SECTION A

ITEM 1:

Government is introducing a new project of constructing parish schools but requires that every parish should have the land in rectangular form with dimensions exceeding 120 m by 90 m.

A certain parish in one community of Namisindwa district gazetted a rectangular piece of land whose area is 15,000 m^2 with a perimeter of 500 m. However, the community is not sure whether they can also benefit from the government project and the government is taking records of those parishes ready to provide dimensions in order to be sure of the boundaries for purposes of surveying.

Task:

Help the Community to know whether they can also benefit from the government project.

ITEM 2:

A certain producer wishes to produce up to a certain number of units that can enable him achieve the maximum profit. He employed two people to help him with the model which relates the profit and the amount produced. The first model (model-1) was $p(x) = x(500 - x) - 60{,}000$, where x is the units produced and $p(x)$ is the profit in US Dollars from x units.

The second model (model-2) was given by $20{,}000 - (x - 200)^2 = 0$. This model assumes two arbitrary solutions α and β. However, this model requires improvement to another one but still in quadratic form such that the new arbitrary solutions are $\frac{\beta}{\alpha-1}$ and $\frac{\alpha}{\beta-1}$ in order to use.

The producer wants to know maximum profit using the first model (model-1) and the correct improved second model (model-2) in form of a quadratic equation.

Task:

Help the producer to:

(a). express the profit model-1 in the form $a(x + b)^2 + c$ and hence determine the maximum profit and units needed to be produced.

(b). form, without solving for α and β, the new model-2 equation such that $\frac{\beta}{\alpha-1}$ and $\frac{\alpha}{\beta-1}$ are the only two solutions to enable further analysis.

ITEM 3:

A certain parent knows that saving his pin-code with $\left(\frac{\sqrt{2}}{1+\sqrt{2}}\right)\log_4 x^2$ containing one of the digits of the pin-code as $\boldsymbol{x}$ is more secure because the children have access to his phone. His correct pin-code in other words is

$\mathbf{23xx5}$ where is the value got by solving the logarithmic equation $\left(\frac{\sqrt{2}}{1+\sqrt{2}}\right)\log_4 x^2 = 2-\sqrt{2}$.
One day, the parent sent his son with the above details so that he can find out the correct pin and withdraw some money but the boy failed to discover the correct pin.

Task:

As a mathematics student, help the boy to come up with the correct pin-code in order to complete the task of the parent.

ITEM 4:

A company that participates in international trade is planning to import three types of goods from a certain foreign country. The goods are A, B and C. The Company knows that with US Dollars \$3,500, all the three types of goods can be imported. With \$5,800, it can import 2, 1 and 2 good(s) of type A, B and C altogether respectively; and with \$6,700, it can import 3, 2 and 1 good(s) of type A, B and C altogether respectively. The company is not sure of how much in Uganda shillings, it takes each good to be imported.

Hint: $\$1 = \text{Ugshs. } 3{,}900$.

Task:

Help the company to determine the amount of money needed to import each of the goods. (Leave your answer in Ugshs.).

ITEM 5:

In a physics experiment, the voltage, V across a certain non-linear resistor is modeled by the equation: $V = 5\left(\frac{\sqrt{8x}}{2^x}\right)$ where x is the time in seconds.

Task:

(a). Simplify the expression for V in terms of a **single power of 2** and a **surd**, using the laws of indices and surds.

(b). Given that $V = 5$, show that $\log_2 x = 2x - 3$.

(c). Hence, or otherwise, determine the value of $\log_2 \sqrt{8x}$ at this time

ITEM 6:

A bakery sells three types of pastry boxes, Small, Medium and Large. Each box contains a different number of croissants, muffins, and cookies.

- A small box contains: 2 croissants, 1 muffin, 3 cookies
- A medium box contains: 3 croissants, 2 muffins, 2 cookies
- A large box contains: 4 croissants, 3 muffins, 5 cookies.

One morning the bakery sold: 26 croissants, 17 muffins and 27 cookies.

Task:

Determine the number of small boxes, medium boxes and large boxes sold that day.

