2019

JEE MAIN

COMPLETE PHYSICS





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Mc Graw Hill Education

JEE MAIN

COMPLETE PHYSICS

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To Our Readers...

How to Crack the JEE

This book contains 21 chapters which cover the complete **Physics** syllabus prescribed for **JEE (Main)**. Each chapter begins with a **Review of Basic Concepts** following by **Solved Examples**. Tips for quick solution to problems are given to save time. Then MCQs are given in Section I which is followed by **Answer Key** and **Complete Solutions** of all questions. MCQs based on passage form Section II. Section III is **Assertion-Reason Type Questions** with solutions. Finally in Section IV are given the questions asked (on the specific chapter) in various competitive examinations such as AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced). Complete solutions of these questions are provided at the end of the chapter. Questions of 2015 and 2016 with their solutions have been added at end of the book.

How to Use this Book

Students are advised to follow the following steps for a high score in JEE (Main).

STEP ONE

Begin each chapter by reading the **Review of Basic Concepts**. Grasp the fundamentals of the concepts given.

STEP TWO

Attempt to solve the examples given after the concept. If you fail to solve them, read and comprehend the concepts again and then try once more. If you fail to solve the question even after two attempts, read the detailed solution of the question and note where you made a mistake in the earlier two attempts. These solved examples are graded according to their difficulty level.

STEP THREE

In order to save time, read and keep a separate record of the tips and applications given after the solved examples.

STEP FOUR

All MCQs in Section III should be treated as exercises. These questions are graded in order of the level of difficulty. Students are advised to answer these questions without reading the solutions. If your answer does not match with the answer key given at the end of the questions, try again. If you fail to obtain the correct answer even after two attempts, read the detailed solution of the question given after the answer key. Note the mistakes you had made in the previous attempts.

STEP FIVE

Now you are ready to test yourself in the chapter you have studied. In Section IV are MCQs asked in previous years' competitive examinations with **Answer Key** and complete **Solutions**. Answer all the questions without, at first, looking at the solutions. Calculate your total score and compare it with the following Grades Table:

		Grade
95%	correct	A^{++}
90%	correct	A^{+}
85%	correct	A
75%	correct	B^{+}
70%	correct	B
Below 70%	correct	Poor

N K BAJAJ

Trend Analysis

Chapter No.	Topic	Marks Allotted			
		2014	2015	2016	2017
1.	Physics and Measurement	4	0	4	0
2.	Kinematics	8	4	4	4
3.	laws of Motion	4	8	4	4
4.	Work, Energy and Power	8	0	4	8
5.	Rotational Motion	4	8	8	8
6.	Gravitation	4	4	4	4
7.	Solids and Fluids	8	8	0	12
8.	Heat and Thermodynamics	8	4	12	4
9.	Kinetic Theory of Gases	4	4	4	4
10.	Oscillation and Waves	8	12	12	8
11.	Electrostatics	8	12	8	12
12.	Current Electricity	8	4	0	12
13.	Magnetic Effects of Current, and Magnetism	8	12	12	8
14.	Electromagnetic Induction and Alternating Currents	4	12	4	4
15.	Electromagnetic Waves	4	4	4	0
16.	Ray Optics	4	8	8	4
17.	Wave Optics	4	4	4	4
18.	Dual Nature of Matter and Radiation	4	4	4	4
19.	Atoms and Nuclei	8	4	4	8
20.	Electronics Devices	4	4	12	4
21.	Communication Systems	4	0	4	4
	Total Marks	120	120	120	120

About JEE Main

1. Introduction and Scheme of Examination

The Joint Entrance Examination from the year 2013 for admission to the undergraduate programmes in Engineering is being held in two parts, JEE-Main and JEE-Advanced. Only the top 1,50,000 candidates (including all categories) based on performance in JEE Main will qualify to appear in the JEE Advanced examination. Admissions to IITs will be based only on category-wise All India Rank (AIR) in JEE Advanced, subject to condition that such candidates are in the top 20 percentile categories. Admission to NITs will be based on 40% weightage for performance in Class XII board marks (normalized) and the remainder 60% weightage would be given to performance in JEE Main and a combined All India Rank (AIR) would be decided accordingly.

In case any State opts to admit students in the engineering Colleges affiliated to state Universities where States require separate merit list to be provided based on relative weightages adopted by the states, then the merit list shall be prepared with such relative weightages as may be indicated by States.

2. Eligibility Criteria and List of Qualifying Examinations for JEE(Main) Exam

The minimum academic qualification for appearing in JEE(Main) is that the candidate must have passed in final examination of 10+2 (Class XII) or its equivalent referred to as the qualifying examination (see below).

However, admission criteria in the concerned institution/university will be followed as prescribed by concerned university/institution and as per the guidelines & criteria prescribed by AICTE.

Qualifying Examinations

List of Qualifying Examinations

(i) The +2 level examination in the 10+2 pattern of examination of any recognized Central/State Board of Secondary Examination, such as Central Board of Secondary Education, New Delhi, and Council for Indian School Certificate Examination, New Delhi

(ii) Intermediate or two-year Pre-University Examination conducted by a recognized Board/University.

(iii) Final Examination of the two-year course of the Joint Services Wing of the National Defence Academy.

(iv) Any Public School/Board/University Examination in India or in foreign countries recognized by the Association of Indian Universities as equivalent to 10+2 system.

(v) H.S.C. Vocational Examination.

(vi) A pass grade in the Senior Secondary School Examination conducted by the National Open School with a minimum of five subjects.

(vii) 3- or 4-year diploma recognized by AICTE or a State Board of Technical Education.

3. Pattern of Examination

Subject combination for each paper and type of questions in each paper are given below:

	Subjects	Type of Questions	Duration
Paper 1	Physics, Chemistry & Mathematics	Objective type questions with equal weightage to Physics, Chemistry & Mathematics	3 Hours
Paper 2	Mathematics – Part I	Objective type questions	3 Hours
	Aptitude Test – Part II &	Objective type questions	
	Drawing Test – Part III	Questions to Test Drawing Aptitude	

Requirement of papers for different courses is given in the table below:

Course	Papers
B.E/B.TECH	Paper – 1
B.ARCH/B. PLANNING	Paper – 2

Scoring and Negative Marking

There will be objective type questions with four options having single correct answer. For each incorrect response, one fourth (1/4) of the total marks allotted to the question would be deducted. No deduction from the total score will, however, be made if no response is indicated for an item in the answer sheet.

Syllabus

Unit-I: Physics and Measurement

Physics, technology and society, S I units, Fundamental and derived units. Least count, accuracy and precision of measuring instruments, errors in measurement, significant figures.
Dimensions of Physical quantities, dimensional analysis and its applications.

Unit-II: Kinematics

Frame of reference. Motion in a straight line: Position-time graph, speed and velocity. Uniform and non-uniform motion, average speed and instantaneous velocity

Uniformly accelerated motion, velocity-time, position-time graphs, relations for uniformly accelerated motion.

Scalars and Vectors, Vector addition and Subtraction, Zero Vector, Scalar and Vector products, Unit Vector, Resolution of a Vector. Relative Velocity, Motion in a plane, Projectile Motion, Uniform Circular Motion.

Unit-III: Laws of Motion

Force and Inertia, Newton's First Law of motion; Momentum, Newton's Second Law of motion; Impulse; Newton's Third Law of motion. Law of conservation of linear momentum and its applications, Equilibrium of concurrent forces.

Static and Kinetic friction, laws of friction, rolling friction.

Dynamics of uniform circular motion: Centripetal force and its applications.

Unit-IV: Work, Energy and Power

Work done by a constant force and a variable force; kinetic and potential energies, work-energy theorem, power.

Potential energy of a spring, conservation of mechanical energy, conservative and non-conservative forces; elastic and inelastic collisions in one and two dimensions.

Unit-V: Rotational Motion

Centre of mass of a two-particle system, centre of mass of a rigid body; basic concepts of rotational motion; moment of a force, torque, angular momentum, conservation of angular momentum and its applications; moment of inertia, radius of gyration. Values of moments of inertia for simple geometrical objects, parallel and perpendicular axes theorems and their applications.

Rigid body rotation, equations of rotational motion.

Unit-VI: Gravitation

The universal law of gravitation.

Acceleration due to gravity and its variation with altitude and depth.
Kepler's laws of planetary motion.

Gravitational potential energy; gravitational potential.

Escape velocity. Orbital velocity of a satellite. Geo-stationary satellites.

Unit-VII: Properties of Solids and Liquids

Elastic behaviour, Stress-strain relationship, Hooke's Law, Young's modulus, bulk modulus, modulus of rigidity.

Pressure due to a fluid column; Pascal's law and its applications. Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, Reynolds number. Bernoulli's principle and its applications.

Surface energy and surface tension, angle of contact, application of surface tension-drops, bubbles and capillary rise.

Heat, temperature, thermal expansion; specific heat capacity, calorimetry; change of state, latent heat.

Heat transfer-conduction, convection and radiation, Newton's law of cooling.

Unit-VIII: Thermodynamics

Thermal equilibrium, zeroth law of thermodynamics, concept of temperature. Heat, work and internal energy. First law of thermodynamics.

Second law of thermodynamics: reversible and irreversible processes. Camot engine and its efficiency.

Unit-IX: Kinetic Theory of Gases

Equation of state of a perfect gas, work done on compressing a gas.

Kinetic theory of gases-assumptions, concept of pressure. Kinetic energy and temperature: rms speed of gas molecules; Degrees of freedom, Law of equipartition of energy, applications to specific heat capacities of gases; Mean free path, Avogadro's number.

Unit-X: Oscillations and Waves

Periodic motion-period, frequency, displacement as a function of time. Periodic functions. Simple harmonic motion (S.H.M.) and its equation; phase; oscillations of a spring-restoring force and force constant; energy in S.H.M.-kinetic and potential energies; Simple pendulum-derivation of expression for its time period; Free, forced and damped oscillations, resonance.
Wave motion. Longitudinal and transverse waves, speed of a wave. Displacement relation for a progressive wave. Principle of superposition of waves, reflection of waves, Standing waves in strings and organ pipes, fundamental mode and harmonics, Beats, Doppler effect in sound.

Unit-XI: Electrostatics

Electric charges: Conservation of charge, Coulomb's law—forces between two point charges, forces between multiple charges; superposition principle and continuous charge distribution.

Electric field: Electric field due to a point charge, Electric field lines, Electric dipole, Electric field due to a dipole, Torque on a dipole in a uniform electric field.

Electric flux, Gauss's law and its applications to find field due to infinitely long, uniformly charged straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell.

Electric potential and its calculation for a point charge, electric dipole and system of charges; Equipotential surfaces, Electrical potential energy of a system of two point charges in an electrostatic field.

Conductors and insulators, Dielectrics and electric polarization, capacitor, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, Energy stored in a capacitor.

Unit-XII: Current Electricity

Electric current, Drift velocity, Ohm's law, Electrical resistance, Resistances of different materials, V-I characteristics of Ohmic and nonohmic conductors, Electrical energy and power, Electrical resistivity, Colour code for resistors; Series and parallel combinations of resistors; Temperature dependence of resistance.

Electric Cell and its Internal resistance, potential difference and emf of a cell, combination of cells in series and in parallel.

Kirchhoff's laws and their applications. Wheatstone bridge, Metre bridge.

Potentiometer-principle and its applications.

Unit-XIII: Magnetic Effects of Current and Magnetism

Biot-Savart law and its application to current carrying circular loop.

Ampere's law and its applications to infinitely long current carrying straight wire and solenoid.

Force on a moving charge in uniform magnetic and electric fields. Cyclotron.

Force on a current-carrying conductor in a uniform magnetic field. Force between two parallel current-carrying conductors-definition of ampere. Torque experienced by a current loop in uniform magnetic field; Moving coil galvanometer, its current sensitivity and conversion to ammeter and voltmeter.

Current loop as a magnetic dipole and its magnetic dipole moment. Bar magnet as an equivalent solenoid, magnetic field lines; Earth's magnetic field and magnetic elements. Para-, dia- and ferro- magnetic substances
Magnetic susceptibility and permeability, Hysteresis, Electromagnets and permanent magnets.

Unit-XIV: Electromagnetic Induction and Alternating Currents

Electromagnetic induction; Faraday's law, induced emf and current; Lenz's Law, Eddy currents. Self and mutual inductance.

Alternating currents, peak and rms value of alternating current/ voltage; reactance and impedance; LCR series circuit, resonance; Quality factor, power in AC circuits, wattless current.

AC generator and transformer.

Unit-XV: Electromagnetic Waves

Electromagnetic waves and their characteristics. Transverse nature of electromagnetic waves.

Electromagnetic spectrum (radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays). Applications of e.m. waves.

Unit-XVI: Optics

Reflection and refraction of light at plane and spherical surfaces, mirror formula, Total internal reflection and its applications, Deviation and Dispersion of light by a prism, Lens Formula, Magnification, Power of a Lens, Combination of thin lenses in contact, Microscope and Astronomical Telescope (reflecting and refracting) and their magnifying powers.

Wave Optics: wavefront and Huygens' principle, Laws of reflection and refraction using Huygen's principle. Interference, Young's double slit experiment and expression for fringe width, coherent sources and sustained interference of light. Diffraction due to a single slit, width of central maximum. Resolving power of microscopes and astronomical telescopes, Polarisation, plane polarized light; Brewster's law, uses of plane polarized light and Polaroids.

Unit-XVII: Dual Nature of Matter and Radiation

Dual nature of radiation. Photoelectric effect, Hertz and Lenard's observations; Einstein's photoelectric equation; particle nature of light.

Matter waves: wave nature of particle, de Broglie relation. Davisson-Germer experiment.

Unit-XVIII: Atoms and Nuclei

Alpha-particle scattering experiment; Rutherford's model of atom; Bohr model, energy levels, hydrogen spectrum.

Composition and size of nucleus, atomic masses, isotopes, isobars; isotones. Radioactivity-alpha, beta and gamma particles/rays and their properties; radioactive decay law. Mass-energy relation, mass defect; binding energy per nucleon and its variation with mass number, nuclear fission and fusion.

Unit-XIX: Electronic Devices

Semiconductors; semiconductor diode: I-V characteristics in forward and reverse bias; diode as a rectifier; I-V characteristics of LED, photodiode, solar cell, and Zener diode; Zener diode as a voltage regulator. Junction transistor, transistor action, characteristics of a transistor; transistor as an amplifier (common emitter configuration) and oscillator. Logic gates (OR, AND, NOT, NAND and NOR). Transistor as a switch.

Unit-XX: Communication Systems

Propagation of electromagnetic waves in the atmosphere; Sky and space wave propagation, Need for modulation, Amplitude and Frequency Modulation, Bandwidth of signals, Bandwidth of Transmission medium, Basic Elements of a Communication System (Block Diagram only).

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PHYSICS AND MEASUREMENT

Chapter 1

REVIEW OF BASIC CONCEPTS

1. The SI System of Units

The internationally accepted standard units of the fundamental physical quantities are given in Table 1.1.

Table 1.1 *Fundamental SI Units*

Physical Quantity	*Name of the Unit*	*Symbol*
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol
Angle in a plane	radian	rad
Solid angle	steradian	sr

2. Dimensions of Physical Quantities

The dimensions of a physical quantity are the powers to which the fundamental units of mass (M), length (L) and time (T) must be raised to represent the unit of that quantity. The dimensional formula of a physical quantity is an expression that tells us how and which of the fundamental quantities enter into the unit of that quantity.

In *mechanics*, the dimensional formula is written in terms of the dimensions of mass, length and time (M, L and T). In *heat* and *thermodynamics*, in addition to M, L and T, we need to mention the dimension of temperature in kelvin (K). In *electricity and magnetism*, in addition to M, L and T, we need to mention the dimension of current or charge per unit time (A or QT^{-1}).

EXAMPLE 1 Find the dimensional formula of (a) velocity, (b) acceleration, (c) force, (d) work, (e) energy and (f) pressure.

SOLUTION

(a) Velocity $(v) = \dfrac{\text{distance}}{\text{time}} = \dfrac{dx}{dt} = \dfrac{[\text{L}]}{[\text{T}]}$

$= [\text{L T}^{-1}] = [\text{M}^0 \text{LT}^{-1}]$

(b) Acceleration $(a) = \dfrac{dv}{dt} = \dfrac{[\text{M}^0 \text{ L T}^{-1}]}{[\text{T}]} = [\text{M}^0 \text{ L T}^{-2}]$

(c) Force $(F) = ma = [\text{M}] \times [\text{M}^0 \text{ L T}^{-2}] = [\text{M L T}^{-2}]$

(d) Work $(W) = Fx = [\text{M L T}^{-2}] \times [\text{L}] = [\text{M L}^2 \text{ T}^{-2}]$

(e) Energy = work $= [\text{M L}^2 \text{ T}^{-2}]$

(f) Pressure $= \dfrac{\text{force}}{\text{area}} = \dfrac{[\text{MLT}^{-2}]}{[\text{L}^2]} = [\text{ML}^{-1} \text{ T}^{-2}]$

Table 1.2 gives the dimensional formulae of some important physical quantities.

3. Principle of Homogeneity of Dimensions

Consider a simple equation,

$$A + B = C.$$

If this is an equation of Physics, i.e. if A, B and C are physical quantities, then this equation says that one physical quantity A, when added to another physical quantity B, gives a third physical quantity C. This equation will have no meaning in Physics if the nature (i.e. the dimensions) of the quantities on the left-hand side of the equation is not the same as the nature of the quantity on the right-hand side. For example, if A is a length, B must also be a length and the result of addition of A and B must express a length. In other words, the dimensions of both sides of a physical equation must be identical. This is called the *principle of homogeneity of dimensions*.

Table 1.2 *Dimensional Formulae of some Physical Quantities*

Physical Quantity	*Dimensional Formula*	*Physical Quantity*	*Dimensional Formula*
Area	$M^0L^2T^0$	Heat energy	ML^2T^{-2}
Volume	$M^0L^3T^0$	Entropy	$ML^2T^{-2}K^{-1}$
Density	$ML^{-3}T^0$	Specific heat	$M^0L^2T^{-2}K^{-1}$
Velocity	M^0LT^{-1}	Latent heat	$M^0L^2T^{-2}$
Acceleration	M^0LT^{-2}	Molar specific heat	$ML^2T^{-2}K^{-1}\ mol^{-1}$
Momentum	MLT^{-1}	Thermal conductivity	$MLT^{-3}K^{-1}$
Angular momentum	ML^2T^{-1}	Wien's constant	M^0LT^0K
Force	MLT^{-2}	Stefan's constant	$ML^0T^{-3}K^{-4}$
Energy, work	ML^2T^{-2}	Boltzmann's constant	$ML^2T^{-2}K^{-1}$
Power	ML^2T^{-3}	Molar gas constant	$ML^2T^{-2}K^{-1}\ mol^{-1}$
Torque, couple	ML^2T^{-2}	Electric charge	TA or Q
Impulse	MLT^{-1}	Electric current	A or QT^{-1}
Frequency	$M^0L^0T^{-1}$	Electric potential	$MLT^{-2}Q^{-1}$ or $ML^2T^{-3}A^{-1}$
Angular frequency	$M^0L^0T^{-1}$	Electric field	$MLT^{-2}Q^{-1}$ or $MLT^{-3}A^{-1}$
Angular acceleration	$M^0L^0T^{-2}$	Capacitance	$M^{-1}L^{-2}T^4A^2$ or $M^{-1}L^{-2}T^2Q^2$
Pressure	$ML^{-1}T^{-2}$	Inductance	$ML^2T^0Q^{-2}$ or $ML^2T^{-2}A^{-2}$
Elastic modulii	$ML^{-1}T^{-2}$	Resistance	$ML^2T^{-1}Q^{-2}$ or $ML^2T^{-3}A^{-2}$
Stress	$ML^{-1}T^{-2}$	Magnetic flux	$ML^2T^{-1}Q^{-1}$ or $ML^2T^{-2}A^{-1}$
Moment of inertia	ML^2T^0	Magnetic flux density or Magnetic induction field	$ML^0T^{-1}Q^{-1}$ or $ML^0T^{-2}A^{-1}$
Surface tension	ML^0T^{-2}	Permeability	MLQ^{-2} or $MLT^{-2}A^{-2}$
Viscosity	$ML^{-1}T^{-1}$	Permittivity	$M^{-1}L^{-3}T^2Q^2$ or $M^{-1}L^{-3}T^4A^2$
Gravitational constant	$M^{-1}L^3T^{-2}$	Planck's constant	ML^2T^{-1}

EXAMPLE 2 The distance x travelled by a body varies with time t as

$x = at + bt^2$, where a and b are constants.

Find the dimensions of a and b.

SOLUTION The dimensions of each term on the right hand of the given equation must be the same as those of the left hand side. Hence

Dimensions of at = dimensions of x

or $$[a] = \frac{[x]}{[t]} = \frac{[\mathrm{L}]}{[\mathrm{T}]} = [\mathrm{LT}^{-1}] = [\mathrm{M}^0\mathrm{LT}^{-1}]$$

Dimensions of bt^2 = dimensions of x

or $$[b] = \frac{[x]}{[t^2]} = \frac{[\mathrm{L}]}{[\mathrm{T}^2]} = [\mathrm{LT}^{-2}] = [\mathrm{M}^0\,\mathrm{LT}^{-2}]$$

EXAMPLE 3 The pressure P, volume V and temperature T of a gas are related as

$$\left(P + \frac{a}{V^2}\right)(V - b) = cT$$

where $a, b,$ and c are constants. Find the dimensions of $\frac{a}{b}$.

SOLUTION Dimensions of $\frac{a}{V^2}$ = dimensions of P.

$\therefore$ Dimensions of a = dimensions of PV^2

Also dimensions of b = dimensions of V

$\therefore$ Dimensions of $\frac{a}{b} = \frac{[PV^2]}{[V]} = [PV] = [ML^{-1}T^{-2}] \times [L^3]$

$= [ML^2T^{-2}]$

Note

1. Trigonometric functions (sin, cos, tan, cot, etc.) are dimensionless. The arguments of these functions are also dimensionless.
2. Exponential functions are dimensionless. Their exponents are also dimensionless.

EXAMPLE 4 When a plane wave travels in a medium, the displacement y of a particle located at x at time t is given by

$$y = a \sin(bt + cx)$$

where a, b and c are constant. Find the dimensions of $\frac{b}{c}$.

SOLUTION Terms bt and cx must be dimensionless. Hence

$$[b] = \frac{1}{[t]} = [T^{-1}]$$

and

$$[c] = \frac{1}{[x]} = \frac{1}{[L]} = [L^{-1}]$$

$$\therefore \quad \left[\frac{b}{c}\right] = [LT^{-1}] = [M^0 LT^{-1}]$$

Note that the dimensions of a are the same as those of y.

EXAMPLE 5 In the expression

$$P = \frac{a^2}{b} e^{-ax}$$

P is pressure, x is a distance and a and b are constants. Find the dimensional formula for b.

SOLUTION $\left[\frac{a^2}{b}\right] = [P]$

Also ax is dimensionless. Hence $[a] = [L^{-1}]$.

$$\therefore \quad [b] = \frac{[a^2]}{[P]} = \frac{[L^{-2}]}{[ML^{-1}T^{-2}]} = [M^{-1}L^{-1}T^2]$$

The principle of homogeneity of dimensions can also be used to find the dependence of a physical quantity on other physical quantities.

EXAMPLE 6 In the expression

$$L = \frac{a + bx}{ct}$$

L is magnitude of angular momentum, x is a length, t is time and a, b, and c are constants. The dimensions of $\frac{a}{c}$ are the same as those of

(a) torque (b) moment of inertia
(c) work (d) Impulse

SOLUTION $L = \frac{a}{ct} + \frac{bx}{ct}$

$$\therefore \quad \left[\frac{a}{c}\right] = [Lt] = [mvr] \times [t]$$

$$= [M] \times [LT^{-1}] \times [L] \times [T]$$

$$= [ML^2T^0]$$

The correct choice is (b).

EXAMPLE 7 Einstein's photoelectric equation is

$$K_{max} = h\nu - W_0$$

where K_{max} is the maximum kinetic energy of photoelectrons, ν is the frequency of incident radiation, W_0 is the work function of the metal and h is Planck's constant. Find the dimension formula of (a) h and (b) W_0.

SOLUTION The dimensions of $h\nu$ and W_0 are the same as those of K_{max}.

(a) $\therefore \quad [h\nu] = [K_{max}]$

$$\Rightarrow \quad [h] = \left[\frac{K_{max}}{\nu}\right] = \left[\frac{ML^2T^{-2}}{T^{-1}}\right] = [ML^2T^{-1}]$$

(b) $$[W_0] = [K_{max}] = [ML^2T^{-2}]$$

EXAMPLE 8 If pressure P is given by

$$P = \frac{a - \ell^2}{bt}$$

where ℓ is a length, t is time and a and b are constants, find the dimensional formula of b.

SOLUTION $[a] = [\ell^2] = [L^2]$

Also $[Pbt] = [a]$

$$\Rightarrow \quad [ML^{-1}T^{-2}] \times [b] \times [T] = [L^2]$$

$$\Rightarrow \quad [ML^{-1}T^{-1}] \times [b] = [L^2]$$

$$\Rightarrow \quad [b] = \frac{[L^2]}{[ML^{-1}T^{-1}]} = [M^{-1}L^3T]$$

EXAMPLE 9 Find the dimensional formula of (a) gravitation constant (G), (b) coefficient of viscosity (η), (c) specific heat capacity (s).

SOLUTION (a) From Newton's law of gravitation

$$F = \frac{G m_1 m_2}{r^2}$$

$$\therefore \quad [G] = \frac{[F] \times [r^2]}{[m_1] \times [m_2]} = \frac{[MLT^{-2}] \times [L^2]}{[M^2]}$$

$$= [M^{-1}L^3T^{-2}]$$

(b) The resistive force on a spherical body of radius r, moving with a velocity v in a fluid of coefficient of viscosity η is given by

$$F = 6\pi\eta r v$$

$$\therefore \quad [\eta] = \frac{[F]}{[r]\times[v]} = \frac{[MLT^{-2}]}{[L]\times[LT^{-1}]} = [ML^{-1}T^{-1}]$$

(c) $Q = ms\Delta T$

where Q is heat energy, m is the mass of the body and ΔT is the change in temperature.

$$\therefore \quad [s] = \frac{[Q]}{[m\Delta T]} = \frac{[ML^2T^{-2}]}{[MK]} = [M^0L^2T^{-2}K^{-1}]$$

EXAMPLE 10 In a standing wave, the displacement of a particle at x at time t is given by

$$y(x, t) = a \sin(bx)\cos(ct)$$

where a, b, and c are constants. Find the dimensions of $\frac{b}{c}$ are the same as those of

(a) wavelength (b) wave velocity

(c) $\frac{1}{\text{wavelength}}$ (d) $\frac{1}{\text{wave velocitiy}}$

SOLUTION Since bx and ct must be dimensionless, dimension of b is $[L^{-1}]$ and c is $[T^{-1}]$. Therefore

$$\left[\frac{b}{c}\right] = \frac{[L^{-1}]}{[T^{-1}]} = \frac{1}{[LT^{-1}]} = \frac{1}{\text{velocity}}$$

EXAMPLE 11 When a liquid of coefficient of viscosity η and density ρ flows through a pipe of radius r, the flow remains streamline if its velocity does not exceed a value v_c given by

$$v_c = \frac{k\eta}{\rho r}$$

where k is a constant called Reynold's constant. Show that k is dimensionless.

SOLUTION

$$k = \frac{v_c\,\rho\, r}{\eta}$$

$$\therefore \quad [k] = \frac{[v_c]\times[\rho]\times[r]}{[\eta]} = \frac{[LT^{-1}]\times[ML^{-3}]\times[L]}{[ML^{-1}T^{-1}]} = [M^0L^0T^0]$$

EXAMPLE 12 Find the dimensional formula for (a) electric field E and (b) magnetic field B in terms of M, L, T and A where A represents dimension of electric current.

SOLUTION

(a) $$E = \frac{F}{q} \quad \Rightarrow \quad [E] = \frac{[F]}{[q]}$$

$$\text{Current} = \frac{\text{charge}}{\text{time}}$$

or $$I = \frac{q}{t} \Rightarrow q = It \Rightarrow [q] = [AT]$$

$$\therefore \quad [E] = \frac{[MLT^{-2}]}{[AT]} = [MLT^{-3}A^{-1}]$$

(b) The force experienced by a charge q moving with a velocity v in a magnetic field **B** is given by

$$\mathbf{F} = q(v \times \mathbf{B})$$

Magnitude of **F** is $F = qvB\sin\theta$. Since $\sin\theta$ is dimentionless

$$[B] = \frac{[F]}{[q]\times[v]} = \frac{[MLT^{-2}]}{[AT]\times[LT^{-1}]} = [ML^0T^{-2}A^{-1}]$$

EXAMPLE 13 The dimensions of $\frac{E}{B}$ are the same as those of

(a) current (b) $\frac{1}{\text{current}}$

(c) velocity (d) $\frac{1}{\text{velocity}}$

SOLUTION Lorentz force is given by

$$\mathbf{F} = \mathbf{q}[\mathbf{E} + v \times \mathbf{B}]$$

Hence dimensions **E** are the same as those of $v \times \mathbf{B}$.

$$\therefore \quad [E] = [v]\times[B] \quad (\because \sin\theta \text{ is dimensionless})$$

$$\Rightarrow \quad \left[\frac{E}{B}\right] = [v], \text{ which is choice (c).}$$

EXAMPLE 14 Find the dimensional formula of (a) electrical permittivity (ε) and (b) magnetic permeability (μ).

SOLUTION The force between two charges q_1 and q_2 separated by a distance r in a medium of permittivity ε is

$$F = \frac{q_1\, q_2}{4\pi\,\varepsilon\, r^2}$$

$$\therefore \quad [\varepsilon] = \frac{[q_1]\times[q_2]}{[F]\times[r^2]}$$

$$= \frac{[AT]\times[AT]}{[MLT^{-2}]\times[L^2]}$$

$$= [M^{-1}L^{-3}T^4A^2]$$

(b) The magnetic field inside a long solenoid of length ℓ having N turns carrying a current I is given by

$$B = \frac{\mu N I}{\ell}$$

where μ is the permeability of the core. Therefore

$$[\mu] = \frac{[B]\times[\ell]}{[I]}$$

$$= \frac{[ML^0T^{-2}A^{-1}]\times[L]}{[A]}$$

$$= [MLT^{-2}A^{-2}]$$

EXAMPLE 15 The dimensions of $\mu\varepsilon$ are the same as those of

(a) current (b) $\frac{1}{(\text{current})^2}$

(c) velocity (d) $\frac{1}{(\text{velocity})^2}$

SOLUTION The velocity of an electromagnetic wave travelling in a medium of permeability μ and permittivity ε is given by

$$v = \frac{1}{\sqrt{\mu\varepsilon}}.$$

Hence the correct choice is (d).

EXAMPLE 16 Find the dimensions of $\varepsilon_0 E^2$.

SOLUTION The energy density u (energy per unit volume) of the electric field when an electromagnetic wave travels in a vacuum is given by

$$u = \frac{1}{2}\varepsilon_0 E^2$$

$$\therefore \quad [\varepsilon_0 E^2] = [u] = \frac{[\text{energy}]}{[\text{volume}]}$$

$$= \frac{(ML^2T^{-2})}{[L^3]}$$

$$= [ML^{-1}T^{-2}]$$

EXAMPLE 17 Find the dimensional formula of (a) electrical resistance and (b) electrical conductivity.

SOLUTION

(a) $V = IR$. Therefore $[R] = \frac{[V]}{[I]}$ (1)

Now $V = \frac{W}{q}$.

Therefore $[V] = \frac{[W]}{[q]} = \frac{[ML^2T^{-2}]}{[AT]}$

$$= [ML^2T^{-3}A^{-1}] \quad (2)$$

Using (2) in (1) we get

$$[R] = \frac{[ML^2T^{-3}A^{-1}]}{[A]}$$

$$= [ML^2T^{-3}A^{-2}]$$

(b) Electrical resistivity $\rho = \frac{RA}{\ell}$. Electrical conductivity is defined as

$$\sigma = \frac{1}{\rho} = \frac{\ell}{RA}; \quad A = \text{area}$$

$$\therefore \quad [\sigma] = \frac{[\ell]}{[R]\times[A]} = \frac{[L]}{[ML^2T^{-3}A^{-2}]\times[L^2]}$$

$$= [M^{-1}L^{-3}T^3A^2]$$

EXAMPLE 18 Find the dimensional formula of (a) capacitance C and (b) inductance L.

SOLUTION (a) $Q = CV$

$$\therefore \quad [C] = \frac{[Q]}{[V]} = \frac{[AT]}{[ML^2T^{-3}A^{-1}]}$$

$$= [M^{-1}L^{-2}T^4A^2]$$

(b) $\phi = LI$, ϕ = magnetic flux (**B**.**A**)

$$\therefore \quad [L] = [\phi]\times[I]^{-1} = [B]\times[A]\times[I]^{-1}$$

$$= [ML^0T^{-2}A^{-1}]\times[L^2]\times[A^{-1}]$$

$$= [ML^2T^{-2}A^{-2}]$$

EXAMPLE 19 If T denotes dimension of time period, the dimensions of CL are

(a) T^{-2} (b) T^{-1}

(c) T (c) T^2

SOLUTION The angular frequency of an LC circuit is given by

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad LC = \frac{1}{\omega^2} = \frac{1}{4\pi^2\nu^2} = \frac{T^2}{4\pi^2}$$

So the correct choice is (d).

EXAMPLE 20 Find the dimensions of $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$.

SOLUTION $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \sqrt{\frac{L}{R^2C}} = \sqrt{\frac{L/R}{RC}}$

$\frac{L}{R}$ is the time constant of an *LR* circuit and *RC* is the time constant of an *RC* circuit. Hence *Q* is dimensionless.

EXAMPLE 21 The time period (t) of a simple pendulum may depend upon m the mass of the bob, l the length of the string and g the acceleration due to gravity. Find the dependence of t on m, l and g.

SOLUTION

Let $t \propto m^a l^b g^c$

or $t = k\, m^a l^b g^c$,

where k is a dimensionless constant.

Writing the dimensions of each quantity, we have

$$[\mathrm{T}] = [\mathrm{M}]^a\,[\mathrm{L}]^b\,[\mathrm{LT}^{-2}]^c$$

or $$[\mathrm{M}^0\,\mathrm{L}^0\,\mathrm{T}] = [\mathrm{M}^a\mathrm{L}^{b+c}\mathrm{T}^{-2c}]$$

According to the principle of homogeneity of dimensions, the dimensions of all the terms on either side of this equation must be the same. Equating the powers of M, L and T, we have

$a = 0$, $b + c = 0$ and $-2c = 1$,

which give $b = \frac{1}{2}$ and $c = -\frac{1}{2}$.

Hence $t = k\, m^0\, l^{1/2}\, g^{(-1/2)}$

$\Rightarrow$ $t = k\sqrt{\frac{l}{g}}$

Thus t is independent of the mass of the bob and is directly proportional to $\sqrt{l}$ and inversely proportional to $\sqrt{g}$.

EXAMPLE 22 Assuming that the time period T of a planet depends upon its mean distance r from the sun, gravitation constant G and mass m of the sun, show, by using dimensional considerations that $T \propto r^{3/2}$.

SOLUTION Let $T \propto r^a\, G^b\, m^c$

or $T = k\, r^a\, G^b\, m^c$

Substituting dimensions,

$$[\mathrm{M}^0\,\mathrm{L}^0\,\mathrm{T}] = [\mathrm{L}]^a \times [\mathrm{M}^{-1}\,\mathrm{L}^3\,\mathrm{T}^{-2}]^b \times [\mathrm{M}]^c$$
$$= [\mathrm{M}^{-b+c}] \times [\mathrm{L}^{a+3b}] \times [\mathrm{T}^{-2b}]$$

Equating powers of M, L and T, we get

$-b + c = 0$, $a + 3b = 0$, and $-2b = 1$.

These equations give $a = \frac{3}{2}$, $b = -\frac{1}{2}$ and $c = -\frac{1}{2}$. Hence

$$T = k\, r^{3/2}\, G^{-1/2}\, m^{-1/2}$$

Thus $T \propto r^{3/2}$. This is Kepler's law of periods.

EXAMPLE 23 The volume of a liquid flowing per second (Q) through a uniform pipe depends upon r the radius of the pipe, η the coefficient of viscosity of the liquid and pressure gradient $\left(\frac{p}{\ell}\right)$ between the ends of the pipe. Dimensional considerations show that Q is proportional to

(a) r (b) r^2
(c) r^3 (d) r^4

SOLUTION $Q = k\left(\frac{p}{\ell}\right)^a\, r^b\, \eta^c$

$$\therefore \quad [\mathrm{L}^3\,\mathrm{T}^{-1}] = \frac{[\mathrm{ML}^{-1}\,\mathrm{T}^{-2}]^a}{[\mathrm{L}]^a} \times [\mathrm{L}^b] \times [\mathrm{ML}^{-1}\,\mathrm{T}^{-1}]^c$$
$$= [\mathrm{M}^{a+c}] \times [\mathrm{L}^{-2a+b-c}] \times [\mathrm{T}^{-2a-c}]$$

$\therefore$ $a + c = 0$, $-2a + b - c = 3$ and $-2a - c = 1$. These equations give $a = 1$, $b = 4$ and $c = -1$. Thus

$$Q = k\,\frac{pr^4}{\ell\eta}$$

$\therefore$ $Q \propto r^4$, which is choice (d).

EXAMPLE 24 The speed (v) of transverse waves travelling on a string depends on tension (T) with which the string is stretched and mass per unit length (μ) of the string. Using dimensional considerations, obtain the expression for v in terms of T and μ.

SOLUTION Let $v = k\, T^a\, \mu^b$

$$\therefore \quad [\mathrm{M}^0\,\mathrm{LT}^{-1}] = [\mathrm{MLT}^{-2}]^a \times \left[\frac{\mathrm{M}}{\mathrm{L}}\right]^b$$
$$= [\mathrm{M}^{(a+b)}] \times [\mathrm{L}^{(a-b)}] \times [\mathrm{T}^{-2a}]$$

Equating powers of M, L and T,

$a + b = 0$, $a - b = 1$, $-2a = -1$. Solving these equations we get $a = +\frac{1}{2}$, $b = -\frac{1}{2}$. Hence

$$v = k\, T^{\frac{1}{2}}\, \mu^{-\frac{1}{2}} = k\sqrt{\frac{T}{\mu}}$$

where k is a dimensionless constant.

4. Significant Figures

The number significant figure in any measurement indicates the degree of precision of that measurement. The degree of precision is determined by the least count of the measuring instrument. Suppose a length measured by a metre scale (of least count = 0.1 cm) is 1.5 cm, then it has two significant figures, namely 1 and 5. Measured with a vernier callipers

(of least count = 0.01 cm) the same length is 1.53 cm and it then has three significant figures. Measured with a screw gauge (of least count = 0.001 cm) the same length may be 1.536 cm which has four significant figures.

It must be clearly understood that we cannot increase the accuracy of a measurement of changing the unit. For example, suppose a measurement of mass yields a value 39.4 kg. It is understood that the measuring instrument has a least count of 0.1 kg. In this measurement, three figures 3, 9 and 4 are significant. If we change 39.4 kg to 39400 g or 39400000 mg, we cannot change the accuracy of measurement. Hence 39400 g or 39400000 mg still have three significant figures; the zeros only serve to indicate only the magnitude of measurement.

Estimation of Appropriate Significant Figures in Calculations

The importance of significant figures lies in calculation to find the result of addition or multiplication of measured quantities having a different number of significant figures. The least accurate quantity determines the accuracy of the sum or product. The result must be rounded off to the appropriate digit.

Rules for Rounding off

The following rules are used for dropping figures that are not significant

1. If the digit to be dropped is less than 5, the next (preceding) digit to be retained is left unchanged. For example, if a number 5.34 is to be rounded off to two significant figures, the digit to be dropped is 4 which is less than 5. Hence the next digit, namely 3, is not changed. The result of the indicated rounding-off is therefore, 5.3.
2. If the digit to be dropped is more than 5, the preceding digit to be retained is increased by 1. For examples 7.536 is rounded off as 7.54 to three significant figures.
3. If the digit to be dropped happens to be 5, then
   (a) the preceding digit to be retained is increased by 1 if it is odd, or
   (b) the preceding digit is retained unchanged if it is even.

   For example, 6.75 is rounded off to 6.8 to two significant figures and 4.95 is rounded off to 5.0 but 3.45 is rounded off to 3.4.

Rule for Finding Significant Figures

1. For addition and subtraction, we use the following rule.

   Find the sum or difference of the given measured quantities and then round off the final result such that it has the same number of digits after the decimal place as in the least accurate quantity (i.e., the quantity which has the least number of significant figures)

EXAMPLE 25 Four objects have masses 2.5 kg, 1.54 kg, 3.668 kg and 5.1278 kg. Find the total mass up to appropriate significant figures.

SOLUTION

$M = 2.5 + 1.54 + 3.668 + 5.1278 = 12.8358$ kg

In this example, the least accurate quantity is 2.5 kg. This mass is accurate only up to the first decimal place in kg. Hence the final result much be rounded off to the first decimal place in kg. The correct result up to appropriate significant figures is $M = 12.8$ kg.

2. We use the following rule to determine the number of significant figures in the result of multiplication and division of various physical quantities.

   Do not worry about the number of digits after the decimal place. Round off the result so that it has the same number of significant figures as in the least accurate quantity.

EXAMPLE 26 A man runs 100.2 m in 10.3 s. Find his average speed up to appropriate significant figure.

SOLUTION

$$\text{Average speed } (v) = \frac{100.2\,\text{m}}{10.3\text{s}} = 9.728155 \text{ ms}^{-1}$$

The distance 100.2 m has four significant figures but the time 10.3 s has only three. Hence the value of the result must be round off to three significant figures. The correct result is $v = 9.73$ ms^{-1}

5. Least Counts of Some Measuring Instruments

1. Least count of metre scale = 1 mm = 0.1 cm
2. Vernier constant (or least count) of vernier callipers = value of 1 main scale division – value of 1 vernier scale division = 1 M.S.D. – 1 V.S.D

   Let the value of 1 M.S.D = a unit

   If n vernier scale divisions coincide with m main scale divisions, then value of

$$1.\text{V.S.D} = \frac{m}{n} \text{ of } 1 \text{ M.S.D} = \frac{ma}{n} \text{ unit}$$

$$\therefore \quad \text{Least count} = a - \frac{ma}{n} = \left(1 - \frac{m}{n}\right) a \text{ unit}$$

3. Least count of a micrometer screw is found by the formula

$$\text{Least count} = \frac{\text{Pitch of screw}}{\text{Total number of divisions on circular scale}}$$

where pitch = lateral distance moved in one complete rotation of the screw.

EXAMPLE 27 In a vernier callipers, 19 divisions of the main scale exactly coincide with 20 divisions of the vernier scale. If the smallest division of the main scale is 1.0 mm, find the vernier constant of the callipers.

SOLUTION Value of 1 M.S.D. = 1.0 mm

20 divisions of the vernier scale = 19 M.S.D.

$$= 19 \times 1.0\,\text{mm} = 19\,\text{mm}$$

$$\therefore \quad \text{Value of 1 V.S.D.} = \frac{19}{20}\,\text{mm}$$

Least count or vernier constant = 1 M.S.D. – 1 V.S.D

$$= 1.0\ \text{mm} - \frac{19}{20}\,\text{mm}$$

$$= \frac{1}{20}\ \text{mm} = 0.05\ \text{mm}$$

$$= 0.005\ \text{cm}$$

EXAMPLE 28 A physical quantity $X = \dfrac{A^2B}{c^{1/3}\sqrt{D}}$ is calculated by using measured quantities A, B, C and D.

If the errors in the measurement of A, B, C and D are 1%, 2%, 3% and 4% respectively, find the percentage error in the measurement of X.

SOLUTION Given $X = \dfrac{A^2B}{c^{1/3}\sqrt{D}}$

$$\therefore \quad \frac{\Delta X}{X} = 2\,\frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{1}{3}\frac{\Delta C}{C} + \frac{1}{2}\frac{\Delta D}{D}$$

$$= 2 \times 1\% + 2\% + \frac{1}{3} \times 3\% + \frac{1}{2} \times 4\%$$

$$= 2\% + 2\% + 1\% + 2\%$$

$$= 7\%$$

EXAMPLE 29 The length and breadth of a rectangular lamina are $L = (2.6 \pm 0.1)$ cm and $B = (1.8 \pm 0.2)$ cm. Find the area of the lamina up to appropriate significant figures stating the error limits.

SOLUTION Area is $A = 2.6 \times 1.8 = 4.68\ \text{cm}^2$

Now $A = L \times B$

$$\therefore \quad \frac{\Delta A}{A} = \frac{\Delta L}{L} + \frac{\Delta B}{B}$$

$$= \frac{0.1}{2.6} + \frac{0.2}{1.8}$$

$$= 0.038 + 0.111$$

$$= 0.149$$

$$\therefore \quad \Delta A = 0.149 \times A = 0.149 \times 4.68$$

$$= 0.697\ \text{cm}^2$$

Since the error is in the first decimal place, we round off $\Delta A = 0.7\ \text{cm}^2$. The value of A cannot be correct upto the second decimal place. So $A = 4.7\ \text{cm}^2$. The result of the measurement is written as

$$A = (4.7 \pm 0.7)\,\text{cm}^2$$

EXAMPLE 30 The error in the measurement of diameter of a sphere is 1%. The percentage error in the measurement of the volume is

(a) 1% (b) 2%
(c) 3% (d) 4%

SOLUTION

$$V = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = \frac{\pi}{6}D^3 \quad \left(\because R = \frac{D}{2}\right)$$

$$\therefore \quad \frac{\Delta V}{V} = \frac{3\Delta D}{D} = 3 \times 1\% = 3\%$$

EXAMPLE 31 Two resistors $R_1 = (2.2 \pm 0.1)\,\Omega$ and $R_2 = (8.0 \pm 0.2)\,\Omega$ are connected in series. The resistance R_s of the series combination is

(a) $(10.2 \pm 0.1)\,\Omega$ (b) $(10.2 \pm 0.2)\,\Omega$
(c) $(10.2 \pm 0.3)\,\Omega$ (d) $(10.2 \pm 0.4)\,\Omega$

SOLUTION $R_s = R_1 + R_2 = 2.2 + 8.0 = 10.2\,\Omega$

$\Delta R_s = \Delta R_1 + \Delta R_2 = 0.1 + 0.2 = 0.3\,\Omega$

$\therefore \quad R_s = (10.2 \pm 0.3)\,\Omega$. So the correct choice is (c).

EXAMPLE 32 In Ex. 31 above, if the resistors R_1 and R_2 are connected in parallel, the resistance R_p of the parallel combination is

(a) $(1.725 \pm 0.171)\,\Omega$ (b) $(1.73 \pm 0.17)\,\Omega$
(c) $(1.7 \pm 0.2)\,\Omega$ (d) $(1.725 \pm 0.2)\,\Omega$

SOLUTION

$$R_p = \frac{R_1R_2}{R_1 + R_2} = \frac{R_1R_2}{R_s} = \frac{2.2 \times 8.0}{10.2} = 1.725\,\Omega$$

$$\therefore \quad \frac{\Delta R_p}{R_p} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_s}{R_s}$$

$$= \frac{0.1}{2.2} + \frac{0.2}{8.0} + \frac{0.3}{10.2}$$
$$= 0.045 + 0.025 + 0.029$$
$$= 0.099$$

$\therefore \quad \Delta R_p = 0.099 \times R_p = 0.099 \times 1.725 = 0.171 = 0.2\,\Omega$

$\therefore \quad R_p = (1.7 \pm 0.2)\,\Omega$

The correct choice is (c).

6. Order of Accuracy: Proportionate Error

The order of accuracy of the result of measurements is determined by the least counts of the measuring instruments used to make those measurements. Suppose a length x is measured with a metre scale, then the error in x is $\pm \Delta x$, where Δx = least count of metre scale = 0.1 cm. If the same length is measured with vernier callipers of least count 0.01 cm, then Δx = 0.01 cm.

Fractional or proportionate error is defined as $\frac{\Delta x}{x}$.

Maximum percentage error = $\frac{\Delta x}{x} \times 100$.

1. **Error in Sum:** Suppose a quantity is given by
   $a = x + y$
   Then $\Delta a = \Delta x + \Delta y$ is the maximum error and
   $$\frac{\Delta a}{a} = \frac{\Delta x + \Delta y}{(x + y)}$$
2. **Error in Difference:** If $a = x - y$, then the maximum error is
   $\Delta a = \Delta x + \Delta y$
   We take the worst case in which errors add up.
   $$\frac{\Delta a}{a} = \frac{\Delta x + \Delta y}{(x - y)}$$
3. **Error in Product and Division:** Suppose we determine the value of a physical quantity u by measuring three quantities x, y and z whose true values are related to u by the equation
   $u = x^{\alpha} y^{\beta} z^{-\gamma}$
   Let the expected small errors in the measurement of quantities x, y and z be respectively $\pm \delta x$, $\pm \delta y$ and $\pm \delta z$ so that the error in u by using these observed quantities is $\pm \delta u$. The numerical values of δx, δy and δz are given by the least count of the instruments used to measure them.
   Taking logarithm of both sides we have
   $$\log u = \alpha \log x + \beta \log y - \gamma \log z$$
   Partial differentiation of the above equation gives
   $$\frac{\Delta u}{u} = \alpha \frac{\Delta x}{x} + \beta \frac{\Delta y}{y} - \gamma \frac{\Delta z}{z}$$
   The proportional or relative error in u is $\Delta u/u$. The values of Δx, Δy and Δz may be positive or negative and in some cases the terms on the right hand side may counteract each other. This effect cannot be relied upon and it is necessary to consider the worst case which is the case when all errors *add* up giving an error Δu given by the equation:
   $$\left(\frac{\Delta u}{u}\right)_{\max} = \alpha \frac{\Delta x}{x} + \beta \frac{\Delta y}{y} + \gamma \frac{\Delta z}{z}$$
   Thus to find the maximum proportional error in u, multiply the proportional errors in each factor (x, y and z) by the numerical value of the power to which each factor is raised and then add all the terms so obtained.
   The sum thus obtained will give the maximum proportional error in the result of u. When the proportional error of a quantity is multiplied by 100, we get the percentage error of that quantity.

EXAMPLE 33 In an experiment for determining density (ρ) of a rectangular metal block, a student makes the following measurements.

Mass of block (m) = 39.3 g
Length of block (x) = 5.12 cm
Breadth of block (y) = 2.56 cm
Thickness of block (z) = 0.37 cm

The uncertainty in the measurement of m is $\pm$ 0.1 g and in the measurement of x, y and z is $\pm$ 0.01 cm. Find the value of ρ (in g cm^{-3}) up to appropriate significant figures, stating the uncertainty in the value of ρ.

SOLUTION

$$\rho = \frac{m}{x\,y\,z} = \frac{39.3}{5.12 \times 2.56 \times 0.37} = 8.1037 \text{ g cm}^{-3}$$

$$\left(\frac{\Delta \rho}{\delta}\right)_{\max} = \frac{\Delta m}{m} + \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$
$$= \frac{0.1}{39.3} + \frac{0.01}{5.12} + \frac{0.01}{2.56} + \frac{0.01}{0.37}$$
$$= 0.0353$$

$\therefore \quad \Delta\rho = 0.0353 \times \rho = 0.0353 \times 8.1037$
$= 0.286 \text{ gcm}^{-3}$

Round off error $\Delta\rho$ to the first significant figure as $\Delta\rho = 0.3$ gcm^{-3}.

Hence ρ = 8.1037 gm^{-3} is not accurate to the fourth decimal place. In fact, it is accurate only up to the first decimal place. Hence the value of ρ much be rounded off as 8.1 and the result of the measurement is written as

$$\rho = (8.1 \pm 0.3) \text{ g cm}^{-3}$$

An Important Tip

To find the dimensional formula of the required quantity, recall any relation which relates that quantity with other quantities whose dimensions we already know. For example, to find the dimensional formula for capacitance C, we can use relation

$Q = CV$ or $C = \dfrac{\varepsilon A}{d}$ or $U = \dfrac{1}{2}\ CV^2$.

Similarly, to find the dimensional formula for magnetic field B, we can use relation $F = q\ v\ B$ or $F = BIL$. or $B = \mu nI$ (here n = Number of turns per unit length).

EXAMPLE 34 Find the dimensional formula of Bohr magneton.

SOLUTION Bohr magneton $= \dfrac{e\,h}{4\,\pi\,m_e}$

$$= \frac{[\mathrm{AT}] \times \left[\mathrm{ML^2\,T^{-1}}\right]}{\mathrm{M}}$$

$$= [\mathrm{M^0\,L^2\,T^0\,A}]$$

The SI unit of Bohr magneton is ampere (metre)^2 (or Am^2)

Note

1. Errors always add; they never cancel other.
2. The quantity which is raised to the highest power contributes the maximum error and hence it must be measured to a high degree of accuracy.

Applications

1. For a simple pendulum $T = 2\pi\sqrt{\dfrac{\ell}{g}} \Rightarrow \dfrac{\Delta T}{T} = \dfrac{1}{2}\dfrac{\Delta \ell}{\ell}$
2. For a sphere of radius r,

   Surface area $A = 4\pi r^2 \Rightarrow \dfrac{\Delta A}{A} = \dfrac{2\Delta r}{r}$

   Volume $V = \dfrac{4}{3}\pi r^3 \Rightarrow \dfrac{\Delta V}{V} = \dfrac{3\Delta r}{r}$
3. Acceleration due to gravity $g = \dfrac{GM}{R^2}$

   $\Rightarrow \quad \dfrac{\Delta g}{g} = \dfrac{2\Delta R}{R} + \dfrac{\Delta M}{M}$
4. For resistances connected in series

   $R_s = R_1 + R_2 \Rightarrow \dfrac{\Delta R_s}{R_s} = \dfrac{\Delta R_1 + \Delta R_2}{R_1 + R_2}$
5. For resistances connected in parallel

   $\dfrac{1}{R_p} = \dfrac{1}{R_1} + \dfrac{1}{R_2} \quad \Rightarrow \quad -\dfrac{\Delta R_p}{R_p^2} = -\dfrac{\Delta R_1}{R_1^2} - \dfrac{\Delta R_2}{R_2^2}$

   $\Rightarrow \dfrac{\Delta R_p}{R_P^2} = \dfrac{\Delta R_1}{R_1^2} + \dfrac{\Delta R_2}{R_2^2}$
6. Kinetic energy K and linear momentum p are related as

   $K = \dfrac{p^2}{2m} \quad \Rightarrow \quad \dfrac{\Delta K}{K} = \dfrac{2\Delta p}{p}$

SECTION 1

Multiple Choice Questions with One Correct Choice

Level A

1. Which one of the following is *not* a unit of length?
   (a) angström (b) light year
   (c) fermi (d) radian
2. The unit of impulse is the same as that of
   (a) moment of force
   (b) linear momentum
   (c) rate of change of linear momentum
   (d) force
3. Which pair of quantities has dimensions different from the other three pairs?
   (a) Impulse and linear momentum
   (b) Planck's constant and angular momentum
   (c) Moment of inertia and moment of force
   (d) Young's modulus and pressure
4. The dimensions of the coefficient of viscosity are
   (a) $\mathrm{ML^2T^{-2}}$ (b) $\mathrm{MLT^{-1}}$
   (c) $\mathrm{ML^{-1}T^{-1}}$ (d) $\mathrm{ML^{-1}T^{-2}}$
5. The dimensions of surface tension are
   (a) $\mathrm{ML^0T^{-2}}$ (b) $\mathrm{MLT^{-2}}$
   (c) $\mathrm{ML^{-1}T^{-2}}$ (d) $\mathrm{ML^{-2}T^{-2}}$
6. The SI unit of the universal gravitational constant G is
   (a) $\mathrm{Nm\ kg^{-2}}$ (b) $\mathrm{Nm^2\ kg^{-2}}$
   (c) $\mathrm{Nm^2\ kg^{-1}}$ (d) $\mathrm{Nm\ kg^{-1}}$

7. The dimensions of the coefficient of thermal conductivity are
(a) $MLT^{-3}K^{-1}$ (b) $MLT^{-2}K^{-1}$
(c) $MLT^{-1}K^{-1}$ (d) $MLT^{-2}K^{-2}$

8. The SI unit of Stefan's constant is
(a) $W\,s^{-1}\,m^{-2}\,K^{-4}$ (b) $J\,s\,m^{-2}\,K^{-4}$
(c) $J\,s^{-1}\,m^{-2}\,K^{-1}$ (d) $Wm^{-2}\,K^{-4}$

9. What is the physical quantity whose dimensions are ML^2T^{-2}?
(a) kinetic energy (b) pressure
(c) momentum (d) power

10. Which one of the following has the dimensions of $ML^{-1}T^{-2}$?
(a) torque (b) surface tension
(c) viscosity (d) stress

11. The dimensions of angular momentum are
(a) MLT^{-1} (b) ML^2T^{-1}
(c) $ML^{-1}T$ (d) ML^0T^{-2}

12. The gravitational force F between two masses m_1 and m_2 separated by a distance r is given by $F = \dfrac{Gm_1m_2}{r^2}$ where G is the universal gravitational constant. What are the dimensions of G?
(a) $M^{-1}L^3T^{-2}$ (b) ML^3T^{-2}
(c) ML^2T^{-3} (d) $M^{-1}L^2T^{-3}$

13. Time period T of a simple pendulum may depend on m, the mass of the bob, l, the length of the string and g, the acceleration due to gravity, i.e.
$$T \propto m^a\, l^b\, g^c$$
What are the values of a, b and c?
(a) $0, \dfrac{1}{2}, -\dfrac{1}{2}$ (b) $0, -\dfrac{1}{2}, \dfrac{1}{2}$
(c) $\dfrac{1}{2}, 0, -\dfrac{1}{2},$ (d) $-\dfrac{1}{2}, 0, \dfrac{1}{2}$

14. The volume V of water passing any point of a uniform tube during t seconds is related to the cross-sectional area A of the tube and velocity u of water by the relation
$$V \propto A^{\alpha}\, u^{\beta}\, t^{\gamma}$$
which one of the following will be true?
(a) $\alpha = \beta = \gamma$ (b) $\alpha \neq \beta = \gamma$
(c) $\alpha = \beta \neq \gamma$ (d) $\alpha \neq \beta \neq \gamma$

15. The frequency n of vibrations of uniform string of length l and stretched with a force F is given by
$$n = \frac{p}{2l}\sqrt{\frac{F}{m}}$$
where p is the number of segments of the vibrating string and m is a constant of the string. What are the dimensions of m?
(a) $M\,L^{-1}\,T^{-1}$ (b) $M\,L^{-3}\,T^0$
(c) $M\,L^{-2}\,T^0$ (d) $M\,L^{-1}\,T^0$

16. The dimensions of specific heat capacity are
(a) $MLT^{-2}K^{-1}$ (b) $ML^2T^{-2}K^{-1}$
(c) $M^0L^2T^{-2}K^{-1}$ (d) $M^0LT^{-2}K^{-1}$

17. What are the dimensions of latent heat?
(a) ML^2T^{-2} (b) $ML^{-2}T^{-2}$
(c) M^0LT^{-2} (d) $M^0L^2T^{-2}$

18. What are the dimensions of Boltzmann's constant?
(a) $MLT^{-2}K^{-1}$ (b) $ML^2T^{-2}K^{-1}$
(c) $M^0LT^{-2}K^{-1}$ (d) $M^0L^2T^{-2}K^{-1}$

19. The dimensions of potential difference are
(a) $ML^2T^{-3}A^{-1}$ (b) $MLT^{-2}A^{-1}$
(c) $ML^2T^{-2}A$ (d) $MLT^{-2}A$

20. What are the dimensions of electrical resistance?
(a) $ML^2T^{-2}A^2$ (b) $ML^2T^{-3}A^{-2}$
(c) $ML^2T^{-3}A^2$ (d) $ML^2T^{-2}A^{-2}$

21. The dimensions of electric field are
(a) $MLT^{-3}A^{-1}$ (b) $MLT^{-2}A^{-1}$
(c) $MLT^{-1}A^{-1}$ (d) MLT^0A^{-1}

22. The dimensions of magnetic field are
(a) $ML^0T^{-1}A^{-1}$ (b) $M^0LT^{-1}A^{-1}$
(c) $MLT^{-2}A^{-1}$ (d) $ML^0T^{-2}A^{-1}$

Level B

23. The quantities L/R and RC (where L, C and R stand for inductance, capacitance and resistance respectively) have the same dimensions as those of
(a) velocity (b) acceleration
(c) time (d) force

24. The equation of state of a real gas can be expressed as $\left(P + \dfrac{a}{V^2}\right)(V - b) = cT$ where P is the pressure, V the volume, T the absolute temperature and a, b and c are constants. What are the dimensions of a?
(a) $M^0L^3T^{-2}$ (b) ML^5T^{-2}
(c) $M^0L^3T^0$ (d) $ML^{-2}T^5$

25. The equation of state for n moles of an ideal gas is
$$PV = nRT$$
where R is the universal gas constant and P, V and T have the usual meanings. What are the dimensions of R?
(a) $M^0LT^{-2}K^{-1}\,mol^{-1}$
(b) $ML^2T^{-2}K^{-1}\,mol^{-1}$
(c) $M^0L^2T^{-2}K^{-1}\,mol^{-1}$
(d) $ML^{-2}T^{-2}K^{-1}\,mol^{-1}$

26. According to the quantum theory, the energy E of a photon of frequency ν is given by
$$E = h\nu$$
where h is Planck's constant. What is the dimensional formula for h?
(a) $M L^2 T^{-2}$ (b) $M L^2 T^{-1}$
(c) $M L^2 T$ (d) $M L^2 T^2$

27. What is the SI unit of Planck's constant?
(a) watt second (b) watt per second
(c) joule second (d) joule per second

28. The dimensions of Planck's constant are the same as those of
(a) energy (b) power
(c) angular frequency (d) angular momentum

29. When a wave traverses a medium, the displacement of a particle located at x at time t is given by
$$y = a \sin (bt - cx)$$
where a, b and c are constants of the wave. The dimensions of b are the same as those of
(a) wave velocity (b) amplitude
(c) wavelength (d) wave frequency

30. In Q. 28, the dimensions of $\frac{b}{c}$ are the same as those of
(a) wave velocity (b) wavelength
(c) wave amplitude (d) wave frequency

31. The van der Waal equation for n moles of a real gas is
$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT$$
where P is the pressure, V is the volume, T is the absolute temperature, R is the molar gas constant and a, b are van der Waal constants. The dimensions of a are the same as those of
(a) PV (b) PV^2
(c) P^2V (d) P/V

32. In Q. 30, the dimensions of b are the same as those of
(a) P (b) V
(c) PV (d) nRT

33. In Q. 30, the dimensions of nRT are the same as those of
(a) energy (b) force
(c) pressure (d) specific heat

34. In Q. 30, the dimensional formula for ab is
(a) ML^2T^{-2} (b) ML^4T^{-2}
(c) ML^6T^{-2} (d) ML^8T^{-2}

35. If velocity (V), acceleration (A) and force (F) are taken as fundamental quantities instead of mass (M), length (L) and time (T), the dimensions of Young's modulus would be
(a) FA^2V^{-2} (b) FA^2V^{-3}
(c) FA^2V^{-4} (d) FA^2V^{-5}

36. The dimensions of permittivity (ε_0) of vacuum are
(a) $M^{-1} L^{-3} T^4 A^2$ (b) $ML^{-3} T^2 A^2$
(c) $M^{-1} L^3 T^4 A^2$ (d) $ML^3 T^2 A^2$

37. What are the dimensions of permeability (μ_0) of vacuum?
(a) $MLT^{-2} A^2$ (b) $MLT^{-2} A^{-2}$
(c) $ML^{-1} T^{-2} A^2$ (d) $ML^{-1} T^{-2} A^{-2}$

38. The dimensions of $1/\sqrt{\mu_0 \varepsilon_0}$ are the same as those of
(a) velocity (b) acceleration
(c) force (d) energy

39. What are the dimensions of magnetic flux?
(a) $ML^2 T^{-2} A^{-1}$ (b) $ML^2 T^{-2} A^{-2}$
(c) $ML^{-2} T^{-2} A^{-1}$ (d) $ML^{-2} T^{-2} A^{-2}$

40. The dimensions of self inductance are
(a) $ML^2 T^{-2} A^{-1}$ (b) $ML^2 T^{-2} A^{-2}$
(c) $ML^{-2} T^{-2} A^{-1}$ (d) $ML^{-2} T^{-2} A^{-2}$

41. The dimensions of capacitance are
(a) $M^{-1} L^{-2} TA^2$ (b) $M^{-1} L^{-2} T^2 A^2$
(c) $M^{-1} L^{-2} T^3 A^2$ (d) $M^{-1} L^{-2} T^4 A^2$

42. Frequency (n) of a tuning fork depends upon length (l) of its prongs, density (ρ) and Young's modulus (Y) of its material. Then frequency and Young's modulus will be related as
(a) $n \propto \sqrt{Y}$ (b) $n \propto Y$
(c) $n \propto \frac{1}{\sqrt{Y}}$ (d) $n \propto \frac{1}{Y}$

43. The dimensions of $\frac{1}{2}\varepsilon_0 E^2$ (ε_0 = permittivity of free space and E = electric field) are
(a) MLT^{-1} (b) ML^2T^{-2}
(c) $ML^{-1} T^{-2}$ (d) $ML^2 T^{-1}$

44. Of the following quantities, which one has dimensions different from the remaining three
(a) Energy per unit volume
(b) Force per unit area
(c) Product of voltage and charge per unit volume
(d) Angular momentum

45. If the time period t of a drop of liquid of density d, radius r, vibrating under surface tension s is given by the formula $t = \sqrt{d^a r^b s^c}$ and if $a = 1$, $c = -1$, then b is
(a) 1 (b) 2
(c) 3 (d) 4

46. A pair of physical quantities having the same dimensional formula is
    (a) angular momentum and torque
    (b) torque and energy
    (c) entropy and power
    (d) power and angular momentum

47. In the measurement of a physical quantity $X = \frac{A^2 B}{C^{1/3} D^3}$. The percentage errors introduced in the measurements of the quantities A, B, C and D are 2%, 2%, 4% and 5% respectively. Then the minimum amount of percentage of error in the measurement of X is contributed by:
    (a) A (b) B
    (c) C (d) D

48. Which of the following has the dimensions $ML^{-1}T^{-1}$?
    (a) Surface tension
    (b) Coefficient of viscosity
    (c) Bulk modulus
    (d) Angular momentum

49. Pressure gradient dp/dx is the rate of change of pressure with distance. What are the dimensions of dp/dx?
    (a) $ML^{-1}T^{-1}$ (b) $ML^{-2}T^{-2}$
    (c) $ML^{-1}T^{-2}$ (d) $ML^{-2}T^{-1}$

50. If E, M, J and G respectively denote energy, mass, angular momentum and gravitational constant, then $\frac{EJ^2}{M^5G^2}$ has the dimensions of
    (a) length (b) angle
    (c) mass (d) time

51. If e, ε_0, h and c respectively represent electronic charge, permittivity of free space, Planck's constant and speed of light, then $\frac{e^2}{\varepsilon_0 hc}$ has the dimensions of
    (a) current (b) pressure
    (c) angular momentum (d) angle

52. If L, R, C and V respectively represent inductance, resistance, capacitance and potential difference, then the dimensions of $\frac{L}{RCV}$ are the same as those of
    (a) current (b) $\frac{1}{\text{current}}$
    (c) charge (d) $\frac{1}{\text{charge}}$

53. If E and B respectively represent electric field and magnetic induction field, then the ratio $\frac{E}{B}$ has the dimensions of
    (a) displacement (b) velocity
    (c) acceleration (d) angle

54. If C and V respectively represent the capacitance of a capacitor and the potential difference between its plates, then the dimensions of CV^2 are
    (a) ML^2T^{-2} (b) $ML^3T^{-2}I^{-1}$
    (c) $ML^2T^{-1}I^{-1}$ (d) $M^0L^0T^0$

55. If h and e respectively represent Planck's constant and electronic charge, then the dimensions of $\left(\frac{h}{e}\right)$ are the same as those of
    (a) magnetic field (b) electric field
    (c) magnetic flux (d) electric flux

56. If energy E, velocity V and time T are chosen as the fundamental units, the dimensional formula for surface tension will be
    (a) EV^2T^{-2} (b) $EV^{-1}T^{-2}$
    (c) $EV^{-2}T^{-2}$ (d) $E^2V^{-1}T^{-2}$

57. A gas bubble from an explosion under water oscillates with a period proportional to $P^a d^b E^c$ where P is the static pressure, d is the density of water and E is the energy of explosion. Then a, b and c respectively are
    (a) $\frac{-5}{6}, \frac{1}{2}, \frac{1}{3}$ (b) $\frac{1}{2}, \frac{-5}{6}, \frac{1}{3}$
    (c) $\frac{1}{3}, \frac{1}{2}, \frac{-5}{6}$ (d) 1, 1, 1

58. The error in the measurement of the radius of a sphere is 1%. The error in the measurement of the volume is
    (a) 1% (b) 3%
    (c) 5% (d) 8%

59. If the error in the measurement of the volume of a sphere is 6%, then the error in the measurement of its surface area will be
    (a) 2% (b) 3%
    (c) 4% (d) 7.5%

60. The percentage errors in the measurements of the length of a simple pendulum and its time period are 2% and 3% respectively. The maximum error in the value of the acceleration due to gravity obtained from these measurements is
    (a) 5% (b) 1%
    (c) 8% (d) 10%

61. The moment of inertia of a body rotating about a given axis is 6.0 kg m^2 in the SI system. What is the value of the moment of inertia in a system of units in which the unit of length is 5 cm and the unit of mass is 10 g?
    (a) 2.4×10^3 (b) 2.4×10^5
    (c) 6.0×10^3 (d) 6.0×10^5

62. A quantity X is given by $\varepsilon_0 L \frac{\Delta V}{\Delta t}$ where ε_0 is the permittivity of free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of
(a) resistance (b) charge
(c) voltage (d) current

63. The coefficient of viscosity (η) of a liquid by the method of flow through a capillary tube is given by the formula

$$\eta = \frac{\pi}{8}\frac{R^4}{l}\frac{P}{Q}$$

where R = radius of the capillary tube,
l = length of the tube,
P = pressure difference between its ends, and
Q = volume of liquid flowing per second.
Which quantity must be measured most accurately?
(a) R (b) l
(c) P (d) Q

64. The mass m of the heaviest stone that can be moved by the water flowing in a river depends on v, the speed of water, density (d) of water and the acceleration due to gravity (g). Then m is proportional to
(a) v^2 (b) v^4
(c) v^6 (d) v^8

65. The speed (v) of ripples depends upon their wavelength (λ), density (ρ) and surface tension (σ) of water. Then v is proportional to
(a) $\sqrt{\lambda}$ (b) λ
(c) $\frac{1}{\lambda}$ (d) $\frac{1}{\sqrt{\lambda}}$

66. The period of revolution (T) of a planet moving round the sun in a circular orbit depends upon the radius (r) of the orbit, mass (M) of the sun and the gravitation constant (G). Then T is proportional to
(a) $r^{1/2}$ (b) r
(c) $r^{3/2}$ (d) r^2

67. If the velocity of light (c), gravitational constant (G) and planck's constant (h) are chosen as fundamental units, the dimensions of time in the new system will be
(a) $c^{-5/2}G^2h^{-1/2}$ (b) $c^{-3/2}G^{-2}h^2$
(c) $c^2G^{-2}h^{1/2}$ (d) $c^{-5/2}G^{1/2}h^{1/2}$

68. The amplitude of a damped oscillator of mass m varies with time t as

$$A = A_0 e^{(-at/m)}$$

The dimensions of a are
(a) ML^0T^{-1} (b) M^0LT^{-1}
(c) MLT^{-1} (d) $ML^{-1}T$

69. A student measures the value of g with the help of a simple pendulum using the formula

$$g = \frac{4\pi^2 L}{T^2}$$

The errors in the measurements of L and T are ΔL and ΔT respectively. In which of the following cases is the error in the value of g the minimum?
(a) $\Delta L = 0.5$ cm, $\Delta T = 0.5$ s
(b) $\Delta L = 0.2$ cm, $\Delta T = 0.2$ s
(c) $\Delta L = 0.1$ cm, $\Delta T = 1.0$ s
(d) $\Delta L = 0.1$ cm, $\Delta T = 0.1$ s

70. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with a uncertainty of ± 0.01 mm. Take $g = 9.8$ m/s^2 (exact). The Young's modulus obtained from the reading is
(a) $(2.0 \pm 0.3) \times 10^{11}$ N/m^2
(b) $(2.0 \pm 0.2) \times 10^{11}$ N/m^2
(c) $(2.0 \pm 0.1) \times 10^{11}$ N/m^2
(d) $(2.0 \pm 0.05) \times 10^{11}$ N/m^2

71. In a vernier callipers, one main scale division is x cm and n divisions of the vernier scale coincide with $(n - 1)$ divisions of the main scale. The least count (in cm) of the callipers is
(a) $\left(\frac{n-1}{n}\right)x$ (b) $\frac{nx}{(n-1)}$
(c) $\frac{x}{n}$ (d) $\frac{x}{(n-1)}$

72. In the relation

$$P = ae^{-\frac{bx}{kT}}$$

where x is a distance, k is Boltzmann constant, T is the absolute temperature and a and b are constants, the dimensions of $\frac{a}{b}$ are the same as those of
(a) area (b) $\frac{1}{\text{area}}$
(c) volume (d) $\frac{1}{\text{volume}}$

73. The intensity I of a wave falls with distance x as

$$I = ae^{-bx}$$

where a and b are constants. The dimensional formula of ab is

(a) $[ML^{-1}T^{-2}]$ (b) $[ML^2T^{-3}]$

(c) $[ML^{-1}T^{-3}]$ (d) $[MLT^{-3}]$

74. The distance x moved by a particle in time t is given by

$$x = a(1 - e^{-bt})$$

The dimensions of ab are the same as those of

(a) velocity (b) momentum

(c) force (d) impulse

75. The energy E of a particle at position x at time t is given by

$$E = \frac{a}{t(b + x^2)}$$

where a and b are constants. The dimensional formula of a is

(a) MLT^{-1} (b) ML^2T^{-1}

(c) ML^3T^{-1} (d) ML^4T^{-1}

76. Two resistors of resistances $R_1 = (100 \pm 1)\,\Omega$ and $R_2 = (200 \pm 2)\,\Omega$ are connected in parallel. The resistance of the parallel combination is

(a) $(66.67 \pm 0.67)\,\Omega$ (b) $(66.67 \pm 0.7)\,\Omega$

(c) $(67 \pm 1)\,\Omega$ (d) $(66.7 \pm 0.7)\,\Omega$

77. The time period T of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where L is the length of the pendulum and g is the acceleration due to gravity. The value of L is measured to be 100.0 cm using a metre scale of least count 0.1 cm. The time for 20 oscillations is measured to be 40.0s using a stop-watch of least count 0.1s. The calculated value of g is

(a) $(9.87 \pm 0.06)\,ms^{-2}$ (b) $(9.9 \pm 0.1)\,ms^{-2}$

(c) $(9.86 \pm 0.05)\,ms^{-2}$ (d) $(9.872 \pm 0.059)\,ms^{-2}$



Answers

Level A

1. (d)	2. (b)	3. (c)	4. (c)
5. (a)	6. (b)	7. (a)	8. (d)
9. (a)	10. (d)	11. (b)	12. (a)
13. (a)	14. (b)	15. (d)	16. (c)
17. (d)	18. (b)	19. (a)	20. (b)
21. (a)	22. (d)		

Level B

23. (c)	24. (b)	25. (b)	26. (b)
27. (c)	28. (d)	29. (d)	30. (a)
31. (b)	32. (b)	33. (a)	34. (d)
35. (c)	36. (a)	37. (b)	38. (a)
39. (a)	40. (b)	41. (d)	42. (a)
43. (c)	44. (d)	45. (c)	46. (b)
47. (c)	48. (b)	49. (b)	50. (b)
51. (d)	52. (b)	53. (b)	54. (a)
55. (c)	56. (c)	57. (a)	58. (b)
59. (c)	60. (c)	61. (b)	62. (d)
63. (a)	64. (c)	65. (d)	66. (c)
67. (d)	68. (a)	69. (d)	70. (b)
71. (c)	72. (b)	73. (c)	74. (a)
75. (d)	76. (d)	77. (a)	



Solutions

Level A

1. Choices (a), (b) and (c) are units of length. Choice (d) the radian is a unit of angle in a plane.

2. Impulse = force × time

$$\frac{d\mathbf{p}}{dt} \times dt$$

$$= d\mathbf{p}$$

= change in momentum

Hence the correct choice is (b).

3. The dimensions of moment of inertia are ML^2T^0 and of moment of force are ML^2T^{-2}. All other pairs in (a), (b) and (d) have identical dimensions.

4. The viscous force acting on a spherical body of radius r moving with a speed v in a fluid of coefficient of viscosity η is given by

$$F = 6\pi\eta r v \text{ or } \eta = \frac{F}{6\pi r v}$$

$\therefore$ Dimensions of η

$$= \frac{\text{dimension of } F}{\text{dimension of } r \times \text{dimension of } v}$$

$$= \frac{MLT^{-2}}{L \times LT^{-1}} = ML^{-1}T^{-1}$$

Hence the correct choice is (c).

5. Surface tension = force/length = $MLT^{-2}/L = ML^0T^{-2}$. Hence the correct choice is (a).

6. From Newton's law of gravitation, the force of attraction F between two bodies of masses m_1 and m_2, separated by a distance r is given by

$$F = G\,\frac{m_1 m_2}{r^2};$$

G = gravitational constant

or $$G = \frac{Fr^2}{m_1 m_2}$$

$$\therefore \quad \text{Unit of } G = \frac{\text{unit of } F \times \text{unit of } r^2}{\text{unit of } m_1 m_2}$$

$$= \frac{\text{newton} \times (\text{metre})^2}{(\text{kg})^2} = \text{N m}^2\,\text{kg}^{-2}$$

Hence, the correct choice is (b).

7. The coefficient of thermal conductivity is defined as the rate of flow of heat energy per unit area of cross-section per unit temperature gradient. The dimensions of rate of flow of heat are $ML^2\,T^{-2}/T = ML^2\,T^{-3}$. The dimensions of area are L^2 and the dimensions of temperature gradient are = dimensions of temperature/dimension of length = KL^{-1}. Hence the dimensions of the coefficient of thermal conductivity are

$$\frac{ML^2\,T^{-3}}{L^2 \times KL^{-1}} = MLT^{-3}\,K^{-1}$$

Thus, the correct choice is (a).

8. From Stefan's law, the total energy emitted per second by a unit area of a black body is proportional to the fourth power of its absolute temperature, i.e.

$$E \propto T^4 \quad \text{or} \quad E = \sigma T^4$$

where σ is the Stefan's constant. Thus

$$\sigma = \frac{E}{T^4}$$

$\therefore$ SI unit of σ

$$= \frac{\text{SI unit of energy per second}}{\text{SI unit of area} \times (\text{SI unit of temperature})^4}$$

$$= \frac{\text{watt}}{(\text{metre})^2 \times (\text{kelvin})^4} = \text{W m}^{-2}\,\text{K}^{-4}$$

Hence the correct choice is (d).

9. The dimensions of energy are $ML^2\,T^{-2}$. The dimensions of pressure, momentum and power are $ML^{-1}\,T^{-2}$, MLT^{-1} and $ML^2\,T^{-3}$ respectively. Thus the correct choice is (a).

10. $ML^{-1}\,T^{-2}$ are the dimensions of force per unit area. Out of the four choices, stress is the only quantity that is force per unit area. Hence the correct choice is (d).

11. The angular momentum **L** of a particle with respect to point whose position vector is **r** is given by

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

where **p** is the linear momentum of the moving particle.

$\therefore$ Dimensions of **L** = dimension of **r** $\times$ dimensions of **p** = $L \times MLT^{-1} = ML^2T^{-1}$

Thus the correct choice is (b).

12. Since $G = \frac{Fr^2}{m_1 m_2}$, the dimensions of G are

$$(G) = \frac{\text{dimensions of } F \times \text{dimensions of } r^2}{\text{dimensions of } m_1 m_2}$$

$$= \frac{MLT^{-2} \times L^2}{M^2} = M^{-1}L^3T^{-2}$$

Thus, the correct choice is (a).

13. The dimensions of the two sides of proportionality are

$$T = M^a\,L^b\,(LT^{-2})^c$$

where LT^{-2} are the dimensions of g.

$\therefore$ $$T = M^a\,L^b\,L^c\,T^{-2c} = M^a\,L^{b+c}\,T^{-2c}$$

Equating the powers of dimensions on both sides, we have $a = 0$, $b + c = 0$ and $-2c = 1$

which give $c = -\frac{1}{2}$, $b = \frac{1}{2}$ and $a = 0$.

Thus, the correct chioce is (a).

14. The dimensions of the two sides of proportionality are

$$L^3 = L^{2\alpha}(LT^{-1})^{\beta}\,T^{\gamma} = L^{2\alpha+\beta}\,T^{\gamma-\beta}$$

Equating the powers of dimensions on both sides, we have

$$2\alpha + \beta = 3$$

$$\gamma - \beta = 0$$

which give $\beta = \gamma$ and $\alpha = \frac{1}{2}(3 - \beta)$, i.e. $\alpha \neq \beta = \gamma$.

Thus, the correct choice is (b).

15. Squaring both sides of the given relation, we get

$$n^2 = \frac{p^2}{4l^2} \cdot \frac{F}{m} \quad \text{or} \quad m = \frac{p^2 F}{4 l^2 n^2}$$

$\therefore$ dimensions of m

$$= \frac{\text{dimensions of } F}{\text{dimensions of } l^2 \times \text{dimensions of } n^2}$$

($\because$ p is a dimensionless number)

$$= \frac{\mathrm{MLT}^{-2}}{\mathrm{L}^2 \times (\mathrm{T}^{-1})^2} = \mathrm{ML}^{-1}\mathrm{T}^0$$

Hence, the correct choice is (d).

16. The heat energy content H of a body of mass m at temperature θ is given by $H = ms\theta$ where s is the specific heat. Therefore

$$s = \frac{H}{m\theta}$$

$\therefore$ Dimensions of s

$$= \frac{\text{dimensions of heat energy}}{\text{dimension of mass} \times \text{dimension of temperature}}$$

$$= \frac{\mathrm{ML}^2\mathrm{T}^{-2}}{\mathrm{M} \times \mathrm{K}} = \mathrm{M}^0\mathrm{L}^2\mathrm{T}^{-2}\mathrm{K}^{-1}$$

Thus, the correct choice is (c).

17. Latent heat L is the amount of heat energy H required to change the state of a unit mass without producing any change in temperature. Thus

$$L = \frac{H}{m}$$

$\therefore$ Dimensions of $L = \dfrac{\mathrm{ML}^2\mathrm{T}^{-2}}{\mathrm{M}}$

$$= \mathrm{L}^2\mathrm{T}^{-2} = \mathrm{M}^0\mathrm{L}^2\mathrm{T}^{-2}$$

Thus, the correct choice is (d).

18. According to the law of equipartition of energy, the energy per degree of freedom of a gas atom or molecule at a temperature θ kelvin is given by

$$E = \frac{1}{2}\, k\theta \quad \text{or} \quad k = \frac{2E}{\theta}$$

where k is the Boltzmann's constant.

$\therefore$ Dimensions of $k = \dfrac{\text{dimensions of } E}{\text{dimension of } \theta}$

$$= \frac{\mathrm{ML}^2\mathrm{T}^{-2}}{\mathrm{K}} = \mathrm{ML}^2\mathrm{T}^{-2}\mathrm{K}^{-1}$$

19. The potential difference V between two points is the amount of work done in moving a unit charge from one point to the other.

Thus, $$V = \frac{\text{work done}}{\text{charge moved}} = \frac{W}{q}$$

$\therefore$ Dimensions of $V = \dfrac{\mathrm{ML}^2\mathrm{T}^{-2}}{\mathrm{Q}} = \mathrm{ML}^2\,\mathrm{T}^{-2}\,\mathrm{Q}^{-1}$

$$= \mathrm{ML}^2\,\mathrm{T}^{-3}\,\mathrm{A}^{-1} \quad (\because \mathrm{Q} = \mathrm{AT})$$

Hence, the correct choice is (a).

20. From Ohm's law, resistance R is given by

$$R = \frac{\text{potential difference}}{\text{current}}$$

$\therefore$ Dimensions of $R = \dfrac{\mathrm{ML}^2\mathrm{T}^{-3}\mathrm{A}^{-1}}{\mathrm{A}} = \mathrm{ML}^2\mathrm{T}^{-3}\mathrm{A}^{-2}$

Thus, the correct choice is (b).

21. Force F experienced by a charge q in an electric field E is given by

$$F = qE \text{ or } E = \frac{F}{q}$$

$\therefore$ Dimensions of $E = \dfrac{\text{dimensions of } F}{\text{dimensions of } Q} = \dfrac{\mathrm{MLT}^{-2}}{\mathrm{AT}}$

$$= \mathrm{MLT}^{-3}\,\mathrm{A}^{-1}.$$

22. The force F, experienced by a charge q moving with speed v perpendicular to the direction of a uniform magnetic induction field B is given by

$$F = qvB \text{ or } B = \frac{F}{qv}$$

$\therefore$ Dimensions of $B = \dfrac{\mathrm{MLT}^{-2}}{\mathrm{Q} \times \mathrm{LT}^{-1}} = \mathrm{ML}^0\mathrm{T}^{-1}\mathrm{Q}^{-1}$

$$= \mathrm{ML}^0\mathrm{T}^{-2}\mathrm{A}^{-1} \quad (\because \mathrm{Q} = \mathrm{AT})$$

Hence, the correct choice is (d).

Level B

23. L/R is the time constant of an L-R circuit and CR is the time constant of a C-R circuit. The dimension of the time constant is the same as that of time. Hence the correct choice is (c).

24. From the principle of homogeneity of dimensions, the dimensions of $\dfrac{a}{V^2}$ must be the same as those of P. Therefore dimensions of a = dimensions of $P \times$ dimensions of V^2

$= \mathrm{ML}^{-1}\,\mathrm{T}^{-2} \times (\mathrm{L}^3)^2 = \mathrm{ML}^{-1}\,\mathrm{T}^{-2} \times \mathrm{L}^6 = \mathrm{ML}^5\,\mathrm{T}^{-2}$.

Thus, the correct choice is (b).

25. Since $R = \dfrac{PV}{nT}$, the dimensions of R are given by

$$(R) = \frac{\text{dimensions of } P \times \text{dimensions of } V}{\text{dimensions of } n \times \text{dimensions of } T}$$

$$= \frac{\mathrm{ML}^{-1}\,\mathrm{T}^{-2} \times \mathrm{L}^3}{\mathrm{mol} \times \mathrm{K}} = \mathrm{ML}^2\,\mathrm{T}^{-2}\,\mathrm{mol}^{-1}\,\mathrm{K}^{-1}$$

Thus the correct choice is (b).

26. Dimensions of $h = \dfrac{\text{dimension of } E}{\text{dimension of } \nu}$

$$= \frac{ML^2T^{-2}}{T^{-1}} = ML^2\,T^{-1}$$

Thus, the correct choice is (b).

27. Unit of $h = \dfrac{\text{unit of } E}{\text{unit of } \nu} = \dfrac{\text{joule}}{(\text{second})^{-1}}$ = joule second.

Thus, the correct choice is (c).

28. The correct choice is (d).

29. Since the argument of a sine function (or any trigonometric function) must be dimensionless, bt and cx are dimensionless. Since bt is dimensionless, the dimensions of b = dimensions of $1/t = T^{-1}$, which are the dimensions of frequency. Hence, the correct choice is (d).

30. Dimensions of bt = dimensions of cx, as they are both dimensionless.

$\therefore$ Dimensions of $\dfrac{b}{c}$ = dimensions of $\dfrac{x}{t} = \dfrac{L}{T} = LT^{-1}$.

Hence, the correct choice is (a).

31. From the principle of homogeneity, the dimensions of $\dfrac{a}{V^2}$ must be the same as those of P, i.e. dimensions of $\dfrac{a}{V^2}$ = dimensions of P

$\therefore$ dimensions of a = dimensions of PV^2. Hence, the correct choice is (b).

32. The correct choice is (b).

33. The dimensions of nRT = dimensions of PV

$$= ML^{-1}\,T^{-2} \times L^3 = ML^2\,T^{-2}$$

which are the dimensions of energy. Hence, the correct choice is (a)

34. The dimensions of $\dfrac{ab}{V^2}$ are the same as those of PV.

$\therefore$ Dimensions of ab = dimensions of $(PV) \times V^2 = ML^2\,T^{-2} \times L^6 = ML^8\,T^{-2}$

Hence, the correct choice is (d).

35. Dimensions of Young's modulus Y are $ML^{-1}\,T^{-2}$. The dimension of V, A and F in terms of M, L and T are

$$(V) = LT^{-1},\ (A) = LT^{-2}$$

and $(F) = MLT^{-2}$

Let $(Y) = (V^a\,A^b\,F^c)$

Putting dimensions of Y, V, A and F. we have

$$(ML^{-1}\,T^{-2}) = (LT^{-1})^a \times (LT^{-2})^b \times (MLT^{-2})^c$$

or $M^1\,L^{-1}\,T^{-2} = M^c\,L^{a+b+c}\,T^{-a-2b-2c}$

Equating powers of M, L and T we have

$$c = 1,\ a + b + c = -1$$

and $-a - 2b - 2c = -2$

which give $a = -4$, $b = 2$ and $c = 1$.

Thus $(Y) = (FA^2V^{-4})$

Thus, the correct choice is (c).

36. According to Coulomb's law of electrostatics, force F between two charges q_1 and q_2 a distance r apart in vacuum, is given by

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

or $$\varepsilon_0 = \frac{1}{4\pi F} \cdot \frac{q_1 q_2}{r^2}$$

$\therefore$ Dimensions of $\varepsilon_0 = \dfrac{Q^2}{MLT^{-2} \times L^2}$

$$= M^{-1}\,L^{-3}\,T^2\,Q^2 = M^{-1}L^{-3}T^4A^2 \quad \left(\because A = \frac{Q}{T}\right)$$

The correct choice is (a).

37. The force per unit length between two long wires carrying currents I_1 and I_2 a distance r apart in vacuum, is given by

$$f = \frac{\mu_0}{4\pi} \cdot \frac{I_1 I_2}{r} \quad \text{or} \quad \mu_0 = \frac{4\pi r f}{I_1 I_2}$$

$\therefore$ Dimensions of $\mu_0 = \dfrac{L \times MLT^{-2} \times L^{-1}}{A^2}$

$$= MLT^{-2}\,A^{-2}$$

Therefore, the correct choice is (b).

38. Dimensions of

$$\frac{1}{\sqrt{\mu_0\,\varepsilon_0}} = \frac{1}{\left(MLT^{-2}\,A^{-2} \times M^{-1}L^{-3}T^4A^2\right)^{\frac{1}{2}}}$$

$$= \frac{1}{\left(L^{-2}T^2\right)^{\frac{1}{2}}} = LT^{-1}$$

which are the dimensions of velocity. Hence the correct choice is (a).

39. The magnetic flux ϕ linked with a circuit of area A in a magnetic induction field B is given by

$$\phi = BA\cos\theta$$

where θ is the angle between the field and area vectors.

$\therefore$ Dimensions of ϕ = dimensions of BA

($\because \cos\theta$ is dimensionless)

$= ML^0T^{-2}A^{-1} \times L^2 = ML^2T^{-2}A^{-1}$

Thus, the correct choice is (a).

40. The self inductance L of a coil in which the current varies at a rate $\frac{dI}{dt}$ is given by

$$e = -L\frac{dI}{dt}$$

where e is the e.m.f. induced in the coil. Now, the dimensions of e.m.f. are the same as those of potential difference, namely, $ML^2T^{-3}A^{-1}$.

Now, $$L = -\frac{e}{\frac{dI}{dt}}$$

$\therefore$ Dimensions of L

$$= \frac{\text{dimensions of } e}{\text{dimensions of } I / \text{dimensions of } t}$$

$$= \frac{ML^2T^{-3}A^{-1}}{A/T} = ML^2T^{-2}A^{-2}$$

Thus, the correct choice is (b).

41. When a capacitor of capacitance C is charged to a potential difference V, the charge Q on the capacitor plates is given by

$$Q = CV \text{ or } C = \frac{Q}{V}$$

$\therefore$ Dimensions of $C = \frac{\text{dimensions of } Q}{\text{dimensions of } V}$

$$= \frac{AT}{ML^2T^{-3}A^{-1}} = M^{-1}L^{-2}T^4A^2$$

Hence, the correct choice is (d).

42. Let $n \propto l^a \rho^b Y^c$

Putting dimensions of all the quantities, we have

$$(T^{-1}) \propto L^a (ML^{-3})^b (ML^{-1}T^{-2})^c$$

Equating powers of M, L and T on both sides, we get

$$b + c = 0,\ a - 3b - c = 0 \text{ and } -2c = -1$$

which give $a = -1$, $b = -\frac{1}{2}$ and $c = \frac{1}{2}$. Thus

$$n \propto l^{-1} \rho^{1/2} Y^{1/2}$$

Hence, the correct choice is (a).

43. We know that

$$F = \frac{q_1 q_2}{(4\pi\varepsilon_0) r^2} \text{ and } E = \frac{F}{q}$$

$\therefore$ $$\varepsilon_0 = \frac{q_1 q_2}{4\pi F r^2}$$

Hence $$\frac{1}{2}\varepsilon_0 E^2 = \frac{q_1 q_2}{8\pi F r^2} \times \frac{F^2}{q^2} = \frac{q_1 q_2}{8\pi q^2} \times \frac{F}{r^2}$$

$\therefore$ Dimensions of $\frac{1}{2}\varepsilon_0 E^2$ = dimensions of $\frac{F}{r^2}$

$$= \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

Hence, the correct choice is (c).

44. Energy per unit volume, force per unit area and product of voltage and charge density all have dimensions of ML^2T^{-2}, but the dimensions of angular momentum are ML^2T^{-1}. Hence, the correct choice is (d).

45. Given $t = d^{a/2} r^{b/2} s^{c/2}$. Substituting dimensions, we have

$$(T) = (ML^{-3})^{a/2} (L)^{b/2} (MT^{-2})^{c/2}$$

$$= M^{(a+c)/2} \cdot L^{(-3a/2 + b/2)} T^{-c}$$

Equating powers of L, we have, $-\frac{3}{2}a + \frac{b}{2} = 0$. Given $a = 1$.

$\therefore$ $$-\frac{3}{2} + \frac{b}{2} = 0 \quad \text{or} \quad b = 3,$$

which is choice (c).

46. Both torque and energy have the dimensions of force $\times$ distance. Hence the correct choice is (b).

47. Given $X = \frac{A^2 B}{C^{1/3} D^3}$

Taking logarithm of both sides, we have

$$\log X = 2\log A + \log B - \frac{1}{3}\log C - 3\log D$$

Partially differentiating, we have

$$\frac{\Delta x}{x} = 2\frac{\Delta A}{A} + \frac{\Delta B}{B} - \frac{1}{3}\frac{\Delta C}{C} - 3\frac{\Delta D}{D}$$

Percentage error in $A = 2\frac{\Delta A}{A} = 2 \times 2\% = 4\%$

Percentage error in $B = \frac{\Delta B}{B} = 2\%$

Percentage error in $C = \frac{1}{3}\frac{\Delta C}{C} = \frac{1}{3} \times 4\%$

$$= \frac{4}{3}\%$$

Percentage error in $D = 3\ \dfrac{\Delta D}{D} = 3 \times 5\% = 15\%$

We find that the minimum percentage error is contributed by C. Hence the correct choice is (c).

48. The correct choice is (b)

49. The correct choice is (b)

50. Dimensions of J and G are ML^2T^{-1} and $M^{-1}L^3\ T^{-2}$ respectively. The correct choice is (b).

51. Dimensions of ε_o and h are $M^{-1}L^{-3}T^4A^2$ and ML^2T^{-1} respectively. The correct choice is (d).

52. RC has the dimensions of time (T). V has the dimensions of emf which has the dimensions of $L\dfrac{dI}{dt}$. The correct choice is (b).

53. The force **F** on a particle of charge q moving with a velocity **v** in **E** and **B** fields is given by

$$\mathbf{F} = q\ (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Hence the dimensions of E are the same as those of vB. The correct choice is (b).

54. Energy stored in a capacitor of capacitance C having a potential difference V between its plates is given by

$$U = \frac{1}{2} CV^2$$

Hence, the dimensions of CV^2 = dimensions of energy. Hence the correct choice is (a).

55. Dimensions of $\left(\dfrac{h}{e}\right) = \dfrac{(ML^2T^{-1})}{AT} = ML^2\ T^{-2}\ A^{-1}$

Dimensions of $B = MT^{-2}\ A^{-1}$

Magnetic flux = $B \times$ area. The correct choice is (c).

56. Let surface tension $\sigma = E^a V^b T^c$. Using the dimensions of σ, E, V and T and equating powers of M, L and T, find the values of a, b and c. The correct choice is (c).

57. The correct choice is (a).

58. $V = \dfrac{4}{3}\pi r^3$. Taking logarithm of both sides, we have

$$\log V = \log 4 + \log \pi + 3 \log r - \log 3$$

Differentiating, we get

$$\frac{\delta V}{V} = 3\frac{\delta r}{r} = 3 \times 1\% = 3\%$$

59. $\dfrac{\delta V}{V} = 3\dfrac{\delta r}{r}$ or $6\% = 3\dfrac{\delta r}{r}$ or $\dfrac{\delta r}{r} = 2\%$.

Now surface area $s = 4\pi r^2$ or $\log s = \log 4\pi + 2 \log r$

$$\therefore \quad \frac{\delta s}{s} = 2\frac{\delta r}{r} = 2 \times 2\% = 4\%.$$

60. $T = 2\pi\sqrt{\dfrac{l}{g}}$ or $g = 4\pi^2\ \dfrac{l}{T^2}$

or $\log g = \log (4\pi^2) + \log l - 2 \log T$. The maximum error in g is

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T} = 2\% + 2 \times 3\% = 8\%$$

61. The dimensions of moment of inertia are (ML^2). We have

$$n_1(u_1) = n_2(u_2)$$

or $\quad n_1(M_1L_1^2) = n_2(M_2L_2^2)$

$$\therefore \quad n_2 = \frac{n_1(M_1L_1^2)}{(M_2L_2^2)} = n_1\left(\frac{M_1}{M_2}\right)\left(\frac{L_1}{L_2}\right)^2$$

Given $n_1 = 6.0$, $M_1 = 1$ kg, $L_1 = 1$ m, $M_2 = 10$ g and $L_2 = 5$ cm. Therefore,

$$n_2 = 6.0 \times \left(\frac{1\text{ kg}}{10\text{ g}}\right) \times \left(\frac{1\text{ m}}{5\text{ cm}}\right)^2$$

$$= 6.0 \times \left(\frac{1000\text{ g}}{10\text{ g}}\right) \times \left(\frac{100\text{ cm}}{5\text{ cm}}\right)^2$$

$$= 6.0 \times 100 \times (20)^2 = 2.4 \times 10^5$$

62. The capacitance of a parallel plate capacitor is given by $C = \varepsilon_0 A/d$. Hence the dimensions of $\varepsilon_0 L$ are the same as those of capacitance.

$$\therefore \text{ Dimension of } \varepsilon_0 L\frac{\Delta V}{\Delta t}$$

$$= \frac{\text{dimension of } C \times \text{dimensions of } V}{\text{time}}$$

$$= \frac{\text{dimension of } Q}{\text{time}} \qquad (\because\ Q = CV)$$

Hence the correct choice is (d).

63. The correct choice is (a).

The maximum permissible error in η is given by the relation

$$\frac{\Delta\eta}{\eta} = 4\frac{\Delta R}{R} + \frac{\Delta l}{l} + \frac{\Delta P}{P} + \frac{\Delta Q}{Q}$$

It is clear that the error in the measurement of R is magnified four times on account of the occurrence of R^4 in the formula. Hence the radius (R) of the capillary tube must be measured most accurately. Thus the quantity which is raised to the highest power needs the most accurate measurement.

64. Take $m \propto v^a\ d^b\ g^c$ and show that $a = 6$.

65. Take $v \propto \lambda^a\ \rho^b\ \sigma^c$ and show that $a = -\dfrac{1}{2}$.

66. Take $T \propto r^a\ M^b\ G^c$ and show that $a = \dfrac{3}{2}$.

67. The correct choice is (d).

68. The exponent is a dimensionless number. Hence at/m is dimensionless. Therefore,

$$\text{Dimension of } a = \frac{\text{dimension of } m}{\text{dimension of } t} = \frac{[\text{M}]}{[\text{T}]} = [\text{M L}^0\text{T}^{-1}]$$

69. The proportionate error in the measurement of g is

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$$

Hence Δg will be minimum if ΔL and ΔT are minimum. Thus the correct choice is (d).

70. $$Y = \frac{FL}{Al} = \frac{4MgL}{\pi d^2 l} \qquad (1)$$

where

$M = 1.0$ kg (exact), $g = 9.8$ ms^{-2} (exact)

$L = 2$ m (exact), $l = 0.8$ mm $= 0.8 \times 10^{-3}$ m

$\Delta l = \pm 0.05$ mm, $d = 0.4$ mm $= 0.4 \times 10^{-3}$ m

$\Delta d = \pm 0.01$ mm

Substituting the values of M, g, L, d and l in Eq. (1) we get

$$Y = 2.0 \times 10^{11}\ \text{Nm}^{-2}$$

From Eq. (1) the proportionate uncertainty in Y is given by

$$\frac{\Delta Y}{Y} = \frac{\Delta M}{M} + \frac{\Delta g}{g} + \frac{\Delta L}{L} + \frac{2\Delta d}{d} + \frac{\Delta l}{l}$$

Since the values of M, g and L are exact, $\Delta M = 0$, $\Delta g = 0$ and $\Delta L = 0$. Hence

$$\frac{\Delta Y}{Y} = \frac{2\Delta d}{d} + \frac{\Delta l}{l} = \frac{2 \times 0.01\,\text{mm}}{0.4\,\text{mm}} + \frac{0.05\,\text{mm}}{0.8\,\text{mm}}$$

$$= 0.05 + 0.0625 = 0.1125$$

$$\therefore \quad \Delta Y = 0.1125 \times Y = 0.1125 \times 2.0 \times 10^{11}$$

$$= 0.225 \times 10^{11}\ \text{Nm}^{-2}$$

Since the value of Y is correct only up to the first decimal place, the value of ΔY must be rounded off to the first decimal place. Thus $\Delta Y = 0.2 \times 10^{11}$ Nm^{-2}. Therefore, the result of the experiment is

$$Y + \Delta Y = (2.0 \pm 0.2) \times 10^{11}\ \text{Nm}^{-2}$$

Hence the correct choice is (b).

71. Vernier constant = value of 1 M.S.D – value of 1 V.S.D.

Now n V.S.D = $(n - 1)$ M.S.D = $(n - 1)\ x$ cm

$$\therefore \quad 1\ \text{V.S.D} = \left(\frac{n-1}{n}\right) x\ \text{cm}$$

$$\therefore \quad \text{V.C.} = x\ \text{cm} - \left(\frac{n-1}{n}\right) x\ \text{cm} = \frac{x}{n}\ \text{cm}$$

Hence the correct choice is (c).

72. Since the exponential function is dimensionless,

$$[a] = [P] = [\text{ML}^{-1}\text{T}^{-2}]$$

Since the exponent is also dimensionless,

$$[b] = \frac{[k\,\text{T}]}{[x]} = \frac{[\text{ML}^2\text{T}^{-2}\text{K}^{-1}] \times [\text{K}]}{[\text{L}]}$$

$$= [\text{MLT}^{-2}]$$

$$\therefore \quad \left[\frac{a}{b}\right] = \frac{[\text{ML}^{-1}\text{T}^{-2}]}{[\text{MLT}^{-2}]} = \text{L}^{-2}$$

So the correct choice is (b).

73. The intensity of a wave is defined as the rate at which the energy crosses a unit area held normal to it. Hence

$$I = \frac{\text{energy}}{\text{area} \times \text{time}} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{L}^2] \times [\text{T}]}$$

$$= [\text{MT}^{-3}]$$

$$[a] = [I] = [\text{MT}^{-3}]$$

$$[b] = \left[\frac{1}{x}\right] = \left[\frac{1}{\text{L}}\right] = [\text{L}^{-1}]$$

$$\therefore \quad [ab] = [\text{MT}^{-3}] \times [\text{L}^{-1}]$$

$$= [\text{ML}^{-1}\text{T}^{-3}], \text{ which is choice (c).}$$

74. $[a] = [x] = [\text{L}]$

$[b] = [t^{-1}] = [\text{T}^{-1}]$

$$\therefore \quad [ab] = [\text{L}] \times [\text{T}^{-1}] = [\text{LT}^{-1}]$$

So the correct choice is (a).

75. $[b] = [x^2] = [\text{L}^2]$

$a = Et\,[b + x^2]$

$$\therefore \quad [a] = [E] \times [t] \times [b]$$

$$= [\text{ML}^2\text{T}^{-2}] \times [\text{T}] \times [\text{L}^2]$$

$$= [\text{ML}^4\text{T}^{-1}], \text{ which is choice (d)}$$

76. Refer also to Example 32 on page 1.8.

$$R_p = \frac{R_1 R_2}{R_1 + R_2}$$

$$= \frac{100 \times 200}{100 + 200} = 66.67\ \Omega$$

From $\dfrac{1}{R_p} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$, we get

$$\frac{\Delta R_p}{R_p^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\Rightarrow \quad \Delta R_p = \left(\frac{R_p}{R_1}\right)^2 \Delta R_1 + \left(\frac{R_p}{R_2}\right)^2 \Delta R_2$$

$$= \left(\frac{66.67}{100}\right)^2 \times 1 + \left(\frac{66.67}{200}\right)^2 \times 2.$$

$$= 0.444 + 0.222$$

$$= 0.666$$

$$= 0.7\ \Omega$$

$\therefore$ $R_p = 66.7\ \Omega$ and the result with appropriate significant figure is written as

$$R_p = (66.7 + 0.7)\Omega$$

So the correct choice is (d).

77. $T = 2\pi\sqrt{\frac{L}{g}} \quad \Rightarrow \quad g = \frac{4\pi^2 L}{T^2}$.

If the pendulum completes n oscillations in t seconds, then $T = \frac{t}{n}$. In terms of measured quantities, we have

$$g = \frac{4\pi^2 L n^2}{t^2} \qquad (1)$$

Given L = 1.0 m, n = 20 and t = 40.0s. Substituting these values in (1) we get $g = 9.872\ \text{ms}^{-2}$. From (1), we get

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta t}{t}$$

$$= \frac{0.1\,\text{cm}}{100.0\,\text{cm}} + \frac{2 \times 0.1\text{s}}{40.0\text{s}}$$

$$= 0.001 + 0.005 = 0.006$$

$\therefore$ $\Delta g = 0.006 \times 9.872 = 0.059 \simeq 0.06\ \text{ms}^{-2}$

Therefore, the value of g is written as

$$g = (9.87 \pm 0.06)\ \text{ms}^{-2}$$

The correct choice is (a).

2 SECTION — Multiple Choice Questions Based on Passage

Questions 1, 2 and 3 are based on the following passage.

Passage I

In the study of physics, we often have to measure the physical quantities. The numerical value of a measured quantity can only be approximate as it depends upon the least count of the meaasuring instrument used. The number of significant figures in any measurement indicates the degree of precision of that measurement. The importance of significant figures lies in calculation. A mathematical calculation cannot increase the precision of a physical measurement. Therefore, the number of significant figures in the sum or product of a group of numbers cannot be greater than the number that has the least number of significant figures. A chain cannot be stronger than its weakest link.

1. A bee of mass 0.000087 kg sits on a flower of mass 0.0123 kg. What is the total mass supported by the stem of the flower upto appropriate significant figures?
   (a) 0.012387 kg (b) 0.01239 kg
   (c) 0.0124 kg (d) 0.012 kg
2. The radius of a uniform wire is r = 0.021 cm. The value π is given to be 3.142. What is the area of cross-section of the wire up to appropriate significant figures.
   (a) 0.0014 cm^2 (b) 0.00139 cm^2
   (c) 0.001386 cm^2 (d) 0.0013856 cm^2
3. A man run 100.5 m is 10.3 s. His average speed up to appropriate significant figures is
   (a) 9.76 ms^{-1} (b) 9.757 ms^{-1}
   (c) 9.7573 ms^{-1} (d) 9.8 ms^{-1}



Solutions

1. Total mass = 0.000087 + 0.0123 = 0.012387 kg. The mass of the bee is accurate up to sixth decimal place in kg, whereas the mass of the flower is accurate only upto the fourth decimal place. Hence the sum must be rounded off to the fourth decimal place. Therefore the correct choice is (c).
2. $A = \pi r^2 = 3.142 \times (0.021)^2 = 0.00138562\ \text{cm}^2$. Now, there are only two significant figures in

0.021 cm. Hence the result must be rounded off to two significant figure as $A = 0.0014\ \text{cm}^2$, which is choice (a).

3. Average speed $= \dfrac{100.5\ \text{m}}{10.3\ \text{s}} = 9.7573\ \text{ms}^{-1}$

The distance has four significant figures but the time has only three. Hence the result must be rounded off to three significant figure to $9.76\ \text{ms}^{-1}$. Thus the correct choice is (a).

SECTION 3 Assertion-Reason Type Questions

In the following questions, **Statement-1 (Assertion)** is followed by **Statement-2 (Reason)**. Each question has the following four options out of which only **one** choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. **Statement-1**
   The order of accuracy of measurement depends on the least count of the measuring instrument.
   Statement-2
   The smaller the least count the greater is the number of significant figures in the measured value.
2. **Statement-1**
   The dimensional method cannot be used to abtain the dependence of the work done by a force **F** on the angle θ between force **F** and displacement $\boldsymbol{x}$.
   Statement-2
   All trignometric functions are dimensionless.
3. **Statement-1**
   The mass of an object is 13.2 kg. In this measurement there are 3 significant figures.
   Statement-2
   The same mass when expressed in grams as 13200 g has five significant figures.

Solutions

1. The correct choice is (b).
2. Work done is $W = Fx \cos\theta$. Since $\cos\theta$ is dimensionless, the dependence of W on θ cannot be determined by the dimensional method. Hence the correct choice is (a)
3. The correct choice is (c). The degree of accuracy (and hence the number of significant figures) of a measurement cannot be increased by changing the unit.

SECTION 4 Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. The dimensions of $\frac{1}{2}\varepsilon_0 E^2$ (ε_0 = permittivity of free space and E = electric field) are
   (a) MLT^{-1} (b) ML^2T^{-2}
   (c) $\text{ML}^{-1}\text{T}^{-2}$ (b) ML^2T^{-1} **[2000]**

2. A quantity X is given by $\varepsilon_0 L\dfrac{\Delta V}{\Delta t}$ where is the ε_0 is the permittivity of free space, L is the length, ΔV is a potential difference and is Δt a time interval. The dimensional formula for X is the same as that of
   (a) resistance (b) charge
   (c) voltage (d) current **[2001]**

3. The dimensions of $\frac{1}{\mu_0 \varepsilon_0}$, where the symbols have their usual meaning are
   (a) $[L^{-1}T]$ (b) $[L^2T^2]$
   (c) $[L^2T^{-2}]$ (d) $[LT^{-1}]$ **[2003]**
4. The physical quantities not having the same dimensions are
   (a) torque and work
   (b) momentum and Planck's constant
   (c) stress and Young's modulus
   (d) speed and $(\mu_0\varepsilon_0)^{-1/2}$ **[2003]**
5. In the expression
   $$P = \left(\frac{\alpha^2}{\beta}\right) e^{-\alpha Z / k\theta}$$
   P is pressure, Z is distance, k is Boltzmann constant and θ is the temperature. The dimensional formula for β is
   (a) $[M\ L^3\ T^{-2}]$ (b) $[M\ L^2\ T^{-2}]$
   (c) $[M\ L\ T^{-1}]$ (d) $[M^0\ L^2\ T^{-1}]$ **[2004]**
6. Which of the following denotes the dimensions ML^2/Q^2, where Q denotes the electric charge?
   (a) H/m^2 (b) Weber (Wb)
   (c) Wb/m^2 (d) Henry(H) **[2006]**
7. A student measure the value of g with the help of a simple pendulum susing the formula
   $$g = \frac{4\pi^2 L}{T^2}$$
   The errors in the measurements of L and T are ΔL and ΔT respectively. In which of the following cases is the error in the value of g the minimum?
   (a) $\Delta L = 0.5$ cm, $\Delta T = 0.5$ s
   (b) $\Delta L = 0.2$ cm, $\Delta T = 0.2$ s
   (c) $\Delta L = 0.1$ cm, $\Delta T = 1.0$ s
   (d) $\Delta L = 0.1$ cm, $\Delta T = 0.1$ s **[2006]**
8. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with a uncertainty of ± 0.01 mm. Take $g = 9.8\ m/s^2$ (exact). The Young's modulus obtained from the reading is
   (a) $(2.0 \pm 0.3) \times 10^{11}\ N/m^2$
   (b) $(2.0 \pm 0.2) \times 10^{11}\ N/m^2$
   (c) $(2.0 \pm 0.1) \times 10^{11}\ N/m^2$
   (d) $(2.0 \pm 0.05) \times 10^{11}\ N/m^2$ **[2007]**
9. In a vernier callipers, one main scale division is x cm and n divisions of the vernier scale coincide with $(n - 1)$ divisions of the main scale. The least count (in cm) of the callipers is
   (a) $\left(\frac{n-1}{n}\right)x$ (b) $\frac{nx}{(n-1)}$
   (c) $\frac{x}{n}$ (d) $\frac{x}{(n-1)}$ **[2007]**
10. Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different lengths of the pendulum and/ or record time for different number of oscillations. The observations are shown in the table.
    Least count for length = 0.1 cm
    Least count for time = 0.1 s

Student	*Length of the pendulum (cm)*	*Number of oscillations (n)*	*Total time for (n) oscillations (s)*	*Time period (s)*
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	6.0

    If E_I,E_{II} and E_{III} are the percentage arrors in g. i.e. $\left(\frac{\Delta g}{g} \times 100\right)$ for student I, II and III, respectively,
    (a) $E_1 = 0$ (b) E_I is minimum
    (c) $E_I = E_{II}$ (d) E_{II} is minimum **[2008]**
11. Two full turns of the circular scale of screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of –0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is
    (a) 3.38 mm (b) 3.32 mm
    (c) 3.73 mm (d) 3.67mm **[2008]**
12. A body of mass $m = 3.513$ kg is moving along the x-axis with a speed of 5.00 ms^{-1}. The magnitude of its momentum is recorded as
    (a) 17.56 kg ms^{-1} (b) 17.57 kg ms^{-1}
    (c) 17.6 kg ms^{-1} (d) 17.565 kg ms^{-1} **[2008]**
13. The dimension of magnetic field in M, L, T and C (Coulomb) is given as
    (a) $MT^{-1}C^{-1}$ (b) $MT^{-2}C^{-1}$
    (c) $MLT^{-1}C^{-1}$ (d) $MT^{-2}C^{-2}$ **[2008]**

14. In an experiment, the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (= 0.5º), then the least count of the instrument is :
    (a) one degree (b) half degree
    (c) one minute (d) half minute **[2009]**
15. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are
    (a) 5, 1, 2 (b) 5, 1, 5
    (c) 5, 5, 2 (d) 4, 4, 2 **[2010]**
16. A Vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is
    (a) 0.02 mm (b) 0.05 mm
    (c) 0.1 mm (d) 0.2 mm **[2010]**
17. A screw gauge gives the following reading when used to measure the diameter of a wire. Main scale reading : 0 mm
    Circular scale reading : 52 divisions
    Given that 1 mm on main scale corresponds to 100 divisions of the circular scale. The diameter of wire from the above data is:
    (a) 0.052 cm (b) 0.026 cm
    (c) 0.005 cm (d) 0.52 cm **[2011]**
18. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0. 5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is
    (a) 0.9% (b) 2.4%
    (c) 3.1% (d) 4.2% **[2011]**
19. In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi \ell d^2}\right)$ by using Searle's method, a wire of length $L = 2$ m and diameter $d = 0.5$ mm is used. For a load $M = 2.5$ kg, an extension $\ell = 0.25$ mm in the length of the wire is observed. Quantities d and ℓ are measured using a screw gauge and a micrometer, respectively. They have same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement
    (a) due to errors in the measurement of d and ℓ are the same.
    (b) due to errors in the measurement of d is twice that due to the error in the measurement of ℓ.
    (c) due to errors in the measurement of ℓ is twice that due to the error in the measurement of d.
    (d) due to the error in the measurement of d is four times that due to the error in the measurement of ℓ. **[2012]**
20. The diameter of a cylinder is measured using a Vernier callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The 24th division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is:
    (a) 5.112 cm (b) 5.124 cm
    (c) 5.136 cm (d) 5. 148 cm **[2013]**
21. The physical quantities not having the same dimensions are
    (a) torque and work
    (b) momentum and Planck's constant
    (c) pressure and Young's modulus
    (d) speed and $(\mu_0 \in_0)^{-1/2}$ **[2014]**
22. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005)m, the gas in the tube is
    (**Useful information:** $\sqrt{167RT} = 640\ J^{1/2}\ mole^{-1/2}$; $\sqrt{140RT} = 590\ J^{1/2}\ mole^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)
    (a) Neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$
    (b) Nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$
    (c) Oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$
    (d) Argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$ **[2014]**

Answers

1. (c)	2. (d)	3. (c)	4. (b)
5. (a)	6. (d)	7. (d)	8. (b)
9. (c)	10. (b)	11. (a)	12. (c)
13. (a)	14. (c)	15. (a)	16. (d)
17. (a)	18. (c)	19. (a)	20. (b)
21. (b)	22. (d)		

Solutions

1. We know that

$$F = \frac{q_1 q_2}{(4\pi\varepsilon_0) r^2} \text{ and } E = \frac{F}{q}$$

$$\therefore \quad \varepsilon_0 = \frac{q_1 q_2}{4\pi F r^2}$$

Hence $$\frac{1}{2}\varepsilon_0 E^2 = \frac{q_1 q_2}{8\pi F r^2} \times \frac{F^2}{r^2}$$

$$= \frac{q_1 q_2}{8\pi q^2} \times \frac{F}{r^2}$$

$\therefore$ Dimensions of $\frac{1}{2}\varepsilon_0 E^2$ = dimensions of

$$\frac{F}{r^2} = \frac{M L T^{-2}}{L^2} = ML^{-1}T^{-2}$$

2. The capacitance of a parallel plate capacitor is given by $C = \varepsilon_0 A/d$. Hence the dimensions of $\varepsilon_0 L$ are the same as those of capacitance.

$\therefore$ Dimension of $\varepsilon_0 L \frac{\Delta V}{\Delta t}$

$$= \frac{\text{dimension of } C \times \text{ dimensions of } V}{\text{time}}$$

$$= \frac{\text{dimension of } Q}{\text{time}} \qquad (\because\ Q = CV)$$

= dimension of current

3. The velocity of an electromagnetic wave is

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\therefore \quad \frac{1}{\mu_0 \varepsilon_0} = v^2 = [LT^{-1}]^2 = [L^2 T^{-2}]$$

4. Momentum = $mv = [ML\ T^{-1}]$

Planck's constant $h = \frac{E}{V} = \frac{[ML^2\ T^{-2}]}{T^{-1}} = [ML^2\ T^{-1}]$

5. The exponent $\alpha Z/k\theta$ is dimensionless. Hence

$$[\alpha] = \left[\frac{k\theta}{Z}\right] = \left[\frac{JK^{-1} \times K}{m}\right]$$

$$= J\ m^{-1}$$

$$= [M\ L\ T^{-2}]$$

Dimensions of $\frac{\alpha^2}{\beta}$ are the same as those of P. Hence

$$[\beta] = \left[\frac{\alpha^2}{P}\right] = \frac{[M L T^{-2}]^2}{[M L^{-1} T^{-2}]}$$

$$= [M\ L^3\ T^{-2}]$$

6. The energy stored in an inductor is given by

$$U = \frac{1}{2} LI^2$$

or $$L = \frac{2U}{I^2}$$

$\therefore$ Dimensions of inductance

$$(L) = \frac{\text{dimension of energy}}{\text{dimensions of (current)}^2}$$

$$= \frac{ML^2\ T^{-2}}{I^2} = \frac{ML^2\ T^{-2}}{(QT^{-1})^2} \qquad \left(\because I = \frac{Q}{T}\right)$$

$$= ML^2Q^{-2}$$

All other choices have dimensions different from ML^2Q^{-2}.

7. The proportionate error in the measurement of g is

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$$

Hence Δg will be minimum if ΔL and ΔT are minimum. Thus the correct choice is (d).

8. $$Y = \frac{FL}{Al} = \frac{4MgL}{\pi d^2 l} \qquad (1)$$

Where

$M = 1.0$ kg (exact), $g = 9.8$ ms^{-2} (exact)

$L = 2$ m (exact), $l = 0.8$ mm $= 0.8 \times 10^{-3}$ m

$\Delta l = \pm 0.05$ mm, $d = 0.4$ mm $= 0.4 \times 10^{-3}$m

$\Delta d = \pm 0.01$ mm

Substituting the values of M, g, L, d and l in Eq. (1) we get

$$Y = 2.0 \times 10^{11}\ \text{Nm}^{-2}$$

From Eq. (1) the proportionate uncertainty in Y is given by

$$\frac{\Delta}{Y} = \frac{\Delta M}{M} + \frac{\Delta g}{g} + \frac{\Delta L}{L} + \frac{2\Delta d}{d} + \frac{\Delta l}{l}$$

Since the values of M, g and L are exact, $\Delta M = 0$, $\Delta g = 0$ and $\Delta L = 0$. Hence

$$\frac{\Delta Y}{Y} = \frac{2\Delta d}{d} + \frac{\Delta l}{l} = \frac{2 \times 0.01\,\text{mm}}{0.4\,\text{mm}} + \frac{0.05\,\text{mm}}{0.8\,\text{mm}}$$
$$= 0.05 + 0.0625$$
$$= 0.1125$$
$$\therefore\ \Delta Y = 0.1125 \times Y = 0.1125 \times 2.0 \times 10^{11}$$
$$= 0.225 \times 10^{11}\ \text{Nm}^{-2}$$

Since the value of Y is correct only up to the first decimal place, the value of ΔY must be rounded off to the first decimal place. Thus $\Delta Y = 0.2 \times 10^{11}\ \text{Nm}^{-2}$. Therefore, the result of the experiment is

$Y + \Delta Y = (2.0 \pm 0.2) \times 10^{11}\ \text{NM}^{-2}$

9. Vernier constant = value of 1 M.S.D – value of 1 V.S.D.

Now n V.S.D = $(n-1)$ M.S.D = $(n-1)\,x$ cm

$$\therefore \quad 1\ \text{V.S.D} = \left(\frac{n-1}{n}\right) x\ \text{cm}$$

$$\therefore \quad \text{V.C.} = x\ \text{cm} - \left(\frac{n-1}{n}\right) x\ \text{cm} = \frac{x}{n}\ \text{cm}$$

10. $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow g = \frac{4\pi^2 l}{T^2}$ Therefore,

$$\frac{\Delta g}{g} = \frac{\Delta t}{l} + \frac{2\Delta T}{T}$$

For student I, $E_I = \frac{\Delta g}{g} 100 = \left(\frac{0.1}{64.0} + \frac{2 \times 0.1}{128.0}\right) \times 100$

$$= \frac{5}{16}\,\%$$

For student II, $E_{II} = \left(\frac{0.1}{64} + \frac{2 \times 0.1}{64}\right) \times 100$

$$= \frac{15}{32}\,\%$$

For student III, $E_{III} = \left(\frac{0.1}{20.0} + \frac{2 \times 0.1}{36}\right) \times 100$

$$= \frac{19}{18}\,\%$$

Thus, the precentage error is minimum for student I.

11. Pitch of screw = $\frac{1\ \text{mm}}{2} = 0.5$ mm

Least count = $\frac{0.5\ \text{mm}}{50} = 0.01$ mm

Measured diameter = 3 mm + 35 × 0.01 mm
= (3 + 0.35) mm = 3.35 mm

Corrected diameter = measured diameter – zero error
= 3.35 –(–0.03) = 3.38 mm

12. Momentum $p = m \times v = 3.513 \times 5.00 = 17.565$ kg ms^{-1}. The total number of significant figures in the result of multiplication or division is equal to the number of signficiant figures in the least accurate quantity. The velocity v has the least number of significant figures (equal to three). Hence, the result must be rounded off up to three significant figures as $p = 17.6$ kg ms^{-1}.

13. $F = q\,v\,B \sin\theta$. Therefore, dimensions of B are

$$[B] = \frac{\text{dimension of } F}{\text{dimension of } qv} = \frac{\text{M L T}^{-2}}{\text{CL T}^{-1}} = [\text{MT}^{-1}\,\text{C}^{-1}]$$

14. Value of 1 main scale division = 0.5º

30 vernier scale divisions = 29 main scale divisions
= 29 × 0.5º

$\therefore$ Value of 1 vernier scale division = $\frac{29}{30} \times 0.5^\circ$

Least count = value of 1 m.s.d – value of 1 v.s.d.

$$= 0.5^\circ - \frac{29}{30} \times 0.5^\circ = \frac{0.5^\circ}{30} = \frac{0.5}{30} \times 60\,\text{min} = 1\ \text{min}$$

15. 23.023 has 5 significant figures (s.f.), $0.0003 = 3 \times 10^{-4}$ has 1 s.f. and 2.1×10^{-3} has 2 s.f.

16. Least count = 1 M.S.D. – 1 V.S.D.
= 1 M.S.D. – $\frac{16}{20}$ M.S.D.
= $\frac{4}{20}$ M.S.D.
= $\frac{4}{20} \times 1$ mm = 0.2 mm,

which is choice (d).

17. Least count (L.C.) = $\frac{1\,\text{mm}}{100} = 0.01\,\text{mm} = 0.001$ cm

Diameter of wire = main scale reading + (circular scale reading) × L.C.
= 0 + 52 × 0.001
= 0.052 cm

18. Least count of screw gauge
$$= \frac{\text{pitch}}{\text{number of divisions on circular scale}}$$
$$= \frac{0.5\,\text{mm}}{50} = 0.01\ \text{mm}$$

$\therefore$ diameter of ball = 2.5 mm + 20 × 0.01 mm = 2.7 mm

Density $\rho = \frac{\text{M}}{\frac{4\pi}{3} r^3}$

Maximum relative error in ρ is

$$\therefore \quad \frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{3\Delta r}{r}$$
$$= 2\% + 3 \times \frac{0.01}{2.7} \times 100$$
$$= 2\% + 1.1\% = 3.1\%$$

19. Least count = $\frac{0.5\text{ mm}}{100} = 0.005$ mm

$$\frac{\Delta Y}{Y} = \frac{\Delta \ell}{\ell} + \frac{2\Delta d}{d}$$

$$\frac{\Delta \ell}{\ell} = \frac{0.005\,\text{mm}}{0.25\,\text{mm}} = 0.02$$

$$\frac{2\Delta d}{d} = \frac{2 \times 0.005\,\text{mm}}{0.5\,\text{mm}} = 0.02$$

So, the correct choice is (a).

20. Least count = value of 1 MSD – value of VSD

$$= 0.05\text{ cm} - \frac{2.45}{50}\text{ cm}$$

$$= 0.05 - 0.049 = 0.001\text{ cm}$$

Diameter = 5.10 cm + 24 × 0.001 cm

= 5.10 cm + 0.024 cm

= 5.124 cm

21. Momentum = $mv = [\text{M L T}^{-1}]$

Planck's constant $h = \frac{E}{\nu}$

$$= \frac{[\text{M L}^2\,\text{T}^{-2}]}{[\text{T}^{-1}]} = [\text{M L}^2\,\text{T}^{-1}]$$

22. Speed of sound $v = \sqrt{\frac{\gamma RT}{M}}$. For a closed pipe, $\lambda = 4L$ (fundamental mode). Also $v = \nu\lambda$. Thus

$$v = \nu \times 4L$$

$$\Rightarrow \quad \sqrt{\frac{\gamma RT}{M}} = \nu \times 4L$$

$$\Rightarrow \quad L = \frac{1}{4\nu}\sqrt{\frac{\gamma RT}{M}} \qquad (1)$$

For Neon: $M = 20 \times 10^{-3}$kg. Neon in monatomic gas for which $\gamma = 1.67$. Substituting the given values in (1), we have

$$\left[\text{since } \sqrt{167\,RT} = 640\text{ J}^{1/2}\text{ mol}^{-1/2} \text{ and } \sqrt{\frac{10}{20}} = \sqrt{\frac{1}{2}} = \frac{7}{10}\right]$$

$$L = \frac{1}{4 \times 244}\sqrt{\frac{1.67\,RT}{2 \times 10^{-2}}}$$

$$= \frac{1}{4 \times 244}\sqrt{\frac{167\,RT}{2}}$$

$$= \frac{1}{4 \times 244} \times 640 \times \frac{7}{10} = 0.459\text{ m}$$

Similarly, for nitrogen $\gamma = 1.4$, $L = 0.363$ m, for oxygen $\gamma = 1.4$, $\lambda = 0.340$ m and for argon ($\gamma = 1.67$), $L = 0.348$ m. It is given that $L = (0.350 \pm 0.005)$m. Since the error in L is $\pm$ 0.005, the third digit (after the decimal point) in the value of L must be either zero or 5. Hence $L = 0.348$ m must be rounded off as $L = 0.350$ m. So the correct choice is (d).

KINEMATICS

Chapter 2

REVIEW OF BASIC CONCEPTS

1. Scalar and Vector Quantities

A scalar quantity has only magnitude but no direction, such as distance, speed, mass, area, volume, time, work, energy, power, temperature, specific heat, charge, potential, etc.
A vector quantity has both magnitude and direction, such as displacement, velocity, acceleration, force, momentum, torque, electric field, magnetic field, etc.

2. Parallelogram Law of Vector Addition

The procedure of finding the resultant of two vectors is known as the *parallelogram law of vector addition and may be stated as follows*.

If the two vectors are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.

Figure 2.1 show two vectors **A** and **B** of magnitudes A and B inclined at an angle α. The magnitude R of the resultant vector **R** is given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\alpha}$$

The angle β which the resultant vector **R** subtends with vector **A** is given by

$$\tan\beta = \frac{B\sin\alpha}{A + B\cos\alpha} \quad \text{or} \quad \sin\beta = \frac{B\sin\alpha}{R}$$

In vector notation, the resultant vector is written as **R** = **A** + **B**.

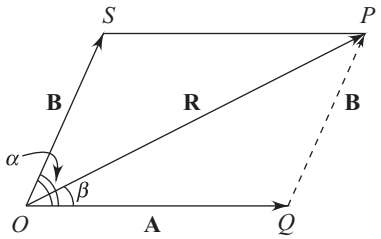


Fig. 2.1

Special Cases

(i) *When the two vectors are in the same direction*, i.e. $\alpha = 0°$, then $R = \sqrt{A^2 + B^2 + 2AB\cos 0°} = A + B$. Therefore, the magnitude of the resultant is equal to the sum of the magnitudes of the two vectors. Also $\tan\beta = 0$ or $\beta = 0$, i.e. the direction of the resultant is the same as that of either vector.

(ii) *When the two vectors are in opposite directions*, i.e., $\alpha = 180°$, then $R = \sqrt{A^2 + B^2 + 2AB\cos 180°} = A - B$. Also $\beta = 0$

(iii) *When the two vectors are at right angles to each other*, i.e. $\alpha = 90°$, then

$R = \sqrt{A^2 + B^2 + 2AB\cos 90°} = \sqrt{A^2 + B^2}$. Also $\tan\beta = \frac{B}{A}$.

Note: $R_{max} = A + B$ and $R_{min} = A - B$

EXAMPLE 1 The following pairs of forces act on a particle at an angle θ which can have any value

(a) 2 N and 3 N (b) 3 N and 3 N
(c) 2 N and 6 N and (d) 3 N and 8 N

The resultant of which pair cannot have a magnitude of 4 N?

SOLUTION

For pair (a) : $R_{min} = A - B = 3 - 2 = 1$ N
and $R_{max} = A + B = 3 + 2 = 5$ N
For pair (b) : $R_{min} = 0$, $R_{max} = 6$ N
For pair (c) : $R_{min} = 4$ N, $R_{max} = 8$ N
For pair (d) : $R_{min} = 5$ N, $R_{max} = 11$ N

Hence the correct answer is (d).

EXAMPLE 2 The magnitude of the resultant of two vectors of the same magnitude is equal to the magnitude of either vector. Find the angle between the two vectors.

SOLUTION Given $A = B$ and $R = A$ or B.

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\Rightarrow \quad A = \sqrt{A^2 + A^2 + 2A^2\cos\theta} = \sqrt{2A^2(1+\cos\theta)}$$

$$\Rightarrow \quad A^2 = 2A^2(1+\cos\theta) \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$$

3. Triangle Law of Vector Addition

The parallelogram law of vector addition yields the triangle law of vector addition. In Fig. 2.1, vector QP = vector OS = **B**. In triangle OQP, vector OP = **R**. Hence, the triangle law of vector addition may be stated as follows:

If the two vectors are represented in magnitude and direction by the two sides of a triangle taken in the same order, then their resultant is represented in magnitude and direction by the third side of the triangle taken in the opposite order.

4. Subtraction of Vectors

Suppose we wish to subtract a vector **B** from a vector **A**. Since

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

the subtraction of vector **B** from vector **A** is equivalent to the addition of vector – **B** to vector **A**. Hence the procedure to find (**A** – **B**) is as follows:

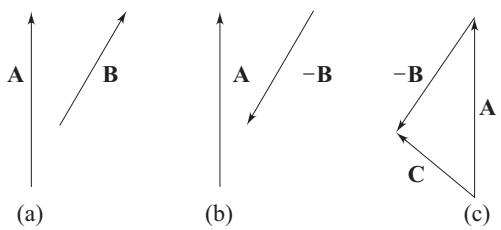


Fig. 2.2

Choose a convenient scale and draw the vectors **A** and **B** as shown in Fig. 2.2 (a). If **B** is to be subtracted from **A**, draw the vector negative of **B**, i.e. draw the vector – **B** [see Fig. 2.2 (b)]. Now shift the vector – **B** parallel to itself so that the tail of – **B** is at the head of **A**. Vector **C** is the sum of vectors **A** and – **B**, i.e. [see Fig. 2.2 (c)]

$$\mathbf{C} = \mathbf{A} + (-\mathbf{B}) = \mathbf{A} - \mathbf{B}$$

5. The Unit Vector

A vector whose magnitude is unity is called a unit vector. A unit vector is represented by $\hat{\mathbf{A}}$ where

$$\hat{\mathbf{A}} = \frac{\mathbf{A}}{A}$$

The unit vectors along x, y and z axes of a rectangular co-ordinate system are represented by $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ respectively.

6. Resolution of a Vector into Rectangular Components

Consider a vector **A** in the x–y plane making an angle θ with the x-axis. The x and y components of **A** are $\mathbf{A}_x$ and $\mathbf{A}_y$. The magnitudes of $\mathbf{A}_x$ and $\mathbf{A}_y$ are (Fig. 2.3)

$$A_x = A\cos\theta, \text{ along } x\text{-direction}$$

and $$A_y = A\sin\theta, \text{ along } y\text{-direction}$$

Thus $$\mathbf{A}_x = A_x\,\hat{\mathbf{i}} = (A\cos\theta)\,\hat{\mathbf{i}}$$

and $$\mathbf{A}_y = A_y\,\hat{\mathbf{j}} = (A\sin\theta)\,\hat{\mathbf{j}}$$

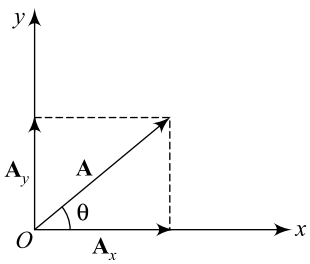


Fig. 2.3

From parallelogram law

$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y = (\mathbf{A}\cos\theta)\,\hat{\mathbf{i}} + (\mathbf{A}\sin\theta)\,\hat{\mathbf{j}}$$

The magnitude of **A** in terms of the magnitudes of its rectangular components is

$$A = \sqrt{A_x^2 + A_y^2}$$

Also $$\tan\theta = \frac{A_y}{A_x}$$

EXAMPLE 3 Resolve **A** into x and y components. The magnitude of **A** is 4 units.

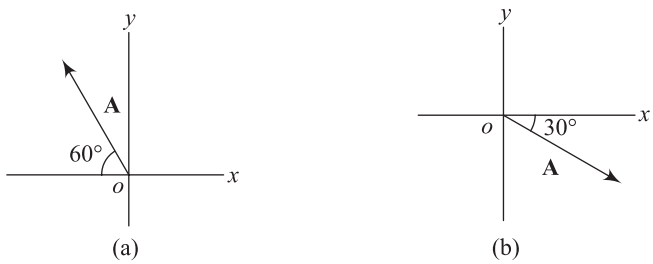


Fig. 2.4

SOLUTION

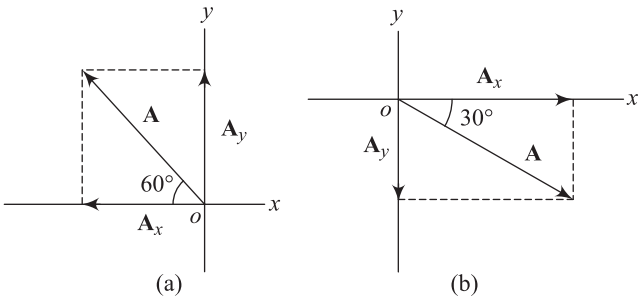


Fig. 2.5

In Fig. 2.5(a): $\mathbf{A}_x = -\mathbf{A}\cos 60° = -\ 4 \times \frac{1}{2} = -2$ units

$$\mathbf{A}_y = +\mathbf{A}\sin 60° = +\ 4 \times \frac{\sqrt{3}}{2}$$

$$= +\ 2\sqrt{3}\ \text{units}$$

In Fig. 2.5(b): $\mathbf{A}_x = +\mathbf{A}\cos 30° = +\ 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$ units

$$\mathbf{A}_y = -\mathbf{A}\sin 30° = -\ 4 \times \frac{1}{2} = -2\ \text{units}$$

Note The magnitude of a component along + x direction or + y direction is taken to be positive while the magnitude of a component along – x direction or – y direction is taken to be negative.

7. Addition of Vectors Using Components

Two or more vectors are added by using components as follows:

(a) Resolve each vector into its rectangular components.

(b) Add the magnitudes of all the x-components taking into account their signs.

R_x = sum of x-components of all the vectors

(c) Similarly R_y = sum of y-components of all the vectors

(d) Magnitude of resultant is

$$R = \sqrt{R_x^2 + R_y^2}$$

(e) The angle θ which the resultant subtends with the x-axis is given by

$$\tan\theta = \frac{R_y}{R_x}$$

EXAMPLE 4 Find the resultant of two vectors **A** = 4 units and **B** = 3 units shown in Fig. 2.6 by using

(a) parallelogram law and

(b) components method.

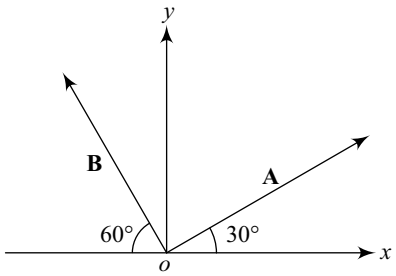


Fig. 2.6

SOLUTION

(a) Angle between the two vectors is (Fig. 2.7)

$$\theta = 90°$$

$$\therefore \quad R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{4^2 + 3^2 + 2 \times 4 \times 3\cos 90°}$$

$$= 5\ \text{units}$$

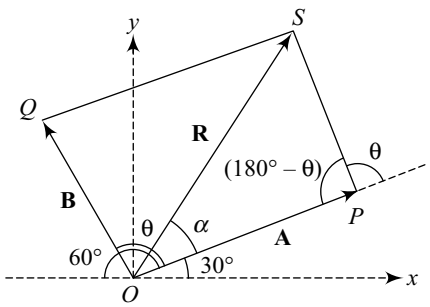


Fig. 2.7

$$\frac{R}{\sin(180° - \theta)} = \frac{B}{\sin\alpha}$$

$$\Rightarrow \quad \sin\alpha = \frac{B\sin\theta}{R} = \frac{3 \times \sin 90°}{5} = \frac{3}{5}$$

$$\therefore \quad \alpha = \sin^{-1}\left(\frac{3}{5}\right) \quad \Rightarrow \quad \alpha \simeq 37°$$

(b) The x and y components of **A** and **B** are [see Fig. 2.8]

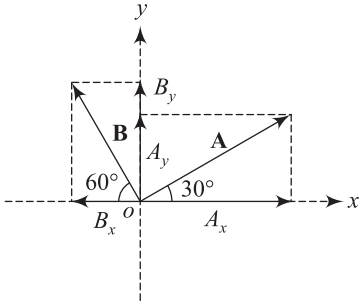


Fig. 2.8

$$A_x = A\cos 30° = 4\times\frac{\sqrt{3}}{2}$$
$$= 2\sqrt{3} \text{ units along } +x \text{ direction}$$
$$A_y = A\sin 30° = 4\times\frac{1}{2}$$
$$= 2 \text{ units along } +y \text{ direction}$$

and $B_x = -B\cos 60° = -3\times\frac{1}{2}$
$= -3/2$ units along $-x$ direction

and $B_y = B\sin 60° = 3\times\frac{\sqrt{3}}{2}$ units along $+y$ direction

$\therefore$ $R_x = A_x + B_x = \left(2\sqrt{3} - 3/2\right)$ along $+x$ direction

$$R_y = A_y + B_y = \left(2+\frac{3}{2}\sqrt{3}\right)\text{along}+y\text{direction}$$

$$R^2 = R_x^2 + R_y^2 = \left(2\sqrt{3}-\frac{3}{2}\right)^2 + \left(2+\frac{3\sqrt{3}}{2}\right)^2$$
$$= 25$$

$\Rightarrow$ $R = 5$ units

The angle subtended by the resultant with the x-axis is given by

$$\tan\alpha = \frac{R_y}{R_x} = \frac{\left(2+\frac{3\sqrt{3}}{2}\right)}{\left(2\sqrt{3}-\frac{3}{2}\right)} = \frac{4.6}{1.96} = 2.347$$

giving $\alpha \simeq 67°$ above the x-axis. Therefore, the angle which the resultant subtends with **A** is 67° – 30° = 37°.

Note

The components method is more useful if more than two vectors have to be added.

8. Scalar or Dot Product

The scalar (or dot) product of two vectors **A** and **B** is defined as the product of the magnitudes of **A** and **B** and the cosine of the angle between them, i.e.

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$$

The scalar product is represented by putting a dot between the two vectors. The scalar product of two vectors is a scalar quantity.

Properties of the Scalar Product

(i) The scalar product is commutative, i.e. $\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$

(ii) $\mathbf{A}\cdot\mathbf{A} = A^2$ or $A = \sqrt{\mathbf{A}\cdot\mathbf{A}}$. This is true because in this case $\theta = 0°$.

(iii) If two vectors **A** and **B** are perpendicular to each other, then $\theta = 90°$ and $\mathbf{A}\cdot\mathbf{B} = AB\cos 90° = 0$. Note that $\mathbf{A}\cdot\mathbf{B}$ can be zero when neither **A** nor **B** is zero.

(iv) For unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ along the three field axes x, y and z, we have

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$$
and $$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0$$

(v) $\mathbf{A}\cdot\mathbf{B} = (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}})\cdot(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}})$
$= A_xB_x + A_yB_y + A_zB_z$

Some Examples of Scalar Product

(i) Work done is $W = \mathbf{F}\cdot\mathbf{s}$ where **F** is the force vector and **s** is the displacement vector.

(ii) Power consumed is $P = \mathbf{F}\cdot\mathbf{v}$ where **F** is the force vector and **v** is the velocity vector.

(iii) Electric current is $I = \mathbf{J}\cdot\mathbf{A}$ where **J** is the current density vector and **A** is the area vector.

(iv) Magnetic flux is $\Phi = \mathbf{B}\cdot\mathbf{A}$ where **B** is the magnetic field vector and **A** is the area vector.

9. Vector or Cross Product

If the smaller angle between two vectors **A** and **B** is θ, then the vector or cross product of vectors **A** and **B** is defined as

$$\mathbf{A}\times\mathbf{B} = (AB\sin\theta)\,\hat{\mathbf{n}}$$

where A and B are the magnitudes of vectors **A** and **B** and $\hat{\mathbf{n}}$ is a unit vector perpendicular to the plane containing **A** and **B**. The vector product of two vectors **A** and **B** is equal to a vector **C**, i.e.

$$\mathbf{A}\times\mathbf{B} = \mathbf{C}$$

The magnitude of vector **C** is given by

$$C = AB\sin\theta$$

The direction of **C** is perpendicular to the plane formed by **A** and **B** and is given by the right hand screw rule.

Properties of a Vector Product

(i) Vector product is anticommutative, i.e. $(\mathbf{A}\times\mathbf{B}) = -(\mathbf{B}\times\mathbf{A})$

(ii) $\mathbf{A}\times\mathbf{A} = 0$, i.e. the vector product of a vector by itself is zero. This is because, in this case, $\theta = 0$, and hence $\sin\theta = 0$.

Therefore $\mathbf{A}\times\mathbf{A} = AA\sin\theta = 0$

Hence, the condition for two vectors to be parallel ($\theta = 0°$) or antiparallel ($\theta = 180°$) is that their vector product should be zero.

If $\mathbf{A} \times \mathbf{B} = 0$, it means either (i) $\mathbf{A}$ is zero or, (ii) $\mathbf{B} = 0$ or (iii) the angle θ between them is 0° or 180°.

(iii) The distributive law holds for both scalar and vector products, i.e.

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$$

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$$

(iv) $(\lambda \mathbf{A}) \times \mathbf{B} = \mathbf{A} \times (\lambda \mathbf{B}) = \lambda(\mathbf{A} \times \mathbf{B})$; λ a real number.

(v) $|\mathbf{A} \times \mathbf{B}|^2 = |\mathbf{A}|^2 |\mathbf{B}|^2 - (\mathbf{A} \cdot \mathbf{B})^2$.

(vi) $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are the three mutually perpendicular unit vectors at the origin O and along OX, OY and OZ respectively; the right-hand rule gives:

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}, \quad \hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$$

$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}, \quad \hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

(vii) $$(\mathbf{A} \times \mathbf{B}) = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{\mathbf{i}}\ (A_yB_z - A_zB_y) + \hat{\mathbf{j}}\ (A_zB_x - A_xB_z) + \hat{\mathbf{k}}\ (A_xB_y - A_yB_x)$$

Some Examples of Vector Product

(i) Torque = $\mathbf{r} \times \mathbf{F}$, where $\mathbf{r}$ is the position vector and $\mathbf{F}$ is the force vector.

(ii) Linear velocity = $\omega \times \mathbf{r}$ where ω is the angular frequency vector and $\mathbf{r}$ is the position vector.

(iii) Angular momentum = $\mathbf{r} \times \mathbf{p}$ where $\mathbf{r}$ is the position vector and $\mathbf{p}$ is the linear momentum vector.

EXAMPLE 5 Find the angle between vectors $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$.

SOLUTION $\mathbf{A} \cdot \mathbf{B} = AB \cos\theta$

Magnitude of $\mathbf{A}$ is $A = \sqrt{1^2 + 1^2} = \sqrt{2}$

Magnitude of $\mathbf{B}$ is $B = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$$\therefore \quad (\hat{\mathbf{i}} + \hat{\mathbf{j}}) \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}}) = \sqrt{2} \times \sqrt{2} \cos\theta$$

$$\Rightarrow \quad \hat{\mathbf{i}} \cdot \hat{\mathbf{i}} - \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = 2 \cos\theta \qquad (\because\ \hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{i}} = 0)$$

$$\Rightarrow \quad 1 - 1 = 2 \cos\theta \text{ gives } \cos\theta = 0$$

or $\theta = 90°$

EXAMPLE 6 Two vectors $\mathbf{A}$ and $\mathbf{B}$ are inclined at an angle θ. Using definition of scalar product, show that the magnitude of the resultant of vectors $\mathbf{A}$ and $\mathbf{B}$ is given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

SOLUTION $\mathbf{R} = \mathbf{A} + \mathbf{B}$. Therefore,

$$\mathbf{R} \cdot \mathbf{R} = (\mathbf{A} + \mathbf{B}) \,.\, (\mathbf{A} + \mathbf{B}) = \mathbf{A} \,.\, \mathbf{A} + \mathbf{A} \,.\, \mathbf{B} + \mathbf{B} \,.\, \mathbf{A} + \mathbf{B} \,.\, \mathbf{B}$$

$$\Rightarrow \quad R^2 = A^2 + 2\ \mathbf{A} \,.\, \mathbf{B} + B^2 = A^2 + B^2 + 2\ AB \cos\theta$$

$$\Rightarrow \quad R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

EXAMPLE 7 Given $\mathbf{A} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$ and $\mathbf{B} = 5\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$. Find the magnitude and direction of $(\mathbf{A} \times \mathbf{B})$.

SOLUTION $$(\mathbf{A} \times \mathbf{B}) = \left(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}\right) \times \left(5\hat{\mathbf{i}} + 4\hat{\mathbf{j}}\right)$$

$$= 10\,\hat{\mathbf{i}} \times \hat{\mathbf{i}} + 8\,\hat{\mathbf{i}} \times \hat{\mathbf{j}} - 15\,\hat{\mathbf{j}} \times \hat{\mathbf{i}} - 12\,\hat{\mathbf{j}} \times \hat{\mathbf{j}}$$

$$\left(\because \hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = 0 \text{ and } \hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}\right)$$

$$= 0 + 8\hat{\mathbf{k}} + 15\hat{\mathbf{k}} - 0$$

$$= 23\ \hat{\mathbf{k}}$$

Thus the magnitude of $(\mathbf{A} \times \mathbf{B})$ is 23 units and its direction is along + z axis.

EXAMPLE 8 Find the unit vector in the direction of vector

$$\mathbf{A} = 3\,\hat{\mathbf{i}} + \hat{\mathbf{j}} - 5\,\hat{\mathbf{k}}.$$

SOLUTION $$\hat{\mathbf{A}} = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{A}$$

where $|\mathbf{A}| = A$ is the magnitude of $\mathbf{A}$ which is

$$A = \sqrt{(3)^2 + (1)^2 + (-5)^2} = \sqrt{35}$$

$$\therefore \quad \hat{\mathbf{A}} = \frac{3}{\sqrt{35}}\,\hat{\mathbf{i}} + \frac{1}{\sqrt{35}}\,\hat{\mathbf{j}} - \frac{5}{\sqrt{35}}\,\hat{\mathbf{k}}$$

EXAMPLE 9 Given $\mathbf{A} = 3\,\hat{\mathbf{i}} + \hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}$ and $\mathbf{B} = \hat{\mathbf{i}} + 3\,\hat{\mathbf{j}} - 5\,\hat{\mathbf{k}}$, find the unit vector in the direction of (a) $\mathbf{A} + \mathbf{B}$ and (b) $\mathbf{A} - \mathbf{B}$.

SOLUTION (*a*) $\mathbf{C} = \mathbf{A} + \mathbf{B}$

$$= (3\,\hat{\mathbf{i}} + \hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}) + (\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}} - 5\,\hat{\mathbf{k}})$$

$$= 4\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} - \hat{\mathbf{k}}$$

$$\therefore \quad \hat{\mathbf{C}} = \frac{\mathbf{C}}{|\mathbf{C}|} = \frac{4\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{(4)^2 + (4)^2 + (-1)^2}}$$

$$= \frac{1}{\sqrt{33}}\left(4\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}\right)$$

(b) $\mathbf{D} = \mathbf{A} - \mathbf{B} = \left(3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 4\hat{\mathbf{k}}\right) - \left(\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}\right)$

$= 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$

$\therefore \quad \hat{\mathbf{D}} = \dfrac{\mathbf{D}}{|\mathbf{D}|} = \dfrac{2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 9\hat{\mathbf{k}}}{\sqrt{(2)^2 + (-2)^2 + (9)^2}}$

$= \dfrac{1}{\sqrt{89}}\left(2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 9\hat{\mathbf{k}}\right)$

EXAMPLE 10 If $|\mathbf{A} + \mathbf{B}| = |\mathbf{A} - \mathbf{B}|$, the angle between vectors **A** and **B** is

(a) zero (b) 45°
(c) 90° (d) 180°

SOLUTION $|\mathbf{A} + \mathbf{B}|$ = magnitude of the resultant of **A** and **B**

$= \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$|\mathbf{A} - \mathbf{B}|$ = magnitude of the resultant of **A** and $-\mathbf{B}$

$= \sqrt{A^2 + B^2 - 2\,AB\cos\theta}$

Given $|\mathbf{A} + \mathbf{B}| = |\mathbf{A} - \mathbf{B}|$

$\Rightarrow \sqrt{A^2 + B^2 + 2\,AB\cos\theta} = \sqrt{A^2 + B^2 - 2\,AB\cos\theta}$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90°$

EXAMPLE 11 The sum (i.e. resultant) of vectors **A** = 4 units and **B** = 8 units is perpendicular to **A**. The angle θ between **A** and **B** is

(a) 60° (b) 120°
(c) 135° (d) 145°

SOLUTION Let $\mathbf{C} = \mathbf{A} + \mathbf{B}$. Given that **C** is perpendicular to **A**. Therefore

$\mathbf{A} \cdot \mathbf{C} = 0$

$\Rightarrow \quad \mathbf{A} \cdot (\mathbf{A} + \mathbf{B}) = 0$

$\Rightarrow \quad \mathbf{A} \cdot \mathbf{A} + \mathbf{A} \cdot \mathbf{B} = 0$

$\Rightarrow \quad A^2 + AB\cos\theta = 0$

$\Rightarrow \quad A + B\cos\theta = 0 \Rightarrow \cos\theta = -\dfrac{A}{B} = -\dfrac{4}{8} = -\dfrac{1}{2}$

Hence $\theta = 120°$

EXAMPLE 12 Find the angle θ between $\mathbf{A} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{B} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$.

SOLUTION $\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$

$\Rightarrow \quad \cos\theta = \dfrac{\mathbf{A} \cdot \mathbf{B}}{AB}$

Now $\mathbf{A} \cdot \mathbf{B} = \left(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right) \cdot \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}\right)$

$= 2 + 6 + 15 = 23 \quad \left(\because \hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = 0\right)$

Also $A = \sqrt{(1)^2 + (2)^2 + (3)^2} = \sqrt{14}$

and $B = \sqrt{(2)^2 + (3)^2 + (5)^2} = \sqrt{38}$

$\therefore \quad \cos\theta = \dfrac{23}{\sqrt{14} \times \sqrt{38}} = \dfrac{23}{\sqrt{532}}$

$\Rightarrow \quad \theta = \cos^{-1}\left(\dfrac{23}{\sqrt{532}}\right)$

EXAMPLE 13 If $\mathbf{A} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{B} = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$, then the angle between **A** and **B** is

(a) zero (b) 90°
(c) 60° (d) 120°

SOLUTION $\mathbf{B} = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$

$= 3\left(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right) = 3\mathbf{A}$

It follows that the magnitude of **B** is 3 times that of **A** but the direction of **B** is the same as that of **A**. Hence the correct choice is (a).

EXAMPLE 14 If $\mathbf{A} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, $\mathbf{B} = 2\hat{\mathbf{i}} + \hat{\mathbf{k}}$ and $\mathbf{C} = \hat{\mathbf{j}} - \hat{\mathbf{k}}$, find $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$.

SOLUTION $\mathbf{B} \times \mathbf{C} = \left(2\hat{\mathbf{i}} + \hat{\mathbf{k}}\right) \times \left(\hat{\mathbf{j}} - \hat{\mathbf{k}}\right)$

$= 2\,\hat{\mathbf{i}} \times \hat{\mathbf{j}} - 2\hat{\mathbf{i}} \times \hat{\mathbf{k}} + \hat{\mathbf{k}} \times \hat{\mathbf{j}} - \hat{\mathbf{k}} \times \hat{\mathbf{k}}$

$= 2\,\hat{\mathbf{k}} + 2\hat{\mathbf{j}} - \hat{\mathbf{i}} - 0$

$= -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \left(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right) \cdot \left(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right)$

$= -1 + 4 + 6 = 9$

EXAMPLE 15 If $\mathbf{A} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{B} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$, the $\mathbf{A} \cdot (\mathbf{A} \times \mathbf{B})$ is

(a) 15 (b) $\sqrt{29}$
(c) 45 (d) zero

SOLUTION If $\mathbf{C} = \mathbf{A} \times \mathbf{B}$, then **C** is perpendicular to both **A** and **B**

$\mathbf{A} \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{A} \cdot \mathbf{C} = 0$, since **C** is perpendicular to **A**.

EXAMPLE 16 Three vectors **AB**, **BC** and **CA** are represented by the three sides of a triangle as shown in Fig. 2.9. Show that

$\mathbf{AB} + \mathbf{BC} + \mathbf{CA} = 0$

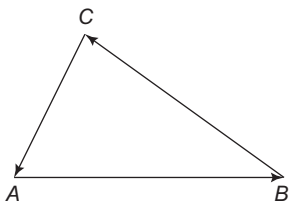


Fig. 2.9

SOLUTION From triangle law,

$$\mathbf{AB} + \mathbf{BC} = \mathbf{AC} = -\mathbf{CA}$$

$$\therefore \quad \mathbf{AB} + \mathbf{BC} + \mathbf{CA} = 0$$

Note If three vectors are represented by the three sides of a triangle taken in the same order, their resultant is zero.

10. Position Vector and Displacement Vector

The position vector of a particle describes its instantaneous position with respect to the origin of the chosen frame of reference. It is a vector joining the origin to the particle and is denoted by vector **r**.

For one-dimensional motion (say along x-axis), $\mathbf{r} = x\hat{\mathbf{i}}$, where x is the distance of the particle from origin O.

For two-dimensional motion (say in the x–y plane), $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$, where (x, y) are the x and y coordinates of the particle.

For three-dimensional motion, $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$.

Displacement vector

If $\mathbf{r}_1$ is the position vector of a particle at time t_1, and $\mathbf{r}_2$ at time t_2, then the displacement vector is given by

$$\mathbf{s} = \mathbf{r}_2 - \mathbf{r}_1$$

Vector **s** is the resultant of vectors $\mathbf{r}_2$ and $-\mathbf{r}_1$ (Fig. 2.10).

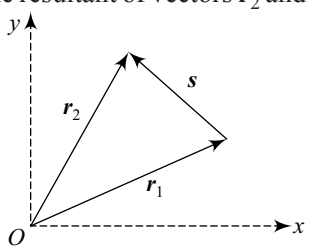


Fig. 2.10

The displacement vector is a vector joining the initial and the final positions of the particle after a given interval of time and its direction is from the initial to the final position.

EXAMPLE 17 A particle moves from A to B along a circle of radius R. Find the path length and the magnitude of the displacement in terms of R and θ. [see Fig. 2.11(a)]

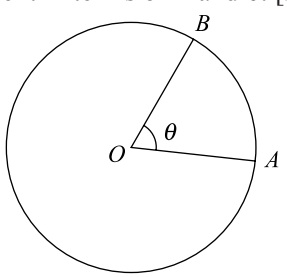


Fig. 2.11(a)

SOLUTION Path length = arc $AB = R\theta$

As shown in Fig. 2.11(b), the magnitude of displacement is

$$s = AB = AC + CB$$

$$= 2\,AC$$

$$\Rightarrow \quad s = 2R \sin(\theta/2)$$

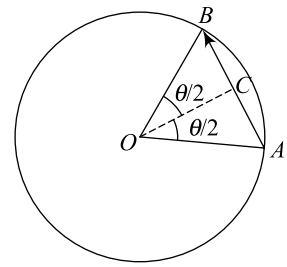


Fig. 2.11(b)

11. Instantaneous Velocity and Average Velocity

The rate of change of displacement with time at a given instant is called instantaneous velocity and is given by

$$v = \frac{dx}{dt}$$

The average velocity in a given time interval is defined as

$$v_{av} = \frac{\text{total displacement}}{\text{time interval}}$$

EXAMPLE 18 The position of a particle moving along x-axis is given by $x = 2t - 3t^2 + t^3$, where x is in metre and t in second.

(a) Find the velocity of the particle at $t = 2$ s.

(b) Find the average speed of the particle in the time interval from $t = 2$ s to $t = 4$ s.

SOLUTION

(a) $x = 2t - 3t^2 + t^3$

$$v = \frac{dx}{dt} = 2 - 6t + 3t^2$$

$$\therefore \quad v \text{ at } (t = 2 \text{ s}) = 2 - 6 \times 2 + 3 \times (2)^2 = 2 \text{ ms}^{-1}$$

(b) Position at $t = 2$ s is

$$x_1 = 2 \times 2 - 3 \times (2)^2 + (2)^3 = 0$$

Position at $t = 4$ s is

$$x_2 = 2 \times 4 - 3 \times (4)^2 + (4)^3 = 24 \text{ m}$$

$\therefore$ Displacement is $x_2 - x_1 = 24 - 0 = 24$ m. Time interval $= 4 - 2 = 2$ s. Therefore,

$$\text{Average velocity} = \frac{24}{2} = 12 \text{ ms}^{-1}$$

12. Instantaneous Acceleration

The rate of change of velocity with time at a given instant is called instantaneous acceleration and is given by

$$a = \frac{dv}{dt}$$

13. Equations of One Dimensional Motion with Constant Acceleration

Let x_0 be the position of a particle at time $t = 0$ and let u be its velocity at $t = 0$. It is given a constant acceleration a for time t. As a result it moves in a straight line to a position x and acquires a velocity v.

The particle suffers a displacement $s = x - x_0$ in time t. The equations of motion of the particle are

$$v = u + at \qquad (2.1)$$

$$x = x_0 + ut + \frac{1}{2}at^2$$

or
$$s = ut + \frac{1}{2}at^2 \qquad (2.2)$$

and
$$v^2 - u^2 = 2\,a(x - x_0)$$

or
$$v^2 - u^2 = 2as \qquad (2.3)$$

1. While solving numerical problems of bodies moving in a straight horizontal direction, we will consider only the magnitudes of u, v, a and s and take care of their direction by assigning positive or negative sign to the quantity. For example, $+a$ will mean acceleration $-a$ will mean retardation (or deceleration).
2. In the case of a body falling vertically under gravity or projected vertically upwards, we use the following sign conventions.

   Quantities directed vertically upwards are taken to be positive and those directed vertically downwards are taken to be negative. Since the acceleration due to gravity is directed downward for a body moving vertically up or falling vertically down, we take $a = -g = -9.8\ \text{ms}^{-2}$ in Eqs. (2.1), (2.2) and (2.3).
3. Displacement in the nth second is given by

$$s_n = \text{displacement in } n \text{ seconds} - \text{displacement in } (n-1) \text{ seconds}$$

$$= u_n + \frac{1}{2}a(n)^2 - u(n-1) - \frac{1}{2}a(n-1)^2$$

$$\Rightarrow \qquad s_n = u + \frac{a}{2}(2n-1) \qquad (2.4)$$

Applications

(i) If a body moving with constant acceleration, starts from A with initial velocity u and reaches B with a velocity v, then the velocity midway between A and B is

$$v' = \sqrt{\frac{u^2 + v^2}{2}}$$

(ii) A body starting from rest has an acceleration a for a time t_1 and comes to rest under a retardation b for a time t_2. If s_1 and s_2 are the distances travelled in t_1 and t_2,

(a) $\dfrac{s_1}{s_2} = \dfrac{b}{a} = \dfrac{t_1}{t_2}$

(b) Total distance travelled $(s_1 + s_2)$

$$= \frac{1}{2}\left(\frac{ab}{a+b}\right)T^2, \text{ where } T = t_1 + t_2.$$

(c) Maximum velocity attained is

$$v_{\max} = \left(\frac{ab}{a+b}\right)T$$

(d) Average velocity over the whole trip is

$$v_{\text{av}} = \frac{v_{\max}}{2}.$$

(iii) At time $t = 0$ a body is thrown vertically upwards with a velocity u at time $t = T$, another body is thrown vertically upwards with the same velocity u. The two bodies will meet at time

$$t = \frac{T}{2} + \frac{u}{g}$$

(iv) A body is dropped from rest and at the same time another body is thrown downward with a velocity u from the same point, then
(a) the acceleration of each body is g,
(b) their relative velocity is always u,
(c) their separation will be x after a time $t = \dfrac{x}{u}$.

(v) From the top of a building, body A is thrown upwards with a certain speed, body B is thrown downwards with the same speed and body C is dropped from rest from the same point. If t_1, t_2 and t_3 are their respective times of reach the ground, then

$$t_3 = \sqrt{t_1 t_2}$$

(vi) A body of mass m is dropped from a height h on a heap of sand. If it penetrates a depth x in the sand,
(a) the average retardation in sand is given by

$$a = \frac{gh}{x}$$

because loss in PE (mgh) = work done against the resistive force of sand (max).
(b) total average force exerted by sand is

$$F = mg + ma = m(g + a)$$

(vii) A body is thrown vertically upward with a velocity u. If the resistive force due to air-friction produces a constant acceleration (or retardation) a ($< g$)
(a) the net acceleration during upward motion $= g + a$,
(b) the net acceleration during downward motion = $g - a$,
(c) the maximum height attained is

$$h = \frac{u^2}{2(g+a)}$$

(d) the time taken to reach the maximum height is

$$t_1 = \sqrt{\frac{2h}{(g+a)}} = \frac{u}{(g+a)}$$

(e) the time of descent is

$$t_2 = \sqrt{\frac{2h}{(g-a)}} = \frac{u}{\left(g^2 - a^2\right)^{1/2}}$$

(f) the speed with which the body hits the ground is

$$v = \sqrt{2(g-a)h} = u\sqrt{\frac{(g-a)}{(g+a)}}$$

Note

Note that t_1 is less than t_2.

Some useful tips

(i) If a body, starting from rest, moves with a constant acceleration, the distances covered by it in 1s, 2s, 3s …… are in the ratio $1^2 : 2^2 : 3^2$ … = 1 : 4 : 9 ……

(ii) If a body starts from rest and moves with a contant acceleration, the distance covered by it in the lst, 2nd, 3rd….seconds are in the ratio 1 : 3 : 5…

(iii) A body is projected upward with a certain speed. If air resistance is nelected, the speed with which it hits the ground = the speed of projection.

(iv) If a body is projected upwards with a velocity u, the maximum height attained is proportional to u^2 and the time of ascent is proportional to u.

(v) For a freely falling body,
(a) velocity ∝ time
(b) distance fallen ∝ (time)^2
(c) velocity ∝ $\sqrt{\text{distance fallen}}$

14. Graphical Representation

1. *Displacement – time (x – t) graphs (Fig. 2.12)*

Fig. 2.12(a) : Body at rest
Fig. 2.12(b) : Body in uniform motion
Fig. 2.12(c) : Body subjected to acceleration ($a > 0$)
Fig. 2.12(d) : Body subjected to retardation ($a < 0$)
Fig. 2.12(e) : Body accelerating and then decelerating

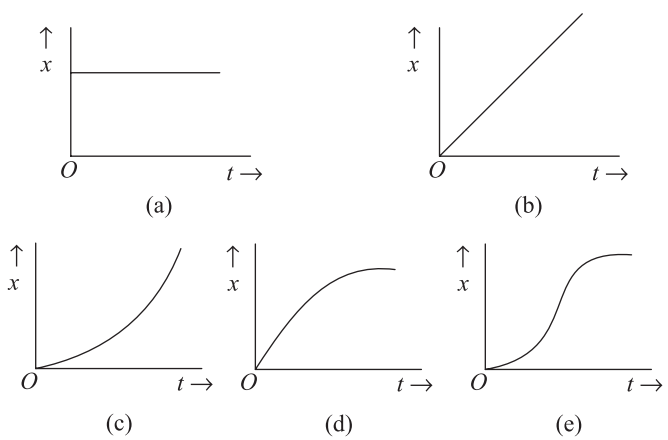


Fig. 2.12

Note

The slope of $x - t$ graph gives velocity for uniform motion [Fig. 2.16(b)]. For non-uniform motion [Fig. 2.16(c), (d) and (e)], the slope of the tangent to the curve at a point gives velocity at that instant.

2. Velocity-time ($v - t$) graphs for uniformly accelerated motion (Fig. 2.13)

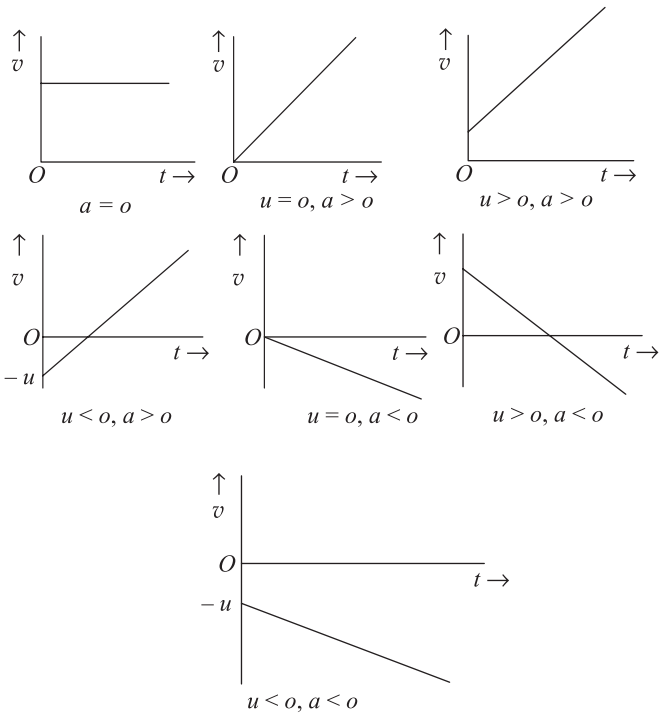


Fig. 2.13

> **Note**
> Acceleration = slope of ($v - t$) graph
> Displacement = area under ($v - t$) graph

3. Acceleration – time ($a - t$) graph [Fig. 2.14]

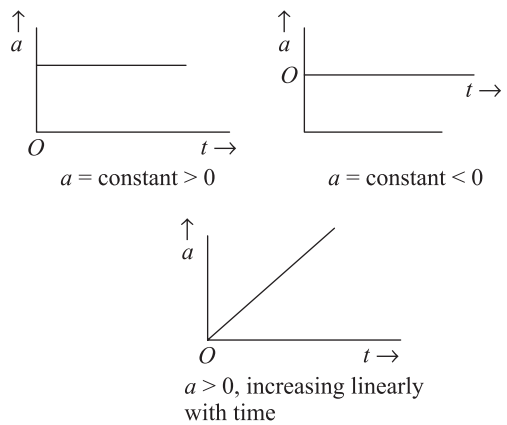


Fig. 2.14

4. ($a - t$), ($v - t$), ($x - t$) graphs for free fall [Fig 2.15]

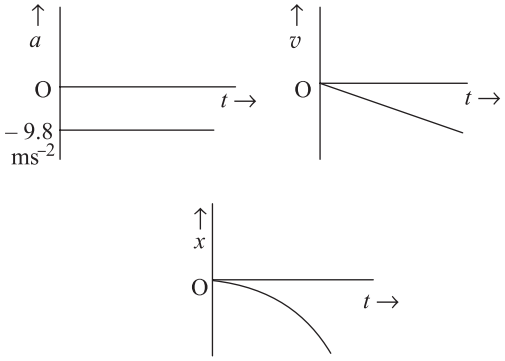


Fig. 2.15

EXAMPLE 19 The displacement x (in metres) of a body varies with time t (in seconds) as

$$x = -\frac{2}{3}t^2 + 16t + 2$$

How long does the body take to come to rest?

SOLUTION $v = \dfrac{dx}{dt} = -\dfrac{4}{3}t + 16$

$$0 = -\frac{4}{3}t + 16 \quad \Rightarrow \quad t = 12 \text{ s}$$

EXAMPLE 20 From the top of a building 40 m tall, a ball a thrown vertically upwards with a velocity of 10 ms $^{-1}$. (a) After how long will the ball hit the ground? (b) After how long will the ball take to pass through the point from where it was projected? (c) With what velocity will it hit the ground? Take $g = 10$ ms^{-2}.

SOLUTION

(a) $s = -40$ m, $u = +10$ ms $^{-1}$, $a = -10$ ms^{-2}

Now $\quad s = ut + \dfrac{1}{2}\, at^2$

$$\Rightarrow \quad -40 = 10t + \frac{1}{2} \times (-10)t^2$$

$$\Rightarrow \quad t^2 - 2t - 8 = 0$$

$$\Rightarrow \quad (t + 2)\,(t - 4) = 0$$

$\Rightarrow \quad t = -2$ s or 4 s. The negative value of t is not possible.

Hence the ball will hit the ground after 4 s.

(b) $s = 0$, $u = +10\ \text{ms}^{-1}$ and $a = -10\ \text{ms}^{-2}$.
Therefore $0 = 10t - 5t^2 \Rightarrow t = 2$ s

(c) $v = u + at = 10 - 10 \times 4 = 10 - 40 = -30\ \text{ms}^{-1}$.
The negative sign indicates the velocity v is directed downwards.

EXAMPLE 21 Figure 2.16 shows the velocity – time graph of a body moving in a straight line. Find (a) the distance travelled by the body in 20 s, (b) the displacement of the body in 20 s and (c) the average velocity in the time interval $t = 0$ to $t = 20$ s.

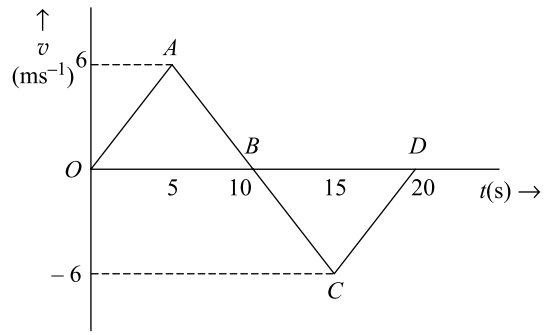


Fig. 2.16

SOLUTION

(a) Distance = area under speed – time graph which is shown in Fig. 2.17.

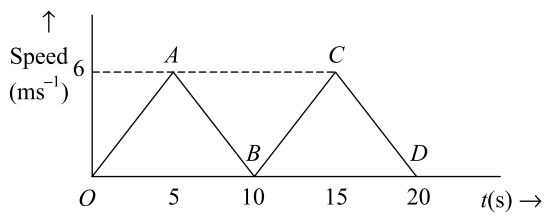


Fig. 2.17

$\therefore$ Distance travelled in 20 s = area of OAB + area of BCD

$$= \frac{1}{2} \times 6 \times 10 + \frac{1}{2} \times 6 \times 10 = 60 \text{ m}$$

(b) Displacement in 20 s = area of OAB – area of BCD in Fig. 2.16
$= 30 - 30 = 0$

(c) Average velocity $= \dfrac{\text{displacement}}{\text{time}} = 0$

EXAMPLE 22 A body moving in a straight line covers a distance of 14 m in the 5th second and 20 m in the 8th second. How much distance will it cover in the 15th second?

SOLUTION $S_n = u + \dfrac{a}{2}(2n - 1)$

$$S_5 = u + \frac{9a}{2} \quad (1)$$

$$S_8 = u + \frac{15a}{2} \quad (2)$$

Solving (1) and (2), we get $a = 2\ \text{ms}^{-2}$ and $u = 5\ \text{ms}^{-1}$.

$$\therefore \quad S_{15} = u + \frac{29a}{2} = 5 + \frac{29 \times 2}{2} = 34 \text{ m}$$

EXAMPLE 23 A police jeep moving at a constant speed v on a straight road is chasing a thief riding on motor-cycle. The thief starts from rest when the police jeep is at a distance x away and accelerates at a constant rate a. The police will be able to catch the thief if

(a) $v \geq \sqrt{ax}$ (b) $v \geq \sqrt{2ax}$

(c) $v \geq 2\sqrt{ax}$ (d) $v \geq \sqrt{\dfrac{ax}{2}}$

SOLUTION Suppose the thief is caught at a time t after the starting of the motorcycle. The distance travelled by the motorcycle in this time t is

$$S = \frac{1}{2}at^2 \quad \text{(i)}$$

During this time, the jeep must travel a distance

$$S + x = vt \quad \text{(ii)}$$

Using (i) and (ii)

$$\frac{1}{2}at^2 + x = vt$$

$$\Rightarrow \quad at^2 - 2vt + 2x = 0$$

The roots of this quadratic equation are

$$t = \frac{v \pm \sqrt{v^2 - 2ax}}{a}$$

Since t must be real and positive, we must have

$$v^2 \geq 2ax \Rightarrow v \geq \sqrt{2ax}$$

EXAMPLE 24 A balloon is rising vertically upwards with a constant velocity of 10 ms^{-1}. When it is at a height of 45 m above the ground, a stone is dropped from it. Find the height of the balloon when the stone hits the ground. Take $g = 10\ \text{ms}^{-2}$.

SOLUTION At time $t = 0$ (when the stone is dropped), its velocity is $u = +10\ \text{ms}^{-1}$ (vertically upwards). Displacement $s = -45$ m (vertically downwards) and $a = -g = -10\ \text{ms}^{-2}$

Using $S = ut + \dfrac{1}{2}at^2$, we have

$$-45 = 10t + \frac{1}{2}(-10)t^2$$

$$\Rightarrow \qquad -45 = 10t - 5t^2$$

$$\Rightarrow \qquad t^2 - 2t - 9 = 0$$

The two roots of t are

$$t = \frac{1 \pm \sqrt{4+36}}{2} = \frac{1 \pm 6.32}{2}$$

$$= -2.7 \text{ s or } 3.7 \text{ s}$$

The negative value of t is not possible. Therefore $t = 3.7$ s. During this time the balloon has moved up a distance = ut = 10 × 3.7 = 37 m. Hence the balloon is at a height of 45 + 37 = 82 m above the ground.

EXAMPLE 25 A ball is dropped from the top of a tower. In the last second of its motion, the ball covers a distance of 9/25 times the height of the tower. Find the height of the tower.

SOLUTION Let h be the height of the tower. If t is the time taken by the ball to reach the ground, then

$$h = \frac{1}{2} gt^2 \qquad \text{(i)}$$

The distance covered in $(t - 1)$ seconds is

$$h' = \frac{1}{2} g(t-1)^2$$

Given $$h' = h - \frac{9}{25} h = \frac{16h}{25}$$

$$\therefore \qquad \frac{16h}{25} = \frac{1}{2} g(t-1)^2 \qquad \text{(ii)}$$

Dividing (ii) by (i)

$$\frac{16}{25} = \frac{(t-1)^2}{t^2}$$

or $$\frac{4}{5} = \frac{t-1}{t} \Rightarrow t = 5\text{s}$$

Using $t = 5$ s in Eq (i), we get

$$h = \frac{1}{2} \times 9.8 \times (5)^2 = 122.5 \text{ m}$$

15. Relative Velocity in One Dimension

If two bodies A and B are moving in a straight line with velocities v_A and v_B respectively, the relative velocity of A with respect to B is defined as

$$v_{AB} = v_A - v_B$$

The relative velocity of B with respect to A will be

$$v_{BA} = v_B - v_A$$

EXAMPLE 26 A police van moving on a highway with a speed of 10 ms^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 30 ms^{-1}. If the muzzle speed of the bullet is 140 ms^{-1}, with what speed will the bullet hit the thief's car?

SOLUTION Speed of police van is v_V = 10 ms^{-1}, speed of thief's car is v_C = 30 ms^{-1}. Relative velocity of bullet with respect to van is v_{BV} = 140 ms^{-1}. Let v_B be the velocity of the bullet. $v_{BV} = v_B - v_V$. Hence $v_B = v_{BV} + v_V$ = 140 + 10 = 150 ms^{-1}. The bullet will hit the car with a speed

$$v_{BC} = v_B - v_C = 150 - 30 = 120 \text{ ms}^{-1}.$$

EXAMPLE 27 From the top of a tower 60 m tall, a body is thrown vertically down with a velocity of 10 ms^{-1}. At the same time, another body is thrown vertically upward from the ground with a velocity of 20 ms^{-1}. (a) After how long will the two bodies meet? (b) At what height above the ground do they meet? Take g = 10 ms^{-2}.

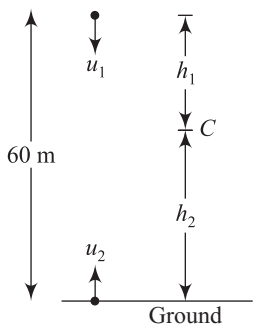


Fig. 2.18

SOLUTION

(a) Suppose the bodies meet at C and let t be the time at which they meet.

For body 1: $s = -h_1$, $u_1 = -10$ ms^{-1}, $a = -10$ ms^{-2}

$$\therefore \qquad -h_1 = -10t + \frac{1}{2} \times (-10) t^2$$

which gives $$h_1 = 5t\,(t+2) \qquad (1)$$

For body 2: $s = +h_2$, $u_2 = +20$ ms^{-1}, $a = -10$ ms^{-2}

$$\therefore \qquad h_2 = 20t - 5t^2 = 5t\,(-t+4) \qquad (2)$$

Adding (1) and (2), $h_1 + h_2 = 30t$ or $60 = 30t$ $\Rightarrow$ $t = 2$ s.

(b) Using $t = 2$ s in Eq. (2), $h_2 = 5 \times 2\,(4-2) = 20$ m

Alternative method

Relative velocity of body 1 with respect to body 2 is

$$u_{12} = u_1 - u_2 = -10 - (20) = -30 \text{ ms}^{-1}$$

Relative displacement of body 1 with respect of body 2 is

$$s_{12} = -h_1 - (h_2) = -(h_1 + h_2) = -60 \text{ m}$$

Relative acceleration is $a_{12} = a_1 - a_2 = -g - (-g) = 0$. Using

$$s = ut + \frac{1}{2} at^2, \text{ we have}$$

$$-60 = -30\,t \quad \Rightarrow \quad t = 2 \text{ s.}$$

$$h_2 = 20t - 5t^2 = 20 \times 2 - 5 \times (2)^2 = 20 \text{ m}$$

EXAMPLE 28 The driver of train A moving at a speed of 30 ms^{-1} sights another train B moving on the same track at a speed of 10 ms^{-1} in the same direction. He immediately applies brakes and achieves a uniform retardation of 2 ms^{-2}. To avoid collision, what must be the minimum distance between trains A and B when the driver of A sights B?

SOLUTION Relative initial velocity A w.r.t. B is,

$$u_{AB} = u_A - u_B = 30 - 10 = 20 \text{ ms}^{-1}$$

Relative retardation of A w.r.t B is

$$a_{AB} = a_A - a_B = -2 - 0 = -2 \text{ ms}^{-2}$$

To avoid collision, the relative final velocity of A w.r.t. B = v_{AB} must be zero. Minimum relative displacement of A w.r.t. B is s_{AB} which is found by using the relation $v^2 - u^2 = 2\,as$ which gives

$$0 - (20)^2 = 2 \times (-2)\, s_{AB} \quad \Rightarrow \quad s_{AB} = 100 \text{ m}$$

EXAMPLE 29 The driver of a train moving at a speed v_1 sights another train at a distance d ahead of him, moving in the same direction with a slower speed v_2. He immediately applies brakes to achieve a constant retardation a. There will be no collision if d is greater than

(a) $\dfrac{(v_1 - v_2)^2}{a}$ (b) $\dfrac{(v_1^2 - v_2^2)}{a}$

(c) $\dfrac{(v_1 - v_2)^2}{2a}$ (d) $\dfrac{(v_1^2 - v_2^2)}{2a}$

SOLUTION The two trains will not collide if the initial relative velocity $u = (v_1 - v_2)$ reduces to zero relative velocity ($v = 0$) in a minimum distance $S = d_{\min}$ under a retardation ($-a$). Using $v^2 - u^2 = 2a\,S$, we have

$$0 - (v_1 - v_2)^2 = 2(-a)\, d_{\min}$$

$$\Rightarrow \qquad d_{\min} = \frac{(v_1 - v_2)^2}{2a}$$

So the correct choice is (c).

16. Solving Problems Involving Non-uniform Acceleration

(a) *Finding velocity and displacement if the dependence of acceleration on time is given.*

Use $a(t) = \dfrac{dv}{dt}$ which gives $dv = a\,dt$. Substitute the given expression for a in terms of t and integrate both sides.

$$\int_u^v dv = \int_0^t a(t)\,dt$$

and obtain the expression for $v(t)$.

To find displacement x, use $v(t) = \dfrac{dx}{dt}$ which gives $dx = v(t)\,dt$.

Substitute the expression for v and integrate both sides

$$\int_{x_0}^{x} dx = \int_0^t v(t)\,dt$$

EXAMPLE 30 A particle starts from rest at $x = 0$. Its acceleration at time $t = 0$ is 5 ms^{-2} which varies with time as shown in Fig. 2.19. Find (i) the maximum speed of the particle and (ii) its displacement in time interval from $t = 0$ to $t = 2$ s.

SOLUTION

(i) Slope of graph is $m = -\dfrac{5}{6}$ ms^{-2} per second. Intercept $c = 5$ ms^{-2}. Using $y = mx + c$, the acceleration a varies with time as

$$a = -\frac{5}{6}t + 5$$

$$\Rightarrow \qquad \frac{dv}{dt} = -\frac{5}{6}t + 5$$

Integrating $\displaystyle\int_0^v dv = \int_0^t \left(-\frac{5}{6}t + 5\right) dt$

$$\Rightarrow \qquad v = -\frac{5}{12}t^2 + 5t \qquad (1)$$

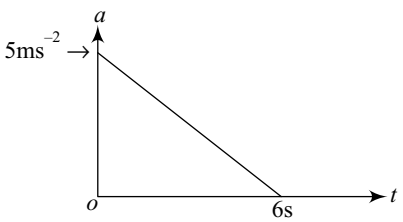


Fig. 2.19

It follows from graph that deceleration becomes zero at $t = 6$ s. Hence v will be maximum at $t = 6$ s. Using $t = 6$ s in Eq. (1) gives $v_{\max} = 15$ ms^{-1}.

(ii) From $v = \dfrac{dx}{dt}$, we have $dx = v\,dt$. Integrating

$$\int_0^x dx = \int_0^t v\,dt = \int_0^2 \left(-\frac{5}{12}t^2 + 5t\right) dt$$

$$\Rightarrow \qquad x = \left| -\frac{5}{12} \times \frac{t^3}{3} + 5 \times \frac{t^2}{2} \right|_0^2$$

$$= -\frac{5}{12} \times \frac{2^3}{3} + 5 \times \frac{2^2}{2} = \frac{80}{9} \text{ m}$$

(b) *Finding velocity and displacement if the dependence of acceleration on displacement is given*

Use $$a(x) = \frac{dv}{dt} = \frac{dx}{dt} \times \frac{dv}{dx} = v\,\frac{dv}{dx}$$

$$\left(\because v = \frac{dx}{dt}\right)$$

or $$v\,dv = a(x)\,dx$$

Substitute the given expression of $a(x)$ in terms of x and integrate.

$$\int_{v}^{v} v\,dv = \int_{x_0}^{x} a(x)\,dx$$

Hence we get an expression for $v(x)$ in terms of x.

To find displacement, we use $v(x) = \frac{dx}{dt} \Rightarrow \frac{dx}{v(x)} = dt$ and integrate

$$\int_{x_0}^{x} \frac{dx}{v(x)} = \int_{0}^{t} dt$$

where v is the expression obtained above.

EXAMPLE 31 A particle is moving along the x-axis with an acceleration $a = 2x$ where a is in ms^{-2} and x is in metre. If the particle starts from rest at $x = 1$ m, find its velocity when it reaches the position $x = 3$ m.

SOLUTION $a = v\,\frac{dv}{dx} \Rightarrow 2x = v\,\frac{dv}{dx}$

$$v\,dv = 2x\,dx$$

$$\therefore \quad \int_{0}^{v} v\,dv = 2\int_{1}^{3} x\,dx$$

$$\Rightarrow \quad \frac{v^2}{2} = 2\left|\frac{x^2}{2}\right|_1^3 = 3^2 - 1 = 8$$

which gives $v = 4$ ms^{-1}.

(c) *Finding velocity and displacement if the dependence of acceleration on velocity is given*

$$a(v) = \frac{dv}{dt} \quad \Rightarrow \quad \frac{dv}{a(v)} = dt$$

Integrating

$$\int_{u}^{v} \frac{dv}{a(v)} = \int_{0}^{t} dt$$

Hence we obtain the expression for $v(t)$.

$$v(t) = \frac{dx}{dt} \quad \Rightarrow \quad dx = v(t)\,dt$$

Integrating

$$\int_{x_0}^{x} dx = \int_{0}^{t} v(t)\,dt$$

EXAMPLE 32 A particle initially (i.e. at $t = 0$) moving with a velocity u is subjected to a retarding force which decelerates it at a rate $a = -k\sqrt{v}$ where v is the instantaneous velocity and k is a positive constant. (a) Find the time taken by the particle to come to rest. (b) Find the distance the particle travels during this time.

SOLUTION

(a) $a = \frac{dv}{dt} \quad \Rightarrow \quad -k\sqrt{v} = \frac{dv}{dt} \quad \Rightarrow \quad \frac{dv}{\sqrt{v}} = -k\,dt$

Integrating

$$\int_{u}^{v} \frac{dv}{\sqrt{v}} = -k\int_{0}^{t} dt$$

$$\Rightarrow \quad 2\left(v^{1/2} - u^{1/2}\right) = -kt$$

Putting $v = 0$, we get $t = \frac{2\sqrt{u}}{k}$

(b) To find the stopping distance, we use

$$a = \frac{dv}{dt} = v\frac{dv}{dx}$$

$$\Rightarrow -k\sqrt{v} = v\frac{dv}{dx} \quad \Rightarrow \quad dx = -\frac{\sqrt{v}\,dv}{k}$$

Integrating

$$\int_{0}^{x} dx = -\frac{1}{k}\int_{u}^{0} \sqrt{v}\,dv$$

$$\Rightarrow \quad x = \frac{2u^{3/2}}{3k}$$

EXAMPLE 33 The velocity v (in ms^{-1}) of a particle moving in a straight line varies with its displacement x (in metre) as

$$v = \sqrt{9 - x^2}$$

The maximum acceleration of the particle in ms^{-2} is

(a) 3 (b) 6

(c) 9 (d) 18

SOLUTION $a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v\,\frac{dv}{dx}$ $\left(\because v = \frac{dx}{dt}\right)$

$$= \frac{1}{2}\frac{d}{dx}\left(v^2\right)$$

Given $v = \sqrt{9-x^2} \Rightarrow v^2 = 9 - x^2$

$$a = \frac{1}{2}\frac{d}{dx}\left[9-x^2\right] = \frac{1}{2}\times(-2x) = -x$$

$\therefore$ Since v must be real, $x^2 < 9 \Rightarrow x < \pm 3$.

$\therefore$ $a_{max} = 3\ \text{ms}^{-2}$

So the correct choice is (a).

EXAMPLE 34 The acceleration a of a particle moving with an initial velocity u varies with distance x as $a = k\sqrt{x}$ where k is a constant. The distance covered by the particle when its velocity becomes $3u$ is given by

(a) $\left(\frac{3u^2}{k}\right)^{2/3}$ (b) $\left(3ku^2\right)^{4/3}$

(c) $\left(\frac{6u^2}{k}\right)^{2/3}$ (d) $\left(\frac{u^2}{3k}\right)^{2/3}$

SOLUTION $a = \frac{dv}{dt} = \frac{dv}{dx}\times\frac{dx}{dt} = v\frac{dv}{dx}$

$\Rightarrow$ $v\,dv = adx = k\sqrt{x}\,dx$

Integrating

$$\int_u^{3u} v dv = k\int\sqrt{x}\,dx$$

$$\Rightarrow \left|\frac{v^2}{2}\right|_u^{3u} = \frac{k\,x^{3/2}}{3/2}$$

$$\Rightarrow \frac{1}{2}(9u^2 - u^2) = \frac{2k}{3}x^{3/2}$$

$$\Rightarrow 4u^2 = \frac{2k}{3}x^{3/2}$$

$$\Rightarrow x = \left(\frac{6u^2}{k}\right)^{2/3}$$

EXAMPLE 35 The velocity v of a particle moving in a straight line varies with distance x as $v = k\sqrt{x}$ where k is a positive constant. The distance x varies with time t as

(a) $x \propto \sqrt{t}$ (b) $x \propto t$

(c) $x \propto t^{3/2}$ (d) $x \propto t^2$

SOLUTION $v = k\sqrt{x}$

Acceleration $a = \frac{dv}{dt} = \frac{dv}{dx}\times\frac{dx}{dt}$

$$= v\frac{dv}{dx}$$

$$= k\sqrt{x}\times\frac{d}{dx}\left(k\sqrt{x}\right)$$

$$= k\,x^{1/2}\times\frac{k}{2}x^{-1/2} = \frac{k^2}{2}$$

$\therefore$ $dv = \frac{k^2}{2}dt$

Integrating

$$\int dv = \frac{k^2}{2}\int dt$$

$\Rightarrow$ $v = \frac{k^2}{2}t$

$\Rightarrow$ $\frac{dx}{dt} = \frac{k^2}{2}t$

$\Rightarrow$ $dx = \frac{k^2}{2}t\,dt$

Integrating

$$\int dx = \frac{k^2}{2}\int t\,dt$$

$\Rightarrow$ $x = \frac{k^2t^2}{2}$

or $x \propto t^2$

17. Motion in Two Dimensions: Projectile Motion

(i) *A body projected horizontally with a velocity u from a height h.* [Fig. 2.20] Horizontal and vertical distances covered in time t are

$$x = ut \tag{2.5}$$

$$y = \frac{1}{2}gt^2 \tag{2.6}$$

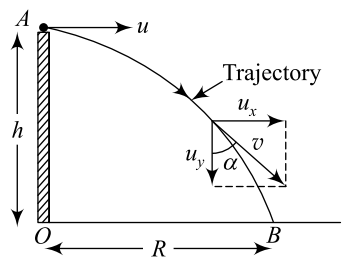


Fig. 2.20

Differentiating Eqs. (2.5) and (2.6) w.r.t time t, we get the horizontal and vertical velocities.

$$v_x = \frac{dx}{dt} = u \tag{2.6}$$

$$v_y = \frac{dy}{dt} = gt \tag{2.7}$$

Equation to trajectory

Eliminating t from Eqs. (2.5) and (2.6), we get

$$y = \left(\frac{g}{2u^2}\right)x^2$$

Since $y \propto x^2$, the trajectory of the body is parabolic.

Time of flight (t_f)

Putting $y = h$ and $t = t_f$ in Eq. (2.6), we get $t_f = \sqrt{\frac{2h}{g}}$

Horizontal range (R)

Putting $t = t_f$ and $x = R$ in Eq. (2.5), we get

$$R = u\,t_f = u\sqrt{\frac{2h}{g}}$$

Magnitude of resultant velocity at time t is

$$v = \left(u^2 + g^2\,t^2\right)^{1/2}$$

The angle α which the resultant velocity vector subtends with the vertical is given by

$$\tan\alpha = \frac{u}{gt}$$

(ii) *A body projected from the ground with a velocity u at an angle θ with the horizontal*. (Fig. 2.21)

The horizontal and vertical distances covered in time t are

$$x = (u\cos\theta)t \qquad (2.8)$$

and

$$y = (u\sin\theta)t - \frac{1}{2}gt^2 \qquad (2.9)$$

Horizontal and vertical components of the velocity at time t are

$$v_x = u\cos\theta \qquad (2.10)$$

and

$$u_y = u\sin\theta - gt \qquad (2.11)$$

Magnitude of resultant velocity at time t is

$$v = \left(v_x^2 + v_y^2\right)^{1/2}$$

The angle α subtended by the resultant velocity vector with the horizontal is given by

$$\tan\alpha = \frac{v_y}{v_x}$$

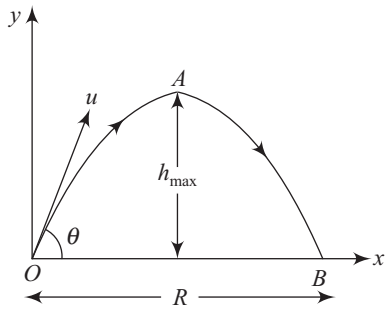


Fig. 2.21

Equation of trajectory

Eliminating t from Eqs. (2.8) and (2.9), we get

$$y = (\tan\theta)x - \frac{gx^2}{2u^2\cos^2\theta}$$

Time of flight (t_f)

Putting $y = 0$ and $t = t_f$ in Eq. (2.9), we get

$$t_f = \frac{2u\sin\theta}{g}$$

Maximum height attained (h_{max})

Put $t = t_f/2$ and $y = h_{max}$ in Eq. (2.9), we get

$$h_{max} = \frac{u^2\sin^2\theta}{2g}$$

Horizontal range (R)

Putting $t = t_f$ and $x = R$ in Eq. (2.8) we get

$$R = \frac{u^2\sin(2\theta)}{g}$$

(iii) *A body projected from a height h with a velocity u at an angle θ with the horizontal.* (Fig. 2.22)

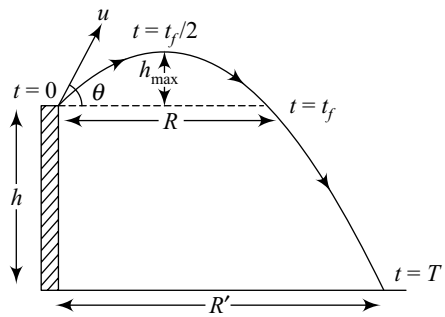


Fig. 2.22

If T is the total time of flight, then we have

$$h = u_y T - \frac{1}{2}gT^2$$

$$= (u\sin\theta)T - \frac{1}{2}gT^2$$

which gives $$T = \frac{u\sin\theta}{g} + \left(\frac{u^2\sin^2\theta}{g^2} + \frac{2h}{g}\right)^{1/2}$$

The horizontal range is $R' = (u\cos\theta)T$

Applications

(i) The horizontal range is the same for angles θ and $(90° - \theta)$.

(ii) The horizontal range is maximum for $\theta = 45°$. $R_{max} = u^2/g$

(iii) When horizontal range is maximum, $h_{max} = \frac{R_{max}}{4}$

(iv) At the point of projection, KE = $\frac{1}{2}mu^2$, PE = 0. Total energy $E = \frac{1}{2}mu^2$.

(v) At the highest point, KE = $\frac{1}{2}mu^2\cos^2\theta$

and PE = total energy – KE

$$= \frac{1}{2}mu^2 - \frac{1}{2}mu^2\cos^2\theta = \frac{1}{2}mu^2\sin^2\theta.$$

(vi) To find R and h_{max} from the equation of trajectory $y = ax - bx^2$

where a and b are constants, Refer to Fig. 2.23.

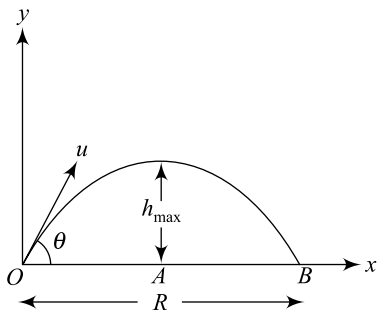


Fig. 2.23

(a) At O and B, $y = 0$. Putting $y = 0$ in the above equation, we have $0 = ax - bx^2 \Rightarrow x = 0$, $x = a/b$. Therefore $R = \frac{a}{b}$.

(b) At A, $y = h_{max}$ and $x = \frac{R}{2} = \frac{a}{2b}$. Using these values in $y = ax - bx^2$, we get $h_{max} = \frac{a^2}{4b}$.

(vii) If A and B are two points at the same horizontal level on a trajectory at a height h from the ground, (see Fig. 2.24), then

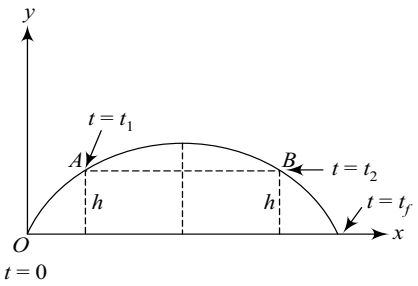


Fig. 2.24

(a) $t_f = \frac{2u\sin\theta}{g} = t_1 + t_2$

(b) $h = \frac{1}{2}gt_1t_2$

(c) Average velocity during time interval $(t_2 - t_1)$ is $v_{av} = u\cos\theta$
($\because$ during this interval, the vertical displacement is zero)

(viii) *Velocity and Direction of Motion of Projectile at any Height.* Let P be a point on the trajectory of a projectile at a height h and let v be the velocity of the projectile at that height. If α is the angle which the velocity vector makes with the horizontal, then the horizontal and vertical components of the velocity are given by $v_x = u_x$ = constant

or $v\cos\alpha = u\cos\theta$ (i)

and $v_y^2 = u_y^2 - 2gh$

or $(v\sin\alpha)^2 = (u\sin\theta)^2 = -2gh$ (ii)

Squaring Eq. (i) and then adding to Eq. (ii), we get

$v^2 = v_x^2 + v_y^2 = u_2 - 2gh$ or $v = (u^2 - 2gh)^{1/2}$

This gives the speed of the projectile at height h. The direction of the velocity vector (i.e., direction of motion) is obtained by taking the square root of Eq. (ii) and then dividing by Eq. (i). We get

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\left(u^2\sin^2\theta - 2gh\right)^{1/2}}{u\cos\theta}$$

(ix) *Time of Flight and Range of a Projectile on an Inclined Plane*

Consider an inclined plane OAB making an angle α with the horizontal (Fig. 2.25). Let a body be projected with a velocity u at an angle θ with the horizontal. Let us choose the x-axis along the plane OA and y-axis perpendicular to the plane OA. Let the body hit the inclined plane at point P so that $R = OP$ is the range on the inclined plane. The x and y components of the velocity of the projectile are

$v_x = u\cos(\theta - \alpha)$ and $v_y = u\sin(\theta - \alpha)$

The x and y components of acceleration due to gravity are $-g \sin \alpha$ and $-g \cos \alpha$ respectively, as shown in Fig. 2.25. Let T_f be the time of flight on the inclined plane. Since the net vertical displacement in time T_f is zero (i.e., $h = 0$), we have

$$0 = v_y T_f - \frac{1}{2} g \cos\alpha \, T_f^2$$

or $$0 = v_y - \frac{1}{2} g \cos\alpha \, T_f$$

or $$0 = u \sin(\theta - \alpha) - \frac{1}{2} g \cos\alpha \, T_f$$

or $$T_f = \frac{2u \sin(\theta - \alpha)}{g \cos\alpha} \qquad \text{(iii)}$$

During this time, the horizontal component of velocity $u \cos\theta$ remains constant. Hence, horizontal distance OQ is

$$OQ = (u \cos\theta) T_f$$

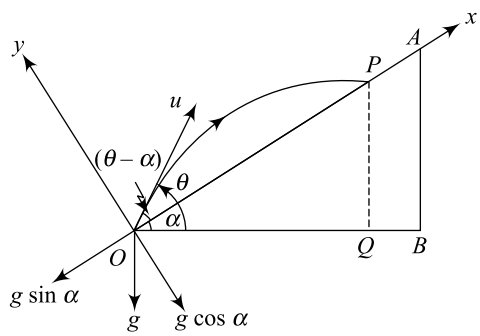


Fig. 2.25

∴ Range of the projectile on the inclined plane is

$$R = OP = \frac{OQ}{\cos\alpha} = \frac{(u \cos\theta) T_f}{\cos\alpha} \qquad \text{(iv)}$$

Using Eq. (iii) in Eq. (iv), we get

$$R = \frac{2u^2 \sin(\theta - \alpha)\cos\theta}{g \cos^2\alpha}$$

EXAMPLE 36 A ball is thrown horizontally with a velocity of 5 ms^{-1} from the top of a building of height 80 m. Take $g = 10\ ms^{-2}$ and find (a) the time taken by the ball to hit the ground, (b) the distance from the base of the building of the point where the ball hits the ground and (c) the magnitude and direction of the velocity of the ball at $t = 2$ s.

SOLUTION (a) $t_f = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 80}{10}} = 4$ s

(b) $R = ut_f = 5 \times 4 = 20$ m

(c) $v = \sqrt{u^2 + g^2 t^2} = \sqrt{(5)^2 + (10)^2 \times (2)^2}$

$= \sqrt{425} = 20.6\ ms^{-1}$

If α is the angle which the velocity vector v subtends with the vertical, then

$$\tan\alpha = \frac{u}{gt} = \frac{5}{10 \times 2} = \frac{1}{4}$$

$$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{1}{4}\right) = 14°$$

EXAMPLE 37 Two bodies are projected horizontally in opposite directions with velocities u_1 and u_2 from the top of a building. If their velocitices become perpendicular to each other after falling through a distance h, then

(a) $h = \sqrt{2}\,\frac{u_1 u_2}{g}$ (b) $h = \frac{2u_1 u_2}{g}$

(c) $h = \frac{u_1 u_2}{\sqrt{2} g}$ (d) $h = \frac{u_1 u_2}{2g}$

SOLUTION Let u_1 be along the positive x-axis and u_2 along the negative x-axis. Their velocities at any time t are

$$v_1 = u_1 \hat{\mathbf{i}} + (gt)\hat{\mathbf{j}}$$

$$v_2 = -u_2 \hat{\mathbf{i}} + (gt)\hat{\mathbf{j}}$$

Let t be the time at which v_1 becomes perpendicular to v_2, then

$$v_1 \cdot v_2 = 0$$

$$\Rightarrow \quad \left[u_1 \hat{\mathbf{i}} + (gt)\hat{\mathbf{j}}\right] \cdot \left[-u_2 \hat{\mathbf{i}} + (gt)\hat{\mathbf{j}}\right] = 0$$

$$\Rightarrow \quad -u_1 u_2 + (gt)^2 = 0$$

$$\Rightarrow \quad t = \frac{1}{g}(u_1 u_2)^{1/2}$$

$$\therefore \quad h = \frac{1}{2} g t^2 = \frac{1}{2} g \times \frac{u_1 u_2}{g^2} = \frac{u_1 u_2}{2g}$$

EXAMPLE 38 A building P is 150 m tall and is at a distance of 100 m from another building Q. A ball is thrown horizontally from the top of building P such that it enters a window in building Q. The height of the window above the ground is 25 m. The speed (in ms^{-1}) with which the ball is thrown must be (take $g = 10\ ms^{-2}$).

(a) 20 (b) 30

(c) 40 (d) 50

SOLUTION Refer to Fig. 2.26. It follows from the figure that the horizontal range is $R = 100$ m and the vertical displacement $h = (150 - 25) = 125$ m

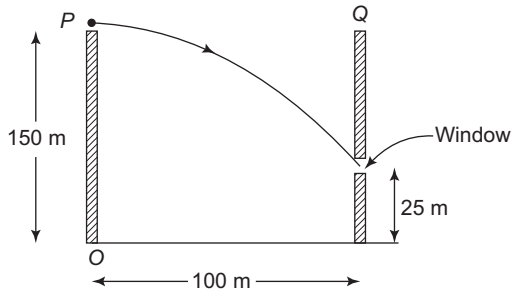


Fig. 2.26

Time taken to fall through this distance is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times125}{10}} = 5\text{ s}$$

Now $R = ut$

$$\Rightarrow \quad u = \frac{R}{t} = \frac{100}{5} = 20\text{ m s}^{-1}$$

EXAMPLE 39 A body is projected horizontally from the top of a building. It strikes the ground after a time t with its velocity vector making an angle θ with the horizontal. The speed with which the body is projected is

(a) $\frac{gt}{\sin\theta}$ (b) $\frac{gt}{\cos\theta}$

(c) $\frac{gt}{\tan\theta}$ (d) $\frac{gt}{\cot\theta}$

SOLUTION Let u be the speed with which the body is projected and let v_x and v_y be the horizontal and vertical components of the velocity at time t. Then

$$\tan\theta = \frac{v_y}{v_x} = \frac{v_y}{u} \qquad (\because v_x = u \text{ remains constant})$$

$$\Rightarrow \quad u = \frac{v_y}{\tan\theta} \qquad \text{(i)}$$

Now, since the initial y component (u_y) is zero, we have

$$-v_y = 0 - gt$$

$$\Rightarrow \quad v_y = gt \qquad \text{(ii)}$$

Using (ii) in (i), we get

$$u = \frac{gt}{\tan\theta}\text{, which is choice (c)}$$

EXAMPLE 40 A body is projected horizontally with a speed u from the top A of an plane ABC inclined at an angle $\theta = 30°$ with the horizontal. It strikes the plane at point P. The distance AP is given by

(a) $\frac{u^2}{2g}$ (b) $\frac{3}{2}\frac{u^2}{g}$

(c) $\frac{2}{3}\frac{u^2}{g}$ (d) $\frac{4}{3}\frac{u^2}{g}$

SOLUTION

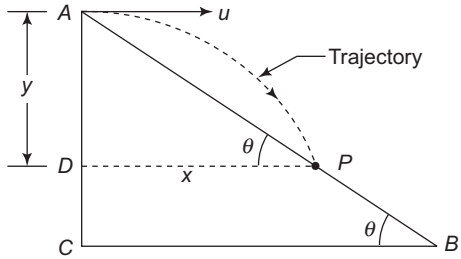


Fig. 2.27

If the time taken by the body to strike the plane at P is t (Fig. 2.27), then the horizontal and vertical distances travelled by it are

$$x = ut \qquad \text{(i)}$$

and $$y = \frac{1}{2}gt^2 \qquad \text{(ii)}$$

Eliminating t from (i) and and (ii),

$$y = \frac{gx^2}{2u^2}$$

Now $$\tan\theta = \frac{AD}{DP} = \frac{y}{x}$$

$$\Rightarrow \quad y = x\tan\theta\text{. Thus}$$

$$x\tan\theta = \frac{gx^2}{2u^2}$$

$$\Rightarrow \quad x = \frac{2u^2\tan\theta}{g}$$

Distance $$AP = \sqrt{x^2 + y^2}$$

$$= \sqrt{x^2 + x^2\tan^2\theta}$$

$$= x\sqrt{1+\tan^2\theta}$$

$$= x\sec\theta$$

$$= \frac{2u^2\tan\theta\sec\theta}{g}$$

or $$AP = \frac{2u^2\sin\theta}{g\cos^2\theta}$$

For $\theta = 30°$, $AP = \frac{2u^2\sin30°}{g\cos^2 30°} = \frac{4u^2}{3g}$

EXAMPLE 41 A body is projected with a speed u at an angle α with the horizontal. The speed of the body when its velocity vector makes an angle β with the vertical is

(a) $\frac{u\cos\alpha}{\cos\beta}$ (b) $\frac{u\cos\alpha}{\sin\beta}$

(c) $\frac{u\sin\alpha}{\cos\beta}$ (d) $\frac{u\sin\alpha}{\sin\beta}$

SOLUTION Let the speed of the body be v when its velocity vector makes an angle β with the vertical (Fig 2.28)

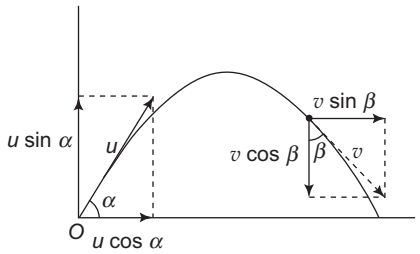


Fig. 2.28

Horizontal component of u is $u \cos \alpha$ and the horizontal component of v is $v \sin \beta$. Since the horizontal component of velocity remains constant,

$\therefore \quad u \cos \alpha = v \sin \beta$

$\Rightarrow \quad v = \dfrac{u \cos \alpha}{\sin \beta}$, which is choice (b)

EXAMPLE 42 A body is projected from the ground with a velocity $\mathbf{u} = \left(6\hat{\mathbf{i}} + 8\hat{\mathbf{j}}\right)$ ms^{-1}. Find (a) the maximum height attained by it, (b) the time of flight and (c) the horizontal range. Take $g = 10$ ms^{-2}.

SOLUTION Given $\mathbf{u} = \left(6\hat{\mathbf{i}} + 8\hat{\mathbf{j}}\right)$ ms^{-1}

$\therefore$ Speed of projection is $u = \sqrt{(6)^2 + (8)^2} = 10$ ms^{-1}

Also $u_x = 6$ ms^{-1} and $u_y = 8$ ms^{-1}. If θ is the angle of projection with the horizontal (x-axis), then

$$u_x = u \cos \theta = 6 \text{ ms}^{-1}$$

and $$u_y = u \sin \theta = 8 \text{ ms}^{-1}$$

These equations give $\tan \theta = \dfrac{8}{6} = \dfrac{4}{3}$ which gives $\sin \theta = \dfrac{4}{5}$.

(a) $h_{\max} = \dfrac{u^2 \sin^2 \theta}{2g} = \dfrac{(10)^2 \times \left(\frac{4}{5}\right)^2}{2 \times 10} = 3.2$ m

(b) $t_f = \dfrac{2u \sin \theta}{g} = \dfrac{2 \times 10 \times \frac{4}{5}}{10} = 1.6$ s

(c) $R = \dfrac{u^2 \sin (2\theta)}{g} = \dfrac{2u^2 \sin \theta \cos \theta}{g}$

Since $\sin \theta = \dfrac{4}{5}$, $\cos \theta = \dfrac{3}{5}$. Therefore,

$$R = \frac{2 \times (10)^2 \times \left(\frac{4}{5}\right) \times \left(\frac{3}{5}\right)}{10} = 9.6 \text{ m}$$

EXAMPLE 43 A body is projected with kinetic energy K at an angle of 60° with the vertical. The kinetic energy of the body when it is at the highest point on the trajectory is

(a) zero (b) $\dfrac{3K}{4}$

(c) $\dfrac{K}{2}$ (d) $\dfrac{K}{4}$

SOLUTION Angle with the horizontal is $\theta = 90° - 60° = 30°$. At the highest point, the horizontal component of velocity is $u \cos \theta$ and the vertical component is zero.

Given $\dfrac{1}{2} mu^2 = K$. At the highest point,

$$K' = \frac{1}{2} m (u \cos \theta)^2 = \frac{1}{2} mu^2 \cos^2 \theta$$

$$= K \cos^2 30° = \frac{3K}{4}$$

EXAMPLE 44 A ball is projected with a certain speed from a point on the ground which is at a distance of 30 m from a vertical wall. If the angle of projection is 45° with the horizontal, the ball just clears the top of the wall and strikes the ground at a distance of 10 m from the wall on the other side. The height of wall must be (Take $g = 10$ ms^{-2})

(a) 7.5 m (b) 10.0 m

(c) 12.5 m (d) 15.0 m

SOLUTION Refer to Fig. 2.29.

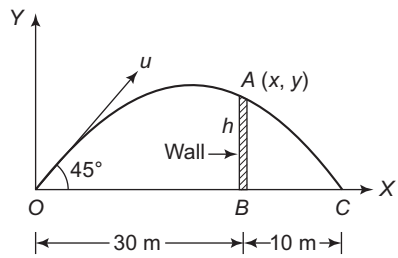


Fig. 2.29

Let h be the height of the wall. Given,
Horizontal range $R = 30 + 10 = 40$ m and $\theta = 45°$

Now $$R = \frac{u^2 \sin (2\theta)}{g}$$

$\Rightarrow \quad 40 = \dfrac{u^2 \sin (90°)}{10} \Rightarrow u = \sqrt{400}$

$= 20$ ms^{-1}

Let the coordinates of the top A of the wall be (x, y). Then

$$x = (u \cos \theta) t \quad \text{(i)}$$

and $$y = (u \sin \theta) t - \frac{1}{2} gt^2 \quad \text{(ii)}$$

where $x = 30$ m and $y = h$. From Eq. (i)

$$t = \frac{x}{u\cos\theta} = \frac{30}{20\cos 45°} = \frac{3}{\sqrt{2}}\text{ s}$$

From Eq (ii),

$$h = (20 \sin 45°) \times \frac{3}{\sqrt{2}} - \frac{1}{2} \times 10 \times \left(\frac{3}{\sqrt{2}}\right)^2$$

$$= 30 - 22.5 = 7.5 \text{ m}$$

EXAMPLE 45 From the top of a building 25 m tall, a ball is thrown with a velocity of 40 ms^{-1} at an angle of 30° with horizontal. Find (a) the time taken by the ball to hit the ground and (b) the distance from the foot of the building of the point where the ball hits the grounds. Take $g = 10\ ms^{-2}$.

SOLUTION Refer to Fig. 2.30.

$$u_x = u\cos\theta = 40 \times \cos 30° = 40 \times \frac{\sqrt{3}}{2} = 34.6\ ms^{-1}$$

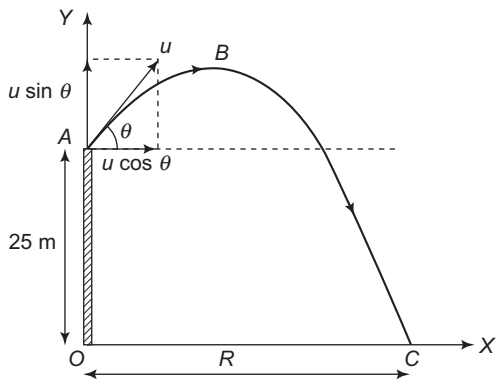


Fig .2.30

$$u_y = u\sin\theta = 40 \sin 30° = 40 \times \frac{1}{2} = 20\ ms^{-1}$$

(a) Let t be the time taken by the ball to go from A to C via B.

Vertical displacement $S = -25$ m (downwards)

Initial vertical velocity $u_y = +20\ ms^{-1}$ (upwards)

acceleration $a = -g = -10\ ms^{-2}$

Using $S = u_y t + \frac{1}{2} at^2$, we have

$$-25 = 20t + \frac{1}{2}(-10)t^2$$

$$\Rightarrow \quad -25 = 20t - 5t^2$$

$$\Rightarrow \quad t^2 - 4t - 5 = 0$$

The positive root of this equation is $t = 5$ s.

(b) Horizontal range $R = u_x t$

$$= 34.6 \times 5 = 173 \text{ m}$$

18. Relative Velocity in Two Dimensions

The relative velocity of a body B with respect to body A is defined as

$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A$$

If vectors $\mathbf{v}_A$ and $\mathbf{v}_B$ are inclined to each other at an angle θ as shown in Fig. 2.31, the relative velocity $\mathbf{v}_{BA}$ is found as follows.

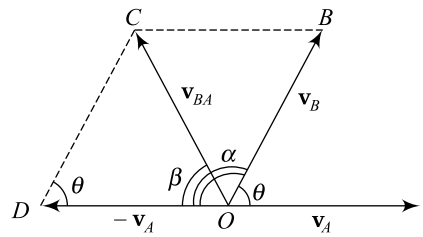


Fig. 2.31

$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A = \mathbf{v}_B + (-\mathbf{v}_A)$$

Thus, the magnitude and direction of vector $\mathbf{v}_{BA}$ can be found by finding the resultant of vectors $\mathbf{v}_B$ and $-\mathbf{v}_A$ which is vector **OC** as shown in Fig. 2.31.

Magnitude of vector $\mathbf{v}_{BA}$ is given by

$$v_{BA} = \left(v_A^2 + v_B^2 + 2v_A v_B \cos\alpha\right)^{1/2}$$

$$= \left(v_A^2 + v_B^2 - 2v_A v_B \cos\theta\right)^{1/2}$$

$$(\because \alpha = 180° - \theta)$$

The angle β which the resultant vector **OC** subtends with vector **OD** is given by

$$\frac{OC}{\sin\theta} = \frac{CD}{\sin\beta}$$

$$\Rightarrow \quad \sin\beta = \frac{CD\sin\theta}{OC} = \frac{v_B \sin\theta}{v_{BA}}$$

Special Case

(i) If vector v_A and v_B are in the same direction, $\theta = 0°$, then $v_{BA} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B} = v_B - v_A$.

(ii) If vector v_A and v_B are in opposite direction, $\theta = 180°$, then $v_{BA} = v_B + v_A$.

Applications

(i) *To cross the river of width d along the shortest path which is PQ*, the boat must move along *PR* making an angle $(90° + \theta)$ with the direction of the stream such that the direction of the resultant velocity **v** is along *PQ*. Angle θ is given by (see Fig. 2.32)

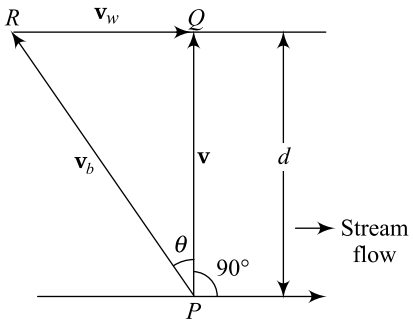


Fig. 2.32

$$\sin\theta = \frac{v_\omega}{v_b}$$

Also $$v = \sqrt{v_b^2 - v_\omega^2}$$

The time taken to cross the river along the shortest path is given by

$$t = \frac{d}{v} = \frac{d}{\sqrt{v_b^2 - v_\omega^2}}$$

(b) *To cross the river in the shortest time*, the boat should move along PQ. The shortest time is given by

$$t = \frac{d}{v_b}$$

At this time, the boat will reach the point R on the opposite bank of the river at a distance x from the point Q (Fig. 2.33). From the Figure, we have

$$x = d \tan\theta,$$

but $$\tan\theta = \frac{\mathbf{v}_w}{\mathbf{v}_b}$$ Therefore,

$$x = d\left(\frac{\mathbf{v}_w}{\mathbf{v}_b}\right) \qquad (2.12)$$

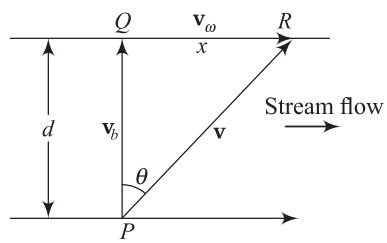


Fig. 2.33

(ii) *Holding an Umbrella to Project from Rain* Let $\mathbf{v}_r$ be the velocity of the rain falling vertically downward and $\mathbf{v}_m$ the velocity of a man walking from north to south direction (Fig. 2.34). In order to protect himself from rain, he must hold his umbrella in the direction of the resultant velocity $\mathbf{v}$, which is given by

$$v = \sqrt{v_r^2 + v_m^2}$$

This is the speed with which the rain strikes the umbrella. If θ is the angle subtended by the resultant velocity $\mathbf{v}$ with the vertical, then from triangle ORM', we have

$$\tan\theta = \frac{RM'}{OR} = \frac{v_m}{v_r}$$

or $$\theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$$

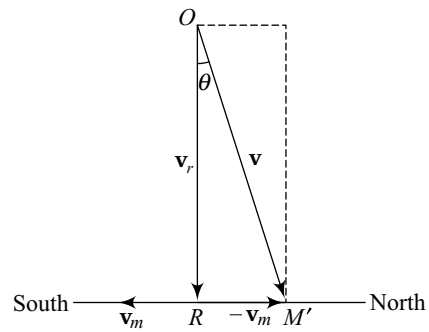


Fig. 2.34

Thus, the man must hold the umbrella at an angle θ with the vertical towards north.

19. Uniform Circular Motion

(i) *For a body moving in a horizontal circle*

The centripetal acceleration of a body of mass m moving in a circle of radius R with a constant speed v (or angular speed ω) is

$$a_c = \omega v = \omega^2 R = \frac{v^2}{R}$$

Centripetal force is

$$f_c = m\, a_c = m\, \omega^2\, R = \frac{mv^2}{R}$$

(ii) *For a body moving in a vertical circle*

The minimum speed to complete the circle when the body is at the top of the circle is $v = \sqrt{Rg}$. The minimum speed to complete the circle when the body is at the bottom of the circle is $v = \sqrt{5Rg}$.

SECTION 1

Multiple Choice Questions with One Correct Choice

Level A

1. The magnitude of the resultant of two vectors of magnitudes 3 units and 4 units in 5 units. What is the angle between two vectors?

   (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

   (c) $\frac{3\pi}{4}$ (d) π

2. The magnitude of the resultant of two equal vectors is equal to the magnitude of either vector. What is the angle between the two vectors?

   (a) 60° (b) 90°

   (c) 120° (d) 150°

3. In Q.2, the angle between the resultant and either vector will be

   (a) 60° (b) 90°

   (c) 120° (d) 150°

4. Given $\mathbf{P} = \mathbf{A} + \mathbf{B}$ and $\mathbf{Q} = \mathbf{A} - \mathbf{B}$. If the magnitudes of vectors **P** and **Q** are equal, what is the angle between vectors **A** and **B**?

   (a) zero (b) $\frac{\pi}{4}$

   (c) $\frac{\pi}{2}$ (d) π

5. If $\mathbf{A} \cdot \mathbf{B} = \frac{1}{2} AB$, what is the angle between **A** and **B**?

   (a) zero (b) 30°

   (c) 60° (d) 90°

6. If $|\mathbf{A} \times \mathbf{B}| = \frac{1}{2} AB$, what is the angle between **A** and **B**?

   (a) zero (b) 30°

   (c) 60° (d) 90°

7. Given $\mathbf{C} = \mathbf{A} \times \mathbf{B}$ and $\mathbf{D} = \mathbf{B} \times \mathbf{A}$. What is the angle between **C** and **D**?

   (a) zero (b) 60°

   (c) 90° (d) 180°

8. If $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors along *x*-axis and *y*-axis respectively, the magnitude of vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ will be

   (a) 1 (b) $\sqrt{2}$

   (c) $\sqrt{3}$ (d) 2

9. In Q.8, the angle subtended by vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ with the *x*-axis is

   (a) 30° (b) 45°

   (c) 0° (d) 75°

10. Given $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{B} = \hat{\mathbf{i}} + \hat{\mathbf{k}}$. What is the value of the scalar product of **A** and **B**?

    (a) 1 (b) $\sqrt{2}$

    (c) $\sqrt{3}$ (d) 2

11. The cross product of vectors **A** and **B** in Q. 10 is

    (a) $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ (b) $\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$

    (c) $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ (d) $\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$

12. The displacement *y* (in metres) of a body varies with time *t* (in seconds) as

    $$y = -\frac{2}{3} t^2 + 16t + 2$$

    How long does the body take to come to rest?

    (a) 8 seconds (b) 10 seconds

    (c) 12 seconds (d) 16 seconds

13. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is *t*, the maximum velocity acquired by the car is given by

    (a) $\frac{\alpha\beta}{\alpha+\beta} t$ (b) $\frac{\alpha+\beta}{\alpha\beta} t$

    (c) $\frac{\alpha^2+\beta^2}{\beta\alpha} t$ (d) $\frac{\alpha^2-\beta^2}{\beta\alpha} t$

14. The distance travelled by the car in Q.35 above is given by

    (a) $\frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2$ (b) $\frac{1}{2}\left(\frac{\alpha+\beta}{\alpha\beta}\right)t^2$

    (c) $\frac{1}{2}\left(\frac{\alpha^2+\beta^2}{\alpha}\right)t^2$ (d) $\frac{1}{2}\left(\frac{\alpha^2-\beta^2}{\beta}\right)t^2$

15. The acceleration of a particle, starting from rest, varies with time according to the relation $a = kt + c$, where *k* and *c* are constants of motion.

    The velocity *v* is the particle after a time *t* will be

    (a) $k\,t + ct$ (b) $(1/2)\,k\,t^2 + c\,t$

    (c) $1/2\,(kt^2 + ct)$ (d) $kt^2 + (1/2)ct$

16. The acceleration of a particle, starting from rest, varies with time according to the relation $a = -s\,\omega^2 \cos \omega t$, where s and ω are constants. The displacement of this particle at a time t will be

    (a) $-\frac{1}{2}\ (s\,\omega^2 \sin \omega t)t^2$ (b) $s\,\omega \sin \omega t$

    (c) $s\,\omega \cos \omega t$ (d) $s \cos \omega t$

17. A balloon is rising vertically upwards at a velocity of 10 ms^{-1}. When it is at a height of 45 m from the ground, a parachutist bails out from it. After 3s he opens his parachute and decelerates at a constant rate of 5 ms^{-2}. What was the height of the parachutist above the ground when he opened his parachute? Take $g = 10\ ms^{-2}$.

    (a) 15 m (b) 30 m

    (c) 45 m (d) 60 m

18. In Q.17, how far is the parachutist from the balloon at $t = 3s$?

    (a) 15 m (b) 30 m

    (c) 45 m (d) 60 m

19. In Q.17, with what velocity does the parachutist hit the ground?

    (a) 10 ms^{-1} (b) 20 ms^{-1}

    (c) 30 ms^{-1} (d) 40 ms^{-1}

20. In Q.17, after how long does the parachutist hit the ground after his exit from the balloon?

    (a) 4 s (b) 5 s

    (c) 6 s (d) 7 s

21. A body moving in a straight line with constant acceleration of 10 ms^{-2} covers a distance of 40 m in the 4th second. How much distance will it cover in the 6th second?

    (a) 50 m (b) 60 m

    (c) 70 m (d) 80 m

22. A body, moving in a straight line with an initial velocity of 5 ms^{-1} and a constant acceleration, covers a distance of 30 m in the 3rd second. How much distance will it cover in the next 2 seconds?

    (a) 70 m (b) 80 m

    (c) 90 m (d) 100 m

23. A body, moving in a straight line, with an initial velocity u and a constant acceleration a, covers a distance of 40 m in the 4th second and a distance of 60 m in the 6th second. The values of u and a respectively are

    (a) 10 ms^{-1}, 5 ms^{-2} (b) 10 ms^{-1}, 10 ms^{-2}

    (c) 5 ms^{-1}, 5 ms^{-2} (d) 5 ms^{-1}, 10 ms^{-2}

24. A body, starting from rest and moving with a constant acceleration, covers a distance s_1 in the 4th second and a distance s_2 in the 6th second. The ratio s_1/s_2 is

    (a) $\frac{2}{3}$ (b) $\frac{4}{9}$

    (c) $\frac{6}{11}$ (d) $\frac{7}{11}$

25. A body, moving in a straight line, covers half the distance with a speed V, the remaining part of the distance was covered with a speed V' for half the time and with a speed V'' for the other half of the time. What is the average speed of the body?

    (a) $\frac{2V(V'+V'')}{(2V+V'+V'')}$ (b) $\frac{V(V'+V'')}{(2V+V'+V'')}$

    (c) $\frac{2V'V''}{(V+V'+V'')}$ (d) $\frac{V'V''}{(V+V'+V'')}$

26. A car is moving on a straight horizontal road with a speed v. When brakes are applied to give a constant retardation a, the car is stopped in a shortest distance S. If the car were moving on the same road with a speed 3 v and the same retardation a is applied, the shortest distance in which the car is stopped will be

    (a) 3 S (b) 6 S

    (c) 9 S (d) 27 S

27. The maximum height attained by a projectile and its horizontal range are equal to each other if the projectile is projected at an angle θ given by

    (a) $\theta = \tan^{-1}(2)$ (b) $\theta = \tan^{-1}(3)$

    (c) $\theta = \tan^{-1}(4)$ (d) $\theta = \tan^{-1}(5)$

28. A person aims a gun at a target located at a horizontal distance of 100 m. If the gun imparts a horizontal speed of 500 ms^{-1} to the bullet, at what height above the target must he aim his gun in order to hit it? Take $g = 10\ ms^{-2}$.

    (a) 10 cm (b) 20 cm

    (c) 50 cm (d) 100 cm

29. When a projectile is at the highest point of its trajectory, the directions of its velocity and acceleration are

    (a) parallel to each other

    (b) anti-parallel to each other

    (c) inclined to each other at an angle of 45°

    (d) perpendicular to each other

30. A ball is projected horizontally with a velocity of 5 ms^{-1} from the top of a building 19.6 m high. How long will the ball take to hit the ground?

    (a) $\sqrt{2}$ s (b) 2 s

    (c) $\sqrt{3}$ s (d) 3 s

31. A bomb is dropped from an aeroplane when it is at a height h directly above a target. If the aeroplane is moving horizontally at a speed v, the distance by which the bomb will miss the target is given by

(a) $2v\sqrt{\frac{h}{g}}$ (b) $v\sqrt{\frac{h}{g}}$

(c) $v\sqrt{\frac{2h}{g}}$ (d) $v\sqrt{\frac{h}{2g}}$

32. An enemy plane is flying horizontally at an altitude of 2 km with a speed of 300 ms^{-1}. An armyman with an anti–aircraft gun on the ground sights the enemy plane when it is directly overhead and fires a shell with a muzzle speed of 600 ms^{-1}. At what angle with the vertical should the gun be fired so as to hit the plane?

(a) 30° (b) 45°

(c) 60° (d) 75°

33. In Q.32, at what minimum altitude should the enemy plane fly to avoid being hit? Take $g = 10\ ms^{-2}$.

(a) 12.5 km (b) 13.5 km

(c) 14.5 km (d) 15.5 km

34. The ceiling of a tunnel is 5 m high. What is the maximum horizontal distance that a ball thrown with a speed of 20 ms^{-1}, can go without hitting the ceiling of the tunnel? Take $g = 10\ ms^{-2}$.

(a) $10\sqrt{3}$ m (b) $20\sqrt{3}$ m

(c) $30\sqrt{3}$ m (d) $40\sqrt{3}$ m

35. From the top of a tower of height 40 m, a ball is projected upwards with a speed of 20 ms^{-1} at an angle of 30° to the horizontal. If $g = 10\ ms^{-2}$, after how long will the ball hit the ground?

(a) 1 s (b) 2 s

(c) 3 s (d) 4 s

36. A projectile has a maximum range of 200 m. What is the maximum height attained by it?

(a) 25 m (b) 50 m

(c) 75 m (d) 100 m

37. A block is placed on the top of a smooth inclined plane of inclination θ kept on the floor of a lift. When the lift is descending with a retardation a, the block is released. The acceleration of the block relative to the incline is

(a) $g \sin\theta$ (b) $a \sin\theta$

(c) $(g - a)\sin\theta$ (d) $(g + a)\sin\theta$

38. A ball is projected vertically upwards with a certain initial speed. Another ball of the same mass is projected at angle of 60° with the vertical with the same initial speed. At the highest point, the ratio of their potential energies will be

(a) 4 : 1 (b) 3 : 2

(c) 2 : 3 (d) 2 : 1

39. A river is flowing from west to east at a speed of 5 m/minute. In what direction should a man on the south bank of the river capable of swimming at 10 m/minute in still water swim to cross the river in the shortest time?

(a) East (b) West

(c) South (d) North

40. In Q. 39 above, in what direction should the man swim so that he crosses the river by swimming the shortest distance?

(a) North (b) 60º west of north

(c) 30º west of north (d) 30º west of south

41. A particle moving in a straight line covers half the distance with a speed of 3 m/s. The other half of the distance is covered in two equal time intervals with speeds of 4.5 m/s and 7.5 m/s respectively. The average speed of the particle during this motion is

(a) 4.0 m/s (b) 5.0 m/s

(c) 5.5 m/s (d) 4.8 m/s

42. A car is travelling at a velocity of 10 km/h on a straight road. The driver of the car throws a parcel with a velocity of $10\sqrt{2}$ km/h when the car is passing by a man standing on the side of the road. If the parcel is to reach the man, the direction of throw makes the following angle with the direction of the car,

(a) 135º (b) 45º

(c) $\tan^{-1}\left(\sqrt{2}\right)$ (d) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Level B

43. If $\mathbf{A}\cdot\mathbf{B}$ = magnitude of $\mathbf{A}\times\mathbf{B}$, then the angle between vectors **A** and **B** is

(a) 30° (b) 45°

(c) 60° (d) 75°

44. Given $\mathbf{A}\cdot\mathbf{B} = 0$ and $\mathbf{A}\times\mathbf{C} = 0$. What is the angle between **B** and **C**?

(a) 45° (b) 90°

(c) 135° (d) 180°

45. The resultant of two vectors **A** and **B** subtends an angle of 45° with either of them. The magnitude of the resultant is

(a) zero (b) $\sqrt{2}\ A$

(c) A (d) $2A$

46. Vector **C** is the sum of two vectors **A** and **B** and vector **D** is the cross product of vectors **A** and **B**. What is the angle between vectors **C** and **D**?

(a) zero (b) 60°
(c) 90° (d) 180°

47. The resultant of two vectors of magnitudes 3 units and 4 units is 1 unit. What is the value of their dot product?
(a) – 12 units (b) – 7 units
(c) – 1 unit (d) zero

48. The resultant of two vectors of magnitudes 3 units and 4 units is 1 unit. What is the magnitude of their cross product?
(a) 12 units (b) 7 units
(c) 1 unit (d) zero

49. Three vectors **A**, **B** and **C** are related as **A** + **B** = **C**. If vector **C** is perpendicular to vector **A** and the magnitude of **C** is equal to the magnitude of **A**, what will be the angle between vectors **A** and **B**?
(a) 45° (b) 90°
(c) 135° (d) 180°

50. The magnitude of the resultant of (**A** + **B**) and (**A** – **B**) is
(a) 2*A* (b) 2*B*
(c) $\sqrt{A^2 + B^2}$ (d) $\sqrt{A^2 - B^2}$

51. In Q. 50, what is the angle between the resultant vector and vector **A**?
(a) zero (b) $\cos^{-1}\left(\frac{A}{B}\right)$
(c) $\cos^{-1}\left(\frac{B}{A}\right)$ (d) $\cos^{-1}\left(\frac{A-B}{A+B}\right)$

52. If $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are unit vectors along *x*, *y* and *z*-axes respectively, the angle θ between the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and vector $\hat{\mathbf{i}}$ is given by
(a) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
(c) $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (d) $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

53. Given that $0.2\,\hat{\mathbf{i}} + 0.6\,\hat{\mathbf{j}} + a\,\hat{\mathbf{k}}$ is a unit vector. What is the value of *a*?
(a) $\sqrt{0.3}$ (b) $\sqrt{0.4}$
(c) $\sqrt{0.6}$ (d) $\sqrt{0.8}$

54. Given $\mathbf{A} = 2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}$ and $\mathbf{B} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$. The component of vector **A** along vector **B** is
(a) $\frac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ (b) $\frac{3}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$
(c) $\frac{5}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ (d) $\frac{7}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

55. In Q. 54 above, what is the component of vector **A** perpendicular to vector **B** and in the same plane as **B**?
(a) $\frac{1}{\sqrt{2}}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$ (b) $\frac{3}{\sqrt{2}}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$
(c) $\frac{5}{\sqrt{2}}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$ (d) $\frac{7}{\sqrt{2}}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$

56. A stone falls from rest. The distance covered by the stone in the last second of its motion equals the distance covered by it during the first three seconds of its motion. How long does the stone take to reach the ground?
(a) 4 s (b) 5 s
(c) 6 s (d) 9 s

57. The acceleration *a* (in ms^{-2}) of a body, starting from rest varies with time *t* (in s) according to the relation
$$a = 3t + 4$$
The velocity of the body at time *t* = 2s will be
(a) 10 ms^{-1} (b) 12 ms^{-1}
(c) 14 ms^{-1} (d) 16 ms^{-1}

58. Two balls of masses m_1 and m_2 are thrown vertically upward with the same speed *u*. If air resistance is neglected, they will pass through their point of projection in the downward direction with a speed
(a) $\frac{m_1 u}{m_2}$ (b) $\frac{m_2 u}{m_1}$
(c) $\frac{(m_1 + m_2)u}{(m_1 - m_2)}$ (d) *u*

59. The velocity-time graph of a stone thrown vertically upward with an initial velocity of 30 ms^{-1} is shown in the Fig. 2.35. The velocity in the upward direction is taken as positive and that in the downward direction as negative. What is the maximum height to which the stone rises?

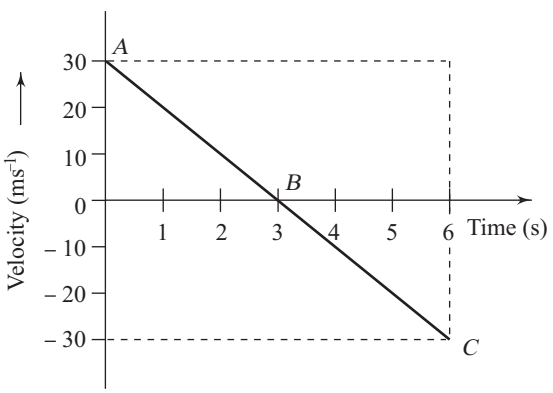


Fig. 2.35

(a) 30 m (b) 45 m
(c) 60 m (d) 90 m

60. At $t = 0$, an arrow is fired vertically upwards with a speed of 98 ms^{-1}. A second arrow is fired vertically upwards with the same speed at $t = 5$ s. Then
    (a) the two arrows will be at the same height above the ground at $t = 10$ s
    (b) the two arrows will reach back their starting points at $t = 20$ s and at $t = 30$ s
    (c) the ratio of the speeds of the first and the second arrows at $t = 20$ s will be 2 : 1
    (d) the maximum height attained by either arrow will be 980 m
61. The area under the velocity-time graph between any two instants $t = t_1$ and $t = t_2$ gives the distance covered in time $\delta t = t_2 - t_1$
    (a) only if the particle moves with a uniform velocity
    (b) only if the particle moves with a uniform acceleration
    (c) only if the particle moves with an acceleration increasing at a uniform rate
    (d) in all cases irrespective of whether the motion is one of uniform velocity, or of uniform acceleration or of variable acceleration.
62. The displacement x of a particle varies with time according to the relation $x = \frac{a}{b}\left(1 - e^{-bt}\right)$. Then
    (a) At $t = 1/b$, the displacement of the particle is nearly (1/3) (a/b)
    (b) The velocity and acceleration of the particle at $t = 0$ are a and ab respectively
    (c) The particle cannot reach a point at a distance x' from its starting position if $x' > a/b$
    (d) The particle will come back to its starting point as $t \to \infty$
63. The variation in the speed of a car during its two hour journey is shown in the graph of Fig. 2.36. The magnitude of the maximum acceleration of the car occurs during the interval

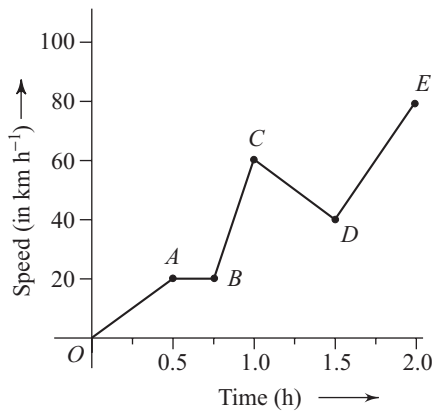


Fig. 2.36

    (a) OA (b) BC
    (c) CD (d) DE
64. Car A is moving with a speed of 36 km h^{-1} on a two-lane road. Two cars B and C, each moving with a speed of 54 km h^{-1} in opposite directions on the other lane are approaching car A. At a certain instant when the distance AB = distance AC = 1 km, the driver of car B decides to overtake A before C does. What must be the minimum acceleration of car B so as to avoid an accident?
    (a) 1 ms^{-2} (b) 2 ms^{-2}
    (c) 3 ms^{-2} (d) 4 ms^{-2}
65. The driver of a train A moving at a speed of 30 ms^{-1} sights another train B moving on the same track towards his train at a speed of 10 ms^{-1}. He immediately applies brakes and achieves a uniform retardation of 4 ms^{-2}. To avoid head-on collision, what must be the minimum distance between the trains?
    (a) 100 m (b) 200 m
    (c) 300 m (d) 400 m
66. A cyclist starts from the centre O of a circular track of radius r =1 km, reaches the edge P of the track and then cycles along the circumference and stops at point Q as shown in Fig. 2.37. If the total time taken is 10 min, what is the average velocity of the cyclist?

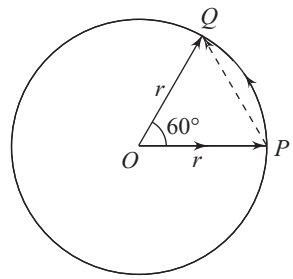


Fig. 2.37

    (a) 3 km h^{-1} (b) 6 km h^{-1}
    (c) 9 km h^{-1} (d) 12 km h^{-1}
67. In Q.66 above, the average speed of the cyclist is approximately equal to
    (a) 12.3 ms^{-1} (b) 13.4 ms^{-1}
    (c) 14.4 ms^{-1} (d) 15.4 ms^{-1}
68. Rain is falling vertically with a speed of 4 ms^{-1}. After some time, wind starts blowing with a speed of 3 ms^{-1} in the north to south direction. In order to protect himself from rain, a man standing on the ground should hold his umbrella at an angle θ given by
    (a) $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ with the vertical towards south
    (b) $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ with the vertical towards north

(c) $\theta = \cot^{-1}\left(\frac{3}{4}\right)$ with the vertical towards south

(d) $\theta = \cot^{-1}\left(\frac{3}{4}\right)$ with the vertical towards north

69. In Q.68 above, with what speed does the rain strike the umbrella?
(a) 3 ms^{-1} (b) 4 ms^{-1}
(c) 5 ms^{-1} (d) 6 ms^{-1}

70. A swimmer can swim in still water with a speed of 5 ms^{-1}. While crossing a river his average speed is 3 ms^{-1}. If he crosses the river in the shortest possible time, what is the speed of flow of water?
(a) 2 ms^{-1} (b) 4 ms^{-1}
(c) 6 ms^{-1} (d) 8 ms^{-1}

71. A body, starting from rest, moves in a straight line with a constant acceleration a for a time interval t during which it travels a distance s_1. It continues to move with the same acceleration for the next time interval t during which it travels a distance s_2. The relation between s_1 and s_2 is
(a) $s_2 = s_1$ (b) $s_2 = 2s_1$
(c) $s_2 = 3s_1$ (d) $s_2 = 4s_1$

72. In Q. 71, if v_1 is the velocity of the body at the end of first time interval and v_2 that at the end of the second time interval, the relation between v_1 and v_2 is
(a) $v_2 = v_1$ (b) $v_2 = 2v_1$
(c) $v_2 = 3v_1$ (d) $v_2 = 4v_1$

73. A body dropped from the top of a tower hits the ground after 4s. How much time does it take to cover the first half of the distance from the top of the tower?
(a) 1 s (b) 2 s
(c) 2 $\sqrt{2}$ s (d) $\sqrt{3}$ s

74. A car, starting from rest, has a constant acceleration a_1 for a time interval t_1 during which it covers a distance s_1. In the next time interval t_2, the car has a constant retardation a_2 and comes to rest after covering a distance s_2 in time t_2. Which of the following relations is correct?
(a) $\frac{a_1}{a_2} = \frac{s_1}{s_2} = \frac{t_1}{t_2}$ (b) $\frac{a_1}{a_2} = \frac{s_2}{s_1} = \frac{t_1}{t_2}$
(c) $\frac{a_1}{a_2} = \frac{s_1}{s_2} = \frac{t_2}{t_1}$ (d) $\frac{a_1}{a_2} = \frac{s_2}{s_1} = \frac{t_2}{t_1}$

75. In Q. 74, if the total distance covered by the car is s, the maximum speed attained by it will be
(a) $\left(2s.\frac{a_1a_2}{a_1+a_2}\right)^{\frac{1}{2}}$ (b) $\left(2s.\frac{a_1a_2}{a_1-a_2}\right)^{\frac{1}{2}}$
(c) $\left(\frac{s}{2}.\frac{a_1a_2}{a_1+a_2}\right)^{\frac{1}{2}}$ (d) $\left(\frac{s}{2}.\frac{a_1a_2}{a_1-a_2}\right)^{\frac{1}{2}}$

76. A car, starting from rest, is accelerated at a constant rate α until it attains a speed v. It is then retarded at a constant rate β until it comes to rest. The average speed of the car during its entire journey is
(a) zero (b) $\frac{\alpha v}{2\beta}$
(c) $\frac{\beta v}{2\alpha}$ (d) $\frac{v}{2}$

77. A simple pendulum is hanging from the ceiling of a compartment of a train. It is observed that the string is inclined towards the rear of the train. If follows that the train is
(a) at rest (b) accelerated
(c) decelerated (d) in uniform motion.

78. The displacement of a body from a reference point, is given by
$$\sqrt{x} = 2t + 3$$
where x is in metres and t in seconds. This shows that the body is
(a) at rest (b) accelerated
(c) decelerated (d) in uniform motion

79. In Q.78, what is the initial velocity of the body?
(a) 2 ms^{-1} (b) 3 ms^{-1}
(c) 6 ms^{-1} (d) 12 ms^{-1}

80. In Q.78, what is the acceleration of the body?
(a) 2 ms^{-2} (b) 3 ms^{-2}
(c) 6 ms^{-2} (d) 8 ms^{-2}

81. A car moving at a speed v is stopped in a certain distance when the brakes produce a deceleration a. If the speed of the car was nv, what must be the deceleration of the car to stop it in the same distance and in the same time?
(a) $\sqrt{n}\,a$ (b) na
(c) n^2a (d) n^3a

82. A car is moving at a certain speed. The minimum distance over which it can be stopped is x. If the speed of the car is doubled, what will be the minimum distance over which the car can be stopped during the same time?
(a) $4x$ (b) $2x$
(c) $x/2$ (d) $x/4$

83. A bullet is fired vertically upwards with an initial velocity of 50 ms^{-1}. It covers a distance h_1 during the first second and a distance h_2 during the last 3

seconds of its upward motion. If $g = 10\ \text{ms}^{-2}$, h_1 and h_2 will be related as

(a) $h_1 = 3h_2$ (b) $h_1 = 2h_2$

(c) $h_1 = h_2$ (d) $h_1 = \frac{h_2}{3}$

84. A ball is thrown vertically downward with a velocity u from the top of a tower. It strikes the ground with a velocity $3u$. The time taken by the ball to reach the ground is given by

(a) $\frac{u}{g}$ (b) $\frac{2u}{g}$

(c) $\frac{3u}{g}$ (d) $\frac{4u}{g}$

85. In Q. 84, the height of the tower is given by

(a) $\frac{u^2}{g}$ (b) $\frac{2u^2}{g}$

(c) $\frac{3u^2}{g}$ (d) $\frac{4u^2}{g}$

86. The distance x covered by a body moving in a straight line in time t is given by the relation

$$2x^2 + 3x = t$$

If v is the velocity of the body at a certain instant of time, its acceleration will be

(a) $-v^3$ (b) $-2v^3$

(c) $-3v^3$ (d) $-4v^3$

87. The distance x covered by a body moving in a straight line in time t is given by

$$x^2 = t^2 + 2t + 3$$

The acceleration of the body will vary as

(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$

(c) $\frac{1}{x^3}$ (d) $\frac{1}{x^4}$

88. The motion of a body is given by the equation

$$\frac{dV(t)}{dt} = 6.0 - 3V(t)$$

where $V(t)$ is the speed (in ms^{-1}) at time t (in second). If the body was at rest at $t = 0$, which of the following statements is wrong?

(a) the terminal speed is $2.0\ \text{ms}^{-1}$

(b) the magnitude of the initial acceleration is $6.0\ \text{ms}^{-2}$

(c) the speed varies with time as $V(t) = 2(1 - e^{-3t})\ \text{ms}^{-1}$

(d) the speed is $2.0\ \text{ms}^{-1}$ when the acceleration is half the initial value.

89. From the top of a tower of height 40 m, a ball is projected upwards with a speed of 20 m/s at an angle of elevation of 30°. The ratio of the total time taken by the ball to hit the ground to its time of flight (time taken to come back to the same elevation) is (Take $g = 10\ \text{m/s}^2$)

(a) 2 : 1 (b) 3 : 1

(c) 3 : 2 (d) 1.5 : 1

90. In Q.89, the horizontal displacement (from the foot of the tower) of the ball is approximately equal to

(a) 50 m (b) 60 m

(c) 70 m (d) 80 m

91. A cannon on a level plain is aimed at an angle α above the horizontal and a shell is fired with a muzzle velocity v_0 towards a vertical cliff a distance R away. Then the height from the bottom at which the shell strikes the side walls of the cliff is

(a) $R \sin \alpha - \frac{1}{2}\frac{gR^2}{v_0^2 \sin^2 \alpha}$

(b) $R \cos \alpha - \frac{1}{2}\frac{gR^2}{v_0^2 \cos^2 \alpha}$

(c) $R \tan \alpha - \frac{1}{2}\frac{gR^2}{v_0^2 \cos^2 \alpha}$

(d) $R \tan \alpha - \frac{1}{2}\frac{gR^2}{v_0^2 \sin^2 \alpha}$

92. It is possible to project a particle with a given velocity in two possible ways so as to make it pass through a point P at a distant r from the point of projection. The product of the times taken to reach this point in the two possible ways is then proportional to

(a) $1/r$ (b) r

(c) r^3 (d) $\frac{1}{r^2}$

93. A gun kept on a straight horizontal road is used to hit a car travelling on the same road away from the gun at a uniform speed of $10\sqrt{2}\ \text{ms}^{-1}$. The car is at a distance of 150 m from the gun when it is fired at an angle of 45° to the horizontal. With what speed should the shell be projected so that it hits the car? Take $g = 10\ \text{ms}^{-2}$.

(a) $20\ \text{ms}^{-1}$ (b) $30\ \text{ms}^{-1}$

(c) $40\ \text{ms}^{-1}$ (d) $50\ \text{ms}^{-1}$

94. In Q.93, what is the distance of the car from the gun when the shell hits it?

(a) 250 m (b) 500 m

(c) 750 m (d) 1000 m

95. A body thrown along a frictionless inclined plane of angle of inclination 30° covers a distance of 40 m along the plane. If the body is projected with the same speed at angle of 30° with the ground, it will have a range of (take $g = 10\ \text{ms}^{-2}$)

(a) 20 m (b) $20\sqrt{2}$ m

(c) $20\sqrt{3}$ m (d) 40 m

96. Which of the following remains constant during the motion of a projectile fired from a planet?

(a) kinetic energy
(b) momentum
(c) vertical component of velocity
(d) horizontal component of velocity

97. A body is projected with kinetic energy K at an angle of 60° with the horizontal. Its kinetic energy at the highest point of its trajectory will be

(a) $2K$ (b) K

(c) $K/2$ (d) $K/4$

98. A body, projected with a certain kinetic energy, has a horizontal range R. The kinetic energy will be minimum at a position of the projectile when its horizontal range is

(a) R (b) $3R/4$

(c) $R/2$ (d) $R/4$

99. At what angle (θ) with the horizontal should a body be projected so that its horizontal range equals the maximum height it attains?

(a) $\theta = \tan^{-1}\left(\sqrt{2}\right)$ (b) $\theta = \tan^{-1}\left(2\sqrt{2}\right)$

(c) $\theta = \tan^{-1}\left(2\sqrt{3}\right)$ (d) $\theta = \tan^{-1}(4)$

100. The horizontal distance x and the vertical height y of a projectile at a time t are given by

$$x = at \text{ and } y = bt^2 + ct$$

where a, b and c are constants. What is the magnitude of the velocity of the projectile 1 second after it is fired?

(a) $[a^2 + (2b + c)^2]^{1/2}$ (b) $[2a^2 + (b + c)^2]^{1/2}$

(c) $[2a^2 + (2b + c)^2]^{1/2}$ (d) $[a^2 + (b + 2c)^2]^{1/2}$

101. In Q.100, the angle (θ) with the horizontal at which the projectile is projected is given by

(a) $\theta = \tan^{-1}\left(\frac{a}{b}\right)$ (b) $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

(c) $\theta = \tan^{-1}\left(\frac{a}{c}\right)$ (d) $\theta = \tan^{-1}\left(\frac{c}{a}\right)$

102. In Q.100, the acceleration due to gravity is given by

(a) $-2a$ (b) $-2b$

(c) $-2c$ (d) $-ac + b$

103. In Q. 100, the magnitude of the initial velocity of the projectile is given by

(a) $(a^2 + b^2)^{1/2}$ (b) $(b^2 + c^2)^{1/2}$

(c) $(a^2 + c^2)^{1/2}$ (d) $(b^2 - 4ac)^{1/2}$

104. Four projectiles are projected with the same speed at angles 20°, 35°, 60° and 75° with the horizontal. The range will be the longest for the projectile whose angle of projection is

(a) 20° (b) 35°

(c) 60° (d) 75°

105. A projectile thrown at an angle of 30° with the horizontal has a range R_1 and attains a maximum height h_1. Another projectile thrown, with the same speed, at an angle 30° with the vertical, has a range R_2 and attains a maximum height h_2. The relation between R_1 and R_2 is

(a) $R_1 = \frac{R_2}{2}$ (b) $R_1 = R_2$

(c) $R_1 = 2R_2$ (d) $R_1 = 4R_2$

106. In Q.105 what is the relation between h_1 and h_2?

(a) $h_1 = \frac{h_2}{4}$ (b) $h_1 = \frac{h_2}{3}$

(c) $h_1 = \frac{h_2}{2}$ (d) $h_1 = h_2$

107. The maximum height attained by a projectile is increased by 10% by increasing its speed of projection, without changing the angle of projection. The percentage increase in the horizontal range will be

(a) 20% (b) 15%

(c) 10% (d) 5%

108. In Q.107, what is the percentage increase in the time of flight of the projectile?

(a) 20% (b) 15%

(c) 10% (d) 5%

109. The maximum height attained by projectile is increased by 10% by changing the angle of projection, without changing the speed of projection. The percentage increase in the time of flight will be

(a) 20% (b) 15%

(c) 10% (d) 5%

110. The speed of projection of projectile is increased by 5%, without changing the angle of projection. The percentage increase in the range will be

(a) 2.5% (b) 5%

(c) 7.5% (d) 10%

111. In Q. 110, what is the percentage increase in the maximum height attained by the projectile?
(a) 2.5% (b) 5%
(c) 7.5% (d) 10%

112. In Q. 110, what is the percentage increase in the time of flight of the projectile?
(a) 2.5% (b) 5%
(c) 7.5% (d) 10%

113. A projectile attains a certain maximum height when projected from earth. If it is projected at the same angle and with the same initial speed from the moon, where the acceleration due to gravity is one–sixth that on earth, by what factor will the maximum height of the projectile increase?
(a) $\sqrt{3}$ (b) 3
(c) $\sqrt{6}$ (d) 6

114. A projectile has a range R and time of flight T. If the range is doubled (by increasing the speed of projection, without changing the angle of projection), the time of flight will become
(a) $\frac{T}{\sqrt{2}}$ (b) $\sqrt{2}\,T$
(c) $\frac{T}{2}$ (d) $2\,T$

115. A projectile has the same range R when the maximum height attained by it is either h_1 or h_2. Then R, h_1 and h_2 will be related as
(a) $R = \sqrt{h_1 h_2}$ (b) $R = 2\sqrt{h_1 h_2}$
(c) $R = 3\sqrt{h_1 h_2}$ (d) $R = 4\sqrt{h_1 h_2}$

116. A body is projected horizontally from a point above the ground. The motion of the body is described by the equations
$$x = 2t$$
and
$$y = 5t^2$$
where x and y are the horizontal and vertical displacements (in m) respectively at time t. The trajectory of the body is
(a) a straight line (b) a circle
(c) an ellipse (d) a parabola

117. What is the velocity with which the body in Q. 116 is projected?
(a) 2 ms^{-1} (b) 2.5 ms^{-1}
(c) 3.5 ms^{-1} (d) 5 ms^{-1}

118. What is the magnitude of the velocity of the body in Q. 116, 0.2 second after it is projected?
(a) $\sqrt{2}$ ms^{-1} (b) $2\sqrt{2}$ ms^{-1}
(c) $3\sqrt{2}$ ms^{-1} (d) $4\sqrt{2}$ ms^{-1}

119. The velocity of the body in Q. 116 at time $t = 0.2$s is inclined with the vertical at an angle of
(a) 22.5° (b) 30°
(c) 45° (d) 60°

120. A body is projected at time $t = 0$ from a certain point on a planet's surface with a certain velocity at a certain angle with the planet's surface (assumed horizontal). The horizontal and vertical displacements x and y (in metres) respectively vary with time t (in seconds) as
$$x = 10\sqrt{3}\,t$$
$$y = 10\,t - t^2$$
What is the magnitude and direction of the velocity with which the body is projected?
(a) 20 ms^{-1} at an angle of 30° with the horizontal
(b) 20 ms^{-1} at an angle of 60° with the horizontal
(c) 10 ms^{-1} at an angle of 30° with the horizontal
(d) 10 ms^{-1} at an angle of 60° with the horizontal

121. What is the value of acceleration due to gravity on the planet's surface in Q. 120?
(a) 1.0 ms^{-2} (b) 2 ms^{-2}
(c) 4 ms^{-2} (d) 9.8 ms^{-2}

122. What is the maximum height which the body in Q.120 will attain?
(a) 25 m (b) 50 m
(c) 75 m (d) 100 m

123. What is the total time of flight of the body in Q.120?
(a) 2 s (b) 5 s
(c) 10 s (d) 20 s

124. What is the distance between the point from where the body in Q. 120 is projected and the point where it strikes the planet's surface?
(a) $25\sqrt{3}$ m (b) $50\sqrt{3}$ m
(c) $75\sqrt{3}$ m (d) $100\sqrt{3}$ m

125. Which one of the following statements is NOT true about the motion of a projectile?
(a) The time of flight of a projectile is proportional to the speed with which it is projected.
(b) The horizontal range of a projectile is proportional to the square–root of the speed with which it is projected.
(c) For a given speed of projection, the angle of projection for maximum range is 45°.
(d) At maximum height, the acceleration due to gravity is perpendicular to the velocity of the projectile.

126. A plumb line is hanging from the ceiling of a train. If the train moves along a horizontal track with a uniform acceleration a, the plumb line gets inclined to the vertical at an angle

(a) $\tan^{-1}\left(\frac{a}{g}\right)$ (b) $\tan^{-1}\left(\frac{g}{a}\right)$

(c) $\sin^{-1}\left(\frac{a}{g}\right)$ (d) $\cos^{-1}\left(\frac{g}{a}\right)$

127. Displacement (x) of a particle is related to time (t) as
$$x = at + bt^2 - ct^3$$
where a, b and c are constants of motion. The velocity of the particle when its acceleration is zero is given by

(a) $a + \frac{b^2}{c}$ (b) $a + \frac{b^2}{2c}$

(c) $a + \frac{b^2}{3c}$ (d) $a + \frac{b^2}{4c}$

128. The angle between vectors $\mathbf{A} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\mathbf{B} = 6\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$ is

(a) zero (b) 45º

(c) 90º (d) 180º

129. A vector **A** is along the positive z-axis and its vector product with another vector **B** is zero, then vector **B** could be

(a) $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ (b) $4\hat{\mathbf{i}}$

(c) $\hat{\mathbf{j}} + \hat{\mathbf{k}}$ (d) $-7\hat{\mathbf{k}}$

130. **A** is a vector which when added to the resultant of vectors $(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ and $(\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ yields a unit vector along the y–axis. Then vector **A** is ?

(a) $-3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 6\hat{\mathbf{k}}$ (b) $3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 6\hat{\mathbf{k}}$

(c) $3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ (d) $3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

131. The angle between two vectors **A** and **B** is θ. Vector **R** is the resultant of the two vectors. If **R** makes an angle $\frac{\theta}{2}$ with **A**, then

(a) $A = 2B$ (b) $A = \frac{B}{2}$

(c) $A = B$ (d) $AB = 1$

132. A body is thrown vertically up with a velocity u. It passes three points A, B and C in its upward journey with velocities $\frac{u}{2}$, $\frac{u}{3}$ and $\frac{u}{4}$ respectively. The ratio $\frac{AB}{BC}$ is

(a) $\frac{20}{7}$ (b) 2

(c) $\frac{10}{7}$ (d) 1

133. A body is thrown vertically up with a velocity u. It passes a point at a height h above the ground at time t_1 while going up and at time t_2 while falling down. Then the relation between u, t_1 and t_2 is

(a) $t_1 + t_2 = \frac{2u}{g}$ (b) $t_2 - t_1 = \frac{2u}{g}$

(c) $t_1 + t_2 = \frac{u}{g}$ (d) $t_2 - t_1 = \frac{u}{g}$

134. In Q. 133 above, the relation between t_1, t_2 and h is

(a) $t_1 t_2 = \frac{2h}{g}$ (b) $t_1 t_2 = \frac{h}{g}$

(c) $(t_1 + t_2)^2 = \frac{2h}{g}$ (d) $(t_1 + t_2)^2 = \frac{h}{g}$

135. A body dropped from a height H above the ground strikes an inclined plane at a height h above the ground. As a result of the impact, the velocity of the body becomes horizontal. The body will take the maximum time to reach the ground if

(a) $h = \frac{H}{4}$ (b) $h = \frac{H}{2\sqrt{2}}$

(c) $h = \frac{H}{2}$ (d) $h = \frac{H}{\sqrt{2}}$

136. A body is dropped from the roof of a multi-storeyed building. It passes the ceiling of the 15th storey at a speed of 20 ms^{-1}. If the height of each storey is 4 m, the number of storeys in the building is (take $g = 10\ ms^{-2}$ and neglect air resistance)

(a) 20 (b) 25

(c) 30 (d) 35

137. Two balls A and B are projected from the same location simultaneously. Ball A is projected vertically upwards and ball B at 30º to the vertical. They reach the ground simultaneously. The velocities of projection of A and B are in the ratio

(a) $\sqrt{3} : 1$ (b) $1 : \sqrt{3}$

(c) $\sqrt{3} : 2$ (d) $2 : \sqrt{3}$

138. A body is projected with a velocity $\mathbf{v} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ ms^{-1}. The maximum height attained by the body is (take $g = 10\ ms^{-2}$)

(a) 0.8 m (b) 8 m

(c) 80 m (d) 800 m

139. In Q. 138 above, the time of flight of the body is

(a) 0.8 s (b) 1.0 s

(c) 4.0 s (d) 8.0 s

140. A body is projected with a velocity **u** at an angle θ with the horizontal. The velocity of the body will become perpendicular to the velocity of projection after a time t given by

(a) $\frac{2u\sin\theta}{g}$ (b) $\frac{u\sin\theta}{g}$

(c) $\frac{2u}{g\sin\theta}$ (d) $\frac{u}{g\sin\theta}$

141. A body is projected at an angle θ with the horozontal. When it is at the highest point, the ratio of the potential and kinetic energies of the body is

(a) $\tan\theta$ (b) $\tan^2\theta$

(c) $\cot\theta$ (d) $\cot^2\theta$

142. The equation of motion of a projectile is $y = ax - bx^2$ where a and b are constants of motion. The horizontal range of the projectile is

(a) $\frac{a}{b}$ (b) $\frac{\sqrt{a}}{2b}$

(c) $\frac{a^2}{2b}$ (d) $\frac{a^2}{4b}$

143. In Q. 142 above, the greatest height attained by the projectile is

(a) $\frac{a}{b}$ (b) $\frac{\sqrt{a}}{2b}$

(c) $\frac{a^2}{2b}$ (d) $\frac{a^2}{4b}$

144. In Q. 142 above, the time of flight of the projectile is

(a) $a\sqrt{\frac{2}{bg}}$ (b) $\frac{a}{\sqrt{bg}}$

(c) $\sqrt{\frac{2a}{bg}}$ (d) $\sqrt{\frac{a}{2bg}}$

145. The initial velocity of a particle is $\mathbf{u} = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ ms^{-1}. It is moving with uniform acceleration $\mathbf{a} = (0.4\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}})$ ms^{-2}. The magnitude of its velocity after 10 s is

(a) 3 ms^{-1} (b) 4 ms^{-1}

(c) 5 ms^{-1} (d) 10 ms^{-1}

146. The horizontal and vertical displacements of a projectile at time 't' are $x = 36t$ and $y = 48t - 4.9t^2$ respectively where x and y are in metre and t is in second. Initial velocity of the projectile in m/s is:

(a) 15 (b) 30

(c) 45 (d) 60

147. The displacement x of a particle moving in one dimension is related to time t by the equation

$$t = \sqrt{x} + 3$$

where x is in metres and t in seconds. The displacement of the particle when its velocity is zero is

(a) zero (b) 4 m

(c) 1 m (d) 0.5 m

148. A particle initially (i.e, at $t = 0$) moving with a velocity u is subjected to a retarding force, as a result of which it decelerates at a rate

$$a = -k\sqrt{v}$$

where v is the instantaneous velocity and k is a positive constant. The time T taken by the particle to come to rest is given by

(a) $T = \frac{2\sqrt{u}}{k}$ (b) $T = \frac{2u}{k}v$

(c) $T = \frac{2u^{3/2}}{k}$ (d) $T = \frac{2u^2}{k}$

149. A particle starts from rest. Its acceleration at time $t = 0$ is 5 ms^{-2} which varies with time as shown in Fig. 2.38. The maximum speed of the particle will be

(a) 7.5 ms^{-1} (b) 15 ms^{-1}

(c) 30 ms^{-1} (d) 7.5 ms^{-1}

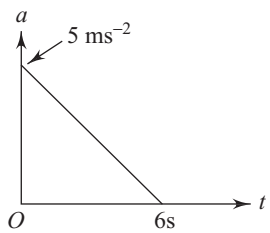


Fig. 2.38

150. Figure 2.38 shows the variation of velocity (v) of a body with position (x) from the origin O. Which of the graphs shown in Fig. 2.39 correctly represents the variation of the acceleration (a) with position (x)?

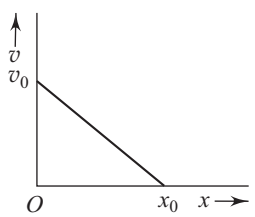


Fig. 2.39

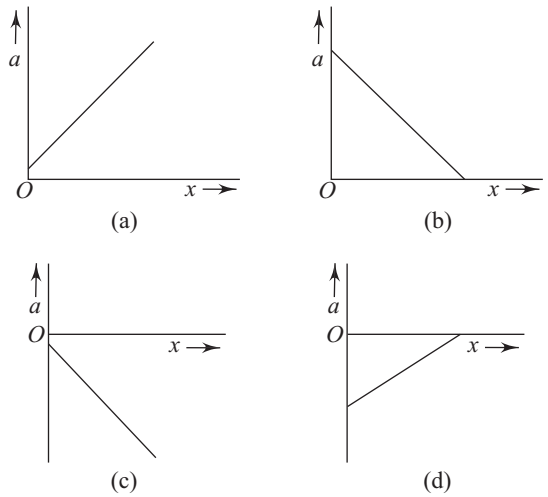


Fig. 2.40

151. The velocity (v) of a body moving along the postive x-direction varies with displacement (x) from the origin as $v = k\sqrt{x}$, where k is a constant. Which of the graphas shown in Fig. 2.41 correctly represents the displacement-time ($x - t$) graph of the motion?

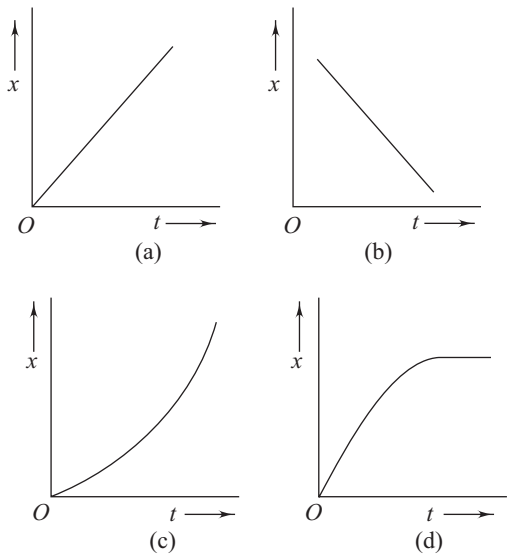


Fig. 2.41

152. The velocity (v) of an object, moving in a straight line, varies with time (t) as show in Fig. 2.42.

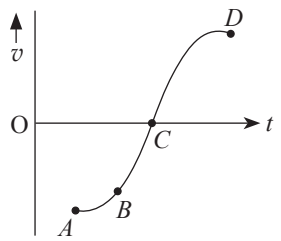


Fig. 2.42

The magnitude of the acceleration is the greatest a point

(a) A (b) B

(c) C (d) D

153. The position (x) of an object, moving in a straight line, varies with time (t) as shown in Fig. 2.43.

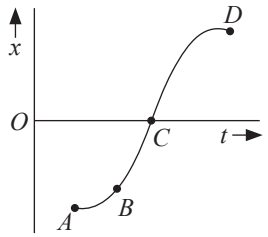


Fig. 2.43

The magnitude of the acceleration is the greatest at point.

(a) A (b) B

(c) C (d) D

154. An object has velocity v_1 at time t and velocity v_2 at time $(t + \Delta t)$ as shown in Fig. 2.44. If $v_1 = v_2 = v$, the magnitude of the average acceleration of the object in the time interval Δt is

(a) zero (b) $\dfrac{v \sin\theta}{\Delta t}$

(c) $\dfrac{2v \sin(\theta/2)}{\Delta t}$ (d) $\dfrac{v \cos\theta}{\Delta t}$

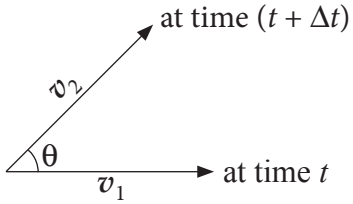


Fig. 2.44

155. A stone is dropped from the top of a building of height h. After 1s another stone is dropped from the balcony 25 m below the top of the building. If both the stones strike the ground at the same time, what is value of h? Take $g = 10\ \text{ms}^{-2}$.

(a) 30 m (b) 45 m

(c) 60 m (d) 75 m

156. The x and y components of the displacement of a particle moving in the x–y plane at time t are given by

$x = at + bt^2$ and $y = ct$

where a, b and c are constants. Then the acceleration of the particle

(a) is zero
(b) is constant = $2b$
(c) increases with time t
(d) decreases with time t.

157. A train 75m long moving at a speed of 54 kmh^{-1}, starts overtaking another train of length 125m moving at a speed of 36 kmh^{-1} on a parallel track. How long will the first train take to completely overtake the second train?
(a) 7.5 s (b) 12.5 s
(c) 20 s (d) 40 s

158. A person throws two balls vertically upward with the same velocity, one after the other He throws the second ball at time when the first ball is at the highest point. If he throws the balls in an interval of 1s, what is the maximum height attained by each ball?
(a) 3.7 m (b) 4.9 m
(c) 9.8 m (d) 19.6 m

159. A person throws up four balls, each with a velocity of 10 ms^{-1}, at regular intervals, in such a way that when one ball is in his hand, the other three balls are in air. At the instant when one ball just leaves his hand, the heights (in metre) above his hand of the other three balls are (take $g = 10\ ms^{-2}$)
(a) 3.75, 5.0 and 3.75 (b) 2.5, 7.5 and 4.5
(c) 3.0, 5.0, 7.0 (d) 4.5, 6.0, 4.5

160. Water drops from a leaky tap are falling, at regular time intervals, on the ground 125 cm below the tap. The first drop strikes the ground when the sixth drop just begins to fall. What is the height from the ground of the third drop when the first drop strikes the ground? Take $g = 10\ ms^{-2}$.
(a) 50 cm (b) 60 cm
(c) 70 cm (d) 80 cm

161. The displacement x of a particle moving in a straight line varies with time t as
$$x^2 = a + bt^2$$
where a and b are constants. The acceleration of the particle at time t is given by
(a) $\frac{a}{x} - \frac{bt^2}{x^3}$ (b) $\frac{a}{x} - \frac{bt}{x^2}$
(c) $\frac{b}{x} - \frac{b^2t^2}{x^3}$ (d) $\frac{a}{x} - \frac{at}{x^2}$

162. The velocity v of an object, moving in a straight line varies with time t as
$$v = at - bt^2$$
where a and b are constants. The average velocity of the object in the time interval $t = 0$ s to $t = 2$ s is
(a) $\frac{1}{2}(3a - 4b)$ (b) $\frac{2}{3}(a - 4b)$
(c) $(a - 2b)$ (d) $(2a - 3b)$

163. If a person stands stationary on an escalator, it takes time t_1 to take him from the ground floor to the first floor. If the escalator is stationary and he walks up, it takes time t_2. How much time would it take him if he walks up the moving escalator?
(a) $(t_1 + t_2)$ (b) $\frac{1}{2}(t_1 + t_2)$
(c) $\frac{t_1 t_2}{t_1 + t_2}$ (d) $\sqrt{(t_1^2 + t_2^2)}$

164. Two buildings A and B are 20 m apart. Building A has a window at a height of 50 m above the ground and building B has a window at a height of 30.4 m above the ground. With what speed must a man throw a ball horizontally from the window of building A so that it enters the window of building B?
(a) 5 ms^{-1} (b) 10 ms^{-1}
(c) 15 ms^{-1} (d) 20 ms^{-1}

165. Two stations A and B are 80 km apart and are situated on a straight road. A car leaves station A for station B every 15 minutes. A man drives a car at a speed of 80 kmh^{-1} from station A and at the same time another man drives a car at a speed of 80 kmh^{-1} from station B. How many cars will he meet before reaching station A?
(a) 4 (b) 5
(c) 6 (d) 7

166. A body is projected from the ground with a velocity u at an angle θ with the horizontal. The average velocity of the body between its point of projection and the highest point of its trajectory is
(a) $\frac{u}{2}(1 + \cos\theta)$ (b) $\frac{u}{2}(1 + \cos^2\theta)^{1/2}$
(c) $\frac{u}{2}(1 + 2\cos^2\theta)^{1/2}$ (d) $\frac{u}{2}(1 + 3\cos^2\theta)^{1/2}$

167. A body is projected from the ground with a speed u at an angle θ with the horizontal. When the body is at the highest point of the trajectory, its speed is $\sqrt{2/3}$ times its speed when it is at half the maximum height. The value of θ is
(a) 30° (b) 45°
(c) 60° (d) $\sin^{-1}\left(\frac{2}{3}\right)$

Answers

Level A

1. (b)	2. (c)	3. (a)	4. (c)
5. (c)	6. (b)	7. (d)	8. (b)
9. (b)	10. (a)	11. (d)	12. (c)
13. (a)	14. (a)	15. (b)	16. (d)
17. (b)	18. (c)	19. (a)	20. (b)
21. (b)	22. (c)	23. (d)	24. (d)
25. (a)	26. (c)	27. (c)	28. (b)
29. (d)	30. (b)	31. (c)	32. (a)
33. (b)	34. (b)	35. (d)	36. (b)
37. (d)	38. (a)	39. (d)	40. (c)
41. (d)	42. (a)		

Level B

43. (b)	44. (b)	45. (b)	46. (c)
47. (a)	48. (d)	49. (a)	50. (a)
51. (a)	52. (a)	53. (c)	54. (c)
55. (a)	56. (b)	57. (c)	58. (d)
59. (b)	60. (c)	61. (d)	62. (c)
63. (b)	64. (a)	65. (b)	66. (b)
67. (a)	68. (b)	69. (c)	70. (b)
71. (c)	72. (b)	73. (c)	74. (d)
75. (a)	76. (d)	77. (b)	78. (b)
79. (d)	80. (d)	81. (c)	82. (a)
83. (c)	84. (b)	85. (d)	86. (d)
87. (c)	88. (b)	89. (a)	90. (c)
91. (c)	92. (b)	93. (d)	94. (a)
95. (c)	96. (d)	97. (d)	98. (c)
99. (d)	100. (a)	101. (d)	102. (b)
103. (c)	104. (b)	105. (b)	106. (b)
107. (c)	108. (d)	109. (d)	110. (d)
111. (d)	112. (b)	113. (d)	114. (b)
115. (d)	116. (d)	117. (a)	118. (b)
119. (c)	120. (a)	121. (b)	122. (a)
123. (c)	124. (d)	125. (b)	126. (a)
127. (c)	128. (a)	129. (d)	130. (a)
131. (c)	132. (a)	133. (a)	134. (a)
135. (c)	136. (a)	137. (c)	138. (a)
139. (a)	140. (d)	141. (b)	142. (a)
143. (d)	144. (a)	145. (d)	146. (d)
147. (a)	148. (a)	149. (b)	150. (d)
151. (c)	152. (c)	153. (d)	154. (c)
155. (b)	156. (b)	157. (d)	158. (b)
159. (a)	160. (d)	161. (c)	162. (a)
163. (c)	164. (b)	165. (b)	166. (d)
167. (b)			

Solutions

Level A

1. The magnitude R of the resultant vector **R** of two vectors **A** and **B** inclined at an angle θ is given by $R^2 = A^2 + B^2 + 2AB\cos\theta$

$$\cos\theta = \frac{R^2 - A^2 - B^2}{2AB} = \frac{(5)^2 - (3)^2 - (4)^2}{2\times 3\times 4} = 0$$

$\therefore \quad \theta = \frac{\pi}{2}$. Hence the correct choice is (b).

2. $R^2 = A^2 + B^2 + 2AB\cos\theta$. It is given that $R = A = B$. Putting these values we have

$$A^2 = A^2 + A^2 + 2A^2\cos\theta$$

or $\cos\theta = -\frac{1}{2}$ which gives $\theta = 120°$

Hence the correct choice is (c).

3. From the triangle law of vector addition, it is clear that the vectors **R**, **A** and **B** in Q.2 must be represented by the three sides of an equilateral triangle because the magnitudes of the three vectors are equal. Hence the angle α between vectors **R** and **A** is 60°. Alternatively, the value of α is given by

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} = \frac{\sin\theta}{1+\cos\theta} \quad (\because A = B)$$

$$= \frac{\sin 120°}{1+\cos 120°} = 1.732$$

which gives $\alpha = 60°$. Hence the correct choice is (a).

4. Let θ be the angle between **A** and **B**. Then magnitudes of vectors **P** and **Q** are given by

$$P^2 = A^2 + B^2 + 2AB\cos\theta$$

and $$Q^2 = A^2 + B^2 - 2AB\cos\theta$$

Since $P = Q$, it follows that $\cos\theta = 0$ or $\theta = \pi/2$. Hence correct choice is (c).

5. Given $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = \frac{1}{2}AB$ or $\cos\theta = \frac{1}{2}$. Therefore $\theta = 60°$. Hence correct choice is (c).

6. Given $|\mathbf{A}\times\mathbf{B}| = AB\sin\theta = \frac{1}{2}AB$ or $\sin\theta = \frac{1}{2}$. Therefore $\theta = 30°$. Hence the correct choice is (b).

7. Since $\mathbf{A}\times\mathbf{B} = -\mathbf{B}\times\mathbf{A}$, $\mathbf{C} = -\mathbf{D}$, *i.e.* vectors **C** and **D** are in opposite directions. Hence the correct choice is (d).

8. Let $|\hat{\mathbf{i}}|$ and $|\hat{\mathbf{j}}|$ represent the magnitudes of vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ respectively. Since $|\hat{\mathbf{i}}|$ and $|\hat{\mathbf{j}}|$ are unit vectors, $\hat{\mathbf{i}} = 1$ and $\hat{\mathbf{j}} = 1$. Therefore, the magnitude of vector $\hat{\mathbf{i}} + \hat{\mathbf{j}} = \sqrt{(1)^2 + (1)^2} = \sqrt{2}.$

Thus, the correct choice is (b).

9. The angle subtended by vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ with the x–axis is given by

$$\tan\theta = \frac{|\hat{\mathbf{i}}|}{|\hat{\mathbf{j}}|} = \frac{1}{1} = 1$$

or $\theta = 45°$ which is choice (b).

10. $\mathbf{A}\cdot\mathbf{B} = (\hat{\mathbf{i}}+\hat{\mathbf{j}})\cdot(\hat{\mathbf{i}}+\hat{\mathbf{k}}) = \hat{\mathbf{i}}\cdot\hat{\mathbf{i}} + \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} + \hat{\mathbf{j}}\cdot\hat{\mathbf{i}} + \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = 1 + 0 + 0 + 0 = 1$

Thus, the correct choice is (a).

11. $\mathbf{A}\times\mathbf{B} = (\hat{\mathbf{i}}+\hat{\mathbf{j}})\times(\hat{\mathbf{i}}+\hat{\mathbf{k}}) = \hat{\mathbf{i}}\times\hat{\mathbf{i}} + \hat{\mathbf{i}}\times\hat{\mathbf{k}} + \hat{\mathbf{j}}\times\hat{\mathbf{i}} + \hat{\mathbf{j}}\times\hat{\mathbf{k}}$

$= 0 - \hat{\mathbf{j}} - \hat{\mathbf{k}} + \hat{\mathbf{i}}$ which is choice (d).

12. Given $y = -\frac{2}{3}t^2 + 16t + 2$. Comparing it with $s = ut + \frac{1}{2}at^2$, we have $u = 16$ ms^{-1} and $\frac{a}{2} = -\frac{2}{3}$ or $a = -\frac{4}{3}$ ms^{-2}. For a body to come to rest, the final velocity $v = 0$. Using these values of u, v and a in relation

$$v = u + at$$

we have $$0 = 16 - \frac{4}{3}t$$

or $$t = 12 \text{ s}$$

Hence the correct choice is (c).

13. Let t_1 be the time during which the car accelerates at a rate α. The velocity at the end of time t_1 will be

$$v \text{ at } t_1 = u + \alpha t_1 = 0 + \alpha t_1 = \alpha t_1 \qquad (\because\ u = 0)$$

The time during which the car decelerates is $t_2 = t - t_1$. For this time t_2, the initial velocity is αt_1 and the final velocity is zero and the acceleration is $-\beta$. Therefore

$$0 = \alpha t_1 - \beta(t - t_1)$$

which gives $t_1 = \frac{\beta t}{\alpha+\beta}$. Therefore, maximum velocity = v at $t_1 = \alpha t_1 = \frac{\alpha\beta t}{\alpha+\beta}$.

Hence the correct choice is (a).

14. The distance travelled in time t_1 is

$$s_1 = ut_1 + \frac{1}{2}\alpha t_1^2 = \frac{1}{2}\alpha t_1^2 \qquad (\because\ u = 0)$$

$$= \frac{1}{2}\alpha\left(\frac{\beta}{\alpha+\beta}\right)^2 t^2 \qquad \left(\because\ t_1 = \frac{\beta t}{\alpha+\beta}\right)$$

Also, the distance travelled in time t_2 is given by

$$-2\beta s_2 = 0^2 - \left(\frac{\alpha\beta}{\alpha+\beta}\right)^2 t^2$$

or $$s_2 = \frac{1}{2}\beta\left(\frac{\alpha}{\alpha+\beta}\right)^2 t^2$$

Therefore, $$s_1 + s_2 = \frac{1}{2}\alpha\left(\frac{\beta}{\alpha+\beta}\right)^2 t^2 + \frac{1}{2}\beta\left(\frac{\alpha}{\alpha+\beta}\right)^2 t^2$$

$$= \frac{1}{2}\frac{\alpha\beta}{\alpha+\beta}t^2$$

Hence the correct choice is (a).

15. Since $a = \frac{dv}{dt}$, we have $v = \int a\,dt = \int (kt + c)\,dt$

$= \frac{1}{2}kt^2 + ct$. Hence the correct choice is (b).

16. The velocity of the particle at time t is given by

$$v = \int a\,dt$$

$$= \int(-s\omega^2\cos\omega t)\,dt$$

$$= -s\omega\sin\omega t$$

Now, the displacement is given by

$$x = \int v\,dt = -s\int\omega\sin\omega t\,dt$$

$$= s\cos\omega t$$

Thus, the correct choice is (d).

17. When the parachutist bails out, he shares the velocity of the balloon and has an upward velocity of 10 ms^{-1}, *i.e.* $u = +10\ ms^{-1}$. Also $g = -10\ ms^{-2}$ (acting downwards). The displacement in $t = 3s$ is given by

$$s = ut + \frac{1}{2} gt^2$$
$$= 10 \times 3 + \frac{1}{2} \times (-10) \times (3)^2$$
$$= -15\ m$$

Since the displacement is negative, it is directed downwards. So the height from the ground when he opened his parachute = 45 – 15 = 30 m. Thus, the correct choice is (b).

18. In time 3s, the balloon has risen through 30m (as the velocity of the balloon is 10 ms^{-1} upwards). Hence the parachutist is now 30 + 15 = 45 m away from the balloon. Thus, the correct choice is (c).

19. The velocity of the parachutist 3s after he bails out is

$$v = u + gt$$
$$= 10 + (-10) \times 3 = -20\ ms^{-1}$$
(directed downwards)

At $t = 3s$, his initial velocity is $u = -20\ ms^{-1}$ and to hit the ground, his displacement $s = -30$ m (see solution of Q. 40). Now $a = +5\ ms^{-2}$ (directed upwards). The time taken to hit the ground is given by

$$s = ut + \frac{1}{2} gt^2$$

or $$-30 = -20\, t + \frac{5}{2} t^2$$

or $$t^2 - 8t + 12 = 0$$

or $$(t - 6)(t - 2) = 0$$

which gives $t = 6$ s or 2 s. If $t = 6$ s, then the velocity with which he hits the ground is $v = u + at = -20 + 5 \times 6 = 10\ ms^{-1}$. This is positive, *i.e.* v is directed upwards, which is not possible. Thus the correct answer is $t = 2$ s, in which case, the velocity with which he hits the ground is

$$v = -20 + 5 \times 2 = -10\ ms^{-1}$$

which is negative as it should be. Thus, the correct choice is (a).

20. The total time the parachutist takes (after his exit from the balloon) to hit the ground is = 3 s + 2 s = 5 s which is choice (b).

21. The distance covered in the nth second is given by

$$s_n = u + a\left(n - \frac{1}{2}\right)$$

$$\therefore \quad s_4 = u + a\left(4 - \frac{1}{2}\right) = u + \frac{7}{2}a$$

or $$40 = u + \frac{7}{2} \times 10 \text{ or } u = 40 - 35$$
$$= 5\ ms^{-1}$$

$$\therefore \quad s_6 = 5 + 10 \times \left(6 - \frac{1}{2}\right)$$

which gives $s_6 = 60$ m.

Thus, the correct choice is (b).

22. Now, $s_n = u + a\left(n - \frac{1}{2}\right)$. Therefore,

$30 = 5 + a\left(3 - \frac{1}{2}\right)$ which gives $a = 10\ ms^{-2}$.

$$\therefore \quad s_4 = 5 + 10\left(4 - \frac{1}{2}\right) = 40\ m$$

and $$s_5 = 5 + 10\left(5 - \frac{1}{2}\right) = 50\ m$$

$\therefore\ s = s_4 + s_5 = 40 + 50 = 90$ m. Thus the correct choice is (c).

23. Given $$40 = u + \frac{7a}{2}$$

and $$60 = u + \frac{11a}{2}$$

These equations give $u = 5\ ms^{-1}$ and $a = 10\ ms^{-2}$. Thus, the correct choice is (d).

24. Since $u = 0$, we have

$$s_1 = 0 + \frac{7a}{2} = \frac{7a}{2}$$

and $$s_2 = 0 + \frac{11a}{2} = \frac{11a}{2}$$

$$\therefore \quad s_1/s_2 = 7/11, \text{ which is choice (d).}$$

25. Let the total distance be 2 s and let t_1 and t_2 be the times taken to traverse the first half and the second half of the distance respectively. Then $t_1 = s/V$. For the second half of the distance, the distance s_1 is covered in time $\frac{t_2}{2}$ with speed V' is $s_1 = V' \frac{t_2}{2}$ and the distance s_2 is covered in time $\frac{t_2}{2}$ with speed V' is $s_2 = V'' \frac{t_2}{2}$, so that $s = s_1 + s_2 = V' \frac{t_2}{2} + V'' \frac{t_2}{2} = (V' + V'') \frac{t_2}{2}$, which gives

$$t_2 = \frac{2s}{(V' + V'')}$$

$\therefore$ Total time taken = $t_1 + t_2$

$$= \frac{s}{V} + \frac{2s}{V' + V''} = \frac{s(2V + V' + V'')}{V(V' + V'')}$$

Hence average speed = $\dfrac{\text{total distance}}{\text{total time}}$

$$= \frac{2s \times V(V' + V'')}{s(2V + V' + V'')} = \frac{2V(V' + V'')}{(2V + V' + V'')}$$

Thus, the correct choice is (a).

26. The shortest stopping distance S is given by

$$0 - v^2 = -2aS$$

or $S = \dfrac{v^2}{2a}$. Thus, for a given value of a, $S \propto v^2$. If v is increased by a factor of 3, S will increase by a factor $(3)^2 = 9$. Hence the correct choice is (c).

27. The maximum height ($u^2 \sin^2 \theta/2g$) and the horizontal range ($= u^2 \sin 2\theta/g$) are equal if $\sin^2 \theta/2 = \sin 2\theta = 2 \sin \theta \cos \theta$ or $\tan \theta = 4$ or $\theta = \tan^{-1}(4)$. Hence the correct choice is (c).

28. The bullet covers a distance of 100 m in time $t = 100/500 = 0.2$ s. The vertical downward distance moved in time $t = 0.2$ s is $\dfrac{1}{2} gt^2 = \dfrac{1}{2} \times 10 \times (0.2)^2 =$ 0.2 m = 20 cm. Hence the correct choice is (b).

29. At the highest point of the trajectory, the velocity of the projectile is parallel to the ground but its acceleration is directed vertically downwards. Hence the correct choice is (d).

30. The time taken to reach the ground is independent of the horizontal velocity of projection. Since the initial downward velocity is zero (the initial velocity has only a horizontal component), the time of flight is given by

$$h = \frac{1}{2} gt^2$$

which gives $$t = \sqrt{\frac{2h}{g}} = \left(\frac{2 \times 19.6}{9.8}\right)^{1/2} = 2 \text{ s}$$

Hence the correct choice is (b).

31. The time taken by the bomb to hit the ground is given by (as shown above)

$$t = \sqrt{\frac{2h}{g}}$$

Now, the horizontal speed of the aeroplane (and also of the bomb) is v.

$\therefore$ Horizontal range = horizontal velocity $\times$ t

$$= vt = v\sqrt{\frac{2h}{g}}$$

Hence the correct choice is (c).

32. Let G be the position of the gun and E that of the enemy plane flying horizontally with speed $u = 300$ ms^{-1}, when the shell is fired with a speed $v_0 = 600$ ms^{-1} in a direction θ with the horizontal (Fig. 2.50). The horizontal component of v_0 is

$$v_x = v_0 \cos \theta$$

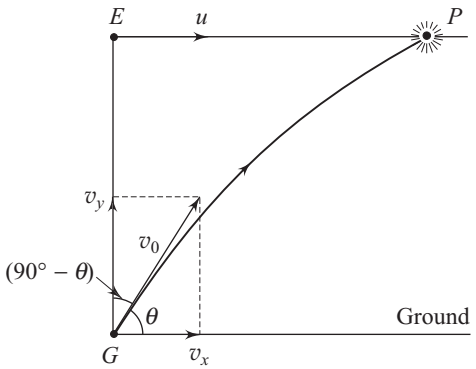


Fig. 2.50

Let the shell hit the plane at point P and let t be the time taken for the shell to hit the plane. It is clear that the shell will hit the plane, if the horizontal distance EP travelled by the plane in time t = the distance travelled by the shell in the horizontal direction in the same time, *i.e.*, $u \times t = v_x \times t$ or $u = v_x = v_0 \cos \theta$

or $\cos \theta = \dfrac{u}{v_0} = \dfrac{300}{600} = 0.5$ or $\theta = 60°$. Therefore angle with the vertical = $90° - \theta = 30°$. Hence the correct choice is (a).

33. To avoid being hit, the plane should have a minimum altitude = maximum height attained by the shell which is

$$h_{\max} = \frac{v_0^2 \sin^2 \theta}{2g} = \frac{(600)^2 \times \sin^2 (60°)}{2 \times 10}$$

$$= 13.5 \times 10^3 \text{ m} = 13.5 \text{ km}.$$

which is choice (b).

34. Given $h_{\max} = 5$ m, $u = 20$ ms^{-1} and $g = 10$ ms^{-2}. We know that

$$h_{\max} = \frac{u^2 \sin^2 \theta}{2g}$$

or $\sin \theta = \dfrac{1}{u} \times \sqrt{2gh_{\max}} = \dfrac{1}{20} \times \sqrt{2 \times 10 \times 5} = 0.5$

which gives $\theta = 30°$. Therefore

$$R_{max} = \frac{u^2 \sin 2\theta}{g} = \frac{(20)^2 \times \sin 60°}{10} = 20\sqrt{3} \text{ m}$$

Hence the correct choice is (b).

35. The horizontal range $R = AC$ is

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{20 \times 20 \times \sin 60°}{10}$$

$$= 20\sqrt{3} \text{ m}$$

At C, the velocity of the ball is again 20 ms^{-1} directed down as shown in Fig. 2.52.

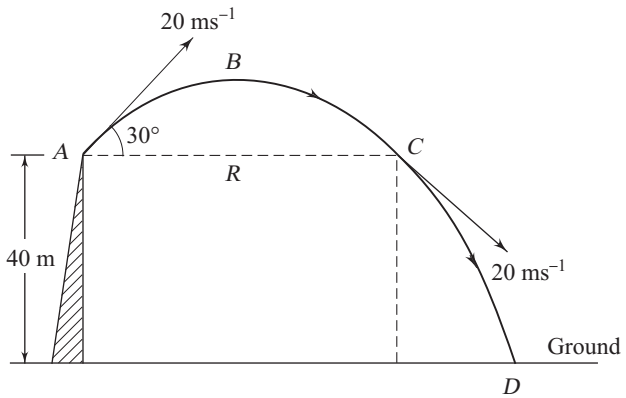


Fig. 2.52

The downward vertical component of this velocity is 20 sin 30° = 10 ms^{-1}. The ball will hit the ground at D after travelling a vertical distance $h = 40$ m. If t_1 is the time taken for this, then

$$h = ut_1 + \frac{1}{2} gt_1^2$$

or $\quad 40 = 10\, t_1 + 5\, t_1^2$

or $\quad t_1^2 + 2\, t_1 - 8 = 0$

The positive root of this quadratic equation is $t_1 = 2$s. Now, the time of flight from A to C via B is

$$t_2 = \frac{2u \sin\theta}{g} = \frac{2 \times (20) \times \sin 30°}{10}$$

$$= \frac{2 \times 20 \times \sin 30°}{10} = 2 \text{ s}$$

$\therefore$ Total time taken = $t_1 + t_2 = 2\text{s} + 2\text{s} = 4\text{s}$. Hence the correct choice is (d).

36. For maximum range $\theta = 45°$. Hence $R_{max} = u^2/g$ and $h_{max} = u^2/4g$. Thus $h_{max} = R_{max}/4 = 200/4 = 50$ m. Hence the correct choice is (b).

37. When the lift is descending with a retardation (negative acceleration) a, the effective value of g is $g_{eff} = g + a$. The component of this acceleration along the inclined plane is $g_{eff} \sin\theta = (g + a) \sin\theta$. Hence the correct choice is (d).

38. The maximum height attained by the first ball is

$$h_1 = \frac{u^2}{2g}$$

where u is the initial speed of projection. The maximum height attained by the second ball is ($\because \theta = 90° - 60° = 30°$)

$$h_2 = \frac{u^2 \sin^2(30^\circ)}{2g} = \frac{u^2}{8g}$$

Now, PE of ball 1 at height $h_1 = mgh_1$ and that of ball 2 at height $h_2 = mgh_2$. Therefore, the ratio of potential energies = $\frac{h_1}{h_2} = \frac{u^2}{2g} \times \frac{8g}{u^2} = 4$. Hence the correct choice is (a).

39. Refer to Fig. 2.55, where **u** is velocity of water current. Let **v** be the velocity of the man and let $AB = w$ be the width of the river. The river is crossed with a velocity equal to the component of **v** along AB, which is $v \cos\theta$, where θ is the angle between the north direction and the direction of velocity **v**.

$\therefore$ Time taken to cross the river = $\frac{w}{v\cos\theta}$.

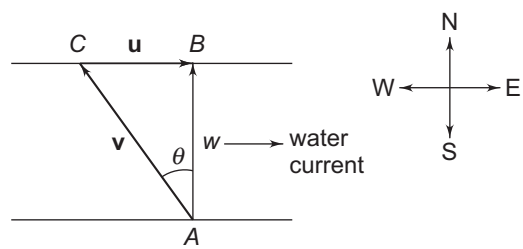


Fig. 2.55

This time is the shortest if $\theta = 0°$. Hence the man should swim in the north direction, which is choice (d).

40. The distance will be the shortest if the resultant of velocities **u** and **v** is along AB. Thus,

$$\sin\theta = \frac{u}{v} = \frac{5\text{m/min}}{10\text{m/min}} = \frac{1}{2}$$

or $\quad \theta = 30°$,

i.e., He should swim 30° west of north, which is choice (c).

41. Let the total distance be S. The time taken to travel the first half, *i.e.,* $\frac{S}{2}$ is

$$t_1 = \frac{S/2}{3} = \frac{S}{6}$$

Let t_2 be the time taken to cover a distance S_1 with speed 4.5 m/s and t_3 that to cover distance S_2 with speed 7.5 m/s.

Then $S_1 = 4.5\ t_2$ and $S_2 = 7.5\ t_3$

Now $S_1 + S_2 = \frac{S}{2}$ and $t_2 = t_3$ (given).

Therefore $\frac{S}{2} = S_1 + S_2$

$= (4.5 + 7.5)t_2$ or $t_2 = \frac{S}{24}$.

$\therefore$ Total time taken $t_1 = t_1 + t_2 = \frac{S}{6} + \frac{S}{24} = \frac{5S}{24}$

$\therefore$ Average speed $= \frac{\text{total distance}}{\text{total time}} = \frac{S}{5S/24}$

$= \frac{24}{5} = 4.8$ m/s,

which is choice (d).

42. In Fig. 2.56 v_c represents the velocity of the car and v_r that of the parcel. M is the position of the man. From parallelogram law, the direction of the resultant velocity v_r must be along the direction along which the man is standing. It follows from Fig. 2.56 that angle θ is given by

$$\sin\theta = \frac{v_c}{v_p} = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}} \text{ or } \theta = 45°$$

Hence the correct choice is (a).

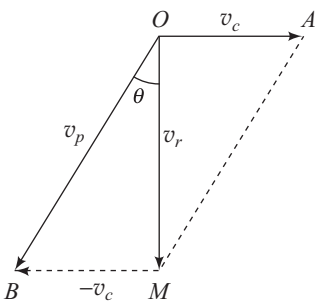


Fig. 2.56

Level B

43. Given $AB\cos\theta = AB\sin\theta$ or $\tan\theta = 1$ which gives $\theta = 45°$. Hence the correct choice is (b).

44. Since $\mathbf{A}\cdot\mathbf{B} = 0$, it follows that **A** is perpendicular to **B**. Also $\mathbf{A}\times\mathbf{C} = 0$. Therefore **A** is perpendicular to **C**. Hence **B** is perpendicular to **C**. Therefore, the correct choice is (b).

45. As shown in Fig. 2.45, the angle θ between vectors **A** and **B** is 90°. Also $A = B$. Therefore, the magnitude of the resultant is given by

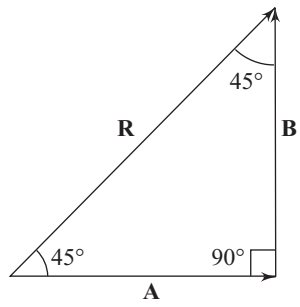


Fig. 2.45

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$
$$= A^2 + A^2 + 2A^2\cos 90°$$
$$= 2A^2$$

or $R = \sqrt{2}\ A$. Hence the correct choice is (b).

46. Vector **C** lies in the plane containing vectors **A** and **B**, and vector **D** is perpendicular to both **A** and **B**. Hence **D** must be perpendicular to **C**. Hence the correct choice is (c).

47. Let θ be the angle between the two vectors. The resultant is given by

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

Putting the values of R, A and B we get

$$(1)^2 = (3)^2 + (4)^2 + 2\times 3\times 4\times\cos\theta$$

or $\cos\theta = -1$ or $\theta = 180°$

Now $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = 3\times 4\times\cos 180° = -12$

Hence the correct choice is (a).

48. The magnitude of $\mathbf{A}\times\mathbf{B} = AB\sin\theta = 3\times 4\times\sin 180° = 0$. Hence the correct choice is (d).

49. Since $\mathbf{A} + \mathbf{B} = \mathbf{C}$, vector **C** is the resultant of vectors **A** and **B**. Using the triangle law of vector addition (see Fig. 2.46), we have $\theta = 45°$ ($\because A = C$)

Thus, the correct choice is (a).

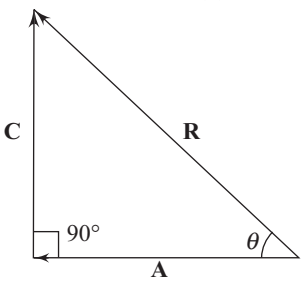


Fig. 2.46

50. The resultant **R** of vectors (**A** + **B**) and (**A** – **B**) is
R = **(A** + **B)** + **(A** – **B)** = 2**A**

$\therefore$ The magnitude of the resultant = 2*A*. Hence the correct choice is (a)

51. Since **R** = 2**A**, **R** is parallel to **A**. Hence the correct choice is (a).

52. $\cos\theta = \dfrac{(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}).\hat{\mathbf{i}}}{(1^2+1^2+1^2)^{1/2}\times 1} = \dfrac{\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}+\hat{\mathbf{j}}\cdot\hat{\mathbf{i}}+\hat{\mathbf{k}}\cdot\hat{\mathbf{i}}}{(1+1+1)^{1/2}\times 1}$

$= \dfrac{1+0+0}{\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$(\because\ \hat{\mathbf{j}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0 \text{ and } \hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = 1)$

Hence the correct choice is (a).

53. Here $(0.2)^2 + (0.6)^2 + a^2 = 1$ or $a^2 = 1 - 0.04 - 0.36$, $= 0.6$ or $a = \sqrt{0.6}$. So the correct choice is (c).

54. The component of vector **A** along vector **B** = $(\mathbf{A}\cdot\mathbf{B})\hat{\mathbf{B}}$ where $\hat{\mathbf{B}} = \dfrac{\mathbf{B}}{B}$ where *B* is the magnitude of vector **B**. Now

$$(\mathbf{A}\cdot\mathbf{B}) = \left(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}\right)\cdot\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$$
$$= 2\,\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} + 2\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} + 3\hat{\mathbf{j}}\cdot\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}$$
$$= 2 + 0 + 0 + 3 = 5$$

Also $\hat{\mathbf{B}} = \dfrac{\mathbf{B}}{B} = \dfrac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{1^2+1^2}} = \dfrac{1}{\sqrt{2}}(\hat{\mathbf{i}}+\hat{\mathbf{j}})$

Thus, the answer is $\dfrac{5}{\sqrt{2}}\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$ which is choice (c).

55. Since $\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)\cdot\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right) = 0$, vector $\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right)$ is perpendicular to vector $\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$. Let $\hat{\mathbf{i}}-\hat{\mathbf{j}}$ = **C**. Now

$(\mathbf{A}\cdot\mathbf{C}) = \left(2\hat{\mathbf{i}}+3\hat{\mathbf{i}}\right)\cdot\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right)$

The required component is

$$(\mathbf{A}\cdot\mathbf{C})\ \frac{\mathbf{C}}{C} = \left(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}\right)\cdot\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right)\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}}{|\hat{\mathbf{i}}-\hat{\mathbf{j}}|}$$
$$= -\frac{1}{\sqrt{2}}\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right) = \frac{1}{\sqrt{2}}\left(\hat{\mathbf{j}}-\hat{\mathbf{i}}\right)$$

$\left(\because\ \text{Magnitude of } \hat{\mathbf{i}}-\hat{\mathbf{j}} = \sqrt{1+1} = \sqrt{2}\right)$

Thus, the correct choice is (a).

56. Let *n* seconds be the time taken by the stone to reach the ground. The distance covered in the last second = distance covered in *n* seconds – distance covered in $(n-1)$ seconds $= \dfrac{1}{2}gn^2 - \dfrac{1}{2}g(n-1)^2$. The distance covered in the first 3 seconds $= \dfrac{1}{2}g\times(3)^2$. It is given that

$$\frac{1}{2}gn^2 - \frac{1}{2}g(n-1)^2 = \frac{1}{2}g\times(3)^2$$

or $$n^2 - (n-1)^2 = 9$$

which gives *n* = 5s. Hence the correct choice is (b).

57. Since $a = \dfrac{dv}{dt}$, we have $v = \displaystyle\int_0^t a\,dt = \int_0^t (3t+4)\,dt$

$$= \frac{3t^2}{2} + 4t.$$

At t = 2s, $v = \dfrac{3}{2}\times(2)^2 + 4\times 2 = 14\ \text{ms}^{-1}$.

Hence the correct choice is (c).

58. For a given initial speed, the maximum height attained and the downward speed at the point of projection are independent of the masses of balls. Hence the correct choice is (d).

59. Given $u = 30\ \text{ms}^{-1}$. From Fig. 2.35, the acceleration a = slope of line $AB = \dfrac{30\ \text{ms}^{-1}}{-3s} = -10\ \text{ms}^{-2}$. The maximum height is reached when final velocity $v = 0$. Using the values of u, v and a in relation $v^2 - u^2 = 2as$, we have

$$0 - 30\times 30 = 2\times -10\times s$$

or $$s = \frac{30\times 30}{20} = 45\ \text{m}$$

Hence the correct choice is (b).

60. The two arrows will be at the same height at $t = n$ if $98 - 4.9n^2 = 98\,(n-5) - 4.9\,(n-5)^2$. This gives n = 12.5 s. The time to reach the highest point is given by $0 = 98 - 9.8\,t$ i.e., t = 10 s. Therefore, time of flight is 2 × 10 s or 20 s. The speed of the first arrow at t = 20 s is 98 ms^{-1} again while that of the second arrow is $(98 - 9.8\times 0.5)\ \text{ms}^{-1}$ *i.e.* 49 ms^{-1}. The maximum heights attained are 490 m each.

Hence the correct choice is (c)

61. The area under the velocity–time graph always gives the distance covered as it is given by the integral $\displaystyle\int_{t_1}^{t_2} v\,dt$ which is true irrespective of whether the body has a uniform velocity or a uniform or non-uniform acceleration. Hence the correct choice is (d).

62. Velocity of the particle is given by

$$v = \frac{dx}{dt} = \frac{d}{dt}\left\{\frac{a}{b}\left(1 - e^{-bt}\right)\right\} = ae^{-bt}$$

Acceleration of the particle is given by

$$\alpha = \frac{dv}{dt} = \frac{d}{dt}\left(ae^{-bt}\right) = -abe^{-bt}$$

At $t = 1/b$, the displacement of the particle is

$$x = \frac{a}{b}\left(1 - e^{-1}\right) \simeq \frac{a}{b}\left(1 - \frac{1}{3}\right)$$

$$\simeq \frac{2}{3}\frac{a}{b} \qquad \left(\because\ e^{-1} \simeq \frac{1}{3}\right)$$

Hence choice (a) is wrong. At $t = 0$, the values v and α respectively are $v = ae^{-0} = a$ and $\alpha = -abe^{-0} = -ab$. Hence choice (b) is also wrong. The displacement x is maximum when $t \to \infty$, *i.e.*, $x_{\max} = \frac{a}{b}\left(1 - e^{-\infty}\right) = \frac{a}{b}$. Hence choice (c) is correct.

63. The acceleration is given by the slope of the speed—time graph. The slope of the graph is maximum during the interval BC. Hence the correct choice is (b).

64. Let us suppose that cars A and B are moving in the positive x–direction. Then car C is moving in the negative x–direction. Therefore, v_A = + 36 km h^{-1} = + 10 ms^{-1}, v_B = + 54 km h^{-1} = + 15 ms^{-1} and v_C = – 54 km h^{-1} = – 15 ms^{-1}. The relative velocity B with respect to A is $v_{BA} = v_B - v_A = 15 - 10 = 5$ ms^{-1}. The relative velocity of C with respect to A is $v_{CA} = v_C - v_A = -15 - 10 = -25$ ms^{-1}. At time $t = 0$, the distance between A and B = distance between A and C = 1 km = 1000 m. The car C will cover a distance AC = 1000 m and just reach car A at a time t given by

$$t = \frac{AC}{|v_{CA}|} = \frac{1000 \text{ m}}{25 \text{ ms}^{-1}} = 40 \text{ s}$$

Car B will overtake car A just before car C does and avoid an accident, if it acquires a minimum acceleration a such that it covers a distance $s = AB$ = 1000 m in time t = 40 s, travelling at a relative speed $u = v_{BA}$ = 5 ms^{-1}. Putting these values in relation

$$s = ut + \frac{1}{2}at^2$$

We get $$1000 = 5 \times 40 + \frac{1}{2} \times a \times (40)^2$$

which gives a = 1 ms^{-2} which in choice (a).

65. The relative speed of train A with respect to train B = 30 + 10 = 40 ms^{-1}. The minimum distance now is given by

$$0 - (40)^2 = 2 \times 4 \times s$$

which gives s = 200 m which is choice (b).

66. Average velocity = $\dfrac{\text{net displacement}}{\text{total time}}$

The displacement of the cyclist in moving from P to Q along the circumference = shortest distance between P and Q = straight line distance PQ (shown by broken lines in the figure) in the direction from P to Q. The net displacement in moving from O to P and then from P to Q = resultant of vector displacement OP and PQ. From the triangle law of vector addition, the magnitude of the net displacement = OQ = 1 km. Now, the time taken = 10 min = $\frac{1}{6}$ h. Therefore, average velocity is 1 km/$\frac{1}{6}$ h = 6 km h^{-1}, which is choice (b).

67. Average speed = $\dfrac{\text{total distance}}{\text{total time}}$.

Total distance = O to P (along radius) + P to Q (along the arc)

$$= r + \frac{r\pi}{3} \qquad \left(\because 60° = \frac{\pi}{3} \text{ rad}\right)$$

$$= 1 \text{ km} + 1 \text{ km} \times \frac{3.14}{3} = 2.05 \text{ km}$$

$$\therefore \text{ Average speed} = \frac{2.05 \text{ km}}{\frac{1}{6}\text{h}} \simeq 12.3 \text{ km h}^{-1}$$ which is choice (a).

68. Velocity of rain (v_r) = 4 ms^{-1} vertically downwards. Velocity of wind (v_w) = 3 ms^{-1} from north to south direction. A rain drop is acted upon by two velocities v_r and v_w as shown in Fig. 2.47. From the triangle law, the resultant velocity of the rain drop is $v = OW$. In order to protect himself from rain, he must hold his umbrella at an angle θ with the vertical (towards north) given by

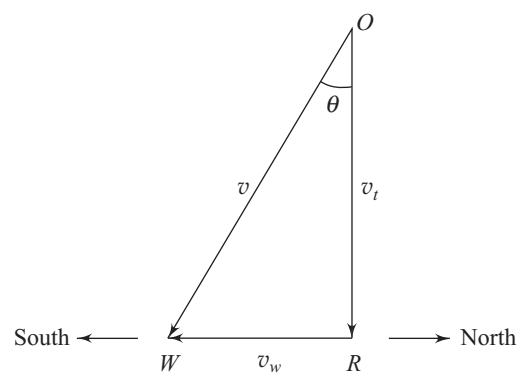


Fig. 2.47

$$\tan\theta = \frac{RW}{OR} = \frac{v_w}{v_r} = \frac{3}{4}$$

Thus, the correct choice is (b).

69. The magnitude v of the resultant velocity gives the speed with which the rain strikes the umbrella, which is given by

$$v = [v^2_r + v^2_w]^{1/2} = [16 + 9]^{1/2} = 5 \text{ ms}^{-1}$$

Hence the correct choice is (c).

70. In order to cross the river in the shortest time, the resultant velocity v of the swimmer must be perpendicular to the velocity v_w of water, as shown in Fig. 2.48. It follows from the figure that $v_s^2 = v^2 + v^2_w$ or $v^2_w = v_s^2 - v^2 = 25 - 9 = 16$ or $v_w = 4 \text{ ms}^{-1}$ which is choice (b).

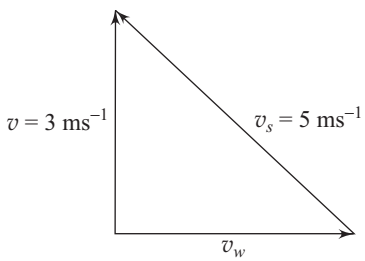


Fig. 2.48

71. Since the initial velocity is zero, the distance travelled in the first time interval t is

$$s_1 = 0 + \frac{1}{2}\, at^2 = \frac{1}{2}\, at^2$$

The velocity of the body at the end of this time interval is $v = 0 + at = at$. This is the initial velocity for the next time interval t during which the body travels a distance.

$$s_2 = ut + \frac{1}{2}\, at^2 = at^2 + \frac{1}{2}\, at^2 = \frac{3}{2}\, at^2 \quad (\because\ u = at)$$

$\therefore\ s_2 = 3\, s_1$. Thus the correct choice is (c).

72. Here $v_1 = 0 + at = at$ and $v_2 = v_1 + at = at + at = 2\, at$. Therefore, $v_2 = 2v_1$. Hence the correct choice is (b).

73. Let h be the height of the tower. Then $h = \frac{1}{2}\, gt^2 = \frac{1}{2}\, g(4)^2 = 8g$.

The time t taken to fall through $\frac{h}{2} = 4g$ is given by

$\frac{h}{2} = \frac{1}{2}\, gt^2$ or $4g = \frac{1}{2}\, gt^2$ or $t^2 = 8$ or $t = 2\sqrt{2}$ s.

Thus, the correct choice is (c).

74. Since the initial velocity of the car is zero, its velocity at the end of the first time interval t_1 is $v = 0 + a_1 t_1 = a_1 t_1$. This is the initial velocity for the next time interval t_2. Since the final velocity is zero, we have, from $v = u + at$,

$$0 = a_1\, t_1 - a_2\, t_2 \qquad (\because\ u = a_1\, t_1)$$

Now, the distance covered in the first time interval t_1 is given by

$$2a_1 s_1 = v^2 - u^2 = a_1^2 t_1^2$$

$(\because\ v = a_1 t_1$ and $u = 0)$

or $$s_1 = \frac{1}{2}\, a_1 t_1^2 \qquad \text{(i)}$$

The distance covered in the next time interval t_2 is given by

$$-2a_2\, s_2 = 0 - a_1^2 t_1^2$$

$(\because\ v = 0$ and $u = a_1 t_1$ now)

or $$s_2 = \frac{1}{2}\, \frac{a_1^2}{a_2}\, t_1^2 = \frac{1}{2}\, \frac{a_2^2\, t_2^2}{a_2}$$

$(\because\ a_1 t_1 = a_2 t_2)$

or $$s_2 = \frac{1}{2}\, a_2 t_2^2 \qquad \text{(ii)}$$

From (i) and (ii) we get

$$\frac{s_1}{s_2} = \frac{a_1\, t_1^2}{a_2\, t_2^2} = \frac{a_2}{a_1}\, \frac{a_1^2\, t_1^2}{a_2^2\, t_2^2} = \frac{a_2}{a_1} \ (\because\ a_1 t_1 = a_2 t_2)$$

Thus we have $\frac{s_2}{s_1} = \frac{a_1}{a_2} = \frac{t_2}{t_1}$ which is choice (d).

75. The maximum speed v attained by the car = speed it attains at the end of time interval t_1 during which it is accelerated. As shown above, this speed is $v = a_1 t_1 = a_2 t_2$.

Now $$s_1 = \frac{1}{2}\, a_1 t_1^2 = \frac{v^2}{2a_1} \quad (\because\ v = a_1 t_1)$$

and $$s_2 = \frac{1}{2}\, a_2 t_2^2 = \frac{v^2}{2a_2} \quad (\because\ v = a_2 t_2)$$

$\therefore$ $$s = s_1 + s_2 = \frac{v^2}{2}\left(\frac{1}{a_1} + \frac{1}{a_2}\right)$$

or $$v = \left[2s.\, \frac{a_1\, a_2}{a_1 + a_2}\right]^{1/2}$$

Hence the correct choice is (a).

76. The distance s_1 covered by the car during the time it is accelerated is given by $2\alpha s_1 = v^2$, which gives $s_1 = v^2/2\alpha$. The distance s_2 covered during the time the

car is decelerated is, similarly given by $s_2 = v^2/2\beta$. Therefore, the total distance covered is

$$s = s_1 + s_2 = \frac{v^2}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) \qquad \text{(i)}$$

If t_1 is the time of acceleration and t_2 that of deceleration, then $v = \alpha t_1 = \beta t_2$ or $t_1 = v/\alpha$ and $t_2 = v/\beta$. Therefore, the total time taken is

$$t = t_1 + t_2 = v\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) \qquad \text{(ii)}$$

From (i) and (ii), the average speed of the car is given by

$$\frac{\text{total distance}}{\text{total time}} = \frac{s}{t} = \frac{v}{2}$$

Hence the correct choice is (d).

77. If the train is at rest or in uniform motion, its acceleration is zero. The only acceleration acting on the bob is the acceleration due to gravity. Hence the string will be vertical. Hence choice (a) and (d) are not possible. The string is inclined along the direction of the resultant of the acceleration **a** of train acting horizontally and the acceleration due to gravity **g** acting vertically. In Fig. 2.49, vector $\mathbf{a}_r$ is the resultant of vectors **a** and **g** (use triangle law). Hence, the string is inclined opposite to the direction of the acceleration of the train. Thus the correct choice is (b).

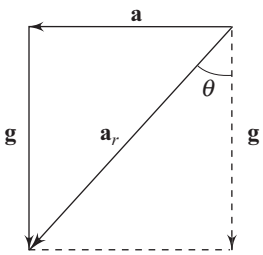


Fig. 2.49

78. Squaring both sides, we have

$$x = 12t + 4t^2 + 9$$

Since displacement x changes with time t, the body cannot be at rest. The velocity of the body is given by

$$v = \frac{dx}{dt} = 12 + 8t$$

Since the velocity v changes with time t, the body is not in uniform motion; it is accelerated because v increases with t. Hence the correct choice is (b).

79. We have seen that $v = 12 + 8t$. Comparing it with $v = u + at$ we find that $u = 12$ ms^{-1}. Hence the correct choice is (d).

80. Now $v = 12 + 8t$. Comparing it with $v = u + at$, we find that $a = 8$ ms^{-2}. Alternatively, acceleration a is given by

$$a = \frac{dv}{dt} = \frac{d}{dt}(12 + 8t) = 8$$

or $a = 8$ ms^{-2}. Hence the correct choice is (d).

81. The distance over which the car can be stopped is given by $2as = v^2$ or $a = v^2/2s$. If v becomes nv, the value a^* of a to stop the car in the same distance is $a^* = (nv)^2/2s = n^2v^2/2s$. Thus $a^* = n^2a$. Hence the correct choice is (c).

82. Let a be the maximum retardation the brakes of the car can produce. Then $2ax = v^2$. If v is doubled to $2v$, then the minimum distance x^* is given by $2ax^* = (2v)^2 = 4v^2$. Thus $x^* = 4x$. Hence the correct choice is (a).

83. The total time taken by the bullet to reach the highest point (where its velocity becomes zero) is given by $0 = u - gt$ or $t = u/g = 50/10 = 5$ s. The distance it covers in the first 1 second of its upward motion is $h_1 = ut - \frac{1}{2}gt^2 = 50 \times 1 - \frac{1}{2} \times 10 \times (1)^2 = 50 - 5 = 45$ m.

Now, the velocity of the bullet at the end of first 2 seconds is $v = u - gt = 50 - 10 \times 2 = 30$ ms^{-1}. This is the initial velocity for the last 3 seconds of upward motion. The distance covered in the last 3 seconds is $h_2 = 30 \times 3 - \frac{1}{2} \times 10 \times (3)^2 = 90 - 45 = 45$ m. Thus $h_1 = h_2$, which is choice (c).

84. From $v = u + gt$, we have $3u = u + gt$ or $2u = gt$ or $t = 2u/g$. Hence the correct choice is (b).

85. From $2gh = v^2 - u^2$, we have $2gh = (3u)^2 - u^2 = 8u^2$ or $h = 4u^2/g$. Hence the correct choice is (d).

86. Differentiating $2x^2 + 3x = t$ with respect to t we have

$$4x\frac{dx}{dt} + \frac{3dx}{dt} = 1 \qquad \text{(i)}$$

Now $\frac{dx}{dt} = v$. Therefore, $4xv + 3v = 1$ or $4x + 3 = 1/v$. Differentiating Eq. (i) with respect to time t, we have

$$4\left(\frac{dx}{dt}\right)^2 + 4x\frac{d^2x}{dt^2} + 3\frac{d^2x}{dt^2} = 0$$

or $$4v^2 + 4xa + 3a = 0$$

or $$a = -\frac{4v^2}{4x + 3} \qquad \text{(ii)}$$

where $a = \frac{d^2x}{dt^2}$ is the acceleration. But $4x + 3 = 1/v$.

Using this in Eq. (ii) we get $a = -4v^3$. Hence the correct choice is (d).

87. Given, $x^2 = t^2 + 2t + 3$. Differentiating, we have

$$2x\frac{dx}{dt} = 2t + 2 \quad \text{or} \quad x\frac{dx}{dt} = t + 1 \quad \text{(i)}$$

Differentiating again, we have

$$\left(\frac{dx}{dt}\right)^2 + x\frac{d^2x}{dt^2} = 1.$$

$$\text{or } x^2\left(\frac{dx}{dt}\right)^2 + x^3a = x^2 \quad \text{(ii)}$$

where $a = \dfrac{d^2x}{dt^2}$ is the acceleration. Using (i) in (ii) we have

$$(t + 1)^2 + x^3a = x^2$$

$$\text{or } t^2 + 2t + 1 + x^3a = x^2 \quad \text{(iii)}$$

But it is given that $x^2 = t^2 + 2t + 3$. Using this in Eq. (iii) we get $a = 2/x^3$. Hence the correct choice is (c).

88. Given $\dfrac{dV(t)}{dt} = 6.0 - 3V(t)$ (i)

Since at $t = 0$, $V(0) = 0$ (given), the acceleration at $t = 0$ is

$$a(0) = \left.\frac{dV(t)}{dt}\right|_{t=0} = 6.0 - 0 = 6.0 \text{ ms}^{-2}$$

Thus, choice (b) is correct. To check whether choice (c) is correct, differentiating $V(t)$, given in choice (c), with respect to t, we get

$$\frac{dV(t)}{dt} = \frac{d}{dt}\left[2\left(1 - e^{-3t}\right)\right]$$

$$= 2\frac{d}{dt}(1 - e^{-3t}) = 6e^{-3t}$$

$$= 6 - 6(1 - e^{-3t}) = 6 - 3V(t)$$

$$(\because V(t) = 2(1 - e^{-3t})]$$

which is Eq. (i). Hence choice (c) is also correct. Now let us check if choice (a) is correct or not. Terminal speed is the speed at time $t \to \infty$. Putting $t \to \infty$ in

$$V(t) = 2(1 - e^{-3t}) \quad \text{(ii)}$$

We have

$$V(\infty) = 2(1 - e^{-\infty}) = 2(1 - 0) = 2 \text{ ms}^{-1}$$

Hence, choice (a) is correct. Finally, let us check choice (d). The acceleration will be half the initial value, *i.e.,* it will be equal to $6.0/2 = 3.0$ ms^{-1} at time t^* given by [use Eq. (i)]

$$3.0 = 6.0 - 3V(t^*)$$

$$\text{or } V(t^*) = 1$$

$$\text{or } 2(1 - e^{-3t^*}) = 1$$

$$\text{or } e^{-3t^*} = 0.5$$

$$\text{or } -3t^* \ln e^{-1} = \ln(0.5)$$

$$\text{or } -3t^* = -0.693$$

$$\text{or } t^* = 0.231 \text{ s}$$

Putting this value of t in Eq. (ii) we have

$$V(\text{at } t = 0.231 \text{ s}) = 2(1 - e^{-3 \times 0.231})$$

$$= 2(1 - e^{-0.693}) = 2(1 - 0.5)$$

$$= 1.0 \text{ ms}^{-1}$$

so choice (d) is wrong.

89. The time of flight $t_f = \dfrac{2u\sin\theta}{g} = \dfrac{2 \times 20 \times 0.5}{10} =$ 2 s. The initial downward velocity = 20 sin 30° = 10 ms^{-1}. The time taken to fall through a height of 40 m with velocity 10 ms^{-1} is given by

$$40 = 10 \times t + \frac{1}{2} \times 10 \times t^2$$

$$\text{or } t^2 + 2t - 8 = 0$$

which gives $t = 2$s. Hence the total time to hit the ground $= t_f + t = 2 + 2 = 4$ s. Thus, the correct choice is (a).

90. The horizontal distance covered in 4s $= 20 \times \cos 30° \times 4 \simeq 70$ m. Hence the correct choice is (c).

91. The time taken to move a horizontal distance R is $t = R/(v_0 \cos \alpha)$. Therefore, the vertical distance moved in this time is given by

$$h = u_0t - \frac{1}{2}gt^2$$

$$= v_0 \sin\alpha \times \frac{R}{v_0\cos\alpha} - \frac{1}{2}g\left(\frac{R}{v_0\cos\alpha}\right)^2$$

$$= R\tan\alpha - \frac{gR^2}{2v_0^2\cos^2\alpha}$$

which is choice (c).

92. The range of a projectile is $r = 2v_0^2\cos\theta\sin\theta/g$, the two possible angles of projection are θ and $(90° - \theta)$. The time of flight corresponding to these two angles are

$$t_1 = \frac{2v_0\sin\theta}{g} \text{ and}$$

$$t_2 = \frac{2v_0\sin(90° - \theta)}{g} = \frac{2v_0\cos\theta}{g}$$

so that $t_1 t_2 = \dfrac{4v_0^2 \sin\theta\cos\theta}{g^2} = \dfrac{2r}{g}$. Thus $t_1 t_2 \propto r$. Hence the correct choice is (b).

93. Let at $t = 0$, O and A are the positions of the gun and the car (Fig. 2.51). Let us say that at time $t = t_0$, the shell and the car reach B simultaneously so that the shell hits the car when it is at a distance OB from the gun. Let u be the speed of projection of the shell. Then, initial horizontal component of velocity of the shell = $u \cos 45° = \dfrac{u}{\sqrt{2}}$, and initial vertical component = $u \sin 45° = u/\sqrt{2}$. Therefore, the time of flight = $\dfrac{2u}{\sqrt{2}\,g} = \sqrt{2}\,u/g$. The car takes this time to cover the distance AB while the shell covers the distance OB in this time. Now $OB = OA + AB = 150 \text{ m} + AB$. Distance AB is given by

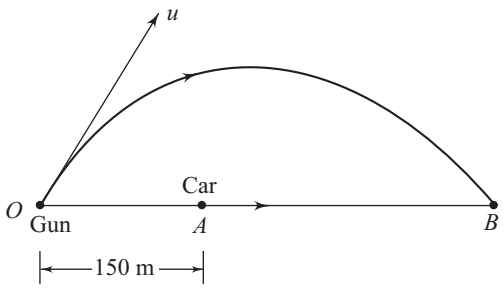


Fig. 2.51

$$AB = 10\sqrt{2} \times \sqrt{2} \times \frac{u}{g} = 20\,u/g$$

and $$OB = \frac{u}{\sqrt{2}} \times \frac{\sqrt{2}\,u}{g} = u^2/g$$

$$\therefore \quad \frac{u^2}{g} = 150 + \frac{20u}{g}$$

or $u^2 - 20u - 1500 = 0$ $(\because g = 10 \text{ ms}^{-2})$

The positive root of this quadratic equation is $u = 50 \text{ ms}^{-1}$. Hence the correct choice is (d).

94. The distance of the car from the gun when the shell hits is

$$OB = \frac{u^2}{g} = \frac{50 \times 50}{10} = 250 \text{ m, which is choice (a).}$$

95. Let u be the initial speed with which the body is thrown along the inclined plane. As shown in Fig. 2.53, the effective deceleration is given by

$$a = g \sin\theta$$
$$= g \sin 30° = \frac{g}{2} = 5 \text{ ms}^{-2}$$

The body stops after covering a distance, say, s along the plane, which is given by $-2as = 0 - u^2$ or $u = \sqrt{2as} = \sqrt{2 \times 5 \times 40} = 20 \text{ ms}^{-1}$. A projectile projected at angle $\theta = 30°$ with this speed will have a range of

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{20 \times 20 \times \sin 60°}{10} = 20\sqrt{3} \text{ m}$$

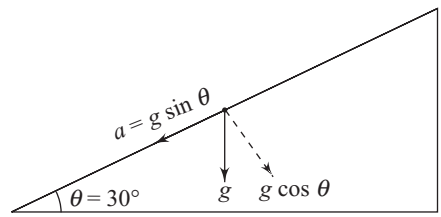


Fig. 2.53

Hence the correct choice is (c).

96. Since the velocity of the projectile changes continuously, both kinetic energy and momentum undergo a change with time. Only the vertical component of velocity changes due to gravity; the horizontal component always remains constant. Hence the correct choice is (d).

97. At the highest point, the velocity has only the horizontal component $v_x = v \cos\theta = v \cos 60° = v/2$. Now kinetic energy $\dfrac{1}{2}\,mv^2$ is proportional to v^2. Since the velocity is reduced to half, the kinetic energy becomes one–fourth, *i.e.,* $K/4$. Hence the correct choice is (d).

98. Kinetic energy is minimum when the projectile is at the highest point of its trajectory. At the highest point, its range = half the horizontal range. Hence the correct choice is (c).

99. Now $R = \dfrac{u^2 \sin 2\theta}{g}$ and $h_{max} = \dfrac{u^2 \sin^2\theta}{2g}$. Given $R = h_{max}$. Therefore $\sin 2\theta = \dfrac{\sin^2\theta}{2}$ or $2 \sin\theta\cos\theta = \dfrac{\sin^2\theta}{2}$ or $\tan\theta = 4$. Hence the correct choices is (d).

100. The horizontal component of velocity is

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(at) = a \qquad \text{(i)}$$

The vertical component of velocity is

$$v_y = \frac{dy}{dt} = \frac{d}{dt}\left(bt^2 + ct\right) = 2\,bt + c \qquad \text{(ii)}$$

The value of v_y at $t = 1$ s is $(2b + c)$. Therefore, the magnitude of velocity at $t = 1$ s is

$$v = (v_x^2 + v_y^2)^{1/2} = [a^2 + (2b + c)^2]^{1/2}$$

Thus, the correct choice is (a).

101. If a projectile is projected with an initial velocity u at an angle θ with the horizontal, the horizontal and vertical components of its velocity at time t are given by

$$v_x = u \cos \theta \quad \text{(iii)}$$

and $$v_y = u \sin \theta - gt \quad \text{(iv)}$$

Comparing (iii) and (iv) with (i) and (ii) above we have $u \cos \theta = a$ and $u \sin \theta = c$. Dividing, we get, $\tan \theta = c/a$. Hence the correct choice is (d).

102. Comparing (iv) with (ii) we have $g = -2b$. Hence the correct choice is (b).

103. We have seen above that $u \cos \theta = a$ and $u \sin\theta = c$. Squaring and adding we get: $u^2 = a^2 + c^2$ or $u = (a^2 + c^2)^{1/2}$. Hence the correct choice is (c).

104. Range $R = u^2 \sin 2\theta/g$. For the same u, $R \propto \sin 2\theta$. Since $\sin 2\theta$ is the largest for $\theta = 35°$, the correct choice is (b).

105. The range is the same for θ and $(90° - \theta)$. Hence $R_1 = R_2$ for $\theta = 30°$ or $60°$. Thus the correct choice is (b).

106. Since $h_{max} = \dfrac{u^2 \sin^2 \theta}{2g}$ and u is the same for both projectiles, we have

$$\frac{h_1}{h_2} = \frac{\sin^2 30°}{\sin^2 60°} = \frac{1}{3} \text{ or } h_1 = \frac{h_2}{3}.$$

Thus the correct choice is (b).

107. We know that $h = \dfrac{u^2 \sin^2 \theta}{2g}$. The increase δh in h when u changes by δu can be obtained by partially differenting this expression. Thus

$$\delta h = \frac{2u \delta u \sin^2 \theta}{2g}$$

$$\therefore \quad \frac{\delta h}{h} = \frac{2\delta u}{u}$$

Since h is increased by 10%, $\dfrac{\delta h}{h} = 0.1$. Therefore,

$$\frac{2\delta u}{u} = 0.1.$$

Now, range $R = \dfrac{u^2 \sin 2\theta}{g}$. Therefore,

$$\delta R = \frac{2u\, \delta u \sin 2\theta}{g}$$

$$\therefore \quad \frac{\delta R}{R} = \frac{2\delta u}{u} = \frac{\delta h}{h} = 0.1$$

Thus $\delta R = 0.1\ R$. Hence R also increases by 10%. Thus, the correct choice is (c).

108. The time of flight is $T = \dfrac{2u \sin \theta}{g}$. Therefore

$$\delta T = \frac{2\delta u \sin \theta}{g}$$

which give $$\frac{\delta T}{T} = \frac{\delta u}{u}$$

But $\dfrac{\delta u}{u} = 0.05$. Therefore $\dfrac{\delta T}{T} = 0.05$

or $\delta T = 0.05\ T$. Hence T increases by 5%. Thus, the correct choice is (d).

109. Given $h = \dfrac{u^2 \sin^2 \theta}{2g}$. Differentiating partially we get

($\because$ v_0 = constant) $\delta h = \dfrac{u^2}{2g}\ 2 \sin \theta \cos \theta\ \delta\theta$. Thus

$$\frac{\delta h}{h} = \frac{2\cos\theta\, \delta\theta}{\sin\theta} = 0.1 \text{ (given).}$$

Therefore, $\dfrac{\cos\theta\, \delta\theta}{\sin\theta} = 0.05$. We also have $T = \dfrac{2u \sin\theta}{g}$ which gives $\delta T = \dfrac{2u \cos\theta\, \delta\theta}{g}$. Thus $\dfrac{\delta T}{T} = \dfrac{\cos\theta\, \delta\theta}{\sin\theta}$. But $\dfrac{\cos\theta\, \delta\theta}{\sin\theta} = 0.05$. Therefore, $\dfrac{\delta T}{T} = 0.05$ or $\delta T = 0.05\ T$. Hence T increases by 5%. Thus the correct choice is (d).

110. We have seen above that $\dfrac{\delta R}{R} = \dfrac{2\delta u}{u} = 2 \times 0.05 = 0.1$ $\left(\because \dfrac{\delta u}{u} = 0.05\right)$ or $\delta R = 0.1\ R$. Hence, R will increase by 10%. Thus, the correct choice is (d).

111. We have seen that $\dfrac{\delta h}{h} = \dfrac{2\delta u}{u} = 2 \times 0.05 = 0.1$ or $\delta h = 0.1\ h$. Hence h increases by 10%. Thus, the correct choice is (d).

112. Now $\frac{\delta T}{T} = \frac{\delta u}{u}$ = 0.05 or δT = 0.05 T. Thus, T increases by 5%. Hence, the correct choice is (b).

113. Let h_e and h_m be the maximum heights attained by the projectile when projected from earth and moon respectively. Now

$$h_e = \frac{u^2 \sin^2\theta}{2g_e} \text{ and } h_m = \frac{u^2 \sin^2\theta}{2g_m} \text{ which give}$$

$\frac{h_m}{h_e} = \frac{g_e}{g_m}$ = 6. Hence the correct choice is (d).

114. Now $R = \frac{2u^2 \sin\theta\cos\theta}{g^2}$ and $T^2 = \frac{4u^2 \sin^2\theta}{g^2}$. From these two equations we have $T^2 = 2R \tan\theta$ or $T \propto \sqrt{R}$. Hence the correct choice is (b).

115. The range of a projectile is the same for two angles of projection θ and 90° – θ. For these two angles of projection, the maximum heights are

$$h_1 = \frac{u^2 \sin^2\theta}{2g} \text{ and}$$

$$h_2 = \frac{u^2 \sin^2(90° - \theta)}{2g} = \frac{u^2 \cos^2\theta}{2g}$$

$$\therefore \quad h_1 h_2 = \frac{u^4 \sin^2\theta\cos^2\theta}{4g^2} .$$

$$\text{Also } R^2 = \frac{4u^4 \sin^2\theta\cos^2\theta}{g^2}.$$

Which give $R^2 = 16\, h_1 h_2$ or $R = 4\sqrt{h_1 h_2}$. Hence the correct choice is (d).

116. Given $x = 2t$ and $y = 5t^2$. Eliminating t we get $y = \frac{5x^2}{4}$, i.e. $y \propto x^2$. Hence, the trajectory of the body is parabolic. Thus, the correct choice is (d).

117. Let u be the velocity with which the body is projected and g be the acceleration due to gravity. Then, we have

$$x = u\,t \text{ and } y = -\frac{1}{2}\, gt^2$$

Thus, the magnitude of the velocity of projection is the coefficient of t in the expression for x. Now, given that $x = 2t$. Hence u = 2 ms^{-1} which is choice (a).

118. The x and y components of velocity are

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(2t) = 2 \text{ ms}^{-1}$$

and

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(5t^2) = 10t$$

At t = 0.2 s, $v_y = 10 \times 0.2 = 2$ ms^{-1}. The magnitude of velocity at t = 0.2 s is

$$v = (v_x^2 + v_y^2)^{1/2} = \sqrt{4+4} = 2\sqrt{2} \text{ ms}^{-1}.$$

Hence the correct choice is (b).

119. The angle of the velocity vector with the vertical at time t is given by

$$\tan\alpha = \frac{u}{|g|t}$$

We have seen that u = 2 ms^{-1}. Comparing $y = -\frac{1}{2}\, gt^2$ with $y = 5t^2$, we get, $-\frac{1}{2}\, g = 5$ or $g = -10$ ms^{-2}. Therefore, at t = 0.2 s,

$$\tan\alpha = \frac{2}{|-10| \times 0.2} = 1$$

which gives $\alpha = 45°$. Hence the correct choice is (c).

120. We know that the position coordinates x and y are given by

$$x = (u\cos\theta)t \qquad \text{(i)}$$

and

$$y = (u\sin\theta)\, t - \frac{1}{2}\, gt^2 \qquad \text{(ii)}$$

Comparing Eq. (i) with $x = 10\sqrt{3}\ t$, we have, $u\cos\theta = 10\sqrt{3}$ ms^{-1}. Also, comparing Eq. (ii) with $y = 10t - t^2$, we have, $u\sin\theta$ = 10 ms^{-1}. These equations give $u^2 = 10^2 + 10^2 \times 3 = 400$ or u = 20 ms^{-1} and $\tan\theta = \frac{1}{\sqrt{3}}$ which gives $\theta = 30°$. Hence the correct choice is (a).

121. Comparing $y = 10t - t^2$ with Eq. (ii) above, we get, – 1 $= -\frac{g}{2}$ or g = 2 ms^{-2}. Hence the correct choice is (b).

122. We have seen above that u = 20 ms^{-1}, $\theta = 30°$ and g = 2 ms^{-2}. Now, the maximum height attained is

$$h = \frac{u^2 \sin^2\theta}{2g} = \frac{(20)^2 \times \sin^2 30°}{2 \times 2} = 25 \text{ m}$$

Hence the correct choice is (a).

123. The time of flight is

$$T = \frac{2u\sin\theta}{g} = \frac{2\times 20\times \sin 30^\circ}{2} = 10 \text{ s}$$

Thus, the correct choice is (c).

124. The horizontal range is

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(20)^2 \times \sin 60^\circ}{2} = 100\sqrt{3} \text{ m}$$

Hence, the correct choice is (d).

125. Since $R \propto u^2$, choice (b) is incorrect.

126. When the train is at rest or moving with a uniform velocity, the plumb line hangs vertically along *OB* (Fig. 2.54). If the train moves with an acceleration *a*, the plumb line gets inclined along *OC*, the direction of the resultant of accelerations *a* and *g*. It is clear from the figure that $\tan\theta = a/g$. Hence the correct choice is (a).

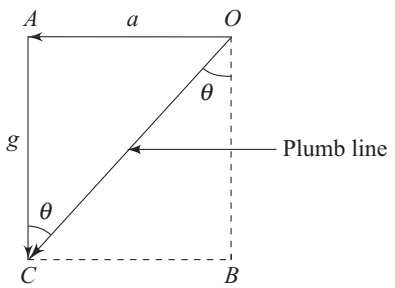


Fig. 2.54

127. Velocity is $v = \frac{dx}{dt} = a + 2bt - 3ct^2$ (i)

Acceleration is $a = \frac{d^2x}{dt^2} = 2b - 6ct$

Acceleration is zero at time *t* given by $0 = 2b - 6ct$ or $t = \frac{b}{3c}$. Putting this value of *t* in Eq. (i).

We have $$v = a + 2b\cdot\frac{b}{3c} - 3c\times\frac{b^2}{9c^2}$$

$$= a + \frac{b^2}{3c}$$

Thus, the correct choice is (c).

128. We know that $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$ (i)

$$\mathbf{A}\cdot\mathbf{B} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}})\cdot(6\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}})$$

$$= 3\times 6 + 4\times 8 + 5\times 10$$

$$= 100 \quad \text{(ii)}$$

The magnitudes of **A** and **B** are

$$A = \sqrt{(3)^2 + (4)^2 + (5)^2} = \sqrt{50} \quad \text{(iii)}$$

and $$B = \sqrt{(6)^2 + (8)^2 + (10)^2} = \sqrt{200} \quad \text{(iv)}$$

Using (ii), (iii) and (iv) in (i), we have

$$100 = \sqrt{50} \times \sqrt{200}\cos\theta = 100\cos\theta$$

or $\cos\theta = 1$ or θ = zero, which is choice (a).

129. The vector product of two non-zero vectors is zero if they are in the same direction. Hence, vector **B** must be parallel to vector **A**, i.e. along $\pm z$-axis. Thus the correct choice is (d).

130. Given $\mathbf{A} + (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) + (\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 1\hat{\mathbf{j}}$

or $\mathbf{A} = -3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 6\hat{\mathbf{k}}$, which is choice (a).

131. The angle α which the resultant **R** makes with **A** is given by

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

given $\alpha = \frac{\theta}{2}$. Hence

$$\tan\left(\frac{\theta}{2}\right) = \frac{B\sin\theta}{A + B\cos\theta}$$

or $$\frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = \frac{2B\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{A + B\cos\theta}$$

which gives $A + B\cos\theta = 2B\cos^2\left(\frac{\theta}{2}\right)$

or $A + B\left[2\cos^2\left(\frac{\theta}{2}\right) - 1\right] = 2B\cos^2\left(\frac{\theta}{2}\right)$

which gives $A = B$, which is choice (c).

132. $$-2g(AB) = \left(\frac{u}{3}\right)^2 - \left(\frac{u}{2}\right)^2$$

$$= \frac{u^2}{9} - \frac{u^2}{4} = -\frac{5u^2}{36} \quad \text{(i)}$$

and $$-2g(BC) = \left(\frac{u}{4}\right)^2 - \left(\frac{u}{3}\right)^2$$

$$= \frac{u^2}{16} - \frac{u^2}{9} = -\frac{7u^2}{144} \quad \text{(ii)}$$

Dividing (i) by (ii), we get $\frac{AB}{BC} = \frac{20}{7}$ which is choice (a).

133. We know that $h = ut - \frac{1}{2} gt^2$

or $t^2 - \frac{2u}{g} t + \frac{2h}{g} = 0$

The roots of this quadratic equation are t_1 and t_2. The sum of the roots is $t_1 + t_2 = \frac{2u}{g}$. Hence the correct choice is (a).

134. Product of roots is $t_1 t_2 = \frac{2h}{g}$ which is choice (a).

135. The time t_1 taken by the body to strike the inclined plane is given by

$$t_1 = \sqrt{\frac{2(H-h)}{g}}$$

The time t_2 taken by the body to reach the ground after striking the plane is

$$t_2 = \sqrt{\frac{2h}{g}}$$

Total time $t = t_1 + t_2 = \sqrt{\frac{2(H-h)}{g}} + \sqrt{\frac{2h}{g}}$

Time t will be maximum if $\frac{dt}{dh} = 0$, *i.e.,* if

$$\frac{d}{dh}\left[\sqrt{\frac{2(H-h)}{g}} + \sqrt{\frac{2h}{g}}\right] = 0$$

or $$\sqrt{\frac{2}{g}}\left[-\frac{1}{2}(H-h)^{-1/2} + \frac{1}{2}h^{-1/2}\right] = 0$$

or $\frac{1}{\sqrt{H-h}} = \frac{1}{\sqrt{h}}$ or $H - h = h$ or $h = \frac{H}{2}$.

Hence the correct choice is (c).

136. Let n be the number of storeys above the 15th storey. Then height fallen is $h = 4n$ metres.

Using $v^2 - u^2 = 2gh$, we have

$$(20)^2 - 0 = 2 \times 10 \times 4n$$

which gives $n = \frac{400}{80} = 5$. Therefore, the total number of storeys = 15 + 5 = 20, which is choice (a).

137. $t_A = \frac{2u_A}{g}$ and $t_B = \frac{2u_B \sin\theta}{g} = \frac{2u_B \sin 60°}{g}$

$= \frac{\sqrt{3}u_B}{g}$

Given $t_A = t_B$. Therefore

$$\frac{2u_A}{g} = \frac{\sqrt{3}u_B}{g} \quad \text{or} \quad \frac{u_A}{u_B} = \frac{\sqrt{3}}{2}$$

Hence the correct choice is (c).

138. $\mathbf{v} = (3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})$ ms^{-1}. The magnitude of velocity is

$v = \sqrt{(3)^2 + (4)^2} = 5$ ms^{-1}. The angle subtended by the velocity vector with the horizontal (x-axis) is given by

$\tan\theta = \frac{4}{3}$ which gives $\sin\theta = \frac{4}{5}$.

Now, $h_{\max} = \frac{v^2 \sin^2\theta}{2g} = \frac{(5)^2 \times \left(\frac{4}{5}\right)^2}{2 \times 10} = 0.8$ m

Hence the correct choice is (a).

139. $T_f = \frac{2v\sin\theta}{g} = \frac{2 \times 5 \times 4}{5 \times 10} = 0.8$ s. Hence the correct choice is (a).

140. Velocity of projection is $\mathbf{v}_0 = (u\cos\theta)\hat{\mathbf{i}} + (u\sin\theta)\hat{\mathbf{j}}$. At time t, the velocity of the body is

$$\mathbf{v} = (u\cos\theta)\,\hat{\mathbf{i}} + (u\sin\theta - gt)\,\hat{\mathbf{j}}$$

The dot product of $\mathbf{v}_0$ and $\mathbf{v}$ is

$$\mathbf{v}_0 \cdot \mathbf{v} = u^2\cos^2\theta + u\sin\theta(u\sin\theta - gt)$$

or $$\mathbf{v}_0 \cdot \mathbf{v} = u^2 - (u\sin\theta)\,gt \qquad \text{(i)}$$

Since v is perpendicular to $\mathbf{v}_0$, $\mathbf{v}_0 \cdot \mathbf{v} = 0$. Using this in (i), we have

$$0 = u^2 - (u\sin\theta)\,gt \quad \text{or} \quad t = \frac{u}{g\sin\theta}$$

Hence the correct choice is (d).

141. $\text{PE} = mg\,h_{\max} = mg\left(\frac{u^2\sin^2\theta}{2g}\right) = \frac{mu^2\sin^2\theta}{2}$

$\text{KE} = \frac{1}{2} m(u\cos\theta)^2$. Hence

$$\frac{\text{PE}}{\text{KE}} = \tan^2\theta$$

Thus, the correct choice is (b).

142. Given $y = ax - bx^2$. Comparing this equation with

$$y = (\tan\theta)x - \frac{gx^2}{2u^2\cos^2\theta}$$

We have $a = \tan\theta$ and $b = \dfrac{g}{2u^2\cos^2\theta}$

Horizontal range $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{2u^2\sin\theta\cos\theta}{g}$

$$= \frac{2u^2\cos^2\theta}{g}(\tan\theta) = \frac{a}{b}$$

Hence the correct choice is (a).

143. $h_{max} = \dfrac{u^2\sin^2\theta}{2g} = \dfrac{u^2\cos^2\theta}{2g}(\tan^2\theta)$

$$= \frac{2u^2\cos^2\theta}{4g}(\tan^2\theta) = \frac{a^2}{4b}$$

Hence the correct choice is (d).

144. $t_f = \dfrac{2u\sin\theta}{g} = \dfrac{2u\cos\theta}{g}(\tan\theta)$ (i)

Now $b = \dfrac{g}{2u^2\cos^2\theta}$ or $u\cos\theta = \sqrt{\dfrac{g}{2b}}$ (ii)

Using (ii) in (i), we get

$t_f = a\sqrt{\dfrac{2}{bg}}$, which is choice (a).

145. $\mathbf{v} = \boldsymbol{u} + \boldsymbol{at} = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) + (0.4\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}}) \times 10$

$= (8\hat{\mathbf{i}} + 6\hat{\mathbf{j}})$ ms^{-1}

The x and y components of v and $v_x = 8$ ms^{-1} and $v_y = 6$ ms^{-1}. The magnitude of v is

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(8)^2 + (6)^2} = 10$ ms^{-1}

Hence the correct choice is (d).

146. Given: $x = 36t$ (i)

and $y = 48t - 4.9t^2$ (ii)

For a body projected with a velocity u at an angle θ with the horizontal, the x and y displacements at time t are given by

$x = (u\cos\theta)t$ (iii)

and $y = (u\sin\theta)t - \dfrac{1}{2}gt^2$ (iv)

Comparing (i) with (iii) and (ii) with (iv), we have

$u\cos\theta = 36$

and $u\sin\theta = 48$

Squaring and adding, we have

$u^2(\cos^2\theta + \sin^2\theta) = (36)^2 + (48)^2$

or $u^2 = 3600$ or $u = 60$ ms^{-1}, which is choice (d).

147. Given $t = \sqrt{x} + 3$. Squaring, we have

$x = t^2 - 6t + 9$ (i)

velocity $v = \dfrac{dx}{dt} = \dfrac{d}{dt}(t^2 - 6t + 9) = 2t - 6$ (ii)

Find t from Eq. (ii) when $v = 0$. Use this value of t is Eq. (i). The correct choice is (a).

148. Given $a = -k\sqrt{v}$ or $\dfrac{dv}{dt} = -k\sqrt{v}$. Thus $v^{-1/2}\,dv = -k\,dt$

Integrating, we get $2v^{1/2} = -kt + c$. Using the given initial condition ($v = u$ at $t = 0$), we get $c = 2\sqrt{u}$. Thus, we have

$2(v^{1/2} - u^{1/2}) = -kt$

Now, use $t = T$ and $v = 0$. The correct choice is (a).

149. The slope of the line is $m = -\dfrac{5}{6}$ ms^{-2} per second and its intercept is $c = 5$ ms^{-2}. Using $y = mx + c$, the acceleration a (in ms^{-2}) as a function of time t is given by

$a = -\dfrac{5}{6}t + 5$

or $\dfrac{dv}{dt} = -\dfrac{5}{6}t + 5$

or $v = \displaystyle\int_0^t \left(\frac{5}{6}t + 5\right)dt$

or $v = -\dfrac{5}{12}t^2 + 5t + k$ (i)

where k is the constant of integration. Since the particle starts from rest, $v = 0$ at $t = 0$. Using this in (i) we ger $k = 0$. Hence

$v = -\dfrac{5}{6}t^2 + 5t$ (ii)

It follows from the graph that the deceleration becomes zero at $t = 6$ s. Hence, the speed of the particle will be maximum at $t = 6$ s. Putting $t = 6$ s in Eq. (ii), we have

$v_{max} = -\dfrac{5}{12} \times (6)^2 + 5 \times 6$

$= -15 + 30 = 15$ ms^{-1}

Hence the correct choice is (b).

150. The slope of the given v versus x graph is $m = -\dfrac{v_0}{x_0}$ and intercept is $c = +v_0$. Hence v varies with x as

$$v = -\left(\frac{v_0}{x_0}\right) x + v_0 \qquad \text{(i)}$$

where v_0 and x_0 are constants of motion. Differentiating with respest to time t, we have

$$\frac{dv}{dt} = -\left(\frac{v_0}{x_0}\right) \frac{dx}{dt}$$

or $$a = -\left(\frac{v_0}{x_0}\right) v \qquad \text{(ii)}$$

Using Eq. (i) in Eq. (ii), we get

$$a = -\left(\frac{v_0}{x_0}\right)\left(-\frac{v_0}{x_0}x + v_0\right)$$

or $$a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

Thus, the graph of a versus x is a straight line having a positive slope $= \left(\dfrac{v_0}{x_0}\right)^2$ and negative intercept $= -\dfrac{v_0^2}{x_0}$. Hence the correct choice is (d).

151. Given $v = k\sqrt{x} \Rightarrow v^2 = k^2 x$. Differentiating, we have

$$2v\frac{dv}{dt} = k^2\frac{dx}{dt} = k^2 v \qquad \left(\because v = \frac{dx}{dt}\right)$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{k^2}{2}$$

$$\Rightarrow \quad \int dv = \frac{k^2}{2}\int dt$$

$$\Rightarrow \quad v = \frac{k^2 t}{2}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{k^2 t}{2}$$

$$\Rightarrow \quad \int dx = \frac{k^2}{2}\int t\,dt$$

$$\Rightarrow \quad x = \frac{k^2}{4}t^2$$

Thus $x \propto t^2$. Hence the correct choice is (c)

152. The slope of the $(v - t)$ graph gives the acceleration. The slope a point on a curve is the slope of the tangent at that point. At point A, the slope is nearly zero. At point B, the slop is small and positive. At point D, the slope is again nearly zero. The slope at point C is the largest. Hence the magnitude of acceleration is the greatest at point C. So the correct choice is (c).

153. The slope of the $(x - t)$ graph gives the velocity. So the velocity of the object is nearly zero at point A. It slowly increases to a small positive value at B and becomes large at point C, becoming nearly zero at point D. Since the time interval between A and C is higher than between C and D, the rate of change of velocity with time (which is acceleration) is the maximum at point D. So the correct choice is (d).

154. Average acceleration

$$a = \frac{\text{change in velocity}}{\text{time intervel}}$$

$$= \frac{\boldsymbol{v}_2 - \boldsymbol{v}_1}{\Delta t}$$

$$= \frac{\boldsymbol{v}_2 + (-\boldsymbol{v}_1)}{\Delta t}$$

$$= \frac{\Delta \boldsymbol{v}}{\Delta t}$$

where $\Delta\boldsymbol{v} = \boldsymbol{v}_2 + (-\boldsymbol{v}_1)$ is the resultant of vectors $\boldsymbol{v}_2$ and $-\boldsymbol{v}_1$.

From parallelogram low, the resultant $\Delta\boldsymbol{v}$ of $\boldsymbol{v}_2$ and $-\boldsymbol{v}_1$ is shown in Fig. 2.57.

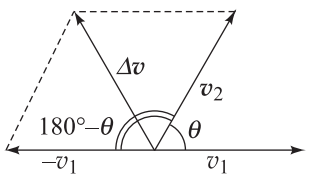


Fig. 2.57

The magnitude of $\Delta\boldsymbol{v}$ is given by

$$\Delta v = \sqrt{v_1^2 + v_2^2 + 2v_1 v_2 \cos(180° - \theta)}$$

$$= \sqrt{v^2 + v^2 - 2v^2\cos\theta}$$

$$= \sqrt{2v^2(1 - \cos\theta)}$$

$$= \sqrt{4v^2 \sin^2\left(\frac{\theta}{2}\right)}$$

$$= 2v\sin\left(\frac{\theta}{2}\right)$$

$$\therefore \qquad a = \frac{\Delta v}{\Delta t} = \frac{2v\sin\left(\dfrac{\theta}{2}\right)}{\Delta t}$$

So the correct choice is (c).

155. $h = \frac{1}{2} gt^2$ (1)

$h - 25 = \frac{1}{2} g (t-1)^2$ (2)

Subtracting (2) from (1), we get

$$25 = \frac{1}{2} g (2t-1)$$

$$= \frac{1}{2} \times 10 \times (2t-1)$$

which gives $t = 3$s. Using $t = 3$s in (1), we get $h = 45$ m.

156. The x and y components of the velocity are

$$v_x = \frac{dx}{dt} = a + 2bt$$

and $$v_y = \frac{dy}{dt} = c$$

The x and y components of the acceleration are

$$a_x = \frac{dv_x}{dt} = 2b$$

$$a_y = \frac{dv_y}{dt} = 0$$

$$\therefore \quad a = \sqrt{a_x^2 + a_y^2} = \sqrt{4b^2 + 0} = 2b$$

So the correct choice is (b).

157. $L_1 = 75$ m, $v_1 = 54$ kmh^{-1} = 15 ms^{-1}, $L_2 = 125$ m and $v_2 = 36$ kmh^{-1} = 10 ms^{-1}.

Relative velocity of overtaking = $v_1 - v_2 = 15 - 10 = 5$ ms^{-1}.

The relative distance covered = $L_1 + L_2 = 75 + 125 = 200$ m

$$\therefore \quad \text{Time taken} = \frac{200}{5} = 40 \text{ s}$$

158. Let u be the initial velocity with which he throws the ball. This ball reaches the highest point in time $t = 1$s. At this point $v = 0$ and we have

$$0 = u - gt$$

$$\Rightarrow \quad u = gt = 9.8 \times 1 = 9.8 \text{ ms}^{-1}$$

If h is the maximum height attained, then

$$h = ut - \frac{1}{2} gt^2$$

$$\Rightarrow \quad h = u \times 1 - \frac{1}{2} \times (9.8) \times (1)^2$$

$$= 9.8 - 4.9 = 4.9 \text{ m}$$

159. It he throws a ball at time $t = 0$, it will return to his hand at time t given by (since the net displacement is zero)

$$0 = ut - \frac{1}{2} gt^2$$

$$= t\left(u - \frac{gt}{2}\right)$$

which gives $t = \frac{2u}{g} = \frac{2 \times 10}{10} = 2$s. So he throws the balls at an interval of $\frac{2\text{s}}{4} = 0.5$s.

Let us say that the 4th ball is just about to leave his hand at time $t = 0$. Then 3rd ball was thrown 0.5 s earlier *i.e.*, at $t_3 = 0.5$s. In this time this ball rose to a height

$$h_3 = ut_3 - \frac{1}{2} g (t_3)^2$$

$$= 10 \times 0.5 - \frac{1}{2} \times 10 \times (0.5)^2 = 3.75 \text{ m}$$

The second ball was thrown $t_2 = 2 \times 0.5 = 1.0$ s earlier. In this time, it attains a height

$$h_2 = ut_2 - \frac{1}{2} g (t_2)^2$$

$$= 10 \times 1.0 - \frac{1}{2} \times 10 \times (1.0)^2 = 5 \text{ m}$$

Similarly for the first ball $t_1 = 3 \times 0.5 = 1.5$s and

$$h_1 = ut_1 - \frac{1}{2} g (t_1)^2$$

$$= 10 \times 1.5 - \frac{1}{2} \times 10 \times (1.5)^2 = 3.75 \text{ m}$$

So the correct choice is (a).

160. The time t taken by the first drop to strike the ground is given by

$$h = \frac{1}{2} gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1.25}{10}} = 0.5 \text{ s}$$

At this time, the sixth drop just emerges from the tap. Hence the time interval between consecutive drops = $\frac{0.5}{5} = 0.1$ s. Therefore, the time for which the third drop has fallen is $t_3 = 0.1 + 0.1 + 0.1 = 0.3$ s. The height through which the third drop falls in 0.3 s is

$$h_3 = \frac{1}{2} g (t_3)^2 = \frac{1}{2} \times 10 \times (0.3)^2$$

$$= 0.45 \text{ m} = 45 \text{ cm}$$

The height of this drop above the ground when the sixth drop just begins to fall = 125 – 45 = 80 cm.

161. $x^2 = a + bt^2$. Differentiating w.r.t time t,

$$2x \frac{dx}{dt} = 2bt$$

$$\Rightarrow \qquad \frac{dx}{dt} = \frac{bt}{x}$$

$$\Rightarrow \qquad v = \frac{bt}{x}$$

$$\text{Acceleration} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{bt}{x}\right)$$

$$= \frac{b}{x} - \frac{bt}{x^2}\frac{dx}{dt}$$

$$= \frac{b}{x} - \frac{bt}{x^2}\frac{bt}{x}$$

$$= \frac{b}{x}\left(1 - \frac{bt^2}{x^2}\right)$$

So the correct choice is (c).

162. $\text{Average velocity} = \dfrac{\text{Total displacement}}{\text{time interval}}$

$$= \frac{\int_0^2 v\,dt}{\int_0^2 dt}$$

$$= \frac{\int_0^2 (at - bt^2)\,dt}{(2-0)}$$

$$= \frac{1}{2}\left[\frac{a}{2}(2^2 - 0^2) - \frac{b}{3}(2^3 - 0^3)\right]$$

$$= \frac{1}{2}\left(2a - \frac{8}{3}b\right)$$

$$= \frac{1}{3}(3a - 4b)$$

163. Let L be the length of the escalator. The speed of escalator is

$$v_E = \frac{L}{t_1}$$

The speed of the person relative to the escalator is

$$v_{PE} = \frac{L}{t_2}$$

Now $v_{PE} = v_P - v_E$, where v_P is the speed of the person relative to the ground.

$$v_P = v_E + v_{PE}$$

$$= \frac{L}{t_1} + \frac{L}{t_2} = L\left(\frac{1}{t_1} + \frac{1}{t_2}\right)$$

$$= L\left(\frac{t_1 + t_2}{t_1 t_2}\right)$$

$\therefore$ Required time is $t = \dfrac{L}{v_P} = \dfrac{t_1 t_2}{t_1 + t_2}$. So the correct choice is (c).

164. The vertical downward displacement of the stone is $h = 50 - 30.4 = 19.6$ m. Therefore, the time taken by the ball must be

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 19.6}{9.8}} = 2\,\text{s}$$

If u is the required horizontal speed, then horizontal range is $R = 20$ m (the separation between the two buildings). Hence

$$R = ut \Rightarrow u = \frac{R}{t} = \frac{20\,\text{m}}{2\,\text{s}} = 10\ \text{ms}^{-1}$$

165. The distance between two consecutive cars leaving station is (15 min = $\frac{1}{4}h$)

$$x = 80\ \text{km h}^{-1} \times \frac{1}{4}\,\text{h} = 20\ \text{km}$$

The man meets the first car after a time

$$t_1 = \frac{80}{80 + 80} = \frac{1}{2}\,\text{h}$$

He will meet the next after a time

$$t_2 = \frac{20}{80 + 80} = \frac{1}{8}\,\text{h}$$

In the remaining $\frac{1}{2}$h, the number of cars he will meet is

$$n = \frac{\frac{1}{2}}{\frac{1}{8}} = 4$$

Therefore, the total number of cars he will meet before reaching station A = 4 + 1 = 5.

166. Refer to Fig. 2.58. A is the highest point on the trajectory.

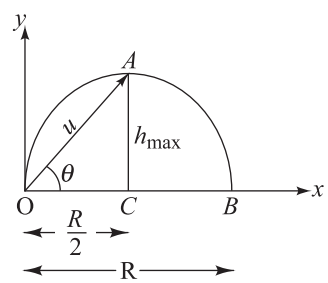


Fig. 2.58

$$\text{Average velocity} = \frac{\text{displacement } OA}{\text{time taken}}$$

$$= \frac{\sqrt{h_{\max}^2 + \frac{R^2}{4}}}{\frac{1}{2}t_f} \quad (1)$$

where $h_{\max} = \frac{u^2 \sin^2\theta}{2g}$,

$$R = \frac{u^2 \sin^2 2\theta}{g},$$

and $t_f = \frac{2u \sin\theta}{g}$

Using these in Eq. (1) and simplifying, we get

$$\text{Average velocity} = \frac{u}{2}(1 + 3\cos^2\theta)^{1/2}$$

So the correct choice is (d).

167. $h_{\max} = \frac{u^2 \sin^2\theta}{2g} \Rightarrow g\,h_{\max} = \frac{u^2 \sin^2\theta}{2}$.

At the highest point, the speed is

$$v_x = u\cos\theta$$

$$\Rightarrow \quad v_x^2 = u^2\cos^2\theta. \quad (1)$$

The speed of the body at half the maximum height is

$$v^2 = u^2 - 2g \times \frac{h_{\max}}{2} = u^2 - g\,h_{\max} \quad (2)$$

It is given that $v_x = v \times \sqrt{\frac{2}{3}}$. Substituting (1) and (2) in this equation, we get

$$u^2\cos^2\theta = \frac{2}{3}\left(u^2 - \frac{u^2\sin^2\theta}{2}\right)$$

$$\Rightarrow \quad 3\cos^2\theta = 2 - \sin^2\theta$$

$$\Rightarrow \quad 3(1 - \sin^2\theta) = 2 - \sin^2\theta$$

$$\Rightarrow \quad \sin^2\theta = \frac{1}{2}$$

$$\Rightarrow \quad \sin\theta = \frac{1}{\sqrt{2}}$$

which gives $\theta = 45°$. So the correct choice is (b).

SECTION 2

Multiple Choice Questions Based on Passage

Questions 1 and 2 are based on the following passage.

Passage I

A particle initially (i.e. at time $t = 0$) moving with a velocity u is subjected to a retarding force, as a result of which it decelerates at a rate

$$a = -k\sqrt{v}$$

where v is the instantaneous velocity and k is a positive constant.

1. The particle comes to rest in a time

   (a) $\frac{2\sqrt{u}}{k}$ (b) $\frac{\sqrt{u}}{k}$

   (c) $2k\sqrt{u}$ (d) $k\sqrt{u}$

2. The distance covered by the particle before coming to rest is

   (a) $\frac{u^{3/2}}{k}$ (b) $\frac{2u^{3/2}}{k}$

   (c) $\frac{3u^{3/2}}{2k}$ (d) $\frac{2u^{3/2}}{3k}$

Solutions

1. Given $a = -kv^{1/2}$ or $\frac{dv}{dt} = -kv^{1/2}$

   Thus $v^{-1/2}dv = -k\,dt$

   Integrating, we have

   $$\int_u^v v^{-1/2}dv = -k\int_0^t dt$$

   or $2(v^{1/2} - u^{1/2}) = -kt + c$ (i)

   where c is the constant of integration. Given that at $t = 0$, $v = u$. Using this in (i) we get $c = 0$.

   Using this value of c in (i),

   we have $2(v^{1/2} - u^{1/2}) = -kt$ (ii)

   Let τ be the time taken by the particle to come to rest. Then, $v = 0$ at $t = \tau$. Using this in (ii), we get

   $2(0 - u^{1/2}) = -k\tau$ or $\tau = \frac{2u^{1/2}}{k}$ (iii)

   Hence the correct choice is (a).

2. To find the distance s covered in this time, we use Eq. (i) to get

$$v^{1/2} = u^{1/2} - \frac{kt}{2}$$

Squaring, we have

$$v = u - ktu^{1/2} + \frac{k^2t^2}{4}$$

But $v = \frac{dx}{dt}$

Therefore, $\frac{dx}{dt} = u - kt\,u^{1/2} + \frac{k^2t^2}{4}$

Integrating from $t = 0$ to $t = \tau$, we have

$$x = \left| ut - \frac{ku^{1/2}t^2}{2} + \frac{k^2t^3}{12} \right|_0^{\tau}$$

or $$x = u\tau - \frac{1}{2}ku^{1/2}\tau^2 + \frac{1}{12}k^2\tau^3 \quad \text{(iv)}$$

Substituting the value of τ from (iii) in (iv), we get

$$x = \frac{2u^{3/2}}{k} - \frac{4u^{3/2}}{2k} + \frac{8u^{3/2}}{12k}$$

or $x = \frac{2u^{3/2}}{3k}$, which is choice (d).

Questions 3 to 6 are based on the following passage.

Passage II

The vertical displacement y of a projectile varies with the horizontal displacement x as

$$y = ax - bx^2$$

where a and b are constants.

3. The trajectory of the projectile is a
   (a) straight line (b) circle
   (c) parabola (d) hyperbola
4. The horizontal range of the projectile is
   (a) a (b) b
   (c) $\sqrt{\frac{a}{b}}$ (d) $\frac{a}{b}$
5. The maximum height attained by the projectile is
   (a) $\frac{a}{b}$ (b) $\frac{2a}{b}$
   (c) $\frac{a^2}{2b}$ (d) $\frac{a^2}{4b}$
6. The time of flight of the projectile is
   (a) $\frac{a}{\sqrt{bg}}$ (b) $a\sqrt{\frac{2}{bg}}$
   (c) $\frac{2a}{\sqrt{bg}}$ (d) $2a\sqrt{\frac{2}{bg}}$

Solutions

3. The given equation

$$y = ax - bx^2 \quad \text{(i)}$$

is the equation of a parabola. Hence the correct choice is (c).

4. The value of y is zero at $x = 0$ and $x = R$ (horizontal range). Putting $y = 0$ and $x = R$ in Eq. (i), we get $R = a/b$, which is choice (d).

5. Differentiating Eq. (i) with respect to time t, we have

$$\frac{dy}{dt} = a\frac{dx}{dt} - 2\,bx\frac{dx}{dt}$$

$$\Rightarrow \quad v_y = av_x - 2\,bxv_x = (a - 2bx)v_x \quad \text{(ii)}$$

At the maximum height $v_y = 0$. Using this in Eq. (ii), we get $(a - 2\,bx) = 0$ or $x = a/2b$. Putting this value of x in Eq. (i), we have (since $y = h_{max}$ at this value of x)

$$h_{max} = a\left(\frac{a}{2b}\right) - b\left(\frac{a}{2b}\right)^2 = \frac{a^2}{4b}$$

Hence the correct choice is (d).

6. The time t to reach the maximum height is given by

$$h_{max} = \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2h_{max}}{g}} = \sqrt{\frac{2a^2}{4bg}} = \frac{a}{\sqrt{2bg}}$$

Therefore, the time of flight is

$$t_f = 2t = a\sqrt{\frac{2}{bg}}, \text{ which is choice (b).}$$

Questions 7 to 13 are based on the following passage.

Passage III

The position vector **r** with respect to the origin of a particle varies with time t as

$$\mathbf{r} = (at)\,\hat{\mathbf{i}} + (bt - ct^2)\,\hat{\mathbf{j}}$$

where a, b and c are constants.

7. The trajectory of the particle is a
   (a) straight line (b) circle
   (c) parabola (d) none of these
8. The magnitude of the initial velocity of the particle is
   (a) $\sqrt{a^2 + c^2}$ (b) $\sqrt{b^2 + c^2}$
   (c) $\sqrt{a^2 + b^2}$ (d) $(a + b - 2c)$
9. The angle θ with the horizontal along which the particle is projected is given by

(a) $\sin\theta = \frac{b}{c}$ (b) $\cos\theta = \frac{a}{c}$

(c) $\tan\theta = \frac{b}{a}$ (d) $\tan\theta = \frac{2c}{\sqrt{a^2+b^2}}$

10. The time of flight of the particle is

(a) $\frac{b}{c}$ (b) $\frac{ab}{c}$

(c) $\frac{c}{a}$ (d) $\frac{b}{ac}$

11. The acceleration due to gravity at that place is

(a) 2 a (b) 2 b

(c) 2 c (d) none of these

12. The maximum height to which the particle rises is

(a) $\frac{2b^2}{c}$ (b) $\frac{b^2}{2c}$

(c) $\frac{b^2}{4c}$ (d) $\frac{4b^2}{c}$

13. The horizontal range of the particle is

(a) $\frac{ab}{c}$ (b) $\frac{ac}{b}$

(c) $\frac{bc}{a}$ (d) abc



Solutions

7. Comparing Eq. $\mathbf{r} = (at)\,\hat{\mathbf{i}} + (bt - ct^2)\,\hat{\mathbf{j}}$ with Eq. $\boldsymbol{r} = x\hat{\mathbf{i}} + x\hat{\mathbf{j}}$, we get

$$x = at \quad \text{(i)}$$

and $$y = bt - ct^2 \quad \text{(ii)}$$

Eliminating t from (i) and (ii), we get

$y = \left(\frac{b}{a}\right)x - \left(\frac{c}{a^2}\right)x^2$ which is the equation of a parabola.

Hence the correct choice is (c).

8. From Eqs. (i) and (ii), we have

$$v_x = \frac{dx}{dt} = a \quad \text{(iii)}$$

$$v_y = \frac{dy}{dt} = b - 2ct \quad \text{(iv)}$$

Putting $t = 0$ in Eq. (iii), the initial values of v_x and v_y are a and b, respectively. The initial speed of the particle is

$u = \sqrt{a^2+b^2}$, which is choice (c).

9. If the particle is projected with an initial velocity u at an angle θ with the horizontal, then the horizontal displacement x and vertical displacement y at time t are

$$x = (u\cos\theta)t \quad \text{(v)}$$

and $$y = (u\sin\theta)t - \frac{1}{2}gt^2 \quad \text{(vi)}$$

Comparing Eqs. (v) and (vi) with Eqs. (iii) and (iv) we have $u\cos\theta = a$ and $u\sin\theta = b$ which give $\tan\theta = bla$, which is choice (c).

10. When $t = t_f$, $y = 0$. Putting $y = 0$ and $t = t_f$ in Eq. (ii), we get $0 = t_f(b - ct_f)$ gives $t_f = 0$ and $t_f = b/c$. But $t_f = 0$ is not possible. Hence the correct choice is (a).

11. Comparing Eq. (vi) with Eq. (ii), we get $g = 2c$, which is choice is (c).

12. Now $y = h_{\max}$ when $t = \frac{1}{2}$ $t_f = \frac{b}{2c}$. Putting $y = h_{\max}$ and $t = b/2c$ in Eq. (ii), we get

$$h_{\max} = b \times \frac{b}{2c} - c\left(\frac{b}{2c}\right)^2 = \frac{b^2}{4c}$$

13. $R = (u\cos\theta)t_f$ (vii)

Now $u = \sqrt{a^2+b^2}$, $\tan\theta = \frac{b}{a}$ which gives

$\cos\theta = \frac{a}{\sqrt{a^2+b^2}}$ and $t_f = \frac{b}{c}$. Putting these values in Eq. (vii), we get $R = \frac{ab}{c}$. Hence the correct choice is (a).

SECTION 3

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four options out of which only ONE choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1

(b) Statement-1 is true, Statement-2 is true but Statement-2 is NOT the correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

1. **Statement-1**

   A body moving in a straight line may have non-zero acceleration at an instant when its speed is zero.

   Statement-2

   If a body is thrown vertically upwards, its speed at the instant when it reaches the highest point is zero but its acceleration is 9.8 ms^{-2}.

2. **Statement-1**

   A body moving in a straight line with a constant speed must have a zero acceleration.

   Statement-2

   A body moving along a curve with a constant speed may have a zero acceleration.

3. **Statement-1**

   A wooden ball and a steel ball of the same mass, released from the same height in air, do not reach the ground at the same time.

   Statement-2

   The apparent weight of a body in a meduim depends on the density of the body relative to that of the meduim.

4. **Statement-1**

   If the displacement-time graph of the motion of a body is a straight line parallel to the time axis, then it follows that the body is at rest.

   Statement-2

   Velocity is equal to the rate of change of displacement.

5. **Statement-1**

   If the velocity-time graph of the motion of a body is a curve, then the body is either uniformly accelerated or uniformly retarded.

   Statement-2

   The slope of the velocity-time graph gives the acceleration.

6. **Statement-1**

   A body is projected horizontally with a velocity u from the top of a building of height h. It hits the ground after a time $t = \sqrt{2h/g}$.

   Statement-2

   The vertical and horizontal motions can be treated independently.

7. **Statement-1**

   A body is projected from the ground with kinetic energy K at an angle of 60° with the horizontal. If air resistance is neglected, its kinetic energy when it is at the highest point of its trajectory will be $K/4$.

   Statement-2

   At the highest point of the trajectory, the directions of the velocity and acceleration of the body are perpendular to each other.

8. **Statement-1**

   One end of a string of length R is tied to a stone of mass m and the other end to a small pivot on a frictionless vertical board. The stone is whirled in a vertical circle with the pivot as the centre. The minimum speed the stone must have, when it is at the topmost point on the circle, so that the string does not slack is $\sqrt{gR}$.

   Statement-2

   At the topmost point on the circle, the centripetal force is provided partly by tension in the string and partly by the weight of the stone.

9. **Statement-1**

   The maximum range on an inclined plane when a body is projected upwards from the base of the plane is less than that when it is projected downwards from the top of the same plane with the same speed.

   Statement-2

   The maximum range along an inclined plane is independent of the angle of inclination of the plane.

10. **Statement-1**

    In projectile motion, the velocity of the body at a point on it trajectory is equal to the slope at that point.

    Statement-2

    The velocity vector at a point is always along the tangent to the trajectory at that point.

11. **Statement-1**

    In a uniform circular motion, the centripetal force is always perpendicular to the velocity vector.

    Statement-2

    Then the force does no work on the body and its kinetic energy remains constant.

12. **Statement-1**

    In a non-uniform circular motion, the particle has two accelerations–one along the tangent to the circle and the other towards the centre of the circle.

    Statement-2

    In a non-uniform circular motion, the magnitude and the direction of the velocity vector both change with time.

13. **Statement-1**

    In a non-uniform circular motion, the acceleration of the particle is equal to the sum of the tangential acceleration and the centripetal acceleration.

    Statement-2

    The two accelerations are perpendicular to each other.

14. **Statement-1**

    In a uniform circular motion, the kinetic energy of the body remains constant.

    Statement-2

    The momentum of the body does not change with time.

15. **Statement-1**

    In a uniform circular motion, the acceleration is always directed towards the centre of the circle.

    Statement-2

    Otherwise the speed of the body moving along the circle will change with time.



Solutions

1. The correct choice is (b).

2. The correct choice is (c). The velocity of a body moving along a curve continuously changes because its direction of motion is changing. Hence a body moving along a curve with a constant speed has acceleration called centripetal acceleration.

3. The correct choice is (a). The effective acceleration due to gravity in a meduim is given by

$$g_{\text{eff}} = g\left(1 - \frac{\rho}{\sigma}\right)$$

where ρ = density of the medium and σ = density of the body.

4. The correct choice is (a). If the displacement-time graph is parallel to the time axis, then, rate of change of displacement is zero.

5. The correct choice is (d). If the velocity-time graph is a curve, the slope of the graph is not constant.

6. The correct choice is (a). The time taken by the body to hit the ground is the same as if it was dropped from that height and fell freely under gravity.

7. The correct choice is (b). If m is the mass of the body and u its velocity of projection, the initial kinetic energy is

$$K = \frac{1}{2}mu^2$$

At the highest point, the horizontal velocity is (u cos 60°) and vertical velocity is zero. Hence the kinetic energy at the highest point is

$$K' = \frac{1}{2}m(u\cos 60°)^2 = \frac{1}{4} \times \frac{1}{2}mu^2 = \frac{K}{4}$$

At the highest point of the trajectory, the velocity of the body is horizontal (parallel to the ground) but its acceleration is g directed vertically downwards.

8. The correct choice is (a). When the stone is at the topmost point A on the circle, the centripetal force is provided by ($mg + T$) as shown in Fig. 2.59.

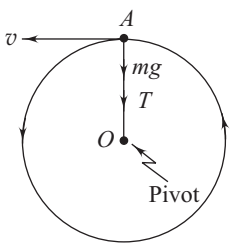


Fig. 2.59

Thus $$\frac{mv^2}{R} = mg + T$$

When the stone is at A, the string will not slack if tension $T = 0$, which gives $\frac{mv^2}{R} = mg \quad \Rightarrow v = \sqrt{Rg}$.

9. The range along the inclined plane when a body is projected with velocity u at an angle θ with the horizontal is given by

$$R = \frac{2u^2 \sin(\theta - \alpha)\cos\theta}{g\cos^2\alpha}$$

$$= \frac{u^2}{g\cos^2\alpha}[\sin(2\theta - \alpha) - \sin\alpha]$$

where α is the angle of inclination of the plane. Range R will be maximum if $\sin(2\theta - \alpha) = 1$ or $2\theta - \alpha = 90°$ in which case

$$R_{max} = \frac{u^2}{g\cos^2\alpha}[1 - \sin\alpha] = \frac{u^2}{g(1+\sin\alpha)}$$

If the body is projected downwards from the top of the same inclined plane, the maximum range will be

$$R'_{max} = \frac{u^2}{g(1-\sin\alpha)}$$

Thus $R'_{max} > R_{max}$. Since the range R depends on angle α, Statement 2 is false. Hence the correct choice is (c).

10. The correct choice is (d). At the highest point on the trajectory, the slope is zero but velocity is $u\cos\theta$.
11. The correct choice is (a).
12. The correct choice is (a).
13. The correct choice is (d). The acceleration of the particle is given by

$$a = \sqrt{a_c^2 + a_t^2}$$

where a_c = centripetal acceleration and a_t = tangential acceleration.

14. The correct choice is (c). The speed of the body remains constant but the momentum changes with time because the direction of the velocity vector changes with time.
15. The correct choices is (a). If the acceleration vector is not directed towards the centre of the circle, it will have a component along the tangent, as a result the speed of the body will change and the motion no longer remains uniform.

4 SECTION — Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. A ball is projected with kinetic energy E at an angle of 45º to the horizontal. The kinetic energy of the ball at the highest point of its trajectory will be

   (a) E (b) $\frac{E}{\sqrt{2}}$

   (c) $\frac{E}{2}$ (d) zero **[2002]**

2. Two forces are such that the sum of their magnitudes is 18 N and their resultant whose magnitude is 12 N, is perpendicular to the smaller force. Then the magnitudes of the forces are

   (a) 12 N, 6 N (b) 13 N, 5 N

   (c) 10 N, 8 N (d) 16 N, 2 N **[2002]**

3. From a building two balls A and B are thrown such that A is thrown vertically upward and B vertically downwards, both with the same speed. If v_A and v_B are their respective velocities on reaching the ground, then

   (a) $v_B > v_A$

   (b) $v_A = v_B$

   (c) $v_A > v_B$

   (d) their velocities depend on their masses. **[2002]**

4. If a body loses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest?

   (a) 1 cm (b) 2 cm

   (c) 3 cm (d) 4 cm **[2002]**

5. A car moving with a speed of 50 km/h can be stopped by brakes after at least 6 m. If the same car is moving with a speed of 100 km/h, the minimum stopping distance is

   (a) 12 m (b) 18 m

   (c) 24 m (d) 6 m **[2003]**

6. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30º with the horizontal. How far from the throwing point will the ball be at a height of 10 m from the ground? [Take g = 10 m/s^2]

   (a) 5.20 m (b) 4.33 m

   (c) 2.60 m (d) 8.66 m **[2003]**

7. The coordinates of a moving particle at any time t are $x = \alpha t^3$ and $y = \beta t^3$ where α and β are constants. The speed of the particle at time t is

   (a) $t\sqrt{\alpha^2+\beta^2}$ (b) $3t\sqrt{\alpha^2+\beta^2}$

   (c) $t^2\sqrt{\alpha^2+\beta^2}$ (d) $3t^2\sqrt{\alpha^2+\beta^2}$

   [2003]

8. Three forces start acting simultaneously on a particle moving with velocity $\vec{v}$. Those forces are represented in magnitude and direction by the three

sides of a triangle *ABC* as shown in the figure. The particle will now move with velocity.

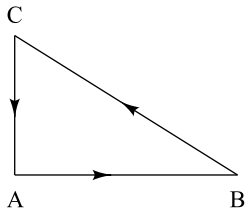


Fig. 2.60

(a) $\vec{v}$ (remaining unchanged)
(b) greater than $\vec{v}$
(c) less than $\vec{v}$
(d) $|\vec{v}|$ in the direction of the largest force $\overrightarrow{BC}$ **[2003]**

9. A ball is released from rest from the top of a tower of height h meter. It takes T seconds to reach the ground. What was the height of the ball from the ground in $T/3$ seconds?
(a) $\frac{h}{9}$ (b) $\frac{7h}{9}$
(c) $\frac{8h}{9}$ (d) $\frac{17h}{18}$ **[2004]**

10. If $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$, then the angle between $\vec{A}$ and $\vec{B}$ is
(a) π (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$ **[2004]**

11. A projectile can have the same range for two angles of projection. If T_1 and T_2 are the times of flight in two cases, then the product $T_1\,T_2$ is proportional to
(a) $\frac{1}{R^2}$ (b) $\frac{1}{R}$
(c) R (d) R^2 **[2004]**

12. Which of the following statements is false for a particle moving in a circle with a constant angular speed?
(a) The velocity vector is tangent to the circle
(b) The acceleration vector is tangent to the circle
(c) The acceleration vector points towards the centre of the circle
(d) The velocity and acceleration vectors are perpendicular to each other. **[2004]**

13. A ball is thrown from a point with a speed u at an angle θ to the horizontal. From the same point and at the same instant, a person starts running with a constant speed $u/2$ to catch the ball. Will he be bale to catch the ball? If yes, what should be the value of θ?
(a) Yes, 60º (b) Yes, 30º
(c) No (d) Yes, 45º **[2004]**

14. The relation between time t and distance x is $t = ax^2 + bx$ where a and b are constants. The acceleration is
(a) $-2abv^2$ (b) $2bv^3$
(c) $-2av^3$ (d) $2av^2$ **[2005]**

15. A car, starting from rest, accelerates at a rate f through a distance S, then continues at constant speed for time t and then decelerates at a rate $f/2$ to come to rest. If the total distance travelled is 15 S, then
(a) $S = ft^2$ (b) $S = \frac{1}{6}ft^2$
(c) $S = \frac{1}{2}ft^2$ (d) $S = \frac{1}{4}ft^2$ **[2005]**

16. A particle is moving eastwards with a velocity of 5 ms^{-1}. In 10 s the velocity changes to 5 ms^{-1} northwards. The average acceleration in this time is
(a) $\frac{1}{\sqrt{2}}ms^{-2}$ towards north-east
(b) $\frac{1}{2}ms^{-2}$ towards north
(c) zero
(d) $\frac{1}{\sqrt{2}}ms^{-2}$ towards north-west **[2005]**

17. A parachutist after bailing out falls 50 m without friction. When he opens the parachute, the decelerates at 2 ms^{-2}. He reaches the ground with a speed of 3 ms^{-1}. At what height did he bail out?
(a) 91 m (b) 182 m
(c) 293 m (d) 111 m **[2005]**

18. A particle located at $x=0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$, where α is a constant. The displacement x of the particle varies with time t as
(a) $t^{1/2}$ (b) t^3
(c) t^2 (d) t **[2006]**

19. The velocity of a particle is given by
$$v = v_0 + gt + ft^2$$
where v_0, g and f are constants. If its position is $x = 0$ at $t = 0$, then its displacement at time $t = 1$ s is
(a) $v_0 + 2g + 3f$ (b) $v_0 + \frac{g}{2} + \frac{f}{3}$
(c) $v_0 + g + f$ (d) $v_0 + \frac{g}{2} + f$ **[2007]**

20. A particle is projected at 60º to the horizontal with a kinetic energy K. The kinetic energy at the highest point is
(a) K (b) zero
(c) $K/4$ (d) $K/2$ **[2007]**

21. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describe $(x_1 - x_2)$ as a function of time t?

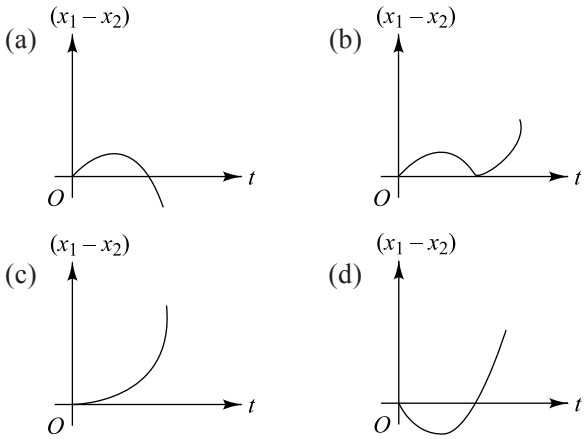


Fig. 2.61 **[2008]**

22. A projectile is projected with a velocity u at an angle θ with the horizontal. For a fixed θ, which of the graphs shown in the following figure shows the variation of range R versus u?

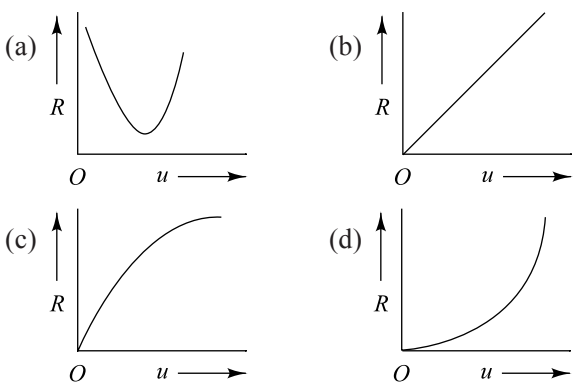


Fig. 2.62 **[2008]**

23. A particle has an initial velocity of $3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ and an acceleration of $0.4\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}}$. Its speed after 10 s is :
    (a) 7 units (b) 8.5 units
    (c) 10 units (d) $7\sqrt{2}$ units **[2009]**

24. A particle is moving with velocity $v = K(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$, where K is a constant. The general equation for its path is
    (a) $y = x^2$ + constant (b) $y^2 = x$ + constant
    (c) xy = constant (d) $y^2 = x^2$ + constant
    [2010]

25. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. the acceleration of 'P' when $t = 2$ s is nearly.

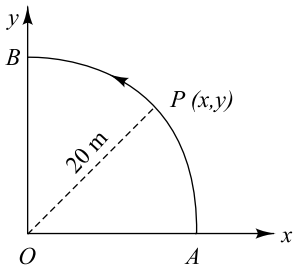


Fig. 2.63

    (a) 14 m/s^2 (b) 13 m/s^2
    (c) 12 m/s^2 (d) 7.2 m/s^2 **[2010]**

26. A particle is moving in a circle of radius R with a uniform speed v. The acceleration $\vec{a}$ of the particle at a point $P(R,\theta)$ on the circle is (here θ is measured from the x-axis).

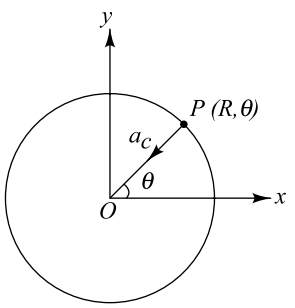


Fig. 2.64

    (a) $\frac{v^2}{R}\hat{\mathbf{i}} + \frac{v^2}{R}\hat{\mathbf{j}}$
    (b) $-\frac{v^2}{R}\cos\theta\hat{\mathbf{i}} + \frac{v^2}{R}\sin\theta\hat{\mathbf{j}}$
    (c) $-\frac{v^2}{R}\sin\theta\hat{\mathbf{i}} + \frac{v^2}{R}\cos\theta\hat{\mathbf{j}}$
    (d) $-\frac{v^2}{R}\cos\theta\hat{\mathbf{i}} - \frac{v^2}{R}\sin\theta\hat{\mathbf{j}}$ **[2010]**

27. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by:

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be:
    (a) 2 s (b) 4 s
    (c) 8 s (d) 1 s **[2011]**

28. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is :

(a) $\pi\frac{v^4}{g^2}$ (b) $\frac{\pi}{2}\frac{v^4}{g^2}$

(c) $\pi\frac{v^2}{g^2}$ (d) $\pi\frac{v^4}{g}$ **[2011]**

29. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal accelerations is

(a) $\frac{m_1 r_1}{m_2 r_2}$ (b) $\frac{m_1}{m_2}$

(c) $\frac{r_1}{r_2}$ (d) 1 **[2012]**

30. A projectile is given an initial velocity of $(\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ ms^{-1}, where $\hat{\mathbf{i}}$ is along horizontal and $\hat{\mathbf{j}}$ is along the vertical. If g = 10 ms^{-2}, the equation of the trajectory is

(a) $y = 2x - 5x^2$ (b) $y = x - 5x^2$

(c) $4y = 2x - 5x^2$ (d) $y = 2x + 5x^2$ **[2013]**

Answers

1. (c)	2. (b)	3. (b)	4. (a)
5. (c)	6. (d)	7. (d)	8. (a)
9. (c)	10. (a)	11. (c)	12. (b)
13. (a)	14. (c)	15. none	16. (d)
17.(c)	18. (c)	19. (b)	20. (c)
21. (d)	22. (d)	23. (d)	24. (d)
25. (a)	26. (d)	27. (a)	28. (a)
29. (c)	30. (a)		

Solutions

1. At the highest point, the velocity has only horizontal component $v_x = u_x = u\cos\theta = u\cos 45^\circ = u/\sqrt{2}$.

K.E. $= \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2}\left(\frac{1}{2}mu^2\right) = \frac{E}{2}$

$\left(\because E = \frac{1}{2}mu^2\right)$

2. Given $A + B = 18$ (i)

$$12 = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad \text{(ii)}$$

and $\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$

$\Rightarrow \tan 90^\circ = \frac{B\sin\theta}{A + B\cos\theta} \Rightarrow A + B\cos\theta = 0$

$\therefore \cos\theta = -\frac{A}{B}$

Using this in (ii), $12 = \sqrt{B^2 - A^2}$

$\Rightarrow 144 = B^2 - A^2 = (B + A)(B - A)$

Using (i), $144 = 18(B - A) \Rightarrow B - A = \frac{144}{18} = 8$ (iii)

From (i) and (iii) we get A = 5N and B = 13N

6. Given $u_A = u_B = u$. Acceleration $a = -g$ and displacement $s = -h$ (for both balls) where h = height of the building. Using $v^2 = u^2 + 2as$, we have

$v_A^2 = u^2 + 2 \times (-g) \times (-h) = u^2 + 2gh$

and $v_B^2 = u^2 + 2gh$

Hence $v_A = v_B$

4. Let v be the velocity of body at A when it enters the block

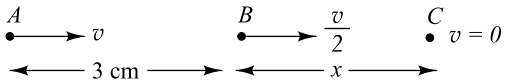


Fig. 2.60

Given AB = 3 cm. Let a be the retardation (assumed constant). Then from $v^2 - u^2 = 2aS$ we have

$\left(\frac{v}{2}\right)^2 - v^2 = 2a \times 3 \quad \Rightarrow a = -\frac{v^2}{8}$

For distance $BC = x$, $u = \frac{v}{2}$ $v = 0$ and $a = \frac{v^2}{8}$

Hence $(0)^2 - \left(\frac{v}{2}\right)^2 = 2 \times \left(-\frac{v^2}{8}\right) \times x$

$\Rightarrow x$ = 1 cm

5. $v^2 = u^2 + 2as \Rightarrow 0 = u^2 + 2as \Rightarrow u^2 = -2as$. Thus $s \propto u^2$. If u is doubled, s becomes four times = 4 × 6 = 24 m

6. The ball will be at point P at a height of 10 m from the ground

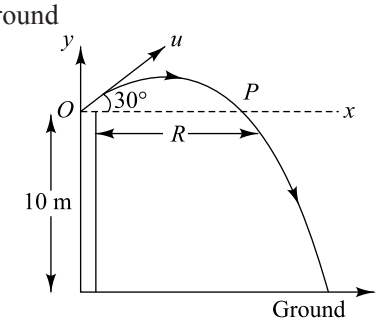


Fig 2.65

$$OP = R = \frac{u^2 \sin(2\theta)}{g}$$

$$= \frac{(10)^2 \sin 60^\circ}{10} = 8.66 \text{ m}$$

7. $v_x = \dfrac{dx}{dt} = 3\alpha t^2$

$v_y = \dfrac{dy}{dt} = 3\beta t^2$

$v = \sqrt{v_x^2 + v_y^2} = 3t^2\sqrt{\alpha^2 + \beta^2}$

8. The three forces are represented by the three sides of triangle ABC taken in order. From triangle law, their resultant is zero. Hence no net force acts on the particle. From Newton's first law of motion, its velocity remains unchanged.

9. $s = -h$, $t = T$, $u = 0$ and $a = -g$.

Using $s = ut + \frac{1}{2}at^2$, we have

$$-h = 0 - \frac{1}{2}gT^2 \Rightarrow T = \sqrt{\frac{2h}{g}} \qquad \text{(i)}$$

If the ball travels a distance h' in time $t = T/3$

$$-h' = 0 - \frac{1}{2}g\left(\frac{T}{3}\right)^2 \Rightarrow h' = \frac{1}{2}g \times \frac{T^2}{9} \qquad \text{(ii)}$$

Using (i) in (ii) we get $h' = \dfrac{h}{9}$

$\therefore$ Distance of the ball from the ground

$$= h - h' = h - \frac{h}{9} = \frac{8h}{9}$$

10. $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$ (given)

$\Rightarrow (\vec{A} \times \vec{B}) - (\vec{B} \times \vec{A}) = \vec{0}$

$\Rightarrow (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{B}) = \vec{0} \quad \left[\because (\vec{B} \times \vec{A}) = -(\vec{A} \times \vec{B})\right]$

$\Rightarrow 2(\vec{A} \times \vec{B}) = \vec{0}$

$\therefore \quad 2AB \sin\theta = 0 \Rightarrow \theta = 0 \text{ or } \pi$

11. A projectile has the same range for angles of projection θ and $(90^\circ - \theta)$. Therefore,

$$T_1 = \frac{2u\sin\theta}{g}$$

and $$T_2 = \frac{2u\sin(90^\circ - \theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\therefore \quad T_1 T_2 = \frac{4u^2 \sin\theta\cos\theta}{g} = \frac{2u^2\sin(2\theta)}{g^2} = \frac{2R}{g}$$

Hence $T_1 T_2 \propto R$

12. For uniform circular motion, the velocity vector is tangential to the circle and acceleration vector is directed towards the centre (centripetal acceleration). If the acceleration vector were along the tangent to the circle, it would change the angular speed of the particle. Hence statement (b) is false.

13. The person will catch the ball if the horizontal range = distance covered by the person in the time of flight, i.*e.*, if

$$R = \frac{u}{2} \times t_f$$

$$\Rightarrow \quad \frac{u^2 \sin(2\theta)}{g} = \frac{u}{2} \times \frac{2u\sin\theta}{g}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

14. Given $t = ax^2 + bx$. Differentiating w.r.t. time t,

$$1 = 2ax\frac{dx}{dt} + b\frac{dx}{dt}$$

$$v = \frac{dx}{dt} = \frac{1}{(2ax + b)} \qquad \text{(i)}$$

Acceleration $$A = \frac{dv}{dt} = \frac{d}{dt}\left[\frac{1}{2(ax+b)}\right]$$

$$= \frac{-2a}{(2ax+b)^2}\frac{dx}{dt}$$

Using (i) we have

$$S = \frac{-2a}{(2ax+b)} = -2av^3$$

15. Let t_1 be the time for which the car accelerates at rate f from rest. The distance travelled in time t_1 is

$$S_1 = \frac{1}{2}f t_1^2 \Rightarrow S = \frac{1}{2}f t_1^2 \qquad \text{(i)}$$

Velocity of car at time t_1 is $v_1 = ft_1$. The distance travelled in time t by the car moving at speed v_1 is

$$S_2 = v_1 t = f t_1 t \qquad \text{(ii)}$$

Let t_2 be the time for which the decelerates at rate $f/2$ to come to rest. The distance S_3 travelled in time t_2 is given by

$$2 \times \left(-\frac{f}{2}\right) S_3 = 0 - v_1^2 = -f^2 t_1^2$$

$$\Rightarrow \quad S_3 = f t_1^2 \qquad \text{(iii)}$$

From (i) and (ii) $S_3 = 2S_1 = 2S$

$S_1 + S_2 + S_3 = 15\ S$ (given)

$S + f t_1 t + 2\ S = 15\ S$

$$\Rightarrow \quad f t_1 t = 12\ S \qquad \text{(iv)}$$

Dividing (i) and (iv), we get

$$\frac{12S}{S} = \frac{f t_1 t}{\frac{1}{2}f t_1^2} = \frac{2t}{t_1}$$

which gives $t_1 = \dfrac{1}{6}$. Using this in (i), we get

$$S = \frac{1}{2} f t_1^2 = \frac{1}{2} f \left(\frac{t}{6}\right)^2 = \frac{1}{72} f t^2$$

Hence none of the given choice is correct.

16. Change in velocity is

$$\Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$$
$$= \mathbf{v}_2 + (-\mathbf{v}_1)$$

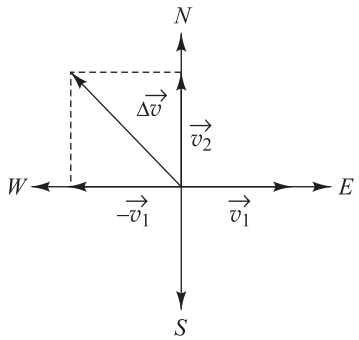


Fig 2.66

Vector $\Delta\mathbf{v}$ is the resultant of vectors $\mathbf{v}_2$ and $-\mathbf{v}_1$. The magnitude of $\Delta\mathbf{v}$ is

$$\Delta v = \sqrt{(-v_1)^2 + v_2^2} = \sqrt{(-5)^2 + (5)^2} = 5\sqrt{2} \text{ ms}^{-1}$$

$\therefore$ Average acceleration $\frac{\Delta v}{t} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}} \text{ms}^{-2}$.

From the figure, it is clear $\Delta\vec{v}$ is directed towards north-west.

17. Let H be the height from the ground when the parachutist bails out. The velocity after falling through h =50 m is.

$v = \sqrt{2gh} = \sqrt{2 \times (9.8) \times (50)} = \sqrt{980}$ ms^{-1}

directed downwards. After opening the parachute, he falls a distance $(H - h)$. For this distance, initial velocity $u = \sqrt{980}$ ms^{-1}, final velocity $v = 3$ ms^{-1} and acceleration $a = -2$ ms^{-2}.

$$\therefore \quad H - h = \frac{v^2 - u^2}{2a} = \frac{9 - 980}{-4} = \frac{971}{4} \simeq 243 \text{ m}$$

$$\Rightarrow \quad H = 243 + h = 243 + 50 = 293 \text{ m}$$

18. Given $v = \alpha\sqrt{x}$ Since $v = \frac{dx}{dt}$, we have

$$\frac{dx}{dt} = \alpha\sqrt{x}$$

or $$\frac{dx}{\sqrt{x}} = \alpha\, dt$$

Integrating, we have

$$\int_0^x \frac{dx}{\sqrt{x}} = \alpha \int_0^t dt$$

Which gives $-2\sqrt{x} = \alpha t$ or $x = \frac{\alpha^2 t^2}{4}$. Hence x is proportional to t^2.

19. $v = \frac{dx}{dt}$. Thus

$$\frac{dx}{dt} = v_0 + gt + ft^2$$

$$\Rightarrow \quad dx = v_0\, dt + gt\, dt + ft^2\, dt$$

Integrating we have

$$\int_0^x dx = \int_0^t (v_0 dt + gt\, dt + ft^2\, dt)$$

$$\Rightarrow \quad x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}$$

At $$t = 1 \text{ s}, x = v_0 + \frac{g}{2} + \frac{f}{3}$$

20. Let u be the velocity of projection. The component of u along the horizontal is $u_x = u \cos\theta$, where $\theta = 60^\circ$. Given $\frac{1}{2}mu^2 = K$. At the highest point the vertical component $u_y = u \sin\theta = 0$, but the horizontal component remains the same. Hence, kinetic energy at the highest point is

$$K' = \frac{1}{2} m u_x^2 = \frac{1}{2} m (u\cos\theta)^2$$
$$= \frac{1}{2} mu^2 \cos^2 60^\circ$$
$$= \frac{1}{4}\left(\frac{1}{2} mu^2\right) = \frac{K}{4}$$

21. Let a be the constant acceleration of the first body and v be the constant velocity of the second body. Then

$$x_1 = \frac{1}{2} at^2 \quad \text{and} \quad x_2 = vt$$

$$\therefore \quad \Delta x = x_1 - x_2 = \frac{1}{2}\, at^2 - vt = \left(\frac{at}{2} - v\right) t$$

Thus $\Delta x = 0$ at $t = 0$ and at $t = \frac{2v}{2}$. After $t = 2v/a$, Δx increase with t. Between $t = 0$ and $t = 2v/a$, Δx will be maximum if $\frac{d}{dt}(\Delta x) = 0$ and $\frac{d^2}{dt^2}(\Delta x)$ is negative and minimum if $\frac{d}{dt}(\Delta x) = 0$ and $\frac{d^2}{dt^2}(\Delta x)$ is positive

$$\frac{d}{dt}(\Delta x) = \frac{d}{dt}\left(\frac{1}{2} at^2 - vt\right) = at - v$$

Thus Δx will be maximum or minimum at $t = v/a$.

Now $\frac{d^2}{dt^2}(\Delta x) = \frac{d}{dt}(at - v) = a$ which is positive.

Hence Δx $(= x_1 - x_2)$ is minimum at $t = v/a$. So the correct graph is (d).

22. For a fixed θ, $R \propto u^2$. So The correct graph is (d).

23. $\mathbf{v} = \mathbf{u} + \mathbf{at} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) + (0.4\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}}) \times 10$

$= 7\hat{\mathbf{i}} + 7\hat{\mathbf{j}}$

$|\mathbf{v}| = \sqrt{7^2 + 7^2} = 7\sqrt{2}$ units

24. Given $\vec{v} = K(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$. Hence

$v_x = Ky$ and $v_y = Kx$. Therefore,

$$\frac{dx}{dt} = Ky \quad \text{and} \quad \frac{dy}{dt} = Kx$$

Now $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{Kx}{Ky} = \frac{x}{y}$$

$$\Rightarrow \qquad y\,dy = x\,dx$$

Integrating we get $\frac{y^2}{2} = \frac{x^2}{2} + c$

where c = constant of integration. Hence $y^2 = x^2 + 2c$.

25. $s = t^3 + 5$

$\therefore$ Speed $v = \frac{ds}{dt} = 3t^2$

Tangential acceleration $(a_t) = \frac{dv}{dt} = 6t$

At $t = 2$ s, $v = 3 \times (2)^2 = 12$ ms^{-1} and $a_t = 6 \times 2 = 12$ ms^{-2}.

Centripetal acceleration $(a_c) = \frac{v^2}{R} = \frac{(12)^2}{20} = \frac{144}{20}$ ms^{-2}

$\therefore$ Acceleration of P is

$$a = \sqrt{a_t^2 + a_c^2}$$

Substituting the values of a_t and a_c, we get $a = 14$ ms^{-2}.

26. Centripetal acceleration is

$$a_c = \frac{v^2}{R}$$

directed towards centre O. The x and y components of a_c are $-a_c \cos\theta$ and $-a_c \sin\theta$. Hence

$$\vec{a} = -a_c\left[\cos\theta(\hat{\mathbf{i}}) + \sin\theta(\hat{\mathbf{j}})\right]$$

$$= -\frac{v^2}{R}\left[\cos\theta(\hat{\mathbf{i}}) + \sin\theta(\hat{\mathbf{j}})\right]$$

27. $\frac{dv}{dt} = -2.5\sqrt{v}$

$$\Rightarrow \qquad \frac{dv}{\sqrt{v}} = -2.5\,dt$$

Integrating, we get

$$2\sqrt{v} = -2.5t + C \qquad \text{(i)}$$

At $t = 0$, $v = 6.25$ ms^{-1}. Using this in (i) we get $C = 5$.

Thus $2\sqrt{v} = -2.5t + 5$

For $v = 0$, $-2.5t + 5 = 0 \Rightarrow t = 2$s

28. Maximum horizontal range $(R_{max}) = \frac{v^2}{g}$. Therefore,

the required total area $= \pi (R_{max})^2 = \frac{\pi v^4}{g^2}$

29. $v_1 = \frac{2\pi r_1}{t}$ and $v_2 = \frac{2\pi r_2}{t}$

$\therefore \qquad \frac{v_1}{v_2} = \frac{r_1}{r_2}$

$$a_1 = \frac{v_1^2}{r_1} \quad \text{and} \quad a_2 = \frac{v_2^2}{r_2}$$

$\therefore \qquad \frac{a_1}{a_2} = \left(\frac{v_1}{v_2}\right)^2 \times \frac{r_2}{r_1}$

$$= \left(\frac{r_1}{r_2}\right)^2 \times \frac{r_2}{r_1} = \frac{r_1}{r_2}$$

30. $v_x = 1$ m s^{-1} and $v_y = 2$ m s^{-1}

Now $x = v_x\, t = t$ (i)

and $y = v_y\, t - \frac{1}{2} g\, t^2 \Rightarrow y = 2\, t - 5\, t^2$ (ii)

Using (i) in (ii),

$$y = 2x - 5x^2$$

LAWS OF MOTION

Chapter 3

REVIEW OF BASIC CONCEPTS

1. Newton's First Law of Motion

Newton's first law of motion states that *every body continues in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by an external unbalanced force.*

2. Newton's Second Law of Motion

Newton's second law of motion states that *the rate of change of linear momentum of a body is directly proportional to the applied force and the change takes place in the direction in which the force acts.*

Linear Momentum Newton defined linear momentum as the product of the mass and the velocity of a body.

$$\mathbf{p} = m\mathbf{v}$$

where $\mathbf{v}$ is the velocity at a certain instant of time. Differentiating this equation with respect to time, we get

$$\frac{d\mathbf{p}}{dt} = m\frac{d\mathbf{v}}{dt} \qquad (\because m \text{ is constant})$$

$$\boldsymbol{F} = m\mathbf{a}$$

where $\mathbf{a} = \dfrac{d\mathbf{v}}{dt}$ is the acceleration produced.

Note

1. Force = slope of momentum–time ($p - t$) graph.
2. Change in momentum = area under the force–time ($F - t$) graph.

EXAMPLE 1 The velocity of a body of mass 2 kg changes from $\mathbf{v}_1 = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})\,\text{ms}^{-1}$ to $\mathbf{v}_2 = (-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})\,\text{ms}^{-1}$ in 3 s. Find (a) the magnitude of the change in momentum of the body and (b) the magnitude of the force applied.

SOLUTION

(a) Change in momentum = final momentum – initial momentum

or
$$\Delta\mathbf{p} = m\mathbf{v}_2 - m\mathbf{v}_1$$
$$= m[(-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})]$$
$$= m(-5\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}})$$

$\therefore$
$$|\Delta\mathbf{p}| = 2\sqrt{(-5)^2 + (-1)^2 + (4)^2}$$
$$= 2 \times \sqrt{42} = 12.96 \text{ kg ms}^{-1}$$

(b) $$|\mathbf{F}| = \frac{|\Delta\mathbf{p}|}{\Delta t} = \frac{12.96}{3} = 4.32 \simeq 4.3 \text{ N}$$

EXAMPLE 2 A particle of mass 1 g is moving along the positive x-axis under the influence of a force.

$$F = -\frac{k}{x^2}$$

where $k = 10^{-3}$ Nm^2. When the particle is at $x = 1.0$ m, its velocity $v = 0$. Find (a) the magnitude of its velocity when, it reaches $x = 0.5$ m and (b) its position when its speed is 1 ms^{-1}.

SOLUTION

$$F = ma = m\frac{dv}{dt} = m\frac{dv}{dx}\cdot\frac{dx}{dt} = mv\frac{dv}{dx}$$

Given, $F = -\dfrac{k}{x^2}$. Therefore

$$-\frac{k}{x^2} = mv\frac{dv}{dx} \Rightarrow v\,dv = -\frac{k}{mx^2}dx$$

Integrating, we have

$$\int v\,dv = -\frac{k}{m}\int x^{-2}\,dx$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{k}{mx} + c \qquad \text{(i)}$$

where c is the constant of integration. Given $v = 0$ when $x = 1.0$ m. Using this in eq. (i), we get $c = -\frac{k}{m}$. Equation (i) becomes

$$\frac{v^2}{2} = \frac{k}{mx} - \frac{k}{m}$$

$$\Rightarrow \quad v = \left[\frac{2k}{m}\left(\frac{1}{x} - 1\right)\right]^{1/2}$$

$$v = \left[\frac{2\times 10^{-3}}{10^{-3}}\left(\frac{1}{x} - 1\right)\right]^{1/2}$$

$[\because\ k = 10^{-3}\ \text{Nm}^2$ and $m = 10^{-3}$ kg$]$

$$\Rightarrow \quad v = \left[2\left(\frac{1}{x} - 1\right)\right]^{1/2}$$

(a) When $x = 0.5$ m, $v = \left[2\left(\frac{1}{0.5} - 1\right)\right]^{1/2} = \sqrt{2}\ \text{ms}^{-1}$

(b) When $v = 1\ \text{ms}^{-1}$, $1 = \left[2\left(\frac{1}{x} - 1\right)\right]^{1/2} \Rightarrow x = 0.67$ m

3. Newton's Third Law of Motion

Newton's third law of motion states that *whenever one body exerts a force on a second body, the second body exerts an equal and opposite force on the first, or, to every action there is an equal and opposite reaction*. The action and reaction forces act on different body.

4. Law of Conservation of Linear Momentum

The *law of conservation of linear momentum* may be stated as '*when no net external force acts on a system consisting of several particles, the total linear momentum of the system is conserved, the total linear momentum being the vector sum of the linear momentum of each particle in the system*'.

Recoil of a Gun

The gun and the bullet constitute a two-body system. Before the gun is fired, both the gun and the bullet are at rest. Therefore, the total momentum of the gun-bullet system is zero. After the gun is fired, the bullet moves forward and the gun recoils backwards. Let m_b and m_g be the masses of the bullet and the gun. If $\mathbf{v}_b$ and $\mathbf{v}_g$ are their respective velocities after firing, the total momentum of the gun-bullet system after firing is $(m_b\,\mathbf{v}_b + m_g\,\mathbf{v}_g)$. From the law of conservation of momentum, the total momentum after and before the gun is fired must be the same, i.e.

$$m_b\,\mathbf{v}_b + m_g\,\mathbf{v}_g = 0$$

or $$\mathbf{v}_g = -\frac{m_b\,\mathbf{v}_b}{m_g}$$

The negative sign indicates that the gun recoils in a direction opposite to that of the bullet. In terms of magnitudes, we have

$$v_g = \frac{m_b\,v_b}{m_g}$$

5. Impulse

Consider a collision between two bodies A and B moving in the same straight line. Let Δt be the duration of the collision, i.e. the time for which the bodies were in contact during which time the transfer of momentum took place. We assume that the bodies continue moving in the same straight line after the collision with velocities different from their initial velocities.

Impulse of a force is the product of the average force and the time for which the force acts and it is equal to the change in momentum of the body during that time. Impulse is a vector and is measured in kg m s^{-1} or N s.

$$\mathbf{I} = \mathbf{F}_{av}\,\Delta t = \Delta\mathbf{p}$$

EXAMPLE 3 A ball of mass m is moving with a velocity $\mathbf{v}$ towards a rigid vertical wall. After striking the wall, the ball deflects through an angle θ without change in its speed. Obtain the expression for the impulse imparted to the ball.

SOLUTION Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the initial and final velocities of the ball [Fig. 3.1(a)]

Impulse = change in momentum

$$= m\mathbf{v}_2 - m\mathbf{v}_1 = m(\mathbf{v}_2 - \mathbf{v}_1) = m[\mathbf{v}_2 + (-\mathbf{v}_1)]$$

or $$\text{Impulse} = m\Delta\mathbf{v}$$

where $\Delta\mathbf{v} = \mathbf{v}_2 + (-\mathbf{v}_1)$ is the resultant of $\mathbf{v}_2$ and $-\mathbf{v}_1$ [Fig. 3.1(b)].

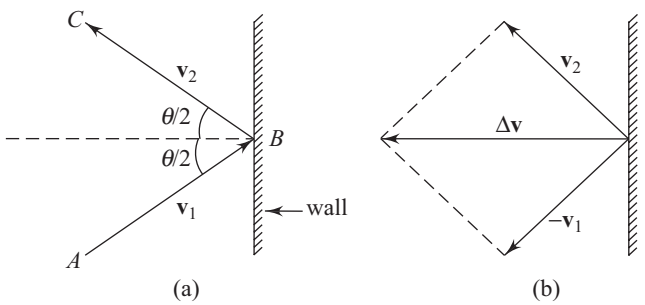


Fig. 3.1

Magnitude of $\Delta\mathbf{v}$ is

$[\because$ magnitude of $\mathbf{v}_1$ = magnitude of $\mathbf{v}_2 = v]$

$$\Delta v = \sqrt{v_1^2 + v_1^2 + 2v_1 v_2 \cos\theta}$$

$$= \sqrt{v^2 + v^2 + 2v^2 \cos\theta} = \sqrt{2v^2(1+\cos\theta)}$$

$$= 2v\cos\left(\frac{\theta}{2}\right)$$

The direction of impulse is perpendicular to the wall and away from it.

6. Contact Forces

(1) *Normal Reaction*

The force exerted by one body when placed on the surface of another body is known as contact force. If the two surfaces in contact are perfectly smooth (i.e., frictionless), then the contact force acts only perpendicular (normal) to their surface of contact and is known as *normal reaction* (R).

If a block of mass m is placed on a horizontal frictionless surface [Fig. 3.2 (a)], the normal reaction $R = mg$. If the block is placed on an inclined plane of inclination α [Fig. 3.2 (b)], the normal reaction $R = mg\cos\alpha$

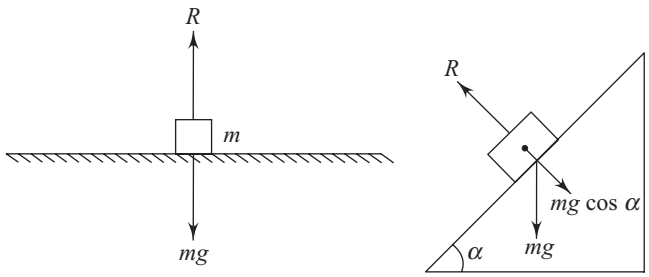


Fig. 3.2

If there is friction between the surfaces of contact, then the component of the contact force perpendicular to their surface gives the normal reaction and the other component which acts along the tangent to the surface of contact gives the force of friction. The normal reaction, tension and friction are examples of contact force.

(2) *Tension*

The force in a string is called tension (T). If the string is massless, the tension has the same magnitude at all points of the string. Tension in the string always acts away from the body to which it is attached. If the string passes over a frictionless pulley and its ends are attached to two bodies, the tension in the entire string has the same magnitude and its direction is towards its point of contact with the pulley.

7. Friction

Friction is the force which comes into play when one body slides or rolls over the surface of another body and acts in a direction tangential to the surfaces in contact and opposite to the direction of motion of the body.

The maximum (or limiting) force of friction when a body just begins to slide over the surface of another body is called the *limiting friction*. The force of friction just before one body begins to slide over another is called the *limiting* or *static friction* (f_s). The coefficient of limiting or static friction (μ_s) is defined as

$$\mu_s = \frac{f_s}{R}$$

where R is the normal reaction, i.e. the normal force pressing the two surfaces together.

The force necessary to maintain a body in uniform motion over the surface of another body, after motion has started, is called the *kinetic* or *sliding friction* (f_k). The *coefficient of kinetic friction* (μ_k) is defined as

$$\mu_k = \frac{f_k}{R}$$

Note that μ_k is always less than μ_s.

Angle of Friction Angle of friction is the angle between the resultant of the force of limiting friction (f) and the normal reaction (R). In Fig. 3.3, θ is the angle of friction, which is given by

$$\tan\theta = \frac{f}{R} = \mu$$

$$\theta = \tan^{-1}(\mu)$$

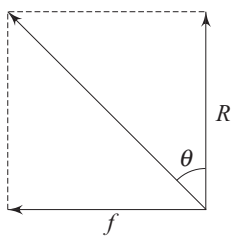


Fig. 3.3

Angle of Repose Suppose a body is placed on an inclined plane. The angle of inclination is gradually increased until the body just begins to slide along the plane. When this happens the angle of inclination α of the inclined surface with the horizontal is called the angle of repose (see Fig. 3.4). It follows from the figure that

Force of limiting friction (f) = $mg\sin\alpha$

Force of normal reaction (R) = $mg\cos\alpha$

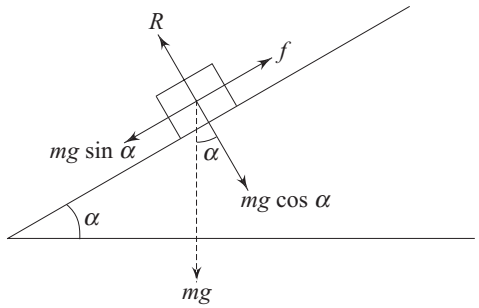


Fig. 3.4

Therefore, $\tan \alpha = \dfrac{f}{R} = \mu = \tan \theta$

or $\alpha = \theta = \tan^{-1}(\mu)$

8. Solving Problems in Mechanics by Free Body Diagram Method

In mechanics, we often have to handle problems which involve a group of bodies connected to one another by strings, pulleys, springs, etc. They exert forces on one another. Furthermore, there are frictional forces and the force of gravity acting on each body in the group. To solve such complicated problems, it is always convenient to choose one body in the group, find the magnitude and the direction of the forces acting on this body by all the remaining bodies in the group. Then we find the resultant of all the forces acting on the body to obtain the net force exerted on it. We then use the laws of motion to determine the dynamics of the body. We apply this procedure to all other bodies in the group one by one. It is useful to draw a separate diagram for each body, showing the directions of the different forces acting on it. Such a diagram is called the *free body diagram* (F.B.D.) of the body.

(1) Two masses tied to a string going over a frictionless pulley Consider two bodies of masses m_1 and m_2 ($m_1 > m_2$) connected by a string which passes over a pulley, as shown in Fig. 3.5(a). When the bodies are released, the heavier one moves downwards and the lighter one moves up.

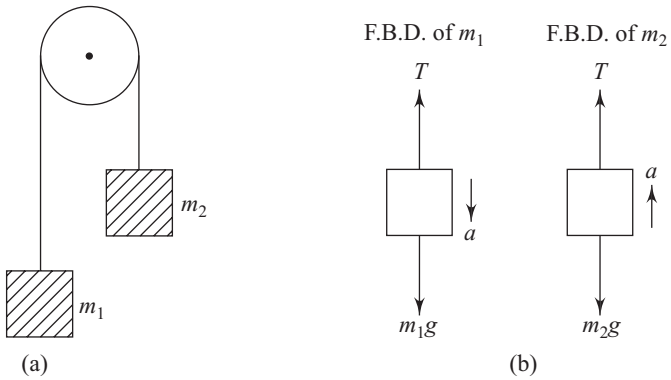


Fig. 3.5

Net force in the direction of motion of m_1 is $m_1g - T$. Therefore, the equation of motion of m_1 is

$$m_1g - T = m_1a \quad \text{(i)}$$

Net force in the direction of motion of m_2 is $(T - m_2g)$. Therefore, the equation of motion of m_2 is

$$T - m_2g = m_2a \quad \text{(ii)}$$

From (i) and (ii) we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

and $$T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g$$

(2) Two masses in contact Figure 3.6(a) shows two blocks of masses m_1 and m_2 placed in contact on a horizontal frictionless surface. A force F is applied to mass m_1. As a result, the masses move with a common acceleration a. To find a and the contact force on m_2, we draw the free body diagrams as shown in Figs. 3.6(b) and (c).

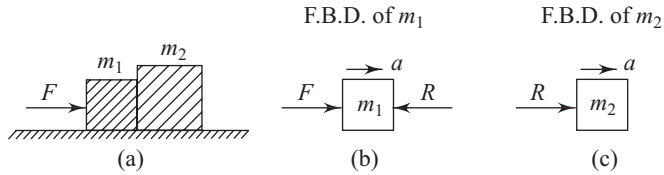


Fig. 3.6

R = normal reaction force between the blocks. From Figs. 3.6(b)and (c), we get

$$F - R = m_1a \quad \text{(i)}$$

and $$R = m_2a \quad \text{(ii)}$$

Adding (i) and (ii) we get

$$F = (m_1 + m_2)a$$

$$\Rightarrow \quad a = \frac{F}{(m_1 + m_2)}$$

Contact force on m_2 is

$$F_2 = m_2\, a = \frac{m_2F}{(m_1 + m_2)}$$

(3) Three masses in contact Figure 3.7(a) shows three blocks of masses m_1, m_2, and m_3 placed in contact on a horizontal frictionless surface. A force F is applied to m_1. As a result, the three masses move with a common acceleration a. To find a and the contact forces on m_2 and m_3, we draw the free body diagrams as shown in Figs. 3.7(b) (c) and (d).

