P425/1

PURE MATHEMATICS

PAPER 1

July\Aug 2022

3Hours

ERETA EDUCATION CONSULTS

JOINT MOCK EXAMINATIONS 2022

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 Hours

INSTRUCTIONS TO CANDIDATES

- Answer **all** the **eight** questions in section A and any **five** from B.
- Any additional question(s) answered will **not** be marked.
- All necessary working **must** be shown clearly
- Begin each answer on a fresh page.
- Silent non programmable scientific calculators and Mathematical tables with a list of formula may be used.

SECTION A (40 MARKS)

1. Given that $\propto$ and β are roots of the equation $2x^2 - 11x + 15 = 0$ without solving the equation, find the possible value of $\propto - \beta$, hence form aquadratic equation with roots $\propto$ $and - \beta$ $(\propto > \beta)$. (05marks)

2. Solve the equation$3\sin x + \cos 2x = 2$, for $0 \leq x \leq 2\pi$. (05marks)

3. The first, fourth and eighth terms of an A.P form a G.P. If the first term is 9, find the common ratio of the GP and the common difference of an A.P. (05marks)

4. Find the equation of a circle with diameter AB, where $A(-1,6)$, $B(1,12)$. (05marks)

5. $\int_0^1 x^3 e^{x2} dx$ (05marks)

6. Determine the Cartesian equation of a line passing through points $A\ (2,5,4)$ and $B\ (5,3,7)$. (05marks)

7. Given the parametric equation $y = \tan\theta$, $x = sec^2\theta$, prove that $\frac{d^2y}{dx^2} = \frac{-1}{4}\ cot^3\theta$. (05marks)

8. Given that the complex number Z and its complex X conjugate $\bar{z}$ satisfy $3Z\bar{z} + 2i\ \bar{z} = 11 + \frac{10}{3}i$ find the possible values of Z. (05marks)

SECTION B(60 MARKS)

9 a) The first term of an arithmetic progression is -11, the last term is 44 and the sum of the terms of the progression is 198. Find;

(i) The number of terms in the progression

(ii) The common difference (06marks)

b) John deposits shs. 3,000,000 at the beginning of every year in a macro – finance bank starting 2015, how much would he collect at the each of 2020 if the bank offers compound interest of 12.5% per annum and the no withdrawal is made within the period. (06marks)

10 a) Solve the equation $\tan x + \sec x = 3\cos x$ $0 \leq \times \leq 360^0$ (04marks)

b) Express $7\cos x + 24 \sin x$ in the form $R \cos(x - \propto)$

Find (i) the maximum value of $7\cos x + 24 \sin x$

(ii) the value of X between -180 and +180 inclusive for which

$7\cos x + 24 \sin x = 2$ (08marks)

11 a) The area bounded by the curve $y = 1 + \sin x$, the coordinates axes and the line $X = \pi/_2$ is rotated about the X- axis through 360⁰, show that the volume generated is $\frac{\pi}{4}(3\pi + 8)$ cubic unit. (06marks)

b) Differentiate with respect to X

(i) $(\sin x)^{\cos x} + (\cos x)^{\sin x}$ (03marks)

(ii) $y = X^{-x}$ and find the value of $\frac{dy}{dx}$ when $x = 2$ (03marks)

12 a) Given that $\frac{a}{b} = \frac{c}{d} = k$, showthat $K = \frac{a+c}{b+d}$ Hence solve the equation

$\frac{x+4z}{4} = \frac{y+z}{6} = \frac{3x+y}{5}$ and $4x + 2y + 5z = 30$ (07marks)

b) Prove that $log_c^{ab} = log_c^a + log_c^b$. Hence solve the equation

$\log_3(x-2) + \log_3(x+3) = 3$ (05marks)

13 a) Given that $y = \sqrt{\frac{1+\cos x}{1-\cos x}}$, show that $\frac{dy}{dx} = \frac{-1}{1-\cos x}$ (06marks)

b) Given that $f(x) = 4x^2 - 8x + 13$. Express $f(x)$ in the form $a + b(x+c)^2$, hence find the minimum value of $f(x)$, starting the value of X which it occurs. (06marks)

14. a) Solve the differential equation $X\frac{dy}{dx} + 2y = x^2$ Given that $y(1) = 1$. (5marks)

b) A machine depreciates at a rate proportional to the current value. Initially the machine is valued at shs. 2.5 million, 5 years later, it was valued at shs.1.875 million. If θ is the value of the machine after t years from a differential equation and solve it to find

(i) the value of the machine after 15 years

(ii) the number of years it will take the machine to be valued at shs. 0.5 million. (07marks)

15. a) A curve is given parametrically by $= 3\left(\frac{1}{P^2} + \frac{2}{P} + 1\right)$ and $y = 6\left(\frac{1+P}{P}\right)$ show that the curve is a parabola and find its focus. (05marks)

b) i) Find the equation of the tangent to the parabola $y^2 = 4ax$ at the $T(at^2, 2at)$

(ii) the tangent to the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ intersect at R, find the coordinates of R. (07marks)

16. a) Find the Cartesian equation of the plane through points $A(2, 1, 3)$, $B(7, 2, 3)$ and $C(5, 3, 5)$. (06marks)

b) Find the point of intersection of the line $\frac{x+3}{2} = \frac{y-5}{-1} = \frac{z-2}{3}$ with the plane $2x + 7y + 5z - 3 = 0$. (06marks)

END