

UACE MATHEMATICS SAMPLE ITEMS

PURE MATHEMATICS

ITEM 1

A community group in Gulu is managing the costs for drilling three boreholes (A, B, and C). The total cost was UGX 25,000,000. The cost of Borehole B was UGX 1,000,000 less than Borehole A. The combined cost of Boreholes A and C was three times the cost of Borehole B. Let the costs of drilling boreholes A, B and C be a , b , and c in UGX respectively.

Tasks:

- Formulate a system of three linear simultaneous equations representing the information given.
- By Using Row reduction, calculate the individual cost of drilling each borehole (a , b and c).
- If the cost per meter drilled was UGX 250,000 for all boreholes, determine the depth of Borehole A.

ITEM 2

A certain city is experiencing social challenges with two distinct population groups having different experiences of urban life, the city planners are trying to understand if there is a common problem affecting both groups, they are modelling the wellbeing of two groups using two separate quadratic equations. $x^2 + px + q = 0$ p represents access to social services and q represents employment opportunities.

and $x^2 + mx + k = 0$. The collective wellbeing is given by, where m represents educational attainment and healthcare access, k represents safety quality.

The social challenge exists if $(q - k)^2 = (m - p)(pk - mq)$.

Task.

As a data analyst identify whether the common social challenge exist or not.

ITEM 3

A certain city in Uganda is investing in a new smart traffic management system designed to optimize traffic flow and reduce congestion especially during the rush hours. The system caters road density, vehicle capacity and pedestrian movement, the engineers have developed some indicator related to the efficiency of the traffic flow on the major roads. The indicator θ , which relates to a specific traffic cycle parameter which is considered when the following equation is satisfied. $4 + \tan^2 \theta = \sec \theta (7 - \sec \theta)$. The system needs to be designed in way such that an optimal traffic flow is achieved.

Task.

Help to determine the indicator values (θ) at which the system may be efficient.

ITEM 4

The city is facing a challenge in urban planning where by balancing economic development and preservation of green spaces, to model this relationship between economic development and preservation of the green space, the city planner comes up with a simplified systems of logarithms with two indicators x and y , he believes that for optimal urban health, the indicies should satisfy the following two logarithmic relationships.

$$\log_{10} x - \log_{10} y = 1.0 \text{ and } \log_{10} x - \log_{10} y = \log_{10} 2.5$$

Task

Use the above system of simultaneous equations to obtain the indicators of sustainability economic development

ITEM 5

A treasure map uses a coordinate grid to help adventures locate hidden treasures in Kassanda district. The grid is labelled with coordinates (x, y) where each point represents a specific location. Two equations are used for safety of the treasures but the travelers have no knowledge of how to identify the coordinates. These equations are $5^{x+2} + 7^{y+1} = 3468$ and $7^y = 5^x - 76$

Task

Help them to identify the location of the treasures.

ITEM 6

Your class teacher loses her phone after being stolen by thugs while coming to school. He decides to go to police so that the phone can be tracked using GPS coordinates. The phone's location at different times is recorded and the police are trying to find its movement pattern. The tracking device uses a logical operation given simultaneously as $2^x + 3^y = 5$ and $2^{x+3} - 3^{y+2} = 23$. When they got so close to the phone, they could not tell the coordinates to locate the phone with ease. The coordinates are in meters and one point gives the position of those with the tracking device, the other point gives the position of those with the phone.

Task

- (a) Help them find the coordinates.
- (b) How far is the phone from the police tracking it?

ITEM 7

Kawalya is designing a triangular garden plot in his backyard. He wants to calculate the area of the garden, analyze how sunlight hits it throughout the day using a model, and simplify expressions related to the light distribution over the garden using trigonometric identities. The triangular garden is formed by three points: A, B, and C. Maya measures the lengths of two sides and the included angle as follows:

- $AB=12$ meters
- $AC=15$ meters
- $\text{Angle } BAC=65^\circ$

The sunlight intensity at a certain location in the garden over time is modeled by the function: $Y(x) = 5 \sin x + 12 \cos x$ where x is the time in hours after sunrise. Maya is also working with the expressions $\mathbf{M} = b + c$ And $\mathbf{W} =$

$a \cos \frac{(B-C)}{2} \csc \frac{A}{2}$ Where A, B and C are angles of a triangle, she wants to verify that M matches with W using trigonometric identities.

Task: ,

- Using the given dimensions, calculate the area of triangle ABC. Give your answer to 2 decimal places.
- Express the function $Y(x)$ in the form $R \sin(x + \alpha)$ and hence interpret the greatest and least values of $Y(x)$.
- Help Maya to verify that **M** matches with **W**.

ITEM 8

Kwagala sells photographs at art fairs with small, medium and large sizes. She sell **3** more small photos than twice the difference between large from medium photos. It is also noted that when he sells **2** times small photos and **3** times medium photos, their sum is equal to **11** minus the number of large photos. She knows that small photos cost \$10, medium \$30 and large \$40. She wants to determine how many of each size to sell to cover her booth rental cost of \$150. Additionally, she monitors her stock by the equation $Y = \frac{x+1}{x^2+3}$, then considers the discriminant of a suitable quadratic equation to determine the range of possible values of y which would give real stock value of x .

Task

Help Kwagala determine;

- How many of each size he should sell to cover her both rental cost.
- The range of possible values of y which would give real stock value of x .

ITEM 9

A new road (Road 1) is being constructed in Kampala, represented by the equation $y = 2x + 3$. It will intersect an existing road (Road 2), represented by the equation $3x + 2y = 12$. A traffic light needs to be installed at the intersection point. Another planned road (Road 3) needs to be parallel to Road 1 and pass through the point $(4, 1)$. A fourth road (Road 4) must be perpendicular to Road 2 and pass through the same point $(4, 1)$.

Tasks:

- Calculate the coordinates of the intersection point of Road 1 and Road 2 where the traffic light will be placed.
- Determine the equation of the line representing the planned Road 3 and Road 4.
- Calculate the acute angle between Road 1 and Road 2 at their intersection point. **Give your answer in degrees.**

ITEM 10

A surveyor is mapping a triangular piece of land in the hilly region of Kabale. The vertices are marked as P, Q and R. The distance PQ is measured as 120 meters, and the distance PR is 150 meters. The angle at P, angle QPR , is measured as 75° . The surveyor needs to find the length of the third side QR and the area of the land. The Surveyor also had to test whether it's true that

$$\cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

Task.

- Determine the length of the side QR to the nearest meter and angle $\angle PQR$ to 1 decimal place.
- Calculate the area of the triangular piece of land PQR.
- If angle QPR was actually measured as $(45^\circ + 30^\circ)$, apply an appropriate formula to find the exact value of $\cos 75^\circ$.
- Is it true that $\cos 18^\circ$ is equivalent to the given surd? Justify.

ITEM 11

A bus heading to Masaka passes through Mpigi town. At time, t hours, past noon, the distance of the bus from Mpigi to Masaka is expressed as $x = 7 \sin t - 4 \cos^2 t + 2$. It's known that 1 degree is $\frac{\pi}{180}$

Task: Help:

- The manager wants to know when the bus was in Mpigi give the answer to 2 decimal places
- Identify the time the bus will be behind Mpigi by a distance of 5km.

ITEM 12

The remote village of Kijungu relies heavily on its local well for water due to climate change and increasing population, the wells water output is becoming very low, an engineer has developed a model to predict the wells daily water output in liters based on the factor x which accounts for drought conditions increased demand and underground water table depletion, the model is given by the equation $3^{2x+1} + 3 = 10(3^x)$

Task.

As mathematics students help in identifying the stress factor x at which the wells water output meets the condition described by the above equation.

ITEM 13

A certain farmer in the village has bought land which he plans to fence, he has contracted a person to fence it and the contractor has been informed that the area of the land is $6300m^2$. This land is rectangular in nature and that the length is $20m$ more than the width.

Task.

- By completing the squares determine the actual dimensions of the piece of land.
- Determine the maximum value of the expression obtained from above.

ITEM 14

An airline offers three ticket classes (Economy, Premium and Business) on a flight. The revenue from ticket sales must cover fixed costs while maximizing profit.

On a given flight, The sum of all tickets sold is 200 passengers.

When Economy tickets cost is \$300, Premium \$600, and Business \$900.

Total revenue that was collected was exactly \$120,300.

Support:

the number of Economy tickets is equal to twice the number of Premium tickets minus a 4 times the number of the Business ticket plus 6.

