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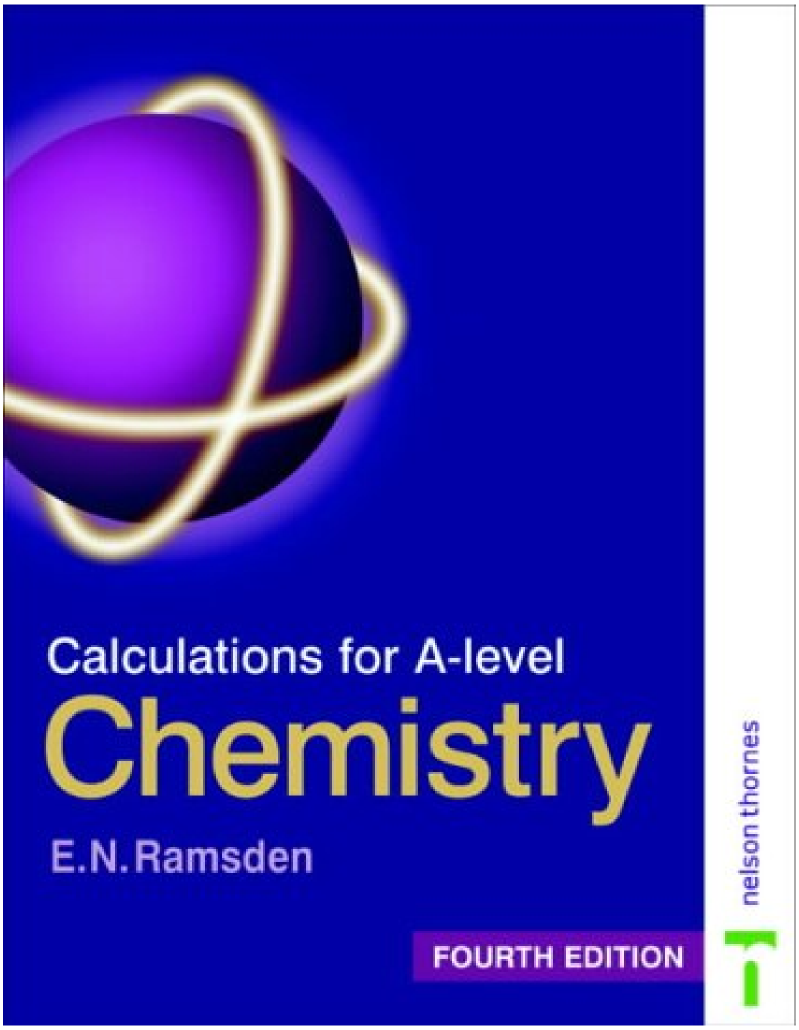

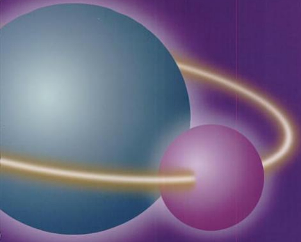

Calculations for A-level

Physics

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Basic ideas

1
How to approach a calculation

In this chapter a question of the kind found in A-level physics examination papers is answered. The question concerns an electric circuit calculation and, although the physics for such calculations is not discussed until Chapter 20 is reached, you can learn a lot from the question and its answer. The answer shows how you can present a calculation so that it is easily understood by an examiner or by other people. The comments which follow the calculation will prepare you for answering all the other kinds of calculations you can meet.

Example 1 – an A-level question

A 6.0 V, 3.0 W light bulb is connected in series with a resistor and a battery as in Fig. 1.1. The battery's EMF is 6.0 V and its internal resistance is 4.0 Ω.

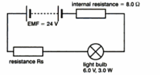

Fig. 1.1 Circuit diagram for worked example 1

The resistance R_s is chosen so that the bulb is run under the conditions for which it was designed, i.e. at a power of 3.0 W.

Calculate:
(a) the current that flows through the bulb under the design conditions
(b) the resistance of the bulb under these conditions
(c) the total resistance of the circuit
(d) the value of the resistance R_s.