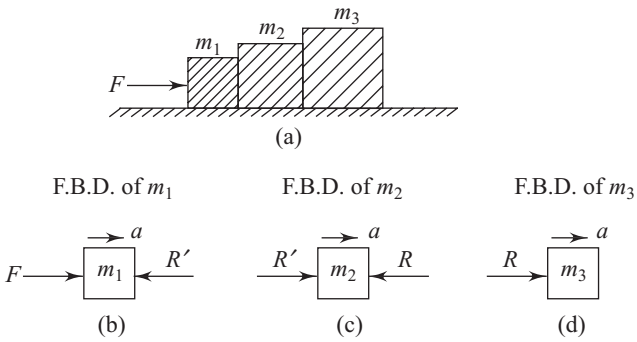


Fig. 3.7

R' = contact force on m_2 = reaction force between m_1 and m_2

R = contact force on m_3 = reaction force between m_2 and m_3

It follows from Figs. 3.7(b), (c) and (d) that

$$F - R' = m_1 a \quad \text{(i)}$$

$$R' - R = m_2 a \quad \text{(ii)}$$

and $$R = m_3 a \quad \text{(iii)}$$

Adding (i), (ii) and (iii) we get

$$a = \frac{F}{(m_1 + m_2 + m_3)}$$

Contact force on m_2 is $F_2 = R'$ which from (ii) is given by

$$F_2 = R' = R + m_2 a$$

Using (iii) we have

$$F_2 = m_3 a + m_2 a$$

$$= (m_2 + m_3)a$$

$$\Rightarrow \quad F_2 = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$$

Contact force on m_3 is

$$F_3 = R = m_3 a = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$

(4) Two masses connnected with a string Figure 3.8(a) shows two blocks of masses m_1 and m_2 connected with a string and lying on a horizontal frictionless surface. A force F is applied to m_2. As a result, the masses move with a common acceleration a. To find a and force exerted on m_1, we draw the free body diagrams as shown in Figs. 3.8(b) and (c). T is the tension in the string.

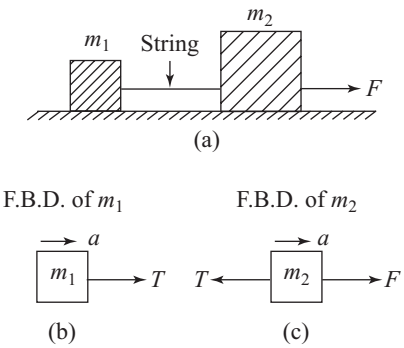


Fig. 3.8

It follows from Figs. 3.8(b) and (c) that

$$T = m_1 a \quad \text{(i)}$$

and $$F - T = m_2 a \quad \text{(ii)}$$

Adding (i) and (ii) we get

$$a = \frac{F}{m_1 + m_2}$$

Tension in the string is

$$T = m_1 a = \frac{m_1 F}{m_1 + m_2}$$

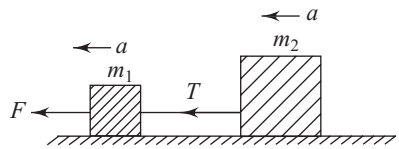


Fig. 3.9

If force F is applied on mass m_1 as shown in Fig. 3.9, then

$$a = \frac{F}{m_1 + m_2}$$

Tension in the string is

$$T = \frac{m_2 F}{m_1 + m_2}$$

(5) Three masses connected by strings Figure 3.10 (a) shows three blocks of masses m_1, m_2 and m_3 connected by two strings and placed on a horizontal frictionless surface. A force F is applied to m_1. As a result, the blocks move with a common acceleration a. To find a and the forces acting on m_2 and m_3, we draw free body diagrams as shown in Fig. 3.10(b) and (c) and (d). T is the tension in the string between m_1 and m_2 and T' is the tension in the string between m_2 and m_3.

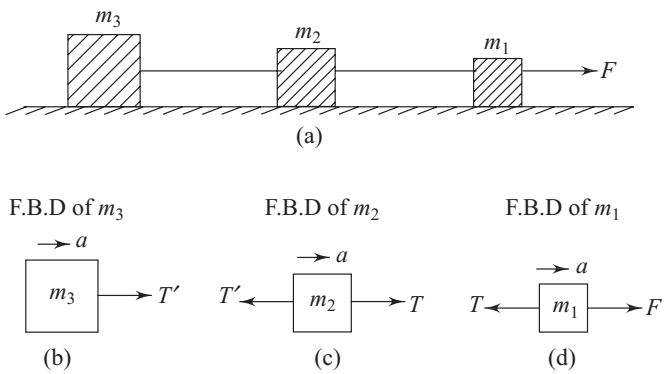


Fig. 3.10

It follows from Figs. 3.10(b), (c) and (d) that

$$T' = m_3 a \quad \text{(i)}$$

$$T - T' = m_2 a \quad \text{(ii)}$$

and $$F - T = m_1 a \quad \text{(iii)}$$

Adding (i), (ii) and (iii), we get

$$a = \frac{F}{(m_1 + m_2 + m_3)}$$

The tension in the string between m_1 and m_2 is T, which is obtained by adding (i) and (ii).

$$T = (m_2 + m_3)a = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$$

The tension in the string between m_2 and m_3 is T', which from (i) is given by

$$T' = m_3 a = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$

(6) Two masses connected by a string and suspended from a support Two blocks of masses m_1 and m_2 are connected by two strings and suspended from a support as shown in Fig. 3.11(a). Mass m_2 is pulled down by a force F. The tension T in the string between m_1 and m_2 and tension T' in the string between m_1 and the support can be found from the free body diagrams as shown in Fig. 3.11(b) and (c).

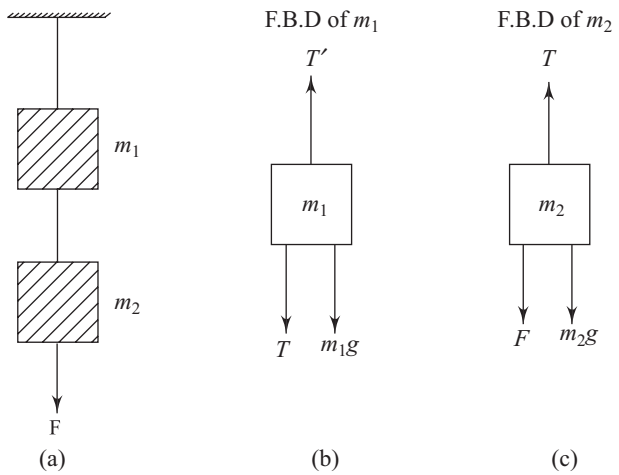


Fig. 3.11

$$T' = T + m_1 g \quad \text{(i)}$$

$$T = F + m_2 g \quad \text{(ii)}$$

Using (ii) and (i), we get

$$T' = F + (m_1 + m_2)g$$

(7) Two blocks connected by a string passing over a frictionless pulley fixed at the edge of a horizontal table Consider a block of mass m_1 lying on a frictionless table connected through a pulley to another block of mass m_2 hanging vertically (Fig. 3.12). When the system is released, let acceleration of the blocks be a.

From free body diagrams, the equations of motion of m_1 and m_2 are

$$T = m_1 a \quad \text{(i)}$$

and $$m_2 g - T = m_2 a \quad \text{(ii)}$$

Adding (i) and (ii), we get

$$a = \frac{m_2 g}{(m_1 + m_2)}$$

Also $$T = m_1 a = \frac{m_1 m_2 g}{(m_1 + m_2)}$$

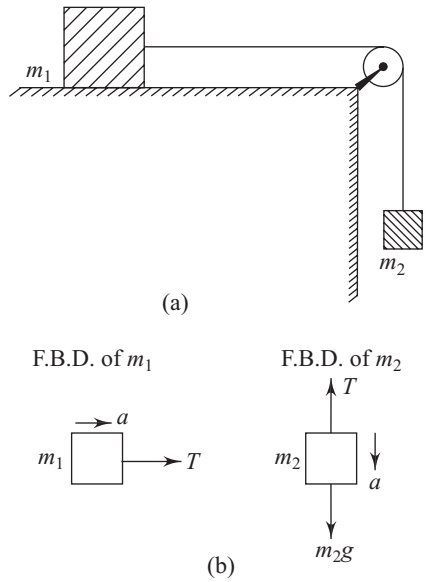


Fig. 3.12

If the table top is frictionless, the blocks will move even if $m_2 < m_1$.

If μ is the coefficient of friction between block m_1 and the table, the force of friction is

$$f = \mu R = \mu m_1 g$$

From F.B.D. of m_1 (Fig. 3.13), Eq. (i) becomes

$$T - f = m_1 a \quad \Rightarrow \quad T - \mu m_1 g = m_1 a$$

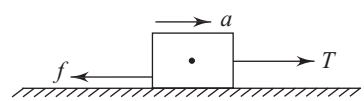


Fig. 3.13

(8) Two blocks connected by a string passing over a frictionless pulley fixed at the top of an inclined plane Let T be the tension in the string. Since the pulley is frictionless, the tension is the same throughout the string (Fig. 3.14). There are the following two cases:

(a) Mass m_1 moving up along the incline with acceleration a [Fig. 3.14]

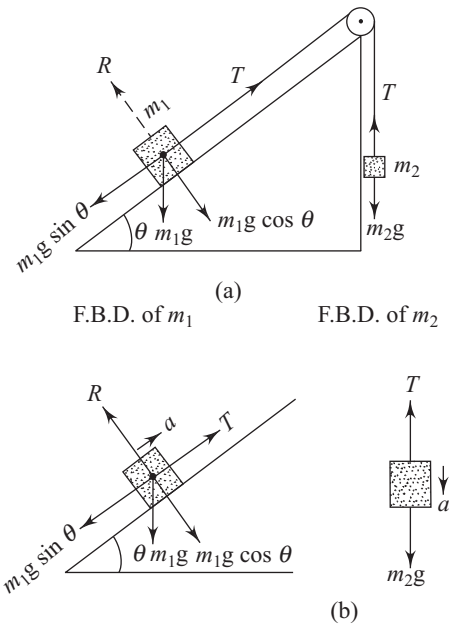


Fig. 3.14

The equations of motion of m_1 and m_2 [see Fig. 3.14(b)]

$$T - m_1 g \sin\theta = m_1 a \quad \text{(i)}$$

$$m_2 g - T = m_2 a \quad \text{(ii)}$$

which give $$a = \frac{(m_2 - m_1 \sin\theta) g}{(m_1 + m_2)}$$

and $$T = m_2 (g - a)$$

If μ is the coefficient of friction between m_1 and the inclined plane, the frictional force $f = \mu R = \mu m_1 g \cos\theta$ will act down the plane because the block m_1 is moving up the plane. In this case, Eq. (i) is replaced by

$$T - m_1 g \sin\theta - f = m_1 a$$

$$\Rightarrow \quad T - m_1 g \sin\theta - \mu m_1 g \cos\theta = m_1 a$$

(*b*) *Mass m_1 moves down the incline with acceleration a*

In this case, we get $m_1 g \sin\theta - T = m_1 a$ and $T - m_2 g = m_2 a$ which give

$$a = \frac{(m_1 \sin\theta - m_2) g}{(m_1 + m_2)}$$

and $T = m_2 (g + a)$

(9) Two blocks connected by a string passing over a frictionless pulley fixed at the top of a double inclined plane Let the block of mass m_1 move up along the inclined plane of angle of inclination θ_1, and the block of mass m_2 move down the inclined plane of angle of inclination θ_2 (Fig. 3.15). Let T be the tension in the string. Then, for m_1 and m_2, we have

$$T - m_1 g \sin\theta_1 = m_1 a$$

and $$m_2 g \sin\theta_2 - T = m_2 a$$

Eliminating T, we get

$$a = \frac{(m_2 \sin\theta_2 - m_1 \sin\theta_1) g}{(m_1 + m_2)}$$

Also $$T = m_1 (a + g \sin\theta_1)$$

$$= m_2 (g \sin\theta_2 - a)$$

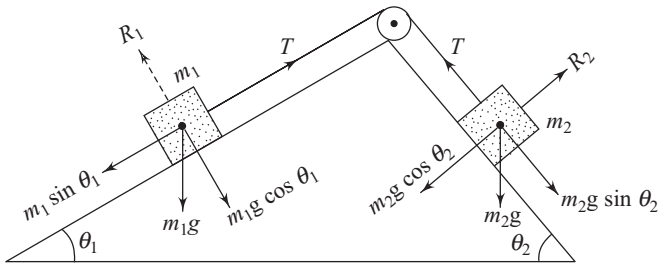


Fig. 3.15

(10) One blocks are placed on top of the another A block of mass m_1 is placed on another block of mass m_2, which is lying on a horizontal frictionless surface. The coefficient of friction between the blocks is μ.

Case 1: The maximum force that can be applied on the lower block so that the upper block does not slip [Fig. 3.16(a)]

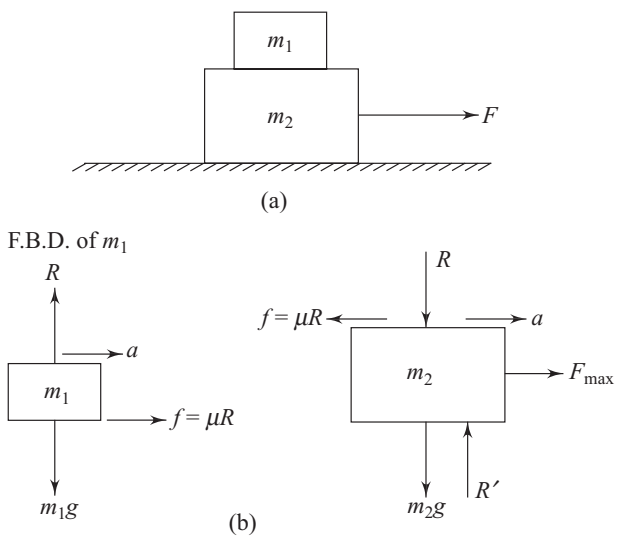


Fig. 3.16

F_{max} = maximum value of force F so that block m_1 does not slip of block m_2

f = frictional force on m_1 due to m_2

$= \mu R = \mu m_1 g$

Due to friction, m_2 will try to drag m_1 to the right. Hence frictional force f acts towards left. From F.B.D. of m_1,

$$f = m_1 a \Rightarrow \mu m_1 g \quad \Rightarrow \mu g = a \qquad \text{(i)}$$

Here, a is the acceleration of each block.

R' = normal reaction on m_2 by the horizontal surface.

From F.B.D. of m_2, we have

$$R + m_2 g = R' \qquad \text{(ii)}$$

and $$F_{max} - f = m_2 a$$

$$\Rightarrow \qquad F_{max} - \mu R = m_2 a \qquad \text{(iii)}$$

$$\Rightarrow \qquad F_{max} - \mu m_1 g = m_2 a$$

Using (i) in (iii), we get

$$F_{max} = (m_1 + m_2)\, \mu g$$

(a) If $F > F_{max}$, m_1 will begin to slide on m_2 and then their accelerations will be different.

(b) If $F < F_{max}$, m_1 and m_2 move together without any relative motion between them.

Case 2: The maximum force that can be applied on the upper block so that it does not slip on the lower block. [Fig. 3.17(a)]

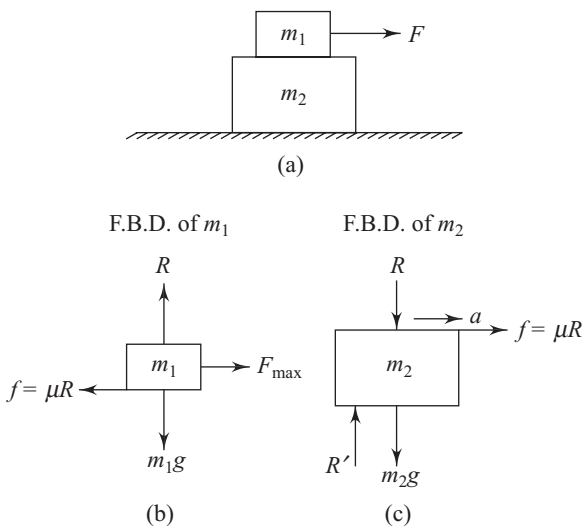


Fig. 3.17

F_{max} = maximum value of force F so that block m_1 just begins to slide on block m_2

Block m_1 tries to drag block m_2 toward right due to frictional force $f = \mu R = \mu m_1 g$. The frictional force exerted by m_1 on m_2 will be towards right.

R' = normal reaction on m_2 by the horizontal surface. If a is the acceleration of blocks towards right, from F.B.D. of m_1 we have

$$F_{max} - \mu R = m_1 a \text{ and } R = m_1 g$$

or $$F_{max} - \mu m_1 g = m_1 a \qquad \text{(i)}$$

From F.B.D. of m_2,

$$f = m_2 a \Rightarrow \mu_1 m_1 g = m_2 a$$

$$\Rightarrow \qquad a = \frac{\mu m_1 g}{m_2} \qquad \text{(ii)}$$

Using (ii) in (i), we get

$$F_{max} = \frac{\mu(m_1 + m_2) m_1 g}{m_2}$$

(a) If $F < F_{max}$, the blocks move together without any relative motion.

(b) If $F > F_{max}$, the blocks slide relative to each other and then their accelerations are different.

EXAMPLE 4 A block of mass m = 1 kg is pulled by a force F = 10 N at an angle $\theta = 60°$ with a horizontal surface as shown in Fig. 3.18. Find the acceleration of the block if

(a) the surface is frictionless and

(b) the coefficient of kinetic friction between the surface and the block is μ = 0.2. Take g = 10 ms^{-2}.

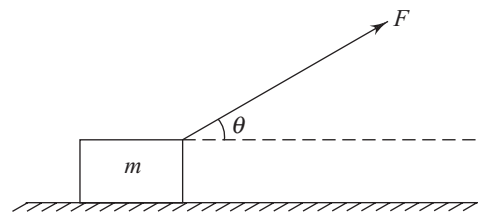
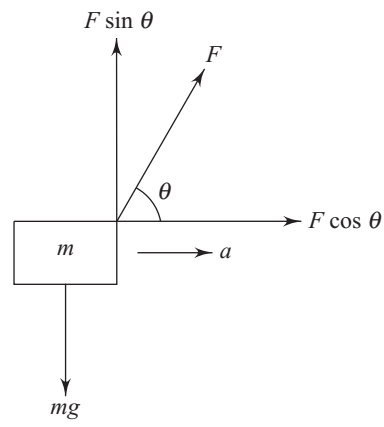
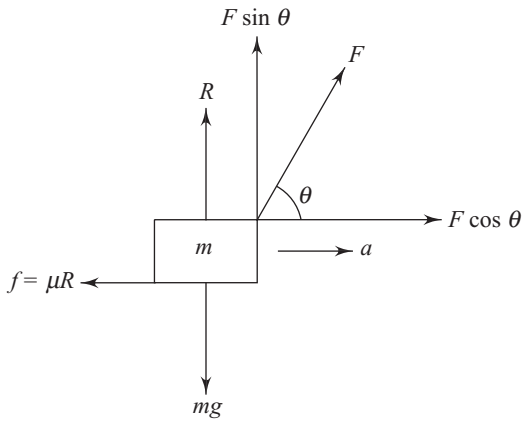


Fig. 3.18

SOLUTION The free body diagrams of the block in the two cases are shown in Fig. 3.19.





(b)

Fig. 3.19

(a) From Fig. 3.19(a)

$$F\cos\theta = ma$$

$$\Rightarrow \quad a = \frac{F\cos\theta}{m} = \frac{10\times\cos 60°}{1} = 5 \text{ ms}^{-2}$$

(b) From Fig. 3.19(b)

$$F\cos\theta - f = ma$$

$$\Rightarrow \quad F\cos\theta - \mu mg = ma$$

$$\Rightarrow \quad a = \frac{F\cos\theta - \mu mg}{m}$$

$$= \frac{10\cos 60° - 0.2\times 1\times 10}{1}$$

$$= 3 \text{ ms}^{-2}$$

From Fig. 3.19(b) we also have $F \sin\theta + R = mg$ or $F\sin\theta = mg - R$. Since $F \sin\theta < mg$, the block does not move upwards.

EXAMPLE 5 Two blocks of masses $m_1 = 2$ kg and $m_2 = 3$ kg are suspended from a rigid support by means of strings *AB* and *CD* as shown in Fig. 3.20. String *AB* has negligible mass and string *CD* has mass 0.5 kg/m. Each string has length 50 cm. Find the tension (a) at mid-point *P* of string *AB* and (b) at point *Q* of string *CD* where *CQ* = 20 cm. Take $g = 10 \text{ ms}^{-2}$.

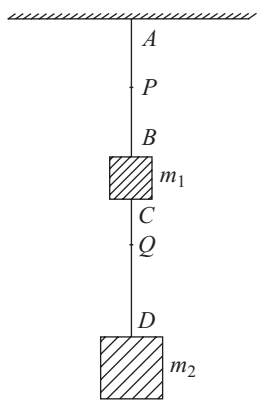


Fig. 3.20

SOLUTION Mass of string *CD* is $m = 0.5 \times 0.5 = 0.25$ kg Since string *AB* is massless, the tension in *AB* is the same at every point.

(a) Total mass below point $P = m_1 + m + m_2$

$$= 2 + 0.25 + 3 = 5.25 \text{ kg}$$

$\therefore$ Tension at $P = 5.25 \times 10 = 52.5$ N

(b) Total mass below point Q = mass of length $QD + m_2$

$$= 0.5 \times 0.3 + 3 = 3.15 \text{ kg}$$

$\therefore$ Tension at $Q = 3.15 \times 10 = 31.5$ N

EXAMPLE 6 A block of mass $m = 100$ g is placed on an inclined plane of inclination $\theta = 30°$ as shown in Fig. 3.21. There is no friction between the block and the inclined plane. What minimum acceleration *a* should be given to the system to the left so that the block does not slide down the plane?

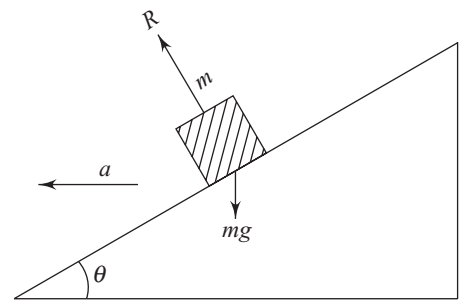


Fig. 3.21

SOLUTION Figure 3.22 shows the forces acting on the block.

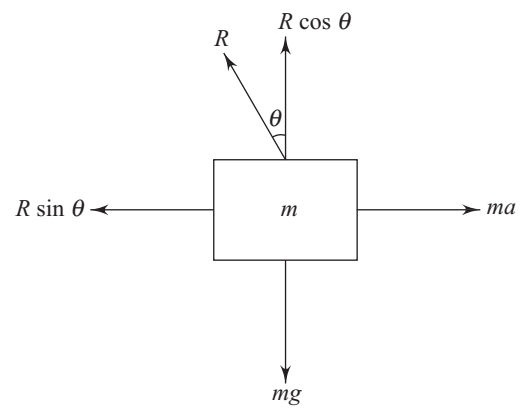


Fig. 3.22

$$R\cos\theta = mg$$

$$R\sin\theta = ma$$

$\therefore$

$$a = g\tan\theta$$

$$= g\tan 30° = \frac{g}{\sqrt{3}}$$

EXAMPLE 7 A pendulum of bob of mass m = 100 g is suspended from the ceiling of the compartment of a train. If the train has the acceleration a as shown in Fig. 3.23, the string makes an angle $\theta = 60°$ with the vertical. Find the value of a.

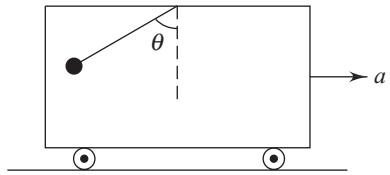


Fig. 3.23

SOLUTION Figure 3.24 shows the free body diagram of the bob.

Force on the bob in the direction of motion of the train = $T \sin \theta$. Hence the equation for horizontal direction is

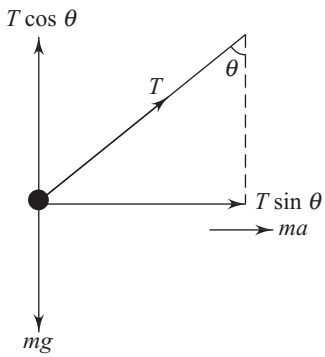


Fig. 3.24

$$T \sin \theta = ma \qquad \text{(i)}$$

For equilibrium along the vertical direction

$$T \cos \theta = mg \qquad \text{(ii)}$$

Dividing (i) and (ii), we get

$$\tan \theta = \frac{a}{g} \quad \Rightarrow \quad a = g \tan \theta = 9.8 \times \tan 60°$$

$$= 9.8 \times \sqrt{3} \simeq 17 \,\text{ms}^{-2}$$

EXAMPLE 8 Two blocks of masses $m_1 = 1.5$ kg and $m_2 = 2$ kg are attached to each other by strings and pulleys as shown in Fig. 3.25. Assume that pulleys are massless and frictionless and strings are massless. The system is released. If the table is frictionless, find the accelerations of m_1 and m_2 and tensions in the strings. Take $g = 10$ ms^{-2}.

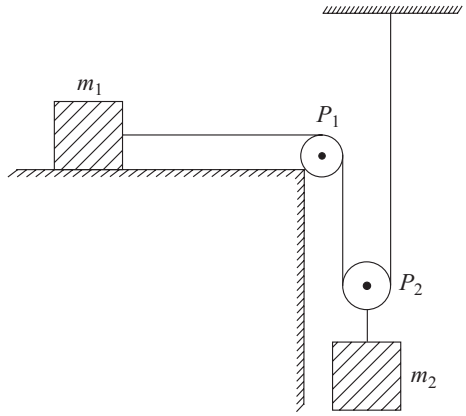


Fig. 3.25

SOLUTION Let a_1 and a_2 be the acceleration of m_1 and m_2 respectively. Let x_1 and x_2 be the distances moved by m_1 and m_2 in a time t. Since the total length of the string remains unchanged, it follows that if m_1 moves a distance x_1 to the right, m_2 will descend by a distance $x_2 = \frac{x_1}{2}$ or $x_1 = 2x_2$. Differentiating twice w.r.t. time we get

$$\frac{d^2x_1}{dt^2} = 2\frac{d^2x_2}{dt^2} \Rightarrow a_1 = 2a_2 \qquad \text{(i)}$$

Figure 3.26 shows the free body diagrams of m_1 and m_2. Here T_1 = tension in string attached to m_1 and T_2 = tension in string attached to m_2.

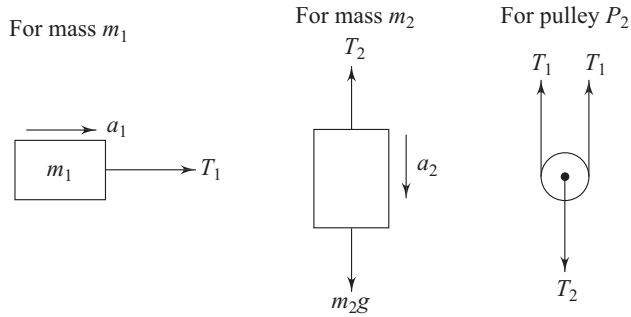


Fig. 3.26

For block m_1

$$T_1 = m_1a_1 \qquad \text{(i)}$$

For block m_2

$$m_2g - T_2 = m_2a_2 \qquad \text{(ii)}$$

Since the pulley is massless and frictionless $T_2 = 2T_1$ (iii)

Also $a_1 = 2a_2$. (iv)

Using (iii) and (iv) in (ii), we have

$$m_2 g - 2T_1 = \frac{m_2\, a_1}{2} \qquad \text{(v)}$$

Eliminating T_1 from (i) and (v), we get

$$a_1 = \frac{2m_2 g}{m_2 + 4m_1} = \frac{2 \times 2 \times 10}{2 + 4 \times 1.5} = 5 \text{ ms}^{-2}$$

$$\therefore \quad a_2 = \frac{a_1}{2} = \frac{5}{2} = 2.5 \text{ ms}^{-2}$$

From eq. (i),

$$T_1 = m_1 a_1 = 1.5 \times 5 = 7.5 \text{ N}$$

$$\therefore \quad T_2 = 2T_1 = 2 \times 7.5 = 15 \text{ N}$$

EXAMPLE 9 Two blocks of masses $m_1 = 100$ g and $m_2 = 5$ kg with m_1 placed on m_2 are connected to a frictionless and massless pulley as shown in Fig. 3.27. The string connecting them is also massless. The coefficient of static friction between m_1 and m_2 is $\mu = 0.5$. There is no friction between m_2 and the horizontal surface. Find the maximum horizontal force F that can be applied on m_1 so that it does not slide on m_2.

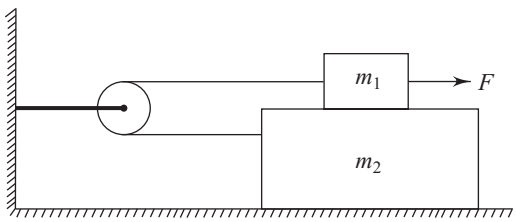


Fig. 3.27

SOLUTION Frictional force between m_1 and m_2 is $f = \mu R_1 = \mu m_1 g$. If T is the tension in the string, the free body diagrams of m_1 and m_2 are as shown in Fig. 3.28.

It follows from Fig. 3.28(a) that

$$R_1 = m_1 g \qquad \text{(i)}$$

and

$$F - f - T = 0 \qquad \text{(ii)}$$

From Fig. 3.28(b), we have

$$T - f = 0 \Rightarrow T = f \qquad \text{(iii)}$$

and

$$R_2 = R_1 + m_2 g \qquad \text{(iv)}$$

Using (iii) in (ii)

$$F - f - f = 0 \Rightarrow F = 2f = 2\mu m_1 g$$

Block m_1 will not slide on block m_2 if F is less than a maximum value $F_{\max}$ given by

$$F_{\max} = 2\mu m_1 g$$

$$= 2 \times 0.5 \times 0.1 \times 10 = 1 \text{ N}$$

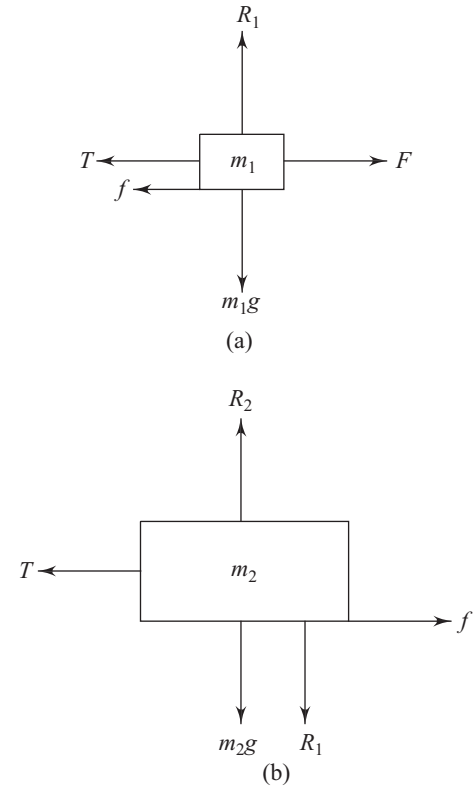


Fig. 3.28

EXAMPLE 10 A boy of mass $m = 50$ kg is standing on a weighing machine placed on the floor of an elevator. What is the weight of the boy when the elevator is (a) at rest, (b) moving up with an acceleration $a = 2.2$ ms^{-1} and (c) moving down with an acceleration 2.2 ms^{-2}.

SOLUTION The weighing machine reads the reaction R exerted by it on the boy. Fig. 3.29 shows the free body diagrams in the three cases.

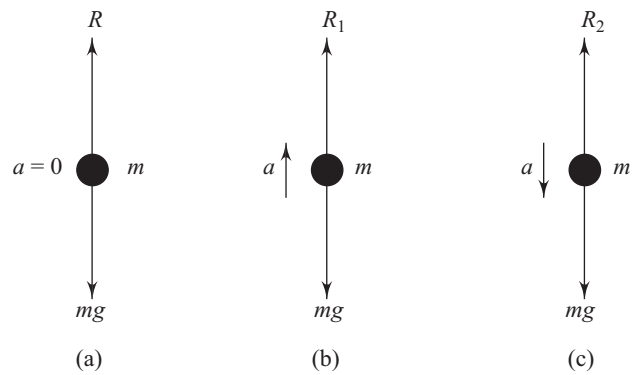


Fig. 3.29

(a) $R = mg = 50 \times 9.8 = 490$ N, the true weight of the boy.

(b) $R_1 - mg = ma$

$\Rightarrow R_1 = m(g + a) = 50 \times (9.8 + 2.2) = 600$ N

(c) $mg - R_2 = ma$

$\Rightarrow R_2 = m(g - a) = 50 \times (9.8 - 2.2) = 380$ N

Note If the elevator is moving up or down with a uniform velocity $a = 0$ then the reading of the machine gives the actual weight.

EXAMPLE 11 A uniform cord AB of mass $M = 2$ kg and length $L = 100$ cm is pulled at ends A and B will forces $F_1 = 4$ N and $F_2 = 3$ N as shown in Fig. 3.30(a). Find the tension at point P at a distance $x = 20$ cm from end A.

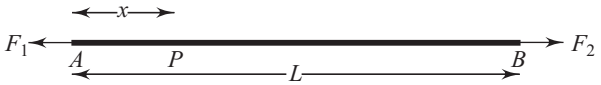


Fig. 3.30(a)

SOLUTION Since $F_1 > F_2$, the cord will accelerate to the left. Let a be the acceleration. To find tension T at P we consider the sections AP and PB of the cord.

$$\text{Mass of part } AP \text{ is } m_1 = \frac{Mx}{L}$$

$$\text{Mass of part } PB \text{ is } m_2 = \frac{M}{L}(L - x)$$

Figure 3.30(b) shows the free body diagrams of parts AP and PB.

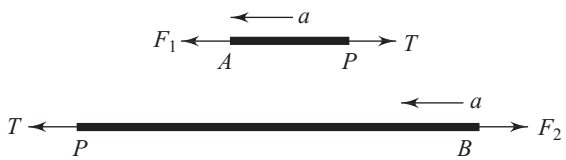


Fig. 3.30(b)

For part AP: $F_1 - T = m_1 a$ (i)

For part PB: $T - F_2 = m_2 a$ (ii)

Dividing (i) and (ii) we get

$$\frac{F_1 - T}{T - F_2} = \frac{m_1}{m_2} = \frac{\frac{Mx}{L}}{\frac{M}{L}(L - x)} = \frac{x}{L - x}$$

which gives $$T = \frac{F_1(L - x) + F_2 x}{L}$$

$$= \frac{4(1 - 0.2) + 3 \times 0.2}{1}$$

$$= 3.8 \text{ N}$$

EXAMPLE 12 A monkey of mass $m = 30$ kg is climbing up a rope with an acceleration $a = 5$ ms^{-2} relative to the rope. The rope passes over a frictionless fixed pulley and has a block of mass $M = 15$ kg at the other end as shown in Fig. 3.31(a). Find (a) acceleration of the rope, (b) acceleration of monkey and (c) tension in the rope. Take $g = 10$ ms^{-2}.

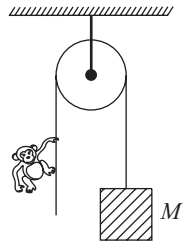


Fig. 3.31(a)

SOLUTION Let A be the acceleration of the block in the upward direction and T be the tension in the rope. The rope will have acceleration A in the downward direction. Hence the monkey will have a net acceleration $(A - a)$ in the downward direction. Figure 3.31(b) shows the free body diagrams of the monkey and the block.

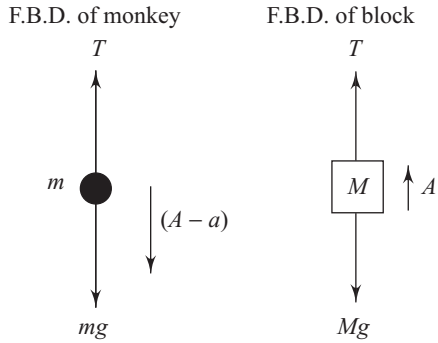


Fig. 3.31(b)

For monkey: $mg - T = m(A - a)$ (i)

For block: $T - Mg = MA$ (ii)

Adding (i) and (ii), we get

$$mg - Mg = m(A - a) + MA$$

$$\Rightarrow A = \frac{m(g + a) - Mg}{M + m}$$

$$= \frac{30(10 + 5) - 15 \times 10}{15 + 30}$$

$$= 6.7 \text{ ms}^{-2}$$

Acceleration of monkey $= A - a = 6.7 - 5$

$$= 1.7 \text{ ms}^{-2}$$

Tension in the rope $T = M(g + A)$

$$= 15 \times (10 + 6.7) \simeq 250 \text{ N}$$

EXAMPLE 13 A block of mass m = 2 kg is held stationary against a wall by applying a horizontal force F on it as shown in Fig. 3.32(a). If the coefficient of friction between the block and the wall is $\mu = 0.25$, find the minimum value of F required to hold the block against the wall. Take $g = 10\ \text{ms}^{-2}$.

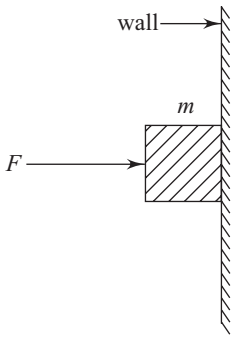


Fig. 3.32(a)

SOLUTION Let R be the normal reaction exerted on the block by the wall and f be the frictional force. Figure 3.32(b) shows the forces on the block.

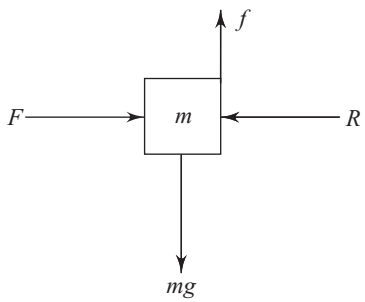


Fig. 3.32(b)

Since the block is held stationary, no net force acts on it. Hence

$$f = mg$$

and $$F = R$$

For no slipping, $f \le \mu R$ or $mg \le \mu F$ or $F \ge \dfrac{mg}{\mu}$

$$\therefore \quad F_{\min} = \frac{mg}{\mu} = \frac{2\times 10}{0.25} = 80\ \text{N}$$

EXAMPLE 14 A block of mass m = 2 kg is held in contact with a block of mass M = 10 kg by applying a horizontal force F on it as shown in Fig. 3.33(a). Block M is lying on a horizontal frictionless surface. The coefficient of friction between the blocks is $\mu = 0.4$. Find the minimum value of F required to hold m against M. Take $g = 10\ \text{ms}^{-2}$.

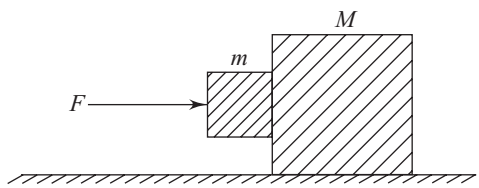


Fig. 3.33(a)

SOLUTION Since the two blocks are always in contact, they will have the same acceleration, say a. Figure 3.33(b) show the free body diagrams of the blocks.

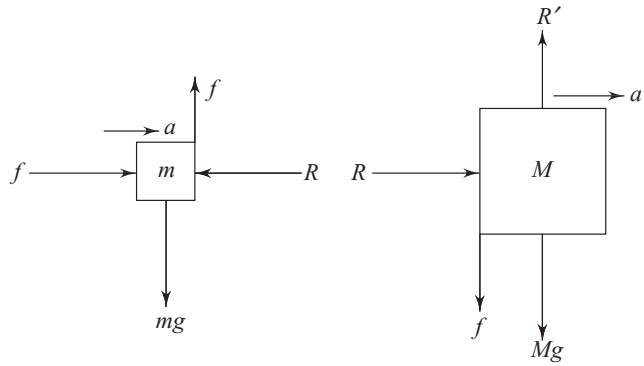


Fig. 3.33(b)

For block m : $$F - R = ma \qquad \text{(i)}$$

and $$f = mg \qquad \text{(ii)}$$

For block M : $$R = Ma \qquad \text{(iii)}$$

and $$Mg + f = R' \qquad \text{(iv)}$$

Eliminating a from (i) and (iii), we get

$$R = \frac{MF}{M+m}$$

For no slipping, $f \le \mu R$

or $$mg \le \frac{\mu MF}{M+m}$$

or $$F \ge \frac{mg(M+m)}{\mu M}$$

$$\therefore \quad F_{\min} = \frac{mg(M+m)}{\mu M}$$

$$= \frac{2\times 10(10+2)}{0.4\times 10} = 60\ \text{N}$$

EXAMPLE 15 A block P of mass m = 1 kg is placed over a plank Q of mass M = 6 kg placed over a smooth horizontal surface as shown in Fig. 3.34. Block P is given a velocity $v = 2\ \text{ms}^{-1}$ to the right. If the coefficient of friction between P and Q is $\mu = 0.3$, find the acceleration of Q relative to P.

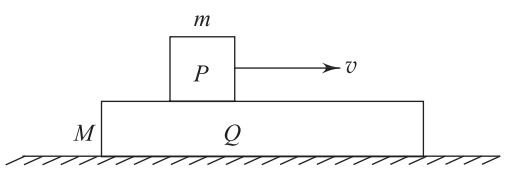


Fig. 3.34

SOLUTION Frictional force between P and Q is $f = \mu mg$ which will retard P and accelerate Q.

Retardation of P is $a_P = -\dfrac{f}{m} = -\dfrac{\mu mg}{m} = -\mu g$

Acceleration of Q is $a_Q = \dfrac{f}{M} = \dfrac{\mu mg}{M}$

$\therefore$ Acceleration of Q relative to P is

$$a_{QP} = a_Q - a_P = \frac{\mu mg}{M} - (-\mu g)$$

$$= \mu g\left(1 + \frac{m}{M}\right)$$

$$= 0.3 \times 10\left(1 + \frac{1}{6}\right)$$

$$= 3.5 \text{ ms}^{-2}$$

EXAMPLE 16 A block of mass $m = 500$ g is placed on the top of an inclined of inclination $\theta = 30°$ kept on the floor of a lift which is moving up with an acceleration $a = 2$ ms^{-2}. Find the coefficient of friction between the block and the incline so that the block moves down with a constant velocity.

SOLUTION $g_{eff} = g + a$

The block will move down the plane with a constant velocity if no net force acts on it, i.e.

Force down the plane = frictional force

$$mg_{eff} \sin\theta = \mu mg_{eff} \cos\theta$$

$$\Rightarrow \quad \mu = \tan\theta = \tan 30° \simeq 0.58$$

EXAMPLE 17 A cube of mass $m = 1$ kg is placed on a wedge of mass $M = 2$ kg as shown in Fig. 3.35(a). There is no friction between the cube and the wedge. Find the minimum coefficient of friction between the wedge and the horizontal surface so that the wedge does not move.

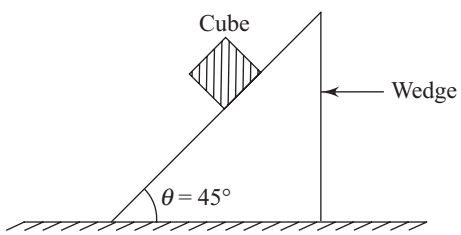


Fig. 3.35(a)

SOLUTION Figure 3.35(b) shows the horizontal force F_x and the vertical force F_y exerted by the cube on the wedge.

$F_x = (mg \cos\theta) \sin\theta$ and $F_y = (mg \cos\theta) \cos\theta$

Weight of the wedge = Mg acting vertically down wards. Hence Net horizontal force on the wedge is

$$F = mg \cos\theta \sin\theta$$

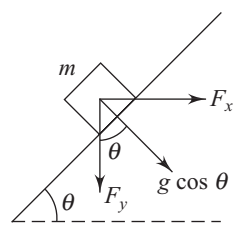


Fig. 3.35(b)

Net vertical force on the wedge is

$$N = Mg + mg \cos^2\theta$$

$\therefore$ $$\mu_{min} = \frac{F}{N} = \frac{m \cos\theta \sin\theta}{M + m\cos^2\theta}$$

$$= \frac{1 \times \cos 45° \times \sin 45°}{2 + 1 \times \cos^2 45°} = 0.2$$

EXAMPLE 18 In Example 17 above there is no friction between the cube and the wedge and between the wedge and the horizontal surface below. If the wedge moves towards the right with an acceleration $a = \dfrac{1}{\sqrt{2}}$ ms^{-2}, find the acceleration of the cube relative to the wedge when the cube is released.

SOLUTION Let A be the acceleration of the cube relative to the wedge as the cube moves down the plane. Its acceleration when the wedge moves to the right with acceleration a is $(A\cos\theta - a)$ directed towards the left. For dynamic equilibrium,

$$m(A\cos\theta - a) = Ma$$

$$\Rightarrow \quad A = \frac{(M+m)a}{m\cos\theta}$$

$$= \frac{(2+1) \times 1/\sqrt{2}}{1 \times \cos 45°}$$

$$= 3 \text{ ms}^{-2}$$

EXAMPLE 19 A block of mass m is lying on the floor of a lift. With what acceleration a should the lift descend so that the block exerts a force $mg/3$ on the floor of the lift?

(a) $\dfrac{g}{3}$ (b) $\dfrac{2g}{3}$

(c) g (d) $\dfrac{3g}{2}$

SOLUTION Refer to Fig. 3.36.

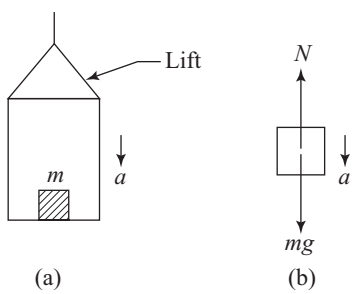


Fig. 3.36

Figure 3.36(b) shows the free body diagram of the block. It is clear that

$$mg - N = ma$$

For $$N = \frac{mg}{3},$$

$$mg - \frac{mg}{3} = ma$$

$$\Rightarrow \quad a = \frac{2g}{3}$$

EXAMPLE 20 A block of mass m is placed on a frictionless inclined plane of inclination θ. The inclined plane has its base fixed on the floor of a lift which is going up with a constant acceleration a. When the block is released, it will slide down the plane with acceleration

(a) $(g + a) \sin \theta$ (b) $(g - a) \sin \theta$

(c) $(g + a) \cos \theta$ (d) $(g - a) \cos \theta$

SOLUTION Refer to Fig. 3.37.

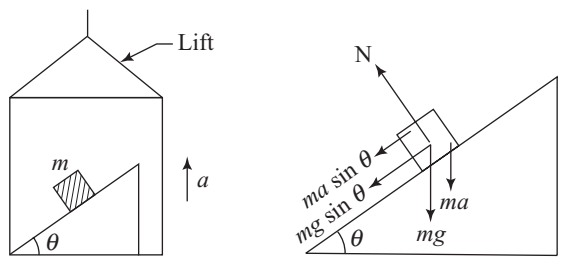


Fig. 3.37

Figure 3.37(b) shows the free body diagram of the block. The forces acting on the block are:

(i) weight mg downwards

(ii) normal reaction N perpendicular to the inclined plane

(iii) reaction force ma downwards (pseudo force).

The components of mg and ma parallel to the inclined plane are $mg \sin \theta$ and $ma \sin \theta$ respectively. The net force on the block sliding down the plane is

$$F = mg \sin \theta + ma \sin \theta = m(g + a) \sin \theta$$

$\therefore$ Acceleration of the block $= \frac{F}{m} = (g + a) \sin \theta$

EXAMPLE 21 In Example 22 above, the normal force acting on the block is

(a) $m(g + a) \sin \theta$ (b) $m(g - a) \sin \theta$

(c) $m(g + a) \cos \theta$ (d) $m(g - a) \cos \theta$

SOLUTION The components of mg and ma perpendicular to the inclined plane are $mg \cos \theta$ and $ma \cos \theta$ respectively [see Fig. 3.37(b)]. Since the block does not move perpendicular to the plane, the total force on it must be zero, i.e.

$$N - mg \cos \theta - ma \cos \theta = 0$$

$$\Rightarrow \quad N = m(g + a) \cos \theta$$

EXAMPLE 22 A block B of mass m placed on a horizontal frictionless surface is tied to a point A on a vertical wall by means of a massless string going over a frictionless moveable pulley P as shown in Fig. 3.38(a). A force F is applied to the pulley as shown. The acceleration of the block is

(a) $\frac{2F}{m}$ (b) $\frac{F}{m}$

(c) $\frac{F}{2m}$ (c) zero

SOLUTION Refer to Fig. 3.38.

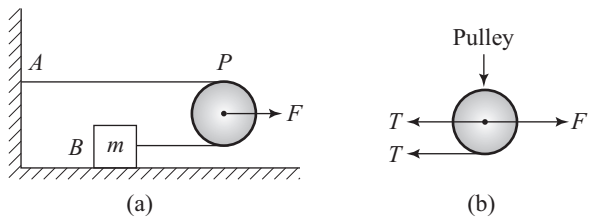


Fig. 3.38

Figure 3.38(b) shows the free body diagram of the pulley. The forces acting on the pulley are:

(i) F towards right

(ii) tension T towards left by portion PA of the string

(iii) tension T towards left by portion PB of the string

Since the mass of the pulley is zero, we have

$$F - T - T = 0$$

$$\Rightarrow \quad T = \frac{F}{2}$$

Now, for the block, the only horizontal force is tension T acting towards right, its acceleration is

$$a = \frac{T}{m} = \frac{F}{2m}$$

EXAMPLE 23 A block of mass m is suspended by strings AB and CB making angles α and β with the horizontal as shown in Fig. 3.39(a). If the strings have negligible mass, the tension in AB is

(a) $\frac{mg \cos \beta}{\sin(\alpha + \beta)}$ (b) $\frac{mg \sin \beta}{\sin(\alpha + \beta)}$

(c) $\frac{mg \cos \alpha}{\cos(\alpha + \beta)}$ (d) $\frac{mg \sin \alpha}{\cos(\alpha + \beta)}$

SOLUTION Refer to Fig. 3.39

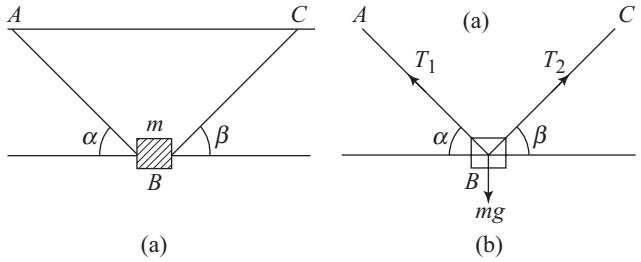


Fig. 3.39

Figure 3.39 (b) shows the free body diagram of the block. The horizontal components of T_1 and T_2 give

$$T_1 \cos \alpha = T_2 \cos \beta \quad \text{(i)}$$

For the vertical components, we have

$$T_1 \sin \alpha + T_2 \sin \beta = mg \quad \text{(ii)}$$

Eliminating T_2 from (i) and (ii), we get

$$T_1 = \frac{mg \cos \beta}{\sin (\alpha + \beta)}$$

EXAMPLE 24 Block A of mass m and block B of mass $2m$ are placed on a fixed triangular wedge by means of a massless string and a frictionless pulley as shown in the figure. The coefficient of friction between block A and the wedge is 2/3 and that between block B and the wedge is 1/3. If the blocks are released from rest, find the acceleration of block A.

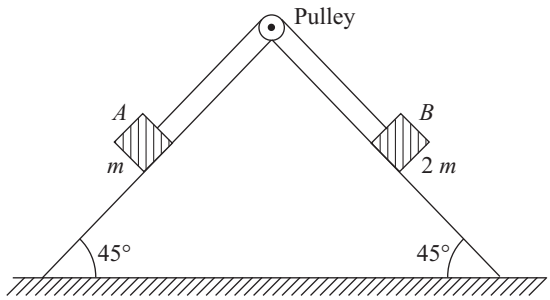


Fig. 3.40(a)

SOLUTION Case (a): Let us assume that block A moves up the plane and block B moves down the plane. The free body diagrams of the blocks are as follows.

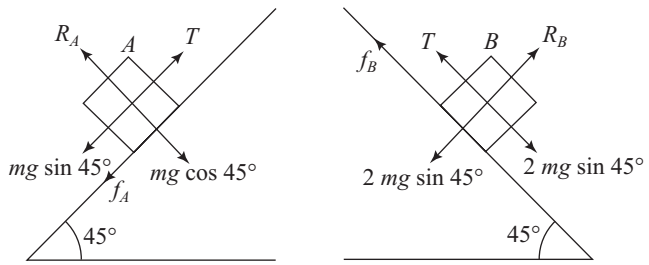


Fig. 3.40(b)

The equations of motion of blocks A and B are $T - mg \sin 45° - \mu_A mg \cos 45° = ma$, where $\mu_A = 2/3$ and $2\mu g \sin 45° - \mu_B 2mg \cos 45° - T = 2ma$, where $\mu_B = 1/3$.

Adding these equations and solving we get $a = -\frac{g}{9\sqrt{2}}$

Case (b): If we assume that block A moves down and block B moves up, we would get $a = -\frac{7g}{9\sqrt{2}}$. Thus in both cases, the acceleration has a negative value which implies that the blocks will decelerate. This is not possible because the blocks start from rest. Hence when the blocks are released, they move with zero acceleration. Thus acceleration of block $A = 0$.

9. Solving Problems in Mechanics by Constraint Relation Method

(1) *Constraint Relation for a Moveable Pulley*

Consider two blocks 1 and 2 attached at the ends of a string going over a moveable pulley. Let x_p be the displacement of the pulley and x_1 and x_2 be the displacements of the blocks.

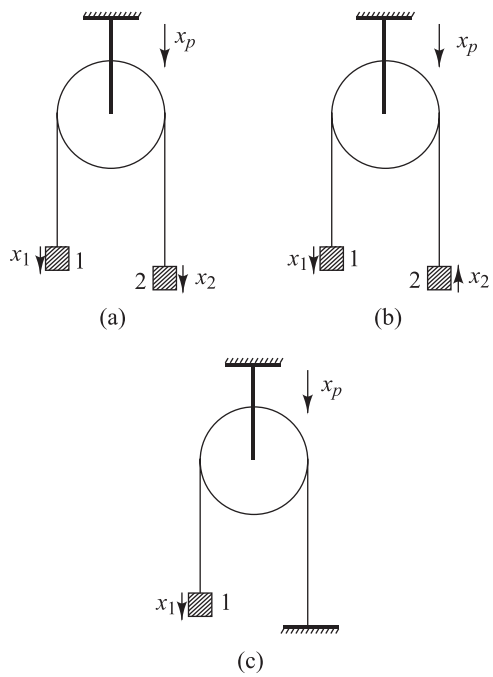


Fig. 3.41

The constraint relation is as follows. The displacement of the pulley = average displacement of the blocks, i.e.

(a) If the blocks are displaced in the direction of the displacement of the pulley, the constraint relation is [Fig. 3.41 (a)]

$$x_p = \frac{x_1 + x_2}{2}$$

(b) If block 1 moves up and block 2 moves down as shown in Fig. 3.41(b), the constraint relation is

$$x_p = \frac{x_1 - x_2}{2}$$

(c) If one ends of the string is connected to a fixed end, as shown in Fig. 3.41(c), then $x_2 = 0$ and the constraint relation is

$$x_p = \frac{x_1 + 0}{2} = \frac{x_1}{2} \quad \text{or} \quad x_1 = 2x_p$$

This result can also be obtained as proved in Example 25 below.

EXAMPLE 25 A block B of mass m placed on a horizontal frictionless surface is tied to a point A on a vertical wall by means of a masslass string going over a frictionless moveable pulley P as shown in Fig. 3.42. Show that if the pulley is displaced by a distance x_p, the block will be displaced by $2x_p$.

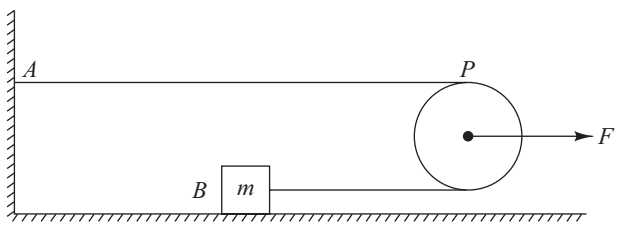


Fig. 3.42

SOLUTION Suppose the pulley is displaced to P' and block to B' (see Fig. 3.43)

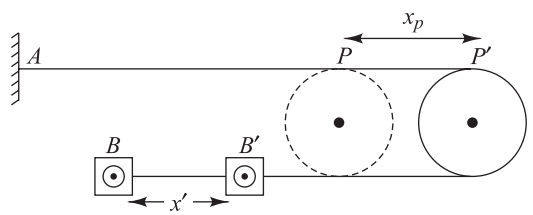


Fig. 3.43

Since the total length of the string remains the same

$$AP + PB = AP + PP' + P'P + PB'$$

Hence $$AP + PB' + B'B = AP + 2PP' + PB'$$

$$B'B = 2PP'$$

$$\Rightarrow \qquad x = 2x_p$$

EXAMPLE 26 If a force F is applied to the pulley as shown in Fig. 3.42, find (a) the acceleration of the block and (b) the acceleration of the pulley.

SOLUTION Refer to the solution of Example 22 on page 3.15.

(a) The acceleration of the block is $a = \frac{F}{2m}$

(b) Since the displacement of the pulley = half the displacement of the block, the acceleration of the pulley is

$$a_p = \frac{a}{2} = \frac{F}{4m}$$

EXAMPLE 27 Figure 3.44 shows two blocks 1 and 2 connected by means of strings to two pulleys P_1 and P_2. Pulley P_1 is fixed and pulley P_2 is moveable. Show that, if block 1 is moved down by a distance x_1, the block 2 moves up by a distance $x_2 = \frac{x_1}{2}$.

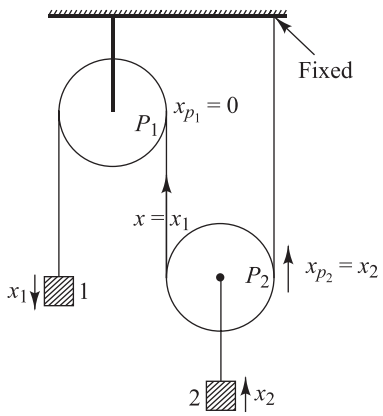


Fig. 3.44

SOLUTION As pulley P_1 is fixed $x_{p_1} = 0$. Therefore, if block 1 is moved down by x_1, the other end of the string must move up by x_1, because

$$x_{p_1} = 0 = \frac{x_1 + x}{2} \quad \Rightarrow \quad x = -x_1$$

Since the other end of the string going over pulley P_2 is fixed,

$$x_{P_2} = \frac{x_1 + 0}{2} = \frac{x_1}{2}$$

which gives $x_1 = 2x_{P_2} \Rightarrow x_1 = 2x_2 \Rightarrow x_2 = \frac{x_1}{2}$

EXAMPLE 28 Figure 3.45 shows two blocks 1 and 2 connected by means of strings to two pulleys P_1 and P_2. Pulley P_1 is fixed and pulley P_2 is moveable. Show that, if block 1 is moved up by x_1, the block 2 moves down by $x_2 = 2x_1$.

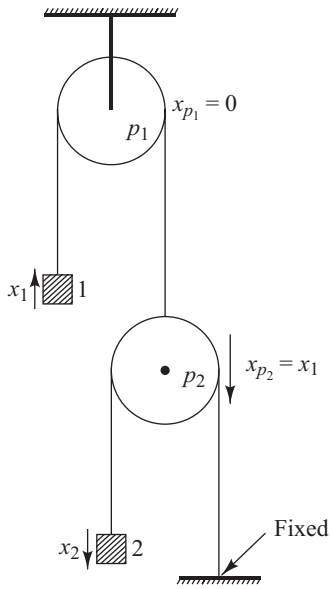


Fig. 3.45

SOLUTION As pulley P_1 is fixed, pulley P_2 will move down by x_1. Hence

$$x_{P_2} = x_1$$

For pulley 2,

$$x_{P_2} = -x_1 = \frac{x_2 + 0}{2} \quad \Rightarrow \quad x_2 = -2x_1.$$

The negative sign shows that if block 1 moves up, block 2 will move down.

(2) *Constraint Relation for a Moveable Wedge (or Inclined Plane)*

Consider a block of mass m moving with a certain velocity on a wedge (inclined plane) of inclination θ as shown in Fig. 3.46(a). The wedge in moved with a velocity u as shown.

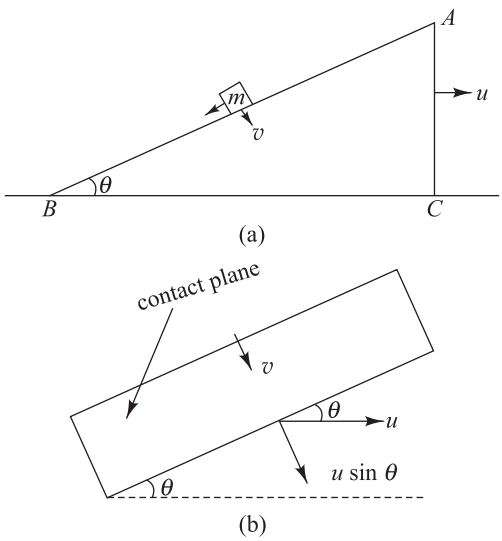


Fig. 3.46

To find the velocity v of the block perpendicular to the contact plane AB, we use the following constraint relation: *The relative velocity of the block (with respect to the wedge) in a direction perpendicular to the contact plane is always zero*. i.e. velocity v of the block perpendicular to AB = component of velocity v of the block perpendicular to AB. From Fig. 3.46 (b) it follows that

$$v = u \sin \theta$$

Note : If the wedge is immoveable or is at rest, $u = 0$ then $v = 0$, i.e. the block cannot move perpendicular to AB. This happens because the component $mg \cos \theta$ of the weight of the block balances with the normal reaction.

(3) *Constraint Relation when the Distance Between Two Points Always Remains Constant*

In cases when the distance between two points always remains constant, we use the following constraint relation. *The relative velocity of one point of an object with respect to any other point on the same object in the direction of the line joining them always remains equal to zero*, i.e. the velocity of one point on the object = the components of velocity of any other point along the line joining them. This is illustrated by the following examples 29 and 30.

EXAMPLE 29 A rod AB of length L is leaning on a wall and the floor at an angle θ as shwon in Fig. 3.47(a). The end A is moved with a constant velocity u to the left. Find the velocity v with which the end B moves downwards.

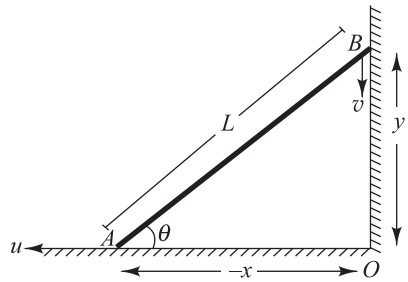


Fig. 3.47(a)

SOLUTION Using the constraint relation, (since the distance between points A and B always remains constant = L), we have [see Fig. 3.47(b)]

Velocity of B along BA = velocity of A along BA.

or $v \sin \theta = u \cos \theta$

$\Rightarrow$ $v = u \cot \theta$

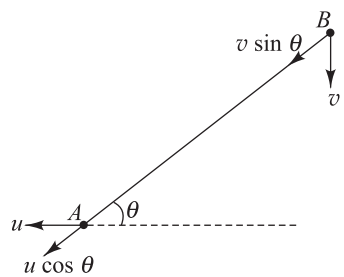


Fig. 3.47(b)

Alternative Method

$$x^2 + y^2 = L^2$$

Differentiating w.r.t. time t, we have (since L = constant)

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

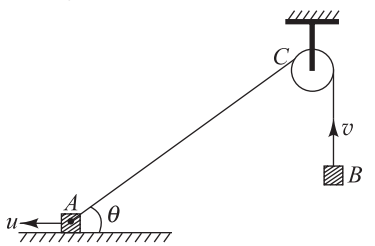
$$\Rightarrow \qquad u = -v\ \frac{y}{x} \quad \left(\because\ \frac{dx}{dt} = u,\ \frac{dy}{dt} = v\right)$$

$$\Rightarrow \qquad = +v\ \frac{y}{-x}$$

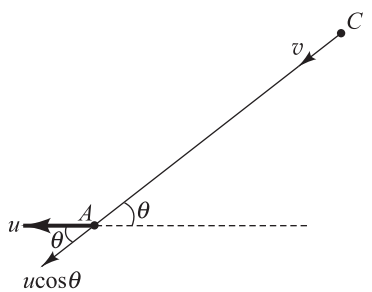
$$\Rightarrow \quad u = v\tan\theta \qquad \left(\because \tan\theta = \frac{OB}{OA}\right)$$

$$\Rightarrow \quad v = u\cot\theta$$

EXAMPLE 30 A block A placed one horizontal frictionless surface is tied to another block B by means of an inextensible string going over a pulley as shown in Fig. 3.48(a). The block A is moved towards left with a velocity u. Find the velocity with which block B moves upwards.



(a)



(b)

Fig. 3.48

SOLUTION Since the distance between points A and C remains constant [see Fig. 3.48(b)] and the pulley is fixed, velocity of point C along $CA = v$, the velocity of block B upwards.

Velocity of point C along CA = velocity of point A along CA

or $$v = u\cos\theta$$

10. Centripetal Acceleration

If a body moves in a circle at a constant speed, it is said to be in uniform circular motion. In such a motion, the magnitude of the velocity (i.e. speed) is constant but the direction of the velocity vector is continually changing. Thus the velocity is changing with time. Hence the motion of the body is accelerated (see Fig. 3.49). The acceleration is directed towards the centre of the circle and is called centripetal acceleration. The magnitude of the centripetal acceleration is given by

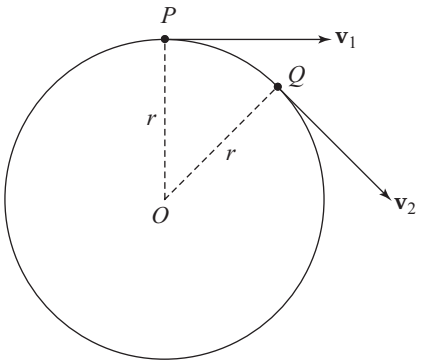


Fig. 3.49

$$a_c = \omega v$$

where ω is the angular velocity (or angular frequency) and v is the speed along the circle. Since $v = r\omega$, we have

$$a_c = \omega v = \omega^2 r = \frac{v^2}{r}$$

where r is the radius of the circular path. The angular frequency is related to time period T and frequency ν as

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

Therefore, centripetal acceleration is also given by (since $v = 2\pi r/T$)

$$a_c = \omega v = \frac{4\pi^2 r}{T^2} = 4\pi^2 r\nu^2$$

11. Banking of Round Tracks

When a car (or some other vehicle) negotiates a curved level road, the centripetal force required to keep the car in motion around the curve is provided by the friction between the road and the tyres. The weight of the car is supported by the normal reaction due to the earth. If the surface of the road is very rough, it provides a large amount of friction and hence the car can successfully negotiate the bend with a fairly high speed. If F is the total frictional force between the tyres and the road, then

$$F = \frac{mv^2}{R}$$

when m is the mass of the car, v its speed around the curve and R is the radius of the curved track. The higher the

value of F, the faster is the speed at which the bend can be negotiated. If μ is the coefficient of friction between the tyres and the road, then $F \le \mu N$; N = normal reaction = mg. The maximum speed which friction can sustain is

$$v \le v_{max} = (\mu Rg)^{1/2}$$

The large amount of friction between the tyres and the road would damage the tyres. To minimize the wearing out of tyres the road bed is banked, i.e. the outer part of the road is raised a little so that the road slopes towards the centre of the curved track. Suppose a car of mass m is moving around a banked track in a circular path of radius R as shown in Fig. 3.50. Let N_1 and N_2 be the reaction at each tyre due to the road. Then the total reaction is $N = N_1 + N_2$ acting in the middle of the car. If θ is the angle of the banking, the vertical component $N \cos \theta$ supports the weight mg of the car while the horizontal component $N \sin \theta$ provides the necessary centripetal force.

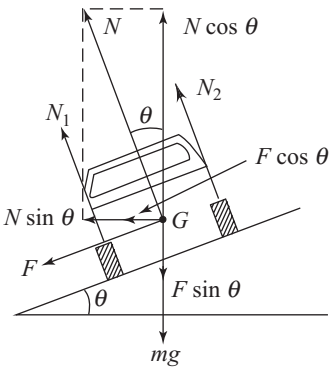


Fig. 3.50 Car on banked curved road

Thus

$$N \sin \theta = \frac{mv^2}{R} - F \cos \theta$$

and $$N \cos \theta = mg + F \sin \theta$$

Also $$F = \mu N$$

where F is the force of friction acting radially inwards on the car. These equations give

$$\tan \theta = \frac{v^2 - \mu Rg}{Rg + \mu v^2} \text{ and } v^2 = Rg \frac{\mu + \tan \theta}{1 - \mu \tan \theta}$$

The first equation determines the proper banking angle for given v, R and μ, and the second equation the maximum speed at which the car can successfully negotiate the curve for given R, μ and θ.

For given θ and R, there is an optimum (best) speed for negotiating a banked curve at which there will be the least wear and tear, i.e. when friction is not needed at all (μ = 0). If $\mu = 0$, this speed is

$$v = (Rg \tan \theta)^{1/2}$$

The car will not skid if the angle of banking of the track satisfies the relation

$$\tan \theta = \frac{v^2}{Rg}$$

12. A Cyclist Negotiating a Curved Level Road

While negotiating a curved level (unbanked) road, a cyclist has to lean inwards which provides the necessary centripetal force which prevents him from falling down. Figure 3.51 shows a cyclist leaning at an angle θ with the vertical. N is the normal reaction which is given by

$$N = mg$$

where m is the mass of the cyclist plus the bicycle. The force of friction between the road and the tyres is

$$F = \mu N = \mu mg$$

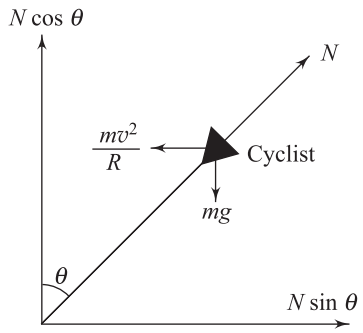


Fig. 3.51

The cyclist will skid if the centripetal force mv^2/R exceeds the frictional force F, i.e. if

$$\frac{mv^2}{R} > \mu mg$$

where R is the radius of the curved road. Thus skidding occurs if

$$v > \sqrt{\mu gR}$$

13. Motion in a Vertical Circle

Figure 3.52 shows an object of mass m whirled with a constant speed v in a vertical circle of centre O with a string of length R. When the object is at top A of the circle, let us say that the tension (force) in the string is T_1. Since the weight mg acts vertically downwards towards the centre O, we have,

Force towards centre, $$F = T_1 + mg = \frac{mv^2}{R}$$

or $$T_1 = \frac{mv^2}{R} - mg \quad \text{(i)}$$

At the point B, where OB is horizontal, the weight mg has no component along OB. Thus, if the tension in the string is T_2 at B, we have

Force towards centre,

$$F = T_2 = \frac{mv^2}{R} \quad \text{(ii)}$$

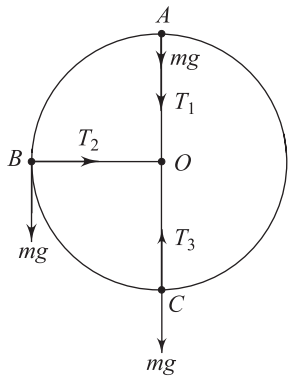


Fig. 3.52

At *C*, the lowest point of motion, the weight *mg* acts in the opposite direction to the tension T_3 in the string. Thus at *C* we have,

Force towards centre, $F = T_3 - mg = \dfrac{mv^2}{R}$

or $$T_3 = \frac{mv^2}{R} + mg$$

From (i), (ii) and (iii), we see that the maximum tension occurs at lowest point *C* of the motion. Here the tension T_3 must be greater than *mg* by $\dfrac{mv^2}{R}$ to keep the object in a circular path. The minimum tension is given by (i) when the object is at the highest point *A* of the motion. Here part of the required centripetal force is provided by the weight and the rest by T_1.

In order to keep a body of mass *m* in a circular path, the centripetal force, at the highest point *A*, must at least be equal to the weight of the body. Thus

$$\frac{mv_A^2}{R} = mg \quad \text{or} \quad v_A^2 = Rg \quad \text{or} \quad v_A = \sqrt{Rg}$$

gives the minimum speed the body must have at the highest point so that it can complete the circle. Then the minimum speed v_C the body must have at the lowest point *C* is given by

$$v_C^2 = v_A^2 + 2 \times 2\, Rg$$

where we have used $v^2 = u^2 + 2gh$, with $h = 2R$. Thus

$\therefore$ $$v_C^2 = Rg + 4\, Rg = 5\, Rg$$

or $$v_C = \sqrt{5Rg}$$

The tension at this point is given by

$$T_1 = m\left(\frac{v_C^2}{R} + g\right) = m\,(5g + g)$$

$$= 6\, mg$$

EXAMPLE 31 A body of mass $m = 20$ g is attached to an elastic spring of length $L = 50$ cm and spring constant $k = 2$ Nm^{-1}. The system is revolved in a horizontal plane with a frequency $\nu = 30$ rev/min. Find the radius of the circular motion and the tension in the spring.

SOLUTION Angular velocity

$$\omega = 2\pi\nu = 2\pi \times \frac{30}{60}$$

$$= \pi \text{ rad s}^{-1}.$$

For an elastic spring force $F = kx$ where x is the extension.

Radius of circular motion $r = L + x$.

Centripetal force = $mr\omega^2 = F$

Or $$m(L + x)\omega^2 = kx$$

$\Rightarrow$ $$x = \frac{mL\omega^2}{k - m\omega^2}$$

$$= \frac{0.02 \times 0.5 \times (3.14)^2}{2 - 0.02 \times (3.14)^2}$$

$$\simeq 0.05 \text{ m}$$

$\therefore$ $$r = L + x = 0.5 + 0.05$$

$$= 0.55 \text{ m}$$

Tension $$F = kx = 2 \times 0.05 \simeq 0.1 \text{ N}$$

EXAMPLE 32 A liquid of mass *M* and density ρ is filled in a tube *AB* of length *L*. The tube is rotated about end *A* with angular velocity ω. Obtain the expression for the force exerted at the other end *B*.

SOLUTION

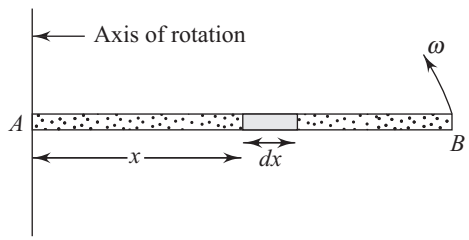


Fig. 3.53

Mass of element of length *dx* is $m = \dfrac{M}{L}dx$

Centripetal force on element = $\displaystyle\int_o^L m\omega^2 x$

$$= \frac{M\omega^2}{L}\int_o^L x\,dx$$

$$= \frac{1}{2}ML\omega^2$$

EXAMPLE 33 A conical pendulum has a string of length $l = 50$ cm and bob of mass $m = 200$ g. The bob is revolved in a horizontal circle of radius $r = 20$ cm. If the string makes an angle $\theta = 60°$ with the vertical, find (a) the tension in the string and (b) the speed of the bob around the circle. Take $g = 10$ ms^{-2}.

SOLUTION Refer to Fig. 3.54,

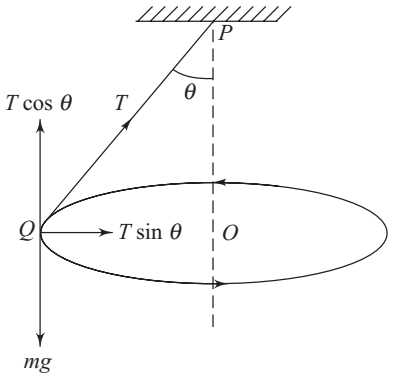


Fig. 3.54

$OQ = r$ and $PQ = l$

Force towards centre $O = T \sin\theta$

i.e. $$\frac{mv^2}{r} = T\sin\theta \quad \text{(i)}$$

Also $$mg = T\cos\theta \quad \text{(ii)}$$

(a) From (ii) $$T = \frac{mg}{\cos\theta} = \frac{0.2\times 10}{\cos 60^\circ} = 4 \text{ N}$$

(b) Dividing (i) by (ii) we have

$$\frac{v^2}{rg} = \tan\theta$$

$$\Rightarrow \quad v = \sqrt{rg\tan\theta}$$

$$= \sqrt{0.2\times 10\times \tan 60^\circ}$$

$$= 1.86 \text{ ms}^{-1}$$

EXAMPLE 34 A coin of mass $m = 10$ g is placed at a distance of 30 cm from the centre of a disc. The disc is rotated at 30 rev/min about a vertical axis passing through its centre. What should be the minimum value of the coefficient of friction between the coin and the disc so that the coin does not skid off the disc?

SOLUTION For no slipping, frictional force > centripetal force, i.e.

$$\mu mg > m\omega^2 r$$

$$\Rightarrow \quad \mu > \frac{\omega^2 r}{g}$$

$$\Rightarrow \quad \mu > \frac{(2\pi\nu)^2 r}{g}$$

$\nu = 30$ rev/min $= \frac{30}{60} = 0.5$ Hz,

$r = 0.3$ m and $g = 9.8$ ms^{-2}.

$$\therefore \quad \mu_{\min} = \frac{4\pi^2\nu^2 r}{g}$$

$$= \frac{4\times(3.14)^2\times(0.5)^2\times 0.3}{9.8}$$

$$= 0.3$$

EXAMPLE 35 A small sphere of mass $m = 500$ g moving on the inner surface of a large hemispherical bowl of radius $R = 5$ m describes a horizontal circle at a distance $OC = 2.5$ m below the centre O of the bowl as shown in Fig. 3.55. Find the force exerted by the sphere on the bowl and the time period of revolution of the sphree around the circle. Take $g = 10$ ms^{-2}.

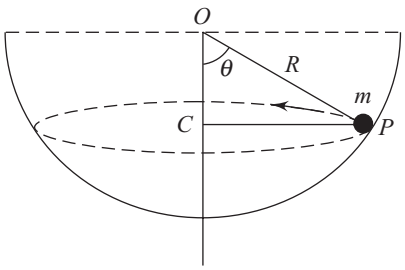


Fig. 3.55

SOLUTION Given $OP = 5$ m and $OC = 2.5$ m. Therefore $\cos\theta = \frac{OC}{OP} = \frac{1}{2} \Rightarrow \theta = 60^\circ$.

Radius of circle is $r = CP = OP\sin\theta = 5\sin 60^\circ = \frac{5\times\sqrt{3}}{2}$ m

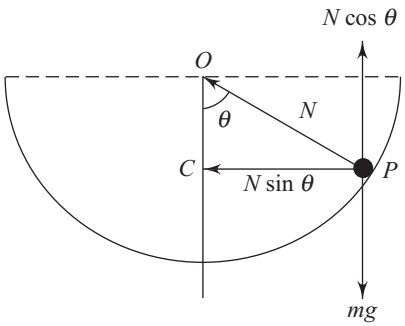


Fig. 3.56

Figure 3.56 shows the forces acting on the sphere, N is the normal reaction.

Net force towards centre C of the circle $= N\sin\theta$

$$\Rightarrow \quad mr\omega^2 = N\sin\theta \quad \text{(i)}$$

Also $$mg = N\cos\theta \quad \text{(ii)}$$

From (ii), $$N = \frac{mg}{\cos\theta} = \frac{0.5\times 10}{\cos 60^\circ} = 10 \text{ N}$$

From (i), $$m\omega^2 R\sin\theta = N\sin\theta$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{N}{mR}} = \sqrt{\frac{10}{0.5 \times 5}} = 2 \text{ rad s}^{-1}$$

$$\therefore \text{ Time period } \qquad T = \frac{2\pi}{\omega} = \pi \text{ second} = 3.14 \text{ s}$$

EXAMPLE 36 A steel metre rod of mass $m = 1.5$ kg rests with its upper end against a smooth vertical wall and its lower end on rough horizontal floor. What should be the minimum coefficient of friction between the ground and the rod so that it can be inclined at an angle of 30° with the floor without slipping?

SOLUTION Refer to Fig. 3.57. Length of rod $AB = 1$ m

N_1 = normal reaction of the wall

N_2 = normal reaction of the floor

The frictional force (f) between the rod and the floor acts along AD. The weight mg of the rod acts at its mid-point (centre of mass) C so that $AC = AB/2$.

For translational equilibrium,

$$N_2 = mg \quad \text{and} \quad N_1 = f$$

Taking moments of forces about A, we have for rotational equilibrium

$$N_2 \times 0 - mg \times AD + N_1 \times BE = 0$$

$$\Rightarrow \quad 0 - N_2 \times AD + f \times BE = 0 \; (\therefore \; mg = N_2, \; N_1 = f)$$

$$\Rightarrow \qquad \frac{f}{N_2} = \frac{AD}{BE} = \frac{AC\cos 30°}{AB \sin 30°}$$

$$\Rightarrow \qquad \mu = \frac{1}{2} \times \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{2} = 0.87$$

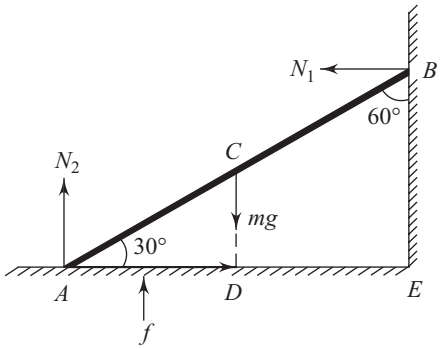


Fig. 3.57

1 SECTION — Multiple Choice Questions with One Correct Choice

Level A

1. Two masses m and $2m$ are joined to each other by means of a frictionless pulley as shown in Fig. 3.58. When the mass $2m$ is released, the mass m will ascend with an acceleration of

   (a) $\frac{g}{3}$ (b) $\frac{g}{2}$

   (c) g (d) $2g$

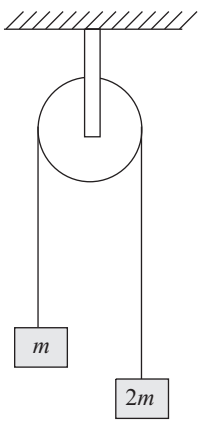


Fig. 3.58

2. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force F is applied at the free end of the rope, the net force exerted on the block will be

   (a) $\frac{FM}{(M+m)}$ (b) $\frac{Fm}{(M+m)}$

   (c) $\frac{FM}{(M-m)}$ (d) F

3. A block is pulled along a horizontal frictionless surface by a rope. The tension in the rope will be the same at all points on it
   (a) if and only if the rope is not accelerated
   (b) if and only if the rope is massless
   (c) if either the rope is not accelerated or is massless
   (d) always

4. A block is placed on the top of a smooth inclined plane of inclination θ kept on the floor of a lift. When the lift is descending with a retardation a, the block is released. The acceleration of the block relative to the incline is

(a) $g \sin \theta$ (b) $a \sin \theta$

(c) $(g - a) \sin \theta$ (d) $(g + a) \sin \theta$

5. A block takes twice as much time to slide down a rough 45° inclined plane as it takes to slide down an identical smooth 45° inclined plane. The coefficient of kinetic friction between the block and the rough inclined plane is

(a) 0.25 (b) 0.50

(c) 0.75 (d) 1.0

6. The upper half of an inclined plane of inclination θ is perfectly smooth while the lower half is rough. A block starting from rest at the top of the plane will again come to rest at the bottom if the coefficient of friction between the block and the lower half of the plane is given by

(a) $\mu = 2 \tan \theta$ (b) $\mu = \tan \theta$

(c) $\mu = \dfrac{2}{\tan \theta}$ (d) $\mu = \dfrac{1}{\tan \theta}$

7. A boy of mass m stands on one end of a wooden plank of length L and mass M. The plank is floating on water. If the boy walks from one end of the plank to the other end at a constant speed, the resulting displacement of the plank is given by

(a) $\dfrac{mL}{M}$ (b) $\dfrac{ML}{m}$

(c) $\dfrac{mL}{(M+m)}$ (d) $\dfrac{mL}{(M-m)}$

8. Ball A of mass m_1 moving with a velocity v undergoes a head–on collision with ball B of mass m_2 at rest. After collision, ball A continues moving in its original direction with half its original speed. The speed of ball B after collision will be

(a) $\dfrac{m_1 v}{2m_2}$ (b) $\dfrac{m_2 v}{2m_1}$

(c) $\dfrac{m_1 v}{m_2}$ (d) $\dfrac{m_2 v}{m_1}$

9. A shell of mass $2m$ fired with a speed u at an angle θ to the horizontal explodes at the highest point of its trajectory into two fragments of mass m each. If one fragment falls vertically, the distance at which the other fragment falls from the gun is given by

(a) $\dfrac{u^2 \sin 2\theta}{g}$ (b) $\dfrac{3u^2 \sin 2\theta}{2g}$

(c) $\dfrac{2u^2 \sin 2\theta}{g}$ (d) $\dfrac{3u^2 \sin 2\theta}{g}$

10. A truck carrying sand is moving on a smooth horizontal road with a uniform speed u. If a mass Δm of sand leaks in a time Δt from the bottom of the truck, the force needed to keep the truck moving at its uniform speed u is given by

(a) $\dfrac{\Delta mu}{\Delta t}$ (b) $\dfrac{\Delta mu}{2\Delta t}$

(c) $\dfrac{\Delta mu^2}{\Delta t}$ (d) zero

11. A block of mass m_2 lying on a horizontal frictionless surface is connected to a block of mass m_1 by means of string which passes over a frictionless pulley as shown in Fig. 3.59. If $m_1 > m_2$, the common acceleration of the masses is given by (g is the acceleration due to gravity)

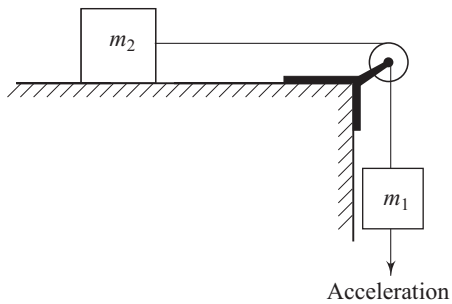


Fig. 3.59

(a) $\dfrac{m_1}{m_2} g$ (b) $\dfrac{m_2}{m_1} g$

(c) $\dfrac{m_1}{(m_1 + m_2)} g$ (d) $\dfrac{m_2}{(m_1 + m_2)} g$

12. A force F is applied horizontally to block A of mass m_1 which is in contact with a block B of mass m_2, as shown in Fig. 3.60. If the surfaces are frictionless, the force exerted by A on B is given by

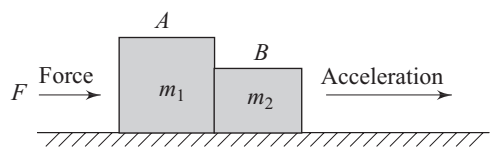


Fig. 3. 60

(a) $\dfrac{m_1}{m_2} F$ (b) $\dfrac{m_2}{m_1} F$

(c) $\dfrac{m_1 F}{(m_1 + m_2)}$ (d) $\dfrac{m_2 F}{(m_1 + m_2)}$

13. Two skaters A and B of mass 50 kg and 70 kg, respectively, stand facing each other, 6 metres apart on a horizontal smooth surface. They pull a rope

stretched between them. How far has each moved when they meet?
(a) Both have moved 3 m.
(b) A moves 4 m and B moves 2 m.
(c) A moves 2.5 m and B moves 3.5 m.
(d) A moves 3.5 m and B moves 2.5 m.

14. Which of the following statements is correct?
(a) A body has a constant velocity but a varying speed.
(b) A body has a constant speed but a varying acceleration.
(c) A body having a constant speed cannot have an acceleration.
(d) A body having a constant speed can have a varying velocity.

15. A person is sitting facing the engine in a moving train. He tosses a coin. The coin falls behind him. This shows that the train is
(a) moving forward with a finite acceleration
(b) moving forward with a finite retardation
(c) moving backward with a uniform speed
(d) moving forward with a uniform speed.

16. N bullets each of mass m kg are fired with a velocity v ms^{-1}, at the rate of n bullets per second, upon a wall. The reaction offered by the wall to the bullets is given by

(a) $nNmv$ (b) $\frac{Nmv}{n}$

(c) $\frac{nNm}{v}$ (d) $\frac{nNv}{m}$

17. A block A is released from the top of smooth inclined plane and slides down the plane. Another block B is dropped from the same point and falls vertically downwards. Which one of the following statements will be true if the friction offered by air is negligible?
(a) Both blocks will reach the ground at the same time.
(b) Block A reaches the ground earlier than block B.
(c) Both blocks will reach the ground with the same speed.
(d) Block B reaches the ground with a higher speed than block A.

18. A block is released from the top of an inclined plane of height h and angle of inclination θ. The time taken by the block to reach the bottom of the plane is given by

(a) $\sqrt{\frac{2h}{g}}$ (b) $\sin\theta\sqrt{\frac{2h}{g}}$

(c) $\frac{1}{\sin\theta}\cdot\sqrt{\frac{2h}{g}}$ (d) $\frac{1}{\cos\theta}\cdot\sqrt{\frac{2h}{g}}$

19. A shell, initially at rest, suddenly explodes into two equal fragments A and B. Which one of the following is observed?
(a) A and B move in the same direction at the same speed.
(b) A and B move at right angles to each other at the same speed.
(c) A and B move in opposite directions at the same speed.
(d) A and B move in any direction at the same speed.

20. A bomb at rest explodes into a large number of tiny fragments. The total momentum of all the fragments
(a) is zero
(b) depends on the total mass of all the fragments
(c) depends on the speeds of various fragments
(d) is infinity

21. A block of mass M is resting on an inclined plane as shown in Fig. 3.61. The inclination of the plane to the horizontal is gradually increased. It is found that when the angle of inclination is θ the block just begins to the slide down the plane. What is the minimum force F applied parallel to the plane that would just make the block move up the plane?

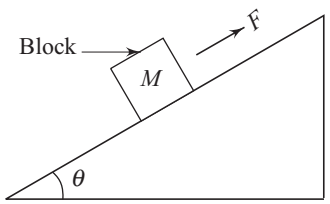


Fig. 3.61

(a) $Mg\sin\theta$ (b) $Mg\cos\theta$
(c) $2Mg\cos\theta$ (d) $2Mg\sin\theta$

22. The driver of a car moving at a speed of 20 ms^{-1} sees a child standing in the middle of the road. He immediately applies brakes to bring the car to rest in 5 s, just in time to save the child. If the mass of the car is 940 kg that of the driver 60 kg, what is the magnitude of the retarding force on the vehicle?
(a) 1000 N (b) 2000 N
(c) 3000 N (d) 4000 N

23. In Q.22, how far was the child from the car when the brakes were applied?
(a) 50 m (b) 75 m
(c) 100 m (d) 125 m

24. A body of mass 200 g is moving with a velocity of 5 ms^{-1} along the positive x–direction. At time $t = 0$, when the body is at $x = 0$, a constant force of 0.4 N

directed along the negative x–direction is applied to the body for 10 s. What is the position (x) of the body at $t = 2.5$ s?

(a) $x = 1.0$ m (b) $x = 1.25$ m

(c) $x = 1.5$ m (d) $x = 1.75$ m

25. In Q.24, what is the velocity of the body at $t = 2.5$ s?

(a) 7.5 ms^{-1} (b) 6.25 ms^{-1}

(c) 5.0 ms^{-1} (d) zero

26. In Q.24, what is the position (x) of the body at $t = 30$ s?

(a) $x = -350$ m (b) $x = -400$ m

(c) $x = -450$ m (d) $x = -500$ m

27. In Q. 24, what is the speed of the body at $t = 30$ s?

(a) 10 ms^{-1} (b) 15 ms^{-1}

(c) 20 ms^{-1} (d) 25 ms^{-1}

28. A train starts from rest with a constant acceleration $a = 2$ ms^{-2}. After 5 s, a stone is dropped from the window of the train. If the window is at a height of 2 m from the ground, what is the magnitude of the velocity of the stone 0.2 s after it was dropped? Take $g = 10$ ms^{-2}.

(a) $4\sqrt{6}$ ms^{-1} (b) 10 ms^{-1}

(c) $2\sqrt{26}$ ms^{-1} (d) 12 ms^{-1}

29. In Q.28, the angle, which the resultant velocity vector makes with the horizontal is given by

(a) $\theta = \tan^{-1}(0.1)$ (b) $\theta = \tan^{-1}(0.2)$

(c) $\theta = \tan^{-1}(0.3)$ (d) $\theta = \tan^{-1}(0.4)$

30. In Q.28, the acceleration of the stone after it is dropped is given by

(a) $g + a$ (b) $\sqrt{g^2 + a^2}$

(c) a (d) g

31. A rope which can withstand a maximum tension of 400 N is hanging from a tree. If a monkey of mass 30 kg climbs on the rope, in which of the following cases will the rope break? Take $g = 10$ ms^{-2} and neglect the mass of the rope.

(a) The monkey climbs up with a uniform speed of 5 ms^{-1}.

(b) The monkey climbs up with a uniform acceleration of 2 ms^{-2}.

(c) The monkey climbs up with a uniform acceleration of 5 ms^{-2}.

(d) The monkey climbs down with a uniform acceleration of 5 ms^{-2}.

32. A stream of a liquid of density ρ flowing horizontally with a speed v gushes out of a tube of radius r and hits at a vertically wall nearly normally. Assuming that the liquid does not rebound from the wall, the force exerted on the wall by the impact of liquid is given by

(a) $\pi r \rho v$ (b) $\pi r \rho v^2$

(c) $\pi r^2 \rho v$ (d) $\pi r^2 \rho v^2$

33. Two blocks of masses $m_1 = 5$ kg and $m_2 = 6$ kg are connected by a light string passing over a light frictionless pulley as shown in Fig. 3.62. The mass m_1 is at rest on the inclined plane and mass m_2 hangs vertically. If the angle of incline $\theta = 30°$, what is the magnitude and direction of the force of friction on the 5 kg block. Take $g = 10$ ms^{-2}.

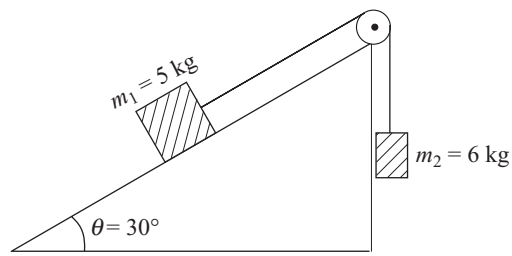


Fig. 3.62

(a) 35 N up the plane (b) 35 N down the plane

(c) 85 N up the plane (d) 85 N down the plane

34. A horizontal force of 300 N pulls two blocks of masses $m_1 = 10$ kg and $m_2 = 20$ kg which are connected by a light inextensible string and lying on a horizontal frictionless surface (Fig. 3.63). What is the acceleration of each mass?

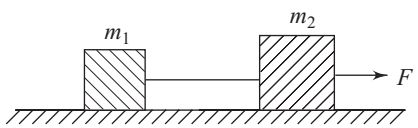


Fig. 3.63

(a) 10 ms^{-2} (b) 15 ms^{-2}

(c) 30 ms^{-2} (d) zero

35. In Q.34, the tension in the string is

(a) 100 N (b) 200 N

(c) 300 N (d) zero

36. In Q.34, the force on mass m_1 is

(a) 100 N (b) 200 N

(c) 300 N (d) zero

37. In Q.34, what will be the tension in the string if the force F is applied to mass m_1 as shown in Fig. 3.64?

(a) 100 N (b) 200 N

(c) 300 N (d) zero

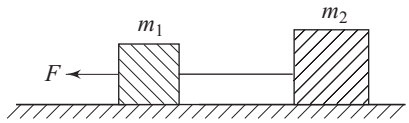


Fig. 3.64

38. In Q.37, the force exerted on mass m_2 is
(a) 100 N (b) 200 N
(c) 300 N (d) zero

Level B

39. A shell explodes into three fragments of equal masses. Two fragments fly off at right angles to each other with speeds of 9 ms^{-1} and 12 ms^{-1}. What is the speed of the third fragment?
(a) 9 ms^{-1} (b) 12 ms^{-1}
(c) 15 ms^{-1} (d) 18 ms^{-1}

40. A cricket ball of mass 150 g is moving with a velocity of 12 ms^{-1} and is hit by a bat so that the ball is turned back with a velocity of 20 ms^{-1}. The force of the blow acts for 0.1 s. What is the average force exerted on the ball by the bat?
(a) 18 N (b) 30 N
(c) 48 N (d) 60 N

41. Two billiard balls each of mass 50 g, moving in opposite directions each with a speed 6 ms^{-1}, collide and rebound with the same speed. The impulse imparted to each ball due to the other is
(a) 0.3 Ns (b) 0.6 Ns
(c) 0.9 Ns (d) 1.2 Ns

42. A ball of mass m is moving towards a batsman at a speed v. The batsman strikes the ball and deflects it by an angle θ without changing its speed. The impulse imparted to the ball is given by
(a) $mv\cos(\theta)$ (b) $mv\sin(\theta)$
(c) $2mv\cos\left(\frac{\theta}{2}\right)$ (d) $2mv\sin\left(\frac{\theta}{2}\right)$

43. Figure 3.65 shows the position–time (x–t) graph of one-dimensional motion of a body of mass 0.4 kg. What is the time interval between consecutive impulses received by the body?
(a) 2 s (b) 4 s
(c) 8 s (d) 16 s

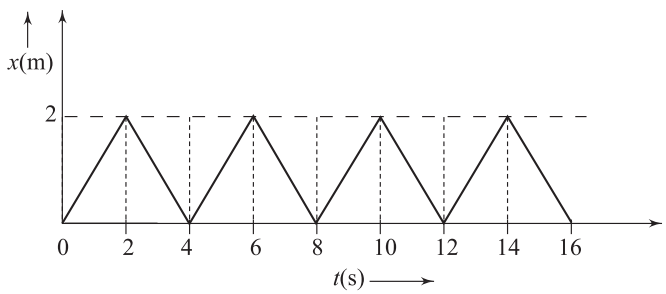


Fig. 3.65

44. In Q.43, what is the magnitude of each impulse?
(a) 0.2 Ns (b) 0.4 Ns
(c) 0.8 Ns (d) 1.6 Ns

45. An aeroplane of mass M requires a speed v for take–off. The length of the runway is s and the coefficient of friction between the tyres and the ground is μ. Assuming that the plane accelerates uniformly during the take–off, the minimum force required by the engine of the plane for take–off is given by
(a) $M\left(\frac{v^2}{2s}+\mu g\right)$ (b) $M\left(\frac{v^2}{2s}-\mu g\right)$
(c) $M\left(\frac{2v^2}{s}+2\mu g\right)$ (d) $M\left(\frac{2v^2}{s}-2\mu g\right)$

46. A block of mass m is projected up an inclined plane of inclination θ with an initial velocity u. If the coefficient of kinetic friction between the block and the plane is μ, the distance up to which the block will rise up the plane, before coming to rest, is given by
(a) $\frac{u^2\mu}{2g\sin\theta}$ (b) $\frac{u^2\mu}{2g\cos\theta}$
(c) $\frac{u^2}{4g\sin\theta}$ (d) $\frac{u^2}{4g\cos\theta}$

47. A block of mass 5 kg is resting on an inclined plane. The inclination of the plane to the horizontal direction is gradually increased. It is found that, when the angle of inclination is 30°, the block just begins to slide down the plane. The coefficient of sliding friction μ_s between the block and the plane is
(a) $\mu_s = \sin 30°$ (b) $\mu_s = \cos 30°$
(c) $\mu_s = \tan 30°$ (d) $\mu_s = \cot 30°$

48. In Q.47, what minimum force must be applied parallel to the plane that would just make the block move up the plane? Take g = 10 ms^{-2}.
(a) 25 N (b) 50 N
(c) $25\sqrt{3}$ N (d) $50\sqrt{3}$ N

49. A block of mass 10 kg is placed at a distance of 5 m from the rear end of a long trolley as shown in Fig. 3.66. The coefficient of friction between the block and the surface below is 0.2. Starting from rest, the trolley is given a uniform acceleration of 3 ms^{-2}. At what distance from the starting point will the block fall off the trolley? Take g = 10 ms^{-2}.
(a) 15 m (b) 20 m
(c) 25 m (d) 30 m

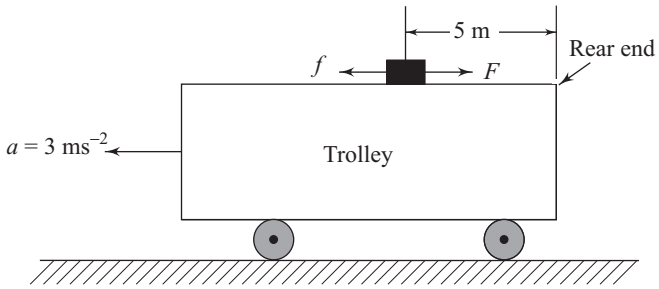


Fig. 3.66

50. Two blocks of equal masses $m_1 = m_2 = 3$ kg, connected by a light string, are placed on a horizontal surface which is not frictionless. If a force of 20 N is applied in the horizontal direction on a block, the acceleration of each block is 0.5 ms^{-2}. Assuming that the frictional forces on the two blocks are equal, the tension in the string will be
    (a) 10 N (b) 20 N
    (c) 40 N (d) 60 N
51. In Q.50, the frictional force on each block is
    (a) 6.5 N (b) 7.5 N
    (c) 8.5 N (d) 9.5 N
52. A body slides down an inclined plane of inclination θ. The coefficient of friction down the plane varies in direct proportion to the distance moved down the plane ($\mu = k\,x$). The body will move down the plane with a
    (a) constant acceleration = $g \sin \theta$
    (b) constant acceleration = $(g \sin \theta - \mu\, g \cos \theta)$
    (c) constant retardation = $(\mu\, g \cos \theta - g \sin \theta)$
    (d) variable acceleration that first decreases from $g \sin \theta$ to zero and then becomes negative.
53. A jet of water with a cross-sectional area a is striking against a wall at an angle θ to the normal and rebounds elastically. If the velocity of water in the jet is v, the normal force acting on the wall is,
    (a) $2\,a\,v^2\,\rho \cos \theta$ (b) $a\,v^2\,\rho \cos \theta$
    (c) $2\,a\,v\,\rho \cos \theta$ (d) $a\,v\,\rho \cos \theta$
54. A given object takes n times as much time to slide down a 45° rough incline as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is given by
    (a) $\mu_k = 1/(1 - n^2)$ (b) $\mu_k = 1 - 1/n^2$
    (c) $\mu_k = \sqrt{1/\left(1-n^2\right)}$ (d) $\sqrt{\left(1-1/n^2\right)}$
55. A body is sliding down a rough inclined plane of angle of inclination θ for which the coefficient of friction varies with distance x as $\mu(x) = kx$, where k is a constant. Here x is the distance moved by the body down the plane. The net force on the body will be zero at a distance x_0 given by
    (a) $\dfrac{\tan \theta}{k}$ (b) $k \tan \theta$
    (c) $\dfrac{\cot \theta}{k}$ (d) $k \cot \theta$
56. A body is moving down a long inclined plane of angle of inclination θ. The coefficient of friction between the body and the plane varies as $\mu = 0.5\,x$, where x is the distance moved down the plane. The body will have the maximum velocity when it has travelled a distance x given by
    (a) $x = 2 \tan \theta$ (b) $x = \dfrac{2}{\tan \theta}$
    (c) $x = \sqrt{2}\ \cot \theta$ (d) $x = \dfrac{\sqrt{2}}{\cot \theta}$
57. An object is kept on a smooth inclined plane of 1 in l. The horizontal acceleration to be imparted to the inclined plane so that the object is stationary relative to the incline is given by
    (a) $g\sqrt{l^2 - 1}$ (b) $g\,(l^2 - 1)$
    (c) $\dfrac{g}{\sqrt{l^2 - 1}}$ (d) $\dfrac{g}{l^2 - 1}$
58. An insect is crawling up a hemispherical bowl of radius R. If the coefficient of friction is 1/3, the insect will be able to go up to height h equal to (take $3/\sqrt{10} = 0.95$)
    (a) $\dfrac{R}{5}$ (b) $\dfrac{R}{10}$
    (c) $\dfrac{R}{20}$ (d) $\dfrac{R}{30}$
59. Two blocks of masses 5 kg and 3 kg are placed in contact on a horizontal frictionless surface as shown in Fig. 3.67. A force of 4 N is applied mass 5 kg as shown. The acceleration of the mass 3 kg will be
    (a) $\dfrac{4}{5}$ ms^{-2} (b) $\dfrac{4}{3}$ ms^{-2}
    (c) 2 ms^{-2} (d) 0.5 ms^{-2}

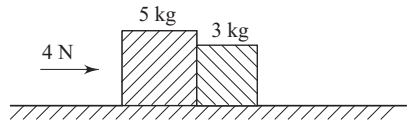


Fig. 3.67

60. In Q.59, what force is exerted on mass 3 kg?
    (a) 0.5 N (b) 1.0 N
    (c) 1.5 N (d) 4 N
61. In Q.59, if a force of 4 N is applied on mass 3 kg, the acceleration of mass 5 kg will be
    (a) $\frac{4}{3}$ ms^{-2} (b) $\frac{4}{5}$ ms^{-2}
    (c) 2 ms^{-2} (d) 0.5 ms^{-2}
62. In Q.61, what force is exerted on mass 5 kg?
    (a) 2.5 N (b) 3 N
    (c) 3.5 N (d) 4 N
63. Three blocks of masses $m_1 = 1$ kg, $m_2 = 2$ kg and $m_3 = 3$ kg are placed in contact on a horizontal frictionless surface as shown in Fig. 3.68. A force $F = 12$ N is applied to mass m_1 as shown. The acceleration of the system is

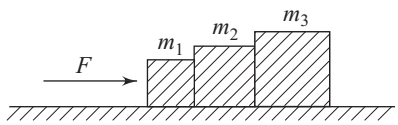


Fig. 3.68

    (a) 12 ms^{-2} (b) 6 ms^{-2}
    (c) 4 ms^{-2} (d) 2 ms^{-2}
64. In Q. 63, the contact force acting on mass m_2 is
    (a) 12 N (b) 10 N
    (c) 8 N (d) 6 N
65. In Q.63, the contact force acting on mass m_3 is
    (a) 12 N (b) 10 N
    (c) 8 N (d) 6 N
66. Three blocks of masses $m_1 = 1$ kg, $m_2 = 2$ kg and $m_3 = 3$ kg are connected by massless strings and placed on a horizontal frictionless surface as shown in Fig. 3.69. A force $F = 12$ N is applied to mass m_1 as shown. The acceleration of the system is
    (a) 12 ms^{-2} (b) 6 ms^{-2}
    (c) 4 ms^{-2} (d) 2 ms^{-2}

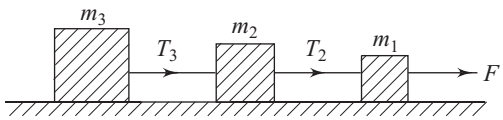


Fig. 3.69

67. In Q.66, what is tension T_2 between masses m_1 and m_2?
    (a) 12 N (b) 10 N
    (c) 8 N (d) 6 N
68. In Q.66, what is tension T_3 between masses m_2 and m_3?
    (a) 12 N (b) 10 N
    (c) 8 N (d) 6 N
69. A ball of mass m is connected to a ball of mass M by means of a massless spring. The balls are pressed so that the spring is compressed. When released, ball of mass m moves with acceleration a. The magnitude the acceleration of mass M will be
    (a) $\frac{ma}{(M+m)}$ (b) $\frac{Ma}{(M+m)}$
    (c) $\frac{ma}{M}$ (d) $\frac{Ma}{m}$
70. A boy, while catching a ball, experiences an impulse of 6 Ns. If the mass of the ball is 200 g, what was the speed of the ball before it was caught?
    (a) 10 ms^{-1} (b) 20 ms^{-1}
    (c) 30 ms^{-1} (d) 40 ms^{-1}
71. A car moving at a speed v is stopped by a retarding force F in a distance s. If the speed of the car were $3v$, the force needed to stop it within the same distance s will be
    (a) 3 F (b) 6 F
    (c) 9 F (d) 12 F
72. A car moving at a speed v is stopped by a retarding force F in a distance s. If the retarding force were $3F$, the car will be stopped in a distance
    (a) $\frac{s}{3}$ (b) $\frac{s}{6}$
    (c) $\frac{s}{9}$ (d) $\frac{s}{12}$
73. Two blocks of masses m and M are placed on a horizontal frictionless table connected by a spring as shown in Fig. 3.70. Mass M is pulled to the right with a force F. If the acceleration of mass m is a, the acceleration of mass M will be
    (a) $\frac{(F-ma)}{M}$ (b) $\frac{(F+ma)}{M}$
    (c) $\frac{F}{M}$ (d) $\frac{am}{M}$

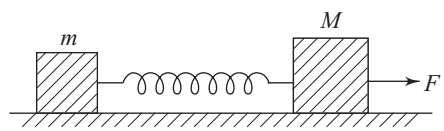


Fig. 3.70

74. A boy wants to climb down a rope. The rope can withstand a maximum tension equal to two–thirds the weight of the boy. If g is the acceleration due to

gravity, the minimum acceleration with which the boy should climb down the rope should be

(a) $\frac{g}{3}$ (b) $\frac{2g}{3}$

(c) g (d) zero

75. A mass m is suspended from a rigid support P by means of a massless string as shown in Fig. 3.71. A horizontal force F is applied at point O of the string. The system is in equilibrium when the string makes an angle θ with the vertical. Then the relation between the tension T, force F and angle θ is

(a) $F = T \sin \theta$ (b) $F = T \cos \theta$

(c) $F = \frac{T}{\sin\theta}$ (d) $F = \frac{T}{\cos \theta}$

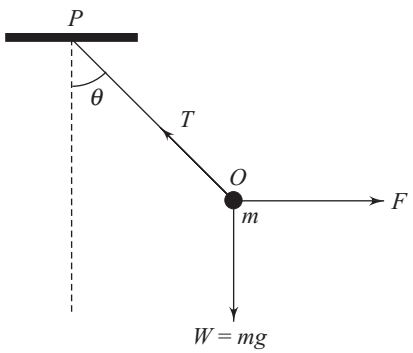


Fig. 3.71

76. In Q. 75, what is the relation between the weight $W = mg$, T and θ?

(a) $W = T \sin \theta$ (b) $W = T \cos \theta$

(c) $W = \frac{T}{\sin\theta}$ (d) $W = \frac{T}{\cos \theta}$

77. In Q. 75, the relation between T, W and F is

(a) $T^2 = WF$ (b) $T = \frac{W^2}{F}$

(c) $T = \frac{F^2}{W}$ (d) $T^2 = W^2 + F^2$

78. In Q. 75, the relation between F, W and θ is

(a) $F = W \tan \theta$ (b) $F = W \cot \theta$

(c) $F = W \sin \theta$ (d) $F = W \cos \theta$

79. Two masses $m_1 = 6$ kg and $m_2 = 4$ kg are connected by means of a string which passes over a frictionless pulley as shown in Fig. 3.72. When the masses are released, what is the acceleration of the masses? Take $g = 10$ ms^{-2}.

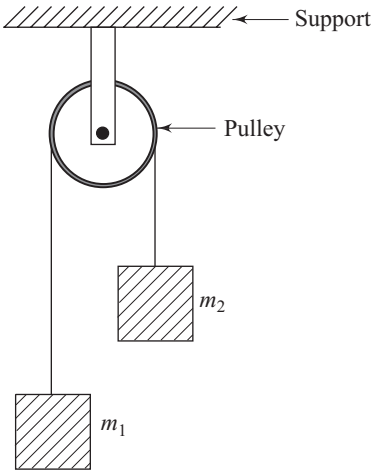


Fig. 3.72

(a) 2 ms^{-2} (b) 4 ms^{-2}

(c) 8 ms^{-2} (d) 10 ms^{-2}

80. In Q.79, what is the tension in the string?

(a) 40 N (b) 48 N

(c) 56 N (d) 60 N

81. The support of the pulley in Q.79 is attached to the ceiling of a lift. What will be the tension in the string if the lift starts ascending with a constant acceleration $a = 5$ ms^{-2}?

(a) 24 N (b) 48 N

(c) 60 N (d) 72 N

82. In Q. 81, what will be the tension in the string if the lift starts descending with a constant acceleration of $a = 5$ ms^{-2}?

(a) 24 N (b) 48 N

(c) 60 N (d) 72 N

83. The support of the pulley in Q. 79 is attached to the ceiling of a compartment of a train. What will be the tension in the string if the train moves in the horizontal direction with a constant acceleration of 5 ms^{-2}?

(a) 24 N (b) 48 N

(c) 60 N (d) 72 N

84. A block, released from rest from the top of a smooth inclined plane of angle of inclination θ_1, reaches the bottom in time t_1. The same block, released from rest from the top of another smooth inclined plane of angle of inclination θ_2, reaches the bottom in time t_2. If the two inclined planes have the same height, the relation between t_1 and t_2 is

(a) $\frac{t_2}{t_1}=\left(\frac{\sin\theta_1}{\sin\theta_2}\right)^{1/2}$ (b) $\frac{t_2}{t_1}=\frac{\sin^2\theta_1}{\sin^2\theta_2}$

(c) $\frac{t_2}{t_1}=\frac{\sin\theta_1}{\sin\theta_2}$ (d) $\frac{t_2}{t_1}=1$

85. A thick uniform rope of mass 6 kg and length 3 m is hanging vertically from a rigid support. The tension in the rope at a point 1 m from the support will be (Take $g = 10\ \text{ms}^{-2}$)
(a) 20 N (b) 30 N
(c) 40 N (d) 60 N

86. A block released from rest from the top of a smooth inclined plane of inclination 45° takes t seconds to reach the bottom. The same block released from rest from top of a rough inclined plane of the same inclination of 45° takes $2t$ seconds to reach the bottom. The coefficient of friction is
(a) $\sqrt{0.5}$ (b) $\sqrt{0.75}$
(c) 0.5 (d) 0.75

87. A block, released from rest from the top of a smooth inclined plane of inclination θ, has a speed v when it reaches the bottom. The same block, released from the top of a rough inclined plane of the same inclination θ, has a speed v/n on reaching the bottom, where n is a number greater than unity. The coefficient of friction is given by
(a) $\mu=\left(1-\frac{1}{n^2}\right)\tan\theta$
(b) $\mu=\left(1-\frac{1}{n^2}\right)\cot\theta$
(c) $\mu=\left(1-\frac{1}{n^2}\right)^{1/2}\tan\theta$
(d) $\mu=\left(1-\frac{1}{n^2}\right)^{1/2}\cot\theta$

88. Two blocks of masses $M = 5$ kg and $m = 3$ kg are placed on a horizontal surface as shown in Fig. 3.73. The coefficient of friction between the blocks is 0.5 and that between the block M and the horizontal surface is 0.7. What is the maximum horizontal force F that can be applied to block M so that the two blocks move without slipping? Take $g = 10\ \text{ms}^{-2}$.

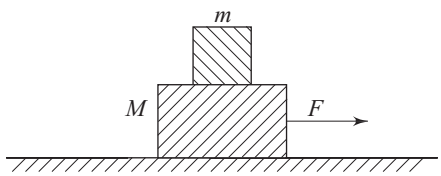


Fig. 3.73

(a) 4 N (b) 16 N
(c) 24 N (d) 96 N

89. A box is gently placed on a horizontal conveyer belt moving with a speed of 4 ms^{-1}. If the coefficient of friction between the box and the belt is 0.8, through what distance will the block slide without slipping? Take $g = 10\ \text{ms}^{-2}$
(a) 0.6 m (b) 0.8 m
(c) 1.0 m (d) 1.2 m

90. Two blocks of masses $m_1 = 4$ kg and $m_2 = 6$ kg are connected by a string of negligible mass passing over a frictionless pulley as shown in Fig. 3.74. The coefficient of friction between block m_1 and the horizontal surface is 0.4. When the system is released, the masses m_1 and m_2 start accelerating. What additional mass m should be placed over mass m_1 so that the masses $(m_1 + m)$ slide with a uniform speed?

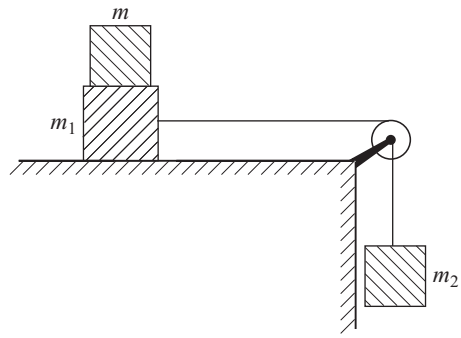


Fig. 3.74

(a) 9 kg (b) 10 kg
(c) 11 kg (d) 12 kg

91. Two blocks of masses $m_1 = m_2 = m$, are connected by a string of negligible mass which passes over a frictionless pulley fixed on the top of an inclined plane as shown in Fig. 3.75. When the angle of inclination $\theta = 30°$, the mass m_1 just begins to move up the inclined plane. What is the coefficient of friction between block m_1 and the inclined plane?

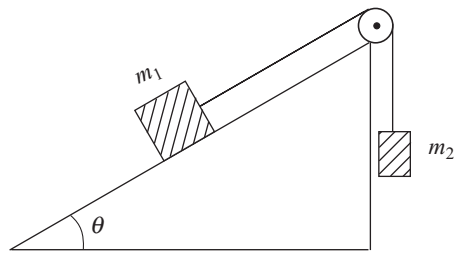


Fig. 3.75

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2\sqrt{3}}$

92. In Q. 91 above, if $\mu = 1/2\sqrt{3}$ and $\theta = 30°$, what should be the ratio m_1/m_2 of the masses so that mass m_1 just begins to slide down the plane?
(a) 4 (b) 3
(c) 2 (d) 1

93. A block is resting on an inclined plane. The angle of inclination is gradually increased. The block just begins to slide down the plane when the angle of inclination is 30°. What is the coefficient of friction between the inclined surface and the block?
(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{1}{2\sqrt{3}}$ (d) $\frac{1}{3\sqrt{3}}$

94. In Q. 93, if $\theta = 30°$ and $\mu = 1/\sqrt{3}$, the downward acceleration of the block will be
(a) zero (b) $1/\sqrt{3}$ ms^{-2}
(c) $1/2\sqrt{3}$ ms^{-2} (d) $1/3\sqrt{3}$ ms^{-2}

95. A boy of mass m is sliding down a vertical pole by pressing it with a horizontal force f. If μ is the coefficient of friction between his palms and the pole, the acceleration with which he slides down will be
(a) g (b) $\frac{\mu f}{m}$
(c) $g + \frac{\mu f}{m}$ (d) $g - \frac{\mu f}{m}$

96. A boy of mass 40 kg is climbing a vertical pole at a constant speed. If the coefficient of friction between his palms and the pole is 0.8 and $g = 10$ ms^{-2}, the force that he is applying on the pole is
(a) 300 N (b) 400 N
(c) 500 N (d) 600 N

97. A block of mass m is lying on a another block of mass M, lying on a horizontal frictionless surface as shown in Fig. 3.76. If the coefficient static friction between the two blocks is μ_s, the minimum horizontal force F that must be applied to block of mass m so that it moves over block of mass M is
(a) zero (b) mg
(c) mg/μ_s (d) $\mu_s mg$

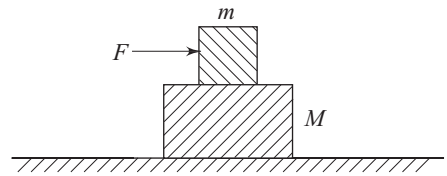


Fig 3.76

98. In Q. 97, if a force F ($> \mu_s mg$) is applied to block m, the acceleration with which block M will move on the horizontal surface is given by (here μ_k is the coefficient of kinetic friction between block M and the horizontal surface).
(a) $\frac{\mu_k mg}{M}$ (b) $\frac{\mu_s Mg}{m}$
(c) $\frac{\mu_s}{\mu_k}\frac{mg}{M}$ (d) $\frac{\mu_s}{\mu_k}\frac{Mg}{m}$

99. A block of mass 5 kg is lying on a rough horizontal surface. The coefficients of static and kinetic friction between the block and the surface respectively are 0.7 and 0.5. A horizontal force just sufficient to move the block is applied to it. If the force continues to act even after the block has started moving, the acceleration of block will be (take $g = 10$ ms^{-2})
(a) 1 ms^{-2} (b) 2 ms^{-2}
(c) 3 ms^{-2} (d) 4 ms^{-2}

100. A body projected along an inclined plane of angle of inclination 30° stops after covering a distance x_1. The same body projected with the same speed stops after covering a distance x_2, if the angle of inclination of the inclined plane is increased to 60°. The ratio x_1/x_2 is
(a) 1 (b) $\sqrt{2}$
(c) $\sqrt{3}$ (d) 2

101. A spring is compressed between two blocks of masses m_1 and m_2 placed on a horizontal frictionless surface as shown in Fig. 3.77. When the blocks are released, they have initial velocity of v_1 and v_2 as shown. The blocks travel distances x_1 and x_2 respectively before coming to rest. The ratio x_1/x_2 is

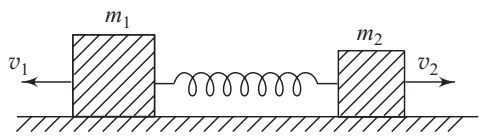


Fig. 3.77

(a) $\frac{m_1}{m_2}$ (b) $\frac{m_2}{m_1}$
(c) $\sqrt{\frac{m_1}{m_2}}$ (d) $\sqrt{\frac{m_2}{m_1}}$

102. A smooth inclined plane of angle of inclination 30° is placed on the floor of a compartment of a train moving with a constant acceleration a. When a block is placed on the inclined plane, it does not slide down or up the plane. The acceleration a must be

(a) g (b) $\frac{g}{2}$

(c) $\frac{g}{\sqrt{2}}$ (d) $\frac{g}{\sqrt{3}}$

103. A block released on a rough inclined plane of inclination $\theta = 30°$ slides down the plane with an acceleration $g/4$, where g is the acceleration due to gravity. What is the coefficient of friction between the block and the inclined plane?

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{2\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

104. A block of mass m placed on a rough inclined plane of inclination $\theta = 30°$ can be just prevented from sliding down by applying a force F_1 up the plane and it can be made to just slide up the plane by applying a force F_2 up the plane. If the coefficient of friction between the block and the inclined plane is $1/2\sqrt{3}$, the relation between F_1 and F_2 is

(a) $F_2 = F_1$ (b) $F_2 = 2F_1$

(c) $F_2 = 3F_1$ (d) $F_2 = 4F_1$

105. A person standing in a stationary lift drops a coin from a certain height h. It takes time t to reach the floor of the lift. If the lift is rising up with a uniform acceleration a, the time taken by the coin, dropped from the same height h, to reach the floor will be

(a) t (b) $t\sqrt{\frac{a}{g}}$

(c) $t\left(1+\frac{a}{g}\right)^{1/2}$ (d) $t\left(1-\frac{a}{g}\right)^{1/2}$

106. In Q. 105, if the lift is descending with an acceleration a, the time taken by the coin to reach the floor will be

(a) t (b) $t\sqrt{\frac{a}{g}}$

(c) $t\left(1+\frac{a}{g}\right)^{1/2}$ (d) $t\left(1-\frac{a}{g}\right)^{1/2}$

107. A block is lying on a horizontal frictionless surface. One end of a uniform rope is fixed to the block which is pulled in the horizontal direction by applying a force F at the other end. If the mass of the rope is half the mass of the block, the tension in the middle of the rope will be

(a) F (b) $\frac{2F}{3}$

(c) $\frac{3F}{5}$ (d) $\frac{5F}{6}$

108. When a force F acts on a body of mass m, the acceleration produced in the body is a. If three equal forces $F_1 = F_2 = F_3 = F$ act on the same body as shown in Fig. 3.78 the acceleration produced is

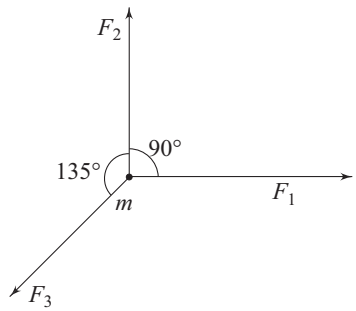


Fig. 3.78

(a) $\left(\sqrt{2}-1\right)a$ (b) $\left(\sqrt{2}+1\right)a$

(c) $\sqrt{2}\ a$ (d) a

109. A shell of mass m is at rest initially. It explodes into three fragments having masses in the ratio $2:2:1$. The fragments having equal masses fly off along mutually perpendicular directions with speed v. What will be the speed of the third (lighter) fragment?

(a) v (b) $\sqrt{2}\ v$

(c) $2\sqrt{2}\ v$ (d) $3\sqrt{2}\ v$

110. The linear momentum p of a body of mass 2 kg varies with time t as

$$p = 3t^2 + 4$$

where p and t are in SI units. It follows that the body is moving with a

(a) constant speed
(b) constant acceleration
(c) variable acceleration
(d) variable retardation.

111. In Q. 110, what is the acceleration of the body at $t = 2$s?

(a) 3 ms^{-2} (b) 6 ms^{-2}

(c) 9 ms^{-2} (d) 12 ms^{-2}

112. In Q. 110, the force acting on the body varies with time t as

(a) $t^{1/2}$ (b) t

(c) $t^{3/2}$ (d) t^2

113. An elastic spring has a length l_1 when it is stretched with a force of 2 N and a length l_2 when it is stretched with a force of 3 N. What will be the length of the spring if it is stretched with force of 5N?

(a) $l_1 + l_2$ (b) $\frac{1}{2}(l_1 + l_2)$

(c) $3l_2 - 2l_1$ (d) $3l_1 - 2l_2$

114. A shell is fired from a cannon with a speed of $100\ \text{ms}^{-1}$ at an angle 30° with the vertical (y-direction). At the highest point of its trajectory, the shell explodes into two fragments of masses in the ratio 1 : 2. The lighter fragment moves vertically upwards with an initial speed of $200\ \text{ms}^{-1}$. What is the speed of the heavier fragment at the time of explosion?

(a) $125\ \text{ms}^{-1}$ (b) $150\ \text{ms}^{-1}$

(c) $175\ \text{ms}^{-1}$ (d) $200\ \text{ms}^{-1}$

115. A balloon of mass M is rising up with an acceleration a. If a mass m is removed from the balloon, its acceleration becomes

(a) $\frac{Ma + mg}{M - m}$ (b) $\frac{Ma + mg}{M + m}$

(c) $\frac{ma + Mg}{M - m}$ (d) $\frac{ma + Mg}{M + m}$

116. A uniform rope is hanging vertically from the ceiling such that its free end just touches the horizontal floor of a room. The upper end of the rope is then released. At any instant during the fall of the rope, the total force exerted by it on the floor is n times the weight of that part of the rope which is on the floor at that time. What is the value of n?

(a) 1 (b) 2

(c) 3 (d) 4

117. The distance x (in metre) travelled by a body in time t (in second) is given by

$$\frac{d^2x}{dt^2} = 2t - t^2$$

How much distance does the body travel before reversing its direction of motion?

(a) $\frac{3}{4}$ m (b) $\frac{(3)^2}{4}$ m

(c) $\frac{(3)^3}{4}$ m (d) $\frac{(3)^4}{4}$ m

118. A long horizontal rod has a bead which can slide along its length, and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is

(a) $\sqrt{\frac{\mu}{\alpha}}$ (b) $\frac{\mu}{\sqrt{\alpha}}$

(c) $\frac{1}{\sqrt{\mu\alpha}}$ (d) infinitesimal

119. A force vector $\mathbf{F} = 6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$ newton applied to a body accelerates it by $1\ \text{ms}^{-2}$. What is the mass of the body?

(a) $10\sqrt{2}$ kg (b) $2\sqrt{10}$ kg

(c) 10 kg (d) 20 kg

120. A uniform chain of length L is lying on the horizontal surface of a table. If the coefficient of friction between the chain and the table top is μ, what is the maximum length of the chain that can hang over the edge of the table without disturbing the rest of the chain on the table?

(a) $\frac{L}{1+\mu}$ (b) $\frac{\mu L}{1+\mu}$

(c) $\frac{L}{1-\mu}$ (d) $\frac{\mu L}{1-\mu}$

121. In Q. 120 above, if $\mu = 0.25$, what is the maximum percentage of the length of the chain that can hang over the edge of the table without disturbing the rest of the chain on the table?

(a) 8% (b) $\frac{40}{3}$ %

(c) 20% (d) 25%

122. A block of mass m is lying on a horizontal surface of coefficient of friction μ. A force F is applied to the block at an angle θ with the horizontal. The block will move with a minimum force F if

(a) $\mu = \tan\theta$ (b) $\mu = \cot\theta$

(c) $\mu = \sin\theta$ (d) $\mu = \cos\theta$

123. In Q. 122 above, the minimum F is given by

(a) $\frac{\mu m g}{\mu - 1}$ (b) $\frac{\mu m g}{\mu + 1}$

(c) $\frac{\mu^2 mg}{\sqrt{1-\mu^2}}$ (d) $\frac{\mu m g}{\sqrt{1+\mu^2}}$

124. The coefficients of static and kinetic friction between a body and the surface are 0.75 and 0.5 respectively. A force is applied to the body to make it just slide with a constant acceleration which is

(a) $\frac{g}{4}$ (b) $\frac{g}{2}$

(c) $\frac{3g}{4}$ (d) g

125. Two blocks A and B are connected to each other by a string and a spring of force constant k, as shown in

Fig. 3.79. The string passes over a frictionless pulley as shown. The block B slides over the horizontal top surface of a stationary block C and the block A slides along the vertical side of C both with the same uniform speed. The coefficient of friction between the surfaces of the blocks B and C is μ. If the mass of block A is m, what is the mass of block B?

(a) $\frac{m}{\sqrt{\mu}}$ (b) $\frac{m}{\mu}$

(c) $\sqrt{\mu}\, m$ (d) $\mu\, m$

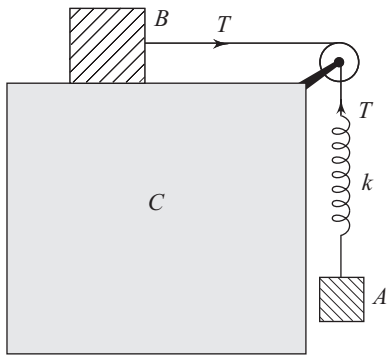


Fig. 3.79

126. In Q. 125 above, the energy stored in the spring is

(a) $\frac{m^2 g^2}{2k}$ (b) $\frac{m^2 g^2}{k}$

(c) $\frac{\mu\, m^2 g^2}{2k}$ (d) μmgk

127. A trolley of mass M is attached to a block of mass m by a string passing over a frictionless pulley as shown in Fig. 3.80. If the coefficient of friction between the trolley and the surface below is μ, what is the acceleration of the trolley and the block system, when they are released?

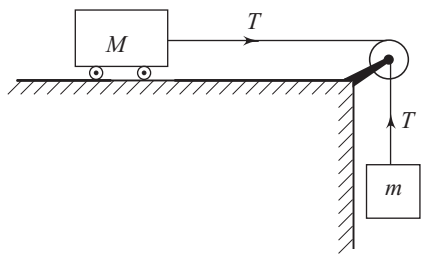


Fig. 3.80

(a) $\left(\frac{m-M}{m+M}\right)g$ (b) $\frac{m}{M}g$

(c) $\left(\frac{\mu m - M}{m+M}\right)g$ (d) $\left(\frac{m-\mu M}{m+M}\right)g$

128. In Q. 127 above, the tension T in the string is

(a) $\frac{mMg(1+\mu)}{m+M}$ (b) $\frac{mMg(1-\mu)}{M-m}$

(c) μmg (d) $(m + \mu M)\, g$

129. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are:

(a) up the incline while ascending and down the incline while descending

(b) up the incline while ascending as well as descending

(c) down the incline while ascending and up the incline while descending

(d) down the incline while ascending as well as descending

130. An insect crawls up a hemispherical surface very slowly (see Fig. 3.81). The coefficient of friction between the insect and the surface is 1/3. If the line joining the center of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by

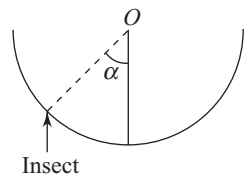


Fig. 3.81

(a) $\cot \alpha = 3$ (b) $\tan \alpha = 3$

(c) $\sec \alpha = 3$ (d) $\operatorname{cosec} \alpha = 3$

131. The pulleys and strings shown in Fig. 3.82 are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be

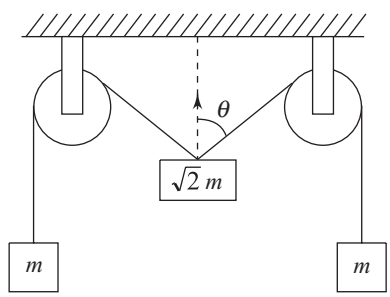


Fig. 3.82

(a) 0° (b) 30°

(c) 45° (d) 60°

132. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as

shown in Fig. 3.83. The force on the pulley by the clamp is given by

(a) $\sqrt{2}\ Mg$ (b) $\sqrt{2}\ mg$

(c) $\sqrt{(M+m)^2+m^2}\ g$ (d) $\sqrt{(M+m)^2+M^2}\ g$

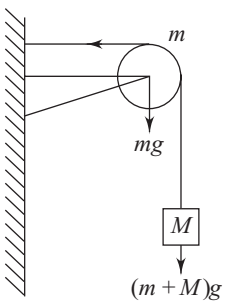


Fig. 3.83

133. A block of weight 200 N is pulled along a rough horizontal surface at a constant speed by a force of 100 N acting at an angle of 30° above the horizontal. The coefficient of friction between the block and the surface is
(a) 0.43 (b) 0.58
(c) 0.75 (d) 0.85

134. Bullets of mass 0.03 kg each hit a plate at the rate of 200 bullets per second with a velocity of 50 ms^{-1} and reflect back with a velocity of 30 ms^{-1}. The average force (in newton) acting on the plate is
(a) 120 (b) 180
(c) 300 (d) 480

135. A body of mass M kg is on the top point of a smooth hemisphere of radius 5 m. It is released to slide down the surface of the hemisphere. It leaves the surface when its velocity is 5 m/s. At this instant the angle made by the radius vector of the body with the vertical is: (Acceleration due to gravity = 10 ms^{-2})
(a) 30° (b) 45°
(c) 60° (d) 90°

136. A stationary body of mass 3 kg explodes into three equal pieces. Two of the pieces fly off at right angles to each other, one with a velocity $2\hat{\mathbf{i}}$ m/s and the other with a velocity $3\hat{\mathbf{j}}$ m/s. If the explosion takes place in 10^{-5} sec, the average force acting on the third piece in newton is:
(a) $(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}) \times 10^{-5}$ (b) $-(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}) \times 10^{5}$
(c) $(3\hat{\mathbf{j}}-2\hat{\mathbf{i}}) \times 10^{5}$ (d) $(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}) \times 10^{-5}$

137. A string can withstand a tension of 25 N. What is the greatest speed at which a body of mass 1 kg can be whirled in a horizontal circle using a 1 m length of a string?
(a) 2.5 ms^{-1} (b) 5.0 ms^{-1}
(c) 7.5 ms^{-1} (d) 10 ms^{-1}

138. A body of mass 0.5 kg is whirled in a vertical circle at an angular frequency of 10 rad s^{-1}. If the radius of the circle is 0.5 m, what is the tension in the string when the body is at the top of the circle? Take g = 10 ms^{-2}.
(a) 10 N (b) 20 N
(c) 30 N (d) 40 N

139. In Q.138, what will be the tension in the string when the body is at the bottom of the circle?
(a) 10 N (b) 20 N
(c) 30 N (d) 40 N

140. A cyclist is moving with a speed of 6 ms^{-1}. As he approaches a circular turn on the road of radius 120 m, he applies brakes and reduces his speed at a constant rate of 0.4 ms^{-2}. The magnitude of the net acceleration of the cyclist on the circular turn is
(a) 0.5 ms^{-2} (b) 1.0 ms^{-2}
(c) 2.0 ms^{-2} (d) 4.0 ms^{-2}

141. A simple pendulum of bob mass m swings with an angular amplitude of 40°. When its angular displacement is 20°, the tension in the string is
(a) mg cos 20°
(b) mg sin 20°
(c) greater than mg cos 20°
(d) greater than mg sin 20°

142. A car moves at a speed of 36 km h^{-1} on a level road. The coefficient of friction between the tyres and the road is 0.8. The car negotiates a curve of radius R. If g = 10 ms^{-2}, the car will skid (or slip) while negotiating the curve if value of R is
(a) 10 m (b) 13 m
(c) 14 m (d) 16 m

143. A train has to negotiate a curve of radius 200 m. By how much should the outer rails be raised with respect to the inner rails for a speed of 36 km h^{-1}. The distance between the rails is 1.5 m. Take g = 10 ms^{-2}.
(a) 7.5 cm (b) 10 cm
(c) 12.5 cm (d) 15 cm

144. A train rounds an unbanked circular bend of radius 50 m at a speed of 54 km h^{-1}. If g = 10 ms^{-2}, the angle of banking required to prevent wearing out of rails is given by
(a) $\theta = \tan^{-1}(0.15)$ (b) $\theta = \tan^{-1}(0.25)$
(c) $\theta = \tan^{-1}(0.35)$ (d) $\theta = \tan^{-1}(0.45)$

145. One end of a string of length R is tied to a stone of mass m and the other end to a small pivot on a frictionless vertical board. The stone is whirled in a vertical circle with the pivot as the centre. The minimum speed the stone must have, when it is at the topmost point on the circle, so that the string does not slack is given by

(a) $\sqrt{gR}$ (b) $\sqrt{mgR}$

(c) $\sqrt{2gR}$ (d) $\sqrt{2mgR}$

146. In Q.145, the minimum speed the stone must have, when it is at the lowermost point on the circle, so that the stone can complete the circle is given by

(a) $\sqrt{4gR}$ (b) $\sqrt{4mgR}$

(c) $\sqrt{5gR}$ (d) $\sqrt{5mgR}$

147. The pilot of an aircraft, who is not tied to his seat, can loop a vertical circle in air without falling out at the top of the loop. What is the minimum speed required so that he can successfully negotiate a loop or radius 4 km? Take $g = 10\ \text{ms}^{-2}$.

(a) $100\ \text{ms}^{-1}$ (b) $200\ \text{ms}^{-1}$

(c) $300\ \text{ms}^{-1}$ (d) $400\ \text{ms}^{-1}$

148. A disc of radius $r = 20$ cm is rotating about its axis with an angular speed of 20 rad s^{-1}. It is gently placed on a horizontal surface which is perfectly frictionless (Fig. 3.84). What is the linear speed of point A on the disc?

(a) $1\ \text{ms}^{-1}$ (b) $2\ \text{ms}^{-1}$

(c) $3\ \text{ms}^{-1}$ (d) $4\ \text{ms}^{-1}$

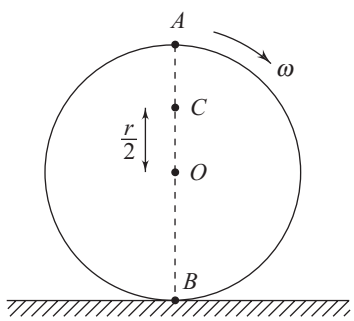


Fig. 3.84

149. In Q.148, the linear speed of point B on the disc will be

(a) $1\ \text{ms}^{-1}$ (b) $2\ \text{ms}^{-1}$

(c) $3\ \text{ms}^{-1}$ (d) $4\ \text{ms}^{-1}$

150. In Q.148, what is the linear speed of point C on the disc?

(a) $1\ \text{ms}^{-1}$ (b) $2\ \text{ms}^{-1}$

(c) $3\ \text{ms}^{-1}$ (d) $4\ \text{ms}^{-1}$

151. The combined mass of a rider and a motorcycle is 200 kg. What is the necessary frictional force if the rider is to negotiate a curve of 80 m radius at a speed of 72 km h^{-1}? Take $g = 10\ \text{ms}^{-2}$.

(a) 500 N (b) 750 N

(c) 1000 N (d) 1250 N

152. In Q.151, the angle at which rider must lean to avoid falling is given by

(a) $\theta = \tan^{-1}(0.2)$ (b) $\theta = \tan^{-1}(0.3)$

(c) $\theta = \tan^{-1}(0.4)$ (d) $\theta = \tan^{-1}(0.5)$

153. A car, moving at a speed of 54 km h^{-1}, is to go round a curved road of radius 30 m. If the curved road is not banked, what must be the coefficient of friction between the tyres and the road for the car to negotiate the curve? Take $g = 10\ \text{ms}^{-2}$.

(a) zero (b) 0.25

(c) 0.50 (d) 0.75

154. A string of length $L = 1$ m is fixed at one end and carries a mass of 100 g at the other end. The string makes $\sqrt{5}/\pi$ revolutions per second about a vertical axis passing through its second end. What is the angle of inclination of the string with the vertical? Take $g = 10\ \text{ms}^{-2}$.

(a) 30° (b) 45°

(c) 60° (d) 75°

155. In Q.154, the tension in the string is

(a) 2 N (b) 3 N

(c) 4 N (d) 5 N

156. In Q.154, what is the linear speed of the mass?

(a) $\sqrt{10}\ \text{ms}^{-1}$ (b) $\sqrt{15}\ \text{ms}^{-1}$

(c) $2\sqrt{5}\ \text{ms}^{-1}$ (d) $5\ \text{ms}^{-1}$

157. A simple pendulum of length $r = 1$ m and bob mass 100 g is swinging with an angular amplitude of 60°. What is the tension in the string when the bob passes through the equilibrium position? Take $g = 10\ \text{ms}^{-2}$.

(a) 1 N (b) 2 N

(c) 3 N (d) 4 N

158. A spring which obey's Hooke's law extends by 1 cm when a mass is hung on it. It extends by a further 3 cm when the attached mass is moved in a horizontal circle making 2 revolutions per second. What is the length of the unstretched spring? Take $g = \pi^2\ \text{ms}^{-2}$.

(a) 18 cm (b) 19 cm

(c) 20 cm (d) 21 cm

159. In Q.158, the angle of inclination of the spring to the vertical is given by

(a) $\theta = \cos^{-1}\left(\frac{1}{4}\right)$ (b) $\theta = \cos^{-1}\left(\frac{1}{3}\right)$

(c) $\theta = \cos^{-1}\left(\frac{1}{2}\right)$ (d) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

160. A boy is seated on top of a hemispherical mound of ice of radius R. He is given a little push and he starts sliding down the ice. If ice is frictionless, the boy will leave the ice at a point whose height is

(a) $\frac{3R}{4}$ (b) $\frac{2R}{\sqrt{3}}$

(c) $\frac{2R}{3}$ (d) $\frac{R}{3}$

161. A boy whirls a stone in a horizontal circle 2 m above the ground by means of a string 1.25 m long. The string breaks and the stone flies off horizontally, striking the ground 10 m away. What is the magnitude of the centripetal acceleration during circular motion? Take $g = 10\ \text{ms}^{-2}$?

(a) $100\ \text{ms}^{-2}$ (b) $200\ \text{ms}^{-2}$

(c) $300\ \text{ms}^{-2}$ (d) $400\ \text{ms}^{-2}$

162. Refer again to Q.145. If $v_B = \sqrt{5gr}$, what will be the kinetic energy of the stone when it is at the topmost point of the circle? B is the lowermost point of the circle.

(a) $\frac{1}{2}\ mgR$ (b) mgR

(c) $\frac{3}{2}\ mgR$ (d) $\frac{5}{2}\ mgR$

163. In Q.145, if $v_A = \sqrt{gR}$, what will be the kinetic energy of the stone when the string becomes horizontal? A is the topmost point of the circle.

(a) $\frac{1}{2}\ mgR$ (b) mgR

(c) $\frac{3}{2}\ mgR$ (d) $2\ mgR$

164. A body is resting on top of a hemispherical mound of ice of radius R. If ice is frictionless, what minimum horizontal velocity must be imparted to the body so that it leaves the mound without sliding over it?

(a) $\sqrt{\frac{gR}{2}}$ (b) $\sqrt{gR}$

(c) $\sqrt{2gR}$ (d) $2\sqrt{gR}$

165. A car is negotiating a curved road of radius r. If the coefficient of friction between the tyres and the road is μ, the car will skid if its speed exceeds

(a) $\sqrt{\mu rg}$ (b) $\sqrt{2\mu rg}$

(c) $\sqrt{3\mu rg}$ (d) $2\sqrt{\mu rg}$

166. The over-bridge of a river is in the form of a circular arc of radius of curvature 10 m. If $g = 10\ \text{ms}^{-2}$, what is the highest speed at which a motor cyclist can cross the bridge without leaving the ground?

(a) $10\ \text{ms}^{-1}$ (b) $10\sqrt{2}\ \text{ms}^{-1}$

(c) $10\sqrt{3}\ \text{ms}^{-1}$ (d) $20\ \text{ms}^{-1}$

167. The over-bridge of a river is in the form of a circular arc of radius of curvature R. If m is the combined mass of the motorcycle and the rider crossing the bridge at a speed v, the thrust on the bridge at the highest point will be

(a) $\frac{mv^2}{R}$ (b) mg

(c) $\frac{mv^2}{R} - mg$ (d) $\frac{mv^2}{R} + mg$

168. The blocks A and B of masses 2 m and m are connected as shown in Fig. 3.85. The spring has negligible mass. The string is suddenly cut. The magnitudes of accelerations of masses 2 m and m at that instant are

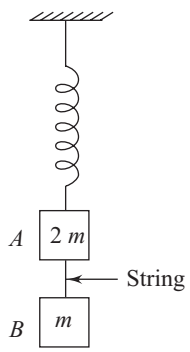


Fig. 3.85

(a) g, g

(b) $g, \frac{g}{2}$

(c) $\frac{g}{2}, g$

(d) $\frac{g}{2}, \frac{g}{2}$

169. A force F hits a block of mass m = 35 g initially at rest. The duration of the impact is 5 ms. It the force varies with time as shown in Fig. 3.86, the speed of the block immediately after impact is

(a) $1\ \text{ms}^{-1}$ (b) $2\ \text{ms}^{-1}$

(c) $3\ \text{ms}^{-1}$ (d) $4\ \text{ms}^{-1}$

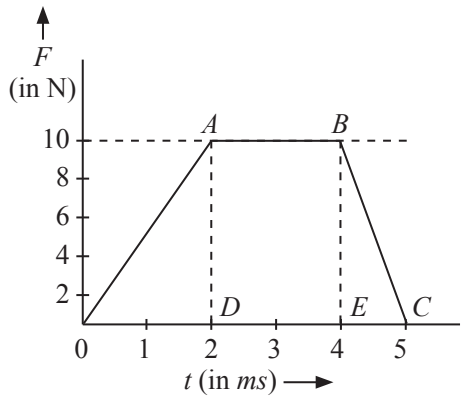


Fig. 3.86

170. A rope of mass m is attached to a block of mass M lying on a horizontal surface. The block is pulled along the surface by applying a force F on the free end of the rope. If μ is the coefficient of friction between block and the surface, the force exerted by the rope on the block is

(a) $\frac{m}{(M+m)}(F-\mu mg)$ (b) $\frac{M}{(M+m)}(F-\mu Mg)$

(c) $\frac{m}{M}(F+\mu Mg)$ (d) $\frac{M}{m}(F+\mu Mg)$

171. Figure 3.87 shows an elevator and blocks A and B. The block A has mass m and block B has mass M. If the elevator is descending will an acceleration a, the force exerted by block A or block B is

(a) $\frac{m}{(M+m)}(g-a)$

(b) $\frac{M}{(M+m)}(g+a)$

(c) $m(g-a)$

(d) $M(g+a)$

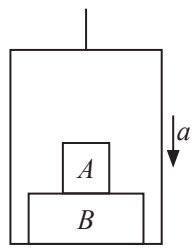


Fig. 3.87

172. Three blocks A, B and C of masses m_1, m_2 and m_2 are connected by a string passing over a fixed frictionless pulley as shown in Fig. 3.88. If $(m_2 + m_3) > m_1$, and $m_1 + m_2 + m_3 = M$, the tension in the string connecting blocks B and C is

(a) $\frac{2m_1m_2\,g}{M}$

(b) $\frac{2m_1m_3\,g}{M}$

(c) $\frac{2m_2m_3g}{M}$

(d) zero

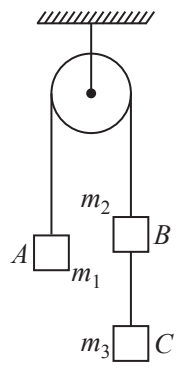


Fig. 3.88

173. A man of mass M stands on the floor of a box of mass m as shown in Fig. 3.89. He raises himself and the box with an acceleration $a = g/3$ by means of a rope going over a fixed frictionless pulley. If the mass of the rope is negligible compared to $(M + m)$ and if $M = 2m$, the tension in the rope will be

(a) 2 mg

(b) $\frac{2mg}{3}$

(c) mg

(d) $\frac{4mg}{3}$

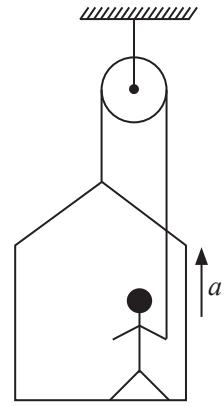


Fig. 3.89

174. In Q. 173 above, the normal reaction (or contact force) between the man and the box is

(a) $\frac{mg}{3}$ (b) $\frac{2mg}{3}$

(c) $\frac{4mg}{3}$ (d) zero

175. Figure 3.90 shows a block of mass m placed on a horizontal surface. The coefficient of static friction between the block and the surface is μ. The maximum force F that can be applied at point O such that the block does not slip on the surface is

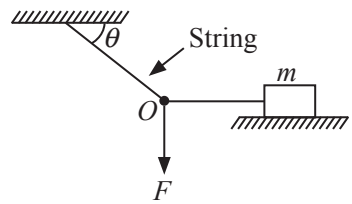


Fig. 3.90

(a) $\mu\, mg \sin\theta$ (b) $\mu\, mg \cos\theta$

(c) $\mu\, mg \tan\theta$ (d) $\mu\, mg$

Answers

(Level A)

1. (a)	2. (a)	3. (c)	4. (d)
5. (c)	6. (a)	7. (c)	8. (a)
9. (b)	10. (d)	11. (c)	12. (d)
13. (d)	14. (d)	15. (a)	16. (a)

17. (c)	18. (c)	19. (c)	20. (a)
21. (d)	22. (d)	23. (a)	24. (b)
25. (d)	26. (a)	27. (b)	28. (c)
29. (b)	30. (d)	31. (c)	32. (d)
33. (b)	34. (a)	35. (a)	36. (b)
37. (b)	38. (a)		

Level B

39.(c)	40. (c)	41. (b)	42. (c)
43. (a)	44. (c)	45. (a)	46. (c)
47. (c)	48. (b)	49. (a)	50. (a)
51. (c)	52. (d)	53. (a)	54. (b)
55. (a)	56. (a)	57. (c)	58. (c)
59. (d)	60. (c)	61. (d)	62. (a)
63. (d)	64. (b)	65. (d)	66. (d)
67. (b)	68. (d)	69. (c)	70. (c)
71. (c)	72. (a)	73. (a)	74. (a)
75. (a)	76. (b)	77. (d)	78. (a)
79. (a)	80. (b)	81. (d)	82. (a)
83. (b)	84. (c)	85. (c)	86. (d)
87. (a)	88. (d)	89. (c)	90. (c)
91. (c)	92. (a)	93. (b)	94. (a)
95. (d)	96. (c)	97. (d)	98. (a)
99. (b)	100. (c)	101. (b)	102. (d)
103. (c)	104. (c)	105. (c)	106. (d)
107. (d)	108. (a)	109. (c)	110. (c)
111. (b)	112. (b)	113. (c)	114. (a)
115. (a)	116. (c)	117. (b)	118. (a)
119. (a)	120. (b)	121. (c)	122. (a)
123. (d)	124. (a)	125. (b)	126. (a)
127. (d)	128. (a)	129. (b)	130. (a)
131. (c)	132. (d)	133. (b)	134. (d)
135. (c)	136. (b)	137. (b)	138. (b)
139. (c)	140. (a)	141. (c)	142. (a)
143. (a)	144. (d)	145. (a)	146. (c)
147. (b)	148. (d)	149. (d)	150. (b)
151. (c)	152. (d)	153. (d)	154. (c)
155. (a)	156. (b)	157. (b)	158. (d)
159. (a)	160. (c)	161. (b)	162. (a)
163. (c)	164. (b)	165. (a)	166. (a)
167. (d)	168. (c)	169. (a)	170. (a)
171. (c)	172. (c)	173. (b)	174. (c)



Solutions

1. The net force acting on mass $m = \frac{2mg - mg}{3} = \frac{mg}{3}$.

   Therefore, the acceleration of mass $m = \frac{mg}{3m} = \frac{g}{3}$.

   Hence the correct choice is (a).

2. The total mass of the block-rope system = $M + m$.

   Therefore, the acceleration of the block–rope system $= \frac{F}{M+m}$. Thus, the net force acting on the block = acceleration × mass = $\frac{FM}{M+m}$. Hence the correct choice is (a).

3. The tension in the rope depends on the acceleration of the block–rope system and the mass of the rope. The tension will be the same at all points on the rope if either the acceleration of the rope is zero or the mass of the rope is negligible compared to the mass of the block. Hence the correct choice is (c).

4. When the lift is descending with a retardation (negative acceleration) a, the effective value of g is $g_{\text{eff}} = g + a$. The component of this acceleration along the inclined plane is $g_{\text{eff}} \sin\theta = (g + a)\sin\theta$. Hence the correct choice is (d).

5. The accelerations of the block sliding down a smooth and rough 45° inclined planes respectively are

   $$a_1 = g \sin 45° = \frac{g}{\sqrt{2}}$$

   and $a_2 = g(\sin 45° - \mu_k \cos 45°) = \frac{g}{\sqrt{2}}(1 - \mu_k)$

   where μ_k is the coefficient of kinetic friction. Now, we know that the square of the time of slide is inversely proportional to the acceleration. Therefore

   $$\frac{t_2^2}{t_1^2} = \frac{a_1}{a_2} = \frac{1}{1-\mu_k}$$

   Since $t_2 = 2t_1$, we have $4 = \frac{1}{1-\mu_k}$ or $\mu_k = 0.75$.

   Hence the correct answer is (c).

6. The acceleration of the block while it is sliding down the upper half of the inclined plane is $g \sin\theta$. If μ is the coefficient of kinetic friction between the block and the lower half of the plane, the retardation of the block while it is sliding down the lower half $= -(g \sin\theta - \mu g \cos\theta)$. For the block to come to rest at the bottom of the inclined plane, the acceleration

in the first half must be equal to the retardation in the second half, i.e.

$$g \sin \theta = -(g \sin \theta - \mu g \cos \theta)$$

or $$\mu \cos \theta = 2 \sin \theta$$

$$\mu = 2 \tan \theta$$

Hence the correct choice is (a).

7. Before the boy starts walking on the plank, both the boy and the plank are at rest. Therefore, the total momentum of the boy–plank system is zero. If the boy walks with a speed v on the plank and as a result if the speed of the plank in the opposite direction is V, then the total momentum of the system is $mv - (M + m)V$. From the principle of conservation of momentum, we have

$$mv - (M + m)V = 0$$

or $$\frac{V}{v} = \frac{m}{(M+m)}$$

Since the distance moved is proportional to speed, the displacement L' of the plank is given by

$$\frac{L'}{L} = \frac{V}{v} = \frac{m}{(M+m)}$$

or $$L' = \frac{mL}{(M+m)}$$

Hence the correct choice is (c).

8. The law of conservation of momentum gives

$$m_1 v = m_1 \frac{v}{2} + m_2 v'$$

where v' is the speed of ball B after collision. Thus

$$v' = \frac{m_1 v}{2m_2}$$

Hence the correct choice is (a).

9. At the highest point of trajectory, the projectile has only a horizontal velocity which is $u \cos \theta$. After explosion, the fragment falling downwards has no horizontal velocity. If u' is the horizontal velocity of the other fragment, the law of conservation of momentum gives

$$(2m)\, u \cos \theta = m \times 0 + mu'$$

which gives $$u' = 2u \cos \theta$$

Now, the time taken to reach the highest point (as well as the time taken to fall down from this point) is $\frac{u \sin \theta}{g}$. Therefore, the horizontal distance travelled by the other fragment is

$$u \cos \theta \times \frac{u \sin \theta}{g} + 2\, u \cos \theta \times \frac{u \sin \theta}{g}$$

$$= \frac{u^2 \sin 2\theta}{2g} + \frac{u^2 \sin 2\theta}{g} = \frac{3u^2 \sin 2\theta}{2g}$$

Hence the correct choice is (b).

10. The force exerted by the leaking sand on the truck
= rate of change of momentum

$$= \frac{\Delta mu}{\Delta t}$$

The sand falling vertically downward will exert this force on the truck in vertically upward direction. This perpendicular force can do no work on the truck. Since friction is absent, no force is needed to keep the truck moving at a constant speed in the horizontal direction. Hence the correct choice is (d).

11. For masses m_1 and m_2, we have

$$T = m_2 a$$

$$m_1 g - T = m_1 a$$

Adding the two equations, we get

$$a = \frac{m_1 g}{(m_1 + m_2)}$$

Hence, the correct choice is (c).

12. The contact force exerted by A on B is $F_2 = \frac{m_2 F}{(m_1 + m_2)}$, which is choice (d).

13. Let m_A and m_B be the masses of skaters A and B and a_A and a_B their respective accelerations, when they pull at each other. From Newton's third law, action and reaction forces are equal in magnitude, i.e.

$$m_A a_A = m_B a_B$$

or $$m_A \frac{v_A}{t} = m_B \frac{v_B}{t}$$

or $$m_A v_A = m_B v_B$$

or $$m_A^2 v_A^2 = m_B^2 v_B^2 \qquad \text{(i)}$$

where v_A and v_B are their respective speeds and t is the time taken for them to meet. Let s_A and s_B be the distances travelled by them when they meet, we have,

$$2a_A s_A = v_A^2 \quad \text{and} \quad 2a_B s_B = v_B^2$$

Using these equations in Eq. (i), noting that $m_A a_A = m_B a_B$, we get $\frac{s_A}{s_B} = \frac{m_B}{m_A} = \frac{70}{50} = \frac{7}{5}$. Since $s_A + s_B = 6$ m; $s_A = 3.5$ m and $s_B = 2.5$ m. Hence, the correct choice is (d).

14. If a body has a constant velocity, both its speed and its direction of motion are constant. Hence, choice (a) is incorrect. A body having a constant speed can have an acceleration, called centripetal acceleration ($= v^2/R$) which is not variable. Hence, choices (b) and (c) are also incorrect. A body having a constant speed can have a varying velocity, e.g. a body moving in a circle with a constant speed; its direction of motion and hence its velocity is continuously changing with time. Hence, the only correct choice is (d).

15. As long as the coin is in the hand of the person, it shares the acceleration of the train; it has the inertia of motion. When he tosses the coin, it falls behind him opposite to the direction of accelerated motion but now it no longer shares the acceleration of the train. Hence the correct choice is (a).

16. The reaction force offered by the wall to the bullets = the force exerted by bullets on the wall (third law of motion) = the rate of change of momentum of bullets (second law of motion). Now, total mass of n bullets $= Nm$. Momentum of n bullets $= Nmv$. If n bullets are fired per second, the change of momentum per second $= nNmv$. Hence, the correct choice is (a).

17. Since $v = \sqrt{2gh}$, the correct choice is (c).

18. The correct choice is (c).

19. Since the shell is at rest, the initial momentum is zero. After it explodes, the total momentum of the equal fragments A and B must be zero, which is possible only if they fly off in opposite directions with equal speeds. Hence the correct choice is (c).

20. Since the bomb is at rest, its momentum is zero. From the principle of conservation of momentum, it follows that, after explosion, the total momentum of all the fragments must be zero. Hence the correct choice is (a).

21. The component of weight Mg of the block along the inclined plane $= Mg \sin \theta$. The minimum frictional force to be overcome is also $Mg \sin \theta$. To make the block just move up the plane the minimum force applied must overcome the component $Mg \sin \theta$ of gravitational force as well as the frictional force $Mg \sin \theta$. Hence the correct choice is (d).

22. Now, $u = 20$ ms^{-1}, $t = 5$s and $v = 0$ (since the car is brought to rest). The acceleration of the car is

$$a = \frac{v-u}{t} = \frac{0-20}{5} = -4 \text{ ms}^{-2}$$

The negative sign indicates retardation and its magnitude is 4 ms^{-2}.

$\therefore$ Retardation force = mass $\times$ acceleration

$$= (940 + 60) \times 4 = 4000 \text{ N}$$

Hence the correct choice is (d).

23. Now $2as = v^2 - u^2$. Therefore

$$s = \frac{v^2 - u^2}{2a} = \frac{0 - (20)^2}{2 \times (-4)} = 50 \text{ m}$$

Hence the correct choice is (a).

24. Given $u = +5$ ms^{-1} along positive x–direction

$F = -0.4$ N along negative x–direction

$m = 200$ g $= 0.2$ kg

The acceleration $a = \frac{F}{m} = \frac{-0.4}{0.2} = -2$ ms^{-2}. The negative sign shows that the motion is retarded. The position of the body at time t is given by

$$x = x_0 + ut + \frac{1}{2} at^2$$

At $t = 0$, the body is at $x = 0$. Therefore, $x_0 = 0$. Hence

$$x = ut + \frac{1}{2} at^2$$

Since the force acts during the time interval from $t = 0$ to $t = 10$ s, the motion is decelerated only between $t = 0$ and $t = 10$ s. The position of the body at $t = 2.5$ s is given by

$$x = 5 \times 2.5 + \frac{1}{2} \times (-2) \times (2.5)^2$$
$$= 1.25 \text{ m}$$

Hence the correct choice is (b).

25. The velocity of the body at $t = 2.5$ s is

$$v = u + at = 5 + (-2) \times (2.5)$$
$$= 5 - 5 = 0$$

Thus, the correct choice is (d).

26. During the first ten seconds (i.e. from $t = 0$ to $t = 10$ s) the motion is decelerated. During this time $a = -2$ ms^{-2}. Putting $u = 5$ ms^{-1}, $a = -2$ ms^{-2} and $t = 10$ s in equation $x = ut + \frac{1}{2} at^2$. We have

$$x_1 = 5 \times 10 + \frac{1}{2} \times (-2) \times (10)^2$$
$$= -50 \text{ m} \quad \text{(i)}$$

The velocity of the body at $t = 10$ s is

$$v = u + at = 5 + (-2) \times 10$$
$$= -15 \text{ ms}^{-1}$$

During the remaining 20 seconds, i.e. from $t = 10$ s to $t = 30$ s, the acceleration $a = 0$, because the force ceases to act after $t = 10$ s. The velocity of the body remains constant at -15 ms^{-1} during the last 20 seconds. The distance covered by the body during the last 20 seconds is

$$x_2 = -15 \times 20 = -300 \text{ m}$$

$\therefore$ Position of the body at $t = 30$ s is

$$x = x_1 + x_2 = -50 - 300 = -350 \text{ m}$$

Thus, the correct choice is (a).

27. The magnitude of the velocity (i.e. speed) of the body at $t = 30$ s is 15 ms^{-1}. Hence, the correct choice is (b).

28. Given $u = 0$, $a = 2$ ms^{-2}. Since the stone is located in the train, the acceleration of the stone is $a = 2$ ms^{-2}. At time $t = 5$s, the velocity of the stone is $v = u + at = 0 + 2 \times 5 = 10$ ms^{-1}. Before the stone is dropped, its motion is accelerated with the train. But, the moment it is dropped, its acceleration due to the motion of the train ceases. Therefore, after the stone is dropped, it has the following two motions:

(a) a uniform motion with velocity 10 ms^{-1} parallel to the ground, i.e.

$$v_x = 10 \text{ ms}^{-1} \text{ (the horizontal velocity)}$$

(b) an accelerated motion vertically downwards due to gravity. In time $t = 0.2$ s, the vertical velocity of the stone is $v_y = 0 + gt = 10 \times 0.2 = 2$ ms^{-2}.

The resultant velocity of stone at $t = 0.2$ s is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(10)^2 + (2)^2}$$
$$= = \sqrt{104} = 2\sqrt{26} \text{ ms}^{-1}$$

Hence the correct choice is (c).

29. The angle, which the resultant velocity vector, makes with the horizontal is given by

$$\tan \theta = \frac{v_y}{v_x} = \frac{2}{10} = 0.2$$

Hence, the correct choice is (b).

30. After the stone is dropped, the horizontal velocity v_x remains unchanged because the acceleration is zero along the horizontal direction. The only acceleration of the stone is the acceleration due to gravity. Hence the correct choice is (d).

31. Mass of monkey (m) = 30 kg, $g = 10$ ms^{-2}. If the monkey climbs up the rope with a uniform acceleration a, the tension in the rope is $T = m(g + a)$. If he climbs down with a uniform acceleration a, the tension is $T = m(g - a)$. In choice (a), since the speed is uniform acceleration $a = 0$. Therefore

$$T = mg = 30 \times 10 = 300 \text{ N}$$

In case (b), the tension is

$$T = m(g + a) = 30 \times (10 + 2)$$
$$= 360 \text{ N}$$

In case (c), the tension is

$$T = m(g + a) = 30 \times (10 + 5)$$
$$= 450 \text{ N}$$

In case (d), the tension is

$$T = m(g - a) = 30 \times (10 - 5)$$
$$= 150 \text{ N}$$

Since the rope can withstand a maximum tension of 400 N, the rope will break only in case (c). Hence, the correct choice is (c).

32. Cross–sectional area of tube (A) = πr^2. Since the speed of the liquid is v, the volume of liquid flowing out per second = $Av = \pi r^2 v$. Mass of liquid flowing out per second = $\pi r^2 v \rho$. Therefore,

Initial momentum of liquid per second

= mass of liquid flowing per second × speed of liquid

$$= \pi r^2 \rho v^2$$

This is the rate at which momentum is imparted to wall on impact. Since the liquid does not rebound after impact, the momentum after impact is zero. Hence, the rate of change of momentum = $\pi r^2 \rho v^2$. From Newton's second law, the force exerted on the wall = rate of change of momentum = $\pi r^2 \rho v^2$. Hence, the correct choice is (d).

33. Weight of mass $m_2 = 6 \times 10 = 60$ N. The weight of m_2 provides the tension. Thus

$$T = 60 \text{ N}$$

Opposing this force along the plane is the component $F_1 = m_1 g \sin \theta$ of the force $m_1 g$. Now $F_1 = m_1 g \sin \theta = 5 \times 10 \times \sin 30° = 25$ N. Since F_1 is less

than T and is, therefore, insufficient to balance T (see Fig. 3.91), the force of friction (f) down the plane is necessary to keep block m_1 at rest. Thus, f must act down the plane. Since mass m_1 is at rest, the net force on m_1 along the plane must be zero. Thus

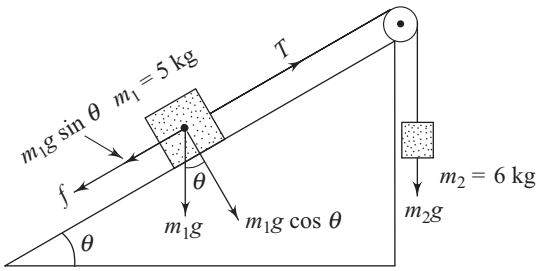


Fig. 3.91

$$T - m_1 g \sin 30° - f = 0$$

or $$f = T - m_1 g \sin 30°$$
$$= 60 - 5 \times 10 \times \sin 30°$$
$$= 60 - 25 = 35 \text{ N}$$

Hence, the correct choice is (b).

34. The acceleration of each mass is

$$a = \frac{F}{m_1 + m_2} = \frac{300}{10 + 20} = 10 \text{ ms}^{-2}$$

Hence, the correct choice is (a).

35. The tension in the string is

$$T = m_1 a = 10 \times 10 = 100 \text{ N}$$

Thus, the correct choice is (a).

36. The force on mass m_1 is

$$F_1 = \frac{m_2 F}{m_1 + m_2} = \frac{20 \times 300}{10 + 20} = 200 \text{ N}$$

Hence, the correct choice is (b).

37. If the force is applied to mass m_1, the acceleration remains the same; it does not depend on whether force F is applied to m_2 or m_1. Thus $a = 10$ ms^{-2}. The tension in the string, in this case, is

$$T = m_2 a = 20 \times 10 = 200 \text{ N}$$

Hence, the correct choice is (b).

38. The force exerted on m_2 is

$$F_2 = \frac{m_1 F}{m_1 + m_2} = \frac{10 \times 300}{10 + 20} = 100 \text{ N}$$

Hence, the correct choice is (a).

(Level B)

39. The momentum of the shell before explosion is zero. The total momentum of the three fragments after explosion must also be zero. If m is the mass of each fragment and $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ their velocities, then their momenta are

$$\mathbf{p}_1 = m\,\mathbf{v}_1$$
$$\mathbf{p}_2 = m\,\mathbf{v}_2$$
$$\mathbf{p}_3 = m\,\mathbf{v}_3$$

where $\mathbf{v}_1 = 9$ ms^{-1} along, say, the x–direction and $\mathbf{v}_2 = 12$ ms^{-1} along the y-direction (Fig. 3.92). The resultant of $\mathbf{p}_1$ and $\mathbf{p}_2$ has a magnitude given by

$$p = (p_1^2 + p_2^2)^{1/2} = m(v_1^2 + v_2^2)^{1/2}$$

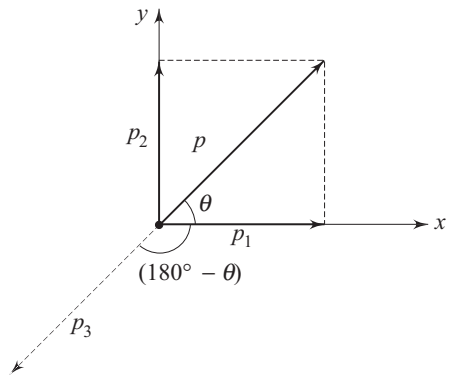


Fig. 3.92

The direction of the resultant vector $\mathbf{p}$ is at an angle θ with the x-axis. Since the total momentum is zero, we have $\mathbf{p}_3 + \mathbf{p} = 0$ or $\mathbf{p}_3 = -\mathbf{p}$. Therefore, the magnitude of $\mathbf{p}_3$ is equal to that of $\mathbf{p}$ but its direction is opposite $(180° - \theta)$ with x–axis. Therefore, magnitude of $\mathbf{p}_3$ is

$$p_3 = \text{magnitude of } \mathbf{p}$$
$$= m(v_1^2 + v_2^2)^{1/2}$$
$$= m \times (9^2 + 12^2)^{1/2}$$
$$= 15\,m \text{ kg ms}^{-1}$$

$\therefore$ Speed of the third fragme]nt $= \frac{p_3}{m} = \frac{15\,m}{m} =$ 15 ms^{-1}. Hence, the correct choice is (c).

40. Mass of the ball = 0.15 kg. Initial momentum of the ball = 0.15 kg × 12 ms^{-1} = 1.8 kg ms^{-1}. Final momentum of the ball = 0.15 kg × (– 20 ms^{-1}) = – 3 kg ms^{-1}. Change in momentum = 1.8 – (– 3) = 4.8 kg ms^{-1}. This is the impulse of the force exerted by the bat. Now, impulse = average force × time of impact.

$\therefore$ Average force $= \frac{\text{impulse}}{\text{time}} = \frac{4.8}{0.1} = 48$ N. Hence, the correct choice is (c).

41. Mass of a ball (m) = 0.05 kg, initial velocity each ball (u) = 6 ms^{-1} and final velocity of each ball = – 6 ms^{-1}. Change in momentum of each ball = $mu - mv = m\,(u - v) = 0.05 \times \{6 - (-6)\} = 0.6$ kg ms^{-1}.

Now, impulse = change in momentum = 0.6 kg ms^{-1} = 0.6 Ns. Hence, the correct choice is (b).

42. The ball moving along AB with velocity $\mathbf{v}$ is deflected along BC with velocity $\mathbf{v}'$. The magnitude of vectors $\mathbf{v}$ and $\mathbf{v}'$ is the same = v [see Fig. 3.93 (a)].

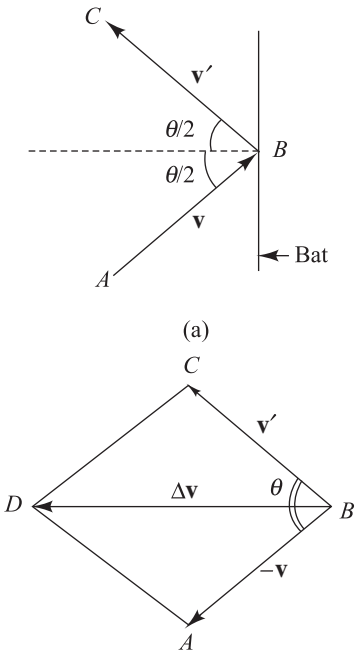


Fig. 3.93

Change in velocity is $\Delta\mathbf{v} = \mathbf{v}' - \mathbf{v} = \mathbf{v}' + (-\mathbf{v})$, i.e. $\Delta\mathbf{v}$ is the resultant of vectors $\mathbf{v}'$ and $-\mathbf{v}$. As shown in Fig. 3.89 (b), the magnitude of $\Delta\mathbf{v}$ is given by

$$\Delta v = \sqrt{v^2 + v^2 + 2v^2\cos\theta}$$

$$= \sqrt{2v^2(1+\cos\theta)}$$

$$= 2\,v\cos\left(\frac{\theta}{2}\right)$$

$\therefore$ Change in momentum = $2\,mv\cos\left(\frac{\theta}{2}\right)$. Hence the correct choice is (c)

43. The slope of the graph between $t = 0$ and $t = 2$ s is constant and positive. Therefore, the body moves from position $x = 0$ to $x = 2$ m during the time interval from $t = 0$ to 2 s. Between $t = 2$ s and $t = 4$ s, the slope of the graph is constant but negative. This implies that at $t = 2$s, the velocity of the body is reversed and it retraces its path and returns to $x = 0$ at $t = 4$ s; and so on. Thus, the body receives impulses at $t = 0$, 2 s, 4 s, ..., etc. Therefore, the interval between two consecutive impulses is 2 s. Hence, the correct choice is (a).

44. Between $t = 0$ and $t = 2$s, the speed of the body is v = slope of the (x–t) graph between $t = 0$ and $t = 2$s, i.e.

$$v = \frac{(2-0)\text{m}}{(2-0)\text{s}} = 1\ \text{ms}^{-1}$$

At $t = 2$ s, the velocity of the body is reversed and it moves in the opposite direction with a speed = – 1 ms^{-1}. Therefore,

$$\begin{aligned}\text{Impulse} &= \text{change in momentum}\\ &= mv - (-mv) = 2\,mv\\ &= 2 \times 0.4\ \text{kg} \times 1\ \text{ms}^{-1}\\ &= 0.8\ \text{kg ms}^{-1} = 0.8\ \text{Ns}\end{aligned}$$

Hence, the correct choice is (c).

45. The required force is to (i) accelerate the plane from rest to a speed v over a distance s and (ii) to overcome the force of friction (= $\mu R = \mu Mg$). The acceleration a required to impart a speed v in a distance s is given by $v^2 - u^2 = 2as$. Since, $u = 0$, we have $v^2 = 2\,as$ or $a = v^2/2s$. The force needed to produce this acceleration is

$$F_1 = \text{mass} \times \text{acceleration} = \frac{Mv^2}{2s}$$

The force needed to overcome the force of friction is

$$F_2 = \mu Mg$$

$\therefore$ Total force needed = $F_1 + F_2 = M\left(\frac{v^2}{2s} + \mu g\right)$

Hence, the correct choice is (a).

46. Refer to Fig. 3.94. Since the block is projected upwards, it rises after overcoming two forces: (i) the component $mg\sin\theta$ of the weight mg and (ii) the force of friction $F = mg\sin\theta$, both acting downwards. Therefore, the total downward acceleration is

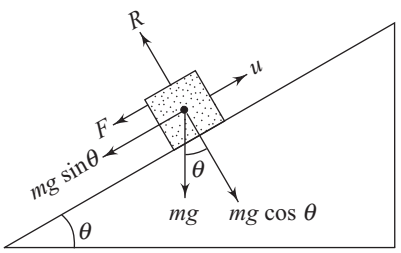


Fig. 3.94

$$a = -g\sin\theta - g\sin\theta = -2g\sin\theta$$

Let s be the distance moved up the plane before the block comes to rest. Then, from $v^2 - u^2 = 2as$, we have

$$0 - u^2 = 2 \times (-2g\sin\theta) \times s$$

or $$s = \frac{u^2}{4g\sin\theta}$$

Hence, the correct choice is (c).

47. The weight $W = mg$ of the block can be resolved into two rectangular components: one along the plane ($W \sin \theta$) and the other perpendicular to it ($W \cos \theta$). Let R be the magnitude of the normal reaction and f_s be the force of sliding friction (see Fig. 3.95).

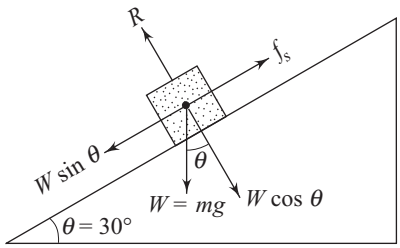


Fig. 3.95

When these forces are in equilibrium, the block just begins to slide, i.e.

$$f_s = W \sin \theta$$

Also $$R = W \cos \theta$$

$\therefore$ Coefficient of sliding friction is $\mu_s = \dfrac{f_s}{R} = \dfrac{W \sin \theta}{W \cos \theta}$

$= \tan \theta = \tan 30°$.

Hence, the correct choice is (c).

48. The minimum force F, applied parallel to the plane, that would just make the block move up the plane, must not only overcome the force of friction f_s but also the component $mg \sin \theta$ of the weight along the plane. Therefore,

$$F = f_s + mg \sin \theta$$

But $f_s = mg \sin \theta$. Therefore, $F = 2\ mg \sin \theta = 2 \times 5 \times 10 \times \sin 30° = 50$ N.

Hence the correct choice is (b).

49. Since the block is placed on the trolley, the acceleration of the block = acceleration of the trolley $a = 3\ \text{ms}^{-2}$. Therefore, the force acting on the block is

$$F = ma = 10 \times 3 = 30 \text{ N}$$

The weight mg of the block is balanced by the normal reaction R. As the trolley accelerates in the forward direction, it exerts a reaction force $F = 30$ N on the block in the direction, as shown in the Fig. 3.63 on page 3.28. The force of friction will oppose this force and will act in a direction opposite to that of F. The force of limiting friction f is given by

$$\mu = \frac{f}{R} = \frac{f}{mg}$$

or $$f = \mu mg = 0.2 \times 10 \times 10 = 20 \text{ N}$$

Thus, the block is acted upon by two forces-force $F = 30$ N towards the right and frictional force $f = 20$ N towards the left. The net force on the block towards the right, i.e. towards the rear end of the trolley is

$$F' = F - f = 30 - 20 = 10 \text{ N}$$

Due to this force, the block experiences an acceleration towards the rear end which is given by

$$a' = \frac{F'}{m} = \frac{10}{10} = 1\ \text{ms}^{-2}$$

Let t be the time taken for the block to fall from the rear end of the trolley. Clearly, the block has to travel a distance $S' = 5$ m to fall off the trolley. Since the trolley starts from rest, initial velocity $u = 0$. Now t can be obtained from the relation

$$s = ut + \frac{1}{2}\ at^2$$

Putting $s = 5$ m, $u = 0$ and $a = a' = 1\ \text{ms}^{-2}$, we get $t = \sqrt{10}$ s.

The distance covered by the trolley in time $t = \sqrt{10}$ s is ($\because u = 0$)

$$s' = ut + \frac{1}{2}\ at^2 = 0 + \frac{1}{2} \times 3 \times 10 = 15 \text{ m}$$

Hence, the correct choice is (a).

50. Let f be the frictional force on each block.

$$m_1 a = T - f \qquad \text{(i)}$$

and $$m_2 a = F - T - f \qquad \text{(ii)}$$

Subtracting the two equations, we have

$$(m_1 - m_2)\ a = 2T - F$$

Since $m_1 = m_2$, we get $0 = 2T - F$ or $T = \dfrac{F}{2} = \dfrac{20}{2} =$ 10 N

Hence, the correct choice is (a).

51. Putting $T = 10$ N in Eq. (i) above, we have

$$f = T - m_1 a$$
$$= 10 - 3 \times 0.5 = 8.5 \text{ N}$$

Hence, the correct choice is (c).

52. Force of friction is

$$f = \mu R = \mu\ mg \cos \theta$$

When the body slides down, the downward force along the plane = component $mg \sin \theta$ of the weight mg. Since the force of friction acts upwards along the plane, the effective downward force = $mg \sin \theta - \mu mg \cos \theta = mg\ (\sin \theta - \mu \cos \theta)$

$\therefore$ Acceleration = force/mass = $g(\sin \theta - \mu \cos \theta) = g\ (\sin \theta - kx \cos \theta)$. Hence, the acceleration varies with x and decreases as x increases. Thus, the correct choice is (d).

53. The mass of water stream striking against the wall in 1 second = $a v\rho$. Hence, the change in its momentum per second is $(av\rho)v - (-av\rho)v = 2a\rho v^2$. The normal component of the rate of change of momentum and, therefore, force is $2a\rho v^2 \cos\theta$. Hence the correct choice is (a).

54. The square of the time of slide is inversely proportional to the acceleration. The accelerations in the two cases are

$$a_1 = g \sin 45° = \frac{g}{\sqrt{2}} \text{ and}$$

$$a_2 = (g \sin 45° - \mu_k g \cos 45°)$$

$$= \frac{g}{\sqrt{2}}(1 - \mu_k)$$

$$\therefore \qquad \frac{t_2^2}{t_1^2} = n^2 = \frac{a_1}{a_2} = \frac{1}{1-\mu_k}$$

or $$\mu_k = 1 - \frac{1}{n^2}.$$

Hence, the correct choice is (b).

55. Refer to Fig. 3.96. The net downward force on the body at a distance x is

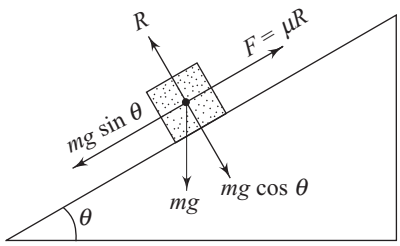


Fig. 3.96

$$f(x) = mg \sin\theta - \mu mg \cos\theta$$
$$= mg(\sin\theta - \mu\cos\theta)$$
$$= mg(\sin\theta - kx\cos\theta)$$

$\therefore f(x) = 0$ at a value of $x = x_0$ given by

$$\sin\theta - kx_0\cos\theta = 0$$

which gives $$x_0 = \frac{\tan\theta}{k}$$

Thus, the correct choice is (a).

56. The acceleration of the body down the plane is $g \sin\theta - \mu g \cos\theta = g(\sin\theta - \mu\cos\theta) = g(\sin\theta - 0.5x\cos\theta)$. Therefore, the body will first accelerate up to $x < 2\tan\theta$. The velocity will be maximum at $x = 2\tan\theta$, because for $x > 2\tan\theta$, the body starts decelerating. Hence, the correct choice is (a).

57. Refer to Fig. 3.97. Given $AB = 1$, $AC = l$, so that $BC = \sqrt{l^2 - 1}$. Thus $\tan\theta = AB/BC = 1/\sqrt{l^2-1}$. A horizontal acceleration a imparted to the inclined plane has a component $a\cos\theta$ down the plane. If this equals the component $g\sin\theta$ of the g along the plane, the object will appear stationary relative to the incline, i.e. if

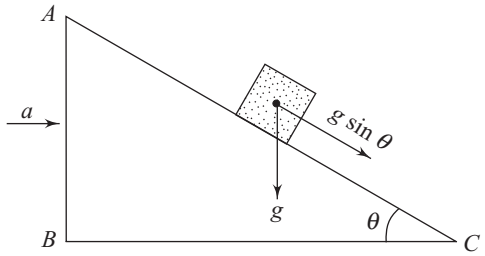


Fig. 3.97

$$a\cos\theta = g\sin\theta$$

or $$a = g\tan\theta = \frac{g}{\sqrt{l^2-1}}$$

Hence, the correct choice is (c).

58. Refer to Fig. 3.98.

Let A be the position of the insect when it has reached the maximum height h. Now $OA = OC = R$. The insect will crawl up the bowl until the component $mg\sin\theta$ of his weight down the plane equals the force $F = \mu mg\cos\theta$ of limiting friction (the insect will slip down if $mg\sin\theta$ exceeds $\mu mg\cos\theta$). Thus $mg\sin\theta = \mu mg\cos\theta$ or $\tan\theta = \mu = 1/3$.

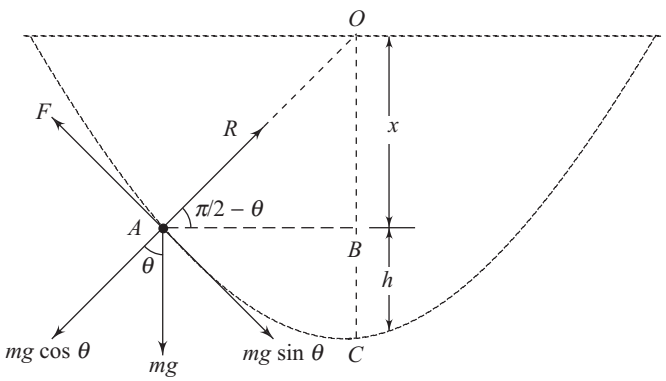


Fig. 3.98

Now in triangle OAB, $\tan\left(\frac{\pi}{2} - \theta\right) = \frac{OB}{AB}$.

Let $OB = x$, then $AB = \sqrt{R^2 - x^2}$. Thus

$$\cot\theta = \frac{x}{\sqrt{R^2 - x^2}}$$

or $$3 = \frac{x}{\sqrt{R^2 - x^2}} \quad (\because \tan\theta = \frac{1}{3})$$

which gives $x = \frac{3R}{\sqrt{10}} = 0.95\,R$. Therefore, $h = R - x = R - 0.95\,R = 0.05\,R$. Hence, the correct choice is (c).

59. The acceleration a is
$$a = \frac{F}{m_1 + m_2} = \frac{4}{5+3} = 0.5 \text{ ms}^{-2}$$
which is the same for both masses. Hence, the correct choice is (d).

60. The force on mass of 3 kg = $m_2 a$ = 3 kg × 0.5 ms^{-2} = 1.5 N. Hence the correct choice is (c).

61. The acceleration a, which is the same for both masses will still be 0.5 ms^{-2}. Hence the correct choice is (d).

62. Force exerted on mass 5 kg = 5 kg × 0.5 ms^{-2} = 2.5 N. Hence the correct choice is (a).

63. The acceleration of the system is
$$a = \frac{\text{net force}}{\text{total mass}} = \frac{12}{1+2+3} = 2 \text{ ms}^{-2}$$
Hence the correct choice is (d).

64. The contact force on mass m_2 is
$$F_2 = (m_2 + m_3)a = (2 + 3) \times 2 = 10 \text{ N}$$
Hence the correct choice is (b).

65. The contact force on mass m_3 is
$$F_3 = m_3 a = 3 \times 2 = 6 \text{ N}$$
Hence the correct choice is (d).

66. The acceleration of the system is
$$a = \frac{\text{net force}}{\text{total mass}} = \frac{12}{1+2+3} = 2 \text{ ms}^{-2}$$
Hence the correct choice is (d).

67. Tension T_2 between m_1 and m_2 is
$$T_2 = (m_2 + m_3)a = (2 + 3) \times 2 = 10 \text{ N}$$
Hence the correct choice is (b).

68. Tension T_3 between m_2 and m_3 is
$$T_2 = m_3 a = 3 \times 2 = 6 \text{ N}$$
Hence the correct choice is (d).

69. When the balls are released, the force experienced by mass m is F = its mass × its acceleration = ma. This is the force exerted by mass M on mass m. From Newton's third law, the mass m will exert an equal force F on mass M. Thus, force on $m_2 = F = ma$. Therefore, the acceleration of M is
$$a' = \frac{F}{M} = \frac{ma}{M}$$
Hence the correct choice is (c).

70. Impulse = change in momentum. Let v be the speed of the ball before it was caught and m its mass. Since the ball is brought to rest after it is caught, the change in momentum = mv. Now m = 0.2 kg. Therefore, impulse is 0.2 v. Given, impulse = 6 Ns. Hence
$$v = \frac{6}{0.2} = 30 \text{ ms}^{-1}$$
Thus, the correct choice is (c).

71. Let m be the mass of the car and a be the deceleration produced by force F, then $F = ma$, where a is given by
$$2as = v^2 \text{ or } a = \frac{v^2}{2s}$$
Therefore, $F = \frac{mv^2}{2s}$. Thus $F \propto v^2$. If v is increased by 3 times, F will increase by 9 times. Hence, the correct choice is (c).

72. Now, $F = \frac{mv^2}{2s}$, which implies that $s \propto \frac{1}{F}$, i.e. s is inversely proportional to F. Thus, the correct choice is (a).

73. Force acting on mass m is $f = ma$. Mass m will pull mass M to the left with a force $f = ma$. Hence, the net acting on mass $M = F - f = F - ma$. Therefore, acceleration of mass $M = \frac{F - ma}{M}$. Hence the correct choice is (a).

74. Let a be the minimum acceleration with which the boy must climb down the rope. Then $mg - T = ma$ or $T = mg - ma$ is the maximum tension. Now, $T = \frac{2}{3}\,mg$. Therefore,
$$\frac{2}{3}\,mg = mg - ma$$
which gives $a = g/3$. Hence the correct choice is (a).

75. The mass is in equilibrium at point O under the action of the concurrent forces F, T and $W = mg$. Therefore, as shown in Fig. 3.99, The horizontal component $T \sin\theta$ of tension T must balance with force F and

the vertical component $T \cos \theta$ must balance with weight $W = mg$. Thus

$$F = T \sin \theta \quad \text{(i)}$$

and $$W = T \cos \theta \quad \text{(ii)}$$

Thus, the correct choice is (a).

76. From Eq. (ii) above, it follows that the correct choice is (b).

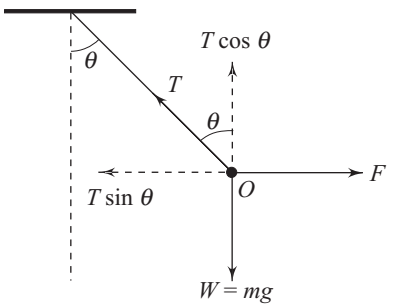


Fig. 3.99

77. Squaring Eqs. (i) and (ii) and adding we get $T^2 = W^2 + F^2$. Hence the correct choice is (d).

78. Dividing (i) by (ii) we get $F = W \tan \theta$, which is choice (a).

79. The common acceleration of the masses is

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)} = \frac{(6-4)\times 10}{(6+4)} = 2 \text{ ms}^{-2}$$

Hence the correct choice is (a).

80. The tension in the string is

$$T = \frac{2m_1 m_2 g}{(m_1 + m_2)} = \frac{2\times 6\times 4\times 10}{(6+4)} = 48 \text{ N}$$

Hence the correct choice is (b).

81. If the lift rises upwards with an acceleration a, the effective value of g is $g_{\text{eff}} = g + a$. Hence

$$T = \frac{2m_1 m_2}{(m_1 + m_2)} \times (g + a) = \frac{2\times 6\times 4}{10} \times (10 + 5)$$
$$= 72 \text{ N}$$

Thus, the correct choice is (d).

82. In this case, $g_{\text{eff}} = g - a$. Hence

$$T = \frac{2m_1 m_2}{(m_1 + m_2)} \times (g - a) = 24 \text{ N}$$

Thus, the correct choice is (a).

83. Since the acceleration of the train is perpendicular to the acceleration due to gravity, the acceleration vector of the train has no component along the vertical direction. Hence, in this case, $g_{\text{eff}} = g$. Thus, the correct choice is (b).

84. Let h be the height of each inclined plane. Then, the distances along the plane are $s_1 = \frac{h}{\sin\theta_1}$ and $s_2 = \frac{h}{\sin\theta_2}$ respectively. The accelerations of the block are $a_1 = g \sin \theta_1$ and $a_2 = g \sin \theta_2$ respectively. Now, since the block is released from rest, the velocity of the block when it reaches the bottom of the planes is $v_1^2 = 2a_1 s_1$ and $v_2^2 = 2a_2 s_2$ respectively. But $v_1 = a_1 t_1$ and $v_2 = a_2 t_2$ or $a_1^2 t_1^2 = 2a_1 s_1$ and $a_2^2 t_2^2 = 2a_2 s_2$. These equations give

$$\frac{t_2^2}{t_1^2} = \frac{a_1}{a_2} \cdot \frac{s_2}{s_1} = \frac{g\sin\theta_1}{g\sin\theta_2} \cdot \frac{h}{\sin\theta_2} \cdot \frac{\sin\theta_1}{h}$$
$$= \frac{\sin^2\theta_1}{\sin^2\theta_2}$$

$$\Rightarrow \quad \frac{t_2}{t_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

Hence, the correct choice is (c).

85. Let m be the mass of the rope and l its length. The tension T at a distance x from the support = weight of length $(l - x)$ of the rope $= \frac{mg}{l} \times (l - x)$ or

$$T = \frac{6\times 10}{3} \times (3 - 1) = 40 \text{ N}$$

Hence the correct choice is (c).

86. The acceleration of the block sliding down the smooth inclined plane is $a_1 = g \sin \theta$ and down the rough inclined plane is $a_2 = g \sin \theta - \mu g \cos \theta$. Given $t_1 = t$ and $t_2 = 2t$. If the length of the inclined plane is s, we have

$$s = \frac{1}{2} a_1 t_1^2 = \frac{1}{2} a_2 t_2^2$$

or $$a_1 t_1^2 = a_2 t_2^2$$

or $$g \sin\theta \times t^2 = (g \sin\theta - \mu g \cos\theta) \times (2t)^2$$

or $$\sin\theta = 4(\sin\theta - \mu \cos\theta)$$

which gives $$\mu = \frac{3}{4} \tan\theta = \frac{3}{4} \quad (\because \theta = 45°)$$

Hence the correct choice is (d).

87. We use the relation $v^2 - u^2 = 2as$. Since $u = 0$, we have $v^2 = 2as$.

Now $v_1^2 = 2a_1 s$ or $v^2 = 2g \sin\theta \times s$

and $v_2^2 = 2\, a_2 s$ or $\frac{v^2}{n^2} = 2(g \sin\theta - \mu g \cos\theta) \times s$

Dividing, we get or $n^2 (\sin\theta - \mu \cos\theta) = \sin\theta$

which gives $\mu = \left(1 - \frac{1}{n^2}\right) \tan\theta$, which is choice (a).

88. The force of friction between block m and block $M = \mu_1 mg$, where μ_1 is the coefficient of friction between the two blocks. Now, the force of friction between block M (with block m on top of it) and the horizontal surface $= \mu_2 (M + m)g$, where μ_2 is the coefficient of friction between block M and the surface. The maximum force F applied to block M must be enough to overcome this force of friction and the force due to acceleration of the system. If the acceleration of the system is a then this force $= (M + m)a$. Thus

$$F = (M + m)a + \mu_2 (M + m)g \quad \text{(i)}$$

Now, since the force on block m is $\mu_1 mg$, its acceleration is

$$a = \frac{\text{force on mass } m}{\text{mass } m} = \frac{\mu_1 mg}{m} = \mu_1 g \quad \text{(ii)}$$

Using (ii) in (i) we get

$$F = \mu_1 (M + m)g + \mu_2 (M + m)g$$
$$= (\mu_1 + \mu_2)(M + m)\, g$$
$$= (0.5 + 0.7) \times (5 + 3) \times 10$$
$$= 96 \text{ N}$$

Hence the correct choice is (d).

89. The force of friction acting on the box $= \mu mg$, where m is the mass of the box. This force produces an acceleration $a = \mu mg/m = \mu g$ in the box. The box will slide on the belt, without slipping, till it attains a speed (v) = the speed of the belt. The distance s moved by the box is given by

$$v^2 - u^2 = 2as$$

Since $u = 0$, $s = \frac{v^2}{2a} = \frac{v^2}{2\mu g} = \frac{4 \times 4}{2 \times 0.8 \times 10} = 1$ m.

Hence the correct choice is (c).

90. When the masses are accelerating, there is a tension in the string. When a mass m is added to m_1 such that the acceleration is zero, the system of masses $(m_1 + m)$ will slide on the surface with a uniform speed and then there is no tension in the string. This will happen if the downward force $m_2 g$ equals the force of friction $\mu(m_1 + m)g$ on blocks m_1 and m, i.e. if $\mu(m_1 + m)g = m_2 g$ or $m = \frac{m_2}{\mu} - m_1 = \frac{6}{0.4} - 4 =$ 11 kg. Hence the correct choice is (c).

91. The block m_1 will just begin to move up the plane if the downward force $m_2 g$ due to mass m_2 trying to pull the mass m_1 up the plane just equals the force $(m_1 g \sin\theta + \mu m_1 g \cos\theta)$ trying to push the mass m_1 down the plane, i.e. when

$$m_2 g = m_1 g (\sin\theta + \mu \cos\theta)$$

Now, it is given that $m_1 = m_2 = m$ and $\theta = 30°$. Therefore, we have

$$1 = \sin 30° + \mu \cos 30°$$

which gives $\mu = \frac{1}{\sqrt{3}}$. Hence the correct choice is (c).

92. The block m_1 will just begin to move down the plane if the downward force $(m_1 g \sin\theta - \mu m_1 g \cos\theta)$ on m_1 just equals the upward force $m_2 g$ acting on m_1 due to m_2, i.e. if

$$m_2 g = m_1 g (\sin\theta - \mu \cos\theta)$$

or $$\frac{m_1}{m_2} = \frac{1}{\sin\theta - \mu\cos\theta} = \frac{1}{\sin 30° - \frac{1}{2\sqrt{3}} \cos 30°}$$

$= 4$, which is choice (a).

93. The block will just begin to slide when the force of limiting friction ($mg \sin\theta$) = force of normal reaction $\mu mg \cos\theta$, i.e. if $mg \sin\theta = \mu mg \cos\theta$ or $\mu = \tan\theta$. Therefore, $\mu = \tan 30° = 1/\sqrt{3}$. Hence the correct choice is (b).

94. Downward acceleration is

$$a = g (\sin\theta - \mu \cos\theta)$$
$$= g \times \left(\sin 30° - \frac{1}{\sqrt{3}} \times \cos 30°\right)$$
$$= g \times \left(\frac{1}{2} - \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}\right) = 0$$

Hence the correct choice is (a).

95. Normal reaction $R = f$. Therefore, force of friction $= \mu R = \mu f$. The net downward force $F = mg - \mu f$. Hence, the acceleration $a = \frac{F}{m} = \frac{mg - \mu f}{m} = g - \frac{\mu f}{m}$. Hence the correct choice is (d).

96. As the boy is climbing the pole at a constant speed (no acceleration), the force of friction must be just balanced by his weight, i.e. $\mu R = mg$ or $R = \frac{mg}{\mu} = \frac{40 \times 10}{0.8} = 500$ N. Hence the correct choice is (c).

97. Block m will move over block M if the force F applied to m exceeds the force of static friction between the two blocks. The minimum F must be just enough to overcome static friction. Thus

$F_{\min}$ = coefficient of friction × normal reaction of M on $m = \mu_s mg$.

Hence the correct choice is (d).

98. If $F > \mu_s mg$, the block m will start moving on block M. It will, therefore, exert a force on block M due to kinetic friction between m and M. This force $f_k = \mu_k mg$. Thus, the acceleration of block $M = \dfrac{f_k}{M} = \mu_k \dfrac{mg}{M}$. Hence the correct choice is (a).

99. Given m = 5 kg, μ_s = 0.7 and μ_k = 0.5. The force applied to the block sufficient to move it = force of static friction, i.e. $F = \mu_s mg = 0.7 \times 5 \times 10 = 35$ N. Force responsible for producing acceleration of the block is

$$f = \text{applied force} - \text{force of dynamic friction}$$
$$= F - \mu_k mg$$
$$= 35 - 0.5 \times 5 \times 10$$
$$= 35 - 25 = 10 \text{ N}$$

$\therefore$ Acceleration $a = \dfrac{f}{m} = \dfrac{10}{5} = 2 \text{ ms}^{-2}$

Hence the correct choice is (b).

100. Let the initial speed be u. Final speed v = 0 in both cases. The retardation for $\theta_1 = 30°$ is $a_1 = g \sin \theta_1$ and for $\theta_2 = 60°$ is $a_2 = g \sin \theta_2$. Now, using $v^2 - u^2 = 2ax$, we have

$$u^2 = 2a_1x_1$$
$$= 2a_2x_2$$

Thus $$\frac{x_1}{x_2} = \frac{a_2}{a_1} = \frac{g\sin\theta_2}{g\sin\theta_1} = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$$

Hence the correct choice is (c).

101. From the principle of conservation of momentum, we have

$$m_1v_1 = m_2v_2 \text{ or } \frac{v_1}{v_2} = \frac{m_2}{m_1} \quad \text{(i)}$$

When the spring is released, it exerts an equal and opposite force F on each block. Let a_1 and a_2 be the accelerations of blocks m_1 and m_2 respectively. Then

$$F = m_1a_1 = m_2a_2 \text{ or } \frac{a_2}{a_1} = \frac{m_1}{m_2} \quad \text{(ii)}$$

Also $v_1^2 = 2a_1x_1$ and $v_2^2 = 2a_2x_2$, which give

$$\frac{x_1}{x_2} = \frac{v_1^2}{v_2^2}.\frac{a_2}{a_1} = \left(\frac{m_2}{m_1}\right)^2 \times \left(\frac{m_1}{m_2}\right) = \frac{m_2}{m_1}$$

[Use Eqs. (i) and (ii)]

Hence the correct choice is (b).

102. Refer to Fig. 3.100. The component of acceleration vector **a** along the plane is $a \cos \theta$. The component of acceleration due to gravity **g** along the plane is $g \sin \theta$. The block will stay at rest if $a \cos \theta = g \sin \theta$ or $a = g \tan \theta$

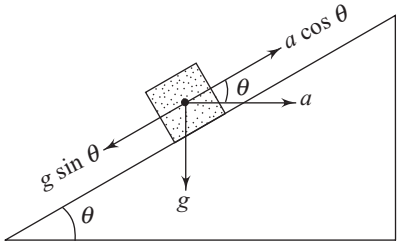


Fig. 3.100

Now $\theta = 30°$. Therefore, $a = g \tan 30° = \dfrac{g}{\sqrt{3}}$. Hence the correct choice is (d).

103. The acceleration of block moving down the inclined plane is

$$a = g \sin \theta - \mu g \cos \theta$$

Putting $a = \dfrac{g}{4}$ and $\theta = 30°$, we get $\mu = \dfrac{1}{2\sqrt{3}}$, which is choice (c).

104. The force F_1 required to prevent the block from sliding down is

$$F_1 = mg \sin \theta - \mu mg \cos \theta \quad \text{(i)}$$

The force F_2 required to make the block move up the plane is

$$F_2 = mg \sin \theta + \mu mg \cos \theta \quad \text{(ii)}$$

From Eqs. (i) and (ii) we get

$$F_2 + F_1 = 2 mg \sin \theta$$

and $$F_2 - F_1 = 2 \mu mg \cos \theta$$

Dividing the two equations, we get

$$\frac{F_2 + F_1}{F_2 - F_1} = \frac{\tan \theta}{\mu} = \frac{\tan 30°}{1/2\sqrt{3}} = 2$$

which gives $F_2 = 3 F_1$. Hence the correct choice is (c).

105. We have, $h = \dfrac{1}{2} gt^2$. When the lift is rising up with an acceleration a, the effective acceleration is $g' = g + a$ and t' is given by $h = \dfrac{1}{2} g't'^2$. Thus

$$\frac{1}{2} g't'^2 = \frac{1}{2} gt^2$$

or $$(g + a)t'^2 = gt^2$$

or $$t' = t\left(1 + \frac{a}{g}\right)^{1/2}$$ which is choice (c).

106. In this case, the effective acceleration of the coin is $g' = g - a$. Thus the correct choice is (d).

107. Let M be the mass of the block and m that of the rope. The acceleration of the block–rope system is

$$a = \frac{F}{(M+m)}$$

Therefore, the tension at the middle point of the rope will be

$$T = \left(M + \frac{m}{2}\right)a = \frac{\left(M + \frac{m}{2}\right)F}{(M+m)}$$

Given, $m = \frac{M}{2}$. Therefore, $T = \frac{5F}{6}$. Hence the correct choice is (d).

108. The acceleration $a = \frac{F}{m}$. The resultant of F_1 and F_2 has magnitude F' given by (see Fig. 3.101).

$F' = (F_1^2 + F_2^2)^{1/2} = \sqrt{2}\,F\ (\because F_1 = F_2 = F)$

The direction $\mathbf{F}'$ is opposite to that of $\boldsymbol{F}_3$.

$\therefore$ Net force on body $= F' - F_3$

$= \sqrt{2}\ F - F$

$= (\sqrt{2}\ - 1)\,F$

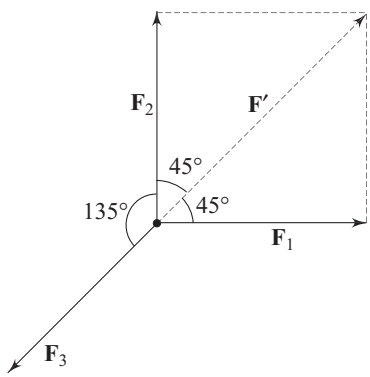


Fig. 3.101

$\therefore$ Acceleration $= (\sqrt{2}\ - 1)\ \frac{F}{m} = (\sqrt{2}\ - 1)a$. Hence the correct choice is (a).

109. The mass of two fragments of equal masses $= \frac{2m}{5}$ each. The mass of the lighter fragment $= \frac{m}{5}$. The momenta of heavier fragments are $p = \frac{2mv}{5}$. The resultant of momenta p and p is

$$p' = (p^2 + p^2)^{1/2} = \sqrt{2}\ p$$

From the principle of conservation of momentum, the momentum of the third (lighter) fragment of mass $\frac{m}{5}$ must be $\sqrt{2}\,p$ but opposite in direction. Thus, if V is the speed of the lighter fragment, we have

$$\frac{mV}{5} = \sqrt{2}p = \sqrt{2}\,\frac{2mv}{5}$$

or $$V = 2\sqrt{2}\ v$$

Hence the correct choice is (c).

110. Now, $mv = 3t^2 + 4$. Since $m = 2$ kg, $v = \frac{3}{2}\ t^2 + 2$. The acceleration is $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{3}{2}t^2 + 2\right) = 3t$. Thus, the acceleration of the body is increasing with time. Hence the correct choice is (c).

111. At $t = 2$s, acceleration $a = 3 \times 2 = 6\ \text{ms}^{-2}$. Hence the correct choice is (b).

112. We know from Newton's second law that force F is given by

$$F = \frac{dp}{dt} = \frac{d}{dt}\left(3t^2 + 4\right) = 6t$$

Hence the correct choice is (b).

113. For an elastic spring, the relation between force F and extension x is

$$F = kx$$

where k is the force constant of the spring. Let l_0 be the original length of the spring, then $F = k(l - l_0)$ where l is the spring length when stretched by a force F. We are given that

$$2 = k(l_1 - l_0) \quad \text{(i)}$$

and $$3 = k(l_2 - l_0) \quad \text{(ii)}$$

Dividing (ii) by (i) we have

$$\frac{3}{2} = \frac{l_2 - l_0}{l_1 - l_0}$$

Which gives $l_0 = 3l_1 - 2l_2$. Using this value of l_0 in either (i) or (ii) we get $k = \frac{1}{l_2 - l_1}$.

When a stretching force of 5 N is applied, let l_3 be the length of the spring. Then

$$5 = k(l_3 - l_0)$$

Substituting the values of l_0 and k, and solving we get

$$l_3 = 3l_2 - 2l_1$$

Hence the correct choice is (c).

114. Refer to Fig. 3.102. The velocity of the shell at the highest point is $v = u \sin \theta = 100 \times \sin 30° = 50$ ms^{-1} parallel to the positive x-direction. Let m be the mass of the shell. Then the mass of the lighter fragment is $m_1 = \frac{m}{3}$ and its momentum is $p_1 = m_1 v_1$; where $v_1 = 200$ ms^{-1}. The direction of p_1 is vertically upwards. The mass of the heavier fragment is $m_2 = \frac{2m}{3}$ and its momentum is $p_2 = m_2 v_2$, where v_2 is the speed of the heavier fragment at the time of explosion. Let momentum vector p_2 substend an angle α with the x-direction as shown. From the law of conservation of momentum, the component $p_2 \sin \alpha$ of p_2 along y-direction must balance with p_1 and the component $p_2 \cos \alpha$ must balance with p, i.e.

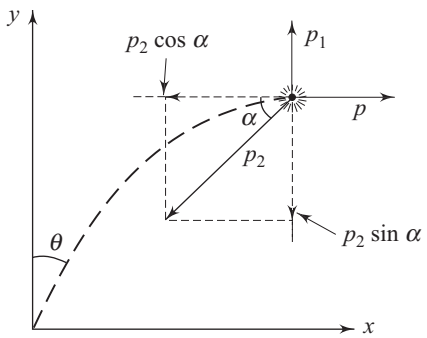


Fig. 3.102

$p_2 \sin \alpha = p_1$ or $m_2 v_2 \sin \alpha = m_1 v_1$

or $\frac{2mv_2}{3} \sin \alpha = \frac{mv_1}{3}$

or $2v_2 \sin \alpha = v_1$ (i)

and $p_2 \cos \alpha = p$ or $m_2 v_2 \cos \alpha = mv$

or $\frac{2mv_2}{3} \cos \alpha = mv$ or

$2 v_2 \cos \alpha = 3v$ (ii)

Squaring and adding (i) and (ii) we have

$$4v_2^2 = v_1^2 + 9v^2$$

or $$2v_2 = (v_1^2 + 9v^2)^{1/2}$$

Now $v = 50$ ms^{-1} and $v_1 = 200$ ms^{-1} (given). Using these values, we get $v_2 = 125$ ms^{-1}. Hence the correct choice is (a).

115. The forces acting on the balloon are its weight acting downwards and upthrust F acting upwards. Thus

$$F - Mg = Ma \quad \text{(i)}$$

When mass m is removed, we have

$$F - (M - m)\, g = (M - m)\, a' \quad \text{(ii)}$$

where a' is the new acceleration. Eliminating F from (i) and (ii) and simplifying we get

$$a' = \frac{Ma + mg}{M - m}$$

which is choice (a).

116. Let m be the mass per unit length of the rope. Let x be the part of the rope on the floor at time t. The weight of this part is

$$F_1 = mgx$$

Now, if a small part dx falls on the floor in time dt, the force exerted by it is

$$F_2 = \text{rate of change of momentum}$$

$$= \frac{(m\, dx)v}{dt}$$

Now $\frac{dx}{dt} = v$, where v is the velocity of that part of the rope at that instant. But $v^2 = 2gx$. Hence $F_2 = mv^2 = m \times (2gx) = 2mgx$. Total force $F = F_1 + F_2 = mgx + 2\, mgx = 3mgx = 3F_1$

Hence the correct choice is (c).

117. Given $\frac{d^2x}{dt^2} = 2t - t^2$ or $\frac{dv}{dt} = 2t - t^2$. Integrating

we get $$v = t^2 - \frac{t^3}{3} = t^2 \left(1 - \frac{t}{3}\right)$$

The body will reverse its direction of motion at a time t when $v = 0$, i.e. at $t = 3$ s. Now, since $v = \frac{dx}{dt}$, we have $\frac{dx}{dt} = t^2 - \frac{t^3}{3}$

Integrating, we have

$$x = \frac{t^3}{3} - \frac{t^4}{12} = \frac{t^3}{3}\left(1 - \frac{t}{4}\right)$$

$$\therefore \quad x \text{ (at } t = 3 \text{ s)} = \frac{(3)^3}{3}\left(1 - \frac{3}{4}\right) = \frac{(3)^2}{4} \text{ m}$$

Hence the correct choice is (b).

118. The radius of the circular motion of the bead is $r = L$. The linear acceleration of the bead is $a = \alpha r = \alpha L$. If m is the mass of the bead, then

Force acting on the bead = $ma = m\, \alpha\, L$

$\therefore$ Reaction force acting on the bead is $R = m\, \alpha\, L$

The bead starts slipping when frictional force between the bead and the rod becomes equal to centrifugal force acting on the bead, i.e.

$$\mu R = \frac{mv^2}{r}$$

or $\mu m \alpha L = mr\omega^2 = mL\omega^2$ $(\because v = r\omega)$

or $\mu\alpha = \omega^2 = (\alpha t)^2$ $(\because \omega = \alpha t)$

or $\mu\alpha = \alpha^2 t^2$ or $t = \sqrt{\dfrac{\mu}{\alpha}}$,

which is choice (a).

119. The magnitude of the force is

$$F = \sqrt{\mathbf{F}\cdot\mathbf{F}} = \left[\left(6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}}\right)\cdot\left(6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}}\right)\right]^{1/2}$$

$$= \{(6)^2 + (8)^2 + (10)^2]^{1/2} = (200)^{1/2} = 10\sqrt{2} \text{ N}$$

$\therefore$ Mass $= \dfrac{F}{a} = \dfrac{10\sqrt{2}\text{N}}{1\text{ms}^{-2}} = 10\sqrt{2}$ kg. Hence the correct choice is (a).

120. let M be the mass of the chain and L its length. If a length l hangs over the edge of the table, the force pulling the chain down is $\dfrac{Ml}{L}g$. The force of friction between the rest of the chain of length $(L - l)$ and the table is $\dfrac{\mu M(L-l)}{L}g$.

For equilibrium, the two forces must be equal, i.e.

$$\frac{Ml}{L}g = \frac{\mu M(L-l)}{L}g$$

or $l = \mu(L - l)$

or $l = \dfrac{\mu L}{1+\mu}$

Thus, the correct choice is (b).

121. $\dfrac{l}{L} = \dfrac{\mu}{1+\mu} = \dfrac{0.25}{1+0.25} = \dfrac{1}{5}$ or 20%, which is choice (c).

122. Refer to Fig. 3.103. Vertical component of F is $F\sin\theta$ and the horizontal component is $F\cos\theta$. Thus

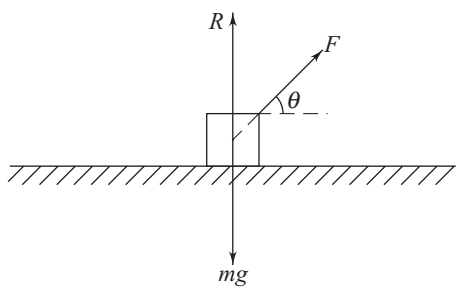


Fig. 3.103

$$R + F\sin\theta = mg$$

or $R = mg - F\sin\theta$

Frictional force $\mu R = \mu(mg - F\sin\theta)$. Also

$\mu(mg - F\sin\theta) = F\cos\theta$

or $F = \dfrac{\mu mg}{(\mu\sin\theta + \cos\theta)}$ (i)

F will be minimum if the denominator is maximum, i.e. if

$$\frac{d}{d\theta}(\mu\sin\theta + \cos\theta) = 0$$

or $\mu\cos\theta - \sin\theta = 0$ or $\mu = \tan\theta$, which is choice (a).

123. Now $\tan\theta = \mu$. Therefore, $\cos\theta = \dfrac{1}{\sqrt{1+\mu^2}}$ and $\sin\theta = \dfrac{\mu}{\sqrt{1+\mu^2}}$

Using these in Eq. (i) above and simplifying, we get

$$F = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

Hence the correct choice is (d).

124. Force required to accelerate the body of mass m is $F = (\mu_s - \mu_k)mg = (0.75 - 0.5)mg = 0.25\,mg$

$\therefore$ Acceleration $= \dfrac{F}{m} = 0.25\,g$, which is choice (a).

125. Since the blocks slide at the same uniform speed, no net force acts on them. If M is the mass of block B, then the tension in the string is $T = \mu Mg$. Also $T = mg$. Equating the two, we get $\mu M = m$ or $M = \dfrac{m}{\mu}$, which is choice (b).

126. Extension in the spring $x = \dfrac{F}{k} = \dfrac{mg}{k}$. Therefore, potential energy stored in the spring is

$$\text{PE} = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = \frac{m^2g^2}{2k}$$

Hence the correct choice is (a).

127. If the acceleration of the block and trolley system is a, then we have

$$mg - T = ma \quad \text{(i)}$$

and $T - \mu Mg = Ma$ (ii)

Eliminating T, we get

$a = \left(\dfrac{m - \mu M}{m + M}\right)g$, which is choice (d).

128. Dividing Eq. (i) by Eq. (ii) and simplifying we find that the correct choice is (a).

129. When a cylinder rolls up or down an inclined plane, its angular acceleration is always directed down the plane. Hence the frictional force acts up the inclined plate when the cylinder rolls up or down the plane. Thus, the correct choice is (b).

130. As shown in Fig. 3.104, the insect will crawl without slipping if the value of α is not greater than that given by the condition: force of friction $f = mg \sin \alpha$. Now $f = \mu N$, where N is the normal reaction. Thus

$$\mu N = mg \sin \alpha$$

or $$\mu\, mg \cos \alpha = mg \sin \alpha$$

or $$\cot \alpha = \frac{1}{\mu} = 3,$$ which is choice (a).

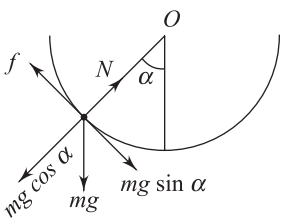


Fig. 3.104

131. Let T be the tension in the string. When the system is in equilibrium, then for the two equal masses m, we have

$$T = mg \qquad \text{(i)}$$

and for the mass $\sqrt{2}\, m$, we have

$$2T \cos \theta = \sqrt{2}\ mg \qquad \text{(ii)}$$

Dividing (ii) by (i), we get $\cos \theta = \frac{1}{\sqrt{2}}$ or $\theta = 45°$, which is choice (c).

132. The force F on the pulley by the clamp is given by the resultant of two forces: tension $T = Mg$ acting horizontally and a force $(m + M)g$ acting vertically downwards. Thus

$$F = \sqrt{(Mg)^2 + \{(m+M)\,g\}^2} = [M^2 + (m+M)^2]^{1/2}\, g$$

which is choice (d).

133. Refer to Fig. 3.105. Since the block moves with a constant velocity, no net force acts on it. Therefore, the horizontal component $F \cos \theta$ of force F must balance with the friction force, i.e. $f_r = F \cos \theta$. Also

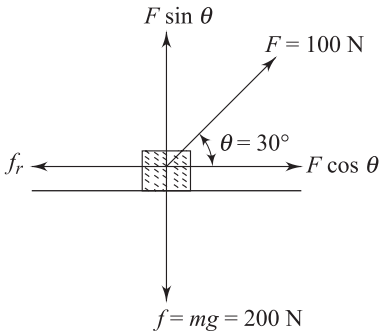


Fig. 3.105

$$f_r = \mu\,(mg - F \sin \theta)$$
$$= \mu\,(f - F \sin \theta)$$

$\therefore$ $\mu\,(f - F \sin \theta) = F \cos \theta$

or $\mu\,(200 - 100 \sin 30°) = 100 \cos 30°$

or $$\mu \left(200 - 100 \times \frac{1}{2}\right) = 100 \times 0.866 = 86.6$$

or $$\mu = \frac{86.6}{150} = 0.58,$$

which is choice (b).

134. Change of momentum of one bullet
$$= m\,(v - u)$$
$$= 0.03 \times \{50 - (-30)\}$$
$$= 2.4 \text{ kg ms}^{-1}$$

Average force = rate of change of momentum of 200 bullets
$$= 200 \times 2.4 = 480 \text{ N},$$

which is choice (d).

135. Let the body leave the surface at point B as shown in Fig. 3.106. When the body is between points A and B, we have

$$Mg \cos \theta - N = \frac{Mv^2}{r}$$

When the body leaves the surface at point B, the normal reaction N becomes zero. Thus

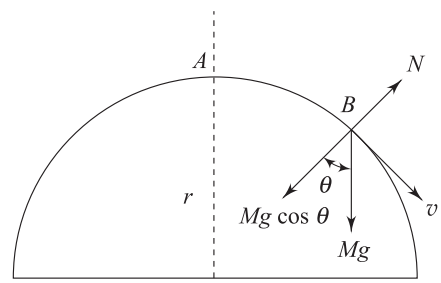


Fig. 3.106

$$Mg \cos \theta = \frac{Mv^2}{r}$$

or $$\cos \theta = \frac{v^2}{rg} = \frac{(5)^2}{5 \times 10} = \frac{1}{2} \quad \text{or} \quad \theta = 60°$$

Hence the correct choice is (c).

136. Mass of each piece (m) = 1 kg. Initial momentum = 0. Final momentum = $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3$. From the principle of conservation of momentum, we have

$$\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0$$

or $$\mathbf{p}_3 = -(\mathbf{p}_1 + \mathbf{p}_2) = -(m\mathbf{v}_1 + m\mathbf{v}_2)$$

$$= -m(\mathbf{v}_1 + \mathbf{v}_2)$$

$$= -1 \text{ kg} \times (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \text{ ms}^{-1}$$

$$= -(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \text{ kg ms}^{-1}$$

Force $$F = \frac{\mathbf{p}_3}{t} = \frac{-(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \text{ kgms}^{-1}}{10^{-5}\text{s}}$$

$$= -(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times 10^5 \text{ newton}$$

Hence the correct choice is (b).

137. Since the body is whirled in a horizontal circle, the gravity, acting vertically downwards, has no effect on the motion. If v is the greatest speed with which the body can be whirled, the maximum centripetal force (or tension) in the string is mv^2/R, which must balance a force of 25 N. Thus

$$25 = \frac{mv^2}{R} = \frac{1 \times v^2}{1}$$

which gives $v = 5$ ms^{-1}, which is choice (b).

138. It is clear from Fig. 3.107, that when the body at the top point A of the circle, its weight mg and tension T_1 in the string act downwards towards the centre O of the circle and the sum of the two provides the necessary centripetal force. Thus

$$T_1 + mg = mR\omega^2$$

or $$T_1 = m(R\omega^2 - g)$$

$$= 0.5 \times (0.5 \times 10^2 - 10)$$

$$= 20 \text{ N}$$

Thus, the correct choice is (b)

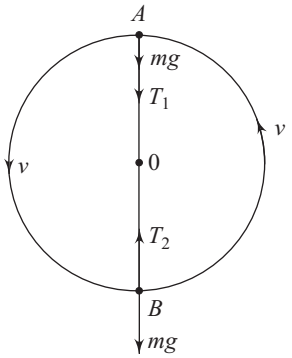


Fig. 3.107

139. When the body is at the bottom of the circle (point B in Fig. 3.107), the tension T_2 is opposite to weight mg and the difference ($T_2 - mg$) provides the necessary centripetal force. Therefore, we have

$$T_2 = m(R\omega^2 + g)$$

$$= 0.5 \times (0.5 \times 10^2 + 10) = 30 \text{ N}$$

Thus, the correct choice is (c).

140. Referring to Fig. 3.108, the cyclist is moving on a straight road from A to B with a velocity $v = 6$ ms^{-1}.

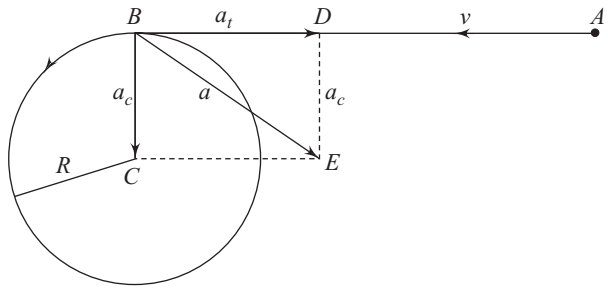


Fig. 3.108

As he approaches the circular turn, he decelerates at rate $\mathbf{a}_t$, represented by vector **BD**. The magnitude of declaration is $a_t = 0.4$ ms^{-2}. At point B, two accelerations $\mathbf{a}_t$ and $\mathbf{a}_c$, the centripetal acceleration directed towards the centre C act on the cyclist. Now $a_c = \frac{v^2}{R} = \frac{(6)^2}{120} = 0.3$ ms^{-2}. Using the law of parallelogram of vector addition, vector **B E** gives the resultant acceleration a whose magnitude is ($\because$ **DE** = $\mathbf{a}_c$)

$$a = (a_t^2 + a_c^2)^{1/2} = \{(0.4)^2 + (0.3)^2\}^{1/2} = 0.5 \text{ ms}^{-2}$$

Hence the correct choice is (a).

141. The tension in the string is given by

$$T = mg\cos\theta + \frac{mv^2}{r}$$

where r is the length of the string and v, the velocity of the bob when its angular displacement is θ. When the angular displacement is maximum, i.e. when $\theta = 40°$, $v = 0$. Tension at $\theta = 20°$ is given by

$$T = mg \cos 20° + \frac{mv^2}{r}$$

where v is the velocity when $\theta = 20°$. Hence the correct choice is (c).

142. Speed of car $(v) = 36 \text{ km h}^{-1} = 10 \text{ ms}^{-1}$. The maximum centripetal force that friction can provide is

$$f_{\max} = \mu mg = \frac{mv^2}{R}$$

or $$R_{\min} = \frac{v^2}{\mu g} = \frac{10 \times 10}{0.8 \times 10} = 12.5 \text{ m}$$

This is the minimum radius the curve must have for the car to negotiate it without sliding at a speed of 10 ms^{-1}. Hence the correct choice is (a).

143. Speed of train $(v) = 36 \text{ km h}^{-1} = 10 \text{ ms}^{-1}$

Radius of the curve $(R) = 200$ m

Distance between rails $(x) = 1.5$ m

Let the outer rails be raised by a height h with respect to the inner rails so that the angle of banking is θ (Fig. 3.109).

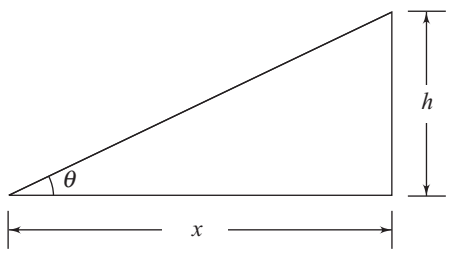


Fig. 3.109

Then $$\tan \theta = \frac{h}{x} = \frac{v^2}{Rg}$$

or $$h = \frac{xv^2}{Rg} = \frac{1.5 \times (10)^2}{200 \times 10}$$

$$= 0.075 \text{ m} = 7.5 \text{ cm}$$

Thus, the correct choice is (a).

144. Now $v = 54 \text{ km h}^{-1} = 15 \text{ ms}^{-1}$, $R = 50$ m. The required angle of banking is given by

$$\tan \theta = \frac{v^2}{Rg} = \frac{15 \times 15}{50 \times 10} = 0.45$$

Thus, the correct choice is (d)

145. Refering to Fig. 3.110, when the stone is at the topmost point A, the net force towards the centre is

$$T_A + mg = \frac{mv_A^2}{R} \quad \text{(i)}$$

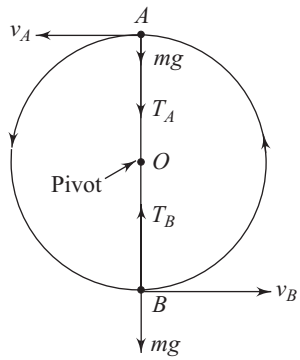


Fig. 3.110

When the stone is at the lowermost point B, the net force towards the centre is

$$T_B - mg = \frac{mv_B^2}{R} \quad \text{(ii)}$$

The relation between v_A and v_B can be obtained from the principle of conservation of energy. Let the gravitational potential energy be zero at the lowermost point B. Then

KE at B + PE at B = KE at A + PE at A

or $$\frac{1}{2} mv_B^2 + 0 = \frac{1}{2} mv_A^2 + mg \times AB$$

$$= \frac{1}{2} mv_A^2 + mg \times 2R$$

or $$v_B^2 = v_A^2 + 4gR \quad \text{(iii)}$$

Now, when the stone is at A, the string will not slack if the whole of centripetal force is provided by the weight mg, i.e. $T_A = 0$. Putting $T_A = 0$, we have

$$mg = \frac{mv_A^2}{R} \text{ or } v_A = \sqrt{gR}$$

Hence, the correct choice is (a).

146. The minimum speed of the stone when it is at its lowermost position B, so that the stone can complete the circle is obtained from Eq. (iii) above by putting $v_A = \sqrt{gR}$ which gives

$$v_B = \sqrt{5gR}$$

Hence, the correct choice is (c).

147. The pilot does not drop down when he is at the top of the loop because his weight mg is less than the centripetal force $m\,v^2/R$ required to keep him in the

loop. The rest of the centripetal force is balanced by the reaction of the seat. Hence, he is stuck to the seat without being tied to it. If the speed of the aircraft is reduced so that $mg > mv^2/R$, he will fall off from his seat. Therefore, the minimum speed $v_{\min}$ required to successfully negotiate the vertical loop is given by

$$mg = \frac{mv_{\min}^2}{R}$$

or $$v_{\min} = \sqrt{gR} = \sqrt{10 \times 4000}$$
$$= 200 \text{ ms}^{-1}$$

Thus, the correct choice is (b).

148. Since the surface is perfectly frictionless, the disc will not roll on the surface; it will simply keep on rotating at point B where it is placed. Now

Linear speed = distance from centre × angular speed

Given, $r = 20$ cm $= 0.2$ m and $\omega = 20$ rad s^{-1}.

$\therefore$ Linear speed at point $A = OA \times \omega = r\omega = 0.2 \times 20 = 4$ ms^{-1}. Thus, the correct choice is (d).

149. Linear speed at point $B = BO \times \omega = r\omega = 0.2 \times 20 = 4$ ms^{-1}. Hence, the correct choice is (d).

150. Linear speed at point $C = CO \times \omega = \frac{1}{2} r\omega = 2$ ms^{-1}, which is choice (b).

151. Given, $m = 200$ kg, $R = 80$ m and $v = 72$ km h$^{-1} = 20$ ms^{-1}. The necessary frictional force is given by

$$F = \frac{mv^2}{R} = \frac{200 \times (20)^2}{80} = 1000 \text{ N}$$

Thus, the correct choice is (c).

152. The angle at which the rider must lean is given by

$$\tan\theta = \frac{v^2}{Rg} = \frac{(20)^2}{80 \times 10} = 0.5$$

Hence, the correct choice is (d).

153. The maximum centripetal force that the friction can provide is

$$F = \mu mg = \frac{mv^2}{R}$$

or $$\mu = \frac{v^2}{Rg} = \frac{15 \times 15}{30 \times 10}$$
$$= 0.75 \ (\because 54 \text{ km h}^{-1} = 15 \text{ ms}^{-1})$$

Hence, the correct choice is (d).

154. Given, $m = 100$ g $= 0.1$ kg, $n = \frac{\sqrt{5}}{\pi}$ Hz and $L = 1$ m.

Referring to Fig. 3.111, tension T can be resolved into two perpendicular components: $T \sin\theta$ and $T\cos\theta$. The horizontal component $T \sin\theta$ provides the centripetal force for circular motion and the vertical component $T\cos\theta$ balances with the weight mg. Thus, since

$R = CB = L \sin\theta$,

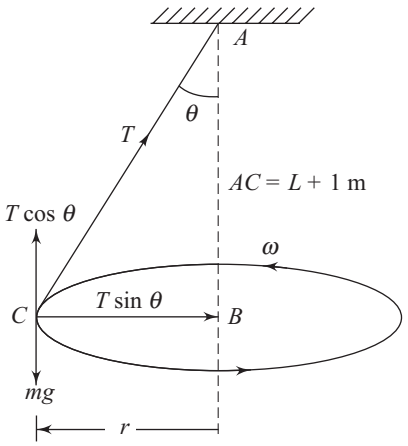


Fig. 3.111

$$T \sin\theta = \frac{mv^2}{R} = m\omega^2 R = m\omega^2 L \sin\theta \quad \text{(i)}$$

and $$T\cos\theta = mg \quad \text{(ii)}$$

From (i) we have $T = m\omega^2 L = m\omega^2 \quad (\because L = 1 \text{ m})$

Using this in Eq. (ii) we get,

$$m\omega^2 \cos\theta = mg$$

or $$\cos\theta = \frac{g}{\omega^2} = \frac{g}{4\pi^2 n^2} = \frac{10}{4\pi^2 \times \frac{5}{\pi^2}}$$
$$= 0.5$$

which gives $\theta = 60°$. Thus the correct choice is (c).

155. $T = m\omega^2 = 0.1 \times 4\pi^2 \times \frac{5}{\pi^2} = 2$ N, which is choice (a).

156. Linear speed $v = \omega R = 2\pi n L \sin\theta$

$$= 2\pi \times \frac{\sqrt{5}}{\pi} \times 1 \times \sqrt{1 - \cos^2 60°}$$

$$= \sqrt{15} \text{ ms}^{-1}$$

Hence, the correct choice is (b).

157. When the bob passes through the equilibrium position O, the tension in the string is given by

$$T = mg + \frac{mv^2}{r}$$

where v is the speed of the bob at O. Now, the potential energy of the bob at the extreme position $A = mgh$ (see Fig. 3.112) which is converted into kinetic energy $\frac{1}{2}\ mv^2$ when it reaches O. Therefore

$$\frac{1}{2}\ mv^2 = mgh$$

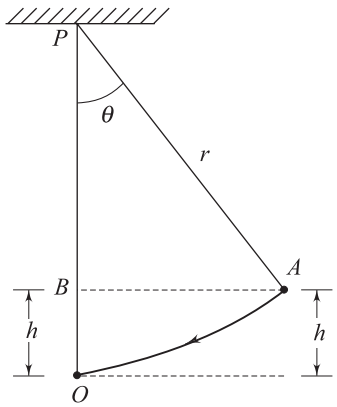


Fig. 3.112

or $v^2 = 2gh$. Using this we get $T = mg + \frac{2mgh}{r}$. $PB = r\cos\theta$. Therefore, $h = PO - PB = r - r\cos\theta = r(1 - \cos\theta)$. Using this value of h, the tension is

$$T = mg + 2\ mg\ (1 - \cos\theta)$$

$$= mg\ \{1 + 2\ (1 - \cos\theta)\}$$

Given, $m = 100$ g $= 0.1$ kg, $r = 1$ m and $\theta = 60°$. Putting these values, we get $T = 2$ N. Hence the correct choice is (b).

158. According to Hookes' law, the stretching force $F = kx$, where k is the force constant and x, the extension of the spring. A force mg stretches the spring by 1 cm. When the mass is describing the horizontal circle, total stretching $= 1 + 3 = 4$ cm. Hence

$$T = 4\ mg$$

Referring to Fig. 3.113, the horizontal component $T\sin\theta$ provides the necessary centripetal force for circular motion, i.e.

$$T\sin\theta = \frac{mv^2}{r} = m\omega^2 r$$

Given, $\omega = 2\pi n = 2\pi \times 2 = 4\pi$ rad s^{-1}. Let L cm be the length of the unstretched spring. Then $AC = (L + 4)$ cm and $r = (L + 4)\sin\theta$.

$$\therefore \quad T\sin\theta = m \times (4\pi)^2 \times (L + 4)\sin\theta$$

$$\text{or } L + 4 = \frac{T}{16\pi^2 m} = \frac{4mg}{16\pi^2 m} \qquad (\because\ T = 4\ mg)$$

$$= \frac{g}{4\pi^2} = \frac{\pi^2}{4\pi^2} = 0.25 \text{ m} = 25 \text{ cm}$$

or $L = 25 - 4 = 21$ cm. Hence the correct choice is (d).

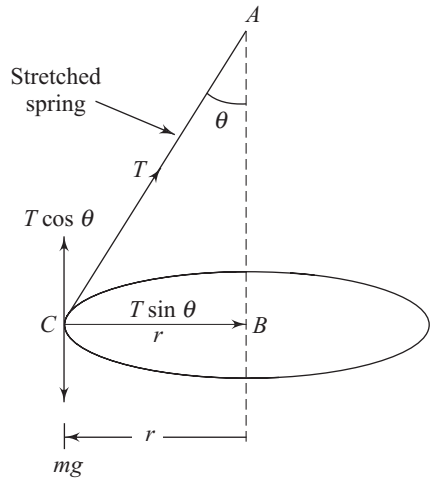


Fig. 3.113

159. We have seen above that $T = 4mg$. Referring to Fig. 3.101, we have

$$T\cos\theta = mg$$

$$\text{or} \qquad \cos\theta = \frac{mg}{T} = \frac{mg}{4mg} = \frac{1}{4}$$

Hence the correct choice is (a).

160. As the boy is given a little push, his initial speed can be taken to be zero. Suppose he leaves the mound at point P at a height h (Fig. 3.114). The forces acting on him are his weight mg and the normal reaction F. It is clear that at point P

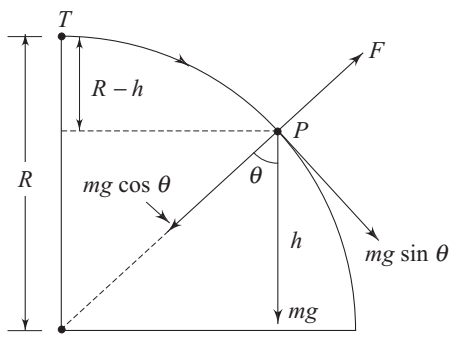


Fig. 3.114

$$mg\cos\theta - F = \frac{mv^2}{R} \qquad \text{(i)}$$

where v is the speed of the boy at P. This is the speed with which he leaves the ice. Energy conservation gives

$$\frac{1}{2}\ mv^2 = mg\ (R - h) \qquad \text{(ii)}$$

Using Eqs (i) and (ii) we get,

$$mg\cos\theta - F = 2mg\left(\frac{R-h}{R}\right) \qquad \text{(iii)}$$

As the boy leaves the mound, the normal reaction F vanishes. Thus, putting $F = 0$ in Eq. (iii), we have

$$\cos\theta = 2\left(\frac{R-h}{R}\right)$$

But $\cos\theta = \frac{h}{R}$. Therefore, we get $\frac{h}{R} = 2 - \frac{2h}{R}$ or $h = \frac{2}{3}\ R$. Hence, the correct choice is (c).

161. Given, $h = 2$ m, $R = 1.25$ m and horizontal distance $s = 10$ m. When the string breaks, the stone is projected in the horizontal direction, which means that there is no initial vertical velocity.

From $s = ut + \frac{1}{2}\ gt^2$, we have ($\because\ u = 0$),

$$h = \frac{1}{2}\ gt^2 \qquad \text{(i)}$$

The horizontal distance travelled in time t is

$$s = vt \qquad \text{(ii)}$$

where v is the velocity of the stone in the horizontal direction which is the same as its velocity in circular motion.

Eliminating t from (i) and (ii) we get

$$v^2 = \frac{gs^2}{2h}$$

Now, centripetal acceleration is

$$a_c = \frac{v^2}{R} = \frac{gs^2}{2hR} = \frac{10\times100}{2\times2\times1.25}$$

$$= 200\ \text{ms}^{-2}$$

Thus, the correct choice is (b).

162. Refer again to the solution to Q. 161. We have

$$v_B^2 = v_A^2 + 4gR$$

Putting $v_B = \sqrt{5gR}$, we get $v_A = \sqrt{gR}$. Therefore, kinetic energy at $A = \frac{1}{2}\ mv_A^2 = \frac{1}{2}\ mgR$. Hence the correct choice is (a).

163. Kinetic energy at $A = \frac{1}{2}\ mv_A^2 = \frac{1}{2}\ mgR$. When the string becomes horizontal, the stone falls through a height R and gains kinetic energy = potential energy lost in falling through height $R = mgR$. Hence, the kinetic energy when the string is horizontal = $\frac{1}{2}\ mgR + mgR = \frac{3}{2}\ mgR$. Thus, the correct choice is (c).

164. Refer also to the solution of Q.160 again. The horizontal velocity v must be such that the centripetal force equals the weight of the body, i.e. $\frac{mv^2}{R} = mg$ or $v = \sqrt{gR}$.

165. The car will skid if the normal reaction $F = \mu\ mg$ is less than the centripetal force mv^2/r or if $\frac{mv^2}{r} > \mu\ mg$ or $v > \sqrt{\mu rg}$. Hence, the correct choice is (a).

166. The motor cyclist can leave the ground only at the highest point on the bridge. At this point, the centripetal force is mv^2/R. He will not leave the ground if the centripetal force equals the weight mg. Thus $\frac{mv^2}{R} = mg$ or $v = \sqrt{gR} = \sqrt{10\times10} = 10\ \text{ms}^{-1}$.

Hence, the correct choice is (a)

167. Thrust at the highest point = centripetal force + weight

$$= \frac{mv^2}{R} + mg$$

Hence, the correct choice is (d).

168. When the system is in equilibrium, the spring force = 3 mg. When the string is cut, the net force on block $A = 3\ mg - 2\ mg = mg$. Hence the acceleration of this block at this instant is

$$a = \frac{\text{force on block A}}{\text{mass of block A}} = \frac{mg}{2m} = \frac{g}{2}$$

When the string is cut, the block B falls freely with an acceleration equal to g. Hence the correct choice is (c).

169. Impulse I = area under the $F - t$ graph

$$= AD\times\frac{1}{2}(AB + OC)$$

$$= 10\,\text{N}\times\frac{1}{2}(2+5)\text{ms}$$

$$= 35\times10^{-3}\ \text{Ns}$$

Impulse = change in momentum , *i.e.*

$$I = mv - mu = mv\ (\because\ u = 0)$$

$$\Rightarrow \qquad v = \frac{I}{m} = \frac{35\times10^{-3}\,\text{Ns}}{35\times10^{-3}\,\text{kg}} = 1\,\text{ms}^{-1}$$

So the correct choice is (*a*)

170. The free - body diagrams of the block and the rope are shown in Fig. 3.115.

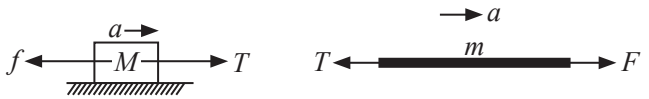


Fig. 3.115

For block : $T - f = Ma$ where $f = \mu Mg$

$$\therefore\ T - \mu Mg = Ma \qquad (1)$$

For rope : $F - T = ma$ (2)

Dividing (1) by (2) and simplifying, we find that the correct choice is (a).

171. Let R be the force exerted by A or B. The free-body diagrams are (since B applies an equal and opposite force R or A) as shown in Fig. 3.116

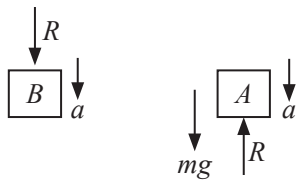


Fig. 3.116

For block A : $mg - R = ma$

$$\Rightarrow \qquad R = m\,(g - a)$$

So the correct choice is (c).

172. Since $(m_2 + m_3) > m_1$, blocks B and C will move down will an acceleration and block A will move up with the same acceleration. Let T_1 be the tension in the string connecting A and B and T_3 be the tension in the string connecting B and C. The free body diagrams of A, B and C are shown in Fig. 3.117.

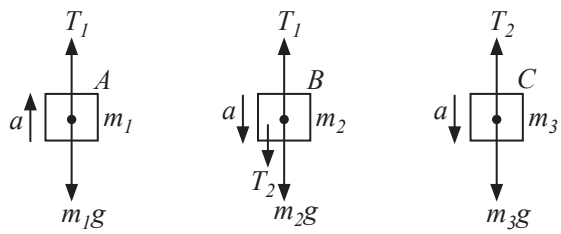


Fig. 3.117

For block A : $T_1 - m_1 g = m_1 a$ (1)

For block B : $T_2 + m_2 g - T_1 = m_2 a$ (2)

For block C : $m_3 g - T_2 = m_3 a$ (3)

Adding (1) and (2) and simplifying, we get

$$a = \frac{T_2 + (m_1 - m_2)g}{(m_1 + m_2)} \qquad (4)$$

Using (4) in (3) and simplifying, we get

$$T_2 = \frac{2m_2 m_3 g}{(m_1 + m_2 + m_3)} = \frac{2m_2 m_3 g}{M}$$

So the correct choice is (c).

173. Let R be the normal reaction exerted downward by man on the floor of the box. The box, in turn, will exert a force R upward on the man. The free body diagrams of the box and the man are shown in Fig. 3.118. Let T be the tension in the rope.

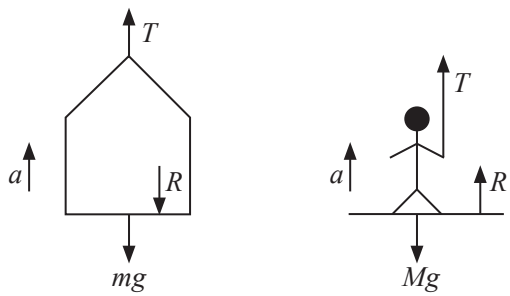


Fig. 3.118

For box : $T - R - mg = ma$ (1)

For man : $T + R - Mg = Ma$ (2)

Adding (1) and (2) we get

$$T = \frac{1}{2}(m + M)(g + a)$$

Putting $M = 2m$ and $a = g/3$, we get $T = \dfrac{2mg}{3}$. The correct choice is (a).

174. Subtracting (1) and (2) we get.

$$R = \frac{1}{2}(M - m)(g + a)$$

$$= \frac{1}{2}(2m - m)\left(g + \frac{g}{3}\right)$$

$$= \frac{2mg}{3}$$

So the correct choice in (b).

175. Let T_1 and T_2 be the tensions in the string as shown in Fig. 3.119.

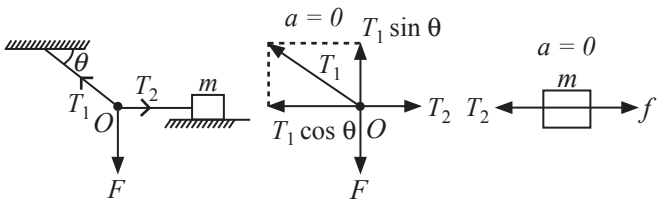


Fig. 3.119

Here frictional force $f = \mu mg$. Since the system is in static equilibrium, no net force acts at point O and on block m. Hence, acceleration of O and m is zero.

For point O : $T_2 = T_1 \cos\theta$ and

$$: F = T_1 \sin\theta \quad (1)$$

For block : $T_2 = f$

$$\Rightarrow \quad : T_1 \cos\theta = \mu mg \quad (2)$$

Dividing (1) and (2) we get

$$F = \mu mg \tan\theta \quad (3)$$

If force F exceeds the value given by (3), the block will begin to slide on the surface. So the correct choice is (c).

2 SECTION Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage.

Passage I

A block of masses m is initially at rest on a frictionless horizontal surface. A time-dependent force $F = at - bt^2$ acts on the body, where a and b are positive constants.

1. The magnitude of the force is maximum at time t_1 given by

   (a) $\frac{a}{b}$ (b) $\frac{2a}{b}$

   (c) $\frac{a}{2b}$ (d) $\frac{a}{\sqrt{2}b}$

2. The maximum force F_{max} is given by

   (a) $\frac{a^2}{2b}$ (b) $\frac{a^2}{4b}$

   (c) $\frac{2a^2}{b}$ (d) $\frac{4a^2}{b}$

3. The maximum impulse I_{max} imparted to the block is given by

   (a) $\frac{a^3}{3b^2}$ (b) $\frac{a^3}{6b^2}$

   (c) $\frac{a^3}{9b^2}$ (d) $\frac{a^3}{12b^2}$

4. The maximum velocity v_{max} attained by the block is

   (a) $\frac{a^3}{4mb^2}$ (b) $\frac{a^3}{8mb^2}$

   (c) $\frac{a^3}{12mb^2}$ (d) $\frac{a^3}{16mb^2}$

Solutions

1. The force is maximum when $\frac{dF}{dt} = 0$ and $\frac{d^2F}{dt^2} < 0$.

   Now $\frac{dF}{dt} = \frac{d}{dt}(at - bt^2) = a - 2bt$

   Putting $\frac{dF}{dt} = 0$ and $t = t_1$, we get

   $0 = a - 2\,bt_1 \Rightarrow t_1 = \frac{a}{2b}$

   Also $\frac{d^2F}{dt^2} = \frac{d}{dt}(a - 2\,bt) = -2\,b$, which is negative.

   Hence the correct choice is (c).

2. $F_{max} = at_1 - bt_1^2 = a \times \frac{a}{2b} - b \times \left(\frac{a}{2b}\right)^2 = \frac{a^2}{4b}$. Hence the correct choice is (b).

3. Maximum impulse is given by

$$I_{max} = \int_0^{t_1} F dt$$

$$= \int_0^{t_1} (at - bt^2) dt$$

$$= \frac{at_1^2}{2} - \frac{bt_1^3}{3}$$

$$= \frac{a}{2}\left(\frac{a}{2b}\right)^2 - \frac{b}{3}\left(\frac{a}{2b}\right)^3 = \frac{a^3}{12b^2}$$

   Hence the correct choice is (d).

4. Now impulse = change in momentum = $mv - 0 = mv$

   $\therefore v_{max} = \frac{I_{max}}{m} = \frac{a^3}{12mb^2}$, which is choice (c).

Questions 5 to 8 are based on the following passage.

Passage II

A body of mass m is initially at rest. A periodic force $F = a\cos(bt + c)$ is applied to it, where a, b and c are constants.

5. The time period T of the force is

   (a) $\frac{1}{b}$ (b) $\frac{2\pi}{b}$

   (c) $2\pi\sqrt{\frac{a}{b}}$ (d) $2\pi\sqrt{\frac{b}{a}}$

6. The maximum velocity of the body is

   (a) $\frac{a}{mb}$ (b) $\frac{b}{mc}$

   (c) $\frac{c}{ma}$ (d) $\frac{b+c}{ma}$

7. The smallest value of t after $t = 0$ when the velocity of the body becomes zero is given by

   (a) $t_1 = \frac{\pi}{a}$ (b) $t_1 = \frac{\pi - a}{c}$

   (c) $t_1 = \frac{\pi - c}{b}$ (d) $t_1 = \frac{\pi}{b}$

8. The distance travelled by the body from time $t = 0$ to $t = t_1$ is given by

   (a) $\frac{a}{mb^2}\cos c$ (b) $\frac{2a}{mb^2}\sin c$

   (c) $\frac{a^2}{mb}\cos c$ (d) $\frac{2a^2}{mb}\sin c$

Solutions

5. F will repeat itself at values of t given by $\cos(bt + c) = +1$, i.e.

   $bt + c = 0, 2\pi, 4\pi, \cdots$

   $\Rightarrow \quad t = -\frac{c}{b}, \frac{2\pi - c}{b}, \frac{4\pi - c}{b}, \cdots$

   The smallest time interval is $T = \frac{2\pi}{b}$. Hence the correct choice is (b).

6. From Newton's second law of motion,

   $$F = \frac{dp}{dt} = m\frac{dv}{dt}$$

   Thus $m\frac{dv}{dt} = a\cos(bt + c)$

   $\Rightarrow \quad dv = \frac{a}{m}\cos(bt + c)dt$

   $$\therefore v = \frac{a}{m}\int_0^t \cos(bt + c)dt = \frac{a}{mb}\sin(bt + c) \qquad \text{(i)}$$

   Since the maximum value of $\sin(bt + c) = 1$, $v_{max} = \frac{a}{mb}$.

   Hence the correct choice is (a).

7. From Eq (i) it follows that $v = 0$ at values of t given by $\sin(bt + c) = 0$ or $(bt + c) = 0, \pi, 2\pi, \ldots$ or $t = -\frac{c}{b}, \frac{\pi - c}{b}, \frac{2\pi - c}{b}$. Therefore,

   $t_1 = \frac{\pi - c}{b} - \left(-\frac{c}{b}\right) = \frac{\pi}{b}$, which is choice (d).

8. Now $v = \frac{dx}{dt} \Rightarrow dx = v\,dt$. Therefore, the distance moved between $t = 0$ and $t = t_1$ is

   $$x = \int_0^{t_1} v\,dt = \frac{a}{mb}\int_0^{t_1} \sin(bt + c)dt$$

   $$= -\frac{a}{mb^2}\cos(bt_1 + c)$$

   $$= -\frac{a}{mb^2}\cos\left[b \times \frac{\pi}{b} + c\right]$$

   $$= -\frac{a}{mb^2}\cos(\pi + c) = \frac{a\cos c}{mb^2}$$

   Hence the correct choice is (a).

Questions 9 to 11 are based on the following passage.

Passage III

Three masses $m_1 = m$, $m_2 = 2\,m$ and $m_3 = 3\,m$ are hung on a string passing over a frictionless pulley as shown in Fig. 3.120. The mass of the string is negligible. The system is then released.

9. If a_1, a_2 and a_3 are the accelerations of masses m_1, m_2 and m_3 respectively, then

   (a) $a_1 < a_2 < a_3$

   (b) $a_1 > a_2 > a_3$

   (c) $a_1 > a_2 = a_3$

   (d) $a_1 = a_2 = a_3$

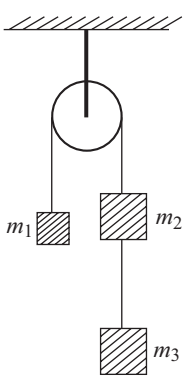
10. The tension in the string between masses m_2 and m_3 is

    (a) mg

    (b) $3\,mg$

    (c) $4\,mg$

    (d) $\frac{5mg}{3}$



3. 120

11. The tension in the string between masses m_1 and m_2 is

(a) 4 mg (b) $\frac{2mg}{3}$

(c) $\frac{5mg}{3}$ (d) 2 mg

Solutions

9. When the masses are released, mass m_1 moves upward and masses m_2 and m_3 move downward with a common acceleration given by

$$a = \frac{(m_2 + m_3 - m_1)g}{(m_1 + m_2 + m_3)} = \frac{(2m + 3m - m)g}{(m + 2m + 3m)} = \frac{2g}{3}$$

The correct choice is (d).

10. Let T be the tension in the string between m_1 and m_2 and T' be the tension in the string between m_2 and m_3 [see Fig. 3.121 (a)]. Figure 3.121 (b) shows the free-body diagram of mass m_3.

$$m_3 g - T' = m_3 a$$

$$\Rightarrow \quad T' = m_3(g - a) = 3\ m \times \left(g - \frac{2g}{3}\right) = mg$$

Hence the correct choice is (a).

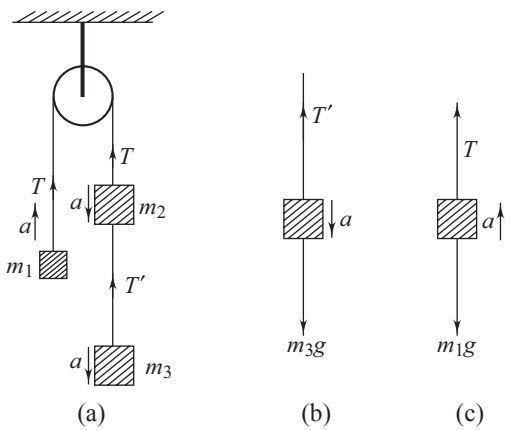


Fig. 3.121

11. Figure 3.113(c) shows the free-body diagram of mass m_1.

$$T - m_1 g = m_1 a$$

$$\Rightarrow \quad T = m_1(g + a) = m \times \left(g + \frac{2g}{3}\right) = \frac{5mg}{3}$$

Hence the correct choice is (c).

Questions 12 to 14 are based on the following passage.

Passage IV

Two blocks of masses $m_1 = 3\ m$ and $m_2 = 2\ m$ are suspended from a rigid support by two inextensible uniform wires A and B. Wire A has negligible mass and wire B has a mass $m_3 = m$, as shown in Fig. 3.122. The whole system of blocks, wires and the support have an upward acceleration a.

12. The tension at the mid-point C of wire B is

(a) $\frac{1}{2}\ m(g + a)$

(b) $\frac{3}{2}\ m(g - a)$

(c) $\frac{3}{2}\ m(g + a)$

(d) $\frac{5}{2}\ m(g + a)$

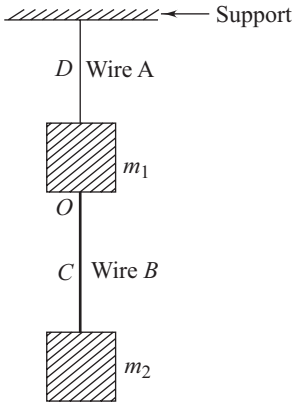
13. The tension at point O of wire B is

(a) $3m(g + a)$

(b) $3m(g - a)$

(c) $2m(g + a)$

(d) $2m(g - a)$



3. 122

14. The tension at the mid-point D of wire A is

(a) $2m(g + a)$ (b) $4m(g - a)$

(c) $6m(g + a)$ (d) $8m(g - a)$

Solutions

12. Refer to Fig. 3.123. Let T be the tension at the mid-point C of wire B. Then

$$T - \left(m_2 + \frac{m_3}{2}\right)g = \left(m_2 + \frac{m_3}{2}\right)a$$

$$\Rightarrow \quad T = \left(m_2 + \frac{m_3}{2}\right)(g + a)$$

$$= \left(2m + \frac{m}{2}\right)(g + a)$$

$$= \frac{5}{2}\ m(g + a),$$

which is choice (d).

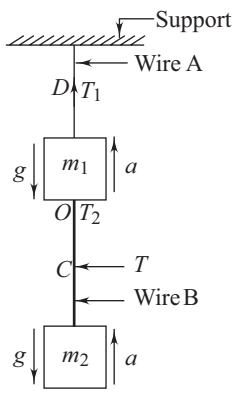


Fig. 3.123

13. Let T_1 be the tension in wire A. Since this wire has negligible mass, the tension is the same ($= T_1$) at every point on this wire. Let T_2 be the tension at point O of wire B. Then, we have for wire A.

$$T_1 - T_2 - m_1 g = m_1 a \qquad \text{(i)}$$

where T_2 is given by

$$T_2 - (m_2 + m_3)g = (m_2 + m_3)a$$

$$\Rightarrow \qquad T_2 = (m_2 + m_3)(g + a)$$

$$= (2m + m)(g + a) = 3m(g + a)$$

Hence the correct choice is (a).

14. Putting $T_2 = 3m(g + a)$ in Eq. (i), we get

$$T_1 = 6\, m(g + a).$$

Hence the correct choice is (c).

SECTION 3

Assertion-Reason Type Questions

In the following questions, **Statement-1 (Assertion)** is followed by **Statement-2 (Reason)**. Each question has four choices out of which only **one** choice is correct

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true but Statement-2 is not the correct explanation for Statement-1.

(c) Statement-1 is true; Statement-2 is false.

(d) Statement-1 is false; Statement-2 is true.

1. **Statement-1**

A block is pulled along a horizontal frictionless surface by a thick rope. The tension in the rope will not always be the same at all points on it.

Statement-2

The tension in the rope depends on the acceleration of the block-rope system and the mass of the rope.

2. **Statement-1**

A truck moving on a horizontal surface with a uniform speed u is carrying sand. If a mass Δm of the sand 'leaks' from the truck in a time Δt, the force needed to keep the truck moving at its uniform speed is $u\, \Delta m/\Delta t$.

Statement-2

Force = rate of change of momentum.

3. **Statement-1**

Two blocks of masses m and M are placed on a horizontal surface as shown in Fig. 3.124. The coefficient of friction between the two blocks is μ_1 and that between the block M and the horizontal surface is μ_2. The maximum force that can be applied to block M so that the two blocks move without slipping is $F = (\mu_1 + \mu_2)(M + m)g$.

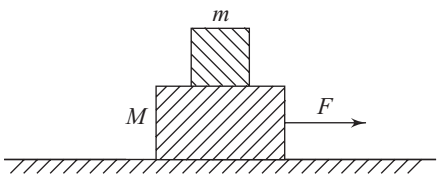


Fig. 3.124

Statement-2

Maximum force = total mass × maximum acceleration.

4. **Statement-1**

A shell of mass m is at rest initially. It explodes into three fragments having masses in the ratio 2:2:1. The fragments having equal masses fly off along mutually perpendicular directions with speed v. The speed of the third (lighter) fragment will be $2\sqrt{2}\, v$.

Statement-2

The momentum of a system of particles is conserved if no external force acts on it.

5. **Statement-1**

The maximum value of force F such that the block shown in Fig. 3.125 does not move is $\mu\, mg/\cos\theta$, where μ is the coefficient of friction between the block and the horizontal surface.

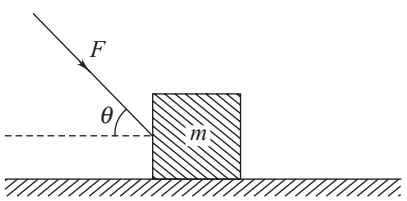


Fig. 3.125

Statement-2

Frictional force = coefficient of friction × normal reaction.

6. **Statement-1**

   A ball of mass m is moving towards a batsman at a speed v. The batsman strikes the ball and deflects it by an angle θ without changing its speed. The impulse imparted to the ball is zero.

   Statement-2

   Impulse = change in momentum

7. **Statement-1**

   A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.

   Statement-2

   For every action there is an equal and opposite reaction.

8. **Statement-1**

   When a ball dropped from a certain height hits the floor, it exerts a force equal to the rate of change of momentum.

   Statement-2

   The floor does not move because the action and reaction forces, being equal and opposite, cancel each other.



Solutions

1. The correct choice is (a).
2. The correct choice is (d). The force exerted by the leaking sand on the truck = rate of change of momentum = $u\ \Delta m/\Delta t$. The sand falling vertically downwards will exert this force on the truck in the vertically upward direction. This perpendicular force can do no work on the truck. Since the truck moves with a uniform velocity, the force exerted just overcomes the frictional force.
3. The correct choice is (c). The force of friction between block m and block $M = \mu_1 mg$, where μ_1 is the coefficient of friction between the two blocks. Now, the force of friction between block M (with block m on top of it) and the horizontal surface = $\mu_2(M + m)g$, where μ_2 is the coefficient of friction between block M and surface. The maximum force F applied to block M must be enough to overcome this force of friction and the force due to acceleration of the system. If the acceleration of the system is a then this force = $(M + m)a$. Thus

$$F = (M + m)a + m_2(M + m)g \qquad \text{(i)}$$

   Now, since the force on block m is $\mu_1 mg$, its acceleration a is

$$a = \frac{\text{force on mass } m}{\text{mass } m} = \frac{\mu_1 mg}{m} = \mu_1 g \qquad \text{(ii)}$$

   Using (ii) in (i) we get

$$F = \mu_1(M + m)g + \mu_2(M + m)g$$
$$= (\mu_1 + \mu_2)\,(M + m)g$$

4. The correct choice is (a). The mass of two fragments of equal masses = $\frac{2m}{5}$ each. The mass of the lighter fragment = $\frac{m}{5}$. The momenta of heavier fragments are $p = \frac{2mv}{5}$. The resultant of momenta p and p is

$$p' = (p^2 + p^2)^{1/2} = \sqrt{2}\,p$$

   From the principle of conservation of momentum, the momentum of the third (lighter) fragment of mass $\frac{m}{5}$ must be $\sqrt{2}\,p$ but opposite in direction. Thus, if V is the speed of the lighter fragment, we have

$$\frac{mV}{5} = \sqrt{2}\,p = \sqrt{2}\,\frac{2mv}{5}$$

   or $$V = 2\sqrt{2}\,v$$

5. The correct choice is (a). The component of F parallel to the horizontal surface is $F\cos\theta$. F will be maximum when $F\cos\theta$ just overcomes the frictional force $f = \mu mg$. Thus

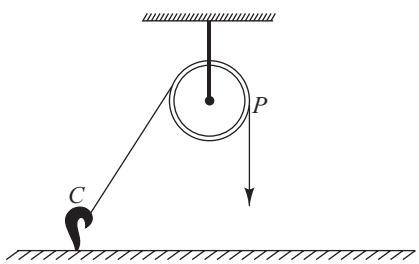
$$F_{\max}\cos\theta = \mu mg \Rightarrow F_{\max} = \frac{\mu mg}{\cos\theta}$$

6. The correct choice is (d). Refer to the solution of Q.44 of section I.
7. Statement-1 follows the Newton's first law of motion also called the law of inertia. The dishes are not dislodged even when the cloth is suddenly pulled because the dishes have the inertia of rest. Statement-2 is Newton's third law of motion, it does not explain statement-1. Hence the correct choice is (b).
8. The assertion is true but the reason is not correct because action and reaction forces do not act on the same body and hence do not cancel each other. Hence the correct choice is (c).

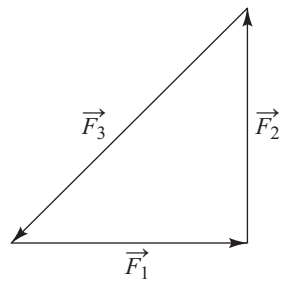
SECTION 4

Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

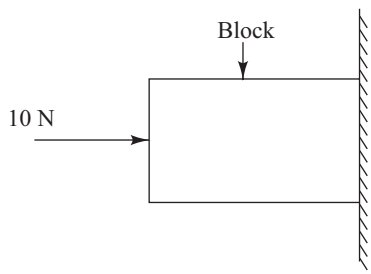
1. The minimum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is
   (a) 60 (b) 30
   (c) 15 (d) 25 **[2002]**
2. A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The accelerations of the ball as observed by the man in the lift and man standing stationary on the ground respectively are
   (a) g, g (b) $(g-a), (g-a)$
   (c) $(g-a), g$ (d) a, g **[2002]**
3. When forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ are acting on a particle of mass m such that $\vec{F}_2$ and $\vec{F}_3$ are mutually perpendicular, then the particle remains stationary. If force $\vec{F}_1$ is now removed, the magnitude of acceleration of the particle will be
   (a) $\frac{F_1}{m}$ (b) $\frac{F_2F_3}{mF_1}$
   (c) $\frac{F_2-F_3}{m}$ (d) $\frac{F_2}{m}$ **[2002]**
4. The speeds of two identical cars are u and $4u$ at a given instant. The ratio of the respective distances at which the two cars are stopped from that instant is
   (a) 1:1 (b) 1:4
   (c) 1:8 (d) 1:16 **[2001]**
5. One end of a massless rope, which passes over a massless and frictionless pulley P is tied to a hook C while the other end is free. The rope can bear a maximum tension of 360 N. With that maximum accleration (in ms^{-2}) can a man of mass 60 kg climb on the rope?
   (a) 16 (b) 6
   (c) 4 (d) 8 **[2002]**



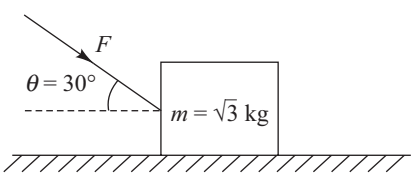
6. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $\frac{g}{8}$, then the ratio of masses is
   (a) 8:1 (b) 9:7
   (c) 4:3 (d) 5:3 **[2002]**
7. A spring balance is attached to the ceilling of a lift. A man hangs his bag on the spring and the balance reads 49 N, when the lift is stationary. If the lift moves downwards with an acceleration 5 ms^{-2}, the reading of the balance will be
   (a) 24 N (b) 74 N
   (c) 15 N (d) 49 N **[2003]**
8. Three forces start acting simultaneously on a particle moving with a velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangles as shown in the figure. The particle will now move with velocity
   (a) less then $\vec{v}$ (b) greater $\vec{v}$
   (c) equal to $\vec{v}$ (d) equal to zero **[2003]**



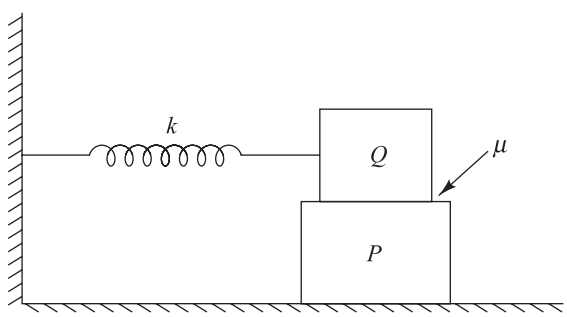
9. A horizontal force of 10 N is necessary to just hold a block stationary against a wall as shown in the figure. If the coefficient of friction between the block and the wall is 0.2, the weight of the block is
   (a) 20 N (b) 50 N
   (c) 100 N (d) 2 N **[2003]**



10. A marble block of mass 2 kg lying on ice when given a velocity of 6 ms^{-1} is stopped by friction in 10 s. The coefficient of friction between the block and ice is
    (a) 0.02 (b) 0.03
    (c) 0.06 (d) 0.01 **[2003]**
11. A block of mass M is pulled along a horizontal frictionless surface by a rope is mass m. If a force F is applied at the free end of the rope, the force exerted by the rope on the block is
    (a) $\frac{mF}{M+m}$ (b) $\frac{mF}{M-m}$
    (c) F (d) $\frac{MF}{M+m}$ **[2003]**
12. A light spring balance B_2 hangs from the hook of another light spring balance B_1. A block of mass M kg is hung from B_2. Choose the correct statement from the following.
    (a) Both B_1 and B_2 read M kg each
    (b) B_2 reads M kg and B_1 reads zero
    (c) The readings of B_1 and B_2 can have any values but the sum of the readings will be M kg
    (d) Both B_1 and B_2 will read $\frac{M}{2}$ kg. **[2003]**
13. A rocket with a of mass 3.5×10^4 kg is blasted upwards with an initial acceleration of 10 ms^{-2}. The initial thrust of the blast is
    (a) 3.5×10^5 N (b) 7.0×10^5 N
    (c) 14.0×10^5N (d) 1.75×10^5 N **[2003]**
14. What is the maximum value of the force F such that the block shown in the arrangement does not move? The coefficient of friction between the block and the horizontal surface is 0.5. (Take g = 10 ms^{-2})
    (a) 20 N (b) 10 N
    (c) 12 N (d) 15 N **[2003]**



15. A car is moving in a circular path of radius 500 m with a speed of 30 ms^{-1}. If the speed is increased at the rate of 2ms^{-2}, the resultant acceleration will be
    (a) 2 ms^{-2} (b) 2.5 ms^{-2}
    (c) 2.7 ms^{-2} (d) 4 ms^{-2} **[2003]**
16. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most?
    (a) one (b) two
    (c) three (d) four **[2004]**
17. A string passing over a light frictionless pulley carries two masses m_1 = 5 kg and m_2 = 4.8 kg at its ends hanging vertically. When the masses are released, their acceleration (in ms^{-2}) will be
    (a) 0.2 (b) 9.8
    (c) 5 (d) 4.8 **[2004]**
18. A block rests on a rough inclined plane making an angle of 30º with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) is (take g = 10 ms^{-2})
    (a) 2.0 (b) 4.0
    (c) 1.6 (d) 2.5 **[2004]**
19. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes in a plane. It follows that
    (a) Its velocity is constant
    (b) its acceleration is constant
    (c) its kinetic energy is constant
    (d) it moves in a straight line **[2004]**
20. A block P of mass m is placed on a horizontal frictionless surface. Another block Q of same mass is kept on P and connected to a rigid wall by means of a spring of spring constant k as shown in the figure. The two blocks move together, without slipping, performing simple harmonic motion of amplitude A. If μ is the coefficient of static friction between blocks P and Q, the maximum value of the force of friction between P and Q is
    (a) mg (b) $\frac{kA}{2}$
    (c) kA (d) zero **[2004]**



21. A smooth block is released from rest on a 45º rough incline and then slides a distance d. The time taken to slide is n times the time it takes to slide the same distance on a perfectly smooth 45º incline. the coefficient of kinetic friction is

(a) $\mu_k = 1 - \frac{1}{n^2}$ (b) $\mu_k = \sqrt{1 - \frac{1}{n^2}}$

(c) $\mu_k = \sqrt{\frac{1}{1 + \frac{1}{n^2}}}$ (d) $\mu_k = \frac{1}{1 + n^2}$

[2005]

22. The upper half of an inclined plane with inclination θ is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of friction of the lower half is

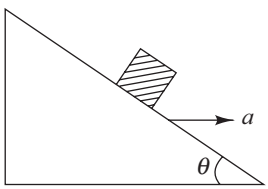
(a) 2 sin θ (b) 2 cos θ

(c) 2 tan θ (d) tan θ **[2005]**

23. A block is kept on a frictionless inclined surface of inclination θ as shown in the figure. The incline is given an acceleration a to keep the block stationary. Then a is

(a) $\frac{g}{\tan\theta}$ (b) g cosec θ

(c) g (d) $g \tan \theta$ **[2005]**



24. A particle of mass 0.3 kg is subjected to force $F = -kx$ where $k = 15$ N m^{-1}. What will be its initial acceleration (in ms^{-2}) if it is released from point 20 cm away from the origin?

(a) 3 (b) 15

(c) 5 (d) 10 **[2005]**

25. A car is moving on a straight road with a speed of 100 ms^{-1}. If the coefficient of friction between the types and road is 0.5, the distance at which the car can be stopped is

(a) 800 m (b) 1000 m

(c) 100 m (d) 400 m **[2005]**

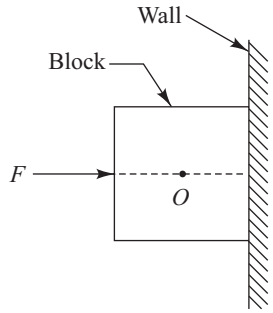
26. A block of mass m is held stationary against a wall by applying a horizontal force F on the block. Which of the following statement is false?

(a) The frictional force acting on the block is $f = mg$

(b) The normal reaction force acting on the block is $N = F$

(c) No net torque acts on the block

(d) N does not produce any torque. **[2005]**



27. A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1 s, the force of the blow exerted by the ball on the hand of the player is equal to

(a) 30 N (b) 300 N

(c) 150 N (d) 3 N **[2006]**

28. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes up to 2 m height further, find the magnitude of the force. Consider $g = 10$ m/s^2.

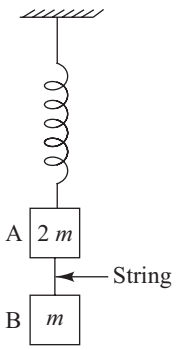
(a) 20 N (b) 22 N

(c) 4 N (d) 16 N **[2006]**

29. The blocks A and B of masses $2m$ and m are connected as shown in the figure. The spring has negligible mass. The string is suddenly cut. The magnitudes of accelerations of masses $2m$ and m at that instant are

(a) g, g (b) $g, \frac{g}{2}$

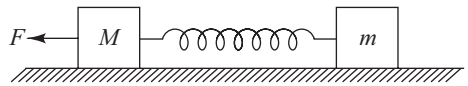
(c) $\frac{g}{2}, g$ (d) $\frac{g}{2}, \frac{g}{2}$ **[2006]**



30. A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k as shown in the figure. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then a constant force F starts acting on the block of mass M to pull it. Find the force on the block of mass m.

(a) $\frac{mF}{M}$ (b) $\frac{(M+m)F}{M}$

(c) $\frac{mF}{(m+M)}$ (d) $\frac{MF}{(m+M)}$ **[2007]**



31. A particle moves in the $x-y$ plane under the influence of a force such that its linear momentum is

$$\vec{p}(t) = A[\hat{\mathbf{i}}\ \cos\ (kt) - \hat{\mathbf{j}}\sin(kt)]$$

Where A and K are constants. The angle between the force and momentum is

(a) 0º (b) 30º

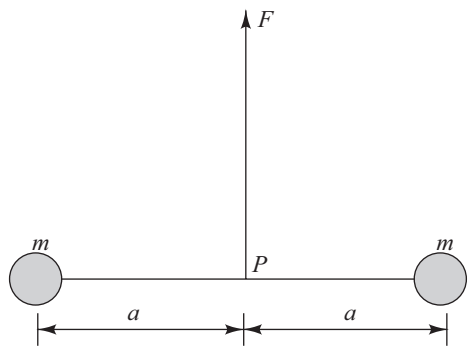
(c) 45º (d) 90 **[2007]**

32. Two particles of mass m each are tied at the ends of a light string of length $2a$. The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance 'a' from the centre P as shown in the figure. Now the mid-point of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them becomes $2x$, is

(a) $\frac{F}{2m}\frac{a}{\sqrt{a^2-x^2}}$ (b) $\frac{F}{2m}\frac{x}{\sqrt{a^2-x^2}}$

(c) $\frac{F}{2m}\frac{x}{a}$ (d) $\frac{F}{2m}\frac{\sqrt{a^2-x^2}}{x}$

[2007]



33. A body of mass m = 3.513 kg is moving along the x-axis with a speed of 5.00 ms^{-1}. The megnitude of its momentum (in kg ms^{-1}) is recorded as

(a) 17.6 (b) 17.565

(c) 17.56 (d) 17.57 **[2008]**

34. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then

(a) at θ =30º, the block will start sliding down the plane

(b) the block will remain at rest on the plane up to certain θ and then it will topple

(c) at θ = 60º, the block will start sliding down the plane and continue to do so at higher angles

(d) at θ = 60º, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ. **[2009]**

35. A piece of wire is bent in the shape of parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now acclerated parallel to the x-axis with a constant accleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is

(a) $\frac{a}{gk}$ (b) $\frac{a}{2gk}$

(c) $\frac{2a}{gk}$ (d) $\frac{a}{4gk}$ **[2009]**

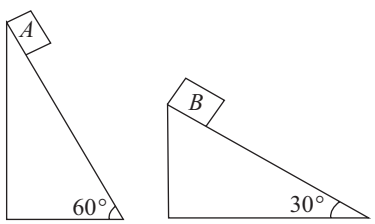
36. Two fixed frictionless inclined planes making an angle 30º and 60º with horizontal are shown in the figure. Two block A and B are place on the two planes, What is the relative vertical acceleration of A with respect to B?

(a) 4.9 ms^{-2} in horizontal direction

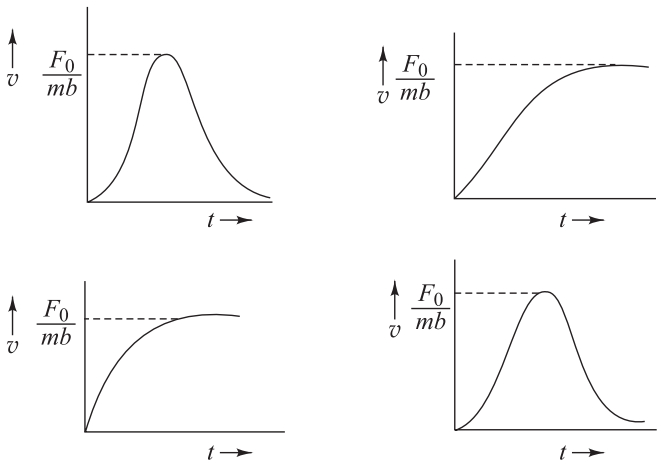
(b) 9.8 ms^{-2} in vertical direction

(c) zero

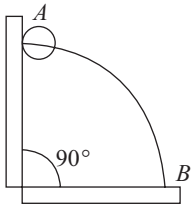
(d) 4.9 ms^{-2} in vertical direction **[2010]**



37. A particle of mass m is at rest at the origin at time t = 0. It is subjected to a force. $F(t) = F_0e^{-bt}$ in the x direction where F_0 and b are constants. Which of the following graphs represents the variation of the velocity (v) of the particle with time t?



38. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from *A* to *B*, the force it applies on the wire is



(a) always radially outwards
(b) always radially inwards
(c) radially outwards initially and radially inwards later.
(d) radially inwards initially and radially outwards later. **[2014]**

Answers

1. (b)	2. (c)	3. (a)	4. (d)
5. (c)	6. (b)	7. (a)	8. (d)
9. (d)	10. (c)	11. (d)	12. (a)
13. (a)	14. (b)	15. (c)	16. (c)
17. (a)	18. (a)	19. (c)	20. (b)
21. (a)	22. (c)	23. (d)	24. (d)
25. (b)	26. (d)	27. (a)	28. (b)
29. (c)	30. (c)	31. (d)	32. (b)
33. (a)	34. (b)	35. (b)	36. (d)
37. (c)	38. (d)		

Solutions

1. $v = \sqrt{\mu Rg} = \sqrt{0.6 \times 150 \times 10} = 30 \text{ ms}^{-1}$

2. Apparent weight in the lift = $m(g - a)$. Therefore, apparent accleration inside the lift is $(g - a)$. If the man is standing stationary on the ground, the acceleration of the falling ball is g. So the correct choice is (c).

3. Since the particle remains stationary, the resultant of $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ is zero, i.e.

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

$$\Rightarrow \quad \vec{F}_1 = -(\vec{F}_2 + \vec{F}_3)$$

Thus magnitude of $\vec{F}_1$ is equal to the magnitude of $(\vec{F}_2 + \vec{F}_3)$ but opposite in direction. Hence if $\vec{F}_1$ is removed, the magnitude of the force on the particle = magnitude of $(\vec{F}_2 + \vec{F}_3)$ = –magnitude of $\vec{F}_1$.

$$\therefore \quad \text{Acceleration } \vec{a} = -\frac{\vec{F}_1}{m}$$

Magnitude of acceleration = $-\frac{F_1}{m}$ but its direction is opposite to $\vec{F}_1$.

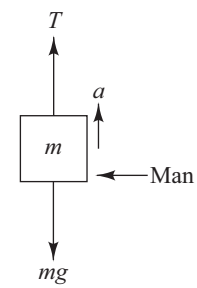
4. Since the cars are identical, their retardation a due to frictional force will be the same. From $v^2 - u^2 = 2as$, we have

$$0 - u^2 = -2as_1 \Rightarrow s_1 = \frac{u^2}{2a}$$

and $$0 - (4u)^2 = -2as_2 \Rightarrow s_2 = \frac{16u^2}{2a}$$

$$\therefore \quad \frac{s_1}{s_2} = \frac{1}{16}$$

5. From the free body diagram of the man



$$T - mg = ma$$

$$\Rightarrow \quad a = \frac{T - mg}{m}$$

$$\therefore \quad a = \frac{T_{\max} - mg}{m}$$

$$= \frac{360 - 60 \times 10}{60} = -4 \text{ ms}^{-2}$$

Negative acceleration implies climbing up the rope. The rope will break even if the man does not climb up the rope. However, choice (c) will be correct if the man were to climb down the rope.

6. Refer to Fig. 3.5 on page 3.4 of this chapter. The acceleration of the blocks is

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \qquad \ldots(i)$$

Given $a = \frac{g}{8}$. Using this in Eq (i), we get

$$\frac{m_1}{m_2} = \frac{9}{7}$$

7. Mass of bag is $m = \frac{49}{9.8} = 5$ kg

Aparent weight of the bag when the lift moves down an acceleration a is

$$W' = m\,(g - a) = 5 \times (9.8 - 5) = 24 \text{ N}$$

8. The resultant of three forces which can be represented in magnitude and direction by the three sides of triangle taken in the same order is zero. This follows from the triangle law of vector addition. From Newton's first law, if no net force acts on a particle, its velocity remains unchanged in magnitude and direction. Hence the correct choice is (d)

9. It follows from the free body diagram shown in the figure that

$W = f$

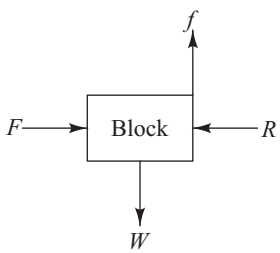
and $F = R$

But $f = \mu R$. Therefore

$W = \mu R = \mu F$

$= 0.2 \times 10$

$= 2$ N



10. Frictional force $f = \mu mg$. Therefore retardation is

$$a = -\frac{f}{m} = -mg$$

From $v = u + at$, we have

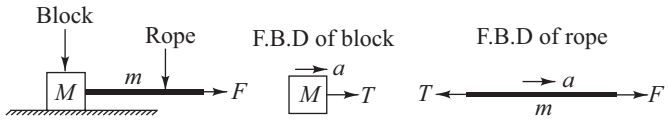
$$0 = 6 - a \times 10 \Rightarrow a = \frac{6}{10}$$

$$\therefore \text{ f} = m \times \frac{6}{10}$$

or $\mu mg = m \times \frac{6}{10}$

$$\therefore \mu = \frac{6}{10g} = \frac{6}{10 \times 10} = 0.06$$

11. Refer to the following figure.



Acceleration of block-rope system is

$$a = \frac{F}{M + m} \qquad \ldots(i)$$

From free body diagrams of block and rope, we have

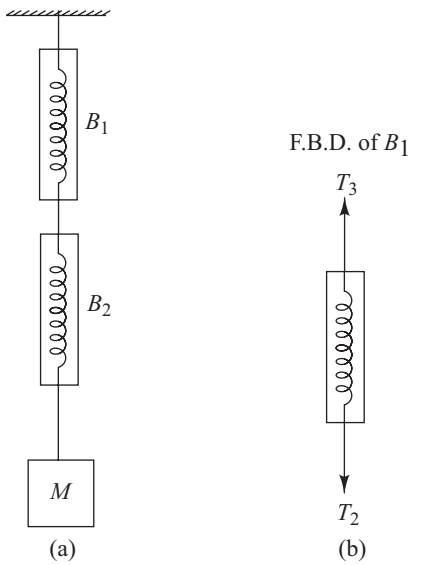
$$T = Ma$$

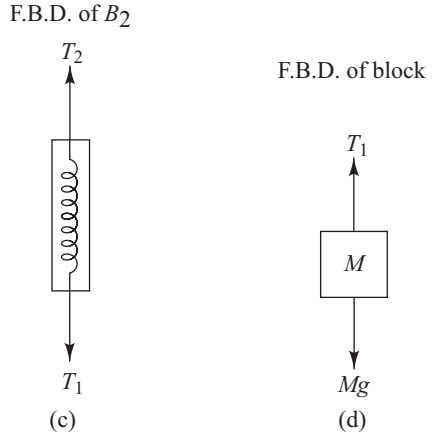
and $F - T = ma \qquad \ldots(ii)$

Using (i) in (ii) we have

$$T = \frac{MF}{M + m}, \text{ which is choice } (d)$$

12. Refer to the following figure.





T_1 = reading of B_3 and T_2 = reading of B_1.

For equilibrium

$$T_1 = Mg$$

$$T_2 = T_1$$

and $$T_3 = T_2$$

Hence $T_1 = T_2 = Mg$. So both B_1 and B_2 will read M kg.

13. Initial thrust = ma

$$= (3.5 \times 10^4) \times 10 = 3.5 \times 10^5 \text{ N}$$

14. The horizontal component of F parallel to the surface is $F \sin \theta$. Hence maximum value of F is given by

$$F \sin \theta = \mu mg$$

or $$F \sin 60^\circ = 0.5 \times \sqrt{3} \times 10$$

or $$F \frac{\sqrt{3}}{2} = 0.5 \times \sqrt{3} \times 10$$

Which gives $F = 10$ N.

15. Centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{(30)^2}{500} = 1.8 \text{ ms}^{-2}$$

Tangential acceleration is

$a_t = 2 \text{ms}^{-2}$

Resultant acceleration a = $\sqrt{a_c^2 + a_t^2}$

$$= \sqrt{(1.8)^2 + (2)^2} = 2.7 \text{ ms}^{-2}$$

16. Momentum of one bullet = mv. If n bullets are fired per second, the momentum imparted to the gun per second = nmv. From Newton's second law, force = rate of change of momentum, i.e.

$$F = nmv$$

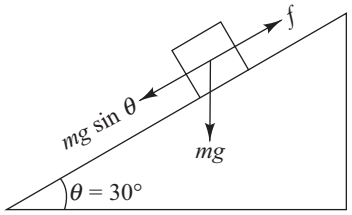
$$\Rightarrow \quad n = \frac{F}{mv} = \frac{144}{0.04 \times 1200} = 3$$

17. Refer to Fig 3.5 on page 3.4 of this chapter. The common acceleration of the masses is

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) g$$

$$= \left(\frac{5 - 4.8}{5 + 4.8} \right) \times 9.8 = 0.2 \text{ ms}^{-2}$$

18. Angle of repose is $\alpha = \tan^{-1}(\mu) = \tan^{-1}(0.8) = 39.7^\circ$. Since $\theta < \alpha$, the frictional force f on the block is less than the limiting friction and is given by ($\because$ the block is at rest)



$f = mg \sin \theta$

$\Rightarrow 10 = m \times 10 \times \sin 30^\circ \Rightarrow m = 2.0$ kg

19. Power $P = \vec{F} \cdot \vec{v} = Fv \cos \theta$. Since $\theta = 90^\circ$, $P = 0$, i.e. $\frac{dW}{dt} = 0.$ Hence the work done by the force is constant. Therefore, kinetic energy of the particle remains constant.

20. Let a be the acceleration at a time t of the blocks executing SHM. The force on the blocks due to acceleration is

$$F = (m + m)\, a = 2\, ma$$

$\therefore$ $$F_{\max} = 2\, m\, a_{\max} \qquad (1)$$

Now, the acceleration is maximum when the blocks are at the extreme position of maximum displacement, i.e.

$$F_{\max} = kA$$

Equating (1) and (2), we get

$$a_{\max} = \frac{kA}{2m}$$

$\therefore$ Maximum force of friction = $ma_{\max}$

$$= m \times \frac{kA}{2m} \times \frac{kA}{2} \qquad (2)$$

21. The acceleration of the block sliding on a smooth inclined plane of inclination θ is

$$a_1 = g \sin \theta \qquad \text{(i)}$$

$\therefore$ Distance moved in time t_1 is

$$S_1 = \frac{1}{2} a_1 t_1^2 \qquad \text{(ii)}$$

The acceleration on a rough inclined plane is

$$a_2 = g \sin \theta - \mu_k g \cos \theta \qquad \text{(iii)}$$

Distance moved in time t_2 is

$$s_2 = \frac{1}{2} a_2 t_2^2 \qquad \text{(iv)}$$

Given $s_1 = s_2 = d$ and $t_2 = nt_1$. Using these in (ii) and (iv), we have

$$a_1 t_1^2 = a_2 (nt_1)^2$$

$$\Rightarrow \quad a_1 = n^2 a_2 \Rightarrow \frac{a_1}{a_2} = n^2 \qquad \text{(v)}$$

Dividing (i) and (iii)

$$\frac{a_1}{a_2} = \frac{g \sin \theta}{g \sin \theta - \mu_k g \cos \theta}$$

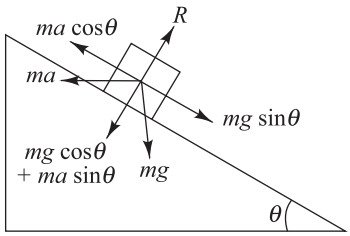
$$\Rightarrow \quad n^2 = \frac{\sin 45^\circ}{\sin 45^\circ - \mu_k \cos 45^\circ} = \frac{1}{1 - \mu_k}$$

$$\Rightarrow \quad \mu_k = 1 - \frac{1}{n^2}$$

22. Refer to the solution of Question 6 on page 3.21.The correct choice is (c).

23. If the inclined plane is given an acceleration a to the right, the block experiences a force ma to the left, where m is the mass of the block.

The following figure shows the forces acting on the block.



The block with remain stationary if no net force acts on it down the plane, i.e. if

$mg \sin \theta - ma \cos \theta = 0$

which gives $a = g \tan \theta$.

24. Acceleration $a = \dfrac{F}{m} = \dfrac{-kx}{m}$

$= \dfrac{-15 \times 0.2}{0.3} = -10 \text{ ms}^{-2}$

25. Magnitude of acceleration = 10 ms^{-2}

Frictional force $f = \mu mg$. Therefore retardation is

$$a = -\frac{f}{m} = -\mu g = -0.5 \times 10 = -5 \text{ ms}^{-2}$$

Using $v^2 - u^2 = 2as$, we have

$0 - (100)^2 = 2 \times -5 \times s \Rightarrow s = 1000$ m

26. Since the block is held stationary, it is in translational as well as rotational equilibrium. Hence no net force and no net torque acts on the block. No net force will act on the block if $f = mg$ and $N = F$. No net torque will act on the block, if torque by frictional force f about centre O = counter torque by normal reaction N about centre O. Hence choice (d) is false.

27. Given $m = 150 \times 10^{-3}$ kg, = 20 ms^{-1} and $t = 0.1$ s. Let F be the force of the impact. Now, impulse = force × time of impact = $F \times t$. Also impulse = change in momentum = mv. Equating them we have $F \times t = mv$

or $$F = \frac{m \times v}{t} = \frac{(150 \times 10^{-3}) \times 20}{0.1} = 30 \text{ N}$$

28. A velocity of the ball just after it is released from the hand is

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 2} = \sqrt{40} \text{ ms}^{-1}$$

Let a be the acceleration imparted to the ball during the time the hand was moving. During this time, the distance moved is $s = 0.2$ m. The value of a is given by

$$v^2 - u^2 = 2as$$

or $40 - 0 = 2 \times a \times 0.2$

which gives $a = 100$ ms^{-2}. If F is the magnitude of the applied force, then $F - mg = ma$ or $F = m(g + a)$ $= 0.2 \times (10 + 100) = 22$ N.

29. When the system is in equilibrium, the spring force = 3 mg. When the string is cut, the net force on block A = 3 mg – 2 mg = 1 mg. Hence the acceleration of this block at this instant is

$$a = \frac{\text{force on block A}}{\text{mass of block A}} = \frac{mg}{2m} = \frac{g}{2}$$

When the string is cut, the block B falls freely with an acceleration equal to g.

30. The common acceleration of the blocks is

$$a = \frac{F}{(m+M)}$$

$\therefore$ Force on block of mass $m = ma = \dfrac{mF}{(m+M)}$

31. Force $\vec{F} = \dfrac{d\vec{p}}{dt}$

$= \dfrac{d}{dt}[A\{\hat{i} \cos (kt) - \hat{j} \sin (kt)]$

$= Ak[-\hat{i} \sin (kt) - \hat{j} \cos (kt)]$

Now, $\vec{F} \cdot \vec{p} = Ak[-\hat{i} \sin (kt) - \hat{j} \cos kt]$.

$A[\hat{i} \cos (kt) - \hat{j} \sin (kt)]$.

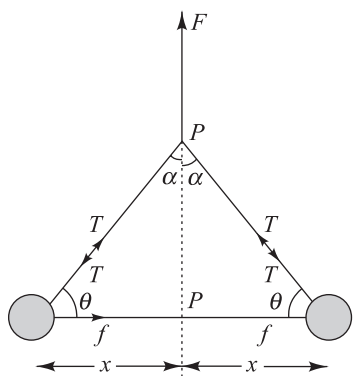
$= A^2K[-\sin (kt) \cos (kt) + \cos (kt) \sin kt]$

$= 0 (\because \hat{i}.\hat{i} = \hat{j}.\hat{j} = 1 \text{ and } \hat{i}.\hat{j} = 0)$

Hence the angle between $\vec{F}$ and $\vec{p}$ is 90º.

32. Refer to the figure. Let f be the force producing the acceleration of each mass. It follows from the figure that $F = T \cos \alpha + T \cos \alpha = 2T \cos \alpha = 2T \sin \theta$ ($\because$ $\alpha = 90^\circ - \theta$)

$\Rightarrow$ $$T = \frac{F}{2 \sin \theta} \qquad \text{(i)}$$



Also $T \cos \theta = f = ma$ (2)

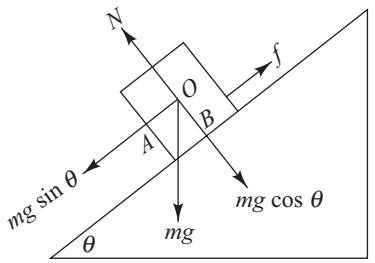
Using (1) and (2), we get

$$a = \frac{F\cos\theta}{2m\sin\theta} = \frac{F}{2m\tan\theta}$$

$$= \frac{Fx}{2m\sqrt{a^2 - x^2}}$$

33. $p = mv = 3.513 \times 5.00 = 17.565$ kg ms^{-1}. Since the speed $v = 5.00$ ms^{-1} has 3 significant figures, The result of multiplication must have 3 significant figures. Hence the correct choice is (a).

34.



The block will just begin to slide if the downward force $mg \sin\theta$ just overcomes the frictional force, i.e. if $mg \sin\theta = \mu N = \mu\, mg \cos\theta \Rightarrow \tan\theta = \mu = \sqrt{3}$ $\Rightarrow \theta = 60°$

The block will topple if the torque due to normal reaction N about O just exceeds the torque due to $mg \sin\theta$ about O, i.e.

$N \times OA = mg \sin\theta \times OB$

$$\Rightarrow \qquad mg\cos\theta \times 5 \text{ cm} = mg\sin\theta \times \frac{15}{2}\text{ cm}$$

$$\Rightarrow \qquad \tan\theta = \frac{2}{3} \Rightarrow \theta = 34°.$$

Since θ for toppling is less than θ for sliding, the correct choice is (b).

35. For the bead to stay at rest,

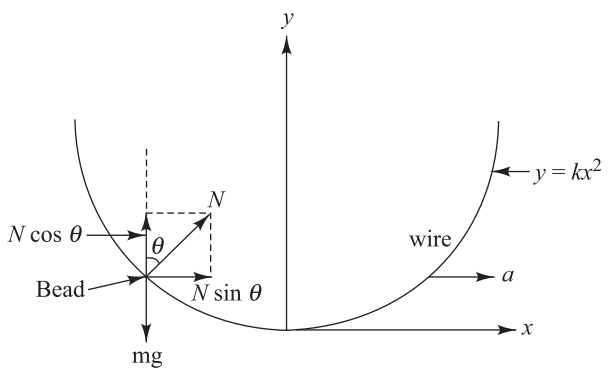
$N\cos\theta = mg$

$N\sin\theta = ma$

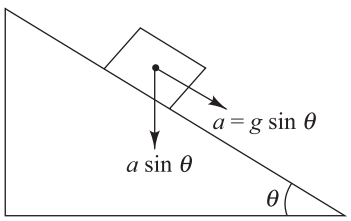
which gives $\tan\theta = \frac{a}{g}$. Now

$\tan\theta$ = slope of the curve = $\frac{dy}{dx} = \frac{d}{dx}(kx^2) = 2kx$

$$\therefore \quad 2kx = \frac{a}{g} \Rightarrow x = \frac{a}{2gk}$$



36. Accleration along the inclined plane is $a = g \sin\theta$. Vertical component of a is $a \cos(90° - \theta) = a \sin\theta$ $= g \sin^2\theta$.



For block A, vertical acceleration is is $g \sin^2(60°)$ $= \frac{3g}{4}$ and for block B the vertical acceleration is $g \sin^2(30°) = \frac{g}{4}$. Therefore, the relative vertical acceleration of A with respect to $B = \frac{3g}{4} - \frac{g}{4} = \frac{g}{2}$ $= 4.9$ ms^{-2}.

37. $F = F_0 e^{-bt} \Rightarrow ma = F_0 e^{-bt}$

$$\therefore \qquad m\frac{dv}{dt} = F_0 e^{-bt}$$

$$\Rightarrow \qquad dv = \frac{F_0}{m} e^{-bt}\, dt$$

Integrating, we get

$$v = \frac{F_0}{m}\left|\frac{e^{-bt}}{-b}\right|_0^t$$

$$\Rightarrow \qquad v = \frac{F_0}{mb}(1 - e^{-bt})$$

It is clear that $v = 0$ at $t = 0$ and $\frac{F_0}{mb}$ as $t \to \infty$. So the correct graph is (c).

38. Initially the bead exerts an inward radial force (centripetal force) on the wire and the wire exerts a normal reaction N radially outwards. At a certain instant during the motion, the normal reaction N becomes zero. After that instant, the normal reaction N will act radially outwards. So the correct choice is (d).

WORK, ENERGY AND POWER

Chapter 4

REVIEW OF BASIC CONCEPTS

1. Work Done by a Force

(a) Work done by a constant force

When a constant force **F** acting on a body produces a displacement **S**, then the work done by the force is given by

$$W = \mathbf{F}\cdot\mathbf{S} = FS\cos\theta$$

where θ is the angle between the force vector **F** and the displacement vector **S** [see Fig. 4.1]. F and S are the magnitudes of **F** and **S** respectively.

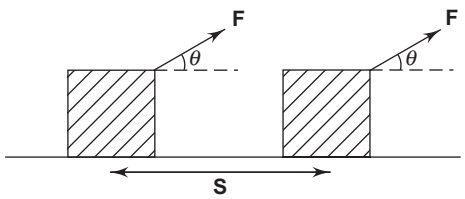


Fig. 4.1

(i) If θ is acute, $\cos\theta$ is positive. Hence work is positive for acute θ. In this case the force increases the speed of the body.

(ii) If $\theta = 90°$, $W = 0$, i.e. if the force is perpendicular to displacement work done by the force is zero.

(iii) If θ is obtuse, W is negative. In this case the force decreases the speed of the body.

(iv) If $\theta = 0$, i.e. force **F** is in the same direction as displacement **S**, then $W = FS$.

(v) If $\theta = 180°$, force **F** is opposite to **S** (example frictional force), $W = -FS$. Work done by frictional and viscous force is always negative.

(b) Work done by a variable force

Suppose a force **F** is not constant but depends on the position vector **r** of the body, then the work done by the force **F** in moving the body from a position r_1 to a position r_2 is given by

$$W = \int_{r_1}^{r_2} \mathbf{F}\cdot\mathbf{dr}$$

EXAMPLE 1 A block of mass $m = 5$ kg slides down from the top of an inclined plane of inclination $\theta = 30°$ with the horizontal. The coefficient of sliding friction between the block and the plane is 0.25. The length of the plane is 2 m. Find the work done by the (a) gravitational force, (b) frictional force and (c) normal reaction if the block slides to the the bottom of the plane.

SOLUTION Refer to Fig. 4.2.

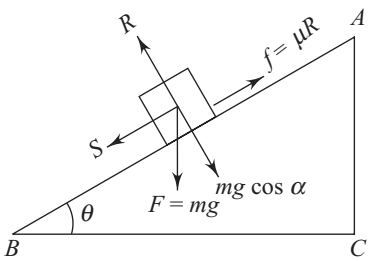


Fig. 4.2

Displacement $S = AB = 2$ m from A to B.

(a) Angle between **F** and **S** is $(90° - \theta) = 90° - 30° = 60°$

$\therefore$ Work done by gravitational force is

$$W_1 = FS\cos 60° = mgS\cos 60°$$

$$= 5\times 9.8\times 2\times\frac{1}{2} = 49 \text{ J}$$

(b) Work done by frictional force is

$$W_2 = fS\cos 180°$$

$$= -fS = -\mu RS$$

$$= -\mu mg\,(\cos\theta)S$$
$$= -0.25 \times 5 \times 9.8 \times \cos 30° \times 2$$
$$= -21.2 \text{ J}$$

(c) Since the normal reaction R is perpendicular to displacement S, work done by normal reaction is

$$W_3 = RS \cos 90° = 0$$

EXAMPLE 2 A block of mass $m = 2$ kg is raised vertically upwards by means of a massless string through a distance of $S = 4$ m with a constant acceleration $a = 2.2$ ms^{-2}. Find the work done by (a) tension and (b) gravity. Also find the net work done on the block.

SOLUTION

(a) From the free body diagram (Fig. 4.3)

$T - mg = ma$

$\Rightarrow T = m(a + g)$

$= 2 \times (2.2 + 9.8)$

$= 24$ N

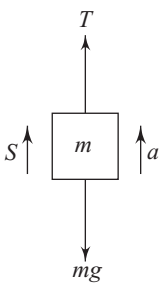


Fig. 4.3

$\therefore$ Work done by tension is ($\because$ T and S are in the same direction)

$W_1 = TS \cos 0°$

$= 24 \times 4 \times 1 = 96$ J

(b) Since the gravitational force mg and displacement S are in opposite directions, work done by gravity is

$$W_2 = mgS \cos 180°$$
$$= -2 \times 9.8 \times 4 = -78.4 \text{ J}$$

(c) Net work done $W = W_1 + W_2 = 96 - 78.4 = 17.6$ J

EXAMPLE 3 A block of mass $m = 2$ kg is suspended by a light string from the ceiling of a lift. The lift starts moving down with an acceleration $a = 1.8$ ms^{-2}. Find the work done by the tension in the string during the first 5 seconds.

SOLUTION Tension $T = m(g - a) = 2 \times (9.8 - 1.8) = 16$ N

Distance moved in $t = 5$ s is

$$S = \frac{1}{2}at^2 = \frac{1}{2} \times 1.8 \times (5)^2 = 22.5 \text{ m}$$

Since the tension and displacement are in opposite directions, the work done by tension is

$$W = TS \cos 180° = -TS$$
$$= -16 \times 22.5 = -360 \text{ J}$$

EXAMPLE 4 A constant force $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ newton displaces a body from position $\mathbf{r}_1 = (4\hat{\mathbf{i}} - 5\hat{\mathbf{j}})$ metre to $\mathbf{r}_2 = (\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ metre. Find the work done by the force.

SOLUTION

Displacement $\mathbf{S} = \mathbf{r}_2 - \mathbf{r}_1$

$$= (\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) - (4\hat{\mathbf{i}} - 5\hat{\mathbf{j}}) = -3\hat{\mathbf{i}} + 8\hat{\mathbf{j}}$$

$\therefore$ $W = \mathbf{F} \cdot \mathbf{S} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \cdot (-3\hat{\mathbf{i}} + 8\hat{\mathbf{j}})$

$= -6 + 24 = 18$ J

$[\because \hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = 1 \text{ and } \hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = 0]$

EXAMPLE 5 A body of mass $m = 0.5$ kg travels in a straight line with a velocity $v = 5x^{3/2}$ where v is in ms^{-1} and x is in metre. Find the work done in displacing the body from $x = 0$ to $x = 2$ m.

SOLUTION

Acceleration $a = \dfrac{dv}{dt} = \dfrac{d}{dt}(5x^{3/2})$

$$= 5 \times \frac{3}{2} x^{1/2} \frac{dx}{dt}$$
$$= \frac{15}{2} x^{1/2} \times (5x^{3/2}) \qquad \left[\because \frac{dx}{dt} = v\right]$$
$$= \frac{75}{2} x^2$$

$\therefore$ Work done $W = \displaystyle\int_{x=0}^{x=2} F dx = \int_0^2 ma dx$

$$= 0.5 \times \frac{75}{2} \int_0^2 x^2 dx$$
$$= 0.5 \times \frac{75}{2} \times \left|\frac{x^3}{3}\right|_0^2$$
$$= \frac{0.5 \times 75}{2 \times 3}(8 - 0)$$
$$= 50 \text{ J}$$

EXAMPLE 6 The displacement x of a particle of mass m moving in a straight line varies with time t as $x = kt^{3/2}$ under the action of a force F where k is a constant. The work done by the force is proportional to

(a) $\sqrt{t}$ (b) t

(c) $t^{3/2}$ (d) t^2

SOLUTION $x = kt^{3/2}$

Velocity $v = \dfrac{dx}{dt} = \dfrac{3}{2} kt^{1/2}$ (i)

Acceleration $a = \dfrac{dv}{dt} = \dfrac{3}{2} k \times \dfrac{1}{2} \times t^{-1/2} = \dfrac{3k}{4} t^{-1/2}$

Force $F = ma = \dfrac{3km}{4} t^{-1/2}$ (ii)

Work done $W = \int F dx$ (iii)

From Eq (i) $dx = \frac{3}{2} k\, t^{\frac{1}{2}}\, dt$ (iv)

Using (ii) and (iv) in (iii) we have

$$W = \int \frac{3km}{4} t^{-1/2} \times \frac{3}{2} kt^{1/2}\, dt$$

$$= \frac{9k^2 m}{8} \int dt = \frac{9k^2 m}{8} t$$

Hence $W \propto t$.

EXAMPLE 7 The force F on a particle of mass m moving in a straight line varies with its velocity v as $F = \frac{k}{v}$ where k is a constant. The work done by the force in time t is

(a) $\left(\frac{kt}{mv}\right)^2$ (b) $\frac{kt}{v}$

(c) $\frac{kt^2}{m}$ (d) kt

SOLUTION $F = \frac{k}{v} \Rightarrow Fv = k \Rightarrow F\frac{dx}{dt} = k$

$\therefore$ $Fdx = k\, dt$

Work done is $W = \int F\, dx = k \int_0^t dt = kt.$

So the correct choice is (d).

EXAMPLE 8 A uniform chain of mass M and Length L has a part l of its length hanging over the edge of the table. If the friction between the chain and the edge is neglected, the work done to pull the length l on the table is

(a) $\frac{Mgl^2}{L}$ (b) $\frac{Mgl^2}{2L}$

(c) $\frac{MgL^2}{l}$ (d) $\frac{MgL^2}{2l}$

SOLUTION Mass per unit length = $\frac{M}{L}$. Mass of part of length l is $m = \frac{Ml}{L}$. Force (weight) of this part is

$$F = mg = \frac{Mlg}{L}$$

Work done $W = \int_0^l Fdl = \frac{Mg}{L} \int_0^l ldl = \frac{Mgl^2}{2L}$

2. Energy

Energy can be defined as the capacity or ability to do work and is measured by the amount of work a body can do. So, energy is measured in the same units as work, namely, joule. Like work, energy is a scalar quantity.

Energy can exist in various forms, such as heat energy, electrical energy, sound energy, light energy, chemical energy, nuclear energy, mechanical energy, etc. We will be mainly concerned with mechanical energy. Mechanical energy is of two types, *kinetic* and *potential.*

Kinetic Energy: Energy due to Motion A moving object can do work on another object when it strikes it. In other words, an object in motion has the ability to do work and, by definition, has energy. *The energy possessed by a body by virtue of its motion is called kinetic energy.*

An initially motionless body can move and acquire a velocity only if a force acts on it. The work done by the force in causing the body to move measures the kinetic energy (written as KE) of the moving body, i.e.

$$\text{KE} = W$$

The kinetic energy of a body of mass m, moving with a velocity v is given by

$$\text{KE} = \frac{1}{2}\, mv^2$$

This relation holds even if the force is variable, i.e. if the force varies both in magnitude and direction.

Work-Energy Principle Suppose a body of mass m moves with an initial velocity u. A force F acts on it, as a result of which it acquires a final velocity v. The work done by the force is given by

$$W = \int_u^v F dx = m\int_u^v a dx = m\int_u^v \frac{dv}{dt} dx$$

or $$W = m\int_u^v v\, dv = \frac{1}{2} m\left(v^2 - u^2\right) \quad \left(\because \frac{dx}{dt} = v\right)$$

$$= \frac{1}{2}\, mv^2 - \frac{1}{2}\, mu^2$$

= final KE – initial KE

= change in KE

Thus, *the work done by a force in displacing a body measures the change in its kinetic energy.* This is the work-energy principle.

Thus, when a force does work on a body, its kinetic energy increases; the increase in kinetic energy being equal to the amount of work done. The converse of this is also true. When the kinetic energy of a body is decreased by a retarding force, the decrease is equal to the work done by the body against the retarding force. Thus kinetic energy and work are equivalent quantities and are, therefore, measured in the same units, namely, joule.

Potential Energy: Energy due to Position or Configuration An object can have energy not only by

virtue of its motion, but also because of its position or configuration. *The energy possessed by a body owing to its position or configuration is called potential energy.* For example, a wound watch spring has potential energy on account of its wound state or configuration of the coils. As the spring unwinds, it does work to move the hands of the watch. Thus, a wound spring has the *potentiality* to do work.

Gravitational Potential Energy An object held at a position above the surface of the earth has potential energy by virtue of its position. When it falls from that position, it can do work. The potential energy of an object held above the earth is called *gravitational potential energy*. To calculate the energy stored in a body which has been lifted above the earth's surface against the gravitational force, we have to calculate the amount of work done in carrying it there.

Consider a body of mass m. It is lifted vertically to a height h above the earth by applying a force F vertically upward. The force F must be just enough to overcome the gravitational attraction, i.e.

$$F = mg$$

where g is the acceleration due to gravity at that place. For bodies not too far above the surface of the earth, the value of g is practically constant. Hence the work done by a constant force F in displacing a body by a height h can be calculated by the product $F \times h = mgh$. Thus gravitational potential energy of a body of mass m at a height h above the surface of the earth is mgh. The gravitational potential energy on the surface of the earth is taken to be zero.

$$\text{Gravitational PE} = mgh \quad \text{or} \quad U = mgh$$

Potential Energy of a Spring. Consider a perfectly elastic spring. One end of the spring is fixed to a rigid wall and other end is fixed to a block which is placed on a frictionless horizontal surface as shown in Fig. 4.4. We assume that the mass of the spring is negligible compared to the mass of the block.

If we stretch the spring by a distance x, the spring will exert a force on us during stretching. This force is due to the reaction of the spring and is called the *restoring force* which is proportional to the displacement x and acts in a direction opposite to the displacement, i.e.

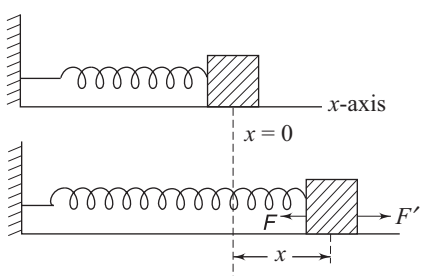


Fig. 4.4

$$F \propto -x$$

or $$F = -kx$$

where k is the force constant of the spring. The negative sign indicates that the force acts in a direction opposite to displacement.

To stretch a spring by a displacement x, we must exert a force F' on it, equal but opposite to the force F exerted by the spring on us. Therefore, the applied force is

$$F' = -F = kx$$

Notice that F' is a variable force as it depends on x. Therefore, the work done by the applied force in stretching the spring through a distance x is given by

$$W = \int_0^x F'\,dx$$

$$= \int_0^x (kx)dx = k\int_0^x x\,dx$$

$$= k\left|\frac{x^2}{2}\right|_0^x = \frac{1}{2}kx^2$$

It is evident that the work done in compressing the spring by an amount x is also given by $W = \frac{1}{2}kx^2$. The work done in stretching or compressing a spring is stored in it in the form of potential energy which is due to the changed configuration of the coils of the spring. Hence the potential energy of a massless elastic spring of force constant k when it is stretched or compressed by an amount x is given by

$$U = W = \frac{1}{2}kx^2$$

3. Conservative and Non-Conservative Forces

(a) Conservative force

A force is conservative if

(i) the work done by it on a body in moving it from one position to another depends only on the initial and final positions of the body and not on the path followed by it between the two positions.

or

(ii) the net work done by the force on a body that moves through any closed path is zero.

The above two conditions are equivalent. Examples of conservative forces are gravitational force, electrostatic force and spring force.

(b) Non-conservative force

A force is non-conservative if

(i) The work done by it on a body in moving it from one position to another depends on the path followed by the body between the two positions.

or

(ii) The work done by the force on a body that moves through a closed path is non zero.

Examples of non-conservative forces are frictional and viscous forces.

4. Conservative Force and Potential Energy

For a conservative force F that depends upon position r, there is a potential energy function U which also depends on r. When a conservative force does positive work, the potential of the system decreases, i.e.

Work done = decrease in potential energy

or $$Fdx = -dU$$

or $$F = -\frac{dU}{dx}$$

Hence *the negative derivative of the potential energy function with respect to position gives the conservative force acting on the system.*

5. Principle of Conservation of Energy

The total energy of an isolated system remains constant; it may change from one form to another.

EXAMPLE 9 An elastic spring of negligible mass has a force constant $k = 4$ Nm^{-1}. One end of the spring is fixed to the wall and the other end touches a block of mass $m = 250$ g placed on a horizontal surface. The spring is compressed by an amount $x = 5$ cm as shown in Fig. 4.5. The coefficient of friction between the block and the horizontal surface is $\mu = 0.2$. If the system is released, the find the speed of the block when it leaves the spring.

SOLUTION Loss in P.E. of spring + work done against friction = gain in K.E.

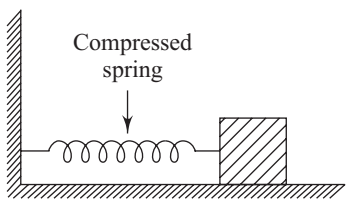


Fig. 4.5

or $$\frac{1}{2}kx^2 + \mu mgx = \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}\times 4\times(0.05)^2 + 0.2\times 0.250\times 10\times 0.05 = \frac{1}{2}\times 0.250\times v^2$$

which gives $v = 0.5$ ms^{-1}

EXAMPLE 10 A bullet moving with a speed of 100 ms^{-1} travels a distance of 2 cm in a plank of wood before coming to rest. How much distance will the same bullet travel in the same plank before coming to rest if it were moving with a speed of 200 ms^{-1} ?

SOLUTION Since the bullet and the plank are the same, the resistive force F exerted on the bullet is the same in the two cases. The kinetic energy is spent in doing work against friction. Hence

$$\frac{1}{2}mv_1^2 = Fx_1$$

and $$\frac{1}{2}mv_2^2 = Fx_2$$

These equations give

$$x_2 = x_1 \times \frac{v_2^2}{v_1^2} = 2 \text{ cm} \times \left(\frac{200}{100}\right)^2 = 8 \text{ cm}$$

EXAMPLE 11 A bullet moving with initial kinetic energy K enters a block of wood at A. It loses 1/4 its initial kinetic energy after travelling a distance $AB = 3$ cm in the block. How much further distance will it penetrate before coming to rest? Assume that the resistance offered by the block is constant.

SOLUTION Refer to Fig. 4.6.

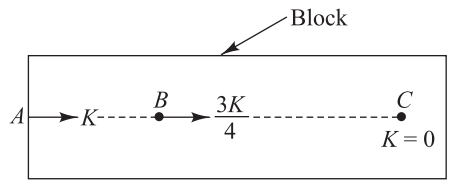


Fig. 4.6

Let F be the constant resistance force. For distance AB, we have change in kinetic energy = work done, i.e.

$$K - \frac{3K}{4} = F \times AB$$

$$\Rightarrow \quad F = \frac{K}{4AB}$$

Let us suppose that the bullet comes to rest at C. For distance BC, we have

$$\frac{3K}{4} - 0 = F \times BC$$

$$\Rightarrow \quad BC = \frac{3K}{4F} = \frac{3K}{4}\times\frac{4AB}{K} = 3AB$$

$$\Rightarrow \quad BC = 3 \times 3 \text{ cm} = 9 \text{ cm}$$

EXAMPLE 12 A body of mass $m = 250$ g tied to a string is whirled in a vertical circle of radius $r = 1$ m. Find the minimum speed the body must have to complete the circle when the body is (a) at the top of the circle and (b) at the

bottom of the circle. Also find the tension in the string when the body is at the bottom of the circle. Take $g = 10\ \text{ms}^{-2}$.

SOLUTION

(a) When, the body is at top A of the circle, the net force towards centre O is $TA + mg$. Hence [see Fig. 4.7]

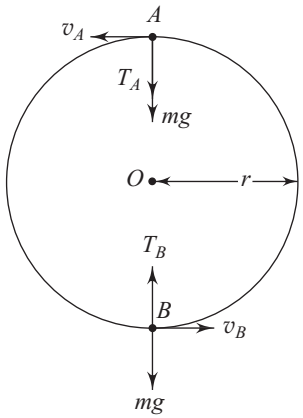


Fig. 4.7

$$T_A + mg = \frac{mv_A^2}{r}$$

Where v_A = velocity of the body at top A

and T_A = tension in the string when the body is at A.

For v_A to be minimum, $T_A \simeq 0$ which gives

$$v_A = \sqrt{gr} = \sqrt{10 \times 1} = 3.16\ \text{ms}^{-}$$

(b) Let v_B be the minimum velocity when the body is at bottom B of the circle so that it can complete the circle, then from conservation of energy, increase in kinetic energy as the body moves from A to B = decrease in gravitational potential energy. Hence

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = mg \times AB = mg \times 2r$$

$$\Rightarrow \quad v_B^2 = 4gr + v_A^2 = 4gr + gr \qquad (\because\ v_A = \sqrt{gr}\ )$$

$$\therefore \quad v_B = \sqrt{5gr} = \sqrt{5 \times 10 \times 1} = 7.07\ \text{ms}^{-1}$$

(c) Net force towards centre O when the body is at B is $(T_B - mg)$. Hence

$$T_B - mg = \frac{mv_B^2}{r}$$

$$\Rightarrow \quad T_B = \frac{mv_B^2}{r} + mg$$

$$= 5\ mg + mg \qquad (\because\ v_B = \sqrt{5gr}\ )$$

$$= 6\ mg$$

$$= 6 \times 0.250 \times 10 = 15\ \text{N}$$

EXAMPLE 13 A small block of mass m, starts from rest at A and slides on a frictionless track which ends in a circular loop of radius r. If $h = 6r$, find the speed of the block when it reaches C as shown in Fig. 4.8(a). What is the force exerted on the block by the track when it is at C? Also find the minimum height h so that the block is able to complete the circle.

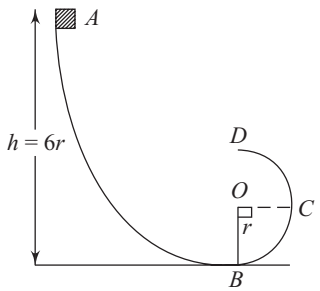


Fig. 4.8(a)

SOLUTION Let v be the speed of the block when it reaches C. From conservation of energy, gain in K.E. = loss in P.E., i.e.

$$\frac{1}{2}mv^2 - 0 = mgh - mg \times OB = mgh - mgr$$

$$\frac{1}{2}v^2 = g \times 6r - gr = 5gr$$

$$\Rightarrow \quad v = \sqrt{10gr}$$

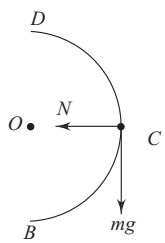


Fig. 4.8(b)

When the block is at C, the track exerts a normal reaction N on the block. Since the block is moving in a circular path, the necessary centripetal force for circular motion is provided by the normal reaction (Fig. 4.8(b)).

$$\therefore \quad N = \frac{mv^2}{r} = \frac{m \times 10gr}{r} = 10\ mg$$

Thus the track exerts a force on the block equal to 10 times the weight of the block.

To complete the circle, the minimum speed at D must be $v_{\text{min}} = \sqrt{5gr}$. Hence

$$mgh_{\text{min}} = \frac{1}{2}mv_{\text{min}}^2 \Rightarrow h_{\text{min}} = 2.5r.$$

EXAMPLE 14 The force between two point charges q_1 and q_2 separated by a distance r is $F = \dfrac{kq_1q_2}{r^2}$

where k is a constant. Find the potential energy of the system of charges.

SOLUTION $F = -\dfrac{dU}{dr} \Rightarrow dU = -Fdr$. Integrating

$$U = -\int_0^r Fdr = -kq_1q_2\int_0^r r^{-2}dr$$

$$\Rightarrow \quad U = \frac{kq_1q_2}{r}$$

EXAMPLE 15 One end of an elastic spring of natural length L and spring constant k is fixed to a wall and the other end is attached to a block of mass m lying on a horizontal frictionless table (Fig. 4.9). The block is moved to a position A so that the spring is compressed to half its natural length and then released. What is the velocity of the block when it reaches position B which is at a distance $\dfrac{3L}{4}$ from the wall.

(a) $\dfrac{L}{4}\sqrt{\dfrac{k}{m}}$ (b) $\dfrac{L}{2}\sqrt{\dfrac{2k}{m}}$

(c) $\dfrac{L}{4}\sqrt{\dfrac{3k}{m}}$ (d) $\dfrac{L}{2}\sqrt{\dfrac{k}{m}}$

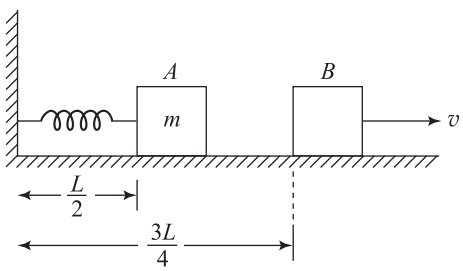


Fig. 4.9

SOLUTION At position A, the compression of the spring is $\dfrac{L}{2}$. At position B, the compression is $L - \dfrac{3L}{4} = \dfrac{L}{4}$. Therefore, loss of potential energy as the block moves from A to B is given by

$$\frac{1}{2}k\left(\frac{L}{2}\right)^2 - \frac{1}{2}k\left(\frac{L}{4}\right)^2 = \frac{3kL^2}{32}$$

From conservation of energy, loss in P.E. = gain in K.E., i.e.

$$\frac{3kL^2}{32} = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \frac{L}{4}\sqrt{\frac{3k}{m}}$$

EXAMPLE 16 In Ex. 15 above, the speed of the block when it leaves the spring is

(a) $\dfrac{L}{2}\sqrt{\dfrac{k}{m}}$ (b) $L\sqrt{\dfrac{k}{m}}$

(c) $\dfrac{3L}{2}\sqrt{\dfrac{k}{m}}$ (d) $2L\sqrt{\dfrac{k}{m}}$

SOLUTION The block will leave the spring when it acquires its natural length L when its P.E. is zero. Hence the loss of P.E = $\dfrac{1}{2}k\left(\dfrac{L}{2}\right)^2$. If v is the velocity when the block leaves the spring, then its K.E. = $\dfrac{1}{2}mv^2$. Hence

$$\frac{1}{2}k\left(\frac{L}{2}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \frac{L}{2}\sqrt{\frac{k}{m}}$$

EXAMPLE 17 A uniform chain of mass M and length L has a part $L/3$ hanging over the edge of a table. If the friction between the table and the chain is neglected, the kinetic energy of the chain as it completely falls off the edge of the table is

(a) $\dfrac{2MgL}{3}$ (b) $\dfrac{4MgL}{9}$

(c) $\dfrac{3}{2}MgL$ (d) MgL

SOLUTION Mass per unit length of the chain is $\dfrac{M}{L}$. Consider a small element of length dx at a distance x below the edge of the table. The mass of this element is $m = \left(\dfrac{M}{L}\right)dx$. The potential energy of the hanging part of the chain is

$$U = \int_0^{L/3} \frac{Mg}{L}x\,dx$$

$$= \frac{Mg}{L}\left|\frac{x^2}{2}\right|_0^{L/3} = \frac{MgL}{18}$$

Let us take the potential energy to be zero when the chain is on the table. Therefore, the total initial P.E. of the chain is

$$U_i = 0 + U = \frac{MgL}{18}$$

If the full chain (of length L) were to fall, the P.E. would be

$$U_f = \int_0^L \frac{Mg}{L}x\,dx = \frac{MgL}{2}$$

$\therefore$ Loss in P.E. = $U_f - U_i = \dfrac{MgL}{2} - \dfrac{MgL}{18} = \dfrac{4}{9}\,MgL$

From law of conservation of energy,

Gain in K.E. = Loss in P.E. = $\frac{4}{9}\ MgL$

Since the initial K.E. is zero, the K.E. as the chain slips off the edge = $\frac{4}{9}\ MgL$

5. Power

The rate of doing work is called power, i.e.

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

The faster a given amount of work is done, the greater is the power of the agent that does the work.

In the SI system, the unit of power is the watt (symbol W). *Power is said to be 1 watt when 1 joule of work is done in 1 second,* i.e.

$$1\ \text{W} = 1\ \text{Js}^{-1}$$

Since the watt is a small unit for the measurement of power, larger units, namely kilowatt (kW) and megawatt (MW) are often used.

$$1\ \text{kW} = 1000\ \text{W} = 10^3\ \text{W}$$

$$1\ \text{MW} = 1{,}000{,}000\ \text{W} = 10^6\ \text{W}$$

The power of an agent can also be expressed in terms of the force applied and the velocity of the object on which the force is applied. Now, power P is given by

$$P = \frac{W}{t} = \frac{\mathbf{F}\cdot\mathbf{S}}{t}$$

$$= \mathbf{F}\cdot\mathbf{v}\ \left(\because\ \frac{\mathbf{S}}{t} = \text{rate of change of displacement} = \mathbf{v}\right)$$

Power is a scalar quantity as it is the ratio of two scalars W and t, or a scalar product of two vectors **F** and **v**.

EXAMPLE 18 An engine pulls a car of mass 1000 kg on a level road at a constant velocity of 5 ms^{-1}. If the frictional force is 500 N, what power does the engine generate? What extra power must the engine supply to maintain the same speed up an inclined plane having a gradient of 1 in 10?

SOLUTION Since the car moves at a constant velocity, its acceleration is zero. Hence the engine has to do work only to overcome the frictional force f.

$\therefore$ Power = $f \times v = 500 \times 5 = 2500\ \text{W} = 2.5\ \text{kW}$

For an inclined plane having a gradient of 1 in 10, $\sin\theta = \frac{1}{10}$. To maintain the same speed up the inclined plane, the engine has to do extra work against the force $mg \sin\theta$. Therefore,

Extra power = $mg \sin\theta \times v$

$$= 1000 \times 9.8 \times \frac{1}{10} \times 5 = 4900\ \text{W}$$

$$= 4.9\ \text{kW}$$

EXAMPLE 19 An electric pump on the ground floor of a building takes 10 minutes to fill a tank of volume 2000 litre with water. If the tank is 40 m above the ground and the efficiency of the pump is 40%, how much electric power is consumed by the pump in filling the tank? Take $g = 10\ \text{ms}^{-2}$.

SOLUTION Volume of tank V = 2000 litre

$$= 2000 \times 10^{-3}\ \text{m}^3 = 2\ \text{m}^3$$

Mass of water $m = \rho V = 1000 \times 2$

$$= 2 \times 10^3\ \text{kg}$$

Work done to lift this mass to a height h = 40 m is

$$W = mgh = 2 \times 10^3 \times 10 \times 40 = 8 \times 10^5\ \text{J}$$

$$\text{Power needed} = \frac{W}{t} = \frac{8 \times 10^5}{10 \times 60} = \frac{4}{3} \times 10^3\ \text{W}$$

If P is the total power consumed, the useful power available = 40% if $P = 0.4\ P$. Hence

$$0.4\ P = \frac{4}{3} \times 10^3 \quad \Rightarrow \quad P = 3.33 \times 10^3\ \text{W}$$

$$= 3.33\ \text{kW}$$

EXAMPLE 20 A constant power P is supplied to a car of mass m = 3000 kg. The velocity of the car increases from $u = 2\ \text{ms}^{-1}$ to $v = 5\ \text{ms}^{-1}$ when the car travels a distance x = 117 m. Find the value of P. Neglect friction.

SOLUTION $P = Fv = mav \quad \Rightarrow \quad a = \frac{P}{mv}$

Now $$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{v\,dv}{dx}.$$

$\therefore$ $$v\ \frac{dv}{dx} = \frac{P}{mv}$$

$\Rightarrow$ $$v^2 dv = \frac{P}{m} dx$$

$\therefore$ $$\int_u^v v^2 dv = \frac{P}{m} \int_0^x dx$$

$\Rightarrow$ $$\frac{1}{3}\left(v^3 - u^3\right) = \frac{Px}{m}$$

$\Rightarrow$ $$P = \frac{m\left(v^3 - u^3\right)}{3x}$$

$$= \frac{3000 \times \left[(5)^3 - (2)^3\right]}{3 \times 117}$$

$$= 1000\ \text{W} = 1\ \text{kW}$$

EXAMPLE 21 A car of mass m starts from rest at time $t = 0$ and is driven on a straight horizontal road by the engine which exerts a constant force F. If friction is negligible, the car acquires kinetic energy E at time t and develops a power P. Which of the following is/are correct ?

(a) $E \propto t$ (b) $E \propto t^2$

(c) $P \propto t$ (d) $P \propto t^2$

SOLUTION Since F is constant, acceleration $a = \dfrac{F}{m}$ is constant. At time t, the velocity of the car is

$$v = u + at = 0 + at = at = \frac{Ft}{m}.$$

$\therefore$ Kinetic energy E at time

$$t = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{Ft}{m}\right)^2 = \left(\frac{F^2}{m}\right)t^2.$$

Since F is constant, $E \propto t^2$.

Power P at time $t = Fv = F \times \dfrac{Ft}{m} = \left(\dfrac{F^2}{m}\right)t$

Thus $P \propto t$. So the correct choices are (b) and (c).

6. Collisions

Elastic Collisions: *If there is no change of kinetic energy during a collision it is called an elastic collision.* The collision between subatomic particles is generally elastic. The collision between two steel or glass balls is nearly elastic.

Inelastic Collisions: *If there is a loss of kinetic energy during a collision, it is called an inelastic collision.* Since there is always some loss of kinetic energy in any collision, collisions are generally inelastic. If the loss is negligibly small, the collision is very nearly elastic. Perfectly elastic collisions are not possible. If two bodies stick together, after colliding, the collision is perfectly inelastic, e.g. a bullet striking a block of wood and being embedded in it. The loss of kinetic energy usually results in heat or sound energy.

In may be remembered that *the total momentum remains conserved in both elastic and inelastic collisions.* Further, since the interacting forces become effectively zero after the collision, *the potential energy remains the same both before and after the collision.*

One-dimensional or Head-on Collision

Consider two bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 in the same straight line (with $u_1 > u_2$) colliding with each other. Let v_1 and v_2 be their respective velocities after the collision. If velocities u_1, u_2, v_1 and v_2 are all along the same straight line, the collision is known as one-dimensional or head-on collision (Fig. 4.10)
From the law of conservation of momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

If momentum along positive x-axis is taken to be positive, the momentum along the negative x-axis is taken to be negative.

Two-dimensional or Oblique Collision

If the velocities of the colliding bodies are not along the same straight line, the collision is known as two-dimensional or oblique collision (Fig. 4.11)

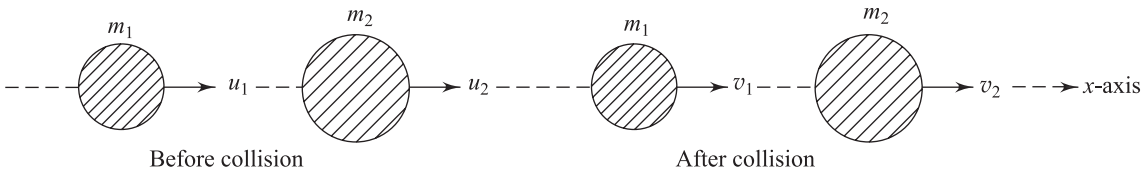


Fig. 4.10

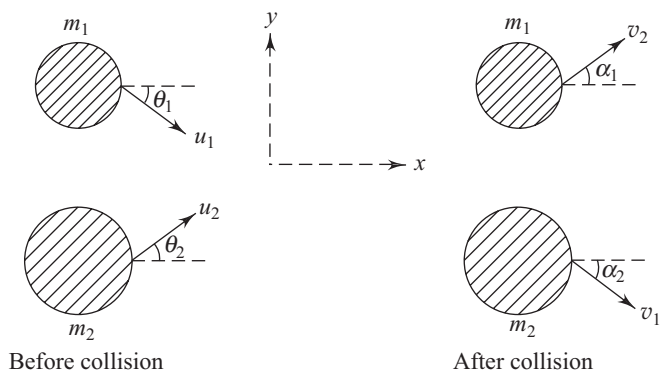


Fig. 4.11

In this case, we apply the law of conservation of momentum separately for x and y components of momenta. The components of momentum along the positive x-axis and positive y-axis are taken to be positive and components of momentum along negative x-axis and negative y-axis are taken to be negative.

Momentum conservation of x-components gives

$$m_1u_1 \cos \theta_1 + m_2u_2 \cos \theta_2 = m_1v_1 \cos \alpha_1 + m_2v_2 \cos \alpha_2$$

Momentum conservation of y-components gives

$$-m_1u_1 \sin \theta_1 + m_2u_2 \sin \theta_2 = m_1v_1 \sin \alpha_1 - m_2v_2 \sin \alpha_2$$

Cofficient of Restitution

Newton proved experimentally that, when two bodies collide, the ratio of the relative velocity after collision to the relative velocity before collision is constant for the two bodies. This constant is known as *coefficient of restitution* and is denoted by letter e.

$$e = \frac{\text{velocity of separation after collision}}{\text{velocity of approach before collision}}$$

or
$$e = -\frac{\mathbf{v}_2 - \mathbf{v}_1}{\mathbf{u}_2 - \mathbf{u}_1}$$

(i) For a perfectly elastice collision, $e = 1$.

(ii) For a perfectly in inelastic collision, $e = 0$, because the two bodies stick together and hence $\mathbf{v}_2 = \mathbf{v}_1$.

(iii) Perfectly elastic or perfectly inelastic collisions do not occur in nature. Hence, for any collision, e lies between 0 and 1.

(iv) For a head-on collision (Fig. 4.10)

$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}} = -\frac{v_2 - v_1}{u_2 - u_1}$$

(v) For an oblique collision (Fig. 4.11)

$$\text{Velocity of approach} = u_1 \cos \theta_1 - u_2 \cos \theta_2$$

$$\text{Velocity of separation} = v_2 \cos \alpha_2 - v_1 \cos \alpha_1$$

$$\therefore \quad e = \frac{v_2 \cos \alpha_2 - v_1 \cos \alpha_1}{u_1 \cos \theta_1 - u_2 \cos \theta_2}$$

Velocities after Head-on Elastic Collision

Refer to Fig. 4.10 again. From the law of conservation of momentum, we have

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad (1)$$

The coefficient of restitution is defined as

$$e = \frac{v_1 - v_2}{u_2 - u_1}$$

$$\Rightarrow \quad v_1 - v_2 = e\,(u_2 - u_1) \qquad (2)$$

Eliminating v_2 from (1) and (2), we get

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{m_2\,(1+e)}{m_1 + m_2}\right)u_2 \qquad (3)$$

Using (3) in (2), we get

$$v_2 = \left(\frac{m_1\,(1+e)}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2 \qquad (4)$$

Perfectly Elastic Collision

For perfectly elastic collision, $e = 1$. Putting $e = 1$ in Eqs. (3) and (4) we get

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \qquad (5)$$

and
$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 \qquad (6)$$

Special cases

(i) *If both bodies have the same mass, then*

$$m_1 = m_2 = m$$

In this case,

$$v_1 = u_2$$

and
$$v_2 = u_1$$

This means that *in a one-dimensional elastic collision between two bodies of equal mass, the bodies merely exchange their velocities after the collision.*

(ii) *If one of the bodies, say m_2, is initially at rest*, then

$$u_2 = 0$$

In this case,

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$

and
$$v_2 = \left(\frac{2m_1u_1}{m_1 + m_2}\right)$$

If, in addition, $m_1 = m_2 = m$, these equations give

$$v_1 = 0 \quad v_2 = u_1$$

Thus, *if a body suffers a one-dimensional elastic collision with another body of the same mass at rest, the first body is stopped dead, but the second begins in move with the velocity of the first.*

However, if the body at rest, namely B, is much more massive than the colliding body A, i.e. $m_2 \gg m_1$, such that m_1 is negligibly small, then

$$v_1 = -u_1$$

and
$$v_2 \to 0$$

Thus, *if a very light body suffers an elastic collision with a very heavy body at rest, the velocity of the lighter body is reversed on collision, while the heavier body remains practically at rest.* A common example of this type of collision is the dropping of a hard steel ball on a hard concrete floor. The ball rebounds and regains its original height from where it was dropped while the much more massive ground remains at rest.

Finally, if the body at rest is much lighter than the colliding body, i.e. if $m_2 \ll m_1$, we have

$$v_1 \simeq u_1 \quad v_2 \simeq 2u_1$$

i.e. *the velocity of the massive body remains practically unchanged on collision with the lighter body at rest and the lighter body acquires nearly twice the initial velocity of the massive body.*

(iii) *Kinetic energy delivered by incident body to a stationary body in perfectly elastic head-on collision.*

Kinetic energy of m_1 before collision is $K_i = \frac{1}{2}m_1u_1^2$

and after collision is $K_f = \frac{1}{2}m_1v_1^2$. Therefore

$$\frac{K_i}{K_f} = \frac{u_1^2}{v_1^2}$$

$$\text{or} \quad \frac{K_i - K_f}{K_i} = 1 - \frac{v_1^2}{u_1^2}$$

The fractional decrease in kinetic energy of m_1 is

$$\frac{\Delta K}{K_i} = 1 - \frac{v_1^2}{u_1^2}$$

If $u_2 = 0$, $\frac{v_1}{u_1} = \frac{m_1 - m_2}{m_1 + m_2}$. Therefore,

$$\frac{\Delta K}{K_i} = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

Note

The fraction of kinetic energy lost by mass m_1 is maximum if $m_1 = m_2$ and minimum if $m_2 \to \infty$.

(iv) *Change in kinetic energy of a system in a perfectly inelastic head-on collision.*

In a perfectly inelastic collision, the two stick together after the collision. Hence $v_1 = v_2$ and $e = 0$.

Putting $e = 0$ in Eqs. (1) and (2), we get

$$v_1 = \left(\frac{m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2}{m_1 + m_2}\right)u_2$$

and $$v_2 = \left(\frac{m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2}{m_1 + m_2}\right)u_2$$

when mass m_2 is stationary, $u_2 = 0$. Then

$$v_1 = \left(\frac{m_1}{m_1 + m_2}\right)u_1 \tag{7}$$

and $$v_2 = \left(\frac{m_1}{m_1 + m_2}\right)u_1$$

Notice that $v_1 = v_2 = v$ (say)

Total K.E. of the system before collision is

$$K_i = \frac{1}{2}m_1u_1^2$$

and after the collision is

$$K_f = \frac{1}{2}(m_1 + m_2)v^2$$

$\therefore$ Loss in K.E. of the system is

$$K_i - K_f = \frac{1}{2}m_1u_1^2 - \frac{1}{2}(m_1 + m_2)v^2 \tag{8}$$

From Eq. (7)

$\frac{v}{u_1} = \left(\frac{m_1}{m_1 + m_2}\right)$. Using this in Eq. (8) we get

$$K_i - K_f = \frac{m_1m_2u_1^2}{2(m_1 + m_2)}$$

In general, if $u_2 \neq 0$, we have

$$K_i - K_f = \left(\frac{m_1m_2}{2(m_1 + m_2)}\right)(u_1 - u_2)^2 \tag{9}$$

Oblique Impact on a Fixed Horizontal Plane

Consider a body of mass m moving with a velocity u making an angle α with the normal ON to a fixed horizontal floor as shown in Fig. 4.12. After collision with the horizontal plane, the body is deflected with a velocity v making an angle β with the normal. Since the horizontal plane is fixed, it remains at rest. Hence the impact takes place along the normal. The normal component of u is $u\cos\alpha$ along $-y$ direction and the normal component of v is $v\cos\beta$ along the $+y$ direction. Now

$$e = -\frac{\text{velocity of separation}}{\text{velocity of approach}}$$

$$e = -\frac{(v\cos\beta)\hat{\mathbf{j}}}{(u\cos\alpha)(-\hat{\mathbf{j}})} = \frac{v\cos\beta}{u\cos\alpha}$$

$$\Rightarrow \quad v\cos\beta = eu\cos\alpha \tag{1}$$

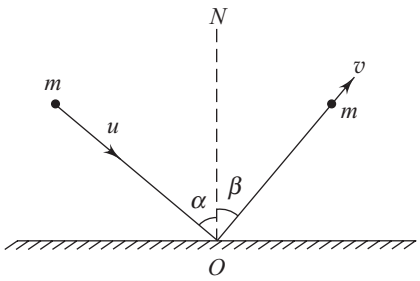


Fig. 4.12

Since the impulsive force acts along the normal, the momentum along the normal is not conservved. Since the component of the impulsive force along the horizontal is zero, the momentum along the horizontal is conserved. Hence

$$u\sin\alpha = v\sin\beta \tag{2}$$

From Eqs. (1) and (2), we get

$$v = (e^2\cos^2\alpha + \sin^2\alpha)^{1/2}\, u \tag{3}$$

$$\text{and} \quad \tan\beta = \frac{\tan\alpha}{e} \tag{4}$$

For a perfectly elastic collision, e =1 and Eqs. (3) and (4) give

$$v = u$$

and

$$\beta = \alpha$$

i.e. for a perfectly elastic collision, the body rebounds from a fixed surface with the same speed and at the same angle on the other side of the normal.

Direct Impact on a Fixed Plane

If the body falling normally on a fixed plane rebounds after impact, then, in this case $\alpha = \beta = 0$. Using this in Eqs. (3) we get

$$v = eu$$

EXAMPLE 22 A body is dropped from rest from a height h = 5.0 m. After rebounding twice from a horizontal floor, to what height will it rise if the coefficient of restitution is 0.8?

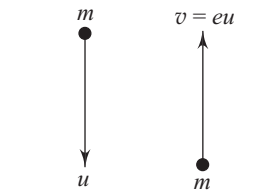


Fig. 4.13

SOLUTION Speed of the body just before first impact with floor = $\sqrt{2gh}$

Speed just after first impact = $e\sqrt{2gh}$. This is also the speed just before the second impact. Therefore, speed just after second impact = $e^2\sqrt{2gh}$. This is the initial speed for the upward motion of the body after the second impact, i.e. $u = e^2\sqrt{2gh}$. Therefore, height attained after two impacts is

$$h_2 = \frac{u^2}{2g} = \frac{1}{2g}\left(e^2\sqrt{2gh}\right)^2 = e^4 h$$

$$= (0.8)^4 \times 5 = 2.05 \text{ m}$$

Note

(1) Height attained after n impacts is $h_n = e^{2n}h$

(2) Speed of rebound after nth impact is $v_n = e^n\sqrt{2gh}$

(3) Total distance travelled before the body comes to rest = $h\left(\frac{1+e^2}{1-e^2}\right)$.

EXAMPLE 23 A steel ball of mass m moving with velocity u_1 undergoes a perfectly elastic head-on collision with another identical steel ball moving with velocity u_2. Show that, after the collision, they merely exchange their velocities.

SOLUTION Refer to Fig. 4.10 again. From conservation of momentum,

$$mu_1 + mu_2 = mv_1 + mv_2$$

$$\Rightarrow \quad u_1 + u_2 = v_1 + v_2 \tag{i}$$

From the definition of coefficient of restitution,

$$v_2 - v_1 = e\,(u_1 - u_2)$$

For a perfectly elastic collision, e = 1. Hence

$$v_2 - v_1 = u_1 - u_2 \tag{ii}$$

From (i) and (ii) we get

$$v_1 = u_2 \text{ and } v_2 = u_1$$

EXAMPLE 24 A steel ball of mass m moving with a velocity u undergoes a perfectly elastic oblique collision with another indentical steel ball initially at rest. Show that, after the collision, they move at right angles to each other.

SOLUTION Refer to Fig. 4.14

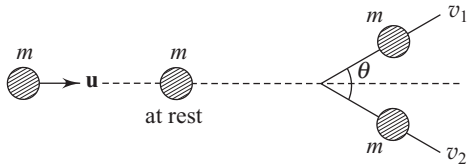


Fig. 4.14

From conservation of momentum,

$$m\mathbf{u} = m\mathbf{v}_1 + m\mathbf{v}_2$$

$$\Rightarrow \quad \mathbf{u} = \mathbf{v}_1 + \mathbf{v}_2 \qquad \text{(i)}$$

Taking the scalar product of $\boldsymbol{u}$ with itself, we have

$$\mathbf{u} \cdot \mathbf{u} = (\mathbf{v}_1 + \mathbf{v}_2) \cdot (\mathbf{v}_1 + \mathbf{v}_2)$$

$$\Rightarrow \quad u^2 = v_1^2 + 2\mathbf{v}_1 \cdot \mathbf{v}_2 + v_2^2 \qquad \text{(ii)}$$

Since kinetic energy is also conserved in an elastic collision, we have

$$\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$\Rightarrow \quad u^2 = v_1^2 + v_2^2 \qquad \text{(iii)}$$

Using (iii) in (ii), we get

$$2\,\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$$

$$\Rightarrow \quad \mathbf{v}_1 \cdot \mathbf{v}_2 = 0$$

$$\Rightarrow \quad v_1 v_2 \cos\theta = 0$$

$$\Rightarrow \quad \cos\theta = 0 \quad \Rightarrow \quad \theta = 90°$$

EXAMPLE 25 Two steel balls of the same mass m moving in opposite directions with the same speed u collide head-on. If the collision is perfectly elastic, predict the result of the collision.

SOLUTION $m_1 = m_2 = m$, $u_1 = u$ and $u_2 = -u$. Let v_1 and v_2 be their velocities after collision.

Total momentum before collision $= m_1u_1 + m_2u_2 = m(u - u) = 0$

Total momentum after collision $= mv_1 + mv_2 = m(v_1 + v_2)$

From conservation of momentum,

$$0 = m(v_1 + v_2) \Rightarrow v_2 = -v_1$$

Since $e = 1$, we have

$$v_2 - v_1 = u_1 - u_2 = u - (-u) = 2u$$

Putting $v_1 = -v_2$, we get $v_2 = u$. Also $v_1 = -u$. Thus, after the collision, the two balls move in opposite directions with equal speeds, each equal to u but their directions are reversed.

EXAMPLE 26 A ball mass 2 kg moving with a velocity of 8 ms^{-1} collides head-on with another ball of mass 4 kg moving with a velocity of 2 ms^{-1} moving in the same direction. The collision is elastic and the coefficient restitution is $e = 0.5$.

(a) Find the velocities of the balls after the collision.

(b) Calculate the loss of kinetic energy due to collision.

SOLUTION Refer to Fig 4.10 again.

(a) Given $m_1 = 2$ kg, $m_2 = 4$ kg, $u_1 = 8$ ms^{-1}, $u_2 = 2$ ms^{-1} and $e = 0.5$

From conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow \quad 2 \times 8 + 4 \times 2 = 2v_1 + 4v_2$$

$$\Rightarrow \quad 12 = v_1 + 2v_2 \qquad \text{(i)}$$

Since $e = 0.5$, we have

$$v_2 - v_1 = e\,(u_1 - u_2) = 0.5 \times (8 - 2) = 3 \qquad \text{(ii)}$$

Eliminating v_2 from (i) and (ii) we get $v_1 = 2$ ms^{-1}. Using this in (i) or (ii), we get $v_2 = 5$ ms^{-1}

(b) Kinetic energy before collision is

$$K_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

$$= \frac{1}{2} \times 2 \times (8)^2 + \frac{1}{2} \times 4 \times (2)^2 = 72 \text{ J}$$

Kinetic energy after collision is

$$K_f = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$= \frac{1}{2} \times 2 \times (2)^2 + \frac{1}{2} \times 4 \times (5)^2 = 54 \text{ J}$$

$\therefore$ Loss of K.E. $= K_i - K_f = 72 - 54 = 18$ J

EXAMPLE 27 In Example 27, what is the loss of kinetic energy if the ball of mass 4 kg is moving towards the mass of mass 2 kg, their speeds being the same?

SOLUTION In this case $u_2 = -2$ ms^{-1}. Equations (i) and (ii) become

$$4 = v_1 + 2v_2 \qquad \text{(iii)}$$

and $$v_2 - v_1 = 5 \qquad \text{(iv)}$$

Equations (iii) and (iv) give $v_1 = -2$ ms^{-1} and $v_2 = 3$ ms^{-1}

$$K_i = \frac{1}{2} \times 2\,(8)^2 + \frac{1}{2} \times 4 \times (-2)^2 = 72 \text{ J}$$

$$K_f = \frac{1}{2} \times 2 \times (-2)^2 + \frac{1}{2} \times 4 \times (3)^2 = 22 \text{ J}$$

$\therefore$ Loss of K.E $= 72 - 22 = 50$ J

EXAMPLE 28 Two blocks B and C of masses 1 kg and 2 kg respectively are connected by a massless elastic spring of spring constant 150 Nm^{-1} and placed on a horizontal frictionless surface as shown in Fig. 4.15. A third block A of mass 1 kg moves with a velocity of

3 ms^{-1} along the line joining B and C and collides with B. If the collision is perfectly elastic and the natural length of the spring is 80 cm, find the minimum separation between blocks B and C.

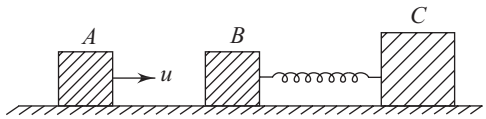


Fig. 4.15

SOLUTION Given $m_A = m_B = 1$ kg, $m_C = 2$ kg and $u = 3$ ms^{-1}. Block A will collide with block B. Since they have equal masses and the collision is perfectly elastic, A will come to rest and B will move to the right with a velocity u. Block B will compress the spring. Hence block C will accelarate and block B will retard until both B and C move with the same velocity. Let this common velocity be v. Since no external force acts, the momentum of B and C is conserved, i.e.

$$m_B\, u = (m_B + m_C)\, v$$

$$\Rightarrow \qquad 1 \times 3 = (1 + 2)\, v \quad \Rightarrow \quad v = 1 \text{ ms}^{-1}$$

If x is the maximum compression, then from the principle of conservation of energy,

$$\frac{1}{2}\, m_A u^2 = \frac{1}{2}\,(m_B + m_C)\, v^2 + \frac{1}{2}\, kx^2$$

$$\Rightarrow \quad \frac{1}{2} \times 1 \times (3)^2 = \frac{1}{2} \times (1 + 2) \times (1)^2 + \frac{1}{2} \times 150 \times x^2$$

which gives $x = 0.2$ m = 20 cm

$\therefore$ Minimum separation between B and C = 80 cm – 20 cm = 60 cm

EXAMPLE 29 A block of $m_1 = m$ is moving on a frictionless horizontal surface with velocity $u_1 = 2u$ towards another block of mass $m_2 = 3m$ moving on the same surface with velocity $u_2 = u$ in the same direction. A massless spring of force constant k is attached to m_2 as shown in Fig. 4.16. When block m_1 collides with the spring, show that the maximum compression of the spring is given by $x = \frac{u}{2}\sqrt{\frac{3m}{k}}$.

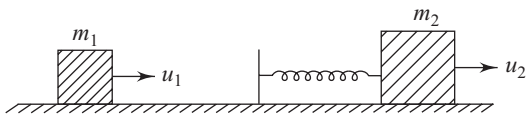


Fig. 4.16

SOLUTION When block m_1 collides with spring, it begins to get compressed. As a result m_2 gains speed. The compression of the spring is maximum at the instant when the relative velocity of m_1 with respect to m_2 is zero, i.e. when both m_1 and m_2 have equal velocities. Let v be the common velocity of the blocks. From conservation of momentum,

$$m_1 u_1 + m_2\, u_2 = (m_1 + m_2)v$$

$$\Rightarrow \quad 2mu + 3mu = (m + 3m)v$$

$$\Rightarrow \qquad v = \frac{5u}{4}$$

From the law of conservation of energy

Loss in K.E. = gain in P.E of spring

If x is the maximum compression, then

$$\left(\frac{1}{2} m_1\, u_1^2 + \frac{1}{2} m_2\, u_2^2\right) - \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2} kx^2$$

$$\Rightarrow \qquad \frac{1}{2} \times m \times (2u)^2 + \frac{1}{2} \times 3m \times u^2 - \frac{1}{2}\,(m + 3m) \times \left(\frac{5u}{4}\right)^2 = \frac{1}{2}\, kx^2$$

$$\Rightarrow \qquad \frac{3}{4}\, mu^2 = kx^2 \Rightarrow x = \frac{u}{2}\sqrt{\frac{3m}{k}}$$

7. Useful Formulae and Tips

1. A body of mass m is dropped from a height h. Due to the friction of air, it will hit the ground with a speed less than $\sqrt{2gh}$. If v is the speed with which it hits the ground, the work done by friction is

$$W_f = \frac{1}{2}\, mv^2 - mgh = \frac{1}{2}\, m(v^2 - 2gh)$$

If friction is absent, $W_f = 0$, then $v = \sqrt{2gh}$.

2. Two block A and B of masses m_1 and m_2 are released from the same height at the same time. Block A slides along an inclined plane of inclination θ and block B falls vertically downwards (Fig. 4.17)

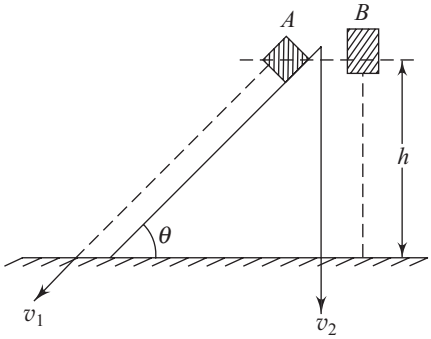


Fig. 4.17

If the inclined plane is frictionless, gain in KE = loss in PE, i.e.

$$\frac{1}{2} m_1 v_1^2 = m_1 gh \quad \Rightarrow \quad v_1 = \sqrt{2gh}$$

If the air friction is neglected.

$$\frac{1}{2} m_2 v_2^2 = m_2 gh \quad \Rightarrow \quad v_2 = \sqrt{2gh}$$

Thus both block will hit the ground with the same speed independent of the mass. But the times taken to reach the ground will be different.

For block A, $t_1 = \sqrt{\dfrac{2h}{g \sin^2 \theta}}$

For block B, $t_2 = \sqrt{\dfrac{2h}{g}}$

3. If a block of mass m in contact with a spring compressed by a distance x is released, the block will leave the spring with a velocity v determined from

$$\frac{1}{2} kx^2 = \frac{1}{2} mv^2$$

which gives $v = \sqrt{\dfrac{k}{m}}\, x$, where k is the spring constant.

4. If a block of mass m moving with speed u comes in contract with a relaxed spring of spring constant k, its velocity v when the spring is compressed by an amount x is obtained from.

$$\frac{1}{2} mu^2 = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$$

which gives $\quad v = \left(\dfrac{k\, x^2}{m} - u^2 \right)^{1/2}$

5. If two springs of spring constants k_1 and k_2 are stretched by the same force F, then $F = k_1 x_1 = k_2 x_2$. Potential energy stored is

$$U_1 = \frac{1}{2} k_1 x_1^2 = \frac{1}{2} Fx_1$$

and $\qquad U_2 = \dfrac{1}{2} k_2 x_2^2 = \dfrac{1}{2} Fx_2$

which give $\quad \dfrac{U_1}{U_2} = \dfrac{x_1}{x_2} = \dfrac{k_2}{k_1}$

6. If the two springs are stretched by the same amount x, then $F_1 = k_1 x$ and $F_2 = k_2 x$.

$U_1 = \dfrac{1}{2} k_1 x^2$ and $U_2 = \dfrac{1}{2} k_2 x^2$.

$$\frac{U_1}{U_2} = \frac{k_1}{k_2} = \frac{F_1}{F_2}$$

7. A chain has a length L and mass M. A part L/n is hanging at the edge of the table. The length of the chain lying on the table is $(L - L/n)$. Then work done against gravity to pull the the hanging part on the table $= \dfrac{MgL}{2n^2}$

8. If a body of mass m moving with velocity v is stoped in a distance x by a retarding force F, then

$$\frac{1}{2} mv^2 = Fx$$

(a) If two bodies of masses m_1 and m_2 moving with the same velocity are subjected to the same retarding force, the ratio of the stopping distance is

$$\frac{x_1}{x_2} = \frac{m_1}{m_2}$$

(b) If the two bodies are moving with equal kinetic energy and are stopped by the same retarding force, then

$$x_1 = x_2$$

(c) If the two bodies are moving with equal linear momentum and are stopped by the same force, then.

$$\frac{p^2}{2m} = Fx$$

and $\quad \dfrac{x_1}{x_2} = \dfrac{m_2}{m_1}$

SECTION 1

Multiple Choice Questions with One Correct Choice

(Level A)

1. A raindrop of radius r falls from a certain height h above the ground. The work done by the gravitational force is proportional to
   (a) r (b) r^2
   (c) r^3 (d) r^4

2. A pendulum has a length l. Its bob is pulled aside from its equilibrium position through an angle α and then released. The speed of the bob when it passes through the equilibrium position is given by
   (a) $\sqrt{2gl}$ (b) $\sqrt{2gl\cos\alpha}$
   (c) $\sqrt{2gl(1-\cos\alpha)}$ (d) $\sqrt{2gl(1-\sin\alpha)}$

3. A small ball is pushed from a height h along a smooth hemispherical bowl. With what speed should the ball be pushed so that it just reaches the top of the opposite end of the bowl? The height of the top of the bowl is R.
   (a) $\sqrt{2gh}$ (b) $\sqrt{2gR}$
   (c) $\sqrt{2g(R+h)}$ (d) $\sqrt{2g(R-h)}$

4. Two particles of masses m and $4m$ have linear momenta in the ratio of 2 : 1. What is the ratio of their kinetic energies?
   (a) $\sqrt{2}$ (b) 2
   (c) 4 (d) 16

5. Two particles of masses m and $4m$ have kinetic energies in the ratio of 2:1. What is the ratio of their linear momenta?
   (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$
   (c) $\frac{1}{4}$ (d) $\frac{1}{16}$

6. A moving bullet hits a solid target resting on a frictionless surface and gets embedded in it. What is conserved in this process?
   (a) momentum and kinetic energy
   (b) kinetic energy alone
   (c) momentum alone
   (d) neither momentum nor kinetic energy

7. A bullet is fired from a rifle which recoils after firing. The ratio of the kinetic energy of the rifle to that of the bullet is
   (a) zero (b) one
   (c) less than one (d) more than one

8. A bullet is fired at a plank of wood with a speed of 200 ms^{-1}. After passing through the plank, its speed reduces to 180 ms^{-1}. Another bullet, of the same mass and size but moving with a speed of 100 ms^{-1} is fired at the same plank. What would be the speed of this bullet after passing through the plank? Assume that the resistance offered by the plank is the same for both the bullets?
   (a) 48 ms^{-1} (b) 49 ms^{-1}
   (c) 50 ms^{-1} (d) 51 ms^{-1}

9. A particle of mass m has half the kinetic energy of another particle of mass $m/2$. If the speed of the heavier particle is increased by 2 ms^{-1}, its new kinetic energy equals the original kinetic energy of the lighter particle. The ratio of the original speeds of the lighter and heavier particles is
   (a) 1 : 1 (b) 1 : 2
   (c) 1 : 3 (d) 1 : 4

10. In Q. 9, what is the original speed of the heavier particle?
    (a) $2(1+\sqrt{2})$ ms^{-1}
    (b) $2(1-\sqrt{2})$ ms^{-1}
    (c) $(2\sqrt{2}+1)$ ms^{-1}
    (d) $(2\sqrt{2}-1)$ ms^{-1}

11. Two identical cylindrical vessels, with their bases at the same level, each contains a liquid of density ρ. The height of the liquid in one vessel is h_1 and that in the other is h_2. The area of either base is A. What is the work done by gravity in equalizing the levels when the vessels are interconnected?
    (a) $A\rho g(h_1-h_2)^2$ (b) $A\rho g(h_1+h_2)^2$
    (c) $A\rho g\left(\frac{h_1-h_2}{2}\right)^2$ (d) $A\rho g\left(\frac{h_1+h_2}{2}\right)^2$

12. The bob of a pendulum is released from a horizontal position A as shown in Fig. 4.18. The length of the

pendulum is 2 m. If 10% of the initial energy of the bob is dissipated as heat due to the friction of air, what would be the speed of the bob when it reaches the lowermost point *B*? Take $g = 10\ \text{ms}^{-2}$.

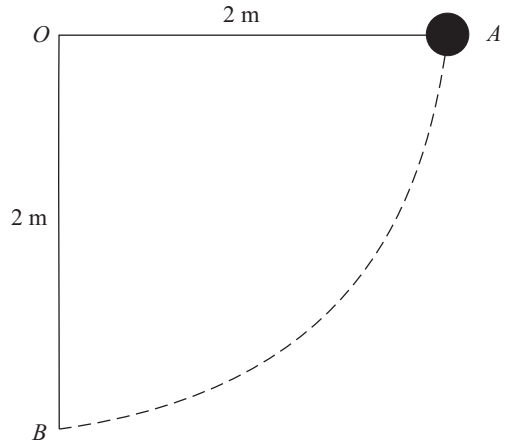


Fig. 4.18

(a) $3\ \text{ms}^{-1}$ (b) $4\ \text{ms}^{-1}$
(c) $5\ \text{ms}^{-1}$ (d) $6\ \text{ms}^{-1}$

13. A wooden block of mass 0.9 kg is suspended from the ceiling of a room by thin wires. A bullet of mass 0.1 kg moving horizontally with a speed of $10\ \text{ms}^{-1}$ strikes the block and sticks to it. What is the height to which the block rises? Take $g = 10\ \text{ms}^{-2}$.
(a) 2.5 m (b) 5.0 m
(c) 7.5 m (d) 10.0 m

14. In Q. 13, what is the loss in kinetic energy of the system due to impact?
(a) 450 J (b) 400 J
(c) 350 J (d) 300 J

15. Two identical balls marked 2 and 3, in contact with each other and at rest on a horizontal frictionless table, are hit head-on by another identical ball marked 1 moving initially with a speed v as shown in Fig. 4.19. What is observed, if the collision is elastic?

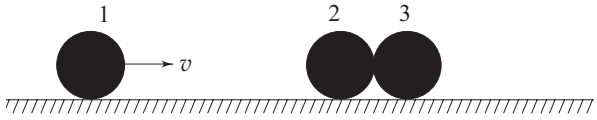


Fig. 4.19

(a) Ball 1 comes to rest and balls 2 and 3 roll out with speed $\frac{v}{2}$ each.
(b) Balls 1 and 2 come to rest and ball 3 rolls out with speed v.
(c) Balls 1, 2 and 3 roll out with speed $\frac{v}{3}$ each.
(d) Balls 1, 2 and 3 come to rest.

16. The bob *A* of a pendulum released from a height *h* hits head–on another bob *B* of the same mass of an identical pendulum initially at rest. What is the result of this collision? Assume the collision to be elastic (see Fig. 4.20).

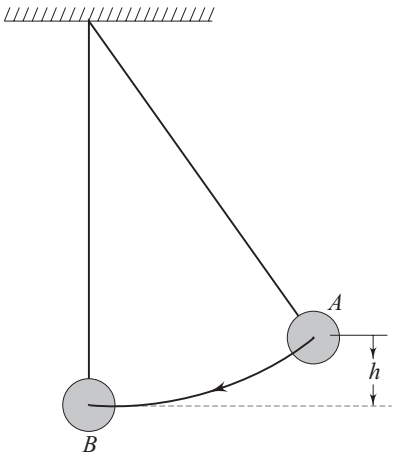


Fig. 4.20

(a) Bob *A* comes to rest at *B* and bob *B* moves to the left attaining a maximum height *h*.
(b) Bobs *A* and *B* both move to the left, each attaining a maximum height $\frac{h}{2}$.
(c) Bob *B* moves to the left and bob *A* moves to the right, each attaining a maximum height $\frac{h}{2}$.
(d) Both bobs come to rest.

(Level B)

17. A steel ball falls from a height *h* on a floor for which the coefficient of restitution is *e*. The height attained by the ball after two rebounds is
(a) eh (b) e^2h
(c) e^3h (d) e^4h

18. Two balls marked 1 and 2 of the same mass *m* and a third ball marked 3 of mass *M* are arranged over a smooth horizontal surface as shown in Fig. 4.21. Ball 1 moves with a velocity v_1 towards balls 2 and 3. All collisions are assumed to be elastic. If $M < m$, the number of collisions between the balls will be
(a) one (b) two
(c) three (d) four

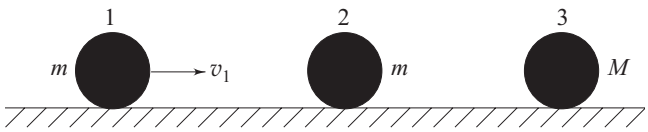


Fig. 4.21

19. In Q. 18, if $M > m$, the number of collisions between the balls will be

(a) one (b) two
(c) three (d) four

20. The distance x moved by a body of mass 0.5 kg by a force varies with time t as

$$x = 3t^2 + 4t + 5$$

where x is expressed in metre and t in second. What is the work done by the force in the first 2 seconds?

(a) 25 J (b) 50 J
(c) 75 J (d) 100 J

21. A body of mass m is dropped from a height h above the ground. The velocity v of the body when it has lost half its initial potential energy is given by

(a) $v = \sqrt{gh}$ (b) $v = \sqrt{2gh}$
(c) $v = \sqrt{\frac{gh}{2}}$ (d) $v = 2\sqrt{gh}$

22. A body of mass m is thrown vertically upwards with a velocity v. The height h at which the kinetic energy of the body is half its initial value is given by

(a) $h = \frac{v^2}{g}$ (b) $h = \frac{v^2}{2g}$
(c) $h = \frac{v^2}{3g}$ (d) $h = \frac{v^2}{4g}$

23. A car of mass m moving at a speed v is stopped in a distance x by the friction between the tyres and the road. If the kinetic energy of the car is doubled, its stopping distance will be

(a) $8x$ (b) $4x$
(c) $2x$ (d) x

24. A body of mass m is dropped from a certain height. It has a velocity v when it is at a height h above the ground. Which of the following will remain constant during the free fall?

(a) $v^2 + 2gh$ (b) $v^2 - 2gh$
(c) $v + \sqrt{\frac{gh}{2}}$ (d) $v - 2\sqrt{gh}$

25. A body of mass m moving with a speed v suffers a perfectly inelastic collision with another body of M at rest. The speed of the composite body will be

(a) $\left(\frac{m+M}{m}\right) v$ (b) $\left(\frac{m}{m+M}\right) v$
(c) $\left(\frac{M}{m+M}\right) v$ (d) $\left(\frac{m+M}{M}\right) v$

26. In Q. 25, the ratio of the final kinetic energy of the system to the initial kinetic energy is

(a) $\frac{m}{m+M}$ (b) $\frac{M}{m+M}$
(c) $\frac{m+M}{m}$ (d) $\frac{m+M}{M}$

27. A ball of mass m moving horizontally at a speed v collides with the bob of a simple pendulum at rest. The mass of the bob is also m. If the collision is perfectly inelastic, the height to which the two balls rise after the collision will be given by

(a) $\frac{v^2}{g}$ (b) $\frac{v^2}{2g}$
(c) $\frac{v^2}{4g}$ (d) $\frac{v^2}{8g}$

28. In Q. 27, the ratio of the kinetic energy of the system immediately after the collision to that before the collision will be

(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 4

29. In Q.27, if the collision is perfectly elastic, the bob of the pendulum will rise to a height of

(a) $\frac{v^2}{g}$ (b) $\frac{v^2}{2g}$
(c) $\frac{v^2}{4g}$ (d) $\frac{v^2}{8g}$

30. A body of mass m_1 moving at a constant speed undergoes an elastic collision with a body of mass m_2 initially at rest. The ratio of the kinetic energy of mass m_1 after the collision to that before the collision is

(a) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$ (b) $\left(\frac{m_1 + m_2}{m_1 - m_2}\right)^2$
(c) $\left(\frac{2m_1}{m_1 + m_2}\right)^2$ (d) $\left(\frac{2m_2}{m_1 + m_2}\right)^2$

31. In Q. 30, the ratio of the kinetic energies of masses m_2 and m_1 after the collision is

(a) $\frac{m_1 m_2}{(m_1 - m_2)^2}$ (b) $\frac{\sqrt{2} m_1 m_2}{(m_1 - m_2)^2}$

(c) $\frac{2 m_1 m_2}{(m_1 - m_2)^2}$ (d) $\frac{4 m_1 m_2}{(m_1 - m_2)^2}$

32. A ball P of mass 2 kg undergoes an elastic collision with another ball Q at rest. After collision, ball P continues to move in its original direction with a speed one-fourth of its original speed. What is the mass of ball Q?
(a) 0.9 kg (b) 1.2 kg
(c) 1.5 kg (d) 1.8 kg

33. A ball of mass m moving with a velocity v undergoes an oblique elastic collision with another ball of the same mass m but at rest. After the collision, if the two balls move with the same speeds, the angle between their directions of motion will be
(a) 30° (b) 60°
(c) 90° (d) 120°

34. In Q. 33, if the two balls move with different speeds after the collision, the angle between their directions of motion will be
(a) less than 90° (b) more than 90°
(c) exactly 90° (d) exactly 180°

35. A box is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to
(a) $t^{1/2}$ (b) $t^{3/4}$
(c) $t^{3/2}$ (d) t^2

36. A bullet, incident normally on a wooden plank, loses one-tenth of its speed in passing through the plank. The least number of such planks required to stop the bullet is
(a) 5 (b) 6
(c) 7 (d) 8

37. A bullet is fired normally on an immovable wooden plank. It loses 25% of its momentum in penetrating a thickness of 3.5 cm. The total thickness penetrated by the bullet is
(a) 8 cm (b) 10 cm
(c) 12 cm (d) 14 cm

38. A bullet is fired normally on an immovable wooden plank. It loses 25% of its kinetic energy in penetrating a thickness x of the plank. What is the total thickness penetrated by the bullet?
(a) $2x$ (b) $4x$
(c) $6x$ (d) $8x$

39. A uniform rod of mass m and length l is made to stand vertically on one end. The potential energy of the rod in this position is
(a) $\frac{mgl}{4}$ (b) $\frac{mgl}{3}$
(c) $\frac{mgl}{2}$ (d) mgl

40. If the rod in Q. 39 is held inclined at an angle of 60° with the vertical, what will be the potential energy of the rod in this position?
(a) $\frac{mgl}{4}$ (b) $\frac{mgl}{3}$
(c) $\frac{mgl}{2}$ (d) mgl

41. A body, having kinetic energy k, moving on a rough horizontal surface, is stopped in a distance x. The force of friction exerted on the body is
(a) $\frac{k}{x}$ (b) $\frac{\sqrt{k}}{x}$
(c) $\frac{k}{\sqrt{x}}$ (d) kx

42. A body of mass m, having momentum p, is moving on a rough horizontal surface. If it is stopped in a distance x, the coefficient of friction between the body and the surface is given by
(a) $\mu = \frac{p^2}{2gm^2x}$ (b) $\mu = \frac{p^2}{2mgx}$
(c) $\mu = \frac{p}{2mgx}$ (d) $\mu = \frac{p}{2gm^2x}$

43. A uniform chain of mass M and length L is held on a horizontal frictionless table with $\frac{1}{n}$ th of its length hanging over the edge of the table. The work done is pulling the chain up on the table is
(a) $\frac{M g l}{n}$ (b) $\frac{M g l}{2n}$
(c) $\frac{M g l}{n^2}$ (d) $\frac{M g l}{2n^2}$

44. A body of mass m = 1 kg is dropped from a height h = 40 cm on a horizontal platform fixed to one end of an elastic spring, the other being fixed to a base, as shown in Fig. 4.22. As a result the spring

is compressed by an amount $x = 10$ cm. What is the force constant of the spring. Take $g = 10$ ms^{-2}.

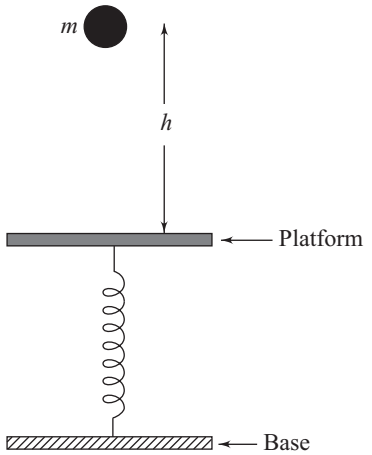


Fig. 4.22

(a) 600 Nm^{-1} (b) 800 Nm^{-1}
(c) 1000 Nm^{-1} (d) 1200 Nm^{-1}

45. A block of wood of mass M is suspended by means of a thread. A bullet of mass m is fired horizontally into the block with a velocity v. As a result of the impact, the bullet is embedded in the block. The block will rise to vertical height given by

(a) $\frac{1}{2g}\left(\frac{mv}{M+m}\right)^2$ (b) $\frac{1}{2g}\left(\frac{mv}{M-m}\right)^2$

(c) $\frac{1}{2g}\frac{mv^2}{(M+m)}$ (d) $\frac{1}{2g}\frac{mv^2}{(M-m)}$

46. A moving particle of mass m makes a head-on collision with a particle of mass $2m$ initially at rest. If the collision is perfectly elastic, the percentage loss of energy of the colliding particle is

(a) 50% (b) 66.7%
(c) 88.9% (d) 100%

47. A body of mass m moving with a velocity v in the x-direction collides with a body of mass M moving with a velocity V in the y-direction. They stick together during collision. Then

(a) the magnitude of the momentum of the composite body is $\sqrt{(mv^2)+(MV)^2}$
(b) the composite body moves in a direction making a angle $\theta = \tan^{-1}\left(\frac{MV}{mv}\right)$ with the x-axis.
(c) the loss of kinetic energy as a result of collision is $\frac{1}{2}\frac{Mm}{(M+m)}(V^2+v^2)$
(d) all the above choices are correct.

48. A body of mass 2 kg moving with a velocity $\left(\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}}\right)$ ms^{-1} collides with another body of mass 3 kg moving with a velocity $\left(2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$ ms^{-1}. If they stick together, the velocity in ms^{-1} of the composite body is

(a) $\frac{1}{5}\left(8\hat{\mathbf{i}}+7\hat{\mathbf{j}}-3\hat{\mathbf{k}}\right)$ (b) $\frac{1}{5}\left(-4\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}}\right)$

(c) $\frac{1}{5}\left(8\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)$ (d) $\frac{1}{5}\left(-4\hat{\mathbf{i}}+7\hat{\mathbf{j}}-3\hat{\mathbf{k}}\right)$

49. A body of mass 2 kg has an initial velocity $\mathbf{v}_i = \left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$ ms^{-1}. After collision with another body its velocity becomes $\mathbf{v}_f = \left(5\hat{\mathbf{i}}+6\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$ ms^{-1}. If the impact time is 0.02 s, the average force of impact on the body (in newton) is

(a) 50 $\left(4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$ (b) 50 $\left(4\hat{\mathbf{i}}-5\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)$

(c) 100 $\left(4\hat{\mathbf{i}}+5\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)$ (d) 100 $\left(4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$

50. A body falls from a height h on a horizontal surface and rebounds. Then it falls again, and again rebounds and so on. If the restitution coefficient is $\frac{1}{3}$, the total distance covered by the body before it comes to rest is

(a) $\frac{h}{4}$ (b) $\frac{5h}{4}$

(c) $2h$ (d) $3h$

51. In Q. 50 above, the total time taken by the body to come to rest is

(a) $\sqrt{\frac{2h}{g}}$ (b) $2\sqrt{\frac{2h}{g}}$

(c) $3\sqrt{\frac{2h}{g}}$ (d) $4\sqrt{\frac{2h}{g}}$

52. A body P strikes another body Q of mass that is p times that of body P and moving with a velocity that is $\frac{1}{q}$ of the velocity of body P. If body P comes to rest, the coefficient of restitution is

(a) $\frac{p+q}{p-q}$ (b) $\frac{p-q}{q(p-1)}$

(c) $\frac{p-q}{q(p-1)}$ (d) $\frac{p+q}{q(p-1)}$

53. Two equal spheres A and B lie on a smooth horizontal circular groove at opposite ends of a diameter. Sphere A is projected along the groove and at the end of time T impinges on sphere B. If e is the coefficient of restitution, the second impact will occur after a time equal to

(a) T (b) eT

(c) $\frac{2T}{e}$ (d) $2\,eT$

54. Two masses M and m (with $M > m$) are connected by means of a pulley as shown in Fig. 4.23. The system is released. At the instant when mass M has fallen through a distance h, the velocity of mass m will be

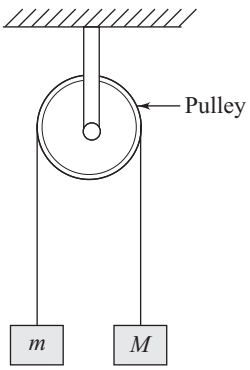


Fig. 4.23

(a) $\sqrt{2gh}$ (b) $\sqrt{\frac{2gh\,M}{m}}$

(c) $\sqrt{\frac{2gh\,(M-m)}{(M+m)}}$ (d) $\sqrt{\frac{2gh\,(M+m)}{(M-m)}}$

55. A mass m, lying on a horizontal frictionless surface is connected to mass M as shown in Fig. 4.24. The system is now released. The velocity of mass m when mass M as descended a distance h is

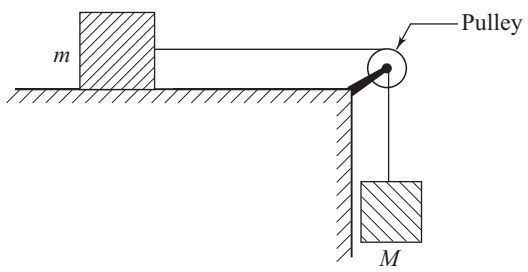


Fig. 4.24

(a) $\sqrt{\frac{2\,Mgh}{m}}$ (b) $\sqrt{\frac{2\,mgh}{M}}$

(c) $\sqrt{\frac{2\,Mgh}{(M+m)}}$ (d) $\sqrt{2gh}$

56. An escalator is moving downwards with a uniform speed u. A man of mass m is running upwards on it at a uniform speed v. If the height of the escalator is h, the work done by the man in going up the escalator is

(a) zero (b) mgh

(c) $\frac{mghu}{(v-u)}$ (d) $\frac{mgh\,v}{(v-u)}$

57. A uniform rope of length L is lying in a lump on a frictionless table. A small part of the rope is held hanging through a hole in the table just below the lump. The system is then released. The speed of the end of the rope as it leaves the hole is

(a) $\sqrt{2gL}$ (b) $2\sqrt{\frac{gL}{3}}$

(c) $\sqrt{\frac{2\,gL}{3}}$ (d) zero

58. The potential energy (in joule) of a body of mass 2 kg moving in the $x - y$ plane is given by

$$U = 6x + 8y$$

where the position coordinates x and y are measured in metre. If the body is at rest at point (6 m, 4 m) at time $t = 0$, it will cross the y-axis at time t equal to

(a) 1 s (b) 2 s

(c) 3 s (d) 4 s

59. In Q. 58 above, the speed of the body when it crosses the y-axis is

(a) zero (b) 5 ms^{-1}

(c) 10 ms^{-1} (d) 20 ms^{-1}

60. A bullet of mass m is fired horizontally with a velocity v on a wooden block of mass M suspended from a support and gets embedded in it. The kinetic energy of the bullet + block system is

(a) $\frac{1}{2}\,mv^2$ (b) $\frac{1}{2}\,(M+m)v^2$

(c) $\frac{Mmv^2}{2(M+m)}$ (d) $\frac{m^2v^2}{2(M+m)}$

61. A rubber ball is dropped from a height of 5 m on a planet where the acceleration due to gravity is not known. On bouncing it rises to 1.8 m. The ball loses its velocity on bouncing by a factor of

(a) $\frac{16}{25}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{9}{2}$

62. An isolated particle of mass m is moving in a horizontal plane ($x - y$), along the x-axis, at a certain height above the ground. It suddenly explodes into two fragments of masses $m/4$ and $3m/4$. An instant later, the smaller fragment is at $y = +15$ cm. The larger fragment at this instant is at

(a) $y = -5$ cm (b) $y = +20$ cm
(c) $y = +5$ cm (d) $y = -20$ cm

63. A particle, which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is (see Fig. 4.25)

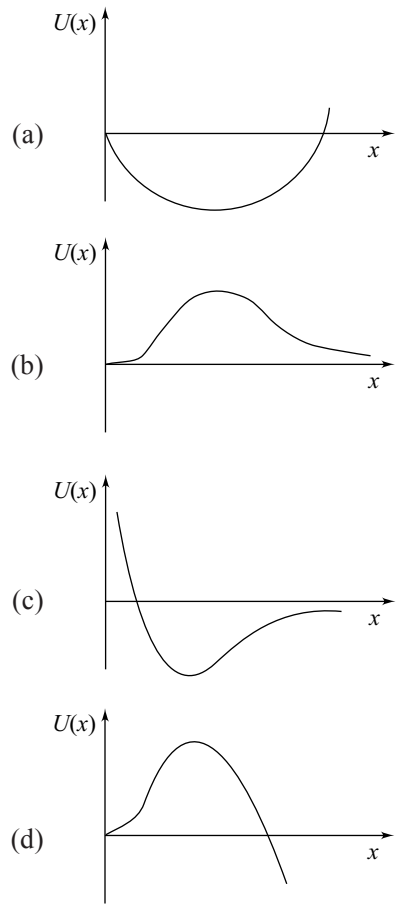


Fig. 4.25

64. A body of mass 6 kg is acted upon by a force which causes a displacement in it given by $x = \frac{t^2}{4}$ metre where t is the time in second. The work done by the force is 2 seconds is

(a) 12 J (b) 9 J
(c) 6 J (d) 3 J

65. A ladder 2.5 m long and of weight 150 N has its centre of mass 1 m from its bottom. A weight of 40 N is attached to the top end. The work required to raise the ladder from the horizontal position to the vertical position is

(a) 190 J (b) 250 J
(c) 285 J (d) 475 J

66. A body of mass 5 kg rests on a rough horizontal surface of coefficient of friction 0.2. The body is pulled through a distance of 10 m by a horizontal force of 25 N. The kinetic energy acquired by it is (take $g = 10\ ms^{-2}$)

(a) 200 J (b) 150 J
(c) 100 J (d) 50 J

67. A body is moving up an inclined plane of angle θ with an initial kinetic energy E. The coefficient of friction between the plane and the body is μ. The work done against friction before the body comes to rest is:

(a) $\frac{\mu \cos\theta}{E\cos\theta + \sin\theta}$ (b) $\mu E \cos\theta$

(c) $\frac{\mu E \cos\theta}{\mu\cos\theta - \sin\theta}$ (d) $\frac{\mu E \cos\theta}{\mu\cos\theta + \sin\theta}$

68. A particle falls from a height h on a fixed horizontal plate and rebounds. If e is the coefficient of restitution, the total distance travelled by the particle before it stops rebounding is

(a) $\frac{h(1+e^2)}{(1-e^2)}$ (b) $\frac{h(1-e^2)}{(1+e^2)}$

(c) $\frac{h(1-e^2)}{2(1+e^2)}$ (d) $\frac{h(1+e^2)}{2(1-e^2)}$

69. A force F = $(3x + 4)$ newton displaces an object of mass 10 kg from $x = 0$ to $x = 4$m. If the speed of the object at $x = 0$ is $2ms^{-1}$, its speed at $x = 4$m will be

(a) $3ms^{-1}$ (b) $\sqrt{3}\ ms^{-1}$
(c) $2\sqrt{3}\ ms^{-1}$ (d) $4ms^{-1}$

70. The potential energy U of a particle of mass m=3kg moving in the x-direction is given by

$$U = 3(x-1) - (x-3)^3$$

when U is in joule and x is in metre. Which of the graphs shown in Fig. 4.26 best represents the variation of the acceleration A of the particle with position x ?

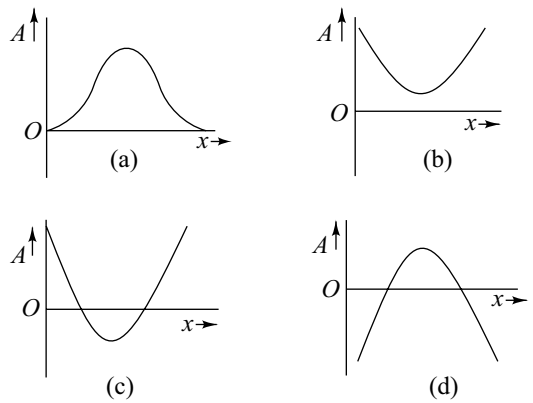


Fig. 4.26

71. A block of wood of mass M is moving at a speed V and a bullet of mass m is moving towards it wil a velocity v as shown in Fig. 4.27.



Fig. 4.27

If the bullet gets embedded in the block after impact, the value of v so that the block + bullet system is stopped dead after impact must be

(a) $\frac{MV}{M+m}$ (b) $\frac{mV}{M+m}$

(c) $\frac{MV}{M}$ (d) $\frac{mV}{M}$

72. A steel ball of mass m tied to a light string of length L is released from rest when the string is horizontal. The tension in the string when the ball is at a height $\frac{L}{2}$ above the lowest position (see fig. 4.28) is

(a) $\frac{1}{2}mg$ (b) $\sqrt{2}\,mg$

(c) $\frac{3}{2}mg$ (d) $2\,mg$

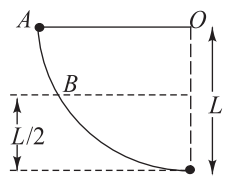


Fig. 4.28

73. A steel ball of mass m tied to a light string of length L in released from rest when the string is horizontal. It strikes a steel block of mass $M = 3m$ which is initially at rest at the bottom as shown in Fig.4.29. If the collision is elastic, to what height h will the ball rise after the collision ?

(a) $\frac{2L}{3}$ (b) $\frac{L}{2}$

(c) $\frac{L}{3}$ (d) $\frac{L}{4}$

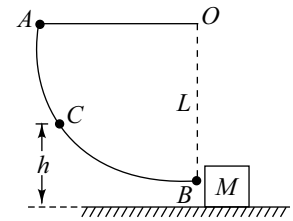


Fig. 4.29

74. When a force acts on a body of mass m, its position x varies with time t as

$$x = \frac{kt^3}{3}$$

where k is a constant. The work done by the force in the first 2 seconds is

(a) $2\,mk^2$ (b) $4\,mk^2$

(c) $8\,mk^2$ (d) $16\,mk^2$

75. A pump of power P is used to pump water in a certain pipe at a certain rate. To pump twice as much water through the same pipe in the same time, the power of the pump must be increased to

(a) $2P$ (b) $4P$

(b) $8P$ (d) $16P$

76. A force acts on a car initially at rest. The engine of the car drives it with a constant power P . If the car acquires a velocity v when it has travelled a distance x, then v is proportional to

(a) $x^{1/3}$ (b) $x^{2/3}$

(c) $x^{3/2}$ (d) x^3

77. When a force acts on a body of mass m, its position x varies with time t as

$$x = at^4 + bt + c$$

where a, b and c are constants. The work done by the force during the first second of the motion is

(a) $ma\,(2a + c)$ (b) $2ma\,(a + 2b)$

(c) $3ma\,(2a + 3b)$ (d) $4ma\,(2a + b)$

78. The kinetic energy K of a particle moving in a circle of radius r is given by

$$K = \frac{kx^2}{2}$$

where k is a constant and x is the distance moved along the arc. The net force on the particle is

(a) kx (b) $\frac{kx^2}{r}$

(c) $kx\left(1+\frac{x^2}{r^2}\right)^{1/2}$ (d) $kx\left(1-\frac{x^2}{r^2}\right)^{1/2}$

79. A block begins to move on a rough horizontal surface with an initial velocity u. If it loses $\frac{1}{4}$ of its initial kinetic energy in time t, the coefficient of kinetic friction between the block and the surface is

(a) $\frac{u}{gt}$ (b) $\frac{\sqrt{3}u}{2gt}$

(c) $\frac{u}{2gt}\left(\sqrt{3}-1\right)$ (d) $\frac{u}{2gt}\left(2-\sqrt{3}\right)$

80. Ball A moving with momentum p undergoes a one dimensinal collision with a stationary ball B of the same mass. During the collision ball B imparts an impulse I to ball A. The coefficient of restitution is

(a) $\frac{2I}{p}$ (b) $\frac{2I}{p}-1$

(c) $\frac{I}{p}+1$ (d) $\frac{2I}{p}+1$

81. A hydrogen filled balloon of mass M has a rope of length l of negligible mass attached to it with a man of mass m at the other end of the rope. The whole system is in equilibruim in mid-air. The man climbs up the rope and reaches the balloon. As a result the balloon descends by a height h. The work done by the man in reaching the balloon is

(a) $(M + m)gh$ (b) $(M – m)\, g\, (l – h)$

(c) Mgh (d) mgl

82. A ball of mass m moving with a velocity v_1 strikes normally a massive rock of mass M ($>> m$) moving towards the ball with a velocity v_2. If the collision is perfectly elastic, the magnitude of the velocity of the ball after impact is

(a) $v_1 + v_2$ (b) $v_1 – v_2$

(c) $v_1 + 2v_2$ (d) $v_1 – 2v_2$

Answers

(Level A)

1. (c)	2. (c)	3. (d)	4. (d)
5. (a)	6. (c)	7. (c)	8. (b)
9. (b)	10. (a)	11. (c)	12. (d)
13. (b)	14. (a)	15. (b)	16. (a)

(Level B)

17. (d)	18. (b)	19. (c)	20. (c)
21. (a)	22. (d)	23. (c)	24. (a)
25. (b)	26. (a)	27. (d)	28. (b)
29. (b)	30. (a)	31. (d)	32. (b)
33. (c)	34. (c)	35. (c)	36. (b)
37. (a)	38. (b)	39. (c)	40. (a)
41. (a)	42. (a)	43. (d)	44. (c)
45. (a)	46. (c)	47. (d)	48. (a)
49. (d)	50. (b)	51. (b)	52. (d)
53. (c)	54. (c)	55. (c)	56. (d)
57. (b)	58. (b)	59. (c)	60. (d)
61. (b)	62. (a)	63. (d)	64. (d)
65. (b)	66. (b)	67. (d)	68. (a)
69. (c)	70. (c)	71. (c)	72. (c)
73. (d)	74. (c)	75. (c)	76. (a)
77. (d)	78. (c)	79. (d)	80. (b)
81. (d)	82. (c)		

Solutions

(Level A)

1. Mass of the drop m = volume × density of water = $\frac{4\pi}{3}r^3\rho$, where ρ is the density of water. Work done by gravitational force is

$$W = mgh = \frac{4\pi}{3}r^3\rho gh$$

Thus $W \propto r^3$. Hence the correct choice is (c).

2. As shown in Fig. 4.26, the height attained by the bob when the string subtends an angle α with the vertical is

$$h = l – l\cos\alpha = l\,(1 – \cos\alpha)$$

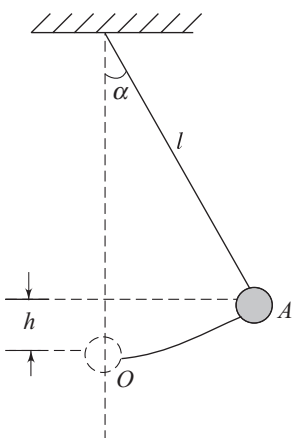


Fig. 4.26

Its potential energy at the highest point $A = mgh$, where m is the mass of the bob. Let v be the speed of the bob when it passes through O. Its kinetic energy at $O = \frac{1}{2}\ mv^2$. From the principle of conservation of energy, we have

$$\frac{1}{2}\ mv^2 = mgh$$

or $$v = \sqrt{2gh} = \sqrt{2gl(1-\cos\alpha)}$$

Hence the correct choice is (c).

3. From the principle of conservation of energy, we have

$$\frac{1}{2}\ mv^2 + mgh\ = 0 + mgR$$

which gives $v = \sqrt{2g(R-h)}$. Hence the correct choice is (d).

4. Given $p_1 = m_1v_1 = 2p$ and $p_2 = m_2v_2 = p$, so that

$$\frac{m_1v_1}{m_2v_2} = 2$$

The ratio of their kinetic energies is

$$\frac{(\text{KE})_1}{(\text{KE})_2} = \frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_2v_2^2} = \frac{m_1^2v_1^2}{m_2^2v_2^2}\cdot\frac{m_2}{m_1}$$

But $m_2 = 4\ m_1$ and $\frac{m_1v_1}{m_2v_2} = 2$. Therefore,

$$\frac{(\text{KE})_1}{(\text{KE})_2} = (2)^2 \times 4 = 16$$

Hence the correct choice is (d).

5. Given $(\text{KE})_1 = \frac{1}{2}\ m_1v_1^2 = 2K$

and $(\text{KE})_2 = \frac{1}{2}\ m_2v_2^2 = K$, so that

$$\frac{m_1v_1^2}{m_2v_2^2} = 2$$

or $$\frac{m_1^2v_1^2}{m_2^2v_2^2} = \frac{2m_1}{m_2}$$

or $$\frac{p_1}{p_2} = \sqrt{\frac{2m_1}{m_2}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}} \qquad (\because\ m_2 = 4\ m_1)$$

Hence the correct choice is (a)

6. The correct choice is (c).

7. Let M be the mass of the rifle and m that of the bullet and let V and v be their respective speeds. From the principle of conservation of momentum, we have $MV = mv$. The ratio of their kinetic energies is

$$\frac{K_r}{K_b} = \frac{\frac{1}{2}MV^2}{\frac{1}{2}mv^2} = \frac{MV^2}{mv^2} = \frac{(MV)^2}{(mv)^2}\cdot\frac{m}{M} = \frac{m}{M}$$

Since $m < M$, $K_r < K_b$. Hence the correct choice is (c).

8. Let m be the mass of each bullet. Since the resistance offered by the plank is the same for the two bullets, the amount of work done by the plank is the same for the two bullets. From work–energy principle, the decrease in the kinetic energy is the same for the two bullets.

Decrease in KE of first bullet $= \frac{1}{2}\ mu_1^2 - \frac{1}{2}\ mv_1^2$

$$= \frac{1}{2}\ m(200)^2 - \frac{1}{2}\ m(180)^2 \qquad \text{(i)}$$

If v_2 is the speed of the second bullet after passing through the plank, then

Decrease in KE of second bullet $= \frac{1}{2}mu_2^2 - \frac{1}{2}\ mv_2^2$

$$= \frac{1}{2}\ m(100)^2 - \frac{1}{2}\ mv_2^2 \qquad \text{(ii)}$$

Equating (i) and (ii) we have,

$$\frac{1}{2}\ m(200)^2 - \frac{1}{2}\ m(180)^2 = \frac{1}{2}\ m(100)^2 - \frac{1}{2}\ mv_2^2$$

which gives $v_2^2 = 2400$ or $v = 49\ \text{ms}^{-1}$

Hence the correct choice is (b).

9. Let v and v' be the original speeds of the heavier and the lighter particles respectively. We then have

$$\frac{1}{2}\ mv^2 = \frac{1}{2}\times\left\{\frac{1}{2}\left(\frac{m}{2}\right)v'^2\right\}$$

$$\therefore \qquad v^2 = \frac{1}{4}\ v'^2$$

or $$v' = 2v$$

Hence the correct choice is (b).

10. When the heavier particle is speeded up by 2.0 m s^{-1}, its kinetic energy becomes $\frac{1}{2}\ m(v+2)^2$. Since this equals the original kinetic energy of the lighter particle, we have

$$\frac{1}{2}\, m(v+2)^2 = \frac{1}{2}\,(m/2)(4v^2)$$

$$v^2 + 4 + 4v = 2v^2$$

or $$v^2 - 4v - 4 = 0$$

$$v = \frac{4 \pm \sqrt{16+16}}{2} = \frac{4 \pm 2\sqrt{8}}{2} = 2 \pm 2\sqrt{2}$$

The positive root is $v = 2 + 2\sqrt{2} = 2(1 + \sqrt{2})$. Hence the correct choice is (a).

11. The work done by gravity equals the change in the potential energy of the system after the vessels are interconnected. We may regard the liquid in each vessel as equivalent to a point mass kept at their respective centres of mass. Remembering that the mass of the liquid is given by $(Ah\rho)$ and that the PE of a mass at a height h in earth's gravity is mgh, we have

$$\text{Total PE at start} = (\text{A}h_1\rho)g\,\frac{h_1}{2} + (\text{A}h_2\rho)g\,\frac{h_2}{2}$$

$$= \frac{A\rho g}{2}\,(h_1^2 + h_2^2)$$

After the vessels are connected, the height of liquid in each vessel is $(h_1 + h_2)/2$.

Hence PE after connection

$$= \left\{A\rho\left(\frac{h_1+h_2}{2}\right)g\left(\frac{h_1+h_2}{2}\right)\right\}$$

$$= \frac{A\rho g}{4}\,(h_1+h_2)^2$$

$$\text{Change in PE} = \frac{A\rho g}{4}\,\{(h_1+h_2)^2 - 2(h_1^2+h_2^2)\}$$

$$= -\frac{A\rho g}{4}\,(h_1-h_2)^2$$

$$= -A\rho g\left(\frac{h_1-h_2}{2}\right)^2$$

This must be equal to the work done 'by' gravity on the liquid. Thus the work done 'by' gravity is

$$A\rho g\left(\frac{h_1-h_2}{2}\right)^2$$

Hence the correct choice is (c).

12. PE at $A = mgh$. Since 10% of this energy is lost,

KE at point $B = mgh \times \frac{90}{100} = 0.9\, mgh$. Therefore,

$$\frac{1}{2}\, mv^2 = 0.9\, mgh$$

or $$v^2 = 1.8\, gh = 1.8 \times 10 \times 2 = 36$$

which gives $v = 6$ ms^{-1}. Hence the correct choice is (d).

13. Mass of block $(M) = 0.9$ kg, mass of bullet $(m) = 0.1$ kg, initial velocity of bullet $(u) = 100$ ms^{-1} and initial velocity of block $(U) = 0$.

$\therefore$ Momentum of bullet and block before impact = $mu + MU = mu$. Let v be the velocity of the bullet + block after impact. Then, the momentum after impact = $(m + M)v$. From the principle of conservation of momentum, we have

$$mu = (m+M)v$$

or $$v = \frac{mu}{(M+m)} = \frac{0.1 \times 100}{(0.9+0.1)} = 10 \text{ ms}^{-1}$$

$$\text{KE of bullet + block} = \frac{1}{2}\,(m+M)v^2 \qquad \text{(i)}$$

Let the bullet + block system rise to a height h. At this height,

$$\text{PE of bullet + block} = (m+M)\,gh \qquad \text{(ii)}$$

Equating (i) and (ii), we get

$$h = \frac{v^2}{2g} = \frac{(10)^2}{2 \times 10} = 5 \text{ m}$$

Hence the correct choice is (b).

14. Initial KE $= \frac{1}{2}\, mu^2$

Final KE $= \frac{1}{2}\,(m+M)v^2$

$\therefore$ Loss in KE $= \frac{1}{2}\, mu^2 - \frac{1}{2}\,(m+M)v^2$

$$= \frac{1}{2} \times 0.1 \times (100)^2 - \frac{1}{2}\,(0.1+0.9) \times (10)^2$$

$$= 450 \text{ J}$$

Hence the correct choice is (a).

15. The system consists of three identical balls marked 1, 2 and 3. Let m be the mass of each ball. Before the collision,

KE of the system = KE of 1 + KE of 2 + KE of 3

$$= \frac{1}{2}\, mv^2 + 0 + 0 = \frac{1}{2}\, mv^2$$

Case (a) After the collision,

KE of the system = KE of 1 + KE of 2 + KE of 3

$$= 0 + \frac{1}{2}\, m\left(\frac{v}{2}\right)^2 + \frac{1}{2}\, m\left(\frac{v}{2}\right)^2$$

$$= \frac{1}{4}\, mv^2$$

Case (b)

KE of the system = KE of 1 + KE of 2 + KE of 3

$$= 0 + 0 + \frac{1}{2}\, mv^2 = \frac{1}{2}\, mv^2$$

Case (c)

KE of the system = KE of 1 + KE of 2 + KE of 3

$$= \frac{1}{2}\, m\left(\frac{v}{3}\right)^2 + \frac{1}{2}\, m\left(\frac{v}{3}\right)^2 + \frac{1}{2}\, m\left(\frac{v}{3}\right)^2$$

$$= \frac{1}{6}\, mv^2$$

Case (d)

KE of the system = 0

Now, in an elastic collision, the kinetic energy of the system remains unchanged. Hence choice (b) is the only possible result of the collision.

16. Suppose the bob A acquires a velocity v on reaching the bob B. In a head-on elastic collision between two bodies of the same mass when one of them is at rest, the velocities are exchanged after the collision. Hence the bob A will come to rest at the lowermost position (occupied by B before collision) and the bob B will move to the left attaining a maximum height h. Hence the correct choice is (a).

(Level B)

17. The velocity attained after a fall through a height h is given by

$$v^2 = 2gh$$

Thus $h \propto v^2$. The velocity after first rebound is ev. Therefore, the height attained after first rebound = e^2h. Velocity after second rebound is e^2v. Hence the height attained after second rebound is e^4h. Thus the correct choice is (d).

18. The first collision will be between balls 1 and 2. Since both have the same mass, after the collision ball 1 will come to rest and ball 2 will move with speed v_1. This ball will collide with the stationary ball 3. After this second collision, let v_2 and v_3 be the speeds of balls 2 and 3 respectively. Since the collisions are elastic, v_2 and v_3 are given by

$$v_2 = \left(\frac{m - M}{m + M}\right) v_1 \qquad \text{(i)}$$

and

$$v_3 = \left(\frac{2m}{m + M}\right) v_1 \qquad \text{(ii)}$$

If $M < m$, it follows from (i) and (ii) that $v_2 < v_3$ and both have the same direction. Therefore, ball 2 cannot collide with ball 3 again. Hence there are only two collisions. Thus, the correct choice is (b).

19. If $M > m$, we have from Eq. (i)

$$v_2 = -\left(\frac{M - m}{M + m}\right) v_1$$

The negative sign indicates that, after the second collision, ball 2 will move in opposite direction towards the ball 1 which is at rest after the first collision. Therefore, ball 2 will make another collision with ball 1.

Hence, in this case, there are three collisions in all between the balls. Thus the correct choice is (c).

20. Velocity $(v) = \frac{dx}{dt} = \frac{d}{dt}\,(3t^2 + 4t + 5) = 6t + 4.$

Acceleration is $a = \frac{dv}{dt} = \frac{d}{dt}\,(6t + 4) = 6\ \text{ms}^{-2}.$

Therefore, applied force is $F = ma = 0.5 \times 6 = 3$ N. Now $t = 2$s, the distance moved is $x = 3 \times (2)^2 + 4 \times 2 + 5 = 25$ m ($\because$ $u = 4\text{ms}^{-1}$ and $x = 5$ m at $t = 0$)

$\therefore$ Work done $W = Fx = 3 \times 25 = 75$ J. Hence the correct choice is (c).

21. Initial PE = mgh. Now, gain in KE = loss in PE. Thus

$$\frac{1}{2}\, mv^2 = \frac{1}{2}\, mgh$$

or

$$v = \sqrt{gh}$$

Hence the correct choice is (a).

22. Initial KE = $\frac{1}{2}\, mv^2$. Now, gain in PE = loss in KE. Thus

$$mgh = \frac{1}{4}\, mv^2$$

or

$$h = \frac{v^2}{4g}$$

Hence the correct choice is (d).

23. If a is the deceleration due to the force of friction f, then

$$2ax = v^2$$

or

$$\frac{1}{2}\, mv^2 = max$$

or

$$\text{KE} = fx \qquad (\because f = ma)$$

Thus if KE is doubled, x is also doubled. Hence the correct choice is (c).

24. The total energy = KE + PE remains constant during the free fall, i.e.

$$mgh + \frac{1}{2}\, mv^2 = \text{constant}$$

or $$gh + \frac{v^2}{2} = \text{constant}$$

Hence the correct choice is (a).

25. Initial momentum of the system = mv, since body of mass M is at rest. After the inelastic collision, the bodies stick together and the mass of the composite body is $(m + M)$. If V is the speed of the composite body, its momentum will be $(m + M)V$. From the principle of conservation of momentum, we have

$$mv = (m + M)V$$

or $$V = \left(\frac{m}{m+M}\right)v$$

Hence the correct choice is (b).

26. Initial KE = $\frac{1}{2}\, mv^2$. Final KE = $\frac{1}{2}\, (m + M)V^2$. Therefore,

$$\frac{\text{Final KE}}{\text{Initial KE}} = \left(\frac{m+M}{m}\right)\frac{V^2}{v^2} = \frac{m}{m+M}$$

Hence the correct choice is (a).

27. In a perfectly inelastic collision, two bodies stick together. After the collision, the speed of the ball and the bob (sticking together) is $v' = v/2$. The height to which they will rise is given by

$$v' = \sqrt{2gh'}$$

or $$h' = \frac{v'^2}{2g} = \frac{v^2}{8g}$$

Hence the correct choice is (d).

28. Mass of the ball and the bob sticking together is $m' = 2m$. KE after collision = $\frac{1}{2}\, m'\, v'^2 = \frac{1}{2} \times 2m \times \left(\frac{v}{2}\right)^2$ $= \frac{1}{4}\, mv^2$. KE before collision = $\frac{1}{2}\, mv^2$. Therefore, their ratio is 1:2. Hence the correct choice is (b).

29. In an elastic collision between two bodies of the same mass with one of them initially at rest, the moving body is brought to rest and the other moves with the same speed. Thus the ball will come to rest and the bob of the pendulum acquires a speed v. At this speed, it will rise to height h given by $h = v^2/2g$. Hence the correct choice is (b).

30. Let u_1 be the speed of mass m_1 before the collision. Here $u_2 = 0$. Therefore, the speeds of masses m_1 and m_2 after the collision respectively are

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$

and $$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1$$

$\therefore$ KE of m_1 after collision = $\frac{1}{2}\, m_1 v_1^2$

$= \frac{1}{2}\, m_1 \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 u_1^2$. KE of m_1 before collision

$= \frac{1}{2}\, m_1 u_1^2$. The ratio of the two is $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$.

Hence the correct choice is (a).

31. KE of m_2 after collision = $\frac{1}{2}\, m_2 v_2^2$

$= \frac{1}{2}\, m_2 \left(\frac{2m_1}{m_1 + m_2}\right)^2 u_1^2$.

KE of m_1 after collision = $\frac{1}{2}\, m_1 v_1^2$

$= \frac{1}{2}\, m_1 \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 u_1^2$. Dividing the two, we find that the correct choice is (d).

32. Since the collision is elastic, both momentum and kinetic energy are conserved. If m is mass of ball Q and v' its speed after the collision, the law of conservation of momentum gives

$$2v = 2 \times \frac{v}{4} + mv'$$

where v is the original speed of ball P. Thus

$$mv' = \frac{3v}{2} \text{ or } v' = \frac{3v}{2m} \qquad \text{(i)}$$

The law of conservation of energy gives

$$\frac{1}{2} \times 2 \times v^2 = \frac{1}{2} \times 2 \times \left(\frac{v}{4}\right)^2 + \frac{1}{2}\, mv'^2$$

or $$mv'^2 = \frac{15v^2}{8} \qquad \text{(ii)}$$

Using (i) in (ii) we get $m = 1.2$ kg. Hence the correct choice is (b).

33. Refer to Fig. 4.27.

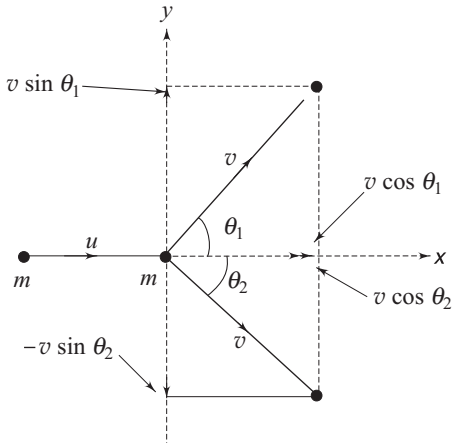


Fig. 4.27

Taking the components of the velocities along x and y–axes and using the law of conservation of momentum for x and y components we have

$$mu = mv\cos\theta_1 + mv\cos\theta_2 \quad \text{(i)}$$

and $$0 = mv\sin\theta_1 - mv\sin\theta_2 \quad \text{(ii)}$$

From (ii) we get $\sin\theta_1 = \sin\theta_2$ or $\theta_1 = \theta_2$. Using $\theta_1 = \theta_2 = \theta$ in (i) we have

$$mu = 2\,mv\cos\theta$$

or $$\cos\theta = \frac{u}{2v} \quad \text{(iii)}$$

Since the collision is elastic, kinetic energy is also conserved, i.e.

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

or $$u^2 = 2v^2 \text{ or } u = \sqrt{2}\,v \quad \text{(iv)}$$

Using (iv) and (iii) we have $\cos\theta = \frac{1}{\sqrt{2}}$ or $\theta = 45°$.

Thus $\theta_1 + \theta_2 = 2\theta = 90°$. Hence the correct choice is (c).

34. Refer to Fig. 4.27 again. Let $\mathbf{v}_1$ be the velocity of the ball moving along direction θ_1 and $\mathbf{v}_2$ be the velocity of the ball moving along θ_2.

From momentum conservation,

$$m\mathbf{u} = m\mathbf{v}_1 + m\mathbf{v}_2$$

$$\Rightarrow \quad \mathbf{u} = \mathbf{v}_1 + \mathbf{v}_2$$

The magnitude of $\mathbf{u}$ is

$$u = \left[v_1^2 + v_2^2 + 2v_1v_2\cos(\theta_1 + \theta_2)\right]^{1/2} \quad \text{(i)}$$

Since kinetic energy is also conversed,

$$\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$\Rightarrow \quad u^2 = v_1^2 + v_2^2 \quad \text{(ii)}$$

Using (ii) in (i) we have

$$v_1^2 + v_2^2 = v_1^2 + v_2^2 + 2v_1v_2\cos(\theta_1 + \theta_2)$$

$$\Rightarrow \quad \cos(\theta_1 + \theta_2) = 0 \quad \Rightarrow \quad \theta_1 + \theta_2 = 90°$$

35. Power $P = Fv = mav = m\,\frac{dv}{dt}\cdot v$ or $v\,dv = \frac{P}{m}dt$.

Integrating, we have

$$\int v\,dv = \frac{P}{m}\int dt \qquad (\because P = \text{constant})$$

or $$\frac{v^2}{2} = \frac{Pt}{m}$$

or $$v = \sqrt{\frac{2P}{m}}\,t^{1/2}$$

or $$\frac{dx}{dt} = \sqrt{\frac{2P}{m}}\,t^{1/2}$$

or $$dx = \sqrt{\frac{2P}{m}}\,t^{1/2}\,dt$$

Integrating again, we have

$$\int dx = \sqrt{\frac{2P}{m}}\int t^{1/2}dt$$

or $$x = \frac{2}{3}\sqrt{\frac{2P}{m}}\,t^{3/2}$$

i.e. $x \propto t^{3/2}$. Hence the correct choice is (c).

36. Let v be the speed of the bullet incident on the first plank. Its speed after it passes the plank $= \frac{9v}{10}$. If x is the thickness of the plank, the deceleration a due to the resistance of the plank is given by

$$2ax = v^2 - \left(\frac{9v}{10}\right)^2 = \frac{19v^2}{100} \quad \text{(i)}$$

Suppose the bullet is stopped after passing through n such planks. Then the distance covered by the bullet is $s = nx$. Thus, we have

$$v^2 - 0 = 2as = 2anx$$

or $$n = \frac{v^2}{2ax} = \frac{v^2 \times 100}{19v^2}$$ [use Eq. (i) above]

$$= \frac{100}{19} = 5.26$$

Thus the minimum number of planks required is 6. Hence the correct choice is (b).

37. Let u cms^{-1} be the speed of the bullet. Since the mass of the bullet remains unchanged, its speed becomes $v = \frac{3u}{4}$ cms^{-1} after it penetrates a distance $x = 3.5$ cm. The retardation a due to the resistance of the wooden is given by

$$u^2 - v^2 = 2ax$$

or $$u^2 - \left(\frac{3u}{4}\right)^2 = 2a \times 3.5$$

which gives $a = \frac{u^2}{16}$ cms^{-2}. The bullet will come to rest when its velocity $v' = 0$. If x' is the thickness penetrated by the bullet, then

$$u^2 - v'^2 = 2ax'$$

or $$x' = \frac{u^2}{2a}. \text{ But } a = \frac{u^2}{16} \text{ cms}^{-2}. \text{ Therefore}$$

$$x' = \frac{u^2 \times 16}{2u^2} = 8 \text{ cm}$$

Hence the correct choice is (a).

38. Since the wood offers a constant deceleration and hence a constant retardation force, the bullet will lose the remaining 75% of its kinetic energy after penetrating a further distance of $3x$. Therefore, the total distance penetrated by the bullet before it comes to rest = $x + 3x = 4x$. Hence the correct choice is (b).

39. The potential energy in the vertical position = work done in raising it from horizontal position to vertical position. In doing so, the mid–point of the rod is raised through a height $h = l/2$. Since the entire mass of the rod can be assumed to be concentrated at the mid–point (centre of mass), the work done = $mgh = mgl/2$. Hence the correct choice is (c).

40. Refer to Fig. 4.28. $AD = AB = l$. In the inclined position, let the centre of mass C of the rod be at a height h above the ground, so that $AC = l/2$. In triangle ACE, we have

$$h = AC \sin 30° = \frac{l}{2} \sin 30° = \frac{l}{4}$$

$$\therefore PE = mgh = \frac{mgl}{4}, \text{ which is choice (a).}$$

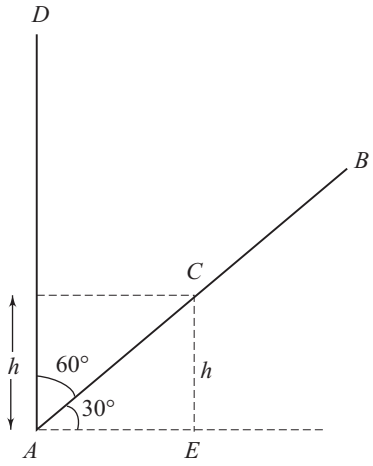


Fig. 4.28

41. Let f be the force of friction and m be the mass of the body. The retardation $a = f/m$. If v is the initial speed of the body, then

$$2ax = v^2$$

or $$max = \frac{1}{2} mv^2 = k$$

But $ma = f$. Therefore $fx = k$ or $f = k/x$. Hence the correct choice is (a).

42. Force of friction = μmg. Therefore, retardation $a = \mu mg/m = \mu g$. Also $2ax = v^2$ or $2am^2x = m^2v^2$. But $p = mv$. Therefore,

$$2\, am^2x = p^2$$

But $a = \mu g$. Therefore, $2\, \mu g\, m^2x = p^2$ or $\mu = \frac{p^2}{2gm^2x}$. Hence the correct choice is (a).

43. The mass per unit length of the chain $m = \frac{M}{L}$. The mass of the hanging portion of the chain is $m' = \frac{mL}{n}$. This mass can be assumed to be concentrated at the centre of the hanging portion of the chain which is a distance of $x = \frac{L}{2n}$ from the edge of the table. Therefore, the work done in pulling the hanging portion of the chain on to the table top is

$$W = m'gx = \frac{mL}{n} \times g \times \frac{L}{2n} = \frac{mgL^2}{2n^2} = \frac{MgL}{2n^2}$$

Hence the correct choice is (d).

44. Since the platform is depressed by an amount x, the total work done on the spring is $mg\,(h + x)$. This work is stored in the spring in the form of potential energy $\frac{1}{2} kx^2$. Equating the two, we have

$$\frac{1}{2}\ kx^2 = mg\ (h + x)$$

or $$k = \frac{2m\ g(h + x)}{x^2}$$

Given, $h = 0.4$ m, $x = 0.1$ m, $m = 1$ kg and $g = 10\ \text{ms}^{-2}$. Substituting these values, we get $k = 1000\ \text{Nm}^{-1}$. Hence the correct choice is (c).

45. Let V be the velocity of the block with the bullet embedded in it at the time of impact. Then from the principle of conservation of momentum, we have

$$mv = (M + m)\ V$$

or $$V = \frac{mv}{(M + m)} \qquad \text{(i)}$$

If the block, with the bullet embedded in it, rises to a vertical height h, then from the principle of conservation of energy, we have

$$\frac{1}{2}\ (M + m)\ V^2 = (M + m)\ gh$$

or $$V = \sqrt{2gh} \qquad \text{(ii)}$$

Using (ii) in (i), we get

$$\sqrt{2gh} = \frac{mv}{(M + m)}$$

Squaring this equation, we find that h is given correctly by choice (a).

46. Percentage loss of energy $= \dfrac{4mM}{(M + m)^2} \times 100$

$$= \frac{4m \times 2m}{(2m + m)^2} \times 100$$

$$= \frac{800}{9} = 88.9\%$$

Hence the correct choice is (c).

47. Refer to Fig. 4.29. Here $\mathbf{p} = m\mathbf{v}$ and $\mathbf{P} = M\mathbf{V}$. The resultant of $\mathbf{p}$ and $\mathbf{P}$ is

$$p_r = \sqrt{p^2 + P^2} = \sqrt{(mv)^2 + (MV)^2}$$

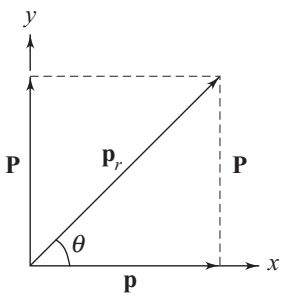


Fig. 4.29

which is choice (a). The angle which the resultant momentum p_r subtends with the x-axis is given by

$$\tan\theta = \frac{P}{p} = \frac{MV}{mv}\text{, which is choice (b).}$$

$$\text{Loss of KE} = \left(\frac{1}{2}mv^2 + \frac{1}{2}MV^2\right) - \frac{1}{2}\left[\frac{m^2v^2 + M^2V^2}{(M + m)}\right]$$

$$= \frac{1}{2}\frac{Mm}{(M + m)}\ (V^2 + v^2)\text{, which is choice (c).}$$

Hence the correct choice is (d).

48. $\mathbf{p}_1 = 2\text{ kg}\ \left(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}\right)\ \text{ms}^{-1} = \left(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 6\hat{\mathbf{k}}\right)\ \text{kg ms}^{-1}$

$\mathbf{p}_2 = 3\text{ kg}\ \left(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right)\ \text{ms}^{-1} = \left(6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right)\ \text{kg ms}^{-1}$

Resultant momentum is

$$\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2$$

$$= \left(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 6\hat{\mathbf{k}}\right) + \left(6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right)$$

$$= \left(8\hat{\mathbf{i}} + 7\hat{\mathbf{j}} - 3\hat{\mathbf{k}}\right)\ \text{kg ms}^{-1}$$

Total mass $(m) = 2 + 3 = 5$ kg. Therefore, the velocity of composite body is

$$\mathbf{v} = \frac{\mathbf{P}}{m} = \frac{1}{5}\left(8\hat{\mathbf{i}} + 7\hat{\mathbf{j}} - 3\hat{\mathbf{k}}\right)\text{ms}^{-1}$$

Hence the correct choice is (a).

49. Change in momentum $= m\ (\mathbf{v}_f - \mathbf{v}_i) = 2\text{ kg}\ \left(5\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + \hat{\mathbf{k}}\right)\text{ms}^{-1} - 2\text{ kg}\ \left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)\text{ms}^{-1} = 2\ \left(4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}}\right)\ \text{kg ms}^{-1}$.

$$\text{Average force} = \frac{\text{change in momentum}}{\text{time of impact}}$$

$$= \frac{2\left(4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}}\right)}{0.02\,\text{s}} = 100\left(4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}}\right)$$

Hence the correct choice is (d).

50. Total distance $= h + 2\ e^2\ h + 2\ e^4\ h + ...$

$$= h + 2\ e^2\ h\ (1 + e^2 + ...)$$

$$= h + \frac{2e^2h}{1 - e^2} = h\left(\frac{1 + e^2}{1 - e^2}\right)$$

$$= h\left[\frac{1 + \left(\frac{1}{3}\right)^3}{1 - \left(\frac{1}{3}\right)^2}\right] = \frac{5h}{4}$$

Hence the correct choice is (b).

51. Total time $= \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}} + 2e^2\sqrt{\frac{2h}{g}} + \cdots$

$$= \sqrt{\frac{2h}{g}} + \sqrt{\frac{2h}{g}}\left(e + e^2 + \cdots\right)$$

$$= \sqrt{\frac{2h}{g}} + \sqrt{\frac{2h}{g}}\,\frac{e}{(1-e)} = \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$$

$$= \sqrt{\frac{2h}{g}}\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right) = 2\sqrt{\frac{2h}{g}}$$

Hence the correct choice is (b).

52. Given $m_Q = p\, m_P$ and $v_Q = v_P/q$. From the principle of conservation of momentum, we have (since body P comes to rest after collision)

$$m_P\, v_P + m_Q\, v_Q = m_Q\, v$$

where v is the velocity of body Q after collision. Thus

$$m_P\, v_P + p\, m_P\, \frac{v_P}{q} = p\, m_P v.$$

which gives $$\frac{v}{v_P} = \frac{p+q}{pq} \qquad \text{(i)}$$

Now, the coefficient of restitution is given by

$$e = \frac{v}{v_p - v_Q} = \frac{v}{v_p - \frac{v_p}{q}}$$

which gives $$\frac{v}{v_P} = \frac{e}{q}\,(q-1) \qquad \text{(ii)}$$

Equating (i) and (ii), we get $e = \frac{p+q}{q(p-1)}$ which is choice (d).

53. Refer to Fig. 4.30. If sphere A is projected with velocity v, the time taken by it to strike B is equal to $\frac{\pi r}{v} = T$ or $\pi r = Tv$. Now, the coefficient of restitution is given by

$$e = \frac{v_B - v_A}{v}$$

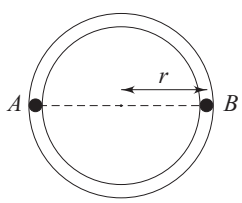


Fig. 4.30

where v_A and v_B are the velocities of A and B after the collision. Thus, $v_B - v_A = ev$.
The spheres travel with this relative velocity. It is clear that one will overtake the other after travelling a distance $= 2\pi r$.

$\therefore$ Time taken $= \frac{2\pi r}{v_B - v_A} = \frac{2\pi r}{ev} = \frac{2Tv}{ev} = \frac{2T}{e}$

(since $\pi r = Tv$).

Hence the correct choice is (c).

54. If mass m falss through a distance h, mass m rises up through the same distance h. Let v be the common velocity of the masses when this happens. Now, loss in PE = gain in KE, i.e.

$$Mgh - mgh = \frac{1}{2}\,(M+m)\,v^2$$

which gives $v = \sqrt{\frac{2gh(M-m)}{(M+m)}}$, which is choice (c).

55. When M has descended a distance h, loss of PE $= Mgh$. If v is the common velocity of the masses, gain in KE $= \frac{1}{2}\,(M+m)\,v^2$. Hence

$$\frac{1}{2}\,(M+m)\,v^2 = Mgh$$

or $$v = \sqrt{\frac{2Mgh}{(M+m)}}$$, which is choice (c).

56. Relative speed of man with respect to escalator $= (v-u)$.
$\therefore$ Actual displacement of man per second $= (v-u)$.
Hence, the actual displacement of man in going up the escalator of height h is $\frac{v\,h}{(v-u)}$. Therefore,

Work done $= mg \times \frac{v\,h}{(v-u)}$, which is choice (d)

57. Let the mass of the rope be M. If a small part of length x hangs through the hole, it weight

$$= \frac{Mx}{L}(g)$$

and its acceleration is $a = \frac{Mx}{L}\frac{g}{M} = \frac{xg}{L}$

$\therefore$ Force on the remaining part of length $(L-x)$ = mass of part of length $(L-x) \times$ acceleration (a)

$$= \frac{M}{L}\,(L-x) \times \frac{xg}{L} = \frac{Mg}{L^2}\,(Lx - x^2)$$

If the rope falls through a distance dx, the work done by gravity is

$$dW = \frac{Mg}{L^2}\,(Lx - x^2)\,dx$$

∴ Total work done is

$$W = \int_0^L dW = \frac{Mg}{L^2}\int_0^L \left(Lx - x^2\right) dx$$

$$= \frac{Mg}{L^2}\left|\frac{Lx^2}{2} - \frac{x^3}{3}\right|_0^L = \frac{MgL}{6}$$

Since the centre of mass of the rope falls through a distance $\frac{L}{2}$, decrease in PE = $\frac{MgL}{2}$. From the principle of conservation of energy, we have

Decrease in PE – Work done (W) = Increase in KE

or $$\frac{MgL}{2} - \frac{MgL}{6} = \frac{1}{2}Mv^2$$

which gives $v = \sqrt{\frac{4gL}{3}}$, which is choice (b).

