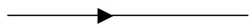
GEOMETRICAL OPTICS

Optics is a branch of physics that studies the behaviour and properties of light, including its interactions with matter. The field encompasses a wide range of phenomena, from the basic principles of reflection and refraction to complex behaviours of lenses, mirrors and optical instruments.

Rays and beams.

A ray of light is the path that shows the direction of light.

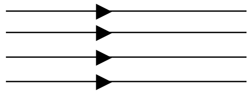
It is indicated by a straight line with an arrow on it.



A beam of light is the collection of light rays.

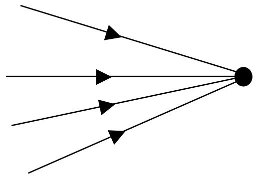
A beam of light can be parallel, convergent or divergent.

A parallel beam: This is a collection of parallel rays of light.



Examples of parallel beams includes light from the sun.

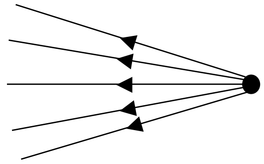
A convergent beam. This is a beam in which rays from different directions collect at a single point.



Examples of convergent beams includes beams reflected by concave mirrors, beams refracted by convex lenses.

A divergent beam: This is a beam in which light rays emerge from a single point to different

\



Examples of convergent beams includes beams reflected by convex mirrors, beams refracted by concave lenses, beams from torches, beams from car headlamps etc.

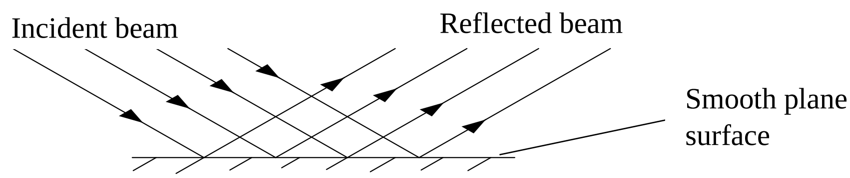
REFLECTION OF LIGHT

Reflection of light is the process by which light bounces off a reflecting surface.

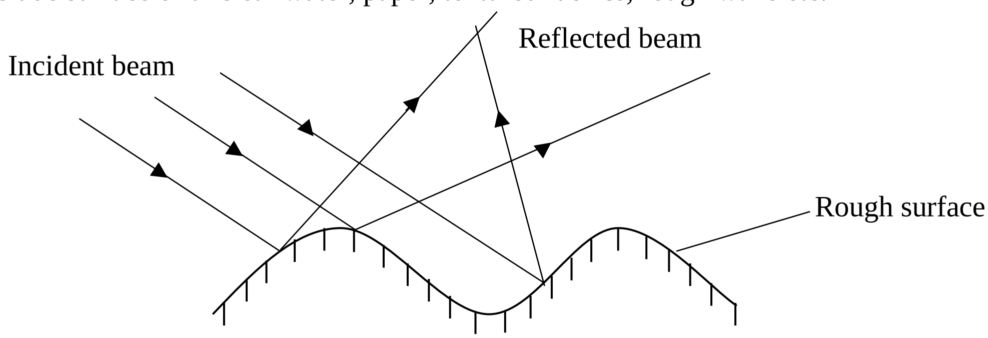
Examples of reflecting surfaces include mirrors, water surface, glass surface etc.

Types of reflection of light

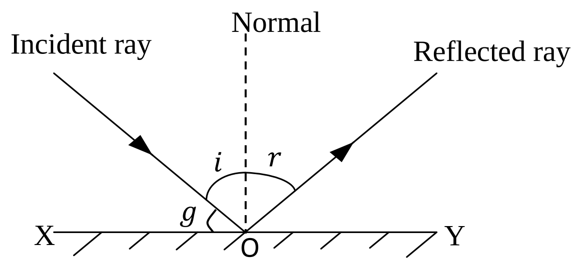
1. **Regular(specular) reflection:** This is the type of reflection in which a parallel beam of light incident on a smooth reflecting surface is reflected as a parallel beam of light. Examples of smooth surfaces include plane mirrors, surface of clear water, polished metal etc.



2. **Irregular or diffuse reflection:** This is the type of reflection in which a parallel beam of light incident on a rough reflecting surface is scattered in different directions. Examples of smooth surfaces include surface of unclear water, paper, textured fabrics, rough walls etc.



Reflection of light in plane mirrors



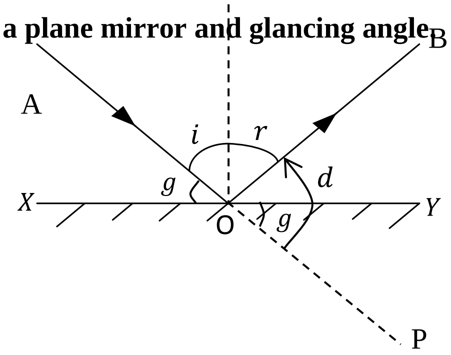
XOY- mirror surface
i- angle of incidence $\quad$ r- angle of reflection
g- glancing angle

Laws of reflection of light

Law 1: The incident ray, the reflected ray the normal at the point of incidence all lie in the same plane.

Law 2: The angle of incidence is equal to the angle of reflection ($i = r$).

Deviation of light at a plane mirror and glancing angle



Deviation of light is the change in the direction of light after striking a reflecting surface. It is the angle from the direction of the incident ray to the reflected ray (Angle POB). It is denoted by d.

Glancing angle is the angle between the incident ray and the mirror surface. It is denoted by angle $\boldsymbol{g}$**.**

$$d = g + \angle YOB$$

But $\angle YOB = 90° - r$

$$d = g + (90° - r)$$

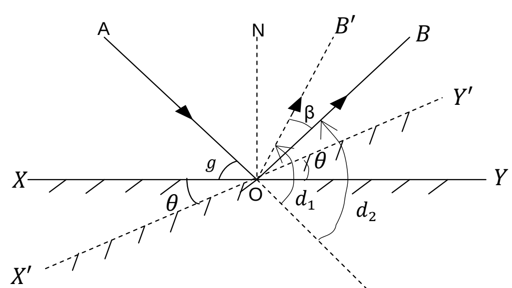
By law 2 of reflection of light, $i = r$

$$d = g + (90° - i)$$
$$i + g = 90° \Rightarrow g = 90° - i$$
$$d = g + g$$
$$\boldsymbol{d = 2g}$$

Therefore, the angle of deviation is twice the glancing angle.

Angle of rotation of the reflected ray by a rotated mirror when the direction of the incident ray is kept fixed.

Consider a ray AO incident on a plane mirror and reflected along OB.



When the mirror is in position XOY, the glancing angle= g

The angle of deviation, $d_1 = 2g$.

When the mirror is rotated anticlockwise through angle θ to position $X'OY'$, keeping the direction of the incident ray fixed, the reflected ray moves to position OB'.

The new glancing angle is $(g + \theta)$.

The new angle of deviation, $d_2 = 2(g + \theta)$

The angle of rotation of the reflected ray, $\beta = d_2 - d_1$

$$\beta = 2(g + \theta) - 2g$$

$$\beta = 2\theta$$

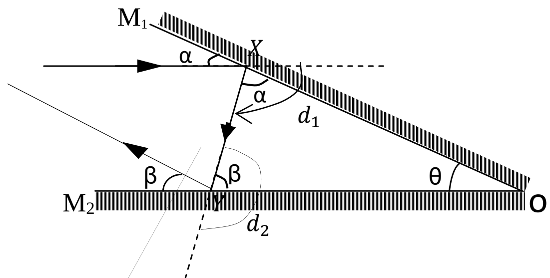
Therefore, if the direction of the incident ray on a plane mirror is kept fixed and the mirror is rotated, the reflected ray rotates through an angle twice the angle the angle of rotation of the mirror.

Assignment: Light is made incident on a plane mirror. If the direction of the incident ray is kept constant, show that when the mirror is rotated through an angle θ clockwise, the reflected ray rotates through angle 2θ. **(Skip one page)**

Deviation of light after successive reflections at two inclined plane mirror.

Case I

Consider two plane mirrors OM_1 and OM_2 inclined at angle θ . Suppose a ray AX is incident on mirror OM_1 at a glancing angle α and the reflected ray XY makes a glancing β with mirror OM_2. the glancing angles at each of the mirrors.



When light is incident on mirror OM_1, the glancing angle $= \alpha$.

The deviation, $d_1 = 2\alpha$ (clockwise)

After reflection at mirror OM_2, the glancing angle $= \beta$.

The deviation, $d_2 = 2\beta$ (clockwise)

The net total deviation, $d = d_1 + d_2$

$d = 2\alpha + 2\beta = 2(\alpha + \beta)$ (clockwise)

From triangle XOY, $\alpha + \beta + \theta = 180°$

$$\alpha + \beta = 180° - \theta$$

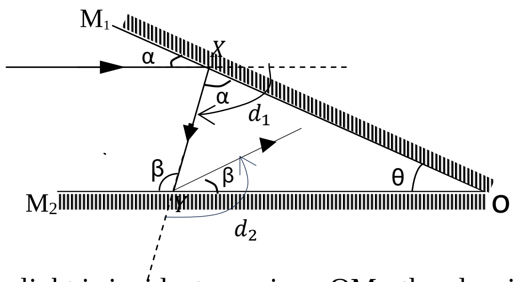
$$d = 2(180° - \theta)$$

$d = 360° - 2\theta$ (Clockwise)

$d = 360° - (360° - 2\theta)$ (Anti clockwise)

$d = 2\theta$ (Anti clockwise)

Case II



When light is incident on mirror OM_1, the glancing angle $= \alpha$.