Task

Using knowledge of solving simultaneous equations using reduction to row echelon form, help the manager know the number of tickets in each class.

ITEM 15

Aerospace engineers are analyzing the stability of a satellite's elliptical orbit. The orbital path is modelled by the quadratic equation:

$$f(t) = t^2 - (2e + 4)t + (e^2 + 4e + 3) = 0$$

where:

- Roots (α, β) represent critical stability points in the orbit
- Parameter e represents orbital eccentricity $(0 < e < 1)$

For stability calculations, engineers need:

- a. Sum of cubes $\alpha^3 + \beta^3$
- b. Sum of reciprocals squared $(\frac{1}{\alpha^2} + \frac{1}{\beta^2})$

Task:

Use knowledge of mathematics to Derive expressions for a) and b) in terms of e

ITEM 16

In a certain pharmaceutical company a certain product is produced by using a process function which depends on the thickness t , of the tablet in mm produced given $f(t) = 3t^2 + 4t + 1$, at a certain instant when $f(t) = 0$, tablets of thickness V and U were produced. However the company started facing a problem of varying in thickness of tablets produced due to mechanical problems. In the production machine. However the company has no money to import a better machine but after analysis, it has been observed that tablets produced differ from the previous ones by 2mm.

Your father is a process engineer in this company and he is stuck on what to do and has approached you.

Task

As attendant of mathematics use mathematical concepts to come up with a new process function that can be used by your father to continue tablet production using the available machine.

ITEM 17

In a certain chemical processing company product X is formed by combining reactants A, B and C according to the reaction equation below. $aA + bB + cC \rightarrow X$. Where a, b, and c are different ratios that vary with time. The company produces product X in shifts morning, evening and nights shifts. The table below shows how different shifts combined reactants A, B and C to produce x on Friday.

Shift	Ratios used			Quality of X produced in grams
	A	B	C	
Morning	3	-2	-1	5
Evening	1	3	-1	4
Night	2	-1	4	13

Your Aunt is working with the company and would like to know the quantities of each reactants used previous da and has approached you for assistance.

Task

As a student of mathematics use your mathematical knowledge to help your Aunt to know the amount of A, B and C used on that Friday.

ITEM 18

In the school games and sports champions, there were a lot of competitions among different schools; this was shown by different techniques among the players. At a certain occasion during netball semifinals, Jenifer threw a ball and it took a parabolic path given by the equation $f(x) = 2\sec^2 x - 3 - \tan x$ directly to the target in the last minutes, this created joy and happiness in the team including the team coach. On their way back, the coach asked Jennifer to explain to the team the direction the ball took to hit the target but she couldn't explain clearly.

Task:

As a student of mathematics use mathematical concepts to help the team identify any four possible directions the ball would have taken to hit the target from Jenifer's hand (Hint: The ball hits the target when $f(x) = 0$).

ITEM 19

In final MDD competitions at BFA results are fed in the computer by data entrant basing on the function $f(x) = Q(x) + h(x)$ The entrant entered the above function and the computer displayed $f(x) = \frac{x+2}{(x+1)(x-3)}$ where x the number of students is. Due to the increase in the number of students, the entrant wants to edit the function entered by changing $Q(x)$ to a new function but because of absence of electrical power the computer couldn't display and he doesn't remember the functions he put initially.

Task:

As a mathematics student, use mathematical concepts to come up with the function $Q(x)$ the entrant placed in the system initially.

ITEM 20

The National Water and Sewerage Corporation (NWSC) is designing a water distribution system for three neighbouring communities in Kampala. Each community has different water requirements and infrastructure constraints. A consultant engineer working on this project wants to determine the optimal flow rates for each community. The water distribution system is modelled by the following equations

$$X+2Y+Z=2400 \text{ (Total available water supply in litres per minute)}$$

$$3X+Y+3Z=3900 \text{ (pressure balancing equation)}$$

$3X+4Y+2Z=5100$ (Flow optimization equation) Where X , Y and Z represent the flow rate in litres per minute in communities A, B and C respectively The polynomial equation $P(X) = X^3 - 7X^2 + 14X - 8$ models the operational efficiency of the pumping systems

Task:

- Help the engineer to determine the optimal flow for each community
- If the community A's water requirements increase by 200 litres per minute. What adjustments should be made to the other communities' supply to maintain the system balance?
- Determine all the possible value of X where the efficiency of the pumping system is zero

ITEM 21

The Entebbe Municipal Council is planning to develop a new neighborhood with roads, residential areas, and a community center. The planning department

uses a coordinate system where each unit represents 100 meters. An urban planner, needs to design the road network and determine optimal locations for the key facilities. A road P is planned to connect points A(2, 3) and B(8, 15) and another road Q will connect points C(4, 5) and D(12, 9)

Task:

- a. Help the urban planner to determine the length of the road P in kilometers
- b. Establish whether the two roads P and Q intersect, and if so, at what point and angle between the two roads will they meet.
- c. If the parametric equations $x = \frac{1+t}{1-t}$ and $y = \frac{2t^3}{1-t}$ can give the curve in (c), use them to find its second derivative.

ITEM 22

Toyota Motors Uganda limited have started a plant to manufacture electric vehicle models. The Engineers need to calculate exact dimensions for specific components to ensure proper fitment. The chief Engineer has given you measurements for a triangular mounting bracket for battery bracket design which involves surds. The diagonal support brace of the bracket needs to be exactly $(7\sqrt{50} + 3\sqrt{98} - 5\sqrt{72})$ long. Two sides of the triangular bracket are $a = (6 + 3\sqrt{5})\text{cm}$ while the third side is $d\text{cm}$ long.)cm and $b = (9 - 2\sqrt{5})\text{cm}$ long The Engineer was told that you are a good mathematician with deep understanding of surds, logarithms and indices. He wishes to solve for the unknown variable, x in different mathematical equation models developed as shown below to produce more fitments.

Model one: $\sqrt{x + 5} + \sqrt{x + 4} = \sqrt{6x + 40}$

Model two: $2^{2(x+1)} - 5(2^x) + 1 = 0$

Model three: $\log_3 x + 6 \log_x 3 = 5$

Task: Help the Engineer:

- a. Express the diagonal support length in the form $a\sqrt{2}$ hence, state the value a
- b. Find the length of the third side of the triangular bracket, d if the perimeter of the entire triangular bracket is 25 cm and Hence find the value of $\frac{a}{b} + \frac{a}{d}$ in simplified form.

- c. Solve for x in each of the three models to make more fits for the best electronic vehicles.

ITEM 23

The Entebbe Municipal Council is planning to develop a new neighborhood with roads, residential areas and community center. The planning department uses a coordinate system where each unit represents 100 metres. An urban planner needs to design road network and determine the optimal locations for key facilities. Road A is planned to connect points P(2,3) and R(8,5) and another road B will connect points S(4,5) and N(12,9)

TASKS:

- a. Help the planner to determine;
 - i. The length of the road A in kilometers
 - ii. How steep road A is and the linear equation that models the path of the road.
- b. Establish whether the two roads A and B intersect, and if so, at what point, and the angle between the two roads? (Correct your angle to 1 decimal place)
- c. If a community center needs to be located at a point that is equidistant from points A, B and C, find the co-ordinates of this point.

ITEM 23

The Uganda National Roads Authority (UNRA) is studying traffic patterns on the Kampala Jinja highway. They've developed a mathematical model to predict traffic density at different times of the day. The traffic density function (Number of vehicles per kilometer) at time t hours after 6:00AM is given by:

$$D(t) = \frac{20t^2 + 30t + 15}{(t^2 + 1)(t + 3)}$$

TASKS:

As a traffic Engineer, you need to analyze this model to optimize traffic flow and plan for future road extensions.

- a) Represent the function as a sum of partial fractions.
- b) By partial fraction decomposition, determine the time of the day when the traffic intensity is at its minimum

ITEM 24

A renewable Energy Company is installing solar panels on buildings in Budduda district. To maximize energy collection, they need to determine optimal installation angles based on the building's orientation and the sun's position through the year. The company engineer needs to know the ideal mounting angles for solar project at school. He finds out that the amount of solar energy collected by the panels is modelled by the function;

$E(\theta) = 800 \sin \theta - 200 \sin 2\theta + 100$ where E is the energy collected in Watts-hours per square metre and θ is the angle of the panels from the horizontal in radians and the shadow length L of a vertical pole of height, (h) at time, t hours afternoon is given by; $L = h \tan\left(\frac{\pi}{12t} + \frac{\pi}{4}\right)$

Tasks

- If the school building has a roof with a pitch of 35° facing south, and the optimal angle for solar panel is 15° relative to the roof surface, determine the absolute angle of the panels from the horizontal.
- Establish the maximum Energy that can be collected, and at what value of θ ?
- At what time will the shadow be equal to the height of the pole?