Answer

(a) Power $P = V \times I$

$\therefore \quad 3.0 = 6.0 \times I$

$\therefore \quad I = \dfrac{3.0}{6.0} = 0.50\,\text{A}$

(b) Resistance $R = \dfrac{V}{I} = \dfrac{6.0}{0.50} = 12\,\Omega$

(c) Total resistance $= \dfrac{EMF}{I} = \dfrac{12}{0.50} = 24\,\Omega$

(d) Resistance R_s = total resistance − bulb resistance − internal resistance

$= 24 - 12 - 8 = 4\,\Omega$

Reading the question

Don't be surprised if you have to read a question a few times to understand it. At first you discover which branch of physics is concerned, perhaps electricity or mechanics. You ask yourself whether you have met a similar question before. 'Do I recall one or more formulae likely to fit the question?' 'What have I got to work out?'

Read the question until it makes sense

The question in our worked example should make you think of a light bulb that would normally be used with 6.0 volts across it and would produce 3.0 joules of heat and light energy per second, i.e. its power output is 3.0 watts. The formulae that might come into your mind are $P = V \times I$, $P = V^2/R$ and $P = I^2 R$, where P is the power of the bulb, V the voltage across it, I the current through it and R its resistance.

Diagrams

A diagram contains information that can be seen at a glance and it can be easier to work from than the lengthy wording of a question. If a diagram is not provided with a question or asked for, a quick sketch may be worthwhile. Note that the question in our example would have been complete without the diagram.

Diagrams can be very helpful

For diagrams you will need to be familiar with some symbols such as those for the battery, light bulb and resistors that are shown in Fig. 1.1. The lines drawn with arrows on them to represent forces are another example.

Symbols for electric circuits are listed on page 319.

Symbols for quantities and units

You will find in this and other physics books that there is a well-established set of abbreviations for most physics quantities. F for force and R for resistance are examples. You soon become familiar with them. V is used for potential difference (or voltage) and I is used for electric current ('Intensité de courant' in French). Greek symbols are often used; the symbol λ (pronounced lambda) for wavelength is an example.

Using upper case and lower case letters can distinguish between two similar quantities, e.g. masses M and m in the formula $F = \dfrac{GMm}{r^2}$ in which M could be the mass of the earth and m the mass of a satellite pulled towards the earth with a force F. Subscripts serve the same purpose, e.g. m_1 and m_2 for two masses. R_s was used for the series resistance in our worked example (1). In Example 3 of Chapter 4 abbreviations H_A and H_B are used for two horizontal forces, similarly V_A and V_B for two vertical forces.

For some quantities many different symbols are used. You will find d, s, x, r and other letters used for distances.

Whatever letters you decide upon you must state what they are being used for.

State what your symbols stand for

You have much less choice with units. The international agreement known by the name of 'Système International' (SI) fixes the units you MUST use and the accepted abbreviations for them. Examples are the ampere for current and its abbreviation A, the metre (m) for measuring distances (lengths) and kilogram (kg) for a mass. A list of the SI units you will meet is given in Table I on page 319. Note that the SI unit for resistance measurements is the ohm and its abbreviation is Ω (the Greek letter omega). We have already had an example: $8.0\,\Omega$ for the internal resistance of the battery.

It is customary to write units in the singular. So we see 3 metre rather than 3 metres. You should adopt this practice in your calculations. Otherwise 'metres' might be mistaken for 'metre s' (interpreted as 'metre second'). However, in text, the plural is acceptable because it makes more comfortable reading. For example '3 joules of heat' was mentioned above. When abbreviations are used 3 joule is written as 3 J and we certainly do not put an s after the J here. The J s (joule second) applies to quite a different quantity.

Write units in the singular in all calculations
3 volt or 3 V, not 3 volts

All the formulae you learn should work with SI units and all formulae used in this book work with SI units.

Formulae work with SI units

Multiples of SI units such as the kilometre (km, a thousand times a metre) and megawatt (a million times a watt) may be used when stating the size of a large quantity. Submultiples of the SI units such as the milliampere (a thousandth of an ampere) or a centimetre (one hundredth of a metre) may be convenient for describing small quantities.

1 kilovolt = 1000 × 1 volt = 1000 volt,
or 1 kV = 1000 V
and
1 milliampere = 1 ampere/1000
or $\frac{1}{1000}$ ampere, or 1 mA = $\frac{1}{1000}$ A

A list of the multiples and submultiples that you can use is shown in Table II on page 319.

