

## SECTION A: (NUMERICAL METHODS)

(Compulsory)

### ITEM I

Pearl Construction Company was awarded a contract of constructing valley dams in the semi-arid regions of Uganda. Before construction work began, a team of surveyors started surveying sites for the dams to see the possible spots where these dams could be located and possible their surface areas.

They discovered a serious important mineral 'Gold' and picked interest of whether it covers major spots they had surveyed. They decided to report the news to Uganda mining board who used drones with E-cameras to cover the Gold sites and produced images showing that the mineral is situated along the path  $f(x) = x^3 - 6x + 1$  but were not sure the exact location but the range was  $0.1 \leq x \leq 1$ , where  $x$  is in kilometres. The board approached the department of mathematics to one of the prestigious universities and among their requests include;

Confirmation that indeed Gold exists between  $x = 0.1km$  and  $1Km$ .

How to get the very approximate value of  $x$  close to the Gold site once.

How they can get the exact spot(s) where Gold could be.

In their pursuit of the sight of Gold, the computer system generated two iterative formula(e) where one is believed to give a convergent solution to the Gold position and this formula was to be checked using the Newton-Raphson iterative formula twice starting with  $x = 0.2$  as the close value to Gold mine.

To estimate the areas of the valley dams and see if they would interfere with the Gold mines.

Where by the model  $f(x) = \left(\frac{2x+1}{x^2+x}\right)$  represents the required curved surfaces designed for the dams,  $x$  is in kilometres, and  $0.2 \leq x \leq 1.0$ . The two iterative formulae were;

$$x_{n+1} = \sqrt{6x_n - 1} \text{ ----- 1}$$

and  $x_{n+1} = \frac{1}{6} (x_n^3 + 1) \text{ ----- 2}$  giving answers to 3 decimal places.

### Tasks

The department of mathematics has approached you, help them to;

Confirm that Gold mine is between  $x=1.0$  and  $x=0.1km$

Identify which iterative formula is appropriate. Confirm it using the Newton – Raphson iterative formula.

Use 5 (five) ordinates and estimate for them the area of valley dams, which should not exceed  $1.546km^2$ , otherwise Gold mines would be encroached. Help them to know whether they should continue with dam's construction or not.

**SECTION B:**  
**PART I: PROBABILITY AND STATISTICS**  
*(Attempt only one Item.)*

**ITEM 2**

Uganda is facing a significant challenge of Malaria which kills many people especially the children. In an effort to reduce the incidence and severity of malaria cases, a project is being undertaken to produce a special syrup that contains 30% zinc, 50% water, 8% iron and the rest calcium so that the immune function in children bodies is supported. The team responsible has discovered that once a child takes the syrup the nutritional contents provide different immune supports with proportions of 15% for zinc, 1% for water, 10% for iron and 3% for calcium. The total cost of making the syrup went to 5 dollars but research shows that the target population cannot afford and therefore the government of Uganda has provided the following conditions to the syrup manufacturers to first meet them and later take charge of giving syrup to its population freely.

- If a sample of 15 people is taken and probability of a person healing after taking syrup is 0.4 such that the probability that;
  - (i) More than 9 will recover is atleast 0.033,
  - (ii) Between 5 and eight inclusive is atleast 0.383,
- If a sample of 100 people is taken and probability of a person healing after taking the syrup is 0.6 such that the probability that 68 or more people heal is atleast 0.062.

**Task**

You are approached as a mathematician,

- a) Present the probability that the immune system will improve after taking the syrup.
- b) If a patient's immunity is sustained after taking the syrup, present the chance that it was basically because of the presence of zinc.
- c) Show whether the government should take control of the distribution of syrup or not.

**ITEM 3**

Kacwekano Agricultural Centre is having plans to recruit local farmers from the surrounding areas to reduce on the rate of unemployment and improve on the livelihoods of people. However it is facing a challenge of the selection criteria considering that some communities have short people 'BATWA' and it was very hard to predict the age (years) of someone against his height (cm), yet one of the conditions on selection criteria is to recruit majority of the youth in the age bracket of 28-35 years and few adults of about 38 years who would be like role models to the youth.

The recruiting panel came up with two options through their statistical research from selected ten (10) families.

**PART II**  
**MECHANICS**  
(Attempt only one item)

**ITEM 4**

Kipro is training for the upcoming Olympic Games that are to take place in Tokyo 2026. Complaints have always been presented that the poor performance in cycling sport is due to strong winds in the training tracks in Uganda yet in Tokyo winds are not so strong and hence fail to compete favorably. A research about the speeds of wind is being conducted through the training whereby Kipro's speed is considered as follows;

When he rides due south at  $20\text{kmh}^{-1}$ , the winds appear to be blowing in the direction  $240^\circ$ . When he reduces his speed to  $15\text{kmh}^{-1}$  but without changing the direction the winds appears to blow in the direction  $210^\circ$ . The association of riders failed to know exactly where the winds are blowing and at what speed actually.