58. Given $U = 6x + 8y$ joule and mass $m = 2$ kg. Force along x-axis is

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}(6x + 8y) = -6 \text{ newton}$$

Force along y-axis is

$$F_y = -\frac{dU}{dy} = -\frac{d}{dy}(6x + 8y) = -8 \text{ newton}$$

Therefore, the x and y components of acceleration are

$$a_x = \frac{F_x}{m} = \frac{-6}{2} = -3 \text{ ms}^{-2}$$

and $$a_y = \frac{F_y}{m} = \frac{-8}{2} = -4 \text{ ms}^{-2}$$

∴ Resultant acceleration

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-3)^2 + (-4)^2} = 5 \text{ ms}^{-2}$$

The x and y coordinates of the body at time t are

$$x = x_0 + \frac{1}{2}a_x t^2 = 6 - \frac{1}{2}\times 3 \times t^2$$

$$= \left(6 - \frac{3}{2}t^2\right) \text{ metre}$$

and $$y = y_0 + \frac{1}{2}a_y t^2 = 4 - \frac{1}{2}\times 4 \times t^2$$

$$= (4 - 2t^2) \text{ metre}$$

The body will cross the y-axis when $x = 0$, i.e. at time t given by $\left(6 - \frac{3}{2}t^2\right) = 0$ or $t = 2$ s. Hence the correct choice is (b).

59. $v_x = a_x t = -3 \text{ ms}^{-2} \times 2 \text{ s} = -6 \text{ ms}^{-1}$

$v_y = a_y t = -4 \text{ ms}^{-2} \times 2 \text{ s} = -8 \text{ ms}^{-1}$

∴ $$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-6)^2 + (-8)^2} = 10 \text{ ms}^{-1}$$

Hence the correct choice is (c).

60. Initial momentum (p) = momentum of bullet + momentum of block = $mv + 0 = mv$. From the principle of conservation of momentum, final momentum of bullet + block system of mass ($M + m$) = Initial momentum p. Now

$$\text{KE} = \frac{p^2}{2\times(M\times m)} = \frac{m^2v^2}{2(M+m)}$$

Hence the correct choice is (d).

61. A ball dropped from a height h_1 on reaching the planet's surface will have a velocity given by

$$v_1 = \sqrt{2gh_1}$$

Let v_2 be the velocity with which the ball bounces. It will attain a height h_2 given by

$$v_2 = \sqrt{2gh_2}$$

∴ $$\frac{v_2}{v_1} =$$

or $$1 - \frac{v_2}{v_1} = 1 - 0.6 \text{ or } \frac{v_1 - v_2}{v_1} = 0.4 = \frac{2}{5}$$

Hence the correct choice is (b).

62. Let m' and M be the masses of the lighter and heavier fragments respectively. Since the particle is moving along the x-axis, the y-component of momentum will be zero immediately after and before explosion, i.e.

$$m'v_y + MV_y = 0$$

where v_y and V_y are the velocities of the lighter and heavier fragments respectively immediately after explosion. Thus

$$V_y = -\left(\frac{m'}{M}\right)v_y = -\left(\frac{m/4}{3m/4}\right)v_y = -\frac{1}{3}v_y$$

Since y = + 15 cm, the direction of v_y is along the positive y-axis and that of V_y will be along the negative y-axis. An instant later (say, at time t), it is given that

$$y = 15 \text{ cm} = v_y t$$

∴ $$Y = V_y t = -\frac{1}{3}v_y t = -\frac{1}{3}y$$

$$= -\frac{1}{3}\times 15 \text{ cm} = -5 \text{ cm}$$

Hence the correct choice is (a).

63. The potential energy of the particle is given by

$$U = -\int F dx = -\int (-kx + ax^3)\, dx$$

or $$U = k\frac{x^2}{2} - a\frac{x^4}{4} = \frac{x^2}{4}\left(2k - ax^2\right) \qquad \text{(i)}$$

From Eq. (i) it follows that $U = 0$ at two values of x which are $x = 0$ and $x = \sqrt{2k/a}$. Hence graphs (b) and (c) are not possible. Also U is maximum or minimum at a value of x given by $\frac{dU}{dx} = 0$, i.e.

$$0 = \frac{d}{dx}\left(\frac{kx^2}{2} - \frac{ax^4}{4}\right)$$

$$= kx - ax^3 = x(k - ax^2)$$

or $$x = \sqrt{k/a}\,.$$

At this value of x, U is maximum if $\frac{d^2U}{dx^2} < 0$,

Now $$\frac{d^2U}{dx^2} = \frac{d}{dx}(kx - ax^3) = k - 3ax^2.$$

At $x = \sqrt{k/a}$,

$$\frac{d^2U}{dx^2} = k - 3a\,\frac{k}{a} = k - 3k = -2k,$$

which is negative.

Hence U is maximum at $x = \sqrt{k/a}$.

Hence graph (a) is also not possible. Also U is negative for $x > \sqrt{2k/a}$. Therefore, the correct graph is (d).

64. The velocity of the body at time t is given by

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(\frac{t^2}{4}\right) = \frac{t}{2}$$

$\therefore$ At $t = 0$, $v = u = 0$ and at $t = 2$ s, $v = 1$ ms^{-1}, Now, work done = increase in KE

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}mv^2 - 0$$

$$= \frac{1}{2}mv^2 = \frac{1}{2} \times 6 \times (1)^2 = 3 \text{ J}$$

65. Work done = increase in potential energy in (i) raising the weight 150 N of the ladder through a height 1 m and (ii) raising a weight 40 N through 2.5 m

$$= 150 \text{ N} \times 1 \text{ m} + 40 \text{ N} \times 2.5 \text{ m}$$

$$= 250 \text{ Nm} = 250 \text{ J}$$

Hence the correct choice is (b).

66. Friction force = $\mu\, mg = 0.2 \times 5 \times 10 = 10$ N. Effective force F = applied force – frictional force = 25 – 10 = 15 N. Kinetic energy = work done by force F in pulling the body through a distance S (= 10 m) = $15 \times 10 = 150$ J, which is choice (b).

67. The retardation is given by (see Fig. 4.31)

$$a = g(\mu \cos\theta + \sin\theta) \qquad \text{(i)}$$

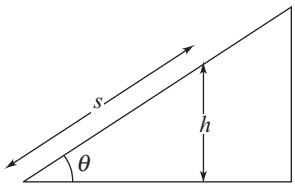


Fig. 4.31

Let u be the initial velocity of the body. If it is stopped after moving a distance s up the plane, then

$$u^2 = 2as$$

$\therefore$ Kinetic energy = $E = \frac{1}{2}\,mu^2$

$$= \frac{1}{2}\,m \times 2as = mas \qquad \text{(ii)}$$

Now, work done against friction is

$$W = \text{gain in PE} = mgh$$

It is clear from the figure that $h = s \sin\theta$. Therefore,

$$W = mgs \sin\theta \qquad \text{(iii)}$$

From (i), we have

$$g = \frac{a}{(\mu\cos\theta + \sin\theta)} \qquad \text{(iv)}$$

Also $$\mu = \tan\theta = \frac{\sin\theta}{\cos\theta} \quad \text{or} \quad \sin\theta = \mu\cos\theta \qquad \text{(v)}$$

Using (iv) and (v) in (iii), we have

$$W = \frac{mas(\mu\cos\theta)}{(\mu\cos\theta + \sin\theta)} \qquad \text{(vi)}$$

Using (ii) in (vi), we get

$$W = \frac{\mu\, E\cos\theta}{(\mu\cos\theta + \sin\theta)}\,,$$ which is choice (d).

68. The total distance travelled is

$$S = h + 2e^2 h + 2e^4 h + 2e^6 h + \ldots$$

$$= h + 2h(e^2 + e^4 + e^6 + \ldots)$$

$$= h + 2h\left(\frac{e^2}{1 - e^2}\right)$$

$$= h\left[1+\frac{2e^2}{1-e^2}\right]=\frac{h(1+e^2)}{(1-e^2)}$$

Hence the correct choice is (a).

69. Work done by the force is

$$W=\int f dx \int_0^4 (3x+4)dx$$

$$=\left|\frac{3x^2}{2}+4x\right|_0^4 = 40 \text{ J}$$

From workenergy principle, work done = change in kinetic energy.

$$W=\Delta k=\frac{1}{2}m\,(v^2-u^2)$$

$$40=\frac{1}{2}\times 10\times(v^2-2^2)$$

$$\Rightarrow \quad v=2\sqrt{3}\ \text{ms}^{-1},\ \text{which is choice (c)}$$

70. $$F=-\frac{dU}{dx}=-\frac{d}{dx}\left[3(x-1)-(x-3)^3\right]$$

$$=-3+3\,(x-3)^2$$

$\therefore$ Acceleration is

$$A=\frac{F}{m}=\frac{-3+3\,(x-3)^2}{3}$$

$$=(x-3)^2-1 \qquad (1)$$

From Eq. (1), it follows that $A=0$ when $(x-3)^2-1=0$ which gives $x=2$m and $x=4$m. Also A is minimum $=-1$ ms^{-2} at $x=3$m. Hence correct graphs is (c).

71. Let $+mv$ be the momentom of the bullet before impact. Then momentum of the block $=-MV$. The total momentum of the system before impact $=(mv-MV)$. Since the block + bullet system is brought to rest after impact, its momentum after impact is zero. From conservation of momentum,

$mv-MV=0 \Rightarrow v=\dfrac{MV}{m}$, which is choice (c).

72. Refer to Fig 4.32.

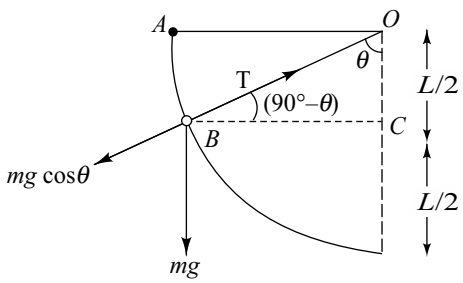


Fig. 4.32

The equation of motion when the ball is at position B is

$$\frac{mv^2}{L}=T-mg\cos\theta \qquad (1)$$

Where v is the speed of the ball at point B. From conservation of energy,

Total energy at A = total energy at B

$$mgL+0=\frac{1}{2}mv^2+mg\left(\frac{L}{2}\right)$$

$$\Rightarrow \quad mv^2=mgL \qquad (2)$$

Using (2) in (1), we have

$$mg=T-mg\cos\theta$$

$$\Rightarrow \quad T=mg\,(1+\cos\theta)$$

In triangle OBC

$$\sin(90^0-\theta)=\frac{OC}{OB}=\frac{L/2}{L}=\frac{1}{2}$$

$$\Rightarrow \quad \cos\theta=\frac{1}{2}$$

$$\therefore \quad T=mg\left(1+\frac{1}{2}\right)=\frac{3}{2}\,mg$$

So the correct choice is (c).

73. Let v be the speed of the ball just before impact. From conservation of energy,

total energy at A = total energy at B

$$mg\,L+0=0+\frac{1}{2}mu^2$$

$$\Rightarrow \quad u^2=2gL$$

Since the collision is elastic, the ball rebounds after the impact with a speed v given by

$$v=\left(\frac{M-m}{M+m}\right)u$$

$$=\left(\frac{3m-m}{3m+m}\right)u=\frac{u}{2}$$

If the ball to a height h, then from the conservation of energy,

total energy at C = total energy at B

$$mg\,h+0=0+\frac{1}{2}mv^2$$

$$\Rightarrow \quad h=\frac{v^2}{2g}=\frac{u^2}{8g} \qquad \left(\because v=\frac{u}{2}\right)$$

$$= \frac{2gL}{8g} \qquad (\because u^2 = 2gL)$$

$$= \frac{L}{4}$$

So that correct choice is (d).

74. Given $x = \frac{kt^3}{3}$

$$v = \frac{dx}{dt} = kt^2$$

$$a = \frac{dv}{dt} = 2kt$$

$$F = ma = 2mkt$$

$\therefore$ Work done in interval $t = 0$ to $t = 2$s *is*

$$W = \int F dx$$

$$= 2mk\int_0^2 t dx$$

$$= 2mk\int_0^2 t\frac{dx}{dt}dt$$

$$= 2mk\int_0^2 tv\, dt$$

$$= 2mk^2\int_0^2 t^3\, dt \qquad (\because \quad v = 2kt)$$

$$= 8mk^2$$

So the correct choice is (c)

75. If A is the cross-sectional area of the pipe, the volume of water flowing a small distance Δx in time $\Delta t = A\Delta x$. The Volume flowing per second $= \frac{A\Delta x}{\Delta t} = Av$, Where v is the speed of water. Mass of water flowing per second $= \rho Av$ where ρ is the density of water. Therefore, increase in kinetic energy per second (which is power)

$$= \frac{1}{2}mv^2$$

$$= \frac{1}{2}\rho Av \times v^2$$

or $$P = \frac{1}{2}\rho Av^3$$

In order to doube the mass of water flowing per second, v must be doubled. Since $P \propto v^3$, power P must be increased $(2)^3 = 8$ times. So the correct choice is (c).

76. $P = Fv = mav = m\frac{dv}{dt} \times v$

$$\Rightarrow \quad v\frac{dv}{dt} = \frac{P}{m}$$

$$\Rightarrow \quad v\frac{dv}{dx} \times \frac{dx}{dt} = \frac{P}{m}$$

$$\Rightarrow \quad v^2\frac{dv}{dx} = \frac{P}{m} \qquad \left(\because \frac{dx}{dt} = v\right)$$

$$\Rightarrow \quad v^2\, dv = \frac{P}{m}dx$$

Integrating, we have

$$\int_o^v v^2 dv = \frac{P}{m}\int_o^x dx \qquad (\because P = \text{constant})$$

$$\Rightarrow \quad \frac{v^3}{3} = \frac{Px}{m}$$

$$\Rightarrow \quad v = \left(\frac{3Px}{m}\right)^{1/3}$$

So the correct choice is (a).

77. Given $x = at^4 + bt + c$.

$$v = \frac{dx}{dt} = 4\, at^3 + \text{b}$$

$$\frac{dv}{dt} = 12\, at^2$$

$$P = \frac{dW}{dt}$$

$$\Rightarrow \quad Fv = \frac{dW}{dt}$$

$$\Rightarrow \quad m\frac{dv}{dt}\, v = \frac{dW}{dt} \qquad \left(\because \quad acc. = \frac{dv}{dt}\right)$$

or $$\frac{dW}{dt} = m \times 12at^2 \times (4\, at^3 + b)$$

$$= 48\, ma^2\, t^5 + 12\, mabt^2$$

$$\Rightarrow \quad dW = 48\, ma^2\, t^5\, dt + 12\, mabt^2\, dt$$

Integrating, we get

$$W = 48ma^2\int_0^1 t^5 dt + 12mab\int_0^1 k^2 dt$$

$$= 8\, ma^2 + 4\, mab$$

$w = 4ma\,(b + 2a)$, which is choice (d).

ALTERNATIVE METHOD

From work-energy principle,

work done = Change in K.E.

= Final K.E. – Initial K.E.

$$= \frac{1}{2}\ mv^2 \text{ at } t = 1s - \frac{1}{2}\ mv^2 at\ t = 0$$

$$= \frac{1}{2}\ m(4a + b)^2\ - \frac{1}{2}\ mb^2$$

$$= \frac{1}{2}m\left[(4a+b)^2 - b^2\right]$$

$$= 4ma(2a + b)$$

78. Given $\frac{mv^2}{2} = \frac{kx^2}{2}$

$$\Rightarrow \quad mv^2 = kx^2 \qquad (1)$$

Differentiating w.r.t. time t,

$$2mv\frac{dv}{dt} = 2kx\frac{dx}{dt} = 2\ kx\ v$$

$$\Rightarrow \quad m\frac{dv}{dt} = kx$$

$$\Rightarrow \quad m\ a_t = kx$$

where a_t is tangential acceleration. Therefore, tangential force is

$$F_t = kx$$

Radial (centripetal) force is

$$F_r = \frac{mv^2}{r} = \frac{kx^2}{r} \qquad \text{[use Eq. (1)]}$$

$$\therefore \quad \text{Net force} = \sqrt{F_t^2 + F_r^2}$$

$$= \left(k^2x^2 + \frac{k^2x^4}{r^2}\right)^{1/2}$$

$$= kx\left(1 + \frac{x^2}{r^2}\right)^{1/2}$$

So the correct choice is (c).

79. $K_i = \frac{1}{2}\ mu^2$. Let v be the velocity of the block when it has lost $\frac{1}{4}$ th of its kinetic energy, then its new K.E. is $3K_i / 4$. Therefore,

$$\frac{1}{2}\ mu^2 = \frac{3}{8}\ mv^2 \Rightarrow v = \frac{\sqrt{3}u}{2}$$

Frictional force is $f = \mu mg$. Therefore, retardation $a = -\frac{f}{m} = -\mu g$. From $v = u + at$. we get

$$\frac{\sqrt{3}u}{2} = u - \mu gt$$

$$\Rightarrow \quad \mu = \frac{u}{2gt}\left(2 - \sqrt{3}\right), \text{ which is choice (d).}$$

80. Let m be the mass of each ball. Let u_1 and u_2 be the velocities A and B before collision and v_1 and v_2 are collision. Then

$$u_1 = \frac{p}{m} \text{ and } u_2 = 0 \text{ (given)}$$

Impulse = change in momentum. It is given that

I = change in momentum of B.

$$= mv_2 - 0$$

$$\Rightarrow \quad v_2 = \frac{I}{m}$$

Therefore,

$$v_1 = \frac{p - I}{m}$$

Now $$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$= \frac{\left(\frac{I}{m} - \frac{p - I}{m}\right)}{\left(\frac{p}{m} - 0\right)} = \frac{2I}{p} - 1$$

So the correct choice is (b).

81. Net height gained by the man = $(l - h)$. Therefore, gain in potential energy of man = $mg(l - h)$. The weight mg of the man pulls the balloon down by a height h. Hence work done by this force on the balloon = mgh. This is equal to the gain in potential energy of the balloon. Therefore, the work done by man in reaching the balloon is

W = increase in P.E. of man + increase in P.E. of balloon = $mg\ (l - h) + mgh = mgl$. So the correct choice is (d).

82. For a perfectly elastic one-dimensional collision, the velocity of the ball after the colision is

$$v_1 = \left(\frac{m - M}{m + M}\right)u_1 + \frac{2M(-u_2)}{m + M}$$

Since $M >> m$,

$$v_1 = -u_1 - 2u_2 = -(u_1 + 2u_2).$$

The magnitude of v_1 is

$$|\ v_1\ | = u_1 + 2u_2$$

SECTION 2 Multiple Choice Questions Based on Passage

So the correct choice is (c).

Questions 1 to 2 are based on the following passage.

Passage I

A light rod of length L having a body of mass M attached to its end hangs vertically. It is turned through 90° so that it is horizontal and then released.

1. The centripetal acceleration when the rod makes an angle θ with the vertical is
   (a) $g \cos \theta$ (b) $2g \cos \theta$
   (c) $g \sin \theta$ (d) $2g \sin \theta$
2. The tension in the rod when it makes an angle θ with the vertical is
   (a) $Mg \cos \theta$ (b) $2\ Mg \cos \theta$
   (c) $3\ Mg \cos \theta$ (d) zero

Solutions

1. The rod is released from the horizontal position OA. Let OB be the position of the rod when it makes an angle θ with the vertical. (Fig. 4.33).

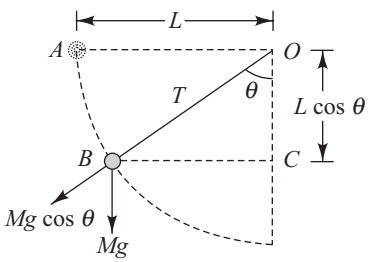


Fig. 4.33

The loss of PE when the body falls from A to $B = Mg \times OC = MgL \cos \theta$. If v is the velocity of the body at B, then

$$\frac{1}{2} Mv^2 = MgL \cos \theta \text{ or } v^2 = 2gL \cos \theta \qquad (1)$$

$$\text{centripetal acceleration} = \frac{v^2}{L} = \frac{2gL\cos\theta}{L} = 2g \cos \theta,$$

which is choice (b).

2. The centripetal force when the body is at B is

$$F_c = \frac{Mv^2}{L}$$

Thus, we have

$$T - Mg \cos \theta = \frac{Mv^2}{L} \qquad (2)$$

Using (1) in (2), we get

$$T - Mg \cos \theta = \frac{M}{L} \times 2gL \cos \theta = 2\ Mg \cos \theta$$

or $$T = 3\ Mg \cos \theta$$

Thus the correct choice is (c).

Questions 3 to 5 are based on the following passage.

Passage II

A small roller coaster starts at point A with a speed u on a curved track as shown in Fig. 4.34. The friction between the roller coaster and the track is negligible and it always remains in contact with the track.

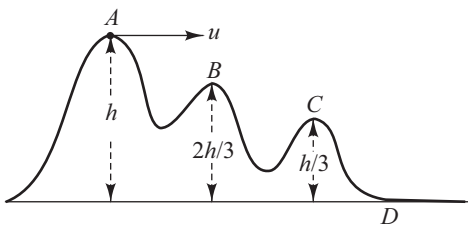


Fig. 4.34

3. The speed of the roller coaster at point B on the track will be
   (a) $(u^2 + gh)^{1/2}$ (b) $\left(u^2 + \frac{2gh}{3}\right)^{1/2}$
   (c) $(u^2 + 2gh)^{1/2}$ (d) $\left(u^2 + \frac{3gh}{2}\right)^{1/2}$
4. The speed of the roller coaster at point C on the track will be
   (a) $\left(u^2 + \frac{gh}{3}\right)^{1/2}$ (b) $\left(u^2 + \frac{2gh}{3}\right)^{1/2}$
   (c) $\left(u^2 + \frac{4gh}{3}\right)^{1/2}$ (d) $(u^2 + 2gh)^{1/2}$

5. The speed of the roller coaster at point D on the track will be
   (a) $(u^2 + gh)^{1/2}$ (b) $(u^2 + 2gh)^{1/2}$
   (c) $(u^2 + 3gh)^{1/2}$ (d) $(u^2 + 4gh)^{1/2}$



Solutions

3. Total energy at A = KE + PE = $\frac{1}{2}mu^2 + mgh$. If v_b is the speed at point B, the total energy at $B = \frac{1}{2}mv_b^2 + mg(2h/3)$. From the principle of conservation of energy, we have
   $\frac{1}{2}mu^2 + mgh = \frac{1}{2}mv_b^2 + \frac{2mgh}{3}$ which gives
   $v_b = \left(u^2 + \frac{2gh}{3}\right)^{1/2}$, which is choice (b).
4. Similarly, the speed at point C is given by
   $\frac{1}{2}mu^2 + mgh = \frac{1}{2}mv_c^2 + \frac{mgh}{3}$ which gives
   $v_c = \left(u^2 + \frac{4gh}{3}\right)^{1/2}$, which is choice (c).
5. At point D, the energy is entirely kinetic. If the speed of the roller coaster at point D is v_d, then we have
   $\frac{1}{2}mv_d^2 = mgh + \frac{1}{2}mu^2$
   or $v_d = (u^2 + 2gh)^{1/2}$, which is choice (b).

Questions 6 to 8 are based on the following passage.

Passage III

The displacement x of a particle moving in one dimension, under the action of a constant force is related to time t by the equation

$$t = \sqrt{x} + 3$$

where x is in metre and t is in second.

6. The displacement of the particle when its velocity is zero is
   (a) zero (b) 1 m
   (c) 2 m (d) 3 m
7. The acceleration of the particle
   (a) increases with time
   (b) decreases with time
   (c) increases with time up to $t = 3$ s and then decreases with time.
   (d) remains constant at 2 ms^{-2}.
8. The work done by the force in first 6 s is
   (a) 1 J (b) 3 J
   (c) 6 J (d) zero



Solutions

6. Given $t = \sqrt{x} + 3$ or $\sqrt{x} = t - 3$ or
   $$x = (t-3)^2 \quad (1)$$
   Differentiating (1) with respect to t, we get
   $$\frac{dx}{dt} = 2(t-3)$$
   or $$v = 2(t-3) \quad (2)$$
   From (2) it follows that $v = 0$ at $t = 3$ s. Using $t = 3$ s in (1), we get $x = 0$. Thus, the displacement of the particle is zero when its velocity is zero. Thus, the correct choice is (a).
7. From Eq. (2), we have
   $$a = \frac{dv}{dt} = \frac{d}{dt}[2(t-3)] = 2 \text{ ms}^{-2}.$$
   Hence the correct choice is (d).
8. From Eq. (2), the initial velocity, i.e., velocity at $t = 0$ is
   $$v_0 = 2(0-3) = -6 \text{ ms}^{-1}$$
   Final velocity, i.e., velocity at $t = 6$ s is
   $$v = 2(6-3) = 6 \text{ ms}^{-1}$$
   Work done = final KE – initial KE
   $$= \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m(v^2 - v^2{}_0)$$
   $$= \frac{1}{2}m[(6)^2 - (-6)^2] = 0, \text{ which is choice (d).}$$

Questions 9 to 12 are based on the following passage.

Passage IV

The kinetic energy of a particle moving along a circle of radius R depends on distance (s) as $K = as^2$ where a is a constant.

9. The centripetal force is given by
   (a) $\frac{as^2}{2R}$ (b) $\frac{as^2}{R}$
   (c) $\frac{2as^2}{R}$ (d) $\frac{4as^2}{R}$

10. The speed of the particle around the circle is

(a) $2s\left(\frac{a}{m}\right)^{1/2}$ (b) $s\left(\frac{a}{m}\right)^{1/2}$

(c) $s\left(\frac{2a}{m}\right)^{1/2}$ (d) $s\left(\frac{a}{2m}\right)^{1/2}$

11. The tangential force acting on the particle is

(a) mas (b) $2mas$

(c) as (d) $2as$

12. The net force acting on the particle is

(a) $2as\left(1+\frac{s}{R}\right)$ (b) $as\left(1+\frac{s^2}{R^2}\right)^{1/2}$

(c) $2as\left(1+\frac{s^2}{R^2}\right)^{1/2}$ (d) zero

Solutions

9. Given KE $(K) = \frac{1}{2}mv^2 = as^2$. Therefore, the centripetal force is

$$f_c = \frac{mv^2}{R} = \frac{2\times\left(\frac{1}{2}mv^2\right)}{R} = \frac{2as^2}{R},$$

which is choice (c).

10. The speed v of the particle around the circle is given by

$$\frac{1}{2}mv^2 = as^2 \quad \text{or} \quad v = s\left(\frac{2a}{m}\right)^{1/2}$$

Hence the correct choice is (c).

11. The tangential acceleration is

$$a_t = \frac{dv}{dt} = \frac{d}{dt}\left[s\left(\frac{2a}{m}\right)^{1/2}\right] = \left(\frac{2a}{m}\right)^{1/2}\frac{ds}{dt}$$

But $\frac{ds}{dt} = v = s\left(\frac{2a}{m}\right)^{1/2}$.

Therefore,

$$a_t = \left(\frac{2a}{m}\right)^{1/2} \times s\left(\frac{2a}{m}\right)^{1/2} = \frac{2as}{m}$$

$\therefore$ Tangential force is $f_t = ma_t$

$$= m \times \frac{2as}{m} = 2as, \text{ which is choice (d).}$$

12. Net force acting on the particle is

$$f = (f_c^2 + f_t^2)^{1/2}$$

$$= \left[\left(\frac{2as^2}{R}\right)^2 + (2as)^2\right]^{1/2}$$

$$= 2as\left[1+\frac{s^2}{R^2}\right]^{1/2}$$

Thus the correct choice is (c).

Questions 13 to 15 are based on the following passage.

Passage V

A conical pendulum consists of a string of length L fixed at one end carrying a body of mass m at the other end. The mass is revolved in a circle in the horizontal plane about a vertical axis passing through the fixed end of the string. The angular frequency of revolution of the body is ω. The string makes an angle θ with the vertical axis.

13. The tension in the string is

(a) $\frac{m\omega^2}{L}$ (b) $\frac{L\omega^2}{m}$

(c) $m\omega^2 L$ (d) $m\omega L^2$

14. The angle of inclination of the string with the vertical is given by

(a) $\cos\theta = \frac{g}{\omega^2 L}$ (b) $\sin\theta = \frac{g}{\omega^2 L}$

(c) $\cos\theta = \frac{\omega^2 L}{g}$ (d) $\sin\theta = \frac{\omega^2 L}{g}$

15. The linear speed of the body is

(a) ωL (b) $\omega L \sin\theta$

(c) $\omega L \cos\theta$ (d) $\omega L \tan\theta$

Solutions

13. Let T be tension in the string. Figure 4.35 shows the forces acting on the system. Tension T can be resolved into two mutually perpendicular components. The horizontal component $T\sin\theta$ provides the centripetal force for circular motion and the vertical component $T\cos\theta$ balances the weight mg.

Thus

$$T\cos\theta = mg \qquad (1)$$

and $$T\sin\theta = \frac{mv^2}{r} = m\omega^2 r$$

But $r = L\sin\theta$. Therefore,

$$T\sin\theta = m\omega^2 L\sin\theta \text{ or } T = m\omega^2 L \qquad (2)$$

Hence the correct choice is (c).

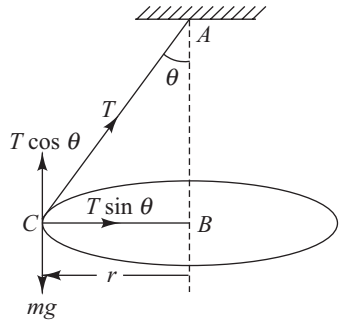


Fig. 4.35

14. From (1), we have $\cos\theta = \dfrac{mg}{T}$ (3)

    Using (2) in (3), we get

    $$\cos\theta = \frac{mg}{m\omega^2 L} = \frac{g}{\omega^2 L}$$, which is choice (a).

15. Linear velocity is $v = \omega r = \omega L \sin\theta$, which is choice (b).

3 SECTION

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

1. **Statement-1**

   A simple pendulum of length l is displaced from its mean position O to position A so that the string makes an angle θ_1 with the vertical and then released. If air resistances is neglected, the speed of the bob when the string makes an angle θ_2 with the vertical is $v = \sqrt{2gl(\cos\theta_2 - \cos\theta_1)}$.

   Statement-2

   The total momentum of a system is conserved if no external force acts on it.

2. **Statement-1**

   A uniform rod of mass m and length l is held at an angle θ with the vertical. The potential energy of the rod in this position is $\frac{1}{2}\ mg\ l \cos\theta$.

   Statement-2

   The entire mass of the rod can be assumed to be concentrated at its centre of mass.

3. **Statement-1**

   A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

   Statement-2

   The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

4. **Statement-1**

   A man carrying a bucket of water and walking on a rough level road with a uniform velocity does no work while carrying the bucket.

   Statement-2

   The work done on a body by a force **F** in giving it a displacement **S** is defined as

   $$W = \mathbf{F} \cdot \mathbf{S} = FS\cos\ \theta$$

   where θ is the angle between vectors **F** and **S**.

5. **Statement-1**

   A crane P lifts a car up to a certain height in 1 min. Another crane Q lifts the same car up to the same height in 2 min. Then crane P consumes two times more fuel than crane Q.

Statement-2

Crane P supplies two times more power than crane Q.

6. **Statement-1**

Two inclined frictionless tracks of different inclinations θ_1 and θ_2 meet at A from where two blocks P and Q of different masses m_1 and m_2 are allowed to slide down from rest, one on each track as shown in Fig. 4.36. Then blocks P and Q will reach the bottom with the same speed.

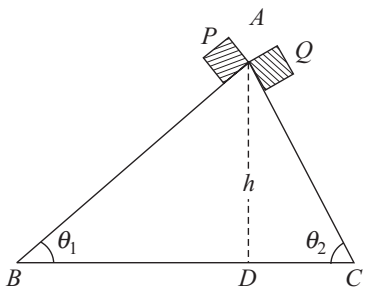


Fig. 4.36

Statement-2

Blocks P and Q have equal accelerations down their respective tracks.

7. **Statement-1**

In Q.6 above, block P will take a longer time to reach the bottom than block Q.

Statement-2

Block Q has a greater acceleration down the track than block P.

8. **Statement-1**

Comets move around the sun in highly elliptical orbits. The work done by the gravitational force of the sun on a comet over a complete orbit is zero.

Statement-2

The gravitational force is conservative.

9. **Statement-1**

The total energy of a system is always conserved irrespective of whether external forces act on the system.

Statement-2

If external forces act on a system, the total momentum and energy will increase.

10. **Statement-1**

The rate of change of the total linear momentum of a system consisting of many particles is proportional to the vector sum of all the internal forces due to inter-particle interactions.

Statement-2

The internal forces can change the kinetic energy of the system of particles but not the linear momentum of the system.

11. **Statement-1**

An elastic spring of force constant k is stretched by a small length x. The work done in extending the spring by a further length x is $2\,kx^2$.

Statement-2

The work done in extending an elastic spring by a length x is proportional to x^2.

12. **Statement-1**

Two identical balls B and C lie on a horizontal smooth straight groove so that they are touching. A third identical ball A moves at a speed v along the groove and collides with B (see Fig. 4.37). If the collisions are perfectly elastic, then after the collision, balls A and B will come to rest and ball C moves with velocity v to the right.



Fig. 4.37

Statement-2

In an elastic collision, linear momentum and kinetic energy are both conserved.

13. **Statement-1**

Two bodies A and B of masses m and $2\,m$ respectively are placed on a smooth floor. They are connected by a spring. A third body C of mass m moves with a velocity u and collides elastically with A as shown in Fig. 4.38. At a certain instant t_0 after the collision, it is found that the velocities of A and B are the same $= u/3$.

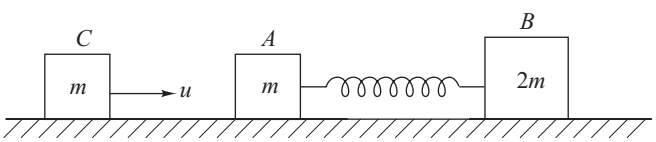


Fig. 4.38

Statement-2

In an elastic collision, the kinetic energy of the system is conserved.

14. **Statement-1**

In an inelastic collision between two bodies, the total energy does not change after the collision but the kinetic energy of the system decreases.

Statement-2

The loss of kinetic energy appears as heat in the system.

15. **Statement-1**

In a collision between two bodies, each body exerts an equal and opposite force on the other at each instant of time during the collision.

Statement-2

The total energy of the system is conserved.

16. **Statement-1**

The term 'collision' between two bodies does not necessarily mean that the two bodies actually strike against each other.

Statement-2

In physics, a collision is said to take place if the one body influences the motion of the other.

17. **Statement-1**

In an inelastic collision, the two colliding bodies stick to each other after the collision and move with a common velocity.

Statement-2

There is a loss of total kinetic energy in an inelastic collision.

18. **Statement-1**

In a collision between two bodies, the linear momentum of each body remains constant.

Statement-2

If no external force acts, the total linear momentum of a system is conserved.

19. **Statement-1**

In an elastic collision between two bodies, the energy of each body is conserved.

Statement-2

The total energy of an isolated system is conserved.

20. **Statement-1**

The total energy of a system is always conserved irrespective of whether external forces act on the system.

Statement-2

The total energy of an isolated system is always conserved.

21. **Statement-1**

A body P of mass M moving with speed u collides head-on and elastically with a body Q of m initially at rest. If $m \ll M$, body Q will have a maximum speed equal to 2 u after the collision.

Statement-2

In an elastic collision, the momentum and kinetic energy are both conserved.

Solutions

1. The correct choice is (b). It is clear from Fig. 4.39 that $PQ = l \cos \theta_1$ and $PR = l \cos \theta_2$. Therefore, $h_1 = l - l \cos \theta_1 = l(1 - \cos \theta_1)$ and $h_2 = l(1 - \cos \theta_2)$.

Let m be the mass of the bob and v be its speed when it reaches position B. Then, from the principle of conservation of energy, K.E. at B = loss of P.E. as the bob moves from A to B.

Hence

$$\frac{1}{2} mv^2 = mgh_1 - mgh_2$$

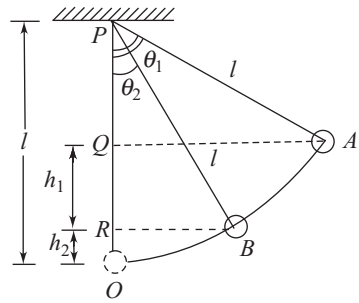


Fig. 4.36

$$= mg[l(1 - \cos \theta_1) - l(1 - \cos \theta_2)]$$

$$= mg\, l\, (\cos \theta_2 - \cos \theta_1)$$

$$\Rightarrow \qquad v = \sqrt{2gl(\cos\theta_2 - \cos\theta_1)}$$

2. The correct choice is (a). Let C be the centre of mass of the rod AB so that $AC = l/2$. Let h be the height of C above the ground. In triangle ACD, we have $CD = AC \sin (90° - \theta)$ (see Fig. 4.40).

Or $h = \frac{l}{2} \cos \theta$. Since the entire mass of the rod can be assumed to be concentrated at the centre of mass, therefore, potential energy = work done to raise the rod from horizontal position on the ground to the position shown in the figure = $mgh = \frac{l}{2} mgl \cos \theta$.

3. Statement-1 is true. The decrease in mechanical energy is smaller when the block is made to go up on the inclined surface because some part of the kinetic energy is converted into gravitational potential energy. Statement-2 is false. The coefficient of friction does not depend on the angle of inclination of the plane. Hence the correct choice is (c).

4. The correct choice is (a). Since the velocity is uniform, the man exerts no net force on the bucket in the direction of motion. The only force he exerts on the bucket is against gravity (to overcome) the weight mg of the bucket) and this force is perpendicular to the displacement (i.e. $\theta = 90°$). Hence $W = FS \cos 90° = 0$.

5. The two cranes do the same amount of work = mgh. Hence they consume the same amount of fuel. Crane P does the same amount of work in half the time. Hence crane P supplies two times more power than crane Q. Thus the correct choice is (d).

6. The acceleration of blocks P and Q respectively are

$$a_1 = \frac{m_1 g \sin\theta_1}{m_1} = g \sin\theta_1$$

and $$a_2 = \frac{m_2 g \sin\theta_2}{m_2} = g \sin\theta_2$$

Since $\theta_2 > \theta_1$; $a_2 > a_1$. The potential energy of block P at $A = m_1gh$. When it reaches the bottom B, its kinetic energy is $\frac{1}{2} m_1 v_1^2$ where $v_1$is its speed when it reaches B. Now P.E at A = K.E. at B. Hence

$m_1gh = \frac{1}{2} m_1 v_1^2 \Rightarrow v_1 = \sqrt{2gh}$.

Similarly $m_2gh = \frac{1}{2} m_2 v_2^2 \Rightarrow v_2 = \sqrt{2gh} = v_1$.

Hence the correct choice is (c).

7. The correct choice is (a). If t_1 and t_2 are the times taken by P and Q to reach the bottom, then

$$v_1 = u_1 + a_1 t_1 = a_1 t_1 \qquad (\because u_1 = 0)$$

and $$v_2 = u_2 + a_2 t_2 = a_2 t_2 \qquad (\because u_2 = 0)$$

Now $v_1 = v_2$. Hence $a_1 t_1 = a_2 t_2$.

Thus $$\frac{t_1}{t_2} = \frac{a_2}{a_1}$$

Since $a_2 > a_1$; $t_1 > t_2$.

8. The correct choice is (a). For a conservation force, the work done in moving a body from one point to another does not depend on the nature of the path and the work done over a closed path is zero, irrespective of the nature of the path.
9. Statement-1 is false; the total energy of an isolated system is conserved. Statement-2 is true. Hence the correct choice is (d).
10. Statement-1 is false and Statement-2 is true. The rate of change of momentum is proportional to the net external force acting on the system. Hence the correct choice is (d).
11. The correct choice is (d). Potential energy stored in the spring when it is extended by x is $U_1 = \frac{1}{2} kx^2$

Potential energy stored in the spring when it is further extended by x is

$$U_2 = \frac{1}{2} k(x + x)^2 = 2kx^2$$

$\therefore$ Work done = gain in potential energy = $U_2 - U_1$ = $2\,kx^2 - \frac{1}{2} kx^2 = \frac{3}{2} kx^2$

12. The correct choice is (a). Linear momentum will be conserved if A comes to rest and B and C move to the right with a velocity $v/2$ each or A, B and C all move to the right with velocity $v/3$ each. It is easy to see that in these two cases, the kinetic energy is not conserved. Hence the only result of the collision is the one given in Statement-1.
13. The correct choice is (b). Let C collide with A at $t = 0$. Since the collision in elastic and A and C have equal masses, C will come to rest and A will move to the right with velocity u and at this instant the spring is uncompressed and B is at rest. Hence at $t = 0$, the momentum of the system = mu. When A moves to the right, it compresses the spring and as a result B beings to move to the right. Let v be the common velocity of A and B at time t_0. From the principle of conservation of linear momentum, we have

Momentum of C before collision = momentum of A after collision + momentum of B after collision or $mu = mv + (2m)\, v \Rightarrow v = \frac{u}{3}$.

14. The correct choice is (a). The total energy (which includes all forms of energy) is conserved in any process.
15. The correct choice is (b). Statement-1 follows from Newton's third law of motion.
16. The correct choice is (a).
17. The correct choice is (d). The two colliding body need not get stuck after an inelastic collision.
18. The correct choice is (d). Since the velocities of the two bodies change due to collision, the linear momentum of each body will change but the total linear momentum of the system of two bodies is conserved.
19. The correct choice is (d). Due to change in velocity, the energy of each body changes on collision but the total energy of the system of two bodies is conserved.
20. The correct choice is (d). If an external force acts on a system, it is accelerated which will increase the total energy.
21. The correct choice is (a). If v and V are the velocities of Q and P after the collision, then from conservation of momentum and kinetic energy, we have

$$Mu = mv + MV \quad \Rightarrow \quad M(u - V) = mv \quad (1)$$

$$\frac{1}{2} Mu^2 = \frac{1}{2} mv^2 + \frac{1}{2} MV^2$$

$$\Rightarrow \qquad M(u - V)(u + V) = mv^2 \qquad (2)$$

From Equations (1), (2), we get

$$v = \frac{2Mu}{(M + m)} = \frac{2u}{\left(1 + \frac{m}{M}\right)}$$

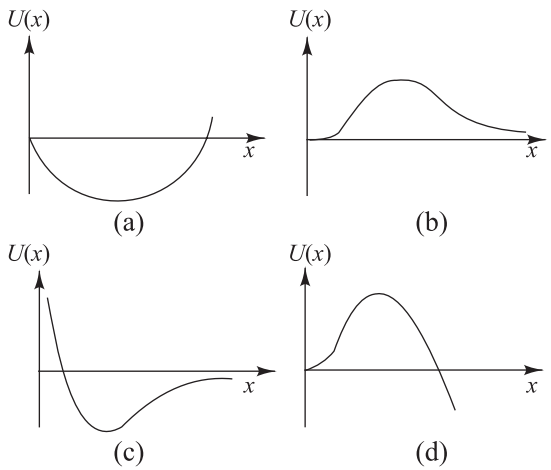
If $M \gg m$, then v is maximum equal to $2u$

(since $\frac{m}{M} \rightarrow$ zero).

SECTION 4

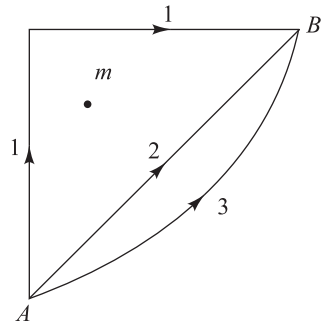
Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. A spring of force consttant 800 Nm^{-1} has an extension of 5 cm, The work done in extending it from 5 cm to 15 cm is
   (a) 16 J (b) 8 J
   (c) 32 J (d) 24 J **[2002]**
2. A particle, which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle



[2002]

3. Consider the following statements
   A. Linear momentum of a system of particles is zero.
   B. Kinetic energy of a system of particles is zero.
   (a) A does not imply B and B does not imply A.
   (b) A implies B but B does not imply B.
   (c) A does not imply B but B implies A.
   (d) A implies B and B implies A. **[2003]**
4. A spring of spring constant 5×10^3 Nm^{-1} is stretched initially by 5 cm from the unstretched position. Then the work required to stretch by another 5 cm is
   (a) 12.50 J (b) 18.75 J
   (c) 25.00 J (d) 6.25 J **[2003]**
5. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to
   (a) $t^{3/4}$ (b) $t^{3/2}$
   (c) $t^{1/4}$ (d) $t^{1/2}$ **[2003]**
6. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 (as shown in the figure) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3.
   (a) $W_1 > W_3 > W_2$ (b) $W_1 = W_2 = W_3$
   (a) $W_1 < W_3 < W_2$ (b) $W_1 < W_2 < W_3$
   [2003]



7. A particle move in a straight line with retardation proportional to its displacement. It loss of kinetic energy for any displacement x is proportional to
   (a) x^2 (b) e^x
   (c) x (d) $\log_e x$ **[2004]**
8. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table? Take $g = 10$ ms^{-2}.
   (a) 7.2 J (b) 3.6 J
   (c) 120 J (d) 1200 J **[2004]**
9. A force $\mathbf{F} = (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ newton is applied to a particle which displaces it from the origin to a point $\mathbf{r} = (2\hat{\mathbf{i}} - \hat{\mathbf{j}})$ metre. The work done on the particle is
   (a) –7J (b) +7J
   (c) +10 J (d) +13 J **[2004]**
10. A body of mass m accelerates uniformly from rest to velocity v_1 in time t_1. The instantaneous power delivered to the body as a function of time t is

(a) $\frac{mv_1t}{t_1}$ (b) $\frac{mv_1^2t}{t_1^2}$

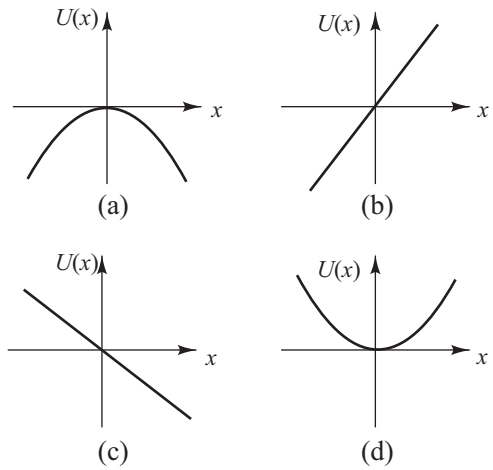
(c) $\frac{mv_1t^2}{t_1}$ (d) $\frac{mv_1^2t}{t_1}$ **[2004]**

11. A wire fixed at the upper end stretches by length l by applying force F. The work done is stretching is

(a) $\frac{1}{2}Fl$ (b) Fl

(c) $2Fl$ (d) $2\sqrt{2}Fl$ **[2004]**

12. A particle at the origin is under the influence of a force $F = kx$, where k is a positive constant. If the potential energy U is zero at $x = 0$, the variation of potential energy with the coordinate x is represented by



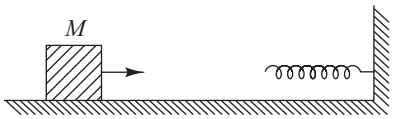
[2004]

13. A ball of mass 20 kg is stationary at the top of the hill of height 100 m. It rolls down the smooth surface of the hill to the ground and then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is (take $g = 10\ \text{ms}^{-2}$)

(a) $40\ \text{ms}^{-1}$ (b) $20\ \text{ms}^{-1}$

(c) $10\ \text{ms}^{-1}$ (d) $10\sqrt{30}\ \text{ms}^{-1}$ **[2005]**

14. A block of mass M moving on a frictionless horizontal surface collides with spring of spring constant k and compresses it by length L. The maximum momentum of the block after the collision is



(a) $L\sqrt{Mk}$ (b) $\frac{KL^2}{2M}$

(c) zero (d) $\frac{ML^2}{k}$ **[2005]**

15. A mass m moves with velocity v and collides with another identical mass initially at rest. After the collision the first mass moves with a velocity $v/\sqrt{3}$ in a direction perpendicular to the initial direction of motion. If the collision is inelastic, the speed of the second mass after collision is

(a) v (b) $\sqrt{3}v$

(c) $\frac{2v}{\sqrt{3}}$ (d) $\frac{v}{\sqrt{3}}$ **[2005]**

16. A particle of mass 0.3 kg is subjected to a force $F = -kx$ with $k = 15\ \text{Nm}^{-1}$. What will be its initial acceleration if it is released from a point 20 cm away from the origin?

(a) $10\ \text{ms}^{-2}$ (b) $5\ \text{ms}^{-2}$

(c) $15\ \text{ms}^{-2}$ (d) $3\ \text{ms}^{-2}$ [....]

17. A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle of 45° with the initial vertical direction is

(a) $\frac{Mg}{\sqrt{2}}$ (b) $Mg(\sqrt{2-1})$

(c) $Mg(\sqrt{2+1})$ (d) $Mg\sqrt{2}$ **[2006]**

18. A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is (take $g = 10\ \text{ms}^{-2}$)

(a) 1.25 J (b) 0.5 J

(c) –0.5 J (d) –1.25 J **[2006]**

19. A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is $4\ \text{ms}^{-1}$. The kinetic energy of the other mass is

(a) 192 J (b) 96 J

(c) 144 J (d) 288 J **[2006]**

20. The potential energy of 1 kg particle free to move along the x-axis is given by

$$v(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)\text{joule}$$

The total mechanical energy of the particle is 2 J. Then the maximum speed (in m/s) is

(a) $\frac{1}{\sqrt{2}}$ (b) 2

(c) $\frac{3}{\sqrt{2}}$ (d) $\sqrt{2}$ **[2006]**

21. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes an uncompressed spring and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10,000 N/m. The spring compresses by

(a) 5.5 cm (b) 2.5 cm

(c) 11.0 cm (d) 8.5 cm **[2007]**

22. An athlete in the Olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range

(a) 20,000 J – 50,000 J (b) 2,000 J – 5,000 J

(c) 200 J – 500 J (d) 2×10^5 J – 3×10^5 J **[2008]**

23. A block of mass 0.5 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is

(a) 0.34 J (b) 0.16 J

(c) 1.00 J (d) 0.67 J **[2008]**

24. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and 4 k, respectively (see Figure I). The other ends are attached to identical supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (Figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacement x and y are measured with respect to the equilibrium position of the block B. The ratio $\frac{x}{y}$ is

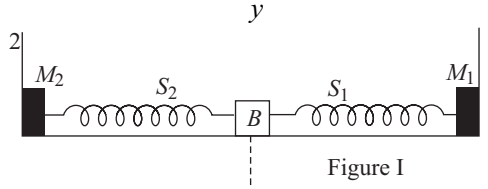


Figure I

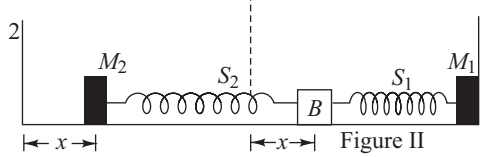
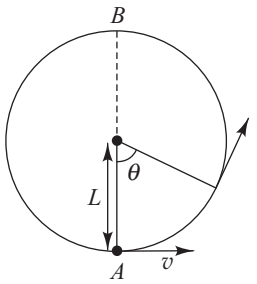


Figure II

(a) 4 (b) 2

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$ **[2008]**

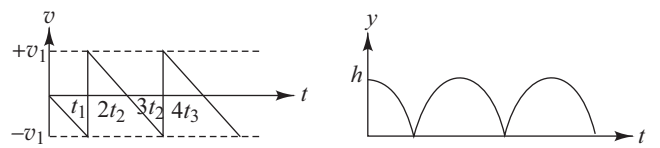
25. A bob of mass m is suspended by a massless string of length L. The horizontal velocity v at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A satisfies



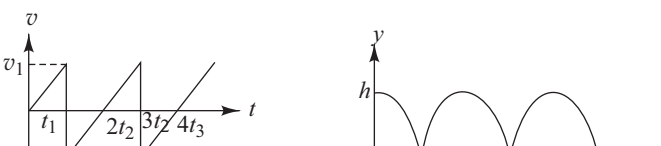
(a) $\theta = \frac{\pi}{4}$ (b) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

(c) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (d) $\frac{3\pi}{4} < \theta < \pi$ **[2008]**

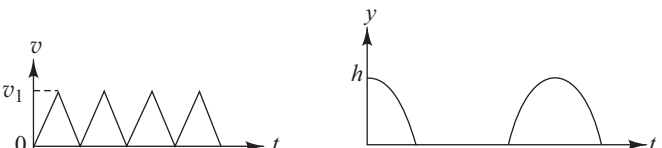
26. Consider a rubber ball falling freely from a height h = 4.9 m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as a function of time will be:



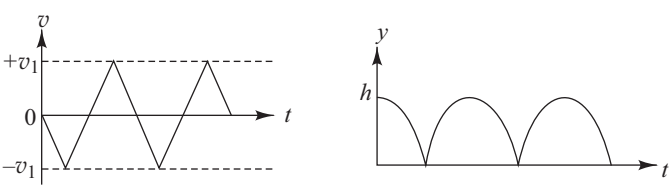
(a)



(b)



(c)



(d)

[2009]

27. **Statement-1:** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2: Principle of conservation of momentum holds true for al kinds of collisions.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.

(b) Statement-1 is ture, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.

(c) Statement-1 is false, Statement-2 is true.

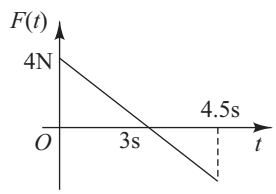
(d) Statement-1 is true, Statement-2 is false. **[2010]**

28. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecules is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is

(a) $\frac{b^2}{6a}$ (b) $\frac{b^2}{2a}$

(c) $\frac{b^2}{12a}$ (d) $\frac{b^2}{4a}$ **[2010]**

29. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is



(a) 4.50 J (b) 7.50 J

(c) 5.06 J (d) 14.06 J **[2010]**

30. This question has two **statements** 1 and 2. Of the four choices given after the statements, choose the one that best describes the two statements.
If two spring S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on S_1 than that on S_2.
Statement 1 : If stretched by the same amount, work done on S_1 will be more than that on S_2.
Statement 2 : k_1 is less than k_2.

(a) Statement 1 is false, statement 2 is true.

(b) Statement 1 is true, statement 2 is false.

(c) Statement 1 is true, statement 2 is true and statement 2 is the correct explanation for statement 1.

(d) Statement 1 is true, Statement 2 is true but statement 2 is not the correct explanation of statement 1.

[2012]

31. The question has Statement-I and Statement-II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-I : A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f = \left(\frac{m}{M+m}\right)$.

Statement-II: Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(a) Statement-I is true, Statement-II is true, Statement-II is not the correct explanation of Statement-I.

(b) Statement-I is true, Statement-II is false.

(c) Statement-I is false, Statement-II is true.

(d) Statement-I is true, Statement-II is true, Statement-II is the correct explanation of Statement-I.

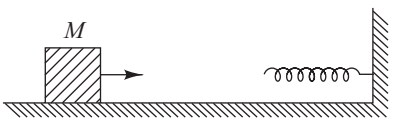
[2003]

32. The work done on a particle of mass m by a force, $K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{\mathbf{i}} + \frac{y}{(x^2+y^2)^{3/2}}\hat{\boldsymbol{j}}\right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point $(a, 0)$ to the point $(0, a)$ along a circular path of radius a about the origin in the x–y plane is:

(a) $\frac{2K\pi}{a}$ (b) $\frac{K\pi}{a}$

(c) $\frac{K\pi}{2a}$ (d) 0 **[2013]**

33. A block of mass 4 kg moving on a frictionless horizontal surface collides with spring of force constant 4 Nm^{-1} and compresses it by length 50 cm. The maximum momentum (in kg ms^{-1}) of the block after the collision is



(a) 2 (b) 4

(c) 1 (d) 0.5 **[2013]**

Answers

1. (b)	2. (d)	3. (c)	4. (b)
5. (b)	6. (b)	7. (a)	8. (b)

9. (b)	10. (b)	11. (a)	12. (a)
13. (a)	14. (a)	15. (c)	16. (a)
17. (b)	18. (d)	19. (d)	20. (c)
21. (a)	22. (b)	23. (d)	24. (c)
25. (d)	26. (a)	27. (a)	28. (d)
29. (c)	30. (a)	31. (c)	32. (d)
33. (a)			

Solutions

1. Here $x_1 = 5$ cm $= 0.05$ m and $x_2 = 15$ cm $= 0.15$ m

$$W = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}\times 800 \times [0.15)^2 - (0.05)^2]$$

$$= 8 \text{ J}$$

2. Refer to the solution of Q.65 on page 4.32 of this chapter.

3. The kinetic energy of a system of particles is zero only if the velocity of every particles is zero. Linear momentum of a system of particles is zero if the vector sum of linear momenta of individual particles is zero which is possible even if velocities are non-zero. Hence statement B implies statement A but does not imply B.

4. Here $x_1 = 5$ cm $= 0.05$ m and $x_2 = 10$ cm $= 0.1$ m

$$W = \frac{1}{2}k(x_2^2 - x_1^2)$$

$$= \frac{1}{2}\times 5 \times 10^3\,[(0.1)^2 - (0.5)^2] = 18.75 \text{ J}$$

5. $P = Fv = mav = m\dfrac{dv}{dt}\,v = m\dfrac{dv}{dx}.\dfrac{dx}{dt}\,v = mv^2\dfrac{dv}{dx}$

$$\therefore \quad v^2 dv = \frac{P}{m}dx$$

Integrating (assuming that the body starts from rest), we have

$$\int_0^v v^2 dv = \frac{P}{m}\int_0^x dx \qquad (\because P = \text{constant})$$

$$\Rightarrow \qquad \frac{v^3}{3} = \frac{Px}{m}$$

$$\Rightarrow v = \left(\frac{3P}{m}\right)^{1/3} = x^{1/3} = k\,x^{1/3} \text{ where } k = \left(\frac{3P}{m}\right)^{1/3} =$$

constant

$$\Rightarrow \qquad \frac{dx}{dt} = k\,x^{1/3}$$

$$\Rightarrow \qquad x^{-1/3}\,dx = k\,dt$$

Integrating $\int_0^x x^{-1/3}dx = k\int_0^t dt$

$$\Rightarrow \qquad \frac{x^{2/3}}{2/3} = kt \;\Rightarrow\; x = \left(\frac{2k}{3}\right)^{3/2} \times t^{3/2}$$

Hence $\quad x \propto t^{3/2}$

6. Gravitational force is conservative. The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle. Hence the correct choice is (b).

7. $a = -kx$ where k is a positive constant

or $\quad \dfrac{dv}{dt} = -kx \Rightarrow \dfrac{dv}{dx}.\dfrac{dx}{dt} = -kx$

or $\quad vdv = -kxdx \qquad \left(\because v = \dfrac{dx}{dt}\right)$

Integrating,

$$\int_u^v vdv = -k\int_0^x xdx$$

$$\frac{1}{2}(v^2 - u^2) = \frac{kx^2}{2}$$

where u = initial velocity and v = final velocity. If m is the mass of the particle, then

$$\text{Initial K.E. is } K_i = \frac{1}{2}mu^2$$

$$\text{Final K.E. is } K_f = \frac{1}{2}mv^2$$

$$\text{Loss in K.E.} = k_i - k_f = m\,(u^2 - v^2)$$

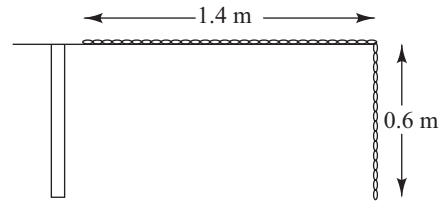
Using (i) in (ii)

$$\text{Loss in K.E.} = \frac{km}{2}x^2$$

Thus loss in K.E. $\propto x^2$

8. Mass per unit length of the chain is

$$\frac{M}{L} = \frac{4}{2} = 2 \text{ kgm}^{-1}$$



Mass of hanging portion is

$$m = 2 \times 0.6 = 1.2 \text{ kg}$$

This mass can be assumed to be concentrated at the centre of the hanging portion of the chain which is at a distance of $x = 0.3$ from the edge of the table.

$\therefore$ Work done $= mgx = 1.2 \times 10 \times 0.3 = 3.6$ J

9. $\mathbf{W = F.\ r}$

$= (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}).(2\hat{\mathbf{i}} - \hat{\mathbf{j}})$

$= 10 - 3 = 7$ J

$(\because \hat{\mathbf{i}}.\hat{\mathbf{i}} = \hat{\mathbf{j}}.\hat{\mathbf{j}} = 1 \text{ and } \hat{\mathbf{i}}.\hat{\mathbf{j}} = \hat{\mathbf{k}}.\hat{\mathbf{j}} = 0)$

10. If a is the uniform acceleration. then

$$v_1 = 0 + at_1$$

$$\Rightarrow \quad a = \frac{v_1}{t_1}$$

At an instant of time t, the velocity v of the body is

$$v = 0 + at = \frac{v_1 t}{t_1}$$

Instantaneous power $P = Fv = mav$

Using (i) and (ii), we have

$$P = m \times \frac{v_1}{t_1} \times \frac{v_1 t}{t_1} = \frac{m v_1^2 t}{t_1^2}$$

Note Choice (a) has dimensions of momentum and choice (d) has dimensions of energy. Choice (b) is the only one which has dimensions of power (ML^2T^{-3})

11. $W = \frac{1}{2}kx^2 = \frac{1}{2}kl^2 = \frac{1}{2}(kl) \times l$

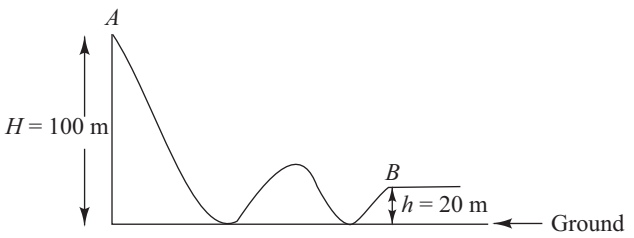
$$= \frac{1}{2}F \times l \qquad (\because F = kx = kl)$$

12. Potential energy function is

$$U(x) = -\int_0^x F\, dx = -k \int_0^x x\, dx = \frac{1}{2}kx^2$$

The value of $U(x)$ is always negative for both positive and negative values of x. Thus the variation of potential energy with x is an inverted parabola as shown in choice (a).

13. Refer to the following figure.



Velocity at $A = 0$. Let v be the velocity at B. From the law of conservation of energy, loss of P.E. = gain in K.E. i.e.

$$mg\,(H - h) = \frac{1}{2}mv^2$$

$$\Rightarrow \quad 10(100 - 20) = \frac{1}{2}v^2 \quad \Rightarrow \quad v = 40 \text{ ms}^{-1}$$

14. The momentum is maximum when kinetic energy is maximum. This happens when the entire potential energy of the compressed spring is transferred to the block, i.e. when

$$\frac{1}{2}\,kL^2 = \frac{1}{2}\,Mv^2$$

here v is the maximum velocity imparted to the block. Thus

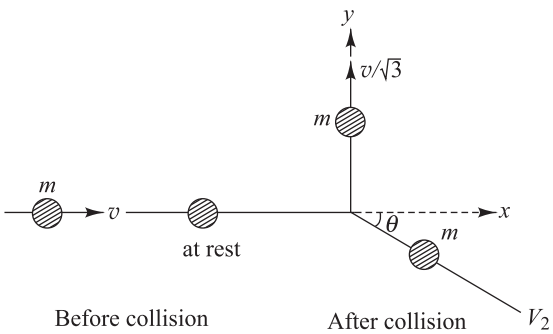
$$Mv^2 = kL^2$$

$$\Rightarrow \quad M^2v^2 = MkL^2$$

$$\Rightarrow \quad p^2 = MkL^2$$

$$\Rightarrow \quad p = L\sqrt{Mk}$$

15. Refer to the following figure.



Conservation of x and y components of momentum gives

$$mv = mv_2 \cos\theta \Rightarrow v = v_2 \cos\theta \qquad \text{(i)}$$

and $m\frac{v}{\sqrt{3}} = -mv_2 \sin\theta \quad \Rightarrow \quad \frac{v}{\sqrt{3}} = -v_2 \sin\theta$ (ii)

Squarring (i) and (ii) and adding we get

$$v_2 + \frac{v^2}{3} = v_2^2$$

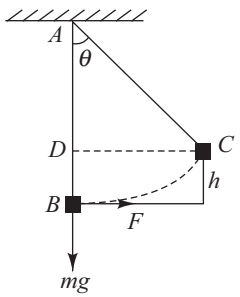
$$\Rightarrow \quad v_2 = \frac{2v}{\sqrt{3}}$$

Note This question is wrong as it violates the law of conservation of energy. If we calculate the total initial kinetic energy and final kinetic energy, we get $k_i = \frac{1}{2}mv^2$ and $k_f = \frac{1}{2}mv^2 \times \frac{5}{3}$, i.e. $k_f > k_i$ which is not possible.

16. $F = -kx = -15 \times 0.2 = -3$ N. Therefore

$$|a| = \frac{|F|}{m} = \frac{3}{0.3} = 10 \text{ ms}^{-2}$$

17. Let l be the length of the string. The mass is moved from B to C by a horizontal force F until $\theta = 45°$. The work done against the force of gravity is



$$W_g = Mgh = MgDB$$
$$= Mg\,(AB - AD)$$
$$= Mg\,(l - l\cos\theta)$$
$$= Mg\,l\,(1 - \cos\theta)$$

This work is done against the force of gravity. Hence, by sign convention, it is negative. Thus

$$W_g = -Mg\,l\,(1 - \cos\theta)$$

The work done by the applied horizontal force **F** is

$$F_a = F \times \text{horizontal distance moved}$$
$$= F \times CD = F\,l\sin\theta$$

$\therefore$ Total work done is

$$W = W_g + W_a$$
$$= -Mg\,l\,(1 - \cos\theta) + F\,l\sin\theta$$

Since the mass is at rest at positions B and C, the change in kinetic energy is zero. From work energy principle, work done = change in kinetic energy, i.e.

$$-Mg\,l\,(1 - \cos\theta) + Fl\,\text{Sin}\,\theta = 0$$

or
$$F = \frac{Mg(1-\cos\theta)}{\sin\theta} = \frac{Mg(1-\cos 45°)}{\sin 45°}$$

$$= \frac{Mg\left(1 - \frac{1}{\sqrt{2}}\right)}{1/\sqrt{2}} = Mg(\sqrt{2} - 1)$$

18. The vertical height to which the particle rises is

$$h = \frac{u^2}{2g} = \frac{(5)^2}{2\times 10} = 1.25 \text{ m}$$

Work done $W_g = mgh = (100\times 10^{-3})\times 10\times 1.25 = 1.25$ J

Since the work W_g is done against the force of gravity, it is negative. Hence the correct choice is (d), i.e. W_g = –1.25 J.

19. Let v be the speed of the 4 kg mass just after the explosion. Since the bomb was at rest, its momentum is zero. From the conservation of momentum, we have

$$4\times v - 12\times 4 = 0 \quad \Rightarrow \quad v = 12 \text{ ms}^{-1}$$

$\therefore$ K.E. of 4 kg mass $= \frac{1}{2}\times 4\times (12)^2 = 288$ J

20. $V = \frac{x^4}{4} - \frac{x^2}{2}$

$\therefore \quad \frac{dV}{dx} = x^3 - x$

V is maximum or minimum if $\frac{dV}{dx} = 0$, i.e. if $x^3 - x =$ 0 or $x(x^2 - 1) = 0$ which gives $x = 0$, 1 and –1. Now

$$\frac{d^2V}{dx^2} = 3x^2 - 1$$

V is maximum if $\frac{d^2V}{dx^2}$ is negative and V is minimum if $\frac{d^2V}{dx^2}$ is positive. For $x = 0$, $\frac{d^2V}{dx^2} = -1$.

Hence for $x = 0$, V is maximum. For $x = \pm 1$, $\frac{d^2V}{dx^2} = 2$. Hence for $x = \pm 1$, V is minimum, Now

$$V = \frac{x^4}{4} - \frac{x^2}{2}$$

Therefore, the minimum potential energy is (put $x = \pm 1$)

$$V_{\min} = \frac{(\pm 1)^4}{4} - \frac{(\pm 1)^2}{2} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} \text{ joule}$$

Maximum kinetic energy is $K_{\max} = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}v_{\max}^2$ ($\because$ $m = 1$ kg).

Given, total energy $E = 2$ joule. From $E = V + K$, we have

$$2 = \frac{1}{4} + \frac{1}{2}v_{\max}^2$$

which gives $v_{\max} = \frac{3}{\sqrt{2}}$ ms^{-1}.

21. Let x be the compression of the spring. The kinetic energy of the block $\left(=\frac{1}{2}mv^2\right)$ is used up in

(i) imparting potential energy $\left(=\frac{1}{2}kx^2\right)$ to the spring and

(ii) doing work ($=fx$) against friction. Thus

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + fx$$

$$\frac{1}{2}\times 2\times (4)^2 = \frac{1}{2}\times 10{,}000\times x^2 + 15x$$

$\Rightarrow \quad 5000x^2 + 15x - 16 = 0$

The two roots are $x = 0.055$ m and –0.058 m. Since The negative value of x is not permissible,
$x = 0.055$ m = 5.5 cm

22. Average speed of athlete $v = \dfrac{100\,\text{m}}{10\,\text{s}} = 10\ \text{ms}^{-1}$. Now kinetic energy $K = \dfrac{1}{2}mv^2 \Rightarrow m = \dfrac{2K}{v^2}$, where m is the mass of the athlete. Thus

$$m = \frac{2K}{(10)^2} = \frac{K}{50}$$

Choice (a), (c) and (d) give absurd values of m. Choice (b) gives m in the range 40 kg to 100 kg. So the correct choice is (b).

23. Let v be the velocity of the composite body. From conservation of momentum, we have

$$0.5 \times 2 = (0.5 + 1.0)v \ \Rightarrow\ v = \frac{2}{3}\ \text{ms}^{-1}$$

Initial K.E. is $K_i = \dfrac{1}{2} \times 0.5 \times (2)^2 = 1.0$ J

Final K.E. is $K_f = \dfrac{1}{2}(1.0 + 0.5) \times \left(\dfrac{2}{3}\right)^2 \text{J} = \dfrac{1}{3}\text{J}$

$\therefore$ Loss of energy = $1.0 - \dfrac{1}{3} = 0.67$ J

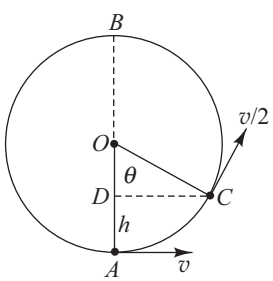
24. Potential energy stored in spring S_1 when the block B is moved through a distance x is $U_1 = \dfrac{1}{2}k_1x^2 = \dfrac{1}{2}kx^2$. When the block is released, it moves to the left, compressing the spring S_2 through a distance y. The potential energy stored in spring S_2 when its compression is y is $U_2 = \dfrac{1}{2}k_2y^2 = \dfrac{1}{2}(4\,k)y^2 = 2ky^2$. Since y is maximum compression of spring S_2, from conservation of energy, we have $U_1 = U_2$, i.e.

$$\frac{1}{2}\ kx^2 = 2\,k\,y^2$$

which gives $\dfrac{y}{x} = \dfrac{1}{2}$.

25. Refer to the figure. Here $OA = OB = OC = L$ and $OD = OC \cos\theta = L\cos\theta$. Therefore

$h = OA - OD = L - L\cos\theta$



or $h = L(1 - \cos\theta)$. From conservation of energy, total energy at A = total energy at C, i.e.

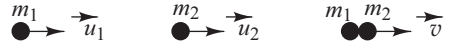
$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + mgL(1 - \cos\theta)$$

$$v^2 = \frac{8gL}{3}(1 - \cos\theta) \qquad (1)$$

The minimum velocity the bob must have at A so as to reach B is $v = \sqrt{5gL}$. Putting this in Eq. (1), we get $\cos\theta = -\dfrac{7}{8}$. Therefore θ lies between $\dfrac{3\pi}{4}$ and π.

26. While falling the velocity of the ball is negative as it is directed downwards. After the collision, the velocity is reversed and is positive as it is now directed upwards. Hence the correct choice is (a).

27. The principle of conservation of mementum holds for both elastic as well as inelastic collisions. In a completely inelastic collision, the two particles stick together.



Before collision After collision

$$m_1\vec{u}_1 + m_2\vec{u}_2 = (m_1 + m_2)\vec{v}$$

$$\Rightarrow\quad \vec{v} = \frac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1 + m_2} = \frac{\vec{P}_1 + \vec{P}_2}{m_1 + m_2}$$

$\because$ K.E. after collision = $\dfrac{1}{2}(m_1 + m_2)v^2$, which can be zero if $\vec{v} = 0$ which is possible if

$$\vec{P}_1 = -\vec{P}_2$$

This is not possible for two particles moving in the same direction. Hence the particles do not lose all their kinetic energy. So the correct choice is (a).

28. $U(x) = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$ (1)

$\therefore\quad U(x = \infty) = 0$

Force $F = -\dfrac{dU}{dx} = -\dfrac{d}{dx}\left(\dfrac{a}{x^{12}} - \dfrac{b}{x^6}\right)$

$$\Rightarrow\quad F = -\left(-\frac{12a}{x^{13}} + \frac{6b}{x^7}\right) \qquad (2)$$

At equilibrium $F = 0$. Putting $F = 0$ in Eq. (2) we get

$$x^6 = \frac{2a}{b}$$

Putting $x^6 = \dfrac{2a}{b}$ in Eq. (1), we get

U at equilibrium $= \dfrac{a}{\left(\dfrac{2a}{b}\right)^2} - \dfrac{b}{\dfrac{2a}{b}} = -\dfrac{b^2}{4a}$

$\therefore \quad D = [U \text{ at } x = \infty - U \text{ at equilibrium}]$

$= 0 - \left(-\dfrac{b^2}{4a}\right) = \dfrac{b^2}{4a}$, which is choice (d).

29. Slope of graph $= -\dfrac{4 \text{ N}}{3 \text{ s}} = -\dfrac{4}{3} \text{ Ns}^{-1}$. Therefore,

$F = -\dfrac{4}{3}t + 4$. Now change in momentum $= \int F\, dt$

$\Rightarrow \quad m\,v - 0 = \int_0^{4.5\text{s}} \left(-\dfrac{4}{3}t + 4\right) dt$

or $\quad 2v = \left|-\dfrac{2t^2}{3} + 4t\right|_0^{4.5s} = 4.5$

Which gives $v = 2.25 \text{ ms}^{-1}$. Therefore,

K.E. $= \dfrac{1}{2}mv^2 = \dfrac{1}{2} \times 2 \times (2.25)^2$

$\simeq 5.06$ J, which is choice (c).

30. $F = k_1 x_1 = k_2 x_2$

$W_1 = \dfrac{1}{2}k_1 x_1^2 = \dfrac{1}{2k_1}(k_1 x_1)^2 = \dfrac{F^2}{2k_1}$

Similarly $\quad W_2 = \dfrac{F^2}{2k_2}$

$\therefore \quad \dfrac{W_1}{W_2} = \dfrac{k_2}{k_1}$

It is given that $W_1 > W_2$. Hence $k_2 > k_1$. So Statement 2 is true.

If the springs are stretched by the same amount x, then $W_1 = \dfrac{1}{2}k_1 x^2$ and $W_2 = \dfrac{1}{2}k_2 x^2$. Hence

$$\frac{W_1}{W_2} = \frac{k_1}{k_2}$$

Since $k_1 < k_2$; $W_1 < W_2$, so statement 1 is false. The correct choice is (a).

31. The maximum loss of kinetic energy occurs when the collision is perfectly inelastic, i.e. when the particles get stuck together after the collision. So Statement-II is correct.

From conservation of momentum,

$$mv + 0 = (m + M)\, V$$

$\Rightarrow \quad V = \dfrac{mv}{(m+M)}$

Here V is the velocity of the composite body.

Total K.E. before collision is

$$k_i = \frac{1}{2}mv^2 + 0 = \frac{1}{2}mv^2$$

Total K.E. after collision is

$$K_f = \frac{1}{2}(m+M)V^2 = \frac{1}{2}(m+M) \times \left(\frac{mv}{m+M}\right)^2 = \frac{1}{2}\frac{m^2v^2}{(m+M)}$$

$\therefore$ Loss of K.E. $= K_i - K_f$

$$= \frac{1}{2}mv^2 - \frac{1}{2}\frac{m^2v^2}{(m+M)} = \frac{1}{2}mv^2\left[1 - \frac{m}{(m+M)}\right] = \frac{M}{(m+M)} \times \frac{1}{2}mv^2$$

$\therefore \quad f = \dfrac{M}{(m+M)}$. So Statement-I is false.

32. $W = \int \vec{F}.\vec{dr}$

$= \int F_x dx + \int F_y\, dy$

$= K \int_{(a,0)}^{(0,a)} \dfrac{x dx}{(x^2+y^2)^{3/2}} + K \int_{(a,0)}^{(0,a)} \dfrac{y dy}{(x^2+y^2)^{3/2}}$

$= -\dfrac{K}{2}\left|(x^2+y^2)^{-1/2}\right|_{(a,0)}^{(0,a)} - \dfrac{K}{2}\left|(x^2+y^2)^{-1/2}\right|_{(a,0)}^{(0,a)}$

$= -0 - 0$

$= 0$

33. The momentum is maximum when kinetic energy is maximum. This happens when the entire potential energy of the compressed spring is transferred to the block, i.e. when

$$\frac{1}{2}kL^2 = \frac{1}{2}Mv^2$$

here v is the maximum velocity imparted to the block and L is the length of the spring. Thus

$$Mv^2 = kL^2$$

$\Rightarrow \quad M^2v^2 = MkL^2$

$\Rightarrow \quad p^2 = MkL^2$

$\Rightarrow \quad p = L\sqrt{Mk}$

$= 0.5 + \sqrt{4 \times 4}$

$= 2 \text{ kg ms}^{-1}$

ROTATIONAL MOTION

Chapter 5

REVIEW OF BASIC CONCEPTS

1. Centre of Mass of Discrete Particles

For a system of particles, the centre of mass is defined as that point where the entire mass of the system is imagined to be concentrated, for considerations of its translational motion.

If $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$, ... $\mathbf{r}_n$ are the position vectors of masses m_1, m_2, m_3, ...m_n respectively, the position vector of the centre of mass of the system is

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3 + \cdots + m_n\mathbf{r}_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

$$= \frac{\sum_{N=1}^{n} m_n\mathbf{r}_n}{\sum_{N=1}^{n} m_n} = \frac{\sum_{N=1}^{n} m_n\mathbf{r}_n}{M}$$

where M is the total mass of the system of particles.

$\mathbf{r}_{CM}$ is the weighted average of all the position vectors of the particles of the system, the contribution of each particle being proportional to its mass.

For a system consisting of two particles, the centre of mass is

$$\mathbf{r}_{cm} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}$$

If the masses are equal, i.e. $m_1 = m_2$, then

$$\mathbf{r}_{CM} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$$

which means that the centre of mass lies exactly in the middle of the line joining the two masses.

EXAMPLE 1 Three particles of masses $m_1 = m$, and $m_2 = 2m$ and $m_3 = 3m$ are placed at the corners of an equilateral triangle of side a as shown in Fig. 5.1. Locate the centre of mass of the system.

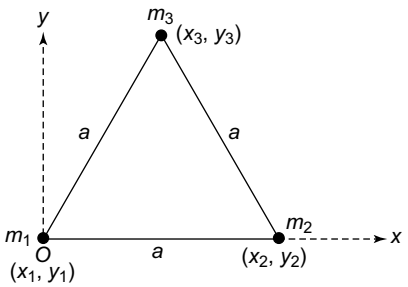


Fig. 5.1

SOLUTION Take any one particle to be located at the origin O. The x and y coordinates of m_1, m_2, and m_3 respectively are $x_1 = 0$ and $y_1 = 0$, $x_2 = a$ and $y_2 = 0$ and $x_3 = \frac{a}{2}$ and $y_3 = \frac{\sqrt{3}a}{2}$

The x and y coordinates of the centre of mass are

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{m \times 0 + 2m \times a + 3m \times \frac{a}{2}}{m + 2m + 3m} = \frac{7a}{12}$$

and

$$y_{CM} = \frac{m_1\, y_1 + m_2\, y_2 + m_3\, y_3}{m_1 + m_2 + m_3}$$

$$= \frac{m \times 0 + 2m \times 0 + 3m \times \frac{\sqrt{3}a}{2}}{m + 2m + 3m}$$

$$= \frac{\sqrt{3}a}{4}$$

EXAMPLE 2 Four particles of masses 1 kg, 1 kg, 2 kg, and 2 kg, are placed at the corners of a square of side 12 cm as shown in Fig. 5.2. Find the position vector of the centre of mass of the system.

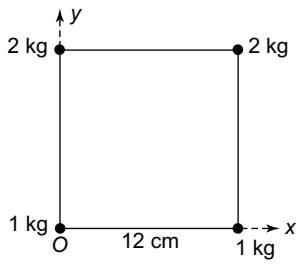


Fig. 5.2

SOLUTION $x_{CM} = \dfrac{1\times0+1\times12+2\times12+2\times0}{1+1+2+2}$

$= 6$ cm

$y_{CM} = \dfrac{1\times0+1\times0+2\times12+2\times12}{1+1+2+2}$

$= 8$ cm

The position vector of the centre of mass is

$$\mathbf{r}_{CM} = \left(6\hat{\mathbf{i}}+8\hat{\mathbf{j}}\right) \text{ cm}$$

2. Centre of Mass of a Body having Continuous Distribution of Mass

If a body has continuous distribution of mass, the position of its centre of mass is determined by dividing the body into a very large number of extremely small elements. If dm is the mass of the element and it is at a distance x and y from the origin of chosen coordinate system, then x and y coordinates of the centre of mass are given by

$$x_{CM} = \frac{\int x\, dm}{\int dm}$$

and

$$y_{CM} = \frac{\int y\, dm}{\int dm}$$

EXAMPLE 3 Locate the centre of mass of a uniform rod of mass M and length L.

SOLUTION We assume that the rod lies along the x-axis with one end at origin O.

Mass per unit length of the rod $= \dfrac{M}{L}$

Mass of element of length dx is [Fig 5.3]

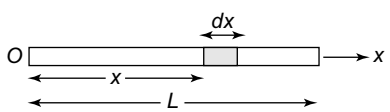


Fig. 5.3

$$dm = \frac{M}{L}\, dx$$

The x-coordinate of the centre of mass is

$$x_{CM} = \frac{\int_0^L x\, dm}{\int_0^L dm} = \frac{1}{M}\int_0^L \frac{M}{L}\, x\, dx = \frac{L}{2}$$

The centre of mass is at the centre of the rod.

EXAMPLE 4 A non-uniform rod of length L is lying along the x-axis with one end at origin O as shown in Fig. 5.4. The linear mass density (i.e. mass per unit length) λ varies with x as $\lambda = a + bx$, where a and b are constants. Find the distance of the centre of mass from origin O.

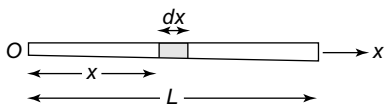


Fig. 5.4

SOLUTION Mass of element is $dm = \lambda\, dx = (a + bx)\, dx$

$$x_{CM} = \frac{\int_0^L x\, dm}{\int_0^L dm} \qquad \text{(i)}$$

$$\int_0^L x\, dm = \int_0^L (a+bx)\, x\, dx$$

$$= a\left|\frac{x^2}{2}\right|_0^L + b\left|\frac{x^3}{3}\right|_0^L$$

$$= \frac{aL^2}{2} + \frac{bL^3}{3} \qquad \text{(ii)}$$

$$\Rightarrow \quad \int_0^L x\, dm = \frac{L^2}{6}(3a+2bL)$$

And

$$\int_0^L dm = \int_0^L (a+bx)\, dx = aL + \frac{bL^2}{2}$$

$$= \frac{L}{2}(2a+bL) \qquad \text{(iii)}$$

Using (ii) and (iii) in (i), we get

$$x_{CM} = \frac{L(3a+2bL)}{3(2a+bL)}$$

EXAMPLE 5 Locate the centre of mass of a uniform semicircular ring (or wire) of radius R and linear mass density λ.

SOLUTION Let us take the centre of the ring at origin O. Consider a small element of arc length dl of the ring. Let θ be the angle which the radius vector of the element makes with the x-axis as shown in Fig. 5.5.

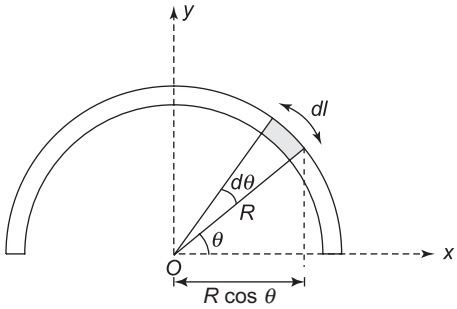


Fig. 5.5

Let $d\theta$ be the angle subtended by the element at the centre. Then $dl = R\,d\theta$. Mass of the element is

$$dm = \lambda dl = \lambda R d\theta$$

The x and y components of radius vector R are $x = R\cos\theta$ and $y = R\sin\theta$. Then

$$x_{CM} = \frac{\int x\,dm}{\int dm} = \frac{\lambda R^2 \int_0^{\pi} (\cos\theta)\,d\theta}{\lambda R \int_0^{\pi} d\theta}$$

$$= \frac{\lambda R^2}{\lambda R \pi} |\sin\theta|_0^{\pi}$$

$$= \frac{R}{\pi}(\sin\pi - \sin 0) = 0$$

$$y_{CM} = \frac{\int y dm}{\int dm} = \frac{\lambda R^2 \int_0^{\pi} (\sin\theta)\,d\theta}{\lambda R \int_0^{\pi} d\theta}$$

$$= \frac{\lambda R^2}{\lambda R \pi} |-\cos\theta|_0^{\pi}$$

$$= -\frac{R}{\pi}(\cos\pi - \cos 0) = \frac{2R}{\pi}$$

Thus, the centre of mass is at a distance of $\frac{2R}{\pi}$ from origin O on the y-axis. By symmetry, the x-coordinate of centre of mass is $x = 0$ (i.e. at O).

EXAMPLE 6 A thin uniform semi-circular disc (or plate) has mass M and radius R. The y-coordinate of the centre of mass of the disc is

(a) $\frac{4R}{3\pi}$ (b) $\frac{2R}{3\pi}$

(c) $\frac{2R}{\pi}$ (d) $\frac{3R}{2\pi}$

SOLUTION We can solve this problem by using the result obtained above for a semi-circular ring. Let us take the centre of the disc at origin O (Fig. 5.6).

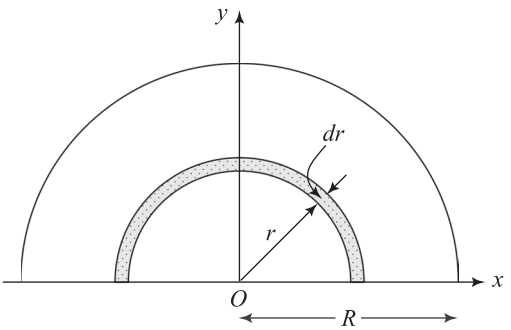


Fig. 5.6

Divide the disc into a large number of very small elements, each of width dr. Consider one such element at a distance r from O.

Area of shaded part = $\pi r\,dr$

Area of semicircular disc = $\frac{\pi R^2}{2}$

$\therefore$ Mass per unit area = $\frac{M}{\frac{\pi R^2}{2}} = \frac{2M}{\pi R^2}$

Mass of the element is $dm = \frac{2M}{\pi R^2} \times \pi r\,dr = \frac{2M r\,dr}{R^2}$

Now, as shown in Example 5 above, this element can be treated as a ring of radius r. The y-coordinate of this element is $\frac{2r}{\pi}$. Hence the y-coordinate of the centre of mass of the semicircular disc is

$$y_{CM} = \frac{1}{M}\int_0^R \frac{2r}{\pi}\,dm$$

$$= \frac{1}{M}\int_0^R \frac{2r}{\pi} \times \frac{2M r\,dr}{R^2}$$

$$= \frac{4}{\pi R^2}\int_0^R r^2\,dr$$

$$= \frac{4}{\pi R^2} \times \frac{R^3}{3} = \frac{4R}{3\pi}, \text{ which is choice (a).}$$

By symmetry, the x-coordinate of the centre of mass is $x = 0$ (i.e. at origin O).

3. Finding Centre of Mass of a System when a Part of its Mass is Removed

Consider of system of mass M. If a mass m is removed, the remaining mass = $M - m$ which may be written as $M + (-m)$. Then the x and y coordinates of the centre of mass of the remaining portion are given by

$$x_{CM} = \frac{Mx - mx'}{M - m}$$

and

$$y_{CM} = \frac{My - my'}{M - m}$$

where x and y are the coordinates of the centre of mass of the complete part and x' and y' are the coordinates of the centre of mass of the removed part.

EXAMPLE 7 Four particles, each of mass 1 kg, are placed at the corners of a square of side 12 cm. If mass m_3 is removed, find the shift in the centre of mass of the system (see Fig. 5.7).

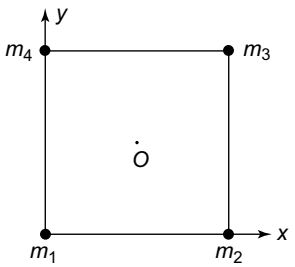


Fig. 5.7

SOLUTION As shown in Example 2, the x and y coordinates of the original system are $x_{CM} = 6$ cm and $y_{CM} = 6$ cm

If mass m_3 is removed, the x and y coordinates of the centre of mass of the remaining system are

$$x'_{CM} = \frac{1\times 0 + 1\times 12 \times 1\times 0}{1+1+1} = 4 \text{ cm}$$

$$y'_{CM} = \frac{1\times 0 + 1\times 0 + 1\times 12}{1+1+1} = 4 \text{ cm}$$

Thus $\mathbf{r} = 6\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$

and $\mathbf{r}' = 4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

$\therefore$ shift $\Delta\mathbf{r} = \mathbf{r} - \mathbf{r}' = (6\hat{\mathbf{i}} + 6\hat{\mathbf{j}}) - (4\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$

$= 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

EXAMPLE 8 From a uniform thin disc of radius R and mass M, a circular portion of radius $r = R/2$ is removed as shown in Fig. 5.8(a). Find the centre of mass of the remaining part of the disc.

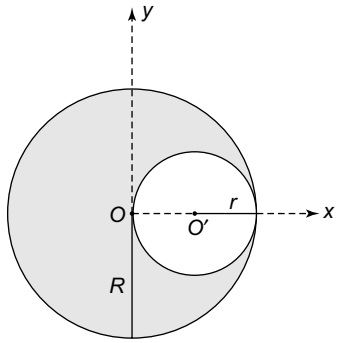


Fig. 5.8(a)

SOLUTION Let the centre of the disc be at origin O. The centre of mass of the complete disc will be at O (by symmetry) and its x and y coordinates are (0, 0). Mass per unit area of the disc = $\frac{M}{\pi R^2}$. Therefore, mass of removed portion is

$$m = \frac{M}{\pi R^2} \times \pi r^2 = \frac{M}{\pi R^2} \times \pi \left(\frac{R}{2}\right)^2 = \frac{M}{4}.$$

The x and y coordinates of the centre of mass of the removed portion are

$$x' = r = \frac{R}{2} \text{ and } y' = 0$$

The x and y coordinates of the remaining portion (shown shaded) are

$$x_{CM} = \frac{Mx - mx'}{M - m} = \frac{M\times 0 - \frac{M}{4}\times\frac{R}{2}}{M - \frac{M}{4}} = -\frac{R}{6}$$

$$y_{CM} = \frac{My - my'}{M - m} = \frac{M\times 0 - \frac{M}{4}\times 0}{M - \frac{M}{4}} = 0$$

The negative sign of x_{CM} indicates that the centre of mass of the remaining portion is located at a distance $R/6$ towards the left of the center O of the complete disc.

Alternative Method

It is clear that centre of mass of the remaining portion of the disc will shift to the left of O. Let G be the centre of mass of the remaining portion of the disc (Fig. 5.8(b). Let $OG = x$. Equating the moments of Mg and mg about G, we have

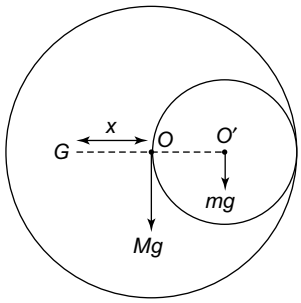


Fig. 5.8(b)

$$Mg \times OG = mg \times O'G$$

$$\Rightarrow \quad M \times x = \frac{M}{4} \times \left(\frac{R}{2} + x\right)$$

$$\Rightarrow \quad x = \frac{R}{6}$$

Thus the centre of mass shifts by $R/6$ to the left of O.

EXAMPLE 9 From a thin uniform square lamina of side a a square of side $a/2$ is removed from its corner as shown in Fig. 5.9. Find the centre of mass of the remaining portion (shown shaded) of the lamina.

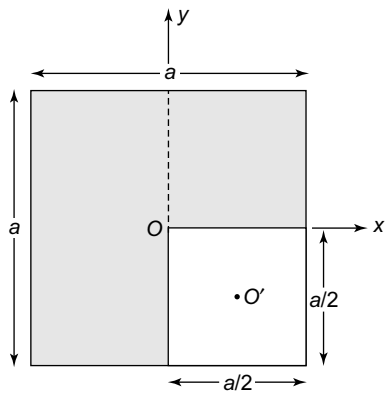


Fig. 5.9

SOLUTION Let M be the mass of the complete lamina. Mass of the removed portion is $m = \frac{M}{4}$. The centre of mass of the complete lamina is taken to be at origin O ($x = 0$, $y = 0$). The coordinates of the centre of mass O' of the removed portion are $x' = \frac{a}{4}$ and $y' = -\frac{a}{4}$.

The x and y coordinates of the centre of mass of the remaining portion are

$$x_{CM} = \frac{Mx - mx'}{M - m} = \frac{M \times 0 - \frac{M}{4} \times \frac{a}{4}}{M - \frac{M}{4}} = -\frac{a}{12}$$

and

$$y_{CM} = \frac{My - my'}{M - m} = \frac{M \times 0 - \frac{M}{4} \times -\frac{a}{4}}{M - \frac{M}{4}} = +\frac{a}{12}$$

4. Velocity and Acceleration of Centre of Mass of a System of Particles

If $\mathbf{r}_1$, $\mathbf{r}_2$, ... $\mathbf{r}_n$ are the position vectors of masses m_1, m_2, ... m_n, respectively, the position vector of the centre of mass of the system of particles is given by

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + \cdots + m_n\mathbf{r}_n}{m_1 + m_2 + \cdots + m_n}$$

The velocity of the centre of mass is given by

$$\mathbf{v}_{CM} = \frac{d\mathbf{r}_{CM}}{dt} = \frac{m_1\frac{d\mathbf{r}_1}{dt} + m_2\frac{d\mathbf{r}_2}{dt} + \cdots + m_n\frac{d\mathbf{r}_n}{dt}}{m_1 + m_2 + \cdots + m_n}$$

$$\Rightarrow \quad \mathbf{v}_{CM} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \cdots + m_n\mathbf{v}_n}{m_1 + m_2 + \cdots + m_n}$$

The acceleration of the centre of mass is given by

$$\mathbf{a}_{CM} = \frac{d\mathbf{v}_{CM}}{dt} = \frac{m_1\frac{d\mathbf{v}_1}{dt} + m_2\frac{d\mathbf{v}_2}{dt} + \cdots + m_n\frac{d\mathbf{v}_n}{dt}}{m_1 + m_2 + \cdots + m_n}$$

$$\Rightarrow \quad \mathbf{a}_{CM} = \frac{m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + \cdots + m_n\mathbf{a}_n}{m_1 + m_2 + \cdots + m_n}$$

or

$$\mathbf{a}_{CM} = \frac{\mathbf{F}_1 + \mathbf{F}_2 + \cdots + \mathbf{F}_n}{m_1 + m_2 + \cdots + m_n} = \frac{\sum \mathbf{F}_{ext}}{M}$$

If $\sum \mathbf{F}_{ext} = 0$, then $\mathbf{a}_{CM} = 0$, i.e $\mathbf{v}_{CM}$ = constant. Hence if *no net external force acts on a system, its centre of mass will remain at rest or will move with a constant velocity*.

EXAMPLE 10 A boy of mass $m = 50$ kg stands at the end A of a flat plank AB of wood of mass $M = 100$ kg and length $l = 10$ m floating in the still water in a lake. The end B of the plank is at a distance of 30 m from the shore of the lake as shown in Fig. 5.10. The boy walks a distance of 6 m on the plank towards the shore. How far is the boy from the shore now? Neglect viscosity of water.

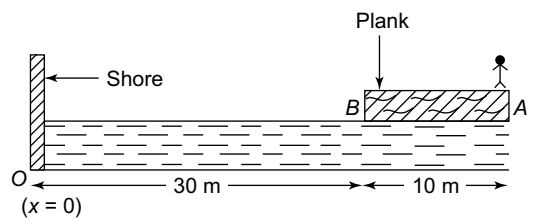


Fig. 5.10

SOLUTION Initially the centre of mass of the system (plank + boy) is at rest. To walk, the boy exerts a force in the backward direction. The plank in turns exerts

a reaction force on the boy in the forward direction. These forces are internal to the system. Since no external force acts, the centre of mass of the system remains at rest even when the boy walks on the plank.

Let the shore be at the origin O ($x = 0$). Initially let x be the distance of the centre of mass of the plank from O. Then the distance of the centre of mass of the system (plank + boy) from O will be

$$x_{CM} = \frac{M \times x + m \times (30+10)}{M+m} = \frac{100x + 50 \times 40}{100+50}$$

$$= \frac{100x + 2000}{150}$$

Since the boy moves towards the shore and the centre of mass of the system has to remain at rest, the plank will move away from the shore. If x' is the distance moved by the plank, the distance of the centre of mass from the shore when the boy walks 6 m on the plank is given by

$$x'_{\text{CM}} = \frac{100(x+x') + 50(40-6+x')}{100+50}$$

$$= \frac{100x + 150x' + 1700}{150}$$

Since $x_{\text{CM}} = x'_{\text{CM}}$,

$$\frac{100x + 2000}{150} = \frac{100x + 150x' + 1700}{150}$$

$$\Rightarrow \quad x' = \frac{300}{150} = 2 \text{ m}$$

$\therefore$ Distance of the boy from the shore = 40 – 6 + 2 = 36 m.

EXAMPLE 11 A car of mass 1000 kg is moving with a velocity of 10 ms^{-1} towards another car of mass 1500 kg moving with a velocity of 15 ms^{-1} in the same direction. Find the velocity of the centre of mass of the two cars.

SOLUTION

$$V_{\text{CM}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{1000 \times 10 + 1500 \times 15}{1000 + 1500}$$

$$= 13 \text{ ms}^{-1}$$

5. Momentum Conservation and Centre of Mass Motion

We have seen that

$$\sum \mathbf{F}_{\text{ext}} = M\mathbf{a}_{\text{CM}} = M\frac{d\mathbf{v}_{\text{CM}}}{dt} = \frac{d}{dt}(M\mathbf{v}_{\text{CM}})$$

$$\Rightarrow \quad \sum \mathbf{F}_{\text{ext}} = \frac{d\mathbf{P}}{dt}$$

where $\mathbf{P} = M\mathbf{v}_{CM}$ is the *total linear momentum of the system of particles which is equal to the product of the total mass of the system and the velocity of the centre of mass.*

If $\sum \mathbf{F}_{\text{ext}} = 0$, $\frac{d\boldsymbol{P}}{dt} = 0 \quad \Rightarrow \quad \mathbf{P} = \text{constant}.$

Thus, if no net external force acts on a system, the total linear momentum of the system remains constant; the total linear momentum being the vector sum of linear momentum of individual particles, i.e.

$$\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2 + \cdots + \mathbf{P}_n$$

EXAMPLE 12 A boy of mass $m = 40$ kg is standing on a stationary long plank of mass $M = 260$ kg floating on still water in a lake. He starts running with a velocity $v = 6$ ms^{-1} relative to the plank. Find the velocity of the boy relative to a stationary observer on the bank of the lake.

SOLUTION Since the system (boy + plank) is initially at rest, its momentum is zero. Since no external force acts on the system, the momentum of the system will remain zero. Let us assume that the boy runs in the positive x-direction.

Velocity of boy relative to the plank is

$$\mathbf{v}_{\text{bp}} = v\hat{\mathbf{i}}$$

Now $\mathbf{v}_{\text{bp}} = \mathbf{v}_{\text{b}} - \mathbf{v}_{\text{p}}$

where $\mathbf{v}_{\text{b}}$ = velocity of boy relative to ground

and $\mathbf{v}_{\text{p}}$ = velocity of plank relative to ground

Hence $\mathbf{v}_{\text{b}} = \mathbf{v}_{\text{bp}} + \mathbf{v}_{\text{p}}$

$$= v\hat{\mathbf{i}} + v_{\text{p}}\hat{\mathbf{i}}$$

$$= (v + v_{\text{p}})\hat{\mathbf{i}}$$

Total momentum of plank + boy = 0

$$\Rightarrow \quad M\mathbf{v}_{\text{p}} + m\mathbf{v}_{\text{b}} = 0$$

$$\Rightarrow \quad Mv_{\text{p}}\hat{\mathbf{i}} + mv_{\text{b}}\hat{\mathbf{i}} = 0$$

$$\Rightarrow Mv_p\hat{\mathbf{i}} + m(v + v_p)\hat{\mathbf{i}} = 0$$

which gives $v_p = -\frac{mv}{M+m}$

$\therefore$ Velocity of boy is $\mathbf{v}_b = \left(v - \frac{mv}{M+m}\right)\hat{\mathbf{i}} = \frac{Mv}{M+m}\hat{\mathbf{i}}$

$$= \left(\frac{260 \times 6}{260+40}\right)\hat{\mathbf{i}}$$

$$= 5.2\,\hat{\mathbf{i}} \text{ ms}^{-1}$$

Hence the velocity of boy relative to a stationary observer is 5.2 ms^{-1} in the direction along which the boy is running.

6. Torque

If a force **F** acts on a particle *P* whose position vector with respect to the origin of an inertial reference frame is **r**, the torque $\vec{\tau}$ acting on the particle with respect to the origin is defined as [Fig. 5.11]

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

In terms of magnitudes,

$$\tau = r\,F \sin\theta = F\,(r \sin\theta) = Fr_{\perp}$$

where θ is the angle between vectors **r** and **F**.

Torque $\boldsymbol{\tau}$ is a vector quantity. Its magnitude is given by $\tau = rF \sin\theta$; its direction is normal to the plane containing vectors **r** and **F** and can be determined by the *right–hand screw rule.*

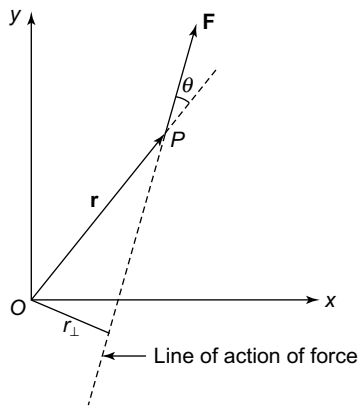


Fig. 5.11

Unit of Torque Torque has the same dimensions as those of work (both being force times distance) viz. ML^2T^{-2}. The two are, however, very different quantities. Work is a scalar, torque is a vector. To distinguish between the two we express work in joules and torque in newton–metre (N m).

Couple

Two equal antiparallel forces having different lines of action constitute a couple.

The moment of couple or torque = $Fr_{\perp}$ (Fig. 5.12)

= magnitude of either force × perpendicular distance between the two antiparallel forces

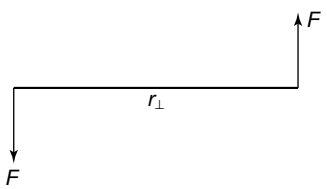


Fig. 5.12

Work done by torque

If a force *F* acts on a rigid body at perpendicular $r_{\perp}$ from the axis of rotation, the work done by the force in rotating the body through an angle $\Delta\theta$ is given by

$$\Delta W = Fr_{\perp}\Delta\theta = \tau\,\Delta\theta$$

= magnitude of torque × angular displacement

$$\text{Power} = \frac{dW}{dt} = \tau\,\frac{d\theta}{dt} = \tau\omega$$

where ω is the angular velocity.

EXAMPLE 13 A force $\mathbf{F} = \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right)$ newton acts on a particle whose position vector with respect to origin *O* is $\vec{\mathbf{r}} = \left(4\hat{\mathbf{i}} - 5\hat{\mathbf{j}}\right)$ metre. Find the magnitude and direction of the torque.

SOLUTION

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

$$= \left(4\hat{\mathbf{i}} - 5\hat{\mathbf{j}}\right) \times \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right)$$

$$= 4\hat{\mathbf{i}} \times 2\hat{\mathbf{i}} + 12\hat{\mathbf{i}} \times \hat{\mathbf{j}} - 10\hat{\mathbf{j}} \times \hat{\mathbf{i}} - 15\hat{\mathbf{j}} \times \hat{\mathbf{j}}$$

$$= 0 + 12\hat{\mathbf{k}} + 10\hat{\mathbf{k}} - 0$$

$$= 22\ \hat{\mathbf{k}} \text{ newton metre}$$

The magnitude of torque is 22 Nm and its direction is along the positive *z*-axis.

EXAMPLE 14 A rectangular plate *OPQR* of dimensions 2 m × 3 m lies in the *x*-*y* plane as shown in Fig 5.13. A force $\mathbf{F} = \left(3\hat{\mathbf{i}} + 5\hat{\mathbf{j}}\right)$ newton is applied at point *Q*. Find the torque of **F** (a) about origin *O*, (b) about point *P* and (c) about *x*-axis, *y*-axis and *z*-axis.

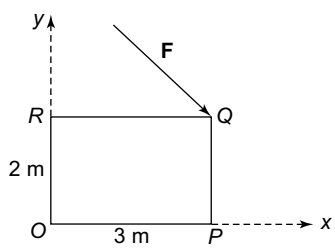


Fig. 5.13

SOLUTION

(a) $\mathbf{r} = \overrightarrow{OQ} = \left(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}\right)$ metre

Torque about *O* is

$$\tau_O = \mathbf{r} \times \mathbf{F}$$

$$= \left(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}\right) \times \left(3\hat{\mathbf{i}} + 5\hat{\mathbf{j}}\right)$$

$$= 9\hat{\mathbf{i}} \times \hat{\mathbf{i}} + 15\hat{\mathbf{i}} \times \hat{\mathbf{j}} + 6\hat{\mathbf{j}} \times \hat{\mathbf{i}} + 10\hat{\mathbf{j}} \times \hat{\mathbf{j}}$$

$$= 0 + 15\hat{\mathbf{k}} - 6\hat{\mathbf{k}} + 0 = 9\hat{\mathbf{k}} \text{ Nm}$$

(b) Torque about P is

$$\boldsymbol{\tau}_P = \overrightarrow{PQ} \times \mathbf{F}$$

$$= 2\hat{\mathbf{j}} \times \mathbf{F}$$

$$= 2\hat{\mathbf{j}} \times (3\hat{\mathbf{i}} + 5\hat{\mathbf{j}}) = -6\hat{\mathbf{k}} \text{ Nm}$$

(c) Torque about x-axis is

$$\boldsymbol{\tau}_x = \boldsymbol{\tau}_O \cdot \hat{\mathbf{i}} = 9\hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = 0$$

Torque about y-axis is

$$\boldsymbol{\tau}_y = \tau_O \cdot \hat{\mathbf{j}} = 9\hat{\mathbf{k}} \cdot \hat{\mathbf{j}} = 0$$

Torque about z-axis is

$$\boldsymbol{\tau}_z = z_O \hat{\mathbf{k}} = 9\hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 9 \text{ Nm}$$

7. Angular Momentum

The angular momentum **L** of a particle P with respect to the origin of an inertial reference frame is defined as [Fig. 5.14]

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

where **r** is the position vector of the particle and **p** its linear momentum. In terms of magnitudes,

$$L = rp \sin\theta = r_\perp \times p$$

where θ is the angle between vectors **r** and **p**. The dimensions of angular momentum are ($ML^2\ T^{-1}$) and its SI unit is kg m^2s^{-1}.

The direction of **L** is perpendicular to the plane containing the vectors **r** and **p** and its sense is given by the *right–hand rule*.

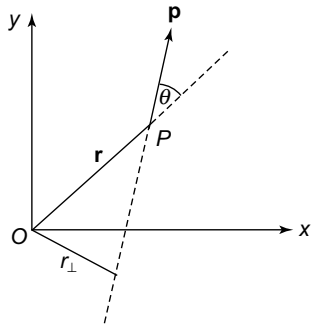


Fig. 5.14

EXAMPLE 15 A body of mass $m = 200$ g is moving parallel to the x-axis with a velocity $\mathbf{v} = 30$ cms^{-1} in the x-y plane as shown in Fig. 5.15. Calculate the magnitude of its angular momentum about origin O at any time t.

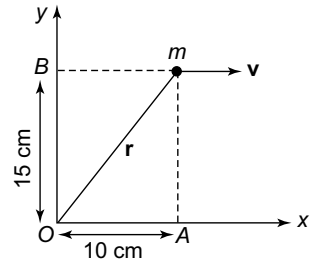


Fig. 5.15

SOLUTION Magnitude of angular momentum

$$= r_\perp p$$

$$= r_\perp mv$$

$$= OB \times m \times v$$

$$= 0.15 \times 0.2 \times 0.3$$

$$= 9 \times 10^{-3} \text{ kg m}^2\text{s}^{-1}$$

EXAMPLE 16 A body of mass m = 200 g is projected with a velocity $u = 5$ ms^{-1}at an angle $\theta = 30°$ with the horizontal. Calculate the magnitude of the angular momentum of the body about the point of projection when it is at the highest point of its trajectory. Take $g = 10$ ms^{-2}.

SOLUTION Refer to Fig. 5.16. At the highest point A, the body has only horizontal velocity

$$v = u \cos\theta$$

$$h_{\max} = \frac{u^2 \sin^2\theta}{2g}$$

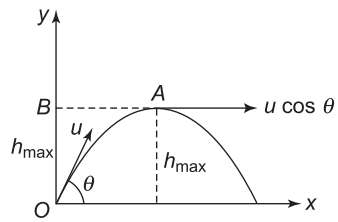


Fig. 5.16

Magnitude of angular momentum of the body about O when it is at point A is

$$L = mv \times OB$$

$$= mv \times h_{\max}$$

$$= mu\cos\theta \times \frac{u^2 \sin^2\theta}{2g}$$

$$= \frac{mu^3 \sin^2\theta \cos\theta}{2g}$$

$$= \frac{0.2 \times (5)^3 \times \sin^2(30°)\cos(30°)}{2 \times 10}$$

$$= \frac{0.2 \times 125 \times \frac{1}{4} \times \frac{\sqrt{3}}{2}}{20}$$

$$= 0.27 \text{ kg m}^2\text{s}^{-1}$$

EXAMPLE 17 A particle of mass m is moving in a horizontal circle of radius r with kinetic energy K and time period T. The magnitude of the angular momentum of the particle about the centre of the circle will be

(a) $L = KT$ (b) $L = \frac{KT}{2}$

(c) $L = \frac{KT}{\pi}$ (d) $L = \frac{KT}{2\pi}$

SOLUTION Speed of particle is $v = r\omega = \frac{2\pi r}{T}$.

Kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{2\pi r}{T}\right)^2$

which gives $K = \frac{2\pi^2 mr^2}{T^2}$ (i)

Magnitude of angular momentum is

$$L = mvr = m\left(\frac{2\pi r}{T}\right)r = \frac{2\pi mr^2}{T} \quad \text{(ii)}$$

From (i) and (ii) we find that $L = \frac{KT}{\pi}$, which is choice (c).

EXAMPLE 18 A body of mass m and length l is whirled in a horizontal circle with a uniform speed v. The tension in the string is T. The length of the string is gradually reduced such that the angular momentum of the body about the centre of the circle remains constant. At the instant of time when the length of the string is $l/2$, the tension in the string will be

(a) T (b) $2T$

(c) $4T$ (d) $8T$

SOLUTION For uniform motion in a horizontal circle, the tension in the string is equal to the centripetal force.

$$T = \frac{mv^2}{r} = \frac{mv^2}{l} \quad (\because r = l)$$

Angular momentum about the centre of the circle is

$$L = mvr = mvl = \text{constant} = k \text{ (given)}$$

$$\therefore \quad v = \frac{k}{ml}$$

$$\therefore \quad T = \frac{m}{l}\left(\frac{k}{ml}\right)^2 = \frac{k^2}{ml^3}$$

When $l' = l/2$,

$$T' = \frac{k^2}{m\left(\frac{l}{2}\right)^3} = \frac{8k^2}{ml^3} = 8T$$

8. Relation between Torque and Angular Momentum

In linear motion, the relation between force **F** and linear momentum **p** is

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

In rotational motion, the relation between torque τ and angular momentum **L** is

$$\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$$

which states that *the torque acting on a particle is equal to the rate of change of angular momentum*.

9. Angular Impulse

In linear motion, impulse **I** is defined as

$$\mathbf{I} = \mathbf{F}\,dt$$

$$= \frac{d\mathbf{p}}{dt}\,dt$$

$$= d\mathbf{p} = \text{change in linear momentum}$$

In rotational motion, angular impulse **J** is defined as

$$\mathbf{J} = \boldsymbol{\tau}\,dt$$

$$= \frac{d\mathbf{L}}{dt}\,dt$$

$$= d\mathbf{L} = \text{change in angular momentum}$$

10. Law of Conservation of Angular Momentum

If no external torque acts, the total angular momentum of a body or a system of particles is conserved.

We have seen that the rate of change of angular momentum of a particle is equal to the torque produced by the total force. If, in a certain situation, the torque itself vanishes, then it follows that the angular momentum of the particle will remain constant. This is the law of conservation of the angular momentum of a particle. One trivial situation is when the force vanishes. Then the torque vanishes too. The particle then moves freely in a straight line in accordance with Newton's first law in which case both linear and angular momenta are conserved.

A general situation is when the torque vanishes without the force itself vanishing. The torque $\boldsymbol{\tau}$ will vanish if the component $\mathbf{F}_{\perp}$ (the angular component) of $\mathbf{F}$ vanishes but the radial component $\mathbf{F}_{\text{II}}$ does not. The radial component $\mathbf{F}_{\text{II}}$ is the component of $\mathbf{F}$ along the radius (or position) vector $\mathbf{r}$. Hence, *if the force acting on the particle is purely radial (i.e. if it is directed along or against its position vector) then the torque acting on the particle vanishes and its angular momentum is conserved and so is its areal velocity.*

11. Moment of Inertia

The moment of inertia of a rigid body about a particular axis may be defined as the sum of the products of the masses of all the particles constituting the body and the squares of their respective distances from the axis of rotation, i.e.

$$I = m_1 r_1^2 + m_2 r^2{}_2 + m_3 r_3^2 + \cdots + m_n r_n^2$$

$$= \sum_{\text{N}=1}^{n} m_n r_n^2$$

Its value depends upon the particular axis about which the body rotates and the way the mass is distributed in the body with respect to the axis of rotation.

In the case of a body which does not consist of separate, discrete particles but has a continuous and homogeneous distribution of matter in it, the summation is replaced by integration, so that

$$I = \int r^2 dm$$

where dm is the mass of an infinitesimally small element of the body at a distance r from the axis of rotation.

Moment of inertia is a scalar quantity. Its SI unit is kg m^2 and its dimensions are (ML^2).

12. Radius of Gyration

The radius of gyration of a body about its axis of rotation may be defined as the distance from the axis of rotation at which, if the entire mass of the body were concentrated, its moment of inertia about the given axis would be the same as with its actual distribution of mass. It is usually denoted by the letter K. If M is the mass of the body, its moment of inertia I in terms of its radius of gyration K can be written

$$I = MK^2$$

13. Moment of Inertia and Rotational Kinetic Energy

Kinetic energy of a rotating body is related to moment of inertia as

$$\text{KE} = \frac{1}{2} I\omega^2$$

where ω is the angular velocity (or frequency) of the body.

14. Moment of Inertia and Torque

The magnitude of torque is given by

$$\tau = I\alpha$$

where α is the angular acceleration of the body.

15. Moment of Inertia and Angular Momentum

The magnitude of angular momentum of a rotating body is given by

$$L = I\omega$$

16. Parallel Axes Theorem

If M is the total mass of a body and h the distance between two parallel axes, then according to parallel axes theorem

$$I = I_{\text{CM}} + Mh^2$$

17. Perpendicular Axes Theorem

The theorem of perpendicular axes for a body of plane lamina states that the moment of inertia of a plane lamina about an axis perpendicular to the plane of the lamina is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its own plane and intersecting each other at the point where the perpendicular axis passes through it.

If I_x and I_y are the moments of inertia of a plane lamina about the perpendicular axes x and y respectively which lie in the plane of the lamina and intersect each other at O, then the moment of inertia I of the lamina about an axis passing through O and perpendicular to its plane is given by

$$I = I_x + I_y$$

18. Expressions for moment of inertia of bodies of regular shapes about particular axes of rotation:

	Shape of body	*Axis of Rotation*	*Expression for Moment of Inertia*
1.	Circular ring of mass M and radius R	(i) through centre, perpendicular to plane of ring	MR^2
		(ii) any diameter	$(1/2)\,MR^2$
		(iii) any tangent in the plane of ring	$(3/2)\,MR^2$
		(iv) any tangent perpendicular to plane of ring	$2\,MR^2$
2.	Circular disc of mass M and radius R	(i) through centre, perpendicular to plane of disc	$(1/2)\,MR^2$
		(ii) any diameter	$(1/4)\,MR^2$
		(iii) tangent in the plane of the disc	$(5/4)\,MR^2$
		(iv) tangent perpendicular to plane of disc	$(3/2)\,MR^2$
3.	Sphere of mass M and radius R	(i) any diameter	$(2/5)\,MR^2$
		(ii) any tangent plane	$(7/5)\,MR^2$
4.	Hollow sphere of mass M and radius R	(i) any diameter	$\frac{2}{3}MR^2$
		(ii) any tangent plane	$\frac{5}{3}MR^2$
5.	Cylinder of mass M, radius R and length L	(i) own axis	$(1/2)\,MR^2$
		(ii) through centre perpendicular to length	$M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$
		(iii) through end faces and $\perp$ to length	$M\left(\frac{R^2}{4}+\frac{L^2}{3}\right)$
6.	One dimensional rod of mass M and length L	(i) centre of rod and $\perp$ to length	$ML^2/12$
		(ii) one end and $\perp$ to length	$ML^2/3$
7.	Rectangular lamina of mass M, length L and breadth B	(i) length of lamina and in its plane	$MB^2/3$
		(ii) breadth of lamina and in its plane	$ML^2/3$
		(iii) centre of lamina and parallel or to length or breadth in its plane	$\frac{MB^2}{12}$ or $\frac{ML^2}{12}$
		(iv) centre of lamina and $\perp$ to its plane	$M\left(\frac{L^2+B^2}{12}\right)$
		(v) centre of length and $\perp$ to its plane	$M\left(\frac{L^2}{12}+\frac{B^2}{3}\right)$
		(vi) centre of breadth and $\perp$ to its plane	$M\left(\frac{L^2}{3}+\frac{B^2}{12}\right)$

19. Kinematics of Rotational Motion with Constant Angular Acceleration

Consider a body rotating with an initial angular velocity ω_0. It is given a constant angular acceleration α (by applying a constant torque) for a time t. As a result it acquires a final angular velocity ω and suffers an angular displacement θ in time t. The equations of rotational motion are

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

and $$2\alpha\theta = \omega^2 - \omega_0^2$$

EXAMPLE 19 The moment of inertia of a uniform circular disc of mass M and radius R about an axis passing through its centre and perpendicular to its plane is $\frac{1}{2}MR^2$. Find the moment of inertia of the disc

(a) about any diameter

(b) about an axis passing through a point on the edge of the disc and perpendicular to the disc

(c) about a tangent in the plane of the disc.

SOLUTION The plane of the disc is the x-y plane.

(a) Using perpendicular axes theorem [Fig. 5.17(a)]

$$I_x + I_y = I_z$$

Now $$I_z = I_C = \frac{1}{2}MR^2 \text{ (given).}$$

From symmetry $I_y = I_x$.

$$\therefore \quad 2I_x = \frac{1}{2}MR^2 \quad \Rightarrow \quad I_x = I_y = \frac{1}{4}MR^2$$

(b) Using parallel axes theorem [Fig. 5.17(b)]

$$I_{AB} = I_z + M(CD)^2$$

$$= I_C + MR^2$$

$$= \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

(c) Using parallel axes theorem [Fig. 5.17(c)]

$$I_{EF} = I_y + MR^2$$

$$= \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$$

EXAMPLE 20 A thin uniform disc of mass M and radius R has concentric hole of radius r. Find the moment of inertia of the disc about an axis passing through its centre and perpendicular to its plane.

SOLUTION Mass per unit area of the disc is [Fig. 5.18]

$$m = \frac{M}{\pi\left(R^2 - r^2\right)}$$

Mass of the disc if it was complete (i.e. without hole) is

$$M_1 = m \times \pi R^2 = \frac{M}{\pi\left(R^2 - r^2\right)} \times \pi R^2$$

$$= \frac{M R^2}{R^2 - r^2}$$

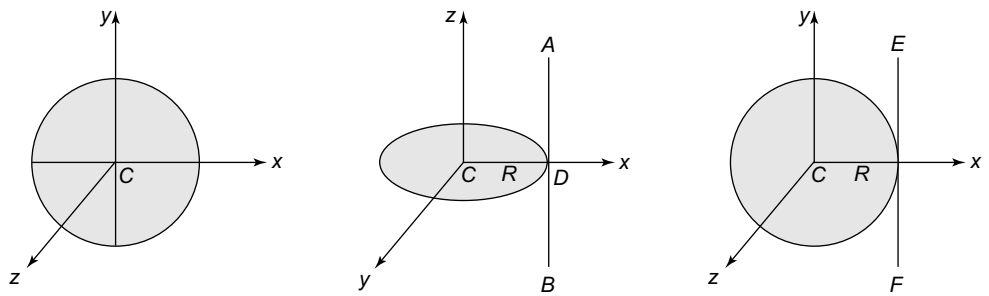


Fig. 5.17

Mass of the removed portion is

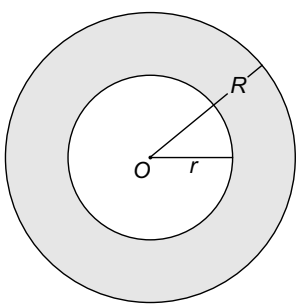


Fig. 5.18

$$M_2 = m \times \pi r^2 = \frac{Mr^2}{R^2 - r^2}$$

Since the two portions are concentric, the moment of inertia of the given disc about the given axis is

$$I = \frac{1}{2} M_1 R^2 - \frac{1}{2} M_2 r^2$$

$$= \frac{1}{2}\left[\frac{MR^4}{R^2 - r^2} - \frac{Mr^4}{R^2 - r^2}\right]$$

$$= \frac{M}{2}\left[\frac{R^4 - r^4}{R^2 - r^2}\right] = \frac{1}{2} M\left(R^2 + r^2\right)$$

EXAMPLE 21 Find the moment of inertia of a thin uniform rod of mass M and length L about an axis passing through its centre and making an angle θ with the rod.

SOLUTION Divide the rod into a very large number of extremely small elements each of length dx. Consider one such element at a distance x from the centre O of the rod (Fig. 5.19).

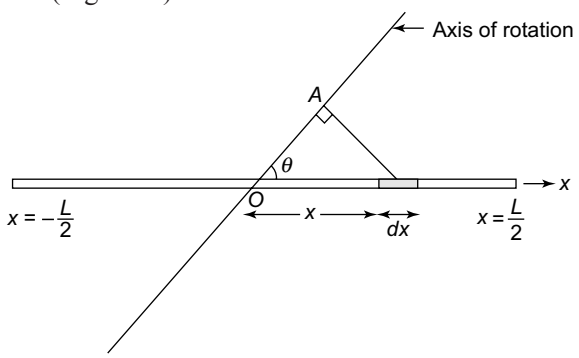


Fig. 5.19

Mass of element is $dm = \frac{M}{L} dx$

Perpendicular distance of the element from the axis of rotation is

$$r = OA = x \sin\theta$$

Moment of inertia of the rod about the given axis is

$$I = \int dm\, r^2 = \int \frac{M}{L} dx \times (x \sin\theta)^2$$

$$= \frac{M \sin^2\theta}{L} \int_{x=-\frac{L}{2}}^{x=+\frac{L}{2}} x^2 dx$$

$$= \frac{M \sin^2\theta}{L} \left|\frac{x^3}{3}\right|_{-L/2}^{+L/2}$$

$$= \frac{ML^2}{12} \sin^2\theta$$

If $\theta = 90°$, $I = \frac{ML^2}{12}$

EXAMPLE 22 The radius of gyration K of a hollow sphere of mass M and radius R about a certain axis is equal to R. Find the distance of that axis from the centre of the sphere.

SOLUTION Let x be the distance of the axis from the centre of the sphere [Fig. 5.20]. From parallel axes theorem

$$I_{XY} = I_{AB} + Mx^2$$

$$\Rightarrow \quad MK^2 = \frac{2}{3} MR^2 + Mx^2$$

Given $K = R$. Hence

$$R^2 = \frac{2}{3} R^2 + x^2 \Rightarrow x = \frac{R}{\sqrt{3}}$$

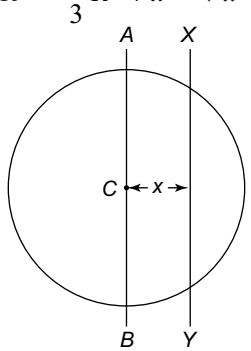


Fig. 5.20

EXAMPLE 23 Figure 5.21 shows a section (a part) of circular disc of radius R. The mass of the section is M. Find the moment of inertia of the section of the disc about an axis passing through its centre O and perpendicular to its plane.

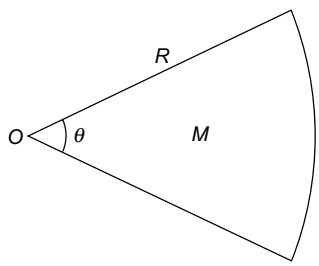


Fig. 5.21

SOLUTION Area of the section

$$A = \pi R^2 \times \frac{\theta}{2\pi} = \frac{R^2\theta}{2}$$

where θ is in radian [Fig 5.22].

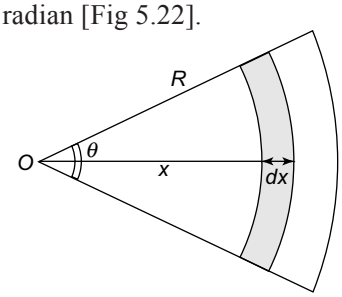


Fig. 5.22

$$\text{Mass per unit area} = \frac{M}{A} = \frac{2M}{R^2\theta}$$

$$\text{Area of strip is } dA = \left(\pi(x+dx)^2 - \pi x^2\right) \times \frac{\theta}{2\pi} \simeq 2\pi x dx \times \frac{\theta}{2\pi} = (xdx)\theta$$

$$\text{Mass of strip is } dm = \frac{2M}{R^2\theta} \times dA = \frac{2M}{R^2\theta} \times (xdx)\theta = \frac{2M}{R^2} xdx$$

∴ Moment of inertia of the section about the given axis is

$$I = \int dm x^2 = \frac{2M}{R^2} \int_0^R x^3 dx = \frac{1}{2} MR^2$$

EXAMPLE 24 Two particles of masses $m_1 = 1$ kg and $m_2 = 2$ kg are connected by a rigid bar of length $L = 1.2$ m of negligible mass. The system rotates about an axis perpendicular to the rod and at a distance x from mass m_1. Find the value of x for which the moment of inertia about the given axis is minimum. what is the minimum moment of inertia?

SOLUTION From Fig 5.23, it follows that

$$I = m_1 x^2 + m_2 (L-x)^2 \quad \text{(i)}$$

I will be minimum if $\frac{dI}{dx} = 0$ and $\frac{d^2I}{dx^2} > 0$

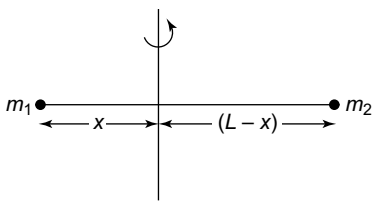


Fig. 5.23

Differentiating (i)

$$\frac{dI}{dx} = 2m_1 x + 2m_2 (L-x)(-1) = 0 \quad \text{(ii)}$$

$$\Rightarrow \quad x = \frac{m_2 L}{m_1 + m_2} = \frac{2 \times 1.2}{(1+2)} = 0.8 \text{ m}$$

Differentiating (ii)

$$\frac{d^2I}{dx^2} = 2m_1 + 2m_2, \text{ which is positive for any value of } x.$$

Substituting x in (i), we have

$$I_{\min} = m_1\left(\frac{m_2 L}{m_1+m_2}\right)^2 + m_2\left(L - \frac{m_2 L}{m_1+m_2}\right)^2 = \frac{m_1 m_2 L^2}{m_1+m_2} = \frac{1 \times 2 \times (1.2)^2}{1+2} = 0.96 \text{ kg m}^2$$

EXAMPLE 25 A giant wheel of radius 2.0 m and mass 100 kg is initially at rest. (a) What torque should be applied to it so that it acquires frequency of 300 r.p.m. in 10 s? (b) Find the kinetic energy when it is rotating at 300 r.p.m.

SOLUTION Given $\omega_0 = 0$, $\nu = 300$ r.p.m. $= \frac{300}{60}$ = 5 Hz. Therefore

$$\omega = 2\pi\nu = 10\pi \text{ rad s}^{-1} \text{ and } t = 10 \text{ s}$$

Using $\omega = \omega_0 + \alpha t$, we have $10\pi = 0 + 10\alpha$

$\Rightarrow \quad \alpha \quad \Rightarrow \quad \pi \text{ rad s}^{-2}$

(a) Torque required is $\tau = I\alpha = \left(\frac{1}{2} MR^2\right)\alpha$

$$= \left(\frac{1}{2} \times 100 \times 2^2\right) \times \pi = 200\pi = 628 \text{ Nm}$$

(b) K.E. $= \frac{1}{2} I\omega^2 = \frac{1}{2} \times \left(\frac{1}{2} \times 100 \times 2^2\right) \times (10\pi)^2$

$$= 10^4 \pi^2 \simeq 9.9 \times 10^4 \text{ J}$$

EXAMPLE 26 A stationary horizontal uniform disc of mass M and radius R is free to rotate about an axis passing through its centre and perpendicular to its plane. A torque $\tau = a\theta + b$ is applied to it, where θ is the angular displacement and a and b are positive constants. Obtain the expression for the angular velocity of the disc as a function of θ.

SOLUTION $\tau = I\alpha$

or $$\alpha = \frac{\tau}{I}$$

$$\Rightarrow \quad \frac{d\omega}{dt} = \frac{a\theta + b}{I}$$

$$\Rightarrow \quad \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \frac{a\theta+b}{I}$$

$$\Rightarrow \quad \omega\, d\omega = \left(\frac{a\theta+b}{I}\right) d\theta = \frac{a}{I}\theta\, d\theta + \frac{b}{I} d\theta$$

Integrating

$$\int_0^\omega \omega d\omega = \frac{a}{I}\int_0^\theta \theta d\theta + \frac{b}{I}\int_0^\theta d\theta$$

$$\Rightarrow \qquad \frac{\omega^2}{2} = \frac{a}{I}\frac{\theta^2}{2} + \frac{b\theta}{I}$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{1}{I}\left(a\theta^2 + 2b\theta\right)}$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{2}{MR^2}\left(a\theta^2 + 2b\theta\right)}$$

$$\left(\because I = \frac{1}{2}MR^2\right)$$

EXAMPLE 27 A thin uniform rod AB of mass M and length L is hinged at one end A to the horizontal floor. Initially it stands vertically. It is allowed to fall freely in a vertical plane.

(a) What is the angular acceleration of the rod when it is at an angle θ with the vertical?

(b) With what linear speed will the end B hit the floor?

SOLUTION

(a) The entire mass of the rod acts at its centre of mass C. $AC = L/2$. The magnitude of the torque due to weight Mg is

$$\tau = Mg \times r_\perp$$

$$= Mg \times AD = Mg \times \frac{L}{2}\sin\theta$$

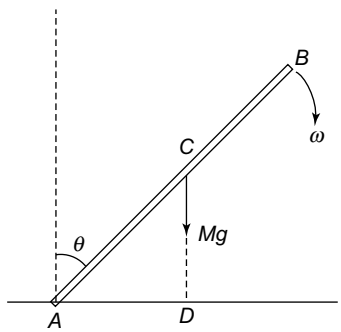


Fig. 5.24

Moment of inertia of the rod about A is $I = \frac{ML^2}{3}$

$$\therefore \quad \text{Angular acceleration } \alpha = \frac{\tau}{I} = \frac{MgL\sin\theta \times 3}{2 \times ML^2}$$

$$= \frac{3g\sin\theta}{L}$$

(b) When the end B hits the floor, the vertical distance through which C falls is $L/2$. From the law of conservation of energy,

Loss in P.E. = gain in K.E

$$Mg \times \frac{L}{2} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{3g}{L}}$$

Linear speed of end $B = L\omega = \sqrt{3gL}$

EXAMPLE 28 A particle of mass m is released from rest at point P located at a distance x_0 from origin O on the x-axis as shown in Fig. 5.25. It falls vertically along the negative y-axis.

(a) Find the magnitude and direction of the torque acting on the particle at time t when it reaches point Q whose position vector with respect to O is **r**.

(b) Find the magnitude of the angular momentum of the particle about O at this time t.

(c) Show that, in this example, $\tau = \frac{dL}{dt}$

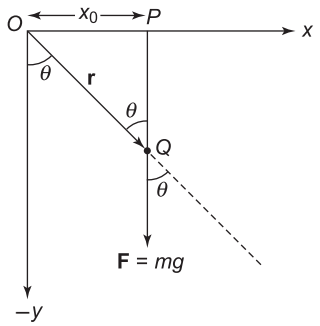


Fig. 5.25

SOLUTION

(a) The torque is due to force of gravity $\mathbf{F} = m\mathbf{g}$. The magnitude of the torque of **F** about O is

$$\tau = rF\sin\theta = r\,mg\sin\theta$$

$$= rmg\,\frac{x_0}{r} \qquad \left(\because \sin\theta = \frac{x_0}{r}\right)$$

$$\Rightarrow \qquad \tau = mg\,x_0$$

From right hand rule, the direction of the torque is into the page .

(b) The magnitude of angular momentum about O is

$$L = rp\sin\theta = rmv\sin\theta$$

From $v = u + at$ we have $v = 0 + gt = gt$. Therefore

$$L = rm \times gt \times \frac{x_0}{r} = mg\,x_0 t$$

(c) $$\frac{dL}{dt} = \frac{d}{dt}\left(mgx_0 t\right) = mgx_0 = \tau$$

20. Rolling Motion without Slipping

(1) *Total Kinetic Energy*

The total kinetic energy of a body which is moving as well as rotating is equal to the sum of translational K.E. and rotational K.E., i.e.

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

where m = mass of the body, v = linear velocity of its centre of mass, I = moment of inertia of the body about an axis passing through its centre of mass and ω = angular velocity of rotation.

(2) *Instantaneous Velocity of a Point on a Rolling Body*

Consider a wheel of radius $R(= AC = BC)$ rolling without slipping on a horizontal rough surface (Fig. 5.26).

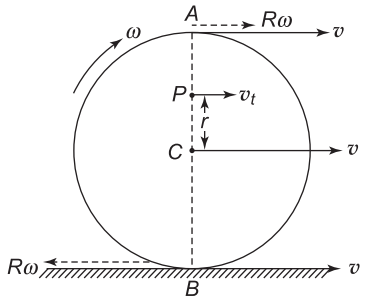


Fig. 5.26

Every point on the wheel has instantaneous velocity. For a point P at a distance r from the centre of mass C, the instantaneous velocity is the vector sum of velocity $\mathbf{v}$ of the centre of mass and tangential velocity $\mathbf{v}_t$ of point P relative to the centre of mass, i.e.

$$\mathbf{v}_P = \mathbf{v} + \mathbf{v}_t$$

where $v_t = r\omega$. Vector $\mathbf{v}_t$ is directed along the tangent to the circle of radius r about C.

For point A, $v_t = R\omega$ and $v = r\omega$. Since these velocities are in the same direction, the instantaneous velocity of A is

$$v_A = v + v_t = R\omega + R\omega = 2R\omega$$

For point B in contact with the horizontal surface,

$$v_B = v + v_t = R\omega - R\omega = 0$$

If a body rolls on a surface without slipping, the instantaneous velocity of the point of contact with the surface is zero.

(3) *A Body Rolling without Slipping on a Rough Horizontal Surface*

Horizontal force F is applied at the centre of mass of a body (disc, ring, cylinder or sphere) of mass M and radius R (Fig. 5.27) on a rough horizontal surface. If f is the frictional force, and the body rolls without slipping $a_{CM} = \alpha R$.

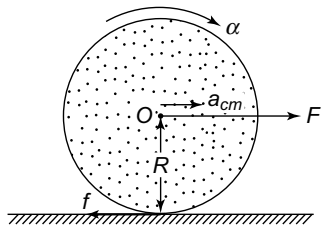


Fig. 5.27

$$F - f = Ma_{CM} \qquad \text{(i)}$$

$$\tau = fR = I\alpha \Rightarrow f = \frac{I\alpha}{R} = \frac{Ia_{CM}}{R^2} \qquad \text{(ii)}$$

From (i) and (ii), acceleration of centre of mass is

$$a_{CM} = \frac{F}{M\left(1 + \frac{I}{MR^2}\right)}$$

and

$$f = \frac{F}{\left(1 + \frac{MR^2}{I}\right)}$$

For a disc $I = \frac{1}{2}MR^2 \Rightarrow a_{CM} = \frac{2F}{3M}$

For a ring $I = MR^2 \Rightarrow a_{CM} = \frac{F}{2M}$

For a solid cylinder $I = \frac{1}{2}MR^2 \Rightarrow a_{CM} = \frac{2F}{3M}$

For a hollow cylinder $I = MR^2 \Rightarrow a_{CM} = \frac{F}{2M}$

For a solid sphere $I = \frac{2}{5}MR^2 \Rightarrow a_{CM} = \frac{5F}{7M}$

For a hollow sphere $I = \frac{2}{3}MR^2 \Rightarrow a_{CM} = \frac{3F}{5M}$

If the force is applied tangentially to the body as shown in Fig. 5.28, then

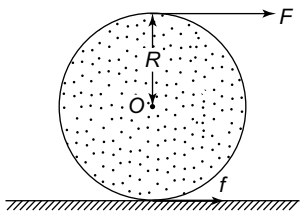


Fig. 5.28

$$a_{CM} = \frac{2F}{M\left(1+\frac{I}{MR^2}\right)}$$

and $$f = \frac{F\left(1-\frac{I}{MR^2}\right)}{1+\frac{I}{MR^2}}$$

(4) *A Body Rolling without slipping on a Rough Inclined Plane*

A body (ring, disc, cylinder or sphere) of mass M and radius R is rolling (without slipping) down a rough inclined plane of inclination θ (Fig. 5.29).

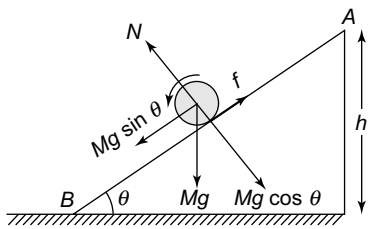


Fig. 5.29

For linear motion parallel to the plane

$$Mg \sin\theta - f = Ma \qquad \text{(i)}$$

where a = linear acceleration of the centre of mass.
For rotational motion about the axis through the centre of mass

$$\tau = I\alpha$$

$$\Rightarrow \qquad Rf = I\alpha \Rightarrow$$

$$f = \frac{I\alpha}{R} = \frac{Ia}{R^2} \qquad (\because a = R\alpha) \text{ (ii)}$$

I is the moment of inertia about the centre of mass.
Using (i) and (ii), we get

$$a = \frac{g\sin\theta}{\left(1+\frac{I}{MR^2}\right)}$$

For a ring $I = MR^2 \quad \Rightarrow \quad a = \frac{g\sin\theta}{2}$

For a disc $I = \frac{1}{2}MR^2 \quad \Rightarrow \quad a = \frac{2g\sin\theta}{3}$

For a solid cylinder $I = \frac{1}{2}MR^2 \quad \Rightarrow \quad a = \frac{2g\sin\theta}{3}$

For a hollow cylinder $I = MR^2 \quad \Rightarrow \quad a = \frac{g\sin\theta}{2}$

For a hollow sphere $I = \frac{2}{3}MR^2 \quad \Rightarrow \quad a = \frac{3g\sin\theta}{5}$

For a solid sphere $I = \frac{2}{5}MR^2 \quad \Rightarrow \quad a = \frac{5g\sin\theta}{7}$

Frictional force is $f = \frac{Ia}{R^2} = \frac{Mg\sin\theta}{\left(1+\frac{MR^2}{I}\right)}$

Condition for rolling without slipping

To prevent slipping, $f \le \mu N$, where μ is the coefficient of static friction between the body and the plane and $N = Mg\cos\theta$ is the normal reaction.
Hence to avoid slipping,

$$\frac{Mg\sin\theta}{1+\frac{MR^2}{I}} \le \mu\, Mg\cos\theta$$

$$\Rightarrow \qquad \mu \ge \frac{\tan\theta}{\left(1+\frac{MR^2}{I}\right)}$$

EXAMPLE 29 A solid cylinder of mass M and radius R is released from rest from top A of an inclined plane of height h and inclination θ as shown in Fig. 5.30. The cylinder rolls without slipping. Find (a) the speed at which it reaches bottom B of the plane and (b) the time it takes to reach B.

SOLUTION The acceleration of the cylinder is

$$a = \frac{g\sin\theta}{1+\frac{I}{MR^2}} = \frac{2g\sin\theta}{3} \qquad \left(\because I = \frac{1}{2}MR^2\right)$$

Distance travelled is $s = AB = \frac{h}{\sin\theta}$

(a) Using $v^2 - u^2 = 2as$, we have

$$v^2 - 0 = 2 \times \frac{2\,g\sin\theta}{3} \times \frac{h}{\sin\theta}$$

$$\Rightarrow \qquad v = \sqrt{\frac{4\,gh}{3}}$$

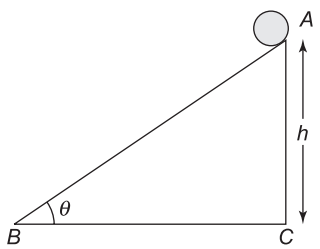


Fig. 5.30

Speed v can also be found from the law of conservation of energy. As the cylinder moves from A to B, it loses P.E. and gains K.E.

Loss in P.E. = gain in K.E.

or $$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

Now $$I = \frac{1}{2}MR^2 \text{ and } \omega = \frac{v}{R}. \text{ Therefore}$$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2} \times \frac{1}{2}MR^2 \times \left(\frac{v}{R}\right)^2$$

$$= \frac{3}{4}Mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{4gh}{3}}$$

(b) From $v = u + at$, we have

$$\sqrt{\frac{4gh}{3}} = 0 + \frac{2g\sin\theta}{3}t$$

$$\Rightarrow \quad t = \sqrt{\frac{3h}{g}} \cdot \frac{1}{\sin\theta}$$

EXAMPLE 30 A billiard ball has mass $M = 250$ g and radius $R = 2.5$ cm and is initially at rest. A rod held horizontal at a height h above centre C hits the ball. The ball begins to roll without slipping. Find the value of h [see Fig. 5.31].

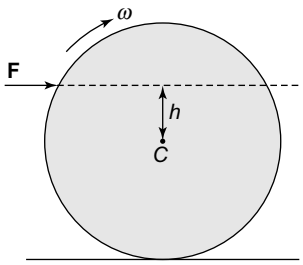


Fig. 5.31

SOLUTION The horizontal force **F** imparts a linear impulse

$$I = \int F dt = \text{change in linear momentum}$$

$$\Rightarrow \quad I = Mv - 0 = Mv = MR\omega \qquad \text{(i)}$$

where v is the velocity of the centre of mass of the ball. Since it rolls without slipping, $v = R\omega$, where ω is the angular velocity.

The torque due to **F** imparts an angular impulse

$$J = I\,h = \text{change in angular momentum}$$

$$= I\omega - 0 = I\omega$$

or $$Ih = \left(\frac{2}{5}MR^2\right)\omega \qquad \left(\because I = \frac{2}{5}MR^2\right) \text{ (ii)}$$

Dividing (ii) by (i)

$$h = \frac{\frac{2}{5}MR^2\omega}{MR\omega} = \frac{2R}{5} = \frac{2 \times 2.5 \text{ cm}}{5} = 1.0 \text{ cm}$$

EXAMPLE 31 A turntable of radius $R = 10$ m is rotating making 98 revolutions in 10 s with a boy of mass $m = 60$ kg standing at its centre. He starts running along a radius. Find the frequency of the turntable when the boy is 4 m from the centre. The moment of inertia of the turntable about is axis 1000 kg m^2.

SOLUTION Initial moment of inertia of the system is

M_1 = M.I. of turntable + M.I. of boy at the centre

$= 1000 + 0 = 1000$ kg m^2

Initial frequency $\nu_1 = 9.8$ rev/sec

Final moment of the system is

M_2 = M.I. of turntable + M.I. of boy at a distance 4 m from the centre of turn table

$= 1000 + 60 \times (4)^2 = 1960$ kg m^2

Since no external torque acts, the angular momentum of the system is conserved, i.e.

$$I_2\omega_2 = I_1\omega_1 \Rightarrow I_2\nu_2 = I_1\nu_1$$

$$\Rightarrow \quad \nu_2 = \frac{I_1\nu_1}{I_2} = \frac{1000 \times 9.8}{1960} = 5 \text{ rev/s}$$

$$= 5 \text{ Hz}$$

EXAMPLE 32 A uniform rod AB of length $L = 1$ m is sliding along two mutually perpendicular surfaces OP and OQ as shown in Fig. 5.32. When the rod subtends an angle $\theta = 30°$ with OQ, the end B has a velocity $\sqrt{3}$ ms^{-1}. Find the velocity of end A at that time.

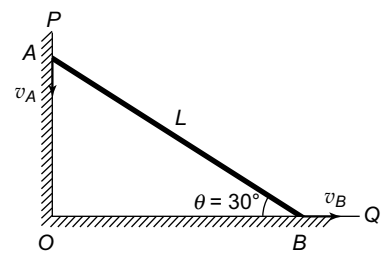


Fig. 5.32

SOLUTION $OB = x$, $OA = y$ and $x^2 + y^2 = L^2$ and $x = L\cos\theta$

Differentiating $x^2 + y^2 = L^2$ with respect to t we have

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\Rightarrow \qquad 2xv_B + 2yv_A = 0$$

$$\Rightarrow \qquad v_A = -\frac{x}{y}\, v_B$$

$$\therefore \qquad |v_A| = \frac{x}{y}\, v_B = v_B \cot\theta$$

$$= \sqrt{3} \times \cot 30°$$

$$= \sqrt{3} \times \frac{1}{\sqrt{3}} = 1 \text{ ms}^{-1}$$

EXAMPLE 33 A rope is wound around a hollow cylinder of mass $M = 3$ kg and radius $R = 40$ cm. If the rope is pulled with a force $F = 30$ N, find (a) the angular acceleration of the cylinder and (b) the linear acceleration of the rope.

SOLUTION For a hollow cylinder $I = MR^2$

(a) Torque on cylinder is $\tau = FR \Rightarrow I\alpha = FR$.

Therefore

$$\alpha = \frac{FR}{I} = \frac{FR}{MR^2} = \frac{F}{MR} = \frac{30}{3 \times 0.4}$$

$$= 25 \text{ rad s}^{-2}$$

(b) Linear acceleration of rope is

$$a = R\alpha = 0.4 \times 25 = 10 \text{ ms}^{-2}$$

EXAMPLE 34 A uniform rod AB of mass $M = 0.4$ kg and length $L = 1$ m lies on a horizontal frictionless table with its end A pivoted to the table. A ball of mass $m = 0.2$ kg moving along the surface of the table with velocity $u = 4$ ms^{-1} perpendicular to the rod collides with the free end B of the rod. If the collision is elastic, find (a) the velocity of the ball immediately after the collision and (b) the angular velocity of the rod after collision.

SOLUTION Refer to Fig. 5.33.

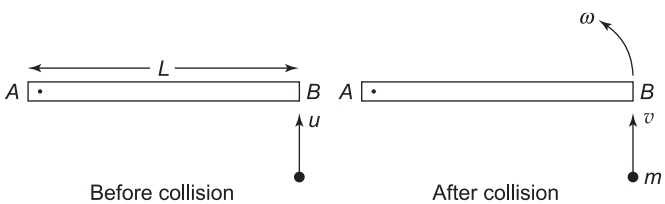


Fig. 5.33

(a) Let v be the velocity of the ball just after collision. Since the collision is perfectly elasitc, $e = 1$, i.e.

Velocity of approach = velocity of separation

or $$u = \omega L - v$$

$$\Rightarrow \qquad \omega = \frac{u + v}{L} \qquad \text{(i)}$$

Since there is no external torque, the angular momentum about A is conserved, i.e.

$$mu\, L = mvL + I\omega$$

$$= mvL + \frac{ML^2}{3}\, \omega$$

$$\Rightarrow \qquad \omega = \frac{3(u - v)m}{ML} \qquad \text{(ii)}$$

From (i) and (ii), we get

$$v = \left(\frac{3m - M}{3m + M}\right) u \qquad \text{(iii)}$$

$$= \left(\frac{3 \times 0.2 - 0.4}{3 \times 0.2 + 0.4}\right) \times 4 = 0.8 \text{ ms}^{-1}$$

(b) Using (iii) in (i), we get

$$\omega = \frac{6\, mu}{(3\, m + M)\, L}$$

$$= \frac{6 \times 0.2 \times 4}{(3 \times 0.2 + 0.4) \times 1} = 4.8 \text{ rad s}^{-1}$$

EXAMPLE 35 A uniform rod AB of mass M and length L is hinged at one end A. It is released from rest at a horizontal position.

The linear acceleration of the centre of mass as it falls is

(a) g (b) $\frac{g}{2}$

(c) $\frac{g}{4}$ (d) $\frac{3g}{4}$

SOLUTION Refer to Fig. 5.34.

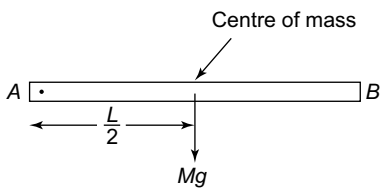


Fig. 5.34

$$\text{Torque about } A = Mg \times \frac{L}{2} \quad \Rightarrow \quad I\alpha = \frac{MgL}{2}$$

or $$\alpha = \frac{MgL}{2I} = \frac{3g}{2L} \qquad \left(\because I = \frac{ML^2}{3}\right)$$

Linear acceleration of centre of mass is

$$a_{CM} = \frac{L}{2} \times \alpha = \frac{L}{2} \times \frac{3g}{2L} = \frac{3g}{4}$$

EXAMPLE 36 A uniform rod of mass M and length l lies on a horizontal frictionless surface. A particle of mass m moving at a speed u perpendicular to the length of the rod strikes it at a distance $l/3$ from the centre and stops after the collision. The angular velocity of the rod about its centre just after the collision is

(a) $\frac{mu}{Ml}$ (b) $\frac{2mu}{Ml}$

(c) $\frac{3mu}{Ml}$ (d) $\frac{4mu}{Ml}$

SOLUTION Refer to Fig. 5.35.

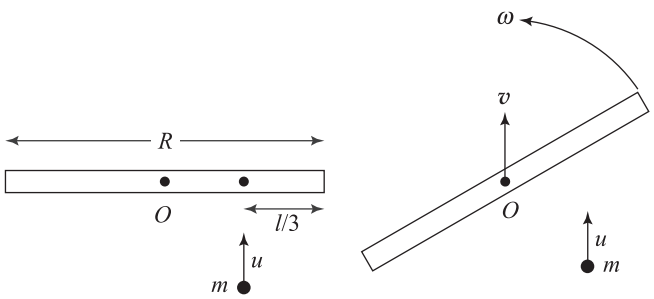


Fig. 5.35

Let **v** be the velocity of the centre O of the rod and ω its angular velocity about O. Since no net force acts on the system, the linear momentum of the system is conserved. Hence

$$mu = MV \Rightarrow V = \frac{mu}{M}$$

Since no net external torque acts on the system, the angular momentum of the system is also conserved. Since the rod is initially at rest, the angular momentum of the system before collision.

$$L_i = mu\left(\frac{l}{3}\right) = \frac{mu\,l}{3} \qquad \text{(i)}$$

After the collision, the angular momentum of the system is

$$\mathbf{L}_f = \mathbf{L}_{CM} + M(\boldsymbol{r} \times \boldsymbol{v})$$

Since **r** is parallel to **v**, $\mathbf{r} \times \mathbf{v} = 0$, Thus

$$\mathbf{L}_f = \mathbf{L}_{CM}$$

Since $L_f = L_i$, we have

$$L_i = L_{CM} = I\omega = \frac{Ml^2}{12}\omega \qquad \text{(ii)}$$

Using (i) in (ii)

$$\frac{mul}{3} = \frac{Ml^2\omega}{12}$$

$$\Rightarrow \quad \omega = \frac{4\,mu}{Ml}$$

So the correct choice is (d).

EXAMPLE 37 A solid sphere of mass M and radius R lies on a horizontal rough surface. A horizontal force F is applied at the centre of the sphere. If the sphere rolls without slipping, the acceleration of the centre of the sphere will be

(a) $\frac{F}{M}$ (b) $\frac{3F}{5M}$

(c) $\frac{2F}{3M}$ (d) $\frac{5F}{7M}$

SOLUTION Let f be the frictional force. Since the sphere rolls without slipping, the acceleration of its centre is (Fig. 5.36)

$$a = \alpha R$$

where α is the angular acceleration. The equation of motion is

$$F - f = Ma \qquad \text{(i)}$$

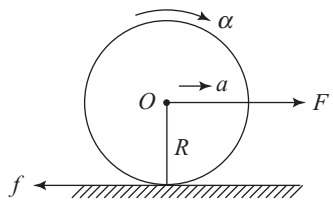


Fig. 5.36

Torque on the sphere is $\tau = fR$. Also $\tau = I\alpha$. i.e., $fR = I\alpha$.

Since $\alpha = \frac{a}{R}$,

$$f = \frac{I\alpha}{R} = \frac{Ia}{R^2} \qquad \text{(ii)}$$

From (i) and (ii), we have

$$F - \frac{Ia}{R^2} = Ma$$

$$\Rightarrow \quad a = \frac{F}{M\left(1 + \frac{I}{MR^2}\right)} \qquad \text{(iii)}$$

For a solid sphere, $I = \frac{2}{5}MR^2$. Using this in (iii) we get

$$a = \frac{5F}{7M}$$

EXAMPLE 38 In Example 37 above, the frictional force between the sphere and the surface is

(a) $\frac{2F}{7}$ (b) $\frac{2F}{5}$

(c) $\frac{2F}{3}$ (d) $\frac{F}{2}$

SOLUTION Using (iii) in (ii) in the above example, we get

$$f = \frac{F}{\left(1+\frac{MR^2}{I}\right)}$$

Putting $I = \frac{2}{5}MR^2$, we get $f = \frac{2F}{7}$

EXAMPLE 39 A solid sphere of mass M and radius R lies on a horizontal rough surface. A force F is applied tangentially on the topmost point A of the sphere as shown in Fig. 5.37. If the sphere rolls without slipping, the acceleration of its centre is

(a) $\frac{F}{M}$ (b) $\frac{3F}{5M}$

(c) $\frac{8F}{7M}$ (d) $\frac{10F}{7M}$

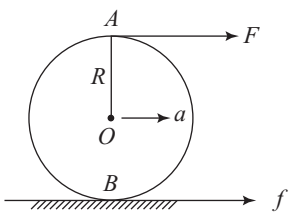


Fig. 5.37

SOLUTION Torque of force F on the sphere is $\tau = fR$. As this torque rotates the sphere, the point B of contact has a tendency to slip towards the left. To prevent slipping, the frictional force must act towards the right as shown in Fig. 5.37. Since there is no slipping, the angular acceleration about the centre O is

$$\alpha = \frac{a}{R}$$

where a is the linear acceleration of the centre O of the sphere. For the linear motion of the centre, the equation of motion is

$$F + f = Ma \qquad \text{(i)}$$

Now, clockwise torque due to F is $\tau_1 = F \times OA = FR$
Anticlockwise torque due to f is $\tau_2 = f \times OB = fR$
Net torque is $\tau = \tau_1 - \tau_2 = FR - fR = R(F - f)$ (ii)

Also $\tau = I\alpha = \left(\frac{2}{5}MR^2\right) \times \frac{a}{R} = \frac{2}{5}MaR$ (iii)

Equating (ii) and (iii), we get

$$F - f = \frac{2}{5}Ma \qquad \text{(iv)}$$

Adding (i) and (iv), we get

$2F = \frac{7}{5}Ma \Rightarrow a = \frac{10F}{7M}$, which is choice (d).

EXAMPLE 40 In Example 39 above, the frictional force between the sphere and the surface is

(a) $\frac{2F}{3}$ (b) $\frac{3F}{5}$

(c) $\frac{3F}{7}$ (d) F

SOLUTION Putting $a = \frac{10F}{7M}$ in Eq. (i) in the above Example, we have

$$F + f = M \times \frac{10F}{7M} = \frac{10F}{7}$$

$$\Rightarrow \qquad f = \frac{3F}{7}$$

So the correct choice is (c).

EXAMPLE 41 Two blocks of masses m_1 and m_2 ($m_1 > m_2$) are connected by a massless string going over a frictionless pulley of radius R and moment of inertia I about its axis. The system of blocks is released. If the string does not slip on the pulley and $m_1 = m$ and $m_2 = 2m$, the acceleration of the blocks is given by [see Fig. 5.38(a)]

(a) $a = \frac{g}{1+\frac{I}{mR^2}}$ (b) $a = \frac{g/2}{1+\frac{I}{2mR^2}}$

(c) $a = \frac{g/3}{1+\frac{I}{3mR^2}}$ (d) $a = \frac{g/4}{1+\frac{I}{4mR^2}}$

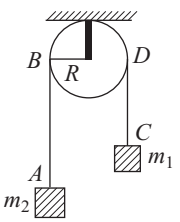


Fig. 5.38(a)

SOLUTION The tensions in strings AB and CD will be different because the pulley is not massless (it has a finite moment of inertia). Let T_1 and T_2 be the tensions in CD and AB. The free body diagrams of m_1 and m_2 are shown in the Fig. 5.38(b)

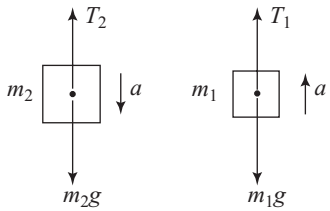


Fig. 5.38(b)

The equations of motion of m_1 and m_2 are

$$T_1 - m_1g = m_1a$$

$$\Rightarrow \quad T = m_1(a + g) \qquad \text{(i)}$$

$$m_2g - T_2 = m_2a$$

$$\Rightarrow \quad T_2 = m_2(g - a) \qquad \text{(ii)}$$

Clockwise torque on pulley by T_2 is

$$\tau_2 = T_2 R$$

Anticlockwise torque on pulley by T_1 is

$$\tau_1 = T_1 R$$

Net torque is $\quad \tau = \tau_1 - \tau_2 = R(T_1 - T_2)$

Also $\quad \tau = I\alpha = \dfrac{Ia}{R} \qquad (\because\ a = R\alpha)$

$$\Rightarrow \quad a = \frac{\tau R}{I} = \frac{R^2(T_1 - T_2)}{I} \qquad \text{(iii)}$$

Using (i) and (ii) in (iii) we have

$$a = \frac{R^2}{I}[m_1(a + g) - m_2(g - a)]$$

which gives $\quad a = \dfrac{(m_1 - m_2)\, g\, R^2}{I + (m_1 + m_2)\, R^2}$

Putting $m_1 = m$ and $m_2 = 2\ m$, we get

$$a = \frac{mgR^2}{I + 3mR^2}$$

$$\Rightarrow \quad a = \frac{g}{3 + \dfrac{I}{mR^2}}$$

1 SECTION — Multiple Choice Questions with One Correct Choice

(Level A)

1. A shell fired from a gun at an angle to the horizontal explodes in mid–air. Then the centre of mass of the shell fragments will move
   (a) vertically down
   (b) horizontally
   (c) along the same parabolic path along which the 'intact' shell, was moving
   (d) along tangent to the parabolic path of the 'intact' shell, at the point of explosion.
2. A child is standing at one end of a long trolley moving with a speed v on a smooth horizontal track. If the child starts running towards the other end of the trolley with a speed u, the centre of mass of the system (trolley + child) will move with a speed
   (a) zero (b) $(v + u)$
   (c) $(v - u)$ (d) v
3. Choose the only incorrect statement from the following:
   (a) The position of the centre of mass of a system of particles does not depend upon the internal forces between particles.
   (b) The centre of mass of a solid may lie outside the body of the solid.
   (c) A body tied to a string is whirled in a circle with a uniform speed. If the string is suddenly cut, the angular momentum of the body will change from its initial value.
   (d) The angular momentum of a comet revolving around a massive star, remains constant over the entire orbit.
4. A loaded spring gun of mass M fires a 'shot' of mass m with a velocity v at an angle of elevation θ. The gun is initially at rest on a horizontal frictionless surface. After firing, the centre of mass of the gun–shot system
   (a) moves with a velocity $v\ m/M$
   (b) moves with velocity $\dfrac{vm}{M}\cos\theta$ in the horizontal direction
   (c) remains at rest
   (d) moves with a velocity $\dfrac{v(M - m)}{(M + m)}$ in the horizontal direction.
5. If a man of mass M jumps to the ground from a height h and his centre of mass moves a distance x in the time taken by him to 'hit' the ground, the average force acting on him (assuming his retardation to be constant during his impact with the ground) is

(a) $M g h/x$ (b) $M g x/h$

(c) $M g (h/x)^2$ (d) $M g (x/h)^2$

6. Four particles of masses m, m, $2m$ and $2m$ are placed at the four corners of a square of side a as shown in Fig. 5.39. The (x, y) co-ordinates of the centre of mass are

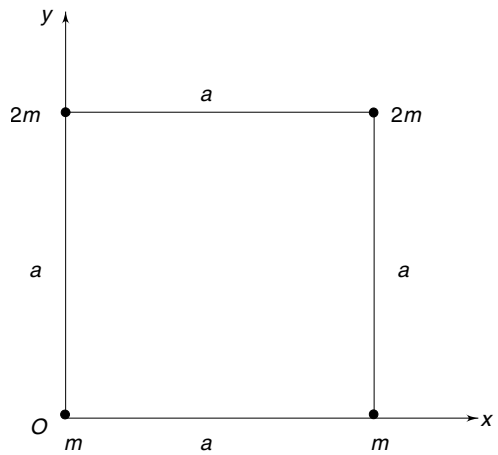


Fig. 5.39

(a) $\left(\frac{a}{2}, 2a\right)$ (b) $\left(\frac{a}{2}, a\right)$

(c) $\left(\frac{a}{2}, \frac{2a}{3}\right)$ (d) $\left(a, \frac{a}{3}\right)$

7. A carpet of mass M, made of an inextensible material, is rolled along its length in the form of a cylinder of radius R and kept on a rough floor. If the carpet is unrolled, without sliding, to a radius $R/2$, the decrease in potential energy is

(a) $\frac{1}{2}\ MgR$ (b) $\frac{5}{8}\ MgR$

(c) $\frac{3}{4}\ MgR$ (d) $\frac{7}{8}\ MgR$

8. Three point masses m_1, m_2 and m_3 are located at the vertices of an equilateral triangle of side a. What is the moment of inertia of the system about an axis along the altitude of the triangle passing through m_1?

(a) $(m_1 + m_2)\frac{a^2}{4}$ (b) $(m_2 + m_3)\frac{a^2}{4}$

(c) $(m_1 + m_3)\frac{a^2}{4}$ (d) $(m_1 + m_2 + m_3)\frac{a^2}{4}$

9. A molecule consists of two atoms, each of mass m, separated by a distance a. The moment of inertia of the molecule about its centre of mass is

(a) $2\ ma^2$ (b) ma^2

(c) $\frac{1}{2}\ ma^2$ (d) $\frac{1}{4}\ ma^2$

10. In Q. 9, if k is the average rotational kinetic energy of the molecule at room temperature, its frequency of rotation is

(a) $\frac{1}{\pi a}\sqrt{\frac{k}{m}}$ (b) $\frac{1}{2\pi a}\sqrt{\frac{k}{m}}$

(c) $\frac{1}{\pi a}\sqrt{\frac{2k}{m}}$ (d) $\frac{1}{2\pi a}\sqrt{\frac{2k}{m}}$

11. The moment of inertia of a solid sphere of mass M and radius R, about an axis through its centre, is $\frac{2}{5}\ MR^2$. The moment of inertia about an axis tangential to the surface of the sphere will be

(a) $\frac{4}{5}\ MR^2$ (b) MR^2

(c) $\frac{6}{5}\ MR^2$ (d) $\frac{7}{5}\ MR^2$

12. The moment of inertia of a uniform circular disc of mass M and radius R about any of its diameters is $\frac{1}{4}\ MR^2$. What is the moment of inertia of the disc about an axis passing through its centre and normal to the disc?

(a) MR^2 (b) $\frac{1}{2}\ MR^2$

(c) $\frac{3}{2}\ MR^2$ (d) $2\ MR^2$

13. In Q.18, what is the moment of inertia of the disc about an axis passing through a point on its edge and normal to the disc?

(a) MR^2 (b) $\frac{1}{2}\ MR^2$

(c) $\frac{3}{2}\ MR^2$ (d) $2\ MR^2$

14. When W joule of work is done on a flywheel, its frequency of rotation increases from ν_1 Hz to ν_2 Hz. The moment of inertia of the flywheel about its axis of rotation is given by

(a) $\frac{W}{2\pi^2\left(\nu_2^2 - \nu_1^2\right)}$ (b) $\frac{W}{2\pi^2\left(\nu_2^2 + \nu_1^2\right)}$

(c) $\frac{W}{4\pi^2\left(\nu_2^2 - \nu_1^2\right)}$ (d) $\frac{W}{4\pi^2\left(\nu_2^2 + \nu_1^2\right)}$

15. Two discs of moments of inertia I_1 and I_2 about their respective axes, rotating with angular frequencies ω_1 and ω_2 respectively, are brought into contact face to face with their axes of rotation coincident. The angular frequency of the composite disc will be

(a) $\frac{I_1\omega_1 - I_2\omega_2}{I_1 - I_2}$ (b) $\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$

(c) $\frac{I_2\omega_1 + I_1\omega_2}{I_1 + I_2}$ (d) $\frac{I_2\omega_1 - I_1\omega_2}{I_1 - I_2}$

(Level B)

16. A solid cylinder of mass M and radius R rolls down an inclined plane of height h. The angular velocity of the cylinder when it reaches the bottom of the plane will be

(a) $\frac{2}{R}\sqrt{gh}$ (b) $\frac{2}{R}\sqrt{\frac{gh}{2}}$

(c) $\frac{2}{R}\sqrt{\frac{gh}{3}}$ (d) $\frac{1}{2R}\sqrt{gh}$

17. In Q.22, the rotational kinetic energy of the cylinder when it reaches the bottom of the plane will be

(a) Mgh (b) $\frac{Mgh}{2}$

(c) $\frac{Mgh}{3}$ (d) $\frac{Mgh}{4}$

18. A light-weight boy holds two heavy dumb-bells of equal mass with outstretched arms while standing on a turn-table which is rotating at an angular frequency ω_1 when the dumb-bells are at a distance r_1 from the axis of rotation. The boy suddenly pulls the dumb-bells towards his chest until they are at a distance r_2 from the axis of rotation. The new angular frequency of rotation ω_2 of the turn-table will be equal to

(a) $\omega_1 \frac{r_2}{r_1}$ (b) $\omega_1 \frac{r_1^2}{r_2^2}$

(c) $\omega_1 \frac{r_1}{r_2}$ (d) $\omega_1 \frac{r_2^2}{r_1^2}$

19. A cord is wound around the circumference of a bicycle wheel (without tyre) of diameter 1 m. A mass of 2 kg is tied to the end of the cord and it is allowed to fall from rest. The weight falls 2 m in 4 s. The axle of the wheel is horizontal and the wheel rotates with its plane vertical. The angular acceleration produced is (Take $g = 10\ \text{ms}^{-2}$)

(a) 0.5 rad s^{-2} (b) 1.0 rad s^{-2}

(c) 2.0 rad s^{-2} (d) 4.0 rad s^{-2}

20. In Q. 19, the moment of inertia of the wheel about the horizontal axis is

(a) 10 kg m^2 (b) 20 kg m^2

(c) 30 kg m^2 (d) 40 kg m^2

21. If the earth were to suddenly contract to half its present size, without any change in its mass, the duration of the new day will be

(a) 6 hours (b) 12 hours

(c) 18 hours (d) 30 hours

22. Two circular discs have masses in the ratio of 1 : 2 and radii in the ratio of 2 : 1. The ratio of their moments of inertia about their diameter is

(a) 1 : 1 (b) 2 : 1

(c) 4 : 1 (d) 8 : 1

23. A circular ring of mass M and radius R is rotating about its axis at an angular frequency ω. Two blocks, each of mass m, are gently placed on the opposite ends of a diameter of the ring. The angular frequency of the ring becomes ω'. The ratio ω'/ω is

(a) $\frac{M}{(M+2m)}$ (b) $\frac{2M}{(M+2m)}$

(c) $\frac{2m}{M}$ (d) $\frac{M}{2m}$

24. A solid sphere rolls down from the top of an inclined plane. Its velocity on reaching the bottom of the plane is v. When the same sphere slides down from the top of the plane, its velocity on reaching the bottom is v'. The ratio v'/v is

(a) $\sqrt{\frac{3}{5}}$ (b) 1

(c) $\sqrt{\frac{7}{5}}$ (d) $\frac{3}{\sqrt{5}}$

25. A circular disc rolls down an inclined plane without slipping. What fraction of its total energy is translational?

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

26. A sphere rolls down an inclined plane without slipping. What fraction of its total energy is rotational?

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$

(c) $\frac{4}{7}$ (d) $\frac{5}{7}$

27. A circular disc is rolling down an inclined plane without slipping. If the angle of inclination is 30°, the acceleration of the disc down the inclined plane is

(a) g (b) $\frac{g}{2}$

(c) $\frac{g}{3}$ (d) $\frac{\sqrt{2}}{3}\ g$

28. A block of mass M is released from the top of an inclined plane. Its velocity on reaching the bottom of the plane is v. A circular disc of the same mass M rolls down the incline plane from the top. Its velocity on reaching the bottom is v'. The ratio v'/v will be

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{\frac{2}{3}}$

(c) 1 (d) $\frac{2\sqrt{2}}{3}$

29. Two circular loops A and B of radii R and $2R$ respectively are made of the same wire. Their moments of inertia about the axis passing through the centre and perpendicular to their plane are I_A and I_B respectively. The ratio I_A/I_B is

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

30. Two circular loops A and B are made of the same wire and their radii are in the ratio $1 : n$. Their moments of inertia about the axis passing through the centre and perpendicular to their plane are in the ratio $1 : m$. The relation between m and n is

(a) $m = n$ (b) $m = n^2$
(c) $m = n^3$ (d) $m = n^4$

31. A small coin is placed at a distance r from the centre of a gramophone record. The rotational speed of the record is gradually increased. If the coefficient of friction between the coin and the record is μ, the minimum angular frequency of the record for which the coin will fly off is given by

(a) $\sqrt{\frac{2\mu g}{r}}$ (b) $\sqrt{\frac{\mu g}{2r}}$

(c) $\sqrt{\frac{\mu g}{r}}$ (d) $2\sqrt{\frac{\mu g}{r}}$

32. In Q. 31, what would be the minimum angular frequency at which two identical coins, placed one on top of the other, at the same location on the record, will fly off?

(a) $\sqrt{\frac{\mu g}{r}}$ (b) $\sqrt{\frac{2\mu g}{r}}$

(c) $2\sqrt{\frac{\mu g}{r}}$ (d) $2\sqrt{\frac{2\mu g}{r}}$

33. Three particles, each of mass m, are placed at the corners of a right angled triangle as shown in Fig. 5.40. If $OA = a$ and $OB = b$, the position vector of the centre of mass is (here **i** and **j** are unit vectors along x and y axes respectively).

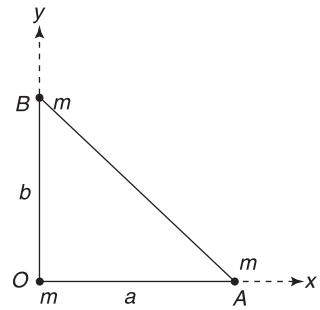


Fig. 5.40

(a) $\frac{1}{3}\ (a\mathbf{i} + b\mathbf{j})$ (b) $\frac{1}{3}\ (a\mathbf{i} - b\mathbf{j})$

(c) $\frac{2}{3}\ (a\mathbf{i} + b\mathbf{j})$ (d) $\frac{2}{3}\ (a\mathbf{i} - b\mathbf{j})$

34. Three particles each of mass m, are placed at the corners of an equilateral triangle of side a, as shown in Fig. 5.41. The position vector of the centre of mass is

(a) $\frac{a}{2}(\mathbf{i} + \mathbf{j}/\sqrt{3})$ (b) $\frac{a}{2}(\mathbf{3\ i} + \mathbf{j})$

(c) $\frac{a}{2}(\mathbf{3\ i} + \sqrt{3}\,\mathbf{j})$ (d) $\frac{a}{2}(\mathbf{3\ i} + \mathbf{j}/\sqrt{3})$

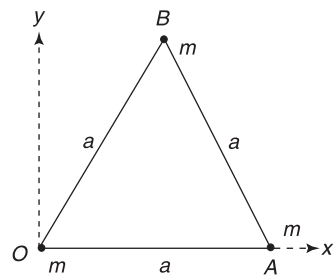


Fig. 5.41

35. A cylinder of mass m and radius r is rotating about its axis with a constant speed v. Its kinetic energy is

(a) $2\ mv^2$ (b) mv^2

(c) $\frac{1}{2}\ mv^2$ (d) $\frac{1}{4}\ mv^2$

36. A circular disc of mass m and radius r is rolling on a horizontal surface with a constant speed v. Its kinetic energy is

(a) $\frac{1}{4}mv^2$ (b) $\frac{1}{2}mv^2$

(c) $\frac{3}{4}mv^2$ (d) mv^2

37. Two solid spheres A and B, each of radius R, are made of materials of densities ρ_A and ρ_B respectively. Their moments of inertia about a diameter are I_A and I_B respectively. The ratio I_A/I_B is

(a) $\sqrt{\frac{\rho_A}{\rho_B}}$ (b) $\sqrt{\frac{\rho_B}{\rho_A}}$

(c) $\frac{\rho_A}{\rho_B}$ (d) $\frac{\rho_B}{\rho_A}$

38. A cylinder, released from the top of an inclined plane, rolls without sliding and reaches the bottom with speed v_r. Another identical cylinder, released from the top of the same inclined plane, slides without rolling and reaches the bottom with speed v_s. Then

(a) $v_r > v_s$ (b) $v_r < v_s$

(c) $v_r = v_s$ (d) $v_r = v_s = 0$

39. A sphere of mass M and radius R is released from the top of an inclined plane of inclination θ. The minimum coefficient of friction between the plane and the sphere so that it rolls down the plane without sliding is given by

(a) $\mu = \tan\theta$ (b) $\mu = \frac{2}{3}\tan\theta$

(c) $\mu = \frac{2}{5}\tan\theta$ (d) $\mu = \frac{2}{7}\tan\theta$

40. Three thin metal rods, each of mass M and length L, are welded to form an equilateral triangle. The moment of inertia of the composite structure about an axis passing through the centre of mass of the structure and perpendicular to its plane is

(a) $\frac{ML^2}{2}$ (b) $\frac{ML^2}{4}$

(c) $\frac{ML^2}{8}$ (d) $\frac{ML^2}{12}$

41. Four thin metal rods, each of mass M and length L, are welded to form a square $ABCD$ as shown in Fig. 5.42. What is the moment of inertia of the composite structure about a line which bisects rods AB and CD and perpendicular to the plane of the structure?

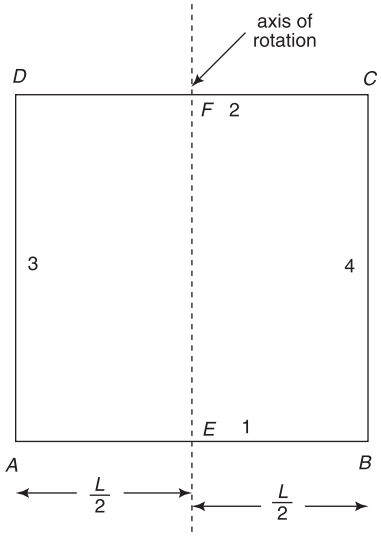


Fig. 5.42

(a) $\frac{ML^2}{6}$ (b) $\frac{ML^2}{3}$

(c) $\frac{ML^2}{2}$ (d) $\frac{2ML^2}{3}$

42. A thin uniform metallic triangular sheet of mass M has sides $AB = BC = L$. What is its moment of inertia about axis AC lying in the plane of the sheet? (See Fig. 5.43)

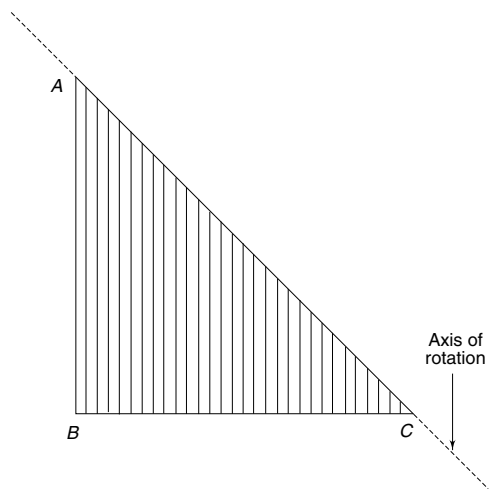


Fig. 5.43

(a) $\frac{ML^2}{12}$ (b) $\frac{ML^2}{6}$

(c) $\frac{ML^2}{3}$ (d) $\frac{2ML^2}{3}$

43. In the rectangular lamina $ABCD$ shown in Fig. 5.44, $a = AB = BC/2$. The moment of inertia of the lamina is the minimum along the axis passing through

(a) *BC* (b) *AB*

(c) *HF* (d) *EG*

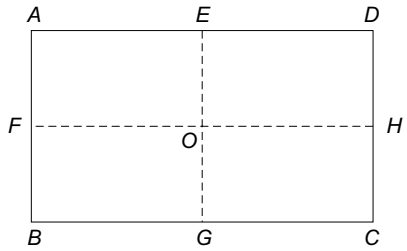


Fig. 5.44

44. The track shown in Fig. 5.45 ends in a circular track of radius r with centre at O. A small solid sphere of mass m rolls from rest without slipping from a point A at a height $h = 6r$ from the level ground. What is the speed of the sphere when it reaches a point B at height r above the level ground?

(a) $\sqrt{10gr}$ (b) $\sqrt{\frac{50}{7}gr}$

(c) $\sqrt{\frac{22}{7}gr}$ (d) zero

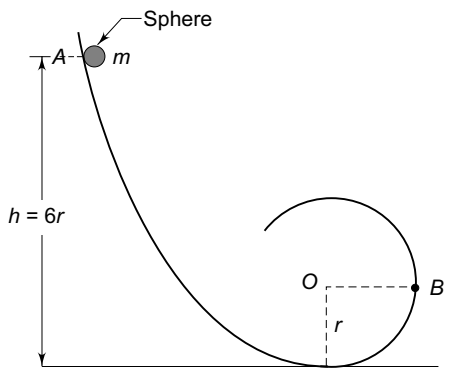


Fig. 5.45

45. In Q. 44 above, the horizontal force acting on the sphere when it reaches B is

(a) 10 mg (b) mg

(c) $\frac{50}{7}$ mg (d) $\frac{22}{7}$ mg

46. A solid sphere is rotating about a diameter at an angular velocity ω. If it cools so that its radius reduces to $\frac{1}{n}$ of its original value, its angular velocity becomes

(a) $\frac{\omega}{n}$ (b) $\frac{\omega}{n^2}$

(c) $n\omega$ (d) $n^2\omega$

47. In Q. 46 above, if the original rotational kinetic energy of the sphere is K, its new value will be

(a) $\frac{K}{n^2}$ (b) $\frac{K}{n^4}$

(c) n^2K (d) n^4K

48. A solid sphere is rotating about its diameter. Due to increase in room temperature, its volume increases by 0.5%. If no external torque acts, the angular speed of the sphere will

(a) increase by nearly $\frac{1}{3}$ %

(b) decrease by nearly $\frac{1}{3}$ %

(c) increase by nearly $\frac{1}{2}$ %

(d) decrease by nearly $\frac{2}{3}$ %

49. A cylinder of mass M has a length L that is $\sqrt{3}$ times its radius R. What is the ratio of its moment of inertia about its own axis and that about an axis passing through its centre and perpendicular to its axis?

(a) 1 (b) $\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

50. A uniform rod of length L is suspended from one end such that it is free to rotate about an axis passing through that end and perpendicular to the length. What minimum speed must be imparted to the lower end so that the rod completes one full revolution?

(a) $\sqrt{2gL}$ (b) $2\sqrt{gL}$

(c) $\sqrt{6gL}$ (d) $2\sqrt{2gL}$

51. The height of a solid cylinder is four times its radius. It is kept vertically at time $t = 0$ on a belt which is moving in the horizontal direction with a velocity $v = 2.45\ t^2$ where v is in ms^{-1} and t is in second. If the cylinder does not slip, it will topple over at time t equal to

(a) 1 s (b) 2 s

(c) 3 s (d) 4 s

52. A thin uniform rod AB of mass M and length L is hinged at one end A to the horizontal floor. Initially it stands vertically. It is allowed to fall freely on the floor in the vertical plane. The angular velocity of the rod when its end B strikes the floor is

(a) $\sqrt{\frac{g}{L}}$ (b) $\sqrt{\frac{2g}{L}}$

(c) $\sqrt{\frac{3g}{L}}$ (d) $2\sqrt{\frac{g}{L}}$

53. The moment of inertia of a thin rod of mass M and length L about an axis passing through the point at a distance $L/4$ from one of its ends and perpendicular to the rod is

(a) $\frac{7ML^2}{48}$ (b) $\frac{ML^2}{12}$

(c) $\frac{ML^2}{9}$ (d) $\frac{ML^2}{3}$

54. A circular disc of radius R is free to oscillate about an axis passing through a point on its rim and perpendicular to its plane. The disc is turned through an angle of 60° and released. Its angular velocity when it reaches the equilibrium position will be

(a) $\sqrt{\frac{g}{3R}}$ (b) $\sqrt{\frac{2g}{3R}}$

(c) $\sqrt{\frac{2g}{R}}$ (d) $2\sqrt{\frac{2g}{R}}$

55. A massless and inextensible cord is wound round the circumference of a circular ring of mass M and radius R. The ring is free to rotate about an axis passing through its centre and perpendicular to its plane. A mass m is attached at the free end of the cord and is at rest. The angular speed of the ring when mass m has fallen through at height h is

(a) $\sqrt{\frac{2gh}{R^2}}$ (b) $\sqrt{\frac{2mgh}{MR^2}}$

(c) $\sqrt{\frac{2mgh}{(M+m)R^2}}$ (d) $\sqrt{\frac{2mgh}{(M+2m)R^2}}$

56. A circular portion of diameter R is cut out from a uniform circular disc of mass M and radius R as shown in Fig. 5.46. The moment of inertia of the remaining (shaded) portion of the disc about an axis passing through the centre O of the disc and perpendicular to its plane is

(a) $\frac{15}{32}MR^2$ (b) $\frac{7}{16}MR^2$

(c) $\frac{13}{32}MR^2$ (d) $\frac{3}{8}MR^2$

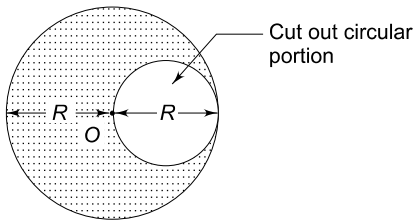


Fig. 5.46

57. The moment of inertia of a hollow sphere of mass M and internal and external radii R and $2R$ about an axis passing through its centre and perpendicular to its plane is

(a) $\frac{3}{2}MR^2$ (b) $\frac{13}{32}MR^2$

(c) $\frac{31}{35}MR^2$ (d) $\frac{62}{35}MR^2$

58. A man, standing on a turn-table, is rotating at a certain angular frequency with his arms outstretched. He suddenly folds his arms. If his moment of inertia with folded arms is 75% of that with outstretched arms, his rotational kinetic energy will

(a) increase by 33.3% (b) decrease by 33.3%

(c) increase by 25% (d) decrease by 25%

59. Two blocks of masses 1 kg and 2 kg are suspended at the end of a light string passing over a frictionless pulley of mass 4 kg and radius 10 cm. When the masses are released, the acceleration of the system is

(a) $\frac{g}{9}$ (b) $\frac{g}{7}$

(c) $\frac{g}{5}$ (d) $\frac{g}{3}$

60. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in second) applied tangentially. The force is then withdrawn. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion is reversed, is very nearly equal to

(a) $5\frac{1}{2}$ (b) $8\frac{1}{2}$

(c) $11\frac{1}{2}$ (d) $14\frac{1}{2}$

61. Four forces are applied to a wheel of radius 20 cm as shown in Fig. 5.47. The net torque produced by the forces is

(a) 5.4 Nm anticlockwise
(b) 1.8 Nm clockwise
(c) 2.0 Nm clockwise
(d) 5.4 Nm clockwise

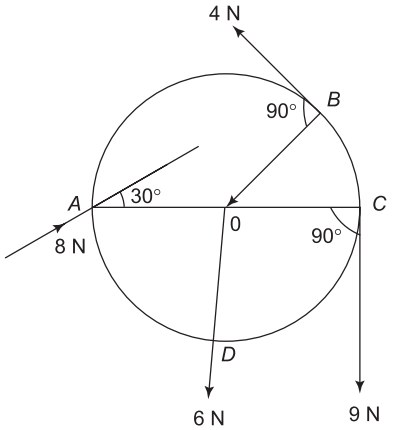


Fig. 5.47

62. A smooth uniform rod of length L and mass M has two identical beads of negligible size, each of mass m, which can slide freely along the rod. Initially the two beads are at the centre of the rod and the system is rotating with angular velocity ω_0 about its axis perpendicular to the rod and passing through its mid point (see Fig. 5.48). There are no external forces. When the beads reach the ends of the rod, the angular velocity of the system is

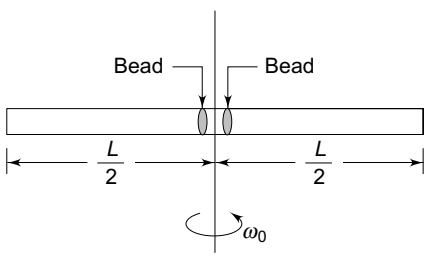


Fig. 5.48

(a) $\dfrac{M\omega_0}{M+3m}$ (b) $\dfrac{M\omega_0}{M+6m}$

(c) $\dfrac{(M+6m)\omega_0}{M}$ (d) ω_0

63. I_1, I_2, I_3 and I_4 are respectively the moments of inertia of a thin square plate $ABCD$ of uniform thickness about axes 1, 2, 3 and 4 which are in the plane of the plate. The moment of inertia of the plate about an axis passing through the centre O and perpendicular to the plane of the plate is (Fig. 5.49)

(a) $2(I_1 + I_2)$ (b) $2(I_3 + I_4)$
(c) $I_1 + I_3$ (d) $I_1 + I_2 + I_3 + I_4$

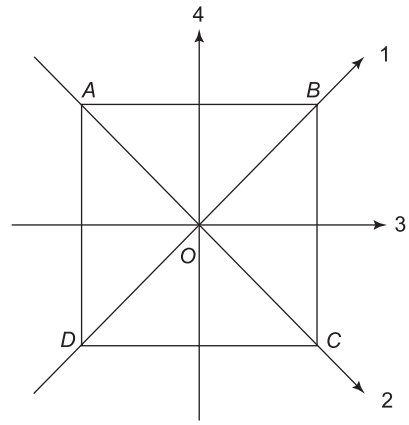


Fig. 5.49

64. A mass m is moving with a constant velocity along a line parallel to the x-axis, away from the origin. Its angular momentum with respect to the origin

(a) is zero (b) remains constant
(c) goes on increasing (d) goes on decreasing

65. A smooth sphere A is moving on a frictionless horizontal surface with angular speed ω and centre of mass velocity v. It collides elastically and head-on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B respectively. Then

(a) $\omega_A < \omega_B$ (b) $\omega_A = \omega_B$
(c) $\omega_A = \omega$ (d) $\omega_B = \omega$

66. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown in Fig. 5.50. The magnitude of angular momentum of the disc about the origin O is

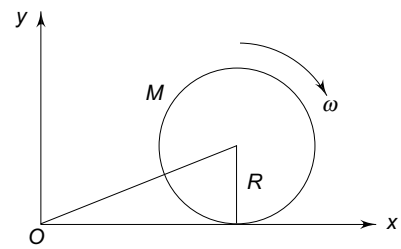


Fig. 5.50

(a) $\dfrac{1}{2}\ MR^2\ \omega$ (b) $MR^2\ \omega$

(c) $\dfrac{3}{2}\ MR^2\ \omega$ (d) $2\ MR^2\ \omega$

67. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown in Fig. 5.51. The moment of inertia of the loop about the axis XX' is

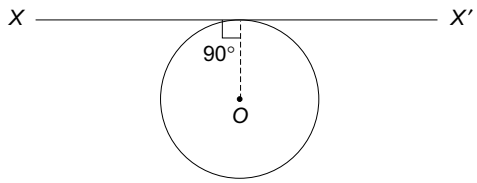


Fig. 5.51

(a) $\frac{\rho L^3}{8\pi^2}$ (b) $\frac{\rho L^3}{16\pi^2}$

(c) $\frac{5\rho L^3}{16\pi^2}$ (d) $\frac{3\rho L^3}{8\pi^2}$

68. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is

(a) $\frac{M\omega^2 L}{2}$ (b) $M\omega^2 L$

(c) $\frac{M\omega^2 L}{4}$ (d) $\frac{M\omega^2 L^2}{2}$

69. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown in Fig. 5.52. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is

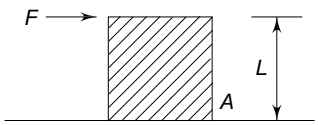


Fig. 5.52

(a) infinitesimal (b) $mg/4$

(c) $mg/2$ (d) $mg\,(1-\mu)$

70. Two particles A and B, initially at rest, move towards each other under a mutual force of attraction. At the instant when the speed of A is V and that *of* B is $2V$, the speed of the centre of mass of the system is

(a) 0 (b) V

(c) $1.5V$ (d) $3V$

71. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the center of mass is

(a) 30 m/s (b) 20 m/s

(c) 10 m/s (d) 5 m/s

72. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are:

(a) **up** the incline while ascending and **down** the incline while descending

(b) **up** the incline while ascending as well as descending

(c) **down** the incline while ascending and **up** the incline while descending

(d) **down** the incline while ascending as well as descending.

73. The angular momentum of a particle moving in a circular orbit with a constant speed remains conserved about

(a) any point on the circumference of the circle

(b) any point inside the circle

(c) any point outside the circle

(d) the centre of the circle

74. The angular velocity of a body changes from ω_1 to ω_2 without applying a torque but by changing the moment of inertia about its axis of rotation. The ratio of the corresponding radii of gyration is

(a) $\omega_1 : \omega_2$ (b) $\sqrt{\omega_1} : \sqrt{\omega_2}$

(c) $\sqrt{\omega_2} : \sqrt{\omega_1}$ (d) $\omega_2 : \omega_1$

75. Moment of inertia of uniform horizontal solid cylinder of mass M about an axis passing through its edge and perpendicular to the axis of the cylinder if its length is 6 times its radius R is:

(a) $\frac{39MR^2}{4}$ (b) $\frac{39MR^2}{8}$

(c) $\frac{49MR^2}{8}$ (d) $\frac{49MR^2}{4}$

76. If A is the areal velocity of a planet of mass M, its angular momentum is

(a) M (b) $2MA$

(c) A^2M (d) AM^2

77. One end of a thin uniform rod of length L and mass M_1 is rivetted to the centre of a uniform circular disc of radius 'r' and mass M_2 so that both are coplanar. The centre of mass of the combination from the centre of the disc is:

(Assume that the point of attachment is at the origin)

(a) $\frac{L(M_1 + M_2)}{2M_1}$ (b) $\frac{LM_1}{2(M_1 + M_2)}$

(c) $\frac{2L(M_1 + M_2)}{M_1}$ (d) $\frac{2LM_1}{(M_1 + M_2)}$

78. A body of mass 'm' is tied to one end of a spring and whirled round in a horizontal plane with a constant angular velocity. The elongation in the spring is one centimeter. If the angular velocity is doubled, the elongation in the spring is 5 cm. The original length of the spring is:
(a) 16 cm (b) 15 cm
(c) 14 cm (d) 13 cm

79. A particle performs uniform circular motion with an angular momentum L. If the angular frequency of the particle is doubled, and kinetic energy is halved, its angular momentum becomes:
(a) $4L$ (b) $2L$
(c) $\frac{L}{2}$ (d) $\frac{L}{4}$

80. A uniform rod of length 1 metre is bent at its mid-point to make 90° angle. The distance of the centre of mass from the centre of the rod is
(a) 36.1 cm (b) 25.2 cm
(c) 17.7 cm (d) zero

81. A mass is whirled in a circular path with constant angular velocity and its angular momentum is L. If the string is now halved keeping the angular velocity the same, the angular momentum is
(a) $\frac{L}{4}$ (b) $\frac{L}{2}$
(c) L (d) $2L$

82. A solid metallic sphere of radius R having moment of inertia equal to I about its diameter is melted and recast into a solid disc of radius r of a uniform thickness. The moment of inertia of the disc about an axis passing through its edge and perpendicular to its plane is also equal to I. The ratio r/R is
(a) $\frac{2}{\sqrt{15}}$ (b) $\frac{2}{\sqrt{10}}$
(c) $\frac{2}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{2}}$

83. The mass per unit length of a non-uniform rod OP of length L varies as

$$m = k\,\frac{x}{L}$$

where k is a constant and x is the distance of any point on the rod from end O. The distance of the centre of mass of the rod from end O is
(a) $\frac{L}{3}$ (b) $\frac{2L}{3}$
(c) $\frac{L}{2}$ (d) $\frac{2L}{\sqrt{3}}$

84. A uniform thin rod of mass M and length L is hinged by a frictionless pivot at its end O, as shown Fig. 5.53. A bullet of mass m moving horizontally with a velocity v strikes the free end of the rod and gets embedded in it. The angular velocity of the system about O just after the collision is

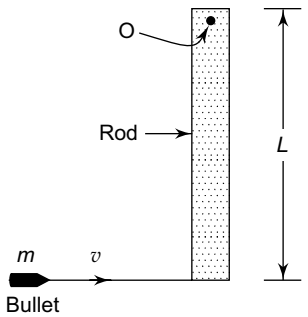


Fig. 5.53

(a) $\frac{mv}{L(M+m)}$ (b) $\frac{2mv}{L(M+2m)}$
(c) $\frac{3mv}{L(M+3m)}$ (d) $\frac{mv}{LM}$

85. A gramophone record of mass M and radius R is rotating at an angular velocity ω. A coin of mass m is gently placed on the record at a distance $r = R/2$ from its centre. The new angular velocity of the system is
(a) $\frac{2\omega M}{(2M+m)}$ (b) $\frac{2\omega M}{(M+2m)}$
(c) ω (d) $\frac{\omega M}{M}$

86. Two blocks of masses M and $2M$ are hanging from the ends AB of a uniform rod of mass $3M$ and length $3L$ as shown in Fig. 5.54. Assuming that the centre of the rod is at the origin (0, 0) and the strings have negligible mass, the x and y coordinates of the centre of mass of the system are
(a) $\frac{L}{2}, -\frac{3L}{4}$ (b) $\frac{L}{3}, -\frac{4L}{5}$
(c) $\frac{L}{4}, -\frac{5L}{6}$ (d) $\frac{L}{4}, -\frac{3L}{5}$

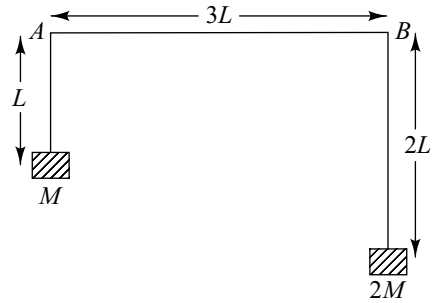


Fig. 5.54

87. A man of mass m is standing at end A of a stationary plank AB of wood of mass M and length L floating on a pond of water. If the man walks to the other end B, the plank will.

(a) remain stationary

(b) move through a distance $\frac{mL}{M}$ opposite to the direction of motion of the man

(c) move through a distance $\frac{mL}{2(M+m)}$ in the direction of motion of the man

(d) move through a distance $\frac{mL}{(M+m)}$ opposite to the direction of motion of the man.

88. In Q. 87 above, if the man walks with an average velocity v, the average velocity of the plank will be

(a) zero

(b) $\frac{vm}{M}$ to the left

(c) $\frac{vm}{m+M}$ to the left

(d) v to the right

89. A non-uniform bar AB of length L = 50.0 cm has a linear density $\lambda = 4x + 5$ where λ is in kg m^{-1} and x is in metre. The distance of the centre of mass of the bar from its midpoint is

(a) 1.4 cm (b) 2.6 cm

(c) 3.0 cm (d) 3.8 cm

90. A block of mass m is hung from a pulley of mass M and radius R as shown in Fig. 5.55. If $M = 4m$, the acceleration of the block when the system is released is

(a) $\frac{g}{4}$ (b) $\frac{g}{3}$

(c) $\frac{g}{2}$ (d) $\frac{2g}{3}$

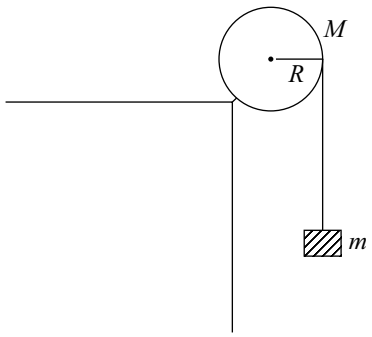


Fig. 5.55

91. A block of mass m is hung from a pulley mass M and radius R and then released from rest. Initially the block is at height h from the floor. The speed of the block when it strikes the floor if $M = 4m$ will be (see Fig. 5.56)

(a) $\sqrt{2gh}$ (b) $\sqrt{gh}$

(c) $\sqrt{\frac{2gh}{3}}$ (d) $\sqrt{\frac{gh}{2}}$

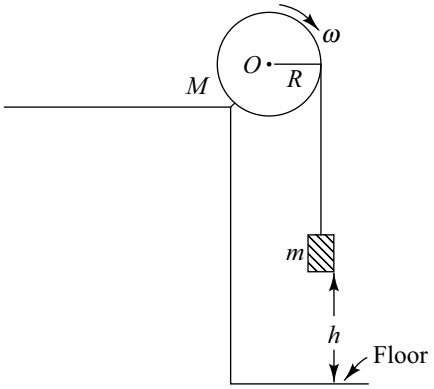


Fig. 5.56

92. A solid cylinder of mass M and radius R rolls (without slipping) down an inclined plane of inclination θ. The minimum coefficient of friction μ between the cylinder and the plane so that it rolls without slipping must be

(a) $\tan\theta$ (b) $\frac{\tan\theta}{2}$

(c) $\frac{\tan\theta}{3}$ (d) $\frac{\tan\theta}{4}$

93. In Q.92 above, the time taken by the block to hit the floor is

(a) $\sqrt{\frac{2h}{g}}$ (b) $\sqrt{\frac{3h}{g}}$

(c) $\sqrt{\frac{h}{g}}$ (d) $\sqrt{\frac{6h}{g}}$

94. A boy of mass m stands at the edge of a uniform disc of mass M and radius R rotating at angular frequency ω about an axis passing through its centre and perpendicular to its plane. The boy walks towards the centre of the disc and stops when he is at a distance r from the centre of the disc. If friction is neglected and $r = R/2$ and $M = 10\ m$, the angular frequency will be

(a) $\frac{8\omega}{7}$ (b) $\frac{5\omega}{3}$

(c) $\frac{9\omega}{5}$ (d) 2ω

95. A uniform bar AB of mass m and length L is fixed to a vertical wall. The bar can rotate about end A in the vertical plane. A supporting string has one end fixed to the well and the other end to the midpoint of the bar making an angle θ with it as shown in Fig. 5.57. A block of mass M hangs from end B of the bar. The system is in static equilibrium. If $\theta = 30°$ and $M = m$, the tension in the string is

(a) 2 mg (b) 4 mg

(c) 6 mg (d) 8 mg

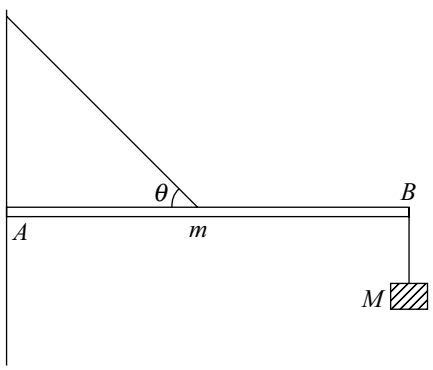


Fig. 5.57

96. In Q. 94 above, what is the contact force F in the case $M = m$ and $\theta = 30°$?

(a) $4\sqrt{2}\ mg$ (b) $3\sqrt{3}\ mg$

(c) $3\sqrt{5}\ mg$ (d) $2\sqrt{7}\ mg$

97. A block of mass $m_1 = m$ is attached to a block $m_2 = 2m$ by means of a light string going over a pulley which is a solid disc of mass $M = 4m$ and radius R. Initially the block of mass m_1 is on the floor and the block of mass m_2 is held at a height h above the floor as shown in Fig. 5.58. The block of mass m_2 is now released. The speed of this block just before it hits the floor is

(a) $\sqrt{\frac{2gh}{3}}$ (b) $\sqrt{\frac{gh}{2}}$

(c) $\sqrt{\frac{2gh}{5}}$ (d) $\frac{1}{2}\sqrt{3gh}$

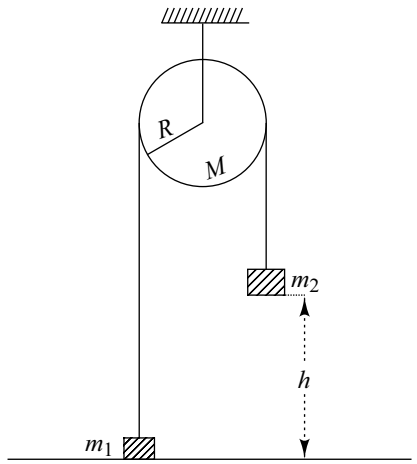


Fig. 5.58

98. A uniform disc of radius R is rolling (without shipping) with angular velocity ω. The instantaneous linear acceleration of the topmost point of the disc is (a_{cm} is the acceleration of the centre of mass of the disc)

(a) zero (b) $\frac{a_{cm}}{2}$

(c) a_{cm} (d) $2a_{cm}$

99. A uniform disc of mass M and radius R is attached to a block of mass m by means of a light string and a light pulley fixed at the top of an inclined plane of inclination θ. The string is wrapped around the disc. The disc rolls down the incline. If $M = 6m$ and $\theta = 30°$, the acceleration of the centre of mass of the disc is [see Fig. 5.59].

(a) $\frac{g}{6}$ (b) $\frac{g}{13}$

(c) $\frac{g}{16}$ (d) g

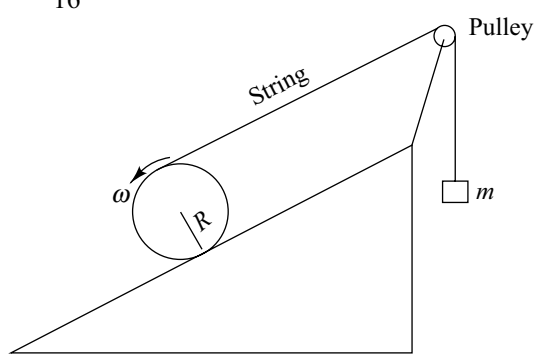


Fig. 5.59

100. Two uniform rods *AB* and *CD* of mass *M* and length 2*L* meet at right angles at their mid-points *O* to form a rigid assembly that can rotate about an axis passing through *O* and perpendicular to the page. Four tiny steel balls, each of mass *m* are fixed at *A*, *B*, *C* and *D* as shown in Fig. 5.60. The moment of inertia of the system about the given axis is

(a) $\frac{2}{3}(M+6m)L^2$ (b) $\frac{2}{3}(M+2m)L^2$

(c) $\frac{1}{12}(2M+\frac{m}{4})L^2$ (d) $(2M+4m)L^2$

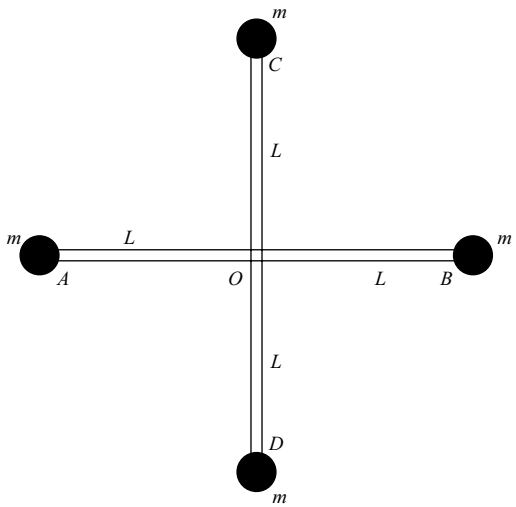


Fig. 5.60

101. Two objects of masses m_1 and m_2 are lying at rest a certain distance apart on the *x*-axis. A constant force *F* acts on m_2 at time $t = 0$ as shown in Fig. 5.61. The velocity of the centre of mass of the system at time *t* will be ($m_2 > m_1$).

(a) zero (b) $\frac{Ft}{m_2}$

(c) $\frac{Ft}{(m_1+m_2)}$ (d) $\frac{Ft}{(m_2-m_1)}$

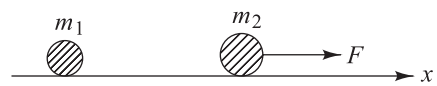


Fig. 5.61

102. Two particles of the same mass move in the *x* – *y* plane with velocities $2\hat{\mathbf{i}}$ ms^{-1} and $3\hat{\mathbf{j}}$ ms^{-1}. The first particle is given an acceleration $(2\hat{\mathbf{i}} + 2\hat{\mathbf{j}})ms^{-2}$ and the second particle in given an acceleration $(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}})ms^{-2}$. The centre of mass of the system of two particles will move along a

(a) straight line (b) circle

(c) ellipse (d) parabola

103. Three rods, each of mass *M* and length *L*, are welded together to form an equilateral triangle as shown in Fig. 5.62. The moment of inertia of the triangular frame about an axis passing through its centroid *O* and perpendicular to its plane is

(a) $\frac{ML^2}{12}$ (b) $\frac{ML^2}{6}$

(c) $\frac{ML^2}{3}$ (d) $\frac{ML^2}{2}$

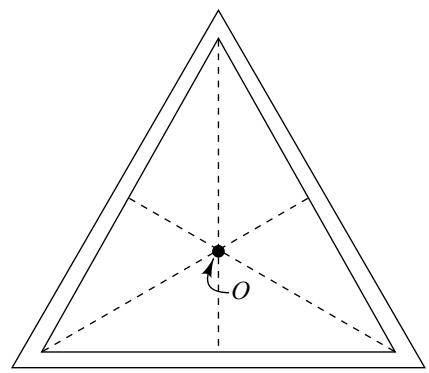


Fig. 5.62

104. A thin square plate has mass *M* and side *L*. Four holes, each of radius *R*, are cut out as shown in Fig. 5.63. The ratio of moment of inertia of the remaining portion to the moment of inertia of the complete plate is

(a) 0.10 (b) 0.18

(c) 0.26 (d) 0.32

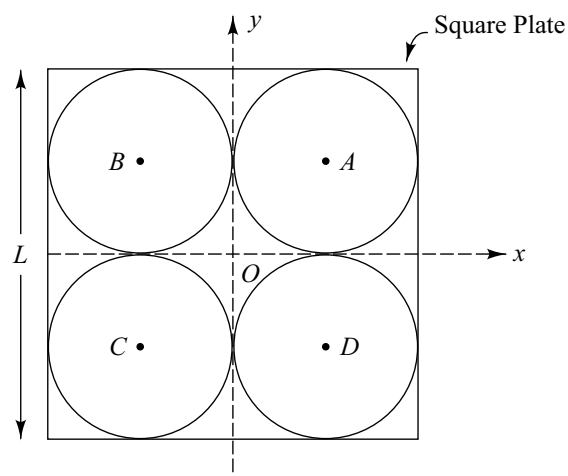


Fig. 5.63

Answers

(Level A)

1. (c)	2. (d)	3. (c)	4. (c)
5. (a)	6. (c)	7. (d)	8. (b)
9. (c)	10. (a)	11. (d)	12. (b)
13. (c)	14. (a)	15. (b)	

(Level B)

16. (c)	17. (c)	18. (b)	19. (a)
20. (b)	21. (a)	22. (b)	23. (a)
24. (c)	25. (d)	26. (a)	27. (c)
28. (b)	29. (d)	30. (c)	31. (c)
32. (a)	33. (a)	34. (a)	35. (d)
36. (c)	37. (c)	38. (b)	39. (d)
40. (a)	41. (d)	42. (a)	43. (c)
44. (b)	45. (c)	46. (d)	47. (c)
48. (b)	49. (a)	50. (c)	51. (a)
52. (c)	53. (a)	54. (b)	55. (c)
56. (c)	57. (d)	58. (a)	59. (c)
60. (a)	61. (b)	62. (b)	63. (c)
64. (b)	65. (c)	66. (c)	67. (d)
68. (a)	69. (c)	70. (a)	71. (c)
72. (b)	73. (d)	74. (c)	75. (d)
76. (b)	77. (b)	78. (b)	79. (d)
80. (c)	81. (a)	82. (a)	83. (b)
84. (c)	85. (a)	86. (c)	87. (d)
88. (c)	89. (a)	90. (b)	91. (c)
92. (c)	93. (d)	94. (a)	95. (c)
96. (d)	97. (c)	98. (d)	99. (b)
100. (a)	101. (c)	102. (a)	103. (d)
104. (c)			

Solutions

(Level A)

1. Since there no external forces, the centre of mass will continue to maintain its original motion. Hence the correct choice is (c).
2. The speed of the centre of mass of the system will remain unchanged (= v). The speed of the centre of mass of a system can change only if a net external force acts on it. The forces involved (such as the action and reaction and frictional forces), when the child runs on the trolley, are internal to the (trolley + child) system.
3. The only incorrect statement is (c). Since no external torque acts on the body even after the string is cut, the angular momentum will remain unchanged.
4. Since there is not external force acting on the gun–shot system, the centre of mass of the system continues to remain at rest. Hence the correct choice is (c).
5. Since the centre of mass moves through a distance x, the average force F is given by

$$Fx = Mgh$$

or $$F = \frac{Mgh}{x}$$

Hence the correct choice is (a).

6. The (x, y) co-ordinates of the centre of mass are

$$x = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

and $$y = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$$

It is easy to show that $x = \frac{a}{2}$ and $y = \frac{2a}{3}$, which is choice (c).

7. The centre of mass of the whole carpet is originally at a height R above the floor. When the carpet unrolls itself and has a radius $R/2$, the centre of mass is at a height $R/2$ but the mass left over unrolled is $\frac{M(R/2)^2}{R^2} = \frac{M}{4}$. Hence the decrease in P.E. is

$$MgR - \frac{M}{4}g \cdot \frac{R}{2} = \frac{7}{8}\ MgR.$$

Hence the correct choice is (d).

8. Refer to Fig. 5.64. The moment of inertia about AD is

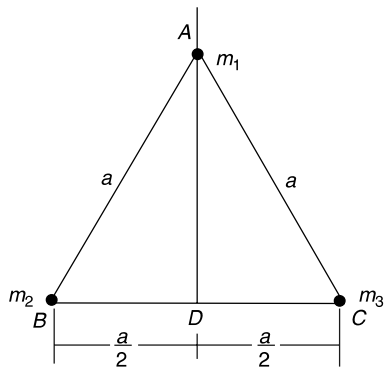


Fig. 5.64

$I = m_1 \times (\text{distance of } m_1 \text{ from } AD)^2$
$+ m_2 \times (\text{distance of } m_2 \text{ from } AD)^2$
$+ m_3 \times (\text{distance of } m_3 \text{ from } AD)^2$
$= m_1 \times 0 + m_2 \times (BD)^2 + m_3 \times (CD)^2$

$$= 0 + m_2 \times \left(\frac{a}{2}\right)^2 + m_3 \times \left(\frac{a}{2}\right)^2$$

$$= (m_2 + m_3)\ \frac{a^2}{4}$$

Hence the correct choice is (b).

9. Since the two atoms have the same mass, the centre of mass is at a distance of $a/2$ from each atom. Therefore, the moment of inertia of the molecule about its centre of mass is

$$I = m\left(\frac{a}{2}\right)^2 + m\left(\frac{a}{2}\right)^2 = \frac{ma^2}{2}$$

Hence the correct choice is (c).

10. Kinetic energy is $k = \frac{1}{2}\ I\omega^2$, which gives

$$\omega = \sqrt{\frac{2k}{I}} = \sqrt{\frac{2k}{ma^2} \times \frac{2}{1}} = \frac{2}{a}\sqrt{\frac{k}{m}}$$

$\therefore$ Frequency $\nu = \frac{\omega}{2\pi} = \frac{1}{\pi a}\sqrt{\frac{k}{m}}$, which is choice (a).

11. Given

$$I_c = \frac{2}{5}\ MR^2$$

Using the parallel axes theorem, the moment of inertia about an axis tangential to the sphere will be

$$I = I_c + MR^2 = \frac{2}{5}\ MR^2 + MR^2$$

$$= \frac{7}{5}\ MR^2$$

Hence the correct choice is (d).

12. Let us consider two perpendicular diameters, one along the x-axis and the other along the y-axis. Then

$$I_x = I_y = \frac{1}{4}\ MR^2$$

According to the perpendicular axes theorem, the moment of inertia of the disc about an axis passing through the centre is

$$I_c = I_x + I_y = \frac{1}{4}\ MR^2 + \frac{1}{4}\ MR^2$$

$$= \frac{1}{2}\ MR^2$$

which is choice (b).

13. Since the disc is uniform, its centre of mass coincides with its centre. Therefore, the moment of inertia of the disc about an axis passing through its centre of mass and normal to its plane is

$$I_{CM} = I_C = \frac{1}{2}\ MR^2$$

According to the theorem of parallel axes, the moment of inertia of the disc about an axis passing through a point on its edge and normal to its plane is given by

$$I_e = I_{CM} + Mh^2$$

$$= \frac{1}{2}\ MR^2 + MR^2 \qquad (\because\ h = R)$$

$$= \frac{3}{2}\ MR^2$$

Hence the correct choice is (c).

14. Work done = increase in kinetic energy or

$$W = \frac{1}{2}\ I\omega_2^2 - \frac{1}{2}\ I\omega_1^2$$

$$= \frac{I}{2}\ (\omega_2^2 - \omega_1^2)$$

$$= 2\pi^2\, I\,(\nu_2^2 - \nu_1^2) \qquad (\because\ \omega = 2\pi\nu)$$

or $$I = \frac{W}{2\pi^2\left(\nu_2^2 - \nu_1^2\right)}$$

Hence the correct choice is (a).

15. The angular frequency of the composite system can be obtained by using the principle of conservation of angular momentum.
Total initial angular momentum of the two discs = $I_1\omega_1 + I_2\omega_2$
Since the two discs are brought into contact face to face (one on top of the other) and their axes of rotation coincide, the moment of inertia I_c of the composite system will be equal to the sum of their individual moments of inertia, i.e.

$$I_c = I_1 + I_2$$

If ω_c is the angular frequency of the composite system, the final angular momentum of the system is

$$I_c\omega_c = (I_1 + I_2)\omega_c$$

Since no external torque acts on the system,
Final angular momentum = Initial angular momentum
or $(I_1 + I_2)\omega_c = I_1\omega_1 + I_2\omega_2$

or $$\omega_c = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

(Level B)

16. Let M be the mass of the cylinder and R be its radius. When it is at the top of the inclined plane of height h, its potential energy is Mgh (Fig. 5.65). As it rolls down the inclined plane, it moves along the plane and also rotates about an axis passing through its axis perpendicular to the plane of the figure, which shows its section in the plane of the paper. It thus acquires both kinetic energy of translation $\left(\frac{1}{2}Mv^2\right)$ and kinetic energy of rotation $\left(\frac{1}{2}I\omega^2\right)$ where v is its linear velocity and ω its angular velocity when it reaches the bottom of the plane. I is its moment of inertia about the axis mentioned above which is given by

$$I = \frac{1}{2}MR^2$$

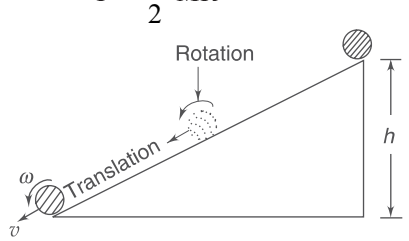


Fig. 5.65

From the law of conservation of energy, we have

Potential energy = Translational kinetic energy + Rotational kinetic energy

or $$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

or $$Mgh = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$$

$$(\because\ v = R\omega)$$

$$= \frac{3}{4}MR^2\omega^2$$

or $$\omega^2 = \frac{4gh}{3R^2}$$

or $$\omega = \frac{2}{R}\sqrt{\frac{gh}{3}}$$, which is choice (c).

17. The rotational kinetic energy = $\frac{1}{2}I\omega^2$.

Substituting for ω^2 and I, we have

Rotational kinetic energy = $\frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{4gh}{3R^2}$ = $\frac{Mgh}{3}$

Hence the correct choice is (c).

18. Let the mass of each dumb-bell be m. Then,

Total initial angular momentum = $mr_1^2\omega_1 + mr_1^2\omega_1$ = $2mr_1^2\omega_1$

When the boy pulls the dumb-bells towards his chest, let the new value of period be T_2. It is given that

$$r_2 = 10 \text{ cm}$$

$$\omega_2 = \frac{2\pi}{T_2}$$

Final angular momentum = $2mr_2^2\omega_2$

From the principle of conservation of angular momentum,

Initial angular momentum = Final angular momentum

or $$2mr_1^2\omega_1 = 2mr_2^2\omega_2$$

$$\frac{\omega_1}{\omega_2} = \frac{r_2^2}{r_1^2}$$

Hence the correct choice is (b).

19. We first find the linear acceleration a by using the relation

$$s = ut + \frac{1}{2}at^2$$

or $$2 = 0 + \frac{1}{2} \times a \times (4)^2$$

which gives $a = \frac{1}{4}$ ms^{-2}. Now $R = 0.5$ m. The angular acceleration α is

$$\alpha = \frac{a}{R} = \frac{1}{4 \times 0.5} = \frac{1}{2} = 0.5 \text{ rad s}^{-2}$$

Hence the correct choice is (a).

20. The torque produced by the force of the falling weight is

$$\tau = mg \times \text{moment arm} = mgR$$

$$= 2 \times 10 \times 0.5 = 10 \text{ Nm}$$

Now $$I = \frac{\tau}{\alpha} = \frac{10}{0.5} = 20 \text{ kg m}^2$$

Hence the correct choice is (b).

21. Let M be the mass and R the initial radius of the earth. If ω is the angular velocity of the rotation of the earth, the duration T of the day is

$$T = \frac{2\pi}{\omega}$$

Let R' be the radius of the earth after contraction and ω' its angular velocity. From the conservation of angular momentum, we have

$$I\omega = I'\omega'$$

where $I\left(=\frac{2}{5}MR^2\right)$ and $I'\left(=\frac{2}{5}MR'^2\right)$ are the moments of inertia of the earth before and after contraction, respectively.

$$\therefore \quad \frac{2}{5}MR^2\omega = \frac{2}{5}MR'^2\omega'$$

or $$\omega' = \frac{R^2\omega}{R'^2} = 4\omega \qquad (\because R' = R/2)$$

The duration T' of the new day will be (since $T = 2\pi/\omega$)

$$T' = \frac{2\pi}{\omega'} = \frac{2\pi}{4\omega} = \frac{T}{4}$$

$$T' = \frac{24 \text{ hours}}{4} = 6 \text{ hours}$$

22. $I_1 = \frac{1}{4}M_1R_1^2$ and $I_2 = \frac{1}{4}M_2R_2^2$. Therefore,

$$\frac{I_1}{I_2} = \frac{M_1}{M_2}\cdot\frac{R_1^2}{R_2^2} = \frac{1}{2}\times(2)^2 = 2$$

Hence the correct choice is (b).

23. From the law of conservation of angular momentum, we have

$$I\omega = I'\omega'$$

Here $I = MR^2$ and $I' = (M + 2m)R^2$. Therefore

$$\frac{\omega'}{\omega} = \frac{I}{I'} = \frac{M}{(M+2m)}$$

Hence the correct choice is (a).

24. For rolling :

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}\times\left(\frac{2}{5}MR^2\right)\times\frac{v^2}{R^2}$$

$$= \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2$$

$$\left(\because I = \frac{2}{5}MR^2 \text{ and } \omega = \frac{v}{R}\right)$$

For sliding :

$Mgh = \frac{1}{2}Mv'^2$. Therefore $\frac{1}{2}Mv'^2 = \frac{7}{10}Mv^2$

or $$\frac{v'}{v} = \sqrt{\frac{7}{5}}$$, which is choice (c).

25. Rotational kinetic energy is

$$(\text{KE})_r = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$$

$$= \frac{1}{4}MR^2\omega^2 \qquad (\because I = \frac{1}{2}MR^2)$$

where M is the mass of the disc and R its radius.

Translational kinetic energy is

$$(\text{KE})_t = \frac{1}{2}Mv^2 = \frac{1}{2}M(R\omega)^2 = \frac{1}{2}MR^2\omega^2$$

$$(\because v = R\omega)$$

Total energy, $\text{KE} = (\text{KE})_r + (\text{KE})_t = \frac{3}{4}MR^2\omega^2$

$$\therefore \quad \frac{(\text{KE})_t}{\text{KE}} = \frac{2}{3}$$

Hence the correct choice is (d).

26. $(\text{KE})_r = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 = \frac{1}{5}MR^2\omega^2$

$$(\because I = \frac{2}{5}MR^2)$$

$$(\text{KE})_t = \frac{1}{2}Mv^2 = \frac{1}{2}MR^2\omega^2$$

Total $\text{KE} = \frac{7}{10}MR^2\omega^2$

$$\therefore \quad \frac{(\text{KE})_r}{\text{KE}} = \frac{2}{7}$$

Hence the correct choice is (a).

27. Downward force $F = Mg\sin\theta$. The effective mass of the rolling disc is $M_{\text{eff}} = M + \frac{I_{\text{CM}}}{R^2}$, where I_{CM} is the moment of inertia about its centre of mass, which is

$$I_{\text{CM}} = \frac{1}{2}MR^2$$

$$\therefore \quad M_{\text{eff}} = M + \frac{1}{2}\frac{MR^2}{R^2} = \frac{3M}{2}$$

$$\therefore \quad \text{Acceleration } a = \frac{F}{M_{\text{eff}}} = \frac{Mg\sin\theta}{3M/2}$$

$$= \frac{2}{3}g\sin 30° = \frac{g}{3}$$

Hence the correct choice is (c).

28. The acceleration of the block sliding down the plane is

$$a = g\sin\theta$$

where θ is the angle of inclination. If l is the length of the inclined plane, the velocity of the block on reaching the bottom is given by

$$v^2 = 2\,al = 2g\sin\theta \times l$$

or $$v = \sqrt{2gl\sin\theta}$$

The acceleration of the disc rolling down the plane is (as shown above)

$$a' = \frac{2}{3}\,g\sin\theta$$

Therefore, the velocity of the disc on reaching the bottom is given by

$$v'^2 = 2a'l = \frac{4}{3}\,gl\sin\theta \text{ or } v' = 2\sqrt{\frac{gl\sin\theta}{3}}$$

$$\therefore \quad \frac{v'}{v} = \sqrt{\frac{2}{3}}$$

Hence the correct choice is (b).

29. Let μ be the mass per unit length of the wire. The mass of loop A is $M_A = 2\pi R\mu$ and mass of loop B is $M_B = 4\,\pi R\mu$. Their moments of inertia respectively are

$$I_A = M_A R_A^2 = 2\,\pi R\mu \times R^2 = 2\pi\,\mu\,R^3$$

and $$I_B = M_B R_B^2 = 4\,\pi R\mu \times (2R)^2 = 16\,\pi\mu R^3$$

$$\therefore \quad \frac{I_A}{I_B} = \frac{1}{8}$$

Hence the correct choice is (d).

30. Here $M_A = 2\pi\mu R$ and $M_B = 2\pi n\mu R$. Their moments of inertia are

$$I_A = M_A R_A^2 = 2\pi\mu R \times R^2 = 2\pi\mu R^3$$

and $$I_B = M_B R_B^2 = 2\pi n\mu R \times (nR)^2 = 2\pi n^3\mu R^3$$

$$\therefore \quad \frac{I_B}{I_A} = n^3; \text{ but } \frac{I_B}{I_A} = m \text{ (given)}$$

Thus, $m = n^3$. Hence the correct choice is (c).

31. Let m be the mass of the coin. It will fly off when the centripetal force $mr\omega^2$ just exceeds the force of friction μmg. The minimum ω is given by

$$mr\omega^2 = \mu mg$$

or $$\omega = \sqrt{\frac{\mu g}{r}}$$

Hence the correct choice is (c).

32. The minimum angular frequency is independent of the mass. Hence the correct answer is still $\sqrt{\mu g/r}$ which is choice (a).

33. The (x, y) co-ordinates of the masses at O, A and B respectively are (refer to Fig. 5.40 on page 5.25)

$(x_1 = 0, y_1 = 0)$, $(x_2 = a, y_2 = 0)$ and $(x_3 = 0, y_3 = b)$

The (x, y) co-ordinates of the centre of mass are

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{m\times 0 + m\times a + m\times 0}{m+m+m} = \frac{a}{3}$$

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{m\times 0 + m\times 0 + m\times b}{m+m+m} = \frac{b}{3}$$

The position vector of the centre of mass is $x_{CM}\,\mathbf{i} + y_{CM}\,\mathbf{j}$

$$= \frac{a}{3}\,\mathbf{i} + \frac{b}{3}\,\mathbf{j} = \frac{1}{3}(a\mathbf{i} + b\mathbf{j}),$$

which is choice (a).

34. Refer to Fig. 5.41 on page 5.25. The (x, y) co-ordinates of the masses at O, A and B respectively are

$$(x_1 = 0, y_1 = 0),\ (x_2 = a, y_2 = 0),\ \left(x_3 = \frac{a}{2}, y_3 = \frac{a\sqrt{3}}{2}\right)$$

Therefore, the (x, y) co-ordinates of the centre of mass are

$$x_{CM} = \frac{m\times 0 + m\times a + m\times a/2}{m+m+m} = \frac{a}{2}$$

$$y_{CM} = \frac{m\times 0 + m\times a + m\times a\sqrt{3}/2}{m+m+m} = \frac{a}{2\sqrt{3}}$$

$\therefore$ Position vector of centre of mass is $\frac{a}{2}\left(\mathbf{i} + \frac{\mathbf{j}}{\sqrt{3}}\right)$. Hence the correct choice is (a).

35. The kinetic energy (which is rotational) is $\frac{1}{2}I\omega^2$. The moment of inertia $I = \frac{1}{2}mr^2$ and $\omega = \frac{v}{r}$. Therefore, KE $= \frac{1}{2} \times \frac{1}{2}\,mr^2 \times \left(\frac{v}{r}\right)^2 = \frac{1}{4}\,mv^2$, which is choice (d).

36. The kinetic energy of a rolling disc consists of two parts: translational energy $= \frac{1}{2}\,mv^2$ and rotational energy $= \frac{1}{2}\,I\omega^2$.

$$\therefore \quad \text{KE} = \frac{1}{2}\,mv^2 + \frac{1}{2}\,I\omega^2$$

$$= \frac{1}{2}\ mv^2 + \frac{1}{2} \times \left(\frac{1}{2}mr^2\right) \times \left(\frac{v}{r}\right)^2$$

$$(\because I = \frac{1}{2}\ mr^2)$$

$$= \frac{1}{2}\ mv^2 + \frac{1}{4}\ mv^2 = \frac{3}{4}\ mv^2$$

Hence the correct choice is (c).

37. Mass of sphere A, $M_A = \frac{4}{3}\ \pi R^3 \rho_A$, mass of sphere B, $M_B = \frac{4}{3}\ \pi R^3 \rho_B$. Now, $I_A = \frac{2}{5}\ M_A R^2$ and $I_B = \frac{2}{5}\ M_B R^2$. Therefore,

$$\frac{I_A}{I_B} = \frac{M_A}{M_B} = \frac{\rho_A}{\rho_B}$$

Hence the correct choice is (c).

38. When the cylinder rolls without sliding, the acceleration down the plane is

$$a_r = \frac{2}{3}\ g \sin\theta$$

When the cylinder slides without rolling, the acceleration is

$$a_s = g \sin\theta$$

where θ is the inclination of the plane.

If h is the height of the inclined plane, their speeds on reaching the bottom are given by

$$v_r = \sqrt{2a_r h} \text{ and } v_s = \sqrt{2a_s h}$$

Since $a_s > a_r$, it follows that $v_s > v_r$, which is choice (b).

39. When the sphere rolls down the plane, its acceleration is given by

$$a = \frac{g \sin\theta}{1 + \frac{I}{MR^2}}$$

Now, the moment of inertia of the sphere about its diameter is

$$I = \frac{2}{5}\ MR^2$$

Therefore, $$a = \frac{g \sin\theta}{1 + \frac{2}{5}} = \frac{5}{7}\ g \sin\theta \qquad \text{(i)}$$

For rolling without sliding, the frictional force f provides the necessary torque τ which is given by

$$\tau = \text{force} \times \text{moment arm} = fR$$

But $\tau = I\alpha$, where α is the angular acceleration of the sphere. Thus, $I\alpha = fR$. Also, linear acceleration $a = \alpha R$. Therefore,

$$f = \frac{I\alpha}{R} = \frac{Ia}{R^2} = \frac{2}{5}\ Ma \left(\because I = \frac{2}{5}MR^2\right)$$

Now, force of friction = $\mu \times$ normal reaction = $\mu Mg \cos\theta$. Thus $\mu Mg \cos\theta = \frac{2}{5}\ Ma$

or $$a = \frac{5}{2}\ \mu g \cos\theta \qquad \text{(ii)}$$

Equating (i) and (ii) we have

$$\frac{5}{7}\ g \sin\theta = \frac{5}{2}\ \mu g \cos\theta$$

or $$\mu = \frac{2}{7}\ \tan\theta$$

Hence the correct choice is (d).

40. Given $PQ = QR = RP = L$. The centre of mass is located at centroid C which cuts lines PS, QT and UR in the ratio $2:1$. Let $h = CS = CT = UC$. In $\Delta\ PQS$, we have (see Fig. 5.66)

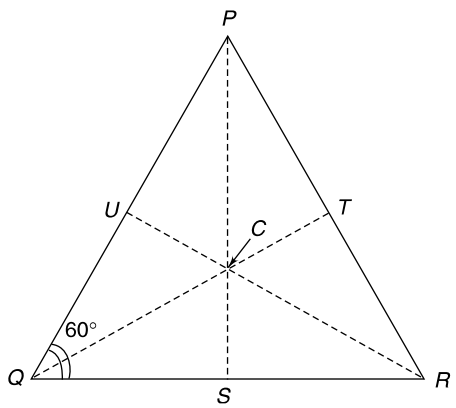


Fig. 5.66

$$PS = PQ \sin 60° = L \sin 60° = \frac{\sqrt{3}}{2}\ L.$$

$$\therefore \quad h = \frac{PS}{3} = \frac{1}{3} \times \frac{\sqrt{3}}{2}\ L = \frac{L}{2\sqrt{3}}$$

Since the structure consists of three identical rods, its moment of inertia about an axis passing through its centre of mass C and perpendicular to its plane is, from parallel axes theorem,

$$I_c = 3\ (I + Mh^2)$$

where I is the moment of inertia of each rod about the axis passing through its centre and perpendicular to its length, which is given by

$$I = \frac{M L^2}{12}$$

Also $$Mh^2 = M\left(\frac{L}{2\sqrt{3}}\right)^2 = \frac{M L^2}{12}$$

$$\therefore \quad I_c = 3\left(\frac{ML^2}{12} + \frac{ML^2}{12}\right)$$

$$= 3 \times \frac{ML^2}{6} = \frac{ML^2}{2}$$

Hence the correct choice is (a).

41. Refer to Fig. 5.43 on page 5.26. Moment of inertia is a scalar quantity. So the moment of inertia of the structure is the sum of the moments of inertia of the four rods about the specified axis of rotation, i.e.,

$$I = I_1 + I_2 + I_3 + I_4$$

where I_1 = moment of inertia of rod 1 about an axis passing through its centre E and perpendicular to its plane = $\frac{ML^2}{12}$,

I_2 = moment of inertia of rod 2 about an axis passing through its centre F and perpendicular to its plane = $\frac{ML^2}{12}$,

I_3 = moment of inertia of rod 3 about a parallel axis at a distance $\frac{L}{2}$ from it = $M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{4}$, and

I_4 = moment of inertia of rod 4 about a parallel axis at a distance $\frac{L}{2}$ from it = $\frac{ML^2}{4}$.

$$\therefore \quad I = \frac{ML^2}{12} + \frac{ML^2}{12} + \frac{ML^2}{4} + \frac{ML^2}{4}$$

$$= \frac{2}{3} ML^2, \text{ which is choice (d).}$$

42. Refer to Fig. 5.67. It is clear from the figure that the moment of inertia of triangular sheet $ABC = \frac{1}{2} \times$ moment of inertia of a square sheet $ABCD$ about its diagonal AC or $I_t = \frac{1}{2} I_s$. Now, mass of square sheet $= M + M = 2M$. Therefore,

$$I_s = (2M)\frac{L^2}{12} = \frac{ML^2}{6}$$

$$\therefore \quad I_t = \frac{I_s}{2} = \frac{ML^2}{12}$$

Hence the correct choice is (a).

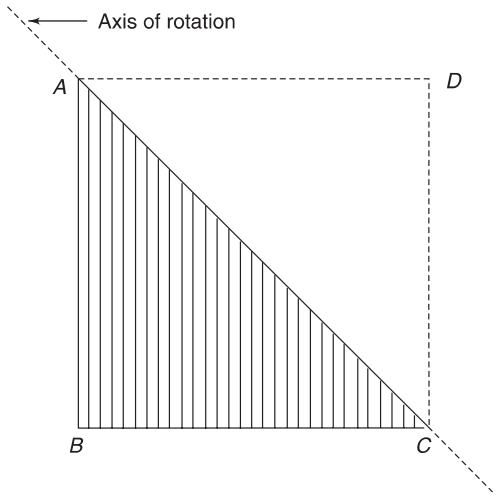


Fig. 5.67

43. $$I_{BC} = \frac{m(AB)^2}{3} = \frac{ma^2}{3}$$

$$I_{AB} = \frac{m(BC)^2}{3} = \frac{4}{3} ma^2$$

$$I_{HF} = \frac{m(AB)^2}{12} = \frac{ma^2}{12}$$

$$I_{EG} = \frac{m(BC)^2}{12} = \frac{ma^2}{3}$$

Thus, the moment of inertia about HF is the minimum, which is choice (c).

44. Let v be the speed of the sphere when it reaches B. Then, loss in PE = gain in translation KE + gain in rotational KE, i.e.

$$mg\,(6r - r) = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

or $$5\,mgr = \frac{1}{2} mv^2 + \frac{1}{2} \times \frac{2}{5} mr^2 \times \frac{v^2}{r^2}$$

$$= \frac{1}{2} mv^2 + \frac{1}{5} mv^2 = \frac{7}{10} mv^2$$

or $v = \sqrt{\frac{50gr}{7}}$, which is choice (b).

45. Horizontal force = $\frac{mv^2}{r} = \frac{m}{r} \times \frac{50gr}{7} = \frac{50mg}{7}$

Hence the correct choice is (c).

46. $I\omega = I_1\omega_1$. $I = \frac{2}{5}\ mr^2$, $I_1 = \frac{2}{5}m\left(\frac{r}{n}\right)^2 = \frac{I}{n^2}$. Hence

$$I\omega = \frac{I_1}{n^2}\ \omega_1$$

or $\omega_1 = n^2\omega$, which is choice (d).

47. $K = \frac{1}{2}\ I\omega^2$, $K_1 = \frac{1}{2}\ I_1\ \omega_1^2 = \frac{1}{2}\times\frac{I}{n^2}\ (n^2\omega)^2$

$$= \left(\frac{1}{2}I\omega^2\right)\ n^2 = n^2K.$$

Hence the correct choice is (c).

48. $V = \frac{4}{3}\ \pi r^3$ or $\log V = \log\left(\frac{4\pi}{3}\right) + 3\log r$.

Differentiating, we have

$$\frac{\delta V}{V} = 3\frac{\delta r}{r} \text{ or } \frac{\delta r}{r} = \frac{1}{3}\frac{\delta V}{V} = \frac{1}{3}\times 0.5\% = \frac{1}{6}\%$$

Since no external torque acts, $I\omega$ = constant or $\frac{2}{5}\ mr^2\omega$ = constant or $r^2\omega$ = constant (c)

or $2\log r + \log\omega = \log c$. Differentiating, we have

$$\frac{2\delta r}{r} + \frac{\delta\omega}{\omega} = 0$$

or $$\frac{\delta\omega}{\omega} = -2\frac{\delta r}{r} = -2\times\frac{1}{6}\% = -\frac{1}{3}\%$$

The negative sign indicates that ω decreases. Hence the correct choice is (b).

49. $I_1 = \frac{1}{2}\ MR^2$, $I_2 = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)$

$$= M\left(\frac{R^2}{4} + \frac{\left(\sqrt{3}R\right)^2}{12}\right) = \frac{1}{2}MR^2$$

$\therefore \frac{I_1}{I_2} = 1$, which is choice (a).

50. In one full revolution the increase in PE = MgL, where M is the mass of the rod. Therefore,

$$MgL = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$$

or $$\omega = \sqrt{\frac{6g}{L}}\ .\ \text{Now } v = L\omega = L\sqrt{\frac{6g}{L}} = \sqrt{6gL}$$

Hence the correct choice is (c).

51. The cylinder will topple when the torque mgr equals the torque $ma\ \frac{h}{2}$ (see Fig. 5.68)

or $$a = \frac{2gr}{h} = \frac{g}{2} \qquad (\because\ h = 4r) \qquad \text{(i)}$$

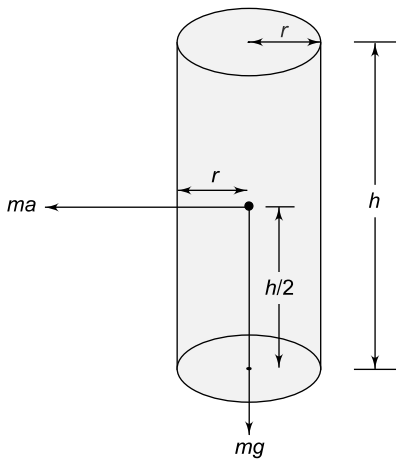


Fig. 5.68

Now $$v = 2.45\ t^2$$

$\therefore$ $$a = \frac{dv}{dt} = \frac{d}{dt}\left(2.45t^2\right) = 4.9t \qquad \text{(ii)}$$

Equating (i) and (ii), we get $t = \frac{g}{2\times 4.9} = \frac{9.8}{9.8} = 1$ s.

Hence the correct choice is (a).

52. Loss in PE = gain in rotational KE. As the centre of mass of the rod falls through a distance $L/2$, the loss in PE = $\frac{MgL}{2}$. Gain in KE = $\frac{1}{2}\ I\omega^2 = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$

Equating the two, we have

$$\frac{MgL}{2} = \frac{ML^2\omega^2}{6}$$

or $$\omega = \sqrt{\frac{3g}{L}}\ \text{, which is choice (c).}$$

53. Using the parallel axes theorem, $I = I_{CM} + Mx^2$ where x is the distance of the axis of rotation from the C.M. (centre of mass) of the rod, which is $x = \frac{L}{2} - \frac{L}{4} = \frac{L}{4}$. Also $I_{C.M.} = \frac{ML^2}{12}$. Hence

$I = \frac{ML^2}{12} + \frac{ML^2}{16} = \frac{7ML^2}{48}$, which is choice (a).

54. PE at $\theta = 60°$ is $Mgh\ (1 - \cos\theta)$ where h is the distance between the axis of rotation and the centre of mass of the disc. Gain in KE when the disc reaches the equilibrium position $= \frac{1}{2}\ I\omega^2$ where $I = I_{C.M.} + Mx^2 = \frac{1}{2}\ MR^2 + MR^2 = \frac{3}{2}\ MR^2$. Here x is the distance between the centre of mass and the axis of rotation, i.e. $x = R$.
Now PE = KE gives

$$MgR\ (1 - \cos 60°) = \frac{1}{2}\ I\omega^2 = \frac{3}{4}\ MR^2\ \omega^2 \qquad (\because\ h = R)$$

which gives $\omega = \sqrt{\frac{2g}{3R}}$, which is choice (b).

55. Loss in PE = gain in rotational KE. Thus

$$mgh = \frac{1}{2}\ (I + mR^2)\ \omega^2 = \frac{1}{2}\ (MR^2 + mR^2)\ \omega^2$$

$$= \frac{1}{2}\ R^2\ (M + m)\ \omega^2$$

or $\omega = \sqrt{\frac{2mgh}{(M + m)\ R^2}}$.

Hence the correct choice is (c).

56. Moment of inertia of complete disc about O is $I = \frac{1}{2}\ MR^2$. Mass of the cut-out part is $m = \left(\frac{M}{4}\right)$. The moment of inertia of the cut-out portion about its own centre $I_0 = \frac{1}{2}\ mr^2 = \frac{1}{2}\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2 = \frac{1}{32}\ MR^2$ because $r = R/2$. From the parallel axes theorem, the moment of inertia of the cut out portion about O is

$$I_c = I_0 + mr^2 = \frac{1}{32}\ MR^2 + \left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2$$

$$= \frac{3}{32}\ MR^2$$

$\therefore$ Moment of inertia of the shaded portion about O is $I_s = I - I_c = \frac{1}{2}\ MR^2 - \frac{3}{32}\ MR^2 = \frac{13}{32}\ MR^2$, which is choice (c).

57. We obtain the given hollow sphere as if a solid sphere of radius R has been removed from a solid sphere of radius $2R$. The mass of the given hollow sphere is (here ρ is the density of the material of the sphere)

$$M = M_1 - M_2$$

where $M_1 = \frac{4}{3}\ \pi\ (2R)^3\rho$ and $M_2 = \frac{4}{3}\ \pi R^3\rho$ are the masses of spheres of radii $2R$ and R respectively.

$$\therefore \qquad M = \frac{28}{3}\ \pi R^3\rho \qquad \text{(i)}$$

The moment of inertia of the given hollow sphere is

$$I = \frac{2}{5}\ M_1\ (2R)^2 - \frac{2}{5}\ M_2R^2$$

$$= \frac{2}{5} \times \frac{4}{3}\ \pi\ (2R)^3\ \rho(2R)^2 - \frac{2}{5} \times \frac{4}{3}\left(\pi R^3\rho\right)R^2$$

$$= \frac{2}{5}\ (32 - 1)\ \frac{4}{3}\ \pi R^5\rho \qquad \text{(ii)}$$

Using (i) in (ii), we get $I = \frac{62}{35}\ MR^2$, which is choice (d).

58. Let I_1 and ω_1 be the moment of inertia and angular frequency when his arms are outstretched and I_2 and ω_2 those when his arms are folded. Then

$$I_1\omega_1 = I_2\omega_2$$

Given $I_2 = \frac{3}{4}\ I_1$. Hence $I_1\omega_1 = \frac{3}{4}\ I_1\omega_2$ or $\omega_2 = \frac{4}{3}\ \omega_1$.

Initial KE is $K_1 = \frac{1}{2}\ I_1\omega_1^2$ and final KE is

$$K_2 = \frac{1}{2}\ I_2\ \omega_2^2 = \frac{1}{2} \times \frac{3I_1}{4} \times \left(\frac{4\omega_1}{3}\right)^2 = \frac{4}{3}\left(\frac{1}{2}\ I_1\omega_1^2\right)$$

$$= \frac{4}{3}\ K_1$$

$\therefore$ Percentage increase in KE

$$= \frac{K_2 - K_1}{K_1} \times 100 = \frac{\frac{4}{3}\ K_1 - K_1}{K_1} \times 100$$

$$= \frac{100}{3} = 33.3\%$$

Hence the correct choice is (a).

59. Refer to Fig. 5.69. When the masses are released, the torque is

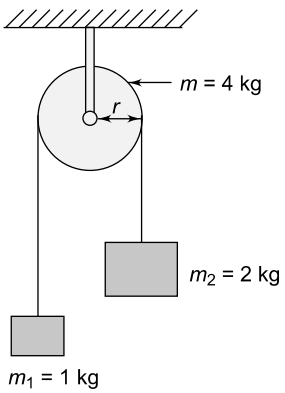


Fig. 5.69

$$\tau = (m_2 g - m_1 g) \times r$$
$$= (m_2 - m_1)\, gr \qquad \text{(i)}$$

But $\tau = I\alpha = I\,\dfrac{a}{r}$ $\qquad (\because a = r\alpha)$

where I = M.I. due to m_1 + M.I. due to m_2 + M.I. of pulley

$$= m_1 r^2 + m_2 r^2 + \frac{1}{2}\, mr^2$$

$$= \left(m_1 + m_2 + \frac{m}{2}\right) r^2$$

$$\therefore \qquad \tau = \left(m_1 + m_2 + \frac{m}{2}\right) ar \qquad \text{(ii)}$$

Equating (i) and (ii), we get

$$a = \frac{(m_2 - m_1)\, g}{\left(m_1 + m_2 + \frac{m}{2}\right)}$$

Putting m_1 = 1 kg, m_2 = 2 kg and m = 4 kg, we get $a = \dfrac{g}{5}$. Hence the correct choice is (c).

60. Torque $I\alpha = F \times r$ or $10\alpha = (20t - 5t^2) \times 2$ or $\alpha = (4t - t^2)$ rad s^{-2}. But $\alpha = \dfrac{d\omega}{dt}$. Therefore

$$\frac{d\omega}{dt} = 4t - t^2$$

Integrating, we have

$$\omega = \left(2t^2 - \frac{t^3}{3}\right) = t^2\left(2 - \frac{t}{3}\right) \text{ rad s}^{-1}$$

The pulley reverses its direction when ω = 0 momentarily, i.e. when $\left(2 - \dfrac{t}{3}\right) = 0$ or t = 6 s.

Now $\omega = \dfrac{d\theta}{dt}$. Thus

$$\frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$$

or $\qquad \theta = \dfrac{2t^3}{3} - \dfrac{t^4}{12} = \dfrac{t^3}{3}\left(2 - \dfrac{t}{4}\right)$ rad

For t = 6 s, $\theta = \dfrac{(6)^3}{3}\left(2 - \dfrac{6}{4}\right) = 36$ rad

One full rotation corresponds to $\theta = 2\pi$ rad. Therefore, number of rotations

$$= \frac{36}{2\pi} = \frac{36}{2 \times 3.14} = 5.7$$

Thus the closest choice is (a).

61. Magnitude of torque = $Fr \sin\theta$ where θ is the angle between the force vector and the radius vector. The torques are (see Fig. 5.48 on page 5.29)

$\tau_1 = 8\text{ N} \times 0.2\text{ m} \times \sin 30° = 0.8$ Nm (clockwise)

$\tau_2 = 4\text{ N} \times 0.2\text{ m} \times \sin 90° = 0.8$ Nm (anticlockwise)

$\tau_3 = 9\text{ N} \times 0.2\text{ m} \times \sin 90° = 1.8$ Nm (clockwise)

$\tau_4 = 6\text{ N} \times 0.2\text{ m} \times \sin 0° = 0$

$\therefore$ Net torque = + 0.8 Nm – 0.8 Nm + 1.8 Nm + 0 = 1.8 Nm clockwise. Hence the correct choice is (b).

62. From the principle of conservation of angular momentum, $I_0\omega_0 = I\omega$, where I_0 and ω_0 are the moment of inertia and angular velocity when the beads are at the centre of the rod and I and ω those when the beads are at the ends of the rod.

$$I_0 = \frac{ML^2}{12} \text{ and } I = \frac{ML^2}{12} + \frac{mL^2}{4} + \frac{mL^2}{4} = \frac{L^2}{12}\,(M + 6m)$$

$$\therefore \qquad \frac{ML^2}{12}\,\omega_0 = \frac{(M + 6m)\,\omega L^2}{12}$$

or $\qquad \omega = \dfrac{M\omega_0}{(M + 6m)}$

Hence the correct choice is (b).

63. Because of symmetry about axes 1 and 2, $I_1 = I_2$. Similarly, $I_3 = I_4$. From perpendicular axes theorem, it follows that the moment of inertia of the plate about an axis passing through the centre and perpendicular to the plane of the plate is

$$I = I_1 + I_2 = I_3 + I_4 = 2I_1 = 2I_3 \qquad (\because I_1 = I_2,\ I_3 = I_4)$$

or $\qquad I_1 = I_3$.

Thus $\qquad I = I_1 + I_2 = I_3 + I_4 = I_1 + I_3$.

Hence the correct choice is (c).

64. Refer to Fig. 5.70. The angular momentum of the mass at point $P(x, y)$ about origin O is defined as

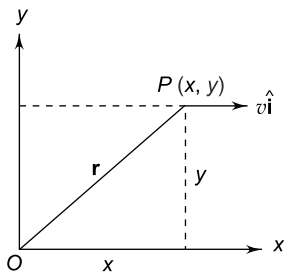


Fig. 5.70

$$\mathbf{L} = m\mathbf{r} \times \mathbf{v} = m\left(x\hat{\mathbf{i}} + y\hat{\mathbf{j}}\right) \times \left(v\hat{\mathbf{i}}\right)$$

$$= myv\left(-\hat{\mathbf{k}}\right) \qquad \left(\because \hat{\mathbf{i}} \times \hat{\mathbf{i}} = 0 \text{ and } \hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}\right)$$

Now m and v are constants. Also y remains constant as the mass moves parallel to the x-axis. Hence L remains constant. Thus the correct choice is (b).

65. Since there is no friction between the sphere and the horizontal surface and also between the spheres themselves, there will be no transfer of angular momentum from sphere A to sphere B due to the collision. Since the collision is elastic and the spheres have the same mass, the sphere A only transfers its linear velocity v to sphere B. Sphere A will continue to rotate with the same angular speed ω at a fixed location. Hence the correct choice is (c).

66. Refer to Fig. 5.71. Let $OC = R_C$ and let $\mathbf{v}_c$ be the velocity of the centre of mass of the disc. The linear momentum of the centre of mass is $\mathbf{p}_c = M\mathbf{v}_c$

If $\mathbf{L}_c$ is angular momentum of the disc about C, then the angular momentum about origin O is $\mathbf{L}_0 = \mathbf{L}_c + \mathbf{R}_c \times \mathbf{p}_c$

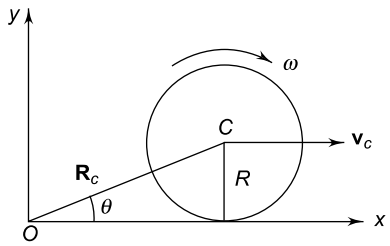


Fig. 5.71

$\therefore$ Magnitude $L_o = I_c\omega + R_c \times Mv_c \sin\theta$

$$= \frac{1}{2} MR^2\omega + MR_c v_c \sin\theta \qquad \left(\because I_c = \frac{1}{2} MR^2\right)$$

$$= \frac{1}{2} MR^2\omega + MR \times R\omega \qquad (\because R_c \sin\theta = R \text{ and } v_c = R\omega)$$

$$= \frac{3}{2} MR^2\omega$$

Hence the correct choice is (c).

67. Let m be the mass of the loop and r its radius. The moment of inertia of the loop about an axis passing through the centre O is (Fig. 5.72)

$$I_O = \frac{1}{2} mr^2$$

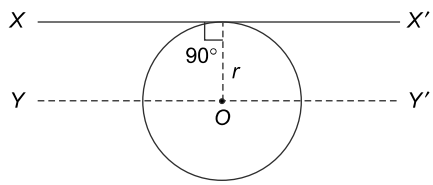


Fig. 5.72

From the parallel axes theorem, the moment of inertia about XX' is

$$I = I_O + mr^2 = \frac{1}{2} mr^2 + mr^2 = \frac{3}{2} mr^2$$

The mass of the loop, $m = \rho L$ and radius $r = L/2\pi$. Hence

$$I = \frac{3}{2} \times \rho L \times \left(\frac{L}{2\pi}\right)^2 = \frac{3\rho L^3}{8\pi^2}$$

Thus the correct choice is (d).

68. The entire mass of the liquid can be regarded to be concentrated at the centre of mass of the tube which is at a distance of $r = \frac{L}{2}$ from the axis of revolution. The force exerted by the liquid at the other end of the tube is the centripetal force of a mass M revolving in a circle of radius $r = \frac{L}{2}$. Thus

$$F_c = \frac{Mv^2}{r} = \frac{M(r\omega)^2}{r} = Mr\omega^2 = \frac{ML\omega^2}{2}$$

Hence the correct choice is (a).

69. Torque due to F about A is $\tau_1 = FL$

Since the weight mg acts through the centre of mass of the block (which is at a distance of $L/2$ from the side of the block) the torque due to weight mg about A is

$$\tau_2 = mg\left(\frac{L}{2}\right)$$

The minimum force required to topple the block is obtained when τ_1 is slightly greater than τ_2, i.e.

$$(\tau_1)_{\min} = \tau_2 \text{ or } F_{\min} L = mg\left(\frac{L}{2}\right) \text{ or } F_{\min} = \frac{mg}{2}$$

Hence the correct choice is (c).

70. Since no external force acts on the system, the centre of mass will remain at rest. Hence the correct choice is (a).

71. The velocity v_{CM} of the centre of mass can be obtained by using the principle of conservation of linear momentum,

$$MV = (M + m)\, v_{\text{CM}}$$

or
$$v_{\text{CM}} = \frac{MV}{(M+m)} = \frac{10\text{ kg} \times 14\text{ ms}^{-1}}{(10+4)\text{ kg}}$$

$$= 10\text{ ms}^{-1}$$

Hence the correct choice is (c).

72. When a cylinder rolls up or down an inclined plane, its angular acceleration is always directed down the plane. Hence the frictional force acts up the inclined plane when the cylinder rolls up or down the plane. Thus, the correct choice is (b).
73. The correct choice is (d).
74. The magnitude of angular momentum of a rotating body is given by $L = I\omega$. If no torque acts, the angular momentum is conserved, i.e. $I\omega$ = constant. Hence $I_1\,\omega_1 = I_2\,\omega_2$. If K_1 and K_2 are the corresponding radii of gyration, then $I_1 = MK_1^2$ and $I_2 = MK_2^2$. Hence

$$MK_1^2\,\omega_1 = MK_2^2\,\omega_2$$

or $$\frac{K_1}{K_2} = \frac{\sqrt{\omega_2}}{\sqrt{\omega_1}}$$, which is choice (c).

75. Given: $l = 6R$. From parallel axes theorem, the moment of inertia about the given axis is given by

$$I = M\left(\frac{R^2}{4} + \frac{l^2}{3}\right)$$

$$= M\left[\frac{R^2}{4} + \frac{(6R)^2}{3}\right]$$

$$= M\left(\frac{R^2}{4} + \frac{36R^2}{3}\right) = \frac{49\,MR^2}{4}$$

Hence the correct choice is (d).

76. Areal velocity $A = \dfrac{\text{area swept by radius vector}}{\text{time taken}}$

Assuming that the orbit of the planet is a circle of radius R, then

$$A = \frac{\pi R^2}{T}$$

Now, time period $T = \dfrac{2\pi}{\omega}$. Hence

$$A = \frac{\pi R^2}{2\pi/\omega} = \frac{R^2\omega}{2}$$

or $$\omega = \frac{2A}{R^2}$$

Angular momentum $L = I\omega = (MR^2) \times \dfrac{2A}{R^2} = 2\,MA$

Hence the correct choice is (b).

77. The mass of the rod can be considered to be concentrated at its centre ($x = L/2$) where $x = 0$ is the origin. Hence

$$R_{CM} = \frac{M_1 \times L/2 + 0}{M_1 + M_2} = \frac{LM_1}{2(M_1 + M_2)}$$

Hence the correct choice is (b).

78. Let L cm be the original length of the spring and k be the spring constant. Then

$$m(L + x_1)\,\omega_1^2 = kx_1$$

and $$m(L + x_2)\,\omega_2^2 = kx_2$$

Dividing, we get

$$\left(\frac{L + x_1}{L + x_2}\right) \times \left(\frac{\omega_1}{\omega_2}\right)^2 = \frac{x_1}{x_2} \qquad \text{(i)}$$

Given $x_1 = 1$ cm, $x_2 = 5$ cm and $\omega_2 = 2\omega_1$. Using these in (i) and solving, we get $L = 15$ cm, which is choice (b).

79. $L = I\omega = \dfrac{\frac{1}{2}I\omega^2}{\frac{1}{2}\omega} = \dfrac{2K}{\omega}$, where $K = \dfrac{1}{2}\,I\omega^2$ is the kinetic energy. If ω is doubled and K is halved, the value of L becomes one-fourth. Hence the correct choice is (d).
80. Rod POQ of length $l = 100$ cm is bent at its mid-point O so that $\angle POQ = 90°$ (see Fig. 5.73). The mass of part PO of length $l/2$ can be taken to be concentrated at its mid-point A whose coordinates are $(0, l/4)$ and of part OQ of length $l/2$ at its mid-point B whose coordinates are $(l/4, 0)$. The centre of mass of these two equal masses is at mid-point C between A and B. The coordinates of C are $(l/8, l/8)$.

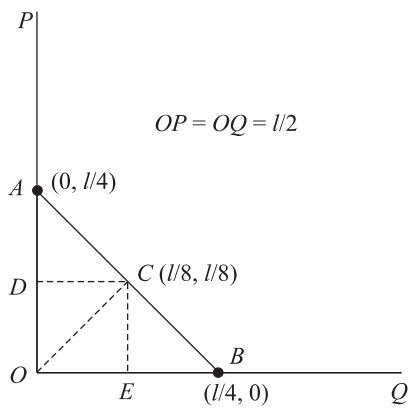


Fig. 5.73

$$\therefore \quad OC = \sqrt{(OE)^2 + (CE)^2} = \sqrt{\left(\frac{l}{8}\right)^2 + \left(\frac{l}{8}\right)^2}$$

$$= \frac{l}{\sqrt{32}} = \frac{100\text{ cm}}{\sqrt{32}} = 17.7\text{ cm},$$

which is choice (c).

81. $L = mr^2\,\omega$. For given m and ω, $L \propto r^2$. If r is halved, the angular momentum L becomes one-fourth. Hence the correct choice is (a).

82. Let M be the mass of the sphere. The mass of the disc will also be M. The moment of inertia of the sphere about its diameter is

$$I_s = \frac{2}{5} MR^2$$

The moment of inertia of the disc about its edge and perpendicular to its plane is (using parallel axes theorem)

$$I_d = I_{\text{cm}} + Mh^2 = \frac{1}{2} Mr^2 + Mr^2$$

$$= \frac{3}{2} Mr^2$$

Given $I_s = I_d$. Hence, we have

$$\frac{2}{5} MR^2 = \frac{3}{2} Mr^2$$

which gives $\frac{r}{R} = \frac{2}{\sqrt{15}}$, which is choice (a).

83. Consider a rod OP of length L lying along the x-axis with O as the origin (Fig. 5.74). Consider a small element AB of length dx at a distance x from O.

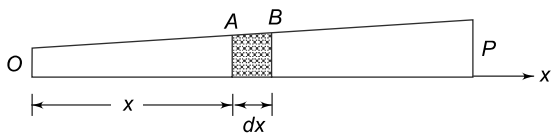


Fig. 5.74

Mass of element $AB\ (= dm) = mdx = \frac{k}{L}\ (xdx)$

The distance of the centre of mass from O is given by

$$x_{\text{CM}} = \frac{\int (d\,M)\,x}{\int (d\,M)} = \frac{\frac{k}{L}\int_0^L x^2\,d\,x}{\frac{k}{L}\int_0^L x\,d\,x}$$

$$= \frac{\left|\frac{x^3}{3}\right|_0^L}{\left|\frac{x^2}{2}\right|_0^L} = \frac{L^3/3}{L^2/2} = \frac{2\,L}{3}$$

The correct choice is (b).

84. Let ω be the angular velocity acquired by the system (rod + bullet) immediately after the collision. Since no external torque acts, the angular momentum of the system is conserved. Thus

$$mvL = I\omega \qquad (1)$$

where I is the moment of inertia of the system about an axis passing through O and perpendicular to the rod. Thus

I = M.I. of rod about O + M.I. of bullet stuck at its lower end about O

$$= \frac{1}{3}\ ML^2 + mL^2 = \frac{1}{3}\ (M + 3m)L^2 \quad (2)$$

Using Eq. (1) in Eq. (2), we have

$$m\,vL = \frac{1}{3}\ (M + 3m)\,L^2\omega$$

or $$\omega = \frac{3mv}{L\,(M + 3m)}$$

Hence the correct choice is (c).

85. The initial angular momentum of the rotating record is

$$L = I\omega$$

where $$I = \frac{1}{2}\ MR^2.$$

Let ω' be the angular velocity of the record when the coin of mass m is placed on it at a distance r from its centre. The angular momentum of the system becomes

$$L' = (I + mr^2)\omega'$$

Since no external torque acts on the system, the angular momentum is conserved, i.e.

$$L' = L \text{ or } (I + mr^2)\omega' = I\omega$$

or $$\omega' = \frac{I\,\omega}{I + m\,r^2} = \frac{\frac{1}{2}\,M\,R^2\,\omega}{\frac{1}{2}\,M\,R^2 + m\,r^2}$$

or $$\omega' = \frac{\omega}{\left(1 + \frac{2\,m\,r^2}{M\,R^2}\right)}$$

Putting $r = R/2$, we find that the correct choice is (a).

86. Since the rod is uniform, its centre of mass is at its midpoint. So the entire mass $3M$ of the rod can be assumed to be concentrated at its centre (0, 0). Now refer to Fig. 5.75.

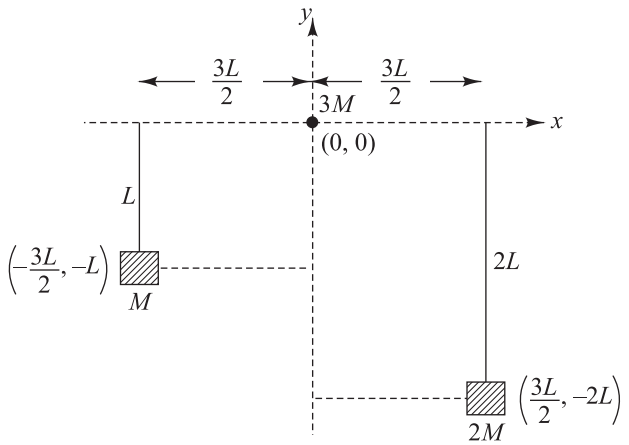


Fig. 5.75

The x and y coordinates of the centre of mass are

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{M \times (-3L/2) + 2M(3L/2) + 3M \times 0}{M + 2M + 3M}$$

$$= \frac{L}{4}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$= \frac{M \times (-L) + 2M \times (-2L) + M \times 0}{M + 2M + 3M}$$

$$= -\frac{5L}{6}.$$

So the correct choice is (c).

87. Since no external force acts on the man-plank system, the centre of mass of the system cannot accelerate. Since the system is initially at rest, the centre of mass cannot move. Let the midpoint of the plank be at $x = 0$, which is also the location of the centre of mass of the plank (Fig. 5.76)

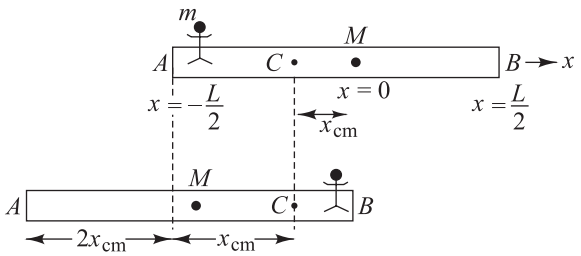


Fig. 5.76

The centre of mass C of the system when the man is at end A is

$$x_{cm} = \frac{m \times \left(-\frac{L}{2}\right) + M \times 0}{m + M} = -\frac{m}{2(m+M)}$$

The negative sign shows that the centre of mass is to the left of $x = 0$.

The centre of mass of the system when the man reaches end B is

$$x_{cm} = \frac{m \times \left(\frac{L}{2}\right) + M \times 0}{m + M} = \frac{mL}{2(m+m)}$$

to the right of $x = 0$.

Since the position of the centre of mass cannot change, this means that the plank itself must have moved through a distance $= 2x_{cm} = \frac{mL}{(m+M)}$ to the left. So the correct choice in (d).

88. The time taken by the man to move from A to B is

$$t = \frac{AB}{v} = \frac{L}{v}$$

Since the plank moves through a distance $d = \frac{mL}{(m+M)}$ to the left, the average speed of the plank will be

$$V = \frac{d}{t} = \frac{mv}{(m+M)} \text{ to the left.}$$

So the correct choice is (c).

89. Let the bar lie along the x-axis with its end A at $x = 0$ (see Fig. 5.77)

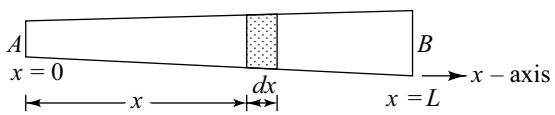


Fig. 5.77

Mass of the bar is

$$M \int dm = \int \lambda dx$$

$$= \int_0^L (4x + 5) dx$$

$$= \left| \frac{4x^2}{2} + 5x \right|_0^L$$

$$= \frac{4L^2}{2} + 5L$$

$$= \frac{4(0.5)^2}{2} + 5 \times 0.5 \quad (\because L = 0.5\text{m})$$

$$= 3 \text{ kg}$$

The x coordinate of the centre of mass of the bar is determined from the equation

$$x_{cm} = \frac{1}{M}\int x\,dm$$

$$= \frac{1}{M}\int_0^L (4x+5)x\,dx$$

$$= \frac{1}{M}\left|\frac{4x^3}{3} + \frac{5x^2}{2}\right|_0^L$$

$$= \frac{1}{M}\left[\frac{4L^3}{3} + \frac{5L^2}{2}\right]_0^L$$

$$= \frac{1}{3}\left[\frac{4}{5}\times(0.5)^3 + \frac{5}{2}\times(0.5)^2\right]$$

$$= \frac{19}{72} \text{ m} = 0.264\text{m} = 26.4\,\text{cm}$$

The x coordinate of the midpoint of the bar is $x_0 = 25.0$ cm.

$\therefore$ Distance of the centre of mass from the midpoint is $x_{cm} - x_0 = 26.4 - 25.0 = 1.4$ cm. Note that the centre of mass of the bar is to the right of its midpoint as expected. So the correct choice in (a).

90. The free body diagrams of the block and pulley are shown in Fig. 5.78.

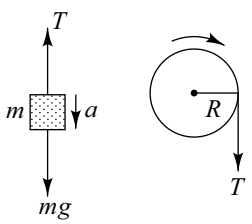


Fig. 5.78

The equation of motion of block is

$$mg - T = ma \qquad (1)$$

where T is the tension in the string. The torque acting on the pulley is

$$\tau = I\alpha$$

where $I = \frac{1}{2} MR^2$ and $\alpha = \frac{a}{R}$. Thus

$$\tau = \frac{1}{2} MR^2 \times \frac{a}{R} = \frac{1}{2} MRa$$

Now $\tau = RT$. Therefore

$$RT = \frac{1}{2} MRa$$

$$\Rightarrow \quad T = \frac{1}{2} Ma \qquad (2)$$

Using (2) in (1), we have

$$mg - \frac{1}{2} Ma = ma$$

$$\Rightarrow \quad a = \left(\frac{2m}{M+2m}\right)g$$

Putting $M = 4m$, we get $a = \frac{g}{3}$, which is choice (b).

91. The initial gravitational potential energy of the block in transformed into translational kinetic energy of the block and rotational kinetic energy of the pulley. If v is the speed of the block just before it hits the floor, then from conservation of energy, we have

$$0 + mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + 0$$

$$\Rightarrow \quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}\times\left(\frac{1}{2}MR^2\right)\times\left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{4}Mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{mgh}{\left(\frac{m}{2} + \frac{M}{4}\right)}}$$

Putting $M = 4m$, we get

$$v = \sqrt{\frac{2gh}{3}}, \quad \text{which in choice (c).}$$

92. Refer to Fig. 5.29 on page 5.17. The acceleration of the cylinder down the incline is

$$a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

The frictional force is

$$f = \frac{Ia}{R^2} = \frac{I}{R^2}\times\frac{g\sin\theta}{\left(1+\frac{I}{MR^2}\right)} = \frac{Mg}{\left(1+\frac{MR^2}{I}\right)}$$

To avoid slipping $f < \mu N$ where $N = Mg\cos\theta$

$$\therefore \quad \frac{Mg\sin\theta}{\left(1+\frac{MR^2}{I}\right)} < \mu Mg\cos\theta \qquad (1)$$

For a solid cylinder, $I = \frac{1}{2} MR^2$. Using this in (1), we get

$$\frac{1}{3}\sin\theta < \mu \cos\theta$$

or $$\mu < \frac{\tan\theta}{3}$$

$\therefore$ $$\mu_{\min} = \frac{\tan\theta}{3}$$, which in choice (c)

93. We have seen in Qs. 90 and 91 that $a = \frac{g}{3}$ and $v = \sqrt{\frac{2gh}{3}}$. Using $v = u + at$, we have

$$-\sqrt{\frac{2gh}{3}} = 0 + \left(-\frac{g}{3}\right)t$$

$$\Rightarrow \quad t = \sqrt{\frac{6h}{g}}$$

Alternatively, using $s = ut + \frac{1}{2} gt^2$, we have

$$-h = 0 + \frac{1}{2}\left(-\frac{g}{3}\right)t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{6h}{g}}$$

94. Refer to Fig. 5.79.

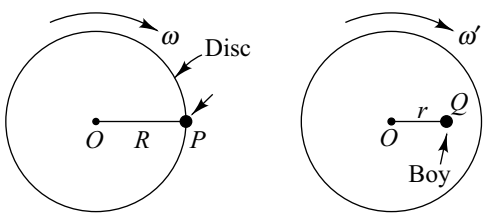


Fig. 5.79

Since no external torque acts as the boy walks from P to Q, the angular momentum of the disc + boy system is conserved, i.e

$$I\,\omega = I'\,\omega' \qquad (1)$$

where I' = moment of the system when the boy is at P, I' = moment of inertia of the system when the boy is at Q and ω' = new angular frequency.

$$I = I_0 + mR^2$$

$$= \frac{1}{2} MR^2 + mR^2$$

$$\Rightarrow \quad I = \left(\frac{M}{2} + m\right)R^2 \qquad (2)$$

$$I' = I_0 + mr^2$$

$$\Rightarrow \quad I' = \frac{1}{2} MR^2 + mr^2 \qquad (3)$$

Using (2) and (3) we have

$$\left(\frac{M}{2} + m\right)R^2\omega = \left(\frac{MR^2}{2} + mr^2\right)\omega'$$

$$\Rightarrow \quad \omega' = \frac{\left(\frac{M}{2} + m\right)R^2\omega}{\left(\frac{MR^2}{2} + mr^2\right)} \qquad (4)$$

Putting $r = R/2$ and $M = 10m$ in (3) we get $\omega' = \frac{8\omega}{7}$. So the correct choice is (a').

95. Let $\vec{F}$ be the contact (reaction) force exerted by the wall or end A of the bar. Since the frictional force between the wall and the bar is present, force $\vec{F}$ will not be normal to the wall. Let F_x and F_y be the horizontal and vertical components of F. The horizontal and vertical components of tension T in the string are $T_x = T\cos\theta$ and $T_y = T\sin\theta$. Figure 5.80 shows the forces acting on the bar.

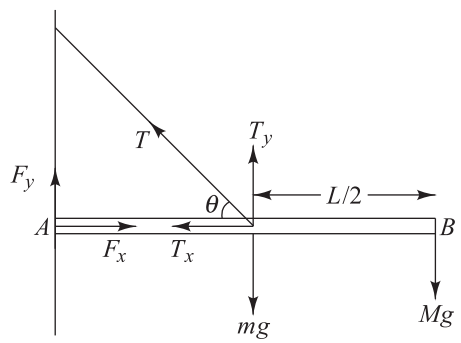


Fig. 5.80

The bar is in translational as well as rotational equilibrium. For translational equilibrium, the net horizontal and vertical force must be zero, i.e.

$$F_x - T_x = 0 \Rightarrow F_x - T\cos\theta = 0 \qquad (1)$$

and $$F_y + T_y - mg - Mg = 0$$

or $$F_y + T\sin\theta - (m + M)\,g = 0 \qquad (2)$$

For rotational equilibrium, the net clockwise torque about A = net anti-clockwise torque about A, i.e.

$$Mg \times L + mg \times \frac{L}{2} = T_y \times \frac{L}{2}$$

$$\Rightarrow \left(M + \frac{m}{2}\right)g = \frac{T\sin\theta}{2}$$

$$\Rightarrow \qquad T = \frac{(2M+m)g}{\sin\theta} \qquad (3)$$

Notice that we do not need Eqs. (1) and (2) to find T. Putting $M = m$ and $\theta = 30°$ in Eq. (3), we get $T = 6mg$. So the correct choice is (c).

96. Refer to the solution of Q. 95 above. From Eq. (1),

$F_x = T\cos\theta = 6mg \times \cos 30°$

$= 3\sqrt{3}mg$

From Eq. (2), we have

$$F_y = (m+M)g - T\sin\theta$$

$$= (m+M)g - \frac{(2M+m)g}{\sin\theta}\times\sin\theta$$

$$= -Mg = -mg \qquad (\because M = m)$$

$$\Rightarrow \qquad F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(3\sqrt{3}\,mg)^2 + (-mg)^2}$$

$$= mg \times \sqrt{27+1}$$

$$= \sqrt{28}mg = 2\sqrt{7}mg$$

So the correct choice is (d).

97. Initially m_2 is at rest at a height h above the floor and m_1 and pulley are also at rest. Hence the total initial energy of the system is

$$E_i = 0 + m_2 gh = m_2 gh$$

When m_2 is released, it begins to fall and m_1 begins to rise. Let v be the speed of m_2 just before it hits the floor. At this instant the speed of m_1 rising up is also v and height of m_1 at this instant is also h. therefore, the final total of the system is

E_f = P.E. of m_1 at height h + K.E. of m_1 and m_2 + rotational K.E. of pulley

$$= m_1 gh + \frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 + \frac{1}{2}I\omega^2$$

$$= m_1 gh + \frac{1}{2}v^2(m_1+m_2) + \frac{1}{2}\times\left(\frac{1}{2}MR^2\right)\times\left(\frac{v}{R}\right)^2$$

$$= m_1 gh + \frac{1}{2}v^2\left(m_1 + m_2 + \frac{M}{2}\right)$$

From conservation of energy,

$E_f = E_i$

$$\Rightarrow \qquad m_1 gh + \frac{1}{2}v^2\left(m_1 + m_2 + \frac{M}{2}\right) = m_2 gh$$

$$\Rightarrow \qquad v = \left[\frac{2(m_2 - m_1)gh}{m_1 + m_2 + \frac{M}{2}}\right]^{1/2} \qquad (1)$$

Putting $m_1 = m$, $m_2 = 2m$ and $M = 4m$ in (1), we get

$v = \sqrt{\frac{2gh}{5}}$. So the correct choice in (c).

98. Refer to Fig. 5.81. Every point on the disc has instantaneous velocity. The velocity of the centre of mass (C) of the disc is $v_{cm} = R\vec{\omega}$. For a point P at a distance r from C, the velocity is

$$\vec{v}_p = \vec{v}_{cm} + \vec{v}_t$$

where $\vec{v}_t$ is the tangential velocity of point P. The magnitude of $\vec{v}_p$ is $v_p = v_{cm} + v_t = R\omega + R\omega$.

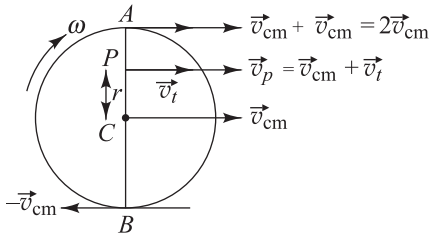


Fig. 5.81

For the topmost point A, $\vec{v}_A = \vec{v}_{cm} + \vec{v}_t$ at

$\underline{A} = R\vec{\omega} + (AC)\vec{\omega} = R\vec{\omega} + R\vec{\omega} = 2R\vec{\omega}$. Thus $\vec{v}_A = 2\vec{v}_{cm}$

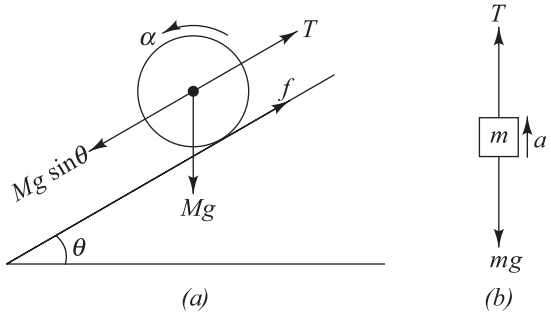
Differentiating,

$$\frac{d\vec{v}_A}{dt} = 2\frac{d\vec{v}_{cm}}{dt}$$

or $$\vec{a}_A = 2\vec{a}_{cm}$$

So the correct choice is (d). Note that the instantaneous velocity at the point of contact (point B) with the surface is $\vec{v}_B = \vec{v}_{cm} - \vec{v}_{cm} = 0$.

99. Figures 5.82. (a) and (b) show the free body diagrams of the disc and the block.



(a) *(b)*

Fig. 5.82

T is the tension in the string and f is the frictional force between the disc and the incline. The equations of motion of the disc and the block are

$$Mg \sin\theta - T - f = Ma_{cm} \quad (1)$$

$$T - mg = ma \quad (2)$$

We have seen above that $a = 2\,a_{cm}$. Therefore,

$$T - mg = 2\,ma_{cm} \quad (3)$$

Net torque on the disc is

$$\tau = fR - TR$$

$$I\alpha = fR - TR$$

$$\Rightarrow \frac{1}{2}MR^2 \times \frac{a_{cm}}{R} = fR - TR$$

$$\Rightarrow \quad f = T + \frac{1}{2}Ma_{cm} \quad (4)$$

Putting (4) in (1) we get

$$Mg\sin\theta - T - T - \frac{1}{2}Ma_{cm} = Ma_{cm}$$

$$\Rightarrow \quad Mg \sin\theta - 2\,T = \frac{3}{2}Ma_{cm} \quad (5)$$

Using (3) in (5) we get

$$Mg \sin\theta - 2\,(mg + 2ma_{cm}) = \frac{3}{2}Ma_{cm}$$

$$\Rightarrow \quad a_{cm} = \left[\frac{(M\sin\theta - 2m)}{\left(\frac{3}{2}M + 4m\right)}\right]g \quad (6)$$

Putting $M = 6m$ and $\theta = 30°$ in (6), we get $a_{cm} = \frac{g}{13}$.

So the correct choice is (b).

100. Moment of inertia of the system about the indicated axis = moment of inertia of rod AB of mass M and length $2L$ about O + moment of inertia of rod CD of mass M and length $2L$ about O + 4 × (moment of inertia of each ball of mass m about O)

$$= \frac{1}{12}M(2L)^2 + \frac{1}{12}M(2L)^2 + 4 \times mL^2$$

$$= \frac{2}{3}ML^2 + 4mL^2$$

$$= \frac{2}{3}(M + 6m)L^2$$

So the correct choice is (a).

101. $u_1 = 0$, $u_2 = 0$. Acceleration of m_2 is

$$a = \frac{F}{m_2}$$

$\therefore$ Velocity of m_2 at time t is $v_2 = u_2 + at = 0 + \frac{Ft}{m_2}$

or $v_2 = \frac{Ft}{m_2}$.

The velocity of the centre of mass is

$$v_{cm} = \frac{m_1u_1 + m_2v_2}{m_1 + m_2} = \frac{0 + m_2 \times \frac{Ft}{m_2}}{m_1 + m_2}$$

$$= \frac{Ft}{m_1 + m_2}, \text{ which is choice (c)}$$

102. $v_{cm} = \frac{mv_1 + mv_2}{m + m}$

$$= \frac{1}{2}(v_1 + v_2)$$

$$\boldsymbol{v}_{cm} = \frac{1}{2}(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\ \text{ms}^{-1}$$

$$\boldsymbol{a}_{cm} = \frac{m\boldsymbol{a}_1 + m\boldsymbol{a}_2}{m + m}$$

$$= \frac{1}{2}(a_1 + a_2)$$

$$= \frac{1}{2}\left[(2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) + (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\right]$$

$$= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\ \text{ms}^{-2}$$

It is clear that v_{cm} and a_{cm} are parallel vectors. Thus the acceleration of the centre of mass is along its velocity. Hence it follows a straight line trajectory.

103. Refer to Fig. 5.83.

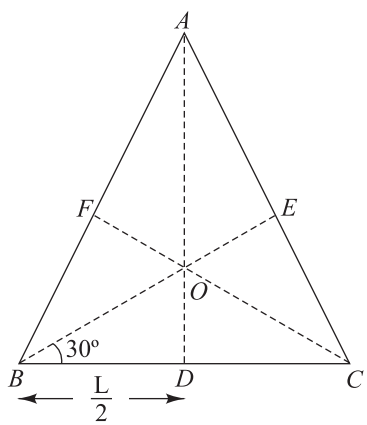


Fig. 5.83

$AB = BC = AC = L$, $OD = OE = OF = x$ (say).

$$x = OD = BD \tan 30°$$

$$= \frac{L}{2\sqrt{3}}$$

Using the parallel axes theorem, the moment of inertia of rod BC about $O = \left(\frac{ML^2}{12} + Mx^2\right)$

$$= \frac{ML^2}{12} + \frac{ML^2}{12} = \frac{ML^2}{6}$$

Moment of inertia of the complete frame about $O = 3 \times \frac{ML^2}{6} = \frac{ML^2}{2}$. So the correct choice is (d).

104. Mass per unit are of the plate is $\sigma = \frac{M}{L^2}$

Mass of each hole is $m = \sigma \times \pi R^2 = \frac{\pi MR^2}{L^2}$

Since $L = 4R$, $\quad m = \frac{\pi M}{16}$

Distance of the centre of a hole and centre O of the plate is

$$x = AO = \sqrt{2}\,R$$

Using parallel-axes theorem, the moment of inertia of a hole about the z-axis is

$$I = \frac{1}{2}mR^2 + mx^2$$

$$= m\left(\frac{R^2}{2} + 2R^2\right)$$

$$= \frac{\pi M}{32} \times \frac{5R^2}{2} = \frac{5\pi MR^2}{32}$$

Moment of four holes about the z-axiz $= 4I = \frac{5\pi MR^2}{8}$

Moment of inertia of the complete square plate is $I_1 = \frac{ML^2}{6} = \frac{M}{6} \times (4R)^2 = \frac{8}{3}MR^2$. Therefore, moment of inertia of the remaining portion about the z-axis is

$$I_2 = \frac{8}{3}MR^2 - \frac{5\pi}{8}MR^2 = \left(\frac{8}{3} - \frac{5\pi}{8}\right)MR^2$$

The required ratio $= \frac{I_2}{I_1} = 1 - \frac{15\pi}{64} \simeq 0.26$

So the correct choice is (c).

SECTION 1 Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage.

Passage I

A hollow sphere of mass M and radius R is initially at rest on a horizontal rough surface. It moves under the action of a constant horizontal force F as shown in Fig. 5.84.

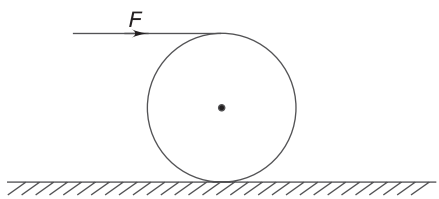


Fig. 5.84

1. The frictional force between the sphere and the surface
   (a) retards the motion of the sphere
   (b) makes the sphere move faster
   (c) has no effect on the motion of the sphere
   (d) is independent of the velocity of the sphere.
2. The linear acceleration of the sphere is
   (a) $a = \frac{10F}{7M}$ (b) $a = \frac{7F}{5M}$
   (c) $a = \frac{6F}{5M}$ (d) $a = \frac{F}{M}$
3. The frictional force between the sphere and the surface is
   (a) $\frac{F}{2}$ (b) $\frac{F}{3}$
   (c) $\frac{F}{4}$ (d) $\frac{F}{5}$

Solutions

1. If the horizontal force F is applied at the centre of mass of the sphere, then the frictional force opposes the translational motion of the sphere. If force F is applied above the centre of mass, the torque due to frictional force tends to rotate the sphere faster. Hence, in this case, frictional force f acts in the direction of motion, as shown in Fig. 5.85. Thus the correct choice is (b).

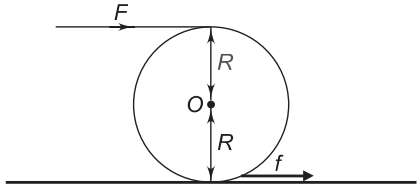


Fig. 5.85

2. Let a and α be the linear and angular accelerations of the sphere, respectively. For translational motion,

$$F + f = Ma \qquad (1)$$

The magnitude of the net torque acting on the sphere $= FR - fR$. Hence, for rotational motion the equation is

$$FR - fR = Ia = \frac{Ia}{R} \qquad (\because\ a = aR)$$

For a hollow sphere, $I = \frac{2}{3}MR^2$. Hence

$$FR - fR = \frac{2}{3}MR^2 \times \frac{a}{R} = \frac{2}{3} = MRa$$

$$\Rightarrow \qquad F - f = \frac{2}{3}Ma \qquad (2)$$

Equations (1) and (2) give $a = \frac{6F}{5M}$, which is choice (c)

3. From Eqs. (1) and (2) we get $f = \frac{Ma}{6} = \frac{F}{5}$. Hence the correct choice is (d).

Questions 4 to 7 are based on the following passage.

Passage II

Four solid spheres each of mass m and radius r are located with their centres on four corners of a square $ABCD$ of side a as shown in Fig. 5.86.

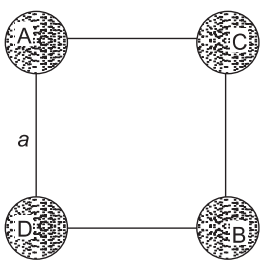


Fig. 5.86

4. The moment of inertia of sphere A about diagonal AB is

(a) $\frac{2}{3}mr^2$ (b) $\frac{2}{5}mr^2$

(c) $m\left(r^2 + \frac{a^2}{4}\right)$ (d) $m\left(\frac{a^2}{4} - r^2\right)$

5. The moment of inertia of sphere C about diagonal AB is

(a) $\frac{2}{5}mr^2$ (b) $\frac{2}{5}m(2r^2 + 3a^2)$

(c) $\frac{m}{5}(5r^2 + 3a^2)$ (d) $\frac{m}{10}(4r^2 + 5a^2)$

6. The moment of inertia of the system of four spheres about diagonal AB is

(a) $\frac{m}{5}(8r^2 + 5a^2)$ (b) $\frac{m}{5}(7r^2 + 4a^2)$

(c) $\frac{m}{5}(5r^2 + 8a^2)$ (d) $\frac{m}{5}(3r^2 + 5a^2)$

7. The moment of inertia of the system of four spheres about side AD is

(a) $\frac{2m}{5}(2r^2 + 5a^2)$ (b) $\frac{m}{5}(7r^2 + 5a^2)$

(c) $\frac{2m}{5}(4r^2 + 5a^2)$ (d) $\frac{m}{5}(3r^2 + 5a^2)$

Solutions

Refer to Fig. 5.87.

The moment of inertia of spheres A and B about their common diameter $AB = \frac{2}{5}mr^2$ each. Also the moment of inertia of spheres C and D about an axis passing through their centre and parallel to $AB = \frac{2}{5}mr^2$ each. The distance of this axis (shown by broken lines) from the diagonal $AB = a/\sqrt{2}$. From the parallel axes theorem, the moment of inertia of spheres C and D about diagonal AB is

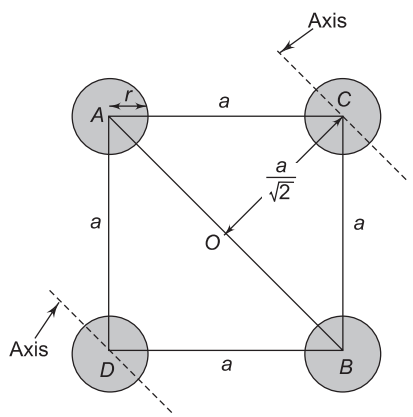


Fig. 5.87

$$\frac{2}{5}mr^2 + m(CO)^2 = \frac{2}{5}mr^2 + m\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{2}{5}mr^2 + \frac{ma^2}{2}$$

4. The correct choice is (b).
5. The correct choice is (d).

6. The moment of inertia of the system of four spheres about diagonal AB is

I_{AB} = MI of A about AB + MI of B about AB + MI of C about AB + MI of D about AB

$$= \frac{2}{5}mr^2 + \frac{2}{5}mr^2 + \frac{2}{5}mr^2 + \frac{1}{2}ma^2 + \frac{2}{5}mr^2 + \frac{1}{2}ma^2$$

$$= \frac{8}{5}mr^2 + ma^2 = m\left(\frac{8r^2}{5} + a^2\right)$$

The correct choice is (a).

7. Moment of inertia of sphere A about side AD = moment of inertia of sphere D about side $AD = \frac{2}{5}mr^2$. Using the parallel axes theorem, moment of inertia of sphere C about AD = moment of inertia of sphere B about $AD = \frac{2}{5}mr^2 + ma^2$. Hence the moment of inertia of the system of four spheres about side AD is

I_{AD} = MI of A about AD + MI of D about AD + MI of B about AD + MI of C about AD

$$= \frac{2}{5}mr^2 + \frac{2}{5}mr^2 + \frac{2}{5}mr^2 + ma^2 + \frac{2}{5}mr^2 + ma^2$$

$$= \frac{8}{5}mr^2 + 2ma^2 = m\left(\frac{8r^2}{5} + 2a^2\right)$$

The correct choice is (c).

Questions 8 to 10 are based on the following passage.

Passage III

A small sphere rolls down without slipping from the top of a track in a vertical plane. The track has a elevated section and a horizontal part. The horizontal part is 1.0 m above the ground and the top of the track is 2.4 m above the ground. (See Fig. 5.88)

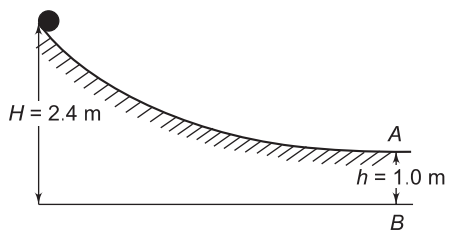


Fig. 5.88

8. If $g = 10\ ms^{-2}$, the horizontal velocity when the sphere reaches point A is

(a) $\sqrt{5}\ ms^{-1}$ (b) $2\sqrt{5}\ ms^{-1}$

(c) $\sqrt{7}\ ms^{-1}$ (d) $2\sqrt{7}\ ms^{-1}$

9. If $g = 10\ ms^{-2}$, the time taken by the sphere to fall through $h = 1.0$ m is

(a) $\frac{1}{\sqrt{5}}$ s (b) $\frac{2}{\sqrt{5}}$ s

(c) 0.1 s (d) 0.2 s

10. If $g = 10\ ms^{-2}$, the distance of the point at which the sphere falls on the ground with respect to point B (which is vertically below the end A of the track) is

(a) 1.0 m (b) 1.4 m

(c) 2.0 m (d) 2.8 m

Solutions

8. The loss in potential energy when the sphere moves from the top of the track to point A = gain in total kinetic energy (translational and rotation), i.e.

$$Mg(H-h) = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

where $I = \frac{2}{5}MR^2$ and $\omega = \frac{v}{R}$. Thus

$$Mg(H-h) = \frac{1}{2}Mv^2 + \frac{1}{2} \times \frac{2}{5}MR^2 \times \left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2$$

or $$v = \left[\frac{10(H-h)g}{7}\right]^{1/2}$$

$$= \left[\frac{10 \times (2.4-1.0) \times 10}{7}\right]^{1/2} = 2\sqrt{5}\ ms^{-1}$$

The correct choice is (b).

9. Since the vertical component of velocity is zero, the time of flight is

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1.0}{10}} = \frac{1}{\sqrt{5}}$ s, which is choice (a).

10. Horizontal range = $vt = 2\sqrt{5} \times \frac{1}{\sqrt{5}} = 2.0$ m, which is choice (c).

Questions 11 to 14 are based on the following passage.

Passage IV

Two blocks of masses $m_1 = 3\ m$ and $m_2 = m$ are attached to the ends of a string which passes over a frictionless fixed pulley (which is a uniform disc of mass $M = 2\ m$ and radius R) as shown in Fig. 5.89. The masses are then released.

11. The acceleration of the system is

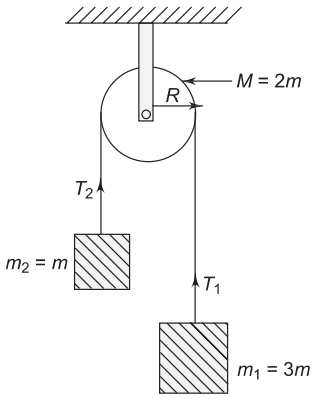


Fig. 5.89

(a) $2\,g$

(b) $\frac{2g}{3}$

(c) $\frac{2g}{5}$

(d) $\frac{3g}{7}$

12. Tension T_1 is

(a) $\frac{mg}{5}$ (b) $\frac{3mg}{5}$

(c) $\frac{7mg}{5}$ (d) $\frac{9mg}{5}$

13. Tension T_2 is

(a) $\frac{9mg}{5}$ (b) $\frac{7mg}{5}$

(c) mg (d) $\frac{3mg}{5}$

14. The magnitude of torque on the pulley is

(a) $2\,mgR$ (b) $\frac{2mgR}{3}$

(c) $\frac{2mgR}{5}$ (d) $3\,mgR$

Solutions

Refer to Fig. 5.90. Since the pulley has a finite mass, the tensions T_1 and T_2 will not be equal. If a is the acceleration of the system, the equations of motion of masses

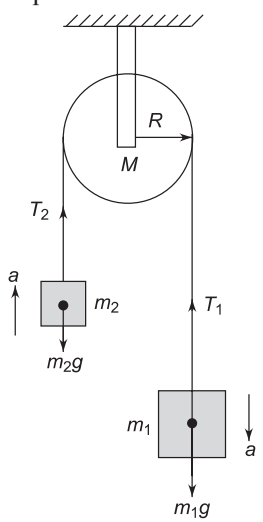


Fig. 5.90

$$m_1g - T_1 = m_1a \quad \Rightarrow \quad 3mg - T_1 = 3ma \qquad (1)$$

and $$T_2 - m_2g = m_2a \quad \Rightarrow \quad T_2 - mg = ma \qquad (2)$$

The resultant tension $(T_1 - T_2)$ exerts a torque on the pulley which is given by

$$\tau = (T_1 - T_2)R \qquad (3)$$

Also, $\tau = I\alpha$ where $I = \frac{1}{2}MR^2$ and α is the angular acceleration which is given by

$$\alpha = \frac{a}{R}$$

$$\therefore \qquad \tau = \frac{1}{2}MR^2 \times \frac{a}{R} = \frac{1}{2}MRa = mRa \qquad (4)$$

$(\because M = 2m)$

From Eqs. (3) and (4) we get

$$T_1 - T_2 = ma \qquad (5)$$

11. Using Eqs. (1) and (2) and (5) and solving for a we get $a = \frac{2g}{5}$. Hence the correct choices is (c).

12. From Eqs. (1) and (2) we get $T_1 = 3m(g - a) = 3m\left(g - \frac{2g}{5}\right) = \frac{9mg}{5}$; which is choice (d).

13. Eqs. (1) and (2) give $T_2 = m(g + a) = m\left(g + \frac{2g}{5}\right) = \frac{7mg}{5}$ so choice (b) is correct.

14. From Eq. (4) $\tau = mRa = mR \times \frac{2g}{5} = \frac{2mgR}{5}$, which is choice (c).

Questions 15 to 18 are based on the following passage.

Passage V

A small solid sphere of mass m rolls without slipping on the track shown in Fig. 5.91. The radius of the circular part of the track is R. The sphere starts from rest from point P at a height $H = 4.5\,R$ above the bottom.

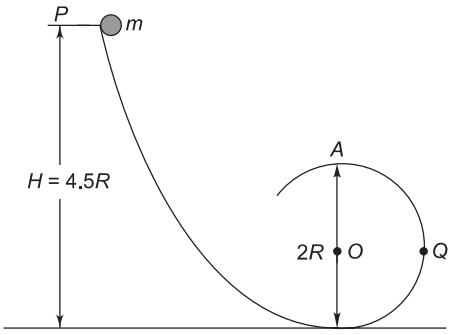


Fig. 5.91

15. The speed of the sphere when it reaches point Q on the track is

(a) $\sqrt{2gR}$ (b) $\sqrt{3gR}$

(c) $\sqrt{5gR}$ (d) $\sqrt{7gR}$

16. The horizontal force acting on the sphere when it is at point Q is
    (a) mg (b) $2\,mg$
    (c) $3\,mg$ (d) $5\,mg$
17. The magnitude of the force acting on the sphere when it is at point Q is
    (a) $\sqrt{4.5}\,mg$ (b) $\sqrt{5}\,mg$
    (c) $\sqrt{26}\,mg$ (d) $3\sqrt{3}\,mg$
18. What is the minimum value of height H so that the sphere can reach the top A of the circle?
    (a) $2.4\,R$ (b) $2.5\,R$
    (c) $2.6\,R$ (d) $2.7\,R$



Solutions

15. Loss of P.E. at Q = Gain in K.E.

$$mg(H-R) = \frac{1}{2}\,mv^2 + \frac{1}{2}\,I\omega^2$$

$$\Rightarrow mg(4.5R-R) = \frac{1}{2}\,mv^2 + \frac{1}{2}\times\left(\frac{2}{5}mR^2\right)\times\left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2}\,mv^2 + \frac{1}{5}\,mv^2 = \frac{7}{10}\,mv^2$$

which gives $v = \sqrt{5gR}$. Hence the correct choice is (c).

16. At point Q the velocity is directed tangentially. Thus, the horizontal force acting on the sphere at point Q is the centripetal force directed towards the centre O of the circular part of the track and is given by

$F_h = \dfrac{mv^2}{R} = \dfrac{m}{R}\times(5\,gR) = 5\,mg$, which is choice (d).

17. The vertical force acting on the sphere is

$$F_v = \text{weight of the sphere} = mg$$

$\therefore$ Net force $F = \sqrt{F_h^2 + F_v^2} = \sqrt{26}\,mg$

Thus the correct choice is (c).

18. The sphere will rise to point A if it has a minimum speed at A which satisfies

$$\frac{mv_A^2}{R} = mg \quad\Rightarrow\quad v_A = \sqrt{Rg}$$

The minimum value of H is given by

$$mg\,(H_{\min} - 2R) = \frac{7}{10}mv_A^2 = \frac{7}{10}\,mRg$$

$\Rightarrow\quad H = 2R + 0.7R = 2.7R$, which is choice (d).

SECTION 3

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true but Statement-2 is not the correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

1. **Statement-1**

   A solid cylinder of mass M and radius R rolls down an inclined plane of height h. The rotational kinetic energy of the cylinder when it reaches the bottom of the plane is $Mgh/3$.

   Statement-2

   The total energy of the cylinder remains constant throughout its motion.

2. **Statement-1**

   Two bodies A and B initially at rest, of masses $2m$ and m respectively move towards each other under mutual gravitational force of attraction. At the instant when the speed of A is v and that of B is $2v$, the speed of the centre of mass of system is $4v/3$.

   Statement-2

   The speed of the centre of mass of a system changes if an external force acts on the system.

3. **Statement-1**

   The angular momentum of a particle moving in a circular orbit with a constant speed remains conserved about any point on the circumference of the circle.

Statement-2

If no net torque acts, the angular momentum of a system is conserved.

4. **Statement-1**

Two blocks of masses M and m (with $M > m$) are connected by a spring of negligible mass and placed on a horizontal frictionless surface. An impulse gives a velocity V to the heavier block in the direction of the lighter block. The velocity of the centre of mass is

$$v_{CM} = \frac{MV}{(M+m)}$$

Statement-2

The principle of conservation of linear momentum is applicable to the centre of mass motion.

5. **Statement-1**

A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is $ML\,\omega^2$.

Statement-2

The entire mass of the liquid can be regarded as being concentrated at the centre of mass of the tube.

6. **Statement-1**

A thin wire of length L and mass m is bent into a circular loop of radius r as shown in Fig. 5.92. The moment of inertia of the loop about the XX' is $3mL^2/8\pi^2$.

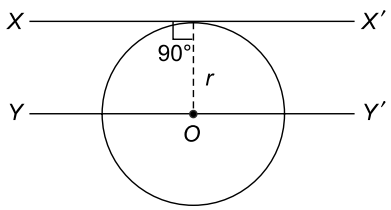


Fig. 5.92

Statement-2

According to the parallel axes theorem, the moment of inertia of the loop about XX' = moment of inertia about $YY' + mr^2$.

7. **Statement-1**

A thin uniform rod AB of mass M and length L is hinged at one end A to the horizontal floor. Initially it stands vertically. It is allowed to fall freely on the floor in the vertical plane. The angular velocity of the rod when its ends B strikes the floor is $\sqrt{3g/L}$.

Statement-2

The angular momentum of the rod about the hinge remains constant throughout its fall to the floor.

8. **Statement-1**

For a particle moving in a circle with a constant speed, the direction of the centripetal force depends on whether the particle is moving clockwise or anticlockwise along the circle.

Statement-2

The centripetal force is directed radially towards the centre of the circle.

9. **Statement-1**

A body tied to a string is moved in a circle with a uniform speed. If the string suddenly breaks, the angular momentum of the body becomes zero.

Statement-2

The torque on the body equals the rate of change of angular momentum.

10. **Statement-1**

A solid cylinder and a solid sphere have the same mass M and the same radius R. If torques of equal magnitude are applied to them for the same time, the sphere will acquire greater angular speed.

Statement-2

For a given torque, the angular acceleration is inversely proportional to the moment of inertia.

11. **Statement-1**

If a disc rotating about its axis with a certain angular speed is gently placed on a horizontal frictionless surface, it will roll along the surface with the same angular speed.

Statement-2

No torque acts on the disc if the friction is absent.

12. **Statement-1**

Two solid spheres of the same radius, one made of steel and the other of aluminium are released from rest from the top of an inclined plane at the same time. The two spheres will reach the bottom at the same time.

Statement-2

The linear acceleration down the plane is independent of the mass of the sphere.

13. **Statement-1**

A sphere and a cylinder slide without rolling from rest from the top of an inclined plane. They will reach the bottom with the same speed.

Statement-2

Bodies of all shapes, masses and sizes slide down a plane with the same acceleration.

14. **Statement-1**

If a cylinder rolling with angular speed ω suddenly breaks up into two equal halves of the same radius, the angular speed of each piece becomes 2ω.

Statement-2

If no external torque acts, the angular momentum of a system is conserved.

15. **Statement-1**

Friction is necessary for a body to roll on a surface.

Statement-2

Friction provides the necessary tangential force and torque.

16. **Statement-1**

A sphere is rolling on a rough surface in the direction of the arrow as shown in Fig. 5.93. The force of friction at the point of contact will be in the direction of the arrow.

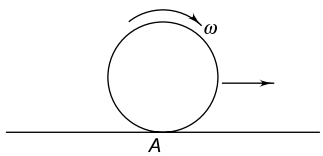


Fig. 5.93

Statement-2

Friction opposes motion.

17. **Statement-1**

During perfect rolling, the force of friction becomes zero.

Statement-2

The speed at the point of contact becomes zero.

18. **Statement-1**

During rolling, the acceleration of the point of contact is non zero.

Statement-2

The direction of the velocity changes with time.

Solutions

1. The correct choice is (a).

From the law of conservation of energy, we have Potential energy = Translational kinetic energy + Rotational kinetic energy

$$\text{or} \qquad Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$\text{or} \qquad Mgh = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$$

$$= \frac{3}{4}MR^2\,\omega^2$$

$$\text{or} \qquad \omega^2 = \frac{4gh}{3R^2}$$

Now the rotational kinetic energy = $\frac{1}{2}I\omega^2$.

Substituting for ω^2 and I, we have

$$\text{Rotational kinetic energy} = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{4gh}{3R^2}$$

$$= \frac{Mgh}{3}$$

2. The correct choice is (d). Mutual gravitational force is internal to the system. Since no external force acts on the system, the centre of mass (which is initially at rest) will remain at rest.

3. The correct choice is (d). Since the centripetal force is radial (directed towards the centre of the circle), the torque due to this force is zero about the centre. Hence, angular momentum remains conserved only about the centre of the circle.

4. The correct choice is (a). From the principle of conservation of linear momentum, we have

$$MV = (M + m)v_{CM}$$

$$\text{which gives } v_{CM} = \frac{MV}{(M+m)}$$

5. The correct choice is (d). The entire mass of the liquid can be regarded as being concentrated at the centre of mass of the tube which is at a distance of $r = \frac{L}{2}$ from the axis of revolution. The force exerted by the liquid at the other end of the tube is the centripetal force of a mass M revolving in a circle or radius $r = \frac{L}{2}$.

Thus

$$F_c = \frac{Mv^2}{r} = \frac{M(r\omega)^2}{r} = Mr\omega^2 = \frac{ML\omega^2}{2}$$

6. The correct choice is (a). The moment of inertia of the loop about an axis passing through the centre O is

$$I_O = \frac{1}{2}mr^2$$

From the parallel axes theorem, the moment of inertia about XX' is

$$I = I_O + mr^2 = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$$

Now $L = 2\pi r$ or $r = L/2\pi$. Therefore,

$$I = \frac{3m}{2} \times \left(\frac{L}{2\pi}\right)^2 = \frac{3mL^2}{8\pi^2}$$

7. The correct choice is (c). Loss in P.E. = gain in rotational K.E. As the centre of mass of the rod falls through a distance $L/2$, the loss in P.E. $= \frac{MgL}{2}$. Gain in K.E. $= \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$

Equating the two, we have

$$\left(\frac{MgL}{2}\right) = \left(\frac{ML^2\omega^2}{6}\right) \quad \Rightarrow \quad \omega = \sqrt{\frac{3g}{L}}$$

8. The correct choice is (d).

9. The correct choice is (d). Since no external torque acts on the body even after the string breaks, the angular momentum will remain unchanged.

10. The correct choice is (a). For cylinder, $I_c = \frac{1}{2}MR^2$ and for sphere $I_s = \frac{2}{5}MR^2$. Also $\tau = I\alpha$

or $\alpha = \frac{\tau}{I}$. For a given τ, α is inversely proportional to I. Hence

$$\frac{\alpha_s}{\alpha_c} = \frac{I_c}{I_s}$$

Since $I_c > I_s$; $\alpha_s > \alpha_c$. Since radius R is the same for both, $a_s > a_c$. Since time is the same, $\omega_s > \omega_c$.

11. The correct choice is (d). Since no torque acts, the disc will not roll on the surface; it will simply keep on rotating at the point where it is placed.

12. The correct choice is (a). The linear acceleration is

$$a = \frac{g\sin\theta}{\left(1 + \frac{I}{MR^2}\right)} = \frac{5g\sin\theta}{7}$$

$$\left(\because I = \frac{2}{5}MR^2\right)$$

Thus the acceleration is independent of the mass of the sphere. Hence the two spheres of the same radius will reach the bottom at the same time.

13. The correct choice is (a). If a body slides down an inclined plane, its acceleration is

$$a = g(\sin\theta - \mu\cos\theta)$$

which depends only on g, θ and μ.

14. The correct choice is (a). If I is the moment of inertia of the complete cylinder, the moment of inertia of each piece becomes $I/2$. Since $L = I\omega$ is constant, the angular speed of each piece becomes 2ω.

15. The correct choice is (a).

16. The correct choice is (a). The direction of the linear velocity at A, the point of contact is to the left (opposite to the direction of the arrow). Since friction opposes motion, the direction of the frictional force at A will be in the direction of the arrow, i.e. in the direction along which the sphere is rolling.

17. The correct choice is (a). The effect of frictional force is to decrease the speed of the body at the point of contact. When speed is zero, perfect rolling begins and the force of friction becomes zero. Hence no work is done.

18. The correct choice is (a). Since the body is rotating while it is rolling, the direction of the velocity is changing with time. Hence the instantaneous acceleration of the point of contact is not zero.

SECTION 4

Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

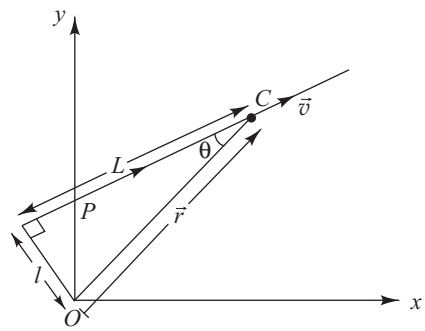
1. Two identical particles move towards each other with velocity $2v$ and v, respectively. The velocity of the centre of mass is
   (a) v (b) $\frac{v}{3}$
   (c) $\frac{v}{2}$ (d) zero **[2002]**

2. The initial angular velocity of circular disc of mass M is ω_1. When two small spheres, each of mass m, are placed gently on two diametrically opposite points on the edge of the disc, the angular velocity of the disc becomes
   (a) $\left(\frac{M+m}{M}\right)\omega_1$
   (b) $\left(\frac{M+m}{m}\right)\omega_1$
   (c) $\left(\frac{M}{M+4m}\right)\omega_1$
   (d) $\left(\frac{M}{M+2m}\right)\omega_1$ **[2002]**

3. A solid sphere, a hollow sphere and a ring are released from the top of a frictionless inclined plane so that they slide down the plane. Then
   (a) the solid sphere will have maximum acceleration
   (b) the ring will have maximum acceleration
   (c) the hollow sphere will have maximum acceleration
   (d) all of them will have the same acceleration **[2002]**

4. The moment of inertia of a circular wire of mass M and radius R about its diameter is
   (a) $\frac{1}{2}MR^2$ (b) MR^2
   (c) $2MR^2$ (d) $\frac{MR^2}{4}$ **[2002]**

5. A particle of mass m moves along a line PC with velocity v as shown in the figure. What is the magnitude of angular momentum of the particles about O?
   (a) mvL (b) mvl
   (c) mvr (d) zero **[2002]**



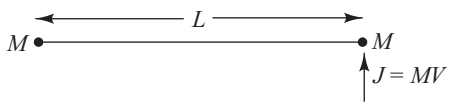
6. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the center of mass is
   (a) 30 m/s (b) 20 m/s
   (c) 10 m/s (d) 5 m/s **[2002]**

7. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are:
   (a) **Up** the incline white ascending and **down** the incline while descending
   (b) **Up** the incline while ascending as well as descending.
   (c) **down** the incline while ascending and **up** the incline while descending
   (d) **down** the incline while ascending as well as descending. **[2002]**

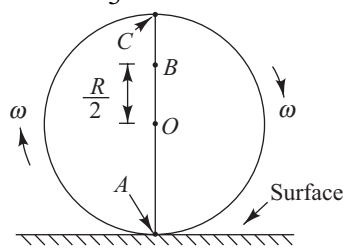
8. A circular disc X of radius R is made from an iron plate of thickness t and another circular disc Y of radius 4 R is made from an iron disc of thickness $t/4$. Then the relation between moments of inertia I_x and I_y is
   (a) $I_y = 32I_x$ (b) $I_y = 16I_x$
   (c) $I_y = I_x$ (d) $I_y = 64I_x$ **[2003]**

9. A particle performing uniform circular motion has angular momentum L. If the angular frequency is doubled and its kinetic energy is halved, then the new angular momentum is
   (a) $\frac{L}{4}$ (b) $2L$
   (c) $4L$ (d) $\frac{L}{2}$ **[2002]**

10. A force $\vec{F}$ acts on a particle having a position vector $\vec{r}$ and $\vec{\tau}$ is the torque of this force about the origin. Then
    (a) $\vec{r}.\vec{\tau}=0$ and $\vec{F}.\vec{\tau}\neq 0$
    (b) $\vec{r}.\vec{\tau}\neq 0$ and $\vec{F}.\vec{\tau}=0$
    (c) $\vec{r}.\vec{\tau}\neq 0$ and $\vec{F}.\vec{\tau}\neq 0$
    (d) $\vec{r}.\vec{\tau}=0$ and $\vec{F}.\vec{\tau}=0$ **[2003]**
11. The angular momentum of a particle moving in a circular orbit with a constant speed remains conserved about
    (a) any point on the circumference of the circle
    (b) any point inside the circle
    (c) any point outside the circle
    (d) the centre of the circle **[2003]**
12. Consider a body, shown in the figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse $J = M\,V$ is imparted to the body at one of its ends, it is angular velocity will be



    (a) $\frac{V}{4L}$ (b) $\frac{2V}{L}$
    (c) $\frac{V}{3L}$ (d) $\frac{V}{L}$ **[2003]**
13. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass the same, which one of the following will not be affected?
    (a) Moment of inertia
    (b) Angular momentum
    (c) Angular velocity
    (d) Rotational kinetic energy **[2004]**
14. One solid sphere A of diameter d_A and another hollow sphere B of diameter d_B have the same mass and the same outer radius. Their moments of inertia about diameter are I_A and I_B respectively. Then
    (a) $I_A = I_B$ (b) $I_A > I_B$
    (c) $I_A < I_B$ (d) $\frac{I_A}{I_B}=\frac{d_A}{d_B}$ **[2004]**
15. A man, standing on a turn-table, is rotating at a certain angular frequency with his arms outstretched. He suddenly folds his arms. If his moment of inertia with folded arms is 75% of that with outstretched arms, his rotational kinetic energy will
    (a) increase by 33.3% (b) decrease by 33.3%
    (c) increase by 25% (d) decrease by 25%
    [2004]
16. A uniform disc of radius R is rolling (without slipping) on a horizontal surface with an angular speed as shown in the figure. O is the centre of the disc, points A and C are located on its rim and point B is at a distance $R/2$ from O. During rolling, the points A, B and C lie on the vertical diameter at a certain instant of time. If v_A, v_B and v_C are the linear speeds of points A, B and C respectively at that instant, then
    (a) $v_A = v_B = v_C$ (b) $v_A > v_B > v_C$
    (c) $v_A = 0,\ v_A = \frac{4}{3}v_B$ (d) $v_A = 0,\ v_C = 2v_B$



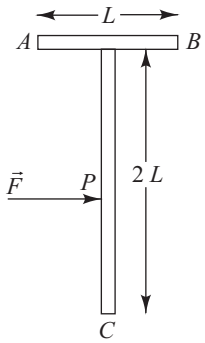
    [2004]
17. A particle is moving in the $x-y$ plane with a constant velocity along a line parallel to the x-axis, away from the origin. The magnitude of its angular momentum about the origin.
    (a) is zero (b) remains constant
    (c) goes on increasing (d) goes on decreasing
    [2004]
18. An annular ring with inner and outer raddii R_1 and R_2 is rolling without slipping with a uniform angular speed. A particle is situated on the inner part of the ring and another particle of the same mass is situated on the outer part. The ratio of the force experienced by the two particles $\left(\frac{F_1}{F_2}\right)$ is
    (a) $\frac{R_2}{R_1}$ (b) $\left(\frac{R_1}{R_2}\right)^2$
    (c) 1 (d) $\frac{R_1}{R_2}$ **[2005]**
19. A body A of mass M while falling vertically downwards under gravity breaks into two bodies : a body B of mass $M/3$ and a body C of mass $2M/3$. The centre of mass of bodies B and C taken together.
    (a) shifts towards body B
    (b) shifts towards body C
    (c) does not shift
    (d) depends on the height at which body A breaks.
    [2005]
20. The moment of inertia of a uniform semicircular disc of mass M and radius R about an axis perpendicular

to the plane of the disc and passing through the centre is

(a) $\frac{1}{4}MR^2$ (b) $\frac{2}{5}MR^2$

(c) MR^2 (d) $\frac{1}{2}MR^2$ **[2005]**

21. A T-shaped object with dimensions shown in the figure, is lying on a smooth horizontal floor. A force $\vec{F}$ is applied at a point P parallel to AB such that the object has only translatory motion without rotation. The distance of point P from C is



(a) $\frac{2L}{3}$

(b) $\frac{3L}{2}$

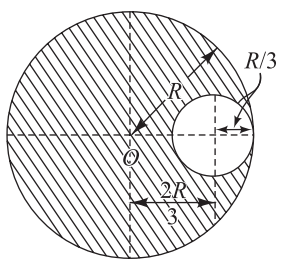
(c) $\frac{4L}{3}$

(d) L **[2005]**

22. A thin uniform disc has mass M and radius R. A circular hole is radius $R/3$ is made in the disc as shown in the figure. The moment of inertia of the remaining portion of disc about an axis passing through O and perpendicular to the plane of the disc is

(a) $\frac{1}{9}MR^2$ (b) $\frac{2}{9}MR^2$

(c) $\frac{1}{3}MR^2$ (d) $\frac{4}{9}MR^2$



[2005]

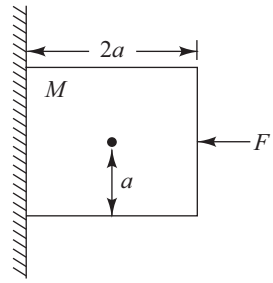
23. A cubical block is held stationary against a rough wall by applying force F as shown in the figure. The incorrect statement among the following is

(a) frictional force, $f = mg$

(b) normal force, $N = F$

(c) normal force does not apply torque

(d) force F does not apply torque.



[2005]

24. A solid metallic sphere of radius R having moment of inertia equal to I about its diameter is melted and recast into a solid disc of radius r of a uniform thickness. The moment of inertia of the disc about an axis passing through its edge and perpendicular to its plane is also equal to I. The ratio r/R is

(a) $\frac{2}{\sqrt{15}}$ (b) $\frac{2}{\sqrt{10}}$

(c) $\frac{2}{\sqrt{5}}$ (d) $\frac{2}{\sqrt{2}}$ **[2006]**

25. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should be second particle be moved, so as to keep the centre of mass at the same position?

(a) $\frac{m_1}{m_2}d$ (b) d

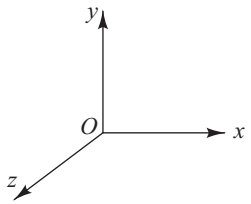
(c) $\frac{m_2}{m_1}d$ (d) $\frac{m_1}{m_2+m_2}d$ **[2006]**

26. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter. The ring now rotates with an angular velocity ω' equal to

(a) $\frac{\omega m}{(m+M)}$ (b) $\frac{\omega m}{(m+2M)}$

(c) $\frac{\omega(m+2M)}{m}$ (d) $\frac{\omega(m-2M)}{(m+2M)}$ **[2006]**

27. A force $-F\hat{\mathbf{k}}$ acts on O, the origin of the coordinate system shown in the figure. The torque about the point (1,–1) is



(a) $F(\hat{\mathbf{i}}+\hat{\mathbf{j}})$ (b) $-F(\hat{\mathbf{i}}-\hat{\mathbf{j}})$

(c) $F(\hat{\mathbf{i}}-\hat{\mathbf{j}})$ (d) $-F(\hat{\mathbf{i}}+\hat{\mathbf{j}})$ **[2006]**

28. Four point masses, each of value m, are placed at the corners of square $ABCD$ of side l. The moment of inertia of this system about an axis passing through A and parallel to BD is

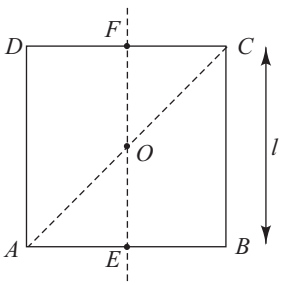
(a) $3ml^2$ (b) ml^2

(c) $2ml^2$ (d) $\sqrt{3}ml^2$ **[2006]**

29. For the given uniform square lamina $ABCD$, whose centre is O

(a) $\sqrt{2}I_{AC} = I_{EF}$ (b) $I_{AD} = 3I_{EF}$

(c) $I_{AC} = I_{EF}$ (d) $I_{AC} = \sqrt{2}I_{EF}$ **[2007]**



30. A circular disc of radius R is removed from a bigger circular disc of radius $2R$ such that the circumferences of the discs touch. The centre of mass of the new disc is $\frac{\alpha}{R}$ from the centre of the bigger disc. The value of α is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{6}$ (d) $\frac{1}{4}$ **[2007]**

31. A round uniform body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is

(a) $\frac{g\sin\theta}{1+\frac{I}{MR^2}}$ (b) $\frac{g\sin\theta}{1+\frac{MR^2}{I}}$

(c) $\frac{g\sin\theta}{1-\frac{I}{MR^2}}$ (d) $\frac{g\sin\theta}{1-\frac{MR^2}{I}}$ **[2007]**

32. Angular momentum of the particle rotating with a central force is constant due to

(a) constant force

(b) constant linear momentum

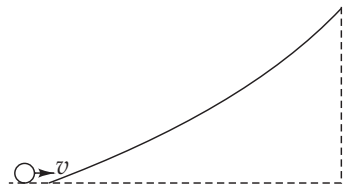
(c) zero torque

(d) constant torque **[2007]**

33. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum length $h = \frac{3v^2}{4g}$, with respect to the initial position. The object is (see the figure)

(a) ring (b) solid sphere

(c) hollow sphere (d) disc **[2007]**

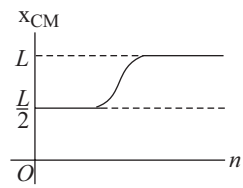


34. Consider a uniform square plate of side 'a' and mass 'm'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

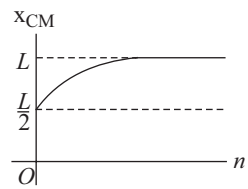
(a) $\frac{2}{3}ma^2$ (b) $\frac{5}{6}ma^2$

(c) $\frac{1}{12}ma^2$ (d) $\frac{7}{12}ma^2$ **[2008]**

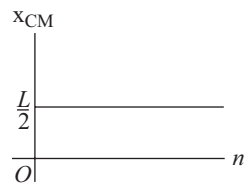
35. A thin rod of length 'L' is lying along the x-axis at $x = 0$ and $x = L$. Its linear density (mass/length) varies with x as $k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against 'n' which of the following graphs best approximates the dependence of x_{CM} on n?



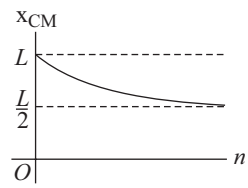
(a)



(b)



(c)



(d)

[2008]

36. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0º. Then

(a) at $\theta = 30^\circ$, the block will start sliding down the plane

(b) the block will remain at rest on the plane up to certain θ and then it will topple

(c) at $\theta = 60^\circ$, the block will start sliding down the plane and continue to do so at higher angles

(d) at, $\theta = 60^\circ$ the block will start sliding down the plane and on further increasing , it will topple at certain θ.

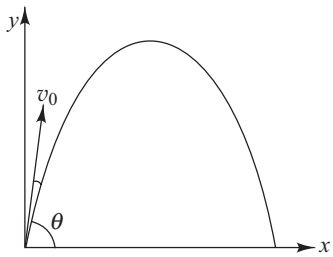
[2009]

37. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to maximum height of:

(a) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (b) $\frac{1}{6}\frac{l^2\omega^2}{g}$

(c) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{6}\frac{l\omega}{g}$ **[2009]**

38. A small particle of mass m is projected at an angle with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is



(a) $-mgv_0t^2\cos\theta\,\hat{\mathbf{j}}$

(b) $-mgv_0\,t\cos\theta\,\hat{\mathbf{k}}$

(c) $-\frac{1}{2}mgv_0\,t^2\cos\,\theta\,\hat{\mathbf{k}}$

(d) $\frac{1}{2}mgv_0\,t^2\cos\,\theta\,\hat{\mathbf{i}}$

where $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are unit vectors along x, y and z-axes respectively. **[2010]**

39. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of insect, the angular speed of the disc

(a) remains unchanged

(b) continuously decreases

(c) continuously increases

(d) first increases and then decreases **[2011]**

40. A mass m hangs with the help of string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is

(a) $3g/2$ (b) g

(c) $2g/3$ (d) $g/3$ **[2011]**

41. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kgm^2, the number of rotations made by the pulley before its direction of motion is reversed is:

(a) less than 3

(b) more than 3 but less than 6

(c) more than 6 but less than 9

(d) more than 9 **[2011]**

42. Adiatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by:

(a) $\frac{(m_1+m_2)^2n^2h^2}{2\pi^2m_1^2m_2^2r^2}$ (b) $\frac{n^2h^2}{2\pi^2(m_1+m_2)r^2}$

(c) $\frac{8\pi^2n^2h^2}{(m_1+m_2)r^2}$ (d) $\frac{(m_1+m_2)n^2h^2}{8\pi^2m_1m_2r^2}$

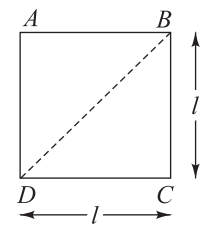
[2012]

43. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

(a) $\frac{r\omega_0}{3}$ (b) $\frac{r\omega_0}{2}$

(c) $r\omega_0$ (d) $\frac{r\omega_0}{4}$ **[2013]**

44. Four point masses, each of value m, are placed at the corners of square $ABCD$ of side l. The moment of inertia of this system about an axis passing through A and parallel to BD is



(a) $3\,ml^2$ (b) ml^2

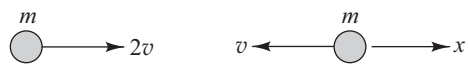
(c) $2\,ml^2$ (d) $\sqrt{3}\,ml^2$ **[2014]**

Answers

1. (c)	2. (c)	3. (d)	4. (a)
5. (b)	6. (c)	7. (b)	8. (d)
9. (a)	10. (d)	11. (d)	12. (d)
13. (b)	14. (c)	15. (a)	16. (c)
17. (b)	18. (d)	19. (c)	20. (d)
21. (c)	22. (d)	23. (c)	24. (a)
25. (a)	26. (b)	27. (a)	28. (a)
29. (c)	30. (a)	31. (a)	32. (c)
33. (d)	34. (a)	35. (b)	36. (b)
37. (b)	38. (c)	39. (d)	40. (c)
41. (b)	42. (d)	43. (b)	44. (a)

Solutions

1. Let m be the mass of each particle. Velocity along + x-axis is taken to be positive and along – x-axis as negative



$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

$$\therefore\ v_{CM} = \frac{m \times 2v + m \times (-v)}{m + m} = \frac{v}{2}$$

2. Since no external torque acts, the angular momentum is conserved. Hence

$$I_1\,\omega_1 = I_2\,\omega_2 \qquad \text{(i)}$$

If R is the radius of the disc, $I_1 = \frac{1}{2}MR^2$ and $I_2 = \frac{1}{2}Mr^2 + mR^2 + mR^2 = \frac{1}{2}(M + 4m)R^2$. Using these in (i), we get

$$\omega_2 = \left(\frac{I_1}{I_2}\right)\omega_1 = \left(\frac{M}{M + 4m}\right)\omega_1$$

3. Since there is no friction, rolling will not take place. The acceleration of a body sliding down an inclined plane of inclination θ is $g \sin\theta$, which is independent of the mass of the body. Hence the correct choice is (*d*).

4. Suppose the ring lies in the x-y plane with its O. The moment of inertia about the z-axis passing through the origin O is

$$I_{CM} = I_z = MR^2$$

If I_x and I_y are the moment of inertia about x and y diameters, then from perpendicular axes theorem,

$$I_x + I_y = I_z$$

From symmetry $I_y = I_x$. Therefore

$$I_x + I_x = I_z$$

$$\Rightarrow 2I_x = MR^2 \quad \Rightarrow \quad I_x = \frac{1}{2}MR^2$$

5. Refer to the figure of Q.5. The angular momentum of the particle about O is

$$\vec{L} = m(\vec{r} \times \vec{v})$$

Magnitude of $\vec{L}$ is $L = mrv \sin\theta$

$$= mv\,(r \sin\theta)$$

$$= mvl$$

6. The velocity v_{CM} of the centre of mass can be obtained by using the principle of conservation of linear momentum.

$$M\,V = (M + m)\,v_{CM}$$

$$\text{Or} \quad v_{CM} = \frac{MV}{(M + m)} = \frac{10\,\text{kg} \times 14\,\text{ms}^{-1}}{(10 + 4)\,\text{kg}}$$

$$= 10\ \text{ms}^{-1}$$

7. When a cylinder rolls up or down in inclined plane, its angular acceleration is always directed down the plane. Hence the frictional force acts up the inclined plate when the cylinder rolls up or down the plane.

8. If ρ is the density of iron, then the masses of discs are

$$M_x = \pi R^2 \times t \times \rho$$

$$\text{and} \qquad M_y = \pi\,(4R)^2 \times \frac{t}{4} \times \rho$$

$$I_x = \frac{1}{2}M_xR^2 = \frac{1}{2} \times R^3 t\rho$$

$$\text{and} \qquad I_y = \frac{1}{2}M_y(4R)^2 = \frac{1}{2} \times 64\pi R^3 t\rho$$

$$\therefore \qquad \frac{I_y}{I_x} = 64$$

9. Angular momentum $L = mvr = mr^2\omega \qquad (\because\ v = r\omega)$

where r = radius of the circle, m = mass of the particle and ω = angular velocity (or angular frequency)

Kinetic energy is $K = \frac{1}{2}\,mv^2 = \frac{1}{2}\,m(r\omega)^2 = \frac{1}{2}\,mr^2\omega^2$

$$\text{Now} \qquad L = mr^2\omega = \frac{mr^2\omega^2}{\omega} = \frac{2K}{\omega}$$

If K is halved and ω is doubled, the new angular momentum will be 4 L.

10. $\vec{\tau} = \vec{r} \times \vec{F}$. Hence $\vec{\tau}$ is perpendicular to both $\vec{r}$ and $\vec{F}$ Therefore, $\vec{r}.\vec{\tau} = 0$ and $\vec{F}.\vec{\tau} = 0$
11. The correct choice is (*d*).
12. Applying the conservation of angular momentum about the centre of rod, we have

$$I_{CM} \times \omega = J \times \frac{L}{2}$$

$$\Rightarrow \quad 2 \times M\left(\frac{L}{2}\right)^2 \times \omega = M\,V \times \frac{L}{2}$$

$$\Rightarrow \quad \omega = \frac{V}{L}$$

13. Since in free space, no torque acts on the sphere, its angular momentum will remain unchanged, i.e. $L = I\omega$ = constant. $I = \frac{2}{5}MR^2$ will increase if R is increased. Hence ω will decrease. Rotational K.E. = $\frac{1}{2}I\omega^2$ = $I\omega \times \omega = L\omega$ will also decrease as L remains constant.
14. $I_A = \frac{2}{5}MR^2$ and $I_B = \frac{2}{3}MR^2$. Therefore $\frac{I_A}{I_B} = \frac{3}{5} \Rightarrow I_A < I_B$
15. Let I_1 and ω_1 be the moment of inertia and angular frequencey when his arms are outretched and I_2 and $I_2\omega_2$ those when his arms are folded. Then

$I_1\ \omega_1 = I_2\omega_2$

Given $I_2 = \frac{3}{4}I_1$. Hence $I_1\omega_1 = \frac{3}{4}\ I_2\ \omega_2$

or $\omega_2 = \frac{4}{3}\omega_1$.

Initial KE is $K_1 = \frac{1}{2}I_1\omega_1^2$ and final KE is

$$K_2 = \frac{1}{2}I_2\omega_2^2 = \frac{1}{2} \times \frac{3I}{4} \times \left(\frac{4\omega_1}{3}\right)2$$

$$= \frac{4}{3}\left(\frac{1}{2}I_1\omega_1^2\right) = \frac{4}{3}K_1$$

$\therefore$ Percentage increase in KE = $\frac{K_2 - K_1}{K_1} \times 100$

$$= \frac{\frac{4}{3}K_1 - K_1}{K_1} \times 100 = \frac{100}{3} = 33.3\%$$

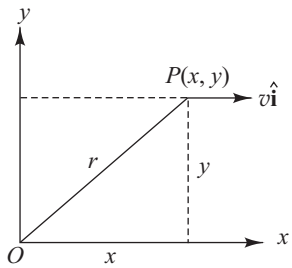
16. The disc is rolling about the point *O*. Thus the axis of rotation passes through the point *A* and is perpendicular to the plane of the disc. From the relation $v = r\omega$ where r is the distance of the point on the rim about the axis of rotation, we have

$v_A = 0, v_B = (A\ B)\ \ \omega = \frac{3R\omega}{2}$

and $v_c = (A\ C)\ \omega = 2R\omega$

Hence $\frac{v_B}{v_c} = \frac{3R\omega}{2} \times \frac{1}{2R\omega} = \frac{3}{4}$. Thus ther correct choice is (c).

17. The angular momentum of the particle at point P (x, y) about origin O is given by



$L = m\ \mathbf{r} \times v$

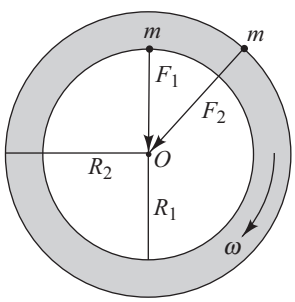
$= m\ (x\hat{\boldsymbol{i}} + y\,\hat{\boldsymbol{j}}) \times v\hat{\boldsymbol{i}}$

$= -myv\ \hat{\boldsymbol{k}}$

$(\because\ \hat{\boldsymbol{i}} \times \hat{\boldsymbol{i}} = 0\ and\ \hat{\boldsymbol{j}} \times \hat{\boldsymbol{i}} = -k)$

Now, mass m and velocity v are constant. Also y remain constant as the particle moves parallel to the x-axis. Hence L remains constant. Thus the correct choice is (*b*).

18. The particles experience radial force directed towards centre *O*. The centripetal forces experienced by the particals are



$$F_1 = mR_1\omega^2$$

and $$F_2 = mR_2\omega^2$$

$\therefore$ $$\frac{F_1}{F_2} = \frac{R_1}{R_2}$$

19. The acceleration of bodies *B* and *C* is the same, equal to *g*. Therefore, the acceleration of the centre of mass of *B* and *C* is

$$a_{CM} = \frac{m_A a_A + m_B a_B}{m_A + m_B}$$

$$= \frac{\frac{M}{3} \times g + \frac{2M}{3} \times g}{\frac{M}{3} + \frac{2M}{3}} = g$$

Hence acceleration before splitting = acceleration after splitting. So the centre of mass of bodies *B* and C taken together does not shift. The correct choice is (c).

20. Since the mass of the semicircular disc is *M*, the mass of the complete disc would be

$M' = M + M = 2M$

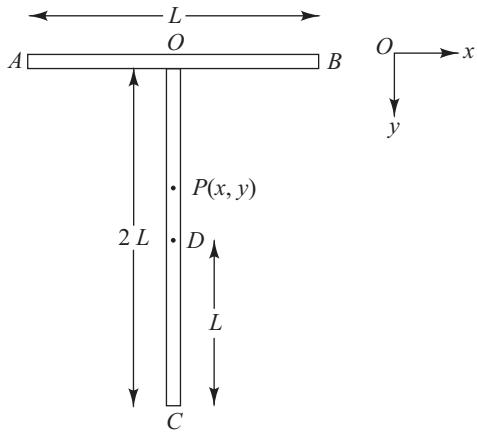
Therefore, the moment of inertia of the complete disc about the given axis is

$$I' = \frac{1}{2}M'R^2 = \frac{1}{2}\times 2M \times R^2 = MR^2$$

Let *I* be the moment of inertia of the semicircular disc. Since the complete disc is combination of two semicircular discs, it follows that

$$I' = \frac{I'}{2} = \frac{1}{2}MR^2$$

21. Since there is no rotation, the torque about the centre of mass of the object is zero. Hence $\vec{F}$ must be applied at the centre of mass *P* of the object. Let us take centre *O* of portion *AB* at the origin (0,0). If m_1 is the mass of part *AB* and m_2 the mass of part *OC*, then if the object has uniform linear mass density, $m_2 = 2m_1$. (Then since the mass of *AB* can be assumed to be concentrated at *O* and the mass of *OC* at *D*)



$$OP = \frac{m_1 \times 0 + m_2 \times L}{m_1 \times m_2} = \frac{2m_1 L}{3m_1} = \frac{2L}{3}$$

$$\therefore \quad PC = OC - OP = 2L - \frac{2L}{3} = \frac{4L}{3}.$$

22. Mass per unit area of the disc = $\frac{M}{\pi R^2}$. Therefore mass of the removed portion (hole of radius R/3) is

$$m = \frac{M}{\pi R^2} \times \pi\left(\frac{R}{3}\right)^2 = \frac{M}{9}$$

The moment of inertia of the complete disc about an axis passing through its centre *O* and perpendicular to its plane is

$$I = \frac{1}{2}MR^2$$

Using the parallel axes theorem, the moment of inertia of the removed portion of the disc about the axis passing through *O* and perpendicular to the plane of the disc is

I' = MI of mass *m* about $O' + m \times OO'$

$$= \frac{1}{2}\, m\left(\frac{R}{3}\right)^2 + m \times \left(\frac{R}{3}\right)^2$$

$$= \frac{1}{2} \times \frac{M}{9} \times \frac{R^2}{9} + \frac{M}{9} \times \frac{4R^2}{9} = \frac{1}{18} M R^2$$

Therefore, the moment of inertia of the remaining portion of the disc about $O = I - I' = \frac{1}{2}M R^2 - \frac{1}{18}M R^2$

$= \frac{4}{9}M R^2$.

23. Since the block is held stationary, it must satisfy the conditions of translational and rotational equilibrium. Hence $F = N$ and $f = mg$. Force *F* and weight *mg* do not produce any torque as their lines of action pass through the centre of mass. Hence (c) is the only incorrect statement.

24. Let *M* be the mass of the sphere. The mass of the disc will also be *M*. The moment of inertia of the sphere about its diameter is

$$I_s = \frac{2}{5}M R^2$$

The moment of inertia of the disc about its edge and perpendicular to its plane is (using parallel axes theorem)

$$I_d = I_{cm} + M h^2 = \frac{1}{2}Mr^2 + Mr^2$$

$$= \frac{3}{2}Mr^2$$

Given $I_s = I_d$. Hence, we have

$$\frac{2}{5}MR^2 = \frac{3}{2}Mr^2$$

which gives $\frac{r}{R} = \frac{2}{\sqrt{15}}$

25. If x_1 and x_2 are the positions of masses m_1 and m_2, the position of the centre of mass given by

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

If x_1 changes by Δx_1 and x_2 changes by Δx_2, the change in x_{cm} will be

$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} \qquad (1)$$

Given Δx_{cm} and $\Delta x_1 = d$. Using these values in Eq. (1), we get $m_1 d + m_2 \Delta x_2 = 0$ or

$$\Delta x_2 = \frac{m_1 d}{m_2}$$

$\therefore$ Distance moved by $m_2 = \dfrac{m_1 d}{m_2}$

26. The angular momentum of the ring before the two objects attached is

$L = mR^2\omega$

After the two objects are attached, the angular momentum becomes

$L' = (m + 2M)R^2\omega'$

Since no external torque acts, the angular momentum is conserved, i.e. $L = L'$ or

$mR^2\omega = (m + 2M)R^2\omega'$

or $\omega' = \dfrac{m\omega}{(m + 2M)}$

27. $r = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

$= 1 \times \hat{\mathbf{i}} + (-1) \times \hat{\mathbf{j}} + 0 \times \hat{\mathbf{k}} = (\hat{\mathbf{i}} - \hat{\mathbf{j}})$

$\vec{F} = -F\hat{\mathbf{k}}$

Torque $= \mathbf{r} \times \mathbf{F}$

$= (\hat{\mathbf{i}} - \hat{\mathbf{j}}) \times (-F\hat{\mathbf{k}})$

$= F(-\hat{\mathbf{i}} \times \hat{\mathbf{k}} + \hat{\mathbf{j}} \times \hat{\mathbf{k}})$

$= F(\hat{\mathbf{j}} + \hat{\mathbf{i}})$

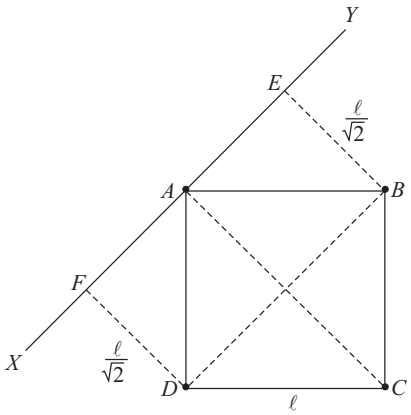
$(\because \hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$ and $\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}})$

28. The moment of inertia of the system about axis XY is

$I = m$ at $A \times 0 + m$ at $B \times (BE)^2 + m$ at $C \times (AC)^2 + m$ at $D \times (DF)^2$

$= m \times 0 + m \times \left(\dfrac{l}{\sqrt{2}}\right)^2 + m \times (\sqrt{2}l)^2$

$= 0 + \dfrac{ml^2}{2} + 2ml^2 + \dfrac{ml^2}{2} = 3ml^2$



29. The moment of inertia of a square lamina about an axis EF passing through its centre is given by

$I_{EF} = \dfrac{Ml^2}{12}$; M = mass of lamina

It is clear from the figure that the moment of inertia of triangular sheet ABC about $AC = \dfrac{1}{2} \times$ moment of inertia of the square sheet about it diagonal AC, i.e.

$I_{AC} = 2 \times$ (M.I. of triangular sheet ABC about AC)

$= 2 \times \left(\dfrac{M}{2} \times \dfrac{l^2}{12}\right)$ ($\because$ mass to triangular sheet $= M/2$)

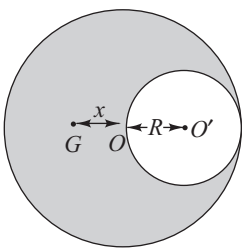
$= \left(\dfrac{Ml^2}{12}\right)$

Thus $I_{EF} = I_{AC}$, which is choice (c).

30. Let σ be the mass per unit area of the disc.

Mass of the disc $M = \sigma \times 4\pi (2R)^2 = 16\pi\sigma R^2$

Mass of the removed part $m_1 = 4\pi\sigma R^2 = \dfrac{M}{4}$



$\therefore$ Mass of the remaining part $m_2 = M - m_1 = \text{M} - \dfrac{M}{4} = \dfrac{3M}{4}$

Let G be the centre of mass of the remaining part of the disc. (see the figure). The centre of the complete disc is O and O' is the centre of the removed part of the disc. From the definition of the centre of mass, we have

$$x_{cm} = \frac{m_1 \times OO' + m_2 \times OG}{m_1 + m_2}$$

since $x_{cm} = 0$, we get

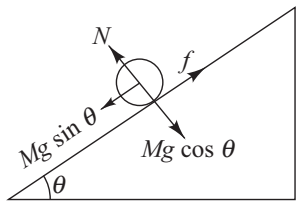
$$m_1 \times OO' + m^2 \times OG = 0 \qquad (1)$$

Let O be at the origin ($x = 0$), then $OG = -x$ and $OO' = R$. Then from Eq. (1), we have

$$\frac{M}{4} \times R + \frac{3M}{4} \times (-x) = 0 \implies x = \frac{R}{3} \qquad (2)$$

In the question it is given that $x = \dfrac{\alpha}{R}$ which is wrong because this equation is dimensionally inconsistent. The correct relation should be $x = \alpha R$. Comparing this equation with Eq.(2), we get $\alpha = \dfrac{1}{3}$ which is choice (*a*).

31. It is clear that



$Mg \sin \theta - f = Ma_{CM}$ (1)

where f is the frictional force and a_{CM} is the accleration of the centre of mass of the body. The torque τ for rolling is provided by the frictional force f and is given by

$\tau = f \times R = I\alpha$ (2)

Where α is the angular acceleration of the body. Also

$a_{CM} = R\alpha$ (3)

Using (3) in (2), we get $f = \dfrac{a_{CM} I}{R^2}$

Using (4) in (1) we get (4)

$$Mg \sin \theta - \frac{a_{CM} I}{R^2} = M a_{CM}$$

$$\Rightarrow \quad a_{CM} = \frac{g \sin\theta}{1 + \dfrac{I}{MR^2}}$$

32. Torque $\tau = \left|\vec{r} \times \vec{F}\right| = rF \sin \theta$. Since the force is directed towards the centre, θ = 180º. Therefore torque = 0. Hence the correct choice is (c).

33. From the principle of conservation of mechanical energy, we have

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = mg \times \frac{3v^2}{4g}$$

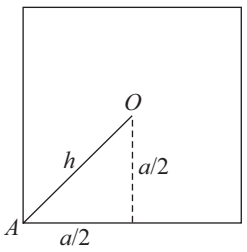
On solving, we get $I = \dfrac{mR^2}{2}$. Hence the object is a disc.

34. From parallel axes theorem the moment of inertia about the axis passing through A and perpendicular to the plane of the square plate is

$$I_A = I_{CM} + mh^2$$

$$= \frac{mR^2}{6} + m\left[\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right]$$

$$= \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{2ma^2}{3}.$$



35. $x_{CM} = \dfrac{\int x\,dm}{\int dm}$

Now $dm = k\left(\dfrac{x}{L}\right)^n dx$. Therefore

$$x_{CM} = \frac{\int_0^L k\left(\frac{x}{L}\right)^n x\,dx}{\int_0^L k\left(\frac{x}{L}\right)^n dx}$$

$$= \frac{\int_0^L x^{(n+1)} dx}{\int_0^L x^n dx} = \frac{(n+1)L}{(n+2)}$$

If $n = 0$, $x_{CM} = \dfrac{L}{2}$, if $n \to \infty$, x_{CM} approaches L.

Between $n = 0$ and $n = \infty$, x_{CM} increases non-linearly with increase in the value of n. Hence the correct choice is (*b*).

36. The block will just begin to slide if the downward force $mg \sin \theta$ just overcomes the frictional force, i.e. if $mg \sin \theta = \mu N = \mu\, mg \cos \theta \Rightarrow \tan \theta = \mu = \sqrt{3} \Rightarrow \theta = 60^\circ$.

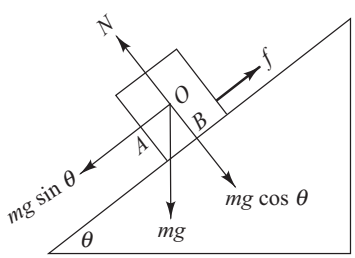
The block will topple if the torque of due to normal reaction N about O just exceeds the torque due to $mg \sin \theta$ about 0, i.e.

$N \times OA = mg \sin\theta \times OB$

$\Rightarrow mg \cos\theta \times 5 \text{ cm} = mg \sin\theta \times \dfrac{15}{2}\text{cm}$

$\Rightarrow \tan\theta = \dfrac{2}{3} \Rightarrow \theta \simeq 34^\circ.$

Since θ for toppling is less than θ for sliding, the correct choice is (b).



37. $mgh = \frac{1}{2} I\omega^2$

$\Rightarrow mgh = \frac{1}{2} \times \left(\frac{ml^2}{3}\right) \times \omega^2$

$\Rightarrow h = \frac{l^2\omega^2}{6g}$

38. $\vec{L} = m(\vec{r} \times \vec{v})$

The horizontal and vertical distances travelled by the particle in time t are

$x = (v_0 \cos\theta)\, t$

and $y = (v_0 \sin\theta)t - \frac{1}{2} gt^2$

$\therefore \quad \vec{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$

$= [(v_0 \cos\theta)t]\hat{\mathbf{i}} + \left[(v_0 \sin\theta)\, t - \frac{1}{2} gt^2\right]\hat{\mathbf{j}}$

The horizontal and vertical velocities at time t are

$v_x = \frac{dx}{dt} = v_0 \cos\theta$

and $v_y = \frac{dy}{dt} = v_0 \sin\theta - gt$

$\therefore \quad \vec{v} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = (v_0 \cos\theta)\hat{\mathbf{i}} + (v_0 \sin\theta - gt)\hat{\mathbf{j}}$

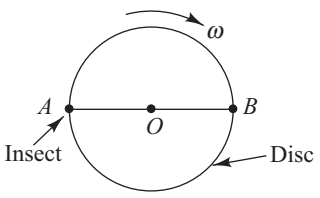
$\therefore \quad \vec{L} = m(\vec{r} \times \vec{v})$

$= m\left[\{(v_0 \cos\theta)t\,\hat{\mathbf{i}}\} + \left\{(v_0 \sin\theta)t - \frac{1}{2} gt^2\right\}\hat{\mathbf{j}}\right.$

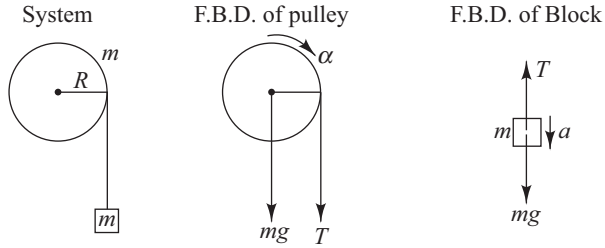
$\left. \times\{(v_0 \cos\theta)\hat{\mathbf{i}} + (v_0 \sin\theta - gt)\hat{\mathbf{j}}\}\right]$

$= \left(-\frac{1}{2} m g\, v_0\, t^2 \cos\theta\right)\hat{\mathbf{k}}$

39. Since no external torque acts on the system (insect + disc), the total angular momentum remains constant, i.e. $I\omega$ = constant. As the insect moves from A to O, the moment of inertia of the system about the given axis decreases. Hence ω increases. As the insect moves from O to B, the moment of inertia of the system decreases. Hence ω increases. Thus, the correct choice is (d).



40. Refer to the following figure.



Torque acting on pulley is

$\tau = TR \qquad (1)$

Also $\tau = I\alpha = \frac{1}{2} mR^2 \times \frac{a}{R} = \frac{mRa}{2} \qquad (2)$

From (1) and (2) $TR = \frac{mRa}{2} \Rightarrow T = \frac{ma}{2}$

From F.B.D. of block

$mg - T = ma$

$\Rightarrow \quad mg - \frac{ma}{2} = ma \quad \Rightarrow \quad a = \frac{2g}{3}$

41. Torque $\tau = FR = (20t - 5t^2) \times 2$ Nm

$= (40\, t - 10\, t^2)$ Nm

Also $\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I} = \frac{(40t - 10t^2)}{10}$

$= (4t - t^2)$ rad s^{-2}

or $\frac{d\omega}{dt} = 4t - t^2$

$\Rightarrow \quad \int d\omega = \int (4t - t^2)dt$

$\Rightarrow \quad \omega = 2t^2 - \frac{t^3}{3}$

The moment the direction of motion is reversed, $\omega = 0$ at that moment. This happens at time t given by

$0 = 2t^2 - \frac{t^3}{3} \quad \Rightarrow \quad t = 6s$

$0 = \int_0^6 \omega\, dt = \int_0^6 \left(2t^2 - \frac{t^3}{3}\right) dt$

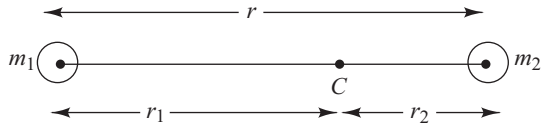
$= \left|\frac{2t^3}{3}\right|_0^6 - \left|\frac{t^4}{12}\right|_0^6$

$= \frac{2}{3}(6)^3 - \frac{1}{12}(6)^4$

$\Rightarrow \quad \theta = 36$ rad

$\therefore$ Number of rotation in $6s = \dfrac{36}{2\pi} = 5.7$, which is more than 3 and less than 6. So the correct choice is (b)

42. Let C be the centre of mass of the diatomic molecule.



$$r_1 = \frac{m_2 r}{m_1 + m_2}$$

and $$r_2 = \frac{m_1 r}{m_1 + m_2}$$

Rotational kinetic energy is

$$K = \frac{1}{2} I\omega^2 \qquad (1)$$

From Bohr's quantization condition,

$$L = \frac{nh}{2\pi}$$

or $$I\omega = \frac{nh}{2\pi}$$

$\Rightarrow$ $$\omega = \frac{nh}{2\pi I} \qquad (2)$$

Using (2) in (1),

$$K = \frac{1}{2} \times \frac{n^2 h^2}{4\pi^2 I} \qquad (3)$$

Also $$I = m_1 r_1^2 + m_2 r_2^2$$

$$= m_1 \times \left(\frac{m_2 r}{m_1 + m_2}\right)^2 + m_2 \times \left(\frac{m_1 r}{m_1 + m_2}\right)^2$$

$$= \frac{m_1 m_2 r^2}{(m_1 + m_2)} \qquad (4)$$

Using (4) in (3),

$$K = \frac{m^2 h^2 (m_1 + m_2)}{8\pi^2 m_1 m_2 r^2}$$

43. Since no external torque acts, the angular momentum about the point of contact is conserved, i.e.

$I\omega_0 = mvr + I\omega$

where $I = mr^2$, the moment of inertia about the centre of mass.

$\therefore$ $$mr^2 \omega_0 = mvr + mr^2 \times \frac{v}{r} \qquad \left(\because \omega = \frac{v}{r}\right)$$

$$= 2mrv$$

$\Rightarrow$ $$v = \frac{r\omega_0}{2}$$

44. Refer to the solution of question 28.

GRAVITATION

Chapter 6

REVIEW OF BASIC CONCEPTS

1. Newton's Law of Gravitation

Newton's law of universal gravitation states as follows:

'Any two particles of matter anywhere in the universe attract each other with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them, the direction of the force being along the line joining the particles, i.e. (Fig. 6.1)

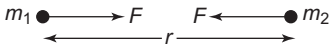


Fig. 6.1

$$F \propto \frac{m_1 m_2}{r^2}$$

where F is the magnitude of the force of attraction between two particles of masses m_1 and m_2 separated by a distance r.

In the form of an equation the law is written as

$$F = \frac{G m_1 m_2}{r^2}$$

where G is a constant called the universal gravitation constant. The value of this constant is to be determined experimentally and is found to be

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}.$$

2. Gravitational Force due to Multiple Masses

If a system consists of more than two masses, the gravitational force experienced by a given mass due to all other masses is obtained from the principle of superposition which states that '*the gravitational force experienced by one mass is equal to the vector sum of the gravitational forces exerted on it by all other masses taken one at a time.*'

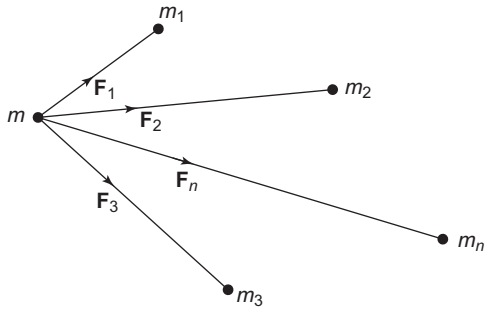


Fig. 6.2

The gravitational force on mass m due to masses m_1, m_2, m_3, ... m_n is given by (Fig. 6.2)

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots + \mathbf{F}_n$$

Note

1. Gravitational force is alway attractive.
2. Gravitational force between two masses does not depend on the medium between them.
3. Gravitational force acts along the straight line joining the centres of the two bodies.

EXAMPLE 1 Two bodies A and B of masses $m_1 = 1$ kg and $m_2 = 16$ kg respectively are placed 1.0 m apart. A third body C of mass $m = 3$ kg is placed on the line joining A and B. At what distance from A should C be placed so that it experiences no gravitational force?

SOLUTION Let x metre be the distance between A and C (Fig. 6.3)

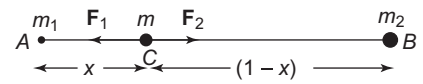


Fig. 6.3

Force exerted by A on C is

$$F_1 = \frac{G m_1 m}{x^2} \text{ directed towards } A$$

Force exerted by B on C is

$$F_2 = \frac{G m_2 m}{(1-x)^2} \text{ directed towards } B$$

C will experience no force if $F_1 = F_2$, i.e.

$$\frac{G m_1 m}{x^2} = \frac{G m_2 m}{(1-x)^2}$$

$$\frac{m_2}{m_1} = \frac{(1-x)^2}{x^2}$$

$$16 = \frac{(1-x)^2}{x^2}$$

$$4 = \frac{1-x}{x}$$

which gives $x = 0.2$ m. Note that if body C is placed to the left of body A or to the right of body B, it will experience a finite gravitational force.

EXAMPLE 2 Three bodies, each of mass m, are placed at the vertices of an equilateral triangle of side a as shown in Fig. 6.4(a). Find the magnitude and direction of the force experienced by the body at vertex A.

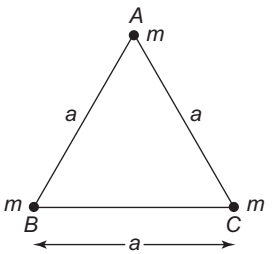


Fig. 6.4(a)

SOLUTION The forces exerted on the body at A by bodies at B and C are shown in Fig. 6.4(b).

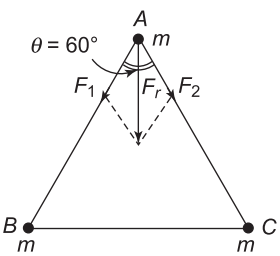


Fig. 6.4(b)

$$F_1 = F_2 = \frac{Gm^2}{a^2} = F \text{ (say)}$$

The magnitude of the resultant force is

$$F_r = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{F^2 + F^2 + 2F^2\cos 60^\circ}$$

$$= \sqrt{3}\,F = \sqrt{3}\,\frac{Gm^2}{a^2},$$

directed vertically downwards.

EXAMPLE 3 A uniform sphere of radius R and mass M exerts a force F on a body of mass m placed at point P at a distance $2R$ from the centre of the sphere. If a spherical cavity of radius $R/2$ is made in the sphere as shown in Fig. 6.5, the force of attraction exerted by the sphere with cavity in it on the same body placed at the same point P will now be

(a) $\frac{3F}{5}$ (b) $\frac{5F}{7}$

(c) $\frac{7F}{9}$ (d) $\frac{F}{2}$

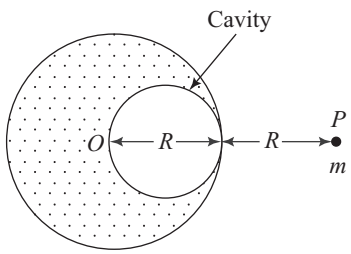


Fig. 6.5

SOLUTION $F = \frac{GMm}{(2R)^2} = \frac{GMm}{4R^2}$

Mass per unit volume $= \frac{M}{\frac{4\pi}{3}R^3}$

$\therefore$ Mass removed to make the cavity is

$$M' = \frac{M}{\frac{4\pi}{3}R^3} \times \frac{4\pi}{3}\left(\frac{R}{2}\right)^3 = \frac{M}{8}$$

The force of attraction the body at P now will be

F' = force due to complete sphere of mass M

– force due to removed part of mass $\frac{M}{8}$

$$= \frac{GmM}{4R^2} - \frac{G m \times \frac{M}{8}}{\left(\frac{3R}{2}\right)^2}$$

$$= \frac{GmM}{4R^2} - \frac{GmM}{18R^2} = \frac{7}{36}\frac{GmM}{R^2}$$

or $$F' = \frac{7}{9} \times \frac{GmM}{4R^2} = \frac{7F}{9}$$

EXAMPLE 4 A block of mass m is hung on a spring of spring constant k and of negligible mass. The spring extends by x on the surface of the earth. When taken to a height $h = R/2$, the spring will extend by (R = radius of earth)

(a) x (b) $\frac{2x}{3}$

(c) $\frac{3x}{8}$ (d) $\frac{4x}{9}$

SOLUTION $\frac{GmM}{R^2} = mg$ (M = mass of earth)

But $mg = kx$. Therefore

$$\frac{GmM}{R^2} = kx$$

$$\Rightarrow \quad x = \frac{GMm}{kR^2} \qquad \text{(i)}$$

At a height h,

$$\frac{GmM}{(R+h)^2} = mg' = kx'$$

$$\Rightarrow \quad x' = \frac{GmM}{k(R+h)^2} \qquad \text{(ii)}$$

From (i) and (ii)

$$x' = x \times \left(\frac{R}{R+h}\right)^2$$

$$= x \times \left(\frac{R}{R+R/2}\right)^2 = \frac{4x}{9}$$

3. Acceleration due to Gravity

Considering the earth as an isolated mass, a force is experienced by a body near it. This force is directed towards the centre of the earth and has a magnitude mg, where g is the acceleration due to gravity.

$$F = mg = \frac{GmM}{R^2}$$

where M is the mass of the earth and R its radius (nearly constant for a body in the vicinity of the earth)

$$\therefore \quad g = \frac{GM}{R^2}$$

All bodies near the surface of the earth fall with the same acceleration which is directed towards the centre of the earth.

4. Variation of g

1. Variation with altitude The acceleration due to gravity of a body at a height h above the surface of the earth is given by

$$g_h = g\left(\frac{R}{R+h}\right)^2$$

where g is the acceleration due to gravity on the surface of the earth. If h is very small compared to R, we can use binomial expansion and retain terms of order h/R. We then get

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

Thus, the acceleration due to gravity decreases as the altitude (h) is increased.

2. Variation with depth The acceleration due to gravity at a depth d below the surface of the earth is given by

$$g_d = g\left(1 - \frac{d}{R}\right)$$

This equation shows that the acceleration due to gravity decreases with depth. At the centre of the earth where $d = R$, $g_d = 0$. Thus the acceleration due to gravity is maximum at the surface of the earth, decreases with increase in depth and becomes zero at the centre of the earth.

3. Variation with Latitude Due to the rotation of earth about its axis, the value of g varies with latitude, i.e. from one place to another on the earth's surface. At poles, the effect of rotation on g is negligible. At the equator, the effects of rotation on g is the maximum. In general, the value of acceleration due to gravity at a place decreases with the decrease in the latitude of the place.

The accelration due to gravity at a place on earth where the latitude is ϕ is given by

$$g_\phi = g - \omega^2 R \cos^2 \phi;$$

ω = angular velocity of rotation of earth.

At equator, $\phi = 0 \quad \Rightarrow \quad g_e = g - R\omega^2$ (minimum)

At poles, $\phi = 90° \quad \Rightarrow \quad g_p = g$ (maximum)

$$g_\phi = g - 0.0337 \cos^2 \phi$$

Thus the value of g varies slightly from place to place on earth. Variation of g with altitude and depth is shown in Fig. 6.6.

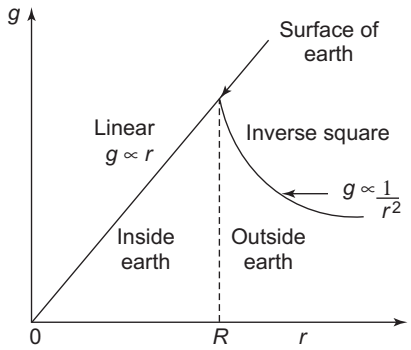


Fig. 6.6 *Variation of g (gravitational acceleration)*

EXAMPLE 5 A body weighs 63 N on the surface of the earth. How much will it weigh at a height equal to half the radius of the earth?

SOLUTION $W = mg = 63$ N

$$W' = mg' = mg\left(\frac{R}{R+h}\right)^2$$

$$\therefore \quad W' = W\left(\frac{R}{R+h}\right)^2$$

$$= 63 \times \left(\frac{R}{R+R/2}\right)^2 = 28 \text{ N}$$

EXAMPLE 6 Assuming the earth to be a sphere of uniform mass density, find the percentage decrease in the weight of a body when it is taken to the end of a tunnel 32 km below the surface of the earth. Radius of earth = 6400 km.

SOLUTION $g' = g\left(1-\frac{d}{R}\right)$

$$= g\left(1-\frac{32}{6400}\right) = \frac{199\,g}{200}$$

Decrease in weight $= mg - mg'$

$$= mg\left(1-\frac{199}{200}\right) = \frac{mg}{200}$$

Percentage decrease $= \frac{mg/200}{mg} \times 100 = 0.5\%$

EXAMPLE 7 At what depth below the surface of the earth will the value of acceleration due to gravity become 90% of its value at the surface? $R = 6.4 \times 10^6$ m.

SOLUTION $g' = 0.9\,g.$ $\qquad g' = g\left(1-\frac{d}{R}\right)$

$$\therefore \quad 0.9 = 1 - \frac{d}{R}$$

$$\Rightarrow \quad d = 0.1\,R = 6.4 \times 10^5 \text{ m}$$

5. Gravitational Field Intensity

Just as the region around a magnet has magnetic field and the region around a charge has electric field, the region around a mass has gravitational field.

The gravitational field intensity (or simply gravitational field) at a point is defined as the gravitational force experienced by a unit mass placed at that point.

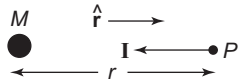


Fig. 6.7

Consider the gravitational field of a body of mass M. To find the strength of the field at a point P at a distance r from M, we place a small mass m at P. The gravitational force exerted m by M is

$$F = \frac{GMm}{r^2}$$

By definition, the gravitational field (intensity) of M at P is given by

$$I = \frac{F}{m} = \frac{GM}{r^2}$$

I is a vector quantity. In vector form

$$\mathbf{I} = -\frac{GM}{r^2}\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector directed from M to P, i.e radially away from M. The negative sign indicates that $\mathbf{I}$ directed radially inwards towards M.

The SI unit of I is N kg^{-1}.

In three dimensions, if mass M is located at the origin, the magnetic field at a $P(x, y, z)$ is given by

$$\mathbf{I} = -\frac{GM}{r^2}\hat{\mathbf{r}}$$

where $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ represents the position of point P with respect to mass M at the origin.

For a many body system, the principle of superposition holds for gravitational field (intensities) just as it holds for gravitational forces, i.e.

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \cdots + \mathbf{I}_n$$

where $\mathbf{I}_1, \mathbf{I}_2, \ldots \mathbf{I}_n$ are the gravitational field intensities at a point due to bodies of masses $M_1, M_2, \ldots M_n$.

For continuous mass distributions (i.e rigid bodies), we divide the body into an infinitely large number of infinitesimally small elements. Then the gravitational field intensity is given by

$$\mathbf{I} = \int d\,\mathbf{I}$$

Gravitational Field due to some continuous Mass Distributions

1. Gravitational field due to a circular ring of mass M and radius R at a point at a distance r from the centre and on the axis of the ring is given by

$$I = \frac{GMr}{(R^2 + r^2)^{3/2}}$$

2. Gravitational field due to a thin spherical shell of mass M and radius R at a point P at a distance $r > R$ from the centre of shell,

$$I = \frac{GM}{r^2} \text{ (outside the shell)}$$

Inside the shell, $I = 0$

3. Gravitational field of a solid sphere of mass M and radius R is

$$I = \frac{GM}{r^2} \text{ for } r > R$$

$$I = \frac{GM}{R^2} \text{ for } r = R$$

$$I = \frac{GMr}{R^3} \text{ for } r < R$$

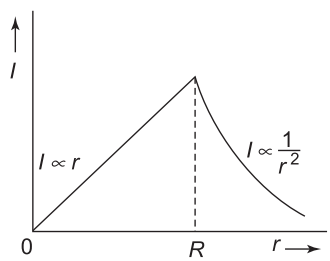


Fig. 6.8

Figure 6.8 shows the variation of the gravitational field of a solid sphere.

6. Gravitational Potential Energy

Gravitational potential energy of a system of two masses M and m held a distance r apart is defined as the amount of work done to bring the masses from infinity to their respective locations along any path and without any acceleration.

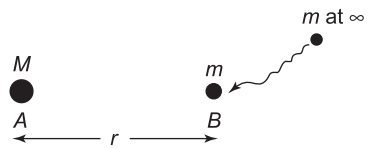


Fig. 6.9

Work done to bring mass M from infinity to A is $W_1 = 0$.
Work done to bring mass m from $r = \infty$ to $r = r$ is

$$W_2 = \int_0^r \mathbf{F} \cdot d\mathbf{r}$$

$$= \int_\infty^r F\, dr \cos\theta$$

Since mass M will attract mass m, angle θ between $\mathbf{F}$ and $d\mathbf{r}$ is zero. Hence

$$W_2 = \int_\infty^r \frac{GMm}{r^2} dr$$

$$= GMm \int_\infty^r r^{-2} dr = -\frac{GMm}{r}$$

Total work $W = W_1 + W_2 = -\dfrac{GMm}{r}$

$\therefore$ Gravitational potential energy of the system is

$$U = W = -\frac{GMm}{r}$$

The zero of potential energy is assumed to be at $r = \infty$. The negative sign indicates the potential energy is negative for any finite separation between the masses and increases to zero at infinite separation.

Expression for Increase in Gravitational Potential Energy

If the body m is moved away from M, the potential energy of the system increases.

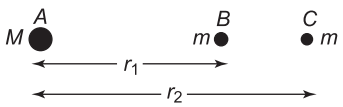


Fig. 6.10

$$\text{P.E. at } B = -\frac{GMm}{r_1}$$

$$\text{P.E. at } C = -\frac{GMm}{r_2}$$

$$\therefore \text{ Increase in P.E.} = -\frac{GMm}{r_2} - \left(-\frac{GMm}{r_1}\right)$$

$$= GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

If the body of mass M is the earth, then the increase in gravitational P.E. when a body of mass m is taken from the surface of the earth

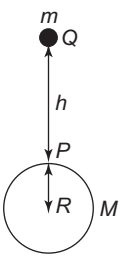


Fig. 6.11

to a height h above the surface is given by (see Fig. 6.11), R = radius of the earth.

$$\Delta U = \text{P.E. at } Q - \text{P.E at } P$$

$$= -\frac{GMm}{(R+h)} - \left(-\frac{GMm}{R}\right)$$

$$= \frac{GmMh}{R(R+h)}$$

If $h << R$; $\quad \Delta U = \dfrac{GmMh}{R^2} = mgh \quad \left(\because \dfrac{GM}{R^2} = g\right)$

7. Gravitational Potential

Gravitational potential at a point P in the gravitational field of a body of M is defined as the amount of work done to bring a unit mass from infinity to that point along any path and without any acceleration, i.e., (Fig. 6.12)

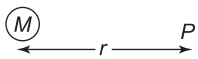


Fig. 6.12

$$V = \frac{W}{m} = -\frac{GMm}{r \times m}$$

$$\Rightarrow \qquad V = -\frac{GM}{r}$$

Potential V is a scalar quantity. Hence the gravitational potential at a point P due to a number of masses $m_1, m_2, \ldots m_n$ at distances $r_1, r_2, \ldots r_n$ respectively from P is given by

$$V = V_1 + V_2 + \cdots + V_n$$

$$= -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2} + \cdots + \frac{m_n}{r_n}\right)$$

The SI unit of V is J kg^{-1}.

Relation between Gravitational Field Intensity (I) and Gravitational Potential (V)

Gravitational field intensity and gravitational potential at a point are related as

$$I = -\frac{dV}{dr}$$

Gravitational Potential due to a Spherical Shell

(i) At a point outside the shell, $V = -\dfrac{GM}{r}$ (for $r > R$) where M is the mass and R is the radius of the shell.

(ii) At a point on the surface of the shell, $V = -\dfrac{GM}{R}$

(iii) At a point inside the shell, $V = -\dfrac{GM}{R}$ (for $r < R$)

Figure 6.13 shows the variation V with r for a spherical shell.

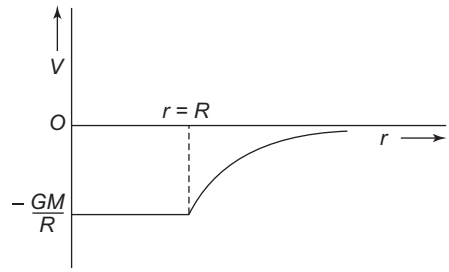


Fig. 6.13

Gravitational Potential due to a Solid Sphere of Mass M and Radius R

(i) For points outside the sphere ($r > R$), $V = -\dfrac{GM}{r}$

(ii) For points inside the sphere ($r < R$),

$$V = -\frac{3GM}{R^3}\left(\frac{R^2}{2} - \frac{r^2}{6}\right)$$

(ii) At the centre of the sphere ($r = 0$)

$$V = -\frac{3GM}{2R}$$

(iv) On the surface of the sphere ($r = R$)

$$V = -\frac{GM}{R}$$

EXAMPLE 8 Two masses m_1 = 100 kg and m_2 = 8100 kg are held 1 m apart.

(a) At what point on the line joining them is the gravitational field equal to zero? Find the gravitational potential at that point.

(b) Find the gravitational potential energy of the system. Given $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.

SOLUTION Gravitational field at P due to m_1 is [Fig. 6.14]

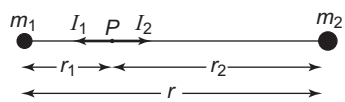


Fig. 6.14

$$I_1 = \frac{Gm_1}{r_1^2} \text{ directed towards } m_1$$

Gravitational field at P due to m_2 is

$$I_2 = \frac{Gm_2}{r_2^2} \text{ directed towards } m_2$$

Gravitational field at P will be zero if

$$I_1 = I_2 \quad \Rightarrow \quad \frac{Gm_1}{r_1^2} = \frac{Gm_2}{r_2^2}$$

or $$\frac{m_2}{m_1} = \left(\frac{r_2}{r_1}\right)^2$$

$$\Rightarrow \qquad 81 = \left(\frac{1-r_1}{r_1}\right)^2 \quad (\because\ r_2 = r - r_1 \text{ and } r = 1 \text{ m})$$

$$\therefore \qquad 9 = \frac{1-r_1}{r_1} \quad \Rightarrow \quad r_1 = 0.1 \text{ m}$$

Gravitational potential at P is

$$V = V_1 + V_2 = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2}\right)$$

$$= -6.67 \times 10^{-11} \times \left(\frac{100}{0.1} \times \frac{8100}{0.9}\right)$$

$$= -6.67 \times 10^{-7} \text{ J kg}^{-1}$$

(b) Gravitational potential energy of the system is

$$\text{G.P.E.} = -\frac{G\, m_1\, m_2}{r}$$

$$= -\frac{6.67 \times 10^{-11} \times 100 \times 8100}{1}$$

$$= -5.4 \times 10^{-5} \text{ J}$$

EXAMPLE 9 Three equal masses, each equal to m, are kept at the vertices of an equilateral triangle of side a. Find the gravitational field and gravitational potential at the centroid of the triangle.

SOLUTION Refer to Fig. 6.15.

The centroid O divides the lines AD, BE and CF in the ratio 2:1. Also $AO = BO = CO = r$ (say). Now $AO = \frac{2}{3}\ AD$

and $$AD = \sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}\, a}{2}$$

$\therefore$ $$AO = \frac{2}{3} \times \frac{\sqrt{3}\, a}{2} = \frac{a}{\sqrt{3}}, \text{ i.e. } r = \frac{a}{\sqrt{3}}$$

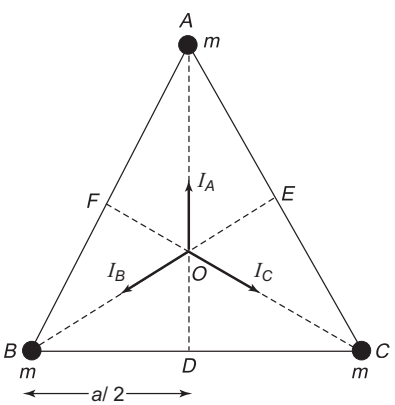


Fig. 6.15

The gravitational fields at O due masses m at A, B and C are $I_A = I_B = I_C = \frac{GM}{r^2} = I$. Their directions are shown in Fig. 6.14. The angle between any two of them is $\theta = 120°$.

The resultant of I_B and I_C is

$$I' = \sqrt{I^2 + I^2 + 2I^2 \cos 120°} = I$$

directed vertically down.

I' will cancel with I_A. Hence the net gravitational field at O is zero.

The gravitational potential at O is

$$V = V_1 + V_2 + V_3$$

$$= -\frac{Gm}{r} - \frac{Gm}{r} - \frac{Gm}{r}$$

$$= -\frac{3Gm}{r} = -3\sqrt{3}\,\frac{Gm}{a} \quad \left(\because r = \frac{a}{\sqrt{3}}\right)$$

EXAMPLE 10 The gravitational potential at a height h = 1600 km above the surface of the earth is -4.0×10^7 Jkg^{-1}. Assuming the earth to be a sphere of radius R = 6400 km, find the value of the acceleration due to gravity at that height.

SOLUTION Let $r = R + h$. Then

$$V = -\frac{GM}{r}$$

and $$g' = \frac{GM}{r^2}$$

$\therefore$ $$\frac{g'}{|V|} = \frac{GM/r^2}{GM/r} = \frac{1}{r} = \frac{1}{R+h}$$

$\Rightarrow$ $$g' = \frac{|V|}{(R+h)} = \frac{4.0 \times 10^7 \text{ J kg}^{-1}}{(6.4 + 1.6) \times 10^6 \text{ m}}$$

$$= 5.0 \text{ J kg}^{-1} \text{ m}^{-1} = 5.0 \text{ ms}^{-2}$$

EXAMPLE 11 Two particles of masses m_1 and m_2 are initially at rest at an infinite distance apart. They begin to move towards each other due to gravitational attraction. Find the ratio of their accelerations and speeds when the separation between them becomes r.

SOLUTION Since no external force acts on the system, the acceleration of their centre of mass is zero, i.e.

$$a_{CM} = \frac{m_1\, a_1 + m_2 a_2}{m_1 + m_2}$$

$\Rightarrow$ $$0 = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$$

$\Rightarrow$ $$m_1 a_1 = -m_2\, a_2$$

$\Rightarrow$ $$\frac{a_1}{a_2} = -\frac{m_2}{m_1}$$

The negative sign indicates that they move in opposite directions.

Let v_1 and v_2 be the speeds of two masses when they are at a distance r. Due to gravitational attraction, they gain speed as they approach each other. Hence their kinetic energy increases and gravitational potential energy (G.P.E) decreases. From the conservation of energy,

$$\text{Loss in G.P.E.} = \text{gain in K.E.}$$

$$(\text{G.P.E.})_i - (\text{G.P.E.})_f = (\text{K.E.})_f - (\text{K.E.})_i$$

$$\Rightarrow \quad 0 - \left(-\frac{Gm_1m_2}{r}\right) = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - 0$$

$$\Rightarrow \quad \frac{G\,m_1\,m_2}{r} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \qquad \text{(i)}$$

Since no external force acts, the total momentum of the system is conserved, i.e., $p_i = p_f$ or

$$0 = m_1v_1 - m_2v_2 \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$v_1 = \left(\frac{2Gm_2^2}{r(m_1+m_2)}\right)^{1/2}$$

and

$$v_2 = \left(\frac{2G\,m_1^2}{r(m_1+m_2)}\right)^{1/2}$$

$$\therefore \quad \frac{v_1}{v_2} = \frac{m_2}{m_1}$$

EXAMPLE 12 Infinite number of tiny spheres, each of mass m are placed on the x-axis at distances r, $2r$, $4r$, … etc from origin O. The magnitude of gravitational field intensity at O is

(a) $\frac{4Gm}{3r^2}$ (b) $\frac{2GM}{3r^2}$

(c) $\frac{GM}{r^2}$ (d) infinity

SOLUTION Since gravitational force is attractive, the direction of individual intensities will be towards O. Therefore, their magnitudes simply add up. Thus

$$I = I_1 + I_2 + I_3 + \cdots$$

$$= \frac{Gm}{r^2} + \frac{Gm}{(2r)^2} + \frac{Gm}{(4r)^2} + \cdots$$

$$= \frac{Gm}{r^2}\left(1 + \frac{1}{2^2} + \frac{1}{4^2} + \cdots\right)$$

$$= \frac{Gm}{r^2}\left(\frac{1}{2^0} + \frac{1}{2^2} + \frac{1}{2^4} + \cdots\right)$$

$$= \frac{Gm}{r^2} \times \frac{1}{\left(1 - \frac{1}{2^2}\right)} = \frac{4Gm}{3r^2}$$

EXAMPLE 13 In Example 11 above, the gravitational potential at origin O is

(a) $-\frac{Gm}{r}$ (b) $-\frac{2Gm}{r}$

(c) $-\frac{4Gm}{r}$ (d) infinity

SOLUTION Potential is a scalar quantity. Therefore, the gravitational potential at O is

$$V = V_1 + V_2 + V_3 + \cdots$$

$$= -\frac{Gm}{r} - \frac{Gm}{2r} - \frac{Gm}{4r} - \cdots$$

$$= -\frac{Gm}{r}\left(1 + \frac{1}{2} + \frac{1}{4} + \cdots\right)$$

$$= -\frac{Gm}{r}\left(\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \cdots\right)$$

$$= -\frac{Gm}{r} \times \frac{1}{\left(1 - \frac{1}{2}\right)} = -\frac{2Gm}{r}$$

EXAMPLE 14 The gravitational field due to a mass distribution is given by

$$I = \frac{k}{x^2}$$

along the $+x$ direction, where k is a constant. If the gravitational potential at infinity is taken to be zero, its value at a distance x will be

(a) $\frac{k}{x}$ (b) $\frac{k}{2x}$

(c) $\frac{k}{x^2}$ (d) $\frac{k}{2x^2}$

SOLUTION $I = -\frac{dV}{dx} \Rightarrow dV = -I\,dx$

$$\therefore \quad V \text{ at } x = -\int_0^x I\,dx = -k\int_0^x \frac{dx}{x^2}$$

$$= +k\left|\frac{1}{x}\right|_0^x = \frac{k}{x}$$

EXAMPLE 15 The gravitational potential at point P whose position coordinate is r (due to a certain mass distribution) is given by

$$V = \frac{a}{\sqrt{b^2 + r^2}}$$

where a and b are constants. The magnitude of the gravitational field at P will be

(a) $\frac{rV}{a}$ (b) $\frac{2rV^2}{3a}$

(c) $\frac{rV^3}{a^2}$ (d) $\frac{2rV^3}{3a^2}$

SOLUTION $E = -\frac{dV}{dr} = -\frac{d}{dr}\left[\frac{a}{(b^2 + r^2)^{1/2}}\right]$

$$\Rightarrow \quad E = \frac{ar}{(b^2 + r^2)^{3/2}} \qquad \text{(i)}$$

Now $(b^2 + r^2)^{1/2} = \frac{a}{V}$

$$\therefore \quad (b^2 + r^2)^{3/2} = \frac{a^3}{V^3} \qquad \text{(ii)}$$

Using (ii) in (i) we get

$$E = \frac{rV^3}{a^2}$$

EXAMPLE 16 Two particles, each of mass m, are moving in a circle of radius r under the action of their mutual gravitational force. The speed of each particle round the circle will be proportional to

(a) $\frac{1}{\sqrt{r}}$ (b) $\frac{1}{r}$

(c) $\frac{1}{r^{3/2}}$ (d) $\frac{1}{r^2}$

SOLUTION The centripetal force for circular motion is provided by the gravitational force between the particles. Since the gravitational force is attractive, the particles will be diametrically opposite to each other. If the speed of each particle is v, then for a given particle,

$$\frac{mv^2}{r} = \frac{Gm^2}{(2r)^2}$$

$$\Rightarrow \quad v = \frac{1}{2}\sqrt{\frac{Gm}{r}}$$

So $v \propto \frac{1}{\sqrt{r}}$

EXAMPLE 17 The density of a solid sphere of mass M and radius R varies with distance r from its centre as $\rho = \frac{k}{r}$ where k is a constant. The gravitational field due to this sphere at a distance $2R$ from its centre is given by

(a) $\frac{\pi k G}{2}$ (b) $\frac{\pi^2 k G}{R}$

(c) $\frac{kG}{MR^2}$ (d) $\frac{3}{2}\frac{MG}{k}$

SOLUTION Refer to Fig. 6.16.

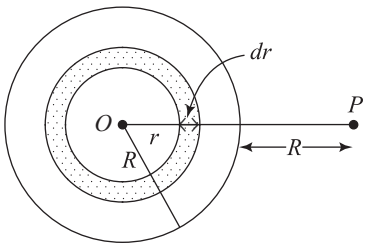


Fig. 6.16

To find gravitational field at point P, we divide the sphere into concentric shells of a very small thickness dr. The mass of each tiny shell can be assumed to be concentrated at centre O. Thus the mass M of the whole sphere can be assumed to be at O. The gravitational field due to this mass M at point P is

$$I = \frac{GM}{(2R)^2} = \frac{GM}{4R^2} \qquad \text{(i)}$$

To find M, consider a shell of radius r and thickness dr. The volume of this shell is

$$dV = (4\pi r^2)\, dr$$

Therefore, the mass of this shell is

$$dM = \rho\, dV = \left(\frac{k}{r}\right) \times (4\pi r^2\, dr)$$

$$= 4\pi k\, r\, dr$$

The mass of the whole sphere is

$$M = \int_0^R dM = 4\pi k \int_0^R r\, dr$$

$$= 4\pi k \frac{R^2}{2} = 2\pi k R^2 \qquad \text{(ii)}$$

Using (ii) in (i), we get

$$I = \frac{G}{4R^2} \times 2\pi k R^2 = \frac{\pi k G}{2}$$

EXAMPLE 18 A solid sphere of mass M and radius R is surrounded by a concentric shell of equal mass M and radius $3R$. The gravitational field at a point P_1 at a distance $r_1 = 2R$ from the centre is I_1 and a point P_2 at distance $r_2 = 4R$ from the centre is I_2. The ratio $\frac{I_2}{I_1}$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{9}$ (d) $\frac{9}{25}$

SOLUTION Refer to Fig. 6.17.

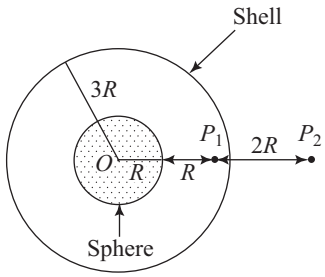


Fig. 6.17

Since point P_1 is inside the shell, the gravitational field at P_1 due to the shell is zero. The field at P_1 due to the solid sphere is

$$I_1 = \frac{GM}{r_1^2} = \frac{GM}{(2R)^2} = \frac{GM}{4R^2}$$

Point P_2 is outside the shell as well as the sphere. The mass M of the sphere and the mass M of the shell can be assumed to be concentrated at O so that the total mass at O is $2M$. The gravitational field due to mass $2M$ at point P_2 is

$$I_2 = \frac{G(2M)}{r_2^2} = \frac{2GM}{(4R)^2} = \frac{GM}{8R^2}$$

$$\Rightarrow \qquad \frac{I_2}{I_1} = \frac{1}{2}$$

8. Escape Velocity

The escape velocity is the minimum velocity with which a body must be projected in order that it may escape the earth's gravitational pull. The magnitude of the escape velocity is given by

$$v_e = \sqrt{\frac{2MG}{R}}$$

where M is the mass of the earth and R its radius. Substituting the known values of G, M and R, we get v_e = 11.2 kms^{-1}. The escape velocity is independent of the mass of the body. The expression for the escape velocity can be written in terms of g as

$$v_e = \sqrt{2gR}$$

The escape velocity is independent of the mass of the body and the direction of projection.

9. Satellites

A body moving in an orbit around a much larger and massive body is called a satellite. The moon is the natural satellite of the earth.

Orbital Velocity Let us assume that a satellite of mass m goes around the earth in a circular orbit of radius r with a uniform speed v. If the height of the satellite above the earth's surface is h, then $r = (R + h)$, where R is the mean radius of the earth. The centripetal force $\frac{mv^2}{r}$ necessary to keep the satellite in its circular orbit is provided by the gravitational force $\frac{GmM}{r^2}$ between the earth and the satellite. This means that

$$\frac{mv^2}{r} = G\frac{mM}{r^2}$$

where M is the mass of the earth. Thus

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+h)}}$$

Now the acceleration due to gravity on earth's surface is given by

$$g = \frac{GM}{R^2}$$

or $$GM = gR^2$$

Substituting for GM we get

$$v = R\sqrt{\frac{g}{(R+h)}}$$

If the satellite is a few hundred kilometres above the earth's surface (say 100 to 300 km), we can replace $(R + h)$ by R. The error involved in this approximation is negligible since the radius of the earth $(R) = 6.4 \times 10^6$ m. Thus, we may write

$$v = \sqrt{gR} = \sqrt{9.8 \times 6.4 \times 10^6}$$

$$= 7.9 \times 10^3 \text{ ms}^{-1} \simeq 8 \text{ km s}^{-1}$$

Periodic Time The periodic time T of a satellite is the time it takes to complete one revolution and it is given by (since $r = R + h$)

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

If $h \ll R$, we have $v = \sqrt{gR}$. Hence

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} \simeq 2\pi\sqrt{\frac{R}{g}}$$

Total Energy of a Satellite

Total energy E = K.E. + P.E.

$$= \frac{1}{2}mv^2 - \frac{GmM}{r}$$

$$= \frac{GmM}{2r} - \frac{GmM}{r} \qquad \left(\because v = \sqrt{\frac{GM}{r}}\right)$$

$$\Rightarrow \qquad E = -\frac{GmM}{2r} = -\frac{GmM}{2(R+h)}$$

Note

1. $E = \dfrac{\text{P.E.}}{2}$
2. $E = -$ (K.E.)
3. The total energy is negative which implies that the satellite is bound by the gravitational field of the earth. The binding energy = $\dfrac{GmM}{2r}$. This energy must be given to the orbiting satellite to escape to infinity.

Angular Momentum

The magnitude of angular momentum of a satellite is given by

$$L = mvr$$

$$= m\sqrt{\frac{GM}{r}}\, r$$

$$\Rightarrow \qquad L = m\sqrt{GMr}$$

Geostationary Satellites A geostationary satellite is a particular type used in communication. A number of communication satellites are launched which remain in fixed positions at a specified height above the equator. They are called *geostationary* or *synchronous satellites* used in international communication.

For a satellite to appear fixed at a position above a certain place on the earth, it must corotate with the earth so that its orbital period around the earth is exactly equal to the rotational period of the earth about its axis of rotation.

This condition is satisfied if the satellite is put in orbit at a height of about 36,000 km above the earth's surface.

10. Kepler's Laws of Planetary Motion

Johannes Kepler formulated three laws which describe planetary motion. They are as follows:

1. *Law of orbits* Each planet revolves about the sun in an elliptical orbit with the sun at one of the focii of the ellipse. The orbit of a planet is shown in Fig. 6.18(a) in which the two focii F_1 and F_2, are far apart. For the planet earth, F_1 and F_2 are very close together. In fact, the orbit of the earth is practically circular.
2. *Law of areas* A line drawn from the sun to the planet (termed the radius) sweeps out equal areas in equal intervals of time. In Fig. 6.18(b) P_1, P_2, P_3 and P_4 represent positions of a planet at different times in its orbit and S, the position of the sun.

   According to Kepler's second law, if the time interval between P_1 and P_2 equals the time interval between P_3 and P_4, then area A_1 must be equal to area A_2. Also the planet has the greater speed in its path from P_1 to P_2 than in its path from P_3 to P_4.
3. *Law of periods* The squares of the periods of the planets are proportional to the cubes of their mean distances from the sun. If T_1 represents the period of a planet about the sun, and r_1 its mean distance, then

$$T_1^2 \propto r_1^3$$

If T_2 represents the period of a second planet about the sun, and r_2 its mean distance, then for this planet

$$T_2^2 \propto r_2^3$$

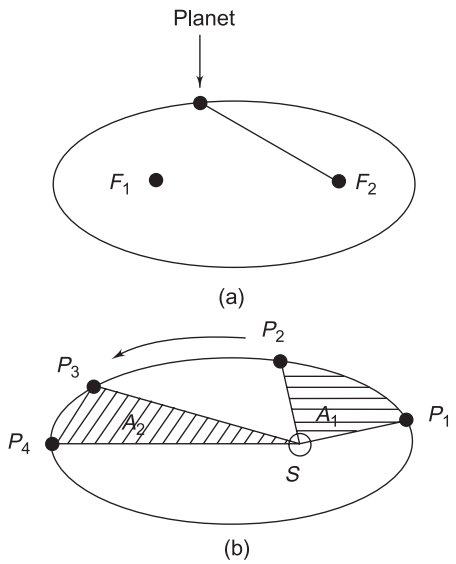


Fig. 6.18

These two relations can be combined since the factor of proportionality is the same for both. Thus

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

EXAMPLE 19 The mass of Jupiter is 318 times that of the earth and its radius is 11.2 times that of the earth. Calculate the escape velocity from Jupiter's surface. Given the escape velocity from earth'surface = 11.2 km s^{-1}.

SOLUTION For Jupiter : $v_J = \sqrt{\dfrac{2M_J G}{R_J}}$

For Earth : $v_J = \sqrt{\dfrac{2M_E G}{R_E}}$

$$\therefore \quad \frac{v_J}{v_E} = \sqrt{\frac{M_J}{M_E} \times \frac{R_E}{R_J}} = \sqrt{318 \times \frac{1}{11.2}} = 5.33$$

$$\therefore \quad v_J = 5.33\, v_E = 5.33 \times 11.2 = 59.7 \text{ km s}^{-1}$$

EXAMPLE 20 Calculate the escape velocity of a body at a height 1600 km above the surface of the earth. Radius of earth = 6400 km.

SOLUTION Work required to move a body of mass m from $r = R + h$ to $r = \infty$ is

$$W = \int_{R+h}^{\infty} F dr = GmM \int_{R+h}^{\infty} \frac{dr}{r^2}$$

$$= \frac{GmM}{R+h}$$

If v_e is the escape velocity, then

$$\frac{1}{2} m v_e^2 = \frac{GmM}{R+h}$$

$$\Rightarrow \quad v_e = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2gR^2}{R+h}} \quad \left(\because g = \frac{GM}{R^2}\right)$$

Given $R = 6.4 \times 10^6$ m, and $h = 1.6 \times 10^6$ m and $g = 9.8$ ms^{-2}

$$\therefore \quad v_e = \sqrt{\frac{2 \times 9.8 \times (6.4 \times 10^6)^2}{(6.4 + 1.6)10^6}}$$

$$= 10 \times 10^3 \text{ ms}^{-1} = 10 \text{ km s}^{-1}$$

EXAMPLE 21 A rocket is launched vertically from the surface of the earth with an initial velocity equal to half the escape velocity. Find the maximum height attained by it in terms of R where R is the radius of the earth. Ignore atmospheric resistance.

SOLUTION On the surface of the earth, the total energy of the rocket is

$$E_i = \text{K.E.} + \text{P.E.} = \frac{1}{2} m v^2 - \frac{GmM}{R}$$

At the highest point, $v = 0$. If h is the maximum height attained, the energy of the rocket at height h is

$$E_f = -\frac{GmM}{(R+h)}$$

From conservation of energy, $E_i = E_f$, i.e.,

$$\frac{1}{2} m v^2 - \frac{GmM}{R} = -\frac{GmM}{(R+h)}$$

$$\Rightarrow \quad v = \sqrt{\frac{2GM}{R} \times \frac{h}{R+h}} = v_e \sqrt{\frac{h}{R+h}}$$

Given $v = \frac{v_e}{2}$. Hence $\frac{v_e}{2} = v_e \sqrt{\frac{R}{R+h}} \quad \Rightarrow \quad h = \frac{R}{3}$

EXAMPLE 22 A body is dropped from a height h equal to $\frac{R}{2}$, where R is the radius of the earth. Show that it will hit the surface of the earth with a speed $v = v_e/\sqrt{3}$, where v_e is the escape velocity from the surface of the earth.

SOLUTION Total energy of the body at height h is

$$E_i = -\frac{GmM}{(R+h)} \qquad \text{(i)}$$

Total energy when it hits the surface of the earth is

$$E_f = \frac{1}{2} m v^2 - \frac{GmM}{R} \qquad \text{(ii)}$$

From conservation of energy, $E_i = E_f$, i.e.

$$-\frac{GmM}{(R+h)} = \frac{1}{2} m v^2 - \frac{GmM}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{2GM}{R}} \times \sqrt{\frac{h}{(R+h)}} = v_e \sqrt{\frac{R}{(R+h)}}$$

For $h = \frac{R}{2}$, $v = \frac{v_e}{\sqrt{3}}$.

EXAMPLE 23 Show that the minimum energy required to launch a satellite of mass m from the surface of the earth in a circular orbit at an altitude $h = R$, where R is the radius of the earth is $\frac{3mgR}{4}$ where M is the mass of the earth.

SOLUTION Total energy of the satellite orbiting the earth is

$$E_1 = -\frac{GmM}{2r} = -\frac{GmM}{2(R+h)} = -\frac{GmM}{4R}$$

$(\because h = R)$

Total energy when the satellite was at rest on the surface of the earth is

$$E_2 = \text{K.E.} + \text{P.E.} = 0 - \frac{GmM}{R} = -\frac{GmM}{R}$$

$\therefore$ Minimum energy required is

$$E_{\min} = E_1 - E_2$$

$$= -\frac{GmM}{4R} - \left(-\frac{GmR}{R}\right)$$

$$= \frac{3GmM}{4R} = \frac{3}{4} mgR$$

EXAMPLE 24 A satellite of mass $m = 100$ kg is in a circular orbit at a height $h = R$ above the surface of the earth where R is the radius of the earth. Find

(a) the acceleration due to gravity at any point on the path of the satellite,

(b) the gravitational force on the satellite and

(c) the centripetal force on the satellite.

SOLUTION

(a) $g' = g\left(\frac{R}{R+h}\right)^2 = 9.8\times\left(\frac{R}{R+R}\right)^2$

$= \frac{9.8}{4} = 2.45 \text{ ms}^{-2}$

(b) Gravitational force on satellite is

$F_g = mg' = 100 \times 2.45 = 245 \text{ N}$

(c) Centripetal force

$F_c = \frac{mv^2}{r} = \frac{GmM}{r^2} \qquad \left(\because g = \frac{GM}{R^2}\right)$

$= \frac{GmM}{R^2}\times\frac{R^2}{r^2}$

$= mg\times\left(\frac{R}{R+h}\right)^2 = mg' = 245 \text{ N}.$

Note

Since $F_g = F_c$, the satellite is a freely falling body and is, therefore, weightless.

EXAMPLE 25 A body projected vertically upwards from the surface of the earth with a certain velocity rises to a height of 10 m. How high will it rise if it is projected with the same velocity vertically upwards from a planet whose density is one-third that of the earth and radius half that of earth? Ignore atmospheric resistance.

SOLUTION Since the kinetic energy of the body is the same in both the cases, loss in K.E. = gain in P.E. will be equal, i.e.,

$mg_p h_p = mg_e h_e$

$\Rightarrow \quad h_p = h_e \times \frac{g_e}{g_p}$

Now $g = \frac{GM}{R^2} = \frac{G}{R^2}\times\frac{4\pi}{3}R^3\rho = \frac{4\pi}{3}GR\rho$

$\therefore \quad \frac{g_e}{g_p} = \frac{R_e}{R_p}\times\frac{\rho_e}{\rho_p} = 2\times 3 = 6$

$\therefore \quad h_p = 6h_e = 6\times 10 = 60 \text{ m}$

EXAMPLE 26 A satellite of mass 2000 kg is orbiting the earth at an altitude $R/2$, where R is the radius of the earth. What extra energy must be given to the satellite to increase its altitude to R? Given $R = 6.4\times10^6$ m.

SOLUTION In the first case, $r_1 = R + \frac{R}{2} = \frac{3R}{2}$ and in the second case, $r_2 = R + R = 2R$.

Required energy = $E_2 - E_1$

$= -\frac{GmM}{2r_2} - \left(-\frac{GmM}{2r_1}\right)$

$= -\frac{GmM}{4R} + \frac{GmM}{3R}$

$= \frac{GmM}{12R}$

$= \frac{mgR}{12} \qquad \left(\because g = \frac{GM}{R^2}\right)$

$= \frac{2000\times9.8\times\left(6.4\times10^6\right)}{12}$

$= 1.04\times10^8 \text{ J}$

EXAMPLE 27 Two bodies of masses m_1 and m_2 are held at a distance r apart. Show that at the point where the gravitational field due to them is zero, the gravitational potential is given by

$$V = -\frac{G}{r}\left(m_1 + m_2 + 2\sqrt{m_1 m_2}\right)$$

SOLUTION Let the gravitational field be zero at point P (Fig. 6.19). Then

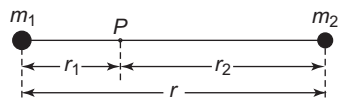


Fig. 6.19

$\frac{G m_1}{r_1^2} = \frac{G m_2}{r_2^2}$

$\Rightarrow \quad \frac{r_1}{r_2} = \frac{\sqrt{m_1}}{\sqrt{m_2}}$

$\Rightarrow \quad \frac{r_1}{r - r_1} = \frac{\sqrt{m_1}}{\sqrt{m_2}} \quad \Rightarrow \quad r_1 = \frac{r\sqrt{m_1}}{\left(\sqrt{m_1}+\sqrt{m_2}\right)}$ (i)

Also $r_2 = r - r_1 = r - \frac{r\sqrt{m_1}}{\left(\sqrt{m_1}+\sqrt{m_2}\right)}$

$= \frac{r\sqrt{m_2}}{\left(\sqrt{m_1}+\sqrt{m_2}\right)}$ (ii)

Gravitational potential at P is

$V = -\frac{Gm_1}{r_1} - \frac{Gm_2}{r_2}$ (iii)

Using (i) and (ii) in (iii) and simplifying, we get

$$V = -\frac{G}{r}\left(m_1 + m_2 + 2\sqrt{m_1 m_2}\right)$$

EXAMPLE 28 The distance of a planet from the sun is 10 times that of the earth. Find the period of revolution of the planet around the sun.

SOLUTION From Kepler's law of periods, $T^2 \propto r^3$. Therefore

$$\frac{T_p^2}{T_e^2} = \frac{r_p^3}{r_e^3} = (10)^3 = 1000$$

$$\therefore \quad T_p = T_e \times \sqrt{1000}$$

$$= 1 \text{ year} \times 31.6 = 31.6 \text{ years}$$

EXAMPLE 29 A satellite is revolving in a circular orbit close to the surface of the earth with a speed v. What minimum additional speed must be imparted to it so that it escapes the gravitational pull of the earth? Radius of earth $= 6.4 \times 10^6$ m.

SOLUTION $v = \sqrt{gR}$ and $v_e = \sqrt{2gR}$

$\therefore$ Additional speed required is

$$v_e - v = \left(\sqrt{2} - 1\right)\sqrt{gR}$$

$$= 0.414 \times \sqrt{9.8 \times 6.4 \times 10^6}$$

$$= 3.28 \times 10^3 \text{ ms}^{-1}$$

$$= 3.28 \text{ km s}^{-1}$$

EXAMPLE 30 A body of mass m is placed at the centre of a spherical shell of radius R and mass M. Find the gravitational potential on the surface of the shell.

SOLUTION Gravitational potential on the surface of the shell due to body of mass m is

$$V_b = -\frac{Gm}{R}$$

Gravitational potential on the surface of the shell due to shell itself is

$$V_s = -\frac{GM}{R}$$

$$\therefore \quad V = V_b + V_s = -\frac{G}{R}(m + M)$$

EXAMPLE 33 A tunnel is drilled from the surface of the earth to its centre. A body of mass m is dropped into the tunnel. Find the speed with which the body hits the bottom of the tunnel. The mass of earth is M and its radius is R.

SOLUTION Let v be the required speed. Gain in K.E. = loss in P.E. = P.E. at the surface – P.E. at the centre. The potential energy at the centre of the sphere $= -\frac{3}{2}\frac{GmM}{R}$. Hence

$$\frac{1}{2}mv^2 = -\frac{GmM}{R} - \left(-\frac{3}{2}\frac{GmM}{R}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}} = \sqrt{gR} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

SECTION 1 Multiple Choice Questions with One Correct Choice

Level A

1. The acceleration due to gravity g on earth is 9.8 ms^{-2}. What would the value of g for a planet whose size is the same as that of earth but the density in twice that of earth?
   (a) 19.6 ms^{-2} (b) 9.8 ms^{-2}
   (c) 4.9 ms^{-2} (d) 2.45 ms^{-2}
2. If the radius of the earth suddenly decreases to 80% of its present value, the mass of the earth remaining the same, the value of the acceleration due to gravity will
   (a) remain unchanged
   (b) become (9.8×0.8) ms^{-2}
   (c) increase by 36%
   (d) increase by about 56%
3. The mass of a planet is 1/10$^{\text{th}}$ that of earth and its diameter is half that of earth. The acceleration due to gravity at the planet will be
   (a) 1.96 ms^{-2} (b) 3.92 ms^{-2}
   (c) 9.8 ms^{-2} (d) 19.6 ms^{-2}
4. The escape velocity of a body projected vertically upwards from the surface of the earth is v. If the body is projected in a direction making an angle θ with the vertical, the escape velocity would be
   (a) v (b) $v \cos \theta$
   (c) $v \sin \theta$ (d) $v \tan \theta$
5. A small planet is revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force between the planet and the star were proportional to $R^{-5/2}$, then T would be proportional to

(a) $R^{3/2}$ (b) $R^{3/5}$

(c) $R^{7/2}$ (d) $R^{7/4}$

6. If g is the acceleration due to gravity on the surface of the earth, the gain in potential energy of an object of mass m raised from the earth's surface to a height equal to the radius R of the earth is

(a) $mgR/4$ (b) $mgR/2$

(c) mgR (d) $2\,mgR$

7. Two satellites of the same mass are orbiting round the earth at heights of R and $4R$ above the earth's surface: R being the radius of the earth. Their kinetic energies are in the ratio of

(a) 4 : 1 (b) 3 : 2

(c) 4 : 3 (d) 5 : 2

8. A satellite is orbiting the earth in a circular orbit of radius r. Its period of revolution varies as

(a) $\sqrt{r}$ (b) r

(c) $r^{3/2}$ (d) r^2

9. If the gravitational force of attraction between any two bodies were to vary as $1/r^3$ instead of $1/r^2$, the period of revolution of a planet round the sun would vary as

(a) $\sqrt{r}$ (b) r

(c) $r^{3/2}$ (d) r^2

10. If both the mass and the radius of the earth decrease by 1%, the value of the acceleration due to gravity will

(a) decrease by 1% (b) increase by 1%

(c) increase by 2% (d) remain unchanged.

11. A geostationary satellite is orbiting the earth at a height of $6R$ above the surface of the earth; R being the radius of the earth. What will be the time period of another satellite at a height 2.5 R from the surface of the earth?

(a) $6\sqrt{2}$ hours (b) $6\sqrt{2.5}$ hours

(c) $6\sqrt{3}$ hours (d) 12 hours

Level B

12. The masses and radii of the earth and moon are M_1, R_1 and M_2, R_2 respectively. Their centres are a distance d apart. The minimum speed with which a particle of mass m should be projected from a point midway between the two centres so as to escape to infinity is given by

(a) $2\left[\dfrac{G(M_1+M_2)^2}{md}\right]^{1/2}$ (b) $2\left[\dfrac{G(M_1+M_2)}{d}\right]^{1/2}$

(c) $2\left[\dfrac{G(M_1-M_2)^2}{md}\right]^{1/2}$ (d) $2\left[\dfrac{G(M_1-M_2)}{d}\right]^{1/2}$

13. The angular momentum of the earth revolving round the sun is proportional to R^n where R is the distance between the earth and the sun. The value of n is

(a) 0.5 (b) 1.0

(c) 1.5 (d) 2.0

14. A satellite is moving around the earth in a stable circular orbit. Which one of the following statements will be wrong for such a satellite?

(a) It is moving at a constant speed.
(b) Its angular momentum remains constant.
(c) It is acted upon by a force directed away from the centre of the earth which counter- balances the gravitational pull of the earth.
(d) It behaves as if it were a freely falling body.

15. Astronauts in a stable orbit around the earth are said to be in a weightless condition. The reason for this is that

(a) the capsule and its contents are falling freely at the same rate
(b) there is no gravitational force acting on them
(c) the gravitational force of the earth balances that of the sun
(d) there is no atmosphere at the height at which they are orbiting.

16. The escape velocity from the earth is v_e. What is the escape velocity from a planet whose radius is twice that of the earth and mean density is the same as that of the earth?

(a) $v_e/2$ (b) v_e

(c) $2\,v_e$ (d) $4\,v_e$

17. Choose the wrong statement. The escape velocity of a body from a planet depends upon

(a) the mass of the body
(b) the mass of the planet
(c) the average radius of the planet
(d) the average density of the planet

18. Choose the wrong statement. The orbital velocity of a body in a stable orbit around a planet depends upon

(a) the average radius of the planet
(b) the height of the body above the planet
(c) the acceleration due to gravity
(d) the mass of the orbiting body

19. Choose the correct statement. In planetary motion
    (a) the speed along the orbit remains constant
    (b) the angular speed remains constant
    (c) the total angular momentum remains constant.
    (d) the radius of the orbit remains constant.

20. An object weighs W newton on earth. It is suspended from the lower end of a spring balance whose upper end is fixed to the ceiling of a space capsule in a stable orbit around the earth. The reading of the spring balance will be
    (a) W (b) less than W
    (c) more than W (d) zero.

21. If M is the mass of the earth, R its radius (assumed spherical) and G the universal gravitational constant, then the amount of work that must be done on a body of mass m so that it completely escapes from the gravity of the earth, is given by
    (a) $\frac{GmM}{R}$ (b) $\frac{GmM}{2R}$
    (c) $\frac{3GmM}{2R}$ (d) $\frac{3GmM}{4R}$

22. A rocket is fired from the earth to the moon. The distance between the earth and the moon is r and the mass of the earth is 81 times the mass of the moon. The gravitational force on the rocket will be zero, when its distance from the moon is
    (a) $\frac{r}{20}$ (b) $\frac{r}{15}$
    (c) $\frac{r}{10}$ (d) $\frac{r}{5}$

23. Assuming that the earth is a sphere of radius R, at what altitude will the value of the acceleration due to gravity be half its value at the surface of the earth?
    (a) $h = \frac{R}{2}$ (b) $h = \frac{R}{\sqrt{2}}$
    (c) $h = \left(\sqrt{2}+1\right)R$ (d) $h = \left(\sqrt{2}-1\right)R$

24. Assuming that the earth is a sphere of uniform mass density, what is the percentage decrease in the weight of a body when taken to the end of a tunnel 32 km below the surface of the earth? Radius of earth = 6400 km.
    (a) 0.25% (b) 0.5%
    (c) 0.75% (d) 1%

25. An extremely small and dense neutron star of mass M and radius R is rotating at an angular frequency ω. If an object is placed at its equator, it will remain stuck to it due to gravity if
    (a) $M > \frac{R\omega}{G}$ (b) $M > \frac{R^2\omega^2}{G}$
    (c) $M > \frac{R^3\omega^2}{G}$ (d) $M > \frac{R^2\omega^3}{G}$

26. Two small and heavy spheres, each of mass M, are placed a distance r apart on a horizontal surface. The gravitational field intensity at the mid–point of the line joining the centres of the spheres is
    (a) zero (b) $\frac{GM^2}{r^2}$
    (c) $\frac{GM^2}{2r^2}$ (d) $\frac{GM^2}{4r^2}$

27. In Q. 26, the gravitational potential at the mid–point of the line joining the centres of the spheres is
    (a) zero (b) $-\frac{GM}{r}$
    (c) $-\frac{2GM}{r}$ (d) $-\frac{4GM}{r}$

28. Three particles, each of mass m, are placed at the vertices of an equilateral triangle of side a. The gravitational field intensity at the centroid of the triangle is
    (a) zero (b) $\frac{Gm^2}{a^2}$
    (c) $\frac{2Gm^2}{a^2}$ (d) $\frac{3Gm^2}{a^2}$

29. In Q. 28, the gravitational potential at the centroid is
    (a) zero (b) $-3\sqrt{3}\frac{Gm}{a}$
    (c) $-2\sqrt{3}\frac{Gm}{a}$ (d) $-\sqrt{3}\frac{Gm}{a}$

30. The distance between the sun and the earth is r and the earth takes time T to make one complete revolution around the sun. Assuming the orbit of the earth around the sun to be circular, the mass of the sun will be proportional to
    (a) $\frac{r^2}{T}$ (b) $\frac{r^2}{T^2}$
    (c) $\frac{r^3}{T^2}$ (d) $\frac{r^3}{T^3}$

31. A rocket is launched vertically from the surface of the earth of radius R with an initial speed v. If

atmospheric resistance is neglected, the maximum height attained by the rocket is given by

(a) $h = \dfrac{R}{\left(\dfrac{2gR}{v^2} - 1\right)}$ (b) $h = \dfrac{R}{\left(\dfrac{2gR}{v^2} + 1\right)}$

(c) $h = R\left(\dfrac{2gR}{v^2} - 1\right)$ (d) $h = R\left(\dfrac{2gR}{v^2} + 1\right)$

32. The escape velocity of a body on the earth's surface is v_e. A body is thrown with a speed 3 v_e. Assuming that the sun and planets do not influence the motion of the body, its speed at infinity would be

(a) zero (b) v_e

(c) $\sqrt{2}\, v_e$ (d) $2\sqrt{2}\, v_e$

33. A body is released from a height equal to the radius (R) of the earth. The velocity of the body when it strikes the surface of the earth will be

(a) $\sqrt{gR}$ (b) $\sqrt{2gR}$

(c) $2\sqrt{2gR}$ (d) $2\sqrt{gR}$

34. A satellite of mass m is orbiting the earth at a height h from its surface. If M is the mass of the earth and R its radius, the kinetic energy of the satellite is

(a) $\dfrac{GmM}{(R+h)^2}$ (b) $\dfrac{GmM}{2(R+h)^2}$

(c) $\dfrac{GmM}{(R+h)}$ (d) $\dfrac{GmM}{2(R+h)}$

35. In Q. 34 the potential energy of the satellite is given by

(a) $-\dfrac{GmM}{(R+h)^2}$ (b) $-\dfrac{GmM}{2(R+h)^2}$

(c) $-\dfrac{GmM}{(R+h)}$ (d) $-\dfrac{GmM}{2(R+h)}$

36. How much energy must be spent to pull the satellite in Q.34 out of the earth's gravitational field?

(a) $\dfrac{2GmM}{(R+h)^2}$ (b) $\dfrac{GmM}{2(R+h)^2}$

(c) $\dfrac{2GmM}{(R+h)}$ (d) $\dfrac{GmM}{2(R+h)}$

37. How much energy would be spent to pull the satellite in Q. 34 out of the earth's gravitational field if the earth shrank suddenly to half its present size?

(a) $\dfrac{GmM}{2(R+h)^2}$ (b) $\dfrac{GmM}{4(R+h)^2}$

(c) $\dfrac{GmM}{2(R+h)}$ (d) $\dfrac{GmM}{4(R+h)}$

38. The radius of the earth is R. For a satellite to appear stationary, it must be placed in orbit around the earth at a height of about (given R = 6380 km)

(a) 5.6 R (b) 6.6 R

(c) 7.6 R (d) 8.6 R

39. What is the minimum energy required to launch a satellite of mass m from the surface of the earth of radius R in a circular orbit at an altitude of 2R?

(a) $\dfrac{5GmM}{6R}$ (b) $\dfrac{2GmM}{3R}$

(c) $\dfrac{GmM}{2R}$ (d) $\dfrac{GmM}{3R}$

40. Two stars, each of mass m and radius R are approaching each other for a head–on collision. They start approaching each other when their separation is $r >> R$. If their speeds at this separation are negligible, the speed with which they collide would be

(a) $v = \sqrt{Gm\left(\dfrac{1}{R} - \dfrac{1}{r}\right)}$ (b) $v = \sqrt{Gm\left(\dfrac{1}{2R} - \dfrac{1}{r}\right)}$

(c) $v = \sqrt{Gm\left(\dfrac{1}{R} + \dfrac{1}{r}\right)}$ (d) $v = \sqrt{Gm\left(\dfrac{1}{2R} + \dfrac{1}{r}\right)}$

41. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the escape velocity from the earth of radius R. What is the height of the satellite above the surface of the earth?

(a) $\dfrac{R}{2}$ (b) R

(c) 3 R (d) 6 R

42. In Q 41, if the satellite is stopped suddenly in its orbit and allowed to fall freely on the earth, the speed with which it will hit the surface of the earth will be

(a) $\sqrt{gR}$ (b) $\sqrt{2gR}$

(c) $\sqrt{3gR}$ (d) $2\sqrt{gR}$

43. A body is projected vertically upward from the surface of the earth with a velocity equal to half the escape velocity. If R is the radius of the earth, the maximum height attained by the body is

(a) $\dfrac{R}{6}$ (b) $\dfrac{R}{3}$

(c) $\dfrac{2R}{3}$ (d) R

44. Infinite number of masses, each of mass m, are placed along a straight line at distances of r, 2r, 4r, 8r, etc. from a reference point O. The gravitational field intensity at point O will be

(a) $\frac{5Gm}{4r^2}$ (b) $\frac{4Gm}{3r^2}$

(c) $\frac{3Gm}{2r^2}$ (d) $\frac{2Gm}{r^2}$

45. In Q. 44, the magnitude of the gravitational potential at point O will be

(a) $\frac{Gm}{2r}$ (b) $\frac{Gm}{r}$

(c) $\frac{3Gm}{2r}$ (d) $\frac{2Gm}{r}$

46. A meteor of mass M breaks up into two parts. The mass of one part is m. For a given separation r the mutual gravitational force between the two parts will be the maximum if

(a) $m = \frac{M}{2}$ (b) $m = \frac{M}{3}$

(c) $m = \frac{M}{\sqrt{2}}$ (d) $m = \frac{M}{2\sqrt{2}}$

47. A body of mass m is raised to a height h above the surface of the earth of mass M and radius R until its gravitational potential energy increases by $\frac{1}{3}\ mgR$. The value of h is

(a) $\frac{R}{3}$ (b) $\frac{R}{2}$

(c) $\frac{mR}{(M+m)}$ (d) $\frac{mR}{M}$

48. A satellite of mass m is moving in a circular orbit of radius R above the surface of a planet of mass M and radius R. The amount of work done to shift the satellite to a higher orbit of radius $2R$ is (here g is the acceleration due to gravity on planet's surface)

(a) mgR (b) $\frac{mgR}{6}$

(c) $\frac{mMgR}{(M+m)}$ (d) $\frac{mMgR}{6(M+m)}$

49. The change in the gravitational potential energy when a body of mass m is raised to a height nR above the surface of the earth is (here R is the radius of the earth)

(a) $\left(\frac{n}{n+1}\right) mgR$ (b) $\left(\frac{n}{n-1}\right) mgR$

(c) $nmgR$ (d) $\frac{mgR}{n}$

50. A body of mass m is dropped from a height nR above the surface of the earth (here R is the radius of the earth). The speed at which the body hits the surface of the earth is

(a) $\sqrt{\frac{2gR}{(n+1)}}$ (b) $\sqrt{\frac{2gR}{(n-1)}}$

(c) $\sqrt{\frac{2gRn}{(n-1)}}$ (d) $\sqrt{\frac{2gRn}{(n+1)}}$

51. Two balls A and B are thrown vertically upwards from the same location on the surface of the earth with velocities $2\sqrt{\frac{gR}{3}}$ and $\sqrt{\frac{2gR}{3}}$ respectively, where R is the radius of the earth and g is the acceleration due to gravity on the surface of the earth. The ratio of the maximum height attained by A to that attained by B is

(a) 2 (b) 4

(c) 8 (d) $4\sqrt{2}$

52. Two solid spheres of radii r and $2r$, made of the same material, are kept in contact. The mutual gravitational force of attraction between them is proportional to

(a) $\frac{1}{r^4}$ (b) $\frac{1}{r^2}$

(c) r^2 (d) r^4

53. A comet is moving in a highly elliptical orbit round the sun. When it is closest to the sun, its distance from the sun is r and its speed is v. When it is farthest from the sun, its distance from the sun is R and its speed will be

(a) $v\left(\frac{r}{R}\right)^{1/2}$ (b) $v\left(\frac{r}{R}\right)$

(c) $v\left(\frac{r}{R}\right)^{3/2}$ (d) $v\left(\frac{r}{R}\right)^{2}$

54. The value of the acceleration due to gravity at the surface of the earth of radius R is g. It decreases by 10% at a height h above the surface of the earth. The gravitational potential at this height is

(a) $-\frac{gR}{\sqrt{10}}$ (b) $-\frac{2gR}{\sqrt{10}}$

(c) $-\frac{3gR}{\sqrt{10}}$ (d) $-\frac{4gR}{\sqrt{10}}$

55. Two stars of masses m and $2m$ are co-rotating about their centre of mass. Their centres are at a distance r apart. If r is much larger than the sizes of the stars, their common period of revolution is proportional to

(a) r (b) $r^{3/2}$

(c) r^2 (d) r^3

56. In Q. 55 above, the kinetic energies of stars of masses m and $2m$ are in the ratio

(a) $1 : \sqrt{2}$ (b) $\sqrt{2} : 1$

(c) $1 : 2$ (d) $2 : 1$

57. In Q. 55 above, the angular momenta of the stars of masses m and $2m$ about their centre of mass are in the ratio

(a) $1 : 2$ (b) $2 : 1$

(c) $1 : 4$ (d) $4 : 1$

58. A uniform sphere of mass M and radius R exerts a force F on a small mass m situated at a distance of $2R$ from the centre O of the sphere. A spherical portion of diameter R is cut from the sphere as shown in Fig. 6.20. The force of attraction between the remaining part of the sphere and the mass m will be

(a) $\frac{7F}{9}$ (b) $\frac{2F}{3}$

(c) $\frac{4F}{9}$ (d) $\frac{F}{3}$

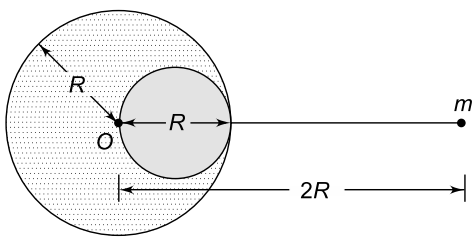


Fig. 6.20

59. The centres of a ring of mass m and a sphere of mass M of equal radius R, are at a distance $\sqrt{8}\,R$ apart as shown in Fig. 6.21. The force of attraction between the ring and the sphere is

(a) $\frac{2\sqrt{2}}{27}\frac{GmM}{R^2}$ (b) $\frac{GmM}{8R^2}$

(c) $\frac{GmM}{9R^2}$ (d) $\frac{\sqrt{2}}{9}\frac{GmM}{9R^2}$

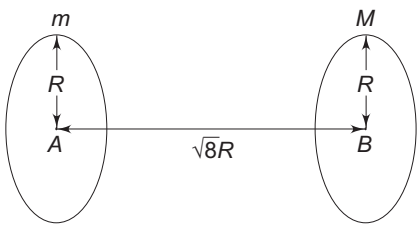


Fig. 6.21

60. A solid sphere of uniform density and radius 4 units is located with its centre at origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at $A\,(-2, 0, 0)$ and $B\,(2, 0, 0)$ respectively are taken out of the solid sphere leaving behind spherical cavities as shown in Fig. 6.22. Choose the incorrect statement from the following.

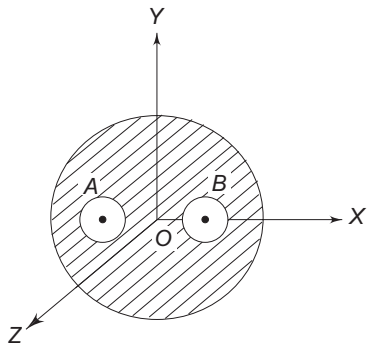


Fig. 6.22

(a) The gravitational force due to this object at the origin is zero.

(b) The gravitational force at point $B(2, 0, 0)$ is zero.

(c) The gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$.

(d) The gravitational potential is the same at all points of the circle $y^2 + z^2 = 4$.

61. Two bodies of masses m_1 and m_2 are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Their relative velocity of approach at a separation distance r between them is

(a) $\left[\frac{2G(m_1 + m_2)}{r}\right]^{1/2}$

(b) $\left[\sqrt{\frac{2G}{r}}\frac{(m_1 + m_2)}{2}\right]^{1/2}$

(c) $\left[\frac{r}{2G(m_1 m_2)}\right]^{1/2}$

(d) $\left(\frac{2G}{r}m_1 m_2\right)^{1/2}$

62. Two objects of masses m and $4m$ are at rest at infinite separation. They move towards each other under mutual gravitational attraction. Then, at a separation r, which of the following is true?

(a) The total energy of the system is not zero.

(b) The force between them is not zero.

(c) The centre of mass of the system is at rest.

(d) All the above are true.

63. A satellite is launched into a circular orbit of radius R around the earth. A second satellite is launched into an orbit of radius 1.01 R. The period of the second satellite is longer than that of the first by approximately

(a) 0.5% (b) 1.0%
(c) 1.5% (d) 3.0%

64. If the distance between the earth and the sun were half its present value, the number of days in a year would have been

(a) 64.5 (b) 129
(c) 182.5 (d) 730

65. An artificial satellite moving in a circular orbit around the earth has a total (kinetic + potential) energy E_0. Its potential energy is

(a) $-E_0$ (b) $1.5\,E_0$
(c) $2\,E_0$ (d) E_0

66. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. Which of the following statements is correct?

(a) The acceleration of S is always directed towards the centre of the earth.
(b) The angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant.
(c) The total mechanical energy of S remains constant.
(d) The linear momentum of S remains constant in magnitude.

67. A simple pendulum has a time period T_1 when on the earth's surface, and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2/T_1 is

(a) 1 (b) $\sqrt{2}$
(c) 4 (d) 2

68. An ideal spring with spring-constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is

(a) $4\,Mg/k$ (b) $2\,Mg/k$
(c) Mg/k (d) $Mg/2k$

69. A geo-stationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbiting a few hundred kilometres above the earth's surface (R_{Earth} = 6400 km) will approximately be

(a) (1/2) h (b) 1 h
(c) 2 h (d) 4 h

70. A mass M is divided into two parts xm and $(1 - x)m$. For a given separation, the value of x for which the gravitational attraction between the two pieces becomes maximum is

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$
(c) 1 (d) 2

71. The height of the point vertically above the earth's surface at which the acceleration due to gravity becomes 1% of its value at the surface is (R is the radius of the earth)

(a) 8 R (b) 9 R
(c) 10 R (d) 20 R

72. A body is projected up with a velocity equal to $\frac{3}{4}$ of the escape velocity from the surface of the earth. The height it reaches is: (Radius of the earth = R)

(a) $\frac{10R}{9}$ (b) $\frac{9R}{7}$
(c) $\frac{9R}{8}$ (d) $\frac{10R}{3}$

73. Two bodies of masses $M_1 = m$ and $M_2 = 4m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

(a) zero (b) $-\frac{4Gm}{r}$
(c) $-\frac{6Gm}{r}$ (d) $-\frac{9Gm}{r}$

74. The radius of the earth is R and g is the acceleration due to gravity on its surface. What should be the angular speed of the earth so that bodies lying on the equator may appear weightless?