ITEM 25

The chairperson of ward B who is suffering from blood pressure is given a sleeping pill. This has caused an initial level of 4mg of a drug per litre of his blood. After hours, the number of mg per litre in the blood is N, where $N = 4(0.76)^t$

TASK:

- Help the doctor to estimate the number of mg per litre after four (4) hours. (Correct your answer to one (1) d.p)
- The doctor also wants to know how long it'll take the amount of the drug per litre of blood to reduce to half its initial value?
- If the mg per litre of blood increases to thrice the initial amount, the patient has to be put on drip, advise the doctor, after how long can this happen?

ITEM 26

John has applied for a job in a construction company and He has been selected among those to sit for interviews. Among the areas to be considered, is how

good one's knowledge of trigonometry is. John has tried the task below and failed. Given that; $\tan \theta + \sin \theta = X$ **and** $\tan \theta - \sin \theta = Y$ then; $(x^2 - y^2)^2 = 16xy$.

TASK:

As a student who has done trigonometry, help John prepare for interview.

ITEM 27

During a physics experiment, a student investigates how the angle of rotation of a mirror affects the direction of a reflected light beam. The relationship between the mirror's rotation and the reflected beam's angle x (in degrees) is given by the equation $\tan 3x = \cot x$ for $0^\circ \leq x \leq 180^\circ$

Task ;

Solve for all possible values of x that satisfy the equation within the given range.

ITEM 28

Root analysis is a major concept when studying quadratic equations , Without obtaining the roots themselves and stating a valid condition predict the type of roots in the following equations;

- i) $2x^2 + 3x + 7 = 0$
- ii) $x^2 - 7x = 12$
- iii) $25x^2 + 4 = 20x$

ITEM 29

The robot's Motion in a factory is modeled by an explicit function; $y = \frac{(x+1)}{\sqrt{(x^2-1)}}$ where x and y are its horizontal and vertical displacements from the factory's loading dock. show that the tangent-slope function $(\frac{dy}{dx})$ of the robot at any point from the dock is given by; $\frac{-1}{(x+1)^{\frac{1}{2}}(x-1)^{\frac{3}{2}}}$

ITEM 30

A certain farmer in the village has bought land which he plans to fence, he has contracted a person to fence it and the contractor has been informed that the area of the land is $6300m^2$. this land is rectangular in nature and that the length is $20m$ more than the width .

Task.

- a. By completing the squares determine the actual dimensions of the peace of land.
- b. Determine the maximum value of the expression obtained from above.

ITEM 31

Alex and Francis are friends that always have daily arguments starting from the premier league teams they support that is, Manchester united and Arsenal respectively. Recently they were moving around school and entered a S.4 classroom where they found an expression on the chalkboard reading $\sin 3A$, immediately the argument started when Alex claimed that the handwriting looked familiar and Francis said it was a lie. They later moved to the next classroom of S.3 and found another expression on the white board reading $3 \sin A - 4 \sin^3 A$, another argument came in when Francis said that the two statements look different but mean the same thing, “no way, it cannot be, lets bet on it buddy”, were the statements of Alex.

As they were still arguing, Francis received a notification on his phone from BBC news showing that Iran was to launch a nuclear missile over Israel whose speed is given by the expression $s = 3 \cos \theta + \sin \theta$ where s is the speed in kilometres per hour (km/h) and θ the angle of projection.

Tasks

Using the knowledge of trigonometry help these two;

- a) Without using calculators or tables, settle the argument whether the statements mean the same thing or not
- b) Determine the speed for the missile in km/h if it is to cover 5km in 300 minutes
- c) Calculate the possible angles of projecting the missile in the range $0^\circ \leq \theta \leq 360^\circ$ if it is to move at the speed obtained in (b) above

ITEM 32

David Witboi, an award-winning technology innovator and celebrated calculus champion, is recognized for his groundbreaking work in applying advanced mathematics and Genetic Algorithms to solve real-world problems. Passionate about merging technology with engineering science, David has launched a project aimed at creating an intelligent traffic management system capable of predicting and optimizing vehicle movement on busy highways.

Leveraging his expertise in data modeling, machine learning, and applied calculus, he has developed a predictive mathematical model that estimates

traffic density (vehicles per kilometer) at various times of the day. This model serves as the foundation for a future AI-driven Smart Traffic Control Platform, designed to assist governments and city planners in making informed decisions about road design, expansion, and congestion management.

The model is expressed by the traffic density function:

$$\triangleright Y(x) = \frac{x^4 + x^3 - 6x^2 - 13x - 6}{(x-3)(x+2)(x+1)}$$

where x represents the number of hours after 6:00 AM.

Tasks

As part of his research and system optimization process, David seeks to perform a detailed calculus-based analysis of the model to ensure its reliability and efficiency in real-world traffic forecasting.

- a) Determine the type of fraction represented by the traffic density function $Y(x)$. Justify your answer clearly.
- b) By partial fraction decomposition, derive a fully simplified expression for $Y(x)$.

ITEM 33

You are a student of mathematics in Uganda, and you're working on a project to optimize transportation systems in the city. Your project involves using mathematical models to improve route efficiency and reduce energy consumption. Part 1: Vehicle Navigation:

You've developed a navigation system that uses the function

$$y = \tan^{-1} \frac{(1-x)}{(1+x)}$$

To calculate the optimal turning angles for vehicles on the road, you're to determine the rate of change of the turning angle with respect to the vehicle's position, ensuring smooth and efficient navigation.

Part 2: Energy Consumption:

As part of your project, you're also analysing the integral

$$\int \frac{dx}{x^2 + 2x + 2}$$

which represents the total energy expended by vehicles. You are to estimate the total energy consumption and identify areas for improvement.

Task.

Present your findings to the city authorities and propose ways to implement your optimized transportation system.

ITEM 34

The classroom is abuzz with a heated debate between two students, Alex and Sam, regarding a trigonometric identity. The teacher, Mrs. Nambalirwa, has just presented the equation:

$$\frac{\sin 8x \cos x - \sin 6x \cos 3x}{\cos 2x \cos x - \sin x \sin 4x} = \tan 2x$$

Alex claims that the equation is true and can be proven using mathematical identities, while Sam insists that it's false and can't be proven. The debate is getting intense, with both students presenting their arguments. Mrs. Nambalirwa intervenes, and tasks you to step in and help the class.

Task.

You, being the math whiz in the class, are tasked with resolve the argument.

ITEM 35

The Town Council Committee is planning to construct two new roads in the city centre to ease traffic congestion. Before construction begins, engineers must determine the exact alignment of these roads to ensure smooth traffic flow and proper connectivity.

- The first proposed road will pass through two points on the map: one located 10 km east and 10 km north, and another at 60 km east and 40 km north.
- The second road will connect a point 20 km east and 50 km north to another point 30 km east and 90 km north.

To ensure safety and efficiency, engineers must calculate the angle of inclination (θ) between the two roads. For optimal road design, this angle should not be too sharp, and the space between the roads should not be below 30°

Additionally, due to emergency response requirements, the town plans to construct a hospital along the first road at a point located 20 km east and 10 km south. However, for quick emergency access, the distance from this point to the road should not exceed 10 km. If this condition is not met, the hospital

will be relocated to a backup site located 5 km east and 6 km north on the map.

Task:

Help the committee:

- a) Compute the equations of the two roads and determine if the inclination angle between them is safe.
- b) Assess whether the initially proposed hospital location lies within the required 10 km from the first road. If not, confirm if the backup location is suitable.

ITEM 36

A landscaping team is redesigning a section of a public park and needs to determine the exact distance and angles between two tall, decorative trees Tree A and Tree B in order to plan the placement of walking paths, benches, and a central monument. However, due to a large flowerbed and a small pond situated between the two trees, it is impractical to measure the distance between them directly.

To solve this, a team member named Grace walks to a point C within the park, where both trees are clearly visible. She hires a team of land surveyors, and the following measurements are recorded:

- The distance from Tree A to point C is 50 meters
- The distance from Tree B to point C is 65 meters
- The angle between the two lines of sight from point C (i.e, $\angle ACB$) is 70°

Grace now needs to calculate the distance between Tree A and Tree B (side AB), as well as the remaining two angles of the triangle formed by points A, B, and C. These calculations will assist the team in achieving a symmetrical and mathematically accurate layout for the planned features.