The deviation, $d_1 = 2\alpha$ (clockwise)

After reflection at mirror OM_2, the glancing angle $= \beta$.

The deviation, $d_2 = 2\beta$ (Anti clockwise)

The net total deviation, $d = d_2 - d_1$

$$d = 2\beta - 2\alpha = 2(\beta - \alpha) \text{ (Anti clockwise)}$$

From triangle XOY, $\alpha + \theta = \beta$

$$\beta - \alpha = \theta$$

$d = 2\theta$ (Anti clockwise)

The two cases yield the same result.

In general, no matter the angle of incidence on the first mirror, the total deviation after two successive reflections in the mirrors (once in each mirror) is constant and equal to twice the angle between the mirrors.

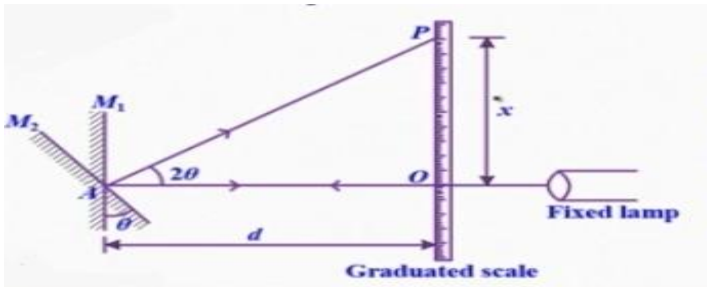
Assignment: Two plane mirrors are inclined at an angle β. Show that the deviation suffered by light after successive reflections in the mirrors once in each mirror is 2β. **(1 page)**

The knowledge of inclined plane mirrors and rotation of plane mirrors is used in;

(i) Optical lever in mirror galvanometer.
(ii) A sextant.

Optical lever in mirror galvanometer

An optical lever in mirror galvanometer consists of a small plane mirror M_1 which is rigidly attached to a system that rotates when current flows through it, a lamp which is the source of light and a graduated scale as shown in the diagram below.



Operation

Light from the lamp is incident normally on the mirror at A in position M_1, the beam is reflected back along the same path and a spot is obtained at O on the graduated scale placed just besides the lamp.

When current, I is passed through the system, the mirror rotates through an angle θ from position M_1 to M_2 and the spot of light is deflected through a distance, x to position P on the scale.

The reflected ray turns through an angle which is twice the angle of rotation of the mirror. Therefore the reflected ray rotates through angle 2θ.

Lengths x and d are measured and θ is then got from $\theta = \frac{d}{2x}$

Since current $I \propto \theta$, the magnitude of current through the instrument can then be determined.

Theory

When the mirror rotates through angle, θ , the reflected ray rotates through angle 2θ

From the illustration;

$$\tan 2\theta = \frac{d}{x}$$

Since 2θ is a small angle in radians, $\tan 2\theta \approx 2\theta$

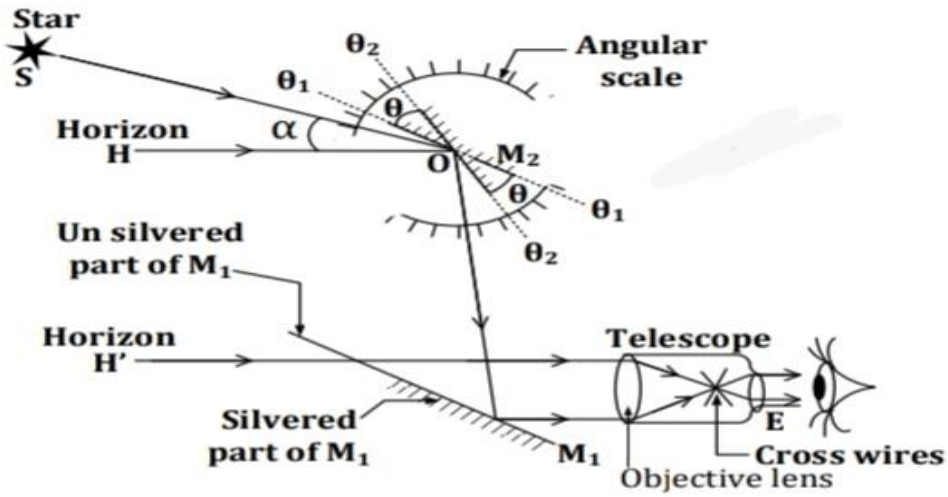
Thus $2\theta = \frac{d}{x} \quad \Rightarrow \theta = \frac{d}{2x}$

Question: describe the operation of an optical lever in a mirror galvanometer (UNEB 2020 2(a))

A sextant

The sextant is an instrument used in navigation to determine the angle of elevation of the sun or stars .

It essentially consists of two mirrors; a fixed mirror M_1 which is half silvered and a fully silvered mirror M_2 which can be rotated about its axis at O. The observer views the images through a fixed telescope at E directed towards mirror M_1.



Light from the horizon H' is viewed directly through the un silvered part of the fixed mirror M_1, until the image of the horizon H' is in sharp focus at the centre of cross wires of the eyepiece(E) of the telescope.

Mirror M_2 is rotated about O so as to focus the image of the horizon H after two successive reflections in mirror M_2 and then M_1 until the image of H coincides with that of H' as seen through the telescope. At this point, mirrors M_1 and M_2 are parallel.

The angular position θ_1 of mirror M_2 on the circular scale is noted.

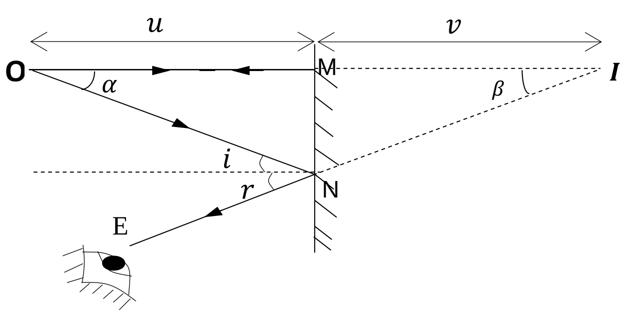
Mirror M_2 is again rotated slowly until the image of the star S coincides with the image of horizon H' at the centre of the crosswires as viewed through the telescope. The new angular position θ_2 of mirror M_2 on the circular scale is noted.

The angle of rotation of mirror M_2 is obtained from $\theta = |\theta_2 - \theta_1|$

The angle of elevation of the star is $\alpha = 2\theta$.

Images formed by plane mrrors.

Consider a point object O placed infront of a plane mirror M.



A ray OM incident normally on the mirror is reflected back along the same path.

A ray ON incident on the mirror at N is reflected along NE. The reflected rays MO and NE appear to come from point I behind the mirror, which is the image position.

$< \alpha =< i$ (alternate angles)

$< i =< r$ (laws of reflection)

$< r =< \beta$ (corresponding angles)

Therefore; $< \alpha =< \beta$

$$\tan \alpha = \tan \beta$$

$$\frac{MN}{OM} = \frac{MN}{IM}$$

$$IM = OM$$

Since $OM = u$ and $IM = v$

Hence $u = v$ (The object distance is equal to the image distance).

Therefore, the image formed is far behind the mirror as the object is infront.

Properties of images formed by plane mirrors.

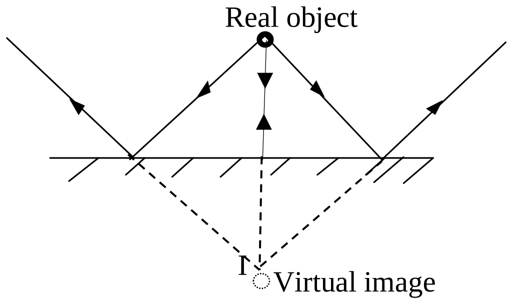
- ✓ The images are virtual (images cannot be formed on the screen)
- ✓ The image distance is equal to the object distance.
- ✓ The images are formed far behind the mirror as the object is infront.
- ✓ The image is of the same size as the object.
- ✓ The image is erect (upright).
- ✓ The images are laterally inverted (they are turned through 180^0 in the mirror)

Real and virtual images

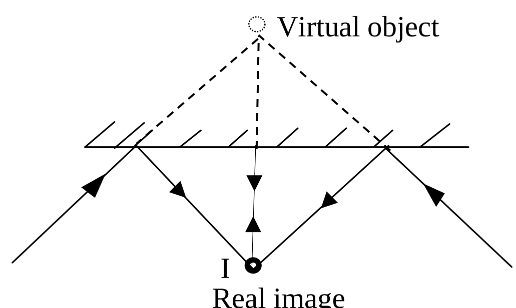
A real image is the one that is formed by actual intersection of light rays and can be formed on the screen.

A virtual image is the one that is formed by apparent intersection of light rays and cannot be formed on the screen.