ITEM 7:
A company models its profit, $\boldsymbol{P}$ (in thousands of dollars) as a quadratic function of time $\boldsymbol{t}$ (in years since the product launch), given by $P(t) = -2t^2 + 10t - 8$. Let the roots of the equation $P(t) = 0$ be α and β.
Task:
(i). Interpret what the roots α and β represent in the context of the company's profit.
(ii). Calculate the roots of the equation.
(iii). Find the time at which the company achieves maximum profit.
(iv). Determine the maximum profit and explain how it relates to α and β.

ITEM 8:
Jude has just been recruited as a cashier at a busy shopping mall. He must use a computerised shelf to access change from the shelf. The shelf is designed in a way that he inputs a four-digit pascode $XYZT$ in that order, on his PC in order for the shelf to open. The manager gives him a hint on how to get started "that he must determine the remainder $Xp^2 + Yp + Z$ when a polynomial $f(p) = p^5 + 2p^4 - 16p^3 + p^2 + 19p + 7$ is divided by $p^3 + 2p^2 - 15p$ and T is the positive root of the equation $p^3 + 2p^2 - 15p = 0, p \neq 0$."
Task:
As a student of mathematics,
(a). help Jude get access to the shelf.
(b). remainder when $f(p) = p^5 + 2p^4 - 16p^3 + p^2 + 19p + 7$ is divided by $p - 4$.

ITEM 9:
A construction company is building a ramp to make a building accessible. Two ramps are used to move from one floor to the next floor. The ramps are constructed in such a way that no gap is left between them and are joined by a square horizontal slab at their ends. Each ramp rises vertically along the width of the building and at an inclination, $\boldsymbol{x}$, to the horizontal and has a width of 1 m. The company uses a function, $f(x)$, to determine the angle of inclination of the ramp to the horizontal. The function, $f(x)$, is equal to sum of eight times the cosine of the angle of inclination and fifteen times the sine of angle of inclination.
Task:
Determine:

(i). angle of inclination of the ramp to the horizontal if it is equal to a tenth of the angle for which $f(x)$ is a maximum.
(ii). the height between two adjacent floors if it's equal to the a fifth of the maximum value of $f(x)$.
(iii). the length of each ramp.
(iv). the dimensions of the slab used to join the ramps.
(v). the width of the room (assuming the thickness of the walls is negligible).
(vi). values of x in the range $0° \leq x \leq 360°$, for which $f(x)$ is zero.

ITEM 10:
A health practitioner wants to study about the growth of certain bacteria and find out how their numbers vary with time. A colony of the bacteria is grown in a dish starting from 3:00 am in the laboratory and it is found out that their population doubles each hour that passes. He managed to get the machine that counts their numbers at 9:00 a.m, when he realized that at that time, the population was one million.

Task:
Help the health practitioner to determine the:
(a). population of the bacteria at:
  (i). 12:00 noon.
  (ii). 11:30 a.m.
  (iii). 7:00 p.m.
(b). population at:
  (i). the start of the growth.
  (ii). 8:15 a.m.
(c). time at which the population was:
  (i). 32,000,000.
  (ii). about 353553.

ITEM 11:
On his return from Japan, Moses came with a special and beautiful design of a rectangular foundation of a building known as *golden design* using a special ratio known as the *golden ratio*.

The golden ratio = Perimeter divided by twice the length

He wishes to design his rectangular foundation of width 24 m using this design and is to cover it with smaller golden rectangular tiles each of width of $\frac{\sqrt{12}}{4}$ m. He never got the ratio, but he was given a sample golden rectangle of dimensions $\left(\frac{\sqrt{5}+1}{2}\right)$ cm by 1 cm to calculate the ratio.

Task:
As an architect help Moses to determine the

(a). Golden ratio in its simplest form.
(b). Exact length of:
  (i). the foundation in the simplest form.
  (ii). each tile in the simplest form.
(c). Number of tiles to be used to cover the foundation

ITEM 12:
A wheel chat ramp is to be constructed on a storeyed building. Mrs. Dibya, the owner of the building, wishes that the outer view of one side of the ramp is a right-angled triangle of base $\boldsymbol{x}$ metres, length $\boldsymbol{y}$ metres and area $\mathbf{84\ m^2}$ and of angle of inclination $\boldsymbol{\theta}$ for safety.
Task:
Help Mrs. Dibya to:
(a). Prove that if $x = 18(\sec\theta + \tan\theta)$ and $y = \frac{2\pi}{3}(\sec\theta - \tan\theta)$, then the area required for the side view of the ramp will he achieved.
(b). Determine the base and beight of the triangular side view, if $x + y = 31$.
(c). Determine the length and angle of the ramp.