Additionally, the team has agreed that if the distance between the two trees is less than 60 meters, they will install a paved walkway or

If the distance is 60 meters or more, they will cement the area for durability

Task:

Help the team determine:

- (i) Whether they will pave or cement the distance between the two trees.
- (ii) The angles between the trees and point C.

ITEM 37

A new shop attendant at a clothing and stationery store was reviewing some recent sales summaries. According to the records, the combined cost of one book, one shirt, and one trouser is shs. 55,000. Another record shows that two books, two shirts, and three trousers cost shs. 140,000, while four books, three shirts, and two trousers cost shs. 145,000. Later that day, a customer walked in and inquired whether she could purchase five books, four shirts, and six trousers with a budget of shs. 250,000. She added that if the budget is not enough, she would consider reducing the books by 2 and trousers by three. However, she is unsure which option her budget can accommodate.

Task:

Help the shop attendant:

- a) Calculate the cost of each individual item.
- b) Advise the customer on which of the two options fits within her shs. 250,000 budget.

ITEM 38

Kiira Motors Corporation, Uganda's vehicle manufacturer, is designing components for their new electric vehicle model. The engineers need to calculate exact dimensions for specific components to ensure proper fitment.

You are an engineering intern at Kiira Motors working on the battery bracket design. The chief engineer has given you measurements for a triangular mounting bracket that involves surds. The diagonal support brace of the bracket needs to be exactly $\sqrt{50}$ cm long. Two sides of the triangular bracket are $(5 + 3\sqrt{2})$ cm and $(7 - 2\sqrt{2})$ cm long.

Tasks:

- a) Express the length the bracket diagonal support in the simplified form $\frac{a}{\sqrt{b}}$ where a and b are integers and b is not divisible by a perfect square.
- b) Establish the exact length of material needed for both sides combined.
- c) If the perimeter of the entire triangular bracket must be exactly 25 cm, calculate the length of the third side.

ITEM 39

Ms. Nabukalu, a maize farmer in Masaka, has discovered that the yield of her crop (measured in bags per acre), denoted by Y , depends on the amount of a specific fertilizer applied, denoted by x (measured in kilograms per acre). The relationship between the yield and the amount of fertilizer is modeled by the quadratic equation:

$$Y(x) = -0.5x^2 + 20x + 50.$$

She aims to maximize her yield but is also mindful of the cost associated with using the fertilizer.

Tasks:

- Help Ms. Nabukalu to determine the amount of the fertilizer that will give her the maximum maize yield.
- Calculate the maximum possible yield in bags per acre.
- If the cost of the fertilizer is UGX 1,500 per kg. Ms. Nabukalu wants the yield to be at least 200 bags per acre. Formulate a quadratic inequality to represent this situation.
- By solving the inequality in part (c) above, determine the range of fertilizer amounts (in kg per acre) she can use to achieve a yield of at least 200 bags per acre.

ITEM 40

An information security analyst designs a system where encryption keys are generated using interactions of periodic function $\sin(2x + 30) = 0.8$ where x is an acute angle. If in addition the analyst combines two sine waves to create a specific audio effect. The combined signal is modeled by; $Y = 5 \sin x + 6 \operatorname{cosec} x - 17$. This signal is used to modulate volume for background alarms.

Tasks:

- Determine the possible values of x for the periodic function
 - Determine the x values at which the second waves cross the horizontal axis, Take $-180^\circ \leq x \leq 180^\circ$
- Validate the following identities;

- $$\sec^2 x = \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \sin x}$$

ii. $\frac{\tan x}{1-\cot x} + \frac{\cot x}{1-\tan x} = 1 + \tan x + \cot x$

ITEM 41

The Uganda National Roads Authority (UNRA) is studying traffic patterns on the Kampala-Jinja highway. They have developed a mathematical model to predict traffic density at different times of the day.

The traffic density function (vehicles per kilometer) at time x hours after 6:00 AM is given by;

$$Y(x) = \frac{x^3 - 3}{(x - 2)(x^2 + 1)}$$

Tasks:

As a traffic engineer, you need to analyze this model to optimize traffic flow and plan for future road expansions.

- a) Determine what type of fraction $Y(x)$ represents. Justify your answer.
- b) Express $Y(x)$ as the sum of a polynomial and a proper rational fraction.
- c) By partial fraction decomposition write a complete simplified expression for $Y(x)$

ITEM 42

Abitex organized a show at Theater Laboniter which filled to capacity of 350. The theater charges \$4.50 for children, \$7.50 for students, and \$12.50 of adults. There are half as many adults as there are students. The total ticket sales was \$2415, Abitex wants to know how many children, students, and adults attended.

Task:

- (a) Explain to Abitex on how he can know the number of children, students and adults attended
- (b) Based on your explanation, determine how many children, students and adults attended

ITEM 43

Mr. Jookun is a farmer and his son has joined A' level this year offering Mathematics, Economics and Geography. The Mathematics teacher gave homework and whoever doesn't present the work with correct solution will be given a punishment next day, and the farmer's son is not good at all health-wise. Mr. Jookun is so much worried and has contacted you to help his son solve the task below.

- a. If $\log_5 x = n$ and $\log_{15} x = m$ then, prove that: $\frac{n+m}{n-m} = \log_3 75$
- b. Without using calculators, find the value of: $\frac{\log 0.8 - \log 32 + \log 8}{\log 0.7 + \log 7 - \log 49}$ given that $\log 2 = 0.303103$

ITEM 44

Musa can differentiate $y = 5x^2 - 13x + 9$ by the rule of nx^{n-1} to obtain $\frac{dy}{dx} = 10x - 13$ but he cannot do it from first principles, take him through to obtain the same answer.

ITEM 45

An economist is studying the relationship between investment I and national income Y. The relationship involves a complex function where a particular term is given by $h(Y) = \frac{Y^3 + 2Y^2 - Y + 5}{Y^2 + Y - 2}$

Before proceeding with the economic analysis, the economist needs to simplify this expression.

Tasks:

Help the economist to:

- (a) Identify $h(Y)$ as a proper or improper rational function. Justify your answer.
- (b) Express $h(Y)$ as the sum of a polynomial and a proper rational function.
 - (c) Take the proper rational fraction part obtained in (b) and decompose it into its partial fractions.
- (c) Combine the results from (b) and (c) to write the complete simplified expression for $h(Y)$.

ITEM 46

MTN Uganda has been tracking its mobile money subscribers over the past 5 years. The number of active users has been growing exponentially. The company started with 850,000 active users in 2020 and has experienced a constant annual growth rate.

You are a data analyst at MTN Uganda tasked with analyzing this growth to help the company plan for future infrastructure needs. In 2023, the number of active users was recorded as 1,309,500.

Tasks:

- a) Utilizing the knowledge of logarithms, determine the annual growth rate of MTN mobile money subscribers. Express your answer as a percentage to 1 decimal place.
- b) Predict the number of active subscribers MTN Uganda will have in 2025.
- c) How many years would it take for the number of subscribers to reach 5 million users?
- d) How can exponential growth models can be used by telecommunications companies for planning resource allocation.

ITEM 47

A farmer in Mbarara has 800 square meters of land for growing beans and maize. Based on local market prices, she makes a profit of UGX 5,000 per square meter of beans and UGX 3,000 per square meter of maize. Due to crop rotation requirements, she must plant at least 200 square meters of beans and at least 300 square meters of maize. The farmer has approached you he needs help of how he can maximize her profits while meeting all the constraints.

Tasks:

- a) Help the farmer to represent all the constraints as inequalities including the total profit
- b) Determine the maximum possible profit and the corresponding areas of land to be allocated for each crop.
- c) If the profit per square meter for maize increases to UGX 4,000, how would this change your recommendation? Show your workings.

ITEM 48

A small-scale manufacturer in Jinja produces handcrafted furniture. Their workshop makes tables and chairs from locally sourced wood. Each table

requires 5 hours of labor and 8 square meters of wood, while each chair requires 3 hours of labor and 2 square meters of wood. The workshop has 60 hours of labor and 72 square meters of wood available per week.

Tasks

- Obtain all the constraints as inequalities that can help the production manager to determine the optimal production strategy.
- The profit is UGX 70,000 per table and UGX 25,000 per chair. Model the total weekly profit as a function of the number of table and the number of chairs
- Illustrate the feasible region defined by the constraints and use it to find the optimal strategy that maximizes the profit.