(a)



(b)



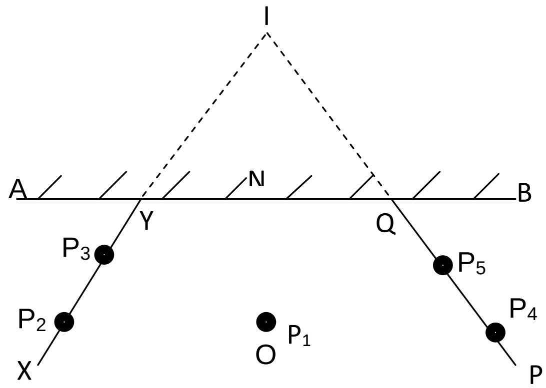
In (a), rays from the real object incident on the plane mirror are reflected and appear to come from Point I, which is behind the mirror. This image cannot be received on the screen and therefore is virtual.

However, a real image can also be formed by the plane mirror if a beam of light that converges at O is incident on the mirror as in (b). The reflected rays converge at point I infront of the mirror to form a real image.

In both cases, the image and the object are at equal distances from the mirror.

Location of the image formed by plane mirrors.

(a) Using pins.



A white sheet of paper is stuck on a soft board. The plane mirror is placed on the paper and its outline AB is traced.

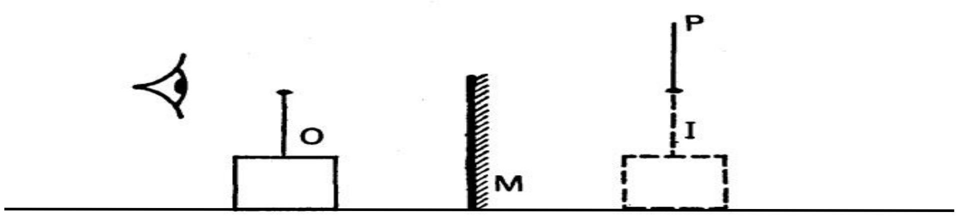
Pin P_1 is stuck infront of the mirror at O. With the mirror on its outline, pins P_2 and P_3 are stuck while looking through the mirror at side AN so that the pins appear to be in line with the image P_1.

The pins P_2 and P_3 and the mirror are removed and line XY is drawn through the marks of P_2 and P_3 to meet AB at Y.

The mirror placed back along its outline and the experiment is repeated with pins P_4 and P_5 while looking through the mirror on side NB. Line PQ is drawn to pass through the marks of the pins.

Lines XY and PQ are extended to meet at I. Point I is the location of the image.

(b) Using the no parallax method.



An object pin O is placed infront of the mirror M and the image pin P is positioned behind the mirror.

The image pin is moved towards and away from the mirror until it coincides with the image I of pin O in the mirror and there there is no parallax between P and I

The position of pin P gives the location of the image formed by the mirror.

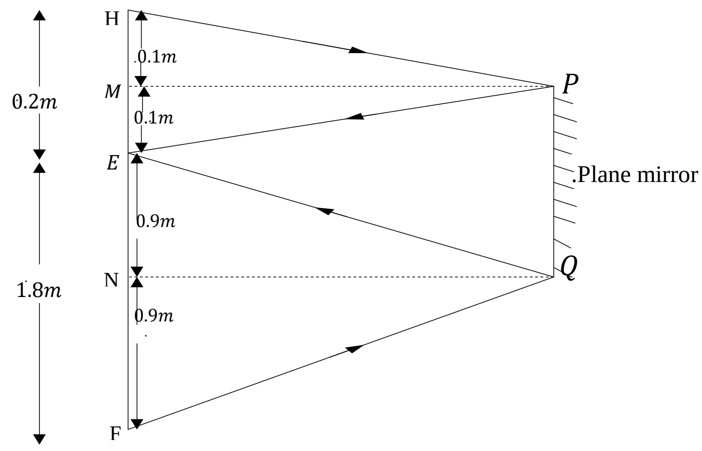
Note: No parallax means that at the point of coincidence, there is no relative motion between the pins when the observer moves his head.

Minimum vertical length of a plane mirror

Question: A man 2m tall whose eye level is 1.8m above the ground looks at his image in a vertical mirror. What must be the minimum vertical length of the mirror so that the man can see the whole of himself completely in the mirror. **(skip 1 page)**

solution

Let F, E and H represent the feet, eye and head levels of the man respectively.



Length of mirror (PQ)=MN

Rays from the man's head(H) are reflected from the top part of the mirror (P) and are incident into his eyes(E). Rays from the feet(F) are reflected from the bottom part of the mirror(Q) into the eyes.

Since angle FQE is equal to angle NQE, then FN=NE= $\frac{1}{2} \times 1.8 = 0.9m$

Since angle HPM is equal to angle MPE, then HM=ME= $\frac{1}{2} \times 0.2 = 0.1m$

$$MN = ME + NE$$

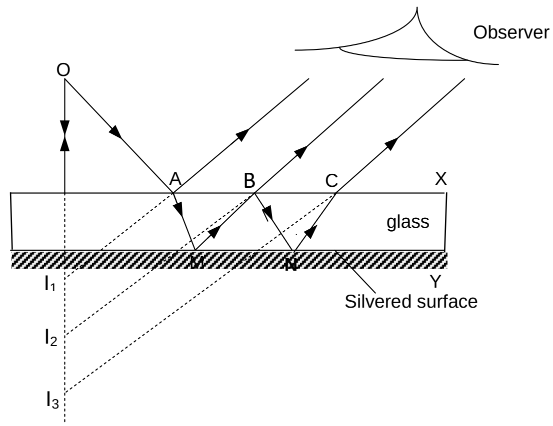
$$MN = 0.9 + 0.1 = 1.0m$$

The minimum length of the mirror should be $1.0m$.

It can be noted that the minimum vertical length of a plane mirror required for a person to see the whole of himself completely is half the half the height of the person.

Assignment: Show that for a man of height, **h** standing upright, the minimum length of a vertical plane mirror in which he can see the whole of himself completely is $\frac{h}{2}$. **(Skip 1 page)**

Formation of images by thick plane mirrors



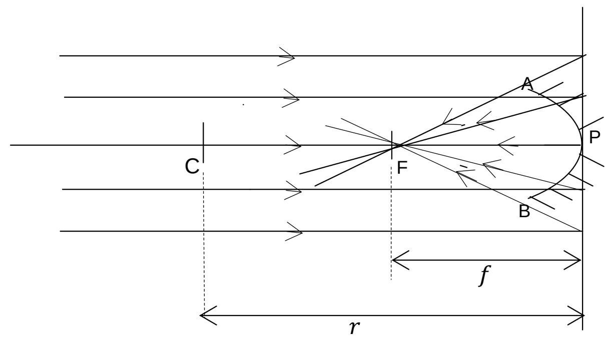
- A thick plane has two plane surfaces X and Y, reflection takes place at the two surfaces.
- Light from the object incident on the mirror at A is partially reflected and transmitted. The reflected light leads to I_1
- The transmitted light is reflected at the silvered surface at M, it undergoes partial reflection and transmission at B. The transmitted light appears to originate from I_2.
- Successive total internal reflections and transmission of light at surface X continuously occurs and this leads to formation of multiple images.

Note: Thick plane mirrors form multiple images and its distant images are faint. This is because at each reflection, some of the energy of the incident light is absorbed. Therefore, the transmitted and reflected light are less intense than the incident light.

REFLECTION OF LIGHT AT CURVED MIRRORS

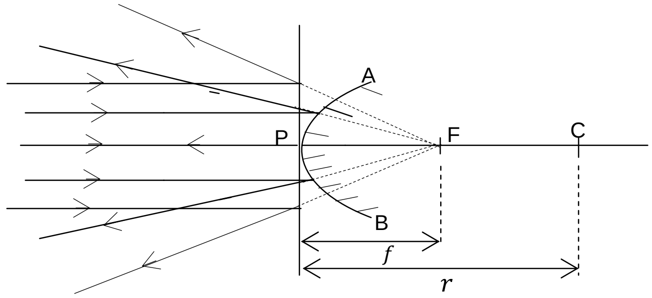
Curved mirrors are obtained from spherical surfaces. They are also referred to as spherical mirrors. Curved mirrors are of two types ie concave mirrors(converging mirror) and convex mirrors(diverging mirror).

(i) **Concave (Converging) mirror:** It is the part of a sphere whose centre (C) is in front of its reflecting surface. The reflecting surface of a concave mirror curves outwards.



A concave mirror converges a wide beam of light incident on it to a single point called the principal focus or the focal point(F). This is why a concave is referred to as a converging mirror.

(ii) **Convex (diverging) mirror:** It is the part of a sphere whose centre (C) is behind its reflecting surface. The reflecting surface of a convex mirror curves inwards.



When a wide beam of light is incident on a convex mirror, it is reflected such that the reflected light appears to diverge from the focal point. Therefore a convex mirror is referred to as a diverging mirror.