(a) $\sqrt{\frac{g}{R}}$ (b) $\sqrt{\frac{2g}{R}}$
(c) $\sqrt{\frac{g}{2R}}$ (d) $2\sqrt{\frac{g}{R}}$

75. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 (as shown in Fig. 6.23) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3.

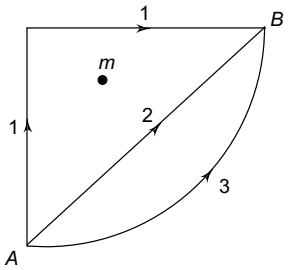


Fig. 6.23

(a) $W_1 > W_3 > W_2$ (b) $W_1 = W_2 = W_3$

(c) $W_1 < W_3 < W_2$ (d) $W_1 < W_2 < W_3$

76. A binary star system consists of two stars of masses M_1 and M_2 revolving in circular orbits of radii R_1 and R_2 respectively. If their respective time periods are T_1 and T_2, then

(a) $T_1 > T_2$ if $R_1 > R_2$ (b) $T_1 > T_2$ if $M_1 > M_2$

(c) $T_1 = T_2$ (d) $\dfrac{T_1}{T_2} = \left(\dfrac{R_1}{R_2}\right)^{3/2}$

77. A uniform bar AB of mass M and length L has a particle of mass m held at a distance a from end A as shown in Fig. 6.24. The gravitational force exerted by the bar on mass m in given by

(a) $\dfrac{GmM}{\left(\dfrac{L}{2}+a\right)^2}$ (b) $\dfrac{GmM}{a(L+a)}$

(c) $\dfrac{GmM}{L(L+a)}$ (d) $\dfrac{GmM}{L(L+2a)}$

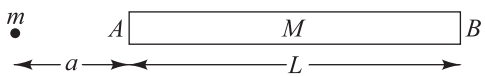


Fig. 6. 24

78. A satellite of mass m is moving in a circular orbit of radius r around the earth of mass M and radius R. The amount of work needed (or energy spent) to shift the satellite into a new orbit of radius $2r$ will be

(a) $\dfrac{GmM}{2(R+r)}$ (b) $\dfrac{GmM}{4(R+r)}$

(c) $\dfrac{GmM}{2r}$ (d) $\dfrac{GmM}{4r}$

79. A comet of mass m is in an elliptical orbit around the SUN (mass M) a shown in Fig. 6.25. A is the position of the comet when it is closest to the SUN and $\mathbf{v}_1$ is its velocity at A. B is the position of the comet when it is farthest from the sun and $\mathbf{v}_2$ is its velocity at B.

The ratio $\dfrac{v_1}{v_2}$

(a) $\dfrac{r_1}{r_2}$ (b) $\dfrac{r_2}{r_1}$

(c) $\sqrt{\dfrac{r_1}{r_2}}$ (d) $\sqrt{\dfrac{r_2}{r_1}}$

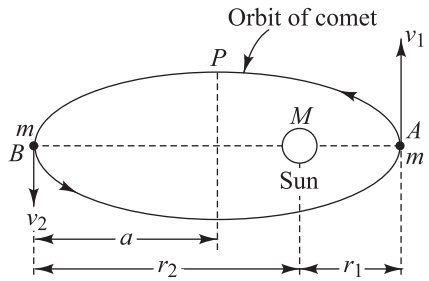


Fig. 6.25

80. If $m << M$ and a is the semi-major axis of the ellipe, the speed v_1 when the comet is at A is given by

(a) $2\sqrt{\dfrac{GM\ r_2}{r_1\ a}}$ (b) $\sqrt{\dfrac{2GM\ r_2}{r_1\ a}}$

(c) $\sqrt{\dfrac{GM\ r_2}{r_1\ a}}$ (d) $\sqrt{\dfrac{GM\ r_2}{2r_1\ a}}$

81. The speed v_2 of the comet at B is given by

(a) $\sqrt{\dfrac{GM\ r_1}{2r_2\ a}}$ (b) $\sqrt{\dfrac{GM\ r_1}{r_2\ a}}$

(c) $\sqrt{\dfrac{2GM\ r_1}{r_2\ a}}$ (d) $2\sqrt{\dfrac{GM\ r_1}{r_2\ a}}$

82. In Q.81 above, the magnitude of the angular momentum of the comet about the centre of the sun when the comet is at A is proportional to

(a) r_1 (b) r_2

(c) $\dfrac{r_1 r_2}{(r_1+r_2)}$ (d) $\dfrac{r_1^2}{(r_1+r_2)}$

83. The speed of the comet when it is a point P marked in Fig. 6.25 is

(a) $\sqrt{\dfrac{GM}{(r_1+r_2)}}$ (b) $\sqrt{\dfrac{2GM}{(r_1+r_2)}}$

(c) $2\sqrt{\dfrac{GM}{(r_2-r_1)}}$ (d) $\sqrt{\dfrac{GM}{(r_2-r_1)}}$

84. The earth has mass M and radius R. An object of mass m is dropped from a distance of $3R$ from the centre of the earth. The object strikes the surface of the earth with a speed $v = \sqrt{kgR}$, where g is acceleration due to gravity or the surface of the earth. The value of k

is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{3}{4}$ (d) $\frac{4}{3}$

85. A planet revolves around the sun is an elliptual orbit. The maximum and minimum distances of the planet from the sun are a and b respectively. The time period of revolution of the planet is proportional to

(a) $a^{3/2}$ (b) $b^{3/2}$

(c) $(a+b)^{3/2}$ (d) $(a-b)^{3/2}$

86. A solid sphere of mass M and a ring of mass m have their centres lying on the x–axis separated by a distance $2\sqrt{2}\,r$ where r is the radius of the ring as shown in Fig. 6.26. The gravitational force exerted by the sphere on the ring is

(a) $\frac{2\sqrt{2}\,GMm}{27\,r^2}$ (b) $\frac{GMm}{9\sqrt{2}\,r^2}$

(c) $\frac{GMm}{8r^2}$ (d) $\frac{\sqrt{2}\,GMm}{9r^2}$

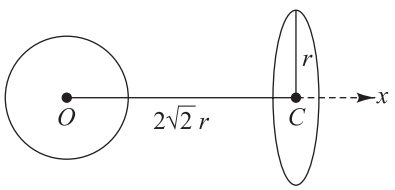


Fig. 6.26

87. There identical spheres A, B and C each of mass M and radius R are placed touching each other as shown in Fig. 6.27. The magnitude of the gravitational force on any sphere due to the other two spheres is $\frac{kGM^2}{R^2}$ where the value of k is

(a) 1 (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{\sqrt{3}}{4}$ (d) 3

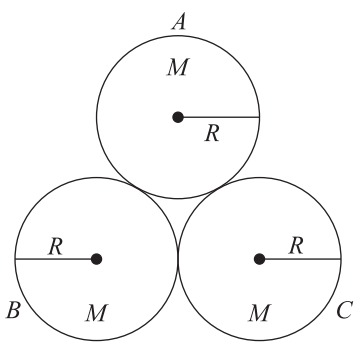


Fig. 6.27

88. A tunnel is dug along the diameter of the earth. An object is held in the tunnel at a distance x from the centre of the earth. The magnitude of the gravitational force on the object is proportional to

(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$

(c) x (d) x^2

89. A thin rod of mass M and length L is bent in the form of a circle. The gravitational potential at the centre of the circular rod is

(a) zero (b) $-\frac{GM}{2\pi L}$

(c) $-\frac{GM}{L}$ (d) $-\frac{2\pi GM}{L}$

90. A uniform solid sphere of mass M and radius R has its centre at the origin O of the coordinate system. Two spherical cavities A and B, each of radius $r = R/4$ are made such that their coordinates are $(2r,0,0)$ and $(-2r,0,0)$ respectively as shown in Fig. 6.28. The gravitational field at O due to the remaining part of the sphere is

(a) zero (b) $\frac{7GM}{R^2}$

(c) $\frac{9GM}{R^2}$ (d) $\frac{GM}{R^2}$

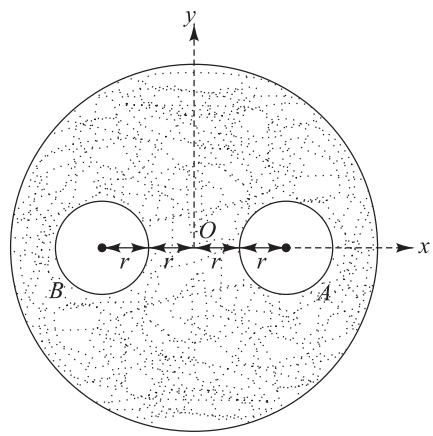


Fig. 6.28

Answers

Level A

1. (a)	2. (d)	3. (b)	4 . (a)
5. (d)	6. (b)	7. (d)	8. (c)

9. (d) 10. (b) 11. (a)

Level B

12. (b)	13. (a)	14. (c)	15. (a)
16. (c)	17. (a)	18. (d)	19. (c)
20. (d)	21. (a)	22. (c)	23. (d)
24. (b)	25. (c)	26. (a)	27. (d)
28. (a)	29. (b)	30. (c)	31. (a)
32. (d)	33. (a)	34. (d)	35. (c)
36. (d)	37. (c)	38. (a)	39. (a)
40. (b)	41. (b)	42. (a)	43. (b)
44. (b)	45. (d)	46. (a)	47. (b)
48. (b)	49. (a)	50. (d)	51. (c)
52. (d)	53. (b)	54. (c)	55. (b)
56. (c)	57. (c)	58. (a)	59. (a)
60. (b)	61. (a)	62. (d)	63. (c)
64. (b)	65. (c)	66. (a)	67. (d)
68. (b)	69. (c)	70. (a)	71. (b)
72. (b)	73. (d)	74. (a)	75. (b)
76. (c)	77. (b)	78. (d)	79. (b)
80. (a)	81. (d)	82. (c)	83. (b)
84. (d)	85. (c)	86. (a)	87. (c)
88. (c)	89. (d)	90. (a)	



Solutions

Level A

1. Volume of earth (V) = $\frac{4\pi}{3} R^3$. Therefore, density of earth is $\rho = \frac{M}{V}$ or $M = V\rho = \frac{4\pi}{3} R^3\rho$

   Now $g = \frac{GM}{R^2} = \frac{4\pi G\rho R}{3}$. Since R is the same for both planets, if ρ is doubled, the value of g is also doubled. Hence the correct choice is (a).

2. Now $g = \frac{GM}{R^2}$. If R reduces to $R' = 0.8\ R$, the value of g becomes

$$g' = \frac{GM}{R'^2} = \frac{GM}{0.64R^2}$$

$$= \frac{g}{0.64} = \frac{9.8\ \text{ms}^{-2}}{0.64}$$

Increase in value of $g = \frac{g}{0.64} - g = \frac{0.36\,g}{0.64}$

$\therefore$ Percentage increase = $\frac{0.36\,g}{0.64\,g} \times 100 = 56.25\%$

Hence the correct choice is (d).

3. For earth $g = \frac{GM}{R^2}$

   For planet $g' = \frac{GM'}{R'^2}$

$$\therefore \frac{g'}{g} = \frac{M'}{M} \times \frac{R^2}{R'^2} = \frac{1}{10} \times (2)^2 = \frac{2}{5}$$

Thus $g' = \frac{2g}{5} = \frac{2}{5} \times 9.8 = 3.92\ \text{ms}^{-2}$

Hence the correct choice is (b).

4. The escape velocity is independent of the direction of projection. Hence the correct choice is (a).

5. Since the gravitational force provides the necessary centripetal force for circular motion, we have

$$\frac{mv^2}{R} \propto R^{-5/2}$$

or $\frac{mv^2}{R} = kR^{-5/2}$, where k is a constant.

Therefore $v = \sqrt{\frac{kR^{-3/2}}{m}}$

Period of revolution $T = \frac{2\pi R}{v} = 2\pi\sqrt{\frac{m}{k} \times R^{7/2}}$ or $T \propto R^{7/4}$. Hence the correct choice is (d).

6. If M is the mass of the earth, the gain in potential energy is given by

$$\int_R^{2R} \frac{GmM}{x^2}\,dx = -GmM\left|\frac{1}{x}\right|_R^{2R} = \frac{GmM}{2R}$$

$$= \frac{GM}{2R^2} \cdot mR = \frac{mgR}{2} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

Hence the correct choice is (b). Remember, the expression $PE = mgh$ is an approximate one which is valid if $h << R$.

7. Let a satellite of mass m revolve in an orbit at a height r from the centre of the earth. If the speed of the satellite is v, then

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \text{ or } mv^2 = \frac{GMm}{r}$$

where M is the mass of the earth. The kinetic energy of the satellite is given by

$$\frac{1}{2}\ mv^2 = \frac{GMm}{2r}$$

The ratio of the kinetic energies of the two satellites is ($\because$ their masses are equal)

$$\frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{GMm}{2r_1} \cdot \frac{2r_2}{GMm} = \frac{r_2}{r_1}$$

But $r_1 = R + R = 2R$ and $r_2 = R + 4R = 5R$. Therefore, the ratio $r_2/r_1 = 5/2$. Hence the correct answer is (d).

8. The speed the satellite is given by

$$v = \sqrt{\frac{GM}{r}}$$

Therefore, its period of revolution is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\sqrt{GM}} \cdot r^{3/2}$$

Hence the correct choice is (c).

9. If the gravitational force were to vary as $1/r^3$, the speed of the planet round the sun would be given by

$$v = \sqrt{\frac{GM}{r^2}} \text{ instead of } v = \sqrt{\frac{GM}{r}}$$

Therefore, the period of revolution of the planet would be given by

$$T = \frac{2\pi r}{v} = \frac{2\pi r^2}{\sqrt{GM}}$$

Hence the correct choice is (d).

10. The acceleration due to gravity is given by

$$g = \frac{GM}{R^2}$$

$$\therefore \quad \frac{\Delta g}{g} = \frac{\Delta M}{M} - \frac{2\Delta R}{R}$$

$$= (-1\%) - (-2 \times 1\%)$$

$$= +1\%$$

i.e. the value of g will increase by 1%. Hence the correct choice is (b).

11. The time period of satellite orbiting at a distance r from the centre of the earth is given by

$$T^2 = \frac{4\pi^2 r^3}{GM^2}$$

where M is the mass of the earth. Therefore, the ratio of the time periods of two satellites at distance r_1 and r_2 respectively from the centre of the earth is

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

or $$T_2 = T_1 \left(\frac{r_2}{r_1}\right)^{3/2}$$

For the geostationary satellite T_1 = 1 day = 24 hours and $r_1 = 6R + R = 7R$.

For the other satellite, $r_2 = 2.5\ R + R = 3.5\ R$. Therefore

$$T_2 = 24 \times \left(\frac{3.5R}{7R}\right)^{3/2} = 24 \times \left(\frac{1}{2}\right)^{3/2}$$

$$= 6\sqrt{2} \text{ hours.}$$

Hence the correct choice is (a).

Level B

12. Let P be a particle of mass m situated midway between the centres of the earth and the moon (Fig. 6.29). The potential energy of particle P due to earth is

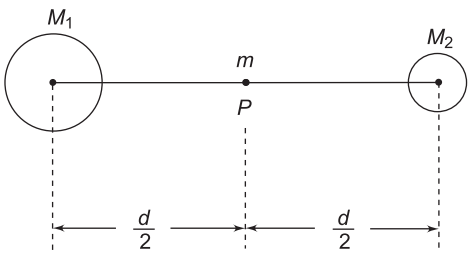


Fig. 6.29

$$-\frac{GmM_1}{r} = -\frac{GmM_1}{d/2} = -\frac{2GM_1m}{d}$$

and that due to moon $= -\dfrac{2GM_2m}{d}$

$\therefore$ Total potential energy $= -\dfrac{2Gm}{d}(M_1 + M_2)$

If the particle P is projected with a velocity v, its kinetic energy $= \dfrac{1}{2}\ mv^2$. Therefore, the total initial energy of the particle is

$$E_i = -\frac{2Gm}{d}\ (M_1 + M_2) + \frac{1}{2}\ mv^2$$

If the particle is to escape to infinity, its final potential and kinetic energy will be zero. Thus the total energy $E_f = 0$. From the principle of conservation of energy,

$$E_i = E_f$$

or $-\frac{2Gm}{d}(M_1 + M_2) + \frac{1}{2}mv^2 = 0$

which gives $v = 2\left[\frac{G(M_1 + M_2)}{d}\right]^{\frac{1}{2}}$

Hence the correct choice is (b). Notice that v is independent of the mass m of the particle. This is the minimum value of the velocity for the particle to escape to infinity.

13. We know that

$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$

which gives $v = \sqrt{\frac{GM}{R}}$

Now, angular momentum $L = mvR = m \times \sqrt{\frac{GM}{R}} \times R$

$= m\sqrt{GM}\ R^{1/2}$.

or $L \propto R^{1/2}$. Hence the correct choice is (a).

14. Choices (a), (b) and (d) are all correct. Hence the choice (c) is the correct answer.

15. The correct choice is (a).

16. For earth : $v_e = \sqrt{\frac{2M_eG}{R_e}}$

For planet : $v_p = \sqrt{\frac{2M_pG}{R_p}}$

Therefore, $\frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e} \cdot \frac{R_e}{R_p}}$ (i)

If ρ_p and ρ_e are the respective average densities of the planet and the earth, then

$$M_P = \frac{4\pi}{3} R_p^3 \rho_p$$

and $M_e = \frac{4\pi}{3} R_e^3 \rho_e$

Therefore, $\frac{M_p}{M_e} = \frac{R_p^3}{R_e^3}$ $(\because \rho_p = \rho_e)$ (ii)

Using (ii) in (i) we get

$$\frac{v_p}{v_e} = \frac{R_p}{R_e} = 2 \qquad (\because R_p = 2R_e)$$

or $v_p = 2v_e$. Hence the correct choice is (c).

17. Statement (a) is wrong.

18. Statement (d) is wrong.

19. The orbits of planets are elliptical. The speed of the planet and its angular speed (or angular frequency) keep changing. Since no net torque acts on the planet, its angular momentum remains constant. Hence the correct choice is (c).

20. Since the object in the space capsule is in a state of weightlessness (or zero gravity), the reading of the spring balance will be zero. Hence the correct choice is (d).

21. The gravitational force acting on the body is

$$F = \frac{GmM}{r^2}$$

The work done by the body against the gravitational pull of the earth in moving upward through a small distance dr is

$$dW = Fdr = \frac{GmM}{r^2}\ dr$$

Therefore, the total work done by the body in escaping from the earth, i.e. in moving to an infinite distance, is given by

$$W = \int_R^\infty dW = GmM \int_R^\infty \frac{dr}{r^2}$$

$$= GmM \left| -\frac{1}{r} \right|_R^\infty = \frac{GmM}{R}$$

Hence the correct choice is (a).

22. Let the rocket be at a distance x from the moon when the gravitational force on it is zero. Its distance from earth $= r - x$. Gravitational force on the rocket due to earth is

$$F_e = \frac{GmM_e}{(r-x)^2}$$

where m is the mass of the rocket. Gravitational force on the rocket due to moon is

$$F_m = \frac{GmM_m}{x^2}$$

Since the two forces are in opposite directions, the net force on the rocket will be zero if $F_e = F_m$. Equating the two we get

$$\frac{r-x}{x} = \sqrt{\frac{M_e}{M_m}} = \sqrt{81} = 9$$

which gives $x = \frac{r}{10}$. Hence the correct choice is (c).

23. Now

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2}$$

or $\frac{1}{2} = \frac{R^2}{(R+h)^2}$

which gives $h = (\sqrt{2} - 1)R$, which is choice (d).

24. Acceleration due to gravity at depth d is given by

$$g' = g\left(1 - \frac{d}{R}\right) = g\left(1 - \frac{32}{6400}\right) = \frac{199g}{200}$$

If m is the mass of the body, its weight on the surface of the earth is mg and at the end of the tunnel it is mg'. Therefore, decrease in weight is

$$mg - mg' = m\left(g - \frac{199g}{200}\right) = \frac{mg}{200}$$

$$\therefore \text{Percentage decrease} = \frac{mg/200}{mg} \times 100 = 0.5\%$$

Hence the correct choice is (b).

25. An object of mass m, placed at the equator of the star, will experience two forces: (i) an attractive force due to gravity towards the centre of the star and (ii) an outward centrifugal force due to the rotation of the star. The centrifugal force arises because the object is in a rotating (non–inertial) frame; this force is equal to the inward centripetal force but opposite in direction. Force on object due to gravity is

$$F_g = \frac{GmM}{R^2}$$

Centrifugal force on the object is

$$F_c = mR\omega^2$$

The object will remain stuck to the star and not fly off if

$$F_g > F_c$$

$$\text{or} \quad \frac{GmM}{R^2} > mR\omega^2$$

$$\text{or} \quad M > \frac{R^3\omega^2}{G}$$

Hence the correct choice is (c).

26. Gravitational field intensity at a point is defined as the gravitational force experienced by a unit mass placed at that point. Since the spheres have the same mass, the gravitational forces exerted by each sphere on a unit mass placed at the mid-point will be equal and opposite. Hence the gravitational field intensity at the mid-point is zero. Thus the correct choice is (a).

27. Gravitational potential at the mid-point is

$$V = -\frac{GM}{r/2} - \frac{GM}{r/2} = -\frac{4MG}{r}$$

Hence the correct choice is (d).

28. Given $AB = BC = AC = a$ (see Fig. 6.30). The perpendiculars from A, B and C on opposite sides meet at the centroid O, which bisect the sides AB, BC and AC. Let $r = AO = BO = CO$. Centroid also divides the lines AD, BE and CF in the ratio 2 : 1, i.e.

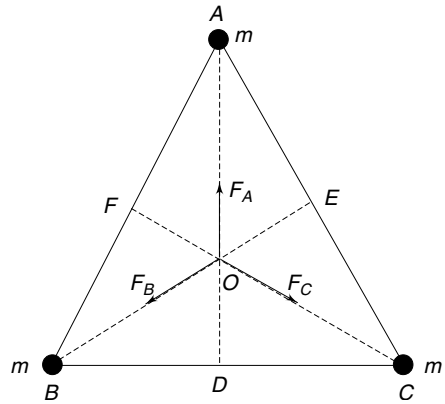


Fig. 6.30

$$AO = \frac{2}{3}AD,\ BO = \frac{2}{3}BE,\ CO = \frac{2}{3}CF.$$

In triangle ABD, $AD = a \sin 60° = \frac{\sqrt{3}a}{2}$.

Similarly, $BE = CF = \frac{\sqrt{3}a}{2}$.

$$\therefore \quad r = AO = OB = OC = \frac{2}{3} \times \frac{\sqrt{3}a}{2} = \frac{a}{\sqrt{3}}$$

The gravitational field intensity at point O is the net force exerted on a unit mass placed at O due to three equal masses m at vertices A, B and C. Since the three masses are equal and their distances from O are also equal, they exert forces F_A, F_B and F_C of equal magnitude. Their directions are shown in the figure. It follows from symmetry of forces that their resultant at point O is zero. Hence the correct choice is (a).

29. Refer to Fig. 6.25 again. Gravitational potential at O is

$$V = -\frac{Gm}{r} - \frac{Gm}{r} - \frac{Gm}{r}$$

$$= -\frac{3Gm}{r} = -\frac{3Gm}{a/\sqrt{3}} = -3\sqrt{3}\frac{Gm}{a}$$

Hence the correct choice is (b).

30. Let m and M be the masses of the earth and the sun respectively and v the speed of the earth in circular orbit. To keep the earth in circular orbit, the gravitational force $\frac{GmM}{r^2}$ must balance the

centripetal force $\frac{mv^2}{r}$, i.e.

$$\frac{GmM}{r^2} = \frac{mv^2}{r}$$

or $$M = \frac{v^2 r}{G}$$

Also $v = \frac{2\pi r}{T}$. Using this, we get $M = \frac{4\pi^2 r^3}{T^2 G}$ or $M \propto \frac{r^3}{T^2}$.

Hence the correct choice is (c).

31. On the surface of the earth, the total energy is

$$\text{KE} + \text{PE} = \frac{1}{2}\, mv^2 - \frac{GmM}{R}$$

where m is the mass of the rocket and M that of earth. At the highest point, $v = 0$ and the energy is entirely potential.

$$\text{PE} = -\frac{GmM}{(R+h)}$$

where h is the maximum height attained. From the law of conservation of energy, we have

$$\frac{1}{2}\, mv^2 - \frac{GmM}{R} = -\frac{GmM}{(R+h)}$$

which gives $$\frac{R+h}{h} = \frac{2gR}{v^2} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

or $$h = \frac{R}{\left(\frac{2gR}{v^2} - 1\right)}$$

Hence the correct choice is (a).

32. If v_i and v_f are respectively the initial and final speeds of the body, we have, from the law of conservation of energy,

$$\frac{1}{2}\, mv_i^2 - \frac{GmM}{R} = \frac{1}{2}\, mv_f^2 \qquad \text{(i)}$$

where m is the mass of the body and M is the mass of the earth and R its radius. The escape velocity is given by

$$\frac{1}{2}\, mv_e^2 = \frac{GmM}{R} \qquad \text{(ii)}$$

Using (ii) in (i) gives

$$\frac{1}{2}\, mv_i^2 - \frac{1}{2}\, mv_e^2 = \frac{1}{2}\, mv_f^2$$

or $$v_f = (v_i^2 - v_e^2)^{1/2} \qquad \text{(iii)}$$

Given $v_i = 3v_e$. Therefore, $v_f = 2\sqrt{2}\ v_e$. Hence the correct choice is (d).

33. Since the initial velocity of the body is zero, its total energy is

$$E = -\frac{GmM}{r} \qquad \text{(i)}$$

where m is the mass of the body, M the mass of the earth and r its distance from the centre of the earth. When the body reaches the earth, let its velocity be v and its distance from the centre of the earth is the earth's radius R. Therefore, the energy now is

$$E = \frac{1}{2}\, mv^2 - \frac{GmM}{R} \qquad \text{(ii)}$$

Equating (i) and (ii) we get

$$v^2 = 2\, GM \left(\frac{1}{R} - \frac{1}{r}\right)$$

Also $g = \frac{GM}{R^2}$. Therefore $GM = gR^2$. Using this in above equation we get

$$v = R\left[2g\left(\frac{1}{R} - \frac{1}{r}\right)\right]^{1/2}$$

Now $r = 2R$ (given). Therefore

$$v = R\left[2g\left(\frac{1}{R} - \frac{1}{2R}\right)\right]^{1/2} = \sqrt{gR}$$

Hence the correct choice is (a).

34. Distance of the satellite from the centre of the earth $r = R + h$. If v is the speed of the satellite in its orbit, then

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

or $$\frac{1}{2}\, mv^2 = \frac{GmM}{2r}$$

or $$\text{KE} = \frac{GmM}{2(R+h)}$$

Hence the correct choice is (d)

35. Potential energy $= -\frac{GmM}{r} = -\frac{GmM}{(R+h)}$. Hence the correct choice is (c).

36. Total energy of the satellite = KE + PE = $\frac{GmM}{2(R+h)} - \frac{GmM}{(R+h)} = -\frac{GmM}{2(R+h)}$. Hence the total energy needed to pull the satellite out of the earth's gravitational

field is $\frac{GmM}{2(R+h)}$, which is choice (d).

37. If the earth were to shrink to half its size, the height of the satellite from the surface of the earth would become $\left(h+\frac{R}{2}\right)$. Its distance from the centre of the earth would be $\left(h+\frac{R}{2}+\frac{R}{2}\right) = (h + R)$, the same as the original distance. Hence the kinetic, potential and total energy of the satellite will be the same. Hence the correct choice is (c).

38. For a geostationary satellite, $h = 35870$ km.

$\therefore \quad \frac{h}{R} = \frac{35870}{6380} = 5.6$, which is choice (a)

39. Consider a satellite of mass m moving with a speed v at an altitude r (measured from the centre of the earth). Then

$$\text{Kinetic energy (KE)} = \frac{1}{2}\ mv^2$$

$$\text{Gravitational potential energy (PE)} = -\ \frac{GmM}{r}$$

where M is the mass of the earth.

For a satellite in circular orbit, we have

$$\frac{mv^2}{r} = \frac{GmM}{r^2} \quad \text{or} \quad v^2 = \frac{GM}{r}$$

$$\text{or} \quad \frac{1}{2}\ mv^2 = \frac{GmM}{2r}$$

$$\text{i.e.} \quad \text{KE} = \frac{GmM}{2r}$$

Thus the KE of a satellite in a circular orbit is numerically half its PE but opposite in sign. The total energy of the satellite in orbit is

$$E = \text{KE} + \text{PE}$$

$$= \frac{GmM}{2r} - \frac{GmM}{r} = -\frac{GmM}{2r}$$

It is given that $r = 2R + R = 3R$, where R is the radius of the earth.

$$\therefore \quad E = -\frac{GmM}{6R}$$

Now PE on the surface of the earth $= -\frac{GmM}{R}$

$\therefore$ Minimum energy required

$$(E_{\min}) = -\frac{GmM}{6R} - \left(-\frac{GmM}{R}\right)$$

$$= \frac{5GmM}{6R}$$

Hence the correct choice is (a).

40. The speeds of stars at separation r are negligible. Therefore, their energy is entirely potential at this separation (since KE = 0)

$$E_1 = (\text{PE at } r) = -\frac{Gm_1m_2}{r} = -\frac{Gm^2}{r}$$

As the stars approach each other under gravitational attraction, they begin to acquire speed and hence kinetic energy at the expense of potential energy. When they eventually collide, the separation between their centres is

$$r = R + R = 2R$$

At $r = 2R$, the total energy is

$$E_2 = \text{PE at } (r = 2R) + \text{KE at } (r = 2R)$$

$$= -\frac{Gm^2}{2R} + \frac{1}{2}\ mv^2 + \frac{1}{2}\ mv^2$$

or $$E_2 = -\frac{Gm^2}{2R} + mv^2$$

From the principle of conservation of energy, $E_1 = E_2$, i.e.

$$-\frac{Gm^2}{r} = -\frac{Gm^2}{2R} + mv^2$$

which gives $v = \sqrt{Gm\left(\frac{1}{2R}-\frac{1}{r}\right)}$

Hence the correct choice is (b).

41. If M is the mass of the earth, the escape velocity is

$$v_e = \sqrt{\frac{2GM}{R}}$$

For a satellite of mass m and orbital radius r (= its distance from the centre of the earth), the orbital speed v is given by

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

$$\text{or} \quad v = \sqrt{\frac{GM}{r}}$$

$$\text{But} \quad v = \frac{1}{2}\ v_e = \frac{1}{2}\sqrt{\frac{2GM}{R}} = \sqrt{\frac{GM}{2R}}$$

$$\therefore \quad \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{2R}}$$

or $r = 2R$. Height above earth $= 2R - R = R$. Hence the correct choice is (b).

42. Potential energy of the satellite in its orbit is

$$E_1 = -\frac{GmM}{r} = -\frac{GmM}{2R} \qquad (\because r = 2R)$$

The kinetic energy is zero because the satellite is stopped. Potential energy of the satellite on the surface of the earth is

$$E_2 = -\frac{GmM}{R}$$

$$\therefore \text{ Loss of PE} = E_1 - E_2 = -\frac{GmM}{2R} - \left(-\frac{GmM}{R}\right)$$

$$= \frac{GmM}{2R}$$

This is converted into kinetic energy. If v is the speed with which the satellite hits the surface of the earth, then from the law of conservation of energy, we have

$$\frac{1}{2}\, mv^2 = \frac{GmM}{2R}$$

$$\text{or} \qquad v^2 = \frac{GM}{R} = gR \qquad \left(\because\ g = \frac{GM}{R^2}\right)$$

Hence the correct choice is (a).

43. $$\text{Initial KE} = \frac{1}{2}\, mv^2 = \frac{1}{2}\, m \left(\frac{v_e}{2}\right)^2$$

$$= \frac{1}{8}\, mv_e^2$$

$$= \frac{GmM}{4R} \qquad \left(\because v_e = \sqrt{\frac{2MG}{R}}\right)$$

$$\text{Initial PE} = -\frac{GmM}{R}$$

$$\therefore \quad \text{Total initial energy} = \frac{GmM}{4R} - \frac{GmM}{R}$$

$$= -\frac{3GmM}{4R} \qquad \text{(i)}$$

If the body comes to rest at a height r from the centre of the earth, its final energy will be given be

$$\text{Final energy} = -\frac{GmM}{r} \qquad \text{(ii)}$$

Equating (i) and (ii) we get $r = 4R/3$. Maximum height attained $= r - R = \frac{4R}{3} - R = \frac{R}{3}$. Hence the correct choice is (b).

44. The gravitational field intensity at point O will be

$$I = Gm\left[\frac{1}{r^2} + \frac{1}{(2r)^2} + \frac{1}{(4r)^2} + \frac{1}{(8r)^2} + \cdots\right]$$

$$= \frac{Gm}{r^2}\left[1 + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + \cdots\right]$$

$$= \frac{Gm}{r^2}\left[\frac{1}{2^0} + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdots\right]$$

$$= \frac{Gm}{r^2}\left(\frac{1}{1-\frac{1}{2^2}}\right) = \frac{Gm}{r^2}\left(\frac{1}{1-\frac{1}{4}}\right) = \frac{4Gm}{3r^2}$$

Hence the correct choice is (b).

45. The gravitational point, in magnitude, at point O is

$$V = Gm\left[\frac{1}{r} + \frac{1}{2r} + \frac{1}{4r} + \frac{1}{8r} + \cdots\right]$$

$$= \frac{Gm}{r}\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\right]$$

$$= \frac{Gm}{r}\left[\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\right]$$

$$= \frac{Gm}{r}\left(\frac{1}{1-\frac{1}{2}}\right) = \frac{2Gm}{r}$$

Hence the correct choice is (d).

46. Mass of the second part $= M - m$. Gravitational force between the two parts is

$$F = \frac{G(M-m)m}{r^2} = \frac{G}{r^2}\,(Mm - m^2)$$

F will be maximum if $\frac{dF}{dm} = 0$ and $\frac{d^2F}{dm^2}$ is negative.

Now, $\frac{dF}{dm} = \frac{G}{r^2}\,(M - 2m)$. Setting $\frac{dF}{dm} = 0$, we get

$M - 2m = 0$ or $m = \frac{M}{2}$. Now

$$\frac{d^2F}{dm^2} = -\frac{2G}{r^2}, \text{ which is negative.}$$

Hence the correct choice is (a).

47. $$\text{PE on the surface of earth} = -\frac{GMm}{R}$$

PE at a height h above the surface of earth

$$= -\frac{GMm}{(R+h)}$$

$$\therefore \text{ Increase in PE} = -\frac{GMm}{(R+h)} - \left(-\frac{GMm}{R}\right)$$

$$= GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$= GMm\left[\frac{h}{R(R+h)}\right]$$

$$= \frac{gRmh}{(R+h)} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

$\therefore$ PE will increase by $\frac{1}{3}mgR$ at a value of h given by

$$\frac{gRmh}{(R+h)} = \frac{1}{3}mgR$$

or $\frac{h}{R+h} = \frac{1}{3}$ or $h = \frac{R}{2}$, which is choice (b).

48. PE at a distance r from the centre of the planet

$$= -\frac{GMm}{r}$$

$$\text{Initial PE} = -\frac{GMm}{R+R} = -\frac{GMm}{2R}$$

$$\text{Final PE} = -\frac{GMm}{R+2R} = -\frac{GMm}{3R}$$

Now, work down = increase in PE

$$= \frac{GMm}{R}\left(\frac{1}{2}-\frac{1}{3}\right) = \frac{GMm}{6R}$$

$$= \frac{1}{6}mgR \qquad \left(\because g = \frac{GM}{R^2}\right)$$

Hence the correct choice is (b).

49. Change in PE $= \frac{GMm}{R} - \frac{GMm}{(n+1)R} = \left(\frac{n}{n+1}\right)mgR$

Hence the correct choice is (a).

50. From the principle of conservation of energy, we have

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{(R+nR)}$$

which gives $v^2 = \frac{2nR}{(n+1)}\frac{GM}{R^2} = \frac{2nRg}{(n+1)}$

$$\left(\because g = \frac{GM}{R^2}\right)$$

Hence the correct choice is (d).

51. If h is the maximum height attained, then we have

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{(R+h)}$$

which gives $v^2 = \frac{2ghR}{(R+h)} \qquad \left(\because g = \frac{GM}{R^2}\right)$

For ball A, we have $\frac{4gR}{3} = \frac{2gh_AR}{(R+h_A)}$

$$\Rightarrow \qquad h_A = 4R$$

For ball B, we have $\frac{2gR}{3} = \frac{2gh_BR}{(R+h_B)}$

$$\Rightarrow \qquad h_B = \frac{R}{2}$$

$\therefore \frac{h_A}{h_B} = 8$, which is choice (c).

52. If ρ is the density of the material of each sphere, then the mass of the sphere of radius r is $M_1 = \frac{4\pi}{3}r^3\rho$ and the mass of the sphere of radius $2r$ is

$M_2 = \frac{4\pi}{3}(2r)^3\rho$.

Distance between their centres is $d = r + 2r = 3r$.

Now $F = \frac{GM_1M_2}{d^2} = \frac{G\times\left(\frac{4\pi}{3}\right)r^3\rho\times\frac{4\pi}{3}(2r)^3\rho}{9r^2}$

which gives $F \propto r^4$, which is choice (d).

53. The angular momentum of the planet is constant over the entire orbit. Hence $mvr = mVR$ or $V = v\left(\frac{r}{R}\right)$, which is choice (b).

54. $g_h = \frac{Gm}{(R+h)^2}$

Also $g = \frac{Gm}{R^2}$. Thus $\frac{g_h}{g} = \frac{R^2}{(R+h)^2}$. Given

$g_h = \frac{90g}{100}$

$\therefore \frac{R^2}{(R+h)^2} = \frac{9}{10}$ or $(R+h) = \frac{\sqrt{10}R}{3}$

Potential energy $= -\frac{GM}{(R+h)} = -\frac{GMR^2}{R^2(R+h)}$

$$= -\frac{gR^2}{(R+h)} = -\frac{3gR}{\sqrt{10}}$$

Hence the correct choice is (c).

55. The distance x of the star of mass m from the centre of mass is given by

$$\frac{m}{x} = \frac{2m}{(r-x)}$$

which gives $x = \frac{r}{3}$. The orbital speed v_1 of the star of mass $m_1 = m$ is given by (here $m_2 = 2m$)

$$\frac{Gm_1m_2}{r^2} = \frac{m_1v_1^2}{x} = \frac{m_1v_1^2}{r/3}$$

which gives $v_1 = \sqrt{\frac{Gm_2}{3r}} = \sqrt{\frac{2Gm}{3r}}$ (i)

$\therefore$ Time period (T) of $m = \frac{2\pi x}{v_1} = \frac{2\pi r}{3} \times \sqrt{\frac{3r}{2GM}}$

$= \pi\sqrt{\frac{2}{3Gm}}\ (r)^{3/2}$

or $T \propto r^{3/2}$, which is choice (b).

56. The orbital speed v_2 of the star of mass $m_2 = 2m$ is given by

$$\frac{G m_1 m_2}{r^2} = \frac{m_2 v_2^2}{(r-x)} \text{ or } \frac{G m(2m)}{r^2} = \frac{2m v_2^2}{2r/3}$$

or $v_2 = \sqrt{\frac{2Gm}{3r}} = v_1$ [see Eq. (i)]

Now $K_1 = \frac{1}{2}\ m_1 v_1^{\,2} = \frac{1}{2}\ m v_1^{\,2}$ and $K_2 = \frac{1}{2}\ m_2 v_2^{\,2} = \frac{1}{2}\ (2m)\ v_2^{\,2}$

$\therefore\ \frac{K_1}{K_2} = \frac{v_1^2}{2v_2^2} = \frac{1}{2}$, since $v_1 = v_2$. Hence the correct choice is (c).

57. Angular momentum of m_1 is $L_1 = m_1\ v_1\ x = \frac{m v_1 r}{3}$

Angular momentum of m_2 is $L_2 = m_2\ v_2\ (r - x)$

$= \frac{2m v_2 \times 2r}{3}$

$\therefore\ \frac{L_1}{L_2} = \frac{1}{4}$ (since $v_1 = v_2$), which is choice (c).

58. The force of attraction between the complete sphere and mass m is

$$F = \frac{G m M}{(2R)^2} = \frac{G m M}{4R^2} \quad \text{(i)}$$

Mass of complete sphere is $M = \frac{4\pi}{3} R^3 \rho$. Mass of the cut out portion is $m_0 = \frac{4\pi}{3}\left(\frac{R}{2}\right)^3 \rho$. Thus, $m_0 = \frac{M}{8}$. The distance between the centre of the cut out portion and mass $m = 2R - \frac{R}{2} = \frac{3R}{2}$.

Hence the force of attraction between the cut out portion and mass m is

$$f = \frac{G m_0 m}{(3R/2)^2} = \frac{G(M/8)m}{9R^2/4} = \frac{GmM}{4R^2} \times \frac{2}{9}$$

Using (i), we get $f = \frac{2F}{9}$. Therefore, the force of attraction between the remaining part of the sphere and mass $m = F - f = F - \frac{2F}{9} = \frac{7F}{9}$ which is choice (a).

59. Refer to Fig. 6.31. Let μ be the mass per unit length of the ring. $L = 2\pi R$ is the length of the ring. Consider a small element of length dx of the ring located at C. Then

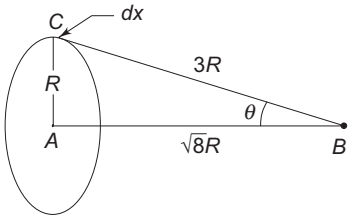


Fig. 6.31

Force along BC is $f = \frac{G M \mu\, dx}{(3R)^2}$. Therefore, force along BA is $dF = f \cos\theta = \frac{G M \mu\, dx}{9R^2}\ \frac{\sqrt{8}R}{3R}$

$= \frac{\sqrt{8}}{27}\ \frac{GM\ \mu dx}{R^2}$

$\therefore$ Total force $= \frac{\sqrt{8}}{27}\frac{GM}{R^2}\int \mu\, dx = \frac{\sqrt{8}}{27}\frac{GMm}{R^2}$

because $\int \mu\, dx = \mu \times L = m$, the mass of the ring. Hence the correct choice is (a).

60. The distance of each cavity from the centre O is the same. Since the two cavities are symmetrical with respect to the centre O and the mass of the sphere can be regarded as being concentrated at the centre O, the gravitational force due to the sphere is zero at the centre. Hence choice (a) is correct. For the same reason, the gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$ whose radius is 6 units and at all points of the circle $y^2 + z^2 = 4$ whose radius is 2 units. Hence choices (c) and (d) are also correct. But the gravitational force at point B cannot be zero.

61. Initially when the two masses are at an infinite distance from each other, their gravitational potential energy is zero. When they are at a distance r from each other the gravitational P.E. is

$$\text{PE} = -\frac{G m_1 m_2}{r}$$

The minus sign indicates that there is a decrease in P.E. This gives rise to an increase in kinetic energy. If

v_1 and v_2 are their respective velocities when they are a distance r apart, then, from the law of conservation of energy, we have

$$\frac{1}{2} m_1 v_1^2 = \frac{G m_1 m_2}{r} \text{ or } v_1 = \sqrt{\frac{2Gm_2}{r}}$$

and $$\frac{1}{2} m_2 v_2^2 = \frac{G m_1 m_2}{r} \quad \text{or} \quad v_2 = \sqrt{\frac{2Gm_1}{r}}$$

Therefore, their relative velocity of approach is

$$v_1 + v_2 = \sqrt{\frac{2Gm_2}{r}} + \sqrt{\frac{2Gm_1}{r}}$$

$$= \sqrt{\frac{2G}{r}(m_2 + m_1)}$$

Hence the correct choice is (a).

62. At a finite separation, the total kinetic energy of the system of two masses and the force between them are both finite. Since the two masses are at rest initially and there is no external force, the centre of mass cannot move. Hence the correct choice is (d).

63. According to Kepler's law of period, $T^2 = kR^3$ where k is a constant. Taking logarithm of both sides, we have

$$2 \log T = \log k + 3 \log R$$

Differentiating, we get

$$2 \frac{\delta T}{T} = 0 + 3 \frac{\delta R}{R}$$

or $$\frac{dT}{T} = \frac{3}{2} \frac{\delta R}{R} = \frac{3}{2} \times \left(\frac{1.01R - R}{R}\right) \times 100$$

$$= 1.5\%$$

Hence the correct choice is (c).

64. According to Kepler's law of period,

$$\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{R_1}{R_1/2}\right)^{3/2}$$

$$= (2)^{3/2} = 2\sqrt{2}$$

$$\therefore \quad T_2 = \frac{T_1}{2\sqrt{2}} = \frac{365 \text{ days}}{2\sqrt{2}} = 129 \text{ days.}$$

65. For a satellite, we have

$$\text{Kinetic energy} = \frac{G m M}{2r}$$

$$\text{Potential energy} = -\frac{G m M}{r}$$

$$\text{Total energy } E_0 = \text{KE} + \text{PE} = \frac{G m M}{2r} - \frac{G m M}{r}$$

$$= -\frac{G m M}{2r} = \frac{\text{PE}}{2}$$

or PE = $2E_0$. Hence the correct choice is (c).

66. For elliptical orbit, the earth is at one focus of the ellipse. For spherical bodies, the gravitational force is central (or radial). Hence statement (a) is correct. The gravitational force exerts no torque on the satellite. Hence the angular momentum of S remains constant in magnitude as well as direction. Hence choice (b) is incorrect. For elliptical orbit, the distance of the satellite from the earth varies periodically. Hence potential energy, kinetic energy and linear momentum vary periodically. Hence choices (c) and (d) are also incorrect.

67. The acceleration due to gravity at a height h above the surface of the earth is given by

$$g_2 = g_1 \left(\frac{R}{R+h}\right)^2$$

where g_1 is the value at the surface of the earth. Now

$$T_2 = 2\pi \sqrt{\frac{l}{g_2}} \text{ and } T_1 = 2\pi \sqrt{\frac{l}{g_1}}$$

$$\therefore \quad \frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}} = \frac{R+h}{R}$$

$$= \frac{R+R}{R} = 2 \qquad (\because h = R)$$

Hence the correct choice is (d).

68. Let x be the extension in the spring when it is loaded with mass M. The change in gravitational potential energy = Mgx. This must be the energy stored in the spring which is given by $\frac{1}{2} kx^2$. Thus

$$\frac{1}{2} kx^2 = Mg\,x \text{ or } x = \frac{2Mg}{k}, \text{ which is choice (b).}$$

69. For a satellite of mass m moving with a velocity v in a circular orbit of radius r around the earth of mass M, we have

$$\frac{mv^2}{r} = \frac{G m M}{r^2} \text{ or } v = \sqrt{\frac{GM}{r}}$$

Now $$v = \frac{2\pi r}{T}. \text{ Thus } \frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

or $$T \propto r^{3/2}.$$

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} \qquad (1)$$

Given r_2 = 6400 km and r_1 = 36000 km. For a geostationary satellite T_1 = 24 h. Using these values in (1), we have get $T_2 = 24 \times \left(\frac{64}{360}\right)^{3/2} = 1.8$ h.

Hence the closest choice is (c).

70. Given $m_1 = xm$ and $m_2 = (1-x)m$. For a separation r between them, the force of attraction is

$$F = \frac{Gm_1m_2}{r^2} = \frac{G}{r^2}\, xm(1-x)\, m$$

$$= \frac{Gm^2}{r^2}\,(x - x^2)$$

For a given r, F will be maximum if $\frac{dF}{dx} = 0$ and $\frac{d^2F}{dx^2} < 0$, i.e.

$\frac{d}{dx}(x - x^2) = 0$ or $1 - 2x = 0$ or $x = \frac{1}{2}$.

Now $\frac{d^2F}{dx^2} = \frac{Gm^2}{r^2}(-2) = -\frac{2Gm^2}{r^2}$, which is negative. Hence the correct choice is (a).

71. $g_h = \frac{gR^2}{(R+h)^2}$. Given $g_h = \frac{g}{100}$. Therefore, we have

$$\frac{gR^2}{(R+h)^2} = \frac{g}{100}$$

or $R + h = 10\,R$ or $h = 9\,R$, which is choice (b).

72. Escape velocity $v_e = \sqrt{\frac{2GM}{R}}$. Velocity of projection $v = \frac{3}{4}\, v_e = \frac{3}{4}\sqrt{\frac{2GM}{R}}$. The total energy of the body when it is projected is

$$E_i = \text{KE} + \text{PE}$$

$$= \frac{1}{2}\, mv^2 - \frac{GmM}{R}$$

$$= \frac{1}{2}m \times \frac{9}{16} \times \frac{2GM}{R} - \frac{GmM}{R}$$

$$= \frac{9}{16}\frac{GmM}{R} - \frac{GmM}{R} = -\frac{7}{16}\frac{GmM}{R}$$

Let h be the maximum height attained by the body. The distance of the body from the centre of the earth is $r = R + h$. At this height, the total energy of the body is

$$E_f = \text{KE} + \text{PE}$$

$$= 0 - \frac{GmM}{r} = -\frac{GmM}{r} = -\frac{GmM}{(R+h)}$$

From the principle of conservation of energy, $E_i = E_f$, i.e.

$$-\frac{7}{16}\frac{GmM}{R} = -\frac{GmM}{(R+h)}$$

or $7\,(R + h) = 16R$ or $7h = 9R$ or $h = \frac{9R}{7}$, which is choice (b).

73. If the gravitational field is zero at a point at a distance x from M_1, then

$$\frac{GM_1}{x^2} = \frac{GM_2}{(r-x)^2}$$

or $\frac{x}{(r-x)} = \sqrt{\frac{M_1}{M_2}} = \sqrt{\frac{m}{4m}} = \frac{1}{2}$

which gives $x = \frac{r}{3}$. Therefore, $r - x = \frac{2r}{3}$. The gravitational potential at $x = \frac{r}{3}$ is

$$U = -\frac{GM_1}{x} - \frac{GM_2}{(r-x)}$$

$$= -\frac{Gm}{r/3} - \frac{G(4m)}{2r/3} = -\frac{9Gm}{r}$$

Hence the correct choice is (d).

74. At the equator, the value of g is

$$g' = g - R\omega^2$$

where ω is the angular speed of the earth. For bodies to appear weightless at the equator, $g' = 0$, i.e.

$$g - R\omega^2 = 0$$

which gives $\omega = \sqrt{\frac{g}{R}}$. Hence the correct choice is (a).

75. Gravitational force is conservative. The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle. Hence the correct choice is (b).

76. In a binary star system, the two stars move under their mutual gravitational force. Therefore, their angular velocities and hence their time periods are equal. Thus the correct choice is (c).

77. Refer to Fig. 6.32.

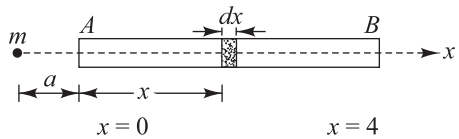


Fig. 6.32

Consider a small element of length dx of the bar at a distance x from end A. The mass of this element $dm = \frac{M}{L}\,dx$. The force exerted by this element on mass m is

$$dF = \frac{G\,m\,dM}{(a+x)^2} = \frac{G\,m\,M\,dx}{L(a+x)^2}$$

The gravitational force exerted by the complete bar on mass m will be

$$F = \int dF = \frac{GmM}{L}\int_0^L \frac{dx}{(a+x)^2}$$

$$= -\frac{GmM}{L}\left|\frac{1}{a+x}\right|_0^L$$

$$= -\frac{GmM}{L}\left[\frac{1}{a+L} - \frac{1}{a}\right]$$

$$= \frac{GmM}{a(a+L)}$$

So the correct choice is (b).

78. Total energy of the satellite when its orbital radius is r is

$$E_1 = \text{K.E.} + \text{P.E.}$$

$$= \frac{GmM}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

Total energy of the satellite for orbital radius $2r$ is

$$E_2 = \text{K.E.} + \text{P.E.}$$

$$= \frac{GmM}{2(2r)} - \frac{GMm}{2r} = -\frac{GMm}{4r}$$

The amount of work needed is

$$W = E_2 - E_1$$

$$= -\frac{GMm}{4r} - \left(-\frac{GMm}{2r}\right) = \frac{GMm}{4r}$$

79. The angular momentum of the comet about the centre of the SUN is conserved. If L_1 is the magnitude of the angular momentum when the comet is at A and L_2 that at B, then,

$$L_1 = L_2$$

i.e. $mv_1r_1 = mv_2r_2$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{r_2}{r_1}$$

So the correct choice is (b).

80. The total energy of the comet of mass m in an elliptical orbit with semi-major axis a around the sun of mass M is given by

$$E = -\frac{GmM}{2a}$$

Since $2a = r_1 + r_2$,

$$E = -\frac{GmM}{(r_1 + r_2)}$$

So the kinetic energy of the comet at A is given by

K.E. at A = total energy – P.E. at A

or $$\frac{1}{2}mv_1^2 = -\frac{GmM}{(r_1+r_2)} - \left(-\frac{GmM}{r_1}\right)$$

$$= -\frac{GmM\ r_2}{r_1(r_1+r_2)} = \frac{2GmMr_2}{r_1 a}$$

$$\Rightarrow \quad v_1 = 2\sqrt{\frac{GMr_2}{r_1 a}}$$, which is choice (a).

81. The kinetic energy of the comet at B is given by

$$\frac{1}{2}mv_2^2 = -\frac{GmM}{(r_1+r_2)} - \left(-\frac{GmM}{r_2}\right)$$

$$= \frac{GmMr_1}{r_2(r_1+r_2)} = \frac{2GmMr}{r_2 a}$$

$$\Rightarrow \quad v_2 = 2\sqrt{\frac{GMr_1}{r_2 a}}$$

So the correct choice is (d).

Note

We notice that

$$\frac{v_1}{v_2} = \frac{2\sqrt{\frac{GM\ r_2}{r_1 a}}}{2\sqrt{\frac{GM\ r_1}{r_2 a}}} = \frac{r_2}{r_1}$$

This result we have obtained above by using the principle of conservation of angular momentum.

82. $$L_1 = mv_1r_1$$

$$= m \times 2\sqrt{\frac{GMr_2}{r_1 a}} \times r_1$$

$$= m\sqrt{\frac{2GMr_1r_2}{(r_1+r_2)}} \qquad \left[\because a = \frac{1}{2}(r_1+r_2)\right]$$

Therefore $L_1 \propto \frac{r_1 r_2}{(r_1+r_2)}$, which is choice (c).

83. We know that the sum of the distances of any point on ellipse from the two foci = $2a = (r_1 + r_2)$. Since point P is equidistant from the two foci, one of which is at the centre of the SUN, the distance of P from

the centre of the SUN is equal to $a = \frac{1}{2}(r_1 + r_2)$.

Therefore, the kinetic energy of the comet when it is at P is

$$E_p = \text{Total energy at } P - \text{P.E. at } P$$

$$= -\frac{G m M}{(r_1 + r_2)} - \left(-\frac{G m M}{\frac{1}{2}(r_1 + r_2)} \right)$$

$$= \frac{G m M}{(r_1 + r_2)}$$

or $$\frac{1}{2} m v_p^2 = \frac{GM m}{r_1 + r_2}$$

$$\Rightarrow \qquad v_p = \sqrt{\frac{2GM}{(r_1 + r_2)}}$$

So the correct choice is (b).

84. Total energy when the object is dropped is

$$E_1 = \text{K.E.} + \text{P.E.}$$

$$= 0 - \frac{GMm}{3R} = -\frac{GMm}{3R}$$

Total energy when the object strickes the earth's surface is

$$E_2 = \text{K.E.} + \text{P.E.}$$

$$= \frac{1}{2} m v^2 - \frac{GMm}{R}$$

From conservation of energy, $E_1 = E_2$, i.e.

$$-\frac{GmM}{3R} = \frac{1}{2} m v^2 - \frac{GMm}{R}$$

$$v = \sqrt{\frac{\frac{4}{3}GM}{R}}$$

$$= \sqrt{\frac{4}{3} gR} \qquad \left(\because \quad g = \frac{GM}{R^2} \right)$$

which gives $k = \frac{4}{3}$. So the correct choice is (d).

85. According to kepler's law, the square of the time period is proportional to the cube of the semi-major axis of the ellipse, i.e. $T^2 \propto \left(\frac{a+b}{2} \right)^3$. So the correct choice is (c).

86. The entire mass of the sphere can be assumed to be concentrated at its centre of mass O. Consider a small element of the ring at A. Let dm be the mass of the element (see Fig. 6.33)

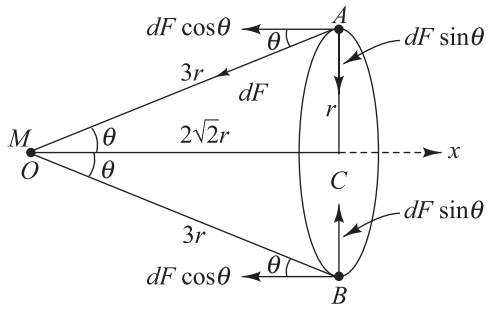


Fig. 6.33

The gravitational force exerted by M on dm is

$$dF = \frac{GM m}{r_1 + r_2} = \frac{GM\, dm}{9r^2}$$

This force can be resolved into x and y components as shown. Now consider another element at B. We find that x ompoments (namely $dF\cos\theta$) add up but y-components (namely of $dF\sin\theta$) cancel each other. Hence the x components of all elements add up and y-components cancel. Hence, the total force exerted by the sphere on the ring is

$$F = \int dF\cos\theta = \int \frac{GM\, dm}{9r^2} \times \frac{2\sqrt{2}}{3}$$

$$= \frac{2\sqrt{2}\, GM}{27r^2} \int dm = \frac{2\sqrt{2}\; G M m}{27 r^2}$$

So the correct choice is (a).

87. The mass of a sphere can be assumed to be concentrated at its centre. So the given system can be regarded as system of three particles, each of mass M located at the three vertices of an equilateral triangle of side $2R$ as shown in Fig. 6.34.

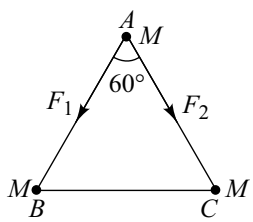


Fig. 6.34

$$AB = BC = AC = 2R.$$

$$F_1 = F_2 = \frac{GM^2}{(2R)^2} = \frac{GM^2}{4R^2} = F\,(\text{say})$$

The resultant force on A has magnitude

$$F_r = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{F^2 + F^2 + 2F^2 \cos 60°}$$

$$= \sqrt{3}F = \frac{\sqrt{3}\,GM^2}{4R^2}$$

So the correct choice is (c).

88. Let M and R respectively be the mass and radius of the earth and let m be the mass of the object (Fig. 6. 35) Gravitational force on the object is

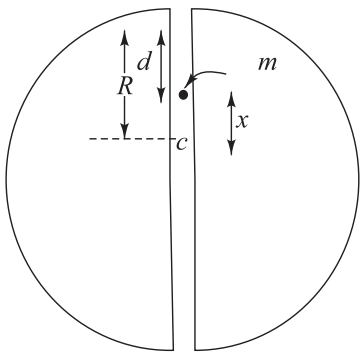


Fig. 6.35

$$F = mg_d$$

$$= mg\left(1 - \frac{d}{R}\right)$$

$$= m \times \frac{MG}{R^2} \times \left(\frac{R-d)}{R}\right)$$

$$= \frac{G\,m\,M\,x}{R^3}$$

So the correct choice is (c).

89. Radius of the circular rod is $R = \frac{L_1}{2\pi}$. Since every point on the circle is at the same distance R from its centre and since potential is a scalar quantity, the gravitation potential at the centre is

$V = -\frac{GM}{R} = -\frac{2\pi GM}{L}$, which is choice (d).

90. Mass of complete sphere is $M = \frac{4\pi}{3}R^3\rho$. Mass of each spherical cavity is

$$m = \frac{4\pi}{3}\left(\frac{R}{4}\right)^3 \rho = \frac{M}{64}$$

Gravitational at O due to cavity A is

$$I_A = \frac{GM}{(2r)^2}$$

$$= \frac{GM \times 16}{64 \times 4 \times R^2} = \frac{GM}{16R^2}$$

directed to the left.

Gravitational field at O due to cavity B is

$$I_B = \frac{GM}{16R^2} \text{ directed to the right.}$$

If I is gravitational field at O due to the complete sphere, then the gravitational field at O due to the remaining part is

$$I_r = I - I_A + I_B$$

Now $I_A = I_B$ and $I = 0$ (because $x = 0$ see Q. 89 above), $I_r = 0$. So the correct choic is (a).

2 SECTION — Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage.

Passage I

A satellite of mass m is revolving in a circular orbit of radius r around the earth of mass M. The speed of the satellite in its orbit is one-fourth the escape velocity from the surface of the earth.

1. The height of the satellite above the surface of the earth is (R = radius of earth)
   (a) $2R$ (b) $3R$
   (c) 5R (d) $7R$

2. The magnitude of angular momentum of the satellite is
   (a) $m\sqrt{GMR}$ (b) $\frac{m}{2}\sqrt{GMR}$
   (c) $\frac{m}{2\sqrt{2}}\sqrt{GMR}$ (d) $2m\sqrt{GMR}$

3. If the total energy of the satellite is E, its potential energy is
   (a) $-E$ (b) E
   (c) $2E$ (d) $-2E$

Solutions

1. $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+h)}}$ (1)

   $v = \frac{v_e}{4} = \frac{1}{4}\sqrt{\frac{2GM}{R}}$ (2)

   Equations (1) and (2) give $h = 7R$, which is choice (d)

2. $L = mvR = m\sqrt{\frac{GM}{8R}}R = \frac{m}{2\sqrt{2}}\sqrt{GMR}$ $(\because h = 7R)$

   The correct choice is (c).

3. The correct choice is (c).

Questions 4 to 6 are based on the following passage.

Passage II

Considering the earth as an isolated mass, a force is experienced by a body at any distance from it. This force is directed towards the centre of the earth and has a magnitude *mg*, where *m* is the mass of the body and *g* is the acceleration due to gravity. The value of the acceleration due to gravity decreases with increase in the height above the surface of the earth and with increase in the depth below the surface of the earth. Even on the surface of the earth, the value of *g* varies from place to place and decreases with decrease in the latitude of the place.

4. Assuming the earth to be a sphere of uniform mass density, which of the graphs shown in Fig. 6.34 represents the variation *g* with distance *r* from the centre of the earth (*R* = radius of earth)

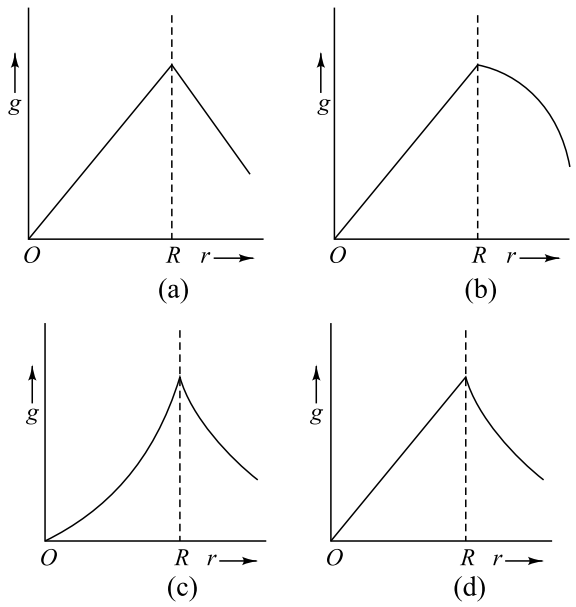


Fig. 6.34

5. Assuming the earth to be a sphere of uniform mass density, the weight of a body when it is taken to the end of a tunnel 32 km below the surface will (radius of earth = 6400 km)
   (a) decrease by 0.5% (b) decrease by 1%
   (c) increase by 0.5% (d) increase by 1%
6. If a tunnel is dug along a diameter of the earth and a body is dropped from one end of the tunnel,
   (a) it will fall and come to rest at the centre of the earth where its weight becomes zero.
   (b) it will emerge from the other end of the tunnel.
   (c) it will execute simple harmonic motion about the centre of the earth.
   (d) it will accelerate till it reaches the centre and decelerate after that eventually coming to rest at the other end of the tunnel.

Solutions

4. At a height *h* above the surface of the earth,

   $$g_h = \frac{g_0 R^2}{(R+h)^2}$$

   At a depth *d* below the surface of the earth

   $$g_d = g_0\left(1 - \frac{d}{R}\right)$$

   where g_0 = acceleration due to gravity at the surface of the earth. Hence the correct choice is (d).

5. $g = \left(1 - \frac{d}{R}\right)g_0 = \left(1 - \frac{32}{6400}\right)g_0 = \frac{199g_0}{200}$

   $\therefore$ Decrease in weight = $mg_0 - mg$

   $$= mg_0\left(1 - \frac{199}{200}\right) = \frac{mg_0}{200}$$

   Hence the correct choice is (a).

6. The correct choice is (c).

Questions 7 to 9 are based on the following passage.

Passage III

The escape velocity on a planet or a satellite is the minimum velocity with which a body must be projected from that planet so that it escapes the gravitational pull of the planet and goes into outer space. We obtain the expression for the escape velocity by equating the work required to move the body from the surface of the planet to infinity with the initial kinetic energy given to the body. The escape velocity from a planet of mass *M* and radius *R* is given by

$$v_e = \sqrt{\frac{2MG}{R}} = \sqrt{2gR}$$

where g is the acceleration due to gravity on the surface of the planet and G is the gravitation constant.

7. The mass of Jupiter is about 319 times that of the earth and its radius is about 11 times that of the earth. The ratio of the escape velocity on Jupiter to that on earth is
   (a) $\sqrt{29}$ (b) 29
   (c) $\frac{1}{\sqrt{29}}$ (d) $\frac{1}{29}$
8. If R is the radius of the earth and g the acceleration due to gravity on its surface, the escape velocity of a body projected from a satellite orbiting the earth at a height $h = R$ from the surface of the earth will be
   (a) $\sqrt{gR}$ (b) $\sqrt{2gR}$
   (c) $\sqrt{3gR}$ (d) $2\sqrt{gR}$
9. A body is dropped from a height equal to half the radius of the earth. If v_e is the escape velocity on earth and air resistance is neglected, it will strike the surface of the earth with a speed
   (a) $\frac{v_e}{\sqrt{2}}$ (b) $\frac{v_e}{2}$
   (c) $\frac{v_e}{\sqrt{3}}$ (d) $\frac{v_e}{3}$



Solutions

7. $\frac{v_J}{v_E} = \sqrt{\frac{M_J}{M_E} \times \frac{R_E}{R_J}} = \sqrt{319 \times \frac{1}{11}} = \sqrt{29}$

Hence the correct choice is (a).

8. The escape velocity at a height h is given by

$$v'_e = \sqrt{2g'(R+h)}$$

where g' is the acceleration due to gravity at height h,

$$g' = g\left(\frac{R}{R+h}\right)^2$$

For $h = R$, we get $v'_e = \sqrt{gR}$, which is choice (a).

9. The correct choice is (c). Use conservation of energy, i.e.

Total energy at h ($= R/2$)

= Total energy when the body strikes the earth

$$\Rightarrow \quad -\frac{GmM}{(R+h)} = \frac{1}{2}mv^2 - \frac{GmM}{R}$$

which gives $v = v_e/\sqrt{3}$

SECTION 3

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only *one* choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. Statement-1

A body is projected up with a velocity equal to half the escape velocity from the surface of the earth. If R is the radius of the earth and atmospheric resistance is neglected, it will attain a height $h = R/3$.

Statement-2

The gravitational potential is $-GM/R$ on the surface of the earth and it increases with height; M being the mass of the earth.

2. Statement-1

The total energy (kinetic + potential) of a satellite moving in a circular orbit around the earth is half its potential energy.

Statement-2

The gravitational force obeys the inverse square law of distance.

3. Statement-1

Two bodies of masses $m_1 = m$ and $m_2 = 3m$ are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual

gravitational attraction. Their relative velocity of approach when they are at a separation r is

$$v = \sqrt{\frac{2Gm}{r}}$$

Statement-2

The gain in the kinetic energy of each body equals the loss in its gravitational potential energy.

4. **Statement-1**

An astronaut inside a massive space-ship orbiting around the earth will experience a finite but small gravitational force.

Statement-2

The centripetal force necessary to keep the space-ship in orbit around the earth is provided by the gravitational force between the earth and the space-ship.

5. **Statement-1**

The escape velocity varies with altitude and latitude of the place from where it is projected.

Statement-2

The escape velocity depends on the gravitational potential at the point of projection.

6. **Statement-1**

A comet orbits the sun in a highly elliptical orbit. Its potential energy and kinetic energy both change over the orbit but the total energy remains constant throughout the orbit.

Statement-2

For a comet, only the angular momentum remains constant throughout the orbit.

7. **Statement-1**

The acceleration due to gravity decreases due to rotation of the earth.

Statement-2

A body on the surface of the earth also rotates with it in a circular path. A body in a rotating (non-inertial) frame experiences an outward centrifugal force against the inward force of gravity.



Solutions

1. The correct choice is (b). Use $v_e = \sqrt{\frac{2Gm}{R}}$ and total energy at r $(= R + h)$ = total initial energy, i.e. $-\frac{GmM}{r} = \frac{1}{2}mv^2 - \frac{GmM}{R}$

2. The correct choice is (a). The centripetal force needed for circular motion is provided by the gravitational force. Since the gravitational force obeys the inverse square law of distance, the orbital velocity of the satellite is given by

$$v = \sqrt{\frac{GM}{r}}$$

where M = mass of earth and r = orbital radius.

Therefore

$$\text{Kinetic energy} = \frac{1}{2}mv^2 = \frac{GmM}{2r}$$

where m = mass of the satellite. From the inverse square law of distance, we find that the potential of the satellite is given by

$$\text{Potential energy} = -\frac{GmM}{r}$$

$\therefore$ Total energy E = K.E. + P.E.

$$= \frac{GmM}{2r} + \left(-\frac{GmM}{r}\right)$$

$$= -\frac{GmM}{2r} = \frac{\text{P.E.}}{2}$$

3. The correct choice is (d). Initially when the two masses are at an distance from each other, their gravitational potential energy is zero. When they are at a distance r from each other the gravitational P.E. is

$$\text{P.E} = -\frac{Gm_1m_2}{r}$$

The minus sign indicates that there is a decrease in P.E. This gives rise to an increase in kinetic energy. If v_1 and v_2 are their respective velocities when they are at a distance r apart, then, from the law of conservation of energy, we have

$$\frac{1}{2}m_1v_1^2 = \frac{Gm_1m_2}{r} \text{ or } v_1 = \sqrt{\frac{2Gm_2}{r}}$$

and $\frac{1}{2}m_1v_2^2 = \frac{Gm_1m_2}{r}$ or $v_2 = \sqrt{\frac{2Gm_1}{r}}$

Therefore, their relative velocity of approach is

$$v = v_1 + v_2 = \sqrt{\frac{2Gm_2}{r}} + \sqrt{\frac{2Gm_1}{r}}$$

$$= \sqrt{\frac{2G}{r}(m_2 + m_1)}$$

Putting $m_1 = m$ and $m_2 = 3m$, we get $v = 2\sqrt{\frac{2Gm}{r}}$

4. The correct choice is (b). Because the centripetal force equals the gravitational force exerted by the earth on the space-ship, the astronaut does not experience any gravitational force of the earth. The only force of gravity that an astronaut in an orbiting space-ship experiences is that which is due to the gravitational force exerted by the space-ship. Since the space-ship is very massive, this force is finite but very small.
5. The correct choice is (a). The gravitational potential at a point varies with the altitude and latitude of the place.
6. The correct choice is (c).
7. The correct choice is (a).

4 SECTION Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

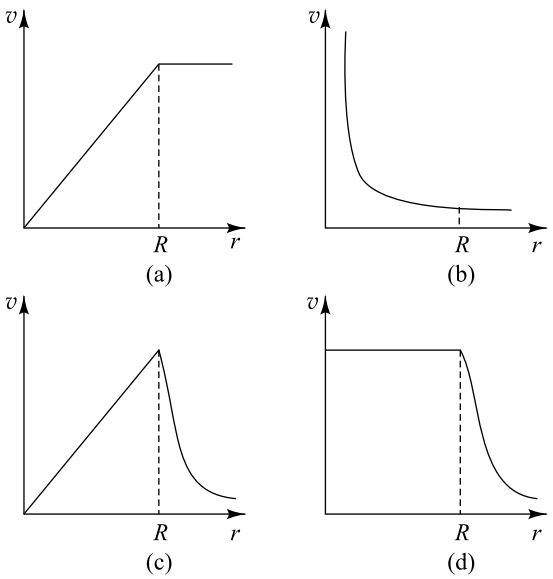
1. If suddenly the gravitational force of attraction between the earth and a satellite revolving around it becomes zero, then the satellite will
   (a) continue to move in the original orbit with the same velocity
   (b) move tangentially to the original orbit with the same velocity
   (c) become stationary it its orbit
   (d) move towards the earth **[2002]**
2. The energy required to move a body of mass m from an orbit of radius 2 R to an orbit of radius 3 R is.
   (a) $\frac{GmM}{12R}$ (b) $\frac{GmM}{3R}$
   (c) $\frac{GmM}{8R}$ (d) $\frac{GmM}{6R}$ **[2002]**
3. The escape velocity of a body depends on its mass m as
   (a) m^0 (b) m^1
   (c) m^2 (d) m^3 **[2002]**
4. The kinetic energy needed to project a body of mass m from the earth's surface (radius R) to infinity is
   (a) $\frac{mgR}{2}$ (b) $2mgR$
   (c) mgR (d) $\frac{mgR}{4}$ **[2002]**
5. An ideal spring with spring-constant k is hung from the ceiling and a block of Mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension of spring is.
   (a) 4 Mg/K (b) 2 Mg/K
   (c) Mg/K (d) $Mg/2\ K$ **[2002]**
6. A geo-stationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbiting a few hundred kilometeres above the earth's surface (R_{Earth} = 6400 km) will be approximately be
   (a) $\frac{1}{2}$ h (b) 1 h
   (c) 2 h (d) 4 h **[2002]**
7. The time period of a satellite of earth is 5 h. If the separation between the earth and the satellite is increased to 4 times the previous value, the new period will be
   (a) 10 h (b) 80 h
   (c) 40 h (d) 20 h **[2003]**
8. Two spherical bodies of mass M and 5M and radii R and 2R respectively are released in free space with initial separation between their centres equal to 12R. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is
   (a) 2.5 R (b) 4.5 R
   (c) 7.5 R (d) 1.5 R **[2003]**
9. The escape velocity of a body projected vertically upwards from the surface of the earth is 11 km s^{-1}. If the body is projected at angle 45° with the vertical, the escape velocity will be
   (a) $11\sqrt{2}$ km s^{-1} (b) 22 km s^{-1}
   (c) 11 km s^{-1} (d) $\frac{11}{\sqrt{2}}$ km s^{-1} **[2003]**
10. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is

(a) gx (b) $\frac{gR}{(R-x)}$

(c) $\frac{gR^2}{(R+x)}$ (d) $\left[\frac{gR^2}{(R+x)}\right]^{1/2}$ **[2004]**

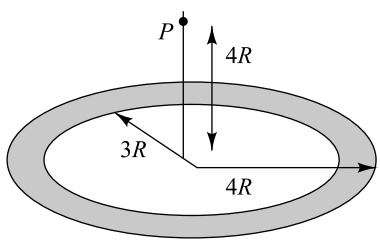
11. The time period of an earth satellite in a circular orbit is independent of
    (a) the mass of the satellite
    (b) the radius of the orbit
    (c) both the mass of the satellite and the radius of the orbit
    (d) neither the mass of the satellite nor the radius of the orbit **[2004]**
12. If g is the acceleration due to gravity on earth's surface, the gain in potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth is
    (a) $2\ mgR$ (b) $\frac{1}{2}mgR$
    (c) $\frac{1}{4}mgR$ (d) mgR **[2004]**
13. Suppose the gravitational force varies inversely as the n^{th} power of distance, then the time period of a planet in circular orbit of radius R around the sun will be proportional to
    (a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$
    (c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$ **[2004]**
14. The average density of the earth
    (a) does not depend on the g
    (b) is a complex function of g
    (c) is directly proportional to g
    (d) is inversely proportional to g **[2005]**
15. The change in the value of g at a height h above the surface of the earth is the same as that at a depth d below the surface of the earth. If both h and d are much smaller than the radius of the earth, then which of the following is correct?
    (a) $d = \frac{h}{2}$ (b) $d = \frac{3h}{2}$
    (c) $d = 2h$ (d) $d = h$ **[2005]**
16. A particle of mass 10 g is kept on the surface of sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere. Take $G = 6.67 \times 10^{-11}\ \text{Nm}^2\ \text{Kg}^{-2}$
    (a) 13.34×10^{-10}J (b) 3.33×10^{-10}J
    (c) 6.67×10^{-10}J (d) 6.67×10^{-9}J **[2005]**
17. If g_E and g_M are the acceleration due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio to $\frac{\text{electonic charge on the moon}}{\text{electronic charge on the earth}}$ be
    (a) 1 (b) zero
    (c) $\frac{g_E}{g_M}$ (d) $\frac{g_M}{g_E}$ **[2007]**
18. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.
    Statement-1:
    For a mass M kept at the centre of a cube of side 'a', the flux of gravitational field passing through its sides is $4\pi GM$.
    And
    Statement-2:
    If the direction of a field due to a point source is radial and its dependence on the distance 'r' from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.
    (a) Statement-1 is true, Statement-2 is false
    (b) Statement-1 is false, Statement-2 is true
    (c) Statement-1 is true, Statement-2 is true; Statement-2 is correct explanation for Statement-1.
    (d) Statement-1 is true, Statement-2 is true; Statement-2 is not correct explanation for Statement-1.
    [2008]
19. A planet in a distant solar system is 10 time more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be
    (a) 0.11 km s^{-1} (b) 1.1 km s^{-1}
    (c) 11 km s^{-1} (d) 110 km s^{-1} **[2008]**
20. The height at which acceleration due to gravity becomes $g/9$ (where g = the acceleration due to gravity on the surface of the earth) in terms or R, the radius of the earth is
    (a) $\frac{R}{2}$ (b) $\sqrt{2}R$
    (c) $2\ R$ (d) $\frac{r}{\sqrt{2}}$ **[2009]**

21. A spherically symmetric gravitational system of particles has a mass density $\rho = \begin{cases} \rho_0 \text{ for } r \le R \\ 0 \text{ for } r > R \end{cases}$ where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by



[2009]

22. A thin uniform annular disc (see the figure) of mass M has outer radius 4 R and inner radius 3 R. The work required to take a unit mass from point P on its axis to infinity is



(a) $\frac{2GM}{7R}(4\sqrt{2}-5)$ (b) $-\frac{2GM}{7R}(4\sqrt{2}-5)$

(c) $\frac{GM}{4R}$ (d) $\frac{2GM}{5R}(\sqrt{2}-1)$

[2010]

23. Two bodies of masses m and $4m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

(a) zero (b) $-\frac{4Gm}{r}$

(c) $-\frac{6Gm}{r}$ (d) $-\frac{9Gm}{r}$ **[2011]**

24. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be

(a) 6.4×10^{11} Joules (b) 6.4×10^{8} Joules

(c) 6.4×10^{9} Joules (d) 6.4×10^{10} Joules

[2012]

25. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$.

(a) $\frac{2GmM}{3R}$ (b) $\frac{GmM}{2R}$

(c) $\frac{GmM}{3R}$ (d) $\frac{5GmM}{6R}$ **[2013]**

26. A planet of radius $R = \frac{1}{10} \times$ (radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = 6×10^6 m and the acceleration due to gravity of Earth is 10 ms^{-2})

(a) 96 N (b) 108 N

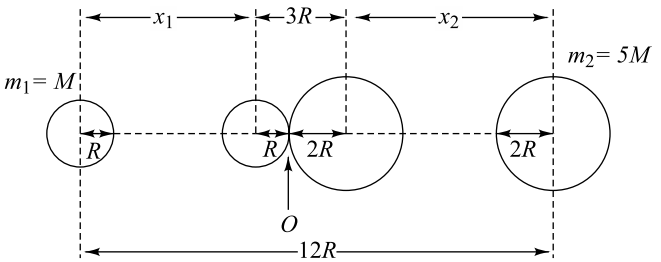
(c) 120 N (d) 150 N **[2014]**

Answers

1. (b)	2. (a)	3. (a)	4. (c)
5. (b)	6. (c)	7. (c)	8. (c)
9. (c)	10. (d)	11. (a)	12. (b)
13. (a)	14. (c)	15. (c)	16. (c)
17. (a)	18. (c)	19. (d)	20. (c)
21. (c)	22. (a)	23. (d)	24. (d)
25. (d)	26. (b)		

Solutions

1. If the gravitational force becomes zero. then there will be no force to provide the necessary centripetal force for the circular motion. Hence the satellite move along the tangent to a point in the orbit when the gravitational force vanishes.
2. Total energy of the body orbiting the earth is
$$E = -\frac{GmM}{2r}$$
where m = mass of body, M = mass of earth and r = radius of the orbit
At $r = 2R$, $\quad E_1 = -\dfrac{GmM}{4r}$
At $r = 3R$, $\quad E_2 = -\dfrac{GmM}{6r}$
$\therefore$ Energy required is
$$E = E_2 - E_1 = -\frac{GmM}{6R} - \left(-\frac{GmM}{4R}\right) = \frac{GmM}{12R}$$
So, the correct choice is (a)
3. Escape velocity $v_e = \sqrt{2gR}$ which is independent of the mass of the body. So the correct choice is (a)
4. The minimum kinetic energy required is
$$\text{K. E} = \frac{1}{2}\,\text{m}v_e^2$$
where v_e is the escape velocity, Now $v_e = \sqrt{2gR}$.
Hence $\quad \text{K. E} = \dfrac{1}{2}\, m \times 2gR = mgR$
5. Let x be the extension in the spring when it is loaded with mass M. The change in gravitational potential energy = Mgx. This must be the energy stored in the spring which is given by $\frac{1}{2}kx^2$. Thus $\frac{1}{2}kx^2 = Mgx$ or $x = \dfrac{2Mg}{k}$
6. $T \propto r^{3/2}$. Therefore,
$$\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} \qquad (1)$$
Given r_2 = 6400 km and r_1 = 36000 km. For a geostationary satellite T_1 = 24 h. Using these values in (1), we have get $T_2 = 24 \times \left(\dfrac{64}{360}\right)^{3/2}$ =1.8. h.
7. According to Kepler's law of periods $T^2 \propto r^3$. Therefore
$$\frac{T_2^2}{T_1^2} = \left(\frac{r_2}{r_1}\right)^3 = (4)^3 = 64$$
$\therefore \quad T_2 = T_1\sqrt{64} = 8\ T_1 = 8 \times 5\ \text{h} = 40\ \text{h}$
8. Suppose the two bodies collide at O. If the smaller body $m_1 = M$ moves a distance x_1 to reach O, then the bigger body moves a distance $x_2 = (9R - x_1)$ to reach O.



Since no net external force acts, the acceleration of the centre of mass the system is zero, i.e. $a_{CM} = 0$. Now
$$a_{CM} = \frac{m_1a_1 + m_2a_2}{m_1 + m_2}$$
Putting $a_{CM} = 0$, $m_1 = M$, and $m_2 = 5\ M$ we get
$$Ma_1 + 5Ma_2 = 0 \quad \Rightarrow\ a_1 = -5\ a_2$$
The negative sign shows that the two bodies move in opposite directions. If t is the time after which they collide,
$$x_1 = \frac{1}{2}a_1t^2 \quad \text{and} \quad x_2 = \frac{1}{2}a_2t^2$$
Dividing we get
$$x_1 = \frac{a_1}{a_2} = \frac{|5a_2|}{a_2} = 5 \quad \Rightarrow \quad x_1 = 5\ x_2$$
or $\quad x_1 = 5 \times (9R - x_1) \quad \Rightarrow \quad x_1 = 7.5\ R$
9. The escape velocity is independent of the angle of projection. Hence it will remain equal to km s^{-1}.
10. Radius of circular orbit is $r = R + x$. If v is the orbital speed, then
centripetal force = gravitational force
or $\quad \dfrac{mv^2}{r} = \dfrac{GmM}{r^2}$
$$\Rightarrow \quad v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{(R+x)}} \qquad \left(\because g = \frac{GM}{r^2}\right)$$
Notice that choices (a), (b) and (c) do not give the dimensions of speed.
11. The time period of the satellite is given by
$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$
where r = radius of orbit and M = mass of earth. Thus the time period is independent of the mass of the satellite.

12. Gain in P.E. $= -\dfrac{GmM}{(R+h)} - \left(\dfrac{GmM}{R}\right)$

$$= -\frac{GmM}{2R} + \frac{GmM}{R} \qquad (\because\ h = R)$$

$$= \frac{GmM}{2R} = \frac{1}{2}mgR \qquad \left(\because\ g = \frac{GM}{R^2}\right)$$

13. The necessary centripetal force for the circular motion of the planet around the sun is provided by the gravitational force exerted by the sun, i.e.

$$\frac{mv^2}{R} = \frac{GmM}{R^n} \Rightarrow v = \left(\frac{Gm}{R^{(n-1)}}\right)^{1/2}$$

where M = Mass of sun and m = mass of planet. Now the time period is

$$T = \frac{2\pi R}{v} = 2\pi R \times \left(\frac{R^{(n-1)}}{GM}\right)^{1/2} = \frac{2\pi}{\sqrt{Gm}} \times \left[R^{\frac{(n+1)}{2}}\right]^{1/2}$$

$$T \propto R^{\frac{(n+1)}{2}}$$

14. $\rho = \dfrac{M}{V} \Rightarrow M = \rho V = \rho \times \dfrac{4\pi}{3} R^3$

Now $\quad g = \dfrac{GM}{R^2} = \dfrac{G}{R^2} \times \dfrac{4\pi}{3} R^3 \rho = \dfrac{4\pi}{3} G\rho R$

$\Rightarrow \quad \rho = \dfrac{3g}{4\pi GR}$. Hence $\rho \propto g$

15. $g_h = g\left(1 - \dfrac{2h}{r}\right) \quad \Rightarrow \quad g - g_h = \dfrac{2hg}{R}$

$g_d = g\left(1 - \dfrac{d}{r}\right) \quad \Rightarrow \quad g - g_d = \dfrac{gd}{R}$

Given $g - g_h = g - g_d \ \Rightarrow \ \dfrac{2hg}{R} = \dfrac{gd}{R} \ \Rightarrow \ d = 2h$

16. Mass of particle m = 10 g = 10^{-2} kg, mass of sphere M = 100 kg and radius of sphere R = 10 cm = 0.1 m. The work needed to take the particle from $r = R$ to $r = \infty$ is given by

$W = \int_R^\infty \mathbf{F} \cdot d\mathbf{r} = \int_R^\infty F dr \cos 180°$ ($\because$ Force and displacement are in opposite directions.)

$$= -\int_R^\infty F dr = -\int_R^\infty \frac{GmM}{r^2} dr$$

$$= -GmM \int_R^\infty r^{-2} dr$$

$$= \frac{GmM}{R} = \frac{6.67 \times 10^{-11} \times 10^{-2} \times 100}{0.1} = 6.67 \times 10^{-10}\ \text{J}$$

17. The charge of an electron is the same anywhere in the universe. Hence t he correct choice is (a).

18. Statement-2 is Gauss's theorem in gravitation. Gauss's theorem holds for any fields which obeys $1/r^2$ dependence. Just as electric field intensity due to a charge Q at a distance r from it is given by

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

The gravitational field intensity due to a mass M at at a distance r from it is given by

$$I = \frac{GM}{r^2}$$

The mass M plays the same role in gravitation as charge Q does in electrostatics. Further constant G is analogous to constant $1/4\pi\varepsilon_0$. From Gauss's theorem in electrostatics, electric flus through a closed surface is given by

$$\phi_e = \frac{Q}{\varepsilon_0}$$

where Q is the charge enclosed in the surface.

Replaced Q by M, and ε_0 by $1/4\pi G$, the gravitational flux through a closed surface will be

$$\phi_g = \frac{M}{1/4\pi G} = 4\pi GM$$

where M is the mass enclosed in the surface. Hence statements 1 and 2 are true and Statement 2 is the correct explanation for Statement-1. So the correct choice is (c).

19. $v_e = \sqrt{\dfrac{GM_e}{R_e}}$ and $v_p = \sqrt{\dfrac{GM_p}{R_p}}$

$$\therefore \qquad v_\text{p} = v_\text{e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}}$$

$$= (11\ \text{km s}^{-1}) \times \sqrt{10 \times 10}$$

$$= 110\ \text{km s}^{-1}$$

20. $g_h = g\left(\dfrac{R}{R+h}\right)^2$

$$\frac{g}{9} = g\left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \qquad \frac{1}{3} = \frac{R}{R+h} \qquad \Rightarrow \qquad h = 2R$$

21. If M is the total mass of the system of particles, the orbital speed of the test mass is

$$v = \sqrt{\frac{GM}{r}}$$

For $r \le R$, $v = \sqrt{\dfrac{GM \times \frac{4\pi}{3} r^3 \rho_0}{r}}$ which gives $v \propto r$,

i.e. v increases linearly with r up to $r = R$. Hence choices (b) and (d) are wrong.

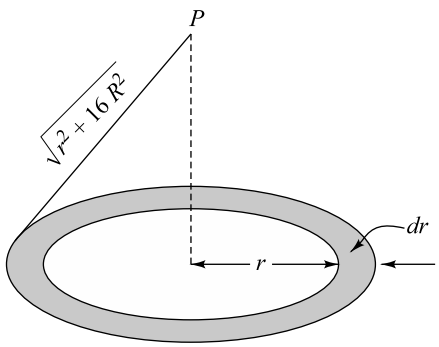
For $r > R$, the whole mass of the system is

$M = \dfrac{4\pi}{3} R^3 \rho_0$, which is constant. Hence for $r > R$.

$$v = \sqrt{\frac{GM}{r}}$$

i.e. $v \propto \dfrac{1}{\sqrt{r}}$. Hence the correct choice is (c).

22. By definition, the work, required to take a unit mass from P to infinity $= -V_p$, where V_p is the gravitational potential at P due to the disc. To find V_p, we divide the disc into small elements, each of thickness dr. Consider one such element at a distance r from the centre of the disc, as shown in the figure.



Mass of the element, $dm = \dfrac{M(2\pi r dr)}{\pi(4R)^2 - \pi(3R)^2}$

or $$dm = \frac{2Mrdr}{7R^2}$$

$$V_p = \int_{3R}^{4R} \frac{Gdm}{\sqrt{r^2 + 16r^2}}$$

$$= -\frac{2MG}{7R^2} \int_{3R}^{4R} \frac{rdr}{(r^2 + 16r^2)^{1/2}}$$

Putting $r^2 + 16\,R^2 = x^2$, we get $2\,rdr = 2x\,dx$ or $rdr = x\,dx$.

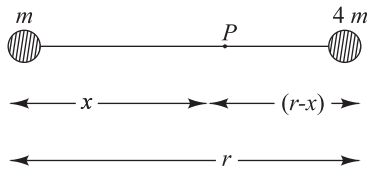
When $r = 3\,R$, $x = \sqrt{9R^2 + 16R^2} = 5R$

When $r = 4\,R$, $x = \sqrt{16R^2 + 16R^2} = 4\sqrt{2}\,R$

$$\therefore\ V_p = -\frac{2MG}{7R^2} \int_{5R}^{4\sqrt{2}R} dx = -\frac{2MG}{7R^2}(4\sqrt{2} - 5)R$$

Hence $-V_p = \dfrac{2GM}{7R}(4\sqrt{2} - 5)$, which is choice (a).

23. Let P be the point where the gravitational field is zero.



Then $$\frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$$

$$\Rightarrow \quad \frac{1}{x^2} = \frac{4}{(r-x)^2}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{4}{r-x} \quad \Rightarrow \quad x = \frac{r}{3}$$

Gravitational potential at P is

$$V = V_1 + V_2$$

$$= -\frac{Gm}{x} - \frac{G(4m)}{(r-x)}$$

$$= -\frac{Gm}{r/3} - \frac{4Gm}{2r/3} = -\frac{9Gm}{r}$$

24. $E = 0 - \left(-\dfrac{GmM}{R}\right)$

$$= \frac{GmM}{R}$$

$$= mgr \qquad \left(\because g = \frac{GM}{R^2}\right)$$

$$= 1000 \times 10 \times 6400 \times 10^3$$

$$= 6.4 \times 10^{10}\ \text{J}$$

25. The speed of the satellite in its circular orbit of radius r is

$$v = \sqrt{\frac{Gm}{r}} = \sqrt{\frac{Gm}{R+h}} = \sqrt{\frac{Gm}{3R}} \qquad (\because h = 2R)$$

Total energy of the satellite in its circular orbit is

$$E_1 = \text{K.E} + \text{P.E}$$

$$= \frac{1}{2}mv^2 - \frac{GmM}{r}$$

$$= \frac{1}{2}m \times \frac{GM}{3R} - \frac{GmM}{3R} = -\frac{GmM}{6R}$$

Total energy when the satellite was on the launching pad is

$$E_2 = \text{K.E} + \text{P.E}$$

$$= 0 - \frac{GmM}{R} = -\frac{GmM}{R}$$

$\therefore$ Minimum energy required is

$$E_{\min} = E_1 - E_2 = -\frac{GmM}{6R} - \left(\frac{-GmM}{R}\right) = \frac{5\,GmM}{6R}$$

26. At a depth r below the surface of a planet,

$$g_r = \frac{4\pi G\rho r}{3}$$

If F is the force needed to keep the wire at rest, then

F = weight of the wire = mg_r

$$\therefore \quad F = \int_{\frac{4R}{5}}^{R} (\lambda dr)\left(\frac{4\pi G\rho r}{3}\right)$$

$$= \frac{4\pi G\rho r}{3}\left|\frac{r^2}{2}\right|_{\frac{4R}{5}}^{R}$$

$$\Rightarrow \quad F = \frac{4\pi G\rho r}{3} \times \frac{9R^2}{50} \tag{1}$$

On the surface of the earth,

$$g_e = \frac{GM_e}{R_e^2} \Rightarrow G = \frac{g_e R_e^2}{M_e} \tag{2}$$

Also $$\rho = \frac{M_e}{\frac{4\pi}{3}R_e^3} \tag{3}$$

Given $$R = \frac{R_e}{10} \tag{4}$$

Using (2), (3) and (4) in (1), we get

$$F = \frac{9g_e R_e \lambda}{5\times 10^3}$$

$$= \frac{9\times 10\times (6\times 10^6)\times 10^{-3}}{5\times 10^{+3}} = 108\,\text{N}$$

SOLIDS AND FLUIDS

Chapter 7

REVIEW OF BASIC CONCEPTS

PART (A) Fluids

1. Fluid Pressure

Pressure is defined as the force exerted normally on a unit area of the surface of a fluid and is given by

$$P = \frac{F}{A}$$

In the SI system, the unit of pressure is newton per square metre (Nm^{-2}) which is also called Pascal (Pa). Thus

$$1 \text{ Pa} = 1 \text{ Nm}^{-2}$$

2. Pascal's Law

Blaise Pascal (1623–1662), a French scientist, discovered a principle which tells us how force (or pressure) can be transmitted in a fluid. Pascal's law states that *pressure in a fluid in equilibrium is the same eveywhere* (if the effect of gravity can be neglected).

The pressure difference between two points in a fluid is either zero (if they are at the same horizontal level) or is a definite quantity depending on their height difference. So if the pressure at some point in a fluid is changed, there will be an equal change in pressure at any other point. Thus fluids, especially liquids (because they are incompressible) are ideal for transmitting pressure. This fact is used in hydraulic machines, such as hydraulic press, hydraulic brakes, hydraulic jacks, etc.

3. Density and Relative Density

The density of a substance is defined as the mass per unit volume of the substance. The SI unit of density is kilogram per cubic metre (kg m^{-3}).

$$1 \text{ g cm}^{-3} = 1000 \text{ kg m}^{-3}$$

The relative density of a substance is the ratio of its density to that of water, i.e.

$$\text{Relative density} = \frac{\text{density of the substance}}{\text{density of water}}$$

Being a ratio of two similar quantities, relative density is just a number; it has no units.

4. Atmospheric Pressure

Like all gases, air also has weight and hence exerts pressure. Just as water pressure is caused by the weight of water, the weight of all the air above the earth causes an atmospheric pressure. The atmosphere exerts this pressure not only on the earth's surface, but also on the surface of all objects on the earth including living beings.

The atmospheric pressure at sea level is

$$P_0 = 1.01 \times 10^5 \text{ Pa}$$

5. Hydrostatic Pressure

Pressure exists everywhere within a fluid. The hydrostatic pressure at a depth h below the surface of a fluid is given by

$$P = h \rho g$$

where ρ is the density of the fluid and g, the acceleration due to gravity. The pressure is the same at all points at the same horizontal level. The pressure at any point in a fluid contained in a vessel is independent of the shape or size of the vessel.

6. Gauge Pressure

The pressure at any point in a fluid is equal to the sum of the atmospheric pressure P_0 acting on its surface and the hydrostatic pressure $h\rho g$ due to the weight of the fluid above that point which is at a depth h below the surface of the fluid. This is called the gauge pressure which is given by

$$P = P_0 + h\rho g$$

Since liquids are incompressible, the density of a liquid is constant throughout the liquid.

EXAMPLE 1 A large tank with a square base of side 1.0 m is divided into two compartments by a vertical partition in the middle as shown in Fig. 7.1. There is a small hinged door of size 2.0 cm × 2.5 cm at the bottom of partition.Water is filled in one compartment and oil of relative density 1.5 in the other both to the same height h = 2.0 m. Find the force necessary to keep the door closed.

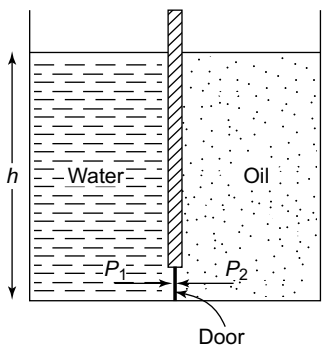


Fig. 7.1

SOLUTION Density of water (ρ_1) = 1000 kg m^{-3}, density of oil (ρ_2) = 1500 kgm^{-3} and area of the door (A) = 2.0 × 2.5 = 5 cm^2 = 5 × 10^{-4} m^2.

Lateral pressure by water on the face of the door is

$$p_1 = h\rho_1 g$$

Total pressure $P_1 = P_0 + h\rho_1 g$

(P_0 = atmospheric pressure)

Total pressure on the door due to oil is

$$P_2 = P_0 + h\rho_2 g$$

$\therefore$ Net pressure difference = $P_2 - P_1 = (\rho_2 - \rho_1)\, hg$

Net force = $(P_2 - P_1)\, A = (\rho_2 - \rho_1)\, hgA$

$= (1500 - 1000) \times 2.0 \times 9.8 \times 5 \times 10^{-4}$

= 4.9 N

EXAMPLE 2 A horizontal tube OP of length L and of uniform cross-sectional area A is open at end O and has a small hole at the other end P. The tube is filled with a liquid of density ρ and then rotated about the axis passing through O with an angular velocity ω. Find the pressure exerted by the liquid at end P at the instant when length $L/2$ of the liquid is left in the tube.

SOLUTION Consider a small element of the liquid of length dr at a distance r from O (Fig. 7.2). Mass of element is $m = A\rho dr$.

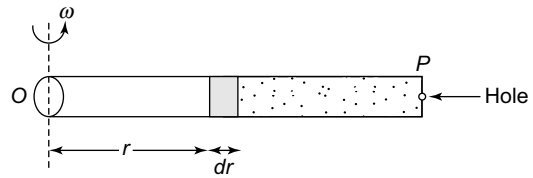


Fig. 7.2

Outwards force (centrifugal force) acting on the element is

$$dF = mr\omega^2 = A\rho\omega^2\, rdr$$

Integrating from $r = L/2$ to $r = L$, we have

$$F = A\rho\omega^2 \int_{L/2}^{L} r\, dr = A\rho\omega^2 \left|\frac{r^2}{2}\right|_{L/2}^{L}$$

$$= \frac{3}{8} A\rho\omega^2 L^2$$

$\therefore$ Pressure at $P = \dfrac{F}{A} = \dfrac{3}{8}\rho\omega^2 L^2$

EXAMPLE 3 A U-tube contains mercury in both sides of its arms. A glycerine column of length 10 cm is introduced in one of the arms. Oil is poured in the other arm until the upper surfaces of oil and glycerine are at the same horizontal level. Find the length of the oil column. Given density of glycerine = 1300 kgm^{-3}, density of oil = 800 kg m^{-3}, density of mercury = 13600 kg m^{-3}.

SOLUTION Since glycerine is denser than oil, the level of mercury in the left arm will be higher than that in the right arm (Fig. 7.3).

Pressure at $A = P_0 + h_0\rho_0 g + h_m\rho_m g$ (i)

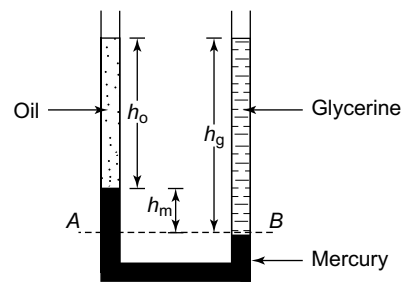


Fig. 7.3

Pressure at $B = P_0 + h_g\rho_g g$ (ii)

Since pressure at the same horizontal level is the same (Pascal's law), equating (i) and (ii) we get

$$h_0\rho_0 + h_m\rho_m = h_g\rho_g$$

$$\Rightarrow \quad h_0\rho_0 + (h_g - h_0)\rho_m = h_g\rho_g$$

$$\Rightarrow \quad h_0 \times 800 + (0.1 - h_0) \times 13600$$

$$= 0.1 \times 1300 (\because h_g = 10 \text{ cm} = 0.1 \text{ m})$$

$$\Rightarrow \quad h_0 = 0.096 \text{ m} = 9.6 \text{ cm}$$

EXAMPLE 4 A cylindrical vessel is filled to a height h by a liquid of density ρ. A tight-fitting frictionless piston of mass m is placed on the liquid as shown in Fig. 7.4. If P_0 is the atmospheric pressure and a the cross-sectional area of the piston, the pressure P of the liquid just below the piston is

(a) P_0 (b) $P_0 - \frac{mg}{a}$

(c) $P_0 + \frac{mg}{a}$ (d) $P_0 - h\rho g + \frac{mg}{a}$

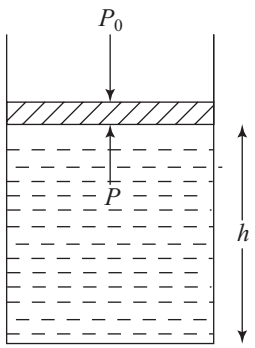


Fig. 7.4

SOLUTION The forces acting on the piston are

(i) force $P_0 a$ due to atmospheric pressure acting downwards

(ii) weight mg of the piston acting downwards

(iii) force Pa due to liquid below the piston.

Since the piston is in equilibrium,

$$Pa = P_0 a + mg$$

$$\Rightarrow \quad P = P_0 + \frac{mg}{a}$$

7. Pressure Difference in an Accelerated Liquid

Consider a liquid of density ρ in a container. If the container is given an acceleration a, say, to the right, the liquid miniscus will no longer remain horizontal, it will be inclined at an angle θ as shown in Fig. 7.5.

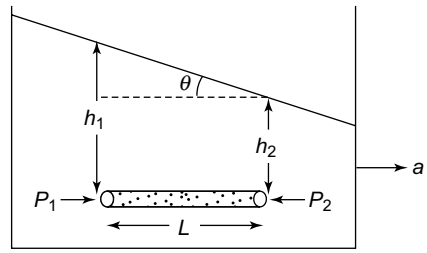


Fig. 7.5

Consider a small element (shown shaded) of length L of the liquid. Let ΔA be the cross-sectional area of the element. The mass of the element is $m = \rho L \Delta A$. When an acceleration a is given as shown, the equation motion of the element is

$$P_1 \Delta A - P_2 \Delta A = ma = (\rho L \Delta A)\, a$$

where P_1 and P_2 are the pressures at the ends of the element.

Thus $$P_1 - P_2 = \rho L a \qquad \text{(i)}$$

$$\Rightarrow \quad (h_1 - h_2)\,\rho = \rho L a$$

$$\Rightarrow \quad h_1 - h_2 = \frac{La}{g}$$

It follows from (i) that $P_1 > P_2$.

$$\tan\theta = \frac{h_1 - h_2}{L} = \frac{a}{g}$$

EXAMPLE 5 A liquid is contained in a rectangular vessel fastened on a cart. A constant force is applied to the cart. As a result, the level becomes inclined at an angle of 30° with the horizontal. What acceleration is produced by the force?

SOLUTION $$\tan\theta = \frac{a}{g}$$

$$\Rightarrow \quad \tan 30° = \frac{a}{g} \Rightarrow a = 9.8 \times \tan 30° = \frac{9.8}{\sqrt{3}}$$

$$= 5.66 \text{ ms}^{-2}$$

EXAMPLE 6 A uniform U-tube containing a liquid of density ρ is moved with a horizontal acceleration a as shown in Fig. 7.6(a). The length of liquid column PQ is L. The difference $(h_1 - h_2)$ in the liquid columns will be equal to

(a) $\frac{La}{g}$ (b) $\frac{Lg}{a}$

(c) $L\sqrt{\frac{a}{g}}$ (d) $L\sqrt{\frac{g}{a}}$

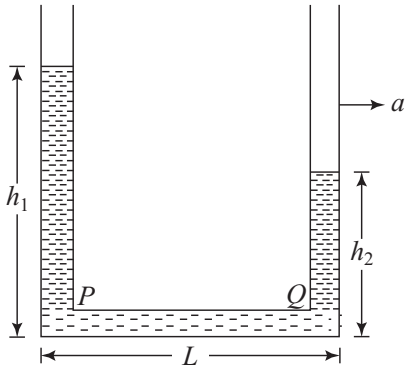


Fig. 7.6(a)

SOLUTION Let A be the cross-sectional area of the U - tube. Figure 7.6(b) shows the forces acting on column PQ

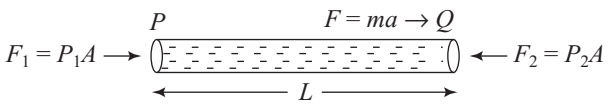


Fig. 7.6(b)

Mass of liquid in PQ is $m = \rho LA$. Force on liquid in PQ due to acceleration is $F = ma = (\rho LA)a$.

Force at P due to column h_1 is $F_1 = P_1A = \rho g h_1 A$

Force at Q due to column h_2 is $F_2 = P_2A = \rho g h_2 A$

The equation of motion is

$$\Rightarrow \quad F_1 - F_2 = F$$

$$\Rightarrow \quad \rho g\, h_1 A - \rho g\, h_2\, A = (\rho LA)\, a$$

$$\Rightarrow \quad h_1 - h_2 = \frac{La}{g}$$

8. Archimedes' Principle

Archimedes' principle states as follows:

'When a solid body is wholly or partly immersed in a fluid, it experiences an upward thrust or buoyant force equal to the weight of the fluid displaced by it.' The word 'fluid' includes both liquids and gases. The principle is a general one and holds for solids of any shape and for all fluids.

Law of Floatation The necessary condition for a body to float in a fluid is that the weight of the fluid displaced by it must be equal to the weight of the body. This is the law of floatation.

9. Applications

1. Buoyant force = weight of the liquid displaced by the immersed portion of the body = $V\rho g$ where V is the volume immersed, ρ is the density of the liquid.

   Case (a). If the density (σ) of the body is less than that of the liquid ($\sigma < \rho$). Then the body will float with volume V_0 outside liquid and V_i inside liquid such that

   $$V_i \rho g = V \sigma g$$

   $$\Rightarrow \quad \frac{V_i}{V} = \frac{\sigma}{\rho} \text{ and } \frac{V_0}{V} = \frac{\rho - \sigma}{\sigma}$$

   Case (b). If $\sigma = \rho$, the body will float completely immersed, i.e. $V_0 = 0$.

   Case (c). If $\sigma > \rho$, the body will sink to the bottom.

2. Tension T in the string tied to a body immersed in a liquid ($\sigma > \rho$).

   Case (a) The system is at rest (Fig. 7.7).

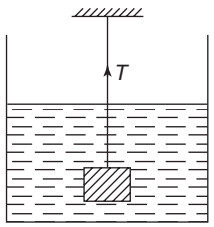


Fig. 7.7

$$T = V \sigma\, g - V\rho g = V\,(\sigma - \rho)\, g$$

   Case (b) When the system is accelerating, then

$$T' = V\,(\sigma - \rho)\, g'$$

   (i) If the system moves up with acceleration a,

$$g' = g + a \text{ and } T' = V\,(\sigma - \rho)\,(g + a)$$

   (ii) If the system moves down with acceleration $a\ (< g)$

$$g' = g - a \text{ and } T' = V\,(\sigma - \rho)\,(g - a)$$

   (iii) If the system falls freely, $a = g$ and $T' = 0$

3. $\sigma < \rho$ and the body is kept immersed in a liquid by a string fixed at the bottom of the beaker (Fig. 7.8).

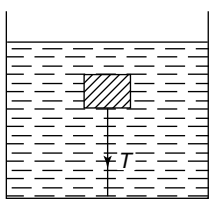


Fig. 7.8

   (i) If the system is at rest,

$$T = V\,(\rho - \sigma)\, g$$

   (ii) If the system moves up with acceleration a,

$$T' = V\,(\rho - \sigma)\, g' \text{ where } g' = (g + a)$$

(iii) If the system moves down with acceleration a ($< g$)

$$T' = V(\rho - \sigma)\, g' \text{ where } g' = (g - a)$$

(iv) If the system falls freely, $T' = 0$

4. A toy boat carrying an object is floating in water in a beaker. The object is droped into water.
   (a) If the object is made of wood (of density less than that of water), it will float and the level of water in the beaker remains unchanged.
   (b) If object is denser than water, it will sink and the water level will fall.
5. A piece of ice is floating in a liquid. If the ice melts, the level of water
   (a) remains unchanged if the liquid is water,
   (b) rises if the relative density if the liquid is greater than I,
   (c) falls if the relative density of the liquid is less than 1.
6. A piece of ice with an object embedded in it is floating in water. If all the ice melts, the water level.
   (a) falls if the object sinks in water
   (b) remains unchanged if the object floats in water.
7. An object of density ρ and volume V floats at the interface of two liquids 1 and 2 of densities ρ_1 and ρ_2 with volume V_1 in liquid 1 and V_2 in liquid 2 (Fig. 7.9) Then, for equilibrium ($\rho_1 < \rho < \rho_2$)

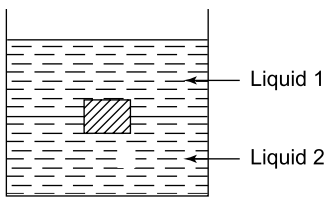


Fig. 7.9

Weight of body = buoyant force

i.e. $V\rho g = V_1 \rho_1 g + V_2 \rho_2 g$ (i)

Also $V_1 + V_2 = V$ (ii)

Eqs (i) and (ii) give

$$V_1 = \frac{V(\rho_2 - \rho)}{(\rho_2 - \rho_1)},\ V_2 = \frac{V(\rho - \rho_1)}{(\rho_2 - \rho_1)}$$

and $$\frac{V_2}{V_1} = \frac{(\rho - \rho_1)}{(\rho_2 - \rho)}$$

EXAMPLE 7 A cubical metal block of edge 5 cm is suspended from a support by a massless string and immersed in water in a beaker as shown in Fig. 7.10(a). The relative density of metal is 7. Find the tension in the string when the whole system

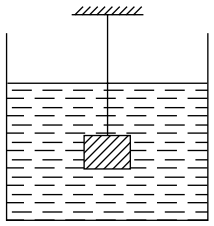


Fig. 7.10(a)

(a) is at rest
(b) moves up with an acceleration of 2 ms^{-2}
(c) moves down with an acceleration of 2 ms^{-2}.
Take $g = 10$ ms^{-2}.

SOLUTION

Density of water (ρ) = 1000 kg m^{-3}

Density of block (σ) = 7000 kg m^{-3}

Volume of block (V) = 5 cm × 5 cm × 5 cm = 125 cm^3 = 1.25×10^{-4} m^3

Weight of block $W = mg = \sigma Vg$

Upthrust U = weight of water displaced = ρVg

Figure 7.10(b) shows the free body diagrams.

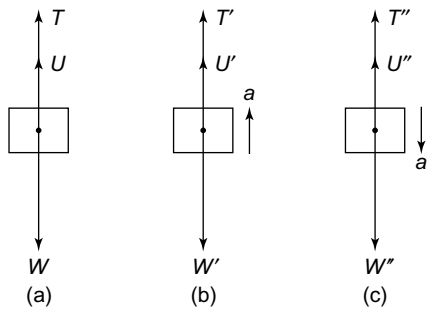


Fig. 7.10(b)

(a) From Fig. 7.9(a)

$$T + U = W$$

$$\Rightarrow \quad T = W - U = \sigma Vg - \rho Vg$$

$$= (\sigma - \rho)\, Vg$$

$$= (7000 - 1000) \times (1.25 \times 10^{-4}) \times 10$$

$$= 7.5 \text{ N}$$

(b) In this case $g_{eff} = g + a$ [Fig. 7.9(b)]

$$\therefore \quad T' = W' - U' = \sigma V g_{eff} - \rho V g_{eff}$$

$$= (\sigma - \rho)\, V(g + a)$$

$$= (7000 - 100) \times (1.25 \times 10^{-4}) \times (10 + 2)$$

$$= 9.0 \text{ N}$$

(c) In this case $g_{eff} = g - a$ [Fig. 7.9(c)]

$$\therefore \quad T'' = (\sigma - \rho)\, V\, (g - a)$$
$$= (7000 - 1000) \times (1.25 \times 10^{-4}) \times (10 - 2)$$
$$= 6.0 \text{ N}$$

EXAMPLE 8 A solid block of density ρ_1 is held inside a liquid of density ρ_2 (with $\rho_1 < \rho_2$) by means of a string fixed to the bottom of the beaker as shown in Fig. 7.11(a). The system is placed at the floor of a lift. If the lift is at rest, the tension in the string is T. The tension in the string when the lift moves upwards with an acceleration a will be

(a) T (b) $\dfrac{Tg}{a}$

(c) $T\left(1 + \dfrac{a}{g}\right)$ (d) $T\left(1 - \dfrac{a}{g}\right)$

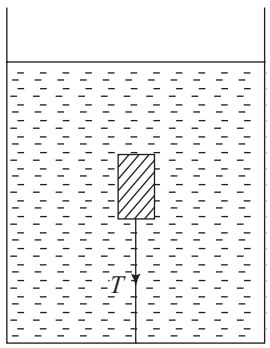


Fig. 7.11(a)

SOLUTION Let V be the volume of the block. Its mass is $m = \rho_1 V$ and its weight = $mg = \rho_1 Vg$.

Buoyant force on the block is

$$U = \text{weight of liquid displaced}$$
$$= \rho_2\, V\, g$$

Figure 7.11(b) shows the free body diagrams of the block. When the lift is at rest,

$$U = T + mg$$
$$\Rightarrow \quad \rho_2\, V\, g = T + \rho_1\, Vg$$
$$\Rightarrow \quad T = (\rho_2 - \rho_1)\, Vg \qquad \text{(i)}$$

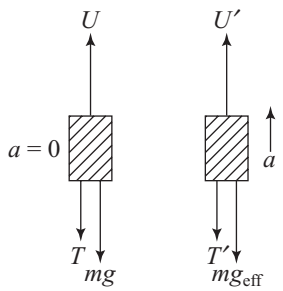


Fig. 7.11(b)

Let T' be the tension in the string when the lift moves up with acceleration a, then $g_{eff} = g + a$. Therefore, in this case

$$T' = U' - mg_{eff}$$
$$= \rho_2 V g_{eff} - \rho_1 V g_{eff}$$
$$= (\rho_2 - \rho_1) V g_{eff}$$
$$= \frac{T}{g}\, g_{eff} \qquad \text{[using Eq. (i)]}$$
$$= \frac{T}{g}(g + a) = T\left(1 + \frac{a}{g}\right)$$

EXAMPLE 9 A uniform cylinder of density σ and length L floats vertically completely immersed in two liquids with length L_1 immersed in a liquid of density ρ_1 and the remaining length L_2 immersed in the other liquid of density ρ_2 as shown in Fig. 7.12. The ratio $\dfrac{L_1}{L_2}$ will be ($\rho_1 > \rho_2$) equal to

(a) $\dfrac{\sigma - \rho_2}{\sigma - \rho_1}$ (b) $\dfrac{\sigma - \rho_1}{\rho_2 - \sigma}$

(c) $\dfrac{\sigma - \rho_2}{\rho_1 - \sigma}$ (d) $\dfrac{\sigma}{\rho_1 - \rho_2}$

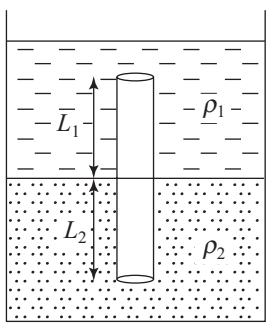


Fig. 7.12

SOLUTION Mass of the body is $m = \sigma AL$, where A is the cross-sectional area of the cylinder. Weight of cylinder is $W = mg = \sigma ALg$.

Weight of volume of liquid of density ρ_1 displaced by the length L_1 is

$$W_1 = \rho_1\, A\, L_1\, g$$

Weight of volume of liquid of density ρ_2 displaced by length L_2 is

$$W_2 = \rho_2\, A\, L_2\, g$$

According to the law of floatation, the weight of the cylinder must be equal to the total weight of the liquids displaced, i.e.

$$W = W_1 + W_2$$

or $$\sigma\, ALg = \rho_1\, A\, L_1\, g + \rho_2\, AL_2\, g$$

$$\Rightarrow \qquad \sigma L = \rho_1 L_1 + \rho_2 L_2 \qquad \text{(i)}$$

Now $L_1 + L_2 = L$ (ii)

From (ii), $L_2 = L - L_1$. Using this in Eq. (i),

$$\sigma L = \rho_1 L_1 + \rho_2 (L - L_1)$$

$$\Rightarrow \qquad L_1 = \left(\frac{\sigma - \rho_2}{\rho_1 - \rho_2}\right) L$$

Similarly, using $L_1 = L - L_2$ in Eq. (i), we get

$$L_2 = \left(\frac{\rho_1 - \sigma}{\rho_1 - \rho_2}\right) L$$

$$\therefore \qquad \frac{L_1}{L_2} = \frac{\sigma - \rho_2}{\rho_1 - \sigma}$$

EXAMPLE 10 A solid sphere of diameter D and density σ has a spherical cavity of diameter d inside it. If it just floats completely immersed in a liquid of density ρ, then the ratio D/d is

(a) $\left(\frac{\sigma}{\sigma - \rho}\right)^{1/2}$ (b) $\left(\frac{\sigma - \rho}{\sigma}\right)^{1/2}$

(c) $\left(\frac{\sigma}{\sigma - \rho}\right)^{1/3}$ (d) $\left(\frac{\sigma - \rho}{\sigma + \rho}\right)$

SOLUTION Let R be the radius of the sphere and r that of the cavity. Mass of the sphere with cavity is

$$m = \frac{4\pi}{3} R^3 \sigma - \frac{4\pi}{3} r^3 \sigma$$

$$\therefore \text{ Weight of sphere} = mg = \frac{4\pi}{3} (R^3 - r^3) \sigma g \qquad \text{(i)}$$

The volume of liquid displaced = volume of the sphere

$$= \frac{4\pi}{3} R^3$$

$$\text{Weight of liquid displaced} = \frac{4\pi}{3} R^3 \rho g \qquad \text{(ii)}$$

Equating (i) and (ii), we get

$$\frac{4\pi}{3} (R^3 - r^3) \sigma g = \frac{4\pi}{3} R^3 \rho g$$

$$\Rightarrow \qquad (R^3 - r^3)\, \sigma = R^3 \rho$$

$$\Rightarrow \qquad \frac{R^3}{r^3} = \frac{\sigma}{\sigma - \rho}$$

$$\Rightarrow \qquad \frac{R}{r} = \left(\frac{\sigma}{\sigma - \rho}\right)^{1/3}$$

$$\Rightarrow \qquad \frac{D}{d} = \frac{2R}{2r} = \left(\frac{\sigma}{\sigma - \rho}\right)^{1/3}$$

EXAMPLE 11 A cubical block of wood (density = 800 kg m^{-3}) of side 5 cm floats on the surface of water with its lower face just touching a vertical spring fixed at the bottom of the container. When a body of mass m = 75 g is placed on top of the block, it floats in water with its top face in level with water. Find the value of spring constant k. Take g = 10 ms^{-2}. [see Fig. 7.13(a)]

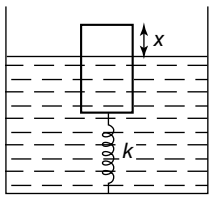


Fig. 7.13(a)

SOLUTION

Let x cm be the height of the block above the surface of water. From the law of floatation,

upthrust = weight of the block (W)

or weight of water displaced = weight of the block, i.e.

$$(5 - x) \times 10^{-2} \times (5 \times 10^{-2})^2 \times 1000\, g = (5 \times 10^{-2})^3 \times 800\, g$$

which gives x = 1 cm

When a body of mass m = 75 g = 0.075 kg is placed on the block, it depresses by x = 1cm = 1×10^{-2} m. If k is the spring constant, the force in the spring is $f = kx = 10^{-2} k$. The upthrust now is

$U = (5 \times 10^{-2})^3 \times 1000 \times g = 1.25$ N ($\because g = 10$ ms^{-2})

Figure 7.13(b) shows the free body diagram. For equilibrium

$$W + mg = U + f \qquad \text{(i)}$$

Now $W = (5 \times 10^{-2})^3 \times 800 \times 10 = 1$ N,

$f = 10^{-2} k$,

m = 0.075 kg and U = 1.25 N.

Using these values in (i),

$$1 + 0.075 \times 10 = 1.25 + 10^{-2} k$$

$$\Rightarrow \qquad k = 50 \text{ Nm}^{-1}$$

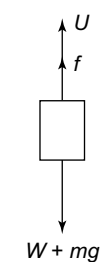


Fig. 7.13(b)

EXAMPLE 12 A cylinder of radius R and height h and mass M is suspended by a string in a liquid of density ρ where it stays vertical with its upper surface at a depth h_1 below the surface of the liquid. Find the force at the bottom of the cylinder.

SOLUTION

Pressure at top of cylinder is (Fig. 7.14)

$$P_1 = h_1 \rho g$$

Pressure at the bottom of cylinder is

$$P_2 = h_2 \rho g$$

If A is the cross-sectional area of the cylinder, the net force at the bottom is

$$F = (p_2 - p_1)\, A = (h_2 - h_1)\, \rho A g$$
$$= h \rho A g = V \rho g$$

where V = volume of cylinder.

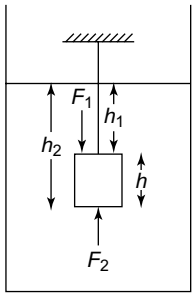


Fig. 7.14

EXAMPLE 13 In Example 9 above, what will be the force at the bottom if a hemispherical portion of radius R is removed from the bottom of the cylinder. The volume of the remaining part of the cylinder is V' and the top of the cylinder is now at a depth h' below the liquid surface as shown in Fig. 7.15.

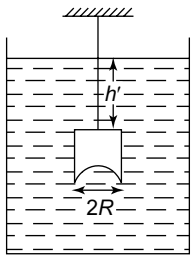


Fig. 7.15

SOLUTION From Archimedes' principle,

Force at bottom = force at top + $V' \rho g$,

$$= (h' \times \pi R^2) \times \rho g + V' \rho g$$
$$= \rho g\, [V' + \pi R^2 h']$$

EXAMPLE 14 A spring balance reads 10 kg when a bucket of water is suspended from it. What will be the reading of the spring balance when

(a) an ice cube of mass 1.5 kg is put into the bucket?

(b) an iron piece of 7.2 kg suspended from another spring is immersed with half its volume inside water in the bucket?

Relative density of iron is 7.2.

SOLUTION

(a) When an ice cube of mass 1.5 kg is put into the bucket, the total mass suspended from the spring balance = 10 kg + 1.5 kg = 11.5 kg. Hence the balance will read 11.5 kg.

(b) Density of iron = 7.2×10^3 kg m^{-3}. Therefore volume of iron piece is

$$V = \frac{\text{mass}}{\text{density}} = \frac{7.2}{7.2 \times 10^3} = 10^{-3} \text{ m}^3$$

Volume of iron immersed in water = $\frac{V}{2}$

$\therefore$ Weight of water displaced = $\frac{V}{2} \times 1000 \times g$ newton

$$= \frac{10^{-3}}{2} \times 1000 \times g = 0.5\, g$$

= weight of 0.5 kg

This is the buoyant force on the iron piece. Hence, according to Newton's third law, the iron piece will exert an equal force on water in the downward direction. Hence the balance will now read = 10 kg + 0.5 kg = 10.5 kg

10. Viscosity

When a fluid flows , there exists a relative motion between the layers of the fluid. Internal force acts which destroys this relative motion. This force is called viscous force. The viscous force between two layers of a fluid is given by

$$F = -\eta A \frac{dv}{dx}$$

Where A is the area of the layer, $\frac{dv}{dx}$ is the velocity gradient and η is called the coefficient of viscosity of the fluid.

The SI unit of η is Nsm^{-2} which is called poiseuilli (Pl) or pascal second (Pa-s). The dimensional formula of η is [ML^{-1} T^{-1}].

11. Stokes' Law

The viscous force experienced by a small spherical body of radius r moving with a small velocity v through a homogeneous fluid of coefficient of viscosity η is given by

$$F = 6 \pi \eta r v$$

This relation is called Stokes' law.

12. Terminal Velocity

If a body is released in a viscous fluid, it is accelerated due to gravity and its velocity begins to increase. Hence viscous force on it also increases. A stage is reached when the velocity is such that the viscous force F becomes equal to ($W - U$), where W is the weight of the body and U is the upthrust (Fig. 7.16). Then no net force acts on the body and it falls with a constant velocity called the terminal velocity (v_t). For a spherical body of radius r and density ρ falling in a fluid of density σ and coefficient of viscosity η, the terminal velocity is given by

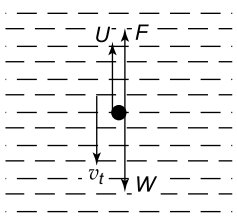


Fig. 7.16

$$v_t = \frac{2(\rho - \sigma) r^2 g}{9 \eta}$$

For a body moving in a given fluid, $v_t \propto r^2$

Note For a liquid the coefficient of viscosity decreases with increase in temperature whereas for a gas it increases with increase in temperature. Hence the terminal velocity of a body falling in a liquid increases with increase in temperature.

13. Poiseuilli's Formula

The volume of a liquid flowing per second through a capillary tube of radius r when its ends are maintained at a pressure difference p is given by

$$Q = \frac{\pi p r^4}{8\eta l}$$

Where l is the length of the tube and η is the coefficient of viscosity of the liquid.

Capillaries Connected in Series

If two capillaries of lengths l_1 and l_2 and radii r_1 and r_2 are connected in series across constant pressure difference p, then the fluid resistance R is given by

$$R = R_1 + R_2 = \frac{8\eta l_1}{\pi r_1^4} + \frac{8\eta l_2}{\pi r_2^4}$$

As the volume of liquid flowing per second is the same through both capillaries.

$$Q = Q_1 = Q_2 = \frac{p}{R_1 + R_2}$$

If p_1 and p_2 are the pressure differences across individual capillaries, then

$$p = p_1 + p_2$$

Capillaries Connected in Parallel

If two capillaries are connected in parallel across constant pressure difference p, then the fluid resistance is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \text{ or } R = \frac{R_1 R_2}{R_1 + R_2}$$

where $$R_1 = \frac{8\eta l_1}{\pi r_1^4} \text{ and } R_2 = \frac{8\eta l_2}{\pi r_2^4}$$

The volume of liquid flowing per second through the capillary of radius r_1 is

$$Q_1 = \frac{p}{R_1}$$

For the capillary of radius r_2, we have

$$Q_2 = \frac{p}{R_2}$$

EXAMPLE 15 A vertical U-tube is filled with a liquid of density $\rho = 1200$ kg m^{-3} a shown in Fig. 7.17. The tube is rotated about a vertical axis with angular velocity ω such that the difference in levels of the liquid in the two arms is 25 cm. If $x_1 = 0.6$ m and $x_2 = 0.4$ m, find the value of ω. Take $g = 10$ ms^{-2}.

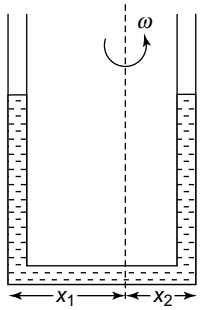


Fig. 7.17

SOLUTION Consider a small element AB of the liquid of length dx at a distance x from the axis of rotation (Fig. 7.18). Due to centripetal force, the liquid rises to height h_1 in the left arm and to a height h_2 in the right arm.

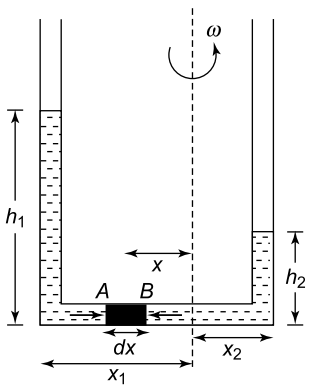


Fig. 7.18

Pressure at A is $p_1 = h_1 \rho g$

Pressure at B is $p_2 = h_2 \rho g$

Pressure difference $\Delta p = p_1 - p_2 = (h_1 - h_2)\, \rho g$

If a is the cross-sectional area of the tube, the net force on element AB is $F = a\Delta p = (h_1 - h_2)\, a\rho g$ (i)

Mass of element $dm = \rho a dx$

$\therefore$ Centripetal force is

$$F_e = \int_{x_2}^{x_1} dm\, x\omega^2 = \omega^2 \rho a \int_{x_2}^{x_1} x dx$$

$$= \frac{1}{2}\, \omega^2 \rho a\, (x_1^2 - x_2^2) \quad \text{(ii)}$$

Equating (i) and (ii), we get

$$h_1 - h_2 = \frac{\omega^2}{2g}\, (x_1^2 - x_2^2)$$

$$\Rightarrow \quad 0.25 = \frac{\omega^2}{2 \times 10}\, [(0.6)^2 - (0.4)^2]$$

$$\Rightarrow \quad \omega = 5 \text{ rad s}^{-1}$$

14. Streamline or Laminar Flow

Streamline or Laminar flow is the flow in which every particle of the liquid follows the same path and has exactly the same velocity in magnitude and direction as that of the preceding particle at a given point in the flow.

The actual path followed by the particles in a regular flow is called a streamline, which can be straight or curved. The tangent at a point on a streamline gives the direction of the liquid flow at that point.

15. Critical Velocity and Reynold's Number

The liquid flow remains steady or streamline if its velocity does not exceed a limiting value called the critical value, which is given by

$$v_c = \frac{k\eta}{\rho r}$$

where η = coefficient of viscosity of the liquid, ρ = density of the liquid, r = radius of the pipe in which the liquid flows and k is a dimensionless constant called Reynold's number.

If the velocity of the liquid exceeds the critical velocity, the flow becomes irregular causing the liquid to flow in a disorderly fashion. Such a flow is called turbulent flow.

The value of k is usually very high. If k is less than 2000, the flow is streamline. If the value of k exceeds 2000, the flow becomes turbulent.

16. Equation of Continuity of Flow

If a_1 and a_2 are the areas of cross-section at two sections of a tube of a variable cross-section and v_1 and v_2 are the velocities of flow crossing these sections, then

$$a_1 v_1 = a_2 v_2 \text{ or } av = \text{constant}$$

This means that smaller the area of cross-section, higher is of the liquid flow and *vice versa*. This is called equation of continuity of flow and it holds only if the flow is streamline.

17. Bernoulli's Theorem

Bernoulli's theorem states that *the total energy of an incompressible and non-viscous liquid in a streamline flow remains constant throughout the flow; the total energy being the sum of pressure energy, potential energy and kinetic energy of the liquid.*

$$PV + mgh + \frac{1}{2}mv^2 = \text{constant}$$

or $$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant} \qquad \left(\because \rho = \frac{m}{V}\right)$$

18. Velocity of Efflux

A liquid is filled up to a height H in a vessel which has a small hole at a depth h below the surface of the liquid (Fig. 7.19).

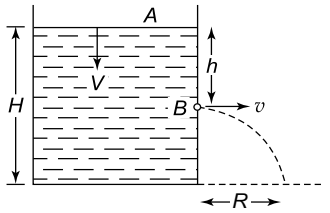


Fig. 7.19

When the hole is unplugged, the velocity v with which the liquid comes out of the hole is called the velocity of efflux. Let V be the velocity with which the free surface of the liquid falls in the vessel. Applying Bernoulli's theorem to points A and B

$$P_A + \frac{1}{2}\rho V^2 + \rho gH = P_B + \frac{1}{2}\rho v^2 + \rho g(H-h)$$

Since $P_A = P_B = P_0$ (atmospheric pressure) and $AV = av$ (equation of continuity) where A = cross-sectional area of the vessel and a = cross-sectional area of hole, we have

$$P_0 + \frac{1}{2}\rho\frac{a^2v^2}{A^2} + \rho gH = P_0 + \frac{1}{2}\rho v^2 + \rho g(H-h)$$

which gives $$v = \left[\frac{2gh}{\left(1-\frac{a^2}{A^2}\right)}\right]^{1/2}$$

Since $A >> a$, $v = \sqrt{2gh}$

The time taken by the liquid emerging from the hole to hit the ground is

$$t = \sqrt{\frac{2(H-h)}{g}}$$

$\therefore$ Horizontal Range $R = vt = 2\sqrt{h(H-h)}$

EXAMPLE 16 A cylindrical vessel of a large cross-sectional area has two tiny holes A and B and lies on a horizontal floor. Hole A is at a depht h_1 below the free surface of liquid in the vessel and hole B is at a height h_2 above the bottom of the vessel. When the holes are unplugged, the streams emerging from A and B strike the floor at the same point. The ratio h_1/h_2 is

(a) $2\sqrt{2}$ (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 1

SOLUTION Refer to Fig. 7.20.

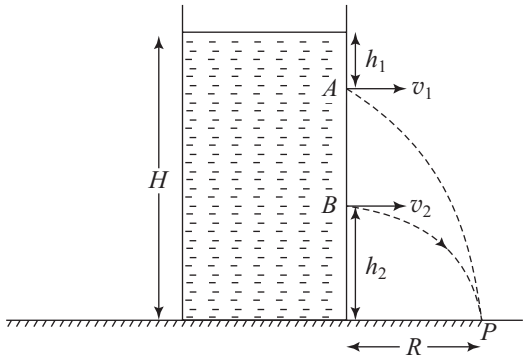


Fig. 7.20

Velocity of efflux is

$$v_1 = \sqrt{2g h_1}$$

and $$v_2 = \sqrt{2g(H - h_2)}$$

The times taken by the liquid emerging from A and B to hit the floor are

$$t_1 = \sqrt{\frac{2(H - h_1)}{g}}$$

and $$t_2 = \sqrt{\frac{2h_2}{g}}$$

Since the two streams strike the floor at the same point P, $R_1 = R_2$ or $v_1 t_1 = v_2 t_2$ or

$$\sqrt{2g h_1} \times \sqrt{\frac{2(H - h_1)}{g}} = \sqrt{2g(H - h_2)} \times \sqrt{\frac{2h_2}{g}}$$

$$\Rightarrow \quad h_1 (H - h_1) = (H - h_2) h_2$$

$$\Rightarrow \quad [H - (h_1 + h_2)]\ [h_1 - h_2] = 0$$

which gives $H = h_1 + h_2$ (which is not possible) and $h_1 = h_2$. So the correct choice is (d).

EXAMPLE 17 A cylindrical vessel of cross-sectional area A is filled with a liquid to a height h and placed on a horizontal surface. The vessel has a small hole of cross-sectional area a on its side near the bottom. The hole is unplugged. If $a << A$, the minimum coefficient of friction between the vessel and the floor so that it just begins to slide on the surface is

(a) $\mu = \dfrac{a}{A}$ (b) $\mu = \dfrac{2a}{A}$

(c) $\mu = \sqrt{\dfrac{a}{A}}$ (d) $\mu = \sqrt{\dfrac{2a}{A}}$

SOLUTION Refer to Fig. 7.21.

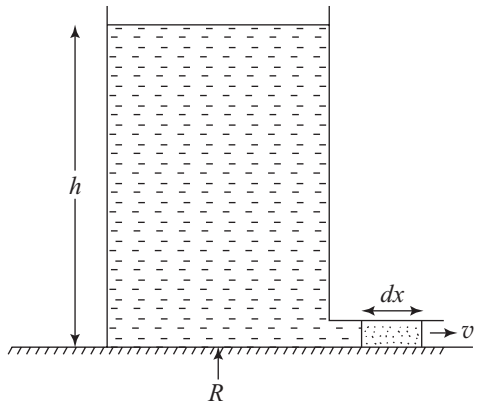


Fig. 7.21

velocity of efflux is $v = \sqrt{2g h}$

From Newton's second law, the backward force exerted on the vessel by the emerging stream of liquid is

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv) = v\frac{dm}{dt} \quad (\because\ v = \text{constant})$$

Let dx be the distance moved by the liquid in time dt. Volume of liquid flowing out in time $dt = a\ dx$. Mass of liquid flowing out in time $dt = \rho\ a\ dx$. Therefore, rate of flow of mass is

$$\frac{dm}{dt} = \rho a \frac{dx}{dt} = \rho a v \qquad \text{(i)}$$

$$\therefore \quad F = v \times \rho a v = \rho a v^2 = \rho a \times 2gh = 2\rho g h a$$

Frictional force $f = \mu R = \mu M g$

(M = mass of liquid in vessel)

$$= \mu \times \rho A h \times g$$

The vessel will just begin to slide if $F = f$, i.e.

$$2\rho g h a = \mu \rho A h g$$

$$\Rightarrow \quad \mu = \frac{2a}{A}$$

Note

1. Horizontal range is maximum when $h = \dfrac{H}{2}$ and $R_{max} = H$.
2. The horizontal range is the same if the hole is at a depth h below the surface of water or at a height h above the bottom of the vessel.

EXAMPLE 18 Figure 7.22 shows a stream of water emerging from the opening of a tap. As the water falls through a height $h = PQ$, the cross-sectional area of the stream decreases from A to a. Obtain the expression for the rate of flow of water through the opening of the tap.

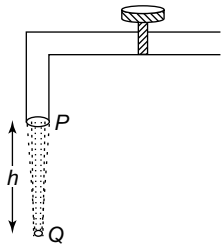


Fig. 7.22

SOLUTION

Let V be the velocity of water at P and v at Q. From the equation of continuity, $AV = av$. From Bernoulli's theorem,

$$P_P + \rho gh + \frac{1}{2}\rho V^2 = P_Q + \frac{1}{2}\rho g\,(0) + \frac{1}{2}\rho v^2$$

Now $P_P = P_Q = P_0$ (atmospheric pressure).

Therefore,

$$P_0 + \rho gh + \frac{1}{2}\rho V^2 = P_0 + \frac{1}{2}\rho v^2$$

$$\Rightarrow \qquad V^2 = v^2 - 2gh \qquad \text{(i)}$$

From $AV = av$, $v = \dfrac{AV}{a}$. Using this in (i) we get

$$V = \left[\frac{2gha^2}{A^2 - a^2}\right]^{1/2}$$

$$\therefore \quad \text{Rate of flow} = AV = aA\left[\frac{2gh}{A^2 - a^2}\right]^{1/2}$$

19. Surface Tension and Surface Energy

Surface tension is the force acting per unit length of an imaginary line on a liquid surface; the direction of the force being perpendicular to the line and tangential to the liquid surface.

The SI unit of surface tension is N m^{-1} and its dimensional formula is $[M L^{\circ} T^{-2}]$. Consider a frame $ABCD$ having a wire PQ of length L which can slide along sides AB and CD. The frame is dipped in a liquid (e.g. soap solution) and taken out. We get a film of liquid within $PBCQ$ (Fig. 7.23). Since the film has two surfaces each of length L, the force due to surface tension acting on wire PQ is

$$F = \sigma \times 2L = 2\sigma L$$

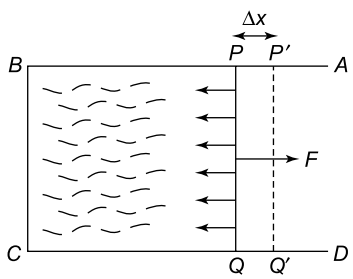


Fig. 7.23

This force is directed inwards to the left and has to be applied to the right to hold the wire PQ in place. Hence, if the area of the film has to be increased, work has to be done against the force of surface tension. This work is stored as potential energy called surface energy. Work done to move the wire from a position PQ to a position $P'Q'$ is

$$\Delta W = F\Delta x = \sigma\ (2L\,\Delta x) = \sigma\,\Delta A$$

where ΔA = increase in the surface area of the film. Thus work done = surface tension × increase in surface area of the film. Another SI unit of surface tension is Jm^{-2}.

EXAMPLE 19 A U-shaped wire frame is dipped in a soap solution and removed. A thin film is formed in the frame. A light slider supports a weight of 1.5×10^{-2} N, which includes the weight of the slider (Fig. 7.24). The length of the slider is 30 cm. Find the surface tension of soap solution.

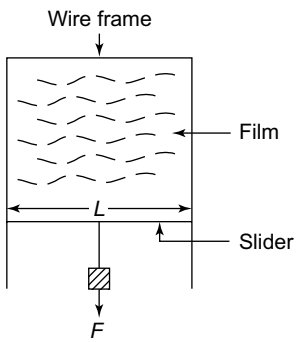


Fig. 7.24

SOLUTION $F = 2\sigma L \Rightarrow \sigma = \dfrac{F}{2L} = \dfrac{1.5 \times 10^{-2}}{2 \times 0.3}$

$= 2.5 \times 10^{-2}$ N m^{-1}

20. Excess Pressure

When the free surface of a liquid is curved, there is a difference of pressure between the liquid side and the vapour side of the surface.

(i) Excess pressure inside a liquid drop of radius r is given by

$$p = \frac{2\sigma}{r}$$

where σ is the surface tension of the liquid.

(ii) Excess pressure inside a liquid bubble of radius r is given by

$$p = \frac{4\sigma}{r}$$

(iii) Excess pressure inside an air bubble of radius r in a liquid of surface tension σ is given by

$$p = \frac{2\sigma}{r}$$

If the pressure outside is P, the total pressure inside bubble $= P + \dfrac{2\sigma}{r}$

21. Work Done in Blowing a Soap Bubble

Suppose the radius of a soap bubble is increased from r_1 to r_2 by blowing. Then, since there are two surfaces of the film, initial energy $= 2 \times (4\pi r_1^2\sigma)$ and final energy $= 2 \times (4\pi r_2^2\sigma)$

$\therefore$ Work done = final energy – initial energy

or $$W = 8\pi\sigma (r_2^2 - r_1^2)$$

(i) Work done in forming a bubble of radius r is (since $r_2 = r$ and $r_1 = 0$)

$$W = 8\pi\sigma r^2$$

(ii) Work done in doubling the radius of a bubble from $r_1 = r$ to $r_2 = 2r$ is

$$W = 8\pi\sigma[(2r)^2 - r^2] = 24\pi\sigma r^2$$

(iii) Work done in splitting a drop of radius R into n identical drops, each of radius r, is obtained as follows:

$$\text{Initial surface area} = 4\pi R^2$$

$$\text{Final surface area} = n \times 4\pi r^2$$

$\therefore$ Work done is $W = 4\pi(nr^2 - R^2)\sigma$

Since the volume remains unchanged,

$$\frac{4}{3}\pi r^3 n = \frac{4}{3}\pi R^3$$

or $n^{1/3} r = R$. Hence

$$W = 4\pi R^2 (n^{1/3} - 1)\sigma$$

EXAMPLE 20 Calculate the work done (or energy needed) to split a spherical drop of mercury of diameter 1 cm into 8 identical drops. The surface tenstion of mercury $= 0.035$ N m^{-1}.

SOLUTION As shown above

$$(8^{1/3})r = R \Rightarrow r = \frac{R}{2}$$

Surface area of the big drop $= 4\pi R^2$

Surface area of 8 small drops $= 8 \times 4\pi r^2$

$$= 8 \times 4\pi \times \left(\frac{R}{2}\right)^2 = 8\pi R^2$$

$\therefore$ Increase in surface area $\Delta A = 8\pi R^2 - 4\pi R^2$

$$= 4\pi R^2$$

Work done $= \sigma \times \Delta A$

$$= 0.035 \times 4 \times 3.14 \times (0.5 \times 10^{-2})^2 = 1.1 \times 10^{-5} \text{ J}$$

EXAMPLE 21 Eight spherical droplets, each of radius r of a liquid of density ρ and surface tension σ coalesce to form one big drop. If s is the specific heat capacity of the liquid, find the rise in the temperature of the liquid in this process.

SOLUTION In this process, energy is evolved as heat because there is a decrease in surface area. Radius of big drop is $R = 2r$. Decrease in surface area is

$$\Delta A = 8 \times 4\pi r^2 - 4\pi R^2$$

$$= 32\pi r^2 = 4\pi(2r)^2$$

$$= 16\pi r^2$$

$\therefore$ Energy evolved is $Q = \sigma \Delta A = 16\pi\sigma r^2$

Mass of big drop is $m = \dfrac{4}{3}\pi R^3 \rho = \dfrac{32}{3}\pi r^3 \rho$

If ΔT is the rise in temperature, then

$$Q = m s \Delta T$$

$$\Rightarrow \quad 16\pi\sigma r^2 = \frac{32}{3}\pi r^3 \rho s \Delta T$$

$$\Rightarrow \quad \Delta T = \frac{3\sigma}{2r\rho s}$$

22. Angle of Contact

The shape of meniscus of water in a narrow glass tube is concave upwards [Fig. 7.25 (a)] while the shape of meniscus of mercury in a narrow glass tube is convex upwards [Fig. 7.25 (b)]

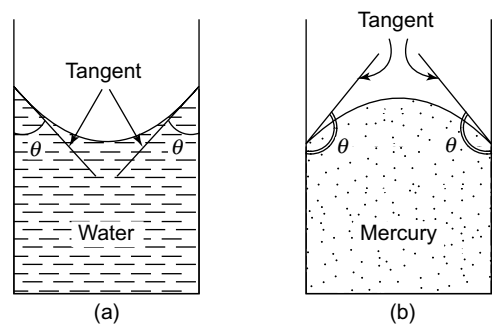


Fig. 7.25

The angle of contact (θ) between a liquid and a solid surface is defined as the angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid.

The value of angle of contact depends upon

(i) the nature of the liquid and solid in contact,

(ii) the nature of the medium above the free surface of the liquid and

(iii) the temperature of the liquid.

> **Note** Generally the angle of contact for liquids which wet glass is acute and obtuse for liquids which do not wet glass.

23. Capillarity

The rise or fall of a liquid in a capillary tube is known as capillarity. The height to which a liquid of surface tension σ and density ρ rises in a capillary tube of radius r is given by

$$h = \frac{2\sigma \cos\theta}{\rho r g}$$

where θ is the angle of contact. For pure water and clean glass, $\theta \simeq 0°$ in which case $\cos\theta \simeq \cos 0° = 1$ and we have

$$h = \frac{2\sigma}{\rho r g}$$

For mercury and glass, $\theta \simeq 140°$ so that $\cos\theta$ is negative. Hence mercury falls in a capillary tube, i.e. the level of mercury in the capillary tube is lower than the level outside.

> **Note** Surface tension of a liquid decreases with increase in temperature.

EXAMPLE 22 A narrow glass tube of diameter 1.0 mm is dipped vertically in a container of water. The surface tension of water is 0.07 N m^{-1} and the angle of contact with glass is zero. (a) Find the height to which water rises in the tube. (b) To what height will water rise if the tube is held slanting making an angle of 60° with the vertical? Also find the length which the water occupies in the tube. Take g = 10 m s^{-2}.

SOLUTION

(a) $h = \dfrac{2\sigma\cos\theta}{r\rho g} = \dfrac{2 \times 0.07 \times \cos 0°}{(0.5 \times 10^{-3}) \times 10^{3} \times 10}$

$= 2.8 \times 10^{-2}$ m = 2.8 cm

(b) The vertical height h of water column will remain the same = 2.8 cm (Fig. 7.26).

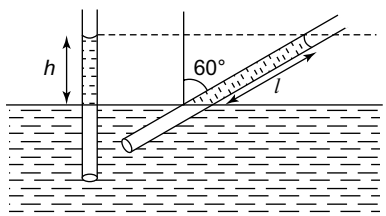


Fig. 7.26

Length of water column is $l = \dfrac{h}{\cos 60°} = 2h = 5.6$ cm

PART (A)

Solids

24. Elasticity

The ability of a body to regain its original shape and size when the deforming force is withdrawn, is known as elasticity. A body which completely regains its original shape and size after the removal of the deforming force is said to be *perfectly elastic*. A body which retains its deformed shape and shows no tendency at all to regain its original shape, after the removal of the external force, is said to be *perfectly plastic*. In fact, no material body can be perfectly elastic or perfectly plastic.

25. Stress

When a deforming force is applied to a body, it reacts to the applied force by developing a reaction (or restoring) force which, from Newton's third law, is equal in magnitude and opposite in direction to the applied force. *The reaction force per unit area of the body which is called into play due to the action of the applied force is called stress.* The mangnitude of the stress is equal to the magnitude of the applied force divided by the area of the body, but the direction of the stress is opposite to that of the applied force. Stress is measured in units of force per unit area, i.e. Nm^{-2}. Thus

$$\text{Stress} = \frac{F}{A}$$

where F is the applied force and A is the area over which it acts.

26. Strain

When a deforming force is applied to a body, it may suffer a change is size or shape. Strain is defined as the ratio of the change in size or shape to the original size or shape of the body. Strain is a number; it has no units or dimensions.

The ratio of the change in length to the original length is called *longitudinal strain*. The ratio of the change in volume to the original volume is called *volume strain*. The strain resulting from a change in shape is called *shearing strain*.

27. Hookes' Law

Hookes' law states that, within the elastic limit, the stress developed in a body is proportional to the strain produced

in it. Thus, the ratio of stress to strain is a constant. This constant is called the modulus of elasticity. Thus

$$\text{Modulus of elasticity} = \frac{\text{stress}}{\text{strain}}$$

Since strain has no unit, the unit of the modulus of elasticity is the same as that of stress, namely, Nm^{-2}.

28. Young's Modulus

Suppose that a rod of length L and a uniform cross-sectional area A is subjected to a longitudinal pull. In other words, two equal and opposite forces are applied at its ends.

As a result of applying the deforming forces, there is an extension in length which we denote by ΔL. The strain is given by

$$\text{Strain} = \frac{\Delta L}{L}$$

Since the deformation involves a change in length, the strain is called *longitudinal strain* or *linear strain*. Since the length increases ($\Delta L > 0$) it is also called *extensional strain* or *tensile strain*. The stress is given by

$$\text{Stress} = \frac{F}{A}$$

The stress in the present case is called linear stress, tensile stress, or extensional stress. If the direction of the forces is reversed so that ΔL is negative, we speak of *compressional strain* and *compressional stress*. If the elastic limit is not exceeded, then from Hooke's law

$$\text{Stress} \propto \text{strain}$$

or $$\text{Stress} = Y \times \text{strain}$$

or $$Y = \frac{\text{stress}}{\text{strain}} = \frac{F}{A} \cdot \frac{L}{\Delta L}$$

where Y, the constant of proportionality, is called the *Young's modulus* of the material of the rod and may be defined as the *ratio of the linear stress to linear strain, provided the elastic limit is not exceeded.* Since strain has no unit, the unit of Y is Nm^{-2}.

29. Bulk Modulus

Solids, liquids and gases can be deformed by subjecting them to a uniform normal pressure P in all directions.

Stress and pressure have the same dimension (force per unit area), but pressure is not the same thing as stress. Pressure is the force per unit area acting on the surface of a system, *the force being everywhere perpendicular to the surface* so that, for a uniform pressure, the force per unit area is the same.

Pressure is a particular kind of stress which changes only the volume of the substance and not its shape. The substance may be a solid, liquid or gas. A small increase in pressure ΔP applied to a substance decreases its volume from, say, V to $V - \Delta V$ so that ΔV is the small decrease in volume. The volume strain is given by

$$\text{Volume strain} = -\frac{\Delta V}{V}$$

The bulk modulus is defined as the ratio of the excess pressure and the corresponding volume strain, i.e.

$$B = \frac{\Delta P}{\left(-\frac{\Delta V}{V}\right)} = -\frac{\Delta P V}{\Delta V}$$

If ΔP is positive, ΔV will be negative and vice versa. The negative sign in our definition of bulk modulus B ensures that B is always positive. The SI unit of B is Nm^{-2}. The reciprocal of B is known as *compressibility*.

The bulk modulus of a gas depends on the pressure. Under isothermal conditions (i.e. when the temperature is kept constant), the bulk modulus of a gas is equal to its pressure P. Under adiabatic conditions (i.e. when heat is not allowed to leave or enter the system), the bulk modulus is equal to γP, where $\gamma (= C_p/C_v)$ is the ratio of the molar heat capacities of the gas at constant pressure and constant volume. Thus

Isothermal bulk modulus = P

Adiabatic bulk modulus = γP.

30. Shear Modulus or Modulus of Rigidity

Shear is a particular kind of stress which only solids can withstand. The solid is deformed by changing its shape without changing its size. The body does not move or rotate as a whole: there is a relative displacement of its contiguous layers.

Consider a solid in the form of a rectangular cube as in Fig. 7.27.

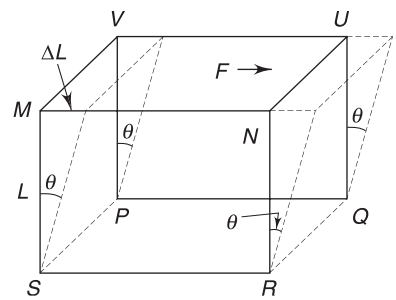


Fig. 7.27

Suppose the lower face PQRS is held fixed and a force F is applied parallel to the upper face MNUV. If we do this to a liquid, it will begin to flow; it will not remain in equilibrium. In other words, liquids cannot withstand this kind of shear force. But when it is applied to a solid, it will remain in equilibrium because no net force and no net torque act on it. There is a couple produced by this

force and an equal and opposite force coming into play on the lower face. As a result of this, the lines joining the two faces turn through an angle θ. We say that the face MNRS is sheared through an angle θ (measured in radians). The angle θ is called the *shear strain or the angle of shear* and is a measure of the degree of deformation.

If A is the area of the face MNUV, the ratio F/A is the shearing stress. It is found that for small deformation, the shearing stress is proportional to the shear strain, i.e.

$$\frac{F}{A} \propto \theta$$

$$\frac{F}{A} = \eta\theta$$

$$\eta = \frac{F}{A\theta}$$

The quantity η is called the *shear modulus* or the modulus of rigidity. Referring to Fig. 7.23, if θ is small,

$$\theta \simeq \tan\theta = \frac{\Delta L}{L}$$

so that $$\eta = \left(\frac{F}{A}\right)\cdot\frac{L}{\Delta L}$$

If a rod or wire of length l and radius r is fixed at one end and a torque τ is aplied at the other end, the rod or wire is twisted about its axis. If θ is the angle of twist, the torque is related to modulus of rigidity (η) by the relation

$$\tau = \frac{\eta\pi r^4\theta}{2l}$$

31. Poisson's Ratio

When a wire is stretched with a force, apart from an increase in its length, there is a slight decrease in its diameter, i.e. both shape and volume change under longitudinal stress.

The ratio of the decrease ΔD in diameter to the original diameter D is called *lateral strain*, i.e. strain at right angles to the deforming force. Thus

$$\text{Lateral strain} = \frac{\text{decrease in diameter}}{\text{original diameter}} = \frac{\Delta D}{D}$$

Also $$\text{Longitudinal strain} = \frac{\text{Increase in length}}{\text{original length}} = \frac{\Delta L}{L}$$

It is found experimentally that within the elastic limit, the lateral strain is proportional to the longitudinal strain. The ratio of the two is called Poisson's ratio and is denoted by σ. Hence,

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\Delta D/D}{\Delta L/L}$$

or $$\sigma = \frac{\Delta D}{D}\cdot\frac{L}{\Delta L}$$

Since it is a ratio between two types of strain, σ is dimensionless. Theoretically, one can show that it must be less than 0.5. For most solids it lies between 1/4 and 1/3, and for rubber it is very nearly 0.5.

32. Energy Stored in a Strained Wire: Strain Energy

If a wire is stretched, the potential energy stored per unit volume is given by

$$U = \frac{1}{2}(S\times\varepsilon) = \frac{1}{2}(\varepsilon^2\times Y) = \frac{1}{2}\left(\frac{S^2}{Y}\right)$$

Where S = strees, ε = strain and Y = Young's modulus of the material of the wire.

33. Thermal Stresses

If a metal rod fixed rigidly at its ends is heated or cooled, then due to expansion or contraction, tensile or compressive stress is set up in the rod. These stresses are called thermal stresses. If a rod of length L is free to expand or contract and its temperature is changed by ΔT, the change in its length is given by

$$\Delta L = \alpha L\,\Delta T$$

where α is the coefficient of linear expansion of the rod. Now

$$\Delta L = \frac{FL}{AY}$$

Using this, we get

$$F = \alpha AY\Delta T$$

Thus, the thermal stress in the rod is

$$\frac{F}{A} = \alpha Y\Delta T$$

Similarly, if a fluid is contained in a vessel such that its volume cannot change, then a change in temperature results in a change in pressure. The thermal stress is then given by

$$\Delta P = \gamma B\Delta T$$

where B is the bulk modulus of the fluid and γ is its coefficient of volume expansion or contraction.

EXAMPLE 23 A steel wire (original length = 2 m, diameter = 1mm) and a copper wire (original length = 1m, diameter = 2 mm) are loaded as shown in Fig. 7.28. Find the ratio of extension of steel wire to that of copper wire. Young's

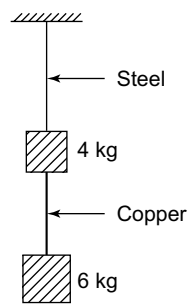


Fig. 7.28

modulus of steel = 2 $\times 10^{11}$ Nm^{-2} and that of copper = 1×10^{11} Nm^{-2}.

SOLUTION $Y = \dfrac{FL}{A\Delta L}$

$$\Rightarrow \qquad \Delta L = \frac{FL}{YA} = \frac{4FL}{Y \times \left(\pi d^2\right)}$$

For steel wire $F_s = (6 + 4) \times 9.8 = 98$ N
For copper wire $F_e = 6 \times 9.8 = 58.8$ N

$$(\Delta L)_s = \frac{4F_s\, L_s}{Y_s \times \left(\pi d_s^2\right)} \text{ and } (\Delta L)_c = \frac{4F_c\, L_c}{Y_c \times \left(\pi d_c^2\right)}$$

$$\therefore \quad \frac{(\Delta L)_s}{(\Delta L)_c} = \frac{F_s}{F_c} \times \frac{L_s}{L_c} \times \frac{Y_c}{Y_s} \times \left(\frac{d_c}{d_s}\right)^2$$

$$= \frac{98}{58.8} \times \frac{2}{1} \times \frac{1}{2} \times (2)^2$$

$$= \frac{20}{3}$$

EXAMPLE 24 A rubber eraser 3 cm × 1 cm × 8 cm is clamped at one end with 8 cm edge vertical as shown in Fig. 7.29. A horizontal force F = 2.1 N is applied on the free face. Calculate

(a) the shear angle and
(b) the horizontal displacement Δx of the top face.
Given sheal modulus of rubber = 1.4×10^5 Nm^{-2}

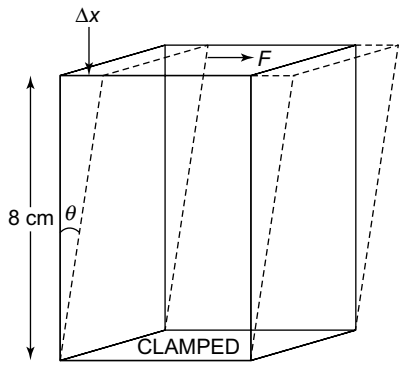


Fig. 7.29

SOLUTION

(a) Area of top face (A) = 3 cm × 1 cm = 3 cm^2
$= 3 \times 10^{-4}$ m^2

$$\text{Shearing stress} = \frac{F}{A} = \frac{2.1}{3 \times 10^{-4}} = 0.7 \times 10^4 \text{ Nm}^{-2}$$

$$\therefore \text{ Angle of shear } (\theta) = \frac{\text{shearing stress}}{\text{shear modulus}}$$

$$= \frac{0.7 \times 10^4}{1.4 \times 10^5} = 0.05 \text{ rad}$$

(b) $\tan\theta = \dfrac{\Delta x}{8\text{ cm}} \Rightarrow \theta = \dfrac{\Delta x}{8\text{ cm}}$ ($\because$ θ is small)

$$\therefore \qquad \Delta x = 8 \text{ cm} \times 0.05 = 0.4 \text{ cm} = 4 \text{ mm}$$

SECTION 1

Multiple Choice Questions with One Correct Choice

Level A

1. A block of wood floats in a liquid with four-fifths of its volume submerged. If the relative density of wood is 0.8, what is the the density of the liquid in units of kg m^{-3}?
   (a) 750 (b) 1000
   (c) 1250 (d) 1500

2. An ice cube floats on water in a beaker with $\frac{9}{10}^{\text{th}}$ of its volume submerged under water. What fraction of its volume will be submerged if the beaker of water is taken to the moon where the gravity is 1/6$^{\text{th}}$ that on the earth?
   (a) $\frac{9}{10}$ (b) $\frac{27}{50}$
   (c) $\frac{2}{3}$ (d) zero

3. A block of wood floats in a liquid in a beaker with 3/4$^{\text{th}}$ of its volume submerged under the liquid. If the beaker is placed in an enclosure that is falling freely under gravity, the block will
   (a) float with 3/4$^{\text{th}}$ of its volume submerged
   (b) float completely submerged
   (c) float with any fraction of its volume submerged
   (d) sink to the bottom.

4. A cubical block of steel of each side equal to l is floating on mercury in a vessel. The densities of steel and mercury are ρ_s and ρ_m. The height of the block above the mercury level is given by

   (a) $l\left(1+\frac{\rho_s}{\rho_m}\right)$ (b) $l\left(1-\frac{\rho_s}{\rho_m}\right)$

   (c) $l\left(1+\frac{\rho_m}{\rho_s}\right)$ (d) $l\left(1-\frac{\rho_m}{\rho_s}\right)$

5. A cube of ice is floating in water contained in a vessel. When the ice melts, the level of water in the vessel

   (a) rises
   (b) falls
   (c) remains unchanged
   (d) falls at first and then rises to the same height as before.

6. A cube of ice is floating in a liquid of relative density 1.25 contained in a beaker. When the ice melts, the level of the liquid in the beaker

   (a) rises
   (b) falls
   (c) remains unchanged
   (d) falls at first and then rises to the same height as before.

7. A piece of ice, with a stone frozen inside it, is floating in water contained in a beaker. When the ice melts, the level of water in the beaker

   (a) rises
   (b) falls
   (c) remains unchanged
   (d) falls at first and then rises to the same height as before.

8. A cubical vessel of height 1 m is full of water. What is the amount of work done in pumping water out of the vessel? Take $g = 10\ \text{ms}^{-2}$.

   (a) 1250 J (b) 2500 J
   (c) 5000 J (d) 1000 J

9. The dimensions of viscosity in terms of M, L and T are

   (a) $ML^{-1}T^{-1}$ (b) $ML^{-1}T^{-2}$
   (c) $ML^{-2}T^{-1}$ (d) $ML^{-2}T^{-2}$

10. Eight spherical rain drops of the same mass and radius are falling down with a terminal speed of 6 cm s^{-1}. If they coalesce to form one big drop, what will be its terminal speed? Neglect the buoyancy due to air.

    (a) 1.5 cms^{-1} (b) 6 cms^{-1}
    (c) 24 cms^{-1} (d) 32 cms^{-1}

11. Water flows steadily through a horizontal pipe of a variable cross-section. If the pressure of water is p at a point where the velocity of flow is v, what is the pressure at another point where the velocity of flow is $2v$; ρ being the density of water?

    (a) $p-\frac{3}{2}\rho v^2$ (b) $p+\frac{3}{2}\rho v^2$
    (c) $p-2\rho v^2$ (d) $p+2\rho v^2$

12. Water stands at a depth H in a tank whose side walls are vertical. A hole is made in one of the walls at a height h below the water surface. The stream of water emerging from the hole strikes the floor at a distance R from the tank, where R is given by

    (a) $R=\sqrt{h(H-h)}$ (b) $R=\sqrt{h(H+h)}$
    (c) $R=2\sqrt{h(H-h)}$ (d) $R=2\sqrt{h(H+h)}$

13. In Q. 12, R is maximum if

    (a) $h=\frac{H}{4}$ (b) $h=\frac{H}{3}$
    (c) $h=\frac{H}{2}$ (d) $h=H$

14. The cylindrical tube of a spray pump has a radius R, one end of which has n fine holes, each of radius r. If the speed of flow of the liquid in the tube is V, the speed of ejection of the liquid through the holes is

    (a) $\frac{V}{n}\left(\frac{R}{r}\right)^{1/2}$ (b) $\frac{V}{n}\left(\frac{R}{r}\right)$
    (c) $\frac{V}{n}\left(\frac{R}{r}\right)^{3/2}$ (d) $\frac{V}{n}\left(\frac{R}{r}\right)^{2}$

15. In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the lower and upper surfaces of the wing are v and $\sqrt{2}\ v$ respectively. If the density of air is ρ and the surface area of the wing is A, the dynamic lift on the wing is given by

    (a) $\frac{1}{\sqrt{2}}\rho v^2 A$ (b) $\frac{1}{2}\rho v^2 A$
    (c) $\sqrt{2}\ \rho v^2 A$ (d) $2\rho v^2 A$

16. A stone of relative density k is released from rest on the surface of a lake. If viscous effects are ignored, the stone sinks in water with an acceleration of

    (a) $g(1-k)$ (b) $g(1+k)$
    (c) $g\left(1-\frac{1}{k}\right)$ (d) $g\left(1+\frac{1}{k}\right)$

17. Two identical cylindrical vessels, each of base area A, have their bases at the same horizontal level. They

contain a liquid of density ρ. In one vessel the height of the liquid is h_1 and in the other $h_2 > h_1$. When the two vessels are connected, the work done by gravity in equalizing the levels is

(a) $2\rho Ag(h_2 - h_1)^2$ (b) $\rho Ag(h_2 - h_1)^2$

(c) $\frac{1}{2}\rho Ag(h_2 - h_1)^2$ (d) $\frac{1}{4}\rho Ag(h_2 - h_1)^2$

18. A cylindrical jar has radius r. To what height h should it be filled with a liquid so that the force exerted by the liquid on the sides of the jar equals the force exerted on the bottom?

(a) $h = \frac{r}{2}$ (b) $h = r$

(c) $h = 2r$ (d) $h = 4r$

19. A rectangular tank is filled to the brim with water. When a hole at its bottom is unplugged, the tank is emptied in time T. If the tank is half–filled with water, it will be emptied in time

(a) $\frac{T}{\sqrt{2}}$ (b) $\frac{T}{\sqrt{3}}$

(c) $\frac{T}{2}$ (d) $\frac{T}{2\sqrt{2}}$

20. Two capillary tubes A and B of radii r_a and r_b and lengths l_a and l_b respectively are held horizontally. The volume of water flowing per second through tube A is Q_a when the pressure difference across its ends is maintained at P. When the same pressure difference is maintained across tube B, the volume of water flowing per second through it is Q_b. The ratio Q_a/Q_b is

(a) $\frac{l_b}{l_a}\left(\frac{r_a}{r_b}\right)$ (b) $\frac{l_b}{l_a}\left(\frac{r_a}{r_b}\right)^2$

(c) $\frac{l_b}{l_a}\left(\frac{r_a}{r_b}\right)^3$ (d) $\frac{l_b}{l_a}\left(\frac{r_a}{r_b}\right)^4$

21. Two capillary tubes A and B of equal radii $r_a = r_b = r$ and equal lengths $l_a = l_b = l$ are held horizontally. When the same pressure difference P is maintained across each tube, the rate of flow of water in each is Q. If the tubes are connected in series and the same pressure difference P is maintained across the combination, the rate of flow through the combination will be

(a) $\frac{Q}{2}$ (b) Q

(c) $2Q$ (d) $\frac{Q}{\sqrt{2}}$

22. In Q. 21, $r_a = r_b = r$ and $l_a = 2l$ and $l_b = l$. If a pressure difference P is maintained across tube A, the rate of flow of water in it is Q. If the tubes are connected in series and the same pressure difference P is maintained across the combination, the rate of flow through the combination will be

(a) $\frac{Q}{3}$ (b) $\frac{2Q}{3}$

(c) Q (d) $\frac{4Q}{3}$

23. If the surface tension of soap solution is σ, what is the work done in blowing soap bubble of radius r?

(a) $\pi r^2\sigma$ (b) $2\pi r^2\sigma$

(c) $4\pi r^2\sigma$ (d) $8\pi r^2\sigma$

24. The work done to break up a drop of a liquid of radius R and surface tension σ into eight drops, all of the same size, is

(a) $4\pi\sigma R^2$ (b) $2\pi\sigma R^2$

(c) $\frac{1}{2}\pi\sigma R^2$ (d) $\frac{1}{4}\pi\sigma R^2$

25. A soap bubble of radius r is blown up to form a bubble of radius $2r$ under isothermal conditions. If σ is the surface tension of soap solution, the energy spent in doing so is

(a) $3\pi\sigma r^2$ (b) $6\pi\sigma r^2$

(c) $12\pi\sigma r^2$ (d) $24\pi\sigma r^2$

26. The excess pressure across a soap bubble of radius r is $p = 4\sigma/r$, where σ is the surface tension of soap solution. What is the excess pressure across an air bubble of the same radius r formed inside a container of soap solution?

(a) $\frac{\sigma}{r}$ (b) $\frac{2\sigma}{r}$

(c) $\frac{4\sigma}{r}$ (d) $\frac{2\sqrt{2}\sigma}{r}$

27. A small drop of water of surface tension σ is squeezed between two clean glass plates so that a thin layer of thickness d and area A is formed between them. If the angle of contact is zero, the force required to pull the plates apart is

(a) $\frac{\sigma A}{d}$ (b) $\frac{2\sigma A}{d}$

(c) $\frac{4\sigma A}{d}$ (d) $\frac{8\sigma A}{d}$

28. Water rises to a height h in a capillary tube of area of cross-section a. To what height will water rise in a capillary tube of area of cross-section $4a$?

(a) $\frac{h}{4}$ (b) $\frac{h}{2}$

(c) $2h$ (d) $4h$

29. Water rises to a height h in a capillary tube held vertically in a beaker containing water. If the capillary tube is inclined at an angle 30° with the water surface, the height to which water rises will be

(a) $\frac{h}{2}$ (b) h

(c) $\frac{2h}{\sqrt{3}}$ (d) $2h$

Level B

30. A spherical small ball of density ρ is gently released in a liquid of density $\sigma(\rho > \sigma)$. The initial acceleration of the free fall of the ball will be

(a) $\left(\frac{\rho+\sigma}{\rho}\right) g$ (b) $\left(\frac{\rho-\sigma}{\sigma}\right) g$

(c) $\left(\frac{\rho-\sigma}{\rho}\right) g$ (d) g

31. The time period of a simple pendulum is T. The pendulum is oscillated with its bob immersed in a liquid of density σ. If the density of the bob is ρ and viscous effect is neglected, the time period of the pendulum will be

(a) $\left(\frac{\rho}{\rho-\sigma}\right)^{1/2} T$ (b) $\left(\frac{\sigma}{\rho-\sigma}\right)^{1/2} T$

(c) $\left(\frac{\rho}{\sigma}\right)^{1/2} T$ (d) $\left(\frac{\sigma}{\rho}\right)^{1/2} T$

32. A wooden block of mass m and density ρ is tied to a string; the other end of the string is fixed to the bottom of a tank. The tank is filled with a liquid of density σ with $\sigma > \rho$. What is the tension in the string.

(a) $\left(\frac{\sigma-\rho}{\sigma}\right) mg$ (b) $\left(\frac{\sigma-\rho}{\rho}\right) mg$

(c) $\frac{\rho mg}{\sigma}$ (d) $\frac{\sigma mg}{\rho}$

33. A balloon of mass m contains water of mass M. If it is completely immersed in water, the apparent mass of the balloon with water in it will be

(a) $M + m$ (b) $M - m$

(c) M (d) m

34. A wooden ball of density σ is released from the bottom of a tank which is filled with a liquid of density ρ ; $(\rho > \sigma)$ up to a height h_1. The ball rises in the liquid, emerges from its surface and attains a height h_2 in air. If viscous effects are neglected, the ratio h_2/h_1 is

(a) $\left(\frac{\rho}{\sigma}+1\right)$ (b) $\left(\frac{\rho}{\sigma}-1\right)$

(c) $\frac{\rho}{\sigma}$ (d) $\frac{\sigma}{\rho}$

35. If W is the amount of work done in blowing a bubble of volume, V, what will be the amount of work done to blow a bubble of volume $8V$?

(a) $2W$ (b) $4W$

(c) $8W$ (d) $16W$

36. Water is flowing through a tube of radius r with a speed v. If this tube is joined to another tube of radius $r/2$, what is the speed of water in the second tube?

(a) $\frac{v}{4}$ (b) $\frac{v}{2}$

(c) $2v$ (d) $4v$

37. Equal masses of two substances of densities ρ_1 and ρ_2 are mixed together. The density of the mixture would be

(a) $\frac{1}{2}(\rho_1 + \rho_2)$ (b) $\sqrt{\rho_1\rho_2}$

(c) $\frac{\rho_1\rho_2}{(\rho_1+\rho_2)}$ (d) $\frac{2\rho_1\rho_2}{(\rho_1+\rho_2)}$

38. Equal volumes of two substances of densities ρ_1 and ρ_2 are mixed together. The density of the mixture would be

(a) $\frac{1}{2}(\rho_1 + \rho_2)$ (b) $(\rho_1 + \rho_2)$

(c) $\sqrt{\rho_1\rho_2}$ (d) $\frac{\rho_1\rho_2}{(\rho_1+\rho_2)}$

39. A small sphere of volume V falling in a viscous fluid acquires a terminal velocity v_t. The terminal velocity of a sphere of volume $8V$ of the same material and falling in the same fluid will be

(a) $\frac{v_t}{2}$ (b) v_t

(c) $2v_t$ (d) $4v_t$

40. A spherical steel ball released at the top of a long column of glycerine of length L, falls through a distance $L/2$ with accelerated motion and the remaining distance $L/2$ with a uniform velocity. If

t_1 and t_2 denote the times taken to cover the first and second half and W_1 and W_2 the work done against gravity in the two halves, then

(a) $t_1 < t_2$; $W_1 > W_2$ (b) $t_1 > t_2$; $W_1 < W_2$

(c) $t_1 = t_2$; $W_1 = W_2$ (d) $t_1 > t_2$; $W_1 = W_2$

41. A capillary tube of radius r is immersed in water and water rises in it to a height h. The mass of water in the capillary tube is 5 g. Another capillary tube of radius $2r$ is immersed in water. The mass of water that will rise in this tube is

(a) 2.5 g (b) 5.0 g

(c) 10 g (d) 20 g

42. When a capillary tube of radius r is immersed in a liquid of density ρ, the liquid rises to a height h in it. If m is the mass of the liquid in the capillary tube, the potential energy of this mass of the liquid in the tube is

(a) $mgh/4$ (b) $mgh/2$

(c) mgh (d) $2mgh$

43. When water flows at a rate Q through a capillary tube of radius r placed horizontally, a pressure difference p develops across the ends of the tube. If the radius of the tube is doubled and the rate of flow halved, the pressure difference becomes

(a) $\frac{p}{32}$ (b) $\frac{p}{8}$

(c) p (d) $8p$

44. A liquid flows through a pipe of varying diameter. The velocity of the liquid is 2 ms^{-1} at a point where the diameter is 6 cm. The velocity of the liquid at a point where the diameter is 3 cm will be

(a) 1 ms^{-1} (b) 4 ms^{-1}

(c) 8 ms^{-1} (d) 16 ms^{-1}

45. Which one of the following statements is correct? When a fluid passes through the narrow part of non-uniform pipe,

(a) its velocity and pressure both increase

(b) its velocity and pressure both decrease

(c) its velocity decreases but its pressure increases

(d) its velocity increases but its pressure decreases

46. Water stands at a height H in a tall cylinder (see Fig. 7.30). Two holes A and B are made on the sides of the cylinder. If hole A is at a height h above the ground, what is the height of hole B above the ground so that the two streams of water emerging from holes A and B strike the ground at the same point?

(a) $2h$ (b) $H/2$

(c) $H - h$ (d) $H - h/2$

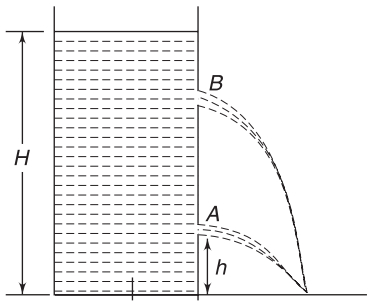


Fig. 7.30

47. Two water droplets coalesce to form a large drop. In this process,

(a) energy is liberated

(b) energy is absorbed

(c) energy is neither liberated nor absorbed

(d) a small amount of mass is converted into energy in accordance with Einstein's mass–energy equivalence relation $E = mc^2$.

48. A solid iron ball and a solid aluminium ball of the same diameter are released together on a deep lake. Which ball will reach the bottom first?

(a) Aluminium ball

(b) Iron ball

(c) Both balls will reach the bottom at the same time

(d) The aluminium ball will never reach the bottom and will remain suspended in the lake

49. Under a constant pressure head, the rate of streamlined volume flow of a liquid through a capillary tube is Q. If the length of the capillary tube is doubled and the diameter of the bore is halved, the rate of flow would become

(a) $Q/4$ (b) $Q/8$

(c) $Q/16$ (d) $Q/32$

50. What are the dimensions of stress?

(a) MLT^{-2} (b) $ML^{-1}T^{-2}$

(c) $ML^{-2}T^{-1}$ (d) $ML^{0}T^{-1}$

51. Which one of the following physical quantities does NOT have the dimensions of force per unit area?

(a) Stress (b) Strain

(c) Young's modulus (d) Pressure

52. A thick uniform rubber rope of density 1.5 $g\,cm^{-3}$ and Young's modulus 5×10^6 Nm^{-2} has a length of 8 m. When hung from the ceiling of a room, the increase in length of the rope due to its own weight will be

(a) 9.6×10^{-2} m (b) 19.2×10^{-3} m

(c) 9.6×10^{-3} m (d) 9.6 m

53. Two wires A and B of the same material have their lengths in the ratio of 1 : 2 and their diameters in the ratio of 2 : 1. If they are stretched with the same

force, the ratio of the increase in the length of A to that of B will be

(a) 1 : 2 (b) 4 : 1
(c) 1 : 8 (d) 1 : 4

54. A metal bar of length L and area of cross-section A is rigidly clamped between two walls. The Young's modulus of its material is Y and the coefficient of linear expansion is α. The bar is heated so that its temperature increases by θ °C. Then the force exerted at the ends of the bar is given by

(a) $YL\,\alpha\,\theta$ (b) $YL\,\alpha\,\theta/A$
(c) $YA\,\alpha\,\theta$ (d) $Y\,\theta\alpha/LA$

55. A wire of length L is stretched by a length l when a force F is applied at one end. If the elastic limit is not exceeded, the amount of energy stored in the wire is given by

(a) $F \times l$ (b) 1/2 $(F \times l)$
(c) Fl^2/L (d) 1/2 $F\,l^2/L$

56. When a force is applied at one end of an elastic wire, it produces a strain ε in the wire. If Y is the Young's modulus of the material of the wire, the amount of energy stored per unit volume of the wire is given by

(a) $Y \times \varepsilon$ (b) 1/2 $(Y \times \varepsilon)$
(c) $Y \times \varepsilon^2$ (d) 1/2 $(Y \times \varepsilon^2)$

57. A wire, suspended vertically from one end, is stretched by attaching a weight of 20 N to the lower end. The weight stretches the wire by 1 mm. How much energy is gained by the wire?

(a) 0.01 J (b) 0.02 J
(c) 0.04 J (d) 1.0 J

58. A uniform steel wire of length 4 m and area of cross-section 3×10^{-6} m^2 is extended by 1 mm by the application of a force. If the Young's modulus of steel is 2.0×10^{11} Nm^{-2} the energy stored in the wire is

(a) 0.025 J (b) 0.050 J
(c) 0.075 J (d) 0.100 J

59. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied to each?

(a) length = 0.5 m, diameter = 0.5 mm
(b) length = 1.0 m, diameter = 1.0 mm
(c) length = 2.0 m, diameter = 2.0 mm
(d) length = 3.0 m, diameter = 3.0 mm

60. A steel wire of diameter 2 mm has a breaking strength of 4×10^5 N. The breaking strength of similar steel wire of diameter 1 mm will be

(a) 4×10^5 N (b) 2×10^5 N
(c) 1×10^5 N (d) 0.5×10^5 N

61. A copper wire of length 1.0 m and a steel wire of length 0.5 m having equal cross-sectional areas are joined end to end. The composite wire is stretched by a certain load which stretches the copper wire by 1 mm. If the Young's modulii of copper and steel are 1.0×10^{11} Nm^{-2} and 2.0×10^{11} Nm^{-2}, the total extension of the composite wire is

(a) 1.25 mm (b) 1.50 mm
(c) 1.75 mm (d) 2.0 mm

62. Two springs of equal lengths and equal cross-sectional areas are made of materials whose Young's modulii are in the ratio of 3 : 2. They are suspended and loaded with the same mass. When stretched and released, they will oscillate with time periods in the ratio of

(a) $\sqrt{3} : \sqrt{2}$ (b) 3 : 2
(c) $3\sqrt{3} : 2\sqrt{2}$ (d) 9 : 4

63. A wire of length L and cross-sectional area A is made of a material of Young's modulus Y. The work one in stretching the wire by an amount x is given by

(a) $\dfrac{YA\,x^2}{L}$ (b) $\dfrac{YA\,x^2}{2L}$
(c) $\dfrac{YA\,L^2}{x}$ (d) $\dfrac{YA\,L^2}{2x}$

64. A solid sphere of radius R and made of a material of bulk modulus K is completely immersed in a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid, the fractional change in the radius of the sphere, $\delta R/R$ is given by

(a) $\dfrac{Mg}{KA}$ (b) $\dfrac{Mg}{2KA}$
(c) $\dfrac{Mg}{3KA}$ (d) $\dfrac{Mg}{4KA}$

65. A composite wire of a uniform cross-section of 5.5×10^{-5} m^2 consists of a steel wire of length 1.5 m and a copper wire of length 2.0 m. By how much will it stretch when it is loaded with a mass of 200 kg? Young's modulus of steel is 2×10^{11} Nm^{-2} and that of copper is 1×10^{11} Nm^{-2}. Take $g = 10$ ms^{-2}.

(a) 1 mm (b) 2 mm
(c) 3 mm (d) 4 mm

66. The density of water at the surface of the ocean is ρ. If the bulk modulus of water is B, what is the density of ocean water at a depth where the pressure is nP_0, where P_0 is the atmospheric pressure?

(a) $\frac{\rho B}{B-(n-1)P_0}$ (b) $\frac{\rho B}{B+(n-1)P_0}$

(c) $\frac{\rho B}{B-nP_0}$ (d) $\frac{\rho B}{B+nP_0}$

67. One end of a uniform rod of mass M and cross-sectional area A is suspended from rigid support and an equal mass M is suspended from the other end. The stress at the mid-point of the rod will be

(a) $\frac{2Mg}{A}$ (b) $\frac{3Mg}{2A}$

(c) $\frac{Mg}{A}$ (d) zero

68. Two rods of different materials, having coefficients of linear expansion α_1 and α_2 and Young's modulii Y_1 and Y_2, are fixed between two rigid walls. The rods are heated to the same temperature. There is no bending of rods. If $\alpha_1 : \alpha_2 = 2 : 3$, the thermal stresses developed in the two rods will be equal provided $Y_1 : Y_2$ is equal to

(a) 2 : 3 (b) 1 : 1

(c) 3 : 2 (c) 4 : 9

69. Two spherical soap bubbles formed in vacuum have diameters 3.0 mm and 4.0 mm. They coalesce to form a single spherical bubble. If the temperature remains unchanged, the diameter of the bubble so formed will be

(a) 5.0 mm (b) 5.8 mm

(c) 6.2 mm (d) 7.0 mm

70. A spherical liquid drop of diameter D breaks up into n identical spherical drops. If the surface tension of the liquid is σ, the change in energy in this process is

(a) $\pi\sigma D^2 (n^{1/3} - 1)$ (b) $\pi\sigma D^2 (n^{2/3} - 1)$

(c) $\pi\sigma D^2 (n - 1)$ (d) $\pi\sigma D^2 (n^{4/3} - 1)$

71. A liquid of density ρ and surface tension σ rises to a height h in a capillary tube of diameter d. The weight of the liquid in the capillary tube is

(a) $2\pi\sigma h$ (b) $\frac{2\pi\sigma h^2}{d}$

(c) $\pi\sigma d$ (d) $\frac{\pi\sigma d^2\rho}{h}$

72. In Q. 72 above, the potential energy of the liquid in the capillary tube is

(a) $h\rho g$ (b) $\frac{2\pi\sigma^2}{\rho g}$

(c) $2\pi\sigma^2\rho g$ (d) $\frac{2\pi\sigma^2}{\rho g h}$

73. A needle of length l and density ρ will float on a liquid of surface tension σ if its radius r is less than or equal to

(a) $\sqrt{\frac{2\sigma}{\pi\rho lg}}$ (b) $\sqrt{\frac{2\sigma l}{\pi\rho g}}$

(c) $\sqrt{\frac{\sigma}{\pi\rho g}}$ (d) $\sqrt{\frac{2\sigma}{\pi\rho g}}$

74. A film of water is formed between two straight parallel wires, each 10 cm long and at a separation of 0.5 cm. The work that must be done to increase the separation between the wires by 1 mm is (surface tension of water = 7.0×10^{-2} Nm^{-1})

(a) 7.0×10^{-5} N (b) 1.4×10^{-5} N

(c) 7.0×10^{-7} N (d) 1.4×10^{-7} N

75. Two separate air bubbles of radii r_1 and r_2 ($r_2 > r_1$) formed of the same liquid come together to form a double bubble. The radius of the internal film surface common to both bubbles is

(a) $\frac{r_1 r_2}{r_2 - r_1}$ (b) $\frac{r_1 r_2}{r_2 + r_1}$

(c) $\frac{1}{2}(r_1 + r_2)$ (d) $(r_2 - r_1)$

76. Find the depth at which an air bubble of radius 0.7 mm will remain in equilibrium in water. Given, surface tension of water = 7.0×10^{-2} Nm^{-1}. Take $g = 10$ ms^{-2}.

(a) 2 cm (b) 3 cm

(c) 4 cm (d) 5 cm

77. If a number of identical droplets of water, each of radius r, coalesce to form a single drop of radius R, the resulting rise in the temperature of water is given by (here ρ is the density of water, s its specific heat and σ its surface tension)

(a) $\frac{\sigma}{\rho s}\left(\frac{1}{r} - \frac{1}{R}\right)$ (b) $\frac{3\sigma}{\rho s}\left(\frac{1}{r} - \frac{1}{R}\right)$

(c) $\frac{\sigma}{\rho s}\left(\frac{1}{r} + \frac{1}{R}\right)$ (d) $\frac{3\sigma}{\rho s}\left(\frac{1}{r} + \frac{1}{R}\right)$

78. Work W is required to be done to form a spherical bubble of volume V from a given soap solution. How much work is needed to form a spherical bubble of volume $2V$?

(a) $2W$ (b) $\sqrt{2}\ W$

(c) $(2^{1/3})\ W$ (d) $(4^{1/3})\ W$

79. A circular wire frame of radius R is dipped in a soap solution of surface tension σ. When it is taken out a

thin soap film is formed inside the frame. The force on the frame will be

(a) $\pi\sigma R$ (b) $2\pi\sigma R$

(c) $4\pi\sigma R$ (d) $8\pi\sigma R$

80. A thin loop of a thread floats on a soap film formed inside a wire frame which is kept horizontal. When the film is pierced, the loop arranges in the form of a circle of radius r. If σ is the surface tension of the soap solution, the tension in the wire will be

(a) $\pi\sigma R$ (b) $2\pi\sigma R$

(c) $4\pi\sigma R$ (d) $8\pi\sigma R$

81. A ring of external and internal radii r_1 and r_2 just touches the horizontal surface of a liquid of surface tension σ. The force required to pull the ring away from the surface is

(a) $2\pi(r_1 + r_2)\sigma$ (b) $2\pi(r_1 - r_2)\sigma$

(c) $4\pi(r_1 + r_2)\sigma$ (d) $4\pi(r_1 - r_2)\sigma$

82. A liquid is kept in a cylindrical vessel. When the vessel is rotated about its axis, the liquid rises at its sides. If the radius of the vessel is 0.05 m and the frequency of rotation is 2 revolutions per second, the difference in the heights of the liquid at the centre and at the sides of the vessel will be (take $g = 10\ \text{ms}^{-2}$ and $\pi^2 = 10$)

(a) 2 cm (b) 4 cm

(c) 1 cm (d) 8 cm

83. A cylindrical tank of height H is completely filled with water. On its vertical side there are two tiny holes, one above the middle at a height h_1 and the other below the middle at a depth h_2. If the jets of water from the holes meet at the same point at the horizontal plane through the bottom of the tank then the ratio $\frac{h_1}{h_2}$ is

(a) 1 (b) 2

(c) 3 (d) 4

84. A cylindrical vessel of radius r is filled with a liquid to a height h such that the force exerted by the liquid on the sides of the vessel is equal to that exerted at the bottom. The ratio $\frac{h}{r}$ is

(a) 1 (b) 2

(c) 3 (d) 4

85. A liquid flows through two capillary tubes A and B connected in series. The length and radius of B are twice those of A. The ratio of the pressure difference across A to that across B is

(a) 8 (b) 4

(c) 2 (d) 1

86. A tiny sphere of mass m and density x is dropped in a tall jar of glycerine of density y. When the sphere acquires terminal velocity, the magnitude of the viscous force acting on it is

(a) $\frac{mgx}{y}$ (b) $\frac{mgy}{x}$

(c) $mg\left(1-\frac{y}{x}\right)$ (d) $mg\left(1+\frac{x}{y}\right)$

87. Two capillary tubes of the same length l and radii r and $2r$ are fitted to the bottom of a vessel with pressure head p in parallel with each other. What should be the radius of the single tube of the same length l that can replace the two so that the rate of flow is not affected?

(a) $(17)^{1/4}\,r$ (b) $17\,r$

(c) $8.5\,r$ (d) $\sqrt{17}\,r$

88. A long metal rod of length l and relative density σ is held vertically with its lower end just touching the surface of water. The speed of the rod when it just sinks in water is given by

(a) $\sqrt{2gl}$ (b) $\sqrt{2gl\sigma}$

(c) $\sqrt{2gl\left(1-\frac{1}{2\sigma}\right)}$ (d) $\sqrt{2gl(2\sigma-1)}$

89. A sphere of relative density σ and diameter D has concentric cavity of diameter d. It will just float on water in a tank if the ratio D/d is

(a) $\frac{\sigma}{(\sigma-1)}$ (b) $\frac{(\sigma+1)}{\sigma}$

(c) $\left(\frac{\sigma}{\sigma-1}\right)^{1/3}$ (d) $\left(\frac{\sigma+1}{\sigma}\right)^{1/3}$

90. A large block of ice 5 m thick has a vertical hole drilled through it and is floating in the middle of a lake. The minimum length of the rope required to scoop up bucket full of water through the hole is (the relative density of ice = 0.9)

(a) 1 m (b) 0.9 m

(c) 0.5 m (d) 0.45 m

91. A body floats in a liquid contained in a beaker. The whole system falls freely under gravity. The upthrust on the body due to the liquid is

(a) zero

(b) equal to the weight of the liquid displaced

(c) equal to the weight of the body in air

(d) equal to the weight of the immersed portion of the body.

92. A closed compartment containing gas is moving with some acceleration in horizontal direction.

Neglect the effect of gravity. Then the pressure in the compartment is
(a) the same everywhere
(b) lower in the front side
(c) lower in the rear side
(d) lower in the upper side

93. Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is steady. The cross-sectional area of the stream 0.15 m below the tap is (take $g = 10$ ms^{-2})
(a) 5.0×10^{-4} m^2 (b) 1.0×10^{-5} m^2
(c) 5.0×10^{-5} m^2 (d) 2.0×10^{-5} m^2

94. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both the holes are the same. Then R is equal to
(a) $\frac{L}{\sqrt{2\pi}}$ (b) $2\pi L$
(c) L (d) $\frac{L}{2\pi}$

95. A uniform wire of cross-sectional area A and Young's modulus Y is stretched within the elastic limit. If S is the stress in the wire, the elastic energy density stored in the wire in terms of the given parameters is
(a) $\frac{S}{2Y}$ (b) $\frac{2Y}{S^2}$
(c) $\frac{S^2}{2Y}$ (d) $\frac{S^2}{Y}$

96. One end of a uniform wire of length L and of weight W is attached rigidly to a point in the ceiling and a weight W_1 is suspended from its lower end. If S is the area of cross-section of the wire, the stress in the wire at a height $3L/4$ from its lower end is
(a) W_1/S (b) $\left(W_1 + \frac{W}{4}\right)\Big/S$
(c) $\left(W_1 + \frac{3W}{4}\right)\Big/S$ (d) $(W_1 + W)/S$

97. A uniform wire (Young's modulus 2×10^{11} Nm^{-2}) is subjected to a longitudinal tensile stress of 5×10^7 Nm^{-2}. If the overall volume change in the wire is 0.02%, the fractional decrease in the radius of the wire is
(a) 1.5×10^{-4} (b) 1.0×10^{-4}
(c) 0.5×10^{-4} (d) 0.25×10^{-4}

98. An elastic spring of unstretched length L and force constant k is stretched by a small length x. It is further stretched by another small length y. The work done in the second stretching is
(a) $\frac{1}{2}\ ky^2$ (b) $\frac{1}{2}\ k(x^2 + y^2)$
(c) $\frac{1}{2}\ k(x + y)^2$ (d) $\frac{1}{2}\ ky(2x + y)$

99. A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass is M. It is suspended by a string in a liquid of density ρ where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is (Fig. 7.31)
(a) Mg (b) $Mg - V\rho g$
(c) $Mg + \pi R^2 h\rho g$ (d) $\rho g\,(V + \pi R^2 h)$

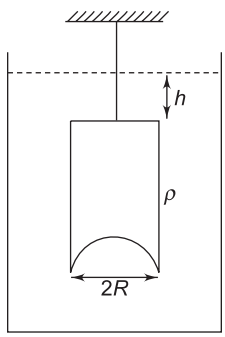


Fig. 7.31

100. Figure 7.32 shows a graph of the extension (Δl) of a wire of length 1 m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross-sectional area of the wire is 10^{-6} m^2, the Young's modulus of the material of the wire is
(a) 2×10^{11} N/m^2 (b) 2×10^{-11} N/m^2
(c) 3×10^{12} N/m^2 (d) 3×10^{-12} N/m^2

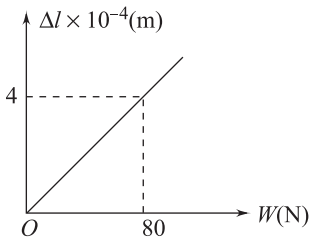


Fig. 7.32

101. A capillary tube is immersed vertically in water and the height of the water column is x. When this arrangement is taken into a mine of depth d, the

height of the water column is y. If R is the radius of the earth, the ratio $\frac{x}{y}$ is

(a) $\left(1-\frac{d}{R}\right)$ (b) $\left(1+\frac{d}{R}\right)$

(c) $\left(\frac{R-d}{R+d}\right)$ (d) $\left(\frac{R+d}{R-d}\right)$

102. The length of an elastic string is a meter when the longitudinal tension is 4 N and b metre when the tension is 5 N. The length of the string (in metre) when the longitudinal tension is 9 N is

(a) $a-b$ (b) $5b-4a$

(c) $2b-\frac{a}{2}$ (d) $4a-3b$

103. The Poisson's ratio of a material is 0.4. If a force is applied to a wire of this material, there is a decrease of cross-sectional area by 2%. The percentage increase in its length is:

(a) 3% (b) 2.5%

(c) 1% (d) 0.5%

104. A water barrel having water upto a depth d is placed on a table of height h. A small hole is made on the wall of the barrel at its bottom. If the stream of water coming out of the hole falls on the ground at a horizontal distance 'R' from the barrel, then the value of d is

(a) $\frac{4h^2}{R}$ (b) $\frac{2h^2}{R}$

(c) $\frac{R^2}{4h}$ (d) $\frac{R^2}{2h}$

105. Tanks A and B open at the top contain two different liquids upto certain height in them. A hole is made to the wall of each tank at a depth 'h' from the surface of the liquid. The area of the hole in A is twice that of in B. If the liquid mass flux through each hole is equal, then the ratio of the densities of the liquids respectively, is:

(a) $\frac{2}{1}$ (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

106. A light rod of length L is suspended from a support horizontally by means of two vertical wires A and B of equal length as shown in Fig. 7.33. The cross-sectional area of A is half that of B and the Young's modulus of A is twice that of B. A weight W is hung as shown. The value of x so that W produces equal stress in wires A and B is

(a) $\frac{L}{3}$ (b) $\frac{L}{2}$

(c) $\frac{2L}{3}$ (d) $\frac{3L}{4}$

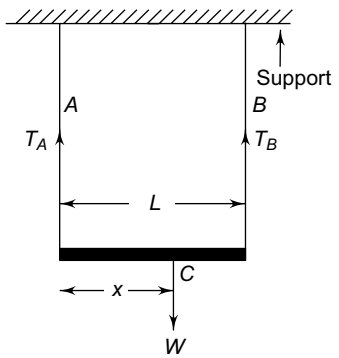


Fig. 7.33

107. In Q. 106 above, the value of x at which W produces equal strain in wires A and B is

(a) $\frac{L}{4}$ (b) $\frac{L}{2}$

(c) $\frac{2L}{3}$ (d) $\frac{4L}{5}$

108. A wire breaks when subjected to a stress S. If ρ is the density of the material of the wire and g, the acceleration due to gravity, then the length of the wire so that it breaks by its own weight is

(a) ρgS (b) $\frac{\rho g}{S}$

(c) $\frac{gS}{\rho}$ (d) $\frac{S}{\rho g}$

109. A stone of mass m is attached to one end of a wire of cross-sectional area A and Young's modulus Y. The stone is revolved in a horizontal circle at a speed such that the wire makes an angle θ with the vertical. The strain produced in the wire will be.

(a) $\frac{mg\cos\theta}{AY}$ (b) $\frac{mg}{AY\cos\theta}$

(c) $\frac{mg\sin\theta}{AY}$ (d) $\frac{mg}{AY\sin\theta}$

110. The density of a metal at a normal pressure is ρ. Its density when it is subjected to an excess pressure p is ρ'. If B is the bulk modulus of the metal, the ratio ρ'/ρ is

(a) $\frac{1}{1-p/B}$ (b) $1+\frac{p}{B}$

(c) $\frac{1}{1+p/B}$ (d) $1+B/p$

111. A block of mass M is suspended from a wire of mass m, cross-sectional area A and length L. If all the energy stored in the wire is converted into heat, the rise in the temperature of the wire is (Y = Young's modulus and s = specific heat capacity of the material of the wire).

(a) $\frac{(Mg)^2 L}{2YAms}$ (b) $\frac{MgLms}{2YA}$

(c) $\frac{2YAL}{(Mg)^2 ms}$ (d) none of these

112. A rubber cord of mass M, length L and cross-sectional area A is hung vertically from a ceiling. The Young's modulus of rubber is Y. If the change in the diameter of the cord due to its own weight is neglected, the increase in its length due to its own weight is

(a) $\frac{MgL}{AY}$ (b) $\frac{MgL}{2AY}$

(c) $\frac{2MgL}{AY}$ (d) $\frac{MgL}{4AY}$

113. The volume of a wire remains unchanged when the wire is subjected to a certain tension. The Poisson's ratio of the material of the wire is

(a) 0.25 (b) 0.3

(c) 0.4 (d) 0.5

114. A large container (with open top) of negligible mass and uniform cross-sectional area A has a small hole of cross-sectional area a in its side wall near the bottom. The container is keep on a smooth horizontal platform and contains a liquid of density ρ and mass m. If the liquid starts flowing out of the hole at time $t = 0$, the initial acceleration of the container is

(a) $\frac{ga}{A}$ (b) $\frac{gA}{a}$

(c) $\frac{2ga}{A}$ (d) $\frac{gA}{2a}$

115. In Q. 114 above, the velocity of the liquid when 75% of the liquid has drained out is

(a) $\sqrt{\frac{3mg}{4A\rho}}$ (b) $\sqrt{\frac{2mg}{A\rho}}$

(c) $2\sqrt{\frac{mg}{A\rho}}$ (d) $\sqrt{\frac{mg}{2A\rho}}$

116. A cylindrical tank having cross-sectional area A is filled with water to a height of 2.0 m. A circular hole of cross-sectional area a is opened at a height of 75 cm from the bottom. If $\frac{a}{A} = \sqrt{0.2}$, the velocity with which water emerges from the hole is ($g = 9.8$ ms^{-2}).

(a) 4.9 ms^{-1} (b) 4.95 ms^{-1}

(c) 5.0 ms^{-1} (d) 5.5 ms^{-1}

117. A small spherical ball of radius r falls freely under gravity through a distance h before entering a tank of water. If, after entering the water, the velocity of the ball does not change, then h is proportional to

(a) r^2 (b) r^3

(c) r^4 (d) r^5

118. A wide vessel of uniform cross-section with a small hole in the bottom is filled with 40 cm thick layer of water and 30 cm thick layer of kerosene. The relative density of kerosene is 0.8. The inital velocity of flow of water streaming out of the hole is (take g = 10 ms^{-2})

(a) $\frac{2}{\sqrt{5}}$ ms^{-1} (b) $\frac{4}{\sqrt{5}}$ ms^{-1}

(c) $\frac{6}{\sqrt{5}}$ ms^{-1} (d) $\frac{8}{\sqrt{5}}$ ms^{-1}

119. A constant pressure P is applied on all sides of a sphere at a certain temperature. By what amount should the temperature of the sphere by raised in order to bring its volume to the value it had before the pressure was applied? The coefficient of volume expansion of the material of the sphere is α and its compressibility is σ.

(a) $\frac{\sigma P}{\alpha}$ (b) $\frac{\alpha P}{\sigma}$

(c) $\alpha\sigma P$ (d) $\frac{P}{\alpha\sigma}$

120. The normal density of a metal is ρ and its bulk modulus is B. The increase in the density of a block of the metal when an excess pressure P is applied to it normally on all sides is

(a) $\frac{\rho P}{B}$ (b) $\frac{\rho B}{P}$

(c) $\frac{P}{\rho B}$ (d) ρPB

121. When a rubber cord is loaded, its length increases by 0.1%. If Poisson's ratio of rubber is 0.4, the percentage decrease in the diameter of the cord is

(a) 0.04% (b) 0.08%

(c) 0.10% (d) 0.12%

122. If a rubber cube is taken 50 m deep in a lake, its volume reduces by 0.5%. The bulk modulus of rubber is nearly.

(a) 10^6 Pa (b) 10^7 Pa

(c) 10^8 Pa (d) 10^9 Pa

123. A large number of identical droplets of a liquid, each of radius r, coalesce to form a bigger drop of radius R. It σ is the surface tension of the liquid and ρ its density and if 60% of the energy released is used up in increasing the kinetic energy of the big drop, the velocity of drop will be

(a) $\left[\frac{3.6\sigma}{\rho rR}(R-r)\right]^{1/2}$ (b) $\left[\frac{2.4\sigma}{\rho rR}(R-r)\right]^{1/2}$

(c) $\left[\frac{1.2\sigma}{\rho rR}(R-r)\right]^{1/2}$ (d) $\left[\frac{0.6\sigma}{\rho rR}(R-r)\right]^{1/2}$

124. In Q. 123 above, if 8 drops coalesce and if s is the specific heat capacity of the liquid and the remaining 40% of energy released is used up in heating the big drop, the rise in its temperature is

(a) $\frac{\sigma}{\rho sR}$ (b) $\frac{0.4\sigma}{\rho sR}$

(c) $\frac{0.8\sigma}{\rho sR}$ (d) $\frac{1.2\sigma}{\rho sR}$

125. A small steel ball of radius r experiences a viscous force F when it is falling in a jar of glycerine with terminal velocity v. The viscons force experienced by a steel ball of radius $r/2$ falling in glycerine with terminal velocity $v/2$ is

(a) F (b) $\frac{F}{2}$

(c) $\frac{F}{4}$ (d) $\frac{F}{8}$

126. A small copper ball of mass m falls in a viscous liquid with terminal velocity v. Another spherical copper ball of mass M falls in the same liquid with terminal velocity nv where n is a number. The ratio $\frac{M}{m}$ is

(a) $\sqrt{n}$ (b) n

(c) $n^{3/2}$ (d) n^2

127. A liquid flows steadily through two capillary tubes A and B connected in series. The lengths of A and B are L and $2L$ and their radii are r and $2r$. The ratio of pressure differences between the ends of tubes A and B is

(a) 1 (b) 2

(c) 4 (d) 8

128. A liquid flows steadily through a series combination of three capillary tubes of radii r, $2r$ and $3r$, all of the same length L. If the pressure difference across the combination is 28 cm of mercury, the pressure difference (in cm of Hg) across the tube of radius $2r$ is very nearly equal to

(a) 1.6 (b) 3.2

(c) 7.0 (d) 9.6

129. A cubical block of wood of side l floats at the interface between oil of relative density n and water with its lower face $x = l/4$ below the interfaces as shown in Fig. 7.34. The relative density of wood is

(a) $\frac{1}{2}(n+1)$ (b) $\frac{1}{3}(2n+1)$

(c) $\frac{1}{4}(3n+1)$ (d) $(1-n)$

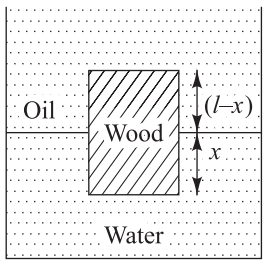


Fig. 7.34

130. A small cork ball of density σ is immersed in water of density ρ ($> \sigma$) to a depth h and released. If $\rho = 4\sigma$, the height up to which the ball will rise above the surface of water is

(a) h (b) $2h$

(c) $3h$ (d) $4h$

Answers

(Level A)

1. (b)	2. (a)	3. (c)	4. (a)
5. (c)	6. (a)	7. (b)	8. (c)
9. (a)	10. (c)	11. (a)	12. (c)
13. (c)	14. (d)	15. (b)	16. (c)
17. (d)	18. (b)	19. (a)	20. (d)
21. (a)	22. (b)	23. (d)	24. (a)
25. (d)	26. (b)	27. (b)	28. (b)
29. (b)			

(Level B)

30. (c)	31. (a)	32. (b)	33. (d)
34. (b)	35. (b)	36. (d)	37. (d)
38. (a)	39. (d)	40. (d)	41. (c)
42. (b)	43. (a)	44. (c)	45. (d)
46. (c)	47. (a)	48. (b)	49. (d)

50. (b)	51. (b)	52. (a)	53. (c)
54. (c)	55. (b)	56. (d)	57. (a)
58. (c)	59. (a)	60. (c)	61. (a)
62. (a)	63. (b)	64. (c)	65. (a)
66. (a)	67. (b)	68. (c)	69. (a)
70. (a)	71. (c)	72. (b)	73. (d)
74. (d)	75. (a)	76. (c)	77. (b)
78. (d)	79. (c)	80. (b)	81. (a)
82. (a)	83. (a)	84. (a)	85. (a)
86. (c)	87. (a)	88. (c)	89. (c)
90. (c)	91. (a)	92. (b)	93. (c)
94. (a)	95. (c)	96. (c)	97. (d)
98. (d)	99. (d)	100. (a)	101. (a)
102. (b)	103. (b)	104. (c)	105. (d)
106. (c)	107. (b)	108. (d)	109. (b)
110. (a)	111. (a)	112. (b)	113. (d)
114. (c)	115. (d)	116. (c)	117. (c)
118. (d)	119. (a)	120. (b)	121. (a)
122. (c)	123. (a)	124. (d)	125. (c)
126. (c)	127. (d)	128. (a)	129. (c)
130. (c)			



Solutions

(Level A)

1. Let the volume of the block be V m^3.

$$\text{Volume of block under liquid} = \frac{4V}{5} \text{ m}^3$$

$$\therefore \quad \text{Volume of liquid displaced} = \frac{4V}{5} \text{ m}^3$$

Now let the density of the liquid be ρ kg m^{-3}.

Mass of liquid displaced

= (volume of liquid displaced) × (density of liquid)

$$= \frac{4V}{5} \rho \text{ kg}$$

$$\text{Weight of liquid displaced} = \frac{4V}{5} \times \rho \times g \text{ newton}$$

$$\text{Relative density of wood} = 0.8$$

$$\therefore \quad \text{Density of wood} = 0.8 \times 1000$$

$$= 800 \text{ kg m}^{-3}$$

$\therefore$ Mass of the block = $800 \times V$ kg

Weight of the block = $800 \times V \times g$ newton

From the law of flotation,

Weight of block = weight of liquid displaced

$$\text{or} \qquad 800 \times V \times g = \frac{4V}{5} \times \rho \times g$$

$$\text{or} \qquad \rho = 800 \times \frac{5}{4}$$

$$= 1000 \text{ kg m}^{-3}$$

Hence the correct choice is (b).

2. Refer to the solution of Q.1 above. Notice that, although the buoyant force on the ice cube depends on the value of g, the fraction of the cube submerged under a liquid is independent of the value of g and depends only on the density of the body relative to that of the liquid on which it floats. Hence the correct choice is (a).

3. When the beaker, with the block floating in the liquid, is falling freely under gravity, both its weight and upthrust will be zero. Hence the block will float with any fraction of its volume submerged. Thus the correct choice is (c).

4. Volume of block = l^3. Let h be the height of the block above the surface of mercury. Volume of mercury displaced = $(l - h)l^2$.

$\therefore$ Weight of mercury displaced = $(l - h)l^2 \rho_m g$.

This is equal to the weight of the block which is $\rho_s l^3 g$.

Thus $\qquad (l - h)l^2 \rho_m g = \rho_s l^3 g$

which gives $\qquad h = l\left(1 + \frac{\rho_s}{\rho_m}\right)$

Hence the correct choice is (a).

5. The weight of the floating ice is equal to the weight of water displaced by it. So, when the ice melts, the volume of water produced by the melting ice is equal to the volume of water displaced by the ice cube. Hence the correct choice is (c).

6. The density of the liquid is $\rho = 1.25$ g cm^{-3} which is higher than the density of water which is 1.0 g cm^{-3}. This means that for x grams of ice, the volume of the liquid displaced is $x/\rho = x'$(say). But x grams of ice, on melting, will produce x cm^3 of water. Since $x > x'$, the level of liquid will rise. The volume of the liquid displaced by ice cube is less than the volume of the water formed when ice melts. Hence the correct choice is (a).

7. Let the mass of ice be m_1 and the mass of stone be m_2. The mass of the displaced water is equal to $(m_1 + m_2)$. If ρ is the density of water, the volume of water displaced is

$$V = (m_1 + m_2)/\rho$$

When the ice melts, additional volume of water obtained is m_1/ρ. The stone sinks in water, and displaces a volume of water equal to its own volume, which is m_2/ρ_s where ρ_s is the density of stone. Thus the total volume of extra water is $V' = \dfrac{m_1}{\rho} + \dfrac{m_2}{\rho_s}$

Since $\rho_s > \rho$, $\dfrac{1}{\rho_s} < \dfrac{1}{\rho}$

$\therefore$ $V' < V$. Therefore, level of water in beaker decreases.

8. Side of the cube l = 1 m. Base area of the cubical vessel = l^2. Mass of water contained in a height $dx = l^2 dx\rho$. Weight of this water = $l^2 dx\rho g$. Therefore, work done in pumping out water up to a height x is

$$dW = \rho g l^2 x dx$$

Therefore, total work done in pumping out water up to a height l is

$$W = \rho g l^2 \int_0^l x\,dx = \frac{1}{2}\ \rho g l^2\ \left|x^2\right|_0^l = \frac{1}{2}\ \rho g l^4$$

$$= \frac{1}{2} \times 1000 \times 10 \times (1)^4 = 5000 \text{ J}$$

Hence the correct choice is (c).

9. The coefficient of viscosity η is given by

$$F = -\eta A\ \frac{dv}{dx}$$

where F is the viscous force, A the area and dv/dx the velocity gradient. Thus, numerically

$$\eta = \frac{F}{A\,dv/dx}$$

$\therefore$ Dimensions of η

$$= \frac{\text{dimensions of } F}{\text{dimensions of } A \times \text{dimensions of } dv/dx}$$

$$= \frac{\text{MLT}^{-2}}{\text{L}^2 \times \text{LT}^{-1}/\text{L}} = \text{ML}^{-1}\text{T}^{-1}$$

Hence the correct choice is (a).

10. Let r be the radius of each small drop. The volume of each drop = $\dfrac{4}{3}\ \pi r^3$. Let R be the radius of the big drop formed when 8 small drops coalesce. The volume of the big drop will be 8 times that of the small drop. Hence

$$\frac{4}{3}\ \pi R^3 = 8 \times 4\pi\ \frac{r^3}{3}$$

or $R^3 = 8r^3$

or $R = 2r$

i.e. the radius of the big drop is twice the radius of each small drop. Now, if the buoyancy of the drop due to air is neglected, the terminal speed of the small drop is given by

$$v_t = \frac{2}{9}\frac{\rho r^2 g}{\eta} \qquad \text{(i)}$$

The terminal speed of the big drop will be

$$V_t = \frac{2}{9}\frac{\rho R^2 g}{\eta} \qquad \text{(ii)}$$

Dividing (i) and (ii), we have

$$\frac{V_t}{v_t} = \frac{R^2}{r^2} = \frac{(2r)^2}{r^2} = 4$$

or $V_t = 4v_t = 4 \times 6 = 24 \text{ cm s}^{-1}$

11. According to Bernoulli's theorem, for a horizontal flow at a height h from ground level,

$$p_1 + \frac{1}{2}\ \rho v_1^2 + \rho g h = p_2 + \frac{1}{2}\ \rho v_2^2 + \rho g h$$

or $$p_2 = p_1 + \frac{1}{2}\ \rho\,(v_1^2 - v_2^2)$$

$$= p + \frac{1}{2}\ \rho\{v^2 - (2v)^2\}$$

$$= p - \frac{3}{2}\ \rho v^2$$

Hence the correct choice is (a).

12. Let h be the depth of the hole below the free surface of water. (see Fig. 7.35).

According to Torricelli's theorem, the velocity of efflux v of water through the hole is given by

$$v = \sqrt{2gh} \qquad \text{(i)}$$

The height through which water falls is

$$S = H - h$$

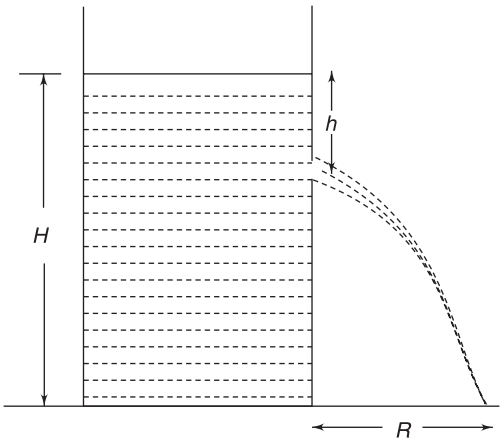


Fig. 7.35

If t is the time taken by water to strike the floor, then

$$S = \frac{1}{2}\, gt^2$$

or $$H - h = \frac{1}{2}\, gt^2$$

giving $$t = \sqrt{\frac{2(H-h)}{g}} \qquad \text{(ii)}$$

The distance R where the emerging stream strikes the floor is given by

$$R = vt$$

Substituting for v and t from Eqs. (i) and (ii), we get

$$R = \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}} = 2 \times \sqrt{h(H-h)}$$

Hence the correct choice is (c).

13. R will be maximum at value of h for which $\frac{dR}{dh} = 0$ and $\frac{d^2R}{dh^2} < 0$

Now $$R = 2(hH - h^2)^{1/2}$$

Differentiating with respect to h, we have

$$\frac{dR}{dh} = 2 \times \frac{1}{2}\, (hH - h^2)^{-1/2}\, (H - 2h) = \frac{(H-2h)}{\left(hH - h^2\right)^{1/2}} \qquad \text{(i)}$$

It is clear that $\frac{dR}{dh}$ will be zero at a value of h given by

$$H - 2h = 0$$

i.e. $$h = \frac{H}{2}$$

To find out whether $\frac{d^2R}{dh^2}$ is negative at this value of h, we differentiate Eq. (i) with respect to h to get

$$\frac{d^2R}{dh^2} = -\frac{2h}{(H-h)^{1/2}} \left\{1 + \frac{1}{4}\frac{(H-2h)}{h(H-h)}\right\}$$

Putting $h = H/2$, we have

$$\left(\frac{d^2R}{dh^2}\right)_{h=H/2} = -\sqrt{2H}$$

which is negative. Hence R will be maximum at $h = H/2$. Thus for range R to be maximum the hole must be exactly in the middle of the tank.

14. Cross-sectional area of tube $(A) = \pi R^2$. Cross-sectional area of each hole $= \pi r^2$. Therefore, cross-sectional area of n holes $(a) = \pi nr^2$. If v is the speed of ejection of the liquid through the holes, then from the continuity of flow, we have

$$av = AV$$

or $$v = \frac{AV}{a} = \frac{\pi R^2 V}{\pi nr^2} = \frac{V}{n}\left(\frac{R}{r}\right)^2$$

Hence the correct choice is (d).

15. Let p_1 and p_2 be the air pressures on the upper and lower surfaces of the wing. Then from Bernoulli's theorem we have

$$p_1 + \frac{1}{2}\, \rho v_1^2 = p_2 + \frac{1}{2}\, \rho v_2^2$$

where $v_1 = \sqrt{2}\, v$ and $v_2 = v$. Therefore, the pressure difference is

$$p_2 - p_1 = \frac{1}{2}\, \rho (v_1^2 - v_2^2) = \frac{\rho}{2}\, (2v^2 - v^2) = \frac{\rho v^2}{2}$$

$\therefore$ Force of dynamic lift = pressure difference $\times$ surface area of wing

$$= \frac{1}{2}\, \rho v^2 A,$$

which is choice (b).

16. If m is the mass of the stone and V its volume, the weight of the water displaced by it $= \rho Vg = \rho' \times \frac{m}{\rho} \times g = \frac{mg}{k}$, where ρ' is the density of water. Therefore, $k = \rho/\rho'$. Thus, the buoyant force acting upwards is mg/k whereas the weight mg of the stone acts vertically downwards. Therefore, the net force in the downward direction $= mg - mg/k = mg\left(1 - \frac{1}{k}\right)$. If a is the acceleration of the sinking stone, then

$$ma = mg\left(1-\frac{1}{k}\right)$$

or $$a = g\left(1-\frac{1}{k}\right)$$

Hence the correct choice is (c).

17. After the levels in the two vessels become equal, the increase in height of the liquid in one vessel is $\frac{1}{2}(h_2 - h_1)$ with the same decrease in height in the other. Thus, effectively a slab of liquid $\frac{1}{2}(h_2 - h_1)$ in thickness falls a vertical distance equal to its thickness under the action of gravity. Therefore,

Work done by the gravity is

$$W = m\,g \times \frac{1}{2}\,(h_2 - h_1)$$

where mass of the slab m is given by

$$m = \rho \times V = \rho \times \frac{1}{2}\,(h_2 - h_1) \times A$$

Therefore $$W = \frac{1}{2}\,\rho\,(h_2 - h_1) \times A \times g \times \frac{1}{2}(h_2 - h_1)$$

$$= \frac{1}{4}\,\rho\,A\,g\,(h_2 - h_1)^2$$

18. Let a cylinder with radius r be filled with a homogeneous liquid of density ρ up to a height h.
Pressure at the bottom of the cylinder, $p_1 = \rho\,g\,h$
Pressure at the top of the liquid surface, $p_2 = 0$
Average pressure on the sides of the cylinder,

$$p = \frac{p_1 + p_2}{2} = \rho\,g\,h/2$$

Force on the sides of the vessel = average pressure × area = $\frac{\rho g h}{2} \times 2\,\pi\,r\,h$

$$= \pi\,r\,\rho\,gh^2$$

Force on the bottom of the vessel = $p_1\,\pi\,r^2$
$= \rho\,g\,h\,\pi\,r^2$
Two forces will be equal if

$$\pi\,r\,\rho\,g\,h^2 = \rho\,g\,h\,\pi\,r^2$$

or $$h = r$$

19. The velocity of efflux through a hole is given by

$$v = \sqrt{2gh}$$

where h is the height of the tank. If the hole is at the top of the tank, v is obviously zero. Therefore, the average velocity of efflux $v_a = \frac{1}{2} \times \sqrt{2gh} = \sqrt{\frac{gh}{2}}$.
If the volume of water in the tank, when it is full, is V and A the cross-sectional area of the hole, then time taken by the tank to be emptied is

$$T = \frac{V}{Av_a} = \frac{\sqrt{2}V}{A\sqrt{gh}}$$

When the tank is half–full, V is replaced by $\frac{V}{2}$ and h is replaced by $\frac{h}{2}$. Therefore, time taken now is

$$T' = \frac{\sqrt{2}V/2}{A\sqrt{gh/2}} = \frac{V}{A\sqrt{gh}} = \frac{T}{\sqrt{2}}$$

Hence the correct choice is (a).

20. The volume of a liquid flowing per second through a capillary tube of length l and radius r, when a pressure difference P is maintained across its ends is given by

$$Q = \frac{\pi P r^4}{8\eta l}$$

where η is the coefficient of viscosity of the liquid. Since P is kept the same for tubes A and B, the correct choice is (d).

21. A series combination of two identical capillary tubes, each of length l and radius r, is equivalent to a tube of length $2l$ and radius r. Since Q is inversely proportional to l, if l is doubled, Q becomes half. Hence the correct choice is (a).

22. Since pressure difference across a tube is directly proportional to its length and the lengths of tubes A and B are in the ratio 2 : 1, the pressure difference across A and B is $P_a = \frac{2P}{3}$ and $P_b = \frac{P}{3}$. If l' is the equivalent length of the series combination, we have

$$Q' = Q_a + Q_b$$

or $$\frac{Pr^4}{8\eta l'} = \frac{2P}{3} \times \frac{r^4}{8\eta(2l)} + \frac{P}{3} \times \frac{r^4}{8\eta l}$$

or $$\frac{1}{l'} = \frac{1}{3l} + \frac{1}{3l} = \frac{2}{3l}$$

or $$l' = \frac{3l}{2}$$

$\therefore$ $$Q' = \frac{Pr^4}{8\eta(3l/2)} = \frac{2}{3}\frac{Pr^4}{8\eta l}$$

$$= \frac{2}{3}\,Q,$$ which is choice (b).

23. Since a bubble has two surfaces in contact with air, the total surface area of the bubble = $2 \times (4\pi r^2) = 8\pi r^2$. Now work done = surface tension × increase in surface area

$$= \sigma \times (8\pi r^2 - 0) = 8\pi r^2 \sigma$$

Hence the correct choice is (d).

24. Volume of big drop = $\frac{4}{3}\pi R^3$. If r is the radius of each tiny drop, the total volume of eight drops = $8 \times \frac{4\pi}{3} r^3 = \frac{32}{3}\pi r^3$. Since the total volume remains unchanged, we have

$$\frac{32}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

which gives $r = \frac{R}{2}$. Now, surface area of big drop = $4\pi R^2$.

Total surface area of eight drops = $32\pi r^2 = 8\pi R^2$ ($\because r = R/2$).

$\therefore$ Increase in surface area = $8\pi R^2 - 4\pi R^2 = 4\pi R^2$.

$\therefore$ Work done = surface tension × increase in surface area

$$= 4\pi\sigma R^2$$

Hence the correct choice is (a).

25. Surface area of bubble of radius $r = 4\pi r^2$. Surface area of bubble of radius $2r = 4\pi(2r)^2 = 16\pi r^2$. Therefore, increase in surface area = $16\pi r^2 - 4\pi r^2 = 12\ \pi r^2$. Since a bubble has two surfaces, the total increase in surface area = $24\ \pi r^2$.

$\therefore$ Energy spent = work done = $24\ \pi\sigma r^2$

Hence the correct choice is (d).

26. For an air bubble is formed inside a soap solution, there is only one surface in contact with soap solution. Hence the correct choice is (b).

27. An extremely thin layer of a liquid can be regarded as a collection of a large number of semi–spherical drops. Hence the excess pressure across a thin layer of a liquid is $\frac{\sigma}{r}$ instead of $\frac{2\sigma}{r}$ as in the case of a spherical drop, where $r = d/2$. Therefore, excess pressure is

$$p = \frac{\sigma}{r} = \frac{\sigma}{d/2} = 2\,\frac{\sigma}{d}$$

$\therefore$ Force due to surface tension pushing the two plates together is

F = excess pressure × area of layer

$$= \frac{2\sigma A}{d}$$

Hence the correct choice is (b).

28. Area of cross-section $a = \pi r^2$. Therefore $r = \sqrt{a/\pi}$. In terms of a, the height to which a liquid rises in a capillary tube, is given by

$$h = \frac{2\sigma\cos\theta}{r\rho g} = \frac{2\sqrt{\pi}\sigma\cos\theta}{\sqrt{a}\rho g}$$

Thus, h is inversely proportional to $\sqrt{a}$. If a is increased 4 times, h will decrease by a factor of 2. Hence the correct choice is (b).

29. The vertical height does not depend on the angle of inclination. Hence the correct choice is (b).

(Level B)

30. Let m be the mass of the ball and V its volume. Its mass $m = \rho V$. The weight of the ball is

$$W = mg = \rho Vg$$

The volume of the liquid displaced = V. If σ is the density of the liquid, the weight of the liquid displaced is the upthrust U it experiences.

$$U = V\sigma g$$

$\therefore$ The net downward force acting on the body is

$$F = W - U = (\rho - \sigma)\ Vg$$

The initial acceleration is

$$a = \frac{F}{m} = \frac{(\rho-\sigma)Vg}{\rho V} = \left(\frac{\rho-\sigma}{\rho}\right)g$$

Hence the correct choice is (c).

31. Refer to solution of Q. 30. The net downward force acting on the bob is

$$F = (\rho - \sigma)\ Vg = \left(\frac{\rho-\sigma}{\rho}\right) mg \qquad (\because m = \rho V)$$

or $$mg' = \left(\frac{\rho-\sigma}{\rho}\right) mg$$

or $$g' = \left(\frac{\rho-\sigma}{\rho}\right) g$$

This is the effective acceleration due to gravity.

$\therefore$ $$T' = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{\rho l}{(\rho-\sigma)g}}$$

$$= \left(\frac{\rho}{\rho-\sigma}\right)^{1/2} T \qquad \left(\because T = 2\pi\sqrt{\frac{l}{g}}\right)$$

Hence the correct choice is (a).

32. Volume of the block = $\frac{m}{\rho}$. Now upthrust = weight of water displaced = weight of volume m/ρ of liquid = mass of volume m/ρ of liquid $\times g = \frac{m\sigma g}{\rho}$. This is the upward force on the block due to buoyancy. The downward force on the block = its weight = mg. The tension in the string is the net upward force on the block which is

$$T = \frac{m\sigma g}{\rho} - mg = \left(\frac{(\sigma-\rho)}{\rho}\right) mg$$

Hence the correct choice is (b).

33. Since, loss of weight = weight of water displaced, the apparent mass of the balloon when completely immersed in water will be m, which is choice (d).

34. Weight of the ball $W = mg = \sigma Vg$. Upthrust $U = \rho Vg$. Therefore, the net upward force acting on the ball is

$$F = U - W = (\rho - \sigma)\, Vg$$

Now, mass of the ball is $m = \sigma V$. Therefore, upward acceleration of the ball while it is rising in the liquid is

$$a = \frac{F}{m} = \frac{(\rho-\sigma)Vg}{\sigma V} = \left(\frac{\rho-\sigma}{\sigma}\right)g$$

Velocity of the ball on reaching the surface of water is

$$v = \sqrt{2ah_1} \qquad \text{(i)}$$

This is the initial upward velocity of the ball in air. If it rises to a height h_2 in air, we have

$$v = \sqrt{2gh_2} \qquad \text{(ii)}$$

Equating (i) and (ii), we have $a\, h_1 = g\, h_2$

or $$\frac{h_2}{h_1} = \frac{a}{g} = \frac{\rho-\sigma}{\sigma} = \left(\frac{\rho}{\sigma} - 1\right)$$

Hence the correct choice is (b).

35. Work done to blow a bubble of radius r is

$$W = 4\pi\sigma r^2$$

In terms of volume $V = \frac{4\pi}{3} r^3$, we have

$$W = 4\pi\sigma\left(\frac{3V}{4\pi}\right)^{2/3}$$

Now, work done to blow a bubble of volume $8\,V$ is

$$W' = 4\pi\sigma\left(\frac{3\times 8V}{4\pi}\right)^{2/3}$$

$$= 4\pi\sigma\left(\frac{3V}{4\pi}\right)^{2/3} \times (8)^{2/3} = 4\ W$$

Hence the correct choice is (b).

36. From the equation of continuity of flow, we have

$$a_1 v_1 = a_2 v_2$$

or $$\pi r_1^2 v_1 = \pi r_2^2 v_2$$

or $$\frac{v_2}{v_1} = \frac{r_1^2}{r_2^2} = \frac{r^2}{(r/2)^2} = 4$$

or $$v_2 = 4v_1 = 4v.$$

Hence the correct choice is (d).

37. The density of the mixture is

$$\rho = \frac{\text{total mass}}{\text{total volume}} = \frac{m_1 + m_2 + m_3 + \cdots + m_n}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2} + \frac{m_3}{\rho_3} + \cdots + \frac{m_n}{\rho_n}}$$

For two substances mixed together, we have

$$\rho = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}} = \frac{(m_1 + m_2)\,\rho_1\rho_2}{(m_1\rho_2 + m_2\rho_1)}$$

Given $m_1 = m_2 = m$. Therefore, $\rho = \frac{2\rho_1\rho_2}{(\rho_2 + \rho_1)}$. Hence the correct choice is (d).

38. The density of the mixture is

$$\rho = \frac{m_1 + m_2}{V_1 + V_2}$$

$$= \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$$

where V_1 and V_2 are the volumes of the two substances.

Given $V_1 = V_2 = V$. Therefore, $\rho = \frac{1}{2}\ (\rho_1 + \rho_2)$. Hence the correct choice is (a).

39. We know that

$$v_t = \frac{2}{9}\frac{(\rho-\sigma)\,gr^2}{\eta} = kr^2,$$

where $$k = \frac{2}{9}\frac{(\rho-\sigma)\,g}{\eta}$$

Now $$V = \frac{4\pi}{3}\ r^3.$$

Therefore $r^2 = \left(\frac{3V}{4\pi}\right)^{2/3}$. Thus

$$v_t = k\left(\frac{3}{4\pi}\right)^{2/3} \times V^{2/3}$$

$\therefore$ Terminal velocity of the sphere of volume 8 V will be

$$V_t' = k\left(\frac{3}{4\pi}\right)^{2/3} \times (8V)^{2/3} = 4v_t \qquad [\because (8)^{2/3} = 4]$$

Hence the correct choice is (d).

40. The average velocity in the first half of the distance $= \frac{0+v}{2} = \frac{v}{2}$; while in the second half, the average velocity is v. Therefore, $t_1 > t_2$. The work done against gravity in both halves = $mgh = mgL/2$. Hence the correct choice is (d).

41. Mass of water in first tube is

$$m = \pi r^2 h\rho$$

Now, surface tension $\sigma = \frac{h\rho gr}{2} = \frac{h'\rho gr'}{2}$

where h' is the height to which water rises in the second tube and r' its radius. Since $r' = 2r$, $h' = h/2$. Therefore, the mass of water in the second capillary tube is

$$m' = \pi r'^2 h'\,\rho = \pi (2r)^2 \frac{h}{2}\,\rho$$

$$= 2\pi r^2\, h\,\rho = 2m = 2 \times 5 = 10 \text{ g}$$

Hence the correct choice is (c).

42. The mass of the liquid in a column between x and $x + dx$ is

$$dm = \pi r^2\,\rho\,dx$$

Therefore, the potential energy of the liquid in a column of height h is

$$\int_0^h \left(\pi r^2\right) x\,\rho g\,dx = \pi r^2\,\rho g \int_0^h x\,dx$$

$$= \pi r^2\,\rho g\,\frac{h^2}{2} = (\pi r^2 h\,\rho)\,gh/2$$

$$= mgh/2$$

Hence the correct choice is (b).

43. If the length of capillary tube is l, the pressure difference is given by

$$p = \frac{8\eta lQ}{\pi r^4}$$

where η is the coefficient of viscosity of water. If r becomes $2r$ and Q becomes $Q/2$, we have

$$p' = \frac{8\eta l(Q/2)}{\pi(2r)^4} = \frac{8\eta lQ}{\pi r^4} \cdot \frac{1}{32} = \frac{p}{32}$$

Hence the correct choice is (a).

44. From the equation of continuity of flow, the velocities v_1 and v_2 of the liquid at points where the areas of cross-section of the pipe are a_1 and a_2 respectively are related as

$$a_1 v_1 = a_2 v_2$$

$$\therefore \quad v_2 = \frac{a_1 v_1}{a_2} = \frac{\pi r_1^2 v_1}{\pi r_2^2}$$

$$= v_1 \left(\frac{r_1}{r_2}\right)^2$$

$$= 2 \times \left(\frac{6}{3}\right)^2 = 8 \text{ ms}^{-1}$$

Hence the correct choice is (c).

45. Consider two points A and B in the pipe at the same horizontal level. At point A, let a_1 be the area of cross-section of the pipe, v_1 the velocity of fluid flow and p_1 the fluid pressure. Let a_2, v_2 and p_2 be the corresponding quantities at point B. Then from Bernoulli's theorem, we have

$$p_1 + \frac{1}{2}\,\rho\, v_1^2 = p_2 + \frac{1}{2}\,\rho v_2^2$$

$$\text{or} \quad (p_1 - p_2) = \frac{1}{2}\,\rho\,(v_2^2 - v_1^2) \qquad \text{(i)}$$

If $a_2 < a_1$, it follows from continuity equation $a_1 v_1 = a_2 v_2$, that $v_1 > v_2$. From Eq. (i), it follows that $p_2 < p_1$. Hence, at the narrow part of the pipe, the fluid velocity increases but its pressure decreases, which is choice (d).

46. Refer to solution of Q. 12. We have seen that the range is given by

$$R = 2\sqrt{h(H-h)}$$

Notice that R remains the same if h and $(H - h)$ are interchanged. Thus, if hole A is at a height h above the ground, hole B must be at a distance $(H - h)$ from the ground for R to remain the same. Hence the correct choice is (c).

47. Refer to the solution of Q. 24. When a big drop breaks up into smaller drops, the total surface area of the smaller drops is more that the surface area of the big drop. The increase in the surface area can be brought about by supplying energy. Thus a big drop has to absorb energy to break up into smaller drops.

On the other hand, when smaller drops coalesce to form a big drop, there is a decrease in the surface area. Hence energy is liberated in this process, which is choice (a).

48. Since both balls have the same volume, they experience the same upthrust. Since the density of iron is greater than that of aluminium, the iron ball has a greater mass and therefore a greater weight; it therefore accelerates more and will reach the bottom before the aluminium ball. Hence the correct choice is (b).

49. The rate of flow is given by

$$Q = \frac{\pi P r^4}{8\eta l}$$

If l is doubled and r is halved, the rate of flow will become $\left(\frac{1}{2}\right)^4 \times \frac{Q}{2} = \frac{Q}{32}$. Hence the correct choice is (d).

50. Stress = $\frac{\text{force}}{\text{area}}$. Therefore, dimensions of stress are

$$\frac{\text{dimensions of force}}{\text{dimensions of area}} = \frac{\text{MLT}^{-2}}{\text{L}^2} = \text{ML}^{-1}\text{T}^{-2}$$

Hence the correct choice is (b).

51. The correct choice is (b).

52. The weight of the rope can be assumed to act at its mid-point. Now, the extension l is proportional to original length L. If the weight of the rope acts at its mid-point, the extension will be that produced by half the rope. So, replacing L by $L/2$ in the expression for Young's modulus, we have

$$Y = \frac{FL/2}{Al} = \frac{FL}{2Al}$$

$$\therefore \quad l = \frac{FL}{2AY} = \frac{gL^2\rho}{2Y}$$

($\because F = mg$ and $m = \rho V = \rho AL$)

Now $\rho = 1.5$ g cm^{-3} = 1500 kg m^{-3}, therefore

$$l = \frac{10 \times (8)^2 \times 1500}{2 \times 5 \times 10^6} = 9.6 \times 10^{-2} \text{ m}$$

Hence the correct choice is (a).

53. Here $l_1 = \frac{FL_1}{\pi r_1^2 Y}$ and $l_2 = \frac{FL_2}{\pi r_2^2 Y}$. Therefore,

$$\frac{l_1}{l_2} = \frac{L_1}{L_2} \times \left(\frac{r_2}{r_1}\right)^2$$

Given $L_1 = 2L_1$ and $r_2 = r_1/2$. Thus $\frac{l_1}{l_2} = \frac{1}{2} \times \frac{1}{(2)^2}$ $= \frac{1}{8}$. Hence the correct choice is (c).

54. The coefficient of linear expansion is defined as

$$\alpha = \frac{\text{increase in length}}{\text{original length} \times \text{temp.rise}} = \frac{l}{L\theta}$$

$\therefore$ Increase in length $l = \alpha L \theta$. Now

$$Y = \frac{FL}{Al}$$

or $$F = \frac{YAl}{L} = \frac{YA\alpha L\theta}{L} = YA\alpha\theta$$

Hence the correct choice is (c).

55. The force F can be assumed to act at the mid-point of the wire. Therefore, the average force responsible for extension is $F/2$. Thus, the work done by the force $\frac{F}{2}$ to produce an extension l in the wire = force × extension = $\frac{1}{2}$ $(F \times l)$. Hence the correct choice is (b).

56. Energy stored per unit volume = $\frac{1}{2}$ (stress × strain). But stress = Young's modulus × strain. Therefore energy stored per unit volume = $\frac{1}{2}$ $Y\varepsilon^2$, which is choice (d).

57. We have seen above that work done by a force F to produce an extension l in the wire = $\frac{1}{2}$ $F \times l$. Thus, the energy gained by the wire = $\frac{1}{2}$ $F \times l = \frac{1}{2} \times 20 \times (1 \times 10^{-3}) = 10^{-2}$ J = 0.01 J. Hence the correct choice is (a).

58. Refer to solution of Q. 56. Energy stored per unit volume = $\frac{1}{2}$ $Y \times \varepsilon^2$. Now

Strain (ε) = $\frac{l}{L} = \frac{1\text{mm}}{4\text{m}} = \frac{1 \times 10^{-3}\text{m}}{4\text{m}} = 2.5 \times 10^{-4}$

$\therefore$ Energy stored per unit volume = $\frac{1}{2} \times 2.0 \times 10^{11}$ $\times (2.5 \times 10^{-4})^2 = 6.25 \times 10^3$ Jm^{-3}.

Volume of the wire = $\pi r^2 L = AL = 3 \times 10^{-6} \times 4 = 1.2 \times 10^{-5}$ m^3. Therefore, Energy stored in the wire

$= 6.25 \times 10^3 \times 1.2 \times 10^{-5} = 7.5 \times 10^{-2}$ J = 0.075 J. Hence the correct choice is (c).

59. We know that

$$F = \frac{YAl}{L} = \frac{Yl\pi r^2}{L}$$

or $$l = \frac{FL}{\pi r^2 Y}$$

Since Y and F are the same for all wires, $l \propto \frac{L}{r^2}$ or $l \propto \frac{L}{d^2}$. It is easy to see that $\frac{L}{d^2}$ is the highest for the first wire. Hence the correct choice is (a).

60. The breaking strength is proportional to the square of the diameter ($F \propto d^2$). Since the diameter is reduced to half, the breaking strength is reduced to one-fourth. Hence the correct choice is (c).

61. When a wire of length L, cross-sectional area A and Young's modulus Y is stretched with a force F, the extension l in the wire is given by

$$l = \frac{FL}{AY}$$

Since F and A are the same for the two wires, we have

For copper wire $\quad l_c = \frac{FL_c}{AY_c}$

For steel wire, $\quad l_s = \frac{FL_s}{AY_s}$

$\therefore \quad l_s = l_c \frac{L_s}{L_c} \cdot \frac{Y_c}{Y_s}$

$$= 1 \text{ mm} \times \left(\frac{0.5\,\text{m}}{1.0\,\text{m}}\right) \times \left(\frac{1.0 \times 10^{11}\,\text{Nm}^{-2}}{2.0 \times 10^{11}\,\text{Nm}^{-2}}\right)$$

$$= 0.25 \text{ mm}$$

$\therefore$ Total extension = 1 + 0.25 = 1.25 mm.

Hence the correct choice is (a).

62. Young's modulus $Y = \frac{F}{A} \cdot \frac{L}{l}$

Force constant $k = \frac{F}{l} = \frac{YA}{L}$

where l is the extension in the spring of original length L and cross-sectional area A when a force $F = Mg$ is applied. Now, the time period of vertical oscillations is given by

$$T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{ML}{YA}}$$

$$\therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{Y_2}{Y_1}} = \sqrt{\frac{3}{2}}$$

Hence the correct choice is (a).

63. Work done per unit volume of the wire

$$= \frac{1}{2} \text{ (stress} \times \text{strain)}.$$

$$Y = \frac{\text{stress}}{\text{strain}}$$

$\therefore \quad \text{stress} = Y \times \text{strain}$

$\therefore$ Work done per unit volume

$$= \frac{1}{2} \times Y \times (\text{strain})^2 = \frac{1}{2} \times Y \times \left(\frac{x}{L}\right)^2$$

But, volume of the wire = $A \times L$

$\therefore$ Work done $= \frac{1}{2} \times Y \times \left(\frac{x}{L}\right)^2 \times A \times L$

$$= \frac{YAx^2}{2L}$$

Hence the correct choice is (b).

64. Pressure exerted by the piston on the liquid when a mass M is placed on the piston, $P = Mg/A$. This pressure is exerted by the liquid equally in all directions. Therefore, the surface of the sphere experiences a force P per unit area. The stress on the sphere is $P = Mg/A$. Now, the volume of the sphere is

$$V = \frac{4\pi R^3}{3}$$

Due to stress, the change in the volume of the sphere is

$$\Delta V = \Delta\left(\frac{4\pi R^3}{3}\right) = \frac{4\pi}{3} \cdot 3R^2\,\Delta R$$

$$= 4\pi R^2 \Delta R$$

$\therefore \quad$ Volume strain $\frac{\Delta V}{V} = \frac{3\Delta R}{R}$

By definition, bulk modulus

$$K = \frac{\text{stress}}{\text{strain}} = \frac{Mg/A}{3\Delta R/R}$$

or $$\frac{\Delta R}{R} = \frac{Mg}{3KA}$$

Hence the correct choice is (c).

65. Since the steel and copper parts of the wire are of the same cross-sectional area (A) and are loaded with the same weight $F = Mg$, they are under the same stress (F/A). Therefore,

$$\frac{Y_s l_s}{L_s} = \frac{Y_c l_c}{L_c} = \frac{F}{A} = \frac{Mg}{A}$$

or $$l_s = \frac{Mg}{A}\cdot\frac{L_s}{Y_s} \quad \text{and} \quad l_c = \frac{Mg}{A}\cdot\frac{L_c}{Y_c}$$

$\therefore$ Extension of the composite wire is

$$l = l_s + l_c = \frac{Mg}{A}\left(\frac{L_s}{Y_s} + \frac{L_c}{Y_c}\right)$$

Substituting the given values, we find that $l = 10^{-3}$ m = 1 mm. Hence the correct choice is (a).

66. Pressure at the surface of the ocean = P_0, the atmospheric pressure. Pressure at a depth = nP_0 (given).

$\therefore$ Increase in pressure $(\Delta P) = nP_0 - P_0$
$= (n-1)P_0$

Let V be the volume of a certain mass M of water at the surface, so that $M = \rho V$. The decrease in volume under pressure ΔP is

$$\Delta V = \frac{V\,\Delta P}{B}$$

$\therefore$ Volume of the same mass M of water at the given depth is

$$V' = V - \Delta V = V - \frac{V\,\Delta P}{B} = V\left(1 - \frac{\Delta P}{B}\right)$$
$$= \frac{V}{B}\,(B - \Delta P)$$

$\therefore$ Density of water at that depth is

$$\rho' = \frac{M}{V'} = \frac{\rho V}{V'} = \frac{\rho V}{\frac{V}{B}(B - \Delta P)}$$
$$= \frac{\rho B}{B - \Delta P} = \frac{\rho B}{B - (n-1)P_0}$$

Hence the correct choice is (a).

67. Since the rod is uniform, half its weight can be taken to act at its mid-point. Therefore, stress at mid-point is

$$\frac{\text{weight of suspended mass + weight of half the rod}}{\text{cross-sectional area}}$$

$$= \frac{Mg + \frac{1}{2}Mg}{A} = \frac{3Mg}{2A}$$, which is choice (b).

68. Young's modulus $Y = \dfrac{\text{stress}}{\text{strain}} = \dfrac{\sigma}{\varepsilon}$, where $\varepsilon = \dfrac{l}{L}$.

Now, if the temperature of a rod is increased by θ, the increase in its length due to thermal expansion is

$$l = \alpha L \theta$$

$\therefore$ Strain $\varepsilon = \dfrac{l}{L} = \alpha\theta$. Now stress is

$$\sigma = Y\varepsilon = Y\alpha\theta$$

For the two rods, the stress is

$$\sigma_1 = Y_1\alpha_1\,\theta$$

and $$\sigma_2 = Y_2\alpha_2\,\theta$$

But $\sigma_1 = \sigma_2$ (given). Hence $Y_1\alpha_1\,\theta = Y_2\alpha_2\,\theta$ or

$\dfrac{Y_1}{Y_2} = \dfrac{\alpha_2}{\alpha_1} = \dfrac{3}{2}$. Hence the correct choice is (c).

69. Since the bubbles are in vacuum, the pressure of air inside them are $P_1 = \dfrac{4\sigma}{r_1}$ and $P_2 = \dfrac{4\sigma}{r_2}$, where $r_1 = 3.0$ mm and $r_2 = 4.0$ mm. Since the temperature remains unchanged, we have from Boyle's law

$$P_1 V_1 + P_2 V_2 = PV$$

or $$\frac{4\sigma}{r_1}\cdot\frac{4}{3}\pi r_1^3 + \frac{4\sigma}{r_2}\cdot\frac{4}{3}\pi r_2^3 = \frac{4\sigma}{r}\cdot\frac{4}{3}\pi r^3 \qquad \text{(i)}$$

where r is the radius of the single bubble formed.

From (i), we get $r^2 = r_1^2 + r_2^2$ or $r = \sqrt{r_1^2 + r_2^2}$

$= \sqrt{(3.0)^2 + (4.0)^2} = 5.0$ mm, which is choice (a).

70. Refer to the solution of Q. 24. Initial energy $= 4\pi R^2\sigma = \pi D^2\sigma$ where R is the radius of the big drop. Final energy

$= (4\pi r^2)n\sigma = \pi d^2 n\sigma$, where $d = 2r$, is the radius of each tiny drop.

Now $R^3 = nr^3$ or $D^3 = nd^3$ or $D = n^{1/3}\,d$
or $d = D/n^{1/3}$.

Therefore, final energy $= \dfrac{\pi D^2 n\sigma}{n^{2/3}} = \pi D^2\,\sigma\,n^{1/3}$.

Hence, change in energy $= \pi D^2\,\sigma\,n^{1/3} - \pi D^2\sigma$
$= \pi D^2\,\sigma\,(n^{1/3} - 1)$,

which is choice (a).

71. Weight of the liquid in the capillary tube (W) = mg = $\pi r^2\,h\rho g$.

Now $$\sigma = \frac{h\rho r g}{2}$$ or $h\rho g r = 2\sigma$.

So, $W = 2\pi\sigma r = \pi\sigma d$. ($\because d = 2r$)

Hence the correct choice is (c).

72. Since the mass of the liquid in the capillary tube can be considered to be concentrated at the centre of mass which is at a height $h/2$, the potential energy is

$$\text{PE} = mg\frac{h}{2} = \pi r^2 h\rho g\frac{h}{2} = \frac{\pi(rhg)^2\rho}{2g} \quad (1)$$

Now $$\sigma = \frac{h\rho rg}{2} \text{ or } rhg = \frac{2\sigma}{\rho} \quad (2)$$

Using (2) in (1), we have

$$\text{PE} = \frac{\pi}{2g}\left(\frac{2\sigma}{\rho}\right)^2\rho = \frac{2\pi\sigma^2}{\rho g}$$

Hence the correct choice is (b).

73. Force per unit area exerted on the needle due to liquid is

$$P_1 = \frac{\sigma}{r}$$

Pressure exerted by needle on the liquid is

$$P_2 = \frac{mg}{l(2r)} = \frac{\pi r^2 l\rho g}{2lr} = \frac{\pi r\rho g}{2}$$

For equilibrium, $P_1 = P_2$, i.e.

$$\frac{\sigma}{r} = \frac{\pi r\rho g}{2}$$

or $$r = \sqrt{\frac{2\sigma}{\pi\rho g}}$$

Hence the correct choice is (d).

74. $F = 2\sigma l$. Work done is $W = FS = 2\sigma lS = 2 \times 7.0 \times 10^{-2} \times 0.1 \times 0.001 = 1.4 \times 10^{-7}$ N, which is choice (d).

75. Let r be the radius of the common surface and σ the surface tension of the liquid. When the bubbles come together, the difference of the excess pressures on either side of the common surface must be equal, i.e.

$$\frac{4\sigma}{r_1} - \frac{4\sigma}{r_2} = \frac{4\sigma}{r}$$

which gives $r = \dfrac{r_1 r_2}{r_2 - r_1}$, which is choice (a).

76. The bubble will be in equilibrium at a depth h if excess pressure $\dfrac{2\sigma}{r} = h\rho g$ or $h = \dfrac{2\sigma}{r\sigma g}$. Given $\sigma = 7.0 \times 10^{-2}$ Nm^{-1}, $r = 0.7 \times 10^{-3}$ m, $\rho = 10^3$ kg m^{-3} and $g = 10$ ms^{-2}. Using these values, we get $h = 4 \times 10^{-2}$ m = 4 cm, which is choice (c).

77. Refer also to the solution of Q. 70. Work done is

$$W = 4\pi R^2 (n^{1/3} - 1)\sigma = 4\pi R^2\left(\frac{R}{r} - 1\right) \quad (1)$$

($\because R = n^{1/3} r$)

Heat produced $$Q = ms\Delta T = \frac{4}{3}\pi R^3\rho s\Delta T \quad (2)$$

Equating (1) and (2), we find that choice (b) gives the correct expression for ΔT.

78. $W = 8\pi r^2\sigma$. Now $V = \dfrac{4}{3}\pi r^3$. Hence

$$W = 8\pi\sigma\left(\frac{3V}{4\pi}\right)^{2/3}, \text{ i.e. } W \propto V^{2/3}$$

Hence the correct choice is (d).

79. Total length of wire is $L = 2\pi R$. Since the film is in air, force on frame $= 2\sigma \times L = 2\sigma \times 2\pi R = 4\pi\sigma R$. Hence the correct choice is (c).

80. Tension $= \sigma \times 2\pi r = 2\pi\sigma r$, which is choice (b).

81. Force $= 2\pi r_1\sigma + 2\pi r_2\sigma = 2\pi(r_1 + r_2)\sigma$ which is choice (a).

82. Using Bernoulli's theorem, we have

$$\frac{1}{2}\rho v^2 = p = \rho gh$$

or $$h = \frac{v^2}{2g} \quad (1)$$

Now $v = r\omega = r(2\pi\nu)$. Using this in (1), we get

$$h = \frac{2\pi^2\nu^2 r^2}{g}$$

Given, $\nu = 2$ rev. per second, $r = 0.05$ m, $g = 10$ ms^{-2} and $\pi^2 = 10$. Using these values, we get $h = 0.02$ m = 2 cm, which is choice (a).

83. The horizontal range of a jet of water emerging from a hole at a height h below the surface of water is given by

$$R = 2\sqrt{h(H - h)}$$

The upper hole is at a height $\left(\dfrac{H}{2} + h_1\right)$ from the bottom and the lower hole is at a height $\left(\dfrac{H}{2} - h_2\right)$ from the bottom. Their depths from the surface are respectively $\left(\dfrac{H}{2} - h_1\right)$ and $\left(\dfrac{H}{2} + h_2\right)$. The horizontal ranges will be equal if

$$2\sqrt{\left(\frac{H}{2}+h_1\right)\left(\frac{H}{2}-h_1\right)} = 2\sqrt{\left(\frac{H}{2}+h_2\right)\left(\frac{H}{2}-h_2\right)}$$

which gives $h_1 = h_2$. Hence the correct choice is (a).

84. Pressure at the bottom = $h\rho g$. Therefore, the force exerted at the bottom is $F_1 = h\rho g \times \pi r^2$.

Now, since the mass of the liquid can be regarded as being concentrated at its centre of mass which is at a height $h/2$, the pressure on the sides = $\frac{h}{2}\ \rho g$.

Therefore, force exerted on sides is

$$F_2 = \frac{h}{2}\ \rho g \times 2\ \pi rh. \text{ Given } F_1 = F_2, \text{ or}$$

$$h\rho g \times \pi r^2 = \frac{h}{2}\ \rho g \times 2\ \pi rh$$

which gives $r = h$. Hence the correct choice is (a).

85. $Q_1 = \frac{\pi P_1 r_1^4}{8\eta l_1}$ and $Q_2 = \frac{\pi P_2 r_2^4}{8\eta l_2}$.

Since the tubes are connected in series, $Q_1 = Q_2$

$$\text{or} \quad \frac{\pi P_1 r_1^4}{8\eta l_1} = \frac{\pi P_2 r_2^4}{8\eta l_2}$$

$$\text{or} \quad \frac{P_1}{P_2} = \left(\frac{l_1}{l_2}\right)\times\left(\frac{r_2}{r_1}\right)^4 = \left(\frac{1}{2}\right)\times 2^4 = 8$$

Hence the correct choice is (a).

86. When the sphere acquires terminal velocity, the upward viscous force equals the apparent weight in glycerine. Therefore, the magnitude of the viscous force is (here r is the radius of the sphere)

$$F = \frac{4}{3}\ \pi r^3\,(x-y)g = \frac{m}{x}\ (x-y)g = mg\left(1-\frac{y}{x}\right)$$

Hence the correct choice is (c).

87. Radius R of the single tube is given by

$$\frac{\pi p R^4}{8\eta l} = \frac{\pi p r^4}{8\eta l} + \frac{\pi p (2r)^4}{8\eta l}$$

or $R^4 = r^4 + 16r^4$ or $R = (17)^{1/4}\ r$ which is choice (a).

88. Let the densities of metal and water be ρ and ρ_0 respectively and let x be the length of the rod immersed in water at an instant of time t. Then, acceleration at that instant = apparent weight divided by mass of the rod, i.e.

$$\frac{dv}{dt} = \frac{\pi r^2 l\rho g - \pi r^2 x\rho_0 g}{\pi r^2 l\rho} = g - \frac{gx\rho_0}{l\rho}$$

$$= g\left(1-\frac{x}{\sigma l}\right)$$

$$\text{or} \quad \frac{dv}{dx}\cdot\frac{dx}{dt} = g\left(1-\frac{x}{\sigma l}\right)$$

$$\text{or} \quad v\,\frac{dv}{dx} = g\left(1-\frac{x}{\sigma l}\right)$$

Integrating, we have

$$\frac{v^2}{2} = g\left|x-\frac{x^2}{2\sigma l}\right|_0^l = g\left(l-\frac{l}{2\sigma}\right)$$

$$\text{or} \quad v = \sqrt{2gl\left(1-\frac{1}{2\sigma}\right)} \text{ which is choice (c).}$$

89. Let ρ be the density of the sphere and ρ_0 that of water. Volume of metal = $\frac{4}{3}\ \pi(R^3 - r^3)$ and volume of water displaced = $\frac{4}{3}\ \pi R^3$. From the principle of floatation, we have

$$\frac{4}{3}\ \pi\,(R^3 - r^3)\,\rho g = \frac{4}{3}\ \pi R^3\,\rho 0\,g$$

$$\text{or} \quad (R^3 - r^3)\ \frac{\rho}{\rho_0} = R^3$$

$$\text{or} \quad (R^3 - r^3)\sigma = R^3$$

$$\text{which gives} \quad \frac{R}{r} = \left(\frac{\sigma}{\sigma-1}\right)^{1/3}$$

Hence the correct choice is (c).

90. Let h be the height of the block and A its area of cross-section. Weight of ice block = $Ahg \times 0.9$. If x is the height of the block immersed in water, then the weight of water displaced = Axg. Equating the two we get $x = 0.9\ h$. Height above the surface of water = $h - 0.9\ h = 0.1\ h = 0.1 \times 5$ m = 0.5 m. Hence the correct choice is (c).

91. For a body falling freely under gravity, the effective value of g is zero. Hence upthrust is zero. Hence the correct choice is (a).

92. Due to frictional force (which acts in a direction opposite to the direction of acceleration) on the rear face, the pressure in the rear side will be increased. Hence the pressure in the front side will be lowered. Hence the correct choice is (b).

93. The equation of continuity of flow is $v_1\, a_1 = v_2\, a_2$ where $v_1 = 1.0$ ms^{-1}, $a_1 = 10^{-4}$ m^2, v_2 = velocity of stream at $h = 0.15$ m below the tap. The value of v_2 is given by

$$v_2^2 = v_1^2 + 2gh$$
$$= 1.0 + 2 \times 10 \times 0.15 = 4.0$$

or $\quad v_2 = 2.0$ ms^{-1}.

Now $\quad a_2 = \dfrac{v_1 a_1}{v_2} = \dfrac{1.0 \times 10^{-4}}{2.0} = 5.0 \times 10^{-5}$ m^2

Hence the correct choice is (c).

94. The ratio of volume of water flowing out per second is given by

$$\frac{V_1}{V_2} = \frac{v_1 a_1}{v_2 a_2} = \frac{v_1 (L)^2}{v_2 (\pi R^2)} \qquad (1)$$

The velocities of water flowing out are given by

$$v_1 = \sqrt{2gy} \text{ and } v_2 = \sqrt{2g(4y)}$$

$$\therefore \quad \frac{v_1}{v_2} = \frac{\sqrt{2gy}}{\sqrt{8gy}} = \frac{1}{2} \qquad (2)$$

Using (2) in (1), we have

$$\frac{V_1}{V_2} = \frac{1}{2}\,\frac{L^2}{\pi R^2}$$

Given $\quad V_1 = V_2$. Therefore $1 = \dfrac{1}{2}\,\dfrac{L^2}{\pi R^2}$

or $\quad R = \dfrac{L}{\sqrt{2\pi}}$ which is choice (a).

95. Let the strain be ε. Then $Y = \dfrac{S}{\varepsilon}$ or $\varepsilon = \dfrac{S}{Y}$. Now energy density $= \dfrac{1}{2}$ stress $\times$ strain $= \dfrac{1}{2} \times S \times \dfrac{S}{Y}$ $= \dfrac{S^2}{2Y}$. Hence the correct choice is (c).

96. Since the wire is uniform, i.e. its mass per unit length is constant over its entire length L, the total downward weight = the weight of the suspended mass + weight of length $\dfrac{3L}{4}$ of the wire or $F = W_1 + \dfrac{3W}{4}$.

$$\therefore \quad \text{Stress} = \frac{\text{force}}{\text{area}} = \frac{F}{S} = \frac{W_1 + \frac{3}{4}W}{S}$$

Hence the correct choice is (c).

97. $\dfrac{\Delta L}{L} = \dfrac{\text{Stress}}{Y} = \dfrac{5 \times 10^7}{2 \times 10^{11}} = 2.5 \times 10^{-4}$. Given $\dfrac{\Delta V}{V}$ $= \dfrac{0.02}{100} = 2 \times 10^{-4}$

Now, $\quad V = \pi r^2 L$. Therefore,

$$\frac{\Delta V}{V} = \frac{\Delta(\pi r^2 L)}{\pi r^2 L} = \frac{2\pi r \Delta r L + \pi r^2 L \Delta L}{\pi r^2 L}$$
$$= 2\frac{\Delta r}{r} + \frac{\Delta L}{L}$$

or $\quad \dfrac{2\Delta r}{r} = \dfrac{\Delta V}{V} - \dfrac{\Delta L}{L} = 2 \times 10^{-4} - 2.5 \times 10^{-4}$
$$= -0.5 \times 10^{-4}$$

or $\quad \dfrac{\Delta r}{r} = -0.25 \times 10^{-4}$

$\therefore$ Fractional decrease in $r = 0.25 \times 10^{-4}$ which is choice (d).

98. Potential energy stored in the spring when it is extended by x is

$$U_1 = \frac{1}{2}\, kx^2$$

Potential energy stored in the spring when it is further extended by y is

$$U_2 = \frac{1}{2}\, k(x+y)^2$$

$$\therefore \text{ Work done} = U_2 - U_1 = \frac{1}{2}\, k(x+y)^2 - \frac{1}{2}\, kx^2$$
$$= \frac{1}{2}\, ky(2x+y)$$

Hence the correct choice is (d).

99. From Archimedes' principle, $F_b - F_t$ = upthrust = weight of volume V of the liquid. Here F_b and F_t denote the force exerted by the liquid on the bottom and the top of the cylinder respectively. Upthrust = ρVg and $F_t = (\pi R^2 h)\, \rho g$. Hence

$$F_b = (\pi R^2 h)\rho g + \rho V g = \rho g\, (\pi R^2 h + V)$$

Hence the correct choice is (d).

100. Stress $= \dfrac{\text{force}}{\text{area}} = \dfrac{W}{A}$. Strain $= \dfrac{\Delta l}{l}$. Young's modulus is given by

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{W/A}{\Delta l/l} = \frac{W}{\Delta l} \times \frac{l}{A} \qquad \text{(i)}$$

Now, slope of graph is $\dfrac{\Delta l}{W} = \dfrac{4 \times 10^{-4}}{80}$ m/N. Using this in (i), we get (given $l = 1$ m and $A = 10^{-6}$ m^2)

$$Y = \frac{80}{4 \times 10^{-4}} \times \frac{1}{10^{-6}} = 2 \times 10^{11} \text{ N/m}^2$$

101. On earth's surface, $\sigma = \dfrac{x\rho r g}{2}$

In the mine, $\sigma = \dfrac{y\rho r g_d}{2}$

Dividing, we get $\dfrac{x}{y} = \dfrac{g_d}{g} = \dfrac{g\left(1-\dfrac{d}{R}\right)}{g} = 1 - \dfrac{d}{R}$

Hence the correct choice is (a).

102. If L is the initial length, then the increase in length by a tension F is given by

$$l = \frac{FL}{\pi r^2 Y}$$

Hence $$a = L + l = L + \frac{4L}{\pi r^2 Y} = L + 4c \qquad (1)$$

and $$b = L + \frac{5L}{\pi r^2 Y} = L + 5c \qquad (2)$$

where $c = \dfrac{L}{\pi r^2 Y}$. Solving (1) and (2) for L and c, we get $L = 5a - 4b$ and $c = b - a$. For $F = 9$ N, we have

$$x = L + \frac{9L}{\pi r^2 Y}$$

$$= L + 9c = (5a - 4b) + 9(b - a)$$

$$= 5b - 4a$$

Hence the correct choice is (b).

103. Poisson's ratio is defined as

$$\sigma = \frac{\Delta d/d}{\Delta l/l} \quad \text{or} \quad \frac{\Delta l}{l} = \frac{\Delta d/d}{\sigma} \qquad (i)$$

where d is the diameter and l is the length of the wire. The area of cross-section is

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

or $$\log A = \log\left(\frac{\pi}{4}\right) + 2\log d$$

Differentiating, we have

$$\frac{\Delta A}{A} = 2\frac{\Delta d}{d}$$

or $$\frac{\Delta d}{d} = \frac{1}{2}\frac{\Delta A}{A} = \frac{1}{2} \times 2\% = 1\%$$

$$\left(\because \frac{\Delta A}{A} = 2\%, \text{given}\right)$$

Using this in (i) we have

$$\frac{\Delta l}{l} = \frac{1\%}{0.4} = 2.5\% \qquad (\because \sigma = 0.4)$$

Hence the correct choice is (b).

104. Velocity of efflux (v) = $\sqrt{2gd}$. Time taken

$t = \sqrt{\dfrac{2h}{g}}$

$\therefore$ Range R = velocity × time = $vt = \sqrt{2gd} \times \sqrt{\dfrac{2h}{g}}$

$= \sqrt{4dh}$

or $R^2 = 4\,dh$ or $d = \dfrac{R^2}{4h}$, which is choice (c).

105. According to the equation of continuity $A_1 v_1 = A_2 v_2$, we have

$$\frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{2A_1}{A_1} = 2 \qquad (\because A_2 = 2A_1)$$

Now mass flux = rate of mass of liquid flowing per unit area

$$= \frac{\text{mass}}{\text{area}} \div \text{time}$$

$$= \frac{\text{mass of liquid}}{\text{area} \times \text{time}}$$

$$= \frac{\text{mass}}{\text{volume}} \times \frac{\text{distance}}{\text{time}}$$

$$= \text{density} \times \text{velocity of flow}$$

$$= \rho v$$

Therefore, $m_1 = \rho_1 v_1$ and $m_2 = \rho_2 v_2$. Given $m_1 = m_2$. Hence

$$\rho_1 v_1 = \rho_2 v_2$$

or $$\frac{\rho_1}{\rho_2} = \frac{v_2}{v_1} = \frac{1}{2} \qquad \left(\because \frac{v_1}{v_2} = 2\right)$$

Thus, the correct choice is (d).

106. Let T_A and T_B be the tensions in wires A and B respectively. If a_A and a_B are the respective cross-sectional areas, then

$$\text{Stress in wire } A = \frac{T_A}{a_A}$$

$$\text{Stress in wire } B = \frac{T_B}{a_B}$$

The stress in wires A and B will be equal if

$$\frac{T_A}{a_A} = \frac{T_B}{a_B} \text{ or } \frac{T_A}{T_B} = \frac{a_A}{a_B} = \frac{1}{2} \text{ (given).}$$

Since the system is in equilibrium, the moments of forces T_A and T_B about C will be equal (see Fig. 7.33), i.e.

$$T_A \times x = T_B \times (L - x)$$

or $$\frac{T_A}{T_B} = \frac{L-x}{x} \quad \text{or} \quad \frac{1}{2} = \frac{L-x}{x}$$

which gives $x = \dfrac{2L}{3}$. Hence the correct choice is (c).

107. Strain $= \dfrac{\text{Stress}}{\text{Young's modulus}} = \dfrac{T}{aY}$

Strain in wires A and B will be equal if

$$\frac{T_A}{a_A Y_A} = \frac{T_B}{a_B Y_B}$$

or $$\frac{T_A}{T_B} = \frac{Y_A}{Y_B} \times \frac{a_A}{a_B} = 2 \times \frac{1}{2} = 1$$

which gives $T_A = T_B$. Equating moments about C, we have

$$T_A \times x = T_B \times (L - x)$$

which gives $x = L - x$ or $x = \dfrac{L}{2}$ ($\because T_A = T_B$). Hence the correct choice is (b).

108. Let L be the required length of wire. If A is its area of cross-section and ρ is the density of its material, the weight of the wire is

$$W = mg = AL\,\rho g$$

$$\therefore \quad \text{Stress} = \frac{\text{weight}}{\text{area}} = \frac{AL\rho g}{A} = L\rho g$$

or $$S = L\rho g \Rightarrow L = S/\rho g$$

109. Refer to Fig. 7.36. For vertical equilibrium

$$T\cos\theta = mg$$

or $$T = \frac{mg}{\cos\theta}$$

If L is the original length of the wire, the increase in length is

$$l = \frac{TL}{AY}$$

$$\therefore \quad \text{Strain} = \frac{l}{L} = \frac{T}{AY} = \frac{mg}{AY\cos\theta}$$

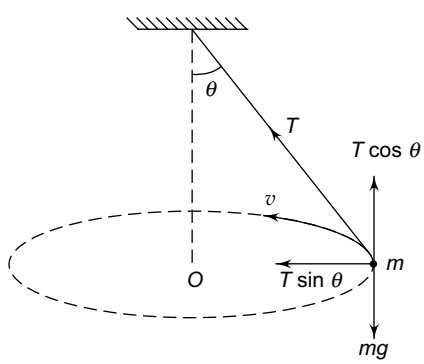


Fig. 7.36

110. Let V be the volume of the metal at normal pressure.

From $B = -\dfrac{p}{\Delta V/V}$, the decrease in the volume when the metal is subjected to an excess pressure p is given by

$$|\Delta V| = \frac{pV}{B}$$

$\therefore$ New volume of metal is $V' = V - \dfrac{pV}{B} = V\left(1 - \dfrac{p}{B}\right)$

Now mass of the metal is $m = \rho V$. Therefore, its new density is

$$\rho' = \frac{m}{V'} = \frac{\rho V}{V\left(1 - \dfrac{p}{B}\right)} = \frac{\rho}{1 - \dfrac{p}{B}}$$

or $$\frac{\rho'}{\rho} = \frac{1}{1 - p/B}$$, which is choice (a).

111. Stress $= \dfrac{Mg}{A}$, strain $= \dfrac{\Delta L}{L}$, volume $= AL$

Energy stored in wire is

U = energy stored per unit volume × volume

$$= \left(\frac{1}{2}\text{stress} \times \text{strain}\right) \times \text{volume}$$

$$= \frac{1}{2} \times \left(\frac{Mg}{A} \times \frac{\Delta L}{L}\right) \times AL$$

$$= \frac{1}{2} Mg\Delta L$$

$$= \frac{(Mg)^2 L}{2AY} \qquad \left(\because \Delta L = \frac{MgL}{AY}\right)$$

If ΔT is the rise in temperature, then $U = ms\Delta T$, i.e.

$$\frac{(Mg)^2 L}{2AY} = ms\Delta T$$

$$\Rightarrow \quad \Delta T = \frac{(Mg)^2 L}{2AYms}$$, which is choice (a).

112. Let M be the mass of the rubber cord, L its length, A its cross-sectional area (assumed constant). Let us first find the elongation dl of an element AB of length dy at a distance y from the fixed end (Fig. 7.37). The force due to the weight of the cord is maximum at $P(y = 0)$ and zero at $Q(y = L)$. Therefore, the force acting at the element is

$$F = \frac{Mg}{L}(L - y) \qquad (1)$$

This force is responsible for the elongation of element AB. Now

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{dl/dy} = \frac{Fdy}{Adl}$$

or $$dl = \frac{Fdy}{AY} \qquad (2)$$

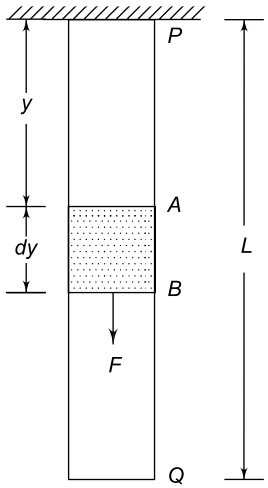


Fig. 7.37

Using Eq. (1) in Eq. (2), we get

$$dl = \frac{Mg}{LAY}(L - y)dy$$

To obtain the total elongation l of the cord, we integrate from $y = 0$ to $y = L$. Thus

$$l = \int dl = \frac{Mg}{LAY}\int_0^L (L - y)dy$$

$$= \frac{Mg}{LAY}\left| Ly - \frac{y^2}{2} \right|_0^L$$

$$= \frac{Mg}{LAY}\left(L^2 - \frac{L^2}{2} \right) = \frac{MgL}{2AY}$$

The correct choice is (b).

113. Poisson's ratio σ is defined as

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\Delta r/r}{\Delta l/l}$$

where r is the radius of the wire and l its length and Δr is the change in r and Δl the change in l when the wire is subjected to tension.

Volume of wire before elongation is

$$V_1 = \pi r^2 l$$

Volume of wire after elongation is

$$V_2 = \pi(r - \Delta r)^2 (l + \Delta l)$$

Given $V_1 = V_2$. Thus

$$\pi r^2 l = \pi(r - \Delta r)^2 (l + \Delta l)$$

$$= \pi[r^2 - 2r(\Delta r) + (\Delta r)^2] (l + \Delta l)$$

$$= \pi r^2(l + \Delta l) - 2\ \pi r\ \Delta r\ (l + \Delta l) + \pi(\Delta r)^2 (l + \Delta l)$$

Since Δr and Δl are very small, terms of order $\Delta r\ \Delta l$ and $(\Delta r)^2$ and higher can be ignored. Then, we have

$$\pi r^2 l = \pi r^2 l + \pi r^2 \Delta l - 2\ \pi r l\ \Delta r$$

or $$r\Delta l = 2\ l\Delta r \quad \text{or} \quad \frac{\Delta l}{l} = 2\ \frac{\Delta r}{r}$$

$$\therefore \qquad \sigma = \frac{\Delta r/r}{\Delta l/l} = \frac{1}{2} = 0.5,$$

which is choice (d).

114. Let h be the initial height of the liquid of density ρ in the container of cross-sectional area A. The mass of the liquid in the container initially is (Fig. 7.38)

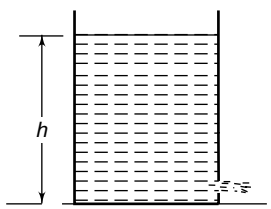


Fig. 7.38

$$m = Ah\rho$$

From Torricelli's theorem, the velocity of the liquid flowing out of the hole is

$$v = \sqrt{2gh}$$

$\therefore$ Volume of liquid flowing out per unit time = av. Hence the mass of liquid flowing out per unit time = ρav. Therefore, the momentum carried per unit time by the liquid flowing out is = (mass per unit time) × velocity = $(\rho av)v = \rho av^2$.

This is the rate of change of momentum of the liquid flowing out which is the force with which the liquid flows out at $t = 0$.

$$\therefore \quad \text{Initial acceleration} = \frac{\text{force}}{\text{mass}} = \frac{\rho av^2}{Ah\rho} = \frac{av^2}{Ah}$$

$$= \frac{a \times 2gh}{Ah} \quad (\because v = \sqrt{2gh})$$

$$= \frac{2ga}{A}, \text{ which is choice (c).}$$

115. When 75% of the liquid has drained out, the height of the liquid in the container will be $h' = h/4$. For this height,

Velocity of liquid flowing out (v')

$$= \sqrt{2gh'}$$

$$= \sqrt{2g \times \frac{h}{4}} = \sqrt{\frac{gh}{2}}$$

Now, $h = \dfrac{m}{A\rho}$. Hence

$$v' = \sqrt{\frac{gm}{2A\rho}}$$

116. Refer to Fig. 7.39. Let v be the velocity of efflux and V be the velocity with which the water level in the tank falls. From the equation of continuity of flow, we have $a\,v = AV$ or

$$V = \frac{av}{A} \qquad (1)$$

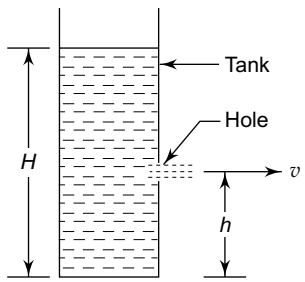


Fig. 7.39

From Bernoulli's theorem, we have

$$P_0 + \frac{1}{2}\,\rho V^2 + \rho gH = P_0 + \frac{1}{2}\,\rho v^2 + \rho gh$$

which gives $v^2 = V^2 + 2g\,(H - h)$ (2)

Using (1) in (2), we get

$$v^2 = \frac{2g\,(H-h)}{1-\left(\dfrac{a}{A}\right)^2} = \frac{2 \times 9.8 \times (2.0 - 0.75)}{1 - 0.2} = 25$$

which gives $v = 5.0$ ms^{-1}. Hence the correct choice is (c).

117. As indicated in the question as the ball enters the water it has already attained the terminal velocity. Terminal velocity is reached when the viscous force (acting upwards) balances the weight mg of the ball. Therefore,

$$6\,\pi\,\eta\,rv = mg$$

where $6\,\pi\eta\,rv$ is the viscous force on the ball. Here v its terminal velocity and η, the coefficient of viscosity of water. If ρ is the density of the material of the ball,

we have $m = \dfrac{4}{3}\,\pi r^3\rho,\ \therefore\ 6\,\pi\eta\,rv = \dfrac{4}{3}\,\pi r^3\rho g$

or $v = \dfrac{2r^2\rho g}{9\eta}$ (1)

Now $v^2 - 0 = 2gh \quad \Rightarrow \quad h = \dfrac{v^2}{2g}$ (2)

From (1) and (2) we find that $h \propto r^4$, which is choice (c).

118. Let ρ_w and ρ_k be the densities of water and kerosene. The initial weight of the liquid in the vessel = $h_w\,\rho_w\,ag + h_k\,\rho_k\,ag$ where h_w and h_k are the thicknesses of water and kerosene layers and a is the cross-sectional area of the vessel. Let this weight be equivalent to water layer of thickness h, then

$$h\rho_w\,ag = h_w\rho_w ag + h_k\rho_k ag$$

or $$h = \left\{h_w + h_k\left(\frac{\rho_k}{\rho_w}\right)\right\}$$

$$= 0.4 + 0.3 \times 0.8 = 0.64 \text{ m}$$

From Torricelli's theorem, the velocity of efflux is

$$v = \sqrt{2gh}$$

$$= \sqrt{2 \times 10 \times 0.64} = \frac{8}{\sqrt{5}}\ \text{ms}^{-1}$$

Hence the correct choice is (d).

119. Let V be the original volume of the sphere. The decrease in volume under excess pressure P is

$$\Delta V = \frac{VP}{B} = \sigma VP \qquad \left(\because \sigma = \frac{1}{B}\right)$$

where B is the bulk modulus. Let ΔT be the increase in temperature to compensate for this increase in volume.

Then

$$\Delta V = \alpha V\,\Delta T$$

Equating the two expression of ΔV, we get $\Delta T = \dfrac{\sigma P}{\alpha}$

So the correct choice is (*a*).

120. The decrease in the density of the block is

$$\Delta V = \frac{VP}{B}$$

If M is the mass of the block, its density is $\rho = \dfrac{M}{V}$.

Due to decrease ΔV in its volume, the increase in its density is

$$\Delta\rho = \frac{M}{\Delta V} = \frac{M\,B}{VP} = \frac{\rho B}{P}$$

So the correct choice is (b).

Note

You can easily guess the correct choice in Question 119 and 120 using dimensional consideration.

The dimension P = dimensions of B = dimensions of $\frac{1}{\sigma}$.

121. Poisson's ratio = $\frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

or $\sigma = -\frac{\Delta D/D}{\Delta L/L}$

$\therefore \quad \left|\frac{\Delta D}{D}\right| = \sigma \times \frac{\Delta L}{L} = 0.4 \times 0.1\%$

$= 0.04\%$

$\therefore$ So the correct choice is (a).

122. Excess pressure at depth $h = 50$ m is

$P = h\rho g = 50 \times 1000 \times 9.8 = 4.9 \times 10^5$ Pa

Volume strain $\frac{\Delta V}{V} = -0.5\% = -0.5 \times 10^{-2}$

$\therefore$ Bulk modulus $= -\frac{P}{\frac{\Delta V}{V}} = -\frac{4.9 \times 10^5}{-0.5 \times 10^{-2}} \simeq 10^8$ Pa

So the correct choice is (c).

123. If n droplets coalesce, the energy released is

$E = (n \times 4\pi r^2 - 4\pi R^2)\,\sigma$

Since there is no change in volume in this process,

$n \times \frac{4\pi}{3} r^3 = \frac{4\pi}{3} R^3$

$\Rightarrow \quad n = \frac{R^3}{r^3}$

$\therefore \quad E = \left(\frac{R^3}{r^3} \times 4\pi r^2 - 4\pi R^2\right)\sigma$

$= 4\pi R^2 \left(\frac{R}{r} - 1\right)\sigma$

60% of $E = 0.6 \times 4\pi R^2 \left(\frac{R}{r} - 1\right)\sigma$

Mass of big drop is $M = \frac{4\pi}{3}\rho R^3$

If v is the velocity of the drop.

$\frac{1}{2} M v^2 = 0.6 \times 4\pi R^2 \left(\frac{R}{r} - 1\right)\sigma$

$\Rightarrow \quad \frac{1}{2} \times \frac{4\pi}{3} \rho R^3 v^2 = 0.6 \times 4\pi R^2 \left(\frac{R}{r} - 1\right)\sigma$

$\Rightarrow \quad v = \left[\frac{3.6\sigma}{\rho r R}(R - r)\right]^{1/2}$

So the correct choice is (a).

124. If $n = 8$, then $R = (8)^{1/3} r = 2r$.

40% of $E = 0.4 \times 4\pi R^2 (2 - 1)\,\sigma = 1.6\pi R^2 \sigma$

If ΔT is the rise in temperature of the drop,

$Ms\,\Delta T = 1.6\,\pi R^2\,\sigma$

$\Rightarrow \quad \frac{4\pi}{3} R^3 \rho s \Delta T = 1.6\,\pi R^2\,\sigma$

$\Rightarrow \quad \Delta T = \frac{1.2\sigma}{\rho s R}$

So the correct choice is (d).

125. $F = 6\,\pi\eta r v$

$F' = 6\pi\eta\left(\frac{r}{2}\right) \times \left(\frac{v}{2}\right) = \frac{1}{4} \times 6\pi\eta r v = \frac{F}{4}$

So the correct choice is (c).

126. Let r be the radius of the ball of mass m and R be the radius of the ball with mass M.

Terminal velocity $\propto r^2$. Therefore

$\frac{nv}{v} = \frac{R^2}{r^2}$

$\Rightarrow \quad \frac{R}{r} = n^{1/2}$

Now $\frac{M}{m} = \frac{\frac{4\pi}{3}\rho R^3}{\frac{4\pi}{3}\rho r^3} = \frac{R^3}{r^3} = \left(\frac{R}{r}\right)^3 = n^{3/2}$

So the correct choice is (c).

127. The volume of a liquid flowing per second is through a capillary tube is given by

$Q = \frac{\pi p r^4}{8\eta L}$

where η is the coefficient of viscosity of the liquid and p is the pressure difference between the ends of the tube. Since the liquid does not accumulate anywhere in the tube, $Q_1 = Q_2$ or

$\frac{\pi p_1 r_1^4}{8\eta L_1} = \frac{\pi p_2 r_2^4}{8\eta L_2}$

$$\Rightarrow \quad \frac{p_1}{p_2} = \left(\frac{L_1}{L_2}\right)\times\left(\frac{r_2}{r_1}\right)^4$$

$$= \left(\frac{L}{2L}\right)\times\left(\frac{2r}{r}\right)^4$$

$$= \frac{1}{2}\times 2^4 = 2^3 = 8$$

So the correct choice is (d).

128. $Q_1 = Q_2 = Q_3$. Therefore,

$$\frac{\pi p_1 r^4}{8\eta L} = \frac{\pi p_2 (2r)^4}{8\eta L} = \frac{\pi p_3 (3r)^4}{8\eta L}$$

$$\Rightarrow \quad p_1 = 16\, p_2 = 81\, p_3$$

$$\therefore \quad p_1 = 16\, p_2$$

and $$p_3 = \frac{16}{81} p_2$$

Given $p_1 + p_2 + p_3 = 28$ cm of Hg

$$\Rightarrow \; 16p_2 + p_2 + \frac{16}{81} p_2 = 28$$

$$\Rightarrow \quad \left(16 + 1 + \frac{16}{81}\right) p_2 = 28$$

$$\Rightarrow \quad p_2 = \frac{81\times 28}{1393} = 1.6 \text{ cm of Hg}$$

So the correct choice is (a).

129. Total upward buoyant force on the block is

U = buoyant force due to oil + buoyant force due to water

$$= \rho_0 (l - x)\, l^2 g + \rho_w x l^2 g$$

For floating U = weight of the block = $\rho_b l^3 g$

$$\therefore \quad \rho_b l^3 g = \rho_0 (l - x)\, l^2 g + \rho_w\, x\, l^2 g$$

$$\Rightarrow \quad \frac{\rho_b}{\rho_w} = \frac{1}{l}\left[\frac{\rho_0}{\rho_w}(l - x) + x\right].$$

$$= \frac{1}{l}[n(l - x) + x]$$

Putting $x = \frac{l}{4}$, we get

$$\frac{\rho_b}{\rho_w} = \frac{1}{4}(3n + 1)$$

So the correct choice is (c).

130. Let V be the volume of the ball. The upward force when the ball is completely immersed in water is

$$U = \rho V g$$

Downward force acting on the ball is its weight

$$W = \sigma V g$$

$\therefore$ Net upward force is

$$F = U - W = (\rho - \sigma)\, V g$$

Mass of the ball is $m = \sigma V$. Therefore, upward acceleration is

$$a = \frac{F}{m} = \frac{(\rho - \sigma) V g}{\sigma V} = \left(\frac{\rho - \sigma}{\sigma}\right) g \qquad (1)$$

If u is the velocity of the ball when it emerges from water, then

$$0 - u^2 = -2ah$$

$$\Rightarrow \quad u = \sqrt{2ah}$$

If H is the maximum height attained,

$$0 - u^2 = 2 \times (-g)\, H$$

$$\Rightarrow \quad H = \frac{u^2}{2g} = \frac{2ah}{2g} = \frac{ah}{g} \qquad (2)$$

Using (1) in (2) we get

$$H = \left(\frac{\rho - \sigma}{\sigma}\right) h$$

Putting $\rho = 4\sigma$, we get $H = 3h$, which is choice (c).

2 SECTION — Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage.

Passage I

One end of a string of length L and cross-sectional area A is fixed to a support and the other end is fixed to a bob of mass m. The bob is revolved in a horizontal circle of radius r with an angular velocity ω such that the string makes an angle θ with the vertical.

1. The angular velocity ω is equal to

   (a) $\sqrt{\frac{g\sin\theta}{r}}$ (b) $\sqrt{\frac{g\cos\theta}{r}}$

   (c) $\sqrt{\frac{g\tan\theta}{r}}$ (d) $\sqrt{\frac{g\cot\theta}{r}}$

2. The tension T in the string is

   (a) $\frac{mg}{\cos\theta}$ (b) $\frac{mg}{\sin\theta}$

   (c) $\frac{mg}{\tan\theta}$ (d) $m(g^2 + r^2\omega^4)^{1/2}$

3. The increase ΔL in length of the string is

   (a) $\frac{TL}{AY}$ (b) $\frac{MgL}{AY\cos\theta}$

   (c) $\frac{MgL}{AY\sin\theta}$ (d) $\frac{MgL}{AY}$

4. The stress in the string is

   (a) $\frac{mg}{A}$ (b) $\frac{mg}{A}\left(1-\frac{r}{L}\right)$

   (c) $\frac{mg}{A}\left(1+\frac{r}{L}\right)$ (d) $\frac{mg}{A}\left(\frac{r}{L}\right)$

Solutions

Refer to Fig. 7.40. The vertical component $T\cos\theta$ of tension T balances the weight mg and the horizontal component $T\sin\theta$ provides the necessary centripetal force. Thus

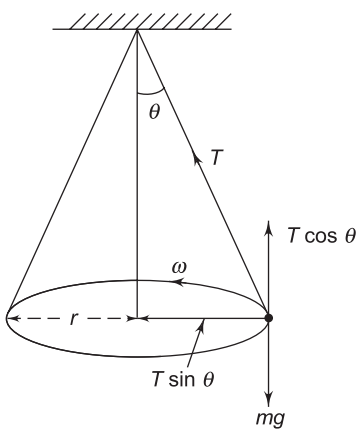


Fig. 7.40

$$T\cos\theta = mg \quad (1)$$

$$T\sin\theta = mr\omega^2 \quad (2)$$

1. Dividing (1) and (2)

$$\tan\theta = \frac{r\omega^2}{g}$$

or $$\omega = \sqrt{\frac{g\tan\theta}{r}}$$

Hence the correct choice is (c).

2. From Eqs. (1) and (2) we find that the correct choices are (a) and (d).

3. Stress $= \frac{T}{A}$. Also strain $\frac{\Delta L}{L} = \frac{\text{stress}}{Y}$

$$\therefore \quad \Delta L = \frac{TL}{AY} = \frac{mg}{\cos\theta}\times\frac{L}{AY}$$

Hence the correct choices are (a) and (b)

4. The correct choice is (a).

Questions 5 to 7 are based on the following passage.

Passage II

Two blocks of masses m and $M = 2\,m$ are connected by means of a metal wire of cross-sectional area A passing over a frictionless fixed pulley as shown in Fig. 7.41. The system is then released.

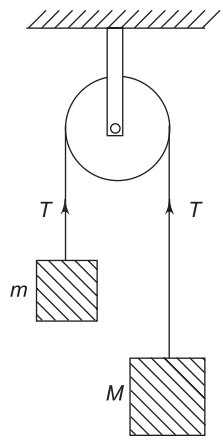


Fig. 7.41

5. The common acceleration of the blocks is

   (a) g

   (b) $\frac{g}{3}$

   (c) $\frac{2g}{3}$

   (d) $\frac{3g}{2}$

6. The stress produced in the wire is

   (a) $\frac{mg}{A}$ (b) $\frac{2mg}{3A}$

   (c) $\frac{3mg}{4A}$ (d) $\frac{4mg}{3A}$

7. If $m = 1$ kg, $A = 8\times10^{-9}$ m^2, the breaking stress $= 2\times10^9$ Nm^{-2} and $g = 10$ ms^{-2}, the maximum value of M for which the wire will not break is

   (a) 4 kg (b) 6 kg

   (c) 8 kg (d) 10 kg

Solutions

If a is the common acceleration of the blocks and T the tension in the wire, then the equations of motion of the blocks are

$$Mg - T = Ma \quad (1)$$

and $$T - mg = ma \quad (2)$$

5. Adding Eq. (1) and Eq. (2), we get

$$a = \frac{(M-m)g}{(M+m)}$$

For $M = 2\,m$, we get $a = g/3$, which is choice (b).

6. From Eqs. (1) and (2), $T = m(g + a) = m(g + g/3) = 4\,mg/3$.

$$\therefore \quad \text{Stress} = \frac{T}{A} = \frac{4mg}{3A}, \text{ which is choice (d).}$$

7. Breaking stress is the maximum stress the wire can withstand. From Eqs. (1) and (2)

$$T = \frac{2mMg}{(M+m)}$$

$$\therefore \text{ Breaking stress} = \frac{T}{A} = \frac{2mg}{A\left(1+\frac{m}{M_{\max}}\right)}.$$

Using the given values, we get $M_{\max} = 4$ kg, which is choice (a).

Questions 8 to 11 are based on the following passage.

Passage III

A large container of negligible mass is open at the top and has a uniform cross-sectional area A. It has a small hole of cross-sectional area a ($<< A$) in its side wall near the bottom. The containar is kept on a horizontal frictionless floor and contains a liquid of density ρ and mass m filled up to a height h from the bottom.

8. The initial rate of change of momentum of the liquid flowing out from the hole is
   (a) ρagh (b) ρAgh
   (c) $2\rho agh$ (d) $2\rho Agh$
9. The initial force with which the liquid flows out from the hole is
   (a) $\left(\frac{a}{A}\right)mg$ (b) $\left(\frac{A}{a}\right)mg$
   (c) $\left(\frac{2a}{A}\right)mg$ (d) $\left(\frac{2A}{a}\right)mg$
10. The initial acceleration of the container is
   (a) zero (b) g
   (c) $\frac{Ag}{2a}$ (d) $\frac{2ag}{A}$
11. The velocity of efflux when 75% of the liquid has drained out is
   (a) $\sqrt{\frac{gh}{2}}$ (b) $\sqrt{gh}$
   (c) $\sqrt{\frac{3gh}{4}}$ (d) $\sqrt{\frac{3gh}{2}}$



Solutions

8. Pressure at the bottom (hole) is $P = h\rho g$.
   $\therefore$ Force at the hole is
   F = pressure $P \times$ area a of the hole = $h\rho ga$
   From Torricelli's theorem, the velocity of the liquid flowing out of the hole is

$$v = \sqrt{2gh}$$

$\therefore$ Volume of liquid flowing out per unit time = av. Hence, the mass of liquid flowing out per unit time = ρav. Therefore, the momentum carried per unit time by the liquid flowing out is = (mass per unit time) $\times$ velocity = $(\rho av)v = \rho av^2$.

Thus, the rate of change of momentum of the liquid coming out of the hole = $\rho a v^2 = \rho a \times 2gh = 2\rho agh$, which is choice (c).

9. Force = rate of change of momentum = $2\rho agh = \left(\frac{2a}{A}\right)mg$ ($\because\ m = \rho Ah$). Hence the correct choice is (c).

10. Initial acceleration of container is due to the backward reaction force exerted on the container by the liquid emerging from the hole. The action force = rate of change of momentum of the emerging water = $\left(\frac{2a}{A}\right)mg$ which equal and opposite to the reaction force on the container. Hence the backward acceleration of the container = $\frac{\text{backward force}}{\text{mass}} = \left(\frac{2a}{A}\right)g$, which is choice (d).

11. When 75% of the liquid has drained out, the height of the liquid in the container will be $h' = h/4$. For this height,

Velocity of liquid flowing out (v') = $\sqrt{2gh'} = \sqrt{2g \times \frac{h}{4}} = \sqrt{\frac{gh}{2}}$, which is choice (a).

Questions 12 to 14 are based on the following passage.

Passage IV

A cylindrical tank of cross-sectional area A rests on a platform of height H from the ground as shown in Fig. 7.42. It has a hole of cross-sectional area a on the side wall at the bottom. The tank is filled with water to a height h. The plug is removed from the hole.

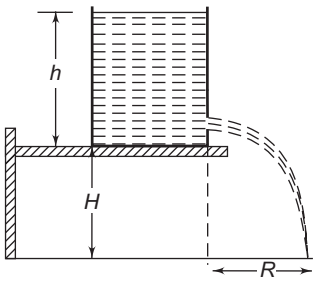


Fig. 7.42

12. The initial speed with which the water hits the ground is

(a) $\sqrt{2gh}$ (b) $[2g(H^2 + h^2)^{1/2}]^{1/2}$

(c) $\sqrt{2g(H+h)}$ (d) $\sqrt{2gH}$

13. The stream of water strikes the ground at a distance R given by

(a) $\sqrt{hH}$ (b) $\sqrt{2hH}$

(c) $2\sqrt{hH}$ (d) $\sqrt{(h^2 + H^2)}$

14. The time taken to empty the tank to one-fourth of its original volume of water is

(a) $\frac{A}{a}\sqrt{\frac{h}{2g}}$ (b) $\frac{A}{a}\sqrt{\frac{2h}{g}}$

(c) $\frac{A}{a}\sqrt{\frac{H}{2g}}$ (d) $\frac{a}{A}\sqrt{\frac{2H}{g}}$

Solutions

12. Velocity of efflux from the hole is $v_x = \sqrt{2gh}$. This is the horizontal velocity, which remains unchanged. Vertical velocity on striking the ground is $v_y = \sqrt{2gH}$ (use $u^2 = v^2 + 2as$).

The resultant speed with which the water strikes the ground is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{2g(H+h)}$$

Hence the correct choice is (c).

13. Time taken by stream to strike the ground is

$$t = \sqrt{\frac{2H}{g}}.$$

$\therefore R = v_x \times t = \sqrt{2gh} \times \sqrt{\frac{2H}{g}} = 2\sqrt{hH}$, which is choice (c).

14. As water flows through the orifice, the level of water in the tank keeps falling with time. If x is the height of the water level at an instant of time t, then the instantaneous speed of water flowing out is

$$v_x = \sqrt{2gx}$$

If a is the cross-sectional area of the orifice, the volume of water flowing per unit time at that instant $= av_x = a\sqrt{2gx}$. This must be equal to the rate at which the volume of water in the tank decreases at that instant which is given by $-A\,dx/dt$, where A is the cross-sectional area of the tank. The negative sign indicates that height x of water decreases with time. Thus

$$a\sqrt{2gx} = -A\frac{dx}{dt}$$

or
$$dt = -\frac{A\,dx}{a\sqrt{2gx}}$$

$$= -\frac{A}{a\sqrt{2g}}\,x^{-1/2}\,dx \qquad (1)$$

The time t taken to empty the tank to one-fourth its original volume is obtained by integrating (1) from $x = h$ to $x = h/4$. Thus

$$t = -\frac{A}{a\sqrt{2g}}\int_h^{h/4} x^{-1/2}\,dx$$

$$= -\frac{A}{a\sqrt{2g}}\left|\frac{x^{1/2}}{1/2}\right|_h^{h/4}$$

$$= -\frac{2A}{a\sqrt{2g}}\left[\left(\frac{h}{4}\right)^{1/2} - h^{1/2}\right]$$

$$= \frac{A}{a}\sqrt{\frac{h}{2g}}, \text{ which is choice (a).}$$

Questions 15 to 17 are based on the following passage.

Passage V

A cylindrical tank is open at the top and has cross-sectional area a_1. Water is filled in it up to a height h. There is a hole of cross-sectional area a_2 at its bottom. Given $a_1 = 3a_2$.

15. The initial velocity with which the water falls in the tank is

(a) $\sqrt{2gh}$ (b) $\sqrt{gh}$

(c) $\sqrt{\frac{gh}{2}}$ (d) $\frac{1}{2}\sqrt{gh}$

16. The initial velocity with which the water emerges from the hole is

(a) $\frac{1}{2}\sqrt{gh}$ (b) $\sqrt{2gh}$

(c) $\frac{3}{2}\sqrt{gh}$ (d) $2\sqrt{2gh}$

17. The time taken to empty the tank is

(a) $\sqrt{\frac{2h}{g}}$ (b) $4\sqrt{\frac{h}{g}}$

(c) $6\sqrt{\frac{2h}{g}}$ (d) $8\sqrt{\frac{2h}{g}}$

Solutions

15. Let v_1 be the initial velocity of water in the tank and v_2 the initial velocity of flow of water from the hole, then from continuity of flow (Fig.7.43),

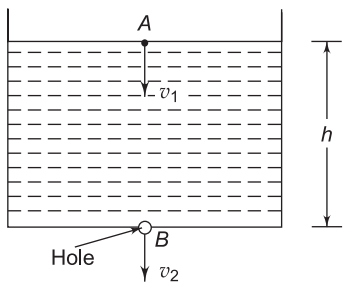


Fig. 7.43

$$a_1 v_1 = a_2 v_2$$

$$\Rightarrow \qquad v_1 = \left(\frac{a_2}{a_1}\right) v_2 = \frac{v_2}{3} \qquad (1)$$

Applying Bernoulli's theorem at A and B,

$$P_0 + \rho gh + \frac{1}{2}\rho v_1^2 = P_0 + \frac{1}{2}\rho v^2{}_2$$

$$\Rightarrow \qquad v_1^2 = v_2^2 - 2gh \qquad (2)$$

Using (1) in (2), we get $v_1 = \frac{1}{2}\sqrt{gh}$, which is choice (d).

16. $v_2 = 3v_1 = \frac{3}{2}\sqrt{gh}$. Hence the correct choice is (c).

17. As h decreases with time, we have

$$v_1 = -\frac{dh}{dt}$$

$$\Rightarrow \qquad dt = -\frac{-dh}{v_1} = -\frac{2}{\sqrt{g}} \times h^{-1/2}\, dh$$

The time taken to empty the tank is obtained by integrating from $h = h$ to $h = 0$. Thus

$$t = -\frac{2}{\sqrt{g}} \int_h^0 h^{-1/2}\, dh$$

$$= -\frac{2}{\sqrt{g}}(0 - 2\sqrt{h}) = 4\sqrt{\frac{h}{g}}$$

Hence the correct choice is (b).

Questions 18 to 20 are based on the following passage.

Passage VI

A container of large uniform cross-sectional area A resting on a horizontal surface, is filled with two non-mixing and non viscous liquids of densities d and $2d$, each to a height $H/2$ as shown in Fig.7.44. A tiny hole of cross-sectional area a ($<< A$) is punched on the vertical side of the container at a height $h = H/4$.

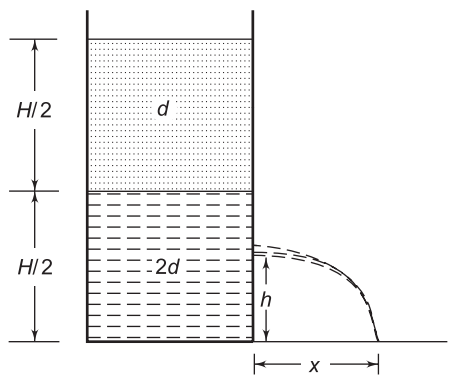


Fig. 7.44

18. The initial speed of efflux of the liquid from the hole is

(a) $2\sqrt{2gH}$ (b) $2\sqrt{gH}$

(c) $\sqrt{2gH}$ (d) $\sqrt{gH}$

19. The horizontal range x of the liquid initially is

(a) H (b) $\frac{H}{\sqrt{2}}$

(c) $\frac{H}{2}$ (d) $\frac{H}{2\sqrt{2}}$

20. The height h of the hole from the ground for which the horizontal range x is maximum is

(a) $\frac{2H}{3}$ (b) $\frac{3H}{2}$

(c) $\frac{3H}{4}$ (d) $\frac{H}{2}$

Solutions

18. Let v_1 be the initial velocity of the fall of liquid level in the container and v_2 the velocity of efflux from the hole. Then $Av_1 = av_2$ or $v_1 = \left(\frac{a}{A}\right) v_2$. since $a << A$, $v_2 \simeq 0$. Applying Bernoulli's theorem at a point on the top of the liquid and at the hole, we have

$$P_0 + \frac{1}{2} dv_1^2 + dg\left(\frac{H}{2}\right) + (2d)\, g\left(\frac{H}{2}\right)$$

$$= P_0 + \frac{1}{2}\,(2d)\, v_2^2 + (2d)\, gh$$

Putting $v_1 = 0$, we get

$$v_2 = \left[\left(\frac{3}{2}H - 2h\right)g\right]^{1/2}$$

Putting $h = H/4$, we get $v_2 = \sqrt{gH}$ which is choice (d).

19. Time of fall $t = \sqrt{\frac{2h}{g}}$ .Horizontal range is

$$x = v_2\, t = \left[\left(\frac{3}{2}H - 2h\right)g\right]^{1/2} \times \sqrt{\frac{2h}{g}}$$

$$= [h(3H - 4h)]^{1/2} \qquad (1)$$

putting $h = H/4$, we get $x = \frac{H}{\sqrt{2}}$, which is choice (b).

20. To determine the height h at which the range x is maximum, we differentiate (1) with respect to h and set $\frac{dx}{dh} = 0$, i.e.

$$\frac{d}{dh}\,[3Hh - 4h^2]^{1/2} = 0$$

or $$\frac{1}{2}\,[3Hh - 4h^2]^{-1/2} \times (3H - 8h) = 0$$

which gives $3H - 8h = 0$ or $h = \frac{3}{8}\,H$. Notice that $(3Hh - 4h^2)^{-1/2}$ cannot be zero since this would give x = infinity which is not possible. Hence x is maximum (= x_m) at a value of h given by

$$h = \frac{3}{8}\,H$$

Using this value of h in (1), we get

$$x_m = \left[\frac{3}{8}H\left(3H - \frac{12}{8}H\right)\right]^{1/2} = \frac{3}{4}\,H$$

Hence the correct choice is (c).

SECTION 3

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only *one* choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. **Statement-1**
The length of an elastic string of initial length L is a metre when the tension is 4 N and b metre when the tension is 5 N. The length of the string (in metre) when the tension is 9 N is $(a + b - L)$

Statement-2
The extension of an elastic string is proportional to the initial length of the string.

2. **Statement-1**
Steel is more elastic than rubber.

Statement-2
The Young's modulus of steel is greater than that of rubber.

3. **Statement-1**
The stretching of an elastic spring is determined by the shear modulus of the material of the spring.

Statement-2
For a given stretching force, the amount of stretching depends on the force constant of the spring.

4. **Statement-1**

   Figure 7.45 shows that stress-strain curves for two different types of rubber. Rubber *A* rather than rubber *B* should be used as a car tyre.

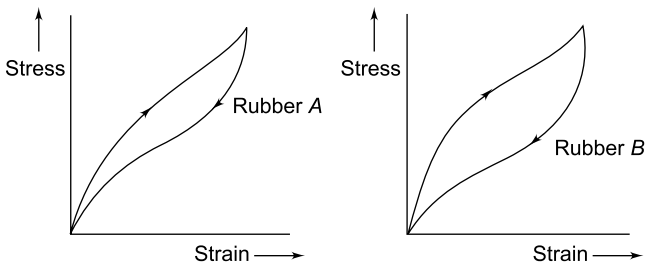


Fig. 7.45

   Statement-2

   Rubber *A* dissipates larger amount heat energy than rubber *B*.

5. **Statement-1**

   Two wires *A* and *B* have the same cross-sectional area and are made of the same material but the length of wire *A* is twice that of *B*. For a given load, the extension of *A* will be twice that of *B*.

   Statement-2

   For a given load the extension of a wire is proportional to its length.

6. **Statement-1**

   Two wires *A* and *B* have the same cross-sectional area and are made of the same material but the length of wire *A* is the twice that *B*. For a given load, the strain in wire *A* is twice that in *B*.

   Statement-2

   For a given load, the extension in a wire is proportional to its length.

7. **Statement-1**

   Two wires *A* and *B* have equal lengths and are made of the same material but the diameter of wire *A* is twice that of *B*. For a given load, the extension of *B* will be four times that of *A*.

   Statement-2

   For a given load, the extension of a wire is inversely proportional to its area of cross-section.

8. **Statement-1**

   Two wires *A* and *B* are made of the same material. The length of wire *A* is twice that of *B* but the diameter of *A* is half that of *B*. For a given load, the strain produced in *B* will be twice that in *A*.

   Statement-2

   For a given load, the extension produced in a wire is directly proportional to its length and inversely proportional to the area of cross-section.

9. **Statement-1**

   When a material is subjected to a tensile (stretching) stress, the restoring forces are caused by interatomic attraction.

   Statement-2

   Restoring force is called into play in an elastic material by an inherent property of the material and not due to interatomic attraction.

10. **Statement-1**

    When a material is subjected to a compressional stress, the restoring forces are caused by interatomic repulsion.

    Statement-2

    The atoms of a material never repel.

11. **Statement-1**

    To keep a piece of paper horizontal, we should blow over it and not under it.

    Statement-2

    In a steady flow of a fluid, the total energy of a given mass of a fluid is conserved.

12. **Statement-1**

    If we plug a running water tap with our fingers, fast jets of water gush through the opening between in fingers.

    Statement-2

    The pressure of water at the opening increases due to the decrease in the area.

13. **Statement-1**

    When a fluid flows out of a small hole in the sides of a vessel, a backward force is exerted on the vessel.

    Statement-2

    The total energy of a given mass of a fluid in motion is conserved.

14. **Statement-1**

    If a liquid is a vessel is stirred and left to itself, the motion disappears after sometime.

    Statement-2

    The moving liquid exerts equal and opposite reaction force.

15. **Statement-1**

The critical velocity of a liquid flowing through a tube is inversely proportional to the radius of the tube.

Statement-2

The velocity of a liquid flowing through a tube is inversely proportional to the cross-sectional area of the tube.

16. **Statement-1**

The viscous force experienced by a steel ball moving in a liquid is less than that experienced by an aluminuim ball of the same radius moving in the liquid with the same speed.

Statement-2

The density of steel is greater than that of aluminuim.

17. **Statement-1**

The terminal velocity of a small steel ball falling in a liquid is more than that of an identical aluminuim ball falling in the same liquid.

Statement-2

The density of steel is greater than that of aluminuim.

18. **Statement-1**

No net force acts on a body falling in a liquid with a velocity equal to the terminal velocity.

Statement-2

The weight of the body is balanced by the upward buoyant force.

19. **Statement-1**

A very light spinning ball can be held hanging in air by blowing air as shown in Fig. 7.46.

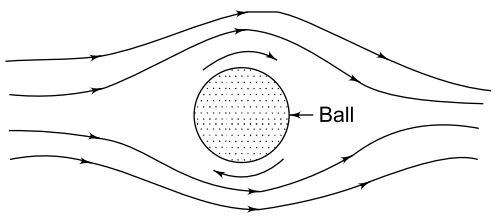


Fig. 7.46

Statement-2

The weight of the ball can be balanced by upthrust due to viscosity of air.

20. **Statement-1**

The uplift of the wing of an aircraft moving horizontally is caused by a pressure difference between the upper and lower faces of the wing.

Statement-2

The velocity of air moving along the upper surface is higher than that along the lower surface.

Solutions

1. The correct choice is (d). If L is the initial length, then the increase in length by a tension F is given by

$$l = \frac{FL}{\pi r^2 Y}$$

Hence $$a = L + l = L + \frac{4L}{\pi r^2 Y} = L + 4c \qquad (1)$$

and $$b = L + \frac{5L}{\pi r^2 Y} = L + 5c \qquad (2)$$

where $c = \frac{L}{\pi r^2 Y}$. Solving (1) and (2) for L and c, we get $L = 5a - 4b$ and $c = b - a$. For $F = 9$ N, we have

$$x = L + \frac{9L}{\pi r^2 Y} = L + 9c$$

$$= (5a - 4b) + 9(b - a) = 5b - 4a$$

2. The correct choice is (a).
3. The correct choice is (b). If a spring is stretched, the total length of the wire of the coil and the volume of the wire, both do not change. Only the shape (or configuration) of the coils of the wire undergoes a change. Hence, the stretching of a spring is not determined by Young's modulus or bulk modulus. It is determined by the shear modulus.
4. The correct choice is (c). The area of the hysteresis loop for rubber A much smaller than that for rubber B. This implies that rubber A dissipates a smaller amount of heat energy than rubber B. Consequently, tyres made of rubber A will not get heated to a high temperature. This prevents wear and tear of tyres.
5. $\Delta L = \frac{FL}{AY}$. Since the two wires are made of the same material, the Young's modulus Y is the same. Since F and A also the same, $\Delta L \propto L$. Hence the correct choice is (a).
6. The correct choice is (d).

$$\text{Strain} = \frac{\Delta L}{L} = \frac{F}{AY}$$

Since F, A and Y are the same for the two wires, the strains in them are equal.

7. Area of cross-section $A = \pi d^2/4$, where d is the diameter of the wire. Thus

$$\Delta L = \frac{4F}{\pi d^2}\cdot\frac{L}{Y} \quad \text{or} \quad \Delta L \propto \frac{1}{d^2}$$

Hence the correct choice is (a).

8. Strain $\frac{\Delta L}{L} = \frac{4F}{\pi d^2 Y}$. Thus strain $\propto \frac{1}{d^2}$.

Hence strain in *B* will be four times that in *A*. Thus the correct choice is (d).

9. The correct choice is (c). When the material is not subjected to any stress, its atoms are in their normal (equilibrium) positions. When a tensile stress is applied, the distance *R* between atoms becomes greater than their equilibrium separation R_0. For $R > R_0$, the interatomic force is attractive and this force provides the restoring force under which the material regains its original shape and size when the stress is removed.

10. The correct choice is (c). When the material is subjected to a compressional stress, *R* becomes less than R_0 and in this case the interatomic force is repulsive which causes the restoring force.

11. The correct choice is (a). If we blow over a piece of paper, the velocity of air moving along the upper surface of the paper is higher than that along the lower surface. From Bernoulli's theorem of conservation of energy of a fluid flow, the air pressure on the upper surface will be less that on the lower surface causing an uplift of the paper.

12. The correct choice is (c). The cross-sectional area of the opening between fingers is very small compared to that of the tap. From continuity of flow, the speed of water emerging from the opening is very high.

13. The correct choice is (c). The water emerging of from the hole has a momentum. From conservation of momentum, the vessel acquires a recoil momentum in the backward direction and hence a backward thrust.

14. The correct choice is (c). The motion stops due to viscosity of the liquid.

15. The correct choice is (c).

16. The correct choice is (d). The viscous force is independent of the density of the body, it depends only on the radius and speed of the body and the viscosity of the fluid in which it moves ($F = 6\pi\eta r v$).

17. The correct choice is (b).

18. The correct choice is (c). The weight of the body is balanced by two upward forces, namely the buoyant force and viscous force.

19. The correct choice is (c). The ball drags some air with it while spinning. Therefore, the velocity of air at the lower surface increases. According to Bernoulli's theorem, the pressure of air at the upper surface becomes less than the atmospheric pressure (P_0) whereas the air pressure at the lower surface become more than P_0. This pressure difference causes a dynamic uplift of the ball.

20. The correct choice is (a).

SECTION 4

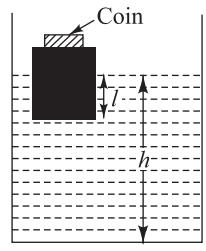
Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. A cylinder of height 20 cm is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near the bottom is (take $g = 10\ ms^{-2}$)

   (a) 10 (b) 20

   (c) 25.5 (d) 5 **[2002]**

2. A wooden block with a coin placed on its top, floats in water as shown in the figure. The distances *l* and *h* are shown. After some time, the coin falls into the water. Then



   (a) *l* decreases and *h* increases

   (b) *l* increases and *h* decreases

   (c) both *l* and *h* increase

   (d) both *l* and *h* decrease **[2002]**

3. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. The elastic energy stored in the wire is

   (a) 0.2 J (b) 10 J

   (c) 20 J (c) 0.1 J **[2003]**

4. A wire fixed at the upper end stretches by l by applying a force F. The work done in stretching is

   (a) $\frac{F}{2l}$ (b) Fl

   (c) $2Fl$ (d) $\frac{Fl}{2}$ **[2004]**

5. A spherical ball of radius R is falling in a viscous fluid of viscosity η with a velocity v. The retarding viscous force acting on the ball is

   (a) directly proportional to R but inversely proportional to v

   (b) directly proportional to both R and v

   (c) inversely proportional to both R and v

   (d) inversely proportional to R but directly proportional to v. **[2004]**

6. If two soap bubbles of different radii are connected by a tube,

   (a) air flows from the bigger bubble to the smaller bubble till their sizes become equal

   (b) air flows from the bigger bubble to the smaller bubble till their sizes are interchanged

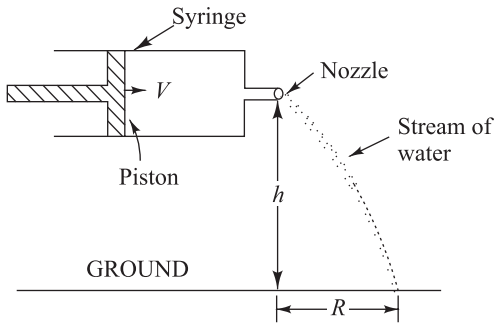
   (c) air flows from the smaller bubble to the bigger bubble

   (d) there is no flow of air. **[2004]**

7. A syringe containing water is held horizontally with its nozzle at a height h above the ground as shown in the figure. The cross-sectional areas of the piston and the nozzle are A and a, respectively. The piston is pushed with a constant speed V. The horizontal range R of the stream of water on the ground is

   (a) $R = V\sqrt{\frac{2h}{g}}$ (b) $R = V\sqrt{\frac{g}{2h}}$

   (c) $R = \frac{aV}{A}\sqrt{\frac{2h}{g}}$ (d) $R = \frac{AV}{a}\sqrt{\frac{2h}{g}}$



8. A small metal sphere of radius r and density ρ falls from rest in a viscous liquid of density σ and coefficient of viscosity η. Due to friction heat is produced. The rate of production of heat when the sphere has acquired the terminal velocity is proportional to

   (a) r^2 (b) r^3

   (c) r^4 (d) r^5 **[2004]**

9. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire apparatus is put in a freely falling elevator, the length of water column in the capillary tube will be

   (a) 8 cm (b) 10 cm

   (c) 4 cm (d) 20 cm **[2005]**

10. If S is the stress and Y is Young's modulus of the material of a wire, the energy stored per unit volume in the wire is

    (a) $2S^2Y$ (b) $\frac{S^2}{2Y}$

    (c) $\frac{2Y}{S^2}$ (d) $\frac{S}{2Y}$ **[2005]**

11. A cylindrical tank having cross-sectional area A is filled with water to a height of 2.0 m. A circular hole of cross-sectional area a is opened at a height of 75 cm from the bottom. If $\frac{a}{A} = \sqrt{0.2},$, the velocity with which water emerges from the hole is ($g = 9.8$ ms^{-2}).

    (a) 4.9 ms^{-1} (b) 4.95 ms^{-1}

    (c) 5.0 ms^{-1} (d) 5.5 ms^{-1} **[2005]**

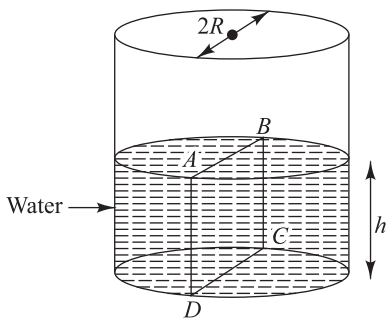
12. A wire elongates by l mm when a load W is hung from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm)

    (a) zero (b) $l/2$

    (c) l (d) $2l$ **[2006]**

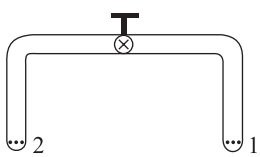
13. The terminal speed of a sphere of gold (density 19.5 $\times 10^3$ kg m^{-3}) in a viscous liquid (density 1.5×10^3 kg m^{-3}) is 0.2 ms^{-1}. What is the terminal speed of a sphere of silver (density 10.5×10^3kg m^{-3}) of the same size in the same liquid?
    (a) 0.1 ms^{-1} (2) 0.2 ms^{-1}
    (c) 0.4 ms^{-1} (d) 0.133 ms^{-1} **[2006]**

14. Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is p, the surface tension of water is T and the atmospheric pressure is P_0. Consider a vertical section $ABCD$ of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude
    (a) $|2P_0Rh + \pi R^2\rho gh - 2\,RT|$
    (b) $|2P_0Rh + R\rho gh^2 - 2\,RT|$
    (c) $|P_0\pi R^2 + R\rho gh^2 - 2\,RT|$
    (d) $|P_0\pi R^2 + R\rho gh^2 + 2\,RT|$



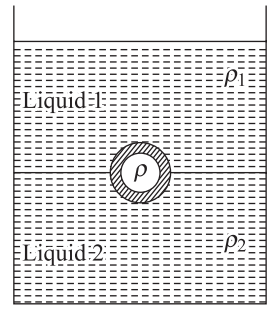
[2007]

15. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. End 1 has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in the figure. Just after opening the valve,



    (a) air from end 1 flows towards end 2. No change in the volume of the soap bubbles
    (b) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases
    (c) no change occurs
    (d) air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases **[2008]**

16. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ, and ρ_2 respectively. A solid ball, made of a material of density ρ_3 is dropped in the jar. It comes to equilibrium in the position shown in the figure.
    Which of the following is true for ρ_1, ρ_2 and ρ_3?

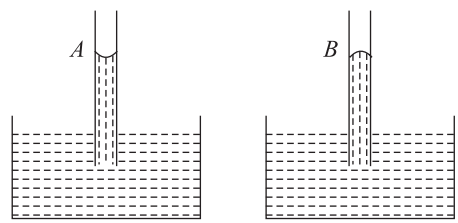


    (a) $\rho_1 < \rho_3 < \rho_2$ (c) $\rho_3 < \rho_1 < \rho_2$
    (c) $\rho_1 > \rho_3 > \rho_2$ (d) $\rho_1 < \rho_2 < \rho_3$ **[2008]**

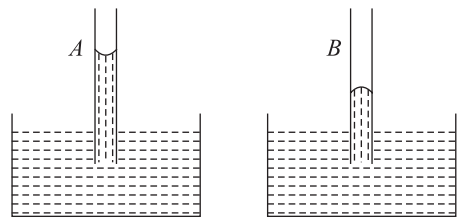
17. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_2 ($\rho_2 < \rho_1$). Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, i.e. $F_{viscous} = -kv^2$ ($k > 0$). The terminal speed of the ball is

    (a) $\dfrac{Vg(\rho_1 - \rho_2)}{k}$ (b) $\sqrt{\dfrac{Vg(\rho_1 - \rho_2)}{k}}$
    (c) $\dfrac{Vg\rho_1}{k}$ (d) $\sqrt{\dfrac{Vg\rho_1}{k}}$ **[2008]**

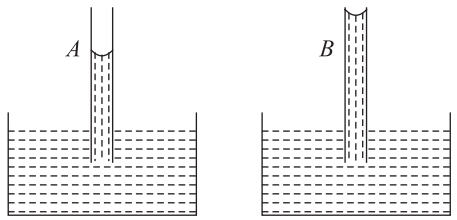
18. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes? **[2008]**



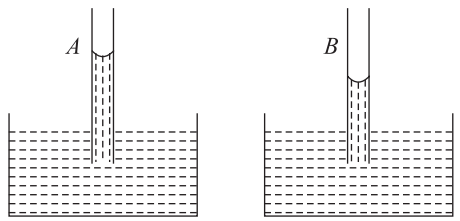
(a)



(b)



(c)



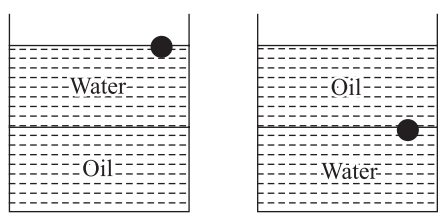
(d)

19. Two wires are made of the same material and have the same volume. However, wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount?

(a) 6 F (b) 9 F

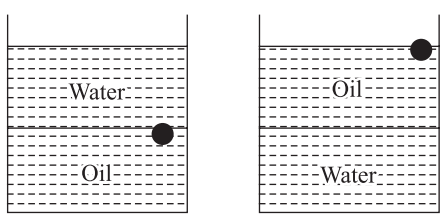
(c) F (d) $4F$ **[2009]**

20. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a misture of this oil and water, which of the following pictures represents its equilibrium position?



(a)

(b)



(c)

(d) **[2010]**

21. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (surface tension of soap solution = 0.03 Nm^{-1})

(a) 4π mJ (b) 0.2π mJ

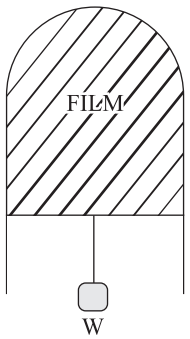
(c) 2π mJ (d) 0.4π mJ **[2011]**

22. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 0.2 m below the tap is close to

(a) 5.0×10^{-3} m (b) 7.5×10^{-3} m

(c) 9.6×10^{-3} m (d) 3.6×10^{-3} m **[2011]**

23. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see the figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is:



(a) 0.0125 Nm^{-1}

(b) 0.1 Nm^{-1}

(1) 0.05 Nm^{-1}

(4) 0.025 Nm^{-1} **[2012]**

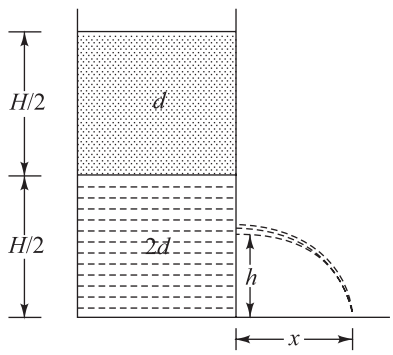
24. A unifom cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is: (Here k is spring constant)

(a) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$ (b) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$

(c) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$ (d) $\frac{Mg}{k}$ **[2013]**

Questions 25 and 26 are based on the following passage

A container of large uniform cross-sectional area A resting on a horizontal surface, is filled with two non-mixing and non viscous liquids of densities d and $2d$, each to a height $H/2$ as shown in the figure. A tiny hole of cross-sectional area a ($<<A$) is punched on the vertical side of the container at a height $h = H/4$.



25. The horizontal range x of the liquid initially is

(a) H (b) $\frac{H}{\sqrt{2}}$

(c) $\frac{H}{2}$ (d) $\frac{H}{2\sqrt{2}}$

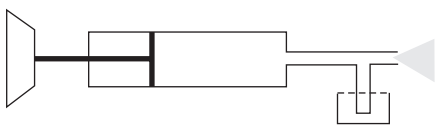
26. The height h of the hole from the ground for which the horizontal range x is maximum is

(a) $\frac{2H}{3}$ (b) $\frac{3H}{2}$

(c) $\frac{3H}{4}$ (d) $\frac{H}{2}$ **[2014]**

Questions 27 and 28 are based on the following paragraph.

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm, respectively. The upper end of the container is open to the atmosphere.



27. If the piston is pushed at a speed of 5 mms^{-1} the air comes out of the nozzle with a speed of

(a) 0.1 ms^{-1} (b) 1 ms^{-1}

(c) 2 ms^{-1} (d) 8 ms^{-1}

28. If the density of air is ρ_a and that of the liquid ρ_l, then for a given piston speed, the rate (volume per unit time) at which the liquid is sprayed will be proportional to

(a) $\sqrt{\frac{\rho_a}{\rho_l}}$ (b) $\sqrt{\rho_a \rho_l}$

(c) $\sqrt{\frac{\rho_l}{\rho_a}}$ (d) ρ_l **[2014]**

Answers

1. (b)	2. (d)	3. (d)	4. (d)
5. (b)	6. (c)	7. (d)	8. (d)
9. (d)	10. (b)	11. (c)	12. (c)
13. (a)	14. (b)	15. (b)	16. (a)
17. (b)	18. (d)	19. (b)	20. (b)
21. (d)	22. (d)	23. (d)	24. (b)
25. (b)	26. (c)	27. (c)	28. (a)

Solutions

1. $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20 \text{ ms}^{-1}$
2. When the coin falls into water, the downward force reduces causing the block to rise thus decreasing l. Since the density of the coin is greater then that of water, the volume of coin will be less than the volume of the liquid displaced, causing a decrease in h. Hence the correct choice is (d).
3. The work done by a force in stretching a wire is stored in the wire as potential energy. The work done by a force F in stretching the wire by a small amount dl is

$$dW = Fdl$$

The work done in stretching a wire by l is

$$W = \int_0^l Fdl$$

Now $F = \frac{YAl}{L}$. Therefore

$$W = \frac{YA}{L}\int_0^l ldl = \frac{YAl^2}{2L} = \frac{1}{2}Fl \left(\because F = \frac{YAl}{L}\right)$$

$$= \frac{1}{2} \times 200 \times (1 \times 10^{-3})$$

$$= 0.1\text{J}$$

4. Refer to the solution of Q. 3 above. The correct choice is (d).
5. According to Stake's law, viscous force is

$F = 6\pi\eta Rv$

So the correct choice is (b)

6. The excess pressure inside a bubble is $p = 4\sigma/R$ where σ = surface tension of liquid and R = radius of bubble. Thus $p \propto \frac{1}{R}$. Hence pressure inside the smaller bubble is higher than that in the bigger babble. So air will flow from the smallar bubble to the bigger bubble.
7. Let v be the horizontal speed of water when it emerges from the nozzle. From the equation of continuity, we have

$$AV = av$$

or $$v = \frac{AV}{a} \qquad (1)$$

Let t be the time taken by the stream of water to strike the ground. The horizontal and vertical distances covered in time t are

$$R = vt \quad (2)$$

$$h = \frac{1}{2}gt^2 \quad (3)$$

From Eq. (3) we have $t = \sqrt{\frac{2h}{g}}$. Using this value in Eq. (2), we get

$$R = v\sqrt{\frac{2h}{g}} \quad (4)$$

Using Eqs. (1) and (4), we have

$$R = \frac{AV}{a}\sqrt{\frac{2h}{g}}$$

8. The terminal velocity is

$$v_t = \frac{2}{9}\frac{(\rho-\sigma)r^2 g}{\eta}$$

The rate of production of heat (or power dissipated) is given by

$$P = fv_t, \text{ where } f = 6\pi\eta r v_t$$

$$\therefore \quad P = 6\pi\eta r\, v_t \times v_t$$

$$= 6\,\pi\eta r\left[\frac{2}{9}\frac{(\rho-\sigma)r^2\, g}{\eta}\right]^2$$

$$= \left[\frac{8\pi g^2}{27\eta}(\rho-\sigma)^2\right]r^5$$

i.e. $P \propto r^5$.

9. The freely falling apparatus is in a weightless condition. Hence, water will rise to the top of the capillary tube. So the length of water column will be 20 cm.

10. Energy stored per unit volume is

$$u = \frac{1}{2}\text{stress} \times \text{strain}$$

$$= \frac{1}{2}\text{stress} \times \frac{\text{stress}}{\text{Young's modulus}}$$

$$= \frac{1}{2}\frac{s^2}{Y}$$

11. Let v be the velocity of efflux and V be the velocity with which the water level in the tank falls. From the equation of continuity of flow, we have $av = A\,V$ or

$$V = \frac{av}{A}$$

From Bernoulli's theorem, we have

$$P_0 + \frac{1}{2}\rho V^2 + \rho g H = P_0 + \frac{1}{2}\rho v^2 + \rho g h$$

which gives $v^2 = V^2 + 2g\,(H - h)$ (2)

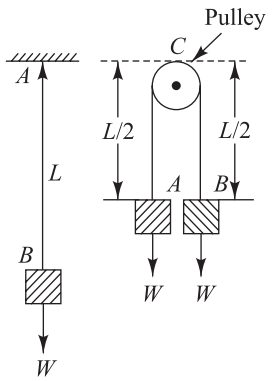
Using (1) in (2), we get

$$V^2 = \frac{2g(H-h)}{1-\left(\frac{a}{A}\right)^2} = \frac{2\times 9.8\times(2.0-0.75)}{1-0.2} = 25$$

which gives $v = 5.0\ \text{ms}^{-1}$.

12. Let L be the length of wire AB and a be the area of cross-section of the wire. The Young's modulus is given by

$$Y = \frac{W/a}{l/L} = \frac{WL}{al}$$



For a given wire, Y is fixed. For small extension, a is constant. Then for a given load, it follows from (1) that L/l = constant, i.e. $l \propto L$. The extension is proportional to the original length of the wire. In the second case $AC = CB = \frac{L}{2}$ since the load on both sides of the pulley is the same. Since the extension is proportional to length, the extension of wire AC of length $L/2$ will be $l/2$. Similarly, the extension of wire BC will be $l/2$. Hence, the total extension is $\frac{l}{2}+\frac{l}{2} = l$, which in choice (c).

13. The terminal speed of a sphere of radius r and density σ in a liquid of density ρ and viscosity η is given by

$$v = \frac{2}{9}.\frac{r^2 g}{\eta}(\sigma-\rho) = K(\sigma-\rho)$$

where $K = \frac{2}{9}.\frac{r^2 g}{\eta}$ is a constant for this problem.

For the gold sphere,

$0.2 = K(19.5 - 1.5)10^3 = 18\times 10^3\, K$ (1)

For the silver sphere,

$v_t = K(10.5 - 1.5)10^3 = 9\times 10^3\, K$ (2)

From (1) and (2) we get

$$\frac{v_t}{0.2} = \frac{9\times10^3 K}{18\times10^3 K} = \frac{1}{2}$$

$$\Rightarrow \qquad v_t = 0.1 \text{ ms}^{-1}$$

14. Let us find the force exerted on water in the right side of section *ABCD*. This force F = force f_1, due to pressure on the left side of section *A BCD* + force f_2 due to surface tension. Now

f_1 = pressure at the center × area of *ABCD*

$$= \left(P_0 + \frac{h}{2}\rho g\right) \times 2Rh \text{ directed towards right}$$

f_2 = surface tension × length *AB*

$= T \times 2R = 2RT$ directed toward left

$$\therefore \qquad F = \left(P_0 + \frac{h}{2}\rho g\right) \times 2Rh - 2RT$$

$$= [2P_0Rh + R\rho gh^2 - 2RT]$$

15. Excess pressure across a bubble is inversely proportional to the radius of the bubble, i.e. $\Delta p \propto \frac{1}{r}$.

Since the value of r of the sub-hemispherical bubble is more than that for a hemispherical bubble, the excess pressure across the bubble at end 2 is less than that at end 1. Hence air from end 1 will flow towards end 2 resulting in a decrease in the volume of the bubble at end 1. Thus the correct choice is (b).

16. Since liquid 1 is above liquid 2, $\rho_1 < \rho_2$. Since the ball sinks in liquid l, $\rho_3 > \rho_1$. Since the ball floats in liquid 2, $\rho_3 < \rho_2$. Hence $\rho_t < \rho_3 < \rho_2$, which is choice (a).

17. The ball will acquire terminal speed v_t when its weight W acting downwards = upthrust U + viscous force F acting upwards, i.e.,

$$W = U + F$$

$$\rho, Vg = \rho_2 Vg - kv_t^2$$

$$v_t = \sqrt{\frac{(\rho_1 - \rho_2)Vg}{k}}$$

18. $h = \frac{2\sigma}{\rho rg}$. The surface tension of soap solution is less than that of water. Hence the value of height h to which the liquid rises in the capillary tube is less for soap solution than for water. Also the angle of contact of soap solution with glass is acute and it wets glass. Hence the shape of its meniscus is concave as for water. So the correct choice is (d)

19. Let L_1 and L_2 be the lengths of wires 1 and 2.

Given $\qquad V_1 = V_2 \Rightarrow AL_1 = 3AL_2 \Rightarrow \frac{L_1}{L_2} = 3.$

If F and F' are the forces applied to 1 and 2,

$$Y = \frac{FL_1}{A\Delta x} = \frac{F'L_2}{3A\Delta x} \Rightarrow \frac{F'}{F} = \frac{3L_1}{L_2} = 3\times3 = 9.$$

Thus $F' = 9F$, which is choice (b)

20. Since the density of oil is less than that of water, oil will float on water. So choices (a) and (c) are wrong. Since the density of the material of the ball is more than that of oil and less than that of water, the ball will sink in oil and float in water. Hence the correct choice is (b).

21. $w = \sigma\Delta A$

$= \sigma \times [8\pi (r_2^2 - r_1^2)]$

$= 0.03 \times [8\pi(5^2 - 3^2) \times 10^{-4}]$

$= 0.384\pi \times 10^{-3}$ J

$\simeq 0.4\pi$ mJ

22. Given u = 0.4 ms–1. The velocity (v) of water at a distance h = 0.2 m below the tap is given by

$$v^2 - u^2 = 2\text{gh}$$

$$\Rightarrow \qquad v = \sqrt{u^2 + 2gh}$$

$$= \sqrt{(0.4)^2 + 2\times10\times0.2}$$

$$= \sqrt{4.16} \text{ sm}^{-1} \simeq 2.04 \text{ sm}^{-1}$$

From equation of continuity of flow, $A_1v_1 = A_2v_2$ which gives

$$\pi r_1^2 \times u = \pi r_2^2 \times v$$

$$\Rightarrow \qquad r_2 = \sqrt{\frac{r_1^2 u}{v}}$$

$$= \sqrt{\frac{(4\times10^{-3})^2 \times 0.4}{2.04}}$$

$$\simeq 1.8 \times 10^{-3} \text{ m}$$

$\therefore$ Diameter $d_2 = 2r_2 = 3.6 \times 10^{-3}$ m

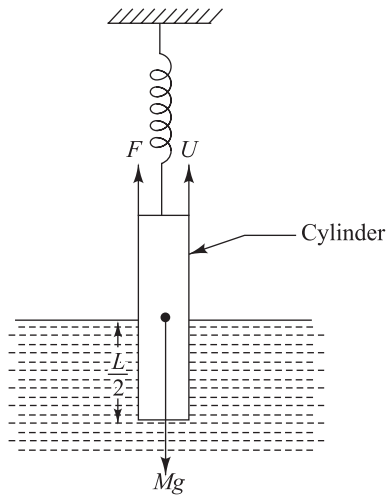
23. $2\sigma L = mg$

$$\Rightarrow \quad \sigma = \frac{mg}{2L} = \frac{1.5\times10^{-2}}{2\times0.3} = 0.025 \text{ Nm}^{-1}$$

24. Upthrust U = weight of liquid displaced

$$= A \times \frac{L}{2} \times \sigma g$$

or $\qquad U = \frac{AL\sigma g}{2}$



Spring force $F = kx_0$.

At equilibrium,

$$Mg = F + U = kx_0 + \frac{AL\sigma g}{2}$$

$$\Rightarrow \qquad x_0 = \frac{Mg}{k}\left(1 - \frac{AL\sigma}{2M}\right)^2$$

So the correct choice is (b).

25. Let v_1 be the initial velocity of the fall of liquid level in the container and v_2 the velocity of efflux from the hole. Then $Av_1 = av_2$ or $v_1 = \left(\frac{a}{A}\right)v_2$. Since $a << A$, $v_1 = 0$. Applying Bernoulli's theorem at a point on the top of the liquid and at the hole, we have

$$P_0 + \frac{1}{2}dv_1^2 + dg\left(\frac{H}{2}\right) + (2d)g\left(\frac{H}{2}\right)$$

$$= P_0 + \frac{1}{2}(2d)dv_2^2 + (2d)gh$$

Putting $v_1 = 0$, we get

$$v_2 = \left[\left(\frac{3}{2}H - 2h\right)g\right]^{1/2}$$

Putting $h = H/4$, we get $v_2 = \sqrt{gH}$ which is choice (d).

Time of fall $t = \sqrt{\frac{2h}{g}}$. Horizontal range is

$$x = v_2 t = \left[\left(\frac{3}{2}H - 2h\right)g\right]^{1/2} \times \sqrt{\frac{2h}{g}}$$

$$= [h(3H - 4h)]^{1/2}$$

Putting $h = H/4$, we get $x = \frac{H}{\sqrt{2}}$, which is choice (b).

26. To determine the height h at which the range x is maximum, we differentiate (1) with respect to h and set $\frac{dx}{dh} = 0$, i.e.

$$\frac{d}{dh}[3Hh - 4h^2]^{1/2} = 0$$

or

$$\frac{1}{2}[3Hh - 4h^2]^{-1/2} \times (3H - 8h) = 0$$

which gives $3H - 8h = 0$ or $h = \frac{3}{8}H$. Notice that $(3Hh - 4h^2)^{-1/2}$ cannot be zero since this would give x = infinity which is not possible. Hence x is maximum ($= x_m$) at a value of h given by

$$h = \frac{3}{8}H$$

Using this value of h in (1), we get

$$x_m = \left[\frac{3}{8}H\left(3H - \frac{12}{8}H\right)\right]^{1/2} = \frac{3}{4}H$$

27. From the equation of continuity of flow,

$$\Rightarrow \qquad a_1 v_1 = a_2 v_2$$

$$\Rightarrow \qquad (\pi r_1^2) v_1 = (\pi r_2^2) v_2$$

$$\Rightarrow \qquad v_2 = v_1 \times \left(\frac{r_1}{r_2}\right)^2$$

$$= 5 \text{ mm s}^{-1} \times \left(\frac{20 \text{ mm}}{1 \text{ mm}}\right)^2$$

$$= 2000 \text{ mm s}^{-1}$$

$$= 2 \text{ ms}^{-1}$$

28. From Bernoulli's principle,

$$P_0 + \frac{1}{2}\rho_a v_a^2 = P_0 + \frac{1}{2}\rho_l v_l^2$$

$$\Rightarrow \qquad v_l^2 = v_a^2 \frac{\rho_a}{\rho_l}$$

$$\Rightarrow \qquad v_l = v_a \sqrt{\frac{\rho_a}{\rho_l}}$$

For given v_a, $v_l \propto \sqrt{\frac{\rho_a}{\rho_l}}$, which is choice (a).

HEAT AND THERMODYNAMICS

Chapter 8

REVIEW OF BASIC CONCEPTS

1. Thermal Expansion

If the temperature of a body is increased, its length, surface area and volume all increase.

(i) Coefficient of Linear Expansion If the body is in the form of a rod, the increase ΔL in its length when the temperature is increased by ΔT is proportional to (*a*) the original length L and (*b*) increase in temperature ΔT, i.e.

$$\Delta L \propto L\Delta T \quad \Rightarrow \quad \Delta L = \alpha L \Delta T$$

Thus $$\alpha = \frac{\Delta L}{L\Delta T}$$

where α is the coefficient of linear expansion of the material of the body. Thus

$$L = L_0 (1 + \alpha \Delta T)$$

(ii) Coefficient of Area Expansion

Similarly $$\beta = \frac{\Delta A}{A\Delta T}$$

where β is called the coefficient of area expansion

$$A = A_0 (1 + \beta \Delta T)$$

(iii) Coefficient of Volume Expansion

$$\gamma = \frac{\Delta V}{V\Delta T}$$

$$V = V_0 (1 + \gamma \Delta T)$$

The SI unit of α, β and γ is $(°C)^{-1}$ or K^{-1}.

(iv) Relation between α, β and γ is

$$\alpha : \beta : \gamma = 1 : 2 : 3$$

i.e. $$\gamma = 2\beta = 3\alpha.$$

(v) Variation of Density with Temperature

$$\text{Density} = \frac{\text{mass}}{\text{volume}} \quad \text{or} \quad \rho = \frac{m}{V}$$

Since mass m remains constant, ρV = constant. Thus

$$\rho V = \rho_0 V_0$$

$$\Rightarrow \qquad \rho = \frac{\rho_0 V_0}{V} = \frac{\rho_0 V_0}{V_0 (1 + \gamma \Delta T)} = \frac{\rho_0}{(1 + \gamma \Delta T)}$$

Thus density of a substance decreases with increase in temperature.

(vi) Thermal Stress

If a rod is held between two rigid supports and its temperature is increased or decreased, the rigid supports prevent the rod from expanding or contracting. As a result, a stress (called thermal stress) is developed in the rod. The change in length of the rod is

$$\Delta L = \alpha L \Delta T$$

$$\therefore \qquad \text{Strain} \left(\frac{\Delta L}{L} \right) = \alpha \Delta T$$

Now $$\text{Young's modulus } (Y) = \frac{\text{stress}}{\text{strain}}$$

$$\therefore \qquad \text{Thermal stress} = \frac{Y \Delta L}{L} = Y \alpha \Delta T$$

EXAMPLE 1 At 20°C, a brass rod has a length 50.0 cm. It is joined to a steel rod of the same length and the same diameter at the same temperature. Find the change in the length of the composite rod when it is heated to 220°C. For brass $\alpha = 2 \times 10^{-5}\ K^{-1}$ and for steel $\alpha = 1 \times 10^{-5}\ K^{-1}$.

SOLUTION

For brass: $(\Delta L)_b = \alpha_b L_b \Delta T = (2 \times 10^{-5}) \times 0.5 \times (220 - 20)$

$$= 2 \times 10^{-3}\ \text{m} = 2\ \text{mm}$$

For steel: $(\Delta L)_s = \alpha_s L_s \Delta T = (1 \times 10^{-5}) \times 0.5 \times (220 - 20)$

$= 1$ mm

$\therefore \quad \Delta L = (\Delta L)_b + (\Delta L)_s = 3$ mm

EXAMPLE 2 A metal wire of cross-sectional area 5×10^{-6} m^2 is held taut at 30°C between two rigid supports with negligible tension in it. Find the tension developed in the wire if it is cooled to –20°C. Given α of metal = 2×10^{-5} K^{-1} and $Y = 1 \times 10^{11}$ N m^{-2}.

SOLUTION

Thermal stress $= Y\alpha\Delta T$

Also $\quad$ stress $= \dfrac{F}{A}$

$\therefore \quad F = YA\alpha\Delta T$

$= (1 \times 10^{11}) \times (5 \times 10^{-6}) \times (2 \times 10^{-5}) \times [30 - (-20)]$

$= 500$ N

EXAMPLE 3 The coefficient of volume expansion of a liquid is 5×10^{-4} K^{-1}. If its temperature is increased by 30°C, find the percentage change in its density.

SOLUTION $\quad \rho = \dfrac{\rho_0}{(1 + \gamma\Delta T)}$

$\Rightarrow \quad \rho(1 + \gamma\Delta T) = \rho_0 \quad \Rightarrow \quad \rho - \rho_0 = -\rho_0 \gamma\Delta T$

or $\quad \dfrac{\rho - \rho_0}{\rho_0} = -\gamma\Delta T = -5 \times 10^{-4} \times 30 = -1.5 \times 10^{-2}.$

The negative sign indicates that the density decreases with increase in temperature.

$\therefore$ Percentage change in density = $(1.5 \times 10^{-2}) \times 100 = 1.5\%$

2. Measurement of Heat

Heat is a form of energy. It is, therefore, measured in energy units. The SI unit of heat is *joule* (J). Another unit commonly used is the *calorie*. A calorie is the amount of heat required to raise the temperature of 1 g of water through 1°C. Experiments have shown that 4.18 J of mechanical work produce one calorie of heat.

Thus

1 calorie = 4.18 joules

or $\quad$ 1 cal = 4.18 J

3. Calorimetry

If two substances having different temperatures are brought into thermal contact, heat energy will flow from the hotter to the colder substance. The flow of heat energy will continue till the temperatures are equalized. We say that the thermal equilibrium has been attained. The common temperature at thermal equilibrium is called the equilibrium temperature. If no heat energy is allowed to escape to the surroundings, the amount of heat energy gained by the initially colder body is equal to the amount of heat energy lost by the initially hotter body, i.e.

Heat gained by one body = heat lost by the other body. Thus is the basic principle of calorimetry. The heat energy Q needed to raise the temperature through ΔT of a mass m of a substance of specific heat capacity s is given by

$$Q = ms\,\Delta T$$

This is the basic heat formula.

Specific heat capacity If $m = 1$ unit and $\Delta T = 1$ unit, then $s = Q$. Hence the specific heat capacity of a substance is the amount of heat required to raise the temperature of a unit mass of the substance through a unit degree. A commonly used unit of s is cal g^{-1} °C^{-1}. In the SI system s is expressed J kg^{-1} K^{-1}. The two units are related as

1 cal g^{-1} °C^{-1} = 4.18 Jg^{-1} °C^{-1}

($\because$ 1 cal = 4.18 J)

or $\quad$ 1cal g^{-1} °C^{-1} = 4180 J kg^{-1} °C^{-1}

Since the size of a degree on the celsius scale is equal to that on the kelvin scale, a temperature *difference* of, say, 1 °C is equal to a temperature difference of 1 K. Thus

1 cal g^{-1} °C^{-1} = 4180 J kg^{-1} K^{-1}

The specific heat capacity of water = 1 cal g^{-1} °C^{-1} or 4180 J kg^{-1} K^{-1}.

Molar specific Heat The molar specific heat C of a substance is the amount of heat energy required to raise the temperature of 1 mole of the substance through 1 K. It is expressed in J mol^{-1} K^{-1}.

Specific heat (s) and molar specific heat (C) are related as

$$s = \frac{C}{m}$$

where m is the number of kilograms per mole in the substance.

4. Two Specific Heats of a Gas

Two specific heats of a gas are of special significance, namely, the specific heat at constant value (C_v) and that at constant pressure (C_p). They are defined as: *The molar specific heat of a gas at constant volume (C_v) is the amount of heat energy required to raise the temperature of 1 mole of the gas through 1* K *when its volume is kept constant. The molar specific heat of a gas at constant pressure (C_p) is the amount of heat energy required to raise the temperature of 1 mole of the gas through 1* K *when its pressure is kept constant.*

The two specific heats of an ideal gas are related as

$$C_p - C_v = R$$

when R is the universal gas constant and its value is

$$R = 8.315 \text{ J mol}^{-1} \text{ K}^{-1}$$

5. Latent Heat

The heat energy supplied to a substance to change from solid to liquid state or from liquid to gaseous state is not registered by a thermometer as the heat energy is used up in bringing about a change of state. Hence it is called latent (or hidden) heat. A substance has two latent heats.

1. The latent heat of fusion (or melting) of a substance is the heat energy required to convert a unit mass of a substance from the solid to the liquid state, without change of temperature. The latent heat of fusion of ice is 3.36×10^5 J kg^{-1} or 80 cal g^{-1}.

2. The latent heat of vaporisation (or boiling) of a substance is the heat energy required to convert a unit mass of the substance from the liquid to the gaseous state, without change of temperature. The latent heat of vaporisation of steam is 2.26×10^6 J kg^{-1} or 540 cal g^{-1}.

The amount of heat energy required to change the state of a mass m of a substance without change in temperature is given by

$$Q = mL$$

where L is the latent heat.

EXAMPLE 4 A copper block of mass 3.35 kg is heated to 500°C and then placed on a large block of ice. What is the maximum mass of ice that can melt? Specific heat capacity of copper = 390 J kg^{-1} K^{-1} and latent heat of fusion of water = 335 J g^{-1}.

SOLUTION The amount of heat energy in copper block at 500°C is

$$Q = ms\Delta T$$
$$= (3.35 \times 390 \times 500) \text{ J}$$

Now $L = 335 \text{ J g}^{-1} = 335 \times 10^3 \text{ J kg}^{-1}$.

The maximum mass of ice that can melt is

$$m = \frac{Q}{L} = \frac{3.35 \times 390 \times 500}{335 \times 10^3} = 1.95 \text{ kg}$$

EXAMPLE 5 16 g of oxygen is heated at constant volume from 25°C to 35°C. Find the amount of heat energy required. Given $C_v = 20$ J mol^{-1} K^{-1}.

SOLUTION Mass of 1 mole of oxygen = 32 g

$\therefore$ Number of moles in 16 g of oxygen is $n = \frac{16}{32} = \frac{1}{2}$

Heat energy required is

$$Q = n\, C_v\, \Delta T = \frac{1}{2} \times 20 \times (35 - 25) = 100 \text{ J}$$

EXAMPLE 6 290 J of heat energy is required to raise the temperature of 7 g of nitrogen by 40°C. Find molar specific heat of nitrogen at constant pressure.

SOLUTION Number of moles in 7 g of oxygen is

$$n = \frac{7}{28} = \frac{1}{4}$$

$\therefore$ $Q = n\, C_p\, \Delta T$

$\Rightarrow$ $$C_p = \frac{Q}{n\Delta T} = \frac{290}{\frac{1}{4} \times 40} = 29 \text{ J mol}^{-1} \text{ K}^{-1}$$

EXAMPLE 7 200 g of water at 25°C is added to 75 g of ice at 0°C in an insulated vessel. What is the final temperature of the mixture?

SOLUTION Heat energy of 200 g of water at 25°C is

$$Q_1 = ms\Delta T = 200 \times 1 \times 25 = 5000 \text{ cal}$$

Heat energy required to melt 75 g of ice at 0°C is

$$Q_2 = mL = 75 \times 80 = 6000 \text{ cal}$$

Since $Q_1 < Q_2$, the whole of ice will not melt. Hence, the final temperature of mixture is 0°C.

Note

1. If 1 kg of ice at 0°C is mixed with 1 kg of steam at 100°C, the equilibrium temperature is 100°C, 0.665 kg of steam will be left and 1.335 kg of water will be formed.
2. Water of mass m_w at t°C is mixed with ice of mass m_i at 0°C.
   (a) If $m_w = \frac{L_f m_i}{t}$, whole of ice will melt and the final temperature = 0°C.
   (b) If $m_w < \frac{L_f m_i}{t}$, the whole of ice will not melt, final temperature = 0°C and mass of ice melted is $m'_i = \frac{m_w t}{L_f}$. Amount of ice left = $m_i - m'_i$.
   (c) If $m_w > \frac{L_f m_i}{t}$, the whole of ice will melt and final temperature = $\frac{m_w t - L_f m_i}{m_w + m_i} > 0$°C

EXAMPLE 8 According to the theory of specific heat of solids at very low temperatures (close to absolute zero), the specific heat s of a solid varies with absolute temperature T as

$$s = kT^3$$

where k is a constant whose value depends upon the material of the solid. Find the heat energy required to raise the temperature of 200 g of the solid from 1 K to 4 K.

SOLUTION The amount of heat energy required to raise the temperature of mass m of the solid by dT kelvin is given by

$$dQ = msdT$$

Heat energy required to raise the temperature of the solid from $T_1 = 1$ K to $T_2 = 4$ K is

$$Q = \int_{T_1}^{T_2} msdT = mk\int_{T_1}^{T_2} T^3 dT$$

$$= \frac{mk}{4}\left[T_2^4 - T_1^4\right]$$

$$= \frac{0.2 \times k}{4}\left[(4)^4 - (1)^4\right]$$

$$= 12.75\ k \text{ joule}$$

6. Thermal Conductivity

If a steady temperature difference $(T_1 - T_2)$ is to be maintained between the ends of a rod, heat energy must be supplied at a steady rate at one end and the same must be taken out at the other end. Suppose heat energy Q flows through a rod in time t, then the rate of flow of heat Q/t through the rod in the steady state is

(i) proportional to cross-sectional area A of the rod,

(ii) proportional to temperature difference $(T_1 - T_2)$ to be maintained between the ends of the rod and

(iii) inversely proportional to length L of the rod.

$$\frac{Q}{t} \propto \frac{A(T_1 - T_2)}{L}$$

$$\Rightarrow \quad \frac{Q}{t} = \frac{k\,A(T_1 - T_2)}{L}$$

where k is a constant called thermal conductivity of the material of the rod. It is a measure of how quickly heat energy is conducted (or transferred) through the substance. The SI unit of k is J s^{-1} m^{-1} K^{-1} or W m^{-1} K^{-1}. The dimensions of k are [M L T^{-3} K^{-1}]

7. Conduction Through a Composite Slab

Case 1. Two slabs placed one on top of the other

Suppose we have a composite slab made up of two different slabs of materials of thermal conductivities k_1 and k_2, and cross-sectional areas A_1 and A_2 but of equal length L placed one on top of the other as shown in Fig. 8.1.

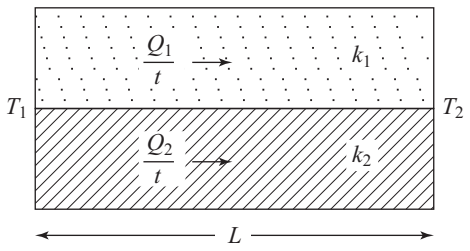


Fig. 8.1

The ends of the composite slab are maintained at temperatures T_1 and T_2 $(T_1 > T_2)$. The rates of heat flow through each slab in the steady state are

$$\frac{Q_1}{t} = \frac{k_1 A_1(T_1 - T_2)}{L} \tag{1}$$

and

$$\frac{Q_2}{t} = \frac{k_2 A_2(T_1 - T_2)}{L} \tag{2}$$

The cross-sectional area of the composite slab is $(A_1 + A_2)$ but its length is L. If k_{eq} is the equivalent thermal conductivity, then the rate of heat flow through the composite slab is

$$\frac{Q}{t} = \frac{k_{eq}(A_1 + A_2)(T_1 - T_2)}{L} \tag{3}$$

Since $\frac{Q}{t} = \frac{Q_1}{t} + \frac{Q_2}{t}$, we get from Eqs. (1), (2) and (3)

$$k_{eq}(A_1 + A_2) = k_1 A_1 + k_2 A_2$$

$$\Rightarrow \quad k_{eq} = \frac{k_1 A_1 + k_2 A_2}{A_1 + A_2} \tag{4}$$

If $A_1 = A_2$, then $k_{eq} = \frac{1}{2}(k_1 + k_2)$.

Case 2. Two slabs placed in contact one after the other

Suppose we have two slabs of different lengths L_1 and L_2 but of the same cross-sectional area A placed in contact as shown in Fig. 8.2.

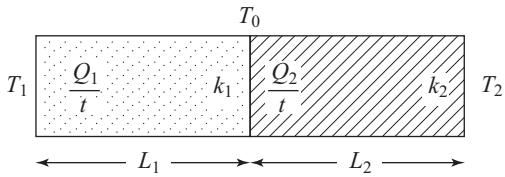


Fig. 8.2

The ends of the composite slab are maintained at temperatures T_1 and T_2 $(T_1 > T_2)$. Let T_0 be the temperature of the junction. In the steady state, $\frac{Q_1}{t} = \frac{Q_2}{t}$, i.e.

$$\frac{k_1 A(T_1 - T_0)}{L_1} = \frac{k_2 A(T_0 - T_2)}{L_2}$$

$$\Rightarrow \quad \frac{k_1}{L_1}(T_1 - T_0) = \frac{k_2}{L_2}(T_0 - T_2)$$

$$\Rightarrow \quad T_0 = \frac{\frac{k_1 T_1}{L_1} + \frac{k_2 T_2}{L_2}}{\frac{k_1}{L_1} + \frac{k_2}{L_2}} \tag{5}$$

The rate of flow of heat through the composite slab is

$$\frac{Q}{t} = \frac{Q_1}{t} \quad \text{or} \quad \frac{Q_2}{t}$$

$$\Rightarrow \quad \frac{Q}{t} = \frac{k_1 A(T_1 - T_0)}{L_1} \tag{6}$$

Using (5) in (6) and simplifying, we get

$$\frac{Q}{t} = \frac{k_1 k_2 A(T_1 - T_2)}{k_1 L_2 + k_2 L_1} \tag{7}$$

The length of the composite slab is $(L_1 + L_2)$ but its cross-sectional area is A. If k_{eq} is the equivalent thermal conductivity of the composite slab,

$$\frac{Q}{t} = \frac{k_{eq} A (T_1 - T_2)}{(L_1 + L_2)} \tag{8}$$

From Eqs. (7) and (8), we get

$$k_{eq} = \frac{k_1 k_2 (L_1 + L_2)}{k_1 L_2 + k_2 L_1} \tag{9}$$

If $L_1 = L_2$, $k_{eq} = \dfrac{2k_1 k_2}{(k_1 + k_2)}$

and $$T_0 = \frac{(k_1 T_1 + k_2 T_2)}{(k_1 + k_2)}$$

8. Thermal Resistance

Finding the equivalent thermal conductivity of a composite slab becomes much easier if we use the concept of thermal resistance. Just as electrical resistance is defined as

$$R = \frac{V}{I} = \frac{\text{potential difference}}{\text{rate of flow of charge}},$$

thermal resistance is defined as

$$R = \frac{\text{temperature difference}}{\text{rate of flow of heat}} = \frac{T_1 - T_2}{Q/t}$$

$$\Rightarrow \quad R = \frac{(T_1 - T_2)L}{k A(T_1 - T_2)} = \frac{L}{k A}$$

(a) If two slabs are joined in series as shown in Fig. 8.2, the equivalent thermal resistance of the composite slab is

$$R_{eq} = R_1 + R_2$$

$$\Rightarrow \quad \frac{L_1 + L_2}{k_{eq} A} = \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A}$$

$$\Rightarrow \quad k_{eq} = \frac{k_1 k_2 (L_1 + L_2)}{(L_1 k_2 + L_2 k_1)}, \text{ which is Eq. (9).}$$

(b) If the two slabs are joined in parallel as shown in Fig. 8.1, then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \quad \frac{k_{eq}(A_1 + A_2)}{L} = \frac{k_1 A_1}{L} + \frac{k_2 A_2}{L}$$

$$\Rightarrow \quad k_{eq} = \frac{(k_1 A_1 + k_2 A_2)}{(A_1 + A_2)}, \text{ which is Eq. (4).}$$

EXAMPLE 9 A cylindrical metal boiler of radius 10 cm and thickness 3.14 cm is filled with water and placed on an electric heater. If the water boils at the rate of 50 g s^{-1}, estimate the temperature of the filament. Thermal conductivity of metal = 1.13×10^2 W m^{-1} K^{-1} and latent heat of vaporisation = 2.26×10^3 J g^{-1}.

SOLUTION $Q = ML_v$

$$\therefore \quad \frac{Q}{t} = (50 \text{ g s}^{-1}) \times (2.26 \times 10^3 \text{ J g}^{-1})$$
$$= 50 \times 2.26 \times 10^3 \text{ Js}^{-1}$$

Base area of boiler $A = \pi r^2 = \pi \times (0.1)^2 = \pi \times 10^{-2}$ m^2

Thickness of metal $L = 3.14$ cm $= 3.14 \times 10^{-2}$ m

$$\frac{Q}{t} = \frac{k A(T_f - T_w)}{L}$$

$$\Rightarrow 50 \times 2.26 \times 10^3 = \frac{(1.13 \times 10^2) \times (\pi \times 10^{-2}) \times (T_f - T_w)}{3.14 \times 10^{-2}}$$

$$\Rightarrow \quad T_f - T_w = 1000°\text{C}$$

$$\therefore \quad T_f = 1000 + T_w = 1000 + 100 = 1100°\text{C}$$

EXAMPLE 10 A steel rod ($L_1 = 10$ cm, $A_1 = 0.02$ m^2 and $k_1 = 50$ J s^{-1} m^{-1} K^{-1}) is welded to a silver rod ($L_2 = 20$ cm, $A_2 = 0.01$ m^2, $k_2 = 400$ J s^{-1} m^{-1} K^{-1}) as shown in Fig. 8.3. The ends of the composite rod are maintained at 300°C and 0°C. The rod is covered with an insulating material so that the heat loss from the sides is negligible. Compute the temperature of the steel-silver junction in the steady state.

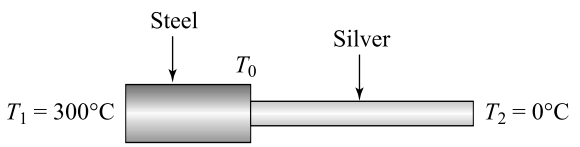


Fig. 8.3

SOLUTION In the steady state, $\dfrac{Q_1}{t} = \dfrac{Q_2}{t}$, i.e.

$$\frac{k_1 A_1 (T_1 - T_0)}{L_1} = \frac{k_2 A_2 (T_0 - T_2)}{L_2}$$

$$\Rightarrow \quad \frac{50 \times 0.02 \times (300 - T_0)}{0.1} = \frac{400 \times 0.01 \times (T_0 - 0)}{0.2}$$

$$\Rightarrow \quad T_0 = 100°\text{C}$$

EXAMPLE 11 A steel rod ($L_1 = 10$ cm, $A_1 = 0.02$ m^2, $k_1 = 50$ W m^{-1} K^{-1}) and a brass ($L_2 = 10$ cm, $A_2 = 0.02$ m^2, $k_2 = 110$ W m^{-1} K^{-1}) are soldered as shown in Fig. 8.4. The ends of the composite rod are maintained at 403 K and 273 K. The sides of compound rod is covered with an insulating material. Calculate the rate of flow through the compound rod in the steady state.

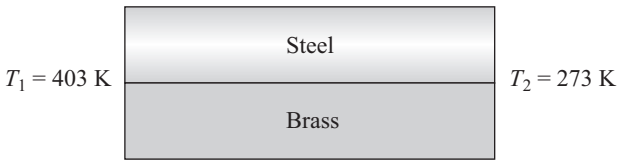


Fig. 8.4

SOLUTION Equivalent thermal conductivity of the composite rod is

$$k_{eq} = \frac{1}{2}(k_1 + k_2) = \frac{1}{2}(50 + 110)$$

$$= 80 \text{ W m}^{-1} \text{ K}^{-1}$$

$$\therefore \quad \frac{Q}{t} = \frac{k_{eq} \times (A_1 + A_2)(T_1 - T_2)}{0.1}$$

$$= \frac{80 \times 0.04 \times (403 - 273)}{0.1}$$

$$= 4.16 \times 10^3 \text{ Js}^{-1}$$

EXAMPLE 12 A metal cylinder of radius r and thermal conductivity $k_1 = 2k$ is surrounded by a cylindrical metallic shell of inner radius r and outer radius $2r$ having thermal conductivity $k_2 = k$ (Fig. 8.5). The ends of this composite system are maintained at constant temperatures T_1 and T_2 ($T_1 > T_2$). Find the equivalent thermal conductivity of the system.

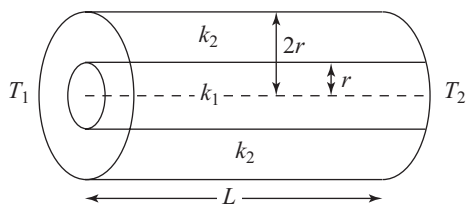


Fig. 8.5

SOLUTION The given system is equivalent to a system consisting of a rod of length L and cross-sectional area $A_1 = \pi r^2$ of conductivity k_1 and a rod of the same length L and cross-sectional area $A_2 = \pi[(2r)^2 - r^2] = 3\pi r^2$ and conductivity k_2 placed one on top of the other (in parallel). In the steady state

$$\frac{Q}{t} = \frac{Q_1}{t} + \frac{Q_2}{t}$$

$$= \frac{k_1 A_1 (T_1 - T_2)}{L} + \frac{k_2 A_2 (T_1 - T_2)}{L}$$

$$\Rightarrow \quad \frac{Q}{t} = \frac{k_1 (\pi r^2)(T_1 - T_2)}{L} + \frac{k_2 (3\pi r^2)(T_1 - T_2)}{L} \qquad \text{(i)}$$

Cross-sectional area of the composite system is $A = \pi(2r)^2 = 4\pi r^2$. If k_{eq} is the equivalent conductivity, then

$$\frac{Q}{t} = \frac{k_{eq} (4\pi r^2)(T_1 - T_2)}{L} \qquad \text{(ii)}$$

From (i) and (ii) we find that

$$4\, k_{eq} = k_1 + 3\, k_2$$

$$\Rightarrow \quad k_{eq} = \frac{k_1 + 3k_2}{4} = \frac{2k + 3k}{4} = \frac{5k}{4}$$

EXAMPLE 13 A cylindrical metallic shell has inner radius $r_1 = 2$ cm and outer radius $r_2 = 4$ cm and it has a length $L = 50$ cm. The inner and outer surfaces are maintained at $T_1 = 0°$C and $T_2 = 100°$C. The thermal conductivity of metal is 69.3 Wm^{-1}K^{-1}. Find the rate of flow of heat from the outer to the inner surface.

SOLUTION Refer to Fig. 8.6.

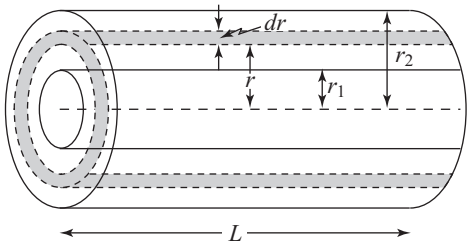


Fig. 8.6

Consider a small element of thickness dr at a distance r from the axis of the shell. Let dT be the temperature between the inner and outer surfaces of the element. If H is the rate of heat flow in the element,

$$H = \frac{k \times 2\pi r L \times dT}{dr}$$

$$\Rightarrow \quad \frac{dr}{r} = \frac{2\pi k L\, dT}{H}$$

Integrating

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k L}{H} \int_{T_1}^{T_2} dT$$

$$\Rightarrow \quad \ln\left(\frac{r_2}{r_1}\right) = \frac{2\pi k L}{H} \times (T_2 - T_1)$$

$$\Rightarrow \quad \ln\left(\frac{4\text{ cm}}{2\text{ cm}}\right) = \frac{2 \times 3.14 \times 69.3 \times 0.5}{H} \times (100 - 0)$$

$$0.693 = \frac{2 \times 3.14 \times 69.3 \times 0.5 \times 100}{H}$$

$$\Rightarrow \quad H = 3.14 \times 10^4 \text{ Js}^{-1}$$

9. Radiation

All bodies emit heat from their surfaces at all temperatures. The heat radiated by a body is called *radiant heat or thermal radiation.* Thermal radiations are electromagnetic waves which travel in space with a velocity equal to that of light.

(1) Black Body A perfect black body is one which completely absorbs radiations of every wavelength incident on it. A good absorber of radiations is also a good emitter of radiations. Consequently, a black body, when heated to a suitably high temperature, will emit radiations of all wavelengths. Such radiation is called *black body radiation.*

(2) Emissive Power The emissive power (e) of a body is the amount of heat energy emitted per second from a unit area of a radiating surface. The SI unit of e is $Js^{-1}\ m^{-2}$ or Wm^{-2}.

(3) Absorptive Power The absorptive power of a body is the ratio of radiant energy absorbed by it to the total amount of radiant energy incident on it. It is denoted by a and is a fraction. Since, by definition, a black body completely absorbs all radiations, $a = 1$ for a black body.

(4) Emissivity ($\in$) of a body is defined as the ratio of its emissive power (e) to that of a black body (E),

$$\text{i.e.} \qquad \varepsilon = \frac{e}{E}$$

For a black body, $\varepsilon = 1$.

(5) Kirchhoff's Law At any given temperature and for radiations of the same wavelength, the ratio of the emissive power to the absorptive power is the same for all substances, i.e.

$$\frac{e}{a} = \text{constant}$$

(6) Stefan's Law The total energy emitted per second by a unit area of a body is proportional to the fourth power of its absolute temperature, i.e.

$$E \propto T^4$$

$$\text{or} \qquad E = \varepsilon \sigma T^4$$

where σ is a constant known as *Stefan's constant.* Its value is

$$\sigma = 5.735 \times 10^{-8}\ Wm^{-2}\ K^{-4}$$

For a black body, $E = \sigma T^4$

(7) Newton's Law of Cooling The rate of loss of heat by a body is directly proportional to the excess of its temperature over that of its surroundings, provided this temperature difference is small, i.e.

$$\frac{dQ}{dt} \propto (T - T_0)$$

where T is the temperature of the body and T_0 that of the surroundings. If m is the mass of the body, s its specific heat and dT the change in temperature in time dt, then $dQ = msdT$. Therefore,

$$\frac{dQ}{dt} = ms\ \frac{dT}{dt} = -k(T - T_0)$$

where $-k$ is a constant of proportionality. Thus

$$\frac{dT}{(T - T_0)} = -\frac{k}{ms}\ dt$$

Integrating between limits T_1 and T_2, we have

$$\left|\log_e (T - T_0)\right|_{T_1}^{T_2} = -\frac{k}{ms}\ t$$

$$\therefore \quad t = \frac{ms}{k}\ \log_e \left(\frac{T_1 - T_0}{T_2 - T_0}\right) = K \log_e \left(\frac{T_1 - T_0}{T_2 - T_0}\right)$$

This expression gives the time taken by a body to cool from T_1 to T_2 when placed in a medium of temperature T_0. An approximate formula is

$$\frac{T_1 - T_2}{t} = \frac{1}{K}\left(\frac{T_1 + T_2}{2} - T_0\right)$$

(8) Wien's Displacement Law As the temperature of a black body increases, the maximum intensity of emission shifts (or is displaced) towards shorter wavelengths. In other words,

$$\lambda_m T = b = \text{constant}$$

where λ_m is the wavelength at which maximum emission takes place at absolute temperature T. The value of constant b is

$$b = 2.89 \times 10^{-3}\ \text{mK (metre kelvin)}$$

EXAMPLE 14 The surface area of the skin of a person is 2 m^2. He is sitting in a room where the air temperature is 27°C. If his skin temperature is 37°C, calculate the rate at which his body loses heat. The emissivity of his skin is 0.8 and Stefan's constant $\sigma = 5.7 \times 10^{-8}\ W\ m^{-2}\ K^{-1}$.

SOLUTION $T = 273 + 37 = 310$ K and $T_0 = 273 + 27 = 300$ K

Rate of loss of heat $= \varepsilon A\ \sigma (T - T_0^4)$

$$= 0.8 \times 2 \times (5.7 \times 10^{-8})\ [(310)^4 - (300)^4]$$

$$= 1.03 \times 10^2\ W$$

EXAMPLE 15 Experimental investigations show that the intensity of solar radiation is maximum for a wavelength 475.3 nm. Estimate the surface temperature of the sun. Wien's constant $b = 2.89 \times 10^{-3}$ mK.

SOLUTION According to Wien's law, $\lambda_m T = b$.

$$\therefore \qquad T = \frac{b}{\lambda_m} = \frac{2.89 \times 10^{-3}}{475.3 \times 10^{-9}} = 6080\ \text{K}$$

EXAMPLE 16 A body cools from 80°C to 50°C in 6 minutes in a room where the temperature is 20°C. What is the temperature of the body at the end of next 6 minutes?

SOLUTION

Given $$\log_e\left(\frac{80-20}{50-20}\right) = 6 \text{ K}$$

or $$\log_e(2) = 6 \text{ K} \quad \text{(i)}$$

If the temperature at the end of next 6 minutes is T, then

$$\log_e\left(\frac{50-20}{T-20}\right) = 6 \text{ K} \quad \text{(ii)}$$

From (i) and (ii)

$$\log_e\left(\frac{30}{T-20}\right) = \log_e(2)$$

$$\Rightarrow \quad 30 = 2(T-20)$$

$$\Rightarrow \quad T = 35°\text{C}$$

Alternative Method

We can also use the approximate formula

$$\frac{T_1 - T_2}{t} = \frac{1}{K}\left[\frac{(T_1+T_2)}{2} - T_0\right]$$

Then we have

$$\frac{80-50}{6} = \frac{1}{K}\left[\frac{(80+50)}{2} - 20\right]$$

$$\Rightarrow \quad 5 = \frac{45}{K} \Rightarrow K = 9 \quad \text{(i)}$$

And $$\frac{50-T}{6} = \frac{1}{K}\left[\left(\frac{50+T}{2}\right) - 20\right]$$

$$\Rightarrow \quad \frac{50-T}{3} = \frac{1}{K}[10+T] \quad \text{(ii)}$$

Using (i) and (ii) we get

$$\frac{50-T}{3} = \frac{1}{9}(10+T)$$

$$\Rightarrow \quad T = 35°\text{C}$$

EXAMPLE 17 A body at temperature T is placed in a room where the temperature is T_0. The rate of decrease of temperature is found to be $k(T - T_0)$ where k is a positive constant. Starting from time $t = 0$, the time taken by the body to lose 1/3 of the maximum heat it can lose is

(a) $\frac{1}{k}\ln(2)$ (b) $\frac{1}{k}\ln\left(\frac{1}{2}\right)$

(c) $\frac{1}{k}\ln(3)$ (d) $\frac{1}{k}\ln\left(\frac{3}{2}\right)$

SOLUTION Given $\frac{dT}{dt} = -k(T - T_0)$

Let T_1 be the temperature of the body at $t = 0$, then

$$\int_{T_1}^{T} \frac{dT}{(T-T_0)} = -k\int_0^t dt$$

$$\Rightarrow \quad \ln\left(\frac{T-T_0}{T_1-T_0}\right) = -kt$$

or $$T - T_0 = (T_1 - T_0)\, e^{-kt} \quad \text{(i)}$$

The body will keep on losing heat till its temperature becomes equal to T_0, the room temperature. Therefore, the maximum heat the body can lose is

$$(\Delta Q)_{\max} = ms\,(T_1 - T_0)$$

or $$\frac{(\Delta Q)_{\max}}{ms} = (T_1 - T_0)$$

If the body loses $\frac{1}{3}(\Delta Q)_{\max}$ heat, the decrease in its temperature will be $\frac{(T_1 - T_0)}{3}$. If the body loses $\frac{1}{3}$ of maximum heat in time t_1, then the temperature at t_1 will be

$$T = T_1 - \frac{T_1 - T_0}{3} = \frac{2T_1 + T_0}{3}$$

Using this in Eq. (i), we have

$$\frac{2T_1 + T_0}{3} - T_0 = (T_1 - T_0)\, e^{-kt_1}$$

$$\Rightarrow \quad \frac{2}{3}(T_1 - T_0) = (T_1 - T_0)\, e^{-kt_1}$$

$$\Rightarrow \quad \frac{2}{3} = e^{-kt_1}$$

$$\Rightarrow \quad \frac{3}{2} = e^{kt_1}$$

$$\Rightarrow \quad \ln\left(\frac{3}{2}\right) = kt_1$$

$$\Rightarrow \quad t_1 = \frac{1}{k}\ln\left(\frac{3}{2}\right)$$

10. Thermodynamics

(1) Equation of State

In the case of ideal gases, the equation of state is

$$PV = RT, \text{ for one mole}$$

and $$PV = nRT, \text{ for } n \text{ moles}$$

where P, V and T are the pressure, volume and temperature of the gas, respectively and R is the universal gas constant.

(2) First Law of Thermodynamics

When heat energy is supplied to a system, a part of this energy is used up in raising the temperature of the system (i.e. in increasing the internal energy of the system) and the rest is used up in doing external work against the surroundings. Thus, if ΔQ is the heat energy supplied to a gas and if ΔW is the work done by it, then from the law of conservation of energy, the increase ΔU in the internal energy of the gas must be equal to $(\Delta Q - \Delta W)$ or

$$\Delta Q = \Delta U + \Delta W$$

Here all quantities are measured in energy units (joule). This equation is the mathematical statement of the first law of thermodynamics which may be stated in words as '*if energy is supplied to a system which is capable of doing work, then the quantity of heat energy absorbed by the system will be equal to the sum of the increase in the internal energy of the system and the external work done by it.*' It must be remembered that, according to the accepted convention, ΔQ is positive if heat is supplied to the system and negative if heat is taken out of it and that ΔW is positive when work is done by the system and negative when work is done on the system. ΔU is positive if the temperature of the system increases and negative if the temperature decreases.

(3) Internal Energy

Every system (solid, liquid or gas) possesses a certain amount of energy. This energy is called the *internal energy* and is usually denoted by the symbol U. The internal energy of solid, liquid or gas consists of two parts: (i) kinetic energy due to the motion (translational, rotational and vibrational) of the molecules, and (ii) potential energy due to the configuration (separation) of the molecules.

If the intermolecular forces are extremely weak or absent, then the change in internal energy is given by

$$\Delta U = nC_v \Delta T$$

(4) Isothermal and Adiabatic Processes

If a system is perfectly conducting to the surroundings and the temperature remains constant throughout the process, the process is called *isothermal*. During an isothermal process, the temperature of the system remains constant, but it can absorb heat from or lose heat to the surroundings. The process has to be extremely slow to be isothermal.

During an adiabatic process, the system is completely insulated from the surroundings. It can neither give heat to nor take heat from the surroundings. Sudden processes are adiabatic such as the bursting of a cycle tube, the compression stroke in an internal combustion engine, etc.

For an isothermal process,

$$PV = \text{constant} \quad \Rightarrow \quad P_1 V_1 = P_2 V_2$$

For an adiabatic process

$$PV^{\gamma} = \text{constant} \quad \Rightarrow \quad P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

Also $$TV^{(\gamma - 1)} = \text{constant} \quad \Rightarrow \quad T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)}$$

and $$P^{(1-\gamma)} T^{\gamma} = \text{constant}$$

$$\Rightarrow \quad P_1^{(1-\gamma)} T_1^{\gamma} = P_2^{(1-\gamma)} T_2^{\gamma}$$

where $$\gamma = \frac{C_p}{C_v}$$

(5) Isobaric and Isochoric Processes

If the pressure of the gas is kept constant, the process is called isobaric, i.e. P = constant or $\Delta P = 0$.

In an isochoric process, the volume of the gas is kept constant, i.e. V = constant or $\Delta V = 0$.

(6) Work Done in a Process

The work done by a gas in expansion from volume V_1 to volume V_2 is given by

$$W = \int_{V_1}^{V_2} P dV$$

(1) Work Done in an Isothermal Process When an ideal gas is allowed to expand isothermally (i.e. at constant temperature) work is done by it. For an isothermal process, the equation of state for n moles of ideal gas is

$$PV = nRT \qquad (1)$$

where T is the constant absolute temperature and R is the universal gas constant. The value of R for all gases is $R = 8.31 \text{ J K}^{-1} \text{ mol}^{-1}$. From Eq. (1)

$$P = \frac{nRT}{V}$$

The work done

$$W = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT \ln \left(\frac{V_2}{V_1} \right)$$

or $$W = 2.303\, nRT \log \left(\frac{V_2}{V_1} \right) \qquad (2)$$

where V_1 is the initial volume and V_2, the final volume of gas. In terms of initial and final pressures, P_1 and P_2, Eq. (2) can be written as (since $P_1V_1 = P_2V_2$)

$$W = 2.303\, nRT \log \left(\frac{P_1}{P_2} \right)$$

(2) Work Done in Adiabatic Process When a gas undergoes an adiabatic change, the pressure-volume changes obey the relation

$$PV^{\gamma} = \text{constant} = C \qquad (3)$$

where $\gamma = C_p/C_v$ is the ratio of specific heat of the gas at constant pressure to that at constant volume.

From Eq. (3), we have

$$P = \frac{C}{V^\gamma}, \text{ where } C \text{ is a constant}$$

The work done is given by

$$W = C\int_{V_1}^{V_2} \frac{dV}{V^\gamma} = C\left|\frac{V^{(1-\gamma)}}{(1-\gamma)}\right|_{V_1}^{V_2}$$

$$= \frac{C}{(1-\gamma)}\{V_2^{(1-\gamma)} - V_1^{(1-\gamma)}\}$$

$$= \frac{C}{(1-\gamma)}\left\{\frac{1}{V_2^{(\gamma-1)}} - \frac{1}{V_1^{(\gamma-1)}}\right\} \quad (4)$$

Now since $C = P_1V_1^\gamma = P_2V_2^\gamma$, we have

$$W = \frac{1}{(1-\gamma)}\left[\frac{P_2V_2^\gamma}{V_2^{\gamma-1}} - \frac{P_1V_1^\gamma}{V_1^{\gamma-1}}\right] \text{ or}$$

$$W = \left(\frac{1}{1-\gamma}\right)(P_2V_2 - P_1V_1)$$

$$= \frac{1}{(\gamma-1)}(P_1V_1 - P_2V_2)$$

$$= \frac{nR}{(\gamma-1)}(T_1 - T_2) \quad (5)$$

since $P_1V_1 = nRT_1$ and $P_2V_2 = nRT_2$, T_1 and T_2 being the absolute temperatures before and after the adiabatic change.

(3) Work Done in an Isobaric Process For an isobaric process, pressure P is constant. Therefore, the work done is given by

$$W = P\int_{V_1}^{V_2} dV = P(V_2 - V_1)$$

(4) Work Done in an Isochoric Process In an isochoric process, volume V is constant, i.e. $dV = 0$. Hence, work done $W = 0$.

(7) Indicator Diagram

If the exact relationship between P and V is not known, the above expressions for work done cannot be used. In such cases, the graphical (indicator diagram) method is employed. Indicator diagram (or $P-V$ diagram) is a graph in which pressure (P) is plotted on the y-axis and volume (V) on the x-axis.

Figure 8.7 shows indicator diagrams for expansion, compression and for a closed cyclic process. The initial state (P_1, V_1) is represented by point A and the final state (P_2, V_2) by point B. The intermediate states are represented by points between A and B on the curve AB.

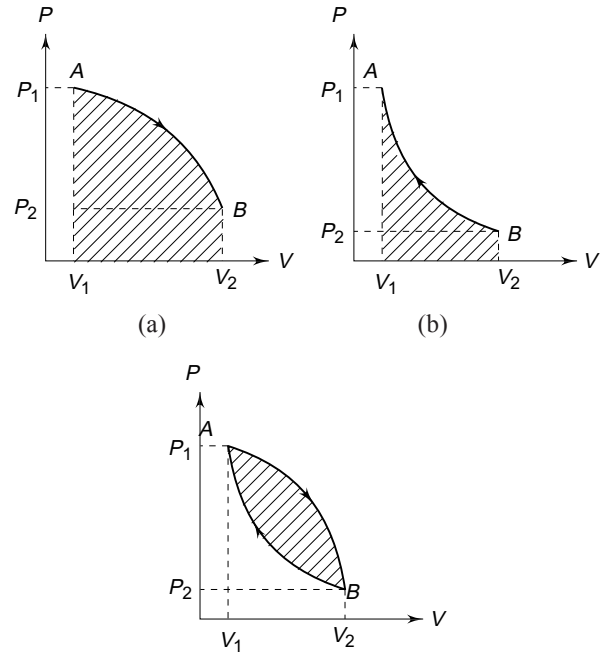


Fig. 8.7

Work done in process $A \rightarrow B$ = area under AB in Fig. 8.7(a).

Work done in process $B \rightarrow A$ = area under AB in Fig. 8.7(b).

Work done in the cyclic process $A \rightarrow B \rightarrow A$ is equal to the area enclosed by the loop.

Note The work done in a process is given by W = area enclosed by $P-V$ curve and volume axis.

(8) Efficiency of an Ideal Heat Engine

The efficiency (η) of an ideal reversible heat engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

where T_1 = absolute temperature of the source which supplies heat

and T_2 = absolute temperature of the sink which takes in the part of heat not converted into useful work,

T_1 is always greater than T_2. If $T_1 = T_2$; $\eta = 0$ implying that an engine working under isothermal conditions can produce no useful work. Complete conversion of heat into work (i.e. $\eta = 1$) is possible only if $T_2 = 0$, i.e. the sink is at absolute zero, which is unattainable.

(9) C_v, C_p and $\gamma = C_p/C_v$ for an Ideal Gas

(1) For a monoatomic gas,

$$C_v = \frac{3R}{2},\ C_p = \frac{5R}{2} \text{ and } \gamma = \frac{5}{3} = 1.67$$

(2) For a diatomic gas,

$$C_v = \frac{5R}{2},\ C_p = \frac{7R}{2} \text{ and } \gamma = \frac{7}{5} = 1.4$$

(3) For a triatomic or polyatomic gas

$$C_v = 3R,\ C_p = 4R \text{ and } \gamma = \frac{4}{3} = 1.33$$

(10) Relation between C_p, C_v and γ

$$C_p - C_v = R \qquad \text{(i)}$$

and $$\frac{C_P}{C_v} = \gamma \qquad \text{or } C_p = \gamma C_v \qquad \text{(ii)}$$

Using (ii) in (i), we have

$$C_v = \frac{R}{\gamma - 1} \text{ and } C_p = \frac{\gamma R}{\gamma - 1}$$

EXAMPLE 18 Four moles of an ideal monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ are mixed with two moles of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$. The value of γ of the mixture will be

(a) $\frac{7}{3}$ (b) $\frac{15}{8}$

(c) $\frac{17}{11}$ (d) $\frac{19}{12}$

SOLUTION *For monoatomic gas*, let

C'_v = molar specific heat at constant volume

C'_p = molar specific heat at constant pressure

For diatomic gas, let

C''_v = molar specific heat at constant volume

C''_p = molar specific heat at constant pressure

For the mixture, let

C_v and C_p be the molar specific heats at constant volume and at constant pressure.

Given $$\frac{C'_p}{C'_v} = \frac{5}{3} \quad \text{and} \quad \frac{C''_p}{C''_v} = \frac{7}{5}$$

Also $$C'_p - C'_v = R = C''_p - C''_v$$

$$\therefore \quad C'_v = \frac{3}{2}R \text{ and } C'_p = \frac{5}{2}R$$

and $$C''_v = \frac{5}{2}R \text{ and } C''_p = \frac{7}{2}R$$

Suppose the temperature of the mixture is increased by dT. The increase in the internal energy of the first gas $= n_1 C'_v dT$. The increase in internal energy of the second gas $= n_2 C''_v dT$ and the increase in internal energy of the mixture $= (n_1 + n_2) C_v dT$. Thus,

$$(n_1 + n_2) C_v dT = n_1 C'_v dT + n_2 C''_v dT$$

or, $$C_v = \frac{n_1 C'_v + n_2 C''_v}{n_1 + n_2} \qquad \text{(i)}$$

$$C_p = C_v + R = \frac{n_1 C'_v + n_2 C''_v}{n_1 + n_2} + R$$

$$= \frac{n_1 (C'_v + R) + n_2 (C''_v + R)}{n_1 + n_2}$$

$$= \frac{n_1 C'_p + n_2 C''_p}{n_1 + n_2} \qquad \text{(ii)}$$

From (i) and (ii), $\gamma = \frac{C_p}{C_v} = \frac{n_1 C'_p + n_2 C''_p}{n_1 C'_v + n_2 C''_v}$

$$= \frac{4 \times \frac{5}{2}R + 2 \times \frac{7}{2}R}{4 \times \frac{3}{2}R + 2 \times \frac{5}{2}R} = \frac{17}{11}$$

EXAMPLE 19 A diatomic gas does 100 J of work when expanded at constant pressure. The heat energy absorbed by the gas in this process is

(a) 200 J (b) 250 J

(c) 300 J (d) 350 J

SOLUTION For a diatomic gas $C_v = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$. The work done in the isobaric process is

$$W = P(V_2 - V_1)$$

$$= nR\,T_2 - nR\,T_1$$

$$= nR(T_2 - T_1)$$

$$\Rightarrow \quad T_2 - T_1 = \frac{W}{nR}$$

Heat absorbed is $\Delta Q = nC_p(T_2 - T_1)$

$$= n\,C_p \times \frac{W}{nR}$$

$$= C_p \times \frac{W}{R}$$

$$= \frac{7R}{2} \times \frac{100}{R} = 350 \text{ J}$$

EXAMPLE 20 5 moles of an ideal gas are compressed to half the initial volume at a constant temperature of 27.0°C. Calculate the work done in the process. Given $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$. Write your result up to appropriate significant figures.

SOLUTION Given $T = 27.0°C = 300$ K. Since T is constant, the process is isothermal

$$\therefore \quad W = nRT \log_e \left(\frac{V_2}{V_1}\right)$$

$$= 5 \times 8.3 \times 300 \times \log_e \left(\frac{1}{2}\right)$$

$$= 5 \times 8.3 \times 300 \times (-0.693)$$

$$= -8628 \text{ J}$$

Since the value of R has two significant figures (*s.f.*), the value of W must be rounded off to 2 *s.f.* as

$$W = -8.6 \times 10^3 \text{ J}$$

The negative sign indicates that work is done on the gas.

EXAMPLE 21 Two moles of a diatomic gas ($\gamma = 1.4$) at 127°C are expanded adiabatically to twice the original volume. Calculate (a) the final temperature and (b) the work done in the process. Given $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$ and $(0.5)^{0.4} = 0.76$.

SOLUTION

(a) $T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)}$

$$\Rightarrow \quad T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (127 + 273) \times (0.5)^{0.4}$$

$$= 400 \times 0.76 = 304 \text{ K}$$

(b) $W = \dfrac{nR}{(\gamma - 1)} (T_1 - T_2)$

$$= \frac{2 \times 8.3}{1.4 - 1} \times (400 - 304) = 3984 \text{ J}$$

EXAMPLE 22 A Carnot's engine working between 0°C and 100°C takes up 746 J of heat from the high temperature reservoir in one cycle. Calculate (a) the work done by the engine, (b) heat rejected to the sink and (c) the efficiency of the engine.

SOLUTION $T_1 = 100°C = 373$ K, $T_2 = 0°C = 273$ K, $Q_1 = 746$ J

(a) $\dfrac{W}{Q_1} = 1 - \dfrac{T_2}{T_1} \quad \Rightarrow \quad W = Q_1 \left(\dfrac{T_1 - T_2}{T_1}\right)$

or $$W = 746 \times \left(\frac{373 - 273}{373}\right)$$

$$= 200 \text{ J}$$

(b) $Q_2 = Q_1 - W = 746 - 200 = 546$ J

(c) $\eta = \dfrac{W}{Q_1} = \dfrac{200}{746} = 0.268 = 26.8\%$

Note $\eta = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{273}{373} = 0.268$

EXAMPLE 23 Figure 8.8 shows the *P-V* diagram of a cyclic process *ABCA*. Calculate the work done in process

(a) *A* to *B* (b) *B* to *C*
(c) *C* to *A* (d) *ABCA*

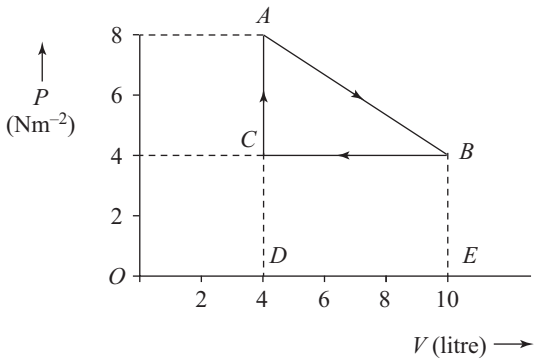


Fig. 8.8

SOLUTION

(a) $W_{A \to B}$ = Area of *ABED*

= Area of ΔABC + area of rectangle *BEDC*

$$= \frac{1}{2} AC \times BC + CD \times DE$$

$$= \frac{1}{2} \times 4 \times 6 \times 10^{-3} + 4 \times 6 \times 10^{-3}$$

($\because$ 1 litre = 10^{-3} m^3)

$$= 36 \times 10^{-3} \text{ J}$$

(b) $W_{B \to C}$ = Area of $BEDC = -24 \times 10^{-3}$ J

Negative sign shows that the work is done on the gas.

(c) $W_{C \to A} = 0$ ($\because$ volume is constant)

(d) $W_{ABCA} = 36 \times 10^{-3} - 24 \times 10^{-3} = 12 \times 10^{-3}$ J

which is equal to area enclosed by the closed loop *ABCA*.

EXAMPLE 24 5 moles of an ideal diatomic gas ($\gamma = 1.4$) are heated at constant pressure. If 280 J of heat energy is supplied to the gas, find (a) the change in internal energy of the gas and (b) the work done by the gas.

SOLUTION

(a) $\Delta U = n\, C_v\, \Delta T = n \dfrac{C_v}{C_p} \times C_p\, \Delta T = \dfrac{n C_p \Delta T}{\gamma}$

Given $\Delta Q = n\, C_p\, \Delta T = 280$ J. Hence

$$\Delta U = \frac{2.80}{1.4} = 200 \text{ J}$$

(b) From first law of thermodynamics

$$\Delta W = \Delta Q - \Delta U = 280 - 200 = 80 \text{ J}$$

EXAMPLE 25 5 moles of an ideal diatomic gas ($\gamma = 1.4$) are heated at constant volume. If 280 J of heat energy is supplied to the gas, find the change in internal energy of the gas and the work done.

SOLUTION Since $\Delta V = 0$, $\Delta W = 0$. Hence from first law of thermodynamics,

$$dU = dQ = 280 \text{ J}$$

EXAMPLE 26 The pressure P of an ideal gas varies with volume V as $P = kV$ where k is a constant. The volume of n moles of the gas is increased from V to mV. Find the work done and the change in internal energy.

SOLUTION $W = \displaystyle\int_V^{mV} P dV = \int_V^{mV} kV dV$

$$= k \left| \frac{V^2}{2} \right|_V^{mV} = \frac{kV^2}{2}(m^2 - 1)$$

From first law of thermodynamics

$$\Delta Q = \Delta U + W$$

$$\Rightarrow \quad nC_p\, \Delta T = \Delta U + W$$

$$\Rightarrow \quad n\gamma C_v\, \Delta T = \Delta U + W \qquad \left(\because\ \gamma = \frac{C_p}{C_v}\right)$$

$$\Rightarrow \quad \gamma \Delta U = \Delta U + W \qquad (\because\ \Delta U = n\, C_v\, \Delta T)$$

$$\Rightarrow \quad \Delta U = \frac{W}{(\gamma - 1)} = \frac{kV^2(m^2 - 1)}{2(\gamma - 1)}$$

EXAMPLE 27 Two moles of a monoatomic gas undergo a cyclic process $ABCA$ as shown in Fig. 8.9. Process $A \to B$ is isobaric, process $B \to C$ is adiabatic and process $C \to A$ is isothermal. Find

(a) pressure and volume at state C

(b) total work done in the process $ABCA$.

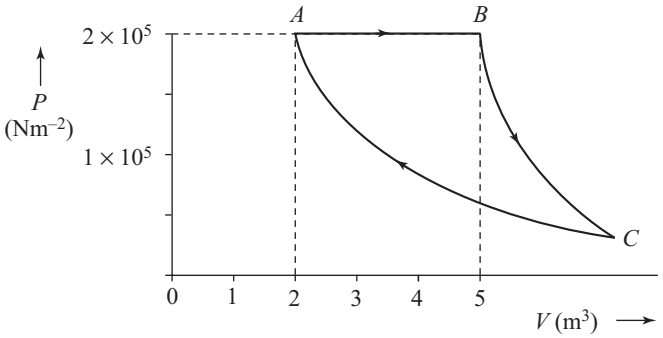


Fig. 8.9

SOLUTION $P_A = P_B = 2 \times 10^5$ Nm^{-2}

$V_A = 2$ m^3, $V_B = 5$ m^3 and

$\gamma = 1.67$ (monoatomic gas)

(a) For adiabatic process $B \to C$

$$P_B\, V_B^{\gamma} = P_C\, V_C^{\gamma} \qquad (1)$$

For isothermal process $C \to A$

$$P_A\, V_A = P_C\, V_C \qquad (2)$$

From (1) and (2), we get $(\because\ P_A = P_B)$

$$V_C = \left(\frac{V_B^{\gamma}}{V_A}\right)^{\frac{1}{\gamma - 1}}$$

Substituting the values of V_A, V_B and γ, we get $V_C = 19.5$ m^3

Also $\quad P_C = \dfrac{P_A V_A}{V_C} = 0.2 \times 10^5$ N m^{-2}

(b) $W = W_{A \to B} + W_{B \to C} + W_{C \to A}$

$$= P_A\,(V_B - V_A) + \frac{1}{(\gamma - 1)}\,(P_B V_B - P_C V_C) + P_A\, V_A \log_e \left(\frac{V_A}{V_C}\right)$$

Substituting the values, we get

$$W = 6 \times 10^5 \text{ N m}^{-2}$$

SECTION 1

Multiple Choice Questions with One Correct Choice

Level A

1. A steel scale measures the length of a copper rod as L cm when both are at 20°C, the calibration temperature for the scale. If the coefficients of linear expansion for steel and copper are α_s and α_c respectively, what would be the scale reading (in cm) when both are at 21°C?
   (a) $L\,\dfrac{(1+\alpha_c)}{(1+\alpha_s)}$ (b) $L\,\dfrac{\alpha_c}{\alpha_s}$
   (c) $L\,\dfrac{\alpha_s}{\alpha_c}$ (d) L

2. In Q. 1 above, what would be the scale reading if the scale was made of copper?
   (a) L (b) $L\,(1+\alpha_c)$
   (c) $L\,(1-\alpha_c)$ (d) none of these

3. A wire of cross-sectional area A at temperature t is held taut with negligible tension between two rigid supports. If the wire is cooled to a temperature $(t-\Delta t)$, what tension is developed in the wire? The coefficient of linear expansion is α and the Young's modulus of the wire is Y.
   (a) $YA\alpha\,\Delta t$ (b) $\dfrac{Y\alpha\Delta t}{A}$
   (c) $\dfrac{A\alpha\Delta t}{Y}$ (d) $\dfrac{YA}{\alpha\Delta t}$

4. Two uniform brass rods A and B of lengths l and $2l$ and radii $2r$ and r, respectively are heated to the same temperature. The ratio of the increase in the length of A to that of B is
   (a) 1 : 1 (b) 1 : 2
   (c) 1 : 4 (d) 2 : 1

5. When a metallic bar is heated from 0°C to 100°C, its length increases by 0.05%. What is the coefficient of linear expansion of the metal?
   (a) 5×10^{-3} °C^{-1} (b) 5×10^{-4} °C^{-1}
   (c) 5×10^{-5} °C^{-1} (d) 5×10^{-6} °C^{-1}

6. Two rods of different materials having coefficients of thermal expansion α_1 and α_2 and Young's moduli Y_1 and Y_2 are fixed between two rigid and massive walls. The rods are heated to the same temperature. If there is no bending of the rods, the thermal stresses developed in them are equal provided
   (a) $\dfrac{Y_1}{Y_2}=\sqrt{\dfrac{\alpha_1}{\alpha_2}}$ (b) $\dfrac{Y_1}{Y_2}=\sqrt{\dfrac{\alpha_2}{\alpha_1}}$
   (c) $\dfrac{Y_1}{Y_2}=\dfrac{\alpha_1}{\alpha_2}$ (d) $\dfrac{Y_1}{Y_2}=\dfrac{\alpha_2}{\alpha_1}$

7. 300 g of water at 25°C is added to 100 g of ice at 0°C. The final temperature of the mixture is
   (a) $-\dfrac{5}{3}$ °C (b) $-\dfrac{5}{2}$ °C
   (c) − 5 °C (d) 0 °C

8. 100 g of ice at 0°C is mixed with 100 g of water at 80°C. The final temperature of the mixture will be
   (a) 0°C (b) 20°C
   (c) 40°C (d) 60°C

9. A drilling machine of power P watts is used to drill a hole in copper block of mass M kg. If the specific heat of copper is s J kg^{-1}°C^{-1} and 40% of the power is lost due to heating of the machine, the rise in the temperature of the block in T seconds will be (in °C)
   (a) $\dfrac{0.6PT}{Ms}$ (b) $\dfrac{0.6P}{MsT}$
   (c) $\dfrac{0.4PT}{Ms}$ (d) $\dfrac{0.4P}{MsT}$

10. A copper block of mass 2 kg is heated to a temperature of 500°C and then placed in a large block of ice at 0°C. What is the maximum amount of ice that can melt? The specific heat of copper is 400 J kg^{-1} °C^{-1} and latent heat of fusion of water is 3.5×10^5 J kg^{-1}.
    (a) $\dfrac{4}{3}$ kg (b) $\dfrac{6}{5}$ kg
    (c) $\dfrac{8}{7}$ kg (d) $\dfrac{10}{9}$ kg

11. How much heat energy in joules must be supplied to 14 grams of nitrogen at room temperature to raise

its temperature by 40°C at constant pressure. Molar mass of nitrogen = 28 and R is the gas constant.

(a) 50 R (b) 60 R
(c) 70 R (d) 80 R

12. The dimensions of the coefficient of thermal conductivity are

(a) $ML^{-1}T^{-2}K^{-1}$ (b) $ML^{-2}T^{-3}K^{-1}$
(c) $ML^{-1}T^{-1}K^{-1}$ (d) $MLT^{-3}K^{-1}$

13. Two different metal rods of the same length have their ends kept at the same temperatures θ_1 and θ_2 with $\theta_2 > \theta_1$. If A_1 and A_2 are their cross-sectional areas and k_1 and k_2 their thermal conductivities, the rate of flow of heat in the two rods will be the same if

(a) $\frac{A_1}{A_2} = \frac{k_1}{k_2}$ (b) $\frac{A_1}{A_2} = \frac{k_2}{k_1}$
(c) $\frac{A_1}{A_2} = \frac{k_1\theta_1}{k_2\theta_2}$ (d) $\frac{A_1}{A_2} = \frac{k_2\theta_2}{k_1\theta_1}$

14. A cylinder of radius R made of a material of thermal conductivity k, is surrounded by a cylindrical shell of inner radius R and outer radius $2R$ made of a material of thermal conductivity k_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is

(a) $k_1 + k_2$ (b) $\frac{k_1 k_2}{k_1 + k_2}$
(c) $\frac{k_1 + 3k_2}{4}$ (d) $\frac{3k_1 + k_2}{4}$

15. Two rods A and B of different materials are welded together as shown in Fig. 8.10. If their thermal conductivities are k_1 and k_2, the thermal conductivity of the composite rod will be

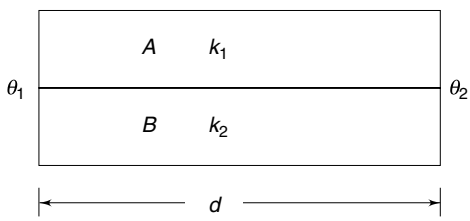


Fig. 8.10

(a) $2(k_1 + k_2)$ (b) $\frac{3}{2}(k_1 + k_2)$
(c) $(k_1 + k_2)$ (d) $\frac{1}{2}(k_1 + k_2)$

16. Two different metal rods of equal lengths and equal areas of cross-section have their ends kept at the same temperatures θ_1 and θ_2. If k_1 and k_2 are their thermal conductivities, ρ_1 and ρ_2 their densities and s_1 and s_2 their specific heats, then the rate of flow of heat in the two rods will be the same if

(a) $\frac{k_1}{k_2} = \frac{\rho_1 s_1}{\rho_2 s_2}$ (b) $\frac{k_1}{k_2} = \frac{\rho_1 s_2}{\rho_2 s_1}$
(c) $\frac{k_1}{k_2} = \frac{\theta_1}{\theta_2}$ (d) $k_1 = k_2$

17. Two rods of different materials having coefficients of thermal expansion α_1, α_2 and Young's modulii Y_1, Y_2 respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2 = 2 : 3$, the thermal stresses developed in the two rods are equal provided $Y_1 : Y_2$ is equal to:

(a) 2 : 3 (b) 1 : 1
(c) 3 : 2 (d) 4 : 9

18. A solid sphere and a hollow sphere of the same material and size are heated to the same temperature and allowed to cool in the same surroundings. If the temperature difference between the body and that of the surroundings is T, then

(a) both spheres will cool at the same rate for all values of T
(b) both spheres will cool at the same rate only for small values of T
(c) the hollow sphere will cool at a faster rate for all values of T
(d) the solid sphere will cool at a faster rate for all values of T

19. A piece of metal floats on mercury. The coefficients of volume expansion of the metal and mercury are γ_1 and γ_2 respectively. If their temperature is increased by ΔT, the fraction of the volume of metal submerged in mercury changes by a factor

(a) $\left(\frac{1+\gamma_2\Delta T}{1+\gamma_1\Delta T}\right)$ (b) $\left(\frac{1+\gamma_2\Delta T}{1-\gamma_1\Delta T}\right)$
(c) $\left(\frac{1-\gamma_2\Delta T}{1+\gamma_1\Delta T}\right)$ (d) $\frac{\gamma_2}{\gamma_1}$

20. The specific heat of helium at constant volume is 12.6 J $mol^{-1}K^{-1}$. What is the specific of hydrogen at constant volume in J $mol^{-1}K^{-1}$? Assume that the temperature of the gas is moderate.

(a) 12.6 (b) 16.8
(c) 18.9 (d) 21

21. The temperature of a liquid does not increase during boiling. The heat energy supplied during this process,
    (a) increases the kinetic energy of the molecules of the liquid
    (b) increases the potential energy of the molecules
    (c) increases both the kinetic and potential energy of the molecules
    (d) is merely wasted since no increase occurs in the total energy of the molecules.

22. In Fig. 8.11, curves *AB* and *CD* represent the relation between pressure *P* and volume *V* of an ideal gas. One of the curves represents an isothermal expansion and the other represents an adiabatic expansion. Which curve represents an adiabatic expansion?

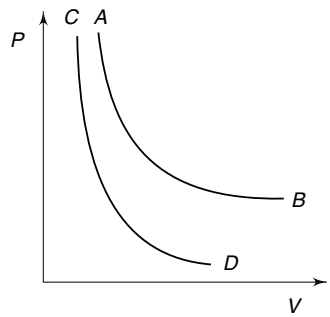


Fig. 8.11

    (a) curve *AB* (b) curve *CD*
    (c) both (d) neither

23. A Carnot's engine working between 27°C and 127°C takes up 800 J of heat from the reservoir in one cycle. What is the work done by the engine?
    (a) 100 J (b) 200 J
    (c) 300 J (d) 400 J

24. In Q. 23, how much heat is rejected to the sink?
    (a) 600 J (b) 500 J
    (c) 400 J (d) 300 J

25. In Q. 23, what is the efficiency of the engine?
    (a) 10% (b) 15%
    (c) 20% (d) 25%

26. A Carnot's engine working between 300 K and 600 K has a work output of 800 J per cycle. How much heat energy is supplied to the engine from the source in each cycle?
    (a) 1400 J (b) 1500 J
    (c) 1600 J (d) 1700 J

27. A Carnot's engine whose sink is at a temperature of 300 K has an efficiency of 40%. By how much should the temperature of the source be increased so as to increase the efficiency to 60%?
    (a) 250 K (b) 275 K
    (c) 300 K (d) 325 K

Level B

28. Two slabs *A* and *B* of different materials but of the same thickness are joined as shown in Fig. 8.12. The thermal conductivities of *A* and *B* are k_1 and k_2, respectively. The thermal conductivity of the composite slab will be
    (a) $(k_1 + k_2)$ (b) $\frac{1}{2}(k_1 + k_2)$
    (c) $\sqrt{k_1 k_2}$ (d) $\frac{2k_1k_2}{(k_1 + k_2)}$

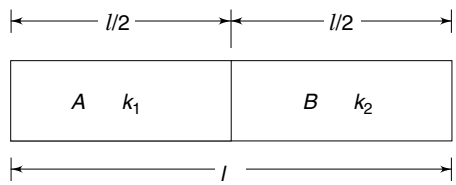


Fig. 8.12

29. A composite slab consists of two slabs *A* and *B* of different materials but of the same thickness placed one on top of the other. The thermal conductivities of *A* and *B* are k_1 and k_2, respectively. A steady temperature difference of 12°C is maintained across the composite slab. If $k_1 = k_2/2$, the temperature difference across slab *A* will be
    (a) 4°C (b) 8°C
    (c) 12°C (d) 16°C

30. Two cylindrical rods of lengths l_1 and l_2, radii r_1 and r_2 have thermal conductivities k_1 and k_2, respectively. The ends of the rods are maintained at the same temperature difference. If $l_1 = 2l_2$ and $r_1 = r_2/2$, the rates of heat flow in them will be the same if k_1/k_2 is
    (a) 1 (b) 2
    (c) 4 (d) 8

31. An ideal monoatomic gas is taken around the cycle *ABCDA* as shown in the *P*-*V* diagram. The work done during the cycle is given by (see Fig. 8.13)
    (a) $\frac{1}{2}PV$ (b) PV
    (c) $2PV$ (d) $4PV$

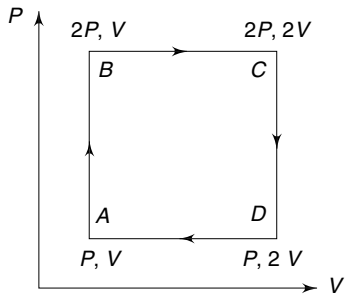


Fig. 8.13

32. How many calories of heat are required to completely evaporate 1 gram of ice at 0°C?
(a) 620 (b) 720
(c) 820 (d) 920

33. How much mechanical work must be done to completely melt 1 gram of ice at 0°C?
(a) 4.2 J (b) 80 J
(c) 336 J (d) 2268 J

34. Two spherical balls *A* and *B* made of the same material, are heated to the same temperature. They are then placed in identical surroundings. If the diameter of *A* is twice that of *B*, ratio of the rates of cooling of *A* and *B* will be
(a) 1 : 1 (b) 2 : 1
(c) 4 : 1 (d) 1 : 4

35. Hydrogen is a diatomic gas. Its molar specific heat a constant volume is very nearly
(a) $\frac{3R}{2}$ (b) $\frac{5R}{2}$
(c) $\frac{7R}{2}$ (d) (a), (b) or (c) depending on the temperature.

36. 743 J of heat energy is needed to raise the temperature of 5 moles of an ideal gas by 2 K at constant pressure. How much heat energy is needed to raise the temperature of the same mass of the gas by 2K at constant volume? Given, molar gas constant $R = 8.3$ J K^{-1} mol^{-1}.
(a) 826 J (b) 743 J
(c) 660 J (d) none of these

37. When an ideal gas at pressure *P*, temperature *T* and volume *V* is isothermally compressed to *V*/*n*, its pressure becomes P_i. If the gas is compressed adiabatically to *V*/*n*, its pressure becomes P_a. The ratio P_i/P_a is ($\gamma = C_p/C_v$)
(a) 1 (b) *n*
(c) n^{γ} (d) $n^{(1-\gamma)}$

38. The ratio of adiabatic bulk modulus and isothermal bulk modulus of a gas is ($\gamma = C_p/C_v$)
(a) 1 (b) γ
(c) $\frac{\gamma}{(\gamma-1)}$ (d) $\frac{(\gamma-1)}{\gamma}$

39. Figure 8.14 shows the *P*—*V* diagram for a fixed mass of an ideal gas undergoing cyclic process *ABCA*. *AB* represents isothermal process.

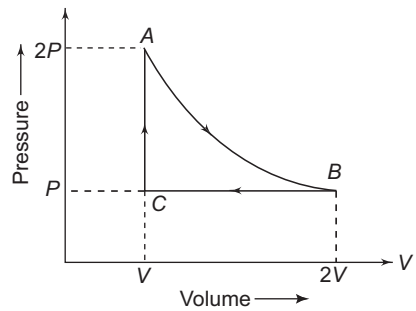


Fig. 8.14

Which of the graphs shown in Fig. 8.15 represents the *P*—*T* diagram of the cyclic process?

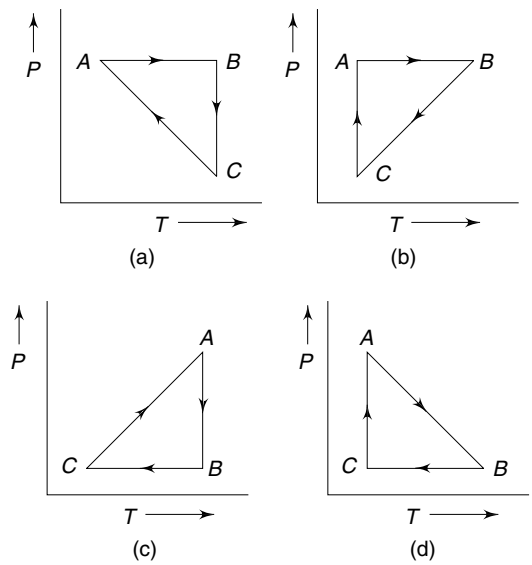


Fig. 8.15

40. In Q. 39, if the temperature at *A* is *T*, what is the temperature at *C*?
(a) 4 *T* (b) 2 *T*
(c) *T* (d) $\frac{T}{2}$

41. In Q. 39, if the point *C* in Fig. 8.15 is at STP, what will be the temperature at *A*?
(a) 0°C (b) 273 K
(c) 100°C (d) 546 K

42. Figure 8.16 shows the *P–V* diagram of a cyclic process. If *dQ* is the heat energy supplied to the system, *dU* is the change in internal energy of the system and *dW* is the work done by the system, then which of the following relations is correct?

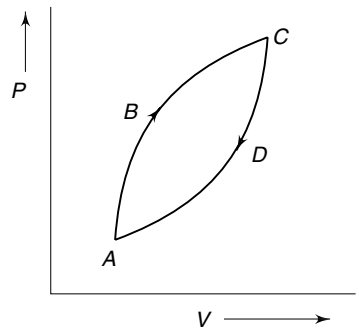


Fig. 8.16

(a) $dQ = dU - dW$ (b) $dU = 0$
(c) $dU = dW$ (d) $dQ = -dW$

43. When two moles of oxygen is heated from 0°C to 10°C at constant volume, its internal energy changes by 420 J. What is the molar specific heat of oxygen at constant volume?
(a) 5.75 J K^{-1} mol^{-1} (b) 10.5 J K^{-1} mol^{-1}
(c) 21 J K^{-1} mol^{-1} (d) 42 J K^{-1} mol^{-1}

44. When 20 J of work was done on a gas, 40 J of heat energy was released. If the initial internal energy of the gas was 70 J, what is the final internal energy?
(a) 50 J (b) 60 J
(c) 90 J (d) 110 J

45. During an adiabatic compression of 5 moles of a gas, 250 J of work was done. The change in the internal energy will be
(a) 50 J (b) – 150 J
(c) 250 J (d) – 250 J

46. In a thermodynamic process, a system absorbs 1 kilocalorie of heat and at the same time does 250 joule of work. What is the change in the internal energy of the system?
(a) 250 J (b) 3950 J
(c) 4200 J (d) 4450 J

47. An ideal gas at pressure *P* is adiabatically compressed so that its density becomes *n* times the initial value. The final pressure of the gas will be ($\gamma = C_p/C_v$),
(a) $n^{\gamma}P$ (b) $n^{-\gamma}P$
(c) $n^{(\gamma-1)}P$ (d) $n^{(1-\gamma)}P$

48. A thin copper wire has a length *L* at 0ºC. When heated to *t*ºC, its length increases by 0.1%. If a thin copper plate of area ($L \times 2L$) is heated through the same range of temperature, its surface area will increase by
(a) 0.1% (b) 0.2%
(c) 0.4% (d) 0.8%

49. Two spheres made of the same material have the same diameter. One sphere is hollow and the other is solid. If they are heated through the same range of temperature,
(a) the hollow sphere will expand more than the solid sphere
(b) the solid sphere will expand more than the hollow sphere
(c) both spheres will expand equally
(d) the hollow sphere will not expand at all.

50. Two rods of lengths L_1 and L_2 are welded together to make a composite rod of length $(L_1 + L_2)$. If the coefficients of linear expansion of the rods are α_1 and α_2 respectively, the effective coefficient of linear expansion of the composite rod will be
(a) $\frac{1}{2}(a_1 + a_2)$ (b) $\sqrt{\alpha_1 \alpha_2}$
(c) $\frac{L_1\alpha_1 + L_2\alpha_2}{L_1 + L_2}$ (d) $\frac{\sqrt{L_1L_2\alpha_1\alpha_2}}{L_1 + L_2}$

51. The coefficient of linear expansion of an inhomogeneous rod changes linearly from α_1 to α_2 from one end to the other end of the rod. The effective coefficient of linear expansion of the rod is
(a) $(\alpha_1 + \alpha_2)$ (b) $\frac{1}{2}(\alpha_1 + \alpha_2)$
(c) $\sqrt{\alpha_1 \alpha_2}$ (d) $(\alpha_1 - \alpha_2)$

52. Three rods of the same length are arranged to form an equilateral triangle. Two rods are made of the same material of coefficient of linear expansion α_1 and the third rod which forms the base of the triangle has coefficient of expansion α_2. The altitude of the triangle will remain the same at all temperatures if the α_1/α_2 is nearly
(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) 4

53. A cylindrical block of wood floats vertically with 80% of its volume immersed in a liquid at 0ºC. When the temperature of the liquid is raised to 62.5ºC, the block just sinks in the liquid. The coefficient of cubical expansion of liquid is
(a) 1×10^{-3} K^{-1} (b) 2×10^{-3} K^{-1}
(c) 3×10^{-3} K^{-1} (d) 4×10^{-3} K^{-1}

54. A certain amount of heat energy is supplied to a monoatomic ideal gas which expands at constant pressure. What fraction of the heat energy is converted into work?

(a) 1 (b) $\frac{2}{3}$

(c) $\frac{2}{5}$ (d) $\frac{5}{7}$

55. Two blocks of the same thickness and thermal conductivities k_1 and k_2 are placed in contact. Their outer surfaces are kept at constant temperatures T_1 and T_2. If $T_2 > T_1$, and $k_2 > k_1$ the *t*emperature of the interface in the steady state will be

(a) $\frac{k_2T_2 + k_1T_1}{k_2 + k_1}$ (b) $\frac{k_2T_2 - k_1T_1}{k_2 - k_1}$

(c) $\frac{k_2T_1 + k_1T_2}{k_2 + k_1}$ (d) $\frac{k_2T_1 - k_1T_2}{(k_2 - k_1)}$

56. A metal ball immersed in alcohol weighs W_1 at 0°C and W_2 at 59°C. The coefficient of cubical expansion of metal is less than that of alcohol. If the density of the metal is large compared to that of alcohol, then

(a) $W_1 > W_2$ (b) $W_1 = W_2$

(c) $W_1 < W_2$ (d) $W_2 = \frac{W_1}{2}$

57. Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter rises to 80°C. The mass of steam condensed in kilogram is

(a) 0.13 (b) 0.065

(c) 0.260 (d) 0.135

58. A block of ice at – 10°C is slowly heated and converted to steam at 100°C. Which of the following curves shown in Fig. 8.17 represents the phenomenon qualitatively?

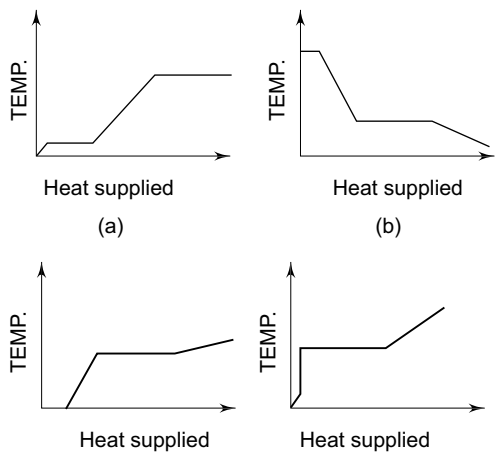


Fig. 8.17

59. Two cylinders *A* and *B* fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of *A* is free to move, while that of *B* is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in *A* is 30 K, then the rise in temperature of the gas in *B* is

(a) 30 K (b) 18 K

(c) 50 K (d) 42 K

60. Two identical containers *A* and *B* fitted with frictionless pistons contains the same ideal gas at the same temperature and the same volume *V*. The mass of the gas in *A* is m_A and that in *B* is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume $2V$. The changes in pressure in *A* and *B* are found to be ΔP and $1.5\,\Delta P$, respectively. Then

(a) $4\,m_A = 9\,m_B$ (b) $2\,m_A = 3\,m_B$

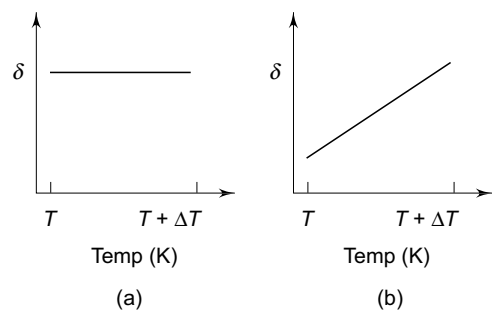
(c) $3\,m_A = 2\,m_B$ (d) $9\,m_A = 4\,m_B$

61. Two monoatomic ideal gases 1 and 2 of molecular masses M_1 and M_2, respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is

(a) $\sqrt{\frac{M_1}{M_2}}$ (b) $\sqrt{\frac{M_2}{M_1}}$

(c) $\frac{M_1}{M_2}$ (d) $\frac{M_2}{M_1}$

62. An ideal gas is initially at temperature *T* and volume *V*. Its volume is increased by ΔV due to an increase in temperature ΔT, pressure remaining constant. The quantity $\delta = \Delta V/(V\,\Delta T)$ varies with temperature as (see Fig. 8.18).



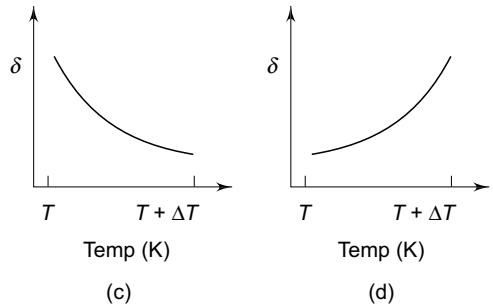


Fig. 8.18

63. A monoatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the lengths of the gas column before and after expansion respectively, then T_1/T_2 is given by

(a) $\left(\frac{L_1}{L_2}\right)^{2/3}$ (b) $\frac{L_1}{L_2}$

(c) $\frac{L_2}{L_1}$ (d) $\left(\frac{L_2}{L_1}\right)^{2/3}$

64. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric and W_3 is purely adiabatic. Then

(a) $W_2 > W_1 > W_3$ (b) $W_2 > W_3 > W_1$

(c) $W_1 > W_2 > W_3$ (d) $W_1 > W_3 > W_2$

65. A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of He gas (molar mass 4) at a temperature $2T$ has a pressure of

(a) $\frac{P}{8}$ (b) P

(c) $2P$ (d) $8P$

66. A vessel contains a mixture of 1 mole of oxygen and two moles of nitrogen at 300 K. The ratio of the rotational kinetic energy per O_2 molecule to that per N_2 molecule is

(a) 1 : 1

(b) 1 : 2

(c) 2 : 1

(d) depends on the moment of inertia of the two molecules.