A symbol placed before a unit to make it into a multiple or submultiple is a 'prefix' and it is fitted to the front of the unit with no gap. A gap must be used when a unit is made up of other units. Thus a newton metre is written as N m. A gap between the m and s in 'm s' causes the unit to read as 'metre second' whereas 'ms' denotes 'millisecond', the 'm' for milli being a prefix.

2 ms means 2 millisecond, but 2 m s means 2 metre second

If the values you put into a formula are measurements in SI units the answer you will get is in SI units. In the worked example the figures given (the data) are all for measurements in whole SI units, namely volts, watts and ohms. Otherwise it is advisable to convert a value given as a multiple or submultiple into a value in whole SI units, as explained in Chapter 3.

You may decide to do a calculation using a multiple or submultiple of a unit but you have to be very sure of what you are doing and of what units will apply to the answer you get for your calculation. It is safer to work in whole SI units.

Whatever units are used you must show the units in your working. For every item you work out you must show the unit. So in our worked example, at the end of the calculation for part (a) of our answer we see the symbol A for ampere. It can be very cumbersome to insert the unit for every quantity every time it is used. So in part (d) of the calculation you see '$= 24 - 12 - 8$' without any Ω signs, but the unit is shown with the final resistance of 4 Ω.

Show the unit with each quantity calculated

Getting an answer

When you have read the question, or even as you are reading it, your aim should be to rewrite the question's information, perhaps first as a labelled diagram and then as one or more equations. In our example the diagram is already there so we move on to choosing an equation for part (a) of the answer.

Equations you might think of that relate the current (I) we want to calculate to the voltage (V), power (P) and resistance (R) are $R = V/I$ and $P = VI$, and you may know $P = V^2/R$ and $P = I^2R$.

Consider relevant formulae

Note that we usually leave out multiplying signs and write $P = VI$ instead of $P = V \times I$ as long as no confusion results. When values are inserted for V and I the × must be used. Otherwise a product like 240×2 would become 2402 instead of 480.

Multiplying signs between symbols can be left out

Regarding the formula $R = V/I$, you can rearrange it to get $V = IR$ or $I = V/R$. This rearranging is called 'transposition' and the rules for this are explained in Chapter 2. Similarly we can get $I = P/V$ as a second formula for I. You might wonder which of these formulae for current is suitable for our part (a) answer. The one to choose is $I = P/V$ or $P = VI$ where I is the current through the bulb, P is the power of the bulb and V is the voltage across the bulb.

So the answer is $I = \frac{3.0}{6.0} = 0.50\,\text{A}$.

Select an appropriate formula

The formula $I = V/R$ would be suitable if V were the voltage across the bulb and R its resistance, but we don't know the value of this resistance. There could be a temptation to put into this formula whatever resistance value is available, namely the 8.0 Ω internal resistance of the battery and this would be quite wrong.

For any formula remember the conditions under which it works

So for part (a) we use $P = VI$ or $I = P/V$.

Rearranging the $P = VI$ formula for P to get the formula $I = P/V$ is very easy and quick to do. Formulae needed in some questions can be more tedious to transpose and, if the values that are to be put into the equation are not too complicated, it is best to start by entering the data in the formula you remember. Transposition is delayed until the calculation has achieved some simplification. In our worked example the equations are simple, as are the values to go into them, so there is little to choose between entering data first or rearranging equations first.

When an equation is complicated consider entering values before rearranging the equation

Write your answer as a series of equations. These may be linked by words of explanation and the symbol '$\therefore$' which stands for 'therefore' or 'it follows that' is particularly useful. It was used in part (a) of our answer. In place of several equations one equation can often be continued through a number of steps, as in part (b), where we see $R = V/I = 6.0/0.50 = 12\,\Omega$ instead of

$$R = V/I$$
$$\therefore \quad R = 6.0/0.5$$
$$\therefore \quad R = 12\,\Omega.$$

Write a calculation as a series of equations

Using your calculator

It is assumed that you have an electronic calculator which has keys for sin, cos and tan (for use with angles) and for the log of a number. These keys and others on such a 'scientific' calculator are essential for A-level physics calculations. Some calculators conform to the 'VPAM' specifications and display not just the last number you have entered or an answer but show all the values and the operations (such as adding and multiplying) that you have keyed in. The answer is then displayed as well when you press the equals key.

