

## SECTION C: MODERN PHYSICS

### CHAPTER 1: CATHODE RAYS & POSITIVE RAYS

#### 1.0: GASES AS CONDUCTORS OF ELECTRICITY

When a gas is free from external influences it has no free charges to act as carriers of electricity but can be a conductor of electricity in various ways:

In all these ways the molecules are made to be ionized by detachment of one or more electrons. E.g. reducing the pressure of a gas and applying a high  $p.d$  across the container having the gas.

Energy must be supplied to molecules in order to ionize them, gas molecules become ionized as a result of collision with negative charges moving rapidly. The pressure of the gas is reduced to enable ions produced to have means free path long enough to allow them gain free energy for ionization.

#### 1.1: CATHODE RAYS

These are streams of fast moving electrons that travel from cathode to anode when a  $p.d$  is connected across the plate.

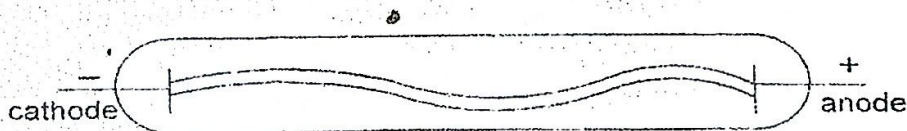
#### DISCHARGE TUBE

Properties of cathode rays can be studied in a discharge tube in which electricity is passed through a gas at low pressure. The electrons are enclosed in a glass tube which is connected to a pressure gauge and a vacuum pump.

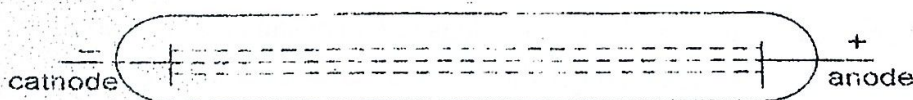
A discharge tube contains 2 metal electrodes approximately 30cm apart between which a  $p.d$  of about 1000V is applied.

Steps leading to the production of a cathode ray.

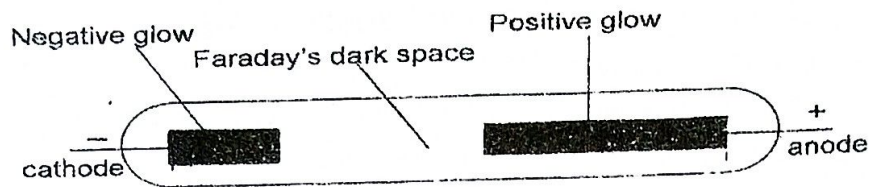
- At a pressure of  $\approx 10$  mmHg, a discharge of blue violet streamers pass between cathode and anode



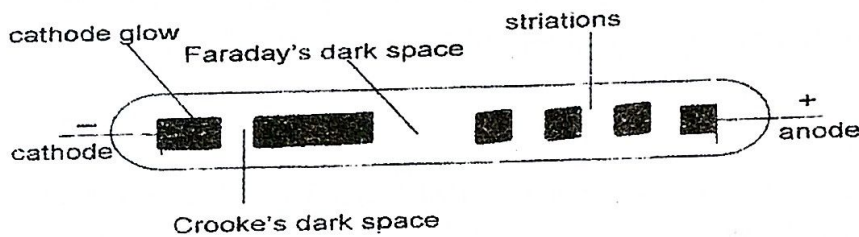
- At a pressure of  $\approx 2$  mmHg, long luminous positive column appears from anode to cathode



- ❖ At a pressure  $\approx 1$  mmHg, a pink positive column and a negative glow appear near the cathode. These two regions are separated by Faraday's dark space



- ❖ At a pressure of 0.1 mmHg, the positive column becomes striated. The negative glow moves away from the cathode, Crookes' dark space appears and the cathode glow appears round the cathode

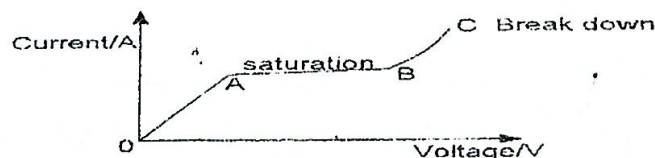


- ❖ At pressure of about 0.01mmHg, Crookes' dark space fills the glass tube and the tube fluoresces due to electron.

#### Limitations of the discharge tube method

- > When cathode rays strike the anode they may produce x-rays which are dangerous
- > A very high *p.d* is needed across the electrodes which can be hazardous to handle
- > The gas is needed at appropriate low gas pressure which is difficult to attain

#### Characteristics of a discharge tube



- ✓ In region OA, as the applied voltage increases the number of electrons reaching the anode increases leading to the increase of the current.
- ✓ In region AB, at saturation the electrons released by collision reach the anode roughly at the same time so that the current through the tube appears constant.

- ✓ In region BC, the number of electrons due to ionization increases rapidly and not all the electrons reach the anode at the same time so the current increases gradually.
- ✓ At break down the number of electrons reaching the anode is so large and current rises sharply and this may damage tube. It can be prevented by connecting a resistance in series with the tube.

#### Applications of a discharge tube

- |  |  |
|--|--|
| (1) lighting fluorescent tubes             | (3) Making Flood lights                |
| (2) In advertisement sign tube, neon signs | (4) Making mercury lamps, sodium lamps |

### 1.1.1: THERMIONIC EMISSION

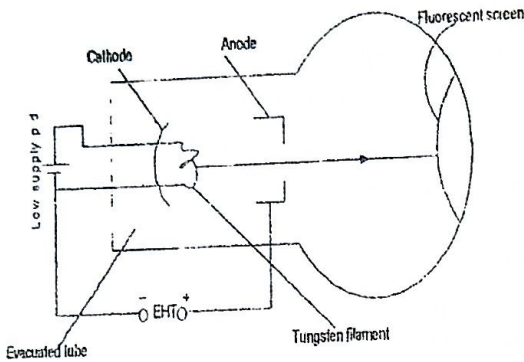
#### Definition

- ❖ Thermionic emission is a process by which electrons are emitted from a hot metal surface.
- ❖ Work function is the minimum energy required to release an electron from the metal surface. The work function of a metal is expressed in electron volts [eV]
- ❖ eV is defined as the kinetic energy gained by an electron in being accelerated by a potential difference of one volt.

### 1.1.2: MECHANISM OF THERMIONIC EMISSION

- ❖ Metals contain free electrons in their lattice that are loosely bound to their parent nuclei.
- ❖ As the temperature of the metal is raised, velocities of the electrons increase, some of the surface electrons acquire sufficient kinetic energy to overcome the electrostatics attraction force of the atomic nuclei and consequently escape from the metal surface.

### 1.1.3: MODERN CATHODE RAY TUBE [PRODUCTION OF CATHODE RAYS]



- ❖ The electrons are focused by the cathode and accelerated by EHT to the fluorescent screen which gives a glow when they strike the screen.
- ❖ It is the beam of fast moving electrons from the cathode which constitute the cathode rays.

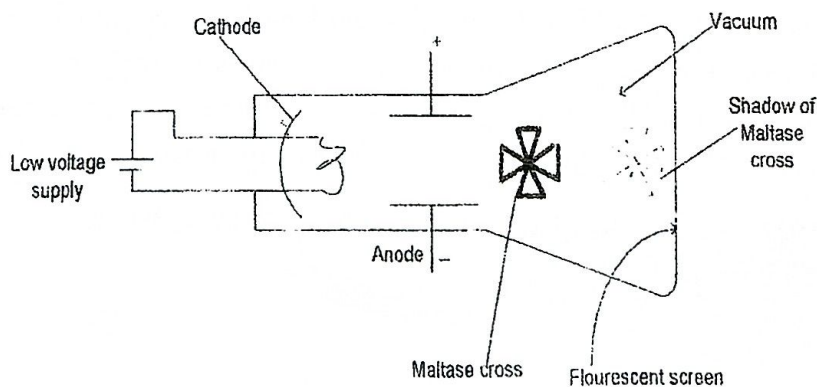
- ❖ The cathode is heated by a low p.d and produces electrons by thermionic emissions.

#### 1.1.4: PROPERTIES OF CATHODE RAYS

- They travel from cathode to anode in a straight line
- They are electrons and carry a negative charge
- They can be deflected in an electric field towards the positive plate
- They can be deflected in a magnetic field towards the North Pole according to Flemings left hand rule.
- They cause certain substances to fluoresce when they collide with them
- They possess kinetic energy which is changed to heat when they are brought to rest
- They can produce x-rays if they are of sufficiently high energy

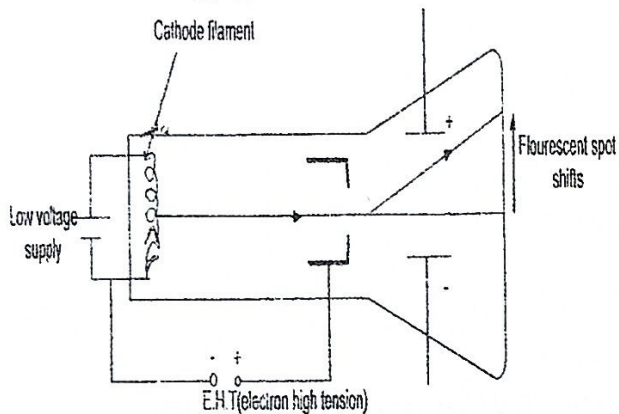
#### 1.1.5: TO STUDY PROPERTIES OF CATHODE RAYS

##### 1: Straight line movement



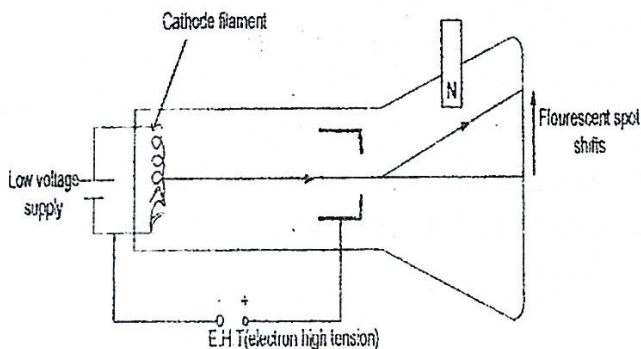
- ❖ Electrons emitted from a heated cathode, are accelerated by the anode and directed towards a maltase cross placed in the center of the a glass tube.
- ❖ A sharp shadow of the maltose cross is cast on a screen at the end of the tube. This shows that cathode rays travel in a straight line

### 2: Carry a negative charge



- ❖ The cathode ray tube is modified to include parallel plates connected to the terminals of a battery as shown below:
- ❖ When cathode rays are produced thermionically and passed through the plate, the fluorescent spot is seen to shift upwards from its initial position before the plates were applied. The spot moves towards the positive plate and away from the negative plate, this shows that cathode rays carry a negative charge.

### 3: Deflection in the magnetic field



The magnetic field is provide by the magnet. The application of the magnetic field makes the fluorescent spot to shift in the direction which depends on the field i.e. the pole nearer the beam. The beam bends towards the North Pole.

## 1.1.6: ELECTRODE DYNAMICS

### 1: Motion in an Electric field

#### a) Speed of an electron

Suppose an electron of charge  $e$  and mass  $m$  is emitted from a hot cathode and accelerated by an electric field of potential  $V_a$  towards the anode, then:

Kinetic energy gained by the electron = work done on an electron by the accelerating p.d  $V_a$

$$\frac{1}{2} mu^2 = e V_a$$

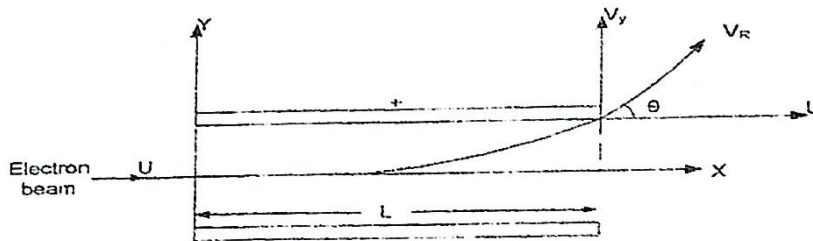
Or

$$u = \sqrt{\frac{2eV_a}{m}}$$

\* Note  $V_a$  must be accelerating p.d and not p.d between the plates

### b) Electron path in an electric field

Consider an electron of charge  $e$  and mass  $m$  entering an electric field horizontally with a speed  $u$ .



Since electric field intensity is force per unit charge, the force on the electron is therefore given by

$$F = Ee \text{-----[1] Where } E \text{ is electric field intensity}$$

$$\text{But also } F = ma \text{----- [2]}$$

$$\text{Equating 1 and 2 } Ma = Ee$$

$$a = \frac{Ee}{m} \text{----- [3]}$$

if the p.d between the plates is  $V$  and their distance is  $d$ , then

$$E = \frac{V}{d}$$

Put into equation 3

$$a = \frac{Ve}{md} \text{----- [4]}$$

$$\text{For vertical motion [} u=0m/s \text{], } y = \frac{1}{2} \frac{Ve}{md} t^2 \text{----- [5]}$$

for horizontal motion [ $a=0m/s$ , constant velocity],  $x = ut$

$$t = \frac{x}{u} \text{----- [6]}$$

put into equation 5

$$y = \frac{1}{2} \frac{Ve}{md} \left(\frac{x}{u}\right)^2$$

$$y = \left(\frac{Ve}{2mdu^2}\right)x^2$$

**Note**

- ❖ When an electron moves horizontally into a uniform electric field, it describes a parabolic path. This parabolic motion is brought by the electric force  $[F=Eq]$  experienced by electrons in the direction of that of the field.
- ❖ The horizontal motion of the electron is not affected by the field. A charge gains energy when it moves in the direction of an electric field

**Formula when the electron just leaves the plate**

After leaving the plates the electron moves in a straight line. Just after passing through the plates, the distance  $x$  moved through is equal to the length of the plate. Therefore  $x=l$

$$y = \left(\frac{Vel^2}{2mdu^2}\right)$$

OR

$$y = \left(\frac{Eel}{2mu^2}\right)l^2$$

For the velocity along the vertical ( $u=0\text{m/s}$ )

$$v = u + at$$

$$V_y = at$$

$$V_y = \frac{ve}{md} \frac{l}{u}$$

where  $t = \frac{l}{u}$

$$V_y = \frac{Ve l}{md u}$$

OR

$$V_y = \frac{Eel}{mu}$$

The resultant velocity  $V_R$  with which the electron leaves the field is given by

$$V_R^2 = v_y^2 + u^2$$

$$V_R = \sqrt{v_y^2 + u^2}$$

The direction with which the electron emerges from the field

$$\tan \theta = \frac{v_y}{u}$$

$$\theta = \tan^{-1} \left(\frac{V_y}{u}\right)$$

**Example**

1. An electron is accelerated from rest through a p.d of 1000V. what is:

- (a) Its kinetic energy in eV
- (b) Its kinetic energy in joules
- (c) Its speed

**Solution**

- a)  $V_a = 1000V$   
 $k.e = 1000eV$   
 b)  $k.e = eV_a$   
 $= 1.6 \times 10^{-19} \times 1000$   
 $k.e = 1.6 \times 10^{-16} J$   
 c)  $k.e = eV_a$

$$\frac{1}{2} mu^2 = 1.6 \times 10^{-19} \times 1000$$

$$u = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 1000}{9.11 \times 10^{-31}}}$$

$m_e = 9.11 \times 10^{-31}$ ,  $m_e$  is mass of the electron  
 $u = 1.874 \times 10^7 m/s$

2. Calculate the speed of a proton which has been accelerated through the p.d of 400V [mass of proton =  $1.67 \times 10^{-27} kg$ ,  $e = 1.6 \times 10^{-19} C$ ]

**Solution**

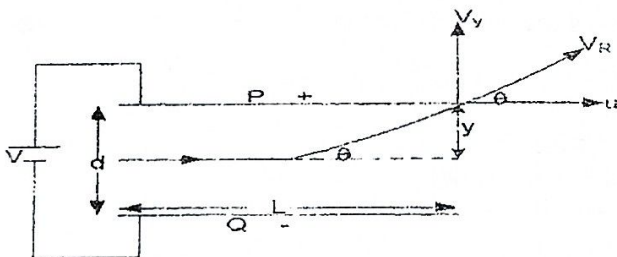
$k.e = eVa$   
 $\frac{1}{2} mu^2 = eVa$   
 $U = \sqrt{\frac{2eVa}{m}}$

$$U = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 400}{1.67 \times 10^{-27}}}$$

$U = 2.769 \times 10^5 m/s$

3. A beam of electrons, moving with velocity of  $1.0 \times 10^7 ms^{-1}$ , enters midway between two horizontal parallel plates P, Q in a direction parallel to the plates. P and Q are 5cm long, 2cm apart and have a p.d V applied between them. Calculate V if the beam is deflected so that it just grazes the edge of the upper plate P

**Solution**

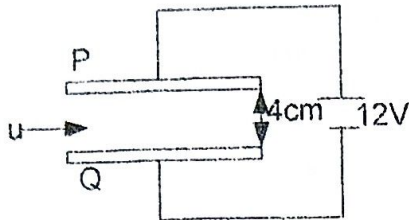


$L = \frac{5}{100} m$   
 $d = \frac{2}{100} m$   
 $u = 1 \times 10^7 ms^{-1}$   
 $y = \frac{1}{100} m$  since the beam is directed midway from  $s = ut + \frac{1}{2} at^2$   
 $y = \frac{1}{2} at^2$  ( $u = 0 m/s$ )  
 but  $a = \frac{ve}{md}$

$y = \frac{vet^2}{2md}$   
 $t = \frac{L}{u}$   
 $y = \frac{veL^2}{2md u^2}$   $m = 9.11 \times 10^{-31} kg$  mass of an electron  
 $\frac{1}{100} = \frac{v \times 1.6 \times 10^{-19} \times (\frac{5}{100})^2}{2 \times 9.11 \times 10^{-31} \times (\frac{2}{100}) \times (1 \times 10^7)^2}$   
 $V = 91.1V$

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4.

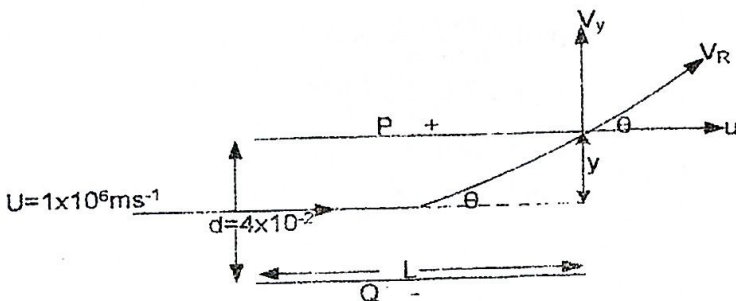


The figure above shows 2 parallel metal plates P and Q each of length 4cm and separated by a distance 4cm. A p.d of 12V is applied between P and Q and a beam of electrons of speed

$u = 1 \times 10^6 \text{ ms}^{-1}$  is directed midway between P and Q at right angles to the electric field between the plates, calculate;

- (i) The angle to the initial direction of the beam at which the beam emerges from the space between plates P and Q
- (ii) The velocity at which electron leaves the plates.

**Solution**



$$d = 4 \times 10^{-2} \text{ m}, \quad u = 1 \times 10^6 \text{ ms}^{-1}, \quad V = 12 \text{ V}, \quad L = 4 \times 10^{-2} \text{ m}$$

$$V_y = at$$

$$V_y = \frac{v_e l}{m d u}$$

$$V_y = \frac{12 \times 1.6 \times 10^{-19} \times 4 \times 10^{-2}}{9.11 \times 10^{-31} \times 4 \times 10^{-2} \times 1 \times 10^6}$$

$$V_y = 2.11 \times 10^6 \text{ ms}^{-1}$$

$$\theta = \tan^{-1} \frac{v_y}{u}$$

$$\theta = \tan^{-1} \frac{2.11 \times 10^6}{10^6}$$

$$\theta = 64.6^\circ$$

$$\text{ii) } V_y = 2.11 \times 10^6 \text{ ms}^{-1}$$

$$u = 1 \times 10^6$$

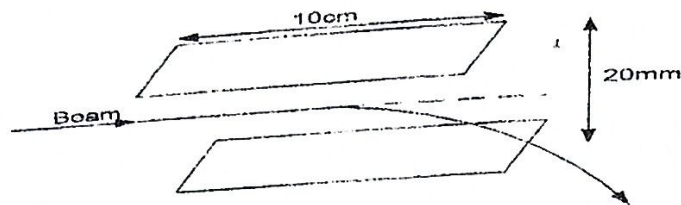
$$V_R = \sqrt{v_y^2 + u^2}$$

$$V_R = \sqrt{(2.11 \times 10^6)^2 + (1 \times 10^6)^2}$$

$$V_R = 2.33 \times 10^6 \text{ m/s}$$

The electron leaves the plates with a velocity of  $2.33 \times 10^6 \text{ m/s}$

5. Two parallel metal sheets of length 10cm are separated by 20mm in a vacuum. A narrow beam of electrons enters symmetrically between them as shown.

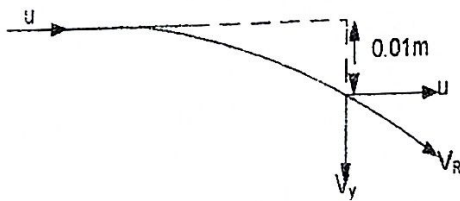


When a p.d of 1000V is applied between the plates the electron beam just misses one of the plates as it emerges.

Calculate the speed of the electrons as they enter the gap [Take the field between the plates to be uniform] [ $\frac{e}{m} = 1.8 \times 10^{11} \text{ Ckg}^{-1}$ ]

**Solution**

Since the beam enters symmetrically  $y = \frac{d}{2}$ ,  $y = \frac{0.02}{2}$ ,  $y = 0.01\text{m}$



$d = 0.02\text{m}$ ,  $L = 0.1\text{m}$        $V = 1000\text{V}$   
 required to get is  $u = \text{m/s}$   
 but specific charge  $\frac{e}{m} = 1.8 \times 10^{11} \text{ Ckg}^{-1}$

$$\text{using } y = \frac{vet^2}{2md}$$

when the beam just emerges  $t = \frac{l}{u}$

$$0.01 = \frac{1000 \times (0.1)^2}{2 \times 0.02 \times u^2} \times \left(\frac{e}{m}\right)$$

$$0.01 = \frac{1000 \times (0.1)^2 \times 1.8 \times 10^{11}}{2 \times 0.02 \times u^2}$$

$$u^2 = 4.5 \times 10^{15}$$

$$u = 6.71 \times 10^7 \text{ ms}^{-1}$$

6. In an evacuated tube, electrons are accelerated through a p.d of 500V. Calculate their final speed and consider whether this depends on the accelerating field being uniform. After this acceleration, the electrons pass through a uniform electric field which is perpendicular to the direction of travel of the electrons as they enter the field. This electric field is produced by applying a potential difference of 10V to two parallel plates which are 0.06m long and 0.02m apart. Assume that the electric field is uniform and confined to the space between the two plates.

Determine the angular deflection of the electron beam produced by the field.

[ $e/m$  for the electron =  $1.76 \times 10^{11} \text{ Ckg}^{-1}$ ]

**Solution**

For the first case

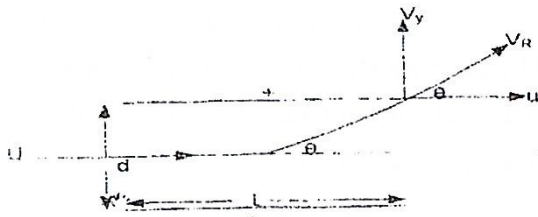
$$\frac{1}{2} mu^2 = eVa$$

$$U^2 = 2Va \frac{e}{m}$$

$$U = \sqrt{2Va \frac{e}{m}}$$

$$U = \sqrt{2 \times 500 \times 1.76 \times 10^{11}}$$

For the second case



$$d = 0.02 \text{ m}, L = 0.06 \text{ m}, V = 10 \text{ V}, \frac{e}{m} = 1.76 \times 10^{11}$$

$$v_y = \frac{veI}{mdu}$$

$$v_y = \frac{10 \times 0.06 \times 1.76 \times 10^{11}}{0.02 \times 1.33 \times 10^7}$$

$$U = 1.33 \times 10^7 \text{ m/s}$$

$$v_y = 3.97 \times 10^5 \text{ m/s}$$

but

$$\Theta = \tan^{-1} \frac{v_y}{u}$$

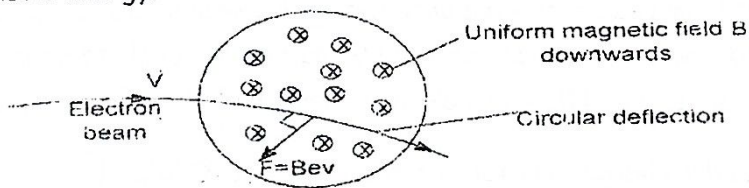
$$\Theta = \tan^{-1} \frac{3.97 \times 10^5}{1.33 \times 10^7}$$

$$\Theta = 1.71^\circ$$

The angular deflection =  $1.71^\circ$

## 2. Motion in a magnetic field

Consider an electron beam, moving with a speed  $V$  which enters a uniform magnetic field of magnitude  $B$  acting perpendicular to the direction of motion. The force  $F$  on an electron then  $F = BeV$ . The direction of the force is perpendicular to both  $B$  and  $V$ , the magnetic force cannot change the energy of the electron. It only deflects the electron but does not change its speed or kinetic energy.



The force  $BeV$  is always normal to the path of the beam. If the field is uniform the force is constant in magnitude and the beam then travels in a circle of radius  $r$ , since  $BeV$  is the centripetal force towards the centre.

$$Bev = \frac{mv^2}{r}$$

$$Be = mv$$

$$r = \frac{mv}{Be}$$

$$r = \frac{\text{momentum}}{Be}$$

If the velocity  $V$  of the electron decreases continuously due to may be collision, its momentum decreases, so from the relation for  $r$  above, the radius of its path decreases and the electron will then tend to spiral instead of moving in a circular path of constant radius.

**Note**

- ❖ When an electron beam having a common velocity enters a uniform magnetic field in a direction normal to the radius  $r$ , in accordance with Flemings left hand rule they experience a magnetic force  $F = Bev$  at right angles to both  $B$  and  $V$  and so the field will provide the centripetal force which makes the electron beam to move in a circular path of constant radius.

**Examples**

1. An electron is moving in a circular path at  $3.0 \times 10^6 \text{ms}^{-1}$  in a uniform magnetic field of flux density  $2.0 \times 10^{-4} \text{T}$ . Find the radius of the path [mass of electron =  $9.1 \times 10^{-31} \text{kg}$ , charge on electron =  $1.6 \times 10^{-19} \text{C}$ ]

**Solution**

$$\begin{aligned}
 V &= 3 \times 10^6 \text{ms}^{-1} \\
 B &= 2 \times 10^{-4} \text{T} \\
 m &= 9.1 \times 10^{-31} \text{kg} \\
 e &= 1.6 \times 10^{-19} \text{C}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \frac{mv^2}{r} &= Bev \\
 r &= \frac{mv}{Be} \\
 r &= \frac{9.1 \times 10^{-31} \times 3 \times 10^6}{2 \times 10^{-4} \times 1.6 \times 10^{-19}} \\
 r &= 8.53 \times 10^{-2} \text{m}
 \end{aligned}$$

2. Electrons accelerated from rest through a potential difference of 3000V enters a region of uniform magnetic field, the direction of the field being at right angles to the motion of the electrons. If the flux density is 0.01T, calculate the radius of the electron orbit.

[Assume that the specific charge  $e/m$  for the electron =  $1.8 \times 10^{11} \text{Ckg}^{-1}$ ]

**Solution**

$$\begin{aligned}
 V_0 &= 3000 \text{V} \\
 \frac{e}{m} &= 1.8 \times 10^{11} \\
 \frac{1}{2} mu^2 &= eVa \\
 U &= \sqrt{2Vax} \frac{e}{m} \\
 U &= \sqrt{2 \times 3000 \times 1.8 \times 10^{11}} \\
 U &= 3.29 \times 10^7 \text{m/s} \\
 \text{But} \\
 \frac{mv^2}{r} &= Bev
 \end{aligned}$$

$$\begin{aligned}
 r &= \frac{mv}{Be} \\
 v &= 3.29 \times 10^7 \text{m/s} \\
 \frac{e}{m} &= 1.8 \times 10^{11} \\
 \frac{m}{e} &= \frac{1}{1.8 \times 10^{11}} \\
 r &= \frac{v}{B} \times \frac{m}{e} \\
 r &= \frac{3.29 \times 10^7}{0.01} \times \frac{1}{1.8 \times 10^{11}} \\
 r &= 1.83 \times 10^{-2} \text{m}
 \end{aligned}$$

3. A beam of protons is accelerated from rest through a potential difference of 2000V and then enters a uniform magnetic field which is perpendicular to the direction of the proton beam. If the flux density is 0.2T, calculate the radius of the path which the beam describes

[proton mass =  $1.7 \times 10^{-27}$  kg electronic charge =  $-1.6 \times 10^{-19}$  C]

**Solution**

$$V_a = 2000V$$

$$\frac{1}{2}mv^2 = eVa$$

$$U = \sqrt{2xVax \frac{e}{m}}$$

$$U = \sqrt{\frac{2 \times 2000 \times 1.6 \times 10^{-19}}{1.7 \times 10^{-27}}}$$

$$U = 6.14 \times 10^5 \text{ m/s}$$

$$Bev = \frac{mv^2}{r}$$

$$Bev = \frac{mv^2}{r}$$

$$r = \frac{mv}{Be}$$

$$r = \frac{v}{B} \times \frac{m}{e}$$

$$r = \frac{6.14 \times 10^5 \times 1.7 \times 10^{-27}}{0.2 \times 1.6 \times 10^{-19}}$$

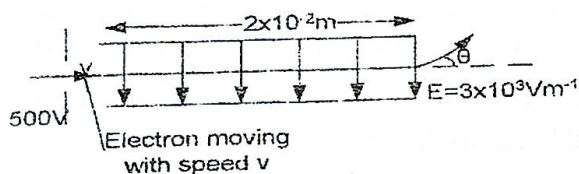
$$r = 3.26 \times 10^{-2} \text{ m}$$

$$u = v = 6.14 \times 10^5 \text{ m/s}$$

**Exercise 1**

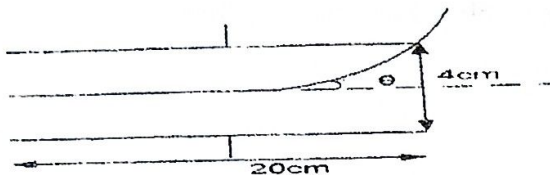
1. Calculate the speed of a proton which has been accelerated through a p.d of 400V. [mass of proton =  $1.67 \times 10^{-27}$  kg, charge on proton =  $1.60 \times 10^{-19}$  C] **Ans** [ $2.77 \times 10^5 \text{ ms}^{-1}$ ]

2.



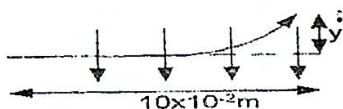
In the figure above, a beam of electrons is accelerated through a p.d of 500V and then enters a uniform electric field of strength  $E = 3 \times 10^3 \text{ Vm}^{-1}$ . Created by two parallel plates each of length  $2 \times 10^{-2} \text{ m}$ . Calculate;

- (a) The speed  $v$  of the electrons as they enter the field  
 (b) The time  $t$  that each electron spends in the field  
 (c) Angle  $\theta$  through which the electrons have been deflected by the time they emerge from the field [e/m of electron =  $1.76 \times 10^{11} \text{ Ckg}^{-1}$ ] **Ans** [ $1.33 \times 10^7 \text{ m/s}$ ,  $1.51 \times 10^{-9} \text{ s}$ ,  $3.4^\circ$ ]
3. (a) In a cathode ray tube electrons are accelerated through a potential difference of 9000V and focused into a narrow beam. Calculate the velocity of electrons in the beam.  
 (b) The same beam along a line midway between the electrostatic deflecting plates 20cm long and 4.0cm apart.



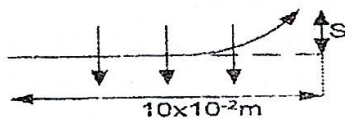
What is the value of the *p.d* between the plates needed to deflect the beam through the greatest angle possible? [ $e=1.6 \times 10^{-19} \text{C}$ ,  $m=9.1 \times 10^{-31} \text{kg}$ ] An [ $5.63 \times 10^7 \text{ms}^{-1}$ , 721V]

4. (a) Electrons are accelerated to a *p.d* of  $3 \times 10^3 \text{V}$  and pass at right angles into a uniform magnetic field of strength  $1.5 \times 10^{-2} \text{T}$ . find the radius of their paths.  
 (b) An identical beam is projected perpendicular into an electric field of  $3 \times 10^5 \text{Vm}^{-1}$ . Calculate the deviation *y* of the electron path at a point  $10 \times 10^{-2} \text{m}$  perpendicular into a field as measured from the point of entry of the beam.



[Specific charge of electron  $e/m = 1.76 \times 10^{11} \text{Ckg}^{-1}$ ] An [ $3.25 \times 10^7 \text{ms}^{-1}$ , 0.25m]

5. (a) Electrons are accelerated by a *p.d* of  $2 \times 10^3 \text{V}$  and passes at right angles into a uniform magnetic field of flux density  $1.0 \times 10^{-2} \text{T}$ . find the radius of the path described.  
 (b) An identical beam of electrons is projected perpendicular into an electric field of intensity  $2.0 \times 10^6 \text{Vm}^{-1}$ .



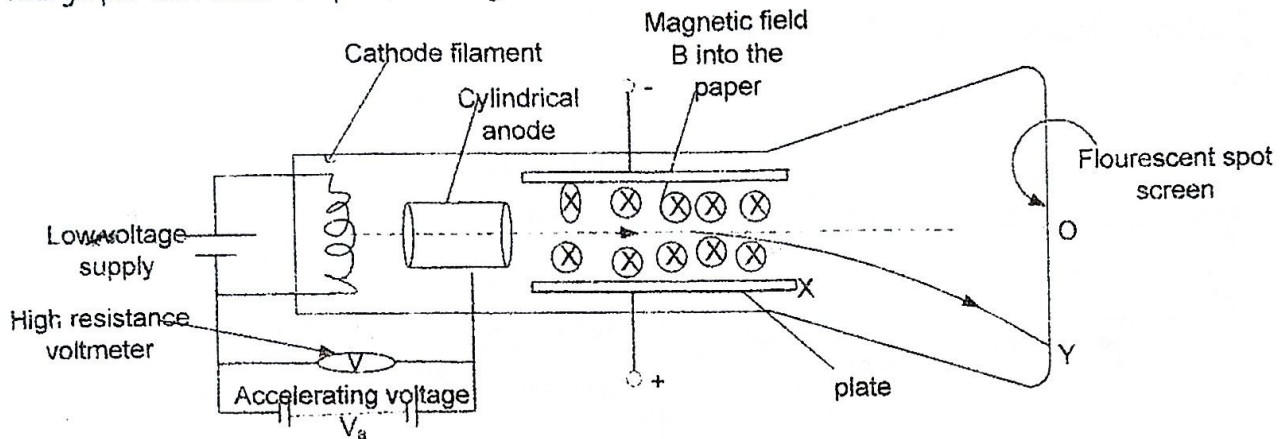
Calculate the deviation *S* of the electrons path at a point  $10 \times 10^{-2} \text{m}$  perpendicular to the field measured from the point of entry of the beam. An [ $1.51 \times 10^{-2} \text{m}$ , 2.5m]

6. A narrow horizontal beam of electrons passes symmetrically between two vertical metal plates mounted one at each side of the beam. The velocity of the electrons is  $3 \times 10^7 \text{ms}^{-1}$ , the plates are 0.03m long and 0.01m apart. It is found that when a battery of 568V is connected to the plates, the electron beam just strikes the end of one the plates. Calculate the value of the specific charge ( $e/m$ ) of the electron. An [ $1.76 \times 10^{11} \text{Ckg}^{-1}$ ]

7. In a cathode ray tube the electrons are accelerated through a *p.d* of 500V and then pass between deflecting plates which are 0.05m long.  
 (i) Find the time it takes an electron to pass between the plates  
 (ii) If a *p.d* across the plates is 10V *d.c* and the plates are 1cm apart, calculate the angle through which electrons are deflected [ $e/m=1.76 \times 10^{11} \text{Ckg}^{-1}$ ] An [ $3.8 \times 10^{-9} \text{s}$ ,  $2.9^\circ$ ]

### 1.1.7: DETERMINATION OF SPECIFIC CHARGE OF AN ELECTRON [e/m] BY J.J THOMSON'S EXPERIMENT

The charge per unit mass or specific charge of an electron can be measured by the apparatus below.



