

P425/2

APPLIED MATHEMATICS

PAPER 2

AUGUST 2017

3 hours

UGANDA ADVANCED CERTIFICATE OF EDUCATION

MOCK 2–2017

APPLIED MATHEMATICS

PAPER 2

3 HOURS

INSTRUCTIONS TO CANDIDATES

Answer **all** the questions in section **A** and any **five** from section **B**.

All necessary working **must** be shown clearly.

Begin each answer on a fresh page.

Additional question(s) answered will **not** be marked.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take **g** to be **9.8 ms²**.

SECTION A: (40 MARKS)

Answer **all** the questions in this section.

1. The distribution function of a continuous random variable X is as follows:

$$F(x) = \begin{cases} 0 & , x < 1 \\ \frac{1}{4}(x-1)^2 & , 1 \leq x \leq 3 \\ 1 & , x > 3 \end{cases}$$

Find:

- (i) $P(1.5 \leq X \leq 2)$. (03 marks)
- (ii) The p.d.f of X . (02 marks)
2. A car of mass 2000 kg has a maximum power of 45 kW up a slope of 1 in 70 against a constant resistance to motion of 300 N. Find the
- (i) driving force of the car engine,
- (ii) acceleration of the car up the slope when its speed is 50 ms^{-1} . (05 marks)
3. The base and height of a triangle were measured as 5 cm and 6 cm with relative errors 0.04 and 0.05 respectively. Find the:
- (i) absolute error in each dimension. (02 marks)
- (ii) range within which the area of the triangle lies. (03 marks)
4. Joan can either use a bus or a taxi to go to work. The chance that she uses a bus is $\frac{1}{4}$. If she uses a bus, the chance that she will be late is $\frac{2}{3}$, otherwise it is $\frac{3}{5}$. Find the chance that she will:
- (i) be late on a particular day.
- (ii) use a taxi given that she is punctual. (05 marks)
5. The bus charges over distances of 4 km and 10 km from town P are shs 2800 and shs 5800 respectively. Estimate the:
- (i) charge that is worth 7 km from P.

(ii) distance from P that is worth shs 5300. (05 marks)

6. Particles of masses 4 kg, 3 kg, 2 kg and 6 kg respectively, act at the corners of a rectangle ABCD in which AB = 18 cm and AD = 12 cm. Find the position of the centre of gravity of the whole system. (05 marks)

7. The marks of 40 students in a test were as follows:

Marks	30 –	40 –	50 –	60 –	70 –	80 –
Frequency	8	5	12	9	6	0

Calculate the:

(i) mode. (02 marks)

(ii) mean mark. (03 marks)

8. An aircraft which can travel at 100 kmh^{-1} in still air, is to be flown from town P to town Q, 340 km apart with Q on the bearing of 060° from P. If there is a steady wind of 40 kmh^{-1} blowing from the west, find the time taken for the flight. (05 marks)

SECTION B: (60 MARKS)

Answer any **five** questions from this section. All questions carry equal marks

9. (a) A particle is projected from a point O with speed u at an angle of elevation θ to the horizontal. Show that its height y above O when it has travelled a distance x horizontally is given by

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}. \quad (04 \text{ marks})$$

(b) Find the velocity and angle of projection of a particle which passes in a horizontal direction just over the top of a wall which is 12 m high at a horizontal distance of 32 m from the point of projection.

(08 marks)

10. (a) Box P contains 3 white and 4 blue beads while box Q contains 5 white and 3 blue beads. A bead is drawn at random from P and put into Q and then a bead is taken from Q and put into P. Find the probability that the bead drawn now from box P is white. (05 marks)
- (b) Bag X contains 4 red and 3 blue pens, while bag Y contains 3 red and 2 blue pens. A bag is selected at random and two pens are drawn from it without replacement. Find the probability of picking:
- (i) pens of different colours. (04 marks)
- (ii) bag Y given that the pens drawn are of the same colour. (03 marks)
11. (a) Use the graphical method to show that the equation $\tan x - 4x = 0$ has a root between 1 and 1.5. (05 marks)
- (b) Show that the equation in (a) above yields Newton Raphson's formula $x_{n+1} = \frac{x_n \tan^2 x_n - \tan x_n + x_n}{\tan^2 x_n - 3}$. Hence find the root of the equation correct to three decimal places. (07 marks)

12. The marks of 100 students were as follows:

Marks	Frequency
Below 5	6
5 and under 10	11
10 and under 15	17
15 and under 25	28
25 and under 30	20
30 and under 35	15
35 and under 40	3

- (a) Calculate the:
- (i) upper quartile mark.
- (ii) variance of the distribution. (06 marks)
- (b) Display the data on a histogram and use it to estimate the mode. (06 marks)
13. (a) If the acceleration of a particle moving with simple harmonic motion is given by $a = -\omega^2 x$, where ω is a constant and x is the distance from the

centre of motion, show that $x = A \sin(\omega t + \varepsilon)$, where ε is a constant and A is the amplitude. (06 marks)

- (b) A particle moving with S.H.M has velocities of 7.5 ms^{-1} and 4 ms^{-1} when at distance of 0.9 m and 0.2 m respectively from the end points of its path. Find the length of its path. (06 marks)

14. (a) The weights of 400 books are normally distributed with mean 50 g and standard deviation 2 g. Find how many books weigh less than 52.48 g. (05 marks)

- (b) A random sample of 100 items taken from a normal population had the following data: $\bar{X} = 82$ and $\sum X^2 = 686800$. Find the:

- (i) unbiased estimate of the population standard deviation.
(ii) 97% confidence interval for the population mean. (07 marks)

15. (a) Use the trapezium rule with six ordinates to estimate $\int_1^3 \sin^2 x \, dx$ correct to three decimal places. (06 marks)

- (b) Find the exact value of $\int_1^3 \sin^2 x \, dx$. (03 marks)

- (c) Find the percentage error in your estimation in (a) above and suggest how it can be reduced. (03 marks)

16. (a) ABCD is a square. Forces of magnitudes 9 N, 5 N and $3\sqrt{2}$ N act along \overrightarrow{AB} , \overrightarrow{BC} and \overrightarrow{BD} respectively. Find the magnitude and direction of their resultant. (05 marks)

- (b) A non uniform ladder AB whose centre of gravity is 2 m from end A is of length 6 m and weight W. The ladder is inclined at angle of θ to the vertical with its end B against a rough vertical wall and end A on a rough horizontal ground with which the coefficients of friction at each point of contact is μ . If the ladder is about to slip when a man of weight 5W ascends two-thirds of the way up the ladder, show that $\tan \theta = \frac{18\mu}{11 - 7\mu^2}$. (07 marks)

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