So to work out 6.0/0.50 in part (b) of the worked example, you remember that 6.0/0.50 is the same as $6.0 \div 0.50$ and key in $6.0 \div 0.50 =$ and the calculator display is exactly as shown in Fig. 1.2 or is the same except that the 1.200^{01} answer is replaced by 12. or by 12.0000 and the number of noughts (zeros) may be different. The differences are the result of the calculator having a number of different 'modes,' i.e. ways of working. The mode we want to use is the 'scientific mode' and it is this mode that gives the display shown in Fig. 1.2.

Advice given in this book for calculator use will apply to the Casio *fx*-83WA calculator. The procedures for switching on this calculator,

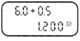

Fig. 1.2 A calculation displayed on a VPAM calculator

selecting the 'scientific mode' and clearing the screen for another calculation are described in Chapter 2.

From the answer displayed as 1.200^{01} you get the expected answer of 12 by multiplying the 1.200 by 10. If the two small figures were 02 you would multiply by 10 a second time, 03 a third time to give 1200. When the small figures are 11, for example, multiplying by 10 eleven times would be inconvenient and this is one reason for keeping the 1.200 and, as explained in Chapter 2, we then write 1.200×10^1 instead of 1.200^{01}.

'I'm stuck'

How often do you hear these despairing words when a calculation question is tried? Even when you have a good knowledge of physics and the appropriate maths you can get stuck.

You might then read the question again and ask yourself:

- Does the question fit what I have been trying to do?
- Is there a diagram I could draw?
- Have I pictured the situation described by the question or have I had in my mind a circuit without a battery or other voltage supply?
- Have I missed an equation that is needed? Perhaps it is in the list provided with the exam paper.
- Are there words in the question that I have disregarded, perhaps 'in series' in our example? If a diagram had not been provided it would have been essential to appreciate that 'in series' meant that the circuit components formed a single loop.

A word like 'series' can make a lot of difference to a calculation. It is a key word. Similar key words often met are 'smooth', 'slowly' and 'steady'. In mechanics questions a 'smooth surface' is one that is so smooth that it cannot provide any force parallel to its surface. So a smooth floor can push upwards and prevent a person falling but cannot provide a force to stop sliding. An object 'raised slowly' means it rises so slowly that it has no kinetic energy and gains only gravitational potential energy. A 'steady speed' means no change in speed.

If at first you don't succeed . . .

Good luck

We all make silly mistakes sometimes, so never get too disappointed. Rough checks are mentioned in Chapter 2 and these will minimise errors. Hurrying encourages errors, of course. Leaving a question and returning to it can waste time but may give you a fresh view of a problem and lead to a successful answer. So take care and good luck with your calculations!

2
Essential mathematics

Expressions and equations

An *expression* is a combination of numbers and symbols. Simple examples are the sum $3 + 2$, the difference $3 - 2$ and the product 3×2.

In any expression the order of multiplication or adding is not important,

e.g. $\mathbf{3 \times 2 = 3 \times 2}$ **and** $\mathbf{2 + 7 = 7 + 2}$.

The order of subtraction DOES matter, e.g. $3 - 2$ is not the same as $2 - 3$.

Alphabetical symbols are used to represent numbers either for convenience or because the number is not yet known.

An equation shows that two expressions have equal size or value, e.g. $3 + 2 = 4 + 1$ or $x = 7$.

A quantity is a number or a measurement. A measurement is a number times a *unit*, e.g. 3 times a metre or 3 metre. (Note that units used in calculations are written in the singular.)

Abbreviations are used for units, e.g. m for metre. The 'Système International' (SI) specifies the symbols to be used for units. Units are discussed in Chapter 3.

A formula is an equation which shows how a quantity on the left may be calculated by inserting values of quantities on the right, e.g. area = length × width or $A = L \times w$.

Note that a × sign is usually omitted if no confusion will result, e.g. $A = Lw$, and

$3a$ means $3 \times a$.

Fractions

A half is obtained by sharing one (equally) between two or dividing 1 by 2. For a half we write $1 \div 2$ or 1/2 or $\frac{1}{2}$. Half is part of a whole one, so it is a fraction.

3 divided by 6 also equals a half, so that $\frac{3}{6} = \frac{1}{2}$. This illustrates that multiplying or dividing the top (the numerator) AND the bottom (the denominator) of a fraction by the same number does not change its value, e.g. $\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$.