67. Three rods of identical cross-sectional area and made from the same metal form the sides of an isosceles triangle ABC, right angled at B. The points A and B are maintained at temperatures T and $\sqrt{2}\ T$, respectively. In the steady state, the temperature of point C is T_C. Assuming that only heat conduction takes place, the ratio T_C/T is

(a) $\frac{1}{2\left(\sqrt{2}-1\right)}$ (b) $\frac{3}{\sqrt{2}+1}$

(c) $\frac{1}{\sqrt{3}\left(\sqrt{2}-1\right)}$ (d) $\frac{1}{\sqrt{2}+1}$

68. Two metallic spheres S_1 and S_2 are made of the same material and have identical surface finish. The mass of S_1 is three times that of S_2. Both the spheres are heated to the same high temperature and placed in the same room having lower temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of S_1 to that of S_2 is

(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{\sqrt{3}}{1}$ (d) $\left(\frac{1}{3}\right)^{1/3}$

69. Three rods made of the same material and having the same cross-section have been joined as shown in Fig. 8.19. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C, respectively. The temperature of the junction of the three rods will be

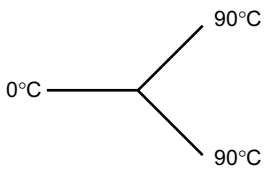


Fig. 8.19

(a) 45°C (b) 60°C

(c) 30°C (d) 20°C

70. In a given process on an ideal gas, $dW = 0$ and $dQ < 0$. Then for the gas

(a) the temperature will decrease

(b) the volume will increase

(c) the pressure will remain constant

(d) the temperature will increase

71. P-V plots for two gases during adiabatic processes are shown in Fig. 8.20. Plots 1 and 2 should correspond respectively to

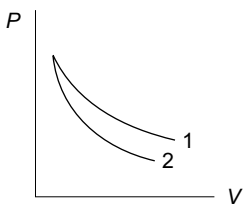


Fig. 8.20

(a) He and O_2 (b) O_2 and He
(c) He and Ar (d) O_2 and N_2

72. Two rods, one made of aluminium and the other made of steel, having initial lengths l_1 and l_2 respectively are connected together to form a single rod of length $(l_1 + l_2)$. The coefficients of linear expansion for aluminium and steel and α_1 an α_2 respectively. If the length of each rod increases by the same amount when their temperature is raised by t°C, then the ratio $l_1/(l_1 + l_2)$
(a) α_1/α_2 (b) α_2/α_1
(c) $\alpha_2/(\alpha_1 + \alpha_2)$ (d) $\alpha_1/(\alpha_1 + \alpha_2)$

73. 2 kg of ice at –20°C is mixed with 5 kg of water at 20°C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water in the vessel. It is given that the specific heat of water and ice are 1 kcal/kg/°C and 0.5 kcal/kg/°C respectively and the latent heat of fusion of ice is 80 kcal/kg.
(a) 7 kg (b) 6 kg
(c) 4 kg (d) 2 kg

74. A uniform metal rod of length L and mass M is rotating with angular speed ω about an axis passing through one of the ends and perpendicular to the rod. If the temperature increases by t°C, then the change in its angular speed is proportional to
(a) $\sqrt{\omega}$ (b) ω
(c) ω^2 (d) $\dfrac{1}{\omega}$

75. One mole of an ideal gas ($\gamma = 1.4$) is adiabatically compressed so that its temperature rises from 27°C to 35°C. The change in the internal energy of the gas is (given R = 8.3 J/mole/K)
(a) –166 J (b) 166 J
(c) –168 J (d) 168 J

76. A sample of an ideal gas has volume V, pressure P and temperature T. The mass of each molecule of the gas is m. The density of the gas is (k is the Boltzmann's constant)
(a) mkT (b) $\dfrac{P}{kT}$
(c) $\dfrac{P}{kVT}$ (d) $\dfrac{mP}{kT}$

77. A gas does 4.5 J of external work during adiabatic expansion. Its temperature falls by 2 K. Its internal energy will
(b) increase by 4.5 J (b) decrease by 4.5 J
(c) decrease by 2.25 J (d) increase by 9.0 J

78. A rod 1 m long and made of material of thermal conductivity 420 W/m/K has one of its ends in melting ice and the other end in boiling water. If its area of cross-section is 10 cm^2, the amount of ice that melts in 1 minute is
(a) 0.125 g (b) 75 g
(c) 7.5 g (d) 450 g

79. The coefficient of real expansion of mercury is 18×10^{-5}/°C. A thermometer has a bulb of volume 10^{-6} m^3 and the cross-sectional area of the stem is 0.002 cm^2. The bulb is completely filled with mercury when the temperature is 0°C. When the temperature rises to 100°C, the length of the mercury column in the stem will be
(a) 9 cm (b) 18 cm
(c) 9 mm (d) 18 mm

80. A closed hollow insulated cylinder is filled with gas at 0°C and also contains an insulated piston of negligible weight and negligible thickness at the middle point. The gas one one side is heated to 100°C. If the piston moves through 5 cm, the length of the hollow cylinder is
(a) 13.65 cm (b) 27.3 cm
(c) 38.6 cm (d) 64.6 cm

81. When the temperature of a rod increases from t to $t + \Delta t$, the moment of inertia increases from I to $I + \Delta I$. The coefficient of linear expansion of the rod is α. The ratio $\dfrac{\Delta I}{I}$ is
(a) $\dfrac{\Delta t}{t}$ (b) $\dfrac{2\Delta t}{t}$
(c) $\alpha \Delta t$ (d) $2\alpha \Delta t$

82. A metal sphere of radius r and specific heat S is rotated about an axis passing through its centre at a speed of n rotations per second. It is suddenly stopped and 50% of its energy is used in increasing its temperature. Then the rise in temperature of the sphere is:
(a) $\dfrac{2\pi^2 n^2 r^2}{5S}$ (b) $\dfrac{\pi^2 n^2}{10 r^2 S}$
(c) $\dfrac{7}{8}\ \pi r^2 n^2 S$ (d) $\dfrac{5(\pi r n)^2}{14S}$

83. 5 moles of Hydrogen $\left(\gamma = \dfrac{7}{5}\right)$ initially at S.T.P. are compressed adiabatically so that its temperature becomes 400°C. The increase in the internal energy of the gas in kilo-joules is: (R = 8.30 J mol^{-1} K^{-1})

(a) 21.55 (b) 41.50
(c) 65.55 (d) 80.55

84. During an adiabatic process, the pressure of a gas is proportional to the cube of its absolute temperature. The value of C_p/C_v for that gas is:
(a) $\frac{3}{5}$ (b) $\frac{4}{3}$
(c) $\frac{5}{3}$ (d) $\frac{3}{2}$

85. A steel rod and a copper rod have lengths L_s and L_c respectively at a certain temperature. It is found that the difference between their lengths remains constants at all temperatures. If α_s and α_c are their respective coefficients of linear expansion, the ratio L_s/L_c is given by
(a) $\left(1+\frac{\alpha_c}{\alpha_s}\right)$ (b) $\left(1+\frac{\alpha_s}{\alpha_c}\right)$
(c) $\frac{\alpha_s}{\alpha_c}$ (d) $\frac{\alpha_c}{\alpha_s}$

86. A uniform metallic circular disc, mounted on frictionless bearings, is rotating at an angular frequency ω about an axis passing through its centre and perpendicular to its plane. The coefficient of linear expansion of the metal is α. If the temperature of the disc is increased by Δt, the angular frequency of rotation of the disc will
(a) remain unchanged (b) increase by $\alpha\omega\Delta t$
(c) increase by $2\alpha\omega\Delta t$ (d) decrease by $2\alpha\omega\Delta t$

87. A metal cube of coefficient of linear expansion α is floating in a beaker containing a liquid of coefficient of volume expansion γ. When the temperature is raised by ΔT, the depth upto which the cube is submerged in the liquid remains unchanged. If the expansion of the beaker is ignored, the relation between α and γ is
(a) $\alpha = \frac{\gamma}{3}$ (b) $\alpha = \frac{\gamma}{2}$
(c) $\alpha = 3\gamma$ (d) $\alpha = 2\gamma$

88. An ideal gas is expanded isothermally from a volume V_1 to volume V_2 and then compressed adiabatically to original volume V_1. The initial pressure is P_1 and the final pressure is P_3. If the net work done is W, then
(a) $P_3 > P_1, W > 0$ (b) $P_3 < P_1, W < 0$
(c) $P_3 > P_1, W < 0$ (d) $P_3 = P_1, W = 0$

89. A thermally insulated rigid container contains an ideal gas at 27ºC. It is fitted with a heating coil of resistance 50 Ω. A current is passed through the coil for 10 minutes by connecting it to a d.c. source of 10 V. The change in the internal energy is
(a) zero (b) 300 J
(c) 600 J (d) 1200 J

90. Two different adiabatic paths for the same gas intersect two isothermals at T_1 and T_2 as shown in the P–V diagram (Fig. 8.21). Then

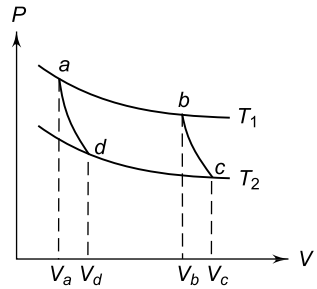


Fig. 8.21

(a) $\frac{V_a}{V_c} = \frac{V_b}{V_d}$ (b) $\frac{V_a}{V_b} = \frac{T_2}{T_1}$
(c) $\frac{V_a}{V_b} = \frac{V_d}{V_c}$ (b) $\frac{V_a}{V_d} = \frac{T_1}{T_2}$

91. Figure 8.22 shows a cyclic process $ABCA$ in the $V-T$ diagram. Which of the diagrams shown in Fig. 8.23 shows the same process on a $P-V$ diagram.

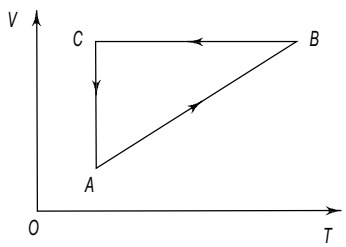


Fig. 8.22

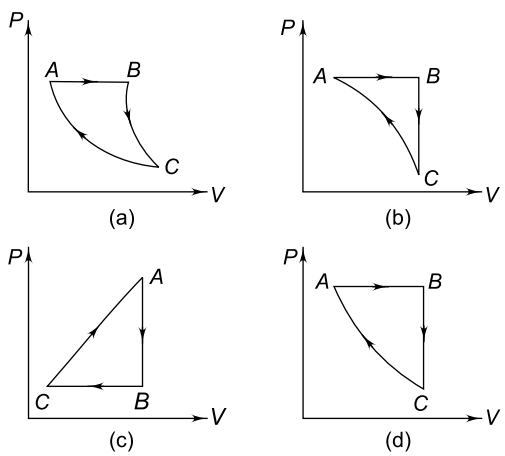


Fig. 8.23

92. An ideal gas ($\gamma = 1.4$) expands from 5×10^{-3} m^3 to 25×10^{-3} m^3 at a constant pressure of 1×10^5 Pa. The heat energy supplied to the gas in this process is
(a) 7 J (b) 70 J
(c) 700 J (d) 7000 J

93. Three moles of an ideal gas are taken through a cyclic process *ABCA* as shown on *T*–*V* diagram in Fig. 8.24. The gas loses 2510 J of heat in the complete cycle. If $T_A = 100$ K and $T_B = 200$ K, The work done by the gas during the process *BC* is

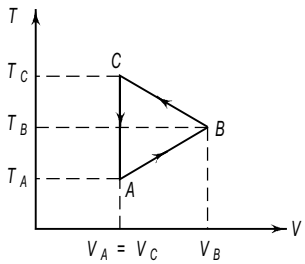


Fig. 8.24

(a) 5000 J (b) – 5000 J
(c) 4000 J (d) – 2500 J

94. Liquid oxygen at 50 K is heated at 300 K at constant pressure of 1 atmosphere. The rate of heating is constant. Which of the graphs shown in Fig. 8.25 represent the variation of temperature (*T*) with time (*t*)?

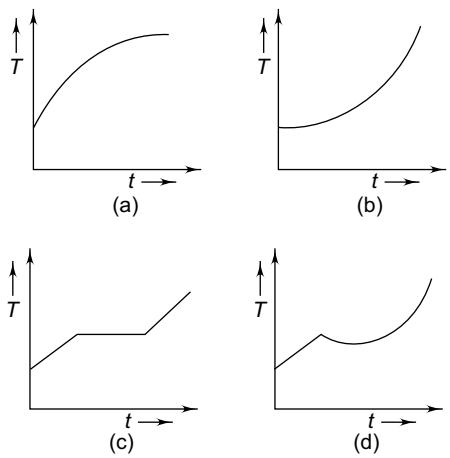


Fig. 8.25

95. One mole of a monoatomic ideal gas is contained in a insulated and rigid container. It is heated by passing a current of 2 A for 10 minutes through a filament of resistance 100 Ω. The change in the internal energy of the gas is
(a) 30 kJ (b) 60 kJ
(c) 120 kJ (d) 240 kJ

96. The *P*–*V* diagram for *n* moles of an ideal gas undergoing a process $A \rightarrow B$ is shown in Fig. 8.26. The maximum temperature of the gas during the process is

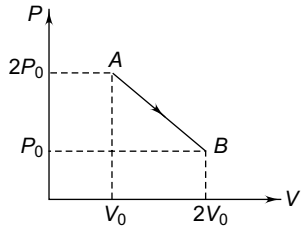


Fig. 8.26

(a) $\frac{9P_0V_0}{4nR}$ (b) $\frac{3P_0V_0}{nR}$
(c) $\frac{2P_0V_0}{nR}$ (d) $\frac{3P_0V_0}{2nR}$

97. Two litres of water (density = 1 g/ml) in an open-lid insulated kettle is heated by an electric heater of power 1 kW. The heat is lost from the lid at the rate of 160 J/s. The time taken for heating water (specific heat capacity 4.2 kJ kg^{-1} K^{-1}) from 20°C to 75°C is
(a) 340 s (b) 550 s
(c) 620 s (d) 760 s

98. A body cools from 75°C to 65°C in 5 minutes in a room where the temperature is 25°C. The temperature of the body at the end of next 5 minutes will be
(a) 55°C (b) 56°C
(c) 57°C (b) 58°C

99. A liquid takes 6 minutes to cool from 80°C to 50°C. If the temperature of the surroundings is 20°C, how long will it take to cool from 60°C to 30°C?
(a) 6 min. (b) 8 min.
(c) 10 min. (d) 12 min.

100. A body initially at 80°C cools to 64°C in 5 minutes and to 52°C in 10 minutes. The temperature of the surroundings is
(a) 15°C (b) 16°C
(c) 20°C (d) 25°C

101. In Q. 100 above, the temperature of the body at the end of 15 minutes will be
(a) 41°C (b) 43°C
(c) 45°C (b) 47°C

102. A sphere, a cube and a thin circular plate have the same mass and are made of the same material. All of them are heated to the same temperature. The rate of cooling is
(a) the maximum for the sphere and minimum for the plate.

(b) the maximum for the sphere and minimum for the cube.

(c) the maximum for the plate and minimum for the sphere.

(d) the same for all the three.

103. In the first experiment, two identical conducting rods are joined one after the other and this combination is connected to two vessels, one containing water at 100°C and the other containing ice at 0°C (see Fig. 8.27). In the second experiment, the two rods are placed one on top of the other and connected to the same vessels. If q_1 and q_2 (in gram per second) are the respective rates of melting of ice in the two cases, then the ratio $\frac{q_1}{q_2}$ is

(a) $\frac{1}{2}$ (b) $\frac{2}{1}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

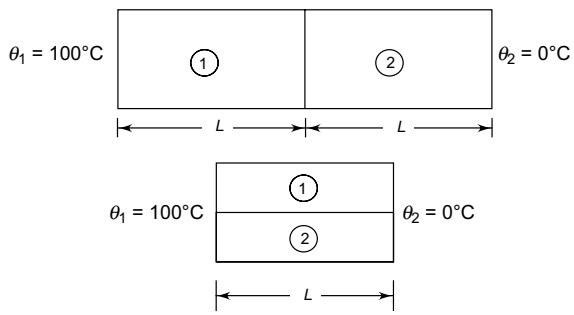


Fig. 8.27

104. Two spheres A and B having radii 3 cm and 5 cm respectively are coated with carbon black on their outer surfaces. The wavelengths of maximum intensity of emission of radiation are 300 nm and 500 nm respectively. The respective powers radiated by them are in the ratio of

(a) $\sqrt{\frac{5}{3}}$ (b) $\frac{5}{3}$

(c) $\left(\frac{5}{3}\right)^2$ (d) $\left(\frac{5}{3}\right)^4$

105. In which of the following processes is the heat transfer promarily due to radiation?

(a) Boiling of water

(b) Land and sea breezes

(c) Heating of a metal rod placed over a flame

(d) Heating of the glass surface of an electric bulb due to current in its filament.

106. A layer of ice at 0 °C of thickness x_1 is floating on a pond of water. L, ρ and k respectively are the latent heat of fusion of water, density of ice and thermal conductivity of ice. If the atmospheric temperature is $-T$ °C, the time taken for the thickness of the layer of ice to increase from x_1 to x_2 is given by

(a) $\frac{\rho L}{2kT}(x_1 + x_2)^2$ (b) $\frac{\rho L}{kT}(x_2 - x_1)^2$

(c) $\frac{\rho L}{2kT}\left(x_2^2 - x_1^2\right)$ (d) $\frac{\rho L}{kT}\left(x_2^2 - x_1^2\right)$

107. Two identical rods AB and CD, each of length L, cross-sectional area A and thermal conductivity k are connected as shown in Fig. 8.28. Ends A, C and D are maintained at temperatures T_1 = 20°C, T_2 = 30°C and T_3 = 40°C, respectively. The temperature at B is

(a) 32 °C (b) 33 °C

(c) 34 °C (d) 35 °C

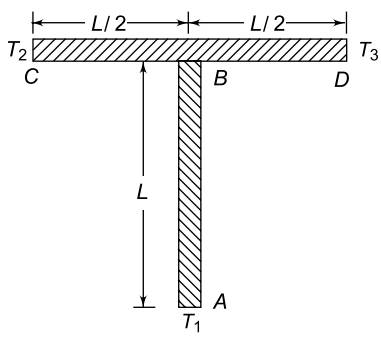


Fig. 8.28

108. Two solid spheres A and B have radii r_1 and r_2 and have similar surface finish. Sphere A is heated to a temperature T_1 kelvin and sphere B is heated to a temperature T_2. If, $r_2 = 2r_1$ and $T_2 = T_1/2$, and they are placed in identical surroundings, the ratio of the rate of loss of heat of A to that of B is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 2 (d) 4

109. A metal cylinder of radius r and thermal conductivity k_1 is surrounded by a cylindrical metallic shell of inner radius r and outer radius $3r$ and thermal conductivity k_2. The equivalent thermal conductivity of the system is

(a) $\frac{k_1 + 2k_2}{3}$ (b) $\frac{k_1 + 4k_2}{6}$

(c) $\frac{k_1 + 8k_2}{9}$ (d) $\frac{k_1 + 3k_2}{2}$

110. When the temperature of a thin copper wire is increased by ΔT; its length L increases by 0.1%. If the temperature of a thin square copper plate of side L is increased by ΔT, its surface area increases by

(a) 0.1% (b) 0.2%

(c) 0.3% (d) 0.4%

111. A pendulum clock having a metal rod keeps correct time at 20°C. When temperature falls to 0°C, it gains 12 seconds per day. The coefficient of linear expansion of the metal is (in per °C)

(a) 1.4×10^{-5} (b) 1.6×10^{-5}

(c) 1.8×10^{-5} (d) 2.0×10^{-5}

112. A metal ball falls freely from a height of 12 m and bounces back to a height of 8 m above the ground. If 10% of the energy dissipated in this process is lost to the surroundings and the rest of energy is absorbed by the ball, the rise in its temperature will be (specific heat capacity of metal = 360 J kg^{-1} k^{-1}. Take $g = 10$ ms^{-2})

(a) 0.1°C (b) 0.5°C

(c) 1.0°C (d) 0.36°C

113. Three insulated metal rods are welded together to form a right-angled isosceles triangle ABC as shown in Fig. 8.29. The three rods have the same cross-sectional area A and are made of the same metal. Ends A and B are maintained at constant temperatures T and $2\sqrt{2}\ T$ respectively. In the steady state, the temperature of end C will be

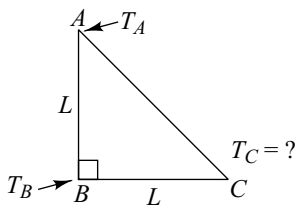


Fig. 8.29

(a) $\frac{2T}{1+\sqrt{2}}$ (b) $\frac{3T}{1+\sqrt{2}}$

(c) $\frac{T}{\sqrt{2}-1}$ (d) $\frac{5T}{1+\sqrt{2}}$

114. In an adiabatic process, the pressure of a monoatomic ideal gas increases by 0.5%. The volume will decrease by

(a) 0.1% (b) 0.2%

(c) 0.3% (d) 0.4%

115. The internal energy of n_1 moles of helium at absolute temperature T is equal to the internal energy of n_2 moles of oxygen at absolute temperature 3 T. Then the ratio $\frac{n_1}{n_2}$ is

(a) $\frac{6}{5}$ (b) $\frac{5}{9}$

(c) 5 (d) $\frac{9}{5}$

116. If 1 mole of hydrogen gas at absolute temperature T is mixed with 2 moles of oxygen gas at absolute temperature $2T$, the temperature of the mixture will be

(a) $\frac{5T}{3}$ (b) $\frac{4T}{3}$

(c) $\frac{3T}{2}$ (d) $\frac{T}{2}$

117. If 8 g of helium is mixed with 32 g of oxygen, the ratio $\gamma (= C_p/C_v)$ of the mixture is

(a) $\frac{39}{23}$ (b) $\frac{27}{19}$

(c) $\frac{21}{13}$ (d) $\frac{17}{11}$

118. In an adiabatic process, the relation between internal energy U, volume V and pressure P of a real gas is

$$U = a + b\,PV$$

where a and b are constants. If a = 3 joule and $b = 2.5$, the ratio of the two specific heats of the gas is

(a) $\frac{7}{5}$ (b) $\frac{5}{3}$

(c) $\frac{6}{5}$ (d) $\frac{3}{2}$

119. Figure 8.30 shows the P-V diagram of an ideal gas. The work done by the gas in the process $ABCDA$ is

(a) PV (b) $\frac{3}{2}PV$

(c) 2 PV (d) 4 PV

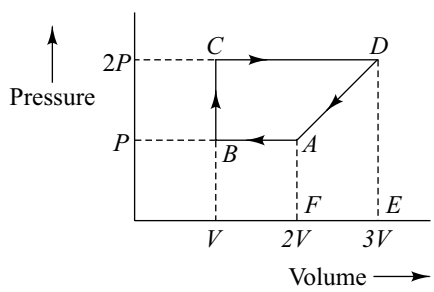


Fig. 8.30

120. Figure 8.31 shows the P-T graph of an ideal gas. If the density of the gas in state A is ρ_A, its density in state B is

(a) $\frac{2}{3}\rho_A$ (b) $\frac{3}{2}\rho_A$

(c) $\frac{3}{4}\rho_A$ (d) $\frac{4}{3}\rho_A$

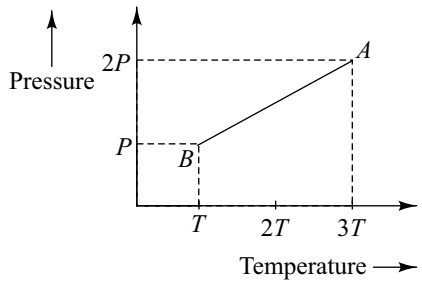


Fig. 8.31

121. Figure 8.32 shows the *P*-*V* graph of *n* moles of an ideal gas undergoing a process $A \rightarrow B$. The maximum temperature of the gas during the process is

(a) $\frac{5P_oV_o}{nR}$ (b) $\frac{3P_oV_o}{4nR}$

(c) $\frac{15P_oV_o}{2nR}$ (d) $\frac{25P_oV_o}{8nR}$

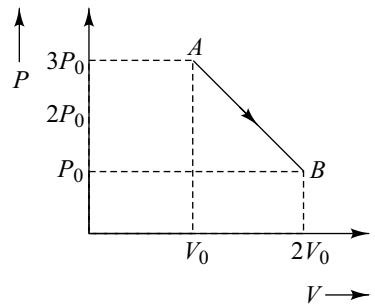


Fig. 8.32

122. Figure 8.33 shows the *P*-*V* diagram for 2 moles of a diatomic ideal gas undergoing a process $A \rightarrow B$. The molar specific heat capacity of the gas is

(a) R (b) $\frac{3R}{2}$

(c) $3R$ (d) $\frac{7R}{2}$

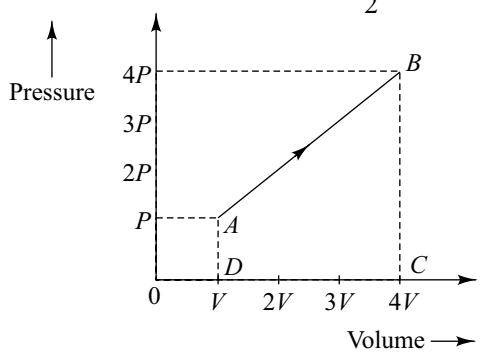


Fig. 8.33

123. In an adiabatic process, the pressure *P* of a gas varies with temperature *T* as $P = k\,T^{5/2}$ where *k* is a constant. The ratio $\gamma (= C_p/C_v)$ of the gas is

(a) $\frac{7}{5}$ (b) $\frac{5}{3}$

(c) $\frac{3}{2}$ (d) $\frac{9}{5}$

124. The pressure *P* of an ideal gas varies with volume *V* as

$$P = a\,V^b$$

where *a* and *b* are constants and $b \neq -1$. The work done to raise the temperature of *n* moles of the gas from *T* to 2*T* is

(a) $\frac{anRT}{(b+1)}$ (b) $\frac{nRT}{a\,(b+1)}$

(c) $\frac{nRT}{(b-1)}$ (d) $\frac{nRT}{(b+1)}$

Answers

Level A

1. (a)	2. (a)	3. (a)	4. (b)
5. (d)	6. (d)	7. (d)	8. (a)
9. (a)	10. (c)	11. (c)	12. (d)
13. (b)	14. (c)	15. (d)	16. (d)
17. (c)	18. (c)	19. (a)	20. (d)
21. (b)	22. (b)	23. (b)	24. (a)
25. (d)	26. (c)	27. (a)	

Level B

28. (d)	29. (b)	30. (d)	31. (b)
32. (b)	33. (c)	34. (c)	35. (d)
36. (c)	37. (d)	38. (b)	39. (c)
40. (d)	41. (d)	42. (b)	43. (c)
44. (a)	45. (c)	46. (b)	47. (a)
48. (b)	49. (c)	50. (c)	51. (b)
52. (c)	53. (d)	54. (c)	55. (a)
56. (b)	57. (a)	58. (a)	59. (d)
60. (c)	61. (b)	62. (c)	63. (d)
64. (a)	65. (c)	66. (a)	67. (b)
68. (d)	69. (b)	70. (a)	71. (b)
72. (c)	73. (b)	74. (b)	75. (b)

76. (d)	77. (b)	78. (c)	79. (a)
80. (d)	81. (d)	82. (a)	83. (b)
84. (d)	85. (d)	86. (d)	87. (b)
88. (c)	89. (d)	90. (c)	91. (d)
92. (d)	93. (b)	94. (c)	95. (d)
96. (a)	97. (b)	98. (c)	99. (d)
100. (b)	101. (b)	102. (c)	103. (d)
104. (c)	105. (d)	106. (c)	107. (a)
108. (d)	109. (c)	110. (b)	111. (a)
112. (a)	113. (d)	114. (c)	115. (c)
116. (a)	117. (d)	118. (a)	119. (b)
120. (a)	121. (d)	122. (d)	123. (b)
124. (d)			



Solutions

Level A

1. The length of 1 cm division of the steel scale at 21°C is

$$(1\text{ cm}) \times [1 + \alpha_s(21 - 20)] = (1 + \alpha_s)\text{ cm}$$

Length of copper rod at 21°C will be

$$(L\text{ cm}) \times [1 + \alpha_c\,(21 - 20)] = L\,(1 + \alpha_c)\text{ cm}$$

$\therefore$ Scale reading at 21°C $= \dfrac{L(1+\alpha_c)}{(1+\alpha_s)}$.

Hence the correct choice is (a).

2. Then $\alpha_s = \alpha_c$. Hence the correct choice is (a).

3. Let l be the length of the wire at temperature t and Δl be the contraction in the length as the temperature falls by Δt. Then

$$\Delta l = \alpha l \Delta t$$

$\therefore$ Strain in the wire $\left(\dfrac{\Delta l}{l}\right) = \alpha \Delta t$. Now, Young's modulus is

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta l/l} = \frac{Fl}{A\Delta l}$$

$\therefore$ Tension in the wires is $F = \dfrac{YA\Delta l}{l} = YA\alpha\Delta t$

Hence the correct choice is (a).

4. The increase in length due to heating is independent of the radius of the rod. The increase in the length of rod A is

$$\Delta l = \alpha l \Delta t$$

and of rod B is

$$\Delta l' = \alpha l' \Delta t = 2\alpha l \Delta t \qquad (\because\ l' = 2l)$$

$$\therefore \quad \frac{\Delta l}{\Delta l'} = \frac{1}{2}$$

Hence the correct choice is (b).

5. Given $\dfrac{\Delta l}{l} = 0.05\% = \dfrac{0.05}{100}$. Now

$$\alpha = \frac{\Delta l}{l\Delta t} = \frac{0.05}{100 \times 100} = 5 \times 10^{-6}\ ^\circ\text{C}^{-1}$$

Hence the correct choice is (d).

6. If a rod of length L and coefficient of thermal expansion α is heated to a temperature t, the increase in the length is given by

$$l = \alpha Lt$$

$\therefore$ Strain $= \dfrac{l}{L} = \alpha t$

and stress $= Y \times$ strain $= Y\,\alpha\,t$

Since the value of t is the same for the two rods, the stresses in them will be equal if

$$Y_1\,\alpha_1\,t = Y_2\,\alpha_2\,t$$

or $\dfrac{Y_1}{Y_2} = \dfrac{\alpha_2}{\alpha_1}$

Hence the correct choice is (d).

7. Let the temperature of the mixture be t°C. Heat lost by water in calories $= 300 \times 1 \times (25 - t) = 7500 - 300t$

Heat in calories required to melt 100 g of ice

$$= 100 \times 80 = 8000$$

Now Heat lost = heat gained

or $7500 - 300t = 8000$

or $t = -\dfrac{5}{3}$ °C

Since t is negative, the water at 25°C cools to 0°C and melts a part of ice at 0°C.

$\therefore$ Heat lost $= 300 \times 1 \times (25 - 0) = 7500$ cal. Hence only a part of the ice melts and resulting temperature is 0°C. Hence the correct choice is (d).

8. The amount of heat required to convert 100 g of ice at 0°C into water at 0°C $= 100 \times 80 = 8000$ calories. This is precisely the amount of heat lost by 100 g of water at 80°C to bring its temperature down to 0°C. Therefore, the temperature of the mixture remains 0°C. Hence the correct choice is (a).

9. Useful power available = 60% of P = 0.6 P watts. Therefore, energy consumed in T seconds = 0.6 PT joules. This energy is used up in heating block. If the rise in temperature $\Delta\theta$, then

$$0.6\,PT = Ms\Delta\theta$$

or $\Delta\theta = \dfrac{0.6PT}{Ms}$

Hence the correct choice is (a).

10. Heat energy in copper block = $2 \times 400 \times 500 = 4 \times 10^5$ J. The amount of ice that melts will be maximum if the entire heat energy of the copper block is used up in melting ice. Now, 3.5×10^5 J of heat energy is needed to melt 1 kg of ice into water. Therefore, the amount of ice melted by 4×10^6 J of heat energy is

$$\frac{4 \times 10^5 \text{ J}}{3.5 \times 10^5 \text{ J kg}^{-1}} = \frac{8}{7} \text{ kg}$$

Hence the correct choice is (c).

11. Number of moles in 14 grams of nitrogen (n) = 14/28 = $\frac{1}{2}$. Since nitrogen is diatomic $C_p = 7R/2$. Therefore, amount of heat energy supplied = $n \times C_p \times \Delta\theta$

$$= \frac{1}{2} \times \frac{7R}{2} \times 40 = 70R \text{ joules}$$

Hence the correct choice is (c).

12. For a bar of cross-sectional area A, length l and having temperatures θ_1 and θ_2 at its two ends, the amount of heat energy Q flowing normally across the bar in time t is given by

$$Q = \frac{kA(\theta_1 - \theta_2)t}{l}$$

where k is the coefficient of thermal conductivity. Therefore,

$$k = \frac{Ql}{A(\theta_1 - \theta_2)t}$$

$$\therefore \text{ Dimension of } k = \frac{\text{dimensions of } Q \times \text{dimensions of } l}{\text{dimensions of } A \times \text{dimensions of } (\theta_1 - \theta_2) \times \text{dimensions of } t}$$

$$= \frac{\text{ML}^2\text{T}^{-2} \times \text{L}}{\text{L}^2 \times \text{K} \times \text{T}} = \text{MLT}^{-3}\text{ K}^{-1}$$

Hence the correct choice is (d).

13. Rate of flow is $\frac{Q}{t} = \frac{kA(\theta_2 - \theta_1)}{l}$

Since $(\theta_2 - \theta_1)$ and l are the same for the two rods, the rate of flow Q/t will be the same if the product kA is the same for the two rods, i.e. if

$$k_1 A_1 = k_2 A_2$$

or $$\frac{A_1}{A_2} = \frac{k_2}{k_1}$$

Hence the correct choice is (b).

14. Let the length of the cylinder be l and let its ends be maintained at temperatures θ_1 and θ_2. Area of the cross-section of the inner cylinder = πR^2. Area of cross-section of outer cylinder = $\pi(2R)^2 - \pi R^2 = 3\pi R^2$.

Rate of flow of heat across inner cylinder is

$$Q_1 = \frac{k_1 \pi R^2 (\theta_1 - \theta_2)}{l} \quad \text{(i)}$$

Rate of flow of heat across the outer shell is

$$Q_2 = \frac{k_2 (3\pi R^2)(\theta_1 - \theta_2)}{l} \quad \text{(ii)}$$

Let the effective thermal conductivity of the compound cylinder be k. The rate of flow of heat across the compound cylinder is

$$Q = \frac{k(4\pi R^2)(\theta_1 - \theta_2)}{l} \quad \text{(iii)}$$

Now $$Q = Q_1 + Q_2 \quad \text{(iv)}$$

Using (i), (ii) and (iii) in (iv), we get $4k = k_1 + 3k_2$

or $$k = \frac{k_1 + 3k_2}{4}$$

Hence the correct choice is (c).

15. If the ends of the rods are maintained at temperatures θ_1 and θ_2, the rate of flow of heat through each rod is given by

$$Q_1 = \frac{k_1 A(\theta_1 - \theta_2)}{d}$$

and $$Q_2 = \frac{k_2 A(\theta_1 - \theta_2)}{d}$$

where A is the cross-sectional area of each rod and d is the length of each rod. Now the cross-sectional area of the composite rod is $2A$. If k is the thermal conductivity of the composite rod, the rate of heat flow is

$$Q = \frac{k(2A)(\theta_1 - \theta_2)}{d}$$

But $Q = Q_1 + Q_2$. Therefore

$$\frac{k(2A)(\theta_1 - \theta_2)}{d} = \frac{k_1 A(\theta_1 - \theta_2)}{d} + \frac{k_2 A(\theta_1 - \theta_2)}{d}$$

or $$k = \frac{1}{2}(k_1 + k_2)$$

Hence the correct choice is (d).

16. Refer to the solution of Q. 13. The correct choice is (d).

17. Let l be the original length of the rods. Let them be heated so that the increase in temperature is t°C.

Increase in length = $\alpha l t$

where α is the coefficient of linear expansion of the rod.

$$\text{Strain} = \alpha l t/l = \alpha t$$

$$\text{Stress} = Y \times \text{strain} = Y \alpha t$$

Stresses are equal for the two rods if

$$Y_1 \alpha_1 t = Y_2 \alpha_2 t$$

or $$\frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$$

Hence the correct choice is (c).

18. The correct choice is (c).
19. The correct choice is (a). It follows from the fact that the volumes of metal and mercury increase to $V_0 (1 + \gamma_1 \Delta T)$ and $V_0 (1 + \gamma_2 \Delta T)$ respectively; V_0 being the initial volume.
20. Helium is a monoatomic gas. For He, $C_v = \frac{3R}{2}$. Hydrogen is a diatomic gas for which $C_v = \frac{5R}{2}$ at moderate temperature. Therefore,

    C_v for hydrogen = C_v for Helium $\times \frac{5}{3} = 12.6 \times \frac{5}{3} =$ 21 J mol^{-1} K^{-1}. Hence the correct choice is (d).
21. The correct choice is (b).
22. For isothermal expansion PV = constant. Partially differentating we have

    $$V \Delta P + P \Delta V = 0$$

    or $$\frac{\Delta P}{\Delta V} = -\frac{P}{V}$$

    For adiabatic expansion PV^γ = constant. Partially differentiating we have

    $$V^\gamma \Delta P + \gamma V^{\gamma-1} \Delta V P = 0$$

    or $$\frac{\Delta P}{\Delta V} = -\frac{\gamma P}{V}$$

    Thus, the slope at any point on the adiabatic curve is γ times that for isothermal curve. Hence curve CD represents adiabatic expansion.
23. $T_1 = 127°C = 127 + 273 = 400$ K, $T_2 = 27°C = 300$ K, $Q = 800$ J.

    The efficiency of the engine is given by

    $$\eta = \frac{W}{Q} = 1 - \frac{T_2}{T_1}$$

    $$\therefore \quad \text{Work done } W = Q\left(1 - \frac{T_2}{T_1}\right)$$

    $$= 800 \times \left(1 - \frac{300}{400}\right) = 200 \text{ J}$$

    Hence the correct choice is (b).
24. Heat rejected to the sink = heat energy supplied – useful work done

    $$= 800 - 200 = 600 \text{ J}$$

    Hence the correct choice is (a).
25. The efficiency of the engine is

    $$\eta = \frac{W}{Q} \text{ which the same as}$$

    $$1 - \frac{T_2}{T_1} = \frac{200}{800} = 0.25$$

    Hence the correct choice is (d).
26. $\frac{W}{Q} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{600} = 0.5$. Therefore $Q = 2W =$ $2 \times 800 = 1600$ J. Hence the correct choice is (c).
27. $T_2 = 300$ K. Now $\eta = 1 - T_2/T_1$. When $\eta = 40\% = 0.4$, the value of T_1 is given by

    $$\frac{T_2}{T_1} = 1 - 0.4 = 0.6$$

    or $$T_1 = \frac{T_2}{0.6} = \frac{300}{0.6} = 500 \text{ K}.$$ When $\eta = 60\% = 0.6$, the value of T_2 should be

    $$T_2' = \frac{300}{0.4} = 750 \text{ K}$$

    $\therefore \; T_2' - T_2 = 750 - 500 = 250$ K, which is choice (a).

Level B

28. The thermal resistance of a slab of length l, area A and thermal conductivity k is given by

    $$R = \frac{l}{kA}$$

    For slab A : $R_1 = \frac{l/2}{k_1 A}$

    For slab B : $R_2 = \frac{l/2}{k_2 A}$

    Since the slabs are joined in series, the thermal resistance of the composite slab is

    $$R_c = R_1 + R_2$$

    or $$\frac{l}{k_c A} = \frac{l/2}{k_1 A} + \frac{l/2}{k_2 A}$$

    or $$\frac{1}{k_c} = \frac{1}{2}\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

    which gives $$k_c = \frac{2k_1 k_2}{(k_1 + k_2)}$$

    Hence the correct choice is (d).
29. Let θ_1 and θ_2 be the temperatures at the two faces of the composite slab and let θ be the temperature at the common face of the slab. If l is the length of each slab and A the area of their face, then, in the steady

state, the rate of flow of heat across A = rate of flow of heat across B, i.e.

$$\frac{k_1 A(\theta_1 - \theta)}{l} = \frac{k_2 A(\theta - \theta_2)}{l}$$

or $k_1(\theta_1 - \theta) = k_2(\theta - \theta_2)$

Now $k_2 = 2k_1$. Therefore

$$(\theta_1 - \theta) = 2(\theta - \theta_2) \quad \text{(i)}$$

Also, $\theta_1 - \theta_2 = 12°\text{C}$ or $\theta_2 = \theta_1 - 12$ (ii)

Using (ii) in (i) we have

$$(\theta_1 - \theta) = 2\{\theta - (\theta_1 - 12)\}$$

or $3(\theta_1 - \theta) = 24$

or $\theta_1 - \theta = 8°\text{C}$

Hence the correct choice is (b).

30. The rate of heat flow in rods A and B are

$$\frac{Q_1}{t} = \frac{k_1 \pi r_1^2 \Delta\theta}{l_1}$$

and $$\frac{Q_2}{t} = \frac{k_2 \pi r_2^2 \Delta\theta}{l_2}$$

$Q_1 = Q_2$, if

$$\frac{k_1 r_1^2}{l_1} = \frac{k_2 r_2^2}{l_2}$$

or $$\frac{k_1}{k_2} = \frac{l_1}{l_2} \times \frac{r_2^2}{r_1^2} = 2 \times (2)^2 = 8$$

Hence the correct choice is (d).

31. Work done = area enclosed by P–V graph

$$= \text{area of } ABCD = AB \times BC$$

$$= (2P - P) \times (2V - V) = PV$$

Hence the correct choice is (b).

32. (i) Heat required to convert 1 g of ice at 0°C to water at 0°C = 80 calories. (ii) Heat required to convert 1 g of water at 0°C to water at 100°C = 1 × 1 × 100 = 100 calories. (iii) Heat required to evaporate 1 g of water at 100°C = 540 calories. Total heat required = 80 + 100 + 540 = 720 calories. Hence the correct choice is (b).

33. Work done = latent heat of fusion of ice = 80 calories = 80 × 4.2 = 336 J. Hence the correct choice is (c).

34. The rate of cooling is directly proportional to surface area if the balls are made of the same material. Since the surface area of A is 4 times that of B, the correct choice is (c).

35. The correct choice is (d).

The molar specific heat at constant volume of a gas is given by

$$C_v = \frac{nR}{2}$$

where n is the number of degrees of freedom. In the temperature range of 250 to 750 K, hydrogen (which is diatomic) has 5 degrees of freedom—three for translational modes and 2 for rotational modes. Hence, in this temperature range, $C_v = 5R/2$. As the temperature is decreased, the rotational modes are suppressed and translational modes survive. Hence at lower temperatures, hydrogen has only three dominant degrees of freedom. Therefore, $C_v = 3R/2$. At higher temperatures, two additional degrees of freedom corresponding to vibrational modes appear, so that hydrogen now has seven degrees of freedom. Consequently, at high temperatures C_v of hydrogen $= 7R/2$.

36. Heat energy needed to raise the temperature of n moles of a gas through ΔT kelvin at constant pressure is

$$Q = n C_p \Delta T = 743 \text{ J (given)}$$

Heat energy needed to raise the temperature of n moles of a gas through ΔT kelvin at constant volume is

$$Q' = n C_v \Delta T = n(C_p - R)\Delta T \quad (\because C_p - C_v = R)$$
$$= n C_p \Delta T - nR\Delta T$$
$$= 743 - 5 \times 8.3 \times 2$$
$$= 743 - 83 = 660 \text{ J}$$

Hence the correct choice is (c).

37. For isothermal process, PV = constant. Therefore

$$P_i V_i = PV$$

or $$P_i \frac{V}{n} = PV$$

or $$P_i = nP \quad \text{(i)}$$

For adiabatic process, PV^γ = constant. Therefore

$$P_a (V_a)^\gamma = PV^\gamma$$

or $$P_a \left(\frac{V}{n}\right)^\gamma = PV^\gamma$$

or $$P_a = n^\gamma P \quad \text{(ii)}$$

From (i) and (ii) we get

$$\frac{P_i}{P_a} = \frac{n}{n^\gamma} = n^{(1-\gamma)}$$

Hence the correct choice is (d).

38. Bulk modulus is defined as

$$B = -\frac{V \Delta P}{\Delta V}$$

For adiabatic process, PV^γ = constant. Differentiating partially, we get

$$\Delta P V^{\gamma} + P\gamma V^{\gamma-1} = 0$$

or $$-\frac{V\Delta P}{\Delta V} = \gamma P$$

Therefore $$B_a = \gamma P$$

Similarly, for isothermal process, PV = constant gives

$$B_i = P$$

Hence $B_a/B_i = \gamma$, which is choice (b).

39. Curve AB represents isothermal change in which the pressure decreases from $2P$ to P and volume increases from V to $2V$, the temperature remaining constant. Since the temperature remains constant between A and B, the P–T graph should be perpendicular to T-axis between A and B. Hence graph (d) is obviously wrong. Diagrams (a) and (b) are also incorrect, because according these graphs, the pressure remains constant between A and B. But the pressure reduces from $2P$ to P between A and B. Hence the only correct choice is diagram (c).

40. Volume at A = Volume at C = V, but pressure at A = $2P$ and at C = P. According to Charles' law, at constant volume, $P \propto T$. Hence the temperature at $A = \frac{T}{2}$, which is choice (d).

41. If point C is at STP, the temperature at C = 273 K. Therefore, temperature at $A = 2 \times 273$ =546 K, which is choice (d).

42. In a cyclic process, the system returns to its initial state. Hence the change in internal energy $dU = 0$. Therefore, choice (b) is correct.

43. Since the volume remains constant, $dW = 0$. Hence

$$dQ = dU = 420 \text{ J}$$

But $$dQ = n\,C_v\,dT = 2 \times C_v \times 10 \quad (\because dT = 10°\text{C} = 10 \text{ K})$$

or $20\,C_v = 420$ or $C_v = 21$ J K^{-1} mol^{-1}, which is choice (c).

44. From the first law of thermodynamics, $dQ = dU + dW$ or $dU = dQ - dW = 40 - 20 = 20$ J. Since heat energy is released by the gas, its internal energy is decreased from 70 J to 70 – 20 = 50 J. Hence the correct choice is (a).

45. In an adiabatic process, $dQ = 0$. Therefore, $dU = -dW$, i.e. change in internal energy = work done. Since work is done on the gas dW is negative. Hence $dU = -(-250) = 250$ J. Thus the internal energy increases by 250 J. Hence the correct choice is (c).

46. Here dQ = 1 kilocalorie = 1000 cal = 1000×4.2 = 4200 J. Now $dU = dQ - dW = 4200 - 250 = 3950$ J. Hence the correct choice is (b).

47. Let m be the mass of the gas and V its initial volume. Then initial density is $\rho = m/V$. Final density $\rho_1 = n\rho$. Let P_1 be the final pressure and V_1 the final volume. Now, for an adiabatic process

$$PV^{\gamma} = P_1 V_1^{\gamma}$$

or $$P\left(\frac{m}{\rho}\right)^{\gamma} = P_1\left(\frac{m}{\rho_1}\right)^{\gamma}$$

or $$\frac{P}{\rho^{\gamma}} = \frac{P_1}{\rho_1^{\gamma}} \quad (\because m = \text{constant})$$

or $$P_1 = P\left(\frac{\rho_1}{\rho}\right)^{\gamma} = P(n)^{\gamma}$$

Hence the correct choice is (a).

48. The surface area of plate is $A = L \times 2L = 2L^2$. Taking logarithm, we have $\log A = \log 2 + 2 \log L$. Differentiating, we get

$$\frac{\delta A}{A} = 2\frac{\delta L}{L} = 2 \times 0.1\% = 0.2\%$$

Hence the correct choice is (b).

49. The correct choice is (c).

50. Total length of the composite rod at 0°C is $L_0 = L_1 + L_2$. When the composite rod is heated to t°C, its length at t°C will be $L = L_1(1 + \alpha_1 t) + L_2(1 + \alpha_2 t) = (L_1 + L_2) + L_1\alpha_1 t + L_2\alpha_2 t$, or

$$L = L_0 + (L_1\alpha_1 + L_2\alpha_2)t$$

$\therefore$ Effective coefficient of expansion of the composite rod is

$$\alpha = \frac{L - L_0}{L_0 t} = \frac{L_1\alpha_1 + L_2\alpha_2}{L_1 + L_2}$$, which is choice (c).

51. Consider a small element of length dx at a distance x from one end of the rod. Let L be the length of the rod. Now the increase in the coefficient of linear expansion per unit length of the rod is $(\alpha_2 - \alpha_1)/L$. Therefore, the value of α at the element located at x is

$$\alpha_x = \alpha_1 + \left(\frac{\alpha_2 - \alpha_1}{L}\right)x$$

$\therefore$ Increase in the length of the element = $\alpha_x\,dx\,\Delta T$, where ΔT is the rise in temperature. Therefore, the increase in the length of rod is

$$\Delta L = \int_0^L \alpha_x\,dx\,\Delta T$$

$$= \Delta T \int_0^L \left\{\alpha_1 + \left(\frac{\alpha_2 - \alpha_1}{L}\right)x\right\}dx$$

$$= \Delta T \left| \alpha_1 x + \left(\frac{\alpha_2 - \alpha_1}{L} \right) \frac{x^2}{2} \right|_0^L$$

$$= \Delta T \left[\alpha_1 L + \left(\frac{\alpha_2 - \alpha_1}{2} \right) L \right]$$

$$= \left(\frac{\alpha_1 + \alpha_2}{2} \right) L \Delta T = \alpha_{\text{eff}} L \Delta T$$

where $\alpha_{\text{eff}} = \frac{1}{2} (\alpha_1 + \alpha_2)$. Hence the correct choice is (b).

52. Let L be the length of each rod at 0°C and h_0 be the altitude AD at 0°C. Then (Fig. 8.34),

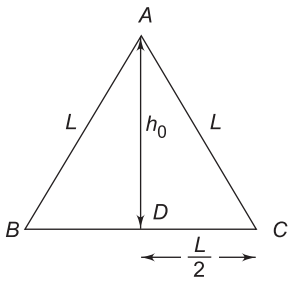


Fig. 8.34

$$h_0 = \left(L^2 - \frac{L^2}{4} \right)^{1/2} \quad (1)$$

When the rods are heated to a temperature t, the altitude becomes

$$h = \left[L^2 (1 + \alpha_1 t)^2 - \frac{L^2}{4} (1 + \alpha_2 t)^2 \right]^{1/2}$$

Since α_1 and α_2 are much less than unity, we can write $(1 + \alpha_1 t)^2 \simeq 1 + 2\alpha_1 t$ and $(1 + \alpha_2 t)^2 \simeq 1 + 2\alpha_2 t$. Thus

$$h = \left[L^2 (1 + 2\alpha_1 t) - \frac{L^2}{4} (1 + 2\alpha_2 t) \right]^{1/2} \quad (2)$$

Equating (1) and (2), we get ($\because$ Given that $h = h_0$)

$$\left(L^2 - \frac{L^2}{4} \right)^{1/2} = \left[L^2 (1 + 2\alpha_1 t) - \frac{L^2}{4} (1 + 2\alpha_2 t) \right]^{1/2}$$

which simplifies to give $\frac{\alpha_1}{\alpha_2} = \frac{1}{4}$, which is choice (c).

53. Weight of cylinder = weight of the liquid displaced, i.e.

$$Ah\rho_b = A \times (0.8\, h) \times \rho_0$$

or $$\rho_0 = \frac{\rho_b}{0.8} = \frac{5\rho_b}{4} \quad (1)$$

Here ρ_0 is the density of the liquid at 0°C. For the block to sink in the liquid at t°C, the density of the liquid must change from ρ_0 to ρ at t°C. Now

$$\rho = \frac{\rho_0}{1 + \gamma t} \quad (2)$$

For the block to sink $\rho = \rho_b$. Using (1) and (2), we have

$$\rho_b = \frac{5\rho_b / 4}{1 + \gamma t}$$

or $$1 + \gamma t = \frac{5}{4}$$

Given $t = 62.5$°C. Using this value, we get $\gamma = 4 \times 10^{-3}$ (°C)$^{-1}$ or K^{-1}, which is choice (d).

54. Heat energy supplied $dQ = C_p\, dT$. Change in internal energy $dU = C_v\, dT$. Therefore, work done $dW = dQ - dU = (C_p - C_v) dT$.

$$\therefore \quad \frac{dW}{dQ} = \frac{(C_p - C_v) dT}{C_p dT}$$

$$= 1 - \frac{1}{\gamma} = 1 - \frac{1}{5/3} = \frac{2}{5}$$

($\because$ $\gamma = 5/3$ for a monoatomic gas)

Hence the correct choice is (c).

55. Let T be the temperature of the interface in the steady state and let d be the thickness of each block. Now, in steady state, the rate of flow of heat through each block must be the same, i.e.

$$\frac{k_1 A (T_1 - T)}{d} = \frac{k_2 A (T - T_2)}{d}$$

or $k_1 (T_1 - T) = k_2 (T - T_2)$

which gives $T = \frac{k_1 T_1 + k_2 T_2}{k_1 + k_2}$, which is choice (a).

56. Let V_0 be the volume of the metal at 0°C and V_t its volume at t°C. At temperature t the upthrust is

$$U_t = V_t \rho_t g$$

where ρ_t is the density of alcohol at temperature t. Now

$$V_t = V_0 (1 + \gamma t)$$

where γ is the coefficient of cubical expansion of alcohol and V_0 is the volume of alcohol displaced at temperature $t = 0$°C. Now the density of alcohol at temperature t is

$$\rho_t = \frac{\rho_0}{1 + \gamma t}$$

where ρ_0 is the density of alcohol at $t = 0°C$. Hence

$$U_t = V_0 (1 + \gamma t) \times \frac{\rho_0 g}{(1+\gamma t)} = V_0 \rho_0 g = U_0$$

where U_0 is the upthrust at 0°C. Since the upthrust is independent of temperature, $W_1 = W_2$. Hence the correct choice is (b).

57. Let the mass of steam condensed be m kg. The latent of vaporisation of water = 556 kcal/kg. Therefore, heat lost by steam = 556 m + m (100 – 80) = 576 m kcal. Heat gained by calorimeter and water = (1.1 + 0.02) (80 – 15) = 72.8 kcal. Now, heat lost = heat gained, i.e.

576 m = 72.8 or m = 0.13 kg, which is choice (a).

58. When heat is supplied, firstly the temperature of ice increases from – 10°C to 0°C. At 0°C, ice starts melting at a constant temperature. When the whole of ice has melted into water, the temperature of water will increase from 0°C to 100°C. At 100°C, again the temperature becomes constant due to the conversion of liquid water into water vapour (steam) at 100°C. Hence the correct graph is (a).

59. Heat is given to the gas in cylinder A at constant pressure while the same amount of heat is given to the gas in cylinder B at constant volume. Heat given to gas in A is

$$Q_A = n\, C_p\, \Delta T_A$$

Heat given to gas in B is $Q_B = nC_v\, \Delta T_B$

Since $Q_A = Q_B$, we have

$$nC_p\, \Delta T_A = nC_v\, \Delta T_B$$

or $$\Delta T_B = \frac{C_p}{C_v}\, \Delta T_A$$

$$= \frac{7}{5} \times 30 \text{ K} = 42 \text{ K}$$

($\because$ for a diatomic gas, $C_P/C_v = 7/5$)

Hence the correct choice is (d).

60. The equation of state for an ideal gas of mass m and molecular mass M is

$$PV = \frac{m}{M}\, RT \qquad \text{(i)}$$

For an isothermal process, T = constant. Differentiating (i) partially at constant T, we get

$$P\Delta V + V\Delta P = 0$$

or $$\Delta P = - P\, \frac{\Delta V}{V} \qquad \text{(ii)}$$

From (i), $P = \frac{mRT}{MV}$. Using this in (ii), we get

$$\Delta P = - \frac{mRT}{M} \qquad (\because \Delta V = 2V - V = V)$$

$$\therefore \qquad \Delta P_A = - \frac{m_A RT}{M} \text{ and } \Delta P_B = - \frac{m_B RT}{M}$$

Hence $$\frac{\Delta P_A}{\Delta P_B} = \frac{m_A}{m_B}$$

Given $\Delta P_B = 1.5\, \Delta P_A$. Therefore, $\frac{1}{1.5} = \frac{m_A}{m_B}$

or 3 $m_A = 2\, m_B$, which is choice (c).

61. The speed of sound in a gas of bulk modulus B and density ρ is given by

$$v = \sqrt{\frac{B}{\rho}}$$

Bulk modulus B is given by $B = - \frac{V\Delta P}{\Delta V}$

Now, for a perfect gas, $PV = nRT$. Differentiating at constant T, we get

$$P\Delta V + V\Delta P = 0 \text{ or } \frac{V\Delta P}{\Delta V} = - P$$

Hence $$v = \sqrt{\frac{P}{\rho}}$$

If m is the mass of the gas and M its molecular mass, then

$$PV = \frac{m}{M}\, RT \text{ or } PM = \frac{mRT}{V} = \rho RT$$

or $$\frac{P}{\rho} = \frac{RT}{M} \text{ or } v^2 = \frac{RT}{M} \text{ or } v = \sqrt{\frac{RT}{M}}$$

Hence $$v_1 = \sqrt{\frac{RT}{M_1}} \text{ and } v_2 = \sqrt{\frac{RT}{M_2}} \text{ which give}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}}, \text{ which is choice (b).}$$

62. For a ideal gas, $PV = nRT$. Since pressure P is kept constant,

$$P\Delta V = nR\Delta T$$

or $$\frac{\Delta V}{\Delta T} = \frac{nR}{P} = \frac{nRV}{nRT} = \frac{V}{T} \qquad \left(\because P = \frac{nRT}{V}\right)$$

or $$\frac{1}{V}\frac{\Delta V}{\Delta T} = \frac{1}{T} \text{ or } \delta = \frac{1}{T}$$

Thus, the value of δ decreases as T is increased. Hence the correct choice is (c).

63. For adiabatic process, $T_1\ V_1^{(\gamma-1)} = T_2\ V_2^{(\gamma-1)}$. Thus

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{(\gamma-1)}$$

For a monoatomic gas, $\gamma = 5/3$. Also $V_2/V_1 = L_2/L_1$. Hence

$$\frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{\left(\frac{5}{3}-1\right)} = \left(\frac{L_2}{L_1}\right)^{2/3}$$, which is choice (d).

64. Since the slope of the P-V graph for adiabatic expansion is γ times that for isothermal expansion, curves AB and AC in Fig. 8.35(a) respectively represent isothermal and adiabatic expansions of the gas from initial volume V_1 to final volume V_2.

As the area under curve AB between volumes V_1 and V_2 is greater than the area under curve AC between V_1 and V_2, it follows that $W_1 > W_3$.

Figure 8.35(b) shows the P-V graph for isobaric (at constant pressure) expansion from initial volume V_1 and pressure P_1 to final volume V_2; the pressure remaining unchanged at P_1. Comparing figures (a) and (b) we find that the area under AD between volumes V_1 and V_2 is greater than the area under curves AB and AC. Hence W_2 is greater than W_1 and W_3. Hence the correct choice is (a).

65. For a gas, $PV = nRT$. Hence

$$(P)_{O_2} = \frac{(1\text{ mole})\,RT}{V} \text{ and } (P)_{He} = \frac{(1\text{ mole})\,R(2T)}{V}$$

$$\therefore \frac{(P)_{He}}{(P)_{O_2}} = 2 \text{ or } (P)_{He} = 2(P)_{O_2}$$, which is choice (c).

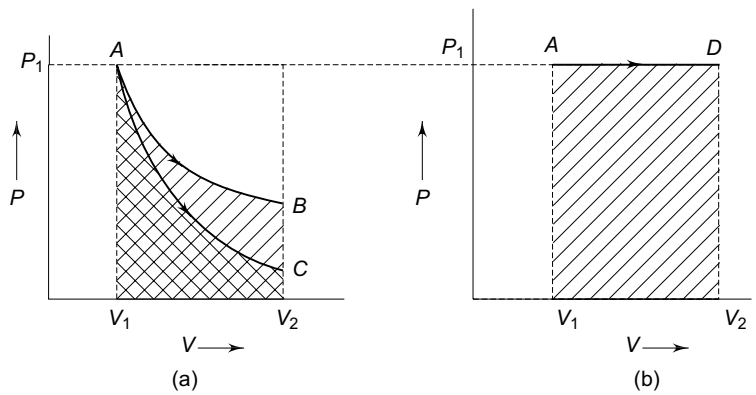


Fig. 8.35

66. Since both the gases are diatomic, each has two degrees of freedom associated with rotational motion. According to the law of equipartition of energy, the rotational kinetic energy per degree of freedom is $(1/2)kT$. Since the temperatures of the two gases are equal, their rotational kinetic energies will be equal. Hence the correct choice is (a).

67. Refer to Fig. 8.36. Since $T_B > T_A$, heat flows from B to A and from B to C. In the steady state, rate of flow of heat from B to C = rate of flow of heat from C to A, i.e. $\frac{Q_2}{t} = \frac{Q_3}{t}$

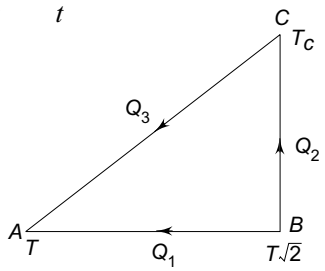


Fig. 8.36

$$\text{or } \frac{kA(T_B - T_C)}{l} = \frac{kA(T_C - T)}{l\sqrt{2}} \quad \left(\because AC = \sqrt{2}\ BC\right)$$

which gives $\frac{T_C}{T} = \frac{3}{\sqrt{2}+1}$ $\left(\because T_B = T\sqrt{2}\right)$, which is choice (b).

68. The rate of loss of heat is given by

$$ms\ \frac{d\theta}{dt} = (4\ \pi r^2)\ \sigma T^4$$

$\therefore$ Initial rate of cooling is $\frac{d\theta}{dt} \propto \frac{r^2}{ms}$. Since $m = \frac{4}{3}\ \pi r^3\ \rho$, $r \propto m^{1/3}$. Hence

$$\frac{d\theta}{dt} \propto \frac{m^{2/3}}{m}$$

$$\propto \frac{1}{m^{1/3}}$$

Therefore $$\frac{\frac{d\theta}{dt}\text{ for } S_1}{\frac{d\theta}{dt}\text{ for } S_2} = \left(\frac{m \text{ of } S_2}{m \text{ of } S_1}\right)^{1/3} = \left(\frac{1}{3}\right)^{1/3},$$

which is choice (d).

69. Let A and l be the area of cross-section and the length of each rod. If k is the coefficient of thermal conductivity and t°C the temperature of the junction O, then the rates at which heat energy enters O from rods A and B are (Fig. 8.37)

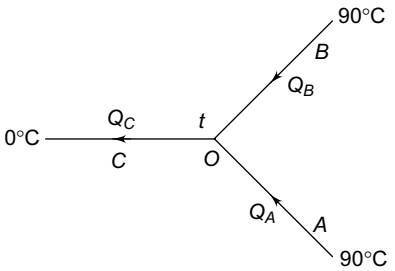


Fig. 8.37

$$Q_A = \frac{kA(90-t)}{l}$$

and $$Q_B = \frac{kA(90-t)}{l}$$

The rate at which heat energy flows in rod C is

$$Q_C = \frac{kA(t-0)}{l}$$

In the steady state, rate at which heat energy enters O = rate at which heat energy leaves O, i.e.

$$Q_A + Q_B = Q_C$$

or $$\frac{kA(90-t)}{l} + \frac{kA(90-t)}{l} = \frac{kA(t-0)}{l}$$

or $$(90-t) + (90-t) = t$$

or $3t = 180$ or $t = 60$°C. Hence the correct choice is (b).

70. From the first law of thermodynamics, we have

$$dU = dQ - dW$$

Given $dW = 0$ and $dQ < 0$. Hence the change in internal energy $dU < 0$. Now, for an ideal gas, the internal energy can decrease only by decrease in temperature. Hence the correct choice is (a).

71. For an adiabatic process, PV^{γ} = constant, Differentiating, we have

$$\gamma P V^{\gamma-1} + \frac{dP}{dV} V^{\gamma} = 0 \text{ or } \frac{dP}{dV} = -\frac{\gamma P}{V}$$

Thus the slope of P-V curve is proportional to γ. Now, for a diatomic gas, γ(= 7/5) is less than that for a monoatomic gas for which γ = 5/3. Therefore, the slope of the P-V curve is less for a diatomic gas than for a monoatomic gas. Hence curve 1 corresponds to diatomic gas and curve 2 to monoatomic gas. Thus, the correct choice is (b).

72. The length of each rod increases by the same amount if $l_1 \alpha_1 = l_2 \alpha_2$ or

$$\frac{l_2}{l_1} = \frac{\alpha_1}{\alpha_2}$$

or $$\frac{l_2}{l_1} + 1 = \frac{\alpha_1}{\alpha_2} + 1$$

or $$\frac{l_1 + l_2}{l_1} = \frac{\alpha_1 + \alpha_2}{\alpha_2}$$

or $$\frac{l_1}{l_1 + l_2} = \frac{\alpha_2}{\alpha_1 + \alpha_2},$$ which is choice (c).

73. Let m kg be the mass of ice melted into water. Heat lost by 5 kg of water = 5 kg × 1 kcal/kg/°C × 20°C = 100 kcal. Heat gained = m kg × 80 kcal/kg + 2 kg × 0.5 kcal/kg/°C × 20°C = 80 m kcal + 20 kcal. Now, heat gained = heat lost. Therefore,

$$80\,m + 20 = 100$$

or m = 1 kg. Therefore, final mass of water = 5 kg + 1 kg = 6 kg. Hence the correct choice is (b).

74. At t°C, the length of the rod becomes $L' = L(1 + \alpha t)$, where α is the coefficient of linear expansion. From the law of conservation of angular momentum, we have

$$I\omega = I'\,\omega'$$

or $$\frac{1}{3} ML^2 \omega = \frac{1}{3} ML'^2 \omega'$$

or $$\frac{\omega'}{\omega} = \left(\frac{L}{L'}\right)^2 = \frac{1}{(1+\alpha t)^2}$$

Now, for a given value of t, $(1 + \alpha t)^{-2}$ is constant, say k.

$\therefore$ $$\frac{\omega'}{\omega} = k$$

or $$\frac{\omega' - \omega}{\omega} = k - 1$$

or $$\omega' - \omega = (k-1)\,\omega$$

i.e. $(\omega' - \omega) \propto \omega$. Hence the correct choice is (b).

75. $\Delta U = C_v \Delta T$. Now $C_p - C_v = R$ or $\frac{C_p}{C_v} - 1 = \frac{R}{C_v}$

or $C_v = \frac{R}{\gamma - 1}$, where $\gamma = \frac{C_p}{C_v}$. Hence

$$\Delta U = \frac{R\Delta T}{(\gamma - 1)} = \frac{8.3 \times 8}{(1.4 - 1)} = 166 \text{ J}$$

Hence the correct choice (b).

76. $PV = \frac{m}{M} RT$. Therefore, the density of the gas is

$$\rho = \frac{m}{V} = \frac{PM}{RT} = \frac{PmN}{RT} = \frac{mP}{kT}$$

Hence the correct choice is (d).

77. $dQ = dU + dW$. In an adiabatic change $dQ = 0$. Hence, $dU = -dW = -4.5$ J. Thus, the correct choice is (b).

78. Given $k = 420$ W/m/K, $A = 10$ cm^2 $= 10 \times 10^{-4}$ m^2 $= 10^{-3}$ m^2, $\theta_1 = 100°$C, $t_2 = 0°$C, $t = 1$ minute $= 60$ s and $l = 1$ m. Using these values in

$$Q = \frac{kA(\theta_1 - \theta_2)t}{l}$$

we get $Q = 2520 \text{ J} = \frac{2520}{4.2} = 600$ cal. Therefore, mass of ice melted is

$$m = \frac{Q}{L} = \frac{600 \text{ cal}}{80 \text{ cal/g}} = 7.5 \text{ g}$$

Hence the correct choice is (c).

79. Increase in volume of mercury when the temperature increases by 100°C is

$v = \gamma_r V \Delta T = (18 \times 10^{-5}) \times (10^{-6}) \times 100 = 18 \times 10^{-9}$ m^3

Now $v = Ah$

or $h = \frac{v}{A}$

$$= \frac{(18 \times 10^{-9} \text{ m}^3)}{(0.002 \times 10^{-4} \text{ m}^2)} = 0.09 \text{ m} = 9 \text{ cm}$$

Hence the correct choice is (a).

80. Let L be the length (in cm) of the hollow cylinder and r its radius. Since the mass of the gas remains unchanged and the pressures of the gas in both sides are equal, we have, from Charles' law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad (1)$$

Given $V_1 = \left(\frac{L}{2} - 5\right) \pi r^2$, $V_2 = \left(\frac{L}{2} + 5\right) \pi r^2$, $T_1 = 0°$C $= 273$ K and $T_2 = 100°$C $= 373$ K. Using these values in (1), we get

$$\frac{\frac{L}{2} - 5}{273} = \frac{\frac{L}{2} + 5}{373}$$

which gives $L = 64.6$ cm. Hence the correct choice is (d).

81. The moment of inertia of a rod of mass M and length L is given by

$$I = k ML^2 \quad (1)$$

where $k = \frac{1}{12}$ if the axis of rotation is through the centre of rod and $k = \frac{1}{3}$ if the axis of rotation is through the end of the rod. Partially differentiating (1), we have

$$\Delta I = 2k ML \Delta L$$

Now $\Delta L = L \alpha \Delta t$. Therefore,

$$\Delta I = 2k ML \times L \alpha \Delta t$$

$$= 2(k ML^2) \alpha \Delta t = 2I \alpha \Delta t$$

or $\frac{\Delta I}{I} = 2 \alpha \Delta t$, which is choice (d).

82. Moment of inertia of the sphere $I = \frac{2}{5} Mr^2$. Given $\omega = n$ rotations per second $= 2\pi n$ rad s^{-1}. The kinetic energy is

$$\text{KE} = \frac{1}{2} I\omega^2 = \frac{1}{2} \times \frac{2}{5} Mr^2 \times (2\pi n)^2$$

$$= \frac{4}{5} M\pi^2 r^2 n^2$$

Since half of KE is converted into heat energy, we have

$$dQ = \frac{1}{2} \times \text{KE} = \frac{2}{5} M\pi^2 r^2 n^2$$

Now $dQ = MSdT$ which gives

$$dT = \frac{dQ}{MS} = \frac{\frac{2}{5} M\pi^2 r^2 n^2}{MS} = \frac{2\pi^2 r^2 n^2}{5S}$$

Hence the correct choice is (a).

83. Given $T_1 = 0°$C $= 273$ K, $T_2 = 400°$C $= 673$ K

Work done $W = \frac{nR}{(\gamma - 1)} (T_2 - T_1) = \frac{5 \times 8.3 \times 400}{\left(\frac{7}{5} - 1\right)}$

$$= 41500 \text{ J} = 41.5 \text{ kJ}$$

By convention, the work done on the gas is taken to be negative, i.e. $W = -41.5$ kJ. From the first laws of thermodynamics $dQ = dU + dW$. For an adiabatic process, $dQ = 0$. Hence $dU = -dW =$

– (– 41.5) = 41.5 kJ. The positive sign of dU implies that the internal energy increases. Hence the correct choice is (b).

84. For an adiabatic process $TP^n = k$ where $n = \dfrac{(1-\gamma)}{\gamma}$, $\gamma = \dfrac{C_p}{C_v}$ and k is a constant. Hence $P = \left(\dfrac{k}{T}\right)^{1/n}$. Since n = constant for a given gas, we have $P \propto T^{-1/n}$. Given $P \propto T^3$. Hence $-\dfrac{1}{n} = 3$

or $-\dfrac{\gamma}{1-\gamma} = 3$ which gives $\gamma = \dfrac{3}{2}$ which is choice (d).

85. When the rods are heated by Δt °C, the increase in length of the steel rod and the copper rod are

$$\Delta L_s = L_s\, \alpha_s\, \Delta t$$

$$\Delta L_c = L_c\, \alpha_c\, \Delta t$$

The difference between their lengths will remain constant at all temperatures if $\Delta L_s = \Delta L_c$, i.e. if

$$L_s\, \alpha_s\, \Delta t = L_c\, \alpha_c\, \Delta t$$

or $$\frac{L_s}{L_c} = \frac{\alpha_c}{\alpha_s}$$

Hence the correct choice is (d).

86. The moment of inertia of the disc about the given axis of rotation is

$$I = \frac{1}{2}\, MR^2 \qquad (1)$$

where M is the mass of the disc and R its radius.

If the disc is heated, it expands. Hence R increases. The resulting increase in I is obtained by partially differentiating (1).

$$\Delta I = \frac{1}{2} \times M \times 2\, R \Delta R \ (\because M = \text{constant})$$

or $$\Delta I = MR\Delta R$$

But $\Delta R = R\alpha\Delta t$. Therefore,

$$\Delta I = MR^2\alpha\Delta t \qquad (2)$$

Now, the angular momentum of the disc is given by

$$J = I\omega \qquad (3)$$

Since no external torque acts, J remains constant. Partially differentiating (3), we have

$$I\Delta\omega + \omega\Delta I = 0$$

or $$\Delta\omega = -\frac{\omega\Delta I}{I} \qquad (4)$$

Using (1) and (2) in (4), we get

$$\Delta\omega = -2\ \alpha\omega\Delta t$$

The negative sign indicates that the angular frequency decreases due to increase in temperature. Hence the correct choice is (d).

87. Let the initial temperature be T and let M be the mass of the cube. Let A_0, ρ_0 and d_0 respectively be the base area of the cube, the density of the material of the cube and the depth up to which it is submerged in the liquid, the upthrust = $A_0\, d_0\, \rho_0\, g$. From the principle of floatation, weight = upthrust, i.e.

$$Mg = A_0 d_0 \rho_0 g$$

or $$M = A_0 d_0 \rho_0 \qquad (1)$$

When the temperature is raised to $(T + \Delta T)$, let A, ρ and d be the base area, density and depth at this temperature. Now, the coefficient of superficial (area) expansion is $\beta = 2\alpha$.

Hence

$$A = A_0(1 + \beta\, \Delta T)$$
$$= A_0\, (1 + 2\ \alpha\Delta T) \qquad (2)$$

Also $$d = \frac{d_0}{(1+\gamma\ \Delta T)} \qquad (3)$$

The upthrust at temperature $(T + \Delta T) = Ad\rho g$. From the principle of floatation we have

$$Mg = Ad\rho g$$

or $$M = A\, d\rho \qquad (4)$$

From (1) and (4), we get

$$A\, d\rho = A_0 d_0 \rho_0$$

or $$A\rho = A_0\rho_0 \qquad (\because d = d_0; \text{ given})$$

Using (2) and (3), we have

$$A_0\, (1 + 2\ a\Delta T) \times \frac{\rho_0}{(1+\gamma\ \Delta T)} = A_0\rho_0$$

which gives $2\alpha = \gamma$. Thus the correct choice is (b).

88. For an isothermal process: PV = constant and for an adiabatic process: PV^γ = constant, where γ is the ratio of the two specific heats (C_p/Cv) of the gas. When a gas is compressed from a volume V to a volume $(V - \Delta V)$, the increase in pressure is

$$(\Delta P)_{\text{adia}} = \frac{\gamma\, \Delta V P}{V} \text{ for an adiabatic compression}$$

and $$(\Delta P)_{\text{iso}} = \frac{\Delta V P}{V} \text{ for an isothermal compression.}$$

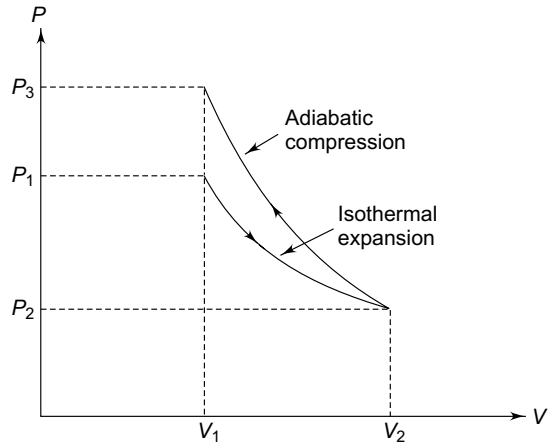


Fig. 8.38

Hence P_3 will be greater than P_1. Therefore, the P–V diagrams of isothermal expansion from V_1, P_1 to V_2, P_2 and adiabatic compression for V_2, P_2 to V_1, P_3 are as shown in Fig. 8.38. Let W_1 and W_2 be the work done in isothermal expansion and adiabatic compression respectively. Therefore, net work done is

$$W = W_1 + (-W_2) = W_1 - W_2$$

Now, the area under the adiabatic curve is more than that under the isothermal curve. Hence $W_2 > W_1$. Therefore, $W < 0$. Hence the correct choice is (c).

89. Heat produced is given by

$$dQ = \frac{V^2 t}{R} = \frac{(10)^2 \times (10 \times 60)}{50} = 1200 \text{ J}$$

Since the container is rigid, the change in volume $dV = 0$. Hence work done $dQ = PdV = 0$. From the first law of thermodynamics, the change in internal energy is $dU = dQ - dW = dQ = 1200$ J. Hence the correct choice is (d).

90. The two adiabatic paths *ad* and *bc* for the gas intersect the two isothermals *ab* and *cd* at temperatures T_1 and T_2 (see Fig. 8.26 on page 8.23). Since points *a* and *d* lie on the same adiabatic path, we have

$$T_1 V_a^{(\gamma-1)} = T_2 V_d^{(\gamma-1)}$$

$$\text{or } \left(\frac{V_a}{V_d}\right)^{(\gamma-1)} = \frac{T_2}{T_1} \quad (1)$$

Since points *b* and *c* also lie on the same adiabatic path,

$$T_1 (V_b)^{(\gamma-1)} = T_2 V_c^{(\gamma-1)}$$

$$\text{or } \left(\frac{V_b}{V_c}\right)^{(\gamma-1)} = \frac{T_2}{T_1} \quad (2)$$

From (1) and (2), we get

$$\left(\frac{V_a}{V_d}\right)^{(\gamma-1)} = \left(\frac{V_b}{V_c}\right)^{(\gamma-1)}$$

$$\text{or } \frac{V_a}{V_d} = \frac{V_b}{V_c}$$

Thus the correct choice is (c).

91. Since the temperature T remains constant along the path CA, P will be inversely proportional to V along this path. Hence, as P increases, V must decrease in a nonlinear fashion. This is represented by the curve CA in Fig. 8.39.

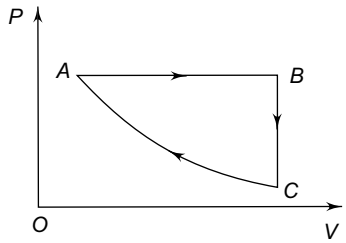


Fig. 8.39

Along the path BC, the volume V is constant. Hence the graph of P against V is a straight line perpendicular to the V-axis. On a P–V diagram, the corresponding path is BC as shown in Fig. 8.39.

For the path AB, V is directly proportional to T pressure remaining constant. The corresponding path AB is, therefore a straight line parallel to the V-axis. Thus the cyclic process on a P–V diagram is represented by choice (d).

92. Work done on the gas is

$$\Delta W = P\Delta V = P(V_f - V_i)$$
$$= 1 \times 10^5 \times (25 - 5) \times 10^{-3}$$
$$= 2000 \text{ J}$$

The internal energy is given by $U = \frac{PV}{(\gamma - 1)}$

$$\therefore \quad U_i = \frac{PV_i}{(\gamma - 1)}, \; U_f = \frac{PV_f}{(\gamma - 1)}.$$

Therefore, change in internal energy is

$$\Delta U = U_f - U_i = \frac{P}{(\gamma - 1)} (V_f - V_i)$$
$$= \frac{1 \times 10^5 \times (25 - 5) \times 10^{-3}}{(1.4 - 1)} = 5000 \text{ J}$$

From the first law of thermodynamics, the heat energy supplied to the gas is

$$\Delta Q = \Delta W + \Delta U = 2000 + 5000$$
$$= 7000 \text{ J}$$

which is choice (d).

93. In process AB, the volume V increases linearly with temperature T. Hence process AB is isobaric (constant pressure). Therefore, work done in this process is

$$W_{AB} = P\Delta V = nR\Delta T \qquad (\because PV = nRT)$$
$$= nR(T_B - T_A)$$
$$= 3 \times 8.3 \times (200 - 100) = 2490 \text{ J}$$

Process CA is isochoric (constant volume). Hence work done in this process $W_{CA} = 0$. Since the whole process $ABCA$ is cyclic, the change in internal energy in the complete cycle is zero, i.e. $\Delta U = 0$. Now, from the first law of thermodynamics, (Given $Q = -2510$ J)

$$Q = \Delta U + W = \Delta U + W_{AB} + W_{BC} + W_{CA}$$

or $\quad -2510 = 0 + 2490 + W_{BC} + 0$

or $\quad W_{BC} = -2510 - 2490 = -5000$ J

The negative sign shows that the work is done by the gas. Thus, the correct choice is (b).

94. At atmospheric pressure, the boiling point of liquid oxygen is greater than 50 K. Therefore, between 50 K and 300 K, liquid oxygen undergoes a change of state. Hence the correct choice is (c).

95. The heat energy supplied is

$$\Delta Q = I^2 R\, t$$
$$= (2)^2 \times 100 \times (10 \times 60)$$
$$= 240 \times 10^3 \text{ J} = 240 \text{ kJ}$$

Since $\Delta V = 0$; work done $\Delta W = 0$. From first law of thermodynamics, $\Delta U = \Delta Q = 240$ kJ. Thus, the correct choice is (d).

96. The equation of straight line AB is

$$P = mV + c \qquad (1)$$

where m is the slope and c is the intercept. For points A and B, we have

$$2P_0 = mV_0 + c$$

and $\quad P_0 = m(2V_0) + c$

These equations give $m = -\dfrac{P_0}{V_0}$ and $c = 3P_0$.

Now $PV = nRT \Rightarrow P = \dfrac{nRT}{V}$. Using this in Eq. (1), we have

$$T = \frac{1}{nR}(mV^2 + cV) \qquad (2)$$

T will be maximum if $\dfrac{dT}{dV} = 0$ and $\dfrac{d^2T}{dV^2} < 0$.

Differentiating Eq. (2) with respect to V and putting $\dfrac{dT}{dV} = 0$, we get

$$2mV + c = 0$$

which gives $\quad V = -\dfrac{c}{2m}$, Using this in Eq (2),

$$T_{\max} = -\frac{c^2}{4nRm} = -\frac{1}{4nR} \times \frac{(3P_0)^2}{-P_0/V_0} = \frac{9P_0V_0}{4nR}$$

Thus, the correct choice is (a).

97. Mass of 2 litres of water = 2 kg. Heat energy needed to raise the temperature of 2 kg of water from 20°C to 75°C is

$$Q = 2 \times (4.2 \times 10^3) \times 55 = 4.62 \times 10^5 \text{ J}$$

If t is the time taken, heat energy supplied by the heater in time t is

$$Q_1 = (\text{power} \times \text{time}) = 1000\, t \text{ joule}$$

Heat energy lost in time t is

$$Q_2 = 160\, t \text{ joule}$$

Heat energy available for heating water is

$$Q = Q_1 - Q_2 = 840\, t$$

Equating $Q = Q'$, we get $t \simeq 550$ s. Thus, the correct choice is (b).

98. We have

$$\log_e\left(\frac{T_1 - T_0}{T_2 - T_0}\right) = Kt$$

or $\quad \log_e\left(\dfrac{75-25}{65-25}\right) = K \times 5$ or $\log_e\left(\dfrac{50}{40}\right) = 5K$

or $\quad \log_e\left(\dfrac{5}{4}\right) = 5K \qquad (1)$

If the temperature at the end of next 5 minutes is T', we have

$$\log_e\left(\frac{65-25}{T'-25}\right) = 5K$$

or $\quad \log_e\left(\dfrac{40}{T'-25}\right) = 5K \qquad (2)$

From Eqs (1) and (2), we get

$$\log_e\left(\frac{40}{T'-25}\right) = \log_e\left(\frac{5}{4}\right)$$

or $\quad \left(\dfrac{40}{T'-25}\right) = \dfrac{5}{4}$

which gives $T' = 57$°C.

99. Given

$$\log_e\left(\frac{80-20}{50-20}\right) = 6K \text{ or } \log_e(2) = 6K \qquad (1)$$

If the body takes t minutes to cool from 60°C to 30°C, then

$$\log_e\left(\frac{60-20}{30-20}\right) = tK \text{ or } \log_e(4) = tK \qquad (2)$$

Dividing (2) by (1), we have

$$\frac{t}{6} = \frac{\log_e(4)}{\log_e(2)} = \frac{2\log_e(2)}{\log_e(2)} = 2$$

or $t = 12$ minutes.

Hence the correct choice is (d).

100. If T_0 is the temperature of the surroundings, then we have

$$\log_e\left(\frac{80-T_0}{64-T_0}\right) = 5K \qquad (1)$$

and $$\log_e\left(\frac{64-T_0}{52-T_0}\right) = 5K \qquad (2)$$

Equating Eqs (1) and (2), we get

$$\frac{80-T_0}{64-T_0} = \frac{64-T_0}{52-T_0}$$

which gives $T_0 = 16$°C, which is choice (b).

101. If T is the temperature after 15 minutes, then

$$\log_e\left(\frac{52-T_0}{T-T_0}\right) = 5K$$

or $$\log_e\left(\frac{52-16}{T-16}\right) = 5K \qquad (3)$$

From Eqs (1) and (3), we get

$$\frac{80-16}{64-16} = \left(\frac{52-16}{T-16}\right)$$

which gives $T = 43$°C, which is choice (b).

102. Since the material is the same, the density is the same. Since the mass is the same and density is the same, all three have the same volume. For the same volume, the surface area of the plate is the largest and of the sphere the smallest. The rate of loss of heat by radiation is proportional to the surface area. Hence the correct choice is (c).

103. If a steady temperature difference $(\theta_1 - \theta_2)$ is maintained between the ends of a conducting rod of length L and cross-sectional area A, the rate of flow of heat through the rod is given by

$$q = \frac{kA(\theta_1-\theta_2)}{L}$$

where k is the coefficient of thermal conductivity of the material of the rod. In the first experiment, the two rods are connected in series. If two rods of equal cross-sectional areas and of lengths L_1 and L_2 and conductivities k_1 and k_2 are joined in series, the equivalent conductivity k_s is given by

$$\frac{L_1+L_2}{k_s} = \frac{L_1}{k_1} + \frac{L_2}{k_2} \qquad (1)$$

For two identical rods, $L_1 = L_2 = L$ and $k_1 = k_2 = k$, in which case, Eq. (1) gives $k_s = k$. Further, when two identical rods are joined in series, the length of the composite rod is $(2L)$ but its cross-sectional area is A, the same as that of each rod. Hence the rate of flow of heat in this case is given by

$$q_1 = \frac{kA(\theta_1-\theta_2)}{(2L)} \qquad (2)$$

In the second case, the two rods are connected in parallel. If two rods of equal lengths and equal cross-section areas and having conductivities k_1 and k_2 are joined in parallel, the equivalent conductivity of the composite rod is given by

$$k_p = k_1 + k_2$$

For two identical rods, $k_1 = k_2 = k$. Hence $k_p = (2k)$. Furthermore, the cross-sectional area of the composite rod is $(2A)$. Therefore, in the second case, the rate of flow of heat is given by

$$q_2 = \frac{(2k)(2A)(\theta_1-\theta_2)}{L} \qquad (3)$$

Dividing (2) by (3), we get $\frac{q_1}{q_2} = \frac{1}{8}$. Now, the rate of melting of ice is proportional to the rate of flow of heat. Hence the correct choice is (d).

104. According to Stefan's law, the power radiated by a black body at absolute temperature T is given by

$$Q = \sigma A T^4 \qquad (1)$$

where A is the surface area of the body and σ is Stefan's constant. According to Wien's displacement law,

$$\lambda_m T = b$$

where λ_m is the wavelength corresponding to maximum emission of radiation and b is Wien's constant. Thus

$$T = \frac{b}{\lambda_m} \qquad (2)$$

Using (2) in (1), we get

$$Q = \sigma A \left(\frac{b}{\lambda_m}\right)^4 = \frac{\sigma b^4 A}{\lambda_m^4}$$

For a sphere of radius r, $A = 4\pi r^2$. Hence

$$Q = \frac{\sigma b^4 4\pi r^2}{\lambda_m^4} = k\frac{r^2}{\lambda_m^4} \qquad (3)$$

where $k = 4\,\pi\sigma b^4$ is a constant. Hence

$$Q_1 = k\,\frac{r_1^2}{\left(\lambda_m^4\right)_1}$$

and $$Q_2 = k\,\frac{r_2^2}{\left(\lambda_m^4\right)_2}$$

$$\therefore \quad \frac{Q_1}{Q_2} = \left(\frac{r_1}{r_2}\right)^2 \frac{\left(\lambda_m^4\right)_2}{\left(\lambda_m^4\right)_1}$$

$$= \left(\frac{3\,\text{cm}}{5\,\text{cm}}\right)^2 \times \left(\frac{500\,\text{nm}}{300\,\text{nm}}\right)^4 = \left(\frac{5}{3}\right)^2$$

Hence the correct choice is (c).

105. The correct choice is (d). Heat transfer in boiling of water and in land and sea-breezes is primarily due to convection. A metal rod placed over a flame is heated primarily by conduction. Heat transfer by convection and conduction takes place through matter (or medium). The electric bulb is evacuated. Hence heat transfer in an electric bulb is not due to conduction or convection.

106. When the temperature of the air is less than 0°C, the cold air near the surface of the pond takes heat (latent) from the water which freezes in the form of layers. Consequently, the thickness of the ice layer keeps increasing with time. Let x be the thickness of the ice layer at a certain time. If the thickness increases by dx in time dt, then the amount of heat flowing through the slab in time dt is given by (see Fig 8.40)

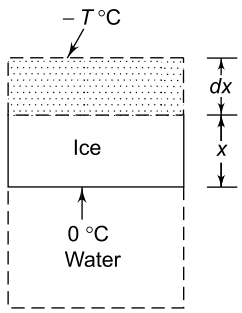


Fig. 8.40

$$Q = \frac{kA[0-(-T)]dt}{x} = \frac{kATdt}{x} \qquad (1)$$

where A is the area of the layer of ice and $-T$ °C is the temperature of the surrounding air. If dm is the mass of water frozen into ice, then $Q = dm \times L$. But $dm = A\rho dx$, where ρ is the density of ice. Hence

$$Q = A\rho L dx \qquad (2)$$

Equating (1) and (2), we have

$$\frac{kATdt}{x} = A\rho L dx \text{ or } dt = \frac{\rho L}{kT} \cdot x dx$$

Integrating, we have

$$\int_0^t dt = \frac{\rho L}{kT}\int_{x_1}^{x_2} x dx$$

or $$t = \frac{\rho L}{kT}\left|\frac{x^2}{2}\right|_{x_1}^{x_2} = \frac{\rho L}{2kT}\,(x_2^2 - x^2{}_1),$$

which is choice (c).

107. Let T_B be the temperature at B. The rate of flow of heat from C towards B is

$$\frac{Q_1}{t} = \frac{k\,A\,(T_2 - T_B)}{L/2}$$

The rate of flow of heat from D towards B is

$$\frac{Q_2}{t} = \frac{k\,A\,(T_3 - T_B)}{L/2}$$

The rate of flow of heat from B towards A is

$$\frac{Q_3}{t} = \frac{k\,A\,(T_B - T_1)}{L}$$

In the steady state, the rate at which heat enters B = rate at which heat leaves B, i.e.

$$\frac{Q_1}{t} + \frac{Q_2}{t} = \frac{Q_3}{t}$$

or $$\frac{2k\,A\,(T_2 - T_B)}{L} + \frac{2k\,A\,(T_3 - T_B)}{L} = \frac{k\,A\,(T_B - T_1)}{L}$$

or $$2(T_2 - T_B) + 2(T_3 - T_B) = T_B - T_1$$

which gives $$T_B = \frac{T_1 + 2T_2 + 2T_3}{5}$$

$$= \frac{20 + 2\times 30 + 2\times 40}{5}$$

$$= 32\ °\text{C},$$ which is choice (a).

108. $\left(\frac{dQ}{dt}\right)_A = \sigma(4\pi r_1^2)\in_1 T_1^4$

and $\left(\frac{dQ}{dt}\right)_B = \sigma(4\pi r_2^2)\in_2 T_2^4$

Since the spheres have the same surface finish, they have the same emissivity, *i.e.*, $\in_1 = \in_2$. Therefore, the ratio of the rate of heat loss by A and B is

$$\left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{r_1}{2r_1}\right)^2 \times \left(\frac{2T_2}{T_2}\right)^4 = 4,$$

which is choice (d).

109. Refer to Example 12 on page 8.6. The correct choice is (c).

110. Surface area of one face of the thin plate $= L \times L = L^2$. Total surface area is $A = 2L^2$.

$$\therefore \quad \frac{\Delta A}{A} = 0 + \frac{2\Delta L}{L} = \frac{2\Delta L}{L} = 2 \times 0.1\% = 0.2\%$$

So the correct choice is (b).

111. $T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow L = \frac{T^2 g}{4\pi^2}$. Therefore,

$$\frac{\Delta L}{L} = \frac{2\Delta T}{T}$$

or $\quad \alpha\Delta\theta = 2 \times \left(\frac{T_{20} - T_0}{T_0}\right)$

or $\quad \frac{T_{20} - T_0}{T_0} = \frac{1}{2}\alpha\Delta\theta = \frac{1}{2} \times \alpha \times 20°\text{C} = 10°\text{C} \times \alpha$

Here T_{20} is the correct time period. Since the length L of the rod decreases when cooled, the time period decreases. So the clock runs fast at 0°C. The time gained in a day (= 24 hrs = 24 × 60 × 60s) is

$10\alpha \times 24$ hrs $= 10\alpha \times (24 \times 60 \times 60\text{s})$. Thus

$$12\text{s} = 10°\text{C}\ \alpha \times 24 \times 60 \times 60\text{s}$$

$$\Rightarrow \quad \alpha = \frac{12\text{s}}{10°\text{C} \times (24 \times 60 \times 60\text{s})}$$

$$= 1.4 \times 10^{-5}\ \text{per°C}$$

So the correct choice is (a).

112. Let m be the mass of the ball. The energy absorbed by it in this process is

$$Q = 90\%\ \text{of}\ mg(h_1 - h_2)$$

$$= 0.9 \times m \times 10\ (12 - 8) = 36\ m\ \text{joule}$$

$$\therefore \quad ms\Delta\theta = 36\ m$$

$$\Rightarrow \quad \Delta\theta = \frac{36}{s} = \frac{36}{360} = 0.1°\text{C}$$

So the correct choice is (a).

113. Since $T_B > T_A$, heat will flow from B to A through both paths BA and BCA. In the steady state, rate of flow of heat in BC = rate of flow of heat in CA

$$\therefore \quad \frac{kA(T_B - T_C)}{L} = \frac{kA(T_C - T_A)}{\sqrt{2}\,L} \quad (\because \quad AC = \sqrt{2}\,L)$$

$$\Rightarrow \quad 2\sqrt{2}\,T - T_C = \frac{T_C - T}{\sqrt{2}}$$

$$\Rightarrow \quad T_C = \frac{5T}{1 + \sqrt{2}}, \text{ which is choice (d).}$$



Note Since the rods are insulated, the loss of heat due to radiation has been neglected. Heat energy is transmitted only through conduction.

114. In a adiabatic process PV^γ = constant (k).

Taking logarithm,

$$\log P + \gamma \log V = \log k$$

Differentiating,

$$\frac{\Delta P}{P} + \gamma\frac{\Delta V}{V} = 0$$

$$\Rightarrow \quad \frac{\Delta V}{V} = -\frac{1}{\gamma}\frac{\Delta P}{P}$$

For a monoatomic gas, $\gamma = \frac{5}{3}$. Therefore

$$\frac{\Delta V}{V} = -\frac{3}{5} \times 0.5\% = -0.3\%$$

The negative sign shows that the volume decreases.

So the correct choice is (c).

115. The internal energy of n moles of a gas at absolute temperature T is given by

$$U = \frac{f}{2}nRT$$

where f is the number of degrees of freedom. Given $U_1 = U_2$, *i.e.*,

$$\frac{f_1}{2}n_1RT_1 = \frac{f_2}{2}n_2RT_2$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{f_2 T_2}{f_1 T_1}$$

For helium, $f_1 = 3$ and for oxygen, $f_2 = 5$.

$$\therefore \quad \frac{n_1}{n_2} = \frac{5 \times 3T}{3 \times T} = 5, \text{ which is choice (c).}$$

116. Let T' be the temperature of the mixture. Then

$$U = U_1 + U_2$$

$$\frac{f}{2}(n_1 + n_2)\,RT' = \frac{f}{2}n_1\,RT + \frac{f}{2}n_2R\,(2T)$$

$$\Rightarrow \quad (n_1 + n_2)\,T' = (n_1 + 2n_2)\,T$$

$$\Rightarrow \quad (1 + 2)\,T' = (1 + 2 \times 2)\,T$$

$$\Rightarrow \quad T' = \frac{5T}{3}, \text{ which is choice (a).}$$

117. For the mixture of gases

$$C_p = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2}$$

and $$C_v = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$$

For helium : $n_1 = \frac{8}{4} = 2,\ C_{v_1} = \frac{3R}{2}, C_{p_1} = \frac{5R}{2}$

For helium : $n_2 = \frac{32}{32} = 1,\ C_{v_2} = \frac{5R}{2}, C_{p_2} = \frac{7R}{2}$

$\therefore$ For the mixture,

$$C_p = \frac{2 \times \frac{5R}{2} + 1 \times \frac{7R}{2}}{2+1} = \frac{8.5R}{3}$$

$$C_v = \frac{2 \times \frac{3R}{2} + 1 \times \frac{5R}{2}}{2+1} = \frac{5.5R}{3}$$

$$\therefore \quad \gamma = \frac{C_p}{C_v} = \frac{8.5}{5.5} = \frac{17}{11}$$

So the correct choice is (d).

118. From the first law, $dQ = dU + PdV$. For an adiabatic process $dQ = 0$. Therefore,

$$dU + PdV = 0$$

$$\Rightarrow \quad d\,(a + bPV) + PdV = 0$$

$$\Rightarrow 0 + bPdV + bVdP + PdV = 0$$

$$\Rightarrow \quad (b + 1)\,PdV + bVdP = 0$$

Dividing by PV,

$$(b+1)\frac{dV}{V} + \frac{bdP}{P} = 0$$

Integrating,

$$(b + 1) \log V + b \log P = \text{constant}$$

$$\Rightarrow \quad V^{b+1}\,P^b = \text{constant}$$

$$\Rightarrow \quad PV^{\frac{b+1}{b}} = \text{constant}$$

For an adiabatic process PV^γ = constant. Hence

$$\gamma = \frac{b+1}{b} = \frac{2.5+1}{2.5} = \frac{7}{5}$$

So the correct choice is (a).

119. $W_{A \to B} = P\,(V - 2V) = -PV$

$W_{B \to C} = 0$

$W_{C \to D} = 2P\,(3V - V) = 4PV$

$W_{D \to A}$ = area under AD up to volume axis

= area of trapezium $ADEF$

$$= EF \times \left(\frac{AF + DE}{2}\right)$$

$$= V \times \left(\frac{P + 2P}{2}\right) = \frac{3}{2}PV$$

Since work is done on the gas to decrease volume, $W_{D \to A}$ is negative,

$\therefore$ Total work done by the gas

$$= -PV + 0 + 4PV - \frac{3}{2}PV$$

$$= \frac{3}{2}PV$$

So the correct choice is (b)

Alternative Method

Work done = area of $ABCDA = \frac{3}{2}PV$

120. $\rho = \frac{M}{V}$ and $PV = nRT$. Therefore,

$$\rho = \frac{MP}{nRT}$$

$$\rho_A = \frac{MP_A}{nRT_A} = \frac{M \times (2P)}{nR\,(3T)} = \frac{2MP}{3nRT} \qquad (1)$$

$$\rho_B = \frac{MP_B}{nRT_B} = \frac{M \times P}{nR \times T} = \frac{MP}{nRT} \qquad (2)$$

From (1) and (2), we get $\rho_B = \frac{2}{3}\rho_A$

So the correct choice is (a)

121. The equation of the straight line AB is

$$P = mV + c \qquad (1)$$

where m is the slope and c is the intercept.

$$P_A = mV_A + c$$

$$\Rightarrow \quad 3P_0 = mV_0 + c \qquad (2)$$

$$P_B = mV_B + c$$

$$\Rightarrow \quad P_0 = 2mV_0 + c \qquad (3)$$

Solving (2) and (3) for m and c we get

$$m = -\frac{2P_0}{V_0} \qquad (4)$$

and $c = 5P_0$ (5)

Equation of state for ideal gas is

$$PV = nRT$$

$$\Rightarrow \quad T = \frac{PV}{nR}$$

Using (1) we have

$$T = \frac{(mV + c)V}{nR} = \frac{1}{nR}(mV^2 + cV) \qquad (6)$$

T will be maximum if $\frac{dT}{dV} = 0$ and $\frac{d^2T}{dV^2} < 0$.

Now $\frac{dT}{dV} = \frac{1}{nR}(2mV + c)$

Setting $\frac{dT}{dV} = 0$, we get $V = -\frac{c}{2m}$.

Also $\frac{d^2T}{dV^2} = \frac{2m}{nR}$ which is negative since m is negative.

Hence $T = T_{\max}$ if $V = -\frac{c}{2m}$. Putting this in (6) we get

$$T_{\max} = \frac{V}{nR}\left(-\frac{c}{2} + c\right) = \frac{Vc}{2nR}$$

$$= \left(-\frac{c}{2m}\right) \times \frac{c}{2nR} \qquad (7)$$

Using (4) and (5) in (7) we get $T_{\max} = \frac{25}{8}\frac{P_0V_0}{nR}$.

So the correct choice is (d)

122. Let C be the molar specific heat capacity of the gas. Then

$$\Delta Q = nC\Delta T$$

$$\Rightarrow \quad C = \frac{1}{n}\frac{\Delta Q}{\Delta T} = \frac{1}{n}\left(\frac{\Delta U + W}{\Delta T}\right)$$

$$= \frac{\Delta U}{n\Delta T} + \frac{W}{n\Delta T}$$

$$\Rightarrow \quad C = C_v + \frac{W}{n\Delta T} \qquad (1)$$

Work done W = area of $ABCD$.

$$= (4V - V) \times \frac{1}{2} \times (P + 4P)$$

$$= \frac{15}{2}PV \qquad (2)$$

Now $T = \frac{PV}{nR}$

$$\therefore \quad \Delta T = T_B - T_A$$

$$= \frac{P_BV_B}{nR} - \frac{P_AV_A}{nR}$$

$$= \frac{1}{nR}(4P \times 4V - P \times V)$$

$$\Rightarrow \quad \Delta T = \frac{15PV}{nR} \qquad (3)$$

For a diatomic gas, $C_v = \frac{5R}{2}$. Using $C_v = \frac{5R}{2}$ and (2) and (3) in (1) we get

$$C = \frac{5R}{2}\frac{\frac{15}{2}PV}{\frac{15\,PV}{nr}} = \frac{5R}{2} + \frac{nR}{2}$$

$$= \frac{R}{2}(5 + n)$$

For $n = 2$, $C = \frac{7R}{2}$. So the correct choice is (d).

123. For an adiabatic process,

$$P^{1-\gamma}T\gamma = \text{constant}$$

or $\frac{T^\gamma}{P^{1-\gamma}} = \text{constant}$

or $p^{1-\gamma} = \frac{T^\gamma}{\text{constant}}$

or $P \propto T^{\frac{\gamma}{\gamma-1}}$

Given $P \propto T^{5/2}$

$$\therefore \quad \frac{\gamma}{\gamma - 1} = \frac{5}{2} \quad \Rightarrow \quad \gamma = \frac{5}{3}$$

So the correct choice is (b).

124. $W = \int_{V_1}^{V_2} PdV = a\int_{V_1}^{V_2} V^b dV \qquad (\because \quad P = aV^b)$

or $W = \frac{a}{(b+1)}(v_2^{b+1} - V_1^{b+1}) \qquad (1)$

$PV = nRT$ gives

$aV_2^b = nRT$ and $aV_2^b = nR(2T)$. Using these in (1), we get

$W = \frac{nRT}{(b+1)}$, which is choice (d).

SECTION 2

Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage.

Passage I

20,000 J of heat energy is supplied to a metal block of mass 500 g at atmospheric pressure. The initial temperature of the block is 30°C. Given specific heat of metal = 400 J kg^{-1} $°C^{-1}$, relative density of metal = 8.0, coefficient of volume expansion of metal = 8×10^{-5} $°C^{-1}$ and atmospheric pressure = 10^5 Pa.

1. The final temperature of the block is
   (a) 120°C (b) 130°C
   (c) 140°C (d) 150°C
2. Work done by the block on the surroundings is
   (a) 0.05 J (b) 0.1 J
   (c) 1.0 J (d) 10 J
3. The change in internal energy is
   (a) zero
   (b) equal to 20,000 J
   (c) slightly greater than zero
   (d) slightly less than 20,000 J.

Solutions

1. $\Delta Q = ms\Delta T$. Therefore,

$$\Delta T = \frac{\Delta Q}{ms} = \frac{20{,}000\,\text{J}}{(0.5\,\text{kg}) \times (400\,\text{J kg}^{-1}\,°\text{C}^{-1})} = 100°\text{C}$$

∴ Final temperature = 100 + 30 = 130°C, which is choice (b).

2. Density of metal (ρ) = 8000 kg m^{-3}. Volume of the block is

$$V = \frac{m}{\rho} = \frac{0.5\,\text{kg}}{8000\,\text{kg m}^{-3}} = \frac{1}{16} \times 10^{-3}\ \text{m}^3$$

$$\therefore \text{Increase in volume} = \Delta V = \gamma V \Delta T$$

$$= (8 \times 10^{-5}) \times \left(\frac{1}{16} \times 10^{-3}\right) \times 100 = \frac{1}{2} \times 10^{-6}\ \text{m}^3$$

Work done $\Delta W = P\Delta V = (10^5) \times \left(\frac{1}{2} \times 10^{-6}\right) = 0.05$ J, which is choice (a).

3. Change in internal energy $\Delta U = \Delta Q - \Delta W = 20{,}000 - 0.05 = 19999.95$ J

Thus the correct choice is (d).

Questions 4 to 6 are based on the following passage.

Passage II

A rod of metal *X* of length 50.0 cm elongates by 0.10 cm when it is heated from 0°C to 100°C. Another rod of metal *Y* of length 80.0 cm elongates by 0.08 cm for the same rise in temperature. A third rod of length 50 cm, made by welding pieces of rods *X* and *Y* placed end to end, elongates by 0.03 cm when its temperature is raised from 0°C to 50°C.

4. The coefficients of linear expansion of metal *X* and of metal *Y* are in the ratio of
   (a) 2 : 1 (b) 3 : 2
   (c) 3 : 1 (d) 4 : 3
5. The length of the rod of metal *X* in the composite piece is
   (a) 10 cm (b) 20 cm
   (c) 30 cm (d) 40 cm
6. The length of the rod of metal *Y* in the composite piece is
   (a) 10 cm (b) 20 cm
   (c) 30 cm (d) 40 cm

Solutions

4. For rod *X*, $\alpha_x = \frac{\Delta L}{L\Delta t} = \frac{0.10}{50.0 \times 100} = 2.0 \times 10^{-6}$ per °C.

For rod *Y*, we get $\alpha_y = 1.0 \times 10^{-6}$ per °C. So the correct choice is (a).

5. In the composite rod, $L_x + L_y = 50.0$ cm. When this rod is heated by 50°C, let the new lengths be L'_x and L'_y. Given $L'_x + L'_y = 50.0 + 0.03 = 50.03$ cm. Here $L'_x = L_x\,(1 + \alpha_x \times 50)$ and $L'_y = L_y(1 + \alpha_y \times 50)$. Therefore,

$$50.03 = L_x + L_y + (L_x\,\alpha_x + L_y\,\alpha_y)\,50$$
$$= 50 + (L_x\,\alpha_x + L_y\,\alpha_y)\,50$$

Substituting the values of α_x and α_y and noting that $L_x + L_y = 50$ cm, we get $L_x = 10$ cm, which is choice (a).

6. $L_y = 50 - L_x = 50 - 10 = 40$ cm, which is choice (d).

Questions 7 to 10 are based on the following passage

Passage III

Three moles of an ideal gas at pressure P_A and temperature T_A are isothermally expanded to twice the original volume. The gas is then compressed at constant pressure to its original volume. Finally the gas is heated at constant volume to its original pressure P_A.

7. Which of the graphs shown in Fig. 8.41 represents the P-V diagram for the complete process?

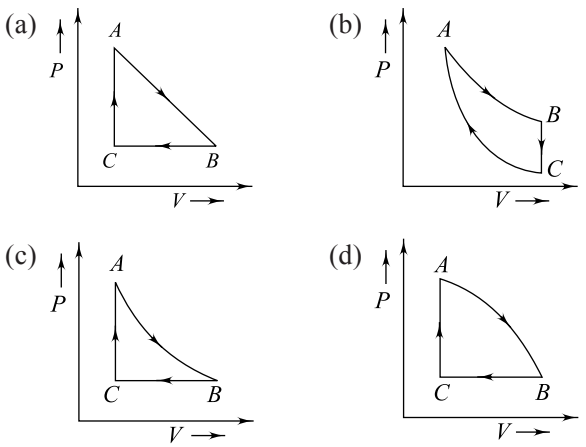


Fig. 8.41

8. Which of the graphs shown in Fig. 8.42 represents the P-T diagram for the complete process?

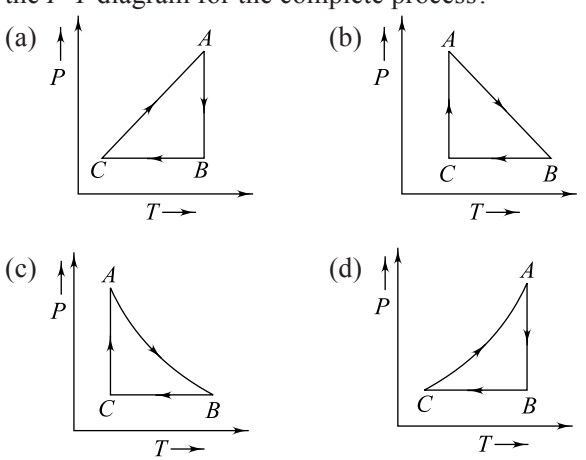


Fig. 8.42

9. The net work done ΔW by the gas during the complete process is (R is the gas constant).
   (a) 0.5 RT_A (b) 0.58 RT_A
   (c) 0.64 RT_A (d) RT_A
10. The net heat energy supplied to the gas in the complete process is
   (a) zero (b) equal to ΔW
   (c) less than ΔW (d) greater than ΔW

Solutions

7. During isothermal process $A \to B$, $P \propto \frac{1}{V}$. During isobaric process ($B \to C$), P = constant and during isochoric process ($C \to A$), V = constant. Hence the correct $P - V$ diagram of the complete process is (c).
8. During isothermal process ($A \to B$), T = constant. During isobaric process ($B \to C$), P = constant and during isochoric process ($C \to A$), $P \propto T$. Hence, the correct P-T diagram for the complete process is (a).
9. For process $A \to B$, $P_A V_A = P_B V_B$ which gives $P_B = \frac{P_A}{2}$ because $V_B = 2V_A$ (given).
   For process $B \to C$, $\frac{V_B}{T_B} = \frac{V_C}{T_C}$. Since $V_B = 2 V_A$ and $V_C = V_A$ and $T_B = T_A$, we get $T_C = \frac{T_A}{2}$. Also $P_C = P_B = \frac{P_A}{2}$.
   For process $C \to A$, $\frac{P_C}{T_C} = \frac{P_A}{T_A}$. Since $P_C = P_A$; $T_A = T_C$.
   Work done is isothermal process $A \to B$ is
   $$W_{AB} = nRT \log_e\left(\frac{V_B}{V_A}\right) = 3\ RT_A \log_e (2) \qquad (2)$$
   Work done in isobaric process $B \to C$ is
   $$W_{BC} = P_B(V_C - V_B) = \frac{P_A}{2}(V_A - 2V_A) = -\frac{P_A V_A}{2} = -\frac{3}{2} RT_A .$$
   Work done is isochoric process $C \to A$ is $W_{CA} = 0$
   $\therefore$ Total work done
   $\Delta W = 3\ RT_A \log_e (2) - \frac{3}{2} RT_A = 3\ RT_A (0.693 - 0.5) = 0.58\ RT_A$
   Thus, the correct choice is (b).
10. Since the process is cyclic, $\Delta U = 0$. From $\Delta Q = \Delta U + \Delta W$, we get $\Delta Q = \Delta W$. So, the correct choice is (b).

Questions 11 to 15 are based on the following passage.

Passage IV

Two moles of an ideal gas at volume V, pressure 2 P and temperature T undergo a cyclic process $ABCDA$ as shown in Fig. 8.43.

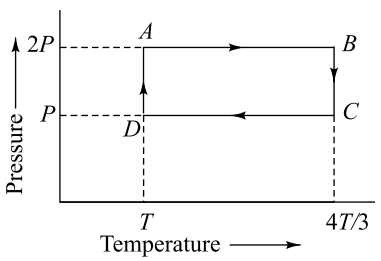


Fig. 8.43

11. The volume (V_B) of the gas in state B is
    (a) $\frac{V}{3}$ (b) $\frac{2V}{3}$
    (c) V (d) $\frac{4V}{3}$
12. The volume (V_C) of the gas in state C is
    (a) $\frac{8V}{3}$ (b) $2\,V$
    (c) $\frac{4V}{3}$ (d) $\frac{2V}{3}$
13. The volume (V_D) of the gas in state D is
    (a) V (b) $2\,V$
    (c) $3\,V$ (d) $4\,V$
14. The net work done (ΔW) in the complete cycle is
    (a) $2\,RT$ (b) $2\,RT\log_e(2)$
    (c) $\frac{4}{3}\,RT$ (d) $\frac{2}{3}\,RT\log_e(2)$
15. The net change (ΔQ) in the heat energy in the complete process is
    (a) zero (b) greater than ΔW
    (c) less than ΔW (d) equal to ΔW.

Solutions

11. For isobaric process $A \to B$,
$$\frac{V_A}{T_A} = \frac{V_B}{T_B} \Rightarrow V_B = \frac{V_A T_B}{T_A} = \frac{V \times 4T/3}{T} = \frac{4V}{3}$$
So, the correct choice is (d).

12. For isothermal process $B \to C$,
$$P_B V_B = P_C V_C \Rightarrow V_C = \frac{P_B V_B}{P_C} = \frac{2P \times 4V/3}{P} = \frac{8V}{3},$$
which is choice (a).

13. For isobaric process $C \to D$,
$$\frac{V_C}{T_C} = \frac{V_D}{T_D} \Rightarrow V_D = \frac{V_D T_D}{T_C} = \frac{8V/3 \times T}{4T/3} = 2V$$
Hence, the correct choice is (b).

14. $$W_{AB} = P_A(V_B - V_A) = 2\,P \times \left(\frac{4V}{3} - V\right) = \frac{2PV}{3}$$
$$W_{BC} = nRT_B \log_e\left(\frac{V_C}{V_B}\right)$$
$$= 2 \times R \times \frac{4T}{3}\log_e\left(\frac{8V/3}{4V/3}\right) = \frac{8}{3}\,RT\log_e(2)$$
$$W_{CD} = P_C(V_D - V_C) = P \times \left(2V - \frac{8V}{3}\right) = -\frac{2PV}{3}$$
$$W_{DA} = nRT_D \log_e\left(\frac{V_A}{V_D}\right)$$
$$= 2 \times R \times T \log_e\left(\frac{V}{2V}\right)$$
$$= 2\,RT\log_e\frac{1}{2} = -2\,RT\log_e(2)$$
Total work done is
$$\Delta W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$
$$= \frac{2PV}{3} + \frac{8}{3}\,RT\log_e(2) - \frac{2PV}{3} - 2\,RT\log_e(2)$$
$$= \frac{2}{3}\,RT\log_e(2),$$ which is choice (d).

15. For a cyclic process, $\Delta U = 0$. Hence $\Delta Q = \Delta W$. So the correct choice is (d).

Questions 16 to 18 are based on the following passage.

Passage V

Two moles of a monoatomic ideal gas occupy a volume V at 27°C. The gas is expanded adiabatically to volume $2\sqrt{2}\,V$. Gas constant $R = 8.3\ \text{JK}^{-1}\ \text{mol}^{-1}$.

16. The final temperature of the gas is
    (a) $\frac{150}{\sqrt{2}}$ K (b) 150 K
    (c) 150°C (d) 13.6°C
17. The change in the internal energy of the gas in this process is
    (a) – 3735 J (b) 1245 J
    (c) – 2490 J (d) + 3735 J
18. The work done by the gas during the process is
    (a) 1245 J (b) 1660 J
    (c) 2490 J (d) 3735 J

Solutions

16. $T_1 = 300$ K, $V_1 = V$, $V_2 = 2\sqrt{2}\,V$.
Let T_2 be the final temperature of the gas. T_2 is obtained from the adiabatic relation
$$T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)}$$
or
$$T_2 = T_1 \times \left(\frac{V_1}{V_2}\right)^{(\gamma-1)}$$
For a monoatomic gas $\gamma = \frac{5}{3}$. Therefore,
$$T_2 = 300 \times \left(\frac{1}{2\sqrt{2}}\right)^{2/3} = 150\ \text{K}$$
So, the correct choice is (b).

17. $\Delta U = nC_v(T_2 - T_1)$
For a monoatomic gas $C_v = \frac{3R}{2}$. Therefore,
$$\Delta U = 2 \times \frac{3}{2} \times 8.3 \times (150 - 300) = -3735\ \text{J}$$

The negative sign implies that the internal energy decreases in this process. So the correct choice is (a).

18. For an adiabatic process, $\Delta Q = 0$. Hence $\Delta W = -\Delta U = 3735$ J, which is choice (d).

Questions 19 to 22 are based on the following passage.

Passage VI

A sample of 2 kg of monoatomic helium (assumed ideal) is taken through the process *ABC* and another sample of 2 kg of the same gas is taken through the process *ADC* as shown in Fig. 8.44. The molecular mass = 4 and $R = 8.3$ J K^{-1} mol^{-1}.

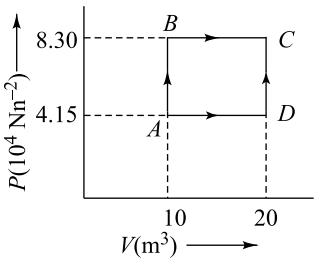


Fig. 8.44

19. The temperature of state *A* is
    (a) 100 K (b) 200 K
    (c) 300 K (d) 415 K
20. The temperature state *B* is
    (a) 150 K (b) 200 K
    (c) 415 K (d) 830 K
21. The temperature of state *C* is
    (a) 150 K (b) 300 K
    (c) 400 K (d) 415 K
22. The temperature of state *D* is
    (a) 830 K (b) 415 K
    (c) 300 K (d) 200 K

Solutions

19. Number of moles of helium is

$$n = \frac{\text{mass in gram}}{\text{molecular mass}} = \frac{2000}{4} = 500$$

From equation state at *A*,

$$P_A V_A = n R T_A \Rightarrow T_A = \frac{P_A V_A}{nR} = \frac{4.15\times10^4\times10}{500\times8.3} = 100 \text{ K}$$

So, the correct choice is (a)

20. For isochoric process $A \rightarrow B$,

$$\frac{T_B}{T_A} = \frac{P_B}{P_A} = \frac{8.3\times10^4}{4.15\times10^4} = 2.$$

Thus $T_B = 2\ T_A = 200$ K, which is choice (b).

21. For isobaric process $B \rightarrow C$,

$$\frac{T_C}{T_B} = \frac{V_C}{V_B} = \frac{20}{10} = 2$$

Thus $T_C = 2\ T_B = 400$ K, which is choice (c).

22. For isobaric process $A \rightarrow D$,

$$\frac{T_D}{T_A} = \frac{V_D}{V_A} = \frac{20}{10} = 2$$

which gives $T_D = 2\ T_A = 200$ K. So, the correct choice is (d).

Questions 23 to 26 are based on the following passage.

Passage VII

One mole of an ideal monoatomic gas is taken round the cyclic process *ABCA* as shown in Fig. 8.45.

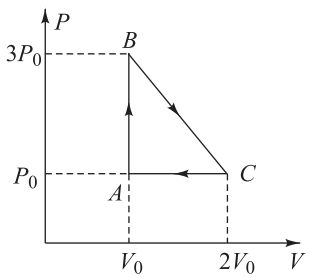


Fig. 8.45

23. The work done by the gas is
    (a) P_0V_0 (b) $2P_0V_0$
    (c) $3P_0V_0$ (d) $4P_0V_0$
24. The heat energy rejected by the gas in the process $C \rightarrow A$ is
    (a) $-\frac{P_0V_0}{2}$ (b) $-\frac{3P_0V_0}{2}$
    (c) $-\frac{5P_0V_0}{2}$ (d) $-3\ P_0V_0$
25. The heat energy absorbed by the gas in the process $A \rightarrow B$ is
    (a) $P_0\ V_0$ (b) $\frac{3P_0V_0}{2}$
    (c) $\frac{5P_0V_0}{2}$ (d) $3\ P_0\ V_0$
26. The net heat energy rejected or absorbed by the gas in the process $B \rightarrow C$ is
    (a) $\frac{P_0V_0}{2}$ (b) $-2\ P_0\ V_0$
    (c) $-\frac{5P_0V_0}{2}$ (d) $3\ P_0\ V_0$

Solutions

23. Work done by the gas in the cyclic process $ABCA$ is
W = area enclosed in the P–V diagram
= area of triangle $ABC = \frac{1}{2} \times AB \times AC$
$= \frac{1}{2}(3P_0 - P_0) \times (2V_0 - V_0) = P_0V_0$,
which is choice (a).

24. In the isobaric process CA, the heat energy rejected by the gas is given by
$$Q_{CA} = nC_p \Delta T = nC_p (T_A - T_C) \quad (1)$$
Using ideal gas equation $PV = nRT$ for points A and C, we have
$$P_A V_A = nRT_A \text{ or } P_0 V_0 = nRT_A \quad (2)$$
and $$P_C V_C = nRT_C \text{ or } P_0 (2V_0) = nRT_C \quad (3)$$
Subtracting (2) from (3), we have
$$P_0V_0 = nR (T_C - T_A)$$
or $$T_A - T_C = -\frac{P_0 V_0}{nR} \quad (4)$$
Using (4) in (1), we get
$$Q_{CA} = -\frac{C_p P_0 V_0}{R}$$
Now, for a monoatomic gas, $C_p = \frac{5R}{2}$. Hence
$Q_{CA} = -\frac{5P_0 V_0}{2}$, which is choice (c). (5)

25. The heat energy absorbed in the isochoric process AB is given by
$$Q_{AB} = nC_v \Delta T = nC_v (T_B - T_A) \quad (6)$$
Using the ideal gas equation for points A and B, we have
$$P_A V_A = nRT_A \text{ or } P_0 V_0 = nRT_A$$
and $$P_B V_B = nRT_B \text{ or } (3 P_0)V_0 = nRT_B$$
which give $T_B - T_A = \frac{2P_0 V_0}{nR}$ (7)
Using (7) in (6), we get
$$Q_{AB} = \frac{2C_v P_0 V_0}{R}$$
Now, for a monoatomic gas, $C_v = \frac{3R}{2}$. Therefore
$Q_{AB} = 3 P_0 V_0$, which is choice (d). (8)

26. From the first law of thermodynamics, the heat energy absorbed by the gas is given by
$$\Delta Q = \Delta U + \Delta W$$
where $\Delta W = W$ = work done by the gas = P_0V_0. Since the process is cyclic, there is no change in the internal energy of the gas, i.e. $\Delta U = 0$. Hence
$$\Delta Q = \Delta W = P_0V_0 \quad (9)$$
If Q_{BC} is the heat absorbed in the process BC, then
$$\Delta Q = Q_{AB} + Q_{BC} + Q_{CA} \quad (10)$$
Using (5), (8) and (9) in (10), we have
$$P_0V_0 = 3P_0V_0 + Q_{BC} - \frac{5P_0V_0}{2}$$
or $$Q_{BC} = \frac{P_0V_0}{2}$$

Questions 27 to 32 are based on the following passage.

Passage VIII

Thermal Radiations

All bodies emit heat energy from their surfaces by virtue of their temperature. This heat energy is called radiant energy or thermal radiation. The heat that we receive from the sun is transferred to us by a process which, unlike conduction or convection, does not require the help of a medium in the intervening space which is almost free of particles. Radiant energy travels in space as electromagnetic waves in the infra-red region of the electromagnetic spectrum. Thermal radiations travel through vacuum with the speed of light. Thermal radiations obey the same laws of reflection and refraction as light does. They exhibit the phenomena of interference, diffraction and polarisation as light does.

The emission of radiation from a hot body is expressed in terms of that emitted from a reference body (called the black body) at the same temperature. A black body absorbs and hence emits radiations of all wavelengths. The total energy E emitted by a unit area of a black body per second is given by
$$E = \sigma T^4$$
where T is the absolute temperature of the body and σ is a constant known as Stefan's constant. If the body is not a perfect black body, then
$$E = \varepsilon \sigma T^4$$
where ε is the emissivity of the body.

27. From Stefan-Boltzman law, the dimensions of Stefan's constant σ are
(a) $ML^{-2}T^{-2}K^{-4}$ (b) $ML^{-1}T^{-2}K^{-4}$
(c) $MLT^{-3}K^{-4}$ (d) $ML^0T^{-3}K^{-4}$

28. What is the SI unit of Stefan's constant?
(a) J $s^{-1}K^{-4}$ (b) W $m^{-1}K^{-4}$
(c) W $m^{-2}K^{-4}$ (d) J $m^{-2}K^{-4}$

29. In which region of the electromagnetic spectrum do thermal radiations lie?
(a) Visible region (b) Infrared region
(c) Ultraviolet region (d) Microwave region

30. Which of the following devices is used to detect thermal radiations?

(a) Constant volume air thermometer

(b) Platinum resistance thermometer

(c) Thermostat

(d) Thermopile

31. When a body A at a higher temperature T_1 is surrounded by another body B at a lower temperature T_2, then the rate of loss of heat from body A will be proportional to

(a) T_1^4 (b) $(T_1 - T_2)^4$

(c) $(T_1 - T_2)$ (d) $(T_1^4 - T_2^4)$

32. The rate at which energy is radiated by a body depends upon

(a) the surface area of the body

(b) the temperature of the body

(c) the nature of the surface of the body

(d) the emissivity of the surface of the body

Solutions

27. Stefan-Boltzmann law states that $E = \sigma T^4$ where E stands for the total energy emitted per unit area per second. Thus, the dimensions of E = dimensions of $\left(\frac{\text{energy}}{\text{area} \times \text{time}}\right) = \frac{ML^2T^{-2}}{L^2T} = MT^{-3}$. Therefore,

Dimensions of $\sigma = \frac{ML^{-3}}{K^4} = MT^{-3}K^{-4}$, which is choice (d).

28. The correct choice is (c).

29. The correct choice is (b).

30. The correct choice is (d).

31. The correct choice is (d).

32. All the four choices are correct.

Questions 33 to 35 are based on the following passage.

Passage IX

Three cylindrical rods A, B and C of equal lengths and equal diameters are joined in series as shown in the following figure. Their thermal conductivities are 2 k, k and 0.5 k, respectively (Fig. 8.46)

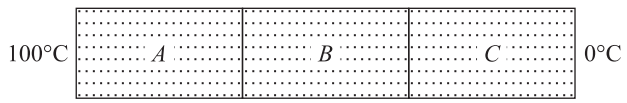


Fig. 8.46

In the steady state, the free ends of rods A and C are at 100°C and 0°C, respectively. Neglect loss of heat from the curved surfaces of rods.

33. The temperature of the junction between rods A and B is

(a) 55.7°C (b) 65.7°C

(c) 75.7°C (d) 85.7°C

34. The temperature of the junction between rods B and C is

(a) 57.1°C (b) 47.1°C

(c) 37.1°C (d) 27.1°C

35. The equivalent thermal conductivity of the combination is

(a) $\frac{7k}{3}$ (b) $\frac{2k}{7}$

(c) $\frac{5k}{3}$ (d) $\frac{3k}{5}$

Solutions

33. In the steady state, the rate of flow of heat is the same for all rods. If T_1 and T_2 are the temperatures at the junction points between A and B and between B and C respectively, then

$$\frac{Q}{t} = \frac{k_A A(100 - T_1)}{d} = \frac{k_B A(T_1 - T_2)}{d} = \frac{k_C A(T_2 - 0)}{d}$$

Given $k_A = 2k$, $k_B = k$ and $k_C = 0.5k$. Hence

$$2(100 - T_1) = (T_1 - T_2) = 0.5(T_2 - 0)$$

which give $200 - 2\,T_1 = T_1 - T_2$ (1)

and $T_1 - T_2 = 0.5\,T_2$ (2)

Equations (1) and (2) give $T_1 = 85.7$°C. So, the correct choice is (d).

34. Putting $T_1 = 85.7$°C in Eq. (1) or (2), we get $T_2 = 57.1$°C, which is choice (a).

35. Since the rods have the same lengths and the same diameters, the equivalent thermal conductivity of the series combination is given by

$$\frac{1}{k_e} = \frac{1}{k_A} + \frac{1}{k_B} + \frac{1}{k_C} = \frac{1}{2k} + \frac{1}{k} + \frac{1}{0.5k}$$

which gives $k_e = \frac{2k}{7}$. So, the correct choice is (b).

SECTION 3

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-1 is true.

1. **Statement-1**

   Figure 8.47 shows $\frac{PV}{T}$ versus P graph for a certain mass of oxygen gas at two temperatures T_1 and T_2. It follows from the graph that $T_1 > T_2$.

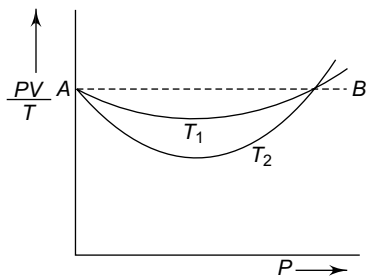


Fig. 8.47

   Statement-2

   At higher temperatures, real gas behaves more like an ideal gas.

2. **Statement-1**

   If two bodies of equal mass and made of the same material at different temperature T_1 and T_2 are brought in thermal contact, the temperature of each body will be $(T_1 + T_2)/2$ when thermal equilibrium is attained.

   Statement-2

   They have the same thermal capacity.

3. **Statement-1**

   The coolant used in a car or a nuclear plant has a high specific heat.

   Statement-2

   The heat energy absorbed by a substance is proportional to its specific heat.

4. **Statement-1**

   The air pressure in a car tyre increases during driving.

   Statement-2

   The pressure of a given mass of a gas is inversely proportional to its volume.

5. **Statement-1**

   A cylinder fitted with a movable piston contains a certain amount of a liquid in equilibrium with its vapour. The temperature of the system is kept constant. If the volume of the vapour is decreased by moving the piston inwards, the vapour pressure will remain unchanged.

   Statement-2

   Vapour, in equilibrium with its liquid, at a constant temperature does not obey Boyle's law.

6. **Statement-1**

   Two vessels A and B of equal capacity are connected to each other by a stopcock. Vessel A contains a gas at 0°C and 1 atmosphere pressure and vessel B is evacuated. If the stopcock is suddenly opened, the final pressure in A and B will be 0.5 atmosphere.

   Statement-2

   If the temperature is kept constant, the pressure of a gas is inversely proportional to its volume.

7. **Statement-1**

   Two vessels A and B are connected to each other by a stopcock. Vessel A contains a gas at 0°C and 1 atmosphere pressure and vessel B is evaluated. The two vessels are thermally insulated from the surroundings. If the stopcock is suddenly opened, there will be no change in the internal energy of the gas.

   Statement-2

   No transfer of heat energy takes place between the system and the surroundings.

8. **Statement-1**

   Two vessels A and B are connected to each other by a stopcock. Vessel A contains a gas at 300 K and

1 atmosphere pressure and vessel B is evacuated. The two vessels are thermally insulated from the surroundings. If the stopcock is suddenly opened, the expanding gas does no work.

Statement-2

Since $\Delta Q = 0$ and $\Delta U = 0$, it follows from the first law of thermodynamics that $\Delta W = 0$.

9. **Statement-1**

   Burns caused by steam at 100°C are move severe then those caused by water at 100°C.

   Statement-2

   Steam has a higher specific heat capacity than water.

10. **Statement-1**

    Heating system based on circulation of steam are more efficient in warming a house than those based on circulation of hot water.

    Statement-2

    The latent heat of steam is high.

11. **Statement 1**

    On a chilly day, the metallic cap of a pen feels much colder than the plastic body although both are at the same (room) temperature.

    Statement 2

    Metal has a higher coefficient of thermal conductivity than plastic.

12. **Statement 1**

    If the earth did not have an atmosphere, it would become extremely cold.

    Statement 2

    Heat energy is transferred through air mainly by convection.

13. **Statement 1**

    Evaporation takes place from the surface of a liquid.

    Statement 2

    The molecules at the surface have less attractive energy than those inside the liquid.

14. **Statement 1**

    Radiation involves transfer of heat by electromagnetic waves.

    Statement 2

    Electromagnetic waves do not require any material medium for propagation.



Solutions

1. The correct choice is (a). The line AB is parallel to the P-axis. This means that PV/T is a constant, independent of pressure. Hence line AB corresponds to an ideal gas for which PV/T = constant. At higher temperatures, a real gas behave more like an ideal gas. Hence T_1 is greater than T_2.
2. The correct choice is (a). Statement-1 is true only if the two bodies have the same thermal capacity which is equal to mass of the body × its specific heat capacity. Since the two bodies have the same mass and are made of the same material, they have the same thermal capacity.
3. The correct choice is (a). If the coolant has a high specific heat, it will remove a large amount of heat from the plant and will prevent it from becoming excessively hot.
4. The correct choice is (c). During driving, the temperature of the tyre and hence of the air in it increases due to friction with the road. Since the volume of the enclosed air is almost constant, an increase in temperature results in increase in pressure ($P \propto T$).
5. The correct choice is (a). If the volume of the vapour is decreased at a constant temperature, a part of the vapour will condense into liquid such that the vapour pressure remains unchanged.
6. The correct choice is (a). Since the two vessels are of equal capacity, the volume occupied by the gas is doubled when the stopclock is opened. Hence, pressure becomes half.
7. The correct choice is (a)
8. The correct choice is (a). Since the system is thermally insulated from the surroundings, no heat flows into the system or out of it, i.e. $\Delta Q = 0$. Since $\Delta U = 0$; $\Delta W = 0$.
9. The correct choice is (c). The latent heat of steam is very high = 2.25×10^6 Jkg^{-1}. This means that 1 kg of steam at 100°C gives out 2.25×10^6 J of heat energy to convert into 1 kg of water at 100°C.
10. The correct choice is (a).
11. The correct choice is (a). On a chilly day, the room temperature is lower than our body temperature. Since metal is a better conductor of heat than plastic, when we touch the metal cap and the plastic body of a pen, heat from our fingers will flow to the metal cap much more quickly than to the plastic body.
12. The correct choice is (c). Thermel radiation from the sun warms the earth during the day. Since air is a poor conductor of heat, the atmosphere acts as a blanket for the earth and keeps the earth warn during the night.
13. The correct choice is (a).
14. The correct choice is (b).

SECTION 4

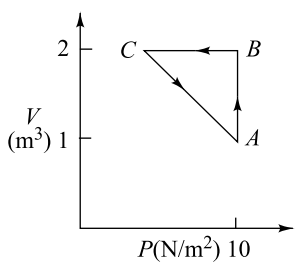
Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. Which statement is incorrect?
   (a) All reversible cycles have the same efficiency
   (b) Reversible cycle has a higher efficiency than an irreversible cycle
   (c) Carnot cycle is a reversible cycle
   (d) Carnot cycle has the maximum efficiency of all cycles **[2002]**

2. Even Carnot engine does not have 100% efficiency because we cannot
   (a) Prevent heat loss due to radiation
   (b) find an ideal source of heat
   (c) attain absolute zero temperature
   (d) eliminate friction **[2002]**

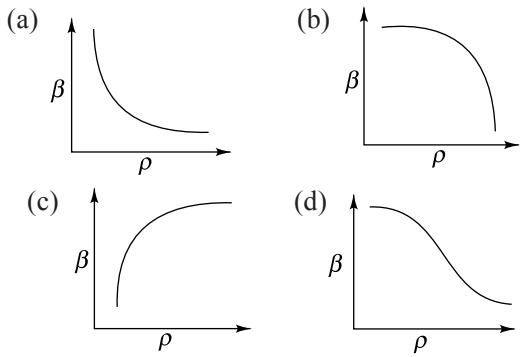
3. If 1 mole of gas of $\gamma = 7/5$ is mixed with 1 mole of a gas of $\gamma = 5/3$, the value of γ the mixture is
   (a) $\frac{7}{5}$ (b) $\frac{5}{2}$
   (c) $\frac{3}{2}$ (d) $\frac{12}{7}$ **[2002]**

4. An ideal gas is taken through the cycle $A \rightarrow B \rightarrow C \rightarrow A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5J, the work done by the gas in the process $C \rightarrow A$ is
   (a) –5 J (b) –10 J
   (c) –15 J (d) –20 J



[2002]

5. Which of the graphs shown in the figure correctly represents the variation of $\beta = -(dV/dp)/V$ with p for an ideal gas at constant temperature?

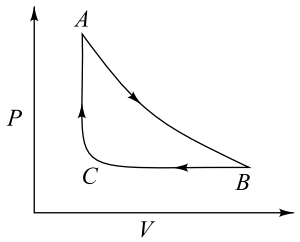


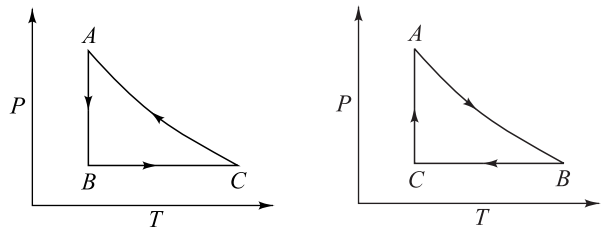
[2002]

6. An ideal black-body at room temperature is thrown into a furnace. It is observed that
   (a) initially it is the darkest body and at later times the brightest
   (b) it is the darkest body at all times
   (c) it cannot be distinguished at all times
   (d) initially it is the darkest body and at later times it cannot be distinguished. **[2002]**

7. Two rods, one made of aluminium and the other made of steel, having initial lengths l_1 and l_2 respectively are connected together to form a single rod of length $(l_1 + l_2)$. The coefficients of linear expansion for aluminium and steel are α_1 and α_2 respectively. If the length of each rod increases by the same amount when their temperature is raised by t°C, then the ratio $l_1/(l_1+l_2)$
   (a) α_1/α_2 (b) α_2/α_1
   (c) $\frac{\alpha_2}{\alpha_1 + \alpha_2}$ (d) $\frac{\alpha_1}{\alpha_1 + \alpha_2}$ **[2003]**

8. The figure given below shows the *P-V* diagram for a fixed mass of an ideal gas undergoing cyclic process. *AB* represents isothermal process and *CA* represents adiabatic process. Which of the graphs shown in the following figure represents the *P-T* diagram of the cyclic process?





[2003]

9. An insulated box containing a diatomic gas of molar mass M is moving with a velocity v. The box is suddenly stopped. The resulting change in temperature is (R is the gast constant)

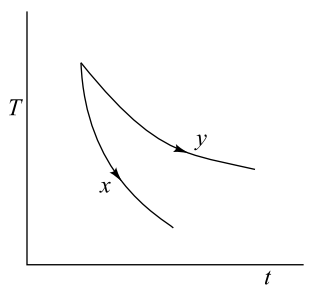
(a) $\frac{Mv^2}{2R}$ (b) $\frac{Mv^2}{3R}$

(c) $\frac{Mv^2}{5R}$ (d) $\frac{2Mv^2}{5R}$ **[2003]**

10. The graph, shown in the following figure, shows the fall of temperature (T) of two bodies x and y, having the same surface area, with time (t) due to emission of radiation. Find the correct relation between emissive power (E) and absorptive power (a) of the two bodies.

(a) $E_x > E_y$; $a_x < a_y$ (b) $E_x < E_y$; $a_x > a_y$

(a) $E_x > E_y$; $a_x > a_y$ (a) $E_x < E_y$; $a_x < a_y$



[2003]

11. "Heat cannot by itself flow from a body at a lower temperature to a body at a higher temperature" is a statement or consequence of

(a) second law of thermodynamics

(b) conservation of momentum

(c) conservation of mass

(d) first law of thermodynamics **[2003]**

12. During an adiabatic process, the pressure of gas is found to be proportional to the cube of its absolute temperature. The ratio C_p/C_v of the gas is

(a) $\frac{4}{3}$ (b) 2

(c) $\frac{5}{3}$ (d) $\frac{3}{2}$ **[2003]**

13. Which of the following parameters does not characterise the thermodynamic state of matter?

(a) Temperature (b) Pressure

(c) Work (d) Volume **[2003]**

14. A Carnot engine takes 3×10^6 cal of heat from a reservoir at 627 ºC and rejects a part of it to a sink at 27 ºC. The work done by the engine is

(a) 4.2×10^6 J (b) 8.4×10^6 J

(c) 16.8×10^6 J (d) zero **[2003]**

15. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$ where $\Delta\theta$ is the difference of the temperature of the body and the surroundings and n is equal

(a) 2 (b) 3

(c) 4 (d) 1 **[2003]**

16. Which of the following statements is correct for any thermodynamic system?

(a) The internal energy changes in all processes.

(b) Internal energy and entropy are state functions.

(c) The change in entropy can never be zero.

(d) The work done in an adiabatic process is always zero. **[2004]**

17. Two thermally insulated vessels 1 and 2 are filled with air at temperature (T_1, T_2), volumes (V_1, V_2) and pressures (P_1,P_2), respectively. If the valve joining the two vessels is opened, the temperature inside the vessel at equilibrium will be

(a) $T_1 + T_2$ (b) $\frac{1}{2}(T_1 + T_2)$

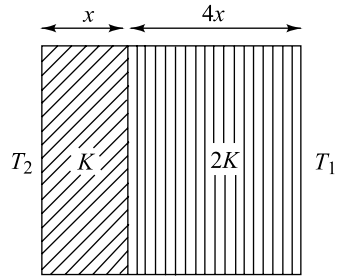
(c) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{(P_1V_1T_2 + P_2V_2T_1)}$ (d) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{(P_1V_1T_1 + P_2V_2T_2)}$

[2004]

18. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively, are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right)f$, with f equal to (see the figure below)

(a) 1 (b) 1/2

(c) 2/3 (d) 1/3



[2004]

19. A metal cube of coefficient of linear expansion α is floating in a beaker containing a liquid of coefficient of volume expansion γ. When the temperature is raised by ΔT, the depth up to which the cube is submerged in the liquid remains unchanged. If the expansion of the beaker is ignored, the relation between α and γ is

(a) $\alpha = \frac{\gamma}{3}$ (b) $\alpha = \frac{\gamma}{2}$

(c) $\alpha = 3\gamma$ (d) $\alpha = 2\gamma$ **[2004]**

20. An ideal gas is expanded isothermally from a volume V_1 to volume V_2 and then compressed adiabatically to original volume V_1. The initial pressure is P_1 and the final pressure is P_3. If the net work done is W, then

(a) $P_3 > P_1, W > 0$ (b) $P_3 < P_1, W < 0$

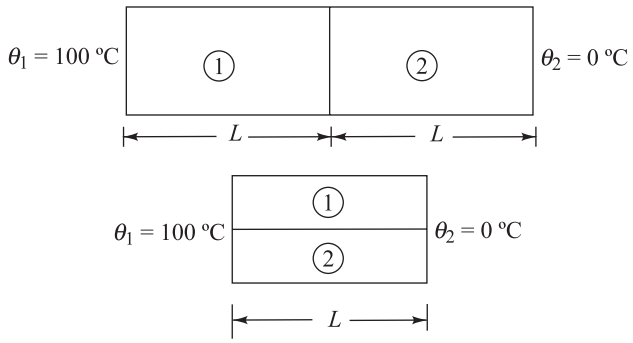
(c) $P_3 > P_1, W < 0$ (d) $P_3 = P_1, W = 0$

[2004]

21. In the first experiment, two identical conducting rods are joined one after the other and this combination is connected to two vessels, one containing water at 100ºC and the other containing ice at 0ºC. In the second experiment, the two rods are placed one on top of the other and connected to the same vessels. If q_1 and q_2 (in gram per second) are the respective rates of melting of ice in the two cases, then the ratio $\frac{q_1}{q_2}$ is

(a) $\frac{1}{2}$ (b) $\frac{2}{1}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$



[2004]

22. Two spheres A and B having radii 3 cm and 5 cm, respectively are coated with carbon black on their outer surfaces. The wavelength of maximum in tensity of emission of radiation are 300 nm and 500 nm respectively. The respective powers radiated by them are in the ratio of

(a) $\sqrt{\frac{5}{3}}$ (b) $\frac{5}{3}$

(c) $\left(\frac{5}{3}\right)^2$ (d) $\left(\frac{5}{3}\right)^4$ **[2004]**

23. Which of the following is incorrect regarding the first law of thermodynamics?

(a) It is not applicable to any cyclic process.

(b) It is a restatement of the principle of conservation of energy.

(c) It introduces the concept of internal energy.

(d) It introduces the concept of entropy. **[2005]**

24. A gaseous mixture consists of 16 g of helium and 16g of oxygen. The ratio $\frac{C_p}{C_v}$ of the mixture is

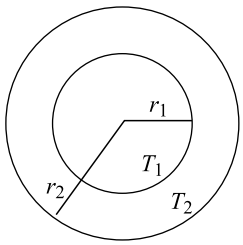
(a) 1.59 (b) 1.62

(c) 1.4 (d) 1.54 **[2005]**

25. The following figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperature T_1 and T_2. The radial rate of flow of heat in the material between the spheres is proportional to

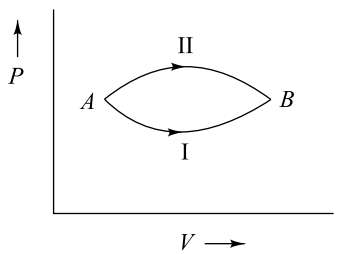
(a) $\frac{(r_2 - r_1)}{r_1 r_2}$ (b) $\text{In}\left(\frac{r_2}{r_1}\right)$

(c) $\left(\frac{r_1 r_2}{(r_2 - r_1)}\right)$ (d) $(r_2 - r_1)$



[2005]

26. A gas is taken from state A to state B via two processes I and II as shown in the figure. If ΔU_1 and ΔU_2 are the changes in internal energy in the processes I and II respectively, then
    (a) $\Delta U_1 = \Delta U_2$
    (b) The relation between ΔU_1 and ΔU_2 cannot be determined
    (c) $\Delta U_2 > \Delta U_1$
    (d) $\Delta U_2 < \Delta U_1$



[2005]

27. Two litres of water (density = 1 g/ml) in an open-lid insulated kettle is heated by an electric heater of power 1 kW. The heat is lost from the lid at the rate of 160 J/s. The time taken for heating water (specific heat capacity 4.2 kJ $kg^{-1}K^{-1}$) from 20ºC to 75ºC is
    (a) 340 s (b) 550 s
    (c) 620 s (d) 760 s **[2005]**
28. In which of the following processes is the heat transfer primarily due to radiation?
    (a) Boilng of water
    (b) Land and sea breezes
    (c) Heating of metal rod placed over a flame
    (d) Heating of the glass surface of an electric bulb due to current in its filament. **[2005]**
29. A spherical body of emissivitye ε, placed inside a perfectly black body (emissivity =1), is maintained at absolute temperature T. The energy radiated by a unit area of the body per second will be (σ is Stefan's constant)
    (a) σT^4 (b) $\varepsilon\,\sigma\,T^4$
    (c) $(1-\varepsilon)\,\sigma\,T^4$ (d) $(1+\varepsilon)\,\sigma\,T^4$

[2005]

30. Assuming the Sun to be a spherical body is radius R at a temperature of T K, evaluate the total radiant power, incident on the Earth, at a distance r from the Sun.
    (a) $\dfrac{r_0^2R^2\sigma T^4}{4\pi r^2}$ (b) $\dfrac{R^2\sigma T^4}{r^2}$
    (c) $\dfrac{4\pi r_0^2R^2\sigma T^4}{r^2}$ (d) $\dfrac{\pi r_0^2R^2\sigma T^4}{r^2}$

    where r_0 is the radius of the Earth and σ is Stafan's constant. **[2006]**
31. Two rigid boxes containing different ideal gases are placed on a table. Box A contains on mole of nitrogen at temperature T_0, while box B contains one mole of helium at temperature $(7/3)T_0$.The boxes are then put into thermal contact with each other, and heat flows between them until the gases reach a common final temperature. (Ignore the heat capacity of boxes). Then the final temperature of the gases, T_f, in terms of T_0 is
    (a) $T_f = \dfrac{3}{2}T_o$ (b) $T_f = \dfrac{5}{2}T_o$
    (c) $T_f = \dfrac{3}{7}T_0$ (d) $T_f = \dfrac{7}{3}T_0$ **[2006]**
32. The work of 146 kJ is performed in order to compress one kilo mole of gas adiabatically and in this process the temperature of the gas increases by 7ºC. The gas is a (R = 8.3 J mol^{-1} K^{-1})
    (a) mixture of monoatomic and diatomic gases
    (b) monoatomic gas
    (c) diatomic gas
    (d) triatomic gas **[2006]**
33. A Carnot engine, having an efficiency of $\eta = 1/10$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is
    (a) 99 J (b) 90 J
    (c) 1 J (d) 100 J **[2007]**
34. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of lengths l_1 and l_2 and thermal conductivities k_1 and k_2, respectively. The temperature at the interface of the section is

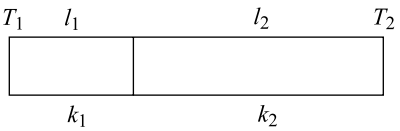


Fig. 8.55

    (a) $(k_1l_2T_1 + k_2l_1T_2)/(k_1l_1 + k_2l_2)$
    (b) $(k_2l_1T_1 + k_1l_2T_2)/(k_2l_1 + k_1l_2)$

(c) $(k_1l_2T_1 + k_2l_1T_2)/(k_1l_2 + k_2l_1)$

(d) $(k_1l_1T_1 + k_2l_2T_2)/(k_1l_1 + k_2l_2)$ **[2007]**

35. If C_p and C_v denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then

(a) $C_p - C_v = R/28$ (b) $C_p - C_v = R/14$

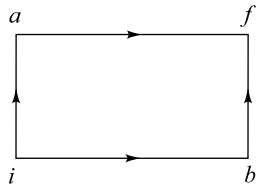
(c) $C_p - C_v = R$ (d) $C_p - C_v = 28R$

[2007]

36. When a system is taken from state *i* to state *f* along the path *iaf*, it is found that $Q = 50$ cal and $W = 20$ cal. Along the path *ibf*, $Q = 36$ cal. W along the path *ibf* is [see the figure]

(a) 6 cal (b) 16 cal

(c) 66 cal (d) 14 cal



[2007]

37. 400 g of ice at 253 K is mixed with 0.05 kg of steam at 100 ºC. Latent heat of vaporisation of stream = 540 cal/g. Latent heat of fusion of ice 80 cal/g. Specific heat of ice = 0.5 cal/g ºC. The resultant temperature of the mixture is

(a) 273 K (b) 300 K

(c) 330 K (d) 373 K **[2007]**

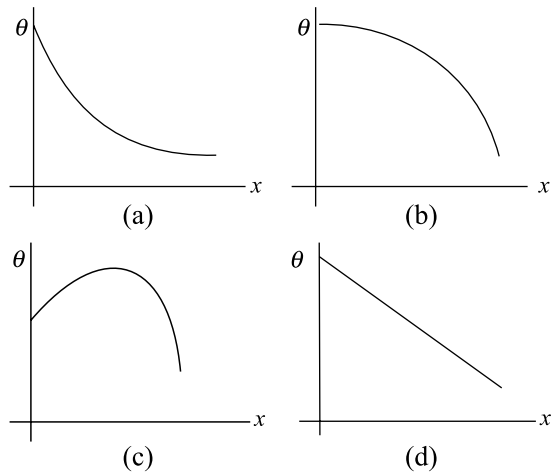
38. An insulated container of gas has two chamber separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1, The other chamber has volume V_2 and contains ideal gas at pressure P_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be

(a) $\dfrac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_1 + P_2V_2T_2}$ (b) $\dfrac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$

(c) $\dfrac{P_1V_1T_1 + P_2V_2T_2}{P_1V_1 + P_2V_2}$ (d) $\dfrac{P_1V_1T_2 + P_2V_2T_1}{P_1V_1 + P_2V_2}$

[2008]

39. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figures?



[2009]

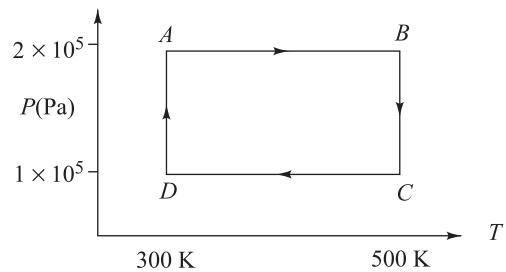
40. One kg of a diatomic gas is at a pressure of 8×10^4 Nm^{-2}. The density of the gas is 4 kg m^{-3}. What is the energy of the gas due to its thermal motion?

(a) 6×10^4 J (b) 7×10^4 J

(c) 3×10^4 J (d) 5×10^4 J **[2009]**

Directions: Questions 41, 42 and 43 are based on the following passage.

Two moles of helium gas are taken over the cycle *ABCDA*, as shown in the *P-T* diagram [see the figure].



41. Assuming the gas to be ideal the work done on the gas in taking it from *A* to *B* is :

(a) 400 *R* (b) 500 *R*

(c) 200 *R* (c) 300 *R* **[2009]**

42. The work done on the gas in taking it from *D* to *A* is:

(a) –690 *R* (b) + 690 *R*

(a) –414 *R* (b) + 414 *R* **[2009]**

43. The network done on the gas in the cycle *ABCDA* is:

(a) 1076 *R* (b) 1904 *R*

(c) zero (d) 276 *R* **[2009]**

44. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle, volume of the gas increases from V to $32V$ the efficiency to the engine is

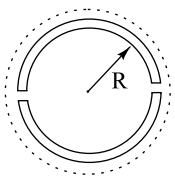
(a) 0.5 (b) 0.75

(c) 0.99 (d) 0.25 **[2010]**

45. A real gas behaves like an ideal gas if its
(a) pressure and temperature are both high
(b) pressure and temperature are both low
(c) pressure is high and temperature is low
(d) pressure is low and temperature is high **[2010]**

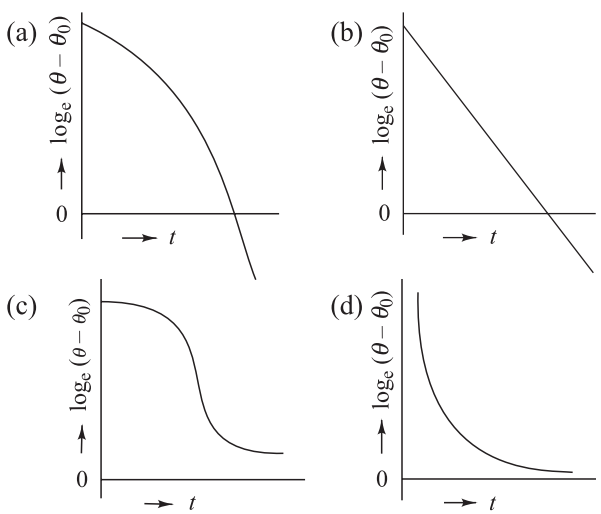
46. 100 g of water is heated from 30ºC to 50ºC. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/kg/K)
(a) 4.2 kJ (b) 8.4 kJ
(c) 84 kJ (d) 2.1 kJ **[2011]**

47. A wooden wheel of radius R is made of two semicircular parts (see the figure). The two parts are held together by a ring made of metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α and its Young's modulus is Y, the force that one part of the wheel applies on the other part is



(a) $2\pi SY\alpha\Delta T$ (b) $SY\alpha\Delta T$
(c) $\pi SY\alpha\Delta T$ (d) $2SY\alpha\Delta T$ **[2012]**

48. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the corect graph between $\log_e(\theta - \theta_0)$ and t

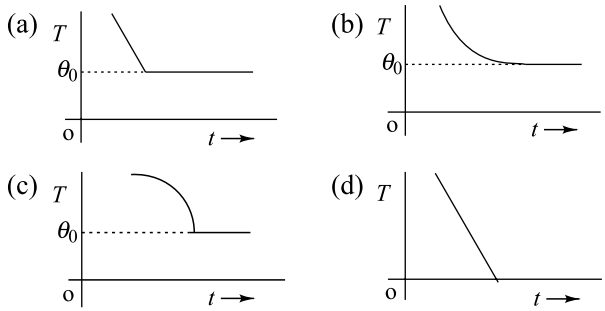


[2012]

49. Three very large plates of same material are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures $2T$ and $3T$ respectively. The temperature of the middle (i.e. second) plate under steady conditions is
(a) $\left(\frac{65}{2}\right)^{1/4} T$ (b) $\left(\frac{97}{4}\right)^{1/4} T$
(c) $\left(\frac{97}{2}\right)^{1/4} T$ (d) $(97)^{1/4}\, T$

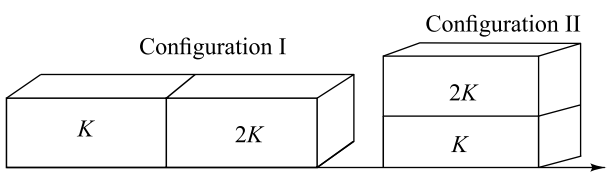
50. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporisation.
(a) $\sqrt{\frac{T}{\rho L}}$ (b) $\frac{T}{\rho L}$
(c) $\frac{2T}{\rho L}$ (d) $\frac{\rho L}{T}$ **[2013]**

51. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closest to



[2013]

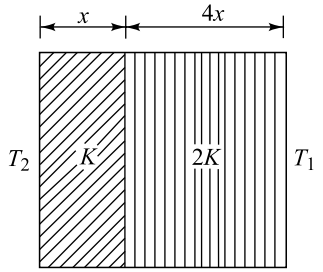
52. Two rectangular blocks, having identical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure. One of the blocks has thermal conductivity K and other $2K$. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9s to transport certain amount of heat from the hot end in the configuration I. The time to transport the same amount of heat in the configuration II is:



(a) 2.0 s (b) 3.0 s

(a) 4.5 s (b) 6.0 s **[2013]**

53. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively, are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right) f$, with f equal to



(a) 1 (b) 1/2

(c) 2/3 (d) 1/3 **[2014]**

54. A diatomic ideal gas is used in Carnot engine as the working substance. If during the adiabatic expansion part of the cycle, volume of the gas increases from V to 32 V the efficiency of the engine is

(a) 0.5 (b) 0.75

(c) 0.99 (d) 0.25 **[2014]**

Questions 55 and 56 are based on the following passage.

In the figure a container is shown to have a moveable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monoatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monoatomic gas are $C_v = \frac{3}{2} R$, $C_P = \frac{5}{2} R$ and those for an ideal diatomic gas are $C_v = \frac{5}{2} R$, $C_P = \frac{7}{2} R$.

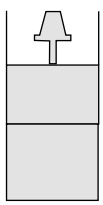


Fig. 8.64

55. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be

(a) 550 K (b) 525 K

(c) 513 K (d) 490 K **[2014]**

56. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be

(a) 250 R (b) 200 R

(c) 100 R (d) –100 R **[2014]**

Answers

1. (a)	2. (c)	3. (c)	4. (a)
5. (a)	6. (a)	7. (c)	8. (a)
9. (c)	10. (c)	11. (a)	12. (d)
13. (c)	14. (b)	15. (d)	16. (b)
17. (c)	18. (d)	19. (b)	20. (c)
21. (d)	22. (c)	23. (d)	24. (b)
25. (c)	26. (a)	27. (b)	28. (d)
29. (b)	30. (c)	31. (a)	32. (c)
33. (b)	34. (c)	35. (a)	36. (a)
37. (a)	38. (a)	39. (d)	40. (d)
41. (a)	42. (d)	43. (d)	44. (b)
45. (d)	46. (b)	47. (d)	48. (a)
49. (c)	50. (c)	51. (b)	52. (a)
53. (d)	54. (b)	55. (d)	56. (d)

Solutions

1. The correct choice is (a). The efficiency of a reversible cycle depends upon the temperature of the source and the sink which will be different for different cycles.

2. The efficiency of Carnot engine is given

$$\eta = 1 - \frac{T_2}{T_1}$$

where T_1 is the absolute temperature of the source and T_2 that of the sink. To have 100% efficiency η must be equal to 1 which is possible if $T_2 = 0$ kelvin which is not attainable.

3. From $C_P - C_v = R$ and $\gamma = \frac{C_p}{C_v}$. we get

$$C_v = \frac{R}{\gamma - 1}$$

For the first gas of $\gamma = \frac{7}{5}$, $C_v = \frac{R}{\left(\frac{7}{5} - 1\right)} = \frac{5R}{2}$

For the second gas of $\gamma = \frac{5}{3}$, $C_v = \frac{R}{\left(\frac{5}{3}-1\right)} = \frac{3R}{2}$

By definition of C_v, the amount of heat required to raise the temperature of 1 mole of the first and second gases by 1K is ($\because Q = n\,C_v\,\Delta T$) is

$$Q_1 = \frac{5R}{2} \text{ and } Q_2 = \frac{3R}{2}$$

$\therefore$ Total heat energy to raise the temperature of 2 moles of the mixture is

$$Q_1 + Q_2 = \frac{5R}{2} + \frac{3R}{2} = 4R$$

For 1 mole of mixture, $\frac{4R}{2} = 2\,R$, i.e. C_v of mixture $= 2R$

The γ of the mixture is given by

$$\frac{R}{\gamma - 1} = 2\,R \quad \Rightarrow \quad \gamma = \frac{3}{2}$$

Alternative Method

Use the formula $\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$

Which gives $\frac{1+1}{\gamma - 1} = \frac{1}{\left(\frac{7}{5}-1\right)} + \frac{1}{\left(\frac{5}{3}-1\right)}$

$\Rightarrow \gamma = \frac{3}{2}$

4. Process A $\rightarrow$ B occurs at constant pressure. Hence the work done in this process is (see the figure).

   $W_{AB} = P\text{dV} = P(V_2 - V_1)$

   $= 10 \times (2 - 1) = 10$ J

   Process $B \rightarrow C$, occurs at constant volume. Hence $W_{BC} = 0$. Given $Q = 5$J, i.e. total work done is $W_t = 5$ J. Therefore, we have

   $W_t = W_{AB} + W_{BC} + W_{CA}$

   or $5 = 10 + 0 + W_{CA}$

   or $W_{CA} = 5$ J, which is choice (a).

5. For an ideal gas, $pV = nRT$. Differentiating, we have since T is kept constant

   $$p\frac{dV}{dp} + V = 0 \text{ or } \frac{dV}{dp} = -\frac{V}{p}$$

   Hence $\beta = -\frac{1}{V}\left(\frac{dV}{dp}\right) = \frac{1}{p}$

   Therefore, choice (a) correctly represents the graph of β versus p.

6. The correct choice is (a) because, initially the black body will absorb radiations as the surroundings, i.e. furnace, are at a higher temperature and, therefore, is the blackest. Later it emits the radiations (having become hot) and is therefore, the brightest.

7. The length of each rod increases by the same amount if $l_1\,\alpha_1 = l_2\,\alpha_2$ or

   $$\frac{l_2}{l_1} = \frac{\alpha_1}{\alpha_2}$$

   or $$\frac{l_2}{l_1} + 1 = \frac{\alpha_1}{\alpha_2} + 1$$

   or $$\frac{l_1 + l_2}{l_1} = \frac{\alpha_1 + \alpha_2}{\alpha_2}$$

   or $$\frac{l_1}{l_1 + _2} = \frac{\alpha_2}{\alpha_1 + \alpha_2}$$

8. Since $A\,B$ is an isolthermal process, the temperature of the gas remains constant between $A\,B$. Hence the P-T diagram must be perpendicular to the T-axis between A and B. Hence the correct choice is (a).

9. Let n be the number of moles of the gas in the box. The kinetic energy of the gas $= n\left(\frac{1}{2}Mv^2\right)$. When the box is suddenly stopped, this energy is used up in changing the internal energy, as a result of which the temperature of the gas rises. The change in internal energy is given by

   $$\Delta U = nC_v\,\Delta T$$

   For a diatomic gas $C_v = \frac{5}{2}$ R.

   Hence $n \times \frac{5R}{2} \times \Delta T = n \times \frac{1}{2}Mv^2$

   or $\Delta T = \frac{Mv^2}{5R}$, which is choice (c)

10. If follows from the figure on page 8.51 that the temperature of body x falls more rapidly with time than that of body y. Hence $E_x > E_y$. Also a good emitter of radiation is also a good absorber of radiation (Kirchhoff's law). Hence $a_x > a_y$. Thus the correct choice is (c).

11. The correct choice is (a)

12. For adiabatic process,

    $$P^{(1-\gamma)}\,T^{\gamma} = \text{constant}$$

    $$\Rightarrow \quad P^{(1-\gamma)} \propto T^{-\gamma}$$

    $$\Rightarrow \quad P \propto T^{\frac{\gamma}{\gamma - 1}}$$

    Given $P \propto T^3$. Hence

    $$3 = \frac{\gamma}{\gamma - 1} \Rightarrow \gamma = \frac{3}{2}$$

13. The correct choice is (c)

14. $T_1 = 627 + 273 = 900$ K, $T_2 = 27 \times 273 = 300$ K

    Efficiency $\frac{W}{Q_1} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{900} = \frac{2}{3}$

$$\therefore \quad W = \frac{2}{3} \times Q_1 = \frac{2}{3} \times (3 \times 10^6 \times 4.2\text{J}) = 8.4 \times 10^6 \text{ J}$$

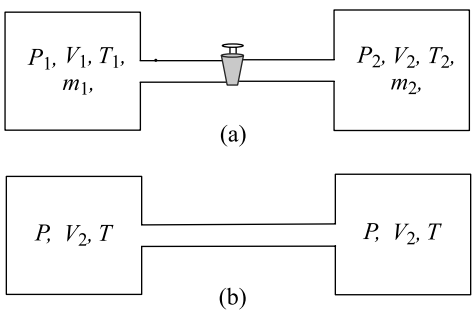
15. According to Newton's law of cooling, the rate of cooling is

$$\frac{d\theta}{dt} = -k(\Delta\theta),\ k = \text{constant}$$

Hence $n = 1$

16. The correct choice is (b). The internal energy does not change in an isothermal process. Entropy can change in an adiabatic process. Work done in an adiabatic process is not zero.

17. Let n_1 and n_2 be the number of moles of the gases in vessels 1 and 2 before the valve is closed [Fig. (a)]. Let P and T be the equilibrium pressure and temperature after the valve is opened [Fig. (b)]. The volume of gas in each vessel = volume of the vessel.



When the valve is closed [Fig. (a)]

$$P_1 V_1 = n_1 RT_1 \qquad \Rightarrow \qquad n_1 T_1 = \frac{P_1 V_1}{R} \qquad \text{(i)}$$

$$P_2 V_2 = n_2 RT_2 \qquad \Rightarrow \qquad n_2 T_2 = \frac{P_2 V_2}{R} \qquad \text{(ii)}$$

The internal energy of gas having f number of degrees of freedom is given by

$$U = \frac{f}{2}(nRT)$$

Since no heat is supplied or taken out of the system and no work is done by on the system, according to the first law of thermodynamics, the internal energy before the valve is opened = internal energy after the valve is opened, i.e.

$$\frac{f}{2}(n_1 R\ T_1) + \frac{f}{2}(n_2\ R\ T_2) = \frac{f}{2}(n_1 + n_2) R\ T$$

$$\Rightarrow \qquad n_1 T_1 + n_2\ T_2 = (n_1 + n_2) T \qquad \text{(iii)}$$

Using (i) and (ii) in (iii), we have

$$\frac{P_1 V_1}{R} + \frac{P_2 V_2}{R} = \left(\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2} \right) \frac{T}{R}$$

$$\Rightarrow \qquad T = \frac{(P_1 V_1 + P_2 V_2) T_1 T_2}{(P_1 V_1 T_2 + P_2 V_2 T_1)}$$

18. Let A be the area of each slab. In the steady state, the rate of heat flow through the composite slab is given by

$$\frac{Q}{t} = \frac{T_2 - T_1}{\frac{l_1}{K_1 A} + \frac{l_2}{K_2 A}} = \frac{A(T_2 - T_1)}{\frac{l_1}{K_1} + \frac{l_2}{K_2}} \qquad (1)$$

Given $l_1 = x$, $l_2 = 4x$, $K_1 = K$ and $K_2 = 2K$. Using these values in (1) we get

$$\frac{Q}{t} = \frac{A(T_2 - T_1)}{\frac{x}{k} + \frac{4x}{2K}} = \left[\frac{A(T_2 - T_1) K}{x} \right] \times \frac{1}{3}$$

Comparing this with the given rate of heat transfer, we get $f = \frac{1}{3}$.

19. Let the initial temperature be T and let M be the mass of the cube. Let A_0, ρ_0 and d_0 respectively be the base area of the cube, the density of the material of the cube and the depth up to which it is submerged in the liquid, the upthrust = $A_0\ d_0\ \rho_0\ g$. From the principle of floatation, weight = upthrust, i.e.,

$$M g = A_0\ d_0\ \rho_0\ g$$

or $$M = A_0\ d_0\ \rho_0 \qquad (1)$$

When the temperature is raised to $(T + \Delta T)$, let A, ρ and d be the base area, density and depth at this temperature. Now, the coefficient of superficial (area) expansion is $\beta = 2\alpha$.

Hence

$$A = A_0 (1 + \beta\ \Delta T)$$
$$= A_0 (1 + 2\ \alpha \Delta T) \qquad (2)$$

Also $$\rho = \frac{\rho_0}{(1 + \gamma\ \Delta T)} \qquad (3)$$

The upthrust at temperature $(T + \Delta T) = A\ d\ \rho\ g$. From the principle of floatation, we have

$$M g = A\ d\rho g$$

or $$M = A\ d\rho$$

From (1) and (4), we get

$$A\ d\rho = A_0 d_0 \rho_0$$

or $$A\rho = A_0 \rho_0 \qquad (\because d = d_0\ ;\ \text{given})$$

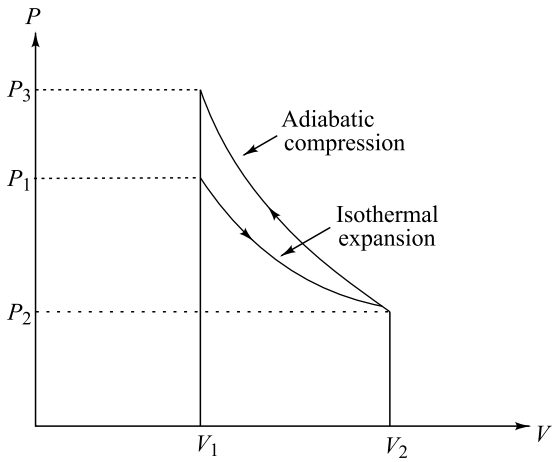
Using (2) and (3), we have

$$A_0 (1 + 2\ a\Delta T) \times \frac{\rho_0}{(1 + \gamma\ \Delta T)} = A_0 \rho_0$$

Which gives $2\alpha = \gamma$.

20. For an isothermal process : PV = constant and for an adiabatic process : PV^γ = constant, where γ is the ratio

of the two specific heats (C_p/C_v) of the gas. When a gas is compressed from a volume V to a volume ($V - \Delta V$), the increase in pressure is $(\Delta P)_{adia} = \dfrac{\gamma \Delta VP}{V}$ for an adiabatic compression and $(\Delta P)_{iso} = \dfrac{\Delta VP}{V}$ for an isothermal compression.



Hence P_3 will be greater than P_1. Therefore, the P–V diagrams of isothermal expansion from V_1, P_1 to V_2, P_2 and adiabatic compression for V_2, P_2 to V_1, P_2 are as shown in the figure. Let W_1 and W_2 be the work done in isothermal expansion and adiabatic compression respectively. Therefore, net work done is

$$W = W_1 + (-W_2) = W_1 - W_2$$

Now, the area under the adiabatic curve is more than that under the isothermal curve. Hence $W_2 > W_1$. Therefore, W< 0. Hence the correct choice is (c).

21. If a steady temperature difference $(\theta_1 - \theta_2)$ is maintained between the ends of a conducting rod of length L and cross-sectional area A, the rate of flow of heat through the rod is given by

$$q = \frac{kA(\theta_1 - \theta_2)}{L}$$

where k is the coefficient of thermal conductivity of the material of the rod. In the first experiment, the two rods are connected in series. If two rods of equal cross-sectional areas and of lengths L_1 and L_2 and conductivities k_1 and k_2 are joined in series, the equivalent conductivity k_2 is given by

$$\frac{L_1 + L_2}{ks} = \frac{L_1}{k_1} + \frac{L_2}{k_2}$$

For two identical rods, $L_1 = L_2 = L$ and $k_1 = k_2 = k$, in which case, Eq. (1) gives $k_2 = k$. Further, when two identical rods are joined in series, the length of the composite rod is ($2L$) but its cross-sectional area is A, the same as that of each rod. Hence the rate of flow of heat in this case is given by

$$q_1 = \frac{kA(\theta_1 - \theta_2)}{(2L)} \qquad (2)$$

In the second case, the two rods are connected in parallel. If two rods of equal lengths and equal cross-section areas and having conductivities k_1 an k_2 are joined in parallel, the equivalent conductivity of the composite rod is given by

$$K_p = k_1 + k_2$$

For two identical rods, $k_1 = k_2 = k$. Hence $k_p = (2k)$. Furthermore, the cross-sectional area of the composite rod is ($2A$). Therefore, in the second case, the rate of flow of heat is given by

$$q_2 = \frac{(2k)(2A)(\theta_1 - \theta_2)}{L} \qquad (3)$$

Dividing (2) by (3), we get $\dfrac{q_1}{q_2} = \dfrac{1}{8}$. Now, the rate of melting of ice is proportional to the rate of flow of heat. So, the correct choice is (*d*).

22. According to Stefan's law, the power radiated by a black body at absolute temperature T is given by

$$Q = \sigma AT^4 \qquad (1)$$

where A is the surface area of the body and σ is Stefan's constant. According to Wien's displacement law,

$$\lambda_m T = b$$

where λ_m is the wavelength corresponding to maximum emission of radiation and b is Wien's constant. Thus

$$T = \frac{b}{\lambda_m} \qquad (2)$$

Using (2) in (1), we get

$$Q = \sigma A\left(\frac{b}{\lambda_m}\right)^4 = \frac{\sigma b^4 A}{\lambda_m^4}$$

For a sphere of radius r, $A = 4\pi r^2$. Hence

$$Q_1 = \frac{\sigma b^4 4\pi r^2}{\lambda_m^4} = k\frac{r^2}{\lambda_m^4} \qquad (3)$$

where $k = 4\pi\sigma b^4$ is a constant. Hence

$$Q_1 = k\,\frac{r_1^2}{(\lambda_m^4)_1}$$

and

$$Q_1 = k\,\frac{r_2^2}{(\lambda_m^4)_2}$$

$$\therefore \quad \frac{Q_1}{Q_2} = \left(\frac{r_1}{r_2}\right)^2 \frac{(\lambda_m^4)_2}{(\lambda_m^4)_1}$$

$$= \left(\frac{3\text{ cm}}{5\text{ cm}}\right)^2 \times \left(\frac{500\text{ nm}}{300\text{ nm}}\right)^4 = \left(\frac{5}{3}\right)^2$$

23. The only incorrect statement is (d). The concept of entropy is related to the second law of thermodynamics.

24. For helium ; $n_1 = \frac{16}{4} = 4$ and $\gamma_1 = \frac{5}{3}$ (monoatomic)

For oxygen; $n_2 = \frac{16}{32} = \frac{1}{2}$ and $\gamma_2 = \frac{7}{5}$ (diatomic)

Refer to the solution of Question 3 of this section.

Use the formula

$$\frac{n_1 + n_2}{(\gamma - 1)} = \frac{n_1}{(\gamma_1 - 1)} + \frac{n_2}{(\gamma_2 - 1)}$$

which gives $\gamma = 1.62$

25. Consider a small element of thickness dx and radius x as shown in the figure. Since the thermal resistance is $R = \frac{1}{KA}$, the thermal resistance of the element is

$$dR = \frac{dx}{k(4\pi x^2)}$$

where k is the thermal conductivity of the material between the two spheres. The thermal resistance of the material is

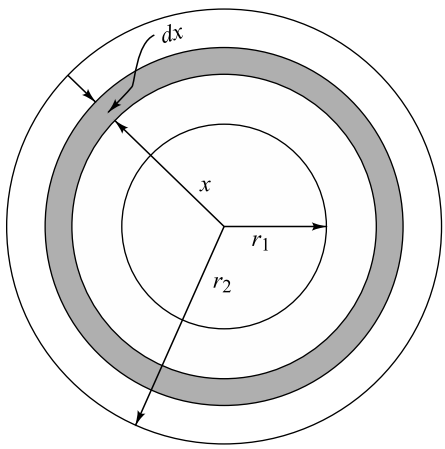
$$R = \int dR = \frac{1}{4\pi k}\int_{r_1}^{r_2} \frac{dx}{x^2}$$

$$= \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{r_2 - r_1}{4\pi k r_1 r_2}$$

The rate of flow of heat is

$$\frac{Q}{t} = \frac{(T_1 - T_2)}{R} = \frac{(T_1 - T_2) \times 4\pi k r_1 r_2}{(r_2 - r_1)}$$

Since $(T_1 - T_2)$ and k are constant, $\frac{Q}{t} \propto \frac{r_1\, r_2}{(r_2 - r_1)}$



26. The change in internal energy is independent of the path of followed by the processes; it depends only on the initial and the final states of the system. Hence $\Delta U_1 = \Delta U_2$.

27. Mass of 2 litres of water = 2 kg. Heat energy needed to raise the temperature of 2 kg of water from 20ºC to 75ºC is

$$Q = 2 \times (4.2 \times 10^3) \times 55 = 4.62 \times 10^5 \text{ J}$$

If t is the time taken, heat energy supplied by the heater in time t is

$$Q_1 = (\text{power} \times \text{time}) = 1000\ t \text{ joule}$$

Heat energy lost in time t is

$$Q_2 = 160\ t \text{ joule}$$

Heat energy available for heating water is

$$Q' = Q_1 - Q_2 = 840 \text{ J}$$

Equating $Q = Q'$, we get $t \simeq 550$ s.

28. The correct choice is (d). Heat transfer in boiling of water and in land and sea-breezes isn primarily due to convection. A metal rod placed over a flame is heated primarily by conduction. Heat transfer by convection and conduction takes place through matter (or medium). The electric bulb is evacuated. Hence heat transfer in an electric bulb is not due to conduction or convection.

29. The correct choice is (b). According to Stefan's law, the energy radiated per second by a unit area of a body of emissivity ε is $\varepsilon\ \sigma\ T_4$, irrespective of the surroundings.

30. Surface area of the Sun = $4\ \pi R^2$. From Stefan's law, the power radiated by the Sun is

$$P_s = \sigma A T^4 = 4\ \pi R^2\ \sigma T^4$$

This power is radiated by the sun in all directions. Therefore, pwer received per unit area on earth's surface is $P_s/4\pi\ r^2$. Hence, the total power incident on the earth is

$$P_e = \frac{P_s}{4\pi r^2} \times \text{surface area of earth}$$

$$= \frac{4\pi R^2 \sigma T^4}{4\pi r^2} \times 4\pi r_0^2$$

$$= \frac{4\pi r_0^2 R^2 \sigma T^4}{r^2}$$

31. The change in internal energy of an ideal gas is given by

$$\Delta U = n\ C_v\ \Delta T = nC_v\ (T_f - T_i)$$

where n = number of moles and T_i = initial temperature.

For a diatomic gas (such as nitrogen) $C_v = \frac{5}{2}R$ and for a monoatomic gas (such as helium) $C_v = \frac{3}{2}R$.

$\therefore$ ΔU for oxygen $= 1\times \frac{5R}{2} \times (T_f - T_o)$

and ΔU for helium $= 1 \times \frac{3R}{2}\left(T_f - \frac{7T_o}{3}\right)$

Since no heat is given to or taken from the surrounding, the total internal energy of the system cannot change. Therefore

$$1 \times \frac{5R}{2} \times (T_f - T_o) + 1 \times \frac{3R}{2}\left(T_f - \frac{7T_o}{3}\right) = 0$$

Which gives $T_f = \frac{3}{2}T_o$, which is choice (a).

32. An adiabatic process, the work is given by

$$W = \frac{nR(T_1 - T_2)}{\gamma - 1}$$

or $146 \times 10^3 = \frac{1000 \times 8.3 \times 7}{(\gamma - 1)}$ $(\because \Delta T = 7^\circ C = 7K)$

which gives $\gamma = 1 + 0.4 = 1.4$. Hence the gas is diatomic

33. The efficiency of a Carnot engine is given by

$$\eta = \frac{\text{Work done per cycle}}{\text{Heat energy supplied per cycle}}$$

$$= \frac{W}{Q_1} = 1 - \frac{T_2}{T_1}$$

where T_1 = temperature of the source and T_2 = temperature of the sink. Given $\eta = 10\% = 0.1$. Hence

$$0.1 = 1 - \frac{T_2}{T_1} \Rightarrow \quad T_2 = 0.9\ T_1$$

The performance of a refrigerator is given by

$$\beta = \frac{\text{Heat absorbed per cycle}}{\text{Work done per cycle}}$$

$$= \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2} = \frac{0.9T_1}{T_1 - 0.9T_1} = 9$$

$\therefore$ $Q_2 = 9W = 9 \times 10 = 90$J

34. Let T_0 be the temperature at the interface and A be the cross-sectional area of each rod.

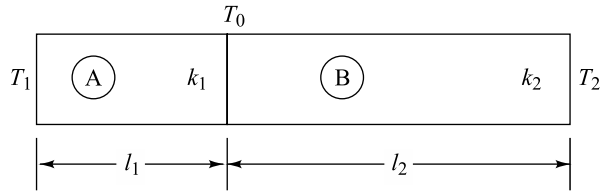


Fig 8. 68

In the steady state, the rate of flow of heat through rod A = rate of flow of heat through rod B, i.e.

$$\frac{Q_1}{t} = \frac{Q_2}{t}$$

$$\Rightarrow \quad \frac{k_1 A(T_1 - T_0)}{l_1} = \frac{k_2 A(T_0 - T_2)}{l_2}$$

$$\Rightarrow \quad k_1\, l_2(T_1 - T_0) = k_2\, l_1(T_0 - T_2)$$

$$\Rightarrow \quad T_0 = \frac{(k_1 l_2 T_1 + k_2 l_1 T_2)}{(k_1 l_2 + k_2 l_1)}$$

35. Let C'_p and C'_v be the molar specific heats of gas at constant pressure and constant volume repectively. Then

$$C'p - C'_v = R \qquad (1)$$

The molecular mass of nitrogen is 28. Therefore, the specific heats per unit mass of nitrogen at constant pressure and at constant volume respectively are

$$C_p = \frac{C'_p}{28} \text{ and } C_v = \frac{C'_v}{28}$$

Using these in Eq. (1), we get $C_p - C_v = \frac{R}{28}$.

36. From the first law of thermodynamics, we have

$$Q = W + \Delta U$$

Where Q = heat energy supplied, W = work done and ΔU = change in internal energy. For the paths *iaf* and *ibf*, we have

$$50 = 20 + \Delta U \qquad (1)$$

and $$36 = W + \Delta U \qquad (2)$$

Eliminating ΔU from Eqs. (1) and (2) we get $W = 6$ cal.

37. Heat required to melt the whole of ice is

$Q_1 = ms\Delta T + mL$

$= 400 \times 0.5 \times 20 + 400 \times 80$ $(\therefore 253\ K = -20^\circ C)$

$= 4000 + 32000 = 36000$ cal

The maximum heat released by steam when the whole of it (= 50g) is converted into water at 0ºC is

$Q_2 = ms\Delta T + mL$

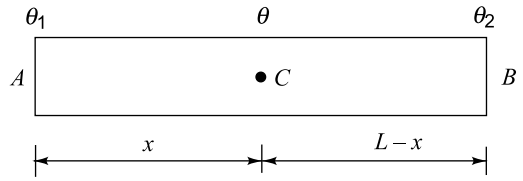
$= 50 \times 1 \times 100 + 540 \times 50$

$= 5000 + 27000 = 32000$ cal

Since Q_2 is less than Q_1, the whole of ice will not melt. Hence the final temperature of the mixture will be 0ºC or 273 K.

38. Refer to the solution of Question 18 of this section. The correct choice is (a).

39. Let θ be the temperature at point C at a distance x from end A of the bar whose ends are kept at temperatures θ_1 and θ_2 with $\theta_1 > \theta_2$. L = length of the bar. [see the figure below]



In the steady state, the rate of flow of heat from A to C = rate of flow of heat from C to B, i.e.

$$\frac{KA(\theta_1-\theta)}{x}=\frac{KA(\theta-\theta_2)}{x}$$

$$\Rightarrow \qquad \theta=\theta_1-\left(\frac{\theta_1-\theta_2}{L}\right)x$$

Thus, the graph of θ versus x is a straight line with a positive intercept and a negative slope.

40. $P = 8 \times 10^4$ N m^{-2}, $V = \dfrac{m}{p}=\dfrac{1}{4}$ m^3.

Thermal energy $=\dfrac{5}{2}\ nRT=\dfrac{5}{2}PV$

$$=\frac{5}{2}\times(8\times10^4)\times\frac{1}{4}=5\times10^4$$

41. $PV = nRT$. Since P = constant in process A to B

$W_{A\to B} = P\Delta V = nR\ \Delta T = 2 \times R \times (500 - 300) = 400\ R$

42. Process D to A is isothermal. Since the pressure increases from D to A, the volume of the gas decreases. hence work is done on the gas. Thus, $W_{D\to A}$ is negative

$$W_{D\to A} = 2.303\ nRT \log 10\left(\frac{P_D}{P_A}\right)$$

$$= -2.303 \times 2 \times R \times 300 \log 10\left(\frac{1\times10^5}{2\times10^5}\right)$$

$$= +414 \text{ R}$$

43. Net work done in the cycle $ABCDA$ is

$$W_{\text{net}} = W_{A\to B} + W_{B\to C} + W_{C\to D} + W_{D\to A}$$

$$= nR\ (T_B - T_A) + nRT_B \text{ In}\left(\frac{P_B}{P_C}\right) + nR\ (T_D - T_C) + nRT_D \text{ In}\left(\frac{P_D}{P_A}\right)$$

$$= 400\ R + nRT_B \text{ In } (2) - 400\ R + nRT_D \text{ In}\left(\frac{1}{2}\right)$$

$$= nR(T_B - T_D) \text{ In } (2)$$

$$= 2 \times R \times (500 - 300) \times 0.693$$

$$= 277\ R.$$ So the closest choice is (d).

44. Efficiency of the engine is

$$\eta = 1 - \frac{T_2}{T_1}$$

For an adiabatic process, $TV^{\gamma-1}$ + constant.

$$T_1V_1^{\gamma-1} = T_2\ V_2^{\gamma-1}$$

$$\Rightarrow \qquad \frac{T_2}{T_1}=\left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

For a diatomic gas $\gamma = \dfrac{7}{5}$. Hence

$$\frac{T_2}{T_1}=\left(\frac{32V}{V}\right)^{\left(\frac{7}{5}-1\right)}=(32)^{2/5}=4$$

$$\therefore \qquad \eta = 1 - \frac{1}{4}=\frac{3}{4}=0.75$$

45. The correct choice is (d)

46. $\Delta Q = ms\ \Delta T = (100 \times 10^{-3}) \times 4184 \times 20 = 8.4 \times 103$ J

$= 9.4$ kJ

Since volume does not change, $\Delta W = 0$. Hence

$\Delta U = \Delta Q - \Delta W = 8.4 - 0 = 8.4$ kJ

47. $\gamma = \dfrac{\text{stress}}{\text{strain}} = \dfrac{F/S}{\Delta L/L}$

$$\Rightarrow \qquad F = \gamma\ \frac{\Delta L}{L}S$$

But $\dfrac{\Delta L}{L} = \alpha\Delta T$

$$F = \gamma\ \alpha\ S\ \Delta\ T$$

Since the wheel has two semicircular parts, the force which one part exerts on the other is

$$F' = 2F = 2\ \gamma\ \alpha\ S\ \Delta\ T$$

48. According to Newton's law of cooling,

$$\frac{d\theta}{dt} = -k\ (\theta - \theta_0)$$

$$\Rightarrow \qquad \frac{d\theta}{\theta-\theta_0} = -kdt$$

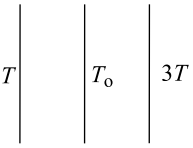
Integrating,

$$\int_{\theta_0}^{\theta}\frac{d\theta}{\theta-\theta_0} = -k\int_0^t dt$$

$$\Rightarrow \qquad \ln(\theta - \theta_0) = -kt + c$$

where c is the constant of integration. Hence the graph of $\ln(\theta - \theta_0)$ vs t is a straight line having a negative slope. So the correct choice is (a).

49. In the steady state, the rate at which the middle plate receives heat energy is equal to the rate at which heat energy is emitted by the other plates. Let A be the area of each plate and T_o be the steady state temperature of the middle plate. Since both sides of the middle plate receive heat energy, the total area of the middle plate receiving energy is $2A$.



From Stefan's law

$$\sigma(2A)(T_o)^4 = \sigma A(2T)^4 + \sigma A(3T)^4$$

$$\Rightarrow \quad 2T_o^4 = 16T^4 + 81T^4 = 97T^4$$

$$\Rightarrow \quad T_o = \left(\frac{97}{2}\right)^{1/4} T$$

50. Heat energy needed to decrease the radius of the drop from R to $(R - \Delta R)$ is

$$\Delta Q = T \times 4\pi[R^2 - (R - \Delta R)^2]$$

$$= T \times 4\pi[R^2 - R^2 - 2R\Delta R)^2] \qquad (\because \Delta R \ll R)$$

$$= 8\pi T R \Delta R \qquad (1)$$

If Δm is the mass of liquid evaporated, then heat energy needed is also given by

$$\Delta Q = (\Delta m)L = (4\pi R^2 \rho \Delta R)L \qquad (2)$$

Equating (1) and (2) we get

$$(4\pi R^2 \rho \Delta R)L = 8\pi T R \Delta R$$

$$\Rightarrow \quad R = \frac{2T}{\rho L}$$

51. Initially the temperature of the metal piece falls rapidly with time because $(\theta - \theta_0)$ is high. The fall in temperature is non-linear. Finally the piece attains the room temperature. So the correct graph is (b).

52. $\frac{Q}{t} = \frac{\Delta T}{R}$; (R = thermal resistance)

$$\Rightarrow \frac{Q}{\Delta T} = \frac{t}{R} \qquad \text{(i)}$$

Since Q and ΔT are the same = $\frac{t}{R}$ constant. Hence

$$\frac{t_1}{R_1} = \frac{t_2}{R_2}$$

Configuration 1 : $R_1 = \frac{l}{KA} + \frac{l}{2KA} = \frac{3l}{2KA}$

Configuration 2 : $\frac{1}{R_2} = \frac{2KA}{l} + \frac{KA}{l} = \frac{3KA}{l}$

$$\Rightarrow \quad R_2 = \frac{l}{3KA}$$

Using these in (i),

$$\frac{9}{3l/2KA} = \frac{t_2}{l/3KA} \quad \Rightarrow t_2 = 2\text{s}$$

53. Refer to the solution of question 18 of this section. The correct choice is (d).

54. Refer to the solution of question 44 of this section. The correct choice is (b).

55. Let T be the equilibrium temperature. Since the partition between then two compartments is regid, the transfer of heat from the lower compartment (which contains a monoatomic gas at 700 K) to the upper compartment takes at constant volume.

$\therefore$ Heat transferred from lower to upper compartment is

$$Q_1 = nC_v \Delta T = 2 \times \frac{3R}{2} \times (700 - T) \qquad (1)$$

Since the position and the top of the upper compartment are frictionless, heat Q_2 is gained by the diatomic gas in the upper compartment at constant pressure, i.e.

$$Q_1 = nC_p \Delta T = 2 \times \frac{7R}{2} \times (T - 400) \qquad (2)$$

Equating (1) and (2),

$$2 \times \frac{3R}{2} \times (700 - T) = 2 \times \frac{7R}{2} \times (T' - 400)$$

$$\Rightarrow \quad 3 \times (700 - T) = 7 \times (T - 400)$$

$$T = 490 \text{ K}$$

56. If the partition between the two compartments is frictionless, the heat exchange between the gases takes place at constant pressure. If T′ is the equilibrium temperature now, then

$$2 \times \frac{5R}{2} \times (700 - T') = 2 \times \frac{7R}{2} \times (T' - 400)$$

$$\Rightarrow \quad T' = 525 \text{ K}$$

Since the temperature of the gas in the lower compartment falls, the work done by the gas is

$$W_1 = -nR\Delta T = -2 \times R \times (700 - 525) = -350R$$

Since the temperature of the gas in the upper compartment increases, the work done by the gas is

$$W_2 = +nR\Delta T = +2 \times R \times (525 - 400) = +250R$$

$\therefore$ Net work done $= W_1 + W_2 = -350R + 250R$

$$= -100R$$

KINETIC THEORY OF GASES

Chapter 9

REVIEW OF BASIC CONCEPTS

1. Assumptions Regarding the Model of an Ideal Gas

(i) A gas consists of very tiny particles called molecules which are identical in all respects.

(ii) The molecules are in random motion and obey Newton's laws of motion.

(iii) The total number of molecules is very large.

(iv) The volume of molecules is negligibly small compared to the volume occupied by the gas.

(v) The molecules do not exert any appreciable force on each other except during collision.

(vi) The collisions of molecules between themselves and with the walls of the container are perfectly elastic. The collisions are of negligible duration.

2. Pressure Exerted by an Ideal Gas

The pressure of a gas in a container is a result of the continuous bombardment of the gas molecules against the walls of the container and is given by

$$P = \frac{1}{3}\frac{mnv_{\text{rms}}^2}{V} = \frac{1}{3}\frac{Mv_{\text{rms}}^2}{V} = \frac{1}{3}\rho v_{\text{rms}}^2 = \frac{2}{3}\frac{U}{V}$$

where m = mass of each molecule, n = number of molecules in the container, v_{rms} = root mean square speed of molecules, V = volume of container, M = mass of gas in the container, ρ = density of the gas and U = internal energy of the gas.

3. Root Mean Square Speed

The root mean square (rms) speed is defined as

$$v_{\text{rms}} = \left[\frac{1}{n}\left(v_1^2 + v_2^2 + v_3^2 + \cdots + v_n^2\right)\right]^{1/2}$$

where $v_1, v_2, v_3, \cdots v_n$ are the speeds of the molecules 1, 2, 3, $\cdots$ n respectively. In terms of P and ρ, v_{rms} is given by

$$v_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$$

where m is the mass of each molecule, T is the absolute temperature of the gas and k is a constant called *Boltzmann's constant*. Its value is

$$k = 1.38 \times 10^{-23}\ \text{J K}^{-1} \text{ per molecule}$$

R is the universal gas constant and its value is

$$R = 8.315\ \text{J K}^{-1}\ \text{mol}^{-1}$$

4. Mean Translational Kinetic Energy

The mean translational kinetic energy of a molecule of a gas is given by

$$E = \frac{1}{2}mv_{\text{rms}}^2$$

In terms of E, the pressure of the gas is given by

$$P = \frac{1}{3}\rho v_{\text{rms}}^2 = \frac{2}{3}\frac{nE}{V} = \frac{2}{3}\frac{U}{V}$$

where $U = nE$ is the total translational kinetic energy of all the n molecules of the gas. It is also called the internal energy of the gas.

5. Avogadro's Law

Equal volumes of all gases at the same temperature and pressure contain an equal number of molecules. This is Avogadro's law. The number of molecules in one mole of any gas is 6.0255×10^{23} which is called the *Avogadro number*. The amount of a substance which contains as many elementary units as there are in 0.012 kg (or 12 g) or carbon-12 is called a *mole*. In other words, a mole of substance is the number of grams equal to its molecular mass in grams.

Thus, the mass of 1 mole of carbon is 12 g or 0.012 kg and that of 1 mole of oxygen is 32 g(0.032 kg).

6. Equation of State of an Ideal Gas

The relationship between pressure P, volume V and absolute temperature T of an ideal gas is called the equation of state. For n moles of a gas, this relation is

$$PV = nRT$$

where R is the molar gas constant.

From Avogadro's law, it follows that one mole of all gases, at the same temperature and pressure, occupies equal volume. Experiments confirm that one mole of any gas occupies 22.4 litres at STP. Consequently, for one mole the ratio $\frac{PV}{T}$ is constant for all gases. This constant is the molar gas constant and can be evaluated as follows:

At STP, $V = 22.4 \text{ litre} = 22.4 \times 10^{-3} \text{ m}^3$

$P = 0.76 \text{ m Hg} = 1.013 \times 10^5 \text{ Nm}^{-2}$

$T = 273 \text{ K}$

$$\therefore \quad R = \frac{PV}{T} = \frac{1.013 \times 10^5 \times 22.4 \times 10^{-3}}{273}$$

$$= 8.315 \text{ J mol}^{-1} \text{ K}^{-1}$$

7. Van der Waal's Equation of State

According to Van der Waal, the true pressure exerted by a gas is greater than P by an amount a/V^2 (where a is a constant) due to attractive forces between molecules and the true volume of the gas is less than V by an amount b (where b is another constant) because molecules themselves occupy a finite space. The Van der Waal's equation of state is

$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT$$

At high pressures, when the molecules are too many and too close together, the correction factors a and b both become important. But at low pressures, when they are not too many and not too close together, a gas behaves like an ideal gas and obeys the equation $PV = nRT$.

8. Degrees of Freedom and Equipartition of Energy

The total number of coordinates or independent quantities required to completely specify the position or configuration of a dynamical system is called the degrees of freedom of the system.

The molecules of a monoatomic gas consist of single atoms. Therefore, the molecules of a monoatomic gas have *three* degrees of freedom corresponding to translational motion. The molecules of a diatomic gas have *five* degrees of freedom-three corresponding to translational motion and two for rotational motion. A polyatomic molecule has *six* degrees of freedom including one of vibrational motion.

The law of equipartition of energy is stated as follows. In any dynamical system with a uniform absolute temperature T, the total energy is distributed equally among all the degrees of freedom and the average energy per degree of freedom per molecule equals $\frac{1}{2}kT$, where $k = 1.38 \times 10^{-23}$ J K^{-1}.

If the molecules of a gas have f degrees of freedom, then kinetic energy per molecule $= f \times \frac{1}{2}kT$. Therefore, kinetic energy per mole is

$$U = Nf \times \frac{1}{2}kT = \frac{f}{2}RT$$

Now $$C_v = \frac{\Delta U}{\Delta T} = \frac{f}{2}R$$

and $$C_p = C_v + R = \frac{f}{2}R + R = \left(\frac{f}{2} + 1\right)R$$

$$\therefore \quad \gamma = \frac{C_p}{C_v} = \frac{\left(\frac{f}{2} + 1\right)R}{\frac{f}{2}R} = 1 + \frac{2}{f}$$

Thus $\gamma = 1 + \frac{2}{3} = \frac{5}{3}$ for a monoatomic gas $= 1 + \frac{2}{5} = \frac{7}{5}$ for a diatomic gas $= 1 + \frac{2}{6} = \frac{4}{3}$ for a triatomic or polyatomic gas

EXAMPLE 1 Calculate the root mean square speed of the molecules of hydrogen gas at S.T.P. Density of hydrogen at S.T.P. is 9×10^{-2} kg m^{-3}.

SOLUTION At S.T.P., pressure $P = 1.01 \times 10^5$ Pa and density $\rho = 9 \times 10^{-2}$ kg m^{-3}.

$$v_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.01 \times 10^5}{9 \times 10^{-2}}}$$

$$= 1840 \text{ ms}^{-1}$$

EXAMPLE 2 Calculate the temperature (in kelvin) at which the root mean square speed of a gas molecule is half its value at 0°C.

SOLUTION $$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

$$\therefore \quad \frac{v'_{\text{rms}}}{v_{\text{rms}}} = \sqrt{\frac{T'}{T}} = \sqrt{\frac{T'}{273}}$$

$$\Rightarrow \qquad \frac{1}{2} = \sqrt{\frac{T'}{273}} \quad \Rightarrow \quad T' = 68.25 \text{ K}$$

EXAMPLE 3 Find the mean translational kinetic energy of an oxygen molecule at 0°C. Given Avogadro number $N = 6.03 \times 10^{23}$ per mole and $R = 8.3$ JK^{-1} mol^{-1}.

SOLUTION $E = \frac{1}{2} mv_{\text{rms}}^2 = \frac{3}{2} kT = \frac{3}{2} \frac{RT}{N}$

$$\therefore \qquad E = \frac{3 \times 8.3 \times 273}{2 \times 6.03 \times 10^{23}} = 5.64 \times 10^{-21} \text{ J}$$

EXAMPLE 4 Calculate the mean translational kinetic energy of 1 mole of hydrogen at S.T.P. Density of hydrogen at S.T.P. is 0.09 kg m^{-3}.

SOLUTION $v_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.01 \times 10^5}{0.09}}$

$$= 1.84 \times 10^3 \text{ ms}^{-1}$$

Mass of 1 mole is

$$m = 22.4 \times 10^{-3} \text{ m}^3 \times 0.09 \text{ kg m}^{-3}$$
$$= 2.016 \times 10^{-3} \text{ kg}$$

$$\therefore \qquad \text{K.E.} = \frac{1}{2} mv_{\text{rms}}^2$$

$$= \frac{1}{2} \times (2.016 \times 10^{-3}) \times (1.84 \times 10^3)^2$$
$$= 3.41 \times 10^3 \text{ J}$$

EXAMPLE 5 Calculate (a) the average translational kinetic energy of the molecules of an ideal gas at 0°C and at 100°C and (b) the energy per mole of the gas at 0°C and 100°C. Given Avogadro's number $N = 6.02 \times 10^{23}$ and Boltzmann's constant $k = 1.38 \times 10^{-23}$ JK^{-1}.

SOLUTION

(a) Average translational K.E. of a molecule of an ideal gas is

$$E = \frac{3}{2} kT,$$

where T = temperature in kelvin

At $T = 0°\text{C} = 273$ K,

$$E = \frac{3}{2} \times (1.38 \times 10^{-23}) \times 273$$
$$= 5.65 \times 10^{-21} \text{ J}$$

At $T = 100°\text{C} = 373$ K,

$$E = \frac{3}{2} \times (1.38 \times 10^{-23}) \times 373$$
$$= 7.72 \times 10^{-21} \text{ J}$$

(b) Number of molecules in 1 mole of a gas is $N = 6.02 \times 10^{23}$

$\therefore$ K.E. of 1 mole at 273 K = $(5.65 \times 10^{-21}) \times (6.02 \times 10^{23}) = 3.40$ J

K.E. of 1 mole at 373 K = $(7.72 \times 10^{-21}) \times (6.02 \times 10^{23}) = 4.65$ J

EXAMPLE 6 The speed of sound in a gas at S.T.P. is 330 ms^{-1} and the density of the gas is 1.3 kg m^{-3}. Find the number of degrees of freedom of a molecule of the gas.

SOLUTION $v = \sqrt{\frac{\gamma P}{\rho}}$

$$\therefore \qquad \gamma = \frac{v^2 \rho}{P} = \frac{(330)^2 \times 1.3}{1.01 \times 10^5} = 1.4$$

If f is the number of degrees of freedom, then

$$f = \frac{2}{\gamma - 1} = \frac{2}{(1.4 - 1)} = 5$$

EXAMPLE 7 The volume of 2 moles of a diatomic ideal gas at 300 K is doubled keeping its pressure constant. Find the change in the internal energy of the gas. Given $R = 8.3$ JK^{-1} mol^{-1}.

SOLUTION According to kinetic theory, there are no internal forces of interaction between the molecules of an ideal gas. This implies that the potential energy is zero. Hence, for an ideal gas, the internal energy is only due to kinetic energy of the molecules.

For n moles of an ideal gas at absolute temperature T, the internal energy is

$$U = \frac{f}{2} nRT$$

where f is the number of degrees of freedom. For a diatomic gas $f = 5$. Therefore

$$U = \frac{5}{2} nRT$$

Since pressure is kept constant, $V \propto T$ (Charles' law), i.e.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\Rightarrow \qquad T_2 = \left(\frac{V_2}{V_1}\right) T_1 = 2T_1 = 600 \text{ K} \qquad (\because V_2 = 2V_1)$$

$$\therefore \qquad \Delta U = U_2 - U_1 = \frac{5}{2} nR(T_2 - T_1)$$

$$= \frac{5}{2} \times 2 \times 8.3 \times (600 - 300)$$
$$= 1.245 \times 10^4 \text{ J}$$

Alternative Method

$$\Delta U = nC_v\Delta T$$

For diatomic gas, $C_v = \frac{5R}{2}$. Therefore

$$\Delta U = 2 \times \frac{5R}{2} \times (600 - 300)$$

$$= 1.245 \times 10^4 \text{ J}$$

SECTION 1 Multiple Choice Questions with One Correct Choice

1. The root mean square speed of gas molecules
   (a) is the same for all gases at the same temperature
   (b) depends on the mass of the gas molecule and its temperature
   (c) is independent of the density and pressure of the gas
   (d) depends only on the temperature and volume of the gas
2. The relation between volume V, pressure P and absolute temperature T of an ideal gas is $PV = xT$ where x is a constant. The value of x depends upon
   (a) the mass of the gas molecule
   (b) the average kinetic energy of the gas molecules
   (c) P, V and T
   (d) the number of gas molecules in volume V
3. The equation of state for n moles of an ideal gas is $PV = n\,RT$, where R is a constant. The SI unit for R is
   (a) J K^{-1} per molecule (b) J kg^{-1} K^{-1}
   (c) J K^{-1} mol^{-1} (d) J K^{-1} g^{-1}
4. Four molecules of a gas have speeds 1, 2, 3 and 4 km s^{-1}. The value of the root mean square speed of the gas molecule is
   (a) $\frac{1}{2}\sqrt{15}$ km s^{-1} (b) $\frac{1}{2}\sqrt{10}$ km s^{-1}
   (c) 2.5 km s^{-1} (d) $\sqrt{15/2}$ km s^{-1}
5. The average kinetic energy of a molecule of a gas at absolute temperature T is proportional to
   (a) $1/T$ (b) $\sqrt{T}$
   (c) T (d) T^2
6. The root mean square speed of the molecules of a gas at absolute temperature T is proportional to
   (a) $1/T$ (b) $\sqrt{T}$
   (c) T (d) T^2
7. The following four gases are at the same temperature. In which gas do the molecules have the maximum root mean square speed?
   (a) Hydrogen (b) Oxygen
   (c) Nitrogen (d) Carbon dioxide
8. If k is the Boltzmann constant, the average kinetic energy of a gas molecule at absolute temperature T is
   (a) $k\,T/2$ (b) $3\,k\,T/4$
   (c) $k\,T$ (d) $3\,k\,T/2$
9. E_0 and E_h respectively represent the average kinetic energy of a molecule of oxygen and hydrogen. If the two gases are at the same temperature, which of the following statements is true?
   (a) $E_0 > E_h$ (b) $E_0 = E_h$
   (c) $E_0 < E_h$
   (d) Nothing can be said about the magnitude of E_0 and E_h as the information given is insufficient.
10. Choose the correct statement from the following:
    (a) The average kinetic energy of a molecule of any gas is the same at the same temperature.
    (b) The average kinetic energy of a molecule of a gas is independent of its temperature.
    (c) The average kinetic energy of 1 g of any gas is the same at the same temperature.
    (d) The average kinetic energy of 1 g of a gas is independent of its temperature.
11. The root mean square speed of the molecules of an enclosed gas is v. What will be the root mean square speed if the pressure is doubled, the temperature remaining the same?
    (a) $v/2$ (b) v
    (c) $2\,v$ (d) $4\,v$
12. The mass of an oxygen molecule is about 16 times that of a hydrogen molecule. At room temperature the rms speed of oxygen molecules is v. The rms speed

of the hydrogen molecules at the same temperature will be

(a) $v/16$ (b) $v/4$
(c) $4v$ (d) $16v$

13. The average kinetic energy of hydrogen molecules at 300 K is E. At the same temperature, the average kinetic energy of oxygen molecules will be

(a) $E/16$ (b) $E/4$
(c) E (d) $4E$

14. Choose the only correct statement from the following:

(a) the pressure of a gas is equal to the total kinetic energy of the molecules in a unit volume of the gas.
(b) the product of pressure and volume of a gas is always constant.
(c) the average kinetic energy of molecules of a gas is proportional to its absolute temperature.
(d) the average kinetic energy of molecules of a gas is proportional to the square root of its absolute temperature.

15. If a gas has f degrees of freedom, the ratio C_p/C_v of the gas is

(a) $\frac{1+f}{2}$ (b) $1+\frac{f}{2}$

(c) $\frac{1}{2}+f$ (d) $1+\frac{2}{f}$

16. At room temperature (27°C) the rms speed of the molecules of a certain diatomic gas is found to be 1920 ms^{-1}. The gas is

(a) H_2 (b) F_2
(c) O_2 (d) Cl_2

17. The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K, the root mean square speed of the gas molecules is v, then at 480 K it will be

(a) $4v$ (b) $2v$

(c) $\frac{v}{2}$ (d) $\frac{v}{4}$

18. Three closed vessels A, B and C are at the same temperature. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of O_2 molecules in vessel A is v_1, that of N_2 molecules in vessel B is v_2, the average speed of O_2 molecules is vessel C is

(a) $\frac{1}{2}(v_1+v_2)$ (b) v_1
(c) $\sqrt{v_1 v_2}$ (d) $\sqrt{\frac{3kT}{M}}$

19. The average translational energy and the rms speed of molecules of a sample of oxygen gas at 300 K are 6.21×10^{-21} J and 484 ms^{-1} respectively. The corresponding values at 600 K are nearly (assuming ideal gas behaviour)

(a) 12.42×10^{-21} J, 968 ms^{-1}
(b) 8.78×10^{-21} J, 684 ms^{-1}
(c) 6.21×10^{-21} J, 968 ms^{-1}
(d) 12.42×10^{-21} J, 684 ms^{-1}

20. A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of He gas (molar mass 4) at a temperature $2T$ has a pressure of

(a) $\frac{P}{8}$ (b) P
(c) $2P$ (d) $8P$

21. A vessel contains a mixture of 1 mole of oxygen and two moles of nitrogen at 300 K. The ratio of the rotational kinetic energy per O_2 molecule to that per N_2 molecule is

(a) 1 : 1 (b) 1 : 2
(c) 2 : 1
(d) depends on the moment of inertia of the two molecules.

22. Two thermally insulated vessels 1 and 2 are filled with air at temperature (T_1, T_2), volume (V_1, V_2) and pressure (P_1, P_2) respectively. If the valve joining the two vessels is opened, the temperature inside the vessel at equilibrium will be

(a) $T_1 + T_2$ (b) $(T_1 + T_2)/2$
(c) $T_1T_2\,(P_1V_1 + P_2V_2)/(P_1V_1T_2 + P_2V_2\,T_1)$
(d) $T_1T_2\,(P_1V_1 + P_2V_2)/(P_1V_1T_1 + P_2V_2T_2)$

23. Which of the following phenomena gives evidence of the molecular structure of matter?

(a) Brownian movement
(b) Diffusion
(c) Evaporation
(d) All the above

24. Choose the correct statement. When the temperature of a gas is increased

(a) the kinetic energy of its molecules increases
(b) the potential energy of its molecules increases

(c) the potential energy decreases and the kinetic energy increases; the total energy remaining unchanged

(d) the potential energy increases and the kinetic energy decreases; the total energy remaining unchanged

25. If the temperature of a gas is increased from 27°C to 927°C, the root mean square speed of its molecules

(a) becomes half (b) becomes doubled

(c) becomes four times (d) remains unchanged

26. At what temperature will oxygen molecules have the same root mean square speed as hydrogen molecules at 200 K?

(a) 527°C (b) 1327°C

(c) 2127°C (d) 2927°C

27. At what temperature would the root mean square speed of a gas molecule have twice its value at 100°C?

(a) 200°C (b) 273°C

(c) 1219°C (d) 1492°C

28. An enclosure of volume V contains a mixture of 8 g of oxygen, 14 g of nitrogen and 22 g of carbon dioxide at absolute temperature T. The pressure of the mixture of gases is (R is universal gas constant)

(a) $\frac{RT}{V}$ (b) $\frac{3RT}{2V}$

(c) $\frac{5RT}{4V}$ (d) $\frac{7RT}{5V}$

29. At what absolute temperature T is the root mean square speed of a hydrogen molecule equal to its escape velocity from the surface of the moon? The radius of moon is R, g is the acceleration due to gravity on moon's surface, m is the mass of a hydrogen molecule and k is the Boltzmann constant

(a) $\frac{mgR}{2k}$ (b) $\frac{2mgR}{k}$

(c) $\frac{3mgR}{2k}$ (d) $\frac{2mgR}{3k}$

30. Two ideal gases at absolute temperatures T_1 and T_2 are mixed. There is no loss of energy in this process. If n_1 and n_2 are the respective number of molecules of the gases, the temperature of the mixture will be

(a) $\frac{n_1T_1 + n_2T_2}{n_1 + n_2}$ (b) $\frac{n_2T_1 + n_1T_2}{n_1 + n_2}$

(c) $T_1 + \frac{n_2}{n_1}T_2$ (d) $T_2 + \frac{n_1}{n_2}T_1$

31. An insulated box containing a diatomic gas of molar mass M is moving with a velocity v. The box is suddenly stopped. The resulting change in temperature is (R is the gas constant)

(a) $\frac{Mv^2}{2R}$ (b) $\frac{Mv^2}{3R}$

(c) $\frac{Mv^2}{5R}$ (d) $\frac{2Mv^2}{5R}$

32. 0.014 kg of nitrogen is enclosed in a vessel at a temperature of 27 °C. The amount of heat energy to be supplied to the gas to double the rms speed of its molecules is approximately equal to

(a) 6350 J (b) 7350 J

(c) 8350 J (d) 9350 J

33. A vessel of volume V contains an ideal gas at absolute temperature T and pressure P. The gas is allowed to leak till its pressure falls to P'. Assuming that the temperature remains constant during leakage, the number of moles of the gas that have leaked is

(a) $\frac{V}{RT}(P + P')$ (b) $\frac{V}{2RT}(P + P')$

(c) $\frac{V}{RT}(P - P')$ (d) $\frac{V}{2RT}(P - P')$



Answers

1.(b)	2. (d)	3. (c)	4. (d)
5. (c)	6. (b)	7. (a)	8. (d)
9. (b)	10. (a)	11. (b)	12. (c)
13. (c)	14. (c)	15. (d)	16. (a)
17. (b)	18. (b)	19. (d)	20. (c)
21. (a)	22. (c)	23. (d)	24. (a)
25. (b)	26. (d)	27. (c)	28. (c)
29. (d)	30. (a)	31. (c)	32. (d)
33. (c)			



Solutions

1. The root mean square speed of the molecules of a gas is given by

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

where m is the mass of the molecule of the gas, T its absolute temperature and k is the Boltzmann's constant. Hence the correct choice is (b).

2. The correct choice is (d).
3. The product PV has dimensions of work and is expressed in joules (J), n is the number of moles and T is expressed in kelvin (K). Thus, dimensions of $R = \dfrac{\text{dimension of } PV}{\text{dimension of } (n \times T)} = \dfrac{\text{J}}{\text{mol K}} = \text{JK}^{-1}\ \text{mol}^{-1}$.

   Hence the correct choice is (c).
4. The rms speed $= \left[\dfrac{1^2 + 2^2 + 3^2 + 4^2}{4}\right]^{1/2}$

   $= \sqrt{\dfrac{15}{2}}\ \text{kms}^{-1}$.

   Hence the correct choice is (d).
5. The average kinetic energy of a molecule of a gas at absolute temperature T is given by

   $$E = \frac{3}{2} kT$$

   Hence the correct choice is (c).
6. The root mean square speed of the molecules of a gas is given by

   $$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

   Hence the correct choice is (b).
7. The rms speed is the maximum for the gas which is the lightest. Hence the correct choice is (a).
8. The correct choice is (d).
9. The average kinetic energy of molecule $= \dfrac{1}{2}\ kT$ which is independent of the molecular mass. Hence the correct choice is (b).
10. The correct choice is (a).
11. The rms speed $= \sqrt{3kT/m}$ which is independent of pressure. Hence the correct choice is (b).
12. $v = \sqrt{3kT/m}$. Therefore, $v_0 = \sqrt{3kT/m_0}$ and $v_h = \sqrt{3kT/m_h}$.

    Thus $\dfrac{v_0}{v_h} = \sqrt{\dfrac{m_h}{m_0}} = \sqrt{\dfrac{1}{16}} = \dfrac{1}{4}$

    or $v_h = 4\ v_0$.

    Hence the correct choice is (c).
13. The correct choice is (c) as explained above.
14. The correct choice is (c)
15. We know that C_v = no. of degrees of freedom × molar gas constant/2

    or $C_v = fR/2$. Also $C_p - C_v = R$.

    Therefore

    $$\frac{C_P}{C_v} = \frac{R}{C_v} + 1 = \frac{2R}{f\,R} + 1 = \frac{2}{f} + 1$$

    Hence the correct choice is (d).
16. We know that $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$ which gives

    $$M = \frac{3\,RT}{v_{\text{rms}}^2} = \frac{3 \times 8.3 \times 300}{(1920)^2}$$

    $$= 2 \times 10^{-3}\ \text{kg} = 2\ \text{g}$$

    Since $M = 2$, the gas is hydrogen.
17. $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$ or $v_{\text{rms}} \propto \sqrt{T}$. Hence the correct choice is (b).
18. For a given M, $v_{av} \propto \sqrt{T}$ only. Therefore, the correct choice is v_1 as the temperatures of vessels A and C are the same.
19. $E \propto T$ and $v_{\text{rms}} \propto \sqrt{T}$. Hence at 600K,

    $$E = 6.21 \times 10^{-21} \times \frac{600}{300}$$

    $$= 12.42 \times 10^{-21}\ \text{J}$$

    and $v_{\text{rms}} = 484 \times \sqrt{\dfrac{600}{300}} \simeq 684\ \text{ms}^{-1}$

    which is choice (d).
20. For a gas, $PV = nRT$. Hence

    $$(P)_{O_2} = \frac{(1\ \text{mole})\,RT}{V}$$

    and $(P)_{\text{He}} = \dfrac{(1\ \text{mole})\,R(2T)}{V}$

    $$\therefore \quad \frac{(P)_{\text{He}}}{(P)_{O_2}} = 2 \text{ or } (P)_{\text{He}} = 2\ (P)_{O_2}$$

21. Since both the gases are diatomic, each has two degrees of freedom associated with rotational motion. According to the law of equipartition of energy, the rotational kinetic energy per degree of freedom is $(1/2)kT$. Since the temperatures of the two gases are equal, their rotational kinetic energies will be equal. Hence the correct choice is (a).

22. According to the kinetic theory, the average kinetic energy (KE) per molecule of a gas = $\frac{3}{2}\ kT$. Let n_1 and n_2 be the number of moles of air in vessels 1 and 2 respectively.

Before mixing, the total KE of molecules in the two vessels is

$$E_1 = \frac{3}{2} n_1 kT_1 + \frac{3}{2} n_2 kT_2$$

$$= \frac{3}{2} k(n_1 T_1 + n_2 T_2)$$

After mixing, the total KE of molecules is

$$E_2 = \frac{3}{2} (n_1 + n_2)\, kT$$

where T is the temperature when equilibrium is established. Since there is no loss of energy (because the vessels are insulated), $E_2 = E_1$ or

$$\frac{3}{2} (n_1 + n_2)kT = \frac{3}{2} k\,(n_1 T_1 + n_2 T_2)$$

or $$T = \frac{n_1 T_1 + n_2 T_2}{(n_1 + n_2)} \qquad (1)$$

Now $P_1 V_1 = n_1 RT_1$ and $P_2 V_2 = n_2 RT_2$ which give

$$n_1 = \frac{P_1 V_1}{RT_1} \text{ and } n_2 = \frac{P_2 V_2}{RT_2}$$

Using these in Eq. (1) and simplifying, we get

$$T = \frac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{(P_1 V_1 T_2 + P_2 V_2 T_1)}$$

23. The correct choice is (d)
24. The correct choice is (a).
25. The root mean square speed is given by

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

i.e. $v_{\text{rms}} \propto T$. Initial temperature T_1 = 27 + 273 = 300 K. Final temperature T_2 = 927 + 273 = 1200 K. Since temperature is increased by 4 times, the speed is doubled. Hence the correct choice is (b).

26. For oxygen:

$$v_{\text{rms}} = \sqrt{\frac{3kT_0}{m_0}}$$

For hydrogen:

$$v_{\text{rms}} = \sqrt{\frac{3kT_h}{m_h}}$$

The value of v_{rms} will be the same if

$$\frac{T_0}{m_0} = \frac{T_h}{m_h}$$

or $$T_0 = \frac{m_0}{m_h} \times T_h = 16 \times 200$$

$$= 3200 \text{ K} = 2927°\text{C}$$

Hence the correct choice is (d).

27. Use $v_{\text{rms}} \propto T$. The correct choice is (c).
28. The pressure exerted by a gas is given by

$$P = \frac{nRT}{V}$$

$$= \frac{\text{mass}}{\text{molecular weight}} \times \frac{RT}{V}$$

Pressure exerted by oxygen $P_1 = \frac{8}{32}\frac{RT}{V} = \frac{1}{4}\frac{RT}{V}$

Pressure exerted by oxygen $P_2 = \frac{14}{28}\frac{RT}{V} = \frac{1}{2}\frac{RT}{V}$

Pressure exerted by carbon dioxide

$$P_3 = \frac{22}{44}\frac{RT}{V} = \frac{1}{2}\frac{RT}{V}$$

From Dalton's law of partial pressures, the total pressure exerted by the mixture is given by

$$P = P_1 + P_2 + P_3$$

$$= \frac{1}{4}\frac{RT}{V} + \frac{1}{2}\frac{RT}{V} + \frac{1}{2}\frac{RT}{V}$$

$$= \frac{5RT}{4V}, \text{ which is choice (c).}$$

29. The root mean square speed is given by

$$v_{\text{rms}} = \frac{\sqrt{3kT}}{m}$$

The escape velocity is given by $v_e = \sqrt{2gR}$

For $v_{\text{rms}} = v_e$, we require

$$\sqrt{\frac{3kT}{m}} = \sqrt{2gR} \text{ or } T = \frac{2mg\,R}{3k}, \text{ which is choice (d).}$$

30. Average kinetic energy per molecule of a perfect gas = $\frac{3}{2}\ kT$.

$\therefore$ Average KE of molecules of the first gas

$$= \frac{3}{2} n_1 kT_1$$

Average KE of molecules of the second gas

$$= \frac{3}{2} n_2 kT_2$$

$\therefore$ Total KE of the molecules of the two gases before they are mixed is

$$K = \frac{3}{2}\ n_1 kT_1 + \frac{3}{2}\ n_2 kT_2$$

$$= \frac{3}{2}(n_1\ T_1 + n_2\ T_2)k \qquad (1)$$

If T is the temperature of the mixture, the kinetic energy of the molecules $(n_1 + n_2)$ in the mixture is

$$K' = \frac{3}{2}(n_1 + n_2)kT \qquad (2)$$

Since there is no loss of energy $K = K'$. Equating (1) and (2) we get

$$n_1\ T_1 + n_2\ T_2 = (n_1 + n_2)\ T \text{ or } T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2}$$

Hence the correct choice is (a).

31. Let n be the number of moles of the gas in the box. The kinetic energy of the gas = $n\left(\frac{1}{2} Mv^2\right)$. When the box is suddenly stopped, this energy is used up in changing the internal energy, as a result of which the temperature of the gas rises. The change in internal energy is given by

$\Delta U = nC_v\ \Delta T$

For a diatomic gas $\quad C_v = \frac{5}{2} R.$

Hence $\quad n \times \frac{5R}{2} \times \Delta T = n \times \frac{1}{2} Mv^2$

or $\quad \Delta T = \frac{Mv^2}{5R}$, which is choice (c)

32. The root mean square speed is related to absolute temperature T as

$$c_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

For a given gas, m is fixed. Therefore, $c_{\text{rms}} \propto \sqrt{T}$. Hence in order to double the root mean square speed, the absolute temperature must be increased to four times the initial value. Initial temperature $T_1 = 273 + 27 = 300$ K. Therefore, final temperature $T_2 = 4\ T_1 = 1200$ K.

Since the volume of the vessel is fixed, $\Delta V = 0$. Hence the heat energy supplied to the gas does no work on the gas; it only increases the internal energy of the molecules.

The increase in internal energy is

$$\Delta U = n\ C_v\ \Delta T$$

Since nitrogen is diatomic $C_v = \frac{5R}{2}$. The number of moles of nitrogen in the vessel is

$$n = \frac{\text{mass in kg}}{\text{molecular mass}} = \frac{0.014 \times 10^3}{28} = 0.5$$

$\therefore \quad \Delta U = 0.5 \times \frac{5R}{2} \times (1200 - 300)$

$$= \frac{0.5 \times 5 \times 8.314 \times 900}{2} = 9353 \text{ J.}$$

So the correct choice is (d)

33. Number of moles present initially is $n = \frac{PV}{RT}$. Let n' be the number of moles of the gas that leaked till the pressure falls to P'. Since volume V of the vessel cannot change and temperature T remains constant during leakage, we have

$$n' = \frac{P'V}{RT}$$

$\therefore$ Number of moles that leaked is

$$\Delta n = n - n' = \frac{PV}{RT} - \frac{P'V}{RT} = \frac{V}{RT}(P - P')$$

So the correct choice is (c).

SECTION 2
Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage.

Passage I

Kinetic Theory of Gases

The molecules of a gas move in all directions with various speeds. The speeds of the molecules of a gas increase with rise in temperature. During its random motion, a fast molecule often strikes against the walls of the container of the gas. The collisions are assumed to be perfectly elastic, i.e. the molecule bounces back with the same speed with which it strikes the wall. Since the number of molecules is very large, billions of molecules strike against the walls of the container every second. These molecules exert a sizeable force on the wall. The force exerted per unit area is the pressure exerted by the gas on the walls. According to the kinetic theory, the pressure of a gas of density ρ at absolute temperature T is given by

$$P = \frac{1}{3}\rho v_{rms}^2$$

where v_{rms} is the root mean square speed of the gas molecule and is given by

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

where m is the mass of a molecule and k is Boltzmann constant.

1. From the relation $v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$, it follows that the constant k should be expressed in units of
   (a) newton per metre per kelvin
   (b) newton per kelvin
   (c) joule per kelvin
   (d) joule per kilogram per kelvin
2. Choose the only correct statement from the following.
   (a) The pressure of a gas is equal to the total kinetic energy of its molecules per unit volume of the gas.
   (b) The product of pressure and volume of a gas is always constant.
   (c) The average kinetic energy of the molecules of a gas is proportional to its absolute temperature.
   (d) The root mean square speed of a molecule is proportional to the absolute temperature of the gas.
3. If the temperature of a gas is increased from 27°C to 927°C, the root mean square speed of its molecules
   (a) becomes half (b) becomes doubled
   (c) becomes four times (d) remains unchanged
4. The root mean square speed of oxygen gas molecule at $T = 320$ K is very nearly equal to (the molar mass of oxygen is $M = 0.0320$ kg per mole and gas constant $R = 8.31$ J mol^{-1} K^{-1})
   (a) 300 ms^{-1} (b) 500 ms^{-1}
   (c) 700 ms^{-1} (d) 900 ms^{-1}

Solutions

1. Squaring we have
$$k = \frac{mv_{rms}^2}{3T} = \frac{\text{unit of energy}}{\text{unit of temperature}}$$
$$= \text{joule per kelvin}$$
2. The correct choice is (c)
3. The correct choice is (b) since v_{rms} is proportional to the square root of absolute temperature
4. $v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\times 8.31\times 320}{0.032}}$
$$= 499.3 \text{ ms}^{-1}$$
Hence the correct choice is (b)

Questions 5 to 7 are based on the following passage.

Passage II

Van der Waals Equation of State

The equation of state $PV = nRT$ holds for an ideal gas. The behaviour of real gases shows departures from an ideal gas behaviour especially at high pressures. The model of an ideal gas is based on a number of assumptions. Van der Waals modified the ideal gas equation $PV = nRT$ by taking into account two of those assumptions which may not be valid. He argued that (i) the volume of the molecules may not be negligible compared to the volume V occupied by the gas and (ii) the attractive forces between the molecules may not be negligible. He said that pressure P in equation $PV = nRT$ is less than the true pressure by an amount p because of attractive forces between the molecules. According to him, the pressure 'defect' p is inversely proportional to the square of volume, i.e.

$$p \propto \frac{1}{V^2}$$

or $$p = \frac{a}{V^2}$$

where a is constant depending on the nature of the gas. Thus the true pressure of the gas is $P' = P + p = P + \frac{a}{V^2}$. He further argued that V is not the true volume of the gas because the molecules themselves occupy a finite volume. According to him, the true volume of the gas is $V' = (V - b)$ where b is a factor depending on the actual volume of the molecules themselves. Thus Van der Waals' equation for real gases is $P'\ V' = nRT$, i.e.

$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT$$

At high pressures, when the molecules are too many and too close together, the correction factors a and b become important.

5. The dimensional formula for ab is
   (a) ML^2T^{-2} (b) ML^4T^{-2}
   (c) ML^6T^{-2} (d) ML^8T^{-2}
6. The correction factors a and b depend upon
   (a) the pressure of the gas
   (b) the volume of the gas
   (c) the temperature of the gas
   (d) the nature of the gas
7. The equation of state $PV = nRT$ holds if the gas has
   (a) low pressure and low density
   (b) low pressure and high density
   (c) high pressure and low density
   (d) high pressure and high density.



Solutions

5. Dimensions of
   ab = dimensions of PV^2 × dimensions of V
   = dimensions of (PV^3)
   = $ML^{-1}T^{-2} \times (L^3)^3 = ML^8T^{-2}$
   Hence the correct choice is (d).
6. The correct choice is (d).
7. The correct choice is (a).

Questions 8 and 9 are based on the following passage.

Passage III

Mean Free Path

During their random motion, the molecules of a gas often come close to each other. When the distance between two molecules is comparable with the diameter of a molecule, the forces between them become very strong. As a result, their individual momenta before and after the encounter are different. When this happens a 'collision' is said to have occurred. The average distance a molecule travels before it suffers a collision with another molecule is called the mean free path (l_c), which can be estimated as follows. Suppose the average speed of a molecule of diameter d is $\bar{v}$. In one second, this molecule sweeps out a volume $\pi d^2\bar{v}$. If it finds any other molecules in this volume, it will suffer collisions with them. If n is the number density (i.e. number per unit volume) of the molecules, then the number of molecules in this volume = $\pi d^2\bar{v}n$. The number of collisions per second = $\nu_c = \pi d^2\bar{v}n$. Therefore, the average time between two collisions (called collision period T_c) is

$$T_c = \frac{1}{\nu_c} = \frac{1}{\pi d^2\bar{v}n}$$

Hence the mean free path (i.e. the average distance the molecule travels between two successive collisions) is

$$lc = \bar{v}\,T_c = \frac{1}{\pi d^2 n}$$

For air at S.T.P. the value of $l_c \simeq 3 \times 10^{-7}$ m.

8. The mean free path of a molecule of a gas depends
   (a) only on its diameter (d)
   (b) only on the number density (n) of the molecules
   (c) on both d and n
   (d) neither on d nor on n.
9. The average collision period in a gas
   (a) increases if the pressure is increased
   (b) decreases if the pressure is increased
   (c) increases if the temperature of the gas is increased.
   (d) decreases if the temperature of the gas is increased.



Solutions

8. The mean free path is given by
   $$l_c = \frac{1}{\pi d^2 n}$$
   Hence the correct choice is (c).
9. The collision period is given by
   $$T_c = \frac{1}{\pi d^2\bar{v}n}$$
   If the pressure is increased, the volume of the gas decreases. Hence number density n increases. Therefore, T_c will decrease. If the temperature of the gas is increased, the average speed $\bar{v}$ of the molecules increases. Hence T_c will decrease. Thus the correct choices are (b) and (d).

SECTION 3
Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside the container
   (a) increases.
   (b) decreases.
   (c) remains the same.
   (d) decreases for some, while increases for others.
   [2002]

2. At what temperature is the rms velocity of a hydrogen molecule equal to that of an oxygen molecule at 47ºC?
   (a) 80 K (b) –73 K
   (c) 3 K (d) 20 K **[2002]**

3. A vessel contains 1 mol of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing 1 mol of He gas (molar mass 4) at temperature $2T$ has pressure of
   (a) $P/8$ (b) $2P$
   (c) P (c) $8P$ **[2002]**

4. One kilogram of diatomic gas is at a pressure of 8×10^4 N/m^2. The density of the gas in 4 kg/m^3. What is the energy of the gas due to its thermal motion?
   (a) 3×10^4 J (b) 5×10^4 J
   (c) 6×10^4 J (d) 7×10^4 J **[2009]**

5. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2, n_3, respectively. Assuming no loss of energy, the final temperature of the mixture is
   (a) $\dfrac{(T_1+T_2+T_3)}{3}$
   (b) $\dfrac{n_1T_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}$
   (c) $\dfrac{n_1T_1^2+n_2T_2^2+n_3T_3^2}{n_1T_1+n_2T_2+n_3T_3}$
   (d) $\dfrac{n_1^2T_1^2+n_2^2T_2^2+n_3^2T_3^2}{n_1T_1+n_2T_2+n_3T_3}$
   [2011]

6. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by
   (a) $\dfrac{(\gamma-1)}{2(\gamma+1)R}Mv^2$K (b) $\dfrac{(\gamma-1)}{2\gamma}Mv^2$K
   (c) $\dfrac{\gamma Mv^2}{2R}$K (d) $\dfrac{(\gamma-1)}{2R}Mv^2$ K
   [2011]

7. Two non- reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4:3. The ratio of their densities is :
   (a) 1 : 4 (b) 1:2
   (c) 6:9 (d) 8:9 **[2013]**

Answers

1. (c) 2. (d) 3. (b) 4. (b)
5. (b) 6. (d) 7. (d)

Solutions

1. The speed of the centre of mass of gas molecules in a container has no effect on the temperature of the gas molecules.

2. $v_{rms}=\sqrt{\dfrac{3RT}{M}}$

   It is given that v_{rms} is the same for both gases. Since R is a constant, it follows that

   $$v_{rms}\propto\sqrt{\frac{T}{M}}$$

   Thus $\dfrac{T}{M}=$ constant

   $\therefore\quad \dfrac{T}{2}=\dfrac{273+47}{32}\quad\Rightarrow\quad T=20$ K

3. $PV=nRT$. Therefore

   $$P_1V=n_1RT_1 \text{ and } P_2V=n_2RT_2$$

   $\therefore\quad \dfrac{P_2}{P_1}=\dfrac{n_2}{n_1}\times\dfrac{T_2}{T_1}$

$$\Rightarrow \quad \frac{P_2}{P} = \frac{1}{1} \times \frac{2T}{T} = 2$$

$$\Rightarrow \quad P_2 = 2P$$

4. Volume $V = \dfrac{\text{mass}}{\text{density}}$

$$= \frac{1\,\text{kg}}{4\,\text{kg m}^{-3}} = \frac{1}{4}\,\text{m}^3$$

Energy of the gas due to thermal motion is

$$U = \gamma PV$$

$$= \frac{5}{2} \times (8 \times 10^4) \times \frac{1}{4} = 5 \times 10^4\,\text{J}$$

5. Let $T_1 < T_2 < T_3$. If T is the final temperature of the mixture, we have

$$n_1 c(T - T_1) + n_2 c\,(T - T_2) = n_3 c\,(T_3 - T)$$

$$\Rightarrow \quad T = \frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$$

7. From energy conservation,

$$\frac{1}{2} M v^2 = n \times C_v \times \Delta T$$

$$\frac{1}{2} M v^2 = 1 \times \frac{R}{\gamma - 1} \times \Delta T$$

$$\Rightarrow \Delta T = \frac{M v^2 (\gamma - 1)}{2R} \text{ kelvin}$$

7. $PV = n\,RT \Rightarrow \dfrac{PM}{\rho} = n\,RT \Rightarrow PM = n\,\rho\,RT$

Since R and T are constant, then for given n

$$\frac{PM}{\rho} = \text{constant}$$

or $\dfrac{P_1 M_1}{\rho_1} = \dfrac{P_2 M_2}{\rho_2}$

$$\Rightarrow \quad \frac{\rho_1}{\rho_2} = \left(\frac{P_1}{P_2}\right) \times \left(\frac{M_1}{M_2}\right) = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$$

OSCILLATIONS AND WAVES

Chapter 10

REVIEW OF BASIC CONCEPTS

PART (A)

Simple Harmonic Motion (SHM)

1. What is Simple Harmonic Motion?

Simple harmonic motion (SHM) is the simplest kind of oscillatory motion in which a body, displaced from its stable equilibrium position, oscillates to and fro about the position when released. If the displacement (x) from the equilibrium position is small, the restoring force (F) acting on the body is given by

$$F = -k\,x$$

where k is a positive constant, known as the force constant. In the SI system k is expressed in *newton per metre* (N m^{-1}). The acceleration (a) of the body is given by

$$a = \frac{F}{m} = -\frac{k}{m}\,x = -\omega^2 x$$

where m is the mass of the body and $\omega = \sqrt{\frac{k}{m}}$

Thus acceleration (a) = – constant × displacement x.

The resulting motion is called simple harmonic motion. Thus, *a simple harmonic motion is a motion in which the acceleration (i) is proportional to the displacement from the equilibrium position and (ii) is directed towards the equilibrium position.*

2. Characteristics of SHM

The displacement x in SHM at time t is given by

$$x = A \sin(\omega t + \phi)$$

where the three constants A, ω and ϕ characterize the SHM, i.e., they distinguish one SHM from another. A SHM can also be described by a cosine function as

$$x = A \cos(\omega t + \delta)$$

We will use the sine function. A cosine function is equally valid.

(1) Amplitude The amplitude of SHM is the maximum (positive or negative) value of the displacement from the equilibrium position. Quantity A (which is the coefficient of the sine or cosine function) is the amplitude of SHM.

(2) Time period The smallest time interval during which the motion repeats itself is called time period or simply period of SHM. The angular frequency ω is related to time period T as

$$\omega = \frac{2\pi}{T}$$

Frequency ν of SHM is the number of complete oscillations completed in 1 second.

Thus, $\nu = \frac{1}{T} = \frac{\omega}{2\pi}$ or $\omega = \frac{2\pi}{T} = 2\,\pi\nu$

Since $\omega = \sqrt{\frac{k}{m}}$, we have

$$\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Angular frequency ω is the coefficient of time t in the sine or cosine function.

(3) Phase The quantity $(\omega t + \phi)$ is called the phase of SHM at time t; it describes the state of motion at that instant. The quantity ϕ is the phase at time $t = 0$ and is called the phase constant or initial phase or epoch of the SHM. The phase constant is the time-independent term in the cosine or sine function.

3. Velocity and Acceleration in SHM

The velocity V or the particle in SHM is given by

$$V = \frac{dx}{dt} = A\,\omega\cos(\omega t + \phi)$$

But $\sin(\omega t + \phi) = \dfrac{x}{A}$

$$\therefore \quad \cos(\omega t + \phi) = \left(1 - \frac{x^2}{A^2}\right)^{1/2}$$

Hence velocity $V = A\,\omega\left(1 - \dfrac{x^2}{A^2}\right)^{1/2} = \omega(A^2 - x^2)^{1/2}$

The acceleration a of the particle in SHM is given by

$$a = \frac{d^2x}{dt^2} = -A\,\omega^2\sin(\omega t + \phi) = -\omega^2 x$$

Notice that, when the displacement is maximum, i.e., when $x = |A|$, the velocity is zero but the acceleration is maximum $= |\omega^2 A|$. But when the displacement is zero ($x = 0$), the velocity is maximum $= |\omega A|$ and the acceleration is zero.

4. Energy in SHM

At any instant of time t, the kinetic energy of the oscillator is given by

$$\text{K.E.} = \frac{1}{2}mV^2 = \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \phi)$$

At any displacement x from the equilibrium position,

$$\text{K.E.} = \frac{1}{2}m\omega^2(A^2 - x^2)$$

because $\cos^2(\omega t + \phi) = 1 - \sin^2(\omega t + \phi) = 1 - \dfrac{x^2}{A^2}$.

At that instant t, the potential energy of the oscillator is given by

$$\text{P.E.} = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 A^2\sin^2(\omega t + \phi)$$

because $k = m\omega^2$ and $x = A\sin(\omega t + \phi)$. In terms of displacement x,

$$\text{P.E.} = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$$

Total energy of the oscillator is

$$E = \text{K.E.} + \text{P.E.}$$
$$= \frac{1}{2}mA^2\omega^2\left[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right]$$
$$= \frac{1}{2}mA^2\omega^2$$

E is also obtained as follows:

$$E = \text{K.E.} + \text{P.E.} = \frac{1}{2}m\omega^2(A^2 - x^2) + \frac{1}{2}m\omega^2x^2)$$
$$= \frac{1}{2}mA^2\omega^2$$

Thus, although the kinetic energy and potential energy of a simple harmonic oscillator both change with time t and displacement x, the total energy is independent of both x and t and hence remains constant. This is expected because friction has been neglected.

Note

1. At the mean position, $x = 0$, K.E. is maximum $= \dfrac{1}{2}mA^2\omega^2$ and P.E. = 0
2. At the extreme position, $x = \pm A$, K.E. = 0 and P.E. is maximum $= \dfrac{1}{2}mA^2\omega^2$.
3. E is constant $= \dfrac{1}{2}mA^2\omega^2$ for all values of x.

Figure 10.1 shows the variation K.E., P.E. and E with t for the case $\phi = 0$.

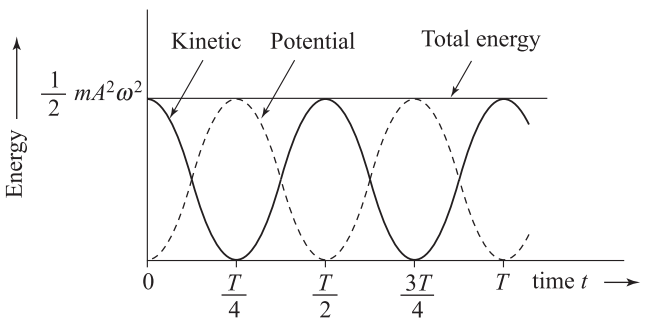


Fig. 10.1

The energy of an oscillator may decrease with time not only by friction (damping) but also due to radiation. The oscllating body imparts periodic motion to the particles of the medium in which it oscillates, thus producing waves.

For example, a tuning fork or a string produces sound waves in the medium which results in a decrease in energy.

EXAMPLE 1 The displacement x of a body varies with time t as

$$x = a \sin(ct) + b \cos(ct)$$

where a, b and c are constants.

(a) Show that the motion of the body is simple harmonic.

(b) Find the amplitude A, time period T and phase constant ϕ of the motion in terms a, b and c.

SOLUTION

(a) Given $x = a \sin(ct) + b \cos(ct)$ (i)

Differentiating (i) w.r.t. time t, we get velocity

$$V = \frac{dx}{dt} = ac \cos(ct) - bc \sin(ct)$$

The acceleration is given by

$$a = \frac{dV}{dt} = \frac{d}{dt}[ac\cos(ct) - bc\sin(ct)]$$

$$= -c^2 [a \sin(ct) + b \cos(ct)]$$

$\Rightarrow$ $a = -c^2 x$ [use Eq. (i)]

Since $a \propto (-x)$, the motion is simple harmonic.

(b) Since the motion is simple harmonic;

$$x = A \sin(\omega t + \phi)$$

$$= A [\sin(\omega t)\cos\phi + \cos(\omega t)\sin\phi]$$

$$x = (A \cos\phi)\sin(\omega t) + (A \sin\phi)\cos(\omega t) \quad \text{(ii)}$$

Comparing (i) and (ii), we get

$$A \cos\phi = a \quad \text{(iii)}$$

$$A \sin\phi = b \quad \text{(iv)}$$

and $$\omega = c \Rightarrow \frac{2\pi}{T} = c \Rightarrow T = \frac{2\pi}{c}$$

Squaring and adding (iii) and (iv), we get

$$A = \sqrt{a^2 + b^2}$$

Dividing (iv) by (iii) we get

$$\tan\phi = \frac{b}{a} \Rightarrow \phi = \tan^{-1}\left(\frac{b}{a}\right)$$

EXAMPLE 2 Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Find the period for each periodic motion. Here k is a positive real constant.

(i) $\sin kt + \cos kt$, (ii) $\sin \pi t + 2\cos 2\pi t + 3 \sin 3\pi t$,

(iii) $\cos\left(2kt + \frac{\pi}{3}\right)$, (iv) $\cos kt + 2 \sin^2 kt$, (v) e^{-kt} and

(vi) $\log kt$.

SOLUTION

(i) $\sin kt + \cos kt$ represents simple harmonic motion since we can write it as $\sqrt{2} \sin\left(kt + \frac{\pi}{4}\right)$ or $\sqrt{2}\cos\left(kt - \frac{\pi}{4}\right)$. The coefficient of t in the argument of the sine or cosine function = $\frac{2\pi}{T}$, where T is the period. Hence $k = \frac{2\pi}{T}$ or $T = \frac{2\pi}{k}$, i.e. the period of the function is $\frac{2\pi}{k}$.

(ii) Each term in $(\sin \pi t + 2\cos 2\pi t + 3\sin 3\pi t)$ represents SHM.

The period T of the term $\sin \pi t$ is given by $\pi = \frac{2\pi}{T}$ or $T = 2$s. The period of the term $2 \cos 2\pi t$ is 1 s, i.e. $T/2$ and the period of the term $3 \sin 3\pi t$ is 2/3 s, i.e. $T/3$. The sum of the three terms, however, does not represent SHM; it represents a periodic motion. By the time the first term completes one cycle, the second term completes two cycles and the third term completes three cycles. This shows that the sum represents a periodic motion with period $T = 2$s.

(iii) $\cos\left(2kt + \frac{\pi}{3}\right)$ represents an SHM whose period T is given by

$$2k = \frac{2\pi}{T}$$

or $$T = \frac{\pi}{k}$$

(iv) $\cos kt + 2 \sin^2 kt = \cos kt + (1 - \cos 2kt) = 1 + \cos kt - \cos 2kt$. The period of $\cos kt$ is $T = \frac{2\pi}{k}$ and that of $\cos 2kt$ is $\frac{\pi}{k} = \frac{T}{2}$. These two terms together have a period $T = \frac{2\pi}{k}$ as explained in (ii). The other term 1 is a constant independent of t and hence does not affect the period of the sum. Hence $(\cos kt + 2\sin^2 kt)$ represents a periodic motion with period $T = \frac{2\pi}{k}$.

(v) e^{-kt} decreases monotonically to zero as $t \to \infty$. It is an exponential function with a negative exponent of e, where $e \simeq 2.71828$. It is non-periodic.

(vi) Function log (kt) increases monotonically with time. Therefore, it never repeats itself and is a non-periodic function.

EXAMPLE 3 An oscillator has a time period T = 4s. If it is oscillating harmonically, find the time period of its kinetic energy.

SOLUTION The kinetic energy of a simple harmonic oscillator varies with time as $\cos^2(\omega t)$, where $\omega = \frac{2\pi}{T}$.

Now $\cos^2(\omega t) = \frac{1}{2}(1 + \cos 2\omega t) = \frac{1}{2}(1 + \cos \omega' t)$,

where $\omega' = 2\omega \Rightarrow \frac{2\pi}{T'} = 2 \times \frac{2\pi}{T} \Rightarrow T' = \frac{T}{2}$.

Hence K.E. varies periodically with period $\frac{T}{2}$.

Note The P.E. also varies periodically with period $\frac{T}{2}$.

EXAMPLE 4 The displacement x (in centimetres) of an oscillating particle varies with time t (in seconds) according to the equation

$$x = 2\cos\left(0.5\pi t + \frac{\pi}{3}\right)$$

Find

(a) the amplitude of oscillation
(b) the time period of oscillation
(c) the maximum velocity of the particle
(d) the maximum acceleration of the particle.

SOLUTION The displacement of the particle is given by

$$x = 2\cos\left(0.5\pi t + \frac{\pi}{3}\right)\text{cm}$$

To find the amplitude and time period of the oscillation, we compare this equation with

$$x = A\cos(\omega t + \delta)$$

(a) Amplitude A = 2 cm

(b) Angular frequency $\omega = 0.5\pi$ rad s^{-1}

or $$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.5\pi} = 4\text{s}$$

(c) Maximum velocity $V_{max} = |A\omega|$
$= 2 \times 0.5\pi = \pi$ cm s^{-1}
$= 3.14$ cm s^{-1}

(d) Maximum acceleration $a_{max} = |-\omega^2 A|$
$= \omega^2 A$
$= (0.5\pi)^2 \times 2$
$= \frac{\pi^2}{2}$ cm s^{-2}
$= 4.935$ cm s^{-2}

EXAMPLE 5 A particle executes SHM of amplitude 25 cm and time period 3s. What is the minimum time required for the particle to move between two points located at 12.5 cm on either side of the mean position?

SOLUTION Let A and B be the two extreme positions of the particle with O as the mean position. Displacements to the right of O are taken as positive while those to the left of O are taken as negative (Fig. 10.2)

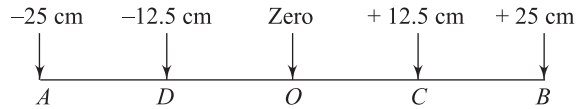


Fig. 10.2

Let the displacement of the particle in SHM be given by

$$x(t) = A\sin(\omega t + \phi) \quad \text{(i)}$$

where $A = 25$ cm and $\omega = \frac{2\pi}{T} = \frac{2\pi}{3}$ rad s^{-1}

Let us suppose that at time $t = 0$, the particle is at extreme position B. Setting $x = A$ and $t = 0$ in Eq. (i) we have

$$A = A\sin\phi$$

giving $\phi = \pi/2$.

Putting $\phi = \pi/2$ in Eq. (i), we get

$$x(t) = A\cos\omega t \quad \text{(ii)}$$

Now let us say that the particle reaches point C at $t = t_1$ and point D at $t = t_2$. At C, the displacement $x(t_1) = +12.5$ cm and at D, it is $x(t_2) = -12.5$ cm (see Fig. 10.2). So from (ii) we have

$$+12.5 = 25\cos\omega t_1$$

and $$-12.5 = 25\cos\omega t_2$$

or $$\cos\omega t_1 = +0.5 \text{ or } \omega t_1 = \pi/3$$

and $$\cos\omega t_2 = -0.5 \text{ or } \omega t_2 = \frac{2\pi}{3}$$

Hence $$\omega(t_2 - t_1) = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$$

$$\therefore \qquad t_2 - t_1 = \frac{\pi}{3\omega} = \frac{T}{6} \qquad \left(\because \omega = \frac{2\pi}{T}\right)$$

$$\text{or} \qquad (t_2 - t_1)_{\min} = \frac{3}{6} = 0.5 \text{ s}$$

Notice that $\cos \omega t_2 = -0.5$ even for $t_2 = \frac{4\pi}{3}$. This value of t_2 does not correspond to the minimum time because this is the time at which the particle, moving to left, reaches A and then returns to D.

EXAMPLE 6 A horizontal platform is executing SHM in the vertical direction with an amplitude of 1.0 m. A block of mass 5 kg is placed on the platform. What is the maximum frequency of platform's SHM so that the block is not detatched from the platform?

SOLUTION The block will not be detatched from the platform, if the frequency of platform's SHM is such that the maximum acceleration of the platform equals the acceleration due to gravity, i.e.

$$A\,\omega^2_{\max} = g$$

$$\Rightarrow \qquad \omega_{\max} = \sqrt{\frac{g}{A}} = \sqrt{\frac{9.8}{1.0}} = 3.13 \text{ rad s}^{-1}$$

$$\therefore \qquad \nu_{\max} = \frac{\omega_{\max}}{2\pi} = \frac{3.13}{2 \times 3.14} \simeq 0.5 \text{ Hz}.$$

EXAMPLE 7 A body of mass m is attached by a string to a suspended spring of spring constant k. Both the string and the spring have negligible mass. The body is pulled down a distance A and released. Assuming that the string remains taut throughout the motion, find

(a) the maximum downward acceleration of the oscillating body and

(b) the maximum amplitude for which the string remains taut.

SOLUTION

(a) As long as the string remains taut, the restoring force will be proportional to displacement from the mean position. The motion of the body is simple harmonic whose amplitude is A and angular frequency is

$$\omega = \sqrt{\frac{k}{m}}$$

The maximum acceleration is $\omega^2 A = \frac{kA}{m}$

(b) If the tension in the string is T, the downward acceleration is

$$a = g - \frac{T}{m}$$

For the string to remain taut $T > 0$. Therefore, the maximum downward acceleration cannot exceed g, i.e.

$$a_{\max} = g$$

$$\Rightarrow \qquad \omega^2 A_{\max} = g$$

$$\Rightarrow \qquad A_{\max} = \frac{g}{\omega^2} = \frac{mg}{k} \qquad \left(\because \omega^2 = \frac{k}{m}\right)$$

5. Expressions for Time Period of Mass-Spring System

(1) *Horizontal Oscillations of a Mass-Spring System*

Consider a block of mass m placed on a horizontal frictionless surface and attached to a spring of negligible mass and spring constant k as shown in Fig. 10.3.

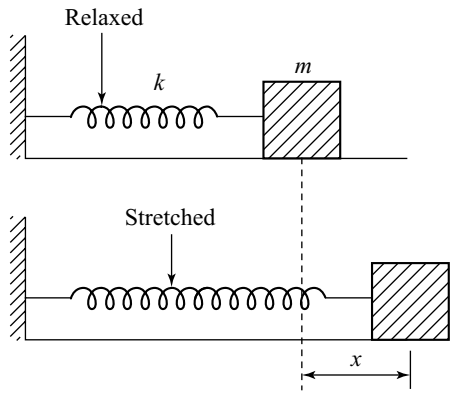


Fig. 10.3

The block is pulled to the right by a small distance x from the equilibrium position and released. The restoring force on the block is $F = -kx$. The acceleration of the block is

$$a = \frac{F}{m} = -\left(\frac{k}{m}\right)x \qquad \text{(i)}$$

Since $a \propto (-x)$, the motion of the block is simple harmonic. Comparing Eq. (i) with $a = -\omega^2 x$, we get

$$\omega^2 = \frac{k}{m} \quad \Rightarrow \quad \omega = \sqrt{\frac{k}{m}}$$

$$\therefore \qquad \text{Time period } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

(2) *Vertical Oscillations of a Mass-spring System*

Consider a massless spring suspended from a support. [Fig. 10.4 (a)]. A block of mass m is attached at the lower end, as a result, the string extends by an amount d given by [Fig. 10.4 (b)].

$$F = kd \quad \Rightarrow \quad mg = kd \qquad \text{(i)}$$

This is the equilibrium state of the system.

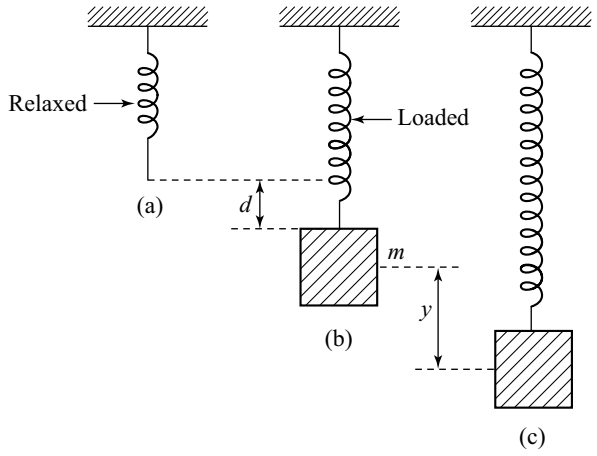


Fig. 10.4

When the body is pulled through a distance y from this position and released [Fig. 10.4 (c)], the restoring force is $F = -ky$ and the acceleration of the block is

$$a = \frac{F}{m} = -\left(\frac{k}{m}\right)y$$

Hence the motion is simple harmonic whose time period is

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \text{(ii)}$$

Equation (i) determines k. Time period given by Eq. (ii) is the same as for horizontal oscillation. It depends only on m and k and is independent of gravity.

(3) Parallel Combination of Springs

Figure 10.5 shows the equilibrium state of a block connected to two springs which are joined in parallel. If the block is pulled down through a distance x, the extension produced in each spring will be x. The restoring forces in the springs are $F_1 = -k_1 x$ and $F_2 = -k_2 x$.

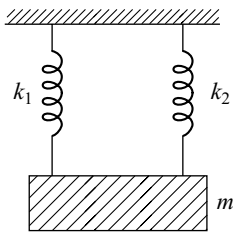


Fig. 10.5

Total restoring force $F = F_1 + F_2 = -(k_1 + k_2)x = -(k_p)x$ where $k_p = k_1 + k_2$ is the effective force constant of the parallel combination. The time period is given by

$$T = 2\pi\sqrt{\frac{m}{k_p}} = 2\pi\sqrt{\frac{m}{(k_1 + k_2)}}$$

(4) Series Combination of Springs

Figure 10.6 shows the equilibrium state of a block connected to two springs which are joined in series. The block is pulled down by a distance x. Let x_1 and x_2 be the extensions produced in the springs. The restoring force in each spring will be the same equal to

$$F = -k_1 x_1 = -k_2 x_2$$

so that $$x_1 = -\frac{F}{k_1} \text{ and } x_2 = -\frac{F}{k_2}$$

Total extension $$x = x_1 + x_2 = -F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$= -F\left(\frac{k_1 + k_2}{k_1 k_2}\right)$$

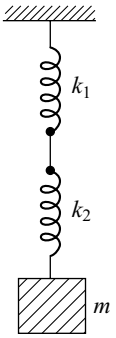


Fig. 10.6

or $$F = -\left(\frac{k_1 k_2}{k_1 + k_2}\right)x = -k_s x$$

where $k_s = \dfrac{k_1 k_2}{k_1 + k_2}$ is the effective force constant of the series combination. The time period is

$$T = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

(5) A Block Connected between Two Springs

Figure 10.7 shows the equilibrium state of a block connected between two springs

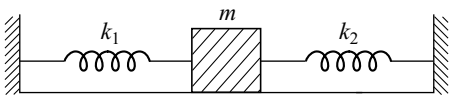


Fig. 10.7

If the block is displaced through a distance x, say to the right, the spring k_1 is extended by x and spring k_2 is compressed by x so that the restoring force exerted by each spring on the block is in the same direction (along the left). If F_1 and F_2 are the restoring forces, $F_1 = -k_1 x$ and $F_2 = -k_2 x$, the total restoring force is

$$F = -(k_1 + k_2)x = -kx$$

where $k = (k_1 + k_2)$ is the effective force constant of the system. The time period is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{(k_1 + k_2)}}$$

EXAMPLE 8 If a spring of force constant k is cut into two equal halves, what is the force constant of each half.

SOLUTION If a force F produces an extension x in the spring, then

$$F = kx \qquad \text{(i)}$$

Since the extension produced by a force is proportional to the length of the spring, if a spring is cut into two equal halves, the same force F will produce an extension $x' = x/2$ in half the spring. If k' is the force constant of half the spring,

$$F = k'\,x' = k'\left(\frac{x}{2}\right) \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get $k' = 2k$.

EXAMPLE 9 Two blocks, of masses m_1 and m_2 , are connected by a spring of force constant k and placed on a horizontal frictionless surface as shown in Fig. 10.8. Equal force F is applied to each block as shown. Find time period of the system when the force is removed.

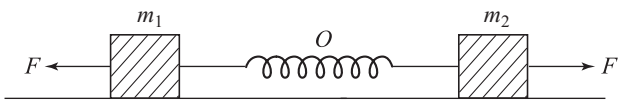


Fig. 10.8

SOLUTION When force F is removed, no external force acts on the system which consists of two blocks and a spring. Hence the centre of mass of the system will remain at rest. The mean position of m_1 and m_2 is the position of the centre of mass of the system when the spring acquires its natural (unstretched) length.

Let m_1 move to the right by x_1 and m_2 move to the left by x_2. If x_0 is the length of the unstretched spring, then

$$x_1 + x_2 = x_0 \qquad \text{(i)}$$

It is clear that x_1 and x_2 will be the amplitudes of m_1 and m_2 respectively. Since the system does not rotate and the centre of mass must not change during the oscillation, the condition

$$m_1 x_1 = m_2 x_2 \qquad \text{(ii)}$$

must be satisfied so that no net torque acts on the system. From (i) and (ii) we get

$$x_1 = \frac{m_2\,x_0}{m_1 + m_2} \qquad \text{(iii)}$$

and

$$x_2 = \frac{m_1\,x_0}{m_1 + m_2} \qquad \text{(iv)}$$

Hence, when the motion begins, the mass m_1 is at a distance x_1 given by (iii) away from its mean position. The force exerted by the spring at this position is

$$F = k\,x_0$$

From (iii), $\quad x_0 = \left(\dfrac{m_1 + m_2}{m_2}\right)x_1$

$$\therefore \qquad F = k\left(\frac{m_1 + m_2}{m_2}\right)x_1$$

The restoring force acting on m_1 is

$$F_r = -F = -k\left(\frac{m_1 + m_2}{m_2}\right)x_1$$

$\therefore$ Acceleration of m_1 is

$$a = \frac{F_r}{m_1} = -k\left(\frac{m_1 + m_2}{m_1\,m_2}\right)x_1$$

$$\Rightarrow \qquad a = -\omega^2\,x_1$$

where $\quad \omega = \sqrt{k\left(\dfrac{m_1 + m_2}{m_1\,m_2}\right)}$

Since $a \propto -x_1$, the motion of the system is simple harmonic. The time period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m_1\,m_2}{k(m_1 + m_2)}}$$

In the special case when $m_1 = m_2 = m$,

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

EXAMPLE 10 One end of a spring of spring constant k is attached to a block of mass m with the other end fixed to the ceiling of a stationary elevator. The block is executing vertical simple harmonic motion of time period T. Suddenly the cable breaks and the elevator begins to fall freely under gravity. Then

(a) the block comes to a stop
(b) the block continutes of execute simple harmonic motion of time period greater than T
(c) the blocks begins to execute periodic but not simple harmonic motion
(d) the block executes simple harmonic motion of amplitude mg/h.

SOLUTION The weight mg of the block stretches the string by an amount x given by

$$mg = kx$$

$$\Rightarrow \qquad x = \frac{mg}{k}$$

This position is the extreme position and the unstretched is in equilibrium. Now

$$T = 2\pi\sqrt{\frac{m}{k}}$$

is independent of g. When the cable breaks the elevator falls freely with acceleration g. Since T is independent

of g, the motion of block remains simple harmonic, with maximum displacement from mean position = x. Hence the amplitude of motion is $x = mg/k$. So the correct choice is (d).

EXAMPLE 11 Figure 10.9 shows a block of mass $m = 2.5$ kg moving towards a spring of spring constant $k = 10\ \text{Nm}^{-1}$ with a constant velocity $u = 1\ \text{ms}^{-1}$. A and B are two rigid vertical walls. The block starts moving from A and strikes the end C of the spring at a distance $x = 50$ cm. If the collision between the block and wall A is perfectly elastic, the time period T (in seconds) of the motion of the block will be

(a) $\left(\frac{\pi}{2}+1\right)$ (b) $(\pi+1)$

(c) $(2\pi+1)$ (d) 2π

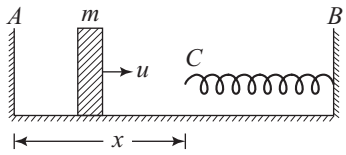


Fig. 10.9

SOLUTION The block moving with kinetic energy $\frac{1}{2}mu^2$ strikes the spring and compresses it until all the kinetic energy is converted into potential energy of the spring. Then the block moves to the left as spring begins to regain its original length BC. At this instant the block loses contact with the spring and begins to move towards wall A with velocity u. The time t_1 taken by the block to strike C and lose contact at C = half the time period of oscillation, i.e.

$$t_1 = \frac{1}{2}\times 2\pi\sqrt{\frac{m}{k}}$$

$$= \pi\sqrt{\frac{2.5}{10}} = \frac{\pi}{2}\ \text{second}$$

The block then travels towards A with velocity u and rebounds with the same velocity u (since the collision is elastic) and agains strikes C after another time t_2 given by

$$t_2 = \frac{2x}{v} = \frac{2\times 0.5}{1} = 1\ \text{second}$$

This whole process is repeated. The block undergoes periodic motion with time period

$$T = t_1 + t_2 = \left(\frac{\pi}{2}+1\right)\text{second}$$

EXAMPLE 12 Figure 10.10 shows a wooden block of mass m_2 attached to a spring of force constant k lying on a horizontal frictionless table. A bullet of mass m_1 moving horizontally with a velocity u hits the block and gets embedded in it. If $m_1 = m$ and $m_2 = 80\ m$ the amplitude of the resulting simple harmonic motion will be

(a) $u\sqrt{\frac{m}{3k}}$ (b) $\frac{u}{3}\sqrt{\frac{m}{k}}$

(c) $\frac{u}{6}\sqrt{\frac{m}{k}}$ (d) $\frac{u}{9}\sqrt{\frac{m}{k}}$

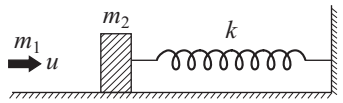


Fig. 10.10

SOLUTION From the principle of conservation of momentum, total momentum before collision = total momentum after collision, i.e.

$$m_1 u = (m_1 + m_2)\, V$$

where V is the common velocity of the bullet-block system. Putting $m_1 = m$ and $m_2 = 80\ m$, we get

$$V = \frac{u}{81}$$

The kinetic energy of the system $= \frac{1}{2}(m_1+m_2)\,V^2$

$$= \frac{1}{2}(m+80m)\frac{u^2}{(81)^2}$$

$$= \frac{1}{2}m\frac{u^2}{81}$$

This is the total energy of the system since the string is unstretched before the collision.

If A is the maximum compression of the spring (which is the amplitude of motion), the potential energy is $\frac{1}{2}kA^2$.

Now Loss in K.E. = gain in P.E.

or $$\frac{1}{2}m\frac{u^2}{81} = \frac{1}{2}kA^2$$

$\Rightarrow$ $$A = \frac{u}{9}\sqrt{\frac{m}{k}},$$ which is choice (d).

EXAMPLE 13 A small trolley of mass 2.0 kg resting on a horizontal frictionless turntable is connected by a light spring to the centre of the table. The relaxed length of the spring is 35 cm. When the turntable is rotated at a speed of 300 rev/min, the length of the spring becomes 40 cm. Find the force constant of the spring.

SOLUTION Mass of trolley (m) = 2.0 kg

Frequency of rotation (ν) = 300 rev. min^{-1} = $\dfrac{300}{60}$ = 5 rev. s^{-1}

$\therefore$ Angular frequency (ω) = $2\pi\nu = 2\pi \times 5 = 10\pi$ rad s^{-1}

The radius of the circle along which the trolley moves is

$$r = 40 \text{ cm} = 0.4 \text{ m}$$

When the table is rotated, the tension in the spring is equal to the centripetal force, i.e.

$$T = \frac{mv^2}{r} = mr\omega^2 = 2.0 \times 0.4 \times (10\pi)^2 = 790 \text{ N}$$

Now, extension produced in the spring by this force

$= 40 - 35$

$= 5 \text{ cm} = 0.05 \text{ m}$

$\therefore$ Force constant of spring = $\dfrac{\text{force}}{\text{extension}} = \dfrac{790\,\text{N}}{0.05\,\text{m}}$

$= 1.58 \times 10^4 \text{ Nm}^{-1}$

$\simeq 1.6 \times 10^4 \text{ Nm}^{-1}$

6. Expressions for Time Period of Some other Systems

(1) A Ball Oscillating in a Concave Mirror

A small spherical steel ball is placed a little away from the centre of a concave mirror whose radius of curvature is R. When the ball is released, it begins to oscillate about the centre.

Place a small steel ball at A, a little away from the centre O of a concave mirror of radius of curvature R (= $OC = AC$) as shown in Fig. 10.11. Let $\angle AOC = \theta$. If m is the mass of the ball, its weight mg acts vertically downwards at A. This force is resolved into two rectangular components: $mg \cos\theta$ (which is balanced by the reaction of the mirror) and $mg \sin\theta$ (which provides the restoring force F). Thus

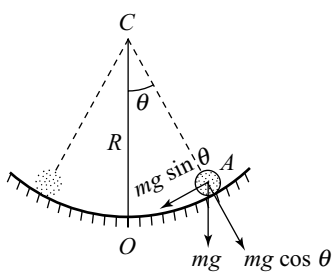


Fig. 10.11

$$F = -mg \sin\theta$$

$$= -mg\,\theta$$

(since θ is small, R being very large).

$\therefore$ $$F = -\frac{mg\,x}{R}$$

($\because x = R\theta$; x being the arc OA)

$$= -Kx$$

where force constant $K = mg/R$. Thus the motion is harmonic and the angular frequency is given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{R}}$$

$\therefore$ $$T = 2\pi\sqrt{\frac{R}{g}}$$

(2) Oscillation of a Liquid in a U-tube

The column of the liquid is displaced through y by gently blowing into the tube (Fig. 10.12). The columns exhibit vertical oscillations. Let L, A and ρ be respectively, the length of the liquid column, area of cross-section of the tube and density of the liquid. We shall neglect viscous effects. Since the right-hand side column is higher by $2y$, with respect to the column on the left-hand side, the mass of this column of liquid is $m = 2A\rho\,y$. The restoring force (which is a gravitational force) is given by

$$F = -mg = -2A\rho gy = -Ky$$

where the force constant $K = 2A\rho g$. The angular frequency of the harmonic oscillation is

$$\omega = \sqrt{\frac{K}{M}}$$

where $M = \rho AL$ is the total mass of the liquid in oscillation. Thus

$$\omega = \sqrt{\frac{2A\rho g}{\rho AL}} = \sqrt{\frac{2g}{L}}$$

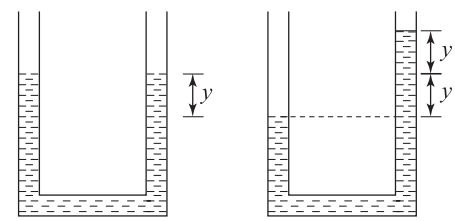


Fig. 10.12 *Oscillations of a liquid column*

The time period of oscillation is

$$T = 2\pi\sqrt{\frac{L}{2g}}$$

It is interesting to note that the period of oscillation does not depend on the density of the liquid or the area of cross-section of the tube.

(3) Oscillation of Floating Vertical Cylindrical Body

A cylindrical piece of wood of height h and density ρ floats in a liquid of density ρ_l. The cylinder is depressed slightly and released.

Let A be the cross-sectional area of the cylinder and M its mass. Fig. 10.13 (a) shows the static equilibrium, the weight of the cylinder being balanced by the weight of the liquid it displaces.

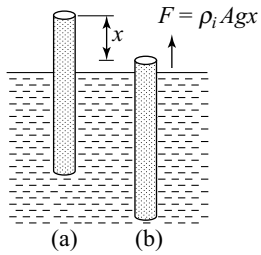


Fig. 10.13

If the cylinder is depressed through a distance x, as shown in Fig. 10.13 (b), the buoyant force on it increases by $\rho_l Agx$, because $\rho_l Ax$ is the mass of the liquid displaced by dipping, g being the acceleration due to gravity. If viscous effects are neglected, the restoring force on the cork is given by

$$F = -\rho_l Agx = -Kx$$

where $K = \rho_l Ag$. Since $F \propto -x$, the motion of the cork is simple harmonic. The time period of the motion is

$$T = 2\pi\sqrt{\frac{M}{K}}$$

where M is the mass of the cork $= Ah\rho_c$. Hence

$$T = 2\pi\sqrt{\frac{Ah\rho_c}{\rho_l Ag}} = 2\pi\sqrt{\frac{h\rho_c}{g\rho_l}}$$

(4) A Simple Pendulum

A simple pendulum consists of a massless inextensible string fixed at one end O and having a small bob at the other end (Fig. 10.14). When the bob is displaced from equilibrium position A to a position B and released, the component $mg\cos\theta$ of its weight balances with tension T and it returns under a restoring force

$$F = -mg\sin\theta$$

If θ is small, $\sin\theta \simeq \theta$. Also $\theta = \dfrac{x}{L}$

$$\therefore \qquad F = -\left(\frac{mg}{L}\right)x$$

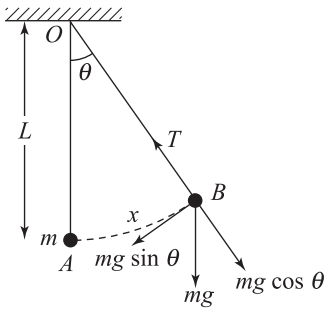


Fig. 10.14

$$\therefore \text{ Acceleration } a = \frac{F}{m} = -\left(\frac{g}{L}\right)x.$$

$$\text{Time period } T = 2\pi\sqrt{\frac{L}{g}}$$

Note

T is independent of the mass of the bob.

(5) A Compound Pendulum

A compound pendulum is a rigid body capable of oscillating about an axis. The pendulum consists of a rod of mass M and length L which is pivoted at O and carries a bob of mass m at the other end as shown in Fig. 10.15. The rod is displaced and released.

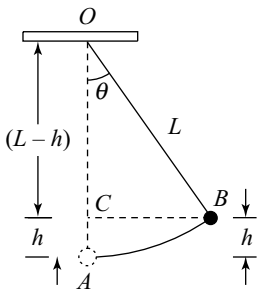


Fig. 10.15

The pendulum tends to return under the influence of a restoring torque

$$\tau = -mgL\sin\theta = -mgL\theta \text{ (for small } \theta)$$

If I is the moment of inertia of the system about the axis passing through O and perpendicular to the plane of the rod, the angular acceleration is

$$\alpha = \frac{\tau}{I} = -\left(\frac{mgL}{I}\right)\theta$$

since $\alpha \propto -\theta$, the motion is simple harmonic whose angular frequency is (compare with $\alpha = -\omega^2\theta$) given by

$$\omega^2 = \frac{mgL}{I} \Rightarrow \omega = \sqrt{\frac{mgL}{I}}$$

Thus $$T = 2\pi\sqrt{\frac{I}{mgL}}$$

where $I = \frac{ML^2}{3} + mL^2 = \frac{1}{3}(M+3m)L^2$. Thus

$$T = 2\pi\sqrt{\frac{(M+3m)L}{3mg}}$$

If $M << m$, $T = 2\pi\sqrt{\frac{L}{g}}$, the same as that of a simple pendulum.

(6) Horizontal Oscillations of a Cylinder-Spring System

A solid cylinder of mass M and radius R is connected to a spring of force constant k as shown in Fig. 10.16.

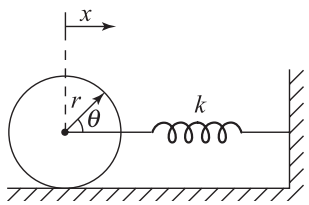


Fig. 10.16

Case (a): *Cylinder slips without rolling.*

The total energy of the system is translational K.E. + P.E. If x is the instantaneous displacement and v the velocity of the cylinder, the total energy is

$$E = \frac{1}{2}Mv^2 + \frac{1}{2}kx^2 \qquad \text{(i)}$$

If friction is neglected, E = constant. Hence $\frac{dE}{dt} = 0$.

Differentiating (i) w.r.t. time t and setting $\frac{dE}{dt} = 0$, we get

$$0 = Mv\frac{dv}{dt} + kx\frac{dx}{dt}$$

$$\Rightarrow \quad 0 = Mva + kxv \quad (\because a = \frac{dv}{dt}, v = \frac{dx}{dt})$$

$$\Rightarrow \quad a = \left(-\frac{k}{M}\right)x$$

$$\therefore \quad T = 2\pi\sqrt{\frac{M}{k}}$$

Case (b): Cylinder rolls without slipping

In this case, the total energy of the system is

$$E = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2$$

$$= \frac{1}{2}Mv^2\left(1 + \frac{I}{MR^2}\right) + \frac{1}{2}kx^2 \qquad \left(\because \omega = \frac{v}{R}\right)$$

Setting $\frac{dE}{dt} = 0$, we get

$$0 = Mv\frac{dv}{dt}\left(1 + \frac{I}{MR^2}\right) + kx\frac{dx}{dt}$$

$$0 = Mva\left(1 + \frac{I}{MR^2}\right) + kxv$$

$$\Rightarrow \quad a = -\left[\frac{k}{\left(M + \frac{I}{R^2}\right)}\right]x$$

$$\therefore \quad T = 2\pi\left[\frac{\left(M + \frac{I}{R^2}\right)}{k}\right]^{1/2}$$

This the general formula. For a solid cylinder, $I = \frac{1}{2}MR^2$ and then we get

$$T = 2\pi\sqrt{\frac{3M}{2k}}$$

(7) Vertical Oscillations Mass-spring System Connected by a Pulley

Case (a)

The block is pulled out through a small distance x and released. If M is the mass of the pulley and R its radius, the total energy of the system is [Fig. 10.17]

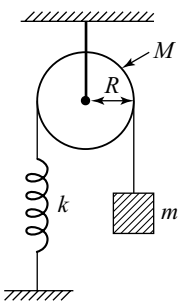


Fig. 10.17(a)

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2$$

where $I = \frac{1}{2}MR^2$ and $\omega = \frac{v}{R}$. Thus

$$E = \frac{1}{2}\left(m + \frac{1}{2}M\right)v^2 + \frac{1}{2}\,kx^2$$

Setting $\frac{dE}{dt} = 0$, we have

$$0 = \left(m + \frac{M}{2}\right)v\ \frac{dv}{dt} + kx\ \frac{dx}{dt}$$

$$\Rightarrow \quad \text{Acceleration } a = \frac{dv}{dt} = -\frac{kx}{(m + \frac{M}{2})} \qquad \left(\because\ v = \frac{dx}{dt}\right)$$

$$= -\omega^2 x$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{k}{m + \frac{M}{2}}}$$

$$\Rightarrow \qquad T = 2\pi\sqrt{\frac{m + \frac{M}{2}}{k}}$$

If the pulley has a mass $M << m$, then

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Case (b)

In the system shown in Fig. 10.17(b), the pulley is a circular disc of mass M and radius R. The string does not slip over the pulley and the spring is elastic and of negligible mass.

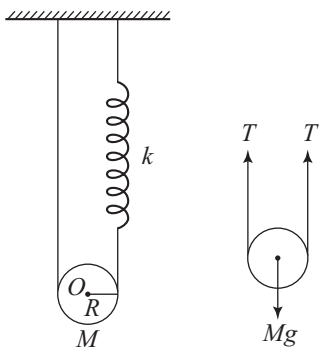


Fig. 10.17(b)

The equilibrium position is the position when the net force acting on it is zero (so that it has no translatory motion) and the torque acting on it is also zero (so that it has no rotatory motion). There will be no torque if the tensions in the two strings are equal. Let this tension be T. The extension in the spring will be

$$x_0 = \frac{T}{k}$$

For translational equilibrium of the pulley,

$$2T = mg$$

$$\Rightarrow \qquad 2\,x_0\,k = mg$$

$$\Rightarrow \qquad x_0 = \frac{mg}{2k} \qquad \text{(i)}$$

If the pulley is pulled down by a small distance x, the total length of string and the spring is $2x$. Since the length of the string is constant, the extension in the spring is $2x$. Total potential energy of the system is

$$\text{P.E.} = \frac{1}{2}\,k\,(x_0 + 2x)^2 - mg\ x$$

$$= \frac{1}{2}\,k\left(\frac{mg}{2k} + 2x\right)^2 - mg\ x$$

$$= \frac{m^2 g^2}{8k} + 2\,kx^2$$

$$\text{Total K.E} = \text{Rotational K.E.} + \text{Translational K.E.}$$

$$= \frac{1}{2}\,I\,\omega^2 + \frac{1}{2}\,Mv^2$$

$$= \frac{1}{2} \times \left(\frac{1}{2}\,MR^2\right) \times \left(\frac{v}{R}\right)^2 + \frac{1}{2}\,Mv^2$$

$$= \frac{1}{4}\,Mv^2 + \frac{1}{2}\,Mv^2 = \frac{3}{4}\,Mv^2$$

$\therefore$ Total energy is

$$E = \text{K.E.} + \text{P.E.}$$

$$= \frac{3}{4}\,Mv^2 + \frac{m^2 g^2}{8k} + 2\,kx^2$$

Since E is not changing with time, $\frac{dE}{dt} = 0$, i.e.

$$\Rightarrow \qquad \frac{3M}{4} \times 2v\,\frac{dv}{dt} + 4\,kx\,\frac{dx}{dt} = 0$$

$$\Rightarrow \qquad \frac{3M}{2}\,v\,\frac{dv}{dt} + 4\,kx\,v = 0$$

$$\Rightarrow \qquad \frac{3M}{2}\,\frac{dv}{dt} + 4\,kx = 0$$

$$\frac{dv}{dt} = -\left(\frac{8k}{3M}\right)x$$

or acceleration $= -\omega^2 x$

where $\omega = \sqrt{\frac{8k}{3M}}$

$$\Rightarrow \qquad T = 2\pi\sqrt{\frac{3M}{8k}}$$

(8) A Block Attached to Three Springs

A block of mass m is connected to three identical springs as shown in Fig. 10.18. The block is pushed towards spring 1 through a small distance so that spring 1 is compressed by x and springs 2 and 3 are extended by $x_2 = x_3 = x \cos 45° = x/\sqrt{2}$. When the block is released, the restoring force acting on it is

$$\vec{F} = -\left(\vec{F_1} + \vec{F_2} + \vec{F_3}\right)$$

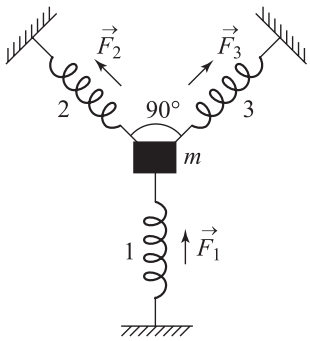


Fig. 10.18

The magnitude of $\vec{F}$ along the vertical direction is

$$F = -(F_1 + F_2 \cos 45° + F_3 \cos 45°)$$

$$= -\left(kx + kx_2 \times \frac{1}{\sqrt{2}} + kx_3 \times \frac{1}{\sqrt{2}}\right)$$

$$= -\left(kx + \frac{kx}{2} + \frac{kx}{2}\right)$$

$$= -2\,kx$$

$\therefore$ Acceleration $a = \dfrac{F}{m} = -\dfrac{2k}{m}x = -\omega^2 x$. Hence

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

7. Important Tips and Additional Formulae

(i) Mass m suspended from a spring of force constant k

$$T = 2\pi\sqrt{\frac{m}{k}}$$

(ii) If a spring of spring constant k is cut into n equal parts, the spring constant of each part becomes $n\,k$. If each part is loaded with a mass m, the time period of each will be

$$T = 2\pi\sqrt{\frac{m}{nk}}$$

(iii) If two springs of spring constants k_1 and k_2 are connected in parallel, the spring constant of the combination is $k = k_1 + k_2$. If the combination is loaded with mass m, then

$$T = 2\pi\left[\frac{m}{(k_1 + k_2)}\right]^{1/2}$$

(iv) If two springs of spring constants k_1 and k_2 are connected in series, the spring constant of the combination is given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

in which case

$$T = \left[\frac{m(k_1 + k_2)}{(k_1 k_2)}\right]^{1/2}$$

(v) If two masses m_1 and m_2 and connected to the ends of a spring of force constant k, the time period of the oscillations of the system is

$$T = 2\pi\sqrt{\frac{\mu}{k}}$$

where $\mu = \dfrac{m_1 m_2}{(m_1 + m_2)}$ is the reduced mass.

(vi) Simple pendulum of length l

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(vii) Liquid column of length L in a U-tube

$$T = 2\pi\sqrt{\frac{L}{2g}}$$

(viii) A pole of mass M and area of cross-section A floating in a liquid of density ρ

$$T = 2\pi\sqrt{\frac{M}{\rho A g}}$$

(ix) LC circuit consisting of inductance L and capacitance C

$$T = 2\pi\sqrt{LC}$$

(x) Simple pendulum immersed in a liquid

$$T = 2\pi\sqrt{\frac{l}{g'}} \text{ where } g' = g\left(1 - \frac{\sigma}{\rho}\right)$$

σ = density of liquid, ρ = density of bob.

(xi) Simple pendulum in a lift moving up with acceleration a

$$T = 2\pi\sqrt{\frac{l}{(g + a)}}$$

If the lift is moving down with acceleration a ($< g$)

$$T = 2\pi\sqrt{\frac{l}{(g - a)}}$$

If the lift is falling freely $a = g$ and $T = \infty$, frequency $\nu = 0$.

(xii) Simple pendulum in a trolley moving with acceleration a in horizontal direction.

$$T = 2\pi\sqrt{\frac{l}{g'}} \text{ where } g' = \sqrt{g^2 + a^2}$$

(xiii) Simple pendulum in a trolley moving down an inclined plane of inclination θ

$$T = 2\pi\sqrt{\frac{l}{g\sin\theta}}$$

(xiv) For a seconds pendulum $T = 2$s. Hence $l = 99$ cm $\simeq$ 1 m.

(xv) An extended body pivoted at point P at a distance h from its centre of mass (Fig. 10.19) and oscillating in the vertical plane.

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

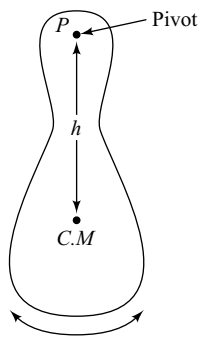


Fig. 10.19

where I = moment of inertia of the body about the pivot.

(a) If the body is a rod of mass m and length l (Fig. 10.20) pivoted at its end, then

$$h = \frac{l}{2} \text{ and } I = \frac{ml^2}{3}.$$

Hence $T = 2\pi\sqrt{\frac{2l}{3g}}$

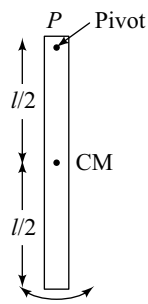


Fig. 10.20

(b) If the body is a ring of mass m and radius r pivoted at a point P on its periphery (Fig. 10.21), then

$h = r$ and $I = mr^2 + mr^2 = 2mr^2$.

Hence $T = 2\pi\sqrt{\frac{2r}{g}}$

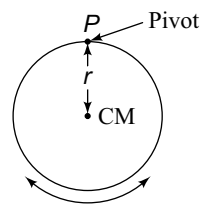


FIg. 10.21

For a disc pivoted at a point on its periphery,

$h = r$ and $I = \frac{1}{2}mr^2 + mr^2$

$= \frac{3}{2}mr^2$.

Hence $T = 2\pi\sqrt{\frac{3r}{2g}}$

(xvi) Some cases of oscillation of spring-mass-pulley system. The mass of pulley is negligible.

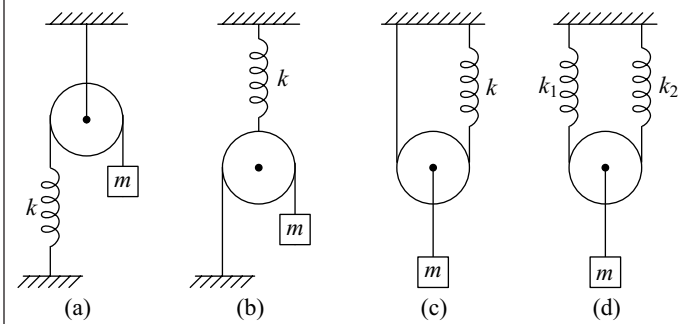


Fig. 10.22

Fig. 10.22 (a) $\Rightarrow T_a = 2\pi\sqrt{\frac{m}{k}}$

Fig. 10.22 (b) $\Rightarrow T_b = 2\pi\sqrt{\frac{4m}{k}}$

Fig. 10.22 (c) $\Rightarrow T_c = 2\pi\sqrt{\frac{m}{4k}}$

Fig. 10.22 (d) $\Rightarrow T_d = 2\pi\sqrt{\frac{m(k_1 + k_2)}{4k_1k_2}}$

(xvii) Horizontal oscillation (without slipping) of a disc, ring, cylinder or sphere of mass M and radius R rolling on a horizontal surface (Fig. 10.23).

$$T = 2\pi\sqrt{\frac{\left(M + \frac{I}{R^2}\right)}{k}}$$

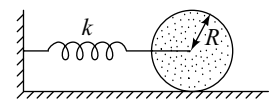


Fig. 10.23

8. Damped Oscillations

Oscillations under the influence of damping (or frictional) force are called damped oscillations. Due to damping the amplitude (and hence energy) of the oscillator keeps on decreasing with time and eventually the oscillator comes to rest. Damping also decreases the frequency of the oscillator.

9. Forced Oscillations and Resonance

The oscillations of a system under the influence of an external periodic force are called forced oscillations. The external

force maintains the oscillations of a damped oscillator. The amplitude of these oscillations remains constant.

If the frequency of the externally applied force is equal to the natural frequency of the oscillator, resonance is said to occur. If damping is small, the amplitude of resonant oscillations will become very large. At resonance, the oscillator absorbs maximum energy supplied by the external force.

PART (B)

Waves

10. Wave Motion

Wave motion involves the transport of energy without any transport of matter. The key word in wave motion is 'disturbance'. In case of mechanical waves, the disturbance is the physical displacement of particles of a medium. In case of electromagnetic waves, the disturbance is a change in electric and magnetic fields.

11. Types of Waves

There are two types of wave motions: (1) transverse and (2) longitudinal.

(1) Transverse Waves In transverse waves the particles of the medium vibrate at right angles to the direction in which the wave propagates. Waves on strings, surface water waves and electromagnetic waves are transverse waves. In electromagnetic waves (which include light waves) the disturbance that travels is not a result of vibrations of particles but it is the oscillation of electric and magnetic fields which takes place at right angles to the direction in which the wave travels.

(2) Longitudinal Waves In longitudinal waves the particles of the medium vibrate along the direction of wave propagation. Sound waves are longitudinal.

12. Characteristics of a Harmonic Wave

(1) Amplitude The amplitude of a wave is the maximum displacement of the particles of the medium from their mean position.

(2) Period The time period of a wave is the period of harmonic oscillations of particles of the medium. The frequency of a wave is the reciprocal of the time period.

(3) Wave Velocity Wave velocity is the distance travelled by the wave in one second.

(4) Wavelength The wavelength is defined as the distance (measured along the direction of propagation of wave) between two nearest particles which are in the same phase of vibration.

13. Displacement Equation for a Travelling Wave

When a plane wave travels in a medium along the positive x-direction, the displacement y of a particle located at x at time t is given by

$$y = A \sin(\omega t - kx)$$

where A = amplitude of the wave, $\omega (= 2\pi\nu)$ is the angular frequency (in rad s^{-1}), ν is the frequency (in Hz) and $k = \left(\frac{2\pi}{\lambda}\right)$ is the angular wave number and λ is the wavelength of the wave.

For a wave travelling along the negative x-direction,

$$y = A \sin(\omega t + kx)$$

The wave velocity is given by

$$v = \nu\lambda = \frac{\omega}{k}$$

14. Phase and Phase Difference

The argument of the sine (or cosine) function which represents a wave is called the phase of the wave. For a wave travelling along the positive x-direction, the phase ϕ at a space point x at time t is given by

$$\phi = \omega t - kx$$

It is clear that the phase changes with time t as well as space point x.

Phase Change with Time

The phase of a given particle (i.e. x fixed) changes with time. As time changes from t to $(t + \Delta t)$, the phase of a particle oscillation changes from ϕ to $(\phi + \Delta\phi)$ where $\Delta\phi$ is given by

$$\Delta\phi = \{\omega(t + \Delta t) - kx\} - \{\omega t - kx\}$$

$$\Rightarrow \quad \Delta\phi = \omega\Delta t = \frac{2\pi}{T}\Delta t$$

where T is the time period of particle oscillation. If $\Delta t = T$, $\phi = 2\pi$.

Phase Change with Position

At a given instant of time t, the phase of particles of the medium varies with position x of the particles. The phase difference at an instant t between two particles separated by x and $(x + \Delta x)$ is given by

$$\Delta\phi = \{\omega t - k(x + \Delta x)\} - (\omega t - kx)$$

$$\Rightarrow \qquad \Delta\phi = -k\Delta x = -\frac{2\pi}{\lambda}\Delta x$$

The minus sign indicates that, for a wave travelling along the positive x-direction, the particles located at higher values of x lag behind in phase. If $\Delta x = \lambda$, $|\Delta\phi| = 2\pi$. Hence *wavelength can be defined as the distance between two particles whose phases differ by* 2π.

EXAMPLE 14 When a plane wave travels in a medium, the displacements of particles are given by

$$y = 0.01 \sin [2\pi (2t - 0.01x)]$$

where x and y are in metre and t in second. Find

(a) the amplitude, wavelength, wave velocity and frequency of the wave,

(b) the phase difference between two positions of the same particle in a time interval of 0.25s and

(c) the phase difference at a given instant of time between two particles 50 m apart.

SOLUTION

(a) The given equation is

$$y = 0.01 \sin (4\pi t - 0.01 \times 2\pi x)$$

Comparing this equation with

$y = A \sin (\omega t - kx)$, we get

$A = 0.01$ m

$\omega = 4\pi \Rightarrow 2\pi\nu = 4\pi \Rightarrow \nu = 2$ Hz

$k = 0.01 \times 2\pi \Rightarrow \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{100} \Rightarrow \lambda = 100$ m

$v = \nu\lambda = 2 \times 100 = 200$ ms^{-1}

(b) $\Delta\phi = 2\pi\nu\Delta t = 2\pi \times 2 \times (0.25) = \pi = 180°$

(c) $\Delta\phi = -\dfrac{2\pi}{\lambda}\Delta x = -\dfrac{2\pi}{100} \times 50 = -\pi = -180°$

The negative sign indicates that the particle at $x = 50$ m lags behind the particle at $x = 0$ by a phase angle of 180°.

EXAMPLE 15 A travelling wave on a string is given by

$$y = 7.5 \sin\left(0.005x + 12t + \frac{\pi}{4}\right)$$

where x and y are in cm and t in second.

(a) Find the displacement and velocity of the particle at a point $x = 1$ cm at $t = 1$s. Is this equal to the wave velocity? Given sin (12.7°) = 0.22.

(b) Locate the points on the string which have the same transverse displacement and velocity as the $x = 1$ cm point has at $t = 2$s, 5s and 11s.

SOLUTION

Given $$y = 7.5 \sin\left(0.005x + 12t + \frac{\pi}{4}\right) \qquad \text{(i)}$$

(a) Putting $x = 1$ cm and $t = 1$s in Eq. (i) we have

$$y = 7.5 \sin\left(0.005 \times 1 + 12 \times 1 + \frac{3.142}{4}\right)$$

$$= 7.5 \sin (12.79 \text{ rad})$$

$$= 7.5 \sin (732.7°) \quad (\because \pi \text{ rad} = 180°)$$

$$= 7.5 \sin (4\pi + 12.7°)$$

$$= 7.5 \sin (12.7°)$$

$$= 7.5 \times 0.22 = 1.65 \text{ cm}$$

Particle velocity is obtained by differentiating (i) w.r.t. time t.

$$V = \frac{dy}{dt} = 7.5 \times 12 \times \cos\left(0.005x + 12t + \frac{\pi}{4}\right)$$

Putting $x = 1$ cm and $t = 1$s,

$$V = 7.5 \times 12 \times \cos 12.7°$$

$$= 87.8 \text{ cm s}^{-1}$$

Comparing Eq. (i) with

$$y = A \sin (kx + \omega t + \phi_0)$$

we get $k = 0.005$ rad cm^{-1} and $\omega = 12$ rad s^{-1} giving

$$v = \frac{\omega}{k} = \frac{12}{0.005} = 2400 \text{ cm s}^{-1}$$

Particle velocity V is not equal to wave velocity v.

(b) By definition of wavelength (λ), all points on the string which are located at $x = \lambda, 2\lambda, \ldots = n\lambda$ ($n = 0, 1, 2, \ldots$) away from $x = 1$ cm have the same displacement and velocity as those at $x = 1$ cm for all values of t.

From $k = 0.005 \Rightarrow \dfrac{2\pi}{\lambda} = 0.005 \Rightarrow \lambda = 12.67$ cm.

Hence the positions of the required points are $x = 1$ cm + (12.67 cm) n = (1 + 12.67 n) cm where n is an integer.

15. Expressions for Wave Velocity in Different Media

1. Velocity of sound in an elastic medium is given by

$$v = \sqrt{\frac{E}{\rho}}$$

where E = modulus of elasticity of the medium and
ρ = density of the medium.

(a) *For gases*:

$$E = \gamma P$$

where $\gamma = C_p/C_v$ is the ratio of the specific heat of the gas at constant pressure and that at constant volume and P is the pressure of the gas.

Thus

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

(b) *For solids*: $E = Y$; the Young's modulus of the solid. Thus

$$v = \sqrt{\frac{Y}{\rho}}$$

(c) *For liquids*: $E = K$, the bulk modulus of the liquid. Thus

$$v = \sqrt{\frac{K}{\rho}}$$

2. The velocity of sound in a gas is independent of the pressure but is directly proportional to the square root of the absolute temperature.

$$\frac{v_t}{v_0} = \sqrt{\frac{T}{T_0}} = \left(\frac{t+273}{273}\right)^{1/2}$$

where t is the temperature in °C.

3. The velocity of sound increases with increase in humidity. Sound travels faster in moist air than in dry air at the same temperature.

4. Velocity of a transverse wave on a stretched string is given by

$$v = \sqrt{\frac{T}{m}}$$

where T = tension in the string and m = mass per unit length of the string. For a string of diameter d and density ρ, we have

$$m = \frac{\pi d^2 \rho}{4}$$

Thus, $$v = \frac{2}{d}\sqrt{\frac{T}{\pi\rho}}$$

EXAMPLE 16 A long uniform steel wire has a diameter of 2.0 mm. What should be the tension in the wire, so that the speed of the transverse wave on it equals the speed of sound at STP (= 320 m s^{-1})? The density of steel is 7800 kg m^{-3}.

SOLUTION The volume of a wire of length L and diameter d is

$$V = \pi r^2 L = \pi d^2 L/4$$

The mass of the wire is

$$M = \text{volume} \times \text{density} = \frac{\pi d^2 L \rho}{4}$$

$\therefore$ Mass per unit length $(m) = \dfrac{M}{L} = \dfrac{\pi d^2 \rho}{4}$

Now $$v = \sqrt{\frac{T}{m}}$$

$\therefore$ $$T = mv^2 = \frac{\pi d^2 \rho v^2}{4}$$

Substituting the values of d, ρ and v and solving we get

$$T = 25.1 \text{ N}$$

EXAMPLE 17 Transverse waves are generated in two uniform steel wires A and B of diameters 10^{-3} m and 0.5×10^{-3} m respectively, by attaching their free end to a vibrating source of frequency 500 Hz. Find the ratio of the wavelengths if they are stretched with the same tension.

SOLUTION The density of a wire of mass M, length L and diameter d is given by

$$\rho = \frac{4M}{\pi d^2 L} = \frac{4m}{\pi d^2}$$

now $$v_A = \sqrt{\frac{T}{m_A}}$$

and $$v_B = \sqrt{\frac{T}{m_B}}$$

$$\frac{v_A}{v_B} = \sqrt{\frac{m_B}{m_A}} = \frac{d_B}{d_A}$$

but $v_A = \nu\lambda_A$ and $v_B = \nu\lambda_B$, ν being the frequency of the source.

Hence $$\frac{\lambda_A}{\lambda_B} = \frac{v_A}{v_B} = \frac{d_B}{d_A} = \frac{0.5 \times 10^{-3}}{10^{-3}} = 0.5$$

EXAMPLE 18 Compare the velocities of sound in hydrogen (H_2) and carbon dioxide (CO_2). The ratio (γ) of specific heats of H_2 and CO_2 is respectively 1.4 and 1.3.

SOLUTION $v_1 = \sqrt{\dfrac{\gamma_1 P}{\rho_1}}$ and $v_2 = \sqrt{\dfrac{\gamma_2 P}{\rho_2}}$

$\therefore$ $$\frac{v_1}{v_2} = \sqrt{\frac{\gamma_1 \rho_2}{\gamma_2 \rho_1}}$$

Since density of a gas is proportional to its molecular weight,

$$\frac{\rho_2}{\rho_1} = \frac{44.01}{2.016} = 21.83$$

$$\therefore \quad \frac{v_1}{v_2} = \left(\frac{1.4}{1.3} \times 21.83\right)^{1/2} = 4.85$$

Velocity of sound in hydrogen is 4.85 times that in carbon dioxide.

EXAMPLE 19 At what temperature will sound travel in hydrogen with the same speed as in oxygen at 927°C?

SOLUTION

$$\frac{v_1}{v_2} = \sqrt{\frac{\gamma_1 \rho_2}{\gamma_2 \rho_1}}$$

Hydrogen and oxygen are both diatomic gases. Hence, $\gamma_1 = \gamma_2$. Also

$$\frac{\rho_2}{\rho_1} = \frac{\text{molecular mass of oxygen}}{\text{molecular mass of hydrogen}}$$

$$= \frac{32}{2} = 16$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{16} = 4$$

$$\frac{v_t}{v_0} = \left(\frac{t+273}{273}\right)^{1/2}$$

For oxygen $$\frac{v_t}{v_2} = \left(\frac{927+273}{273}\right)^{1/2} = \left(\frac{1200}{273}\right)^{1/2} \quad \text{(i)}$$

For hydrogen $$\frac{v_t'}{v_1} = \left(\frac{273+t}{273}\right)^{1/2} \quad \text{(ii)}$$

Dividing (i) and (ii)

$$\frac{v_t}{v_t'} \times \frac{v_1}{v_2} = \left(\frac{1200}{273+t}\right)^{1/2}$$

$$\Rightarrow \quad \frac{v_t}{v_t'} \times 4 = \left(\frac{1200}{273+t}\right)^{1/2}$$

$$\therefore \quad \frac{v_t}{v_t'} = \left(\frac{1200}{273+t}\right)^{1/2}$$

Given $v_t' = v_t$. Hence

$$4 = \left(\frac{1200}{273+t}\right)^{1/2}$$

$$\Rightarrow \quad 16 = \left(\frac{1200}{273+t}\right)$$

$$\Rightarrow \quad t = -198°\text{C}$$

EXAMPLE 20 In a laboratory experiment (room temperature being 15°C) the wavelength of a note of sound of frequency 500 Hz is found to be 0.68 m. If the density of air at STP is 1.29 kg m^{-3}, calculated the ratio of the specific heats of air.

SOLUTION

Speed of sound at 15°C is $v_1 = \nu\lambda = 500 \times 0.68$

$$= 340 \text{ ms}^{-1}$$

Speed of sound at 0°C is $v_0 = 340 \times \sqrt{\frac{273}{273+15}}$

$$= 331 \text{ ms}^{-1}$$

At STP, $\rho_0 = 1.29 \text{ kgm}^{-3}$ and $P_0 = 1.01 \times 10^5 \text{ N m}^{-2}$

Now $$v_0 = \sqrt{\frac{\gamma P_0}{\rho_0}}; \quad \gamma = \frac{C_p}{C_v}$$

$$\Rightarrow \quad \gamma = \frac{v_0^2 \rho_0}{P_0} = \frac{(331)^2 \times 1.29}{1.01 \times 10^5} = 1.39$$

EXAMPLE 21 A uniform thick rope AB of mass 24 kg hangs vesfically from a rigid support. A block of mass 4 kg is attached to the free end of the rope (Fig. 10.24). A transverse pulse of wavelength 5 cm is produced at end A of the rope. The wavelength of the pulse when it reaches point C exactly mid-way from A and B will be

(a) $5\sqrt{6}$ cm (b) 10 cm

(c) $5\sqrt{3}$ cm (d) 5 cm

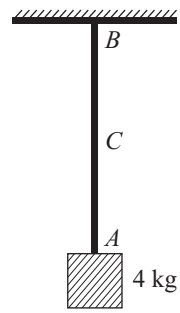


Fig. 10.24

SOLUTION Since the rope has a finite mass, the tension will not be the same at all points on it. At point A, the tension is

$$T_1 = (4 \text{ kg}) \times g = 4g \text{ newton}$$

At point C, the tension is

$$T_2 = (\text{mass of part } AC + 4 \text{ kg}) \times g$$

$$= \left(\frac{24}{2} \text{ kg} + 4 \text{ kg}\right) \times g = 16g \text{ newton}$$

Now $v = \sqrt{\frac{T}{m}}$. Also $v = \nu\lambda$. Thus

$$\nu\lambda = \sqrt{\frac{T}{m}}$$

$$\Rightarrow \quad \lambda = \frac{1}{\nu}\sqrt{\frac{T}{m}}$$

The frequency ν of the pulse cannot change as it depends only on the frequency of the source producing the pulse. As the rope is uniform, the mass per unit length (m) is also constant. Hence $\frac{\lambda}{\sqrt{T}}$ = constant. Therefore

$$\frac{\lambda_1}{\sqrt{T_1}} = \frac{\lambda_2}{\sqrt{T_2}}$$

$$\Rightarrow \quad \lambda_2 = \lambda_1 \times \sqrt{\frac{T_2}{T_1}} = 5\,\text{cm} \times \sqrt{\frac{16g}{4g}}$$

$$= 10 \text{ cm}$$

16. Superposition of Waves (The Superposition Principle)

When a wave reaches a particle of a medium, it imparts a displacement to that particle. If two or more waves arrive at a particle, the resultant displacement of the particle is equal to the vector sum of individual displacements. In the particular case when the waves travelling in the same straight line superpose, the resultant displacement is equal to the algebraic sum of the individual displacements. This is called the principle of superposition.

$$y = y_1 + y_2 + \cdots + y_n$$

The following three cases of superposition are of practical importance.

17. Interference

The superposition of two waves of the same frequency travelling in the same direction in a medium is called interference. Consider two waves of equal amplitude a, equal angular frequency ω and equal angular wave number k but having a phase difference ϕ travelling along the positive x-direction. The displacements y_1 and y_2 of a particle located at x at time t are

$$y_1 = a \sin(\omega t - kx)$$

and $$y_2 = a \sin(\omega t - kx + \phi)$$

According to the superposition principle, the resultant displacement is given by

$$y = y_1 + y_2$$

$$= a\,[\sin(\omega t - kx) + \sin(\omega t - kx + \phi)]$$

Using $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$, we get

$$y = \left(2a\cos\frac{\phi}{2}\right)\sin\left(\omega t - kx + \frac{\phi}{2}\right)$$

$$\Rightarrow \quad y = A\sin\left(\omega t - kx + \frac{\phi}{2}\right)$$

where $A = 2a\cos\left(\frac{\phi}{2}\right)$ is the resultant amplitude. The frequency and wavelength of the resultant wave remain the same as those of individual waves.

(a) *Constructive Interference:* If A is maximum (positive or maximum), the interference is constructive, then $A_{\max} = \pm 2a$. This happens if

$$\cos\left(\frac{\phi}{2}\right) = \pm 1$$

$$\Rightarrow \quad \frac{\phi}{2} = 0, \pi, 2\pi, \ldots$$

$$\Rightarrow \quad \phi = 2n\pi; \qquad (n = 0, 1, 2, \ldots)$$

(b) *Destructive Interference:* If $A = 0$, the interference is destructive. This happens if

$$\cos\left(\frac{\phi}{2}\right) = 0$$

$$\Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$\Rightarrow \quad \phi = (2n+1)\pi; \qquad n = 0, 1, 2, \ldots$$

EXAMPLE 22 Two waves each of amplitude 2 cm and wavelength 5 cm and frequency 10 Hz have a constant phase difference of 60°. Travelling in the same direction, they superpose at a particle of the medium. What is the resultant amplitude of the oscillations of the particle? Also find the frequency and wavelength of the resultant wave.

SOLUTION

Resultant amplitude is $A = 2a\cos\left(\frac{\phi}{2}\right)$

$$= 2 \times (2\text{ cm}) \times \cos\left(\frac{60°}{2}\right)$$

$$= 2\sqrt{3} \text{ cm}$$

The frequency and wavelength of the resultant wave are 10 Hz and 5 cm respectively, the same as those of the individual interfering waves.

EXAMPLE 23 Two waves travelling in a medium are given by

$$y_1 = 2\sin\left(3\pi t - \frac{\pi}{2}x\right)$$

and $$y_2 = 4 \sin\left(6\pi t - \frac{3\pi}{2}x\right)$$

where y_1, y_2 and x are in cm and t in second. The displacement of a particle at $x = 1$ cm at time $t = 1$ s is

(a) -2 cm (b) -3 cm

(c) $+2$ cm (d) $+6$ cm

SOLUTION The displacements of the particle at $x = 1$ cm at $t = 1$ s are given by

$$y_1 = 2 \sin\left(3\pi \times 1 - \frac{\pi}{2} \times 1\right)$$

$$= 2 \sin\left(3\pi - \frac{\pi}{2}\right)$$

$$= 2 \sin\left(\frac{5\pi}{2}\right) = 2 \sin\left(2\pi + \frac{\pi}{2}\right)$$

$$= 2 \sin \frac{\pi}{2} = 2 \text{ cm}$$

and $$y_2 = 4 \sin\left(6\pi \times 1 - \frac{3\pi}{2} \times 1\right)$$

$$= 4 \sin\left(\frac{9\pi}{2}\right) = 4 \sin\left(4\pi + \frac{\pi}{2}\right)$$

$$= 4 \sin \frac{\pi}{2} = 4 \text{ cm}$$

According to superposition principle

$$y = y_1 + y_2 = 2 + 4 = 6 \text{ cm}$$

18. Standing (or Stationary) Waves

Standing (or stationary) waves are produced when two waves of the same frequency travelling in opposite directions in a medium superpose. In actual practice, we do not send two independent waves in a medium in opposite directions. A wave is sent in a finite medium which has its boundaries, for example, a string of a finite length or a rod or a column of gas or liquid. The wave gets reflected at the boundaries and a superposition of the incident and reflected waves occurs continuously, giving rise to standing waves.

When a wave is reflected from a rigid boundary, it undergoes a reversal of amplitude (which implies a phase change of π). Consider a wave travelling in the negative x-direction towards a boundary at $x = 0$, where it is reflected. The particle displacements due to the incident and reflected waves are given by

$$y_i = a \sin(\omega t + kx)$$

and $$y_r = -a \sin(\omega t - kx)$$

From the superposition principle, the resultant displacement is

$$y = y_i + y_r$$

$$= a [\sin(\omega t + kx) - \sin(\omega t - kx)]$$

Using $\sin\alpha - \sin\beta = 2 \sin\left(\frac{\alpha - \beta}{2}\right) \cos\left(\frac{\alpha + \beta}{2}\right)$, we get

$$y = 2a \sin(kx) \cos(\omega t) \quad \text{(i)}$$

$$\Rightarrow \quad y = A \cos \omega t \quad \text{(ii)}$$

where $$A = 2a \sin(kx) \quad \text{(iii)}$$

Equation (i) represents a standing wave. It does not represent a travelling wave since it does not involve the combination $(\omega t \pm kx)$ in the argument of the sine or cosine function. Equation (ii) tells us each particle has a simple harmonic motion and Eq. (iii) tells us that the amplitude of motion is different for different particles (i.e. for different values of x). Such simple harmonic motions of the particles of a medium are called *normal modes*.

Nodes

There are certain points in the medium which are permanently at rest. These points are called nodes. The position of nodes is given by

$$A = 0$$

$$\Rightarrow \quad 2a \sin(kx) = 0$$

$$\Rightarrow \quad \sin kx = 0$$

$$\Rightarrow \quad kx = 0, \pi, 2\pi, \ldots$$

$$\Rightarrow \quad \frac{2\pi}{\lambda}x = 0, \pi, 2\pi, \ldots$$

$$\Rightarrow \quad x = 0, \frac{\lambda}{2}, \lambda, \ldots \text{ and so on}$$

$\therefore$ Distance between two consecutive nodes $= \frac{\lambda}{2}$.

Antinodes

There are certain points in the medium which have maximum (positive or negative) amplitude. These points are called antinodes. Their position is given by

$$A = \pm 1$$

$$\Rightarrow \quad 2a \sin(kx) = \pm 1$$

$$\Rightarrow \quad \sin kx = \pm 1$$

$$\Rightarrow \quad kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$\Rightarrow \quad \frac{2\pi}{\lambda}x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$\Rightarrow \quad x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$$

$\therefore$ Distance between two consecutive antinodes $= \frac{\lambda}{2}$.

Note

1. Exactly mid-way between two nodes is an antinode and vice versa.
2. The distance between a node and the next antinode = $\frac{\lambda}{4}$.
3. There is no transfer of energy along the medium.

EXAMPLE 24 Standing waves are produced by the superposition of two waves

$$y_1 = 5 \sin (3\pi t - 2\pi x)$$

and $$y_2 = 5 \sin (3\pi t + 2\pi x)$$

where y and x are in cm and t in second. Find the amplitude of the particle at $x = 2$ cm.

SOLUTION $y = y_1 + y_2$

$$= 5 [\sin (3\pi t - 2\pi x) + \sin (3\pi t + 2\pi x)]$$

Using $\sin (\alpha + \beta) + \sin (\alpha - \beta) = 2 \sin \alpha \cos \beta$, we get

$$y = 10 \cos (2\pi x) \sin (3\pi t)$$

$\Rightarrow$ $$y = A \sin (3\pi t), \text{ where } A = 10 \cos (2\pi x)$$

Amplitude A at $x = 2$ cm is

$$10 \times \cos (2\pi \times 2) = 10 \cos 4\pi = 10 \text{ cm}$$

Normal Modes of a String Fixed at Both Ends

Consider a uniform string of length L stretched with a tension T and fixed rigidly at its ends at $x = 0$ and $x = L$. The string can vibrate in a number of modes. Figure 10.25 shows the first three harmonics.

(a) *Fundamental Mode (or First Harmonic)*

In this mode, the string vibrates in one segment. [Fig. 10.25 (a)]

$$\frac{\lambda}{2} = L \quad \Rightarrow \quad \lambda = 2L$$

The frequency of the fundamental mode is

$$\nu_1 = \frac{v}{\lambda} = \frac{v}{2L}$$

where $$v = \sqrt{\frac{T}{m}}$$

(b) *Second Harmonic*. [Fig. 10.25 (b)]

For the second harmonic, $\frac{\lambda}{2} + \frac{\lambda}{2} = L \quad \Rightarrow \quad \lambda = L.$

$$\nu_2 = \frac{v}{\lambda} = \frac{v}{L} = 2\nu_1$$

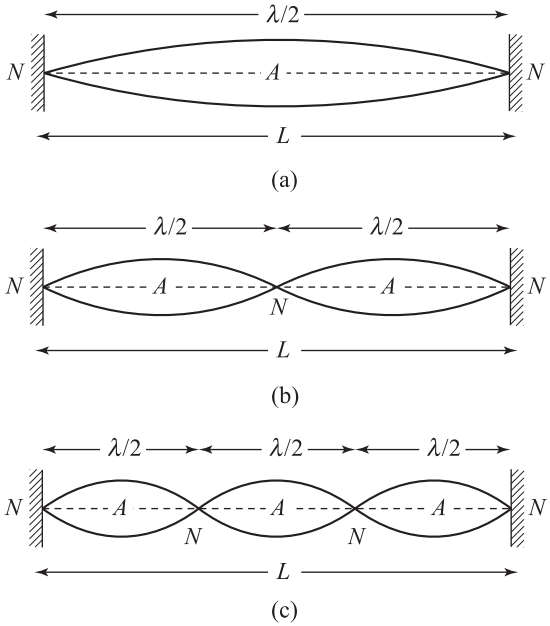


Fig. 10.25

(c) *Third Harmonic*. [Fig. 10.25 (c)]

For the third harmonic, $\frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{2} = L \quad \Rightarrow \quad \lambda = \frac{2L}{3}$

$$\nu_3 = \frac{v}{\lambda} = \frac{3v}{2L} = 3\nu_1$$

In general, for a string vibrating in the nth harmonic, the frequency of vibration is

$$\nu_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

EXAMPLE 25 The transverse displacement of a string fixed at both ends is given by

$$y = 0.06 \sin \left(\frac{2\pi x}{3}\right) \cos (120\pi t) \qquad \text{(i)}$$

where y and x are in metre and t in second. The length of the string is 1.5 m and its mass is 3×10^{-2} kg.

(a) What is the amplitude at point $x = 0.5$ m?

(b) What is the velocity of the particle at $x = 0.75$ m at $t = 0.25$ s?

(c) Write down the equation of the component waves whose superposition gives the vibration given in Eq. (i) above. What is the wavelength, frequency and speed of each wave?

(d) Determine the tension in the string.

(e) Do all points on the string vibrate with the same (i) frequency, (ii) phase and (iii) amplitude?

SOLUTION

(a) Displacement is maximum when $\cos(120\pi t) = 1$.

Hence the amplitude = $0.06 \sin\left(\frac{2\pi x}{3}\right)$.

At $x = 0.5$ m, the amplitude = $0.06 \sin\left(\frac{2\pi}{3} \times 0.5\right)$

$$= 0.06 \sin\frac{\pi}{3} = 0.052 \text{ m}$$

(b) The velocity at point x at time t is obtained by differentiating Eq. (i) with respect to t.

$$V = \frac{dy}{dt} = -(0.06 \times 120\pi) \sin\left(\frac{2\pi x}{3}\right) \sin(120\pi t)$$

Now at $t = 0.25$s, $\sin(120\pi t) = \sin(30\pi) = 0$. Hence at $t = 0.25$s, the velocity is zero for all values of x including $x = 0.75$ m.

(c) The stationary wave in the question may be written as

$$y = 2A \sin\alpha \cos\beta$$

where $A = 0.03$ m, $\alpha = \frac{2\pi x}{3}$ and $\beta = 120\pi t$.

Now $2A \sin\alpha \cos\beta = A \sin(\alpha + \beta) + A \sin(\alpha - \beta)$

$$= 0.03 \sin\left(\frac{2\pi x}{3} + 120\pi t\right) + 0.03 \sin\left(\frac{2\pi x}{3} - 120\pi t\right)$$

i.e. $y = y_1 + y_2$

Hence the two component waves are

$$y_1 = 0.03 \sin\left(\frac{2\pi x}{3} + 120\pi t\right)$$

and $$y_2 = 0.03 \sin\left(\frac{2\pi x}{3} - 120\pi t\right)$$

Let λ be the wavelength, ν the frequency and v the speed of each wave. Then

$\frac{2\pi}{\lambda}$ = coefficient of x in the argument of the sine function = $\frac{2\pi}{3}$

or $\lambda = 3$ m.

Also $\omega = 2\pi\nu$ = coefficient of t in the argument of the sine function

$= 120\pi$

which gives $\nu = 60$ Hz.

Hence $v = \nu\lambda = 60 \times 3 = 180 \text{ m s}^{-1}$

(d) Mass per unit length (m) = $\frac{3.0 \times 10^{-2}}{1.5}$

$= 2.0 \times 10^{-2} \text{ kg m}^{-1}$

We know that $v = \sqrt{\frac{T}{m}}$, where T is tension in the string.

$\therefore \quad T = mv^2 = 2.0 \times 10^{-2} \times (180)^2 = 648$ N

(e) In a stationary wave on a string, all points on the string vibrate with the same frequency and the same phase, but the amplitude is different at different points (i.e. different values of x). The amplitude is zero at nodes and maximum at antinodes.

EXAMPLE 26 A wire of density 9 g cm^{-3} is stretched between two clamps 100 cm apart while being subjected to an extension of 0.05 cm. What is the lowest frequency of transverse vibrations of the wire, assuming that the Young's modulus of the material of the wire = 0.9×10^{11} N m^{-2}.

SOLUTION Young's modulus

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{T/A}{l/L} = \frac{TL}{Al}$$

where T = tension, L = original length = 100 cm, l = extension = 0.05 cm and A = area of cross-section of the wire.

Hence, $$\frac{T}{A} = \frac{Yl}{L} = \frac{0.9 \times 10^{11} \times 0.05}{100} = 4.5 \times 10^7$$

Now mass per unit length $m = \frac{\text{mass}}{\text{length}}$

$$= \frac{\text{volume} \times \text{density}}{\text{length}}$$

$$= \frac{\text{area} \times \text{length} \times \text{density}}{\text{length}}$$

$$= \text{area} \times \text{density}$$

$$= A\rho$$

Density $\rho = 9$ g cm^{-3} = 9000 kg m^{-3}. The lowest frequency is the frequency of the fundamental mode.

$$\nu_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2L}\sqrt{\frac{T}{A\rho}}$$

$$= \frac{1}{2 \times 1} \times \sqrt{\frac{4.5 \times 10^7}{9000}}$$

$$= 35.3 \text{ Hz}$$

EXAMPLE 27 A wire having a linear density of 0.05 g cm^{-1} is stretched between two rigid supports with a tension of 4.5×10^2 N. It is observed that the wire resonates at a frequency of 420 Hz. The next higher frequency at which the wire resonates is 490 Hz. Determine the length of the wire.

SOLUTION Let 420 Hz be the nth harmonic, then 490 Hz is the $(n + 1)$th harmonic. Therefore

$$420 = \frac{n}{2L}\sqrt{\frac{T}{m}} \qquad \text{(i)}$$

and
$$490 = \frac{(n+1)}{2L}\sqrt{\frac{T}{m}} \qquad \text{(ii)}$$

Dividing (i) and (ii) we get $n = 6$. Putting $n = 6$ in (i) we get

$$420 = \frac{6}{2L}\sqrt{\frac{4.5\times10^2}{0.05\times10^{-1}}} \quad \Rightarrow \quad L = 2.14\,\text{m}$$

EXAMPLE 28 A sonometer wire AB of length 110 cm is fixed at both ends. How far from ends A and B should the two bridges be placed below the wire so that the fundamental frequencies of the three segments of the wire are in the ratio 1 : 2 : 3?

(a) 60 cm, 20 cm (b) 50 cm, 30 cm

(c) 45 cm, 35 cm (d) 40 cm, 30 cm

SOLUTION Let L_1, L_2 and L_3 be the lengths of the three segments of wire. Their fundamental frequencies are

$$\nu_1 = \frac{1}{2L_1}\sqrt{\frac{T}{m}}$$

$$\nu_2 = \frac{1}{2L_2}\sqrt{\frac{T}{m}}$$

$$\nu_3 = \frac{1}{2L_2}\sqrt{\frac{T}{m}}$$

which give $\nu_1 L_1 = \nu_2 L_2 = \nu_3 L_3$. Since $\nu_1 : \nu_2 : \nu_3 = 1 : 2 : 3$, we have $\nu_2 = 2\nu_1$ and $\nu_3 = 3\nu_1$. Hence

$$\nu_1 L_1 = 2\nu_1 L_2 \quad \text{and} \quad \nu_1 L_1 = 3\nu_1 L_3 \text{ so that}$$

$$L_2 = L_1\,\frac{\nu_1}{2\nu_1} = \frac{L_1}{2}$$

and
$$L_3 = \frac{L_1}{3}$$

But
$$L_1 + L_2 + L_3 = L$$

$$\Rightarrow \qquad L_1\left(1 + \frac{1}{2} + \frac{1}{3}\right) = 110$$

$$\Rightarrow \qquad L_1 = 60\ \text{cm}$$

Hence
$$L_3 = \frac{60}{3} = 20\ \text{cm}$$

Hence the correct choice is (a).

Normal Modes in Air Columns in a Pipe

A gas column in a pipe can oscillate in a number of modes

Case 1: Closed Pipe

Consider a pipe of length L open at one end and closed at the other. The closed end is a node and the open end is an antinode. Figure 10.26 shows the first three modes of a closed pipe.

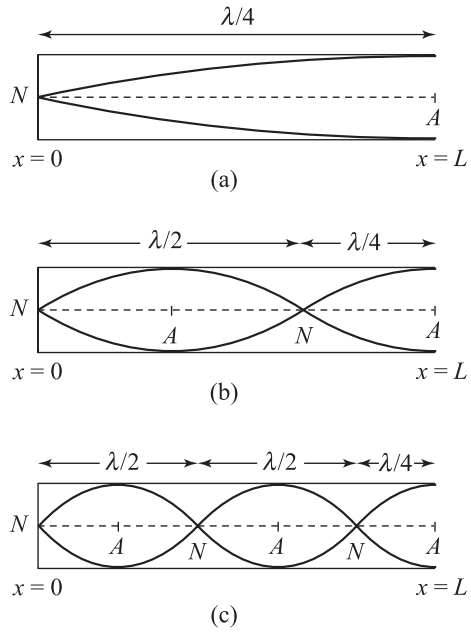


Fig. 10.26

(a) *Fundamental Mode (or First Harmonic)* [Fig. 10.24 (a)]

$$\frac{\lambda}{4} = L \quad \Rightarrow \quad L = 4\lambda$$

$$\therefore \qquad \nu_1 = \frac{v}{\lambda} = \frac{v}{4L}$$

where $v = \sqrt{\dfrac{\gamma P}{\rho}}$ is the speed of sound in the gas.

(b) *Third Harmonic (or First Overtone)* [Fig. 10.24 (b)]

$$\frac{\lambda}{2} + \frac{\lambda}{4} = L \quad \Rightarrow \quad \lambda = \frac{4L}{3}$$

$$\therefore \qquad \nu_3 = \frac{v}{\lambda} = \frac{3v}{4L} = 3\nu_1$$

(c) *Fifth Harmonic (or Second Overtone)* [Fig. 10.24 (c)]

$$\frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{4} = L \quad \Rightarrow \quad \lambda = \frac{4L}{5}$$

$$\therefore \qquad \nu_5 = \frac{v}{\lambda} = \frac{5v}{4L} = 5\nu_1$$

In general, for nth harmonic

$$\nu_n = \frac{nv}{4L} = \frac{n}{4L}\sqrt{\frac{\gamma P}{\rho}}$$

where $n = 1, 3, 5, ...$ etc.

Note In a closed pipe only odd harmonics are present; all the even harmonics are absent.

Case 2: Open Pipe

Figure 10.27 shows the first three modes of an open pipe

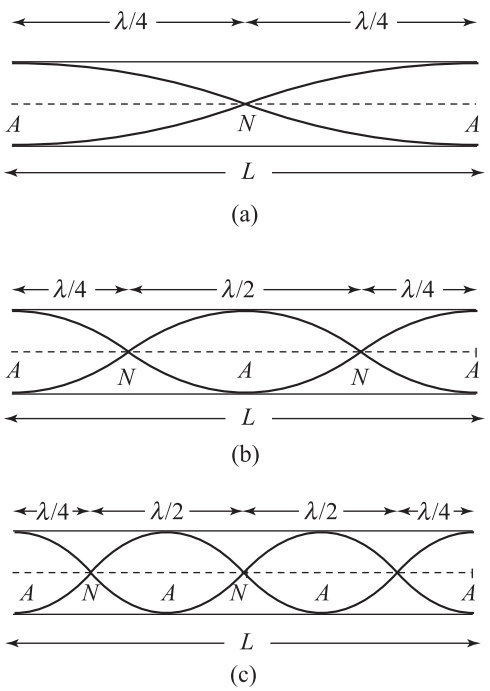


Fig. 10.27

(a) Fundamental Mode (or First Harmonic) [Fig. 10.25 (a)]

$$\frac{\lambda}{4} + \frac{\lambda}{4} = L \quad \Rightarrow \quad \lambda = 2L$$

$$\therefore \qquad \nu_1 = \frac{v}{\lambda} = \frac{v}{2L}$$

(b) Second Harmonic (or First Overtone) [Fig. 10.25 (b)]

$$\frac{\lambda}{4} + \frac{\lambda}{2} + \frac{\lambda}{4} = L \quad \Rightarrow \quad \lambda = L$$

$$\therefore \qquad \nu_2 = \frac{v}{\lambda} = \frac{v}{2L} = 2\nu_1$$

(c) Third Harmonic (or Second Overtone) [Fig. 10.25 (c)]

$$\frac{\lambda}{4} + \frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{4} = L \quad \Rightarrow \quad \lambda = \frac{2L}{3}$$

$$\therefore \qquad \nu_3 = \frac{v}{\lambda} = \frac{3v}{2L} = 3\nu_1$$

For nth harmonic

$$\nu_n = n\nu_1; \qquad n = 1, 2, 3, ...$$

Note In an open pipe, all harmonics (even as well as odd) are present.

End Correction

We have taken the open end of a pipe to be an antinode. This is not strictly true. In fact, the particles of air just at the open end are not perfectly free because of the restriction imposed by the pipe. The true antinode is slightly away from the open end as shown in Fig. 10.28.

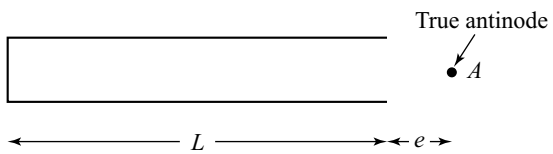


Fig. 10.28

The distance e is called the *end correction*. The effective length of the pipe is $(L + e)$.

EXAMPLE 29 A pipe of length 20 cm is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe if both ends are open? Speed of sound = 340 ms^{-1}.

SOLUTION Let N be the frequency of the source and ν_m that of the mth harmonic of the closed pipe, where $m = 1, 3, 5, ...$ Resonance will occur if

$$N = \nu_m = \frac{mv}{4L}$$

$$\Rightarrow \qquad 430 = \frac{m \times 340}{4 \times 0.2}$$

which gives $m = 1.01 \simeq 1$. The source of frequency 430 Hz will resonantly excite the first harmonic (i.e. fundamental mode) of the closed pipe.

For an open pipe, the condition of resonance with the same source will be

$$N = \nu_n = \frac{nv}{2L}, \quad \text{where } n = 1, 2, 3, \ldots$$

$$\Rightarrow \qquad n = \frac{N \times 2L}{v} = \frac{430 \times 2 \times 0.2}{340} = 0.5$$

which is not an integer. Hence the source will not be in resonance with any harmonic of the open pipe.

EXAMPLE 30 A half-metre long tube open at one end, with a movable piston shows resonance with a tuning fork of frequency 512 Hz when the tube length is 16.0 cm and 49.0 cm. Calculate the speed of sound at the temperature of the experiment and determine the end-correction.

SOLUTION The tuning fork is in resonance at two lengths of the pipe, viz. 16 cm and 49 cm. Since the second resonance length is about three times the first, it is clear that the fork is in resonance with the first harmonic (i.e. the fundamental mode) when $L_1 = 16$ cm and with the third harmonic when $L_2 = 49$ cm. Thus we have

$$N = \nu_1 = \frac{v}{4(L_1 + e)} \qquad \text{(i)}$$

Also
$$N = \nu_3 = \frac{3v}{4(L_2 + e)} \qquad \text{(ii)}$$

where N is the frequency of tuning fork and e is the end-correction. Dividing (ii) by (i) we get

$$3(16.0 + e) = 49.0 + e \quad \text{or} \quad e = 0.5 \text{ cm}$$

Then the speed of sound as obtained from (i) is

$$\begin{aligned} v &= 4N(L_1 + e) \\ &= 4 \times 512 \times (16.0 + 0.5) \\ &= 33792 \text{ cm s}^{-1} = 337.92 \text{ m s}^{-1} \\ &\simeq 338 \text{ m s}^{-1} \end{aligned}$$

19. Beats

The periodic rise and fall of intensity of the wave resulting from the superposition of two waves of different frequencies is called the phenomenon of beats.

Consider two waves of angular frequencies ω_1 and ω_2. For simplicity, we assume that they have equal amplitude a and that the observation point is at $x = 0$. Then

$$y_1 = a \sin \omega_1 t; \quad \omega_1 = 2\pi\nu_1$$

and
$$y_2 = a \sin \omega_2 t; \quad \omega_2 = 2\pi\nu_2$$

Using the superposition principle,

$$\begin{aligned} y &= y_1 + y_2 \\ &= a (\sin \omega_1 t + \sin \omega_2 t) \\ &= 2a \cos\left(\frac{\omega_1 - \omega_2}{2}\right) t \sin\left(\frac{\omega_1 + \omega_2}{2}\right) t \end{aligned}$$

$$\Rightarrow \qquad y = A \sin(\omega_{\text{av}} t); \quad \omega_{\text{av}} = \frac{1}{2}(\omega_1 + \omega_2)$$

where
$$A = 2a \cos\left(\frac{\omega_1 - \omega_2}{2}\right) t$$

Now, intensity is proportional to A^2. Therefore, the resultant intensity is maximum when

$$\cos\left(\frac{\omega_1 - \omega_2}{2}\right) t = \pm 1$$

$$\Rightarrow \qquad \left(\frac{\omega_1 - \omega_2}{2}\right) t = 0, \pi, 2\pi, \ldots$$

$$\Rightarrow \qquad 2\pi\left(\frac{\nu_1 - \nu_2}{2}\right) t = 0, \pi, 2\pi, \ldots$$

$$\Rightarrow \qquad t = 0, \frac{1}{(\nu_1 - \nu_2)}, \frac{2}{(\nu_1 - \nu_2)}, \ldots$$

The time interval between two consecutive maxima is

$$t_b = \frac{1}{\nu_1 - \nu_2}$$

Therefore frequency of maxima is

$$\nu_b = \frac{1}{t_b} = \nu_1 - \nu_2$$

Similarly, we can show that the frequency of minima = ν_b. Hence the frequency of beats is

$$\nu_b = \nu_1 - \nu_2$$

Thus

Beat frequency = difference between the frequencies of interfering waves.

EXAMPLE 31 Two tuning forks A and B produce 10 beats per second when sounded together. On loading fork A with a little wax it is observed that 5 beats per second are produced. If the frequency of fork B is 480 Hz, find the frequency of fork A (a) before loading and (b) after loading.

SOLUTION There are two possibilities (i) $\nu_A < \nu_B$ or (ii) $\nu_A > \nu_B$.

Case (i) $\nu_A < \nu_B$; $\nu_B - \nu_A = \nu_b \Rightarrow 480 - \nu_A = 10 \Rightarrow \nu_A = 470$ Hz.

On loading with a little wax, the frequency of a fork decreases slightly, i.e. ν_A becomes slightly less than 470 Hz. Hence the number of beats per second must increase. But ν_b decreases to 5. Hence ν_A cannot be less than ν_B.

Case (ii) $\nu_A > \nu_B$. In this case $\nu_A = \nu_B + \nu_b = 480 + 10 = 490$ Hz. On loading A, ν_A decreases. Hence $\nu_b = \nu_A - \nu_B$

will decrease. Since ν_b is observed to decrease to 5, ν_A must be greater than ν_B.

(a) Hence before loading, $\nu_A = 490$ Hz

(b) After loading $\nu_A = 490 - 5 = 485$ Hz

EXAMPLE 32 A metal wire of diameter 1.5 mm is held on two knife edges separated by a distance of 50 cm. The tension in the wire is 100 N. The wire vibrating with its fundamental frequency and a vibrating tuning fork together produce 5 beats per second. The tension in the wire is then reduced to 81 N. When the two are excited, beats are heard at the same rate. Calculate (a) the frequency of the fork, and (b) the density of the material of the wire.

SOLUTION

(a) Let N be the frequency of the tuning fork. Then, the frequency of the wire, when the tension is 100 N will be $(N + 5)$ and when the tension is 81 N, it is $(N - 5)$; since in each case 5 beats are heard per second. Hence

$$N + 5 = \frac{1}{2L}\sqrt{\frac{T_1}{m}} = \frac{1}{2 \times 0.5}\sqrt{\frac{100}{m}} = \frac{10}{\sqrt{m}} \quad \text{(i)}$$

$$\text{and } N - 5 = \frac{1}{2L}\sqrt{\frac{T_2}{m}} = \frac{1}{2 \times 0.5}\sqrt{\frac{81}{m}} = \frac{9}{\sqrt{m}} \quad \text{(ii)}$$

Subtracting (ii) from (i) we have

$$10 = \frac{1}{\sqrt{m}} \quad \text{or} \quad m = 0.01 \text{ kg m}^{-1}$$

Using this value of m in (i) or (ii) gives $N = 95$ Hz.

(b) Now $m = \pi r^2 \rho = \dfrac{\pi d^2 \rho}{4}$

Putting $d = 1.5 \times 10^{-3}$ m and $m = 0.01$ kg m^{-1}, we get

$$\rho = 5.7 \times 10^3 \text{ kg m}^{-3}$$

20. Doppler Effect in Sound

The apparent change in frequency of sound heard by an observer due to a relative motion between the observer and the source of sound is called the Doppler effect.

The expressions for the apparent frequencies are as follows:

1. Source approaching a stationary observer

$$\nu_1 = \nu\left(\frac{v}{v - u_s}\right),$$

where ν = real frequency

v = velocity of sound

u_s = velocity of source

2. Source receding from a stationary observer

$$\nu_2 = \nu\left(\frac{v}{v + u_s}\right)$$

3. Observer approaching a stationary source of sound

$$\nu_3 = \nu\left(\frac{v + u_0}{v}\right),$$

where u_0 = velocity of observer

4. Observer receding from a stationary source of sound

$$\nu_4 = \nu\left(\frac{v - u_0}{v}\right)$$

5. Both approaching each other

$$\nu_5 = \nu\left(\frac{v + u_0}{v - u_s}\right)$$

6. Both receding from each other

$$\nu_6 = \nu\left(\frac{v - u_0}{v + u_s}\right)$$

7. Source approaching a receding observer

$$\nu_7 = \nu\left(\frac{v - u_0}{v - u_s}\right)$$

8. Observer approaching a receding source

$$\nu_8 = \nu\left(\frac{v + u_0}{v + u_s}\right)$$

Note

1. If the source of sound moves, the apparent change in frequency is due to change in wavelength; the speed of sound remaining the same.
2. If the observer moves, the apparent change in frequency is due to change in the speed of sound relative to the observer; the wavelength of sound remaining the same.

9. When the source of sound goes past a stationary observer, the apparent change in the frequency of sound is given by

$$\Delta\nu = \frac{2vu_s\nu}{\left(v^2 - u_s^2\right)}$$

If $u_s << v$, then $v^2 - u^2{}_s \simeq v^2$, then

$$\Delta\nu = \frac{2u_s\nu}{v}$$

10. When the observer goes past a stationary source of sound, the apparent change in frequency of sound is given by

$$\Delta\nu = \frac{2u_0\nu}{v}$$

11. *Effect of the Motion of the Medium* The velocity of material or mechanical waves is affected by the motion of the medium. If the medium is moving with a velocity u_m in the direction of propagation of sound, the effective velocity of sound is increased from v to $(v + u_m)$. In this case, v is replaced by $(v + u_m)$ in the above expressions. On the other hand, if the medium is moving with a velocity u_m in a direction opposite to the direction of wave propagation, v is replaced by $(v - u_m)$.

21. Doppler Effect in Light

If a star emitting light of frequency ν goes away from the earth with a speed v, the apparent frequency ν' of the light reaching the earth is given by

$$\nu' = \nu\left(\frac{c}{c+v}\right)$$

where c is the speed of light. Since $\nu = c/\lambda$ and $\nu' = c/\lambda'$, we have

$$\frac{c}{\lambda'} = \frac{c}{\lambda}\left(\frac{c}{c+v}\right)$$

or $$\frac{\lambda'}{\lambda} = 1 + \frac{v}{c}$$

or $$\frac{\lambda' - \lambda}{\lambda} = \frac{v}{c}$$

or $$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

The apparent change in wavelength $\Delta\lambda$ is called the **Doppler shift.** If the wavelength of light reflected from a moving object decreases, the object is moving towards the observer and *vice versa*. The wavelength of light reflected from a galaxy is found to increase. This is called the **red shift** which indicates that the galaxy is receding from us. The red shift indicates that the universe is expanding.

EXAMPLE 33 A train standing at the outer signal of a railway station is blowing a whistle of frequency 500 Hz. The speed of sound is 340 m s^{-1}.

(i) Find the frequency of the sound of the whistle heard by a man standing on the platform when the train (a) approaches the platform with a speed of 20 ms^{-1} and (b) recedes from the platform with a speed of 20 ms^{-1}.

(ii) What is the speed of sound in each case?

(iii) What is the wavelength of sound heard by the man in each case?

SOLUTION ν = 500 Hz, u_s = 20 m s^{-1}, v = 340 m s^{-1}

(i) (a) Train approaching

$$\nu_1 = \frac{\nu v}{v - u_s} = \frac{500 \times 340}{340 - 20} = 531 \text{ Hz}$$

(b) Train receding

$$\nu'_1 = \frac{\nu v}{v + u_s} = \frac{500 \times 340}{340 + 20} = 472 \text{ Hz}$$

(ii) Since the observer is at rest, the speed of sound relative to him is v = 340 m s^{-1} in cases (a) and (b).

(iii) Now, wavelength

$$= \frac{\text{speed of sound relative to observer}}{\text{frequency of sound heard by him}}$$

In case (a) $$\lambda_1 = \frac{v}{\nu_1} = \frac{340}{531} = 0.64 \text{ m}$$

In case (b) $$\lambda'_1 = \frac{v}{\nu'_1} = \frac{340}{472} = 0.72 \text{ m}$$

EXAMPLE 34 A train standing at a platform is blowing a whistle of frequency 500 Hz in still air. The speed of sound in still air is 340 m s^{-1}. (i) What is the frequency of the sound of the whistle heard by a man on a trolley which is moving (a) towards the engine with a speed of 20 m s^{-1} and (b) away from the engine with a speed of 20 m s^{-1}? (ii) What is the speed of sound heard by the man in each case? (iii) What is the wavelength of sound heard by the man in each case?

SOLUTION ν = 500 Hz, u_0 = 20 m s^{-1}, v = 340 m s^{-1}

(i) (a) Observer approaching

$$\nu'_2 = \frac{\nu(v + u_0)}{v} = \frac{500 \times (340 + 20)}{340} = 529 \text{ Hz}$$

(b) Observer receding

$$\nu_2 = \frac{\nu(v - u_0)}{v} = \frac{500 \times (340 - 20)}{340} = 471 \text{ Hz}$$

(ii) Since the observer is moving, the speed of sound heard by him in case (a) is $(v + u_0)$ = 340 + 20 = 360 m s^{-1} and case (b) it is $(v - u_0)$ = 340 – 20 = 320 m s^{-1},

(iii) Wavelength = $\dfrac{\text{speed of sound relative observer}}{\text{frequency of sound heard by him}}$

In case (a) $$\lambda'_2 = \frac{v + u_0}{\nu'_2} = \frac{360}{529} = 0.68 \text{ m}$$

In case (b) $\lambda_2 = \dfrac{v - u_0}{\nu_2} = \dfrac{320}{471} = 0.68$ m

Notice that $\lambda_2 = \lambda'_2$.

EXAMPLE 35 An ambulance blowing a siren of frequency 700 Hz is travelling towards a vertical reflecting wall with a speed of 2 ms^{-1}. If the speed of sound is 350 m s^{-1}, calculate the number of beats heard in one second by

(a) the driver of the ambulance,

(b) a person standing between the ambulance and the wall, and

(c) a person standing behind the ambulance.

SOLUTION

(a) The driver will hear two sounds, one coming directly from the siren and the other reflected at the wall or coming from the acoustic image of the car. ν = frequency of direct sound = 700 Hz. The reflected sound can be imagined to be coming from the mirror image (shown dotted in Fig. 10.29).

The observer (driver) is approaching this image-source which is also approaching him with the same speed. Hence, the frequency of sound heard by him is given by ($u_0 = u_s$)

$$\nu' = \nu \frac{v + u_0}{v - u_s} = 700 \times \frac{350 + 2}{350 - 2} = 708 \text{ Hz}$$

Number of beats heard per second = $\nu' - \nu = 708 - 700 = 8$

$\therefore$ Beat frequency = 8 Hz

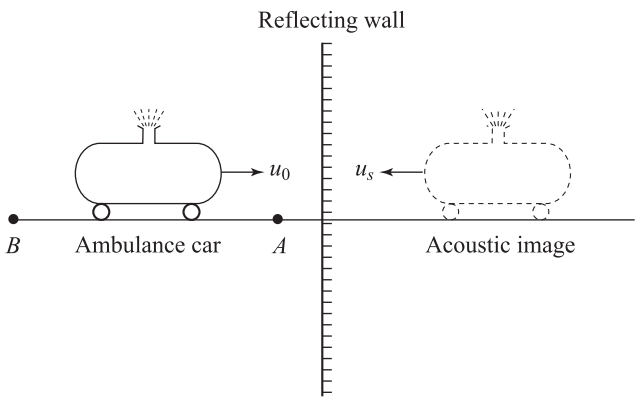


Fig. 10.29

(b) A person standing at A will hear two sounds, one from the siren approaching him and the other from the mirror image of the siren also approaching him with the same speed. Hence he will hear no beats.

(c) A person standing at B will hear two sounds, one from the siren receding from him and the other from the mirror image of the siren approaching him with the same speed.

$$\nu_1 = \nu\left(\frac{v}{v + u_s}\right) = 700 \times \left(\frac{350}{350 + 2}\right) = 696 \text{ Hz}$$

$$\nu_2 = \nu\left(\frac{v}{v - u_s}\right) = 700 \times \left(\frac{350}{350 - 2}\right) = 704 \text{ Hz}$$

$\therefore$ Beat frequency = $\nu_2 - \nu_1 = 704 - 696 = 8$ Hz

22. Intensity and Intensity level of Sound

The intensity of a wave is defined as the rate at which the energy of the wave passes through a unit area held normal to it. The intensity I of a wave of amplitude a, frequency ν, travelling with a speed v in a medium of density ρ is given by

$$I = 2\pi^2 \nu^2 a^2 \rho v$$

Human ear is sensitive to a very large range of intensity of sound. So a logarithmic (rather than an arithmetic) scale is used to define the intensity of sound. The intensity level β of sound is defined as

$$\beta = 10 \log_{10}\left(\frac{I}{I_0}\right) \text{ decibel (or dB)}$$

where $I_0 = 10^{-12}$ Wm^{-2} is the reference intensity. Sounds of intensity $I < I_0$ are barely audible. I_0 is called the threshold intensity and it varies with the frequency of sound. For pure note, $I = 10^{-12}$ Wm^{-2} at frequency of 1 kHz.

Note The intensity level β of a whisper is about 15 dB. For normal conversation β is about 70 dB. If β >120 dB, the sound becomes painful as in the case of a fire cracker or a jet engine.

EXAMPLE 36 The intensity of a sound is 1.0 Wm^{-2}. Find the intensity level (in dB) of this sound.

SOLUTION $\beta = 10 \log_{10}\left(\dfrac{I}{I_0}\right)$

$$= 10 \log_{10}\left(\frac{1.0}{10^{-12}}\right)$$

$$= 10 \log_{10}(10^{12})$$

$$= 10 \times 12 = 120 \text{ dB}$$

EXAMPLE 37. What is the intensity of a sound of intensity level 60 dB?

SOLUTION $60 = 10 \log\left(\frac{I}{I_0}\right)$

$\Rightarrow \quad 6 = \log\left(\frac{I}{I_0}\right)$

$\Rightarrow \quad I = I_0 \times 10^6$

$= 10^{-12} \times 10^6$

$= 10^{-6}\ \text{Wm}^{-2}$

SECTION 1

Multiple Choice Questions with One Correct Choice

Level A

1. A person measures the time period of a simple pendulum inside a stationary lift and finds it to be T. If the lift starts accelerating upwards with an acceleration of $g/3$, the time period of the pendulum will be

   (a) $\sqrt{3}\ T$ (b) $\frac{\sqrt{3}T}{2}$

   (c) $T/\sqrt{3}$ (d) $T/3$

2. A hollow metallic sphere filled with water is hung from a support by a long thread. A small hole is made at the bottom of the sphere and it is oscillated. How will the time period of oscillations be affected as water slowly flows out of the hole?

   (a) The time period will remain unchanged as water is flowing out

   (b) The time period will keep increasing until the sphere is empty

   (c) The time period will keep decreasing until the sphere is empty

   (d) The time period will increase at first, then decrease until the sphere is empty; finally the period will be the same as that when the sphere was full of water

3. A simple pendulum with a brass bob has a time period T. The bob is now immersed in a non–viscous liquid and oscillated. If the density of the liquid is 1/8 that of brass, the time period of the same pendulum will be

   (a) $\sqrt{\frac{8}{7}}\ T$ (b) $\frac{8}{7}\ T$

   (c) $\frac{8^2}{7^2}\ T$ (d) T

4. A simple harmonic motion is given by the equation

   $$x = 3 \sin 3\pi t + 4 \cos 3\pi t$$

   where x is in metres. The amplitude of the motion is

   (a) 3 m (b) 4 m

   (c) 5 m (d) 7 m

5. Figure 10.30 shows three identical springs *A, B, C*. When a 4 kg mass is hung on *A*, it descends by 1 cm. When a 6 kg mass is hung on *C*, it will descend by

   (a) 1.5 cm (b) 3.0 cm

   (c) 4.5 cm (d) 6.0 cm

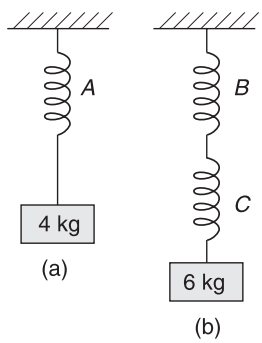


Fig. 10.30

6. A spring of force constant k is cut into two equal halves. The force constant of each half is

   (a) $k/\sqrt{2}$ (b) $k/2$

   (c) k (d) $2k$

7. Two springs of equal lengths and equal cross-sectional areas are made of materials whose Young's modulii are in the ratio of 3 : 2. They are suspended and loaded with the same mass. When stretched and released, they will oscillate with time periods in the ratio of

   (a) $\sqrt{3} : \sqrt{2}$ (b) 3 : 2

   (c) $3\sqrt{3} : 2\sqrt{2}$ (d) 9 : 4

8. Two bodies A and B of equal masses are suspended from two separate springs of force constants k_1 and k_2 respectively. If the two bodies oscillate such that their maximum velocities are equal, the ratio of the amplitudes of oscillation of A and B will be

   (a) k_1/k_2 (b) $\sqrt{k_1/k_2}$

   (c) k_2/k_1 (d) $\sqrt{k_2/k_1}$

9. Two springs A and B have force constants k_1 and k_2 respectively. The ratio of the work done on A to that done on B in increasing their lengths by the same amount is

   (a) k_1/k_2 (b) $\sqrt{k_1/k_2}$

   (c) k_2/k_1 (d) $\sqrt{k_2/k_1}$

10. Two springs A and B have force constants k_1 and k_2 respectively. The ratio of the work done on A to that done on B when they are stretched by the same force is

    (a) k_1/k_2 (b) $\sqrt{k_1/k_2}$

    (c) k_2/k_1 (d) $\sqrt{k_2/k_1}$

11. A spring of force constant k is cut into three equal pieces. If these three pieces are connected in parallel, the force constant of the combination will be

    (a) $k/3$ (b) k

    (c) $3k$ (d) $9k$

12. A simple harmonic motion of amplitude A has a time period T. The acceleration of the oscillator when its displacement is half the amplitude is

    (a) $\frac{4\pi^2 A}{T^2}$ (b) $\frac{2\pi^2 A}{T^2}$

    (c) $-\frac{4\pi^2 A}{T^2}$ (d) $-\frac{2\pi^2 A}{T^2}$

13. A body oscillates harmonically with amplitude 0.05 m. At a certain instant of time its displacement is 0.01 m and acceleration is 1.0 ms^{-2}. What is the period of oscillation?

    (a) 0.1 s (b) 0.2 s

    (c) $\frac{\pi}{10}$ s (d) $\frac{\pi}{5}$ s

14. In Q. 13, what is the maximum velocity of the body?

    (a) 0.25 ms^{-1} (b) 0.5 ms^{-1}

    (c) 0.75 ms^{-1} (d) 1.0 ms^{-1}

15. The displacement x (in centimetres) of an oscillating particle varies with time t (in seconds) as

$$x = 2\cos\left(0.5\pi t + \frac{\pi}{3}\right)$$

    The magnitude of the maximum acceleration of the particle is

    (a) $\frac{\pi}{2}$ cms^{-2} (b) $\frac{\pi}{4}$ cms^{-2}

    (c) $\frac{\pi^2}{2}$ cms^{-2} (d) $\frac{\pi^2}{4}$ cms^{-2}

16. A particle executes SHM of amplitude 25 cm and time periods 3 s. What is the minimum time required for the particle to move between two points located at 12.5 cm on either side of the mean position?

    (a) 0.25 s (b) 0.5 s

    (c) 0.75 s (d) 1.0 s

17. A body executing linear simple harmonic motion has a velocity of 3 cms^{-1} when its displacement is 4 cm and a velocity of 4 cms^{-1} when its displacement is 3 cm. What is the amplitude of oscillation?

    (a) 5 cm (b) 7.5 cm

    (c) 10 cm (d) 12.5 cm

18. In Q. 17, the time period of oscillation is

    (a) 4.12 s (b) 6.28 s

    (c) 9.42 s (d) 12.56 s

19. If Q. 17, if the mass of the oscillator is 50 g, the total energy of the oscillator will be

    (a) 4.25×10^{-5} J (b) 5.25×10^{-5} J

    (c) 6.25×10^{-5} J (d) 7.25×10^{-5} J

20. A particle is executing linear simple harmonic motion of amplitude A. What fraction of the total energy is kinetic when the displacement is half the amplitude?

    (a) $\frac{1}{4}$ (b) $\frac{1}{2\sqrt{2}}$

    (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

21. In Q. 20, at what displacement is the energy of the oscillator half potential and half kinetic?

    (a) $\frac{A}{4}$ (b) $\frac{A}{2}$

    (c) $\frac{A}{\sqrt{2}}$ (d) $\frac{A}{\sqrt{3}}$

22. A horizontal platform is executing simple harmonic motion in the vertical direction of frequency ν. A block of mass m is placed on the platform. What is the maximum amplitude of the platform so that the block is not detached from it?

    (a) $\frac{g}{4\pi^2\nu^2}$ (b) $\frac{mg}{4\pi^2\nu^2}$

    (c) $\frac{g}{2\pi^2\nu^2}$ (d) $\frac{mg}{2\pi^2\nu^2}$

23. A spring of negligible mass having a force constant k extends by an amount y when a mass m is hung from it. The mass is pulled down a little and released. The system begins to execute simple harmonic motion of amplitude A and angular frequency ω. The total energy of the mass-spring system will be

(a) $\frac{1}{2} mA^2\omega^2$ (b) $\frac{1}{2} mA^2\omega^2 + \frac{1}{2} ky^2$

(c) $\frac{1}{2} ky^2$ (d) $\frac{1}{2} mA^2\omega^2 - \frac{1}{2} ky^2$

24. A small trolley of mass 2 kg resting on a horizontal frictionless turntable is connected by a light spring to the centre of the table. The relaxed length of the spring is 35 cm. When the turntable is rotated an angular frequency of 10 rad s^{-1}, the length of the spring becomes 40 cm. What is the force constant of the spring?

(a) 1.2×10^3 Nm^{-1} (b) 1.6×10^3 Nm^{-1}

(c) 2.0×10^3 Nm^{-1} (d) 2.4×10^3 Nm^{-1}

25. Two springs of force constants k_1 and k_2 are connected to a mass m placed on a horizontal frictionless surface as shown in Figs 10.31 (a) and (b). What is the ratio of the time periods of horizontal oscillation in cases (a) and (b) if $k_1 = k_2$?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 2 (d) 4

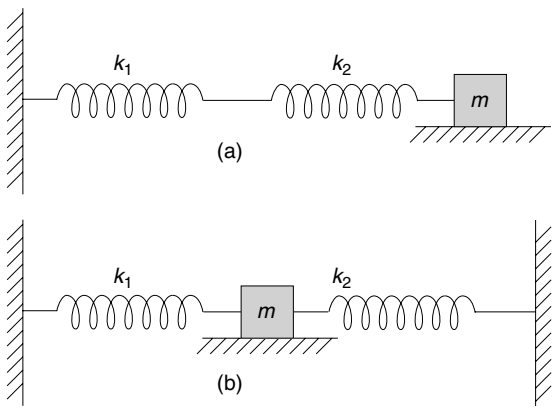


Fig. 10.31

26. Two springs of force constants k_1 and k_2 are connected to a mass m as shown in Figs. 10.32 (a) and (b). What is the ratio of the time periods of vertical oscillation in cases (a) and (b) if $k_1 = k_2$?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 2 (d) 4

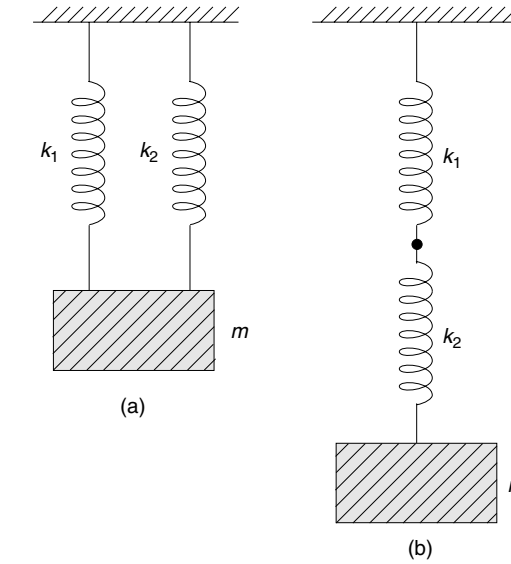


Fig. 10.32

27. A simple pendulum of bob mass m is oscillating with an angular amplitude α_m (in radians). The maximum tension in the string is

(a) mg (b) $mg\,\alpha_m$

(c) $mg\,\alpha_m^2$ (d) $mg\,(1 + \alpha_m^2)$

28. In. Q. 27, the kinetic energy of the bob when it passes through the mean position is (l is the length of the string)

(a) $mgl\,\alpha_m^2$ (b) $mgl\,(1 + \alpha_m^2)$

(c) $mgl\,(1 + \cos\alpha_m)$ (d) $mgl\,(1 - \cos\alpha_m)$

29. A particle is executing simple harmonic motion. Its displacement is given by

$$x = 5 \sin \pi t$$

where x is in cm and t in seconds. How long will the particle take to move from the position of equilibrium to the position of maximum displacement?

(a) 0.5 s (b) 1.0 s

(c) 1.5 s (d) 2.0 s

30. A simple pendulum is moving simple harmonically with a period of 6 s between two extreme position B and C about a point O. If the angular distance between B and C is 10 cm, how long will the pendulum take to move from position C to a position D exactly midway between O and C ?

(a) 0.5 s (b) 1.0 s

(c) 1.5 s (d) 3 s

31. When a mass m is hung from the lower end of a spring of negligible mass, an extension x is produced in the spring. The mass is set into vertical oscillations. The time period of oscillation is

(a) $T = 2\pi\sqrt{\frac{x}{mg}}$ (b) $T = 2\pi\sqrt{\frac{gx}{m}}$

(c) $T = 2\pi\sqrt{\frac{x}{g}}$ (d) $T = 2\pi\sqrt{\frac{x}{2g}}$

32. A small spherical steel ball is placed a little away from the centre of a large concave mirror of radius of curvature $R = 2.5$ m. The ball is then released. What is the time period of the motion? Neglect friction and take $g = 10\ \text{ms}^{-2}$.

(a) $\frac{\pi}{4}$ sec (b) $\frac{\pi}{2}$ sec

(c) π sec (d) 2π sec

Level B

33. Two masses m_1 and m_2 are suspended together by a massless spring of force constant k (see Fig. 10.33). When the masses are in equilibrium, mass m_1 is removed without disturbing the system. The angular frequency of oscillation of mass m_2 is

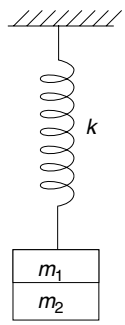


Fig. 10.33

(a) $\sqrt{\frac{k}{m_2}}$ (b) $\sqrt{\frac{k}{m_1}}$

(c) $\sqrt{\frac{k\,m_1}{m_2^2}}$ (d) $\sqrt{\frac{k\,m_2}{m_1^2}}$

34. In Q. 33, the amplitude of oscillation of mass m_2 is

(a) $\frac{m_2 g}{k}$ (b) $\frac{m_1 g}{k}$

(c) $\frac{m_2^2 g}{m_1 k}$ (d) $\frac{m_1^2 g}{m_2 k}$

35. When a wave travels in a medium, the particle displacements are given by

$$y(x, t) = 0.03 \sin \pi (2t - 0.01\, x)$$

where y and x are in metres and t in seconds. The wavelength of the wave is

(a) 10 m (b) 20 m

(c) 100 m (d) 200 m

36. The velocity of the wave in Q. 35 is

(a) $100\ \text{ms}^{-1}$ (b) $200\ \text{ms}^{-1}$

(c) $300\ \text{ms}^{-1}$ (d) $400\ \text{ms}^{-1}$

37. What is the phase difference, at a given instant of time, between two particles 25 m apart, when the wave of Q. 35 above travels in a medium?

(a) $\pi/8$ (b) $\pi/4$

(c) $\pi/2$ (d) π

38. Transverse wave of amplitude 10 cm is generated at one end ($x = 0$) of a long string by a tuning fork of frequency 500 Hz. At a certain instant of time, the displacement of a particle A at $x = 100$ cm is -5 cm and of particle B at $x = 200$ cm is $+5$ cm. What is the wavelength of the wave?

(a) 2 m (b) 3 m

(c) 4 m (d) 5 m

39. How long does the wave of Q. 38 take to travel from A to B?

(a) 10 s (b) 1 s

(c) 10^{-3} s (d) 10^{-2} s

40. Transverse waves of the same frequency are generated in two steel wires A and B. The diameter of A is twice that of B and the tension in A is half that in B. The ratio of the velocities of waves in A and B is

(a) 1 : 2 (b) $1 : \sqrt{2}$

(c) $1 : 2\sqrt{2}$ (d) $3 : 2\sqrt{2}$

41. A sonometer wire, with a suspended mass of $M = 1$ kg, is in resonance with a given tuning fork. The apparatus is taken to the moon where the acceleration due to gravity is 1/6 that on earth. To obtain resonance on the moon, the value of M should be

(a) 1 kg (b) $\sqrt{6}$ kg

(c) 6 kg (d) 36 kg

42. A source of sound vibrates according to the equation $y = 0.05 \cos \pi t$. It sends out waves of velocity $1.5\ \text{ms}^{-1}$. The wavelength of the waves is

(a) 1.5 m (b) 3.0 m

(c) 4.5 m (d) 6.0 m

43. Two identical waves, each of frequency 10 Hz, are travelling in opposite directions in a medium with a speed of 20 cm s^{-1}. The distance between adjacent nodes is

(a) 1.0 cm (b) 1.2 cm

(c) 1.5 cm (d) 2.0 cm

44. Particle displacements (in cm) in a standing wave are given by

$$y(x, t) = 2 \sin (0.1\, \pi x) \cos (100\, \pi t).$$

The distance between a node and the next antinode is

(a) 2.5 cm (b) 5.0 cm

(c) 7.5 cm (d) 10.0 cm

45. An observer moves towards a stationary source of sound with a velocity one–tenth the velocity of sound. The apparent increase in frequency is
(a) zero (b) 5%
(c) 10% (d) 0.1%

46. Two parts of a sonometer wire, divided by a movable knife–edge, differ in length by 1 cm and produce 1 beat per second when sounded together. If the total length of the wire is 100 cm, the frequencies of the two parts of the wire are
(a) 51 Hz, 50 Hz (b) 50.5 Hz, 49.5 Hz
(c) 49 Hz, 48 Hz (d) 49.5 Hz, 48.5 Hz

47. A stone hangs from the free end of a sonometer wire whose vibrating length, when tuned to a tuning fork, is 40 cm. When the stone hangs wholly immersed in water, the resonant length is reduced to 30 cm. The relative density of the stone is
(a) 16/9 (b) 16/7
(c) 16/5 (d) 16/3

48. A cylindrical tube, open at both ends, has fundamental frequency n. If one of the ends is closed, the fundamental frequency will become
(a) $n/2$ (b) n
(c) $2n$ (d) $4n$

49. An organ pipe, open at both ends and another organ pipe, closed at one end, will resonate with each other, if their lengths are in the ratio of
(a) 1 : 1 (b) 1 : 4
(c) 2 : 1 (d) 1 : 2

50. A tuning fork of frequency 340 Hz is sounded above a cylindrical tube 1 m high. Water is slowly poured into the tube. If the speed of sound is 340 ms^{-1}, at what levels of water in the tube will the sound of the fork be appreciably intensified?
(a) 25 cm, 75 cm (b) 20 cm, 80 cm
(c) 15 cm, 85 cm (d) 17 cm, 83 cm

51. A sonometer wire, 65 cm long, is in resonance with a tuning fork of frequency N. If the length of the wire is decreased by 1 cm and it is vibrated with the same tuning fork, 8 beats are heard per second. What is the value of N?
(a) 256 Hz (b) 384 Hz
(c) 480 Hz (d) 512 Hz

52. Two organ pipes, each closed at one end, give 5 beats per second when emitting their fundamental notes. If their lengths are in the ratio of 50 : 51, their fundamental frequencies (in Hz) are
(a) 250, 255 (b) 255, 260
(c) 260, 265 (d) 265, 270

53. Two sources A and B are sounding notes of frequency 680 Hz. A listener moves from A to B with a constant velocity u. If the speed of sound is 340 ms^{-1}, what must be the value of u so that he hears 10 beats per second?
(a) 2.0 ms^{-1} (b) 2.5 ms^{-1}
(c) 3.0 ms^{-1} (d) 3.5 ms^{-1}

54. A hospital uses an ultrasonic scanner of frequency 3.2 MHz to locate tumours in a tissue. What is the wavelength of ultrasonic waves in a tissue in which the speed of the waves is 1.6 km s^{-1}?
(a) 0.25 mm (b) 0.5 mm
(c) 0.75 mm (d) 1 cm

55. A bat flying above a lake emits ultrasonic sound of 100 kHz. When this wave falls on the water surface, it is partly reflected and partly transmitted. What are the wavelengths of the reflected and transmitted waves? The speed of sound in air is 340 m s^{-1} and in water 1450 m s^{-1}.
(a) 6.8 mm and 2.9 cm
(b) 3.4 mm and 1.45 cm
(c) 3.4 mm and 7.8 mm
(d) 6.8 mm and 1.45 cm

56. Standing waves are produced by the superposition of two waves
$$y_1 = 0.05 \sin (3\pi t - 2x)$$
and $$y_2 = 0.05 \sin (3\pi t + 2x)$$
where x and y are expressed in metres and t is in seconds. What is the amplitude of a particle at x = 0.5 m. Given cos (57.3°) = 0.54.
(a) 2.7 cm (b) 5.4 cm
(c) 8.1 cm (d) 10.8 cm

57. The transverse displacement of a string fixed at both ends is given by
$$y = 0.06 \sin \left(\frac{2\pi x}{3}\right) \cos (120\ \pi t)$$
where x and y are in metres and t is in seconds. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg. What is the tension in the string?
(a) 648 N (b) 724 N
(c) 832 N (d) 980 N

58. A pipe of length 20 cm is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 425 Hz source? The speed of sound = 340 ms^{-1}.
(a) First harmonic (b) Third harmonic
(c) Fifth harmonic (d) None

59. A pipe of length 20 cm is open at both ends. Which harmonic mode of the pipe is resonantly excited by a 1700 Hz source? The speed of sound = 340 ms^{-1}.
(a) First harmonic (b) Second harmonic
(c) Third harmonic (d) Fourth harmonic

60. Two tuning forks A and B produce 10 beats per second when sounded together. On slightly loading fork A with a little wax, it was observed that 15 beats are heard per second. If the frequency of fork B is 480 Hz, what is the frequency of fork A before it was loaded?
(a) 465 Hz (b) 470 Hz
(c) 490 Hz (d) 495 Hz

61. In Q. 60, what is the frequency of fork A after it is loaded?
(a) 465 Hz (b) 470 Hz
(c) 490 Hz (d) 495 Hz

62. Two sitar strings A and B are slightly out of tune and produce beats of frequency 6 Hz. When the tension in string A is slightly decreased, the beat frequency is found to be reduced to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B?
(a) 318 Hz (b) 321 Hz
(c) 327 Hz (d) 330 Hz

63. A metal wire of diameter 1 mm is held on two knife edges separated by a distance of 50 cm. The tension in the wire is 100 N. The wire vibrating with its fundamental frequency and a vibrating tuning fork together produce 5 beats per second. The tension in the wire is then reduced to 81 N. When the two are excited, beats are heard at the same rate. What is the frequency of the fork?
(a) 90 Hz (b) 95 Hz
(c) 100 Hz (d) 105 Hz

64. An ambulance blowing a siren of frequency 700 Hz is travelling slowly towards a vertical reflecting wall with a speed of 2 ms^{-1}. The speed of sound is 350 ms^{-1}. How many beats are heard by the driver per second?
(a) 4 (b) 6
(c) 8 (d) 10

65. Two persons A and B, each carrying a source of sound of frequency ν, are standing a few metres apart in a quiet field. A starts moving towards B with a speed u. If v is the speed of sound, the number of beats heard per second by A will be
(a) $\frac{\nu u}{v}$ (b) $\frac{2\nu u}{v}$
(c) $\frac{\nu u}{(v+u)}$ (d) $\frac{\nu u}{(v-u)}$

66. In Q. 65, what is the number of beats heard per second by person B?
(a) $\frac{\nu u}{v}$ (b) $\frac{2\nu u}{v}$
(c) $\frac{\nu u}{(v+u)}$ (d) $\frac{\nu u}{(v-u)}$

67. A submarine is fitted with a SONAR (Sound Navigation and Ranging) system which operates at an ultrasonic frequency of 42 kHz. An enemy submarine is moving towards the SONAR with a speed of 200 ms^{-1}. If the speed of sound in seawater is 1400 ms^{-1}, what is the frequency of sound received back at the SONAR fitted submarine after reflection from the enemy submarine?
(a) 36 kHz (b) 42 kHz
(c) 48 kHz (d) 56 kHz

68. A machine gun is mounted on a tank moving at a speed of 20 ms^{-1} towards a target with the gun pointing in the direction of motion of the tank. The muzzle speed of the bullet equals the speed of sound = 340 ms^{-1}. If, at the time of firing, the target is 500 m away from the tank, then
(a) the sound arrives at the target later than the bullet
(b) the sound arrives at the target earlier than the bullet
(c) both sound and bullet arrive at the target at the same time
(d) the bullet will never arrive at the target.

69. When a wave travels in a medium, the particle displacements are given by
$$y = a \sin 2\pi (bt - cx)$$
where a, b and c are constants. The maximum particle velocity will be twice the wave velocity if
(a) $c = \frac{1}{\pi a}$ (b) $c = \pi a$
(c) $b = ac$ (d) $b = \frac{1}{ac}$

70. Three sound waves of equal amplitudes have frequencies $(\nu - 1)$, (ν) and $(\nu + 1)$. They superpose to give beats. The number of beats produced per second will be
(a) ν (b) $\frac{\nu}{2}$
(c) 2 (d) 1

71. The speed of sound in a gas is v and the root mean square speed of gas molecules is v_{rms}. If the ratio of the specific heats of the gas $\gamma = 1.5$, then the ratio v/v_{rms} is
(a) 1 : 2 (b) 1 : $\sqrt{2}$
(c) 1 : $\sqrt{3}$ (d) 1 : 3

72. The speed of sound in hydrogen at STP is v. The speed of sound in a mixture containing 3 parts of hydrogen and 2 parts of oxygen at STP will be
(a) $\frac{v}{2}$ (b) $\frac{v}{\sqrt{5}}$
(c) $\sqrt{7}\ v$ (d) $\frac{v}{\sqrt{7}}$

73. The speed of sound in hydrogen at STP is v. What is the speed of sound in helium at STP?

(a) $\frac{v}{\sqrt{2}}$ (b) $\frac{v}{2}$

(c) $\sqrt{2}\ v$ (d) $2\ v$

74. Nine tuning forks are arranged in order of increasing frequency. Each tuning fork produces 4 beats per second when sounded with either of its neighbours. If the frequency of the 9th tuning fork is twice that of the first, what is the frequency of the first tuning fork?

(a) 32 Hz (b) 40 Hz

(c) 48 Hz (d) 56 Hz

75. A sonometer wire of length 120 cm is divided into three segments of lengths in the ratio of 1 : 2 : 3. What is the ratio of their fundamental frequencies?

(a) 3 : 2 : 1 (b) 4 : 2 : 1

(c) 5 : 3 : 2 (d) 6 : 3 : 2

76. A tuning fork produces 4 beats per second when sounded with a sonometer of vibrating length 48 cm. It produces 4 beats per second also when the vibrating length is 50 cm. What is the frequency of the tuning fork?

(a) 196 Hz (b) 284 Hz

(c) 375 Hz (d) 460 Hz

77. Two identical strings of a stringed musical instrument are in unison when stretched with the same tension. When the tension in one string is increased by 1%, the musician hears 4 beats per second. What was the frequency of the note when the strings were in unison?

(a) 796 Hz (b) 800 Hz

(c) 804 Hz (d) 808 Hz

78. Two identical flutes produce fundamental notes of frequency 300 Hz at 27°C. If the temperature of the air in one flute is increased to 31°C, the number of beats heard per second will be

(a) 1 (b) 2

(c) 3 (d) 4

79. Radiowaves of frequency 600 MHz are sent by a radar towards an enemy aircraft. The frequency of the radiowaves reflected from the aircraft as measured at the radar station is found to increase by 6 kHz. It follows that the aircraft is

(a) approaching the radar station with a speed 1.5 kms^{-1}

(b) going away from the radar station with a speed 1.5 kms^{-1}

(c) approaching the radar station with a speed 3 kms^{-1}

(d) going away from the radar station with a speed 3 kms^{-1}.

80. Two simple harmonic motions are given by

$$x_1 = \frac{a}{2}\sin \omega t + \frac{\sqrt{3}a}{2}\cos \omega t$$

and $$x_2 = a \sin \omega t + a \cos \omega t$$

What is the ratio of the amplitude of the first motion to that of the second?

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{3}}$

81. In Q. 80 above, the minimum phase difference between the two simple harmonic motions is

(a) 60º (b) 45º

(c) 30º (d) 15º

82. The ends of a rod of length l and mass m are attached to two identical springs as shown in Fig. 10.34. The rod is free to rotate about its centre O. The rod is depressed slightly at end A and released. The time period of the resulting oscillation is

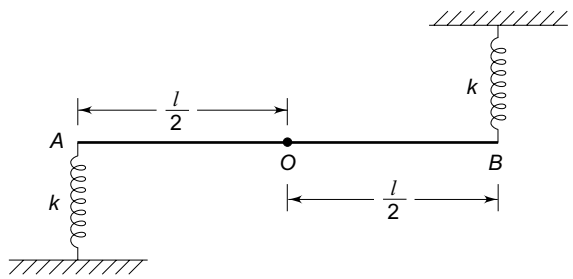


Fig. 10.34

(a) $2\pi\sqrt{\frac{m}{2k}}$ (b) $2\pi\sqrt{\frac{2m}{k}}$

(c) $\pi\sqrt{\frac{2m}{3k}}$ (d) $\pi\sqrt{\frac{3m}{2k}}$

83. A uniform metre scale of length 1 m is balanced on a fixed semi-circular cylinder of radius 30 cm as shown in Fig. 10.35. One end of the scale is slightly depressed and released. The time period (in seconds) of the resulting simple harmonic motion is (Take g = 10 ms^{-2})

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

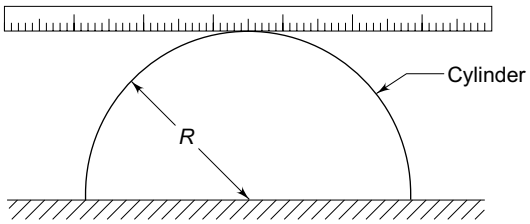


Fig. 10.35

84. The bob of a simple pendulum has a mass m and it is executing simple harmonic motion of amplitude A and period T. It collides with a body of mass m_0 placed at the equilibrium position which sticks to the bob. The time period of the oscillation of the combined masses will be

(a) T (b) $T\sqrt{\dfrac{m+m_0}{m-m_0}}$

(c) $T\sqrt{\dfrac{m}{m-m_0}}$ (d) $T\sqrt{\dfrac{m+m_0}{m}}$

85. In Q. 84 above, the amplitude of the oscillation of the combined masses will be

(a) A (b) $A\sqrt{\dfrac{m}{m+m_0}}$

(c) $A\sqrt{\dfrac{m}{m-m_0}}$ (d) $A\sqrt{\dfrac{m+m_0}{m-m_0}}$

86. A simple pendulum of length l is suspended from the ceiling of a train which is moving in the horizontal direction with a constant acceleration a. The time period of the pendulum is given by $T = 2\pi\sqrt{\dfrac{l}{g_{\text{eff}}}}$ where g_{eff} is given by

(a) g (b) $(g + a)$

(c) $(g - a)$ (d) $\sqrt{g^2 + a^2}$

87. A simple pendulum of length l is suspended from the ceiling of a trolley which is moving, without friction, down an inclined plane of inclination θ. The time period of the pendulum is given by $T = 2\pi\sqrt{\dfrac{l}{g_{\text{eff}}}}$, where g_{eff} is given by

(a) g (b) $g \sin\theta$

(c) $g \cos\theta$ (d) $g \tan\theta$

88. One end of a massless spring of relaxed length 50 cm and spring constant k is fixed on top of a frictionless inclined plane of inclination $\theta = 30^\circ$ as shown in Fig. 10.36. When a mass m = 1.5 kg is attached at the other end, the spring extends by 2.5 cm. The mass is displaced slightly and released. The time period (in seconds) of the resulting oscillation will be

(a) $\dfrac{\pi}{7}$ (b) $\dfrac{2\pi}{7}$

(c) $\dfrac{\pi}{5}$ (d) $\dfrac{2\pi}{5}$

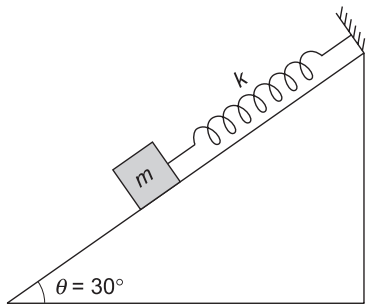


Fig. 10.36

89. Two particles are executing simple harmonic motions of the same amplitude and the same frequency along the same straight line and about the same mean position. If the maximum separation between them is $\sqrt{2}$ times the amplitude, the phase difference between them is

(a) π (b) $\dfrac{\pi}{2}$

(c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{4}$

90. A knife-edge divides a sonometer wire into two parts. The fundamental frequencies of the two parts are ν_1 and ν_2. The fundamental frequency of the sonometer wire when the knife-edge is removed will be

(a) $\nu_1 + \nu_2$ (b) $\dfrac{1}{2}(\nu_1 + \nu_2)$

(c) $\sqrt{\nu_1 \nu_2}$ (d) $\dfrac{\nu_1 \nu_2}{\nu_1 + \nu_2}$

91. A sonometer wire is stretched by a hanging metal bob. Its fundamental frequency is ν_1. When the bob is completely immersed in water, the frequency becomes ν_2. The relative density of the metal is

(a) $\dfrac{\nu_1^2}{\nu_1^2 - \nu_2^2}$ (b) $\dfrac{\nu_2^2}{\nu_1^2 - \nu_2^2}$

(c) $\dfrac{\nu_1}{\nu_1 - \nu_2}$ (d) $\dfrac{\nu_2}{\nu_1 - \nu_2}$

92. The potential energy of a particle executing simple harmonic motion at a distance x from the equilibrium position is proportional to

(a) $\sqrt{x}$ (b) x

(c) x^2 (d) x^3

93. Two blocks A and B, each of mass m, are connected by a massless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length. A third identical block C, also of mass m, moving on the floor with a speed v along the line joining A and B, collides with A (see Fig. 10.37). Then



Fig. 10.37

(a) the kinetic energy of $A - B$ system at maximum compression of the spring is zero.

(b) the kinetic energy of $A - B$ system at maximum compression of the spring is $mv^2/4$.

(c) the maximum compression of the spring is $v\sqrt{m/k}$.

(d) the maximum compression of the spring is $v\sqrt{2m/k}$.

94. The displacement y of a particle executing simple harmonic motion is given by

$$y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000\,t)$$

This expression may be considered to be a result of the superposition of how many simple harmonic motions?

(a) two (b) three

(c) four (d) five

95. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant K. A mass m hangs freely from the free end of the spring. The area of cross-section and Young's modulus of the wire are A and Y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period given by

(a) $2\pi\sqrt{\frac{m}{K}}$ (b) $2\pi\left[\frac{(YA+KL)m}{YAK}\right]^{1/2}$

(c) $2\pi\left(\frac{mYA}{KL}\right)$ (d) $2\pi\left(\frac{mL}{YA}\right)$

96. A rigid cubical block A of mass M and side L is fixed rigidly on to another cubical block B of the same dimensions and of modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small F is applied perpendicular to one of the side faces of A. After the force is withdrawn, block A executes small oscillations, the time period of which is

(a) $2\pi\sqrt{M\eta L}$ (b) $2\pi\sqrt{\frac{M\eta}{L}}$

(c) $2\pi\sqrt{\frac{ML}{\eta}}$ (d) $2\pi\sqrt{\frac{M}{\eta L}}$

97. A particle of mass m moves on the x-axis as follows: it starts from rest at $t = 0$ from the point $x = 0$ and comes to rest at $t = 1$ at the point $x = 1$. No other information is available about its motion at intermediate time $(0 < t < 1)$. If α denotes the instantaneous acceleration of the particle, then

(a) α can remain positive for all t in the interval $0 \le t \le 1$.

(b) $|\alpha|$ cannot exceed 2 at any point in its path.

(c) $|\alpha|$ must be ≥ 4 at some point or points in its path.

(d) α must change sign during the motion, but no other assertion can be made with the information given.

98. A particle free to move along the x-axis has potential energy given by

$U(x) = k[1 - \exp(-x^2)]$ for $-\infty \le x \le +\infty$

where k is a constant of appropriate dimensions. Then

(a) at points away from the origin, the particle is in unstable equilibrium

(b) for any finite nonzero value of x, there is a force directed away from the origin

(c) if its total mechanical energy is $k/2$, it has its minimum kinetic energy at the origin

(d) for small displacements from $x = 0$, the motion is simple harmonic.

99. The period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination α, is given by

(a) $2\pi\sqrt{\frac{L}{g\cos\alpha}}$ (b) $2\pi\sqrt{\frac{L}{g\sin\alpha}}$

(c) $2\pi\sqrt{\frac{L}{g}}$ (d) $2\pi\sqrt{\frac{L}{g\tan\alpha}}$

100. A spring of force constant k is cut into two pieces such that one piece is double the length of the other. Then the longer piece will have a force constant of

(a) $\frac{2}{3}k$ (b) $\frac{3}{2}k$

(c) $3k$ (d) $6k$

101. A wave represented by the equation $y = a\cos(kx - \omega t)$ is superposed with another wave to form a stationary wave such that the point $x = 0$ is a node. The equation of the other wave is
(a) $y' = a\sin(kx + \omega t)$
(b) $y' = -a\cos(kx - \omega t)$
(c) $y' = -a\cos(kx + \omega t)$
(d) $y' = -a\sin(kx - \omega t)$

102. A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced in the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope?
(a) 0.06 m (b) 0.03 m
(c) 0.12 m (d) 0.09 m

103. A tube closed at one end containing air, produces, when excited, the fundamental note of frequency 512 Hz. If the tube is open at both ends, the fundamental frequency that can be excited is (in Hz)
(a) 1024 (b) 512
(c) 256 (d) 128

104. Two sound waves of equal intensity I produce beats. The maximum intensity of sound produced in beats will be
(a) I (b) $4I$
(c) $2I$ (d) $I/2$

105. Two travelling waves
$$y_1 = A\sin[k(x + ct)] \text{ and } y_2 = A\sin[k(x - ct)]$$
are superposed on a string. The distance between adjacent nodes is
(a) $\frac{ct}{\pi}$ (b) $\frac{ct}{2\pi}$
(c) $\frac{\pi}{2k}$ (d) $\frac{\pi}{k}$

106. A train blowing its whistle moves with a constant velocity u away from the observer on the ground. The ratio of the actual frequency of the whistle to that measured by the observer is found to be 1.2. If the train is at rest and the observer moves away from it at the same velocity, the ratio would be given by
(a) 0.51 (b) 1.25
(c) 1.52 (d) 2.05

107. An organ pipe P_1, closed at one end vibrating in its first harmonic and another pipe P_2, open at both ends vibrating in its third harmonic, are in resonance with a given tuning fork. The ratio of the lengths of P_1 and P_2 is
(a) $\frac{8}{3}$ (b) $\frac{1}{6}$
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

108. The extension in a string, obeying Hookes' law, is x. The speed of the wave in the stretched string is v. If the extension in the string is increased to 1.5 x, the speed of the wave in the string will be
(a) 1.22 v (b) 0.61 v
(c) 1.50 v (d) 0.75 v

109. A travelling wave in a stretched string is described by the equation
$$y = A\sin(kx - \omega t)$$
The maximum particle velocity is
(a) $A\omega$ (b) $\frac{\omega}{k}$
(c) $\frac{d\omega}{dk}$ (d) $\frac{x}{t}$

110. A transverse sinusoidal wave of amplitude a wavelength λ and frequency f is travelling on a stretched string. The maximum speed at any point on the string is $v/10$, where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10$ ms^{-1}, then λ and f are given by
(a) $\lambda = 2\pi \times 10^{-2}$ m, $f = \frac{10^3}{2\pi}$ Hz
(b) $\lambda = 10^{-2}$ m, $f = 10^3$ Hz
(c) $\lambda = 10^{-3}$ m, $f = 10^4$ Hz
(d) $\lambda = \frac{10^{-2}}{2\pi}$ m, $f = 2\pi \times 10^3$ Hz

111. The ratio of the speed of sound in nitrogen gas to that in helium gas at 300 K is
(a) $\sqrt{\frac{2}{7}}$ (b) $\sqrt{\frac{1}{7}}$
(c) $\frac{\sqrt{3}}{5}$ (d) $\frac{\sqrt{6}}{5}$

112. Two vibrating strings of the same material but lengths L and $2L$ have radii $2r$ and r, respectively. They are stretched with the same tension. Both the strings vibrate in their fundamental modes, the one of length L with frequency v_1 and the other with frequency v_2. The ratio v_1/v_2 is given by
(a) 2 (b) 4
(c) 8 (d) 1

113. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $A/2$ is T_1 and to go from $A/2$ to A is T_2. Then

(a) $T_1 < T_2$ (b) $T_1 > T_2$

(c) $T_1 = T_2$ (d) $T_1 = 2T_2$

114. The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment, the displacement of the wire is $y_1 = A \sin(\pi x/L) \sin \omega t$ and energy is E_1 and in another experiment its displacement is $y_2 = A \sin(2\pi x/L) \sin 2\omega t$ and energy is E_2. Then

(a) $E_2 = E_1$ (b) $E_2 = 2E_1$

(c) $E_2 = 4E_1$ (d) $E_2 = 16E_1$

115. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that of train A is

(a) 242/252 (b) 2

(c) 5/6 (d) 11/6

116. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is

(a) 25 kg (b) 5 kg

(c) 12.5 kg (d) (1/25) kg

117. A body of mass 1 kg is executing simple harmonic motion. Its displacement x (in cm) at time t (in second) is given by

$$x = 6 \sin\left(100t + \frac{\pi}{4}\right)$$

The maximum kinetic energy of the body is

(a) 6 J (b) 18 J

(c) 24 J (d) 36 J

118. Sound waves of wavelengths 5 m and 6 m produce 30 beats in 3 seconds. The velocity of sound (in ms^{-1}) is

(a) 300 (b) 310

(c) 320 (d) 330

119. Two particles P and Q start from the origin and execute simple harmonic motion along x-axis with the same amplitude and time periods 3 s and 6 s respectively. The ratio of the velocities of P and Q when they meet is

(a) 1 : 2 (b) 2 : 1

(c) 2 : 3 (d) 3 : 2

120. A body is executing simple harmonic motion. At a displacement x, its potential energy is E_1 and at a displacement y, its potential energy is E_2. The potential energy E at a displacement $(x + y)$ is

(a) $E_1 + E_2$ (b) $\sqrt{E_1^2 + E_2^2}$

(c) $E_1 + E_2 + 2\sqrt{E_1 E_2}$ (d) $\sqrt{E_1 E_2}$

121. The frequencies of tuning forks A and B are respectively 3% more and 2% less than the frequency of fork C. When A and B are simultaneously excited 5 beats per second are produced. The frequency (in Hz) of fork A is

(a) 98 (b) 100

(c) 103 (d) 105

122. A body executes simple harmonic motion under the action of a force F_1 with a time period $\frac{4}{5}$ s. If the force is changed to F_2 it executes S.H.M. with time period $\frac{3}{5}$ s. If both the forces F_1 and F_2 act simultaneously in the same direction on the body, its time period in second will be

(a) $\frac{12}{25}$ (b) $\frac{24}{25}$

(c) $\frac{35}{24}$ (d) $\frac{15}{12}$

123. If the displacement (x) and velocity (v) of a particle executing simple harmonic motion are related through the expression $4v^2 = 25 - x^2$, then its time period is

(a) π (b) 2π

(c) 4π (d) 6π

124. The time period of a particle in simple harmonic motion is 8 seconds. At $t = 0$ it is at the mean position. The ratio of the distances travelled by it in the first and second seconds is:

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}-1}$ (d) $\frac{1}{\sqrt{3}}$

125. A radar sends a radio signal of frequency 9×10^9 Hz towards an aircraft approaching the radar. If the reflected wave shows a frequency shift of 3×10^3

Hz, the speed with which the aircraft is approaching the radar, in m/s is (velocity of the radio signal is 3×10^8 m/s)

(a) 150 (b) 100

(c) 50 (d) 25

126. A simple pendulum has a time period of 3.0 s. If the point of suspension of the pendulum starts moving vertically upward with a velocity $v = Kt$ where $K = 4.4\ \text{ms}^{-2}$, the new time period will be (Take $g = 10\ \text{ms}^{-2}$)

(a) $\frac{9}{4}$ s (b) $\frac{5}{3}$ s

(c) 2.5 s (d) 4.4 s

127. A block is kept on a horizontal table. The table is executing simple harmonic motion of time period T in the horizontal plane. The coefficient of static friction between the block and the table is μ. The maximum amplitude of the table for which the block does not slip on the surface of the table is

(a) $\frac{\mu g T}{2\pi}$ (b) $\frac{\mu g T^2}{2\pi^2}$

(c) $\frac{\mu g T^2}{4\pi^2}$ (d) $\mu g T^2$

128. One end of a light spring of force constant k is fixed to a block A of mass M placed on a horizontal frictionless table; the other end of the spring is fixed to a wall (Fig. 10.38). A smaller block B of mass m is placed on block A. The system is displaced by a small amount and released. What is the maximum amplitude of the resulting simple harmonic motion of the system so that the upper block does not slip over the lower block? The coefficient of static friction between the two blocks is μ.

(a) $A_{\max} = \frac{\mu M g}{k}$

(b) $A_{\max} = \frac{\mu m g}{k}$

(c) $A_{\max} = \frac{\mu (M+m) g}{k}$

(d) $A_{\max} = \frac{\mu (M-m) g}{k}$

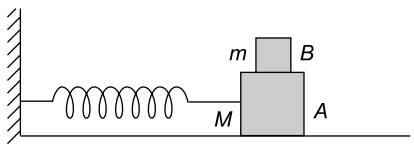


Fig. 10.38

129. A simple pendulum attached to the ceiling of a stationary lift has a time period T. The distance y covered by the lift moving upwards varies with time t as $y = t^2$ where y is in metre and t in second. If $g = 10\ \text{ms}^{-2}$, the time period of the pendulum will be

(a) $\sqrt{\frac{4}{5}}\,T$ (b) $\sqrt{\frac{5}{6}}\,T$

(c) $\sqrt{\frac{5}{4}}\,T$ (d) $\sqrt{\frac{6}{5}}\,T$

130. Transverse waves are generated in two uniform steel wires A and B of diameters 10^{-3} m and 0.5×10^{-3} m respectively, by attaching their free end to a vibrating source of frequency 500 Hz. The ratio of the wavelengths if they are stretched with the same tension is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) 2 (d) $\sqrt{2}$

131. A wire is stretched between two rigid supports with a certain tension. It is observed that the wire resonates in the nth harmonic at a frequency of 420 Hz. The next higher frequency at which the wire resonates is 490 Hz. The value of n is

(a) 2 (b) 4

(c) 6 (d) 8

132. A wire of density ρ, Young's modulus Y and coefficient of linear expansion α is held taut between two rigid supports with negligible tension. If the room temperature falls by ΔT, the speed of transverse waves in the wire is

(a) $\sqrt{\frac{Y\alpha\Delta T}{\rho}}$ (b) $\sqrt{\frac{Y\rho\Delta T}{\alpha}}$

(c) $\sqrt{\frac{Y\alpha}{\rho\Delta T}}$ (d) $\sqrt{\frac{Y\rho}{\alpha\Delta T}}$

133. The fundamental frequency of a sonometer wire increases by 6 Hz if its tension is increased by 44%, keeping the length constant. The frequency of the wire is

(a) 24 Hz (b) 30 Hz

(c) 36 Hz (d) 42 Hz

134. Which of the following functions represents a travelling wave? Here a, b and c are constants.

(a) $y = a \cos(bx) \sin(ct)$

(b) $y = a \sin(bx) \cos(ct)$

(c) $y = a \sin(bx + ct) - a \sin(bx - ct)$

(d) $y = a \sin(bx + ct)$

135. A sonometer wire is vibrating with a frequency of 30 Hz in the fundamental mode. If the length of the wire is increased by 20%, the change in the frequency of the fundamental mode is

(a) 5 Hz (b) 10 Hz

(c) 15 Hz (d) 20 Hz

136. An organ pipe P_1, closed at one end and containing a gas of density ρ_1 is vibrating in its first harmonic. Another organ pipe P_2, open at both ends and containing a gas of density ρ_2 is vibrating in its third harmonic. Both the pipes are in resonance with a given tuning fork. If the compressibility of gases is equal in both pipes, the ratio of the lengths of P_1 and P_2 is

(a) $\frac{1}{3}$ (b) 3

(c) $\frac{1}{6}\sqrt{\frac{\rho_1}{\rho_2}}$ (d) $\frac{1}{6}\sqrt{\frac{\rho_2}{\rho_1}}$

137. A source of sound of frequency 600 Hz is placed inside water. The speed of sound in water is 1500 ms^{-1} and in air it is 300 ms^{-1}. The frequency and wavelength of sound recorded by an observer who is standing in air respectively are

(a) 600 Hz, 0.5 m (b) 600 Hz, 2.5 m

(c) 3000 Hz, 0.4 m (d) 120 Hz, 2 m

138. A rod AB of length L is hung from two identical wires 1 and 2. A block of mass m is hung at point O of the rod as shown in Fig. 10.39. The value of x so that a tuning fork excites the fundamental mode in wire 1 and the second harmonic in wire 2 is

(a) $\frac{L}{5}$ (b) $\frac{L}{4}$

(c) $\frac{L}{3}$ (d) $\frac{2L}{3}$

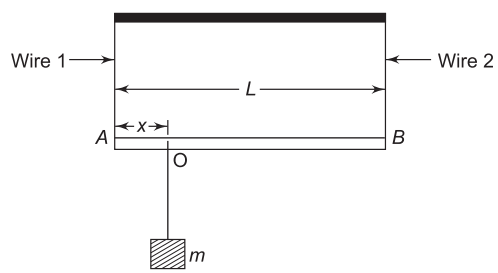


Fig. 10.39

139. Two tuning forks, each of frequency ν, move relative to a stationary observer. One fork moves away from the observer with a speed u while the other fork moves towards him at the same speed. The speed of sound is v. If $u \ll v$, the observer hears beats of frequency

(a) zero (b) $\frac{\nu u}{v}$

(c) $\frac{2\nu u}{v}$ (d) $\frac{\nu u}{2v}$

140. The vibrations of a string of length 60 cm fixed at both ends are represented by

$$y = 4 \sin\left(\frac{\pi x}{15}\right)\cos(96\,\pi t)$$

where x and y are in cm and t in second. The particle velocity at $x = 7.5$ cm and $t = 0.25$ s is

(a) zero (b) 10 cm s^{-1}

(c) 100 cm s^{-1} (d) (4×96) cm s^{-1}

141. A uniform rope of mass M hangs vertically from a rigid support. A block of mass m is attached to the free end of the rope. A transverse pulse of wavelength λ is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope is

(a) $\left(\frac{m+M}{m}\right)\lambda$ (b) $\left(\frac{m+M}{m}\right)^{1/2}\lambda$

(c) $\left(\frac{m\lambda}{M}\right)$ (d) λ

142. A band playing music at a frequency f is moving towards a wall at a speed u. A motorist is following the band with the same speed u. If v is the speed of sound, the beat frequency heard by the motorist is

(a) f (b) $\frac{2fu}{(v+u)}$

(c) $\frac{f(u+v)}{(v-u)}$ (d) $\frac{2fu}{(v-u)}$

143. A block of mass m attached to a spring of force constant k is executing simple hamonic motion of amplitude A. The speed of the block when it is at a distance x from the equilibrium position is

(a) $\sqrt{\frac{kx^2}{m}}$ (b) $\sqrt{\frac{k(A-x)^2}{m}}$

(c) $\sqrt{\frac{k(A^2-x^2)}{m}}$ (d) $\sqrt{\frac{kAx}{m}}$

144. A block of mass m = 3kg is attached to a spring of force constant k = 300 Nm^{-1}. The block is at rest at its equilibrium position. An impulsive force acts on the block which gives it an initial speed of $v = 2$ ms^{-1}. The amplitude of the resulting oscillations is

(a) 5 cm (b) 10 cm

(c) 15 cm (d) 20 cm

145. A block of mass M is attached to a spring of force constant k and is at rest on a horizontal frictionless table with the spring relaxed as shown in Fig. 10.40. A bullet of mass m is fired horizontally with a velocity v on the block and gets embedded in it. As a result of the impact, the block (with bullet embedded in it) begins to execute simple harmonic motion.

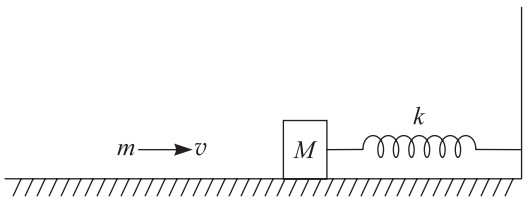


Fig. 10.40

The amplitude of the oscillations is

(a) $\frac{mv}{\sqrt{k\,(M+m)}}$ (b) $\frac{mv}{\sqrt{k\,M}}$

(c) $\frac{Mv}{\sqrt{k\,(M+m)}}$ (d) $\frac{mv}{\sqrt{k\,(M-m)}}$

146. A block of mass M attached to a spring of force constant k is lying on a horizontal frictionless surface as shown in Fig. 10.41 and is executing simple harmonic of amplitude A. When the block is at position $x = A/2$, a ball of plasticine falls from above and sticks to the block. If v_1 is the speed of the block just before the ball hits it and v_2 is the speed of the block + ball system just after the ball hits the block, then the ratio $\frac{v_1}{v_2}$ is

(a) $\frac{M+m}{M}$ (b) $\frac{M+m}{m}$

(c) $\frac{M}{M+m}$ (d) $\frac{M+m}{\sqrt{M\,m}}$

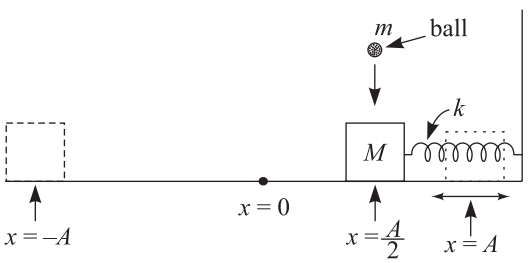


Fig 10.41

147. In Q 146 above, if $M = 2m$, the new amplitude of the system will be

(a) $\sqrt{\frac{3}{4}}\,A$ (b) $\sqrt{\frac{5}{4}}\,A$

(c) $\sqrt{\frac{3}{5}}\,A$ (d) $\sqrt{\frac{5}{8}}\,A$

148. A simple harmonic oscillator has amplitude A and force constant k. Its average potential energy over one time period T is

(a) zero (b) kA^2

(c) $\frac{1}{2}\,kA^2$ (d) $\frac{1}{4}\,kA^2$

149. Two simple harmonic motions $y_1 = A \sin \omega t$ and $y_2 = A \cos \omega t$ are superimposed on a particle of mass m. The total mechanical energy of the particle is

(a) zero (b) $m\omega^2A^2$

(c) $\frac{1}{2}\,m\omega^2A^2$ (d) $\frac{1}{4}\,m\omega^2A^2$

150. The time period of vertical oscillations of the system shown in Fig. 10.42 is (given $k_1 = k_2 = k$ and $k_3 = 2k$)

(a) $2\pi\sqrt{\frac{m}{k}}$ (b) $2\pi\sqrt{\frac{3m}{k}}$

(c) $2\pi\sqrt{\frac{m}{3k}}$ (d) $2\pi\sqrt{\frac{2m}{3k}}$

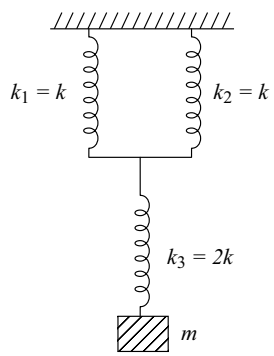


Fig. 10.42

151. An elastic spring of spring constant k hangs vertically with one end fixed to a support and the other end fixed to a block of mass m. The block is held such that the spring is at its natural length. If the block is released, the maximum extension of the spring is

(a) $\frac{mg}{k}$ (b) $\frac{2mg}{k}$

(c) $\frac{mg}{2k}$ (d) mgk

152. A plane wave of amplitude a and wavelength λ travels in a medium. The maximum particle velocity is less than the wave velocity if

(a) $a < \lambda$ (b) $a > \dfrac{\lambda}{2}$

(c) $a < \dfrac{\lambda}{2\pi}$ (d) $a > \dfrac{\lambda}{2\pi}$

153. If the amplitude of sound is halved and its frequency is increased 4 times, the intensity of sound

(a) increases by a factor of 16

(b) increases by a factor of 4

(c) decreases by a factor of 4

(d) remains unchanged.

154. If the intensity level of a sound is increased by 2 dB, the percentage increase in the intensity of sound is nearly equal to

(a) 0.2% (b) 2%

(c) 26% (d) 58%

155. If the intensity of a sound is doubled, its intensity level increases by nearly

(a) 2 dB (b) 3 dB

(c) 10 dB (d) 30 dB

156. A ear C, blowing a horn of frequency 650 Hz, is moving at a speed of 25 ms^{-1} on a straight road towards point P. A person is standing at a point O at distance of 40 m perpendicular to CP as shown in Fig. 10.43. When the car is 30 m away from P, the apparent frequency of sound as heard by the person is (the speed of sound = 345 ms^{-1})

(a) 625 Hz (b) 650 Hz

(c) 675 Hz (d) 690 Hz

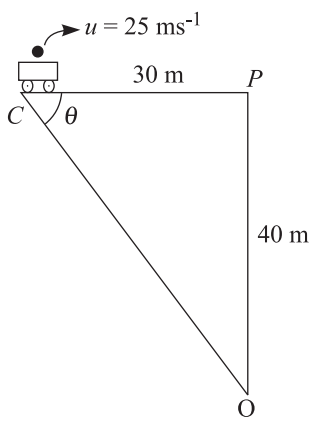


Fig. 10.43

157. The ratio of time periods of vertical oscillation in cases (*i*) and (*ii*) shown in Fig. 10.44 is (pulleys are massless)

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{\sqrt{2}}$

(c) 2 (d) $\sqrt{2}$

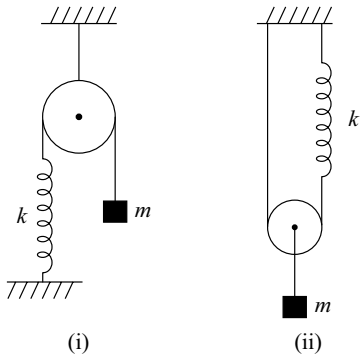


Fig. 10.44

158. Two pulleys of negligible mass are connected to two blocks of mass m_1 and m_2 by two strings of the same material and the same cross-sectional area as shown in Fig. 10.45. The speed of a transverse wave in string AB is v_1 and that in string CD is v_2. The ratio $\dfrac{v_1}{v_2}$ is

(a) 1 (b) 2

(c) $\sqrt{2}$ (d) $\dfrac{1}{\sqrt{2}}$

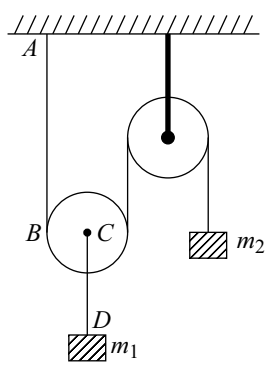


Fig. 10.45

159. The ratio of the intensity of a wave and its energy density is

(a) velocity (b) $\dfrac{1}{\text{velocity}}$

(c) frequency (d) $\dfrac{1}{\text{frequency}}$



Answers

Level A

1. (b)	2. (d)	3. (a)	4. (c)
5. (b)	6. (d)	7. (a)	8. (d)
9. (a)	10. (c)	11. (d)	12. (d)
13. (d)	14. (b)	15. (c)	16. (b)
17. (a)	18. (b)	19. (c)	20. (d)
21. (c)	22. (a)	23. (b)	24. (b)
25. (c)	26. (b)	27. (d)	28. (d)
29. (a)	30. (b)	31. (c)	32. (c)

Level B

33. (a)	34. (b)	35. (d)	36. (b)
37. (b)	38. (a)	39. (c)	40. (c)
41. (c)	42. (b)	43. (a)	44. (b)
45. (c)	46. (b)	47. (b)	48. (a)
49. (c)	50. (a)	51. (d)	52. (a)
53. (b)	54. (b)	55. (b)	56. (b)
57. (a)	58. (a)	59. (b)	60. (b)
61. (a)	62. (a)	63. (b)	64. (c)
65. (a)	66. (d)	67. (d)	68. (a)
69. (a)	70. (d)	71. (b)	72. (d)
73. (a)	74. (a)	75. (d)	76. (a)
77. (b)	78. (b)	79. (a)	80. (b)
81. (d)	82. (c)	83. (c)	84. (d)
85. (b)	86. (d)	87. (c)	88. (a)
89. (b)	90. (d)	91. (a)	92. (c)
93. (b)	94. (b)	95. (b)	96. (d)
97. (c)	98. (d)	99. (a)	100. (b)
101. (c)	102. (b)	103. (a)	104. (b)
105. (d)	106. (b)	107. (b)	108. (a)
109. (a)	110. (a)	111. (b)	112. (d)
113. (a)	114. (c)	115. (b)	116. (a)
117. (b)	118. (a)	119. (b)	120. (c)
121. (c)	122. (a)	123. (a)	124. (c)
125. (c)	126. (c)	127. (c)	128. (c)
129. (b)	130. (a)	131. (c)	132. (a)
133. (b)	134. (d)	135. (a)	136. (d)
137. (a)	138. (a)	139. (c)	140. (a)
141. (b)	142. (d)	143. (c)	144. (d)
145. (a)	146. (a)	147. (a)	148. (d)
149. (b)	150. (a)	151. (b)	152. (c)
153. (b)	154. (d)	155. (b)	156. (d)
157. (c)	158. (b)	159. (a)	



Solutions

Level A

1. $T = 2\pi\sqrt{\frac{l}{g}}$. When the lift accelerates upwards with an acceleration of $g/3$, the effective acceleration is $g' = g + \frac{g}{3} = 4\frac{g}{3}$

   Therefore, the new time period is

   $$T' = 2\pi\sqrt{\frac{l}{g'}}$$

   Thus $$\frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{g}{4g/3}} = \frac{\sqrt{3}}{2}$$

   or $$T' = \frac{\sqrt{3}}{2}\,T.$$

   Hence the correct choice is (b).

2. The length of the pendulum (l) is the distance between the point of support and the centre of mass of the sphere. The centre of mass (CM) of a sphere is at its centre when it is empty and completely filled with water. The CM falls below the centre of the sphere as the water flows out until the sphere is half full after which the CM begins to rise. Hence the effective length of the pendulum (and hence its time period) will increase at first and decrease until the sphere is empty when it becomes equal to the time period when the sphere was full of water. Thus the correct choice is (d).

3. Due to buoyancy, the effective acceleration due to gravity decreases from g to $g' = g - g/8 = 7g/8$. Since $T = 2\pi\sqrt{\frac{l}{g}}$. The new time period will be

   $$T' = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{7g/8}}$$

   Thus $$\frac{T'}{T} = \sqrt{\frac{g}{7g/8}} = \sqrt{\frac{8}{7}} \text{ or } T' = \sqrt{\frac{8}{7}}\,T.$$

   Hence the correct choice is (a).

4. The amplitude of a simple harmonic motion given by

$$x = a \sin \omega t + b \cos \omega t$$

is $A = (a^2 + b^2)^{1/2}$. Hence

$$A = (3^2 + 4^2)^{1/2} = (9 + 16)^{1/2} = 5 \text{ m}$$

Therefore, the correct choice is (c).

5. Let k be the force constant of each spring. The force F needed to extend the spring A by an amount x is

$$F = kx \text{ or } x = \frac{F}{k}$$

In Fig. 10.30b on page 10.29, the springs are connected in series. Therefore, the force constant k' of the combination is given by

$$\frac{1}{k'} = \frac{1}{k} + \frac{1}{k}$$

or $$k' = \frac{k}{2}$$

If a force F' is applied, the combination will extend by

$$x' = \frac{F'}{k'}$$

Thus $$\frac{x'}{x} = \frac{F'}{F} \cdot \frac{k}{k'} = \frac{6}{4} \times \frac{k}{k/2} = 3$$

or $$x' = 3x = 3 \times 1 \text{ cm} = 3 \text{ cm}$$

Hence the correct choice is (b).

6. For a force F, the extension x in a spring of force constant k is given by

$$F = kx \qquad \text{(i)}$$

Now extension is proportional to the length of the spring. When the spring is cut into two equal halves, the extension x' of each half, for a force F will be $x' = x/2$. If k' is the force constant of each half, we have

$$F = k'x' = k' \frac{x}{2} \qquad \text{(ii)}$$

Comparing (i) and (ii) we get $k' = 2k$. Hence the correct choice is (d).

7. Young's modulus $Y = \frac{F}{A} \cdot \frac{L}{l}$

Force constant $$k = \frac{F}{l} = \frac{YA}{L}$$

where l is the extension in the spring of original length L and cross-sectional area A when a force $F = Mg$ is applied. Now, the time period of vertical oscillations is given by

$$T = 2\pi \sqrt{\frac{M}{k}} = 2\pi \sqrt{\frac{ML}{YA}}$$

$$\therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{Y_2}{Y_1}} = \sqrt{\frac{3}{2}}$$

Hence the correct choice is (a).

8. The velocity of an oscillating body is maximum when it is at the equilibrium position where the potential energy is zero and the energy is entirely kinetic. At the extreme positions, the kinetic energy is zero and the energy is entirely potential. Therefore, the kinetic energy at equilibrium position = potential energy at extreme positions = total energy. Since the maximum velocities (i.e. velocities at equilibrium position) are equal for the two equal masses, their kinetic energies are also equal = their potential energies at extreme positions where the displacement is maximum = amplitude. If x_1 and x_2 are amplitudes of bodies A and B, we have

$$\frac{1}{2} k_1 x_1^2 = \frac{1}{2} k_2 x_2^2$$

or $$\frac{x_1}{x_2} = \sqrt{\frac{k_2}{k_1}}$$

Hence the correct choice is (d).

9. $F_1 = k_1 x$, $F_2 = k_2 x$.

Work done $W_1 = \frac{1}{2} k_1 x^2$ and $W_2 = \frac{1}{2} k_2 x^2$

$$\therefore \quad \frac{W_1}{W_2} = \frac{k_1}{k_2}$$

Hence the correct choice is (a).

10. When the springs are stretched by the same force F, the extensions in springs A and B are x_1 and x_2 respectively which are given by

$$F = k_1 x_1 = k_2 x_2 \quad \text{or} \quad \frac{x_1}{x_2} = \frac{k_2}{k_1} \qquad \text{(i)}$$

Work done $W_1 = \frac{1}{2} k_1 x_1^2$ and $W_2 = \frac{1}{2} k_2 x_2^2$

$$\therefore \quad \frac{W_1}{W_2} = \frac{k_1}{k_2} \cdot \frac{x_1^2}{x_2^2} \qquad \text{(ii)}$$

Using (i) and (ii) we get

$$\frac{W_1}{W_2} = \frac{k_1}{k_2} \cdot \frac{k_2^2}{k_1^2} = \frac{k_2}{k_1}$$

Hence the correct choice is (c).

11. If a force F is applied to a spring of force contant k- and the spring extends by an amount x, then

$$F = kx$$

The extension x produced in a spring is proportional to its length. Thus, if the spring is cut into three equal pieces, the same force F will produce an extension $x/3$ in a piece. If k' is the force constant of the piece, we have

$$F = k' x/3$$

Therefore $\frac{k'}{3} = k$ or $k' = 3k$. Thus, the force constant of each piece is $3k$. When springs are connected in parallel, the force constant of the combination is equal to the sum of the individual force constants of the springs so connected. Therefore, the force constant of the combination = $3k + 3k + 3k = 9k$. Hence the correct choice is (d).

12. Acceleration $= -\omega^2 x = -\left(\frac{2\pi}{T}\right)^2 \times \frac{A}{2} = -\frac{2\pi^2 A}{T^2}$, which is choice (d).

13. Displacement $x = +0.01$ m. Therefore, acceleration $a = -1.0\ \text{ms}^{-2}$.

Now $a = -\omega^2 x$

or $-1.0 = -\omega^2 \times 0.01$

which gives, $\omega = 10\ \text{rad s}^{-1}$. Therefore, $T = \frac{2\pi}{\omega} = \frac{\pi}{5}$ s. Hence the correct choice is (d).

14. $V_{\max} = A\omega = 0.05 \times 10 = 0.5\ \text{ms}^{-1}$, which is choice (b).

15. Given $x = 2\cos\left(0.5\pi t + \frac{\pi}{3}\right)$

$\therefore$ Velocity $V = \frac{dx}{dt}$

$$= -2 \times 0.5\ \pi \sin\left(0.5\pi t + \frac{\pi}{3}\right)$$

$\therefore$ Acceleration $a = \frac{dV}{dt}$

$$= -2 \times 0.5\ \pi \times 0.5\ \pi \cos\left(0.5\pi t + \frac{\pi}{3}\right)$$

$\therefore$ Maximum acceleration, $a_{\max} = -2 \times 0.5\ \pi \times 0.5\ \pi$

$$= -\frac{\pi^2}{2}\ \text{cms}^{-2}$$

$\therefore$ Magnitude is $|a_{\max}| = \left|-\frac{\pi^2}{2}\right|$

$$= \frac{\pi^2}{2}\ \text{cms}^{-2}$$

Hence the correct choice is (c).

16. Let A and B be the two extreme positions of the particle with O as the mean position. Displacements to the right of O are taken as positive while those to the left of O are taken as negative

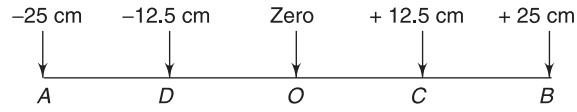


Fig. 10.43

Let the displacement of the particle is SHM be given by

$$x(t) = A \sin(\omega t + \phi) \qquad \text{(i)}$$

where $A = 25$ cm and $\omega = \frac{2\pi}{T} = \frac{2\pi}{3}\ \text{rad s}^{-1}$

Let us suppose that at time $t = 0$, the particle is at extreme position B. Setting $x = A$ at $t = 0$ in Eq. (i) we have

$$A = A \sin\phi$$

giving $\phi = \pi/2$.

Putting $\phi = \pi/2$ in Eq. (i), we get

$$x(t) = A \cos \omega t \qquad \text{(ii)}$$

Now let us say that the particle reaches point C at $t = t_1$ and point D at $t = t_2$. At C, the displacement $x(t_1) = +12.5$ cm and at D, it is $x(t_2) = -12.5$ cm (see Fig. 10.43). So from (ii) we have

$$+12.5 = 25 \cos \omega t_1$$

and $-12.5 = 25 \cos \omega t_2$

or $\cos \omega t_1 = +0.5$ or $\omega t_1 = \pi/3$

and $\cos \omega t_2 = -0.5$ or $\omega t_2 = \frac{2\pi}{3}$

Hence $\omega(t_2 - t_1) = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$

$\therefore$ $t_2 - t_1 = \frac{\pi}{3\omega} = \frac{T}{6}$ $\left(\because \omega = \frac{2\pi}{T}\right)$

or $(t_2 - t_1)_{\min} = \frac{3}{6} = 0.5$ s $(\because T = 3\text{s})$

Notice that $\cos \omega t_2 = -0.5$ even for $t_2 = \frac{4\pi}{3}$. This value of t_2 does not correspond to the minimum time because this is the time at which the particle, moving to left, reaches A and then returns to D.

17. In SHM, the velocity V at a displacement x is given by

$$V = \omega(A^2 - x^2)^{1/2}$$

or $V^2 = \omega^2(A^2 - x^2)$

Now $V = 3\ \text{cm s}^{-1}$ when $x = 4$ cm. Therefore,

$$9 = \omega^2(A^2 - 16) \qquad \text{(i)}$$

Also $V = 4$ cm s^{-1} when $x = 3$ cm. Therefore,

$$16 = \omega^2 (A^2 - 9) \qquad \text{(ii)}$$

Simultaneous solution of Eqs. (i) and (ii) gives

Amplitude $A = 5$ cm

18. Putting $A = 5$ cm in Eq. (i) above we get $\omega = 1$ rad s^{-1}. Therefore, $T = 2\pi/\omega = 2\pi = 6.28$ s. Hence the correct choice is (b).

19. Total energy $E = \frac{1}{2}\ mA^2\omega^2$

$$= \frac{1}{2} \times (50 \times 10^{-3}) \times (5 \times 10^{-2})^2 \times (1)^2$$

$$= 6.25 \times 10^{-5} \text{ J}$$

Hence the correct choice is (c).

20. Kinetic energy (KE) $= \frac{1}{2}\ m\omega^2(A^2 - x^2)$

Potential energy (PE) $= \frac{1}{2}\ m\omega^2 x^2$

Total energy $(E) = \frac{1}{2}\ m\omega^2 A^2$

When $x = A/2$,

$$\text{KE} = \frac{1}{2}\ m\omega^2 \left(A^2 - \frac{A^2}{4}\right) = \frac{3}{8}\ m\omega^2 A^2$$

$$E = \frac{1}{2}\ m\omega^2 A^2$$

$$\therefore \qquad \frac{\text{KE}}{E} = \frac{3}{4}$$

21. The energy will be half kinetic and half potential at a value of x when KE = PE, i.e.

$$\frac{1}{2}\, m\omega^2(A^2 - x^2) = \frac{1}{2}\ m\omega^2 x^2$$

or $$A^2 - x^2 = x^2$$

or $$x = \frac{A}{\sqrt{2}}$$

22. The block will not be detached from the platform, if the amplitude of the platform's SHM is such that the maximum acceleration equals the acceleration due to gravity, i.e.

$$\omega^2 A_{\max} = g$$

or $$A_{\max} = \frac{g}{\omega^2}$$

$$= \frac{g}{4\pi^2 v^2}$$

Hence the correct choice is (a). Notice that $A_{\max}$ is independent of the mass of the block.

23. Let L be the relaxed length of the spring and y the extension produced in it due to force mg so that

$$ky = mg \qquad \text{(i)}$$

The displacement of the mass during oscillation is given by

$$x = A \sin(\omega t + \phi) \qquad \text{(ii)}$$

At the instant when the displacement is x

$$\text{KE of mass} = \frac{1}{2}\ mV^2 = \frac{1}{2}\, m \left(\frac{dx}{dt}\right)^2$$

$$= \frac{1}{2}\ mA^2\ \omega^2 \cos^2(\omega t + \phi) \qquad \text{(iii)}$$

$$\text{PE of spring} = \frac{1}{2}\ k(y + x)^2 = \frac{1}{2}\, k\,(y^2 + 2yx + x^2)$$

$$= \frac{1}{2}\, ky^2 + kyx + \frac{1}{2}\, kx^2$$

Using (i) and (ii) and $\omega = \sqrt{\frac{k}{m}}$, we have

$$\text{PE of spring} = \frac{1}{2}\ ky^2 + mgx$$

$$+ \frac{1}{2}\ m\omega^2 A^2 \sin^2(\omega t + \phi) \qquad \text{(iv)}$$

Taking gravitational PE at the mean position to be zero,

Gravitational PE at $x = -mg\,x$ (v)

Adding (iii), (iv) and (v), we get

Total energy of mass–spring system

$$= \frac{1}{2}\ mA^2\omega^2 \cos^2(\omega t + \phi) + \frac{1}{2}\ ky^2 + mgx$$

$$+ \frac{1}{2}\ mA^2\ \omega^2 \sin^2(\omega t + \phi) - mgx$$

$$= \frac{1}{2}\ mA^2\omega^2 + \frac{1}{2}\ ky^2$$

24. The radius of the circle along which the trolley moves is

$$r = 40 \text{ cm} = 0.4 \text{ m}$$

When the table is rotated, the tension in the spring is equal to the centripetal force, i.e.

$$F = \frac{mv^2}{r} = mr\omega^2 = 2 \times 0.4 \times (10)^2 = 80 \text{ N}$$

The extension in the spring is $x = 40 - 35 = 5$ cm = 0.05 m

$\therefore$ Force constant $k = \dfrac{F}{x} = \dfrac{80}{0.05}$

$= 1.6 \times 10^3$ N m^{-1}

Hence the correct choice is (b).

25. In case (a) the springs are connected in series. Therefore, the spring constant of the combination is given by

$$\frac{1}{k'} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{k} + \frac{1}{k} = \frac{2}{k}$$

$(\because k_1 = k_2 = k)$

or $k' = k/2$. Therefore, for case (a),

$$T_a = 2\pi \sqrt{\frac{m}{k'}} = 2\pi \sqrt{\frac{2m}{k}}$$

In case (b) the spring constant of the combination is

$$k'' = k_1 + k_2 = 2k$$

$$\therefore \quad T_b = 2\pi \sqrt{\frac{m}{k''}} = 2\pi \sqrt{\frac{m}{2k}}$$

Thus $\dfrac{T_a}{T_b} = 2$

Hence the correct choice is (c).

26. In case (a), the springs are connected in parallel. Therefore,

$$k' = k_1 + k_2 = 2k$$

$$\therefore \quad T_a = 2\pi \sqrt{\frac{m}{k'}} = 2\pi \sqrt{\frac{m}{2k}}$$

In case (b), the springs are connected in series. Therefore,

$$T_b = 2\pi \sqrt{\frac{2m}{k}}$$

$$\therefore \quad \frac{T_a}{T_b} = \frac{1}{2}$$

Hence the correct choice is (b).

27. The tension in the string is given by

$$T = mg + \frac{mv^2}{l}$$

where l is the length of the string. Velocity v of the bob is maximum when it passes through the mean position; then the tension T is also maximum. Now

$$v_{\max} = |A\omega| = (l\alpha_m)\omega = l\alpha_m \sqrt{\frac{g}{l}}$$

$(\because A = l\alpha_m)$

$$\therefore \quad v^2_{\max} = l^2\alpha_m^2 \times \frac{g}{l} = lg\alpha_m^2$$

$$\therefore \quad T_{\max} = mg + \frac{m}{l} \times lg\ \alpha_{\mathrm{m}}^2 = mg\ (1 + \alpha_m^2)$$

Hence the correct choice is (d).

28. Refer to Fig. 10.46. At the extreme position B, PE = mgh.

Now $h = AO - AC = l - l \cos \alpha_m = l(1 - \cos \alpha_m)$

KE at O = PE at B

$= mgl\ (1 - \cos \alpha_m)$

Hence the correct choice is (d).

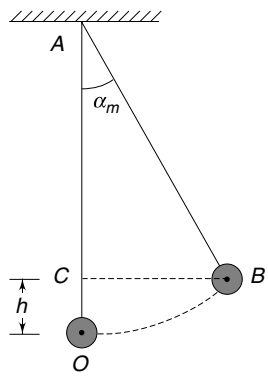


Fig. 10.46

29. Maximum displacement = amplitude = 5 cm

At time $t = 0$, $x = 0$ (equilibrium position). Hence time t taken by the particle to move from $x = 0$ to $x = 5$ cm is given by

$$5 = 5 \sin \pi t$$

or $1 = \sin \pi t$

or $\pi t = \dfrac{\pi}{2}$ or $t = 0.5$ s

30. Given time period $T = 6$ s

$$\text{Amplitude} = OC = OB = \frac{1}{2}\ BC = \frac{10}{2}$$

$= 5$ cm (Fig. 10.47)

$\therefore \quad OD = 2.5$ cm

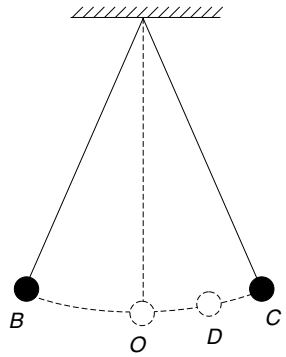


Fig. 10.47

Let the displacement of the pendulum be given by

$$x = A \sin(\omega t + \phi)$$

where $A = 5$ cm

and $$\omega = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3} \text{ rad s}^{-1}$$

Let us suppose that at $t = 0$, the pendulum is at C, i.e., at $t = 0$, $x = A$, so that

$$A = A \sin(\omega \times 0 + \phi)$$

or $$A = A \sin\phi \quad \text{or} \quad \sin\phi = 1 \quad \text{or} \quad \phi = \frac{\pi}{2}$$

Thus the motion of the pendulum is given by $\left(\text{putting } \phi = \frac{\pi}{2}\right)$

$$x = A \sin\left(\omega t + \frac{\pi}{2}\right) = A \cos \omega t = 5 \cos \omega t$$

The value of t for which $x = 2.5$ cm is given by

$$2.5 = 5 \cos \omega t$$

or $$\cos \omega t = \frac{1}{2} \quad \text{or} \quad \omega t = \frac{\pi}{3}$$

Since $$\omega = \frac{\pi}{3}, \quad \text{we have} \quad t = 1 \text{ second.}$$

31. If k is the force constant, we have

$$mg = kx$$

or $$\frac{m}{k} = \frac{x}{g}$$

$$\therefore \quad T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{x}{g}}$$

Hence the correct choice is (c).

32. Let the steel ball be placed at A, a little away from the centre O of a concave mirror of radius of curvature R ($= OC = AC$), as shown in Fig. 10.48. Let $\angle ACO = \theta$. If m is the mass of the ball, its weight $m\,g$ acts vertically downwards at A. This force is resolved into two rectangular components: $m\,g \cos\theta$ (which is balanced by the normal reaction N of the mirror) and $m\,g \sin\theta$ (which provides the restoring force F). Thus

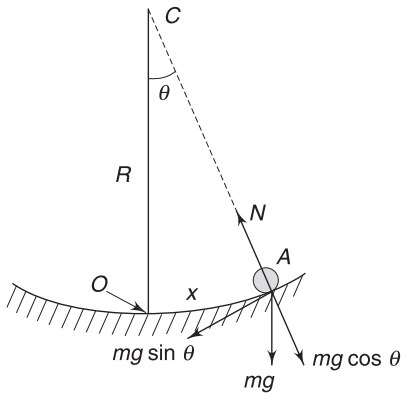


Fig. 10.48

$$F = -m\,g \sin\theta$$

$$= -m\,g\,\theta$$

($\because \theta$ is small; x/R being very small)

$$= -m\,g\,\frac{x}{R}$$

($\because x = R\,\theta$; x being the arc OA)

or $$F = -kx$$

where $k = \frac{mg}{R}$ is the force constant. Hence

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{R}{g}}$$

$$= 2\pi \times \sqrt{\frac{2.5}{10}} = \pi \text{ seconds.}$$

Hence the correct choice is (c).

Level B

33. Let x_1 be the extension produced in the spring when it is loaded with mass m_2 alone and x_2 be the further extension when mass m_1 is added to mass m_2 so that $x = x_1 + x_2$ is the total extension produced by $m_1 + m_2$ (see Fig. 10.49). Thus we have,

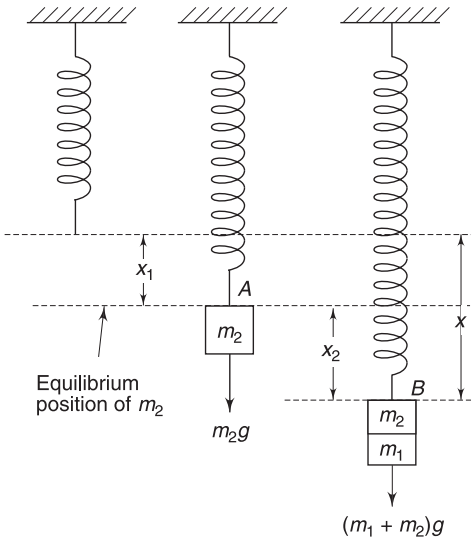


Fig. 10.49

For equilibrium state of m_2

$$m_2 g = k x_1 \qquad \text{(i)}$$

For equilibrium state of $(m_1 + m_2)$

$$(m_1 + m_2)\, g = k\,(x) = k\,(x_1 + x_2) \qquad \text{(ii)}$$

When the mass m_1 is removed, the mass m_2 will move upwards under the unbalanced force $= m_1 g$. Hence. Restoring force (F) on $m_2 = - m_1 g$

Subtracting (i) and (ii) we have

$$m_1 g = k x_2 \qquad \text{(iii)}$$

Hence, restoring force on $m_2 = - k x_2$

$\therefore$ Acceleration of $\quad m_2 = \dfrac{F}{m_2} = - \dfrac{k}{m_2} x_2$

Angular frequency is $\quad \omega = \sqrt{\dfrac{k}{m_2}}$, which is choice (a).