Terms used in curved mirrors

1. **Pole of the mirror(P):** This is the centre (mid point) of the mirror surface.
2. **Aperture of the mirror(APB):** This is the length of the mirror surface.
3. **Centre of curvature (C):** This is the centre of the sphere of which the mirror surface forms part.
4. **Princpal axis (CP)**: This is the imaginary line that joins the centre of curvature and the pole of the mirror.
5. **Radius of curvature (r):** This is the radius of the sphere of which the mirror surface forms part OR Centre of curvature is the distance from the pole of the mirror to the centre of curvature of the mirror.
6. **Paraxial rays:** These are rays close to the principal axis and make small angles with the mirror optical axis.
7. **Marginal rays:** These are rays furthest from the principal axis of the mirror.
8. **Principal focus of a mirror(F):** This is a point on the principal axis at which rays originally parallel and close to the principal axis converge or appear to diverge after reflection from the mirror.
   - **(i)** **Principal focus of a concave mirror:** This is a point on the principal axis where paraxial rays incident on the mirror and parallel to the principal axis converge after reflection by the mirror.
   - **(ii)** **Principal focus of a convex mirror:** This is a point on the principal axis where paraxial rays incident on the mirror and parallel to the principal axis appear to diverge from after reflection by the mirror
9. Focal point (f): This is the distance from the pole of the mirror to the focal point.

Rules of constructing geometrical ray diagrams

These are the rules involved in locating the position of images formed by mirror

(1) A ray parallel to the principal axis is reflected through the focal point.
(2) A ray through the focal point is reflected parallel to the principal axis.
(3) A ray through the centre of curvature is reflected back along its path.
(4) A ray incident on the mirror at the pole is reflected making the same angle with the principal axis.

Steps involved in locating the position of images formed by curved mirrors

Step1: Locate the position of the object. At the object position, draw a ray perpendicular to the principal axis.

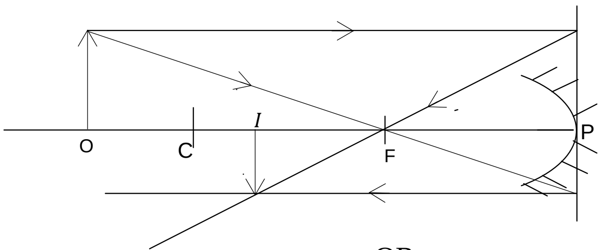
Step 2: Draw a ray from the tip of the object parallel to the principal axis onto the mirror. By rule 1, this ray is reflected through the focal point.

Step 3: Draw a ray from the tip of the object through the focal point. By rule 2, this ray is reflected parallel to the principal axis. Alternatively, a ray can be drawn from the tip of the object through the centre of curvature. This ray is reflected back along the same path through the centre of curvature.

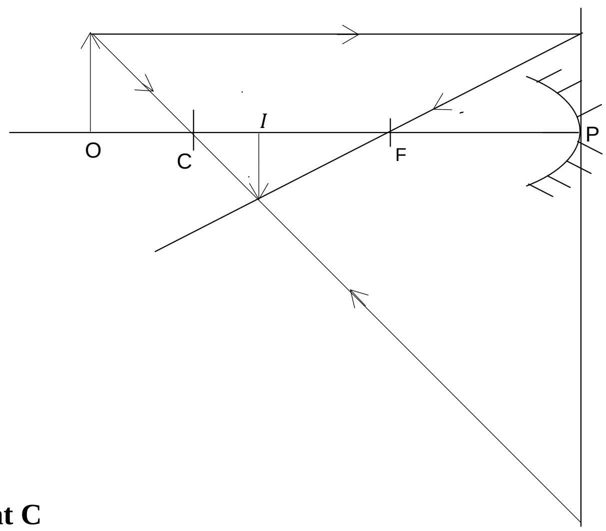
Step 4: The point of intersection of the two reflected rays gives the location of the image. The image is represented by a ray drawn perpendicular to the principal axis with its tip at the point of intersection of the two reflected rays.

Location of the images in concave mirrors

(a) Object beyond C



OR

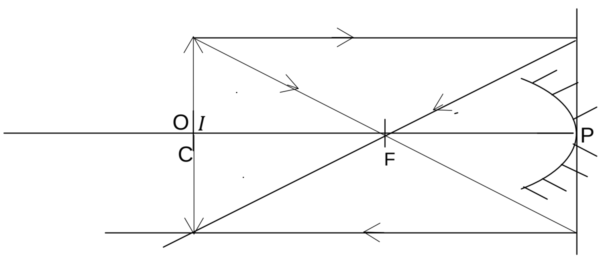


Nature of the image

The image is;

- ✓ real
- ✓ inverted
- ✓ diminished
- ✓ formed between C and F

(b) Object at C



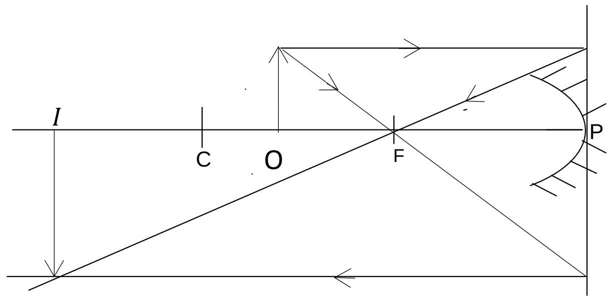
Nature of the image

The image is;

- ✓ real
- ✓ inverted
- ✓ of same size as the object.
- ✓ formed at C

Note: When the object is the centre of curvature, its image is also formed at the centre of curvature.

(c) Object between C and F.

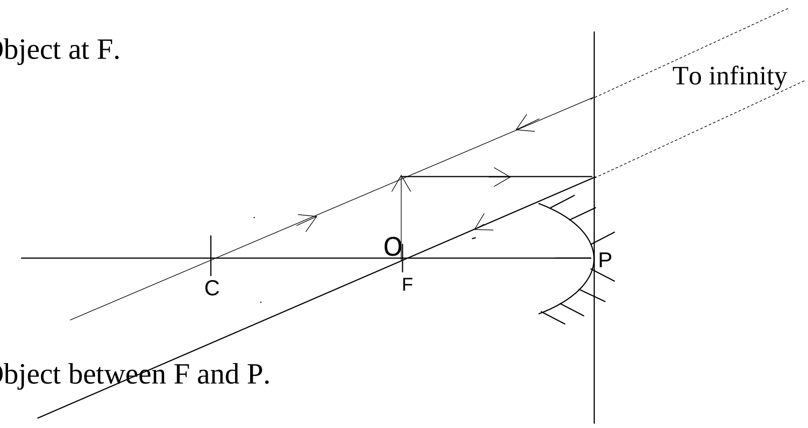


Nature of the image

The image is;

- ✓ real
- ✓ inverted
- ✓ magnified
- ✓ formed beyond C

(d) Object at F.

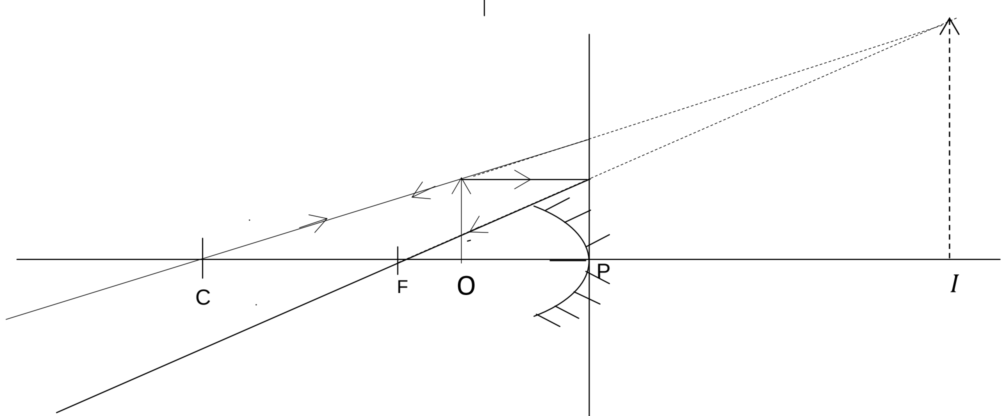


Nature of the image

The image is;

- ✓ virtual
- ✓ Erect(upright)
- ✓ magnified
- ✓ formed beyond at infinity

(e) Object between F and P.



Nature of the image

The image is;

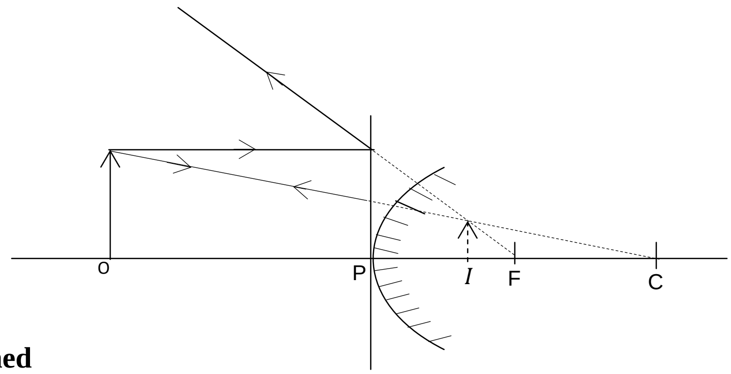
- ✓ virtual
- ✓ Erect(upright)
- ✓ magnified
- ✓ formed behind the mirror

Uses of concave mirrors

- ✓ They are used as shaving mirrors.
- ✓ They are used as solar concentrators in solar panels.

- ✓ They are used by dentists for teeth checkups.
- ✓ They are used in reflecting telescopes to view distant objects.
- ✓ They are used in projectors to concentrate light onto the slide.