- ❖ Electrons are emitted thermionically by the filament and are accelerated towards the cylindrical anode.
- ❖ With no electric and no magnetic fields applied at plate X, the electron beam strikes at the screen at point O which is noted.
- ❖ A magnetic field of flux density, B is applied at X to deflect the electron beam to a point Y which is noted.
- ❖ An electric field of intensity, E is applied at X at right angles to the flux B at X and adjusted until the position of the beam on the screen is restored to point O.
- ❖ The p.d V, the plate separation, d and velocity, u of the movement of the electron beam are noted.

❖ The electric force = magnetic force

$$Beu = eE$$

$u = \frac{E}{B}$  Substituting for u into the equation  $eV_a = \frac{1}{2} mu^2$ , where  $V_a$  is accelerating p.d

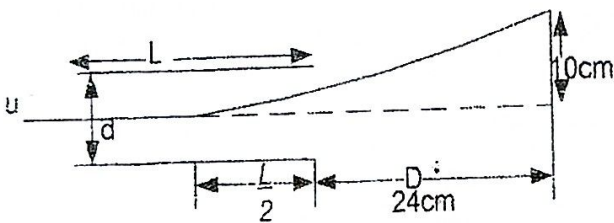
$\frac{e}{m} = \frac{E^2}{2V_a B^2}$ , which gives the charge to mass ratio of an electron

- ❖ The value of E is found from  $E = \frac{V}{d}$  where V is p.d between the deflecting plates d is their separation

**Examples**

1. Two plates are 2cm long and separated by a distance of 0.5cm in a uniform magnetic field of flux density  $4.7 \times 10^{-3} \text{ T}$ . An electron beam incident midway between the plates is deflected by magnetic field through a distance of 10cm on a screen placed 24cm from the end of the plate. When a p.d of 1000V is applied to the plate, the electron is restored to the un deflected position. Calculate the specific charge of the electron

**Solution**



$d = 0.5 \times 10^{-2} \text{ m}$      $L = 2 \times 10^{-2} \text{ m}$

$B = 4.7 \times 10^{-3} \text{ T}$      $V = 1000 \text{ V}$

For no deflection

magnetic force = Electric force

$Beu = E \cdot e$

But  $E = v/d$

$Beu = \frac{ve}{d}$

$u = \frac{v}{Bd}$

$u = \frac{1000}{4.7 \times 10^{-3} \times 0.5 \times 10^{-2}}$

$u = 4.26 \times 10^7 \text{ ms}^{-1}$

but  $\tan \theta = \frac{v_y}{u}$

$\tan \theta = \frac{vel}{mdu/u}$

$\tan \theta = \frac{vel}{mdu^2} \dots \dots \dots [1]$

also  $\tan \theta = \frac{10 \times 10^{-2}}{(D + \frac{L}{2})} \dots \dots \dots [2]$

Equating 1 and 2

$\frac{10 \times 10^{-2}}{(D + \frac{L}{2})} = \frac{vel}{mdu^2}$

$\frac{e}{m} = \frac{10 \times 10^{-2} du^2}{(D + \frac{L}{2}) lv}$

$\frac{e}{m} = \frac{10 \times 10^{-2} \times (4.26 \times 10^7)^2 \times 0.5 \times 10^{-2}}{(24 \times 10^{-2} + 1 \times 10^{-2}) \times 2 \times 10^{-2} \times 1000}$

$\frac{e}{m} = 1.8 \times 10^{11} \text{ Ckg}^{-1}$

2. An electron beam in which the electrons are  $2 \times 10^7 \text{ ms}^{-1}$  enters a magnetic field in a direction perpendicularly to the field direction. It is found that the beam can pass through without change of speed or direction. When an electric field of strength  $2.2 \times 10^4 \text{ Vm}^{-1}$  is applied in the same region at a suitable orientation.

- (i) Calculate the strength of the magnetic field  
 (ii) If the electric field were switched off, what would be the radius of curvature of the electron path. [ $e = 1.6 \times 10^{-19} \text{ C}$ ]

**Solution**

$v = 2 \times 10^7 \text{ ms}^{-1}$      $E = 2.2 \times 10^4 \text{ Vm}^{-1}$

When the beam passes without change of speed or direction then the magnetic force is equal and opposite to the electric force

$Bev = Ee$

$B = \frac{E}{v}$

$B = \frac{2.2 \times 10^4}{2 \times 10^7}$

$B = 1.1 \times 10^{-3} \text{ T}$

If the electric field were switched off, the magnetic force would provide the necessary centripetal force.

$$Bev = \frac{mv^2}{r}$$

$$r = \frac{mv}{Be}$$

$$r = \frac{9.1 \times 10^{-31} \times 2 \times 10^7}{1.1 \times 10^{-3} \times 1.6 \times 10^{-19}}$$

$$r = 1.03 \times 10^{-1} \text{m}$$

### Exercise 2

1. (a) A beam of singly ionized carbon atoms is directed into a region where a magnetic and an electric field are acting perpendicular both to each other and to the beam. The fields have intensities 0.1T and  $10^4 \text{NC}^{-1}$  respectively. If the beam is able to pass undeviated through this region. What is the velocity of the ions

(b) The beam then enters a region where a magnetic field alone is acting. As a result the beam describes an arc of radius 0.75m. Calculate the flux density of this magnetic field. [Mass of carbon atom =  $2.0 \times 10^{-26} \text{kg}$ ,  $e = 1.6 \times 10^{-19} \text{C}$ ]  
**Ans** [ $1 \times 10^5 \text{ms}^{-1}$ ,  $1.7 \times 10^{-2} \text{T}$ ]

2. Radio waves from outer space are used to obtain information about interstellar magnetic fields. These waves are produced by electrons moving in circular orbits. The radio wave frequency is same as the electron orbital frequency. [The mass of an electron is  $9.1 \times 10^{-31} \text{kg}$ , and its charge is  $-1.6 \times 10^{-19} \text{C}$ ]. If waves of frequency 1.2MHz are observed, calculate

(i) The orbital period of the electrons

(ii) The flux density of the magnetic field

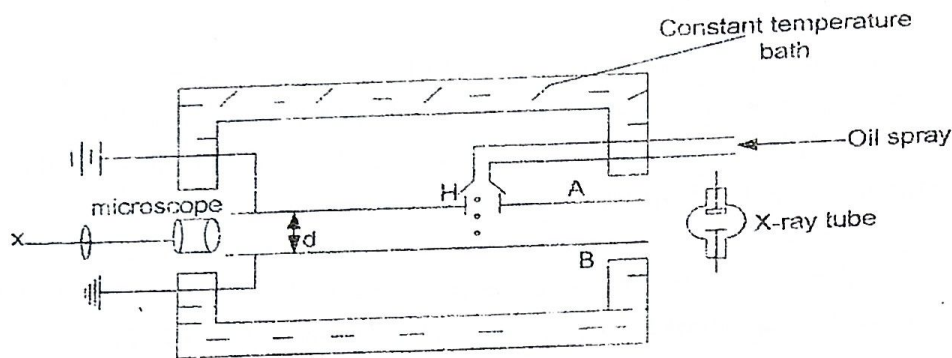
$$\text{Ans} [8.3 \times 10^{-7} \text{s}, 4.3 \times 10^{-5} \text{T}]$$

3. A beam of cathode rays is directed mid way between two parallel metal plates of length 4cm and separation 1cm, the beam is deflected though 10cm on a fluorescent screen placed 20cm beyond the nearest edge of the plate when a p.d of 200V is applied across the plate. If this deflected is annulled by a magnetic field of flux density  $1.14 \times 10^{-3}$  applied normal to electric field between the plates. Find the specific charge of the electrons. **Ans** [ $1.75 \times 10^{11} \text{Ckg}^{-1}$ ]

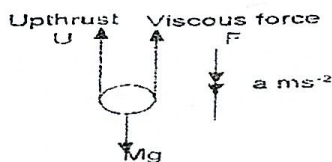
### 1.1.8: MILIKAN'S OIL DROP EXPERIMENT FOR MEASUREMENT OF $e$

When an object falls free in air it experiences a viscous drag which increases its velocity. As the object accelerates downwards, the upward force due to the viscous drag increases and eventually becomes equal to gravitational force. At this stage there is no further acceleration and the object is said to have reached its terminal velocity

#### Experimental procedure



- ❖ The apparatus is arranged as shown above, Millikan used two metal plates A and B with a small hole H in the center of the upper plate.
- ❖ A fine spray was used to atomize the oil and create tiny droplet, when one drop finds its way through the hole H, the drop is observed in a lower power travelling microscope by reflected light when the chamber is brightly illuminated.
- ❖ The drop acquires a charge by friction using x-rays and terminal velocity of selected drop is measured by timing its fall through a known distance by means of the scale on the eye piece of the microscope.



At terminal velocity  $a=0\text{m/s}^2$

$$Mg=U+F$$

$$\frac{4}{3}\pi r^3 \rho_o g = \frac{4}{3}\pi r^3 \rho_a g + 6\pi\eta r v_t$$

$$\frac{4}{3}\pi r^3 g(\rho_o - \rho_a) = 6\pi\eta r v_t \text{ ----- [1]}$$

Where  $v_t$  is terminal velocity

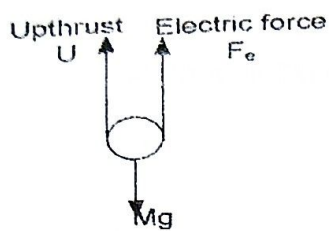
$\rho_o$  is density of oil

$\rho_a$  is density of air

from equation 1

$$r = \left[ \frac{9\eta v_t}{2g(\rho_o - \rho_a)} \right]^{\frac{1}{2}}$$

- ❖ A battery connected across plates A and B provides the field and its intensity  $E = \frac{V}{d}$  is known where  $V$  is p.d between the plates and  $d$  is the distance apart. The p.d across the plates is now varied until the drop remain stationary.



$$Mg = U + Fe$$

$$4/3 \pi r^3 \rho_o g = 4/3 \pi r^3 \rho_a g + EQ$$

$$4/3 \pi r^3 g (\rho_o - \rho_a) = EQ \text{ -----}$$

[2]

Equating equation 1 and equation 2

$$EQ = 6\pi\eta r V_t$$

$$Q = \frac{6\pi\eta r V_t}{E}$$

$$\text{But } E = v/d$$

$$Q = \frac{6\pi\eta r V_t d}{v}$$

$$r = \left[ \frac{9\eta V_t}{2g(\rho_o - \rho_a)} \right]^{1/2}$$

$$\text{Therefore } Q = \frac{6\pi\eta V_t d}{v} \left[ \frac{9\eta V_t}{2g(\rho_o - \rho_a)} \right]^{1/2}$$

### Note

- A constant temperature enclosure surrounded Millikan's apparatus in order to eliminate convection currents. It also served to shield the apparatus from draught hence drop under investigation does not drift side ways. This keeps viscosity and density of air and oil constant.
- Millikan used low- vapour pressure oil because it does not evaporate easily hence size of oil does not change appreciably.
- Stoke's law was assured in fall through a homogeneous medium

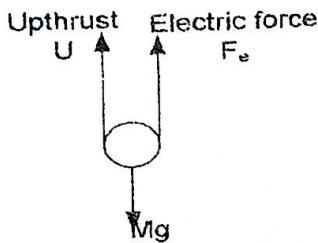
### Results

- Millikan's measured the charges of several oil drops. The results showed that the charges were always integral multiple of  $1.6 \times 10^{-19} \text{C}$  and he concluded that electric charges can never exist in fractions of this amount and that the magnitude of the electronic charge  $e$  is  $1.6 \times 10^{-19}$  i.e Millikan established that charge is quantized.
- Therefore  $Q = ne$  where  $n$  is the number of electrons

### Example

1. Oil droplets are introduced into the space between 2 flat horizontal plates set 5mm apart. The plate voltage is then adjusted exactly to 780V so that one of the droplets is held stationary. Then the plate voltage is switched off and the selected droplet is observed to fall a measured distance of 1.5mm in 11.2s. Given that the density of the oil used is  $900 \text{kgm}^{-3}$  and viscosity of air =  $1.8 \times 10^{-5} \text{Nm}^{-2}$ . Calculate the charge on the droplet and determine the number of electronic charges

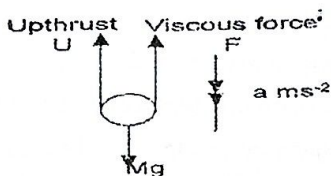
Solution



At the terminal velocity  $a=0$

$$Mg = U + Fe$$

When the plate voltage is switched off there is no electric force but only viscous drag acts



At terminal velocity  $a=0$

$$Mg = U + Fe$$

$$4/3 \pi r^3 \rho_o g = 4/3 \pi r^3 \rho_a g + 6\pi \eta r V_t$$

$$\rho_a = 0 \text{ (negligible)}$$

$$6\pi \eta r V_t = 4/3 \pi r^3 \rho_o g$$

$$r = \left[ \frac{9\eta V_t}{2g\rho_o} \right]^{1/2}$$

[2]

$$\text{but } V_t = \frac{1.5 \times 10^{-3}}{11.2}$$

$$V_t = 1.34 \times 10^{-4} \text{ s}$$

$$4/3 \pi r^3 \rho_o g = 4/3 \pi r^3 \rho_a g + EQ$$

$$EQ = 4/3 \pi r^3 \rho_o g$$

$$Q = \frac{4/3 \pi r^3 \rho_o g}{E}$$

$$\text{But } E = v/d$$

$$Q = \frac{4\pi r^3 \rho_o g d}{3v} \text{----- [1]}$$

But from equation 2

$$\therefore r = \left[ \frac{9 \times 1.8 \times 10^{-5} \times 1.34 \times 10^{-4}}{2 \times 900 \times 9.81} \right]^{1/2}$$

$$r = 1.11 \times 10^{-6} \text{ m}$$

But from Equation 1

$$Q = \frac{4 \times \frac{22}{7} \times (1.11 \times 10^{-6})^3 \times 900 \times 9.81 \times 5 \times 10^{-3}}{3 \times 700}$$

$$Q = 3.23 \times 10^{-19} \text{ C}$$

$$\therefore \text{Electronic charge} = 3.23 \times 10^{-19} \text{ C}$$

Number of charges are obtained from

$$Q = ne$$

$$n = Q/e$$

$$n = \frac{3.23 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$n = 2.01875$$

$$\bullet n \approx 2 \text{ charges}$$

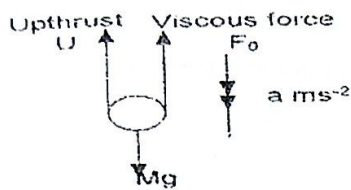
2. In measurement of the electron charge by Millikan's method, p.d of 1.5kV can be applied between horizontal parallel metal plates 12mm apart.

With the field switched off, a drop of oil of mass  $1 \times 10^{-14} \text{ kg}$  is observed to fall with a constant velocity  $400 \mu\text{ms}^{-1}$ . When the field is switched on the drop rises with constant velocity  $80 \mu\text{ms}^{-1}$ .

How many electron charges are there on the drop (Assume that air resistance is proportional to the velocity of the drop and that air buoyancy may be neglected) [electronic charge  $= 1.6 \times 10^{-19} \text{ C}$ ,  $g = 10 \text{ m/s}^2$ ]

**Solution**

when electric forces switched off only the viscous drag acts.



At terminal velocity  $a = 0 \text{ m/s}$

$$Mg = U + F_0$$

$$\frac{4}{3} \pi r^3 \rho_0 g = \frac{4}{3} \pi r^3 \rho_0 g + F_0$$

$$\text{But } \rho_0 = 0$$

$$\frac{4}{3} \pi r^3 \rho_0 g = F_0$$

$$Mg = F_0$$

But from the assumption (viscous force  $\propto$  velocity)

$$F_0 \propto V_0$$

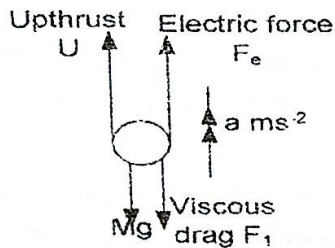
$F_0 = kV_0$  where  $k$  is a constant of proportionality

$$k = \frac{mg}{V_0}$$

$$k = \frac{1 \times 10^{-14} \times 10}{400 \times 10^{-6}}$$

$$k = 2.5 \times 10^{-10}$$

When the field is switched on both the field and drag act but in opposite direction



At terminal velocity  $a = 0$

$$U + F_e = mg + F_1$$

$$U = 0$$

$$F_e = mg + F_1$$

$$F_1 \propto V_1 \text{ and } F_e = EQ$$

$$F_1 = kV_1$$

$$\therefore EQ = mg + kV_1$$

$$\text{Also } E = \frac{V}{d}$$

$$\frac{QV}{d} = mg + kV_1$$

$$Q = \frac{(mg + kV_1)d}{V}$$

$$Q = \frac{(1 \times 10^{-14} \times 10 + 2.5 \times 10^{-10} \times 80 \times 10^{-6}) \times 12 \times 10^{-3}}{1.5 \times 10^3}$$

$$Q = 9.6 \times 10^{-19} \text{ C}$$

Number of charges is obtained from

$$Q = ne$$

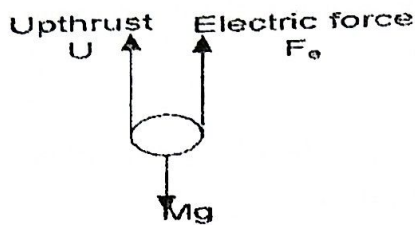
$$n = \frac{Q}{e}$$

$$n = \frac{9.6 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$n = 6 \text{ charges}$$

3. In Millikan's experiment an oil drop of mass  $1.92 \times 10^{-14} \text{ kg}$  is stationary in the space between the two horizontal plates which are  $2 \times 10^{-2} \text{ m}$  apart, the upper plate being earthed and the lower one at a potential of  $-6000 \text{ V}$ . Neglecting the buoyancy of the air. Calculate the magnitude of the charge.

**Solution**



At terminal velocity  $a=0$

$$Mg = U + F_e \text{ ----- [1]}$$

But  $u=0$  [neglecting air buoyancy]

$$EQ = mg$$

$$Q = mg/E$$

But  $E = v/d$

$$Q = \frac{mgd}{v}$$

$$Q = \frac{1.92 \times 10^{-14} \times 9.81 \times 2 \times 10^{-2}}{6000}$$

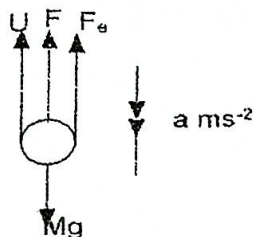
$$Q = 6.28 \times 10^{-19} C$$

4. A small oil drop carrying negative electric charges is falling in air with a uniform speed of  $8 \times 10^{-5} \text{ms}^{-1}$  between the two horizontal parallel plates. The upper plate is maintained at a positive potential relative to the lower one.

Draw a diagram showing all the forces acting on the drop, stating the cause of each force and use the following data to determine the charge on the oil drop and the number electronic charges.

[Radius of drop =  $1.60 \times 10^{-6} \text{m}$ , density of oil =  $800 \text{kgm}^{-3}$ , density of air =  $1.30 \text{kgm}^{-3}$ , viscosity of air =  $1.80 \times 10^{-5} \text{Nm}^{-2}$ , Distance between the plates =  $1 \times 10^{-2} \text{m}$ , p.d between plates =  $2 \times 10^3 \text{V}$ ,  $g = 10 \text{ms}^{-2}$ ]

Solution



- U- upthrust due to air buoyancy (upwards)
- $F_e$ -electric fields created between the plates due to the p.d
- F-viscous drag due to viscosity of air
- Mg-weight of the drop (downwards) due to gravitational pull

[ $r = 1.6 \times 10^{-6}$ ,  $\rho_0 = 800 \text{kgm}^{-3}$ ,  $\rho_a = 1.3 \text{kgm}^{-3}$ ,  $\eta = 1.8 \times 10^{-5} \text{Nm}^{-2}$ ,  $d = 1 \times 10^{-2} \text{m}$ ,  $v = 2 \times 10^{-3}$ ,  $V_p = 8 \times 10^3 \text{ms}^{-1}$ ]

At terminal velocity  $a=0$

$$Mg = U + F_e + F$$

$$\frac{4}{3} \pi r^3 \rho_0 g = \frac{4}{3} \pi r^3 \rho_a g + EQ + 6\pi \eta r v_t$$

$$EQ = \frac{4}{3} \pi r^3 \rho_0 g (\rho_0 - \rho_a) - 6\pi \eta r v_t$$

But  $E = \frac{v}{d}$

$$Q = \frac{4 \pi r^3 \rho_0 g (\rho_0 - \rho_a) - 18 \pi \eta r v_t}{3v}$$

$$Q = \frac{[4 \times \frac{22}{7} \times (1.6 \times 10^{-6})^3 \times 10(800 - 1.3) - 18 \times \frac{22}{7} \times 1.8 \times 10^{-5} \times 1.6 \times 10^{-6} \times 8 \times 10^{-3}] \times 10^{-2}}{3 \times 2 \times 10^3}$$

$$Q = \frac{10^{-2} \times [4 \times \frac{22}{7} \times (1.6 \times 10^{-6})^3 \times 10(800 - 1.3) - 18 \times \frac{22}{7} \times 1.8 \times 10^{-5} \times 1.6 \times 10^{-6} \times 8 \times 10^{-3}]}{3 \times 2 \times 10^3}$$

$$Q = 4.68 \times 10^{-19} C$$

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(b) Such an oil, of mass  $4 \times 10^{-15}$  kg is held stationary when an electric field is applied between the two horizontal plates. If the drop carried six electric charges each of value  $1.6 \times 10^{-19}$  C. Calculate the value of the electric field strength. An[ $4.2 \times 10^4 \text{Vm}^{-1}$ ]

3. An oil drop carrying a charge of  $3e$  falls under gravity with a constant velocity of  $4.6 \times 10^{-4}$  m/s between two metal plates 5mm apart. When a p.d of 4600V is applied to the plates, the drop rises steadily. Calculate:

- Radius of oil drop
- Velocity with which the oil drop rises [density of oil  $900 \text{kgm}^{-3}$ , viscosity of air =  $1.8 \times 10^{-5} \text{Nsm}^{-1}$ ] assume the effect of air buoyancy is negligible An[ $2.05 \times 10^{-6}$  m,  $6.35 \times 10^{-4} \text{ms}^{-1}$ ]

### 1.1.9: IONIC CHARGE AND IT'S RELATION TO FARADAY'S CONSTANT [F] AND ELECTRONIC CHARGE[e]

Faraday's constant [F]

> This is the charge required to liberate one mole of a monovalent element [ion] numerically equals to 96500C

Avogadro's constant [ $N_A$ ]

> This is the number of ions [particles] in one mole of a monovalent ion.

The charge carried by one mole or charge required to liberate one mole of monovalent ion is 96500C.

But in one mole we have  $6.023 \times 10^{23}$  ions.

$$1 \text{ ion} = \frac{96500}{6.023 \times 10^{23}} = 1.6 \times 10^{-19} \text{C}$$

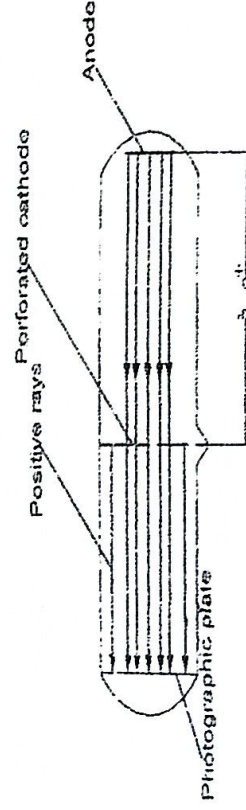
$$1 \text{ ionic charge}(e) = \frac{\text{faraday's constant}}{\text{Avogadro's constant}}$$

$$e = \frac{F}{N_A}$$

$$F = N_A e$$

### 1.1.10: POSITIVE RAYS

These are streams of positively charge particles that pass through a perforated cathode

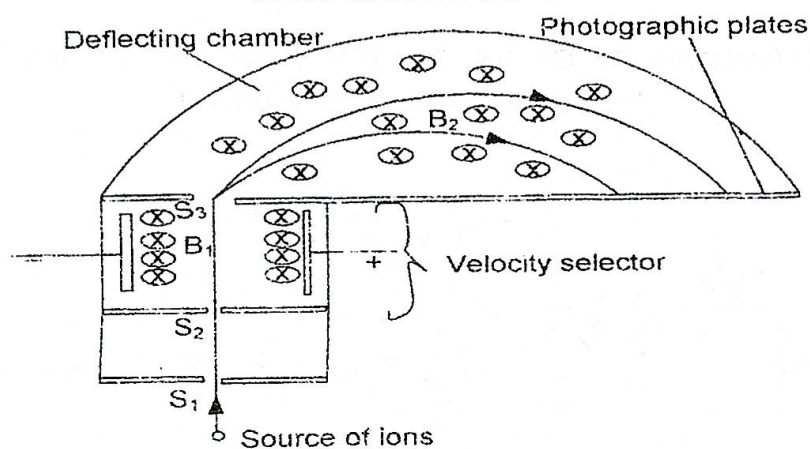


- ❖ Positive rays are produced when cathode rays in a discharge tube collide with gaseous atoms and strip off (knock out) some electrons from the atoms.
- ❖ The positive ions formed are accelerated to the cathode and these streams of positive ions constitute rays.

### 1.1.11: Properties of positive rays

- They are positively charged
- They are deflected in electric and magnetic field in a much smaller extent than cathode rays because they are more massive than cathode rays.
- They cause a fluorescence and affect a photographic plate
- They show a spectrum of different velocities
- They are dependent on the gas in the tube

### 1.1.12 DETERMINATION OF THE SPECIFIC CHARGE OF POSITIVE RAYS USING MASS SPECTROMETER



- ❖ Positive ions from a source are directed through slits S<sub>1</sub> and S<sub>2</sub> into the velocity selector where there are crossed electric field of intensity, E and magnetic field of flux density, B<sub>1</sub>
- ❖ Ions of charge, Q leave the velocity selector undeflected with velocity, u given by  $B_1 Qu = EQ$ , that is  $u = \frac{E}{B_1}$
- ❖ The selected ions pass through S<sub>3</sub> and enter a deflection chamber with a uniform magnetic field of flux density, B<sub>2</sub>
- ❖ The ions move along a semi-circular path and strike the photographic plate where they are detected. The radius, r of the path described is measured and recorded.
- ❖ In a circular path,  $B_2 Qu = \frac{mv^2}{r}$ , that is  $\frac{Q}{m} = \frac{E}{B_2 r}$

❖ On substituting for  $u$ , the charge to mass ratio is got from  $\frac{Q}{m} = \frac{E}{E_1 B_2 r}$

### DIFFERENCE BETWEEN CATHODE RAYS AND POSITIVE RAYS

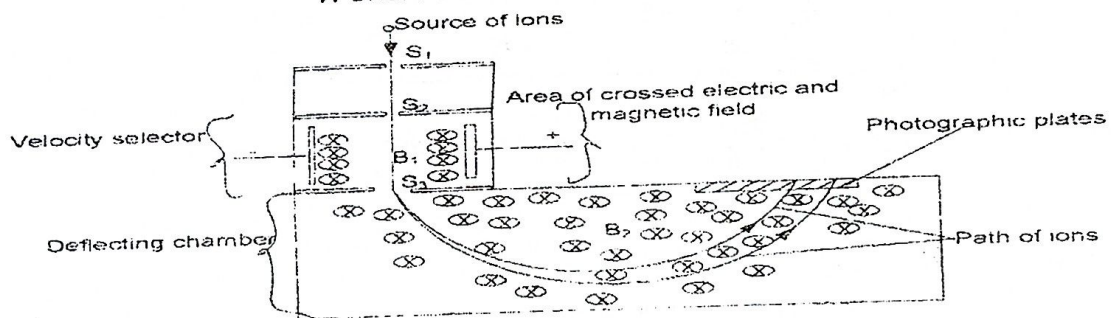
Cathode rays	Positive rays
They are light (less massive)	They are massive
They are negatively charged	They are positively charged
They travel with same velocity	Have a range of velocities
They produce x-rays when they bombard matter	They do not produce x-rays when they bombard matter

### SPECIFIC CHARGE OF AN ION

This is the ratio of charge to mass of an ion

S.I unit is  $C\ kg^{-1}$

### 1.1.12: DETERMINATION OF THE SPECIFIC CHARGE OF IONS USING A BAIN BRIDGE MASS SPECTROMETER



- ❖ Streams of ions from a source is directed through slits  $S_1$  and  $S_2$  into the velocity selector where there are crossed electric field of intensity,  $E$  and magnetic field of flux density,  $B_1$
- ❖ Ions of charge,  $Q$  pass through the selector un deflected with velocity,  $u$  given by  $B_1 Qu = E$  that is  $u = \frac{E}{B_1}$
- ❖ The selected ions pass through  $S_3$  and enter a deflection chamber with a uniform magnetic field of flux density,  $B_2$
- ❖ The ions move along a semi circular path and strike the photographic plate where they are detected. The radius,  $r$  of the path described is measured and recorded.
- ❖ In a circular path,  $B_2 Qu = \frac{mu^2}{r}$ , that is  $\frac{Q}{m} = \frac{E}{B_2 r}$
- ❖ On substituting for  $u$ , the charge to mass ratio is got from  $\frac{Q}{m} = \frac{E}{B_1 B_2 r}$

**Note**

$$r = \frac{mv}{QB_1B_2}$$

$$r \propto \frac{e}{m}$$

Since  $E$ ,  $B_1$  and  $B_2$  are constant,  $r$  depends only on the charge to mass ratio. It follows that the position at which an ion strikes the photographic plate depends on its charge to mass ratio [ions with large  $Q/m$  fall on the near end of the photographic plate].  
Bain bridge mass spectrometer is used to separate different isotopes of a single element.

**Example**

1. A beam of protons is accelerated through a p.d of 10kV and is allowed to enter a uniform magnetic field  $B$  of 0.5T perpendicular to their path. Find the radius of the circle they travel. [mass of proton =  $1.67 \times 10^{-27}$ kg,  $e = 1.6 \times 10^{-19}$ C]

**Solution**

$$B = 0.5T$$

$$m = 1.67 \times 10^{-27} \text{kg}$$

$$V_a = 10 \times 10^3 \text{V}$$

$$\frac{1}{2} mu^2 = eVa$$

$$u = \sqrt{\frac{2eVa}{m}}$$

$$u = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 10 \times 10^3}{1.67 \times 10^{-27}}}$$

$$u = 1.38 \times 10^6 \text{ms}^{-1}$$

In the magnetic field

$$\frac{mv^2}{r} = Beu$$

$$r = \frac{mu}{Be}$$

$$r = \frac{1.6 \times 10^{-27} \times 1.38 \times 10^6}{0.5 \times 1.6 \times 10^{-19}}$$

$$r = 2.9 \times 10^{-2} \text{m}$$

2. In a Bain bridge mass spectrometer singly ionized atoms of  $^{35}\text{Cl}$ ,  $^{37}\text{Cl}$  pass into the deflection chamber with a velocity of  $10^5 \text{ms}^{-1}$ . If the flux density of the magnetic field in the deflecting chamber is 0.08T, calculate the difference in the radii of the path of the ion.

**Solution**

Let  $r_1$  be radius for  $^{35}\text{Cl}$

$r_2$  be radius for  $^{37}\text{Cl}$

$$1u = 1.66 \times 10^{-27} \text{kg}$$

$$35u = (1.66 \times 10^{-27} \times 35) \text{kg}$$

$$37u = (1.66 \times 10^{-27} \times 37) \text{kg}$$

$$\frac{mv^2}{r_1} = Beu$$

$$r_1 = \frac{m_1 u}{Be}$$

$$r_1 = \frac{35 \times 1.66 \times 10^{-27} \times 10^5}{0.08 \times 1.6 \times 10^{-19}}$$

$$r_1 = 0.454 \text{m}$$

Also

$$r_2 = \frac{m_2 u}{Be}$$

$$r_2 = \frac{37 \times 1.66 \times 10^{-27} \times 10^5}{0.08 \times 1.6 \times 10^{-19}}$$

$$r_2 = 0.480 \text{m}$$

$$\text{Difference } r_2 - r_1 = 0.48 - 0.454$$

=0.026m

3. The mass of the singly charged neon isotope.  ${}^{20}_{10}\text{Ne}^+$  is  $3.3 \times 10^{-26}$  kg. A beam of these ions enters a uniform transverse magnetic field of 0.3T and describes a circular orbit of radius 0.22m. What is?

- The velocity of the ions
- The potential difference which has been used to accelerate them to this velocity [ $e=1.6 \times 10^{-19}$ C]

Solution

Magnetic field provides the centripetal force

$$Bev = \frac{mv^2}{r}$$

$$v = \frac{Ber}{m}$$

$$v = \frac{0.3 \times 1.6 \times 10^{-19} \times 0.22}{3.3 \times 10^{-26}}$$

$$v = 3.2 \times 10^5 \text{ m/s}$$

$$\frac{1}{2} mv^2 = eVa$$

$$Va = \frac{mv^2}{2e}$$

$$Va = \frac{3.3 \times 10^{-26} \times (3.2 \times 10^5)^2}{2 \times 1.6 \times 10^{-19}}$$

$$Va = 10560 \text{ V}$$

The accelerating p.d provides the kinetic energy

4. In a mass spectrograph consisting of doubly charged ions, it is required that the radius of the path of the ion with a mass number 72 be exactly 1m. If the electric field intensity across the velocity selector is  $80 \text{ Vm}^{-1}$ . What will be the magnetic field intensity across the deflection chamber [ ${}^1\text{U} = 1.67 \times 10^{-27}$ kg]

Solution

Since its doubly charge  $Q=2e$

$$r = \frac{mE}{QB_1B_2v}$$

but  $B_1=B_2=B$

$$r = \frac{mE}{QB^2}$$

$$r = \frac{72 \times 1.67 \times 10^{-27} \times 80}{2 \times 1.6 \times 10^{-19} B^2}$$

$$B = 5.5 \times 10^{-3} \text{ T}$$

#### Exercise 4

- Singly charged ions having masses close 14U and 15U are accelerated by a p.d of 800V and then passed perpendicular to the lines of force of a uniform magnetic field of flux density

0.2 Wb m<sup>-2</sup>. Calculate the radii of curvature for the path followed by the ions in the magnetic field. An[7.64cm, 7.91cm]

2. In a mass spectrometer, the magnetic flux density in both magnetic field is 0.4T and the electric field in the velocity selector is  $2 \times 10^4 \text{ Vm}^{-1}$ .

(i) What is the velocity of an ion which goes un deviated through the slit system.