34. It is clear that A is the equilibrium position of m_2 and B its maximum displacement position. Hence $AB = x_2$ is the atmplitude of the oscillation of m_2. From Eq. (iii) above, we have

$$x_2 = \frac{m_1 g}{k}$$

Hence the correct choice is (b).

35. The particle displacements are given by

$$y(x, t) = A \sin \left\{\frac{2\pi}{\lambda}(vt - x)\right\}$$

Comparing this with the given equation we have

$\dfrac{2\pi}{\lambda} = 0.01\ \pi \quad$ or $\quad \lambda = 200$ m. Hence the correct choice is (d).

36. $\dfrac{2\pi v}{\lambda} = 2\ \pi \quad$ or $\quad v = \lambda = 200\ \text{ms}^{-1}$. Hence the correct choice is (b).

37. Phase difference $\Delta\phi = \dfrac{2\pi}{\lambda}\ \Delta x = \dfrac{2\pi}{200} \times 25 = \dfrac{\pi}{4}$.

Hence the correct choice is (b).

38. $y = a \sin \left\{\dfrac{2\pi}{\lambda}(vt - x)\right\}$ where $a = 10$ cm

and $\dfrac{v}{\lambda} = \nu = 500$ Hz.

We are given $- 5 = 10 \sin \left\{\dfrac{2\pi}{\lambda}(vt - 100)\right\}$

$\Rightarrow \dfrac{2\pi}{\lambda}\ (vt - 100) = \dfrac{7\pi}{6}$

and $\quad + 5 = 10 \sin \left\{\dfrac{2\pi}{\lambda}(vt - 200)\right\}$

$\Rightarrow \dfrac{2\pi}{\lambda}\ (vt - 200) = \dfrac{\pi}{6}$

These equations give $\lambda = 200$ cm $= 2$ m which is the choice (a).

39. $v = \nu\lambda = 500 \times 2 = 1000\ \text{ms}^{-1}$. Therefore, time taken is

$$t = \frac{2 - 1}{1000} = 10^{-3}\ \text{s}$$

Hence the correct choice is (c).

40. The velocity of transverse waves is given by $v = \sqrt{T/m}$ where T = tension and m = mass per unit length of the wire. If r is the radius of the wire and ρ its density, then $m = \pi r^2 \rho$. Therefore,

$$v = \frac{\sqrt{T}}{r\sqrt{\pi\rho}}. \text{ Thus } v_A = \frac{\sqrt{T_A}}{r_A\sqrt{\pi\rho}}$$

and $\quad v_B = \dfrac{\sqrt{T_B}}{r_B\sqrt{\pi\rho}}$.

Now $\quad \dfrac{v_A}{v_B} = \sqrt{\dfrac{T_A}{T_B}} . \dfrac{r_B}{r_A}$

It is given that $r_A = 2\ r_B$ and $T_A = \dfrac{1}{2}\ T_B$. Hence $\dfrac{v_A}{v_B} = \dfrac{1}{2\sqrt{2}}$. The correct choice is (c).

41. On the earth : $\nu = \dfrac{1}{2l}\sqrt{\dfrac{Mg}{m}} = \dfrac{1}{2l}\sqrt{\dfrac{g}{m}}$, since $M = 1$ kg

On moon $\quad \nu' = \dfrac{1}{2l}\sqrt{\dfrac{Mg/6}{m}} = \dfrac{1}{2l}\sqrt{\dfrac{Mg}{6m}}$

For resonance $\nu = \nu'$ or $\frac{1}{2l}\sqrt{\frac{g}{m}} = \frac{1}{2l}\sqrt{\frac{Mg}{6m}}$

which gives $M = 6$ kg. Hence the correct answer is (c).

42. Angular frequency $\omega = \pi$ or $2\pi\nu = \pi$ or $\nu = 0.5$ Hz. Now $v = 1.5$ ms^{-1}. Therefore $\lambda = v/\nu = 1.5/0.5 = 3$ m. Hence the correct choice is (b).

43. $\lambda = v/\nu = 20/10 = 2$ cm. Distance between adjacent nodes is $\frac{\lambda}{2} = 1$ cm, which is choice (a).

44. $\frac{2\pi x}{\lambda} = 0.1\pi x$ which gives $\lambda = 20$ cm. The distance between a node and the next antinode = $\frac{\lambda}{4} = \frac{20}{4} = 5$ cm. Hence the correct choice is (b).

45. $\nu' = \nu\left(1 + \frac{u_0}{v}\right)$. Given $u_0/v = \frac{1}{10}$. Therefore

$\nu' = \frac{11}{10}\nu$

or $\nu' - \nu = \frac{11}{10}\nu - \nu = \frac{\nu}{10}$. The percentage increase in ν is

$\frac{\nu' - \nu}{\nu} \times 100 = 10\%$ which is choice (c).

46. The lengths of the two parts of the wire are 49.5 cm and 50.5 cm.

47. $\nu = \frac{1}{2l}\sqrt{\frac{Mg}{m}}$ where M = mass of stone. If ρ is the density of the stone and V its volume, then $M = \rho V$. When the stone is wholly immersed in water of density ρ', the effective weight of the stone

$$M'g = (M - V\rho')g = (V\rho - V\rho')g = V\rho'\left(\frac{\rho}{\rho'} - 1\right)g$$

Now $\nu = \frac{1}{2l}\sqrt{\frac{\rho V g}{m}}$

and $\nu' = \frac{1}{2l'}\sqrt{\frac{V\rho'([\rho/\rho'] - 1)g}{m}}$

Given $l = 40$ cm and $l' = 30$ cm. Also $\nu = \nu'$, which gives

$$\frac{\rho}{\rho'} = \frac{16}{7}$$

Hence the correct choice is (b).

48. The value of λ for a pipe open at both ends is $2L$ where L is the length of the pipe. For a closed pipe $\lambda = 4L$. Since λ becomes twice, the frequency reduces to half. Hence the correct choice is (a).

49. The correct choice is (c).

50. Frequency of sound emitted by a closed pipe of length L in the fundamental mode is $\nu = \frac{v}{4L}$. For resonance $\nu = \nu'$ where ν' is the frequency of the tuning fork. Thus $340 = \frac{340}{4L}$ or $4L = 1$ m or $L = 25$ cm. The next resonance occurs at $\frac{4}{3}L = 1$ m or $L = 75$ cm.

Hence the correct choice is (a).

51. $N' = \frac{1}{2(65-1)}\cdot\sqrt{\frac{T}{m}}$. Given $N = \frac{1}{2\times 65}\cdot\sqrt{\frac{T}{m}}$. Therefore

$$\frac{N'}{N} = \frac{2\times 65}{2(65-1)} = \frac{65}{64}$$

Also $N' - N = 8$ or $N' = N + 8$.

$\therefore \quad \frac{N+8}{N} = \frac{65}{64}$ which gives $N = 512$ Hz. Hence the correct choice is (d).

52. $\nu_1 = \frac{1}{4L_1}\sqrt{\frac{\gamma p}{\rho}}$, $\nu_2 = \frac{1}{4L_2}\sqrt{\frac{\gamma p}{\rho}}$. Therefore $\frac{\nu_1}{\nu_2} = \frac{L_2}{L_1} = \frac{50}{51}$. It is given that $\nu_2 = \nu_1 + 5$.

Therefore $\frac{\nu_1}{\nu_1 + 5} = \frac{50}{51}$ which gives $\nu_1 = 250$ Hz.

Hence $\nu_2 = 250 + 5 = 255$ Hz which is choice (a).

53. The listener moves away from A and approaches B. Hence the apparent frequencies are

$$\nu_1 = \nu\left(1 - \frac{u}{v}\right) \text{ and } \nu_2 = \nu\left(1 + \frac{u}{v}\right)$$

$\therefore \quad \nu_2 - \nu_1 = 2\nu u/v$. It is given that $\nu_2 - \nu_1 = 10$.

$v = 340$ ms^{-1} and $\nu = 680$ Hz.

Substituting these values we get

$$10 = \frac{2\times 680\times u}{340}$$

or $u = 2.5$ ms^{-1}.

Hence the correct choice is (b).

54. $\nu = 3.2$ MHz $= 3.2 \times 10^6$ Hz
$v = 1.6$ km^{-1} $= 1.6 \times 10^3$ m s^{-1}

$$\lambda = \frac{v}{\nu} = \frac{1.6 \times 10^3}{3.2 \times 10^6}$$

$$= 0.5 \times 10^{-3} \text{ m} = 0.5 \text{ mm}$$

55. Frequency of sound (ν) = 100 kHz = 10^5 Hz
Speed of sound in air (v_a) = 340 m s^{-1}
Speed of sound in water (v_w) = 1450 m s^{-1}
Since the reflected wave travels in air, its wavelength is

$$\lambda_a = \frac{v_a}{\nu} = \frac{340}{10^5} = 3.4 \times 10^{-3} \text{ m} = 3.4 \text{ mm}$$

Since the transmitted wave travels in water, its wavelengths is

$$\lambda_w = \frac{v_w}{\nu} = \frac{1450}{10^5} = 1.45 \times 10^{-2} \text{ m}$$

$$= 1.45 \text{ cm}$$

56. The resultant displacement is given by
$y = y_1 + y_2 = 0.05 \{\sin(3\pi t - 2x) + \sin(3\pi t + 2x)\}$
Using the trigonometric relation
$\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin\alpha \cos\beta$
we have $y = 0.1 \cos 2x \cdot \sin 3\pi t$
or $y = R \sin 3\pi t$
where R, the amplitude of standing waves, is given by

$$R = 0.1 \cos 2x$$

When $x = 0.5$ m $\cos 2x = \cos(2 \times 0.5 \text{ rad})$

$$= \cos(1 \text{ rad})$$

$$= \cos 57.3° = 0.54$$

$\therefore$ Amplitude R at $x = 0.5$ is $0.1 \times 0.54 = 0.054$ m = 5.4 cm

57. Let λ be the wavelength, ν the frequency and v the speed of each wave. Then

$\frac{2\pi}{\lambda}$ = coefficient of x in the argument of the sine function = $\frac{2\pi}{3}$

or $\lambda = 3$ m.

Also $\omega = 2\pi\nu$ = coefficient of t in the argument of the sine function = $120\ \pi$

which gives $\nu = 60$ Hz.

Hence $v = \nu\lambda = 60 \times 3 = 180$ m s^{-1}

Mass per unit length (m) = $\frac{3.0 \times 10^{-2}}{1.5}$

$$= 2.0 \times 10^{-2} \text{ kg m}^{-1}$$

We know that $v = \sqrt{\frac{T}{m}}$, where T is tension in the string.

$\therefore$ $T = mv^2 = 2.0 \times 10^{-2} \times (180)^2 = 648$ N

58. Let N be the frequency of the source and ν_p that of the pth harmonic of a closed pipe. The source will resonantly excite that harmonic mode of the pipe for which

$$N = \nu_p$$

for any value of $p = 1, 3, 5, \ldots$. Now for a closed pipe we know that

$$\nu_p = \frac{pv}{4L}$$

Therefore, for resonance,

$$N = \frac{pv}{4L}$$

or $$p = \frac{4NL}{v} = \frac{4 \times 425 \times 0.2}{340} = 1$$

Hence the correct choice is (a).

59. In an open pipe, the condition of resonance is

$$N = \nu_p = \frac{pv}{2L} \ ; p = 1, 2, 3, \ldots.$$

or $$p = \frac{2NL}{v} = \frac{2 \times 1700 \times 0.2}{340} = 2$$

Hence, the correct choice is (b).

60. Let ν_1 be the frequency of A and ν_2 that of fork B. There are two possibilities (a) $\nu_1 > \nu_2$ and (b) $\nu_1 < \nu_2$.
(a) $\nu_1 > \nu_2$: $\nu_2 = 480$ Hz, $\nu_b = 10$ Hz.
Now, $\nu_b = \nu_1 - \nu_2$
$\therefore$ $\nu_1 = \nu_2 + \nu_b = 480 + 10 = 490$ Hz

On loading fork A its frequency decreases. In other words, ν_1 decreases.

$\therefore$ $\nu_1 - \nu_2 = \nu_b$ should also decrease. But ν_b increase to 15 beats/s. Hence ν_1 is not greater than ν_2.

(b) $\nu_1 < \nu_2$: In this case $\nu_2 - \nu_1 = \nu_b$.

$\therefore$ $\nu_1 = \nu_2 - \nu_b = 480 - 10 = 470$ Hz

Hence the correct choice is (b).

61. On loading ν_1 decreases, therefore $(\nu_2 - \nu_1) = \nu_b$ increases which is what is observed. Therefore, $\nu_1 = 470$ Hz before it is loaded, and $\nu_1 = \nu_2 - \nu_b = 480 - 15 = 465$ Hz after it is loaded.

Hence the correct choice is (a).

62. $\nu_A = 324$ Hz, $\nu_b = 6$ Hz.

The frequency of string B is

$$\nu_B = \nu_A \pm \nu_B = 324 \pm 6 = 330 \text{ or } 318 \text{ Hz}$$

Now, the frequency of a string is proportional to the square root of tension. Hence, if the tension in A is slightly decreased, its frequency will be slightly reduced, i.e. it will become less than 324 Hz. If the frequency of string B is 330 Hz, the beat frequency would increase to a value greater than 6 Hz if the tension in A is reduced. But the beat frequency is found to decrease to 3 Hz. Hence, the frequency of B cannot be 330 Hz; it is, therefore 318 Hz. When the tension in A is reduced, its frequency becomes $324 - 3 = 321$ Hz which will produce beats of frequency 3 Hz with string B of frequency 318 Hz. Hence the correct choice is (a).

63. Let N be the frequency of the tuning fork. Then, the frequency of the wire, when the tension is 100 N will be $(N + 5)$ and when the tension is 81 N, it is $(N - 5)$; since in each case 5 beats are heard per second. Hence

$$N + 5 = \frac{1}{2L}\sqrt{\frac{T_1}{m}} = \frac{1}{2 \times 0.5}\sqrt{\frac{100}{m}}$$

$$= \frac{10}{\sqrt{m}} \qquad \text{(i)}$$

and $$N - 5 = \frac{1}{2L}\sqrt{\frac{T_2}{m}} = \frac{1}{2 \times 0.5}\sqrt{\frac{81}{m}}$$

$$= \frac{9}{\sqrt{m}} \qquad \text{(ii)}$$

Subtracting (ii) from (i) we have

$$10 = \frac{1}{\sqrt{m}} \quad \text{or} \quad m = 0.01 \text{ kg m}^{-1}$$

Using this value of m in (i) or (ii) gives $N = 95$ Hz.

Hence the correct choice is (b).

64. The driver will hear two sounds, one coming directly from the siren and the other reflected at the wall or coming from the acoustic image of the car. ν = frequency of direct sound = 700 Hz. The reflected sound can be imagined to be coming from the mirror image (shown dotted in Fig. 10.50).

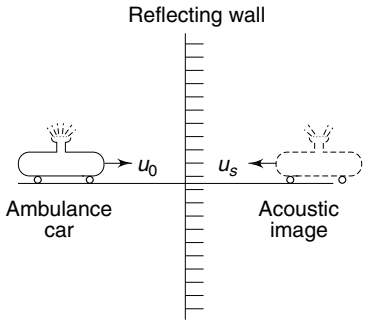


Fig. 10.50

The observer (driver) is approaching his image–source which is also approaching with the same speed. Hence $u_0 = u_s = u$. The frequency of sound heard by him is given by

$$\nu' = \nu\left(\frac{v + u_o}{v - u_s}\right) = \nu\left(\frac{v + u}{v - u}\right)$$

or $$\frac{\nu'}{\nu} = \frac{\left(1 + \frac{u}{v}\right)}{\left(1 - \frac{u}{v}\right)} = \left(1 + \frac{u}{v}\right)\left(1 - \frac{u}{v}\right)^{-1}$$

Since $u << v$, we can expand binomially and retain only the first order terms in u/v. We then get

$$\frac{\nu'}{\nu} = 1 + \frac{2u}{v}$$

or $$\frac{\nu' - \nu}{\nu} = \frac{2u}{v}$$

or $$\nu' - \nu = \frac{2u\nu}{v} = \frac{2 \times 2 \times 700}{350} = 8$$

$\therefore$ Beat frequency $\nu_b = \nu' - \nu = 8$

Hence the correct choice is (c).

65. Person A hears the sound of his own source whose frequency in ν. He also hears the sound of the source carried by person B, towards whom he is moving with a speed u. The apparent frequency of this sound is given by

$$\nu' = \nu\left(1 + \frac{u}{v}\right)$$

or $$\nu' - \nu = \frac{\nu u}{v}$$

$\therefore$ Beat frequency $\nu_b = \nu' - \nu = \frac{\nu u}{v}$. Hence the correct choice is (a).

66. Person B hears the sound of his own source of frequency ν. He also hears the sound of the source carried by person A, who is approaching with a speed u. The apparent frequency of this sound is given by

$$\nu'' = \frac{\nu}{1-\frac{u}{v}} = \frac{\nu v}{v-u}$$

or $$\nu'' - \nu = \nu\left[\frac{v}{v-u} - 1\right] = \frac{\nu u}{v-u}$$

$\therefore$ Beat frequency $\nu_b = \nu'' - \nu = \dfrac{\nu u}{v-u}$, which is choice (d).

67. The frequency of ultrasonic (sound) waves sent out from the SONAR undergoes a change in two steps. (i) Before reflection, the frequency of sound received by the enemy submarine which is approaching the SONAR with a speed $u_0 = 200$ ms^{-1} is given by

$$\nu' = \frac{\nu(v+u_0)}{v} = \frac{42\times10^3\times(1400+200)}{1400}$$

$$= 48\times10^3 \text{ Hz}$$

(ii) The wave of frequency ν' is reflected from the enemy submarine, which acts as a virtual image source of frequency ν', approaching the SONAR with a speed $u_s = 200$ ms^{-1}. Hence the frequency of sound received back at the SONAR will be (observer stationary, source approaching)

$$\nu'' = \frac{v\nu'}{(v-u_s)} = \frac{1400\times48\times10^3}{(1400-200)}$$

$$= 56\times10^3 \text{ Hz} = 56 \text{ kHz}$$

Hence the correct choice is (d).

68. Since only the source of sound, i.e. the gun is in motion, the speed of sound remains unchanged at 340 ms^{-1}. Therefore, the time taken by the sound of firing to arrive at the target 500 m away is

$$t_s = \frac{500}{340} = \frac{25}{17} \text{ s}$$

If the gun points in the direction of motion of the tank, the effective speed of the bullet = 340 + 20 = 360 ms^{-1}. Therefore, the time taken by the bullet to reach the target is

$$t_b = \frac{500}{360} = \frac{25}{18}\text{s}$$

Since $t_s > t_b$, the correct choice is (a).

69. Comparing $y = a \sin 2\pi(bt - cx)$ with

$$y = a \sin\left\{\frac{2\pi}{\lambda}(vt - x)\right\}$$

we have

$$2\pi b = \frac{2\pi v}{\lambda} \quad \text{and} \quad 2\pi c = \frac{2\pi}{\lambda}$$

which give $v = b\lambda$ and $\lambda = \dfrac{1}{c}$. Thus $v = b/c$. Particle velocity is

$$V = \frac{dy}{dt} = \frac{d}{dt}\,[a \sin 2\pi(bt - cx)]$$

$$= 2\pi ab \cos 2\pi(bt - cx)$$

$\therefore$ $V_{\max} = 2\pi ab$. Now $V_{\max} = 2v$, if

$$2\pi ab = \frac{2b}{c}$$

or $$c = \frac{1}{\pi a}$$, which is choice (a).

70. When the three waves superpose at a point, then from the superposition principle, the resultant particle displacement at that point is given by

$$y = y_1 + y_2 + y_3$$

$$= a \sin\{2\pi(\nu - 1)t\} + a \sin(2\pi\nu t) + a \sin\{2\pi(\nu + 1)t\}$$

Now $\sin\{2\pi(\nu - 1)t\} + \sin\{2\pi(\nu + 1)t\}$

$$= 2\cos 2\pi t \sin 2\pi\nu t$$

Therefore,

$$y = a(1 + 2\cos 2\pi t)\sin 2\pi\nu t$$

or $$y = A \sin 2\pi\nu t$$

where $A = a(1 + 2\cos 2\pi t)$ is the resultant amplitude.

Now, the resultant intensity $\propto A^2$. Now A^2 will be maximum when

$$\cos 2\pi t = +1$$

or $$2\pi t = 0, 2\pi, 4\pi, \text{ ... etc.}$$

or $$t = 0, 1\text{s}, 2\text{s}, \text{... etc.}$$

$\therefore$ Time period of beats = time interval between two consecutive maxima = 1 s. Hence the beat frequency is 1 Hz, i.e. one beat is heard per second which is choice (d).

71. The pressure of a gas is given by

$$P = \frac{\rho v_{\text{rms}}^2}{3}$$

$\therefore$ $$v_{\text{rms}} = \sqrt{\frac{3P}{\rho}}$$

Also, the speed of sound in a gas is given by

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

$\therefore$ $$\frac{v}{v_{\text{rms}}} = \sqrt{\frac{\gamma}{3}} = \sqrt{\frac{1.5}{3}} = \frac{1}{\sqrt{2}}$$

Hence the correct choice is (b).

72. Let the density of hydrogen be $\rho' = 2\rho$, then the density of oxygen will be 32 ρ. The density of the mixture will be

$$\rho'' = \frac{3}{5} \times 2\rho + \frac{2}{5} \times 32\,\rho = 14\,\rho$$

Since the pressure is the same, we have

$$\frac{v''}{v} = \sqrt{\frac{\rho'}{\rho''}} = \sqrt{\frac{2\rho}{14\rho}} = \frac{1}{\sqrt{7}}$$

$\therefore$ $v'' = \dfrac{v}{\sqrt{7}}$. Hence the correct choice is (d).

73. The density of helium at STP = 2 times the density of hydrogen at STP. Since $v \propto 1/\sqrt{\rho}$, the speed of sound in helium will be $v/\sqrt{2}$. Hence the correct choice is (a).

74. Let the frequency of the first tuning fork be ν. The frequency of the second will be $(\nu + 4)$ and of the third will be $(\nu + 8)$ and so on. Now $\nu + 8 = \nu + (3 - 1) \times 4$. Therefore, the frequency of the 9th tuning fork $= \nu + (9 - 1) \times 4 = \nu + 32$. It is given that $\nu + 32 = 2\nu$. Hence $\nu = 32$ Hz which is choice (a).

75. The frequency of the fundamental mode is given by

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

$\therefore\ \nu \propto \dfrac{1}{L}$. Hence $\nu_1 : \nu_2 : \nu_3 = \dfrac{1}{L_1} : \dfrac{1}{L_2} : \dfrac{1}{L_3}$.

Now L_1 = 20 cm, L_2 = 40 cm and L_3 = 60 cm. Therefore,

$$\nu_1 : \nu_2 : \nu_3 = \frac{1}{20} : \frac{1}{40} : \frac{1}{60} = 6 : 3 : 2$$

Hence, the correct choice is (d).

76. Let ν_1 be the frequency of the wire when its vibrating length is L_1 = 48 cm and ν_2 when L_2 = 50 cm. Since $\nu \propto \dfrac{1}{L}$; $\nu_1 > \nu_2$. If ν is the frequency of the tuning fork, then

$$\nu_1 = \nu + 4$$

and $$\nu_2 = \nu - 4$$

$\therefore$ $$\frac{\nu_1}{\nu_2} = \frac{\nu + 4}{\nu - 4}$$

But $$\frac{\nu_1}{\nu_2} = \frac{L_2}{L_1} = \frac{50}{48} = \frac{25}{24}$$

Thus $$\frac{\nu + 4}{\nu - 4} = \frac{25}{24}$$

which gives ν = 196 Hz. Hence the correct choice is (a).

77. Let T_1 be the tension in each string when they are in unison. Let T_2 be the tension in each string when they are not in unison; then since $\nu \propto \sqrt{T}$; $\nu_2 > \nu_1$ such that

$$\frac{\nu_2}{\nu_1} = \sqrt{\frac{T_2}{T_1}}$$

Now $T_2 = 1.01\ T_1$. Therefore

$$\frac{\nu_2}{\nu_1} = \sqrt{1.01} = \left(1 + \frac{1}{100}\right)^{1/2}$$

Expanding binomially, we have $\left(1 + \dfrac{1}{100}\right)^{1/2}$

$$\simeq 1 + \frac{1}{200} = \frac{201}{200}.$$ Thus

$$\frac{\nu_2}{\nu_1} = \frac{201}{200}$$

Also, $\nu_2 - \nu_1 = 4$ or $\nu_2 = \nu_1 + 4$. Therefore, we have

$$\frac{\nu_1 + 4}{\nu_1} = \frac{201}{200}$$

which gives ν_1 = 800 Hz. Hence the correct choice is (b).

78. Let v_1 be the speed of sound at 27°C and v_2 at 31°C. Then

$$\frac{v_2}{v_1} = \left(\frac{273 + 31}{273 + 27}\right)^{1/2} = \left(\frac{304}{300}\right)^{1/2}$$

$$= \left(1 + \frac{4}{300}\right)^{1/2} \simeq 1 + \frac{1}{2} \times \frac{4}{300} = \frac{151}{150}$$

Now, frequency $\propto$ speed of sound. Hence

$$\frac{\nu_2}{\nu_1} = \frac{v_2}{v_1} = \frac{151}{150}$$

$\therefore$ $\nu_2 = \nu_1 \times \dfrac{151}{150} = \dfrac{300 \times 151}{150} = 302$ Hz. Hence beat frequency = 302 – 300 = 2 Hz

Thus the correct choice is (b).

79. If the aircraft is approaching the radar station with a speed u, the apparent frequency of radiowaves received by the radar after reflection from the aircraft is given by

$$\nu' = \nu\left(1+\frac{2u}{c}\right)$$

$\therefore$ Apparent increase in frequency is

$$\Delta\nu = \nu' - \nu = \frac{2u\nu}{c}$$

Given, $\nu = 600$ MHz $= 600\times10^6$ Hz and $\Delta\nu = 6$ kHz $= 6\times10^3$ Hz. Thus

$$u = \frac{c\Delta\nu}{2\nu} = \frac{3\times10^8\times6\times10^3}{2\times600\times10^6} = 1.5\times10^3 \text{ ms}^{-1}$$

$$= 1.5 \text{ kms}^{-1}$$

Hence, the correct choice is (a).

80. Amplitude of the first simple harmonic motion is

$$A_1 = \sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}a}{2}\right)^2} = a$$

Amplitude of the second motion is

$$A_2 = \sqrt{a^2+a^2} = \sqrt{2}\,a$$

$\therefore$ $$\frac{A_1}{A_2} = \frac{1}{\sqrt{2}}.$$

Hence the correct choice is (b).

81. We can rewrite the two motions as

$$x_1 = a\left(\frac{1}{2}\sin\omega t+\frac{\sqrt{3}}{2}\cos\omega t\right)$$

$$= a(\cos 60^\circ \sin\omega t + \sin 60^\circ\cos\omega t)$$

or $$x_1 = a\sin(\omega t + 60^\circ) \quad (1)$$

and $$x_2 = \frac{a}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\sin\omega t+\frac{1}{\sqrt{2}}\cos\omega t\right)$$

$$= \frac{a}{\sqrt{2}}(\cos 45^\circ\sin\omega t + \sin 45^\circ\cos\omega t)$$

or $$x_2 = \frac{a}{\sqrt{2}}\sin(\omega t + 45^\circ) \quad (2)$$

From (1) and (2) it follows that the amplitudes are a and $\frac{a}{\sqrt{2}}$ (a result obtained above) the phases are 60º and 45º. Therefore, phase difference = 60º – 45º = 15º, which is choice (d).

82. Let the rod be depressed by a small amount x (Fig. 10.51). Both the springs are compressed by x. When the rod is released, the restoring torque is given by

$$\tau = (kx)\times\frac{l}{2}+(kx)\times\frac{l}{2}$$

$$= (kx)l$$

Now $\tan\theta = \frac{x}{l/2} = \frac{2x}{l}$. Since θ is small, $\tan\theta \simeq \theta$, where θ is expressed in radian. Thus $\theta = 2x/l$ or $x = \theta l/2$. Hence

$$\tau = k\left(\frac{\theta l}{2}\right)\times l = \frac{k\theta l^2}{2}$$

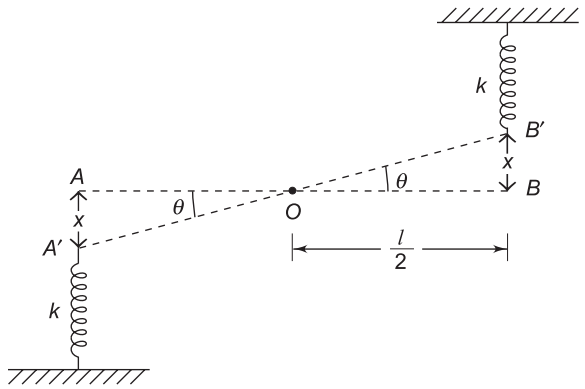


Fig. 10.51

If I is the moment of inertia of the rod about O, then

$$I\frac{d^2\theta}{dt^2} = -\left(\frac{kl^2}{2}\right)\theta$$

or $$\frac{d^2\theta}{dt^2} = -\left(\frac{kl^2}{2I}\right)\theta$$

Since $\frac{d^2\theta}{dt^2} \propto (-\theta)$, the motion is simple harmonic whose angular frequency is given by

$$\omega = \sqrt{\frac{kl^2}{2I}}$$

Now $\omega = \frac{2\pi}{T}$ and $I = \frac{ml^2}{12}$. Therefore, we have

$$\frac{2\pi}{T} = \sqrt{\frac{kl^2}{2}\times\frac{12}{ml^2}} = \sqrt{\frac{6k}{m}}$$

or $$T = \pi\sqrt{\frac{2m}{3k}}$$, which is choice (c).

83. Refer to Fig. 10.52. The magnitude of the restoring torque = force × perpendicular distance

$$= mg\times AB$$

$$= mg\times R\sin\theta$$

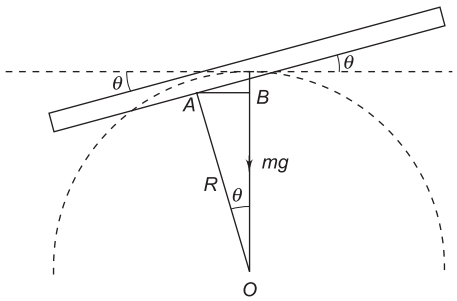


Fig. 10.52

Since θ is small, $\sin\theta \simeq \theta$. Here θ is expressed in radian. The equation of motion of the scale is

$$I\frac{d^2\theta}{dt^2} = -mgR\theta$$

or $$\frac{d^2\theta}{dt^2} = \left(-\frac{mgR}{I}\right)\theta$$

$\therefore$ $$\omega = \sqrt{\frac{mgR}{I}} \text{ or } \frac{2\pi}{T} = \sqrt{\frac{mgR}{I}}$$

or $$T = 2\pi\sqrt{\frac{I}{mgR}}$$. Now $I = \frac{mL^2}{12}$. Hence

$$T = \frac{\pi L}{\sqrt{3gR}}$$

Using the values $L = 1$ m, $g = 10$ ms^{-2} and $R = 0.3$ m, we get $T = \pi/3$ second. Hence the correct choice is (c).

84. Let A_0, T_0 and ω_0 respectively be the amplitude, time period and angular frequency of the combined masses. Let v be the velocity of the bob when it reaches the equilibrium position and v_0 the velocity of the combined masses just after collision. Then, from the principle of conservation of energy, we have

$$\frac{1}{2}mv^2 = \frac{1}{2}(m + m_0)v_0^2 \qquad (1)$$

Now $v = r\omega = r \times \frac{2\pi}{T}$ and $v_0 = r\omega_0 = r \times \frac{2\pi}{T_0}$, where r is the length of the pendulum. Using these in (1), we get

$$\frac{1}{2}m \times \left(\frac{2\pi r}{T}\right)^2 = \frac{1}{2}(m + m_0) \times \left(\frac{2\pi r}{T_0}\right)^2$$

or $$T_0 = T\sqrt{\frac{m + m_0}{m}}$$, which is choice (d).

85. From the principle of conservation of angular momentum, we have

$$mvA = (m + m_0)v_0A_0$$

or $$mr\omega A = (m + m_0)r\omega_0 A_0$$

or $$m\left(\frac{2\pi}{T}\right)A = (m + m_0)\left(\frac{2\pi}{T_0}\right)A_0$$

or $$A_0 = A\left(\frac{T_0}{T}\right)\left(\frac{m}{m + m_0}\right)$$

or $$A_0 = A\sqrt{\frac{m + m_0}{m}}\left(\frac{m}{m + m_0}\right)$$

$$= A\sqrt{\frac{m}{m + m_0}}$$

Hence the correct choice is (b).

86. There are two perpendicular acceleration vectors; g acting vertically downwards and a acting horizontally. The resultant acceleration is given by

$$g_{\text{eff}} = \sqrt{g^2 + a^2}$$

Hence the correct choice is (d).

87. The correct choice is (c). Refer to the solution of Q. 99 on page 10.37.

88. The force which increases the length of the spring by $x = 2.5$ cm is $F = mg\sin\theta$. Therefore, the spring constant is

$$k = \frac{F}{x} = \frac{mg\sin\theta}{x}$$

Now time period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg\sin\theta/x}} = 2\pi\sqrt{\frac{x}{g\sin\theta}}$

Putting $x = 2.5$ cm $= 2.5 \times 10^{-2}$ m, $g = 9.8$ ms^{-2} and $\theta = 30°$, we get $T = \pi/7$ second, which is choice (a).

89. Let the motions be given by $x_1 = A\sin\omega t$ and $x_2 = A\sin(\omega t + \phi)$. Separation between the particles at time t is given by

$$x = x_2 - x_1 = A\sin(\omega t + \phi) - A\sin\omega t \qquad (1)$$

The separation will be maximum when $\frac{dx}{dt} = 0$, i.e. when

$A\omega\cos(\omega t + \phi) - A\omega\cos\omega t = 0$

or $$\cos(\omega t + \phi) = \cos\omega t$$

or $$\omega t + \phi = \pm\omega t \qquad [\because \cos(-\theta) = \cos\theta]$$

The plus sign is not possible because then $\phi = 0$ which implies that the separation x is always zero. Therefore, $\omega t + \phi = -\omega t$ or $\omega t = -\phi/2$. Using this value in (1), we have

$$x_{max} = A \sin\left(-\frac{\phi}{2}+\phi\right) - A \sin\left(-\frac{\phi}{2}\right)$$

$$= 2A \sin\left(\frac{\phi}{2}\right)$$

Given $x_{max} = \sqrt{2}\ A$. Hence

$$\sqrt{2}\ A = 2\ A \sin\left(\frac{\phi}{2}\right)$$

or $\sin\left(\frac{\phi}{2}\right) = \frac{1}{\sqrt{2}}$ or $\frac{\phi}{2} = \frac{\pi}{4}$ or $\phi = \frac{\pi}{2}$. Hence the correct choice is (b).

90. Let L_1 and L_2 be the lengths of the two parts of sonometer wire.

Given $\nu_1 = \frac{1}{2L_1}\sqrt{\frac{T}{m}}$ and $\nu_2 = \frac{1}{2L_2}\sqrt{\frac{T}{m}}$

or $\nu_1 L_1 = \nu_2 L_2 = \frac{1}{2}\sqrt{\frac{T}{m}}$ = constant, say k.

Thus $L_1 = \frac{k}{\nu_1}$ and $L_2 = \frac{k}{\nu_2}$

The fundamental frequency of the complete sonometer wire is

$$\nu = \frac{1}{2(L_1+L_2)}\sqrt{\frac{T}{m}} = \frac{k}{(L_1+L_2)}$$

or $$\frac{1}{\nu} = \frac{L_1}{k}+\frac{L_2}{k} = \frac{1}{\nu_1}+\frac{1}{\nu_2}$$

or $\nu = \frac{\nu_1\nu_2}{\nu_1+\nu_2}$, which is choice (d).

91. Let W_1 be the weight of the bob in air and W_2 when it is immersed in water. Given

$$\nu_1 = \frac{1}{2L}\sqrt{\frac{W_1}{m}} \text{ and } \nu_2 = \frac{1}{2L}\sqrt{\frac{W_2}{m}}$$

$$\therefore \quad \frac{W_1}{W_2} = \frac{\nu_1^2}{\nu_2^2}$$

$$\text{Relative density} = \frac{\text{weight in air}}{\text{loss of weight in water}}$$

$$= \frac{W_1}{W_1 - W_2} = \frac{\nu_1^2}{\nu_1^2 - \nu_2^2}$$

Hence the correct choice is (a).

92. The potential energy of a particle of mass m executing simple harmonic motion of angular frequency ω at a distance x from the equilibrium position is given by

$$\text{P.E.} = \frac{1}{2}\ m\omega^2 x^2 = \frac{1}{2}\ kx^2$$

where $k = m\omega^2$ is constant. Hence the correct choice is (c).

93. Since the spring in massless, the momentum imparted to the system $A - B$ by block C is equally shared between A and B. Since A and B have the same mass, they will have the same velocity; let it be u. From the law of conservation of momentum, we have

$$mv = mu + mu = 2mu$$

which gives $u = v/2$. Let x be the maximum compression of the spring. From the law of conservation of energy, we have

$$\frac{1}{2}\ mv^2 = \frac{1}{2}\ mu^2 + \frac{1}{2}\ mu^2 + \frac{1}{2}\ kx^2$$

or $$v^2 = 2u^2 + \frac{kx^2}{m} = 2\left(\frac{v}{2}\right)^2 + \frac{kx^2}{m}$$

or $$\frac{kx^2}{m} = \frac{v^2}{2}$$

or $$x = v\sqrt{\frac{m}{2k}} \qquad \text{(i)}$$

The kinetic energy is equally shared between masses A and B and the spring is at maximum compression, i.e.

$$\frac{1}{2}\ kx^2 = \frac{1}{2}\ mu^2 + \frac{1}{2}\ mu^2 = mu^2$$

Thus, kinetic energy of $A - B$ at maximum compression $mu^2 = m\left(\frac{v}{2}\right)^2 = \frac{mv^2}{4}$.

Using (i) notice that $\frac{1}{2}\ kx^2 = \frac{mv^2}{4}$. Hence the correct choice is (b).

94. We can write, $4\cos^2\left(\frac{t}{2}\right) = 2 \times 2\cos^2\left(\frac{t}{2}\right)$

$= 2 \times (1 + \cos t)$. Therefore,

$$y = 2 \times (1 + \cos t) \times \sin(1000\ t)$$

$$= 2 \sin(1000\ t) + 2 \cos t \sin(1000\ t)$$

$$= 2\sin(1000\,t) + \sin(1001\,t) + \sin(999\,t)$$

Thus y is a superposition of three simple harmonic motions of angular frequencies 999, 1000 and 1001 rad s^{-1}. Hence the correct choice is (b). But a superposition of two or more simple harmonic motions of different frequency does not produce a simple harmonic motion. The statement of the question is incorrect.

95. If the wire extends by an amount x when a force F is applied to it, then

$$Y = \frac{F/A}{x/L} = \frac{FL}{Ax} \text{ or } F = \left(\frac{YA}{L}\right)x$$

$$= kx;\ k = \frac{YA}{L}$$

Thus the force constant of the wire is k. If K is the force constant of the spring, then the force constant of the series combination of the wire and the spring is given by

$$\frac{1}{k'} = \frac{1}{k} + \frac{1}{K}$$

or
$$k' = \frac{kK}{(k+K)} = \frac{\frac{YA}{L} \times K}{\left(\frac{YA}{L} + K\right)}$$

$$= \frac{YAK}{(YA + KL)} \qquad \text{(i)}$$

The time period of the combination is

$$T' = 2\pi\sqrt{\frac{m}{k'}} \qquad \text{(ii)}$$

Using (i) in (ii) we find that the correct choice is (b).

96. Let the force F produce a deformation x. When this force is withdrawn, the force that tries to restore block A to its equilibrium position is proportional to x and is given by

$$f = -\eta L x$$

$$\therefore \quad \text{Acceleration } \frac{d^2x}{dt^2} = \frac{f}{M} = -\frac{\eta L}{M}x = -\omega^2 x$$

where $\omega = \sqrt{\frac{\eta L}{M}}$. The angular frequency of this simple harmonic frequency is ω. Now $T = \frac{2\pi}{\omega}$.

Therefore

$$T = 2\pi\sqrt{\frac{M}{\eta L}}$$

97. The motion of the particle is simple harmonic. The particle is at one extreme position $x = 0$ at $t = 0$ and at the other extreme position at $x = 1$ unit at $t = 1$s. Therefore, the time period of the motion is $T = 2$ s and its amplitude $A = \frac{1}{2}$ unit. Therefore, the angular frequency is $\omega = 2\pi/T = 2\pi/2 = \pi$ rad s^{-1}. The maximum acceleration is $\alpha_{max} = \omega^2 A = (\pi)^2 \times \frac{1}{2} \simeq 4.93$. Thus, the value of α can exceed 2 and can be greater than 4 at some points in the path of the particle. Further, in simple harmonic motion, α changes sign and cannot remain positive in the time interval $0 \le t \le 1$ s. Hence the correct choice is (c).

98. Figure 10.53 shows the plot of $U(x)$ versus x.

At $x = 0$, potential energy $U(0) = k[1 - \exp(0)] = k(1 - 1) = 0$ and it has a maximum value $= k$ at $x = \pm\infty$ since

$$U(\pm\infty) = k[1 - \exp(-\pm\infty)^2] = k(1 - 0) = k$$

Since the total mechanical energy has a constant value $= (k/2)$, the kinetic energy will be maximum at $x = 0$ and minimum at $x = \pm\infty$. At $x = 0$

$$\left(\frac{dU}{dx}\right)_{x=0} = [2kx \exp(-x^2)]_{\text{at } x=0} = 0$$

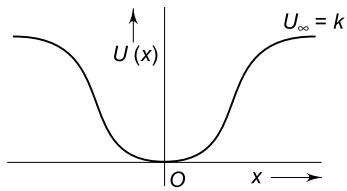


Fig. 10.53

Hence the particle is in stable equilibrium at $x = 0$ (origin) and would oscillate about $x = 0$ (for small displacements) simple harmonically. Hence (d) is the only correct choice.

99. The acceleration of the vehicle down the plane is $g \sin\alpha$. The reaction force acting on the pendulum bob gives it an acceleration $a = g \sin\alpha$ up the plane. This acceleration has two rectangular components $a_x = a\cos\alpha = g\sin\alpha\cos\alpha$ and $\alpha_y = a\sin\alpha = g\sin^2\alpha$ as shown in Fig. 10.54.

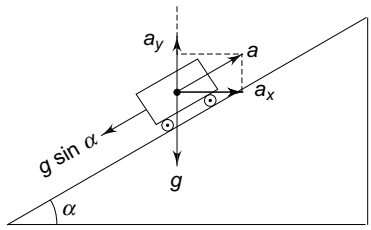


Fig. 10.54

The effective acceleration due to gravity acting on the bob is given by

$$g_{\text{eff}}^2 = a_x^2 + (g - a_y)^2$$
$$= a_x^2 + g^2 + a_y^2 - 2ga_y$$
$$= g^2 \sin^2 \alpha \cos^2 \alpha + g^2 + g^2 \sin^4 \alpha - 2g^2 \sin^2 \alpha$$
$$= g^2 \sin^2 \alpha (\cos^2 \alpha + \sin^2 \alpha) + g^2 - 2g^2 \sin^2 \alpha$$
$$= g^2 (1 - \sin^2 \alpha) = g^2 \cos^2 \alpha$$

or $g_{\text{eff}} = g \cos \alpha$

Now $T = 2\pi \sqrt{\dfrac{L}{g_{\text{eff}}}} = 2\pi \sqrt{\dfrac{L}{g \cos \alpha}}$

Hence the correct choice is (a).

100. The force constant of a spring is inversely proportional to its length. If a spring of length L is cut into two pieces of lengths x and $(L - x)$, such that

$$x = 2(L - x) \text{ or } x = \frac{2L}{3},$$

then the force constant of the spring of length x is related to the force constant k of the complete spring of length L as

$$\frac{k_1}{k} = \frac{L}{x} = \frac{L}{2L/3} = \frac{3}{2} \text{ or } k_1 = \frac{3}{2} k$$

which is choice (b).

101. To form a stationary wave, waves y and y' must travel in opposite directions. Wave $y = a \cos (kx - \omega t)$ travels along the positive x-direction. Waves $y' = -a \cos (kx - \omega t)$ and $y' = -a \sin (kx - \omega t)$ in choices (b) and (d) travel along positive x-direction. Hence choices (b) and (d) are not possible. Choice (a) is also incorrect because at $x = 0$

$$y' = a \sin \omega t \text{ and } y = a \cos (-\omega t) = a \cos \omega t$$

Therefore, the resultant displacement at $x = 0$ which is $y + y' = a \sin \omega t + a \cos \omega t$ is not zero, i.e. these waves do not produce a node at $x = 0$. Choice (c) is correct because at $x = 0$, $y + y' = 0$.

102. Because the rope has a finite mass, the tension in the rope is different at different points on the rope. At the top where the rope is rigidly fixed, the tension = weight of the rope + the weight attached to the free end of the rope = 6 kg + 2 kg = 8 kg wt. Tension at the free end of the rope = 2 kg wt.

Since $v = \sqrt{\dfrac{T}{m}}$, if the tension becomes 4 times, the velocity of the pulse is doubled. Since the frequency cannot change and $\lambda = v/\nu$, if v is doubled, λ is also doubled. Hence the correct choice is (c).

103. For a closed tube $\nu = \dfrac{v}{4L}$. For an open tube $\nu' = \dfrac{v}{2L}$.

Hence $\nu' = 2v = 2 \times 512 = 1024$ Hz. Thus the correct choice is (a).

104. When two waves of amplitudes a_1 and a_2 superpose to produce beats, the resultant amplitude of the maxima of intensity is

$$A = a_1 + a_2$$

Now, intensity $\propto$ (amplitude)^2. Since the two waves have the same intensity, their amplitudes are equal, i.e. $a_1 = a_2 = a$. Thus $A = 2a$. Therefore, $A^2 = 4a^2$ or $I_{\max} = 4I$. hence the correct choice is (b).

105. Distance between adjacent nodes (or antinodes) = $\lambda/2$. Also

$\dfrac{2\pi}{\lambda}$ = coefficient of x in the argument of the sine function = k

or $\lambda = \dfrac{2\pi}{k}$. Hence, the distance between adjacent nodes = $\dfrac{\pi}{k}$.

106. If the train is going away from the observer, the apparent frequency is

$$\nu_1 = \frac{\nu v}{v + u} = \frac{\nu}{1 + \dfrac{u}{v}} \quad \text{(i)}$$

It is observed that $\nu = 1.2\ \nu_1$. In the second case, the apparent frequency is

$$\nu_2 = \frac{\nu(v - u)}{v} = \nu\left(1 - \frac{u}{v}\right)$$

or $$\frac{\nu}{\nu_2} = \frac{1}{\left(1 - \dfrac{u}{v}\right)} \quad \text{(ii)}$$

Now, from (i) we have $\dfrac{\nu}{\nu_1} = 1 + \dfrac{u}{v}$ or $1.2 = 1 + \dfrac{u}{v}$,

or $u = 0.2\ v$ or $\dfrac{u}{v} = 0.2$. Using this in (ii), we get

$\dfrac{\nu}{\nu_2} = \dfrac{5}{4} = 1.25$. Hence the correct choice is (b).

107. For pipe P_1 : $\nu_1 = \dfrac{v}{4L_1}$

For pipe P_2 : $\nu_3 = \dfrac{3v}{2L_2}$

It is given that $\nu_1 = \nu_3$. Therefore, $L_1/L_2 = 1/6$ which is choice (b).

108. The speed of the wave in the string is given by

$$v = \sqrt{\frac{T}{m}}$$

According to Hooke's law, tension (T) $\propto$ extension (x). Hence $v \propto \sqrt{x}$. Therefore

$$\frac{v'}{v} = \frac{\sqrt{1.5x}}{\sqrt{x}} = \sqrt{1.5} = 1.22$$

Hence the correct choice is (a).

109. Particle velocity $V = \frac{dy}{dt} = \frac{d}{dt}$ $[A \sin (kx - \omega t)] = A\omega \cos (kx - \omega t)$

Hence $V_{max} = A\omega$, which is choice (a).

110. Standing waves are formed on the string. Particle displacements are given by

$$y = a \sin\left(\frac{2\pi x}{\lambda}\right) \cos (2\pi ft)$$

Particle velocity $V = \frac{dy}{dt}$

$$= (-2\pi fa) \sin\left(\frac{2\pi x}{\lambda}\right) \sin (2\pi ft)$$

$\therefore$ $(V)_{max} = 2\pi fa$

Given $(V)_{max} = \frac{v}{10}$. Therefore,

$$2\pi fa = \frac{v}{10} \text{ or } f = \frac{v}{20\pi a}$$

$$= \frac{10\ \text{ms}^{-1}}{20\pi \times 10^{-3}\ \text{m}} = \frac{10^3}{2\pi}\ \text{Hz}$$

Now $\lambda = \frac{v}{f} = \frac{10\ \text{ms}^{-1}}{10^{-3}/2\pi\ \text{Hz}} = 2\pi \times 10^{-2}\ \text{m}$

Hence the correct choice is (a).

111. For a gas, adiabatic elasticity $E = \gamma P$ where $\gamma = C_p/C_v$ and P is the pressure. The speed of sound in the gas is given by

$$v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

where M is the molecular mass and R is the gas constant. Thus (since T is the same)

$$\frac{v_N}{v_{He}} = \sqrt{\frac{M_{He}}{M_N}} = \sqrt{\frac{4}{28}} = \sqrt{\frac{1}{7}}$$

112. Frequency of the fundamental mode is given by

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}};$$

T = tension and m = mass per unit length

$\therefore$ $\frac{\nu_1}{\nu_2} = \frac{(1/2L)\sqrt{T/m_1}}{(1/4L)\sqrt{T/m_2}} = 2\sqrt{\frac{m_2}{m_1}}$

Now $m_1 = \frac{M_1}{L} = \frac{\pi(2r)^2 L\rho}{L} = 4\pi r^2\rho$

and $m_2 = \frac{M_2}{2L} = \frac{\pi(r)^2(2L)\rho}{2L} = \pi r^2\rho$

Hence $\frac{\nu_1}{\nu_2} = 2 \times \sqrt{\frac{\pi r^2\rho}{4\pi r^2\rho}} = 1$

113. In simple harmonic motion, the speed of the particle is the maximum at the mean position $x = 0$, decreases as it moves towards the extreme position becoming zero at the extreme position $x = A$. Hence the particle will take shorter time to move from $x = 0$ to $x = \frac{A}{2}$ than to move from $x = \frac{A}{2}$ to $x = A$. Thus, the correct choice is (a).

114. Given $y_1 = A \sin\left(\frac{\pi x}{L}\right) \sin \omega t = A(x) \sin \omega t$. If m is the mass of the string, its energy is

$$E_1 = \frac{1}{2} m \omega^2 \int_0^L A^2(x)\, dx$$

$$= \frac{1}{2} m \omega^2 A^2 \int_0^L \sin^2\left(\frac{\pi x}{L}\right) dx$$

$$= \frac{1}{2} m \omega^2 A^2 \int_0^L \frac{1}{2}\left\{1 - \cos\left(\frac{2\pi x}{L}\right)\right\} dx$$

$$= \frac{1}{4} m \omega^2 A^2 \left| x - \frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\right|_0^L$$

$$= \frac{1}{4} m \omega^2 A^2 (L - 0) = \frac{1}{4} m \omega^2 A^2 L$$

Similarly, $E_2 = \frac{1}{2} m(2\omega)^2 A^2 \int_0^L \sin^2\left(\frac{2\pi x}{L}\right) dx$

$$= 2m \omega^2 A^2 \int_0^L \frac{1}{2}\left\{1 - \cos\left(\frac{4\pi x}{L}\right)\right\} dx$$

$$= m \omega^2 A^2 L$$

$\therefore$ $\frac{E_1}{E_2} = \frac{1}{4}$ or $E_2 = 4E_1$, which is choice (c).

115. For train A: $\nu_A = \nu\left(\frac{v+u_A}{v}\right)$ or $5.5 = 5\left(\frac{v+u_A}{v}\right)$

$\Rightarrow u_A = 0.1\ v$

For train B: $\nu_B = \nu\left(\frac{v+u_B}{v}\right)$ or $6.0 = 5\left(\frac{v+u_B}{v}\right)$

$\Rightarrow u_B = v/5 = 0.2\ v$

$\therefore \quad \frac{u_B}{u_A} = \frac{0.2v}{0.1v} = 2$, which is choice (b).

116. Let L be the length of the wire between the bridges and let m be the mass per unit length of the wire.

Five antinodes on a length L implies that $L = \frac{5}{2}\ \lambda_1$ or $\lambda_1 = \frac{2L}{5}$. Thus in this case we have

$$\nu_1 = \frac{v_1}{\lambda_1} = \frac{5}{2L}\sqrt{\frac{T_1}{m}}, \text{ where } T_1 = M_1 g$$

Three antinodes on a length L implies that $L = \frac{3}{2}\ \lambda_2$ or $\lambda_2 = \frac{2L}{3}$. In this case, we have

$$\nu_2 = \frac{v_2}{\lambda_2} = \frac{3}{2L}\sqrt{\frac{T_2}{m}}, \text{ where } T_2 = M_2 g$$

Given $\nu_1 = \nu_2$, $M_1 = 9$ kg and $M_2 = M$. Therefore, we have

$$\frac{5}{2L}\sqrt{\frac{9g}{m}} = \frac{3}{2L}\sqrt{\frac{Mg}{m}}$$

which gives $M = 25$ kg. Hence the correct choice is (a).

117. Velocity of the body at time t is

$$v = \frac{dx}{dt}$$
$$= (6 \times 100)\cos\left(100t + \frac{\pi}{4}\right) \text{ cm s}^{-1}$$

$\therefore \quad v_{max} = 600 \text{ cm s}^{-1} = 6 \text{ ms}^{-1}$

$$\text{Maximum KE} = \frac{1}{2} m v^2_{max} = \frac{1}{2} \times 1 \times (6)^2 = 18 \text{ J}$$

Hence the correct choice is (b).

118. Beat frequency $\nu_b = \frac{30}{3} = 10$ per second. Now

$$\nu_b = \nu_1 - \nu_2 = \frac{v}{\lambda_1} - \frac{v}{\lambda_2}$$

or $$v = \frac{\lambda_1 \lambda_2 \nu_b}{(\lambda_1 - \lambda_2)} = \frac{5 \times 6 \times 10}{(6-5)} = 300 \text{ ms}^{-1}$$

Hence the correct choice is (a).

119. Since the particles start from $x = 0$ and have the same amplitude but different time periods, they will meet again at $x = 0$ where their velocities are maximum equal to $a\omega_1$ and $a\omega_2$, i.e.

$$\frac{v_1}{v_2} = \frac{\omega_1}{\omega_2} = \frac{2\pi}{T_1} \times \frac{T_2}{2\pi} = \frac{T_2}{T_1} = \frac{6}{3} = 2$$

Hence the correct choice is (b).

120. $$E_1 = \frac{1}{2}\ m\omega^2 x^2 \quad \text{or} \quad \sqrt{E_1} = x\sqrt{\frac{1}{2}m\omega^2} \qquad (1)$$

$$E_2 = \frac{1}{2}\ m\omega^2 y^2 \quad \text{or} \quad \sqrt{E_2} = y\sqrt{\frac{1}{2}m\omega^2} \qquad (2)$$

$$E = \frac{1}{2}\ m\omega^2 (x+y)^2$$

$$\text{or} \quad \sqrt{E} = (x+y)\sqrt{\frac{1}{2}m\omega^2} \qquad (3)$$

From (1), (2) and (3) it follows that

$$\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$$

or $$E = E_1 + E_2 + 2\sqrt{E_1 E_2}$$

which is choice (c).

121. Let the frequency of fork C be n. then $n_A = n + 0.03\ n = 1.03\ n$ and $n_B = n - 0.02\ n = 0.98\ n$. The beat frequency is

$$n_b = n_A - n_B$$

or $$5 = 1.03\ n - 0.98\ n = 0.05\ n$$

which gives $n = 100$ Hz. Hence $n_A = 1.03 \times 100 = 103$ Hz which is choice (c).

122. Let the body be displaced by a distance x. If the restoring force is F_1, then the angular frequency of the resulting simple harmonic motion is given by

$$\omega_1^2 = \frac{K}{m} = \frac{Kx}{mx} = \frac{F_1}{mx} \qquad \text{(i)}$$

where m is the mass of the body. For force F_2, we have

$$\omega_2^2 = \frac{F_2}{m\,x} \qquad \text{(ii)}$$

If F_1 and F_2 act simultaneously, then

$$\omega^2 = \frac{F_1 + F_2}{mx} \qquad \text{(iii)}$$

From (i), (ii) and (iii) we get

$$\omega^2 = \omega_1^2 + \omega_2^2$$

or $$\left(\frac{2\pi}{T}\right)^2 = \left(\frac{2\pi}{T_1}\right)^2 + \left(\frac{2\pi}{T_2}\right)^2$$

or $$\frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2} = \frac{1}{\left(\frac{4}{5}\right)^2} + \frac{1}{\left(\frac{3}{5}\right)^2}$$

or $$\frac{1}{T^2} = \frac{25}{16} + \frac{25}{9} = \frac{25 \times 25}{16 \times 9} = \frac{25 \times 25}{144}$$

or $$T = \sqrt{\frac{144}{25 \times 25}} = \frac{12}{25} \text{ s},$$

which is choice (a).

123. Given $4v^2 = 25 - x^2$. Differentiating with respect to time t, we have

$$8v \frac{dv}{dt} = 0 - 2x \frac{dx}{dt}$$

or $$8va = -2x\, v \qquad \left(\because a = \frac{dv}{dt}, v = \frac{dx}{dt}\right)$$

where a is the acceleration. Thus

$$a = -\left(\frac{1}{4}\right)x \qquad \text{(i)}$$

For a simple harmonic motion

$$a = -\omega^2 x \qquad \text{(ii)}$$

Comparing (i) and (ii), we have

$$\omega = \frac{1}{2} \quad \text{or} \quad \frac{2\pi}{T} = \frac{1}{2}$$

or $T = 4\pi$, which is choice (c).

124. Since the particle is at the mean position at $t = 0$, the phase of the motion is zero. Therefore, the displacement of the particle is given by

$$x = A \sin(\omega t) = A \sin\left(\frac{2\pi t}{T}\right)$$

$$= A \sin\left(\frac{\pi t}{4}\right) \qquad (\because T = 8 \text{ s}) \quad \text{(i)}$$

The distance travelled in the first second is [put $t = 1$ s in (i)]

$$x_1 = A \sin\left(\frac{\pi}{4}\right) = \frac{A}{\sqrt{2}}$$

The distance travelled in first two seconds is [put $t = 2$ s in (i)]

$$x' = A \sin\left(\frac{\pi}{2}\right) = A$$

$\therefore$ Distance travelled in the IInd second is

$$x_2 = x' - x_1 = A - \frac{A}{\sqrt{2}} = A\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\therefore \quad \frac{x_1}{x_2} = \frac{\frac{A}{\sqrt{2}}}{A\left(1 - \frac{1}{\sqrt{2}}\right)} = \frac{1}{\sqrt{2} - 1}$$

Hence the correct choice is (c).

125. The apparent shift in the frequency is given by

$$\Delta\nu = \frac{2u\nu}{c}$$

or $$u = \frac{c\Delta\nu}{2\nu} = \frac{(3 \times 10^8) \times (3 \times 10^3)}{2 \times (9 \times 10^9)}$$

$$= 50 \text{ ms}^{-1}$$

Hence the correct choice is (c).

126. Original time period is

$$T_1 = 2\pi \sqrt{\frac{l}{g}} \qquad (1)$$

When the pendulum is moving upwards, the effective value of g is

$$g_{\text{eff}} = g + a$$

where a is the acceleration of the pendulum which is given by

$$a = \frac{dv}{dt} = \frac{d}{dt}(Kt) = K = 4.4 \text{ ms}^{-2}$$

$$\therefore \; g_{\text{eff}} = g + a = 10 + 4.4 = 14.4 \text{ ms}^{-2}.$$

Therefore, the new time period is

$$T_2 = 2\pi \sqrt{\frac{l}{g_{\text{eff}}}} \qquad (2)$$

From (1) and (2), we get

$$\frac{T_2}{T_1} = \sqrt{\frac{g}{g_{\text{eff}}}} = \sqrt{\frac{10}{14.4}} = \frac{1}{1.2}$$

or $$T_2 = \frac{T_1}{1.2} = \frac{3}{1.2} = 2.5 \text{ s}$$

Hence the correct choice is (c).

127. Refer to Fig. 10.55. Let the mass of the block be m and let, at a certain instant of time, the direction of acceleration a of the table (executing simple harmonic motion) be along the positive x-direction. As a result, the block will experience a force ma directed along the negative x-axis. Consequently, the force of friction μmg will act along the positive x-axis. The weight mg of the block will be balanced by the normal reaction R. The block will not slip on

the surface of the table, if the acceleration a of the motion of the table is such that

$$\mu mg \geq ma \text{ or } \mu g \geq a$$

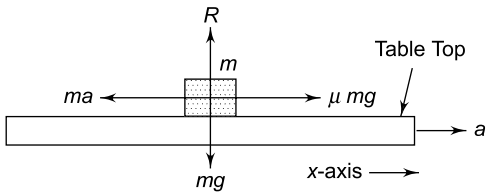


Fig. 10.55

Therefore, for no slipping, the table can have a maximum acceleration $a_{max} = \mu g$. We know that, for a simple harmonic motion, $a_{max} = \omega^2 A$, where ω is the angular frequency and A the amplitude of the motion of the table. Therefore, the maximum amplitude is given by

$$\omega^2 A_{max} = \mu g$$

$$\text{or} \qquad A_{max} = \frac{\mu g}{\omega^2} = \frac{\mu g T^2}{4\pi^2},$$

which is choice (c).

128. The angular frequency of the system is

$$\omega = \left[\frac{k}{(M+m)}\right]^{1/2} \tag{1}$$

The upper block of mass m will not slip over the lower block of mass M if the maximum force on the upper block f_{max} does not exceed the frictional force μmg between the two blocks. Now

$$f_{max} = ma_{max} = m\omega^2 A_{max} \tag{2}$$

where a_{max} is the maximum acceleration and A_{max} is the maximum amplitude. Using (1) in (2), we get

$$f_{max} = \frac{mk\, A_{max}}{(M+m)}$$

For no slipping, $f_{max} = \mu mg$

$$\text{or} \quad \frac{mk\, A_{max}}{(M+m)} = \mu mg \text{ or } A_{max} = \frac{\mu(M+m)g}{k},$$

which is choice (c).

129. Given $y = t^2$. The velocity of the lift varies with t as

$$v = \frac{dy}{dt} = 2t$$

$\therefore$ Acceleration $a = \dfrac{dv}{dt} = 2 \text{ ms}^{-2}$, directed upwards, Hence

$$T' = 2\pi\sqrt{\frac{l}{g+a}}$$

and $$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\therefore \qquad \frac{T'}{T} = \sqrt{\frac{g}{g+a}} = \sqrt{\frac{10}{(10+2)}} = \sqrt{\frac{5}{6}}$$

The correct choice is (b)

130. The density ρ of a wire of mass M, length L and diameter d is given by

$$\rho = \frac{4M}{\pi d^2 L} = \frac{4m}{\pi d^2}$$

Now $$v_A = \sqrt{\frac{T}{m_A}} \quad \text{and} \quad v_B = \sqrt{\frac{T}{m_B}}$$

$$\therefore \qquad \frac{v_A}{v_B} = \sqrt{\frac{m_B}{m_A}} = \frac{d_B}{d_A}$$

but $v_A = n\lambda_A$ and $v_B = n\lambda_B$, n being the frequency of the source.

Hence $$\frac{\lambda_A}{\lambda_B} = \frac{v_A}{v_B} = \frac{d_B}{d_A} = \frac{0.5\times10^{-3}}{10^{-3}} = 0.5$$

which is choice (a).

131. The frequency of the nth hormonic is 420 Hz. The frequency of the $(n+1)$th harmonic is 490 Hz. Therefore,

$$420 = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

and $$490 = \frac{(n+1)}{2L}\sqrt{\frac{T}{m}}$$

Dividing, we have

$$\frac{490}{420} = \frac{(n+1)}{n} \text{ which gives } n = 6.$$

132. Let L be the original length of the wire. The contraction l in length when the temperature falls by ΔT is given by

$$l = L\alpha\Delta T$$

Strain in wire is $\dfrac{l}{L} = \alpha\Delta T$ (1)

Since the wire is rigidly fixed at its ends, it cannot contract, producing a tension T in the wire which caused the strain l/L. If A is the cross-section area of the wire, we have

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{T/A}{l/L} = \frac{T\,L}{Al} \text{ or } T = \frac{YAl}{L} \tag{2}$$

The mass per unit length of the wire is

$$m = A\rho \tag{3}$$

The speed of transverse wave is given by

$$v = \sqrt{\frac{T}{m}} \tag{4}$$

Using (1), (2) and (3) in (4) we get

$$v = \sqrt{\frac{Y\alpha\Delta T}{\rho}}$$, which is choice (a).

133. The fundamental frequency of the vibration of a wire of length L, mass m per unit length and under tension T is given by

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} \tag{1}$$

If the tension is increased by 44%, the new tension is

$$T' = T + \frac{44}{100} \times T = T + 0.44\ T$$
$$= 1.44\ T$$

Since L is kept constant, the new fundamental frequency is

$$\nu' = \frac{1}{2L}\sqrt{\frac{T'}{m}} = \frac{1}{2L}\sqrt{\frac{1.44T}{m}} = \frac{1.2}{2L}\sqrt{\frac{T}{m}}$$

Comparing this with Eq. (1), we have

$$\nu' = 1.2\ \nu$$

Given $\nu' - \nu = 6$ Hz. Hence $\nu + 6 = 1.2\ \nu$

which gives $\nu = \frac{6}{0.2} = 30$ Hz, which is choice (b).

134. A travelling wave is characterized by wave functions of the type $y = f(vt + x)$ or $y = f(vt - x)$ where f stands for sine or cosine function. Hence the correct choice is (d). Choices (a), (b) and (c) represent a stationary or standing wave.

135. If the length is increased by 20%, the new length is

$$L' = L + \frac{20L}{100} = 1.2\ L$$

The original frequency is

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

Since T and m are the same, the new frequency will be

$$\nu' = \frac{1}{2L'}\sqrt{\frac{T}{m}} = \frac{1}{1.2 \times 2L}\sqrt{\frac{T}{m}} = \frac{\nu}{1.2}$$

Now $\nu = 30$ Hz. Therefore, $\nu' = \frac{30}{1.2} = 25$ Hz. Thus, the frequency decreases by 5 Hz. Hence the correct choice is (a).

136. For a closed pipe, $\nu_n = \frac{nv}{4L}$; $n = 1, 3, 5, \ldots$ etc

For an open pipe, $\nu'_n = \frac{nv}{2L}$; $n = 1, 2, 3, \ldots$ etc

where $v = \sqrt{\frac{\gamma P}{\rho}}$.

For the closed pipe vibrating in the first harmonic ($n = 1$), we have

$$\nu_1 = \frac{1}{4L_1}\sqrt{\frac{\gamma P}{\rho_1}} \tag{1}$$

For the closed pipe vibrating in the third harmonic ($n = 3$), we have

$$\nu'_3 = \frac{3}{2L_2}\sqrt{\frac{\gamma P}{\rho_2}} \tag{2}$$

Given $\nu_1 = \nu'_3$. Equating (1) and (2), we get

$$\frac{L_1}{L_2} = \frac{1}{6}\sqrt{\frac{\rho_2}{\rho_1}}\ .$$

Hence the correct choice is (d).

137. The frequency of sound is a characteristic of its source. Hence frequency of sound is the same in air as is water. Therefore, the observer in air will receive a sound of frequency 600 Hz. If λ_a and λ_w are the wavelengths in air and water resptively, then

$$\nu = \frac{v_a}{\lambda_a} = \frac{v_w}{\lambda_w}$$

which gives $\lambda_a = \frac{v_a}{\nu} = \frac{300}{600} = 0.5$ m

The wavelength is water is

$$\lambda_w = \frac{v_w}{\nu} = \frac{1500}{600} = 2.5 \text{ m}$$

Hence the correct choice is (a).

138. Let T_1 and T_2 be the tensions in wires 1 and 2 respectively. Let m be the mass per unit length of each wire and let l be the length of each wire. Given

$$\nu = \frac{1}{2l}\sqrt{\frac{T_1}{2m}} = \frac{2}{2l}\sqrt{\frac{T_2}{m}}$$

which gives $\frac{T_1}{T_2} = 4$

For rotational equilibrium of the rod about O, we have

$$T_1 \times AO = T_2 \times BO \text{ or } T_1 \times x = T_2 (L - x),$$

which gives $\frac{T_1}{T_2} = \frac{(L-x)}{x}$. But $\frac{T_1}{T_2} = 4$. Hence

$$4 = \frac{(L-x)}{x}$$

which gives $x = \frac{L}{5}$. Hence the correct choice is (a).

139. When the source moves towards from the stationary observer, its apparent frequency is

$$\nu' = \nu \times \frac{v}{(v-u)} \quad (1)$$

When the source moves away from the stationary observer, its apparent frequency is

$$\nu'' = \nu \times \frac{v}{(v+u)} \quad (2)$$

When both the forks are moving relative to stationary observer, the number of beats heard by him per second = $\nu' - \nu''$

Since $u_s << v$, Eqs (1) and (2) may be simplified as follows.

$$\nu' = \nu \left(1 - \frac{u}{v}\right)^{-1} \cong \nu \left(1 + \frac{u}{v}\right)$$

$$\nu'' = \nu \left(1 + \frac{u}{v}\right)^{-1} \cong \nu \left(1 - \frac{u}{v}\right)$$

where terms of order u^2/v^2 have been neglected in the binomial expansion. Thus,

$$\nu' - \nu'' = \nu \left(1 + \frac{u}{v}\right) - \nu \left(1 - \frac{u}{v}\right) = 2\nu \frac{u}{v}$$

Thus the correct choice is (c)

140. Particle velocity

$$V = \frac{dy}{dt} = \frac{d}{dt}\left[4 \sin\left(\frac{\pi x}{15}\right) \cos(96\pi t)\right]$$

$$= -4 \times 96\pi \sin\left(\frac{\pi x}{15}\right) \sin(96\pi t)$$

At $x = 7.5$ cm and $t = 0.25$ s, the particle velocity is

$$V = -4 \times 96\pi \sin\left(\frac{\pi \times 7.5}{15}\right) \sin(96\pi \times 0.25)$$

$$= -(4 \times 96) \sin(0.5\pi) \sin(24\pi)$$

$$= 0 \qquad [\because \sin(24\pi) = 0]$$

The correct choice is (a).

141. The tension at the lower end of the rope is $T = mg$. The speed of pulse at this point is $v = \sqrt{\frac{T}{\mu}}$, μ = mass per unit length of the rope. If ν is the frequency of the pulse, then

$$\lambda = \frac{v}{\nu} = \frac{1}{\nu}\sqrt{\frac{T}{\mu}} \quad (1)$$

The tension at the upper end of the rope is $T' = (m + M)\, g$. Let λ' be the wavelength of the pulse when it reaches the upper end of the rope. Then, since frequency ν of the pulse remains the same,

$$\lambda' = \frac{1}{\nu}\sqrt{\frac{T'}{\mu}} \quad (2)$$

From Eqs. (1) and (2), we have

$$\frac{\lambda'}{\lambda} = \sqrt{\frac{T'}{T}} = \sqrt{\frac{(m+M)g}{mg}} = \sqrt{\frac{M+m}{m}}$$

Hence the correct choice is (b).

142. Let u_m = speed of motorist and u_b = speed of the band. The motorist will hear two sounds—one coming directly from the band and the other reflected from the wall. The apparent frequency of the direct sound is given by (the observer is approaching a receding source of sound)

$$f_1 = f\left(\frac{v + u_m}{v + u_b}\right) = f \qquad (\because u_m = u_b = u)$$

The reflected sound can be regarded as coming from the mirror image which is approaching the motorist with a speed v_b. Hence, the apparent frequency of the reflected sound is given by (the observer and the source of sound are both approaching each other)

$$f_2 = f = \left(\frac{v + u_m}{v - u_b}\right) = f\left(\frac{v+u}{v-u}\right)$$

$$\therefore \quad \text{Beat frequency } (= f_2 - f_1) = f\left(\frac{v+u}{v-u}\right) - f$$

$$= \left(\frac{2fu}{v-u}\right)$$

Thus the correct choice is (d).

143. The total energy of the oscillator is constant. At the extreme position ($x=A$), the energy is entirely potential, *i.e.*,

$$E = \text{KE} + \text{P.E.} = 0 + \frac{1}{2}kA^2 = \frac{1}{2}kA^2 \qquad (1)$$

The total energy of the oscillator at position x is

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \qquad (2)$$

where v is the speed of the oscillator at position x. Equaling (1) and (2) we get

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$\Rightarrow \quad v = \sqrt{\frac{k(A^2 - x^2)}{m}}\text{, which is choice (c).}$$

144. The kinetic energy $\frac{1}{2}mv^2$ given to the block at the equilibrium position (where its P.E. = 0) is completely converted into potential energy $= \frac{1}{2}kA^2$ when the block reaches the extreme position (where its K.E. = 0) and its displacement $x = A$, the amplitude of oscillations).

Hence

$$\frac{1}{2}mv^2 = \frac{1}{2}kA^2$$

$$\Rightarrow \quad A = \left[\frac{mv^2}{k}\right]^{1/2}$$

$$= \left[\frac{3\times(2)^2}{300}\right]^{1/2}$$

$$= 0.2 \text{ m} = 20 \text{ cm}$$

So the correct choice is (d).

145. Let V be the velocity of the block + bullet immediately after the collision. From conservation of linear momentum,

$$mv = (M+m)\,V$$

$$\Rightarrow \quad V = \frac{mv}{(M+m)}$$

When the block + bullet system is at the extreme position ($x = A$), all the kinetic energy is transformed into potential energy, *i.e.*,

$$\frac{1}{2}(M+m)V^2 = \frac{1}{2}kA^2$$

$$\Rightarrow \quad (M+m)\times\left[\frac{mv}{(M+m)}\right]^2 = kA^2$$

$$\Rightarrow \quad A = \frac{mv}{\sqrt{k(M+m)}}$$

So the correct choice is (a).

146. If v_1 is the velocity of the block at the instant when it is at $x = A/2$, then from the principle of conservation of energy, we have

$$\frac{1}{2}Mv_1^2 + \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{2}kA^2$$

$$\Rightarrow \quad \frac{1}{2}Mv_1^2 = \frac{1}{2}\times\frac{3}{4}kA^2$$

$$\Rightarrow \quad v_1 = A\sqrt{\frac{3k}{4M}} \qquad (1)$$

Since the ball falls vertically, it does not impart any horizontal momentum to the block.

Hence the horizontal momentum is conserved, *i.e.*

Horizontal momentum of the block before the ball hits the block = horizontal momentum of the block + ball system after the ball sticks to the block, *i.e.*

$$Mv_1 = (M+m)v_2$$

$$\Rightarrow \quad v_2 = \frac{Mv_1}{M+m} \qquad (2)$$

$$\Rightarrow \quad \frac{v_2}{v_1} = \frac{M}{M+m}\text{, which is choice (c)}$$

147. Using (1) in (2), we get

$$v_2 = \frac{M}{(M+m)}\times A\sqrt{\frac{3k}{4M}}$$

$$\Rightarrow \quad v_2 = \frac{A}{(M+m)}\times\sqrt{\frac{3kM}{4}} \qquad (3)$$

If A' is the new amplitude, then from the conservation of energy, we have

$$\frac{1}{2}(M+m)v_2^2 + \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{2}kA'^2$$

Using (3), we have

$$\frac{1}{2}(M+m)\frac{A^2}{(M+m)^2}\times\frac{3kM}{4} + \frac{1}{2}k\frac{A^2}{4} = \frac{1}{2}kA'^2$$

$$\Rightarrow \qquad A' = A\left[\frac{3M}{4(M+m)} + 1\right]^{1/2}$$

Putting $M = 2m$, we get

$$A' = \sqrt{\frac{3}{4}}A \text{ , which is choice (a)}$$

148. The potential energy of the oscillator an time t is given by

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \sin^2(\omega t + \phi)$$

The average potential energy from $t = 0$ to $t = T$ is

$$U_{av} = \frac{\int_0^T U dt}{\int_0^T dt} = \frac{1}{T}\int_0^T \frac{1}{2}kA^2 \sin^2(\omega t + \phi)\, dt$$

$$= \frac{kA^2}{2T}\int_0^T \sin^2(\omega t + \phi)\, dt$$

$$= \frac{kA^2}{4T}\int_0^T [1 - \cos(2\omega t + \phi)]\, dt$$

$$= \frac{kA^2}{4T}(T - 0) = \frac{1}{4}kA^2$$

So the correct is (d).

Note Since $\omega = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega^2$, $U_{av} = \frac{1}{4}m\omega^2 A^2$ which is equal to average kinetic energy over one time period.

149. $y_1 = A \sin \omega t$

$$y_2 = A \cos \omega t = A \sin\left(\omega t + \frac{\pi}{2}\right)$$

Using the superposition principle, the resultant displacement is given by

$$y = y_1 + y_2$$

$$= A\left[\sin \omega t + \sin\left(\omega t + \frac{\pi}{2}\right)\right]$$

$$= 2A \cos\frac{\pi}{4} \sin\left(\omega t + \frac{\pi}{4}\right)$$

$$\Rightarrow \qquad y = A_r \sin\left(\omega t + \frac{\pi}{4}\right) \qquad (1)$$

where $A_r = 2A \cos\frac{\pi}{4} = \sqrt{2}A$ is the resultant amplitude.

It follow from (1) that the resultant motion of the particle is simple harmonic having angular frequency ω. Hence the total energy of the particle is

$$E = \frac{1}{2}m A_r^2 \omega^2$$

$$= \frac{1}{2}m(\sqrt{2}A)^2 \omega^2 = mA^2\omega^2 \text{ , which is choice (b).}$$

150. The spring constant of the parallel combination of springs of spring constants k_1 and k_2 is

$$k_p = k_1 + k_2 = k + k = 2k$$

This combination is in series with the spring of spring constant k_3. So the effective spring constant of the system is.

$$\frac{1}{k_{eff}} = \frac{1}{k_p} + \frac{1}{k_3} = \frac{1}{2k} + \frac{1}{2k} = \frac{1}{k}$$

$$\therefore \qquad k_{eff} = k$$

and $$T = 2\pi\sqrt{\frac{m}{k_{eff}}} = 2\pi\sqrt{\frac{m}{k}}$$

So the correct choice is (a).

151. Let x be the maximum extension of the spring. The block loses potential energy and the spring gains potential energy, *i.e.*

$$mgx = \frac{1}{2}kx^2 \quad \Rightarrow \quad x = \frac{2mg}{k}$$

So the correct choice is (b).

152. $y = a \sin(\omega t - kx)$

Particle velocity is $V = a\omega \cos(\omega t - kx)$

$$\therefore \qquad V_{max} = a\omega = a \times 2\pi\nu$$

Wave velocity $\quad v = \nu\lambda$

$$V_{max} > v \text{ if } a \times 2\pi\nu > \nu\lambda \Rightarrow a > \frac{\lambda}{2\pi}$$

So the correct choice is (c).

153. The intensity of a wave is given by

$$I = 2\pi^2 \nu^2 a^2 \rho v$$

where ρ is the density of the medium and v is the sound speed in that medium.

$$I' = 2\pi^2 (4\nu)^2 \left(\frac{a}{2}\right)^2 \rho v$$

$$= 4 \times 2\pi^2 \nu^2 a^2 \rho v$$

$$= 4\, I$$

So the correct choice is (b).

154. Let I be the intensity of sound and n its intensity level in dB, then

$$n = 10 \log\left(\frac{I}{I_0}\right) \quad (1)$$

If the intensity level is increased by 2 dB, let its intensity become I', then

$$n + 2 = 10 \log\left(\frac{I'}{I_0}\right) \quad (2)$$

Subtracting (1) from (2) we get

$$2 = 10\left[\log\left(\frac{I'}{I_0}\right) - \log\left(\frac{I}{I_0}\right)\right]$$

$$= 10\,[\log I' - \log I_0 - (\log I - \log I_0)$$

$$= 10\,(\log I' - \log I)$$

$$= \log\left(\frac{I'}{I}\right)$$

$$\Rightarrow \log\left(\frac{I'}{I}\right) = 0.2$$

$$\Rightarrow \frac{I'}{I} = 1.585$$

$$\Rightarrow I' = 1.585\, I$$

Percentage increase in intensity is

$$\left(\frac{I' - I}{I}\right) \times 100 = 58.5\%$$

So the correct choice is (d).

155. $\beta_1 = 10 \log\left(\frac{I}{I_0}\right)$

$\beta_2 = 10 \log\left(\frac{2I}{I_0}\right)$

$$\therefore \quad \beta_2 - \beta_1 = 10\left[\log\left(\frac{2I}{I_0}\right) - \log\left(\frac{I}{I_0}\right)\right]$$

$$= 10 \log\left(\frac{2I}{I}\right)$$

$$= 10 \log (2)$$

$$= 10 \times 0.301$$

$$= 3.01 \text{ dB}$$

So the correct choice is (b).

156. $OC = \sqrt{30^2 + 40^2} = 50$ m

The effective speed of source is

$$u_s = 25 \cos\theta = 25 \times \frac{CP}{CO} = \frac{25 \times 30}{50} = 15 \text{ ms}^{-1}$$

$\therefore$ Apparent frequency is

$$\nu_1 = \nu\left(\frac{v}{v - u_s}\right)$$

$$= 650 \times \frac{345}{(345 - 15)} = 690 \text{ Hz}$$

So the correct choice is (d).

157. Case (*i*) Refer to the solution of case (*a*) on page 10.11.

If $M << m$,

$$T_1 = 2\pi \sqrt{\frac{m}{k}}$$

Case (*ii*) Refer to the solution of case (*b*) on page 10.12.

$$\text{P.E.} = \frac{m^2 g^2}{8k} + 2kx^2$$

If the mass of pulley is negligible,

$$\text{K.E.} = \frac{1}{2} mv^2$$

Total energy is

$$\text{E} = \text{P.E.} + \text{K.E.}$$

$$= \frac{m^2 g^2}{8k} + 2kx^2 + \frac{1}{2} mv^2$$

Since E is constant, $\frac{dE}{dt} = 0$, *i.e.*

$$0 = 0 + 4kx \frac{dx}{dt} + mv \frac{dv}{dt}$$

$$\Rightarrow \quad 0 = 4kx + m \frac{dv}{dt} \qquad \left(\because \quad \frac{dx}{dt} = v\right)$$

$$\therefore \text{ Acceleration } a = \frac{dx}{dt} = -\left(\frac{4k}{m}\right) x$$

Thus $\omega = \sqrt{\frac{4k}{m}} = 2\sqrt{\frac{k}{m}}$

and $T_2 = \frac{2\pi}{2}\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{m}{k}}$

$\therefore \quad \frac{T_1}{T_2} = 2$

So the correct choice is (c).

158. Speed of transverse wave in a stretched string is

$$v = \sqrt{\frac{T}{\mu}}$$

where T is the tension in the string and μ is the mass per unit length of the string. Since strings AB and CD have the same density and the same cross-sectional area, they have the same μ. Hence

$$v \propto \sqrt{T}$$

If the tension in CD is T, the tension in string AB is $\frac{T}{2}$.

$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{T_{AB}}{T_{CD}}} = \sqrt{\frac{T/2}{T}} = \frac{1}{\sqrt{2}}$

So the correct choice is (b).

159. Intensity I = Energy per unit area per unit time

or $I = \frac{E}{AT}$

Energy density U is energy per unit volume

or $U = \frac{E}{V} = \frac{E}{AL}$ $(\because V = AL)$

where L is a length.

$\therefore \quad \frac{I}{U} = \frac{E}{AT} \cdot \frac{AL}{E} = \frac{L}{T}$

So the correct choice is (a).

2 SECTION — Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage.

Passage I

A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. When the cylinder is given a small downward push and released, it starts oscillating vertically with a small amplitude.

1. The extension x_0 of the spring when it is in equilibrium is

   (a) $\frac{Mg}{k}$ (b) $\frac{g}{k}(M - LA\sigma)$

   (c) $\frac{g}{k}\left(M - \frac{1}{2}LA\sigma\right)$ (d) $\frac{g}{k}\left(M + \frac{1}{2}LA\sigma\right)$

2. If the cylinder is given a small downward displacement x from the equilibrium position and released, the restoring force F acting on it is

   (a) $-Mgx$ (b) $-(k + A\sigma g)x$

   (c) $= \left(k - \frac{1}{2}A\sigma g\right) - x$ (d) $-\left(k + \frac{1}{2}A\sigma g\right)x$

3. The time period T of the vertical oscillations of the cylinder is

   (a) $2\pi\sqrt{\frac{M}{k}}$

   (b) $2\pi\left[\frac{M}{\left(k + \frac{1}{2}A\sigma g\right)}\right]^{1/2}$

   (c) $2\pi\left[\frac{M}{(k - A\sigma g)}\right]^{1/2}$

   (d) $2\pi\left[\frac{M}{k + A\sigma g}\right]^{1/2}$

Solutions

1. The upthrust on the cylinder with half its length submerged in the liquid is given by

   U = weight of the liquid displaced by a length $L/2$ of the cylinder

$$= A \times \frac{L}{2} \times \sigma \times g = \frac{L}{2} (A\sigma g)$$

Let x_0 be the extension of the spring when it is in equilibrium. Then

$$kx_0 = Mg - \frac{L}{2}(A\sigma g) \quad (1)$$

The correct choice is (c).

2. Let x be the small downward displacement given to the cylinder so that the submerged length of the cylinder is now $\left(\frac{L}{2}+x\right)$ and the extension of the spring is now $(x_0 + x)$. The upthrust now is $\left(\frac{L}{2}+x\right)A\sigma g$ and the force in the spring is $k(x_0 + x)$. Hence, the restoring force on the cylinder is

$$F = \left[k(x_0+x) - \left\{Mg - \left(\frac{L}{2}+x\right)A\sigma g\right\}\right] \quad (2)$$

Using Eq. (1) in Eq. (2), we have

$$F = -(kx + A\sigma gx)$$

or $$F = -(k + A\sigma g)x$$

Thus the correct choice is (b)

3. The acceleration of the cylinder is

$$a = \frac{F}{M} = -\left(\frac{k + A\sigma g}{M}\right)x \quad (3)$$

Comparing Eq. (3) with $a = -\omega^2 x$, where $\omega = 2\pi/T$, we find that the correct choice is (d).

Questions 4 to 6 are based on the following passage.

Passage II

One end of a light spring of force constant k is fixed to a block of mass M placed on a horizontal frictionless surface, the other end of the spring being fixed to a wall. The spring-block system is executing simple harmonic motion of amplitude A and frequency ν. When the block is passing through the equilibrium position, an object of a mass m is gently placed on the block. As a result, the frequency of the system becomes ν' and the amplitude becomes A'.

4. The ratio ν'/ν is

(a) $\left(\frac{M}{M+m}\right)^{1/2}$ (b) $\left(\frac{m}{M+m}\right)^{1/2}$

(c) $\sqrt{\frac{MA}{mA'}}$ (d) $\left[\frac{(M+m)A'}{mA}\right]^{1/2}$

5. If v and v' are the velocities before and after the object is placed on the block, then the ratio v'/v is

(a) $\frac{M}{(M+m)}$ (b) $\left(\frac{M+m}{m}\right)$

(c) $\left(\frac{M+m}{M-m}\right)\frac{A'}{A}$ (d) $\left(\frac{M-m}{M+m}\right)\frac{A}{A'}$

6. The ratio A'/A is

(a) $\left(\frac{M+m}{m}\right)^{1/2}$ (b) $\left[\frac{m}{(M+m)}\right]^{1/2}$

(c) $\left(\frac{M}{M+m}\right)^{1/2}$ (d) $\left[\frac{(M+m)}{M}\right]^{1/2}$

Solutions

4. The frequency of the system before the object is placed on the block is given by

$$\nu = \frac{1}{2\pi}\left(\frac{k}{M}\right)^{1/2}$$

After the object of mass m is placed on the block, the new frequency of the system becomes

$$\nu' = \frac{1}{2\pi}\left[\frac{k}{M+m}\right]^{1/2}$$

$$\therefore \quad \frac{\nu'}{\nu} = \left[\frac{M}{(M+m)}\right],$$

which is choice (a)

5. From conservation of momentum, we have

$$Mv = (M + m)v'$$

or $$\frac{v'}{v} = \frac{M}{(M+m)} .$$

So the correct choice is (a).

6. From the principle of conservation of energy we know that the kinetic energy of the block when it is passing through the equilibrium position = potential energy of the spring when the displacement is equal to the amplitude.

Thus we have

$$\frac{1}{2} Mv^2 = \frac{1}{2} kA^2$$

and $$\frac{1}{2}(M + m)v'^2 = \frac{1}{2} kA'^2$$

$$\therefore \quad \frac{A'}{A} = \left(\frac{v'}{v}\right)\left[\frac{(M+m)}{M}\right]^{1/2}$$

$$= \left(\frac{M}{M+m}\right)\left(\frac{M+m}{M}\right)^{1/2}$$

$$= \left(\frac{M}{M+m}\right)^{1/2},$$

which is choice (c).

Questions 7 to 9 are based on the following passage.

Passage III

When two sound waves travel in the same direction in a medium, the displacements of a particle located at x at time t is given by

$$y_1 = 0.05 \cos (0.50\ \pi x - 100\ \pi t)$$

and $$y_2 = 0.05 \cos (0.46\ \pi x - 92\ \pi t)$$

where y_1, y_2 and x are in metre and t is in second.

7. What is the speed of sound in the medium?
   (a) 332 ms^{-1} (b) 100 ms^{-1}
   (c) 92 ms^{-1} (d) 200 ms^{-1}
8. How many times per second does an observer hear the sound of maximum intensity?
   (a) 4 (b) 8
   (c) 12 (d) 16
9. At $x = 0$, how many times between $t = 0$ and $t = 1$ s does the resultant displacement become zero?
   (a) 46 (b) 50
   (c) 92 (d) 100



Solutions

7. The two displacements can be written as

$$y_1 = A \cos (k_1 x - \omega_1 t) \quad (1)$$

and $$y_2 = A \cos (k_2 x - \omega_2 t) \quad (2)$$

where $A = 0.05$ m, $k_1 = \dfrac{2\pi}{\lambda_1} = 0.50\pi\ m^{-1}$, $\omega_1 = 2\pi\ \nu_1$

$= 100\pi$ rad s^{-1}, $k_2 = \dfrac{2\pi}{\lambda_2} = 0.46\pi\ ms^{-1}$ and $\omega_2 = 2\pi\nu_2$

$= 92\ \pi$ rad s^{-1}. The speed of either wave is

$$v_1 = \nu_1\lambda_1 = 2\pi\nu_1 \times \frac{\lambda_1}{2\pi} = \frac{\omega_1}{k_1}$$

$$= \frac{100\pi}{0.50\pi} = 200\ ms^{-1}$$

or $$v_2 = \nu_2\lambda_2 = \frac{\omega_2}{k_2} = \frac{92\pi}{0.46\pi} = 200\ ms^{-1}.$$

Hence the correct choice is (d).

8. Beat frequency = $\nu_1 - \nu_2$. Now

$$\nu_1 = \frac{\omega_1}{2\pi} = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

and $$\nu_2 = \frac{\omega_2}{2\pi} = \frac{92\pi}{2\pi} = 46 \text{ Hz}$$

$\therefore$ Beat frequency = 50 – 46 = 4 Hz. Hence the correct choice is (a).

9. The resultant displacement is given by

$$y = y_1 + y_2$$

$$= A \cos (k_1 x - \omega_1 t) + A \cos (k_2 x - \omega_2 t)$$

For $x = 0$, we have

$$y = A \cos \omega_1 t + A \cos \omega_2 t$$

$$= 2A \cos \left\{\frac{1}{2}(\omega_1 + \omega_2)t\right\} \times \cos \left\{\frac{1}{2}(\omega_1 - \omega_2)t\right\}$$

$\therefore$ $y = 0.10 \cos (96\ \pi t) \cos (4\ \pi t)$

Between $t = 0$ and $t = 1$ s, cos (96 πt) becomes zero 96 times and cos (4πt) becomes zero 4 times. Hence the resultant displacement y at $x = 0$ becomes zero 100 times between $t = 0$ and $t = 1$ s. The correct choice (d).

Questions 10 to 14 are based on the following passage

Passage IV

The vibrations of a string of length 60 cm fixed at both ends are represented by the equation

$$y = 4 \sin \left(\frac{\pi x}{15}\right) \cos (96\ \pi t)$$

where x and y are in cm and t is in second.

10. The frequency of vibrations of the string is
    (a) 48 Hz (b) 50 Hz
    (c) 96 Hz (d) 100 Hz
11. The maximum displacement of a point at $x = 10$ cm is
    (a) 2 cm (b) 4 cm
    (c) $2\sqrt{3}$ cm (d) zero
12. How many nodes are formed on the string?
    (a) 2 (b) 3
    (c) 4 (d) 5
13. In which harmonic mode is the string vibrating?
    (a) fundamental (b) third
    (c) fourth (d) fifth
14. The velocity of the particle at $x = 7.5$ cm at $t = 0.25$ s is
    (a) zero (b) 320 ms^{-1}
    (c) 60 ms^{-1} (d) 96 ms^{-1}

Solutions

10. $2\pi\nu = 96\pi \rightarrow \nu = 48$ Hz. Thus the correct choice is (a)

11. Displacement is maximum when cos (96 πt) = 1. At x = 10 cm,

$$y_{max} = 4 \sin\left(\frac{\pi \times 10}{15}\right) = 4 \sin\left(\frac{2\pi}{3}\right) = 2\sqrt{3} \text{ cm}$$

The correct choice is (c).

12. At nodes the displacement is always zero. Hence nodes are located at values of x given by

$$\sin\left(\frac{\pi x}{15}\right) = 0 \text{ or } \frac{\pi x}{15} = p\pi$$

where p = 0, 1, 2, 3, … etc.

Thus $x = 15\ p$ = 0, 15, 30, 45 and 60 cm.

Thus the correct choice is (d).

13. The correct choice is (c) because 5 nodes are formed on the string.

14. The velocity of the string at a point x at time t is obtained by differentiating

$$y = 4 \sin\left(\frac{\pi x}{15}\right) \cos(96\pi t)$$

with respect to t.

$$\text{Velocity } \frac{dy}{dt} = -(4 \times 96\pi) \times \sin\left(\frac{\pi x}{15}\right) \sin(96\pi t)$$

At x = 7.5 cm and t = 0.25 s, the velocity is zero because at t = 0.25 s, sin (96 πt) = sin (24 π) = 0.

Hence the correct choice is (a).

Questions 15 to 18 are based on the following passage.

Passage V

The displacement of the medium in a sound wave is given by

$$y_1 = A \cos(ax + bt)$$

where A, a and b are positive constants. The wave is reflected by an obstacle situated at x = 0. The intensity of the reflected wave is 0.64 times that of the incident wave.

15. The wavelength and frequency of the incident wave respectively are

(a) $\frac{2\pi}{a}, \frac{b}{2\pi}$ (b) $\frac{a}{2\pi}, \frac{2\pi}{b}$

(c) $\frac{2}{a}, \frac{b}{2}$ (d) $a, \frac{1}{b}$

16. The equation for the reflected wave is

(a) $y_2 = 0.8\ A \cos(-ax + bt)$

(b) $y_2 = -A \cos(-ax + bt)$

(c) $y_2 = -0.64\ A \cos(-ax + bt)$

(d) $y_2 = -0.8\ A \cos(-ax + bt)$

17. In the standing wave formed due to the superposition of the incident and reflected waves, the maximum value of the particle speed in the medium is

(a) Ab (b) 1.64 Ab

(c) 1.8 Ab (d) 2 Ab

18. The minimum value of the particle speed in the medium is

(a) zero (b) 0.2 Ab

(c) 0.64 Ab (d) 0.8 Ab

Solutions

15. The incident wave is given by

$$y_1 = A \cos(ax + bt) \quad (1)$$

The wave equation is

$$y = A \cos(kx + \omega t) \quad (2)$$

where k is the wave number and ω, the angular frequency. Comparing (1) and (2) we get $k = a$ and $\omega = b$. Hence wavelength $\lambda = \frac{2\pi}{k} = \frac{2\pi}{a}$ and frequency $\nu = \frac{\omega}{2\pi} = \frac{b}{2\pi}$.

The correct choice is (a).

16. Since the intensity of the reflected wave is 0.64 times that of the incident wave, the amplitude A_r of the reflected wave will be $\sqrt{0.64}$ = 0.8 times that of the incident wave, i.e.

$$A_r = 0.8\ A$$

Now, when a wave is reflected by an obstacle, it suffers a reversal of amplitude (which implies a phase change of π radian), i.e. $A_r = -0.8\ A$. Since the incident wave is travelling in the negative x direction, the reflected wave will travel in the positive x direction. Therefore, the equation of the reflected wave can be obtained from Eq. (1) by replacing A by $A_r = -0.8\ A$ and x by $-x$. Thus, the reflected wave is given by

$$y_2 = -0.8\ A \cos(-ax + bt) \quad (3)$$

Thus the correct choice is (d).

17. Differentiating Eq. (1) will respect to time t, we get the expression for the particle speed in the medium due to the incident wave, which is

$$V_1 = \frac{dy_1}{dt} = \frac{d}{dt} \{A \cos(ax + bt)\}$$

$$= -Ab \sin (ax + bt) \qquad (4)$$

$\therefore$ Maximum particle speed due to the incident wave is

$$(V_1)_{max} = Ab$$

Differentiating Eq. (3) with respect to t, the particle speed due to the reflected wave is given by

$$V_2 = \frac{dy_2}{dt}$$

$$= \frac{d}{dt}\,[-0.8\,A \cos (bt - ax)]$$

$$= 0.8\,Ab \sin (bt - ax)$$

$\therefore$ $(V_2)_{max} = 0.8\,Ab$

From the superposition principle, the maximum particle speed in the medium due to both the incident and reflected waves is given by the algebraic sum of the individual maximum particle speeds, i.e.

$$V_{max} = (V_1)_{max} + (V_2)_{max}$$
$$= Ab + 0.8\,Ab = 1.8\,Ab$$

So the correct choice is (c).

18. Since A and b are positive constants, the minimum particle speed is

$$V_{min} = (V_1)_{max} - (V_2)_{max}$$
$$= Ab - 0.8\,Ab = 0.2\,Ab$$

The correct choice is (b).

Questions 19 to 22 are based on the following passage.

Passage VI

The air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 400 Hz. The speed of sound in air is 320 ms^{-1}. The end correction may be neglected. Let P_0 denote the mean pressure at any point in the pipe and ΔP_0 the maximum amplitude of pressure variation.

19. The length L of the air column is

(a) 20 cm (b) 60 cm

(c) 1.0 m (d) 1.4 m

20. The amplitude of pressure variation at the middle of the air column is

(a) ΔP_0 (b) $2\Delta P_0$

(c) $\sqrt{2}\,\Delta P_0$ (d) $\dfrac{\Delta P_0}{\sqrt{2}}$

21. The maximum and minimum pressures at the open end of the pipe respectively are

(a) $P_0 + \Delta P_0, P_0 - \Delta P_0$ (b) $P_0 + \Delta P_0, P_0$

(c) $P_0, P_0 - \Delta P_0$ (d) P_0, P_0

22. The maximum and minimum pressures at the closed end of the pipe respectively are

(a) $P_0 + \Delta P_0, P_0 - \Delta P_0$ (b) $P_0 + \Delta P_0, P_0$

(c) $P_0, P_0 - \Delta P_0$ (d) P_0, P_0

Solutions

19. Figure 10.56 shows the longitudinal displacement y as a function of x for x lying between $x = 0$ and $x = L$, where L is the length of the pipe.

The fundamental frequency of a closed pipe is given by

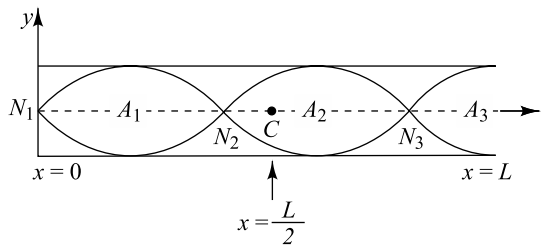


Fig. 10.56

$$\nu = \frac{v}{4L}$$

In a closed pipe, only odd harmonics are present, i.e. the frequency ν_1 of the first overtone is 3 times the fundamental frequency, that of the second overtone ν_2 is 5 times the fundamental frequency and so on. Thus

$$\nu_2 = 5\nu = \frac{5v}{4L}$$

$$L = \frac{5v}{4\nu_2} = \frac{5 \times 320}{4 \times 400} = 1.0 \text{ m}$$

The correct choice is (c).

20. Since the distance between two consecutive nodes is $\dfrac{\lambda}{2}$ and that between a node and the next antinode is $\dfrac{\lambda}{4}$, it follows from Fig. 10.55 that

$$L = \frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{4} = \frac{5\lambda}{4}$$

We know that the pressure variation (excess pressure) is maximum at a node and minimum (equal to zero) at an antinode. Therefore, the pressure variation at a distance x from a node is given by

$$\Delta P = \Delta P_0 \cos\left(\frac{2\pi x}{\lambda}\right) \qquad (1)$$

The centre C of the tube is at $x = \frac{L}{2}$ or $x = \frac{5\lambda}{8}$ $\left(\text{since } L = \frac{5\lambda}{4}\right)$. The second node N_2 is at $x = \frac{\lambda}{2}$.

Therefore, the distance of C from N_2 is $x = \frac{5\lambda}{8} - \frac{\lambda}{2} = \frac{\lambda}{8}$. Using this value of x in Eq. (1) we have

$$\Delta P \text{ at } C = \Delta P_0 \cos\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{8}\right)$$

$$= \Delta P_0 \cos\frac{\pi}{4} = \frac{\Delta P_0}{\sqrt{2}}$$

Thus the correct choice is (d).

21. At an antinode, the pressure variation is zero, i.e. $\Delta P_0 = 0$. Hence, at an antinode

$$P_{max} = P_{min} = P_0$$

So the correct choice is (d).

22. At a node, the pressure variation is maximum equal to ΔP_0. Hence, at a node

$$P_{max} = P_0 + \Delta P_0$$

and $P_{min} = P_0 - \Delta P_0$

Thus the correct choice is (a).

Questions 23 to 25 are based on the following passage.

Passage VIII

A wire of mass 9.8×10^{-3} kg per metre passes over a frictionless pulley fixed at the top of an inclined frictionless plane which makes an angle of 30° with the horizontal. Two masses M_1 and M_2 are tied at the two ends of the wire. Mass M_1 rests on the inclined plane and mass M_2 hangs freely vertically downwards. The whole system is in equilibrium. Now a transverse wave propagates along the wire with a speed of 100 ms^{-1}.

23. The tension T in the wire is

(a) 0.98 N (b) 9.8 N

(c) 98 N (d) 980 N

24. The value of mass M_1 is

(a) 2 kg (b) 5 kg

(c) 10 kg (d) 20 kg

25. The value of mass M_2 is

(a) 5 kg (b) 10 kg

(c) 15 kg (d) 20 kg

Solutions

23. Refer to Fig. 10.57. Let T be the tension in the string when the system is in equilibrium.

It follows from the figure that, at the equilibrium position, the component $M_1 g \cos\theta$ of weight $M_1 g$ balances with the normal reaction N and the other component $M_1 g \sin\theta$ will balance with tension T in the string. Also weight $M_2 g$ of mass M_2 will balance with tension T. Thus

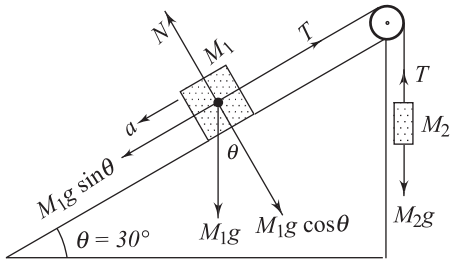


Fig. 10.57

$$M_1 g \sin\theta = T \quad (1)$$

and $$M_2 g = T \quad (2)$$

Now, the speed of a transverse wave in a wire of mass m per unit length and stretched with a tension T is given by

$$v = \sqrt{\frac{T}{m}}$$

or $$T = v^2 m \quad (3)$$

Given $m = 9.8 \times 10^{-3}$ kg m^{-1} and $v = 100$ ms^{-1}. Using these values in Eq. (3), we have

$$T = (100)^2 \times 9.8 \times 10^{-3} = 98 \text{ N}$$

24. From Eq. (1), we get

$$M_1 = \frac{T}{g \sin\theta} = \frac{98}{9.8 \times \sin 30°} = 20 \text{ kg}$$

Thus the correct choice is (d).

25. From Eq. (2), we have

$$M_2 = \frac{T}{g} = \frac{98}{9.8} = 10 \text{ kg, which is choice (b).}$$

Questions 26 to 28 are based on the following passage.

Passage VIII

A source of sound of frequency 90 Hz is moving towards a wall with a speed $u = v/10$, where v is the speed of sound in air.

26. The beat frequency of the sound heard by an observer between the wall and the source is
    (a) 20 Hz (b) 10 Hz
    (c) 5 Hz (d) zero
27. The beat frequency of the sound heard by an observer behind the source is
    (a) $\frac{200}{9}$ Hz (b) 20 Hz
    (c) $\frac{200}{11}$ Hz (d) zero
28. The beat frequency of the sound heard by the observer moving with the source is
    (a) 11 Hz (b) 9.9 Hz
    (c) 10 Hz (d) 20 Hz



Solutions

26. The observer hears two sounds—one coming directly from the approaching source and the other sound after reflection from the wall (which can be considered as coming from the mirror image of the source). The apparent frequency of the approaching source is

$$v' = v\left(\frac{v}{v-u_s}\right) = 90 \times \left(\frac{v}{v-v/10}\right) = 100 \text{ Hz}$$

When the observer is between the wall and the source, the apparent frequency of the sound reflected from the wall is also v'. Therefore, frequency of beats = $v' - v' = 0$. The observer will not hear any beats.

So the correct choice is (d).

27. When the observer is behind the source, i.e. when the source is between the wall and the observer, the apparent frequency of the sound coming directly from the receding source is

$$v'' = v\left(\frac{v}{v+u_s}\right)$$

$$= 90 \times \left(\frac{v}{v+v/10}\right) = \frac{900}{11} \text{ Hz}$$

$$\therefore \quad \text{Beat frequency} = v' - v'' = 100 - \frac{900}{11}$$

$$= \frac{200}{11} \text{ Hz.}$$

Hence the correct choice is (c).

28. If the observer is moving with the source, the frequency of the direct sound is v = 90 Hz. The apparent frequency of the reflected sound is

$$v''' = v\left(\frac{v+u_s}{v-u_s}\right)$$

$$= 90 \times \left(\frac{v+v/10}{v-v/10}\right) = 110 \text{ Hz}$$

$\therefore$ Beat frequency = $v''' - v = 110 - 90 = 20$ Hz

So the correct choice is (d).

SECTION 3

Assertion-Reason Type Questions

In the following questions, Statement-1(Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only *one* choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. **Statement-1**

   If a spring of force constant k is cut into two equal halves, the force constant of each half is $2k$.

   Statement-2

   When an elastic spring is extended by an amount x, the work done is $\frac{1}{2} kx^2$.

2. **Statement-1**

   A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from

$x = 0$ to $x = A/2$ will be less than the time taken for it to go from $x = A/2$ to $x = A$.

Statement-2

In simple harmonic motion, the speed of the particle is the maximum at the mean position $x = 0$ and decreases as it moves towards the extreme position becoming zero at $x = A$.

3. **Statement-1**

A body is executing simple harmonic motion. At a displacement x, its potential energy is E_1 and at a displacement y, its potential energy is E_2. The potential energy at a displacement $(x + y)$ is $E = \sqrt{E_1^2 + E_2^2}$.

Statement-2

For a body executing simple harmonic motion, the potential energy is proportional to the square of its displacement from the mean position.

4. **Statement-1**

The time period of a simple harmonic oscillator depends upon its amplitude and force constant.

Statement-2

The frequency of a simple harmonic oscillator is determined by elasticity and inertia.

5. **Statement-1**

The amplitude and phase constant of a particle in SHM are determined from its initial displacement and initial velocity.

Statement-2

The amplitude and phase constant of SHM depend on the magnitude of the restoring force.

6. **Statement-1**

The displacement of a particle is given by

$$x = a \sin (bt + c)$$

This equation suggests that the time period of motion of $2\pi/b$.

Statement-2

The same value of x is obtained at $t = t$ and at $t' = t + 2\pi/b$.

7. **Statement-1**

The displacement of a simple harmonic oscillator is given by

$$x = A \sin(\omega t + \phi)$$

This equation suggests that the energy of the oscillator remains constant.

Statement-2

The same value of x is obtained after an interval equal to the time period of the oscillator.

8. **Statement-1**

The time period of a simple pendulum is independent of the mass of the bob.

Statement-2

The restoring force does not depend on the mass of the bob.

9. **Statement-1**

For small amplitudes, the motion of a simple pendulum is simple harmonic of time period $T = 2\pi\sqrt{l/g}$. For larger amplitudes, the time period is greater than $2\pi\sqrt{l/g}$.

Statement-2

For larger amplitude, the speed of the bob is greater when it passes through the mean position.

10. **Statement-1**

For an oscillating simple pendulum, the tension in the string is constant at $T = mg$ for all positions of the bob.

Statement-2

The tension in the string will not remain constant at $T = mg$ because the speed of the bob is different at different positions.

11. **Statement-1**

Only longitudinal mechanical waves can propagate in gases.

Statement-2

Gases have only bulk modulus.

12. **Statement-1**

Two sound waves of equal intensity I produced beats. The maximum intensity of sound produced in beats is $4I$.

Statement-2

If two waves of amplitudes a_1 and a_2 superpose, the maximum amplitude of the resultant wave = $a_1 + a_2$.

13. **Statement-1**

A medium must possess elasticity in order to support wave motion.

Statement-2

Restoring force does not exist in a medium which does not have elasticity.

14. **Statement-1**

Solids can support both longitudinal and transverse mechanical waves but only longitudinal mechanical waves can propagate in gases.

Statement-2

Gases do not have shear modulus.

15. **Statement-1**

In standing sound waves, a displacement node is a pressure antinode and vice versa.

Statement-2

In a standing wave, the restoring force is the maximum at a node and minimum at an antinode.

16. Statement-1

Our ears cannot distinguish two notes, one produced by a violin and other by a sitar, if they have exactly the same intensity and the same frequency.

Statement-2

When a musical instrument is played, it produces a fundamental note which is accompanied by a number of overtones called harmonics.

17. **Statement-1**

Doppler's effect does not occur in case of a supersonic source.

Statement-2

A supersonic source produceds a shock wave.

18. **Statement-1**

If a source of sound moves always from a stationary observer, the apparent frequency of sound as heard by the observer is greater than the actual frequency.

Statement-2

The cause of the apparent change in frequency is the change in the wavelength brought about by the motion of the source.

19. **Statement-1**

If an observer moves towards a stationary source of sound, the frequency of the sound as heard by him is greater than the actual frequency.

Statement-2

The apparent increase in frequency is due to the fact that the observer intercepts more waves per second when the moves towards the source.

20. **Statement-1**

If a source of sound is in motion and the observer is stationary, the speed of sound relative to him remains unchanged.

Statement-2

The apparent change in frequency is due to the change in the wavelength brought about by the motion of the source.

21. **Statement-1**

If the observer is in motion and the source of sound is stationary, the speed of sound relative to him is changed.

Statement-2

The wavelength of sound received by the observer does not change due to his motion.

22. Statement-1

The apparent frequency is not the same in the following two cases— (i) source approaching a stationary observer with a certain velocity and (ii) observer approaching a stationary source of sound with the same velocity.

Statement-2

The cause of the apparent change in the frequency is different in the two cases.



Solutions

1. The correct choice is (b). The force required to extend the spring by an amount x is given by

$$F = kx \qquad (1)$$

If the spring cut into two equal halves, the same force F will produce half the extension because the extension is directly proportional to the length of the spring. Hence

$$F = k'x' = k'\frac{x}{2} \qquad (2)$$

From (1) and (2), we get $k' = 2k$.

2. The correct choice is (a).

3. The correct choice is (d).

$$E_1 = \frac{1}{2}\, m\omega^2 x^2 \quad \Rightarrow \quad \sqrt{E_1} = x\sqrt{\frac{m\omega^2}{2}} = xk$$

where $$k = \sqrt{\frac{m\omega^2}{2}} \qquad (1)$$

$$E_2 = \frac{1}{2}\, m\omega^2 y^2 \quad \Rightarrow \quad \sqrt{E_2} = yk \qquad (2)$$

and $$E = \frac{1}{2}\, m\omega^2 (x+y)^2$$

$$\Rightarrow \qquad \sqrt{E} = (x+y)k \qquad (3)$$

From (1), (2) and (3), we get

$$\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$$

$$\Rightarrow \qquad E = E_1 + E_2 + 2\sqrt{E_1 E_2}$$

4. The correct choice is (d).
5. The correct choice is (c).
6. The correct choice is (a). The value of x at $t' = t + 2\pi/b$ is

$$x' = a \sin\left[b\left(t + \frac{2\pi}{b}\right) + c\right]$$

$$= a \sin [bt + 2\pi + c]$$

$$= a \sin (bt + c) = x$$

$$[\because \sin (\theta + 2\pi) = \sin \theta]$$

7. The correct choice is (a).
8. The correct choice is (c). The restoring force when the string makes an angle θ with the vertical is given by $F = -mg \sin \theta$, which depends upon m.
9. The correct choice is (b). Restoring force is $F = -mg \sin \theta$ or $F = -mg_e$ where $g_e = g \sin \theta$. For small oscillations, θ is small so that $\sin \theta \simeq \theta$ (here θ is in radian) and the effective value of g is $g_e\, \theta$. For larger oscillations $g \sin \theta$ is less than $g\theta$ because $\sin \theta < \theta$. Hence T is greater than $2\pi\sqrt{l/g}$.
10. The correct choice is (d). The tension in the string is given by

$$T = mg \cos \theta + \frac{mv^2}{l}$$

where l = length of the pendulum, v its speed when the string makes an angle θ with the vertical.

11. The correct choice is (a). Gases cannot withstand a shearing stress or longitudinal stress. Hence they do not have shear modulus and Young's modulus; they have only bulk modulus.
12. The correct choice is (a). When two waves of amplitudes a_1 and a_2 superpose to produce beats, the resultant amplitude of the maximum of intensity is

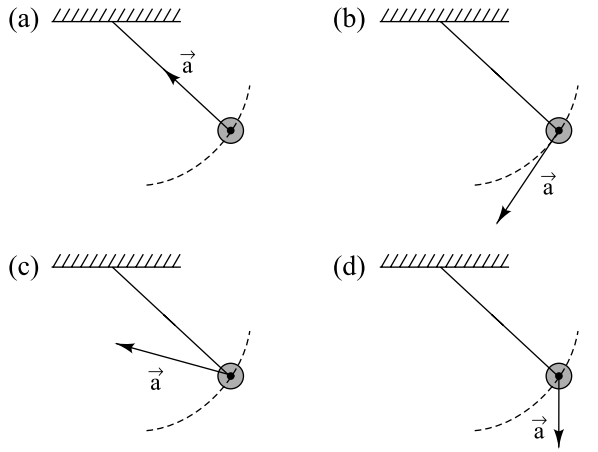
$$A = a_1 + a_2$$

Now, intensity $\propto$ (amplitude)^2. Since the two waves have the same intensity, their amplitudes are equal, i.e. $a_1 = a_2 = a$. Thus $A = 2a$. Therefore, $A^2 = 4a^2$ or $I_{max} = 4I$.

13. The correct choice is (a).
14. The correct choice is (a). Gases cannot withstand a shearing stress. Hence gases do not have any shear modulus; they have only bulk modulus. Solids have Young's modulus, bulk modulus and shear modulus. Therefore, solids can support both transverse and longitudinal waves.
15. The correct choice is (c).
16. The correct choice is (d). When a musical instrument is played, it produced a fundamental note which is accompanied by a number of overtones called harmonics. The number of harmonics is not the same for all instruments. It is the number of harmonics which distinguishes the note produced by a sitar and that produced by a violin.
17. The correct choice is (a). If the source of sound is moving at a speed greater than the speed of sound, then in a given time the source advances more than the wave. The resultant wave motion is a conical wave called a shock wave which produces a sudden and violent sound.
18. The correct choice is (d).
19. The correct choice is (a).
20. The correct choice is (a).
21. The correct choice is (a).
22. The correct choice is (a). In case (i) the speed of sound relative to the observer remains unchanged; the change in frequency is due to a change in wavelength brought about by the motion of the source. In case (ii) the wavelength of sound remains unchanged; the change in frequency is due to a change in the speed of sound relative to the observer.

SECTION 4
Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. A and B are two identical tubes. Tube A has both ends open while tube B has one end closed. The ratio of fundamental frequency of A and B is
   (a) 1 : 2 (b) 1 : 4
   (c) 2 : 1 (d) 4 : 1 **[2002]**
2. Two tuning forks produce 4 beats per second. One fork has a frequency of 288 Hz. A little wax is placed on the prong of the fork of unknown frequency and it then produces 2 beats per second. The frequency of the unknown fork is
   (a) 286 Hz (b) 292 Hz
   (c) 294 Hz (d) 288 Hz **[2002]**
3. A wave $y = a \sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then the equation of the unknown wave is
   (a) $y = a \sin(\omega t + kx)$ (b) $y = -a \sin(\omega t + kx)$
   (c) $y = a \sin(\omega t - kx)$ (d) $y = -a \sin(\omega t - kx)$
   [2002]
4. A string of length 40 cm is tied to two rigid supports. The maximum length (wavelength in cm) of a stationary wave produced in it is
   (a) 20 (b) 80
   (c) 40 (d) 120 **[2002]**
5. When temperature increases, the frequency of a tuning fork
   (a) increases
   (b) decreases
   (c) remains unchanged
   (d) increases or decreases depending on the material. **[2002]**
6. A child is swinging on a swing in sitting position. If he suddenly stands up, the time period of the swing will
   (a) increase
   (b) decrease
   (c) remain unchanged
   (d) increase if the child is tall and decrease if he is short. **[2002]**
7. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector is correctly shown in



8. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train *A* records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train *B* he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train *B* to that of train *A* is
   (a) 242/252 (b) 2
   (c) 5/6 (d) 11/6 **[2002]**
9. A sonometer wire resonates with a given tuning fork forming standing waves with five antidotes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass *M*, the wire resonates with the same tuning fork forming three antidotes for the same positions of the bridges. The value of *M* is
   (a) 25 kg (b) 5 kg
   (c) 12.5 kg (d) (1/25) kg **[2002]**
10. A mass *M* is suspended from a spring of negligible mass. The mass is pulled a little and released so that it executes SHM of time period *T*. If the mass is increased by *m*, the time period becomes $5T/3$. Then the ratio $\frac{m}{M}$ is
    (a) $\frac{3}{5}$ (b) $\frac{25}{9}$
    (c) $\frac{16}{9}$ (d) $\frac{5}{3}$ **[2003]**

11. Two bodies A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities during oscillations are equal, the ratio of amplitudes of A and B is

(a) $\sqrt{\frac{k_1}{k_2}}$ (b) $\frac{k_1}{k_2}$

(c) $\sqrt{\frac{k_2}{k_1}}$ (d) $\frac{k_2}{k_1}$ **[2003]**

12. The length of a simple pendulum is increased by 21%. The percentage increase in the time period is

(a) 11% (b) 21%

(c) 42% (d) 10.5% **[2003]**

13. The displacement of a wave travelling in the x-direction is given by

$$y = 10^{-4} \sin\left(600t - 2x + \frac{\pi}{3}\right)$$

where y and x are in metre and t in second. The speed of the wave (in m s^{-1}) is

(a) 300 (b) 600

(c) 1200 (d) 200 **[2003]**

14. The displacement of a particle varies according to the relation $x = 4(\cos \pi t + \sin \pi t)$. The amplitude of motion is

(a) – 4 (b) 4

(c) $4\sqrt{2}$ (d) 8 **[2003]**

15. A metal wire of linear mass density of 9.8 g m^{-1} is stretched with a tension of 10 kg-wt between two rigid supports 1 m apart. The wire passes at its middle point between the poles of a permanent magnet and it vibrates in resonance when carrying an alternating current of frequency n. The frequency n of the alternating current source is

(a) 50 Hz (b) 100 Hz

(c) 200 Hz (d) 25 Hz **[2003]**

16. A tuning fork of frequency 256 Hz produces 5 beats per second with a vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was

(a) (256 + 2) Hz (b) (256 – 2) Hz

(c) (256 – 5) Hz (d) (256 + 5) Hz **[2003]**

17. A body executes simple harmonic motion. The potential energy (PE), kinetic energy (KE) and total energy (TE) are measured as function of displacement x. Which of the following statements is true?

(a) KE is maximum when $x = 0$

(b) TE is zero when $x = 0$

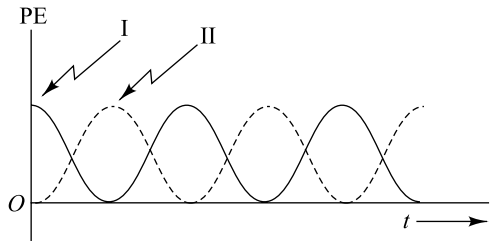
(c) KE is maximum when x is maximum

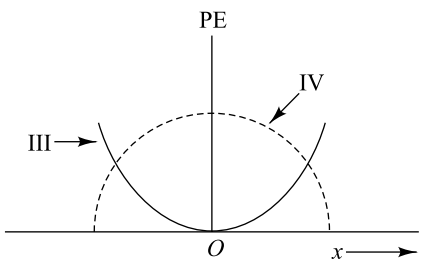
(d) PE is maximum when $x = 0$ **[2003]**

18. For a particle executing simple harmonic motion, the displacement x is given by $x = A \cos \omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x.

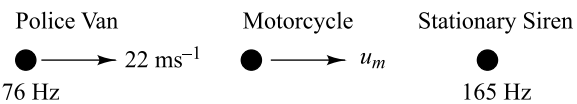
(a) I, III (b) II, IV

(c) II, III (d) I, IV





19. A police van, moving at 22 ms^{-1}, chases a motorcyclist. The policeman sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz, as shown in the figure.



If the motorcyclist does not observe any beats, his speed must be (take the speed of sound = 330 ms^{-1})

(a) 33 ms^{-1} (b) 22 ms^{-1}

(c) zero (d) 11 ms^{-1} **[2003]**

20. In the experiment for the determination of the speed of sound in air using the resonance column, it is observed that 0.1 m of air column resonates with a turning fork in the fundamental mode. When the length of the air column is changed to 0.35 m, the same tuning fork resonates, with the first overtone. What is the end correction?

(a) 0.0125 m (b) 0.025 m

(c) 0.05 m (d) 0.075 m **[2003]**

21. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period in air is t_0. The density of the bob is $\left(\frac{4}{3} \times 400\right)$

kg m^{-3}. If the frictional force (viscosity) is neglected, the relation between t and t_0 is

(a) $t = t_0$ (b) $t = \frac{t_0}{2}$

(c) $t = 2t_0$ (d) $t = 4t_0$ **[2004]**

22. A body at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period with another spring is t_2. If the period of oscillation is T when the two springs are connected in series, then

(a) $T = t_1 + t_2$ (b) $T^2 = t_1^2 + t_2^2$

(c) $\frac{1}{T} = \frac{1}{t_1} + \frac{1}{t_2}$ (d) $\frac{1}{T^2} = \frac{1}{t_1^2} + \frac{1}{t_2^2}$

[2004]

23. A particle is executing simple harmonic motion. When its displacement is x, its total energy is

(a) proportional to x (b) proportional to x^2

(c) proportional to $\sqrt{x}$ (d) independent of x.

[2004]

24. The displacement y of a particle of a medium can be expressed as

$$y = 10^{-6} \sin\left(100t + 20x + \frac{\pi}{4}\right)$$

where y and x are in metre and t is in second. The speed of the wave (in ms^{-1}) is

(a) 2000 (b) 5

(c) 20 (d) 5π **[2004]**

25. A particle of mass m is attached to a spring constant k and has a natural angular frequency ω_0. An external periodic force $F(t)$ proportional to cos (ωt) where $\omega \neq \omega_0$ is applied to the oscillator. In the steady state, the amplitude of the oscillator will be proportional to

(a) $\frac{m}{(\omega_0^2 - \omega^2)}$ (b) $\frac{1}{m(\omega_0^2 - \omega^2)}$

(c) $\frac{1}{m(\omega_0^2 + \omega^2)}$ (d) $\frac{m}{(\omega_0^2 + \omega^2)}$ **[2004]**

26. In forced oscillation of a particle, the amplitude is maximum for an angular frequency ω_1 of the applied force, while the energy is maximum for an angular frequency ω_2 of the applied force, then

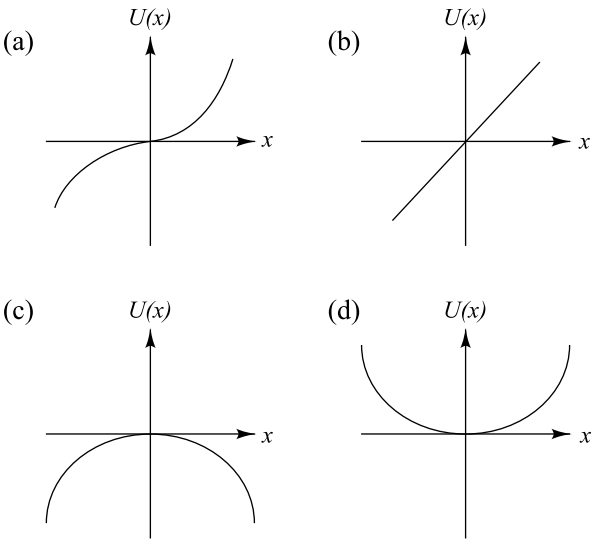
(a) $\omega_1 = \omega_2$

(b) $\omega_1 > \omega_2$

(c) $\omega_1 < \omega_2$

(d) $\omega_1 < \omega_2$, when damping is small and $\omega_1 > \omega_2$ when damping is large. **[2004]**

27. A particle is executing linear harmonic motion about the origin $x = 0$. Which of the graphs shown in the figure represents the variation of the potential energy function $U(x)$ versus x?



28. A simple pendulum has a time period of 3.0 s. If the point of suspension of the pendulum starts moving vertically upward with a velocity of $v = Kt$ where $K = 4.4\ ms^{-2}$, the new time period will be (Take $g = 10\ ms^{-2}$)

(a) $\frac{9}{4}s$ (b) $\frac{5}{3}s$

(c) 2.5 s (d) 4.4 s **[2004]**

29. The function $\sin^2(\omega t)$ represents

(a) a periodic but not simple harmonic motion with a period $\frac{2\pi}{\omega}$

(b) a periodic but not simple harmonic motion with a period $\frac{\pi}{\omega}$

(c) a simple harmonic motion with a period $\frac{2\pi}{\omega}$

(d) a simple harmonic motion with a period $\frac{\pi}{\omega}$

[2005]

30. When two tuning forks 1 and 2 are sounded simultaneously, 4 beats per second are heard. Now some tape is attached to the prong of fork 2. When the forks are sounded again, 6 beats per second are heard. If the frequency of fork 1 is 200 Hz, what was the original frequency of fork 2?

(a) 200 Hz (b) 202 Hz

(c) 196 Hz (d) 204 Hz **[2005]**

31. If a simple harmonic motion is represented by

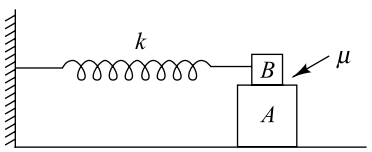
$$\frac{d^2x}{dt^2} + \alpha x = 0$$

its time period is

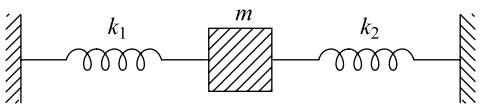
(a) $\frac{2\pi}{\alpha}$ (b) $\frac{2\pi}{\sqrt{\alpha}}$

(c) $2\pi\alpha$ (d) $2\pi\sqrt{\alpha}$ **[2005]**

32. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole at the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would
    (a) first increase and then decrease to the original value
    (b) first decrease and then increase to the original value
    (c) remain unchanged
    (d) increase towards a saturation value. **[2005]**
33. An observer moves towards a stationary source of sound with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency?
    (a) zero (b) 0.5%
    (c) 5% (d) 20% **[2005]**
34. A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed v ms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies up to a maximum of 10,000 Hz, the maximum value of v up to which he can hear the whistle is
    (a) 15 ms^{-1} (b) 30 ms^{-1}
    (c) $15\sqrt{2}$ ms^{-1} (d) $\frac{15}{\sqrt{2}}$ ms^{-1} **[2006]**
35. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is
    (a) 1050 Hz (b) 10.5 Hz
    (c) 105 Hz (d) 1.05 Hz **[2006]**
36. Starting from the origin a body oscillates simple harmonically with a period of 2s. After what time will its kinetic energy be 75% of the total energy?
    (a) $\frac{1}{3}$ s (b) $\frac{1}{12}$ s
    (c) $\frac{1}{6}$ s (d) $\frac{1}{4}$ s **[2006]**
37. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will lose contact with the platform for the first time
    (a) for an amplitude of g^2/ω^2
    (b) at the highest position of the platform
    (c) at the mean position of the platform
    (d) for an amplitude of g/ω^2 **[2006]**
38. A block A of mass m is placed on a frictionless horizontal surface. Another block B of the same mass is kept on A and connected to the wall with the help of spring of force constant k, as shown in the figure. The coefficient of friction between blocks A and B is μ. The blocks move together executing simple harmonic motion of amplitude a. The maximum value of frictional force between A and B is.



    (a) ka (b) $ka/2$
    (c) zero (d) μmg **[2006]**
39. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metres. The time at which the maximum speed first occurs is
    (a) 0.5 s (b) 0.75 s
    (c) 0.125 s (d) 0.25 s **[2007]**
40. A point mass oscillates along the x-axis according to equation $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A \cos(\omega t + \delta)$, then
    (a) $A = x_0$, $\delta = -\pi/4$ (b) $A = x_0\,\omega^2$, $\delta = \pi/4$
    (c) $A = x_0\,\omega^2$, $\delta = -\pi/4$ (d) $A = x_0\,\omega^2$, $\delta = 3\pi/4$
    [2007]
41. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of
    (a) 1000 (b) 10000
    (c) 10 (d) 100 **[2007]**
42. Two springs, of force constant k_1 and k_2, are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes [see the figure below].



    (a) $\frac{f}{2}$ (b) $\frac{f}{4}$
    (c) $4f$ (d) $2f$ **[2007]**
43. A particle of mass m executes simple harmonic motion with amplitude a and frequency ν. The

average kinetic energy during its motion from the position of equilibrium to the extreme position is

(a) $\pi^2 m a^2 \nu^2$ (b) $\frac{1}{4} ma^2 \nu^2$

(c) $4\pi^2 ma^2 \nu^2$ (d) $2\pi^2 ma^2 \nu^2$ **[2007]**

44. A simple pendulum attached to the ceiling of a stationary lift has a time period T. The distance y covered by the lift moving upwards varies with time t as $y = t^2$ where y is in metre and t in second. If $g = 10\ ms^{-2}$, the time period of the pendulum will be

(a) $\sqrt{\frac{4}{5}}T$ (b) $\sqrt{\frac{5}{6}}T$

(c) $\sqrt{\frac{5}{4}}T$ (d) $\sqrt{\frac{6}{5}}T$ **[2007]**

45. The speed of sound is oxygen (O_2) at a certain temperature is 460 ms^{-1}. The speed of sound in helium (He) at the same temperature will be (assume both gases to be ideal)

(a) 330 ms^{-1} (b) 460 ms^{-1}

(c) 500 ms^{-1} (d) 650 ms^{-1} **[2008]**

46. While measuring the speed of sound by performing a resonance column experiment a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, he measures the column length to be x cm for the second resonance. Then

(a) 36 cm > x > 18 cm (b) 18 cm > x

(c) x > 54 cm (d) 54 cm > x > 36 m **[2008]**

47. A wave travelling along the x-axis is described by the equation $y(x, t) = 0.005 \cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are

(a) $\alpha = 12.50\ \pi, \beta = \frac{\pi}{2.0}$ (b) $\alpha = 25.00\ \pi, \beta = \pi$

(c) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$ (d) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$ **[2008]**

48. Three sound waves of equal amplitudes have frequencies $(\nu - 1), \nu$ and $(\nu + 1)$. They superpose to give beats. The number of beats produced per second will be

(a) 2 (b) 1

(c) 4 (d) 3 **[2009]**

49. A motor cycle starts from rest and accelerates along a straight path at 2 m/s^2. At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (Speed of sound = 330 ms^{-1}).

(a) 147 m (b) 196 m

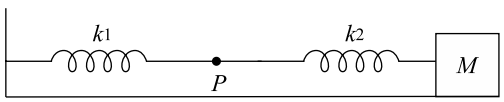
(c) 49 m (d) 98 m **[2009]**

50. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time?

(a) $aT + 2\pi v$ (b) aT/v

(c) $a^2T^2 + 4\pi^2 v^2$ (d) aT/x **[2009]**

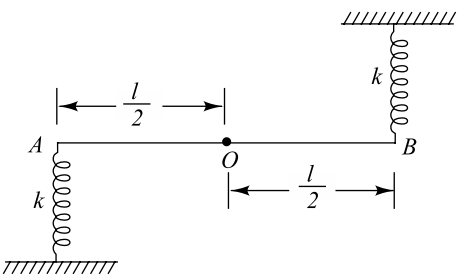
51. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is



(a) $\frac{k_1 A}{k_2}$ (b) $\frac{k_2 A}{k_1}$

(c) $\frac{k_1 A}{k_1 + k_2}$ (d) $\frac{k_2 A}{k_1 + k_2}$ **[2009]**

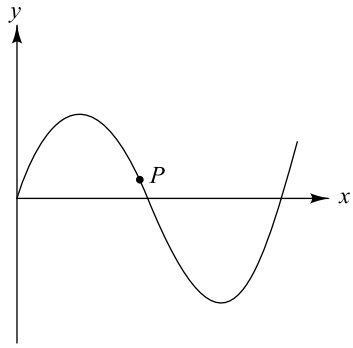
52. The ends of a rod of length l and mass m are attached to two identical springs as shown in the figure. The rod is free to rotate about its centre O. The rod is depressed slightly at end A and released. The time period of the resulting oscillation is



(a) $2\pi\sqrt{\frac{m}{2k}}$ (b) $2\pi\sqrt{\frac{2m}{k}}$

(c) $\pi\sqrt{\frac{2m}{3k}}$ (d) $\pi\sqrt{\frac{3m}{2k}}$ **[2009]**

53. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm s^{-1}. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap-shot of the wave is shown in the figure. The velocity of point P when its displacement is 5 cm is



(a) $\frac{\pi\sqrt{3}}{50}\hat{j}\,\text{ms}^{-1}$ (b) $-\frac{\pi\sqrt{3}}{50}\hat{j}\,\text{ms}^{-1}$

(c) $\frac{\pi\sqrt{3}}{50}\hat{i}\,\text{ms}^{-1}$ (d) $-\frac{\pi\sqrt{3}}{50}\hat{i}\,\text{ms}^{-1}$

[2009]

54. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is
(a) 5 grams (b) 10 grams
(c) 20 grams (d) 40 grams **[2010]**

55. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by $y = 0.02\text{ (m)}\sin\left[2\pi\left(\frac{t}{0.04(\text{s})}-\frac{x}{0.50(\text{m})}\right)\right]$. The tension in the string is
(a) 4.0 N (b) 12.5 N
(c) 0.5 N (d) 6.25 N **[2010]**

56. A mass M attached to a horizontal spring, executes SHM with amplitude A_1. When mass M passes through the mean position, a small mass m is gently placed over it and both of them together move with amplitude A_2. The ratio A_1/A_2 is
(a) $\frac{M}{M+m}$ (b) $\frac{M+m}{M}$
(c) $\left(\frac{M}{M+m}\right)^{1/2}$ (d) $\left(\frac{M+m}{M}\right)^{1/2}$ **[2011]**

57. The transverse displacement $y\,(x,t)$ of a wave on a string is given by $y\,(x,t) = e^{[-(ax^2+bt+2\sqrt{ab}\,xt)]^{1/2}}$ This represents a
(a) wave moving in $+x$ direction with speed $\sqrt{\frac{a}{b}}$
(b) standing wave of frequency $\sqrt{\frac{b}{a}}$
(c) standing wave of frequency $\sqrt{b}$
(d) standing wave of frequency $\frac{1}{\sqrt{b}}$ **[2011]**

58. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0 (X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motions is:
(a) $\pi/2$ (b) $\pi/3$
(c) $\pi/4$ (d) $\pi/6$ **[2011]**

59. If a simple pendulum has significant amplitude (up to a factor of $1/e$ of original) only in the period between $t = 0$ s to t = τ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with b as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds
(a) $0.693/b$ (b) b
(c) $1/b$ (d) $2/b$ **[2012]**

60. A cylindrical tube, open at both ends, has a fundamental frequency, f, in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now
(a) f (b) $f/2$
(c) $3f/4$ (d) $2f$ **[2012]**

61. A student is performing the experiment of resonance column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance Column tube. When the first resonance occurs, the reading of the water level in the column is
(a) 14.0 cm (b) 15.2 cm
(c) 16.4 cm (d) 17.6 cm **[2012]**

62. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of wire if density and elasticity of steel are 7.7×10^3 and 2.2×10^{11} N/m^2 respectively?
(a) 178.2 Hz (b) 200.5 Hz
(c) 770 Hz (d) 188.5 Hz **[2013]**

63. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and

released. Assuming that the system is completely isolated from its surroundings, the piston executes a simple harmonic motion with frequency

(a) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$ (b) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A^2\gamma P_0}}$ (d) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ **[2013]**

64. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to α times its original magnitude, where α equals.

(a) 0.81 (b) 0.729

(c) 0.6 (d) 0.7 **[2013]**



Answers

1. (c)	2. (b)	3. (b)	4. (b)
5. (b)	6. (b)	7. (b)	8. (b)
9. (a)	10. (c)	11. (c)	12. None
13. (a)	14. (c)	15. (a)	16. (c)
17. (a)	18. (a)	19. (b)	20. (b)
21. (c)	22. (b)	23. (d)	24. (b)
25. (b)	26. (a)	27. (d)	28. (c)
29. (b)	30. (c)	31. (d)	32. (a)
33. (d)	34. (a)	35. (c)	36. (c)
37. (d)	38. (b)	39. (a)	40. (d)
41. (d)	42. (d)	43. (a)	44. (b)
45. None	46. (c)	47. (b)	48. (a)
49. (d)	50. (c)	51. (d)	52. (c)
53. (a)	54. (b)	55. (d)	56. (d)
57. (b)	58. (b)	59. None	60. (a)
61. (b)	62. (a)	63. (b)	64. (b)



Solutions

1. $\nu_A = \frac{v}{2L}$ and $\nu_B = \frac{v}{4L}$

$\therefore \quad \frac{\nu_A}{\nu_B} = \frac{2}{1}$

2. Refer to the solution of Example 31 of Section I. The correct choice is (b).
3. Choices (c) and (d) are incorrect as they represent a wave travelling in the same direction as the given wave. Hence they cannot give standing waves. At a node, the resultant displacement is zero. If the other wave is given by choice (1), the resultant displacement is

$y = a \sin(\omega t - kx) + a \sin(\omega t + kx)$

At $x = 0$, $y = a \sin \omega t + a \sin \omega t = 2a \sin \omega t$ which is not zero for all values of t. Hence choice (1) is also incorrect. The superposition of the given wave with the wave given in choice (b) gives

$y = a \sin(\omega t - kx) - a \sin(\omega t + kx)$

At $x = 0$, $y = a \sin \omega t - a \sin \omega t = 0$. Hence the correct choice is (b).

4. The question is wrongly stated. Stationary waves are not travelling waves and hence they do not have a wavelength. Since the frequency of the fundamental mode is minimum and since $v = \nu\lambda$ or $\lambda = \frac{v}{\nu}$, minimum ν corresponds to maximum λ.

$\lambda_{max} = 2L = 2 \times 40 \text{ cm} = 80 \text{ cm}$

5. With the increase in temperature, the length of the prongs increases. Hence the frequency of the fork decreases.
6. When the child stands up, the moment of inertia I of the child-swing system decreases. From the conservation of angular momentum about the point of support, $I\omega$ = constant. If I decreases, ω increases.

Now $\omega = \frac{2\pi}{T}$. Hence time period T will decrease.

7. In the simple harmonic motion of a pendulum, the restoring force vector (and hence the acceleration vector) is tangential to the path of the bob and is directed towards the mean position. Hence the corrected choice is (b).

8. For train: $\nu_A = \nu\left(\frac{v + u_A}{v}\right)$ or $5.5 = 5\left(\frac{v + u_A}{v}\right)$

$\Rightarrow \quad u_A = 0.1v$

For train B: $\nu_B = \nu\left(\frac{v + u_B}{v}\right)$ or $6.0 = 5\left(\frac{v + u_B}{v}\right)$

$\Rightarrow \quad u_B = v/5 = 0.2v$

$\therefore \quad \frac{u_B}{u_A} = \frac{0.2v}{0.1v} = 2.$

9. Let L be the length of the wire between the bridges and let m be the mass per unit length of the wire. Five antinodes on a length L implies that $L = \frac{5}{2}\lambda_1$ or $\lambda_1 = \frac{2L}{5}$. Thus, in this case we have

$$\nu_1 = \frac{v_1}{\lambda_1} = \frac{5}{2L}\sqrt{\frac{T_1}{m}}, \text{ where } T_1 = M_1 g$$

Three antinodes on a length L implies that $L = \frac{3}{2}\lambda_2$ or $\lambda_2 = \frac{2L}{3}$. In this case, we have

$$\nu_2 = \frac{v_2}{\lambda_2} = \frac{3}{2L}\sqrt{\frac{T_2}{m}}, \text{ where } T_2 = M_2 g$$

Given $\nu_1 = \nu_2$, $M_1 = 9$ kg and $M_2 = M$. Therefore, we have

$$\frac{5}{2L}\sqrt{\frac{9g}{m}} = \frac{3}{2L}\sqrt{\frac{Mg}{m}}$$

which gives $M = 25$ kg

10. $T = 2\pi\sqrt{\frac{Mg}{k}}$

$$\frac{5T}{3} = 2\pi\sqrt{\frac{(M+m)g}{k}}$$

Dividing we get

$$\frac{5}{3} = \sqrt{\frac{M+m}{k}} \Rightarrow \frac{25}{9} = \frac{M+m}{m}$$

which gives $\frac{m}{M} = \frac{16}{9}$

11. $V_{max} = A\omega = A\sqrt{\frac{k}{m}}$

$\therefore$ V_{max} of body A $= A_1\sqrt{\frac{k_1}{m}}$

V_{max} of body B $= A_2\sqrt{\frac{k_2}{m}}$

Equating the two, we get $\frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$.

12. Let L be the original length of the pendulum. The new length is $L' = L + 0.21\,L = 1.21\,L$.

$$T' = 2\pi\sqrt{\frac{L'}{g}}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$\therefore$ $\frac{T'}{T} = \sqrt{\frac{L'}{L}} = \sqrt{\frac{1.21\,L}{L}} = \sqrt{1.21} = 1.1$

$\therefore$ $\frac{(T'-T)}{T} \times 100 = 10\%$

No choice given in the question is correct.

13. Comparing the given equation with

$y = A\sin(\omega t - kx + \phi)$

we get $\omega = 600$ s^{-1} and $k = 2$ m^{-1}.

Wave speed $v = \frac{\omega}{k} = \frac{600}{2} = 300$ ms^{-1}

14. If $x = a\cos\omega t + b\sin\omega t$, the amplitude of motion is

$$A = \sqrt{a^2 + b^2}$$

Here $a = b = 4$. Hence $A = \sqrt{4^2 + 4^2} = 4\sqrt{2}$

15. Given $m = 9.8$ g m^{-1} $= 9.8 \times 10^{-3}$ kg m^{-1}, tension $T = 10 \times 9.8 = 98$ N and $L = 1$ m. The wire vibrates in the fundamental mode.

Frequency of vibration is

$$n = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2\times 1}\sqrt{\frac{98}{9.8\times 10^{-3}}} = 50 \text{ Hz}$$

16. $\nu_1 = 256$ Hz. Let ν_2 be the frequency of the piano string. Then

$\nu_2 = 256 \pm 5$

$= 261$ Hz or 251 Hz

The frequency of vibration of a string is proportional to $\sqrt{T}$, where T is the tension. If T is increased slightly ν_2 will also increase. If we take ν_2 to be 261 Hz, then, on increasing T, ν_2 will become greater than 261 Hz. Hence beat frequency will increase. But it is given that the beat frequency decreases to 2 beats per second. Therefore, $\nu_2 = 261$ Hz is not possible. Hence $\nu_2 = 251$ Hz $= (256 - 5)$ Hz before the tension was increased.

17. KE is maximum when $x = 0$ and PE is maximum when x is maximum. TE is always constant. Hence the correct choice is (1).

18. Given $x = A\cos\omega t$. As a function of x, the PE is given by

$$\text{PE} = \frac{1}{2}m\omega^2 x^2$$

At $x = 0$, PE $= 0$. Hence the correct graph is III. As a function of t, the PE is given by

$$\text{PE} = \frac{1}{2}m\omega^2(A\cos\omega t)^2$$

$$= \frac{1}{2}m\omega^2 A^2\cos^2\omega t$$

At $t = 0$, PE is maximum equal to $\frac{1}{2}m\omega^2 A^2$

Hence the correct graph is 1. Thus, the correct choice is (a).

19. Given $v = 330$ ms^{-1} and $u_p = 22$ ms^{-1}. The apparent frequency of the police man's horn of frequency 176 Hz as heard by the motorcyclist is given by

$$\nu_1 = 176\left(\frac{330-u_m}{330-22}\right) = \frac{176}{308}(330-u_m) \qquad \text{(i)}$$

The apparent frequency of the stationery siren of frequency of 165 Hz is as heard by the motorcyclist is given by

$$\nu_2 = 165\left(\frac{330+u_m}{330}\right) \qquad \text{(ii)}$$

Since the motorcyclist does not observe any beats, $\nu_1 = \nu_2$. Equating (i) and (ii) and solving for u_m, we get $u_m = 22$ ms^{-1}.

20. Let ν be the frequency of the tuning fork and e the end correction. Given $L_1 = 0.1$ m $L_2 = 0.35$ m

$$\nu = \nu_1 = \frac{1}{4(L_1+e)}\sqrt{\frac{\gamma P}{\rho}}$$

$$\nu = \nu_3 = \frac{3}{4(L_2+e)}\sqrt{\frac{\gamma P}{\rho}}$$

Equating (i) and (ii), we get

$$\frac{1}{L_1+e} = \frac{3}{L_2+e}$$

or $$\frac{1}{0.1+e} = \frac{3}{0.35+e}$$

which gives $e = 0.025$ m

21. When the pendulum oscillates in air, the time period is

$$t_0 = 2\pi\sqrt{\frac{L}{g}} \qquad \text{(i)}$$

When the pendulum oscillates in water, the time period will be

$$t = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}} \qquad \text{(ii)}$$

where g_{eff} is found a follows. Let V and σ be the volume and density of the bob respectively. If ρ is the density of water, the upthrust is

U = weight of water displaced by the bob

$= \rho V g$

$\therefore$ Apparent weight of bob immersed in water is

$\sigma V g_{\text{eff}} = \sigma V g - \rho V g$

$$\Rightarrow \qquad g_{\text{eff}} = g\left(\frac{\sigma-\rho}{\sigma}\right) \qquad \text{(iii)}$$

Dividing (i) and (ii) we get

$$\frac{t}{t_0} = \sqrt{\frac{g}{g_{\text{eff}}}} = \left(\frac{\sigma}{\sigma-\rho}\right)^{1/2} \qquad \text{[use Eq. (iii)]}$$

$$= \left(\frac{\frac{4}{3}\times 1000}{\frac{4}{3}\times 100 - 1000}\right)^{1/2} = 2$$

$$\Rightarrow \qquad t = 2t_0$$

22. For the first spring,

$$t_1 = 2\pi\sqrt{\frac{m}{k_1}} \qquad \text{(i)}$$

For the second spring,

$$t_2 = 2\pi\sqrt{\frac{m}{k_2}} \qquad \text{(ii)}$$

When the springs are connected in series, the equivalent spring constant k is given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

The time period of series combination is

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \text{(iii)}$$

From (i) and (ii) we have

$$t_1^2 + t_2^2 = 4\pi^2 m\left(\frac{1}{k_1}+\frac{1}{k_2}\right)$$

$$= \frac{4\pi^2 m}{k} = T^2 \qquad \text{[use Eq. (iii)]}$$

23. In simple harmonic motion, the kinetic energy and potential energy both vary with displacement x but the total energy is the same for all values of x if friction is neglected. Hence the correct choice is (d).

24. Here $\omega = 100$ s^{-1} and $k = 20$ m^{-1}

$$\therefore \qquad v = \frac{\omega}{k} = \frac{100}{20} = 5 \text{ ms}^{-1}$$

25. Let $F(t) = F_0 \cos(\omega t)$ where F_0 is the maximum value of the externally applied force. In the steady state, the motion of the forced oscillator is simple harmonic but its frequency is equal to the frequency of the external force (which is ω). Hence

$$x = A\cos(\omega t) \qquad \text{(i)}$$

The equation of the forced oscillator is

$$ma = -m\omega_0^2 x + F(t)$$

$$\Rightarrow \; m\frac{d^2x}{dt^2} = -m\omega_0^2 x + F_0\cos(\omega t) \qquad \text{(ii)}$$

Differentiating (i) w.r.t. time t twice we have

$$\frac{dx}{dt} = -A\omega\sin(\omega t)$$

and $\frac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t)$ (iii)

Using (iii) in (ii) we have [using Eq. (i)]

$-mA\omega^2 \cos(\omega t) = -m\omega_0^2 A \cos(\omega t) + F_0 \cos(\omega t)$

$\Rightarrow mA(\omega_0^2 - \omega^2) = F_0$

$\Rightarrow \quad A = \frac{F_0}{m(\omega_0^2 - \omega^2)}$

Hence $A \propto \frac{1}{m(\omega_0^2 - \omega^2)}$

26. In forced oscillations, the amplitude as well as the energy of the oscillator is maximum at resonance. Resonance occurs if the frequency of the applied force is equal to the frequency of the oscillator, i.e. when $\omega_1 = \omega_2 = \omega_0$. Hence the correct choice is (a).

27. In simple harmonic motion, the force acting on the particle (restoring force) is given by

$$F = -kx$$

where k is a positive constant. Now

$$F = -\frac{dU}{dx}$$

Therefore, $-kx = -\frac{dU}{dx}$

or $dU = kdx$

$\therefore \quad U(x) = \int_0^x kdx = \frac{1}{2}kx^2 + c$

where c is a constant of integration.

In simple harmonic motion, the potential energy of the oscillator is zero at the mean position, i.e. $U(0) = 0$. Hence $c = 0$. Therefore

$$U(x) = \frac{1}{2}kx^2$$

which is the equation of a parabola. Since $U(x)$ is positive for all values (positive and negative) of x, the correct graph is (d).

28. Original time period is

$$T_1 = 2\pi\sqrt{\frac{l}{g}} \qquad (1)$$

When the pendulum is moving upwards, the effective value of g is

$$g_{eff} = g + a$$

where a is the acceleration of the pendulum which is given by

$$a = \frac{dv}{dt} = \frac{d}{dt}(Kt) = K = 4.4\,\text{ms}^{-2}$$

$\therefore \quad g_{eff} = g + a = 10 + 4.4 = 14.4\ \text{ms}^{-2}$

Therefore, the new time period is

$$T_2 = 2\pi\sqrt{\frac{1}{g_{eff}}} \qquad (2)$$

From (1) and (2), we get

$$\frac{T_2}{T_1} = \sqrt{\frac{g}{g_{eff}}} = \sqrt{\frac{10}{14.4}} = \frac{1}{1.2}$$

or $T_2 = \frac{T_1}{1.2} = \frac{3}{1.2} = 2.5\,\text{s}$

29. The function $f(t) = \sin^2(\omega t)$ is never negative for any value of t. Hence it cannot represent a simple harmonic motion.

$$f(t) = \sin^2(\omega t) = \frac{1}{2}[1 - \cos(2\omega t)].$$

Since $\cos\left[2\omega\left(t + \frac{\pi}{\omega}\right)\right] = \cos(2\omega t + 2\pi) = \cos 2\omega t$, it follows that $\cos(2\omega t)$ is periodic with period $= \frac{\pi}{\omega}$. Hence the correct choice is (b).

30. $\nu_2 = \nu_1 \pm 4 = 200 \pm 4 = 196$ or 204 Hz. When some tape is attached to the prong, the frequency of the fork decreases. If $\nu_2 = 204$ Hz, it would decrease when tape is attached to it. Hence, when sounded with $\nu_1 = 200$ Hz, it would produce fewer beats per second. But it is given that the beat frequency increases to 6 beats per second. Hence ν_2 cannot be 204 Hz. Therefore, ν_2 must be 196 Hz.

31. Acceleration $a = \frac{d^2x}{dt^2} = -\alpha x$. Comparing it with $a = -\omega^2 x$, we get

$\omega^2 = \alpha \Rightarrow \omega = \sqrt{\alpha} \Rightarrow \frac{2\pi}{T} = \sqrt{\alpha} \Rightarrow T = \frac{2\pi}{\sqrt{\alpha}}$

32. Refer to the solution of Q.2. on page 10.43. The correct choice is (a).

33. $u_0 = \frac{v}{5} \Rightarrow \frac{u_0}{v} = \frac{1}{5}$

$$\nu' = \nu_0\left(1 + \frac{u_0}{v}\right) = \nu_0\left(1 + \frac{1}{5}\right) = 1.2\ \nu_0$$

$\therefore$ Percentage increase in frequency is

$$\left(\frac{\nu' - \nu_0}{\nu_0}\right) \times 100 = (1.2 - 1) \times 100 = 20\%$$

34. The frequency of sound as heard by the stationary person is

$$\nu' = \left(\frac{v}{v - u_s}\right)$$

$$\therefore \quad 10{,}000 = 9500\left(\frac{300}{300 - u_s}\right)$$

which gives us = 15 ms^{-1}.

35. Let ν_m = 420 Hz be the mth harmonic and ν_n = 315 Hz be the nth harmonic. Then

$$420 = \frac{m}{2L}\sqrt{\frac{T}{\mu}} \qquad (1)$$

and $$315 = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \qquad (2)$$

where μ is the mass per unit length of the string. Dividing (1) and (2), we have

$$\frac{420}{315} = \frac{m}{n} \quad \text{or} \quad \frac{4}{3} = \frac{m}{n}$$

Hence the lowest values of m and n are m = 4 and n = 3, i.e. 420 Hz is the 4th harmonic and 315 Hz is the 3rd harmonic.

$$\therefore \quad 420 = \frac{4}{2L}\sqrt{\frac{T}{\mu}} \quad \text{or} \quad \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{420}{4} = 105$$

Now, the lowest resonant frequency is the fundamental frequency, which is given by

$$\nu_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = 105 \text{ Hz}$$

36. Since the body starts from the origin, its displacement $x = 0$ at time $t = 0$. Hence the phase constant $\phi = 0$. Now, the displacement of a body in simple harmonic motion is given by (sine $\phi = 0$)

$$x = A \sin(\omega t + \phi) = A \sin \omega t \qquad (1)$$

Kinetic energy $$E_k = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

Total energy $$E = \frac{1}{2} m\omega^2 A^2$$

$$\therefore \quad \frac{E_k}{E} = \frac{A^2 - x^2}{A^2}$$

Given $$\frac{E_k}{E} = 75\% = \frac{3}{4}$$. Hence

$$\frac{3}{4} = \frac{A^2 - x^2}{A^2}$$

which gives $x = \frac{A}{2}$. Using this is Eq. (1), we have

$$\frac{A}{2} = A \sin(\omega t)$$

which gives $\sin \omega t = \frac{1}{2}$ or $\omega t = \frac{\pi}{6}$ or $\frac{2\pi t}{T} = \frac{\pi}{6}$

Therefore,

$$t = \frac{T}{12} = \frac{2s}{12} = \frac{1}{6} \text{ s.}$$

37. The coin will lose contact with the platform if the amplitude A of oscillation is such that at the extreme top-most position, the maximum acceleration equals the acceleration due to gravity, i.e. if

$$\omega^2 A = g$$

or $$A = g/\omega^2.$$

38. The blocks will move together as long as the frictional force of block B = mass of block B × maximum acceleration of its S.H.M., i.e.

$$f = m\,\omega^2 a$$

where $$\omega = \sqrt{\frac{k}{(m+m)}} = \sqrt{\frac{k}{2m}}$$

Thus $$f = m \times \frac{k}{2m} \times a$$

$$= ka/2$$

39. $x = 2 \times 10^{-2} \cos \pi t$

The magnitude of velocity of the object is

$$|v| = \left|\frac{dx}{dt}\right| = 2 \times 10^{-2} \times \pi \sin \pi t$$

$\therefore$ $|v|$ is maximum when $\sin \pi t = 1$ which gives

$$\pi t = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$$

For first maximum $\pi t = \frac{\pi}{2}$ or t = 0.5 s

40. Given $x = x_0 \cos(\omega t - \pi/4)$

$\therefore$ Velocity of particle is

$$v = \frac{dx}{dt} = -x_0\,\omega \sin(\omega t - \pi/4)$$

Acceleration of particle is

$$a = \frac{dv}{dt} = -x_0\,\omega^2 \cos(\omega t - \pi/4)$$

$$= x_0\,\omega^2 \cos(\omega t - \pi/4 + \pi)$$

$$= x_0\,\omega^2 \cos(\omega t + 3\pi/4) \qquad (1)$$

It is given that

$$a = A \cos(\omega t + \delta) \qquad (2)$$

Comparing Eqs. (1) and (2), we get

$$A = x_0\,\omega^2 \text{ and } \delta = \frac{3\pi}{4}$$

41. The intensity level in decibel of a sound of intensity I is given by

$$\text{dB} = 10 \log\left(\frac{I}{I_0}\right)$$

where $I_0 = 1 \times 10^{-12}$ Wm^{-2} is the intensity threshold. Now

$$(\text{dB})_1 = 10 \log\left(\frac{I_1}{I_0}\right)$$

and $$(\text{dB})_2 = 10 \log\left(\frac{I_2}{I_0}\right)$$

Given $(\text{dB})_1 - (\text{dB})_2 = 20$ dB. Hence

$$20 = 10 \log\left(\frac{I_1}{I_0}\right) - 10 \log\left(\frac{I_2}{I_0}\right)$$

$$\Rightarrow \qquad 2 = \log\left(\frac{I_1}{I_2}\right)$$

which gives $\frac{I_1}{I_2} = 10^2 = 100$. Hence $\frac{I_2}{I_1} = \frac{1}{100}$, i.e. the intensity decreases by a factor of 100.

42. $f = \frac{1}{2\pi}\left(\frac{k_1 + k_2}{m}\right)^{1/2}$

When k_1 and k_2 are made four times, the new frequency of oscillation becomes

$$f' = \frac{1}{2\pi}\left(\frac{4k_1 + 4k_2}{m}\right)^{1/2}$$

$$= 2 \times \frac{1}{2\pi}\left(\frac{k_1 + k_2}{m}\right)^{1/2} = 2f$$

43. Displacement $x = a \sin(\omega t + \phi)$

Velocity $v = \frac{dx}{dt} = a\omega \cos(\omega t + \phi)$

$\therefore$ Kinetic energy $K = \frac{1}{2} mv^2 = \frac{1}{2} ma^2 \omega^2 \cos^2(\omega t + \phi)$

It T is the time period of the simple harmonic motion, the time taken by the particle to go from the mean position to the extreme position is $T/4$. The average kinetic energy during time $t = 0$ to $t = T/4$ is given by

$$K_{av} = \frac{\int_0^{T/4} K\, dt}{\int_0^{T/4} dt} = \frac{\int_0^{T/4} \frac{1}{2} ma^2\omega^2 \cos^2(\omega t + \phi)\, dt}{T/4}$$

$$= \frac{2ma^2\omega^2}{T} \int_0^{T/4} \cos^2(\omega t + \phi) dt$$

$$= \frac{ma^2\omega^2}{T} \int_0^{T/4} [1 + \cos\{2(\omega t + \phi)\}] dt$$

$$= \frac{ma^2\omega^2}{T}\left(\frac{T}{4} + 0\right)$$

$$= \frac{ma^2(2\pi\nu)^2}{4} = \pi^2 ma^2 \nu^2$$

44. Given $y = t^2$. The velocity of the lift varies with t as

$$v = \frac{dy}{dt} = 2t$$

$\therefore$ Acceleration $a = \frac{dv}{dt} = 2$ ms^{-2}, directed upwards,

Hence $$T' = 2\pi\sqrt{\frac{1}{g + a}}$$

and $$T = 2\pi\sqrt{\frac{1}{g}}$$

$\therefore$ $$\frac{T'}{T} = \sqrt{\frac{g}{g + a}} = \sqrt{\frac{10}{(10 + 2)}} = \sqrt{\frac{5}{6}}$$

45. $v = \sqrt{\frac{\gamma RT}{M}}$

For oxygen (being diatomic) $\gamma = \frac{7}{5}$ and $M = 32$.

For helium (being monoatomic) $\gamma = \frac{5}{3}$ and $M = 4$. Therefore,

$$v_{\text{He}} = 460 \times \sqrt{\frac{5 \times 32 \times 5}{3 \times 4 \times 7}} = 1419 \text{ ms}^{-1}$$

All the four choices given in the question are incorrect.

46. Let v be the speed of sound in winter and v' in summer.

For first resonance: $\frac{v}{4l} = N$, the frequency of tuning fork.

For second resonance: $\frac{3v'}{4x} = N$. Hence

$$\frac{v}{4l} = \frac{3v'}{4x} \Rightarrow x = 3l\left(\frac{v'}{v}\right) = (3 \times 18 \text{ cm})\left(\frac{v'}{v}\right)$$

$$= 54 \text{ cm} \times \left(\frac{v'}{v}\right)$$

As the speed of sound increases with temperature, $v' > v$. Hence $x > 54$ cm.

47. For a wave travelling along the x-axis,

$$y(x, t) = A \cos\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right)$$

The given equation is

$$y(x, t) = 0.005 \cos(\alpha x - \beta t)$$

Equating the coefficients of x and t in the two equations,

we get $$\alpha = \frac{2\pi}{\lambda} = \frac{2\pi}{0.08} = 25\,\pi$$

and $$\beta = \pi$$

48. Let a_0 be the amplitude of each wave. The resultant particle displacement due to superposition of the three waves is given by

$$y = y_1 + y_2 + y_3$$
$$= a_0 \sin\{2\pi(\nu - 1)t\} + a_0 \sin(2\pi\nu t) + a_0 \sin\{2\pi(\nu + 1)t\}$$
$$= 2a_0 \sin(2\pi\nu t)\cos 2\pi t + a_0 \sin(2\pi\nu t)$$
$$= a_0(2\cos 2\pi t + 1)\sin(2\pi\nu t)$$
$$\Rightarrow \quad y = A \sin(2\pi\nu t)$$

where $A = a_0(2\cos 2\pi t + 1)$ is the resultant amplitude. Since intensity $\propto$ (amplitude)^2, the resultant intensity is given by

$$I = I_0(2\cos 2\pi t + 1)^2$$

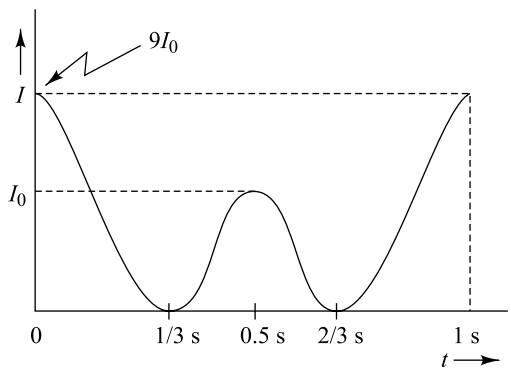
(i) When $\cos 2\pi t = +1 \Rightarrow 2\pi t = 0, 2\pi, \ldots \Rightarrow t = 0$, 1s, ... then $I = 9I_0$

(ii) When $\cos 2\pi t = -1 \Rightarrow, 2\pi t = \pi, 3\pi, \ldots \Rightarrow t = 0.5$s, 1.5s, ... then $I = I_0$

(iii) When $\cos 2\pi t = 0 \Rightarrow 2\pi t = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots \Rightarrow t =$ 0.25s, 0.75s, ... then $I = I_0$

(iv) When $\cos 2\pi t = -\frac{1}{2} \Rightarrow 2\pi t = \frac{2\pi}{3}, \frac{4\pi}{3}, \ldots \Rightarrow t = \frac{1}{3}$ s, $\frac{2}{3}$ s, ... then $I = 0$

So the graph of I versus t (from $t = 0$ to $t = 1$s) is as shown in the figure.



Time period between two consecutive maxima is $T_b = 0.5$. Therefore, frequency of beats is

$$\nu_b = \frac{1}{t_b} = \frac{1}{0.5} = 2 \text{ Hz}$$

Thus 2 beats are produced per second.

49. Let t be the time at which the driver hears the sound of the siren. His speed at time t is $v_0 = 0 + at = 2t$ ms^{-1}. The apparent frequency is 94% of the real frequency (ν). Therefore

$$\frac{94}{100} \times \nu = \nu\left(\frac{v - v_0}{v}\right) \Rightarrow 0.94 = \frac{330 - 2t}{330} \Rightarrow t = 9.9 \text{ s}$$

Distance moved by the motor cycle in time $t = 9.9$ s is $S = \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times (9.9)^2 \simeq 98$ m.

50. If the displacement x is taken from the equilibrium position (as is normally the case) then

$$x = A \sin(\omega t + \phi) \text{ where } \omega = \left(\frac{2\pi}{T}\right)$$

Then $\quad v = \frac{dx}{dt} = A\,\omega\cos(\omega t + \phi)$

and $\quad a = \frac{dv}{dt} = -A\omega^2 \sin(\omega t + \phi)$

Choice (1): $aT + 2\pi v = -A\omega^2 \sin(\omega t + \phi) \times \frac{2\pi}{\omega} + 2\pi A\,\omega\cos(\omega t + \phi)$

Choice (2): $\frac{aT}{v} = -\frac{A\omega^2 \sin(\omega t + \phi)}{A\cos(\omega t + \phi)} \times \frac{2\pi}{\omega}$

$$= 2\,\pi\tan(\omega t + \phi)$$

Choice (3): $a^2T^2 + 4\pi^2 v^2 = A^2\,\omega^2 \sin^2(\omega t + \phi) \times \frac{4\pi^2}{\omega^2} + 4\pi^2 A^2\,\omega^2 \cos^2(\omega t + \phi)$

$= 4\pi^2 A^2\,\omega^2$, which is independent of t.

Choice (4): $\frac{aT}{x} = \frac{-A\omega^2 \sin(\omega t + \phi)}{A\sin(\omega t + \phi)} \times \frac{2\pi}{\omega}$

$= -2\pi\omega$, which is independent of t.

Hence choices (3) and (4) are both correct. But if the displacement x is measured from a reference point P which is a distance x_0 from the equilibrium position of the oscillator, then

$$x = A \sin(\omega t + \phi) + x_0$$
$$v = A\,\omega\cos(\omega t + \phi)$$
$$a = -A\,\omega^2 \sin(\omega t + \phi)$$

In this case,

$$\frac{aT}{x} = \frac{-A\omega^2 \sin(\omega t + \phi)}{[A\sin(\omega t + \phi) + x_0]} \times \frac{2\pi}{\omega}$$

which changes with time. Thus, in the general case, the only correct choice is (c).

51. If a force F is applied to M, say to the right, let A be the distance moved by M. If the system is released, it executes simple harmonic motion of amplitude A. IF A_1 and A_2 are the extensions in springs k_1 and k_2 then $A = (A_1 + A_2)$ and

$$F = k_1 A_1 = k_2 A_2$$

$$\Rightarrow \quad A_1 = \frac{F}{k_1} \quad \text{and} \quad A_2 = \frac{F}{k_2}$$

$$\therefore \quad A = A_1 + A_2 = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$= \frac{F(k_1 + k_2)}{k_1 k_2}$$

$$\Rightarrow \qquad F = \frac{k_1 k_2 A}{(k_1 + k_2)}$$

The amplitude of point P = amplitude of oscillation of spring k_1 which is

$$A_1 = \frac{F}{k_1} = \frac{k_2 A}{(k_1 + k_2)}$$

52. Refer to the solution of Q 82 on page 10.54.

53. In a transverse wave, the particle displacement and particle velocity are perpendicular to the direction of propagation of the wave. Hence choices (c) and (d) are wrong. The particle displacements is given by

$$y = a \sin\left[\frac{2\pi}{\lambda}(vt - x)\right] \qquad (1)$$

Putting y =5 cm and a =10 cm in Eq. (1), we get

$$\sin\left[\frac{2\pi}{\lambda}(vt - x)\right] = \frac{1}{2}$$

$$\Rightarrow \qquad \frac{2\pi}{\lambda}(vt - x) = \frac{\pi}{6}$$

Particle velocity is

$$V = \frac{dy}{dt} = a\left(\frac{2\pi}{\lambda} v\right)\cos\left[\frac{2\pi}{\lambda}(vt - x)\right] \qquad (2)$$

Given a = 0.1 m, v = 0.1 ms^{-1}, a = 0.1 m and $\lambda = 0.5$ m. Also $\frac{2\pi}{\lambda}(vt - x) = \frac{\pi}{6}$. Putting these values in Eq. (2), we get

$$V = \frac{\sqrt{3}\pi}{50} \text{ ms}^{-1} \text{ along the } y\text{-axis.}$$

54. Let l be the length of the string and L be the length of the pipe. The frequency of the string vibrating in the second harmonic is

$$\nu = \frac{2}{2l}\sqrt{\frac{T}{\mu}} \text{ ; } \mu = \text{mass per unit length of the string}$$

The fundamental frequency of the closed pipe is

$$\nu' = \frac{v}{4L}; \; v = \text{speed of sound}$$

For resonance, $\nu = \nu'$, i.e.

$$\frac{1}{l}\sqrt{\frac{T}{\mu}} = \frac{v}{4L}$$

Substituting the values of of l, T, v and L, we get

$$\mu = \frac{1}{50} \text{ kg m}^{-1}$$

$\therefore$ Mass of string = $\mu l = \frac{1}{50} \times 0.5 = 10^{-2}$ kg = 10 grams. So the correct choice is (b).

55. $v = \sqrt{\frac{T}{\mu}}$; μ = mass per unit length of the string.

$$\Rightarrow \qquad T = \mu v^2 = \mu(\nu\lambda)^2$$

Given $\quad y = 0.02 \sin\left(\frac{2\pi t}{0.04} - \frac{2\pi x}{0.5}\right)$

$$\therefore \qquad 2\pi\nu = \frac{2\pi}{0.04} \Rightarrow \nu = 25 \text{ Hz}$$

and $\qquad \frac{2\pi}{\lambda} = \frac{2\pi}{0.5} \Rightarrow \lambda = 0.5$ m

$$\therefore \qquad T = 0.04 \times (25 \times 0.5)^2 = 6.25 \text{ N}$$

56. If V_{max} is the maximum velocity of mass M, then

$$V_{max} = A_1\omega_1 = A_1\sqrt{\frac{k}{M}}$$

If V'_{max} is the maximum velocity of the composite mass $(M + m)$, then

$$V'_{max} = A_2\omega_2 = A_2\sqrt{\frac{k}{M+m}}$$

From conservation of linear momentum,

$$MV_{max} = (M + m)\, V'_{max}$$

$$\Rightarrow \qquad MA_1\sqrt{\frac{k}{M}} = (M+m)A_2\sqrt{\frac{k}{M+m}}$$

$$\Rightarrow \qquad \frac{A_1}{A_2} = \sqrt{\frac{M+m}{M}}$$

57. Since $ax^2 + bt^2 + 2\sqrt{ab}\,xt = [\sqrt{a}x + \sqrt{b}t]^2$ and $\sqrt{-1} = i$, the given equation can be written as

$$y(x,t) = e^{i(\sqrt{a}x + \sqrt{b}t)} \qquad (1)$$

The real part of $y(x, t)$ is

$$R_e y(x,t) = \cos(\sqrt{a}x + \sqrt{b}t)$$

For a wave travelling along $-x$ direction,

$$y(x, t) = A \cos(kx + \omega t) \qquad (2)$$

Comparing (1) and (2), $k = \sqrt{a}$ and $\omega = \sqrt{b}$

$$\therefore \qquad v = \frac{\omega}{k} = \sqrt{\frac{b}{a}}$$

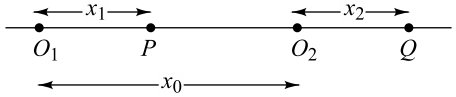
So the correct choice is (b).

58. Let the two simple harmonic motions be given by

$$x_1 = A \sin(\omega t + \phi_1) \quad \text{and}$$

$$x_2 = A \sin(\omega t + \phi_2)$$

Let O_1 and O_2 be their mean positions.



$$PQ = x_0 - x_1 + x_2 = x_0 - (x_1 - x_2)$$

Now $x_1 - x_2 = A\,[\sin(\omega t + \phi_1) - \sin(\omega t + \phi_2)]$

$$= 2A \sin\left(\frac{\phi_1 - \phi_2}{2}\right) \sin\left(\omega t + \frac{\phi_1 + \phi_2}{2}\right)$$

$$\therefore \quad PQ = X_0 - 2A \sin\left(\frac{\phi_1 - \phi_2}{2}\right) \sin\left(\omega t + \frac{\phi_1 + \phi_2}{2}\right)$$

PQ will be maximum when $\sin\left(\omega t + \frac{\phi_1 + \phi_2}{2}\right) = -1$

$$\therefore \quad (PQ)_{\max} = X_0 + 2A \sin\left(\frac{\phi_1 - \phi_2}{2}\right)$$

Given $(PQ)_{\max} = X_0 + A$. Therefore,

$$X_0 + A = X_0 + 2A \sin\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\Rightarrow \quad \sin\left(\frac{\phi_1 - \phi_2}{2}\right) = \frac{1}{2}$$

$$\Rightarrow \quad \frac{\phi_1 - \phi_2}{2} = \frac{\pi}{6} \Rightarrow \phi_1 - \phi_2 = \frac{\pi}{3}$$

59. Viscous drag $= -b\frac{dx}{dt}$ and restoring force $= -kx$. Therefore, the equal equation of the damped oscillator is

$$m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - kx$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \lambda\frac{dx}{dt} + \omega_0^2 x = 0 \qquad (1)$$

where $\lambda = \frac{b}{m}$ and $\omega_0 = \sqrt{\frac{k}{m}}$. Here λ is the damping coefficient. The solution of Eq. (1) is

$$x = A\,e^{\alpha t} \qquad (2)$$

where A and α are constants. Using (2) in (1), we have

$$A\alpha^2 e^{\alpha t} + \lambda A\alpha e^{\alpha t} + \omega_0^2 A\alpha e^{\alpha t} = 0$$

$$\Rightarrow \quad \alpha^2 + \lambda\alpha + \omega_0^2 = 0$$

The roots of this quadratic equation are

$$\alpha = -\frac{\lambda}{2} \pm \frac{1}{2}\sqrt{\lambda^2 - 4\omega_0^2} \qquad (3)$$

Using (3) in (2)

$$x = A \exp\left[-\frac{\lambda}{2} \pm \frac{1}{2}\sqrt{\lambda^2 - 4\omega_0^2}\right] t$$

If $\lambda \ll 0$, or $x = R\,e^{\pm i\omega t} = R\cos\omega t$

where $\omega = \sqrt{\lambda^2 - 4\omega_0^2}$ and $R = e^{-\lambda t/2}$. It is given that when $t = \tau$, $R = \frac{A}{e}$. Hence

$$\frac{A}{e} = A^{-\lambda\tau/2}$$

$$\Rightarrow \quad e^{-1} = e^{-\lambda\tau/2}$$

$$\Rightarrow \quad \frac{\lambda\tau}{2} = 1$$

$$\Rightarrow \quad \tau = \frac{2}{\lambda} = \frac{2m}{b}$$

None of the choices is correct.

60. For an open pipe of length L, the frequency of the fundamental mode is

$$f = \frac{v}{2L}$$

When the pipe is dipped in water so that half of it is in water, it acts as a closed pipe of length $L' = \frac{L}{2}$.

For a closed pipe, the frequency of the fundamental mode is

$$f' = \frac{v}{4L'} = \frac{v}{4 \times \frac{L}{2}} = \frac{v}{2L} = f$$

So the correct choice is (a).

61. End correction $e = 0.3d = 0.3 \times 4 = 1.2$ cm

Wavelength $\lambda = \frac{v}{\nu} = \frac{336}{512} = 0.656$ m $= 65.6$ cm

Now $\quad L + e = \frac{\lambda}{4}$

$$\Rightarrow \quad L = \frac{\lambda}{4} - e = \frac{65.6}{4} - 1.2$$

$$= 16.4 - 1.2 = 15.2 \text{ cm}$$

62. $\nu = \frac{1}{2L}\sqrt{\frac{F}{m}}$; F = tension and m = mass/length

$$= \frac{1}{2L}\sqrt{\frac{F}{A\rho}} = \frac{1}{2L}\sqrt{\frac{\text{stress}}{\rho}}$$

Strain $= 1\% = 10^{-2}$

$\therefore$ Stress $= Y \times$ strain $= 2.2 \times 10^{11} \times 10^{-2}$

$$= 2.2 \times 10^9 \text{ Nm}^{-2}$$

$$\therefore \quad \nu = \frac{1}{2 \times 1.5} \times \sqrt{\frac{2.2 \times 10^9}{7.7 \times 10^3}} = 178.2 \text{ Hz}$$

63.

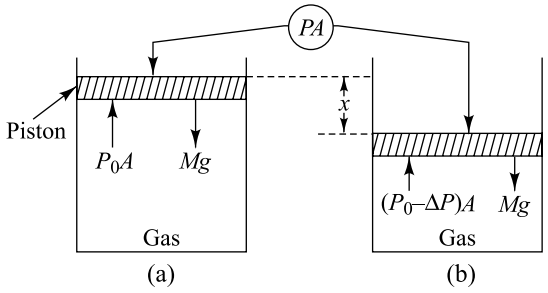


Figure (a) shows the equilibrium position of the piston and the forces acting on it. For equilibrium

$$P_0A + Mg = PA$$

where P = atmospheric pressure. When the piston is depressed by a small distance x (Fig. (b)) and released, the restoring force acting on it is

$$F = [PA - (P_0 - \Delta P)A - Mg]$$
$$= [PA + \Delta PA - (P_0 + Mg)]$$
$$= [PA + \Delta PA - PA] = \Delta PA$$

Acceleration $a = \dfrac{F}{M} = \dfrac{\Delta PA}{M}$ (1)

Now change in volume is $\Delta V = Ax$

Since the process is adiabatic,

$$PV^{\gamma} = \text{constant}$$

Differentiating partially,

$$P\gamma V^{\gamma-1}\Delta V + \Delta P V^{\gamma} = 0$$

$$\Rightarrow \quad \Delta P = -\frac{P\gamma\,\Delta V}{V} = -\frac{P_0\gamma Ax}{V_0} \qquad (2)$$

Using (2) in (1)

$$a = -\left(\frac{\gamma P_0 A^2}{MV_0}\right)x$$

Comparing with $a = -\omega^2 x$, we get

$$\omega = \left(\frac{\gamma P_0 A^2}{MV_0}\right)^{1/2}$$

$$\nu = \frac{1}{2\pi}\left(\frac{\gamma P_0 A^2}{MV_0}\right)^{1/2}$$

So the correct choice is (b).

64. The amplitude of a damped oscillator varies with time t as

$$A = A_0\,e^{-kt}$$

where A_0 = amplitude at $t = 0$ and k = damping constant. It is given that at $t = 5$s, $A = 0.9\,A_0$. Thus

$$0.9\,A_0 = A_0\,e^{-5k} \qquad (1)$$

Also $\alpha\,A_0 = A_0\,e^{-15k}$ (2)

From Eq. (1), $e^{-5k} = 0.9$. Therefore $e^{-15k} = (0.9)^3$

$\therefore \quad \alpha = (0.9)^3 = 0.729$

ELECTROSTATICS

Chapter 11

REVIEW OF BASIC CONCEPTS

1. Coulomb's Law

On the basis of his measurements, Coulomb arrived at a law, known after his name as Coulomb's law, which states that the *magnitude of the electric force between two charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distance between them,* i.e.

$$F \propto \frac{q_1 q_2}{r^2}$$

or
$$F = k \frac{q_1 q_2}{r^2}$$

In the SI system, k is written as $1/4\pi\varepsilon_0$ where ε_0 is called the permittivity of vacuum and its value is

$$\varepsilon_0 = 8.854 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$$

Then
$$k = \frac{1}{4\pi\varepsilon_0} \simeq 9 \times 10^9\ \text{Nm}^2\ \text{C}^{-2}$$

The force F is attractive for unlike charges ($q_1 q_2 < 0$) and repulsive for like charges ($q_1 q_2 > 0$).

Coulomb's Law in vector form

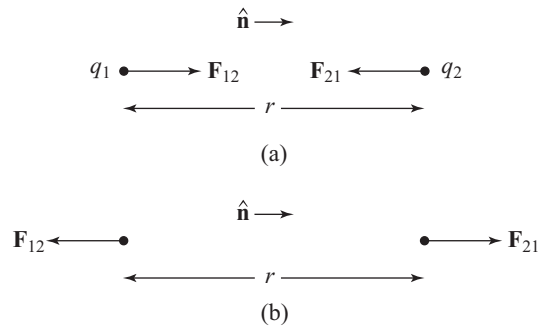


Fig. 11.1

Case (a): Unlike charges ($q_1 q_2 < 0$) [Fig. 11.1(a)]
Force exerted on q_2 by q_1 is

$$\mathbf{F}_{12} = \frac{q_1 q_2 \hat{\mathbf{n}}}{4\pi\varepsilon_0 r^2}$$

where $\hat{\mathbf{n}}$ is a unit vector directed from q_1 to q_2.
Force exerted by q_1 on q_2 is

$$\mathbf{F}_{21} = -\frac{q_1 q_2 \hat{\mathbf{n}}}{4\pi\varepsilon_0 r^2}$$

Case (b): Like charges ($q_1 q_2 > 0$) [Fig. 11.1(b)]

$$\mathbf{F}_{12} = -\frac{q_1 q_2 \hat{\mathbf{n}}}{4\pi\varepsilon_0 r^2}$$

$$\mathbf{F}_{21} = \frac{q_1 q_2 \hat{\mathbf{n}}}{4\pi\varepsilon_0 r^2}$$

2. Relative Permittivity (or Dielectric Constant)

Relative permittivity of a medium is defined as the ratio of the permittivity of the medium to permittivity of vacuum, i.e.

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

ε_r is also called the dielectric constant (K) of the medium. Thus $K = \dfrac{\varepsilon}{\varepsilon_0}$ or $\varepsilon = K\varepsilon_0$. By definition K for air = 1. If charges q_1 and q_2 are situated in a medium other than air or vacuum, the magnitude of force between them is given by

$$F = \frac{q_1 q_2}{4\pi\varepsilon r^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 K r^2}$$

3. Principle of Superposition

If many charges are present, the total force on a given charge is equal to the vector sum of the individual forces exerted on it by all other charges taken one at a time.

EXAMPLE 1 Two point charges $q_1 = +9\ \mu C$ and $q_2 = (-1)\ \mu C$ are held 10 cm apart. Where should at third charge $+Q$ be placed from q_2 on the line joining them so that charge Q does not experience any net force?

SOLUTION Charge Q will not experience any net force if the forces exerted on it by charges q_1 and q_2 are equal and in opposite directions.

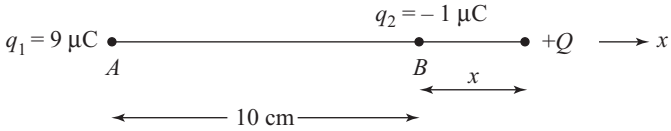


Fig. 11.2

It follows from Fig. 11.2 that charge Q will not experience forces in opposite direction if it lies at any point between AB. Let x be the distance of Q from q_2. Then forces exerted on Q by q_1 and q_2 respectively are

$$\mathbf{F}_1 = \frac{q_1 Q\,\hat{\mathbf{i}}}{4\pi\varepsilon_0 (0.1+x)^2} = \frac{9\times10^{-6} Q\,\hat{\mathbf{i}}}{4\pi\varepsilon_0 (0.1+x)^2}$$

and $$\mathbf{F}_2 = -\frac{q_2 Q\,\hat{\mathbf{i}}}{4\pi\varepsilon_0 x^2} = \frac{1\times10^{-6} Q\,\hat{\mathbf{i}}}{4\pi\varepsilon_0 x^2}$$

Net force on $Q = \mathbf{F}_1 + \mathbf{F}_2$

Net force on $Q = 0$ if $\mathbf{F}_1 + \mathbf{F}_2 = 0$

$$\Rightarrow \quad \frac{9\times10^{-6} Q\,\hat{\mathbf{i}}}{4\pi\varepsilon_0 (0.1+x)^2} - \frac{1\times10^{-6} Q\,\hat{\mathbf{i}}}{4\pi\varepsilon_0 x^2} = 0$$

$$\Rightarrow \quad 9 = \frac{(0.1+x)^2}{x^2}$$

$$\Rightarrow \quad 3 = \frac{0.1+x}{x}$$

$$\Rightarrow \quad x = 0.05\ \text{m} = 5\ \text{cm}$$

EXAMPLE 2 Two charges, each equal to $-4\ \mu C$, are held a certain distance apart. A charge Q is placed exactly mid-way between them. Find the magnitude and sign of Q so that the system of three charges is in equilibrium.

SOLUTION A system of charges is in equilibrium if no charge of the system experiences any net force.

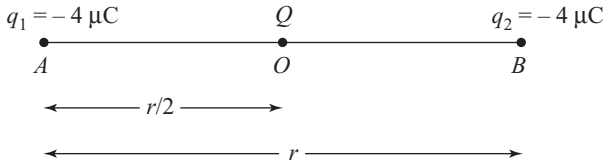


Fig. 11.3

Equilibrium of charge Q at O

Since Q is at the same distance from equal charges q_1 and q_2, it will be equilibrium for any positive or negative value, because it will experience equal and opposite forces.

Equilibrium of charge q_1 at A

If Q is negative, it will repel q_1. Also q_2 will repel q_1. Hence q_1 cannot be in equilibrium if Q is negative. So Q must be positive.

Force exerted on q_1 by Q is

$$\mathbf{F} = \frac{4\times10^{-6} Q\,\hat{\mathbf{i}}}{4\pi\varepsilon_0 \left(\frac{r}{2}\right)^2}$$

Force exerted on q_1 by q_2 is

$$\mathbf{F}' = -\frac{\left(4\times10^{-6}\right)\times\left(4\times10^{-6}\right)\hat{\mathbf{i}}}{4\pi\varepsilon_0 r^2}$$

Net force on q_1 will be zero if $\mathbf{F} + \mathbf{F}' = 0$, i.e. if

$$\frac{4\times10^{-6} Q\,\hat{\mathbf{i}}}{4\pi\varepsilon_0 \left(\frac{r}{2}\right)^2} - \frac{\left(4\times10^{-6}\right)\times\left(4\times10^{-6}\right)\hat{\mathbf{i}}}{4\pi\varepsilon_0 r^2} = 0$$

$$\Rightarrow \quad Q = 1\times10^{-6}\ \text{C} = 1\ \mu C$$

It is easy to check that charge q_2 will also be in equilibrium. Hence the system of three charges will be in equilibrium if $Q = +1\ \mu C$.

EXAMPLE 3 Four point charges, each equal to $q = 4\ \mu C$, are held at the corners of a square $ABCD$ of side $a = 10$ cm. Find the magnitude and sign of a charge Q placed at the centre of the square so that the system of charges is in equilibrium.

SOLUTION

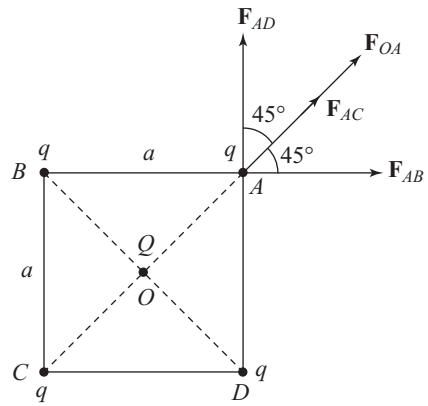


Fig. 11.4

$AC\ (= r) = \sqrt{2}a$. Let us consider the equilibrium of charge q at A (Fig. 11.4)

Force exerted on charge at A by charge at B is

$$\mathbf{F}_{AB} = \frac{kq^2}{a^2}\hat{\mathbf{i}}, \quad \text{where } k = \frac{1}{4\pi\varepsilon_o}$$

Similarly

$$\mathbf{F}_{AD} = \frac{kq^2}{a^2}\hat{\mathbf{j}}$$

$$\mathbf{F}_{AC} = \frac{kq^2}{(a\sqrt{2})^2}(\cos 45°\hat{\mathbf{i}} + \sin 45°\hat{\mathbf{j}})$$

$$= \frac{kq^2}{a^2 2\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

and $$\mathbf{F}_{OA} = \frac{kqQ}{\left(\frac{a}{\sqrt{2}}\right)^2}(\cos 45°\hat{\mathbf{i}} + \sin 45°\hat{\mathbf{j}})$$

$$= \sqrt{2}\frac{k\,q\,Q}{a^2}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

Net force on charge q at A in the x-direction is

$$\mathbf{F}_x = \left(\frac{kq^2}{a^2} + \frac{kq^2}{a^2 2\sqrt{2}} + \frac{\sqrt{2}\,k\,q\,Q}{a^2}\right)\hat{\mathbf{i}}$$

$$\Rightarrow \quad \mathbf{F}_x = \frac{kq}{a^2}\left(q + \frac{q}{2\sqrt{2}} + \sqrt{2}\,Q\right)\hat{\mathbf{i}} = \alpha\,\hat{\mathbf{i}}$$

where $$\alpha = \frac{kq}{a^2}\left(q + \frac{q}{2\sqrt{2}} + \sqrt{2}\,Q\right)$$

Similarly net force on charge q at A in the y-direction is

$$\mathbf{F}_y = \alpha\,\hat{\mathbf{j}}$$

$\therefore$ Resultant force on charge q at A is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{\alpha^2 + \alpha^2} = \sqrt{2}\alpha$$

Charge q will be in equilibrium if $F = 0$, i.e. if

$$\alpha = 0$$

i.e. if $$\frac{kq}{a^2}\left(q + \frac{q}{2\sqrt{2}} + \sqrt{2}\,Q\right) = 0$$

$$\Rightarrow \quad Q = -\frac{q}{4}(1 + 2\sqrt{2})$$

$$= -\frac{4\,\mu C}{4}(1 + 2\sqrt{2}) = -(1 + 2\sqrt{2})\,\mu C$$

4. Electric Field

An electric field exists at any point in the space surrounding a charge. To define the electric field, we place a small positive point charge q_0 at the point in space where the electric field is to be found and we measure the coulomb force **F** at that point. The electric field **E** is then given by

$$\mathbf{E} = \lim_{q_0 \to 0}\frac{\mathbf{F}}{q_0}$$

If a charge q is placed at a point where the electric field due to other charge or charges is **E**, then the charge q will experience a force **F** given by

$$\mathbf{F} = q\mathbf{E}$$

(1) Electric field due to an isolated point charge Electric field at a distance r from a source charge q is given by

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

For a positive charge ($+q$), vector **E** is directed radially outwards from it and for a negative charge ($-q$), **E** is directed radially inwards it. Because electric field **E** is vector quantity, the net electric field due to several charges is given by the vector sum of the electric fields due to the individual charges.

(2) Electric field due to an electric dipole A pair of equal and opposite point charges separated by a certain distance is called an electric dipole.

Case (a): Electric field at a point on the axis of a dipole

Let $2a$ be the separation between point charges $-q$ and $+q$ (Fig. 11.5).

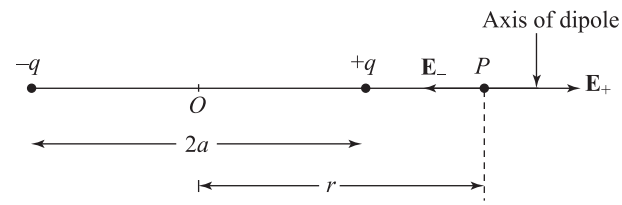


Fig. 11.5

Electric fields at P due to $+q$ and $-q$ respectively are

$$\mathbf{E}_+ = \frac{q\,\hat{\mathbf{i}}}{4\pi\varepsilon_0(r-a)^2}$$

$$\mathbf{E}_- = -\frac{q\,\hat{\mathbf{i}}}{4\pi\varepsilon_0(r+a)^2}$$

Electric field at point P is

$$\mathbf{E}_a = \mathbf{E}_+ + \mathbf{E}_-$$

$$= \frac{2q(2\mathbf{a})r}{4\pi\varepsilon_0(r^2 - a^2)^2} = \frac{2\mathbf{p}\,r}{4\pi\varepsilon_0(r^2 - a^2)^2}$$

where $\mathbf{p} = q(2\mathbf{a})$ is the dipole moment and $2\mathbf{a}$ is the vector distance between charges $-q$ and $+q$. Dipole moment **p** is a vector quantity directed from $-q$ to $+q$.

For a very short dipole ($a \ll r$)

$$\mathbf{E}_a = \frac{2\mathbf{p}}{4\pi\varepsilon_0 r^3}$$

Case (b): Electric field at a point on the perpendicular bisector (equatorial plane) of a dipole

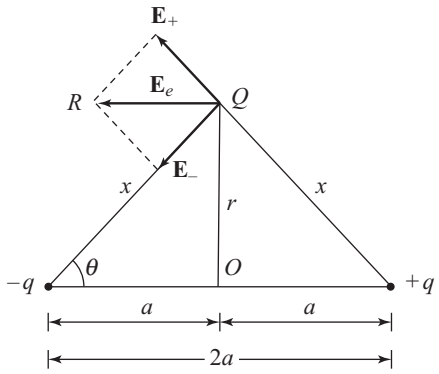


Fig. 11.6

Electric fields at point Q due to $+q$ and $-q$ are (see Fig. 11.6)

$$\mathbf{E}_+ = \frac{q}{4\pi\varepsilon_0 x^2} \quad \text{and} \quad \mathbf{E}_- = \frac{q}{4\pi\varepsilon_0 x^2}$$

The magnitude of the resultant electric field at Q is

$$E_e = E_+ \cos\theta + E_- \cos\theta$$

Using $\cos\theta = \frac{a}{x}$ and $x = \sqrt{r^2 + a^2}$, we get

$$E_e = \frac{q(2a)}{4\pi\varepsilon_0 (r^2 + a^2)^{3/2}} \text{ directed from } Q \text{ to } R$$

In vector form

$$\mathbf{E}_e = -\frac{\mathbf{p}}{4\pi\varepsilon_0 (r^2 + a^2)^{3/2}}$$

For a very short dipole ($a \ll r$)

$$\mathbf{E}_e = -\frac{\mathbf{p}}{4\pi\varepsilon_0 r^3}$$

Note

(i) The direction the electric field at a point on the axial line of a dipole is along the dipole moment.

(ii) The direction of the electric field at a point on the equatorial line of a dipole is antiparallel to the dipole moment.

(3) Electric field due to a uniformly charged conducting rod A conducting rod AB of negligible thickness and length L carries a charge a charge Q uniformly distributed on it. To find the electric field at point P at a distance a from end B (Fig. 11.7), we consider a small element of length dx of the rod located at a distance x from end B.

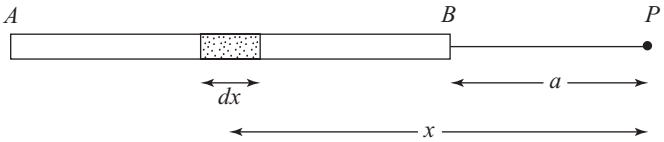


Fig. 11.7

Charge of element is $dq = \frac{Q}{L}\,dx = \lambda\,dx$ where $\lambda = \frac{Q}{L}$ is the linear charge density. The electric field due to the element at point P is

$$dE = \frac{dq}{4\pi\varepsilon_0 x^2} = \frac{\lambda\,dx}{4\pi\varepsilon_0 x^2}$$

Hence $$E = \int dE = \frac{\lambda}{4\pi\varepsilon_0}\int_a^{(L+a)} \frac{dx}{x^2} = \frac{\lambda}{4\pi\varepsilon_0}\left|-\frac{1}{x}\right|_a^{(L+a)}$$

$$\Rightarrow \quad E = -\frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{(L+a)} - \frac{1}{a}\right]$$

$$\Rightarrow \quad E = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{L}{a(L+a)}\right]$$

If Q is positive, $\mathbf{E}$ is directed from left to right.

(4) Electric field due to a uniformly charged ring (or loop) of wire at a point on its axis Consider a ring of radius R carrying a chare Q distributed uniformly on it. To find electric field at a point P on its axis at a distance x from the centre O, consider an element of length dl (Fig. 11.8). The charge of the element is

$$dq = \frac{Q\,dl}{2\pi R}$$

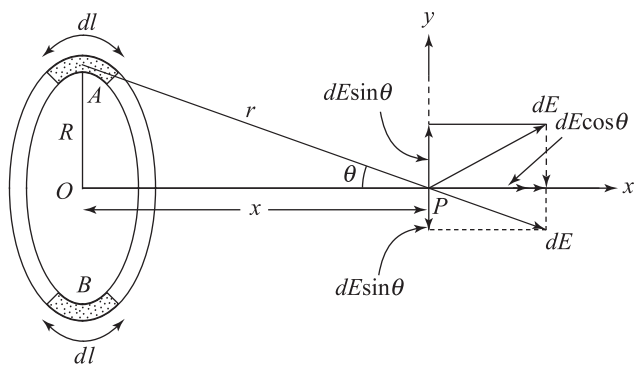


Fig. 11.8

The electric field due to element A is dE given by

$$dE = \frac{dq}{4\pi\varepsilon_0 r^2}$$

There is a similar electric field at P due to element a diametrically opposite point B. The x components of electric fields due to these elements add up while the y components cancel. Hence

$$E = \int dE\cos\theta$$

$$= \int \frac{dq\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$= \frac{Q}{2\pi R}\times\frac{1}{4\pi\varepsilon_0}\times\frac{x}{(R^2+x^2)^{3/2}}\int dl$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qx}{(R^2+x^2)^{3/2}}\right] \quad \left(\because \int dl = 2\pi R\right)$$

The direction of **E** is from O to P if charge Q is positive.

5. Electric Field Lines

Electric field lines of an electrostatic field give a pictorial representation of the field. An electric field line is a curve, the tangent to which at a point gives the direction of the electric field at that point.

Figure 11.9 shows field line patterns around some charge distributions.

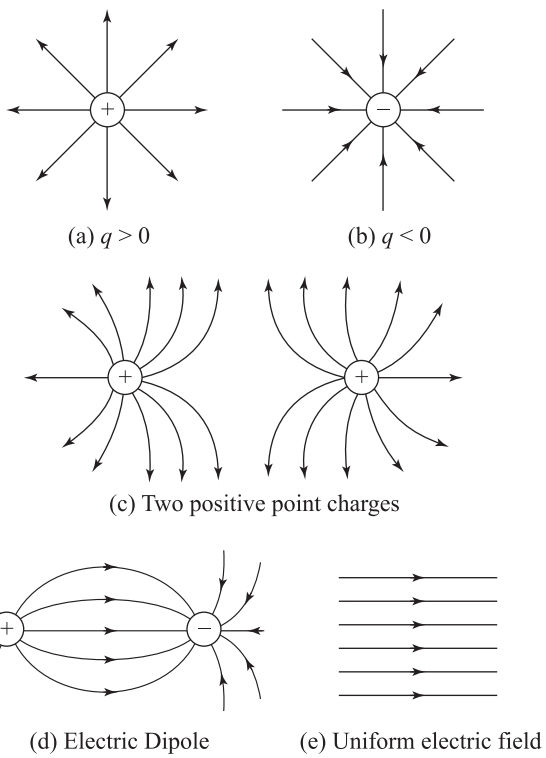


Fig. 11.9

Properties of Electric Field Lines

(i) The tangent to a field line at any point gives the direction of electric field at that point.

(ii) Field lines originate from a positive charge and terminate on a negative charge.

(iii) No two field lines intersect.

(iv) Field lines are closer together in the region where the field is stronger and farther apart where the field is weaker.

(v) The number of field lines originating or ending on a charge is proportional to the magnitude of the charge.

Note Electric field line due to a charge distribution never forms a closed loop. But if the electric field is induced by a time-varying magnetic field, its field line forms a closed loop.

6. Electric Flux

The electric flux through a surface in an electric field is a measure of the number of electric field lines passing through the surface.

For a plane surface of surface area **S** in an electric field **E**, the electric flux ϕ is defined as

$$\phi = \mathbf{E}\cdot\mathbf{S} = ES\cos\theta$$

where **S** is called the area vector, its magnitude is S and its direction is normal to the surface and away from it. Angle θ is the angle between **E** and **S**.

For a curved surface,

$$\phi = \int \mathbf{E}\cdot\mathbf{dS} = \int (\mathbf{E}\cdot\hat{\mathbf{n}})\,dS$$

where $\hat{\mathbf{n}}$ is a unit outward normal to the surface. dS is the surface area of an element of the surface (Fig. 11.10). The SI unit of electric flux is $NC^{-1}\ m^2$ or Vm (volt metre).

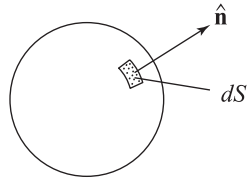


Fig. 11.10

7. Gauss's Law in Electrostatics

Gauss's law states that the electric flux through a closed surface **S** in an electric field **E** is equal to $\frac{q}{\varepsilon_0}$, where q is the net charge enclosed in the surface and ε_0 is electrical permittivity of vacuum.

$$\oint_S \mathbf{E}\cdot\mathbf{dS} = \frac{q}{\varepsilon_0} \qquad (1)$$

Gauss's law is used to obtain the expression for the electric field due to linear, surface and volume charge distributions which are uniform and symmetric so that a proper and convenient closed surface (called the Gaussian surface) can be chosen to evaluate the surface integral in Eq. (1).

Some Important Points about Gauss's Law

(1) Gauss's law holds for any closed surface of any shape or size.

(2) The surface that we choose to evaluate electric flux [i.e. to evaluate the surface integral in Eq. (1)] is called Gaussian surface.

(3) If the Gaussian surface is so chosen that there are some charges outside and some inside the surface, then charge q on the R.H.S. of Eq. (1) is the net charge (taking into account the sign of charges) enclosed inside the surface but electric field **E** on the L.H.S. of Eq. (1) is the electric field due to all the charges both inside and outside the surface.

(4) The exact location of charges inside Gaussian surface does not affect the value of the electric flux.

(5) If Coulomb's law did not hold, Gauss's law also would not hold.

8. Applications of Gauss's Law

(1) Electric field due to a thin infinitely long straight charged rod or wire Electric field at a point at a perpendicular distance r from a thin, infinitely long straight rod or wire carrying a uniform linear charge density λ is given by

$$\mathbf{E} = \frac{\lambda \hat{\mathbf{n}}}{2\pi \varepsilon_0 r}$$

where $\lambda = q/L$ is the charge per unit length of the rod and $\hat{\mathbf{n}}$ is a unit vector pointing away from the rod if q is positive and towords it if q is negative.

(2) Electric field due to a thin sheet of charge Electric field at a perpendicular distance r from a thin, flat and infinite sheet carrying a uniform surface charge density σ is given by

$$\mathbf{E} = \frac{\sigma \hat{\mathbf{n}}}{2\varepsilon_0}$$

where $\sigma = q/A$ is charge per unit area and $\hat{\mathbf{n}}$ is a unit vector pointing away from the sheet if q is positive and towards it if q is negative. Notice that **E** is independent of r, the distance from the sheet.

(3) Electric field due to a thin charged spherical shell Electric field at a distance r from a spherical shell of radius R carrying a surface charge density σ $(= q/4\pi R^2)$ is given by

$$\mathbf{E} = \frac{q\hat{\mathbf{n}}}{4\pi\varepsilon_0 r^2} = \frac{\sigma R^2 \hat{\mathbf{n}}}{\varepsilon_0 r^2} \quad (\text{for } r > R)$$

$$= \frac{\sigma \hat{\mathbf{n}}}{\varepsilon_0} \quad (\text{for } r = R)$$

$$= \text{zero} \quad (\text{for } r < R)$$

where $\hat{\mathbf{n}}$ is a unit vector pointing radially outwards if q is positive and inwards if q is negative.

9. Electric Potential

The electric potential at a point in an electrostatic field is the work per unit charge that is done to bring a small charge in from infinity to that point along any path.

$$V = \lim_{q_0 \to 0} \frac{W}{q_0}$$

(1) Electric potential due to an isolated point charge Electric potential at a point P in the electric field of a point charge is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

where r is the distance of the point P from the charge. This potential is spherically symmetric around the point, i.e. it depends only on r for a given charge q. Since potential is a scalar function, the spherical symmetry means that the potential at a point does not depend upon the direction of that point with respect to the point charge; it only depends on the distance of the point from the charge.

Notice that the potential due to a positive charge $(q > 0)$ is positive, it is negative in the neighbourhood of an isolated negative charge $(q < 0)$.

(2) Electric potential due to two point charges To find the electric potential at a point in the electric field due to two or more charges, we first calculate the potential due to each charge, assuming that all other charges are absent, and then simply add these individual contributions. Since, unlike electric field, electric potential is a scalar, the addition here is the *ordinary* sum, not a *vector* sum.

The potential at any point due to two point charges q_1 and q_2 is, therefore, simply given by

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

where r_1 and r_2 are the distances of the point in the question from charges q_1 and q_2 respectively.

(3) Electric potential due to many point charges The potential at any point due to a system of N point charges is given by

$$V = V_1 + V_2 + \ldots + V_N = \frac{1}{4\pi\varepsilon_0}\sum_{n=1}^{N}\frac{q_n}{r_n}$$

10. Relation between E and V

Electric field is the negative gradient of potential. This means that the potential decreases along the direction of the electric field.

$$E = -\frac{dV}{dr}$$

11. Electric Potential Energy

The electric potential energy of a system of point charges is defined as the amount of work done to assemble this system of charges by bringing them in from an infinite distance. We assume that the charges were at rest when they

were infinitely separated, i.e. they had no initial kinetic energy.

The electric potential energy of two point charges q_1 and q_2 separated by a distance r_{12} as shown in Fig. 11.11 (a) is given by

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{12}}$$

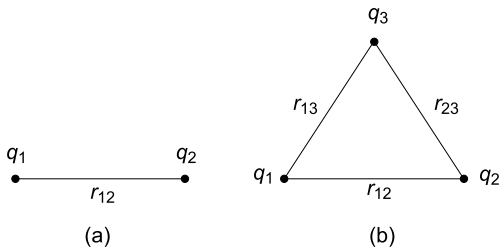


Fig. 11.11

The electric potential energy of a system of three point charges as shown in Fig. 11.11 (b) is given by

$$U = U_{12} + U_{23} + U_{31} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}}\right)$$

This expression can be generalized for any number of charges.

12. The Electron-Volt

The SI unit of potential energy is the *joule*. In atomic physics a more convenient unit called the *electron-volt* (written as eV) is used. *An electron-volt is the potential energy gained or lost by an electron in moving through a potential difference of 1 volt.* Since the magnitude of charge on an electron is 1.6×10^{-19} C,

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

13. Potential Energy of an Electric Dipole in an External Electric Field

When a dipole is placed in a uniform electric field E, as shown in Fig. 11.12, it experiences a torque given by

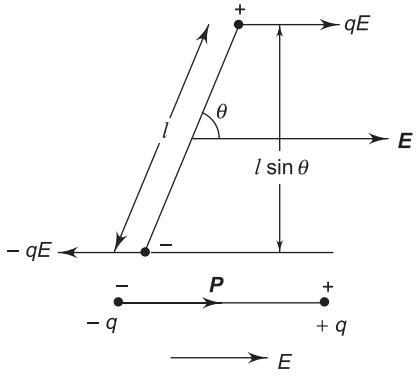


Fig. 11.12

$$\tau = p\,E \sin\theta$$

where θ is the angle between the line joining the two charges and the electric field. In vector form

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$

The torque tends to rotate the dipole to a position where $\theta = 0$, i.e, $\mathbf{p}$ is parallel to $\mathbf{E}$.

The electric potential energy of a dipole is

$$U = -\,\mathbf{p} \cdot \mathbf{E}$$

14. Additional Useful Formulae

(1) Electric field and potential due to a group of charges

(i) Charge q at each vertex of an equilateral triangle of side a (Fig. 11.13).

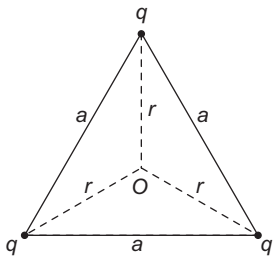


Fig. 11.13

At centroid O, $E_0 = 0$ and $V_0 = \dfrac{3q}{4\pi\varepsilon_0 r}$

where $r = \dfrac{a}{\sqrt{3}}$.

(ii) Charge q at each vertex of a square of side a (Fig. 11.14).

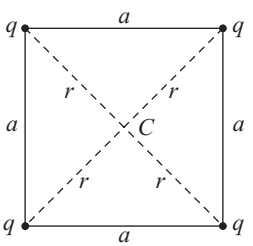


Fig. 11.14

At center C, $E_c = 0$ and $V_c = \dfrac{4q}{4\pi\varepsilon_0 r}$; $r = \dfrac{a}{\sqrt{2}}$

Note In the above two cases, if one of the charges is removed from a vertex, the net electric field at O and C is $E = q/4\pi\varepsilon_0 r^2$, directed towards the empty vertex.

(iii) For Fig. 11.15,

$$V_c = 0 \quad \text{and} \quad E_c = \frac{2\sqrt{2}q}{4\pi\varepsilon_0 r^2}$$

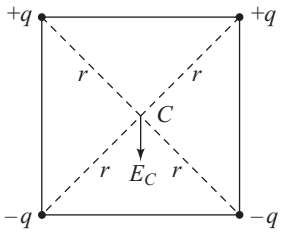


Fig. 11.15

(iv) For Fig. 11.16,

$$V_c = 0$$

$$E_c = 0$$

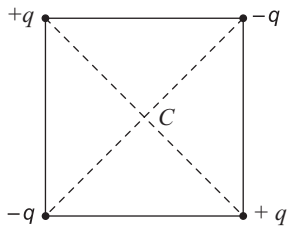


Fig. 11.16

(v) Infinite number of charges, each equal to q, placed on the x-axis at $x = r, x = 2r, x = 4r\ldots$ and so on. Electric field at origin O is

$$E_0 = \frac{q}{4\pi\varepsilon_0 r^2}\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{4^2}+\cdots\right)$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\left(1+\frac{1}{4}+\frac{1}{16}+\cdots\right)$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\times\frac{1}{\left(1-\frac{1}{4}\right)} = \frac{q}{3\pi\varepsilon_0 r^2}$$

Potential at O is

$$V_0 = \frac{q}{4\pi\varepsilon_0 r}\left(1+\frac{1}{2}+\frac{1}{4}+\cdots\right)$$

$$= \frac{q}{4\pi\varepsilon_0 r}\times\frac{1}{\left(1-\frac{1}{2}\right)} = \frac{q}{2\pi\varepsilon_0 r}$$

(vi) A short electric dipole of dipole moment p (Fig. 11.17)

At point A, $E_r = \dfrac{2p\cos\theta}{4\pi\varepsilon_0 r^3}$ and $E_\theta = \dfrac{p\sin\theta}{4\pi\varepsilon_0 r^3}$

Net electric field at A is $E_A = \sqrt{E_r^2 + E_\theta^2}$

$$= \frac{p}{4\pi\varepsilon_0 r^3}(3\cos 2\theta + 1)^{1/2}$$

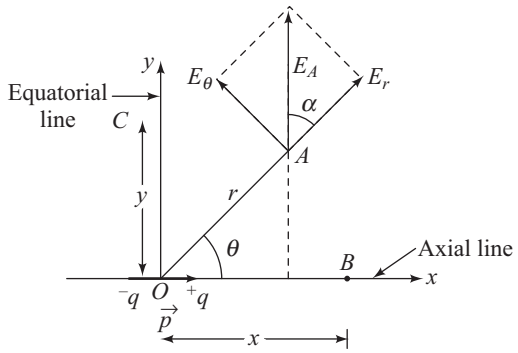


Fig. 11.17

Also $\tan\alpha = \dfrac{1}{2}\tan\theta$. Angle between $\boldsymbol{p}$ and $\boldsymbol{E}_A$ is $(\alpha + \theta)$.

At point B on axial line, $E_B = \dfrac{2p}{4\pi\varepsilon_0 x^3}$ $(\because\ \theta = 0°)$

At point C on equatorial line, $E_C = \dfrac{p}{4\pi\varepsilon_0 y^3}$ $(\because\ \theta = 90°)$

Electric potential at A is $V_A = \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

At point B; $V_B = \dfrac{p}{4\pi\varepsilon_0 r^2}$

At point C; $V_C = 0$

(2) Electric field and potential due to some charge distributions $\left(\text{Linear charge density } \lambda = \dfrac{Q}{L}\right)$

(i) Charge Q distributed uniformly on a rod of length L (Fig. 11.18)

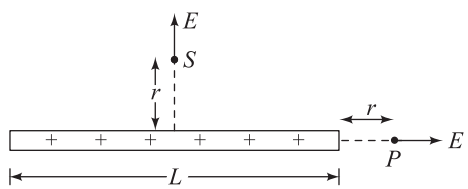


Fig. 11.18

At point P, $E_P = \dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{r(r+L)}\right]$

$$V_P = \frac{Q}{4\pi\varepsilon_0 L}\log_e\left(\frac{r+L}{r}\right)$$

(ii) Charge Q distributed uniformly on a conducting sphere or shell of radius R (Fig. 11.19) $\left(\text{Surface charge densitky } \sigma = \dfrac{Q}{4\pi R^2}\right)$

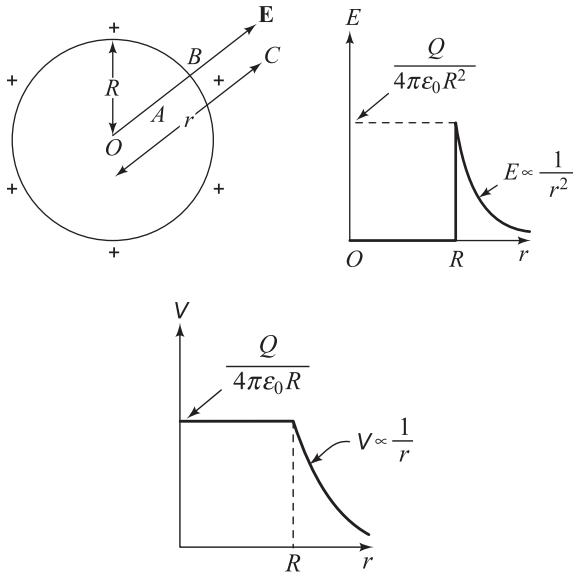


Fig. 11.19

At point C outside the sphere or shell ($r > R$),

$$E_C = \frac{Q}{4\pi\varepsilon_0 r^2}$$

At point B just outside the surface ($r = R$),

$$E_B = \frac{Q}{4\pi\varepsilon_0 R^2}$$

At point A inside the sphere or shell ($r < R$), $E_A = 0$

Potential at center O of sphere or shell, $V_0 = \frac{Q}{4\pi\varepsilon_0 R}$

At points inside ($r < R$), $V_A = \frac{Q}{4\pi\varepsilon_0 R} = V_B$ (at surface)

At point C outside ($r > R$), $V_C = \frac{Q}{4\pi\varepsilon_0 R}$

(iii) Charge Q distributed uniformly on a ring of radius R. (Fig. 11.20) Electric field at a point P on the axis is

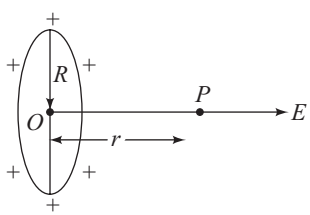


Fig. 11.20

$$E_P = \frac{Q}{4\pi\varepsilon_0} \times \frac{r}{\left(R^2 + r^2\right)^{3/2}}$$

E is maximum at $r = \pm\frac{R}{\sqrt{2}}$ and

$$E_{\max} = \frac{1}{4\pi\varepsilon_0} \times \frac{2Q}{3\sqrt{3}R^2}$$

Electric potential at point P is

$$V_P = \frac{Q}{4\pi\varepsilon_0} \times \frac{1}{\left(R^2 + r^2\right)^{1/2}}$$

V is maximum at $r = 0$ (i.e. at centre O) and

$$V_{\max} = \frac{Q}{4\pi\varepsilon_0 R}$$

(iv) Consider a semi-circular rod of radius R having a charge $+Q$ distributed uniformly on it. Linear charge density is (Fig. 11.21).

$$\lambda = \frac{Q}{\pi R}$$

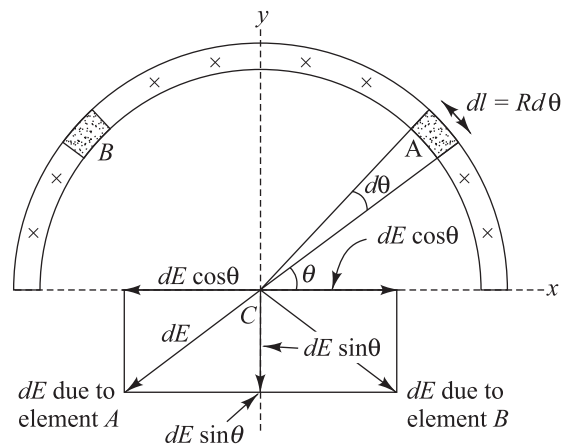


Fig. 11.21

Consider a small element at A of length $dl = Rd\theta$ at A. The charge of the element is $dQ = \lambda dl = \lambda R d\theta$. The electric field due to this element at centre C is

$$dE = \frac{dQ}{4\pi \in_0 R^2} = \frac{\lambda d\theta}{4\pi \in_0 R}$$

The x and y components of dE are $dE\cos\theta$ and $dE\sin\theta$. Now consider an element at a symmetric point B. The x component of electric field of this element will cancel with that of element at A but y component will add up. Hence the electric field at C due to the complete semi-circular rod is

$$E = \int_0^{\pi} dE\sin\theta$$

$$= \frac{\lambda}{4\pi \in_0 R}\int_0^{\pi}\sin\theta\, d\theta$$

$$= \frac{\lambda}{4\pi \in_0 R}\left|-\cos\theta\right|_0^{\pi}$$

or, $$E = \frac{\lambda}{2\pi \in_0 R}$$

directed vertically downwards away from the rod. If the rod carries a negative charge ($-Q$), then the electric field is directed vertically upwards towards the rod.

The electric potential at point C is

$$V = \int \frac{dQ}{4\pi \in_0 R}$$

$$= \frac{\lambda}{4\pi \in_0} \int_0^{\pi} d\theta \qquad (\because \quad dQ = \lambda R d\theta)$$

or $$V = \frac{\lambda}{4 \in_0}$$

The above results also hold for a non-conducting rod.

(3) Potential energy of a system of charges

(i) Charge Q kept at each vertex of an equilateral of side a. Potential energy is (Fig. 11.22).

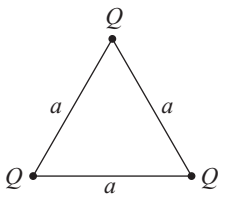


Fig. 11. 22

$$U = \frac{3Q^2}{4\pi \varepsilon_0 a}$$

(ii) Charge Q kept at each vertex of a square of side a (Fig. 11.23)

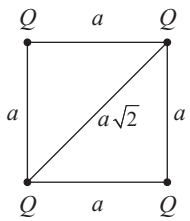


Fig. 11.23

$$U = \frac{4 \times Q^2}{4\pi \varepsilon_0 a} + \frac{2Q^2}{4\pi \varepsilon_0 (a\sqrt{2})}$$

$$= \frac{Q^2}{4\pi \varepsilon_0 a}(4 + \sqrt{2})$$

(iii) An electric dipole of dipole moment **p** placed in uniform electric field **E** with angle θ between **p** and **E** Torque $\tau = \mathbf{p} \times \mathrm{E}$ and potential energy $U = -\mathbf{p}.\mathbf{E}$. The zero of potential energy is taken at $\theta = 90°$.

(a) When $\theta = 0°$, $\tau = 0$, U = minimum = $-pE$ (stable equilibrium)

(b) When $\theta = 90°$, τ = maximum = pE, $U = 0$

(c) When $\theta = 180°$, $\tau = 0$, U = maximum = pE (unstable equilibrium)

(iv) Work done in turning a dipole from angle θ_1 to angle θ_2 is

$$W = pE(\cos\theta_1 - \cos\theta_2)$$

If $\theta_1 = 0°$ and $\theta_2 = 180°$, $W = 2pE$

EXAMPLE 4 An electric dipole AB is placed along the x-axis with its charges placed at distances x_1 and x_2 from a very long thin wire having a uniform linear positive charge density λ as shown in Fig. 11.24. Choose the correct option.

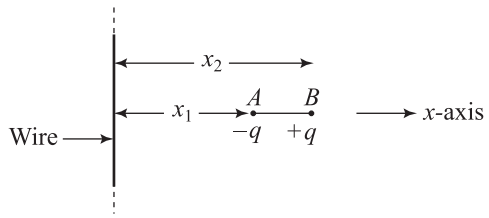


Fig. 11.24

The force experienced by the dipole is

(a) $\frac{\lambda q}{2\pi \varepsilon_0}\left(\frac{1}{x_1} - \frac{1}{x_2}\right)$ along –ve x-direction

(b) $\frac{\lambda q}{4\pi \varepsilon_0}\left(\frac{1}{x_1} - \frac{1}{x_2}\right)$ along +ve x-direction

(c) $\frac{\lambda q}{2\pi \varepsilon_0}(x_2 - x_1)$ along +ve x-direction

(d) $\frac{\lambda q}{2\pi \varepsilon_0}(x_2 + x_1)$ along –ve x-direction

SOLUTION The electric field due to the long wire at a point at a disance x from it is given by

$$E = \frac{\lambda}{2\pi \varepsilon_0 x}$$

As λ is +ve, the direction of E is along +ve x-direction. Hence the force experienced by charge $(-q)$ at A is

$$\mathbf{F}_1 = -\frac{\lambda q \hat{\mathbf{i}}}{2\pi \varepsilon_0 x_1}$$

The force experiened by charge $(+q)$ at B is

$$\mathbf{F}_2 = \frac{\lambda q \hat{\mathbf{i}}}{2\pi \varepsilon_0 x_2}$$

$\therefore$ Net force on the dipole is

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$$

$$= -\frac{\lambda q}{2\pi \varepsilon_0}\left(\frac{1}{x_1} - \frac{1}{x_2}\right)\hat{\mathbf{i}}$$

So the correct choice is (a).

EXAMPLE 5 A metallic solid sphere A of radius r has a charge q distributed uniformly on its surface. It is surrounded by a concentric metallic hollow sphere B of radius R. The electric field at point P at a distance x from O where $r < x < R$ will be

(a) $\dfrac{2q}{4\pi\varepsilon_0 R^2}$ (b) $\dfrac{q}{4\pi\varepsilon_0 (R-x)r}$

(c) $\dfrac{q}{4\pi\varepsilon_0 x^2}$ (d) zero

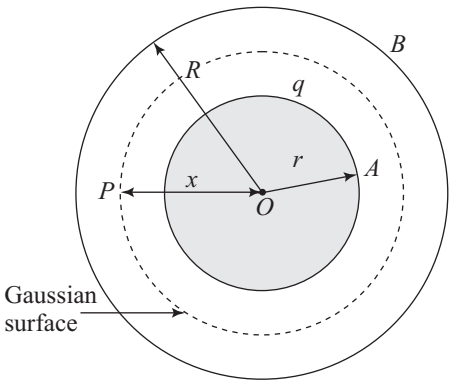


Fig. 11.25

SOLUTION The charge q on A induces a charge $-q$ on the inner side of B and a charge $+q$ on the outer side of B. To find electric field at P we draw a Gaussian spherical surface of radius x, shown dotted in Fig. 11.25. Since point P is inside B, according to Gauss's law, the electric field due to B at P is zero. Now flux through the Gaussian surface due to charge q on A is

$$\phi = \oint \mathbf{E}\cdot d\mathbf{S} = E \times 4\pi x^2$$

From Gauss's law, $\phi = \dfrac{q}{\varepsilon_0}$. Hence

$$\frac{q}{\varepsilon_0} = E \times 4\pi x^2$$

$$\Rightarrow \qquad E = \frac{q}{4\pi\varepsilon_0 x^2}, \text{ which is choice (c).}$$

EXAMPLE 6 In Example 5 above, if a charge Q is given to the spherical shell B, how will the electric field at P be affected?

SOLUTION In this case, the total charge on the outer surface of B will now be $(Q + q)$. Since point P lies inside this sphere, the electric field at P will remain unchanged equal to $\dfrac{q}{4\pi\varepsilon_0 x^2}$.

EXAMPLE 7 Figure 11.26 shows two point charges $+q$ and $+q$ and a metal rod of length L carrying a charge $-q$ with a part $L/3$ of its length inside a box. The electric flux through the box is

(a) $\dfrac{q}{3\varepsilon_0}$ (b) $\dfrac{4q}{3\varepsilon_0}$

(c) $\dfrac{2q}{3\varepsilon_0}$ (d) $\dfrac{5q}{3\varepsilon_0}$

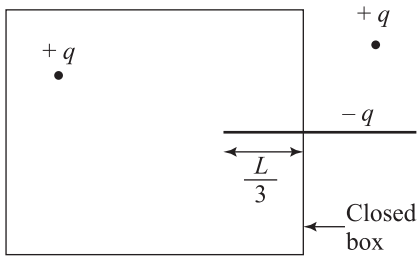


Fig. 11.26

SOLUTION Charge in rod of length $\dfrac{L}{3} = -\dfrac{q}{3}$. According to Gauss's law, the electric flux through a closed surface is equal to $\dfrac{q_{\text{net}}}{\varepsilon_0}$ where q_{net} is the net charge enclosed inside the surface; the charge outside the closed surface does not contribute to the flux. Now

$$q_{\text{net}} = +q - \frac{q}{3} = \frac{2q}{3}$$

Hence $$\phi = \frac{q_{\text{net}}}{\varepsilon_0} = \frac{2q}{3\varepsilon_0}, \text{ which is choice (c).}$$

EXAMPLE 8 Three thin concentric spherical shells 1, 2 and 3 have radii r_1, r_2 and r_3 respectively. Charge $+q$ is given to shell 1 and charge $-2q$ is given to shell 3 as shown in Fig. 11.27(a). If shell 2 is earthed and $r_3 = 2\,r_2$, the charge on the outer surface of shell 2 will be

(a) zero (b) q

(c) $-\dfrac{2q}{3}$ (d) $-\dfrac{q}{3}$

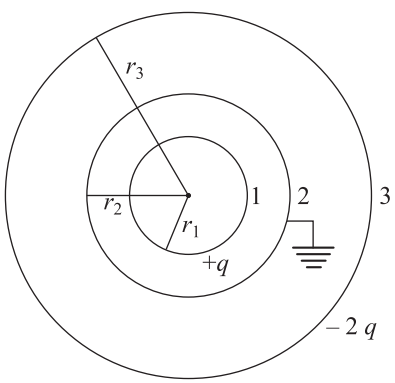


Fig. 11.27(a)

SOLUTION Let q_2 be the charge on the outer surface of shell 2. Charge $+q$ on shell 1 will induce a charge $-q$ on the inner surface of 2. Charge q_2 will induced charge $-q_2$ on the inner surface of 3 and $+q_2$ on its outer surface. Thus the outer surface of 3 will have a charge $(q_2 - 2q)$ as shown in Fig. 11.27(b).

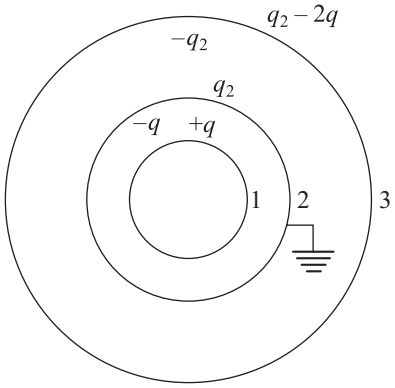


Fig. 11.27(b)

We know that the potential at any point inside a spherical shell is equal to that on its surface. Hence the potential on the outer surface of shell 2 will be

V_2 = potential due to charge $+q$ on 1 + potential due to charge $-q$ on inner surface of 2 + potential due to charge q_2 on outer surface of 2 + potential due to charge $-q_2$ on inner surface of 3 + potential due to charge $(q_2 - 2q)$

$$= \frac{q}{4\pi\varepsilon_0 r_2} - \frac{q}{4\pi\varepsilon_0 r_2} + \frac{q_2}{4\pi\varepsilon_0 r_2} - \frac{q_2}{4\pi\varepsilon_0 r_3} + \frac{(q_2 - 2q)}{4\pi\varepsilon_0 r_3}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_2}{r_2} - \frac{q_2}{r_3} + \frac{(q_2 - 2q)}{r_3}\right]$$

Since shell 2 is earthed, its potential V_2 will be zero. Thus

$$\frac{q_2}{r_2} - \frac{q_2}{r_3} + \frac{(q_2 - 2q)}{r_3} = 0$$

$$\Rightarrow \quad q_2 = \frac{2q\, r_2}{r_3}$$

Given $r_3 = 2\, r_2$. Hence $q_2 = q$, which is choice (b).

EXAMPLE 9 A charge q is placed at the corner of a cube of side l. The electric flux passing through the cube is

(a) $\dfrac{q}{\varepsilon_0}$ (b) $\dfrac{q}{6\varepsilon_0}$

(c) $\dfrac{q}{8\varepsilon_0}$ (d) $\dfrac{q}{32\varepsilon_0}$

SOLUTION It is clear from Fig. 11.28 that eight cubes, each of side l, are required to form a Gaussian surface so that the charge q at the corner of a small cube appears at the centre of the bigger cube. According to Gauss's law, the electric flux through the bigger cube = $\dfrac{q}{\varepsilon_0}$. Hence the electric flux through the given small cube = $\dfrac{q}{8\varepsilon_0}$. So the correct choice is (c).

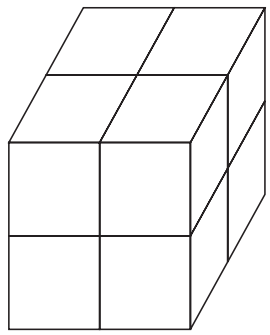


Fig. 11.28

15. Capacitance

When a conductor is given a charge Q, it acquires a potential V which is proportional to the charge given to it, i.e.

$$Q \propto V \quad \text{or} \quad Q = CV \quad \text{or} \quad C = \frac{Q}{V}$$

where C is a constant of proportionality and is called the capacitance which is defined as the amount of charge in coulomb necessary to increase the potential of a conductor by 1 volt. The SI unit of capacitance is farad (symbol F)

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}}$$

The farad is a large unit. More practical units are microfarad (μF) and picofarad (pF).

$$1\ \mu\text{F} = 10^{-6}\ \text{F and } 1\ \text{pF} = 10^{-12}\ \text{F}$$

16. Energy of a Charged Conductor

A charged conductor has electric field in the region around it. If additional similar charge is given to the conductor, work has to be done against the electrical repulsive force. This work is stored in the form of potential energy which resides in the electric field. If a charge Q is given to a conductor of capacitance C, the potential energy in its electric field is given by

$$U = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{1}{2}CV^2 \qquad (\because Q = CV)$$

17. Capacitance of a Single Spherical Conductor

Consider a spherical conductor of radius r having a charge Q. Since the electric field is normal to the surface of the sphere, the lines of force appear to originate from its centre, i.e. the charge Q may be supposed to be concentrated at the centre. Therefore, the potential is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}$$

Since $C = \frac{Q}{V}$, the capacitance of the sphere is given by

$$C = 4\pi\varepsilon_0 r$$

Thus, the greater the radius of the sphere, the higher is its capacitance.

18. Capacitors

Any isolated system of two conducting bodies, of any shape and size, separated by a distance is called a capacitor. If two conductors, carrying equal and opposite charge Q have a potential difference V between them, then

$$Q = CV$$

where C is the capacitance of the capacitor and its value depends on the size, the shape, the separation between the conductors and the nature of the medium between them. If C_0 is the capacitance of the capacitor when the medium is air (or vacuum) and C_m its capacitance when the medium is a dielectric other than air, then the dielectric constant of the medium is given by

$$K = \frac{C_m}{C_0}$$

19. Expressions for Capacitance

1. *Parallel Plate Capacitor*

   The capacitance of a parallel plate capacitor is given by

   $$C = \frac{K\,\varepsilon_0\,A}{d}$$

   where A is the area of each plate and d is the distance between them. K is dielectric constant of the material between the plates. For air or vacuum, $K = 1$.

2. *Spherical Capacitor*

   A spherical capacitor consists of a solid charged sphere of radius a surrounded by a concentric hollow sphere of radius b. Its capacitance is given by

   $$C = 4\pi\,\varepsilon_0\,K\left(\frac{ab}{b-a}\right)$$

3. *Cylindrical Capacitor*

   A cylindrical capacitor consists of two co-axial cylinders and its capacitance is given by

   $$C = \frac{2\pi\,\varepsilon_0\,K\,l}{\log_e\left(\frac{b}{a}\right)}$$

   where l is the length of each cylinder and a and b are the radii of the inner and outer cylinders.

4. If the space between the plates of a parallel plate capacitor is filled with two media of thicknesses d_1 and d_2 having dielectric constants K_1 and K_2, then the capacitance of the capacitor is given by

   $$C = \frac{\varepsilon_0\,A}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$

20. Capacitors in Parallel and Series

In parallel arrangement of capacitors, the potential difference across individual capacitors is the same and the total charge is shared by them in the ratio of their capacitances.

$$Q = Q_1 + Q_2 + Q_3 + \cdots$$

and
$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3} = \ldots$$

$\therefore$
$$C = C_1 + C_2 + C_3 + \ldots$$

In series arrangement of capacitors, the charge on each capacitor is the same and the total potential difference is shared by them in the inverse ratio of their capacitances.

$$Q = C_1\,V_1 = C_2\,V_2 = C_3\,V_3 = \ldots$$
$$V = V_1 + V_2 + V_3 + \cdots$$

Therefore, the effective capacitance of the combination is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

21. Energy Stored in a Capacitor

As in the case of a charged conductor, the energy stored in a capacitor is given by

$$U = \frac{Q^2}{2C} = \frac{1}{2}\,QV = \frac{1}{2}\,CV^2$$

where Q = charge on each plate of the capacitor, V = potential difference between plates and C = capacitance of the capacitor. This potential energy resides in the electric field in the medium between the plates.

22. Loss of Energy on Sharing Charges

If two charged bodies carrying charges Q_1 and Q_2 and having capacitances C_1 and C_2 are connected with each other, then their common potential after the sharing of charges is given by

$$V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

where V_1 and V_2 are the initial potentials of the charged bodies. The loss of energy is given by

$$\Delta E = \frac{1}{2} \frac{C_1 C_2}{(C_1 + C_2)} (V_1 - V_2)^2$$

23. Force between Plates of a Parallel Plate Capacitor

The plates of a capacitor carry equal and opposite charges. Therefore, they exert an attractive force on each other which is given by

$$F = \frac{Q^2}{2K\varepsilon_0 A}$$

The force per unit area of the plates is

$$f = \frac{F}{A} = \frac{Q^2}{2K\varepsilon_0 A^2} = \frac{\sigma^2}{2K\varepsilon_0}$$

where σ is the charge per unit area.

EXAMPLE 10 A parallel plate capacitor of capacitance 10 μF and plate separation 0.5 mm is connected to a 20 V battery.

(a) What is the charge on each plate?
(b) What is the energy stored in the capacitor?
(c) What is the electric field between the plates?
(d) If the battery is disconnected and then the plate separation is doubled, what are the answers to parts (a), (b) and (c) above?
(e) If the battery is kept connected and the plate separation is doubled, then what are the answers to parts (a), (b) and (c) above?

SOLUTION

(a) $Q = CV = (10 \times 10^{-6}) \times 20 = 200 \times 10^{-6}\ \text{C} = 200\ \mu\text{C}$

(b) $U = \frac{1}{2} CV^2 = \frac{1}{2} (10 \times 10^{-6}) \times (20)^2 = 2 \times 10^{-3}\ \text{J}$

(c) $E = \frac{V}{d} = \frac{20}{(0.5 \times 10^{-3})} = 4 \times 10^4\ \text{Vm}^{-1}$

Note The engery is stored in the electric field between the plates

(d) If the battery is disconnected, the charge on the plates remains the same but the potential difference between the plates will charge. If the separation between the plates is doubled capacitance becomes

$$C' = \frac{\varepsilon_0 A}{2d} = \frac{C}{2} = 5\ \mu\text{F} = 5 \times 10^{-6}\ \text{F}$$

and potential difference between the plates becomes

$$V' = \frac{Q}{C'} = \frac{200\ \mu\text{C}}{5\ \mu\text{F}} = 40\ \text{V}$$

$$\therefore \quad U' = \frac{1}{2} C' V'^2$$

$$= \frac{1}{2} \times 5 \times 10^{-6} \times (40)^2 = 4 \times 10^{-3}\text{J}$$

Alternatively, $U' = \frac{Q^2}{2C'} = \frac{(200 \times 10^{-6})^2}{2 \times (5 \times 10^{-6})} = 4 \times 10^{-3}\text{J}$

$$E' = \frac{V'}{d'} = \frac{40}{1 \times 10^{-3}} = 4 \times 10^4\ \text{Vm}^{-1}$$

(e) If the battery is kept connected, the potential difference between the plates always remains equal to the emf of the battery and hence is constant = 20 V. If d is doubled,

Capacitance becomes $C' = \frac{\varepsilon_0 A}{2d} = \frac{C}{2}$

$$= 5\ \mu\text{F} = 5 \times 10^{-6}\ \text{F}$$

Charge becomes $Q'' = C'V = 5 \times 10^{-6} \times 20 = 10^{-4}\ \text{C}$

Energy stored becomes $U'' = \frac{1}{2} C' V^2$

$$= \frac{1}{2} \times 5 \times 10^{-6} \times (20)^2 = 10^{-3}\text{J}$$

Electric field becomes $= \frac{V}{2d} = \frac{20}{20 \times (0.5 \times 10^{-3})}$

$$= 2 \times 10^4\ \text{Vm}^{-1}$$

EXAMPLE 11 Find the charge on each capacitor shown in Fig. 11.29(a). Given C_1 = 2 μF, and C_2 = 2 μF and C_3 = 1 μF.

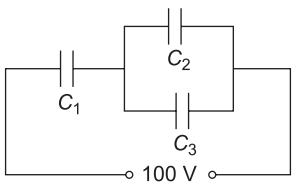


Fig. 11.29(a)

SOLUTION The capacitance of the parallel combination of C_2 and C_3 is $C' = C_2 + C_3 = 3\ \mu\text{F}$

The circuit can be redrawn as shown in Fig. 11.29(b).

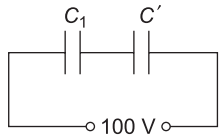


Fig. 11.29(b)

The equivalent capacitance C of series combination of C_1 and C' is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C'}$$

$$\Rightarrow \qquad C = \frac{C_1 C'}{C_1 + C'} = \frac{2 \times 3}{2 + 3} = \frac{6}{5} = 1.2\ \mu\text{F}$$

Let Q_1, Q_2 and Q_3 be the charges on C_1, C_2, and C_3 respectively. Since C_1 and C' are in series, charge on C_1 = charge on $C' = Q_1$. Let V_1 and V' be the potential differences across C_1 and C' respectively. Then

$$Q_1 = C_1 V_1 = C' V' \Rightarrow \frac{V'}{V_1} = \frac{C_1}{C'} = \frac{2}{3}$$

But $V_1 + V' = 100$. Therefore

$$V_1 + \frac{2}{3} V_1 = 100 \Rightarrow V_1 = 60\ \text{V}$$

Therefore $V' = 100 - 60 = 40$ V. Hence

$$Q_1 = C_1 V_1 = 2\ \mu\text{F} \times 60\ \text{V} = 120\ \mu\text{C}$$

Since C_2 and C_3 are in parallel, potential difference V_2 across C_2 = potential difference V_3 across $C_3 = V' = 40$ V. Therefore,

Charge on capacitor $C_2 = C_2 V_2 = 2\ \mu\text{F} \times 40\ \text{V} = 80\ \mu\text{C}$ and Charge on capacitor $C_3 = C_3 V_3 = 1\ \mu\text{F} \times 40\ \text{V} = 40\ \mu\text{C}$

Thus $Q_1 = 120\ \mu\text{C}$, $Q_2 = 80\ \mu\text{C}$ and $Q_3 = 40\ \mu\text{C}$

EXAMPLE 12 In the network shown in Fig 11.29(a) in Example 11, find the total energy stored in the network.

SOLUTION Total (equivalent) capacitance of network is

$$C = 1.2\ \mu\text{F} = 1.2 \times 10^{-6}\ \text{F}.$$

Total potential difference across the network is $V = 100$ V. Therefore, total energy stored is

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times (1.2 \times 10^{-6}) \times (100)^2 = 6 \times 10^{-3}\ \text{J}$$

Notice that

$$U = U_1 + U_2 + U_3 = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 + \frac{1}{2} C_3 V_3^2.$$

EXAMPLE 13 Two capacitors of capacitances C_1 and C_2 have a total capacitance of 20 μF when connected in parallel and a total capacitance 4.8 μF when connected in series. Find C_1 and C_2.

SOLUTION

$$C_1 + C_2 = 20 \qquad (1)$$

$$\frac{C_1 C_2}{C_2 + C_2} = 4.8 \qquad (2)$$

Using (1) in (2) we get

$$\frac{C_1\ C_2}{20} = 4.8 \Rightarrow C_1 C_2 = 96\ \mu\text{F}$$

$$\therefore \qquad C_2 = \frac{96}{C_1}$$

Using this in (2) we get

$$C_1^2 - 20C_1 + 96 = 0$$

The two roots of this equation are $C_1 = 8$ μF and 12 μF. Hence the two capacitors have capacitances of 8 μF and 12 μF.

EXAMPLE 14 Four capacitors of capacitances $C_1 = 1$ μF, $C_2 = 2$ μF, $C_3 = 3$ μF and $C_4 = 4$ μF are connected as shown in Fig. 11.30(a).

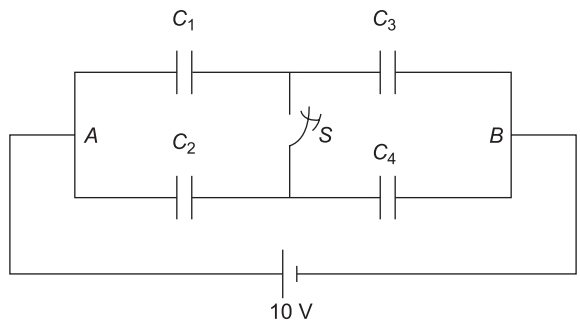


Fig. 11.30(a)

Find the potential difference across C_3 when (a) switch S is open and (b) switch S is closed.

SOLUTION

(a) When switch S is open, the circuit is [see Fig. 11.30(b)]

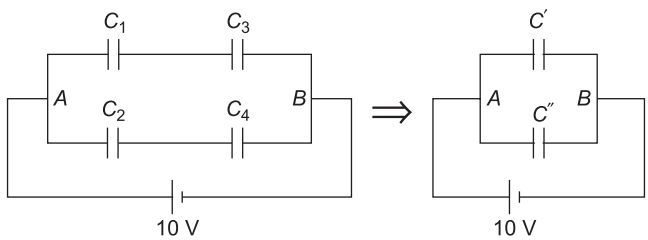


Fig. 11.30(b)

C_1 and C_3 are in series, charge on C_1 = charge on C_3 = Q (say) and potential difference across the combination of C_1 and C_3 is 10 V.

$$\therefore \quad Q = C_1V_1 = C_3V_3 \Rightarrow C_1V_1 = C_3V_3$$

$$\Rightarrow \quad V_1 = 3V_3. \text{ Also } V_1 + V_3 = 10$$

Hence $V_3 = 2.5$ V.

(b) When switch S is closed, the circuit is as shown in Fig. 11.30(c).

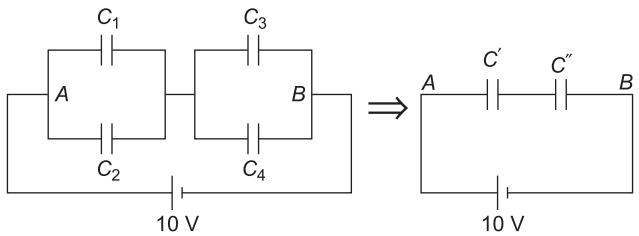


Fig. 11.30(c)

$$C' = 1 + 2 = 3\ \mu\text{F}$$

$$C'' = 3 + 4 = 7\ \mu\text{F}$$

Since C' and C'' are in series, the charge on C' = charge on C''

or $\quad C'V' = C''V'' \Rightarrow 3V' = 7V''$

$$\Rightarrow \quad V' = \frac{7V''}{3}$$

$$\text{Also } \quad V' + V'' = 10 \Rightarrow \frac{7V''}{3} + V'' = 10$$

$$\Rightarrow \quad V'' = 3 \text{ V}$$

Since C_3 and C_4 are in parallel, potential difference across C_3 = potential difference across C_4 = 3 V.

EXAMPLE 15 In Ex. 13 above, find the energy stored in the network of capacitors shown in Fig. 11.30(a) when (a) Switch S is open and (b) switch S is closed.

SOLUTION

(a) Refer to Fig. 11.30(b). The equivalent capacitance between point A and B is

$$C_a = C' + C''$$

$$= \frac{C_1C_3}{C_1 + C_3} + \frac{C_2C_4}{C_2 + C_4}$$

$$= \frac{1\times 3}{(1+3)} + \frac{2\times 4}{(2+4)} = \frac{25}{12}\mu\text{F} = \frac{25}{12}\times 10^{-6}\text{ F}$$

Energy stored is

$$U_a = \frac{1}{2}C_aV^2 = \frac{1}{2}\times\frac{25}{12}\times 10^{-6}\times(10)^2 = 1.04\times 10^{-4}\text{ J}$$

(b) Refer to Fig. 11.30(c). The equivalent capacitance between point A and B is

$$C_b = \frac{C'C''}{C' + C''} = \frac{3\times 7}{3+7} = \frac{21}{10}\mu\text{F} = \frac{21}{10}\times 10^{-6}\text{ F}$$

Energy stored is

$$U_b = \frac{1}{2}C_bV^2 = \frac{1}{2}\times\frac{21}{10}\times 10^{-6}\times(10)^2 = 1.05\times 10^{-4}\text{ J}$$

EXAMPLE 16 The circuit shown in Fig. 11.31 consists of four capacitors C_1, C_2, C_3, and C_4 each of capacitance 4 μF and a battery of emf E = 5V. Find the potential difference between A and B.

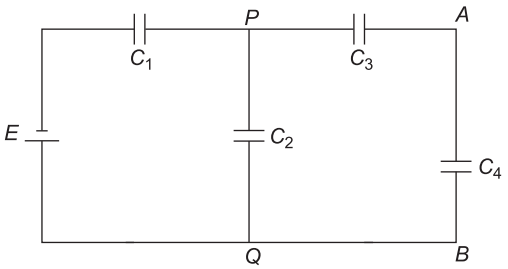


Fig. 11.31

SOLUTION Capacitors C_3 and C_4 are in series, their equivalent capacitance is

$$C' = \frac{C_3C_4}{(C_3 + C_4)} = \frac{4\times 4}{(4+4)} = 2\ \mu\text{F}$$

C' is in parallel with C_2.
Their equivalent capacitance is

$$C'' = C_2 + C' = 4 + 2 = 6\ \mu\text{F}$$

Hence the circuit can be redrawn as shown in Fig. 11.32.

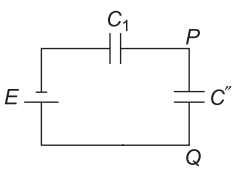


Fig. 11.32

If V_1 is the p.d. across C_1 and V'' across C'', then (since they are in series)

$$C_1V_1 = C''V''$$

$$\Rightarrow \quad 4V_1 = 6V'' \Rightarrow V_1 = \frac{3V''}{2}$$

But $\quad V_1 + V'' = E = 5$. Hence

$$\frac{3V''}{2} + V'' = 5 \Rightarrow V'' = 2\text{ V}$$

Thus the p.d. between P and Q = 2 V. By symmetry of C_3 and C_4, the p.d. between A and B = 1 V.

EXAMPLE 17 Three capacitors C_1 = 1 μF, C_2 = 2 μF and C_3 = 3 μF are connected as shown in Fig. 11.33. Find the equivalent capacitance between points *A* and *B*.

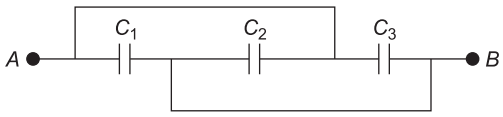


Fig. 11.33

SOLUTION Refer to Fig. 11.34.

Plate 1 of C_1, plate 4 of C_2 and plate 5 of C_3 are connected to point *A*. Plate 2 of C_1, plate 3 of C_2 and plate 6 of C_3 are connected to point *B*. Thus C_1, C_2 and C_3 are connected in parallel. Therefore, the equivalent capacitance C_{eq} between point *A* and *B* is given by

$$C_{eq} = C_1 + C_2 + C_3$$
$$= 1 + 2 + 3 = 6\ \mu F$$

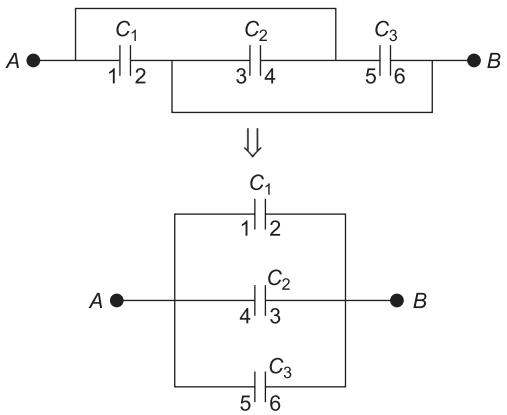


Fig. 11.34

EXAMPLE 18 In the network shown in Fig. 11.35(a), the equivalent capacitance between points *P* and *Q* is

(a) 1 μF (b) 2 μF

(c) 3 μF (d) 4 μF

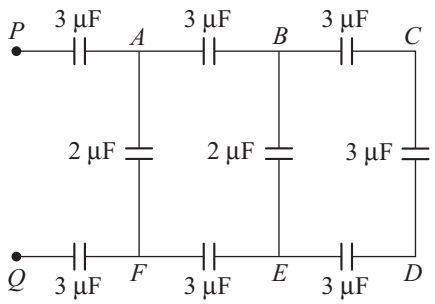


Fig. 11.35(a)

SOLUTION The given network can be simplified as shown in Fig. 11.35(b).

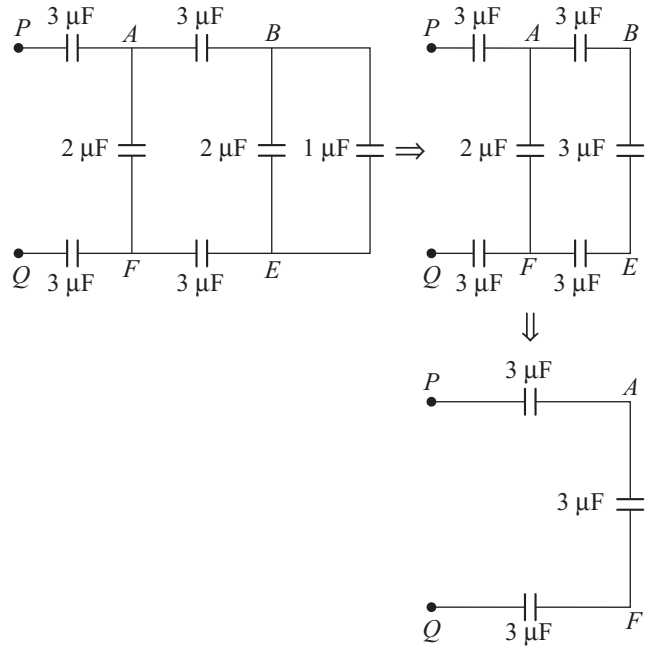


Fig. 11.35(b)

The equivalent capacitance between *P* and *Q* is given by

$$\frac{1}{C_{eq}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1\ \mu F$$

$$\Rightarrow \quad C_{eq} = 1\ \mu F$$

EXAMPLE 19 In the network shown in Fig. 11.36(a), each capacitor has a capacitance *C*. The equivalent capacitance between points *P* and *Q* is

(a) $\frac{4C}{15}$ (b) $\frac{6C}{19}$

(c) $\frac{8C}{21}$ (d) *C*

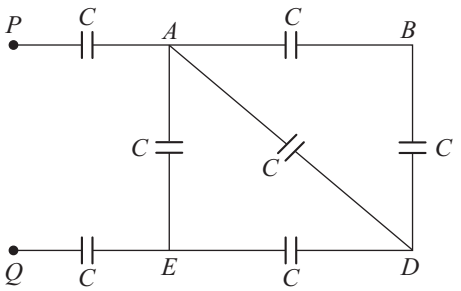


Fig. 11.36(a)

SOLUTION The network can be simplified as shown in Fig. 11.36(b).

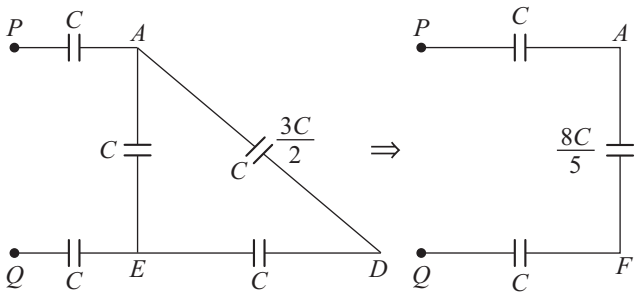


Fig. 11.36(b)

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{5}{8C} + \frac{1}{C}$$

$$\Rightarrow \quad C_{eq} = \frac{8C}{21}$$

EXAMPLE 20 In the circuit shown in Fig. 11.37(a), the charge on the 4 μF capacitor is

(a) zero (b) 15 μC

(c) 20 μC (d) 60 μC

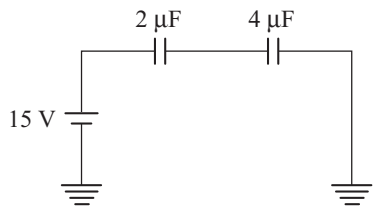


Fig. 11.37(a)

SOLUTION The circuit can be redrawn as shown in Fig. 11.37(b).

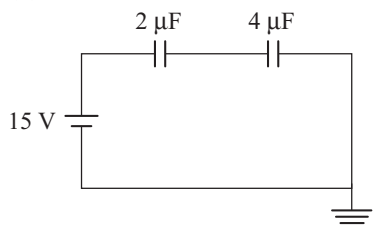


Fig. 11.37(b)

The equivalent capacitance of the series combination is

$$C_{eq} = \frac{2 \times 4}{2 + 4} = \frac{4}{3}\,\mu F$$

In a series combination, the charge on each capacitor is the same.

$$q = C_{eq} \times V$$

$$= \frac{4}{3}\,\mu F \times 15\ V = 20\,\mu C$$

EXAMPLE 21 In the circuit shown in Fig. 11.38(a), the equivalent capacitance between points *A* and *B* is

(a) 3 μF (b) 6 μF

(c) 12 μF (d) 15 μF

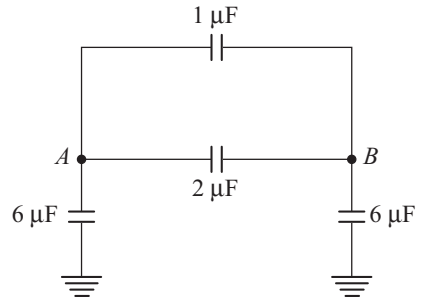


Fig. 11.38(a)

SOLUTION The circuit can be redrawn as shown in Fig. 11.38(b).

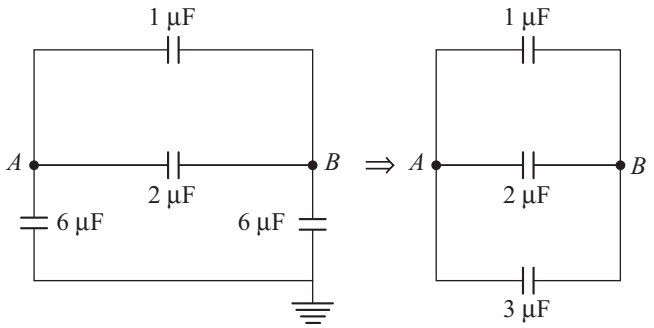


Fig. 11.38(b)

The 6 μF capacitors are connected in series and this combination is in parallel with 1 μF and 2 μF capacitors.

$$C_{eq} = 1 + 2 + 3 = 6\,\mu F$$

24. Wheatstone's Bridge of Capacitors

In the circuit shown in Fig. 11.39(a) in Example 22 below, the network of capacitors form a wheatstone's Bridge. The bridge is balanced if the values of C_1, C_2, C_3 and C_4 satisfy the condition

$$\frac{C_1}{C_2} = \frac{C_3}{C_4}$$

For networks which satisfy this condition, the equivalent capacitance can be found as illustrated in Examples 22, 23 and 24 below.

EXAMPLE 22 Find the equivalent capacitance between *A* and *B* in the circuit shown in Fig. 11.39(a). Given $C_1 = 5\,\mu F$, $C_2 = 20\,\mu F$, $C_3 = 10\,\mu F$, $C_4 = 40\,\mu F$ and $C_5 = 30\,\mu F$.

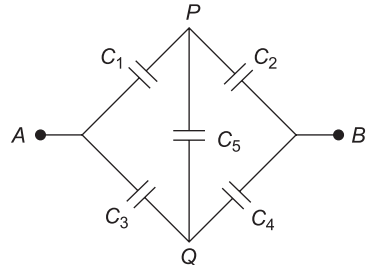


Fig. 11.39(a)

SOLUTION The network shown in the figure is a balanced wheatstone's bridge. If the values of C_1, C_2, C_3 and C_4 are such that the condition

$$\frac{C_1}{C_2} = \frac{C_3}{C_4}$$

is satisfied, the bridge is said to be balanced. The potential at P = potential at Q. Since the potential difference between P and Q is zero (when a battery is connected across A and B), the capacitor C_5 is ineffective as no charge collects on its plates.

Hence the circuit reduces to [see Fig. 11.39(b)]

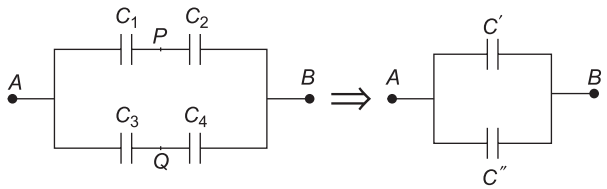


Fig. 11.39(b)

$$C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{5 \times 20}{5 + 20} = 4\ \mu\text{F}$$

$$C'' = \frac{C_3 C_4}{C_3 + C_4} = \frac{10 \times 40}{10 + 40} = 8\ \mu\text{F}$$

$$\therefore \quad C_{AB} = C' + C'' = 12\ \mu\text{F}$$

EXAMPLE 23 Each capacitor in the network shown in Fig. 11.40(a) has a capacitance C. The equivalent capacitance between points A and B is

(a) C (b) 2 C

(c) 3 C (d) 4 C

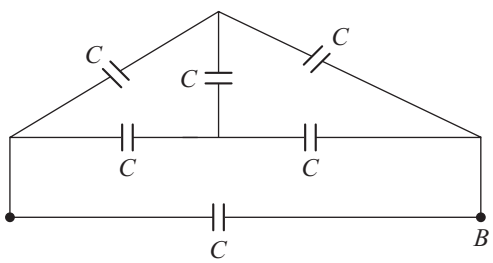


Fig. 11.40(a)

SOLUTION The upper part of the network is a balanced Wheatstone's Bridge. As shown in the above Example 15, the network can be simplified as shown in Fig. 11.40(b).

$$\therefore \quad C_{eq} = C + C = 2\ C$$

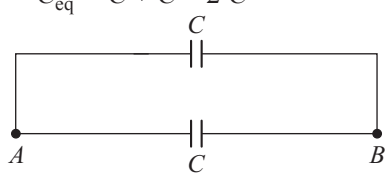


Fig. 11.40(b)

So the correct choice is (b).

EXAMPLE 24 Each capacitor in the network shown in Fig. 11.41(a) has a capacitance C. The equivalent capacitance between points P and Q is

(a) C (b) $\frac{C}{3}$

(c) $\frac{3C}{7}$ (d) $\frac{2C}{5}$

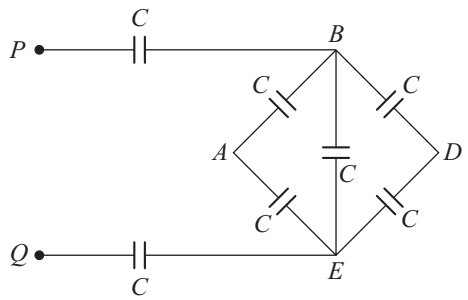


Fig. 11.41(a)

SOLUTION Network $ABDE$ is a balanced Wheatstone's Bridge. As shown above, to find capacitance between A and D, the capacitor between B and E can be ignored. But to find capacitance between B and E, we cannot ignore the capacitor between B and E. Capacitors between B and D and D and E are in series and have a combined capacitance = $C/2$. Capacitors between A and B and A and E have a combined capacitance = $C/2$. Thus the network simplifies to that shown in Fig. 11.41(b).

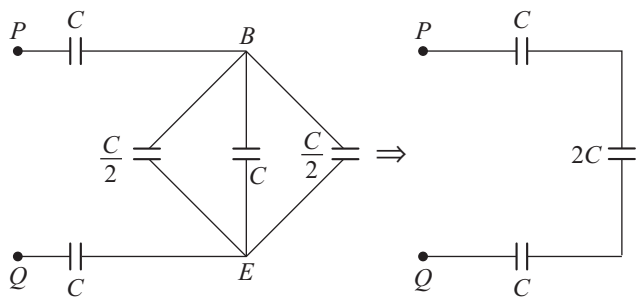


Fig. 11.41(b)

Equivalent capacitance between P and Q is given by

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{C}$$

$$\Rightarrow \quad C_{eq} = \frac{2C}{5}$$

Note For an unbalanced Wheelstone's bridge or for any other more complicated combinations of capacitors, it is not easy to find the equivalent capacitance using the formulae for series and parallel combinations.

For such cases, we should use the following procedure:

(1) Connect an imaginary battery between the points across which the equivalent capacitance is to be found.

(2) Send a positive charge + Q from the positive terminal of the battery and equal negative charge $-Q$ from the negative terminal.

(3) Write the charges on each capacitor plate using the principle of charge conservation. *i.e.*, charges on the two plates must be equal and opposite. Let Q_1, Q_2, ... etc. be the charges on the capacitors in the network and V_1, V_2,... etc. be the respective potential differences.

(4) Use $Q = CV$ for each capacitor. Eliminate Q_1, Q_2, ... etc. and V_1, V_2,... etc. to obtain the equivalent capacitance $C_{eq} = \frac{Q}{E}$, where E is the voltage of the battery

EXAMPLE 25 Find the equivalent capacitance between A and B in the circuit shown in Fig. 11.42(a). Given $C = 5\ \mu F$.

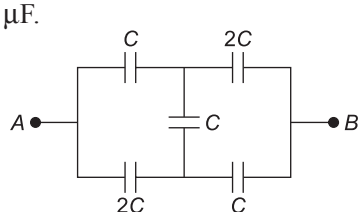


Fig. 11.42(a)

SOLUTION Refer to Fig. 11.42(b).

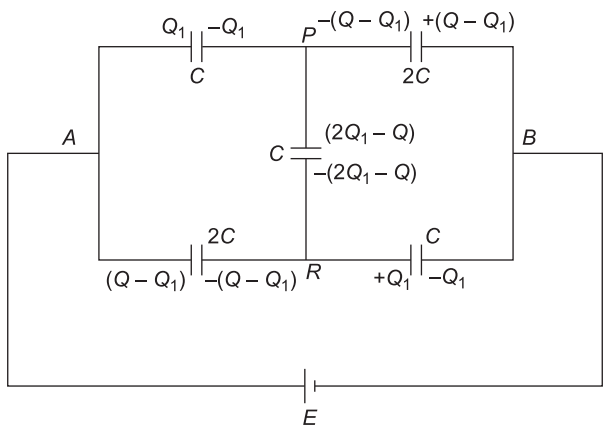


Fig. 11.42(b)

Let us imagine a battery of emf E connected between A and B. Let the positive terminal send a charge $+Q$ and let negative terminal send a charge $-Q$.

Out of charge + Q, charge Q_1 appears on the left plate of C and $-Q_1$ on the right plate. The remaining charge $(Q - Q_1)$ appears on the left plate of $2C$ and $-(Q - Q_1)$ appears on its right plate. Similarly, out of charge $-Q$ sent by the negative terminal of the battery, a charge $-Q_1$ appears on the right plate of C (by symmetry) a charge + Q_1 appears on its left plate. The remaining charge $+(Q - Q_1)$ appears on the right plate of $2C$ and $-(Q - Q_1)$ appears on its left plate. Now, since the right plate of C, the left plate $2C$ and the upper plate of C are all connected (to P), they form an isolated system. Hence the sum of charges on these plates must be zero. Therefore, the charge on the upper plate of C must be $(2Q_1 - Q)$ and the charge on its lower plate must be $-(2Q_1 - Q)$. The distribution of charges on the capacitor plates is shown in the figure using the principle of charge conservation. Then

$$V_A - V_B = (V_A - V_P) + (V_P - V_B)$$

$$\Rightarrow \quad E = \frac{Q_1}{C} + \frac{(Q - Q_1)}{2C}$$

$$\Rightarrow \quad 2CE = 2Q_1 + (Q - Q_1) = Q_1 + Q \qquad (1)$$

Also $$V_A - V_B = (V_A - V_P) + (V_P - V_R) + (V_R - V_B)$$

$$\Rightarrow \quad E = \frac{Q_1}{C} + \frac{(2Q_1 - Q)}{C} + \frac{Q_1}{C}$$

$$\Rightarrow \quad CE = Q_1 + (2Q_1 - Q) + Q_1 = 4Q_1 - Q \qquad (2)$$

Eliminating Q_1 from (1) and (2), we get

$$Q = \frac{7CE}{5}$$

$$\therefore \quad C_{eq} = \frac{Q}{E} = \frac{7C}{5} = \frac{7 \times 5\ \mu F}{5} = 7\ \mu F$$

[$\because$ C = 5 μF (given)]

EXAMPLE 26 The circuit shown in Fig. 11.43 consists of two capacitors $C_1 = 2\ \mu F$ and $C_2 = 4\ \mu F$ and two batteries, each of emf $E = 3V$. Find the charge flowing through points A and B when switch S is closed

SOLUTION When switch S is open, the equivalent capacitance is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{2 \times 4}{(2 + 4)} = \frac{4}{3}\ \mu F$$

Charge on the R.H.S. plate of C_1 and upper plate of C_2 is

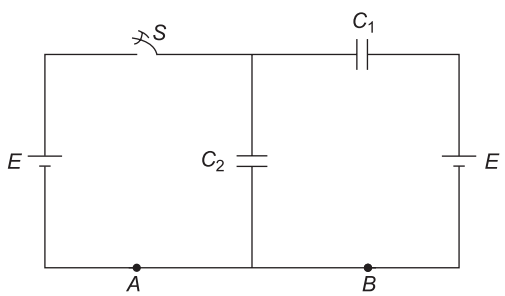


Fig. 11.43

$$Q = CE = \frac{4}{3}\ \mu F \times 3\ V = 4\ \mu C$$

When switch S is closed, the potential difference across C_1 is zero, because the batteries are in opposition. Therefore, charge on the R. H. S. plate of $C_1 = 0$. If the charge flowing through B is q, then

$$Q + q = 0$$

$$\Rightarrow \quad q = -Q = -4\ \mu C$$

Now the charge on the upper plate of $C_2 = C_2E = 4\ \mu F \times 3V = 12\ \mu C$, which is the charge flowing through A.

25. Capacitance of Dielectric Filled Capacitors

(i) Capacitors as shown in Fig. 11.44.

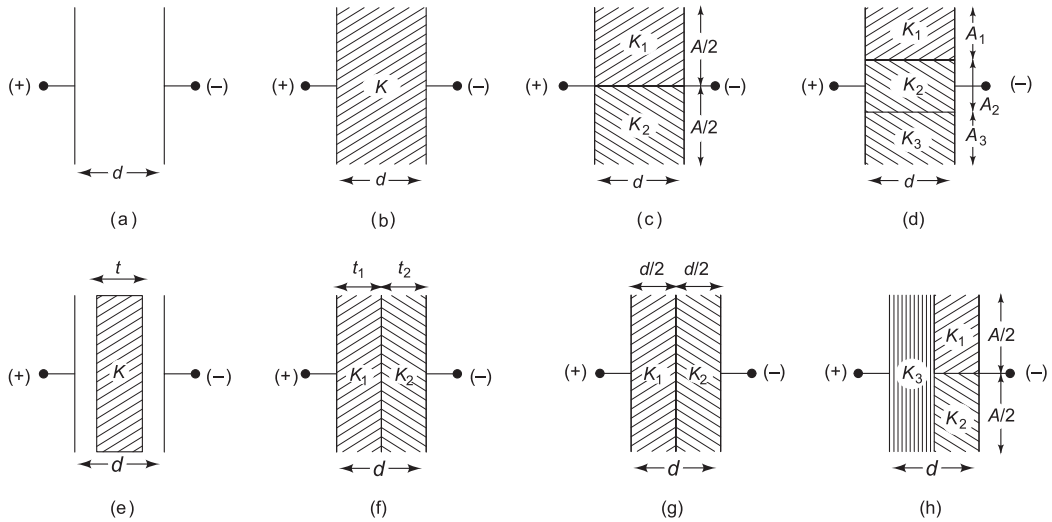


Fig. 11.44

(a) Capacitor with air as dielectric [Fig. 11.45(a)]

$$C_0 = \frac{\varepsilon_0 A}{d}$$

(b) Capacitor completely filled with a dielectric of dielectric constant K [Fig. 11.45(b)]

$$C = KC_0 = \frac{K\varepsilon_0 A}{d}$$

(c) Capacitor filled with two dielectrics as shown in Fig. 11.45(c)

Capacitance of upper dielectric is

$$C_1 = \frac{K_1\varepsilon_0 A/2}{d} = \frac{K_1\varepsilon_0 A}{2d}$$

Capacitance of lower dielectric is $C_2 = \dfrac{K_2\varepsilon_0 A}{2d}$

Since positive plates of C_1 and C_2 are connected together, C_1 and C_2 are in parallel. Hence the capacitance of combination is

$$C = C_1 + C_2 = \left(\frac{K_1 + K_2}{2}\right)\frac{\varepsilon_0 A}{d} = \frac{K_{eq}\varepsilon_0 A}{d}$$

where $K_{eq} = \dfrac{1}{2}(K_1 + K_2)$ is the equivalent dielectric constant

(d) Capacitor filled with three dielectrics as shown in Fig. 11.45(d)

$$C = C_1 + C_2 + C_3 = \frac{K_{eq}\varepsilon_0 A}{d}$$

$$\text{Where } K_{eq} = \frac{K_1A_1 + K_2A_2 + K_3A_3}{(A_1 + A_2 + A_3)}$$

(e) Capacitor partly filled with a dielectric as shown in Fig. 11.45(e)

$$C = \frac{\varepsilon_0 A}{\left[d - t + \dfrac{t}{K}\right]} = \frac{C_0}{\left[1 + \dfrac{t}{d}\left(\dfrac{1}{K} - 1\right)\right]}$$

where $C_0 = \dfrac{\varepsilon_0 A}{d}$. If $t = d$, $C = KC_0$ as in Fig. 11.45(b)

(f) Capacitor filled with two dielectrics as shown in Fig. 11.45(f)

Capacitance of the left capacitor is $C_1 = \dfrac{K_1\varepsilon_0 A}{t_1}$

Capacitance of the right capacitor is $C_2 = \dfrac{K_2\varepsilon_0 A}{t_2}$

Since C_1 and C_2 are in series, the capacitance of the combination is

$$C = \frac{C_1C_2}{C_1 + C_2} = \frac{K_1K_2\varepsilon_0^2 A^2}{t_1t_2\,\varepsilon_0 A\left(\dfrac{K_1}{t_1} + \dfrac{K_2}{t_2}\right)}$$

$$\Rightarrow \qquad C = \frac{K_1 K_2\, \varepsilon_0 A}{(K_1 t_2 + K_2 t_1)} = \frac{K_{eq} \varepsilon_0 A}{d}$$

Where $\dfrac{K_{eq}}{d} = \dfrac{K_1 K_2}{(K_1 t_2 + K_2 t_1)}$

$$\Rightarrow \qquad \frac{K_{eq}}{t_1 + t_2} = \frac{K_1 K_2}{(K_1 t_2 + K_2 t_1)} \qquad (\because\ d = t_1 + t_2)$$

$$\Rightarrow \qquad K_{eq} = \frac{K_1 K_2 (t_1 + t_2)}{(K_1 t_2 + K_2 t_1)}$$

(g) Capacitor filled with two dielectrics as shown in Fig. 11.45(g)

$$C_1 = \frac{2K_1 \varepsilon_0 A}{d},\ C_2 = \frac{2K_2 \varepsilon_0 A}{d}$$

$$\therefore \qquad C = \frac{C_1 C_2}{(C_1 + C_2)} = \frac{K_{eq} \varepsilon_0 A}{d}$$

Where $K_{eq} = \dfrac{2K_1 K_2}{(K_1 + K_2)}$

(h) Capacitor filled with three dielectrics as shown in Fig. 11.45(h)

Capacitors with K_1 and K_2 are in parallel and this combination is in series with the capacitor with K_3

$$C = \frac{K_{eq} \varepsilon_0 A}{d}, \text{ Where } K_{eq} \text{ is given by}$$

$$\frac{1}{K_{eq}} = \frac{1}{(K_1 + K_2)} + \frac{1}{2K_3}$$

26. Charging and Discharging of a Capacitor through a Resistance

Consider a capacitor of capacitance C connected in series to a resistor R and a battery of emf E and negligible internal resistance through a two-way key as shown in Fig. 11.45.

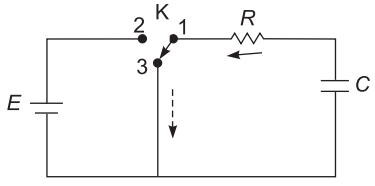


Fig. 11.45

Growth of Charge The battery is introduced in the circuit by connecting terminals 1 and 2 of the key. Initially, i.e. when $t = 0$, the charge on the capacitor plates is zero. As time passes, the charge flows into the capacitor plates and the potential difference q/C (q is the charge at any time t) between the plates rises. The charge on the plates of the capacitor rises till the potential difference between the plates becomes E. The maximum charge collected is $q_0 = CE$. At this stage the current in the circuit becomes zero.

The growth of charge on the capacitor plates as a function of time is given by

$$q = q_0 (1 - e^{-t/RC}) \qquad (1)$$

It is clear that the charge rises exponentially to a steady state maximum value q_0 as shown in Fig. 11.46(a).

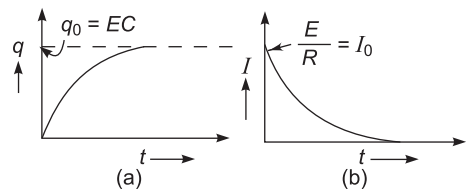


Fig. 11.46

Time Constant The product RC has the dimensions of time. If C is in farad and R in ohm, the product CR will be in seconds. Writing $RC = \tau$ in Eq. (1) we have

$$q = q_0 (1 - e^{-t/\tau})$$

At time $t = \tau$,

$$q = q_0 \left(1 - \frac{1}{e}\right) = q_0 \left(1 - \frac{1}{2.718}\right) = 0.63 q_0$$

Thus the time constant τ of a CR circuit may be defined as the time during which the charge on the capacitor grows from zero to 0.63 of its maximum value q_0. Whether the charge grows quickly or slowly depends on the value of the time constant, i.e. on the values of R and C. If the product CR (i.e. the time constant) is very small, the charge grows quickly.

The behaviour of current as a function of time t is given by

$$I = \frac{dq}{dt} = I_0 e^{-t/RC}$$

where $I_0 = E/R$ is the maximum current. Therefore, the current decreases exponentially from its maximum value I_0 to zero as shown in Fig. 11.46(b).

Decay of Charge When the charge has attained a steady value EC, the battery is short-circuited by connecting the terminals 1 and 3 of the key K. In such a situation the capacitor starts discharging through the resistor, i.e. the

charge on the capacitor starts flowing back through the resistor. The direction of the current is, therefore, reversed.

The decay of charge with time is given by

$$q = q_0\, e^{-t/RC} = q_0\, e^{-t/\tau}$$

Figure 11.47 shows the decay of charge with time.

At time $t = \tau$, $q = q_0\, e^{-1} = 0.368\, q_0$. Thus in a time $t = \tau$, the time constant, the charge on the capacitor decays to 0.368 of its initial value q_0. So the charge decays exponentially with time t. Whether the charge decays slowly or quickly depends on the value of the time constant RC.

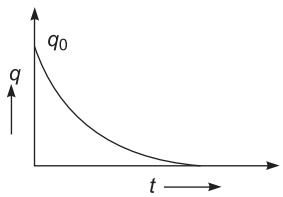


Fig. 11.47

EXAMPLE 27 When the plates of a parallel plate capacitor of capacitance 3.0 μF are connected by a wire of resistance R, the electric field between the plates drops to half of its initial value in 6.9 μs. The value of R is

(a) $\frac{3}{5}\,\Omega$ (b) $\frac{9}{5}\,\Omega$

(c) $\frac{10}{3}\,\Omega$ (d) $\frac{12}{5}\,\Omega$

SOLUTION The voltage between plates is

$$V = \frac{Q}{C} = \frac{Qd}{A\varepsilon_0}$$

Electric field is $E = \frac{V}{d} = \frac{Q}{A\varepsilon_0} = \frac{1}{A\varepsilon_0}(Q_0\, e^{-t/RC})$

$$E = E_0\, e^{-t/RC}$$

where Q_0 is the initial charge and E_0 is the initial electric field. Given that $E = E_0/2$.

$$\frac{E_0}{2} = E_0\, e^{-t/RC}$$

$$\Rightarrow \quad \frac{1}{2} = e^{-t/RC}$$

$$\Rightarrow \quad 2 = e^{t/RC}$$

$$\therefore \quad \ln(2) = \frac{t}{RC}$$

$$\Rightarrow \quad R = \frac{t}{C\ln(2)} = \frac{6.9\,\mu s}{3.0\,\mu F \times 0.69} = \frac{10}{3}\,\Omega$$

EXAMPLE 28 A capacitor of capacitance C is connected to a 6.0 V battery through a resistance of 5 Ω. The potential difference between the plates rises from zero to 3.0 V in 6.9 μs. The value of C is

(a) 1 μF (b) 2 μF

(c) 3 μF (d) 4 μF

SOLUTION During charging, the charge at time t on the capacitor is given by

$$Q = Q_0\,(1 - e^{-t/RC})$$

Therefore, the potential difference at time t between the capacitor plates is

$$V = \frac{Q}{C} = \frac{Q_0}{C}(1 - e^{-t/RC})$$

$$\Rightarrow \quad V = V_0\,(1 - e^{-t/RC})$$

where $V_0 = \frac{Q_0}{C}$ is the final potential difference = 6.0 V, the voltage of the battery. Given $V = 3.0\text{ V} = \frac{V_0}{2}$. Hence

$$\frac{V_0}{2} = V_0\,(1 - e^{-t/RC})$$

$$\Rightarrow \quad 1 - e^{-t/RC} = \frac{1}{2}$$

$$\Rightarrow \quad e^{-t/RC} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{t}{RC} = \ln(2)$$

$$\Rightarrow \quad C = \frac{t}{R\ln(2)} = \frac{6.9\,\mu s}{5 \times 0.69} = 2\,\mu F$$

SECTION 1

Multiple Choice Questions with One Correct Choice

Level A

1. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface to 10 V. The potential at the centre of the sphere is
   (a) zero
   (b) 10 V
   (c) the same as that at a point 5 cm away from the surface
   (d) the same as that at a point 25 cm away from the surface

2. Two equal negative charges $-q$ are fixed at points $(0, a)$ and $(0, -a)$ on the y-axis. A positive charge Q is released from rest at a point $(2a, 0)$ on the x-axis. The charge Q will
   (a) execute simple harmonic motion about the origin
   (b) move to the origin and remain at rest there
   (c) move to infinity
   (d) execute oscillatory but not simple harmonic motion.

3. Four capacitors, each of capacitance 50 μF are connected as shown in Fig. 11.48. If the voltmeter reads 100 V, the charge on each capacitor is
   (a) 2×10^{-3} C (b) 5×10^{-3} C
   (c) 0.2 C (d) 0.5 C

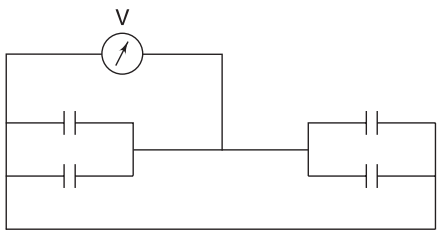


Fig. 11.48

4. Three point charges $4q$, Q and q are placed in a straight line of length l at points distant 0, $l/2$ and l respectively. The net force on charge q is zero. The value of Q is
   (a) $-q$ (b) $-2q$
   (c) $-\frac{1}{2}q$ (d) $4q$

5. Two positive point charges of 12 and 8 microcoulombs respectively are placed 10 cm apart in air. The work done to bring them 4 cm closer is
   (a) zero (b) 3.8 J
   (c) 4.8 J (d) 5.8 J

6. The work done is carrying a charge q once round a circle of radius r with a charge Q at the centre is
   (a) $\frac{qQ}{4\pi\varepsilon_0 r}$ (b) $\frac{qQ}{4\pi\varepsilon_0}\frac{1}{\pi r}$
   (c) $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{2\pi r}\right)$ (d) zero

7. A capacitor of capacitance $C = 2$ μF is connected as shown in Fig. 11.49. If the internal resistance of the cell is 0.5 Ω, the charge on the capacitor plates is
   (a) zero (b) 2 μC
   (c) 4 μC (d) 6 μC

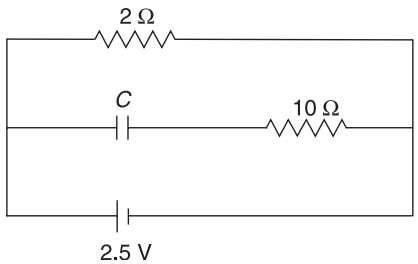


Fig. 11.49

8. A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to
   (a) $-\frac{Q}{2}$ (b) $-\frac{Q}{4}$
   (c) $+\frac{Q}{2}$ (d) $+\frac{Q}{4}$

9. The electric potential V (in volt) varies with x (in metre) according to the relation
   $$V = 5 + 4x^2$$

The force experienced by a negative charge of 2×10^{-6} C located at $x = 0.5$ m is

(a) 2×10^{-6} N (b) 4×10^{-6} N

(c) 6×10^{-6} N (d) 8×10^{-6} N

10. Two parallel plate capacitors of capacitances C and $2C$ are connected in parallel and charged to a potential difference V by a battery. The battery is then disconnected and the space between the plates of capacitor C is completely filled with a material of dielectric constant K. The potential difference across the capacitors now becomes

(a) $\dfrac{V}{K+1}$ (b) $\dfrac{2V}{K+2}$

(c) $\dfrac{3V}{K+2}$ (d) $\dfrac{3V}{K+3}$

11. The force of attraction between the plates of air filled parallel plate capacitor having charge Q and area of each plate A is given by

(a) $\dfrac{2Q^2}{\varepsilon_0 A}$ (b) $\dfrac{Q^2}{\varepsilon_0 A}$

(c) $\dfrac{Q^2}{2\varepsilon_0 A}$ (d) $\dfrac{Q^2}{4\varepsilon_0 A}$

12. In the network shown in Fig. 11.50, $C_1 = 6\ \mu$F and $C = 9\ \mu$F. The equivalent capacitance between points P and Q is

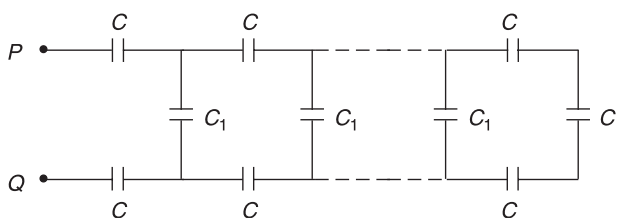


Fig. 11.50

(a) 3 μF (b) 6 μF

(c) 9 μF (d) 12 μF

13. Three capacitors, each of capacitance $C = 3\ \mu$F, are connected as shown in Fig. 11.51. The equivalent capacitance between points P and S is

(a) 1 μF (b) 3 μF

(c) 6 μF (d) 9 μF

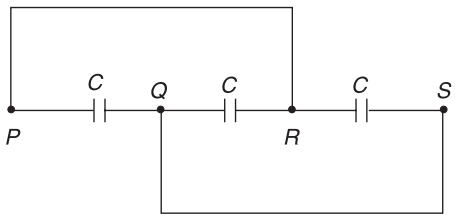


Fig. 11.51

14. A parallel plate capacitor of capacitance 100 pF is to be constructed by using paper sheets of 1.0 mm thickness as dielectric. If the dielectric constant of paper is 4.0, the number of circular metal foils of diameter 2.0 cm each required for this purpose is

(a) 10 (b) 20

(c) 30 (d) 40

15. One thousand spherical water droplets, each of radius r and each carrying a charge q, coalesce to form a single spherical drop. If v is the electrical potential of each droplet and V that of the bigger drop, then

(a) $\dfrac{V}{v} = \dfrac{1}{1000}$ (b) $\dfrac{V}{v} = \dfrac{1}{100}$

(c) $\dfrac{V}{v} = 100$ (d) $\dfrac{V}{v} = 1000$

16. A parallel plate air filled capacitor shown in Fig. 11.52(a) has a capacitance of 2 μF. When it is half filled with a dielectric of dielectric constant $k = 3$ as shown in Fig. 11.52(b), its capacitance becomes

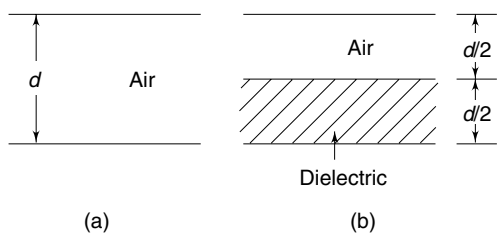


Fig. 11.52

(a) $\dfrac{1}{3}$ μF (b) 1 μF

(c) 3 μF (d) 9 μF

17. A parallel plate air filled capacitor shown in Fig. 11.53(a) has a capacitance of 2 μF. When it is half filled with a dielectric of dielectric constant $k = 3$ as shown in Fig. 11.53(b), its capacitance becomes

(a) 4 μF (b) 3 μF

(c) 1.5 μF (d) 0.5 μF

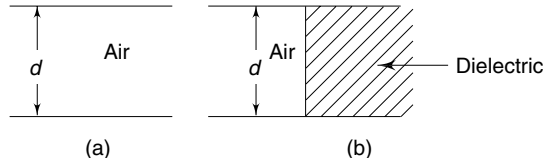


Fig. 11.53

18. Two small identical balls P and Q, each of mass $\sqrt{3}/10$ gram, carry identical charges and are suspended by threads of equal lengths. At equilibrium, they position themselves as shown in Fig. 11.54. What is the charge on each ball. Given $\frac{1}{4\pi\varepsilon_0}$ $= 9 \times 10^9\ \text{Nm}^2\text{C}^{-2}$ and take $g = 10\ \text{ms}^{-2}$.

(a) 10^{-3} C (b) 10^{-5} C
(c) 10^{-7} C (d) 10^{-9} C

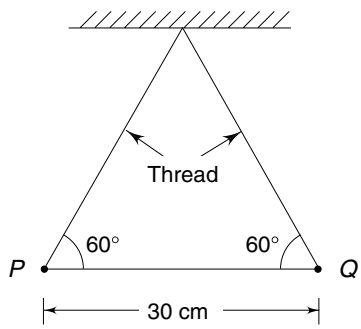


Fig. 11.54

19. Two point charges $q_1 = 2\ \mu\text{C}$ and $q_2 = 1\ \mu\text{C}$ are placed at distances $b = 1$ cm and $a = 2$ cm from the origin on the y and x axes as shown in Fig. 11.55. The electric field vector at point $P(a, b)$ will subtend an angle θ with the x-axis given by

(a) $\tan\theta = 1$ (b) $\tan\theta = 2$
(c) $\tan\theta = 3$ (d) $\tan\theta = 4$

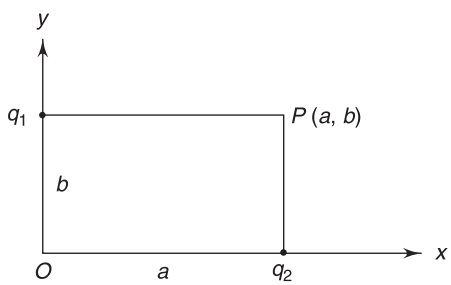


Fig. 11.55

20. An electric dipole placed with its axis in the direction of a uniform electric field experiences
(a) a force but no torque
(b) a torque but no force
(c) a force as well as a torque
(d) neither a force nor a torque

21. An electric dipole placed with its axis inclined at an angle to the direction of a uniform electric field experiences
(a) a force but no torque
(b) a torque but no force
(c) a force as well as a torque
(d) neither a force nor a torque

22. An electric dipole placed in a non-uniform electric field experiences
(a) a force but no torque
(b) a torque but no force
(c) a force as well as a torque
(d) neither a force nor a torque.

23. A cube of side b has a charge q at each of its vertices. What is the electric potential at the centre of the cube?

(a) $\frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$ (b) $\frac{\sqrt{3}q}{\pi\varepsilon_0 b}$
(c) $\frac{2q}{\pi\varepsilon_0 b}$ (d) zero

24. In Q. 23, the Electric field at the centre of the cube is

(a) $\frac{4q}{3\pi\varepsilon_0 b^2}$ (b) $\frac{3q}{\pi\varepsilon_0 b^2}$
(c) $\frac{2q}{\pi\varepsilon_0 b^2}$ (d) zero

25. Two point charges $-q$ and $+q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$ respectively. What is the electric potential at point $(0, 0, z)$?

(a) $\frac{qa}{4\pi\varepsilon_0 z^2}$ (b) $\frac{q}{4\pi\varepsilon_0 a}$
(c) $\frac{2qa}{4\pi\varepsilon_0\left(z^2 - a^2\right)}$ (d) $\frac{2qa}{4\pi\varepsilon_0\left(z^2 + a^2\right)}$

26. In Q. 25, how much work is done in moving a small test charge q_0 from point $(5, 0, 0)$ to a point $(-7, 0, 0)$ along the x-axis?

(a) $\frac{5}{7} \times \frac{q_0 q}{4\pi\varepsilon_0 a}$ (b) $\frac{7}{5} \times \frac{q_0 q}{4\pi\varepsilon_0 a}$
(c) $\frac{1}{12} \times \frac{q_0 q}{4\pi\varepsilon_0 a}$ (d) zero

27. A neutral hydrogen molecule has two protons and two electrons. If one of the electrons is removed we get a hydrogen molecular ion (H_2^+). In the ground state of H_2^+ the two protons are separated by roughly 1.5 Å and the electron is roughly 1 Å from each proton. What is the potential energy of the system?

(a) – 38.4 eV (b) – 19.2 eV

(c) – 9.6 eV (d) zero

28. In a hydrogen atom, the electron and the proton are bound together at a separation of about 0.53 Å. If the zero of potential energy is taken at infinite separation of the electron from the proton, the potential energy of the electron-proton system is

(a) – 54.4 eV (b) – 27.2 eV

(c) – 13.6 eV (d) zero

29. In Q. 28, what is the minimum work required to free the electron from the proton if the kinetic energy of the electron in its orbit is half the potential energy of the electron-proton system?

(a) 2.2×10^{-12} J (b) 2.2×10^{-14} J

(c) 2.2×10^{-16} J (d) 2.2×10^{-18} J

30. In Q. 28, what will be the potential energy of the electron-proton system if the zero of potential energy is taken at a separation of 1.06 Å?

(a) zero (b) – 13.6 eV

(c) – 27.2 eV (d) – 54.4 eV

31. What is the answer to Q. 29 if the zero of potential energy is taken at a separation of 1.06 Å?

(a) zero (b) 1.1×10^{-14} J

(c) 1.1×10^{-16} J (d) 1.1×10^{-18} J

32. What is the equivalent capacitance between A and D of the network shown in Fig. 11.56?

(a) 200 pF (b) 100 pF

(c) $\frac{200}{3}$ pF (d) 50 pF

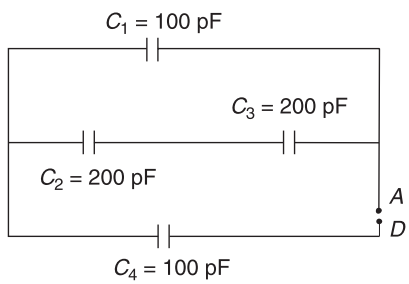


Fig. 11.56

33. Figure 11.57 shows a network of capacitors where the numbers indicate capacitances in microfarad. What must be the value of capacitance C if the equivalent capacitance between points A and B is to be 1 μF?

(a) $\frac{31}{23}$ μF (b) $\frac{32}{23}$ μF

(c) $\frac{33}{23}$ μF (d) $\frac{34}{23}$ μF

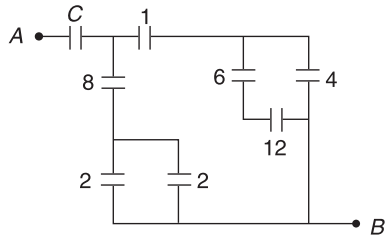


Fig. 11.57

34. A 2 μF capacitor C_1 is charged to a voltage 100 V and a 4 μF capacitor C_2 is charged to a voltage 50 V. The capacitors are then connected in parallel. What is the loss of energy due to parallel connection?

(a) 1.7 J (b) 1.7×10^{-1} J

(c) 1.7×10^{-2} J (d) 1.7×10^{-3} J

35. A positive charge (+ q) is located at the centre of a circle as shown in Fig. 11.58. W_1 is the work done in taking a unit positive charge from A to B and W_2 is the work done in taking the same charge from A to C. Then

(a) $W_1 > W_2$ (b) $W_1 < W_2$

(c) $W_1 = W_2$ (d) $W_1 = W_2 = 0$

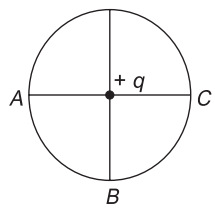


Fig. 11.58

36. Two concentric spheres of radii r_1 and r_2 carry charges q_1 and q_2 respectively. If the surface charge density (σ) is the same for both spheres, the electric potential at the common centre will be

(a) $\frac{\sigma}{\varepsilon_0} \cdot \frac{r_1}{r_2}$ (b) $\frac{\sigma}{\varepsilon_0} \cdot \frac{r_2}{r_1}$

(c) $\frac{\sigma}{\varepsilon_0}(r_1 - r_2)$ (d) $\frac{\sigma}{\varepsilon_0}(r_1 + r_2)$

37. The magnitude of the electric field on the surface of a sphere of radius r having a uniform surface charge density σ is

(a) $\frac{\sigma}{\varepsilon_0}$ (b) $\frac{\sigma}{2\varepsilon_0}$

(c) $\frac{\sigma}{\varepsilon_0 r}$ (d) $\frac{\sigma}{2\varepsilon_0 r}$

38. The electric field due to an extremely short dipole at a distance r from it is proportional to

(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$

(c) $\frac{1}{r^3}$ (d) $\frac{1}{r^4}$

39. The electric potential due to an extremely short dipole at a distance r from it is proportional to

(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$

(c) $\frac{1}{r^3}$ (d) $\frac{1}{r^4}$

40. A soap bubble of radius r is charged to a potential V. If the radius is increased to nr, the potential on the bubble will become

(a) nV (b) n^2V

(c) $\frac{V}{n}$ (d) $\frac{V}{n^2}$

41. If n drops, each of capacitance C, coalesce to form a single big drop, the capacitance of the big drop will be

(a) n^3C (b) nC

(c) $n^{1/2}C$ (d) $n^{1/3}C$

Level B

42. If n drops, each charged to a potential V, coalesce to form a single drop, the potential of the big drop will be

(a) $\frac{V}{n^{2/3}}$ (b) $\frac{V}{n^{1/3}}$

(c) $Vn^{1/3}$ (d) $Vn^{2/3}$

43. If n drops, each of capacitance C and charged to a potential V, coalesce to form a big drop, the ratio of the energy stored in the big drop to that in each small drop will be

(a) $n : 1$ (b) $n^{4/3} : 1$

(c) $n^{5/3} : 1$ (d) $n^2 : 1$

44. A parallel plate capacitor is made by stacking 10 identical metallic plates equally spaced from one another and having the same dielectric between plates. The alternate plates are then connected. If the capacitor formed by two neighbouring plates has a capacitance C, the total capacitance of the combination will be

(a) $\frac{C}{10}$ (b) $\frac{C}{9}$

(c) $9\,C$ (d) $10\,C$

45. Figure 11.59 shows four capacitors connected to an 8 V power supply. What is the potential difference across each 1 μF capacitor?

(a) 1 V (b) 2 V

(c) 3 V (d) 4 V

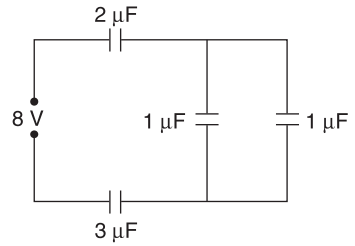


Fig. 11.59

46. Figure 11.60 shows three capacitors connected to a 6 V power supply. What is the charge on the 2 μF capacitor?

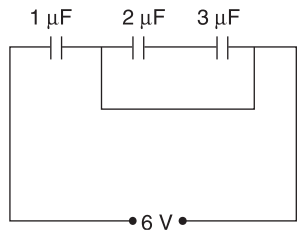


Fig. 11.60

(a) 1 μC (b) 2 μC

(c) 3 μC (d) 4 μC

47. Figure 11.61 shows five capacitors connected across a 12 V power supply. What is the charge on the 2 μF capacitor?

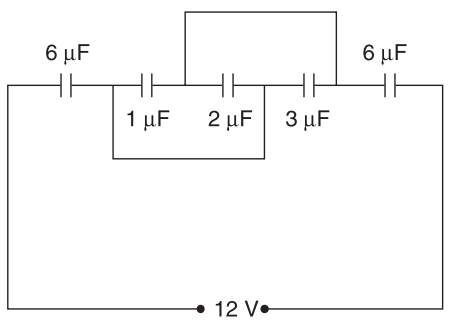


Fig. 11.61

(a) 6 μC (b) 8 μC

(c) 10 μC (d) 12 μC

48. Six charges, each equal to + q, are placed at the corners of a regular hexagon of side a. The electric potential at the point where the diagonals of the hexagon intersect will be given by

(a) zero (b) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a}$

(c) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{6q}{a}$ (d) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{\sqrt{3}q}{2a}$

49. In Q. 48, the electric field at the point of intersection of diagonals is

(a) zero (b) $\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$

(c) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{6q}{a^2}$ (d) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{\sqrt{3}q}{2a^2}$

50. A parallel plate capacitor with air as dielectric is charged to a potential V. It is then connected to an uncharged parallel plate capacitor filled with wax of dielectric constant k. The common potential of both capacitors is

(a) V (b) kV

(c) $(1+k)V$ (d) $\frac{V}{(1+k)}$

51. A capacitor of capacitance C is fully charged by a 200 V supply. It is then discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat 2.5×10^2 J kg^{-1} K^{-1} and of mass 0.1 kg. If the temperature of the block rises by 0.4 K, what is the value of C?

(a) 500 μ F (b) 400 μF

(c) 300 μF (d) 200 μF

52. A charge having magnitude Q is divided into two parts q and $(Q - q)$ which are held a certain distance r apart. The force of repulsion between the two parts will be maximum if the ratio q/Q is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{5}$

53. A charge Q is given to a hollow metallic sphere of radius R. The electric potential at the surface of the sphere is

(a) zero (b) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$

(c) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^2}$ (d) $4\pi\varepsilon_0 Q/R$

54. In Q. 53, the potential at a distance r from the centre of the sphere where $r < R$ is

(a) zero (b) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{(R-r)}$

(c) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R+r}$ (d) $\frac{4\pi\varepsilon_0 Q}{(R-r)}$

55. The electric potential V at any point (x, y, z) in space is given by $V = 4x^2$ volt where x, y and z are all in metre. The electric field at the point (1 m, 0, 2 m) in Vm^{-1} is

(a) 8 along negative x-axis
(b) 8 along positive x-axis
(c) 16 along negative x-axis
(d) 16 along positive x-axis

56. A charge Q is situated at the centre of a cube. The electric flux through one of the faces of the cube is

(a) $\frac{Q}{\varepsilon_0}$ (b) $\frac{Q}{2\varepsilon_0}$

(c) $\frac{Q}{4\varepsilon_0}$ (d) $\frac{Q}{6\varepsilon_0}$

57. Eight dipoles of charges of magnitude q are placed inside a cube. The total electric flux through the cube will be

(a) $\frac{8q}{\varepsilon_0}$ (b) $\frac{16q}{\varepsilon_0}$

(c) $\frac{q}{\varepsilon_0}$ (d) zero

58. The magnitude of the electric field in the annular region of a charged cylindrical capacitor

(a) is the same throughout
(b) is higher near the outer cylinder than near the inner cylinder
(c) varies as $1/r$ where r is the distance from the axis
(d) varies as $1/r^2$ where r is the distance from the axis.

59. n identical capacitors are joined in parallel and the combination is charged to voltage V. The total energy stored is U. The capacitors are now disconnected and joined in series. The total energy stored in the series combination will be

(a) $\frac{U}{n}$ (b) U

(c) nU (d) n^2U

60. Two spheres of radii r and R carry charges q and Q respectively. When they are connected by a wire, there will be no loss of energy of the system if

(a) $qr = QR$ (b) $qR = Qr$

(c) $qr^2 = QR^2$ (d) $qR^2 = Qr^2$

61. Two equal point charges of 1 μC each are located at points $\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$ m and $\left(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)$ m. What is the magnitude of electrostatic force between them?

(a) 10^{-3} N (b) 10^{-6} N
(c) 10^{-9} N (d) 10^{-12} N

62. The introduction of a metal plate between the plates of a parallel plate capacitor increases its capacitance by 4.5 times. If d is the separation of the two plates of the capacitor, the thickness of the metal plate introduced is

(a) $\frac{d}{3}$ (b) $\frac{5d}{9}$
(c) $\frac{7d}{9}$ (d) d

63. If the potential difference between the plates of a capacitor is increased by 20%, the energy stored in the capacitor increases by exactly

(a) 20% (b) 22%
(c) 40% (d) 44%

64. If the potential difference between the plates of a capacitor is increased by 0.1%, the energy stored in the capacitor increases by very nearly

(a) 0.1% (b) 0.11%
(c) 0.144% (d) 0.2%

65. Three capacitors connected in series have an effective capacitance of 2 μF. If one of the capacitors is removed, the effective capacitance becomes 3 μF. The capacitance of the capacitor that is removed is

(a) 1 μF (b) $\frac{3}{2}$ μF
(c) $\frac{2}{3}$ μF (d) 6 μF

66. The effective capacitance of two capacitors of capacitances C_1 and C_2 (with $C_2 > C_1$) connected in parallel is $\frac{25}{6}$ times the effective capacitance when they are connected in series. The ratio C_2/C_1 is

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$
(c) $\frac{5}{3}$ (d) $\frac{25}{6}$

67. Three equal point charges q are placed at the corners of an equilateral triangle. Another charge Q is placed at the centroid of the triangle. The system of charges will be in equilibrium if Q equals

(a) $\frac{q}{\sqrt{3}}$ (b) $-\frac{q}{\sqrt{3}}$
(c) $\frac{q}{3}$ (d) $-\frac{q}{3}$

68. A metallic sphere A of radius a carries a charge Q. It is brought in contact with an uncharged sphere B of radius b. The charge on sphere A now will be

(a) $\frac{aQ}{b}$ (b) $\frac{bQ}{a}$
(c) $\frac{bQ}{a+b}$ (d) $\frac{aQ}{a+b}$

69. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. The potential difference between the surface of the solid sphere and the outer surface of the hollow shell is V. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surfaces is

(a) V (b) $2V$
(c) $4V$ (d) $-2V$

70. Two identical thin rings, each of radius R are coaxially placed at a distance R apart. If Q_1 and Q_2 are the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to the centre of the other is

(a) zero

(b) $\frac{q}{4\pi\varepsilon_0\sqrt{2}R}(Q_1 - Q_2)(\sqrt{2} - 1)$

(c) $\frac{q\sqrt{2}}{4\pi\varepsilon_0 R}(Q_1 + Q_2)$

(d) $\frac{(\sqrt{2}+1)q(Q_1+Q_2)}{\sqrt{2}\,4\pi\varepsilon_0 R}$

71. An electron of mass m_e, initially at rest, moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p, also initially at rest, takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio t_2/t_1 is nearly equal to

(a) 1 (b) $\left(\frac{m_p}{m_e}\right)^{1/2}$
(c) $\left(\frac{m_e}{m_p}\right)^{1/2}$ (d) 1836

72. A metallic solid sphere is placed in a uniform electric field. In Fig. 11.62, which path will the field lines follow?

(a) 1 (b) 2
(c) 3 (d) 4

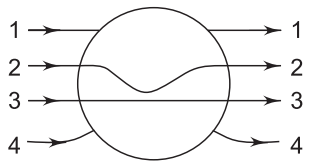


Fig. 11.62

73. A charge $+q$ is fixed at each of the points $x = x_0, x = 3x_0$, $x = 5x_0$ ... upto infinity and a charge $-q$ is fixed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$ ... up to infinity. Here x_0 is a positive constant. The potential at the origin of this system of charges is

(a) zero (b) $\dfrac{q}{4\pi\varepsilon_0 x_0 \ln(2)}$

(c) infinity (d) $\dfrac{q \ln(2)}{4\pi\varepsilon_0 x_0}$

74. Three charges Q, $+q$ and $+q$ are placed at the vertices of a right-angled isosceles triangle as shown in Fig. 11.63. The net electrostatic energy of the configuration is zero if Q is equal to

(a) $\dfrac{-q}{1+\sqrt{2}}$ (b) $\dfrac{-2q}{2+\sqrt{2}}$

(c) $-2q$ (d) $+q$

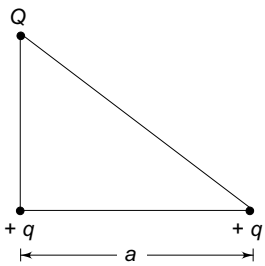


Fig. 11.63

75. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to a potential difference $2V$. The charging battery is then disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

(a) zero (b) $\dfrac{3}{2}\ CV^2$

(c) $\dfrac{25}{6}\ CV^2$ (d) $\dfrac{9}{2}\ CV^2$

76. A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at $x = 0$ and positive plate is at $x = 3d$. The slab is equidistant from the plates. The capacitor is given some charge. As x goes from 0 to $3d$,

(a) the magnitude of the electric field remains the same

(b) the direction of the electric field changes continuously

(c) the electric potential increases continuously

(d) the electric potential increases at first, then decreases and again increases.

77. Two identical metal plates are given positive charges Q_1 and Q_2 ($< Q_1$) respectively. If they are brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is

(a) $\dfrac{Q_1 + Q_2}{2C}$ (b) $\dfrac{Q_1 + Q_2}{C}$

(c) $\dfrac{Q_1 - Q_2}{C}$ (d) $\dfrac{Q_1 - Q_2}{2C}$

78. For the circuit shown in Fig. 11.64, which of the following statements is true?

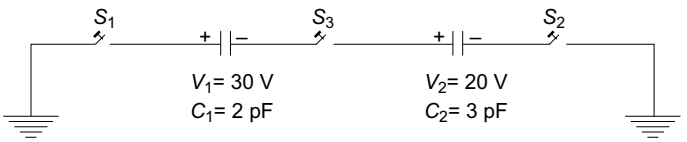


Fig. 11.64

(a) With S_1 closed, $V_1 = 15$ V, $V_2 = 20$ V

(b) With S_3 closed, $V_1 = V_2 = 25$ V

(c) With S_1 and S_2 closed $V_1 = V_2 = 0$

(d) With S_1 and S_3 closed $V_1 = 30$ V and $V_2 = 20$ V

79. A parallel plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials having dielectric constant K_1, K_2 and K_3 as shown in Fig. 11.65. If a single dielectric material is to be used to have the same capacitance C in this capacitor, then its dielectric constant K is given by

(a) $\dfrac{1}{K} = \dfrac{1}{K_1} + \dfrac{1}{K_2} + \dfrac{1}{2K_3}$

(b) $\dfrac{1}{K} = \dfrac{1}{K_1 + K_2} + \dfrac{1}{2K_3}$

(c) $K = \dfrac{K_1 K_2}{K_1 + K_2} + 2K_3$

(d) $K = K_1 + K_2 + K_3$

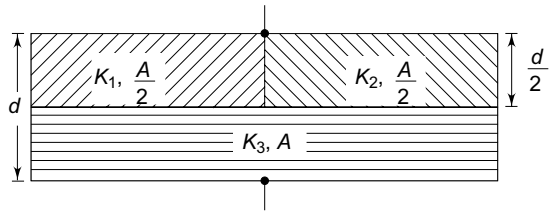


Fig. 11.65

80. A quantity X is given by $\varepsilon_0 L \frac{\Delta V}{\Delta t}$ where ε_0 is the permittivity of free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of
(a) resistance (b) charge
(c) voltage (d) current

81. Consider the situation shown in Fig. 11.66. The capacitor A has a charge q on it whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is

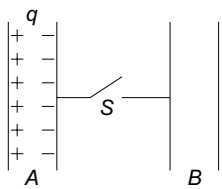


Fig. 11.66

(a) zero (b) $q/2$
(c) q (d) $2q$

82. A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at $x = +1$ cm and C be the point on the y-axis at $y = +1$ cm. Then the potentials at the points A, B and C satisfy:
(a) $V_A < V_B$ (b) $V_A > V_B$
(c) $V_A < V_C$ (d) $V_A > V_C$

83. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to
(a) x (b) x^2
(c) x^3 (d) $1/x$

84. There is a uniform electric field of strength 10^3 Vm^{-1} along the y-axis. A body of mass 1 g and charge 10^{-6} C is projected into the field from the origin along the positive x-axis with a velocity of 10 ms^{-1}. Its speed (in ms^{-1}) after 10 second will be (neglect gravitation)
(a) 10 (b) $5\sqrt{2}$
(c) $10\sqrt{2}$ (d) 20

85. Two identical charges are placed at the two corners of an equilateral triangle. The potential energy of the system is U. The work done in bringing an identical charge from infinity to the third vertex is
(a) U (b) $2U$
(c) $3U$ (d) $4U$

86. A parallel plate capacitor of capacitance 5 μF and plate separation 6 cm is connected to a 1 V battery and charged. A dielectric of dielectric constant 4 and thickness 4 cm is introduced between the plates of the capacitor. The additional charge that flows into the capacitor from the battery is
(a) 2 μC (b) 3 μC
(c) 5 μC (d) 10 μC

87. A capacitor of capacitance 4 μF is charged to 80 V and another capacitor of capacitance 6 μF is charged to 30 V. When they are connected together, the energy lost by the 4 μF capacitor is
(a) 7.8 mJ (b) 4.6 mJ
(c) 3.2 mJ (d) 2.5 mJ

88. The magnitude of electric field at a distance x from a charge q is E. An identical charge is placed at a distance $2x$ from it. Then the magnitude of the force it experiences is
(a) qE (b) $2qE$
(c) $\frac{qE}{2}$ (d) $\frac{qE}{4}$

89. The flux of electric field $\mathbf{E} = 200\,\hat{\mathbf{i}}$ NC^{-1} through a cube of side 10 cm, oriented so that its faces are parallel to the co-ordinate axes is
(a) zero (b) 2 NC^{-1}m^2
(c) 6 NC^{-1}m^2 (d) 12 NC^{-1}m^2

90. Figure 11.67 shows a spherical Gaussian surface and a charge distribution. When calculating the flux of electric field through the Gaussian surface, the electric field will be due to

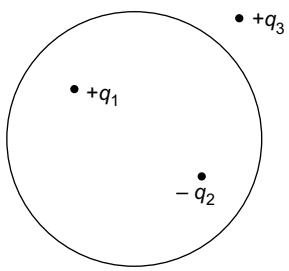


Fig. 11.67

(a) $+q_3$ alone (b) $+q_1$ and $+q_3$
(c) $+q_1, +q_3$ and $-q_2$ (d) $+q_1$ and $-q_2$

91. Three infinite long plane sheets carrying uniform charge densities

$$\sigma_1 = -\sigma,\ \sigma_2 = +2\sigma \text{ and } \sigma_3 = +3\sigma$$

are placed parallel to the x–z plane at $y = a$, $y = 3a$ and $y = 4a$ as shown in Fig. 11.68. The electric field at point P is

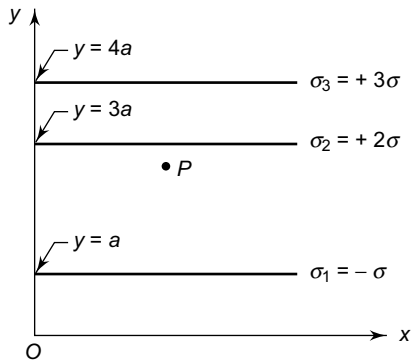


Fig. 11.68

(a) zero (b) $-\dfrac{2\sigma}{\varepsilon_0}\hat{\mathbf{j}}$

(c) $-\dfrac{3\sigma}{\varepsilon_0}\hat{\mathbf{j}}$ (d) $\dfrac{3\sigma}{\varepsilon_0}\hat{\mathbf{j}}$

92. A metallic spherical shell of radius R has a charge $-Q$ distributed uniformly on it. A point charge $+Q$ is placed at the center of the shell. Which graph shown in Fig. 11.69 represents the variation of electric field E with distance r from the centre of the shell?

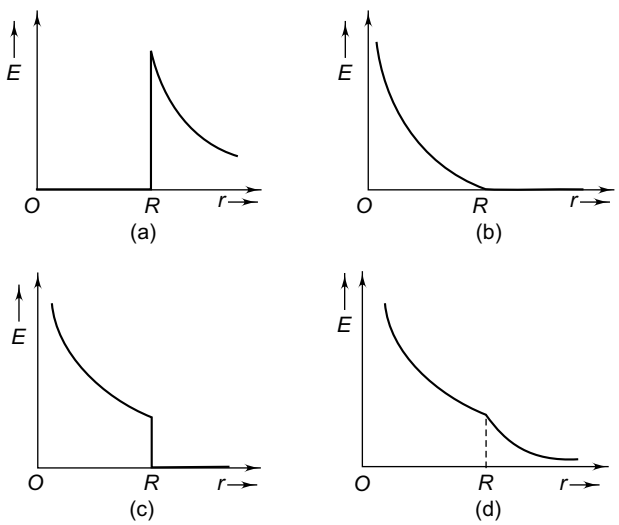


Fig. 11.69

93. A metallic sphere of radius R is charged to a potential V. The magnitude of the electric field at a distance r ($> R$) from the center of the sphere is

(a) $\dfrac{V}{r}$ (b) $\dfrac{Vr}{R^2}$

(c) $\dfrac{VR}{r^2}$ (d) zero

94. Two point charges $q_1 = 1\mu C$ and $q_2 = 2\mu C$ are placed at points A and B 6 cm apart as shown in Fig. 11.70. A third charge $Q = 5$ μC is moved from C to D along the arc of a circle of radius 8 cm as shown. The change in the potential energy of the system is

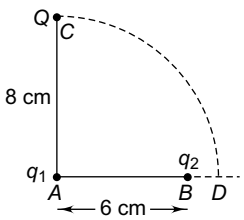


Fig. 11.70

(a) 3.0 J (b) 3.6 J

(c) 5.0 J (d) 7.2 J

95. A partical of mass m and charge $+q$ is midway between two fixed charged particles, each having a charge $+q$ and at a distance $2L$ apart. The middle charge is displaced slightly along the line joining the fixed charges and released. The time period of oscillation is proportional to

(a) $L^{1/2}$ (b) L

(c) $L^{3/2}$ (d) L^2

96. The potential difference between points A and B in the circuit shown in Fig. 11.71 is

(a) 6 V (b) 2 V

(c) 10 V (d) 14 V

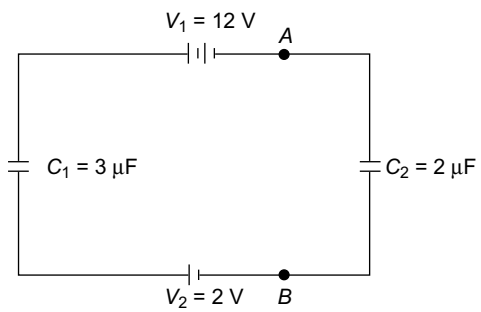


Fig. 11.71

97. An electric field of 200 Vm^{-1} exists in the region between the plates of a parallel plate capacitor of plate separation 5 cm. The potential difference between the plates when a slab of dielectric constant 4 and thickness 1 cm is inserted between the plates is

(a) 7.5 V (b) 8.5 V

(c) 9.0 V (d) 10 V

98. A parallel plate capacitor is maintained at a certain potential difference. When a dielectric slab of thickness 3 mm is introduced between the

plates, the plate separation had to be increased by 2 mm in order to maintain the same potential difference between the plates. The dielectric constant of the slab is

(a) 2 (b) 3
(c) 4 (d) 5

99. A capacitor of capacitance C_1 is charged by connecting it to a battery. The battery is now removed and this capacitor is connected to a second uncharged capacitor of capacitance C_2. If the charge distributes equally on the two capacitors, the ratio of the total energy stored in the capacitors after connection to the total energy stored in them before connection is

(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{3}$

100. Four metal plates numbered 1, 2, 3 and 4 are arranged as shown is Fig. 11.72. The area of each plate is A and the separation between adjacent plates is d. The capacitance of the arrangement is

(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{2\varepsilon_0 A}{d}$
(c) $\frac{3\varepsilon_0 A}{d}$ (d) $\frac{4\varepsilon_0 A}{d}$

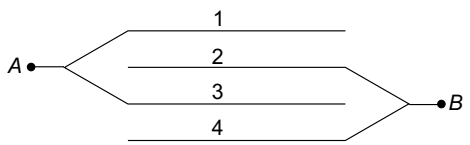


Fig. 11.72

101. Four metal plates numbered 1, 2, 3 and 4 are arranged as shown in Fig. 11.73. The area of each plate is A and the separation between the plates is d. The capacitance of the arrangement is

(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{2\varepsilon_0 A}{d}$
(c) $\frac{3\varepsilon_0 A}{d}$ (d) $\frac{4\varepsilon_0 A}{d}$

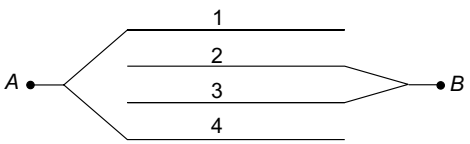


Fig. 11.73

102. The equivalent capacitance between points A and B in the network shown in Fig. 11.74 is ($C_1 = 2$ μF and $C_2 = 3$ μF)

(a) 1 μF (b) 2 μF
(c) 3 μF (d) 4 μF

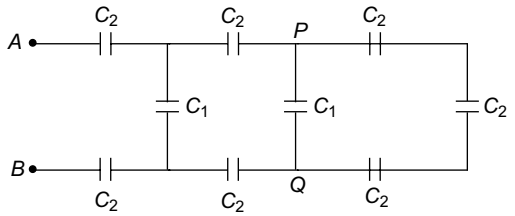


Fig. 11.74

103. A capacitior of capacitance $C_1 = C$ is charged to a voltage V. It is then connected in parallel with a series combination of two uncharged capacitors of capacitances $C_2 = C$ and $C_3 = C$. The charge that will flow through the connecting wires is

(a) $\frac{CV}{3}$ (b) $\frac{2CV}{3}$
(c) CV (d) zero

104. The capacitance of a sphere of radius R_1 is increased 3 times when it enclosed by an earthed sphere of radius R_2. The ratio R_2/R_1 is

(a) 2 (b) $\frac{3}{2}$
(c) $\frac{4}{3}$ (d) 3

105. A parallel plate capacitor of plate area A and plate separation d is charged by a battery of voltage V. The battery is then disconnected. The work needed to pull the plates to a separation $2d$ is

(a) $\frac{AV^2\varepsilon_0}{d}$ (b) $\frac{2AV^2\varepsilon_0}{d}$
(c) $\frac{AV^2\varepsilon_0}{2d}$ (d) $\frac{3AV^2\varepsilon_0}{2d}$

106. One plate of a parallel plate capacitor of plate area A and plate separation d is connected to the positive terminal of a battery of the voltage V. The negative terminal of the battery and the other plate of the capacitor are earthed as shown in Fig. 11.75. The charge that flows from the battery to the capacitor plates is

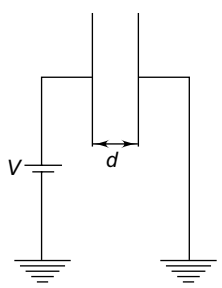


Fig. 11.75

(a) zero (b) $\frac{\varepsilon_0 AV}{d}$

(c) $\frac{Vd}{\varepsilon_0 A}$ (d) $\frac{\varepsilon_0 AV}{2d}$

107. A non-conducting sphere of radius R has a charge Q distributed uniformly throughout its volume. The magnitude of the electric field at a point P inside the sphere at a distance r ($<R$) from the centre is

(a) $\frac{Qr}{4\pi \in_0 R^3}$ (b) $\frac{Qr}{4\pi \in_0 R^3}$

(c) $\frac{Q}{4\pi \in_0 r^2}$ (d) zero

108. In Q. 107 above, the magnitude of the electric field at a point P outside the sphere at a distance r ($>R$) from the centre is

(a) zero (b) $\frac{Qr}{4\pi \in_0 R^3}$

(c) $\frac{Q}{4\pi \in_0 r^2}$ (d) $\frac{Qr^2}{4\pi \in_0 R^4}$

109. A non-conducting sphere of radius R has a charge distributed throughout its volume. The charge density ρ varies with distance r from the centre of the sphere as

$$\rho = \frac{Kr}{R}$$

where K is a constant. The electric field at a point P inside the sphere at a distance r ($< R$) is

(a) $\frac{KR^2}{4r \in_0}$ (b) $\frac{KR^2}{2\pi \in_0 r}$

(c) $\frac{Kr^2}{2\pi \in_0 R}$ (d) $\frac{Kr^2}{4R \in_0}$

110. In Q. 109 above, the electric field at a point P outside the sphere at a distance r ($>R$) is

(a) $\frac{KR^2}{2 \in_0 r}$ (b) $\frac{KR^3}{3\pi \in_0 r^2}$

(c) $\frac{Kr^2}{3\pi \in_0 R}$ (d) $\frac{KR^3}{3 \in_0 r^2}$

111. A solid metal sphere of radius a is carrying at charge $+q$. It is surrounded by a concentric uncharged hollow metal sphere of inner radius b and outer radius c. Choose the only wrong statement from the following.

(a) Electric field is zero at points for which $r < a$.

(b) Electric field is $\frac{q}{4\pi \in_0 r^2}$ for points lying between a and b.

(c) Electric field is zero for points lying between b and c.

(d) Electric field is $\frac{q}{2\pi \in_0 r^2}$ at points for which $r > c$.

112. A conducting spherical shell of inner radius a and outer radius b carries a net charge $+2Q$. It is surrounded by a larger spherical shell (concentric with the inner shell) of inner radius c and outer radius d and carries a net charge $+3Q$ as shown in Fig. 11.76. The electric field at point A at a distance r from centre O is

(a) $\frac{Q}{4\pi \in_0 r^2}$ (b) $\frac{2Q}{4\pi \in_0 r^2}$

(c) $\frac{3Q}{4\pi \in_0 r^2}$ (d) zero

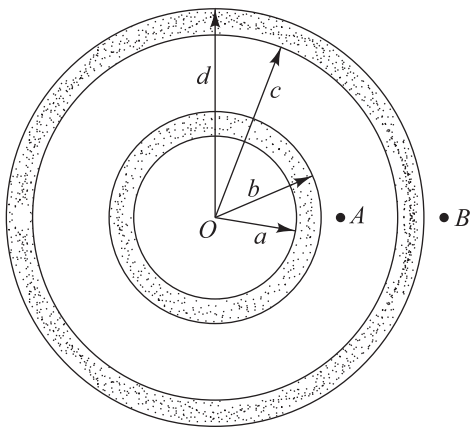


Fig. 11.76

113. In Q. 112 above, the electric field at point B at a distance r from centre O is

(a) zero (b) $\frac{2Q}{4\pi \in_0 r^2}$

(c) $\frac{3Q}{4\pi \in_0 r^2}$ (d) $\frac{5Q}{4\pi \in_0 r^2}$

114. A sphere carrying a charge $+Q$ is fixed. A small sphere carrying a charge $+q$ is placed near the bigger sphere and released from rest. Due to repulsion, the smaller sphere will move away from the bigger sphere with

(a) decreasing velocity and increasing acceleration

(b) increasing velocity and constant acceleration

(c) increasing velocity and decreasing acceleration

(d) increasing velocity and increasing acceleration

115. A thin non-conducting rod of length l carries a positive charge Q uniformly distributed along its length. The rod lies along the y-axis with its mid-point O at the origin. The electric field of the rod at point P at a distance $x = l$ from O is given by

$$E = \frac{kQ}{4\pi \in_0 l^2}$$

where the value of k is

(a) 1 (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{2}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{5}}$

116. Two concentric, conducting, thin spherical shells A and B have radii r and R as shown in Fig 11.77. Shell A carries a charge q and B carries a charge Q. Shell A is supported on an insulating stand. The potential of shell A is

(a) $\frac{1}{4\pi \in_0}\left(\frac{q}{r}+\frac{Q}{R}\right)$ (b) $\frac{1}{4\pi \in_0}\left(\frac{q+Q}{R-r}\right)$

(c) $\frac{1}{4\pi \in_0}\left(\frac{Q}{r}+\frac{q}{R}\right)$ (d) $\frac{1}{4\pi \in_0}\frac{q}{r}$

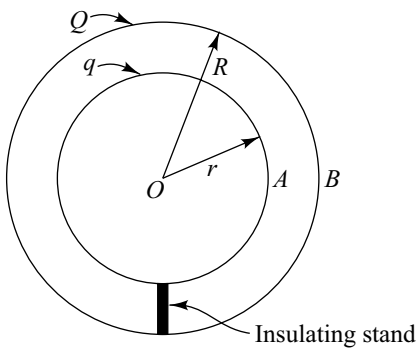


Fig. 11.77

117. In Q. 116 above the potential difference $(V_A - V_B)$ between the inner shell and the outer shell is

(a) $\frac{q}{4\pi \in_0}\left(\frac{1}{r}-\frac{1}{R}\right)$ (b) $\frac{Q}{4\pi \in_0}\left(\frac{1}{r}+\frac{1}{R}\right)$

(c) $\frac{q+Q}{4\pi \in_0}\left(\frac{1}{r}-\frac{1}{R}\right)$ (d) $\frac{q+Q}{4\pi \in_0}\left(\frac{1}{r}+\frac{1}{R}\right)$

118. A long conducting cylinder of radius R has a uniform linear charge density λ. The electric field at a point at a perpendicular distance r from the cylinder is

$$E = \frac{\lambda}{2\pi \in_0 r}$$

The electric potential at that point is

(a) $\frac{\lambda}{2\pi \in_0}$ (b) $-\frac{\lambda R}{2\pi \in_0 r^2}$

(c) $\frac{\lambda}{2\pi \in_0}\ln\left(\frac{r}{R}\right)$ (d) $\frac{\lambda}{2\pi \in_0}\ln\left(\frac{R}{r}\right)$

119. A cylindrical capacitor of length l consists of a solid conducting cylinder of radius a which carries a linear charge density $+\lambda$, surrounded by a concentric outer cylindrical shell of radius b which carries a linear charge density $-\lambda$. The capacitance of the capacitor is

(a) $\frac{2\pi \in_0 l}{\ln\left(\frac{b}{a}\right)}$ (b) $\frac{4\pi \in_0 l}{\ln\left(\frac{b}{a}\right)}$

(c) $\frac{2\pi \in_0 b}{\ln\left(\frac{b}{a}\right)}$ (d) $\frac{4\pi \in_0 a}{\ln\left(\frac{b}{a}\right)}$

120. A spherical capacitor consists of a spherical conducting shell of radius a which is surrounded by a concentric outer spherical shell of radius b. The capacitance of the capacitor is

(a) $4\pi \in_0\left(\frac{ab}{b+a}\right)$ (b) $4\pi \in_0\left(\frac{ab}{b-a}\right)$

(c) $2\pi \in_0\left(\frac{a^2}{b}\right)$ (d) $2\pi \in_0\left(\frac{b^2}{a}\right)$

121. Fig. 11.78 shows two capacitors C_1 = 2μF and C_2 = 4μF connected to a 12V battery and two switches S_1 and S_2. Switch S_2 is initially kept open and S_1 is kept closed until the capacitor C_1 is fully charged. Switch S_1 is now opened and S_2 is closed until electrostatic conditions are restored. In this process, the percentage loss of energy stored in C_1 is

(a) 88.9% (b) 76.5%

(c) 63.4% (d) 55.0%

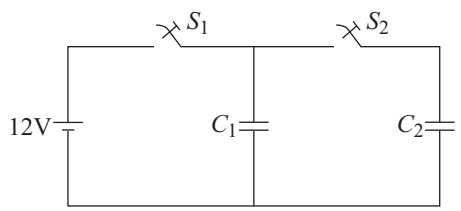


Fig. 11.78

122. Two isolated conducting spheres 1 and 2 each carry a charge $+Q$. Sphere 1 has radius a and sphere B has radius $4a$. If the two spheres are connected by a conducting wire, the charge on sphere 1 is Q_1 and on sphere 2 is Q_2. Then

(a) $Q_1 = Q, \quad Q_2 = Q$

(b) $Q_1 = \frac{2Q}{3}, \quad Q_2 = \frac{4Q}{3}$

(c) $Q_1 = \frac{2Q}{5}, \quad Q_2 = \frac{8Q}{5}$

(d) $Q_1 = \frac{Q}{2}, \quad Q_2 = \frac{3Q}{2}$

123. A solid non-conducting sphere of radius R has a uniform volume charge density ρ. Spherical portion of radius r is removed from the solid sphere as shown in Fig. 11.79. The magnitude of electric field at a point P at a distance $a < r$ from the centre of the cavity is

(a) proportional to a (b) proportional to b
(c) proportional to c (d) zero

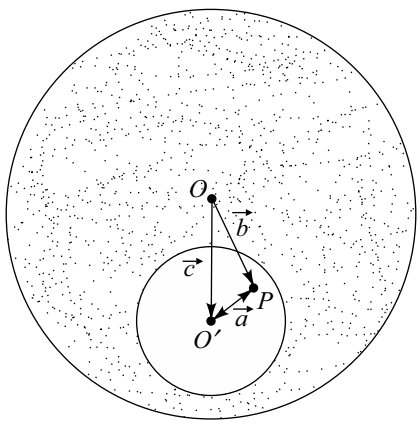


Fig. 11.79

124. A long hollow conducting cylinder A of radius R_1 is kept inside another long hollow conducting cylinder B of radius $R_2 > R_1$. Initially cylinders A and B carry no charge. Which of the following statements is true?

1. When a charge density is given to the outer cylinder A, a potential difference appears between cylinders A and B.
2. When a charge density is given to the inner cylinder B, a potential difference appears between cylinders A and B.
3. When the same charge density is given to both A and B, no potential difference appears between them.

(a) 1 only (b) 2 only
(c) 1 and 3 only (d) 2 and 3 only

125. A negative point charge $-Q$ is placed at distance of r from the surface of an uncharged conducting sphere of radius R. The net charge on the sphere will be

(a) positive
(b) negative
(c) positive if $r < R$ and negative if $r > R$
(d) zero

126. A capacitor of capacitance C_1 is charged to the voltage V of a battery as shown in Fig. 11.80. The fraction of its stored energy dissipated after the switch S is disconnected from 1 and connected to 2 is

(a) $\frac{C_1}{C_1 + C_2}$ (b) $\frac{C_2}{C_1 + C_2}$

(c) $\frac{C_1 C_2}{(C_1 + C_2)^2}$ (d) $\frac{C_1 C_2}{(C_1 - C_2)^2}$

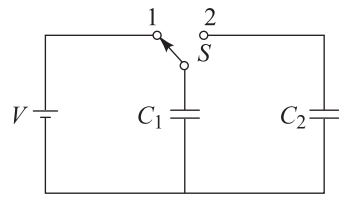


Fig. 11.80

127. Three non-conducting spheres 1, 2 and 3 have radii R, $2R$ and $3R$ respectively. Equal charge Q is uniformly distributed in each sphere. If E_1, E_2 and E_3 are the magnitudes of electric fields at point P at a distance $2R$ from the centre of spheres 1, 2 and 3 respectively as shown in Fig. 11.81, then

(a) $E_1 > E_2 > E_3$ (b) $E_1 < E_2 < E_3$
(c) $E_1 = E_2 = E_3$ (d) $E_2 = E_3 > E_1$

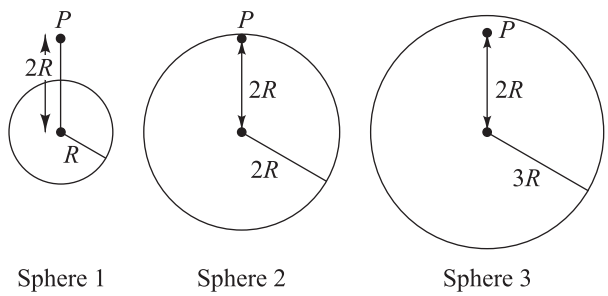


Fig. 11.81

128. In the circuit shown in Fig. 11.82, 7 μC of charge is given to the right plate of the 5 μF capacitor. Then, in the steady state, the charge on the left place of the 4 μF capacitor will be

(a) 1.0 μC (b) 3.5 μC
(c) 2.8 μC (d) 4.0 μC

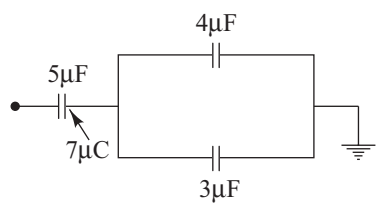


Fig. 11.82

129. The electric potential at a point in three dimensions is given by $V = x^2y + yz$. The magnitude of the electric field at point P (1, 2, 3) is

(a) 6 units (b) 7 units
(c) 8 units (d) $\sqrt{14}$ units

130. The electric potential at a space point P (x, y, z) is

$$V = x^2 + y^2 + z^2$$

The modulus of the electric field at that point is proportional to

(a) $V^{1/2}$ (b) V
(c) $V^{3/2}$ (d) V^2

131. A solid conducting sphere of radius r is surrounded by a conducting hollow spherical shell of radius R with their centres coinciding. When a charge $+Q$ is given to the sphere, a potential difference V appears between the surface of the sphere and the outer surface of the shell. If the shell is now given a charge $-nQ$, the new potential difference between the surface of the sphere and the outer surface of the shell will be

(a) V (b) nV
(c) $(n + 1)\,V$ (d) $(n - 1)\,V$

132. Three concentric metallic spherical shells 1, 2 and 3 have radii R, $2R$ and $3R$ respectively. Each shell is given the same charge Q. Their surface charge densities will be in the ratio of

(a) 3 : 2 : 1 (b) 9 : 4 : 1
(c) 6 : 3 : 2 (d) 36 : 9 : 4

133. The energy stored in the network of capacitors shown in Fig 11.83 is

(a) 40 μJ (b) 80 μJ
(c) 120 μJ (d) 160 μJ

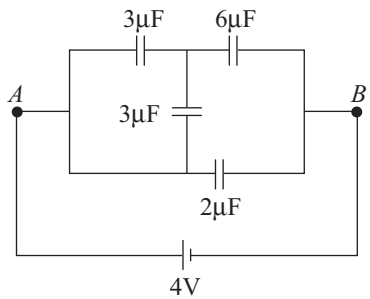


Fig. 11.83

134. In the circuit shown in Fig. 11.84, the charge on the 5 μF capacitor is

(a) 10 μC (b) 20 μC
(c) 30 μC (d) 40 μC

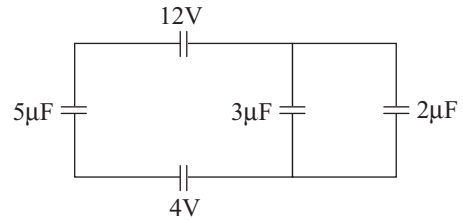


Fig. 11.84

Answers

Level A

1. (b)	2. (d)	3. (b)	4. (a)
5. (d)	6. (d)	7. (c)	8. (b)
9. (d)	10. (c)	11. (c)	12. (a)
13. (d)	14. (a)	15. (c)	16. (c)
17. (a)	18. (c)	19. (b)	20. (d)
21. (b)	22. (c)	23. (a)	24. (d)
25. (c)	26. (d)	27. (b)	28. (b)
29. (d)	30. (b)	31. (a)	32. (c)
33. (b)	34. (d)	35. (d)	36. (d)
37. (a)	38. (c)	39. (b)	40. (c)
41. (d)			

Level B

42. (d)	43. (c)	44. (c)	45. (c)
46. (b)	47. (b)	48. (c)	49. (a)
50. (d)	51. (a)	52. (a)	53. (b)
54. (a)	55. (a)	56. (d)	57. (d)
58. (c)	59. (b)	60. (b)	61. (a)
62. (c)	63. (d)	64. (d)	65. (d)
66. (a)	67. (b)	68. (d)	69. (a)
70. (b)	71. (b)	72. (d)	73. (d)
74. (b)	75. (b)	76. (c)	77. (d)
78. (b)	79. (b)	80. (d)	81. (a)
82. (b)	83. (b)	84. (c)	85. (b)
86. (c)	87. (a)	88. (d)	89. (a)
90. (c)	91. (c)	92. (a)	93. (c)
94. (b)	95. (c)	96. (a)	97. (b)
98. (b)	99. (b)	100. (c)	101. (c)

102. (a)	103. (a)	104. (b)	105. (c)
106. (b)	107. (a)	108. (c)	109. (d)
110. (d)	111. (d)	112. (b)	113. (d)
114. (c)	115. (d)	116. (a)	117. (a)
118. (d)	119. (a)	120. (b)	121. (a)
122. (c)	123. (c)	124. (b)	125. (d)
126. (b)	127. (c)	128. (d)	129. (a)
130. (a)	131. (a)	132. (c)	133. (a)
134. (b)			



Solutions

Level A

1. The potential inside a spherical conductor is constant and is the same as that on the surface. Hence the correct choice is (b).

2. Let the charge Q be at P, with $OP = x$. The resultant force F is along the x-axis directed towards the origin. The charge Q moves to O, and acquires kinetic energy. It will cross O and move to -ve x-axis until it comes to rest. It is again attracted towards O and crosses it and this process continues. Therefore charge Q executes periodic motion (see Fig. 11.85).

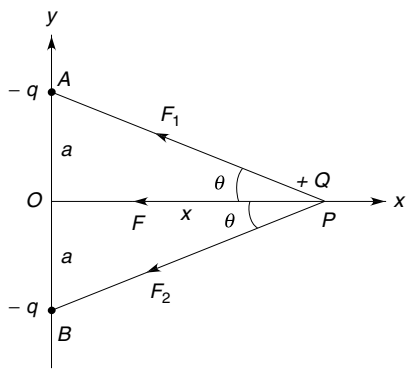


Fig. 11.85

Let $AP = BP = r$. Then

$$F_1 = F_2 = \frac{qQ}{4\pi\varepsilon_0 r^2}$$

The resultant force on Q is

$$F = F_1 \cos\theta + F_2 \cos\theta = \frac{2qQ}{4\pi\varepsilon_0 r^2}\cos\theta$$

$$F = \frac{2qQx}{4\pi\varepsilon_0 r^3} = \frac{2qQ}{4\pi\varepsilon_0}\ \frac{x}{\left(a^2 + x^2\right)^{3/2}}$$

Thus F is not of the form $F = kx$ (where k = constant) and hence the motion is not simple harmonic. Hence the correct choice is (d).

3. Each parallel combination of capacitors is equivalent to a capacitance of 100 μF connected in series. Potential drop across each of them will be 50 V. Charge $Q = CV = 100 \times 10^{-6} \times 50 = 5 \times 10^{-3}$ C
Hence the correct choice is (b).

4. Refer to Fig. 11.86.

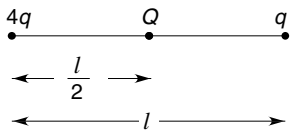


Fig. 11.86

The net force on q will be zero if

$$\frac{q \cdot 4q}{4\pi\varepsilon_0 l^2} + \frac{qQ}{4\pi\varepsilon_0 (l/2)^2} = 0$$

$$4q^2 + 4qQ = 0$$

or $$4q(q + Q) = 0$$

$\therefore$ $$Q = -q$$

Hence the correct choice is (a).

5. Electrostatic potential energy when the charges are 10 cm = 0.1 m apart is

$$W_1 = \frac{q_1 q_2}{4\pi\varepsilon_0 r} = \frac{12\times10^{-6}\times8\times10^{-6}}{4\pi\varepsilon_0 \times 0.1}$$

$$= \frac{96\times10^{-11}}{4\pi\varepsilon_0}$$

Potential energy when the charges are brought 4 cm closer, i.e., when they are 6 cm = 0.06 m apart is

$$W_2 = \frac{12\times10^{-6}\times8\times10^{-6}}{4\pi\varepsilon_0\times0.06} = \frac{16\times10^{-10}}{4\pi\varepsilon_0}$$

$\therefore$ Work done $= W_2 - W_1 = \dfrac{10^{-10}}{4\pi\varepsilon_0}(16 - 9.6)$

$$= 9\times10^9\times10^{-10}\times6.4$$

$$= 5.76\text{ J} \simeq 5.8\text{ J}$$

Hence the correct choice is (d).

6. The work done in carrying a charge round a closed path is zero. Hence the correct choice is (d).

7. When the capacitor is fully charged, no current flows in the 10 Ω resistor. The current in the circuit is

$$I = \frac{2.5}{2 + 0.5} = 1\text{ A}$$

$\therefore$ Potential drop across $2\,\Omega$ resistor $= 2\Omega \times 1\text{A} = 2\text{V}$
This is also the potential drop across the capacitor plates. Therefore, the charge on capacitor plates is

$$Q = CV = 2 \times 10^{-6} \times 2 = 4 \times 10^{-6}\ \text{C} = 4\ \mu\text{C}$$

Hence the correct choice is (c).

8. Refer to Fig. 11.87.

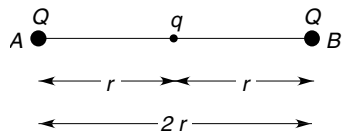


Fig. 11.87

The three charges will be in equilibrium, if no net force acts on each charge. The charge q is in equilibrium because the forces exerted on q by charge Q at A and charge Q at B are equal and opposite. The charge Q at A will be in equilibrium if the forces exerted on it by charge q and charge Q at B are equal and opposite, i.e. if

$$\frac{qQ}{4\pi\varepsilon_0 r^2} = -\frac{Q \times Q}{4\pi\varepsilon_0 (2r)^2}$$

or $$q = -\frac{Q}{4}$$

Similarly, charge Q at B will be in equilibrium if $q = -\frac{Q}{4}$. Hence the correct choice is (b).

9. Electric field $E = -\frac{dV}{dx} = -\frac{d}{dx}(5 + 4x^2) = -8x$

Force on charge $(-q) = -qE = +8qx$

At $x = 0.5$ m, force $= 8 \times 2 \times 10^{-6} \times 0.5 = 8 \times 10^{-6}$ N

Hence the correct choice is (d).

10. Original capacitance of the parallel combination of C and $2C = C + 2C = 3C$. Total charge $Q = 3CV$. When the capacitor C is filled with dielectric, its capacitance becomes KC. Therefore, the capacitance of the combination after the capacitor C is filled with dielectric, $C' = KC + 2C = (K + 2)C$. Since the charge remains the same, $Q = 3CV$, the potential difference across the capacitors will be

$$\frac{Q}{C'} = \frac{3CV}{(K+2)C} = \frac{3V}{K+2}$$

Hence the correct choice is (c).

11. The capacitance of the capacitor is $C = \varepsilon_0 A/x$ where x is the distance between the plates. The energy stored in the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{\varepsilon_0 AV^2}{2x}$$

Differentiating w.r.t x we get

$$\frac{dU}{dx} = \frac{\varepsilon_0 AV^2}{2}\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{\varepsilon_0 AV^2}{2x^2}$$

The force of attraction between the plates is

$$F = -\frac{dU}{dx} = \frac{\varepsilon_0 AV^2}{2x^2} \qquad \text{(i)}$$

Now $$Q = CV = \frac{\varepsilon_0 AV}{x} \quad \text{or} \quad V = \frac{Qx}{\varepsilon_0 A} \qquad \text{(ii)}$$

Using (ii) in (i) we get

$$F = \frac{Q^2}{2\varepsilon_0 A}$$

Hence the correct choice is (c).

12. The last three capacitors on the right, each of capacitance $C = 9\ \mu\text{F}$ are in series and are equivalent to a capacitance C' given by

$$\frac{1}{C'} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{1}{3} \text{ or } C' = 3\ \mu\text{F}.$$

Since C' is in parallel with C_1, the equivalent capacitance of the last part of the network is $C'' = C' + C_1 = 3 + 6 = 9\ \mu\text{F}$. Continuing this process of calculation towards the left, we notice that we are finally left with the combination whose equivalent capacitance is $3\ \mu\text{F}$. Hence the correct choice is (a).

13. The three capacitors can be rearranged as shown in Fig. 11.88. The capacitance between points P and S or between points Q and R = sum of the three capacitances $= 3C = 9\ \mu\text{F}$. Hence the correct choice is (d).

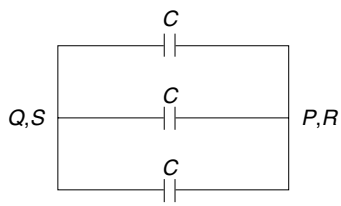


Fig. 11.88

14. $C = 100\ \text{pF} = 100 \times 10^{-12}\ \text{F} = 10^{-10}\ \text{F}$

Let the number of sheets of foils required be n. They will form $(n - 1)$ capacitors. If K is the dielectric constant of the dielectric, the capacitance is given by

$$C = \frac{K\varepsilon_0 (n-1) A}{d}$$

or $$n - 1 = \frac{C d}{K \varepsilon_0 A} = \frac{C d}{K \cdot 4\pi\varepsilon_0} \cdot \frac{4\pi}{\pi r^2}$$

$$= \frac{4 C d}{K \cdot 4\pi\varepsilon_0 r^2}$$

$$= \frac{4 \times 10^{-10} \times 1 \times 10^{-3} \times 9 \times 10^9}{4 \times \left(1.0 \times 10^{-2}\right)^2} = 9$$

or $n = 10$

Hence the correct choice is (a).

15. If R is the radius of the big drop, we have

$$\frac{4\pi R^3}{3} = 1000 \times \frac{4\pi r^3}{3}$$

which gives $R = 10\, r$. The electrical potential of each droplet is

$$v = \frac{q}{4\pi\varepsilon_0 r}$$

and that of the big drop is

$$V = \frac{1000 q}{4\pi\varepsilon_0 R}$$

$$\therefore \quad \frac{V}{v} = \frac{1000 r}{R} = 100 \qquad (\because R = 10\, r)$$

Hence the correct choice is (c).

16. If A is the area of each plate, the capacitance of the air capacitor shown in Fig. 11.52(a) on page 11.25 is

$$C_0 = \frac{\varepsilon_0 A}{d}, \text{ where } C_0 = 2\ \mu\text{F (given).}$$

The capacitance of air capacitor in Fig. 11.53 (b) is

$$C_1 = \frac{\varepsilon_0 A}{d/2} = \frac{2\varepsilon_0 A}{d} = 2 C_0$$

The capacitance of the dielectric filled capacitor in Fig. 11.53 (b) is

$$C_2 = \frac{k\varepsilon_0 A}{d/2} = \frac{2k\varepsilon_0 A}{d} = 2 k C_0$$

where k is the dielectric constant. Now capacitors C_1 and C_2 are in series. Therefore, the capacitance C of the capacitor shown in Fig. 11.52 (b) is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2C_0} + \frac{1}{2kC_0} = \frac{(k+1)}{2C_0 k}$$

or $$C = \frac{2C_0 k}{(k+1)} = \frac{2 \times 2\mu\text{F} \times 3}{(3+1)} = 3\ \mu\text{F}$$

Hence the correct choice is (c).

17. If A is the area of each plate, the capacitance of the air-filled capacitor shown in Fig. 11.53(a) on page 11.26 is

$$C_0 = \frac{\varepsilon_0 A}{d}, \text{ where } C_0 = 2\ \mu\text{F (given).}$$

The capacitance of air capacitor in Fig. 11.54(b) is

$$C_1 = \frac{\varepsilon_0 A/2}{d} = \frac{\varepsilon_0 A}{2d} = \frac{C_0}{2}$$

The capacitance of dielectric filled capacitor in Fig. 11.53(b) is

$$C_2 = \frac{k\varepsilon_0 A/2}{d} = \frac{k\varepsilon_0 A}{2d} = \frac{k C_0}{2}$$

Since C_1 and C_2 are in parallel, the capacitance C of the capacitor shown in Fig. 11.53(b) is

$$C = C_1 + C_2 = \frac{C_0}{2} + \frac{k C_0}{2}$$

$$= \frac{C_0}{2}(1 + k) = \frac{2\mu\text{F}}{2}(1 + 3) = 4\ \mu\text{F}$$

Hence the correct choice is (a).

18. Refer to Fig. 11.89. Let us consider forces on a ball, say, Q. Three forces act on it: (i) tension T in the thread, (ii) force mg due to gravity and (iii) force F due to Coulomb repulsion along +ve x-direction. For equilibrium, the sum of the x and y components of these forces must be zero,

i.e. $\quad T \cos 60° - F = 0$

and $\quad T \sin 60° - mg = 0$

These equations give $F = mg \cot 60° = \frac{\sqrt{3}}{10} \times 10^{-3} \times 10 \times \frac{1}{\sqrt{3}} = 10^{-3}$ N. Now

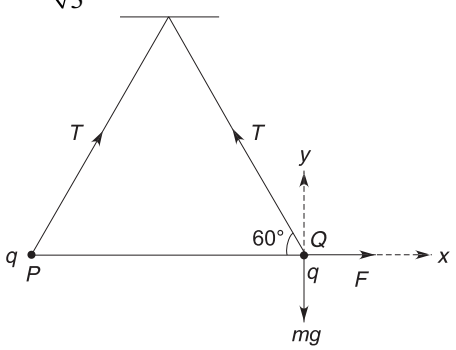


Fig. 11.89

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{r^2}$$

Putting $F = 10^{-3}$ N, $r = 0.3$ m and $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$, we get $q = 10^{-7}$ coulomb.

19. Refer to Fig. 11.90. The electric field E_1 at (a, b) due to q_1 has a magnitude

$$E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{a^2}$$

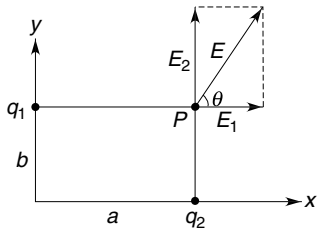


Fig. 11.90

and is directed along + x-axis. The electric field E_2 at (a, b) due to q_2 has a magnitude

$$E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{b^2}$$

and is directed along + y-axis. The angel θ subtended by the resultant field E with the x-axis is given by

$$\tan\theta = \frac{E_2}{E_1} = \frac{q_2}{q_1}\cdot\frac{a^2}{b^2} = \frac{1}{2}\times\left(\frac{2}{1}\right)^2 = 2$$

Hence the correct choice is (b).

20. The correct choice is (d). The electric field E exerts a force qE on charge + q and a force – qE on charge – q of the dipole. Since these forces are equal and opposite, they add upto zero.
21. The correct choice is (b). A torque acts on the dipole which tends to align it along the field.
22. The correct choice is (c). In a non-uniform electric field, a dipole experiences a force which gives it a translational motion and a torque which gives it a rotational motion.
23. The distance of a vertex from the the centre of the cube of side b is $r = \sqrt{3}\ b/2$. Now the potential due to charge q at the centre is $q/4\ \pi\varepsilon_0 r$. Hence the potential due to the arrangement of eight charges (each of magnitude q) at the centre is

$$V = \frac{8q}{4\pi\varepsilon_0 r} = \frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$$

24. We know that electric fields are to be added vectorially. From the symmetry of the eight charges with respect to the centre of the cube, it is evident that the electric fields at the centre due to two opposite charges cancel in pairs (being equal and opposite). Hence the net electric field at the centre of the cube will be zero.
25. Refer to Fig. 11.91. The distance of point P_1 from charge + q is $r_1 = z - a$ and from charge – q is $r_2 = z + a$.

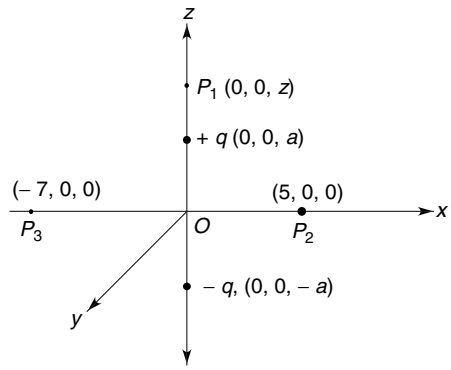


Fig. 11.91

$$\therefore \text{ Potential at } P_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{r_2 - r_1}{r_1 r_2}$$

$$= \frac{2qa}{4\pi\varepsilon_0\left(z^2 - a^2\right)},$$

which is choice (c).

26. Refer to Fig. 11.91 again. Any point on the perpendicular bisector passing through the centre of the dipole is at the same distance from the two charges. Hence the potentials at point $P_2(5, 0, 0)$ and that at point $P_3(-7, 0, 0)$ are zero. Since P_2 and P_3 are at the same potential (zero), the potential difference between them is zero. Hence no work will be done in moving a charge from P_2 to P_3. The answer will not change if the path of the charge is changed because the work done is independent of the path taken.
27. Refer to Fig. 11.92. The total potential energy of the arrangement of charges is the sum of the energies of each pair of charges. The potential energy of the system comprising the three charges q_1, q_2 and q_3 is

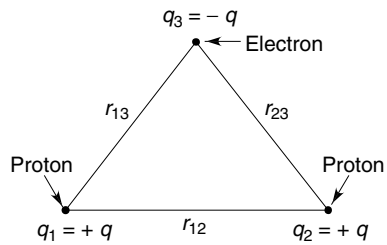


Fig. 11.92

$$U = W_1 + W_2 + W_3$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) \quad \text{(i)}$$

Here $q_1 = q_2 = q = +1.6 \times 10^{-19}$ C (proton), $q_3 = -q = -1.6 \times 10^{-19}$ C (electron), $r_{12} = 1.5$ Å $= 1.5 \times 10^{-10}$ m, $r_{13} = r_{23} = 1$ Å $= 1 \times 10^{-10}$ m and $1/4\ \pi\varepsilon_0 = 9 \times 10^9$ Nm^2C^{-2}. Thus

$$U = -\frac{4}{3}\cdot\frac{q^2 \times 10^{10}}{4\pi\varepsilon_0} \text{ joule}$$

$$= -\frac{4}{3}\cdot\frac{q \times 10^{10}}{4\pi\varepsilon_0} \text{ eV} \quad (\because q = 1.6 \times 10^{-19} \text{ C})$$

$$= -\frac{4 \times 1.6 \times 10^{-19} \times 10^{10} \times 9 \times 10^9}{3}$$

$$= -19.2 \text{ eV}$$

28. Charge on electron $(-e) = -1.6 \times 10^{-19}$ C, charge on proton $(e) = 1.6 \times 10^{-19}$ C, separation $r = 0.53$ Å $= 0.53 \times 10^{-10}$ m. If the zero of potential energy is taken to be at infinite separation, the potential energy of the electron-proton system is

$$U = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r} \text{ joule}$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot\frac{e}{r} \text{ eV}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})}{0.53 \times 10^{-10}} = -27.2 \text{ eV}$$

Hence the correct choice is (b).

29. If the electron was at rest, 27.2 eV of the energy will have to be supplied (or $27.2 \times 1.6 \times 10^{-19}$ J of work will have to be done) to free the electron from the attraction of the proton and remove it to infinity. Since the electron is moving (round the proton) with a kinetic energy = $U/2 = 1/2 \times (-27.2) = -13.6$ eV, the electron itself is supplying an energy of 13.6 eV due to centrifugal action. Hence the minimum amount of work required to free the electron = 27.2 – 13.6 = 13.6 eV = $13.6 \times 1.6 \times 10^{-19} = 2.2 \times 10^{-18}$ J. Hence the correct choice is (d).

30. The potential energy of electron-proton system at a separation of 1.06 Å = half that at a separation of 0.53 Å = half of – 27.2 eV = – 13.6 eV. If the zero of potential energy at a separation of 1.06 Å is taken to be zero (instead of – 13.6 eV), the potential energy of the electron-proton system would be
= – 27.2 – (– 13.6) = – 13.6 eV, which is choice (b).

31. Since the potential energy of the system is now – 13.6 eV, the energy supplied by the electron itself is 13.6 eV by virtue of its orbital motion round the proton. Hence the minimum work to pull the electron from the atom will be zero.

32. The series combination of C_2 and C_3 is equivalent to a capacitance C' given by

$$\frac{1}{C'} = \frac{1}{C_2} + \frac{1}{C_3}$$

or $$C' = \frac{C_2 C_3}{C_2 + C_3} = \frac{200 \times 200}{200 + 200} = 100 \text{ pF}$$

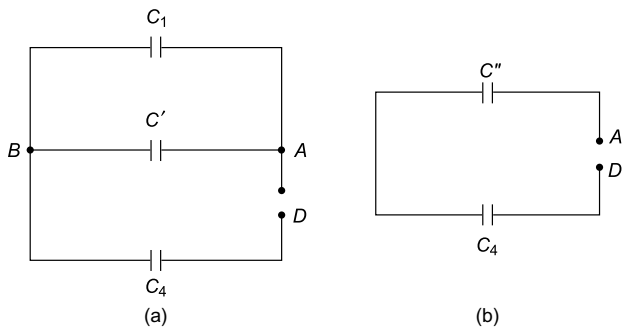


Fig. 11.93

Therefore the circuit reduces to the one shown in Fig. 11.93(a). The equivalent capacitance between points A and B is

$$C'' = C_1 + C' = 100 + 100 = 200 \text{ pF}$$

The circuit may be further simplified to that in Fig. 11.93(b). The equivalent capacitance C of the entire network, i.e., between points A and D, is now that of the series combination of C'' and C_4. Thus

$$\frac{1}{C} = \frac{1}{C''} + \frac{1}{C^4} = \frac{1}{200} + \frac{1}{100}$$

$$= \frac{3}{200} \quad \text{or} \quad C = \frac{200}{3} \text{ pF}$$

Hence the correct choice is (c).

33. The series combination of 6 and 12 is equivalent to 4 and the parallel combination of 2 and 2 is also equivalent to 4. Therefore the network can be simplified as shown in Fig. 11.94.

The parallel combination of 4 and 4 is equivalent to 8 and the series combination of 8 and 4 is equivalent to 8/3. Thus the combination in Fig. 11.94 reduces to that in Fig. 11.95.

The series combination of 1 and 8 in Fig. 11.95 yields 8/9 as shown in Fig. 11.96.

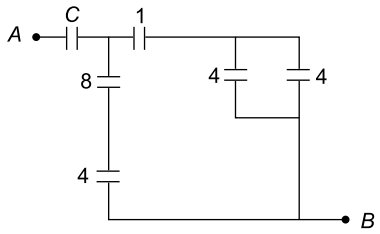


Fig. 11.94

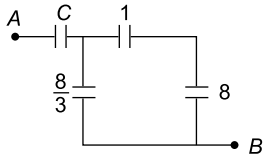


Fig. 11.95

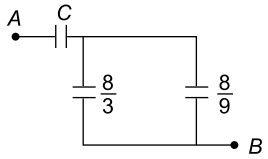


Fig. 11.96

Now 8/3 and 8/9 are in parallel and their equivalent is 32/9. Therefore, the network finally reduces to that in Fig. 11.97. Since the total capacitance between A and B is to be (i.e. 1 μF), we have

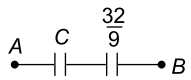


Fig. 11.97

$$1 = \frac{1}{C} + \frac{9}{32}$$

$$\Rightarrow \quad C = \frac{32}{23}\ \mu\text{F}.$$ Hence the correct choice is (b).

34. Charge Q_1 and $C_1 = C_1V_1 = 2 \times 10^{-6} \times 100 = 2 \times 10^{-4}$ C. Charge Q_2 on $C_2 = C_2V_2 = 4 \times 10^{-6} \times 50 = 2 \times 10^{-4}$ C. Total charge $Q = Q_1 + Q_2 = 4 \times 10^{-4}$ C. Total energy before connection is

$$E_1 = \frac{1}{2} C_1V_1^2 + \frac{1}{2} C_2V_2^2$$

$$= \frac{1}{2} \times 2 \times 10^{-6} \times (100)^2 + \frac{1}{2} \times 4 \times 10^{-6} \times (50)^2$$

$$= 1.5 \times 10^{-2}\ \text{J}$$

The common potential difference V after connection is given by

$$C_1V + C_2V = Q$$

or $$V = \frac{Q}{C_1 + C_2}$$

Therefore, total energy after connection is

$$E_2 = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2} \times \frac{Q^2}{(C_1 + C_2)}$$

$$= \frac{1}{2} \times \frac{(4 \times 10^{-4})^2}{(2+4) \times 10^{-6}} = 1.33 \times 10^{-2}\ \text{J}$$

$\therefore$ Loss of energy $= E_1 - E_2 = 0.17 \times 10^{-2}$ J. Hence the correct choice is (d).

35. Points A, B and C are at the same distance from charge $+q$; hence electrical potential is the same at these points, i.e. there is no potential difference between A, B and C. Hence $W_1 = W_2 = 0$.

36. The electric potential at the common centre is

$$V = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2}$$

Now $$\sigma = \frac{q_1}{4\pi r_1^2} = \frac{q_2}{4\pi r_2^2}$$

$\therefore$ $$V = \frac{1}{\varepsilon_0}\left[\frac{q_1 r_1}{4\pi r_1^2} + \frac{q_2 r_2}{4\pi r_2^2}\right] = \frac{\sigma}{\varepsilon_0}(r_1 + r_2)$$

Hence the correct choice is (d).

37. If q is charge on the sphere, the electric field on its surface is

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

But $$\sigma = \frac{q}{4\pi r^2}$$. Therefore, $q = 4\pi r^2\sigma$. Hence

$$E = \frac{4\pi r^2 \sigma}{4\pi\varepsilon_0 r^2} = \frac{\sigma}{\varepsilon_0}$$

Thus the correct choice is (a).

38. The correct choice is (c)

39. The correct choice is (b).

40. If the radius of a bubble is increased by a factor n, its capacitance is also increased by a factor n, i.e. $C' = nC$. Since the charge Q on the bubble remains unchanged, we have

$$Q = CV = C'V'$$

or $$V' = \frac{CV}{C'} = \frac{CV}{nC} = \frac{V}{n}$$

Hence the correct choice is (c).

41. If ρ is the density of a small drop and r its radius, then the mass of each small drop is $m = \frac{4\pi}{3} r^3\rho$. If n such drops coalesce to form a big drop of radius R, then the mass of the big drop is $nm = \frac{4\pi}{3} R^3\rho$. Hence $R = n^{1/3} r$. Now, the capacitance of a sphere is proportional to its radius. Hence the capacitance of the big drop will be $C' = n^{1/3}C$. Hence the correct choice is (d).

Level B

42. If Q is the charge on each small drop, charge on the big drop is $Q' = nQ$. Now $Q' = C'V' = Q = CV$. Therefore

$$\frac{V'}{V} = \frac{Q'}{Q} \times \frac{C}{C'} = \frac{n}{n^{1/3}} = n^{2/3}$$

Hence the correct choice is (d).

43. $E = \frac{1}{2} CV^2$, $E' = \frac{1}{2} C' V'^2$. Therefore,

$$\frac{E'}{E} = \frac{C'}{C} \cdot \frac{V'^2}{V} = n^{1/3} \times (n^{2/3})^2 = n^{5/3}$$

Hence the correct choice is (c).

44. The combination is equivalent to (10 – 1) = 9 capacitors, each of capacitance C connected in parallel. Hence the correct choice is (c).

45. The total capacitance across power supply = $\frac{6}{8}$ μF. The charge on 2 μF capacitor or 3 μF capacitor = $8 \times \frac{6}{8} = 6$ μC. So the charge on each 1 μF capacitor = 3 μC. Therefore, potential difference across each 1 μF capacitor = charge/capacitance = 3 μC/1 μF = 3 volts.

46. Capacitors of capacitances 2 μF and 3 μF are in parallel and this combination is in series with 1 μF capacitor. Thus, we have 1 μF capacitor in series with 5 μF capacitor and the potential difference across this series combination is 6V. Therefore, the potential differences across 5 μF capacitor (which consists of a parallel combination of 2 μF and 3 μF capacitors) is 1 V. Hence the charge on 2 μF capacitor = 2 μF × 1 V = 2 μC, which is choice (b).

47. Capacitors 1 μF, 2 μF and 3 μF are in parallel, their total capacitance is 6 μF. Thus, we have three capacitors in series each of capacitance 6 μF across the 12 V power supply. So, the potential drop across each is 12/3 = 4 V. This is also the potential across 1 μF capacitor and 2 μF capacitor and 3 μF capacitor, because they are in parallel. Therefore, charge on 2 μF capacitor = 2 μF × 4 V = 8 μC. Hence the correct choice is (b).

48. The distance of the point of intersection of diagonals = side of the hexagon = a. The potential at this point due to each charge = $\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{a}$. Therefore, total potential = $\frac{1}{4\pi\varepsilon_0} \cdot \frac{6q}{a}$ which is choice (c).

49. The net electric field at the point of intersection of diagonals is zero because the electric field at this point due to equal charges at opposite corners with cancel each other in pairs.

50. If C is the capacitance of the air-filled capacitor, the total charge on its plates, before connection, is $Q = CV$. After it is connected with an uncharged capacitor, let V' be the common potential and Q_1 be the charge on capacitor C and Q_2 on the other capacitor

$Q_1 = V' C$ and $Q_2 = V' kC$. Also $Q = Q_1 + Q_2$. Therefore,

$$CV = V' C + V' kC$$

or $$V = V' (1 + k)$$

or $$V' = \frac{V}{(1+k)}$$. Hence the correct choice is (d).

51. Energy stored in the capacitor is

$$\frac{1}{2} CV^2 = \frac{1}{2} \times C \times (200)^2 = 2 \times 10^4 \times C \text{ joule}$$

Energy appearing as heat in the block is

$$mc\theta = 0.1 \times 2.5 \times 102 \times 0.4 = 10 \text{ J}$$

Therefore,

$$2 \times 10^4 \times C = 10$$

or $$C = 5 \times 10^{-4} \text{ F} = 500 \text{ μF}$$

52. The force of repulsion between the two parts is given by

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q(Q-q)}{r^2}$$

For F to be maximum, $\frac{dF}{dq} = 0$, i.e.

$$\frac{d}{dq}\left[\frac{1}{4\pi\varepsilon_0} \cdot \frac{q(Q-q)}{r^2}\right] = 0$$

Since r is fixed, we have

$\frac{d}{dq} [q (Q - q)] = 0$ or $1(Q - q) + q (0 - 1) = 0$ or

$$\frac{q}{Q} = \frac{1}{2}$$

Hence the correct choice is (a).

53. For points on the surface of the sphere or outside the sphere, a charged sphere behaves as if the charge is concentrated at its center. Therefore, the potential at the surface of the sphere is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R}$$, which is choice (b).

54. At points inside a charged metallic sphere, i.e. for $r < R$, the potential is zero. Hence the correct choice is (a).

55. $E = -\dfrac{dV}{dx}\hat{\mathbf{i}}$ where $\hat{\mathbf{i}}$ is a unit vector along the positive x-axis. Hence $\mathbf{E}$ at a point whose x-coordinate is $x = 1$ m is

$$\mathbf{E} = -\frac{d}{dx}(4x^2)\hat{\mathbf{i}} = -8x\hat{\mathbf{i}} = -8\hat{\mathbf{i}}\ \text{Vm}^{-1}.$$

The negative sign shows that $\mathbf{E}$ is along the negative x-axis. Hence the correct choice is (a).

56. If a symmetrical closed surface has n identical surfaces and a charge Q is placed at its centre, then the flux through each face = $\dfrac{Q}{n\varepsilon_0}$. For a cube $n = 6$. Hence the correct choice is (d).

57. Since a dipole consists of two equal and opposite charges, the net charge of a dipole is zero. Hence the correct choice is (d).

58. The correct choice is (c).

59. Let C be the capacitance of each capacitor. For parallel combination, the net capacitance is $C_1 = nC$. Also $V_1 = V$. Therefore, the energy stored in the parallel combination is

$$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times nC \times V^2 = \frac{1}{2}nCV^2$$

For series combination, we have $C_2 = C/n$ and $V_2 = nV$. Therefore, the energy stored in the series combination is

$$U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2} \times \frac{C}{n} \times (nV)^2 = \frac{1}{2}nCV^2$$

Hence the correct choice is (b).

60. There will be no loss of energy if the potential of the spheres is the same i.e. if

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{Q}{4\pi\varepsilon_0 R}$$

or $\dfrac{q}{r} = \dfrac{Q}{R}$. Hence the correct choice is (b).

61. $\mathbf{r} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) - (\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ m. The magnitude of $\mathbf{r}$ is

$$r = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1 + 4 + 4} = 3 \text{ m}$$

$$\therefore \quad F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$= \frac{9 \times 10^9 \times 10^{-6} \times 10^{-6}}{(3)^2} = 10^{-3}\ \text{N}$$

Hence the correct choice is (a).

62. Initial capacitance $C = \dfrac{\varepsilon_0 A}{d}$. When a metal plate of thickness t is introduced, the capacitance becomes $C' = \dfrac{\varepsilon_0 A}{(d - t)}$. Given $C' = 4.5\ C$

Thus $$\frac{\varepsilon_0 A}{d - t} = \frac{\varepsilon_0 A}{d} \times \frac{9}{2}$$

which gives $9(d - t) = 2d$ or $t = \dfrac{7d}{9}$ which is choice (c).

63. $U_1 = \dfrac{1}{2}CV^2$, $U_2 = \dfrac{1}{2}C(1.2\ V)^2 = \dfrac{1}{2}CV^2 \times 1.44$

$$\therefore \quad \frac{U_2 - U_1}{U_1} \times 100 = (1.44 - 1) \times 100 = 44\%$$

Thus the correct choice is (d).

64. $U = \dfrac{1}{2}CV^2$. Therefore, $\delta U = CV\,\delta V$. Therefore,

$$\frac{dU}{U} \times 100 = \frac{CV\delta V}{\frac{1}{2}CV^2} \times 100 = \frac{2\delta V}{V} \times 100$$

$$= \frac{2 \times 0.1 \times 100}{100} = 0.2\%$$

Hence the correct choice is (d).

65. Given $\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} = \dfrac{1}{2}$ (1)

and $\dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{1}{3}$ (2)

Using (2) in (1), we have

$$\frac{1}{3} + \frac{1}{C_3} = \frac{1}{2} \text{ which gives } C_3 = 6\ \mu\text{F}$$

Hence the correct choice is (d).

66. Given $C_1 + C_2 = \dfrac{C_1 C_2}{C_1 + C_2} \times \dfrac{25}{6}$

or $$6(C_1 + C_2)^2 = 25\ C_1C_2$$

or $$6C_1^2 + 6C_2^2 + 12\ C_1C_2 = 25\ C_1C_2$$

or $$6C_1^2 + 6C_2^2 - 13\ C_1C_2 = 0$$

Let $C_2 = x\ C_1$. Then, we have

$$6C_1^2 + 6x^2\ C_1^2 - 13\ x\ C_1^2 = 0$$

or $$6x^2 - 13x + 6 = 0$$

which gives $x = \dfrac{3}{2}$ or $\dfrac{2}{3}$. Since $C_2 > C_1$, $x = \dfrac{2}{3}$ is not possible. Hence the correct choice is (a).

67. The system be in equilibrium if the net force on charge q at one vertex due to charges q at the other two vertices is equal and opposite to the force due to charge Q at the centroid, i.e. (here a is the side of the triangle)

$$-\frac{\sqrt{3}\,q^2}{4\pi\varepsilon_0 a^2} = \frac{Qq}{4\pi\varepsilon_0\left(\frac{a}{\sqrt{3}}\right)^2}$$

which gives $Q = -\frac{q}{\sqrt{3}}$. Hence the correct choice is (b).

68. Charge will flow from A to B until their potentials become equal. If charge q flows from A to B, then

$$\frac{Q-q}{4\pi\varepsilon_0 a} = \frac{q}{4\pi\varepsilon_0 b}$$

or $Q - q = \frac{a}{b}\,q$ which gives $q = \frac{bQ}{a+b}$. Hence charge left on $A = Q - q = Q - \frac{bQ}{a+b} = \frac{aQ}{a+b}$.

Hence the correct choice is (d).

69. When any additional negative charge is given to a hollow spherical shell, the potential on its surface falls, but the potential at each point within the shell also falls by the same amount. Hence the potential difference between the given surfaces remains unchanged. Thus the correct choice is (a).

70. Refer to Fig. 11.98.

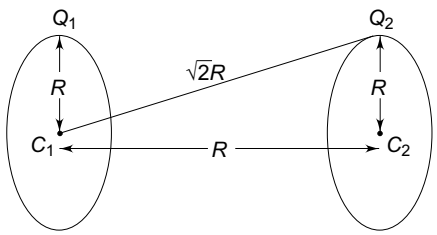


Fig. 11.98

Potential at C_1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}\,R}\right)$$

Potential at C_2 is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}\,R}\right)$$

$\therefore$ Work done $= q\,(V_1 - V_2)$

$$= \frac{q}{4\pi\varepsilon_0}\left[\left(\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}\,R}\right) - \left(\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}\,R}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0\sqrt{2}\,R}\,(Q_1 - Q)\,(\sqrt{2} - 1)$$

Hence the correct choice is (b).

71. Force $F = qE$. Therefore, acceleration $a = qE/m$. Now distance moved in time t is

$$s = \frac{1}{2}\,a t^2 = \frac{1}{2}\left(\frac{qE}{m}\right) t^2.$$

For electron: $s_e = \frac{1}{2}\left(\frac{qE}{m_e}\right) t_1^2$

For proton: $s_p = \frac{1}{2}\left(\frac{qE}{m_p}\right) t_2^2$

Given $s_e = s_p$. Therefore,

$$\frac{t_1^2}{m_e} = \frac{t_2^2}{m_p} \text{ or } \frac{t_2}{t_1} = \left(\frac{m_p}{m_e}\right)^{1/2}$$

Hence the correct choice is (b).

72. The electric field is always perpendicular to the surface of a conductor. On the surface of a metallic solid sphere, the electric field is perpendicular to the surface and directed towards the centre of the sphere. Hence the correct choice is (d).

73. $$V = \frac{1}{4\pi\varepsilon_0}\left\{\frac{q}{x_0} + \frac{q}{3x_0} + \frac{q}{5x_0} + \cdots \text{upto infinity}\right\}$$

$$+ \frac{1}{4\pi\varepsilon_0}\left\{\frac{-q}{2x_0} + \frac{-q}{4x_0} + \frac{-q}{6x_0} + \cdots \text{upto infinity}\right\}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x_0}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} \cdots \text{upto infinity}\right)$$

$$= \frac{q}{4\pi\varepsilon_0 x_0}\ln(1+1) = \frac{q\ln(2)}{4\pi\varepsilon_0 x_0}$$

Hence the correct choice is (d).

74. Since the hypotenuse side of triangle $= \sqrt{2}\,a$, the net electrostatic energy is

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{Qq}{a} + \frac{Qq}{\sqrt{2}\,a} + \frac{qq}{a}\right)$$

For $U = 0$, we require

$$\frac{Qq}{a} + \frac{Qq}{\sqrt{2}\,a} + \frac{q^2}{a} = 0$$

which gives $\quad Q = -q\left(\frac{\sqrt{2}}{\sqrt{2}+1}\right) = \frac{-2q}{2+\sqrt{2}}$

Hence the correct choice is (b).

75. $Q_1 = CV$ and $Q_2 = (2C) \times (2V) = 4CV$. Since the capacitors are connected in parallel such that the plates of opposite polarity are connected together, the common potential is

$$V' = \frac{Q_2 - Q_1}{C_1 + C_2} = \frac{4CV - CV}{C + 2C} = V$$

Equivalent capacitance $C' = C + 2C = 3C$. Therefore, the final energy of the configuration is

$$U' = \frac{1}{2}\ C'V'^2 = \frac{1}{2} \times 3C \times V^2 = \frac{3}{2}\ CV^2,$$

which is choice (b).

76. The insertion of the dielectric slab decreases the electric field without changing its direction. The electric potential increases as we go from the negative to the positive plate. Hence the correct choice is (c).

77. Within the plates electric fields due to charges Q_1 and Q_2 are

$$E_1 = \frac{Q_1}{2\varepsilon_0 A} \text{ and } E_2 = \frac{Q_2}{2\varepsilon_0 A}$$

As these fields are in opposite directions and $Q_1 > Q_2$, the net electric field within the plates is

$$E = E_1 - E_2 = \frac{1}{2\varepsilon_0 A}\ (Q_1 - Q_2)$$

Hence $V = Ed = \frac{d}{2\varepsilon_0 A}\ (Q_1 - Q_2) = \frac{Q_1 - Q_2}{2C}$ which is choice (d). $\left(\because C = \frac{\varepsilon_0 A}{d}\right)$

78. When switch S_3 is closed, the potential difference across C_1 and C_2 will become equal to the average of V_1 and V_2, i.e. (30 + 20)/2 = 25 V. Hence the correct choice is (b).

79. We have $C_1 = \frac{(A/2)\varepsilon_0 K_1}{(d/2)} = \frac{A\varepsilon_0 K_1}{d}$

$$C_2 = \frac{(A/2)\varepsilon_0 K_2}{(d/2)} = \frac{A\varepsilon_0 K_2}{d} \text{ and}$$

$$C_3 = \frac{A\varepsilon_0 K_3}{(d/2)} = \frac{2A\varepsilon_0 K_3}{d}$$

The capacitors C_1 and C_2 are in parallel and their equivalent capacitance is

$$C' = C_1 + C_2 = \frac{A\varepsilon_0}{d}\ (K_1 + K_2)$$

This combination is in series with C_3. Hence the net capacitance is

$$\frac{1}{C''} = \frac{1}{C'} + \frac{1}{C_3} = \frac{d}{A\varepsilon_0(K_1 + K_2)} + \frac{d}{2A\varepsilon_0 K_3}$$

$$= \frac{d}{\varepsilon_0 A}\left[\frac{1}{(K_1 + K_2)} + \frac{1}{2K_3}\right]$$

or $C'' = \frac{A\varepsilon_0 K}{d}$ where $\frac{1}{K} = \frac{1}{(K_1 + K_2)} + \frac{1}{2K_3}$

Hence the correct choice is (b).

80. The capacitance of a parallel plane capacitor is given by $C = \varepsilon_0 A/d$. Hence the dimensions of $\varepsilon_0 L$ are the same as those of capacitance.

$\therefore$ Dimensions of $\varepsilon_0 L\ \frac{\Delta V}{\Delta t}$

$$= \frac{\text{dimension of } C \times \text{dimensions of } V}{\text{time}}$$

$$= \frac{\text{dimension of } Q}{\text{time}} \qquad (\because\ Q = CV)$$

$$= \frac{\text{charge}}{\text{time}} = \text{current}$$

Hence the correct choice is (d).

81. Since the outer plate of B is free, charge cannot flow from A to B. Hence the correct choice is (a).

82. Electric field is the negative gradient of potential, i.e.

$$E = -\frac{dV}{dx}$$

Thus, V decreases as dx increases in the direction of the field. This implies that $V_A > V_B$, which is choice (b).

83. Potential energy of the system when charge Q is at O is

$$U_0 = \frac{qQ}{a} + \frac{qQ}{a} = \frac{2qQ}{a}$$

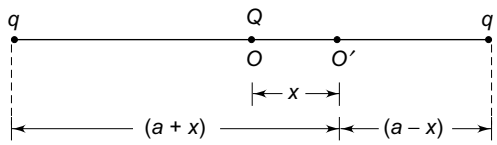


Fig. 11.99

When charge Q is shifted to position O', the potential energy will be (see Fig. 11.99)

$$U = \frac{qQ}{(a + x)} + \frac{qQ}{(a - x)}$$

$$= \frac{qQ(2a)}{(a^2 - x^2)} = \frac{2qQ}{a} \times \left(1 - \frac{x^2}{a^2}\right)^{-1}$$

$$\simeq \frac{2qQ}{a} \times \left(1 + \frac{x^2}{a^2}\right) \qquad (\because\ x << a)$$

$\therefore$ $\Delta U = U - U_0$

$$= \frac{2qQ}{a} \times \left(1 + \frac{x^2}{a^2}\right) - \frac{2qQ}{a} = \frac{2qQ}{a^3}\ (x^2)$$

Hence $\Delta U \propto x^2$ which is choice (b).

84. Given $v_x = 10\ \text{ms}^{-1}$. Since the electric field is directed along the y-axis, the acceleration of the body along the y-direction is

$$a_y = \frac{qE}{m} = \frac{10^{-6} \times 10^3}{10^{-3}} = 1 \text{ ms}^{-2}$$

Therefore, the velocity of the body along the y-axis at time $t = 10$ s is

$$v_y = at = 1 \times 10 = 10 \text{ ms}^{-1}$$

$\therefore$ Resultant velocity $v = \sqrt{v_x^2 + v_y^2}$

$$= \sqrt{(10)^2 + (10)^2} = 10\sqrt{2} \text{ ms}^{-1}$$

Hence the correct choice is (c).

85. Let Q be the magnitude of each charge and a the length of each side of the triangle. The potential energy of the system of two equal charges placed at vertex A and B is U (given). This means that U is the work done in bringing a charge Q from infinity to vertex B. Hence the work done in bringing an identical charge Q from infinity to the third vertex C = work done to overcome the force of repulsion of Q placed at A at a distance a + work done to overcome the force of repulsion of Q placed at B at the same distance $a = U + U = 2U$, which is choice (b).

86. Charge on capacitor plates without the dielectric is

$$Q = CV = (5 \times 10^{-6} \text{ F}) \times 1 \text{ V} = 5 \times 10^{-6} \text{ C} = 5 \text{ μC}$$

The capacitance after the dielectric is introduced is

$$C' = \frac{\varepsilon_0 A}{d - \left(t - \frac{t}{K}\right)} = \frac{\varepsilon_0 A/d}{1 - \left(\frac{t - \frac{t}{K}}{d}\right)}$$

$$= \frac{C}{1 - \left(\frac{t - \frac{t}{K}}{d}\right)} = \frac{5 \text{ μF}}{1 - \left(\frac{4 \text{ cm} - \frac{4 \text{ cm}}{4}}{6 \text{ cm}}\right)}$$

$$= \frac{5 \text{ μF}}{1 - \left(\frac{4-1}{6}\right)} = 10 \text{ μF}$$

$\therefore$ Charge on capacitor plates now will be

$$Q' = C'V = 10 \text{ μF} \times 1 \text{ V} = 10 \text{ μC}$$

Additional charge transferred = $Q' - Q = 10$ μC – 5 μC = 5 μC, which is choice (c).

87. Common potential is $V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

$$= \frac{(4 \times 10^{-6}) \times 80 + (6 \times 10^{-6}) \times 30}{4 \times 10^{-6} + 6 \times 10^{-6}} = 50 \text{ V}$$

$\therefore$ Energy lost by 4 μF capacitor

$$= \frac{1}{2} C_1 V_1^2 - \frac{1}{2} C_1 V^2$$

$$= \frac{1}{2} C_1 (V_1^2 - V^2)$$

$$= \frac{1}{2} \times (4 \times 10^{-6}) \times \{(80)^2 - (50)^2\}$$

$$= 7.8 \times 10^{-3} \text{ J} = 7.8 \text{ mJ}$$

Hence the correct choice is (a).

88. Given $E = \frac{q}{4\pi\varepsilon_0 x^2}$. Hence the magnitude of the electric field at a distance $2x$ from charge q is

$$E' = \frac{q}{4\pi\varepsilon_0 (2x)^2} = \frac{q}{4\pi\varepsilon_0 x^2} \times \frac{1}{4} = \frac{E}{4}$$

Therefore, the force experienced by a similar charge q at a distance $2x$ is

$$F = qE' = \frac{qE}{4}$$

Hence the correct choice is (d).

89. Refer to Fig. 11.100. Let S be the surface area of each face of the cube. The flux through surfaces $ABCD$ and $EFGH$ is zero because these surfaces are parallel to the electric field $\mathbf{E}$ ($\theta = 90°$).

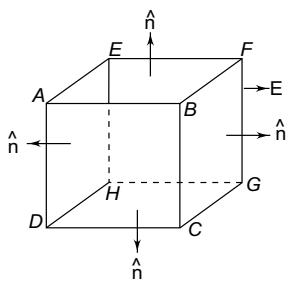


Fig. 11.100

Flux through face $BFGC$ is $\phi_1 = ES \cos 0° = ES$. Flux through face $AEHD$ is $\phi_2 = ES \cos 180° = -ES$. Total flux through the cube = $\phi_1 + \phi_2 = ES - ES = 0$.

Hence the correct choice is (a).

90. The electric flux is given by the surface integral $\int \mathbf{E}.\mathbf{ds}$. Here the electric field $\mathbf{E}$ is due to all the charges, both inside and outside the Gaussian surface. Hence the correct choice is (c).

91. The electric field at a point P due to an infinite long plane sheet carrying a uniform charge density σ is given by

$$E = \frac{\sigma}{2\varepsilon_0}$$

It is independent of the distance of point P from the sheet and is, therefore, uniform. The direction of the electric field is away from the sheet and perpendicular to it if σ is positive and is towards the sheet and perpendicular to it if σ is negative. Hence

$$E_1 = \frac{\sigma}{2\varepsilon_0}\left(-\hat{\mathbf{j}}\right) \text{ along } -\text{ve } y\text{–direction}$$

$$E_2 = \frac{2\sigma}{2\varepsilon_0}\left(-\hat{\mathbf{j}}\right) \text{ along } -\text{ve } y\text{–direction}$$

and $$E_3 = \frac{3\sigma}{2\varepsilon_0}\left(-\hat{\mathbf{j}}\right) \text{ along } -\text{ve } y\text{–direction}$$

From the superposition principle, the net electric field at point P is

$$E = E_1 + E_2 + E_3$$

$$= \frac{\sigma}{2\varepsilon_0}\left(-\hat{\mathbf{j}}\right) + \frac{2\sigma}{2\varepsilon_0}\left(-\hat{\mathbf{j}}\right) + \frac{3\sigma}{2\varepsilon_0}\left(-\hat{\mathbf{j}}\right)$$

$$= -\frac{3\sigma}{\varepsilon_0}\hat{\mathbf{j}}, \text{ which is choice (c).}$$

92. Electric field due to charge $-Q$ on the shell at a distance r from its center is (for $r > R$)

$$E_1 = \frac{Q}{4\pi\varepsilon_0 r^2}$$

directed towards the centre.

Electric field due to charge $+Q$ at the centre at a distance r is

$$E_2 = \frac{Q}{4\pi\varepsilon_0 r^2}$$

directed away from the centre.

$\therefore$ Net electric field E (for $r > R$) $= E_1 - E_2 = 0$. For $r < R$, the electric field due to the shell is zero. In this region, the electric field due to charge $+Q$ at the centre decreases as $1/r^2$. Hence the correct graph is (a).

93. Let the charge on the sphere be Q. Then

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

which gives $Q = 4\pi\varepsilon_0 RV$

The electric field at a distance r is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{4\pi\varepsilon_0 RV}{4\pi\varepsilon_0 r^2} = \frac{RV}{r^2}$$

thus the correct choice is (c).

94. If charge Q is moved from C to D along the arc, the potential energy between pairs (q_1, Q) and (q_1, q_2) will not change as the distance between them remains unchanged ($\because AC = AD$). The potential energy of the pair of charges q_2 and Q will change.

Now, distance $BC = \sqrt{(8)^2 + (6)^2} = 10$ cm and $BD = 8 - 6 = 2$ cm. Therefore, change in P.E. is

$$\Delta U = \frac{q_2 Q}{4\pi\varepsilon_0}\left[\frac{1}{BD} - \frac{1}{BC}\right]$$

$$= (2 \times 10^{-6}) \times (5 \times 19^{-6}) \times (9 \times 10^{-9})\left(\frac{1}{0.02} - \frac{1}{0.1}\right)$$

$$= 3.6 \text{ J, which is choice (b).}$$

95. If the middle charge is displaced by a distance x, the net force acting it, when it is released, is

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{(L+x)^2} - \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{(L-x)^2}$$

$$= \frac{4q^2\, Lx}{4\pi\varepsilon_0\,(L^2 - x^2)^2}$$

For $x << L$, $F = \frac{q^2 x}{\pi\varepsilon_0 L^3} = -kx$

where $k = \frac{q^2}{\pi\varepsilon_0 L^3}$

Now $T = 2\pi\sqrt{\frac{m}{k}}$

So, the correct choice is (c).

96. The batteries are in opppsition as their positive terminals are connected together. Hence the effective voltage is

$$V = V_1 - V_2 = 12 - 2 = 10 \text{ V}$$

As the capacitors C_1 and C_2 are in series, the effective capacitance of the circuit is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

or $$C = \frac{6}{5} = 1.2\ \mu\text{F}.$$

Therefore, charge on capacitors is

$$Q = CV = 1.2\ \mu\text{F} \times 10\text{V} = 12\ \mu\text{C}$$

$\therefore$ Potential difference across A and B = potential difference across capacitor C_2

$$= \frac{Q}{C_2} = \frac{12\,\mu\text{C}}{2\,\mu\text{F}} = 6 \text{ V}$$

97. Potential difference between the plates before the slab is introduced is

$$V = E \times d = 200 \times 0.05 = 10 \text{ V}$$

The capacitance of the capacitor is given by

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{0.05} \text{ or } \varepsilon_0 A = 0.05\ C$$

When a slab of dielectric constant K and thickness t is introduced, the capacitance becomes

$$C' = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)} = \frac{0.05C}{0.05 - 0.01\left(1 - \frac{1}{4}\right)} = \frac{20C}{17}$$

Now $Q = CV = C'V'$. Therefore,

$$V' = \frac{CV}{C'} = \frac{CV}{20C/17} = \frac{17V}{20} = \frac{17 \times 10}{20} = 8.5 \text{ V}$$

98. The capacitance before the introduction of the slab is $C = \dfrac{\varepsilon_0 A}{d}$

If Q is the charge on the plates, the potential difference is

$$V = \frac{Q}{C} = \frac{Qd}{\varepsilon_0 A} \qquad (1)$$

Let d' be the new separation between the plates. When a slab of thickness t and dielectric constant K is introduced, the new capacitance is

$$C' = \frac{\varepsilon_0 A}{d' - t\left(1 - \frac{1}{K}\right)}$$

Since charge Q remains the same, the new potential difference is

$$V' = \frac{Q}{C'} = \frac{Q\left[d' - t\left(1 - \frac{1}{K}\right)\right]}{\varepsilon_0 A} \qquad (2)$$

Given $V' = V$. Equating Eqs. (1) and (2), we get

$$d = d' - t\left(1 - \frac{1}{K}\right) \text{ or } d' - d = t\left(1 - \frac{1}{K}\right)$$

Given $d' = d = 2$ mm and $t = 3$ mm. Thus

$$2 = 3\left(1 - \frac{1}{K}\right)$$

which gives $K = 3$. Hence the correct choice is (b).

99. If Q is the initial charge on capacitor C_1, the initial energy is given by

$$U_i = Q^2/2\ C_1$$

When the two capacitors are connected together, and as the charge is distributed equally, the charge on each capacitor is $Q/2$. Since the potential difference (in a parallel connection) across the two capacitors is also the same, it follows that their capacitances are equal (since $C = Q/V$). Thus $C_1 = C_2 = C$(say). Also, $Q_1 = Q_2 = Q/2$.

Therefore, final energy stored in the two capacitors is

$$U_f = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} = \frac{(Q/2)^2}{2C} + \frac{(Q/2)^2}{2C} = \frac{Q^2}{4C}$$

But $\quad U_i = \dfrac{Q^2}{2C}$

$$\therefore \quad \frac{U_f}{U_i} = \frac{1}{2}, \text{ which is choice (b).}$$

100. Plate 1 is connected to plate 3 and plate 2 is connected to plate 4. Thus, there are three capacitors in parallel, each of capacitance

$$C = \frac{\varepsilon_0 A}{d}$$

as shown in Fig. 11.101. Hence the equivalent capacitance is

$C' = 3C = \dfrac{3\varepsilon_0 A}{d}$, which is choice (c).

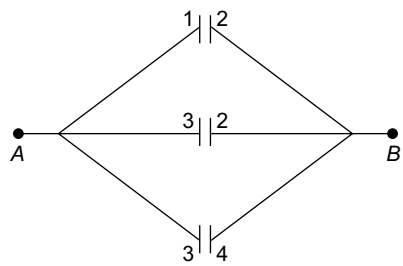


Fig. 11.101

101. The inner plates 2 and 3 are connected together. Hence they act as a single conductor. Since the outer plates 1 and 4 are connected together, there are effectively two capacitors (between plates 1 and 2 and plates 3 and 4) in parallel, each of capacitance $C = \varepsilon_0\ A/d$ as shown in Fig. 11.102. Thus, the equivalent capacitance is

$C' = 2C = \dfrac{2\varepsilon_0 A}{d}$, which is choice (b).

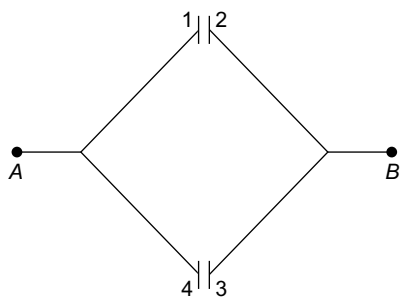


Fig. 11.102

102. The network reduces to that shown in Fig. 11.103. The correct choice is (a).

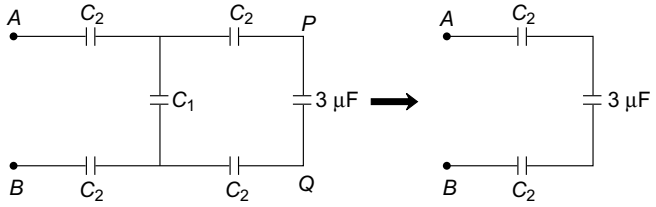


Fig. 11.103

103. Charge on C_1 is $Q_1 = C_1V = CV$. This charge is shared by the three capacitors. The equivalent capacitance of the series combination of C_2 and C_3 is

$$C' = \frac{C_2C_3}{C_2 + C_3} = \frac{C}{2} \qquad (\because\ C_2 = C_3 = C)$$

The common potential of C_1 and C' is

$$V' = \frac{Q_1}{C_1 + C'} = \frac{C_1V}{C_1 + C'}$$

$$= \frac{CV}{C + C/2} = \frac{2V}{3}$$

$\therefore$ Final charge on C_1 is

$$Q_1' = C_1V' = \frac{2CV}{3}$$

$\therefore$ Charge that will flow through the connecting wires is

$Q'' = Q_1 - Q_1' = CV - \frac{2CV}{3} = \frac{CV}{3}$, which is choice (a).

104. $C_1 = 4\pi\varepsilon_0 R_1$

$$C_2 = \frac{4\pi\varepsilon_0(R_1R_2)}{(R_2 - R_1)}$$

Given $C_2 = 3C_1$ Hence

$$\frac{4\pi\varepsilon_0(R_1R_2)}{(R_2 - R_1)} = 3 \times 4\pi\,\varepsilon_0\,R_1$$

which gives $\frac{R_2}{R_1} = \frac{3}{2}$, which is choice (b).

105. Energy stored initially is $U_i = \frac{Q^2}{2C}$. if d is doubled, C becomes $C/2$. Hence, energy stored when d is doubled is $U_f = \frac{Q^2}{C}$

$\therefore$ Work needed is

$$W = U_f - U_i = \frac{Q^2}{C} - \frac{Q^2}{2C} = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \qquad (\because\ Q = CV)$$

Now $C = \frac{A\varepsilon_0}{d}$. Hence

$$W = \frac{A\varepsilon_0V^2}{2d}$$, which is choice (c)

106. The circuit can be redrawn as shown in Fig. 11.104.

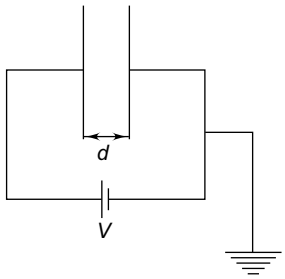


Fig. 11.104

The charge on the capacitor plates is

$$Q = CV = \frac{A\varepsilon_0V}{d}$$

So the correct choice is (b).

107. Volume charge density ρ is given by

$$\rho = \frac{Q}{\frac{4\pi}{3}R^3}$$

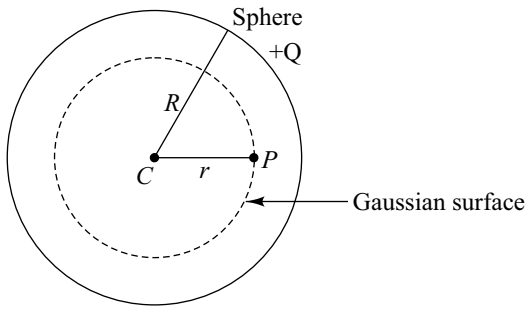


Fig. 11.105

Charge enclosed within the Gaussian surface is

$$q = \rho \times \frac{4\pi}{3}r^3$$

$$= \frac{Q}{\frac{4\pi}{3}R^3} \times \frac{4\pi}{3}R^3 = \frac{Qr^3}{R^3}$$

Electric flux through the Gaussian surface is

$$\phi = \int_s \mathbf{E.ds} = E \times \int_s ds = E \times 4\pi r^2$$

From Gauss's law

$$\phi = \frac{q}{\epsilon_0}$$

$$\Rightarrow \quad E \times 4\pi r^2 = \frac{Qr^3}{\epsilon_0 R^3}$$

$$\Rightarrow \qquad E = \frac{Qr}{4\pi\epsilon_0 R^3}$$

So the correct choice is (a). Since the charge is positive, the direction of **E** at point P is radially outward. If the charge is negative, the direction of **E** will be radially inward. Note that in terms of ρ, $E = \dfrac{\rho r}{3\epsilon_0}$.

108. Refer to Fig. 11.106.

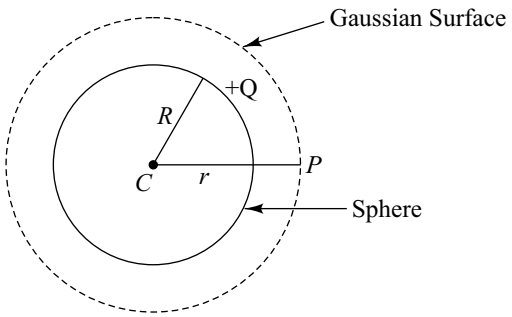


Fig. 11.106

Charge enclosed inside the Gaussian surface is

$$q = \text{charge on the sphere} = Q$$

From Gauss's law

$$\phi = \frac{q}{\epsilon_0}$$

$$\text{or} \quad E \times 4\pi r^2 = \frac{Q}{\epsilon_0} \qquad (\because \quad q = Q)$$

$$\Rightarrow \qquad E = \frac{Q}{4\pi\epsilon_0 r^2}$$

So the correct choice is (c). **E** is radially outward if Q is positive and radially inward if Q is negative.

109. We assume the Gaussian surface of radius r. To find the charge q enclosed inside this surface, we consider a thin spherical shell of radius a and thickness da (see Fig. 11.107).

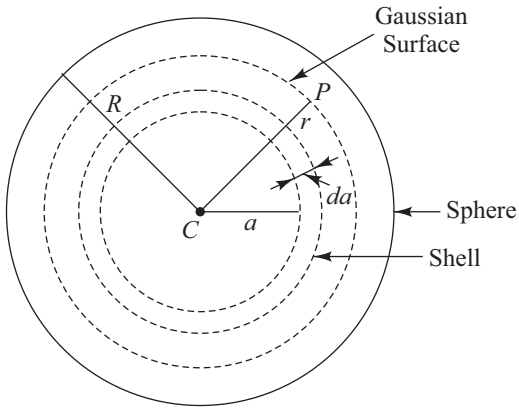


Fig. 11.107

Volume of the shell is $dV = 4\pi a^2 da$. So the charge contained in the shell is

$$dq = \rho \times 4\pi a^2 da$$

$$= \frac{Ka}{R} \times 4\pi a^2 da \qquad (\because \quad r = a)$$

Therefore, the total charge enclosed inside the Gaussian surface is

$$q = \int_{a=0}^{a=r} dq = \frac{4\pi K}{R}\int_0^r a^3 da$$

$$= \frac{4\pi K r^4}{4R} = \frac{\pi K r^4}{R}$$

Electric flux through the Gaussian surface is

$$\phi = EA = E \times 4\pi r^2$$

From Gauss's law,

$$\phi = \frac{q}{\epsilon_0}$$

$$\text{or} \quad E \times 4\pi r^2 = \frac{\pi K r^4}{R\epsilon_0}$$

$$\Rightarrow \qquad E = \frac{Kr^2}{4R\epsilon_0}$$

Hence $E \propto r^2$, which is choice (d).

110. In this case, the charge inside the Gaussian surface = charge on the entire sphere. Since now $r = R$

$$q = \int_{a=0}^{a=R} \frac{KR}{R} \times 4\pi a^2 dR$$

$$= \frac{4\pi K R^3}{3}$$

$$\therefore \quad E \times 4\pi r^2 = \frac{q}{\epsilon_0} = \frac{4\pi K R^3}{3\epsilon_0}$$

$$\Rightarrow \qquad E = \frac{KR^3}{3\epsilon_0 r^2}$$

So the correct choice is (d).

111. At points for which $r < a$, the electric field is zero because there can be no electric field within the body of a conductor.

At points for which r lies between b and c, the inner sphere acts as if all its charge were concentrated at its centre. Hence $E = \dfrac{q}{4\pi\epsilon_0 r^2}$.

At points for which r lies between b and c, the electric field is again zero because these points are in the body of a conductor.

The charge + q on the inner sphere induces a charge – q on the inner surface of the outer sphere and a charge + q on the outer surface of the outer sphere. At points for which $r > c$, the net charge enclosed inside the Gaussian spherical surface of radius r ($> c$) is $+ q + (- q) + (+ q) = + q$. So the electric field for $r > c$ is $E = \dfrac{q}{4\pi \in_0 r^2}$.

So the only incorrect statement is (d).

112. Figure 11.108 shows the distribution of charges on the surfaces of the shells.

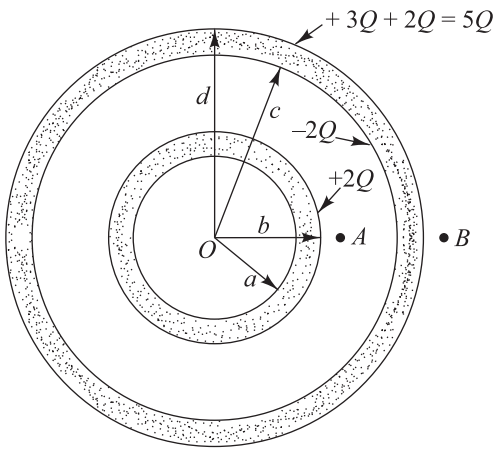


Fig. 11.108

We consider the Gaussian surface to be a sphere of radius $OA = r$. The net charge enclosed inside the surface is

$$q = 0 + 2Q = +2Q$$

because the inner surface of the inner shell has no charge.

From Gauss's law, the electric field at A will be given by

$$\phi = \frac{q}{\in_0}$$

or $$E \times 4\pi r^2 = \frac{+2Q}{\in_0}$$

$$E = \frac{2Q}{4\pi \in_0 r^2}$$

So the correct choice is (b).

113. In this case the radius of the Gaussian surface is $OB = r$. The net charge enclosed inside this surface is

$$q = 5Q + (- 2Q) + 2Q + 0 = 5Q$$

From Gauss's law,

$$E \times 4\pi r^2 = \frac{q}{\in_0} = \frac{5Q}{\in_0}$$

$$\Rightarrow \qquad E = \frac{5Q}{4\pi \in_0 r^2}, \quad \text{which is choice (d).}$$

114. The acceleration of the smaller sphere is

$$a = \frac{F}{m} = \frac{1}{4\pi \in_0} \frac{Qq}{r^2 m}$$

As the smaller sphere moves away, r increases. Hence a decreases. Since a is always positive, the speed of the smaller sphere is always increasing. So the correct choice is (c).

115. Refer to Fig. 11.109. The linear charge density is

$$\lambda = \frac{Q}{l}$$

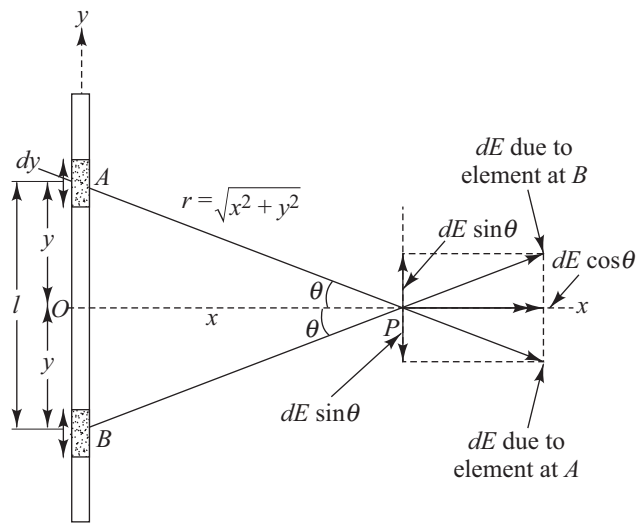


Fig. 11.109

Consider two symmetrically located line elements at A and B, each of small length dy at distance y from origin O. The charge of the element is $dq = \lambda dy$. The electric field at P due to the element at A is

$$dE = \frac{dq}{4\pi \in_0 r^2} = \frac{\lambda dy}{4\pi \in_0 (x^2 + y^2)}$$

It follows from the figure that the x components $dE \cos\theta$ add up but the y components $dE \sin\theta$ cancel each other. Therefore, the electric field at P due to the complete rod is

$$E = \int_{-l/2}^{+l/2} dE \cos\theta = 2\int_0^{l/2} dE \cos\theta$$

$$= \frac{2\lambda}{4\pi \in_0} \int_0^{l/2} \frac{dy}{x^2 + y^2} \times \frac{x}{\sqrt{x^2 + y^2}}$$

$$= \frac{\lambda x}{2\pi \in_0} \int_0^{l/2} \frac{dy}{(x^2 + y^2)^{3/2}}$$

$$= \frac{\lambda x}{2\pi \in_0} \left| \frac{y}{x^2 (x^2 + y^2)^{1/2}} \right|_{y=0}^{y=l/2}$$

or $$E = \frac{\lambda}{2\pi \in_0 x} \left[\frac{l/2}{\left[x^2 + \left(\frac{l}{2} \right)^2 \right]^{1/2}} \right] \quad (1)$$

Putting $x = l$ and $\lambda = \frac{Q}{l}$ in Eq. (1) we get

$$E = \frac{Q}{2\sqrt{5}\pi \in_0 l^2}$$

So the value of $k = \frac{2}{\sqrt{5}}$ which is choice (d).

116. The potential at every point inside the outer shell B is $V_1 = \frac{Q}{4\pi \in_0 R}$, which is also the potential on the surface of inner shell A due to outer shell B. The potential on the surface of shell A due to its own charge is $V_2 = \frac{q}{4\pi \in_0 r}$. Therefore, the total potential of shell A is

$$V_A = V_1 + V_2 = \frac{Q}{4\pi \in_0 R} + \frac{q}{4\pi \in_0 r}$$

$$= \frac{1}{4\pi \in_0} \left(\frac{Q}{R} + \frac{q}{r} \right)$$

So the correct choice is (a).

117. For points outside the outer shell, the two shells behave as if all their charge were concentrated at their common centre O. At a point $x > R$,

$$V = \frac{q + Q}{4\pi \in_0 x}$$

If we put $x = R$, we get the potential on the surface of B. Thus

$$V_B = \frac{q + Q}{4\pi \in_0 R}$$

$$\therefore \quad V_A - V_B = \frac{1}{4\pi \in_0} \left(\frac{q}{r} + \frac{Q}{R} \right) - \frac{1}{4\pi \in_0} \left(\frac{q + Q}{R} \right)$$

$$= \frac{q}{4\pi \in_0} \left(\frac{1}{r} - \frac{1}{R} \right)$$

So the correct choice is (a). Note that the potential difference $(V_A - V_B)$ does not depend on the charge on the outer shell.

118. $$E = -\frac{dV}{dr}$$

$$\Rightarrow \quad dV = -E\, dr$$

$$\Rightarrow \quad V = -\int_R^r E\, dr$$

$$= -\frac{\lambda}{2\pi \in_0} \int_R^r \frac{dr}{r}$$

$$= -\frac{\lambda}{2\pi \in_0} \ln \left(\frac{r}{R} \right)$$

$$= \frac{\lambda}{2\pi \in_0} \ln \left(\frac{R}{r} \right)$$

So the correct choice is (d).

119. The electric field at a distance r from a conducting cylinder is given by

$$E = \frac{\lambda}{2\pi \in_0 r}$$

Therefore, the potential difference between the inner cylinder and the outer cylinder is

$$V = -\int E\, dr$$

$$= -\frac{\lambda}{2\pi \in_0} \int_b^a \frac{dr}{r}$$

$$= \frac{\lambda}{2\pi \in_0} \int_a^b \frac{dr}{r}$$

$$\Rightarrow \quad V = \frac{\lambda}{2\pi \in_0} \ln \left(\frac{b}{a} \right)$$

Now $$C = \frac{Q}{V} = \frac{\lambda l}{\frac{\lambda}{2\pi \in_0} \ln \left(\frac{b}{a} \right)}$$

$$= \frac{2\pi \in_0 l}{\ln \left(\frac{b}{a} \right)}, \quad \text{which is choice (a).}$$

120. Let $+Q$ be the charge on the inner shell and $-Q$ on the outer shell. In the space between the two shells, the electric field is due to the inner shell alone. At a distance r from the centre of the inner shell, the electric field is

$$E = \frac{1}{4\pi \in_0} \frac{Q}{r^2}$$

Let V be the potential difference between the two shells. Then

$$V = -\int_b^a E\, dr = \int_a^b E\, dr$$

or $$V = \frac{Q}{4\pi \in_0} \int_a^b \frac{dr}{r^2}$$

$$= \frac{Q}{4\pi \in_0} \left| -\frac{1}{r} \right|_a^b$$

$$= \frac{Q}{4\pi \in_0} \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{Q(b-a)}{4\pi \in_0 ab}$$

or $$Q = \left[\frac{4\pi \in_0 ab}{(b-a)} \right] V$$

Comparing this equation with $Q = CV$, we get

$$C = 4\pi \in_0 \left(\frac{ab}{b-a} \right)$$, which is choice (b).

121. When C_1 is fully charge, the voltage V_1 across it = voltage of the battery = 12 V. The charge on C_1 is

$$Q = C_1 V_1 = (2 \times 10^{-6}) \times 12 = 24 \times 10^{-6} \text{ C} = 24 \ \mu\text{C}$$

When S_1 is opened and S_2 is closed, charge will flow from C_1 to C_2 until they have the same potential V which is given by

$$V = \frac{Q}{C_1 + C_2} = \frac{24 \mu C}{(2+4) \mu F} = 4 \text{ V}$$

Since C_1 and C_2 are in parallel, they have the same common potential. Now

$$U_i = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times (2 \times 10^{-6}) \times (12)^2 = 1.44 \times 10^{-4} \text{ J}$$

$$U_f = \frac{1}{2} C_1 V^2 = \frac{1}{2} \times (2 \times 10^{-6}) \times (4)^2 = 0.16 \times 10^{-4} \text{ J}$$

$\therefore$ Loss of energy stored in $C_1 = 1.44 \times 10^{-4} - 0.16 \times 10^{-4}$ $= 1.28 \times 10^{-4}$ J. The percentage loss of energy is $\frac{1.28 \times 10^{-4}}{1.44 \times 10^{-4}} \times 100 = 88.9\%$, which is choice (a).

122. The moment the two spheres are connected by a conducting wire, they quickly form an equipotential surface. Charge will flow from sphere 1 (which was at higher potential $= \frac{Q}{4\pi \in_0 a}$) to sphere 2 (which was at lower potential $= \frac{Q}{4\pi \in_0 (4a)}$) until their potentials are equalized.

It Q_1 is charge on sphere 1 and Q_2 on sphere 2, then

$$\frac{Q_1}{4\pi \in_0 (a)} = \frac{Q_2}{4\pi \in_0 (4a)}$$

$$\Rightarrow \quad Q_2 = 4Q_1 \qquad (1)$$

But $Q_1 + Q_2 = Q + Q = 2Q$ (2)

From (1) and (2) we get $5Q_1 = 2Q$ or $Q_1 = \frac{2Q}{5}$ and $Q_2 = 2Q - Q_1 = 2Q - \frac{2Q}{5} = \frac{8Q}{5}$. So the correct choice is (c).

123. Refer to the solution of Q. 106 and to Fig. 11.105 where $\overrightarrow{O'P} = \vec{a}$, $\overrightarrow{OP} = \vec{b}$ and $\overrightarrow{OO'} = \vec{c}$.
Electric field at P due to the complete sphere is

$$\vec{E_1} = \frac{\rho \vec{b}}{3 \in_0}$$

Electric field at P due to cavity is

$$\vec{E}_2 = \frac{\rho \vec{a}}{3 \in_0}$$

Therefore, the net electric field at P is

$$\vec{E} = \vec{E_1} - \vec{E_2}.$$

$$= \frac{\rho \vec{b}}{3 \in_0} - \frac{\rho \vec{a}}{3 \in_0} = \frac{\rho}{3 \in_0} (\vec{b} - \vec{a})$$

$$= \frac{\rho \vec{c}}{3 \in_0} \qquad (\because \ \vec{c} + \vec{a} = \vec{b})$$

So the correct choice is (c). Note that the electric field at any point in the emptied space is finite and constant.

124. When a surface charge density is given to the outer cylinder, the same potential appears on both A and B. Hence statement 1 is false. When a charge density is given to the inner cylinder, an electric field is produced between A and B. Hence a potential difference appears between A and B. So statement 2 is true. When the same charge density $\sigma = Q/4\pi R^2$ is given to A and B, the charge on A is greater than that on B. Hence, a potential difference appears between them. So statement 3 is false. Thus the correct choice is (b).

125. When a negative charge $-Q$ is placed out side a neutral conducting sphere, it will induce a positive charge $+Q$ on the side closer to it and an equal negative charge $-Q$ on the opposite side as shown in Fig. 11.110. Hence the net charge on the sphere will be zero. So the correct choice is (d).

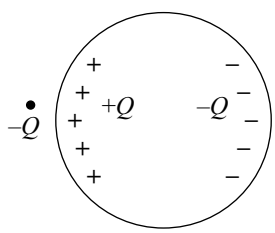


Fig. 11.110

126. Initial energy stored in C_1 is

$$U_i = \frac{1}{2} C_1 V^2$$

when the switch S is turned to position 2, charge will flow from one capacitor to the other until their potentials are equalised. The common potential is

$$V_1 = \frac{q}{C_1 + C_2} = \frac{C_1 V}{C_1 + C_2}$$

Since C_1 and C_2 are in parallel, the equivalent capacitance = $C_1 + C_2$. Therefore, the energy stored now is

$$U_f = \frac{1}{2}(C_1 + C_2) V_1^2 = \frac{1}{2}(C_1 + C_2) \times \left(\frac{C_1 V}{C_1 + C_2} \right)^2$$

$$= \frac{C_1^2 V^2}{2(C_1 + C_2)}$$

$\therefore$ Loss of energy is

$$\Delta U = U_i - U_f$$

$$= \frac{1}{2} C_1 V^2 - \frac{C_1^2 V^2}{2(C_1 + C_2)} = \frac{C_1 C_2 V^2}{2(C_1 + C_2)}$$

Fraction of energy stored in C_1 dissipated is

$$\frac{\Delta U}{U_i} = \frac{C_2}{C_1 + C_2}, \quad \text{which is choice (b)}$$

127. Volume charge densities of spheres 1, 2 and 3 are

$$\rho_1 = \frac{Q}{\frac{4\pi}{3} R^3} = \frac{3Q}{4\pi R^3}$$

$$\rho_2 = \frac{Q}{\frac{4\pi}{3}(2R)^3} = \frac{3Q}{4\pi \times 8R^3} = \frac{\rho_1}{8}$$

$$\rho_3 = \frac{Q}{\frac{4\pi}{3}(3R)^3} = \frac{\rho_1}{27}$$

Now refer to the solution of Q. 127. For points outside the sphere, the entire charge Q can be assumed to be concentrated at its centre. Hence

$$E_1 = \frac{Q}{4\pi \in_0 (2R)^2} = \frac{QR}{4\pi R^3 \in_0} \times \frac{1}{4} = \frac{\rho_1 R}{12 \in_0}$$

For points inside the sphere, $E = \frac{\rho r}{3 \in_0}$, where r is the distance of the point P from the centre of the sphere. Hence

$$E_2 = \frac{\rho_2 \times (2R)}{3 \in_0} = \frac{\rho_1}{8} \times \frac{2R}{3 \in_0} = \frac{\rho_1 R}{12 \in_0}$$

$$E_3 = \frac{\rho_3 \times (2R)}{3 \in_0} = \frac{\rho_1}{27} \times \frac{2R}{3 \in_0} = \frac{2}{81} \frac{\rho_1 R}{\in_0}$$

It is clear that $E_1 = E_2 < E_3$. So the correct choice is (c).

128. Let q μC be the charge on the left plate of the 4 μF capacitor. Then the charge on the left plate of the 3 μF capacitor will be $(7 - q)$ μC. Since the potential difference across the 3 μF capacitor = potential difference across the 4 μF capacitor,

$$\frac{7 - q}{3} = \frac{q}{4}$$

$$\Rightarrow \quad q = 4.0 \ \mu C$$

So the correct choice is (d).

129. The x, y and z components of the electric field are

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}(x^2 y + yz) = -2xy$$

$$E_y = -\frac{dV}{dy} = -\frac{d}{dy}(x^2 y + yz) = -x^2 - z = -(x^2 + z)$$

$$E_z = -\frac{dV}{dz} = -\frac{d}{dz}(x^2 y + yz) = -y$$

The magnitude of the electric field is

$$E = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{4x^2 y^2 + (x^2 + z)^2 + y^2}$$

At space point P (1, 2, 3),

$$E = \sqrt{4 \times 1^2 \times 2^2 + (1^2 + 3)^2 + 2^2}$$

$$= \sqrt{16 + 16 + 4} = 6 \text{ units}$$

So the correct choice is (a).

130. $E_x = -2x$, $E_y = -2y$ and $E_z = -2z$

$$\therefore \quad |E| = \sqrt{E_x^2 + E_y^2 + E_y^2}$$

$$= \sqrt{4x^2 + 4y^2 + 4y^2}$$

$$= 2\sqrt{(x^2 + y^2 + z^2)} = 2\sqrt{V}$$

So the correct choice is (a).

131. V = potential on the surface of the sphere of radius r due to + Q – potential due to + Q at a distance R from the centre of the sphere (which is potential on the outer surface of the shell)

$$= \frac{Q}{4\pi \in_0 r} - \frac{Q}{4\pi \in_0 R}$$

$$\Rightarrow \quad V = \frac{Q}{4\pi \in_0} \left(\frac{1}{r} - \frac{1}{R} \right) \qquad (1)$$

If a charge $-nQ$ is given to the shell, the potential V_1 on the surface of sphere = potential due to + Q on the surface + potential due to $-nQ$ on the surface of the sphere. Therefore,

$$V_1 = \frac{Q}{4\pi \in_0 r} + \frac{-nQ}{4\pi \in_0 R} = \frac{Q}{4\pi \in_0}\left(\frac{1}{r} - \frac{n}{R}\right)$$

Potential on the outer surface of the shell = potential due to + Q at a distance R from + potential due to – nQ on its surface. Therefore,

$$V_2 = \frac{Q}{4\pi \in_0 R} - \frac{nQ}{4\pi \in_0 R} = \frac{Q}{4\pi \in_0 R}(1-n)$$

Therefore, the new potential difference is

$$V' = V_1 - V_2$$

$$= \frac{Q}{4\pi \in_0}\left(\frac{1}{r} - \frac{n}{R}\right) - \frac{Q}{4\pi \in_0 R}(1-n)$$

$$= \frac{Q}{4\pi \in_0}\left(\frac{1}{r} - \frac{1}{R}\right) \qquad (2)$$

From (1) and (2) we find that $V' = V$. So the correct choice is (a).

132. Charge Q on shell 1 induces a charge – Q on the inner surface of shell 2 and a charge + Q on its outer surface, so that the total charge on the outer surface of shell 2 is $Q + Q = 2Q$ [see Fig. 11.111]

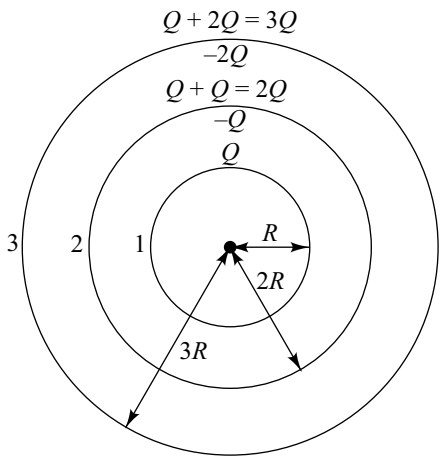


Fig. 11.111

Charge $2Q$ on the outer surface of 2 induced a charge – $2Q$ on the inner surface of 3 and a charge + $2Q$ of its outer surface so that the total charge on the outer surface of 3 is $Q + 2Q = 3Q$. Hence surface charge densities on shells 1, 2 and 3 respectively are

$$\sigma_1 = \frac{Q}{4\pi R^2}$$

$$\sigma_2 = \frac{2Q}{4\pi(2R)^2} = \frac{\sigma_1}{2}$$

and $$\sigma_3 = \frac{3Q}{4\pi(3R)^2} = \frac{\sigma_1}{3}$$

$$\therefore\ \sigma_1 : \sigma_2 : \sigma_3 = 1 : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 2$$

So the correct choice is (c).

133. The circuit can be redrawn as shown in Fig. 11.112.

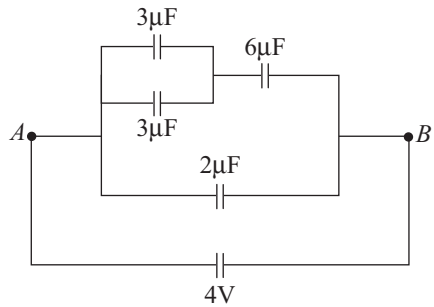


Fig. 11.112

The equivalent capacitance between A and B is

$$C = \frac{6\times 6}{12} + 2 = 5\ \mu F$$

$$\therefore\ \text{Energy stored} = \frac{1}{2}CV^2$$

$$= \frac{1}{2}\times 5\mu F \times (4\,V)^2 = 40\ \mu J$$

134. Since the cells are in opposition, the circuit can be redrawn as shown in Fig. 11.113.

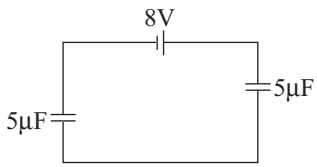


Fig. 11.113

The potential difference across each capacitor = $\frac{8}{2}$ = 4V.

$\therefore$ Charge on 5 μF capacitor = (5 μF) × (4V) = 20 μC

So the correct choice is (b).

SECTION 2

Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage.

Passage I

An infinite number of charges, equal to q, are placed along the x-axis at $x = 1$, $x = 2$, $x = 4$, $x = 8$ .... and so on.