ITEM 13:
In a certain chemical processing company, Product X is formed by combining reactions P, Q and R according to the reaction equation below $pP + qQ + rR = X$, where p, q and r are different ratio that vary with time. The company produces product X in shifts morning, evening and night shifts. The table below shows how different shifts, combined reactants to produce P, Q and R to produce X on Friday

	RATIOS USED			
SHIFT	**P**	**Q**	**R**	**X**
MORNING	**2**	**3**	**1**	**10**
EVENING	**1**	**2**	**3**	**13**
NIGHT	**3**	**1**	**2**	**13**

Your Aunt is working with the company and would like to know the quantities of each reactants used and she has approached you.
TASK:
As a student of mathematics use your knowledge to help your aunt to know the amount of P, Q and R used on Friday.

ITEM 14:
The Uganda National Roads Authority (UNRA) is studying traffic patterns on the Kampala- jinja highway. They have developed a mathematical model to predict traffic density at different times of the day. The traffic density function $D(x)$ at time x hours is given by:

$$D(x) = \frac{x^3 - 8x^2 - 1}{(x+3)(x-2)(x^2+1)}$$

TASK:
As a traffic engineer, you need to analyze this model and present the function as a sum of partial fractions.

ITEM 15:
The Kampala city council is planning to construct two new roads in the city to ease traffic congestion. Before construction begins, engineers need to determine the exact alignment of these roads to ensure smooth traffic flow and proper connectivity. The first proposed road will pass through the coordinate points of kasubi $(10, 35)$ and Kireka $(20, 15)$ while the second road will pass through the points of Banda $(12, 4)$ and kyambogo $(6, 24)$ on a map of Kampala. To ensure roads are well positioned and don't create a sharp turn or difficult intersections engineers must calculate the inclination angle between them. But little do they know, the Bodaboda riders are already planning their shortcuts.
TASK:
Compute the road equations, point of intersection if so and the inclination angle to ensure a safe and efficient road network

ITEM 16:
A renewable energy company is installing solar panels on a building in Tororo district. To maximize energy collection, they need to determine optimal installation angles based on the buildings orientations and suns positions.
The company engineers found out that the solar panels had the following equations
Panel A: $\frac{1+\sin\theta}{1-\sin\theta} = (\sec\theta + \tan\theta)^2$.

Panel B: $\sec\theta + \operatorname{cosec}\theta\cot\theta = \sec\theta\operatorname{cosec}^2\theta$.

Panel C: $\frac{\cot\alpha+\tan\beta}{\cot\beta+\tan\alpha} = \cot\alpha\tan\beta$
TASK:

As a mathematics student prove that the above equations for each panel is related.

ITEM 17:
A bus heading to Masaka passes through Mpigi town. At time, t hours, past noon, the distance of the bus from Mpigi to Masaka is expressed as $x = 7\sin t - 4\cos^2 t + 2$.
Hint:
It's known that 1 degree is $\frac{\pi}{180}$ rad.
Task:
Help:
(a). The manager know when the bus was in Mpigi give answer to **2 d.p**
(b). Identify the time the bus will be behind Mpigi by a distance of **5 km**.

ITEM 18:
The chairperson of ward ***B*** who is suffering from blood pressure is given a sleeping pill. This has caused an initial level of **4 mg** of a drug per litre of his blood. After hours, the number of mg per littre in the blood is $\boldsymbol{N}$, where $\boldsymbol{N = 4(0.76)^t}$.
Task:
(a). Help the doctor to estimate the number of mg per litre after **4 hours**. Answer to **1 d.p**
(b). The doctor also wants to know how long it will take the amount of the drug per litre of blood reduce to half its initial level.
(c). If the mg per litre of blood increases to thrice the initial amount, the patient has to be put on drip, advice the doctor after how long can this happen.

ITEM 19:
Peter has applied for a job in a construction company and he has been selected among those to sit an interview. Among the areas to be considered is how good ones knowledge of trigonometry is. Peter has tried the task below and failed.
Given that $\tan\theta + \sin\theta = x$ and $\tan\theta - \sin\theta = y$ then $(x^2 - y^2)^2 = 16xy$.
Task:
As a student who has done trigonometry help Peter prepare for the interview.