(ii) The source is set to produce singly-charged ions of magnesium isotopes. Mg-24 and Mg-26.

Find the distance between the images formed by them on the photographic plate. [U=1.67x10<sup>7</sup> m/s,  $e=1.6 \times 10^{-19} \text{ C}$ ] An[5x10<sup>4</sup>m/s 5.22x10<sup>-3</sup>m]

3. A velocity selector employs a magnet that produces a flux density of 0.004T and parallel plate capacitor with a plate separation of 1cm for the electric field. What p.d must be applied to the capacitor in order to select charged particles having a speed of  $4.0 \times 10^6 \text{ ms}^{-1}$  An[160V]

4. The following measurement were made in a mass spectrograph for a beam of doubly ionized Neon atoms B=0.005T, r=0.053m, V=2.5x10<sup>4</sup>ms<sup>-1</sup>. Calculate the mass of Neon atom. An[3.4x10<sup>-26</sup>kg]

#### UNEB 2013 Q.8

(a) Explain briefly how positive rays are produced (03marks)

(b) An electron of charge,  $e$  and mass,  $m$ , is emitted from a hot cathode and then accelerated by an electric field towards the anode. If the potential difference between the cathode and anode is  $V$ , show that the speed of the electron,  $U$ , is given by

$$u = \sqrt{\left(\frac{2eV}{m}\right)}$$

(03marks)

(c) An electron starts from rest and moves in an electric field intensity of  $2.4 \times 10^3 \text{ Vm}^{-1}$ . Find the:

(i) Force on the electron. An (3.84x10<sup>-16</sup>N)  
(02marks)

(ii) Acceleration of the electron An (4.22x10<sup>14</sup>ms<sup>-2</sup>)  
(02marks)

(iii) Velocity acquired in moving through a p.d of 90V An (5.6210<sup>6</sup>ms<sup>-1</sup>) (02marks)

(d) A beam of electron each of mass,  $m$ , and charge,  $e$ , is directed horizontal metal plates separated by a distance,  $d$ .

(i) If the p.d between the plates is  $V$ , show that the deflection  $y$  of the beam is given by

$$y = \frac{1}{2m} \left(\frac{eV}{d}\right) x^2$$

Where,  $x$ , is the horizontal distance travelled

(06marks)

(ii) Explain the path of the electron beam as it emerges out of the electric field

(02marks)

UNEB 2012 Q.8

(a) (i) What are cathode rays

(01mark)

(ii) With the aid of a diagram, describe an experiment to show that cathode rays travel in a straight lines

(04marks)

(b) A beam of electrons is accelerated through a potential difference of 500V. The beam enters midway between two similar parallel plates of length 10cm and is 3cm apart. If the potential difference across the plates is 600V, find the velocity of an electron as it leaves the region between the plates.

An [ $2.96 \times 10^7 \text{m/s}$   $\theta = 63.4^\circ$ ]

(08marks)

UNEB 2011 Q.8

(c) Explain why

(i) the apparatus in Millikan's experiment is surrounded with a constant temperature enclosure

(03marks)

(ii) low vapour pressure oil is used

(02marks)

(d) In Millikan's experiment, the radius  $r$  of the drop is calculated from

$$r = \sqrt{\frac{9\eta v}{2\rho g}}$$

Where  $\eta$  is the viscosity of air and  $\rho$  is the density of oil. Identify the symbol  $v$  and describes briefly how it is measured.

(02marks)

UNEB 2010 Q.8

(a) (i) With the aid of a labeled diagram, describe what is observed when a high tension voltage is applied across a gas tube in which pressure is gradually reduce to very low values

(05marks)

(ii) Give two applications of a discharge tubes

(01mark)

(b) Describe Thomson's experiment to determine the specific charge of an electron (06marks)

(c) In Millikan's oil drop experiment, a charged oil drop of radius  $9.2 \times 10^{-7} \text{m}$  and density  $800 \text{kgm}^{-3}$  is held stationary in an electric field of intensity  $4 \times 10^4 \text{Vm}^{-1}$ .

- (i) How many electron charges are on the drop [04marks]
- (ii) Find the electric field intensity that can be applied to move the drop with velocity  $0.005\text{ms}^{-1}$  upwards (density of air  $=1.29\text{kgm}^{-3}$ ,  $\eta=1.8\times 10^{-5}\text{Nsm}^{-1}$ ) [04marks]

An[4.  $2.48\times 10^6\text{Vm}^{-1}$ ]

UNEB 2009 Q.8

- (c) State four differences between cathode rays and positive rays [02marks]
- (b) An electron having energy of  $4.5\times 10^2\text{eV}$  moves at right angles to a uniform magnetic field of flux density  $1.5\times 10^{-3}\text{T}$ . Find the:
- (i) Radius of the path followed by the electrons [04marks]
- (ii) Period of motion [03marks]
- (c) (i) Define the terms Avogadro's constant and Faraday's constant [02marks]
- (ii) Use the Avogadro's constant and faraday constant to calculate the charge on an anion of a mono atomic element An[ $1.6\times 10^{-19}\text{C}$ ] [03marks]

UNEB 2007 Q.9

- (a) What are isotopes [01marks]
- (b) With the aid of a diagram, describe the operation of brain bridge spectrometer in determining the specific charge of ions. [06marks]

UNEB 2006 Q.9

- (a) (i) A beam of electrons, having a common velocity, enters a uniform magnetic field in a direction normal to the field. Describe and explain the subsequent path of the electrons
- (ii) Explain whether a similar path would be followed if a uniform electric field were substitutes for the magnetic field [05marks]
- (b) Describe an experiment to measure the ratio of the charge to mass of an electron [7mk]
- (c) Electrodes are mounted at opposite ends of a low pressure discharge tube and a potential difference of  $1.2\text{kV}$  applied between them. Assuming that the electrons are accelerated from rest, calculate the maximum velocity which they could acquire.  
[specific electron charge  $=-1.76\times 10^{11}\text{Ckg}^{-1}$ ] An[ $2.06\times 10^7\text{ms}^{-1}$ ]  
(05marks)
- (d) (i) Give an account of the stages observed when an electric discharge passes through a gas at pressure varying from atmospheric to about  $0.01\text{mmHg}$  as air is pumped out when the p. d across the tube is maintained at extra high tension. [05marks]
- (ii) State two disadvantages of discharge tubes when used to study cathode rays [01mk]

UNEB 2005 Q.8

- (c) In the measurement of electron charges by Millikan's apparatus, a potential difference of 1.6kV is applied between two horizontal plates 14mm apart. With the *p. d* switched off, an oil drop is observed to fall with constant velocity of  $4 \times 10^{-4} \text{ms}^{-1}$ . When the potential difference is switched on, the drop rises with a constant velocity of  $8 \times 10^{-5} \text{ms}^{-1}$ . If the mass of the oil drop is  $1.0 \times 10^{-14} \text{kg}$ , find the number of electron charges on the drop. [Assume air resistance is proportional to the velocity of the oil drop and neglect the up thrust due to the air]
- [07marks] An[4]

UNEB 2004 Q.8

- (b) A beam of electrons is accelerated through a potential difference of 2000V and is directed mid way between two horizontal plates of length 5.0cm and a separation of 2.0cm. The *p. d* across the plates is 80V.
- (i) Calculate the speed of the electrons as they enter the region between the plates [03marks]
- (ii) Explain the motion of the electrons between the plates
- (iii) Find the speed of the electrons as they emerge from the region between the plates.
- An[ $2.65 \times 10^7 \text{ms}^{-1}$ ,  $2.653 \times 10^7 \text{ms}^{-1}$ ]

UNEB 2003 Q.8

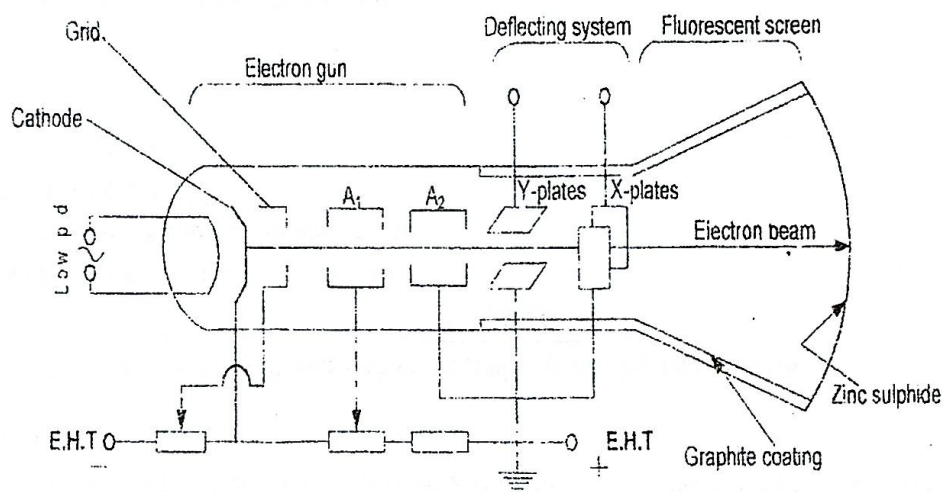
- (b) Explain how Millikan's experiment for measuring the charge of the electron proves that the charge is quantized.
- (c) A beam of positive ions is accelerated through a *p. d* of 1000V into a region of uniform magnetic field of flux density 0.2T. While in the magnetic field it moves in a circle of radius 2.3cm. Derive an expression for the charge to mass ratio of the ions and calculate its value.
- An[ $9.45 \times 10^7 \text{Ckg}^{-1}$ ]

UNEB 2002 Q.9

- (a) (i) What are cathode rays? [01mark]
- (ii) An electron gun operating at  $3 \times 10^3 \text{V}$  is used to project electrons into the space between two oppositely charged parallel plates of length 10cm and separation 5cm, calculate the deflection of the electrons as they emerge from the region between the charged plates when the *p. d* is 1000V.
- An[ $1.66 \times 10^2 \text{m}$ ]
- [04marks]

## CHAPTER 2: ELECTRONIC DEVICES

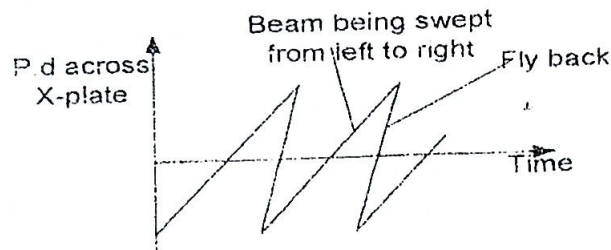
### 2.1.0: THE CATHODE RAY OSCILLOSCOPE (CRO)



- ❖ Cathode is heated and emits electrons thermionically. The electrons are focused and accelerated by the anodes to the screen. Grid controls number of electrons reaching the screen hence brightness of the spot.
- ❖ Y-plates deflect electron beam vertically and X-plates deflect electron beam horizontally.
- ❖ The screen glows to form a spot when struck by electrons. Graphite coating shields electrons from external fields and conducts stray electrons to the earth.

#### Time base

- The time base is connected to the x-plates and provides a saw tooth p.d that sweeps the electron beam from left to right of the screen at constant speed.



- The saw tooth then returns the beam to the initial position at the extreme left of screen almost instantaneously. The time taken for this right to left sweep is called fly-back time.

### 2.1.1: USES OF THE CRO

- ❖ It is used to display wave forms, the signal to be investigate is connected to the y-plate and the time base to the x-plate
- ❖ It measures voltage (AC or DC)
- ❖ Measures frequencies
- ❖ Used to measure phase differences
- ❖ Measures small time intervals

### 2.1.2: Advantages of CRO over a voltmeter

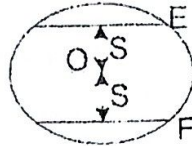
- ❖ It measures both AC and DC voltage unlike a voltmeter measures only D.C voltage unless a rectifier is used
- ❖ It has an instantaneous response since the electron beam behaves as a pointer of negligible inertia.
- ❖ It draws very little current since it has nearly infinite resistance to DC and a very high impedance to AC
- ❖ It has no coil to burn out.

### 2.1.3: APPEARANCE OF ELECTRON SPOT ON THE SCREEN

- ❖ When a signal is not connected to the y-plate and time base switched off, a bright spot is formed on the screen.



- ❖ When the *d.c.* voltage is connected to the y-plate such that the top plate is positive the line is displaced to E. If the lower plate is positive the line is displaced to F. the displacement in either case is proportional to the *d.c.* voltage applied.

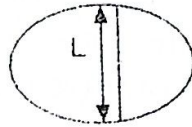


If in the CRO the gain control of the y-deflection amplifier is  $V_g \text{ Vcm}^{-1}$  then

$$V \propto S$$

$$V = V_g S$$

- ❖ When A.C is connected to y-plate and time base switched off. The spot is a vertical line



The length L represents peak to peak voltage

$$2V_0 \propto L$$

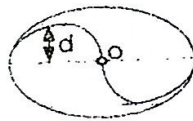
$$2V_0 = V_g L$$

where  $V_0$  is peak voltage

$$V_0 = \frac{V_g L}{2}$$

Also  $V_{r.m.s} = \frac{V_0}{\sqrt{2}}$

- ❖ When the A.C is connected to Y-plate and time base also switched on the a stationary wave is obtained



$$V_0 \propto d$$

$$V_0 = V_g d$$

$$V_{r.m.s} = \frac{V_0}{\sqrt{2}}$$

- ❖ Y-plate off and time base on



Horizontal line formed at the centre of the screen

## Examples

1. If the voltage gain is  $20\text{Vcm}^{-1}$  and an AC voltage connected to Y-plate produces a vertical trace of  $12\text{cm}$  long with time base off. Find the peak value of the voltage and its r.m.s value

Solution

$$2V_0 = V_g L$$

$$V_0 = \frac{20 \times 12}{2}$$

$$V_0 = 120\text{V}$$

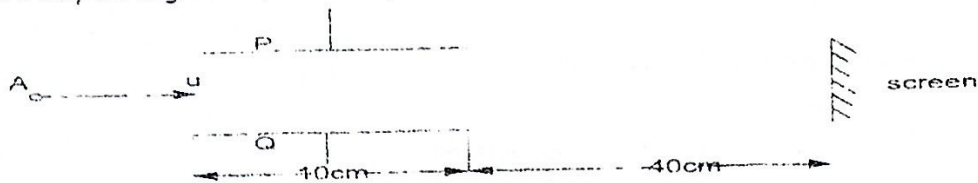
$$\text{Peak value} = 120\text{V}$$

$$V_{r.m.s} = \frac{V_0}{\sqrt{2}}$$

$$V_{r.m.s} = \frac{120}{\sqrt{2}}$$

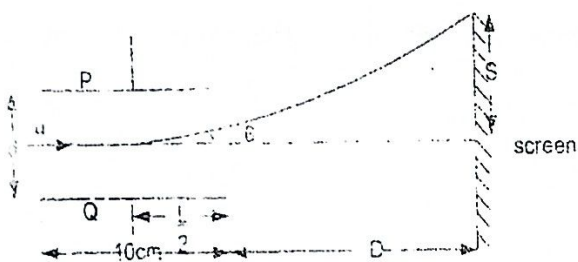
$$V_{r.m.s} = 84.85\text{V}$$

2. The sketch below shows part of the deflecting system of a cathode ray oscilloscope. At the point A, a beam of electrons has a velocity of  $3 \times 10^7 \text{ms}^{-1}$  along the axis of the system. The plates which are  $4\text{cm}$  apart provides a uniform electric field in the space between them. Edge effects may be neglected, P is at a potential of  $+200\text{V}$  with respect to Q



Find the position at which the electron beam strikes the screen ( $e/m = 1.76 \times 10^{11} \text{Ckg}^{-1}$ )

Solution



$$L = 10 \times 10^{-2} \text{m}, d = 4 \times 10^{-2} \text{m}, D = 40 \times 10^{-2} \text{m},$$

$$V = 200, e/m = 1.76 \times 10^{11} \text{Ckg}^{-1}$$

$$\tan \theta = \frac{s}{D + \frac{l}{2}} \quad [1]$$

$$\text{But also } \tan \theta = \frac{v_y}{u} \quad [2]$$

Equating 1 and 2

$$\frac{s}{D + \frac{l}{2}} = \frac{v_y}{u}$$

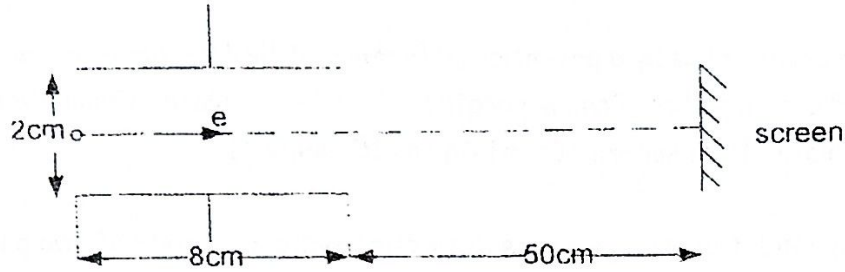
$$S = \frac{v_y}{u} \left( D + \frac{l}{2} \right)$$

$$\text{But } v_y = \frac{vel}{mdu^2} \left( D + \frac{l}{2} \right)$$

$$S = \frac{200 \times 1.76 \times 10^{11} \times 10 \times 10^{-2} \times \left( D + \frac{l}{2} \right)}{1.76 \times 10^{11} \times (3 \times 10^7)^2}$$

$$S = 4.4 \times 10^{-2} \text{m}$$

3. The figure below shows two metal plates  $8\text{cm}$  long and  $2\text{cm}$  apart. A fluorescence screen is placed  $50\text{cm}$  from the one end of the plates. An electron of kinetic energy  $6.4 \times 10^{-16} \text{J}$  is incident midway between the plates



Calculate the p.d which must be applied across the plates to deflect the electron 4.2cm on the screen. Assume that the space through which to electron moves is evacuated.  
 $[e=1.6 \times 10^{-19} \text{C}, m=9.1 \times 10^{-31} \text{kg}]$

**Solution**

$$Ke = \frac{1}{2} mu^2$$

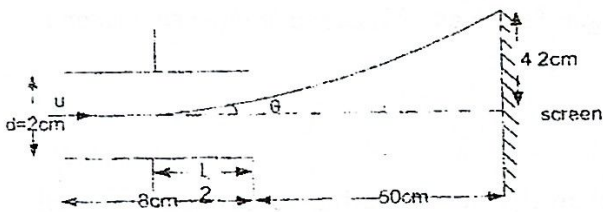
$$6.4 \times 10^{-16} = \frac{1}{2} mu^2$$

$$U = \sqrt{\frac{2 \times 6.4 \times 10^{-16}}{9.1 \times 10^{-31}}}$$

$$U = 3.75 \times 10^7 \text{ms}^{-1}$$

$$d = 2 \times 10^{-2}, D = 50 \times 10^{-2}, L = 8 \times 10^{-2}$$

$$\frac{L}{2} = 4 \times 10^{-2} \text{m}, S = 4.2 \times 10^{-2} \text{m}$$



$$\tan \theta = \frac{S}{D + \frac{L}{2}}$$

$$\tan \theta = \frac{4.2 \times 10^{-2}}{(50 \times 10^{-2} + 4 \times 10^{-2})} \text{----- [1]}$$

Also

$$\tan \theta = \frac{v_y}{u}$$

$$\tan \theta = \frac{vel}{mdu^2}$$

[2]

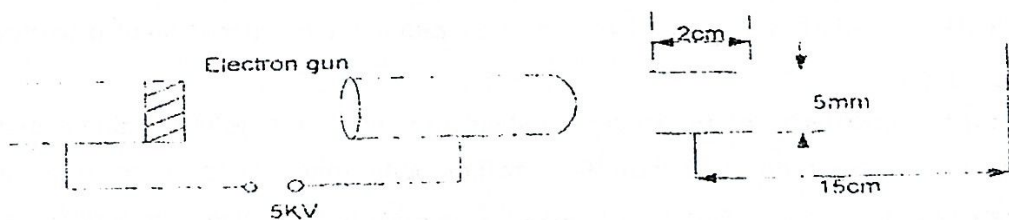
$$\frac{vel}{mdu^2} = \frac{4.2 \times 10^{-2}}{(50 \times 10^{-2} + 4 \times 10^{-2})}$$

$$V = \frac{4.2 \times 10^{-2} \times 9.1 \times 10^{-31} \times 2 \times 10^{-2} \times (3.75 \times 10^7)^2}{(1.6 \times 10^{-19} \times 8 \times 10^{-2})}$$

$$V = 83.98 \text{V}$$

#### Exercise 4

1.



Calculate the deflection sensitivity (deflection of spot in mm per volt potential difference) of the cathode ray tube from the following data.

Electrons are accelerated by a potential difference of 5kV between the cathode and anode. [length of deflection plates = 2cm, separation of deflector plates = 5mm, distance of mid point of deflector plates from screen = 15cm] An  $[6 \times 10^{-2} \text{mmV}^{-1}]$

2. In one type of CRO, the electrostatics deflecting system consists of two parallel metal plates of length 2cm and 0.5cm apart the centre of the plates is situated 15cm from the screen and p.d of 80V is applied between the plates to provide a uniform electric region between the plates at right angles to the field. Calculate.

- (i) Speed with which electrons leave the plates
- (ii) Deflection of electron beam on the screen.  $[e=1.6 \times 10^{-19} \text{C}, m=9.1 \times 10^{-31} \text{kg}]$

An  $[3.11 \times 10^7 \text{ms}^{-1}, 8.76 \times 10^{-3} \text{m}]$

UNEB 2011 Q.8

(a) (i) Describe with the aid of a well labeled diagram, the structure and mode of operation of CRO

[06marks]

(ii) State the advantages of CRO over a moving coil voltmeter

[02marks]

UNEB 2004 Q.8

(a) (i) Describe with the aid of a labeled diagram the main features of a cathode ray oscilloscope (CRO)

[08marks]

(ii) State two uses of a CRO

[01marks]

(iii) The gain control of a CRO is set on  $0.5 \text{Vcm}^{-1}$  and an alternating voltage produces a vertical trace of 2cm along with the time base off. Find the root mean square value of the applied voltage.

An  $[0.354 \text{V}]$

[02marks]

UNEB 2005 Q.9

(b) Describe, with the aid of a diagram, the structure and mode of operation of a cathode ray oscilloscope (CRO)

[06marks]

(c) A CRO has its y sensitivity set to  $20 \text{Vcm}^{-1}$ , a sinusoidal input voltage is suitably applied to give a steady time base switched on so that the electron beam takes 0.01s to traverse the screen.

If the trace seen has a peak -to-peak height of 4cm and contains two complete cycles Find the

(i) r.m.s value of the input voltage

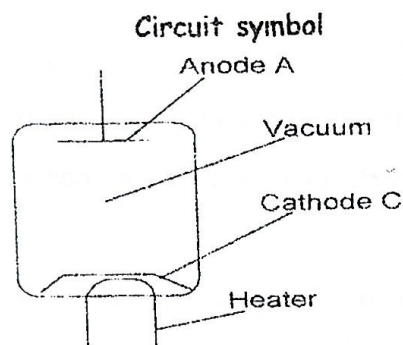
[03marks]

(ii) frequency of the input signal  
[02marks]

An[14.14V, 200Hz]

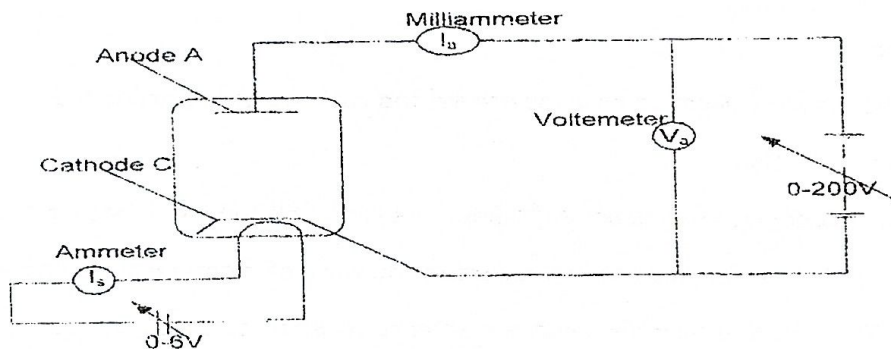
### 2.2.0: THERMIONIC DIODE

A thermionic diode is a device which is used to change alternating voltage to direct voltage. This process is called rectification



A diode consists of cathode (c) and a metal Anode (A). these two elements constitute the electrodes of the valve which are placed inside an evacuated glass envelope

### 2.2.1: DIODE CHARACTERISTICS CIRCUIT



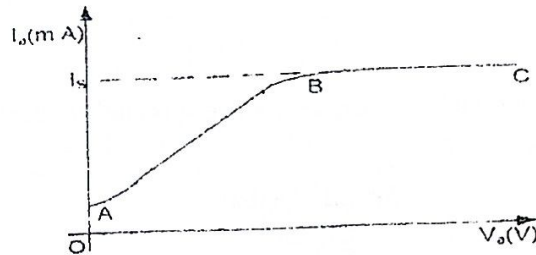
- ❖ When the cathode filament is heated with a low p.d electrons are emitted thermionically.
- ❖ If the anode A is kept at **positive potential** ( $V_a$ ) with respect to the cathode c, some electrons move from cathode to anode and the diode conducts due to **attractive effect** on them.
- ❖ However if anode is at **negative potential** with respect to the cathode, no electrons reach the anode and the diode does not conduct due to the **repulsive effect** on them.

The diode therefore allows current to flow in only one direction.

Anode current ( $I_a$ ) which flows is read from the Milliammeter and the Voltmeter reading gives anode potential ( $V_a$ )

Therefore anode resistance  $R_a = \frac{\Delta V_a}{\Delta I_a}$

### 2.2.2: A graph of $I_a$ against $V_a$ (diode characteristic graph )



- ❖ Along OA, electrons are released with some velocities that some reach the anode. Though the majority have low K.E and do not reach the anode (**space charge**) hence a small current is detected.
- ❖ AB, as voltage increases the number of electrons reaching the anode increases and therefore increase in current.
- ❖ BC, As anode potential is increased further, all the electrons leaving the cathode reach the anode. This is the **saturation region**.

#### Note:

##### Space charge:

This is the cloud of negative charges around the cathode at low anode p.d

##### Space charge limitation:

When the anode potential is not sufficient to attract all the electrons emitted from the cathode, emitted electrons tend to collect in the form of electron cloud above the cathode. This cloud of negative charge electrons constitutes space charge. Space charge exerts a repelling force on electrons being emitted from the cathode there by decreasing the anode current.

##### Saturation

This occurs when the anode potential is increased to a value such that the number of emitted electrons is equal to number of collected electrons

## 2.3.0: A DIODE AS A RECTIFIER

### 2.3.1: RECTIFICATION

Rectification involves converting Alternating current to Direct current.

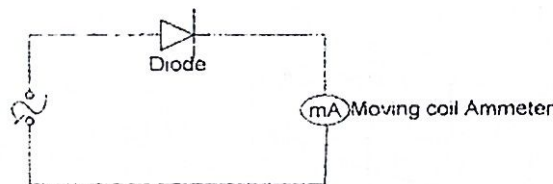
This can be done by use of

❖ Thermionic diodes.

❖ Semiconductor diode

When a rectifier is connected to a supply its supposed to conduct and when it does so its said to be forward biased. And when connected in a reverse way it fails to conduct therefore its said to be reverse-biased.

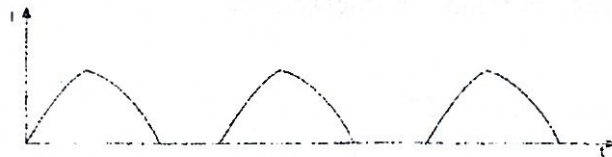
#### a) Half wave Rectification



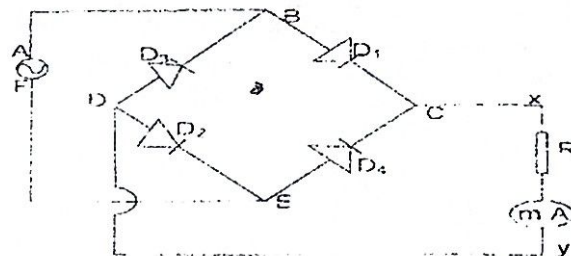
A.c to be measured is first passed through the rectifier which converts it to d.c. The d.c obtained is then measured using a moving coil ammeter.

N.B: The Arrow head in the rectifier symbol shows the direction of flow of current through the circuit.

A graph of  $I$  against  $t$  is drawn

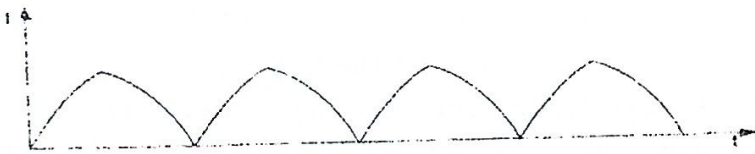


#### b) Full wave rectification



Four diodes are arranged in a bridge network as shown above. If A is positive during the first half cycle, diodes 1 and 2 conduct and current takes the path ABCRDEF

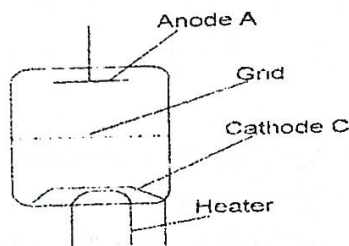
During the next half cycle when F is positive and A is negative diodes  $D_3$  and  $D_4$  conduct while  $D_1$  and  $D_2$  do not conduct in this cycle and current (I) flows through path FECRDBA. The current through R is in the same direction throughout and it can be measured by moving coil ammeter.



UNEB NO 6(e) 1999

With the aid of a circuit diagram, describe the mode of action of a full wave rectifier.

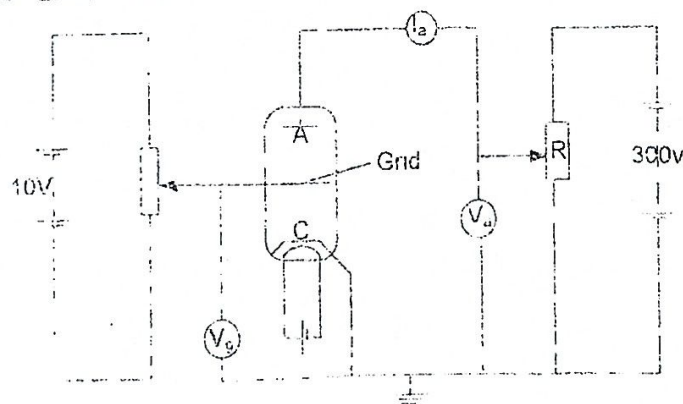
#### 2.4.0: THE TRIODE



It consists of three electrodes, with grid placed between the cathode and anode.

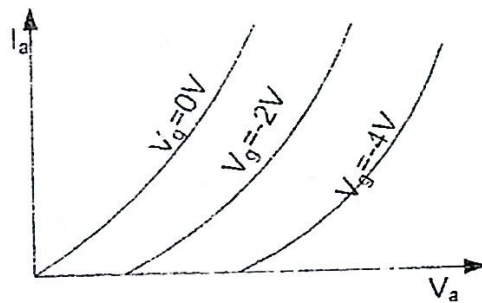
The relationship between the grid potential ( $V_g$ ), Anode potential ( $V_a$ ) and anode current ( $I_a$ ) for a given heating current gives the triode characteristics.

#### 2.4.1: TRIODE CHARACTERISTICS CIRCUIT



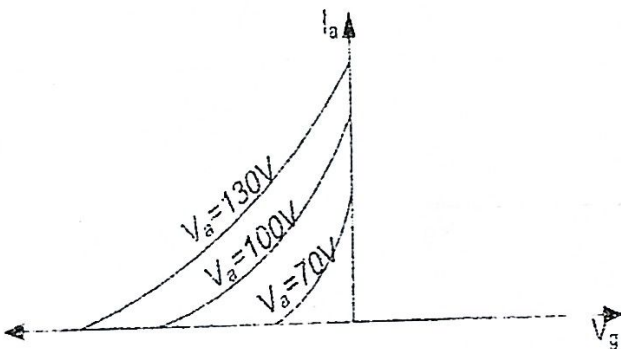
The circuit can be used to generate a set of readings to give a triode characteristics.

### 2.4.2: TYPICAL ANODE CHARACTERISTICS



As the anode voltage increases, the anode current also increases

### 2.4.3: TYPICAL MUTUAL CHARACTERISTICS



resultant electric field intensity at the cathode and hence no electrons move through the grid and hence the anode current is zero ( $I_a = 0$ )

- ❖ As the negative voltage increases and reaches a certain value, the attraction effect of the positive anode overcomes repulsive effect of the grid and electrons now reach the anode.

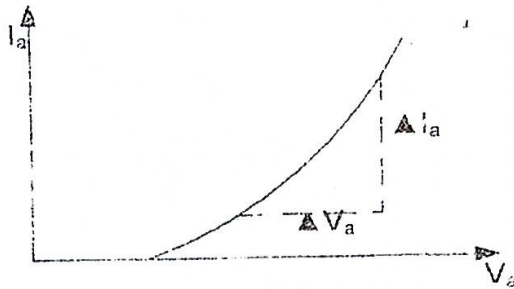
- ❖ When the anode voltage is 70V (for example) the negative voltage on the grid creates the

### 2.4.4: TRIODE CONSTANTS

#### 1: ANODE RESISTANCE ( $R_a$ )

It is defined as  $R_a = \frac{\Delta V_a}{\Delta I_a}$  at constant  $V_g$

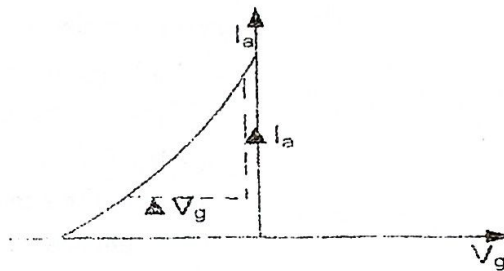
$V_a$  is anode voltage and  $I_a$  is anode current which can be obtained from the straight part of the anode characteristics curve



## 2: MUTUAL CONDUCTANCE ( $g_m$ )

It is defined as  $g_m = \frac{\Delta I_a}{\Delta V_g}$  for constant  $V_a$

$V_g$  is grid voltage



## 3: AMPLIFICATION FACTOR ( $\mu$ )

It is defined as  $\mu = \frac{\Delta V_a}{\Delta V_g}$  for constant  $I_a$

### 2.4.5: RELATION BETWEEN $R_a$ , $g_m$ AND $\mu$

$$R_a = \frac{\Delta V_a}{\Delta I_a}$$

$$\Delta V_a = R_a \Delta I_a$$

$$\text{Also } g_m = \frac{\Delta I_a}{\Delta V_g}$$

$$\Delta V_g = \frac{\Delta I_a}{g_m}$$

$$\text{But } \mu = \frac{\Delta V_a}{\Delta V_g}$$

$$\mu = \frac{R_a \Delta I_a}{\left(\frac{\Delta I_a}{g_m}\right)}$$

$$\mu = R_a \Delta I_a \times \frac{g_m}{\Delta I_a}$$

$$\boxed{\mu = R_a \times g_m}$$

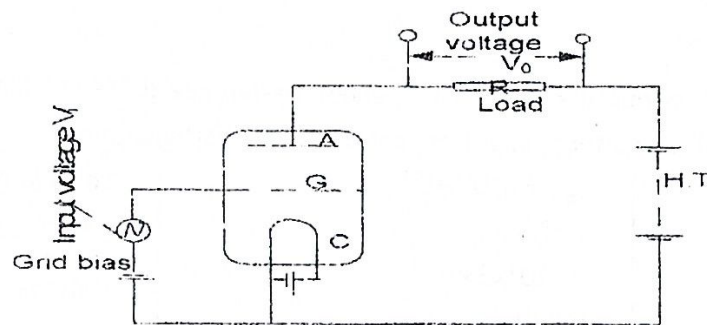
### 2.4.6: USES OF A TRIODE

- It is used as an amplifier in a radio receiver
- It is used as an oscillator in a radio transmitter
- It is used as a detector in a radio receiver

### 2.4.7: TRIODE AS A SINGLE STAGE VOLTAGE AMPLIFIER

The amplifiers are used to boost the level of small voltage /current in radio receivers. Signals in form of alternating currents (voltages) are usually very weak and therefore need amplification. This can be achieved by means of a triode

- Single stage amplification means signals to be amplified, pass through the amplifying circuit only once.