1. The electric potential at the point $x = 0$ due to this set of charges is

   (a) $\frac{q}{\pi\varepsilon_0}$ (b) $\frac{q}{2\pi\varepsilon_0}$

   (c) $\frac{q}{3\pi\varepsilon_0}$ (d) $\frac{q}{4\pi\varepsilon_0}$

2. The electric field at point $x = 0$ is

   (a) $\frac{q}{3\pi\varepsilon_0}$ (b) $\frac{q}{4\pi\varepsilon_0}$

   (c) $\frac{q}{5\pi\varepsilon_0}$ (d) $\frac{q}{6\pi\varepsilon_0}$

3. If the consecutive charges have opposite sign, the electric potential at $x = 0$ would be

   (a) $\frac{q}{3\pi\varepsilon_0}$ (b) $\frac{q}{4\pi\varepsilon_0}$

   (c) $\frac{q}{5\pi\varepsilon_0}$ (d) $\frac{q}{6\pi\varepsilon_0}$

4. If the consecutive charges have opposite sign, the electric field at $x = 0$ would be

   (a) $\frac{q}{3\pi\varepsilon_0}$ (b) $\frac{q}{4\pi\varepsilon_0}$

   (c) $\frac{q}{5\pi\varepsilon_0}$ (d) $\frac{q}{6\pi\varepsilon_0}$

1. The potential at $x = 0$ due to a set of infinite number of charges placed on the x-axis as shown in Fig. 11.101, is

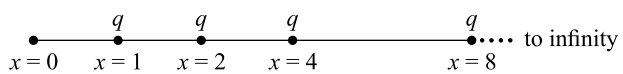


Fig. 11.101

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1}+\frac{q}{2}+\frac{q}{4}+\frac{q}{8}+\cdots\text{to}\infty\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\text{to}\infty\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1-\left(\frac{1}{2}\right)^{\infty}}{1-\frac{1}{2}}\right]t$$

$$= \frac{q}{4\pi\varepsilon_0}\times\frac{(1-0)}{\left(\frac{1}{2}\right)} = \frac{q}{2\pi\varepsilon_0}\text{, which is choice (b).}$$

2. Since the charges are placed along the same straight line, the electric field at $x = 0$ will be directed along the x-axis and its magnitude is given by

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1^2}+\frac{q}{2^2}+\frac{q}{4^2}+\frac{q}{8^2}+\cdots\text{to}\infty\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\cdots\text{to}\infty\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1-\left(\frac{1}{4}\right)^{\infty}}{1-\frac{1}{4}}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\times\frac{(1-0)}{\left(\frac{3}{4}\right)} = \frac{q}{3\pi\varepsilon_0}\text{, which is choice (a).}$$

3. If the consecutive charges have opposite sign, the potential at $x = 0$ is given by

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1}-\frac{q}{2}+\frac{q}{4}-\frac{q}{8}+\frac{q}{16}-\frac{q}{32}\cdots\text{to}\infty\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\left(1+\frac{1}{4}+\frac{1}{16}+\cdots\text{to}\infty\right)\right.$$

$$\left.-\left(\frac{1}{2}+\frac{1}{8}+\frac{1}{32}+\cdots\text{to}\infty\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\left(\frac{1}{1-\frac{1}{4}}\right)-\frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{4}{3}-\frac{1}{2}\times\frac{4}{3}\right] = \frac{q}{6\pi\varepsilon_0}$$

Hence the correct choice is (d).

4. $$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1^2} - \frac{q}{2^2} + \frac{q}{4^2} - \frac{q}{(8)^2} + \frac{q}{(16)^2} - \frac{q}{(32)^2} + \cdots \text{to} \infty\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\left(1 + \frac{1}{16} + \frac{1}{256} + \cdots \text{to} \infty\right) - \left(\frac{1}{4} + \frac{1}{64} + \frac{1}{1024} + \cdots \text{to} \infty\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\left(\frac{1}{1-\frac{1}{16}}\right) - \frac{1}{4}\left(\frac{1}{1-\frac{1}{16}}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{16}{15} - \frac{1}{4} \times \frac{16}{15}\right] = \frac{q}{5\pi\varepsilon_0}$$

Hence the correct choice is (c).

Questions 5 to 7 are based on the following passage.

Passage II

A point particle of mass M is attached to one end of a massless rigid non-conducting rod of length L. Another point particle of the same mass is attached to the other end of the rod. The two particles carry charges $+q$ and $-q$. This arrangement is held in a region of a uniform electric field **E** such that the rod makes a small angle θ (say of about 5°) with the field direction as shown in Fig. 11.114.

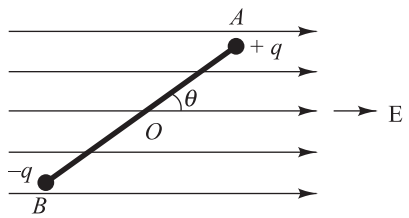


Fig. 11.114

5. The magnitude of the torque acting on the rod is
   (a) $qEL \sin\theta$ (b) $qEL \cos\theta$
   (c) qEL (d) zero
6. When the rod is released, it will rotate with an anular frequency ω equal to
   (a) $\left(\frac{qE}{ML}\right)^{1/2}$ (b) $\left(\frac{2qE}{ML}\right)^{1/2}$
   (c) $\left(\frac{qE}{2ML}\right)^{1/2}$ (d) $\frac{1}{2}\left(\frac{qE}{ML}\right)^{1/2}$
7. The minimum time taken by the rod to align itself parallel to the electric field after it is set free is given by
   (a) $\frac{\pi}{2}\left(\frac{ML}{2qE}\right)^{1/2}$ (b) $2\pi\left(\frac{ML}{qE}\right)^{1/2}$
   (c) $2\pi\left(\frac{2ML}{qE}\right)^{1/2}$ (d) $2\pi\left(\frac{ML}{2qE}\right)^{1/2}$

Solutions

5. A non-conducting rigid rod having equal and opposite charges at the ends is an electric dipole. When it is placed in a uniform electric field, it experiences a torque which tends to align it with the field lines. Referring to Fig. 11.115, the electric forces $F = qE$ each acting at A and B constitute a couple whose torque is given by

τ = force × perpendicular distance

$= F \times AC = F \times AB \sin\theta = qEL \sin\theta$

So the correct choice is (a).

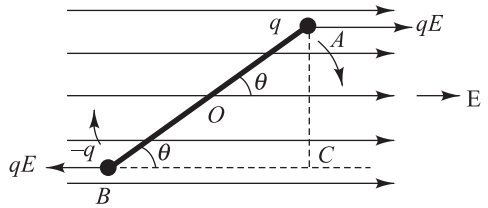


Fig. 11.115

6. Since θ is small, $\sin\theta \simeq \theta$, where θ is expressed in radian. Thus $\tau = qEL\theta$

$\therefore$ Restoring torque $\tau = -qEL\theta$ (1)

If α is the angular acceleration of the rotatory motion, $\tau = I\alpha$

where I is the moment of inertia of the two masses at A and B about an axis passing through the centre O and perpendicular to the rod. Since the rod is massless,

$$I = M \times (AO)^2 + M \times (BO)^2$$

$$= M \times \left(\frac{L}{2}\right)^2 + M \times \left(\frac{L}{2}\right)^2 = \frac{ML^2}{2}$$

Thus $$\tau = \frac{ML^2\alpha}{2} \quad (2)$$

Using Eq. (2) in Eq. (1), we get

$$\alpha = -\left(\frac{2qE}{ML}\right)\theta = -\omega^2\theta$$

where $$\omega = \left(\frac{2qE}{ML}\right)^{1/2}$$, which is choice (b).

7. The time period of oscillation is

$$T = \frac{2\pi}{\omega} = 2\pi \left(\frac{ML}{2qE}\right)^{1/2}$$

Rotating in the clockwise sense, the minimum time taken by the rod to align itself parallel to the electric field is the time it takes to complete one-fourth of angular oscillation, i.e.

$$t_{\min} = \frac{T}{4} = \frac{\pi}{2}\left(\frac{ML}{2qE}\right)^{1/2}$$

So the correct choice is (a).

Questions 8 to 11 are based on the following passage.

Passage III

Two identical particles A and B of mass m carry a charge Q each. Initially particle A is at rest on a smooth horizontal plane and the partical B is projected with a speed v along the horizontal plane from a large distance directly towards the first particle. The distance of closest approach is x.

8. At the closest approach, speed v_1 of particle A is
   (a) $v_1 = v$ (b) $v_1 = \sqrt{2}v$
   (c) $v_1 = v/2$ (d) $v_2 = v/\sqrt{2}$
9. At the closest approach, speed v_2 of particle B is
   (a) $v_2 = v/2$ (b) $v_2 = v/\sqrt{2}$
   (c) $v_2 = v$ (d) $v_2 = 2v$
10. At the distance of closest approach, the total energy of the system is
   (a) $\frac{mv^2}{2}$ (b) $\frac{mv^2}{2} + \frac{Q^2}{4\pi\varepsilon_0 x}$
   (c) $mv^2 + \frac{Q^2}{4\pi\varepsilon_0 x}$ (d) $\frac{mv^2}{4} + \frac{Q^2}{4\pi\varepsilon_0 x}$
11. The distance of closest approach is
   (a) $x = \frac{Q^2}{\pi\varepsilon_0 mv^2}$ (b) $x = \frac{Q^2}{2\pi\varepsilon_0 mv^2}$
   (c) $x = \frac{Q^2}{4\pi\varepsilon_0 mv^2}$ (d) $x = \frac{2Q^2}{\pi\varepsilon_0 mv^2}$



Solutions

8. Initially particle A is at rest and particle B moves towards it with a velocity v from a large distance. Therefore, the initial momentum of the system is

$$p_i = mv + 0 = mv$$

Since the initial distance is large, the particle B exerts negligible repulsive force of the first particle. But, as the distance decreases, the first particle begins to move in the same direction as the second particle under the action of the force of repulsion. The distance between them, therefore, keeps decreasing until it attains a certain minimum value. Let v_1 and v_2 be the velocities of particles 1 and 2 and let t be the time taken by them to acquire these velocities. Then the distances travelled by them will be $x_1 = v_1 t$ and $x_2 = v_2 t$. The separation between them at this time t is

$$x = x_1 - x_2 = (v_1 - v_2)t$$

This separation will be minimum if $dx/dt = 0$. Now

$$\frac{dx}{dt} = v_1 - v_2$$

$\therefore$ For closest approach, $v_1 - v_2 = 0$ or $v_1 = v_2$.

Therefore, the final momentum of the system at the closest approach is

$$p_f = mv_1 + mv_2 = m(v_1 + v_2)$$

From the law of conservation of momentum,

$$p_i = p_f \text{ or } mv = m(v_1 + v_2) \text{ or } v = v_1 + v_2$$

Since the particles are identical, it follows that

$$v_1 = v_2 = \frac{v}{2}.$$

So the correct choice is (c).

9. The correct choice is (a).

10. If x is the distance of closest approach, the final energy of the system is

$$E_f = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{x}$$

or $$= \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{x}$$

or $$E_f = \frac{mv^2}{4} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{x}$$

which is choice (c).

11. The initial energy of the system is

E_i = KE of first particle + KE of second particle + PE due to charge on each particle.

$$= 0 + \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0} \times \frac{Q^2}{\infty}$$

or $$E_i = \frac{1}{2}mv^2$$

From the law of conservation of energy, $E_i = E_f$. Therefore

$$\frac{1}{2}mv^2 = \frac{1}{4}mv^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{x}$$

which gives

$$x = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4Q^2}{mv^2} = \frac{Q^2}{\pi\varepsilon_0 mv^2}$$

Questions 12 to 15 are based on the following passage.

Passage IV

A parallel plate capacitor consists of two metal plates, each of area A, separated by a distance d. A dielectric slab of the same surface area A and thickness t and dielectric constant K is introduced with its faces parallel to the plates as shown in Fig. 11.116.

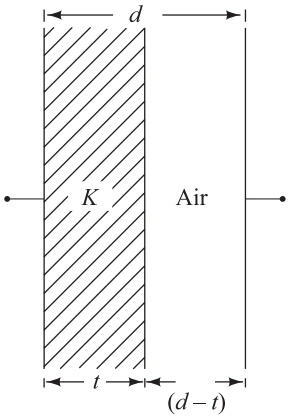


Fig. 11.116

12. The capacitance of the system is

(a) $C = \dfrac{\varepsilon_0 AK}{(d-t)}$

(b) $C = \dfrac{\varepsilon_0 A}{\left(d - \dfrac{t}{K}\right)}$

(c) $C = \dfrac{\varepsilon_0 A}{\left[d + t\left(\dfrac{1}{K} - 1\right)\right]}$

(d) $C = \dfrac{\varepsilon_0 A}{\left[d + t\left(1 - \dfrac{1}{K}\right)\right]}$

13. If $K = 3$, for what value of t/d will the capacitance of the system be twice that of the air capacitor alone ?

(a) $\dfrac{1}{2}$ (b) $\dfrac{2}{3}$

(c) $\dfrac{3}{4}$ (d) $\dfrac{4}{5}$

14. If $K = 3$ and $t/d = 1/2$, the ratio of the energy stored in the system shown in the figure and the air capacitor alone if the charge is the same in both capacitors is

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{3}$

(c) $\dfrac{2}{3}$ (d) $\dfrac{1}{4}$

15. In Q.14 above, the loss of energy is due to

(a) heating of the connecting wires which connect the capacitor with the battery.

(b) the flow of charge from the capacitor to the battery.

(c) leakage of the capacitor.

(d) polarization of the dielectric.

Solutions

12. The system is equivalent to a series combination of two capacitor one of thickness t filled with a dielectric of dielectric constant K and the other of thickness $(d - t)$ with air as dielectric. Their capacitances respectively are

$$C_1 = \frac{\varepsilon_0 KA}{t} \quad (1)$$

and $$C_2 = \frac{\varepsilon_0 A}{(d-t)} \quad (2)$$

The capacitance C of the system is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

or $$C = \frac{C_1 C_2}{C_1 + C_2} \quad (3)$$

Using Eqs. (1) and (2) in Eq. (3) and simplifying we get

$$C = \frac{\varepsilon_0 A}{\left[d + t\left(\dfrac{1}{K} - 1\right)\right]} \quad (4)$$

So the correct chioce is (c).

13. The capacitance of the air capacitor alone is

$$C_a = \frac{\varepsilon_0 A}{d} \quad (5)$$

Dividing Eq. (4) by Eq. (5), we get

$$\frac{C}{C_a} = \frac{1}{\left[1 + \dfrac{t}{d}\left(\dfrac{1}{K} - 1\right)\right]}$$

Given $K = 3$ and $C/C_a = 2$. Thus

$$2 = \frac{1}{\left[1 + \dfrac{t}{d}\left(\dfrac{1}{3} - 1\right)\right]}$$

which gives $\dfrac{t}{d} = \dfrac{3}{4}$, which is choice (c).

14. Putting $K = 3$ and $t/d = \dfrac{1}{2}$ in Eq. (4), we get $C = \dfrac{3\varepsilon_0 A}{d}$.

Now $C_a = \dfrac{\varepsilon_0 A}{d}$. Hence $\dfrac{C_a}{C} = 3$. If Q is the same, $U_a = \dfrac{Q^2}{2C_a}$ and $U = \dfrac{Q^2}{2C}$.

Thus $\dfrac{U}{U_a} = \dfrac{1}{3}$, which is choice (b).

15 The correct choice is (d).

Questions 16 to 18 are based on the following passage

Passage V

A parallel plate capacitor, of plate area A, contains two dielectric slabs of equal dimensions and having dielectric constants K_1 and K_2 as shown in Figs. 11. 117 (a) and (b).

16. The capacitance of the capacitor shown in Fig. 11.117 (a) is

(a) $\frac{\varepsilon_0 A}{2d}(K_1 + K_2)$

(b) $\frac{2\varepsilon_0 A}{d}(K_1 + K_2)$

(c) $\frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$

(d) $\frac{\varepsilon_0 A}{2d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$

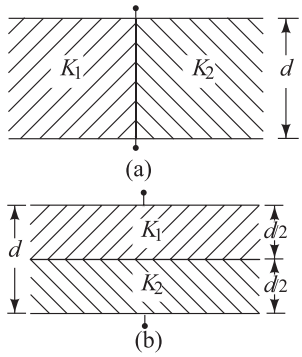


Fig. 11.117

17. The capacitance of the capacitor shown in Fig. 11.117 (b) is

(a) $\frac{2\varepsilon_0 A}{d}(K_1 + K_2)$ (b) $\frac{\varepsilon_0 A}{2d}(K_1 + K_2)$

(c) $\frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$ (d) $\frac{\varepsilon_0 A}{2d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$

18. If $K_1 = 2$ and $K_2 = 3$, the ratio of the capacitances of capacitors in Fig. 11.117 (a) and (b) is

(a) $\frac{9}{4}$ (b) $\frac{4}{9}$

(c) $\frac{24}{25}$ (d) $\frac{25}{24}$

Solutions

16. The system shown in Fig. 11.117 (a) is equivalent to a parallel combination of two capacitors—one of plate area $A/2$ filled with a dielectric constant K_1 and the other of plate area $A/2$ filled with a dielectric of dielectric constant K_2, each having the same thickness d. Their respective capacitances are

$$C_1 = \frac{K_1\varepsilon_0 (A/2)}{d} = \frac{K_1\varepsilon_0 A}{2d}$$

and $C_2 = \frac{K_2\varepsilon_0 (A/2)}{d} = \frac{K_2\varepsilon_0 A}{2d}$

The capacitance of the parallel combination of C_1 and C_2 is

$$C_a = C_1 + C_2 = \frac{\varepsilon_0 A}{2d}(K_1 + K_2) \quad (1)$$

So the correct choice is (a).

17. The system shown in Fig. 11.117 (b) is equivalent to a series combination of the capacitors—one filled with a dielectric of dielectric constant K_1 and of thickness $d/2$ and the other filled with a dielectric of dielectric constant K_2 and of thickness $d/2$, each having the same plate area A. Their respective capacitance are

$$C_1' = \frac{K_1\varepsilon_0 A}{d/2} = \frac{2K_1\varepsilon_0 A}{d}$$

and $C_2' = \frac{K_2\varepsilon_0 A}{d/2} = \frac{2K_2\varepsilon_0 A}{d}$

The capacitance of the series combination of C_1' and C_2' is

$$\frac{1}{C_b} = \frac{1}{C_1'} + \frac{1}{C_2'} = \frac{d}{2\varepsilon_0 A}\left(\frac{1}{K_1 + K_2}\right)$$

$$= \frac{d}{2\varepsilon_0 A}\left(\frac{K_1 + K_2}{K_1 K_2}\right)$$

or $C_b = \frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$ (2)

Thus the correct choice is (c).

18. Using Eqs. (1) and Eqs. (2) we get

$$\frac{C_a}{C_b} = \frac{(K_1 + K_2)^2}{4K_1 K_2}$$

If $K_1 = 2$ and $K_2 = 3$, we have

$\frac{C_a}{C_b} = \frac{(2+3)^2}{4\times 2\times 3} = \frac{25}{24}$, which is choice (d).

Questions 19 to 21 are based on the following passage.

Passage VI

Fig. 11.118 shows a network of seven capacitors. The charge on the 5 μF capacitor is 10 μC.

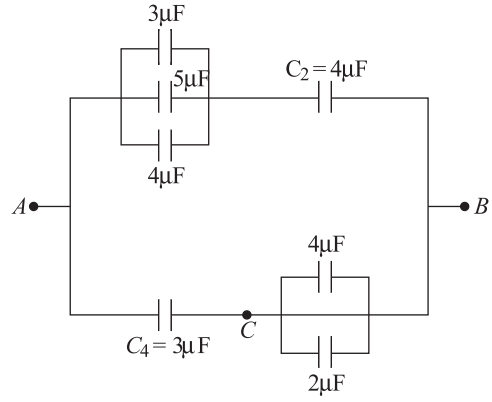


Fig. 11.118

19. The equivalent capacitance between points A and B is
    (a) 2.5 μF (b) 5 μF
    (c) 7.5 μF (d) 10 μF
20. The potential difference between points A and B is
    (a) 2 V (b) 4 V
    (c) 6 V (d) 8 V
21. The potential difference between points A and C is
    (a) $\frac{16}{3}$ V (b) $\frac{12}{5}$ V
    (c) $\frac{8}{3}$ V (d) zero



Solutions

19. The equivalent capacitance C_1 of the parallel combination of 3 μF, 5 μF and 4 μF capacitors is

$$C_1 = 3 + 5 + 4 = 12\ \mu F$$

The equivalent capacitance C_3 of the parallel combination of 4 μF and 2 μF capacitors is

$$C_3 = 4 + 2 = 6\ \mu F$$

Now C_1 and C_2 are in series. Their equivalent capacitance C' is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{12} + \frac{1}{4} = \frac{1}{3}$$

or $C' = 3$ μF. The equivalent capacitance C'' of the series combination of C_3 and C_4 is given by

$$\frac{1}{C''} = \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

or $C'' = 2$ μF. The equivalent capacitance between A and B = capacitance of a parallel combination of C' and C'' = $C' + C'' = 3 + 2 = 5$ μF.

Thus the correct choice is (b).

20. Given, charge on 5 μF capacitor = 10 μC = 10×10^{-6} C.

$\therefore$ Potential difference across 5 μF capacitor

$$= \frac{10 \times 10^{-6}}{5 \times 10^{-6}} = 2\ V$$

As the 3 μF, 4 μF and 5 μF capacitors are joined in parallel, the potential difference across each is the same = 2 V. Therefore,

Charge on 3 μF capacitor = $3 \times 10^{-6} \times 2 = 6 \times 10^{-6}$ = 6 μC

Charge on 4 μF capacitor = $4 \times 10^{-6} \times 2 = 8 \times 10^{-6}$ C = 8 μC

$\therefore$ Total charge flowing through C_1 and C_2 = 10 μC + 6 μC + 8 μC = 24 μC.

$\therefore$ Potential difference across $C_2 = \frac{24\mu C}{4\mu F} = 6$ V

$\therefore$ Total potential differenc across AB = 2 V + 6 V = 8 V

So the correct choice is (d).

21. The equivalent capacitance C_3 and C_4 is $C'' = 2$ μF. Therefore, charge flowing throught C_3 and C_4 = 8 V × 2×10^{-6} F = 16×10^{-6} C = 16 μC. Hence the potential between A and C is = $\frac{16\mu C}{3\mu F}$ = 16/3 V, which is choice (a).

Questions 22 to 25 are based on the following passage.

Passage VII

Two capacitors A and B with capacitances 3 μF and 2 μF are charged to a potential difference of 100 V and 180 V respectively. They are connected to an uncharged 2 μF capacitor C through a switch S as shown in Fig. 11.119.

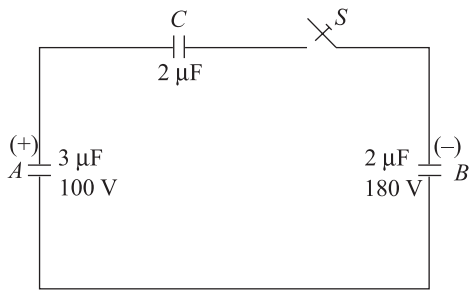


Fig. 11.119

22. When the switch is pressed, the charge flowing through the circuit is
    (a) 180 μC (b) 190 μC
    (c) 200 μC (d) 210 μC
23. When the switch is pressed, the final charge on capacitor A will be
    (a) 60 μC (b) 90 μC
    (c) 120 μC (d) 150 μC
24. When the switch is pressed, the final charge on capacitor B will be
    (a) 150 μC (b) 160 μC
    (c) 180 μC (d) 200 μC
25. When the switch is pressed, the final charge on capacitor C will be
    (a) 90 μC (b) 150 μC
    (c) 210 μC (d) 300 μC

Solutions

26. Refer to Fig. 11.120. Let Q be the charge flowing through the circuit. When the switch is pressed, the voltages developed on capacitors A, B and C are $V_A = \dfrac{Q}{3\times10^{-6}}$ volt, $V_B = \dfrac{Q}{2\times10^{-6}}$ volt and $V_C = \dfrac{Q}{2\times10^{-6}}$ volt.

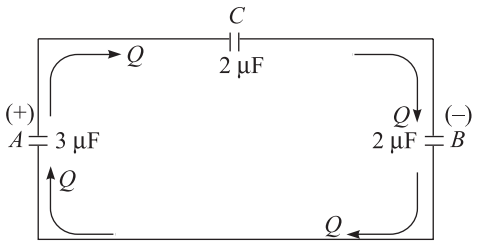


Fig. 11.120

Applying Kirchhoff's law to the loop, we have

$\Delta V_A + \Delta V_B - \Delta V_C = 0$ or

$$\left(100 - \frac{Q}{3\times10^{-6}}\right) + \left(180 - \frac{Q}{2\times10^{-6}}\right) - \frac{Q}{2\times10^{-6}} = 0$$

which gives $Q = 210 \times 10^{-6}$ C = 210 μC. So the correct choice is (d).

23. The initial charge on capacitor A is $(Q_i)_A = (V_i\, C_i)_A$ = 100 V × 3 μF = 300 μC

$\therefore$ Final charge on A is

$(Q_i)_A = (Q_i)_A - Q = 300\,\mu C - 210\,\mu C = 90\,\mu C$, which is chioce (b).

24. Similarly, the final charge on B is

$(Q_f)_B = (Q_i)_B - Q = (180\text{ V}) \times (2\,\mu\text{CF}) - 210\,\mu\text{C}$

$= 360\,\mu\text{C} - 210\,\mu\text{C} = 150\,\mu\text{C}$

So the correct choice is (a).

25. From conservation of charge, we have

$(Q_f)_A + (Q_f)_C = 300\,\mu\text{C}$

$\therefore \quad (Q_f)_C = 300\,\mu\text{C} - (Q_f)_A$

$= 300\,\mu\text{C} - 90\,\mu\text{C} = 210\,\mu\text{C}$,

which is chioce (c).

Questions 26 to 29 are based on the following passage.

Passage VIII

In the circuit shown in Fig. 11.121, emf E_1 = 14 V (internal resistance r_1 = 1 Ω), R_1 = 6 Ω, R_2 = 3.5 Ω, emf E_2 = 12 V (internal resistance r_2 = 0.5 Ω), C_1 = 4 μF and C_2 = 2 μF.

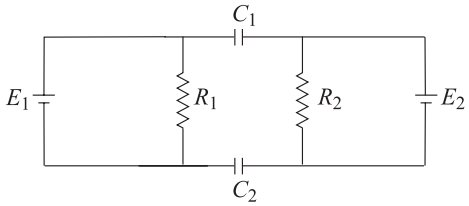


Fig. 121

26. The potential difference acress R_1 is
(a) 2 V (b) 12 V
(c) 14 V (d) 26 V

27. The potential difference across R_2 is
(a) 10.5 V (b) 12.5 V
(c) 15.0 V (d) 26 V

28. The effective capacitance of the circuit is
(a) 2 μF (b) $\frac{2}{3}$ μF
(c) 3 μF (d) $\frac{4}{3}$ μF

29. The charge on capacitor C_2 is
(a) 20 μC (b) 30 μC
(c) 40 μC (d) 60 μC

Solutions

26. Current through R_1 is

$$I_1 = \frac{E_1}{R_1 + r_1} = \frac{14}{6+1} = 2\text{ A}$$

$\therefore$ Potential difference across R_1 is $V_1 = I_1 R_1 = 2 \times 6$ = 12 V, which is choice (b).

27. Current through R_2 is

$$I_2 = \frac{E_2}{R_2 + r_2} = \frac{12}{3.5+0.5} = 3\text{ A}$$

$\therefore$ Potential difference across R_2 is $V_2 = I_2 R_2 = 3 \times 3.5$ = 10.5 V
So the correct choice is (a).

28. Effective capacitance $C = \dfrac{C_1 C_2}{C_1 + C_2} = \dfrac{4\times2}{4+2} = \dfrac{4}{3}\,\mu\text{F}$.
The correct choice is (d).

29. Total voltage $V = V_1 + V_2 = 12 + 10.5 = 22.5$ V

Charge $Q = CV = \dfrac{4}{3} \times 10^{-6} \times 22.5 = 30 \times 10^{-6}\,\text{C} = 30\,\mu\text{C}$
Thus the correct choice is (b).

SECTION 3

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1

(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

1. **Statement-1**

   Figure 11.122 shows the tracks of two charged partices A and B in a uniform electric field between two charged plates. The charge to mass ratio of B is greater than that of A. Neglect the effect of gravity.

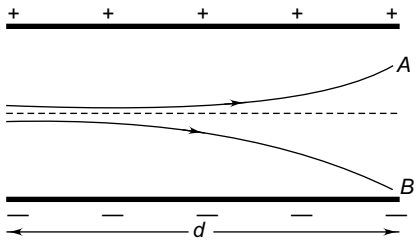


Fig. 11.122

   Statement-2

   The vertical acceleration of particle B is greater than that of particle A.

2. **Statement-1**

   A positive charge + q is located at the centre of a circle as shown in Fig. 11.123. W_1 is the work done in taking a small positive charge + q_0 from A to B and W_2 is the work done in taking the same charge from A to C. Then $W_2 > W_1$

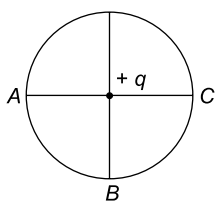


Fig. 11.123

   Statement-2

   Work done = charge × potential difference.

3. **Statement-1**

   A small test charge is initially at rest at a point in an electrostatic field of an electric dipole. When released, it will move along the line of force passing though that point.

   Statement-2

   The tangent at a point on a line of force gives the direction of the electric field at that point.

4. **Statement-1**

   In a non-uniform electric field, a dipole will have translatory as well as rotatory motion.

   Statement-2

   In a non-uniform electric field, a dipole experiences a force as well as a torque.

5. **Statement-1**

   If electric field is zero at a point, the electric potential must also be zero at that point.

   Statement-2

   Electric field is equal to the negative gradient of potential.

6. **Statement-1**

   If electric potential is constant in a certain region of space, the electric field in that region must be zero.

   Statement-2

   Electric field is equal to the negative gradient of potential.

7. **Statement-1**

   If an electron is moved from P to Q, its potential energy increases (see Fig. 11.124)

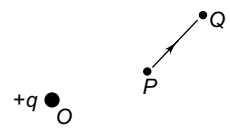


Fig. 11.124

Statement-2

Potential at Q is less than that at P.

8. **Statement-1**

In Q. 7 above, the work done to move an electron from P to Q and then back to P is zero.

Statement-2

Electric field is conservative.

9. **Statement-1**

The work done by the electric field of a nucleus in moving an electron around it in a complete orbit is greater if the orbit is elliptical than if the orbit is circular.

Statement-2

Electric field is conservative.

10. **Statement-1**

Electrons move from a region of higher potential to a region of lower potential.

Statement-2

An electron has less potential energy at a point where the potential is higher and vice versa.

11. **Statement-1**

The electric field is always tangential to the surface of a conductor.

Statement-2

The potential at every point on the surface of a conductor is the same.

12. **Statement-1**

The equipotential surfaces corresponding to a constant electric field along the x-direction are equidistant planes parallel to the y-z plane.

Statement-2

Electric is normal to every point on an equipotential surface

13. **Statement-1**

The electric field in the region around a point charge is uniform.

Statement-2

The equipotential surface of the electric field of a point charge is a sphere with the charge at its centre.

14. **Statement-1**

If a metallic sphere A of radius r carrying a charge Q is brought in contact with an uncharged metallic sphere B of radius $2r$, the charge on sphere A reduces to $Q/3$.

Statement-2

Charge flows from A to B until their potentials are equalised.

15. **Statement-1**

A parallel plate capacitor is charged by a d.c. source supplying a constant voltage V. If the plates are kept connected to the source and the space between the plates is filled with a dielectric, the charge on the plates will increase.

Statement-2

Additional charge will flow from the source to the plates.

16. **Statement-1**

A parallel plate capacitor is charged by a battery of voltage V. The battery is then disonnected. If the space between the plates is filled with a dielectric, the energy stored in the capacitor will decrease.

Statement-2

The capacitance of a capacitor increases due to the introduction of a dielectric between the plates.

17. **Statement-1**

A parallel plate capacitor is charged by a battery. The battery is then disconnected. If the distance between the plates is increased, the energy stored in the capacitor will decrease.

Statement-2

Work has to be done to increase the separation between the plates of a charged capacitor.

18. **Statement-1**

Two adjacent conductors when given the same charge will have a potential difference between them if they are of different shape and size.

Statement-2

The potential to which a conductor is raised depends not only on the amount of charge but also on the shape and size of the conductor.



Solutions

1. The correct choice is (a). Let E be the electric field between the plates; It is directed vertically downwords. If q is the charge of the particle, it will experience a force $F = qE$. Hence its acceleration (in the vertical direction) is

$$a = \frac{F}{m} = \frac{qE}{m}$$

where m is the mass of the particle. If t is time spent by the particle between the plates (i.e in the region of the electric field), the vertical distance travelled (i.e. deflection) of the particle is

$$y = \frac{1}{2}\, at^2 = \frac{qEt^2}{2m}$$

Thus $\frac{q}{m} \propto y$. Since particle B has a higher deflection, it has a higher charge to mass ratio.

2. The correct choice is (d). Points A, B and C are at the same distance from charge + q. Hence electric potential difference between points A, B and C is zero. Hence $W_1 = W_2 = 0$.

3. The correct choice is (d). The test charge will move along the line of force if the line of force is straight (as in the case of a single charge). If the lines of force is curved, the charge will not move along the line of force. The reason is that the line of force does not give the direction of velocity, it gives the direction of the force which is along the tangent to the curve at that point.

4. The correct choice is (a).

5. The correct choice is (d). Since $E = -\, dV/dr$, if $E = 0$, V = constant not necessarily equal to zero.

6. The correct choice is (a).

7. The correct choice is (c). Since charge of an electron is negative, P.E at P and Q is

$$U_P = -\,\frac{eq}{4\pi\varepsilon_0 (OP)}$$

$$U_Q = -\,\frac{eq}{4\pi\varepsilon_0 (OQ)}$$

Since $OQ > OP$, U_Q is less negative than U_P, i.e. $U_Q > U_P$. For the same reason, $V_Q > V_P$.

8. Since charge + q will attract the electron, work is done to move the electron from P to Q is negative because the work is done against the field. To move it from Q back to P an equal positive work is done by the field because electric field is conservative. So the correct choice is (a).

9. The correct choice is (d). The work done by the electric field in moving a charge around a closed path of any shape (circular or elliptical) is zero.

10. The correct choice is (d). Since the electron has a negative charge, it has less energy at a point where the potenial is higher and vice versa. Hence in an electric field an electron moves from a region of lower potential to a region of higher potential.

11. The correct choice is (d). Since E along the surface = 0, $dV/dr = 0$

12. The electric field is always normal to the equipotential surface. Therefore, for a constant electric field in the x-direction, the equipotential surfaces are planes parallel to the y-z plane. Since the field is constant, the equipotential surfaces are equidistant from each other. The correct choice is (a).

13. The correct choice is (d). The electric field in a region around a point charge varies with distance r.

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

14. The correct choice is (a). Charge will flow from A to B until their potentials become equal. If charge q flows from A to B, then

$$\frac{Q-q}{4\pi\varepsilon_0 r} = \frac{q}{4\pi\varepsilon_0 (2r)}$$

which gives $Q - q = \frac{q}{2} \Rightarrow q = \frac{2Q}{3}$. Hence charge left on $A = Q - \frac{2Q}{3} = \frac{Q}{3}$.

15. The correct choice is (a). Since the source supplies a constant voltage V, the potential difference between the plates remains equal to V because the source is not disconnected. The capacitance C increases due to the introduction of the dielectric. Since $Q = CV$, the charge Q on the capacitor plates will increase.
16. The correct choice is (b). The charge Q on the capacitor plates remains unchanged because there is no source to supply extra charge as the battery is disconnected. The capacitance C increases due to the introduction of the dielectric. Now, energy stored $U = Q^2/2C$. Since Q remains unchanged and C increases, U will decrease.
17. The correct choice is (d). The charge Q remains unchanged as the battery is disconnected. The capacitance C decreases if the separation between the plates is increased. Now, energy stored $U = Q^2/2C$. Since Q remains the same and C is decreased, U will increase.
18. The correct choice is (a).

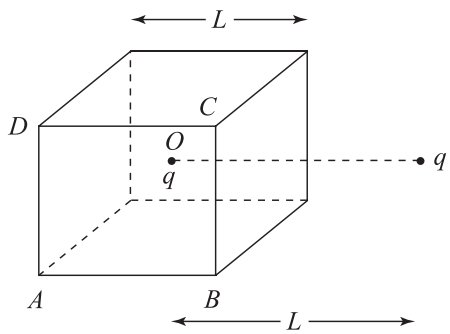
4 SECTION — Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. 2J of work is done in moving a charge of 20 C from one point to another 2 cm apart. The potential difference between the points is

   (a) 0.1 V (b) 8 V

   (d) 2 V (d) 0.5 V **[2002]**

2. A charged particle q is placed at the centre O of a cube of side L. Another equal charge q is placed at a distance L from O. The electric flux through face $ABCD$ is



   (a) $\frac{q}{4\pi\varepsilon_0 L}$ (b) zero

   (c) $\frac{q}{2\pi\varepsilon_0 L}$ (d) $\frac{q}{3\pi\varepsilon_0 L}$ **[2002]**

3. If a parallel combination of n capacitors, each of capacitance C, is connected to a source of voltage V, the energy stored in the system is

   (a) CV (b) $\frac{1}{2}nCV^2$

   (c) CV^2 (d) $\frac{1}{2n}CV^2$ **[2002]**

4. If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium, then the value of q is

   (a) $\frac{Q}{2}$ (b) $-\frac{Q}{2}$

   (b) $\frac{Q}{4}$ (d) $-\frac{Q}{4}$ **[2002]**

5. The capacitance (in F) of a spherical conductor having radius 1 m is

   (a) 1.1×10^{-10} (b) 10^{-6}

   (c) 9×10^{-9} (d) 10^{-3} **[2002]**

6. To identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is

   (a) $\frac{1}{4}C(V_1^2 - V_2^2)$ (b) $\frac{1}{4}C(V_1^2 + V_2^2)$

   (c) $\frac{1}{4}C(V_1 - V_2)^2$ (d) $\frac{1}{4}C(V_1 + V_2)^2$

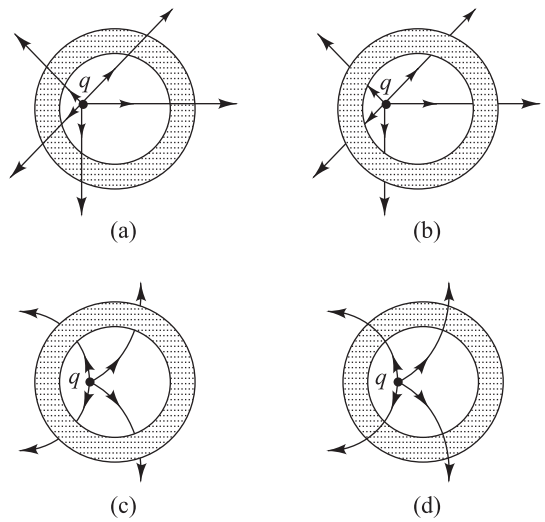
   [2002]

7. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, When it is displaced by a small distance x along the x-axis, is approximately proportional to

   (a) x (b) x^2

   (c) x^3 (d) $1/x$ **[2002]**

8. A metallic shell has a point charge q kept inside its circular cavity. The charge is not exactly at the centre of the cavity. Which of the diagrams correctly represents the electric lines of force?



[2002]

9. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be

   (a) $\dfrac{(\phi_2 - \phi_1)}{\varepsilon_0}$ (b) $\dfrac{(\phi_1 + \phi_2)}{\varepsilon_0}$

   (c) $(\phi_2 - \phi_1)\varepsilon_0$ (d) $(\phi_1 + \phi_2)\varepsilon_0$ **[2003]**

10. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor

    (1) decreases (2) remains unchanged

    (3) becomes infinite (4) increases **[2003]**

11. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P at a distance $R/2$ from the centre of the shell is

    (a) $\dfrac{2Q}{4\pi\varepsilon_0 R}$ (b) $\dfrac{2Q}{4\pi\varepsilon_0 R} - \dfrac{2q}{4\pi\varepsilon_0 R}$

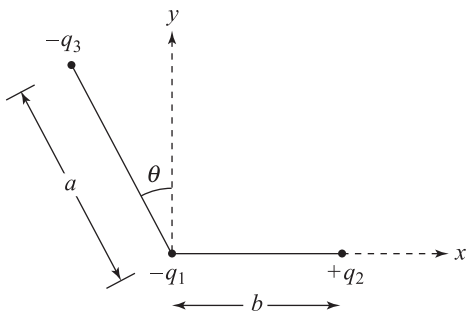
    (c) $\dfrac{2Q}{4\pi\varepsilon_0 R} + \dfrac{q}{4\pi\varepsilon_0 R}$ (d) $\dfrac{2(q+Q)}{4\pi\varepsilon_0 R}$ **[2003]**

12. The work done in placing a charge of 8×10^{-18} C on a capacitor of capacitance 100 μF is

    (1) 16×10^{-32} J (2) 3.1×10^{-26} J

    (3) 4×10^{-10} J (4) 32×10^{-32} J **[2003]**

13. Three charges $-q_1$, $+q_2$ and $-q_3$ are placed as shown in the figure. The x-component of the force on $-q_1$ is proportional to



    (a) $\dfrac{q_2}{b^2} - \dfrac{q_3\cos\theta}{a^2}$ (b) $\dfrac{q_2}{b^2} + \dfrac{q_3\cos\theta}{a^2}$

    (c) $\dfrac{q_2}{b^2} + \dfrac{q_3\sin\theta}{a^2}$ (d) $\dfrac{q_2}{b^2} - \dfrac{q_3\sin\theta}{a^2}$

    [2003]

14. Four charges, each equal to $-Q$, are placed at the corners of a square and a charge q is at its centre. If the system is in equilibrium, the value of q is

    (a) $-\dfrac{Q}{4}(1+2\sqrt{2})$ (b) $\dfrac{Q}{4}(1+2\sqrt{2})$

    (c) $-\dfrac{Q}{2}(1+2\sqrt{2})$ (d) $\dfrac{Q}{2}(1+2\sqrt{2})$ **[2004]**

15. Two spherical conductors A and B having equal radii and carrying equal charges repel each other with a force F when kept a certain distance apart. A third identical spherical conductor but uncharged is brought in contact with B and removed. It is then brought in contact with A and removed. The new force of repulsion between A and B is

    (a) $\dfrac{F}{4}$ (b) $\dfrac{3F}{4}$

    (c) $\dfrac{F}{8}$ (d) $\dfrac{3F}{8}$ **[2004]**

16. A charged particle q is shot towards another charged particle Q which is fixed with an initial speed v. The particle q reaches upto the closest distance r before it is repelled back. What will be the closest distance of approach if the particle q was shot with a velocity $2v$?

(a) r (b) $2r$

(c) $\frac{r}{2}$ (d) $\frac{r}{4}$ **[2004]**

17. A charged oil drop of mass 9.9×10^{-15} kg is suspended in a uniform electric field of 3×10^4 Vm^{-1}. If $g = 10$ ms^{-2}, the charge on the oil drop will be

(a) 3.3×10^{-18} C (b) 3.2×10^{-18} C

(c) 1.6×10^{-18} C (d) 4.8×10^{-18} C

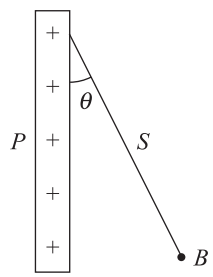
[2004]

18. A fully charged capacitor of capacitance C is discharged through a coil of resistance wire embedded in a thermally insulated block of specific heat capacity s and mass m. If the temperature of the block is raised by ΔT, the potential difference across the capacitor is

(a) $\sqrt{\frac{2mC\Delta T}{s}}$ (b) $\frac{mC\Delta T}{s}$

(c) $\frac{ms\Delta T}{C}$ (d) $\sqrt{\frac{2ms\Delta T}{C}}$ **[2005]**

19. A charged ball B hangs from a silk thread S, which makes an angle θ with a large conducting sheet P as shown in the figure. The surface charge density σ of the sheet is proportional to



(a) $\cos\theta$ (b) $\cot\theta$

(c) $\sin\theta$ (d) $\tan\theta$ **[2005]**

20. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The location of a point on the x-axis at which the net electric field due to the two point charges is

(a) $2L$ (b) $\frac{L}{4}$

(c) $8L$ (d) $4L$ **[2005]**

21. Two thin wire rings, each of radius R, are placed at a distance d apart with their axes coinciding. The charges on the rings are $+q$ and $-q$. The potential difference between the centres of the two rings is

(a) $\frac{qR}{4\pi\varepsilon_0 d^2}$

(b) $\frac{q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2 + d^2}}\right]$

(c) zero

(d) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2 + d^2}}\right]$ **[2005]**

22. An electric dipole is placed at an angle of 30° to a non-uniform electric field. The dipole will experience

(a) a torque as well as a translational force

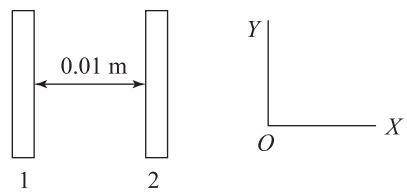
(b) a torque only

(c) a translational force only in the direction of the field

(d) a translational force only in a direction normal to the direction of the field **[2006]**

23. Two insulating plates are both uniformly charged in such a way that the potential difference them is $V_2 - V_1 = 20$ V. (i.e. plate 2 is at a higher potential). The plates are separated by $d = 0.1$ m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate is 1. What is its speed when it hits plate 2?

($e = 1.6 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg.)



(a) 1.87×10^6 ms^{-1} (b) 3.2×10^{-18} ms^{-1}

(c) 2.65×10^6 ms^{-1} (d) 7.02×10^{12} ms^{-1}

[2006]

24. Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire then in equilibrium condition, the ratio of the magnitude of the electric fields at the surfaces of sphere A and B is

(a) 2 : 1 (b) 1 : 4

(c) 4 : 1 (d) 1 : 2 **[2006]**

25. An electric charge 10^{-3} μC is placed at the origin (0, 0) of $X - Y$ co-ordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points A and B will be

(a) 9 volt (b) zero

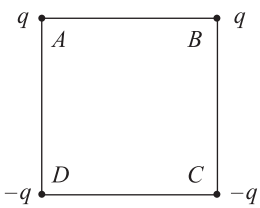
(c) 2 volt (d) 4.5 volt **[2007]**

26. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be

(a) 1 (b) 2

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$ **[2007]**

27. Charges are placed on the vertices of a square as shown in the figure. Let E be the electric field and V the potential at the centre of the square. If the charges on A and B are interchanged with those on D and C respectively, then



(a) E remains unchanged but V will change

(b) both E and V will change

(c) E and V will remain unchanged

(d) V remains unchanged but E will change

28. The potential at a point x due to some charges situated on the x-axis is given by

$$V = \frac{20}{(x^2 - 4)} \text{ volt}$$

where x is measured in μm. The electric fleld E at x = 4 μm is

(a) $\frac{5}{3}$ volt/μm in the –ve x-direction

(b) $\frac{5}{3}$ volt/μm in the +ve x-direction

(c) $\frac{10}{9}$ volt/μm in the –ve x-direction

(d) $\frac{10}{9}$ volt/μm in the +ve x-direction **[2007]**

29. A parallel plate capacitor with a dielectric of dielectric constant K between the plates has a capacitance C and is changed to a voltage V. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is

(a) $\frac{1}{2}(K-1)CV^2$ (b) $\frac{CV^2(K-1)}{K}$

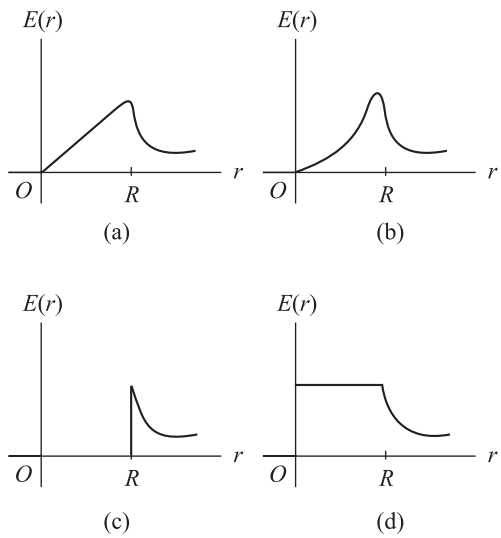
(c) $(K-1)\,CV^2$ (d) zero **[2007]**

30. A parallel plate capacitor with air between the plates has a capacitance of 9pF. The separation between its plates is 'd'. The space between the plates is now filled with two dielectrics. One dielectric has dielectric constant $k_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $k_2 = 6$ and $\frac{2d}{3}$ thickness. Capacitance of the capacitor is now

(a) 40.5 pF (b) 20.25 pF

(c) 1.8 pF (d) 45pF **[2008]**

31. A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field $E(r)$ produced by the shell in the range $0 \le r < \infty$, where r is the distance from the centre of the shell?



32. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then Q/q equals:

(a) 1 (b) $-\frac{1}{\sqrt{2}}$

(c) $-2\sqrt{2}$ (d) –1 **[2009]**

33. *This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.*

Statement-1: For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement-2: The net work done by a conservative force on an object moving along a closed loop is zero.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.

(b) Statement-1 is false, Statement-2 is true.

(c) Statement-1 is true, Statement-2 is false.

(d) Statament-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1. **[2009]**

34. Two points P and Q are maintained at the potentials of 10 V and – 4V, respectively. The work done in the moving 100 electrons from P to Q is:

(a) -2.24×10^{-16} J (b) 2.24×10^{-16} J

(c) -9.60×10^{-17} J (d) 9.60×10^{-17} J **[2009]**

35. Let $\rho(r) = \dfrac{Qr}{\pi R^4}$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point P inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is

(a) $\dfrac{Qr_1^2}{4\pi \in_0 R^4}$ (b) $\dfrac{Qr_1^2}{3\pi \in_0 R^4}$

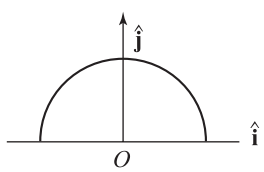
(c) zero (d) $\dfrac{Q}{4\pi\eta_0 r_1^2}$ **[2009]**

36. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be

(a) 1 (b) $\dfrac{1}{2}$

(c) $\dfrac{1}{4}$ (d) 2 **[2010]**

37. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is



(a) $\dfrac{q}{4\pi^2 \varepsilon_0 r^2}\hat{\mathbf{j}}$ (b) $-\dfrac{q}{4\pi^2 \varepsilon_0 r^2}\hat{\mathbf{j}}$

(c) $-\dfrac{q}{2\pi^2 \varepsilon_0 r^2}\hat{\mathbf{j}}$ (4) $\dfrac{q}{2\pi^2 \varepsilon_0 r^2}\hat{\mathbf{j}}$ **[2010]**

38. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\dfrac{5}{4} - \dfrac{r}{R}\right)$ upto $r = R$, and $\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance $r(r < R)$ from the origin is given by

(a) $\dfrac{\rho_0 r}{3\varepsilon_0}\left(\dfrac{5}{4} - \dfrac{r}{R}\right)$ (b) $\dfrac{4\pi\rho_0 r}{3\varepsilon_0}\left(\dfrac{5}{4} - \dfrac{r}{R}\right)$

(c) $\dfrac{\rho_0 r}{4\varepsilon_0}\left(\dfrac{5}{3} - \dfrac{r}{R}\right)$ (d) $\dfrac{4\rho_0 r}{3\varepsilon_0}\left(\dfrac{5}{4} - \dfrac{r}{R}\right)$ **[2010]**

39. Two identically charged spheres are suspended by strings of equal lengths. The strings make an angle of 30º with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the spheres is 1.6 g cm^{-3}, the dielectric constant of the liquid is

(a) 4 (b) 3

(c) 2 (d) 1 **[2010]**

40. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a, b are constants. Then the charge density inside the ball is

(a) $-24\pi a\varepsilon_0 r$ (b) $-6\pi a\varepsilon_0 r$

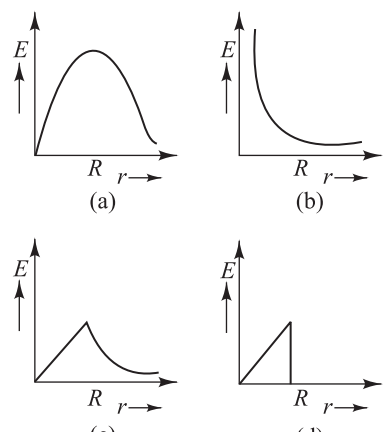
(c) $-24\pi a\varepsilon_0$ (d) $-6a\varepsilon_0$ **[2011]**

41. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d(d<<l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them

(a) $v \propto x^{-1/2}$ (b) $v \propto x^{-1}$

(a) $v \propto x^{1/2}$ (b) $v \propto x$ **[2011]**

42. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as function of distance from the centre. The graph which would correspond to the above will be:



[2012]

43. This questions has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describe the two statements.

An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinite is zero.

Statement-1: When a change 'q' is taken from the centre of the surface of the sphere its potential energy changes by $q\rho/3\varepsilon_0$

Statement-2: The electric field at a distante $r(r < R)$ from the centre of the sphere is $\rho r/3\varepsilon_0$

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true, Statement 2 is the correct explanation of Statement 1.

[2012]

44. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = q/2$ is placed at the origin. If charge q_0 is given a small displacement ($y << a$) along the y-axis, the net force acting on the particle is proportional to

(a) –y (b) $\frac{1}{y}$

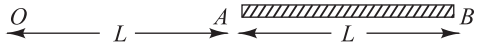
(c) $-\frac{1}{y}$ (d) y **[2013]**

45. Two capacitors C_1 and C_2 are charged to 120 V and 200 V, respectively. It is found that by connecting them together the potential on each one can be made zero. Then

(a) $3C_1 = 5C_2$ (b) $3C_1 + 5C_2 = 0$

(c) $9C_1 = 4C_2$ (d) $5C_1 = 3C_2$ **[2013]**

46. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electricpotential at the point O lying at a distance L from the end A is:



(a) $\frac{3Q}{4\pi\varepsilon_0 L}$ (b) $\frac{Q}{4\pi\varepsilon_0 L \ln 2}$

(c) $\frac{Q \ln 2}{4\pi\varepsilon_0 L}$ (d) $\frac{Q}{8\pi\varepsilon_0 L}$ **[2013]**

47. An electric dipole is placed at an angle of 30º to a non-uniform electric field. The dipole will experience

(a) a torque as well as a translational force

(b) a torque only

(c) a translational force only in the direction of the field

(d) a translational force only in a direction normal to the direction of the filed **[2014]**

48. A parallel plate capacitor with air between the plates has a capacitance of 9pF. The separation between its plates is 'd'. The space between the plates is now filled with two dielectrics. One dielectric has dielectric constant $K_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $K_2 = 6$ and thickness $\frac{2d}{3}$. Capcitance of the capacitor is now

(a) 40.5 pF (b) 20.25 pF

(c) 1.8 pF (d) 45 pF **[2014]**

Answers

1. (a)	2. none	3. (b)	4. (d)
5. (a)	6. (c)	7. (b)	8. (c)
9. (c)	10. (b)	11. (c)	12. (d)
13. (c)	14. (a)	15. (d)	16. (d)
17. (a)	18. (d)	19. (d)	20. (a)
21. (b)	22. (a)	23. (c)	24. (a)
25. (b)	26. (d)	27. (d)	28. (d)
29. (d)	30. (a)	31. (c)	32. (c)
33. (d)	34. (b)	35. (a)	36. (c)
37. (c)	38. (c)	39. (c)	40. (d)
41. (a)	42. (c)	43. (c)	44. (d)
45. (a)	46. (c)	47. (a)	48. (a)

Solutions

1. $\Delta V = \frac{W}{q} = \frac{2}{20} = 0.1$ V

2. Charge outside the cube does not contribute to electric flux. From Gauss's law, electric flux through the cube is $\frac{q}{\varepsilon_0}$. Since a cube has 6 faces, the flux through one face $ABCD$ is $\frac{q}{6\varepsilon_0}$. Hence no choice

given in the question is correct. Moreover, choices (a), (c) and (d) do not have the dimensions of electric flux.

3. Equivalent capacitance is $C_{eq} = nC$. Therefore, energy stored is

$$U = \frac{1}{2}\ C_{eq}\ V^2 = \frac{1}{2}\ nCV^2$$

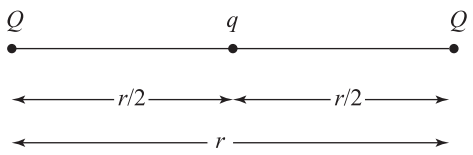
4. Let r be the distance between equal charges Q.

Since q is exactly mid-way between equal charges (Q), charge q will always be in equilibrium. The charge Q will be in equilibrium if

$\mathbf{F}_{Qq} = \mathbf{F}_{QQ} = 0$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{\left(\frac{r}{2}\right)^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{r^2} = 0$$

$$\Rightarrow \quad q = -\frac{Q}{4}$$



5. $C = 4\pi\varepsilon_0 r = \dfrac{1}{9\times10^9} \times 1 = 1.1 \times 10^{-10}$ F.

6. Initial Energy $U_i = U_1 + U_2 = \dfrac{1}{2}\ CV_1^2 + \dfrac{1}{2}\ CV_2^2$

When they are connected, the potential across each is $V = \dfrac{1}{2}\ (V_1 + V_2)$. Final energy is

$$U_f = \frac{1}{2}\ CV^2 + \frac{1}{2}\ CV^2 = CV^2$$

$$= C\left(\frac{V_1 + V_2}{2}\right)^2$$

$\therefore$ Decrease in energy = $Ui - U_f$

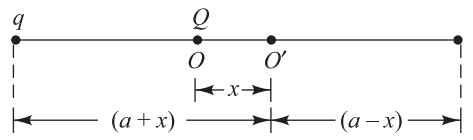
$$= \frac{1}{2}\ C(V_1^2 + V_2^2) - \frac{1}{4}\ C(V_1 + V_2)^2$$

$$= \frac{1}{4}\ C(V_1 - V_2)^2$$

7. Potential energy of the system when charge Q is at O is

$$U_0 = \frac{qQ}{a} + \frac{qQ}{a} = \frac{2qQ}{a}$$

When charge Q is shifted to position O', the potential energy



$$U = \frac{qQ}{(a+x)} + \frac{qQ}{(a-x)} = \frac{qQ(2a)}{(a^2 - x^2)}$$

$$= \frac{2qQ}{a} \times \left(1 - \frac{x^2}{a^2}\right)^{-1}$$

$$\simeq \frac{2qQ}{a} \times \left(1 + \frac{x^2}{a^2}\right) \qquad (x << a)$$

$$\therefore\ \Delta U = U - U_0 = \frac{2qQ}{a}\left(1 + \frac{x^2}{a^2}\right) - \frac{2qQ}{a}$$

$$= \frac{2qQ}{a^3}(x^2)$$

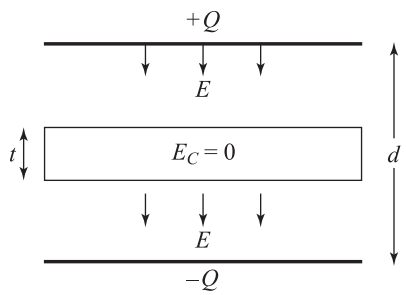
Hence $\Delta U \propto x^2$ Which is choice (b).

8. Because the charge is not located at the centre of the cavity, inside the cavity the lines of force are skewed. Hence choice (a) and (b) are incorrect. Outside the shell, the lines of force are the same as if the charge were located at the centre of the cavity. Also there can be no line of force in the metallice body of the shell. Hence choice (d) also incorrect. Thus, the correct pattern is shown in (c).

9. According to Gauss's law, the net electric flux leaving a closed surface = $\dfrac{q}{\varepsilon_0}$. Hence

$(\phi_2 - \phi_1) = \dfrac{q}{\varepsilon_0} \Rightarrow\ q = \varepsilon_0(\phi_2 - \phi_1)$

10. In electrostatics, the electric field inside a conductor $(E_c) = 0$. The potential between the plates of the capacitor is



$$V = E(d - t) = E_C\, t$$

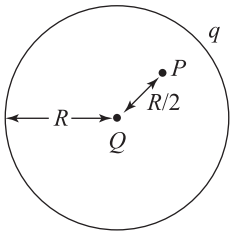
$$= E(d - t) \qquad (\because E_C = 0)$$

$$= \frac{\sigma}{\varepsilon_0}(d - t) = \frac{Q}{A\varepsilon_0}(d - t)$$

$$\therefore\ C = \frac{Q}{V} = \frac{A\varepsilon_0}{(d - t)}$$

If t is negligible compared to d, $C = \dfrac{A\varepsilon_0}{d}$. So the correct choice is (b).

11. Potential at P due to charge q on the shell is



$$V_1 = \frac{q}{4\pi\varepsilon_0 R}$$

Potential at P due to charge Q at the centre of the shell is

$$V_2 = \frac{Q}{4\pi\varepsilon_0 (R/2)} = \frac{2Q}{4\pi\varepsilon_0 R}$$

Therefore, total potential at P is

$$V = V_1 + V_2 = \frac{q}{4\pi\varepsilon_0 R} + \frac{2Q}{4\pi\varepsilon_0 R}$$

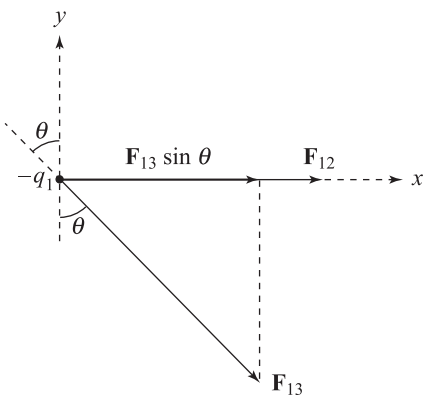
12. $W = \dfrac{Q^2}{2C} = \dfrac{(8\times10^{-18})^2}{2\times(100\times10^{-6})} = 32\times10^{-32}$ J

13. Force exerted on $-q_1$ by $+q_2$ is

$$F_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0 b^2} \text{ along } +x \text{ axis}$$

Force exerted on $-q_1$ by $-q_3$ is

$$F_{13} = \frac{q_1 q_3}{4\pi\varepsilon_0 a^2} \text{ along the direction shown in figure.}$$



The x-component of F_{13} is

$$F_{13}\cos(90° - \theta) = F_{13}\sin\theta$$

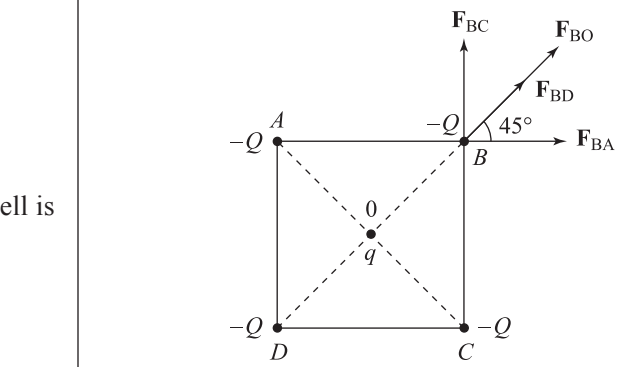
Net force on $-q_1$ is

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 b^2} + \frac{q_1 q_3 \sin\theta}{4\pi\varepsilon_0 a^2}$$

$$\Rightarrow F = \frac{q_1}{4\pi\varepsilon_0}\left(\frac{q_2}{b^2} + \frac{q_3\sin\theta}{a^2}\right)$$

For a given q_1, $F \propto \left(\dfrac{q_2}{b^2} + \dfrac{q_3\sin\theta}{a^2}\right)$

14. The system is in equilibrium if each experiences no force. It is clear that charge q at the centre would be in equilibrium for any value of q.



Let $AB = BC = BD = DA = a$. Then

$DB = \sqrt{2}a$ and $OB = \dfrac{a}{\sqrt{2}}$

$$F_{BA} = F_{BC} = \frac{Q^2}{4\pi\varepsilon_0 a^2}$$

$$F_{BD} = \frac{Q^2}{4\pi\varepsilon_0(\sqrt{2}a)^2} = \frac{Q^2}{4\pi\varepsilon_0(2a)^2}$$

$$F_{BO} = \frac{qQ}{4\pi\varepsilon_0\left(\dfrac{a}{\sqrt{2}}\right)^2} = -\frac{2qQ}{4\pi\varepsilon_0 a^2}$$

Net force on $-Q$ at B will be zero, if

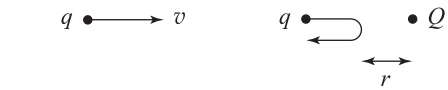
$$F_{BA}\cos45° + F_{BC}\cos45° + F_{BD} + F_{BO} = 0$$

$$\Rightarrow \frac{Q^2}{4\pi\varepsilon_0 a^2\sqrt{2}} + \frac{Q^2}{4\pi\varepsilon_0 a^2\sqrt{2}} + \frac{Q^2}{4\pi\varepsilon_0(2a^2)} - \frac{2qQ}{4\pi\varepsilon_0 a^2} = 0$$

$$\Rightarrow q = \frac{Q}{4}(1+2\sqrt{2})$$

15. The correct choice is (d).

16. Particle q will momentarily come to rest at a distance r from Q and will then be repelled back.



At the closest distance of approach, the kinetic energy of q is converted into potential energy of the system which is q at a distance r from Q. Hence

$$\frac{1}{2}mv^2 = \frac{qQ}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \qquad r = \frac{2qQ}{4\pi\varepsilon_0 mv^2}$$

Hence $r \propto \frac{1}{v^2}$. If v is doubled, r reduces to $\frac{r}{4}$.

17. $mg = qE$

$$\Rightarrow q = \frac{mg}{E} = \frac{9.9\times10^{-15}\times10}{3\times10^4} = 3.3\times10^{-18}\text{ C.}$$

18. $\frac{1}{2}CV^2 = ms\Delta T$

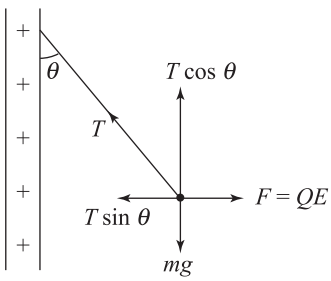
$$\Rightarrow \qquad V = \sqrt{\frac{2ms\Delta T}{C}}$$

19. Electric field due to sheet is

$$E = \frac{\sigma}{2\varepsilon_0}$$

where $\sigma = \frac{Q}{A}$; A = area of sheet

It follows from the figure that



$$T\cos\theta = mg \qquad (1)$$

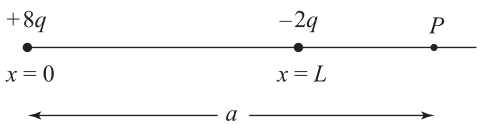
$$T\sin\theta = QE = \sigma AE \qquad (2)$$

Dividing (2) by (1), we get

$$\tan\theta = \frac{\sigma AE}{mg}$$

Since E is uniform and A, m and g are fixed, $\sigma \propto \tan\theta$.

20. It is clear that the net electric field cannot be zero at any point between $x = 0$ and $x = L$. Let the electric field be zero at a point P at a distance a from $x = 0$.



Electric field at P due to charge $+8q$ is

$$\mathbf{E}_1 = \frac{8q\hat{\mathbf{i}}}{4\pi\varepsilon_0 a^2}$$

Electric field at P due to charge $-2q$ is

$$\mathbf{E}_2 = \frac{2q\hat{\mathbf{i}}}{4\pi\varepsilon_0 (a-L)^2}$$

Net electric field at P is $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$. $\mathbf{E} = 0$ if

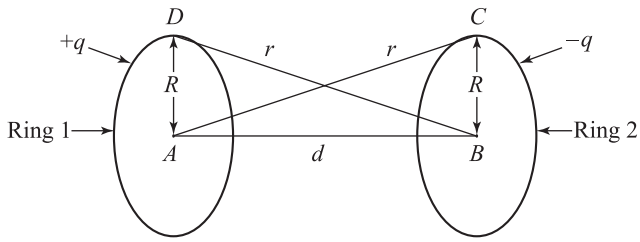
$$\frac{8q\hat{\mathbf{i}}}{4\pi\varepsilon_0 a^2} - \frac{2q\hat{\mathbf{i}}}{4\pi\varepsilon_0 (a-L)^2} = 0$$

$$\Rightarrow \quad \frac{4}{a^2} = \frac{1}{(a-L)^2}$$

$$\Rightarrow \quad \frac{2}{a} = \frac{1}{(a-L)}$$

$$\Rightarrow \quad a = 2L$$

21. Let $AC = BD = r = \sqrt{R^2 + d^2}$



Potential at A is

V_A = potential at A due to charge $+q$ on ring 1 + potential at A due to charge $-q$ on ring 2

$$= \frac{q}{4\pi\varepsilon_0 R} - \frac{q}{4\pi\varepsilon_0 r} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{r}\right)$$

Potential at B is

V_B = potential at B due to charge $-q$ on ring 2 + potential at B due to charge $+q$ on ring 1

$$= \frac{-q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 r} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$$

$$\therefore \quad V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{r}\right) - \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{2}{R} - \frac{2}{r}\right)$$

$$= \frac{q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2 + d^2}}\right]$$

22. The correct choice is (a).

23. Potential difference between plates 1 and 2 is

$V = V_2 - V_1 = 20$ V

Energy gained by the electron is eV. If v is its velocity with which it hits plate 2, then

$$\frac{1}{2}mv^2 = eV$$

or $v = \sqrt{\dfrac{2eV}{m}}$

$= \left[\dfrac{2\times(1.6\times10^{-19})\times 20}{9.11\times10^{-31}}\right]^{1/2}$

$= 2.65\times10^{6}\ \text{ms}^{-1}$.

24. Let Q_1 and Q_2 be the charges on spheres A and B and R_1 and R_2 their respective radii. Their potentials are

$$V_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_1}{R_1} \quad \text{and} \quad V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_2}{R_2}$$

If $V_1 > V_2$, charge will flow from A to B when the spheres are connected by a wire, until their potentials are equalized. If charge q flows from A to B, then

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{(Q_1-q)}{R_1} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(Q_2+q)}{R_2}$$

$$\Rightarrow \frac{(Q_1-q)}{(Q_2+q)} = \frac{R_1}{R_2} \qquad (1)$$

The electric fietlds on the surfaces of the spheres are

$$E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(Q_1-q)}{R_1^2} \quad \text{and} \quad E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(Q_2+q)}{R_2^2}$$

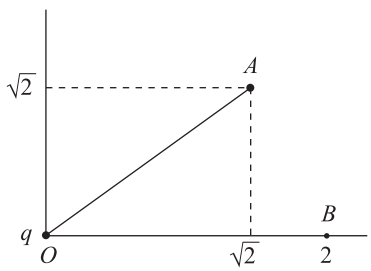
$$\therefore \frac{E_1}{E_2} = \frac{(Q_1-q)}{(Q_2+q)}\cdot\left(\frac{R_2}{R_1}\right)^2 = \frac{R_2}{R_1} = \frac{2}{1} \qquad \text{[use Eq. (1)]}$$

Hence the correct choice is (a). The same result is obtained if $V_2 > V_1$.

25. Refer to the following figure.

$OA = 2$, $OB = 2$

Now $V_A = \dfrac{q}{4\pi\varepsilon_0(OA)}$, $V_B = \dfrac{q}{4\pi\varepsilon_0(OB)}$



$V_A - V_B = 0$ $(\because OA = OB)$

26. Let Q be the charge on the capacitor plates and V be the voltage of the battery.

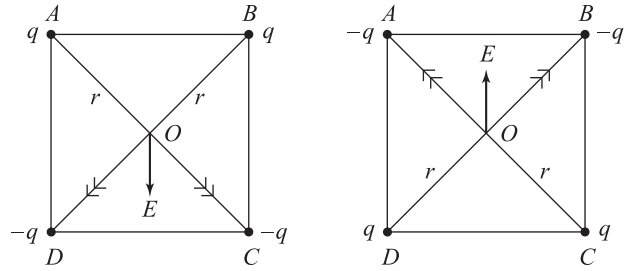
Work done by the battery is W = charge × potential difference

$= Q\times V = CV\times V = CV^2$

Energy stored in the capacitor is $U = \dfrac{1}{2}CV^2$

$$\therefore \frac{U}{W} = \frac{\frac{1}{2}CV^2}{CV^2} = \frac{1}{2}$$

27. Refer to the following figures.



(a) (b)

Case 1: The potential at centre O is [see Fig. (a)]

$$V = V_A + V_B + V_C + V_D = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r}+\frac{q}{r}-\frac{q}{r}-\frac{q}{r}\right] = 0$$

The electric fields at O due to charges at A, B, C and D respectively are

$$E_A = \frac{q}{4\pi\varepsilon_0 r^2} \text{ directed from } O \text{ to } C$$

$$E_B = \frac{q}{4\pi\varepsilon_0 r^2} \text{ directed from } O \text{ to } D$$

$$E_C = \frac{q}{4\pi\varepsilon_0 r^2} \text{ directed from } O \text{ to } C$$

$$E_D = \frac{q}{4\pi\varepsilon_0 r^2} \text{ directed from } O \text{ to } D$$

It is clear that the resultant electric field E at point O is directed vertically downwards.

Case 2: It follows from Fig. (b) that the potential V at point O is still zero but the resultant electric field E is directed vertically upwards.

28. $$E = -\frac{dV}{dx}$$

$$= -\frac{d}{dx}\left[\frac{20}{(x^2-4)}\right]$$

$$= \frac{20\times(2x)}{(x^2-4)^2} \text{ volt}$$

At $x = 4\ \mu\text{m}$, $$V = \frac{20\times(2\times4)}{\left[(4)^2-4\right]^2} \text{ volt}$$

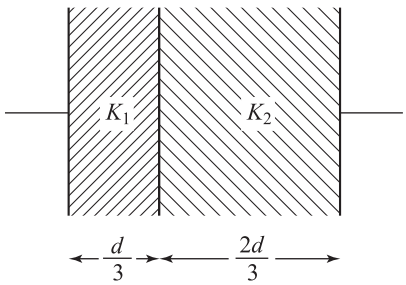
$$= \frac{10}{9} \text{ volt/}\mu\text{m}$$

Since V decreases as x increases, it follows from $E = -dV/dx$ that $\vec{E}$ is along the +ve x-direction.

29. In this process, the energy stored in the capacitor remains unchanged equal to $\dfrac{1}{2}CV^2$ where

$C = \dfrac{K\varepsilon_0 A}{d}$. Hence the net work done by the system is zero.

30. Given $C_0 = \dfrac{\varepsilon_0 A}{d} = 9$ pF.



$$C_1 = \frac{k_1\varepsilon_0 A}{d/3} = \frac{3\varepsilon_0 A}{d/3} = \frac{9\varepsilon_0 A}{d} = 9C_0$$

$$C_2 = \frac{k_2\varepsilon_0 A}{2d/3} = \frac{6\varepsilon_0 A}{2d/3} = \frac{9\varepsilon_0 A}{d} = 9C_0$$

Capacitors C_1 and C_2 are in series. The equivalent capacitance is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{9C_0 \times 9C_0}{18C_0} = \frac{9}{2}C_0$$

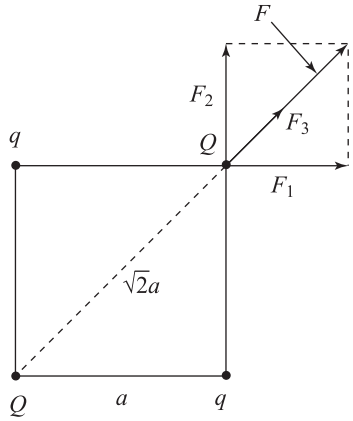
$$= \frac{9}{2} \times 9\text{pF} = 40.5\text{pF}$$

31. Inside a spherical shell (i.e., for $r < R$), the electric field is zero. Outside the shell, i.e., for $r > R$,

$$E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Hence the correct graph is (c).

32. Let a be the side of the square and $k = \dfrac{1}{4\pi\varepsilon_0}$. Assuming that Q and q have the same sign, the forces acting on Q are as shown in the figure.



$$F_1 = \frac{kqQ}{a^2}$$

$$F_2 = \frac{kqQ}{a^2}$$

$$F_3 = \frac{kQ^2}{2a^2}$$

The resultant of F_1 and F_2 is

$$F = \sqrt{F_1^2 + F_2^2} = \frac{\sqrt{2}\,kqQ}{a^2}$$

The net force on Q will be zero if $F + F_3 = 0$, i.e.

$$\frac{\sqrt{2}\,kqQ}{a^2} + \frac{kQ^2}{2a^2} = 0$$

$\Rightarrow \dfrac{Q}{q} = -2\sqrt{2}$, whtich is choice (c). Note that changes Q and q must have opposite sign.

33. Electrostatic field is conservative. For a conservative field, the work done to take a charged particle from a point P to another point Q is independent of the path followed by the particle to go from P to Q. In addition, work done to take the particle from Q to P is equal and opposite in sign. Hence the net work done in moving the particle from P to Q and then back to P is zero. So the correct choice is (d).

34. $q = 100\,e = 100 \times (-1.6 \times 10^{-19}) = -1.6 \times 10^{-17}$ C

$W_{P \to Q} = q\,(V_Q - V_P) = -1.6 \times 10^{-17} \times (-4 - 10) = 2.24 \times 10^{-16}$ J

35. Using Gauss's theorem $\int \vec{E} \cdot \vec{dA} = \dfrac{q}{\varepsilon_0}$, where q is the charge enclosed inside the Gaussian spherical surface of radius r_1,

$$E \times 4\pi r_1^2 = \frac{q}{\varepsilon_0} \qquad (1)$$

$$q = \int_0^{r_1} \rho\, dV$$

$$= \int_0^{r_1} \frac{Qr}{\pi R^4} \times 4\pi r^2 dr \qquad (\because\ dV = 4\pi r^2 dr)$$

$$= \frac{4Q}{R^4}\int_0^{r_1} r^3 dr = \frac{Qr_1^4}{R^4} \qquad (2)$$

Using (2) in (1) we get

$$E = \frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$$, which is choice (a).

36. The charge decays according to the equation

$$Q = Q_0 e^{-t/\tau}$$

where $\tau = RC$ and Q_0 = charge at $t = 0$.

Energy stored is $U = \dfrac{Q^2}{2C} = \dfrac{Q_0^2}{2C} e^{-2t/\tau} = U_0 e^{-2t/\tau}$

where U_0 = initial energy at $t = 0$.

Given $U = \dfrac{U_0}{2}$ at $t = t_1$.

i.e. $\dfrac{U_0}{2} = U_0 e^{-2t_1/\tau} \quad \Rightarrow \quad e^{2t_1/\tau} = 2$

or $\dfrac{2t_1}{\tau} = \ln(2) \quad \Rightarrow \quad t_1 = \dfrac{\tau}{2} \ln(2)$

Given $Q = \dfrac{Q_0}{4}$ at $t = t_2$. Hence

$$\frac{Q_0}{4} = Q_0 e^{-t_2/\tau}$$

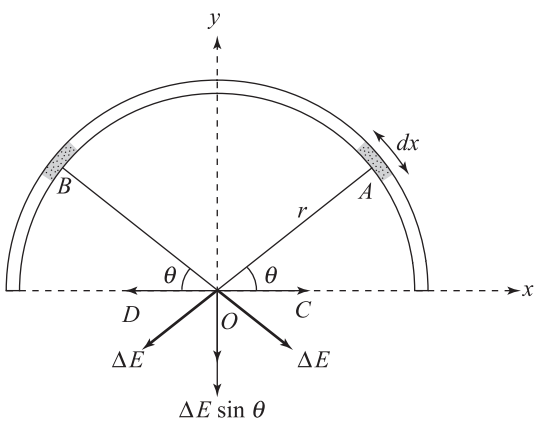
which gives $t_2 = \tau \ln(4) = 2\tau \ln(2)$

$\therefore \quad \dfrac{t_1}{t_2} = \dfrac{1}{4}$, which is choice (c).

37. Linear charge density is

$$\lambda = \frac{q}{\pi r}$$

To find the net electric field at O, we divide the ring into a large number of very small elements each of length dx. The charge of an element is $dq = \lambda dx = \lambda r d\theta$, where $d\theta$ is the angle subtended by the element at centre O. The electric field at O due to the element at A is



$\Delta E = \dfrac{dq}{4\pi \varepsilon_0 r^2}$ directed radially outwards

Similarly, the electric field at O due to an element B symmetrically opposite to A is also ΔE. It is clear that the horizontal components $OC = OD = \Delta E \cos\theta$ cancel but he vertical components = $\Delta E \sin\theta$ add up and are directed along the negative y-direction. Hence the net electric field at O is

$$\mathbf{E} = \int \Delta \mathbf{E} \sin\theta = \frac{\lambda r(-\hat{j})}{4\pi \varepsilon_0 r^2} \int_0^{\pi} \sin\theta \, d\theta \quad (\because dq = \lambda r d\theta)$$

$$= \frac{\lambda \hat{j}}{4\pi \varepsilon_0 r} |-\cos\theta|_0^{\pi}$$

$$= \frac{\lambda \hat{j}}{4\pi \varepsilon_0 r} (\cos\pi - \cos 0)$$

$$= \frac{\lambda \hat{j}}{2\pi \varepsilon_0 r}$$

$$= -\frac{q \hat{j}}{2\pi^2 r^2 \varepsilon_0} \qquad \left(\because \lambda = \frac{q}{\pi r}\right)$$

38. For $r < R$,

$$q = \int_0^r \rho \, dV = \int_0^r \rho_0 \left(\frac{5}{4} - \frac{r}{R}\right) \times 4\pi r^2 dr$$

$$= 4\pi\rho_0 \int_0^r \left(\frac{5}{4} r^2 dr - \frac{r^3}{R} dr\right)$$

$$= 4\pi\rho_0 \left(\frac{5r^3}{12} - \frac{r^4}{4R}\right)$$

For Gauss's law

$$\int \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad E \times 4\pi r^2 = \frac{4\pi\rho_0}{\varepsilon_0} \left(\frac{5r^3}{12} - \frac{r^4}{4R}\right)$$

$$\Rightarrow \qquad E = \frac{\rho_0 r}{4\varepsilon_0} \left(\frac{5}{3} - \frac{r}{R}\right)$$

39. At equilibrium,

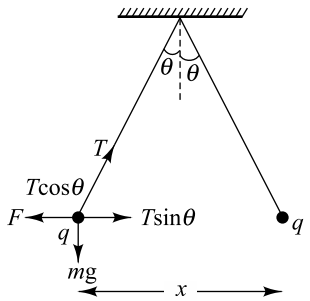
$$F = T \sin\theta$$

and $mg = T \cos\theta$

$$\frac{F}{mg} = \tan\theta \qquad (1)$$

Where F is the force of repulsion between the sphere. If r is the distance between them,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{r^2}$$



When suspended in a liquid of density ρ (since q and hence r remains the same)

$$F_l = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\ , \quad K = \text{dielectric constant of liquid}$$

$$\therefore \ \frac{F}{F_l} = K$$

Due to upthrust (since θ remains the same), the apparent weight of the sphere = $mg\left(1-\frac{\rho}{\sigma}\right)$ where σ = density of the material of the sphere. Hence

$$\frac{F_l}{mg\left(1-\frac{\rho}{\sigma}\right)} = \tan\theta \qquad (2)$$

Equating (1) and (2) we get

$$\frac{F}{F_l} = \left(1-\frac{\rho}{\sigma}\right)$$

$$\Rightarrow \qquad K = \frac{1}{\left(1-\frac{\rho}{\sigma}\right)} = \frac{1}{\left(1-\frac{0.8}{1.6}\right)} = 2$$

40. $\phi = ar^2 + b$

$$E = -\frac{d\phi}{dr} = -2ar$$

From Gusss's law

$$\Rightarrow \qquad \oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \qquad E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \qquad (-2ar) \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \qquad q = -8\pi\varepsilon_0 a\, r^3$$

Charge density $\rho = \dfrac{q}{\frac{4\pi}{3}r^3} = \dfrac{-8\pi\varepsilon_0 a r^3}{\frac{4\pi}{3}r^3}$

$$= -6a\,\varepsilon_0$$

41. As shown in the solution of Q. 39 above,

$$F = mg\tan\theta = \frac{q^2}{4\pi\varepsilon_0 x^2} = mg\tan\theta$$

$$\Rightarrow \quad \tan\theta = \frac{q^2}{4\pi\varepsilon_0 mg\, x^2}$$

If l is the length of the string, $\tan\theta = \dfrac{x/2}{l} = \dfrac{x}{2l}$

$$\therefore \qquad \frac{x}{2l} = \frac{q^2}{4\pi\varepsilon_0 mgx^2}$$

$$\Rightarrow \qquad q^2 = \left(\frac{2\pi\varepsilon_0 mg}{l}\right)x^3$$

$$\Rightarrow \qquad q \propto x^{3/2}$$

$$\therefore \qquad \frac{dq}{dt} \propto \frac{3}{2}x^{1/2}\frac{dx}{dt} = \frac{3}{2}x^{1/2}v \qquad \left(\because v = \frac{dx}{dt}\right)$$

Since $\dfrac{dq}{dt}$ = constant (given), $v \propto x^{-1/2}$

42. For a uniformaly charged sphere, the electric field inside the sphere is directly proportional to R and outside the sphere, it falls as $1/R^2$. Hence the correct graph is (c).

43. Potential energy at the centre of the sphere is

$$U_c = \frac{3}{2}\times\frac{Qq}{4\pi\varepsilon_0 R}$$

and on its surface, the P. E. is

$$U_s = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R}$$

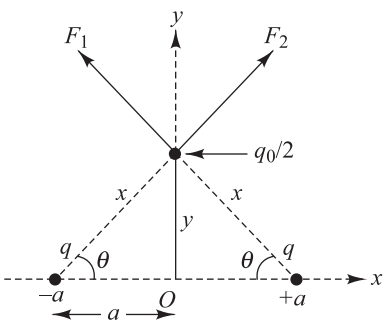
$$\therefore \qquad \Delta U = U_c - U_s = \frac{Qq}{8\pi\varepsilon_0 R}$$

Now $\qquad Q = \dfrac{4\pi}{3}R^3\rho$

$$\therefore \qquad \Delta U = \frac{q}{8\pi\varepsilon_0 R}\times\frac{4\pi}{3}R^3\rho = \frac{\rho R^2 q}{6\varepsilon_0}$$

so the correct choice is (c).

44. $F_1 = F_2 = \dfrac{q\,q_0}{4\pi\varepsilon_0 x^2} = F$



The x-components of F_1 and F_2 cancel and the y-components add up. The net force on $q_0/2$ is

$$F_{net} = 2F\cos\theta = 2F \times \frac{y}{x} = \frac{2q\ q_0}{4\pi\varepsilon_0} \times \frac{y}{x^3}$$

Now $x = (a^2 + y^2)^{1/2}$

$$F_{net} = \frac{2q\ q_0\ y}{4\pi\varepsilon_0 \times (a^2 + y^2)^{3/2}}$$

Since $y << a$

$$F_{net} = \frac{2q\ q_0\ y}{4\pi\varepsilon_0 \times 2a^3}$$

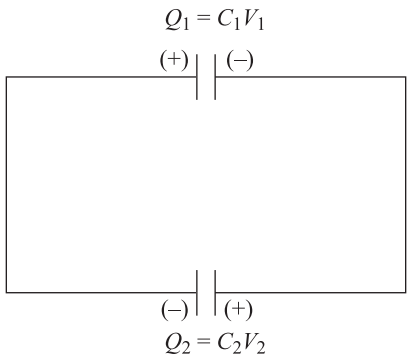
Hence $F_{net} \propto$ y. So the correct choice is (d).

45. The potential on each capacitor will be zero if the capacitors are connected as shown in the figure, i.e.

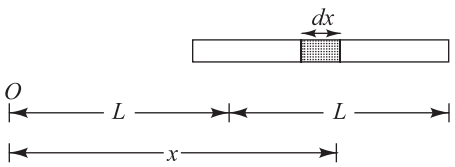
$$Q_1 = Q_2$$

$$\Rightarrow \quad 120\ C_1 = 200\ C_2$$

$$\Rightarrow \quad 3C_1 = 5\ C_2$$



46.



Consisder a small element of length dx at distance x from O. Charge of the element is

$$q = \frac{Qdx}{L}$$

The electric potential at O due to the charged rod is

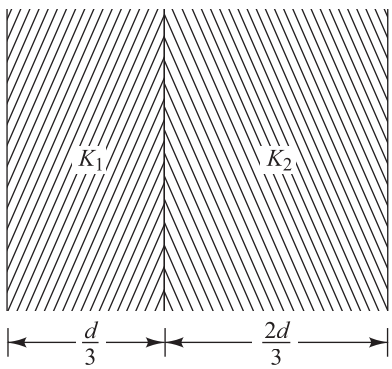
$$V = \int_{x=L}^{x=2L} \frac{1}{4\pi\varepsilon_0}\frac{q}{x} = \int_{x=L}^{x=2L} \frac{1}{4\pi\varepsilon_0}\frac{Q}{L}\frac{d}{x}$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\int_{x=L}^{x=2L}\frac{dx}{x}$$

$$= \frac{Q}{4\pi\varepsilon_0 L}|\ln x|_L^{2L} = \frac{Q\ln(2)}{4\pi\varepsilon_0 L}$$

47. The correct choice is (a)

48. Given $C_0 = \dfrac{\varepsilon_0 A}{d} = 9$ pF



$$C_1 = \frac{k_1\varepsilon_0 A}{d/3} = \frac{3\varepsilon_0 A}{d/3} = \frac{9\varepsilon_0 A}{d} = 9\ C_0$$

$$C_2 = \frac{k_2\varepsilon_0 A}{2d/3} = \frac{6\varepsilon_0 A}{2d/3} = \frac{9\varepsilon_0 A}{d} = 9\ C_0$$

Capacitors C_1 and C_2 are in series. The equivalent capacitance is

$$C = \frac{C_1C_2}{C_1 + C_2} = \frac{9C_0 \times 9C_0}{18C_0} = \frac{9}{2}C_0$$

$$= \frac{9}{2} \times 9\text{pF} = 40.5\text{ pF}$$

CURRENT ELECTRICITY

Chapter 12

REVIEW OF BASIC CONCEPTS

1. Electric Current

The rate of flow of charge is called electric current. If the rate of flow of charge does not change with time, the current is said to be *steady* or *constant* [Fig. 12.1(a)]. For steady current,

$$I = \frac{q}{t}$$

where q is the amount of charge flowing through any cross-section of the conductor in time t.

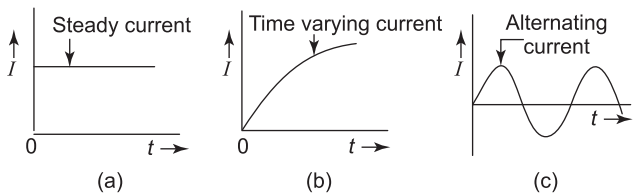


Fig. 12.1

In many situations, the current may vary with time [Fig. 12.1(b) and Fig. 12.1(c)]. In such situations, the current is given by

$$I = \frac{dq}{dt}$$

Convention Regarding Direction of Current

In metallic conductors, the charge is carried by free electrons. In electrolytes, the charge is carried by positive and negative ions. The direction of current is taken to be the direction in which the positive charge moves. A positive charge moving in one direction is equivalent to negative charge moving in the opposite direction (Fig. 12.2).

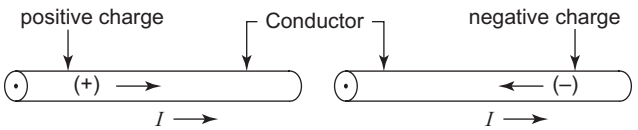


Fig. 12.2

2. Drift Speed of Electrons in a Conductor

A metallic conductor has a large number of free electrons. When a potential difference is applied across the ends of a conductor, an electric field is set up in the conductor which accelerates the electrons in the direction opposite to the direction of the electric field. Due to collisions with the atoms of the conductor, these electrons move with a velocity called the drift velocity $(\vec{v}_d)$ which is defined as the average velocity of the free electrons of the conductor under the influence of the electric field (E) and is given by

$$\vec{v}_d = -\frac{e\vec{E}\tau}{m}$$

where τ is the average time beween two successive collisions and is called the relaxation time. The negative sign indicates that the electrons drift in a direction opposite to that of the field.

Relations Between Drift Speed and Current

The drift speed is related to current I in a conductor as

$$v_d = \frac{I}{neA}$$

where n = number of free electrons per unit volume,
e = magnitude of charge of an electron
A = cross-sectional area of conductor

Note For moderate electric field, the drift-speed of electrons in a conductor is of the of order of a few mms^{-1}.

3. Ohm's Law

Ohm's Law states that the current flowing through a conductor is directly proportional to the potential difference across its ends, provided the physical conditions of the conductor remain the same.

Thus,

$$V \propto I \text{ or } V = RI$$

where R is the resistance of the given conductor.

4. Electrical Resistivity

The resistance of a wire is directly proportional to its length (l) and inversely proportional to its area of cross-section (A), i.e.

$$R \propto \frac{l}{A} \quad \text{or} \quad R = \rho \frac{l}{A}$$

or
$$\rho = \frac{RA}{l} \tag{1}$$

where ρ is a constant of proportionality and its value depends on the material of the wire. The constant ρ is called the *resistivity* of the material of the wire.

Unit of ρ: From Eq. (1) we have

$$\text{Unit of } \rho = \frac{\text{unit of } R \times \text{unit of } A}{\text{unit of } l} = \frac{\text{ohm} \times (\text{metre})^2}{\text{metre}}$$

$$= \text{ohm metre}$$

Hence ρ is measured in ohm metre (or Ω m).

Note that the value of ρ is independent of the length and the area of cross-section of the wire; it depends only on the material of the wire and temperature. Thus, resistivity is a characteristic of the material. The reciprocal of resistivity is called conductivity.

5. Resistors in Series and Parallel

1. *Resistors in series*

   When resistors are joined in series, the total resistance R of the combination is equal to the sum of the individual resistances, i.e.

   $$R = R_1 + R_2 + R_3 + \cdots$$

   The current is the same in all resistors. The total potential difference across the combination is equal to the sum of the potential differences across the individual resistors.

2. *Resistors in parallel*

   When resistors are connected in parallel, the effective resistance of the combination is given by

   $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

   The potential difference is the same across each resistor. The total current is equal to the sum of the currents in the individual resistors.

6. Emf, Terminal Voltage and Internal Resistance of a Cell

The potential difference between the terminals of a cell when it is on an open circuit, i.e. when no current is drawn from it is called its emf (E).

The potential difference between the terminals of a cell when it is in a closed circuit, i.e. when a current is drawn from it is called its terminal voltage (V).

Every cell has a resistance of its own called its internal resistance (r). The value of r depends upon the nature of electrodes, the nature of the electrolyte, size of electrodes and the distance between them.

Figure 12.3 shows a cell of emf E, internal resistance r connected to an external resistance R.

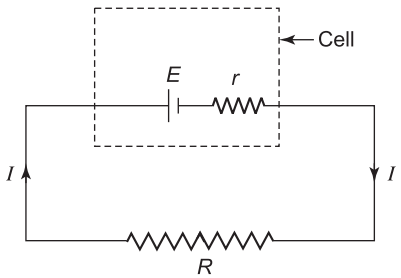


Fig. 12.3

Total resistance of the circuit = $R + r$. The current in the circuit is

$$I = \frac{E}{(R + r)}$$

Potential difference across r is $v = Ir$

Potential difference across R is $V = IR$. V is called the terminal voltage and v is the potential drop across the internal resistance.

Thus
$$E = V + v = I(R + r)$$

Terminal voltage is $V = E - Ir$

7. Grouping of Cells

(a) Cells connected in series

Consider two cells of emfs E_1 and E_2 and internal resistances r_1 and r_2 connected in series as shown in Fig. 12.4.



Fig. 12.4

The equivalent emf and equivalent internal resistance of the combination is given by

$$E_{eq} = E_1 + E_2$$

and $$r_{eq} = r_1 + r_2$$

For *n* cells in series

$$E_{eq} = E_1 + E_2 + \cdots + E_n$$

and $$r_n = r_1 + r_2 + \cdots + r_n$$

(b) Cells connected in parallel

If the cells are connected as shown in Fig. 12.5, then

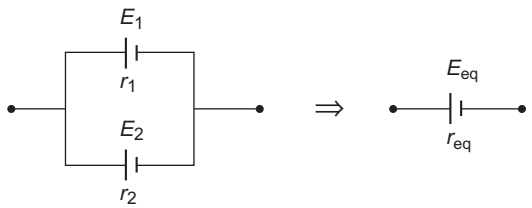


Fig. 12.5

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2}$$

where $$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

For *n* cells in parallel

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \ldots + \frac{E_n}{r_n}$$

and $$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \cdots + \frac{1}{r_n}$$

1. If the individual cells have the same emf *E*, and the number of cells connected in series is *n*, the emf of the combination is *nE* and the total internal resistance is *nr*, where *r* is the internal resistance of each cell.
2. When identical cells are connected in parallel the total emf = *E*, the emf of any one of the cells. Here the sum of the reciprocals of the individual internal resistances is equal to the reciprocal of the total internal resistance.
3. Cells should be connected in series if the external resistance *R* is greater than internal resistance *r*.
4. Cells should be connceted in parallel if $R < r$.
5. Mixed grouping of cells is employed if *R* is comparable with *r*.

   The current in the circuit when *n* cells, each of emf *E*, are connected in series is given by.

$$I = \frac{nE}{R + nr}$$

   The current in the circuit when *n* cells, each of emf *E*, are connected in parallel is given by

$$I = \frac{nE}{nR + r}$$

   If the arrangement consists of *n* rows of cells in parallel each having *m* cells in series, then

$$I = \frac{mnE}{nR + mr}$$

EXAMPLE 1 In a discharge tube, the number of hydrogen ions (protons) drifting through a cross-section is 1.0×10^{18} per second while the number of electrons drifting in the opposite direction is 3.0×10^{18} per second. If the supply voltage is 240 V, what is the effective resistance of the tube.

SOLUTION A proton has a positive charge and an electron has an equal negative charge. Since a positive charge and a negative charge drifting in opposite directions produce current in the same direction,

$$I = (N_p + N_e)e$$
$$= (1.0 \times 10^{18} + 3.0 \times 10^{18}) \times (1.6 \times 10^{-19})$$
$$= 0.64 \text{ A}$$

$$\therefore \quad R = \frac{V}{I} = \frac{240}{0.64} = 375\ \Omega$$

EXAMPLE 2 A potential difference of 0.8 V is maintained between the ends of a metal wire of length 1.0 m. The number density of free electrons in the metal is 8.0×10^{28} per m^3 and the electrical conductivity of the metal is $6.4 \times 10^7\ \Omega^{-1}\ m^{-1}$. Find the drift speed of electrons.

SOLUTION Conductivity is $\sigma = \frac{1}{\rho} = \frac{L}{RA}$

$R = \frac{\rho L}{A} = \frac{L}{\sigma A}$ and $I = \frac{V}{R}$. Therefore,

$$I = \frac{V\sigma A}{L}$$

But $I = enAv_d$. Therefore,

$$enAv_d = \frac{V\sigma A}{L}$$

$$\Rightarrow \qquad v_d = \frac{V\sigma}{Lne}$$

$$= \frac{0.8 \times (6.4 \times 10^7)}{1.0 \times (8.0 \times 10^{28}) \times (1.6 \times 10^{-19})}$$

$$= 4 \times 10^{-3} \text{ ms}^{-1} = 4 \text{ mms}^{-1}$$

EXAMPLE 3 The current I (in ampere) flowing in a wire varies with time t (in second) according to the equation $I = 5t + 3t^2$. Find the amount of charge which passes through a cross-section of the wire in the time interval $t = 2$ s to $t = 4$ s.

SOLUTION $I = \frac{dq}{dt} \quad \Rightarrow \quad dq = Idt = (5t + 3t^2)dt$

Therefore, the amount of charge passing from $t = 2$ s to $t = 4$ s is

$$q = \int_2^4 (5t + 3t^2)\, dt$$

$$= \frac{5}{2}\left|t^2\right|_2^4 + \frac{3}{3}\left|t^3\right|_2^4$$

$$= \frac{5}{2} \times (4^2 - 2^2) + (4^3 - 2^3)$$

$$= \frac{5}{2} \times (16 - 4) + (64 - 8)$$

$$= 86 \text{ coulomb}$$

EXAMPLE 4 A wire of resistance 5 Ω is drawn out so that its length is increased to twice its original length. Find the new resistance of the wire.

SOLUTION If a wire is stretched, its length L increases and its diameter and hence the cross-sectional area A decreases, but the volume of the wire remains the same. Hence AL = constant. Thus

$$A_1L_1 = A_2L_2$$

Given $L_2 = 2L_1$. Therefore $A_2 = \frac{A_1L_1}{L_2} = \frac{A_1}{2}$

Original resistance $R_1 = \frac{\rho L_1}{A_1}$

New resistance $R_2 = \frac{\rho L_2}{A_2}$

Resistivity ρ depends only on the material of the wire and hence it remains the same.

$$\frac{R_2}{R_1} = \frac{L_2}{L_1} \times \frac{A_1}{A_2} = 2 \times 2 = 4$$

$$\Rightarrow \qquad R_2 = 4R_1 = 4 \times 5 = 20\ \Omega$$

Note If a wire of resistance R is stretched to n times its original length, the resistance of the stretched wire = n^2R.

EXAMPLE 5 A wire has a resistance of 9 Ω. It is cut into three equal pieces. Each piece is stretched uniformly to three times its original length. The three streched pieces are connected in parallel. Find the total resistance of the combination.

SOLUTION Resistance each piece is $R = \frac{9}{3} = 3\ \Omega$. The new resistance of each stretched piece = $n^2R = (3)^2 \times 3$ = 27 Ω. The total resistance of the parallel combination is given by

$$\frac{1}{R_p} = \frac{1}{27} + \frac{1}{27} + \frac{1}{27} = \frac{1}{9}$$

$$\Rightarrow \qquad R_p = 9\ \Omega$$

EXAMPLE 6 A wire is stretched to make it 0.1% longer. What is the percentage change in the resistance?

SOLUTION AL = constant $\Rightarrow$ $A_1L_1 = A_2L_2$

Given $L_2 = L_1 + \frac{0.1}{100} \times L_1 = L_1\,(1 + 0.001)$

$$\therefore \qquad A_2 = A_1 \times \frac{L_1}{L_2} = \frac{A_1}{(1 + 0.001)}$$

Now $R_1 = \frac{\rho L_1}{A_1}$ and $R_2 = \frac{\rho L_2}{A_2}$. Therefore

$$\frac{R_2}{R_1} = \frac{L_2}{L_1} \times \frac{A_1}{A_2}$$

$$= (1 + 0.001) \times (1 + 0.001)$$

$$= (1 + 0.001)^2 \simeq 1 + 0.002 = 1.002$$

$$\Rightarrow \qquad R_2 = 1.002\ R_1$$

Change in resistance $\Delta R = R_2 - R_1 = 0.002\ R_1$

Percentage increase in resistance = $\frac{\Delta R}{R_1} \times 100 =$ $0.002 \times 100 = 0.2\ \%$

Simple Method

If $\frac{\Delta L}{L} << 1$, the following simpler method may be used.

$$R = \frac{\rho L}{A}$$

$$\Rightarrow \qquad \log R = \log \rho + \log L - \log A$$

Differentiating ($\because \rho$ = constant)

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} - \frac{\Delta A}{A} \tag{1}$$

Also AL = constant

$$\therefore \qquad \frac{\Delta A}{A} + \frac{\Delta L}{L} = 0 \quad \Rightarrow \quad \frac{\Delta A}{A} = -\frac{\Delta L}{L} \tag{2}$$

Using (2) in (1), we get

$$\frac{\Delta R}{R} = \frac{2\Delta L}{L} = 2 \times 0.1\% = 0.2\%$$

EXAMPLE 7 The resistance network shown in Fig. 12.6 is connected to a battery of emf 30 V and internal resistance 1 Ω. Find the current through the 6 Ω resistance and the terminal voltage of the battery.

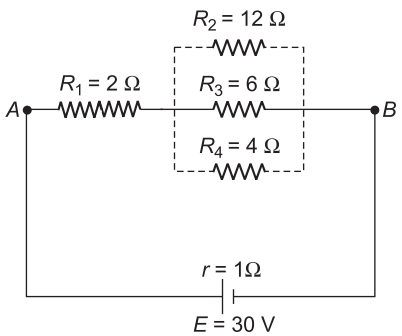


Fig. 12.6

SOLUTION The equivalent resistance of the parallel combination of R_2, R_3 and R_4 is R', which is given by

$$\frac{1}{R'} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4} \quad \Rightarrow \quad R' = 2\ \Omega$$

The circuit can be redrawn as shown in Fig. 12.7.

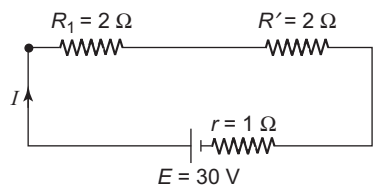


Fig. 12.7

Current in the circuit is

$$I = \frac{E}{R_1 + R' + r} = \frac{30}{2 + 2 + 1} = 6\ \text{A}$$

$\therefore$ Potential difference across R_1 is $V_1 = IR_1 = 6 \times 2 = 12$ V

Potential difference across R' is $V' = IR' = 6 \times 2 = 12$ V, which is same across R_2, R_3 and R_4 since these resistances are in parallel. Hence the current through $R_3 = 6\ \Omega$ is

$$I_3 = \frac{V'}{R_3} = \frac{12}{6} = 2\ \text{A}$$

The potential drop in the internal resistance is

$$v = Ir = 6 \times 1 = 6\ \text{V}$$

$\therefore$ Terminal voltage of the battery is

$$V = E - v = 30 - 6 = 24\ \text{V}$$

EXAMPLE 8 Calculate the steady state current in the 3 Ω resistor in the circuit shown in Fig. 12.8. The internal resistance of the cell is negligible. Also, find the charge on the capacitor.

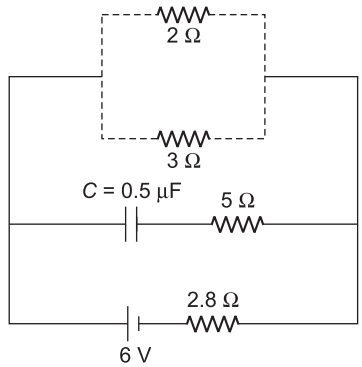


Fig. 12.8

SOLUTION The resistance of the parallel combination of 2 Ω and 3 Ω is

$$R' = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

In the steady state, no current flows through the capacitor and hence through the 5 Ω resistor. Therefore, the circuit reduces to that shown in Fig. 12.9.

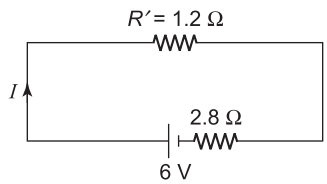


Fig. 12.9

$$\text{Current } I = \frac{6}{(1.2 + 2.8)} = 1.5\ \text{A}$$

$\therefore$ Potential difference across R' is

$$V' = IR' = 1.5 \times 1.2 = 1.8\ \text{V}$$

This is also the p.d. across the 3 Ω resistor. Hence the current through the 3 Ω resistor is $\frac{1.8}{3} = 0.6$ A.

Potential difference across $R' = 1.8$ V. This is also the potential difference between the capacitor plates because in the steady state, no current flows through the 5 Ω resistor. Hence charge on the capacitor is

$$Q = CV = 0.5\ \mu F \times 1.8\ V$$
$$= 0.9\ \mu C$$

EXAMPLE 9 A battery gives a current of 0.5 A when connected across an external resistor of resistance 12 Ω and a current of 0.25 A when connected across an external resistor of resistance 25 Ω. Find the emf and internal resistance of the battery.

SOLUTION $E = I(R + R)$

$\therefore$ $E = 0.5(12 + r)$ (1)

and $E = 0.25(25 + r)$ (2)

Equating (1) and (2),

$$0.5(12 + r) = 0.25(25 + r) \quad \Rightarrow \quad r = 1\ \Omega$$

Using $r = 1\ \Omega$ in either (1) or (2) gives $E = 6.5$ V

EXAMPLE 10 Determine the current drawn from a 12 V supply of internal resistance 0.5 Ω by an infinite network shown in Fig. 12.10. Each resistor has a resistance of 1 Ω.

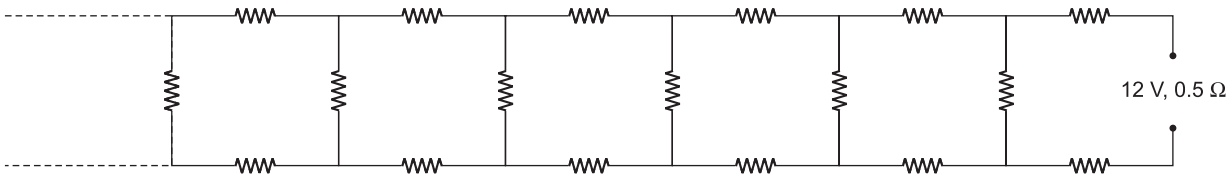


Fig. 12.10

SOLUTION Let R be the total resistance of the network. Since the circuit is infinitely long, its total resistance remains unaffected if one mesh is removed from it, i.e. the resistance of the network to the left of PQ will also be R [see Fig. 12.11]

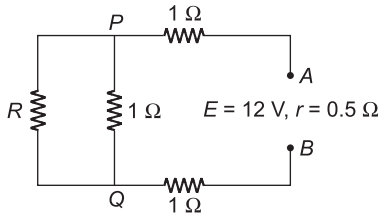


Fig. 12.11

$$R_{PQ} = \frac{R}{R+1}$$

The equivalent resistance between A and B is

$R_{AB} = 2 + \dfrac{R}{R+1}$ which must be equal to R, i.e.

$$2 + \frac{R}{R+1} = R$$
$$\Rightarrow \quad R^2 - 2R - 2 = 0$$

The positive root of this quadratic equation is $R = 2.73\ \Omega$

$$\therefore \text{ Current } I = \frac{E}{R+r} = \frac{12}{(2.73+0.5)} = 3.7\ A$$

EXAMPLE 11 Find the equivalent resistance between points A and B in the network shown in Fig. 12.12. Each resistor has a resistance of 3 Ω.

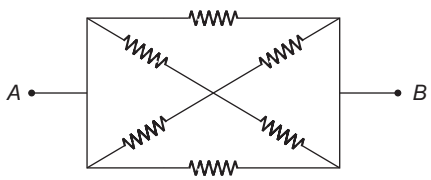


Fig. 12.12

SOLUTION The given circuit can be redrawn as shown in Fig. 12.13.

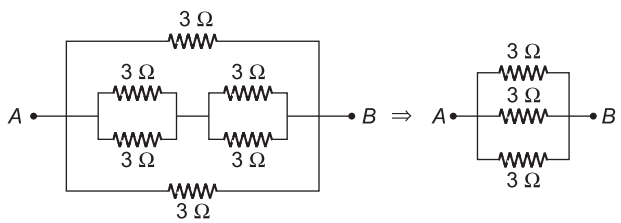


Fig. 12.13

It is clear that

$$\frac{1}{R_{AB}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \quad \Rightarrow \quad R_{AB} = 1\ \Omega$$

EXAMPLE 12 Find the equivalent resistance between points A and B in the network shown in Fig. 12.14, when

(a) switch S is open (b) switch S is closed.

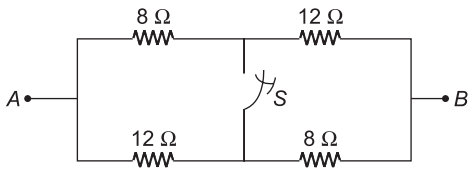


Fig. 12.14

SOLUTION

(a) When switch *S* is open, the circuit is as shown in Fig. 12.15.

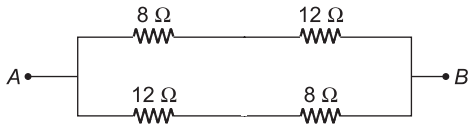


Fig. 12.15

$$R_{AB} = \frac{20 \times 20}{(20 + 20)} = 10\ \Omega$$

(b) When switch *S* is closed, the circuit can be redrawn as shown in Fig. 12.16

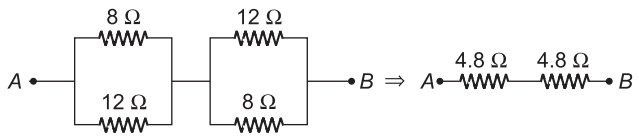


Fig. 12.16

It is clear that

$$R_{AB} = 4.8 + 4.8 = 9.6\ \Omega$$

EXAMPLE 13 Find the value of current *I* in the network shown in Fig. 12.17.

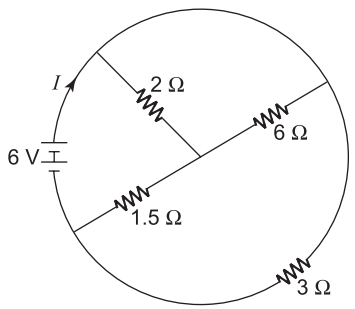


Fig. 12.17

SOLUTION The circuit can be redrawn as shown in Fig. 12.18. The resistance between *A* and *B* is

$$R_1 = \frac{2 \times 6}{(2 + 6)} + 1.5 = 3\ \Omega$$

Resistance between *C* and *D* is

$$R = \frac{3 \times 3}{(3 + 3)} = 1.5\ \Omega$$

$$\therefore \quad \text{Current } I = \frac{V}{R} = \frac{6}{1.5} = 4\ \text{A}$$

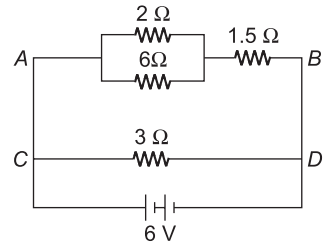


Fig. 12.18

8. Kirchhoff's Laws

First Law or Junction Rule

The algebraic sum of the currents at a junction in a circuit is zero. The currents entering the junction are taken as positive and those leaving the junction are taken as negative. Figure 12.19 shows a part of a circuit having a junction *A*.

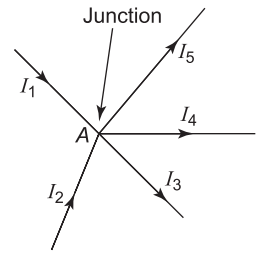


Fig. 12.19

$$I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

$$\Rightarrow \quad I_1 + I_2 = I_3 + I_4 + I_5$$

i.e. sum of currents entering a junction = sum of currents leaving that junction.

Second Law or Loop Rule

In any closed circuit (or loop), the algebriac sum of the potential differernces across the sources of current and across the resistances in the circuit (or loop) is zero. The sources of current are the emfs of the cells and potential differences across the resistance are the voltage drops (*IR*).

Sign Convention for emfs and Voltage drops

Consider the circuit shown in Fig. 12.20. The circuit has three closed loops *abefa*, *bcdeb* and *acdfa*. To use the loop rule, follow the following steps.

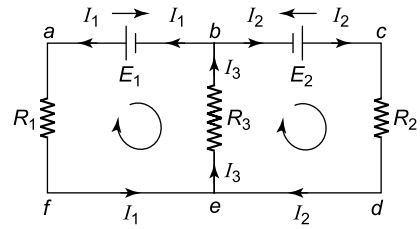


Fig. 12.20

1. Draw a arrow on the top of each cell pointing from the positive to the negative terminal.
2. Choose a closed loop and move in a clockwise direction in that loop.
3. While crossing a cell, if you are moving in the direction of the arrow drawn on the cell, the emf of the cell is taken as positive but if you are moving opposite to the direction of the arrow, the emf is taken as negative. Do not worry about the direction of the current in the branch of the circuit containing that cell.
4. While crossing a resistor, if you are moving in the direction of the current through that resistor, the potential drop (IR) across the resistor is taken as positive but if you are moving opposite to the direction of the current, the potential drop across the resistor is taken as negative.

In the circuit shown in Fig. 12.20, there are two junctions at b and e. Applying the junction rule at either b or e we have

$$I_1 + I_2 = I_3 \quad (1)$$

Applying the loop rule to loops *abefa* and *bcdeb* we have

$$E_1 - I_3R_3 - I_1R_1 = 0 \quad (2)$$

and $$-E_2 + I_2R_2 + I_3R_3 = 0 \quad (3)$$

If the values of E_1, E_2, R_1, R_2 and R_3 are known, the values of I_1, I_2 and I_3 can be obtained by the simultaneous solutions of Eqs. (1), (2) and (3).

Note

1. Select as many loops as the number of unknowns. In the circuit shown in Fig. 12.20. There are two unknowns I_1 and I_2. The third unknown I_3 is determined by using the junction rule [Eq. (1)]. So we select two out of the three loops.
2. If the directions of currents are not given, choose any direction (clockwise or anticlockwise) in a loop and calculate the values of currents. If any current turns out to be negative, it indicates that our choice of the direction of that current is incorrect. So reverse the direction of that current. The magnitude of the current remains the same.

EXAMPLE 14 Figure 12.21 shows currents in a part of an electrical circuit. Find the value of current I.

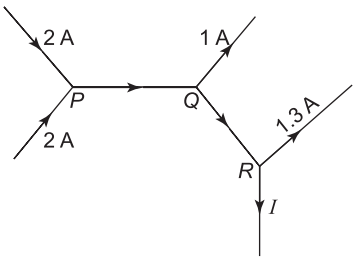


Fig. 12.21

SOLUTION Applying the junction rule at P, the current in branch $PQ = 2 + 2 = 4$ A.

Using the junction rule at Q, the current in branch $QR = 4$ A $- 1$ A $= 3$ A.

Using the junction rule at R, we get

$$I + 1.3 \text{ A} = 3 \text{ A} \quad \Rightarrow \quad I = 1.7 \text{ A}$$

EXAMPLE 15 In the circuit shown in Fig. 12.22, calculate

(a) the values of currents I_1, I_2 and I_3

(b) the potential difference between points B and E

Given $E_1 = 12$ V, $E_2 = 6$ V, $R_1 = 5\ \Omega$, $R_2 = 3\ \Omega$ and $R_3 = 2\ \Omega$

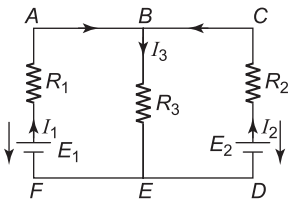


Fig. 12.22

SOLUTION

(a) Applying junction rule at B,

$$I_1 + I_2 = I_3 \quad (1)$$

Applying loop rule to loop *ABEFA*

$$I_3R_3 - E_1 + I_1R_1 = 0$$

$$\Rightarrow \quad 2I_3 - 12 + 5I_1 = 0 \quad (2)$$

Applying loop rule to loop *BCDEB*

$$-I_2R_2 + E_2 - I_3R_3 = 0$$

$$\Rightarrow \quad -3I_2 + 6 - 2I_3 = 0 \quad (3)$$

Using (1) in (2) and (3), we get

$$2(I_1 + I_2) - 12 + 5I_1 = 0$$

$$\Rightarrow \quad 7I_1 + 2I_2 = 12 \qquad (4)$$

and $-3I_2 + 6 - 2(I_1 + I_2) = 0$

$$\Rightarrow \quad 2I_1 + 5I_2 = 6 \qquad (5)$$

Solving (5) and (6) we get $I_1 = \frac{48}{31}$ A , $I_2 = \frac{18}{31}$ A and

$$I_3 = I_1 + I_2 = \frac{66}{31} \text{ A}$$

(b) To find potential difference between B and E, we start from B and go to E.

$$V_B - V_E = I_3 R_3 = \frac{66}{31} \times 2 = 4.26 \text{ V}$$

EXAMPLE 16 Two cells of emfs E_1 = 1.5 V and E_2 = 2.0 V and internal resistances r_1 = 1.0 Ω and r_2 = 1.5 Ω respectively are connected to an external resistor R = 2.5 Ω as shown in Fig. 12.23. Find the potential difference

(a) between points A and B

(b) between points B and C

(c) across R.

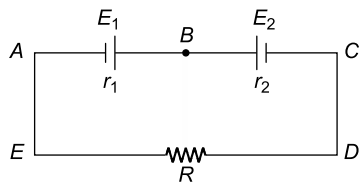


Fig. 12.23

SOLUTION We choose any direction for the current I in the circuit. If the value of I turns out to be negative, then our choice is incorrect, the direction of the current has to be reversed. Let us choose the direction of I as shown in Fig. 12.24.

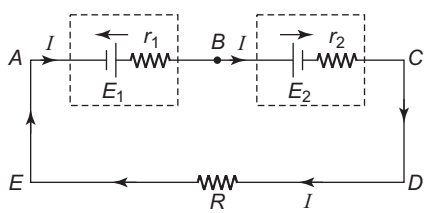


Fig. 12.24

Applying loop rule to loop $ABCDEA$,

$$-E_1 + Ir_1 + E_2 + Ir_2 + IR = 0$$

$$\Rightarrow \quad -1.5 + I \times 1.0 + 2.0 + I \times 1.5 + 2.5I = 0$$

$$\Rightarrow \quad I = -0.1 \text{ A}$$

Since I is negative, our choice for the direction of I is incorrect. So the direction of I will have to be reversed as shown in Fig. 12.25 where now I = + 0.1 A.

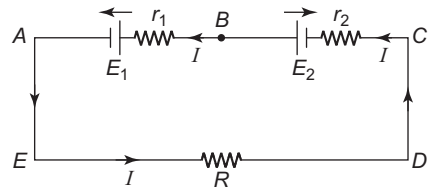


Fig. 12.25

(a) $V_A - V_B = -E_1 - Ir_1 = -1.5 - 0.1 \times 1.0 = -1.6$ V. The negative sign indicates that A is at a lower potential than B.

(b) $V_B - V_C = E_2 - Ir_2 = 2.0 - 0.1 \times 1.5 = 1.85$ V. Point B is at a higher potential that point C.

(c) $V_R = IR = 0.1 \times 2.5 = 0.25$ V.

EXAMPLE 17 Find the value of current I in the circuit shown in Fig. 12.26.

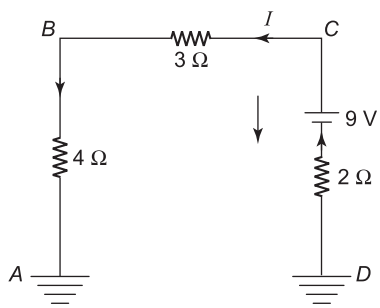


Fig. 12.26

SOLUTION $V_A - V_D = -4I - 3I + 9 - 2I$

$$= -9I + 9$$

But $V_A = V_D$ because points A and D are earthed.

$$\therefore \qquad -9I + 9 = 0 \quad \Rightarrow \quad I = 1 \text{ A}$$

EXAMPLE 18. In the circuit shown in fig 12.27(a), the potential difference between points B and H is:

(a) 2V (b) 3V

(c) 4V (d) 5V

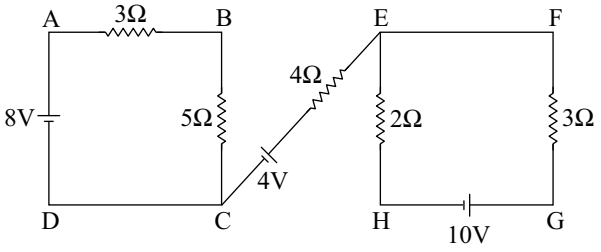


Fig. 12.27 (a)

SOLUTION Let I_1 and I_2 be the currents in the loops ABCD and EFGH respectively. No current flows in the branch CE of the circuit [see fig. 12.27(b)].

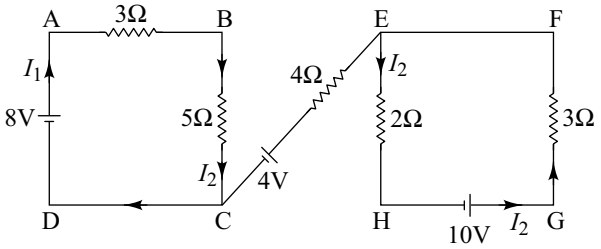


Fig. 12.27 (b)

Applying loop rule to loop ABCDA,

$$3I_1 + 5I_1 - 8 = 0 \quad \Rightarrow \quad I_1 = 1\text{A}$$

Applying loop rule to loop EFGHE,

$$-3I_2 + 10 - 2I_2 = 0 \quad \Rightarrow \quad I_2 = 2\text{A}$$

$$\therefore \quad V_B - V_D = 5I_1 - 4 + 2I_2 = 5 \times 1 - 4 + 2 \times 2 = 5\text{V}$$

So the correct choice is (d).

9. Wheatstone's Bridge

The network of four resistances P, Q, R and S shown in Fig. 12.28 is called Wheatstone's Bridge. A battery is connected between A and C and a galvanometer is connected between B and D. If the values of P, Q, R and S are such that no current flows through the galvanometer, the bridge is said to be balanced. Then points B and D are at the same potential.

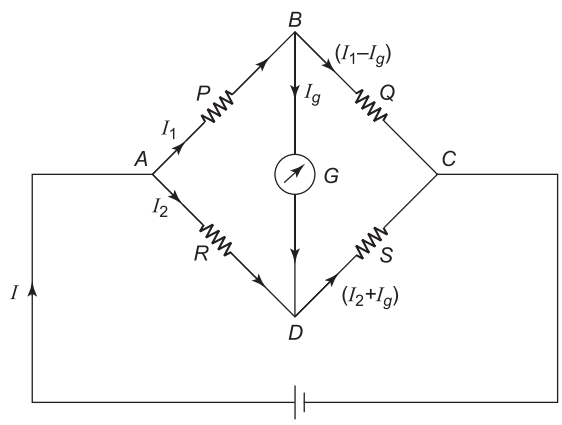


Fig. 12.28

Condition for balanced Wheatstone's Bridge

The currents in the branches of the bridge are shown in the figure. Applying the loop rule to loops $ABDA$ and $BCDB$, we have

$$I_1P + I_gG - I_2R = 0 \qquad \text{(i)}$$

and $$(I_1 - I_g)Q - (I_2 + I_g)S - I_gG = 0 \qquad \text{(ii)}$$

where G is the galvanometer resistance. For a balanced bridge, $I_g = 0$. Putting $I_g = 0$ in (i) and (ii) we get

$$I_1P - I_2R = 0 \quad \Rightarrow \quad \frac{I_1}{I_2} = \frac{R}{P} \qquad \text{(iii)}$$

and $$I_1Q - I_2S = 0 \quad \Rightarrow \quad \frac{I_1}{I_2} = \frac{S}{Q} \qquad \text{(iv)}$$

From (iii) and (iv) we get

$$\frac{P}{Q} = \frac{R}{S} \quad \text{or} \quad \frac{P}{R} = \frac{Q}{S}$$

This is the condition for a balanced bridge.

Figure 12.29 shows a simple form of a Wheatstone's Bridge, called the metre bridge.

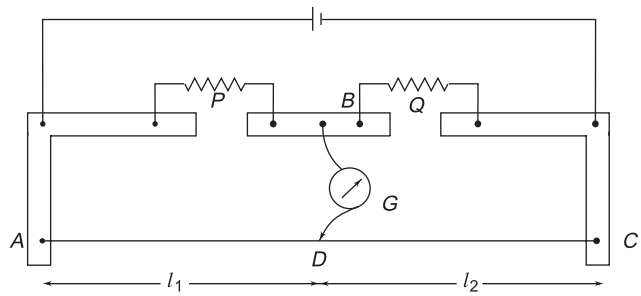


Fig. 12.29

The resistance of the wire of length $AD = l_1$ serves as the resistor R and the resistance of the wire of length $DC = l_2$ serves as the resistor S. If the wire AC is uniform, the resistances of the parts AD and DC of the wire will be proportional to their lengths l_1 and l_2. For a balanced metre bridge, we have

$$\frac{P}{Q} = \frac{R}{S} = \frac{l_1}{l_2}$$

EXAMPLE 19 Find the equivalent resistance between points A and B in the circuits shown in Figs 12.30(a), 12.30(b) and 12.30(c). All resistors have the same resistance R.

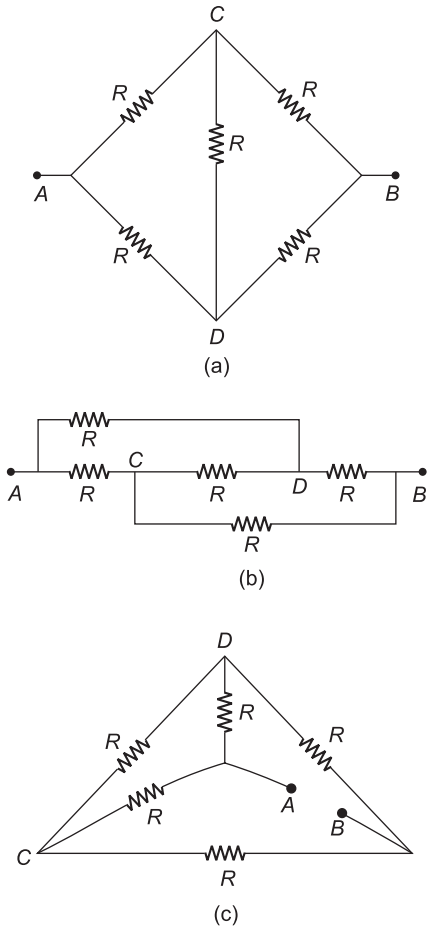


Fig. 12.30

SOLUTION Circuits shown in Figs 12.30(b) and (c) can be redrawn as the circuit shown in Fig. 12.30(a), which is a balanced Wheatstone's Bridge. Hence no current flows through the resistor between C and D because C and D are at the same potential. Hence this resistor is ineffective and circuits (a), (b) and (c) simplify as shown in Fig. 12.30(d). Hence the equivalent resistance between A and B is the resistance of the parallel combination of resistances $2R$ and $2R$.

$$\therefore \qquad R_{AB} = \frac{2R \times 2R}{(2R + 2R)} = R$$

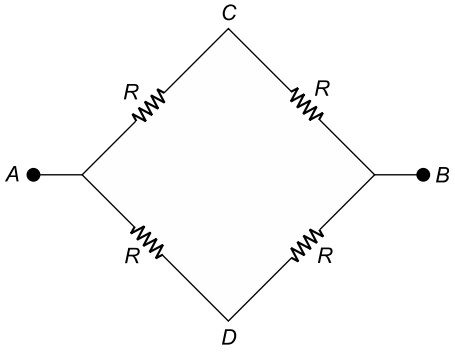


Fig. 12.30 (d)

EXAMPLE 20 Find the equivalent resistance between points A and B in the circuit shown in Fig. 12.31.

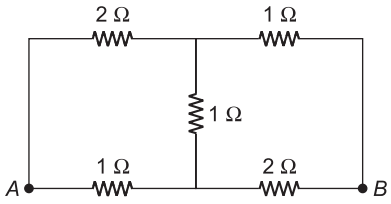


Fig. 12.31

SOLUTION The network of resistances shown in Fig. 12.31 is not a balanced Wheatstone's bridge. In such a case, to find the resistance between A and B, we connect a battery of voltage V across A and B and find the current I drawn from the battery in terms of V and find the equivalent resistance from the relation $V = IR$. Refer to Fig. 12.32.

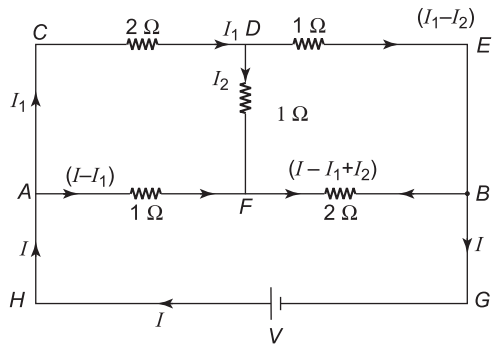


Fig. 12.32

The currents in various branches are shown in Fig. 12.32. Applying loops rule to loops $AFBGHA$, $ACDFA$ and $DEBFD$ we have

$$V - 1(I - I_1) - 2(I - I_1 + I_2) = 0 \qquad \text{(i)}$$

$$2I_1 + I_2 - (I - I_1) = 0 \qquad \text{(ii)}$$

and $$(I_1 - I_2) + 2(I - I_1 + I_2) - I_2 = 0 \qquad \text{(iii)}$$

Simplifying these equations we get

$$3I - 3I_1 + 2I_2 = V \qquad \text{(iv)}$$

$$I - 3I_1 - I_2 = 0 \qquad \text{(v)}$$

and $$2I - 3I_1 + 4I_2 = 0 \qquad \text{(vi)}$$

Eliminating I_1 from (v) and (vi), we have $I_2 = -\frac{I}{5}$.

Using this value of I_1 in (v) we get $I_1 = \frac{2}{5}I$.

Using these values of I_1 and I_2 in (iv) we get

$$7I = 5V \quad \Rightarrow \frac{V}{I} = \frac{7}{5} = 1.4 \quad \Rightarrow \quad R_{AB} = 1.4\ \Omega$$

EXAMPLE 21 Figure 12.33 shows a metre bridge consisting of two resistances R_1 and R_2 connected to a wire AC of length 100 cm. The null point is found to be at a distance $l_1 = 40$ cm from end A. When a 12 Ω resistance is connected in parallel with R_2, the null point shifts to D_2 such that $l_2 = 60$ cm. Find R_1 and R_2.

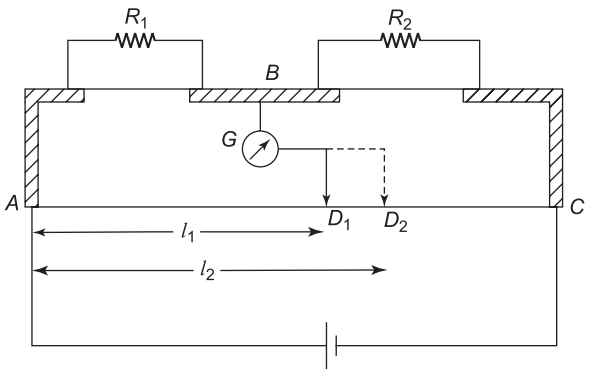


Fig. 12.33

SOLUTION

$$\frac{R_1}{R_2} = \frac{l_1}{100 - l_1} = \frac{40}{100 - 40} = \frac{40}{60} = \frac{2}{3} \quad (1)$$

When a resistance of 12 Ω is connected in parallel with R_2, the resistance of the combination is

$$R_2' = \frac{12R_2}{12 + R_2}$$

$$\therefore \quad \frac{R_1}{R_2'} = \frac{60}{100 - 60} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{R_1 \times (12 + R_2)}{12R_2} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{R_1}{R_2} \times \frac{(12 + R_2)}{12R_2} = \frac{3}{2} \quad (2)$$

Using (1) in (2),

$$\frac{2}{3} \times \frac{(12 + R_2)}{12R_2} = \frac{3}{2} \quad \Rightarrow \quad R_2 = 15\ \Omega$$

and $$R_1 = \frac{2}{3}R_2 = \frac{2}{3} \times 15 = 10\ \Omega$$

10. The Potentiometer

Figure 12.34 shows a potentiometer where AB is a wire of uniform cross-section, V is the driver battery and E is a cell. The emf of the battery is greater than that of the cell.

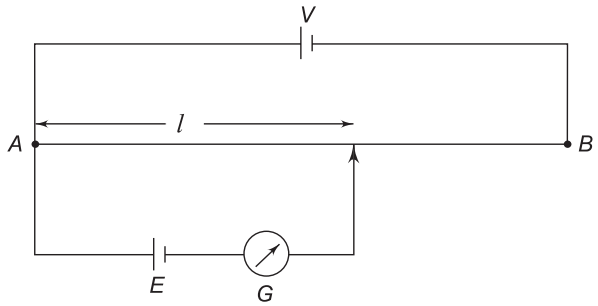


Fig. 12.34

The principle of a potentiometer is based on the fact that the potential difference across any length of the wire is proportional to that length of the wire. If R is the resistance of the potentiometer wire AB and L is its length, then $R = \rho L/A$ where ρ is the resistivity of the material of the wire and A its cross-sectional area. From Ohm's law, $V = IR$ or $V = I\rho L/A = KL$ where $K = I\rho/A$, which is constant for a given wire. Hence

$$V \propto L$$

or $\frac{V}{L} = K$ = potential gradient or fall of potential per unit length of the potentiometer wire. Hence potential difference v across any portion l of the wire is $v = Kl$. The galvanometer will show no deflection if v = emf E of the cell E or $E = v = Kl$. The length l is called the balancing length of the potentiometer wire. At balance point, potential difference across l due to the driver cell V = emf of cell E

Applications of Potentiometer

1. *Comparison of emfs of two cells*
   If l_1 and l_2 are the balancing lengths will cells of emfs E_1 and E_2, then
   $$E_1 = Kl_1 \text{ and } E_2 = Kl_2$$
   $$\therefore \quad \frac{E_1}{E_2} = \frac{Kl_1}{Kl_2} = \frac{l_1}{l_2}$$

2. *Determination of internal resistance of a cell*
   If l_1 is the balancing length will cell of emf E when switch S is open [Fig. 12.35], then
   $$E = Kl_1$$

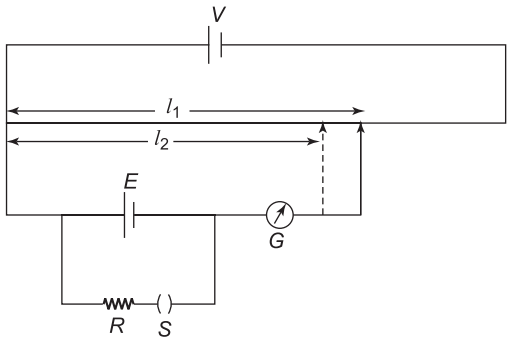


Fig. 12.35

A known resistance R is connected across the cell E, switch S is closed and the new balancing length l_2 is found, then (since a current is now drawn from cell E), the terminal voltage V of E is equal to Kl_2, i.e.

$$V = Kl_2$$

$$\therefore \quad \frac{E}{V} = \frac{Kl_1}{Kl_2} = \frac{l_1}{l_2}$$

Now $E = V + v = IR + Ir$ where I is the current drawn from E and r is its internal resistance. Thus

$$\frac{V+v}{V} = \frac{l_1}{l_2} \Rightarrow 1 + \frac{v}{V} = \frac{l_1}{l_2}$$

$$\Rightarrow \quad \frac{v}{V} = \frac{l_1}{l_2} - 1$$

$$\Rightarrow \quad \frac{Ir}{IR} = \frac{l_1}{l_2} - 1$$

$$\Rightarrow \quad r = R\left(\frac{l_1}{l_2} - 1\right)$$

Knowing the values of R, l_1 and R_2, the value of r is determined.

EXAMPLE 22 A uniform wire of length 400 cm and resistance 2 Ω is used in a potentiometer. The wire is connected in series with a battery of emf 5 V and an external resistance of 3 Ω. With a cell of unknown emf E, the balancing length is found to be 250 cm. Find (a) the potential gradient along the potentiometer wire and (b) the value of E.

SOLUTION Refer to Fig. 12.36. $AB = L = 400$ cm $= 4$ m, $r = 2$ Ω, $R = 3$ Ω and $V = 5$ V, and $AC = l = 250$ cm $= 2.5$ m.

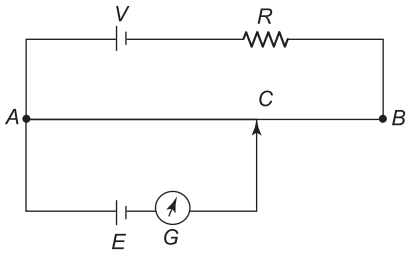


Fig. 12.36

(a) Current in AB due to driver cell is

$$I = \frac{V}{(R+r)} = \frac{5}{(3+2)} = 1 \text{ A}$$

Potential drop across $AB = I \times r$

$$= 1 \times 2 = 2 \text{ V}$$

$\therefore$ Potential gradient along AB is $K = \dfrac{2\text{V}}{4\,\text{m}}$

$$= 0.5 \text{ Vm}^{-1}$$

(b) $E = Kl = 0.5 \times 2.5 = 1.25$ V

EXAMPLE 23 In the potentiometer circuit shown in Fig. 12.37, find the value of l when the galvanometer shows no deflection. The length of wire AB is 100 cm and its resistance is 3 Ω.

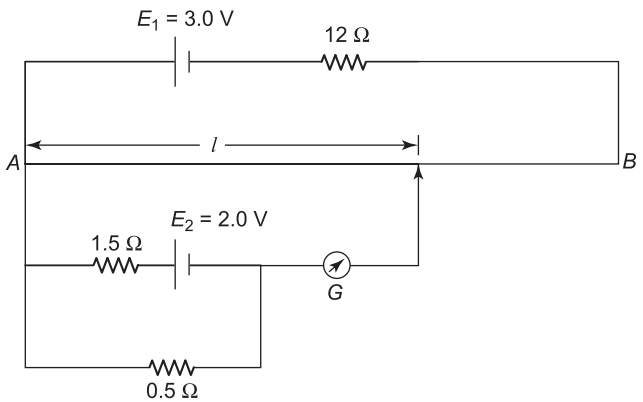


Fig. 12.37

SOLUTION Current flowing in AB due to driver cell of emf 3.0 V is

$$I_1 = \frac{3.0}{(12+3)} = 0.2 \text{ A}$$

$\therefore$ Potential difference across AB due to E_1 is

$$= 0.2 \times 3 = 0.6 \text{ V}$$

Potential difference across l due to E_1 is (l is in cm)

$$V_l = \frac{0.6}{100} \times l \qquad (1)$$

Current through 0.5 Ω resistance due to E_2 is

$$I_2 = \frac{2.0}{(1.5 + 0.5)} = 1.0 \text{ A}$$

$\therefore$ Potential difference across 0.5 Ω resistance

$$= 0.5 \times 1.0 = 0.5 \text{ V.}$$

At balance length l, the potential difference across l due to E_2 is

$$V'_l = 0.5 \text{ V} \qquad (2)$$

At balance point $V_l = V'_l$. Equating (1) and (2), we have

$$\frac{0.6}{100} \times l = 0.5 \quad \Rightarrow \quad l = 83.3 \text{ cm}$$

EXAMPLE 24 The length of a potentiometer wire is 600 cm and it carries a current of 40 mA. For a cell of emf 2V and internal resistance 10 Ω, the balancing length is found to be 500 cm. If a voltmeter is connected across the cell, the balancing length is decreased by 10 cm. Find (a) the resistance of the potentiometer wire, (b) the resistance of voltmeter and (c) the reading of the voltmeter.

SOLUTION Refer to Fig. 12.38. AB = 600 cm, I = 40 mA = 0.04 A, AC = 500 cm and AC' = 490 cm. Let r be the resistance of AB.

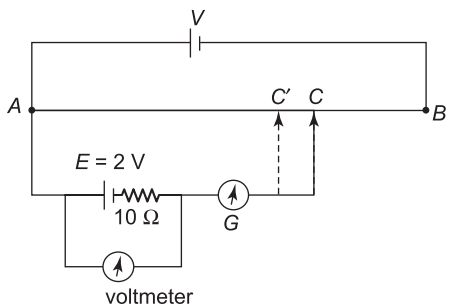


Fig. 12.38

(a) Resistance per cm of $AB = \frac{r}{600}$ Ω cm^{-1}. Therefore, the resistance of AC is $R_{AC} = \frac{r}{600} \times 500 = \frac{5r}{6}$ Ω

$\therefore$ Potential difference across AC due to driver cell is

$$V_{AC} = IR_{AC} = 0.04 \times \frac{5r}{6} \text{ volt} \qquad (1)$$

When the voltmeter is not connected, the potential difference across AC due to cell of emf E is 2 V (because no current is drawn from it at the balance point). Hence

$$V_{AC} = 2$$

$$\Rightarrow \frac{0.04 \times 5r}{6} = 2 \quad \Rightarrow \quad r = 60 \text{ Ω}$$

(b) Let R be the resistance of voltmeter. When it is connected across E, the current drawn from the cell of emf 2 V is

$$I' = \frac{2}{R + 10}$$

$\therefore$ Potential difference across the voltmeter is

$$V_R = I'R = \frac{2R}{R + 10}$$

V_R must be equal to the potential difference across AC' due to the driver cell.

$\therefore$ V_R = resistance of $AC' \times$ current I

$$= \frac{60 \times 490}{600} \times 0.04 = 1.96$$

$$\Rightarrow \frac{2R}{(R + 10)} = 1.96 \quad \Rightarrow \quad R = 490 \text{ Ω}$$

(c) Voltmeter reading is

$$V_R = \frac{2R}{(R + 10)} = \frac{2 \times 490}{(490 + 10)} = 1.96 \text{ V}$$

11. Finding Equivalent Resistance by Using Symmetry in Networks

Let us consider a balanced Wheatstone's bridge again as shown in Fig. 12.39(a).

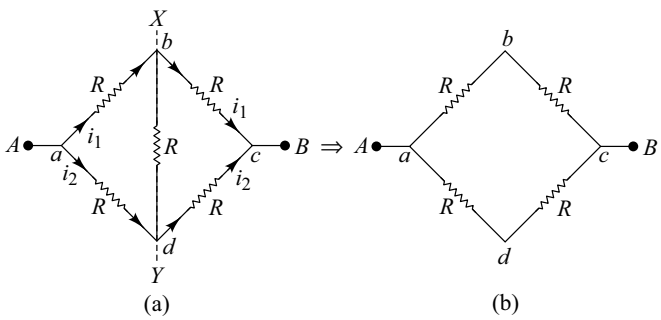


Fig. 12.39

Notice that the network is symmetric about the line XY. By symmetry, we mean that the circuit to the right of XY is just the mirror image of the circuit to the left of XY. In such networks, the potentials and currents must also by symmetrical. Therefore, the current in branch ab is equal to the current in branch bc. Also the current in ad is equal to the current in

dc. Furthermore, the potential at *b* is equal to the potential at *d*. Hence no current flows in the resistor between *b* and *d* (*i.e.,* the resistor which lies in the line of symmetry). This is called the symmetry condition. Since no current flows through resistor *R* between *b* and *d*, this resistor is ineffective and can be removed. So the circuit simplifies to that shown in Fig. 12.39(b). The equivalent resistance between points *A* and *B* is

$$\frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{2R} \quad \Rightarrow \quad R_{eq} = R$$

The same symmetry condition applies also to a network of capacitors as shown in Fig. 12.40.

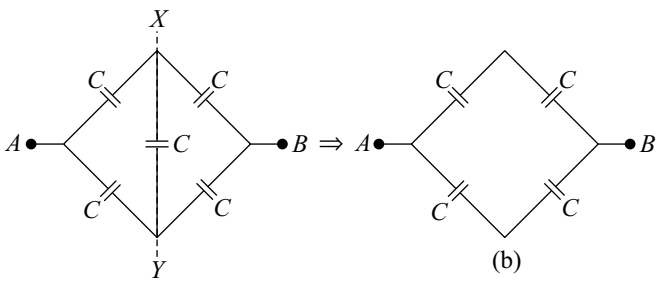


Fig. 12.40

The equivalent capacitance between points *A* and *B* is

$$C_{eq} = \frac{C}{2} + \frac{C}{2} = C$$

Now let us consider a more complicated network as shown in Fig. 12.41(a).

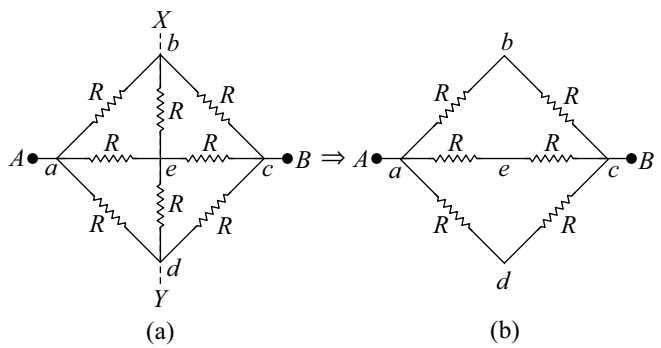


Fig. 12.41

In this case, resistances between *b* and *e* and between *e* and *d* are ineffective (because these resistances lie along the line *XY* of symmetry) and the network simplifies to that shown in Fig. 12.41(b). The equivalent resistance between *A* and *B* is

$$\frac{1}{C_{eq}} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{2R} \quad \Rightarrow \quad C_{eq} = \frac{2R}{3}$$

Fig. 12.42 shows the corresponding network of capacitors.

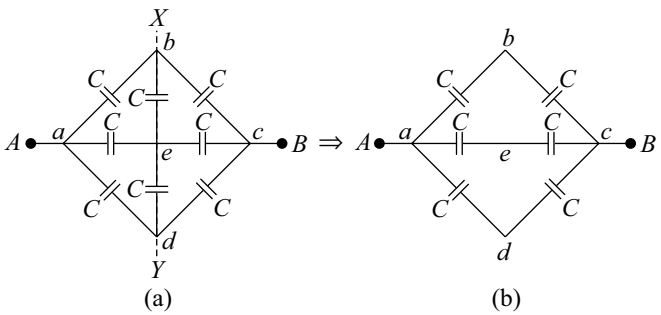


Fig. 12.42

In this case, the equivalent capacitance between *A* and *B* is

$$C_{eq} = \frac{C}{2} + \frac{C}{2} + \frac{C}{2} \quad \Rightarrow \quad C_{eq} = \frac{3C}{2}$$

EXAMPLE 25. Each resistance of the network shown in Fig. 12.43 is equal to *R*. The equivalent resistance between points *A* and *B* is

(a) R (b) $\frac{2R}{3}$

(c) $\frac{4R}{5}$ (d) $\frac{6R}{7}$

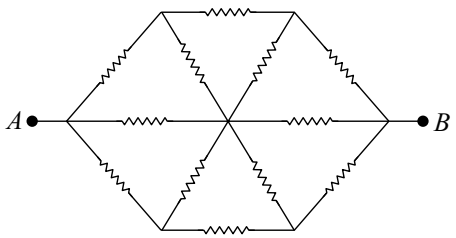
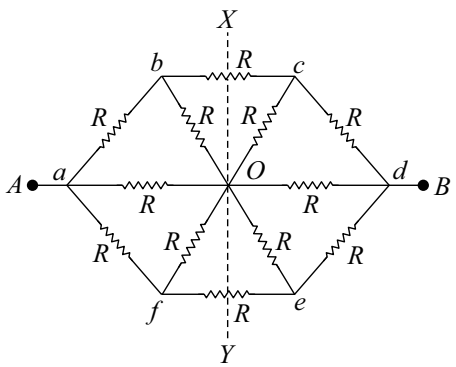
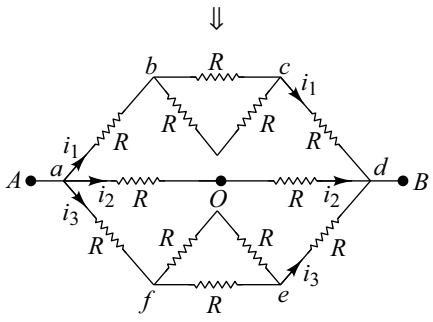


Fig. 12.43

SOLUTION The line of symmetry is *XY* as shown in Fig. 12.44 (a).



(a)



(b)

Fig. 12.44

The current i_1 through ab must be equal to the current through its mirror image cd. The current i_3 through af must be equal to its mirror image ed. Current i_2 entering O must be equal to the current leaving O. The is no mixing of currents from the circuit above AB with the circuit below AB. Hence, the network can be simplified to that shown in Fig. 12.44 (b). It is now easy to find the equivalent resistance between A and B, which turns out to be $4R/5$. So the correct choice is (c).

EXAMPLE 26. Each capacitor in the network shown in Fig. 12.45 has capacitance C. The equivalent capacitance between points A and B is

(a) $\frac{4C}{5}$ (b) $\frac{5C}{4}$

(c) $\frac{7C}{5}$ (d) $\frac{4C}{3}$

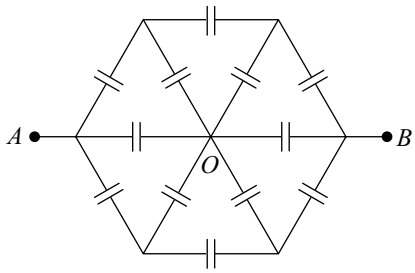


Fig. 12.45

SOLUTION Using symmetry about the perpendicular line passing through O, the above network can be simplified as shown in Fig. 12.46.

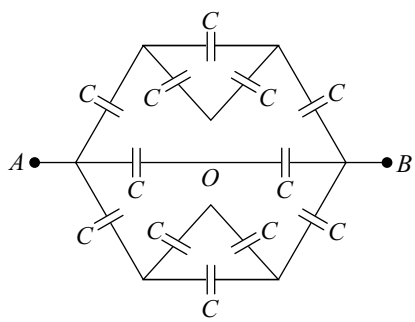


Fig. 12.46

It is now easy to prove that the equivalent capacitance between points A and B is $\frac{5C}{4}$.

11. Ammeter

Ammeter is used for measuring current in a circuit. It is a galvanometer having a very small resistance (called shunt) connected in parallel with it. The ammeter is always connected in series in the circuit the current through which is to be measured.

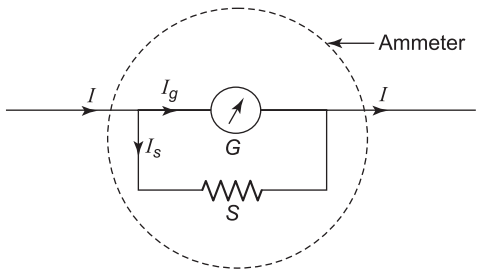


Fig. 12.47

The range of an ammeter is the maximum current it can measure. The value of the shunt resistance determines the range of an ammeter. Figure 12.47 shows a part of a circuit, where

I = current to be measured,

G = galvanometer resistance,

I_g = current through the galvanometer for full scale deflection, and

S = shunt resistance

It is clear that (since p.d. across G = p.d. across S)

$$I_gG = I_sS. \quad \text{Also } I = I_g + I_s$$

$$\therefore \quad I_gG = (I - I_g)S$$

$$\Rightarrow \quad S = \left[\frac{I_g}{(I - I_g)}\right]G$$

EXAMPLE 27 A galvanometer of resistance 20 Ω gives full scale deflection when a current of 1 mA is passed through it

(a) How will you convert it into a ammeter that can read currents upto 1.0 A?

(b) What is the resistance of the ammeter? When it is connected in a circuit, does it read slightly less or more than the actual current in the original circuit?

SOLUTION

(a) Given $I = 1.0$ A, $I_g = 1$ mA $= 00.1$ A and $G = 20$ Ω.

$$S = \left(\frac{I_g}{I - I_g}\right)G = \left(\frac{0.001}{1.0 - 0.001}\right) \times 20 \simeq 0.02\ \Omega$$

The required shunt resistance is 0.02 Ω.

(b) Resistance of the ammeter is

$$R_A = \frac{GS}{G+S} = \frac{20 \times 0.02}{(20+0.02)} \simeq 0.02\ \Omega$$

Since the ammeter is connected in series in the circuit, it will slightly increase the total resistance of the circuit and hence the current in the circuit will slightly decrease. Hence the ammeter will read slightly less than the actual current.

12. Voltmeter

Voltmeter is used for measuring potential difference across a resistor in a circuit. It is a galvanometer having a very high resistance connected in series with it. The voltmeter is always connected in parallel with the resistor across which the potential difference is to be measured.

The range of a voltmeter is the maximum voltage it can measure. The range is determined by the value of high resistance connected in series with it. Figure 12.48 shows a voltmeter, where

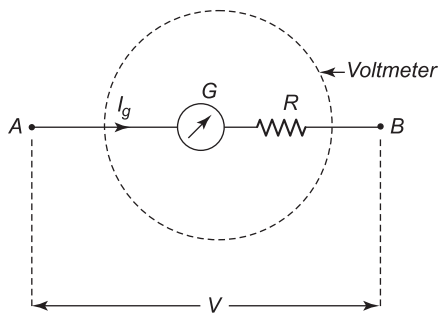


Fig. 12.48

V = voltage to be measured,
I_g = current for full scale deflection
R = required high resistance.

It is clear that

$$V = V_G + V_R$$
$$= I_g G + I_g R$$

$$\Rightarrow \quad \frac{V}{I_g} = G + R \quad \Rightarrow \quad R = \frac{V}{I_g} - R$$

EXAMPLE 28 A galvanometer of resistance 20 Ω gives full scale deflection when a current of 1 mA is passed through it.

(a) How will you convert it into a voltmeter that can read voltages up to 5 V?

(b) What is the resistance of the voltmeter? When it is connected across a resistor in a circuit, will it read slightly less or more than the original voltage?

SOLUTION

(a) Given $G = 20\ \Omega$, $I_g = 1$ mA $= 0.001$ A and $V = 5$ V.

$$R = \frac{V}{I_g} - G = \frac{5}{0.001} - 20 = 4980\ \Omega$$

The required high resistance to be connected in series with the galvanometer is 4980 Ω.

(b) Resistance of voltmeter is

$$R_V = R + G = 4980 + 20 = 5000\ \Omega$$

When the voltmeter is connected across a resistor of resistance $R << R_V$, the current in the main circuit divides between R and R_V. Since $R_V >> R$, a small current will flow through the voltmeter. Hence the current through R decreases slightly. From Ohm's law, the potential differernce across R will decrease slightly. Hence the voltmeter will read slightly less than the actual voltage across R.

EXAMPLE 29 Two resistors of 400 Ω and 600 Ω are connected in series with a 5.0 V battery of negligible internal resistance.

(a) An ammeter of resistance 20 Ω is used to measure the current in the circuit. Find the error in the measurement of current.

(b) A voltmeter of resistance 9600 Ω is used to measure the potential difference across the 400 Ω resistor. Find the error in the measurement of potential difference.

SOLUTION

(a) Actual current without ammeter in the circuit is (Fig. 12.49)

$$I = \frac{5.0}{(400+600)} = 0.005\ \text{A} = 5\ \text{mA}$$

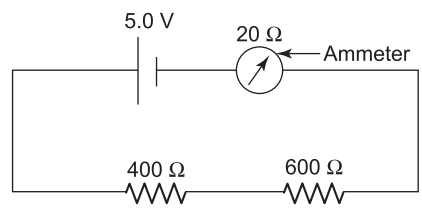


Fig. 12.49

When the ammeter is connected, the total resistance = 400 + 600 + 20 = 1020 Ω. The current indicated by the ammeter will be

$$I' = \frac{5.0}{1020} = 0.0049\ \text{A} = 4.9\ \text{mA}$$

Hence the ammeter reads less than the actual current. Error in the measurement of current = 5.0 mA – 4.9 mA = 0.1 mA

(b) When a voltmeter of resistance 9600 Ω is connected across the 400 Ω resistor as shown in Fig. 12.50, the equivalent resistance of the combination is

$$R = \frac{400 \times 9600}{(400 + 9600)} = 384\ \Omega$$

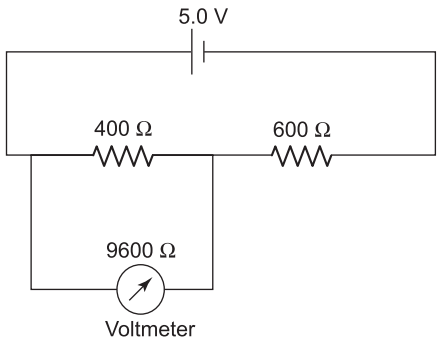


Fig. 12.50

∴ Total resistance of the circuit is

$$R' = 384 + 600 = 984\ \Omega$$

∴ Current in the circuit is

$$I' = \frac{5.0}{984}\text{A}$$

The potential difference indicated by the voltmeter will be

$$V' = \frac{5.0}{984} \times 384 = 1.95\ \text{V}$$

Actual potential difference across the 400 Ω resistance (without voltmeter connected across it) is

$$V = I \times 400 = 0.005 \times 400 = 2.0\ \text{V}$$

Hence the voltmeter reads less than the actual voltage. Error in measurement = 2.0 – 1.95 = 0.05 V

13. Heating Effect of Current

If a current I amperes flows for time t seconds through a resistor R ohms across which a potential difference V volts is maintained, the amount of heat energy H (in joules) delivered is given by

$$H = VIt = I^2 Rt = \frac{V^2 t}{R}$$

To obtain heat in calories, H is divided by $J = 4.2$ joules per calorie is called the mechanical equivalent of heat.

Electrical Power If E is the emf of a source of internal resistance r, the power delivered to an external circuit of resistance R (called the output power) is given by

$$P_{\text{out}} = VI = I^2 R = \frac{V^2}{R} = \frac{E^2 R}{(R+r)^2}$$

$[\because E = I(R + r)]$

Electrical Energy The SI unit of energy is joule. A practical unit for electrical energy is called kilowatt hour (kWh). Since watt = joule per second, watt second is the same as joule. Hence

1 watt hour = 1 watt × (60 × 60) seconds

= 3600 watt seconds

= 3600 joules

∴ 1 kilowatt hour = 1000 watt hours

= 1000 × 3600 joules

$= 3.6 \times 10^6$ joules

Note that watt and kilowatt are units of electrical power and watt hour and kilowatt hour are units of electrical energy.

14. Power-Voltage Rating of Electrical Appliances

Every electrical appliance has a specified power-voltage ($P - V$) rating which determines the resistance of the appliance and the current it will draw. Since $P = VI$, the current that the appliance will draw is given by

$$I = \frac{P}{V}$$

The electrical wiring should be able to withstand this current. The resistance of the appliance is given by

$$R = \frac{V}{I} = \frac{V}{P/V} = \frac{V^2}{P}$$

1. *Power of Electrical Appliances Connected in Parallel*

Let R_1, R_2, R_3, ... be the resistances of the electrical appliances meant to operate at the same voltage V and let P_1, P_2, P_3, ... be their respective electrical powers. Then

$$R_1 = \frac{V^2}{P_1}, R_2 = \frac{V^2}{P_2}, R_3 = \frac{V^2}{P_3}, \ldots$$

or

$$P_1 = \frac{V^2}{R_1}, P_2 = \frac{V^2}{R_2}, P_3 = \frac{V^2}{R_3}, \ldots$$

When the appliances are connected in parallel, their combined resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

∴ Total power consumed is

$$P = \frac{V^2}{R} = V^2 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots\right)$$

$$= \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^2}{R_3} + \cdots$$

or $$P = P_1 + P_2 + P_3 + \cdots$$

2. *Power of Electrical Appliances Connected in Series*

The total resistance R when the appliances are connected in series is given by

$$R = R_1 + R_2 + R_3 + \cdots$$

$\therefore$ Total power consumed is

$$P = I^2R = I^2 (R_1 + R_2 + R_3 + \cdots)$$

$\Rightarrow$ $$P = P_1 + P_2 + P_3 + \cdots$$

EXAMPLE 30 A 1 kW heater is designed to operate on a 200 volt supply. Find (a) the resistance of the heater and (b) by what percentage will its power drop if the supply voltage drops to 160 V?

SOLUTION

(a) $P = 1$ kW $= 1000$ W, $V = 200$ volt. The resistance of the heater is

$$R = \frac{V^2}{P} = \frac{200 \times 200}{1000} = 40\ \Omega$$

(b) Since R remains unchange, the power of the heater when the voltage drops to $V' = 160$ V will be

$$P' = \frac{V'^2}{R} = \frac{160 \times 160}{40} = 640 \text{ W}$$

$\therefore$ Percentage drop in power $= \dfrac{P - P'}{P} \times 100$

$$= \left(\frac{1000 - 640}{1000}\right) \times 100 = 36\%$$

EXAMPLE 31 An electric bulb has a rating of 40 W, 200 V. Can the bulb be safely used across of 300 V supply? If not, what will you do so that it can glow with normal brightness when it is connected across 300 V supply?

SOLUTION Given $P = 40$ W, $V = 200$ volt. The resistance of the bulb is

$$R = \frac{V^2}{R} = \frac{200 \times 200}{40} = 1000\ \Omega$$

If this bulb is connected to a 300 V supply, the power dissipated is

$$P_1 = \frac{V_1^2}{R} = \frac{300 \times 300}{1000} = 90 \text{ W}$$

Since $P_1 >> P$, the bulb will get damaged because now it glows much more brightly and hence cannot be safely used across a 300 V supply. It will glow with normal brightness if it dissipated 40 W. This will happen if the voltage across it is 200 V. To make it glow with normal brightness when connected to 300 V supply, a resistance R_1 must be connected in series with it as shown in Fig. 12.51 so that the voltage drop across R is 200 V and across R_1 is 100 V. The current in the circuit is

$$I = \frac{V}{R + R_1} = \frac{300}{1000 + R_1}$$

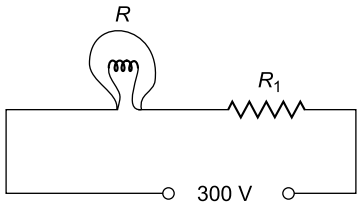


Fig. 12.51

$\therefore$ Voltage drop across $R_1 = IR_1 = \dfrac{300 R_1}{(1000 + R_1)}$ which must be equal to 100 V, i.e.

$$\frac{300 R_1}{(1000 + R_1)} = 100 \Rightarrow R_1 = 500\ \Omega$$

EXAMPLE 32 Two bulbs A and B are rated 40 W, 200 V and 60 W, 200 V respectively. Which bulb will glow more brightly if

(a) the two bulbs are connected in series and this combination is connected across a 200 V supply?

(b) the two bulbs are connected in parallel and this combination is connected across a 200 V supply?

SOLUTION Resistance of bulb A is $R_A = \dfrac{V^2}{P_A}$

$$= \frac{200 \times 200}{40}$$

$$= 1000\ \Omega$$

Resistance of bulb B is $R_B = \dfrac{V^2}{P_B}$

$$= \frac{200 \times 200}{60} = 666.7\ \Omega$$

(a) If the two bulbs are connected in series, the current flowing to each will be the same. Let the current be I.

Power dissipated in A is $P'_A = I^2 R_A$

Power dissipated in B is $P'_B = I^2 R_B$

$\therefore$ $$\frac{P'_A}{P'_B} = \frac{I^2 R_A}{I^2 R_B} = \frac{R_A}{R_B}$$

Since $R_A > R_B$; $P'_A > P'_B$. Hence bulb A will glow more brightly than bulb B.

(b) If the two bulbs are connected in parallel, the potential difference across each bulb will be the same = supply voltage V.

Power dissipated in A is $P_A'' = \dfrac{V^2}{R_A}$

Power dissipated in B is $P_B'' = \dfrac{V^2}{R_B}$

$$\therefore \qquad \frac{P_A''}{P_B''} = \frac{R_B}{R_A}$$

Since $R_B < R_A$; $P_A'' < P_B''$. Hence bulb B will glow more brightly than bulb A.

EXAMPLE 33 An electric cable having a resistance of 0.2 Ω delivers 10 kW of power at 200 V to a factory. Find the efficiency of transmission.

SOLUTION $R = 0.2\ \Omega$, $P = 10\ \text{kW} = 10 \times 10^3\ \text{W}$, $V = 200$ volt

Current in the cable is $I = \dfrac{P}{V} = \dfrac{10 \times 10^3}{200} = 50\ \text{A}$

Power loss in the cable $= I^2R = (50)^2 \times 0.2$

$= 500\ \text{W} = 0.5\ \text{kW}$

$\therefore$ Total power supplied by source = 10 kW + 0.5 kW = 10.5 kW

$$\therefore \text{ Efficiency} = \frac{\text{Power delivered to factory}}{\text{Power supplied by source}}$$

$$= \frac{10\ \text{kW}}{10.5\ \text{kW}} = 0.95 \text{ or } 95\%$$

EXAMPLE 34 100 MW of power from a power station is transmitted to a distant substation through a cable of resistance 10 Ω at (a) 20,000 V and (b) 200 V. Find the power loss in the cable in each case.

SOLUTION $P = 100\ \text{MW} = 100 \times 10^6\ \text{W} = 10^8\ \text{W}$, $R = 10\ \Omega$

(a) $V_1 = 20{,}000$ volt. Current in the cable is

$$I_1 = \frac{P}{V_1} = \frac{10^8}{20{,}000} = 5000\ \text{A}$$

Power loss in cable is $P_1 = I_1^2 R$

$= (5000)^2 \times 10 = 2.5 \times 10^8\ \text{W}$

(b) $V_2 = 200$ volt. Current in the cable is

$$I_2 = \frac{P}{V_2} = \frac{10^8}{200} = 5 \times 10^5\ \text{A}$$

Power loss in cable is $P_2 = I_2^2 R$

$= (5 \times 10^5)^2 \times 10 = 2.5 \times 10^{12}\ \text{W}$

Notice that $P_1 << P_2$. It is for this reason that electrical power from a generating power station is transmitted at a very high voltage of about 30,000 V to a distant substation and not at a low voltage of 200 V. At low voltage the power loss is very high.

EXAMPLE 35 A battery of emf E and internal resistance r is connected to an external resistance R as shown in Fig. 12.52, show that the power drawn from the battery is maximum when $R = r$.

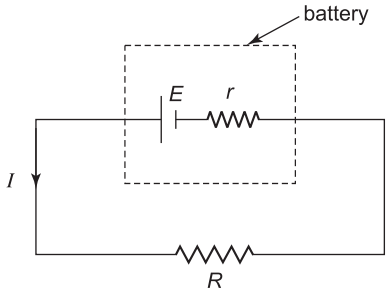


Fig. 12.52

SOLUTION The current in the circuit is

$$I = \frac{E}{(R + r)}$$

The power dissipated in R is

$$P = I^2R = \left(\frac{E}{R + r}\right)^2 R = \frac{E^2 R}{(R + r)^2} \qquad \text{(i)}$$

P is maximum if $\dfrac{dP}{dR} = 0$ and $\dfrac{d^2P}{dR^2} < 0$. Differentiating (i) with respect to R we have

$$\frac{dP}{dR} = E^2\left[\frac{1}{(R + r)^2} - 2R \times (R + r)^{-3}\right]$$

$$= \frac{E^2}{(R + r)^3}\left[(R + r) - 2R\right]$$

$$\frac{dP}{dR} = 0 \text{ if } (R + r) - 2R = 0$$

$$\Rightarrow \qquad R = r.$$

It is easy to check that $\dfrac{d^2P}{dR^2} < 0$ when $R = r$. Hence the power drawn form the battery is maximum when the external resistance is equal to the internal resistance of the battery.

15. Variation of Resistance and Resistivity with Temperature

The resistance of a conductor (and hence its resistivity) depends upon its temperature. For metallic conductors, the

resistance increases with temperature. For small changes in temperature, we have

$$R_2 = R_1[1 + \alpha(t_2 - t_1)]$$

where R_1 is the resistance at a temperature t_1, R_2 is the resistance at temperature t_2 and α is the temperature coefficient of resistance. The variation of resistivity with temperature is given by

$$\rho_2 = \rho_1[1 + \alpha(t_2 - t_1)]$$

For metallic conductors, the temperature coefficient of resistance is positive. Some materials, such as carbon and semi-conductors have a negative temperature coefficient of resistance. The resistance of such materials decreases with increase in temperature.

EXAMPLE 36 The resistance of a heating element is 100 Ω at 27°C. What is the temperature of the element if its resistance is 117 Ω. Temperature coefficient of resistance = 1.7×10^{-4} K^{-1}.

SOLUTION $R_2 = R_1[1 + \alpha(t_2 - t_1)]$

$$\Rightarrow \quad t_2 - t_1 = \frac{R_2 - R_1}{\alpha R_1} = \frac{117 - 100}{(1.7 \times 10^{-4}) \times 100} = 1000°C$$

$$\therefore \quad t_2 = 1000 + t_1 = 1000 + 27 = 1027°C$$

EXAMPLE 37 The resistance of a wire is 3.00 Ω at 0°C and 3.75 Ω at 100°C. Its resistance is measured to be 3.15 Ω at room temperature. Find the room temperature.

SOLUTION $\alpha = \dfrac{R_2 - R_1}{R_1(t_2 - t_1)} = \dfrac{3.75 - 3.00}{3.00(100 - 0)}$

$$= 0.0025 \text{ K}^{-1}$$

$$R_t = R_0(1 + \alpha t) \Rightarrow \frac{R_t - R_0}{R_0} = \alpha t$$

$$\Rightarrow \quad t = \frac{R_t - R_0}{\alpha R_0} = \frac{3.15 - 3.00}{0.0025 \times 3.00} = 20°C$$

SECTION 1

Multiple Choice Questions with One Correct Choice

Level A

1. In the circuit shown in Fig. 12.53, the reading of ammeter A is
   (a) 0.2 A (b) 0.4 A
   (c) 0.6 A (d) 0.8 A

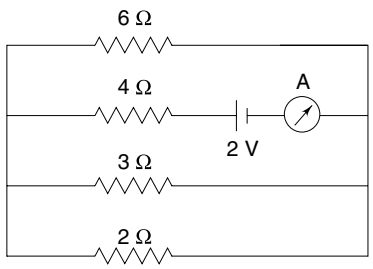


Fig. 12.53

2. Figure 12.54 shows a circuit with two cells in opposition to each other. One cell has an emf of 6 V and internal resistance of 2 Ω and the other cell has an emf of 4 V and internal resistance of 8 Ω. The potential difference across the terminals X and Y is
   (a) 5.4 V (b) 5.6 V
   (c) 5.8 V (d) 6.0 V

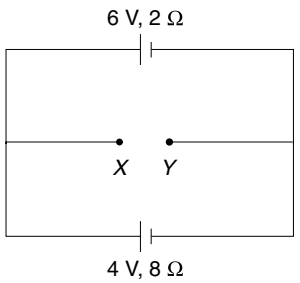


Fig. 12.54

3. A voltmeter having a resistance of 1800 Ω is employed to measure the potential difference across a 200 Ω resistor which is connected to the terminals of a DC power supply having an emf of 50 V and an internal resistance of 20 Ω. What is the percentage decrease in the potential difference across the 200 Ω resistor as a result of connecting the voltmeter across it?
   (a) 1% (b) 5%
   (c) 10% (d) 25%
4. The deflection in a moving coil galvanometer falls from 50 to 10 divisions when a shunt of 12 Ω is connected across it. The resistance of the galvanometer coil is

(a) 24 Ω (b) 36 Ω
(c) 48 Ω (d) 60 Ω

5. A galvanometer of resistance 10 Ω gives full-scale deflection when 1 mA current passes through it. The resistance required to convert it into a voltmeter reading up to 2.5 V is

(a) 24.9 Ω (b) 249 Ω
(c) 2490 Ω (d) 24900 Ω

6. The driver cell of a potentiometer has an emf of 2 V and negligible internal resistance. The potentiometer wire has a resistance of 5 Ω and is 1 m long. The resistance which must be connected in series with the wire so as to have a potential difference of 5 mV across the whole wire is

(a) 1985 Ω (b) 1990 Ω
(c) 1995 Ω (d) 2000 Ω

7. An electric kettle has two coils. When one coil is connected to the ac mains, the water in the kettle boils in 10 minutes. When the other coil is used, the same quantity of water takes 15 minutes to boil. How long will it take for the same quantity of water to boil if the two coils are connected in parallel?

(a) 6 min (b) 12 min
(c) 18 min (d) 24 min

8. A 12 V, 24 W tungsten filament bulb is supplied the current from n cells connected in series. Each cell has an emf 1.5 V and internal resistance of 0.25 Ω. What is the value of n in order that the bulb runs at its rated power?

(a) 6 (b) 8
(c) 12 (d) 16

9. A coil is embedded in a block of ice placed in an insulated box. If a current of 1.0 A is passed through the coil by applying a potential difference of 210 V across it, the amount of ice melted in one hour is

(a) 2.00 kg (b) 2.25 kg
(c) 2.50 kg (d) 2.75 kg

10. An electric bulb has a rating of 500 W, 100 V. It is used in a circuit having a 200 V supply. What resistance must be connected in series with the bulb so that it delivers 500 W?

(a) 10 Ω (b) 20 Ω
(c) 30 Ω (d) 40 Ω

11. Two identical cells of emf 1.5 V each connected in parallel provide a supply to an external circuit consisting of two resistors of 7 Ω each joined in parallel. A very high resistance voltmeter reads the terminal voltage of the cells to be 1.4 V. What is the internal resistance of each cell?

(a) 0.2 Ω (b) 0.3 Ω
(c) 0.4 Ω (d) 0.5 Ω

12. The resistance network shown in Fig. 12.55 is connected to a battery of emf 30 V having an internal resistance of 1 Ω. What is the voltage drop across the 2 Ω resistor?

(a) 3 V (b) 6 V
(c) 12 V (d) 24 V

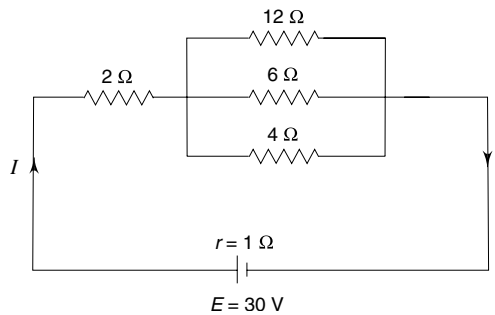


Fig. 12.55

13. In Q. 12, what is the terminal voltage of the battery?

(a) 15 V (b) 18 V
(c) 21 V (d) 24 V

14. In Q. 12, what is the voltage drop across the internal resistance of the battery?

(a) 2 V (b) 4 V
(c) 6 V (d) 8 V

15. What is the steady state current in the 2 Ω resistor shown in Fig. 12.56? The internal resistance of the battery is negligible and the capacitance $C = 0.5\ \mu F$.

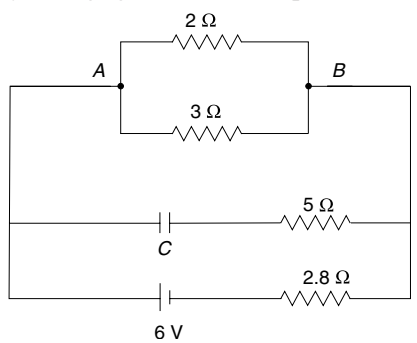


Fig. 12.56

(a) 0.3 A (b) 0.6 A
(c) 0.9 A (d) 1.2 A

16. A battery consists of three cells of emfs 2.2 V, 2.0 V and 1.8 V of internal resistance 0.5 Ω, 0.7 Ω and 0.8 Ω respectively connected in series. The battery is connected to an external resistance of 4.0 Ω via a very low resistance ammeter. What is the ammeter reading?

(a) 0.5 A (b) 1.0 A
(c) 1.5 A (d) 2.0 A

17. A battery of emf 12 V and internal resistance 0.5 Ω is charged by a battery charger which supplies a 132 V dc supply using a series resistance of 11.5 Ω. What is the terminal voltage of the battery during charging?
(a) 15 V (b) 16 V
(c) 17 V (d) 18 V

18. A voltmeter reads 3 V at full-scale deflection and is graded as 6000 Ω/V. What resistance should be connected in series with it so that it reads 12 V at full-scale deflection?
(a) 1.8×10^4 Ω (b) 3.6×10^4 Ω
(c) 5.4×10^4 Ω (d) 7.2×10^4 Ω

19. A DC milliammeter has a resistance of 12 Ω and gives a full scale deflection for a current of 0.01 A. To convert it into a voltmeter giving a full-scale deflection for 3 V, the resistance required to be put in series with the instrument is
(a) 102 Ω (b) 288 Ω
(c) 300 Ω (d) 412 Ω

20. In Fig. 12.57, the galvanometer shows no deflection. What is the resistance *X*?
(a) 7 Ω (b) 14 Ω
(c) 21 Ω (d) 28 Ω

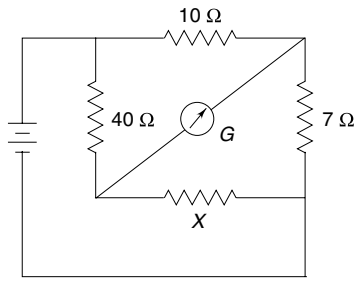


Fig. 12.57

21. The resistance between points *A* and *B* in the circuit shown in Fig. 12.58 is
(a) 4 Ω (b) 6 Ω
(c) 10 Ω (d) 18 Ω

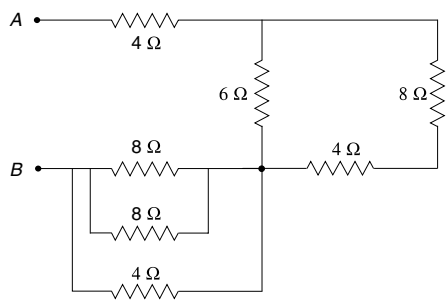


Fig. 12.58

22. The current *I* in the circuit shown in Fig. 12.59 is

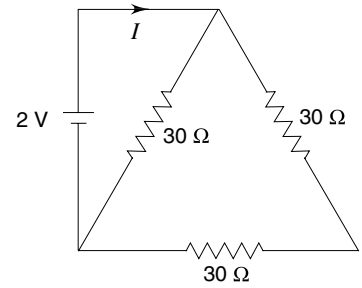


Fig. 12.59

(a) $\frac{1}{45}$ A (b) $\frac{1}{15}$ A
(c) $\frac{1}{10}$ A (d) $\frac{1}{5}$ A

23. The internal resistance of the cell shown in Fig. 12.60 is negligible. On closing the key *K*, the ammeter reading changes from 0.25 A to 5/12 A, then
(a) $R_1 = 10$ Ω
(b) $R_1 = 15$ Ω
(c) the power drawn from the cell decreases
(d) the current through *R* decreases by 40%

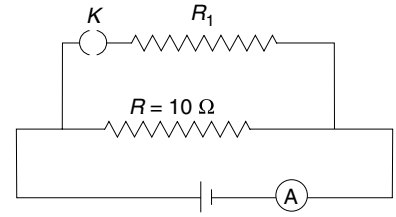


Fig. 12.60

Level B

24. Two equal resistances $R_1 = R_2 = R$ are connected with a 30 Ω resistor and a battery of terminal voltage *V*. The currents in the two branches are 2.25 A and 1.5 A as shown in Fig. 12.61. Then
(a) $R_2 = 15$ Ω (b) $R_2 = 30$ Ω
(c) $V = 36$ V (d) $V = 180$ V

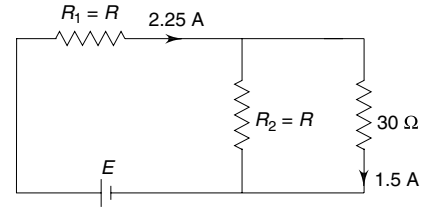


Fig. 12.61

25. Which of the following statements is correct about the circuit shown in Fig. 12.62 where 1 Ω and 0.5 Ω are the internal resistances of the 6 V and 12 V batteries respectively?

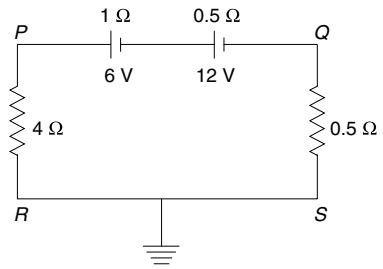


Fig. 12.62

(a) The potential at point P is 6 V.
(b) The potential at point Q is + 0.5 V
(c) If a voltmeter is connected across the 6 V battery, it will read 7 V.
(d) If a voltmeter is connected across the 6 V battery, it will read 5 V.

26. In the circuit shown in Fig. 12.63,
(a) the current through NP is 0.5 A
(b) the value of $R_1 = 30\ \Omega$
(c) the value of $R = 14\ \Omega$
(d) the potential difference across $R = 49$ V

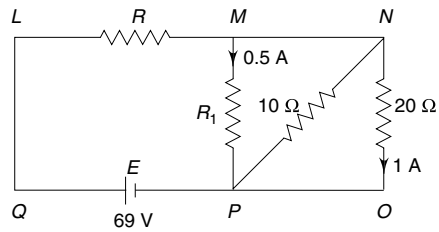


Fig. 12.63

27. Figure 12.64 shows a circuit in which the potential differences across the resistances are given in the diagram. If point Q is grounded, the potential at point S will be
(a) 10 V (b) – 10 V
(c) 40 V (d) – 40 V

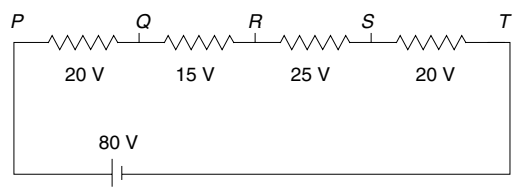


Fig. 12.64

28. Three resistances of 4 Ω each are connected as shown in Fig. 12.65. If the point D divides the resistance into two equal halves, the resistance between points A and D will be
(a) 12 Ω (b) 6 Ω
(c) 3 Ω (d) $\frac{1}{3}$ Ω

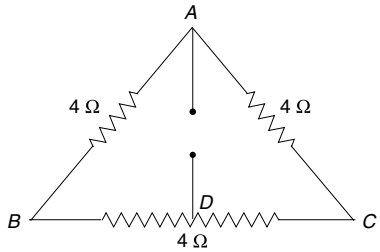


Fig. 12.65

29. The range of a voltmeter is 5 V and its resistance is 5000 Ω (4900 Ω connected in series with a coil of resistance 100 Ω). What additional resistance should be connected in series with it so that its range is doubled?
(a) 5000 Ω (b) 4500 Ω
(c) 4000 Ω (d) 3500 Ω

30. A 100 watt bulb and a 200 watt bulb are designed to operate at 110 V and 220 V respectively. The ratio of their resistances is
(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

31. A constant voltage is applied across a uniform wire, which produces some heat in the wire. If a second wire of the same material but of length twice that of the first wire is used, the heat produced in the second wire will the same as in the first wire if the area of cross-section of the second wire is
(a) increased by a factor of four
(b) decreased by a factor of four
(c) increased by a factor of two
(d) decreased by a factor of two

32. Three unequal resistances connected in parallel are equivalent to a single resistance of 1 Ω. If two resistances are in the ratio of 1:2 and if no resistance is fractional, the largest of three resistances is
(a) 4 Ω (b) 6 Ω
(c) 8 Ω (d) 12 Ω

33. A copper wire of length 50 cm and area of cross-section 10^{-6} m^2 carries a current of 0.5 A. If the resistivity of copper is 1.8×10^{-8} Ωm, the electric field across the wire is
(a) 9 Vm^{-1} (b) 0.9 Vm^{-1}
(c) 0.09 Vm^{-1} (d) 0.009 Vm^{-1}

34. What is the current flowing through the galvanometer in the circuit shown in Fig. 12.66?
    (a) 10 A (b) 1 A
    (c) 0.1 A (d) 0.01 A

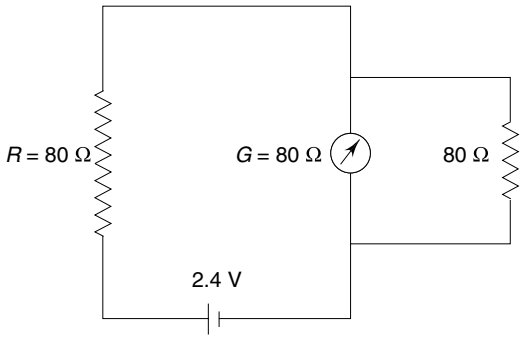


Fig. 12.66

35. In Q. 34, what should be the value of R so that the current in galvanometer reduces to half its earlier value?
    (a) 100 Ω (b) 200 Ω
    (c) 300 Ω (d) 400 Ω
36. A battery of emf E and internal resistance r is connected across a pure resistive device (such as an electric heater) of resistance R. The power output of the device will be maximum if
    (a) $R = r$ (b) $R = \sqrt{2}\ r$
    (c) $R = 2r$ (d) $R = 4r$
37. In Q. 36, the maximum power output is
    (a) $\frac{E^2}{r}$ (b) $\frac{E^2}{2r}$
    (c) $\frac{E^2}{3r}$ (d) $\frac{E^2}{4r}$
38. In Q. 36, if the battery is shorted, the power dissipated inside the battery will be
    (a) $\frac{E^2}{r}$ (b) $\frac{E^2}{2r}$
    (c) $\frac{E^2}{3r}$ (d) $\frac{E^2}{4r}$
39. A 24 V battery of internal resistance 4 Ω is connected to a variable resistor. At what value of the current drawn from the battery is the rate of heat produced in the resistor the maximum?
    (a) 1 A (b) 2 A
    (c) 3 A (d) 4 A
40. An electric bulb has a rating of 100 W, 200 V. If the supply voltage drops to 100 V, what is the total heat and light energy produced by the bulb in 20 minutes?
    (a) 10 k J (b) 20 k J
    (c) 30 k J (d) 40 k J
41. If electric power from 100 MW power station is transmitted to a substation at 20,000 V, the power loss during transmission is P_1. If the same power is transmitted at 200 V, the power loss is P_2. The ratio P_2/P_1 will be
    (a) 10^4 (b) 10^3
    (c) 10^2 (d) 10
42. In the potentiometer circuit shown in Fig. 12.67, the internal resistance of the 6 V battery is 1 Ω and the length of the wire AB is 100 cm. When AD = 60 cm, the galvanometer shows no deflection.
    The emf of cell C is (the resistance of wire AB is 2 Ω)
    (a) 0.7 V (b) 0.8 V
    (c) 0.9 V (d) 1.0 V

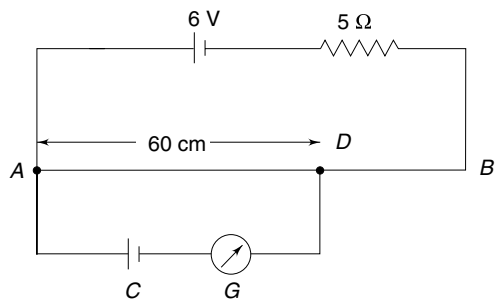


Fig. 12.67

43. The current through a bulb is increased by 1%. Assuming that the resistance of the filament remains unchanged, the power of the bulb (i.e. its wattage) will
    (a) increase by 1% (b) decrease by 1%
    (c) increase by 2% (d) decrease by 2%
44. The voltage across a bulb is decreased by 2%. Assuming that the resistance of the filament remains unchanged, the power of the bulb will
    (a) decrease by 2% (b) increase by 2%
    (c) decrease by 4% (d) increase by 4%
45. Figure 12.68 shows a network of resistances connected to a 2V battery. If the internal resistance of the battery is negligible, current I in the circuit is
    (a) 0.25 A (b) 0.5 A
    (c) 0.75 A (d) 1.0 A

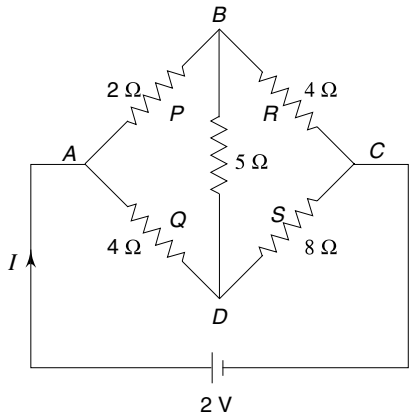


Fig. 12.68

46. Figure 12.69 shows a network of eight resistors numbered 1 to 8, each equal to 2 Ω, connected to a 3 V battery of negligible internal resistance. The current I in the circuit is

(a) 0.25 A (b) 0.5 A

(c) 0.75 A (d) 1.0 A

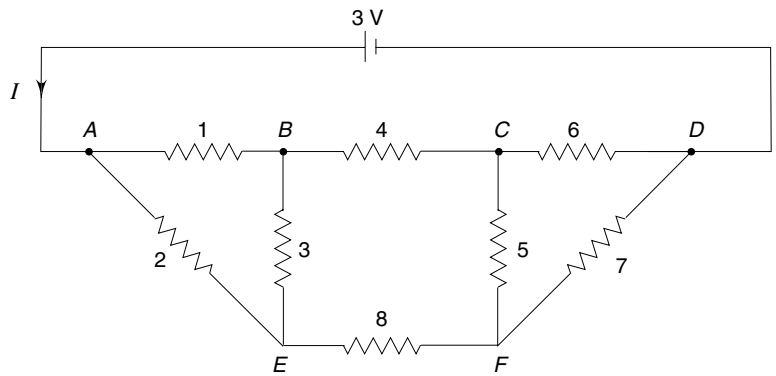


Fig. 12.69

47. Three equal resistors, connected in series with a battery, dissipate P watts of power. What will be the power dissipated if the same resistances are connected in parallel across the same battery?

(a) P (b) 3 P

(c) 9 P (d) 27 P

48. Two electrical devices of power rating 1 kW and 2 kW are connected (a) in series and (b) in parallel. The ratio of the power ratings of the combination in case (a) to that in case (b) is

(a) 1 : 4 (b) 1 : 3

(c) 1 : 2 (d) 1 : 1

49. Two 500 W heaters when connected in series across a source of constant voltage V supply Q joules of heat in a certain time t. When the heaters are connected in parallel across the same source, the amount of heat energy supplied in the same time t will be

(a) $\frac{Q}{4}$ (b) $\frac{Q}{2}$

(c) 2 Q (d) 4 Q

50. A battery of emf E and internal resistance r is connected to a resistor of resistance r_1 and Q joules of heat is produced in a certain time t. When the same battery is connected to another resistor of resistance r_2, the same quantity of heat is produced in the same time t, the value of r is

(a) $\frac{r_1^2}{r_2}$ (b) $\frac{r_2^2}{r_1}$

(c) $\frac{1}{2}(r_1 + r_2)$ (d) $\sqrt{r_1 r_2}$

51. A heater boils a certain quantity of water in time t_1. Another heater boils the same quantity of water in time t_2. If both heaters are connected in series, the combination will boil the same quantity of water in time

(a) $\frac{1}{2}(t_1 + t_2)$ (b) $(t_1 + t_2)$

(c) $\frac{t_1 t_2}{(t_1 + t_2)}$ (d) $\sqrt{t_1 t_2}$

52. A heater boils a certain quantity of water in time t_1. Another heater boils the same quantity of water in

time t_2. If both heaters are connected in parallel, the combination will boil the same quantity of water in time.

(a) $\frac{1}{2}(t_1 + t_2)$ (b) $(t_1 + t_2)$

(c) $\frac{t_1 t_2}{(t_1 + t_2)}$ (d) $\sqrt{t_1 t_2}$

53. In the circuit shown in Fig. 12.70, the ammeter A reads zero. If the batteries have negligible internal resistance, the value of R is

(a) 10 Ω (b) 20 Ω

(c) 30 Ω (d) 40 Ω

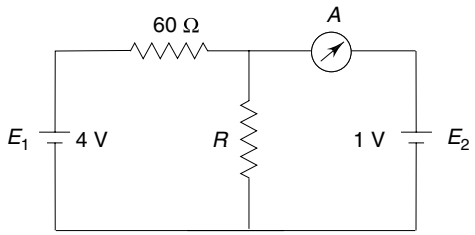


Fig. 12.70

54. A uniform wire of resistance 4 Ω is bent into the form of a circle of radius r. A speciment of the same wire is connected along the diameter of the circle. What is the equivalent resistance between the ends of this wire?

(a) $\frac{4}{(4+\pi)}$ Ω (b) $\frac{3}{(3+\pi)}$ Ω

(c) $\frac{2}{(2+\pi)}$ Ω (d) $\frac{1}{(1+\pi)}$ Ω

55. Figure 12.71 shows currents in a part of an electrical circuit. The current i is

(a) 1 A (b) 1.3 A

(c) 1.7 A (d) 3.7 A

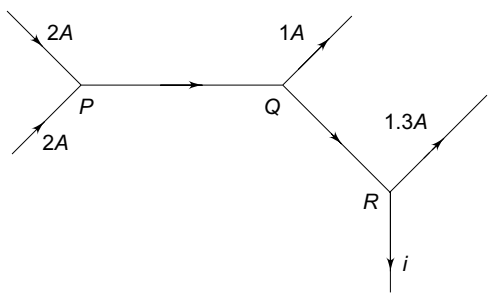


Fig. 12.71

56. A wire of resistance 1 Ω is elongated by 10%. The resistance of the elongated wire is

(a) 1.1 Ω (b) 11.1 Ω

(c) 1.21 Ω (d) 13.1 Ω

57. If each of the resistances shown in the network of Fig. 12.72 is R, the effective resistance between terminals A and B will be

(a) R (b) $2R$

(c) $3R$ (d) $5R$

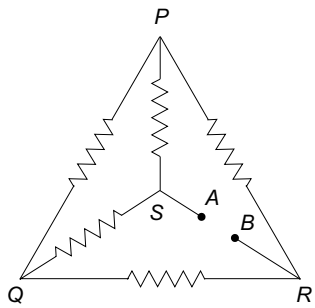


Fig. 12.72

58. What is the potential difference between points C and D in the circuit shown in Fig. 12.73?

(a) 3.6 V (b) 7.2 V

(c) 10.8 V (d) 12 V

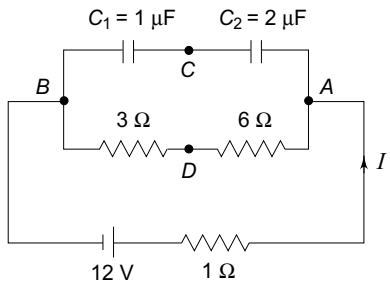


Fig. 12.73

59. In the circuit shown in Fig. 12.74, the effective resistance between A and B is

(a) $\frac{R}{2}$ (b) R

(c) 2 R (d) 4 R

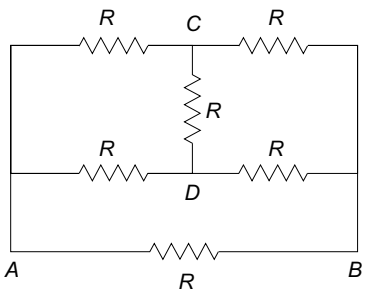


Fig. 12.74

60. A set of n identical resistors, each of resistance R ohm when connected in series has an effective resistance of x ohm. When the resistors are connected

in parallel, the effective resistance is y ohm. What is the relation between R, x and y?

(a) $R = \dfrac{xy}{(x+y)}$ (b) $R = (y-x)$

(c) $R = \sqrt{xy}$ (d) $R = (x+y)$

61. The effective resistance of a number of resistors connected in parallel in x ohm. When one of the resistors is removed, the effective resistance becomes y ohm. The resistance of the resistor that is removed is

(a) $\dfrac{xy}{(x+y)}$ (b) $\dfrac{xy}{(y-x)}$

(c) $(y-x)$ (d) $\sqrt{xy}$

62. You are given 48 cells each of emf 2 V and internal resistance 1 Ω. How will you connect them so that the current through an external resistance of 3 Ω is the maximum?

(a) 8 cells in series, 6 such groups in parallel
(b) 12 cells in series, 4 such groups in parallel
(c) 16 cells in series, 3 such groups in parallel
(d) 24 cells in series, 2 such groups in parallel

63. In Q. 62, the current through the external resistance will be

(a) 1 A (b) 2 A

(c) 3 A (d) 4 A

64. In the electrical circuit shown in Fig. 12.75, the current through the galvanometer will be zero if the value of resistance x is

(a) 4 Ω (b) 8 Ω

(c) 16 Ω (d) 24 Ω

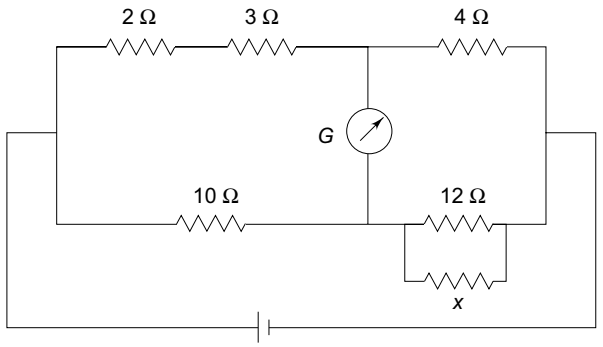


Fig. 12.75

65. The reading of the ammeter in the circuit in Fig. 12.76 is

(a) $\dfrac{3}{5}$ A (b) $\dfrac{4}{5}$ A

(c) $\dfrac{6}{5}$ A (d) $\dfrac{7}{5}$ A

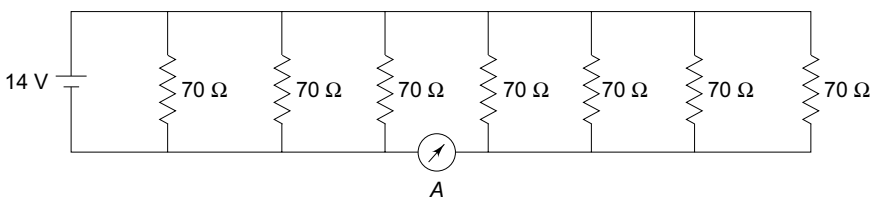


Fig. 12.76

66. In the circuit shown in Fig. 12.77, the resistance of each resistor is R. The equivalent resistance between points X and Y is

(a) $R/2$ (b) R

(c) $4R$ (d) $5R$

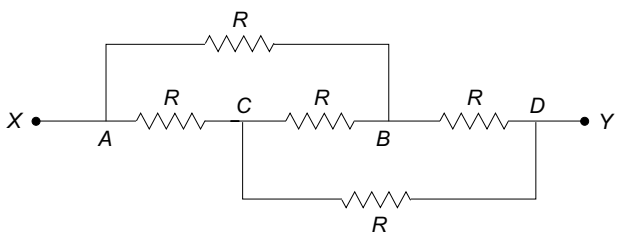


Fig. 12.77

67. Six resistors each of resistance R are connected as shown in Fig. 12.78. What is the effective resistance between points A and B?

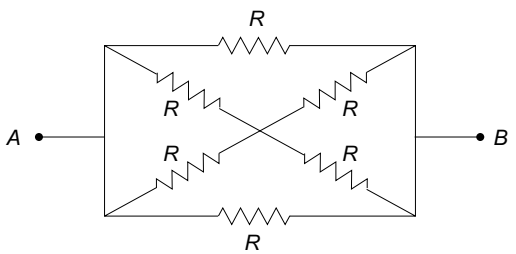


Fig. 12.78

(a) $\frac{R}{3}$ (b) R

(c) $3R$ (d) $6R$

68. A heater is designed to operate with a power of 1000 W in a 100 V line. It is connected, in combination with a resistance of 10 Ω and a resistance R to a 100 V line as shown in Fig. 12.79. What should be the value of R so that the heater operates with a power of 62.5 W?

(a) 10 Ω (b) 62.5 Ω

(c) $\frac{1}{5}$ Ω (d) 5 Ω

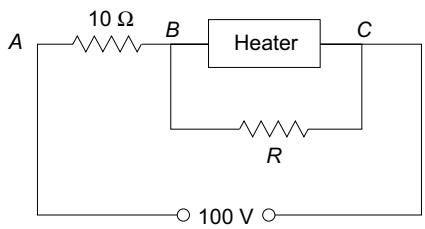


Fig. 12.79

69. A constant voltage is applied between the two ends of a metallic wire. Some heat is developed in it. The heat developed is doubled if

(a) both the length and the radius of the wire are halved

(b) both the length and the radius of the wire are doubled

(c) the radius of the wire is halved

(d) the length of the wire is doubled

70. Eight cells marked 1 to 8, each of emf 5 V and internal resistance 0.2 Ω are connected as shown in Fig. 12.80. What is the reading of the ideal voltmeter V?

(a) 40 V (b) 20 V

(c) 5 V (d) zero

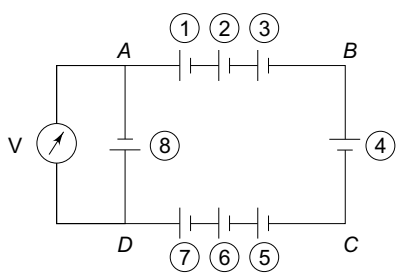


Fig. 12.80

71. A piece of copper and another of germanium are cooled from room temperature to 40 K. The resistance of

(a) each of them decreases

(b) each of them increases

(c) copper increases and of germanium decreases

(d) copper decreases and of germanium increases

72. A steady current flows in a metallic conductor of nonuniform cross-section. The quantity/quantities that remains/remain constant along the length of the conductor is/are

(a) current, electric field and drift speed

(b) drift speed only

(c) current and drift speed

(d) current only

73. In the circuit shown in Fig. 12.81, the current through

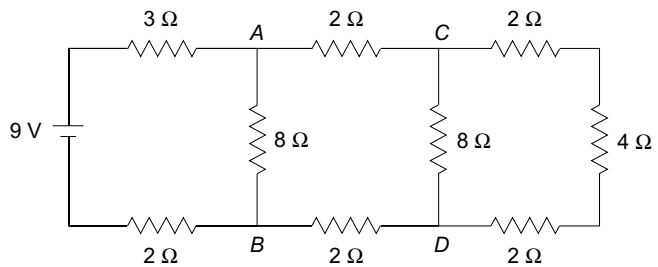


Fig. 12.81

(a) the 3 Ω resistor is 0.50 A

(b) the 3 Ω resistor is 0.25 A

(c) the 4 Ω resistor is 0.50 A

(d) the 4 Ω resistor is 0.25 A

74. A battery of internal resistance 4 Ω is connected to the network of resistances as shown in Fig. 12.82. In order that maximum power can be delivered to the network, the value of R in ohm should be

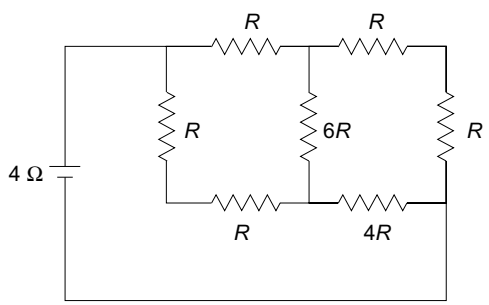


Fig. 12.82

(a) $\frac{4}{9}$ (b) 2

(c) $\frac{8}{3}$ (d) 18

75. In the given circuit, with steady current, the potential drop across the capacitor must be (see Fig. 12.83)

(a) V (b) $V/2$

(c) $V/3$ (d) $2V/3$

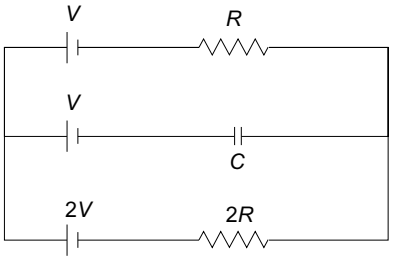


Fig. 12.83

76. A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by ΔT in a time t. A number N of similar cells is now connected in series with a wire of the same material and cross section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t. The value of N is

(a) 4 (b) 6

(c) 8 (d) 9

77. In the given circuit it is observed that the current I is independent of the value of the resistance R_6. Then the resistance values must satisfy (see Fig. 12.84)

(a) $R_1 R_2 R_5 = R_3 R_4 R_6$

(b) $\dfrac{1}{R_5}+\dfrac{1}{R_6}=\dfrac{1}{R_1+R_2}+\dfrac{1}{R_3+R_4}$

(c) $R_1 R_4 = R_2 R_3$

(d) $R_1 R_3 = R_2 R_4 = R_5 R_6$

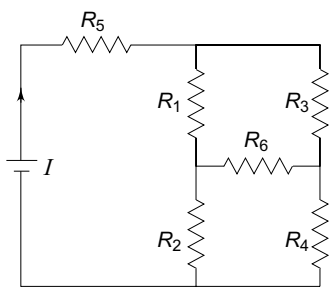


Fig. 12.84

78. The effective resistance between points P and Q of the electrical circuit shown in Fig. 12.85 is

(a) $2Rr/(R + r)$

(b) $8R(R + r)/(3R + r)$

(c) $2r + 4R$

(d) $5R/2 + 2r$

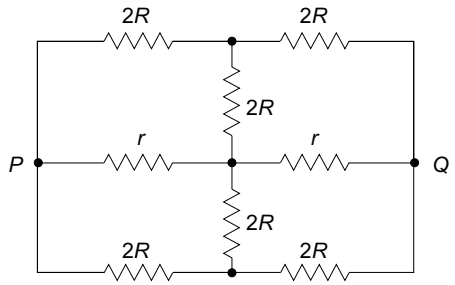


Fig. 12.85

79. A 100 W bulb B_1, and two 60 W bulbs B_2 and B_3, are connected to a 250 V source, as shown in Fig. 12.86. W_1, W_2 and W_3 are the output powers of the bulbs B_1, B_2 and B_3, respectively. Then

(a) $W_1 > W_2 = W_3$

(b) $W_1 > W_2 > W_3$

(c) $W_1 < W_2 = W_3$

(d) $W_1 < W_2 < W_3$

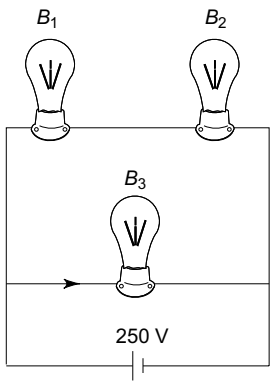


Fig. 12.86

80. In the meter bridge experiment shown in Fig.12.87, the balance length AC corresponding to null deflection of the galvanometers is x. What would be the balance length if the radius of the wire AB is doubled?

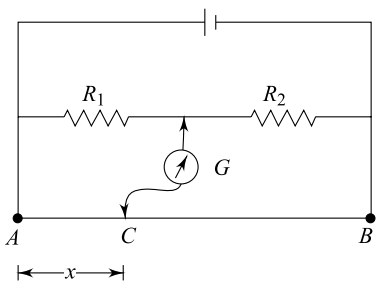


Fig. 12.87

(a) $\dfrac{x}{2}$ (b) x

(c) $2x$ (d) $4x$

81. Three resistances of equal value are connected in four different combinations as shown in Fig. 12.88. Arrange them in increasing order of power dissipation.

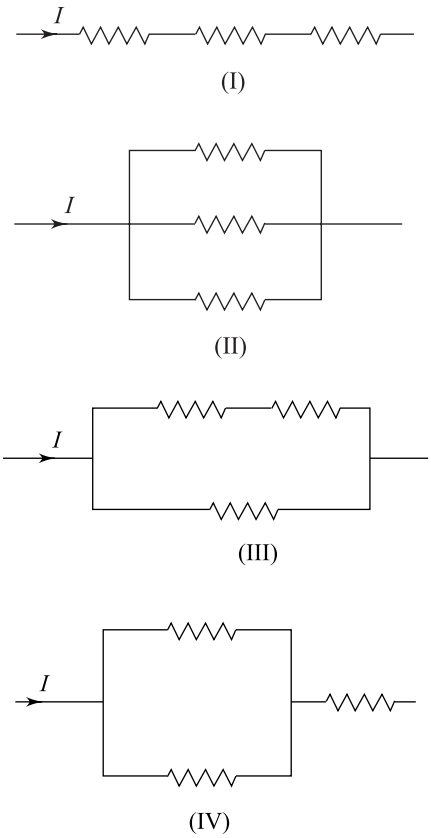


Fig. 12.88

(a) III < II < IV < I (b) II < III < IV < I
(c) I < IV < III < II (d) I < III < II < IV

82. An ideal battery of emf 2V and a series resistance R are connected in the primary circuit of a potentiometer of length 1 m and resistance 5 Ω. The value of R, to give a potential difference of 5 mV across 10 cm of potentiometer wire is
(a) 180 Ω (b) 190 Ω
(c) 195 Ω (d) 200 Ω

83. The range of a voltmeter of resistance 300 Ω is 5 V. The resistance to be connected to convert it into an ammeter of range 5A is
(a) 1 Ω is series (b) 1 Ω is parallel
(c) 0.1 Ω is series (d) 0.1 Ω is parallel

84. A conductor of resistance 3 Ω is stretched uniformly till its length is doubled. The wire is now bent in the form of an equilateral triangle. The effective resistance between the ends of any side of the triangle in ohms is:
(a) $\frac{9}{2}$ (b) $\frac{8}{3}$
(c) 2 (d) 1

85. A wire of resistance 0.1 Ω/cm is bent to form a square $ABCD$ of side 10 cm. A similar wire is connected between corners B and D to form the diagonal BD. The effective resistance of this combination between corners A and C is
(a) 1 Ω (b) 2 Ω
(c) 4 Ω (d) 8 Ω

86. The voltmeter shown in Fig. 12.89 reads 2 V. What resistance should be connected in parallel with the 2 Ω resistor so that the voltmeter reads 3 V?
(a) 1 Ω (b) 2 Ω
(c) 4 Ω (d) 8 Ω

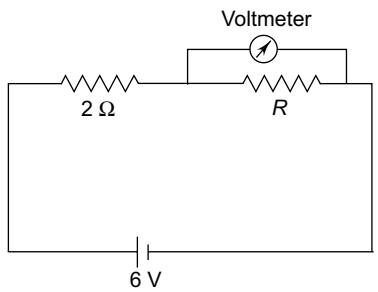


Fig. 12.89

87. Six equal resistances are connected between points A, B and C as shown Fig. 12.90. If R_1, R_2 and R_3 are the net resistances between A and B, between B and C and between A and C respectively, then $R_1 : R_2 : R_3$ will be equal to

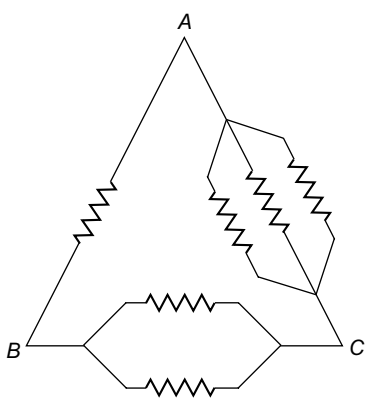


Fig. 12.90

(a) 6 : 3 : 2 (b) 1 : 2 : 3
(c) 5 : 4 : 3 (d) 4 : 3 : 2

88. The potential difference across the 3 Ω resistor shown Fig. 12.91 is

(a) zero (b) 1 V
(c) 3.5 V (d) 7 V

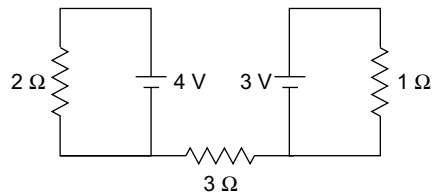


Fig. 12.91

89. A galvanometer of resistance 20 Ω gives full scale deflection when a current of 1 mA is passed through it. It is converted into a voltmeter by connecting a resistance of 4980 Ω in series with it. The maximum potential difference this voltmeter can measure is
(a) 5 mV (b) 0.05 V
(c) 5.0 V (d) 50 V

90. If a wire is stretched to make it 0.1% longer, its resistance will
(a) increase by 0.05% (b) increase by 0.2%
(c) decrease by 0.2% (d) remain unchanged.

91. The effective resistance between points A and B in the network shown in Fig. 12.92 is
(a) 1 Ω (b) 2 Ω
(c) 3 Ω (d) 4 Ω

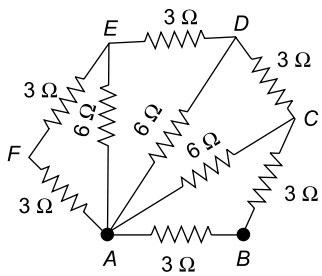


Fig. 12.92

92. A galvanometer together with an unknown resistance in series is connected across two identical batteries each of 1.5 V. When the batteries are connected in series, the galvanometer records a current of 1 A, and when the batteries are in parallel, the current is 0.6 A. What is the internal resistance of the battery?
(a) 1 Ω (b) $\frac{1}{2}$ Ω
(c) $\frac{2}{3}$ Ω (d) $\frac{1}{3}$ Ω

93. In the circuit shown in Fig. 12.93, A and B are two cells of the same emf E and of internal resistances r_A and r_B respectively. L is an ideal inductor and C is an ideal capacitor. The key K is closed. When the current in the circuit becomes steady, what should be the value of R so that the potential difference across the terminals of cell A is zero.
(a) $R = r_A - r_B$ if $r_A > r_B$.
(b) $R = \sqrt{r_A r_B}$
(c) $R = \frac{1}{2}(r_A + r_B)$
(d) For no value of R will the potential difference between the terminals of cell A be equal to zero.

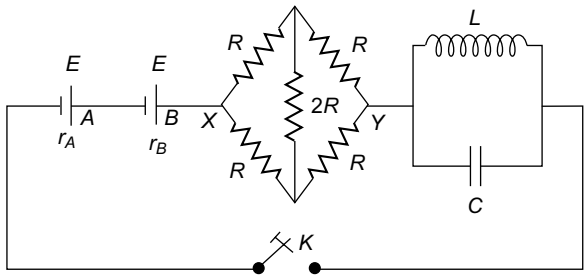


Fig. 12.93

94. Two electric bulbs, each designed to operate with a power of 500 W in a 200 V supply line, are connected in series to a 100 V supply line. The power generated in each bulb is
(a) 31.0 W (b) 31.25 W
(c) 31.5 W (d) 31.75 W

95. Two wires 1 and 2, each of length $2L$ and L are made of the same material. The radius of wire 1 is $2r$ and of wire 2 is r. The current I flows through the composite wire (see Fig. 12.94). Choose the correct statement from the following.
(a) Potential difference across 1 is twice that across 2.
(b) Power dissipated in 1 is four times the power dissipated in 2.
(c) Current densities in 1 and 2 are equal.
(d) Electric field in 1 is four times that in 2.

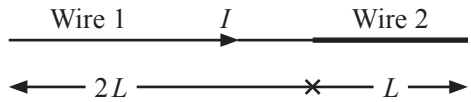


Fig. 12.94

96. In the circuit shown in Fig. 12.95, the resistors of 2 Ω and 3 Ω together dissipte 30 W of power. The current through the 4 Ω resistor is

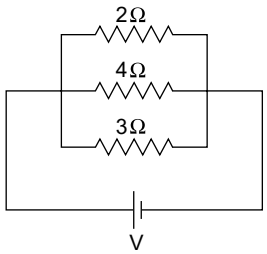


Fig. 12.95

(a) 0.5 A (b) 1.0 A

(c) 1.5 A (d) 2.0 A

97. In the circuit shown in Fig. 12.96, the current in the 1 Ω resistor is 2 A. The power developed in the 3 Ω resistor is

(a) 3 W (b) 9 W

(c) 27 W (d) 81 W

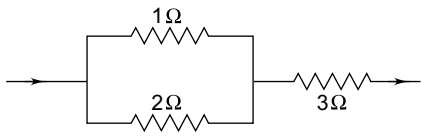


Fig. 12.96

98. Figure 12.97 shows three resistors R_1, R_2 and R_3 connected to points A and B. Point B is grounded and point A is maintained at 30V. What is the current through R_3?

(a) 1 A (b) 2 A

(c) 3 A (d) 4 A

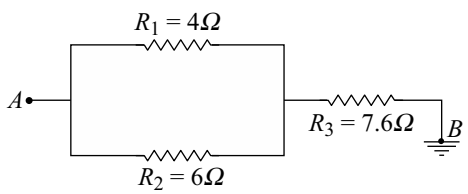


Fig. 12.97

99. Two capacitors C_1 and C_2 are connected to resistor R via two switchs S_1 and S_2 as shown in Fig. 12.98. Capacitor C_1 is initially charged to a voltage V_1 = 15V. Both the switchs are then closed simultaneously. When the steady state is established, the charges on C_1 and C_2 respectively are

(a) 6 μC, 24 μC (b) 9 μC, 36 μC

(c) 12 μC, 48 μC (d) 15 μC, 60 μC

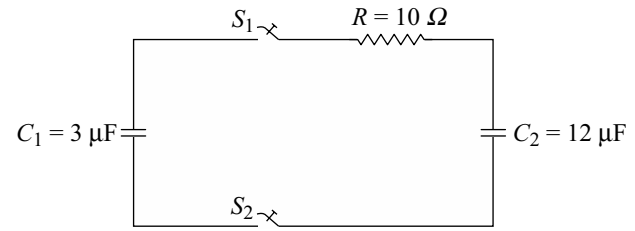


Fig. 12.98

100. Figure 12.99 shows a network of resistors. Points A and D grounded. The current in the 4 Ω resistor is

(a) 1A (b) 2A

(c) 3A (d) 4A

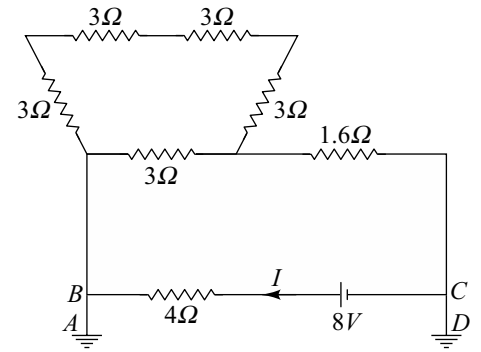


Fig. 12.99

101. What is the potential difference between points A and D in the circuit shown in Fig. 12.100?

(a) 2V (b) 4V

(c) 6V (d) 8V

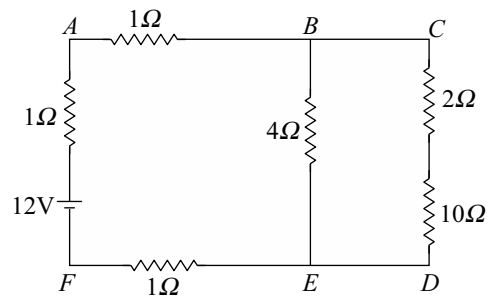


Fig. 12.100

102. Figure 12.101 shows as uncharged capacitor of capacitance C, two resistors of resistances R_1 and R_2 a battery of voltage V and a two-way switch S. Terminal 1 of S is connected to terminal 2. The time required for the charge on the capacitor to reach one-half the final charge is

(a) $R_1C \ln(1/2)$ (b) $\dfrac{R_1C}{\ln\left(\frac{1}{2}\right)}$

(c) $R_1C \ln(2)$ (d) $\dfrac{R_1C}{\ln(2)}$

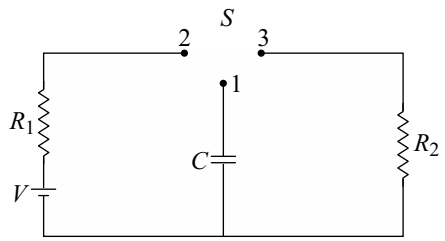


Fig. 12.101

103. In Q.102 above 1 is disconnected from 2 and connected to 3 at time $t = 0$. The time taken by the power dissipated through R_2 to decrease to one half the initial power is

(a) $\frac{R_2C}{2}\ln(2)$ (b) $\frac{2\ln(2)}{R_2C}$

(c) $\frac{R_2C}{2}\ln\left(\frac{1}{2}\right)$ (d) $\frac{2\ln(1/2)}{R_2C}$

104. The equivalent resistance between A and B in the network shown in Fig. 12.102 is

(a) $\frac{4}{3}\,\Omega$ (b) $\frac{6}{5}\,\Omega$

(c) $\frac{7}{5}\,\Omega$ (d) $\frac{8}{7}\,\Omega$

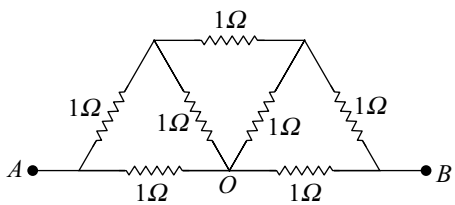


Fig. 12.102

105. An electric current flows through a non-uniform metal rod AB of length L with end A at $x = 0$ and end B at $x = L$. The current density J in the rod varies with x as shown in Fig. 12.103.

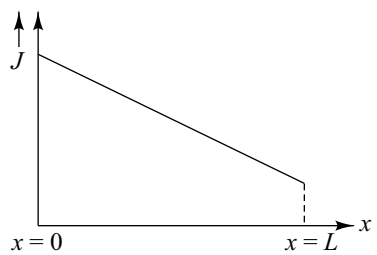


Fig. 12.103

Which of the following statements is correct?

(a) The rod has a smaller radius at A than at B.
(b) The rod has a higher radius at A than at B.
(c) The current at A greater than that at B.
(d) The potential gradient along the rod is constant.

106. The current density $\boldsymbol{J}$ through a plane area has a magnitude 6 Am^{-2}. If $\boldsymbol{J}$ makes a angle of 60° with the area vector of magnitude 3 m^2, the magnitude of the current through the area is

(a) 9A (b) $9\sqrt{3}$A

(c) 12A (d) zero

107. The magnitude of the current density J flowing normally through a circular area of radius R varies with the radial distance $r < R$ is

$$J = 3ar + 2b$$

where a and b are constants. The current through the circular area is

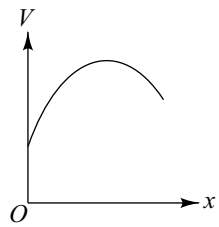
(a) $2R^2(aR + b)$ (b) $2\pi R(aR + b)$

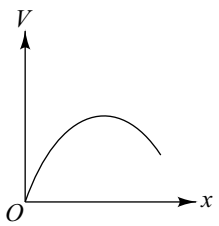
(c) $2\pi R^2(aR + b)$ (d) $\pi R^2(aR + b)$

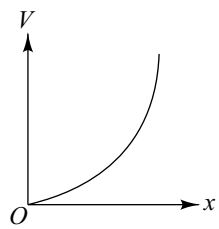
108. A current carrying cylindrical conductor PQ is lying along the x-axis with its end P at $x = 0$. The cylinder has a uniform cross-section and it is made of an alloy whose resistivity varies with x as

$$\rho = ax + b$$

where a and b are positive constants. Which of the graphs shown in Fig. 12.104 represents the variation of potential difference (V) with x?

(a)


(b)


(c)


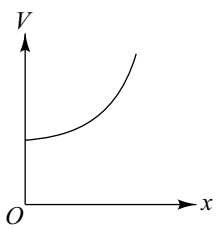
(d)


Fig. 12.104

109. A cylindrical metal rod of radius R carries a current I. The velocity of free electrons drifting at a distance r ($< R$) from the axis of the rod is given by

$$v = v_0\left(1 + \frac{r}{R}\right)$$

where v_0 is a constant. The average drift velocity of the electrons drifting in the rod is

(a) v_0 (b) $\frac{3v_0}{2}$

(c) $\frac{4v_0}{3}$ (d) $\frac{5v_0}{3}$

110. Three resistances R_1, R_2 and R_3 are connected to a three terminals A, B and C and to junction point O as shown in Fig. 12.105. If V_1, V_2 and V_3 are the potentials of terminals A, B and C respectively, the potential of the junction point O will be

(a) $\left(\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)^{-1}$

(b) $\left(\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right)(R_1+R_2+R_3)$

(c) $\left(\frac{V_1}{R_1^2}+\frac{V_2}{R_2^2}+\frac{V_3}{R_3^2}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)^{-2}$

(d) $\left(\frac{V_1}{R_1^2}+\frac{V_2}{R_2^2}+\frac{V_3}{R_3^2}\right)(R_1^2+R_2^2+R_3^2)$

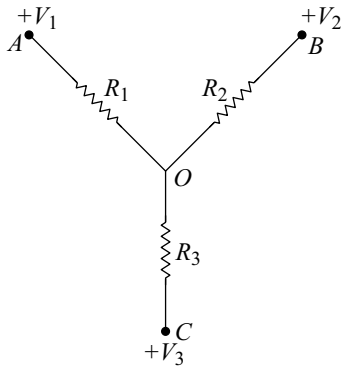


Fig. 12.105

111. Figure 12.106 shows two cells of emfs E_1 and E_2 have negligible internal resistance and connected to two resistances R_1 and R_2 whose values are such that the galvanometer G shows no deflection. The ratio $\frac{E_2}{E_1}$ is

(a) $\frac{R_1}{R_2}$ (b) $\frac{R_1}{R_1+R_2}$

(c) $\frac{R_2}{R_1+R_2}$ (d) $\frac{R_2}{R_1}$

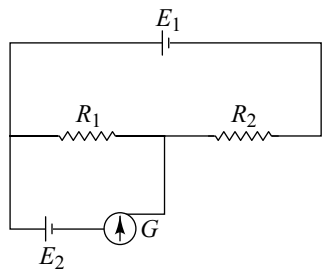


Fig. 12.106

112. In the circuit shown in Fig. 12.107, $R = 9\Omega$. The value of current I is

(a) 1A (b) 2A

(c) 3A (d) 4A

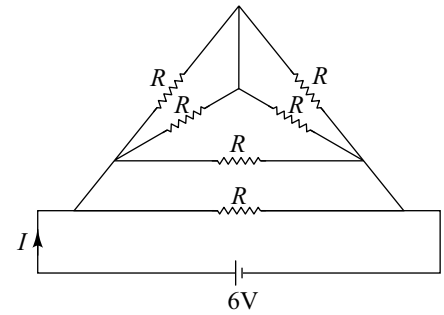


Fig. 12.107

113. The current I in the branch BC of the circuit shown in Fig. 12.108 is

(a) 1A (b) 2A

(c) 3A (d) 4A

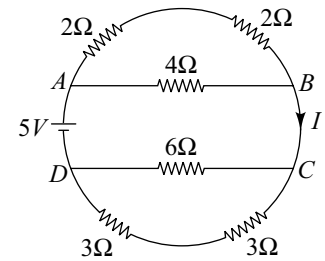


Fig. 12.108

114. A cylindrical metal tube of length L and resistivity ρ has inner radius a and outer radius b. The resistance of the tube between the inner and outer surfaces is

(a) $\frac{\rho b}{2\pi La}$ (b) $\frac{\rho a}{2\pi Lb}$

(c) $\frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$ (d) $\frac{\rho}{2\pi L}\ln\left(\frac{a}{b}\right)$

115. The current I in the circuit shown in Fig. 12.109 is

(a) 5A (b) 6A

(c) 8A (d) 1.2A

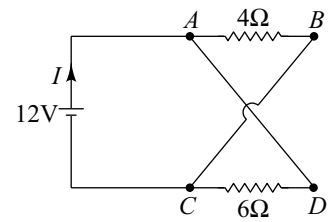


Fig. 12.109

116. The current I in the circuit shown in Fig. 12.110 is
(a) 1A (b) 2A
(c) 3A (d) 4A

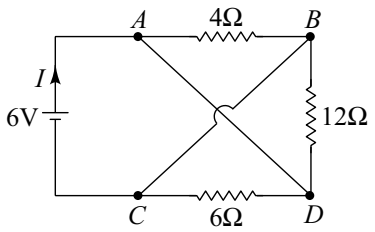


Fig. 12.110

117. The current I in the circuit shown in Fig. 12.111 is
(a) 5A (b) 6A
(c) 8A (d) 3.2A

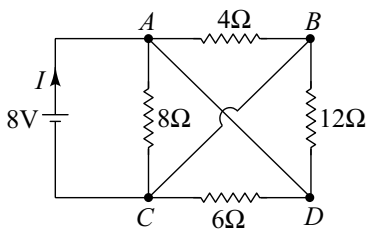


Fig. 12.111

118. Three resistances of 2Ω each are connected to a 2V battery as shown in Fig. 12.112. Then
(a) $I_1 = 1A, I_2 = 1A$ (b) $I_1 = 1A, I_2 = 2A$
(c) $I_1 = 2A, I_2 = 2A$ (d) $I_1 = 2A, I_2 = 1A$

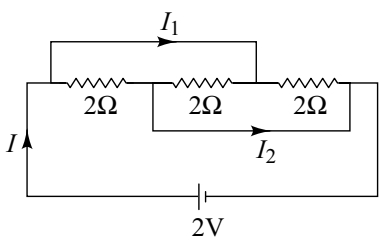


Fig. 12.112

119. The current I flowing through a resistance R varies with time t as shown in Fig. 12.113. The rate of dissipation of heat from the resistor is
(a) $\frac{I_0^2 R}{\sqrt{2}}$ (b) $\frac{I_0^2 R}{2}$
(c) $\frac{I_0^2 R}{3}$ (d) $\frac{I_0^2 R}{2\sqrt{2}}$

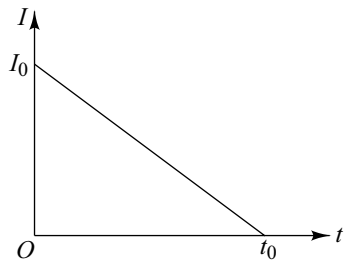


Fig. 12.113

120. A battery of emf E and internal resistance r is connected across a variable external resistance R. Which of the graphs shown in Fig. 12.114 depicts the variation of power P consumed with increase in R?

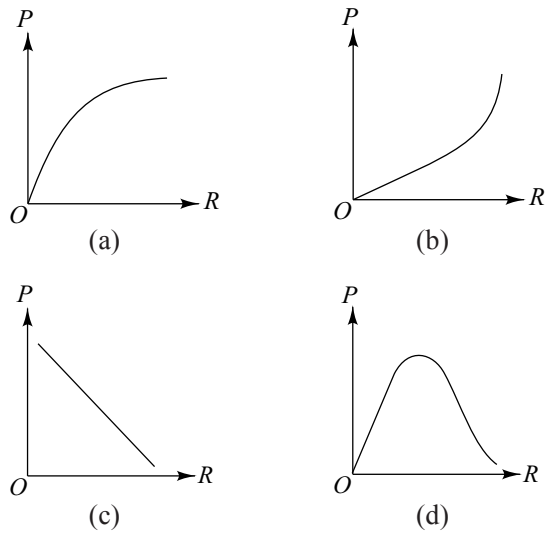


Fig. 12.114

121. Four resistance R_1, R_2, R_3 and R_4 are connected to a battery of voltage V as shown in Fig. 12.115. The resistances consume the same power. If $R_1 = 32\Omega$, the value of R_3 is
(a) 4 Ω (b) 8 Ω
(c) 12 Ω (d) 16 Ω

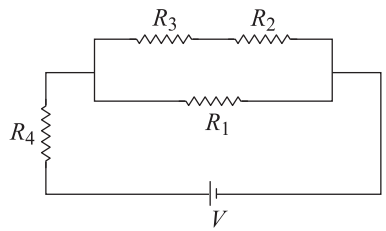


Fig. 12.115

122. In Q. 121 above, the value of R_4 is
(a) $\frac{32}{9}\Omega$ (b) $\frac{32}{7}\Omega$
(c) $\frac{32}{5}\Omega$ (d) $\frac{32}{3}\Omega$

123. What is the maximum number of 100 W bulbs all connected in parallel with a 220 V power supply fitted with a 6 A fuse?
(a) 10 (b) 11
(c) 12 (d) 13

124. Which resistance shown in Fig. 12.116 will have the highest rate of dissipation of heat
(a) 3 Ω (b) 6 Ω
(c) 9 Ω (d) 12 Ω

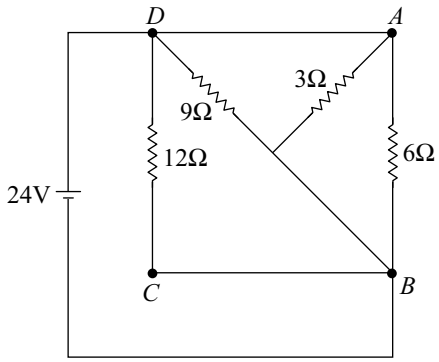


Fig. 12.116

125. In the circuit shown in Fig. 12.117, the capacitance of the capacitor is $C = 5\mu F$. When the steady state is reached, the charge on the capacitor is
(a) 9 μC (b) 18 μC
(c) 24 μC (d) 36 μC

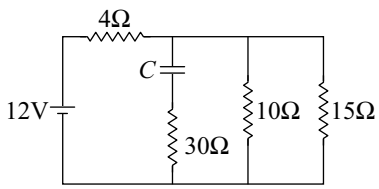


Fig. 12.117

126. Two capacitors C_1 and C_2 and two resistors R_1 and R_2 are connected to a battery as shown in Fig. 12.118. When steady state is reached, the ratio of charge on C_1 to that on C_2 is
(a) 1 : 1 (b) 2 : 1
(c) 1 : 2 (d) 4 : 1

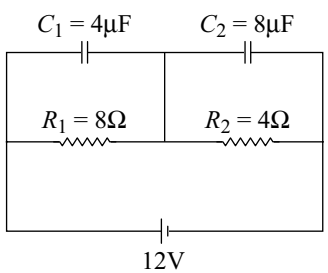


Fig. 12.118

Answers

Level A

1. (b)	2. (b)	3. (a)	4. (c)
5. (c)	6. (c)	7. (a)	8. (c)
9. (b)	10. (b)	11. (d)	12. (c)
13. (d)	14. (c)	15. (c)	16. (b)
17. (c)	18. (c)	19. (b)	20. (d)
21. (c)	22. (c)	23. (c)	

Level B

24. (d)	25. (c)	26. (c)	27. (d)
28. (c)	29. (a)	30. (b)	31. (c)
32. (b)	33. (d)	34. (d)	35. (b)
36. (a)	37. (d)	38. (a)	39. (c)
40. (c)	41. (a)	42. (c)	43. (c)
44. (c)	45. (b)	46. (d)	47. (c)
48. (d)	49. (d)	50. (d)	51. (b)
52. (c)	53. (b)	54. (a)	55. (c)
56. (c)	57. (a)	58. (a)	59. (a)
60. (c)	61. (b)	62. (b)	63. (d)
64. (d)	65. (b)	66. (b)	67. (a)
68. (d)	69. (b)	70. (d)	71. (d)
72. (d)	73. (d)	74. (b)	75. (c)
76. (b)	77. (c)	78. (a)	79. (d)
80. (b)	81. (b)	82. (c)	83. (b)
84. (b)	85. (a)	86. (b)	87. (c)
88. (a)	89. (c)	90. (b)	91. (b)
92. (d)	93. (a)	94. (b)	95. (d)
96. (c)	97. (c)	98. (c)	99. (b)
100. (b)	101. (c)	102. (c)	103. (c)
104. (d)	105. (a)	106. (a)	107. (c)
108. (b)	109. (d)	110. (a)	111. (b)
112. (b)	113. (a)	114. (c)	115. (a)
116. (c)	117. (a)	118. (c)	119. (c)
120. (d)	121. (b)	122. (a)	123. (d)
124. (a)	125. (d)	126. (a)	



Solutions

Level A

1. Resistors of resistances 6 Ω, 3 Ω and 2 Ω are in parallel. Their equivalent resistance r is given by

$$\frac{1}{r} = \frac{1}{6}+\frac{1}{3}+\frac{1}{2}$$

or $r = 1$ Ω. This resistance r is in series with the 4 Ω resistance. Therefore the resistance in series with the ammeter is $R = 1 + 4 = 5$ Ω.

$\therefore$ Current $= \dfrac{2V}{5\Omega} = 0.4$ A. Hence the correct choice is (b).

2. Since the two cells are in opposition, the effective voltage = 6 – 4 = 2 V. The current in the circuit is

$$I = \frac{2}{2+8} = 0.2 \text{ A}$$

$\therefore$ Terminal voltage of 6 V cell = $6 - 2 \times 0.2 = 5.6$ V
Terminal voltage of 4 V cell = $4 + 8 \times 0.2 = 5.6$ V.
Therefore, the potential difference across terminals X and Y is 5.6 V. Hence the correct choice is (b).

3. Current $I = \dfrac{50}{(200+20)} = \dfrac{5}{22}$ A

$\therefore$ Potential drop across 200 Ω resistor

$$(V) = \frac{5}{22} \times 200 = \frac{500}{11} \text{ V}$$

When a voltmeter of resistance 1800 Ω is connected across the 200 Ω resistor, the effective resistance R is given by

$$\frac{1}{R} = \frac{1}{1800} + \frac{1}{200} \text{ which gives } R = 180\,\Omega$$

The current in the circuit becomes

$$I' = \frac{50}{180+20} = \frac{5}{20} \text{ A}$$

The potential drop becomes

$$V' = \frac{5}{20} \times 180 = 45 \text{ V}$$

Difference $V - V' = \dfrac{500}{11} - 45 = \dfrac{5}{11}$ V

Percentage decrease $= \dfrac{5}{11} \times \dfrac{11}{500} \times 100 = 1\%$.

Hence the correct choice is (a).

4. If k is the current sensitivity of the galvanometer, the current in the galvanometer is

$$I_g = 50\,k \qquad \text{(i)}$$

When a shunt S is connected across it the current through the galvanometer becomes

$$I_g' = \frac{I_g S}{G+S} = 10\,k \qquad \text{(ii)}$$

where G is the resistance of the galvanometer. Dividing (i) and (ii) we get

$$\frac{G+S}{S} = 5$$

which gives $G = 4\,S = 4 \times 12 = 48$ Ω. Hence the correct choice is (c).

5. Let R be the resistance to be connected in series with the galvanometer of resistance G in order to convert it into a voltmeter reading up to 2.5 V. Then the current through the galvanometer is

$$I_g = \frac{2.5}{R+G}$$

Putting $I_g = 1 \text{ mA} = 1 \times 10^{-3}$ A and

$G = 10$ Ω, we get $R = 2490$ Ω.

Hence the correct choice is (c).

6. In order to have a potential drop of 5 mV = 5×10^{-3} V across a wire of resistance 5 Ω, the current flowing in the wire should be

$$I = \frac{5 \times 10^{-3}}{5} = 1 \times 10^{-3} \text{ A}$$

If R is the resistance to be connected in series with the wire, then

$$\frac{2}{R+5} = 1 \times 10^{-3}$$

which gives $R = 1995$ Ω. Hence the correct choice is (c).

7. Let H be the amount of heat energy needed to boil the given quantity of water. If R_1 and R_2 are the resistances of coils and V is the applied voltage, then

$$H = \frac{V^2 t_1}{R_1} = \frac{V^2 t_2}{R_2}$$

or $$\frac{R_2}{R_1} = \frac{t_2}{t_1} = \frac{15}{10} = \frac{3}{2}$$

When the coils are connected in parallel, the combined resistance is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

or $$R = \frac{R_1 R_2}{R_1 + R_2}$$

If the water takes t minutes to boil, then

$$\frac{V^2 t}{R} = \frac{V^2 t_1}{R_1}$$

or $$t = t_1 \frac{R}{R_1} = t_1 \times \frac{R_1 R_2}{(R_1 + R_2)} \cdot \frac{1}{R_1}$$

$$= t_1 \times \frac{1}{\left(\frac{R_1}{R_2} + 1\right)} = \frac{10 \times 1}{\left(\frac{2}{3} + 1\right)} = 6 \text{ min}$$

Hence the correct choice is (a).

8. $W = VI$. Therefore $I = W/V = 24/12 = 2$A. Therefore, the terminal voltage of each cell = $1.5 - 0.25 \times 2 = 1.0$ V. Resistance of the bulb is $R = 12/2 = 6\ \Omega$. If n cells are used, then

$$\frac{n \times 1.0}{6} = 2$$

which gives $n = 12$. Hence the correct choice is (c).

9. Heat produced in 1 hour (= 3600 s) is

$$H = VIt = \frac{210 \times 1 \times 3600}{4.2}$$

$$= 180{,}000 \text{ cal.}$$

Now 80 calories of heat is required to melt 1 g of ice. Therefore, the mass of ice melted is

$$\frac{180{,}000}{80} = 2250 \text{ g} = 2.25 \text{ kg}$$

Hence the correct choice is (b).

10. The current flowing in the bulb of 500 W operating at 100 V is

$$I = \frac{500}{100} = 5\text{A}$$

Resistance of the bulb = $\frac{100}{5} = 20\ \Omega$ (say, R_1)

To deliver 500 W, the current in the bulb must remain 5A when it is operated with 200 V supply. The resistance R to be connected in series for this purpose is given by

$$\frac{200}{R + R_1} = 5$$

or $R + R_1 = 40$ or $R + 20 = 40$ or $R = 20\ \Omega$. Hence the correct choice is (b).

11. Resistance of the parallel combination of two 7 Ω resistors is $R = 3.5\ \Omega$. Since the terminal voltage is 1.4 V, the current in the circuit is $I = 1.4/3.5 = 0.4$ A. Therefore, current through each cell = $I/2 = 0.2$ A. Let r be the internal resistance of each cell. The voltage drop across r is $1.5 - 1.4 = 0.1$ V. Therefore, $0.2\ r = 0.1$ or $r = 0.5\ \Omega$. Hence the correct choice is (d).

12. Total resistance of parallel combination is given by

$$\frac{1}{R_1} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4}$$

or $R_1 = 2\ \Omega$. Total resistance in circuit is $R = R_1 + 2 = 4\ \Omega$. Therefore, current in the circuit is

$$I = \frac{E}{R + r} = \frac{30}{4 + 1} = 6 \text{ A}$$

$\therefore$ Potential drop across 2 Ω resistor = $2 \times 6 = 12$ V, which is choice (c).

13. The voltage drop across the 2 Ω resistor is 12 V. Since the resistance of the parallel combination is also 2 Ω, the voltage drop across this combination is also 12 V. Therefore, the total voltage drop across the network = 12 + 12 = 24 V. Thus, the terminal voltage of the battery is 24 V. Hence the correct choice is (d).

14. Voltage drop across the battery = 30 – 24 = 6 V, which is choice (c).

15. The resistance of the parallel combination of 2 Ω and 3 Ω resistors is $R = 1.2\ \Omega$. This resistance is in series with the 2.8 Ω resistor, giving a total effective resistance = 1.2 + 2.8 = 4.0 Ω. In the steady state, no current flows through the capacitor C and hence no current passes through the 5 Ω resistor as it is in series with the capacitor. Hence the current through the circuit = 6/4 = 1.5 A. Therefore, the potential drop across $AB = 1.5 \times 1.2 = 1.8$ V. Hence the current through the 2 Ω resistor = 1.8/2 = 0.9 A.

16. When the three cells are connected in series, the total emf of the battery is $E = 2.2 + 2.0 + 1.8 = 6.0$ V. The total internal resistance of the battery is $r = 0.5 + 0.7 + 0.8 = 2.0\ \Omega$. The ammeter reading when the battery is connected to an external resistance $R = 4\ \Omega$ is

$$I = \frac{E}{R + r} = \frac{6.0}{4.0 + 2.0} = 1.0 \text{ A}$$

17. Given $r = 0.5\ \Omega$ and $R = 11.5\ \Omega$. For charging, the positive terminal of the charger is connected to the positive terminal of the battery. Therefore, during changing, the effective emf is $E = 132 - 12 = 120$ V. Current in the circuit during charging is

$$I = \frac{E}{R+r} = \frac{120}{11.5+0.5} = 10 \text{ A}$$

The terminal voltage of the battery during charging is V = emf of the battery + potential difference between its internal resistance = $12 + Ir = 12 + 10 \times 0.5 = 17$ V.

18. A voltmeter is graded according to its resistance and the voltage it reads at full scale deflection. If a voltmeter has a resistance R ohms and it reads V volts at full scale deflection, it is said to be graded as R/V ohm per volt. It is given that $V = 3.0$ V and the voltmeter is graded as 6000 Ω/V. Hence the resistance of the voltmeter is

$$R = 6000 \ \Omega/\text{V} \times 3.0 \text{ V} = 1.8 \times 10^4 \ \Omega$$

At full-scale deflection, the current through the voltmeter is

$$I = \frac{V}{R} = \frac{3.0}{1.8 \times 10^4} = 1.67 \times 10^{-4} \text{ A}$$

In order to convert this instrument into a voltmeter that reads 12 V at full-scale deflection, the resistance R' that must be connected in series with it is given by (here $V' = 12$ V)

$$R' = \frac{V'}{I} - R = \frac{12}{1.67 \times 10^{-4}} - 1.8 \times 10^4$$
$$= 5.4 \times 10^4 \ \Omega$$

19. $R = \dfrac{V_0}{I_g} - G = \dfrac{3}{0.01} - 12 = 300 - 12 = 288 \ \Omega$

Hence the correct choice is (b).

20. This is a balanced Wheatstone's bridge. Therefore

$$\frac{10}{40} = \frac{7}{X}$$

which gives $X = 28 \ \Omega$. Hence the correct choice is (d)

21. The given circuit is equivalent to that shown in Fig. 12.119. Therefore, the total resistance = 4 + 4 + 2 = 10 Ω. Hence the correct choice is (c).

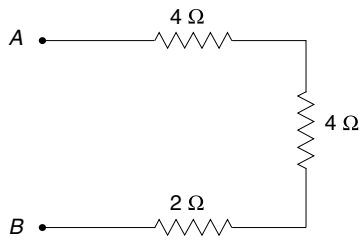


Fig. 12.119

22. The equivalent resistance is given by

$$\frac{1}{R} = \frac{1}{60} + \frac{1}{30} = \frac{3}{60}$$

or $R = 20 \ \Omega$. Therefore, $I = 2/20 = \dfrac{1}{10}$ A, which is choice (c).

23. Initially $I = 0.25$ A

Terminal potential of the battery $V = 10 \times 0.25 = 2.5$ V

When K is closed, $I = \dfrac{5}{12}$ A

Effective resistance R_2 is given by

$$\frac{1}{R_2} = \frac{1}{R_1} + \frac{1}{R} \text{ or } R_2 = \frac{R_1 R}{R_1 + R}$$

$$\therefore \quad \frac{5}{12} = \frac{2.5}{R_2} = \frac{2.5(R_1 + R)}{R R_1}$$

$$50 R_1 = 30 R_1 + 300 \qquad (\because \ R = 10 \ \Omega)$$

or $$R_1 = 15 \ \Omega$$

$$R_2 = \frac{10 \times 15}{25} = 6 \ \Omega$$

Power drawn V^2/R increases since total resistance decreases.

Hence the correct choice is (c).

Level B

24. Using Kirchhoff's Ist law current through R_2 is 2.25 – 1.5 = 0.75 A. Also since R_2 is in parallel with the 30 Ω resistance, R_2 must be 60 Ω since only half the current flows through it compared to the current through 30 Ω resistor. Total resistance in the circuit becomes 60 + 20 = 80 Ω

Potential drop across the battery $V = I R = 2.25 \times 80 = 180$ V.

Hence the correct choice is (d).

25. Total resistance = 4 + 1 + 0.5 + 0.5 = 6 Ω. Net voltage in the circuit is 6 V. Current $I = \dfrac{6}{6} = 1$ A in the anticlockwise direction

$$V_{PR} = 1 \times 4 = 4 \text{ V}$$

Since R is connected to earth, $V_R = 0$. Hence $V_P = 4$ V

$$V_{SQ} = 0.5 \times 1 = 0.5 \text{ V}.$$

S is at a higher potential than Q

$$\therefore \quad V_Q = -0.5 \text{ V}$$

Current is being forced into the 6 V battery in the opposite direction. Hence $V_6 = E + Ir = 6 + 1 \times 1 = 7$ V

Hence the correct choice is (c).

26. Potential difference across MP = p.d. across NO = p.d. across NP (see Fig. 12.55 on page 12.22)
Current across NP, $I_{NP} \times 10 = 20 \times 1$ or $I_{NP} = 2$ A
Across MP, $0.5\ R_1 = 20$ or $R_1 = 40\ \Omega$
Total current = 2 + 0.5 + 1.0 = 3.5 A

$$3.5 = \frac{69}{R + 40/7} \text{ yields } \quad R = 14\ \Omega$$

Hence the correct choice is (c).

27. The potential at Q with respect to R is 15 V and R is at 25 V higher potential than S. Thus Q is 40 V higher than S. When Q is grounded, its potential becomes zero. thus, $V_s = -40$ V.

28. The circuit can be rearranged as shown in Fig. 12.120.
The resistance between points A and D is given by

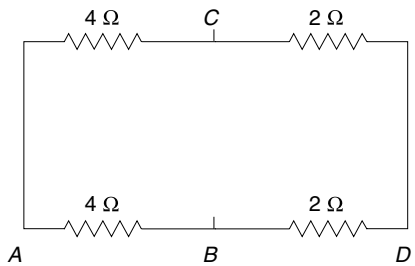


Fig. 12.120

$$\frac{1}{R} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

or $R = 3\ \Omega$. Hence the correct choice is (c).

29. If I is the maximum current that can flow through the coil,

$$100\ I + 4900\ I = 5 \text{ V} \qquad \text{or} \qquad I = \frac{1}{1000} \text{ A}$$

If R is the required resistance,

$100\ I + R'\ I = 10$, which gives $R' = 9900\ \Omega$

Since $R' = R + 4900$, $R = 5000\ \Omega$.

Hence the correct choice is (a).

30. $\frac{P_1}{P_2} = \frac{100}{200} = \frac{1}{2}$. Now $P = V^2/R$. Therefore

$$\frac{P_1}{P_2} = \frac{V_1^2}{R_1} \times \frac{R_2}{V_2^2} = \frac{(110)^2}{R_1} \times \frac{R_2}{(220)^2} = \frac{R_2}{4R_1}$$

But $P_1/P_2 = 1/2$. Therefore, $R_1/R_2 = 1/2$. Hence the correct choice is (b).

31. Heat produced $= \frac{V^2}{R} = \frac{V^2 A}{\rho l}$, where ρ is the specific resistance (or resistivity) of the wire, which is the same for both wires. Therefore, the heat produced will be same in the second wire of length $2\ l$, if its area of cross-section A is doubled. Hence the correct choice is (c).

32. Let the three resistances be X, $2X$ and Y. Then X and $2X$ connected in parallel are equivalent to a single resistance $R_1 = \frac{2}{3} X$.

Also, $\frac{1}{R_1} + \frac{1}{Y} = 1$ or $\frac{3}{2X} + \frac{1}{Y} = 1$

The minimum volume of X which satisfies the given condition is $X = 3$. Hence $Y = 2$. Three resistances are, therefore, 2, 3 and 6 Ω. Hence the correct choice is (b).

33. Potential difference across a wire of length l is

$$V = IR = \frac{I \rho l}{A}$$

$$\therefore \text{ Electric field } E = \frac{V}{l} = \frac{I\rho}{A} = \frac{0.5 \times 1.8 \times 10^{-8}}{10^{-6}}$$

$$= 0.009 \text{ Vm}^{-1}$$

Hence the correct choice is (d).

34. Total resistance $= 80 + \frac{80 \times 80}{160} = 120\ \Omega$. Therefore, current in the circuit is

$$I = \frac{2.4}{120} = 0.02 \text{ A}$$

$\therefore$ Current through galvanometer $I_g = 0.01$ A, which is choice (d).

35. Let the new value of R be R' so that I_g becomes $I'_g = 0.005$ A. Then the total resistance in the circuit becomes

$$R' + \frac{80 \times 80}{160} = R' + 40$$

Now $\qquad I'_g = \frac{2.4}{2(R' + 40)}$

But $I'_g = 0.005$ A. Therefore, $0.005 = \frac{2.4}{2(R' + 40)}$

$\Rightarrow \quad R' = 200\ \Omega$. Hence the correct choice is (b).

36. The current in the circuit is

$$I = \frac{E}{(R + r)}$$

Therefore the power output of the device is given by

$$P = I^2 R = \frac{E^2 R}{(R + r)^2} \qquad \text{(i)}$$

For given values of E and r, power output P will be maximum if $dP/dR = 0$ and $d^2P/dR^2 < 0$. Differentiating (i) with respect to R we get (with E and r fixed)

$$\frac{dP}{dR} = \frac{E^2}{(R+r)^2}\left\{1 - \frac{2R}{(R+r)}\right\} \qquad \text{(ii)}$$

Now $dP/dR = 0$ if

$$1 - \frac{2R}{(R+r)} = 0$$

which gives $R = r$. Thus, P will be either maximum or minimum when $R = r$. To decide whether P is maximum at $R = r$, we find d^2P/dR^2 at $R = r$. If its value is negative, P will be maximum. Differentiating (ii) we have

$$\frac{d^2P}{dR^2} = \frac{2E^2}{(R+r)^3}\left\{\frac{3R}{(R+r)} - 2\right\}$$

$$\therefore \left(\frac{d^2P}{dR^2}\right)_{\text{at } R=r} = -\frac{E^2}{8r^3}$$

which is negative. Hence P is maximum when $R = r$.

37. Putting $R = r$ in Eq. (i) above we get

$$P_{\max} = \frac{E^2}{4r}$$

Hence the correct choice is (d).

38. When the battery is shorted, it cannot deliver any current in resistor R. Hence the power output of the device becomes zero. The current through the battery, in this case is E/r and the power dissipated inside the battery is

$$(\text{current})^2 \times \text{resistance} = \left(\frac{E}{r}\right)^2 \times r = \frac{E^2}{r}$$

39. We have seen above that the rate of heat production in resistor R is maximum when $R = r$, the internal resistance of the battery. Hence $R = 4\ \Omega$. The total resistance $= r + R = 4 + 4 = 8\ \Omega$.

$$\therefore \quad \text{Current} = \frac{\text{emf}}{\text{total resistance}} = \frac{24}{8} = 3\text{ A}$$

Hence the correct choice is (c).

40. Power $P = \dfrac{V^2}{R}$ or $R = \dfrac{V^2}{P} = \dfrac{(200)^2}{100} = 400\ \Omega$ is the resistance of the filament of the bulb. The total energy produced in time $t = 20 \times 60 = 1200$ s when the voltage drops to V' is

$$W = \frac{V'^2 t}{R} = \frac{(100)^2 \times 1200}{400}$$
$$= 3 \times 10^4 \text{ J} = 30 \text{ k J}$$

Hence the correct choice is (c).

41. $P = 100$ MW $= 100 \times 10^6$ W. Let R be the resistance of the cable. At 20,000 V, the current in the cable is

$$I_1 = \frac{100 \times 10^6}{20{,}000} = 5000 \text{ A}$$

$\therefore$ Power dissipated is

$$P_1 = I_1^2 R = (5000)^2\, R = 2.5 \times 10^7\, R$$

At 200 V, the current in the cable is

$$I_2 = \frac{100 \times 10^6}{200} = 5 \times 10^5 \text{ A}$$

$\therefore$ Power dissipated is

$$P_2 = I_2^2 R = 2.5 \times 10^{11}\, R$$

$$\therefore \quad \frac{P_2}{P_1} = \frac{2.5 \times 10^{11}}{2.5 \times 10^7} = 10^4$$

Hence the correct choice is (a).

42. Current in the circuit due to 6 V battery is

$$I = \frac{6}{1+5+2} = \frac{3}{4} \text{ A}$$

Now emf of cell C = potential difference across AD

$$= \frac{3}{4} \times \frac{2 \times 60}{100} = 0.9 \text{ V}$$

43. Power $P = I^2R$. Therefore, $\Delta P = 2I\,\Delta I R$. Hence

$$\frac{\Delta P}{P} = \frac{2\Delta I}{I}$$

Now $\dfrac{\Delta I}{I} = +1\%$. Therefore $\dfrac{\Delta P}{P} = +2\%$. Hence the correct choice is (c).

44. Power $P = \dfrac{V^2}{R}$. Therefore $\Delta P = \dfrac{2V\Delta V}{R}$. Hence

$$\frac{\Delta P}{P} = \frac{2\Delta V}{V}$$

Now $\dfrac{\Delta V}{V} = -2\%$. Therefore $\dfrac{\Delta P}{P} = -2 \times 2 = -4\%$, i.e. the power decreases by 4%. Hence the correct choice is (c).

45. Because $P/Q = R/S$, the circuit is a balanced Wheatstone's bridge. Therefore, potential at B =

potential at D. Hence no current flows through the 5 Ω resistor; it is, therefore, not effective. The equivalent resistance between A and C is the resistance of a parallel combination of $P + R = 2 + 4 = 6\ \Omega$ and $Q + S = 4 + 8 = 12\ \Omega$, which is given by

$$\frac{1}{R} = \frac{1}{6} + \frac{1}{12}$$

or $R = 4\ \Omega$. Therefore current I is

$$I = \frac{2V}{4\Omega} = 0.5\ \text{A}$$

Hence the current choice is (b).

46. Since all resistors are equal, it follows that current in resistor 1 = current in resistor 2 = $I/2$. applying Kirchhoff's loop rule to loop $ABEA$, we find that the current through resistor 3 is zero. Similarly, the current through resistor 5 is zero. These resistors are not effective. Therefore, the equivalent resistance between A and D is the resistance of the parallel combination of resistances of 1, 4, 6 (in series) and resistances of 2, 8, 7 (in series). Thus

$$\frac{1}{R} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

or $R = 3\ \Omega$. Therefore, current $I = 3\text{V}/3\Omega = 1.0$ A, which is choice (d).

47. Let R be the value of each resistance and V, the voltage of the battery. When the three resistances are connected in series, the power dissipated is

$$P = \frac{V^2}{3R}$$

When they are connected in parallel, the power dissipated is

$$P' = \frac{3V^2}{R}$$

$\therefore$ $P' = 9P$. Hence the correct choice is (c).

48. The power ratings are always added irrespective of whether the resistance are connected in series or in parallel. Hence the correct choice is (d).

49. Let R be the resistance of each heater. When they are connected in series, the resistance of the combination is $2R$. Therefore, heat supplied in time t is

$$Q = \frac{V^2 t}{2R}$$

When they are connected in parallel, the resistance of the combination is $R' = R/2$. Therefore, heat supplied in time t is

$$Q' = \frac{V^2 t}{R'} = \frac{2V^2 t}{R}$$

Thus $Q' = 4Q$. Hence the correct choice is (d).

50. In the first case, the current in the circuit is

$$I_1 = \frac{E}{r_1 + r}$$

$$\therefore \quad Q_1 = I^2 r_1 t = \left(\frac{E}{r_1 + r}\right)^2 \times r_1 t \qquad \text{(i)}$$

In the second case,

$$Q_2 = \left(\frac{E}{r_2 + r}\right)^2 \times r_2 t \qquad \text{(ii)}$$

Equating (i) and (ii) we get

$$\frac{r_1}{(r_1 + r)^2} = \frac{r_2}{(r_2 + r)^2}$$

or $$r_1 (r_2 + r)^2 = r_2 (r_1 + r)^2$$

or $$r_1 (r_2^2 + 2r r_2 + r^2) = r_2 (r_1^2 + 2r r_1 + r^2)$$

or $$r^2 (r_1 - r_2) = r_1 r_2 (r_1 - r_2)$$

or $$r = \sqrt{r_1 r_2}$$

Hence the correct choice is (d).

51. Let R_1 and R_2 be the resistances of the heaters. When they are connected in series, the resistance of the combination is $(R_1 + R_2)$. If V is the voltage of the supply, then, if Q is the heat energy needed to boil water,

$$Q = \frac{V^2 t}{(R_1 + R_2)} = \frac{V^2 t_1}{R_1} = \frac{V^2 t_2}{R_2}$$

$$\therefore \quad t = \frac{Q(R_1 + R_2)}{V^2} = \frac{Q R_1}{V^2} + \frac{Q R_2}{V^2} = t_1 + t_2$$

Hence the correct choice is (b).

52. When the two heaters are connected in parallel, the resistance of the combination is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Now $$\frac{1}{t_1} = \frac{V^2}{Q R_1}$$

and $$\frac{1}{t_2} = \frac{V^2}{Q R_2}$$

Also $$\frac{1}{t} = \frac{V^2}{Q}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= \frac{1}{t_1} + \frac{1}{t_2}$$

or $$t = \frac{t_1 t_2}{(t_1 + t_2)}$$

Hence the correct choice is (c).

53. Current due to E_1 is

$$I = \frac{E_1}{R + 60} = \frac{4}{R + 60} \quad \text{(i)}$$

$\therefore$ Potential drop across $R = IR$. Since there is no current due to E_2,

$$IR = E_2 = 1$$

$\therefore$ $I = 1/R$. Using this in Eq. (i) we get

$$\frac{1}{R} = \frac{4}{R + 60}$$

which gives $R = 20\ \Omega$. Hence the correct choice is (b).

54. Circumference of the circle = $2\pi r$. Therefore, the resistance per unit length of the wire = $R/2\pi r$, where $R = 4\ \Omega$ is the resistance of the wire. Now, the length of the specimen connected along the diameter = $2r$. Therefore, the resistance of this specimen is

$$R_1 = \frac{R}{2\pi r} \times 2r = \frac{R}{\pi}$$

Also, the resistance of each semicircle is

$$R_2 = \frac{R}{2}$$

$\therefore$ Equivalent resistance R' across the specimen is given by

$$\frac{1}{R'} = \frac{2}{R} + \frac{2}{R} + \frac{\pi}{R} = \frac{4 + \pi}{R}$$

or $$R' = \frac{R}{4 + \pi} = \frac{4}{4 + \pi}\ \Omega$$

55. From Kirchhoff's first law, the current in branch PQ = 2 + 2 = 4A and in branch QR the current = 4A – 1A = 3A. Hence current i = 3A – 1.3 A = 1.7A. Thus the correct choice is (c).

56. Original resistance $R = \frac{\rho L}{A}$. If L' is the length of the elongated wire and A' its cross-sectional area, the new resistance will be

$$R' = \frac{\rho L'}{A'}$$

Therefore $$\frac{R'}{R} = \frac{L'}{L} \times \frac{A}{A'} = \left(\frac{L'}{L}\right)^2 \times \frac{AL}{A'L'}$$

Given $L' = L$ + 10% of $L' = L + 0.1\ L = 1.1\ L$. Since the volume of the wire cannot change, $A'L' = AL$. Hence

$$\frac{R'}{R} = \left(\frac{L'}{L}\right)^2 = \left(\frac{1.1L}{L}\right)^2 = 1.21$$

or $R' = 1.21R = 1.21 \times 1\ \Omega = 1.21\ \Omega$. Hence the correct choice is (c).

57. Figure 12.121 shows the equivalent circuit which is a balanced Wheatstone's bridge. Hence resistance R between points P and Q is ineffective. Thus the effective resistance between terminals A and B is the resistance of a parallel combination of two resistors, each of value $R + R = 2R$. Thus,

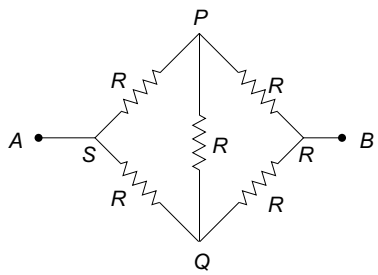


Fig. 12.121

$$\frac{1}{R_{\text{eff}}} = \frac{1}{2R} + \frac{1}{2R} = \frac{1}{R} \quad \text{or} \quad R_{\text{eff}} = R$$

Hence the correct choice is (a).

58. Current in the main circuit is $I = \frac{12\text{V}}{3\ \Omega + 6\ \Omega + 1\ \Omega}$

$$= 1.2\ \text{A}$$

$\therefore$ Potential difference across AB is

$$V_{\text{AB}} = 12 - 1.2 \times 1 = 10.8\ \text{V}$$

Also, potential difference across AD is

$$V_{\text{AD}} = 6 \times 1.2 = 7.2\ \text{V}$$

Effective capacitance is

$$C_{\text{eff}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{1 \times 2}{1 + 2} = \frac{2}{3}\mu\text{F} = \frac{2}{3} \times 10^{-6}\ \text{F}$$

Since the capacitors are in series, charge Q is the same and is given by

$$Q = V_{\text{AB}} \times C_{\text{eff}} = 10.8 \times \frac{2}{3} \times 10^{-6}$$

$$= 7.2 \times 10^{-6}\ \text{C}$$

Therefore, potential difference across AC is

$$V_{\text{AC}} = \frac{Q}{C_2} = \frac{7.2 \times 10^{-6}}{2 \times 10^{-6}} = 3.6\ \text{V}$$

$\therefore$ Potential difference between points C and D is

$$V_{CD} = V_{AD} - V_{AC} = 7.2 - 3.6 = 3.6 \text{ V}$$

Hence the correct choice is (a).

59. The upper part of the circuit is a balanced Wheatstone's bridge. Hence resistance R between CD is ineffective as no current will flow in this branch. The circuit, therefore, reduces to three parallel branches having resistance R, $R + R$ and $R + R$, i.e. R, $2R$ and $2R$. The effective resistance R' is given by

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{2R}$$

which gives $R' = \frac{R}{2}$. Hence the correct choice is (a).

60. For series connection $x = nR$. For parallel connection $y = \frac{R}{n}$. Therefore $xy = nR \times \frac{R}{n} = R^2$. Hence the correct choice is (c).

61. Given $\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_{n-1}} + \frac{1}{R_n} = \frac{1}{x}$ (1)

If the nth resistor is removed, then

$$\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_{n-1}} = \frac{1}{y} \quad (2)$$

Subtracting (2) from (1), we have

$$\frac{1}{R_n} = \frac{1}{x} - \frac{1}{y}$$

which gives $R_n = \frac{xy}{(y-x)}$ which is choice (b).

62. Let m cells be connected in series and n such groups are connected in parallel. If the emf of each cell is E and internal resistance r, then the total emf of m cells in series in mE and the total internal resistance is mr. When n such groups are in parallel, the effective internal resistance is mr/n. Then the current through an external resistance R is

$$I = \frac{mE}{R + \frac{mr}{n}} = \frac{mnE}{nR + mr} \quad (1)$$

$$= \frac{mnE}{\left(\sqrt{nR} - \sqrt{mr}\right)^2 + 2\sqrt{mnRr}}$$

Now, I will be maximum if the denominator is the minimum, i.e. if $nR = mr$. Given $R = 3\ \Omega$ and $r = 1\ \Omega$. Using these values, we have $3n = m$. But $mn = 48$ (given). Therefore $\frac{m \times m}{3} = 48$, which gives $m = 12$. Thus $n = 4$. Hence the correct choice is (b).

63. Substituting the values of m, n, r and E in (1) above, we have

$$I = \frac{12 \times 4 \times 2}{4 \times 3 + 12 \times 1} = 4 \text{ A}$$

Hence the correct choice is (d).

64. The given circuit is a Wheatstone's bridge. The current through the galvanometer will be zero if the bridge is balanced, i.e. if

$\frac{P}{Q} + \frac{R}{S}$ where $P = 2 + 3 = 5\ \Omega$, $Q = 10\ \Omega$ and $R = 4\ \Omega$. The value of S is given by

$$\frac{5}{10} = \frac{4}{S}$$

or $S = 8\ \Omega$. Thus the effective resistance of the parallel combination of 12 Ω and x ohm must be 8 Ω. Therefore

$$\frac{1}{12} + \frac{1}{x} = \frac{1}{8}$$

which gives $x = 24\ \Omega$. Hence the correct choice is (d).

65. Since the seven resistances are in parallel, the effective resistance is $R = 70/7 = 10\ \Omega$. Therefore, the current in the circuit is $I = 14/10 = 7/5$A. The given circuit can be redrawn as shown in Fig. 12.122 where $R_1 = \frac{70}{3}\ \Omega$ and $R_2 = \frac{70}{4}\ \Omega$. The current I_2 is given by

$$I_2 = I \times \frac{R}{R_2}$$

$$= \frac{7}{5} \times \frac{10}{70/4} = \frac{4}{5} \text{ A, which is choice (b).}$$

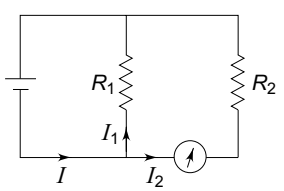


Fig. 12.122

66. Figure 12.123 shows the equivalent Wheatstone's bridge which is balanced. Hence the resistance R between B and C will be ineffective. Thus the equivalent resistance between X and Y is the resistance of the parallel combination of two resistances $(R + R) = 2R$ and $(R + R) = 2R$ which is given by

$$\frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{2R} = \frac{1}{R}$$

or $R_{eq} = R$ which is choice (b).

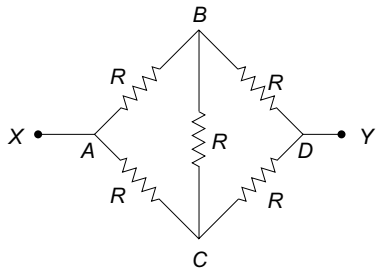


Fig. 12.123

67. Resistances R, $(R + R) = 2R$, $(R + R) = 2R$ and R are in parallel. Hence the effective resistance is given by

$$\frac{1}{R_{eff}} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{2R} + \frac{1}{R}$$

which gives $R_{eff} = \frac{R}{3}$. Hence the correct choice is (a).

68. Resistance of heater $(R_H) = \frac{V^2}{P} = \frac{100 \times 100}{1000} = 10\ \Omega$.

Current in the circuit is

$$I = \frac{100}{10 + \frac{10R}{(10 + R)}}$$

$\therefore$ Current in heater is

$$I_H = I \times \frac{R}{(10 + R)} = \frac{100}{\left[10 + \frac{10R}{(10 + R)}\right]} \times \frac{R}{(10 + R)}$$

$$= \frac{10R}{\left[1 + \frac{R}{(10 + R)}\right](10 + R)} = \frac{5R}{(5 + R)}$$

$\therefore \quad Q = I_H^2 \times 10$

or $$\frac{25R^2}{(5 + R)^2} \times 10 = 62.5 \text{ (given)}$$

which gives $R = 5\ \Omega$. Hence the correct choice is (d).

69. $Q = \frac{V^2}{R}$. But $R = \frac{\rho l}{\pi r^2}$. Therefore,

$$Q = \left(\frac{\pi V^2}{\rho}\right)\frac{r^2}{l}$$

Q is doubled if both l and r are doubled. Hence the correct choice is (b).

70. The emfs of cells connected in reverse polarity cancel each other. Hence cells marked 2, 3 and 4 together cancel the effect of cells marked 5, 6 and 7 and the circuit reduces to that shown in Fig. 12.124. Now cells 1 and 8 are in reverse polarity. Hence the voltmeter reading = 5 – 5 = 0 V. Hence the correct choice is (d).

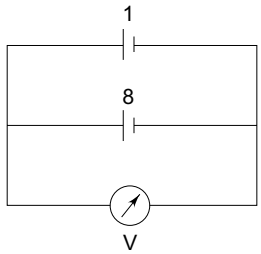


Fig. 12.124

71. The correct choice is (d).

72. The drift speed depends on A, the cross-sectional area of the conductor but the current is independent of A. Hence the correct choice is (d).

73. The equivalent resistance between points A and B to the right of AB is 4 Ω. Therefore, total resistance = 3 + 4 + 2 = 9 Ω. Current I = 9 V/9 Ω = 1 A. This current is equally divided in the 8 Ω resistor between A and B and the remainder 8 Ω resistor. Hence current in AC = 0.5 A. This current is equally divided between the 8 Ω resistor in CD and the circuit to the right of CD. Therefore, current in the 4 Ω resistor = 0.25 A which is choice (d).

74. The given network of resistances is a balanced Wheatstone's bridge. Therefore, no current flows through the 6 Ω resistor and it can be omitted. The equivalent resistance of the network is that of a parallel combination of $R_1 = R + R + R = 3R$ and $R_2 = R + R + 4R = 6R$ which is

$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{3R \times 6R}{3R + 6R} = 2R$$

Now refer to the solution of Question 37. The power delivered will be maximum if the resistance of the emf source = external resistance, i.e. if

$$4\Omega = R' = 2R$$

or R = 2 Ω, which is choice (b).

75. In the steady state, no current flows in the branch containing the capacitor. Thus, the current, say I, flows in the branches containing R and $2R$. Applying Kirchhoff's second rule to the loop $abcdefa$, we have (see Fig. 12.125)

$$2V - I(2R) - IR - V = 0 \text{ or } I = \frac{V}{3R}$$

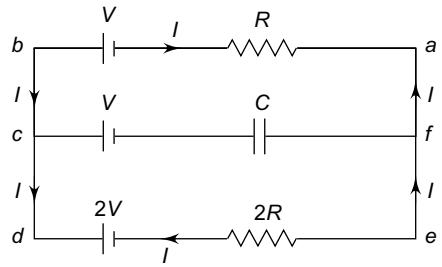


Fig. 12.125

$\therefore$ Potential drop across capacitor

$$= 2V - V - I(2R) = V - \frac{V}{3R} \times 2R$$

$$= V - \frac{2V}{3} = \frac{V}{3}, \text{ which is choice (c).}$$

76. The heat energy generated in time t is $Q = V^2t/R$, where V is the terminal voltage of a single cell. Here $Q = ms\Delta T$ where m is the mass of the wire, s its specific heat and R its resistances.

In the first case: $$ms\Delta T = \frac{(3V)^2 t}{R} \quad (1)$$

Now $m \propto$ length and $R \propto$ length. Hence, in the second case,

$$(2m)\, s\Delta T = \frac{(NV)^2 t}{(2R)} \quad (2)$$

Dividing (2) by (1), we have

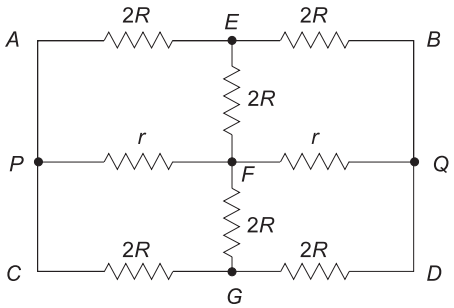
$$2 = \frac{N^2/2}{9} \text{ or } N^2 = 36 \text{ or } N = 6, \text{ which is choice (b).}$$

77. Since no current flows through R_6, resistances R_1, R_2, R_3 and R_4 constitute the four arms of a balanced Wheatstone's bridge. Hence

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \text{ or } R_1 R_4 = R_2 R_3$$

Thus the correct choice is (c).

78. Refer to Fig. 12.126.



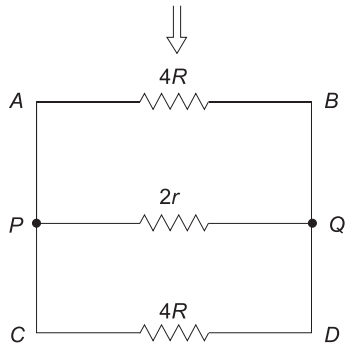


Fig. 12.126

The branches $ABPQ$ and $PQCD$ are a balanced Wheatstone's bridge. Therefore, resistances (each equal to $2R$) between E and F and between F and G do not contribute and the circuit simplies to the one shown in the figure. The effective resistance R_e between P and Q is given by

$$\frac{1}{R_e} = \frac{1}{4R} + \frac{1}{2r} + \frac{1}{4R}$$

which gives $R_e = \frac{2Rr}{(R+r)}$. Hence the correct choice is (a).

79. The resistances of bulbs B_1, B_2 and B_3 respectively are

$$R_1 = \frac{V^2}{W_1} = \frac{(250)^2}{100} = 625\ \Omega$$

$$R_2 = \frac{(250)^2}{60} = 1042\ \Omega = R_3$$

Voltage across B_3 is $V_3 = 250$ V

Voltage across B_1 is $V_1 = \frac{VR_1}{(R_1 + R_2)} = \frac{250 \times 625}{(625 + 1042)}$

$= 93.7$ V

Voltage across B_2 is $V_2 = 250 - 93.7 = 156.3$ V

Power output $W_1 = \frac{V_1^2}{R_1} = \frac{(93.7)^2}{625} = 14$ W

$$W_2 = \frac{(156.3)^2}{1042} = 23 \text{ W}$$

$$W_3 = \frac{(250)^2}{1042} = 60 \text{ W}$$

Hence $W_1 < W_2 < W_3$, which is choice (d).

80. The condition for no deflection of the galvanometer is

$$\frac{R_1}{R_2} = \frac{R_{AC}}{R_{CB}}$$

where R_{AC} and R_{CB} are the resistances of the bridge wire of length AC and CB respectively. If the radius of the wire AB is doubled, the ratio R_{AC}/R_{CB} will remain unchanged. Hence the balance length will remain the same. Thus, the correct choice is (b).

81. Let R be the value of each resistance. The resistances of combinations I, II, III and IV are $3R$, $R/3$, $2R/3$ and $3R/2$ respectively. Now, power dissipation is inversely proportional to resistance. Hence the correct choice is (b).

82. Current in the circuit is $I = \dfrac{E}{(R+r)} = \dfrac{2}{(R+5)}$.

Therefore, the potential difference across the potentiometer wire of length $L = 100$ cm is

$$V = Ir = \frac{2}{(R+5)} \times 5 = \frac{10}{(R+5)} \text{ volt}$$

$\therefore$ Potential difference across $l = 10$ cm of the wire is

$$v = \frac{V \times l}{L} = \frac{10}{(R+5)} \times \frac{10}{100} = \frac{1}{(R+5)} \text{volt}$$

Given $v = 5$ mV $= 5 \times 10^{-3}$ V. Hence, we have

$$5 \times 10^{-3} = \frac{1}{(R+5)}$$

which gives $R = 195\ \Omega$, which is choice (c).

83. A voltmeter is a galvanometers having a high resistance connected in series with it. The current through the galvanometer is

$$I_g = \frac{5\text{V}}{300\Omega} = \frac{1}{60} \text{ A}$$

An ammeter is a galvanometer having a low resistance connected in parallel with it. The shunt resistance S is determined from

$$\frac{I_g}{I} = \frac{S}{G+S}$$

where $G = 300\ \Omega$ (given). For $I = 5$ A, we have

$$\frac{1/60}{5} = \frac{S}{300+S}$$

or $299\ S = 300$ or $S = 300/299 \simeq 1\ \Omega$. Hence the correct choice is (b).

84. The resistance of a conductor of length l, cross-sectional area A and made of a material of resistivity ρ is given by

$$R = \frac{\rho l}{A} = \frac{\rho l^2}{Al} = \left(\frac{\rho}{V}\right) l^2$$

where $V = Al$ is the volume of the conductor. Since ρ is a constant and volume V cannot change if the conductor is stretched, it follows that R is proportional to l^2. Thus if l is doubled, R becomes four times. Hence the new resistance is $3 \times 4 = 12\ \Omega$. So, each side of the equilateral triangle has a resistance of $4\ \Omega$ as shown in Fig. 12.127. Therefore, the effective resistance between the ends of any side of the triangle (such as side AB) is equal to the resistance of a parallel combination of $R_1 = 4\ \Omega$ and $R_2 = 4 + 4 = 8\ \Omega$, which is given by

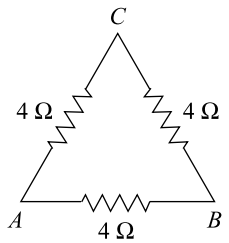


Fig. 12.127

$$R_e = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{4 \times 8}{4 + 12} = \frac{8}{3}\ \Omega$$

Hence the correct choice is (b).

85. Resistance of each side of the square $= 10 \times 0.1 = 1\ \Omega$. As shown in Fig. 12.128, the square forms a Wheatstone's bridge which satisfies the balancing condition. Thus, no current flows along the diagonal BD. Hence the correct choice is (a).

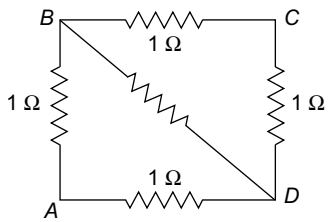


Fig. 12.128

86. The correct choice is (b).

87. Let the value of each resistance be r. The network can be redrawn as shown in the Fig. 12.129.

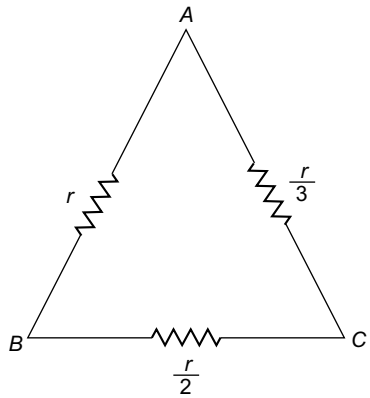


Fig. 12.129

(i) *Net resistance R_1 between points A and B*

The series combination of resistances $\frac{r}{3}$ and $\frac{r}{2}$, which has an equivalent resistance $r_1 = \frac{r}{3} + \frac{r}{2} = \frac{5r}{6}$, is in parallel with resistance r. Hence

$$R_1 = \frac{r \times r_1}{r + r_1} = \frac{r \times \frac{5r}{6}}{r + \frac{5r}{6}} = \frac{5r}{11}$$

(ii) *Net resistance R_2 between points B and C*

The series combination of resistances r and $\frac{r}{3}$, which has an equivalent resistance $r_2 = r + \frac{r}{3} = \frac{4r}{3}$, is in parallel with resistance $\frac{r}{2}$. Hence

$$R_2 = \frac{\frac{r}{2} \times r_2}{\frac{r}{2} + r_2} = \frac{\frac{r}{2} \times \frac{4r}{3}}{\frac{r}{2} + \frac{4r}{3}} = \frac{4r}{11}$$

(iii) *Net resistance R_3 between points A and C*

The series combination of resistances r and $\frac{r}{2}$, which has an equivalent resistance $r_3 = r + \frac{r}{2} = \frac{3r}{2}$, is in parallel with resistance $\frac{r}{3}$. Hence

$$R_3 = \frac{\frac{r}{3} \times r_3}{\frac{r}{3} + r_3} = \frac{\frac{r}{3} \times \frac{3r}{2}}{\frac{r}{3} + \frac{3r}{2}} = \frac{3r}{11}$$

Hence $R_1 : R_2 : R_3 = \frac{5r}{11} : \frac{4r}{11} : \frac{3r}{11} = 5 : 4 : 3.$

Thus the correct choice is (c).

88. The two sub circuits are closed loops. They cannot send any current through the 3 Ω resistor. Hence the potential difference across the 3 Ω resistor is zero, which is choice (a).

89. Given $I = 1$ mA $= 10^{-3}$ A, $G = 20$ Ω and $R = 4980$ Ω.

Now $$I = \frac{V}{R + G}$$

or $$V = I(R + G) = 10^{-3} \times (4980 + 20)$$
$$= 5.0 \text{ V}$$

Hence the correct choice is (c).

90. The mass of a wire of length l, cross sectional area A and density d is given by

$$m = Ald \text{ or } A = \frac{m}{ld}$$

$\therefore$ The resistance of wire of resistivity ρ is

$$R = \frac{\rho l}{A} = \frac{\rho d l^2}{m} = kl^2 \qquad (1)$$

where $k = \rho d/m$ is a constant of the wire. Taking logarithm of both sides of (1) we have

$$\log R = \log k + 2 \log l$$

Differentiating

$$\frac{\delta R}{R} = 0 + \frac{2\delta l}{l} = \frac{2\delta l}{l}$$

Given $\frac{\delta l}{l} = 0.1\%$. Therefore, $\frac{\delta R}{R} = 2 \times 0.1\% = 0.2\%$. Thus, the resistance of the wire increases by 0.2%, which is choice (b).

91. The resistance between points A and E is given by

$$\frac{1}{R_{AE}} = \frac{1}{6} + \frac{1}{6}$$

by giving $R_{AE} = 3$ Ω. The network reduces to that shown in Fig. 12.130 (a). Similarly the resistance A between points A and D in Fig. 12.130 (a) is $R_{AD} = 3$ Ω. The network reduces to that shown in Fig. 12.130 (b). It follows from this figure that the resistance between points A and C is $R_{AC} = 3$ Ω. The network simplifies to that shown in Fig. 12.130 (c). Clearly the effective resistance between points A and B is 2Ω. So, the correct choice is (b).

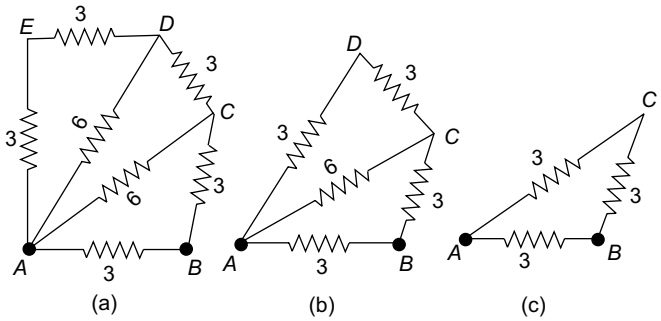


Fig. 12.130

92. Let the internal resistance of each battery be r. Let R be the unknown resistance and G be the resistance of the galvanometer. Let E be the emf of each battery. When the batteries are connected in series, the total emf = $2E = 2 \times 1.5 = 3$ V and total internal resistance is $2r$. The current in the circuit will be

$$I = \frac{3}{R + G + 2r}$$

Given $I = 1$ A. Therefore

$$1 = \frac{3}{R + G + 2r}$$

$$\text{or } R + G = (3 - 2r) \qquad (1)$$

When the batteries are connected in parallel, the total emf = $E = 1.5$ V and the total internal resistance is $r/2$. Hence the current in the circuit will be

$$I' = \frac{1.5}{R + G + \frac{r}{2}}$$

Given $I' = 0.6$ A. Therefore,

$$0.6 = \frac{1.5}{R + G + \frac{r}{2}}$$

$$\text{or } R + G = \left(2.5 - \frac{r}{2}\right) \qquad (2)$$

From Eqs. (1) and (2), we have

$$3 - 2r = 2.5 - \frac{r}{2}$$

which gives $r = \frac{1}{3}$ ohm, which is choice (d)

93. When the key K is inserted, the current starts growing and after some time it acquires a steady value. At this stage, no current flows through the capacitor (because an ideal capacitor offers an infinite resistance to a steady current). All the current flows through the inductor (because an ideal inductor offers zero resistance to a steady current). Now, the network of resistors is a balanced Wheatstone's Bridge. Hence, no current flows through the resistance $2R$. Therefore, this resistance can be ignored. The net resistance between points X and Y = resistance of the parallel combination of $2R$ and $2R$

$$= \frac{2R \times 2R}{(2R + 2R)} = R.$$

Current in the circuit is $I = \frac{E + E}{R + r_A + r_B} = \frac{2E}{R + r_A + r_B}$

Terminal voltage of cell A is $V_A = E - I r_A$. $V_A = 0$, if $E - I r_A = 0 \Rightarrow E = \frac{2E r_A}{R + r_A + r_B} \Rightarrow R = r_A - r_B$, which is choice (a).

94. Resistance of each bulb is $R = \frac{(200)^2}{500} = 80\ \Omega$. When the bulbs are connected in series to a 100 V source, the potential difference across each bulb is $V = 100/2 = 50$ V. Therefore, power generated in each bulb is

$$P = \frac{V^2}{R} = \frac{(50)^2}{80} = 31.25 \text{ W}$$

Hence the correct choice is (b).

95. $R = \frac{\rho L}{\pi r^2}$. Since the two wires are made of the same material, resistivity ρ is the same for wires 1 and 2.

$$R_1 = \frac{\rho(2L)}{\pi r^2} \quad \text{and} \quad R_2 = \frac{\rho(L)}{\pi(2r)^2}$$

$$\therefore \quad \frac{R_1}{R_2} = 8 \quad \Rightarrow \quad R_1 = 8\,R_2$$

Since the current I in the same in wires 1 and 2 (as they are in series).

$$V_1 = IR_1 \quad \text{and} \quad V_2 = IR_2$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{R_1}{R_2} = 8. \text{ So choice (a) is wrong.}$$

Powers in wires are $P_1 = I^2R_1$ and I^2R_2. So

$$\frac{P_1}{P_2} = \frac{R_1}{R_2} = 8. \text{ So choice (b) is also wrong.}$$

Current density $J = \frac{I}{\pi r^2}$.

$$J_1 = \frac{I}{\pi r^2}$$

$$J_2 = \frac{I}{\pi(2r)^2}$$

$$\therefore \quad \frac{J_1}{J_2} = 4. \text{ So choice (c) is also wrong}$$

Electric field $E = \frac{V}{l}$.

$$E_1 = \frac{V_1}{2L} \quad \text{and} \quad E_2 = \frac{V_2}{L}.$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{V_1}{2V_2} = \frac{1}{2} \times 8 = 4$$

So the only correct choice is (d).

96. Given $\frac{V^2}{2} + \frac{V^2}{3} = 30 \quad \Rightarrow \quad V^2 = 36$ or $V = 6$ volt.

So the potential difference across the 4 Ω resistor is 6 V. Therefore, the current through this resistor = 6/4 = 1.5 A, which is choice (c).

97. Since the current through the 1 Ω resistor is 2 A, the current in the 2 Ω resistor will be 1 A. So the current in the 3 Ω resistor = 1 + 2 = 3 A. Hence power developed in the 3 Ω resistor = $(3)^2 \times 3 = 27$ W. So, the correct choice is (c).

98. The equivalent resistance of the parallel combination is

$$R_P = \frac{4 \times 6}{(4+6)} = 2.4\,\Omega$$

Total resistance between points A and B is

$$R = 2.4 + 7.6 = 10\,\Omega$$

Since point B is grounded, $V_B = 0$. Given $V_A = 30$V.

$\therefore$ Potential difference between A and B is

$$V = V_A - V_B = 30 - 0 = 30 \text{ V}$$

$\therefore$ Current through R_3 is

$$I = \frac{30 \text{ V}}{10\,\Omega} = 3\text{A}, \quad \text{which is choice (c).}$$

99. Initial charge as C_1 is $Q = C_1 V_1 = 3\,\mu\text{F} \times 15\text{V} = 45\,\mu\text{C}$. When S_1 and S_2 are closed, this charge will flow from C_1 to C_2 until both the capacitors have the same potential difference V. Let Q_1 and Q_2 are the charges on C_1 and C_2 when steady state is established. Since C_1 and C_2 are in parallel, V is the same in both, *i.e.*,

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

or $$\frac{Q_1}{C_1} = \frac{Q - Q_1}{C_2}$$

$$\Rightarrow \quad \frac{Q_1}{3} = \frac{45 - Q_1}{12}$$

$$\Rightarrow \quad Q_1 = 9\,\mu\text{C}$$

and $Q_2 = Q - Q_1 = 45 - 9 = 36\,\mu$C. So the correct choice is (*b*).

100. Since points A and D are grounded, $V_A = 0$ and $V_D = 0$. Applying Kirchhoff's rule to the part $ABCD$ of the circuit, we have

$$V_A - V_D = -I \times 4 + 8$$

$$\Rightarrow \quad 0 - 0 = -4I + 8 \quad \Rightarrow \quad I = 2\text{A}$$

So the correct choice is (b). The part of the network above BC is redundant because no current flows through it as points C and D are at the same potential which is zero.

101. Figure 12.131 shows the assumed directions of currents.

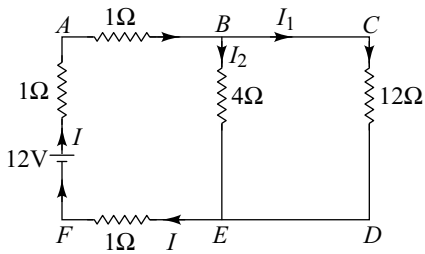


Fig 12.131

From junction rule, $I = I_1 + I_2$

Applying the loop rule to loop $ABEFA$,

$$I \times 1 + I_2 \times 4 + I \times 1 - 12 + I \times 1 = 0$$

$$\Rightarrow \quad 3I + 4I_2 = 12$$

$$\Rightarrow \quad 3(I_1 + I_2) + 4I_2 = 12$$

$$\Rightarrow \quad 3I_1 + 7I_2 = 12 \qquad (1)$$

Applying the loop rule to loop $BCDEB$,

$$12I_1 - 4I_2 = 0$$

$$\Rightarrow \quad I_2 = 3I_1 \qquad (2)$$

Using (2) in (1), we get $I_1 = 0.5$A

$$\therefore \quad V_B - V_D = 12\, I_1 = 12 \times 0.5 = 6\text{V}$$

So the correct choice is (c).

102. The charge on the capacitor at time t is given by

$$Q = Q_0 (1 - e^{-t/\tau_1})$$

where time constant $\tau_1 = R_1 C$ and Q_0 is the final charge, i.e., the charge when the capacitor is fully charged to the voltage of the battery. For Q to become equal to $\frac{Q_0}{2}$, the time t required is given by

$$\frac{Q_0}{2} = Q_0 (1 - e^{-t/\tau_1})$$

$$\Rightarrow \quad 1 - e^{-t/\tau_1} = \frac{1}{2}$$

$$\Rightarrow \quad e^{-t/\tau_1} = \frac{1}{2}$$

$$\Rightarrow \quad e^{-t/\tau_1} = 2$$

$$\Rightarrow \quad \frac{t}{\tau_1} = \ln(2)$$

$$\Rightarrow \quad t = \tau_1 \ln(2) = R_1 C \ln(2)$$

So the correct choice is (c).

103. The current due to discharging of the capacitor decreases with time t as

$$I = I_0 e^{-t/\tau_2}$$

where $I_0 = \frac{V}{R_2}$ is the maximum current and $\tau_2 = R_2 C$.

The power dissipated through R_2 is

$$P = I^2 R_2 = I_0^2 R_2 e^{-2t/\tau_2} = P_0 e^{-2t/\tau_2}$$

where $P_0 = I_0^2 R_2$ is the maximum power dissipated. For P to become $P_0/2$, the time t required is given by

$$\Rightarrow \quad \frac{P_0}{2} = P_0 e^{-2t/\tau_2}$$

$$\Rightarrow \quad e^{-2t/\tau_2} = \frac{1}{2}$$

$$\Rightarrow \quad e^{2t/\tau_2} = 2$$

$$\Rightarrow \quad \frac{2t}{\tau_2} = \ln(2)$$

$$\Rightarrow \quad t = \frac{\tau_2 \ln(2)}{2} = \frac{R_2 C \ln(2)}{2}$$

So the correct choice is (a).

104. Refer to Example 24 on page 12.20. Since the circuit is symmetric about the perpendicular line passing through O, the given network can be simplified as shown in Fig. 12.132.

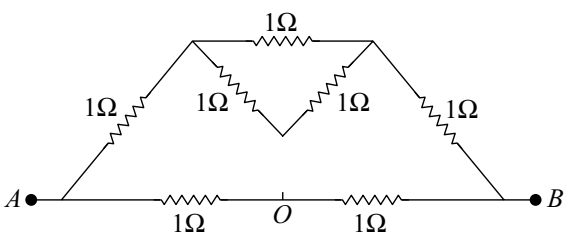


Fig 12.132

It is easy to show that the equivalent resistance between A and B is 8/7Ω, which is choice (d).

105. The current I in a rod is independent of the area of cross-section. So the current at A = current at B and, therefore, choice (c) is wrong. The current density J is given by

$$J = \frac{I}{a} = \frac{I}{\pi r^2}$$

where a is the cross-sectional area. Since I is the same along the rod, J will be higher where a is smaller. From the given graph, it follow that J is greater at $x = 0$ (end A) than at $x = L$ (end B). Hence, the radius r is smaller at A than at B. So choice (a) is correct. Since the cross-sectional area at end A is smaller that at B from Ohm's law, $V = IR$, the potential gradient $\frac{dV}{dx}$ is not constant (because R is not constant) along the rod. So the only correct choice is (a).

106. The current is given by

$$I = \int \mathbf{J} \cdot \mathbf{A}$$

If $\mathbf{J}$ is constant throughout the area, then

$$I = \mathbf{J} \cdot \mathbf{A}$$

where $\mathbf{A}$ is the area vector. Thus

$$I = \text{JA} \cos\theta$$

$$= 6 \times 3 \cos 60°$$

$$= 9\text{A}$$

So the correct choice is (a)

107. Consider a small circular strip of radius r and thickness dr as shown in Fig. 12.133.

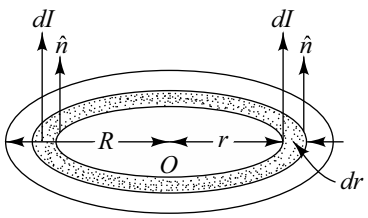


Fig 12.133

The current through the small circular strip is

$$dI = \int \mathbf{J}.\mathbf{dA} = \int J dA \cos\theta$$

where $\boldsymbol{dA} = \hat{\boldsymbol{n}}\, dA$; $\hat{\boldsymbol{n}}$ is a unit vector perpendicular to $\boldsymbol{dA}$. Hence $\theta = 0°$. Also $dA = \pi(r + dr)^2 - \pi r^2 = 2\pi r dr$. Therefore,

$$dI = 2\pi \int (3ar + 2b) \times 2\pi r\, dr \times \cos 0°$$

$$= 2\pi \int_0^R (3ar + 2b)\, r\, dr$$

$$= 6\pi a \int_0^R r^2 dr + 4\pi b \int_0^R r dr$$

$$= 6\pi a \frac{R^3}{3} + 4\pi b \frac{R^2}{2}$$

$$= 2\pi aR^3 + 2\pi bR^2$$

$$= 2\pi R^2 (aR + b)$$

So the correct choice is (*c*).

108. By Ohm's law, the current density J and electric field E are related as

$$J = \sigma E = \frac{E}{\rho} \; ; \qquad (\sigma = \text{conductivity})$$

$$\therefore \qquad E = \rho J = (ax + b)\, J$$

Since the cross-sectional area is constant, J is constant.

Now $$E = -\frac{dV}{dx}$$

$$\Rightarrow \qquad dV = -E\, dx$$

Integrating

$$\int_{V_1}^{V_2} dV = -\int_0^x (ax + b)\, J\, dx$$

$$\Rightarrow \qquad V_2 - V_1 = -J\left(\frac{ax^2}{2} + bx\right)$$

or $$V = V_1 - V_2 = J\left(\frac{ax^2}{2} + bx\right) \qquad (1)$$

At $x = 0$, $V = 0$. Eq. (1) is the equation of a symmetric parabola (as in the case of the trajectory of a projectile). So the correct choice is (b).

109. Consider a small element of thickness dr at a distance r from the axis of the rod as shown in Fig. 12.134.

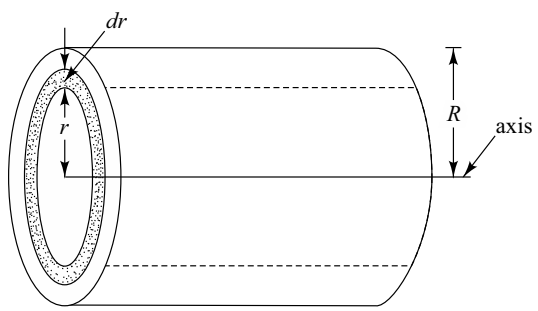


Fig 12.134

The current flowing through the element is

$$dI = en\, dA\, v$$

$$= en \times 2\pi r dr \times v_0 \left(1 + \frac{r}{R}\right)$$

$$= \frac{2\pi en v_0}{R} (R\, dr + r^2 dr)$$

Integrating

$$I = \left(\frac{2\pi en v_0}{R}\right)\left[R\int_0^R r dr + \int_0^R r^2 dr\right]$$

$$= \frac{2\pi en v_0}{R}\left(\frac{R^3}{2} + \frac{R^3}{3}\right)$$

$$= en\, \pi R^2\, v_0 \left(\frac{5}{3}\right)$$

or $$I = en\, A \times \frac{5v_0}{3}$$

Comparing with $I = enA\, v_d$, we get

$$v_d = \frac{5v_0}{3}, \quad \text{which is choice (d).}$$

110. Let us choose the directions of currents I_1, I_2 and I_3 as shown in Fig. 12.135.

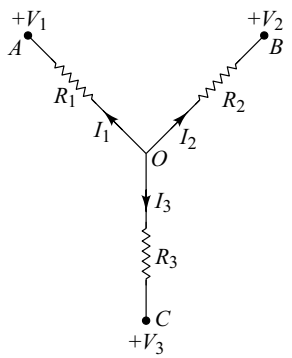


Fig 12.135

Applying junction rule at junction O.

$$I_1 + I_2 + I_3 = 0$$

Let V_0 be the potential at 0. Using Ohm's law, we have

$$V_0 - V_1 = I_1 R_1 \Rightarrow I_1 = \frac{V_0}{R_1} - \frac{V_1}{R_1}$$

$$V_0 - V_2 = I_2 R_2 \Rightarrow I_2 = \frac{V_0}{R_2} - \frac{V_2}{R_2}$$

$$V_0 - V_3 = I_3 R_3 \Rightarrow I_3 = \frac{V_0}{R_3} - \frac{V_3}{R_3}$$

Substituting these values in (1), we have

$$V_0\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right) = 0$$

$$\Rightarrow \qquad V_0 = \frac{\dfrac{V_1}{R_1} + \dfrac{V_2}{R_2} + \dfrac{V_3}{R_3}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

So the correct choice is (a).

111. Current in R_1 and R_2 due to E_1 is

$$I = \frac{E_1}{R_1 + R_2}$$

$\therefore$ Potential difference across R_1 is

$$V_1 = IR_1 = \frac{E_1 R_1}{R_1 + R_2} \quad (1)$$

Since the galvanometer shows no deflection,

$$E_2 = V_1$$

$$\Rightarrow \quad E_2 = \frac{E_1 R_1}{R_1 + R_2}$$

$$\Rightarrow \quad \frac{E_2}{E_1} = \frac{R_1}{R_1 + R_2}$$

So the correct choice is (b).

112. The circuit can be redrawn as shown in Fig. 12.136.

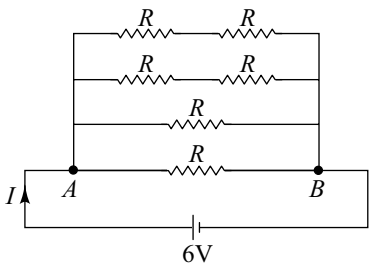


Fig 12.136

The equivalent resistance between A and B is given by

$$\frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R}$$

$$\Rightarrow \quad R_{eq} = \frac{R}{3} = \frac{9\Omega}{3} = 3\Omega$$

$$\therefore \quad I = \frac{6\,\text{V}}{R_{eq}} = \frac{6\,\text{V}}{3\Omega} = 2\text{A}$$

So the correct choice is (b).

113. The given circuit can be redrawn as shown in Fig. 12.137.

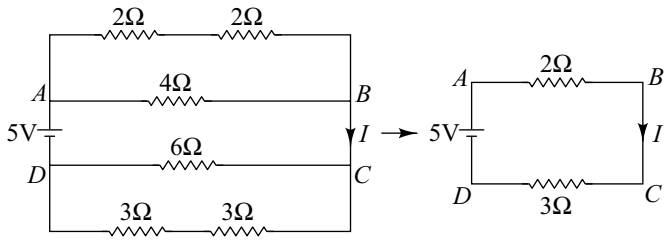


Fig 12.137

Current $I = \dfrac{5\,\text{V}}{2\Omega + 3\Omega} = 1\text{A}$, which is choice (a).

114. Refer to Fig. 12.138.

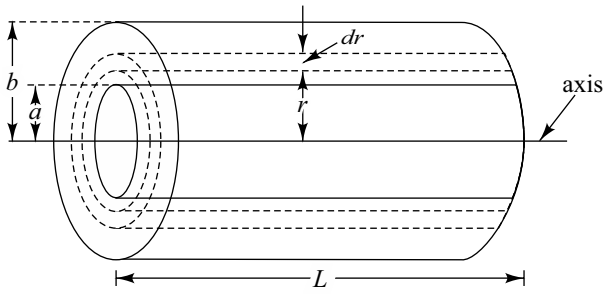


Fig 12.138

Divide the metallic shell into very small elements of very small thickness dr. Consider one such element at a distance r from the axis of the cylindrical shell. The resistance of this element is

$$dR = \frac{\rho\, dr}{2\pi r L}$$

The resistance of the tube between the inner and outer surfaces is

$$R = \int dR = \frac{\rho}{2\pi L}\int_a^b \frac{dr}{r} = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$$

So the correct choice is (c).

115. The given circuit can be redrawn as shown in Fig. 12.139.

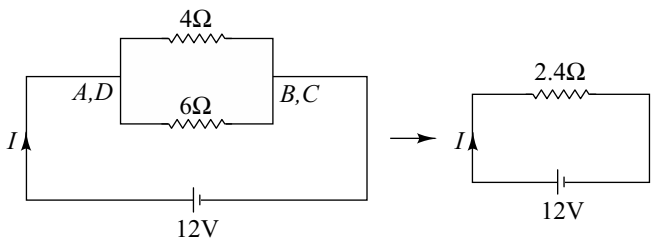


Fig 12.139

$$I = \frac{12\,\text{V}}{2.4\Omega} = 5\text{A}, \text{ which is choice (a).}$$

116. The given circuit can be redrawn as shown in Fig. 12.140.

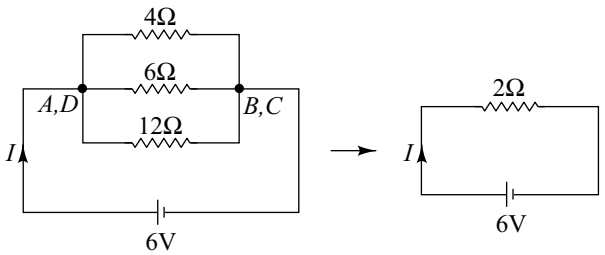


Fig 12.140

$$I = \frac{6\,\text{V}}{2\Omega} = 3\text{A}, \text{ which is choice (c).}$$

117. The given circuit can be redrawn as shown in Fig. 12.141.

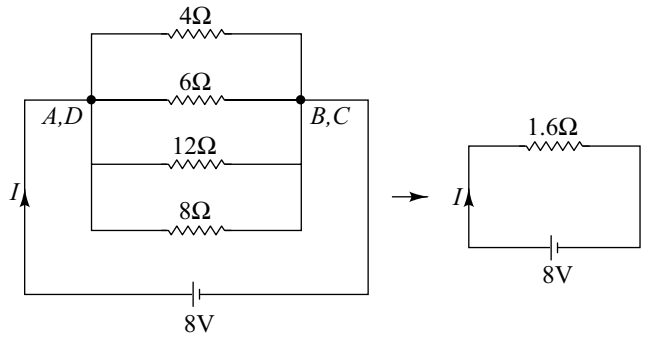


Fig 12.141

$$I = \frac{8\text{V}}{1.6\Omega} = 5\text{A}, \text{ which is choice } (a).$$

118. The three resistances are in parallel. Their equivalent resistance is given by

$$\frac{1}{R_{eq}} = \frac{1}{2}+\frac{1}{2}+\frac{1}{2} \quad \Rightarrow \quad R_{eq} = \frac{2}{3}\Omega$$

$\therefore$ Current $I = \dfrac{2\text{V}}{\frac{2}{3}\Omega} = 3\text{A}$

Since the resistances are in parallel, the potential difference across each resistance is the same = voltage of the battery. Further, since the three resistances are equal, the current through each is the same $= \dfrac{I}{3} = \dfrac{3\text{A}}{3} = 1\text{A}$. So the correct choice is (c).

119. Slope of graph $= -\dfrac{I_0}{t_0}$ and intercept $= I_0$. Therefore,

$$I = -\left(\frac{I_0}{t_0}\right)t + I_0$$

Heat dissipated in time dt is

$$dH = I^2R\,dt$$

$\therefore$ Total heat dissipated in time t_0 is

$$H = \int dH = \int_0^{t_0}\left[I_0 - \left(\frac{I_0}{t_0}\right)t\right]^2 R\,dt$$

$$= \int_0^{t_0} R\left[I_0^2 - \frac{2I_0^2}{t_0}t + \left(\frac{I_0}{t_0}\right)^2 t^2\right]dt$$

$$= I_0^2 Rt_0 - I_0^2 Rt_0 + \frac{I_0^2 Rt_0}{3}$$

$$= \frac{I_0^2 Rt_0}{3}$$

$\therefore$ Rate of dissipation of heat is

$$P = \frac{H}{t_0} = \frac{I_0^2 R}{3}$$

So the correct choice is (c).

120. Current in the circuit [see Fig. 12.142] is

$$I = \frac{E}{R+r}$$

$\therefore$ Power dissipated (consumed) by R is

$$P = I^2R$$

$$\Rightarrow \quad P = \left(\frac{E}{R+r}\right)^2 R$$

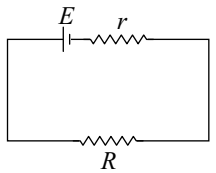


Fig 12.142

Notice that if $R \to 0$, $P = 0$ and if $R \to \infty$, $P \to 0$

So graphs (*b*) and (*c*) are both incorrect. P will be maximum if $\dfrac{dP}{dR} = 0$ and $\dfrac{d^2P}{dR^2} < 0$. Differentiating (1) *w.r.t* R we get

$$\frac{dP}{dR} = \frac{E^2}{(R+r)^2} - \frac{2E^2R}{(R+r)^3} \qquad (2)$$

Putting $\dfrac{dP}{dR} = 0$ in (2) and simplifying we get $R = r$.

It is easy to show that when $R = r$, $\dfrac{d^2P}{dR^2} < 0$. Hence P will be maximum when $R = r$. So the only correct graph is (d).

121. Refer to Fig. 12.143. Let V_A, V_B and V_C be the potentials of points A, B and C respectively.

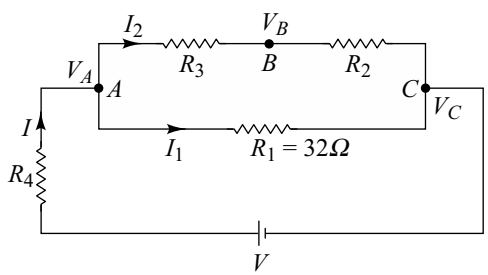


Fig 12.143

Power consumed by R_1, R_2, R_3 and R_4 are

$$P_1 = \frac{(V_A - V_C)^2}{R_1} \quad (1)$$

$$P_2 = \frac{(V_B - V_C)^2}{R_2} \quad (2)$$

$$P_3 = \frac{(V_A - V_B)^2}{R_3} \quad (3)$$

$$P_4 = \frac{(V - V_A)^2}{R_4} \quad (4)$$

Since the current flowing through R_2 and R_3 is the same $= I_2$ and since $P_2 = P_3$, we have

$$I_2^2 R_2 = I_2^2 R_3 \quad \Rightarrow \quad R_2 = R_3$$

Using $R_2 = R_3$ in (2) and (3), we get (because $P_2 = P_3$)

$$V_B - V_C = V_A - V_B$$

or $$V_A = 2V_B - V_C$$

$$\Rightarrow \quad V_A - V_C = 2V_B - V_C - V_C = 2(V_B - V_C) = 2(V_A - V_B).$$

Using this in (1) we get

$$P_1 = \frac{2(V_A - V_B)^2}{R_1} \quad (5)$$

Since $P_1 = P_2$, we have from (5) and (3)

$$\frac{4(V_A - V_B)^2}{R_1} = \frac{(V_A - V_B)^2}{R_3}$$

$$\Rightarrow \quad R_3 = \frac{R_1}{4} = \frac{32\,\Omega}{4} = 8\,\Omega, \text{ which is choice (b).}$$

122. We have seen above that $R_2 = R_3 = 8\Omega$. Since $P_1 = P_3$,

$$I_1^2 R_1 = I_2^2 R_3$$

$$\Rightarrow \quad I_1^2 \times 32 = I_2^2 \times 8$$

$$\Rightarrow \quad 2I_1 = I_2$$

Now $I = I_1 + I_2 = I_1 + 2I_1 = 3I_1$. Since $P_4 = P_1$,

$$I^2 R_4 = I_1^2 R_1$$

$$\Rightarrow \quad (3I_1)^2 R_4 = I_1^2 \times 32$$

$$R_4 = \frac{32}{9}\,\Omega$$

So the correct choice is (a).

123. Current drawn by each bulb is

$$I = \frac{100}{220} = \frac{5}{11}\text{A}$$

If n is the required number of bulbs then the total current is $n \times \frac{11}{5}$ A and this must not exceed 6A, otherwise the fuse will blow.

Hence $$n \times \frac{5}{11} \le 6$$

or $$n \le \frac{66}{5} \quad \text{or} \quad n \le 13.2$$

$\therefore$ $$n_{\max} = 13$$

So the correct choice is (d).

124. One end of each resistor is connected to point A and the other end to point B. Hence all the four resistances are connected in parallel across the battery. The rate dissipation of heat is

$$P = \frac{V^2}{R}$$

Since V is the same across each resistor, $P \propto \frac{1}{R}$.

Therefore, the smallest resistance (= 2Ω) will dissipate maximum heat in a given time. So the correct choice is (a).

125. In the steady state, no current flows through the branch containing the capacitor. Charge on the capacitor is $Q = CV$, where V is the potential difference between the capacitor plates [see Fig. 12.144]

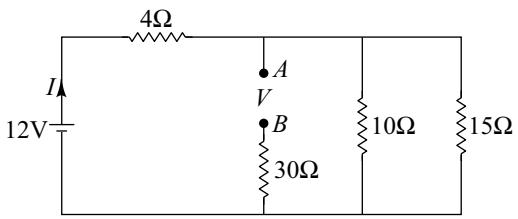


Fig 12.144

The 30Ω resistance carries no current and hence can be ignored. The resistance of circuit is

$$R = 4 + \frac{10 \times 15}{10 + 15} = 10\Omega$$

Current $$I = \frac{12\,\text{V}}{10\Omega} = 1.2\,\text{A}$$

Applying Kirchhoff's voltage law,

$$V_B - V_A = -12 + 4 \times 1.2 = -12 + 4.8 = -7.2\,\text{V}$$

$\therefore$ $$V = V_A - V_B = 7.2\,\text{V}$$

Hence $$Q = CV = 5\,\mu\text{F} \times 7.2\,\text{V} = 36\,\mu\text{C}$$

So the correct choice is (d).

126. In the steady state, no current flows through the capacitors. Therefore, R_1 and R_2 will be in series and their combined resistance is $R = R_1 + R_2 = 8 + 4 = 12\Omega$. Therefore, current through each resistor is

$$I = \frac{12\,\text{V}}{12\Omega} = 1\text{A}$$

Potential difference across R_1 = is

$$V_1 = IR_1 = 1 \times 8 = 8\text{V}$$

and that across R_2 is

$$V_2 = IR_2 = 1 \times 4 = 4\text{V}$$

$\therefore$ Charge on C_1 is $Q_1 = C_1V_1 = 4\ \mu\text{F} \times 8\text{V} = 32\ \mu\text{C}$
Charge on C_2 is $Q_2 = C_2V_2 = 8\ \mu\text{F} \times 4\text{V} = 32\ \mu\text{C}$.
So the correct choice is (a).

2 SECTION — Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage.

Passage I

The circuit shown Fig. 12.145 consists of the following

$E_1 = 3E_2 = 2E_3 = 6$ volt
$R_1 = 2R_4 = 6$ ohm
$R_3 = 2R_2 = 4$ ohm
$C = 5\ \mu\text{F}$

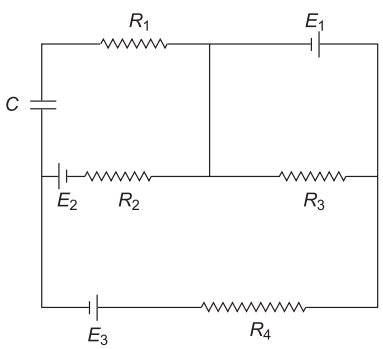


Fig. 12.145

1. The current in resistance R_1 is
   (a) 0.5 A (b) 1.0 A
   (c) 1.5 A (d) zero
2. The current through resistance R_3 is
   (a) 1.5 A (b) 1.2 A
   (c) 0.9 A (d) 0.6 A
3. The current through resistance R_4 is
   (a) 0.3 A (b) 0.25 A
   (c) 0.2 A (d) zero
4. The energy stored in the capacitor is
   (a) 4.8×10^{-6} J (b) 9.6×10^{-6} J
   (c) 1.44×10^{-5} J (d) 1.92×10^{-5} J

Solutions

1. Refer to Fig. 12.146.

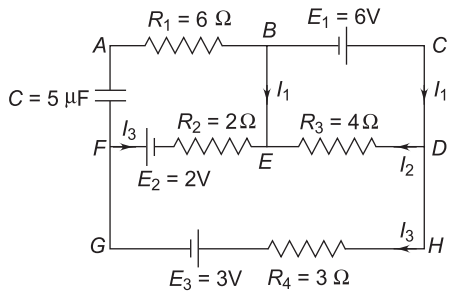


Fig. 12.146

In the steady state, no current flows through the capacitor. Hence the current through R_1 is zero, which is choice (d).

2. Applying Kirchhoff's rule to loop *EBCDE*, we get

$$I_2 R_3 = E_1 \text{ or } I_2 = \frac{E_1}{R_3} = 1.5\text{ A}$$

The correct choice is (a).

3. Applying Kirchhoff's second law to the loop *FDHGF*,we get

$$I_3 R_2 - I_2 R_3 + I_3 R_4 = -E_2 - E_3$$
or $$I_3 (R_2 + R_4) = I_2 R_3 - E_2 - E_3$$
or $$I_3 (2 + 3) = 1.5 \times 4 - 2 - 3$$
or $$5I_3 = 1 \text{ or } I_3 = 0.2\text{ A}$$

So the correct choice is (c).

4. Potential difference between *F* and *E* is

$$V = E_2 + I_3 R_2 = 2 + 0.2 \times 2 = 2.4\text{V}.$$

The points *E* and *A* are at the same potential. Therefore potential difference between *F* and *A* is

2.4 volt. Energy stored in the capacitor = $\frac{1}{2} CV^2$

$= \frac{1}{2} \times 5 \times 10^{-6} \times (2.4)^2 = 14.4 \times 10^{-6}$ J

Hence the correct choice is (c).

Questions 5 to 8 are based on the following passage.

Passage II

The circuit shown in Fig. 12.147, consists of the following $E_1 = 3$ V, $E_2 = 2$ V, $E_3 = 1$ V and $R = r_1 = r_2 = r_3 = R = 1\Omega$.

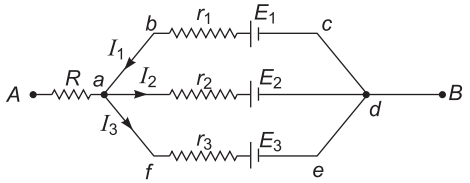


Fig. 12.147

5. Current I_1 through resistance r_1 is
   (a) zero (b) 0.5 A
   (c) 1.0 A (d) 1.5 A
6. Current I_2 through resistance r_2 is
   (a) zero (b) 1.0 A
   (c) 2.0 A (d) 3.0 A
7. Current I_3 through resistance r_3 is
   (a) 1.0 A (b) 1.5 A
   (c) 2.0 A (d) 2.5 A
8. If r_2 is short-circuited and point A is connected to point B, the current through resistor R would be
   (a) 1.0 A (b) 2.0 A
   (c) 3.0 A (d) zero

Solutions

5. Applying Kirchhoff's rule to loops *abcda* and *abcdefa*, we have

$$-I_1r_1 + E_1 - E_2 - I_2r_2 = 0$$

and $$-I_1r_1 + E_1 - E_3 - I_3r_3 = 0$$

which give $E_1 - I_1r_1 = E_2 + i_2r_2 = E_3 + I_3r_3$ (i)

Applying Kirchhoff's first rule to junction a, we have

$$I_1 = I_2 + I_3 \quad \text{(ii)}$$

Using Eq. (ii) in Eq. (i), we get

$E_1 - (I_2 + I_3)\, r_1 = E_3 + I_3r_3$ or $2I_3 + I_2 = 2$ (iii)

Also $E_2 + I_2r_2 = E_3 + I_3r_3$ or $I_3 - I_2 = 1$ (iv)

Equations (ii), (iii) and (iv) give $I_1 = 1$ A. So the correct choice is (c).

6. Equations (ii), (iii) and (iv) give $I_2 = 0$, which choice (a).
7. From Eq. (ii) we get $I_3 = I_1 - I_2 = 1 - 0 = 1$ A. So the correct choice is (a).
8. Since $I_2 = 0$, the potential difference between points a and d = emf $E_2 = 2$ V and remains equal to 2 V even when r_2 is short-circuited. Because the potential difference across a and d remains unchanged, the currents I_1 and I_3 through cells E_1 and E_2 do not change i.e. $I_1 = I_3 = 1$ A, even when point A is connected to point B. Hence the current through R will be zero, which is choice (d).

Questions 9 and 10 are based on the following passage.

Passage III

Figure 12.148 shows a network of six resistors connected a battery of emf 8.5 V and of negligible internal resistance.

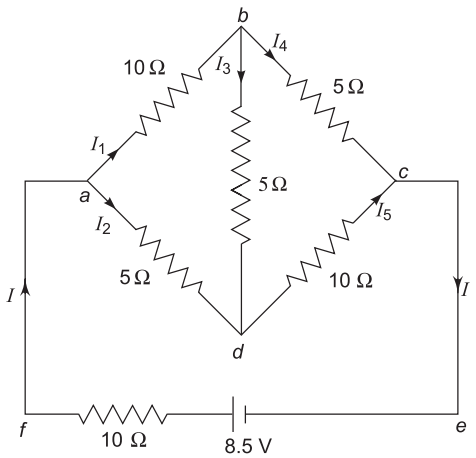


Fig. 12.148

9. In branch *bd*, the current I_3 is
   (a) 0.3 A flowing from b to d
   (b) 0.3 A flowing from d to b
   (c) 0.1 A flowing from b to d
   (d) 0.1 A flowing from d to b.
10. The total resistance of the network is
    (a) 8.0 Ω (b) 8.5 Ω
    (c) 17 Ω (d) 17.5 Ω

Solutions

9. Applying Kirchhoff's junction rule to junctions a, b and d, we have

$$I = I_1 + I_2,\ I_4 = I_1 - I_3 \text{ and } I_5 = I_2 + I_3.$$

Applying Kirchhoff's loop rule to loops *abda*, *bcdb* and *adcefa*, we get

$$10\,I_1 + 5\,I_3 - 5\,I_2 = 0$$

or $$I_2 = 2\,I_1 + I_3 \quad \text{(i)}$$

$$5(I_1 - I_3) - 10(I_2 + I_3) - 5\,I_3 = 0$$

or $$I_1 - 2\,I_2 = 4\,I_3 \quad \text{(ii)}$$

and $$5\,I_2 + 10\,(I_2 + I_3) - 8.5 + 10(I_1 + I_2) = 0$$

or $$2\,I_1 + 5\,I_2 + 2\,I_3 = 1.7 \quad \text{(iii)}$$

Solving Eqs. (i), (ii) and (iii) we get $I_1 = 0.2$ A, $I_2 = 0.3$ A and $I_3 = -0.1$ A. Since I_1 and I_2 are positive, their directions are correct. But the sign of I_3 is negative which indicates that the direction I_3 should be from d to b and not from b to d. The correct choice is (d).

10. Total current is $I = I_1 + I_2 = 0.2 + 0.3 = 0.5$ A. Hence the total resistance of the network is

$$R = \frac{V}{I} = \frac{8.5}{0.5} = 17\ \Omega$$

So the correct choice is (c).

Questions 11 to 14 are based on the following passage.

Passage IV

Figure 12.149 shows a part of the circuit in the steady state. The currents, the values of resistances and emfs of the cells are shown in the figure. The circuit also contains a capacitor of capacitance $C = 4$ μF.

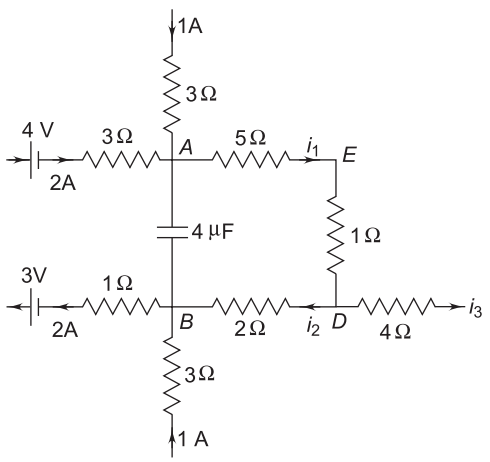


Fig. 12.149

11. The value of current i_1 is
    (a) 1 A (b) 2 A
    (c) 3 A (d) 4 A
12. The value of current i_2 is
    (a) 1 A (b) 2 A
    (c) 3 A (d) 4 A
13. The value of current i_3 is
    (a) 1 A (b) 2 A
    (c) 3 A (d) 4 A
14. The energy stored in the capacitor is
    (a) 2×10^{-4} J (b) 4×10^{-4} J
    (c) 6×10^{-4} J (d) 8×10^{-4} J

Solutions

11. Applying Kirchhoff's junction rule to junction A, we have $i_1 = 2 + 1 = 3$ A, which is choice (c).
12. At junction B, $i_2 + 1 = 2$ or $i_2 = 1$ A, which is choice (a).
13. At junction D, $i_1 = i_2 + i_3 \Rightarrow i_3 = i_1 - i_2 = 3 - 1 = 2$ A. So the correct choice is (b).
14. Potential drop along $AEDB$ is given by
    $V = 5\,i_1 + 1\,i_1 + 2\,i_2$
    $= 15 + 3 + 2 = 20$ V
    $\therefore$ Potential difference across the capacitor $C = 20$ V
    Energy stored in the capacitor $= \frac{1}{2}\,CV^2$
    $= \frac{1}{2} \times 4 \times 10^{-6} \times (20)^2 = 8 \times 10^{-4}$ J
    Thus the correct choice is (d).

Questions 15 to 17 are based on the following passage.

Passage V

In the circuit shown in Fig. 12.150, the 5 Ω resistance develops 45 Js^{-1} of heat due to the flow of current through it.

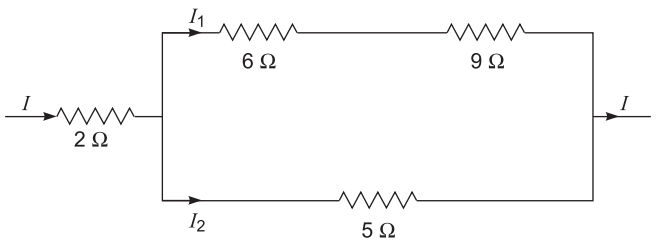


Fig. 12.150

15. The value of current I is
    (a) 1 A (b) 2 A
    (c) 3 A (d) 4 A
16. The power dissipated in the 2 Ω resistor is
    (a) 96 W (b) 48 W
    (c) 32 W (d) 16 W
17. The potential difference across the 6 Ω resistor is
    (a) 2 V (b) 4 V
    (c) 6 V (d) 9 V

Solutions

15. Current through the 5 Ω resistor is

$$I_2 = \frac{15I}{20} = \frac{3I}{4}$$

Current through 6 Ω and 9 Ω resistors is

$$I_1 = \frac{5I}{20} = \frac{I}{4}$$

Heat energy developed per second in the 5 Ω resistors is $I_2^2 \times 5 = 45$ or

$$\left(\frac{3I}{4}\right)^2 \times 5 = 45 \quad \Rightarrow \quad I = 4 \text{ A}.$$

So the correct choice is (d).

16. Power developed in the 2 Ω resistor = $I^2 \times 2 = (4)^2 \times 2 = 32$ W, which is choice (c).

17. Potential difference across the 6 Ω resistor = $I_1 \times 6 = \frac{I}{4} \times 6 = \frac{4}{4} \times 6 = 6$ V, which is choice (c).

Questions 18 to 21 are based on the following passage.

Passage VI

Figure 12.151 shows four resistances $R_1 = 4$ Ω, $R_2 = 6$ Ω, $R_3 = 12$ Ω and $R_4 = 10$ Ω connected with a 120 V battery and a capacitor of capacitance $C = 500$ μF, by means of two switches S_1 and S_2. Switch S_1 is closed so that the capacitor is fully charged. Switch S_1 is then opened and switch S_2 is closed.

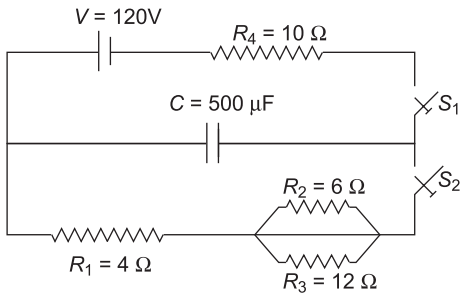


Fig. 12.151

18. The power developed in resistor R_1 is
(a) 1.8 W (b) 3.6 W
(c) 5.4 W (d) 7.2 W

19. The power developed in resistor R_2 is
(a) 0.3 W (b) 0.6 W
(c) 0.9 W (d) 1.2 W

20. The power developed in resistor R_3 is
(a) 0.2 W (b) 0.4 W
(c) 0.6 W (d) 0.8 W

21. The power developed in resistor R_4 is
(a) 3.6 W (b) 1.2 W
(c) 0.4 W (d) zero

Solutions

When switch S_1 is closed, the capacitor is charged to the voltage of the battery. Energy stored in the capacitor is

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 500 \times 10^{-6} \times (120)^2 = 3.6 \text{ J}.$$

When S_1 is opened and switch S_2 is closed, no current flows through R_4. Hence power developed in $R_4 = 0$, i.e. $P_4 = 0$.

Let I_1 be the current through the 4 Ω resistor at an instant of time t. This current divides into I_2 flowing through R_2 and I_3 flowing through R_3 such that $I_1 = I_2 + I_3$. Since the potential difference across R_2 and R_3 is the same, we have, $I_2R_2 = I_3R_3$ or $6I_2 = 12\ I_3$ or $I_2 = 2I_3$. But $I_1 = I_2 + I_3$. Therefore,

$I_1 = 2I_3 + I_3$ or $I_3 = \frac{I_1}{3}$. Similarly $I_2 = \frac{2I_1}{3}$. Thus

$$I_1 : I_2 : I_3 = I_1 : \frac{2I_1}{3} : \frac{I_1}{3} = 1 : \frac{2}{3} : \frac{1}{3} = 3 : 2 : 1$$

Now $P = I^2R$. Therefore,

$$P_1 : P_2 : P_3 = I_1^2 R_1 : I_2^2 R_2 : I_3^2 R_3$$
$$= (3)^2 \times 4 : (2)^2 \times 6 : (1)^2 \times 12$$
$$= 36 : 24 : 12 = 3 : 2 : 1$$

Thus energy $U = 3.6$ J stored in the capacitor is distributed among resistors R_1, R_2 and R_3 in the ratio 3 : 2 : 1.

18. $P_1 = \frac{3.6 \times 3}{6} = 1.8$ W, which is choice (a).

19. $P_2 = \frac{3.6 \times 2}{6} = 1.2$ W, which is choice (d).

20. $P_3 = \frac{3.6 \times 1}{6} = 0.6$ W, which is choice (c).

21. Since no current flows through R_4, $P_4 = 0$, which is choice (d).

SECTION 3
Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by statement-2 (Reason). Each question has the following four choices out of which only *one* choice is correct.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

1. **Statement-1**

   Figure 12.152 shows the current-voltage ($I-V$) graphs for a given metallic wire at two different temperatures T_1 and T_2. It follows from the graphs that T_2 is greater than T_1.

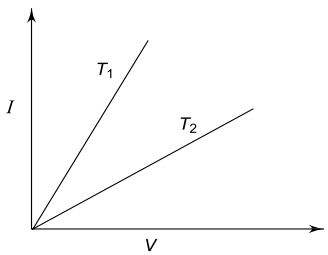


Fig. 12.152

   Statement-2

   The resistance of a metallic conductor increases with increase in temperature.

2. **Statement-1**

   Two bulbs of 60 W and 100 W are connected in series and this combination is connected a d.c. power supply. The potential difference across the 60 W bulb is higher than that across the 100 W bulb.

   Statement-2

   In a series combination, the potential difference across a bulb is proportional to its resistance.

3. **Statement-1**

   A 60 W bulb is connected in series with a room heater and this combination is connected across the mains. If the 60 W bulb is replaced by a 100 W bulb, the heat produced by the heater will be more.

   Statement-2

   The heat produced is inversely proportional to resistance when the resistances are connected in series across the mains.

4. **Statement-1**

   A 60 W bulb is connected in parallel with a room heater and this combination is connected across the mains. If the 60 W bulb is replaced by a 100 W bulb, the heat produced by the heater will remain the same.

   Statement-2

   When resistances are connected in parallel across the mains, the heat produced is inversely proportional to the resistance.

5. **Statement-1**

   If an electric field is applied to a metallic conductor, the free electrons experience a force but do not accelerate; they only drift at a constant speed.

   Statement-2

   The force exerted by the electric field is completely balanced by the Coulomb force between electrons and protons.

6. **Statement-1**

   The drift speed of electrons in metals is small (of the order of a few mm s^{-1}) and the charge of an electron is also very small (= 1.6×10^{-19} C), yet we can obtain a large current in a metal.

   Statement-2

   At room temperature, the thermal speed of electrons is very high (about 10^7 times the drift speed).

7. **Statement-1**

   A wire carrying a current has no electric field around it.

   Statement-2

   A wire carrying current stays electrically neutral because rate of flow of electrons in one direction equals the rate of flow of protons in the opposite direction.

8. **Statement-1**

   In the metre bridge experiment shown in Fig. 12.153, the balance length AC corresponding

to null deflection of the galvanometer is x. If the radius of the wire AB is doubled, the balanced length becomes 4 x.

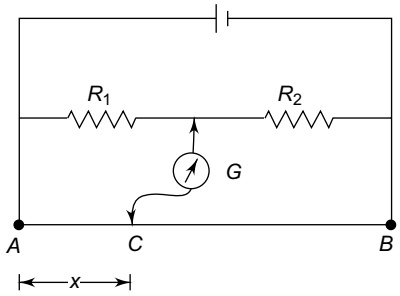


Fig. 12.153

Statement-2

The resistance of a wire is inversely proportional to the square of its radius.

9. **Statement-1**

In the potentiometer circuit shown in Fig. 12.154, E_1 and E_2 are the emfs of cells C_1 and C_2 respectively with $E_1 > E_2$. Cell C_1 has negligible internal resistance. For a given resistor R, the balance length is x. If the diameter of the potentiometer wire AB is increased, the balance length x will decrease.

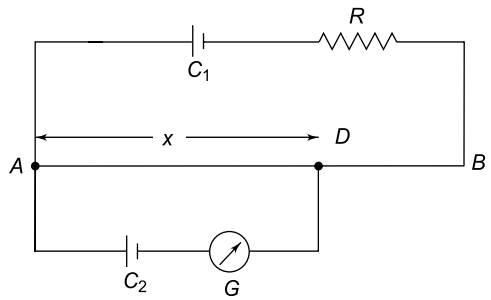


Fig. 12.154

Statement-2

At the balance point, the potential difference between AD due to cell $C_1 = E_2$, the emf of cell C_2.

10. **Statement-1**

In the potentiometer circuit shown in Q.9 above, the wire AB is not changed but the value of resistor R is decreased. Then the balance length x will decrease.

Statement-2

At the balance point, the potential difference between A and D due to cell C_1 = emf E_2 of cell C_2.

Solutions

1. The correct choice is (a). From Ohm's law, the slope of the $I - V$ graph gives the reciprocal of the resistance of the wire. Since the slope of the graph is smaller at temperature T_2, the resistance of the wire is greater at temperature T_2 than at temperature T_1. Hence T_2 is greater than T_1.

2. The correct choice is (a). Since the bulbs are connected in series, the current in each is the same. Therefore, the potential difference across a bulb will be proportional to its resistance R. Now $R \propto 1/P$, where P is the power. Hence the 60 W bulb has a higher resistance than the 100 W bulb.

3. The correct choice is (c). The resistance (R_1) of the 60 W bulb is higher than resistance (R_2) of the 100 W bulb. If R is the resistance of the heater and V the voltage of the mains, the currents through the 60 W bulb and 100 W bulb respectively are

$$I_1 = \frac{V}{R + R_1} \quad \text{and} \quad I_2 = \frac{V}{R + R_2}$$

Since $R_1 > R_2 : I_2 > I_1$. Since the heater and the bulb are connected in series, the current through the bulb = current through the heater. Now, heat produced by the heater is proportional to the square of the current flowing through it.

4. The correct choice is (a). When a bulb and a heater a connected in parallel and this combination is connected across the main, potential difference across each is the same equal to the voltage V of the mains, irrespective of the resistance of the bulb. If R is the resistance of the heater, the rate at which heat is produced will be V^2/R in both cases.

5. The correct choice is (c). The electrons suffer a large number of collisions with the positive ions of the conductor. Although the electric field accelerates an electron between two collisions, it is decelerated by collision. The net acceleration averages out to zero and the electron acquires a constant average speed. The gain in speed between collisions is lost in the next collision.

6. The correct choice is (b). The current in a metal depends not only on the charge of an electron and its drift speed but also on the number of free electrons per unit volume in a metal which is of the order of 10^{29} per m^3.

7. The correct choice is (c). A wire carrying a current stays neutral because as many electrons enter one end of the wire as leave it from the other end. Since

there is no net charge on the wire, there is no electric field around it.

8. The correct choice is (d). The condition for no deflection of the galvanometer is

$$\frac{R_1}{R_2} = \frac{R_{AC}}{R_{CB}}$$

where R_{AC} and R_{CB} are the resistances of the bridge wire of length AC and CB respectively. If the radius of the wire AB is doubled, the ratio R_{AC}/R_{CB} will remain unchanged. Hence the balance length will remain the same.

9. The correct choice is (d). If the diameter of wire AB is increased, its resistance will decrease. Hence, the potential difference between A and B due to cell C_1 will decrease. Therefore, the null point will be obtained at a higher value of x.

10. The correct choice is (a). If the value of R is decreased, the potential difference between A and B due to cell C_1 will increase. Hence the balance length will be smaller than x.

4 SECTION Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. If an ammeter is to be used in place of a voltmeter, then we must connect with the ammeter a
   (a) low resistance in parallel
   (b) high resistance in parallel
   (c) low resistance in series
   (d) high resistance in series. **[2002]**

2. A wire when connected to 220 V mains supply dissipates power P_1. Now the wire is cut into two equal pieces which are connected in parallel to the same supply. If the power dissipated now is P_2, then the ratio $P_2 : P_1$ is
   (a) 1 (b) 4
   (c) 2 (d) 3 **[2002]**

3. In the circuit, the power dissipation is 150 W. Then the value of R is

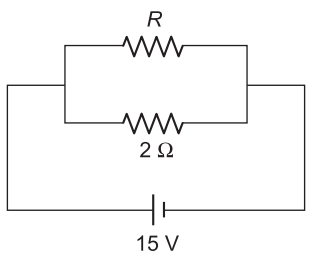


Fig.

   (a) 2 Ω (b) 6 Ω
   (c) 5 Ω (d) 4 Ω **[2002]**

4. A potentiometer has a wire of length 100 cm and the emf of the standard cell used is E volt. It is employed to measure the emf of a battery whose internal resistance is 0.5 Ω. If the balance length obtained is 30 cm from the positive end, the emf of the battery is
   (a) $\frac{30E}{100.5}$
   (b) $\frac{30E}{99.5}$
   (c) $\frac{30(E-0.5i)}{100}$; i = current in potentiometer wire
   (d) $\frac{30E}{100}$ **[2003]**

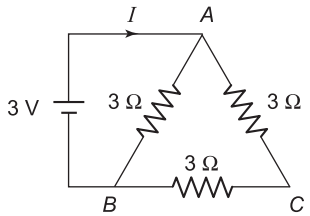
5. A strip of copper and another of germanium are cooled from room temperature to 80 K. The resistance of
   (1) each strip will decrease
   (2) each strip will increase
   (3) copper strip decreasses and of germanium strip increases
   (4) copper strip increases and of germanium strip decreases. **[2003]**

6. The thermo emf of a thermocouple is 25 μV per °C at room temperature. A galvanometer of resistance 40 Ω, capable of detecting current as low as 10^{-5} A is connected with the thermocouple. The smallest temperature difference that can be detected by this system is
   (a) 16°C (b) 12°C
   (c) 8°C (d) 20°C **[2003]**

7. An ammeter of resistance 0.81 Ω reads currents up to 1 A. To increase the range to 10 A, the value of the required shunt is

(a) 0.03 Ω (b) 0.3 Ω
(c) 0.9 Ω (d) 0.09 Ω **[2003]**

8. A 3 V battery of negligible internal resistance is connected in a circuit as shown in the figure. The current I in the circuit will be

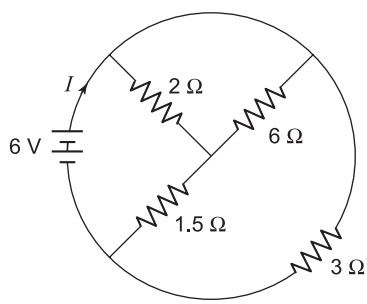


(a) 1 A (b) 1.5 A
(c) 2 A (d) $\frac{1}{3}$ A **[2003]**

9. A 220 V, 1000 W bulb is connected across a 110 V mains supply. The power consumed will be
(a) 750 W (b) 500 W
(c) 250 W (d) 1000 W **[2003]**

10. The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in the diameter of the wire, the change in its resistance will be
(a) 200 % (b) 100 %
(c) 50 % (d) 300 % **[2003]**

11. The current *I* in the network shown in the figure is
(a) 1 A (b) 2 A
(3) 4 A (d) 6 A **[2004]**



12. The resistance of the series combination of two resistances is *S*. When they adre joined in parallel, the total resistance is *P*. If $S = nP$, then the minimum possible value of *n* is
(a) 4 (b) 3
(c) 2 (d) 1 **[2004]**

13. An electric current is passed through a circuit containing two wires of the same material, connected in parallel. If the lengths and radii of the wires are in the ratio of 4/3 and 2/3 respectively, the ratio of the currents in the wires will be
(a) 3 (b) $\frac{1}{3}$
(c) $\frac{8}{9}$ (d) 2 **[2004]**

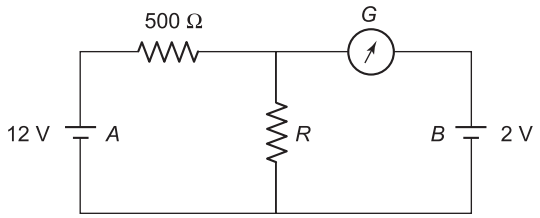
14. In a metre bridge experiment, the null point is obtained at 20 cm from one end of the wire when resistance *X* is balanced against another resistance *Y*. If $X < Y$, then where will be the new position of the null point if one decides to balance a resistance of 4 *X* against *Y*?
(a) 50 cm (b) 80 cm
(c) 40 cm (d) 70 cm **[2004]**

15. The thermistors are usually made of
(a) metals with high temperature coefficient of resistivity
(b) metals with low temperature coefficient of resistivity
(c) metal oxides with high temperature coefficient of resistivity
(d) semiconducting materials having low temperature coefficient of resistivity. **[2004]**

16. The thermo emf of a thermocouple varies with temperature θ of the hot junction as $E = a\theta + b\theta^2$ (in volts) where the ratio $a/b = 700°C$. If the temperature of cold junction is 0°C, the neutral temperature of the thermocouple is
(a) 700°C
(b) 350°C
(c) 1400°C
(d) No neutral temperature is possible of this thermocouple. **[2004]**

17. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 division per millivolt. In order that each division reads 1 V, the resistance (in Ω) needed to be connected in series with the coil will be
(a) 10^3 (b) 10^5
(c) 99995 (d) 9995 **[2005]**

18. In the circuit shown in the figure, the galvanometer *G* shows zero deflection. If batteries *A* and *B* have negligible internal resistance, the value of resistor *R* is
(a) 200 Ω (b) 100 Ω
(c) 500 Ω (d) 1000 Ω **[2005]**



19. Two sources of equal emf are connected in series across an external resistor R. The internal resistances of the two sources are R_1 and R_2 ($R_2 > R_1$). If the potential difference across the source having internal resistance R_2 is zero, then

(a) $R = \dfrac{R_2(R_1 + R_2)}{(R_2 - R_1)}$ (b) $R = R_2 - R_1$

(c) $R = \dfrac{R_1 R_2}{(R_1 + R_2)}$ (d) $R = \dfrac{R_1 R_2}{(R_2 - R_1)}$

[2005]

20. A heater coil is cut into two equal parts and only one part is now used in the heater. The rate of heat generation now will be

(a) doubled (b) four times

(c) one-fourth (d) halved **[2005]**

21. In a potentiometer experiment, the balancing length with a cell is 240 cm. On shunting the cell with a resistance of 2 Ω, the balancing length becomes 120 cm. The internal resistance of cell is

(a) 1 Ω (b) 0.5 Ω

(c) 4 Ω (d) 2 Ω **[2005]**

22. The resistance of hot tungsten filament of a bulb is about 10 times the cold resistance. What will be the resistance of 100 W, 200 V bulb, when not in use?

(a) 40 Ω (b) 20 Ω

(c) 400 Ω (d) 200 Ω **[2005]**

23. A material 'B' has twice the resistivity of 'A'. A circular wire made of 'B' has twice the diameter of a wire made of 'A'. Then for the two wires to have the same resistance, the ratio l_B/l_A of their respective lengths must be

(a) $\dfrac{1}{4}$ (b) 2

(c) 1 (d) $\dfrac{1}{2}$ **[2006]**

24. The Kirchhoff's first law ($\Sigma i = 0$) and second law ($\Sigma iR = \Sigma E$), where the symbols have their usual meanings, are respectively based on

(a) conservation of momentum, conservation of charge

(b) conservation of charge, conservation of energy

(c) conservation of charge, conservation of momentum

(d) conservation of energy, conservation of charge

[2006]

25. A thermocouple is made from two metals, Antimony and Bismuth. If one junction of the couple is kept hot and the other is kept cold, then, an electric current will

(a) not flow through the thermocouple

(b) flow from Antimony to Bismuth at the cold juction

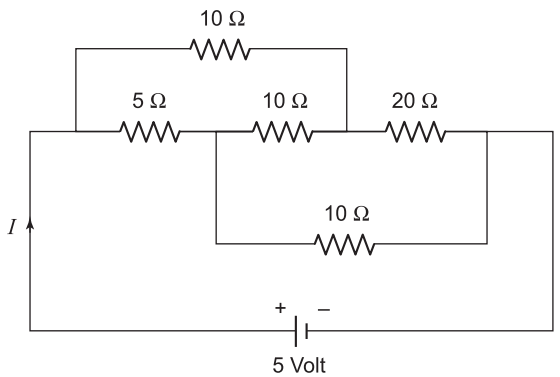
(c) flow from Antimony to Bismuth at the hot junction

(d) flow from Bismuth to Antimony at the cold junction **[2006]**

26. In the circuit shown in the figure the current I drawn from the 5 volt source will be

(a) 0.67 A (b) 0.17 A

(c) 0.33 A (d) 0.5 A **[2006]**



27. In a Wheatstone's bridge, three resistances P, Q and R are connected in the three arms and the fourth arm is formed by two resistances S_1 and S_2 connected in parallel. The condition of the bridge to be balanced will be

(a) $\dfrac{P}{Q} = \dfrac{R(S_1 + S_2)}{2S_1S_2}$ (b) $\dfrac{P}{Q} = \dfrac{R}{S_1 + S_2}$

(c) $\dfrac{P}{Q} = \dfrac{2R}{S_1 + S_2}$ (d) $\dfrac{P}{Q} = \dfrac{R(S_1 + S_2)}{S_1S_2}$

[2006]

28. The resistance of bulb filament is 100 Ω at a temperature of 100°C. If its temperature coefficient of resistance be 0.005 per °C, its resistance will become 200 Ω at a temperature of

(a) 500°C (b) 200°C

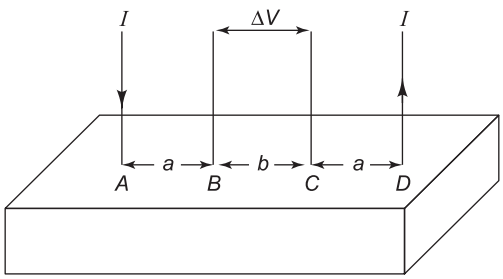
(c) 300°C (d) 400°C **[2006]**

29. An electric bulb is rated 220 volt – 100 watt. The power consumed by it when operated on 110 volt will be
    (a) 25 watt (b) 50 watt
    (c) 75 watt (d) 40 watt **[2006]**
30. The resistance of a wire is 5 ohm at 50°C and 6 ohm at 100°C. The resistance of the wire at 0°C will be
    (a) 2 ohm (b) 1 ohm
    (c) 4 ohm (d) 3 ohm **[2007]**

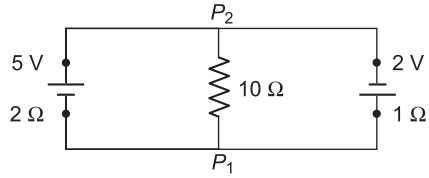
Directions: Questions No. 31 and 32 are based on the following paragraph.

Consider a block of conducting material of resistively 'ρ' shown in the figure. Current 'I' enters at 'A' and leaves from 'D'. We apply superposition principle to find voltage 'ΔV' developed between 'B' and 'C'. The calculation is done in the following steps:

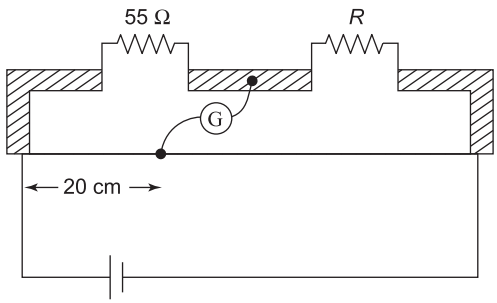
(i) Take current 'I' entering from 'A' and assume it to spread over a hemispherical surface in the block.
(ii) Calculate field $E(r)$ at distance 'r' from A by using Ohm's law $E = \rho J$. Where J is the current per unit area at 'r'.
(iii) From the 'r' dependence of $E(r)$. Obtain the potential $V(r)$ at r.
(iv) Repeat (i), (ii) and (iii) for current 'I' leaving 'D' and superpose results for 'A' and 'D'.



31. ΔV measured between B and C is
    (1) $\frac{\rho I}{2\pi(a-b)}$ (b) $\frac{\rho I}{\pi a} - \frac{\rho I}{\pi(a+b)}$
    (c) $\frac{\rho I}{a} - \frac{\rho I}{(a+b)}$ (d) $\frac{\rho I}{2\pi a} - \frac{\rho I}{2\pi(a+b)}$
    [2008]
32. For current entering at A, the electric field at a distance 'r' from A is
    (a) $\frac{\rho I}{4\pi r^2}$ (b) $\frac{\rho I}{8\pi r^2}$
    (c) $\frac{\rho I}{r^2}$ (d) $\frac{\rho I}{2\pi r^2}$ **[2008]**
33. A 5V battery with internal resistance 2 Ω and a 2V battery with internal resistance 1 Ω are connected to a 10 Ω resistor as shown in the figure. The current in the 10 Ω resistor is



    (a) 0.03 A from P_2 to P_1 (b) 0.27 A from P_1 to P_2
    (c) 0.27 A from P_2 to P_1 (d) 0.03 A from P_1 to P_2
    [2008]
34. The figure shows a metre-bridge set up with null deflection in the galvanometer.



    The value of the unknown resistors R is
    (a) 55 Ω (b) 13.75 Ω
    (c) 220 Ω (d) 110 Ω **[2008]**
35. *This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.*

    Statement 1: The temperature dependence of resistance is usually given as $R = R_0(1 + \alpha\Delta t)$. The resistance of a wire changes from 100 Ω to 150 Ω when its temperature is increased from 27°C to 227°C. This implies that $\alpha = 2.5 \times 10^{-3}$/°C.

    Statement 2: $R = R_0(1 + \alpha\Delta t)$ is valid only when the change in the temperature Δt is small and $\Delta R = (R - R_0) << R_0$.

    (a) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
    (b) Statement-1 is false, Statement-2 is true.
    (c) Statement-1 is true, Statement-2 is false.
    (d) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1. **[2009]**
36. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2, the respective temperature coefficients of their series and parallel combinations are nearly

(a) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$ (b) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$

(c) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$ (d) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$

[2010]

37. A resistor 'R' and 2 μF capacitor in series is connected through a switch to 200V direct supply. Across the capacitor is a neon bulb that lights up at 120V. Calculate the value of R to make the bulb light up 5 a after the switch has been closed. ($\log_{10}$ 2.5 = 0.4)

(a) $1.3 \times 10^4\ \Omega$ (b) $1.7 \times 10^5\ \Omega$

(c) $2.7 \times 10^6\ \Omega$ (d) $3.3 \times 10^7\ \Omega$ **[2011]**

38. If a wire is stretched to make it 0.1% longer, its resistance will

(a) increase by 0.05% (b) increase by 0.2%

(c) decrease by 0.2% (d) decrease by 0.05%

[2011]

39. Two electric bulbs marked 25W – 220 V and 100 W –220 V are connected in series to a 440 V supply. Which of the bulbs will fuse?

(a) Both (b) 100 W

(c) 25 W (d) Neither **[2012]**

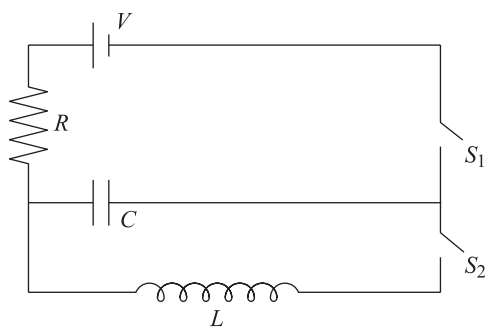
40. Two large vertical and parallel metal plates having a sepration of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical JUST after release. Then X is nearly

(a) 1×10^{-5} V (b) 1×10^{-7} V

(c) 1×10^{-9} V (d) 1×10^{-10} V

[2012]

41. In an LCR circuit as shown below, both switches are open initally. Now switch S_1 is closed and S_2 kept open. (q is charge on the capacitor and $\tau = RC$ is capacitive time constant). Which of the following statement is correct?



(a) At $t = \tau$, q = CV/2

(b) At $t = 2\tau$, $q = CV(1 - e^{-2})$

(c) At $t = \tau/2$, $q = CV(1 - e^{-1})$

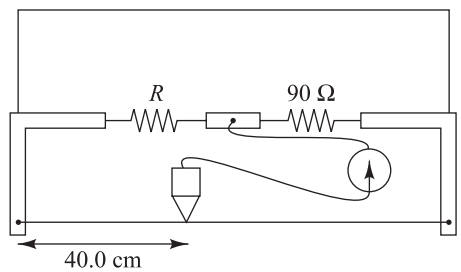
(d) Work done by the battery is half of the energy dissipated in the resistor. **[2013]**

42. The supply voltage to a room is 120 V. The resistance of the lead wires is 6 Ω. A 60-W bulb is already switeched on. What is the decrease of voltage across the bulb, when a 240-W heater is switched on in parallel to the bulb?

(a) 2.9 V (b) 13.3 V

(c) 10.04 V (d) 0 V **[2013]**

43. During an experiment with a metre bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance of 90 Ω, as shown in the figure. The least count of the scale used in the metre bridge is 1 mn. The unknown resistance is



(a) 60 ± 0.15 Ω (b) 135 ± 0.56 Ω

(c) 60 ± 0.25 Ω (d) 135 ± 0.23 Ω

Answers

1. (d)	2. (b)	3. (b)	4. (d)
5. (c)	6. (a)	7. (d)	8. (b)
9. (c)	10. (d)	11. (c)	12. (a)
13. (b)	14. (a)	15. (c)	16. (d)
17. (d)	18. (b)	19. (b)	20. (a)
21. (d)	22. (a)	23. (b)	24. (b)
25. (b)	26. (d)	27. (d)	28. (c)
29. (a)	30. (c)	31. (b)	32. (d)
33. (a)	34. (c)	35. (b)	36. (a)
37. (c)	38. (b)	39. (c)	40. (c)
41. (b)	42. (c)	43. (c)	

Solutions

1. A voltmeter is a galvanometer having a very high resistance connected in series. A voltmeter is always connected in parallel with the resistor across which the potential difference is to be measured. An ammeter is a galvanometer having a very low resistance connected in parallel. So an ammeter has a very low resistance. To use it in place of a voltmeter, a very high resistance must be connected in series with it.

2. Let R be the resistance of the complete wire and V be the voltage of the supply, then

$$P_1 = \frac{V^2}{R}$$

If the wire is cut into two equal pieces, the resistance of each piece = $R/2$. When the two pieces are connected in parallel, their combined resistance is $R_2 = \frac{R}{4}$. Therefore,

$$P_2 = \frac{V^2}{R^2} = \frac{4V^2}{R} = 4P_1$$

$$\therefore \quad \frac{P_2}{P_1} = 4$$

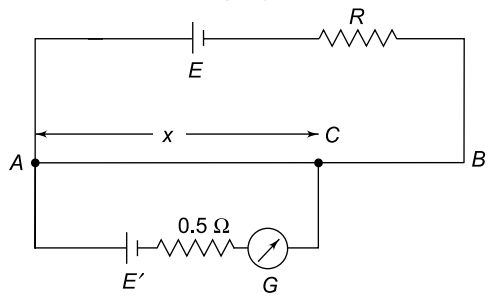
3. The resistance of the parallel combination of R and 2 Ω is

$$R_1 = \frac{R \times 2}{(R+2)}$$

Now $$P = \frac{V^2}{R_1} \quad \Rightarrow \quad 150 = \frac{(15)^2}{2R} \times (R+2)$$

which gives $R = 6\ \Omega$.

4. Refer to the following figure.



Potential gradient along $AB = 100$ cm due to standard cell of emf E is

$$k = \frac{E}{AB} = \frac{E}{100} \text{ volt per cm}$$

Potential difference across AC = 30 cm due to standard cell is

$$V_{AC} = \frac{E}{100} \times AC = \frac{E \times 30}{100} \text{ volt.}$$

At the balance point, the galvanometer shows zero deflection. Hence no current flows through the battery E' and through 0.5 Ω resistance. Therefore.

$$E' = V_{AC} = \frac{30E}{100}$$

5. Copper is a conductor and germanium is a semiconductor. The resistance of a metallic conductor decreases with decrease in temperature while that of a semiconductor increases with decrease in temperature. So the correct choice is (c).

6. Let Δt (in °C) be the smallest temperature difference. Then the emf across the thermocouple is

$$E = 25 \times 10^{-6}\ \text{V (°C)}^{-1} \times \Delta t\ (°\text{C}) = 25 \times 10^{-6}\ \Delta t \text{ volt}$$

Potential drop across the galvanometer = iR

$= 10^{-5} \times 40 = 4 \times 10^{-4}$ volt

$$\therefore \ 25 \times 10^{-6}\ \Delta t = 4 \times 10^{-4} \quad \Rightarrow \quad \Delta t = 16°\text{C}$$

7. $$S = \left(\frac{I_g}{I - I_g}\right) G \quad = \frac{1}{(10-1)} \times 0.81 = 0.09\ \Omega$$

8. Equivalent resistance R between A and B is given by

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{6} \quad \Rightarrow \quad R = 2\ \Omega$$

$$\therefore \text{ Current } I = \frac{V}{R} = \frac{3}{2} = 1.5\ \text{A}$$

9. If R is the resistance of the bulb, then

$$P_1 = \frac{V_1^2}{R} \quad \text{and} \quad P_2 = \frac{V_2^2}{R}$$

$$\therefore \quad \frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^2 = \left(\frac{110}{220}\right)^2 = \frac{1}{4}$$

$$\Rightarrow \quad P_2 = \frac{1}{4} \times P_1 = \frac{1}{4} \times 1000 = 250\ \text{W}$$

10. Given $L_2 = 2L_1$. If the wire is elongated, its volume remains unchanged. If A_1 is the cross-sectional area of the wire before stretching and A_2 after stretching, then

$$A_1 L_1 = A_2 L_2 \quad \Rightarrow \quad \frac{A_1}{A_2} = \frac{L_2}{L_1} = \frac{2L_1}{L_1} = 2$$

If ρ is the resistivity of the material of the wire, then

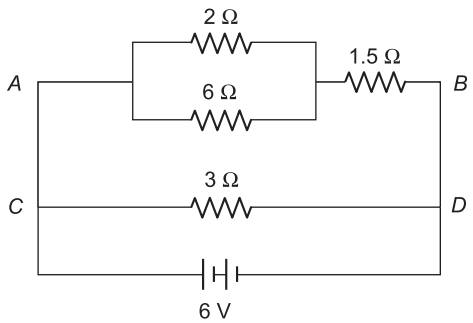
$$R_1 = \frac{\rho L_1}{A_1} \text{ and } R_2 = \frac{\rho L_2}{A_2}$$

$$\therefore \quad \frac{R_2}{R_1} = \left(\frac{L_2}{L_1}\right)\times\left(\frac{A_1}{A_2}\right) = 2 \times 2 = 4$$

$\therefore$ $R_2 = 4R_1$. Therefore percentage increase in resistance is

$$\left(\frac{R_2 - R_1}{R_1}\right)\times 100 = \left(\frac{4R_1 - R_1}{R_1}\right)\times 100 = 300\ \%$$

11. The circuit can be redrawn as shown in the figure. The resistance between A and B is



$$R_1 = \frac{2 \times 6}{(2+6)} + 1.5 = 3\ \Omega$$

Resistance between C and D is

$$R = \frac{3 \times 3}{(3+3)} = 1.5\ \Omega$$

$$\therefore \quad \text{Current } I = \frac{V}{R} = \frac{6}{1.5} = 4\ \text{A}$$

12. Let R_1 and R_2 be the two resistances. Then

$$S = R_1 + R_2$$

and $$P = \frac{R_1 R_2}{(R_1 + R_2)}$$

Given $S = nP$. Hence

$$R_1 + R_2 = n\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$\Rightarrow (R_1 + R_2)^2 = nR_1 R_2 \qquad (1)$$

Putting $R_1 + R_2 = S$ and $R_2 = S - R_1$ in (1) we get

$S^2 = nR_1(S - R_1)$

$\Rightarrow S^2 - nR_1 S + nR_1^2 = 0$

The two roots of this quadratic equation are

$$S = \frac{nR_1}{2} \pm \frac{1}{2}\left[n^2 R_1^2 - 4nR_1^2\right]^{1/2}$$

$$= \frac{nR_1}{2} \pm \frac{\sqrt{n}R_1}{2}(n-4)^{1/2}$$

S is real if $n > 4$. Hence the minimum value of n is 4.

13. Since the wires are made of the same material, their resistivity ρ is the same.

$$R_1 = \frac{\rho L_1}{\pi r_1^2} \quad \text{and} \quad R_2 = \frac{\rho L_2}{\pi r_2^2}$$

$$\therefore \quad \frac{R_2}{R_1} = \left(\frac{L_2}{L_1}\right)\times\left(\frac{r_2}{r_1}\right)^2$$

If the wires are across a source of voltage V,

$$I_1 = \frac{V}{R_1} \quad \text{and} \quad I_2 = \frac{V}{R_2}$$

$$\frac{I_1}{I_2} = \frac{R_2}{R_2} = \left(\frac{L_2}{L_1}\right)\times\left(\frac{r_2}{r_1}\right)^2$$

Given $\frac{L_1}{L_2} = \frac{4}{3}$ and $\frac{r_1}{r_2} = \frac{2}{3}$. Hence

$$\frac{I_1}{I_2} = \frac{3}{4}\times\left(\frac{2}{3}\right)^2 = \frac{1}{3}.$$

14. $$\frac{X}{Y} = \frac{20}{100-20} = \frac{20}{80} \quad \Rightarrow \quad Y = 4X$$

If l is new balance length with resistance $4X$,

$$\frac{4X}{Y} = \frac{l}{(100-l)}$$

Putting $Y = 4X$, we get

$$1 = \frac{l}{(100-l)} \quad \Rightarrow \quad l = 50\ \text{cm}$$

15. Thermistors are made of semiconductors having a high and positive temperature coefficient of resistivity. Pure semiconductors, such as germanium have a high temperature coefficient of resistivity but their resistivity decreases with increase in temperature, i.e. they have a negative temperature coefficient of resistivity. On the other hand, metal oxides are semiconductors having a high and positive temperature coefficient of resistivity. So the correct choice is (c).

16. Neutral temperature θ_N is the value of θ at which E is maximum, i.e. $\theta = \theta_N$ if $\frac{dE}{dt} = 0$.

$$\frac{dE}{dt} = \frac{d}{dt}(a\theta + b\theta)^2 = a + 2b\theta$$

Putting $\frac{dE}{dt} = 0$ and $\theta = \theta_N$, we get

$$\theta_N = -\frac{a}{2b} = -\frac{700}{2} = -350°\text{C}$$

No thermocouple has θ_N less than zero. Hence correct choice is (d).

17. Current for full scale deflection of 150 divisions is

$$I_g = \frac{1\text{ mA}}{10} \times 150 = 15\text{ mA} = 15 \times 10^{-3}\text{ A}$$

Voltage for full scale deflection of 150 divisions is

$$V_g = \frac{1\text{ mV}}{2} \times 150 = 75\text{ mV} = 75 \times 10^{-3}\text{ V}$$

Resistance of galvanometer coil is

$$G = \frac{V_g}{I_g} = \frac{75 \times 10^{-3}}{15 \times 10^{-3}} = 5\ \Omega$$

If each division = 1 V, the required voltage for full scale deflection is

$$V = 1 \times 150 = 150\text{ V}$$

Let R be the required resistance to be connected in series with G, then

$$V = I_g(R + G)$$

$$\Rightarrow \quad 150 = 15 \times 10^{-3}\,(R + 5)$$

$$\Rightarrow \quad R + 5 = 10000 \quad \Rightarrow \quad R = 9995\ \Omega$$

18. Since the current through the galvanometer is zero, the voltage across R is

$$V = \text{emf of battery } B = 2\text{ V} \qquad (1)$$

Now current through R is

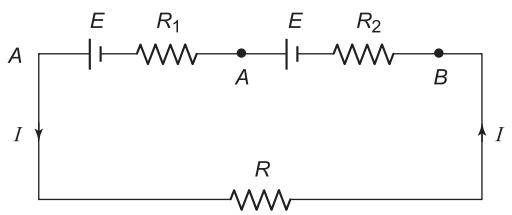
$$I = \frac{12}{(500 + R)}$$

$$\therefore \quad \text{Voltage across } R = IR = \frac{12R}{(500 + R)} \qquad (2)$$

Equating (1) and (2)

$$2 = \frac{12R}{(500 + R)} \quad \Rightarrow \quad R = 100\ \Omega$$

19. The circuit can be drawn as shown in the figure.



Total resistance of the circuit = $R + R_1 + R_2$. Total emf = $2E$.

The current in the circuit is

$$I = \frac{2E}{R + R_1 + R_2} \qquad (1)$$

Potential difference between points A and B is

$$V_A - V_B = E - IR_2$$

Given $V_A - V_B = 0$. Hence $E - IR_2 = 0$ which gives

$$I = \frac{E}{R_2} \qquad (2)$$

Equating (1) and (2),

$$\frac{2E}{R + R_1 + R_2} = \frac{E}{R_2}$$

$$\Rightarrow \quad R = R_2 - R_1$$

20. Let R be the resistance of the complete coil. The rate of heat generation is

$$P_1 = \frac{V^2}{R}$$

If the coil is cut into two equal parts, the resistance of each part = $R/2$. The rate of heat generation with one part is

$$P_2 = \frac{V^2}{R/2}$$

$$\therefore \quad \frac{P_2}{P_1} = 2$$

21. $r = R\left(\frac{l_1 - l_2}{l_2}\right) = 2 \times \left(\frac{240 - 120}{120}\right) = 2\ \Omega$

22. $P = \frac{V^2}{R}$. When the bulb is burning, its filament will be hot and its resistance is

$$R = \frac{V^2}{P} = \frac{200 \times 200}{100} = 400\ \Omega$$

$\therefore$ Resistance when the bulb is not in use, i.e. when the filament is cold = $\frac{400}{10} = 40\ \Omega$.

23. $\rho_A = \dfrac{R_A\left(\pi \frac{d_A^2}{4}\right)}{l_A}$ and $\rho_B = \dfrac{R_B\left(\pi \frac{d_B^2}{4}\right)}{l_B}$

$$\therefore \quad \frac{\rho_A}{\rho_B} = \frac{R_A}{R_B} \times \left(\frac{d_A}{d_B}\right)^2 \times \frac{l_B}{l_A}$$

or $$\frac{l_B}{l_A} = \frac{\rho_A}{\rho_B} \times \frac{R_A}{R_B} \times \left(\frac{d_B}{d_A}\right)^2 = \frac{1}{2} \times 1 \times \left(\frac{2}{1}\right)^2 = 2$$

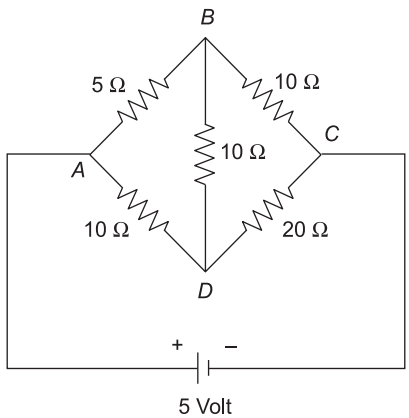
24. Kirchhoff's first law is a statement of conservation of charge. It states that the sum of the currents entering a junction is equal to the sum of the currents leaving it, i.e. the charge must be removed from the junction at the same rate at which it arrives at it.

The product iR is the potential difference across a resistor R. Since potential difference is defined as the amount of work done (or energy spent)

in taking a unit charge from one end of the resistor to the other, Kirchhoff's second law is a statement of conservation of energy. Hence the correct choice is (b).

25. At the hot junction the current flows from the metal which occurs earlier in the thermoelectric series to the metal which occurs later in the series. At the cold junction the current flows from the metal that occurs later in the series to the one which occurs earlier. Since Bismuth occurs earlier than Antimony in the series, at the cold junction, the current will flow from Antimony to Bismuth. Hence the correct choice is (b).

26. The given circuit can be redrawn as shown in the figure.

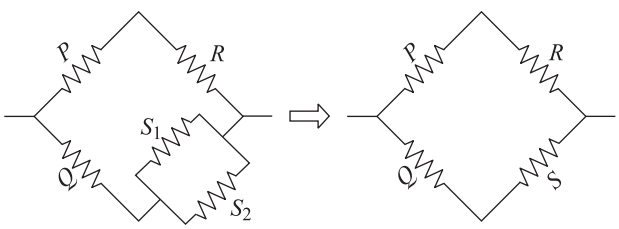


It is a balanced Wheatstone's bridge. The resistor of 10 Ω between points B and D is not effective as no current flows through it and hence it can be ignored. The total resistance is a parallel combination of 5 + 10 = 15 Ω and 10 + 20 = 30 Ω which is

$$R_{\text{eff}} = \frac{15 \times 30}{15 + 30} = 10\ \Omega$$

$$\therefore \quad \text{Current } I = \frac{V}{R_{\text{eff}}} = \frac{5 \text{ volt}}{10 \text{ ohm}} = 0.5 \text{ A}$$

27. The circuit diagram of Wheatstone's bridge is as shown in the figure.



Here $S = \dfrac{S_1 S_2}{S_1 + S_2}$

The balance condition is

$$\frac{P}{Q} = \frac{R}{S} = \frac{R(S_1 + S_2)}{S_1 S_2}$$

28. $R_2 = R_1[1 + \alpha (t_2 - t_1)]$

which gives $t_2 - t_1 = \dfrac{R_2 - R_1}{R_1 \alpha} = \dfrac{200 - 100}{100 \times 0.005} = 200°\text{C}$

$\therefore$ Final temperature $t_2 = t_1 + 200°\text{C} = 100°\text{C} + 200°\text{C} = 300°\text{C}$.

29. $P = \dfrac{V^2}{R}$. Hence $R = \dfrac{V^2}{P} = \dfrac{(220)^2}{100} = 400\ \Omega$

When operated at $V' = 110$ volt, the power will be

$$P' = \frac{V'^2}{R} = \frac{(110)^2}{484} = 25 \text{ watt}$$

30. Let α be the temperature coefficient of resistance of the material of the wire and R_0 its resistance at 0°C. Then

$5 = R_0(1 + 50\ \alpha)$ (1)

and $6 = R_0(1 + 100\ \alpha)$ (2)

Dividing the equations, we have

$$\frac{6}{5} = \frac{1 + 100\alpha}{1 + 50\alpha}$$

$\Rightarrow 6(1 + 50\ \alpha) = 5(1 + 100\ \alpha)$

$$\Rightarrow \alpha = \frac{1}{200} (°\text{C})^{-1}$$

Substituting the value of α in Eq. (1) or Eq. (2) gives $R_0 = 4$ ohm.

31. $E = \rho J = \dfrac{\rho I}{2\pi r^2}$. Also $E = -\dfrac{dV}{dr} \quad \Rightarrow \quad dV = -E dr$

Therefore, potential difference between B and C due to current at A is

$$V = -\int_C^B E\, dr = -\frac{\rho I}{2\pi} \int_{(a+b)}^{a} \frac{dr}{r^2}$$

$$= -\frac{\rho I}{2\pi} \left| -\frac{1}{r} \right|_{(a+b)}^{a} = \frac{\rho I}{2\pi a} - \frac{\rho I}{2\pi (a+b)}$$

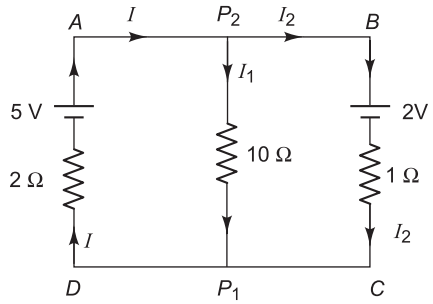
Similarly, potential difference between B and C due to current at D is $V' = \dfrac{\rho I}{2\pi a} - \dfrac{\rho I}{2\pi (a+b)}$. From superposition principle, the potential difference between B and C due to current at A and at D is

$$\Delta V = V + V' = \frac{\rho I}{\pi a} - \frac{\rho I}{\pi (a+b)}.$$

32. $E = \rho J = \dfrac{\rho I}{2\pi r^2}$, Hence the correct choice is (d).

33. $I = I_1 + I_2$ (Refer to the figure)

Applying Kirchhoff's loop rule to the two loops, we have



$$10I_1 + 2I - 5 = 0$$

$$\Rightarrow \quad 10I_1 + 2(I_1 + I_2) - 5 = 0 \qquad (1)$$

and $$2 + I_2 - 10I_1 = 0 \qquad (2)$$

Solving Eqs. (1) and (2), we get $I_1 = \dfrac{1}{32}\text{ A} \simeq 0.03\text{ A}$, from P_2 to P_1. Hence the correct choice is (a).

34. $\dfrac{55}{20} = \dfrac{R}{(100-20)} \quad \Rightarrow \quad R = 220\ \Omega$

35. Relation $R = R_0(1 + \alpha\Delta t)$ is valid for small changes in temperature. So the correct choice is (b).

36. The resistance of a conductor at temperature t°C is given by

$R = R_0(1 + \alpha t)$

where R_0 is the resistance at 0°C

For series combination

$$R_s = R_1 + R_2$$

At 0°C, $$R_s = R_0 + R_0 = 2R_0$$

$$\therefore \quad 2R_0(1 + \alpha_s t) = R_0(1 + \alpha_1 t) + R_0(1 + \alpha_2 t)$$

$$\Rightarrow \quad \alpha_s = \frac{1}{2}(\alpha_1 + \alpha_2)$$

For parallel combination

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

At 0°C, $$\frac{1}{R_p} = \frac{1}{R_0} + \frac{1}{R_0} = \frac{2}{R_0}$$

$$\Rightarrow \quad R_p = \frac{R_0}{2}$$

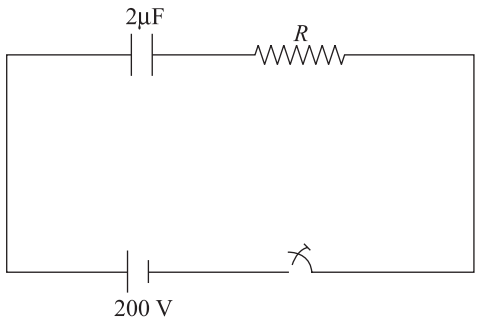
$$\therefore \quad \frac{1}{\frac{R_0}{2}(1 + \alpha_p t)} = \frac{1}{R_0(1 + \alpha_1 t)} + \frac{1}{R_0(1 + \alpha_2 t)}$$

$$\Rightarrow \quad 2(1 + \alpha_p t)^{-1} = (1 + \alpha_1 t)^{-1} + (1 + \alpha_2 t)^{-1}$$

$$\Rightarrow \quad 2(1 - \alpha_p t) = (1 - \alpha_1 t) + (1 - \alpha_2 t) \qquad (\because \alpha t << 1)$$

$$\Rightarrow \quad \alpha_p = \frac{1}{2}(\alpha_1 + \alpha_2)$$

37.



When the switch is closed, the voltage decreases with time t as

$$V = V_0(1 - e^{t/\tau})$$

where $\tau = RC$

Putting V = 120 V, V_0 = 200 V and t = 5s in this equation, we have

$$120 = 200\,(1 - e^{-5/\tau})$$

$$\Rightarrow e^{-5/\tau} = \frac{200 - 120}{200} = \frac{80}{200} = \frac{2}{5}$$

$$\text{or } e^{5/\tau} = \frac{5}{2} = 2.5$$

$$\text{or } \frac{5}{\tau} = \ln\left(\frac{5}{2}\right) = \ln(2.5)$$

$$\text{or } \tau = \frac{5}{\ln(2.5)} = \frac{5}{2.302 \times 0.4}$$

$$\Rightarrow \tau = 5.43$$

$$\text{or} \qquad RC = 5.43$$

$$\Rightarrow \qquad R = \frac{5.43}{C} = \frac{5.43}{2 \times 10^{-6}} = 2.7 \times 10^6\ \Omega$$

38. $R = \dfrac{\rho l}{A}$

$$\Rightarrow \qquad \log R = \log \rho + \log l - \log A$$

Differentiating

$$\frac{\delta R}{R} = 0 + \frac{\delta l}{l} - \frac{\delta A}{A} \qquad (\because \rho = \text{constant})$$

Volume of wire is $V = Al$. Therefore,

$\log V = \log A + \log l$

Differentiating

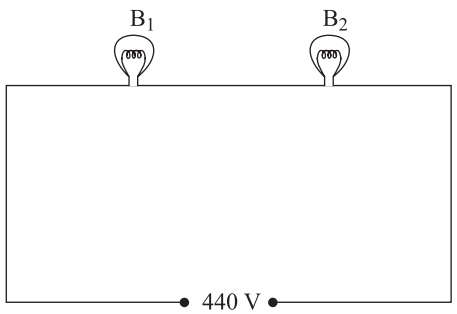
$$0 = \frac{\delta A}{A} + \frac{\delta l}{l} \qquad (\because V = \text{constant})$$

$$\Rightarrow \quad \frac{\delta A}{A} = -\frac{\delta l}{l}$$

$$\therefore \quad \frac{\delta R}{R} = \frac{\delta l}{l} - \left(-\frac{\delta l}{l}\right) = \frac{2\delta l}{l}$$

$$= 2 \times 0.1\% = 0.2\%$$

39.



Resistance of bulb B_1 is

$$R_1 = \frac{220 \times 220}{25}\,\Omega$$

Resistance of bulb B_2 is

$$R_2 = \frac{220 \times 220}{100}\,\Omega$$

$$R_1 + R_2 = 220\left(\frac{220}{25} + \frac{220}{10}\right) = 220 \times 220 \times \frac{1}{20}$$

$$= (220 \times 11)\,\Omega$$

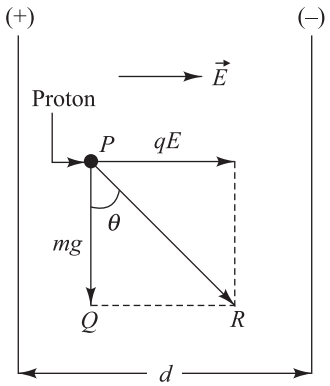
$$\therefore \text{ Current } I = \frac{440}{220 \times 11} = \frac{2}{11} = \frac{40}{220}\text{A}$$

Maximum current that bulb B_1 can withstand is

$$I_{\max} = \frac{P_1}{V_1} = \frac{25}{220}\text{A}$$

Since $I > I_{\max}$, bulb B, will fuse. The maximum current through $B_2 = \frac{100}{220}$ which is greater than $I = \frac{40}{220}\text{A}$. Hence bulb B_2 will not fuse.

40. $\tan\theta = \frac{QR}{PQ} = \frac{qE}{mg}$



Given $\theta = 45°$. Therefore, $\tan 45° = \frac{qE}{mg}$

$$\Rightarrow \quad qE = mg$$

Also $E = \frac{X}{d}$. Hence

$$\frac{qX}{d} = mg$$

$$X = \frac{mgd}{q}$$

$$= \frac{(1.67 \times 10^{-27}) \times 9.8 \times (1 \times 10^{-2})}{1.6 \times 10^{-19}} \simeq 10^{-19}\,\text{V}$$

41. When S_1 is closed and S_2 is kept open, inductor L is not in circuit and we have R and C is series connected to a battery. If S_1 is closed at $t = 0$, the charge on the capacitor at an instant of time t is given by

$q = q_0(1 - e^{-t/\tau}) = CV(1 - e^{-t/\tau})$

where $q_0 = CV$

is the charge on the capacitor when it is fully charged to the voltage V of the battery and $\tau = RC$ = time constant.

(a) At $t = \tau$, $q = CV(1 - e^{-1})$

(b) At $t = 2\tau$, $q = CV(1 - e^{-2})$

(c) At $t = \frac{\tau}{2}$, $q = CV(1 - e^{-1/2})$

So the correct choice is (b).

42. Resistance of bulb is $R_1 = \frac{V^2}{P_1} = \frac{120 \times 120}{60} = 240\,\Omega$

Resistance of heater is $R_2 = \frac{V^2}{P_2} = \frac{120 \times 120}{240} = 60\,\Omega$

Before the heater is switched on the voltage across the bulb is

$$V_1 = \frac{120 \times 240}{(240 + 6)} = 117.07 \text{ V}$$

The resistance of the parallel combination of bulb and heater is

$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{240 \times 60}{(240 + 60)} = 48\ \Omega$$

After the heater is switched on, the voltage across the bulb will be

$$V_2 = \frac{120 \times 48}{(48 + 6)} = 106.67 \text{ V}$$

Decrease in voltage is $V_1 - V_2 = 117.07 - 106.67$

$= 10.4$ V

No choice given in the question is correct. The closest choice is (c).

43. $R = \left(\frac{l}{100 - l}\right) \times 90 = \frac{40 \times 90}{60} = 60\ \Omega$

Since 90 Ω is exact, the fractional error in R is

$$\frac{\Delta R}{R} = \frac{0.1\text{cm}}{40\text{cm}} + \frac{0.1\text{cm}}{60\text{cm}} = \frac{0.1}{40} + \frac{0.1}{60}$$

Since $R = 60\ \Omega$

$$\Rightarrow \Delta R = \frac{0.1}{40} \times 60 + \frac{0.1}{60} \times 60 = 0.15 + 0.1 = 0.25\Omega$$

$\therefore R \pm \Delta R = (60 \pm 0.25)\ \Omega$

The correct choice is (c).

MAGNETIC EFFECTS OF CURRENT, AND MAGNETISM

Chapter 13

REVIEW OF BASIC CONCEPTS

1. Biot-Savart Law

According to Biot-Savart law, the magnetic field $\overrightarrow{dB}$ at a point whose position vector with respect to a current element $\overrightarrow{dl}$ is $\vec{r}$ is given by

$$\overrightarrow{dB} = \frac{\mu_0 I}{4\pi}\frac{(\overrightarrow{dl}\times\vec{r})}{r^3} \qquad (1)$$

where $\mu_0 = 4\pi\times 10^{-7}\ \text{Hm}^{-1}$

2. Ampere's Circuital Law

The loop or line circuit integral of the magnetic field along a closed curve is proportional to the current threading or passing through the closed circuit i.e.

$$\oint \vec{\text{B}}\cdot\vec{\text{dl}} = \mu_0 I$$

where μ_0 is the permeability of free space.
Biot-Savart Law and Ampere's Circuital Law are used to find the magnetic field due to current carrying conductors.

3. Magnetic Field Due to Current Carrying Conductors

(i) *Magnetic field at point P due to an infinitely long wire carrying a current I* (Fig. 13.1)

$B = \frac{\mu_0 I}{2\pi r}$ directed into the page (away from the reader) if the I is upwards and towards the reader if I is downwards. At points Q or S, $B = 0$.

(ii) *Magnetic field at the centre of a circular loop of radius r* (Fig. 13.2)

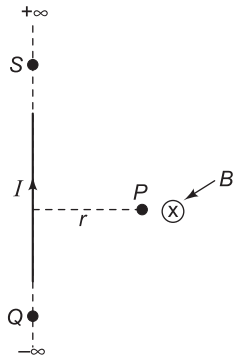


Fig. 13.1

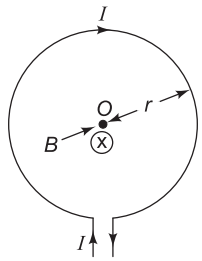


Fig. 13.2

$B = \frac{\mu_0 I}{2r}$ directed into the page if I is clockwise and outside the page if I is anticlockwise.

For a coil of N turns.

$$B = \frac{\mu_0 NI}{2r}$$

(iii) *Magnetic field at the centre of a curved element* (Fig. 13.3).

$$B = \frac{\mu_0 I}{2r}\times\frac{\theta}{2\pi}$$

directed into the page. Here θ is in radian.

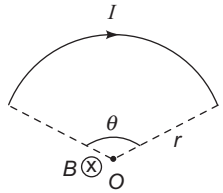


Fig. 13.3

For a semi-circular element ($\theta = \pi$)

$$B = \frac{\mu_0 I}{4r}$$

(iv) *Magnetic field at point P due to a straight wire XY of finite length* (Fig. 13.4)

$$B = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$$

directed into the page.

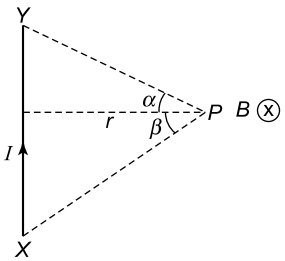


Fig. 13.4

Special Cases

(a) If the conductor XY is of infinite length and point P lies near the centre of the conductor (as in Fig. 13.1), $\alpha = \beta = 90°$ so

$$B = \frac{\mu_0 I}{4\pi r}(\sin 90° + \sin 90°)$$

$$= \frac{\mu_0 I}{2\pi r}$$

(b) If the conductor XY is of infinite length but point P lies near the end X or Y as shown in Fig. 13.5, then $\alpha = 90°$ and $\beta = 0°$, then

$$B = \frac{\mu_0 I}{4\pi r}(\sin 90° + \sin 0°)$$

$$= \frac{\mu_0 I}{4\pi r}$$

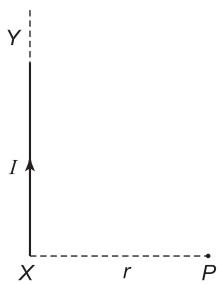


Fig. 13.5

Note For an infinitely long straight conductor carrying a current, the magnetic field near its centre is twice that near one of its ends.

(c) If the conductor XY has a finite length L and point P lies on the right bisector of the conductor, as shown in Fig. 13.6, then

$\alpha = \beta$

and $\sin\alpha = \sin\beta = \dfrac{L/2}{x}$

$$= \frac{L/2}{\sqrt{r^2 + \left(\frac{L}{2}\right)^2}}$$

$$= \frac{L}{\sqrt{4r^2 + L^2}}$$

So $B = \dfrac{\mu_0 I}{2\pi r} \cdot \dfrac{L}{\sqrt{4r^2 + L^2}}$

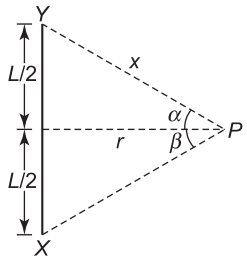


Fig. 13.6

(d) If the point P lies on the straight conductor or on its axis, then $\vec{dl}$ and $\vec{r}$ for each element of the straight conductor are parallel. Therefore, $\vec{dl} \times \vec{r} = 0$. Hence $\vec{B} = 0$ at point P.

(v) *Magnetic field at centre due to a wire PQRST* (Fig. 13.7)

Magnetic field at O due to straight portions PQ and ST is zero and due to semicircular part QRS is

$$B = \frac{\mu_0 I}{4r} \text{ directed into the page.}$$

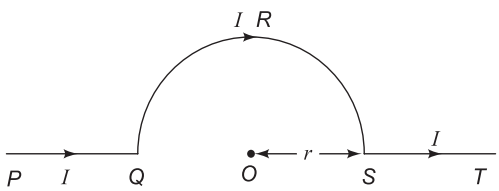


Fig. 13.7

If the current is anticlockwise, B is directed towards the reader.

(vi) *Magnetic field at Centre O of a rectangular coil (Fig. 13.8)*

$$B = \frac{2\mu_0 I}{\pi} \times \frac{\sqrt{a^2 + b^2}}{ab} \text{ directed into the page.}$$

For a square coil ($b = a$)

$$B = \frac{2\sqrt{2}\,\mu_0 I}{\pi a}$$

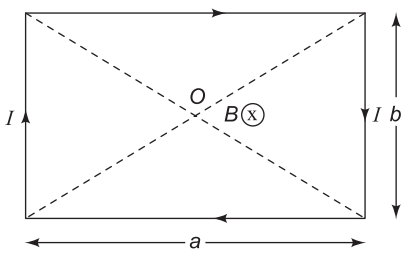


Fig. 13.8

(vii) *Magnetic field due to a hollow metal pipe of radius R carrying current in its walls* (Fig. 13.9)

At point P, $B = \dfrac{\mu_0 I}{2\pi r}$

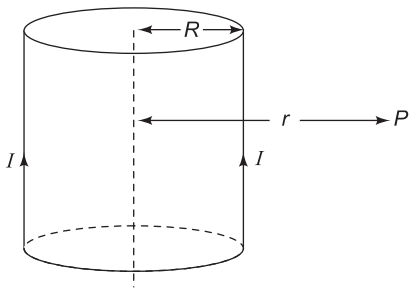


Fig. 13.9

For a solid pipe of radius R (Fig. 13.9)

Inside the pipe at a point at a distance r from the axis,

$$B = \frac{\mu_0 I r}{2\pi R^2}$$

Outside the pipe at a point P at a distance r from the axis,

$$B = \frac{\mu_0 I}{2\pi r}$$

(viii) *Magnetic field on the axis of a circular coil of radius R* (Fig. 13.10)

At point P, $B = \dfrac{\mu_0}{4\pi} \dfrac{2M}{(R^2 + r^2)^{3/2}}$

where $M = IA = I \times \pi R^2$ is the magnetic moment. If current I is anticlockwise $\vec{B}$ is directed from O to P. For clockwise current $\vec{B}$ is from P to O.

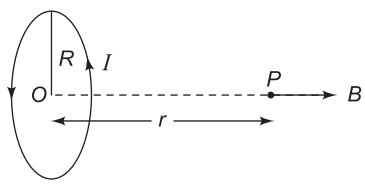


Fig. 13.10

(ix) *Magnetic field due to an electron (charge e) revolving in a circular orbit of radius r with speed v and frequency n* (Fig. 13.11)

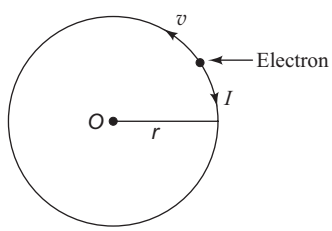


Fig. 13.11

Current along orbit $I = \dfrac{e}{T} = en = \dfrac{ev}{2\pi r}$

The direction of I is opposite to the direction of motion of the electron. Magnetic field at O is

$$B = \frac{\mu_0 I}{4r}$$

where $I = \dfrac{ev}{2\pi r}$ and is directed into the page.

Magnetic moment $M = IA = I \times \pi r^2 = \dfrac{evr}{2}$

(x) *Magnetic field due to a current carrying straight solenoid*

In the middle region $B = \mu_0 nI$; n = no. of turns per unit length.

At the ends of solenoid, $B = \dfrac{\mu_0 nI}{2}$

For a toroid of radius R, $B = \mu_0 nI$, where $n = \dfrac{N}{L} = \dfrac{N}{2\pi R}$; N = total no. of turns. Outside the solenoid, $B = 0$.

EXAMPLE 1 Figure 13.12 shows two stationary and infinitely long bent wires PQR and STU lying in the x-y plane and each carrying a current I as shown. Find the magnitude and direction of the magnetic field at origin O. Given $OQ = OT = a$

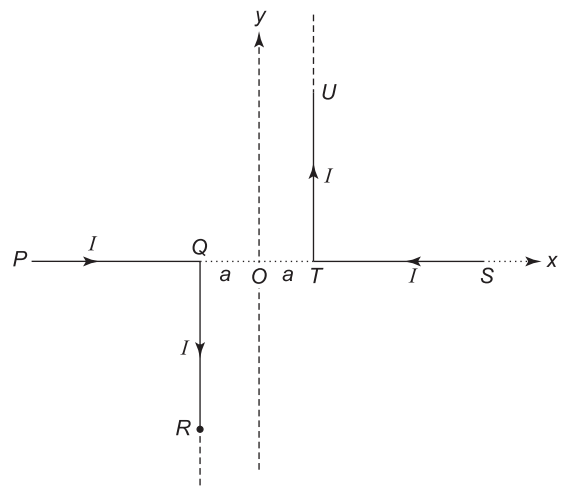


Fig. 13.12

SOLUTION As point O is along the line segments PQ and ST, the magnetic field at O due to PQ and ST is zero. The magnetic field at O due to wires QR and TU respectively are

$$\mathbf{B}_1 = \frac{\mu_0 I\,(\hat{\mathbf{k}})}{4\pi(OQ)} \text{ and } \mathbf{B}_2 = \frac{\mu_0 I\,(\hat{\mathbf{k}})}{4\pi(OT)}$$

both directed along the positive z-axis. The resultant field at O is ($\because OQ = OT = a$)

$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = 2 \times \frac{\mu_0 I}{4\pi a}(\hat{\mathbf{k}}) = \frac{\mu_0 I}{2\pi a}\hat{\mathbf{k}}$$

EXAMPLE 2 Two infinitely long wires carrying equal current I in the opposite direction are placed perpendicular to the x-y plane. One wire is located at point P (0, a, 0) and the other wire at Q (0, $-a$, 0). Find the magnitude and direction of at point $A(x, 0, 0)$.

SOLUTION Refer to Fig. 13.13. Wire 1 carries a current I along the positive z-direction and wire 2 carries a current I along the negative z-direction.

$$OP = OQ = a,\ OA = x,\ PA = QA = r.$$

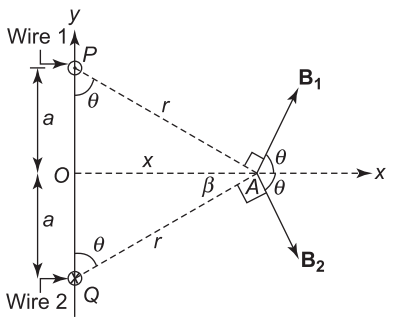


Fig. 13.13

Magnetic field at A due to wire 1 is

$$B_1 = \frac{\mu_0 I}{2\pi(PA)} = \frac{\mu_0 I}{2\pi\sqrt{a^2 + x^2}}$$

According to Biot-Savart law, $\mathbf{B}_1$ is perpendicular to both PA and wire 1 and therefore in the x-y plane. Similarly, magnetic field at A due to wire 2 is

$$B_2 = \frac{\mu_0 I}{2\pi\sqrt{a^2 + x^2}}$$

The y-components of $\mathbf{B}_1$ and $\mathbf{B}_2$ cancel each other but the x-components add up. These components are $B_1 \cos\theta$ and $B_2 \cos\theta$ both along the positive x-direction. Therefore, the resultant magnetic field at A is

$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = (B_1 \cos\theta + B_2 \cos\theta)\hat{\mathbf{i}}$$

$$= \frac{\mu_0 I}{\pi\sqrt{(a^2 + x^2)}} \times \frac{a}{\sqrt{a^2 + x^2}} \times \hat{\mathbf{i}}$$

$$\left(\because \cos\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2 + x^2}}\right)$$

$$= \frac{\mu_0 I\, a}{\pi(a^2 + x^2)}\hat{\mathbf{i}}$$

EXAMPLE 3 Two wires A and B have the same length L and carry equal currents I. Wire A is bent into a circle and wire B is bent into a square. (a) Obtain expression for the magnitude of the magnetic field at (*i*) the centre of the circular loop and (*ii*) the centre of the square. Which wire produces a greater magnetic field at the centre?

SOLUTION

(a) (i) Radius r of wire A when it is bent into a circle is given by

$$2\pi r = L \Rightarrow r = \frac{L}{2\pi}$$

Magnetic field at the centre of the circular loop is

$$B_1 = \frac{\mu_0 I}{2r} = \frac{\mu_0 I \pi}{L} \qquad (1)$$

(ii) Refer to Fig. 13.14. The magnetic field at O due to wire PQ is ($OT = a$)

$$B_{PQ} = \frac{\mu_0 I}{4\pi a}(\sin 45° + \sin 45°)$$

$$= \frac{\mu_0 I}{2\sqrt{2}\,\pi a}$$

$$= \frac{4\,\mu_0 I}{\sqrt{2}\,\pi L} \qquad \left(\because a = \frac{L}{8}\right)$$

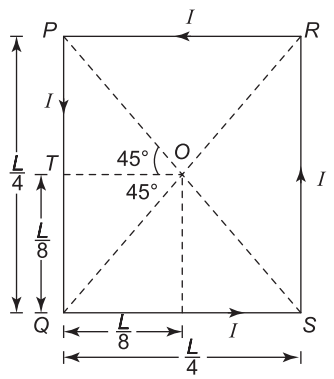


Fig. 13.14

Since centre O of the square is at the same distance from each side of the square and each arm carries the same current, the magnetic field due to each side of the square is of the same and in the same direction. Hence the total magnetic field at O is

$$B_2 = 4B_{PQ} = \frac{16\,\mu_0 I}{\sqrt{2}\,\pi L} = \frac{8\sqrt{2}\,\mu_0 I}{\pi L} \qquad (2)$$

Dividing (2) by (1) we get

$$\frac{B_2}{B_1} = \frac{8\sqrt{2}}{\pi^2} = \frac{8 \times 1.41}{(3.14)^2} = 1.16$$

Hence $B_2 > B_1$. The magnetic field at the centre due to the square loop will be greater than that due to the circular loop.

EXAMPLE 4 Figure 13.15 shows a wire loop *ABCDEA* carrying a current *I* as shown.

Given $AE = ED = a$ and $AB = CD = a/2$.

Find the magnitude and direction of the magnetic field at point *F* where $BF = CF = a/2$.

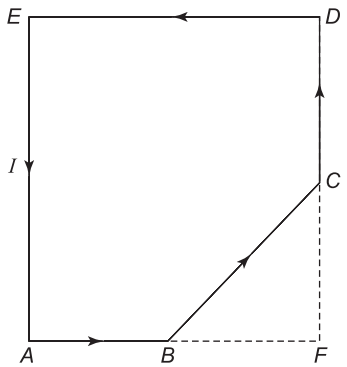


Fig. 13.15

SOLUTION Magnetic field at *F* is

$$\mathbf{B} = \mathbf{B}_{AB} + \mathbf{B}_{BC} + \mathbf{B}_{CD} + \mathbf{B}_{DE} + \mathbf{B}_{EA}$$

Since point *F* lies in line with current elements *AB* and *CD*, $B_{AB} = B_{CD} = 0$

Also $B_{DE} = B_{EA} = \dfrac{\mu_0 I}{4\pi a}\ (\sin 0° + \sin 45°) = \dfrac{\mu_0 I}{4\sqrt{2}\,\pi a}$

directed out of the page and towards the reader.

$$B_{BC} = \frac{\mu_0 I}{4\pi\, BC/2}\ (\sin 45° + \sin 45°)$$

directed into the page and away from the reader. Now

$$BC = \sqrt{BF^2 + FC^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{2}}$$

$$\therefore \qquad B_{BC} = \frac{\mu_0 I}{4\pi\left(\dfrac{a}{2\sqrt{2}}\right)}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\mu_0 I}{\pi a}$$

directed into the page.

Now $B_{DE} + B_{EA} = \dfrac{\mu_0 I}{2\sqrt{2}\pi a}$ directed out of the page.

Since $B_{BC} > B_{DE} + B_{EA}$ the net field, **B** is directed into the page and has a magnitude

$$B = \frac{\mu_0 I}{\pi a} - \frac{\mu_0 I}{2\sqrt{2}\,\pi a} = \frac{\mu_0 I}{\pi a}\left(1 - \frac{1}{2\sqrt{2}}\right)$$

EXAMPLE 5 A wire *ABCDE* is bent as shown in Fig. 13.16. The wire carries a current *I* and the radius of the bent coil *BCD* is *r*. Find the magnitude and direction of the magnetic field at centre *O*.

SOLUTION The straight line segments *AB* and *DE* are collinear with *O*. Hence the magnetic field due to *AB* and *DE* at *O* is zero. Angle subtended at *O* by arc $BCD = 2\pi - \theta$. The magnetic field due to *BCD* at *O* is

$$B = \frac{\mu_0 I}{2r}\left(\frac{2\pi - \theta}{2\pi}\right)$$

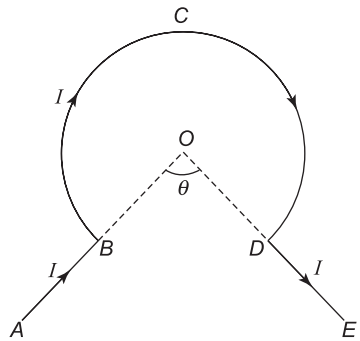


Fig. 13.16

The current through *BCD* is clockwise. Therefore, the direction of the magnetic field at *O* is into the page and away from the reader.

EXAMPLE 6 A wire *ABCDEFA* is bent as shown in Fig. 13.17 and caries a current *I*. The radius of the smaller arc *ABC* is $r_1 = r$ and that of the bigger arc is $r_2 = 2r$. Find the magnitude of the magnetic field at centre *O*.

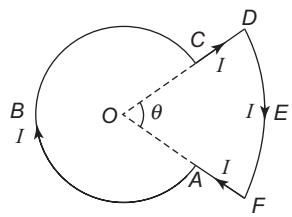


Fig. 13.17

SOLUTION Magnetic field due to arc *ABC* at *O* is

$$B_1 = \frac{\mu_0 I}{2r_1}\left(\frac{2\pi - \theta}{2\pi}\right)$$

Magnetic field due to arc *DEF* at *O* is

$$B_2 = \frac{\mu_0 I}{2r_2} \cdot \frac{\theta}{2\pi}$$

Since B_1 and B_2 are both directed into the page, the total magnetic field at *O* is

$$B = B_1 + B_2 = \frac{\mu_0 I}{2r_1}\left(\frac{2\pi - \theta}{2\pi}\right) + \frac{\mu_0 I}{2r_2}\,\frac{\theta}{2\pi}$$

Putting $r_1 = r$ and $r_2 = 2r$, we get

$$B = \frac{\mu_0 I}{2r}\left(1 - \frac{\theta}{4\pi}\right)$$

EXAMPLE 7 A long straight cylinder of radius *R* carries a current *I* which is uniformly distributed across its cross-section. Find the magnetic field at a point at a distance *r* from the axis of the cylinder in cases (a) $r > R$ and (b) $r < R$.

SOLUTION Figure 13.18 shows the cross-sectional view of the cylinder.

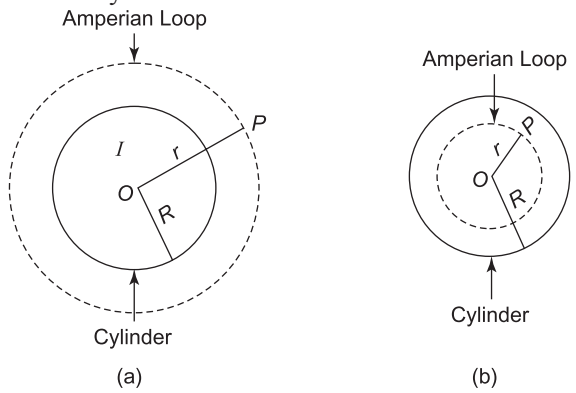


Fig. 13.18

Case (a) $r > R$. For this case, the Amperian loop is a circle of radius r concentric with the cross-section [Fig. 13.18 (a)]. For this loop, $L = 2\pi r$ and the current threading the loop is $i = I$. From Ampere's circuital law.

$$BL = \mu_0 i \Rightarrow B \times 2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

Case (b) $r < R$. For this case, $L = 2\pi r$ [Fig. 13.18 (b)] and the current threading the loop is

i = current per unit cross-sectional area of the cylinder × cross-sectional area of the Amperian loop

$$= \frac{I}{\pi R^2} \times \pi r^2 = \frac{Ir^2}{R^2}$$

From Ampere's law,

$$B \times 2\pi r = \mu_0 i = \frac{\mu_0 I r^2}{R^2}$$

$$\Rightarrow \quad B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r$$

Note

(1) For $r < R$, $B \propto r$ and for $r > R$, $B \propto \frac{1}{r}$. Figure 13.19 shows the variation of B with r.

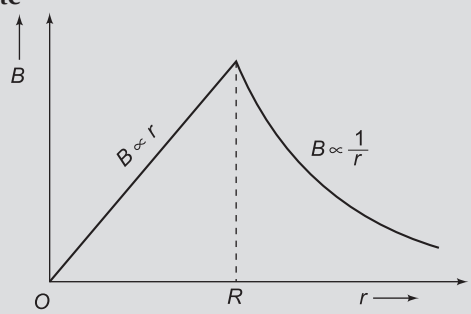


Fig. 13.19

(2) For a hollow cylinder, the current flows along its walls. Therefore, in the case $r < R$ [Fig. 13.18 (b)], no current threads the Amperian loop. Hence $B = 0$ for points inside a hollow cylinder.

EXAMPLE 8 The current density J (current per unit area) in a solid cylinder of radius R varies with distance r from its axis as $J = kr$ where k is a constant. Find the magnetic field at a point P where (a) $r > R$ and (b) $r < R$.

SOLUTION

$$\text{Current } I = \int JdA = \int kr \times (2\pi r dr)$$

Case (a) We take the Amperian loop of radius $r > R$. Since the loop is outside the cylinder, the current through the loop is

$$I = \int_0^R kr \times (2\pi r dr) = 2\pi k \int_0^R r^2 dr = \frac{2\pi\, kR^3}{3}$$

$$\therefore \quad B \times 2\pi r = \mu_0 I = \frac{2\pi\, \mu_0 k R^3}{3} \Rightarrow B = \frac{\mu_0 k R^3}{3r}$$

Case (b) For $r < R$, the current through the Amperian loop is

$$I = \int_0^r kr(2\pi r dr) = \frac{2\pi k r^3}{3}$$

$$\therefore \quad B \times 2\pi r = \mu_0 I = \frac{\mu_0 \times 2\pi k r^3}{3} \Rightarrow B = \frac{\mu_0 k r^2}{3}$$

EXAMPLE 9 Two long wires 1 and 2 carrying equal currents $I_1 = I_2 = 9$ A are held parallel to each other 6 cm apart as shown in Fig. 13.20. Find the magnetic field at (a) point P, (b) point Q and (c) point R.

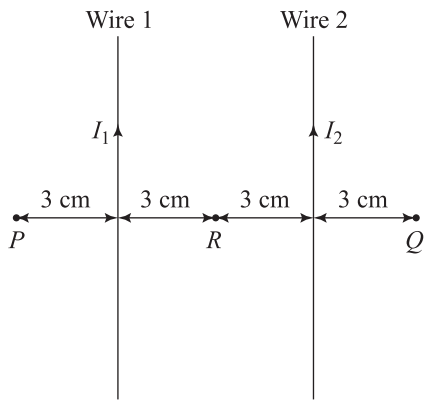


Fig. 13.20

SOLUTION (a) Magnitude of magnetic field at P due to wire 1 is

$$B_1 = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 9}{2\pi \times 3 \times 10^{-2}} = 6 \times 10^{-5}\ \text{T}$$

The direction of the field is perpendicular to plane of the page and towards the reader. The magnitude of magnetic field at P due to wire 2 is

$$B_2 = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 9}{2\pi \times 9 \times 10^{-2}} = 2 \times 10^{-5}\ \text{T}$$

The direction of this field is the same that of B_1. Thus, the net field at P is

$$B_P = B_1 + B_2 = 8 \times 10^{-5}\ \text{T}$$

and its direction is towards the reader.

(b) Similarly, the net field at Q will be 8×10^{-5} T and its direction is perpendicular to the page and away from the reader.

(c) At point R, the magnetic field due to wires 1 and 2 will have equal magnitude but opposite directions. Hence the net magnetic field at R will be zero.

EXAMPLE 10 Two long wires 1 and 2 are kept 8 cm apart and carry currents of $I_1 = 2$ A and $I_2 = 10$ A in opposite directions. At what distance from wire 1 will the resultant magnetic be zero?

(a) 1 cm (b) 2 cm

(c) 3 cm (d) 4 cm

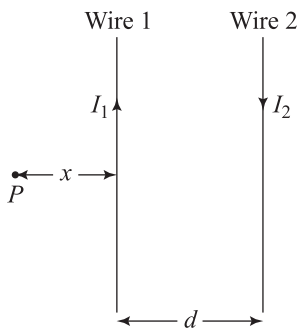


Fig. 13.21

SOLUTION Since the currents are in opposite directions, the resultant magnetic field cannot be zero at any point between the wires; it can be zero at a point to the left of wire 1 or to the right of wire 2. Let the net magnetic field be zero at point P at a distance x from wire 1 (Fig. 13.21). The magnetic field at P due to wire 1 is

$$B_1 = \frac{\mu_0 I_1}{2\pi x} \text{ directed towards the reader.}$$

The magnetic field at P due to wire 2 is

$$B_2 = \frac{\mu_0 I_2}{2\pi (d + x)} \text{ directed away from the reader.}$$

The resultant magnetic field will be zero if $B_1 = B_2$, i.e. if

$$\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi (d + x)}$$

$$\Rightarrow \quad \frac{I_1}{x} = \frac{I_2}{d + x}$$

$$\Rightarrow \quad \frac{2}{x} = \frac{10}{8 + x}$$

$$\Rightarrow \quad x = 2 \text{ cm}$$

EXAMPLE 11 Figure 13.22(a) shows a straight wire AB of length L carrying a current I. The magnitude of magnetic field at point P which is at a perpendicular distance $r = L$ from one end of wire is

(a) $\dfrac{\mu_0 I}{\sqrt{2}\pi L}$ (b) $\dfrac{\mu_0 I}{2\pi L}$

(c) $\dfrac{\mu_0 I}{2\sqrt{2}\pi L}$ (d) $\dfrac{\mu_0 I}{4\sqrt{2}\pi L}$

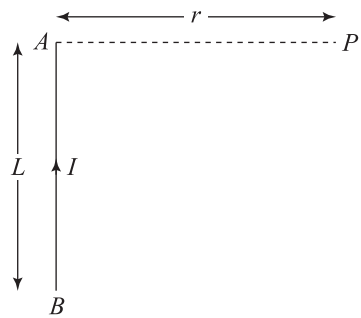


Fig. 13.22(a)

SOLUTION Refer to Fig. 13.22(b).

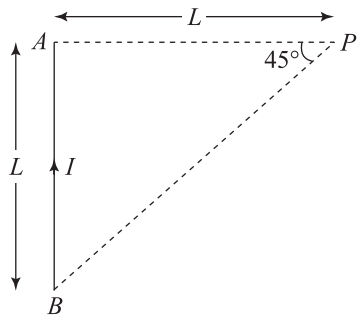


Fig. 13.22(b)

$$B = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$$

Here $\alpha = 45°$ and $\beta = 0°$ and $r = L$.

$$\therefore \quad B = \frac{\mu_0 I}{4\pi L}(\sin 45° + \sin 0°)$$

$$= \frac{\mu_0 I}{4\pi L} \times \frac{1}{\sqrt{2}}$$

So the correct choice is (d).

EXAMPLE 12 Figure 13.23(a) shows a straight wire AB of length L carrying a current I. The magnitude of

magnetic field at point P on the perpendicular bisector of the wire at a distance $r = \frac{L}{2}$ is

(a) $\frac{\sqrt{2}\,\mu_0 I}{\pi L}$ (b) $\frac{\mu_0 I}{\sqrt{2}\,\pi L}$

(c) $\frac{\mu_0 I}{2\sqrt{2}\,\pi L}$ (d) zero

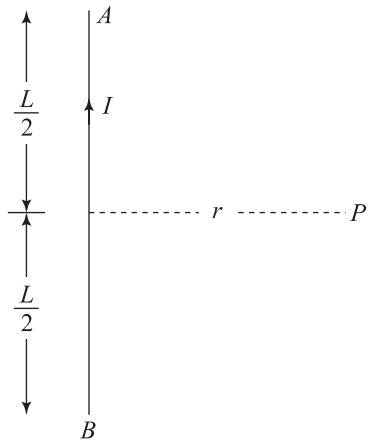


Fig. 13.23(a)

SOLUTION Refer to Fig. 13.23(b).

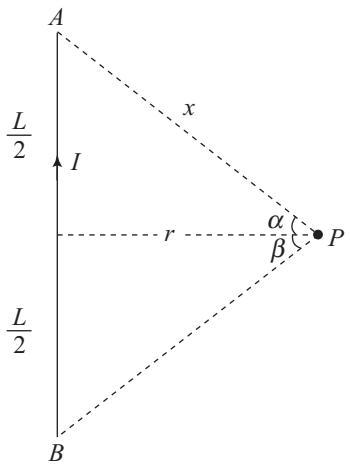


Fig. 13.23(b)

$$B = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$$

Here $\alpha = \beta$,

and $\sin\alpha = \sin\beta = \frac{L/2}{x}$

$$= \frac{L/2}{\sqrt{r^2 + \left(\frac{L}{2}\right)^2}}$$

$$= \frac{L}{\sqrt{4r^2 + L^2}}$$

$$= \frac{1}{\sqrt{2}} \qquad \left(\because\ r = \frac{L}{2}\right)$$

$\therefore$ $$B = \frac{\mu_0 I}{4\pi \times \frac{L}{2}} \times \frac{2}{\sqrt{2}}$$

$$= \frac{\mu_0 I}{\sqrt{2}\,\pi L},$$ which is choice (b).

EXAMPLE 13 A uniform straight wire of length L is turned into a circular wire loop of radius r. The diametrically opposite points P and Q are connected to a battery as shown in Fig. 13.24(a). If I is the current flowing through the battery, the magnetic field at centre O will be

(a) $\frac{\mu_0 I\,\pi}{L}$ (b) $\frac{\mu_0 I}{\pi L}$

(c) $\frac{\mu_0 I}{2\pi L}$ (d) zero

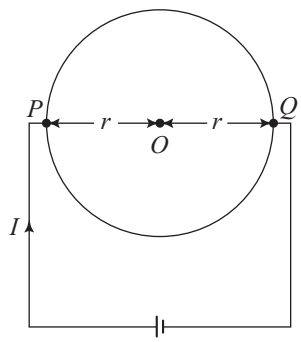


Fig. 13.24(a)

SOLUTION As shown in Fig. 13.24(b), current divides into two equal parts $I_1 = I_2 = I/2$, flowing along semicircular wires PRQ and PSQ. These equal currents produce equal and opposite magnetic fields at centre O. Hence the resultant magnetic field at O will be zero.

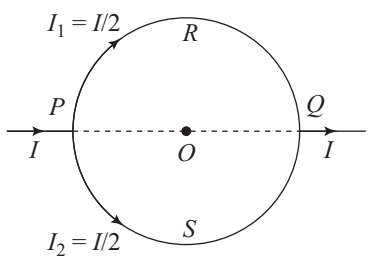


Fig. 13.24(b)

EXAMPLE 14 A uniform straight wire of length L is turned into a square. The points P and R are connected to a battery as shown in Fig. 13.25(a). If current I flows

through the battery, the magnetic field at centre O of the square will be

(a) $\dfrac{8\sqrt{2}\,\mu_0 I}{\pi L}$ (b) $\dfrac{4\sqrt{2}\,\mu_0 I}{\pi L}$

(c) $\dfrac{2\sqrt{2}\,\mu_0 I}{\pi L}$ (d) zero

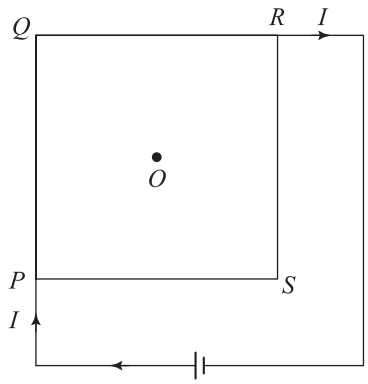


Fig. 13.25(a)

SOLUTION The current divides equally at P and R so that current $I/2$ flows in branch PQR and $I/2$ in branch PSR. Magnetic fields at O due to currents in QR and PS will be equal and opposite and will cancel each other. Similarly currents in PQ and SR will produce equal and oposite fields at O which will cancel each other. Hence the net magnetic field at O will be zero.

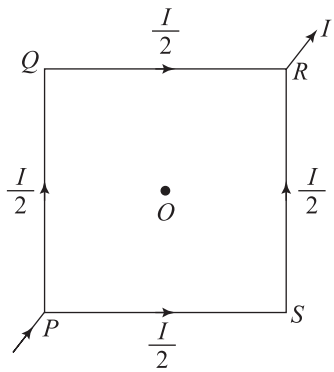


Fig. 13.25(b)

4. Force on a Moving Charge in a Magnetic Field

The force on a charge q moving with a velocity **v** in a magnetic field **B** is given by

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

The direction of **F** is perpendicular to both **v** and **B**. The magnitude F of vector **F** is given by

$$F = qvB \sin\theta$$

where θ is the angle between vectors **v** and **B**.

(1) $\mathbf{F} = 0$ if $\mathbf{v} = 0$, i.e. a charge at rest does not experience any magnetic force.

(2) $\mathbf{F} = 0$ if $\theta = 0$ or 180°, i.e. the magnetic force vanishes if **v** is either parallel or antiparallel to the direction of **B**.

(3) F is maximum $= F_{max}$ if $\theta = 90°$, i.e. if **v** is perpendicular to **B**, the magnetic force has a maximum value given by

$$F_{max} = qvB$$

The direction of the force when $\mathbf{v} \perp \mathbf{B}$ is given by Fleming's left hand rule.

(4) If **v** is perpendicular to both **E** and **B** and **E** is perpendicular to **B**, then $\mathbf{F} = 0$ if $v = \dfrac{E}{B}$.

5. Motion of a Charged Particle in a Magnetic Field

Case (a): If **v** is perpendicular to **B**, the particle describes a circle in the region of the magnetic field because $\mathbf{F} \perp \mathbf{v}$.

(i) The speed along the circular path is constant.

(ii) The kinetic energy is constant.

(iii) Velocity and momentum continually change.

(iv) The radius r of the circular path is given by

$$\frac{mv^2}{r} = qvB$$

$$\Rightarrow r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

where m = mass of particle and K = kinetic energy. If the particle is accelerated through a potential difference V, then $K = qV$.

(v) Time period of revolution is $T = \dfrac{2\pi m}{qB}$

(vi) Frequency of revolution is $v = \dfrac{qB}{2\pi m}$ which is independent of both v and r.

Case (b): If **v** is inclined to **B** at an angle θ, the particle moves in a helical path. The radius of helix is $r = \dfrac{mv \sin\theta}{qB}$, time period $T = \dfrac{2\pi m}{qB}$ and pitch of the helix $= v \cos\theta \times T$

Applications

(i) *The particle moving hori-zontally and entering a region of magnetic field B is as shown in Fig. 13.26. The particle describes a semi-circle of radius.*

$$r = \frac{mv}{qB}$$

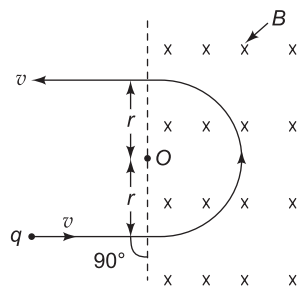


Fig. 13.26

Time spend in the region of magnetic field is

$$t = \frac{T}{2} = \frac{\pi m}{qB}$$

(ii) *If the particle enters the region of magnetic field as shown in Fig. 13.27, then*

$$r = \frac{mv}{qB}$$

and $$t = \frac{2\theta m}{qB}$$

where θ is in radian.

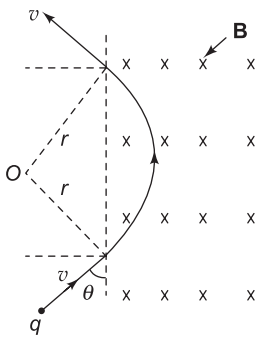


Fig. 13.27

(iii) *In Fig. 13.28, the particle will not be able to hit the wall if $d > r$, i.e.*

$$d > \frac{mv}{qB} \Rightarrow B > \frac{mv}{qd}$$

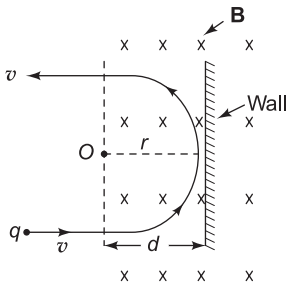


Fig. 13.28

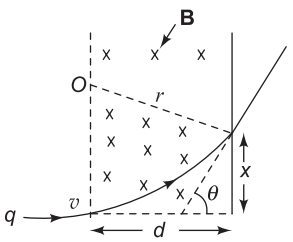


Fig. 13.29

(iv) *If $d < r$, as shown in Fig. 13.29, the deflection θ when the particle leaves the field is given by*

$$\sin\theta = \frac{d}{r} = \frac{qBd}{mv}$$

Linear defection $x = r(1 - \cos\theta)$

EXAMPLE 15 An electron emitted from a hot filament is accelerated through a potential difference of 18 kV and enters a region of a uniform magnetic field of 0.1 T with a certain initial velocity. What is the trajectory of the electron if the magnetic field (a) is transverse to the initial velocity and (b) makes an angle of 30° with the initial velocity? Mass of electron = 9×10^{-31} kg.

SOLUTION

$V = 18 \times 10^3$ V

$$\frac{1}{2}mv^2 = eV \Rightarrow v = \left(\frac{2eV}{m}\right)^{1/2}$$

$$= \left[\frac{2 \times (1.6 \times 10^{-19}) \times (18 \times 10^3)}{9 \times 10^{-31}}\right]^{1/2} = 8 \times 10^7 \text{ ms}^{-1}$$

(a) Since v is $\perp$ to **B**, $\theta = 90°$, the trajectory of the electron is circular having a radius

$$r = \frac{mv}{eB} = \frac{(9 \times 10^{-31}) \times (8 \times 10^7)}{(1.6 \times 10^{-19}) \times 0.1}$$

$$= 4.5 \times 10^{-3} \text{ m} = 4.5 \text{ mm}$$

(b) The trajectory of the electron is helical. The radius of heix is

$$r = \frac{mv_\perp}{eB} = \frac{mv}{eB} \times \sin\theta$$

$$= 4.5 \text{ mm} \times \sin 30° = 2.25 \text{ mm}$$

EXAMPLE 16 A long straight wire lying along the y-axis carries a current of 10 A along the positive y-direction. A proton moving with a velocity of 10^7 ms^{-1} is at a distance 5 cm from the wire at a certain instant. Find the magnitude and direction of the force acting on the proton at that instant if its velocity is directed

(a) along the negative x-direction

(b) along the positive y-direction and

(c) along the positive z-direction

SOLUTION Refer to Fig. 13.30.

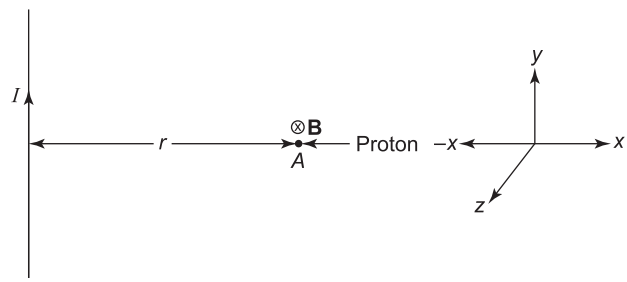


Fig. 13.30

Magnetic field at A is

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.05} = 4 \times 10^{-5} \text{ T}$$

directed inwards along the negative z-direction

(a) $\theta = 90°$. Therefore, force on proton is

$F = qvB \sin\theta$

$= (1.6 \times 10^{-19}) \times 10^7 \times (4 \times 10^{-5}) \times \sin 90°$

$= 6.4 \times 10^{-17}$ N

According to Fleming's L.H. rule, the direction of the force is parallel to the wire and opposite to the direction of current I, i.e. **F** is along the negative y-direction

(b) $\theta = 90°$, $F = qvB = 6.4 \times 10^{-17}$ N. The force is directed towards the wire, i.e. along negative x-direction

(c) $\theta = 180°$. $F = qvB \sin 180° = 0$

EXAMPLE 17 A proton and an α-particle move perpendicular to a uniform magnetic field. The mass of an α-particle is four times that of a proton and its charge is

twice that of a proton. Find the ratio of radii of the circular path followed by them if both

(a) have equal velocities,
(b) have equal kinetic energies and
(c) have equal linear momenta,
(d) are accelerated through the same potential difference.