ITEM 20:
MR Tomsange is a farmer and his son has joined A'Level this year offering Mathematic, Economics and Geography. The Mathematics gave homework and whoever doesn't present the work with collect solution is given a

punishment and his son has poor health. Tomsange is very worried and he has contacted you to help his son solve the task below.

(a). If $\log_5 x = p$ and $\log_{15} x = q$, then $\frac{p+q}{p-q} = \log_3 75$.

(b). Without using calculators find the value of $\frac{\log(0.8)-\log 32+\log 8}{\log(0.7)+\log 7-\log 49}$ given that $\log 2 = 0.30103$.

Task:

Guide the son on how to reach to the solution to the above task so the work and avoids being punished.

ITEM 21:

In January, Monkey See furniture dealers sold off fifty furniture pieces and the selling price of each piece was recorded in millions of shillings as shown below:

01	07	30	28	13	16	29	25	15	24
30	13	11	22	21	12	02	08	26	18
17	12	18	20	14	15	19	16	03	17
24	27	13	10	11	24	16	12	18	13
08	18	23	28	14	04	08	14	30	14

The sales manager wants:

(i). the data to be presented in a frequency table of equal intervals of shs. 5,000,000 starting from shs. 1,000,000, for easy analysis

(ii). to determine graphically the price that was most frequent so as to trade more in that type of furniture in February

(iii). to determine the price of furniture which were sold on average

Task:

Help the sales manager to:

(a). represent the data on a frequency table.

(b). determine the most frequent furniture price using a suitable graph.

(c). determine the average price of furniture sold.

(d). determine the number of furniture pieces whose price is above shs. 1,900,000

ITEM 22:

In order to improve on the performance of mathematics, the head of department laid down strategies, which included a test that was administered and the marks were distributed as shown in the table below:

Marks	< 20	< 26	< 35	< 45	< 55	< 60	< 68	< 80	< 95
Cumulative number of learner	0	6	24	66	110	130	154	174	180

Other strategies included:

- A reward from the administration to be given to a leamer who scores not less than 85 marks
- Remedial lessons to be organized for those who score below average.

Those to get a distinction were limited to only 16 learners.
The performance is inconsistent if the measure of dispersion of all the marks away from the average mark is more than 10 marks.

Task:

(a). Help the Head of Department to determine:
  (i). the number of learners who are to be rewarded
  (ii). the number of learners who are to attend the remedial classes.
  (iii). the minimum mark of a distinction.

(b). Were the learners consistent in their performance?

ITEM 23:

Amy's heavy luggage which is at point, O, is under the action of four forces 1 N, 6 N, $2\sqrt{3}$ N and 3 N acting in the directions given by the bearings 030°, 270°, 060° and 090° respectively. She wants the luggage to be moved in the bearing 120° at a constant rate provided by the net force of magnitude $2\sqrt{3}$ N but the four forces could not achieve her aim.

Task:

By writing each of the forces into vector form, help Amy achieve her aim by determining the magnitude and direction of the fifth force that can be applied onto the loggage.

ITEM 24:

A company wants to understand the job satisfaction levels of its employees. They randomly survey 20 employees from a total of 100 employees in the company. The survey asks about job satisfaction on a scale of 10 to 20. where 10 represents "very dissatisfied" and 20 represents "very satisfied". After gathering the responses, they want to analyze the central tendency and variation in job satisfaction scores. The following data was obtained:
14, 10, 12, 10, 13, 15, 19, 14, 18, 10, 16, 13, 19, 16, 18, 15, 10, 12, 20, 15

Task:

(i). Calculate statistical measures of central tendency (mean, mode and median) to summarize the central tendency of job satisfaction.

(ii). Determine the deviation of the job satisfaction scores from the average score, and what would this tell you about the consistency of employee satisfaction?

ITEM 25:
An association in a certain community carrying out savings came up with a plan to share the interest generated by the end of last month. The table below shows the distribution of interest paid to 100 shareholders by that time.