- The alternating input is supplied in the grid cathode circuit while the out put is taken across a high resistance in series with the anode.
- A triode should not only increase the value of alternating voltage but also give a wave for which is also a replication of the input without any distortion.
- A small negative voltage called a grid bias in the grid cathode is to prevent the distortion.

#### 2.4.8: VOLTAGE GAIN

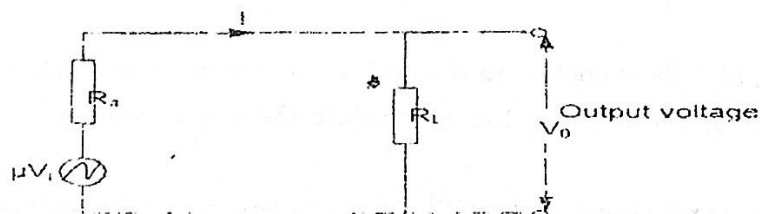
This is the ratio of output voltage  $V_o$  to the input voltages ( $V_i$ )

$$\text{Voltage gain} = \frac{V_o}{V_i}$$

#### 2.4.9: EQUIVALENT CIRCUIT OF TRIODE AS AN AMPLIFIER

To obtain the magnitude of voltage gain, the triode circuit is replaced with an equivalent circuit.

The input voltage  $V_i$  in the grid cathode circuit is equivalent to ( $\mu V_i$ )



Total resistance in the circuit  $= R_a + R_L$

E. M. F of the source  $= \mu V_i$

Therefore  $\mu V_i = I(R_a + R_L)$

$$I = \frac{\mu V_i}{R_a + R_L} \quad [1]$$

Output voltage  $V_o = IR_L$

Therefore  $V_o = \frac{\mu V_i R_L}{(R_a + R_L)}$

$$V_o = \frac{\mu V_i R_L}{R_a + R_L} \quad [2]$$

$$\text{Voltage gain} = \frac{V_0}{V_i}$$

$$\text{Voltage gain} = \frac{\mu V_i R_L}{R_a + R_L} / V_i$$

$$\text{Voltage gain} = \frac{\mu R_L}{R_a + R_L}$$

$$\text{Or } \frac{V_0}{V_i} = \frac{\mu R_L}{R_a + R_L}$$

### Example

1. A triode with mutual conductance of  $3\text{mA V}^{-1}$ , anode resistance  $10^4 \Omega$  and load resistance  $20000 \Omega$  is used as single stage voltage amplifier, calculate the voltage gain.

<p><b>Solution</b></p> <p><math>g_m = 3\text{mA V}^{-1}</math>,</p> <p><math>R_a = 10^4 \Omega</math></p> <p><math>R_L = 20000 \Omega</math>,</p>	<p><math>g_m = 3 \times 10^{-3} \text{AV}^{-1}</math></p> <p><math>\mu = R_a \times g_m</math></p> <p><math>\mu = 10^4 \times 3 \times 10^{-3}</math></p> <p><math>\therefore \mu = 30</math></p>	<p><math>\text{voltage gain} = \frac{\mu R_L}{R_a + R_L}</math></p> <p><math>= \frac{30 \times 20000}{10^4 + 20000}</math></p> <p><b>Voltage gain = 20</b></p>
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2. Calculate the voltage gain for triode whose amplification factor ( $\mu$ ) is 80 and whose anode slope resistance  $R_a$  is  $10^4 \Omega$  when used with an anode load of  $20,000 \Omega$  in a single stage voltage amplifier

<p><b>Solution</b></p> <p><math>\mu = 80</math>,</p> <p><math>R_a = 10^4 \Omega</math></p>	<p><math>R_L = 20000 \Omega</math></p> <p><math>\text{voltage gain} = \frac{\mu R_L}{R_a + R_L}</math></p> <p><math>= \frac{80 \times 20000}{10^4 + 20000}</math></p>	<p><b>Voltage gain = 53.3</b></p>
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3. A triode valve with an anode resistance of  $3 \times 10^3 \Omega$  is used as an amplifier. A sinusoidal alternating signal of amplitude  $0.5\text{V}$  is applied to the grid of the valve. Find the r.m.s value of the output voltage if the amplification factor is 15 and anode load is  $50\text{k} \Omega$

<p><b>Solution</b></p> <p><math>\frac{V_0}{V_i} = \frac{\mu R_L}{R_a + R_L}</math></p> <p><math>V_0 = \left( \frac{\mu R_L}{R_a + R_L} \right) V_i</math></p>	<p><math>V_0 = \frac{15 \times 0.5 \times 50 \times 10^3}{[50 \times 10^3 + 3 \times 10^3]}</math></p> <p><math>V_0 = 7.075\text{V}</math></p> <p><math>\text{r.m.s} = \frac{V_0}{\sqrt{2}}</math></p>	<p><b>r.m.s = 5.003v</b></p>
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### Exercise 5

- A sinusoidal voltage of  $0.2\text{V}$  is applied to the grid of the triode of an amplification factor 10. If the anode resistance of the triode is  $15\text{k}\Omega$ . Calculate the output voltage. **An[0.125V]**
- A triode with mutual conductance of  $4\text{mA V}^{-1}$  and anode resistance  $R_a = 5\text{k}\Omega$  is connected to a load resistance of  $10\text{k}\Omega$ . Estimate the out voltage obtained from an alternating output signal of  $25\text{mV}$ . **An[0.333V]**
- A triode with mutual conductance  $4\text{mA V}^{-1}$  and the anode resistance  $15\text{k}\Omega$  and a load resistance  $30\text{k}\Omega$  is used as a single stage amplifier. Calculate the voltage gain. **An[40]**.

## 2.5.0: THE TRANSISTOR

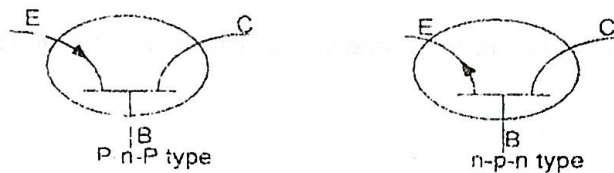
A transistor is made from three layers of p and n-semi conductor called the emitter (E), base (B) and collectors (C). the base is thinner. It can be pnp type or npn type transistor

The junction transistor is called a bipolar transistor because its action is due to two charge carriers i.e the electrons (-) and the holes (+).

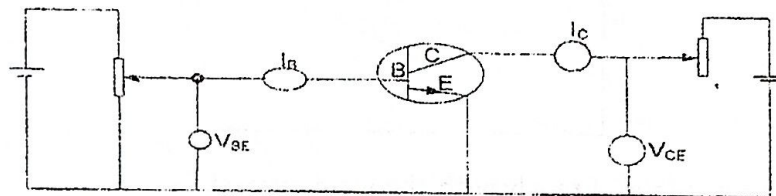
There are two types of junction transistor

- i) n-p-n transistor where the electrons, are the majority charge carriers
- ii) p-n-p transistor where the holes are the majority charge carriers.

Symbols



### 2.5.1: Common - emitter mode (CE mode) for n-p-n transistor

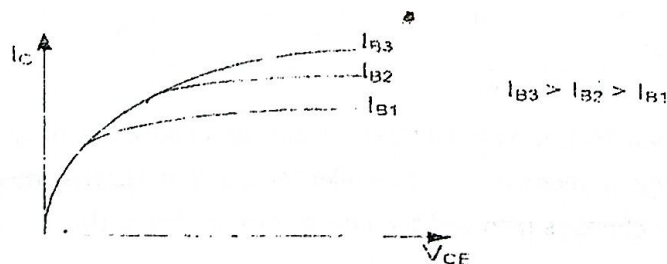


The circuit can be used to obtain three types of characteristics

- (1) Out put characteristics
- (2) Input characteristics
- (3) Transfer characteristic

### 2.5.2: Collector current ( $I_C$ ) Against collector emitter voltage ( $V_{CE}$ )

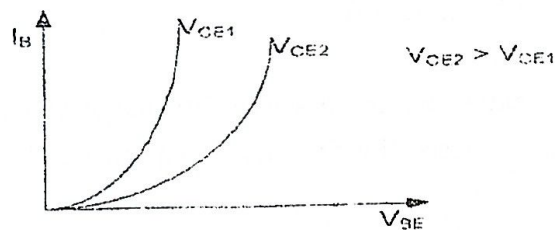
Output characteristics



For small  $V_{CE}$  the output current  $I_C$  increases slightly with  $V_{CE}$ .

At Higher  $V_{CE}$ ,  $I_C$  varies linearly with  $V_{CE}$  for a given base current  $I_B$ . the linear part of the characteristics is used as amplifier circuit so that the output voltage variation is undistorted.

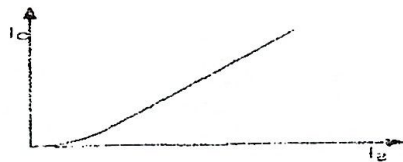
### 2.5.3: $I_B$ Against $V_{BE}$ (Input characteristics)



$I_B$  varies exponentially with  $V_{BE}$  i.e its input characteristics is non linear for a given  $V_{CE}$

Input resistance  $R_B = \frac{\Delta V_B}{\Delta I_B}$

### 2.5.4: A graph of $I_C$ Against $I_B$ (Transfer characteristics)



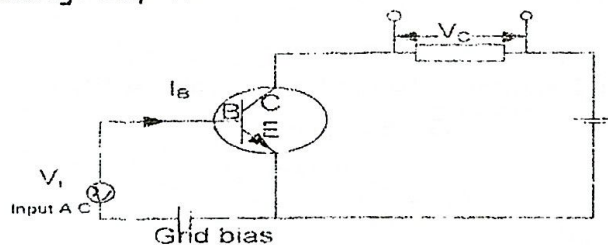
Output current  $I_C$  varies fairly linearly with the input current  $I_B$ .

Current transfer ratio  $B$  or (current gain)

$B = \frac{\text{output current}}{\text{input current}}$

$B = \frac{\Delta I_C}{\Delta I_B}$

### 2.5.5: Transistor as a voltage amplifier



The small A.C voltage  $V_1$  is applied to the base emitter circuit and causes small changes of base current  $\Delta I_B$  which produces large changes  $\Delta I_C$  in the collector current flowing through the load  $R$  which converts these current changes into voltage changes which form the A.C output voltage  $V_0 = \Delta I_C R$ .



- (c) (i) Define space charge as applied to thermionic diode [01marks]  
(ii) Draw anode current-anode voltage curve of thermionic diode for two different filament currents and explain their main feature [06marks]

UNEB 2012 Q 10

- (a) Define the terms below as applied to a triode  
(i) Space charge [01marks]  
(ii) Amplification factor [01marks]  
(iii) Mutual conductance [01marks]
- (b) With the aid of a labeled diagram explain full wave rectification [07marks]
- (c) Derive an expression for the amplification factor  $\mu$  in terms of anode resistance  $R_a$  and mutual conductance  $g_m$  for a triode valve. [03marks]
- (d) A triode with mutual conductance  $3\text{mA V}^{-1}$  and anode resistance of  $10\text{k}\Omega$  is connected to a load resistance of  $20\text{k}\Omega$ . Calculate the amplitude of the output signal, if the amplitude of the input signal is  $25\text{mV}$  An[0.5V] [04marks]
- (e) i) Sketch the output characteristics of a transistor [02marks]  
(ii) Identify on the sketch in e(i) the region over which the transistor can be used as an amplifier. [01mark]

UNEB 2008 Q.10

- (a) Describe the mechanism of thermionic emission [03marks]
- (b) Explain the following terms as applied to a vacuum diode  
(i) space charge limitation [03marks]  
(ii) saturation [01mark]  
(iii) rectification [02marks]
- (c) Sketch the current-potential difference characteristics of thermionic diode for two different operating temperatures and explain their main features [05marks]
- (d) (i) A triode valve with anode resistance of  $3 \times 10^3 \Omega$  is used as an amplifier. A sinusoidal alternating signal of amplitude  $0.5\text{V}$  is applied to the grid of the valve. Find the r.m.s value of the output voltage if the amplification factor is 15 and anode load is  $50\text{k}\Omega$ . An[5.003V] [05marks]
- (ii) Draw an equivalent circuit of a triode as a single stage-amplifier [01marks]

UNEB 2007 Q.8

- (a) Describe briefly the mechanism of thermionic emission [02marks]  
(b) (i) Draw a labeled circuit to show a triode being used as single-stage voltage amplifier

[03marks]

(ii) With the aid of an equivalent circuit of the triode as an amplifier, obtain an expression for the voltage gain [04marks]

(iii) A triode with mutual conductance  $3 \times 10^{-3} \text{AV}^{-1}$  and anode resistance of  $1 \times 10^4 \Omega$  is used as a single-stage amplifier. If the load resistance is  $3 \times 10^4 \Omega$  calculate the voltage gain of the amplifier. An[22.5] [05marks]

- (c) (i) Describe the structure of a junction transistor [02marks]  
(ii) Sketch and describe the collector-current against the collector-emitter voltage characteristics of a junction transistor [03marks]

UNEB 2004 Q.10

- (a) (i) Explain briefly the mechanism of thermionic emission [02marks]  
(ii) Draw labeled diagram of the circuit used to determine the anode current and anode voltage characteristics of thermionic diode [02marks]  
(iii) Sketch the characteristics expected in a)i) at constant filament current and account for its special features [04marks]

UNEB 2003 Q.9

- (a) (i) What is meant by thermionic emission [04marks]  
(ii) Sketch the current-potential difference characteristics of thermionic diode for two different operating temperatures and explain their main features. [05marks]  
(iii) Describe one application of a diode [02marks]

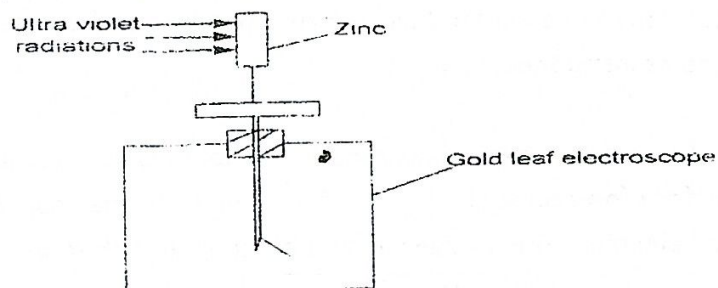
### CHAPTER 3: PHOTOELECTRIC EMISSION

It's defined as a process by which electrons are released from a clean metal surface when irradiated by electromagnetic radiations (light) of high enough frequency (energy).

The electrons emitted this way are called **photo electrons**.

The radiation falling on the metal surface is absorbed by the electrons and becomes internal energy which is sufficient to enable them overcome the inward attraction for the electrons to get loose and fly off the metal surface.

#### 3.1.0: EXPERIMENT TO DEMONSTRATE PHOTO ELECTRIC EFFECT



- ❖ A cleaned zinc plate is placed on a cap of a negatively charged gold leaf electroscope.
- ❖ When ultraviolet radiations are directed on to the plate, the leaf is seen to collapse gradually.

- ❖ This is because the the plate and the cap lost charges (electrons). So the magnitude of the negative charge at the leaf and gold plate decreases thereby decreasing the divergence of the leaf gradually.

Note:

- (1) If the intensity of UV radiation is increased for the positively charged electroscope there is no change on the divergence of the leaf. But for a negatively charged electroscope, the leaf collapses fast since the number of electrons emitted per unit time (photo current) from the zinc plate increases with intensity.
- (2) If infrared radiations are used instead of UV no effect is observed on the leaf divergence because the frequency of the infrared is below threshold frequency for zinc. Hence it cannot eject electrons from the zinc plate no matter how intense it's radiation is.
- (3) When ultraviolet radiations fall on a cleaned zinc plate placed on a cap of a positively charged gold leaf electroscope, there is no change in the divergence of the leaf. This is because the electrons that are emitted photo electrically are attracted back by the positively charged zinc plate. Hence there is no change in the magnitude or sign of charge on the electroscope.

### 3.1.1: EINSTEIN'S PHOTOELECTRIC EQUATION

Einstein proposed that when a photon collides with an electron, it must either be reflected with no reduction in energy or it must give up all its energy to the electron.

The energy of a single photon cannot be shared amongst the electrons i.e no more than one electron can absorb the energy of one photon.

Therefore in any given time the number of electron emitted by a surface is proportional to the number of incident photons i.e to the intensity of the radiation.

Further more , an electron can be emitted as soon as a photon reaches the surface, explaining why photoemission begins instantaneously.

Einstein also reasoned that some energy imparted by a photon is actually used to release an electron from the surface (overcome the binding force) and the rest appears as the kinetic energy of the emitted electron. This is summarized by Einstein's photoelectric equation

$$hf = W_0 + \frac{1}{2} mv_{max}^2$$

where  $h$  = is plank's constant

$hf$  = the energy of each incident photon of frequency

$W_0$  = the work function of the surface

$\frac{1}{2} mv_{max}^2$  = maximum kinetic energy of the emitted electrons



$$W_0 = hf_0 \quad \text{where } f_0 \text{ is threshold frequency}$$

$$\frac{1}{2} mv_{\max}^2 = eV_s \quad V_s \text{ is stopping potential}$$

**Definition**

**Work function of metal ( $W_0$ )**

It is the minimum energy that is needed to just remove an electron from the metal surface

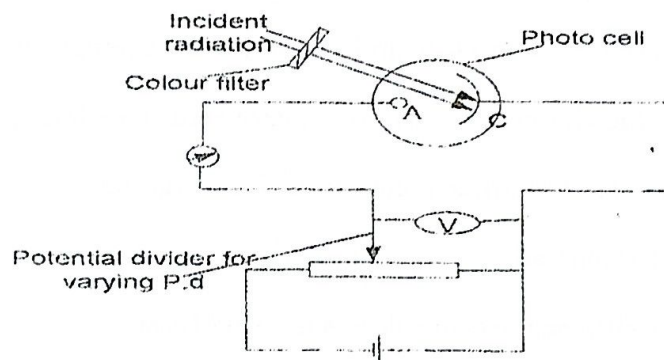
**Threshold frequency ( $f_0$ )**

It is the minimum frequency of the incident radiation below which no electron emission takes place from a metal surface

**Stopping potential ( $V_s$ )**

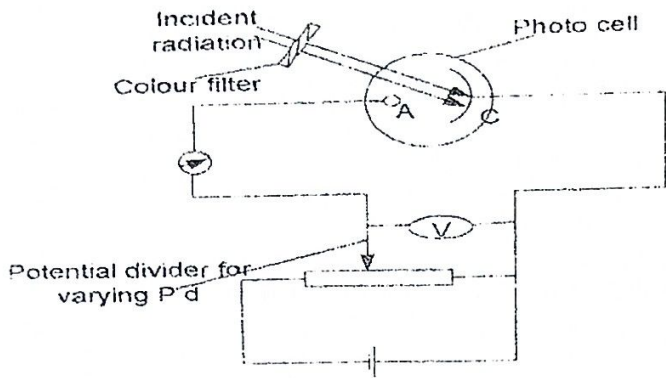
It is the minimum potential which reduces the photo current to zero.

### 3.1.2: AN EXPERIMENT TO MEASURE OF STOPPING POTENTIAL



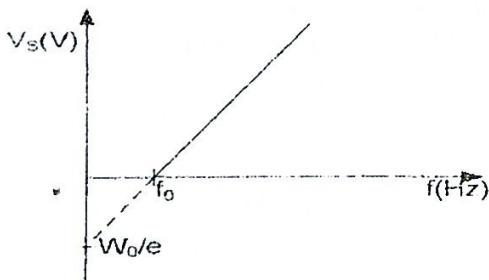
- ❖ The cathode C is made positive with respect to the anode by the potential divider.
- ❖ The beam of radiation is passed through a colour filter on to the cathode.
- ❖ The ammeter gives the photocurrent due to emitted electrons
- ❖ The applied p.d is increased negatively until the ammeter register zero reading.
- ❖ The p.d ( $V_s$ ) for which the photocurrent is zero is recorded from the voltmeter
- ❖ This p.d  $V_s$  is known as the stopping potential

### AN EXPERIMENT TO VERIFY EINSTEIN'S EQUATION OR DETERMINE PLANCK'S CONSTANT



- ❖ The anode A is made negative with respect to the cathode
- ❖ The cathode C is also made positive with respect to the anode.
- ❖ A beam of radiation of known frequency,  $f$  is passed through a colour filter on to the photo cathode.
- ❖ The ammeter gives the photocurrent due to emitted electrons
- ❖ The applied p.d  $V$  is increased negatively until the ammeter register zero reading.
- ❖ The p.d ( $V_s$ ) for which the photocurrent is zero is recorded from the voltmeter
- ❖ The procedure is repeated with other frequencies,  $f$  of radiation.
- ❖ A graph of  $V_s$  against  $f$  is plotted.
- ❖ A straight line graph is obtained and the slope  $s$  is found from it.
- ❖ The plancks constant  $h$  is got from  $h = eS$  where  $e$  is the electronic charge

NOTE



$$hf - W_0 = \frac{1}{2} mv_{\max}^2$$

$$hf = W_0 + eV_s$$

$$eV_s = hf - W_0$$

$$V_s = \frac{h}{e} f - \frac{W_0}{e}$$

$$\text{Slope} = \frac{h}{e}$$

$$\therefore h = e \times \text{slope}$$

Where  $e$  is electronic charge

5

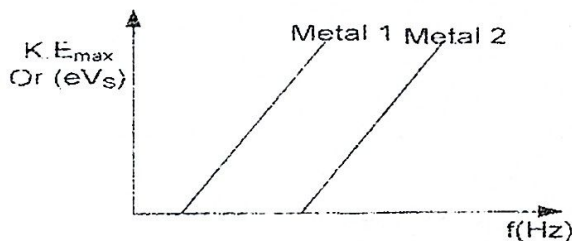
- Photo electric emission is an instantaneous process.

### 3.1.3: LAWS/RESULTS/OBSERVATIONS OF PHOTO ELECTRIC EMISSION

- Law 1- For any given metal surface there is a certain minimum frequency of radiation called threshold frequency below which no photo electrons are emitted.
- Law 2- Photo electrons are emitted with a range of kinetic energies, ranging from zero up to some maximum value and K.E is proportional to the frequency of the incident radiation.
- Law 3- The number of photo electrons emitted per second (photo current) is directly proportional to the intensity of incident radiation for a given frequency.
- Law 4- There is no detectable time lag between irradiation of a metal surface and emission of electrons by the surface.

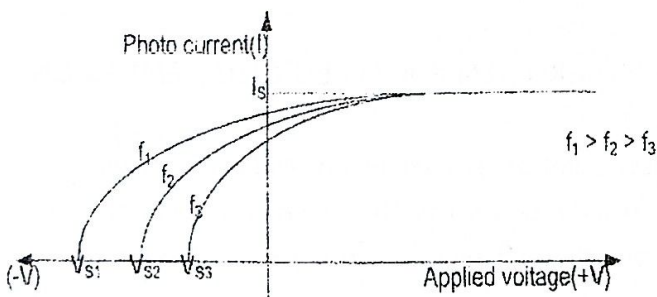
### 3.1.4: Results/observation from the experiment

1: Variation of  $K.E_{max}$  with frequency [keeping applied p.d and intensity constant]



- $K.E_{max}$  is directly proportional to frequency
- For any given surface, there is minimum frequency called threshold frequency below which no electrons are emitted
- The metals have the same slope and  $h$  (plank's constant)

2: Variation of photo current  $I$  with applied voltage for different frequencies but keeping the intensity constant



- $K.E_{max}$  ( $eV_s$ ) increases with frequency
- $I_s$  (saturation current) is independent of frequency

$$I_s = ne$$

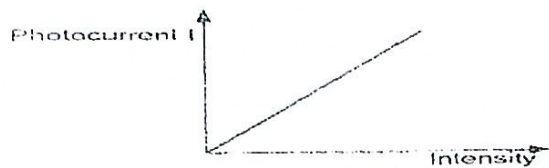
Where  $n$  is the number of photo electrons

3: Variation of photocurrent  $I$  with applied  $p.d$  for different intensities keeping frequency constant



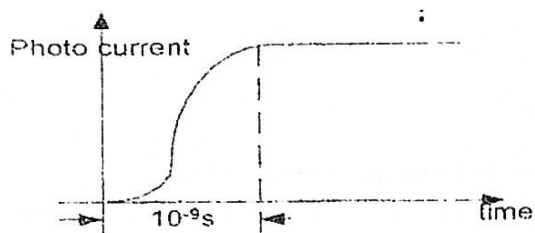
- Photo current increases with intensity
- Max  $k.e$  ( $eV_s$ ) is independent of intensity

4: Variation of photocurrent with intensity when both frequency and applied  $p.d$  are constant



- Photocurrent is directly proportional to intensities
- Max  $K.e$  ( $eV_s$ ) is independent of intensity

5: Variation of photocurrent with time



### 3.1.5: THEORIES OF LIGHT

There are two theories of light

a) Classical wave theory which is considered as wave like propagation of light

It states that radiation is emitted with continuous energy

b) Quantum theory

It states that radiation is emitted in Quanta (packets of light energy).

A photon This is a packet of light energy

### 3.1.6: FAILURES OF CLASSICAL WAVE THEORY TO EXPLAIN PHOTO ELECTRIC EMISSION

❖ Existence of threshold frequency

The theory allows continuous absorption and accumulation of energy; any radiation should eventually be able to provide electrons even if it's below the threshold frequency provided it is intense enough. It therefore predicts no threshold frequency contrary to what was experimentally observed.

❖ Variation of kinetic energy

By the wave theory, an increase in intensity means more energy and hence greater value of maximum kinetic energy of electrons. However, maximum KE depends on frequency of radiation and not.

❖ **Instantaneous emission**

Since there is continuous absorption and accumulation of energy by an electron, the theory predicts a time lag between irradiation and emission of electrons, however such time lag is not experimentally observed

Note: classical theory only accounts for the increase in the number of electrons emitted per second with increasing intensity of the incident radiation.

### 3.1.7: QUANTUM THEORY EXPLANATION OF PHOTOELECTRIC EMISSION

- ❖ Quantum theory considers radiations to be emitted and absorbed in discrete packets or quanta's called photons each of energy  $E = hf$ .
- ❖ Quantum theory considers photoelectric emission to be one electron-one photon affair. Each electron can only absorb one photon. If the photon energy is less than the work function the photon is reflected back, but if the photon energy is greater or equal to the work function, then the photon is absorbed and an electron is released.
- ❖ Increasing intensity increases the number of photons in the beam since each electrons is only allowed to absorb only one photon, the number of electrons emitted per second (photo current) is proportional to the intensity.
- ❖ Quantum theory does not allow continuous absorption and accumulation of energy. Energy transfer process requires very small time [of order  $10^{-9}$ S]. This predicts photoelectric emission to be an instantaneous process.

#### Examples

1. Work function of potassium is 2.25eV. Light having wavelength of 360nm falls on the metal.

Calculate;

(i) Stopping potential

(ii) The speed of the most energetic electron emitted

$$[h=6.60 \times 10^{-34} \text{Js}, c=3 \times 10^8 \text{ms}^{-1}, e=1.6 \times 10^{-19} \text{C}]$$

**Solution**

Work function  $W_0=2.25\text{eV}$

$W_0=2.25 \times 1.6 \times 10^{-19} \text{J}$

$\lambda = 360 \times 10^{-9} \text{m}$

Using Einstein's equation

$$hf = W_0 + \frac{1}{2} m v_{\text{max}}^2$$

$$\text{but also } \frac{1}{2} m v_{\text{max}}^2 = eV_s$$

$$hf = W_0 + eV_s$$

$$h \frac{c}{\lambda} = W_0 + eV_s$$

$$V_s = \frac{h \frac{c}{\lambda} - W_0}{e}$$

$$V_s = \frac{6.6 \times 10^{-34} \times 3 \times 10^8 - 2.25 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$V_s = 1.188 \text{ V}$$

$$\therefore \frac{1}{2} m v_{\text{max}}^2 = eVs$$

$$v_{\text{max}} = \sqrt{\frac{2eVs}{m}}$$

$$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 1.188}{9.1 \times 10^{-31}}}$$

$$v_{\text{max}} = 6.46 \times 10^5 \text{ m/s}$$

2. If a surface has a work function of 3.0 eV

(a) Find the longest wave length of light which will cause the emission of photo electrons on it.

(b) What is the maximum velocity of the photo electrons liberated from the surface having a work function of 4.0 eV by ultraviolet radiations of wave length 0.2 μm.

**Solution**

a)  $W_0 = 3 \times 1.6 \times 10^{-19} \text{ J}$

$$W_0 = hf_0$$

$$hf_0 = 3 \times 1.6 \times 10^{-19}$$

$$h \frac{c}{\lambda_0} = 3 \times 1.6 \times 10^{-19}$$

$$\lambda_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{3 \times 1.6 \times 10^{-19}}$$

$$\lambda_0 = 4.125 \times 10^{-7} \text{ m}$$

Longest wave length of light =  $4.125 \times 10^{-7} \text{ m}$

b) Using Einstein's equation

$$hf = W_0 + \frac{1}{2} m v_{\text{max}}^2$$

$$W_0 = 4 \times 1.6 \times 10^{-19} \text{ J}$$

$$\lambda = 0.2 \times 10^{-6} \text{ m}$$

$$h \frac{c}{\lambda} = W_0 + \frac{1}{2} m v_{\text{max}}^2$$

$$v_{\text{max}} = \sqrt{\frac{2 \times \left( \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{0.2 \times 10^{-6}} - 4 \times 1.6 \times 10^{-19} \right)}{9.1 \times 10^{-31}}}$$

$$v_{\text{max}} = 8.77 \times 10^5 \text{ ms}^{-1}$$

3. Calcium has a work function of 2.7 eV

(a) What is the threshold frequency for calcium

(b) What is the maximum wavelength that will cause emission from calcium.

$$[e = 1.6 \times 10^{-19} \text{ C}, h = 6.6 \times 10^{-34} \text{ Js}, c = 3 \times 10^8 \text{ ms}^{-1}]$$

**Solution**

a)  $W_0 = 2.7 \times 1.6 \times 10^{-19}$

$$hf_0 = 2.7 \times 1.6 \times 10^{-19}$$

$$f_0 = \frac{2.7 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$f_0 = 6.55 \times 10^{14} \text{ Hz}$$

b) Max wavelength is  $\lambda_0$

$$f_0 = \frac{c}{\lambda_0}$$

$$\lambda_0 = \frac{3 \times 10^8}{6.55 \times 10^{14}}$$

$$\lambda_0 = 4.58 \times 10^{-7} \text{ m}$$

## EXERCISE 6

1. Calculate the energy of:

(a) A photon of frequency  $7.0 \times 10^{14} \text{ Hz}$ .

(b) A photon of wavelength  $3 \times 10^{-7} \text{ m}$

$$[h=6.6 \times 10^{-34} \text{Js}, C=3 \times 10^8 \text{ms}^{-1} \text{J}] \quad \text{An}[4.6 \times 10^{-19} \text{J}, 6.6 \times 10^{-19} \text{J}]$$

2. Sodium has a work function of 2.3eV. Calculate:

(a) Its threshold frequency

(b) Maximum velocity of the photoelectrons produced when the sodium is illuminated by light of wavelength  $5 \times 10^{-7} \text{m}$

(c) The stopping potential with light of this wavelength  $[h=6.6 \times 10^{-34} \text{Js}, C=3 \times 10^8 \text{ms}^{-1}, e=1.6 \times 10^{-19} \text{C}, \text{mass of electron } m=9.1 \times 10^{-31} \text{kg}]$  An[ $5.6 \times 10^{14} \text{Hz}, 2.5 \times 10^5 \text{ms}^{-1}, 0.18 \text{V}$ ]

3 Calculate the stopping potential for a platinum surface irradiated with ultraviolet light of wavelength  $1.2 \times 10^{-7} \text{m}$ . The work function of platinum is 6.3eV.  $[h=6.6 \times 10^{-34} \text{Js}, C=3 \times 10^8 \text{ms}^{-1}, e=1.6 \times 10^{-19} \text{C}]$

An[4.0V]

4. Gold has a work function of 4.9eV

(a) Calculate the maximum kinetic energy in joules, of the electrons emitted when gold is illuminated with ultraviolet radiations of frequency  $1.7 \times 10^{15} \text{Hz}$ .

(b) What is the energy in eV

(c) What is the stopping potential for these electrons.  $[h=6.6 \times 10^{-34} \text{Js}, e=1.6 \times 10^{-19} \text{C}]$

An [ $3.4 \times 10^{-19} \text{J}, 2.1 \text{eV}, 2.1 \text{V}$ ]

5. Light of frequency  $6 \times 10^{14} \text{Hz}$ , incident on a metal surface ejects photoelectrons having a kinetic energy  $2 \times 10^{-19} \text{J}$ . Calculate the energy needed to remove an electron from the metal (work function).  $[h=6.6 \times 10^{-34} \text{Js}]$  An[ $1.96 \times 10^{-19} \text{J}$ ]

6. Light of wave length  $0.5 \mu\text{m}$  incident on a metal surface ejects electrons with kinetic energies up to a maximum value of  $2 \times 10^{-19} \text{J}$ . What is the energy required to remove an electron from the metal? If a beam of light causes no electrons to be emitted, however great its intensity what condition must be satisfied by its wavelength?  $[h=6.6 \times 10^{-34} \text{Js}, C=3 \times 10^8 \text{ms}^{-1}]$

An[ $1.96 \times 10^{-19}$ ,

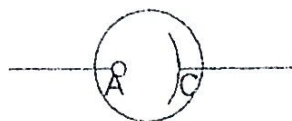
$1.01 \times 10^{-6} \text{m}$ ]

7. The maximum kinetic energy of photo electrons ejected from a tungsten surface by monochromatic light of wavelength 248nm was found to be  $8.6 \times 10^{-20} \text{J}$ . Find the work function of tungsten.  $[h=6.6 \times 10^{-34} \text{Js}, e=1.6 \times 10^{-19} \text{C}, C=3 \times 10^8 \text{ms}^{-1}]$  An[4.45eV]

### 3.1.8: PHOTO CELLS

These are devices that change radiations into current

Symbol



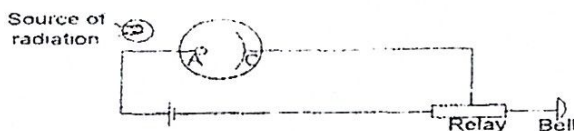
A- Anode  
C- Cathode

When radiation falls on the cathode, electrons are emitted and are collected by the anode if it is positive with respect to the cathode.

### 3.1.9: USES OF PHOTOCELLS

(i) They are used in Burglar alarms

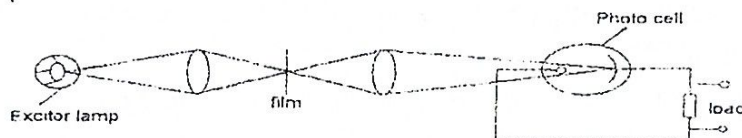
A burglar alarm consists of a photo cell forming a closed circuit and a source of radiation



Light from the source falls on the photocell and maintains a current in the circuit. When someone intersects the radiations, the current is automatically switched off. This automatically operates the relay switch and another circuit containing a bell gets closed, the bell starts ringing and can only stop when it has been switched off.