ITEM 49

There has been a terrorism threat in DRC in the recent months. To curb this problem, the United states government has deployed two teams of soldiers; the Infantry and the Airforce. The Infantry has been divided into two groups. One group is situated at an earth coordinate given simultaneously as; $\log_2 x + 2 \log_4 y = 4$ and $4 \log_{10}(x + y) = 1$ The second group is situated at coordinates given by $\log_{25} 4x^2 = \log_5(3 - x^2)$ And $\log_4(6 - y) = \log_2 y$. At exactly 8:00PM, the Airforce noticed the terrorists are at a distance $-7 \left[\frac{(\sqrt{5}-2)^2 - (\sqrt{5}+2)^2}{8\sqrt{5}} \right]$ kilometers away from the main town, Goma. The Airforce is planning to attack them if they confirm terrorists are more than 5km away from the main town but have not yet confirmed.

Task

- Calculate the coordinates for the location of each of the groups.
- How far are the two groups apart?

ITEM 50

A cinema sells snacks in Combo deals;

Combo A includes 1 tin of popcorn, 1 drink and 1 chocolate bar all at a total cost of UGX.26,250.

Combo B includes 2 tins of popcorn, 1 drink and 2 chocolate bars all at a total cost of UGX.43,750.

Combo C includes 1 tin of popcorn and 2 drinks all at a total cost of UGX.28,000. Your friend wants to buy 3 tins of popcorn, 2 drinks and 1 chocolate bar for both of you but not sure of how much to pay for all of them.

Task

Use your knowledge of mathematics to save your friend from being cheated.

ITEM 51

During your first term holiday in senior five you decide to visit your uncle in the village having taken long not seeing him. He wishes to plant trees around his plot of land whose length is 2 metres longer than its width. The total area of the plot is 35 square metres according to the land title .He plans to use a spacing of 1m from each tree that but doesn't know how much money he will spend on buying the trees .

Hint; Each tree costs UGx 2,500

Task

Help your uncle to determine the total amount of money needed to be spent on buying trees.

ITEM 52

Your class teacher loses her phone after being stolen by thugs while coming to school. She decides to go police so that the phone can be tracked using GPS coordinates. The phone's location at different times is recorded and the police are trying to find its movement pattern. The tracking device uses a logical operation given simultaneously as;

$\log_2 x + 2\log_4 y = 4$ and $xy = 16$ when they got so close to the phone but couldn't tell the coordinates to locate the phone with ease. The coordinates are in metres and one point gives the position of those with the tracking device, the other point gives the position of the phone.

Task

- (a) Help them find the coordinates.
- (b) What is the distance between the two points?

ITEM 53

A robot arm moves such that the positions of its ends are described by

$x = 7 + \sqrt{3} \cos \theta$ and $y = 5 + 7 \sin \theta$ where θ is the rotational angle. The robot operator needs to determine the largest reflex angle of rotation of the robot arm if it is known that $x + y = 13$.

Task

- (a) Show that $49x^2 + 3y^2 = 686x + 30y - 2329$
- (b) Help the robot operator get the required rotational angle.

ITEM 54

Nakato is a mobile money agent in her village, Mukono. When she started in January 2024, she had 50 regular customers. She observed that her customer base seemed to grow exponentially each month. By the end of March 2024 (after 3 months), she had 135 regular customers. She wants to predict her customer growth to plan for liquidity (cash and e-float) and potentially hire an assistant. Assume the growth follows the model $N = N_0 \times K^t$, where N is the number of customers after t months, N_0 is the initial number of customers, and K is the monthly growth factor.

Tasks:

- a) Help Nakato in determining her monthly growth factor,. Express your answer to 3 significant figures.
- b) Determine the number of customers Nakato can expect by the end of December 2024 (after 12 months) if this growth rate continues.
- c) Nakato estimates she needs UGX 10,000 in float per regular customer per month. Using the predicted number of customers for December 2024, calculate the total float she would need. Express this amount using index notation in terms of powers of 10.

ITEM 55

It said that the solutions to the pair of equations below provides with x, y and z representing parameters for given study on effect of water, sunshine and fertilizers respectively to a newly discovered type of plant deep in the Amazon forest.

$$2x + 3y - z = 2 \text{ and } \frac{x - 4}{2} = \frac{2 + y}{-1} = \frac{z - 2}{3}$$

The output that provided negative reactions were to be supplied in minimal quantity and one with positive output supplied in increased amount. Team has modelled a graph function representing the path travelled by a drone after it during its first test flight as $y = 8x - x^2$.

Task:

- a) Which of the two three input may be required to be supplied in large increased quantities?
- b) On graph, sketch the graph showing its path and determine the area it covers during this path from your graph.

ITEM 56

In school that has rectangular water tank where its length is three times its width. If the tank has a surface area of 1800m^2 . The Welfare Teacher wants to know the maximum capacity of water it can hold.

On the tank's surface S5 students found $\sqrt{36.05}$ written on it, they claimed they can find its answer without using calculator but through calculus.

Task:

- a) Determine the maximum capacity the tank can hold.
- b) Confirm the claims above by use of small changes concept to find the answer correct to 4 decimal places.

ITEM 57

A school in Mbarara wants to create a rectangular vegetable garden. They have 80 meters of fencing available. They want the area of the garden to be greater than 300 square meters to grow enough vegetables for the school lunch program. Let the length of the garden be L meters and the width be W meters.

Tasks:

- a) Express the perimeter of the garden in terms of L and W and form an equation using the available fencing.
- b) Using your equation in a) above formulate the area A of the garden purely in terms of L.
- c) Formulate a quadratic inequality representing the condition that the area must be greater than 300 square meters and solve it to determine the

possible range of values for the length (L) of the garden that satisfies both the fencing constraint and the area requirement.

ITEM 58

Mr. Kato owns a rectangular farm near Fort Portal. On a map grid, the corners of his main plot are at $(1, 2)$, $(9, 2)$, $(9, 8)$, and $(1, 8)$. He plans to install two straight irrigation pipes. *Pipe 1* will run from corner A to corner C. *Pipe 2* will run from the midpoint of side AB to the midpoint of side CD. A water source is located at point W $(5, 5)$.

Tasks:

- Determine the coordinates of the midpoints of sides AB and CD.
- Formulate the equation of the line representing Pipe 1 (line AC) and the equation of the line representing Pipe 2.
- Determine the shortest distance from the water source W $(5, 5)$ to Pipe 1 (line AC). Will Pipe 1 pass directly through the water source? Justify your answer.

ITEM 59

Three villages, A, B, and C, are located on a map grid at coordinates A $(2, 1)$, B $(8, 3)$, and C $(4, 7)$. A new health centre needs to be built such that it is equidistant from villages A and B. It must also lie on the line that passes directly between village C and the midpoint of the line segment connecting A and B.

Tasks:

- Determine the coordinates of the midpoint M of the line segment connecting villages A and B.
- Formulate the equation of the perpendicular bisector of the line segment AB. (This line represents all points equidistant from A and B).
- Find the equation of the line passing through village C $(4, 7)$ and the midpoint M calculated in Task 1.
- Calculate the coordinates where the two lines found in b) and c) intersect. This point represents the ideal location for the health centre. Justify why this location satisfies both conditions.

ITEM 60

An Engineer in a chemical engineering plant in Namanve, want to use a chemical with concentration (t) of a product over time t , which is modelled by complex rational functions.

Suppose the rate of change of concentration involves the expression;

$$f(t) = \frac{5t + 3}{(t + 1)(t + 2)}$$

To analyse the long-term behaviour but doesn't know the appropriate techniques to use.

Tasks:

- a) Help the engineer to identify the type of factors in the denominator of (t).
- b) Express (t) as the sum of its partial fractions.

ITEM 61

An electrical engineering student at Makerere University is analysing a signal whose behaviour

over time x is related to the function $g(x) = \frac{2x^2+x-1}{x(x-1)^2}$ This expression needs to be broken down

for further analysis.

Tasks:

- a) Set up the appropriate form for the partial fraction decomposition of (x).
- b) Determine the values of the unknown constants in the partial fraction decomposition.
- c) Write the final partial fraction decomposition of (x).

ITEM 62

A surveyor is mapping a triangular piece of land in the hilly region of Kabale. The vertices are marked as P, Q, and R. The distance PQ is measured as 120 meters, and the distance PR is 150 meters. The angle at P, $\angle QPR$, is measured as 75° . The surveyor needs to find the length of the third side QR and the area of the land.