Location of images in convex mirrors



Nature of image formed

Irrespective of the position of the object, the image formed by the convex mirror is diminished, virtual, and erect.

Uses of convex mirror

- ✓ They are used as driving mirrors. This is because convex mirrors always form erect images and have a wide field of view.
- ✓ They are supermarkets to monitor the activities of customers.
- ✓ They are used at security checkpoints to inspect under vehicles.

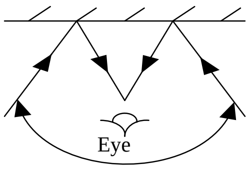
Advantages of convex mirrors over concave mirrors

Convex mirrors have a wide field view than that of convex mirrors.

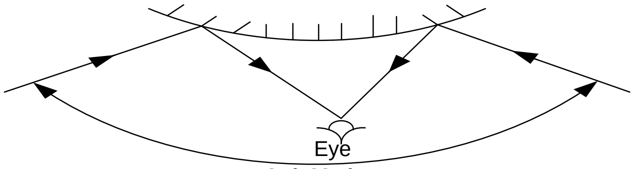
They always form erect images.

Comparison of the field of view of a convex and a plane mirror.

Convex mirrors have got a wide field of view while plane mirrors have a narrow field of view.



Narrow field of view



Wide field of view

Sign convection

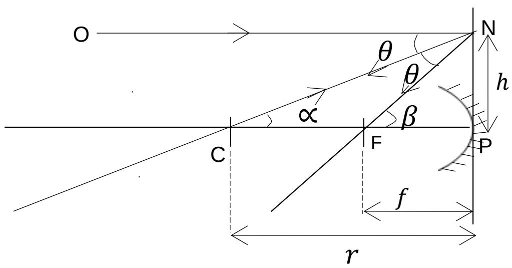
In numerical work; distances corresponding to real images and objects are positive while distances corresponding to virtual images and objects are negative (u positive for real objects and negative for virtual objects while v is positive for real images and negative for virtual images)

Therefore, the focal length of a concave mirror is positive since it has a real principal focus and negative for a convex mirror since a convex mirror has a virtual principal focus.

Relationship between focal length (f) and radius of curvature(r).

(a) Using a concave mirror

Consider a paraxial ray of light ON incident on a concave mirror at a height, h above the principal axis.



$\propto = \theta$ and $2\theta = \beta$ (alternate angles)

Therefore, $2 \propto = \beta$…………(1)

From triangles FPN and CPN, $\tan \propto = \frac{h}{r}$ and $\tan \beta = \frac{h}{f}$

Since $\propto$ and β are small angles in radians; $\tan \propto \approx \propto$ and $\tan \beta \approx \beta$

$\Rightarrow \propto = \frac{h}{r}$ and $\beta = \frac{h}{f}$

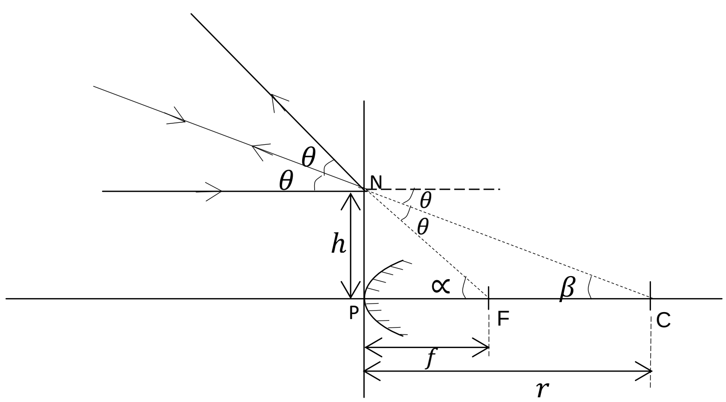
Substituting for $\propto$ and β in (1)

$$2\left(\frac{h}{r}\right) = \frac{h}{f}$$

$$\therefore\ r = 2f$$

(b) Using convex mirror

Consider a paraxial ray of light ON incident on a concave mirror at a height, h above the principal axis.



$\propto = \theta$ and $2\theta = \beta$ (alternate angles)

Therefore, $2 \propto = \beta$…………(1)

From triangles FPN and CPN, $\tan \propto = \frac{h}{-r}$ and $\tan \beta = \frac{h}{-f}$

Since $\propto$ and β are small angles in radians; $\tan \propto \approx \propto$ and $\tan \beta \approx \beta$.

$\Rightarrow \propto = \frac{h}{-r}$ and $\beta = \frac{h}{-f}$

Substituting for $\propto$ and β in (1)

$$2\left(\frac{h}{-r}\right) = \frac{h}{-f}$$

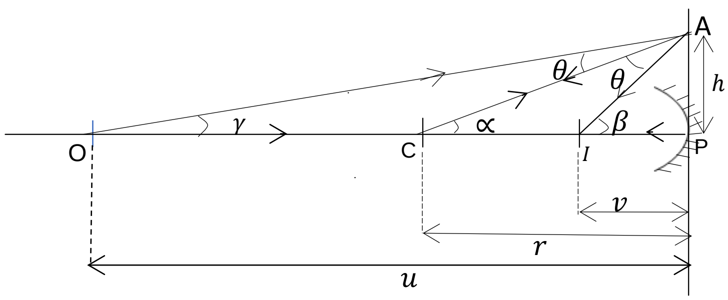
$$\therefore r = 2f$$

The mirror formula

The mirror formula gives the relationship between the object distance(u), image distance(v) and the focal length(f) of the mirror.

(a) Using a point object and a concave mirror.

Consider light from a point object O incident on a concave mirror at a height, h above the principal axis.



Using the exterior angle property on triangles OAC and CAI.

$\gamma + \theta = \propto$ …………(1)

$\propto + \theta = \beta$…………..(2)

Equation (1)−Equation(2)

$\gamma - \propto = \propto - \beta$

$\gamma + \beta = 2\propto$……………(3)

From triangles OAP, CAP and IAP, $\tan \gamma =$, $\tan \propto = \frac{h}{r}$ and $\tan \beta = \frac{h}{r}$

Since $\gamma, \propto$ and β are small angles in radians; $\tan \gamma \approx \gamma$, $\tan \propto \approx \propto$ and $\tan \beta \approx \beta$.

$\Rightarrow \gamma = \frac{h}{u}$, $\propto = \frac{h}{r}$ and $\beta = \frac{h}{f}$

Substituting for $\gamma, \propto$ and β in equation (3)

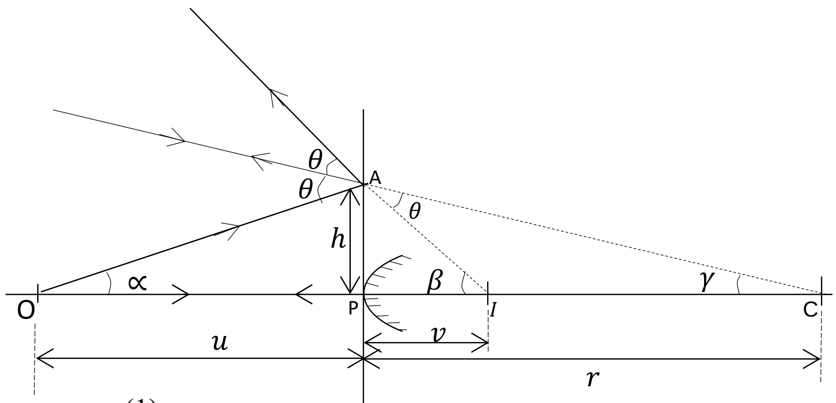
$$\frac{h}{u}+\frac{h}{v}=2\left(\frac{h}{r}\right)$$

$$\frac{1}{u}+\frac{1}{v}=2\left(\frac{h}{2f}\right)$$

$\boldsymbol{\frac{1}{u}+\frac{1}{v}=\frac{1}{f}}$. This is the mirror formula.

(b) Using a point object and a convex mirror

Consider light from a point object O incident on a convex mirror at a height, h above the principal axis.



$\gamma+\theta=\beta$ …………(1)

$\propto+\gamma=\theta$…………(2)

Substituting (2) in (1)

$\gamma+\propto+\gamma=\beta$

$2\gamma+\propto=\beta$……………(3)

From triangles OAP, CAP and IAP, $\tan\propto=\frac{h}{u}$, $\tan\beta=\frac{h}{-v}$ and $\tan\gamma=\frac{h}{-r}$.

Since $\propto$ and β are small angles in radians; $\tan\gamma\approx\gamma$, $\tan\propto\approx\propto$ and $\tan\beta\approx\beta$.