(ii) Reproduction of sound track on a film

A photo cell is used in the reproduction of sound which is recorded on a film in form of a thin transparent strip called a **sound track** on one side of it.



When the film runs, light from lamp goes through the film and falls on a photocell. A variable current is produced which is amplified and fed to a loud speakers to reproduce a sound.

### Example

1. A 100mW beam of light of wave length  $4 \times 10^{-7} \text{m}$  falls on caesium surface of a photocell

(i) How many photons strike the cesium surface per second.

(ii) If 65% of the photons emit photo electrons, find the resulting photo current

(iii) Calculate the kinetic energy of each photon if the work function of caesium is 2.20eV

Solution

(i) Photon energy  $E = hf$  or  $E = h \frac{c}{\lambda}$

$$E = 4.95 \times 10^{-19}$$

$$E = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4 \times 10^{-7}}$$

Power of beam = photon energy  $\times$  number of photons per second  
 $100 \times 10^{-3} = 4.95 \times 10^{-19} \times$  number of photons  
 Number of photons per second =  $2.02 \times 10^{17}$  photons

(ii) Number of electrons emitted per second  $n = 65\%$  of photons

$$n = \frac{65}{100} \times 2.02 \times 10^{17}$$

$$n = 1.31 \times 10^{17} \text{ electrons}$$

$$I = ne$$

$$I = 1.31 \times 10^{17} \times 1.6 \times 10^{-19}$$

$$I = 2 \times 10^{-2} \text{ A}$$

(iii) From Einstein's equation

$$hf = W_0 + \frac{1}{2} m v_{\max}^2$$

$$K.E._{\max} = hf - W_0$$

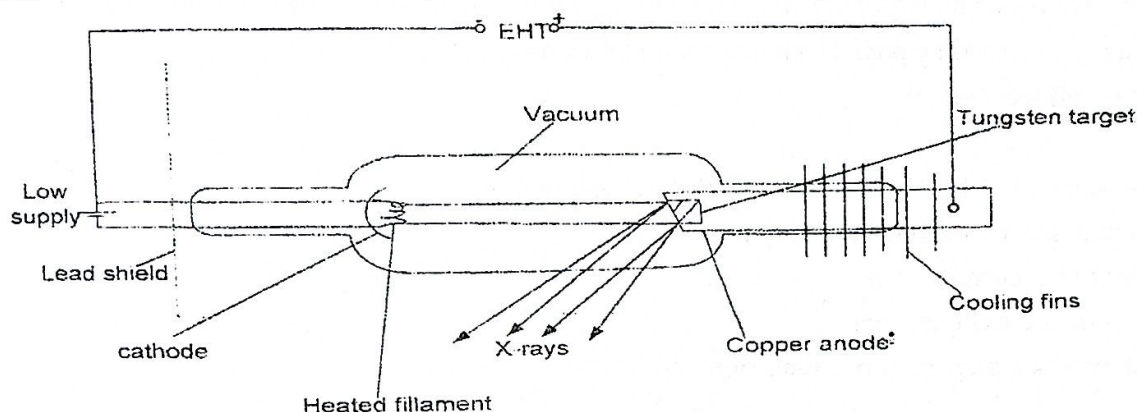
$$K.E._{\max} = 4.95 \times 10^{-19} - 2.2 \times 1.6 \times 10^{-19}$$

$$K.E._{\max} = 1.43 \times 10^{-19} \text{ J}$$

### 3.2.0: X-RAYS

These are electromagnetic radiations of short wavelength ( $\sim 10^{-10} \text{ m}$ ) which travel at a speed of light and produced when fast moving electrons (cathode rays) strike a metal target.

#### 3.2.1: X-RAY TUBE [PRODUCTION OF X-RAYS]



#### Operation

- ❖ The cathode is heated with low voltage and electrons are emitted thermionically.
- ❖ Electrons are accelerated by a high p.d towards the anode.
- ❖ On striking the target, a small percentage of the electrons is converted to X-rays
- ❖ The anode is cooled by the cooling fins.

#### Note

- (1) The energy changes in an x-rays tube are; electrical energy from low voltage source to heat energy used for heating the filament to kinetic energy of electrons and then to heat and x-rays.

- (2) The intensity of x-ray beam increases with the number of electrons hitting the target, therefore intensity is controlled by filament current /heating current or supply voltage.
- (3) The penetrating power (quality) of an x-ray beam is controlled by the accelerating *p.d* between the cathode and the anode
- (4) X-rays with high penetrating power are called hard x-rays while those with low penetrating power are called soft x-rays.
- (5) The x-ray tube is totally evacuated to prevent collision of electrons with gas molecules.

### 3.2.2: PROPERTIES OF X-RAYS

- (1) They travel in straight lines at the velocity of light.
- (2) They cannot be deflected by electric or magnetic field(This is an evidence that they are not charged particles )
- (3) They readily penetrate matter, penetration is least with materials of high density
- (4) They can be reflected but not at very large angles of incidence
- (5) Refractive indices of all materials are very close to unity (one) for x-rays so that very little bending occurs when they pass from one material to another
- (6) They can be diffracted

The following properties 7 to 10 are used to detect x-rays

- (7) They ionize gases through which they pass
- (8) They affect photographic film
- (9) They can produce fluorescence
- (10) They can produce photoelectric emission

### 3.2.3: USES OF X-RAYS

#### Medical uses

- ❖ Used to detect fractures in bones
- ❖ Used to destroy cancer cell
- ❖ Used in detection of lung T.B
- ❖ Used for sterilization of medical equipments

#### Industrial use

- ❖ They are used to locate internal imperfection in welded joints and casting

#### Agric uses

- ❖ Tracing phosphate fertilizers using phosphorus
- ❖ Sterilization of insecticides for pest control
- ❖ X-ray crystallography

- ❖ Used to study crystal structures and determine structure of complex organic molecules

#### Health hazard of x-rays

- ❖ They have harmful effects on human cells which become eminent after sometime

#### Precaution

- ❖ Lead aprons should be worn while dealing with x-rays
- ❖ The brain and other delicate parts of the body should not be exposed to x-rays
- ❖ Unnecessary long time exposure to x-rays should be avoided.

### 3.2.4: X-RAY EMISSION SPECTRA

X-rays spectra have two distinct components

(1) A background of continuous radiation, the minimum wavelength of which depends on the operating voltage of the tube *i.e* the energy of the incident electrons.

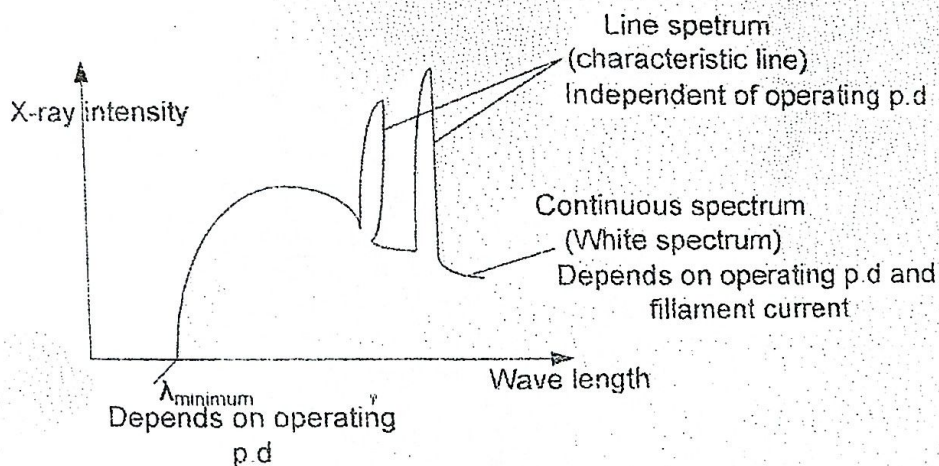
- ❖ At cut off wavelength,  $\lambda_{min}$ . Electrons from the cathode strike the target and lose all their kinetic energy in a single encounter with the target atoms. This results in the production of the most energetic x-ray photons of maximum frequency and corresponding,  $\lambda_{min}$  called cut off wavelength.

From  $E = hf$

$hf_{max} = ev$

$$h \frac{c}{\lambda_{min}} = ev$$

(2) Very intense emission at a few discrete wavelength (an x-ray line spectrum). These wavelength are characteristic of the target material and are independent of the operating voltage.



a) Continuous spectrum

It is formed as a result of multiple collisions of energy electrons with a target atom and these electrons are decelerated. Different amounts of energy are lost, x-rays given off have wavelengths varying from a certain minimum value ( $\lambda_{min}$ ) to infinity.

### b) Line spectrum

When highly energetic electrons penetrate the atom, knock electrons from inner most shells and displace them to higher shells. This puts the atom in an excited state and therefore becomes unstable. The subsequent electron transition from higher energy levels into a vacancy in the lower energy levels causes a high energy x-ray photon of definite wavelength to be emitted whose energy is equal to the difference between the energy levels. This leads to x-ray line spectrum.

### Note

- ❖ The frequency of the x-ray is given by  $E = hf$ . Where E is the difference in the energy levels involved and h is plank's constant.
- ❖ Continuous spectrum is produced due to multiple collisions of electrons with target atoms while
- ❖ Line spectrum is produced by electronic transitions within the atoms as the electrons in them fall back to the lower energy levels.

### Example

1. An x-ray tube operates at 30kV and current through it is 2mA. Calculate
  - (i) The electrical power input
  - (ii) Number of electrons striking the target per second
  - (iii) The speed of electrons when they hit the target
  - (iv) The lower wavelength limit of x-rays emitted

$$[h=6.6 \times 10^{-34} \text{ Js}, e=1.6 \times 10^{-19} \text{ C}, C=3 \times 10^8 \text{ ms}^{-1}, m=9.1 \times 10^{-31} \text{ kg}]$$

#### Solution

(i) Power input = IV

$$\text{Power input} = 2 \times 10^{-3} \times 30 \times 10^3$$

$$\text{Power input} = 60 \text{ Js}^{-1}$$

(ii)  $I = ne$

$$n = I/e$$

$$n = \frac{2 \times 10^{-3}}{1.6 \times 10^{-19}}$$

$$n = 1.25 \times 10^{16} \text{ electrons per second}$$

(iii)  $\frac{1}{2} mu^2 = eV$

$$u = \sqrt{\frac{2ev}{m}}$$

$$u = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 30 \times 10^3}{9.1 \times 10^{-31}}}$$

$$u = 1.03 \times 10^8 \text{ ms}^{-1}$$

(iv)  $hf_{max} = eV$

$$h \frac{c}{\lambda_{min}} = eV$$

$$\lambda_{min} = \frac{hc}{eV}$$

$$\lambda_{min} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 30 \times 10^3}$$

$$\lambda_{\min} = 4.13 \times 10^{-11} \text{ m}$$

2. The p.d between the target and cathode of an x-ray tube is 50kV and current in the tube is 20mA. If only 1% of the total energy is emitted as x-rays.

- (i) What is the maximum frequency of the emitted radiations
- (ii) At what rate must heat be removed from the target in order to keep it a steady temperature.

Solution

$$i) \quad hf_{\max} = eV$$

$$f_{\max} = \frac{1.6 \times 10^{-19} \times 50 \times 10^3}{6.6 \times 10^{-34}}$$

$$f_{\max} = 1.21 \times 10^{19} \text{ Hz}$$

- ii) 1% of power produces x-ray,  
therefore 99% of power produces heat

For a steady temp the rate at which heat is supplied equals to rate at which heat is removed

Rate at which heat is supplied to the target  
99% of power

$$= \frac{99}{100} \text{ of } IV$$

$$= \frac{99}{100} \times 20 \times 10^{-3} \times 50 \times 10^3$$

$$= 990 \text{ J s}^{-1}$$

3. An x-ray tube operated at  $1.8 \times 10^5 \text{ V}$  with target made of a material of S.H.C of  $250 \text{ J kg}^{-1} \text{ K}^{-1}$  and has a mass of 0.25kg. 1% of the electrical power supplied is converted into x-ray and the rest is dissipated as heat in the target. If the temp of the target rises by  $8^\circ \text{C}$  per second. Find

- (i) The number of electrons which strike the target per second
- (ii) The shortest wavelength of x-rays produced

Solution

$$V = 1.8 \times 10^5 \text{ V}, C = 250 \text{ J kg}^{-1} \text{ K}^{-1} \quad m = 0.25 \text{ kg}$$

$$\frac{\Delta\theta}{t} = 8^\circ \text{C s}^{-1}$$

$$i) \quad IVt = mc \Delta\theta$$

$$IV = mc \frac{\Delta\theta}{t}$$

$$I = \frac{mc \Delta\theta}{Vt}$$

$$I = \frac{0.25 \times 250 \times 8}{1.8 \times 10^5}$$

$$I = 2.78 \times 10^{-3} \text{ A}$$

$$\text{Using } I = ne$$

$$n = I/e$$

$$n = \frac{2.78 \times 10^{-3}}{1.6 \times 10^{-19}}$$

$$n = 1.74 \times 10^6 \text{ electrons per second}$$

$$ii) \quad hf_{\max} = eV$$

$$h \frac{c}{\lambda_{\min}} = eV$$

$$\lambda_{\min} = \frac{hc}{eV}$$

$$\lambda_{\min} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 1.8 \times 10^5}$$

$$\lambda_{\min} = 6.88 \times 10^{-12} \text{ m}$$

### Exercise 7

1. Calculate the wavelength of the most energetic x-rays produced by the tube operating at  $1 \times 10^5 \text{ V}$ . [ $h = 6.6 \times 10^{-34} \text{ J s}$ ,  $e = 1.6 \times 10^{-19} \text{ C}$ ,  $C = 3 \times 10^8 \text{ ms}^{-1}$ ]

$$\text{An} [1.24 \times 10^{-11} \text{ m}]$$

2. The current in a water-cooled x-ray tube operating at 60kV is 30mA. 99% of the energy supplied to the tube is converted into heat at the target and removed by water flowing at a rate of  $0.06\text{kg s}^{-1}$  calculate;
- the rate at which energy is being supplied to the tube.
  - The increase in temperature of the cooling water [S.H.C =  $4.2 \times 10^3 \text{J kg}^{-1} \text{ } ^\circ\text{C}^{-1}$ ]

An [ $1.8 \times 10^3 \text{J s}^{-1}$ ,  $7.1^\circ\text{C}$ ]

3. The most energetic x-rays produced by a particle x-ray tube have a wavelength of  $2.1 \times 10^{-11}\text{m}$ . What is the operating p.d of the tube. [ $h=6.6 \times 10^{-34} \text{J s}$ ,  $e=1.6 \times 10^{-19} \text{C}$ ,  $C=3 \times 10^8 \text{ms}^{-1}$ ]

An [59kV]

4. An x-ray tube which is 1% efficient produces x-rays energy at a rate of  $20 \text{J s}^{-1}$ . Calculate the current in the tube if the operating p.d is 50kV An [40mA]

5. Explain how the radiation from an evacuated x-ray tube is affected by changing
- the filament current
  - the filament target p.d
  - the target material

6. State briefly how you would control electrically;
- the intensity
  - the penetrating power of the emitted x-rays.

7. Electrons are accelerated from rest through a potential difference of 10kV in an x-ray tube calculate.

- the resultant energy of the electrons in eV
- the wavelength of the associated electron waves
- The maximum energy and the minimum wavelength of the x-ray radiation generated

[ $h=6.6 \times 10^{-34} \text{J s}$ ,  $e=1.6 \times 10^{-19} \text{C}$ ,  $C=3 \times 10^8 \text{ms}^{-1}$ ,  $m=9.11 \times 10^{-31} \text{kg}$ ]

An [10000eV,  $1.223 \times 10^{-11} \text{m}$ ,  $1.6 \times 10^{18} \text{J}$ ,  $1.24 \times 10^{10} \text{m}$ ]

8. The p.d between the target and cathode of an x-ray tube is 50kV and the current in the tube is 20mA only 1% of the total energy supplied is emitted as x-radiation.
- What is the minimum frequency of the emitted radiation

(b) At what rate must heat be removed from the target in order to keep it at a steady temperature.  $[h=6.6 \times 10^{-34} \text{Js}, e=1.6 \times 10^{-19} \text{C}]$  An $[1.2 \times 10^{19} \text{Hz}, 9.9 \times 10^2 \text{W}]$

9 An x-ray tube works at a d.c.p.d of 50kV. Only 0.4% of the energy of the cathode rays is converted into x-radiation and heat is generated in the target at a rate of 600W. estimate;

(i) Current passed through the tube

(ii) velocity of the electrons striking the target  $[h=9 \times 10^{-31} \text{kg}, e=-1.6 \times 10^{-19} \text{C}]$

An $[12 \text{mA}, 1.33 \times 10^8 \text{ms}^{-1}]$

10. (a) A 900W x-ray tube operates at a d.c.p.d of 30kV. Calculate the minimum wavelength of the x-rays produced

(b) Calculate the current through the tube

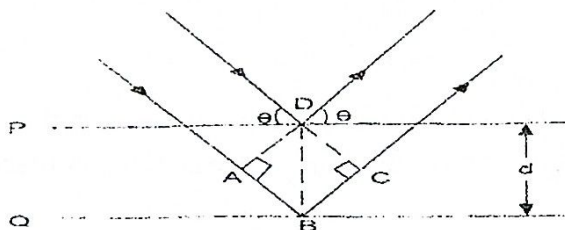
(c) If 99% of the power is dissipated as heat estimate the number of x-ray photons produced per second.  $[h=6.6 \times 10^{-34} \text{Js}, e=1.6 \times 10^{-19} \text{C}, C=3 \times 10^8 \text{ms}^{-1}]$  An $[4.1 \times 10^{11} \text{m}, 30 \text{mA}, 1.9 \times 10^{15} \text{s}^{-1}]$

### 3.3.5: BRAGG'S LAW FOR X-RAY DIFFRACTION

It states that  $2d \sin \theta = n\lambda$  where  $d$  is interplanar separation,  $\theta$  is glancing angle,  $\lambda$  is x-ray wavelength and  $n$  is an interger

#### DERIVATION OF BRAGG'S LAW FOR X-RAY DIFFRACTION

When x-rays are directed to a crystal each atomic plane of a crystal behaves like a reflecting surface.



❖ Constructive interference occurs when the path difference is  $n\lambda$

Where  $n$  is an integer and  $\lambda$  is wavelength the x-rays.

$$\therefore AB + BC = n\lambda$$

$$AB = BC = d \sin \theta$$

$$d \sin \theta + d \sin \theta = n\lambda$$

$$2d \sin \theta = n \lambda$$

generally

$$2d \sin \theta_n = n \lambda \quad \text{Bragg's law}$$

where  $\theta_n$  is glancing angle for the  $n^{\text{th}}$  order maximum

**Note**

The small angle ( $\theta_{\min}$ ) is given when  $n=1$  and it's the first order maxima

$n_{\max}$  occurs when  $\sin \theta = 1$

**Example**

1. A beam of x-rays of wavelength 0.15nm incident on the crystal. The smallest angle at which there is strongly reflected beam is  $15^\circ$ . Calculate the distance between the successive layers between the crystal lattice.

**Solution**

$$\lambda = 0.15 \times 10^{-9} \text{m}, d = ?$$

for smallest angle  $n=1$ ,  $\theta_{\min} = 15^\circ$

from Bragg's law  $2d \sin \theta_n = n \lambda$

$$d = \frac{n \lambda}{2 \sin \theta_n}$$

$$d = \frac{1 \times 0.15 \times 10^{-9}}{2 \sin 15}$$

$$d = 2.898 \times 10^{-10} \text{m}$$

2. A beam of x-rays of wavelength  $8.42 \times 10^{-11} \text{m}$  is incident on a sodium chloride crystal of interplanar separation  $2.82 \times 10^{-10} \text{m}$ . Calculate the first order diffraction angle.

**Solution**

$$\lambda = 8.42 \times 10^{-11} \text{m}, d = 2.82 \times 10^{-10} \text{m}, \theta = ? \text{ For first order diffraction } n=1$$

using Bragg's law  $2d \sin \theta_n = n \lambda$

$$\theta_1 = \sin^{-1} \left( \frac{1 \times 8.42 \times 10^{-11}}{2 \times 2.82 \times 10^{-10}} \right)$$

$$\theta_1 = 8.59^\circ$$

3. A monochromatic x-ray beam of wavelength  $1 \times 10^{-10} \text{m}$  is incident on a set of planes in a crystal of spacing  $2.8 \times 10^{-10} \text{m}$ . What is the maximum order possible in these x-rays.

**Solution**

$n_{\max}$  occurs when  $\sin \theta = 1$

$$2d \sin \theta_n = n_{\max} \lambda$$

$$n_{\max} = \frac{2 \times 2.8 \times 10^{-10}}{1 \times 10^{-10}} = 5.6$$

$n_{\max} \approx 6$  sixth order diffraction

- 4 A monochromatic beam of x-rays of wavelength  $2 \times 10^{-10} \text{ m}$  is incident on a set of cubic plane in a potassium chloride crystal. First order diffraction maxima are observed at a glancing angle of  $18.5^\circ$ . find the density of potassium chloride. If its molecular weight is 74.55.

**Solution**

$$\lambda = 2 \times 10^{-10} \text{ m}, n=1, \theta = 18.5^\circ, d=?$$

$$M = 74.55 \text{ g}, m = 74.55 \times 10^{-3} \text{ kg}$$

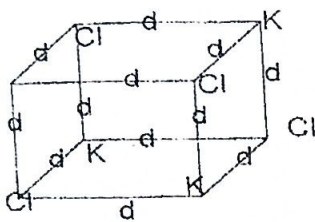
note that molecular weight is measured in grams unless given in kg

Using Bragg's law

$$2d \sin \theta_n = n \lambda$$

$$d = \frac{1 \times 2 \times 10^{-10}}{2 \times \sin 18.5}$$

$$d = 3.15 \times 10^{-10} \text{ m}$$



its set of cubic planes

$$\text{volume of one atoms} = d^3$$

$$= (3.15 \times 10^{-10})^3$$

- 5 Calculate the atomic spacing of sodium chloride if the relative atomic mass of sodium is 23 and that of chlorine is 35.5 [density of sodium chloride  $= 2.18 \times 10^3 \text{ kg m}^{-3}$ ]

**Solution**

$$\text{Mass of one mole} = 23 + 35.5$$

$$= 58.5 \text{ g}$$

$$\begin{aligned} \text{Mass of one molecule of NaCl} &= \frac{58.5 \times 10^{-3}}{N_A} \\ &= \frac{58.5 \times 10^{-3}}{6.02 \times 10^{23}} \\ &= 9.718 \times 10^{-26} \text{ kg} \end{aligned}$$

$26$  kg

Density of one molecule of NaCl

$$= \frac{\text{mass of one molecule of NaCl}}{\text{volume of 1 molecule}}$$

$$2.18 \times 10^3 = \frac{9.718 \times 10^{-26}}{\text{volume of 1 molecule of NaCl}}$$

$$\text{volume of 1 molecule of NaCl} = \frac{9.718 \times 10^{-26}}{2.18 \times 10^3}$$

$$\text{volume of one atoms} = 3.13 \times 10^{-29} \text{ m}^3$$

But one molecule of KCl has two atoms

$\therefore$  volume of the one molecule of KCl

$$= 2 \times 3.13 \times 10^{-29}$$

$$= 6.26 \times 10^{-29} \text{ m}^3$$

Mass of one molecule of KCl =

$$= \frac{\text{molecular weight}}{N_A}$$

$$= \frac{74.55 \times 10^{-3}}{6.02 \times 10^{23}}$$

$$= 1.24 \times 10^{-25} \text{ kg}$$

Density of one molecule of

$$\text{KCl} = \frac{\text{mass of one molecule}}{\text{volume of 1 molecule}}$$

$$\rho = \frac{1.24 \times 10^{-25}}{6.26 \times 10^{-29}}$$

$$\rho = 1.984 \times 10^3 \text{ kg m}^{-3}$$

$$\text{volume of 1 molecule of NaCl} = 4.458 \times 10^{-29} \text{ m}^3$$

volume of 1 molecule of NaCl has two atoms of Na and Cl

volume of 1 atom of either Na or Cl

$$= \frac{4.458 \times 10^{-29}}{2}$$

$$= 2.229 \times 10^{-29} \text{ m}^3$$

$$\therefore d^3 = 2.229 \times 10^{-29} \text{ m}^3$$

$$d = (2.229 \times 10^{-29})^{\frac{1}{3}}$$

$$d = 2.81 \times 10^{-10} \text{ m}$$

### EXERCISES

1. Calculate the smallest glancing angle at which x-rays of wave length  $0.7 \times 10^{-10} \text{ m}$  will be diffracted from a certain crystal which has inter-atomic separation of  $15 \times 10^{-10} \text{ m}$ . what is the highest diffraction order that can be observed from this radiation.  
An [ $13.5^\circ$ ,  $\approx 4$ , fourth order diffraction]

2. A beam of x-rays of frequency  $3.56 \times 10^{18} \text{ Hz}$  is incident on potassium chloride (KCl) crystal and the first order Bragg reflection occurs at  $7.68^\circ$ . The density of KCl is  $1.98 \times 10^3 \text{ kg m}^{-3}$  and its molecular mass is 74.5. Calculate the value of Avogadro's number. An [ $6.02 \times 10^{23} \text{ mol}^{-1}$ ]

### 3.3.6: CONDITION FOR X-RAY DIFFRACTION TO OCCUR

- ❖ Wave length of x-rays must be of the same order as the interplanar spacing.
- ❖ Parallel beam of x-rays must be incident on planes

### 3.3.7: DIFFERENCES BETWEEN CATHOD RAYS AND X-RAYS

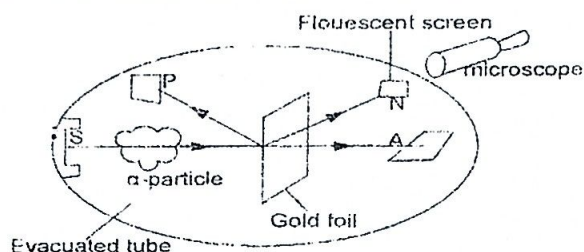
Cathode rays	X-rays
Are fast moving electrons	Are electromagnetic waves
They are negatively charged	They have no charge
Can be deflected by electric and magnetic fields	Can not be deflected by electric and magnetic fields
Have a low penetrating power	Have a high penetrating power
They produce x-rays on striking matter	They eject electrons from matter
They are relatively slower compared to x-rays	Move very fast at a speed of light

### 3.4.0: RUTHERFORD'S MODEL OF THE ATOM

An atom consists of a very small central core called the nucleus with strong electric charge surrounded by electrons of the opposite charge which fill the rest of the atom.

Rutherford's model states: that the positive charge of the atom and nearly all its mass is concentrated in a very small volume at the centre with electrons in motion in a circular orbit around the nucleus.

### 3.4.1: RUTHERFORD'S ALPHA PARTICLE SCATTERING EXPERIMENT



- ❖ Alpha particles from a radioactive source were allowed to strike a thin gold foil placed in the centre of an evacuated vessel and the scattering of alpha particles when they collide with the gold foil was observed from a fluorescent screen mounted on a focal plane of a microscope.
- ❖ Alpha particles produce tiny, but a visible flash of light when they strike a fluorescent screen.
- ❖ Surprisingly, alpha particles not only struck the screen at A but also at N and some were even found to be back scattered to P.
- ❖ The greatest flash was observed at position A.

#### Conclusion

From the experiment, it was observed that:

- ❖ Most of the alpha particles went through the gold foil undeflected. This is because the atom of the foil contains very tiny nuclei, most of the space of an atom is an empty space.
- ❖ Few alpha particles were scattered through angles greater than  $90^\circ$ . This is because the nucleus occupies a very small volume of the atom. Therefore very few alpha particles are incident close to the nucleus and alpha particles incident close to the nucleus are strongly repelled almost backwards.

Question. Explain what is observed when a beam of  $\alpha$ -particles is incident on a gold foil.

### Note

The experiment was done in a vacuum in order to avoid

- Deflection of  $\alpha$ -particles by wind
- Absorption of  $\alpha$ -particles by air which would lead to ionization of the air atoms.

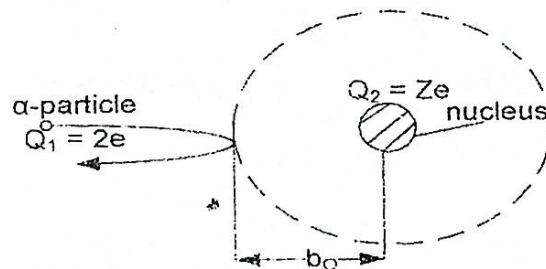
### Structure of the atom

- It is electrically neutral
- The nucleus is at the centre of the atom and is positively charged
- The nucleus contains protons and neutrons
- Orbiting around the nucleus are electrons and are negatively charged
- The atom is largely an empty space
- The nuclear occupies only small volumes of the available space

### 3.4.2: Failure of Rutherford's model of the atom

- (1) An orbiting electron is constantly changing its direction and therefore has an acceleration. In classical physics charges undergoing acceleration emit electromagnetic radiation continuously and therefore they would lose energy. This implies that the electron would spiral towards the nucleus and the atom would collapse and cease to exist within a short time, yet the atom is a stable structure. Therefore Rutherford's model can not explain the stability of the atom.
- (2) Since electrons are continuously accelerating around the nucleus, continuous emission spectra should be emitted by the atom. However experimental observations reveal that it is atomic like spectra which occur.

### 3.4.3: RUTHERFORD'S $\alpha$ -PARTICLE SCATTERING FORMULA



$b_0$  is distance of closest approach

Kinetic energy of alpha particle =  $\frac{1}{2}mv^2$  where  $v$  is speed before collision

Electrostatic potential energy =  $\frac{(2e)(Ze)}{4\pi\epsilon_0 b_0}$

$$\text{At closest distance of approach } \frac{1}{2}mv^2 = \frac{(2e)(Ze)}{4\pi\epsilon_0 b_0}$$

$$\frac{1}{2}mv^2 = \frac{Ze^2}{2\pi\epsilon_0 b_0}$$

$$b_0 = \frac{Ze^2}{\pi\epsilon_0 mv^2}$$

OR 
$$K.e = \frac{Ze^2}{2\pi\epsilon_0 b_0}$$

### Example

1. A beam of 4.7 MeV alpha particle is incident normally on a thin gold foil. What is the closest distance of approach of the alpha particle to the gold nucleus. (Atomic number of gold = 79). What is the significance of this result.

#### Solution

$$\begin{aligned} K.e &= 4.7 \text{ MeV} \\ K.e &= 4.7 \times 10^6 \times 1.6 \times 10^{-19} \text{ J} \\ K.e &= 7.52 \times 10^{-13} \text{ J} \\ K.e &= \frac{Ze^2}{2\pi\epsilon_0 b_0} \\ b_0 &= \frac{Ze^2}{2\pi\epsilon_0 K.e} \end{aligned}$$

$$\begin{aligned} b_0 &= \frac{79 \times (1.6 \times 10^{-19})^2}{2 \times \frac{22}{7} \times 8.85 \times 10^{-12} \times 7.52 \times 10^{-13}} \\ b_0 &= 4.84 \times 10^{-14} \text{ m} \end{aligned}$$

The distance of closest approach is an estimate of the radius of the nucleus.

2. In a head on collision between an alpha particle and a gold nucleus, the minimum distance of approach is  $5 \times 10^{-14} \text{ m}$ . Calculate the energy of the alpha particle in (MeV) (Atomic number of gold = 79)

#### Solution

$$\begin{aligned} Z &= 79 \\ b_0 &= 5 \times 10^{-14} \\ \epsilon_0 &= 8.85 \times 10^{-12} \\ K.e &= \frac{Ze^2}{2\pi\epsilon_0 b_0} \\ K.e &= \frac{79 \times (1.6 \times 10^{-19})^2}{2 \times \frac{22}{7} \times 8.85 \times 10^{-12} \times 5 \times 10^{-14}} \end{aligned}$$

$$K.e = 7.274 \times 10^{-13} \text{ J}$$

$$K.e = \frac{7.274 \times 10^{-13}}{10^6 \times 1.6 \times 10^{-19}}$$

$$K.e = 4.55 \text{ MeV}$$

### Exercise 9

1. An alpha particle with kinetic energy of 5 MeV is in a head on collision with an atom of a gold foil (it is deflected through  $180^\circ$ ). If the atomic number of gold is 79. Calculate the distance of closest approach of alpha particles to the nuclear centre of the atom.

Ans  $(4.55 \times 10^{-14} \text{ m})$

### 3.5.0: BOHR'S THEORY OF HYDROGEN ATOM

#### Definition

A Bohr atom is one with a small central positive nucleus with electrons revolving around it in only certain allowed circular orbits and while in these orbits they do not emit radiations.

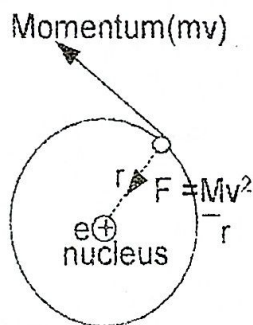
### 3.5.1: POSTULATES OF BOHR

Bohr made the following assumption

- (1) In allowed circular orbits, the angular momentum is a multiple of  $\frac{h}{2\pi}$  where  $h$  is Planck's constant.
- (2) When electrons are orbiting in this allowed orbits they do not emit radiations
- (3) Electromagnetic radiation is emitted when the electron makes a transition between orbits
- (4) In those orbits where the angular momentum is a multiple of  $\frac{h}{2\pi}$  the energy is constant

### 3.5.2:

### EXPRESSION FOR TOTAL ENERGY



From Bohr's assumption  $mvr = \frac{nh}{2\pi}$

$$v = \frac{nh}{2\pi mr} \quad [1]$$

From circular motion

Force on electron  $\frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$

$$mv^2 = \frac{e^2}{4\pi\epsilon_0 r} \quad [2]$$

Put equation(1) into equation(2)

$$m \left( \frac{nh}{2\pi mr} \right)^2 = \frac{e^2}{4\pi\epsilon_0 r}$$

$$m \frac{n^2 h^2}{4\pi^2 m^2 r^3} = \frac{e^2}{4\pi\epsilon_0 r}$$

$$r = \frac{n^2 h^2 \epsilon_0}{\pi m e^2} \quad [3]$$

Also from equation (2)  $mv^2 = \frac{e^2}{4\pi\epsilon_0 r}$

∴

Multiplying both sides by  $\frac{1}{2}$

$$\frac{1}{2} mV^2 = \frac{e^2}{8\pi\epsilon_0 r}$$

$$k.e = \frac{e^2}{8\pi\epsilon_0 r}$$

Also  $p.e = \frac{e}{4\pi\epsilon_0 r} \times -e$

Total energy  $E = K.e + P.e$

$$= \frac{e^2}{8\pi\epsilon_0 r} + \frac{e}{4\pi\epsilon_0 r} \times -e$$

$$E = \frac{-e^2}{8\pi\epsilon_0 r}$$

Putting value of  $r$  in equation (3)

$$E = \frac{-e^2}{8\pi\epsilon_0 \left( \frac{n^2 h^2 \epsilon_0}{\pi m e^2} \right)}$$

$$E = \frac{-e^2 m_e}{8n^2 h^2 \epsilon_0^2}$$

Where  $n$  is quantum number  
 $h$  is Planck constant

$\epsilon_0$  permittivity of free space  
 $m_e$  is mass of the electron  
 $e$  is charge of electron

#### Note

- ❖ Total energy of electron is negative because electrons are bound to the nucleus of the atom and work must be done to remove the electrons from the atom. This work is done against nuclear attraction binding electrons to the atoms.
- ❖ Increasing values of  $r$  are associated with increasing values of  $n$  and therefore with increasing values of  $E$  (less negative).