Tasks:

- Determine the length of the side QR to the nearest meter and angle $\angle PQR$ to 1 decimal place.
- Calculate the area of the triangular piece of land PQR.
- If $\angle QPR$ was actually measured as $(45^\circ + 30^\circ)$, apply an appropriate formula find the exact value of $\cos 75^\circ$.

ITEM 63

A farmer in Mukono wants to create a rectangular enclosure for chickens next to a long, straight existing wall. He has 100 meters of fencing wire available for the other three sides of the rectangle. He wants to maximize the area enclosed for his chickens. Let the side parallel to the wall have length x meters, and the other two sides perpendicular to the wall have length y meters each.

Tasks:

- Help the farmer to express the total length of the fencing used in terms of x and y , and formulate an equation based on the available wire.
- Express the area A of the enclosure ($A = xy$) as a function of only one variable x .
Hence, find the value of x that maximizes the area.
- Determine the maximum possible area of the enclosure and confirm it is a maximum.

ITEM 64

A scientist has a spherical balloon which is being inflated. Its radius r is increasing at a constant rate of 0.1 cm per second. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$. He wants to find the rate at which the volume is increasing when the radius is 5 cm. He also wants to estimate the approximate increase in volume as the radius increases from 5 cm to 5.1 cm.

Tasks:

- Help the scientist to determine the rate at which the volume of the bowl is changing with respect to the radius.
- Determine the rate at which the volume is increasing when the radius $r = 5$ cm.

- c) Estimate the approximate increase in volume (δV) as the radius increases from $r = 5 \text{ cm}$ to $r = 5.1 \text{ cm}$.

ITEM 65

A piece of land is bounded by a river whose shape can be modelled by the curve $y = \sqrt{x}$, the straight line $x = 9$, and the x-axis (representing a straight fence). The coordinates are measured in meters. The owner wants to calculate the area of this piece of land.

Tasks:

- Help the owner to sketch the region bounded by the land.
- Set up the definite integral that represents the area of this region hence use it to calculate the exact area of the piece of land.
- If this area were revolved around the x-axis, it would form a solid shape. Set up the definite integral representing the volume of this solid of revolution hence Calculate this volume.

ITEM 66

Mutesi sells plots of land. The prices of these plots depend on their different locations. The prices of the plots at Senge, Mende and Lusaze are 10 million, 15 million and 20 million shillings respectively. In March this year, the total amount of money collected from the sales of the plots in the different locations is 300 million shillings. The total number of plots sold in March was 18 plots and the number of plots sold in Senge was equal to the total number of those sold in Mende and Lusaze.

Due to high demand of the plots at Lusaze, it was discovered that the number of plots sold in this location keeps on doubling every after one month and those sold from other locations remain at same number each month.

She wants to know the number of plots sold at each of the three locations in March this year and also to estimate the total amount of money she will collect in December this year.

Task

Help Mutesi to determine the:

- Number of plots sold at each location in March

- b. Total amount of money she will collect from the sales of the plots in December

ITEM 67

On his return from Japan, Moses came with a special and beautiful design of a rectangular foundation of a building known as golden design using a special ratio known as the golden ratio. The golden ratio = Perimeter divided by twice the length.

He wishes to design his rectangular foundation of width 24m using this design and is to cover it with smaller golden rectangular tiles each of width $\frac{\sqrt{5}+1}{2}$ m. He never got the ratio, but he was given a sample golden rectangle of dimensions $\frac{\sqrt{5}+1}{2}$ cm by 1cm to calculate the ratio.

Task

As an architect help Moses to determine the:

- Golden ratio in its simplest form
- Exact length of (i) the foundation (ii) each tile, in the simplest forms
- Number of tiles to be used to cover the foundation

ITEM 68

Kampala City Council is planning to construct a new recreational park. The park will be designed as a coordinate system where the main entrance is at the origin O(0,0). Three key facilities are to be located at points A(6,8), B(-4,12), and C(10,-6) representing a restaurant, playground, and parking lot respectively.

The council wants to construct a circular fountain such that it passes through all three facilities. Additionally, they plan to build straight walkways connecting these facilities and want to ensure the walkways meet safety standards for wheelchair accessibility.

Task

Help the Kampala City Council to:

- a. Find the equation of the circle passing through points A, B, and C
- b. Determine the center and radius of the fountain
- c. Calculate the gradient of the line AB and determine if it meets the wheelchair accessibility standard (gradient should not exceed 0.5 in absolute value)
- d. Find the equation of the perpendicular bisector of line segment BC

ITEM 69

A farmer in Nakasongola district wishes to enclose his rectangular piece of land of area 1250cm^2 for goat rearing. One side of the land is bounded by a straight bank of river. He bought this piece of land from another farmer who told him that the distance around the land is 100m. The farmer is planning to buy a barbed wire which is going to be fenced around the land excluding the side bounded by the river. He needs to know the dimensions of the land to help him partition the barbed wire.

Task

Help the farmer to obtain the dimensions of the piece of land to be fenced.

ITEM 70

Two learners A and B were arguing about the size of side of a table. Learner A was saying after measuring the answer was $\cos 15^\circ$ in trigonometric form while learner B was saying it was $\sin 75^\circ$. The teacher got involved into their argument and he told them those two answers are the same. The two learners got worried about the teacher's response.

Task;

As a learner who has studied trigonometry, help the learner's to show that those two answers are equivalent without using a calculator or tables.

ITEM 71

Your teacher presents a challenging problem of determining the vertical height of the flagpole and the horizontal distance between two students, Mia and Sam, who are looking up at the top of the school flagpole. Mia is 1.5 meters tall and stands 10 meters from the base of the flagpole, measuring an angle of elevation of 35° to the top with a clinometer. Sam stands 15 meters away, measuring an angle of elevation of 25° ; however, his height is unknown, although he is taller than Mia. They are on the same side of the flagpole, in a straight line from its base, indicating that Sam is 5 meters farther from the pole than Mia. For safety reasons, the school plans to install a safety net under the flagpole to catch the flag if it falls during the daily flag-raising ceremony, ensuring student safety while maintaining a clean yard. The cost of the net is UGX 50,000 for each meter of the flagpole's height. Both students are on flat ground.

Task:

Help the teacher calculate the cost of the safety net and determine Sam's height.

ITEM 72

It was Sports Day at your school, where students cheered from the sidelines as athletes ran on the field. Three students, Amara, Ben, and Chloe, members of the entrepreneurial club, set up a snack stall near the sidelines to sell to the cheering crowd. Amara made chapatis, Ben made samosas, and Chloe made mandazis. They competed to see who could sell the most and earn the highest sales. However, the busy day created a challenge when they attempted to record only the total sales in their notebooks, instead of the price of each item. Their first sale was to the cheering squad: six chapatis, ten samosas, and three mandazis for UGX 10,500. At midday, the second sale was to the teachers: two chapatis, eight samosas, and seven mandazis for UGX 7,900. Later, the third sale of the day was to the football team: five chapatis, four samosas, and four mandazis for UGX 8,200. When Sports Day ended, Amara, Ben, and Chloe sat with their notebooks. They wanted to find out the prices of one chapati, one samosa, and one mandazi to see who won the competition and plan for the stock for the next day.

Task:

Who won the competition based on sales amount?

ITEM 73

Onyumel runs a maize flour business near Jinja Market. To minimize stock costs, he cultivates maize on a farm close to the Source of the Nile. His initial rectangular plot measures 120 square meters, which is sufficient for his needs, featuring a length that is 5 meters longer than the width, allowing for orderly rows. Each square meter produces 2 kg of maize per season. Seeds are priced at UGX 5,000 per kilogram, with 1 kilogram covering an area of 10 square meters. Fencing costs UGX 1,500 per meter, and he has allocated a budget of UGX 90,000 to keep animals at bay. As demand increases, he plans to expand the land area to 150 m², maintaining a length of 5 meters and a width greater than 5 meters. He considers the implications if fencing prices rise to UGX 2,000 per meter.

Task:

- (a) How many kilograms of maize are planted in the expanded plot?
- (b) What is the difference in the cost of fencing the expanded plot and the first plot?

ITEM 74

In an experiment carried out by senior four students during the chemistry practical lesson in your school to determine how the rate of reaction is affected by concentration, the results obtained were described by the mathematical model below. Experiment model:

$y = x^2 + 3x + 2$, where y is the dependent variable while x is the independent variable of the above experiment.

There is a need to determine the general gradient of the curve obtained such that the tangent lines at $P_1 (-2,3)$ and $P_2 (3,5)$ can easily be defined by their equations.