$\Rightarrow\propto=\frac{h}{u}$, $\beta=\frac{h}{-v}$ and $\gamma=\frac{h}{-r}$

Substituting for $\gamma,\propto$ and β in equation (3)

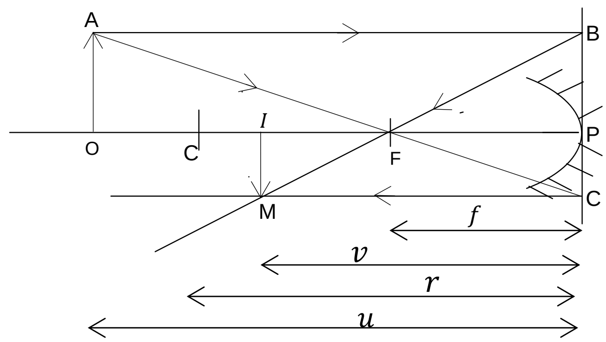
$$2\left(\frac{h}{-r}\right)+\frac{h}{u}=\frac{h}{-v}$$

$$\frac{-2}{2f}+\frac{1}{u}=\frac{-1}{v}$$

$$\boldsymbol{\frac{1}{u}+\frac{1}{v}=\frac{1}{f}}$$

(c) Using a finite object and a finite object.

consider light from a finite object at a point beyond the centre of curvature.



Triangles OFA and FPB are similar.

$\frac{OF}{FP} = \frac{OA}{CP}$…………(1)

Triangles IMF and FPB are also similar.

$\frac{FP}{IF} = \frac{BP}{IM}$…………………(2)

But $OA = BP$ and $CP = IM$

Equations (1) becomes.

$\frac{OF}{FP} = \frac{BP}{IM}$……………..(3)

Equating (2) and (3)

$$\frac{FP}{IF} = \frac{OF}{FP}$$

$$OF = u - f, \qquad FP = f, \qquad IM = v - f$$

$$\frac{f}{v-f} = \frac{u-f}{f}$$

$$f^2 = (u-f)(v-f)$$

$$f^2 = uv - uf - vf + f^2$$

$$vf + uf = uv$$

Dividing through by uvf.

$$\frac{vf}{uvf} + \frac{uf}{uvf} = \frac{uv}{uvf}$$

$$\therefore \frac{\mathbf{1}}{\boldsymbol{u}} + \frac{\mathbf{1}}{\boldsymbol{v}} = \frac{\mathbf{1}}{\boldsymbol{f}}$$

Exercise

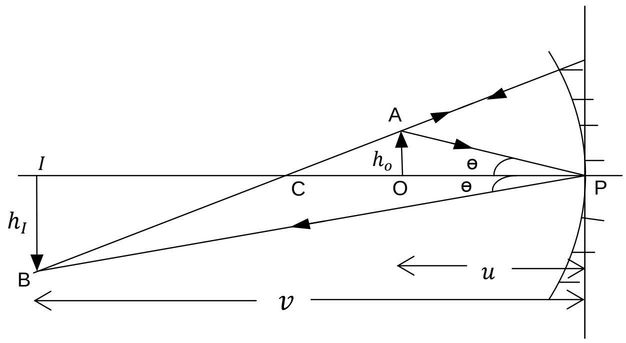
(a) Draw a ray diagram to show formation of the image for real object placed infront of a concave mirror between the focal point and the centre of curvature.

(b) Use a finite object and a convex mirror to derive the mirror formula.**(skip 2 pages).**

Linear magnification

Linear magnification is the ratio of image size to the object size. It is denoted by m.

$$m = \frac{Image\ size(Image\ height)}{Object\ size(Object\ height)}$$



From triangle OAP; $\tan\theta = \frac{h_o}{u}$……………(1)

From triangle IBP; $\tan\theta = \frac{h_I}{v}$……………(2)

Equating (2) and (1)

$$\frac{h_I}{h_o} = \frac{v}{u}$$

But from definition, linear magnification, $m = \frac{h_I}{h_o}$.

$$\therefore m = \frac{v}{u}$$

Thus; $magnification = \frac{Image\ distance(v)}{Object\ distance(u)}$

Therefore linear magnification can also be defined as the ratio of image distance to object distance.

Note: The magnification is positive for a real image and negative for a virtual image.

Relationship between image distance(v), focal length(f) and magnification(m).

From the mirror formula; $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Multiplying through by v

$$\frac{v}{u} + \frac{v}{v} = \frac{v}{f}$$

But $m = \frac{v}{u}$

$$m + 1 = \frac{v}{f}$$

$$\boldsymbol{v = (m + 1)f}$$

Relationship between object distance($\boldsymbol{u}$), focal length($\boldsymbol{f}$) and magnification($\boldsymbol{m}$).

From the mirror formula; $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Multiplying through by u

$$\frac{u}{u} + \frac{u}{v} = \frac{u}{f}$$

But $\frac{u}{v} = \frac{1}{m}$

$$1 + \frac{1}{m} = \frac{u}{f}$$

$$\boldsymbol{u = \left(\frac{1}{m} + 1\right)f}$$

Examples

1. An object 4cm tall is placed 20cm infront of a concave mirror of focal length 15cm. Find the position, size and magnification of the image.
   Solution
   $$u = 20cm, f = 15cm, h_o = 4cm, v = ??, h_I = ??$$
   $$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$
   $$\frac{1}{20} + \frac{1}{v} = \frac{1}{15}$$
   $$v = 60cm$$
   $$\frac{h_I}{h_o} = \frac{v}{u}$$
   $$\frac{h_I}{4} = \frac{60}{20}$$
   $$h_I = 12cm$$
   $$m = \frac{h_I}{h_o} = \frac{12}{4} = 3$$
   Alternatively, $m = \frac{v}{u} = \frac{60}{20} = 3$
   The image is 60cm from the mirror, 12cm high and its magnification is 3.
2. An object is placed 12cm infront of a convex mirror of focal length 18cm. Find the ;
   (i) Position and nature of the image.
   (ii) Magnification.

   Solution
   (i) $u = 12cm, f = -18cm, v = ??$

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$
$$\frac{1}{12}+\frac{1}{v}=\frac{1}{-18}$$
$$\frac{1}{v}=\frac{-5}{36}$$
$$v=-7.2cm$$

The image is formed 7.2cm behind the mirror.

Nature: The image is virtual, diminished and erect.

(ii) $m=\frac{v}{u}$

$$m=\frac{-7.2}{12}=-0.6$$

The magnification is 0.6

3. A concave mirror forms an image which is 2 times smaller that the object. If the focal length of the mirror is 20cm, find the object and image distance.

Solution

$$h_o=2h_I$$
$$\frac{h_I}{h_o}=\frac{1}{2}\Rightarrow m=0.5$$
$$u=\left(\frac{1}{m}+1\right)f$$
$$u=\left(\frac{1}{0.5}+1\right)\times 20$$
$$u=60cm$$

$$v=(m+1)f$$
$$u=(0.5+1)\times 20$$
$$u=30cm$$

Alternatively; $m=\frac{v}{u}$

$$\frac{1}{2}=\frac{v}{60}$$
$$v=30cm$$

4. A concave mirror forms a real image which is three times the size of the object. When the object is moved through a distance, d towards the mirror, the real image formed is now five times the size of the object. If the image shifts through 20cm, find the;
   (i) focal length of the mirror.
   (ii) distance, d.
   (iii) Initial position of the object and image.

   Solution

(i) For case I; $m_1=3$

$$v_1=(m_1+1)f$$
$$v_1=(3+1)f$$

$v_1=4f$……………...(1)

For case II; $m_2=5$

$$v_2 = (m_2 + 1)f$$
$$v_2 = (5 + 1)f$$
$v_2 = 6f$……………….(2)

Shift in the image position= $v_2 - v_1 = 20$

$$6f - 4f = 20$$
$$2f = 20$$
$$f = 10cm$$

(ii) $u_1 = \left(\frac{1}{m_1} + 1\right)f$

$$u_1 = \left(\frac{1}{3} + 1\right)f = \frac{4}{3}f$$
$$u_2 = \left(\frac{1}{m_2} + 1\right)f$$
$$u_2 = \left(\frac{1}{5} + 1\right)f = \frac{6}{5}f$$
$$d = u_1 - u_2$$
$$d = \frac{4}{3}f - \frac{6}{5}f = \frac{2}{15}f$$
$$d = \frac{2}{15} \times 10$$
$$d = 1.333cm$$

(iii) Initial object position; $u_1 = \frac{4}{3}f$

$$u_1 = \frac{4}{3} \times 10 = 13.333cm$$

Initial position of image; $v_1 = 4f$

$$v_1 = 4 \times 10 = 40cm$$

5. A small object placed infront of a spherical mirror gives a real image which is 5 times the size of the object. When the object is moved 12cm towards the mirror, a similarly magnified virtual image is formed. Find the;
   (a) focal length of the mirror and the nature of the mirror.
   (b) final position of the image.

   Solution

   (a) Case I; $m_1 = 5$

$$u_1 = \left(\frac{1}{m_1} + 1\right)f$$
$$u_1 = \left(\frac{1}{5} + 1\right)f = 1.2f$$

Case II; For a virtual image, $m_2 = -5$

$$u_2 = \left(\frac{1}{m_2} + 1\right)f$$
$$u_2 = \left(\frac{1}{-5} + 1\right)f = 0.8f$$
$$u_2 - u_1 = 12$$
$$1.2f - 0.8f = 12$$
$$0.4f = 12$$
$$f = 30cm$$

(b) For the final position of the image; $v_2 = (m_2 + 1)f$

$$v_2 = (-5 + 1) \times 30$$

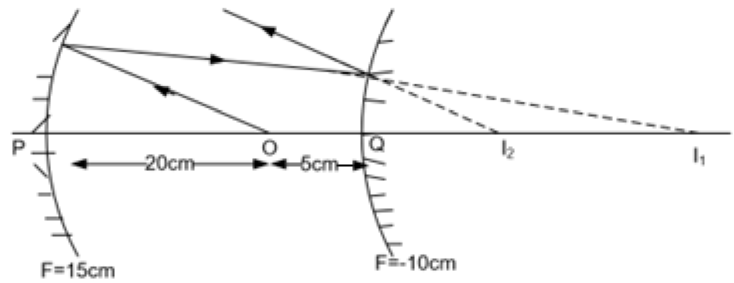
$$v_2 = -120cm$$

After shift of the object, the image is formed 120cm behind the mirror.