A number multiplying a fraction multiplies just the numerator, and a number dividing a fraction multiplies the denominator, e.g.

$$\frac{2}{9} \times 4 = \frac{8}{9} \qquad \frac{1}{2} \div 2 = \frac{1}{2 \times 2} = \frac{1}{4}$$

Note that a fraction 'of' a number means the fraction 'times' the number, e.g. 'a quarter of 3' means $\frac{1}{4} \times 3$ which is $\frac{3}{4}$.

'of' means 'times'

To multiply a fraction by a fraction the numerators are multiplied and the denominators are multiplied, e.g.

$$\frac{9}{10} \times \frac{2}{3} = \frac{18}{30} \left(or\ \frac{6}{10}\right)$$

Simplifying an expression means to rewrite it with smaller numbers or fewer numbers, e.g. $\frac{18}{30}$ above was simplified to $\frac{6}{10}$ and also equals $\frac{3}{5}$.

Reducing two numbers in an expression, as in the 18/30 fraction above, is called 'cancelling'. So too is the removal of two numbers, as in $\frac{9.3 \times 5}{9.7 \times 5}$, which simplifies to $\frac{9.3 \times 1}{9.7 \times 1}$ which equals $\frac{9.3}{9.7}$. Cancelling in equations is discussed later. Cancelling units features in Chapter 3.

Simplifying may be useful when a fraction has been divided by a fraction, e.g.

$$\frac{1/3}{1/2} = \frac{2 \times 1/3}{2 \times 1/2} = \frac{2/3}{1} = \frac{2}{3}$$

The reciprocal of a fraction is obtained by turning the faction upside down. So the reciprocal of 2/3 is 3/2. The reciprocal of a simple number e.g. of 7 (which can be written as 7/1) is 1/7.

The reciprocal of 2/3 is 3/2 and of 7 is 1/7

Percentage

'per' means 'for each' and 'cent' denotes 100, so 50 percent (written as 50%) means 50 for each 100 or 50 out of each hundred, i.e. a fraction 50/100 or a half. So $p\%$ means a fraction, $\frac{p}{100}$, $p\%$ of y means $\frac{p}{100} \times y$, and a fraction $\frac{a}{b} = \frac{p}{100}$, so that

$$p = \frac{a}{b} \times 100 \qquad (2.1)$$

These facts are best recalled by remembering that 50% means $\frac{1}{2}$ *or* $\frac{50}{100}$.

Using brackets

$a \times (b + c)$ or $a(b + c)$ can be rewritten as $ab + ac$. So $3(5 + 2)$, for example, means 3 times the sum of $5 + 2$. The multiplication or other action applied to a bracketed expression applies to everything within the brackets. So $3(5 + 2)$ (which is 3×7 or $21) = 3 \times 5$ plus 3×2 or $15 + 6$.

$$a(b + c) = ab + ac \qquad (2.2)$$

Numbers that are multiplying are called factors and in the expression $ab + ac$ the a is a factor of both ab and ac. It is 'common' to both. Taking out a common factor is the reverse of the process described above. $ab + ac$ becomes $a(b + c)$.

In a fraction such as $\frac{8 + 4x}{2}$ the dividing line shows that 2 divides both the 8 and the $4x$ and the effect is the same as $\frac{8}{2} + \frac{4x}{2}$ which equals $4 + 2x$. But $8 + \frac{4x}{2}$ equals $8 + 2x$.

When two bracketed expressions multiply the rule is

$$(a + b)(c + d) = ac + ad + bc + bd \qquad (2.3)$$

You can test this rule with simple numbers.

Working with + and − signs

If multiplying brackets contain differences we need rules concerning the effect of a − sign before a number, i.e. a negative number. The rule is that two negative numbers give a positive product (− times − gives +), − times + gives −, and + times + gives +. (Note here that a number with no + or − before it is regarded as +.) So using equation 2.3 the expression $(3 - 2)(7 - 3)$ could be written as $21 - 9 - 14 + 6$.

A similar rule applies to sums and differences. For example, the heat in joules required to warm a kilogram of water from temperature $T_1{}^\circ$ to 10° is $4200(10 - T_1)$ and for $T_1 = 9^\circ$ equals 4200, but if $T_1 = -9^\circ$ we have $4200(10 - -9)$ and the − − must give + to give a 19° temperature rise.