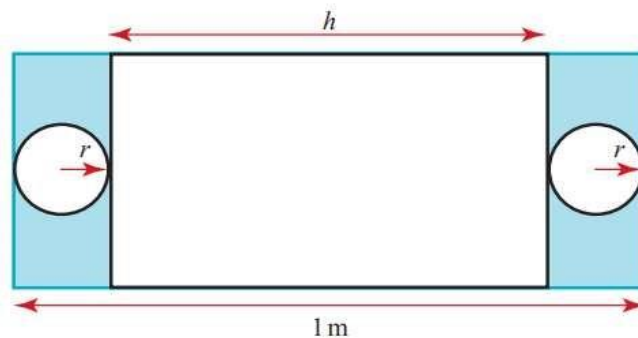
Task:

- a) Help them to obtain the general gradient of the curve obtained and describe the nature of the point where the curve turns.

- b. Given that P_1 and P_2 lie on the curve, and the equations of the tangents at P_1 and P_2 intersect at T . Then describe T and the angle formed by these tangents at their point of intersection based on mathematical calculations.

ITEM 75

The diagram shows the net of a cylindrical container of radius r cm and height h cm. The full width of the metal sheet from which the container is made is 1 m, and the shaded area is waste. The surface area of the container is 1400π cm².



Task:

- i. Write down a pair of simultaneous equations for r and h .
- ii. What are the two possible volumes the container formed can be collect.

ITEM 76

You're a bright and ambitious student from Kampala, Uganda, and you've been invited to a scholarship interview at a prestigious school in the city. The school is renowned for its academic excellence.

As you sit down for the interview, you're greeted by a panel of esteemed teachers, including the head of the mathematics department. They smile warmly and introduce themselves, explaining that the interview will assess your mathematical skills and problem-solving abilities.

The head of the mathematics department hands you a sheet of paper with a challenging problem:

To resolve

$$f(x) = \frac{x^3 + 2x^2 + 61}{(x + 3)^2(x^2 + 4)}$$

Into splitted fractions and ascertain the area covered in the range of 1 to 3.

Task.

Present your response to the panel in not more than 30minutes.

ITEM 77

A biologist is studying the relationship between the population of two bacterial species X and Y, in a controlled environment. The population of this bacteria, denoted as x and y follows the logarithmic relationships

1. The growth rate equation for species X

$$\log_4(6 - x) = \log_2 x$$

2. The interaction equation between species X and Y: $\log_{10} x -$

$$\log_{10} y = \log_{25} 10 \quad \text{and} \quad \log_{10} x + \log_{10} y = 1$$

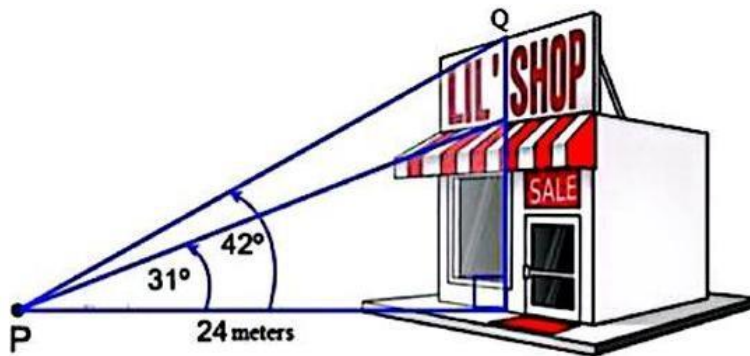
The biologist wants to determine the population values x and y that satisfy these equations

TASK:

Help the biologist to get the value of x and y by forming simultaneous equations

ITEM 78

Simon bought a new shop and wants to order a new sign post for the roof of the building and wanted to know its height. From point P, he finds the angle of elevation of the roof, from ground level, to be 31° and the angle of elevation of the top of the sign post to be 42° . Point P is 24 meters from the building. The design would look as that shown in the diagram below.



Simon was told by the neighbors that the best angle, θ of elevation from P of the top of the sign post could best be determined by solving the equation:

$$14 \tan 2\theta + 2 = 16 \sec \theta. \quad \text{Hint: } 0^\circ \leq \theta \leq 360^\circ$$

Task:

Use the understanding you have acquired in trigonometry to help Simon:

- a. Know the height of the sign post to be made.
- b. Solve for the possible values of θ and suggest if they can provide the best elevation for the desired height of at least 16meters of the sign post.

ITEM 79

A health practitioner wants to study about the growth of certain bacteria and find out how their numbers vary with time. A colony of bacteria is grown in a dish starting from 3:00AM in the laboratory and it is found that their population doubles each hour that passes. He managed to get the machine that counts their numbers at 9:00AM, when he realized that at that time, the population was one million.

TASK:

Help the health practitioner to determine the;

- a) Population of the bacterial at;
 - i. Start of the growth
 - ii. 12:00 noon
 - iii. 11:30AM
 - iv. 7:00PM
 - v. 8:15AM
- b) Time at which the population was:
 - i) 32,000,000
 - ii) About
353553

ITEM 80

"Abbo lost her phone after being stolen by thugs on Her Way Home. She decided to go to police so that Her phone can be tracked using GPS co-ordinates. The phone's location at different times is recorded and the police was trying to find its movement pattern. The tracking device uses the logical operating system that uses simultaneous equations given by;"

$$3^{2x-3y} = 243 \text{ and } \log_y x = 2$$

When they got close to the phone, they could not tell the co-ordinates to easily locate the phone. The co-ordinates are in metres and one point gives the position of those with the tracking device while the other point gives the location of the phone.

TASK

Help the police to;

- a) Determine the co-ordinates of where they were and the stolen phone
- b) Know how far they were from the stolen phone.

ITEM 81

In Kampala's fast growing suburbs, a city tycoon deposits UGX 1500,000 into a business account that compounds annually at 8% per year, hoping to fund a modest dream house; he believes that if the account value after 10 years is above UGX 25000,000, he will proceed with construction, but if it falls short, he will roll the entire balance into the same investment for two additional years before revisiting the plan.

At the same time, he wants to know how long it would take this investment at the same 8% compound rate to reach UGX 32,400,000, and if the required time is more than 18 years, he will abandon that target and switch to a 12-year saving plan instead.

Task:

- i. Determine the accumulated amount after 10 years and decide whether it is enough to buy the dream house or the tycoon should extend for two more years.
- ii. Find the time, correct to the nearest year, required for the investment to exceed UGX 32400000 and state clearly whether he should keep the target or switch to the 12 year plan

ITEM 82

Alice, a farmer in Mbale district is planning to plant sorghum in parallel lines. The first proposed line is to pass through the earth coordinates (10, 50) and (30, 30), while the second line is to pass through the earth coordinates (15, 60) and (25, 25). However, she is not sure whether these lines are actually parallel to each other. To ensure access to sunlight a reasonable distance between these lines is needed. The minimum distance needed between them is 100cm, but Alice does not know if the given coordinates do not meet this requirement.

Task

Help Alice find

- a) The equations of the first and second lines. Convince Alice that these lines are surely parallel to each other.
- b) Whether Alice should increase the distance between these lines.

ITEM 83

A textile manufacturing company has just bought a new technological system that produces curtains. The system was set to produce only the number of curtains (repeated) in the first and second run in a single command to ensure uniform production. The system operator commanded the machine to produce blue and black curtains and the system displayed an operation in form of an equation $\frac{x^2-x+1}{x-1} = K$ where x represents the number of blue curtains and k represents the number of black curtains. The system operator wants to know the number of blue and black curtains that would be produced following the command.

Task;

Help the system operator know how many blue and black curtains that are most likely to be produced by the command.

ITEM 84

Miriam accidentally left the tap not properly closed at **8: 00am**. She has found out that water has been leaking since then. It is known that if the tap starts leaking, loses **1** litre, **1. 5 litres** and **2 litres** of water in the **first,d** and **third minutes** respectively and so on. She wants to know the total volume of water lost by **11: 00am** when realizes this. However, she does not have the mathematical knowledge to finish this task quickly.

Task

Help Miriam to know the total volume of water lost by the tap.

ITEM 85

A pharmaceutical company produces tablets for treating a contagious disease. The company tracks expenses using the function $E = \frac{\sin 8x \cos x - \sin 6x \cos 3x}{\cos 2x \cos x - \sin 3x \sin 4x}$ it also tracks profits using the function $P = \tan 2X$ they are to adopt a better production technology when the expenses (E) are equal to the company profits (P).

Task

Basing on mathematical evidence, find out whether the company should adopt the new production technology.

