MATHEMATICS SCENARIO BASED ITEMS

Topic 1: NUMERICAL CONCEPTS

Sub-topic 1.1: Indices

Scenario 1

A rapidly growing tech startup in Kampala,"ByteHub Uganda," is projecting its user base growth for investors. Their analytics team has modeled the growth using the formula $U(t) = 500 \times 2^{0.5t}$, where U is the number of users and t is the time in months. The board of directors needs to make critical decisions about server capacity and funding rounds based on these projections. They need to know the expected user base 6 months from now and understand how long it will take to reach a milestone of 10,000 users to secure the next phase of venture capital funding. Accurately applying the laws of indices is essential for these calculations, as even a small miscalculation could lead to either costly over-provisioning of resources or catastrophic system failure due to underestimation.

Task:

a)Calculate the projected number of users after 6 months.

b)Determine how many months it will take for the user base to reach 10,000.

c)State what the base and the exponent represent in the context of this model.

Scenario 2

The National Agricultural Research Organisation(NARO) is studying the propagation of a beneficial soil bacterium in a new fertilizer. Under ideal conditions in the lab, a single bacterium divides into two every 30 minutes. The population growth is modeled by $P(t) = P_0 \times 2^{2t}$, where P_0 is the initial population and t is the time in hours. Researchers need to predict the population size after 4 hours starting from an initial colony of 100 bacteria to ensure there are enough bacteria to effectively treat one hectare of farmland. This helps in determining the correct and cost-effective amount of fertilizer to produce for Ugandan farmers, directly impacting crop yields and food security in the region.

Task:

a)Calculate the population of bacteria after 4 hours.

b)If the culture medium can only support 1,000,000 bacteria, how long will it take to reach this capacity from the start?

c)Explain why the exponent in the model is $2t$ and not just t.

Scenario 3

A financial advisor at Stanbic Bank Uganda is explaining the power of compound interest to a young client who wants to save for university. A savings account offers an annual interest rate of 8%, compounded annually. The future value of the investment is given by the formula $A = P(1 + r)^n$, where P is the principal, r is the interest rate, and n is the number of years. The client, who has UGX 500,000 to invest, wants to know how much the investment will be worth in 5 years and how long it will take for her money to double, enabling her to make an informed decision about her financial future and educational goals.

Task:

a)Calculate the future value of the UGX 500,000 investment after 5 years.

b)Determine the number of years it will take for the initial investment to double.

c)State one way the client could decrease the time it takes for her investment to double.

Sub-topic 1.2: Logarithms

Scenario 4

Acoustic engineers are assessing noise pollution at a new factory site in Namanve Industrial Park.They measure the sound intensity at the perimeter wall to be 5×10^{-4} Wm^{-2}. Ugandan law mandates that industrial noise must not exceed 75 dB at residential boundaries, calculated using $P = 10\log_{10}(\frac{I}{L})$, with $L = 10^{-12}$ Wm^{-2}. The factory managers must determine if they are compliant or if they need to invest in expensive sound-dampening measures to avoid fines and legal action from the nearby community, which has already raised concerns about the potential impact on their quality of life.

Task:

a) Calculate the sound level in decibels for the measured intensity.

b) Determine the maximum permissible sound intensity I that corresponds to the 75 dB legal limit.

c) Explain why using a logarithmic scale is more practical than a linear scale for measuring sound intensity in this context.

Scenario 5

A data scientist at a telecommunications company is analyzing the server load for their new mobile money platform. The load on the servers follows the model $L(t) = k \log_2(t + 1)$, where t is the number of concurrent users. During a peak hour, with 1024 users, the load is measured at 50 units. The operations team needs to predict the server load when 4096 users are active simultaneously to decide if the current server infrastructure can handle the anticipated growth during the festive season or if a costly upgrade is necessary to prevent system failure.

Task:

a)Use the given data to find the value of the constant k .

b)Calculate the predicted server load for 4096 concurrent users.

c)State what the value '1' in the term $(t + 1)$ likely represents in this model.

Scenario 6

Public health officials are tracking the spread of an infectious disease in a rural Ugandan district.The effective reproduction number R_t is a crucial metric. If R_t is 2, it means each infected person spreads the disease to 2 others. The growth rate can be analyzed using logarithms. If the number of cases increases from 10 to 160 over a period, officials need to calculate the average R_t to understand the speed of transmission and decide whether to implement lockdowns, which have significant economic and social consequences for the community.

Task:

a)The number of cases follows $C = C_0 X (R_t)^n$, where n is the number of transmission cycles. If cases grow from 10 to 160, and $n=3$, calculate R_t.

b)Express the number of transmission cycles n in terms of C , C_0 , and R_t using logarithms.

c)If R_t is found to be 1.5, is the outbreak growing or shrinking? Explain.

Sub-topic 1.3: Surds

Scenario 7

An architect is designing a uniquely shaped national monument for a Kampala roundabout.The design features a large equilateral triangle with a side length of $8\sqrt{3}$ meters. To source the correct amount of a special, expensive cladding material for the perimeter, the contractor needs the exact perimeter length. Providing a decimal approximation could lead to a costly over-ordering or under-ordering of material, so the exact value in surd form is essential for precise costing and procurement before construction begins.

Task:

a)Calculate the exact perimeter of the equilateral triangle in surd form.

b)Find the area of the triangle, also in surd form.

c)Provide a rational approximation of the perimeter to 2 decimal places for the logistics team, who require numerical values for transport planning.

Scenario 8

A land surveyor is demarcating a rectangular plot of land for a new school in Wakiso District. The plot is $(5\sqrt{2} + 10)$ meters long and $(5\sqrt{2} - 10)$ meters wide. The school administration needs to know the exact area of the land for legal documentation and to plan the placement of buildings and playgrounds. Using surds ensures the legal documents are mathematically precise, avoiding future boundary disputes with neighboring landowners.

Task:

a)Show that the area of the plot is a rational number.

b)Calculate the exact area.

c)If a fence is to be built around the plot, calculate the exact length of the diagonal to determine the amount of fencing material needed for the corners.

Scenario 9

A civil engineer is inspecting a collapsed section of a road embankment.The cross-section of the stable embankment is a right-angled triangle with a vertical height of $4\sqrt{5}$ meters and a base of $2\sqrt{5}$meters. To design a reinforcing support, the engineer needs the exact length of the sloping side (the hypotenuse). Using surds provides the precision required for structural calculations, ensuring the support is designed to the correct specifications to prevent further collapses and ensure public safety.

Task:

a)Calculate the exact length of the hypotenuse of the triangular cross-section.

b)Find the exact perimeter of this triangular cross-section.

c)Rationalize the denominator of the expression for the hypotenuse if it were written as a fraction.