 36$

  He targets to achieve a profit worth **UGX 850,000** in this venture in a period of **T** weeks

- **Venture B:** importing textiles

  The initial capital investment, **I** (in thousands of UGX), required for this venture satisfies;

$$I = 50\ X\ (\frac{12}{3 - \sqrt{5}} - 3\sqrt{5})$$

The projected profit, P (in thousands of UGX) for this venture (B) after 10 – w… period satisfies the expression

$$\sqrt{p+10} = 2x(y-\sqrt{3}), where; y = \sqrt{28+\sqrt{300}}, y > 0$$

To make a most profitable choice, he seeks for your help.

TASK

Help **Mr. James**

a) Prove his logarithmic identity

b) Determine the number of weeks he is to operate **venture A**

c) Determine the capital required for **venture B (in UGX)**

d) Determine the profit He's likely to obtain when he opts for **Venture B (in UGX)**

e) Identify the venture to go for, basing on weekly profit, justify your answer.

ITEM 2

A local community in a rural area has a large water storage tank that supplies water to household through two distribution pipes. The rate at which water flows out of the tank through these pipes depends on how full the tank is. After conducting measurements, the engineers model the rate of water flow, **R(X)** with respect to time using a rational function;

$$R(x) = \frac{5x+3}{(x-1)(x-3)^2}$$

Where R(x) is the rate of water flow (in litres per minute) and x is the number of hours after the start of the day.

The engineer must first decompose it into partial fractions before they determine the volume of the tank.

TASK

By expressing the function as fractions, help the engineer to simplify the integration process by expressing it into partial fraction.

ITEM 3

A city planner in Kampala is studying the age distribution of the population to design better housing, healthcare and educational facilities. The table below shows the population (in

lions) of Kampala across different age groups based on the latest census data collected in ug, 2025.

Age Group	Population in millions
Below 10	2
10 and under 20	8
20 and under 30	10
30 and under 40	14
40 and under 50	10
50 and under 70	5
70 and under 90	1

Draw a histogram to represent the data above.

ITEM 4

A deadly virus attacked a certain country. The initial number of male and female people infected is 14 and 35 respectively. However, this initial number of sick people is thought to be increasing at a rate 30% and 69% per week. As the minister of health was addressing a live presentation on social media, one individual asked through the comment section "What will be the probability of finding a woman contracted by the virus over thousand patients?

TASK.

Kindly show how you as the minister would respond to the question.

ITEM 5

A particle of mass 2kg is acted upon by two forces, F1=3N acting due east and F2=4N acting due north.

a) Calculate the magnitude of the resultant force acting on the particle.
b) Determine the direction of the resultant force relative to the east direction.

SECTION B

PART I

(Attempt any two items)

ITEM 6

A packaging company tracks its daily profits, $f(x)$ for the number of goods, x (in hundreds of kgs) sold in a day.

TASK

The profit function is given by $f(x) = x^4 - 5x^3 + 8x^2 - 4x$

a) Given that a company sells some goods in a day, Determine the number of goods a company can sell to break even at the end of the day

b) Let $\sqrt{\alpha}$ *and* $\sqrt{\beta}$ be the roots of the polynomial; $g(x) = x^2 - 4x + 3$.
The company models the cost function $C(x)$by multiplying $h(x)$ with the linear function, $(x + 2)$, where $h(x)$ is the quadratic equation, whose roots are α^2 *and* β^2
Obtain the simplest expression of $C(x)$

c) Solve the inequality; $\frac{C(x)}{(4-x^2)} \leq 0$

ITEM 7

Ugandan sprinter, Dorcus Ajok is practicing on a curved path. The path of her sprint follows the implicit equation;

$$3x^2 + 2xy + y^2 - 5x + 3y = 6$$

Where x and y measured in meters

TASK

a) Find the gradient function of Ajok's path

b) At the point **(2, -1)** on the track, Determine the equation of the tangent line to her path. Interpret its meaning in the context of her sprint

c) If Ajok stops abruptly at **(2, -1)** and run perpendicularly to her path, Determine the equation of her path

EM 8

Mukasa, a local carpenter in Kampala, has a large rectangular wooden board with dimensions; $(6\sqrt{32})m\ x\ (4\sqrt{8})m$. He wants to use this board to make either rectangular tables or multiple square stools, with the following specifications.

Rectangular table

- Tabletop dimensions: $(5\sqrt{2})m\ x\ (2\sqrt{2})m$.
- Legs and supports: Requires an extra $\mathbf{20}\ \boldsymbol{m^2}$ of the wood per table

Square stool:

- Side length: $(\sqrt{2})m$
- Legs and supports: Requires an extra $\mathbf{4}\ \boldsymbol{m^2}$ of wood per table

Market prices

- 01 table sells for $\boldsymbol{UGX.120,000}$
- 01 stool sells for $\boldsymbol{UGX.15,000}$

Production costs

- Cost of wood per m^2: $\boldsymbol{UGX.500}$
- Labour cost per table: $\boldsymbol{UGX.50,000}$
- Labour cost per stool: $\boldsymbol{UGX.5,000}$

TASK

a) Calculate the total area of the wooden board
b) Determine the total wood needed to make one table (including legs and supports)
c) Find the maximum and minimum number of stools Mukasa can make from the same board
d) Calculate the profit Mukasa will accumulate if he decides to produce
   i. Tables
   ii. Square stools
e) Which one should He produce? And why?

Part II

(Attempt any two items)

adve
Number
sold (y)

TASK.

ITEM 9

In a such village of Arua District, community leaders are preparing for a major cultural festival by forming a youth choir. A total of **5 boys** and 7 **girls** have expressed interest in joining. However, due to space and budget constraints, the choir can contain **only include 6 members.**

To promote fair gender representation, the organizers are considering **four** possible selection options;

a) A choir made up of exactly **3** boys and **3** girls or
b) Choir with at least **4** girls or
c) Choir with at most **2** boys or
d) The majority of the members being girls.

They are unsure how many different ways each option can be formed. To make the selection process easier and more organized, they have decided to go with the option that has the **fewest combinations** so as to avoid confusion and potential conflict.

TASK

Help the organizers by

i) Calculating the number of possible ways, the choir can be formed under each option.
ii) Determining which option should be selected based on the least number of combinations.

ITEM 10

You recently got a job as a senior marketing manager at JMT publishers company limited. The induction and public relations team informed you about the major role of boosting sales in the company and increasing on the online visibility of the company. The company executive director gave you data about the number of books sold and corresponding Television and radio adverts made in the year 2024. The data is summarized in the table below.

Number of adverts(x)	45	56	65	35	72	90	60	79	90	34	56	65
Number of books sold (y)	55	60	75	42	88	98	72	86	88	41	64	78

TASK.

a) As a new marketing manager, advise the top management about the correlation between the adverts and the sales.

b) Represent the data on a relevant graph and establish the relationship between the adverts and the sales of the company. If your target is to sale 120 books, how many adverts should the company make? What about if only 20 adverts were made, how many books are likely to be sold?

c) The quality assurance team would wish to understand the significance of advertising at 10% level, using your expertise, guide the team. (15scores)

ITEM 11

The government of Uganda is analysing crime rates across 5 districts in West Nile region to allocate police resources effectively. Below is the number of reported crime cases in each district over the past year.

DISTRICTS	CRIME RATES
Arua	120
Pakwach	85
Yumbe	150
Nebbi	95
Moyo	50

TASK

Using descriptive statistics, recommend how the government could allocate the resources effectively.