6. A concave mirror P of focal length 15cm faces a convex mirror Q of focal length 10cm placed 25cm from it. An object is placed between P and Q at a point 20cm from P.
   (a) Find the distance from of the final image from Q formed after reflection first in P and then in Q.
   (b) Determine the magnification of the final image formed in (a) above.

Solution

(a)



Considering action of the concave mirror (light from the object O is reflected by the concave to form the image I_1)

$$u = 20cm, f = 15cm, v = ??$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{20} + \frac{1}{v} = \frac{1}{15}$$

$$\frac{1}{v} = \frac{1}{60}$$

$$v = 60cm$$

I_1 is 60cm from the concave mirror P.

Distance of I_1 from behind the mirror is $(60 - 25) = 35cm$

Considering action of the convex mirror Q

Image I_1 acts a virtual object for the convex mirror to form image I_2.

$$u = -35cm, f = -10cm, v = ??$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{-35} + \frac{1}{v} = \frac{1}{-10}$$

$$\frac{1}{v} = \frac{1}{14}$$

$$v = -14cm$$

The final virtual image is formed 14cm behind the convex mirror Q.

(b) When the final image is formed after multiple reflections in mirrors, the effective magnification is equal to the product of the individual magnifications produced by the each of the mirrors.
Required magnification, $m = m_1 \times m_2$

$$m = \frac{60}{20} \times \frac{35}{-14}$$

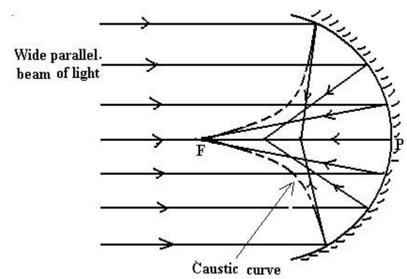
$m = -7.5$ (the negative implies that the final image formed is virtual)
Therefore the magnification of the final image is 7.5.

Exercise 1

1. A 2cm tall object is placed 10cm in front of a concave mirror of focal length 15cm. Find the position, size and nature of the image.
2. A small object is placed on the principal axis of a convex mirror of curvature of 20cm. Determine the position of the image when the object is 15cm from the mirror.
3. A real image is formed 40cm from a spherical mirror, the image being twice the size of the object. What is the radius of the radius of curvature of the mirror. State the type of the mirror.
4. An object is placed perpendicular to the principal axis of a concave mirror of focal length, f at a distance $(f + a)$ and a real image of the object is formed at a distance $(f + b)$. Show that the radius of curvature r of the mirror is given by $\boldsymbol{r = 2\sqrt{ab}}$ **.**
5. A concave mirror forms a real image which is 3 times the linear size of the real object. When the object is displaced a distance x the real image formed is now 4 times its linear size of the object. If the distance between the two images position is 20cm. Find the focal length of the mirror and the value of x.
6. A concave mirror forms on a screen a real image which is three times the size of the object. The screen is moved until the image is five times the size of the object. If the shift of the screen is 30cm, determine the
   (i) focal length of the mirror.
   (ii) Shift of the object.
   (iii) New position of the object.
7. A concave mirror forms an image which half the size of the object. The object is then moved towards the mirror until the image size is three quarters the size of the object. If the image is moved by a distance of 0.8cm. find the;
   (i) focal length of the mirror.
   (ii) initial position of the object.
8. A concave mirror of radius of curvature 20cm faces a convex mirror of radius of curvature 10cm and is 28cm from it. If an object is placed midway between the mirrors, find the nature and position of the image formed by reflection first at the concave mirror and then at the convex mirror.
9. An object placed infront of a concave mirror of focal length f gives a real image of magnification, m. When the object is moved towards the mirror by a distance, d, a similarly magnified virtual image is formed. Show that $f = \frac{dm}{2}$.
10. A concave mirror forms a real image of magnification, m_1 on a screen. When the object is moved through a distance, y towards the mirror, the screen is moved until the magnification of the image is m_2. If the shift of the screen is x and $m_2 > m_1$, show that $x = m_1 m_2 d$.
    (skip 4 pages)

Reflection of a wide beam of light by spherical mirrors

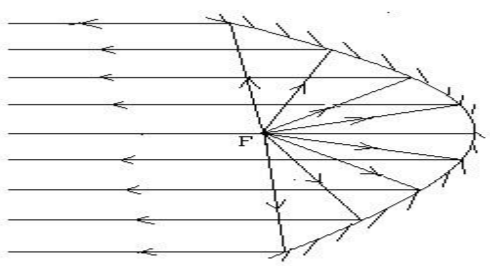
When a wide beam of light, parallel to the principal axis is incident on a concave mirror, the reflected rays are converged to different points on the principal axis. The marginal rays are converged to points close to the mirror while paraxial rays are brought to focus further away from the mirror.

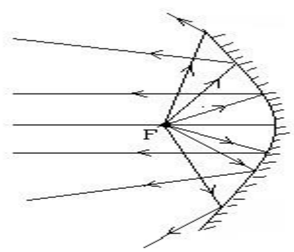


The reflected rays appear to touch a surface known as a caustics surface which has an apex at the principal focus.

Similarly, if a wide parallel beam of light is incident on a convex mirror, the different reflected rays appear to diverge from different points.

Comparison of concave mirrors and parabolic mirrors





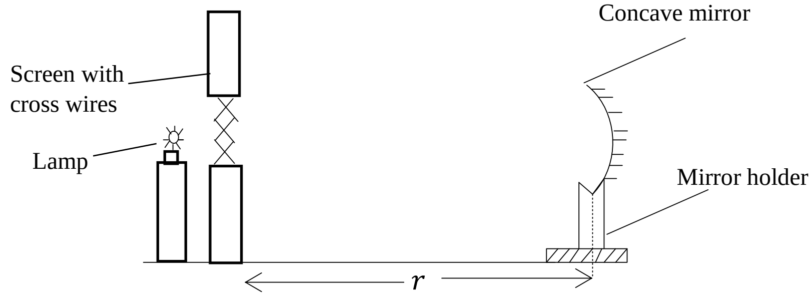
When a small lamp is placed at the principal focus of a parabolic mirror, all rays from the lamp that strike the mirror at points close to and far from the principle axis will be reflected as a beam parallel to the principle axis as in (a) above and the intensity of the reflected beam remains practically undiminished as the distance from the mirror increases.

When the lamp is placed at the principal focus of a concave mirror, only rays from this lamp that strike the mirror at points close to the principle axis will be reflected parallel to the principle axis and those striking at points well away from the principal axis will be reflected in different directions and not as a parallel beam as in (b) above. In this case the intensity of the reflected beam practically diminishes as the distance from the mirror increases.

This why parabolic mirrors are preferred for use in search lights to concave mirrors.

Experiments to determine the focal length of a concave mirror

(a) Using an illuminated object.



The apparatus are arranged as shown above.

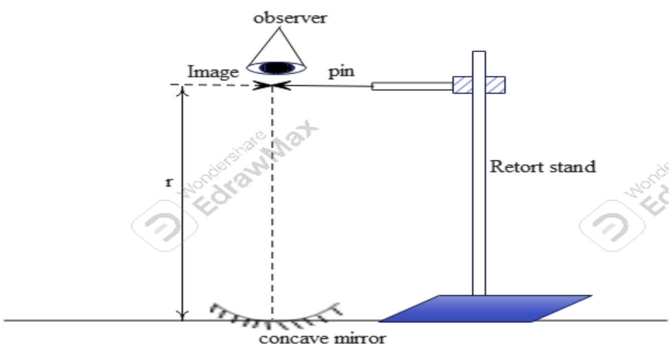
The cross wires are illuminated with light from the lamp and the concave mirror is moved towards and away from the screen until sharp image of the cross wires is formed on the screen.

The distance r between the concave mirror and screen is measured using a metre rule. This distance is the radius of curvature.

The focal length of the mirror is obtained from; $f = \frac{r}{2}$.

Theory: Light from the object (cross wires) is reflected from the mirror and comes back to the screen. Since the object and the image positions coincide, the screen position is therefore the centre of curvature of the mirror. So the distance from the mirror to the screen is the radius of curvature of the mirror.

(b) Using the no parallax method (using a pin and retort stand)



A concave mirror is placed on a horizontal surface with its reflecting surface facing upwards.

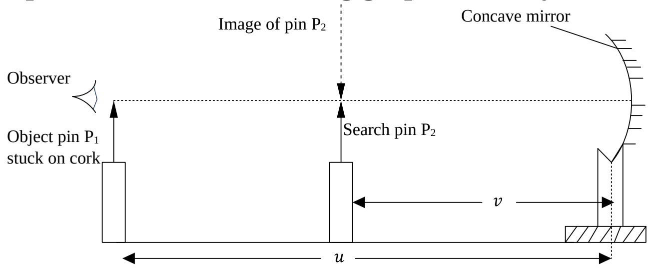
A pin is clamped horizontally on a retort stand with its tip lying along the principal axis.