#### 3.5.3: EMISSION LINE SPECTRA

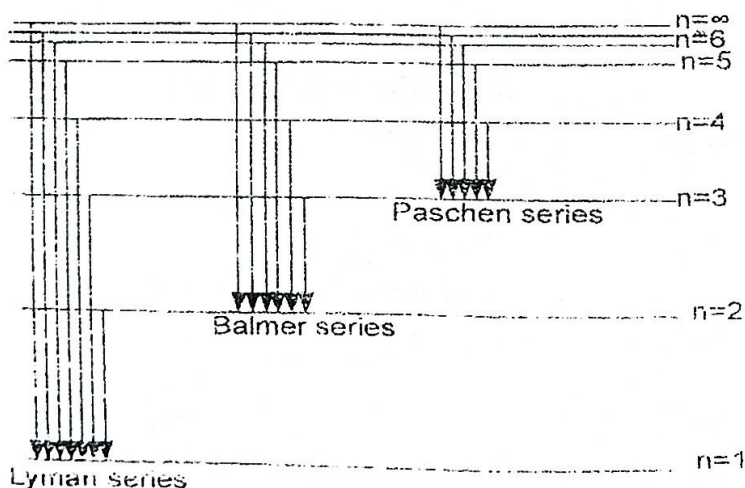
- ❖ When a gas at low pressure is heated to a very high temperature atoms are excited and jump to high energy levels. When electrons fall back to lower energy levels and emit radiations of definite wavelength.
- ❖ Since the frequency is definite for a particular element, then it implies that the energy levels are discrete (quantized). The frequency of the line can also be used to uniquely identify the element. The line formed in the spectrum appears bright against a dark background.

#### Question

1. Explain how line spectra can be used to account for the existence of discrete energy levels in an atom.

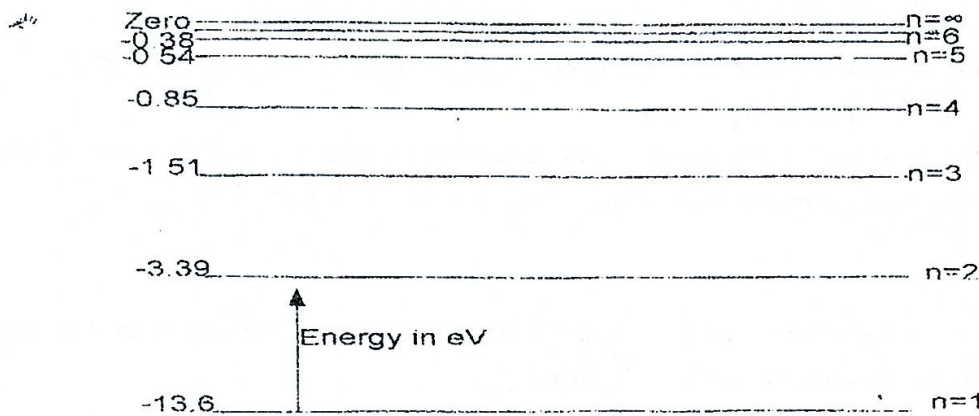
Answer is all the sentences above

#### 3.5.4: MAIN SPECTRAL TRANSITION OF ATOMIC HYDROGEN



The spectrum of atomic hydrogen contains distinct groups of lines. The three most obvious groups are the *Lyman* series, the *Balmer* series and the *Paschen* series. The wavelength of the lines in the Lyman series are in the ultraviolet and each is associated with a transition involving the level with  $n = 1$ . The Balmer series involves transitions to the level with  $n = 2$  and as a result smaller energy differences are involved and the wavelengths are in the visible. The lines of the *Paschen* series are in the infrared.

### 3.5.5: ENERGY LEVELS IN HYDROGEN ATOM



- ❖ The lowest level with  $n = 1$  is called the **ground state**. The electron will always occupy this lowest level unless it absorbs energy. Ground state is also the lowest energy state for the atom.
- ❖ When the atom absorbs energy in some way, the electron may be promoted into one of the higher energy levels, the atom becomes unstable and it is said to be in **Excited state**.
- ❖ The top level with  $n = \infty$  is the **ionization state**. An electron raised to this level will be removed from the atom.

#### Note

All levels have negative energy values because the energy of an electron at rest outside the atom is taken as zero (eV) and work has to be done to remove the electron to infinity.

#### Definition

An electron volt (eV) is the kinetic energy gained by electron in being accelerated through a potential difference of one volt.

### 3.5.6: IONISATION AND EXCITATION POTENTIAL

(1) Ionization energy of an atom is the minimum amount of energy required to remove it's most loosely bound electron when the atom is in it's ground state.

$$\begin{aligned} \text{Ionization energy of hydrogen} &= E_{\infty} - E_0 \\ &= 0 - -13.6 \\ &= 13.6\text{eV} \end{aligned}$$

It follows from definition of an eV that 13.6eV is the kinetic energy gained by an electron in being accelerated through a p.d of 13.6V thus the ionization potential of hydrogen is 13.6V.

(2) Excitation energy of an atom is the energy required to an electron from an atom which is in it's ground state to higher energy level.

For example the first and second excitation energies of hydrogen are 10.2eV and 12.1eV respectively. The corresponding excitation potentials are 10.2V and 12.1V.

#### Note

If the energy absorbed is more than that for ionization then the rest appears as kinetic energy of the electrons from which it's velocity can be calculated.

#### Examples

1. If heat energy absorbed by a hydrogen atom is 1.5eV. Calculate the energy of the excited electron given that ionization energy of hydrogen is 13.6eV.

$$\begin{aligned} \text{K. c of electron} &= 15 - 13.6 \\ \frac{1}{2} mv^2 &= 1.4\text{eV} \end{aligned}$$

2.

$$\begin{aligned} n=3 &\dots\dots\dots -1.50 \\ n=2 &\dots\dots\dots -3.40 \\ n=1 &\dots\dots\dots -13.6\text{eV} \end{aligned}$$

- Calculate: i) first ionization energy  
 ii) second ionization energy  
 iii) state the corresponding excitation potentials

Solution:

$$\begin{array}{l|l} 1^{\text{st}} \text{ ionization energy} = -3.40 - -13.6 & 2^{\text{nd}} \text{ ionization energy} = -1.50 - -13.6 \\ = 10.2\text{eV} & = 12.1\text{eV} \end{array}$$

1<sup>st</sup> and 2<sup>nd</sup> excitation potentials 10.2V and 10.4V

3.



The diagram shows energy levels of mercury

- (i) What is the ionization energy and the corresponding ionization potential, if mercury vapour atom is in the ground state.
- (ii) If mercury vapour atom in a ground state has collision with an electron of energy 5eV. How much energy might be retained by electrons in this case.

**Solution**

$$\begin{aligned} \text{i) Ionization energy} &= 0 - (-10.4) \\ &= 10.4\text{eV} \end{aligned}$$

$$\begin{aligned} \text{ii) first ionization energy} &= -5.5 - (-10.4) \\ &= 4.9\text{eV} \end{aligned}$$

$$\begin{aligned} \text{Since } 5\text{eV} \text{ is more than } 4.9\text{eV} \text{ the electron} \\ \text{retains} &= 5 - 4.9 \\ &= 0.1\text{eV} \end{aligned}$$

4. The energy levels in a mercury atom are -10.4eV, -5.5eV, -3.7eV and -1.6eV

- (i) Find the ionization energy in Joules
- (ii) What is likely to happen if a mercury atom in an unexcited state is bombarded with an electron of energy 4.0eV, 6.7eV or 11.0eV

**Solution**

$$\begin{aligned} \text{ionization energy} &= 0 - (-10.4) \\ &= 10.4\text{eV} \\ &= 10.4 \times 1.6 \times 10^{-19}\text{J} \\ \text{Ionization energy} &= 1.664 \times 10^{-18}\text{J} \end{aligned}$$

$$\begin{aligned} \text{1}^{\text{st}} \text{ ionization energy} &= -5.5 - (-10.4) \\ &= 4.9\text{eV} \end{aligned}$$

\* Since 4.0eV is less than 4.9eV, the atom remain unexcited.

$$\begin{aligned} \text{2}^{\text{nd}} \text{ ionization energy} &= -3.7 - (-10.4) \\ &= 6.7\text{eV} \end{aligned}$$

\* So an electron of 6.7eV excites the atom such that an electron jumps from the ground state to energy level -3.7eV.

For a electron of 11eV, it will cause ionization because its value is greater than that at ground state i.e 10eV.

5.

n=∞	0
n=6	-0.38
n=5	-0.54
n=4	-0.85
n=3	-1.51
n=2	-3.39
n=1	-13.6eV

Calculate the frequency and wavelength of radiations resulting from the following transitions

- a) n = 4 to n = 2      b) n = 2 to n = 1 [h = 6.6 × 10<sup>-34</sup>J, c = 3 × 10<sup>8</sup>ms<sup>-1</sup>]

Solution

(a)

$$hf = E_4 - E_2$$

$$hf = -0.85 - -3.39$$

$$hf = 2.54eV$$

$$f = \frac{2.54 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$f = 6.16 \times 10^{14} \text{ Hz}$$

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{3 \times 10^8}{6.16 \times 10^{14}}$$

$$\lambda = 4.87 \times 10^{-7} \text{ m}$$

(b)

$$hf = E_2 - E_1$$

$$hf = -3.39 - -13.6$$

$$hf = 10.21eV$$

$$f = \frac{10.21 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$f = 2.48 \times 10^{15} \text{ Hz}$$

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{3 \times 10^8}{2.48 \times 10^{15}}$$

$$\lambda = 1.21 \times 10^{-7} \text{ m}$$

6. An electron of energy 20eV comes into collision with the hydrogen atom in it's ground state, the atom is excited into a state of higher state of internal energy and electrons is scattered with a reduced velocity, the atom subsequently returns to it's ground state with emission of a photon of wavelength 1.216 × 10<sup>-7</sup>m. Determine the velocity of the scattered electron (e = 1.6 × 10<sup>-19</sup>C, h = 6.6 × 10<sup>-34</sup>J, m = 9.1 × 10<sup>-31</sup>kg, c = 3 × 10<sup>8</sup>ms<sup>-1</sup>)

Solution

$$E = hf$$

$$E = h \frac{c}{\lambda}$$

$$E = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.216 \times 10^{-7}}$$

$$E = 1.63 \times 10^{-18} \text{ J}$$

$$\text{K.E of scattered electron} = 20eV - 1.63 \times 10^{-18}$$

$$= 20 \times 1.6 \times 10^{-19} - 1.63 \times 10^{-18}$$

$$= 1.57 \times 10^{-18} \text{ J}$$

$$\frac{1}{2} m v^2 = 1.57 \times 10^{-18} \text{ J}$$

$$V = \sqrt{\frac{2 \times 1.57 \times 10^{-18}}{9.1 \times 10^{-31}}}$$

$$V = 1.86 \times 10^6 \text{ms}^{-1}$$

### Exercise 9

1. The ionization potential of the hydrogen atom is 13.6V. Use the data below to calculate

(a) The speed of an electron which could just ionize the hydrogen atom.

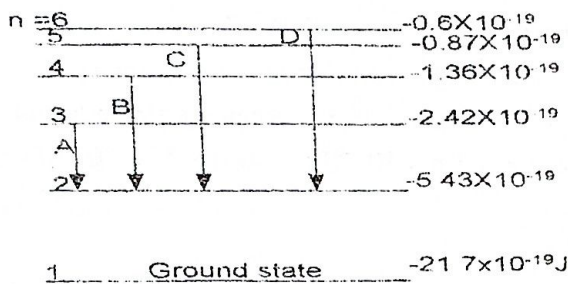
(b) The minimum wavelength which the hydrogen atom can emit

(charge on an electron =  $1.6 \times 10^{-19}$ ,  $m = 9.11 \times 10^{-31}$ kg,  $h = 6.63 \times 10^{-34}$ J s  $c = 3 \times 10^8$ ms $^{-1}$ )

An ( $2.19 \times 10^6$ ms $^{-1}$ ,  $9.14 \times$

$10^8$ m)

2. The figure below representing the lowest energy level of the electron in the hydrogen atom, gives the principle quantum number  $n$  associated with each and the corresponding value of the energy measured in Joules.



- Calculate the wavelength of the lines arising from the transition marked A, B, C, D on the figure.
- The level  $n = 1$  is the ground state of the unexcited hydrogen atom. Explain why hydrogen in its ground state is quite transparent to light emitted by the transitions A, B, C, D and also what happens when  $21.7 \times 10^{-19}$ J of energy is supplied to a hydrogen atom in its ground state.

(take the value of the speed of light in vacuum  $c$  to be  $3 \times 10^8$ ms $^{-1}$  and that of the Planck constant  $h$  to be  $6.63 \times 10^{-34}$ J s). An (661nm, 489nm, 436nm, 412nm)

3. The ionization energy for a hydrogen atom is 13.6eV, if the atom is in its ground state. It is 3.4eV if the atom is in the first excited state. Explain the terms ionization energy and excited state.

Calculate the wavelength of the photon emitted when a hydrogen atom returns to the ground state from the first excited state. Name the part of the electromagnetic spectrum to which this wavelength belongs ( $e = -1.6 \times 10^{-19}$ C,  $h = 6.63 \times 10^{-34}$ J s,  $c = 3 \times 10^8$ ms $^{-1}$ )

4. The energy levels of the hydrogen atom are given by the expression

$$E_n = \frac{-2.16 \times 10^{-18}}{n^2}$$

Where  $n$  is an integer.

(a) What is the ionization energy of the atom

(b) What is the wavelength of the H $\alpha$  line which arises from transition between  $n = 3$  and  $n = 2$  level. ( $h = 6.6 \times 10^{-34}$  Js,  $c = 3 \times 10^8$  ms $^{-1}$ ) An ( $2.16 \times 10^{-18}$  J,  $6.6 \times 10^{-7}$  m)

5. The lowest energy level in a helium atom (the ground state) is -24.6 eV. There are a number of other energy levels, one of which is at -21.4 eV

(a) - Define an eV :

(b) - (i) Explain the significance of the negative signs in the values quoted.

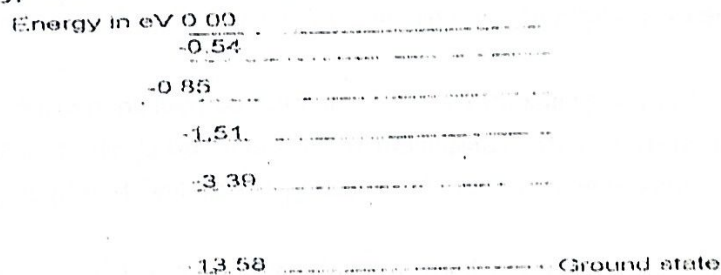
(ii) What is the energy, in J, of a photon emitted when an electron return to the ground state from the energy level at -21.4 eV?

(iii) Calculate the wavelength of the radiation emitted in this transition.

The electronic charge  $e = 1.6 \times 10^{-19}$  C. The speed of electromagnetic radiation

$c = 3 \times 10^8$  ms $^{-1}$ . The Planck's constant  $h = 6.6 \times 10^{-34}$  Js. An ( $5.1 \times 10^{-19}$  J  $3.9 \times 10^{-7}$  m)

6. Some of the energy levels of the hydrogen atom are shown (not to scale) in the diagram



(a) Why are the energy levels labeled with negative energies

(b) State which transition will result in the emission of radiation of wavelength 487 nm. Justify your answer by calculation.

(c) What is likely to happen to a beam of photons of energy (i) 12.07 eV (ii) 5.25 eV when passed through a vapour of atomic hydrogen

7. The diagram below represents the lowest energy levels of the electron in the hydrogen atom, giving the principal quantum number  $n$  associated with each level and the corresponding values of the energy.

3



The diagram shows energy levels of mercury

- (i) What is the ionization energy and the corresponding ionization potential, if mercury vapour atom is in the ground state.
- (ii) If mercury vapour atom in a ground state has collision with an electron of energy 5eV. How much energy might be retained by electrons in this case.

**Solution**

$$\begin{aligned} \text{i) Ionization energy} &= 0 - (-10.4) \\ &= 10.4\text{eV} \end{aligned}$$

$$\begin{aligned} \text{ii) first ionization energy} &= -5.5 - (-10.4) \\ &= 4.9\text{eV} \end{aligned}$$

$$\begin{aligned} \text{Since } 5\text{eV} \text{ is more than } 4.9\text{eV} \text{ the electron} \\ \text{retains} &= 5 - 4.9 \\ &= 0.1\text{eV} \end{aligned}$$

4. The energy levels in a mercury atom are -10.4eV, -5.5eV, -3.7eV and -1.6eV.

- (i) Find the ionization energy in Joules
- (ii) What is likely to happen if a mercury atom in an unexcited state is bombarded with an electron of energy 4.0eV, 6.7eV or 11.0eV

**Solution**

$$\begin{aligned} \text{ionization energy} &= 0 - (-10.4) \\ &= 10.4\text{eV} \\ &= 10.4 \times 1.6 \times 10^{-19}\text{J} \end{aligned}$$

$$\text{Ionization energy} = 1.664 \times 10^{-18}\text{J}$$

$$\begin{aligned} \text{1}^{\text{st}} \text{ ionization energy} &= -5.5 - (-10.4) \\ &= 4.9\text{eV} \end{aligned}$$

\* Since 4.0eV is less than 4.9eV, the atom remain unexcited.

$$\begin{aligned} \text{2}^{\text{nd}} \text{ ionization energy} &= -3.7 - (-10.4) \\ &= 6.7\text{eV} \end{aligned}$$

\* So an electron of 6.7eV excites the atom such that an electron jumps from the ground state to energy level -3.7eV.

❖ For a electron of 11eV, it will cause ionization because its value is greater than that at ground state i.e 10eV.

5.

n=∞	
n=6	-0.38
n=5	-0.54
n=4	-0.85
n=3	-1.51
n=2	-3.39
n=1	-13.6eV

Calculate the frequency and wavelength of radiations resulting from the following transitions

a) n = 4 to n = 2

b) n = 2 to n = 1 [h = 6.6 × 10<sup>-34</sup>J, c = 3 × 10<sup>8</sup>ms<sup>-1</sup>]

Solution

(a)

$$hf = E_4 - E_2$$

$$hf = -0.85 - -3.39$$

$$hf = 2.54eV$$

$$f = \frac{2.54 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$f = 6.16 \times 10^{14} \text{ Hz}$$

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{3 \times 10^8}{6.16 \times 10^{14}}$$

$$\lambda = 4.87 \times 10^{-7} \text{ m}$$

(b)

$$hf = E_2 - E_1$$

$$hf = -3.39 - -13.6$$

$$hf = 10.21eV$$

$$f = \frac{10.21 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$f = 2.48 \times 10^{15} \text{ Hz}$$

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{3 \times 10^8}{2.48 \times 10^{15}}$$

$$\lambda = 1.21 \times 10^{-7} \text{ m}$$

6. An electron of energy 20eV comes into collision with the hydrogen atom in it's ground state, the atom is excited into a state of higher state of internal energy and electrons is scattered with a reduced velocity, the atom subsequently returns to it's ground state with emission of a photon of wavelength 1.216 × 10<sup>-7</sup>m. Determine the velocity of the scattered electron (e = 1.6 × 10<sup>-19</sup>C, h = 6.6 × 10<sup>-34</sup>J, m = 9.1 × 10<sup>-31</sup>kg, c = 3 × 10<sup>8</sup>ms<sup>-1</sup>)

Solution

$$E = hf$$

$$E = h \frac{c}{\lambda}$$

$$E = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.216 \times 10^{-7}}$$

$$E = 1.63 \times 10^{-18} \text{ J}$$

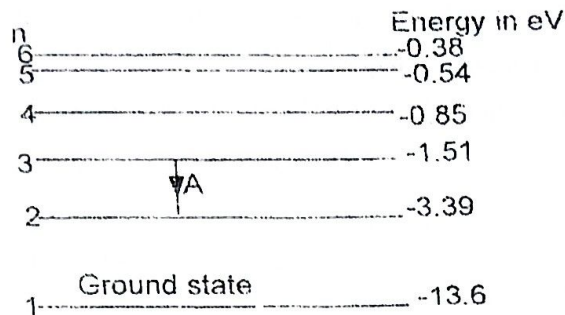
$$\text{K.E of scattered electron} = 20eV - 1.63 \times 10^{-18}$$

$$= 20 \times 1.6 \times 10^{-19} - 1.63 \times 10^{-18}$$

$$= 1.57 \times 10^{-18} \text{ J}$$

$$\frac{1}{2} m v^2 = 1.57 \times 10^{-18} \text{ J}$$

(i) Why are the energies quoted with negative values



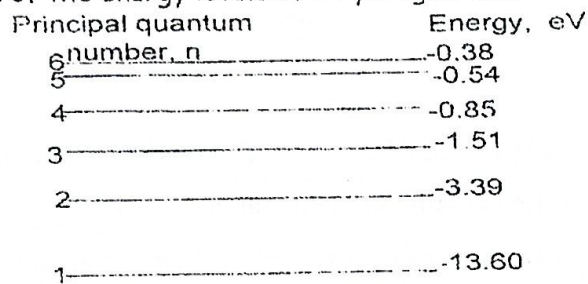
(ii) Calculate the wavelength of the line arising from the transition A, indicating in which region of the electromagnetic spectrum this occurs.

(iii) What happens when 13.6eV of energy is absorbed by a hydrogen atom in its ground state

An ( $6.6 \times 10^{-7}m$ )

UNEB 2013 Q.9

(a) Figure shows some of the energy levels of a hydrogen atom



(i) Why are the energies for the different levels negative (01marks)

(ii) Calculate the wavelength of the line arising from a transition from the third to the second energy level (03marks)

An ( $6.6 \times 10^{-7}m$ )

(iii) Calculate the ionization energy in joules of hydrogen (02marks)

An ( $2.176 \times 10^{-18}J$ )

(b) Explain the physical process in an X-ray tube that accounts for

(i) Cut off wavelength (03marks)

(ii) Characteristic line (04marks)

(c) Calculate the maximum frequency of radiation emitted by an X-ray tube using an accelerating voltage of 33.0kV (03marks)

An ( $8 \times 10^{18}Hz$ )

(d) Derive Bragg's law of X-ray diffraction in crystal (04marks)

UNEB 2013 Q.10

- (a) A beam of  $\alpha$ -particles directed normally to a thin metal foil. Explain why
- (i) Most of the  $\alpha$ -particles passed straight through the foil. (02marks)
  - (ii) Few  $\alpha$ -particles are deflected through angles more than  $90^\circ$ . (02marks)
- (b) Calculate the least distance of approach of a  $3.5\text{MeV}$   $\alpha$ -particles to the nucleus of a gold atom (atomic number of gold = 79)  $\text{An}(6.495 \times 10^{-14}\text{m})$  (04marks)

UNEB 2012 Q.8

- (c) State the laws of photo electric emission (04marks)
- (d) Explain how line emission spectra are produced (03marks)

UNEB 2011 Q.9

- a) (i) Explain how X-rays are produced in an X-ray tube
- (ii) Explain the emission of X-ray characteristic spectra (03marks)
- (iii) Derive the Bragg X-ray diffraction equation (04marks)
- (iv) Under what conditions does X-ray diffraction occur (02marks)

UNEB 2010 Q.10

- c) (i) show that when an alpha particle collides head on with an atom of atomic number. The closest distance of approach to the nucleus,  $Z_0$  is given by  $Z_0 = \frac{Ze^2}{\pi\epsilon_0 m v^2}$
- Where  $e$  is the electronic charge  $\epsilon_0$  is the permittivity of free space,  $m$  is the mass of the alpha particle and  $V$  is the initial speed of the alpha particle (04marks)

UNEB 2010 Q.9

- (c) An X-ray of wavelength  $10^{-10}\text{m}$  is required for the study of it's diffraction in a crystal. Find the least accelerating voltage to be applied to an X-ray tube in order to produce these X-rays. (04marks)

- (d) Sodium has a work function of  $2.0\text{eV}$  and is illuminated by radiation of wavelength  $150\text{nm}$ . Calculate the maximum speed of the emitted electrons  $\text{An}(1.24 \times 10^4\text{V})$

(04marks)

- (e) With aid of a labeled diagram describe how stopping potential of metal can be measured.

UNEB 2009 Q.9d

- (d) Distinguish between continuous and line spectra in an X-ray tube

UNEB 2009 Q.10

(a) (i) Explain the observations made in the Rutherford's alpha particle scattering experiment

(06marks)

(ii) Why is a vacuum necessary in this experiment (01mark)

(b) Distinguish between excitation and ionization energies of an atom (02marks)

(c) Draw a labeled diagram showing the main components of an X-ray tube. (03marks)

(d) An X-ray tube is operated at 50kV and 20mA. If 1% of the total energy supplied is emitted as X-radiation, calculate the:

(i) Maximum frequency of the emitted radiation (3mk)

(ii) Rate at which heat must be removed from the target in order to keep it at a steady temperature

(03marks)

(e) A beam of X-rays of wavelength 0.20nm is incident on a crystal at a glancing angle of  $30^\circ$ . If the inter planar separation is 0.20nm, find the order of diffraction.

(An  $(1.21 \times 10^{19}\text{Hz}, 990\text{W}, n = 1$  (first order

diffraction))

UNEB 2008 Q.8

(a) What is meant by a line spectrum (02marks)

(b) Explain how line spectra accounts for the existence of discrete energy level in atoms (4mk)

(d) Describe with aid of a labeled diagram, the action of an X-ray tube

(e) An X-ray tube is operated at 20kV with an electron current of 16mA in the tube estimate the:

(i) Number of electrons hitting the target per second (02marks)

(ii) Rate of production of heat, assuming 99.5% of the kinetic energy of electrons is converted to heat ( $e = 1.6 \times 10^{-19}\text{C}$ ) An  $(1.0 \times 10^{17}$  electron per second, 318.4W)

02mks)

UNEB 2007 Q.10

(c) Explain X-ray diffraction by crystals and derive Bragg's law (06marks)

(d) The *p.d* between the cathode and the anode of an X-ray tube is  $5 \times 10^4\text{V}$ . If only 0.4% of the kinetic energy of the electrons is converted into X-rays and the rest is dissipated as heat in the target at a rate of 600W. Find the:

(i) Current that flows

(03marks)

- (ii) Speed of the electrons striking the target  
(03marks)

An( $1.21 \times 10^6$  A,  $1.33 \times$

$10^4 \text{ms}^{-1}$ )

UNEB 2006 Q.8

- (a) (i) What is photon (01marks)  
 (ii) Explain, using quantum theory, the experimental observations on the photoelectric effect (06marks)  
 (iii) When light of wavelength 450nm falls on a certain metal, electrons of maximum kinetic energy 0.76eV are emitted. Find the threshold frequency for the metal. (04marks)

An ( $4.83 \times 10^{14}$ Hz)

- (b) Explain, using suitable sketch graphs, how X-ray spectra in an X-ray tube are formed (6mks)  
 (c) A beam of x-rays of wavelength  $8.42 \times 10^{-11}$ m is incident on a sodium chloride crystal of inter planal separation  $2.82 \times 10^{-10}$ m. Calculate the first order diffraction angle (03marks)

An ( $\theta$

=  $8.6^\circ$ )

UNEB 2005 Q.8

- (a) (i) Draw a labeled diagram of an X-ray tube (02marks)  
 (ii) Use the diagram in (a) (i) to describe how X-rays are produced. (03marks)  
 (iii) State one industrial and one biological use of X-rays. (01marks)  
 (b) (i) Sketch a graph of intensity versus wavelength of X-rays from an X-ray tube and describe it's main features. (04marks)  
 (ii) Calculate the maximum frequency of X-rays emitted by an X-ray tube operating a voltage of 34kV  
 An ( $8.34 \times 10^{18}$  Hz) (03marks)

UNEB 2005 Q.9

- (a) (i) State the laws of photo electric emission (04marks)  
 (ii) Write down Einstein's equation for photoelectric emission (02marks)  
 (iii) Ultra-violet light of wavelength  $3.3 \times 10^{-8}$ m is incident on a metal. Given that the work function of the metal is 3.5eV, calculate the maximum velocity of the liberated electron  
 An ( $3.46 \times 10^6 \text{ms}^{-1}$ )

(03marks)

UNEB 2004 Q.9

- (a) Explain the term stopping potential are applied to photoelectric effect.

(b) Explain how intensity and penetrating power of X-rays from an X-ray tube would be affected by changing

(i) the filament current (02marks)

(ii) the high tension potential difference across the tube (02marks)

(c) When a p.d of 60kV is applied across an X-ray tube a current of 30mA flows. The anode is cooled by water flowing at a rate of  $0.06\text{kg s}^{-1}$ . If 99% of the power supplied is converted into heat at the anode, calculate the rate at which the temperature of the water rises.

(S.H.C =  $4.2 \times 10^3 \text{Jkg}^{-1}\text{k}^{-1}$ ) An (7.07K)

(05marks)

(d) (i) Derive Bragg's law of X-ray diffraction (05marks)

#### UNEB 2003 Q.9

(b) (i) What features of an X-ray tube make it suitable for continuous production of X-rays

(03marks)

(ii) Sketch a graph of intensity versus frequency of a radiation produced in an X-ray tube and explain its features (05marks)

(c) A mono chromatic X-ray beam of wavelength  $1 \times 10^{-10}\text{m}$  is incident on a set of planes in a crystal of spacing  $2.8 \times 10^{-10}\text{m}$ . What is the maximum order possible with these X-rays

An (6)

(04marks)

#### UNEB 2003 Q.8

(a) (i) State Rutherford's model of the atom (02marks)

(ii) Explain two main failures of Rutherford's model of the atom (03marks)

#### UNEB 2002 Q.8

(a) What is meant by

(i) Bohr atom (01marks)

(ii) Binding energy of a nucleus (02marks)

(b) The total energy  $E$  of an electron in an atom may be expressed as

$$E = \frac{-mq^2}{8n^2h^2\epsilon_0^2}$$

(i) Identify the quantities  $m, q, n$  and  $h$  in this expression (02marks)

(ii) Explain the physical implication of the fact that  $E$  is always negative (02marks)

(iii) Draw an energy level diagram for hydrogen to indicate emission of ultraviolet visible and infra-red spectral lines (03marks)

(d) The atomic nucleus may be considered to be a sphere of positive charge with a diameter very much less than of the atom. Discuss the experiment evidence which supports this view.

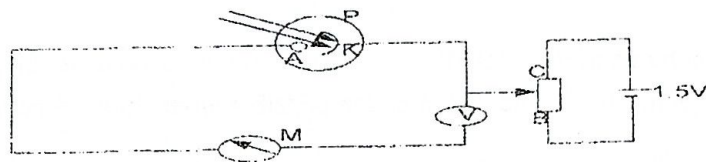
(03marks)

UNEB 2002 Q.9

- (b) (i) Describe a simple experiment to demonstrate photo electric emission (04marks)  
 (ii) Explain why the wave theory of light falls to account for the photoelectric effect (6mk)  
 (iii) Describe an experiment to verify Einstein's equation for the photoelectric effect and explain how plank's constant may be obtained from the experiment (06marks)

UNEB 2001 Q.8

- (a) (i) Write down the Einstein photoelectric equation  
 (ii) Explain how the equation in (i) above accounts for the emission of electrons from metal surface illuminated by radiation  
 (04marks)
- (b)



P is a vacuum photo cell with anode A and cathode K, made from the same metal of work function  $2eV$ . The cathode is illuminated by monochromatic light of constant intensity and of wavelength  $4.4 \times 10^{-7}m$ .

- (i) Describe and explain how the current shown by the micro ammeter M will vary as the slider of the potential divider is moved from B to C. (03marks)
- (ii) What will the reading of the high resistance voltmeter V be when photo-electric emission just ceases? (03marks)
- (c) With the slider set mid-way between B and C, describe and explain how the reading of M would change if:  
 (i) The intensity of the light was increased (03marks)  
 (ii) the wavelength of the light was changed to  $5.5 \times 10^{-7}m$  (06marks)

Solution

(b)(i) As the slider moves from B to C, the cathode will become more positive. Hence more of the photo electrons that are emitted by the cathode are attracted by it. This causes a reduction in

the number of the photo electrons reaching the anode and hence the photo electric current that is measured by the micro-ammeter  $M$  reduces as the slider moves towards  $C$ .

ii) When photo electric emission ceases, it gives stopping potential ( $V_s$ )

$$\begin{array}{l} hf = W + \frac{1}{2} mv^2 \\ hf = W_0 + eV_s \\ V_s = \frac{hf - W_0}{e} \end{array} \quad \left| \quad \begin{array}{l} V_s = \frac{6.6 \times 10^{-34} \times 3 \times 10^8 - 2 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19}} \\ V_s = 0.8125V \end{array} \right.$$

c)(i) with the slider mid way between  $B$  and  $C$  the  $p.d V = \frac{1.5}{2} = 0.75V$  which is less than the stopping potential of  $0.8125V$ . Since increasing the intensity leads to an increase in the number of photo electrons being emitted per second then it implies that the micro-ammeter reading would increase with increasing intensity.

ii)

$$\begin{array}{l} W_0 = hf_0 \\ W_0 = h \frac{c}{\lambda_{max}} \\ \lambda_{max} \times = \frac{hc}{W_0} \end{array} \quad \left| \quad \begin{array}{l} \lambda_{max} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2 \times 1.6 \times 10^{-19}} \\ \lambda_{max} = 6.19 \times 10^7 m \end{array} \right.$$

Since the wavelength of  $5.5 \times 10^{-7}m$  is less than  $\lambda_{max}$  which is the maximum wavelength of the incident radiation that would cause photoelectric emission then it means that there will be photo electric emission. But since the new wavelength less than that of the previous radiation then the kinetic energy of the photo electrons will be less than before. However if the intensity is maintained, the rate of emission of the photo electrons is the same and consequently the reading of  $M$  is unaltered.

#### UNEB 2000 Q.8

- (a) State the laws of photo electric emission (04marks)
- (b) (i) Describe an experiment to determine Planck's constant (05marks)
- (ii) Violet light of wavelength  $0.4\mu m$  is incident on a metal surface of threshold wavelength  $0.65\mu m$ . Find the maximum speed of the emitted electrons (04marks)
- (iii) Explain why light whose frequency is less than the threshold frequency can't cause photo emission (02marks)
- (c) (i) What are X-rays (01marks)
- (ii) Explain how intensity and penetrating power of X-rays produced by an X-ray tube can be varied.

## CHAPTER 4: NUCLEAR STRUCTURE

The nucleus is the central positively charged part of an atom.

Nuclei contain protons and neutrons which are collectively referred to as **nucleons** (nuclear number).

### 4.1.0: ATOMIC NUMBER Z, MASS NUMBER A AND ISOTOPES

**Atomic number Z** of an element is the number of protons in the nucleus of an atom of the element.

**Mass number A** of an atom is the number of nucleons in its nucleus.

**Isotopes** are atoms of the same element which have the same number of protons but different number of neutrons and therefore different mass numbers

**Isotopy:** it is the existence of atoms of the same element with the same atomic number but different mass numbers

Isotopes of an element whose chemical symbol is represented by X can be distinguished by using the symbol  ${}^A_ZX$

Where A is mass number and Z is atomic number

Example of isotopes

Isotopes of Lithium  ${}^7_3\text{Li}$  and  ${}^6_3\text{Li}$

Isotopes of uranium  ${}^{235}_{92}\text{U}$  and  ${}^{238}_{92}\text{U}$

Isotones are nuclei with the same number of neutrons

Isobars are nuclei with the same number of nucleons.

#### 4.1.1: EINSTEIN'S MASS - ENERGY RELATION

Einstein showed from his theory of relativity that mass (m) and energy (E) can be changed from one form to another.