ITEM 86

Mutebi a boda-boda rider in Kampala wants to buy land that is expected to be worth **shs2 million** at the end of the year 2025. He has decided to start saving **shs2**, on day 1, increasing the amount by **shs 500** every day. However, he is not sure whether he will be ready to buy this land by the end of **160 days**. He wants to tighten his spending on food if by this time; he will not be able to fulfill his goal.

Task

Help Mutebi find

- (a) Whether his savings will be ready by the stated time.
- (b) Number of days needed to raise **shs 12 million**.

ITEM 87

The area of Kigezi is infested with tsetse flies and the government is planning to send drones to take aerial photographs in the area, since it is not safe to use vehicles. The drones take clear photographs if the argument of the flight path remains within 0^0 and 160^0 inclusive. Two drones are sent and take different flight paths represented by the expressions

Drone 1; $Z_1 = 4 + .$

Drone 2; $Z_2 = -3 + 3\sqrt{3} i.$

The control panel wants to know the modulus and argument of each path for the drones, and to ensure whether drones will take clear photographs.

Task

Help the panel members complete the task.

ITEM 88

An alternating current (A.C) circuit has an impedance $Z = \frac{(2+3i)^2(1-i)^2}{1-5i}$ and a current $= \frac{2i}{3+4i}$. Engineers are required to obtain the voltage V across the circuit using Ohm's law, $V = IZ$, and the magnitude of this voltage. The current size supplied has to be increased if the voltage size is less than 10.

Task.

Finish the task, and find out whether the current supplied should be increased.

ITEM 89

You have participated in an annual math contest in which all schools within your district have to take part by sending a single representative per school. In the group stages, each participant randomly chooses three questions to answer from the box, selecting a question after the other. Qualified candidates move to the semi-finals, and at this stage, all competitors answer two similar questions. The best two qualify for the final round which declares the winner of the contest.

Task

(a) Suppose you qualified for the semi-finals, through picking and answering the questions below

- i) Given that $\cos 2A - \cos 2B = -p$ and $\sin 2A - \sin 2B = q$, prove that $\sec(A + B) = \frac{1}{q} \sqrt{p^2 + q^2}$
- ii) Show that $\tan\left(\frac{\pi}{4} + x\right) - \tan\left(\frac{\pi}{4} - x\right) = 2 \tan 2x$
- iii) Prove that in any triangle ABC, $\frac{b^2 - a^2}{a^2} = \frac{\sin(B-C)}{\sin(B+C)}$

ITEM 90

In an ecological study focused on movement of new insect species, insects have been identified to be moving following the patterns $24 \operatorname{cosec} \theta = \frac{56}{x^2}$ and $\frac{2}{3}y + 42 \sin \theta = 25$. The model for the new study program to be successful is based on notation of verifying that $\frac{1+\cos x}{\sin x} + \frac{\sin x}{1+\cos x} = 2 \operatorname{cosec} x$.

Task

- (a) Show that the Cartesian equation of the motion of the insect is given by $y = \frac{1}{2}(75 - 54x^2)$
- (b) Verify that the study program is a success.

ITEM 91

A community council is organizing a function at the ssaza playground. The playground has dimensions $(12 + 5i)$ metres by $(9 - \sqrt{19}i)$ metres. They are planning to use tents which they are to hire from a private organization. Each tent occupies an area of $13m^2$, and the company charges *shs 15*, per tent per day. However, the council is not sure about how many tents are to be used and the total expenditure on the tents.

Task

Help the community council to determine the;

- Total area of the playground to be used.
- Number of tents needed for the function.
- Total expenditure on the tents to be used.

ITEM 92

Akello is preparing for the new school term and plans to visit a bookshop to buy: 3 books, 6 pens and 1 mathematical set She has saved up 10,000 shillings for this shopping trip. Before going, she consulted three of her friends who had recently bought school materials from the same bookshop. They shared their purchase details as follows:

- Joan bought 1 book, 1 pen, and 1 set and paid 5,500 shillings.
- Mary bought 3 books, 2 pens, and 2 sets and paid 13,000 shillings.
- Tom bought 5 books, 4 pens, and 2 sets and paid 18,000 shillings.

Akello wants to go to the same bookshop but isn't sure if her 10,000 shillings will be enough.

Task:

Conclude whether her budget of is enough for the 3 books, 6 pens, and 1 set

ITEM 93

A farmer in a certain village normally uses different routes to reach his maize plantation in order to confuse the thieves that always come to steal his maize after learning about his movement. The different routes that he followed is

described by the equation $y = (x + 1)^{\frac{1}{2}}(x + 2)^2$. He was to get the gradient of his path so that he can have a simple equation than the one above, however, he did not know how to get it. Always when he is moving to his garden, he puts on his right circular cone hat whose base radius increases and the volume changes by 2% but the height of the cone hats remain constant. The farmer wanted to know the percentage increase in the circumference of the base but did not know how to get and looking for help.

Task

As a student of mathematics and it happens that this farmer is your relative, help him with all the necessary information that he needs.

ITEM 94

In a bicycle riding competition from one of the villages in west Nile, some young men were to ride from one trading center to another 40km apart. The path to be taken is described by the equation $y = x^2 - 4x + 6$. For them to move smoothly without any interference, they are to pass through the line segment $y = 3$, cut from the curve above. The starter task them to find the area of the segment cut off from the curve by the line above but they did not know to do it. Similarly, one of them fell and rotated enclosing the area given by the curve $y = x^4$ from $y = 3$ to $y = 6$ when it is rotated about $x = 0$ and that one of the participants is to help find the volume formed when the man fell and rotated.

Task

As a student of principle mathematics kindly help in the two situations above so that the challenges being faced by the participants can be put to rest.

ITEM 95

It was discovered that for x to be equals to 2, it must satisfy the equation $\log_x 8 - \log_{x^2} 16 = 1$ and the student were to approve the discovery or disapprove it. The sum of p terms of arithmetic progression is q and the sum of q terms is p , the students were to find the sum of $p+q$ terms but did not know exactly what to do with the letters p and q to help them come out of the situations.

Task

Help to provide the necessary information for students.

ITEM 96

Senior five students of a certain school during their mid- year orientation, were to solve the simultaneous equation $2^x + 4^y = 12$ and $3(2^x) - 2(2^{2y}) = 16$ and thereafter to show that $4^x + 4(3^{2y}) = 100$ as part of their pre assessment into the combinations they were doing. In addition to that, they were told that α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$ and that they were to determine the equation whose roots are $\alpha + \beta$ and $\alpha^3 + \beta^3$. These two scenarios put the mathematics students in a total confusion thinking of changing their combination but was already late. In the process of handling the task above, they were given 30 minutes to consult any other students who may be of help. One of them is your brother and aware that you are good at mathematics and came with his colleagues for help.

Task

As a student of principal mathematics, help your brother and the colleagues out of the confusion.

ITEM 97

During division of polynomials, it was found out that a polynomial $p(x)$ leaves remainders 7 and 1 when divided by $(x-2)$ and $(x+1)$ respectively. Find the remainder when $p(x)$ is divided by $x^2 - x - 2$.

ITEM 98

A student who missed the lesson on differentiation has been given an assignment he is unaware of and has asked you to help him. The question is as below. Differentiate $\tan x$ with respect to x from first principles.

Task:

Help to show this student that the answer is $\sec^2 x$.

ITEM 99

A school disciplinary committee is to be set up comprise of 8 students. The school has chosen 6 girls and 8 boys who are well disciplined from which they have to form the committee from. The Head of Disciplinary Committee he prefers to have either 3 girls or 5 girls on the committee formed

Task:

Help the Head of DC determine the possible number of ways of achieving such from those available.

ITEM 100

During a market research, the model $\sqrt{x + 6} + \sqrt{4 - x} = \sqrt{1 - 3x}$ has been developed for the first new product X put forward by the firm where x represent overall consumer reaction towards the new product.

The company analyst has been tasked with check to see the verified output if it's a negative then the product should not be sold to the public. Unfortunately, the analyst is too sick and asked you the assistant to help out but forgot to give the password to his computer. But left a clue word “ANALYSIS256” to access the three different user's accounts he has.

Task:

- a) Help determine whether the new product should be rolled out for sell or not by the company. Hint: verify your answer.
- b) In how many possible arrangements in a row are there for password to his accounts given that for:
 - i. User account 1, there are no restrictions
 - ii. User account 2, password must start with a number
 - iii. User account 3, “Y” and “5” must start with be together