The rules are

− − gives +   − + or + − gives −   + + gives +

Your calculator

For A-level physics calculations you need an electronic calculator, and it should be a scientific one so that it will handle, for example, the powers, logarithms, sines and cosines explained later in this chapter. This book describes the use of the Casio *fx*-83WA calculator. Other calculators are similar.

Your *fx*-83WA is switched on by pressing the AC/ON key.

The calculator has a number of different ways of working, i.e. different modes, and pressing the MODE key (near top right of the keypad) three times will show you the choice of modes.

Now use the MODE key again and press 1 when 'Comp' is displayed to select that, then 1 when 'Deg' is displayed to select degrees for angles (discussed later), and finally 2 when 'Sci' is displayed to get scientific mode (soon to be explained). In response to your selecting scientific mode you are asked to enter a number; you should enter 4. As a result the four-figure answer of 4.000 is obtained in the following calculation.

Now you can test the calculator by pressing the AC/ON key to clear the screen, then entering, for example, $8 \times 0.5 =$. Your 8×0.5 to be calculated is shown on the screen and the answer is displayed on the right as 4.000, a four-figure answer.

Now try $7 \times 3 =$. Your answer is 2.100^{01}. You expected 21 or 21.00? Well, you are using scientific mode, which will be very useful. Just multiply the 2.100 by 10, i.e. move the decimal point one place to the right. Do this only once as the 1 in the small 01 on the right indicates. You now have 21.00. (2.100^{02} would indicate $2.100 \times 10 \times 10$.)

Dividing, adding and subtracting are achieved in the same way but using the $\div$, $+$ and $-$ keys.

Some care is needed with dividing.

Consider $\frac{9.1}{3.5 \times 4.7}$. This is the same as $\frac{9.1}{3.5}$ DIVIDED by 4.7, as explained above. So the calculator entry should be $9.1 \div 3.5 \div 4.7$.

An example of another difficulty is $\frac{9.34}{2.11 + 3.79}$, where you could unintentionally get the answer for $\frac{9.34}{2.11} + 3.79$. The simplest procedure is to use the calculator for $2.11 + 3.79$ to get 5.90, clear the calculator, then use $9.34 \div 5.90$ to get 1.583. Alternatively, if you know how to use it, the calculator's memory can help.

Brackets are handled by the calculator just as you would expect. For example, entering $6(2 + 3) =$ gives the answer 3.000^{01} meaning 30.00 or 30.

Simple rules for handling equations

If the whole of one side of an equation is multiplied, divided, added to or reduced by any number, then the equation will remain true if the same is done to the other side.

Examples are

- $x + 2 = 5$ gives $x = 5 - 2$ by subtracting 2 from each side, i.e. $x = 3$
- $4x = 8$ gives $x = 2$ when each side is divided by 4
- $6x = 4$ or $x = 4/6$ can be written as $3x = 2$ or $x = 2/3$
- $\frac{x}{2} = 7$ becomes $x = 14$ by multiplying both sides by 2.

Example 1

Calculate the time for which an electric heater must be run to produce 7200 joules of heat if the potential difference across the heater is 12 volts and the current flowing is 2.5 amperes.

Answer

The formula usually learnt is 'heat produced in joules $= VIt$' where V is the potential difference in volts, I is the current in amperes and t is the time in seconds.

$\therefore \quad 7200 = 12 \times 2.5 \times t$

$\therefore \quad 7200 = 30 \times t$

Dividing both sides by 30 (or moving the 30 to the left-hand side where it will divide) we get $\frac{7200}{30} = t$ which can be rewritten as $t = \frac{7200}{30}$.

$\therefore \quad t = 240$ second or 4 minute

(The $\therefore$ symbol denotes the word 'therefore' or 'it follows that.')

Cancelling in an equation

Simplifying or removing a pair of numbers in an equation is called cancelling. In the equation $3(2x + 3) = 3(x + 5)$ the threes cancel when both sides of the equation are divided by 3. If Example 1 above had been $7200 = 12 \times 2 \times t$ and you noticed that 12 divides nicely into 72 you might have divided both sides of the equation by 12 (it would still be true). You would get $600 = 2t$ so that $t = 300$ s.

Simplifying $3.1x + 5 = 4.4 + 5$ to $3.1x = 4.4$ is also an example of cancelling.