The energy  $\Delta E$  produced by a change of mass  $\Delta M$  is given by the relation.

$$\Delta E = \Delta MC^2$$

Where C is the speed of light ( $C = 3 \times 10^8 \text{ms}^{-1}$ )

Energy produced in change of mass of 1kg

$$\Delta E = 1 \times (3 \times 10^8)^2$$

$$\Delta E = 9 \times 10^{16} \text{J}$$

#### 4.1.2: UNIFIED ATOMIC MASS UNIT [U]

It is defined as  $1/12$  of the mass of carbon-12 atom.

The number of molecules in 1mole of carbon 12 is  $6.02 \times 10^{23} \text{mol}^{-1}$

$6.02 \times 10^{23}$  atoms has a mass of 12g of carbon -12

$$6.02 \times 10^{23} \text{ atoms} = 12 \times 10^{-3} \text{kg}$$

$$1 \text{atom} = \frac{12 \times 10^{-3}}{6.02 \times 10^{23}}$$

$$1 \text{atom} = 1.993355482 \times 10^{-26} \text{kg}$$

$$\begin{aligned} 1 \text{unified atomic mass} &= \frac{1}{12} \times 1.993355482 \times 10^{-26} \\ &= 1.661129568 \times 10^{-27} \text{kg} \end{aligned}$$

$$1\text{U} = 1.66 \times 10^{-27} \text{kg}$$

From Einstein's mass - energy relation

$$\Delta E = MC^2$$

$$C = 2.998 \times 10^8 \text{ms}^{-1}$$

$$1\text{U} = 1.661129568 \times 10^{-27} \times (2.998 \times 10^8)^2$$

$$1\text{U} = 1.49302392 \times 10^{-10} \text{J}$$

$$1\text{eV} = 1.602 \times 10^{-19} \text{J}$$

$$1U = \frac{1.49302392 \times 10^{-10}}{1.602 \times 10^{-19}} \text{ eV}$$

$$1U = 931.97 \times 10^6 \text{ eV}$$

$$\boxed{1U = 931 \text{ MeV}}$$

### 4.1.3: MASS DEFECT AND BINDING ENERGY

#### a) MASS DEFECT

❖ It is defined as the mass equivalence of the energy required to split the nucleus into its constituent particles.

The mass of a nucleus is always less than the total mass of its constituent nucleons.

The difference in mass is called the mass defect of the nucleus *i.e.*

$$\boxed{\text{Mass defect} = (\text{mass of nucleons and electrons}) - (\text{mass of atom})}$$

#### Note

The reduction in mass arises because the act of combining the nucleons to form the nucleus causes some of their mass to be released as energy (in form of  $\gamma$ -rays).

Any attempt to separate the nucleons would involve them being given this same amount of energy; it is therefore called the **binding energy** of the nucleus.

#### b) BINDING ENERGY (B.E)

❖ Binding energy of the nucleus is the energy required to break up the nucleus into its constituent nucleons

❖ Binding energy per nucleon is the ratio of the energy needed to split a nucleus into its constituent nucleons to the mass number.

$$\boxed{\text{B.E per nucleon} = \frac{B.E}{\text{Mass number}}}$$

Binding energy per nucleon is very useful in measure of the stability of the nucleus. The higher the binding energy per nucleon the more stable the nucleus is

$$\text{Binding energy (J)} = \text{mass defect (kg)} \times C^2 (\text{ms}^{-1})^2$$

$$\text{Where } 1U = 1.66 \times 10^{-27} \text{ kg}$$

OR

$$\text{Binding energy (MeV)} = \text{mass defect (U)} \times 9.31 (\text{MeV})$$

$$\text{Where } 1U = 931 \text{ MeV}$$

#### Example

1. Given atomic mass of  ${}_{92}^{238}\text{U} = 238.05076 \text{ U}$

mass of neutron = 1.00867U  
 mass of proton = 1.00728U  
 mass of electron = 0.00055U  
 1U = 931MeV

Find: a) mass defect

b) B.E per nucleon for  ${}_{92}^{238}\text{U}$

**Solution**

a) Mass defect = (mass of nucleons and electrons) - (mass of nucleus)

number of protons = 92

number of electrons = 92

number of neutrons = (238 - 92)

= 146

$$\text{Mass defect} = \left( \begin{array}{c} 146 \times 1.00867 \\ + \\ 92 \times 1.00728 \\ + \\ 92 \times 0.00055 \end{array} \right) - (238.05076)$$

= 239.98618 - 38.05076

Mass defect = 1.93542U

b)

B.E per nucleon =  $\frac{B.E}{\text{Mass number}}$

B.E = mass defect  $\times$  931

= 1.93542  $\times$  931

= 1801.87602mev

B.E per nucleon =  $\frac{1801.87602}{238}$

B.E per nucleon = 7.571MeV

2. Given mass of proton = 1.0080U

Mass of neutron = 1.0087U

Mass of alpha particle = 4.0026U

1U = 931MeV

Find:

a) mass defect in (i) U (ii) kg

b) Binding energy in (i) MeV (ii) J

c) Binding energy per nucleon in (i) MeV (ii) J

**Solution**

An alpha particle is a helium nuclei  ${}^4_2\text{He}$

a) Mass defect = (mass of nucleons) - (mass of a tom)

number of protons = 2

number of neutrons = 2

$$\begin{aligned} \text{i) mass defect} &= (2 \times 1.0080 + 2 \times 1.0087) - 4.006 \\ &= 0.0308 \text{U} \end{aligned}$$

ii) mass defect in kg

$$1 \text{U} = 1.66 \times 10^{-27} \text{kg}$$

$$\begin{aligned} \text{Mass defect} &= 0.0308 \times 1.66 \times 10^{-27} \text{kg} \\ &= 5.1128 \times 10^{-29} \text{kg} \end{aligned}$$

$$\begin{aligned} \text{b)(i) Binding energy (MeV)} &= \text{mass defect} \times 931 \text{MeV} \\ &= 0.0308 \times 931 \\ &= 28.6748 \text{MeV} \end{aligned}$$

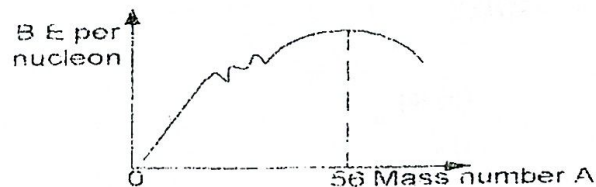
$$\begin{aligned} \text{ii) Binding energy (J)} &= 28.6748 \times 10^6 \times 1.6 \times 10^{-19} \text{J} \\ &= 4.59 \times 10^{-12} \text{J} \end{aligned}$$

$$\begin{aligned} \text{Or Binding energy (J)} &= \text{mass defect} \times C^2 (\text{ms}^{-1})^2 \\ &= 5.1128 \times 10^{-29} \times (3 \times 10^8)^2 \\ &= 4.60 \times 10^{-12} \text{J} \end{aligned}$$

$$\begin{aligned} \text{c)(i) Binding energy per nucleon} &= \frac{B \text{ J}}{\text{Mass number}} \\ &= \frac{28.6748}{4} \text{MeV} \\ &= 7.17 \text{MeV} \end{aligned}$$

$$\begin{aligned} \text{ii) Binding energy per nucleon} &= 7.17 \times 10^6 \times 1.6 \times 10^{-19} \\ &= 1.15 \times 10^{-12} \text{J} \end{aligned}$$

#### 4.1.4: VARIATION OF B.E PER NUCLEON WITH MASS NUMBER



##### Main features of the graph

- ❖ Binding energy per nucleon for very small and large nuclides is small
- ❖ A few peaks for low mass numbers are for lighter nuclei that are comparatively stable
- ❖ The binding energy per nucleon increases sharply to a maximum at mass number 56
- ❖ For  $A > 56$  binding energy per nucleon gradually decreases

## NOTE

The low binding energy per nucleon value for small and high mass number nuclide implies that they are potential sources of nuclear energy because they easily undergo fusion and fission respectively.

### 1.5: Explanation of fusion and fission using the graph

- ❖ During nuclear fusion two light nuclei unite to form a heavier nucleus that has a higher binding energy per nucleon. However the total mass of the heavier nucleus is less the sum of the two light nuclei and the mass difference is accounted for by the energy released.
- ❖ During Nuclear fission, a heavy nucleus splits to form two lighter nuclei having greater binding energy per nucleon. However the total mass of the two daughter nuclei is less than the mass of the heavy nucleus and the difference in mass is accounted for by the release of energy.

### 4.2.0: RADIO-ACTIVITY (RADIOACTIVE DECAY)

This is the spontaneous decay of a heavy nucleus to daughter nuclei with emission of  $\alpha$  particles,  $\beta$ -particle or  $\gamma$ -rays.

Heavy nuclides are generally unstable hence this decay is in attempt to reach a stable state.

Radio-activity is said to be a random process because no particular pattern is followed.

### 4.2.1: TYPES OF IONISING RADIATIONS

#### a) Alpha particles ( $\alpha$ )

They have a mass of 4times that of hydrogen atom and a charge of  $+2e$  where  $e$  is the numerical charge on an electron hence they are Helium nuclei



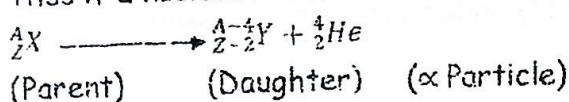
#### Properties

- They have the least penetrating power among the ionizing radiations.
- They are positively charged hence can be deflected by electric and magnetic field
- They are the best ionizers of gases
- They have the shortest range in air among the ionizing radiations
- When emitted, they are emitted with the same speed

Note

When a nucleus undergoes  $\alpha$  - decay it loses four nucleons, two of which are protons, therefore atomic number  $Z$  decreases by two.

Thus if a nucleus  $X$  becomes a nucleus  $Y$  as a result of  $\alpha$  -decay then



E.g Uranium - 238 decays by  $\alpha$  -emission to thorium 234 according to



### b) Beta particle ( $\beta$ )

It is an electron which is moving at a high speed. It is represented as [ ${}_{-1}^0 e$ ]

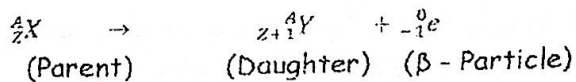
#### Properties

- It has a higher penetrating power than  $\alpha$ particle
- It is negatively charged hence deflected by electric and magnetic field.
- It is a moderate ionizer of gases
- It has a moderate range in air
- $\beta$  particles are emitted by nuclei with various speeds
- It is lighter than  $\alpha$  -particle

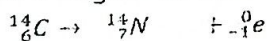
#### Note

$\beta$ -particles are emitted by nuclei which have too many neutrons to be stable. To gain a stable state one of its neutrons should change into a proton and an electron, when this happens the electron is immediately emitted as a  $\beta$ -particle.

Thus when a nucleus undergoes  $\beta$ -decay, its mass number  $A$  does not change and its atomic number  $Z$  increases by one



E.g Carbon-14 decays by  $\beta$ -emission to nitrogen- 14 according to



### c) Gamma rays ( $\gamma$ )

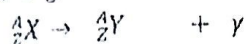
They are electromagnetic waves of very short wave length and they travel with a velocity of light.

#### Properties

- They have the highest penetrating power
- They are electrically neutral hence they can't be deflected by electric or magnetic field
- They are the poorest ionizers of gases
- They can be diffracted and refracted

### Note

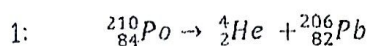
Gamma ray decay involves the release of only energy without the change in atomic mass and atomic number e.g



### 4.2.2: ENERGY OF DISINTEGRATION (Q-value)

It is necessary to investigate whether a given disintegration takes in energy (endothermic) or gives out energy (Exothermic) by considering the total atomic mass of reactant and products when the total mass of reactant is greater than the total mass of products then the reaction is exothermic.

#### Example



Atomic mass of  ${}^{206}_{82}Pb = 205.969U$

Atomic mass of  ${}^4_2He = 4.003U$

Atomic mass of  ${}^{210}_{84}Po = 209.983U$

- State whether the disintegration is endothermic or exothermic and calculate the energy of disintegration.
- Calculate energy of the  $\alpha$ -particle

#### Solution

Mass of reactant = 209.983U

Mass of product = 205.909U + 4.003U

= 209.972U

Since mass of reactant is greater than the total mass of products then its exothermic.

Therefore  ${}^{210}_{84}Po \rightarrow {}^4_2He + {}^{206}_{82}Pb + Q$

Energy of disintegration = mass defect  $\times$  931mev

$$= (209.983 - 209.972) \times 931\text{mev}$$

$$= 0.011 \times 931\text{MeV}$$

$$= 10.24\text{MeV}$$

Note Q-value appears as the kinetic energy of the products

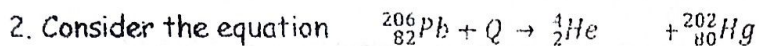
$$K.e_{\alpha} = \frac{M}{M+m_{\alpha}} Q \text{ where}$$

$m_{\alpha}$  is the atomic mass of the  $\alpha$  particle

$M$  is atomic mass of daughter atom

$$K.e_{\alpha} = \frac{206}{206+4} 10.24$$

$$K.e_{\alpha} = 10.05\text{MeV}$$



Atomic mass of Hg = 201.971U

Atomic mass of He = 4.003U

Atomic mass of Pb = 205.969

Calculate i) Q -value

ii) kinetic energy of the  $\alpha$ -particle

Solution

i)  $Q = \text{mass} \times 931\text{MeV}$

$$Q = ((201.971 + 4.003) - 205.969) \times 931\text{mev}$$

$$0.005 \times 931\text{MeV}$$

$$Q\text{-value} = 4.66\text{MeV}$$

ii)  $K.e_{\alpha} = \frac{M}{M+m_{\alpha}} Q$  :

$$K.e_{\alpha} = \frac{202}{202+4} 4.66$$

$$K.e_{\alpha} = 4.57\text{MeV}$$

Generally, A nucleus would tend to be unstable and emit an  $\alpha$ -particle, if the sum of the atomic masses of the products are together less than that of the nucleus and it would be stable if the sum of the atomic masses of the possible reaction products are together greater than the atomic mass of the nucleus.

### EXERCISE 10

1.  ${}^{210}_{84}\text{Po}$  decays to Pb-206 by emission of alpha - particle of single energy

(i) Write down the symbolic equation for the reaction

(ii) Calculate the energy in MeV released in each disintegration

(iii) Explain why this energy does not all appear as kinetic energy of the alpha particle

(iv) Calculate the kinetic energy of the alpha particle

$${}^{210}\text{Po} = 209.93673\text{U}$$

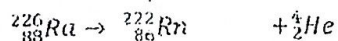
$${}^{206}\text{Pb} = 205.929421\text{U}$$

$${}^4\text{He} = 4.001504\text{U}$$

$$1\text{U} = 931\text{MeV}$$

Ans (5.40MeV, 5.3MeV)

2. Consider the decay process represented by



Calculate the kinetic energy of the alpha particle

$${}^{226}_{88}\text{Ra} = 226.0254\text{U}$$

$${}^{222}_{86}\text{Rn} = 222.0175\text{U}$$

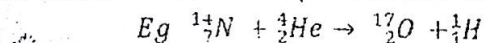
$${}^4_2\text{He} = 4.0026\text{U}$$

An (4.93MeV)

#### 4.2.3: ARTIFICIAL DISINTEGRATION (Nuclear reaction)

This is achieved by bombarding the nuclei with an energetic particle.

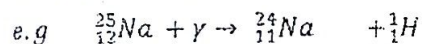
The bombarding particle acquires enough energy by being accelerated in a reasonable speed by use of electric fields except the neutron.



Examples of nuclear reactions

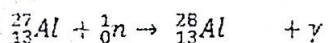
##### 1. Photo disintegration

This is a nuclear reaction in which absorption of  $\gamma$ -ray results into decay of the absorbing nuclei.



##### 2. Neutron radioactive capture

In this reaction the neutron is captured by the parent atom and a  $\gamma$ -ray is emitted



This method is used for production of isotopes in nuclear reactions.

Beta particle as a bombarding particle

Advantage

- > It can be accelerated at a high speed using electric field.

Disadvantages

- > It experiences electrostatic repulsion with shell electrons
- > It is light

Alpha particle as bombarding particle

Advantages

- > It can be accelerated to high speed using electric field
- > It is fairly heavy

Disadvantage

- > It experiences electrostatic repulsion with positive nucleus

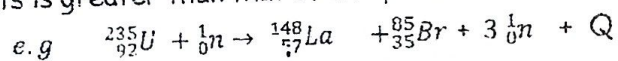
Neutron as a bombarding particle

- This is the best particle for study of nuclear reactions. Being electrically neutral it neither experiences electrostatic repulsion in the shell electrons nor the nucleus.
  - However, it cannot be accelerated to high speeds using electric fields.
- Energetic neutrons for nuclear reactions are obtained from nuclear reactants by the process of fusion.

#### 4.2.4: NUCLEAR FISSION

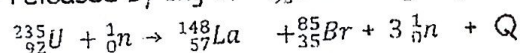
Nuclear fission is the disintegration of a heavy nucleus into two lighter nuclei accompanied by release of energy..

Energy is released by the process because the average binding energy per nucleon of the fission products is greater than that of the parent.



#### Example

Calculate the energy released by 1kg of  ${}_{92}^{235}\text{U}$  under going fission according to



Mass ${}_{92}^{235}\text{U}$	=	235.1U
Mass of ${}_{57}^{148}\text{La}$	=	148.0U
Mass of ${}_0^1\text{n}$	=	1.009U
Mass of ${}_{35}^{85}\text{Br}$	=	84.9U

#### Solution

Mass of reactants	=	235.1 + 1.009
	=	236.109U
Mass of products	=	(148.0 + 84.9 + (3 × 1.009))
	=	235.927U
Energy released	=	mass defect × 931MeV
	=	(236.109 - 235.927) × 931MeV
	=	169.442MeV
Energy released	=	169.442 × 10 <sup>6</sup> × 1.6 × 10 <sup>-19</sup> J
	=	2.71 × 10 <sup>-11</sup> J

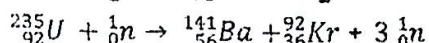
This is the energy released by one atom of uranium - 235

235 × 10 <sup>-3</sup> kg contains	=	6.02 × 10 <sup>23</sup> atom
1 kg contains	=	$\frac{6.02 \times 10^{23}}{235 \times 10^{-3}}$
	=	2.562 × 10 <sup>24</sup> atom
One atom released	=	2.71 × 10 <sup>-11</sup> J
2 562 × 10 <sup>24</sup> atoms	=	2.71 × 10 <sup>-11</sup> × 2 562 × 10 <sup>24</sup> J

$$\begin{aligned} &= 6.943 \times 10^{13} \text{J} \\ \text{Energy released by 1kg of uranium} &= 6.943 \times 10^{13} \text{J} \end{aligned}$$

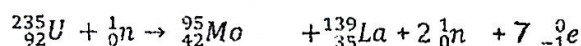
### Exercise

1. Calculate the energy released when 10kg of  $^{235}_{92}\text{U}$  undergoes fission according to;



(mass of  $^{235}\text{U} = 235.04\text{U}$ , of  $^{141}\text{Ba} = 140.91\text{U}$ , of  $^{92}\text{Kr} = 91.91\text{U}$  of  $1\text{n} = 1.01\text{U}$  and  $1\text{U} = 931\text{MeV}$ ,  $N_A = 6.02 \times 10^{23}\text{mol}^{-1}$ ) An ( $7.36 \times 10^{14}\text{J}$  or  $4.77 \times 10^{27}\text{MeV}$ )

2. Atypical fission reaction is as below;



Calculate the total energy released by one gram of uranium - 235 undergoing fission, neglect the masses of the electron

(mass of  $1\text{n} = 1.009\text{U}$ , of  $^{95}\text{Mo} = 94.906\text{U}$  of  $^{139}\text{La} = 138.906\text{U}$  of  $^{235}\text{U} = 235.044\text{U}$ ,  $1\text{U} = 931\text{MeV}$ ). An ( $8.51 \times 10^{10}\text{J}$ )

### Note

The neutrons released in fission can be slowed to a reasonable speed and made to bombard more uranium targets, emission of more neutrons which can still bombard more target thus leading to chain reaction. This is the basis of atomic bombs and nuclear reactants to produce energetic neutrons.

### Application of fission

- > In the production of neutrons
- > In production of atomic bombs

### Condition for fission

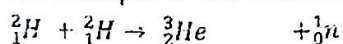
- > It requires an energetic particle like a neutron

### 4.2.5: NUCLEAR FUSION

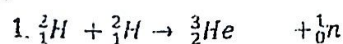
Nuclear fusion is the union of two light nuclei to form a heavier nucleus accompanied by release of energy.

Energy is released in the process.

An example is the fusion of two deuterium nuclei to produce helium -



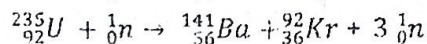
### Examples



$$\text{Energy released by 1kg of uranium} = 6.943 \times 10^{13} \text{ J}$$

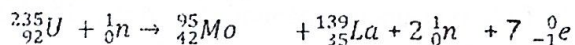
### Exercise

1. Calculate the energy released when 10kg of  ${}^{235}_{92}\text{U}$  undergoes fission according to:



(mass of  ${}^{235}\text{U} = 235.04\text{U}$ , of  ${}^{141}\text{Ba} = 140.91\text{U}$ , of  ${}^{92}\text{Kr} = 91.91\text{U}$  of  $1\text{n} = 1.01\text{U}$  and  $1\text{U} = 931\text{MeV}$ ,  $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$ ) An ( $7.36 \times 10^{14} \text{ J}$  or  $4.77 \times 10^{27} \text{ MeV}$ )

2. Atypical fission reaction is as below;



Calculate the total energy released by one gram of uranium - 235 undergoing fission, neglect the masses of the electron

(mass of  $1\text{n} = 1.009\text{U}$ , of  ${}^{95}\text{Mo} = 94.906\text{U}$  of  ${}^{139}\text{La} = 138.906\text{U}$  of  ${}^{235}\text{U} = 235.044\text{U}$ ,  $1\text{U} = 931\text{MeV}$ ). An ( $8.51 \times 10^{10} \text{ J}$ )

### Note

The neutrons released in fission can be slowed to a reasonable speed and made to bombard more uranium targets, emission of more neutrons which can still bombard more target thus leading to chain reaction. This is the basis of atomic bombs and nuclear reactants to produce energetic neutrons

### Application of fission

- In the production of neutrons
- In production of atomic bombs

### Condition for fission

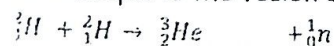
- It requires an energetic particle like a neutron

## 4.2.5: NUCLEAR FUSION

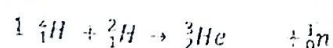
Nuclear fusion is the union of two light nuclei to form a heavier nucleus accompanied by release of energy.

Energy is released in the process.

An example is the fusion of two deuterium nuclei to produce helium -



### Examples



# MISSING 227

## 4.2.6: DETECTION OF IONISING RADIATIONS

In a detector energy transferred from the radiation to the atoms of the detector may cause

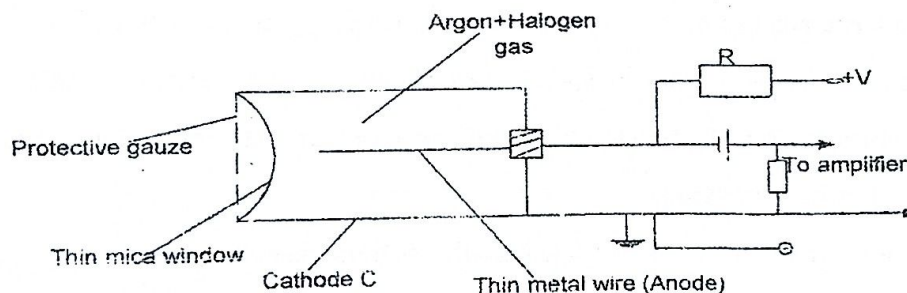
- Ionization of a gas (as in ionization chamber, cloud chamber, G.M tube)
- Fluorescence of a phosphor
- Exposure of a photographic emulsion

The following instruments are used for detecting radiation from a radioactive source.

1. Ionization chambers
2. Scintillation counter
3. Bubble chamber
4. Cloud chamber (Wilson and Diffusion)
5. Solid state detector
6. Photographic plates
7. Electroscope

### 1. THE GEIGER - MULLER TUBE / (GM) TUBE

Gm tube is a very sensitive type of ionization chamber which can detect single ionizing events



#### Operation

- ❖ Ionising radiations enter the G.M tube through the thin mica window and ionise the argon gas atoms, ion pairs are formed
- ❖ The electrons move very fast to the anode due to a high p.d between cathode and anode while the positive ions drift to the cathode
- ❖ Fast moving electrons collide with other argon molecules producing more ion pairs, an avalanche of electrons is obtained.

- ❖ When electrons reach anode, pulse of current is obtained.
- ❖ Voltage pulse is amplified and measure by scaler
- ❖ Positive ions move to cathode.
- ❖ Quenching agent is used to avoid secondary emission of electrons by absorbing kinetic energy of positive ions

#### Note

- The comparatively heavy positive ions move relatively slowly towards the cathode and reaching after the avalanche has occurred. Positive ions have appreciable energy and would cause emission of electrons from the cathode by bombardment causing a second avalanche. This would cause the discharge to persist for some time upsetting the recording of other ionizing particles following the first.
- To prevent a second avalanche due to positive ions, a halogen gas (e.g Bromine) is mixed with the argon gas to form a quenching agent.
- Bromine water acts as a quenching agent so as to prevent secondary electrons to be emitted from the cathode by the positive ions bombarding it.
- An avalanche is a large number of moving ionised particles created as a result of secondary ionisation due to collisions between ions and the gas atoms, when the ions are accelerated by a high enough p.d where each ionisation leads to the formation of more ions pairs which themselves cause further ionisation.
- Time taken by the positive ions to travel towards the cathode is called dead time.

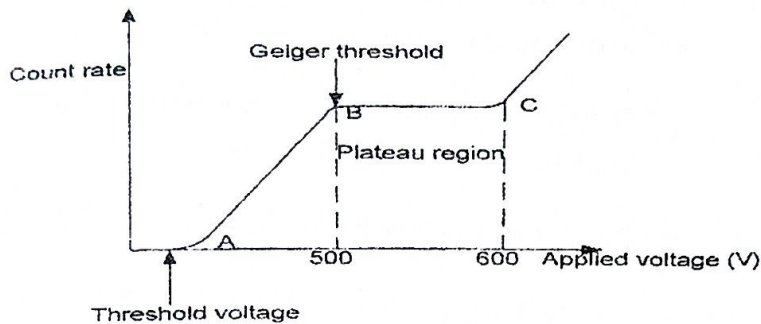
The GM tube may be fitted to a variety of detectors for investigating activity of radioactive sources.

1- Scaler - simply records the total number of pulses

2- Rate meter - records actual count rate  $\left(\frac{dN}{dt}\right)$  and the out put may be fed into a meter

3- Speaker - Amplifier - gives an audible single each time a particle is detected. The signal becomes a continuous crackle when the activity is high.

#### GM tube characteristic curve

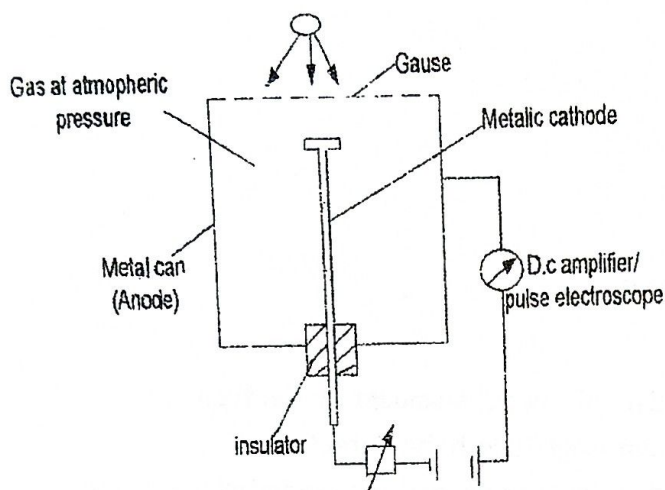


- ❖ Up to the threshold voltage no counts are recorded at all since the amount of electron amplification is not enough to give pulses of sufficient magnitude to be detected.
- ❖ Between A and B, the magnitude of pulse developed in the tube depends on the initial ionization which in turn depends on the energy of the incident ionizing particle. Only some of the freed electrons give pulses of sufficient magnitude to be recorded but their number increases with applied voltage.
- ❖ Between B and C (plateau region), the count rate is almost constant. A full avalanche is obtained along the entire length of the anode and all particles whatever their energy produce detectable pulses.
- ❖ Beyond C, the count rate increases rapidly with voltage due to incomplete quenching. One incident ionizing particle may start a whole train of pulses.

**Note:**

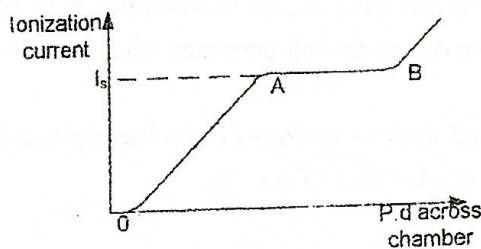
GM tubes should be operated in the plateau region (500 - 600V) preferably in the middle of the region. The sensitivity is then greatest and independent of supply voltage such that every particle that produced ionization is detected.

## 2. THE IONISATION CHAMBER



- ❖ The radiations enter through the thin wire gauze and ionise the gas molecules.
- ❖ The electrons move towards the anode and the positive ions towards the cathode.
- ❖ Current is detected by an electrometer.
- ❖ The pulse per second (\* count rate) gives a measure of the intensity of radiation.

#### Variation of the ionization current with p.d (x-tic curve for ionization chamber)



- Between O and A, the p.d is not large enough to draw all the electrons and positive ions to their respective electrodes. As the p.d increases more ions reach the electrode increasing the current.
- Between A and B, all the ions are attracted to their respective electrodes and there is no recombination. So the current reaches its saturation value ( $I_s$ ) and remains constant as the p.d changes.
- Beyond B, the p.d is large enough to cause secondary ionization. A point is reached when there is rapid multiplication of the ions in the chamber (gas amplification) thereby causing uncontrollable increase in the ionizing current.

#### Note:

- (1) The p.d at which an ionization is operated should be such that the ionization current has its saturation value. Under such condition;
  - (i) The ionization current is independent of fluctuations in supply voltage

(ii) The ionization current is proportional to the rate at which ionization is being produced in the chamber.

(2) Saturation current  $I_s$  is a measure of the rate of primary ionization.

$$I_s = ne$$

Where  $e = 1.6 \times 10^{-19} \text{ C}$ ,  $n$  is the number of primary ion pair produced per second.

### CLOUD CHAMBERS

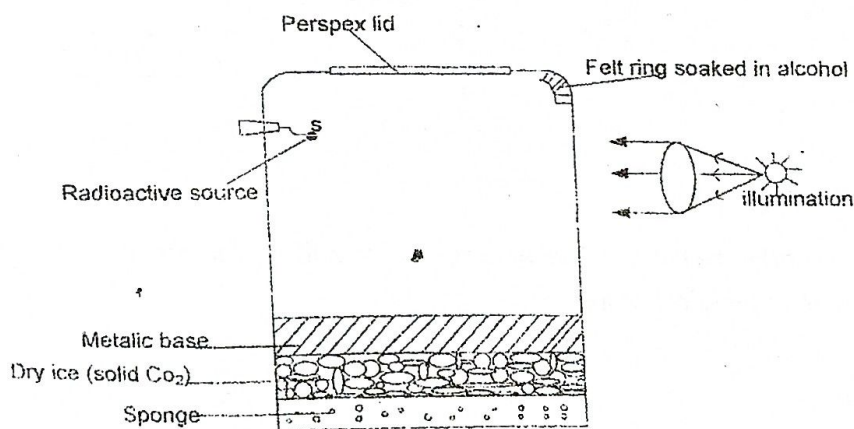
In a cloud chamber a gas is cooled to a temperature slightly below its condensation point. It is said to be super cooled/super saturated. The gas molecules begin to condense on any ionized molecules present in the chamber. When a radiation(ionizing) particle catering the chamber passes through the saturated vapour /super cooled gas, the ions produced serve as centers on which tiny bubbles/ drop lets form (vapor condenses) a line of droplets/bubbles is formed along the track of the particles. Light scatters more from these bubbles/droplets than from the gas background when suitably illuminated. A photograph of the chambers at the right moment reveals the track of the particles. The length of the lines is proportional to particle energy.

The cloud chamber is used to show tracks of the radioactive particles rather than to measure the intensity of the cloud chambers are;

Expansion Wilson cloud chamber and

Diffusion cloud chamber

### 3. DIFFUSION CLOUD CHAMBER



#### Structure and operation

- From the diagram above the base of the chamber is maintained at about  $-80^{\circ}\text{C}$  and the top is at room temperature so that there is a temperature gradient between the top and bottom.

- ❖ The air in the chamber is saturated with alcohol, where the vapour diffuses continuously from the top to the bottom and the air above the metal base becomes supersaturated.
- ❖ Then the radioactive particles cause ionisation of the air molecules
- ❖ The saturated vapour condenses on the ion formed producing tracks which can be seen by looking through the lid, hence radiation is detected.

### 3.0: THE RADIOACTIVE DECAY LAW $[N = N_0 e^{-\lambda t}]$

Radioactive decay is the spontaneous disintegration of unstable radioactive nuclei into more stable nuclei with emission of radiations.