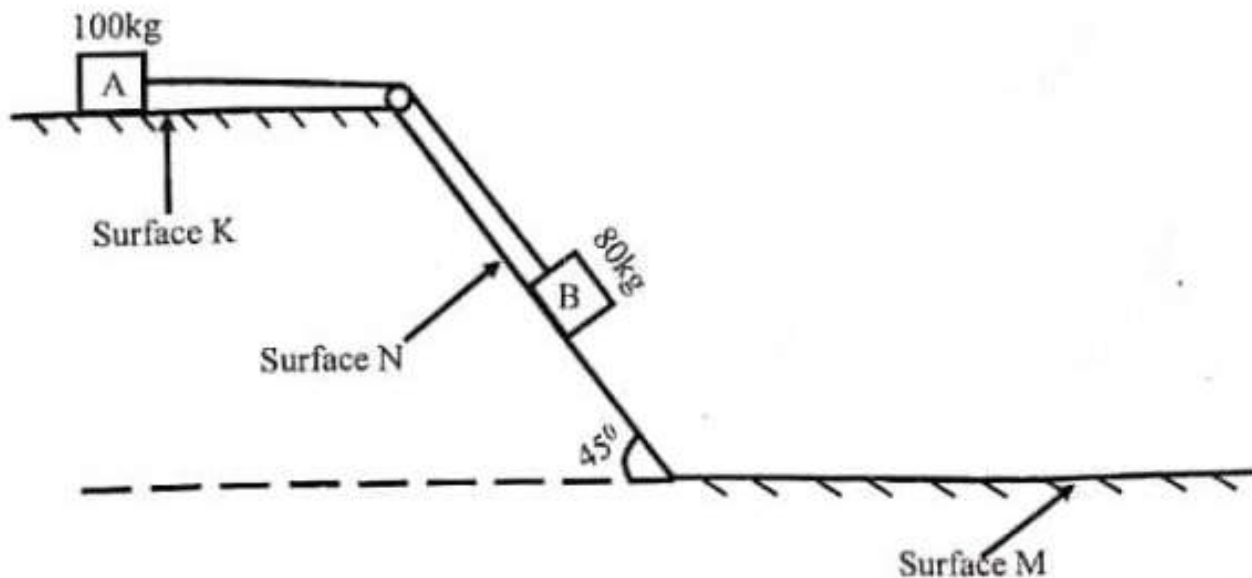
Kipro would request for fresh water after every 2 hours but the bottle would be projected to him at an angle of  $30^\circ$  to the horizontal and a speed of more than  $20\text{kmh}^{-1}$  given to the bottle would be harmful to Kipro. They want to know the horizontal distance moved by Kipro if initially the bottle is projected with a speed of  $20\text{kmh}^{-1}$ .

**Task**

- a) If you have been approached by the association, help them to know the actual speed and direction of wind if it is known that speed of winds in Tokyo don't usually exceed  $5\sqrt{21}\text{kmh}^{-1}$ . Will he compete or not?
- b) Show the horizontal distance Kipro travels every two hours before a bottle of water is given and at what angle and speed he picks the bottle indicating that he will not be harmed by bottle.

**ITEM 5**

Phoenix Company deals in cotton products but is facing a high labour dropouts due to heavy work load and long working hours. To solve this challenge, the board hired a team of engineers who were assigned a duty of designing a plane that can always be used to offload the vehicles and later one strong person would pull the cotton sack along a horizontal surface. The design of the plane should resemble the sketch shown below.



It consists of three planes where surface K is the cabin of a vehicle, surface N is the inclined plane placed at the end of the vehicle and surface M is where a load A would be pulled on. B is the control load that pulls A. After A has reached surface M, B is disconnected and a string is tied on A at an angle of  $60^\circ$  to the horizontal. An acceleration of  $\frac{1}{3} gms^{-2}$  is given to the load for safety purposes and the roughness of surface M should be  $\frac{2}{3}$ . Where g is acceleration due to gravity taken as  $10ms^{-2}$ .

The offloading is done in two phases.

**Phase I:** The surfaces K and N are first made smooth and then acceleration of loads A and B is determined.

**Phase II:** The surfaces K and N are made rough with coefficient of dynamic friction being  $\frac{1}{4}$  and also acceleration of loads A and B is determined.

If the acceleration in Phase I is bigger than that in Phase II, then Phase II is adopted for safety reasons and the Model will be recommended to the company.

#### **Task**

- a) Show that the model will work basing on the two phase options above.
- b) Determine the strength of the person they should hire to pull the load A across surface M.

**End**