While observing from above the mirror, the pin is adjusted vertically until it coincides with its image and there is no parallax between the pin and its image.

The distance of the pin from the mirror is measured and noted as r . This distance is the radius of curvature of the mirror.

The focal length of the mirror is obtained from $f = \frac{r}{2}$.

(c) Using the no parallax method involving graphical analysis



The apparatus are arranged as shown above.

The object pin P_1 is placed at a measured distance, u from the concave mirror with its tip lying along the principal axis of the mirror.

The position of the search pin P_2 placed between P_1 and the mirror is adjusted horizontally until it coincides with its inverted image above the principal axis.

The distance of pin P_2 from the mirror is measured and noted as v.

The procedures are repeated with several other values of u and the corresponding values of v are obtained.

Results are tabulated including values of $(u + v)$ and uv.

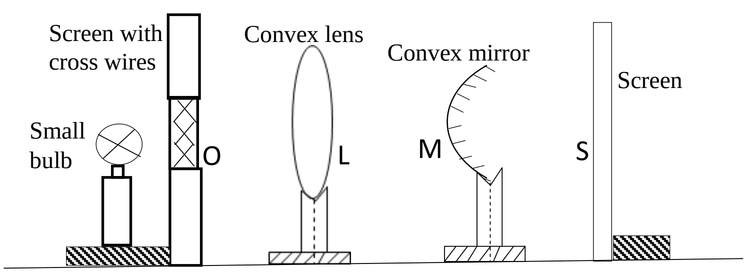
A graph of uv against $(u + v)$ is plotted and its slope is determined.

The slope of the graph is equal to the focal length of the mirror.

Alternatively: A graph of $\frac{1}{u}$ against $\frac{1}{V}$ can be plotted. The intercepts x and y on the axes are read and noted. The focal length of the mirror is obtained from $f = \frac{1}{2}\left(\frac{1}{x} + \frac{1}{y}\right)$

Experiments to determine the focal length of a convex mirror.

(a) Using a convex lens



First without the convex mirror, an illuminated object(O), a convex lens(L) and a screen(S) are arranged coaxially as shown above.

With a suitable distance, **OL** between the object and the lens, the position of the screen is adjusted until a sharp image, **I** is formed on it. Distance **LS** is measured and recorded.

The convex mirror whose focal length is required is placed between the convex lens and the screen(S) with its reflecting surface facing the lens.

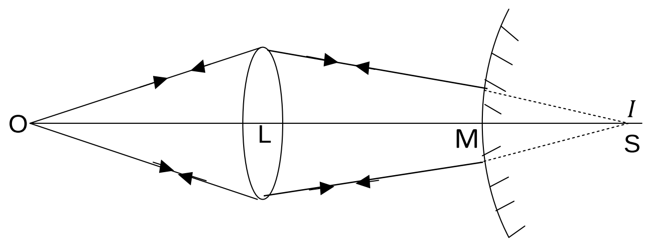
The position of the mirror is adjusted until the image coincides with the object. Distance LM is measured and recorded.

The focal length of the convex mirror is obtained from $\boldsymbol{f} = \frac{-(\boldsymbol{LS}-\boldsymbol{LM})}{\boldsymbol{2}}$

Theory

When the image coincides with the object, it means that light from the object is incident on the convex mirror normally and the reflected rays are reflected back along their original path. This implies that point S is the centre of curvature of the mirror and distance MS is the radius of curvature of the mirror. Hence the focal length of the mirror is $f = \frac{r}{2} = \frac{-MS}{2} = \frac{-(LS-LM)}{2}$.

Ray diagram showing the formation of the final image in the above arrangement.

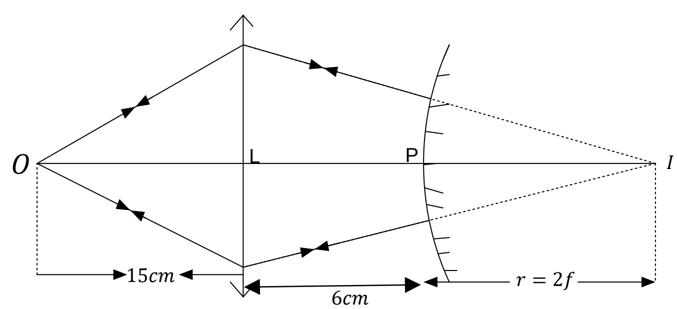


Examples

1. An object O is placed 15cm infront of a convex lens of focal length 10cm forming an image on the screen. A convex mirror placed between the lens and the screen forms a final image which coincide with O. If the convex mirror is 6cm from lens,
   (i) Draw a ray diagram to show how the final image is formed.
   (ii) Determine the focal length of the convex mirror.

Solution

(i)



(ii) Considering action of the convex lens.

$$u = OL = 15cm, f = 10cm, \qquad v = LI$$

$$\frac{1}{10} = \frac{1}{15} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{30}$$

$$v = 30cm$$

$$\therefore\ LI = 30cm$$

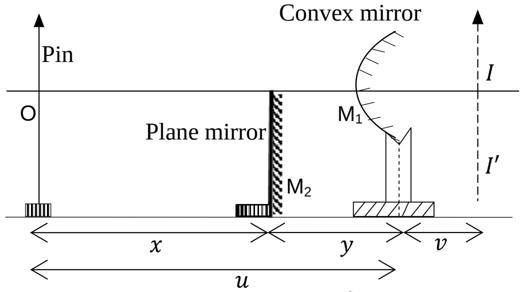
Considering action of the convex mirror.

$$r = -PI$$

$$PI = LI - LP$$
$$PI = 30 - 6 = 24$$
$$r = -24cm$$
$$2f = -24cm$$
$$f = -12cm$$

The focal length of the convex mirror is 12cm.

(b) Using a plane mirror



An object pin O which covers the aperture of the convex mirror M_1 is placed infront of the convex mirror so that the mirror forms a virtual image I.

A plane mirror M_2 is placed between the pin and the mirror so that it covers half the aperture of the mirror. The position of the mirror is adjusted until the image the image, I' of O in the plane mirror coincides with I.

Distances x and y are measured. The object and image distances are obtained from $u = x + y$ and $v = -(x - y)$.

The focal length, f of the convex mirror is obtained from; $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$.

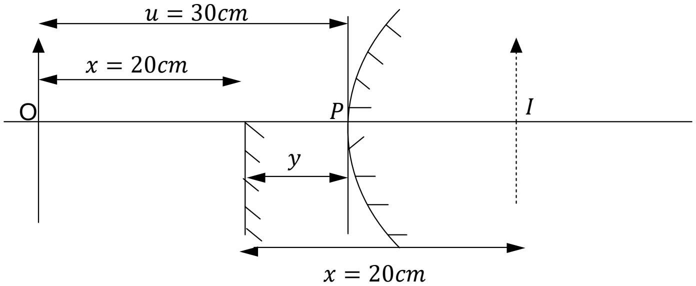
Theory

The two images (I and I') coincide when they are as far behind the plane mirror as the object is in front. In the plane mirror, the object and image distances are equal, therefore $OM_2 = M_2I = x$.

Examples

1. A plane mirror is placed 30cm infront of a convex mirror so that it covers half of the mirror surface. A pin placed 20cm infront of a plane mirror gives an image which coincide with that of the pin in the convex mirror. Find the focal length of the convex mirror.

Solution



$$u = OP = 30cm, \qquad y = 30 - 20 = 20cm$$
$$v = PI = -(x - y) = -(20 - 10) = -10cm$$
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$
$$\frac{1}{f} = \frac{1}{30} + \frac{1}{-10}$$
$$\frac{1}{f} = \frac{-1}{15}$$
$$f = -15cm$$

The focal length of the convex is 15cm.

Exercise

1. An object O placed 40cm infront of a convex lens of focal length 15cm forms a real image on the screen placed beyond the lens. A convex mirror placed between the lens and the screen forms a final image which coincide with object. If the convex mirror is 4cm from lens, determine the focal length of the convex mirror.
2. An illuminated object, a convex lens, and a convex mirror are arranged coaxially in that order. The object is 21cm from the lens and when the mirror is adjusted, the image coincides with the object when the mirror is 10cm from the convex lens. If the focal length of the convex mirror is 16cm, find the focal length of the convex lens.
3. A plane mirror is placed 12cm in front of a convex mirror so that it covers about half of the mirror surface. A pin 24cm in front of the plane mirror gives an image in it, which coincides with that of the pin in the convex mirror. Find the focal length of the convex mirror.
4. A plane mirror is placed at a distance l infront of a convex mirror of focal length f such that it just covers half the aperture of the mirror. A pin placed at a distance d in front of the plane mirror gives an image in it, which coincides with that of the pin in the convex mirror. With the aid of an illustration, show that $2fl = l^2 - d^2$.
5. You are provided with the following apparatus: A screen with cross wires, a lamp, a concave mirror, and a meter rule. Describe the experiment you would carryout to determine the focal length of a concave mirror using the given apparatus.

(skip 3 pages)