The number of atoms decaying per second  $\frac{dN}{dt} \propto N$

Where  $N$  is the number of un-decayed atoms.

$$\frac{dN}{dt} = -\lambda N$$

Where  $\lambda$  is decay constant

$$-\frac{dN}{dt} = \lambda N$$

$$A = -\lambda N$$

Where  $A$  is activity or count rate per second the S.I unit for activity ( $A$ ) is Becquerel ( $Bq$ )

#### Definition

**Activity** is the number of decays per second. OR it is the number of radiations emitted per second.

**Decay constant** is the fraction of radioactive atoms which decay per second.

Given that  $N_0$  = number of radioactive atoms present at time  $t = 0$  (initially)

$N$  = number of un-decayed atoms after time  $t$

$$\text{From } \frac{dN}{dt} = -\lambda N$$

$$\int_{N_0}^N \frac{dN}{N} = \int_0^t -\lambda dt$$

$$[\ln N]_{N_0}^N = [-\lambda t]_0^t$$

$$\ln N - \ln N_0 = -\lambda t$$

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$N = N_0 e^{-\lambda t}$$

#### 4.3.1: HALF LIFE $[t_{1/2}]$

Half life of a radioactive element is the time taken for half of the atoms to decay

If  $N_0$  is the number of original atoms

$$\text{at } t = t_{1/2}, N = \frac{N_0}{2}$$

$$\text{From } N = N_0 e^{-\lambda t}$$

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}}$$

Taking logs to base e on both sides

$$\ln\left(\frac{1}{2}\right) = \ln e^{-\lambda t_{1/2}}$$

$$\ln\left(\frac{1}{2}\right) = -\lambda t_{1/2}$$

$$t_{1/2} = \frac{-\ln(1/2)}{\lambda} = \frac{\ln 2}{\lambda}$$

$$t_{1/2} = \frac{0.693}{\lambda}$$

$$\lambda = \frac{0.693}{t_{1/2}}$$

Note: Activity A at any one given time t is given by  $A = A_0 e^{-\lambda t}$

### Examples

1. A sample of a radioactive material contains  $10^{18}$  atoms. The half life of the material is 2.0 days. Calculate

- The fraction remaining after 5.0 days
- The activity of the sample after 5.0 days

**Solution**

i)  $t_{1/2} = 2 \text{ days}, t = 5 \text{ days},$

$$N_0 = 10^{18} \text{ atoms}$$

$$\text{But } \lambda = \frac{0.693}{t_{1/2}}$$

$$\lambda = \frac{0.693}{2} \text{ day}^{-1}$$

$$\text{But } N = N_0 e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-\frac{0.693}{2} \times 5}$$

$$\frac{N}{N_0} = 0.1768$$

Fraction remaining after 5 days = 0.1768

ii) Activity  $\frac{dN}{dt} = \lambda N$

$$\lambda = \frac{0.693}{2 \times 24 \times 60 \times 60} \text{ s}^{-1}$$

$$\lambda = 4.0104 \times 10^{-6} \text{ s}^{-1}$$

$$\frac{dN}{dt} = \lambda N$$

But from  $\frac{N}{N_0} = 0.1768$

$$N = 0.1768 N_0$$

$$\frac{dN}{dt} = 4.0104 \times 10^{-6} \times 0.1768 \times 10^{10}$$

$$= 7.09 \times 10^{11} \text{ Bq}$$

activity after 5 days =  $7.09 \times 10^{11} \text{ Bq}$

2. Potassium  ${}^{44}_{19}\text{K}$  has half life of 20 minutes and decays to form  ${}^{44}_{20}\text{Ca}$ , a stable isotope of calcium

- How many atoms would there be in 10mg sample of potassium -44
- What would be the activity of the sample?
- What would be the activity be after one hour
- What would the ratio of potassium atoms to calcium atoms be after one hour [ $N_A = 6.0 \times 10^{23} \text{ mol}^{-1}$ ]

**Solution**

i) 44g of potassium =  $6 \times 10^{23}$  atoms

$$10 \times 10^{-3} \text{ g of potassium} = \frac{6 \times 10^{23}}{44} \times 10 \times 10^{-3} \text{ g}$$

$$= 1.364 \times 10^{20} \text{ atoms}$$

10mg of potassium - 44 has  $3.64 \times 10^{20}$  atoms

ii)  $A = -\lambda N$

$$A = \frac{0.693}{t_{1/2}} \times 1.364 \times 10^{20}$$

$$A = \frac{0.693}{20 \times 60} \times 1.364 \times 10^{20}$$

$$A = 7.88 \times 10^{16} \text{ Bq}$$

Activity of the sample =  $7.88 \times 10^{16} \text{ Bq}$

iii) When  $t = 1 \text{ hour}$

$$t = 3600 \text{ s}$$

$$N = N_0 e^{-\lambda t}$$

$$N = 1.364 \times 10^{20} e^{-\frac{0.693}{20 \times 60} \times 3600}$$

$$N = 1.706 \times 10^{19} \text{ atoms}$$

Number of atoms remaining after 1 hour =

$$1.706 \times 10^{19} \text{ atoms}$$

But  $A = -\lambda N$

$$A = \frac{0.693}{20 \times 60} \times 1.706 \times 10^{19}$$

$$= 9.85 \times 10^{15} \text{ Bq}$$

Activity after one hour =  $9.85 \times 10^{15} \text{ Bq}$

iv) Let  $N_K$  = number of potassium atoms present after time  $t$

$N_C$  = number of Calcium atoms present after time  $t$

Then  $N_K + N_C$  = Number of potassium atoms present initially

From  $N = N_0 e^{-\lambda t}$

$$N_K = (N_K + N_C) e^{-\lambda t}$$

$$\frac{N_K}{N_K + N_C} = e^{-\frac{\ln 2}{20 \times 60} \times 3600}$$

$$\frac{N_K}{N_K + N_C} = \frac{1}{8}$$

$$8N_K = N_K + N_C$$

$$7N_K = N_C$$

$$\frac{N_K}{N_C} = \frac{1}{7}$$

$$N_K : N_C = 1:7$$

Ratio would be = 1:7

3. An isotope of krypton  ${}^{87}_{36}\text{Kr}$  has a half-life of 78 minutes. Calculate the activity of  $10 \mu\text{g}$  of  ${}^{87}_{36}\text{Kr}$  [ $N_A = 6 \times 10^{23} \text{ mol}^{-1}$ ]

**Solution**

$87$  of  ${}^{87}_{36}\text{Kr}$  contains  $6 \times 10^{23}$  atoms

$10 \times 10^{-6} \text{ g}$  of  ${}^{87}_{36}\text{Kr}$  contains  $\frac{6 \times 10^{23}}{87} \times 10 \times 10^{-6}$

$$= 6.9 \times 10^{16} \text{ atoms}$$

But  $\frac{dN}{dt} = \lambda N$

$$= \frac{\ln 2}{78 \times 60} \times 6.9 \times 10^{16}$$

$$= 1.022 \times 10^{13} \text{ Bq}$$

4. A sample of radioactive waste has a half-life of 80 years. How long will it take for its activity to fall to 20% of its current value

**Solution**

$$A = \frac{20}{100} A_0 \text{ but } A = A_0 e^{-\lambda t}$$

$$\frac{20}{100} A_0 = A_0 e^{-\left(\frac{\ln 2}{80} t\right)}$$

$$\ln(0.2) = -t \left(\frac{\ln 2}{80}\right)$$

$$t = -80 \frac{\ln 0.2}{\ln 2}$$

$$t = 185.75 \text{ years}$$

it will take  $\approx 186$  years

5. A sample of radioactive material has an activity  $9 \times 10^{12}$  Bq. The material has half life of 80s. how long will it take for the activity to fall to  $2 \times 10^{12}$  Bq

**Solution**

$$A_0 = 9 \times 10^{12} \text{ Bq}, A = 2 \times 10^{12} \text{ Bq}, t = ?$$

$$t_{1/2} = 80$$

$$A = A_0 e^{-\lambda t}$$

$$2 \times 10^{12} = 9 \times 10^{12} e^{-\left(\frac{\ln 2}{80}\right)t}$$

$$\frac{2}{9} = e^{-\left(\frac{\ln 2}{80}\right)t}$$

$$\ln\left(\frac{2}{9}\right) = -\frac{t}{80} \ln 2$$

$$t = \frac{-80 \ln(2/9)}{\ln 2}$$

$$t = 173.594$$

$$\text{Time taken} = 174 \text{ s}$$

6. A radioactive source contains  $1.0 \mu\text{g}$  of plutonium of mass number 239. If the source emits 2300 alpha particles per second. Calculate the half life of plutonium, assume  $[N = N_0 e^{-\lambda t}]$

**Solution**

$$239 \text{ g of plutonium contains} = 6.02 \times 10^{23}$$

$$1 \times 10^{-6} \text{ g of plutonium contains} = \frac{6.02 \times 10^{23}}{239} \times 10^{-6}$$

$$= 2.519 \times 10^{15} \text{ atoms}$$

$$10^{15} \text{ atoms}$$

Since it emits 2300 alpha particles per second, then

$$A = 2300 \text{ s}^{-1}$$

$$A = -\lambda N$$

$$2300 = \lambda \times 2.519 \times 10^{15}$$

$$2300 = \left(\frac{\ln 2}{t_{1/2}}\right) \times 2.519 \times 10^{15}$$

$$t_{1/2} = \frac{2.519 \times 10^{15}}{2300} \ln 2$$

$$t_{1/2} = 7.591 \times 10^{11} \text{ s}$$

7. What mass of radium -227 would have an activity of  $1 \times 10^6$  Bq. The half life of radium-227 is 41 minutes ( $N_A = 6 \times 10^{23} \text{ mol}^{-1}$ )

**Solution**

$$t_{1/2} = 41 \text{ minutes} \quad \text{But } A = -\lambda N$$

$$1 \times 10^6 = \left(\frac{\ln 2}{41 \times 60}\right) N$$

$$N = 3.55 \times 10^9 \text{ atoms}$$

$$\text{But } 6 \times 10^{23} \text{ atoms contains } 227 \text{ g}$$

$$3.55 \times 10^9 \text{ atoms will contain}$$

$$\frac{227}{6 \times 10^{23}} \times 3.55 \times 10^9$$

$$= 1.34 \times 10^{-12} \text{ g}$$

8. A radioactive source has a half life of 20s and an initial activity of  $7 \times 10^{12}$  Bq. Calculate its activity after 50s have elapsed

**Solution**

$$t_{1/2} = 20 \text{ s}, t = 50 \text{ s} \quad A_0 = 7 \times 10^{12}$$

$$A = A_0 e^{-\lambda t}$$

$$A = 7 \times 10^{12} e^{-\frac{\ln 2}{20} \times 50}$$

$$A = 1.24 \times 10^{12} \text{ Bq}$$

9. The half-life of a particular radioactive material is 10 minutes, determine what fraction of a sample of the material will decay in 30 minutes.

**Solution**

$$t_{1/2} = 10 \text{ minutes}, t = 30 \text{ minutes}$$

$$\text{using } N = N_0 e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-\frac{\ln 2}{20} \times 30}$$

$$\frac{N}{N_0} = \frac{1}{8}$$

$$\text{The fraction remaining} = \frac{1}{8}$$

$$\text{The fraction that has decayed} = 1 - \frac{1}{8}$$

$$= \frac{7}{8}$$

10. Find the activity of 1g sample of radium  ${}^{226}_{88}\text{Ra}$  whose half-life is 1620 years

**Solution**

$$226 \text{ g of } {}^{226}_{88}\text{Ra} \text{ contains } 6.02 \times 10^{23} \text{ atoms}$$

$$1 \text{ g of } {}^{226}_{88}\text{Ra} \text{ contains } \frac{6.02 \times 10^{23}}{226} \text{ atoms} = 2.664 \times 10^{21} \text{ atoms}$$

$$\text{But } A = -\lambda N$$

$$A = \frac{\ln 2}{0.693} N, A = \frac{\ln 2}{t_{1/2}} N$$

$$A = \left( \frac{\ln 2}{1620 \times 365 \times 24 \times 3600} \right) \times 2.664 \times 10^{21}$$

$$A = 3.61 \times 10^{10} \text{ s}^{-1}$$

$$\text{Activity} = 3.6 \times 10^{10} \text{ Bq}$$

11. A small volume of a solution which contains a radioactive isotope of sodium had an activity of 12000 disintegration per minute when it was injected into a blood stream of a patient. After 30 hours, the activity of 1.0 cm<sup>3</sup> of the blood was found to be 0.50 disintegration per minute. If the half life of the sodium isotope is taken as 15 hours, estimate the volume of blood in a patient

**Solution**

$$\text{At } t = 0, \text{ activity } A_0 = 12000 \text{ min}^{-1}$$

$$T = 15 \text{ (half life)} \quad A = 6000 \text{ min}^{-1}$$

$$T = 30 \quad A = 3000 \text{ min}^{-1}$$

$$\text{Total activity in the blood stream} = 3000 \text{ min}^{-1}$$

$$\text{Total volume of blood} = \frac{\text{blood in the blood stream}}{\text{activity in } 1 \text{ cm}^3}$$

$$= \frac{3000}{0.5}$$

$$= 6000 \text{ cm}^3$$

$$\therefore \text{Therefore volume of blood in a patient} = 6 \text{ litres}$$

### Calculation on ionization chamber

1. If 32eV is required to produce an ion-pair in air, calculate the current produced when an alpha particle per second from a radium source is stopped inside an ionization chamber, the energy of alpha particles from a radium source is 4.8MeV

#### Solution

32eV produces one ion pair

$$4.8 \times 10^6 \text{ eV will produce} = \frac{1}{32} \times 4.8 \times 10^6$$

$$= 1.5 \times 10^5 \text{ ion pairs}$$

$$\text{But } I = ne$$

$$I = 1.5 \times 10^5 \times 1.6 \times 10^{-19}$$

$$I = 2.4 \times 10^{-14} \text{ A}$$

2. A radioactive source emits  $2 \times 10^5$  alpha particles per second. The particles produce a saturated current of  $1.1 \times 10^{-8}$  in an ionization chamber. If the energy required to produce an ion pair is 32eV. Determine the energy in MeV of an alpha particle emitted by the source.

#### Solution

From  $I = ne$

$$\frac{1.1 \times 10^{-8}}{1.6 \times 10^{-19}} = n$$

$$n = 6.875 \times 10^{10} \text{ ion pairs}$$

One ion pair produces 32eV

$6.875 \times 10^{10}$  ion pairs will produce

$$6.875 \times 10^{10} \times 32 \text{ eV}$$

$$= 2.2 \times 10^{12} \text{ eV}$$

$$\text{Energy of an alpha particle} = \frac{\text{total energy}}{\text{no of } \alpha\text{-particle}}$$

$$= \frac{1.1 \times 10^{12}}{10^5} \text{ eV}$$

$$= 1.1 \times 10^7 \text{ eV}$$

$$= \frac{1.1 \times 10^7}{10^6} \text{ MeV}$$

$$\text{Energy of an } \alpha\text{-particle} = 11 \text{ MeV}$$

3. A radioactive source produces alpha particles each of energy 60MeV. If 20% of the alpha particles enter the ionization chamber, a current of  $0.2 \mu\text{A}$  flows. Find the activity of the alpha source, if the energy needed to make an ion pair in the chamber is 32MeV.

#### Solution

$$I = ne$$

$$\frac{0.2 \times 10^{-6}}{1.6 \times 10^{-19}} = n$$

$$n = 1.25 \times 10^{12} \text{ ion pairs}$$

one ion pair requires 32MeV

$1.25 \times 10^{12}$  ion pairs will require

$$32 \times 1.25 \times 10^{12}$$

$$= 4 \times 10^{13} \text{ MeV}$$

$$\text{Energy of an alpha particle} = \frac{\text{total energy}}{\text{no of } \alpha\text{-particle}}$$

$$60 = \frac{32 \times 1.25 \times 10^{12}}{\text{number of alpha particles}}$$

$$\text{Number of alpha particles} = \frac{32 \times 1.25 \times 10^{12}}{60}$$

$$= 6.667 \times 10^{11} \text{ alpha particles}$$

If A is the activity then

$$\text{Number of particles} = \frac{20}{100} A$$

$$6.667 \times 10^{11} = \frac{20}{100} A$$

$$A = 3.33 \times 10^{12} \text{ s}^{-1}$$

## Examples on carbon dating

1. Wood from a buried ship was found to have a specific activity of  $1.2 \times 10^2 \text{ Bqkg}^{-1}$  due to  $^{14}\text{C}$  whereas a comparable living wood has a specific activity of  $2 \times 10^2 \text{ Bqkg}^{-1}$ . What is the age of the ship? [half life of  $^{14}\text{C} = 5.7 \times 10^3 \text{ years}$ ]

Solution

$$A_0 = 2 \times 10^2 \text{ Bqkg}^{-1}$$

$$A = 1.2 \times 10^2 \text{ Bqkg}^{-1}$$

$$A = A_0 e^{-\lambda t}$$

$$1.2 \times 10^2 = 2 \times 10^2 e^{-\frac{\ln 2}{t_{1/2}} t}$$

$$\frac{1.2}{2} = e^{-\frac{\ln 2}{t_{1/2}} t}$$

$$\ln\left(\frac{1.2}{2}\right) = t \frac{-\ln 2}{(5.7 \times 10^3)}$$

$$t = 4.2 \times 10^3 \text{ years}$$

2. Archeological wood was found to have an activity of 20 units due to  $^{14}\text{C}$ . Recent wood gave an activity of 47.8 units, estimate the age of the wood [half life of  $^{14}\text{C} = 5600 \text{ years}$ ]

Solution

$$A_0 = 47.8, A = 20$$

Using

$$A = A_0 e^{-\lambda t}$$

$$20 = 47.8 e^{-\frac{\ln 2}{5600} t}$$

$$\ln\left(\frac{20}{47.8}\right) = t \frac{-\ln 2}{(5600)}$$

$$t = 7.4 \times 10^3 \text{ years}$$

3. A rock containing  $^{238}_{92}\text{U}$ . Decays to produce a stable isotope of  $^{206}_{82}\text{Pb}$ . Estimate the age of the rock if the ratio of  $^{206}_{82}\text{Pb}$  to  $^{238}_{92}\text{U}$  is 0.6 [half life of  $^{238}_{92}\text{U} = 4.5 \times 10^9 \text{ years}$ ]

Solution

Let  $N_u$  = number of uranium atoms present at time  $t$

$N_{pb}$  = number of lead atoms present at time  $t$

$(N_u + N_{pb})$  = number of uranium atoms present initially

$$\text{From } N = N_0 e^{-\lambda t}$$

$$N_u = (N_u + N_{pb}) e^{-\frac{\ln 2}{4.5 \times 10^9} t}$$

$$\frac{N_u}{N_u + N_{pb}} = e^{-\frac{\ln 2}{4.5 \times 10^9} t}$$

$$\ln\left(\frac{N_u}{N_u + N_{pb}}\right) = \frac{-t \ln 2}{4.5 \times 10^9}$$

$$\ln\left(\frac{N_u + N_{pb}}{N_u}\right) = t \frac{\ln 2}{4.5 \times 10^9}$$

$$\ln\left(1 + \frac{N_{pb}}{N_u}\right) = t \frac{\ln 2}{4.5 \times 10^9}$$

$$\ln(1 + 0.6) = t \frac{\ln 2}{4.5 \times 10^9}$$

$$t = 3.1 \times 10^9 \text{ years}$$

## USES RADIOACTIVITY

- ❖ Treatment of deep-lying tumors
- ❖ Measurement of thickness of metal sheet during manufacture

- ❖ Used to determine the exact position of underground pipes and allows leaks to be detected
- ❖ Radioactive phosphorous is used to assess the different abilities of plants to take up phosphorous from different types of phosphate fertilizer
- ❖ Used in radioactive dating

### Health hazard

The cells of the body may undergo dangerous physical and chemical changes as a result of exposure to the radiation causing:

- ❖ Mutation (genetic changes)
- ❖ Cancer cells

### Precautions

- ❖ Lead aprons should be worn when dealing with radiations
- ❖ Avoid unnecessary exposure to the radiations
- ❖ Delicate parts should not be exposed to the radiations

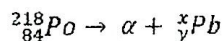
### EXERCISE 12

1. A certain  $\alpha$  - particle track in a cloud chamber has length of 37mm. Given that the average energy required to produce an ion pair in air is  $5.2 \times 10^{-18} \text{J}$  and that  $\alpha$  - particles in air produce on average  $5 \times 10^3$  such ions per mm of track. Find the initial energy of the  $\alpha$  - particle. Express your answer in MeV [ $e = 1.6 \times 10^{-19} \text{C}$ ]

Ans(6.0MeV)

2. A radioactive source has a half-life of 20days. Calculate the activity of the source after 70days have elapsed if its initial activity is  $10^{10} \text{Bq}$  Ans( $8.8 \times 10^8 \text{Bq}$ )

3. The radioactive isotope  ${}_{84}^{218}\text{Po}$  has a half life of 3minutes, emitting  $\alpha$  - particles according to the equation:



(i) What are the values of x and y

(ii) If N atoms of  ${}_{84}^{218}\text{Po}$  emit  $\alpha$  - particles at a rate of  $5.12 \times 10^{-4} \text{s}^{-1}$ , what will be the rate of emission after  $\frac{1}{2}$  hour. Ans( $50 \text{s}^{-1}$ )

4. An isotope of the element radon has a half life of 4days. A sample of radon originally contains  $10^{10}$  atoms.[Take 1day to be  $86 \times 10^3 \text{s}$ ]

Calculate:

- (i) The number of radon atoms remaining after 16 days
- (ii) The radioactive decay constant for radon
- (iii) The rate of decay of the radon sample after 16 days

An( $6.3 \times 10^8$  atoms,  $2 \times 10^{-6} \text{ s}^{-1}$ ,  $1.3 \times 10^3 \text{ Bq}$ )

5. The half life of  $^{30}_{15}\text{P}$  is 2.5 minutes. Calculate the mass of  $^{30}_{15}\text{P}$  which has an activity of  $10^{15} \text{ Bq}$ . ( $N_A = 6.0 \times 10^{23} \text{ mol}^{-1}$ )

An[ $11 \mu\text{g}$ ]

6. The activity of a particular radioactive nuclide falls from  $1 \times 10^{11} \text{ Bq}$  to  $2 \times 10^{10} \text{ Bq}$  in 10 hours, calculate the half life of the nuclide [An  
 $4.3 \text{ hours}$ ]

7. Calculate the activity of  $2 \mu\text{g}$  of  $^{64}_{29}\text{Cu}$ . [half life of  $^{64}_{29}\text{Cu} = 13 \text{ hours}$ ,  
 $N_A = 6.0 \times 10^{23} \text{ mol}^{-1}$ ]

An

[ $2.8 \times 10^{11} \text{ Bq}$ ]

8. The radioactive isotope of iodine  $^{131}\text{I}$  has a half life of 8 days and is used as a tracer in medicine, calculate:

(a) The number of atoms of  $^{131}\text{I}$  which must be present in the patient when she is tested to give a disintegration rate of  $6 \times 10^5 \text{ s}^{-1}$

(b) The number of atoms of  $^{131}\text{I}$  which must have been present in a dose prepared 24 hours before.

[An  $6.0 \times 10^{11}$ ,

$6.5 \times 10^{11}$ ]

9. The activity of a mass of  $^{14}_6\text{C}$  is  $5 \times 10^8 \text{ Bq}$  and the half life is 5570 years. Estimate the number of  $^{14}_6\text{C}$  nuclei present [In2 = 0.69] [An  $1.27 \times 10^{20}$ ]

10. (a) What is meant by the decay constant  $\lambda$  and the half life  $T_{1/2}$  for a radioactive isotope?

Show from first principles that  $\lambda T_{1/2} = 0.69$

(b) At a certain time, two radioactive sources R and S contain the same number of radioactive nuclei. The half life is 2 hours for R and 1 hour for S, calculate

(i) The ratio of the rate of decay of R to that of S at this time

(ii) The ratio of the rate of decay of R to that of S after 2 hours

(iii) The proportion of the radioactive nuclei in S which have decayed in 2 hours

An [1:2, 1:1, 75%]

11. (a) The various isotopes of an element X are distinguished by using the notation  ${}^A_ZX$ . Explain the meaning of A, Z and of the term isotope
- (b) Radioactive sources which might be used in schools are  ${}^{226}\text{Ra}$  which emits  $\alpha$ ,  $\beta$ , and  $\gamma$ -rays and  ${}^{90}\text{Sr}$  which emits  $\beta$ -rays only
- (i) List three safety precautions which need to be taken into account when using such sources.
- (ii) The half-life of the  ${}^{90}\text{Sr}$  is 28 years when its activity falls to 25% of its original value, it should be replaced. After how many years should it be replaced?

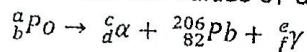
[56 years]

An

- (c) (i) When  ${}^{226}_{88}\text{Ra}$  emits an  $\alpha$ -particle, it decays to Radon (Rn). Write down a balanced equation for this change.
- (ii) Radioactive isotopes have many applications merely by virtue of being isotopes, describe and explain one such application

12. (a) In 420 days, the activity of a sample of polonium Po, fell to one - eighth of its initial value. Calculate the half life of polonium

- (c) Give the numerical values of a, b, c, d, e, f, in the nuclear equation



An[140days, a = 210, b =

84, c = 4, d = 2, e = 0, f = 0]

13. A steel piston ring of mass 16g was irradiated with neutrons until its activity due to the formation of an isotope of iron was 10micro curie. Ten days later after the irradiation, the ring was installed in an engine and after 80 days of continuous use, the crankcase oil was found to have a total activity of  $1.65 \times 10^3$  disintegrations per second. Determine the average mass of iron worn off the ring per day, assuming that all the metal removed from the ring accumulated in the oil and that one curie is equivalent to  $3.7 \times 10^{10}$  disintegration per second.

[half life of the isotope of iron = 45days]

An[4.0mg per day]

14. A tube containing an isotope of radon,  ${}^{222}_{86}\text{Rn}$  is to be implanted in a patient. The radon has an initial activity of  $1.6 \times 10^4 \text{Bq}$ , a half life of 4 days and it decays by a

alpha emission. To provide the correct dose, the tube, containing a freshly for 8 days

(a) What are the protons and nucleon number of the daughter nucleus produced by the daughter of the radon?

(b) Determine;

(i) The decay constant for radon in  $S^{-1}$

(ii) The initial number of radioactive radon atoms in the tube

$$\text{An}[2.0 \times 10^{-6} \text{s}^{-1}, 8.0 \times 10^9]$$

15. At the start of an experiment a mixture of radioactive materials contain  $20 \mu\text{g}$  of a radio isotope A, which has a half-life of 70s and  $40 \mu\text{g}$  of radio isotopes  $\beta$  has a half life of 35s :

(i) After what period of time will the mixture contain equal masses of each isotope. What is the mass of each isotope at this time?

(ii) Calculate the rate at which the atoms of isotope A are decaying when the masses are the same [molar mass of isotope A = 234g,  $N_A = 6 \times 10^{23} \text{mol}^{-1}$

$$\text{An}[70\text{s}, 10 \mu\text{g}, 2.5 \times 10^{14} \text{s}^{-1}]$$

16. The isotope of bismuth of mass number 200 has a half life of  $5.4 \times 10^3 \text{s}$ . It emits alpha particles with an energy of  $8.2 \times 10^{-13} \text{J}$ .

(a) State the meaning of the term half life

(b) Calculate for this isotope;

(i) Decay constant

(ii) The initial activity of  $1 \times 10^{-6}$  mole of the isotope

(iii) the initial power output of this quantity of the isotope

$$[N_A = 6 \times 10^{23} \text{mol}^{-1}]$$

[Hint, power = activity x Energy] [An  $1.3 \times 10^{-4} \text{s}^{-1}$ ,

$$7.7 \times 10^{13} \text{S}^{-1}, 63 \text{W}]$$

17. The radioactive isotope  $^{60}\text{Co}$  decays to  $^{60}\text{Ni}$  which spontaneously decays to give two gamma-ray photons, the half life of  $^{60}\text{Co}$  is 5.27years.

(i) find the activity of 20g of  $^{60}\text{Co}$

(ii) estimate the power obtainable from 20g of  $^{60}\text{Co}$

$$[\text{Mass of } ^{60}\text{Co} = 59.93381 \text{U}, \text{mass of } ^{60}\text{Ni} = 59.93079 \text{U}] [\text{An } 8.35 \times 10^{14} \text{s}^{-1}, 3.76 \times 10^2 \text{J s}^{-1}]$$

looking through the lid

UNEB 2013 Q.10

(d) (i) What is a decay constant

(01mark)

(ii) A sample from fresh wood of a certain species of tree has an activity of 16.0 counts per minute per gram. However, the activity of 5g of dead wood of the same species of tree is 10.0 counts per minute. Calculate the age of the dead wood (Assume half-life of 5730years) An(1.72x10<sup>4</sup>years) (04 marks)

UNEB 2012 Q9

a)(i) What is meant by the terms radioactive decay, half life and decay constant.

(ii) Show that the half life  $t_{1/2}$  of a radio isotope is given by  $t_{1/2} = \frac{0.693}{\lambda}$

Where  $\lambda$  is the decay constant [assume the decay law  $N = N_0 e^{-\lambda t}$ ] [03 marks]

(b) With the aid of a labeled diagram, describe the structure and action of a cloud chamber (05 marks)

(c). A radioactive isotope  ${}_{43}^{99}X$  decays by emission of a gamma ray. The half life of the isotope is 360 minutes. What is the activity of 1mg of the isotope (06 marks)

[An

1.95x10<sup>14</sup>Bq]

d. Explain the term avalanche as applied to an ionization chamber (03 marks)

UNEB 2011 Q10

a)What is meant by unified atomic mass unit (1 mark)

b)(i) Distinguish between nuclear fission and nuclear fusion (2 marks)

ii) State the condition necessary for each of the nuclear reactions in b(i) to occur

c)(i) With the aid of a labeled diagram, describe the operation of an ionization chamber (6 marks)

ii) Sketch the curve of ionization current against applied p.d and explain its main features (4 marks)

d)A typical nuclear reaction is given by  ${}_{92}^{235}U + {}_0^1n \rightarrow {}_{42}^{95}Mo + {}_{57}^{139}La + 2{}_0^1n + 7{}_{-1}^0e$

Calculate the total energy released by 1g of uranium

mass of  ${}_0^1n = 1.009U$  of  ${}_{-1}^0e = 0.00055U$

${}_{42}^{95}Mo = 94.906U$  of  ${}_{57}^{139}La = 138.906U$

${}_{92}^{235}U = 235.044U$   $1U = 1.66 \times 10^{-27}kg$

Ans [8.387x10<sup>10</sup>J]

UNEB 2010 Q 10

a)(i) What is meant by mass defect?

(1 mark)

(ii) Sketch a graph showing how binding energy per nucleon varies with mass number and explain its main features (3 marks)

iii) Find the binding energy per nucleon of  ${}^{56}_{26}\text{Fe}$  given that mass of 1 proton = 1.007825U.

Mass of 1 neutron = 1.008665U, [1U = 931MeV] [Ans 7.7MeV]

b) With the aid of a diagram, explain how an ionization chamber works (6 marks)

### UNEB 2008 Q9

a)(i) Define the term binding energy (1 mark)

(ii) Sketch a graph showing the variation of binding energy per nucleon with mass number (2 marks)

(iii) Use the sketch graph you have drawn in a(ii) to explain how energy is released during fission and fusion (3 marks)

b) Explain why high temperature is required during fusion of nuclides (1 mark)

c) The isotope  ${}^{238}_{92}\text{U}$  emits an alpha particle and forms an isotope of thorium (Th) while the isotope  ${}^{235}_{92}\text{U}$  when bombarded by a neutron, forms  ${}^{144}_{56}\text{Ba}$ ,  ${}^{90}_{36}\text{Kr}$  and neutrons

i) Write the nuclear equations for the reactions of  ${}^{238}_{92}\text{U}$  and  ${}^{235}_{92}\text{U}$  (2 marks)

ii) How does the reaction of  ${}^{235}_{92}\text{U}$  differ from that of  ${}^{238}_{92}\text{U}$  (3 marks)

d) A steel piston ring contains 15g of radioactive iron,  ${}^{54}_{26}\text{Fe}$ . The activity of  ${}^{54}_{26}\text{Fe}$  is  $3.7 \times 10^5$  disintegrations per second. After 100 days of continuous use, the crank case oil was found to have a total activity of  $1.23 \times 10^3$  disintegration per second. Find the:

i) Half life of  ${}^{54}_{26}\text{Fe}$

ii) Average mass of iron worn off the ring per day, assuming that all the metal removed from the ring accumulates in the oil. [Ans  $3.13 \times 10^{-17}\text{s}$ ,  $4.9 \times 10^{-4}\text{g}$ ] (5 marks)

### UNEB 2007 Q9

c) Explain the purpose of each of the following in a Geiger muller tube

i) A thin mica window

ii) Argon gas at low pressure

iii) Halogen gas mixed with argon gas

iv) An anode in the form of a wire (4 marks)

d)(i) What is meant by binding energy per nucleon of a nucleus (1 mark)

ii) Sketch a graph of binding energy per nucleon against mass number for naturally occurring nuclides (1 mark)

iii) State one similarity between nuclear fusion and nuclear fission (1 mark)

e)(i) At a certain time, an alpha-particle detector registers account rate of  $32\text{s}^{-1}$ . Exactly 10 days later the count rate dropped to  $8\text{s}^{-1}$ . Find the decay constant. (4 marks)

[Ans 0.139 per day]

ii) State two industrial uses and two health hazards of radioactivity (2 marks)

### UNEB 2006 Q10

- a) i) What is meant by half life of a radioactive material (1 mark)
- ii) Given the radioactive law  $N_t = N_0 e^{-\lambda t}$ , obtain the relation between  $\lambda$  and half life  $T_{1/2}$
- iii) What are radio isotopes (1 mark)
- iv) The radio isotope  $^{90}_{38}\text{Sr}$  decays by emission of  $\beta$ -particles. The half life of the radio isotope is 28.8 years, determine the activity of 1g of the isotope (5 marks)
- Ans [ $5.1 \times 10^{12} \text{s}^{-1}$ ]
- c) i) With aid of a diagram, describe the structure and action of a Geiger Muller tube (06 marks)
- ii) Sketch the count rate -voltage characteristic of the Geiger muller tube and explain it's main features (3mk)
- (iii) I identify, giving reasons, the suitable range in (b)(ii) of operation of the tube (2mk)

### UNEB 2005 Q10

- a) Define Binding energy of nuclide (1mk)
- b) i) Sketch a graph showing how binding energy per nucleon varies with mass number (1mk)
- (ii) Describe the main features of the graph in (b)(i) (3 marks)
- c) Distinguish between nuclear fission and nuclear fusion; and account for the energy released. (3 marks)
- (i) With the aid of a labeled diagram, the working of the Geiger-Muller tube (5 marks)
- (ii) How would you use a Geiger-Muller tube to determine the half life of a radioactive sample (4 marks)

### UNEB 2004 Q10

- b) Describe with the aid of a labeled diagram the structure and action of diffusion cloud chamber (6 marks)
- c) i) Define the terms radio activity and half life of radioactive substance (2 marks)
- (ii) A radioactive isotope of strontium of mass  $5\mu\text{g}$  has half-life of 28 years, find the mass of the isotope left after 14 years. Ans [ $3.54\mu\text{g}$ ]

### UNEB 2003 Q10

- a) What is meant by the following terms
- i) Nuclear number
- ii) Binding energy (2mk)

b) Calculate the energy released during the decay of  $^{220}_{86}\text{Rn}$  nucleus into  $^{216}_{84}\text{Po}$  and an alpha-particle

Mass of  $^{220}_{86}\text{Ra} = 219.964176\text{U}$

Mass of  $^{216}_{84}\text{Po} = 215.955794\text{U}$

Mass of  $^4_2\text{He} = 4.001566\text{U}$

( $1\text{U} = 931\text{MeV}$ )

[6.35MeV]

Ans

### UNEB 2002 Q10

a) What is meant by

- i) Half life of a radioactive element (1mk)
- ii) Nuclear fission (1mk)
- iii) Nuclear fusion (1mk)

b) An atom of  $^{222}\text{Ra}$  emits an alpha-particle of energy 5.3MeV. Given that the half life of  $^{222}\text{Ra}$  is 3.8 days, use the decay law to calculate the

- i) Decay constant (3mk)
- ii) Amount of energy released by  $3.0 \times 10^{-9}\text{kg}$  of  $^{222}\text{Ra}$  after 3.8 days (5mk)

Ans [ $2.11 \times 10^{-6}\text{s}^{-1}$ ,  $2.16 \times 10^{16}\text{MeV}$ ]

### UNEB 2001 Q9

a) What is meant by the following

- i) An alpha particle (1mk)
- ii) Radioactivity (1mk)

a) Describe the structure and actions of a cloud chamber (6 marks)

d) State four uses of radioactive isotopes (2 marks)

### UNEB 2000 Q9

a) i) Define the term half life and decay constant as applied to radio activity (2 marks)  
ii) State the relationship between half life and decay constant (1 mark)

b) The radio isotope  $^{60}\text{Co}$  decays by emission of  $\beta$ -particles and  $\gamma$ -rays. Its half-life is 5.3 years

i) Find the activity of a source containing 0.1g of  $^{60}\text{Co}$

ii) In what ways do  $\gamma$ -rays differ from  $\beta$ -particles? [Ans  $4.15 \times 10^{12}\text{s}^{-1}$ ]

c) i) What is meant by mass defect in nuclear physics

ii) Calculate the mass defect of  $^{59}_{26}\text{Fe}$ . Given the following information. (1 mark)

Mass of  $^{59}_{26}\text{Fe}$  nucleus = 58.93488u

looking through the lid, hence radiation is detected.

#### 4.3.0: THE RADIOACTIVE -DECAY LAW $(N = N_0 e^{-\lambda t})$

Radioactive d-

Mass of proton = 1.00728u

Mass of neutron = 1.00867u Ans [0.54051U] (4 marks)

d) Describe the structure and action an ionization chamber.