Interest (in thousands)	10 –	30 –	40 –	60 –	80 –	110 –	120 – 130
Cumulative number of share holders	9	20	57	82	97	99	100

The committee of the association was advised to:

- Organize the above information using a frequency distribution table.
- Pay average interest to the shareholders if the standard deviation of the interest does not exceed Ugshs. 20,000.
- Estimate the number of shareholders who were paid Ugshs. 100,000 and above.

Task:

(a). Help the committee to display the above information using a frequency distribution table and come up with the:
  (i). Average interest in Ugshs.
  (ii). Standard deviation in their interest payments.

(b). Using an appropriate statistical graph, determine the number of shareholders who were paid interest of Ugshs. 100,000 and above.

(c). Do you recommend the idea of sharing average interest to shareholders. Give a reason(s) to support your answer.

ITEM 26:
A certain commercial institution realized that there is a lot of errors in typing documents. One day, a meeting was conducted and a member suggested that it was speed causing errors. The administration administered to 12 randomly selected candidates $A, B, C, \ldots, L$ of the institution a text.
The typing errors (X) which represents the number of words typed with errors per minute; and the typing speed (Y) which represents the total number of words typed per minute; for each candidate were record as shown below.

CANDIDATE	A	B	C	D	E	F	G	H	I	J	K	L
ERRORS (X)	12	24	20	10	32	30	28	15	18	40	27	35
SPEED (Y)	130	136	124	120	153	160	155	142	145	172	140	157

The administration is not sure whether it is speed in typing causing errors or not. The institution has a task to look for a technical person who can use the above data to test the relationship between the typing speed and the errors made per minute by a candidate.

Task:

If the institution approached you as a technical person,

(a). Help the administration to display the information on a scatter diagram, draw a line of best and then comment on the relationship between the typing speed and the number of errors made per mimute.

(b). Estimate:

(i). the number of errors likely to be made per minute by a candidate with a typing speed of 150 words per minute.

(ii). the typing speed of a candidate who made 38 errors per minute in typing.

(c). Using the same data, calculate the rank correlation coefficient between the typing speed and the number of errors made per minute. Comment on your result at 1% level of significance.

ITEM 27:

In a certain town, there is a section of the road where many accidents occur and the residents believe it is due to over speeding so they have requested the authorities to build humps along that section, the chairperson of the roads committee has decided to do some research so a checkpoint has been put at that section to measure the speed of 50 vehicles passing that point. They will put humps if the research shows that the majority of vehicles passing that point use a speed greater than the speed limit. The road sign shows a speed limit of 55 km h^{-1} for that section. The results for the 50 vehicles are shown in the table below.

Speed (km h^{-1})	20 – 30	30 - 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Number of vehicles	5	13	20	29	35	40	44	50

TASK:

Assist the chairperson to determine;

(a). the average speed at which vehicles pass the checkpoint.

(b). the standard deviation of the speed at which vehicles pass the checkpoint.
(c). Present a graphical analysis to guide the committee's choice of implementing traffic calming measures.

ITEM 28:
Due to Poor performance in Mathematics that was observed among learners in Pre-Registration examinations, the school administration through academic board came up with strategies to improve on the learner's performance. Some of the strategies are that a prize is given to learners scoring above 78% and a remedial exam be given to learners scoring below the 56%.
Mathematics examination was given to S.5 learners and marks were obtained as below:

Marks	**50 – 54**	**55 – 64**	**65 – 68**	**69 – 74**	**75 – 79**	**80 – 84**
Frequency density	**0.2**	**0.9**	**2.5**	**2.0**	**1.2**	**0.4**

TASK:
(a). Using an appropriate statistical graph, help the administration to determine the number of students to be awarded prizes and those whose who will be given remedial exams.
(b). Determine the:
(i). Average mark scored by the students
(ii). Standard deviation of the marks.

ITEM 29:
A teacher gave a mathematics test and recorded the marks out of 50 for 40 students. The teacher chooses 35 as the assumed mean for easier calculations. He wants to allow students who scored above the middle mark to continue with the subject on merit, advice the last 10% of the students who scored below the middle mark to change the subject, and the rest of the students who scored below the middle mark were given the last chance. The students scored as follows.

12	18	22	25	27	30	32	34	35	36
37	38	39	40	41	42	43	44	45	46
15	17	19	21	23	24	26	28	29	31
33	35	36	38	40	42	44	46	47	48

TASK: