EER: BINOMIAL DISTRIBUTION

1. *A student answers* ***12*** *questions. The chance of getting each question correct is* $\frac{1}{3}$. *Find the probability of obtaining:*

(i) *exactly 7 correct answers.*
(ii) *at least* ***2*** *correct answers.*
(iii) *between* ***3*** *and* ***6*** *incorrect answers.*

[Ans: (i) 0·0477 (ii) 0·9461 (iii) 0·0626]

2*. A random variable* $X \sim Bin(12, 0{\cdot}3)$, *find:*

(i) $P(X \leq 4)$

(ii) $P(2 < X < 8)$
(iii) *the mode of* ***X***

[Ans: (i) 0·7237 (ii) 0·7377 (iii) 3]

3. *The probability of germination of a bean seed is* ***0·45.*** *If* ***10*** *seeds are planted, find the:*

(i) *most likely number of seeds to germinate*
(ii) *chance that less than* ***5*** *seeds will not germinate.*

[Ans: (i) 4 (ii) 0·2616]

4. *A family has* ***5*** *children. The probability of having a boy is* ***0·6.*** *Find the:*

(i) *expected number of girls*
(ii) *probability of having more girls than boys*
(iii) *probability that they are all boys*

[Ans: (i) 2 (ii) 0·3174 (ii) 0·0778]

5. *A biased coin is twice as likely to show heads as tails. If it is tossed* ***5*** *times, find the probability of obtaining:*

(i) *exactly* ***3*** *heads*
(ii) *more heads than tails*
(iii) *not more than* ***4*** *tails*

[Ans: (i) 0·3292 (ii) 0·7901 (iii) 0·9588]

6. *A test has* ***10*** *multiple choice questions each with* ***5*** *suggested answers of which only one is correct. If a student answers the questions randomly, find the probability of passing:*

(i) *exactly* ***3*** *questions.*
(ii) *at most* ***4*** *questions.*

(iii) *more than half of the questions .*

[Ans: (i) 0·2013 (ii) 0·9672 (iii) 0·0064]

7. *Bob played* ***12*** *chess games. The probability that he wins a game is* ***0·75.*** *Find the probability that he will win:*

(i) *exactly* ***8*** *games*
(ii) *at most* ***10*** *games*
(iii) *more than half of the games*

[Ans: (i) 0·1936 (ii) 0·8416 (iii) 0·9456]

8. *A fair coin is tossed* ***10*** *times. Find the probability of obtaining:*

(i) *exactly* ***5*** *heads*
(ii) *at least* ***8*** *heads*
(iii) *at most* ***6*** *heads* ***[Ans: (i) 0·2461 (ii) 0·0547 (iii) 0·8281]***

9. *The probability that a patient recovers from a rare disease is* ***0·4.*** *If* ***6*** *people are infected, find the probability that:*

(i) *exactly* ***3*** *survive*
(ii) *at most* ***2*** *survive*
(iii) *more than half of them survive*

[Ans: (i) 0·2765 (ii) 0·5443 (iii) 0·1792]

10. *A shop sells eggs which are* ***30%*** *rotten. If a packet of one dozen of eggs is sold, find the:*

(i) *expected number of rotten eggs*
(ii) *standard deviation of the number rotten eggs*
(iii) *most likely number of rotten eggs*
(iv) *probability that the packet contains more than* ***4*** *rotten eggs*

[Ans: (i) 3·6 (ii) 1·5875 (iii) 3 (iii) 0·2763]

11. *The chance of hitting a target with a single shot is* ***0·3.***

(a) *How many shots would be fired to be more than* ***95%*** *confident that at least one of them hits the target.*
(b) *If* ***12*** *shots are fired, find the probability that:*
(i) *between* ***2*** *and* ***8*** *shots hit the target*
(ii) *at least* ***8*** *shots miss the target*

[Ans: (a) 9 (b) (i) 0·7377 (ii) 0·7237]

12. *A random variable* ***X*** *is binomially distributed with mean* ***4·8*** *and variance* ***2·88.*** *Find* $P(X < 6)$ ***[Ans: 0·6652]***

13. *A fair dice is rolled* ***5*** *times. Find the probability of obtaining:*

(i) *a* ***2*** *or a* ***4*** *on the first throw*

(ii) *exactly* ***2*** *sixes*

(iii) *at least* ***3*** *sixes* ***[Ans: (i)*** $\frac{1}{3}$ ***(ii) 0·1608 (ii) 0·0354]***

14. *The mean of a binomial distribution is* ***3*** *and standard deviation* ***1·5.*** *Find the:*

(i) *number of trials and probability of success*

(ii) *probability of obtaining at most* ***5*** *successes*

(iii) *probability of the most likely number of successes*

[Ans: (i) 0·25, 12 (ii) 0·9456 (iii) 0·2581]

15. *A random variable* $X \sim Bin(4, p)$.

(i) *Given that* $P(X = 0) = 0{\cdot}4096$, *find the value of* ***P***

(ii) *Given that the standard deviation of* ***X*** *is* ***0·96,*** *find the possible values of* ***P***

[Ans: (i) 0·2 (ii) 0·36, 0·64]

16. *The chance that a student takes Arabic is* ***0·3.*** *Find the least number of students required to be selected in order to be more than* ***95%*** *confident that at least one of them takes Arabic.*

[Ans: 9]

17. *Given that* $X \sim Bin(n, 0{\cdot}2)$, *find the least value of* ***n*** *such that* $P(X \geq 1) > 0{\cdot}87$

[Ans: 10]

18. *A random variable* $X \sim Bin(10, p)$. *Given that the variance of* ***X*** *is* ***1·875*** *and* $p \leq 0{\cdot}5$, *find:*

(i) *the value of* ***p*** ***(ii) E(X)*** ***(iii) P(X < 2)***

[Ans: (i) 0·25 (ii) 2·5 (iii) 0·2440]

19. *Tom played* ***5*** *tennis games. The probability that he wins a game is* ***p.*** *If the probability that he wins exactly four times is twice the probability that he wins exactly two times, find the value of* ***p.***

[Ans: $\frac{2}{3}$ ***]***

20. *In two binomial distributions, the ratio of the number of independent trials is* ***5:6****, the ratio of their mean is* ***2:9*** *and the ratio of their variance is* ***32:45.*** *Find the probability of success for each distribution.*

[Ans: 0·2, 0·75]

21. *A machine produces bolts which are* ***20%*** *defective. Find the probability that in a sample of* ***11*** *bolts produced,*

(i) *at least* ***5*** *are defective*

(ii) *between* ***2*** *and 7 are defective*

[Ans: (i) 0·0504 (ii) 0·3806]

22. *A biased coin is thrice as likely to show heads as tails. If it is tossed* ***9*** *times, find the:*

(i) *expected number of tails.*

(ii) *most likely number of heads.*

(iii) *probability of getting at least* ***6*** *heads.*

[Ans: (i) 2·25 (ii) 7 (ii) 0·8343]

23. *A test has* ***10*** *multiple choice questions each with* ***4*** *suggested answers of which only one is correct. If a student answers the questions randomly, find the probability of:*

(i) *passing at least* ***4*** *questions.*

(ii) *the most likely number of correct answers* ***[Ans: (i) 0·2241 (ii) 0·2816]***

24*. A random variable* $X \sim Bin(11, 0{\cdot}75)$, *find*

(i) $P(X \leq 6)$

(ii) *the mode of* ***X***

[Ans: (i) 0·1146 (ii) 8 or 9]

25. *The chances of a plant to produce pink, red and white flowers are in the ratio* ***1:9:2.*** *Find the probability that in a sample of* ***8*** *plants:*

(i) *at least* ***5*** *produce red flowers.*

(ii) *at least* ***3*** *but less than* ***5*** *produce white flowers*

[Ans: (i) 0·8862 (ii) 0·1302]

26. *The probability that it will be a fine day is* ***0·4.*** *Find the probability that in a given week there are:*

(i) *exactly* ***2*** *fine days.*

(ii) *at least* ***3*** *fine days.*

(iii) *at most* ***4*** *fine days.*

[Ans: (i) 0·2613 (ii) 0·5801 (iii) 0·9037]

27. *In a potato garden, the average number of rotten potatoes was* ***3*** *with a standard deviation of* ***1·5*** *potatoes. Find the chance that the garden contains:*

(i) *exactly* ***4*** *rotten potatoes.*

(ii) *at most* ***5*** *rotten potatoes.*

(iii) *at least* ***2*** *but less than 7 rotten potatoes.*

[Ans: (i) 0·1936 (ii) 0·9456 (iii) 0·8273]

28. *A random variable* $X \sim Bin(4, p)$. *Given that the variance of* ***X*** *is* ***0·84,*** *find the possible values of* ***p*** ***[Ans: 0·3, 0·7]***

29. *One in every five drivers on the road is drunk. If* ***8*** *drivers are tested, find the:*

(i) *most likely number of drunk drivers*

(ii) *probability that at least* ***2*** *but less than* ***6*** *are drunk*

[Ans: (i) 1 (ii) 0·4955]

30. *The chances of a person to have blood group* ***O, A, B*** *and* ***AB*** *are in the ratio* ***2:15:2:1*** *respectively. If* ***6*** *people donated blood, find the probability that:*

(i) *all have blood group* ***A***
(ii) *non has blood group* ***A***
(iii) *at least* ***4*** *have blood group* ***A***

[Ans: (i) 0·1780 (ii) 0·0002 (ii) 0·8306]

31. (a) *A pair of dice is tossed* ***8*** *times, find the probability of getting a sum of* ***5:***
(i) *exactly* ***4*** *times*
(ii) *at least* ***3*** *times.*
(iii) *between* ***2*** *and* ***4*** *inclusive times*
(b) *How many tosses would be required if the probability of getting a sum of* ***5*** *at least once is greater than* ***0·7.***

[Ans: a(i) 0·0067 (ii) 0·0501 (ii) 0·2198 (b) 11]

32. *The chance of hitting a target with a single shot is* ***0·2.*** *Find the least number of shots required to be fired, if the probability that the target is hit at least once is greater than* ***0·95.***

[Ans: 14]

33. *One in every five students in the class gossips. How many students would be selected so that the probability that none of them gossips is less than* ***0·007***

[Ans: 23]

35. *An experiment consists of picking two sweets successively without replacement from a box containing* ***3*** *red and* ***4*** *blue sweets.*
(i) *If* ***H*** *is the event that both sweets picked are of the same colour, find the probability that event* ***H*** *occurred.*
(ii) *If the experiment is repeated* ***6*** *times, find the probability that event* ***H*** *occurred between* ***2*** *and* ***5*** *times*

[Ans: (i) $\frac{3}{7}$ ***(ii) 0·4590]***

36. *The chance that a student takes Arabic but not French is* $\frac{1}{12}$. *If a student takes Arabic, the chance that he takes French is* ***0·9.*** *Find the probability that in a sample of* ***12*** *students chosen at random between* ***3*** *and* ***10*** *of them take both Arabic and French.*

[Ans: 0·6089]

37. *In a family of* ***5*** *children, it is twice as likely to have boys as girls. Find the:*
(i) *probability distribution for the number of boys in the family*
(ii) *expected value and standard deviation of the distribution in* ***(i)*** *above*
(iii) *probability of having at least* ***2*** *but less than* ***4*** *boys.*
(iv) *mostly likely number of boys in the family.*

[Ans: (ii) $\frac{10}{3}$, ***1·0541 (iii)*** $\frac{40}{81}$ ***(iv) 3 or 4]***

38. *Four competitors throw a fair dice in turn. Find the probability that:*
(i) *they all score more than a* ***4***
(ii) *two of them score less than a* ***3***
(iii) *the total score is* ***23***

[Ans: (i) $\frac{1}{81}$ ***(ii)*** $\frac{8}{27}$ ***(iii)*** $\frac{1}{324}$ ***]***

39. *The probability of a person being left-handed is* ***20%.***
(a) *How many people would be selected to be more than* ***95%*** *certain that at least one of them is left-handed*
(b) *Find the probability that in a sample of* ***10*** *people:*
(i) *exactly* ***3*** *are left-handed.*
(ii) *more than half of them are left-handed.*

[Ans: (a) 14 b(i) 0·2013 (ii) 0·0064]

40. *A fair dice is thrown* ***4*** *times and the number of sixes* ***X*** *thrown were recorded as follows:*

x	0	1	2	3	4
$P(X = x)$			$\frac{25}{216}$		$\frac{1}{1296}$

(a) *Copy and complete the above table*
(b) *find:*
(i) *the mode and mean of* ***X***
(ii) $P(X \geq 2)$

[Ans: b(i) 0, $\frac{2}{3}$ ***(ii)*** $\frac{425}{432}$ ***]***

41. *A box contains* ***8*** *red and* ***10*** *blue beads. Six beads are drawn at random from it with replacement. Find the probability of picking:*
(i) *beads of the same colour*
(ii) *exactly four red beads*

[Ans: (i) 0·0371 (ii) 0·1806]