SOLUTION Given $\frac{m_\alpha}{m_p} = 4$ and $\frac{q_\alpha}{q_p} = 2$

(a) $r = \frac{mv}{qB} \Rightarrow r_p = \frac{m_p v}{q_p B}$ and $r_\alpha = \frac{m_\alpha v}{q_\alpha B}$

$$\therefore \frac{r_p}{r_\alpha} = \frac{m_p}{m_\alpha} \times \frac{q_\alpha}{q_p} = \frac{1}{4} \times 2 = \frac{1}{2}$$

(b) $r = \frac{mv}{qB}$

Kinetic energy $K = \frac{1}{2} mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}}$

$$\therefore r = \frac{m}{qB} \times \sqrt{\frac{2K}{m}} = \frac{1}{qB}\sqrt{2\,mK}$$

$$\therefore \frac{r_p}{r_\alpha} = \frac{q_\alpha}{q_p} \times \sqrt{\frac{m_p}{m_\alpha}} = 2 \times \sqrt{\frac{1}{4}} = 1$$

(c) $r = \frac{mv}{qB} = \frac{p}{qB}$

$$\therefore \frac{r_p}{r_\alpha} = \frac{q_\alpha}{q_p} = 2$$

(d) $K = qV$. Therefore,

$$r = \frac{1}{qB}\sqrt{2mqV} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

$$\therefore \frac{r_p}{r_\alpha} = \sqrt{\frac{m_p}{m_\alpha} \times \frac{q_\alpha}{q_p}} = \sqrt{\frac{1}{4} \times 2} = \frac{1}{\sqrt{2}}$$

6. Force on a Current Carrying Conductor in a Magnetic Field

(i) *Force on a straight current carrying conductor in a magnetic field*

If a straight wire of length L carrying a current I is placed in a uniform magnetic field **B**, the force on it is given by (Fig. 13.31)

$$\mathbf{F} = I(\mathbf{L} \times \mathbf{B})$$

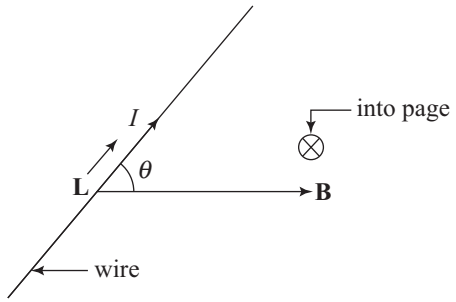


Fig. 13.31

where **L** is a vector whose magnitude is equal to the length of the wire and the direction is the same that of the current. The magnitude of ***F*** is

$$F = I\,L\,B \sin\theta$$

where θ is the angle between vectors **L** and **B**. The direction of **F** is given by the right hand screw rule. In the special case when **L** is perpendiculer to **B**, F is maximum = BIL.

In this case, the direction of **F** can be easily obtained by Fleming's Left Hand rule.

(ii) *Force on a straight conductor placed perpendicular to magnetic field (Fig. 13.32).*

$F = BIL$ upwards if current I is from left to right and downwards if I is from right to left (given by Fleming's Left Hand rule)

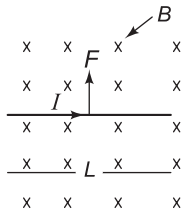


Fig. 13.32

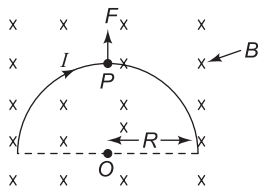


Fig. 13.33

(iii) *Force at point P on a semicircular wire of radius R (Fig. 13.33)*

$F = BI(2R) = 2BIR$ vertically upward for clockwise current and downward for anticlockwise current.

(iv) *Force on a circular wire of radius R (Fig. 13.34)*

Net force $F = F_1 - F_2 = 0$

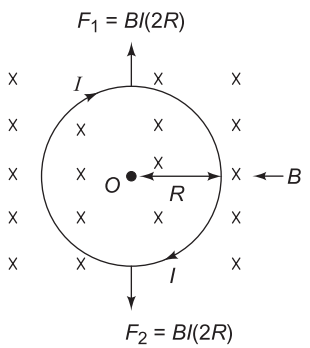


Fig. 13.34

(v) *Force per unit length between two long straight and parallel wires carrying currents I_1 and I_2 and separated by distance r is given by*

$$f = \frac{\mu_0 I_1 I_2}{2\pi r}$$

and is attractive if I_1 and I_2 are in the same direction and repulsive if I_1 and I_2 are in opposite directions. Force on a segment of length l of either wire is $F = f \times l$.

(vi) *Force on a rod carrying a current I_1 placed at a distance x from an infinitely long wire carrying a current I_2 as shown in Fig. 13.35.*

$F = \dfrac{\mu_0 I_1 I_2}{2\pi r} \log_e\left(1 + \dfrac{L}{x}\right)$ vertically upwards.

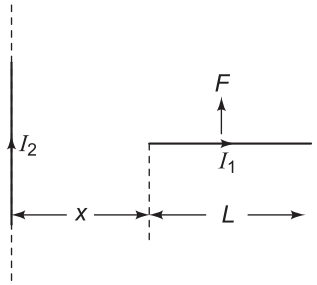


Fig. 13.35

(vii) *Force on a rectangular coil carrying a current I_1 placed at a distance x from an infinitely long wire carrying a current I_2 as shown in Fig. 13.36.*

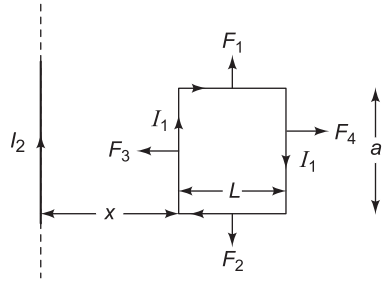


Fig. 13.36

Force F_1 and F_2 being equal and opposite cancel and F_3 and F_4 are given by expression above. Net force on coil is

$$F = F_3 - F_4 = \frac{\mu_0 I_1 I_2 a}{2\pi x} - \frac{\mu_0 I_1 I_2 a}{2\pi(x + L)}$$

$$\Rightarrow \qquad F = \frac{\mu_0 I_1 I_2 a L}{2\pi x(x + L)}$$

directed towards the wire (attractive).

EXAMPLE 18 A uniform wire of length l is shaped into an equilateral triangle PQR which carries a current I as shown in Fig. 13.37. A uniform magnetic field **B** exists parallel to side QR. The magnitude of the force on wire PR is

(a) $\dfrac{\sqrt{3} B I L}{2}$ (b) $\dfrac{3 B I L}{2\sqrt{2}}$

(c) $\dfrac{B I L}{2\sqrt{3}}$ (d) $\dfrac{B I L}{6}$

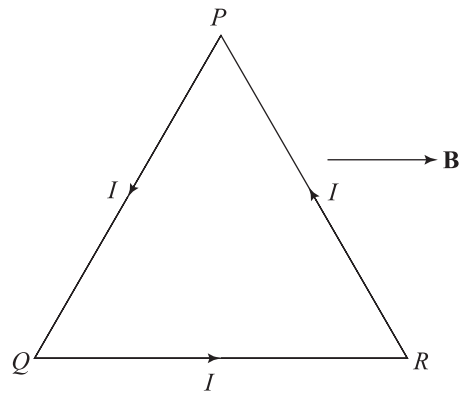


Fig. 13.37

SOLUTION $\mathbf{F} = I(\mathbf{l} \times \mathbf{B}) = I\, l\, B \sin\theta$.

Here $l = \dfrac{L}{3}$ and $\theta = 120°$. Therefore,

$$\therefore \qquad F = I \times \frac{L}{3} \times B \times \sin 120°$$

$$= \frac{B I L}{2\sqrt{3}}$$

EXAMPLE 19 In Example 18 above, find the magnitude of the force on (a) wire PQ and (b) wire QR.

SOLUTION (a) For wire PQ, $\theta = 120°$. Therefore,

$$F = I \times \frac{L}{3} \times B \times \sin 60° = \frac{B I L}{2\sqrt{3}}$$

(b) For wire QR, $\theta = 0°$. Therefore, $F = 0$

EXAMPLE 20 A long wire carries a current of 10 A. A particle of charge q = 2.0 μC travels at a velocity of 5×10^5 ms^{-1} at a perpendicular distance 20 cm from the wire in a direction opposite to the direction of the current in the wire. Find the magnitude and direction of the force experienced by the particle.

SOLUTION Refer to Fig. 13.38. Given I = 10 A, r = 20 cm = 0.2 m, $v = 5 \times 10^5$ ms^{-1} and q = 2.0 μC = 2.0×10^{-6} C.

The magnetic field due to current I at the site of the charged particle is

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7}) \times 10}{2\pi \times 0.2}$$

$$= 1.0 \times 10^{-5} \text{ T}$$

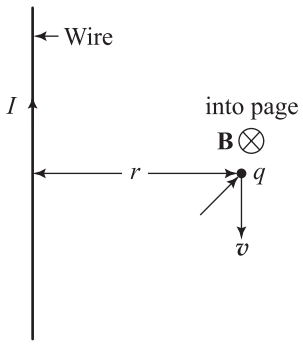


Fig. 13.38

According to Right Hand Thumb rule, the direction of the field is into the page. Hence the charged particle is moving perpendicular to the field, i.e. $\theta = 90°$. The force experienced by the particle is

$$F = q\,v\,B\sin\theta$$
$$= (2.0\times10^{-6})\times(5\times10^{5})\times(1.0\times10^{-5})\times\sin 90°$$
$$= 1.0\times10^{-5}\text{ N}$$

According to Fleming's Left Hand rule, the direction of the force is to the right, i.e. perpendicular to the wire and away from it.

EXAMPLE 21 Two thick and straight conductors *AB* and *CD* are placed horizontally and parallel to each other at a separation of 20 cm. They are connected to battery as shown in Fig. 13.39(a). A straight wire *PQ* of mass 150 g can slide on *AB* and *CD*. If the current I = 2 A, g = 10 ms^{-2} and a magnetic field **B** = 1.5 T is applied as shown in the figure, the minimum coefficient of friction between the wire and the conductors so that the wire is prevented from sliding is

(a) 0.1 (b) 0.2

(c) 0.3 (d) 0.4

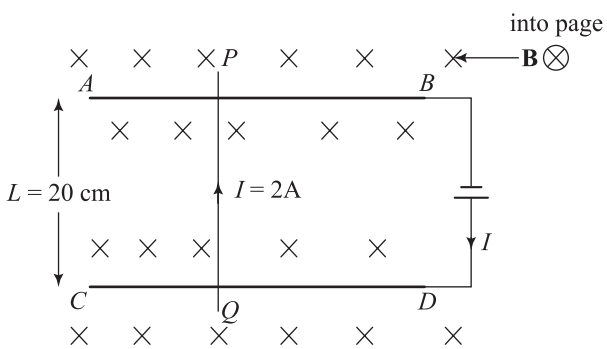


Fig. 13.39(a)

SOLUTION The magnitude of force on wire *PQ* due to the magnetic field is

$$F = B\,I\,L$$

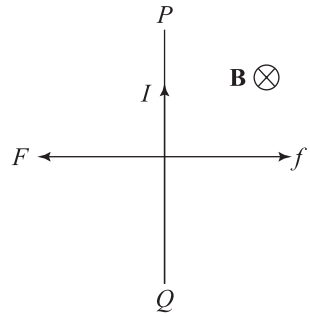


Fig. 13.39(b)

From Fleming's Left Hand rule, the direction of this force is to the left [Fig. 13.39(b)]. If the wire is just prevented from sliding on the conductor, the force of friction (f) which acts to the right must be equal to F. If μ_{min} is the minimum coefficient of friction, then $f = \mu_{min}\,mg$. Thus,

$$\mu_{min}\,mg = B\,I\,L$$

$$\Rightarrow \qquad \mu_{min} = \frac{B\,I\,L}{mg} = \frac{1.5\times2\times0.2}{0.150\times10} = 0.4$$

EXAMPLE 22 In Example 21 above, if the coefficient of friction were one-fourth of μ_{min}, then wire *PQ* will

(a) stay at rest

(b) move to the left with an acceleration of 3.0 ms^{-2}

(c) move to the right will an acceleration of 1.5 ms^{-2}

(d) execute simple harmonic motion.

SOLUTION Given $\mu = \frac{1}{4}\mu_{min} = \frac{0.4}{4} = 0.1.$

Referring to Fig. 13.39(b), the net force on wire *PQ* will be to the left.

$$F_{net} = B\,I\,L - \mu\,mg$$
$$= 1.5\times2\times0.2 - 0.1\times0.15\times10$$
$$= 0.6 - 0.15 = 0.45\text{ N}$$

$\therefore$ Acceleration $a = \frac{F_{net}}{m} = \frac{0.45}{0.15} = 3.0\text{ ms}^{-2}$. So, the correct choice is (b).

EXAMPLE 23 A particle of charge q is revolving in a circle of radius r with a constant speed v. The ratio of the magnitudes of magnetic moment and angular momentum of the particle is

(a) $\frac{q}{\sqrt{2}\,m}$ (b) $\frac{q}{m}$

(c) $\frac{q}{2m}$ (d) $\frac{\sqrt{2}\,q}{m}$

SOLUTION Time period $T = \dfrac{2\pi r}{v}$. The particle passes through a given point on the circle after one complete revolution. Hence the current round the circle is

$$I = \frac{q}{T} = \frac{qv}{2\pi r}$$

Magnetic moment M = current × area enclosed by the circular current

or $$M = I \times \pi r^2 = \frac{qv}{2\pi r} \times \pi r^2 = \frac{qvr}{2}$$

The direction of **M** is perpendicular to the plane of the circular loop.

Angular momentum $L = mvr$

The direcstion of **L** is the same as that of **M** if q is positive.

$$\therefore \quad \frac{M}{L} = \frac{qvr}{2} \times \frac{1}{mvr} = \frac{q}{2m}$$

So the correct choice is (c).

EXAMPLE 24 The battery of a car is connected to the motor by 50 cm long wires which are 1.0 cm apart. If the current in the wires is 200 A, find the force between the wires. Is the force attractive or repulsive.

SOLUTION Force per unit length is

$$f = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{(4\pi \times 10^{-7}) \times 200 \times 200}{2\pi \times (1.0 \times 10^{-2})} = 0.8 \text{ Nm}^{-1}$$

$\therefore \quad F = f \times l = 0.8 \times 0.5 = 0.4$ N

Since the currents in the wires are in opposite direction, the force is repulsive.

EXAMPLE 25 A small rectangular loop $ABCD$ of sides 5 cm and 3 cm carries a current of 5 A. It is placed with its longer side parallel to a long straight conductor PQ of length 5 m at a distance of 2 cm from it as shown in Fig. 13.40. If the current in PQ is 20 A, find the net force on the loop. Is the loop attracted towards PQ or repelled away from it?

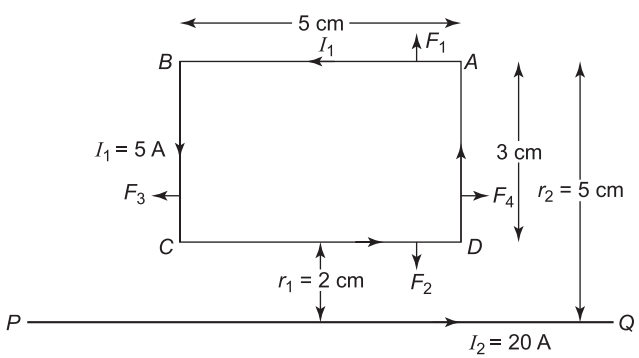


Fig. 13.40

SOLUTION Force exerted by PQ on AB is

$$F_1 = \frac{\mu_0 I_1 I_2}{2\pi r_1} \times AB$$

$$= \frac{(4\pi \times 10^{-7}) \times 5 \times 20 \times 5 \times 10^{-2}}{2\pi \times 5 \times 10^{-2}}$$

$= 2 \times 10^{-5}$ N (repulsive since I_1 and I_2 are in opposite directions)

Force exerted by PQ on CD is

$$F_2 = \frac{\mu_0 I_1 I_2}{2\pi r_2} \times CD$$

$$= \frac{(4\pi \times 10^{-7}) \times 5 \times 20 \times 5 \times 10^{-2}}{2\pi \times 2 \times 10^{-2}}$$

$= 5 \times 10^{-5}$ N (attractive since I_1 and I_2 are in the same direction)

From Fleming's L.H. rule, the magnetic field due to current in PQ is directed outwards (towards the reader) and perpendicular to the plane of the coil. Therefore, forces F_3 and F_4 on BC and AD are equal and opposite and hence cancel each other. Therefore, the net force on coil $ABCD$ is

$$F = F_2 - F_1 = 5 \times 10^{-5} - 2 \times 10^{-5} = 3 \times 10^{-5} \text{ N}$$

(attractive). Hence coil is attracted towards PQ.

EXAMPLE 26 A particle of charge q and mass m moving in region I with a velocity v enters normally a region II of width d where a uniform magnetic field B (directed inwards) exists as shown in Fig. 13.41. There is no magnetic field in regions I and III.

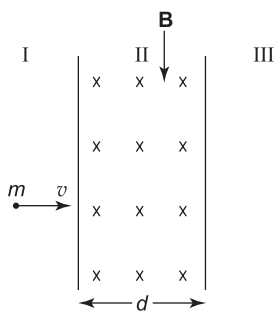


Fig. 13.41

(a) What is the maximum speed (v_{max}) of the particle so that it returns back in region I?

(b) What will happen if $v = \sqrt{2}\, v_{max}$?

SOLUTION

(a) Refer to Fig. 13.42(a). The particle describes a circular path of radius $r = mv/qB$ in region II. It will return to region *I* if it describes a semicircle in region II. This happens if

$$r < d \Rightarrow \frac{mv}{qB} < d \text{ or } v < \frac{qBd}{m}$$

$$\therefore v_{max} = \frac{qBd}{m}$$

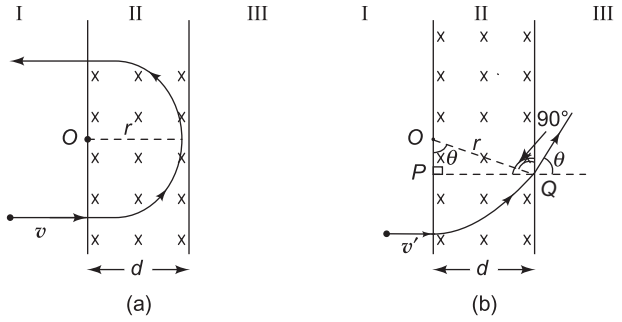


Fig. 13.42

(b) Refer to Fig. 13.32 (b). If $v > v_{max}$, the particle is cross over to region III after describing a circular trajectory in region II with O as the centre. In region III, the particle is moved along the tangent at Q. The particle will suffer a deviation θ.

In triangle OPQ

$$\sin\theta = \frac{PQ}{OQ} = \frac{d}{r}$$

$$\Rightarrow \qquad \sin\theta = \frac{qBd}{mv} = \frac{v_{max}}{v}$$

If $v = \sqrt{2}\, v_{max}$, then $\sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

EXAMPLE 27 A straight horizontal conducting rod of length 50 cm and mass 60 g is suspended by two vertical wires at its ends. A current of 5 A set up in the rod.

(a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero?

(b) What will be the tension in the wires if the direction of the current is reversed, keeping the magnetic field the same?

Ignore the mass of the wires and take $g = 10\ m/s^{-2}$.

SOLUTION Refer to Fig. 13.43.

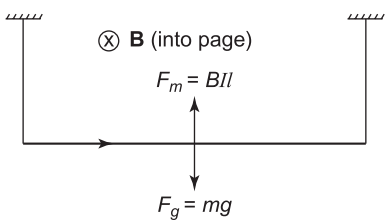


Fig. 13.43

(a) Tension in the wires will be zero if

$$F_m = F_g$$

$$\Rightarrow \qquad BIl = mg$$

$$\Rightarrow \qquad B = \frac{mg}{Il} = \frac{(60\times10^{-3})\times10}{5\times(50\times10^{-2})} = 0.24\ T$$

(b) If current I is reversed, force F_m acts downwards. Hence

Tension $T = BIl + mg$

$= 0.24 \times 5 \times 0.5 + 60 \times 10^{-3} \times 10 = 1.2$ N

7. Torque on a Current Carrying Coil in a Magnetic Field

The torque on a coil of N turns, area A carrying a current I in a magnetic field B is given by

$$\vec{\tau} = \mathbf{M} \times \mathbf{B}$$

Magnitude of torque is $\tau = MB \sin\theta = NIAB \sin\theta$ where $M = NIA$ is the magnetic moment and θ is the angle between the normal to the plane of coil and magnetic field.

The magnitude of torque on a coil in radial magnetic field in moving coil galvanometer is

$$\tau = k\alpha$$

where k is the restoring couple per unit twist and α is the deflection of the coil. For radial magnetic field, $\alpha = 90°$. Then

$$NIAB = k\alpha \Rightarrow I = \frac{k\alpha}{NAB} \quad \text{or } I \propto \alpha$$

Current sensitivity of the galvanometer is

$$C_s = \frac{\alpha}{I} = \frac{NAB}{k}$$

8. Torque on a Bar Magnet in a Magnetic Field

The magnetic dipole moment of a bar magnet of pole strength q and length $(2a)$ is defined as

$$\mathbf{M} = q(2\mathbf{a})$$

It is a vector pointing from the south to the north pole of a magnet.

Force on north pole N of magnet = $q\mathbf{B}$ (in the direction of **B**)

Force on south pole S of the magnet = $-q\mathbf{B}$ (opposite to **B**)

Thus the magnetic field exerts two equal, parallel and opposite forces on the magnet. The two forces, therefore, constitute a coupe which tends to rotate the magnet in the clockwise direction. The arm of the couple is 2 **a**. The torque is given by

$$\tau = \text{arm of the couple} \times \text{force}$$

$$= 2\mathbf{a} \times q\mathbf{B} = q(2\mathbf{a}) \times \mathbf{B}$$

or $$\tau = \mathbf{M} \times \mathbf{B}$$

where $\mathbf{M} = q(2\mathbf{a})$ is called the magnetic moment of the bar magnet. The direction of τ is perpendicular to both **M** and **B**. If **M** and **B** are both in the plane of the paper then the torque τ will be perpendicular to the plane of the paper

and directed into it away from the reader. The magnitude of the torque is

$$\tau = MB \sin \theta$$

where θ is the angle between **M** and **B**.
The SI unit of **M** is Nm T^{-1} or JT^{-1} (joule per tesla).

9. Potential Energy of a Magnetic Dipole

The magnetic potential energy of a magnetic dipole in any orientation θ with an external uniform magnetic field **B** is defined as the work that an external agent must do to turn the dipole from its zero energy position ($\theta = 90°$) to the given position. θ.

$$U = -MB \cos \theta$$

In vector notation,

$$\mathbf{U} = -(\mathbf{M} \cdot \mathbf{B})$$

For stable equilibrium U is minimum. Hence $\theta = 0$ and $\tau = 0$. For unstable equilibrium, U is maximum i.e. $\theta = 180°$. Hence $\tau = 0$

10. Some Useful Tips

1. Magnetic dipole moment of a bar magnet is $M = m \times l$, where m is pole strength and l is the length of the magnet. The value of M depends on the volume of the magnet.
   (a) If a magnet is cut into two equal parts by cutting it by a plane along its length, its volume is halved, Hence the magnetic dipole moment of a piece is halved = $M/2$. The pole strength $m = \frac{M}{l}$ is also halved as length l remains the same.

   (b) If a magnet is cut into two equal parts by cutting it by a plane transverse to its length, the volume and length are both halved. Hence the magnetic moment becomes $M/2$ but pole strength m remains the same.

   (c) If a wire of magnetic dipole moment M and length l is bent as shown in Fig. 13.44, the distance between the pole becomes $\frac{1}{\sqrt{2}}$ and magnetic moment becomes

   $$M' = m \times \frac{l}{\sqrt{2}} = \frac{M}{\sqrt{2}}$$

   If the wire is bent as shown in Fig. 13.45, the magnetic moment becomes

   $$M' = m \times \frac{l}{\sqrt{2}} = \frac{M}{\sqrt{2}}$$

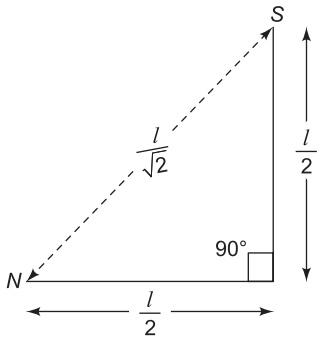


Fig. 13.44

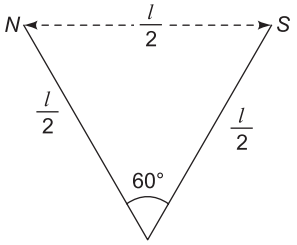


Fig. 13.45

Note

Magnetic dipole moment M is a vector quantity directed from south pole to north pole.

2. A bar magnet placed in a uniform magnetic field experiences no net force but experiences a torque $\vec{\tau} = \boldsymbol{M} \times \boldsymbol{B}$. The magnitude of $\vec{\tau}$ is $\tau = MB \sin \theta$ where θ is the angle between $\boldsymbol{M}$ and $\boldsymbol{B}$.
   (a) when $\theta = 90°$, $\tau_{max} = MB$
   (b) when $\theta = 0°$, $\tau_{min} = 0$ (stable equilibrium)
   (c) when $\theta = 180°$, $\tau_{min} = 0$ (unstable equilibrium)
   Potential energy is $U = -\boldsymbol{M}.\boldsymbol{B} = -MB \cos \theta$. When $\theta = 0°$, P.E is minimum $U_{min} = -\boldsymbol{MB}$. U is max $= U_{max} = MB$ when $\theta = 180°$
   (d) Work done in rotating the magnet from θ_1 to θ_2 is
   $W = MB (\cos \theta_1 - \cos \theta_2)$
   (e) In a non-uniform magnetic field, a bar magnet experiences a force as well as a torque.

3. The time period of a bar magnet oscillating in a uniform magnetic field is $T = 2\pi\sqrt{\frac{I}{MB}}$, where I is the moment of inertia of the bar magnet = $\frac{ml^2}{12}$, m = mass of magnet and l = length of magnet.
   (a) If a bar magnet is cut into two equal parts by cutting along its length, then each part has $M' = M/2$ and $I' = I/2$. Hence $T' = T$.
   (b) If a bar magnet is cut into two equal parts by cutting perpendicular to its length, then each part has $M' = M/2$ and $I' = I/8$. Hence $T' = T/2$.
   (c) If two bar magnets of magnetic moments M_1 and M_2 are placed one on top of the other as shown in Fig. 13.46, then time period is given by (since $I = I_1 + I_2$ and $M = M_1 + M_2$)

$$T_1 = 2\pi\sqrt{\frac{(I_1 + I_2)}{(M_1 - M_2)B}}$$

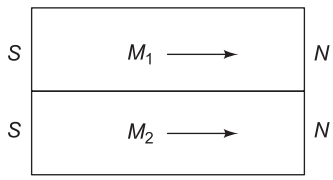


Fig. 13.46

If the magnets are placed as shown in Fig. 13.47, then $I = I_1 + I_2$ but $M = M_1 - M_2$ and

$$T_2 = 2\pi\sqrt{\frac{(I_1 + I_2)}{(M_1 - M_2)B}}$$

T_1 and T_2 are related as $\frac{M_1}{M_2} = \frac{(T_2^2 + T_1^2)}{(T_2^2 - T_1^2)}$

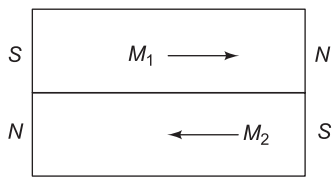


Fig. 13.47

4. Magnetic field due to a bar magnet
   (a) At a point P on axial line (Fig. 13.48)

$$B_a = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2 - l^2)^2} \text{ parallel to } \vec{M} = m \times 2\vec{l}\,.$$

For a very short magnet $(l << r)$, $B_a = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$

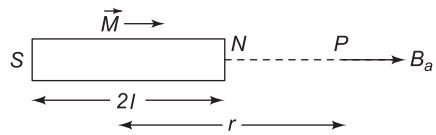


Fig. 13.48

   (b) At a point Q on the equatorial line (Fig. 13.49)

$$B_e = \frac{\mu_0}{4\pi}\frac{M}{(r^2 + l^2)^{3/2}} \text{ antiparallel to } \mathbf{M}$$

For $l << r$,

$$B_e = \frac{\mu_0 M}{4\pi r^3}$$

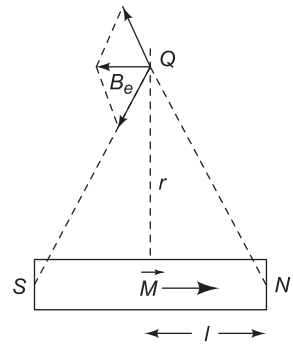


Fig. 13.49

EXAMPLE 28 A closely wound solenoid of 1000 turns and area of cross-section 5 cm^2 carries a current of 3 A. It is suspended through its centre (a) what is the magnetic moment? (b) What is the force and torque acting on the solenoid if a uniform magnetic field of 8×10^{-2} T is set up at an angle of 30° with the axis of the solenoid?

SOLUTION

(a) Magnetic moment $M = NIA = 1000 \times 3 \times (5 \times 10^{-4})$ $= 1.5$ JT^{-1} or Am^2

(b) Since the magnetic field is uniform, the force acting on the solenoid is zero

$$\text{Torque } \tau = MB \sin\theta = 1.5 \times (8 \times 10^{-2}) \times \sin 30°$$
$$= 6 \times 10^{-2} \text{ J}$$

EXAMPLE 29 In a hydrogen atom, the electron moves in a circular orbit of radius 0.5 Å making 10^{16} revolutions per second. Calculate the magnetic moment associated with the orbital motion of electron.

SOLUTION $M = \pi e v r^2 = 3.14 \times (1.6 \times 10^{-19}) \times 10^{16} \times (0.5 \times 10^{-10})^2 = 1.26 \times 10^{-23}$ Am^2

EXAMPLE 30 A bar magnet is suspended at a place where it is acted upon by two magnetic fields which are inclined to each other at an angle of 75°. One of the fields has a magnitude $\sqrt{2} \times 10^{-2}$ T. The magnet attains stable equilibrium at an angle of 30° with this field. Find the magnitude of the other field.

SOLUTION Magnetic field $\mathbf{B}_1$ exerts anticlockwise torque τ_1 to orient **M** along itself and magnetic field $\mathbf{B}_2$ exerts a stockwise torque τ_2 to orient **M** along itself (Fig. 13.50). For equilibrium,

$$\tau_1 = \tau_2$$

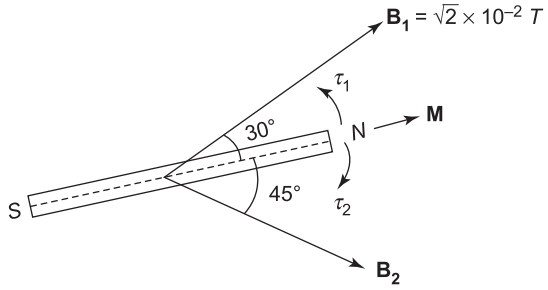


Fig. 13.50

$$MB_1 \sin\theta_1 = MB_2 \sin\theta_2$$

$$\Rightarrow \quad B_2 = \frac{B_1 \sin\theta_1}{\sin\theta_2} = \frac{\sqrt{2}\times 10^{-2}\times \sin 30°}{\sin 45°} = 10^{-2}\ \text{T}$$

EXAMPLE 31 A uniform wire is bent into the shape of an equilateral triangle of side a. It is suspended from a vertex at a place where a uniform magnetic field B exists parallel to its plane. Find the magnitude of the torque acting on the coil when a current I is passed through it.

SOLUTION Area of the coil is ($AB = a$, $BD = a/2$)

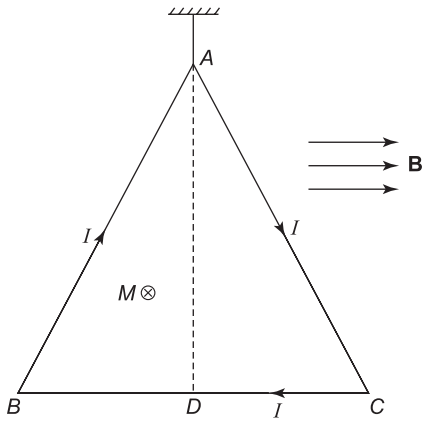


Fig. 13.51

$$A = 2 \times \text{area of triangle } ABD$$

$$= 2 \times \left(\frac{1}{2} \times AD \times BD\right)$$

$$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} a \times \frac{a}{2}$$

$$= \frac{\sqrt{3}}{4} a^2$$

Magnetic moment of the loop is

$$M = IA = I \times \frac{\sqrt{3}}{4} a^2$$

Since the current is clockwise, the direction of vector **M** is perpendicular to the plane of the coil directed inwards as shown in Fig. 13.51. Hence $\theta = 90°$. The magnitude of the torque acting on the coil is

$$\tau = MB \sin\theta = \frac{\sqrt{3}}{4} Ia^2B \sin 90° = \frac{\sqrt{3}}{4} Ia^2B$$

EXAMPLE 32 A wire loop $ABCD$ carrying a current I_2 is placed on a frictionless horizontal table as shown in Fig. 13.52. A long straight wire PQ carrying a current I_1 is placed at a distance a from side $AB = l$. Find the work done by the magnetic field in shifting the wire from position PQ to position $P'Q'$.

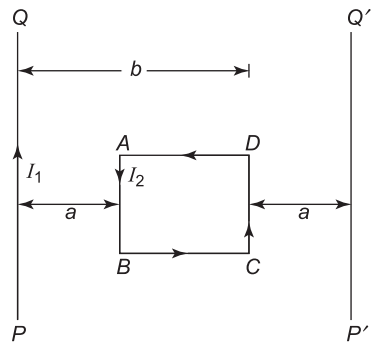


Fig. 13.52

SOLUTION

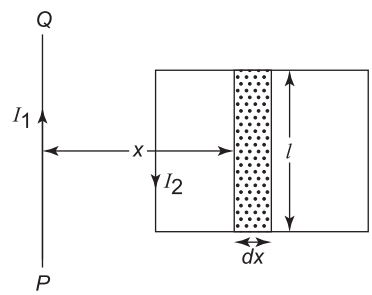


Fig. 13.53

Divide the loop into a large number of elements each of a very small width dx. Consider one such element at a distance x from PQ [see Fig. 13.53]. The magnetic field at any point on the element is

$$B = \frac{\mu_0 I_1}{2\pi x} \text{ directed inwards}$$

Magnetic moment of the element is

$$dM = I_2 \times \text{area of element}$$

$$= I_2 \times l dx$$

Since the current in the coil is anticlockwise, $d\mathbf{M}$ is directed outwards. Hence angle between vectors **B** and $d\mathbf{M}$ is $\theta = 180°$.

Potential energy when the wire is a distance x from the elements is

$$dU = -d\boldsymbol{M}\cdot\mathbf{B} = -dM \times B \cos\theta$$

$$= -I_2\, ldx \times \frac{\mu_0 I_1}{2\pi x} \cos 180°$$

$$= \frac{\mu_0 I_1 I_2 l}{2\pi} \frac{dx}{x}$$

∴ Potential energy of the system when the wire is at position PQ is

$$U_{PQ} = \frac{\mu_0 I_1 I_2 l}{2\pi} \int_a^b \frac{dx}{x} = \frac{\mu_0 I_1 I_2 l}{2\pi} \ln\left(\frac{b}{a}\right)$$

By symmetry, the potential energy of the system when the wire is shifted to position $P'Q'$ is

$$U_{P'Q'} = -U_{PQ}$$

∴ Work done in shifting the wire from position PQ to $P'Q'$ is

$$W = -(U_{P'Q'} - U_{PQ})$$

$$= 2U_{PQ} = \frac{\mu_0 I_1 I_2 l}{\pi} \ln\left(\frac{b}{a}\right)$$

11. Earth's Magnetic Field

The magnetic field of the earth is described in terms of the following three parameters.

(1) Magnetic Declination

The angle between the magnetic meridian at a place and the geographic meridian at that place is called magnetic declination. The magnetic meridian at a place is a vertical plane containing the magnetic axis of a freely suspended small magnet when it has settled in the earth's field. The geographic meridian at a place is a plane containing the place and the earth's axis of rotation.

The magnetic declination is different at different places on the surface of the earth.

(2) Angle of Dip

The angle between the horizontal component and the total magnetic field of the earth is called the angle of dip. This angle is also different at different places on the surface of the earth. Naturally, on the magnetic equator the angle of dip is zero, because the dip needle rests horizontally at the magnetic equator. The needle rests vertically at the two magnetic poles, i.e. the angle of dip is 90° at the magnetic poles. At other places, its value lies between 0° and 90°.

(3) Horizontal Component

The total magnetic field strength B at a place has a horizontal component B_H given by

$$B_H = B \cos\theta$$

where θ is the dip angle at that place. At the magnetic poles, $B_H = B \cos 90° = 0$ and at the magnetic equator $B_H = B \cos 0° = B$. The value of B_H thus differs from place to place on the surface of the earth.

12. Neutral Points

A neutral point is a point in space where the field due to the magnet is completely neutralized by the horizontal component of the earth's magnetic field. If $\mathbf{B}_m$ is the magnetic field due to the magnet and $\mathbf{B}_H$ the horizontal component of the earth's magnetic field, then at the neutral point, $\mathbf{B}_m$ and $\mathbf{B}_H$ are equal in magnitude but opposite in direction.

The magnetic field due to a short dipole at a distance r on the equatorial line is given by

$$B_m = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$$

where m is the magnetic moment of the magnet. On the axial line,

$$B_m = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$$

13. Relation between B and H

Magnetic induction field **B** and magnetic field intensity **H** are related as

$$\mathbf{B} = \mu \mathbf{H}$$

The constant μ is called the *permeability* of the material. If the material is removed leaving a vacuum inside the solenoid, the magnetic field reduces to B_0 given by

$$B_0 = \mu_0 H$$

where μ_0 is the permeability of vacuum.

The ratio μ/μ_0 is called the relative permeability of the material:

$$\mu_r = \frac{\mu}{\mu_0}$$

14. Relation between M and H

Intensity of magnetisation **I** and magnetising field intensity **H** are related as

$$\mathbf{I} = \chi \mathbf{H}$$

where χ is the susceptibility of the material.

Also $$\mu_r = 1 + \chi$$

15. Three Kinds of Magnetic Materials

Different magnetic materials have different χ values which may vary over a very wide range. Depending on the value and sign of χ, magnetic materials are divided into the following three categories.

(1) Diamagnetic Materials

Materials for which χ is small and negative are called *diamagnetic.* In such materials, the individual magnetic

moments of the various atoms tend to cancel out completely. So, in the normal state, the atoms of such substances (in the absence of external field) have no magnetic moment at all. They have a small and negative susceptibility χ. For bismuth $\chi = -0.00015$ and $\mu_r (= 1 + \chi)$ is slightly less than unity. Bismuth, copper, lead, water and sodium chloride are all diamagnetic.

When a sample of a diamagnetic material is placed in a magnetic field **B**, a net magnetic moment is induced in it which is proportional to **B** but opposite in direction. If **B** is non–uniform, the sample tends to move from the region where the field is strong to the region where it is weak.

(2) Paramagnetic Materials

Paramagnetic materials are those whose atoms, in the normal state, already have an intrinsic non-zero magnetic moment, even in the absence of an external magnetic field. Examples are aluminium, sodium, copper chloride and liquid oxygen. Paramagnetics have a small but positive value of χ . The susceptibility χ satisfies the relation

$$\chi = \frac{C}{T}$$

where C is a constant called the *Curie constant* and the above relation is called *Curie's law.*

(3) Ferromagnetic Materials

Materials having a very high (of the other of 1000 or more) and positive susceptibility are called ferromagnetic. Examples are iron, nickel, cobalt and numerous alloys. In such materials there are strong interactions between the individual magnetic moments of the various atoms. Above a certain temperature called the *Curie point,* ferromagnetics become paramagnetics.

1 SECTION Multiple Choice Questions with One Correct Choice

Level A

1. A wire of length L carrying a current I is bent into a circle. The magnitude of the magnetic field at the centre of the circle is

   (a) $\frac{\pi \mu_0 I}{L}$ (b) $\frac{\mu_0 I}{2L}$

   (c) $\frac{2\pi \mu_0 I}{L}$ (d) $\frac{\mu_0 I}{2\pi L}$

2. A part of a long wire carrying a current I is bent into a circle of radius r as shown in Fig. 13.54. The net magnetic field at the centre O of the circular loop is

   (a) $\frac{\mu_0 I}{4r}$ (b) $\frac{\mu_0 I}{2r}$

   (c) $\frac{\mu_0 I}{2\pi r}(\pi + 1)$ (d) $\frac{\mu_0 I}{2\pi r}(\pi - 1)$

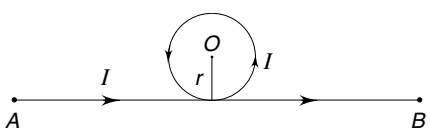


Fig. 13.54

3. A wire is bent into a circular loop of radius R and carries a current I. The magnetic field at the centre of the loop is B. The same wire is now bent into a double loop. If both loops carry the same current I in the same direction, the magnetic field at the centre of the double loop will be

   (a) zero (b) 2 B

   (c) 4 B (d) 8 B

4. In Q. 3, if the currents I in the two loops are in opposite directions, the magnetic field at the centre of the double loop will be

   (a) zero (b) B

   (c) $\frac{B}{4}$ (d) 4 B

5. The magnetic field at the centre of a circular coil of radius r and carrying a current I is B. What is the magnetic field at a distance $x = \sqrt{3}\ r$ from the centre, on the axis of the coil?

   (a) B (b) 2 B

   (c) 4 B (d) 8 B

6. A coil of 50 turns and 10 cm diameter has a resistance of 10 Ω. What must be the potential difference across the coil so as to nullify the earth's magnetic field H = 0.314 G at the centre of the coil?

   (a) 0.5 V (b) 1.0 V

   (c) 1.5 V (d) 2.0 V

7. The magnetic field at the point of intersection of diagonals of a square loop of side L carrying a current I is

   (a) $\frac{\mu_0 I}{\pi L}$ (b) $\frac{2\mu_0 I}{\pi L}$

   (c) $\frac{\sqrt{2}\mu_0 I}{\pi L}$ (d) $\frac{2\sqrt{2}\mu_0 I}{\pi L}$

8. Two identical coils carry equal currents and have a common centre, but their planes are at right angles to each other. What is the magnitude of the resultant magnetic field at the centre if the field due to one coil alone is B?

   (a) zero (b) $B/\sqrt{2}$

   (c) $\sqrt{2}\ B$ (d) $2\ B$

9. The direction of the force experienced by a charged particle moving with a velocity **v** in a uniform magnetic field **B** is

   (a) parallel to **v** and perpendicular to **B**
   (b) parallel to **B** and perpendicular to **v**
   (c) parallel to both **v** and **B**
   (d) perpendicular to both **v** and **B**.

10. When a charged particle moves perpendicular to a uniform magnetic field, its

    (a) energy and momentum both change
    (b) energy changes but momentum remains unchanged
    (c) momentum changes but energy remains unchanged
    (d) energy and momentum both do not change.

11. A particle is projected into a uniform magnetic field acting perpendicular to the plane of the paper. The field points into the paper, indicated by × which represents the tail of the field vector. The trajectory shown could be that of a (see Fig. 13.55)

    (a) neutron (b) proton

    (c) alpha particle (d) electron

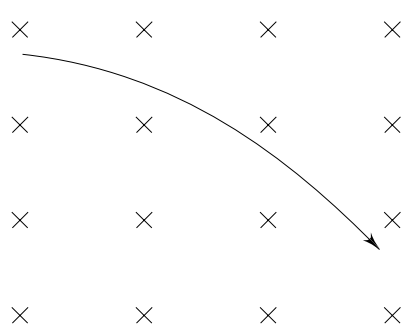


Fig. 13.55

12. A proton with kinetic energy K describes a circle of radius r in a uniform magnetic. An α–particle with kinetic energy K moving in the same magnetic field will describe a circle of radius

    (a) $\frac{r}{2}$ (b) r

    (c) $2\ r$ (d) $4\ r$

13. An α–particle moving with a velocity v in a uniform magnetic field is moving in a circular path at frequency ν called the cyclotron frequency. The cyclotron frequency of a proton moving with a speed $2v$ in the same magnetic field will be

    (a) $\frac{\nu}{4}$ (b) $\frac{\nu}{2}$

    (c) ν (d) $2\ \nu$

14. In the region around a charge at rest, there is

    (a) electric field only
    (b) magnetic field only
    (c) neither electric nor magnetic field
    (d) electric as well as magnetic field.

15. In the region around a moving charge, there is

    (a) electric field only
    (b) magnetic field only
    (c) neither electric nor magnetic field
    (d) electric as well as magnetic field.

16. An electron is accelerated to a high speed down the axis of a cathode ray tube by the application of a potential difference of V volts between the cathode and the anode. The particle then passes through a uniform transverse magnetic field in which it experiences a force F. If the potential difference between the anode and the cathode is increased to 2 V, the electron will now experience a force

    (a) $F/\sqrt{2}$ (b) $F/2$

    (c) $\sqrt{2}\ F$ (d) $2\ F$

17. A magnetic needle is kept in a non-uniform magnetic field. It experiences

    (a) a force as well as a torque
    (b) a force but no torque
    (c) a torque but no force
    (d) neither a force nor a torque

18. A conducting circular loop of radius r carries a constant i. It is placed in a uniform magnetic field B such that B is perpendicular to the plane of the loop. The magnetic force acting on the loop is

    (a) $i\ r\ B$ (b) $2\ \pi\ i\ r\ B$

    (c) zero (d) $\pi\ i\ r\ B$

19. Two long parallel wires separated by a distance R have equal current I flowing in each. The magnetic field of

one exerts a force F on the other. The distance R is increased to $2R$ and the current in each wire is reduced from I to $I/2$. What is the force between them now?

(a) $4F$ (b) F

(c) $F/4$ (d) $F/8$

20. A straight horizontal conducting rod of length 0.5 m and mass 50 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires. What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero? Ignore the mass of the wires and take $g = 10\ \text{ms}^{-2}$.

(a) 0.1 T (b) 0.2 T

(c) 0.3 T (d) 0.4 T

21. In the hydrogen atom the electron moves around the proton with a speed of $2.0 \times 10^6\ \text{ms}^{-1}$ in a circular orbit of radius 5.0×10^{-11} m. What is the equivalent dipole moment?

(a) $2 \times 10^{-24}\ \text{Am}^2$ (b) $4 \times 10^{-24}\ \text{Am}^2$

(c) $8 \times 10^{-24}\ \text{Am}^2$ (d) $16 \times 10^{-24}\ \text{Am}^2$

22. In Q. 21, what is the strength of the magnetic field at the site of the proton (i.e. at the centre of the orbit)?

(a) 12.8 T (b) 6.4 T

(c) 3.2 T (d) 1.6 T

23. In a hydrogen atom, an electron of mass m and charge e is in an orbit of radius r making n revolutions per second. If the mass of the hydrogen nucleus is M, the magnetic moment associated with the orbital motion of the electron is

(a) $\dfrac{\pi n e r^2 m}{M}$ (b) $\dfrac{\pi n e r^2 M}{m}$

(c) $\dfrac{\pi n e r^2 m}{(M+m)}$ (d) $\pi n e r^2$

24. A wire of length l metres carrying a current I amperes is bent in the form of a circle. The magnitude of the magnetic moment is

(a) $\dfrac{l I^2}{2\pi}$ (b) $\dfrac{l I^2}{4\pi}$

(c) $\dfrac{l^2 I}{2\pi}$ (d) $\dfrac{l^2 I}{4\pi}$

25. An electric field of magnitude 5000 V m^{-1} and a magnetic field of magnitude of 0.01 T exist at right angles to each other in a region of space. With what speed must electrons be projected at right angles to both the fields so that they experience no net force?

(a) $5 \times 10^5\ \text{ms}^{-1}$ (b) $3 \times 10^6\ \text{ms}^{-1}$

(c) $5 \times 10^7\ \text{ms}^{-1}$ (d) $3 \times 10^8\ \text{ms}^{-1}$

26. Two circular current carrying coils of radii 3 cm and 6 cm are each equivalent to a magnetic dipole having equal magnetic moments. The currents through the coils are in the ratio of

(a) $\sqrt{2}$: 1 (b) 2 : 1

(c) $2\sqrt{2}$: 1 (d) 4 : 1

27. A proton (mass = 1.7×10^{-27} kg) moves with a speed of $5 \times 10^5\ \text{ms}^{-1}$ in a direction perpendicular to a magnetic field of 0.17 T. The acceleration of the proton is

(a) zero (b) $2 \times 10^{12}\ \text{ms}^{-2}$

(c) $4 \times 10^{12}\ \text{ms}^{-2}$ (d) $8 \times 10^{12}\ \text{ms}^{-2}$

28. An electron of charge e moves in a circular orbit of radius r around the nucleus at a frequency ν. The magnetic moment associated with the orbital motion of the electron is

(a) $\pi \nu e r^2$ (b) $\dfrac{\pi \nu r^2}{e}$

(c) $\dfrac{\pi \nu e}{r^2}$ (d) $\dfrac{\pi e r^2}{\nu}$

29. An electron of charge e moves in a circular orbit of radius r around a nucleus. The magnetic field due to orbital motion of the electron at the site of the nucleus is B. The angular velocity ω of the electron is

(a) $\omega = \dfrac{2\mu_0 e B}{4\pi r}$ (b) $\omega = \dfrac{\mu_0 e B}{\pi r}$

(c) $\omega = \dfrac{4\pi r B}{\mu_0 e}$ (d) $\omega = \dfrac{2\pi r B}{\mu_0 e}$

30. The frequency of the charged particle circulating at right angles to a uniform magnetic field does not depend upon the

(a) speed of the particle (b) mass of the particle

(c) charge of the particle (d) magnetic field

31. The vertical component of the earth's magnetic field is zero at the

(a) magnetic poles (b) magnetic equator

(c) geographic poles (d) 45° latitude

32. The angle of dip is 90° at the

(a) magnetic poles (b) magnetic equator

(c) geographic poles (d) 90° latitude

33. A dip needle free to move in a vertical plane perpendicular to the magnetic meridian will remain

(a) vertical
(b) horizontal
(c) at an angle of 60° to the vertical
(d) at an angle of 45° to the horizontal

34. A sensitive magnetic instrument can be shielded very effectively from outside fields by placing it inside a box of
(a) teak wood
(b) plastic material
(c) a metal of low magnetic permeability
(d) a metal of high magnetic permeability

35. When a material is subjected to a small magnetic field H, the intensity of magnetisation is proportional to
(a) $H^{1/2}$ (b) H
(c) H^2 (d) $H^{-1/2}$

36. The magnetic permeability of a paramagnetic substance is
(a) small and positive
(b) small and negative
(c) large and positive
(d) large and negative.

37. Which of the following has the highest magnetic permeability?
(a) Paramagnetic substances
(b) Diamagnetic substances
(c) Ferromagnetic substances
(d) Vacuum

38. For a paramagnetic material, the dependence of the magnetic susceptibility χ on the absolute temperature T is given by (C is a constant)
(a) $\chi = CT$ (b) $\chi = C/T$
(c) $\chi = CT^2$ (d) $\chi = CT^{-2}$

39. When a ferromagnetic substance is heated to a temperature above its Curie temperature, it
(a) behaves like a paramagnetic substance
(b) behaves like a diamagnetic substance
(c) remains ferromagnetic
(d) is permanently magnetised.

40. The relative permeability of iron is of the order of
(a) zero (b) 10^{-4}
(c) 1 (d) 10^3

41. The relative permeability of a substance A is slightly greater than unity while that of a substance B is slightly less than unity. Then
(a) A is ferromagnetic and B paramagnetic
(b) A is diamagnetic and B paramagnetic
(c) A is paramagnetic and B diamagnetic
(d) A and B are both ferromagnetic

42. A closely wound solenoid of 1000 turns and area of cross-section 1.5×10^{-4} m^2, carrying a current of 2A, is suspended through its centre allowing it to turn in a horizontal plane. What is the magnetic moment associated with the solenoid?
(a) 0.3 Am^2 (b) 0.5 Am^2
(c) 0.75 Am^2 (d) 1.5 Am^2

43. A bar magnet of magnetic moment 2.0 JT^{-1} lies aligned with the direction of a uniform magnetic field of 0.25 T. What is the amount of work required to turn the magnet so as to align its magnetic moment normal to the field direction?
(a) 0.125 J (b) 0.25 J
(c) 0.5 J (d) 1.0 J

44. In Q. 43, what is the work done to turn the magnet so as to align its magnetic moment opposite to the field direction?
(a) 0.25 J (b) 0.5 J
(c) 0.75 J (d) 1.0 J

45. A magnetic dipole is acted upon by two magnetic fields which are inclined to each other at an angle of 75°. One of the fields has a magnitude of $\sqrt{2} \times 10^{-2}$ T. The dipole attains stable equilibrium at an angle of 30° with this field. What is the magnitude of the other field?
(a) 0.01 T (b) 0.02 T
(c) 0.03 T (d) 0.04 T

46. At a certain place on earth a magnetic needle is placed along the magnetic meridian at an angle of 60° to the horizontal. If the horizontal component of the earth's field at the place is 0.20×10^{-4} T, what is the magnitude of the total earth's field at that place?
(a) 0.2×10^{-4} T (b) 0.4×10^{-4} T
(c) 0.8×10^{-4} T (d) 1.6×10^{-4} T

47. A short bar magnet of length 4 cm has a magnetic moment of 4 JT^{-1}. What is the magnitude of the magnetic field at a distance of 2 m from the centre of the magnet on its equatorial line?
(a) 10^{-7} T (b) 5×10^{-6} T
(c) 10^{-6} T (d) 5×10^{-5} T

48. In Q. 47, what is the magnitude of the magnetic field at a distance of 2 m from the centre of the magnet on its axial line?
(a) 10^{-4} T (b) 10^{-5} T
(c) 10^{-6} T (d) 10^{-7} T

49. A toroidal solenoid has 3000 turns and a mean radius of 10 cm. It has a soft iron core of relative permeability 2000. What is the magnitude of the magnetic field in the core when a current of 1 A is passed through the solenoid?
(a) 0.012 T (b) 0.12 T
(c) 1.2 T (d) 12 T

50. A sample of paramagnetic salt contains 2×10^{24} atomic dipoles, each of dipole moment 1.5×10^{-23} JT^{-1}. The sample is placed in a magnetic field of 0.6 T and cooled to a temperature of 4 K. The degree of magnetic saturation achieved is equal to 15%. What is the total dipole moment of the sample for a magnetic field of 0.9 T and a temperature of 3 K?
    (a) 3 J T^{-1} (b) 6 J T^{-1}
    (c) 9 J T^{-1} (d) 12 J T^{-1}
51. A bar magnet of pole strength q and magnetic moment m is divided into two equal pieces by cutting it perpendicular to its length. Then
    (a) q is halved and m is doubled
    (b) q and m both are halved
    (c) q is halved but m remains the same
    (d) q remains the same but m is halved
52. A bar magnet of pole strength q and magnetic moment m is divided into two equal pieces by cutting it along its length. Then
    (a) q is halved and m is doubled
    (b) q and m both are halved
    (c) q is halved but m remains the same
    (d) q remains the same but m is halved
53. A bar magnet of magnetic moment m is placed along the magnetic meridian. If the earth's magnetic field is B, the work required to turn the magnet through an angle θ is
    (a) $m B \sin \theta$ (b) $m B \cos \theta$
    (c) $m B (1 - \cos \theta)$ (d) $m B (1 + \cos \theta)$
54. A small piece of a material is repelled by a strong magnet. The material is
    (a) paramagnetic (b) ferromagnetic
    (c) diamagnetic (d) non-magnetic
55. Choose the correct statement. There will be no force experienced, if
    (a) two parallel wires carry currents in the same direction
    (b) two parallel wires carry currents in the opposite direction
    (c) a positive charge is projected between the pole pieces of a bar magnet
    (d) a positive charge is projected along the axis of a solenoid carrying current

Level B

56. Choose the WRONG statement. The sensitivity of a moving coil galvanometer can be increased by
    (a) increasing the number of turns in the coil
    (b) inserting a soft iron cylinder inside the coil
    (c) increasing the strength of the magnetic field
    (d) using a suspension fibre of a higher restoring couple per unit twist.
57. A potential difference of 600 V is applied across the plates of a parallel plate copacitor. The separation between the plates is 3 mm. An electron projected vertically, parallel to the plates, with a velocity of 2×10^6 m s^{-1} moves undeflected between the plates. What is the magnitude of the magnetic field between the capacitor plates?
    (a) 0.1 T (b) 0.2 T
    (c) 0.3 T (d) 0.4 T
58. Two straight infinitely long and thin parallel wires are held 0.1 m apart and carry a current of 5A each in the same direction. The magnitude of the magnetic field at a point distant 0.1 m from both wires is
    (a) 10^{-5} T (b) $\sqrt{2} \times 10^{-5}$ T
    (c) $\sqrt{3} \times 10^{-5}$ T (d) 2×10^{-5} T
59. When a long wire carrying a steady current is bent into a circular coil of one turn, the magnetic field at its centre is B. When the same wire carrying the same current is bent to form a circular coil of n turns of a smaller radius, the magnetic field at the centre will be
    (a) $\frac{B}{n}$ (b) nB
    (c) $\frac{B}{n^2}$ (d) $n^2 B$
60. A pair of stationary and infinitely long bent wires are placed in the x-y plane as shown in Fig. 13.56. The wires carry a current I = 10 A each as shown.

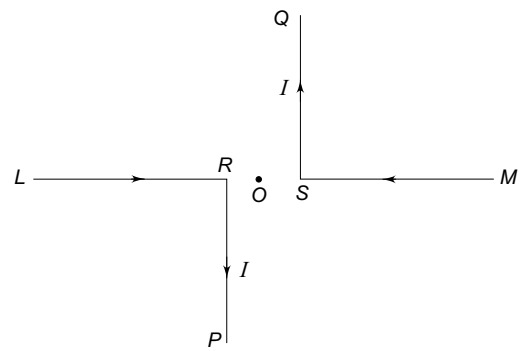


Fig. 13.56

The segments LR and SM are along the x-axis. The segments PR and QS are along the y-axis, such that $OS = OR = 0.02$ m. What is the magnitude and direction of the magnetic field at the origin O?

(a) 100 Wb m^{-2} vertically upward

(b) 10^{-4} Wb m^{-2} vertically downward

(c) 10^{-4} Wb m^{-2} vertically upward

(d) 10^{-2} Wb m^{-2} vertically downward

61. Two long parallel wires P and Q are held perpendicular to the plane of the paper at a separation of 5 m. If P and Q carry currents of 2.5 A and 5 A respectively in the same direction, then the magnetic field at a point mid-way between P and Q is

(a) $\frac{\mu_0}{\pi}$ (b) $\frac{\sqrt{3}\,\mu_0}{\pi}$

(c) $\frac{\mu_0}{2\pi}$ (d) $\frac{3\mu_0}{2\pi}$

62. A proton of mass m and charge $+e$ is moving in a circular orbit a magnetic field with energy 1 MeV. What should be the energy of an α-particle (mass 4 m and charge + 2e) so that it revolves in a circular orbit of the same radius in the same magnetic field?

(a) 1 MeV (b) 2 MeV

(c) 4 MeV (d) 0.5 MeV

63. At a certain place, the horizontal component of earth's magnetic field is $\sqrt{3}$ times the vertical component. The angle of dip at that place is

(a) 30° (b) 45°

(c) 60° (d) 90°

64. A proton, a deuteron and an alpha particle having the same kinetic energy are moving in circular trajectories in a constant magnetic field. If r_p, r_d and r_α denote respectively the radii of trajectories of these particles, then

(a) $r_\alpha = r_p < r_d$ (b) $r_\alpha > r_d > r_p$

(c) $r_\alpha = r_d > r_p$ (d) $r_p = r_d = r_\alpha$

65. Two particles each of mass m and charge q, are attached to the two ends of a light rigid rod of length 2l. The rod is rotated at a constant angular speed about a perpendicular axis passing through its centre. The ratio of the magnitudes of the magnetic moment of the system and its angular momentum about the centre of the rod is

(a) $\frac{q}{2m}$ (b) $\frac{q}{m}$

(c) $\frac{2q}{m}$ (d) $\frac{q}{\pi m}$

66. A charged particle is released from rest in a region of steady and uniform electric and magnetic fields which are parallel to each other. The particle will move in a

(a) straight line (b) circle

(c) helix (d) cycloid

67. A loosely wound helix made of stiff wire is mounted vertically with the lower end just touching a dish of mercury. When a current from the battery is started in the coil through the mercury

(a) the wire oscillates

(b) the wire continues making contact

(c) the wire breaks contact just when the current is passed

(d) the mercury will expand by heating due to passage of current

68. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current I passes through the coil, the magnetic field at the centre is

(a) $\frac{\mu_0 NI}{b}$ (b) $\frac{2\mu_0 NI}{a}$

(c) $\frac{\mu_0 NI}{2(b-a)} \ln \frac{b}{a}$ (d) $\frac{\mu_0 NI}{2(b-a)} \ln \frac{a}{b}$

69. A particle of mass m and charge q moves with a constant velocity v along the positive x direction. It enters a region containing a uniform magnetic field B directed along the negative z direction, extending from $x = a$ to $x = b$. The minimum value of v required so that the particle can just enter the region $x > b$ is

(a) qbB/m (b) $q(b - a)B/m$

(c) qaB/m (d) $q(b + a)B/2m$

70. A magnet of length 10 cm and magnetic moment 1 Am^2, is placed along the side AB of an equilateral triangle ABC. If the length of side AB is 10 cm, the magnetic field at point C is

(a) 10^{-9} T (b) 10^{-7} T

(c) 10^{-5} T (d) 10^{-4} T

71. A magnetized wire of magnetic moment M is bent into an arc of a circle that subtends an angle of 60° at the centre. The equivalent magnetic moment is

(a) $\frac{M}{\pi}$ (b) $\frac{2M}{\pi}$

(c) $\frac{3M}{\pi}$ (d) $\frac{4M}{\pi}$

72. Two poles of the same strength attract each other with a force of magnitude F when placed at two corners of an equilateral triangle. If a north pole of the same strength is placed at the third vertex, it experiences a force of magnitude

(a) $\sqrt{3}\ F$ (b) F

(c) $\sqrt{2}\ F$ (d) $2\ F$

73. A circular coil of radius r having number of turns n and carrying a current I, produces magnetic field of magnitude B at its centre. B can be doubled by

(a) keeping n unchanged and changing I to $\frac{I}{2}$

(b) changing n to $\frac{n}{2}$ and keeping I unchanged

(c) changing n to $2n$ and I to $2I$

(d) keeping I unchanged and changing n to $2n$

74. An electron moves with a speed of 2×10^5 ms^{-1} along the positive x-direction in a magnetic field $\mathbf{B} = (\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$ tesla. The magnitude of the force (in newton) experienced by the electron is (the charge on electron = 1.6×10^{-19} C)
(a) 1.18×10^{-13} (b) 1.28×10^{-13}
(c) 1.6×10^{-13} (d) 1.72×10^{-13}

75. A single charged ion has a mass of 1.13×10^{-23} g. It is accelerated through a potential difference of 500 V and then enters a magnetic field of 0.4 T, moving perpendicular to the field. The radius of its path in the field is
(a) 2.1 cm (b) 2.1 mm
(c) 1.17 m (d) 2.0 m

76. A proton of velocity $(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ ms^{-1} enters a magnetic field of $(2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ tesla. The acceleration produced in the proton is (charge to mass ratio of proton = 0.96×10^8 C kg^{-1})
(a) 2.88×10^8 $(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}})$
(b) 2.88×10^8 $(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$
(c) 2.88×10^8 $(2\hat{\mathbf{i}} + 3\hat{\mathbf{k}})$
(d) 2.88×10^8 $(\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

77. Two short bar magnets of magnetic moments 'M' each are arranged at the opposite corners of a square of side 'd', such that their centres coincide with the corners and their axes are parallel. If the like poles are in the same direction, the magnetic field at any of the other corners of the square is

(a) $\frac{\mu_0}{4\pi} \cdot \frac{M}{d^3}$ (b) $\frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3}$

(c) $\frac{\mu_0}{4\pi} \cdot \frac{M\sqrt{5}}{d^3}$ (d) $\frac{\mu_0}{4\pi} \cdot \frac{3M}{d^3}$

78. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then
(a) the magnetic field at all points inside the pipe is the same but not zero.
(b) the magnetic field at any point inside the pipe is zero.
(c) the magnetic field is zero only on the axis of the pipe.
(d) the magnetic field is different at different points inside the pipe.

79. A power line lies along the east-west direction and carries a current of 10 A. The force per metre due to the earth's magnetic field of 10^{-4} T is
(a) 10^{-5} Nm^{-1} (b) 10^{-4} Nm^{-1}
(c) 10^{-3} Nm^{-1} (d) 10^{-2} Nm^{-1}

80. Two straight and long conductors AOB and COD are perpendicular to each other and carry currents of I_1 and I_2. The magnitude of the magnetic field at a point P at a distance a from point O in a direction perpendicular to the plane $ABCD$ is

(a) $\frac{\mu_0}{2\pi a}(I_1 + I_2)$ (b) $\frac{\mu_0}{2\pi a}(I_1 - I_2)$

(c) $\frac{\mu_0}{2\pi a}(I_1^2 + I_2^2)^{1/2}$ (d) $\frac{\mu_0}{2\pi a} \frac{I_1 I_2}{(I_1 + I_2)}$

81. A 2 MeV proton is moving perpendicular to a uniform magnetic field of 2.5 T. The force on the proton is
(a) 2.5×10^{-10} N (b) 8×10^{-11} N
(c) 2.5×10^{-11} N (d) 8×10^{-12} N

82. A straight section PQ of a circuit lies along the x-axis from $x = -\frac{a}{2}$ to $x = \frac{a}{2}$ and carries a current I. The magnetic field due to the section PQ at point $x = +a$ will be
(a) proportional to a (b) proportional to a^2
(c) proportional to $\frac{1}{a}$ (d) equal to zero

83. Two charged particles M and N enter a region of uniform magnetic field with velocities perpendicular to the field. The paths of particles are shown in Fig. 13.57. The possible reason is
(a) The charge of M is greater than that of N
(b) The momentum of M is greater than that of N
(c) The charge to mass ratio of M is greater than that of N
(d) The speed of M is greater than that of N

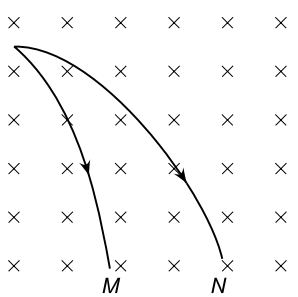


Fig. 13.57

84. The monoenergetic beam of electrons moving along + y direction enters a region of uniform electric and magnetic fields. If the beam goes straight undeflected, then fields **B** and **E** are directed respectively along
    (a) – y axis and – z axis
    (b) + z axis and + x axis
    (c) + x axis and + z axis
    (d) – x axis and – y axis

85. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on its axis at a distance of 4 cm from the centre is 54 μT. The magnetic field (in μT) at the centre of the loop will be
    (a) 250 (b) 150
    (c) 125 (d) 72

86. A wire $ABCDEF$ (with each side of length L) bent as shown in Fig. 13.58 and carrying a current I is placed in a uniform magnetic field B parallel to the positive y–direction. What is the magnitude and direction of the force experienced by the wire?
    (a) BIL along positive z-direction
    (b) BI^2/L along positive z-direction
    (c) BIL along negative z-direction
    (d) BL/I along negative z-direction

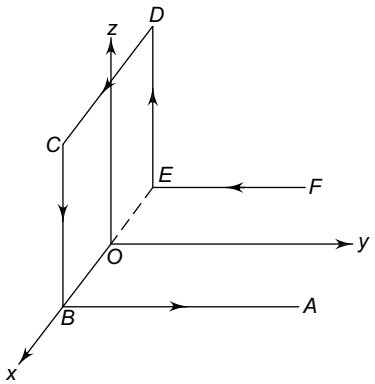


Fig. 13.58

87. A particle of charge q moves with a velocity $\mathbf{v} = a\,\hat{\mathbf{i}}$ in a magnetic field $\mathbf{B} = b\,\hat{\mathbf{j}} + c\,\hat{\mathbf{k}}$ where a, b and c are constants. The magnitude of the force experienced by the particle is
    (a) zero (b) $qa(b + c)$
    (c) $qa(b^2 - c^2)^{1/2}$ (d) $qa(b^2 + c^2)^{1/2}$

88. A current I is flowing through the sides of an equilateral triangle of side a. The magnitude of the magnetic field at the centroid of the triangle is
    (a) $\dfrac{3\mu_0 I}{2\pi a}$ (b) $\dfrac{9\mu_0 I}{2\pi a}$
    (c) $\dfrac{3\sqrt{3}\mu_0 I}{2\pi a}$ (d) zero

89. A particle of mass m and charge q, accelerated by a potential difference V enters a region of a uniform transverse magnetic field B. If d is the thickness of the region of B, the angle θ through which the particle deviates from the initial direction on leaving the region is given by
    (a) $\sin\theta = Bd\left(\dfrac{q}{2mV}\right)^{1/2}$
    (b) $\cos\theta = Bd\left(\dfrac{q}{2mV}\right)^{1/2}$
    (c) $\tan\theta = Bd\left(\dfrac{q}{2mV}\right)^{1/2}$
    (d) $\cot\theta = Bd\left(\dfrac{q}{2mV}\right)^{1/2}$

90. A metal wire of mass m slides without friction on two rails spaced at a distance d apart. The track lies in a vertical uniform magnetic field B. A constant current I flows along one rail, across the wire and back down the other rail. If the wire is initially at rest, the time taken by it to move through a distance x along the track is
    (a) $t = \sqrt{\dfrac{BId}{2xm}}$ (b) $t = \sqrt{\dfrac{2xm}{BId}}$
    (c) $t = \sqrt{\dfrac{BIdm}{2x}}$ (d) $t = \sqrt{\dfrac{2dm}{BIx}}$

91. A particle of charge q and mass m is released from the origin with a velocity $\mathbf{v} = a\,\hat{\mathbf{i}}$ in a uniform magnetic field $\mathbf{B} = b\,\hat{\mathbf{k}}$. The particle will cross the y-axis at a point whose y-coordinate is
    (a) $y = \dfrac{ma}{qb}$ (b) $y = \dfrac{2ma}{qb}$
    (c) $y = -\dfrac{ma}{qb}$ (d) $y = -\dfrac{2ma}{qb}$

92. A U-shaped wire $PQRS$ of mass m carrying a current I is stationary above the surface of the earth in the region of a uniform magnetic field **B** directed into the page as shown in Fig. 13.59. Then

(a) $I = \dfrac{mg}{l_2 B}$ (b) $I = \dfrac{mg}{2l_2 B}$

(c) $I = \dfrac{mg}{(2l_1 + l_2)B}$ (d) $I = \dfrac{mg}{(2l_1 - l_2)B}$

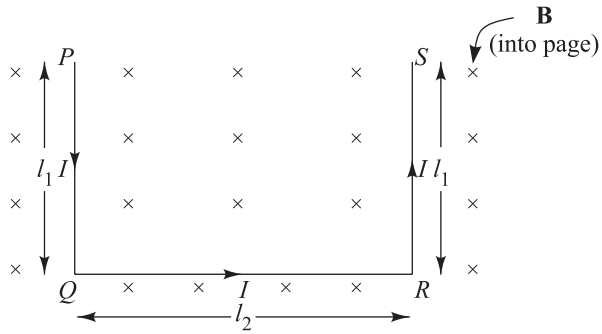


Fig. 13.59

93. A rectangular coil *PQRS* of length $PQ = a$ and breadth $SP = b$ is suspended with is plane parallel to a horizontal magnetic field **B** as shown in Fig. 13.60.

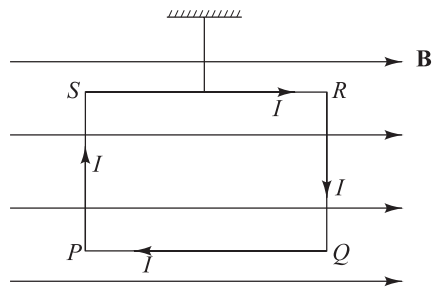


Fig. 13.60

When a current I is passed in the coil as shown, the coil will experience

(a) no net force and no net torque

(b) a net force $F = 2BIa$ will be move it is the direction of **B** but no net torque

(c) a net torque $\tau = BIab$ which will rotate it in the clockwise sense but no net force

(d) a net torque $\tau = BI(a^2 + b^2)$ which will rotate it in the anticlockwise sense and also a net force $F = BI\,(a + b)$ which will be parallel to **B**.

94. A wire is bent into a form *PQRST* and carries a current I as shown in Fig. 13.61. Straight segment *PQ* and *ST* are of equal length l and the semi-circular segment *QRS* has a radius r. The frame is placed in a region of uniform magnetic field **B** directed as shown in the Figure. The magnetic force exerted on the wire frame is

(a) $2BI\,(l + r)$ (b) $2BI\,(l + \pi r)$

(c) $BI\,(2l + r)$ (d) $BI\,(2l + \pi r)$

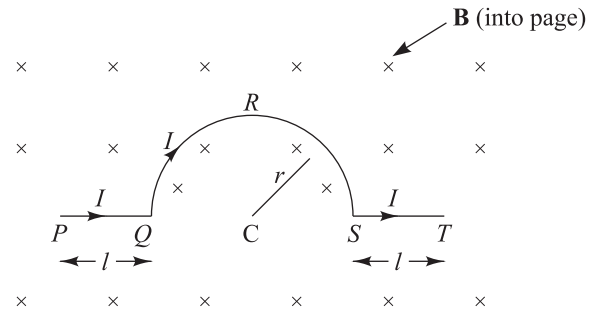


Fig. 13.61

95. A particle of charge $-q$ moves with a velocity **v** in a magnetic field **B** as shown in Fig. 13.62.

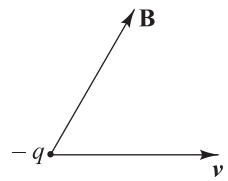


Fig. 13.62

The direction of the magnetic field on the particle will be

(a) to the left

(b) downward in the plane of the page

(c) out of the plane of the page

(d) into the plane of the page

96. Choose the only correct choice out of the four choices (a), (b), (c) and (d) given after the following three statements.

I. The magnetic field lines due to a current carrying wire radiate away from the wire.

II. The kinetic energy of a charged particle can be increased by a magnetic field.

III. A charged particle can move through a region containing only magnetic field without feeling any force.

(a) Only Statement I is true.

(b) Statements II and II are both true

(c) Statements II and III are both true

(d) Only Statement III is true

97. A particle of change q and mass m moves in a circular orbit of radius r in a region of uniform magnetic field **B**. The magnitude the angular momentum of the particle about the centre of the circle is

(a) qBr^2 (b) $mqBr$

(c) qBr (d) $mqBr^2$

98. Two long parallel wires 1 and 2 separated by a distance d carry equal currents I as shown in Fig. 13.63. The magnetic field at the point p which is exactly mid-way between the two wires is

(a) $\frac{2\mu_0 I}{\pi d}$ into the page

(b) $\frac{\mu_0 I}{\pi d}$ out of the page

(c) $\frac{\mu_0 I}{2\pi d}$ into the page

(d) zero

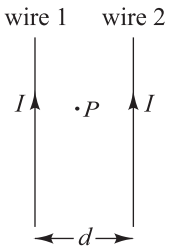


Fig. 13.63

99. In Q. 98 above, which of the four choices (a), (b), (c) and (d) of Q. 98 is correct if the direction of the current in wire 2 is reversed ?

100. A long co-axial cable consists of a solid cylindrical conductor of radius $R_1 = R$ surrounded by a thin conducting cylindrical shell of radius $R_2 = 2R$, The inner cylinder carries a current $I_1 = I$ and the outer cylindrical shell carries a current $I_2 = I/2$, in the same direction. The magnitude of the magnetic field at a point at a distance $r = 3R/2$ from the centre is

(a) $\frac{\mu_0 I}{3\pi R}$ (b) $\frac{2\mu_0 I}{3\pi R}$

(c) $\frac{3\mu_0 I}{2\pi R}$ (d) $\frac{\sqrt{3}\,\mu_0 I}{\pi R}$

101. In Q. 100 above, what is the magnitude of the magnetic field at a point P at a distance $r = 3R$ from centre ?

(a) $\frac{\mu_0 I}{\pi R}$ (b) $\frac{\mu_0 I}{2\pi R}$

(c) $\frac{\mu_0 I}{3\pi R}$ (d) $\frac{\mu_0 I}{4\pi R}$

102. In Q. 101 above, what is the magnitude of the magnetic field at the centre if the direction of current I_2 is reversed?

(a) $\frac{\mu_0 I}{3\pi R}$ (b) $\frac{\mu_0 I}{6\pi R}$

(c) $\frac{\mu_0 I}{9\pi R}$ (d) $\frac{\mu_0 I}{12\pi R}$

103. Figure 13.64 shows a wire $PQRS$ carrying a current I. Portions PQ and RS are straight and QR in a circular arc of radius r subtending an angle α at the centre C. The megnitude of magnetic field due to $PQRS$ at centre C is

(a) $\frac{\mu_0 I}{r}\left(1+\frac{\alpha}{2\pi}\right)$ (b) $\frac{\mu_0 I}{r}\alpha$

(c) $\frac{\mu_0 I\alpha}{2\pi r}$ (d) $\frac{\mu_0 I\alpha}{4\pi r}$

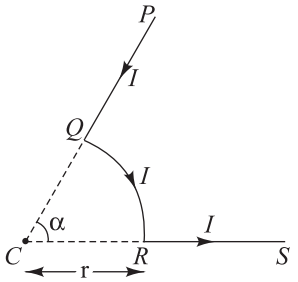


Fig. 13.64

104. A particle of charge $+q$ and mass m, after being accelerated from rest by a voltage V enters a region of a uniform magnetic field in which it describes a circular motion of radius r. How much time does the particle spend in the region of the field?

(a) $2\pi\sqrt{\frac{mr}{2qV}}$ (b) $\frac{1}{\pi r}\sqrt{\frac{2qV}{m}}$

(c) $\pi r\sqrt{\frac{m}{2qV}}$ (d) $2\pi r\sqrt{\frac{m}{2qV}}$

105. A homogeneous cylindrical rod of radius R carries a current I whose current density J (defined as current per unit cross-sectional area) is constant throughout the rod. The magnetic field at a point P at a distance $r = R/2$ from the centre of the rod is

(a) zero (b) $\frac{\mu_0 I}{\pi R}$

(c) $\frac{\mu_0 I}{2\pi R}$ (d) $\frac{\mu_0 I}{4\pi R}$

106. In Q. 105 above the magnetic field at a point P at a distance $r = 2R$ from the centre of the rod is

(a) $\frac{\mu_0 I}{2\pi R}$ (b) $\frac{\mu_0 I}{4\pi R}$

(c) $\frac{\mu_0 I}{8\pi R}$ (d) $\frac{\mu_0 I}{16\pi R}$

107. A non-homogeneous cylinder of radius R carries a current I whose current density varies with the radial distance r from the centre of the rod as $J = \sigma r$ where σ is a constant. The magnetic field at a point P at a distance $r < R$ from the centre of the rod is

(a) $\frac{\mu_0 I r^2}{2\pi R^3}$ (b) $\frac{\mu_0 I r}{2\pi R^2}$

(c) $\frac{\mu_0 I R^2}{2\pi r^3}$ (d) $\frac{\mu_0 I R}{2\pi r^2}$

108. A wire loop $PQTS$ is formed by joining two semicircular wires of radii R and $2R$ in two different ways as shown in Fig. 13.65 (a) and (b). They carry

the same current I. The ratio of magnitudes of magnetic fields at centre C in case (a) to that in case (b) is

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

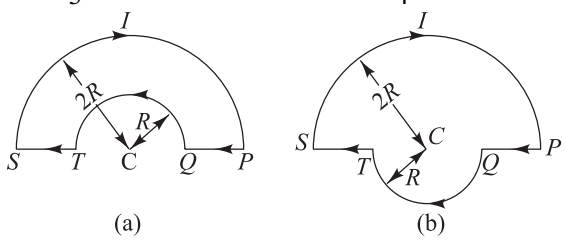


Fig. 13.65

109. In Q. 108 above, if B_1 is magnetic field in case (a) and B_2 in case (b), then

(a) B_1 is directed out of the page and B_2 into the page

(b) B_1 is directed into the page and B_2 out of the page

(c) both B_1 and B_2 are directed into the page

(d) both B_1 and B_2 are directed out of the page

110. Two coils A and B, each having n turns and radius r are held such that coil A lies in the vertical plane and coil B in the horizontal plane with their centres coinciding. The angle of dip at the place is θ. A current I_A has to be passed through coil A and a current I_B through coil B in order to nullify the earth's magnetic field at that place. Then ratio I_A/I_B must be

(a) $\sin\theta$ (b) $\cos\theta$

(c) $\tan\theta$ (d) $\cot\theta$

111. Two charged particles X and Y after being accelerated through the same potential difference V, enter a region of uniform magnetic field B and describe circular paths of radii r_1 and r_2 respectively. Then the ratio

$$\frac{\text{charge to mass ratio of } X}{\text{charge to mass ratio of } Y}\text{, is}$$

(a) $\left(\frac{r_1}{r_2}\right)^{1/2}$ (b) $\left(\frac{r_1}{r_2}\right)$

(c) $\left(\frac{r_1}{r_2}\right)^2$ (d) $\left(\frac{r_2}{r_1}\right)^2$

112. A rectangular loop $PQRS$ carrying a current i is situated near a long straight wire AB. If a steady current I is passed through AB as shown in Fig. 13.66, the loop will

(a) rotate about an axis parallel to AB

(b) move towards AB

(c) move away from AB

(d) remain stationary

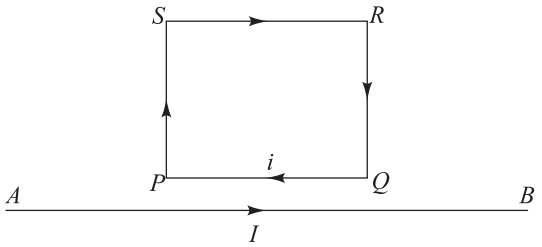


Fig. 13.66

113. Two very long parallel wires, separated by a distance d, carry equal current I in the same direction. At a certain instant of time, a point charge q is at a point P which is equidistant from the two wires, in the plane containing the two wires. If v is the velocity of the charge at this instant is perpendicular to this plane, the force due to magnetic field at P will

(a) accelerate the charged particle

(b) decelerate the charged particle

(c) make the particle oscillate between the two wires

(d) be zero

114. In Q. 113 above, if the two wires carry current I in opposite directions, the force on the charged particle at point P will be (here $B = \mu_0 I/\pi d$)

(a) zero (b) $\frac{1}{2} q\, v\, B$

(c) qvB (d) $2\, qvB$

115. Two particles 1 and 2 of masses m_1 and m_2 and having the same charge q, are moving with velocities v_1 and v_2 respectively in a plane. They describe circular paths of radii r_1 and r_2 in a uniform magnetic field **B** which is directed perpendicular to this plane as shown in Fig. 13.67. Then

(a) $m_1v_1 = m_2v_2$ (b) $m_1v_1 > m_2v_2$

(c) $m_1v_1 < m_2v_2$ (d) $\frac{m_1}{m_2} = \frac{v_1}{v_2}$

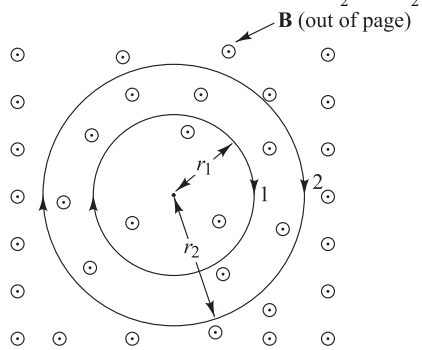


Fig. 13.67

116. A proton moving with a speed u in the $x - y$ plane along the positive x axis enters at $y = 0$ a region of uniform magnetic field **B** directed into the $x - y$ plane as shown in Fig. 13.68. After sometime, the proton leaves the region with a speed v at co-ordinate y. Then

(a) $v = u, y > 0$ (b) $v = u, y < 0$

(c) $v > u, y > 0$ (d) $v > u, y < 0$

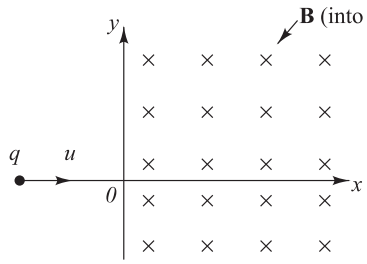


Fig. 13.68

117. A circular wire loop carrying a current I in the anti-clock wise sense is placed in a uniform magnetic field **B** directed into the plane of the coil as shown in Fig. 13.69 The loop will tend to

(a) contract

(b) expand

(c) move towards negative x – direction

(d) move towards positive y – direction

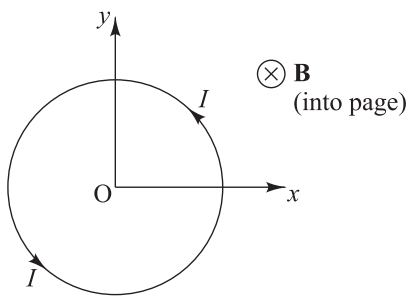


Fig. 13.69

118. Figure 13.70 shows a square wire frame *BCDE*. A current I enters at B and leaves at E. The magnitude of the magnetic field due to the current in the complete frame *ABCDEF* at the centre O of the square is

(a) $\dfrac{\mu_0 I}{4\sqrt{2}\,\pi r}$ (b) $\dfrac{\mu_0 I}{2\sqrt{2}\,\pi r}$

(c) $\dfrac{3\mu_0 I}{4\sqrt{2}\,\pi r}$ (d) zero

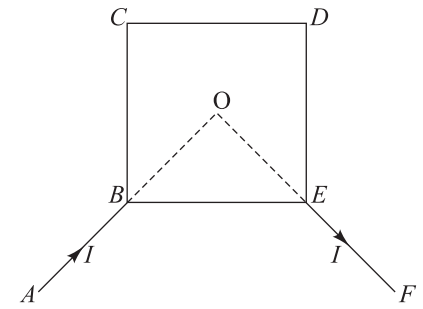


Fig. 13.70

119. Figure 13.71 shows an infinitely long wire *ABCDE* bent such that the part *BCD* of the wire is a semi-circle of radius r. A current I flows in the wire as shown. The magnitude of magnetic field at centre O of the semi-circular part is

(a) $\dfrac{\mu_0 I}{\pi r}$ (b) $\dfrac{\mu_0 I}{4 r}$

(c) $\dfrac{\mu_0 I}{4 r}(2\pi + 1)$ (d) $\dfrac{\mu_0 I}{4 r}(2\pi - 1)$

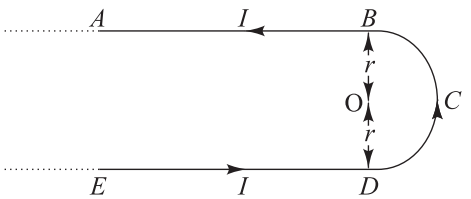


Fig. 13.71

120. In Fig. 13.72, the magnetic field at O is

(a) $\dfrac{\mu_0 I}{4\pi r}(\pi + 1)$ (b) $\dfrac{\mu_0 I}{2\pi r}(2\pi + 1)$

(c) $\dfrac{\mu_0 I}{4\pi r}$ (d) $\dfrac{\mu_0 I}{4 r}$

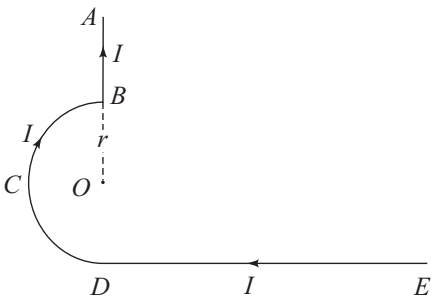


Fig. 13.72

121. An annular wire loop *ABCD* carries a current I as shown in Fig. 13.73. O is the common centre of the curved parts *AD* and *BC*. If $OA = r$ and $OB = 2r$, The magnitude of the magnetic field at O is

(a) $\dfrac{\mu_0 I}{24 r}$ (b) $\dfrac{\mu_0 I}{12 r}$

(c) $\dfrac{\mu_0 I}{8 r}$ (d) $\dfrac{\mu_0 I}{6 r}$

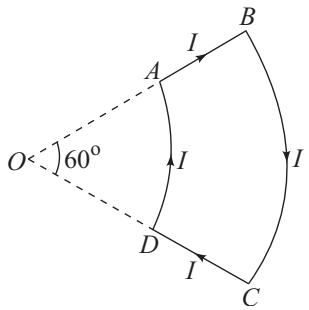


Fig. 13.73

122. The magnetic field at O due to the wire loop $ABCD$ carrying a current I as shown in Fig. 13.74 is

(a) $\dfrac{\mu_0 I}{4r}$ (b) $\dfrac{3\mu_0 I}{8r}$

(c) $\dfrac{5\mu_0 I}{12r}$ (d) $\dfrac{7\mu_0 I}{16r}$

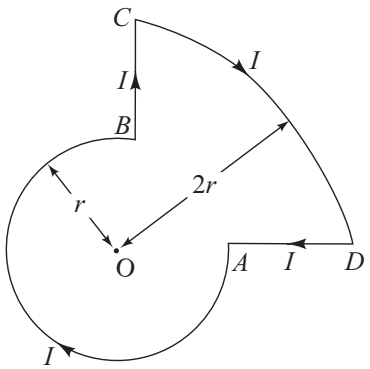


Fig. 13.74

123. The magnitude of the magnetic field at the centroid of a triangular metal loop of side a and carrying a current I is

(a) $\dfrac{3\mu_0 I}{4\pi a}$ (b) $\dfrac{3\sqrt{3}\,\mu_0 I}{2\pi a}$

(c) $\dfrac{3\sqrt{2}\,\mu_0 I}{\pi a}$ (d) $\dfrac{9\mu_0 I}{2\pi a}$

124. A charge q moves with a velocity v in a magnetic field $\mathbf{B} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$. If the acceleration of the particle at time t is $\mathbf{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - n\hat{\mathbf{k}}$, then the value of n is

(a) 1 (b) 2

(c) 3 (d) 4

125. A loop $ABCDEFGHA$ carrying a current I lies in the x–y plane as shown in Fig. 13.75. The unit vector $\hat{\mathbf{k}}$ is directed out of the plane of the page (*i.e.*, x–y plane). The magnetic moment of the current loop is

(a) $a^2 I\,\hat{\mathbf{k}}$ (b) $-\left(\sqrt{3}+2\right) a^2 I\,\hat{\mathbf{k}}$

(c) $-\left(\sqrt{3}+1\right) a^2 I\,\hat{\mathbf{k}}$ (d) $-\sqrt{3}\; a^2 I\,\hat{\mathbf{k}}$

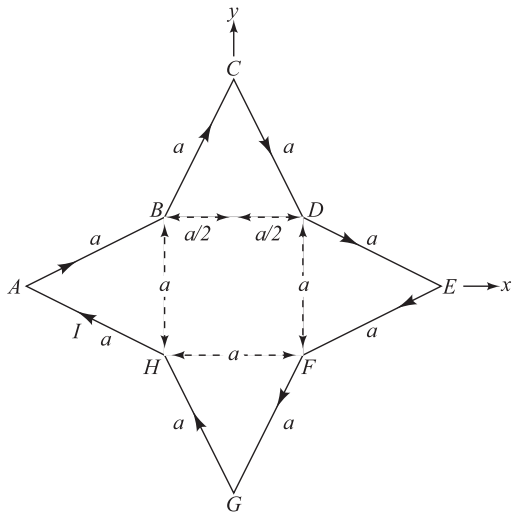


Fig. 13.75

Answers

Level A

1. (a)	2. (c)	3. (c)	4. (a)
5. (d)	6. (a)	7. (d)	8. (c)
9. (d)	10. (c)	11. (d)	12. (b)
13. (d)	14. (a)	15. (d)	16. (c)
17. (a)	18. (c)	19. (d)	20. (b)
21. (c)	22. (a)	23. (d)	24. (d)
25. (a)	26. (d)	27. (d)	28. (a)
29. (c)	30. (a)	31. (b)	32. (a)
33. (a)	34. (d)	35. (b)	36. (a)
37. (c)	38. (b)	39. (a)	40. (d)
41. (c)	42. (a)	43. (c)	44. (d)
45. (a)	46. (b)	47. (b)	48. (b)
49. (d)	50. (c)	51. (d)	52. (b)
53. (c)	54. (c)	55. (d)	

Level B

56. (d)	57. (a)	58. (c)	59. (d)
60. (c)	61. (c)	62. (a)	63. (a)
64. (a)	65. (a)	66. (a)	67. (a)
68. (c)	69. (b)	70. (d)	71. (c)
72. (b)	73. (d)	74. (c)	75. (a)
76. (b)	77. (a)	78. (b)	79. (c)
80. (c)	81. (d)	82. (d)	83. (c)
84. (c)	85. (a)	86. (a)	87. (d)

88. (b)	89. (a)	90. (b)	91. (d)
92. (a)	93. (c)	94. (a)	95. (d)
96. (d)	97. (a)	98. (d)	99. (a)
100. (a)	101. (d)	102. (d)	103. (d)
104. (c)	105. (d)	106. (b)	107. (a)
108. (c)	109. (a)	110. (d)	111. (d)
112. (c)	113. (d)	114. (a)	115. (c)
116. (a)	117. (c)	118. (d)	119. (c)
120. (a)	121. (a)	122. (d)	123. (d)
124. (b)	125. (c)		



Solutions

Level A

1. The radius of the circular loop (r) = $\frac{L}{2\pi}$. Therefore,

$$B = \frac{\mu_0 I}{2r} = \frac{\pi \mu_0 I}{L}$$

Hence the correct choice is (a).

2. Magnetic field at point O due to straight wire AB is

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r} = \frac{\mu_0 I}{2\pi r}$$

and that due to the circular loop is

$$B_2 = \frac{\mu_0 I}{2r}$$

Both these fields are normal to the plane of the loop and directed outside the page. Therefore, net field at point O is

$$B = B_1 + B_2 = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 I}{2r}$$

$$= \frac{\mu_0 I}{2\pi r}(1 + \pi)$$

Hence the correct choice is (c).

3. The radius of the double loop $r = R/2$. Now

$$B = \frac{\mu_0 I}{2R}$$

Magnetic field due to a loop of radius r at the centre of the loop is

$$B_1 = \frac{\mu_0 I}{r} = 2B \qquad (\because\ r = R/2)$$

Similarly for the double loop,

$$B_2 = 2B$$

Since the currents in the two loops are in the same direction, the net magnetic field at the centre = $B_1 + B_2 = 4B$. Hence the correct choice is (c).

4. Since the currents in the two loops are in opposite directions, fields B_1 and B_2 are equal and opposite. Therefore, the net magnetic field at the centre of the double loop = $B_1 - B_2 = 0$. Hence the correct choice is (a).

5. The magnetic field at a distance $x = \sqrt{3}\ r$ is

$$B' = \frac{\mu_0 I n r^2}{2\left(r^2 + x^2\right)^{3/2}} = \frac{\mu_0 I n r^2}{2\left(r^2 + 3r^2\right)^{3/2}}$$

$$= \frac{\mu_0 I n}{16r}$$

But $$B = \frac{\mu_0 I n}{2r}$$

$\therefore\ B' = 8B$. Hence the correct choice is (d).

6. $H = 0.314\ G = 0.314 \times 10^{-4}$ T. Let the current in coil be I. Then the magnetic field at the centre of the coil is

$$B = \frac{\mu_0 I n}{2r} = \frac{4\pi \times 10^{-7} \times I \times 50}{2 \times 5 \times 10^{-2}}$$

$$= 2\pi I \times 10^{-4}\ \text{T}$$

The value of I for which $B = H$ is given by

$2\pi I \times 10^{-4} = 0.314 \times 10^{-4}$

or $I = 0.05$ A

$\therefore$ Potential difference = $IR = 0.05 \times 10 = 0.5$ V. Hence the correct choice is (a).

7. Refer to Fig. 13.76. Here $r = OE = \frac{L}{2}$

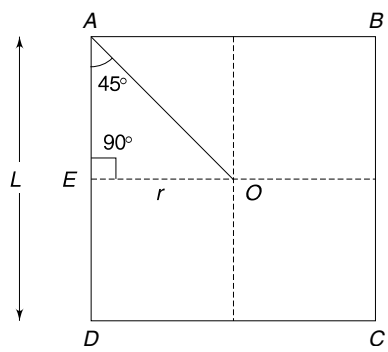


Fig. 13.76

The magnetic field at the centre O due to the current element AE is given by

$$B_{AE} = -\frac{\mu_0 I}{4\pi r} \int_{90°}^{45°} \sin\theta\, d\theta$$

$$= \frac{\mu_0 I}{4\pi r} \left| \cos\theta \right|_{90°}^{45°}$$

$$= \frac{\mu_0 I}{4\pi r} (\cos 45° - \cos 90°)$$

$$= \frac{\mu_0 I}{4\pi r} (\cos 45° - 0) = \frac{\mu_0 I}{4\sqrt{2}\pi r}$$

It is clear that the magnetic field at O due to current element DE is the same as that due to AE. Hence, the magnetic field at O due to one side AD is

$$B_{AD} = \frac{2\mu_0 I}{4\sqrt{2}\pi r} = \frac{\sqrt{2}\mu_0 I}{4\pi r}$$

Since the centre of the square is equidistant from the ends A, B, C and D of each side of the square and each side produces at the centre O the same magnetic field, the field due to the square is 4 times that due to one side. Hence (because $r = L/2$)

$$B = 4B_{AD} = \frac{\sqrt{2}\mu_0 I}{\pi r} = \frac{2\sqrt{2}\mu_0 I}{\pi L}$$

Hence the correct choice is (d).

8. The magnitude of the magnetic field at the centre due to each coil is B. Since the planes of the coils are at right angles to each other, the directions of the fields will be at right angles to each other. Therefore, the resultant field is

$$B_r = \sqrt{B^2 + B^2} = \sqrt{2}\, B$$

Hence the correct choice is (c).

9. The correct choice is (d) since $\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$.
10. Since the force exerted by the magnetic field is perpendicular to the direction of motion of the particle, the speed of the particle cannot change but its velocity changes. Hence the correct choice is (c).
11. Applying the left hand fist rule and noting the charge on an electron is negative, i.e. $\mathbf{F} = -e\,(\mathbf{v} \times \mathbf{B})$, it follows that the particle is an electron, which is choice (d).
12. The radius of the circular orbit is given by

$$r = \frac{\sqrt{2mK}}{qB}$$

The charge of an α–particle is twice that of a proton and its mass is four times the mass of a proton. Therefore $\sqrt{m}/q$ is the same for both. Hence r will the same for both particles. Thus the correct choice is (b).

13. The cyclotron frequency is given by

$$\nu = \frac{qB}{2\pi m}$$

It is independent of the speed of the particle and the radius of its circular path. Now $\nu \propto q/m$. The charge of a proton is half that of an α-particle and the mass of a proton is one-fourth. Therefore, ν will be doubled. Hence the correct choice is (d).

14. The correct choice is (a).
15. The correct choice is (d).
16. The velocity when the potential difference is V is

$$v = \sqrt{\frac{2eV}{m}}$$

and force $F = e\,v\,B$

When the potential difference is doubled, i.e. $V' = 2V$, the velocity is

$$v' = \sqrt{\frac{2eV'}{m}} = \sqrt{\frac{2e \times 2V}{m}} = \sqrt{2}\ v$$

$\therefore$ Force $F' = ev'B = \sqrt{2}\ evB = \sqrt{2}\ F$. Hence the correct choice is (c).

17. The correct choice is (a).
18. The correct choice is (c) because the magnetic field produced by the current in the loop and the external magnetic field are along the same direction.
19. The force per unit length is

$$F = \frac{\mu_0}{4\pi} \times \frac{2I^2}{R}$$

If R is increased to $2R$ and I is reduced to $I/2$, the force per unit length becomes

$$F' = \frac{\mu_0}{4\pi} \times \frac{2(I/2)^2}{2R}$$

$$= \frac{\mu_0}{4\pi} \times \frac{2I^2}{R} \cdot \frac{1}{8} = \frac{F}{8}$$

Hence the correct choice is (d).

20. In order that the tension in the supporting wires is zero the downward gravitational force mg on the rod must be balanced by an upward force BIl due to magnetic field, i.e. $BIl = mg$

or $$B = \frac{mg}{Il} = \frac{50 \times 10^{-3} \times 10}{5 \times 0.5} = 0.2 \text{ T}$$

Hence the correct choice is (b).

21. Speed of electron (v) = 2.0×10^6 ms^{-1}, radius of circular orbit (a) = 5.0×10^{-11} m.

The time period of electron motion in the circular orbit is

$$T = \frac{2\pi a}{v} = \frac{2\pi \times 5.0 \times 10^{-11}}{2.0 \times 10^6}$$

$$= 5\pi \times 10^{-17} \text{ s}$$

Therefore the equivalent current is

$$I = \frac{\text{charge}}{\text{time}} = \frac{e}{T} = \frac{1.6 \times 10^{-19}}{5\pi \times 10^{-17}}$$

$$= \frac{1.6}{5\pi} \times 10^{-2} \text{ A}$$

Equivalent magnetic dipole moment equals current × area of circular orbit

$$= I \times (\pi a^2)$$

$$= \frac{1.6 \times 10^{-2}}{5\pi} \times \pi \times (5.0 \times 10^{-11})^2$$

$$= 8 \times 10^{-24} \text{ Am}^2$$

22. The magnetic field at the centre of the orbit is

$$B = \frac{\mu_0 I}{2a} = \frac{4\pi \times 10^{-7} \times 1.6 \times 10^{-2}}{2 \times 5\pi \times 5.0 \times 10^{-11}}$$

$$= 12.8 \text{ T}$$

23. The charge crossing any point of the path per second, is n times the charge e of the electron. This constitutes the current round the orbit, i.e. $I = ne$. If A is the area of the orbit, the magnetic moment is given by the product

$$IA = ne\,\pi r^2$$

Hence the correct choice is (d).

24. Magnetic moment $m = AI = \pi r^2 I$, where r is the radius of the circular loop. Now, the circumference of the circle = length of the wire, i.e.

$$2\pi r = l$$

or $$r^2 = \frac{l^2}{4\pi^2}$$

Therefore, $$m = \pi r^2 I = \frac{\pi l^2 I}{4\pi^2} = \frac{l^2 I}{4\pi}$$

Hence the correct choice is (d).

25. The required speed is given by

$$v = \frac{E}{B} = \frac{5000}{0.01} = 5 \times 10^5 \text{ ms}^{-1}$$

Hence the correct choice is (a).

26. Magnetic moment $m = IA = I\pi r^2$.

$$m = I_1 \pi r_1^2 = I_2 \pi r_2^2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} = \left(\frac{6}{3}\right)^2 = 4$$

Hence the correct choice is (d).

27. Force $F = qvB$

$$\therefore \text{ Acceleration} = \frac{F}{m} = \frac{qvB}{m}$$

$$= \frac{1.6 \times 10^{-19} \times 5.0 \times 10^5 \times 0.17}{1.7 \times 10^{-27}}$$

$$= 8 \times 10^{12} \text{ ms}^{-2}$$

Hence the correct choice is (d).

28. The charge passing per second through any point of the path is v times the charge of the electron, i.e. $I = ve$. If A is the area of the orbit, the magnetic moment is

$$m = IA = ve\,\pi r^2$$

Hence the correct choice is (a).

29. An electron moving in a circular orbit is equivalent to a current carrying loop. As explained above, the current is

$$I = ve = \frac{e}{T}$$

where T is the time period of the motion of the electron around the nucleus. If v is the speed of the electron,

$$T = \frac{2\pi r}{v}$$

$$\therefore \quad I = \frac{ev}{2\pi r} = \frac{e\omega}{2\pi} \qquad (\because v = r\omega)$$

Now, the magnetic field at the centre of the loop is

$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0 e\omega}{4\pi r}$$

or $$\omega = \frac{4\pi r B}{\mu_0 e}$$

Hence the correct choice is (c).

30. We have

$$\frac{mv^2}{r} = qvB$$

or $$r = \frac{mv}{qB}$$

Frequency $= \frac{v}{2\pi r} = \frac{v}{2\pi} \times \frac{qB}{mv} = \frac{qB}{2\pi m}$ which is independent of v, the speed of the charged particle. Hence the correct choice is (a).

31. The correct choice is (b).
32. The correct choice is (a).
33. The correct choice is (a).
34. The correct choice is (d).
35. The intensity of magnetisation (M) is defined as the magnetic moment per unit volume of the material. The magnitude of M depends upon the magnetisation current which is proportional to H. Hence the correct choice is (b).
36. The correct choice is (a).

37. Ferromagnetic substances have a very high (of the order of 1000 or more) and positive susceptibility (χ). Now $\mu_r = 1 + \chi$. Hence the correct choice is (c).

38. According to Curie's law, the susceptibility χ is related to temperature T as

$$\chi = \frac{C}{T}$$

where C is curie constant. Hence the correct choice is (b).

39. When a ferromagnetic material is heated above its Curie temperature, the thermal motions at a high temperature destroy its magnetism and the material will behave as a paramagnet.

40. The correct choice is (d).

41. Susceptibility (χ) of a paramagnet is small and positive and of a diamagnet is small and negative. Now relative permeability

$$\mu_r = 1 + \chi$$

Hence μ_r is slightly greater than unity for a paramagnet and slightly less than unity for a diamagnet. Hence the correct choice is (c).

42. The magnetic moment of the solenoid is

$$M = I\,NA = 2 \times 1000 \times 1.5 \times 10^{-4} = 0.3 \text{ Am}^2$$

Hence the correct choice is (a).

43. The potential energy of a bar magnet with its magnetic moment **M** inclined at an angle θ with magnetic field **B** is

$$U = -M\,B\cos\theta$$

Potential energy when $\theta = 0$ is

$$U_0 = -MB\cos 0° = -MB$$

Potential energy when $\theta = 90°$ is

$$U' = -M\,B\cos 90° = 0$$

$\therefore$ Work done $= U' - U_0 = 0 - (-MB) = MB = 2.0 \times 0.25 = 0.5$ J. Hence the correct choice is (c).

44. Potential energy when $\theta = 180°$ (i.e. **M** opposite to **B**) is

$$U'' = -MB\cos 180° = MB$$

$\therefore$ Work done $= U'' - U_0 = MB - (-MB)$

$$= 2\,MB = 2 \times 2.0 \times 0.25 = 1.0 \text{ J}.$$

Hence the correct choice is (d).

45. Refer to Fig. 13.77. Let θ_1 (= 30°) be the angle between the magnetic moment vector **M** and the field vector $\mathbf{B}_1$ (= 1.5×10^{-2} T). Then, as shown in Fig. 13.58, the angle between **M** and the other field $\mathbf{B}_2$ will be $\theta_2 = 75° - 30° = 45°$.

The field $\mathbf{B}_1$ exerts a torque $\tau_1 = \mathbf{M} \times \mathbf{B}_1$ on the dipole and the field $\mathbf{B}_2$ exerts a torque $\tau_2 = \mathbf{M} \times \mathbf{B}_2$, where **M** in the magnetic moment of the dipole. Since the dipole is in stable equilibrium, the net torque τ (= $\tau_1 + \tau_2$) must be zero, i.e. the two torques must be equal and opposite. In terms of magnitudes, we have

$$m B_1 \sin\theta_1 = m B_2 \sin\theta_2$$

or
$$B_2 = \frac{B_1 \sin\theta_1}{\sin\theta_2}$$

$$= \frac{\sqrt{2} \times 10^{-2} \times \sin 30°}{\sin 45°} = 0.01 \text{ T}$$

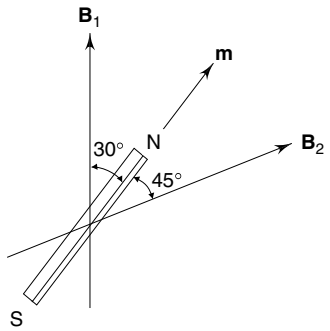


Fig. 13.77

46. $B_H = 0.20 \times 10^{-4}$ T and $\theta = 60°$. We know that $B_H = B\cos\theta$, where B is the magnitude of the total earth's field. Thus,

$$B = \frac{B_H}{\cos\theta} = \frac{0.20 \times 10^{-4}}{\cos 60°} = 0.40 \times 10^{-4} \text{ T}$$

47. Here $2a$ = 4 cm, m = 4 JT^{-1} and r = 2 m. Since $a << r$, the magnetic field at a distance r on the equatorial line is

$$B_m = \frac{\mu_0 M}{4\pi r^3} = \frac{4\pi \times 10^{-7} \times 4}{4\pi \times (2)^3} = 5 \times 10^{-6} \text{ T}$$

48. Since $a << r$, the magnetic field at a distance r on the axis line is

$$B_m = \frac{2\mu_0 M}{4\pi r^3} = 10^{-5} \text{ T}$$

49. The magnetic field in the core is given by

$$B = \mu n I$$

where μ is the permeability of soft iron and n is the number of turns per unit length of the solenoid. Now

$$\mu_r = \frac{\mu}{\mu_0} \text{ and } n = \frac{3000}{2\pi r} = \frac{3000}{2\pi \times 0.1}$$

$\therefore$
$$B = \mu_r \mu_0 n I$$

$$= 2000 \times 4\pi \times 10^{-7} \times \frac{3000}{2\pi \times 0.1} \times 1 = 12 \text{ T}$$

Hence the correct choice is (d).

50. Initial total magnetic moment at temperature T = 4 K is

$$m_1 = 15\% \text{ of } (2.0 \times 10^{24} \times 1.5 \times 10^{-23}) = 4.5 \text{ JT}^{-1}$$

Now from Curie's law, we have $\chi = C/T$

or $$\frac{I}{H} = \frac{C}{T} \quad \text{(i)}$$

where I is magnetization and H is the magnetizing field. If V is the volume of the sample, then by definition, $M = I/V$ and Eq. (i) becomes

$$M = \frac{C}{V}\left(\frac{H}{T}\right) = \text{constant} \times \left(\frac{H}{T}\right) \quad \text{(ii)}$$

If M_2 is the magnetic moment at temperature T_2 = 3 K and field H_2 = 0.9 T, then from Eq. (ii) M_1 and M_2 are related as

$$\frac{M_1}{M_2} = \frac{H_1}{H_2} \cdot \frac{T_2}{T_1}$$

or $$M_2 = M_1 \times \frac{H_2}{H_1} \times \frac{T_1}{T_2}$$

$$= 4.5 \times \frac{0.9}{0.6} \times \frac{4}{3} = 9 \text{ JT}^{-1}$$

51. The correct choice is (d).
52. The correct choice is (b).
53. If a torque τ is applied, the work done is

$$W = \int_0^\theta \tau \, d\theta = \int_0^\theta mB \sin\theta \, d\theta$$

$$= -mB \, |\cos\theta|_0^\theta$$

$$= -mB(\cos\theta - 1) = mB(1 - \cos\theta)$$

Hence the correct choice is (c).

54. The correct choice is (c).
55. The correct choice is (d) since the direction of the field is parallel to the axis of a solenoid. Thus $\theta = 0$. Hence $F = 0$.

Level B

56. The correct choice is (d).
57. Electric field $E = \frac{V}{d}$

where V is the potential difference between the plates and d, the separation between them.

$$d = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$$

$$E = \frac{V}{d} = \frac{600}{3 \times 10^{-3}} = 2 \times 10^5 \text{ V m}^{-1}$$

Since the electron moves undeflected between the plates, the force due to magnetic field must balance the force due to electric field. Thus

$$B\,e\,v = e\,E$$

or $$B = \frac{E}{v} = \frac{2 \times 10^5}{2 \times 10^6} = 0.1 \text{ T}$$

58. Wires A and B carry current I = 5 A each coming out of the plane of the page as shown in Fig. 13.78. The magnitude of magnetic field at point P due wire A is equal to that due to wire B, i.e.

$$B_A = B_B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a}$$

$$= \frac{10^{-7} \times 2 \times 5}{0.1} = 10^{-5} \text{ T}$$

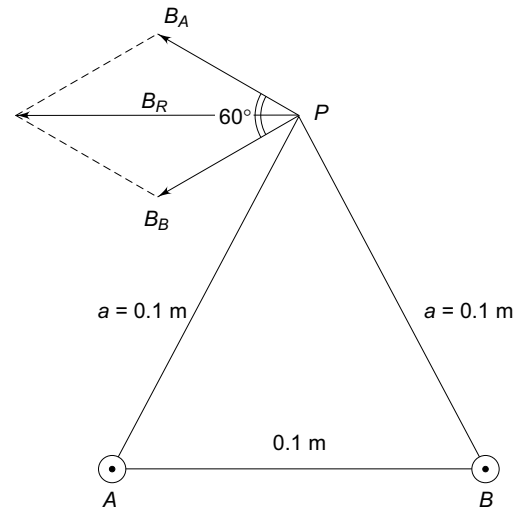


Fig. 13.78

The direction of field B_A is perpendicular to PA and that of field B_B is perpendicular to PB. Therefore, the angle between the two fields is $\theta = 60°$. The magnitude of the resultant field at P is given by

$$B_R^2 = B_A^2 + B_B^2 + 2B_A B_B \cos\theta$$

which gives $$B_R = 2B_A \cos\left(\frac{\theta}{2}\right)$$

$$= 2 \times 10^{-5} \times \frac{\sqrt{3}}{2} = \sqrt{3} \times 10^{-5} \text{ T}$$

Hence the correct choice is (c).

59. $B = \frac{\mu_0 I}{2r}$. For a coil of n turns, $2\pi r = n(2\pi r')$

or $r' = \frac{r}{n}$, where r' is the radius of the coil of n turns.

$$\therefore \quad B' = \frac{n\mu_0 I}{2r'} = \frac{n\mu_0 I}{2r/n} = n^2 B$$

Hence the correct choice is (d).

60. Since point O lies along the segments LR and MS, the magnetic field due to these segments is zero at point

O. As point O is close to R and S, the net magnetic field at O due to segments PR and QS is

$$B = B_P + B_Q$$
$$= \frac{\mu_0 I}{4\pi RO} + \frac{\mu_0 I}{4\pi SO} = \frac{\mu_0 I}{4\pi}\left(\frac{1}{d} + \frac{1}{d}\right)$$
$$= \frac{\mu_0}{4\pi}\left(\frac{2I}{d}\right) = 10^{-7} \times \frac{2 \times 10}{0.02} = 10^{-4}\ \text{Wb m}^{-2}$$

The direction of this field is vertically upward, i.e. outside the plane of the paper. Hence the correct choice is (c).

61. Magnetic field $B = \frac{\mu_0}{4\pi}.\frac{2I_1}{r_1} - \frac{\mu_0}{4\pi}.\frac{2I_2}{r_2}$

Given $I_1 = 2.5$ A, $I_2 = 5$ A and $r_1 = r_2 = 2.5$ m. Using these values, we get $B = -\frac{\mu_0}{2\pi}$. The magnitude of B is $\mu_0/2\pi$. Hence the correct choice is (c).

62. For proton: $r = \frac{mv}{eB}$

For α-particle $r' = \frac{m'v'}{e'B} = \frac{4mv'}{2eB} = \frac{2mv'}{eB}$

Given $r = r'$. Hence $v' = \frac{v}{2}$.

Energy of proton $E = \frac{1}{2}\ mv^2$. Energy of α-particle is

$$E' = \frac{1}{2}\ m'v'^2 = \frac{1}{2} \times 4m \times \left(\frac{v}{2}\right)^2 = \frac{1}{2}\ mv^2 = E$$

Hence $E' = 1$ MeV which is choice (a).

63. $B_H = B \cos\theta$ and $B_V = B \sin\theta$. Hence

$$\frac{B_V}{B_H} = \tan\theta$$

Given $\frac{B_V}{B_H} = \frac{1}{\sqrt{3}}$. Therefore, $\tan\theta = \frac{1}{\sqrt{3}}$, i.e. $\theta = 30°$ which is choice (a).

64. The radius of the circular path is given by

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}, \text{ where } K = \frac{1}{2}\ mv^2.$$

Thus $r \propto \frac{\sqrt{m}}{q}$. Since K and B are the same for the three particles. If m_p is the mass of a proton and q_p its charge, then $m_d = 2m_p$ and $q_d = q_p$ and $m_\alpha = 4\ m_p$ and $q_\alpha = 2q_p$. From these it follows that $r_\alpha = r_p < r_d$, which is choice (a).

65. According to Ampere's Law, the magnetic moment of a current I flowing in a circular path of area of cross-section A is given by

$$\mu_m = IA$$
$$= \frac{q}{T}\ A = \frac{q\pi(2l)^2}{T}$$

It is given that the charge q is moving in a circular path of radius $2l$. Therefore, the time period $T = 2\pi(2l)/v$. Hence

$$\mu_m = \frac{qv}{2\pi\ (2l)} \times \pi(2l)^2 = qvl$$

The angular momentum $L = mv(2l)$. Therefore,

$$\frac{\mu_m}{L} = \frac{qvl}{mv(2l)} = \frac{q}{2m}, \text{ which is choice (a).}$$

66. Due to electric field **E**, the force on a particle of charge q is $\mathbf{F} = q\mathbf{E}$ in the direction of the electric field. Since **E** is parallel to **B**, the velocity **v** of the particle is parallel to **B**. Hence **B** will not affect the motion of the particle since $\mathbf{v} \times \mathbf{B} = 0$. Thus, the correct choice is (a).

67. When a current is passed through the helix, the neighbouring coils of the helix attract each other due to which it contracts. As a result the contact is broken and the coils will recover their original state under the influence of a restoring force. The contract is made again and the process continues. Thus, the wire oscillates. Hence the correct choice is (a).

68. The correct choice is (c). For derivation of the expression, refer to a Textbook of Physics.

69. The radius r of the circular path is given by (see Fig. 13.79)

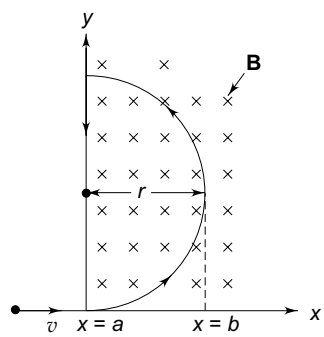


Fig. 13.79

$$\frac{mv^2}{r} = qvB$$

or $\quad v = \dfrac{qB}{m}\,(r)$

$\therefore\ v_{\min} = \dfrac{qB}{m}\,(r_{\min}) = \dfrac{qB}{m}\,(b-a)$, which is choice (b).

70. Let m be the pole strength of each pole of the magnet (see Fig. 13.80). The magnetic field at C due to the N-pole is given by

$$B_1 = \frac{\mu_0}{4\pi}\frac{m}{(AC)^2}$$

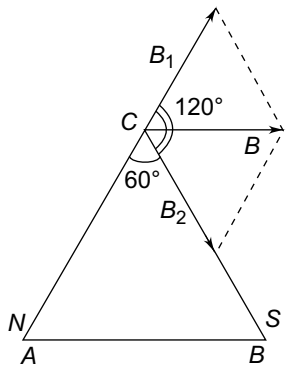


Fig. 13.80

direction along AC away from C. The magnetic field at C due to the S-pole is given by

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{m}{(BC)^2}$$

directed along CB towards B. Since $AC = BC$, $B_1 = B_2$. The resultant magnetic field at C is given by

$$B^2 = B_1^2 + B_2^2 + 2B_1 B_2 \cos 120°$$
$$= B_1^2 + B^2{}_2 - B_1 B_2$$
$$= 2B_1^2 - B_1^2 = B_1^2 \qquad (\because\ B_2 = B_1)$$

or $\quad B = B_1 = \dfrac{\mu_0 m}{4\pi(AC)^2} = \dfrac{\mu_0 m}{4\pi a^2}$

$$= \frac{\mu_0}{4\pi}\cdot\frac{(ma)}{a^3} = \frac{\mu_0}{4\pi}\cdot\frac{M}{a^3} \qquad (1)$$

Given: $M = 1$ A m^2, $a = 10$ cm $= 0.1$ m. Also $\mu_0 = 4\pi \times 10^{-7}$ T A^{-1} m. Substituting these values in (1), we get $B = 10^{-4}$ T, which is choice (d).

71. Let r be the radius of the circle. The length of the arc $= (2\pi r) \times \dfrac{60°}{360°} = \dfrac{\pi r}{3}$. Therefore, the length $2l$ of the magnet is

$$2l = \frac{\pi r}{3} \text{ or } r = \frac{6l}{\pi}$$

If m is the pole strength of each pole of the magnet, the magnetic moment of the arc $= m \times r = m \times \dfrac{6l}{\pi}$

$= \dfrac{3\times(2ml)}{\pi} = \dfrac{3M}{\pi}$.

Hence the correct choice is (c).

72. Refer to Fig. 13.81. Let a north pole be placed at B and a south pole at C so that they attract with a force F. A north pole placed at the third vertex A is repelled with a force F_1 by the north pole at B and attracted with a force F_2 towards the south pole at C. Since all pole strengths are equal, $F_1 = F_2 = F$. The resultant force experienced by the north pole at A is given by

$$F_r^2 = F_1^2 + F_2^2 + 2F_1 F_2 \cos(120°)$$
$$= F^2 + F^2 + 2F^2 \times \left(-\frac{1}{2}\right) = F^2$$

or $F_r = F$, which is choice (b). $\qquad (\because\ F_1 = F_2 = F)$

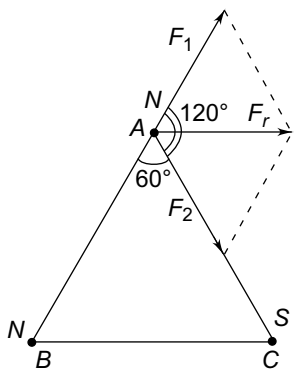


Fig. 13.81

73. The magnetic field at the centre of coil is given by

$$B = \frac{\mu_0 In}{2r}$$

Hence the correct choice is (d).

74. Given $v = (2\times10^5\ \hat{\mathbf{i}})$ ms^{-1}. The force vector is given by

$$\mathbf{F} = q(\mathbf{v}\times\mathbf{B})$$
$$= q\{(2\times10^5\ \mathbf{i})\times(\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 3\hat{\boldsymbol{k}})\}$$
$$= 2\times10^5 \times q\,(-4\hat{\boldsymbol{k}} + 3\hat{\mathbf{j}})$$

Therefore, the y and z components of the force are

$$F_y = 6\times10^5 \times q$$

and $\quad F_z = -8\times10^5\times q$

$\therefore$ Magnitude of force $= \sqrt{F_y^2 + F_z^2}$

$$= q\sqrt{(6\times10^5)^2 + (-8\times10^5)^2}$$

$= q \times 10 \times 10^5$

$= 1.6 \times 10^{-19} \times 10 \times 10^5$

$= 1.6 \times 10^{-13}$ N, which is choice (c).

75. $\frac{1}{2} mv^2 = eV$ or $v = \sqrt{\frac{2eV}{m}}$ (1)

Also $B\,eV = \frac{mv^2}{r}$ or $v = \frac{Ber}{m}$ (2)

Equating (1) and (2), we get

$$r = \frac{1}{B}\sqrt{\frac{2mV}{e}} \quad (3)$$

Given $e = 1.6 \times 10^{-19}$ C (singly charged ion), $B = 0.4$ T, $V = 500$ V and $m = 1.13 \times 10^{-26}$ kg. Using these values in (3), we get $r = 0.021$ m = 2.1 cm, which is choice (a).

76. Given $\mathbf{v} = (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ ms^{-1} and $B = (2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ tesla.

Force experienced by the proton is

$\mathbf{F} = q\,(\mathbf{v} \times \boldsymbol{B}) = q\,(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \times (2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$= q\,(6\hat{\mathbf{i}} \times \hat{\mathbf{j}} + 9\mathbf{i} \times \hat{\mathbf{k}} + 4\hat{\mathbf{j}} \times \hat{\mathbf{j}} + 6\hat{\mathbf{j}} \times \hat{\mathbf{k}})$

$= q\,(6\hat{\mathbf{k}} - 9\hat{\mathbf{j}} + 0 + 6\hat{\mathbf{i}})$

$= 3q\,(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ newton

$\therefore$ Acceleration $= \frac{\boldsymbol{F}}{m} = \frac{3q}{m}(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$= 3 \times (0.96 \times 10^8)\,(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$= 2.88 \times 10^8\,(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ ms^{-2}

Hence the correct choice is (b).

77. Refer to Fig. 13.82. Let us find the net magnetic field at corner C of the square $ABCD$. For the magnet at corner B, the point C is on the axial line at a distance d from the centre of the magnet. For a short magnet, the magnetic field at C is given by

$$B_1 = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$$

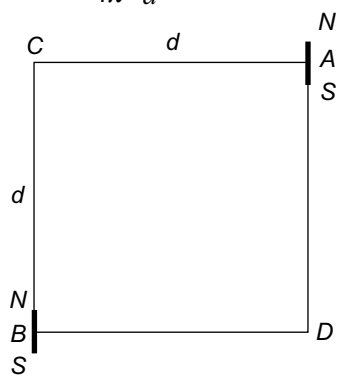


Fig. 13.82

For the magnet at corner A, the point C is on the equatorial line at a distance d from its centre. For a short magnet, the magnetic field at C due to this magnet is given by

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{M}{d^3}$$

Since their like poles are in the same direction, the net magnetic field at C is

$B = B_1 - B_2$

$= \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3} - \frac{\mu_0}{4\pi} \cdot \frac{M}{d^3} = \frac{\mu_0}{4\pi} \cdot \frac{M}{d^3}$

Hence the correct choice is (a).

78. From Ampere's law, we have

$$\oint \boldsymbol{B} \cdot \boldsymbol{dl} = \mu_0 I$$

Since no current exists in the medium (air) inside the pipe $I = 0$. Hence $\mathbf{B} = 0$. Hence the correct choice is (b).

79. Force F on a wire of length $l = BI\,l$. Therefore, force per unit length of the wire $= \frac{F}{l} = BI = 10^{-4} \times 10 = 10^{-3}$ Nm^{-1}, which is choice (c).

80. The magnetic field at point P due to current I_1 in conductor AOB is

$$B_1 = \frac{\mu_0 I_1}{2\pi a}$$

and the magnetic field at point P due to current I_2 in conductor COD is

$$B_2 = \frac{\mu_0 I_2}{2\pi a}$$

Since the two conductors are perpendicular to each other, fields B_1 and B_2 will be perpendicular to each other. Therefore, the resultant field at P is

$$B = (B_1^2 + B_2^2)^{1/2} = \frac{\mu_0}{2\pi a}(I_1^2 + I_2^2)^{1/2}$$

Hence the correct choice is (c).

81. The kinetic energy of proton is

$K = 2$ MeV $= 2 \times 10^6$ eV

$= 2 \times 10^6 \times 1.6 \times 10^{-19}$ J $= 3.2 \times 10^{-13}$ J

$\therefore \quad \frac{1}{2} mv^2 = 3.2 \times 10^{-13}$

Now, mass of proton is $m = 1.67 \times 10^{-27}$ kg. Therefore,

$$v^2 = \frac{2 \times 3.2 \times 10^{-13}}{1.67 \times 10^{-27}} = 3.83 \times 10^{14}$$

or $v = 1.96 \times 10^7$ ms^{-1}. Now force on

proton is

$$F = evB$$
$$= 1.6 \times 10^{-19} \times 1.96 \times 10^{7} \times 2.5$$
$$= 7.84 \times 10^{-12}\ \text{N}$$

Hence the closest choice is (d).

82. The point $x = + a$ lies along the line of the straight section PQ of the circuit. Hence the magnetic field at point $x = a$ is zero.

83. The radius of the circular path of a particle of mass m, charge e moving with a speed v perpendicular to a magnetic field B is given by

$$\frac{mv^2}{r} = evB \text{ or } r = \left(\frac{m}{e}\right)\frac{v}{B}$$

Thus, r is inversely proportional to $\left(\frac{e}{m}\right)$, the charge to mass ratio. Hence the correct choice is (c).

84. The total Lorentz force on the electron is

$$\mathbf{F} = -e\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

The electron will be undeflected if $\mathbf{v} \perp \mathbf{B}$. If $\mathbf{E}$ is along + z-direction, the force $-e\,\mathbf{E}$ will be along $-z$-direction. If $\mathbf{B}$ is along + x direction, force $-e\,(\mathbf{v} \times \mathbf{B})$ will be along + z direction. When $eE = evB$, the electron moves along + y-direction undeflected. Hence the correct choice is (c). Thus, for an electron moving along + y direction, the electric field should be along + z direction and magnetic field along + x direction, then the electron will be undeflected.

85. Given $x = 4$ cm = 0.04 m and $r = 3$ cm = 0.03 m. The magnetic field at a point on the axis of the loop is given by

$$B = \frac{\mu_o I r^2}{2(r^2 + x^2)^{3/2}} \quad (1)$$

Magnetic field at the centre of the coil is given by

$$B_0 = \frac{\mu_o I}{2r} \quad (2)$$

Dividing (1) by (2), we get

$$\frac{B_0}{B} = \frac{r^3}{(r^2 + x^2)^{3/2}}$$

Substituting the values of r and x, we get

$$\frac{B_0}{B} = \frac{125}{27}$$

or $$B_0 = \frac{125}{27}\,B = \frac{125}{27} \times 54\ \mu\text{T} = 250\ \mu\text{T}$$

Hence the correct choice is (a).

86. Wires AB and EF experience no forces since currents in them are parallel to the magnetic field. The forces on BC and DE are equal in magnitude but are directed in opposite directions. Hence their resultant is zero. Only force acting is on CD. Hence the correct choice is (a).

87. $$\mathbf{F} = q(\boldsymbol{v} \times \boldsymbol{B}) = q\{a\,\hat{\mathbf{i}} \times (b\,\hat{\mathbf{j}} + c\,\hat{\mathbf{k}})\}$$
$$= q(ab\,\hat{\mathbf{i}} \times \hat{\mathbf{j}} + ac\,\hat{\mathbf{i}} \times \hat{\mathbf{k}})$$
$$= q(ab\,\hat{\mathbf{k}} - ac\,\hat{\mathbf{j}}) = qa(b\,\hat{\mathbf{k}} - c\,\hat{\mathbf{j}})$$

Magnitude of $F = qa(b^2 + c^2)^{1/2}$

The correct choice is (d).

88. Refer to Fig. 13.83. Let $AB = BC = AC = a$. Let $OD = r$.

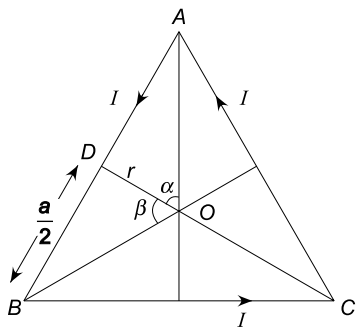


Fig. 13.83

The magnetic field at centroid O due to current I flowing in side AB of the triangle is given by

$$B_{AB} = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$$

It is clear that $\alpha = \beta = 60°$ and

$$OD = r = \frac{AD}{\tan\alpha} = \frac{a/2}{\tan 60°} = \frac{a/2}{\sqrt{3}} = \frac{a}{2\sqrt{3}}$$

$$\therefore \quad B_{AB} = \frac{\mu_0 I}{4\pi} \times \frac{2\sqrt{3}}{a} \times (\sin 60° + \sin 60°)$$

$$= \frac{3\mu_0 I}{2\pi a}$$

By symmetry, the magnetic fields due to current in sides BC and AC = that due to side AB. Hence, the magnetic field at O due to the current in the three sides of triangle ABC is

$$B = B_{AB} + B_{BC} + B_{CA} = 3B_{AB}$$

Hence the correct choice is (b).

89. Refer to Fig. 13.84. Let v be the velocity of the particle. Its kinetic energy is

$$\frac{1}{2}mv^2 = qV \text{ or } v = \left(\frac{2qV}{m}\right)^{1/2} \quad (1)$$

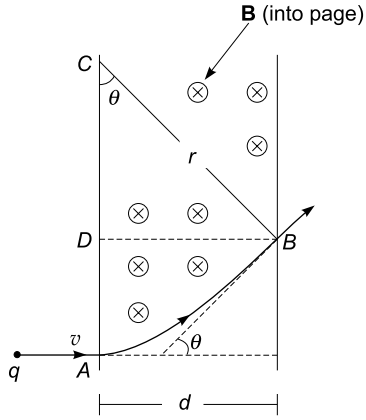


Fig. 13.84

The particle follows a circular path from A to B of radius r which is given by

$$\frac{mv^2}{r} = qvB \text{ or } r = \frac{mv}{qB} \quad (2)$$

Using (1) and (2), we have

$$r = \frac{m}{qB}\left(\frac{2qV}{m}\right)^{1/2} = \frac{1}{B}\left(\frac{2mV}{q}\right)^{1/2}$$

I n triangle BCD, $\sin\theta = \frac{BD}{BC} = \frac{d}{r}$. Therefore,

$\sin\theta = Bd\left(\frac{q}{2mV}\right)^{1/2}$, which is choice (a).

90. Refer to Fig. 13.85. Wire PQ of length d, the spacing between rails carries a current I vertically downwards in a magnetic field pointing towards the reader and perpendicular to the length PQ of the wire. Thus angle θ between I and B is 90°. The force exerted on the wire of length d by the magnetic field is

$F = BId \sin 90° = BId$

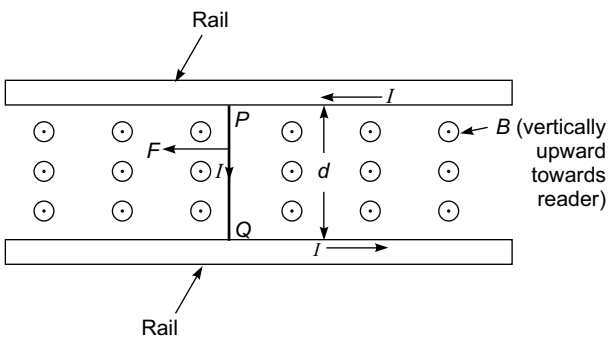


Fig. 13.85

Using Fleming's left hand rule, the direction of the force is to the left. The acceleration of the wire is

$$a = \frac{\text{force}}{\text{mass}} = \frac{F}{m} = \frac{BId}{m}$$

Now $x = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2x}{a}}$. Hence the correct choice is (b).

91. Refer to Fig. 13.86. Since the velocity of the particle is $v = a$ along the positive x-axis and the direction of the magnetic field $B = b$ in the positive z-direction, and the charge of the particle is positive, the path of the particle is a circle as shown in the figure. The radius of the circular path is

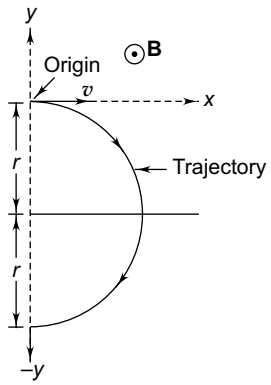


Fig. 13.86

$$r = \frac{mv}{qB} = \frac{ma}{qb}$$

Thus $y = -2r = -\frac{2ma}{qb}$

So the correct choice is (d).

92. Refer to Fig. 13.87. It follows from Fleming's left hand rule that the wire segment PQ experiences a force $F_1 = B\,I\,l_1$ directed to the right and the wire segment SR experiences an equal force $F_1 = B\,I\,l_1$ but directed to the left. These forces cancel each other. The wire segment QR experiences a force $F = B\,I\,l_2$ directed vertically upward. The wire frame will remain stationary in air if F equal the weight mg of the wire frame, *i.e.*

$$B\,I\,l_2 = mg$$

$$\Rightarrow \quad I = \frac{mg}{l_2\,B}, \quad \text{which is choice (a).}$$

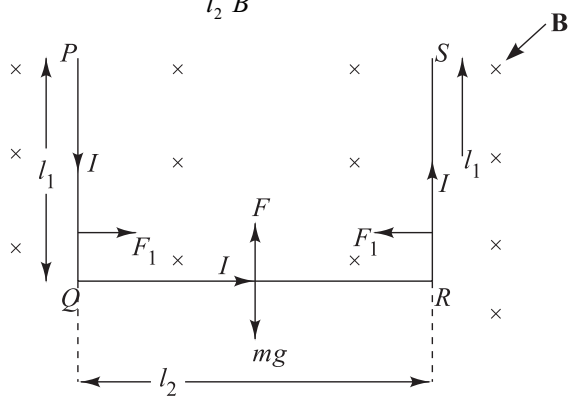


Fig. 13.87

93. Refer to Fig. 13.88.

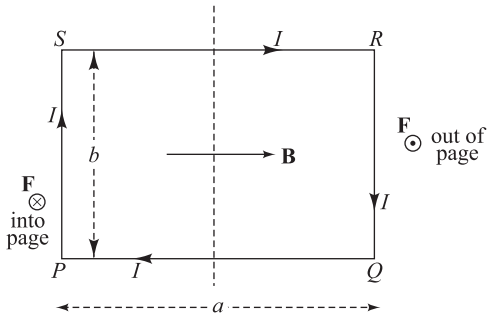


Fig. 13.88

The current element *SR* is parallel to **B**. Therefore $\theta = 0^0$. Hence, force exerted on arm *SR* $= B\,I\,a$ sin 0° = 0. The current element *QP* is antiparallel to *B*. Therefore, θ = 180°. Hence, force exerted on arm *QP* $= B\,I\,a$ sin 180° = 0. From Fleming's left hand rule, the force exerted on arm *PS* is $F = B\,I\,b$ sin 90° $= B\,I\,b$ directed into the page.

The force exerted on arm *RQ* is also $F = B\,I\,b$ but directed out of the page. These equal and opposite forces constitute a couple which exert a torque

τ = magnitude of either force × per perpendicular distance between the antiparallel forces

$= F \times PQ$

$= B\,I\,b\,a$

This torque will rotate the coil in the clockwise sense.

So the correct choice is (c).

94. Current segments *PQ* and *ST* are perpendicular to *B*. Therefore, the magnetic forces on *PQ* and *ST* are

$$F_1 = B\,I\,l \sin 90° = B\,I\,l$$

and $$F_2 = B\,I\,l \sin 90° = B\,I\,l$$

To find the magnetic force on the semi-circular arc *QRS* we divide it into a very large number of extremeley small lengths *dl*. Figure 13.89 shows one such element at *A*. The magnetic force on the element is

dF = I (**dl** × **B**). Since θ = 90°, the magnitude of **dF** is

$dF = IdlB \sin 90°$

$= IdlB = IBrd\theta$ $\quad (\because \quad dl = rd\theta)$

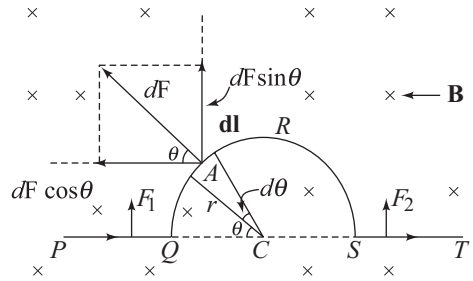


Fig. 13.89

The direction of **dF** is radially away from the centre *C*. The total horizontal component of force on the semi-circle is

$$(F_3)_x = \int dF \cos\theta$$

$$= I\,B\,r \int_0^{\pi} \cos\theta\, d\theta$$

$$= I\,B\,r \left|\sin\theta\right|_0^{\pi} = 0$$

The total vertical component of the force on the semi-circle is

$$(F_3)_y = \int dF \sin\theta$$

$$= I\,B\,r \int_0^{\pi} \sin\theta\, d\theta$$

$$= I\,B\,r \left| -\cos\theta \right|_0^{\pi} = 2IBr$$

∴ The total force on the wire frame is (since F_1, F_2 and $(F_3)_y$ are all directed vertically upward)

$$F = F_1 + F_2 + (F_3)_y$$

$$= BIl + BIl + 2\,BIr$$

$= 2\,BI\,(l + r)$. The direction of **F** is vertically upward.

So the correct choice is (a).

95. $\boldsymbol{F} = -q\,(\boldsymbol{v} \times \mathbf{B})$. The direction of $(\boldsymbol{v} \times \mathbf{B})$ is out of the page. Since $-(\boldsymbol{v} \times \mathbf{B}) = (\mathbf{B} \times \boldsymbol{v})$, we have

$$\mathbf{F} = q\,(\mathbf{B} \times \boldsymbol{v})$$

The direction of $(\mathbf{B} \times \boldsymbol{v})$ is into the page. So the correct choice is (d).

96. Statement I is false. The magnetic field lines due to a current carrying wires encircle the wire in closed loops. Statement II is also false because the magnetic force **F** is always perpendicular to the velocity $\boldsymbol{v}$ of the particle. Therefore power (and hence work done) $= \vec{\mathbf{F}}.\vec{v} = 0$. Since no work is done by the magnetic force, the kinetic energy of the particle cannot change. Statement III is true. Magnetic force is zero if $\boldsymbol{v}$ parallel or antiparallel to **B**. So the only correct choice is (d).

97. The magentic force provides the necessary centripetal force. Therefore,

$$qvB = \frac{mv^2}{r} \quad \Rightarrow \quad mv = qBr$$

Linear momentum $p = mv = qBr$

Angular momentum $l = pr = qBr^2$

So the correct choice is (a).

98. The magnitude of the magnetic field at a point at a distance r from a long wire carrying a current I is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

Therefore, the magnitude of the magnetic field at point P due to current in wire 1 is (since $r = d/2$).

$$B_1 = \frac{\mu_0 I}{2\pi (d/2)} = \frac{\mu_0 I}{\pi d}$$

From the right- hand thumb rule, the direction of the field is into the page. For wire 2,

$$B_2 = \frac{\mu_0 I}{\pi d} \quad \text{out of the page}$$

These two fields have equal magnitude but opposite direction. Hence they cancel each other. So the correct choice (d).

99. In this case, the two fields are both into the page and the magnitude of the field at P is

$$B = B_1 + B_2 = \frac{2\mu_0 I}{\pi d}$$

and its direction is into the page. So the correct choice is (a).

100. Figure 13.90 is the cross-sectional view of the cylinder and the shell.

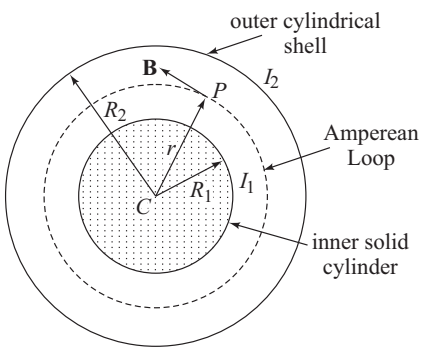


Fig. 13.90

From Ampere's Circuital law

$$\oint \mathbf{B} \cdot \mathbf{dl} = \mu_0 i$$

The current element **dl** at P is parallel to **B** at P. Hence $\theta = 0$ and

$$\oint \mathbf{B} \cdot \mathbf{dl} = \oint B\, dl \cos 0° = B \int dl = B \times 2\pi r$$

The current through the Amperean loop is

$$i = I_1$$

Therefore,

$$B \times 2\pi r = \mu_0 I_1$$

$$\Rightarrow \quad B = \frac{\mu_0 I_1}{2\pi r}$$

Putting $I_1 = I$ and $r = 3R/2$, we get

$$B = \frac{\mu_0 I}{3\pi R}, \quad \text{which is choice is (a).}$$

101. Refer to Fig. 13.91.

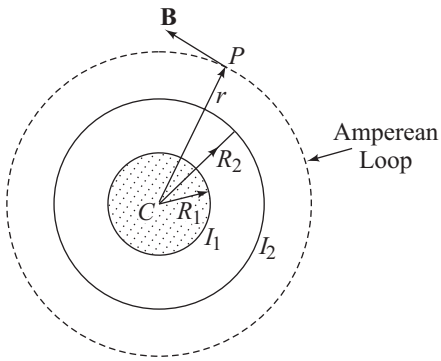


Fig. 13.91

In this case

$$B \times 2\pi r = \mu_0 i = \mu_0 (I_1 + I_2)$$

$$\Rightarrow \quad B = \frac{\mu_0 (I_1 + I_2)}{2\pi r}$$

Putting $I_1 = I$, $I_2 = I/2$ and $r = 3R$, we get

$$B = \frac{\mu_0 I}{12\pi R}$$

So the correct choice is (d).

102. In this case,

$$B \times 2\pi r = \mu_0 i = \mu_0 (I_1 - I_2)$$

$$\Rightarrow \quad B = \frac{\mu_0 (I_1 - I_2)}{2\pi r}$$

Putting $I_1 = I$, $I_2 = I/2$ and $r = 3R$, we get

$$B = \frac{\mu_0 I}{12\pi R}, \quad \text{which is choice (d).}$$

103. The straight portions PQ and RS do not produce magnetic field at C. Only the curved portion QR produces a magnetic field at C. Refer to Fig. 13.92.

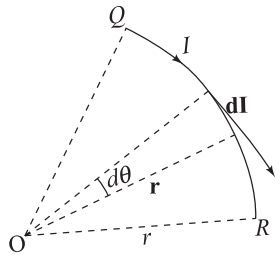


Fig. 13.92

Since the current **dl** is perpendicular to position vector **r**, the magnetic field at *C* by Biot-Savart law is

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin 90°}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{I\times(rd\theta)}{r^2} \qquad (\because\ dl = rd\theta)$$

$$= \frac{\mu_0 I}{4\pi r}d\theta$$

∴ Magnetic field at *C* due to arc *QR* is

$$B = \int dB = \frac{\mu_0 I}{4\pi r}\int_0^{\alpha} d\theta = \frac{\mu_0 I\,\alpha}{4\pi r}$$

So the correct choice is (d).

104. Since the particle describes a circular path, it is obvious that it enters the region of **B** with velocity ***v*** perpendicular to **B** as shown in Fig 13.93.

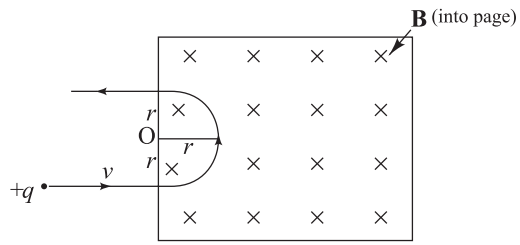


Fig. 13.93

Since the kinetic energy of the particle remains constant (because its velocity vector is always perpendicular to the force vector which is radial), it describes a semi-circle in the region of the magnetic field. Since its K.E. (and hence its speed *v*) remains constant, the time the particle takes to describe the semi-circle is

$$t = \frac{\pi r}{v} \qquad (1)$$

Also, kinetic energy = qV

or $$\frac{1}{2}mv^2 = qV$$

$$\Rightarrow \qquad v = \sqrt{\frac{2qV}{m}} \qquad (2)$$

Using (2) in (1) we get

$$t = \pi r\sqrt{\frac{m}{2qV}},$$ which is choice (c).

105. The current density $J = \dfrac{I}{\pi R^2}$.

For a homogeneous cylinder, the current density *J* is constant and does not depend on the radial distance *r* of the point *P* from centre *O* of the cylinder (see Fig. 13.94)

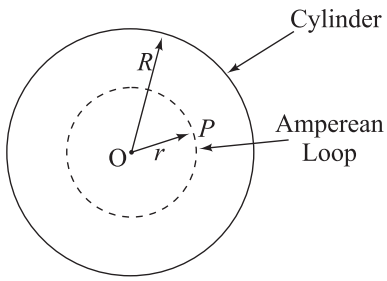


Fig. 13.94

Since the point *P* is inside the cylinder, $r < R$, the current enclosed by the Amperean loop is

$$i = J \times \text{area of Amperean loop}$$

$$= \frac{I}{\pi R^2}\times\pi r^2 = \frac{I r^2}{R^2}$$

From Amperean Circuital law,

$$\oint \mathbf{B}\cdot\mathbf{dl} = \mu_0 i$$

or $$B\times 2\pi r = \mu_0 \times \frac{I r^2}{R^2}$$

$$\Rightarrow \qquad B = \frac{\mu_0 I r}{2\pi R^2} \qquad (1)$$

Putting $r = \dfrac{R}{2}$ in eq. (1), we get

$$B = \frac{\mu_0 I}{4\pi R}$$, which is choice (d).

106. Refer to Fig 13.95.

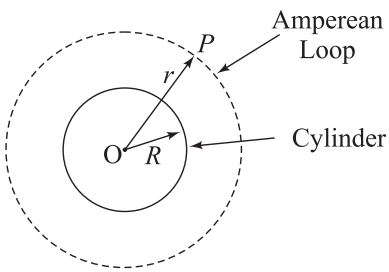


Fig. 13.95

In the case $r > R$, the current enclosed inside the Amperean loop = current on the cylinder, *i.e.* $i = I$. Hence

$$B\times 2\pi r = \mu_0 I$$

$$\Rightarrow \qquad B = \frac{\mu_0 I}{2\pi r}$$

For $r = 2R$,

$$B = \frac{\mu_0 I}{4\pi R}$$, which is choice (b).

107. Since J varies with r, to find the current I through the rod, we divide it into thin cylindrical strips each of a small width dr (see Fig 13.96)

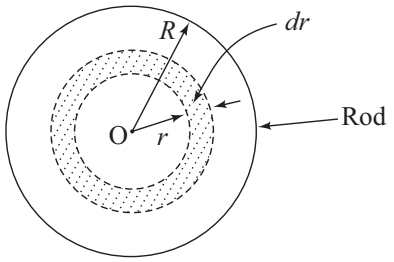


Fig 13.96

Area of strip $dA = (2\pi r)\, dr$. So the current through the strip is

$$dI = JdA = \sigma r \times (2\pi r)dr$$

$$= 2\pi\sigma r^2 dr$$

Therefore, current through the cylinder is

$$I = \int dI = 2\pi\sigma \int_0^R r^2\, dr$$

$$\Rightarrow \qquad I = \frac{2}{3}\pi\sigma R^3 \qquad (1)$$

We consider Amperean loop of radius r. The current through the loop is

$$i = 2\pi\sigma \int_0^r r^2\, dr = \frac{2}{3}\pi\sigma r^3$$

From Ampere's circuital law,

$$B \times 2\pi r = \mu_0 i = \mu_0 \times \frac{2}{3}\pi\sigma r^3$$

$$\Rightarrow \qquad B = \frac{1}{3}\mu_0 \sigma r^2 \qquad (2)$$

From Eq. (1)

$$\sigma = \frac{3I}{2\pi R^3} \qquad (3)$$

Using (3) in (2) we get

$$B = \frac{1}{3}\mu_0 \times \left(\frac{3I}{2\pi R^3}\right) r^2$$

$$\Rightarrow \qquad B = \frac{\mu_0 I r^2}{2\pi R^3}$$

So the correct choice is (a).

Note For points outside the rod, $r > R$, the current enclosed by the Amperean loop = current through the cylinder, *i.e.* $i = I$. So

$$B \times 2\pi r = \mu_0 I$$

$$\Rightarrow \qquad B = \frac{\mu_0 I}{2\pi r}$$

which is the same as the expression for a homogeneous rod for the case $r > R$.

108. The magnetic field due to straight segments PQ and TS in both cases in zero.

Case (a)

The magnetic field at C due to semi-circular loop of radius R is $B = \frac{\mu_0 I}{4R}$ directed out of the page, since the current is clockwise.

The magnetic field at C due to semi-circular loop of radius $2R$ is $B' = \frac{\mu_0 I}{4(2R)} = \frac{\mu_0 I}{8R}$ directed into the page since the current now is anticlockwise.

Since $B > B'$, the net magnetic field at C is

$$B_1 = B - B' = \frac{\mu_0 I}{4}\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{\mu_0 I}{8R}$$

directed out of the page.

Case (b)

In this case, the current in both loops is clockwise. Therefore, the magnetic field due to each semi-circular loop is directed into the page and its magnitude is

$$B_1 = B + B' = \frac{\mu_0 I}{4}\left(\frac{1}{R} + \frac{1}{2R}\right) = \frac{3\mu_0 I}{8R}$$

directed into the page.

Therefor, $\frac{B_1}{B_2} = \frac{1}{3}$, which is choice (c).

109. The correct choice is (a) as explained above.

110. Since the plane of coil A is vertical, it is perpendicular to the magnetic meridian. Hence the magnetic field produced by current I_A can neutralize the horizontal component (B_H) of earth's magnetic field. Similarly, current I_B can neutralize the vertical component (B_V). Hence

$$\frac{\mu_0 I_A n}{2r} = B_H$$

and $\dfrac{\mu_0 I_B n}{2r} = B_V$

Dividing we get, $\dfrac{I_A}{I_B} = \dfrac{B_H}{B_V} = \dfrac{B_H}{B_H \tan\theta} = \cot\theta$

So the correct choice is (d).

111. $r = \dfrac{\sqrt{2mqV}}{qB} = \dfrac{1}{B}\left(\dfrac{2mV}{q}\right)^{1/2}$

$\therefore \quad \dfrac{q}{m} = \dfrac{2V}{B^2 r^2}$

$\therefore \quad \dfrac{\left(\frac{q}{m}\right)_X}{\left(\frac{q}{m}\right)_Y} = \left(\dfrac{r_2}{r_1}\right)^2 \qquad (\because B \text{ and } V \text{ are the same})$

So the correct choice is (d).

112. The forces acting on arms *QR* and *PS* are equal and opposite and hence cancel each other. The current in arm *QP* is antiparallel to current in *AB*. Hence *QP* and *AB* will repal each other. The current in *SR* is parallel to current in *AB*. Hence *SR* and *AB* will attract each other. Since force per unit length is $f = \dfrac{\mu_0 I i}{r}$ it follows that $f \propto \dfrac{1}{r}$.

Since arm *QP* is closer to *AB* than arm *SR*, it follows that the repulsive force on the coil is greater than the attractive force. So the correct choice is (c).

113. Since the currents in the two wires are in the same direction, the magnetic fields due to the current *I* in the wire at point *P* exactly mid-way between them will be equal and opposite. The net magnetic field at point *P* is zero. So the correct choice is (d).

114. If the two wires carry current *I* in opposite directions, the magnetic field at *P* due to current *I* in each wire $= \dfrac{\mu_0 I}{2\pi (d/2)} = \dfrac{\mu_0 I}{\pi d}$ directed perpendicular to the plane of the wires. The net magnetic field at *P* is

$$B' = \frac{\mu_0 I}{\pi d} + \frac{\mu_0 I}{\pi d} = \frac{2\mu_0 I}{\pi d} = 2B$$

Force on charge q is $\mathbf{F} = q(\boldsymbol{v} \times \mathbf{B}') = qvB' \sin\theta = 2\,qvB \sin\theta$.

Since $\mathbf{B}'$ and $\boldsymbol{v}$ are both perpendicular to the plane containing the wires, angle θ between them is zero. Hence $F = 0$ in this case also. So the correct choice is (a).

115. $\dfrac{mv^2}{r} = qvB \quad \Rightarrow \quad mv = qrB$

$\therefore \quad m_1 v_1 = qr_1 B \quad \text{and} \quad m_2 v_2 = qr_2 B$

$\therefore \quad \dfrac{m_1 v_1}{m_2 v_2} = \dfrac{r_1}{r_2} < 1 \quad (\because r_1 < r_2)$

So the correct choice is (c).

116. $\mathbf{F} = q(\mathbf{u} \times \mathbf{B})$. Since force $\mathbf{F}$ is perpendicular to $\mathbf{u}$, it does no work on the particle. Hence the speed of the proton remains unchanged, *i.e.* $v = u$ [see Fig. 13.97] From Fleming's L.H. rule, the force is directed upwards. Hence the proton, after completing a semi-circle in the region of magnetic field emerges at positive *y*-coordinate.

So the correct choice is (a).

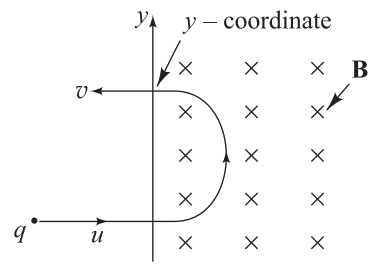


Fig. 13.97

117. Since to loop carries a current, a charge (say q) moves along the circle with a velocity (say v). The velocity v is tangential to the circle at every point. The direction of v gives the direction of the current as shown in Fig 13.98.

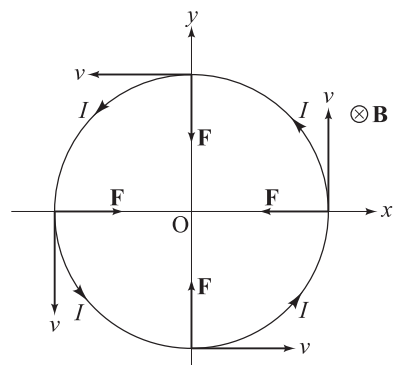


Fig. 13.98

From Fleming's left hand rule, the direction of the force $\mathbf{F}$ exerted by magnetic field is radially inwards towards *O* at every point. Hence the loop tends to contract. Furthermore, since $\mathbf{F}$ is perpendicular to $\boldsymbol{v}$, the force does no work on the loop. Hence it cannot have any translatory motion. Thus the correct choice is (c).

Note The loop will tend to expand if either the current in the loop is clockwise or the magnetic field points out of the plane of the coil.

118. Refer to Fig. 13.99. The magnetic field at O due to parts AB and EF of the frame is zero.

From junction rule.

$$I_1 + I_2 = I \qquad (1)$$

Applying loop rule to loop $BCDEB$, we get (here R = resistance of $BC = CD = DE = BE$)

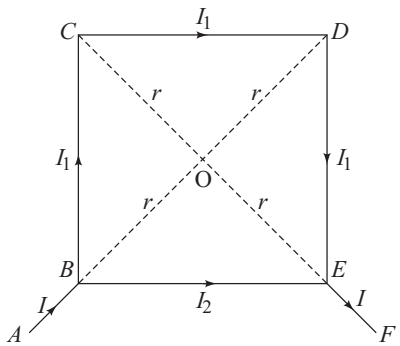


Fig. 13.99

$$3RI_1 = RI_2 \quad \Rightarrow \quad 3I_1 = I_2 \qquad (2)$$

From (1) and (2), we get $I_1 = \dfrac{I}{4}$ and $I_2 = \dfrac{3I}{4}$.

Magnetic field at O due current I_1 in BC is

$$B_{BC} = \frac{\mu_0 I_1}{4\pi r}(\sin 45^0 + \sin 45^0) = \frac{\mu_0 I_1}{2\sqrt{2}\pi r}$$

But $I_1 = I/4$, therefore

$$B_{BC} = \frac{\mu_0 I}{8\sqrt{2}\pi r} \quad \text{directed into the page,}$$

Similarly,

$$B_{CD} = \frac{\mu_0 I}{8\sqrt{2}\pi r} \quad \text{directed into the page,}$$

and $$B_{DE} = \frac{\mu_0 I}{8\sqrt{2}\pi r} \quad \text{directed into the page,}$$

$$\therefore \quad B_{BCDE} = \frac{3\mu_0 I}{8\sqrt{2}\pi r} \qquad (3)$$

directed into the page.

Magnetic field at O due to current I_2 in BE is

$$B_{BE} = \frac{\mu_0 I_2}{2\sqrt{2}\pi r} = \frac{3\mu_0 I}{8\sqrt{2}\pi r} \qquad (4)$$

directed out of the page.

From (3) and (4) it follows that the magnetic field at O due to the complete frame is zero. So the correct choice is (d).

119. Magnetic field at O due to current I in AB is

$$B_{AB} = \frac{\mu_0 I}{4\pi r}(\sin 90° + \sin 0°) = \frac{\mu_0 I}{4\pi r}$$

directed out of the page. Similarly

$$B_{ED} = \frac{\mu_0 I}{4\pi r} \quad \text{directed out of the page.}$$

Magnetic field at O due to current I in semi-circular part BCD is

$$B_{BCD} = \frac{\mu_0 I}{4r} \quad \text{directed out of the page.}$$

$\therefore$ Magnetic field at O due to $ABCDE$ is

$$B = B_{AB} + B_{ED} + B_{BCD}$$

$$= \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4r}$$

$$= \frac{\mu_0 I}{4r}(2\pi + 1) \quad \text{directed out of the page.}$$

So the correct choice is (c).

120. $B_{AB} = 0$

$$B_{DE} = \frac{\mu_0 I}{4\pi r} \quad \text{directed into the page}$$

$$B_{DCB} = \frac{\mu_0 I}{4r} \quad \text{directed into the page}$$

$$\therefore \quad B_{ABCDE} = 0 + \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4r}$$

$$= \frac{\mu_0 I}{4\pi r}(\pi + 1)$$

So the correct choice is (a).

121. $B = \dfrac{\mu_0 I}{2r} \times \dfrac{\theta}{2\pi}$. Here $\theta = 60^0 = \dfrac{\pi}{3}$. Therefore

$$B = \frac{\mu_0 I}{12r}$$

Magnetic field at O due to current I in AD is

$$B_{AD} = \frac{\mu_0 I}{12r} \quad \text{directed out of the page.}$$

Magnetic field at O due to current I in BC is

$$B_{BC} = \frac{\mu_0 I}{12(2r)} = \frac{\mu_0 I}{24 r} \quad \text{directed into the page.}$$

Therefore, Magnetic field at O due to $ABCD$ is

$$B = B_{AD} - B_{BC}$$

$$= \frac{\mu_0 I}{12r} - \frac{\mu_0 I}{24r} = \frac{\mu_0 I}{24r}$$

directed out of the page. The magnetic field at O due to straight segments AB and CD is zero. So the correct choice is (a).

122. Magnetic field at O due to straight segments BC and AD is zero. For the curved part AB, $\theta = \frac{3\pi}{2}$. Therefore,

$$B_{AB} = \frac{3\mu_0 I}{8r} \quad \text{directed into the page.}$$

For the curved part CD, $\theta = \frac{\pi}{2}$. Therefore,

$$B_{CD} = \frac{\mu_0 I}{16r} \quad \text{directed into the page.}$$

Therefore, magnetic field at O due to current I in $ABCD$ is

$$B = B_{AB} + B_{CD}$$

$$= \frac{3\mu_0 I}{8r} + \frac{\mu_0 I}{16r} = \frac{7\mu_0 I}{16r}$$

directed into the page. So the correct choice is (d).

123. Refer to Fig. 13.100.

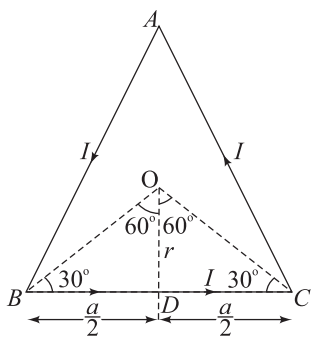


Fig. 13.100

Magnetic field at O due to current I in side BC is

$$B_{BC} = \frac{\mu_0 I}{4\pi r}\left(\sin 60^0 + \sin 60^0\right)$$

$$= \frac{\sqrt{3}\,\mu_0 I}{4\pi r}$$

Now $r = \frac{a}{2}\tan 30^0 = \frac{a}{2\sqrt{3}}$. Therefore,

$$B_{BC} = \frac{\sqrt{3}\,\mu_0 I \times 2\sqrt{3}}{4\pi \times a} = \frac{3\mu_0 I}{2\pi a}$$

directed out of the page. The magnetic field at O due to current I in sides CA and AB is the same as B_{BC} and is directed out of the page. Hence, the magnitude of magnetic field at O due to current I in ABC is

$$B = 3B_{BC} = \frac{9\mu_0 I}{2\pi a}$$

So the correct choice is (d).

124. $\mathbf{F} = q\,(\boldsymbol{v} \times \mathbf{B})$. Hence $\mathbf{F}$ is perpendicular to $\mathbf{B}$. Therefore,

$$\mathbf{F}.\mathbf{B} = 0$$

or $$m\mathbf{a}.\mathbf{B} = 0$$

$$\Rightarrow \quad \mathbf{a}.\mathbf{B} = 0 \quad (\because m \neq 0)$$

$$\Rightarrow (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - n\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) = 0$$

$$\Rightarrow \quad 2 + 6 - 4n = 0$$

$$\Rightarrow \quad n = 2$$

So the correct choice is (b).

125. Magnetic moment $\mathbf{M} = I\mathbf{A}$

where $\mathbf{A}$ is the area vector. Since the current is clockwise the direction of $\mathbf{A}$ is along the negative z-axis, *i.e.*, along $-\hat{\mathbf{k}}$. The magnitude of $\mathbf{A}$ is

A = area of square of side a + area of four equilateral triangles each of side a

= area of $BDFH$ + 4 × area of BCD

$$= a^2 + \frac{4\sqrt{3}\,a^2}{4}$$

$$= a^2\left(\sqrt{3}+1\right)$$

$$\therefore \quad \mathbf{M} = -\left(\sqrt{3}+1\right)a^2 I\,\hat{\mathbf{k}}$$

So the correct choice is (c).

SECTION 2

Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage.

Passage I

Two long parallel wires carrying currents 2.5 amperes and I ampere in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of the paper. The points P and Q are located at a distance of 5m and 2m, respectively, from a collinear point R (see Fig. 13.78).

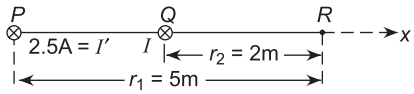


Fig. 78

An electron moving with a velocity of 4×10^5 m/s along the positive x-direction experiences a force of magnitude 3.2×10^{-20} N at the point R.

1. The magnitude of magnetic field at point R is
   (a) 2.5×10^{-7} T (b) 5.0×10^{-7} T
   (c) 5.0×10^{-6} T (d) 2.5×10^{-6} T
2. The magnitude of magnetic field at point R due to current $I' = 2.5$ A in wire P is
   (a) 1×10^{-7} T (b) 2×10^{-7} T
   (b) 3×10^{-7} T (d) 4×10^{-7} T
3. The current I in wire Q is
   (a) 1 A (b) 2 A
   (c) 3 A (d) 4 A

Solutions

1. The magnitude of the force experienced by a particle of charge q moving with a velocity v in a magnetic field B is given by
   $$F = qv\,B \sin\theta$$
   where θ is the angle between v and **B**. Given $F = 3.2 \times 10^{-20}$ N, $v = 4 \times 10^5$ ms^{-1} and $\theta = 90°$. For electron $q = 1.6 \times 10^{-19}$ C. Using these value we get $B = 5 \times 10^{-7}$ T, is choice (b)
2. The magnetic field at point R due to currect I' in wire P is
   $$B_1 = \frac{\mu_0 I'}{2\pi r_1} = \frac{4\pi \times 10^{-7} \times 2.5}{2\pi \times 5} = 1 \times 10^{-7}\ \text{T}$$
   The correct choice is (a).
3. The magnetic field at point R due to currect I in wire Q is
   $$B_2 = \frac{\mu_0 I}{2\pi r_2} = \frac{4\pi \times 10^{-7} \times I}{2\pi \times 2} = I \times 10^{-7}\ \text{T}$$
   Both fields B_1 and B_2 will be in the downward direction, parallel and collinear. Hence, the resultant magnetic field at point R is
   $$B = B_1 + B_2 = (1 + I) \times 10^{-7}\ \text{T}$$
   Now $B = 5 \times 10^{-7}$ T. Therefore,
   $$(1 + I) \times 10^{-7} = 5 \times 10^{-7}$$
   or $1 + I = 5$ or $I = 4$ A.
   So the correct choice is (d).

Questions 4 to 7 are based on the following passage.

Passage II

The region between $x = 0$ and $x = L$ is filled with a uniform, steady magnetic field $B_0\,\hat{\mathbf{k}}$. A particle of mass m, positive charge q and velocity $v_0\,\hat{\mathbf{i}}$ travels along x-axis and enters the region of the magnetic field. Neglect gravity.

4. The force experienced by the charged particle in the magnetic field is
   (a) along the positive y-direction
   (b) along the negative y-direction
   (c) in the x-y plane
   (d) in the y-z plane.
5. If the particle emerges from the region of magnetic field with its final velocity at an angle of 30° to its initial velocity, the value of L is
   (a) $\frac{2mv_0}{qB_0}$ (b) $\frac{mv_0}{qB_0}$
   (c) $\frac{mv_0}{2qB_0}$ (d) $\frac{\sqrt{3}mv_0}{2qB_0}$
6. If the magnetic field now extends up to $x = 2.1\,L$, the final velocity of the particle when it emerges out of the region of magnetic field will be
   (a) $v_0\,\hat{\mathbf{i}}$ (b) $-v_0\,\hat{\mathbf{i}}$
   (c) $v_0\,\hat{\mathbf{j}}$ (d) $-v_0\,\hat{\mathbf{j}}$

7. In Q. 6, the time spent by the particle in the magnetic field is

(a) $t = \dfrac{2\pi m}{qB_0}$ (b) $t = \dfrac{\sqrt{2}\pi m}{qB_0}$

(c) $t = \dfrac{\sqrt{3}\pi m}{2qB_0}$ (d) $t = \dfrac{\pi m}{qB_0}$

Solutions

4. The force experienced by the charged particle is given by

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) = q(v_0\,\hat{\mathbf{i}}) \times (B_0\,\hat{\mathbf{k}})$$
$$= qv_0B_0(\hat{\mathbf{i}} \times \hat{\mathbf{k}})$$
$$= qv_0B_0(-\hat{\mathbf{j}}) \qquad (1)$$

The force is along the negative y-direction, which is choice (b).

5. Refer to Fig. 13.101.

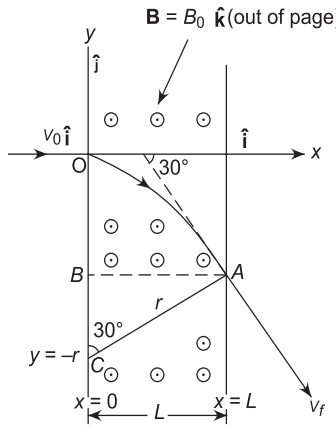


Fig. 13.101

The particle describes a circle of radius

$$r = \frac{mv_0}{qB_0} \qquad (2)$$

Since the particle emerges from the region of the magnetic field with the velocity vector making an angle of 30° with the initial vector, it follows from triangle ABC that

$$AB = AC \sin 30°$$

or $$L = r \sin 30° = \frac{mv_0 \sin 30°}{qB_0} = \frac{mv_0}{2qB_0} \qquad (3)$$

Thus the correct choice is (c).

6. Comparing (2) and (3) we find that $r = 2\ L$. Since the magnetic field now extends up to $x = 2.1\ L$, the particle will continue to move in a circular path till it completes half the circular path and emerges out of the region of the magnetic field with a velocity $-v_0\,\hat{\mathbf{i}}$ moving along the negative x-axis as shown in Fig. 13.102.

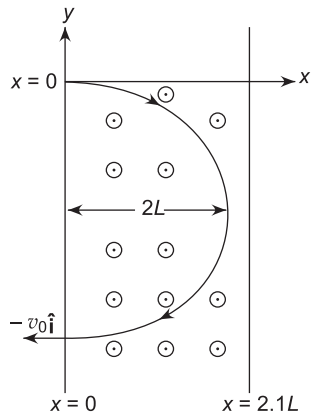


Fig. 13.102

7. Distance travelled by the particle in the magnetic field = half the circumference = πr. Therefore, time spent in the magnetic field is

$$t = \frac{\pi r}{v_0} = \frac{\pi m}{qB_0} \qquad \text{[Use Eq. (2)]}$$

So the correct choice is (d).

Questions 8 to 11 are based on the following passage.

Passage III

A wire loop consists of a straight segment AB and a circular arc ACB of radius r. The segment AB subtends an angle of 60° at the centre O of the circular arc. The wire loop carries a current I in the clockwise direction (Fig. 13.103).

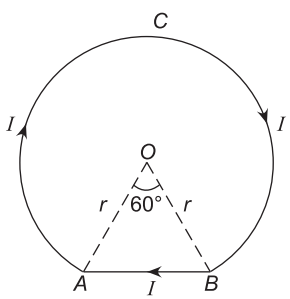


Fig. 13.103

8. The magnetic field B_1 at O due to the straight segment AB is

(a) $\dfrac{\mu_0 I}{2\pi r}$ (b) $\dfrac{\mu_0 I}{2\sqrt{2}\pi r}$

(c) $\dfrac{\mu_0 I}{2\sqrt{3}\pi r}$ (d) $\dfrac{\mu_0 I}{4\pi r}$

9. The magnetic field B_2 at O due to the circular arc ACB is

(a) $\frac{5\mu_0 I}{12r}$ (b) $\frac{\mu_0 I}{2r}$

(c) $\frac{3\mu_0 I}{8r}$ (d) $\frac{7\mu_0 I}{18r}$

10. The net magnetic field B at O due to the whole wire loop is

(a) $B = B_1 + B_2$ (b) $B = B_2 - B_1$

(c) $B = \sqrt{B_1^2 + B_2^2}$ (d) $B = \sqrt{B_2^2 - B_1^2}$

11. The direction of the magnetic field B is

(a) parallel to the plane of the coil

(b) perpendicular to the plane of the coil and directed out of the page

(c) perpendicular to the plane of the coil and directed into the page

(d) inclined at an angle of 60° with the plane of the coil



Solutions

8. As shown in Fig. 13.104, the magnetic field at O due to current I in AB is given by (use Biot-Savart law)

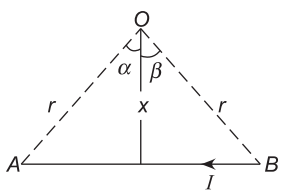


Fig. 13.104

$$B_{AB} = \frac{\mu_0 I}{4\pi x}(\sin\alpha + \sin\beta)$$

Here $\alpha = \beta = 30°$. Also $x = r\cos\alpha = r\cos 30° = \frac{\sqrt{3}r}{2}$.

Therefore,

$$B_1 = \frac{\mu_0 I}{4\pi \times \sqrt{3}r/2} \times (\sin 30° + \sin 30°)$$

$$= \frac{\mu_0 I}{2\sqrt{3}\pi r} \times (0.5 + 0.5) = \frac{\mu_0 I}{2\sqrt{3}\pi r},$$

which is choice (c).

The direction of the field is perpendicular to the plane of the paper directed into the page.

9. The magnetic field at the centre of a complete ($n = 1$ turn) circular loop of radius r and carrying a current I is

$$B = \frac{\mu_0 nI}{2r}$$

Here loop ACB is a fraction of a circle, i.e. $n < 1$. Since ACB subtends an angle (360° – 60°) = 300° at O, hence the fraction n is

$$n = \frac{300°}{360°} = \frac{5}{6}$$

Therefore, magnetic field due to arc ACB is

$$B_2 = \frac{\mu_0 \times \frac{5}{6} \times I}{2r} = \frac{5\mu_0 I}{12r}$$

As the current in ACB is clockwise, the direction of the magnetic field is perpendicular to the plane of the paper and directed into the page.

The correct choice is (a).

10. Since B_1 and B_2 are in the same direction, the net field is $B = (B_1 + B_2)$, which is choice (a).

11. The correct choice is (c).

Questions 12 to 15 are based on the following passage.

Passage IV

A moving coil galvanometer consists of a coil of N turns and area A suspended by a thin phosphor bronze strip in radial magnetic field B. The moment of inertia of the coil about the axis of rotation is I and C is the torsional constant of the phosphor bronze strip. When a current i is passed through the coil, it deflects through an angle θ (in radian).

12. Choose the correct statement from the following. The magnitude of the torque experienced by the coil is independent of

(a) N (b) B

(c) i (d) I

13. The current sensitivity of the galvanometer is increased if

(a) N, A and B are increased and C is decreased.

(b) N and A are increased and B and C are decreased

(c) N, B and C are increased and A is decreased

(d) N, A, B and C are all increased.

14. When a charge Q is passed almost instantly through the coil, the angular speed ω acquired by the coil is

(a) $\frac{NAB}{QI}$ (b) $\frac{BAQ}{NI}$

(c) $\frac{\text{NABQ}}{\text{I}}$ (d) $\frac{NAQI}{B}$

15. In Q. 14, the maximum angular deflection (in radian) of the coil is

(a) $\theta_{max} = \omega\sqrt{\frac{I}{C}}$ (b) $\theta_{max} = \frac{1}{C}\sqrt{I\omega}$

(c) $\theta_{max} = I\sqrt{\frac{\omega}{C}}$ (d) $\theta_{max} = \omega\sqrt{IC}$

Solutions

12. The magnitude of torque experienced by the coil is given by

$$\tau = iNAB \sin \alpha$$

where α is the angle which the normal to the plane of the coil makes with the direction of the magnetic field. If the magnetic field is radial, the plane of the coil is always parallel to the direction of the magnetic field, i.e. $\alpha = 90°$. Hence $\tau = iNAB = Ki$ where

$$K = NAB$$

So the correct choice is (d).

13. Let θ be the angular deflection (in radian) when a current i is passed through the coil. Then, restoring torque = $C\theta$. When the coil is in equilibrium, deflecting torque = restoring torque, i.e.

$$iNAB = C\theta$$

$\therefore$ Current sensitivity is $\frac{\theta}{i} = \frac{NAB}{C}$

Hence the correct choice is (a).

14. If ω is the angular speed acquired by the coil when a charge Q is passed through it for very short time Δt, then

$$\tau = \frac{\text{angular momentum}}{\text{time interval}} = \frac{I\omega}{\Delta t}$$

or $I\omega = \tau \Delta t = Ki\Delta t = KQ \quad \left(\therefore i = \frac{Q}{\Delta t}\right)$

or $I\omega = NABQ$ or $\omega = \frac{NABQ}{I}$, which is choice (c).

15. From the principle of conservation of energy, we have

$$\frac{1}{2} I\omega^2 = \frac{1}{2} C\theta_{\max}^2$$

which gives $\theta_{\max} = \omega\sqrt{\frac{I}{C}}$, which is choice (a).

SECTION 3

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by statement-2 (Reason). Each question has the following four options out of which only one choice is correct.

(a) Statement-1 is True, Statement-2 is True and Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; but Statement-2 is NOT a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

1. **Statement-1**

A non-uniform magnetic field that varies in magnitude from point to point but has a constant direction, is set up in a region of space. If a charged particle enters the region in the direction of the magnetic field, it will be accelerated at non-uniform rate in the region.

Statement-2

The force $\vec{F}$ experienced by a particle of charge q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by $\vec{F} = q(\vec{v} \times \vec{B})$.

2. **Statement-1**

A charged particle moves in a uniform magnetic field for some time. During this time, the kinetic energy of the particle cannot change but its momentum can change.

Statement-2

The magnetic force is always perpendicular to the velocity of the particle.

3. **Statement-1**

A current carrying loop is free to rotate. It is placed in a uniform magnetic field. It will attain equilibrium when its plane is perpendicular to the magnetic field.

Statement-2

The torque on the coil is zero when its plane is perpendicular to the magnetic field.

4. **Statement-1**

An electron moving in the positive x-direction enters a region where uniform electric and magnetic fields exist perpendicular to each other. The electric field is in the negative y-direction. If the electron travels undeflected in this region, the direction of the magnetic field is along the negative z-axis.

Statement-2

If a charged particle moves in a direction perpendicular to a magnetic field, the direction of the force acting on it is given by Fleming's left-hand rule.

5. **Statement-1**

   If a charged particle is released from rest in a region of uniform electric and magnetic fields parallel to each other, it will move in a straight line.

   Statement-2

   The electric field exerts no force on the particle but the magnetic field does.

6. **Statement-1**

   A proton and an alpha particle having the same kinetic energy are moving in circular paths in a uniform magnetic field. The radii of their circular paths will be equal.

   Statement-2

   Any two charged particles having equal kinetic energies and entering a region of uniform magnetic field $\vec{B}$ in a direction perpendicular to $\vec{B}$, will describe circular trajectories of equal radii.

7. **Statement-1**

   Two particles having equal charges and masses m_1 and m_2, after being accelerated by the same potential difference (V), enter a region of uniform magnetic field and describe circular paths of radii r_1 and r_2 respectively. Then

   $$\frac{m_1}{m_2} = \sqrt{\frac{r_1}{r_2}}$$

   Statement-2

   Gain in kinetic energy = work done to accelerate the charged particle through potential difference V.



Solutions

1. The correct choice is (d). If $\vec{v}$ is parallel to $\vec{B}$, $\vec{F}$ = 0. Hence the particle does not experience any force and is, therefore, not accelerated in the region. It will travel undeflected with a constant speed.

2. The correct choice is (a). Since the magnetic force is always perpendicular to the velocity, no work is done by a uniform magnetic field on a charged particle. Hence magnetic force cannot change the magnitude of velocity (i.e. speed); it can only change the direction of velocity. Hence kinetic energy $\left(=\frac{1}{2}mv^2\right)$ remains unchanged but momentum $\vec{p} = m\vec{v}$ will change.

3. The correct choice is (a). The loop will rotate and come to rest when the torque acting on it becomes zero. The magnitude to torque acting on a loop of area A and carrying a current I in a magnetic field B is given by

   $$\tau = B\,I\,A \sin\theta$$

   where θ is the angle between the direction of the magnetic field and the normal to the plane of the coil. It is clear that $\tau = 0$ when $\theta = 0$, i.e. when the plane of the coil is perpendicular to the magnetic field.

4. The correct choice is (a). Because electron has a negative charge, an electric field in the negative y-direction will deflect it in the positive y-direction. It will travel undeflected if the magnetic field imparts an equal deflection in the negative y-direction. Since the magnetic force is perpendicular to the magnetic field and the charge of electron is negative, the direction of the magnetic field (according to Fleming's Left-Hand rule) should be along the negative z-direction.

5. The correct choice is (b). Due to electric field, the force is $\vec{F} = q\vec{E}$ in the direction of $\vec{E}$. Since $\vec{E}$ is parallel to $\vec{B}$, the particle velocity $\vec{v}$ (acquired due to force $\vec{F}$) is parallel to $\vec{B}$. Hence $\vec{B}$ will not exert any force since $\vec{v} \times \vec{B} = 0$ and the motion of the particle is not affected by $\vec{B}$.

6. The correct choice is (c). The radius of the circular path is given by

   $$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB} \text{ ; where } K = \frac{1}{2}mv^2$$

   Since K and B are the same for the two particles, $r \propto \frac{\sqrt{m}}{q}$. Now, the charge of an alpha particle is twice that of a proton and its mass is four times the mass of a proton, $\sqrt{m}/q$ will be the same for both particles. Hence r will be the same for both particles.

7. The correct choice is (d). Kinetic energy $K = qV$.

Therefore $r_1 = \frac{\sqrt{2m_1qV}}{qB}$ and $r_2 = \frac{\sqrt{2m_2qV}}{qB}$

Hence $\frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}} \Rightarrow \frac{m_1}{m_2} = \left(\frac{r_1}{r_2}\right)^2$.

SECTION 4 Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. If in a circular coil A of radius R, current i is flowing and in another coil B of radius $2R$, current $2i$ is flowing, then the ratio of magnetic fields B_A and B_B at their centre is

   (a) 1 (b) 2

   (c) $\frac{1}{2}$ (d) 4 **[2002]**

2. If an electron and a proton having the same momenta, enter perpendicularly to a uniform magnetic field, then

   (a) both will have the same curved path (ignoring the sense of revolution)

   (b) they will move undeflected

   (c) the path of the electron will be move curved than that of the proton

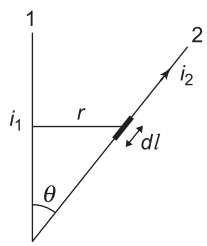
   (d) the path of the proton is more curved. **[2002]**

3. The time period of a charged particle undergoing a circular motion in a uniform magnetic field is independent of its

   (a) speed (b) mass

   (c) charge (d) magnetic field **[2002]**

4. Two wires 1 and 2 carrying currents i_1 and i_2 respectively are inclined at an angle θ as shown in the figure. What is the force on a small element dl of wire 2 at a distance r from wire 1 due to the magnetic field of wire 1?



   (a) $\frac{\mu_0}{2\pi r}(i_1 i_2 dl \tan\theta)$ (b) $\frac{\mu_0}{2\pi r}(i_1 i_2 dl \sin\theta)$

   (c) $\frac{\mu_0}{2\pi r}(i_1 i_2 dl \cos\theta)$ (d) $\frac{\mu_0}{4\pi r}(i_1 i_2 dl \sin\theta)$ **[2002]**

5. A particle of mass M and charge Q moving with velocity v describes a circular path of radius R when subjected to a uniform transverse magnetic field **B**. The work done by the field when the particle completes one full circle is

   (a) $\left(\frac{mv^2}{R}\right)2\pi R$ (b) zero

   (c) $2\pi RBQ$ (d) $2\pi RBvQ$ **[2003]**

6. A particle of charge -1.6×10^{-19} C moving with velocity 10 m s^{-1} along the x-axis enters a region where the magnetic field **B** is along the y-axis and an electric field of magnitude 10^4 V m^{-1} is along the negative z-axis. If the charged particle continuous moving along the x-axis, the magnitude of **B** is

   (a) 10^3 T (b) 10^5 T

   (c) 10^{16} T (d) 10^{-3} T **[2003]**

7. A thin rectangular magnet suspended freely has a time period of oscillation equal to T. Now it is broken into two equal halves (each having half the original length) and one piece is made to oscillate in the same field. If its period of oscillation is T', the ratio T'/T is

   (a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2}$

   (c) 2 (d) $\frac{1}{4}$ **[2003]**

8. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60°. The magnitude of torque needed to maintain the needle in this position will be

   (a) $\sqrt{3}\ W$ (b) W

   (c) $\left(\frac{\sqrt{3}}{2}\right)W$ (d) $2\ W$ **[2003]**

9. The magnetic field lines inside a bar magnet

   (a) are from north pole to south pole of the magnet

   (b) do not exist

   (c) depend on the area of cross section of the magnet

   (d) are from south pole to north pole of the magnet. **[2003]**

10. Curie temperature is the temperature above which,
    (a) a ferromagnetic material becomes paramagnetic
    (b) a paramagnetic material becomes diamagnetic
    (c) a ferromagnetic material becomes diamagnetic
    (d) a paramagnetic material becomes ferromagnetic. **[2003]**

11. A current i flows along an infinitely long straight thin walled tube of radius r. The magnetic field inside the tube is
    (a) infinite (b) zero
    (c) $\frac{\mu_0 i}{2\pi r}$ (d) $\frac{2i}{r}$ **[2004]**

12. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular coil of n turns. The magnetic field at the centre of the coil will be
    (a) nB (b) n^2B
    (c) $2nB$ (d) $2n^2B$ **[2004]**

13. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance 4 cm from the centre is 54 μT. What will be its value at the centre of the loop?
    (a) 250 μT (b) 150 μT
    (c) 125 μT (d) 75 μT **[2004]**

14. Two long conductors, separated by a distance d, carry currents I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one wire is increased two times and its direction is reversed. The distance between wires is also increased to $3d$. The new value of force between them is
    (a) $-2F$ (b) $\frac{F}{3}$
    (c) $-\frac{2F}{3}$ (d) $-\frac{F}{3}$ **[2004]**

15. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2s. The magnet is cut along the length into three equal parts and the three parts are then placed on each other with their like poles together. The time period of this combination will be
    (a) 2s (b) $\frac{2}{3}$ s
    (c) $2\sqrt{3}$ s (d) $\frac{2}{\sqrt{3}}$ s **[2004]**

16. The materials suitable for making electromagnets should have
    (a) high retentivity and high coercivity
    (b) low retentivity and low coercivity
    (c) high retentivity and low coercivity
    (d) low retentivity and high coercivity **[2004]**

17. Two thin long parallel wires separated by a distance d carry a current i in the same direction. They will
    (a) attract each other with a force of $\frac{\mu_0 i^2}{(2\pi d)}$
    (b) repel each other with a force of $\frac{\mu_0 i^2}{(2\pi d)}$
    (c) attract each other with a force of $\frac{\mu_0 i^2}{(2\pi d^2)}$
    (d) repel each other with a force of $\frac{\mu_0 i^2}{(2\pi d^2)}$ **[2005]**

18. Two concentric coils each of radius equal to 2π cm are placed at right angles to each other and carry currents of 3A and 4A. The magnetic field (in Wb m^{-2}) at the centre of the coils will be ($\mu_0 = 4\pi \times 10^{-6}$ Wb A^{-1} m^{-1})
    (a) 12×10^{-5} (b) 10^{-5}
    (c) 5×10^{-5} (d) 7×10^{-5} **[2005]**

19. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity, then
    (a) its velocity will decrease
    (b) its velocity will increase
    (c) it will turn towards right of its initial direction of motion
    (d) it will turn towards left of its initial direction of motion **[2005]**

20. A charged particle of mass m and charge q travels in a circular path of radius r that is perpendicular to magnetic field B. The time taken by the particle to complete one revolution is
    (a) $\frac{2\pi mq}{B}$ (b) $\frac{2\pi q^2 B}{m}$
    (c) $\frac{2\pi qB}{m}$ (d) $\frac{2\pi m}{qB}$ **[2005]**

21. A magnetic needle is kept in a non-uniform magnetic field. It experiences
    (a) a torque but no force
    (b) neither a torque nor a force
    (c) a force as well as a torque
    (d) a force but no torque **[2005]**

22. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a

    (a) ellipse (b) circle

    (c) helix (d) straight line **[2006]**

23. Needles N_1, N_2 and N_3 are made of a ferromagnetic, a paramagnetic and a diamagnetic substance, respectively. A magnet when brought close to them will

    (a) attract N_1 strongly, but repel N_2 and N_3 weakly

    (b) attract all three of them

    (c) attract N_1 and N_2 strongly but repel N_3

    (d) attract N_1 strongly, N_2 weakly and repel N_3 weakly **[2006]**

24. A long solenoid has 200 turns per cm and carries a current i. The magnetic field at its center is 6.28×10^{-2} weber/m^2. Another long solenoid has 100 turns per cm and it carries a current $i/3$. The value of the magnetic field at its center is

    (a) 1.05×10^{-3} weber/m^2

    (b) 1.05×10^{-4} weber/m^2

    (c) 1.05×10^{-2} weber/m^2

    (d) 1.05×10^{-5} weber/m^2 **[2006]**

25. A long straight wire of radius a carries a steady current i. The current is uniformly distributed across its cross-section. The ratio of the magnetic field at $\frac{a}{2}$ and $2a$ is

    (a) $\frac{1}{4}$ (b) 4

    (c) 1 (d) $\frac{1}{2}$ **[2007]**

26. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then

    (a) the magnetic field is zero only on the axis of the pipe

    (b) the magnetic field is different at different points inside the pipe

    (c) the magnetic field at any point inside the pipe is zero

    (d) the magnetic field at all points inside the pipe is the same, but not zero **[2007]**

27. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal field $\vec{E}$ and $\vec{B}$ with a velocity $\vec{v}$ perpendicular to both $\vec{v}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{v}$. Then

    (a) $\vec{v} = \frac{\vec{E} \times \vec{B}}{B^2}$ (b) $\vec{v} = \frac{\vec{B} \times \vec{E}}{B^2}$

    (c) $\vec{v} = \frac{\vec{E} \times \vec{B}}{E^2}$ (d) $\vec{v} = \frac{\vec{E} \times \vec{B}}{E^2}$ **[2007]**

28. A charged particle moves through a magnetic field perpendicular to its direction. Then

    (a) the momentum changes but the kinetic energy is constant

    (b) both momentum and kinetic energy of the particle are not constant

    (c) both, momentum and kinetic energy of the particle are constant

    (d) kinetic energy changes but the momentum is constant. **[2007]**

29. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by

    (a) $\frac{\mu_0}{2\pi}\left(\frac{I_1 + I_2}{d}\right)^{\frac{1}{2}}$ (b) $\frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)^{\frac{1}{2}}$

    (c) $\frac{\mu_0}{2\pi d}(I_1 + I_2)$ (d) $\frac{\mu_0}{2\pi d}(I_1{}^2 + I_2{}^2)$ **[2007]**

30. A horizontal overhead powerline is a height of 4m from the ground and carries a current of 100 A from east to west. The magnetic field directly below it on the ground is ($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})

    (a) 2.5×10^{-7} T northward

    (b) 2.5×10^{-7} T southward

    (c) 5×10^{-6} T northward

    (d) 5×10^{-6} T southward **[2008]**

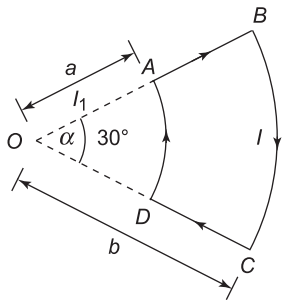
31. Relative permittivity and permeability of a material are ε_r and μ_r, respectively. Which of the following values of these quantities are allowed for diamagnetic material?

    (a) $\varepsilon_r = 1.5$, $\mu_r = 1.5$ (b) $\varepsilon_r = 0.5$, $\mu_r = 1.5$

    (c) $\varepsilon_r = 1.5$, $\mu_r = 0.5$ (d) $\varepsilon_r = 0.5$, $\mu_r = 0.5$ **[2008]**

Questions 32 and 33 are based on the following paragraph.

A current loop *ABCD* is held fixed on the plane of the paper as shown in the figure. The arcs *BC* (radius = *b*) and *DA* (radius = *a*) of the loop are joined by two straight wires *AB* and *CD*. A steady current *I* is flowing in the loop. Angle made by *AB* and *CD* at the origin *O* is 30°. Another straight thin wire with steady current I_1 flowing out of the plane of the paper is kept at the origin.



[2009]

32. The magnitude of the magnetic field (*B*) due to the loop *ABCD* at the origin (*O*) is

(a) $\frac{\mu_0 I}{4\pi}\left[\frac{b-a}{ab}\right]$

(b) $\frac{\mu_0 I}{4\pi}[2(b-a)+\pi/3\,(a+b)]$

(c) zero

(d) $\frac{\mu_0 I(b-a)}{24ab}$ **[2009]**

33. Due to the presence of the current I_1 at the origin:

(a) The magnitude of the net force on the loop is given by

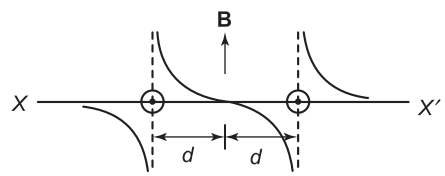
$\frac{I_1 I}{4\pi}\mu_0\,[2(b-a)+\pi/3\,(a+b)]$

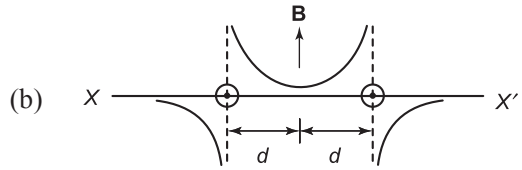
(b) The magnitude of the net force on the loop is given by $\frac{\mu_0 II_1}{24ab}(b-a)$.

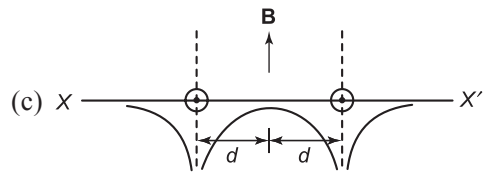
(c) The forces on *AB* and *DC* are zero.

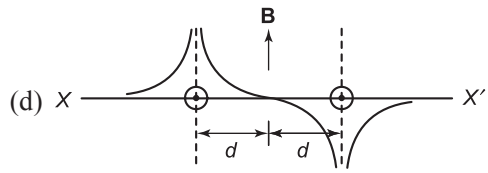
(d) The forces on *AD* and *BC* are zero. **[2009]**

34. Two long parallel wires are at a distance 2*d* apart. They carry steady equal current flowing out of the plane of the paper as shown in the figure. The variation of the magnetic field along the line *XX′* is given by

(a)


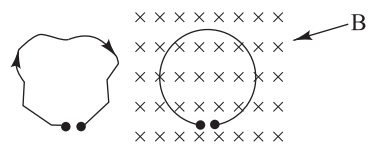
(b)


(c)


(d)


[2010]

35. A thin flexible wire of length *L* is connected to two adjacent fixed points and carries a current *I* in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is



(a) *IBL* (b) $\frac{IBL}{\pi}$

(c) $\frac{IBL}{2\pi}$ (d) $\frac{IBL}{4\pi}$ **[2010]**

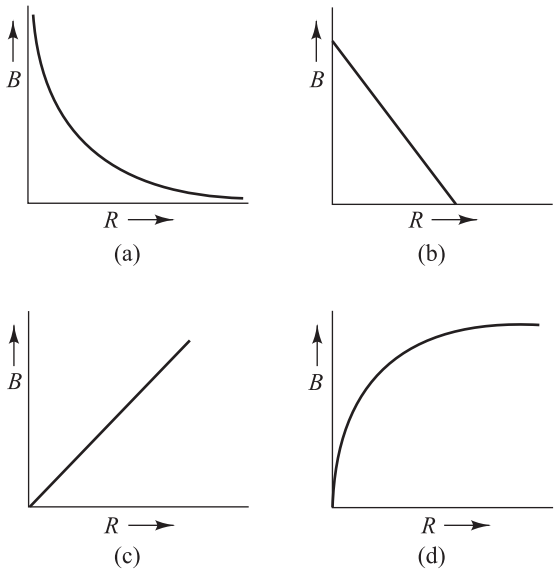
36. A current I flows in an infinitely long wire with cross-section in the form of a semicircular ring of radius *R*. The magnitude of magnetic field along its axis is

(a) $\frac{\mu_0 I}{\pi^2 R}$ (b) $\frac{\mu_0 I}{2\pi^2 R}$

(c) $\frac{\mu_0 I}{2\pi R}$ (d) $\frac{\mu_0 I}{4\pi R}$ **[2011]**

37. A charge *Q* is uniformly distributed over the surface of non-conducting disc of radius *R*. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity *ω*. As a result of this rotation a magnetic field *B* is obtained at the centre of the disc. If we keep both the the amount

of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc, then the variation of the magnetic field at the centre of the disc will be represented by which of the following figures.



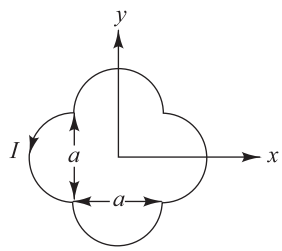
[2012]

38. Proton, deuteron and alpha particle of same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are, respectively, r_p, r_d and r_α. which one of the following relation is correct?

(a) $r_\alpha = r_p = r_d$ (b) $r_\alpha = r_p = r_d$

(a) $r_\alpha = r_p = r_d$ (b) $r_\alpha = r_p = r_d$ **[2012]**

39. A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector k is coming out of the plane of the paper. The magnetic moment of the current loop is

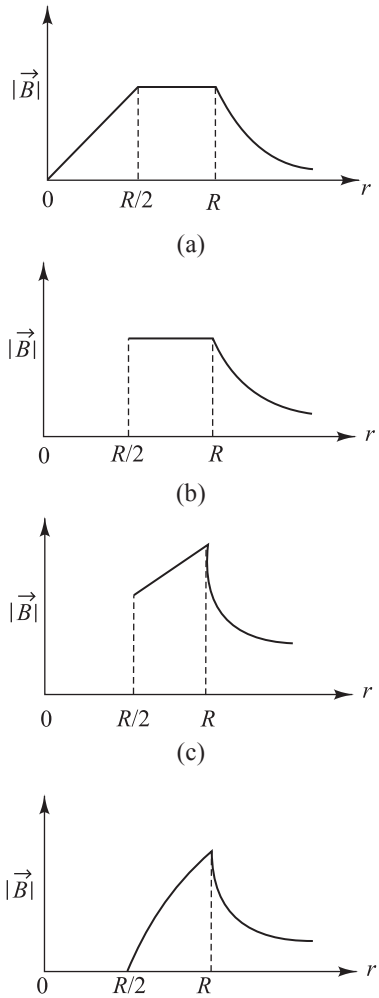


(a) $a^2 I\hat{\mathbf{k}}$ (b) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{\mathbf{k}}$

(c) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{\mathbf{k}}$ (d) $(2\pi+1)a^2 I\hat{\mathbf{k}}$

[2012]

40. An infinitely long hollow conducting cylinder with inner radius $R/2$ and other radius R carries a unifrom current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by



[2012]

41. This question has statement I and statement II. Of the four choices given after the statement, choose the one that best describes the two statements.

Statement-I : Higher the range, greater is the resistance of ammeter.

Statement-II : To increase the range of ammeter, additional shunt needs to be used across it.

(a) Statement-I is true, Statement-II is true, Statement-II is not the correct explanation of Statement-I.

(b) Statement-I is true, Statement-II is false.

(c) Statement-I is false, Statement-II is true.

(d) Statement-I is true, Statement-II is true, Statment-II is the correct explanation of statement-I

[2013]

42. Two short magnets of length 1 cm each have magnetic moment 1.20 A m^2 and 1.00A m^2, respectively. They are placed on a horizontal table parallel to each other with their *N* poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic field at the mid-point O of the line joining their centres is close to (horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb/m^2)

(a) 2.56×10^{-4} Wb/m^2 (b) 3.50×10^{-4} Wb/m^2

(c) 5.80×10^{-4} Wb/m^2 (d) 3.60×10^{-5} Wb/m^2

[2013]

43. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle take place in a plane. It follows that

(a) its velocity remains constant

(b) it moves in a straight line with a constant acceleration

(c) it moves in a circle with a constant acceleration

(d) its kinetic energy remains constant

[2014]



Answers

1. (a)	2. (a)	3. (a)	4. (c)
5. (b)	6. (a)	7. (b)	8. (a)
9. (d)	10. (a)	11. (b)	12. (b)
13. (a)	14. (c)	15. (b)	16. (c)
17. (a)	18. (c)	19. (a)	20. (d)
21. (c)	22. (d)	23. (d)	24. (c)
25. (c)	26. (c)	27. (b)	28. (a)
29. (b)	30. (d)	31. (c)	32. (d)
33. (d)	34. (a)	35. (c)	36. (b)
37. (a)	38. (b)	39. (b)	40. (d)
41. (c)	42. (a)	43. (d)	



Solutions

1. $B_A = \dfrac{\mu_0 i}{2R}$ and $B_B = \dfrac{\mu_0 (2i)}{2(2R)}$

$\therefore \quad \dfrac{B_A}{B_B} = 1$

2. The radius of the circular path is

$$r = \frac{mv}{qB} \quad \Rightarrow \quad r = \frac{p}{qB}; \quad p = mv.$$

Since p, q, B are the same for electron and proton, the value of r will be the same for both. Since the charge of an electron is opposite in sign to that of proton, the sense of revolution will be opposite. So the correct choice is (a).

3. Time period $T = \dfrac{2\pi m}{qB}$. So the correct choice is (a).

4. The component of dl parallel to wire 1 is $dl \cos \theta$. Hence the force on the element is

$$F = B_1 \, i_2 \, dl \cos \theta$$

where B_1 = magnetic field at the element due to current in wire which is given by

$$B_1 = \frac{\mu_0 i_1}{2\pi r}$$

$$\therefore \; F = \frac{\mu_0}{2\pi r} \, (i_1 \, i_2 \, dl \cos \theta)$$

5. When a charged particle describes a circular path, the necessary centripetal force is provided by the magnetic force **F**. Since the velocity **v** is always tangential, vectors **F** and **v** are perpendicular to each other ($\theta = 90°$).

Power = $\mathbf{F} \cdot \mathbf{v} = Fv \cos \theta = Fv \cos 90° = 0$. Since power consumed is zero, the work done is zero.

6. $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

Given $\mathbf{E} = -E\hat{\mathbf{k}}$, $\mathbf{v} = v\hat{\mathbf{i}}$ and $\mathbf{B} = B\hat{\mathbf{j}}$

$\therefore \quad \mathbf{F} = q[-E\hat{\mathbf{k}} + (v\hat{\mathbf{i}}) \times (B\hat{\mathbf{j}})]$

$= q(-E\hat{\mathbf{k}} + vB\hat{\mathbf{k}}) = q\,(-E + vB)\hat{\mathbf{k}}$

Since the particle moves undeflected, $\mathbf{F} = 0$, i.e.

$$q\,(-E + vB)\hat{k} = 0$$

$$\Rightarrow \quad -E + vB = 0$$

$$\Rightarrow \quad B = \frac{E}{v} = \frac{10^4}{10} = 10^3 \text{ T}$$

7. $T = 2\pi \sqrt{\dfrac{I}{MB}}$

where I = moment of inertia of the magnet about the axis of rotation which is

$$I = \frac{mL^2}{12}; \quad L = \text{length of magnet},$$

m = mass of magnet

M = magnetic moment = pL (p = pole strength) and B is the magnetic field.

If the magnet is broken into two parts, each of length $L/2$, the pole strength p remains the same. The magnetic moment of each piece is

$$M' = p \times \frac{L}{2} = \frac{M}{2}$$

Since the mass of each piece is $m' = \frac{M}{2}$, and length $L' = \frac{L}{2}$, the moment of inertia of a piece is

$$I' = \frac{m'(L')^2}{12} = \frac{\frac{m}{2} \times \left(\frac{L}{2}\right)^2}{12} = \frac{mL^2}{8 \times 12} = \frac{I}{8}$$

Time period T' is

$$T' = 2\pi\sqrt{\frac{I'}{M'B}} = 2\pi\sqrt{\frac{I/8}{BM/2}} = \frac{2\pi}{2}\sqrt{\frac{I}{MB}} \qquad (2)$$

Dividing (2) by (1)

$$\frac{T'}{T} = \frac{1}{2}$$

8. $W = MB(1 - \cos\theta)$

$= MB(1 - \cos 60°) = \frac{MB}{2}$

$\Rightarrow \quad MB = 2W$

Magnitude of torque is

$\tau = MB \sin\theta$

$= 2W \sin 60° = 2W \times \frac{\sqrt{3}}{2} = \sqrt{3}W$

9. Magnetic field lines form closed loop. Hence, inside the magnet, they are from south pole to north pole.
10. The correct choice is (a).
11. Consider an amperean loop inside the pipe. Since the current threading the loop is zero, from Ampere's circuital law, the magnetic field at any point inside the hollow pipe carrying a current is zero.
12. The magnetic field at the centre of a coil of n turns and radius R carrying a current I is

$$B = \frac{\mu_0 nI}{2R}$$

For $n = 1, \quad B = \frac{\mu_0 I}{2R}$ (1)

When the wire is bent into a circular coil of n turns, the radius of the coil becomes

$$R' = \frac{L}{2\pi n} = \frac{R}{n} \qquad (\because \quad L = 2\pi R')$$

The magnetic field at the centre of coil will be

$$B' = \frac{\mu_0 nI}{2R'}$$

$$= \frac{\mu_0 nI}{2R/n}$$

$$= \frac{\mu_0 In^2}{2R} = n^2 B$$

13. Magnetic field at a distance x from the centre of the coil of radius R and carrying a current I is

$$B = \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}} \qquad (1)$$

Magnetic field at the centre of the coil is ($\because \quad x = 0$)

$$B_0 = \frac{\mu_0 I}{2R} \qquad (2)$$

Dividing (2) by (1) we get

$$\frac{B_0}{B} = \frac{(R^2 + x^2)^{3/2}}{R^3} \qquad (3)$$

Given $B = 54\ \mu T$, $R = 3\ cm = 3 \times 10^{-2}\ m$ and $x = 4\ cm = 4 \times 10^{-2}\ m$. Substituting these values in Eq. (3), we get

$B_0 = 250\ \mu T$

14. If the length of each conductor is L, then

$$F = \frac{\mu_0 I_1 I_2 L}{2\pi d} \qquad \text{(attractive)}$$

New force is

$$F' = \frac{\mu_0(-2I_1)I_2 L}{2\pi(3d)} \qquad \text{(repulsive)}$$

$$\therefore \quad \frac{F'}{F} = -\frac{2}{3} \quad \Rightarrow \quad F' = -\frac{2F}{3}.$$

15. In a vibration magnetometer, a bar magnet is suspended from a support and oscillated in a horizontal plane. The time period of the torsional oscillations is

$$T = 2\pi\sqrt{\frac{1}{MH}}; \quad I = \frac{mL^2}{12}$$

where m = mass of magnet, L = length of magnet and $M = pL$ is the magnetic moment; p being the pole strength and H is the horizontal component of the earth's magnetic field.

When the magnet is cut along the length into three equal parts, the moment of inertia of each part is

$$I_1 = \frac{1}{12}\left(\frac{m}{3}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{27}\left(\frac{1}{12}mL^2\right)$$

If they are placed one on top of the other with their like poles together, the moment of inertia of the combination is

$$I' = 3I_1 = \frac{1}{9}\left(\frac{1}{12}mL^2\right) = \frac{I}{9}$$

On cutting along the length, pole strength of each part is $p_1 = \frac{p}{3}$. If they are placed one on top of the

other with their like poles together, the pole strength of the combination is

$$p' = 3p_1 = p$$

Magnetic moment of combination is

$$M' = p \times L = M$$

$$\therefore \quad T' = 2\pi\sqrt{\frac{I'}{M'H}} = 2\pi\sqrt{\frac{I}{9MH}}$$

$$= \frac{1}{3} \times T = \frac{1}{3} \times 2\text{s} = \frac{2}{3}\text{ s}$$

16. Electromagnets are made of soft iron because soft iron has low coercivity and high retentivity. As a result, it is easily demagnetized. So, the correct choice is (c).

17. The force per unit length between the two wires is

$$\frac{F}{l} = \frac{\mu_0 i^2}{2\pi d}$$

Since the current in the wires are in the same direction, the force between them is attractive.

18. $B_1 = \dfrac{\mu_0 I_1}{2R} = \dfrac{4\pi \times 10^{-7} \times 3}{2 \times 2\pi \times 10^{-2}} = 3 \times 10^{-5}$ Wb m^{-2}

$$B_2 = \frac{\mu_0 I_2}{2R} = \frac{4\pi \times 10^{-7} \times 4}{2 \times 2\pi \times 10^{-2}} = 4 \times 10^{-5} \text{ Wb m}^{-2}$$

Since the planes of the coils are perpendicular to each other, B_1 and B_2 will be perpendicular to each other. The net field is

$$B = \sqrt{B_1^2 + B_2^2}$$

$$= \sqrt{(3\times10^{-5}) + (4\times10^{-5})^2} = 5 \times 10^{-5} \text{ Wb m}^{-2}$$

19. Force on the electron exerted by magnetic field is

$\mathbf{F}_m = q\,(\mathbf{v} \times \mathbf{B}) = 0$ since $\mathbf{v}$ is parallel to $\mathbf{B}$.

Force on the electron exerted by electric field is

$\mathbf{F}_e = q\,\mathbf{E}$

Since the charge of the electron is negative, force $\mathbf{F}_e$ will be opposite to the direction of motion of the electron. Hence, the motion of the electron will be retarded. So, the correct choice is (a).

20. The necessary centripetal force is provided by the magnetic force qvB. Hence

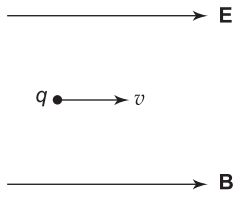
$$\frac{mv^2}{r} = qvB \quad \Rightarrow \quad r = \frac{mv}{qB}$$

Time taken by the particle to complete one revolution is

$$T = \frac{2\pi r}{v} = \frac{2\pi mv}{vqB} = \frac{2\pi m}{qB}$$

21. Since the magnetic field varies with distance, the forces on the poles of the magnet will be unequal. Hence, the magnet will experience a force as well as a torque.

22. Consider a particle of charge q in a region of parallel and uniform electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ as shown in the figure. The electric field exerts a force



$$\mathbf{F}_e = q\,\mathbf{E}$$

in the direction of the field. As a result, the particle is accelerated in the direction of the field. If v is the velocity of the particle at an instant of time, then at that instant, the force experienced by the particle due to the magnetic field is

$$\mathbf{F}_m = q\,(v \times \boldsymbol{B})$$

Since v is parallel to $\mathbf{B}$, $\mathbf{F}_m = 0$. Hence, the particle will keep moving in a straight line in the direction of the electric field if it carries a positive charge and opposite to the direction of the electric field if it carries a negative charge.

23. The correct choice is (d).

24. For the first solenoid

$$B_1 = \mu_0\, n_1\, i_1$$

For the second solenoid

$$B_2 = \mu_0\, n_2\, i_2$$

Dividing we get,

$$\frac{B_2}{B_1} = \frac{n_2}{n_1} \times \frac{i_2}{i_1} = \frac{100}{200} \times \frac{i/3}{i} = \frac{1}{6}$$

or $B_2 = \dfrac{1}{6} B_1 = \dfrac{1}{6} \times 6.28 \times 10^{-2} \simeq 1.05 \times 10^{-2}$ Wb/m^2

25. The magnetic field at a distance r from the axis of the wire of radius a and carrying a current i for the case when r lies between 0 and a is given by

$$B_1 = \frac{\mu_0 ir}{2\pi a}; \qquad 0 \le r \le a$$

$$= \frac{\mu_0 i}{2\pi a} \times \frac{a}{2} \qquad \left(\text{for } r = \frac{a}{2}\right)$$

$$\Rightarrow \qquad B_1 = \frac{\mu_0 i}{4\pi a}$$

For the case $r > a$, the magnetic field is given by

$$B_2 = \frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i}{2\pi(2a)} \qquad \text{(for } r = 2a\text{)}$$

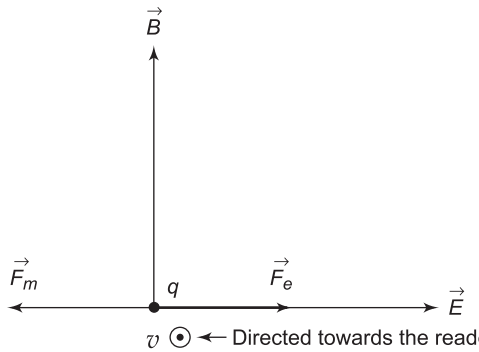
$$\therefore \qquad \frac{B_1}{B_2} = 1$$

26. The magnetic field at any point inside an infinitely long, straight and thin walled pipe carrying a current is zero. Hence the correct choice is (c).

27. The forces acting on the particle by magnetic and electric fields are

$\vec{F}_m = q(\vec{v}\times\vec{B})$ and $\vec{F}_e = q\vec{E}$

The direction of force $\vec{F}_e$ is along the direction of $\vec{E}$ and the direction of force $\vec{F}_m$ is perpendicular to vectors $\vec{v}$ and $\vec{B}$. Since the velocity of the particle remains unchanged, no net force acts on it. Hence, $F_m = F_e$ and the direction of $\vec{F}_m$ must be opposite to that of $\vec{F}_e$, as shown in the figure. From the left-hand screw rule, it follows that $\vec{v}$ should be directed perpendicular to the plane of the page and towards the reader. Thus,



$$q(\vec{v}\times\vec{B}) = -q\vec{E}$$

$$\vec{v}\times\vec{B} = -\vec{E}$$

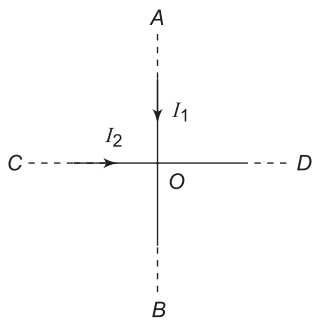
Since $\vec{v}$ is perpendicular to $\vec{B}$, the magnitude of $\vec{v}$ is

$$v = \frac{E}{B}$$

Hence choices (c) and (d) are wrong because they give $v = \frac{B}{E}$. Since $\vec{v}$ is directed towards the reader, it follows that the direction of vector $\vec{v}$ is the same that of vector $(\vec{E}\times\vec{B})$ and not of vector $(\vec{B}\times\vec{E})$. Hence the correct choice is (b).

28. The magnetic force experienced by the charged particle is perpendicular to its velocity. Hence, the force does no work on the particle. Therefore, the speed and hence the kinetic energy of the particle remains unchanged. Since the velocity of the particle is perpendicular to the magnetic field, it will move along a circular path in the region of the magnetic field. Therefore, its velocity and hence linear momentum will continuously change due to change in the direction of motion of the particle moving a circle. Hence the correct choice is (a).

29. The magnetic fields due to wires *AOB* and *COD* at a point *P* at a distance *d* from *O* respectively are (note that point *P* is perpendicular to the plane of the page) [see the figure]



$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

and $$B_2 = \frac{\mu_0 I_2}{2\pi d}$$

Since currents I_1 and I_2 are perpendicular to each other, fields B_1 and B_2 are also perpendicular to each other. Hence the resultant field at point *P* is

$$B = (B_1^2 + B_2^2)^{1/2} = \frac{\mu_0 I}{2\pi r}\ (I_1^2 + I_2^2)^{1/2}$$

30. $$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7})\times100}{2\pi\times4} = 5\times10^{-6}\ \text{T}$$

Using the right hand thumb rule, the direction of the magnetic field will be towards south.

31. For a diamagnetic material, $\mu_r < 1$. For any material $\varepsilon_r > 1$. Hence the correct choice is (c).

32. $B_O = B_{AB} + B_{BC} + B_{CD} + B_{DA}$

$$= 0 - \frac{\mu_0 I}{4\pi b}\times\frac{\pi}{6} + 0 + \frac{\mu_0 I}{4\pi a}\times\frac{\pi}{6}$$

$$= \frac{\mu_0 I}{24}\left(\frac{1}{a} - \frac{1}{b}\right)$$

So the correct choice is (d). Magnetic field at *O* due to arc *BC* is directed into the plane of the page and away from the reader and has been taken to be negative but the magnetic field due to arc *AD* is directed towards the reader and is taken to be positive.

33. The direction of the magnetic field due to current I_1 at wire segments AB and DC is perpendicular to AB as well as to DC. Since the current in AB is opposite to that of in DC and the lengths of AB and DC are equal, the forces on AB and DC are equal and opposite. Hence choice (c) is wrong. The direction of the magnetic field due to current I_1 at arcs AD and BC is tangential to the arcs, i.e. $\theta = 0°$. Hence, force on arc AD and on arc BC is zero. So choice (d) is correct. Since the forces on AB and DC are equal and opposite, the net force on the loop $ABCDA$ is zero. So choices (a) and (b) are incorrect. Hence the only correct choice is (d).

34. Let I be the current in each wire. Let the left wire be located at $x = 0$, then the other wire will be at $x = 2d$. The magnetic field in the space between the wires due to the current in the wires will be in opposite directions. At a distance x from the left wire, the net magnetic field is given by

$$\mathbf{B} = \frac{\mu_0 I}{2\pi x}(\hat{\mathbf{j}}) + \frac{\mu_0 I}{2\pi(2d - x)}(-\hat{\mathbf{j}})$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{x} - \frac{1}{(2d - x)}\right](-\hat{\mathbf{j}})$$

At $x = 0$, $\mathbf{B} = 0$

At $x = 2d$, $\mathbf{B} \to -\infty$ and is along the negative y-direction.

For $x < d$, $\mathbf{B}$ is along the positive y-direction.

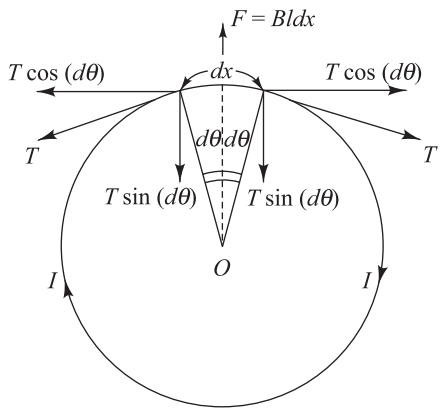
For $x > d$, $\mathbf{B}$ is along the negative y-direction.

For $x < 0$, $\mathbf{B}$ is along the negative y-direction.

For $x > 2d$, $\mathbf{B}$ is along the positive y-direction.

Hence the correct graph is (a).

35. Consider a small element of the wire of length dx. The horizontal components $T\cos(d\theta)$ cancel each other. The vertical components add up to $2T\sin(d\theta)$ because in the limit $d\theta \to 0$, they are collinear. The magnetic force an element dx is $F = B\,Idx$ vetically upward.



For equilibrium,

$2T\sin(d\theta) = BI\,dx = B\,I\,R\,(2\,d\theta)$

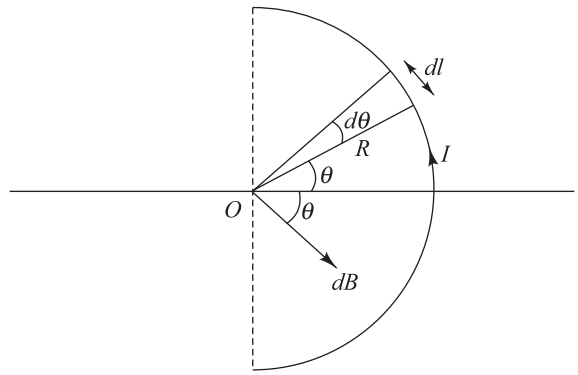
where R is the radius of the ring. For small $d\theta$, $\sin d\theta = d\theta$.

Hence $2\,T\,d\theta = 2\,B\,I\,R\,d\theta$

$$\Rightarrow \qquad T = B\,I\,R = \frac{BIL}{2\pi} \qquad (\therefore\ L = 2\pi R)$$

so the correct choice is (c).

36. Divide the semi-circular ring into a large number of small elements consider an element of length dl.



$dl = R\,d\theta$. Current per unit length is $\lambda = \frac{I}{\pi R}$. Therefore, current flowing through the element is $dI = \lambda dl = \lambda R d\theta$. Magnetic field at O due to the element is

$$dB = \frac{\mu_0 dI}{\pi R} = \frac{\mu_0 \lambda R d\theta}{\pi R}$$

$$= \frac{\mu_0 \lambda d\theta}{\pi}$$

The component of dB along the axis of the semi-circular wire is $dB\cos\theta$. Therefore, the magnetic field due to the complete wire is

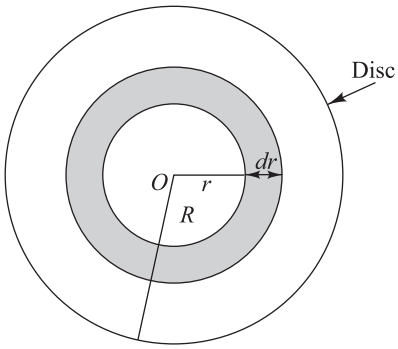
$$B = \int_{-\pi/2}^{\pi/2} dB\cos\theta$$

$$= \frac{\mu_0 \lambda}{\pi}\int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta$$

$$= \frac{\mu_0 \lambda}{2\pi} = \frac{\mu_0 I}{2\pi^2 R}$$

so the correct choice is (b).

37. Charge per unit area of the disc is $\sigma = \frac{Q}{\pi R^2}$. Divide The disc into a large number of concentric circular elements as shown in the figure.



Area of element = $\pi (r + dr)^2 - \pi r^2 = 2\pi\, rdr$

$(\therefore\ dr << r)$

$\therefore$ Charge on the element is dq

$$= \sigma \times 2\pi rdr = \frac{Q}{\pi R^2} \times 2\pi rdr = \frac{2Q}{R^2} \times rdr$$

The magnetic at O due to the element

$$dB = \frac{\mu_0 dq}{2r} \times \frac{\omega}{2\pi} = \frac{\mu_0 \omega dq}{4\pi r} = \frac{\mu_0 \omega \times 2Qrdr}{4\pi R^2 r}$$

$$= \frac{\mu_0 \omega\ Q\ dr}{2\pi R^2}$$

The magnetic field at O due to the complete disc is

$$B = \int_0^R dB = \frac{\mu_0 \omega Q}{2\pi R^2} \int_o^R dr$$

$$= \frac{\mu_0\, \omega Q}{2\pi R}$$

Thus $B \propto \frac{1}{R}$. so the correct graph is (a)

38. $r = \sqrt{\frac{2mk}{3q}}$; k = kinetic energy

$\Rightarrow$ $r = k\sqrt{\frac{m}{q}}$; k = constant = $\sqrt{\frac{2k}{3}}$

$\therefore$ $r_p = k\frac{\sqrt{m_p}}{q}$

$$r_\alpha = k\frac{\sqrt{4m_p}}{2q} = k\sqrt{\frac{m_p}{q}}$$

Hence $r_p = r_\alpha < r_d$, which is choice (b).

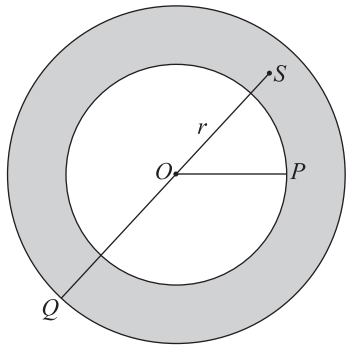
39. Magnetic Moment $\vec{M}$ = current × area of the loop = $I\vec{A}$

$$= I \times \left[a^2 + \pi\left(\frac{a}{2}\right)^2 \times 2 \right] \hat{\mathbf{k}}$$

$$= Ia^2\left(1 + \frac{\pi}{2}\right)\hat{\mathbf{k}}$$

The direction of area vector $\vec{A}$ is along $\hat{\mathbf{k}}$.

40.



$$OP = \frac{R}{2},\ OQ = R,\ OS = r,$$

Inside the cavity

$\left(\text{i.e. for } r \text{ lying between zero and } \frac{R}{2}\right)$; $B = 0$

Outside the cylinder, (i.e. for $r > R$),

$$B = \frac{\mu_0 I}{2\pi r}$$

In the shaded region, $\left(\text{i. e. for } \frac{R}{2} < \text{r} < \text{R}\right)$. From Ampere's circuital law,

$B \times 2\pi r = \mu_0 I = \mu_0 JA$

where $J = \frac{I}{A}$ is the current density and A is the area of the shaded region. Now

$$A = \pi r^2 - \pi\left(\frac{R}{2}\right)^2$$

$$\therefore\ B \times 2\pi r = \mu_0 J\left[\pi r^2 - \frac{\pi R^2}{4}\right]$$

$$\Rightarrow \quad B = \frac{\mu_0 J}{2}\left[\frac{r^2 - R^2/4}{r}\right]$$

$$= \frac{\mu_0 J}{2}\left[r - \frac{R^2}{4r}\right]$$

Hence the correct graph is (d).

41. The resistance S to be connected in parallel with a galvanometer so that it has a range I is

$$S = \frac{I_g G}{I - I_g}$$

where G = galvanometer resistance and I_g= current for full scale deflection. So in order to increase I,

the value of S must be decreased. If an additional resistance S_1 is connected across the galvanometer, the new shunt resistance becomes

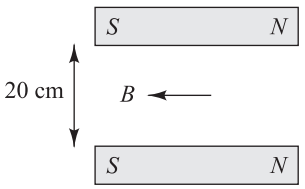
$S_2 = \dfrac{S\ S_1}{S + S_1}$ which is less then S. Hence statement-II is true.

The resistance of the ammeter after S_1 is connected is $R_1 = G + S_1$

The resistance of the ammeter after S_2 is connected is $R_2 = G + S_2$

Since $S_2 < S$; $R_2 < R_1$. Hence statement I is false

42.



Magnetic field due to two short magnetic

$$B = \frac{\mu_0}{4\pi}\frac{M_1}{r^3} + \frac{\mu_0}{4\pi}\frac{M_2}{r^3}$$

$$= \frac{4\pi \times 10^{-7}}{4\pi \times (0.1)^3}[1.2 + 1]$$

$$= 2.2 \times 10^{-4}\ \text{T}$$

The direction of this field is along the horizontal component of the earth's field. Thus,

$B_{net} = 2.2 \times 10^{-4} + 3.6 \times 10^{-5}$

$$= 2.56 \times 10^{-4}\ \text{T}$$

43. The correct choice is (d). The partile move in a circle with a constant speed and hence with a constant kinetic energy.

ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENTS

Chapter 14

REVIEW OF BASIC CONCEPTS

1. Magnetic Flux

The magnetic flux through any surface placed in a magnetic field is determined by the number of field lines that cut through that surface. The magnetic flux through a coil of area A in a uniform magnetic field B is defined as

$$\phi = \mathbf{B} \cdot \mathbf{A} = B\,A \cos\theta$$

where θ is the angle between the normal to the plane of the coil and the magnetic field. If the coil has N turns, the magnetic flux through the coil is given by

$$\phi = N\,B\,A \cos\theta$$

The SI unit of flux is called weber (Wb).

For a curved surface,

$$\phi = \int \mathbf{B}\cdot\mathbf{dA}$$

2. Faraday's Laws of Electromagnetic Induction

The magnitude and direction of induced emf can be determined by the application of two laws of electromagnetic induction: (i) Faraday's law, and (ii) Lenz's law.

Faraday's Law of Electromagnetic Induction

On the basis of various experiments, Faraday found that

1. whenever magnetic flux linked with a circuit changes, an induced emf is produced in the circuit,
2. the induced emf lasts as long as the change in the magnetic flux is taking place, and
3. the magnitude of the induced emf is directly proportional to the rate of change of magnetic flux, i.e.

$$e \propto \frac{d\phi}{dt}$$

Lenz's Law

According to Lenz's law, the direction of the induced emf is such that it always opposes the cause that has produced it.

Thus

$$e = -k\,\frac{d\phi}{dt}$$

where k is a positive constant whose value depends on the system of units. In SI system of units, $k = 1$ and one can write

$$e = -\frac{d\phi}{dt}$$

Magnitude of induced emf is $|e| = \left|\frac{d\phi}{dt}\right|$

If ϕ is the flux through one turn of a coil, then for a coil of N turns

$$|\,e\,| = N\left|\frac{d\phi}{dt}\right|$$

The magnitude of the induced current is given by

$$i = \frac{\text{induced emf}}{\text{total resistance of circuit}} = \frac{1}{R}\left|\frac{d\phi}{dt}\right|$$

The direction of induced current is obtained by Lenz's law.

Flow of Induced Charge

When a current is induced in a circuit due to change in magnetic flux, induced charge q flows through the circuit.

$$q = \int i\,dt = \int \frac{1}{R}\left|\frac{d\phi}{dt}\right|\,dt = \frac{1}{R}\int |d\phi| = \frac{\text{change in flux}}{\text{Resistance}}$$

Heat Dissipation

Heat dissipated due to induced current is

$$H = \int eidt = \int \left|\frac{d\phi}{dt}\right| idt = i\int |d\phi|$$

= induced current × change in flux

Fleming's Right Hand Rule

This rule gives the direction of the induced emf when a conductor moves at right angles to a magnetic field. Hold the thumb and the first two fingers of your right hand mutually perpendicular to each other. Then, if the first finger points in the direction of the magnetic field and the thumb points in the direction of the motion of the conductor, then the second finger gives the direction of the induced emf (and hence of the induced current).

Applications of Lenz's Law

(i) If the magnet is moved towards the coil or coil is moved towards the magnet, the induced current i is anticlockwise Fig. 14.1. The current is clockwise if the magnet is moved away from the coil.

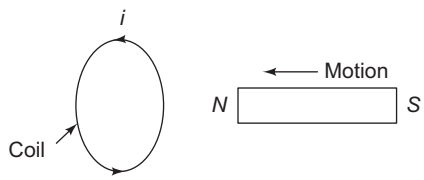


Fig. 14.1

(ii) The induced current i in the coil is anticlockwise if (Fig. 14.2)

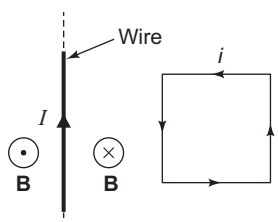


Fig. 14.2

(a) the coil is moved towards the long wire carrying current I

or

(b) the current I increases with time.

The current I is clockwise if

(a) the coil is moved away from wire

or

(b) the current I decreases with time.

(iii) Two coils carrying currents I_1 and I_2 placed with their planes parallel approach each other (Fig. 14.3).

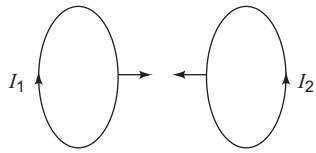


Fig. 14.3

(a) If I_1 and I_2 are both clockwise (or anticlockwise), then both I_1 and I_2 will decrease.

(b) If the currents I_1 and I_2 are in opposite sense, both the currents will increase.

3. Expression for Induced EMF

(i) **Change in flux due to change in magnetic field (B).**

If **B** increases with time, the induced current i is anticlockwise so that it produces a magnetic field pointing outwards (opposite to **B**). The induced emf is (Fig. 14.4)

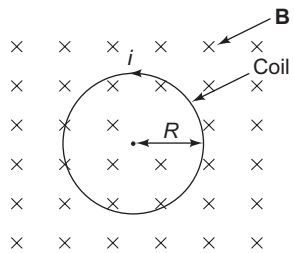


Fig. 14.4

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = \pi R^2 \frac{dB}{dt}$$

If **B** decreases with time, I will be clockwise.

If **B** remains constant but the radius of the coil increases at a rate $\frac{dR}{dt}$, then

$$|e| = B\frac{d}{dt}(\pi R^2) = B \times 2\pi R \frac{dR}{dt}.$$

(ii) **Change in flux due to change in area (A)**

If a rectangular coil $PQRS$ is moved out of a region of uniform magnetic field **B** with a velocity v, the emf induced is (Fig. 14.5(a))

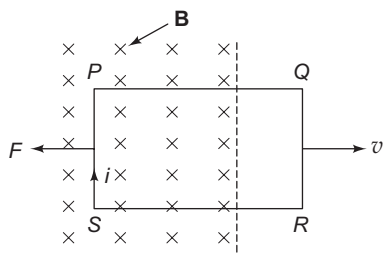


Fig. 14.5(a)

$|e| = Blv$ where $l = PS = QR$

Induced current i is clockwise. If R is the resistance of the coil,

$$i = \frac{e}{R} = \frac{Blv}{R}$$

Force F required to pull the coil out with constant velocity v is

$$F = Bil = \frac{B^2l^2v}{R}$$

Power needed is $P = Fv = \dfrac{B^2l^2v^2}{R}$

= heat dissipated

The current will be anticlockwise, if the coil is pushed into the region of magnetic field.

Note

(a) If the coil is moved within the region of uniform magnetic field, no change in flux takes place and hence no emf is induced.

(b) If the magnetic field is non-uniform and the coil is kept stationary in it, no change in flux occurs and hence no emf is induced.

The above results also hold in the case of rod XY sliding on metallic rails $PQRS$ to the right as shown in Fig. 14.5(b).

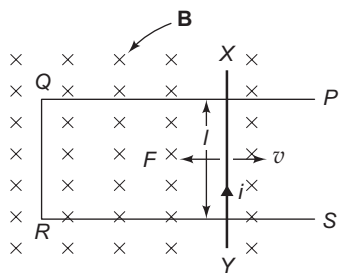


Fig. 14.5(b)

(iii) **Change in flux due to change in orientation (θ) (A.C. generator)**

If a coil of area A, consisting of N turns is rotated in a magnetic field B with angular velocity ω, the emf induced in it is given by

$$e = e_0 \sin\theta = e_0 \sin\omega t$$

where $e_0 = NBA\omega$ is the amplitude (peak value). Thus an alternating emf is produced.

4. Motional Emf

(i) When a rod (or wire) of length l is moved with a velocity v in a magnetic field **B** as shown in Fig. 14.6(a), the emf induced between the ends P and Q of the rod is given by

$$e = Blv$$

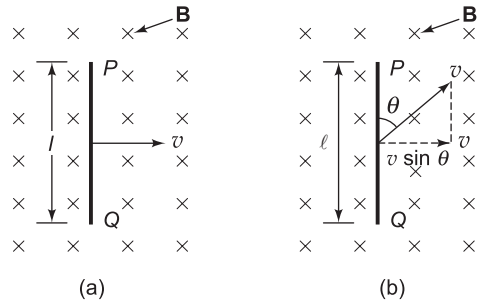


Fig. 14.6

If the rod is moved as shown in Fig. 14.6(b), then

$$e = Blv \sin\theta$$

(ii) When a semicircular rod (or wire) of radius R is moved with a velocity v in a magnetic field **B** as shown in Fig. 14.7, the emf induced between the ends P and Q of the rod is given by $e = Bv\,(2R) = 2BvR$

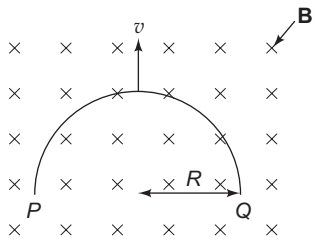


Fig. 14.7

(iii) When a rod PQ of length l pivoted at one end P is rotated with angular velocity ω in a magnetic field **B** as shown in Fig. 14.8(a), the emf induced between its ends is given by

$$e = \frac{1}{2}B\omega l^2$$

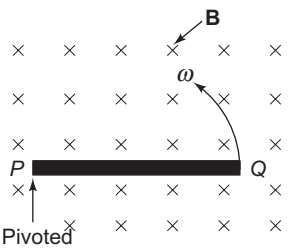


Fig. 14.8(a)

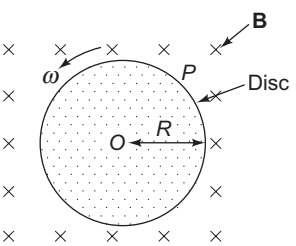


Fig. 14.8(b)

(iv) When a disc of radius R is rotated about its centre with angular velocity ω in a magnetic field **B** as shown in Fig. 14.8(b), the emf induced between its centre O and a point P on its rim is given by

$$e = \frac{1}{2} B\omega R^2$$

5. Electric Motor

When a current is passed through a coil placed in a magnetic field by connecting its end to a source of voltage V, it experiences a torque which rotates it. As a result, an emf e is induced in it. This emf is called back emf as it opposes the applied voltage V (from Lenz's law). If R is the resistance of the coil, then current in it is

$$i = \frac{V - e}{R}$$

Input power = Vi and heat loss = i^2R. Hence output power = $Vi - i^2R = ei$.

Efficiency of motor $\eta = \dfrac{ei}{Vi} = \dfrac{e}{V}$

Some important points about a d.c. motor

(i) Back emf e and hence current i vary sinusoidally even if the source of voltage V is a d.c. (battery).

(ii) When output power is maximum, $e = \dfrac{V}{2}$ and $\eta = 50\%$.

(iii) Initially, i.e. when the motor is switched on, $e = 0$ and initial current = V/R which is very large. So, for safety, a starter is used.

(iv) At full speed, back emf is maximum and current i is minimum.

EXAMPLE 1 A straight metal wire of length L, cross-sectional area a and resistivity ρ is made into a square frame. A uniform magnetic field B is perpendicular to the plane of the frame and is changing at a constant rate dB/dt. The current induced in the frame is

(a) $\dfrac{La}{\rho}\dfrac{dB}{dt}$ (b) $\dfrac{La}{4\rho}\dfrac{dB}{dt}$

(c) $\dfrac{La}{8\rho}\dfrac{dB}{dt}$ (d) $\dfrac{La}{16\rho}\dfrac{dB}{dt}$

SOLUTION Side of frame = $\dfrac{L}{4}$. Area of frame is $A = L^2/16$. Magnitude of induced emf is

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(AB) = A\frac{dB}{dt} = \frac{L^2}{16}\frac{dB}{dt}$$

Resistance of frame is $R = \dfrac{\rho L}{a}$.

$\therefore$ Induced current $= \dfrac{|e|}{R} = \dfrac{La}{16\rho}\dfrac{dB}{dt}$

EXAMPLE 2 A square metal frame $PQRS$ of side 15 cm and resistance 1.0 Ω is moved with a speed of 4/3 cm s^{-1} in a uniform magnetic field B = 2.0 T which is perpendicular to the plane of the frame as shown in Fig. 14.9. The frame is connected to a network of resistances as shown. The current induced in the frame is

(a) 1 mA (b) 2 mA

(c) 3 mA (d) 4 mA

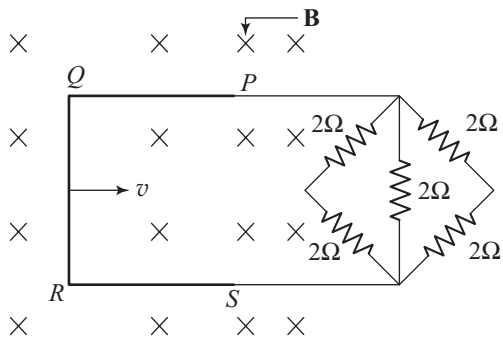


Fig. 14.9

SOLUTION Equivalent resistance of the network between P and S is given by

$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} \quad \Rightarrow \quad R_{eq} = 1\ \Omega$$

Total resistance $R = 1 + 1 = 2\ \Omega$

Magnitude of emf induced in the frame is

$$|e| = B\,l\,v$$

$$= 2.0 \times 0.15 \times \left(\frac{4}{3} \times 10^{-2}\right) = 4 \times 10^{-3}\ \text{V}$$

$\therefore$ Induced current $= \dfrac{|e|}{R} = \dfrac{4 \times 10^{-3}\ \text{V}}{2\ \Omega} = 2 \times 10^{-3}\ \text{A}$

$= 2$ mA

EXAMPLE 3 A square coil $PQRS$ of resistance 2 Ω, 100 turns and side 10 cm is placed in a magnetic field B = 2.0 T. The direction of the magnetic field is perpendicular to the plane of the coil as shown in Fig. 14.10. The work done is pulling the coil completely out of the region of magnetic field in 2.0 s without any acceleration is

(a) 0.01 J (b) 0.1 J

(c) 1.0 J (d) 10 J

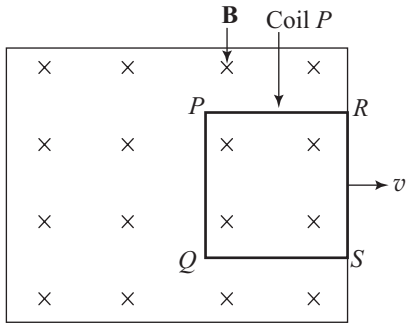


Fig. 14.10

SOLUTION Speed of coil is $v = \dfrac{100\text{ cm}}{2.0\text{ s}} = 5\text{ cm s}^{-1}$

$= 5 \times 10^{-2}\text{ ms}^{-1}$

The emf induced in arm PQ of the coil is

$$|e| = N B l v$$
$$= 100 \times 2.0 \times 0.1 \times 5 \times 10^{-2}$$
$$= 1.0\text{ V}$$

Induced current in the coil is

$$I = \frac{|e|}{R} = \frac{1.0\text{ V}}{2\ \Omega} = 0.5\text{ A}$$

Force on PQ due to magnetic field is

$$F = B I l = 2.0 \times 0.5 \times 0.1 = 0.1\text{ N}$$

From Lenz's law, this force acts to the left on arm PQ. To pull the coil to the right without acceleration, an external force $F = 0.1$ N must be applied to the right. Therefore, work done to pull the coil completely is

$$W = 0.1\text{ N} \times 0.1\text{ m}$$
$$= 0.01\text{ J}$$

So the correct choice is (a).

EXAMPLE 4 A wire PQ of mass $m = 10$ g and length $l = 25$ cm can freely slide on horizontal, smooth and parallel rails placed in a uniform magnetic field $B = 2.0$ T as shown in Fig. 14.11. The ends of the rails are connected by a capacitor $C = 2$ mF. A constant force $F = 1.2 \times 10^{-3}$ N is applied as shown. If the resistance of the rails is zero ohm, the acceleration of wire PQ will be

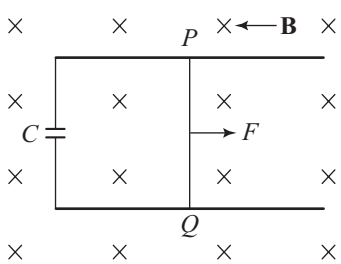


Fig. 14.11

(a) 0.1 ms^{-2} (b) 0.5 ms^{-2}

(c) 1.0 ms^{-2} (d) 2.0 ms^{-2}

SOLUTION Let v be the velocity of the wire at time t. The induced emf is $e = B\ l\ v$. The charge on the capacitor at time t is

$$q = Ce = C\ B\ l\ v$$

$$\therefore \quad \text{Current} \qquad I = \frac{dq}{dt} = C B l \frac{dv}{dt} = C B l a$$

where a = acceleration of the wire. The direction of a (from Lenz's law) is to the left. Force on PQ due to this current is

$$f = B I l = CB^2 l^2 a \text{ towards left.}$$

$\therefore$ Net force on wire is $F_{\text{net}} = F - f = F - CB^2 l^2 a$. The acceleration of wire by this force is

$$a = \frac{F_{\text{net}}}{m} = \frac{F - CB^2 l^2 a}{m}$$

$$\Rightarrow \qquad a = \frac{F}{m + CB^2 l^2}$$

$$= \frac{1.2 \times 10^{-3}}{10^{-2} + 2 \times 10^{-3} \times (2.0)^2 \times (0.25)^2}$$

$$= 0.1\text{ m s}^{-2}$$

EXAMPLE 5 A square coil of resistance 2 Ω, 100 turns and side 10 cm is placed with its plane making an angle of 30° with a uniform magnetic field of 0.1 T. In 0.05 s the coil rotates until its plane becomes parallel to the magnetic field. Calculate the current induced in the coil.

SOLUTION $R = 2\ \Omega$, $N = 100$, $A = 0.1 \times 0.1 = 10^{-2}$ m^2,

$$\theta_1 = 90° - 30° = 60°,$$

$$\theta_2 = 90° - 0 = 90°, B = 0.1\text{ T and } t = 0.05\text{ s}$$

Change in flux $= NBA\ (\cos\theta_2 - \cos\theta_1)$

$$= 100 \times 0.1 \times 10^{-2} \times (\cos 90° - \cos 60°)$$

$$= 0.1 \times \left(0 - \frac{1}{2}\right) = 0.05\text{ Wb}$$

$$\text{Induced emf } e = \frac{\text{change in flux}}{\text{time}} = \frac{0.05}{0.05} = 1\text{ V}$$

$$\text{Induced Current } i = \frac{e}{R} = \frac{1}{2} = 0.5\text{ A}$$

EXAMPLE 6 A solenoid of diameter 0.2 m has 500 turns per metre. At the centre of this solenoid, a coil of 100 turns is wrapped closely around it. If the current in the solenoid changes from zero to 2 A in 1 millisecond, calculate the induced emf developed in the coil.

SOLUTION The magnetic field due to current I in a solenoid having n turns per unit length is

$$B = \mu_0 n I$$

This is the magnetic field threading the coil. The direction of the field is parallel to the axis of the solenoid. Hence angle between the normal to the plane of the coil and the magnetic field is $\theta = 0°$. If A is the cross-sectional area of the coil and N the number of turns in it, then the magnetic flux threading the coil is

$$\phi = NAB \cos\theta = NAB \cos 0° = NAB = N\pi r^2 \mu_0 nI$$

Since the coil is wrapped closely around the solenoid, the radius of the coil (r) = radius of solenoid = 0.1 m.
Change in flux if the current change from $I_1 = 0$ to $I_2 = 2$ A is

$$\Delta\phi = N\pi r^2 \mu_0 n (I_2 - I_1)$$
$$= 100 \times (\pi \times 0.1^2) \times (4\pi \times 10^{-7}) \times 500 \times (2 - 0)$$
$$= 4\pi^2 \times 10^{-4} \text{ Wb}$$

$$\text{Induced emf } e = \left|\frac{\Delta\phi}{\Delta t}\right| = \frac{4\pi^2 \times 10^{-4}}{10^{-3}} = 0.4\pi^2 = 3.95 \text{ V}$$

EXAMPLE 7 The magnetic flux through a coil of resistance 6.5 Ω placed with its plane perpendicular to a uniform magnetic field varies with time t (in second) as

$$\phi = (3t^2 + 5t + 2) \text{ milliweber}$$

Find the induced current in the coil at $t = 10$ s.

SOLUTION $$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(3t^2 + 5t + 2)$$
$$= (6t + 5) \text{ MV.}$$

At $t = 10$ s, $e = (6 \times 10 + 5)$ mV $= 65 \times 10^{-3}$ V

Induced current at $t = 10$ s is

$$I = \frac{e}{R} = \frac{65 \times 10^{-3}}{6.5} = 10^{-2} \text{A}$$

EXAMPLE 8 A metal wheel with 8 metallic spokes, each 60 cm long is rotated at a speed of 100 rev./min in a plane perpendicular to earth magnetic field of 0.3×10^{-4} T. Find the magnitude of the induced emf between the axle and the rim of the wheel.

SOLUTION $\nu = 100$ rev./min. $= \frac{100}{60} = \frac{5}{3}$ rev./s

$l = 0.6$ m. The emf developed between the ends of a spoke is (as $\omega = 2\pi\nu$)

$$e = \frac{1}{2} Bl^2 \omega$$
$$= \frac{1}{2} \times (0.3 \times 10^{-4}) \times (0.6)^2 \times (2\pi \times \frac{5}{3})$$
$$= 1.8\pi \times 10^{-5} \text{ V} = 5.65 \times 10^{-5} \text{ V}$$

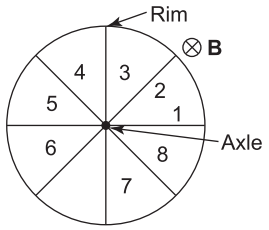


Fig. 14.12

The same emf is induced between the ends of each spoke. It is clear from Fig. 14.12 that the spokes are joined in parallel. Hence the emf between rim and axle = emf across each spoke = 5.65×10^{-5} V.

EXAMPLE 9 An aircraft with a wing span of 50 m is flying with a speed of 1080 kmh^{-1} in the eastward direction at a constant altitude at a place where the vertical component of earth's magnetic field is 2×10^{-5} T. Find the emf developed between the tips of the wing.

SOLUTION $v = 1080$ kmh^{-1} $= 300$ ms^{-1}

$$e = Blv = (2 \times 10^{-5}) \times 50 \times 300 = 0.3 \text{ V}$$

EXAMPLE 10 A circular coil of mean radius r and having N turns is kept in a horizontal plane. A magnetic field B exists in the vertical direction as shown in Fig. 14.13(a). Find the emf induced in the loop

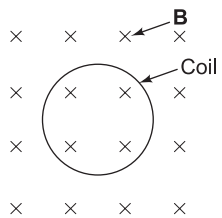


Fig. 14.13(a)

(a) if it is held stationary and the magnetic field is uniform,

(b) if it is held stationary and the magnetic field is non-uniform,

(c) if it is rotated with an angular velocity ω about an axis passing through its centre and perpendicular to its plane, and the magnetic field is uniform.

(d) if it is rotated with an angular velocity ω about its diameter. Assume that the normal to the plane of the coil makes an angle $\theta = 0$ with the magnetic field at time $t = 0$.

(e) if the coil is a square of side L and is rotated about its diameter.

SOLUTION The emf is induced if the magnetic flux through the coil changes with time.

(a) In this case there is no change in magnetic flux with time, hence no emf induced.

(b) In this case also the magnetic flux through the coil does not change with time, hence no emf induced.

(c) In this case, the number of field lines through the coil does not change with time, hence the magnetic flux does not change with time. So no emf is induced in the coil (see Fig. 14.13(b))

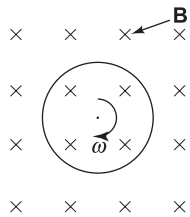


Fig. 14.13(b)

(d) If the coil is rotated about a diameter, as shown in Fig. 14.13(c), there is a change in magnetic flux with time. Hence emf will be induced in the coil. Area of the coil is $A = \pi r^2$. If the normal to the plane of the coil makes an angle $\theta = 0$ with the magnetic field, at times $t = 0$ then at time t, $\theta = \omega t$. The magnetic flux at this time is

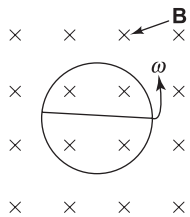


Fig. 14.13(c)

$$\phi = NBA \cos\theta = NBA \cos \omega t$$

$\therefore$ Induced emf is

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA \cos \omega t)$$

$$= \omega NBA \sin \omega t$$

$$= \omega NB \times \pi r^2 \sin \omega t$$

$$= \pi N r^2 B \omega \sin \omega t$$

(e) If a square coil of side L and N turns is rotated about a diameter as shown in 14.13(d), there is a change in magnetic flux with time. Hence emf will be induced in the coil. Area of the coil is $A = L^2$. If the normal to the plane of the coil makes an angle $\theta = 0$ with the magnetic field at time $t = 0$, then at time t, $\theta = \omega t$. The magnetic flux at this time is

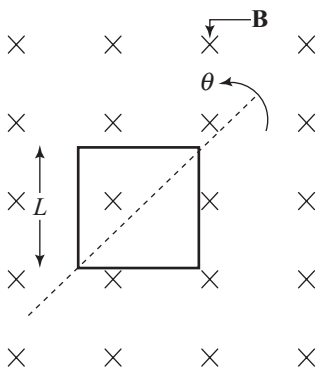


Fig. 14.13(d)

$$\phi = N\,B\,A \cos\theta$$

$$= N\,B\,L^2 \cos(\omega t)$$

$\therefore$ Induced emf is

$$e = -\frac{d\phi}{dt} = NBL^2\,\omega \sin(\omega t)$$

Thus, as in case (d) above, an alternating emf is induced in the coil

EXAMPLE 11 A metal rod PQ of length L is moved with a velocity v making an angle θ with a uniform magnetic field B as shown in Fig. 14.14. Obtain the expression for the emf induced between the ends of the rod.

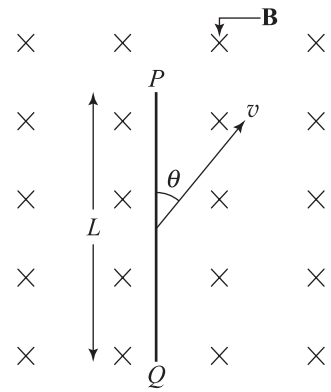


Fig. 14.14

SOLUTION The component of velocity v perpendicular to the length of the rod is

$$v_\perp = v \sin\theta$$

Only the perpendicular component induces an emf in the rod. Since the magnetic field is perpendicular to the plane of motion, the emf induced between the ends of the rod is

$$e = B\,l\,v_\perp = B\,l\,v \sin\theta$$

EXAMPLE 12 A metal rod PQ moves with a velocity v parallal to a very long straight wire CD carrying a current I as shown in Fig. 14.15. The ends P and Q of the rod are at distances a and b from the wire as shown. Obtain the expression for the emf induced between the ends of the rod.

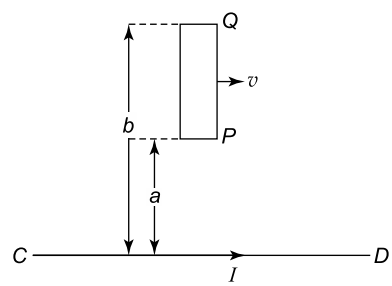


Fig. 14.15

SOLUTION Divide the rod into a large number of very small elements, each of length dx. Consider one such element at distance x from wire CD as shown in Fig. 14.16.

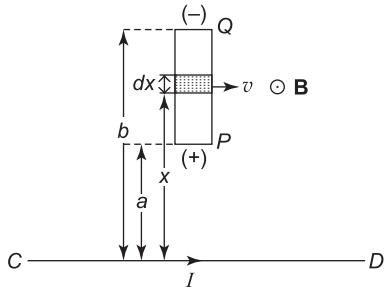


Fig. 14.16

The magnetic field at the element due to current I in wire CD is

$$B = \frac{\mu_0 I}{2\pi x}$$

The direction of the field is upwards perpendicular to velocity v. The magnitude of magnetic field is different at different points on the rod PQ. From Fleming's L.H. rule, the free electrons in the rod will experience force in the direction P to Q. So free electrons move from P to Q. Hence end P acquires a positive charge (due to loss of electrons) and end Q acquires a negative charge (due to gain of electrons).

Force on the element is

$$dF = qvB = qv \times \frac{\mu_0 I}{2\pi x}$$

Therefore, electric field set up in the element is

$$dE = \frac{dF}{q} = \frac{qv\mu_0 I}{2\pi xq} = \frac{\mu_0 Iv}{2\pi x}$$

Now $$dE = -\frac{dV}{dx}$$

$$\Rightarrow \qquad dV = -dE \times dx = -\frac{\mu_0 Iv}{2\pi x} dx$$

where dV is the voltage induced in the element. The voltage induced in the rod PQ is

$$|V| = \frac{\mu_0 Iv}{2\pi} \int_a^b \frac{dx}{x} = \frac{\mu_0 Iv}{2\pi} \ln\left(\frac{b}{a}\right)$$

EXAMPLE 13 A metal rod PQ of length l slides with a velocity v on two parallel rails AB and CD parallel to a long straight wire XY carrying a current I as shown in Fig. 14.17. A resistance R is connected between the rails as shown. The velocity of rod PQ is kept constant by applying force.

(a) Obtain the expression for the current induced in resistance R.

(b) Obtain the expression for the force to be applied on rod PQ to keep its velocity constant at v.

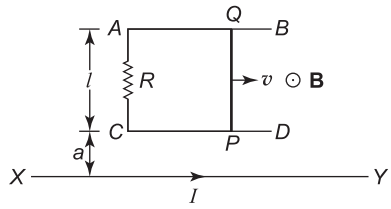


Fig. 14.17

SOLUTION In this case the induced emf is due to change in magnetic flux which is due to the change in the area of $ACPQ$ with time.

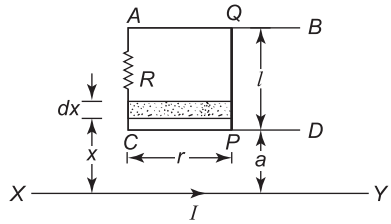


Fig. 14.18

Magnetic flux through an infinitesimal area element of width dx at a distance x from PQ is [Fig. 14.18]

$$d\phi = BdA = Brdx = \frac{\mu_0 Ir}{2\pi x} dx$$

where r is the position of PQ at an instant of time t. Magnetic flux through $ACPQ$ is

$$\phi = \int d\phi = \int_a^{(a+l)} \frac{\mu_0 Ir}{2\pi x} dx$$

$$= \frac{\mu_0 Ir}{2\pi} \int_a^{(a+l)} \frac{dx}{x} = \frac{\mu_0 Ir}{2\pi} \ln\left(\frac{a+l}{a}\right)$$

$$\therefore \text{ Induecd emf } e = \left|\frac{d\phi}{dt}\right| = \frac{\mu_0 I}{2\pi} \ln\left(\frac{a+l}{a}\right)\frac{dr}{dt}$$

$$= \frac{\mu_0 Iv}{2\pi} \ln\left(\frac{a+l}{a}\right) \qquad \left(\because v = \frac{dr}{dt}\right)$$

(a) Current induced in R is

$$i = \frac{e}{R} = \frac{\mu_0 Iv}{2\pi R} \ln\left(\frac{a+l}{a}\right)$$

(b) This current will exert a force on the element of width dx which is given by

$$dF = Bidx = \frac{\mu_0 I}{2\pi x} idx$$

$\therefore$ Force to be applied on PQ is

$$F = \int dF = \frac{\mu_0 Ii}{2\pi}\int_a^{(a+l)} \frac{dx}{x}$$

$$= \frac{\mu_0 I}{2\pi} \times \frac{\mu_0 Iv}{2\pi R} \ln\left(\frac{a+l}{a}\right)\int_a^{(a+l)} \frac{dx}{x}$$

$$= \frac{v}{R}\left[\frac{\mu_0 I}{2\pi}\ln\left(\frac{a+l}{a}\right)\right]^2$$

EXAMPLE 14 A metal rod PQ of length l slides on two parallel rails AB and CD, each rail having a resistance k per unit length. The rod and the rails are in a region of a uniform magnetic field B directed into the plane of the paper as shown in Fig. 14.19. A resistance R is connected between the rails. A variable force F is applied to PQ so that it is accelerated to the right. Obtain the expression for the velocity v of rod PQ when it is at a distance x from R.

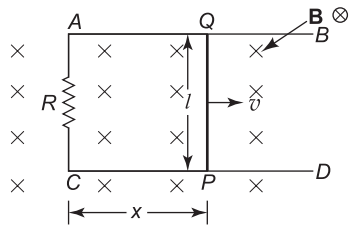


Fig. 14.19

SOLUTION Magnetic flux through $ACPQ$ when the rod is at a distance x from R is

$$\phi = BA = Blx$$

Induced emf at that instant is

$$e = \left|\frac{d\phi}{dt}\right| = Bl\,\frac{dx}{dt} = Blv$$

Resistance of $ACPQ = R + 2kx$. Therefore, current induced in the circuit is

$$i = \frac{e}{(R+2kx)} = \frac{Blv}{(R+2kx)}$$

$$\Rightarrow \qquad v = \frac{i(R+2kx)}{Bl}$$

EXAMPLE 15 A metal rod PQ of mass m and of negligible resistance slides on two parallel metal rails AB and CD separated by a distance l. The rails have negligible resistance and have a resistance R connected between them as shown in Fig. 14.20. The rod and the rails are located in a region of uniform magnetic field direction into the plane of the loop $ACPQ$. The rod is given an initial velocity u. Obtain the expression for the distance x covered by the rod before it comes to rest. Neglect friction between the rod and the rails.

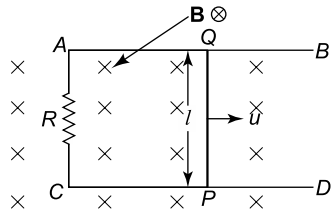


Fig. 14.20

SOLUTION Induced currnet $i = \dfrac{Bvl}{R}$

$$\text{Force } F = Bil = -ma = -m\,\frac{dv}{dt}$$

The negative sign shows that force opposes the acceleration (Lenz's law)

$$\therefore \qquad m\,\frac{dv}{dt} = -B \times \frac{Bvl}{R} \times l$$

$$= -\frac{B^2 vl^2}{R}$$

$$\Rightarrow \qquad \frac{dv}{v} = -\frac{B^2 l^2}{mR}\,dt = -k\,dt$$

where $$k = \frac{B^2 l^2}{mR}$$

Intergrating $$\int_u^v \frac{dv}{v} = -k\int_0^t dt$$

$$\Rightarrow \qquad \ln\left(\frac{v}{u}\right) = -kt$$

$$\frac{v}{u} = e^{-kt}$$

$$\Rightarrow \qquad v = ue^{-kt}$$

$$\therefore \qquad \frac{dx}{dt} = ue^{-kt}$$

The rod comes to rest when $t = \infty$.
Integrating

$$\int_0^x dx = u\int_0^\infty e^{-kt}\,dt$$

$$x = -\frac{u}{k}\left|e^{-kt}\right|_0^\infty = -\frac{u}{k}\,(0-1) = \frac{u}{k}$$

$$\therefore \qquad x = \frac{umR}{B^2 l^2}$$

EXAMPLE 16 A circular coil of radius r has N turns and a resistance R. It is placed with its plane at right angles to a uniform magnetic field B. Find the expression for the amount of charge Q which passes through the coil when it is rotated through an angle of 180° in its plane.

SOLUTION Area of the coil $(A) = \pi r^2$

Since the plane of the coil is normal to the magnetic field, the magnetic flux through the coil = $NBA \cos 0° = NBA$. When the coil is rotated through 180°, the magnetic flux through it will be = $NBA\cos 180° = -NBA$. Therefore, change in flux is

$$\phi = NBA - (-NBA) = 2NBA$$

Magnitude of induced emf is

$$|e| = \frac{d\phi}{dt}$$

$\therefore$ Induced current is $i = \dfrac{|e|}{R} = \dfrac{d\phi}{dt} \times \dfrac{1}{R}$

$$\Rightarrow \quad iR = \frac{d\phi}{dt}$$

$$\frac{dq}{dt} R = \frac{d\phi}{dt}$$

$$\Rightarrow \quad dqR = d\phi$$

$$\Rightarrow \quad dq = \frac{d\phi}{R}$$

$$\Rightarrow \quad Q = \frac{\phi}{R} = \frac{2NBA}{R} = \frac{2NB \times \pi r^2}{R}$$

EXAMPLE 17 Two circular coils A and B of radii a and b respectively (with $b > a$) have their planes perpendicular to the plane of the page. They are separated co-axially by a distance $x = \sqrt{3}\, b$ as shown in Fig. 14.21. A transient current I flows through coil B for a very short time interval. If the resistance of coil A is R obtain the expression for the charge that flows through coil A during the short time interval.

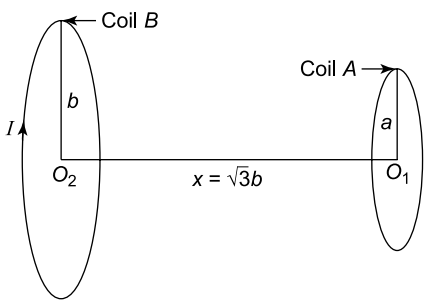


Fig. 14.21

SOLUTION Magnetic field at the centre of coil A due to current I in coil B is

$$B_{AB} = \frac{\mu_0 I b^2}{2(b^2 + x^2)^{3/2}} = \frac{\mu_0 I}{16b} \quad (\because x = \sqrt{3}\, b)$$

Since the magnetic field is along the axis of coil A, it is perpendicular to the plane of A, hence $\theta = 0°$. Therefore, magnetic flux through A is

$$\phi = B_{AB} \times \text{area of coil } A \times \cos 0° = \frac{\mu_0 I \times \pi a^2}{16b}$$

$\therefore$ Induced emf is $|e| = \left|\dfrac{d\phi}{dt}\right|$

$$\Rightarrow \quad IR = \frac{d\phi}{dt}$$

$$\Rightarrow \quad IRdt = d\phi$$

$$\therefore \quad \int Idt = \frac{1}{R}\int d\phi = \frac{\phi}{R}$$

or $$Q = \frac{\phi}{R} = \frac{\mu_0 I \times \pi a^2}{16bR}$$

EXAMPLE 18 A thin non-conducting disc of radius R and mass M is held horizontally and is capable of rotation about an axis passing through its centre and perpendicular to its plane. A charge Q is distributed uniformly over the surface of the disc.

A time-varying magnetic field $B = kt$ (where k is a constant and t is the time) directed perpendicular to the plane of the disc is applied to it. If the disc is stationary initially (i.e. at $t = 0$). Find

(a) The torque acting on the disc.

(b) The angular velocity acquired by the disc as a function of t.

SOLUTION

(a) Area of disc = πR^2

Charge per unit area = $\dfrac{Q}{\pi R^2}$

Area of a small element of width dx at a distance x from the centre of the disc = $2\pi x dx$. Therefore, charge of the element is [Fig. 14.22]

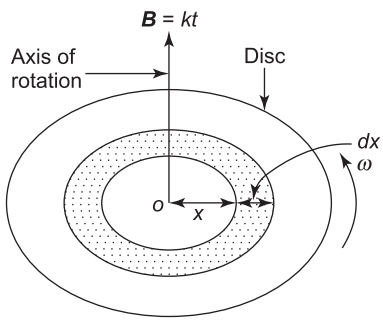


Fig. 14.22

$$dq = \frac{Q}{\pi R^2} \times 2\pi x dx$$

A time-varying magnetic field gives rise to an electric field E. Since

$$E = -\frac{dV}{dl}$$

$$\int dV = -\int E dl = -E \times 2\pi x$$

$$\Rightarrow \quad V = -E \times 2\pi x \qquad (1)$$

where V is emf induced in the element, which is given by

$$V = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA)$$

$$= -\frac{d}{dt}(kt \times \pi x^2) = -\pi k x^2 \qquad (2)$$

From (1) and (2) we get

$$-E \times 2\pi x = -\pi k x^2$$

$$\Rightarrow \quad E = \frac{kx}{2} \qquad (3)$$

Force acting on the element is

$$dF = dq \times E = \frac{Q}{\pi R^2} \times 2\pi x dx \times \frac{kx}{2}$$

$$= \frac{kQ}{R^2} x^2 dx$$

Torque acting on the disc is

$$\tau = \int_0^R x dF = \frac{kQ}{R^2} \int_0^R x^3 dx = \frac{kQR^2}{4}$$

(b) $\tau = I\alpha$, where I is the moment of inertia of the disc about the axis of rotation and α is the angular acceleration

$$I = \frac{1}{2} MR^2 \text{ and } \alpha = \frac{d\omega}{dt} \text{. Hence}$$

$$\frac{kQR^2}{4} = \frac{1}{2} MR^2 \times \frac{d\omega}{dt}$$

$$\Rightarrow \quad d\omega = \frac{kQ}{2M} dt$$

$$\Rightarrow \quad \int_0^\omega d\omega = \frac{kQ}{2M} \int_0^t dt$$

$$\Rightarrow \quad \omega = \frac{kQt}{2M}$$

6. Mutual Inductance

If the current in a coil is i then the flux linked with a neighbouring coil is $\phi = Mi$ where M is the coefficient of mutual inductance. If current i is changing with time, the emf induced in the neighbouring coil is

$$e = -M \frac{di}{dt}$$

Expressions of M in some situations

(i) A small coil of length l, number of turns N_1 wound closely on a long coil of N_2 turns.

$$M = \frac{\mu_0 N_1 N_2 A}{l}\text{; } A = \text{common cross-sectional area}$$

(ii) Two coplanar and concentric coils of radii R and r ($R >> r$) Fig. (14.23)

$$M = \frac{\mu_0 \pi r^2}{2R}$$

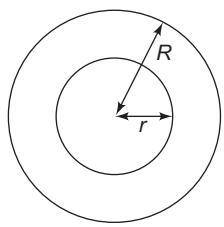


Fig. 14.23

(iii) A small circular coil of radius r at the centre of a large rectangular coil of sides a and b with $a, b >> r$ (Fig. 14.24)

$$M = \frac{2\mu_0 r^2 \sqrt{a^2 + b^2}}{ab}$$

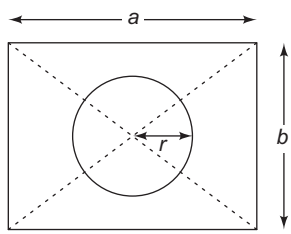


Fig. 14.24

(iv) A rectangular loop of sides a and b placed at a distance x from a long straight wire (Fig. 14.25)

$$M = \frac{\mu_0 a}{2\pi} \log_e \left(\frac{x+b}{x} \right)$$

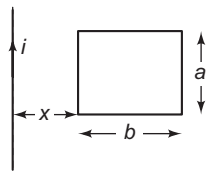


Fig. 14.25

Note If the medium is different from air, μ_0 in above expressions is replaced by $\mu = \mu_0 \mu_r$, where μ_r is the relative permeability of the medium.

7. Self Inductance

If i is the instantaneous current in a coil, flux $\phi = Li$, where L is the self inductance of the coil. Induced emf $e = -L\frac{di}{dt}$.

(i) The self inductance of a coil of N turns, cross-sectional area A and length l is given by

$$L = \frac{\mu_0 N^2 A}{l}$$

(ii) Direction of induced emf is such that it opposes the change in current (Fig. 14.26(a) and (b))

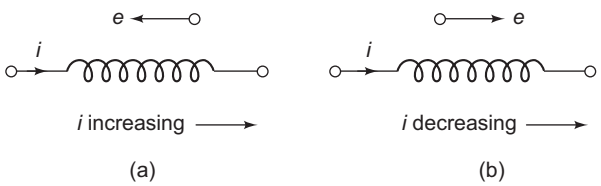


Fig. 14.26

(iii) Energy stored in the inductor $U = \frac{1}{2} Li^2$.

(iv) Inductors in series (Fig. 14.27)

Equivalent inductance is

(a) $L = L_1 + L_2$ (when the flux linked with one coil is not linked with the other, i.e. $M = 0$)



Fig. 14.27

(b) $L = L_1 + L_2 + 2M$ (when flux of one coil is in the same direction as that of the other coil)
$L = L_1 + L_2 - 2M$ (when the fluxes oppose each other)

(v) Inductors in parallel

$$\frac{1}{L} = \frac{1}{L} + \frac{1}{L_2} \text{ (when } M = 0)$$

(vi) $M = k\sqrt{L_1 L_2}$; k = coefficient of coupling.

8. Growth and Decay of Current in a d.c L—R Circuit (Fig. 14.28)

If switch S_1 is closed at $t = 0$, with switch S_2 open, no current flows in the beginning (as the inductor behaves as open switch) [Fig. 14.28]. The current starts increasing and at time t, $i = i_o (1 - e^{-t/\tau})$, where $\tau = \frac{L}{R}$ is the time constant. (see Fig. 14.29)

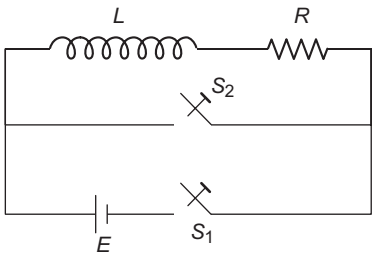


Fig. 14.28

After a long time ($t = \infty$), the current attains a steady value $i_o = E/R$ (now the ideal inductor behaves as a closed switch).

At $t = \tau$, $i = i_o \left(1 - \frac{1}{e}\right) = 0.632\, i_0$.

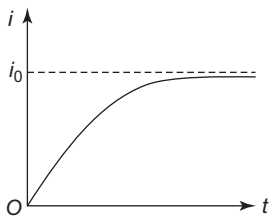


Fig. 14.29

Decay of current: At time $t = 0$, let $i_0 = E/R$ be the current in the circuit. If S_2 is closed (with S_1 open), the current decays as (see Fig. 14.30)

$$i = i_0 e^{-t/\tau}$$

At $t = \tau$, $i = \frac{i_0}{e} = 0.368 i_0$.

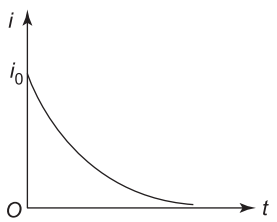


Fig. 14.30

9. Energy Stored in an Inductor

If the current in a coil of self inductance L is increased from zero to a steady value I, the energy stored in the magnetic field of the coil is

$$U = \frac{1}{2} LI^2$$

EXAMPLE 19 A magnetic flux of 5 μWb is linked with a coil when a current of 1 mA flows through it. Find the self inductance of the coil.

SOLUTION $\phi = LI \Rightarrow L = \dfrac{\phi}{I} = \dfrac{5\times10^{-6}}{1\times10^{-3}}$

$= 5 \times 10^{-3}$ H = 5 mH

EXAMPLE 20 An emf of 1 mV is induced in a coil when the current in it changes steadily from 2 A to 4 A in 0.1 s. Find the self inductance of the coil.

SOLUTION $\dfrac{dI}{dt} = \dfrac{4-2}{0.1} = 20\ \text{AS}^{-1}$

$$|e| = L\ \frac{dI}{dt}$$

$$1 \times 10^{-3} = L \times 20 \Rightarrow L = 5 \times 10^{-5}\ \text{H} = 50\ \mu\text{H}$$

EXAMPLE 21 A solenoid 1.0 m long and 0.05 m diameter has 700 turns. Another solenoid of 50 turns is tightly wound over the first solenoid. Find the emf induced in the second solenoid when the current in the first solenoid changes from 0 to 5 A in 0.01 s.

SOLUTION Mutual inductance $M = \dfrac{\mu_0 A N_1 N_2}{l}$

$$= \frac{4\pi\times10^{-7}\times\pi(0.025)^2\times700\times50}{1.0}$$

$= 8.6 \times 10^{-5}$ H

$$|e| = M\ \frac{dI}{dt} = 8.6 \times 10^{-5} \times \frac{5-0}{0.01} = 4.3 \times 10^{-2}\ \text{V}$$

EXAMPLE 22 An ideal inductor of inductance 5 H and a pure resistor of resistance 100 Ω are connected in series to a battery of emf 6 V of negligible internal resistance through a switch. The switch is closed at time $t = 0$

(a) Find the maximum (or steady) value of the current.
(b) What is the time constant τ of the circuit?
(c) How long does it take for the current to rise to 50% of the maximum value?
(d) Find the potential difference across the inductor at $t = 0.1$ s. Given $e^{-2} = 0.135$.

SOLUTION $i = i_0\ (1 - e^{\ t/\tau}\ )$; i_0 = maximum value of i

(a) When $t \to \infty$, $i = i_0 = \dfrac{E}{R} = \dfrac{6}{100} = 0.06$ A

(b) Time constant $\tau = \dfrac{L}{R} = \dfrac{5}{100} = 0.05$ s

(c) $0.5\ i_0 = i_0\ (1 - e^{-t/\tau})$

$$\Rightarrow \qquad \frac{1}{2} = 1 - e^{-t/\tau}$$

$$\Rightarrow \qquad e^{-t/\tau} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \qquad e^{\ t/\tau} = 2 \Rightarrow \frac{t}{\tau} = \ln(2) = 0.693$$

$$\therefore \qquad t = 0.693\ \tau = 0.693 \times 0.05 = 0.0346\ \text{s}$$

(d) $V_L = -L\ \dfrac{di}{dt} = -L\ \dfrac{d}{dt}\ [i_0(1 - e^{-t/\tau})]$

$$= -Li_0\left(-\frac{1}{\tau}\right)e^{-t/\tau}$$

$$= \frac{Li_0}{\tau}\ e^{-t/\tau}$$

$$= \frac{5\times0.06}{0.05} \times e^{-0.1/.05} = 6\times e^{-2}$$

$$= 6\times0.135 = 0.8\ \text{V}$$

EXAMPLE 23 An inductor of inductance 10 H and a resistor of resistance 16 Ω are connected to a 12 V dc source.

(a) Find the final steady current.
(b) How much energy is consumed to attains this steady current?
(c) What is the power dissipated in the resistor at this current?

SOLUTION

(a) $i_0 = \dfrac{E}{R} = \dfrac{12}{16} = \dfrac{3}{4}$ A = 0.75 A

(b) $U = \dfrac{1}{2}\ Li_0^2 = \dfrac{1}{2} \times 10 \times \left(\dfrac{3}{4}\right)^2 = 2.8$ J

(c) $P = i_0 E = \dfrac{3}{4} \times 12 = 9$ W

or $\quad P = i_0^2 R = \left(\dfrac{3}{4}\right)^2 \times 16 = 9$ W

EXAMPLE 24 An inductor of inductance 100 mH and a resistor of resistance 50 Ω are connected in series to a 2 V battery. After some time the current attains a steady value. The battery is now short circuited. Calculate the time required for the current to fall to half the steady value.

SOLUTION L = 100 mH = 0.1 H, R = 50 Ω, E = 2 V.

$$i_0 = \frac{E}{R} = \frac{2}{50} = 0.04\ \text{A}$$

$$\tau = \frac{L}{R} = \frac{0.1}{50} = 0.002\ \text{s}$$

$$\frac{i}{i_0} = e^{-t/\tau}$$

$$\Rightarrow \quad \frac{1}{2} = e^{-t/\tau} \Rightarrow 2 = e^{t/\tau} \Rightarrow \ln 2 = \frac{t}{\tau}$$

$$\therefore \quad t = \tau \ln 2 = 0.002 \times 0.693 = 1.386 \times 10^{-3} \text{ s}$$

10. Transformer

The transformer is a device used for converting a low ac voltage into a high ac voltage and vice versa. The former is called the step-up transformer and the latter the step-down transformer.

A transformer consists of two coils each of which is wound on an iron core. One of the coils is connected to a source of alternating emf. This coil is called the *primary* of the transformer while the other is called the *secondary* of the transformer. Any of the two coils can act as primary while the other as secondary. The alternating emf in one coil induces an alternating emf in the second coil. The presence of an iron core in the primary and secondary makes the flux linkage between the two coils very large. The alternating emf in the coil makes the magnetic flux in the iron also vary periodically. This varying magnetic flux in iron induces an alternating emf in the secondary.

If the magnetic field lines remain confined to the core, then all the field lines threading the primary also go across the secondary. Then the magnetic fluxes across the secondary and primary will be simply proportional to the number of turns in them, i.e.

$$\frac{\phi_s}{\phi_p} = \frac{N_s}{N_p} \text{ or } \phi_s = \left(\frac{N_s}{N_p}\right) \phi_p$$

where N_s is the number of turns in the secondary and N_p is the number of turns in the primary. Now from Faraday's law the emf induced across the secondary is $e_s = -(d\phi_s/dt)$ and that across the primary is $e_p = -(d\phi_p/dt)$.

Thus

$$e_s = -\frac{d}{dt}\left(\frac{N_s}{N_p} \cdot \phi_p\right) = -\frac{N_s}{N_p}\frac{d\phi_p}{dt}$$

or

$$e_s = \frac{N_s}{N_p} e_p$$

From this equation, it follows that if $N_s > N_p$, then $e_s > e_p$, i.e. the voltage across the secondary is greater than the input primary voltage. Such a transformer in which the number of turns in the secondary is more than in the primary is called a *step-up* transformer. But if $N_s < N_p$, then $e_s < e_p$. Such a transformer is called a *step-down* transformer. The former are used in TV, high-voltage power supplies and the latter in radio transmitter sets, battery eliminators, etc.

Usually, there are a number of energy losses in actual transformers. These are: (i) Joule heating (I^2R) losses in the primary and secondary coils due to their resistance (generally, these losses are minimized by using wires of large diameters so that resistance is low); and (ii) the losses in the iron core which include the heating of the core due to eddy currents and power loss due to hysteresis. The eddy currents can be minimized by using laminated iron.

In an ideal transformer, the entire power in the primary is transferred to the secondary. For an ideal transformer,

input power = output power

or

$$e_s I_s = e_p I_p$$

Therefore,

$$\frac{e_s}{e_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

where I_p and I_s are the currents in the primary and the secondary of the transformer.

EXAMPLE 25 A step down transformer is used to reduce the main supply of 220 V to 10 V. If the primary draws 5 A and secondary 88 A current, calculate the efficiency of the transformer.

SOLUTION $e_p = 220$ V, $e_s = 10$ V , $I_p = 5$ A and $I_s = 88$ A

Input power $(P_i) = e_p \times I_p = 220 \times 5 = 1100$ W

Output power $(P_o) = e_s \times I_s = 10 \times 88 = 880$ W

$$\text{Efficiency } \eta = \frac{P_o}{P_i} = \frac{880}{1100} = 0.8 \text{ or } 80\%$$

EXAMPLE 26 A transformer has an efficiency of 75%. The power input is 4 kW at 100 V. If the secondary voltage is 200 V, calculate the currents in the primary and secondary.

SOLUTION

$P_i = e_p I_p = 4$ kW $= 4000$ W, $e_p = 100$ V and $e_s = 200$ V

$$I_p = \frac{P_i}{e_p} = \frac{4000}{100} = 40 \text{ A}$$

$$\eta = \frac{P_o}{P_i} \Rightarrow P_o = \eta P_i = 0.75 \times 4000 = 3000 \text{ W}$$

$$\therefore \quad I_s = \frac{P_o}{e_s} = \frac{3000}{200} = 15 \text{ A}$$

EXAMPLE 27 The primary of a transformer has 400 turns while the secondary has 2000 turns. The power output from the secondary at 1000 V is 12 kW.

(a) Calculate the primary voltage.

(b) If the resistance of the primary is 0.9 Ω and that of the secondary is 5 Ω and the efficiency of the transformer is 90%, calculate the power loss in the primary coil and in the secondary coil.

SOLUTION $N_p = 400$, $N_s = 2000$, $P_o = 12000$ W, $e_s = 1000$ V

(a) $\dfrac{e_p}{e_s} = \dfrac{N_p}{N_s} \quad \Rightarrow \quad e_p = \dfrac{N_p}{N_s} \times e_s = \dfrac{400}{2000} \times 1000$

$= 200$ V

(b) $\eta = \dfrac{P_o}{P_i} = \dfrac{P_o}{e_p I_p}$

$\Rightarrow \qquad I_p = \dfrac{P_o}{\eta e_p} = \dfrac{12000}{0.9 \times 200} = \dfrac{200}{3}$ A

$I_s = \dfrac{P_o}{e_s} = \dfrac{12000}{1000} = 12$ A

Power loss in primary $= I_p^2 \times R_p = \left(\dfrac{200}{3}\right)^2 \times 0.9$

$= 4000$ W

Power loss in secondary $= I_s^2 \times R_s = (12)^2 \times 5$

$= 720$ W

EXAMPLE 28 A power transformer is used to step up an emf of 220 V to 4.4 kV to transmit 6.6 kW of power. If the primary has 1000 turns, find (a) number of turns in the secondary and (b) the current rating of the secondary. Assume that the efficiency of the transformer is 80%.

SOLUTION

(a) $N_s = \dfrac{e_s}{e_p} \times N_p = \dfrac{4400}{220} \times 1000 = 20{,}000$

(b) $I_s = \dfrac{P_o}{e_s} = \dfrac{\eta P_i}{e_s} = \dfrac{0.8 \times 6600}{4400} = 1.2$ A

I_s is called the current rating of the secondary.

11. Alternating Current

If an alternating voltage $V = V_0 \sin \omega t$ is applied across a resistance R, the current I in the circuit is

$$I = \frac{V}{R} = \frac{V_0}{R} \sin \omega t = I_0 \sin \omega t \qquad (1)$$

where $I_0 = V_0/R$, is the *maximum* or *peak value* of the current. It is clear from Eq. (1) that the current I varies sinusoidally with time; its magnitude changes continuously with time and its direction is reversed periodically. A *sinusoidally varying current whose magnitude changes continuously with time and whose direction reverses periodically is called an alternating current* (or simply ac).

The angular frequency ω of an alternating current is related to its time period T and frequency ν as

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

where ω is expressed in radians per second (rad s^{-1}), T in seconds (s) and ν in hertz. (Hz). In terms of T, Eq. (1) reads

$$I = I_0 \sin\left(\frac{2\pi t}{T}\right)$$

Root Mean Square Voltage and Current

The mean value of a periodic function $X(t)$ of time period T over one time period is defined as

$$\overline{X} = \frac{\int_0^T X(t)dt}{\int_0^T dt} = \frac{1}{T}\int_0^T X(t)dt$$

(i) Mean or average value of alternating voltage $V = V_0 \sin(\omega t)$ is

$$\overline{V} = \frac{1}{T}\int_0^T V_0 \sin(\omega t)dt$$

$$= \frac{V_0}{T}\int_0^T \sin(\omega t)dt = -\frac{V_0}{T\omega}\left|\cos(\omega t)\right|_0^T$$

$$= -\frac{V_0}{T\omega}\left|\cos\left(\frac{2\pi t}{T}\right)\right|_0^T$$

$$= \frac{V_0}{2\pi}(\cos 2\pi - \cos 0) = 0$$

Similarly mean value of alternating current $I = I_0 \sin(\omega t)$ over one time period is $\overline{I} = 0$

(ii) Mean square value of alternating voltage $V = V_0 \sin(\omega t)$ is

$$\overline{V^2} = \frac{1}{T}\int V_0^2 \sin(\omega t)dt$$

$$= \frac{V_0^2}{T}\int_0^T \frac{1}{2}(1 - \cos 2\omega t)dt$$

$$= \frac{V_0^2}{T}\left[\frac{1}{2}\int_0^T dt - \frac{1}{2}\int_0^T \cos(2\omega t)dt\right]$$

$$= \frac{V_0^2}{T}\left[\frac{T}{2} - \frac{1}{2}\left|\frac{\sin(2\omega t)}{2\omega}\right|_0^T\right]$$

$$= \frac{V_0^2}{T}\left[\frac{T}{2} - 0\right] = \frac{V_0^2}{2}$$

Root mean square (rms) value of the alternating voltage is

$$V_{\text{rms}} = \sqrt{\overline{V^2}} = \frac{V_0}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \text{peak value of } V$$

Similarly, root mean square (rms) value of alternating current $I = I_0 \sin(\omega t)$ is

$$I_{\text{rms}} = \sqrt{\overline{I^2}} = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \text{peak value of } I$$

EXAMPLE 29 An alternating voltage V (in volt) varies with time t (in second) as

$$V = 100 \sin(50\pi t)$$

Find the peak value, rms value and frequency of the alternating voltage.

SOLUTION Comparing the given equation with

$$V = V_0 \sin(\omega t)$$

We get $V_0 = 100$ V,

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.7 \text{ V}$$

and $\omega = 50\pi$

or $2\pi\nu = 50\pi \Rightarrow \nu = 25$ Hz

EXAMPLE 30 A 100 Ω electric iron is connected to a 200 V, 50 Hz a.c. supply. Find (a) rms value of voltage, (b) peak value of voltage, (c) rms value of current and (d) peak value of current.

SOLUTION If an alternating supply is given to be 200 V, 50 Hz, it implies that the rms value of voltage is 200 V and the frequency is 50 Hz.

(a) $V_{\text{rms}} = 200$ V

(b) $V_0 = \sqrt{2}\, V_{\text{rms}} = \sqrt{2} \times 200 = 282.8$ V.

(c) $I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{200}{100} = 2$ A

(d) $I_0 = \frac{V_0}{R} = \frac{282.8}{100} = 2.828 \simeq 2.83$ A

12. Series LCR Circuit

The applied voltage V divides into three parts, V_L (across L), V_C (across C) and V_R (across R) such that (Fig. 14.31).

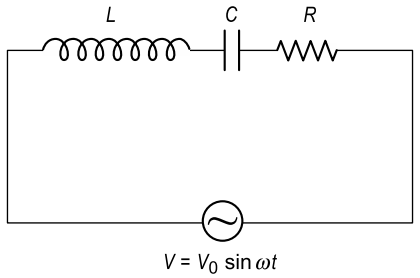


Fig. 14.31

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The current in the circuit is

$$I = I_0 \sin(\omega t - \phi)$$

$$\tan\phi = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$

(i) If $\omega L > \frac{1}{\omega C}$, i.e. $\omega > \frac{1}{\sqrt{LC}}$, then $\tan\phi$ is positive and voltage leads the current.

(ii) If $\omega L < \frac{1}{\omega C}$, i.e. $\omega < \frac{1}{\sqrt{LC}}$, then voltage lags behind current.

(iii) If $\omega L = \frac{1}{\omega C}$, i.e. $\omega = \frac{1}{\sqrt{LC}}$, then $\phi = 0$

This is the case of resonance. Voltage and current are in phase. $Z = R$ (minimum) and current is maximum.

Special Cases

(a) A.C. circuit containing only a pure resistor (Fig. 14.32)

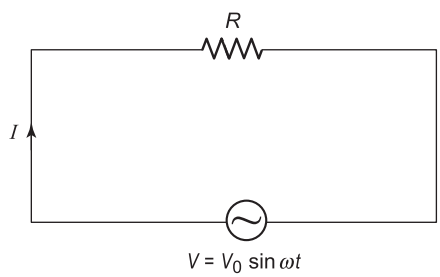


Fig. 14.32

$$V_R = V_0 \sin\omega t$$

$$I = I_0 \sin \omega t$$

where $I_0 = \dfrac{V_0}{R}$

The voltage across R is always in phase with the current in the circuit.

(b) A.C. circuit containing only an ideal inductor (Fig. 14.33)

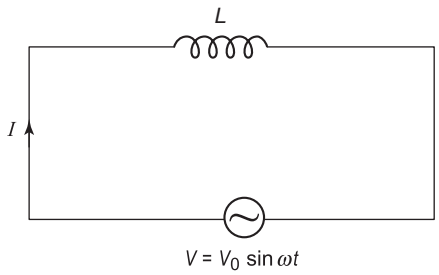


Fig. 14.33

$$V_L = V_0 \sin \omega t$$

$$I = I_0 \sin \left(\omega t - \frac{\pi}{2}\right)$$

where $I_0 = \dfrac{V_0}{X_L} = \dfrac{V_0}{\omega L}$

$X_L = \omega L$ is called inductive reactance. The voltage across the inductance leads the current in the circuit by a phase angle of $\pi/2$.

(c) A.C. circuit containing only an ideal capacitor (Fig. 14.34)

$$V_C = V_0 \sin \omega t$$

$$I = I_0 \sin \left(\omega t + \frac{\pi}{2}\right)$$

where $I_0 = \dfrac{V_0}{X_C} = \omega C V_0$

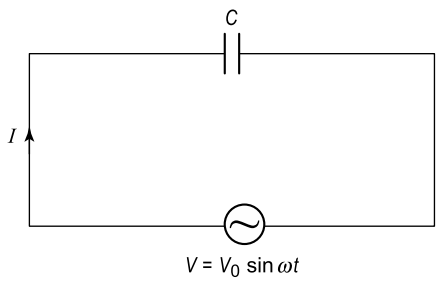


Fig. 14.34

$X_C = \dfrac{1}{\omega C}$ is called capacitative reactance. The voltage across the capacitor lags behind the current in the circuit by a phase angle of $\pi/2$.

(d) A.C. circuit containing an ideal inductor and a pure resistor (Fig. 14.35)

$$V_0 = I_0 Z$$

Where $V_0 = \sqrt{V_R^2 + V_L^2}$

$$V_R = IR,\ V_L = IX_L$$

and $Z = \dfrac{V_0}{I_0} = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$ is called impedance.

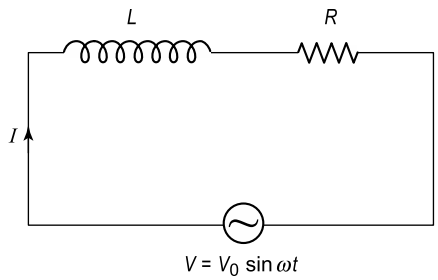


Fig. 14.35

$$I = I_0 \sin (\omega t - \phi)$$

where $\phi = \tan^{-1}\left(\dfrac{\omega L}{R}\right)$ is the phase angle between the voltage and current in the circuit.

(e) A.C. circuit containing an ideal capacitor and a pure resistor (Fig. 14.36)

$$V_0 = I_0 Z$$

where $V_0 = \sqrt{V_R^2 + V_C^2}$

$$V_R = IR,\ V_C = IX_C$$

and $Z = \dfrac{V_0}{I_0} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}$

is called impedance.

$$I = I_0 \sin (\omega t + \phi)$$

where $\phi = \tan^{-1}\left(\dfrac{1}{R\omega C}\right)$ is the phase angle between the voltage and current in the circuit.

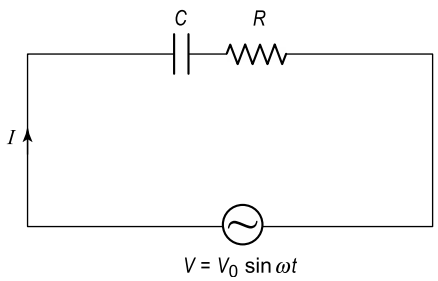


Fig. 14.36

13. Power in LCR Circuit

In a series *LCR* circuit driven by an alternating voltage $V = V_0 \sin \omega t$, the current in the circuit is

$$I = I_0 \sin (\omega t \pm \phi)$$

depending upon whether $X_L < X_C$ or $X_L > X_C$.

$$I_0 = \frac{V_o}{Z};\ Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

and $$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

Instantaneous power supplied to the circuit by the A.C. source is

$$P(t) = VI = V_0 \sin \omega t \times I_0 \sin (\omega t \pm \phi)$$

$$= V_0 I_0 \sin \omega t \times \sin (\omega t \pm \phi)$$

$\therefore$ Average power supplied by the source in one complete cycle is

$$\overline{P} = \frac{1}{T}\int_0^T P(t)\,dt$$

$$= \frac{1}{T} \times V_0 I_0 \int_0^T \sin \omega t\, (\sin \omega t \cos\phi \pm \cos \omega t \sin \phi) dt$$

$$= \frac{V_0 I_0}{T}\left[\cos\phi \int_0^T \sin^2(\omega t)dt \pm \sin\phi \int_0^T \sin(\omega t) \times \cos(\omega t)dt\right]$$

$$= \frac{V_0 I_0}{T}\left[\cos\phi \times \frac{T}{2} \pm 0\right]$$

$$= \frac{V_0 I_0}{2} \cos\phi = \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \cos\phi$$

or $\overline{P} = V_{\text{rms}}\, I_{\text{rms}} \cos \phi$

Power Factor of an A.C. Circuit

The power supplied by the source depends not only on V_{rms} and I_{rms} but also on cos ϕ. The quantity cos ϕ is called the power factor of the A.C. circuit. Now

$$\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

and $$\cos \phi = \frac{1}{(1 + \tan^2 \phi)^{1/2}}$$

$$= \frac{1}{1 + \left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)^2}$$

$$= \frac{R}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}$$

$$= \frac{R}{Z}$$

$\therefore$ Power factor = $\dfrac{\text{resistance}}{\text{impedance}}$

Special Cases

(a) *For an A.C. circuit containing only a resistor,*

$Z = R$ and $\cos \phi = \dfrac{R}{R} = 1 \Rightarrow \phi = 0$ and $\overline{P} = V_{\text{rms}}\, I_{\text{rms}}$

(b) *For an A.C. circuit containing only an inductor or a capacitor,*

$\phi = 90°$. Hence $\overline{P} = 0$

(c) At resonance, $\phi = 0$ for an *LCR* circuit. Hence $\overline{P}$ = maximum, i.e. maximum power is delivered to the circuit form A.C. source.

Wattless Current

For an A.C. circuit containing only a pure inductor or an ideal capacitor $\phi = 90°$. Hence

$$\overline{P} = V_{\text{rms}}\, I_{\text{rms}} \cos 90° = 0$$

Such an A.C. circuit consumes no power. The current flowing through the inductor or capacitor consumes no power and is called wattless current.

Bandwidth and Quality Factor of LCR Circuit

For an *LCR* circuit driven by an alternating voltage $V = V_0 \sin \omega t$, the peak (amplitude) value of the current is given by

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}$$

$\therefore$ $$(I_0)_{\max} = \frac{V_0}{R} \quad \Rightarrow \quad V_0 = (I_0)_{\max} R$$

In terms of $(I_0)_{\max}$, I_0 is given by

$$I_0 = \frac{(I_0)_{\max} R}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}$$

Figure 14.37 shows the variation of I_0 versus ω.

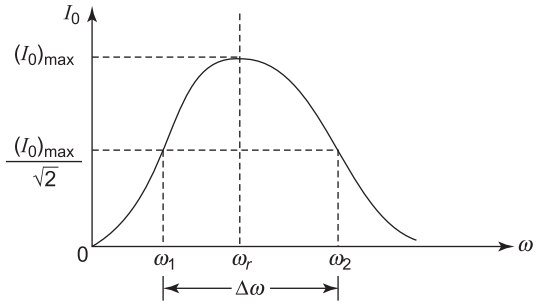


Fig. 14.37

Let ω_1 and ω_2 be the values of ω when $I_0 = \dfrac{(I_0)_{max}}{\sqrt{2}}$, i.e. when

$$\frac{(I_0)_{max}\, R}{\left[R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2\right]^{1/2}} = \frac{(I_0)_{max}\, R}{\sqrt{2}}$$

$$\Rightarrow \qquad R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\Rightarrow \qquad \omega L - \frac{1}{\omega C} = \pm R$$

Case 1: $\omega L - \dfrac{1}{\omega C} = +R \Rightarrow \omega^2 - \dfrac{1}{LC} = \dfrac{R\omega}{L}$

$$\Rightarrow \qquad \omega^2 - \frac{R\omega}{L} - \omega_r^2 = 0, \text{ where } \omega_r = \frac{1}{\sqrt{LC}}$$

The positive root of this quadratic equation is

$$\omega_2 = \frac{R}{2L} + \left(1 + \frac{4\omega_r^2 L^2}{R^2}\right)^{1/2}$$

Case 2: $\omega L - \dfrac{1}{\omega C} = -R$

$$\Rightarrow \qquad \omega^2 + \frac{R\omega}{L} - \omega_r^2 = 0$$

The positive root of this quadratic equation is

$$\omega_1 = -\frac{R}{2L} + \left(1 + \frac{4\omega_r^2 L^2}{R^2}\right)^{1/2}$$

Bandwidth $\Delta\omega = \omega_2 - \omega_1 = \dfrac{R}{L}$

Quality factor (or Q factor) of LCR circuit is defined as

$$Q = \frac{\text{resonant frequency}}{\text{bandwidth}} = \frac{\omega_r}{\Delta\omega}$$

$$= \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Q is a dimensionless number. Figure 14.38 shows the graph of $\overline{P}$ versus ω for some values of Q.

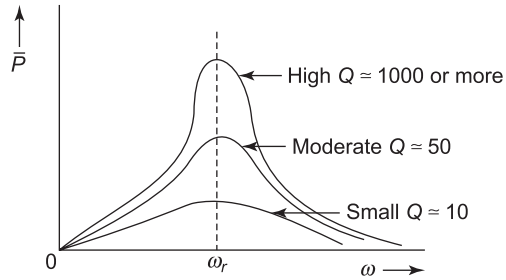


Fig. 14.38

The power peak is sharp for high Q. The resonance is then said to be sharp. Higher the value of Q, the sharper is the resonance and greater is the power absorbed from the source.

EXAMPLE 31 A coil of inductance 0.5 H and a resistor of resistance 100 Ω are connected in series to a 240 V, 50 Hz supply.

(a) Find the maximum current in the circuit.

(b) What is the time lag between voltage maximum and current maximum?

SOLUTION Given $V_{rms} = 240$ V, $\omega = 2\pi\nu = 2\pi \times 50 = 100\pi$ rad s^{-1}, $L = 0.5$ H and $R = 100$ Ω.

$$V = V_0 \sin \omega t$$

$$I = I_0 \sin(\omega t - \phi);$$

$$I_0 = \frac{V_0}{(R^2 + \omega^2 L^2)^{1/2}},$$

$$\tan\phi = \frac{\omega L}{R}$$

(a) $V_0 = \sqrt{2}\, V_{rms} = \sqrt{2} \times 240$ V

$$\therefore I_0 = \frac{\sqrt{2} \times 240}{[(100)^2 + (100\pi \times 0.5)^2]^{1/2}} = 1.82 \text{ A}$$

(b) V is maximum when $\sin \omega t = +1 \Rightarrow \omega t_1 = \dfrac{\pi}{2}$

$\Rightarrow t_1 = \dfrac{\pi}{2\omega}$

I is maximum when $\sin(\omega t - \phi) = +1 \Rightarrow \omega t_2 - \phi = \dfrac{\pi}{2}$

$\Rightarrow t_2 = \dfrac{\pi}{2\omega} - \dfrac{\phi}{\omega}$

$\therefore$ Time lag between voltage maximum and current maximum is

$$\Delta t = t_1 - t_2 = \frac{\pi}{2\omega} - \left(\frac{\pi}{2\omega} - \frac{\phi}{\omega}\right) = \frac{\phi}{\omega}$$

Now $\tan\phi = \dfrac{\omega L}{R} = \dfrac{100\pi \times 0.5}{100} = 1.57$

$$\Rightarrow \qquad \phi = 57.5° = \frac{57.5\times\pi}{180} \text{ rad}$$

$$\therefore \qquad \Delta t = \frac{\phi}{\omega} = \frac{57.5\times\pi}{180\times100\pi} = 3.2\times10^{-3} \text{ s}$$

EXAMPLE 32 A capacitor of capacitance 100 μF and a resistor of resistance 40 Ω are connected in series to a 110 V, 60 Hz supply.

(a) Find the maximum current in the circuit.

(b) What is the time lag between current maximum and voltage maximum?

SOLUTION Given C = 100 μF = 100 × 10^{-6} F, R = 40 Ω, V_{rms} = 110 V, $\omega = 2\pi\nu = 2\pi\times60 = 120$ rad s^{-1}

$$V = V_0 \sin \omega t$$

$$I = I_0 \sin(\omega t + \phi),$$

$$I_0 = \frac{V_0}{\left(R^2 + \frac{1}{\omega^2C^2}\right)^{1/2}}, \tan\phi = \frac{1}{\omega CR}$$

(a) $$I_0 = \frac{\sqrt{2}\times110}{\left[\left((40)^2 + \frac{1}{(120\times10^{-4})^2}\right)\right]^{1/2}} = 3.24 \text{ A}$$

(b) V is maximum when $\sin \omega t = +1 \Rightarrow \omega t_1 = \frac{\pi}{2}$

$$\Rightarrow t_1 = \frac{\pi}{2\omega}$$

I is maximum when $\sin(\omega t + \phi) = +1$

$$\Rightarrow \omega t_2 + \phi = \frac{\pi}{2} \Rightarrow t_2 = \frac{\pi}{2\omega} - \frac{\phi}{\omega}$$

∴ Time lag between current maximum and voltage maximum is

$$\Delta t = t_1 - t_2 = \frac{\phi}{\omega}$$

Now $$\tan\phi = \frac{1}{\omega CR} = \frac{1}{120\pi\times10^{-4}\times40} = 0.663$$

$$\Rightarrow \qquad \phi = 33.5° = \frac{33.5\times\pi}{180} \text{ rad}$$

$$\therefore \qquad \Delta t = \frac{\phi}{\omega} = \frac{33.5\times\pi}{180\times120\pi} = 1.55\times10^{-3} \text{ s}$$

EXAMPLE 33 A series LCR circuit with L = 5 H, C = 80 μF and R = 40 Ω is connected to a variable frequency 230 V a.c. source.

(a) What is the source frequency which drives the circuit at resonance?

(b) What is the impedance of the circuit at resonance?

(c) Find peak value of the current at resonance.

(d) Find the rms potential differences across L, C and R at resonance.

(e) What is the total potential difference across the combination of L and C at resonance.

(f) Find the maximum power transferred to the circuit from the source in one complete cycle.

SOLUTION

(a) $$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5\times80\times10^{-6}}} = 50 \text{ rad } s^{-1}$$

$$\therefore \quad \nu_r = \frac{\omega_r}{2\pi} = \frac{50}{2\pi} = 7.96 \text{ Hz}$$

(b) $$Z = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}$$

$$= R \qquad \left(\because \omega L = \frac{1}{\omega C} \text{ at resonance}\right)$$

$$= 40 \ \Omega$$

(c) $$I_0 = \frac{V_0}{Z} = \frac{\sqrt{2}\times230}{40} = 8.1 \text{ A}$$

(d) $$(V_L)_{rms} = I_{rms}\times X_L = I_{rms}\times\omega_r L$$

$$= \frac{230}{40}\times50\times5 = 1437.5 \text{ V}$$

$$(V_C)_{rms} = I_{rms}\times X_C = I_{rms}\times\frac{1}{\omega_r C}$$

$$= \frac{230}{40}\times\frac{1}{80\times10^{-6}\times50} = 1437.5 \text{ V}$$

$$(V_R)_{rms} = I_{rms}\times R = \frac{230}{40}\times40 = 230 \text{ V}$$

(e) $$(V_{L,C})_{rms} = I_{rms}\times\left(\omega_r L - \frac{1}{\omega_r C}\right)$$

$$= 1437.5 - 1437.5 = 0$$

(f) $$P_{max} = V_{rms}\times I_{rms}\times\cos\phi$$

$$= V_{rms}\times I_{rms} \qquad (\because \phi = 0 \text{ at resonance})$$

$$= \frac{V_{rms}^2}{R} = \frac{230\times230}{40} = 1322.5 \text{ W}$$

EXAMPLE 34 When an alternating voltage of 220 V is applied across a device A, a current of 0.5 A flows through the circuit and it is in phase with the applied voltage. When the same voltage is applied across a device

B, again the same current flows in the circuit but it leads the voltage by $\pi/2$. (a) Name devices *A* and *B*. (b) Compute the current when the same voltage is applied across a series combination of *A* and *B*.

SOLUTION

(a) Device *A* is a resistor and *B* is a capacitor.

(b) Given $V_{rms} = 220$ V, $I_{rms} = 0.5$ A

Resistance of *A* is $R = \dfrac{V_{rms}}{I_{rms}} = \dfrac{220}{0.5} = 440\ \Omega$

Reactance of *B* is $X_C = \dfrac{V_{rms}}{I_{rms}} = \dfrac{220}{0.5} = 440\ \Omega$

Impedance when *A* and *B* are connected in series is

$$Z = \sqrt{R^2 \times X_C^2} = \sqrt{(440)^2 + (440)^2} = 622.3\ \Omega$$

$$\therefore I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{622.3} = 0.35\ \text{A}$$

14. LC Oscillations

In an electrical circuit consisting of an inductance *L* and a capacitance *C*, the charge (and hence current) oscillates harmonically with an angular frequency

$$\omega = \frac{1}{\sqrt{LC}}$$

and time period $T = 2\pi\sqrt{LC}$

The charge and current in the circuit vary with time as

$$q = q_0 \sin(\omega t + \phi) \text{ and } I = I_0 \cos(\omega t + \phi)$$

SECTION 1

Multiple Choice Questions with One Correct Choice

Level A

1. An ideal solenoid of cross-sectional area $10^{-4}\,m^2$ has 500 turns per metre. At the centre of this solenoid, another coil of 100 turns is wrapped closely around it. If the current in the coil changes from 0 to 2 A in 3.14 millisecond, the emf developed in the second coil is
   (a) 1 mV (b) 2 mV
   (c) 3 mV (d) 4 mV
2. A rectangular loop of sides 8 cm and 2 cm with a small break in it is moving out of a region of uniform magnetic field of 0.3 T, directed normal to the loop. What is the emf developed across the break if the velocity of the loop is 1 cms^{-1} in a direction normal to the longer side of the loop?
   (a) 0.06 mV (b) 0.12 mV
   (c) 0.18 mV (d) 0.24 mV
3. In Q. 2, for how long does the induced emf last?
   (a) 2 s (b) 4 s
   (c) 6 s (d) 8 s
4. In Q. 3, what is the emf developed across the break if the velocity of the loop is 1 cm s^{-1} in a direction normal to the shorter side of the loop?
   (a) 0.06 mV (b) 0.12 mV
   (c) 0.18 mV (d) 0.24 mV
5. In Q. 4, how long does the induced emf last?
   (a) 2 s (b) 4 s
   (c) 6 s (d) 8 s
6. A pair of coils has a mutual inductance of 2 H. If the current in the primary changes from 10 A to zero in 0.1 s, the induced emf in the secondary will be
   (a) 100 V (b) 200 V
   (c) 300 V (d) 400 V
7. A motor having an armature of resistance 2 Ω is designed to operate at 220 V mains. At full speed, it develops a back emf of 210 V. What is the current in the armature when the motor is running at full speed?
   (a) 2.5 A (b) 5.0 A
   (c) 7.5 A (d) 10 A
8. In Q. 7, when the motor was switched on, what was the current in the armature if no starter was used?
   (a) zero (b) 5.0 A
   (c) 110 A (d) 220 A
9. In Q. 7, the efficiency of the motor at full speed is very nearly equal to
   (a) 65% (b) 75%
   (c) 85% (d) 95%

10. The resistance of the armature of a generator is 0.2 Ω. It yields an emf of 220 V in an open circuit and a potential difference of 210 V at full load. The current at full load is
    (a) 30 A (b) 40 A
    (c) 50 A (d) 60 A
11. In Q. 10, the power delivered to the external circuit is
    (a) 9.0 kW (b) 9.5 kW
    (c) 10 kW (d) 10.5 kW
12. The primary of a transformer has 400 turns while the secondary has 2000 turns. If the power output from the secondary at 1000 V is 12 kW, what is the primary voltage?
    (a) 200 V (b) 300 V
    (c) 400 V (d) 500 V
13. In Q. 12, if the resistance of the primary is 0.2 Ω and that of the secondary is 2 Ω and the efficiency of the transformer is 80%, the power loss in the primary is
    (a) 1.125 kW (b) 2.25 kW
    (c) 3.375 kW (d) 4.5 kW
14. In Q. 13, the power loss in the secondary is
    (a) 72 W (b) 144 W
    (c) 216 W (d) 288 W
15. The magnitude of the induced emf produced in a coil when a magnet is inserted into it does NOT depend upon the
    (a) number of turns in the coil
    (b) resistance of the coil
    (c) magnetic moment of the magnet
    (d) speed of approach of the magnet
16. A coil of wire is held with its plane horizontal to the earth's surface and a small bar magnet dropped vertically down through it. The magnet will fall with a
    (a) constant acceleration equal to g
    (b) constant acceleration greater than g
    (c) constant acceleration less than g
    (d) non-uniform acceleration less than g
17. An electron moves along the line PQ which lies in the same plane as a circular loop of conducting wire, as shown in Fig. 14.39. What will be the direction of the induced current, if any, in the loop?
    (a) Anti-clockwise
    (b) Clockwise
    (c) Alternating
    (d) No current will be induced in the loop.

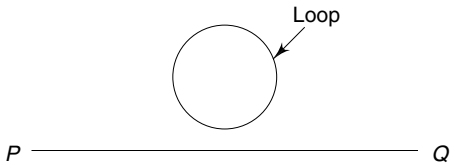


Fig. 14.39

18. A magnet is moved in the direction indicated by an arrow between two coils AB and CD as shown in Fig. 14.40. What is the direction of the induced current in each coil?
    (a) A to B in coil X and C to D in coil Y
    (b) A to B in coil X and D to C in coil Y
    (c) B to A in coil X and C to D in coil Y
    (d) B to A in coil X and D to C in coil Y.

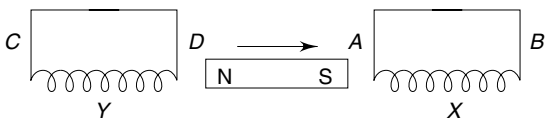


Fig. 14.40

19. Figure 14.41 shows two coils P and Q placed close to each other. When the circuit of coil P is suddenly broken by lifting the key K,
    (a) a current flows from X to Y in coil Q
    (b) a current flows from Y to X in coil Q
    (c) no current flows in coil Q
    (d) an alternating current flows in coil Q.

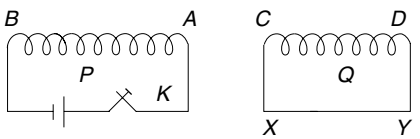


Fig. 14.41

20. A transformer has 200 windings in the primary and 400 windings in the secondary. The primary is connected to an ac supply of 110 V and a current of 10 A flows in it. The voltage across the secondary and the current in it, respectively, are
    (a) 55 V, 20 A (b) 440 V, 5 A
    (c) 220 V, 10 A (d) 220 V, 5 A
21. The magnitude of the emf across the secondary of a transformer does NOT depend upon
    (a) the magnitude of the emf applied across the primary
    (b) the number of the turns in the primary
    (c) the number of turns in the secondary
    (d) the resistances of the primary and the secondary.
22. Electrical energy generated at a power house is delivered to distant places over long transmission cables at a very high ac voltage of about 33,000 volts. The reason for this is that

(a) at high voltages energy is delivered much faster than at low voltages
(b) there is less wastage of energy at high voltages
(c) the high voltage prevents theft of costly transmission cables
(d) it is much easier to generate large amounts of energy at high voltages.

23. Flux ϕ (in weber) in a closed circuit of resistance 10 Ω varies with time t (in seconds) according to the equation

$$\phi = 6t^2 - 5t + 1$$

The magnitude of the induced current in the circuit at $t = 0.25$ s is

(a) 0.2 A (b) 0.6 A
(c) 0.8 A (d) 1.2 A

24. A circuit has a self inductance of 1 henry and carries a current of 2 A. To prevent sparking when the circuit is broken, a capacitor which can withstand 400 volts is used. The least capacitance of the capacitor connected across the switch must be equal to

(a) 12.5 μF (b) 25 μF
(c) 50 μF (d) 100 μF

25. An aeroplane is moving north horizontally, with a speed of 200 ms^{-1}, at a place where the vertical component of the earth's magnetic field is 0.5×10^{-4} T. What is the induced emf set up between the tips of the wings if they are 10 m apart?

(a) 0.01 V (b) 0.1 V
(c) 1 V (d) 10 V

26. A step-down transformer is employed to reduce the main supply of 220 V to 11 V. The primary draws 5 A of current and the secondary 90 A. What is the efficiency of the transformer?

(a) 20% (b) 40%
(c) 70% (d) 90%

27. A 25 kW dc generator produces a potential difference of 250 V. If the resistance of the transmission line is 1 Ω, what percentage of the original power is lost during transmission?

(a) 40% (b) 50%
(c) 60% (d) 75%

28. Two resistances of 10 Ω and 20 Ω and an ideal inductor of inductance 5 H are connected to a 2 V battery through a key K, as shown in Fig. 14.42. The key is inserted at $t = 0$. What is the final value of current in the 10 Ω resistor?

(a) 2/3 A (b) 1/3 A
(c) 1/6 A (d) zero

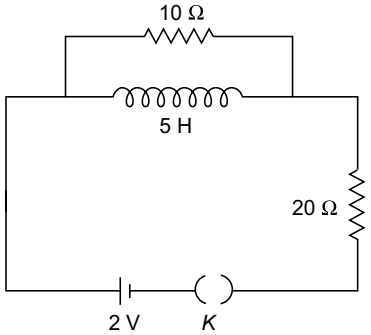


Fig. 14.42

29. What is the final value of the current through the 20 Ω resistor shown in Fig. 14.7.

(a) zero (b) 0.1 A
(c) 2/3 A (d) 1/3 A

30. A 10 Ω electric heater is connected to a 200 V, 50 Hz mains supply. What is the peak value of the potential difference across the heater element?

(a) 220 V (b) $220/\sqrt{2}$ V
(c) 110 V (d) $220\sqrt{2}$ V

31. A choke is used as a resistance in
(a) dc circuits
(b) ac circuits
(c) both ac and dc circuits
(d) full-wave rectifier circuits

32. At resonance, the value of the power factor in an *LCR* series circuit is

(a) zero (b) 1/2
(c) 1 (d) not definite

Level B

33. An ac series circuit contains 4 Ω resistance and 3 Ω inductive reactance. What is the impedance of the circuit?

(a) 1 Ω (b) 5 Ω
(c) 7 Ω (d) $\frac{7}{\sqrt{2}}$ Ω

34. An inductive coil has a resistance of 100 Ω. When an ac signal of frequency 1000 Hz is fed to the coil, the applied voltage leads the current by 45°. What is the inductance of the coil?

(a) 10 mH (b) 12 mH
(c) 16 mH (d) 20 mH

35. An ac source of variable frequency f is connected to an *LCR* series circuit. Which one of the graphs in Fig. 14.43 represents the variation of current I in the circuit with frequency f?

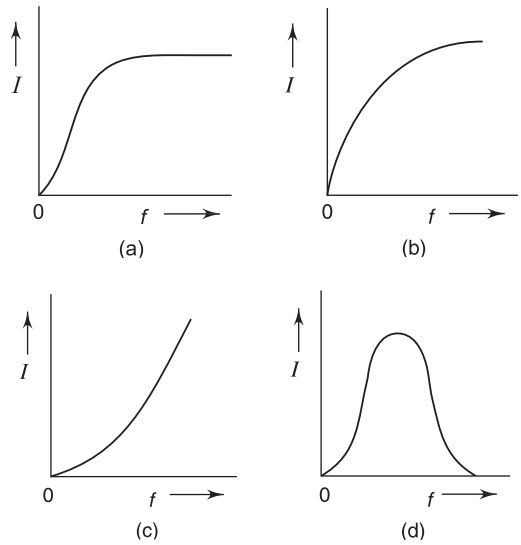


Fig. 14.43

36. Choose the correct statement. In the case of ac circuits, Ohm's law holds for
    (a) peak values of voltage and current
    (b) effective values of voltage and current
    (c) instantaneous values of voltage and current
    (d) all the above.
37. Two circuits 1 and 2 are connected to identical dc sources each of emf 12 V. Circuit 1 has a self inductance L_1 = 10 H and circuit 2 has a self inductance L_2 = 10 mH. The total resistance of each circuit is 48 Ω. The ratio of steady currents in circuits 1 and 2 is
    (a) 1000 (b) 100
    (c) 10 (d) 1
38. In Q. 37, what is the ratio of energy consumed in circuits 1 and 2 to build up the current to the steady state value?
    (a) 1000 (b) 100
    (c) 10 (d) 1
39. In Q. 37, what is the ratio of the power dissipated by circuits 1 and 2 after the steady state is reached?
    (a) 1000 (b) 100
    (c) 10 (d) 1
40. An inductor of self inductance 100 mH and a resistor of resistance 50 Ω are connected to a 2 V battery. The time required for the current to fall to half its steady value is
    (a) 2 millisecond
    (b) 2 ln (0.5) millisecond
    (c) 2 ln (1) millisecond
    (d) 2 ln (2) millisecond
41. Figure 14.44 shows a series *LCR* circuit connected to a variable frequency 200 V source. L = 5 H, $C = 80\,\mu$ F and $R = 40\,\Omega$. What is the source frequency which drives the circuit at resonance?
    (a) 25 Hz (b) $\frac{25}{\pi}$ Hz
    (c) 50 Hz (d) $\frac{50}{\pi}$ Hz

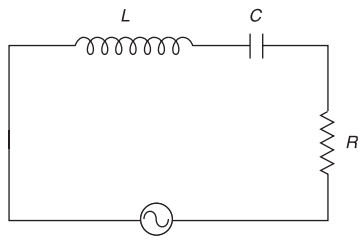


Fig. 14.44

42. In Q. 41, what is the impedance of the circuit at resonance?
    (a) 20 Ω (b) 40 Ω
    (c) 60 Ω (d) 80 Ω
43. In Q. 41, the current amplitude at resonance is
    (a) $\frac{5}{\sqrt{2}}$ A (b) 5 A
    (c) $5\sqrt{2}$ A (d) 10 A
44. In Q. 41, the rms potential drop across the inductor at resonance is
    (a) 1 kV (b) 1.25 kV
    (c) 1.5 kV (d) 1.75 kV
45. In Q. 41, the rms potential drop across the capacitor at resonance is
    (a) 1 kV (b) 1.25 kV
    (c) 1.5 kV (d) 1.75 kV
46. In Q. 41, the rms potential drop across the resistor at resonance is
    (a) $\frac{100}{\sqrt{2}}$ V (b) 100 V
    (c) $\frac{200}{\sqrt{2}}$ V (d) 200 V
47. In an *LCR* circuit, if V is the effective value of the applied voltage, V_R is the voltage across R, V_L is the effective voltage across L, V_C is the effective voltage across C, then
    (a) $V = V_R + V_L + V_C$
    (b) $V^2 = V_R^2 + V_L^2 + V_C^2$
    (c) $V^2 = V_R^2 + (V_L - V_C)^2$
    (d) $V^2 = V_L^2 + (V_R - V_C)^2$

48. L, C and R, respectively represent inductance, capacitance and resistance. Which one of the following combinations has the dimensions of frequency?
    (a) $1/RC$ (b) $1/LC$
    (c) L/R (d) C/L

49. The network shown in Fig. 14.45 is part of a circuit.

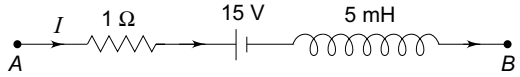


Fig. 14.45

What is the potential difference ($V_B - V_A$) when current I is 5μ A and is decreasing at a rate of 10^{-3} As^{-1}?
    (a) 5 V (b) 10 V
    (c) 15 V (d) zero

50. A capacitor of capacitance 2 μF is charged to a potential difference of 12 V. The charging battery is then removed and the capacitor is connected across an inductor of self inductance 0.6 mH. The current in the circuit at a time when the potential difference across the capacitor is 6 V is
    (a) 0.3 A (b) 0.6 A
    (c) 0.9 A (d) 1.2 A

51. In an inductor, the current I (in ampere) varies with time t (in second) as
$$I = 5 + 16t$$
If the emf induced in the inductor is 10 mV, what is its self inductance?
    (a) 6.25×10^{-4} H (b) 6.25×10^{-3} H
    (c) 7.5×10^{-4} H (d) 7.5×10^{-3} H

52. In Q. 51 above, the power supplied to the inductor at $t = 1$ s is
    (a) 0.021 W (b) 0.21 W
    (c) 2.1 W (d) 21 W

53. A wire in the form of a circular loop of radius r lies with its plane normal to a magnetic field B. If the wire is pulled to take a square shape in the same plane in time t, the emf induced in the loop is given by
    (a) $\frac{\pi Br^2}{t}\left(1-\frac{\pi}{10}\right)$ (b) $\frac{\pi Br^2}{t}\left(1-\frac{\pi}{8}\right)$
    (c) $\frac{\pi Br^2}{t}\left(1-\frac{\pi}{6}\right)$ (d) $\frac{\pi Br^2}{t}\left(1-\frac{\pi}{4}\right)$

54. A uniformly wound solenoid coil of self-inductance 1.8×10^{-4} H and resistance 6 Ω is broken up into two identical coils. These identical coils are then connected across a 12 V battery of negligible resistance. The time constant for the current in the circuit is
    (a) 0.3×10^{-4} s (b) 0.3×10^{-3} s
    (c) 0.3×10^{-2} s (d) 0.3 μs

55. In Q. 54 above, the steady current through the battery is
    (a) 8 μA (b) 8 mA
    (c) 0.8 A (d) 8 A

56. A square loop of side l, mass m and resistance R falls vertically into a uniform magnetic field directed perpendicular to the plane of the coil. The height h through which the loop falls so that it attains terminal velocity on entering the region of magnetic field is given by
    (a) $\frac{mgR}{2Bl}$ (b) $\frac{m^2gR^2}{2B^2l^2}$
    (c) $\frac{mgR^2}{4B^3l^3}$ (d) $\frac{m^2gR^2}{2B^4l^4}$

57. The mutual inductance between two planar concentric rings of radii r_1 and r_2 (with $r_1 > r_2$) placed in air is given by
    (a) $\frac{\mu_0\pi r_2^2}{2r_1}$ (b) $\frac{\mu_0\pi r_1^2}{2r_2}$
    (c) $\frac{\mu_0\pi (r_1+r_2)^2}{2r_1}$ (d) $\frac{\mu_0\pi (r_1+r_2)^2}{2r_2}$

58. A square metal wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity v in a uniform magnetic field B = 2T as shown in Fig. 14.46. The magnetic field is perpendicular to the plane of the loop and directed into the paper. The loop is connected to a network of resistors, each equal to 3 Ω. What should be the speed of the loop so as to have a steady current of 1 mA in the loop?
    (a) 1 cm s^{-1} (b) 2 cm s^{-1}
    (c) 3 cm s^{-1} (d) 4 cm s^{-1}

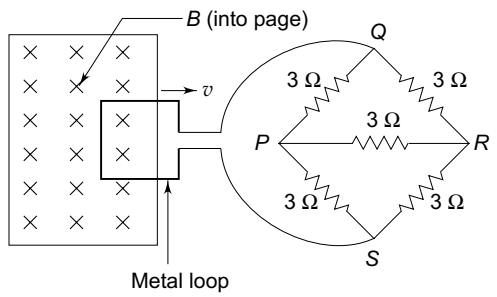


Fig. 14.46

59. If a coil of metal wire is kept stationary in a non-uniform magnetic field,
    (a) an emf and current are both induced in the coil
    (b) a current but no emf is induced in the coil
    (c) an emf but no current is induced in the coil
    (d) neither emf nor current is induced in the coil

60. In an ac circuit the potential differences across an inductance and a resistance connected in series are respectively 16 V and 20 V. The total potential difference across the circuit is
    (a) 20.0 V (b) 25.6 V
    (c) 31.9 V (d) 53.5 V
61. An alternating voltage $V = V_0 \sin \omega t$ is applied across a circuit. As a result a current $I = I_0 \sin(\omega t - \pi/2)$ flows in it. The power consumed per cycle is
    (a) zero (b) $0.5\ V_0 I_0$
    (c) $0.707\ V_0 I_0$ (d) $1.414\ V_0 I_0$
62. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic field B, constant in space and time, pointing perpendicular and into the plane of the loop exists everywhere as shown in Fig. 14.47. The current induced in the loop is
    (a) BLv/R clockwise
    (b) BLv/R anticlockwise
    (c) $2BLv/R$ anticlockwise
    (d) zero

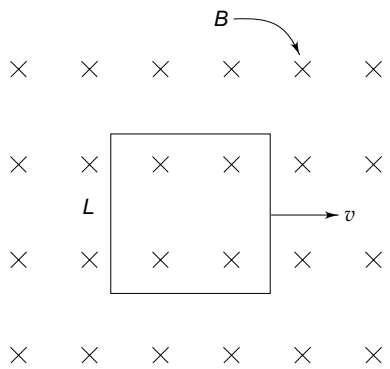


Fig. 14.47

63. A thin circular ring of area A is held perpendicular to a uniform magnetic field B. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of the circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is
    (a) $\frac{BR}{A}$ (b) $\frac{AB}{R}$
    (c) ABR (d) $\frac{B^2 A}{R^2}$
64. A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic field B (Fig. 14.48). At the position MNQ the speed of the ring is v and the potential difference across the ring is
    (a) zero
    (b) $\frac{1}{2}\ Bv\ \pi R^2$ and M is at higher potential
    (c) $\pi R\ Bv$ and Q is at higher potential
    (d) $2\ RBv$ and Q is at higher potential.

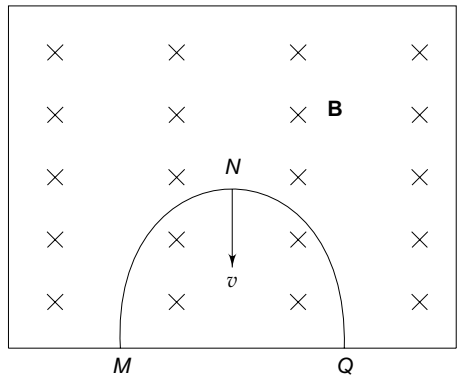


Fig. 14.48

65. A small square loop of wire of side l is placed inside a large square loop of wire of side L ($L >> l$). The loops are coplanar and their centres coincide. The mutual inductance of the system is proportional to
    (a) $\frac{l}{L}$ (b) $\frac{l^2}{L}$
    (c) $\frac{L}{l}$ (d) $\frac{L^2}{l}$
66. A circular loop of radius R, carrying current I, lies in the x-y plane with its centre at the origin. The total magnetic flux through the x-y plane is
    (a) directly proportional to I
    (b) directly proportional to R
    (c) inversely proportional to R
    (d) zero
67. A coil of inductance 8.4 mH and resistance 6 Ω is connected to a 12 V battery. The current in the coil is 1.0 A at approximately the time
    (a) 500 s (b) 20 s
    (c) 35 ms (d) 1 ms
68. A uniform but time varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper as shown in Fig. 14.49. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region
    (a) is zero (b) decreases as $\frac{1}{r}$
    (c) increases as r (d) decreases as $\frac{1}{r^2}$

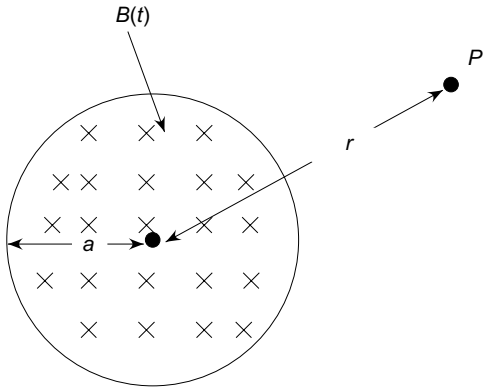


Fig. 14.49

69. Two circular coils can be arranged in any of the three situations shown in Fig. 14.50. Their mutual inductance will be
(a) maximum in situation (A)
(b) maximum in situation (B)
(c) maximum in situation (C)
(d) the same in all situations

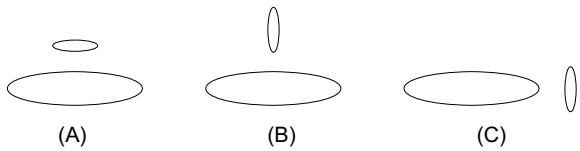


Fig. 14.50

70. A metallic square loop *ABCD* is moving in its own plane with velocity ϑ in a uniform magnetic field perpendicular to its plane as shown in Fig. 14.51. An electric field is induced
(a) in *AD*, but not in *BC*
(b) in *BC*, but not in *AD*
(c) neither in *AD* nor in *BC*
(d) in both *AD* and *BC*

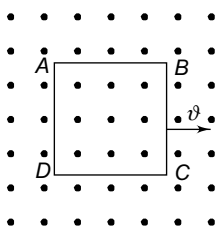


Fig. 14.51

71. As shown in Fig. 14.52, *P* and *Q* are two coaxial conducting loops separated by some distance. When the switch *S* is closed, a clockwise current I_p flows in *P* (as seen by *E*) and an induced current I_{Q1} flows in *Q*. The switch remains closed for a long time. When *S* is opened, a current I_{Q2} flows in *Q*. Then the directions of I_{Q1} and I_{Q2} (as seen by *E*) are:

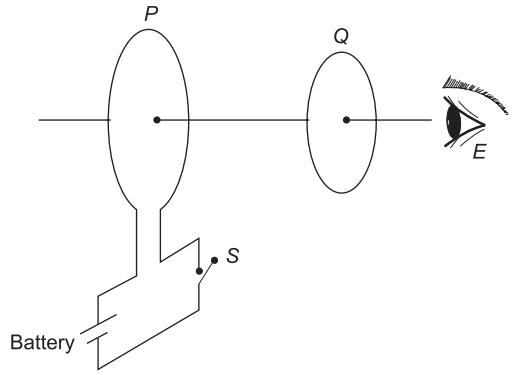


Fig. 14.52

(a) respectively clockwise and anti-clockwise
(b) both clockwise
(c) both anti-clockwise
(d) respectively anti-clockwise and clockwise

72. A short-circuited coil is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be
(a) halved (b) the same
(c) doubled (d) quadrupled

73. If the flux of magnetic induction through a coil of resistance *R* and having *n* turns changes from Φ_1 to Φ_2, then the magnitude of the charge that passes through the coil is
(a) $\dfrac{(\Phi_2 - \Phi_1)}{R}$ (b) $\dfrac{n(\Phi_2 - \Phi_1)}{R}$
(c) $\dfrac{(\Phi_2 - \Phi_1)}{nR}$ (d) $\dfrac{nR}{(\Phi_2 - \Phi_1)}$

74. When an AC source of e.m.f. $E = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the e.m.f. *E* and the current *I* in the circuit is observed to be $\pi/4$, as shown in the Fig. 14.53. If the circuit consists possibly only of *R*-*C* or *R*-*L* or *L*-*C* in series, what will be the relation between the two elements of the circuit?
(a) $R = 1\ \text{k}\Omega$, $C = 10\ \mu\text{F}$
(b) $R = 1\ \text{k}\Omega$, $C = 1\ \mu\text{F}$
(c) $R = 1\ \text{k}\Omega$, $L = 10\ \text{H}$
(d) $R = 1\ \text{k}\Omega$, $L = 1\ \text{H}$

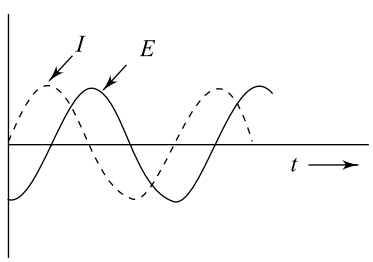


Fig. 14.53

75. Two parallel wires PQ and ST placed a distance w apart are connected by a resistor R as shown in Fig. 14.54 and placed in a magnetic field B which is perpendicular to the plane containing the wires. A rod CD connects the two wires. The power spent to slide the rod CD with a velocity v along the wires is (neglect the resistance of the wires and the rod)

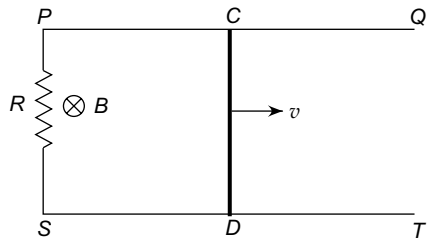


Fig. 14.54

(a) $\frac{Bwv}{R}$ (b) $\frac{Bwv}{R^2}$

(c) $\frac{(Bwv)^2}{R}$ (d) $\left(\frac{Bwv}{R}\right)^2$

76. An air plane, with 20 m wingspread is flying at 250 ms^{-1} parallel to the earth's surface at a place where the horizontal component of earth's magnetic field is 2×10^{-5} T and angle of dip is 60°. The magnitude of the induced emf between the tips of the wings is

(a) $\frac{1}{10}$ V (b) $\frac{\sqrt{2}}{10}$ V

(c) $\frac{\sqrt{3}}{10}$ V (d) $\frac{1}{5}$ V

77. A metallic wheel with 8 metallic spokes each of length r is rotating at an angular frequency ω in a plane perpendicular a magnetic field B. The magnitude of the induced emf between the axle and the rim of the wheel is

(a) $\frac{1}{2}\omega r^2 B$ (b) $2\ \omega r^2 B$

(c) $4\ \omega r^2 B$ (d) $8\ \omega r^2 B$

78. A solenoid of inductance L and resistance R is connected to a battary. The time taken for the magnetic energy to reach $\frac{1}{4}$ of its maximum value is

(a) $\frac{L}{R}\log_e(1)$ (b) $\frac{L}{R}\log_e(2)$

(c) $\frac{L}{R}\log_e(3)$ (d) $\frac{L}{R}\log_e(4)$

79. An LCR series circuit with $R = 100\ \Omega$ is connected to a 200 V, 50 Hz a.c. source. When only the capacitance is removed, the voltage leads the current by 60°. When only the inductance is removed, the current leads the voltage by 60°. The current in the circuit is

(a) $\frac{2}{\sqrt{3}}$ A (b) $\frac{\sqrt{3}}{2}$ A

(c) 1 A (d) 2 A

80. A coil of metal wire is kept stationary with its plane perpendicular to a uniform magnetic field directed along the positive x-axis. The current induced in the coil.

(a) circulates in anti-clockwise direction when viewed from the x-axis.

(b) circulates in clockwise direction when viewed from the x-axis.

(c) is perpendicular to the direction of the magnetic field

(d) is zero

81. A uniform metal rod is moving with a uniform velocity v parallel to a long straight wire carrying a current I. The rod is perpendicular to the wire with its ends at distances r_1 and r_2 (with $r_2 > r_1$) from it. The emf induced in the rod is

(a) zero (b) $\frac{\mu_0 Iv}{2\pi}\log_e\left(\frac{r_2}{r_1}\right)$

(c) $\frac{\mu_0 Iv}{2\pi}\log_e\left(\frac{r_1}{r_2}\right)$ (d) $\frac{\mu_0 Iv}{4\pi}\left(1-\frac{r_1}{r_2}\right)$

82. The current in a coil of self inductance 2.0 H is increasing according to the equation $I = 2\sin(t^2)$ ampere. The amount of energy spent during the period when the current changes from zero to 2 A is

(a) 2 J (b) 4 J

(c) 8 J (d) 16 J

83. In a car spark coil, an emf of 40,000 volts is induced in its secondary when the current in its primary changes from 4 A to zero is 10 μs. The mutual inductance between the primary and the secondary windings of the spark coil is

(a) 0.1 H (b) 0.2 H

(c) 0.3 H (d) 0.4 H

84. A rectangular wire loop of sides a and b is placed in a non-uniform magnetic field which varies with x as $B = kx$ where k is a constant. The magnetic field is directed perpendicular to the plane of the coil as shown in Fig. 14.55. The magnetic flux through the coil is

(a) zero (b) kab^2

(c) $\frac{1}{2}kab^2$ (d) $\sqrt{2}\,kab^2$

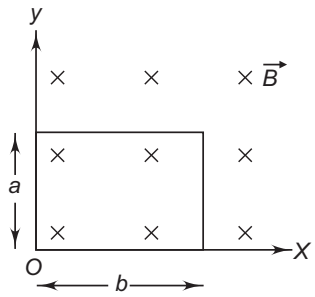


Fig. 14.55

85. A capacitor of capacitance 2 μF is charged to 50 V. The charging battery is then disconnected and a coil of inductance 5 mH is connected across it. Assuming that the coil has negligible resistance, the peak value of the current in the circuit will be
    (a) 1 A (b) 2 A
    (c) 3 A (d) 4 A
86. Figure 14.56 shows two situations in which a magnet falls downwards through a horizontal loop.

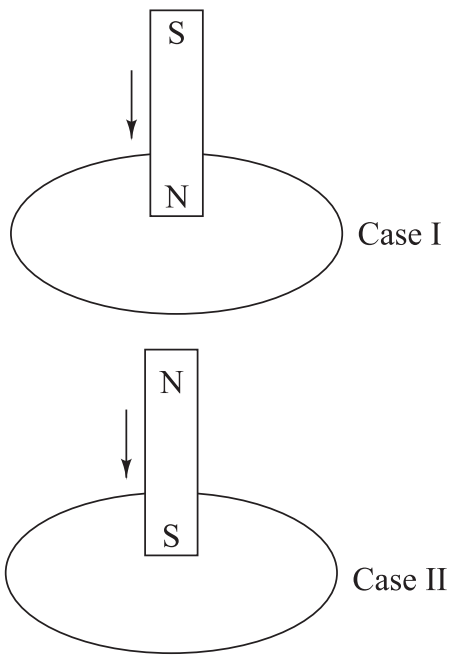


Fig. 14.56

As seen from above, what is the direction of the induced current in cases I and II?
    (a) Counterclockwise in case I and clockwise in case II
    (b) Clockwise in case I and counterclockwise in case II
    (c) Counterclockwise in both the cases I and II
    (d) Clockwise in both the cases I and II.

87. Figure 14.57 shows two situations in which a magnet is pulled upward with a constant velocity and enters a coil at time $t = 0$. The magnet moves completely through the coil at time t.

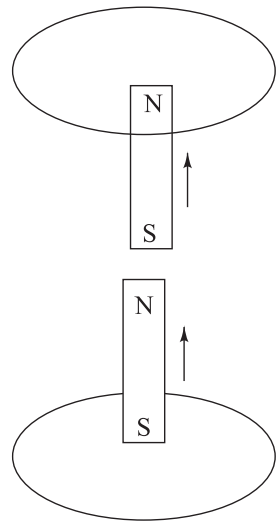


Fig. 14.57

Choose the correct statement from the following. As seen from above, the direction of the induced current is
    (a) always clockwise
    (b) always counterclockwise
    (c) first clockwise and then counterclockwise
    (d) first counterclockwise and then clockwise.
88. A small square wire loop of side $a = 3$ cm has 6 turns and has a resistant of $R = 3$ m Ω. It is placed at a distance of $r = 60$ cm from a long straight wire as shown in Fig. 14.58. If the current I in the wire decreases steadily from 7A to 2A in 1 ms, the induced current in the loop is

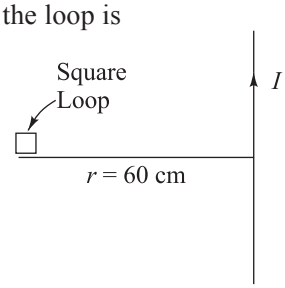


Fig. 14.58

    (a) 3.14 mA in the counterclockwise sense
    (b) 3.14 mA in the clockwise sense
    (c) 3.0 mA in the counterclockwise sense
    (d) 3.0 mA in the clockwise sense.
89. In the circuit shown in Fig. 14.59, the current through the 10 Ω resistor is I_1 when the switch S is open and I_2 when S has been closed for a long time. L is an ideal inductor of inductance of 5 mH. The ratio I_2/I_1 is

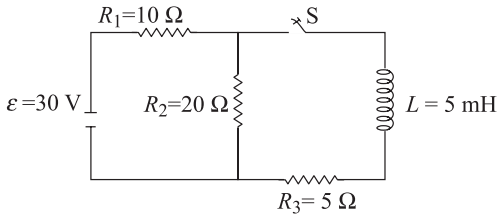


Fig. 14.59

(a) 1 (b) $\frac{5}{3}$

(c) $\frac{15}{7}$ (d) $\frac{30}{11}$

90. Figure 14.60 shows a small circular wire loop of radius r placed on an insulating stand inside a hollow solenoid of radius R. The solenoid has n turns per unit length and carries a current which decreases at a steady rate $\frac{dI}{dt}$. The induced emf in the coil is ε.

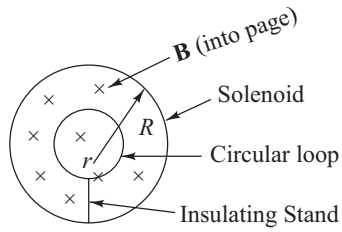


Fig. 14.60

Choose the correct statement from the following.

(a) $\varepsilon = \mu_0 n\pi r^2 \frac{dI}{dt}$; induced current is clockwise

(b) $\varepsilon = \mu_0 n\pi r^2 \frac{dI}{dt}$; induced current is counter clockwise.

(c) $\varepsilon = \mu_0 n\pi R^2 \frac{dI}{dt}$; induced current is clockwise

(d) $\varepsilon = \mu_0 n\pi R^2 \frac{dI}{dt}$; induced current is counter clockwise.

91. A square wire loop, of side a has a long straight wire carrying a current I that passes through the centre of the loop and perpendicular to its plane as shown in Fig. 14.61.

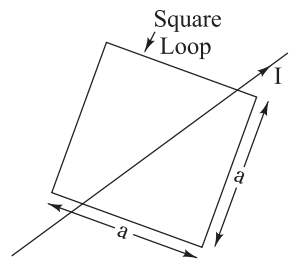


Fig. 14.61

The current induced in the square loop is

(a) $\frac{\mu_0 Ia}{\pi\sqrt{2}}$ (b) $\frac{\mu_0 Ia}{\pi}$

(c) $\frac{\sqrt{2}\,\mu_0 Ia}{\pi}$ (d) zero

92. Figure 14.62 shows a solenoid of inductance L, a resistor of resistance R and a battery of terminal voltage V.

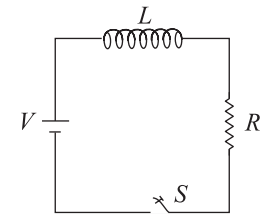


Fig. 14.62

At $t = 0$, the switch S is closed. After how long will the current attain of its final steady state value.

(a) $(\ln 2)\left(\frac{L}{R}\right)$ (b) $2(\ln 2)\left(\frac{L}{R}\right)$

(c) $\left(\ln\frac{3}{4}\right)\left(\frac{L}{R}\right)$ (d) $\left(\ln\frac{1}{4}\right)\left(\frac{L}{R}\right)$

93. In Q. 92 above, if N is the number of turns in the solenoid, what is the magnetic flux per turn of the solenoid when the current attains its maximum value?

(a) $\frac{LV}{NR}$ (b) $\frac{RV}{NL}$

(c) $\frac{NLV}{R}$ (d) $\frac{NRV}{L}$

94. Figure 14.63 shows an electrical circuit containing a two-way switch S. Initially the switch is open. Then terminal 1 is connected to terminal 3. When the current in R_1 attains a maximum (steady-state) value, terminal 1 is disconnected from terminal 2 and quickly connected to terminal 3. The potential difference across 3Ω resistor immediately after 1 is connected to 3 is

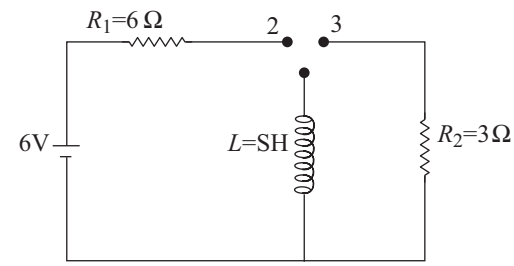


Fig. 14.63

(a) 6V (b) 3V

(c) 2V (d) zero

95. A capacitor of capacitance 2μF is charged to a potential difference of $12\sqrt{2}$ V. The charging battery is then removed and the cpacitor is connected to an inductor of inductance of 5mH. At the time when the potential difference across the capacitor drops to 12V, the current in the circuit is

(a) 0.06 A (b) 0.12A

(c) 0.18 A (d) 0.24 A

96. An inductor of inductance 100 mH is connected in series with a resistor of resistance 0.2Ω. This combination is connected across a 2V battery as shown in Fig. 14.64. The energy stored in the inductor will reduce to 1/9 of its maximum value in time

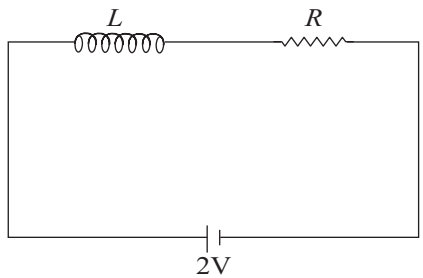


Fig. 14.64

(a) $\frac{1}{2}\ln\left(\frac{3}{2}\right)$ second (b) $\frac{1}{2}\ln(3)$ second

(c) $\frac{1}{9}\ln(2)$ second (d) $2\ln(9)$ second

97. A small circular wire loop of raidus r is placed inside a large square loop $ABCD$ of side a (will $a >> r$). The loops lie in the x-y plane with their centres at the origin O. The mutual inductance of the system is proportional to

(a) $\frac{r^2}{a}$ (b) $\frac{a^2}{R}$

(c) $a\ln\left(\frac{r}{a}\right)$ (d) $r\ln\left(\frac{a}{r}\right)$

98. A small circular loop of radius r is placed inside a large circular loop of radius R (with $R>>r$). The loops are coplanar with their centres coinciding. The mutual inductance of the system is proportional to

(a) $\frac{r}{R}$ (b) $\frac{R}{r}$

(c) $\frac{r^2}{R}$ (d) $\frac{R^2}{r}$

99. A small square loop of side l is placed inside a large square loop of side L (with $L>>l$). The loops are coplanar with their centres coinciding. The mutual inductance of the system is proportional to

(a) $\frac{l}{L}$ (b) $\frac{L}{l}$

(c) $\frac{l^2}{L}$ (d) $\frac{L^2}{l}$

100. A short bar magnet is at rest at time $t = 0$. It is moved towards a solenoid with a constant velocity v up to time $t = t_0$ after which it is moved away from the solenoid [see Fig. 14.65].

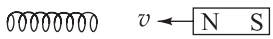


Fig. 14.65

Which of the graphs shown in Fig. 14.66 best represents the variation of induced emf e in the solenoid with time t?

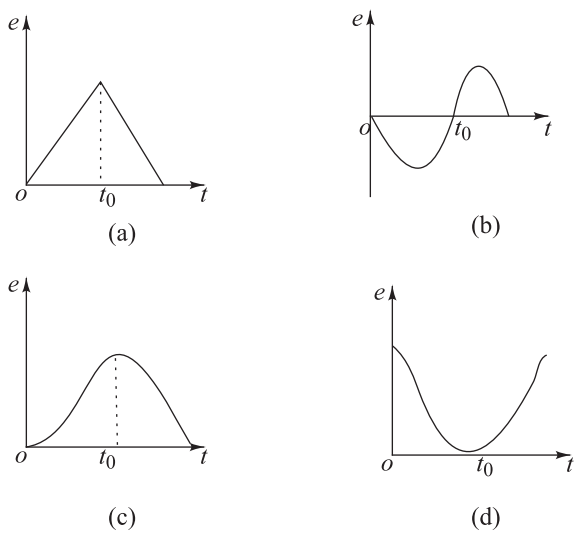


Fig. 14.66

101. An AC voltage source of a fixed peak value V_0 and variable angular frequency ω is connected in series with an inductor of inductance L and a bulb of resistance R (inductance zero). When ω is increased, the brightness of the bulb will

(a) increase (b) decrease

(c) remain unchanged (d) become zero

102. In the circuit shown in Fig. 14.67, the AC voltage source has r.m.s. voltage 20V and angular frequency 100 rad s^{-1}. The peak value of the voltage across the 100 Ω resistor is

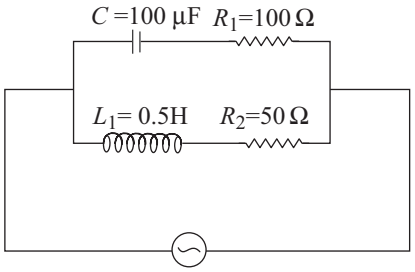


Fig. 14.67

(a) $5\sqrt{2}$ V (b) 10 V

(c) $10\sqrt{2}$ V (d) 20 V

103. In Q. 102 above, what is the peak value of the current through the 50 Ω resistor?
(a) 0.4A (b) 0.6A
(c) 0.8A (d) 1.0A

104. A circuit has an inductance of 100 mH and carries a current of 3A. To prevent sparking when the circuit is switched off, a capacitor is connected across the switch. If the capacitor can withstand a maximum voltage of 300 V, the minimum capacitance it must have should be equal to
(a) 10 μF (b) 20 μF
(c) 60 μF (d) 90 μF

105. When a current is started in the primary of a transformer, an ammeter connected to the secondary shows an instantaneous current *I*. Now if the primary is suddenly rotated through 180º, the instantaneous current
(a) becomes *I*/2
(b) becomes 2*I*
(c) remains unchanged
(d) becomes equal to zero

106. A rod *XY* is connected to the plates of a capacitor. The rod is placed in a region of uniform magnetic field ***B*** directed into the page as shown in fig. 14.68.

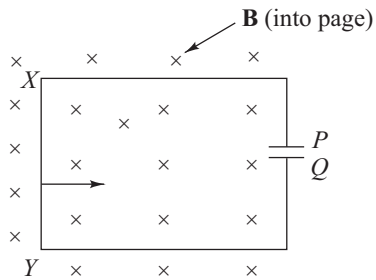


Fig. 14.68

If the rod is pulled out of the region of the magnetic field with a certain velocity *v*, then
(a) plate *P* acquires a positive charge and plate *Q* acquires an equal negative charge
(b) plate *P* acquires a negative charge and plate *Q* acquires an equal positive charge.
(c) both the plates acquire similar charges.
(d) no plate acquires any charge.

107. The variation of magnetic flux ϕ through a coil varies with time *t* as shown in Fig. 14.69.

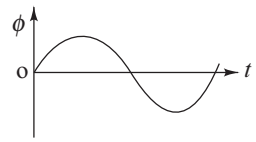


Fig. 14.69

Which graph shown in Fig. 14.70 best represents the variation of induced emf *e* in the coil with time *t*.

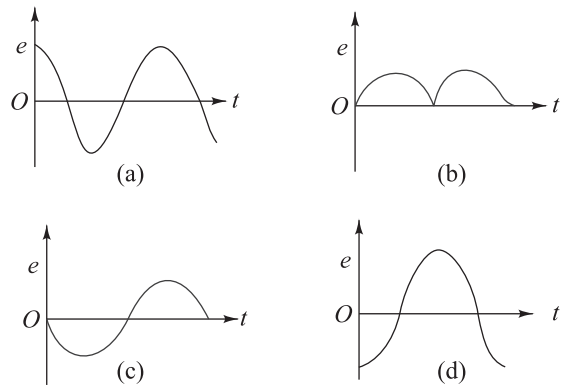


Fig. 14.70

108. Two inductors of inductances L_1 and L_2 are placed far apart so that the mutual inductance between them is negligible. If they are connected in parallel, The equivalent inductance of the combination is
(a) $L_1 + L_2$ (b) $\sqrt{L_1^2 + L_2^2}$
(c) $\sqrt{L_1 L_2}$ (d) $\dfrac{L_1 L_2}{L_1 + L_2}$

109. In the circuit shown in Fig. 14.71, terminal 1 is connected to terminal 2 until steady state is reached. Terminal 1 then disconnected from 2 and connected with terminal 3. The total heat produced in R_2 is
(a) 4 J (b) 8J
(c) 12J (d) 16J

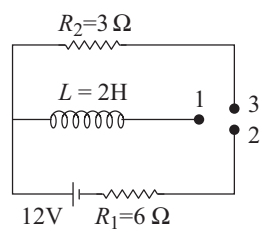


Fig. 14.71

110. Figure 14.72 shows an *L*-*R* circuit connected to a battery of a constant emf *E*. The switch S is closed at time *t* = 0. If *e* denotes the induced emf across the inductor and *i* the current in the circuit at any time *t*,

which of the graphs shown in Fig. 14.73 represents the variation of e with i?

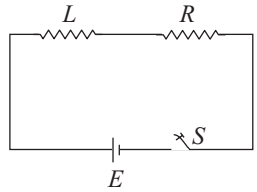


Fig. 14.72

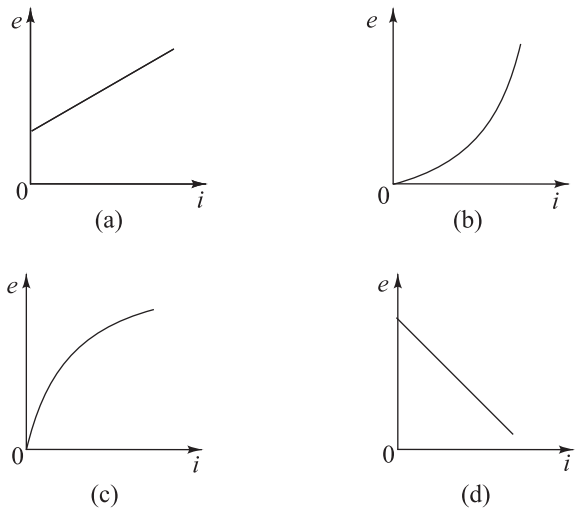


Fig. 14.73

111. A straight wire PQ of length 50 cm and resistance 0.8 Ω slides on parallel metal rails *CD* and *EF* with a velocity of 4 cms^{-1} in a uniform magnetic field of 2 T directed into the page as shown in Fig. 14.74. Two resistances 3 Ω and 2 Ω are connected as shown in the figure. The external force to be applied to *PQ* to keep it moving at a constant velocity of 4 cms^{-1} is

(a) 4×10^{-1} N (b) 2×10^{-2} N

(c) 4×10^{-3} N (d) 2×10^{-4} N

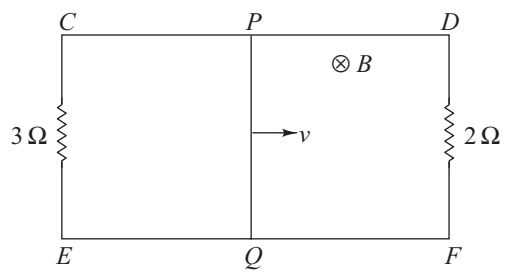


Fig. 14.74

112. A metal wire *PQ* of resistance 6 Ω is connected along a diameter of metal ring of radius 100 cm lying in the *x*-*y* plane. A uniform magnetic field $B = 4$T exists directed along the positive *z*-axis as shown in Fig. 14.75.

The ring is rotated in the *x*-*y* plane about an axis passing through its centre *O* and perpendicular to its plane at an angular frequency of 20 rad s^{-1}. An external resistance of 1Ω is connected between the centre of the ring and its rim. The current through the external resistance is

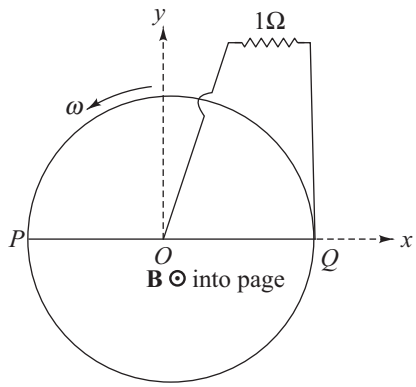


Fig. 14.75

(a) 4A (b) 3A

(c) 2.5A (d) 1.0A

113. The voltage V of an ac source varies will time t as

$$V = 220 \sin(50\pi t) \cos(50\pi t)$$

where V is in volt and t in second. The rms voltage is very nearly equal to

(a) 78 V (b) 89 V

(c) 110 V (d) 155 V

114. In Q.113 above, the frequency of the ac source is

(a) 25 Hz (b) 50 Hz

(c) 100 Hz (d) 200 Hz

115. In mutual inductance between two solenoids is 10 mH. The current I in one solenoid changes with time t as

$$I = 5 \sin (50\pi t)$$

where I is in ampere and t in second. The maximum value of emf (in volt) induced in the other solenoid is

(a) 2.5π (b) 5π

(c) 7.5π (d) 10π

116. In Q. 115 above, the phase between the current in the first solenoid and the emf induced in the second solenoid is

(a) zero (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

117. An inductor and a capacitor are connected to an a.c. voltage source as shown in Fig. 14.76. If I_L is the current through L at a certain instant of time and I_C

is the current through C at that instant, the current drawn from the source at that instant is ($I_L > I_C$)

(a) $I_L + I_C$ (b) $I_L - I_C$

(c) $\frac{I_L I_C}{I_L + I_C}$ (d) $\frac{I_L I_C}{I_L - I_C}$

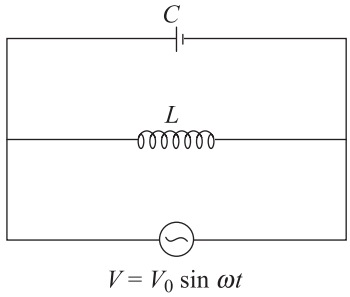


Fig. 14.76

118. In Q. 117, what is the current drawn from the source if C and L were connected to the source as shown in Fig. 14.77?

(a) zero (b) $I_L + I_C$

(c) $I_L - I_C$ (d) $\sqrt{I_L I_C}$

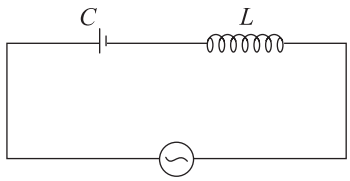


Fig. 14.77

Answers

Level A

1. (d)	2. (d)	3. (a)	4. (a)
5. (d)	6. (b)	7. (b)	8. (c)
9. (d)	10. (c)	11. (d)	12. (a)
13. (a)	14. (d)	15. (b)	16. (d)
17. (a)	18. (d)	19. (b)	20. (d)
21. (d)	22. (b)	23. (a)	24. (b)
25. (b)	26. (d)	27. (a)	28. (d)
29. (b)	30. (d)	31. (b)	32. (c)

Level B

33. (b)	34. (c)	35. (d)	36. (d)
37. (d)	38. (a)	39. (d)	40. (d)
41. (b)	42. (b)	43. (c)	44. (b)
45. (b)	46. (d)	47. (c)	48. (a)
49. (c)	50. (b)	51. (a)	52. (b)
53. (d)	54. (a)	55. (d)	56. (d)
57. (a)	58. (b)	59. (d)	60. (b)
61. (a)	62. (d)	63. (b)	64. (d)
65. (b)	66. (d)	67. (d)	68. (b)
69. (a)	70. (d)	71. (d)	72. (b)
73. (b)	74. (a)	75. (c)	76. (c)
77. (a)	78. (b)	79. (d)	80. (d)
81. (b)	82. (b)	83. (c)	84. (c)
85. (a)	86. (a)	87. (c)	88. (d)
89. (c)	90. (a)	91. (d)	92. (b)
93. (a)	94. (b)	95. (d)	96. (a)
97. (a)	98. (c)	99. (c)	100. (b)
101. (b)	102. (d)	103. (a)	104. (a)
105. (b)	106. (a)	107. (d)	108. (d)
109. (a)	110. (d)	111. (b)	112. (a)
113. (a)	114. (b)	115. (a)	116. (c)
117. (b)	118. (a)		

Solutions

Level A

1. The magnetic field due to the solenoid is

$$B = \mu_0 n I$$

where n is the number of turns per unit length. Since the second coil is wrapped closely around the solenoid, the cross-sectional area of the coil can be taken to be equal to that of the solenoid. Since the initial current and hence the initial magnetic field is zero, the change of flux for a single turn is $\mu_0 nIA$, where A is the cross-sectional area. Therefore, induced emf for a single turn is

$$|e| = \frac{\mu_0 n I A}{t}$$

$$= \frac{4 \times 3.14 \times 10^{-7} \times 500 \times 2 \times 10^{-4}}{3.14 \times 10^{-3}}$$

$$= 4 \times 10^{-5} \text{ V}$$

$\therefore$ Induced emf for 100 turns = $4 \times 10^{-5} \times 100 = 4 \times 10^{-3} = 4$ mV. Hence the correct choice is (d).

2. $B = 0.3$ T, $v = 1$ cms^{-1} $= 0.01$ ms^{-1}, $l = 8$ cm $= 0.08$ m and $b = 2$ cm $= 0.02$ m.

$e = Blv = 0.3 \times 0.08 \times 0.01$

$= 2.4 \times 10^{-4}$ V $= 0.24$ mV

3. The time during which the emf lasts is the time taken by the breadth (b = 2 cm) of the coil to move out of the field. Since the speed of coil is 1 cms^{-1}, this time will be 2 s.

4. $E = Bbv = 0.3 \times 0.02 \times 0.01$

$$= 0.6 \times 10^{-4} \text{ V} = 0.06 \text{ m V}$$

5. Time during which this emf lasts is the time taken by the length (l = 8 cm) to move out of the field = 8 s.

6. Induced emf in the secondary is

$$e_s = -M\frac{dI}{dt} = (-2) \times \left(-\frac{10}{0.1}\right) = 200 \text{ V}$$

Hence the correct choice is (b).

7. R_a = 2 Ω, E_a = 220 V, E_b = 210 V. Current at full speed is

$$I_a = \frac{E_a - E_b}{R_a} = \frac{220 - 210}{2} = 5 \text{ A}$$

Hence the correct choice is (b).

8. When the motor was switched on, E_b = 0 since the initial speed of the armature is zero (it is initially at rest). If no starter is used, the starting current in the armature is

$$I_s = \frac{E_a}{R_a} = \frac{220}{2.0} = 110 \text{ A}$$

which is ruinously large and it will burn the windings of the armature. Hence a starter must be inserted in series with the armature when the motor is first switched on (or started).

9. $\text{Efficiency} = \dfrac{\text{power output}}{\text{power input}}$

Power input is $IE_a = 5.0 \times 220 = 1100$ W. Power loss due to heating of the armature is $I^2R_a = (5.0)^2 \times 2.0 =$ 50 W. Therefore, power output is 1100 – 50 = 1050 W. Hence efficiency is 1050/1100 ≅ 0.95 or 95%.

10. Resistance of armature is 0.2 Ω, potential difference in open circuit is 220 V and potential difference at full load is 210 V.

Current in the circuit is

$$I = \frac{220 - 210}{0.2} = 50 \text{ A}$$

11. Power delivered = 210 × 50 = 10.5 kW, which is choice (d)

12. N_p = 400, N_s = 2000 and E_s = 1000 V. Now

$$\frac{E_p}{E_s} = \frac{N_p}{N_s}$$

or $$E_p = \frac{E_s \times N_p}{N_s} = \frac{1000 \times 400}{2000} = 200 \text{ V}$$

Hence the correct choice is (a).

13. Since the efficiency is 80%, the input power is

$$12 \text{ kW} \times \frac{100}{80} = 15 \text{ kW}$$

∴ Current in the primary is

$$I_p = \frac{\text{input power}}{\text{input voltage}} = \frac{15 \text{ kW}}{200 \text{ V}}$$

$$= \frac{15 \times 1000}{200} = 75 \text{ A}$$

∴ Power loss in primary = $I_p^2 R_p = (75)^2 \times 0.2 =$ 1125 W = 1.125 kW. Hence the correct choice is (a).

14. Current in the secondary is

$$I_s = \frac{\text{output power}}{\text{output voltage}} = \frac{12 \text{ kW}}{1000 \text{ V}}$$

$$= \frac{12 \times 1000}{1000} = 12 \text{ A}$$

∴ Power loss in secondary = $I_s^2 R_s = (12)^2 \times 2 =$ 288 W. Hence the correct choice is (d).

15. The correct choice is (b).

16. The falling magnet induces a current in the coil. From Lenz's law, the direction of the current is such that its magnetic field opposes the motion of the magnet. Hence the correct choice is (d).

17. Since the charge of electron is negative, the moving electron constitutes a current in the direction opposite to the direction of motion of the electron, i.e. the direction of the current will be from Q to P. The magnetic field threading the coil due to the motion of the electron will be directed into the plane of the page, i.e. perpendicular to the plane of coil directed into the page. To oppose this, the current in the coil must be anticlockwise, in accordance with Lenz's law. Hence the correct choice is (a).

18. As the magnet is moving towards coil AB, the magnetic flux linked with it increases. By Lenz's law, the current must flow in the coil in a direction which would tend to oppose the increase in flux, i.e. the current should produce a magnetic field in a direction opposite to the field of the magnet. Thus the end of the coil closer to the magnet should become a

south pole. Hence the current must be clockwise in the coil and flow from B to A.

As the magnet is moving away from coil CD, the magnetic flux linked with it decreases. The current must oppose this decrease and produce a field in the same direction as that of the magnet. The induced current must flow from D to C.

19. The current in the coil P is flowing in an anticlockwise direction. Hence end A is the north pole and end B the south pole. When the key K is lifted, the current in coil P starts decreasing leading to a decrease in magnetic flux through coil Q. By Lenz's law, the induced current in this coil must oppose the decrease in flux. This can happen if the direction of the induced current is such that a south pole is produced at end C and a north pole at end D. Therefore the induced current must flow in clockwise direction, i.e. from Y to X.

20. Voltage across secondary is

$$E_s = E_p \times \frac{N_s}{N_p} = \frac{110 \times 400}{200} = 220 \text{ V}$$

Current in secondary is

$$I_s = I_p \times \frac{E_p}{E_s} = \frac{10 \times 110}{220} = 5 \text{ A}$$

Hence the correct choice is (d).

21. The correct choice is (d).

22. The correct choice is (b).

23. The rate of change of flux gives the induced emf. Thus

$$E = -\frac{d\phi}{dt} = -\frac{d}{dt}(6t^2 - 5t + 1) = -12t + 5$$

At $t = 0.25$ s, $E = -12 \times 0.25 + 5 = -3 + 5 = 2$ V

$\therefore$ Induced current $I = \frac{E}{R} = \frac{2}{10} = 0.2$ A, which is choice (a).

24. The least capacitance is such that the energy stored in the capacitor is equal to that stored in the inductor, i.e.

$$\frac{1}{2} CV^2 = \frac{1}{2} LI^2$$

or
$$C = \frac{LI^2}{V^2} = \frac{1 \times (2)^2}{(400)^2} = 25 \times 10^{-6} \text{ F}$$
$$= 25 \ \mu\text{F}$$

Hence the correct choice is (b).

25. $E = Blv = 0.5 \times 10^{-4} \times 10 \times 200 = 0.1$ V which is choice (b)

26. Power output $= E_s I_s = 11 \times 90 = 990$ W. Power input $= E_p I_p = 220 \times 5 = 1100$ W. Therefore,

$$\text{Efficiency} = \frac{\text{power output}}{\text{power input}} = \frac{990}{1100} = \frac{9}{10}$$
$$= 0.9 \text{ or } 90\%$$

Hence the correct choice is (d).

27. Current in the transmission line is

$$I = \frac{\text{power}}{\text{voltage}} = \frac{25000}{250} = 100 \text{ A}$$

$\therefore$ Power loss $= I^2R = (100)^2 \times 1 = 10000$ W.

Therefore, the percentage of original power lost is

$$\frac{10000}{25000} \times 100 = 40\%$$

Hence the correct choice is (a).

28. Since the resistance of an ideal inductor is zero, the final value of the current in the 10 Ω resistor is zero.

29. Since the resistance of the inductor is zero, the total resistance of the circuit is $R = 20$ Ω. Therefore, current $= V/R = 2/20 = 0.1$ A.

30. Peak value of voltage is $V_0 = \sqrt{2}\ V_{\text{rms}} = \sqrt{2} \times 220$ V. Hence the correct choice is (d).

31. A choke is an inductor. Hence it is used only in ac circuits.

32. The correct choice is (c).

Level B

33. Given $R = 4$ Ω and $\omega L = 3$ Ω. The impedance is

$$Z = (R^2 + \omega^2 L^2)^{1/2} = (16 + 9)^{1/2} = 5 \ \Omega$$

Hence the correct choice is (b).

34. We know that $\tan\delta = \frac{\omega L}{R}$. Therefore

$$L = \frac{R \tan\delta}{\omega} = \frac{100 \times \tan 45^\circ}{2\pi \times 1000}$$
$$\simeq 15.9 \times 10^{-3} \simeq 16 \text{ mH}$$

Hence the correct choice is (c).

35. The current in an LCR circuit is given by

$$I = \frac{V}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}$$

where $\omega = 2\pi f$. Thus I increases with increase in ω upto a value of $\omega = \omega_c$ given by

$$\omega_c L = \frac{1}{\omega_c C}$$

or $$\omega_c = \frac{1}{\sqrt{LC}}$$

when I becomes maximum. At $\omega > \omega_c$, I decreases with increase in ω. Hence the correct graph is (d).

36. The correct choice is (d).

37. $E = 12$ V, $L_1 = 10$ H, $L_2 = 10 \times 10^{-3}$ H and $R = 48\ \Omega$. Steady state current is $I_0 = E/R$ and is independent of the inductance. Hence, the value of the steady state current is the same for both circuits.

$$I_0 = \frac{E}{R} = \frac{12}{48} = 0.25 \text{ A}$$

38. The energy consumed by the circuit to build up the current I_0 is

$$E = \frac{1}{2} L I_0^2$$

For circuit 1,

$$E_1 = \frac{1}{2} L_1 I_0^2 = \frac{1}{2} \times 10 \times (0.25)^2$$

$$= 3.125 \times 10^{-1} \text{ J}$$

For circuit 2,

$$E_2 = \frac{1}{2} L_2 I_0^2 = \frac{1}{2} \times 10 \times 10^{-3} \times (0.25)^2$$

$$= 3.125 \times 10^{-4} \text{ J}$$

$$\therefore \quad \frac{E_1}{E_2} = 1000.$$ Hence the correct choice is (a)

39. Power dissipated at current I_0 is $I_0^2 R$. Since I_0 and R are the same for both the circuits, they dissipate the same power which is

$$P = I_0^2 R = (0.25)^2 \times 48 = 3.0 \text{ W}$$

40. The time constant of the circuit is

$$\tau = \frac{L}{R} = \frac{100 \times 10^{-3}}{50} = 2 \times 10^{-3} \text{ s}$$

$$= 2 \text{ millisecond.}$$

Current at time t is given by

$$I = I_0 e^{-t/\tau}$$

where I_0 is the steady current. Therefore, time for I to fall to $I_0/2$ is

$$e^{-t/\tau} = \frac{1}{2} \text{ or } e^{t/\tau} = 2 \text{ or } t = \tau \ln(2).$$

Hence the correct choice is (d).

41. The resonant angular frequency is

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\left(5.0 \times 80 \times 10^{-6}\right)^{1/2}}$$

$$= 50 \text{ rad s}^{-1}$$

Therefore, the resonant frequency is

$$\nu_r = \frac{\omega_r}{2\pi} = \frac{50}{2\pi} = \frac{25}{\pi} \text{ Hz}$$

42. The impedance is given by

$$Z = \left[R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2 \right]^{1/2}$$

When $$\omega = \omega_r = 1/\sqrt{LC}$$

(i.e. at resonance), $\omega L = 1/\omega C$, and therefore

$$Z = R = 40\ \Omega$$

43. Current amplitude at resonance is

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{R} = \frac{\sqrt{2}\, V_{\text{rms}}}{R} = \frac{\sqrt{2} \times 200}{40} = 5\sqrt{2} \text{ A.}$$

44. The rms current in the circuit is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{200}{40} = 5 \text{ A}$$

$\therefore$ The rms potential drop across L is

$V_{\text{rms}} = I_{\text{rms}} \times \omega_r \times L = 5 \times 50 \times 5 = 1250 \text{ V} = 1.25 \text{ kV}$

45. The rms potential drop across C is

$$V_{\text{rms}} = I_{\text{rms}} \times \frac{1}{\omega_r C} = 5 \times \frac{1}{50 \times 80 \times 10^{-6}} = 1.25 \text{ kV}$$

46. The rms potential drop across R is

$V_{\text{rms}} = I_{\text{rms}} \times R = 5 \times 40 = 200$ V

47. The correct choice is (c).

48. The dimensions of RC are those of ohm × charge / voltage, i.e.

$$\frac{\text{voltage}}{\text{current}} \times \frac{\text{charge}}{\text{voltage}} = \frac{\text{charge}}{\text{charge/time}} = \text{time}$$

Hence the dimensions of $1/RC$ are those of frequency.

49. Since inside the cell, the current is taken to flow from the negative to the positive terminal, we have

$$V_A - IR + E - L \frac{dI}{dt} = V_B$$

or $$V_B - V_A = -IR + E - L \frac{dI}{dt}$$

Since I is decreasing with t, $\frac{dI}{dt}$ is negative. Hence

$$V_B - V_A = -5 \times 10^{-6} \times 1 + 15 - (5 \times 10^{-3}) \times (-10^{-3})$$
$$= 15 \text{ V}$$

Thus the correct choice is (c).

50. We know that $Q = CV$ and $Q = Q_0 \cos \omega t$. Also $Q_0 = CV_0$.

$$\therefore \quad \cos \omega t = \frac{Q}{Q_0} = \frac{V}{V_0} = \frac{6}{12} = \frac{1}{2} \text{ or } \omega t = \frac{\pi}{3}.$$

Now ω is given by

$$\omega = \frac{1}{\sqrt{LC}} \qquad (1)$$

Given $L = 0.6 \times 10^{-3}$ H and $C = 2 \times 10^{-6}$ F. Using these values in Eq. (1) we get $\omega = \frac{10^5}{2\sqrt{3}}$ rad s^{-1}.

$$\text{Now} \qquad I = \frac{dQ}{dt} = \frac{d}{dt}(Q_0 \cos \omega t)$$
$$= -Q_0 \omega \sin \omega t$$

$$\therefore \qquad |I| = Q_0 \omega \sin \omega t = CV_0 \omega \sin \omega t$$
$$= (2 \times 10^{-6}) \times 12 \times \frac{10^5}{2\sqrt{3}} \sin\left(\frac{\pi}{3}\right)$$
$$= 0.6 \text{ A}$$

Hence the correct choice is (b).

51. Now $|e| = L \frac{dI}{dt}$

$$\text{or} \quad 10 \times 10^{-3} = L \times \frac{d}{dt}(5 + 16t) = L \times (16)$$

$$\text{or} \qquad L = 6.25 \times 10^{-4} \text{ H}.$$

Hence the correct choice is (a).

52. Power $= VI = 10 \times 10^{-3} \times (5 + 16t)$

At $t = 1$ s, power $= 10 \times 10^{-3} \times (5 + 16) = 0.21$ W

Hence the correct choice is (b).

53. Induced emf (e)

$$= \frac{\text{magnetic field} \times \text{change in area}}{\text{time}} = \frac{B\Delta A}{t}$$

Since the circumference of the circular loop $= 2\pi r$, the side of the square loop $= \frac{2\pi r}{4} = \frac{\pi r}{2}$.

Therefore,

$$\Delta A = \pi r^2 - \left(\frac{\pi r}{2}\right)^2 = \pi r^2 \left(1 - \frac{\pi}{4}\right)$$

$$\therefore \qquad e = \frac{B(\pi r^2)}{t}\left(1 - \frac{\pi}{4}\right)$$

Hence the correct choice is (d).

54. Self inductance of each coil $= \frac{1}{2} \times (1.8 \times 10^{-4}) = 0.9 \times 10^{-4}$ H. Resistance of each coil $= \frac{6}{2} = 3\ \Omega$. When two such coils are connected in parallel, the self-inductance of the combination is $L = 0.45 \times 10^{-4}$ H and the resistance of the combination is $R = 1.5\ \Omega$.

$$\therefore \text{ Time constant} = \frac{L}{R} = \frac{0.45 \times 10^{-4}}{1.5} = 0.3 \times 10^{-4} \text{ s}$$

Hence the correct choice is (a).

55. Steady current $I_0 = \frac{V}{R} = \frac{12}{1.5} = 8$ A which is choice (d).

56. Velocity $v = \sqrt{2gh}$. Induced emf $e = Blv = Bl\sqrt{2gh}$. Therefore, the induced current in the loop is

$$I = \frac{Bl\sqrt{2gh}}{R}$$

$$\therefore \text{ Force } \quad F = BIl = \frac{B^2 l^2 \sqrt{2gh}}{R}$$

The loop will attain terminal velocity if this force equals mg, i.e. if

$$\frac{B^2 l^2 \sqrt{2gh}}{R} = mg$$

which gives $h = \frac{m^2 g R^2}{2B^4 l^4}$

Hence the correct choice is (d).

57. Magnetic field due to the larger coil at its centre is

$$B = \frac{\mu_0 I}{2r_1}$$

where I is the current in the larger coil. Flux through the inner coil is

$$\phi = B \times \pi r_2^2 = \frac{\mu_0 I}{2r_1} \times \pi r_2^2$$

But $\phi = MI$. Therefore

$$M = \frac{\mu_0 \pi r_2^2}{2r_1}$$

Hence the correct choice is (a).

58. The network *PQRS* is a balanced Wheatstone's bridge. Hence the resistance of 3 Ω between *P* and *R* is ineffective. The net resistance of the network, therefore, is 3 Ω. Total resistance $R = 3\ \Omega + 1\ \Omega = 4\ \Omega$. Now, induced emf is $e = Blv = 2 \times 0.1 \times v = 0.2\ v$.

$\therefore$ Induced current $I = \dfrac{e}{R} = \dfrac{0.2v}{4}$.

Given $I = 1 \times 10^{-3}$ A

Hence $\qquad 1 \times 10^{-3} = \dfrac{0.2v}{4}$

which gives $v = 2 \times 10^{-2}$ ms^{-1} = 2 cm s^{-1}, which is choice (b).

59. If a coil is not moved in a magnetic field, the magnetic flux does not change. Hence no emf or current is induced in the coil. Hence the correct choice is (d).

60. Since the voltage leads the current by a phase angle of 90°, the total potential difference across the circuit is

$V = (V_R^2 + V_L^2)^{1/2} = (20 \times 20 + 16 \times 16)^{1/2} = 25.6$ V, which is choice (b).

61. The phase angle between voltage *V* and current *I* is $\pi/2$. Therefore, power factor $\cos \phi = \cos(\pi/2) = 0$. Hence the power consumed is zero, which is choice (a).

62. Since the magnetic field is constant in time and space and exists everywhere, there is no change in magnetic flux when the loop is moved in it. Hence no current is induced, which is choice (d).

63. Induced charge $q = \dfrac{\text{change of flux}}{\text{resistance}} = \dfrac{\phi_f - \phi_i}{R}$. But final area = 0, therefore, $\phi_f = 0$. Numerically, $\phi_i = BA$. Therefore, $q = BA/R$, which is choice (b).

64. As the ring falls with a velocity *v* the decrease in area with time is

$$\frac{dA}{dt} = -(2R)v$$

$\therefore$ Induced emf, $e = -\dfrac{d\phi}{dt} = -\dfrac{d}{dt}(BA) = -B\dfrac{dA}{dt} = 2RBv$.

From Lenz's law, the induced current in the ring must produce magnetic field in the upward direction. Hence *Q* is at higher potential. Hence the correct choice is (d).

65. Refer to Fig. 14.78. The magnetic field due a current *I* in the large loop at its centre is

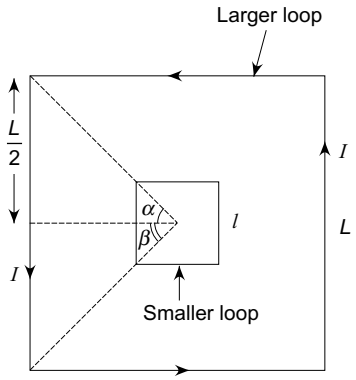


Fig. 14.78

B = 4 times that due to one side

$$= 4 \times \frac{\mu_0}{4\pi}\frac{I}{(L/2)}(\cos\alpha + \cos\beta)$$

$$= \frac{2\mu_0 I}{\pi L}(\cos 45° + \cos 45°)$$

$$= \frac{2\sqrt{2}\mu_0 I}{\pi L} \quad (\because \alpha = \beta = 45°)$$

The magnetic flux that links the larger loop with the smaller loop of side *l* ($l << L$) is

$$\phi_{12} = Bl^2 = \frac{2\sqrt{2}\mu_0 I l^2}{\pi L}$$

$\therefore$ Mutual inductance $M_{12} = \dfrac{\phi_{12}}{I} = \dfrac{2\sqrt{2}\mu_0}{\pi}\left(\dfrac{l^2}{L}\right)$

i.e. $M_{12} \propto \dfrac{l^2}{L}$, which is choice (b).

66. Figure 14.79 shows the field lines (shown as broken curves) of the magnetic field due to the current flowing in the loop. It is clear from the figure that the magnetic flux in the *x*-*y* plane will be zero. Hence the correct choice is (d).

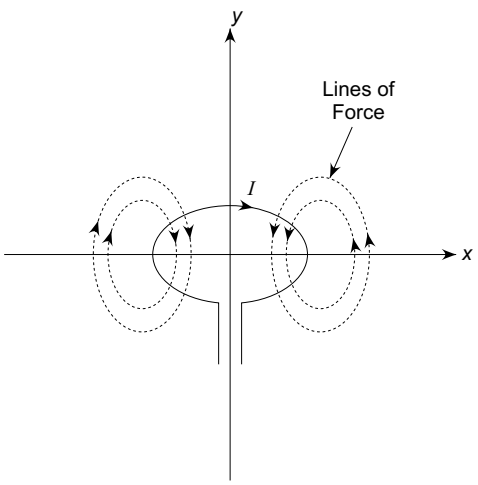


Fig. 14.79

67. The current in the inductor is given by

$I = \frac{V}{R}\ (1 - e^{-t/\tau})$, where $\tau = L/R$.

Given, $\tau = \frac{L}{R} = \frac{8.4\ \text{mH}}{6\ \Omega}$

$= 1.4$ ms (millisecond)

$\therefore\ 1.0 = \frac{12}{6}\ (1 - e^{-t/1.4\ \text{ms}})$

or $e^{-t/1.4\ \text{ms}} = 1 - \frac{1}{2} = \frac{1}{2}$

or $-\frac{t}{1.4\ \text{ms}} = \log_e\left(\frac{1}{2}\right) = -0.693$

or $t = 0.693 \times 1.4\ \text{ms} = 0.97\ \text{ms}$

Hence the correct choice is (d).

68. A time varying magnetic field produces an electric field. The magnitude of the electric field at a distance r from the centre of a circular region of radius a where a time varying field B exists is given by

$$E = \frac{a^2}{2r}\frac{dB}{dt}$$

At $r = a$, $E = (a/2)\ dB/dt$, which the value of E at the edge of the circular region. For $r > a$, E decreases as $1/r$. Hence the correct choice is (b).

69. The mutual inductance between the two coils in orientation (A) is the maximum since the flux linkage in (A) is the maximum as shown in Fig. 14.80.

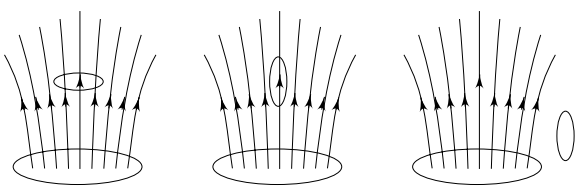


Fig. 14.80

70. Electric field will be induced in both AD and BC, since both are moving perpendicular to the direction of the magnetic field and the flux linked with them is changing with time. Hence the correct choice is (d).

71. Let the switch be closed at time $t = 0$. The current I_p flowing in P grows for a time, say t_0, after which it becomes steady. During this time the magnetic field (due to I_p) (from left to right) increases at the location of circuit Q. According to Lenz's law, the induced current $(I_Q)_1$ should be such that it tries to decrease the magnetic field. Therefore, the magnetic field due to this current must be from right to left. Hence this induced current $(I_Q)_1$ should be anticlockwise (opposite to the direction of I_P). After the switch S is opened, the current I_P takes a finite time to decay to zero and the reverse of the above phenomenon is observed. Hence the induced current $(I_Q)_2$ should be clockwise. Thus the correct choice is (d).

72. The magnitude of the induced voltage is proportional to the rate of change of magnetic flux which, in turn, depends on the number of turns in the coil, i.e. $V \propto n$. The resistance of a wire is given by

$R = \frac{\rho l}{\pi r^2}$ or $R \propto \frac{l}{r^2}$. Here ρ is the resistivity of the material of the wire.

$\therefore$ Power $P = \frac{V^2}{R} \propto \frac{n^2}{l/r^2}$ or $P \propto \frac{(nr)^2}{l}$

$$\therefore \quad \frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^2 \times \left(\frac{r_2}{r_1}\right)^2 \times \left(\frac{l_1}{l_2}\right) \qquad (1)$$

Now, if a wire of length l_1 and radius r_1 is stretched to a length l_2 such that its radius reduced to r_2, then (since the mass of the wire remains constant)

$m = \pi r_1^2 l_1 d = \pi r_2^2 l_2 d$ (d is the density)

or $\frac{l_1}{l_2} = \left(\frac{r_2}{r_1}\right)^2$. Using this in Eq. (1), we get

$$\frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^2 \times \left(\frac{r_2}{r_1}\right)^4$$

Given $\frac{n_2}{n_1} = 4$ and $\frac{r_2}{r_1} = \frac{1}{2}$.

Using these values, we get

$$\frac{P_2}{P_1} = (4)^2 \times \left(\frac{1}{2}\right)^4 = 1,$$

which is choice (b).

73. Induced emf is $|e| = n\ \frac{\Delta\Phi}{\Delta t}$. Now

$\Delta q = I\ \Delta t$

$= \frac{e}{R}\ \Delta t = \frac{n\Delta\Phi}{R\Delta t} \times \Delta t = \frac{n\Delta\Phi}{R}$

$= \frac{n(\Phi_2 - \Phi_1)}{R}$

Hence the correct choice is (b).

74. Given $E = E_0 \sin(100\,t)$. Comparing this with $E = E_0 \sin \omega t$, we have $\omega = 100$ rad s^{-1}. It follows from the figure that the current leads the e.m.f. which is true only for *R*-*C* circuit, and not for *R*-*L* circuit. Hence the circuit does not contain an inductor. Thus choices (c) and (d) are not possible. For *R*-*C* circuit, the phase difference between *E* and *I* is given by

$$\tan \phi = \frac{1}{\omega RC} \qquad \text{(i)}$$

Given $\phi = \pi/4$. Also $\omega = 100$ rad s^{-1}. Using these values in (i), we get

$$\tan\left(\frac{\pi}{4}\right) = \frac{1}{100\,RC} \quad \text{or} \quad RC = \frac{1}{100}$$

This relation between *R* and *C* is satisfied by choice (a) and not choice (b). Hence the correct choice is (a).

75. When wire *CD* is made to slide on wires *PQ* and *ST*, the flux linked with the circuit changes with time and hence an emf is induced in the circuit, which is given by

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = B\,\frac{dA}{dt}$$

If wire *CD* moves a distance *dx* is time *dt*, then $A = wdx$ (here $w = CD$) and

$$|e| = B\,\frac{d}{dt}(wdx) = Bw\,\frac{dx}{dt} = Bwv$$

The induced current is

$$I = \frac{e}{R} = \frac{Bwv}{R}$$

This current is caused by the motion of wire *CD*. From Lenz's law, the current *I* opposes the motion of wire *CD*. Therefore, work has to be done to slide the wire *CD*. Now, the magnetic force on wire *CD* (of length *w*) is

$$F = BIw = B\left(\frac{Bwv}{R}\right)w = \frac{B^2w^2v}{R} \qquad (1)$$

Work done is sliding wire *CD* through a small distance *dx* in time *dt* is

$$dW = Fdx$$

Therefore, the work done per second is

$$P = \frac{dW}{dt} = F\,\frac{dx}{dt} = Fv$$

Using (1), we get

$$P = \frac{B^2w^2v^2}{R}$$

Hence the correct choice is (c).

76. As the air plane is flying horizontally parallel to the earth's surface, the flux linked with it will be due to the vertical component B_V of the earth's field.

Now

$$B_V = B_H \tan\theta = 2 \times 10^{-5} \times \tan 60°$$
$$= 2\sqrt{3} \times 10^{-5}\ \text{Wbm}^{-2}$$

$\therefore$ Induced emf is $|e| = B_V lv = 2\sqrt{3} \times 10^{-5} \times 20 \times 250$

$= \dfrac{\sqrt{3}}{10}$ V, which is choice (c).

77. Refer to Fig. 14.81. Let ν be the frequency of rotation. The time taken for 1 full rotation is $T = 1/\nu$. Therefore, rate of change of area is

$$\frac{A}{T} = \frac{\pi r^2}{T} = \pi r^2 \nu$$

Now, the emf induced between the axle and rim is *e* = *B* × rate of change of area

$$= B \times \pi r^2 \nu = \frac{1}{2} Br^2\omega \quad (\because\ \omega = 2\pi\nu)$$

Since the same emf is produced between the ends of each spoke, and these emfs are in parallel as is evident from Fig. 14.81, the net emf between the axle and the rim of the wheel will be the same as that across each spoke. We notice that all the eight spokes are connected with one end at the rim and the other at the axle. Hence the magnitude of the net emf between the axle and the rim is independent of the number of spokes.

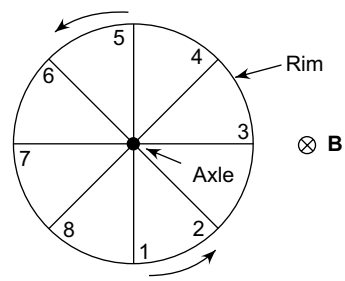


Fig. 14.81

78. The growth of current in an *LR* circuit is given by

$$I = I_0\,(I - e^{-Rt/L}) \qquad (1)$$

where I_0 is the maximum current. The energy stored at time *t* is

$$U = \frac{1}{2} LI^2$$

We are required to find the time at which the energy stored is one-forth the maximum value, i.e. when

$$U = \frac{U_0}{4} \text{ where}$$

$$U_0 = \frac{1}{2} LI_0^2$$

i.e. $$\frac{1}{2} LI^2 = \frac{1}{4}\left(\frac{1}{2} LI_0^2\right) \quad \text{or} \quad I = \frac{I_0}{2}$$

Using this in Eq. (1), we get

$$\frac{I_0}{2} = I_0 (1 - e^{-Rt/L}) \quad \text{or} \quad \frac{1}{2} = 1 - e^{-Rt/L}$$

or $$e^{-Rt/L} = \frac{1}{2} \quad \text{or} \quad -\frac{Rt}{L} = \log_e \left(\frac{1}{2}\right)$$

or $$t = \frac{L}{R} \log_e(2), \text{ which is choice (b).}$$

79. When capacitance is removed, the circuit contains only inductance and resistance. Phase difference θ between the current and voltage is then given by

$$\tan\theta = \frac{\omega L}{R} \quad \text{or} \quad \omega L = R\tan\theta = 100\tan 60°$$

When the circuit contains only capacitance and resistance, the phase difference between the voltage and current is given by

$$\tan\phi = \frac{1}{RC\omega}$$

$$\therefore \quad \frac{1}{C\omega} = R\tan\phi = 100\tan 60°$$

The impedance of the *LCR* circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{C\omega}\right)^2}$$

$$= \sqrt{R^2 + (100\tan 60° - 100\tan 60°)^2}$$

$$= R = 100\ \Omega$$

The current is given by

$$I = \frac{V}{R} = \frac{200}{100} = 2 \text{ A.}$$

Hence the correct choice is (d).

80. If the coil is not moved in a magnetic field, the magnetic flux linked with the coil does not change. Hence no emf or current is induced in the coil. Thus the correct choice is (d).

81. An emf is induced in the rod because it cuts through the lines of force of the magnetic field of the current carrying wire. Consider a small element of length *dr* of the rod at a distance *r* from the wire. The magnetic field due to a current *I* in the wire at a distance *r* from it is (Fig. 14.82)

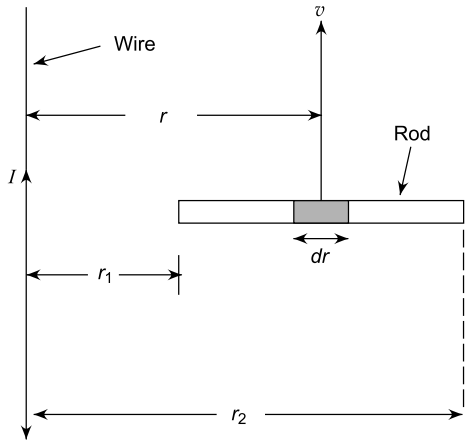


Fig. 14.82

$$B = \frac{\mu_0 I}{2\pi r}$$

The emf induced in the element of length *dr* is

$$de = Bvdr = \frac{\mu_0 I v}{2\pi} \frac{dr}{r}$$

$\therefore$ The emf induced in the whole rod is

$$e = \frac{\mu_0 I v}{2\pi} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu_0 I v}{2\pi} \left| \log_e r \right|_{r_1}^{r_2}$$

or $$e = \frac{\mu_0 I v}{2\pi} \log_e \left(\frac{r_2}{r_1}\right),$$

which is choice (b).

82. $W = \frac{1}{2} LI_0^2$, where I_0 = peak value of I = 2 A. Thus

$$W = \frac{1}{2} \times 2.0 \times (2)^2 = 4 \text{ J}$$

Hence the current choice is (b).

83. $$e = -M\frac{\Delta I}{\Delta t}$$

or $$M = -\frac{e\Delta I}{\Delta I} = -\frac{40{,}000 \times (10 \times 10^{-6})}{(-4-0)}$$

$$= 0.1 \text{ H}$$

The correct choice is (a).

84. Since the magnetic field varies with *x*, we find the flux by considering a small element of the loop of width *dx* and length *a* at a distance *x* from *O*, as shown in Fig. 14.83.

The total magnetic flux is

$$\phi = \int BdA = \int_0^b kx(adx)$$

$$= ka \int_0^b xdx = \frac{1}{2} kab^2$$

so the correct choice is (c).

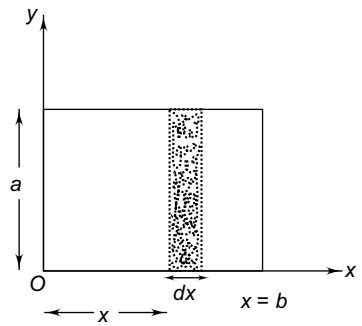


Fig. 14.83

85. $\frac{1}{2}LI_0^2 = \frac{1}{2}CV^2$ ($\because$ there is no loss of energy due to joule heating as $R = 0$). Hence

$$I_0 = V\sqrt{\frac{C}{L}} = 50 \times \sqrt{\frac{2\times10^{-6}}{5\times10^{-3}}} = 1 \text{ A}$$

which is choice (a).

86. From Lenz's law, the direction of the induced current in the coil must be such that it opposes the downward gravitational force of the falling magnet. This can happen if the upper face of the coil in case I develops north polarity so that the magnet is repelled. From clock rule, Ⓝ the current in case I must be counter clockwise. By the same logic, the current in case II must be clockwise. So the correct choice is (a). This can also be explained as follows.

In case I, the magnetic field of the magnet at the centre of the coil (which lies on the axial line of the axis) is directed downwards. By the right hand screw rule, the induced current must be counter-clockwise because then it will produce a magnetic field upwards.

87. At time $t = 0$, the north pole N of the magnet is moved upward towards the loop. From Lenz's law, the lower face of the coil (as seen from above) must acquire north polarity so that the magnet is repelled downwards. Hence the upper face of coil must acquire south polarity. From clock rule, the current induced in the coil must be clockwise Ⓢ. At time t, the magnet moves out of the coil. Since the south pole S of the magnet is going upwards, the upper face of the coil must acquire north polarity to attract it downwards. Hence, as seen from above, the current in the coil is now counter-clockwise Ⓝ. So the correct choice is (c).

88. Since the wire is very small (a = 3 cm = 0.03 m) and the straight wire is at a very large distance (r = 60 cm = 0.6 m) from it, the magnetic field due to the current in the straight wire can be taken to be uniform through the loop and is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

The magnetic flux through the squere loop is

$\phi = N(\mathbf{B}.\mathbf{A}) = NBA\cos\theta$

where $A = a^2$. Since the magnetic field B at the loop is out of the plane of the loop (from right hand thumb rule) and the direction of area vector $\boldsymbol{A}$ is also normal to the plane of the loop, $\theta = 0°$. So

$\phi = NBA\cos 0° = NBA = NBa^2$

$$\phi = \frac{Na^2\mu_0 I}{2\pi r}$$

Therefore, the induced emf is

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{Na^2\mu_0}{2\pi r}\frac{dI}{dt}$$

$$= \frac{6\times(0.03)^2\times\left(4\pi\times10^{-7}\right)\times(2-7)}{2\pi\times(0.6)\times(1\times10^{-3})}$$

$$= 9\times10^{-6}\text{V}$$

$\therefore$ Induced current is

$$I = \frac{\varepsilon}{R} = \frac{9\times10^{-6}}{3\times10^{-3}} = 3\times10^{-3}\text{A} = 3\,\text{mA}$$

The magnetic field B is out of the page. From Lenz's law, the direction of the induced current in the loop must be directed into the page. This is possible if the current in the coil is clockwise. So the correct choice is (d).

89. When switch S is open, the current in the circuit is

$$I_1 = \frac{\varepsilon}{(R_1+R_2)} = \frac{30}{(10+20)} = 1\text{A}$$

Since R_1 and R_2 are in series, this is also the current through R_1.

When switch S is closed, R_2 and R_3 are in parallel and their combined resistance is ($\because$ L is an ideal inductor)

$$R' = \frac{R_2\times R_3}{(R_2+R_3)} = \frac{20\times5}{(20+5)} = 4\,\Omega$$

Therefore, the current in the circuit now is

$$I_2 = \frac{\varepsilon}{(R_1 + R')} = \frac{30}{(10+4)} = \frac{15}{7}\text{ A}$$

$$\therefore \quad \frac{I_2}{I_1} = \frac{15}{7}\text{, which is choice (c).}$$

90. $B = \mu_0 nI$ directed into the page. Magnetic flux through the loop is

$$\phi = BA = (\mu_0 nI) \times \pi r^2$$

Induced emf is

$$\varepsilon = -\frac{d\phi}{dt} = -\mu_0 n\pi r^2\left(-\frac{dI}{dt}\right)$$

$$= \mu_0 n\pi r^2 \frac{dI}{dt}$$

Since the magnetic field due to the solenoid is into the page and is decreasing (because I is decreasing with time) from Lenz's law, the direction of the current in loop should be such that it produces a magnetic field into the page. Hence the induced current in the loop must be clockwise. So the correct choice is (a).

91. The current in the wire produces a magnetic field which produces magnetic flux through the square loop. Since current does not change with time, the magnetic field (and hence the magnetic flux) does not change with time. Hence no emf (and hence no current) is induced in the loop. So the correct choice is (d).

92. $I = I_0(1 - e^{-t/\tau})$

where I_0 is the maximum or final steady-state current and $\tau = \frac{L}{R}$ is the time constant of the $L - R$ circuit.

$I = \frac{3}{4}I_0$ at time t given by

$$\frac{3}{4}I_0 = I_0(1 - e^{-t/\tau})$$

$$\Rightarrow \quad e^{-t/\tau} = -\frac{1}{4}$$

$$\Rightarrow \quad e^{t/\tau} = 4$$

$$\frac{t}{\tau} = \ln 4 = 2(\ln 2)$$

So the correct choice is (b).

93. Maximum current is

$$I_0 = \frac{V}{R}$$

If ϕ is the magnetic flux per unit turn of the solenoid, the total flux is $\Phi = N\phi$ which is equal to $L\,I_0$, i.e.

$$N\phi = LI_0$$

$$\phi = \frac{LI_0}{N} = \frac{LV}{NR}$$

So the correct choice is (a).

94. At the instant when 1 is connected to 2, the inductor L prevents the change in current. At this instant, therefore, the current in the inductor and the rest of the circuit is zero. After the steady state is reached, the inductor has no emf (since $\frac{dI}{dt} = 0$), so the steady state current I_0 is obtained from Ohm'a law,

$$I_0 = \frac{V}{R_1} = \frac{6\text{ V}}{6\Omega} = 1\text{A}$$

When 1 is connected to 3 at, say, $t = 0$, the current in the inductor does not decay instantly; it is equal to $I_0 = 1$A at $t = 0$. The current through the inductor decays with time t as

$$I = I_0\, e^{-t/\tau}$$

where $\tau = \frac{L}{R_2}$. Notice that at $t = 0$, $I = I_0$.

The emf induced in the inductor is

$$\varepsilon = -L\,\frac{dI}{dt}$$

$$= -L\,\frac{dI}{dt}\left[I_0 e^{-t/\tau}\right]$$

$$= -L \times \left(-\frac{1}{\tau}\right) I_0 e^{-t/\tau}$$

At $t = 0$, $e^{-t/\tau} = e^{-0} = 1$. Hence

$$\varepsilon = \frac{LI_0}{\tau}$$

$$= \frac{L}{L/R_2} \times I_0$$

$$= R_2 I_0 = 3\Omega \times 1\text{A} = 3\text{V}$$

Since R_2 is in parallel with L, the potential across $R_2 = \varepsilon = 3$V. So the correct choice is (b).

95. $Q = CV$ and $Q_0 = CV_0$. Also $Q = Q_0 \sin \omega t$ = Thus

$$\sin \omega t = \frac{Q}{Q_0} = \frac{CV}{CV_0} = \frac{V}{V_0} = \frac{12}{12\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Now ω is given by

$$\omega = \frac{L}{\sqrt{LC}}$$

$$I = \frac{dQ}{dt} = \frac{d}{dt}[Q_0 \sin \omega t]$$

$$= \omega\, Q_0 \cos\, \omega t$$

Now $\sin \omega t = \frac{1}{\sqrt{2}}$ because $\cos \omega t = \frac{1}{\sqrt{2}}$. Also $Q_0 = CV_0$.

$$\therefore \quad \text{I} = \frac{1}{\sqrt{LC}} \times CV_0 \times \frac{1}{\sqrt{2}}$$

$$= \sqrt{\frac{C}{L}} \times \frac{V_0}{\sqrt{2}}$$

$$= \sqrt{\frac{2\times10^{-6}}{5\times10^{-3}}} \times \frac{12\sqrt{2}}{\sqrt{2}} = 0.24 \text{ A}$$

So the correct choice is (d).

96. Let I_0 be the maximum current and I be the current at time t when the energy stored in inductor becomes $\frac{1}{9}$ of the maximum energy, then

$$\frac{1}{2}LI^2 = \frac{1}{9}\times\frac{1}{2}LI_0^2 \quad \Rightarrow \quad I = \frac{I_0}{3} \Rightarrow I_0 = 3I$$

Time constant $\tau = \frac{L}{R} = \frac{100\times10^{-3}}{0.2} = \frac{1}{2}$ second

Now $I = I_0\,(1 - e^{-t/\tau})$

$\Rightarrow$ $I = 3I\,(1 - e^{-t/\tau})$

$\Rightarrow$ $e^{-t/\tau} = \frac{2}{3}$

$\Rightarrow$ $e^{t/\tau} = \frac{3}{2}$

$\Rightarrow$ $\frac{t}{\tau} = \ln\left(\frac{3}{2}\right)$

$\Rightarrow$ $t = \tau \ln\left(\frac{3}{2}\right) = \frac{1}{2}\ln\left(\frac{3}{2}\right)$ second

So the correct choice is (a).

97. Refer to Fig. 14.84. Let I be the current in the square loop.

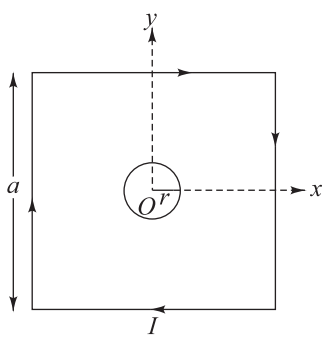


Fig. 14.84

The magnetic field at centre O due to current I in the square loop side a is [see page 13.2 of Chapter 13]

$$B = \frac{2\sqrt{2}\,\mu_0 I}{\pi a}$$

Since $r << a$, the magnetic field can be assumed to be constant throughout the inner circular loop. Therefore, magnetic flux through the circular loop is

$$\phi = BA = B \times \pi r^2$$

$$\phi = \frac{2\sqrt{2}\,\mu_0 I}{\pi a} \times \pi r^2 = \frac{2\sqrt{2}\,\mu_0 I r^2}{a}$$

By definition $\phi = MI$. Hence

$$M = \frac{2\sqrt{2}\,\mu_0 r^2}{a}$$

Thus $M \propto \frac{r^2}{a}$ which is choice (a).

98. Refer to Fig. 14.86. Let I be the current in the outer (larger) loop

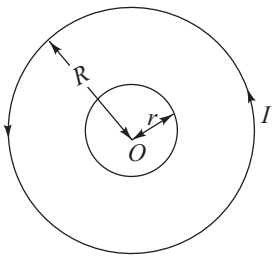


Fig. 14.85

Magnetic field at O due to current I in the outer loop is

$$B = \frac{\mu_0 I}{2R}$$

Magnetic flux through the inner loop is

$$\phi = BA = \frac{\mu_0 I}{2R} \times \pi r^2 \qquad (1)$$

Now $\phi = MI$ (2)

Comparing (1) and (2) we get

$$M = \frac{\mu_0 \pi}{2}\left(\frac{r^2}{R}\right)$$

Thus $M \propto \frac{r^2}{R}$. So the correct choice is (c).

99. Refer to Fig. 14.86. Let I be the current in the outer (larger) loop.

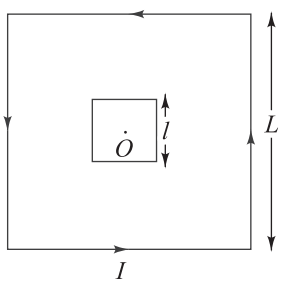


Fig. 14.86

Magnetic field at O due to current I in the outer loop is $B = \frac{2\sqrt{2}\,\mu_0 I}{\pi L}$

Magnetic flux through the inner loop is

$$\phi = BA = \frac{2\sqrt{2}\,\mu_0 I}{\pi L} \times l^2 \qquad \textbf{(1)}$$

Now $\phi = MI$ (2)

Comparing (1) and (2) we find that $M \propto \frac{l^2}{L}$. So the correct choice is (c).

100. As the bar magnet is moved towards the solenoid, the induced emf $e = -d\phi/dt$ is negative and as it is moved away from the solenoid, induced emf is positive. So the correct graph is (b).

101. Impedance $z = \sqrt{R^2 + (\omega L)^2}$ and $I_0 = \frac{V_0}{Z}$. As ω is increased, Z increases. Hence current I_0 decreases. As a result the brightness of the bulb will decrease. So the correct choice is (b).

102. $V_0 = \sqrt{2}\, V_{rms} = \sqrt{2} \times 20$ V.

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times \left(100 \times 10^{-6}\right)} = 100\ \Omega.$$

Let I_1 be the peak value of the current in the upper branch of the circuit. Then

$$I_1 = \frac{V_0}{Z_1} = \frac{V_0}{\left(x_c^2 + R_1^2\right)^{1/2}}$$

$$= \frac{20\sqrt{2}}{\left[(100)^2 + (100)^2\right]^{1/2}}$$

$$= \frac{1}{5}\text{A}$$

$\therefore$ Peak value of voltage across 100 Ω resistor is

$$V_1 = I_1 R_1 = \frac{1}{5} \times 100 = 20\text{V}$$

So the correct choice is (d).

103. $X_L = \omega L = 100 \times 0.5 = 50\,\Omega$

Let I_2 be the peak value of the current in the lower branch of the circuit. Then

$$I_2 = \frac{V_0}{Z_2} = \frac{V_0}{\left(X_L^2 + R_1^2\right)^{1/2}}$$

$$= \frac{20\sqrt{2}}{\left[(50)^2 + (50)^2\right]^{1/2}} = 0.4\text{A}$$

So the correct choice is (a).

104. To Prevent sparking, the energy stored in the induction must at least be equal to that stored in the capacitor, *i.e*.

$$\frac{1}{2}LI^2 = \frac{1}{2}CV^2$$

$$C = \frac{LI^2}{V^2} = \frac{0.1 \times (3)^2}{(300)^2} = 10 \times 10^{-6}\,\text{F} = 10\mu\text{F}$$

So the correct choice is (a).

105. Before the primary is rotated, the current through it changes from zero to i. When the primary is rotated through 180°, the current changes from i, to $-i$. Hence the current now changes by $2i$. Since the change in current is doubled, its rate of change is also doubled. Hence emf induced across the secondary is doubled and the induced current is also doubled. So the correct choice is (b).

106. From Fleming's left hand rule, the free electrons in rod XY will move from X to Y. Hence and X acquires a positive charge (due to loss of electrons) and end Y acquires an equal negative charge (due to gain of electrons). Therefore, the correct choice is (a).

107. The graph shown in Fig. 14.69 is a sine curve. Hence

$$\phi = \phi_0 \sin \omega t = \phi_0 \sin\left(\frac{2\pi t}{T}\right)$$

Induced emf is

$$\text{e} = -\frac{d\phi}{dt} = -\phi_0 \times \frac{2\pi}{T}\cos\left(\frac{2\pi t}{T}\right) = -e_0 \cos\left(\frac{2\pi t}{T}\right)$$

It is clear that the correct choice is (*d*).

108. The inductive reactances are

$X_1 = \omega L_1$ and $X_2 = \omega L_2$

If L_{eq} is the equivalent inductance, then $X_{eq} = \omega L_{eq}$. since the reactances X_1 and X_2 are in parallel,

$$\frac{1}{X_{eq}} = \frac{1}{X_1} + \frac{1}{X_2}$$

$$\Rightarrow \quad \frac{1}{\omega L_{eq}} = \frac{1}{\omega L_1} + \frac{1}{\omega L_2}$$

$$\Rightarrow \quad \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \Rightarrow L_{\text{eq}} = \frac{L_1 L_2}{L_1 + L_2}$$

So the correct choice is (*d*).

109. In the steady state, the current through L is

$$I = \frac{V}{R_1} = \frac{12\,\text{V}}{6\Omega} = 2\text{A}$$

$\therefore$ Energey stored in $L = \frac{1}{2}LI^2$

$$= \frac{1}{2} \times 2 \times (2)^2 = 4\text{J}$$

So the correct choice is (a).

110. $i = \dfrac{E}{R}\left(1 - e^{-t/\tau}\right)$ (1)

where $\tau = \dfrac{L}{R}$

$$\therefore \frac{di}{dt} = \frac{E}{R} \times \left(-\frac{1}{\tau}\right) e^{-t/\tau} = -\frac{E}{L} e^{-t/\tau}$$

Induced emf is

$$e = -L\frac{di}{dt} = E e^{-t/\tau} \quad (2)$$

From (2), $e^{-t/\tau} = \dfrac{e}{E}$. Using this in (1), we have

$$i = \frac{E}{R}\left(1 - \frac{e}{E}\right) = \frac{E}{R} - \frac{e}{R}$$

or $e = E - iR$ (3)

Equation (3) shows that the graph of e against i is a straight line with negative slope and positive intercept. So the correct graph is (d).

111. Motional emf induced between the ends of PQ is

$e = Blv = 2 \times 0.5 \times (4 \times 10^{-2}) = 4 \times 10^{-2}$ V

From Fleming's right hand rule, the direction of the induced current in PQ is from Q to P. The circuit can be assumed to have a cell of emf $e = 4 \times 10^{-2}$ V and internal resistance $r = 0.8\Omega$ connected as shown in Fig. 14.87(a).

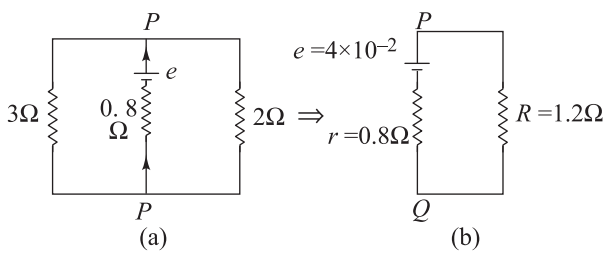


Fig. 14.87

Resistances 3Ω and 2Ω are in parallel. Their combined resistance is

$$R = \frac{3 \times 2}{3 + 2} = 1.2\,\Omega$$

The circuit can be redrawn as shown in Fig. 14.87 (b). Current in the circuit is

$$\mathrm{I} = \frac{e}{R + r} = \frac{4 \times 10^{-2}}{1.2 \times 0.8} = 2 \times 10^{-2}\mathrm{A}$$

Therefore, force acting on wire PQ is

$F = BIL = 2 \times (2 \times 10^{-2}) \times 0.5 = 2 \times 10^{-2}$ N

So the correct choice is (b).

112. The emf inducted between OQ or OP is

$$e = \frac{1}{2} B\omega l^2$$

$$= \frac{1}{2} \times 4 \times 20 \times (0.5)^2 = 10\,\mathrm{V}$$

The circuit can be redrawn as shown in Fig. 14.88 (because OP and OQ are in parallel).

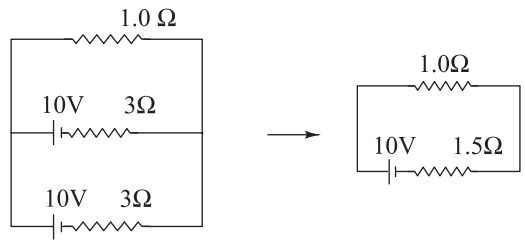


Fig. 14.88

$$\therefore\ I = \frac{10}{1.5 + 1.0} = 4\mathrm{A}$$

So the correct choice is (*a*).

113. $V = 220 \sin(50\,\pi t) \cos(50\,\pi t)$

$= 110 \times [2 \sin(50\,\pi t) \cos(50\,\pi t)]$

or $V = 110 \sin(100\,\pi t)$ (1)

Peak value of voltage is $V_0 = 110$ V. The r.m.s value is

$V_{\mathrm{rms}} = \dfrac{110}{\sqrt{2}} = 77.8 \simeq 78$ V, which is choice (a).

114. From Eq. (1) above,

$$\omega = 100\,\pi$$

$$\Rightarrow \quad 2\pi v = 100\,\pi$$

$$\Rightarrow \quad v = 50\ \mathrm{Hz}$$

So the correct choice is (b).

115. Given $I = 5 \sin(50\,\pi t)$ (1)

The emf induced in the second solenoid is

$$E = -M\frac{dI}{dt}$$

$$= -M \times 5 \times 50\pi \times \cos(50\,\pi t)$$

$$= -E_0 \cos(50\,\pi t)$$

$$E = E_0 \sin\left(50\,\pi t - \frac{\pi}{2}\right) \quad (2)$$

Maximum value of E is

$$E_0 = M \times 5 \times 50\pi$$

$$= (10 \times 10^{-3}) \times 5 \times 50\pi = 2.5\pi \text{ volt}$$

Hence the correct choice is (a).

116. From Eqs. (1) and (2) it follows that the phase difference between I and E is $\frac{\pi}{2}$. So the correct choice is (c).

117. I_L lags behind V by $\pi/2$ and I_c leads V by $\pi/2$. Hence phase difference between I_L and I_c is 180º, *i.e.* they are in opposite phases. So the current drawn from the source is $I = I_L - I_c$, which is choice (b).

118. In Fig. 11.77, L and C are in series. Hence $I_L = I_c$ but they differ in phase by 180°. So $I = I_L - I_C = 0$ which is choice (a).

SECTION 2

Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage.

Passage I

Two long parallel horizontal rails, distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide along the rails without friction (see Fig. 14.89). There is a uniform magnetic field B normal to the plane of the paper and directed into the paper. A variable force F is applied to the rod MN such that as the rod moves, constant current flows through R.

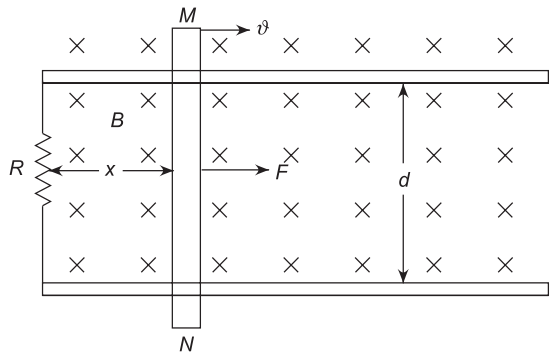


Fig. 14.89

1. The magnitude of the emf induced in the loop is

   (a) $Bvd\left(\frac{2\lambda x}{R}\right)$ (b) $Bvd\left(\frac{R}{2\lambda x}\right)$

   (c) Bvd (d) $\frac{1}{2}Bvd$

2. The current in the loop is

   (a) $\frac{Bvd}{R}$ (b) $\frac{Bvd}{2\lambda x}$

   (c) $\frac{2Bvd}{(R+2\lambda x)}$ (d) $\frac{Bvd}{(R+2\lambda x)}$

3. The velocity of the rod is

   (a) $\frac{B^2d^2}{2\lambda m}\left(1+\frac{2\lambda x}{R}\right)$

   (b) $\frac{B^2d^2}{R}\left(1-\frac{R}{2\lambda x}\right)$

   (c) $\frac{B^2d^2}{2\lambda m}\log_e\left(1-\frac{R}{2\lambda x}\right)$

   (d) $\frac{B^2d^2}{2\lambda m}\log_e\left(1+\frac{2\lambda x}{R}\right)$

Solutions

1. Let the distance from R to MN be x. Then the area of the loop between MN and R is xd and the magnetic flux linked with the loop is Bxd. As the rod moves, the emf induced in the loop is given by

   $$|e| = \frac{d}{dt}(Bxd) = Bd\,\frac{dx}{dt} = Bvd$$

   where v = velocity of MN.

   So the correct choice is (c).

2. The total resistance of the loop between R and MN is $R + 2\,\lambda x$. The current in the loop is given by

   $$I = \frac{|e|}{R+2\lambda x} = \frac{Bvd}{R+2\lambda x}$$

   The correct choice is (d).

3. Force acting on the rod,

   $$F = IBd = \frac{B^2d^2v}{R+2\lambda x}$$

   $$\therefore \quad m\frac{dv}{dt} = \frac{B^2d^2}{R+2\lambda x}\cdot\frac{dx}{dt}$$

or $$dv = \frac{B^2 d^2}{m} \times \frac{dx}{(R + 2\lambda x)}$$

Integrating, we have

$$\int_0^v dv = \frac{B^2 d^2}{m} \int_0^x \frac{dx}{(R + 2\lambda x)}$$

or $$v = \frac{B^2 d^2}{2\lambda m} \log_e \left(\frac{R + 2\lambda x}{R} \right)$$

Hence the correct choice is (d).

Questions 4 to 6 are based on the following passage.

Passage II

A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails without friction. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m, tied to the other end of the string, hangs vertically. A constant magnetic field B exists perpendicular to the table. The system is released from rest. (Fig. 14.90)

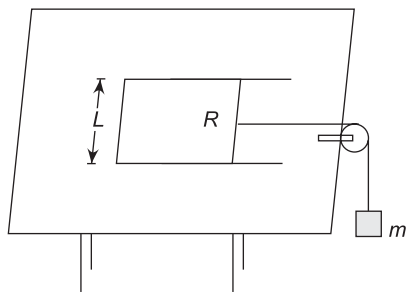


Fig. 14.90

4. The acceleration of the mass m moving in the downward direction is

   (a) g (b) $\frac{B^2 L^2 v}{mR}$

   (c) $\left(g - \frac{B^2 L^2 v}{mR} \right)$ (d) $\left(g + \frac{B^2 L^2 v}{mR} \right)$

5. The terminal velocity acquired by the rod is

   (a) g (b) $\sqrt{gR}$

   (c) $\frac{\sqrt{mgR}}{BL}$ (d) $\frac{mgR}{B^2 L^2}$

6. The acceleration of mass m when the velocity of the rod is half the terminal velocity is

   (a) g (b) $\frac{g}{2}$

   (c) $\frac{g}{3}$ (d) $\frac{g}{4}$

Solutions

4. Refer to Fig. 14.91. Let v be the velocity of the rod along the positive x-direction at an instant of time and let the magnetic field B act perpendicular to the table along the positive y-direction. The emf induced in the rod is $e = BLv$. Therefore, the induced current is

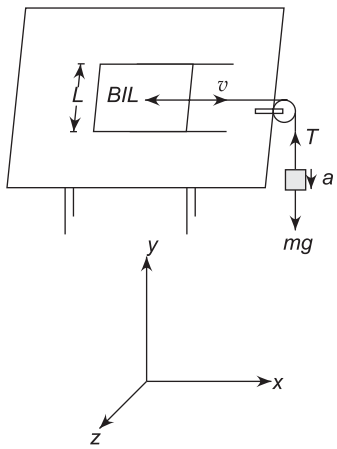


Fig. 14.91

$$I = \frac{e}{R} = \frac{BLv}{R} \quad (1)$$

The rod of length L carrying a current I in magnetic field will experience a force

$$F = BIL \quad (2)$$

along the negative x-direction. Since the rod is massless, this force will also be equal to the tension T in the string acting along the positive x-direction, i.e. $T = F = BIL$.

Let a be the acceleration of mass m moving in the downward direction, then

ma = net force acting on $m = mg - T = mg - F$

or $$a = g - \frac{F}{m} \quad (3)$$

Using (1) and (2) in (3), we have

$$a = g - \frac{BIL}{m} = g - \frac{B \times BL^2 v}{mR}$$

$$= g - \frac{B^2 L^2 v}{mR} \quad (4)$$

So the correct choice is (c).

5. The rod will acquire terminal velocity v_t when $a = 0$. Putting $a = 0$ and $v = v_t$ in Eq. (4) we have

$$0 = g - \frac{B^2 L^2 v_t}{mR} \quad \text{or } v_t = \frac{mgR}{B^2 L^2}$$

The correct choice is (d).

14. When the velocity of the rod is half the terminal velocity, i.e. when

$$v = \frac{v_t}{2} = \frac{mgR}{2B^2L^2},$$

then from Eq. (4), we have

$$a = g - \frac{B^2L^2v_t/2}{mR}$$

$$= g - \frac{B^2L^2}{2mR} \times \frac{mgR}{B^2L^2} = g - \frac{g}{2} = \frac{g}{2}$$

Thus the correct choice is (b).

Questions 7 to 10 are based on the following passage.

Passage III

An infinitesimally small bar magnet of dipole moment **M** is pointing and moving with a speed v in the x-direction. A small closed circular conducting loop of radius a and negligible self inductance lies in the y-z plane with its centre at $x = 0$, and its axis coinciding with the x-axis.

7. The magnitude of magnetic field at a distance x on the axis of the short bar manget is

(a) $\frac{\mu_0 M}{2\pi x}$ (b) $\frac{\mu_0 M}{2\pi x^2}$

(c) $\frac{\mu_0 M}{2\pi x^3}$ (d) $\frac{\mu_0 M}{2\pi x^4}$

8. If $x = 2a$, the magnetic flux through the loop is

(a) $\mu_0 M$ (b) $\frac{\mu_0 M}{2}$

(c) $\frac{\mu_0 M}{4a}$ (d) $\frac{\mu_0 M}{16a}$

9. If $x = 2a$, the emf induced in the loop is

(a) $\frac{3}{16}\frac{\mu_0 Mv}{a^2}$ (b) $\frac{3}{32}\frac{\mu_0 Mv}{a^2}$

(c) $\frac{1}{8}\frac{\mu_0 Mv}{a^2}$ (d) $\frac{1}{16}\frac{\mu_0 Mv}{a^2}$

10. If $x = 2a$, the magnetic moment of the loop is

(a) $\frac{3\pi\mu_0 Mv}{32R}$ (b) $\frac{3\pi\mu_0 Mv}{8R}$

(c) $\frac{\pi\mu_0 Mv}{2R}$ (d) $\frac{3\pi\mu_0 Mv}{4R}$

Solutions

7. Refer to Fig. 14.92. The magnetic field at a distance x on the axis of a magnet of length $2l$ and dipole moment M is given by

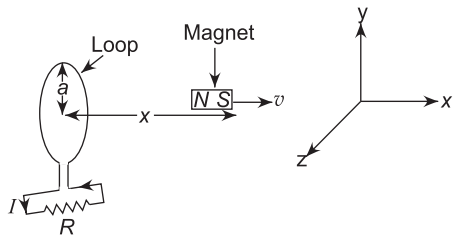


Fig. 14.92

$$B = \frac{\mu_0}{2\pi} \cdot \frac{Mx}{(x^2 - l^2)^2}$$

Since $x >> l$, we have

$$B = \frac{\mu_0 M}{2\pi x^3}$$

So the correct choice is (c).

8. Due to B, the flux through the loop is

$$\phi = BA = B(\pi a^2) = \frac{\mu_0 M}{2\pi x^3} \times \pi a^2 = \frac{\mu_0 Ma^2}{2x^3}$$

If $x = 2a$, we find that the correct choice is (d).

9. Induced emf in the loop is

$$e = -\frac{d\phi}{dt} = -\frac{dx}{dt}\frac{d\phi}{dx} = -v\frac{d\phi}{dx}$$

$$= -\frac{\mu_0 Ma^2 v}{2}\frac{d}{dx}\left(\frac{1}{x^3}\right) = \frac{3}{2}\frac{\mu_0 Ma^2 v}{x^4}$$

Putting $x = 2a$, we get $e = \frac{3\mu_0}{32}\frac{Mv}{a^2}$, which is choice (b).

10. Induced current in the loop is

$$I = \frac{e}{R} = \frac{3}{2}\frac{\mu_0 Ma^2 v}{x^4 R}$$

Magnetic moment of the loop is

$M_0 = I \times$ area enclosed by the loop $= I(\pi a^2)$

$$= \frac{3\pi}{2}\frac{\mu_0 Ma^4 v}{x^4 R}$$

Putting $x = 2a$, we find that the correct choice is (a).

Questions 11 to 14 are based on the following passage.

Passage IV

Two resistances of 10 Ω and 20 Ω and an ideal inductor of inductance 5 H are connected to a battery of 2 V through a key K as shown in Fig. 14.93. If at $t = 0$, K is inserted.

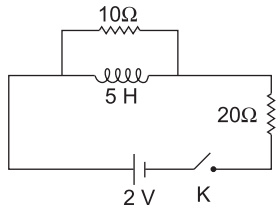


Fig. 14.93

11. The initial current through the battery is
    (a) $\frac{1}{15}$ A (b) $\frac{2}{15}$ A
    (c) 0.2 A (d) 0.4 A
12. The initial potential drop across the inductor is
    (a) $\frac{1}{6}$ V (b) $\frac{1}{3}$ V
    (c) $\frac{2}{3}$ V (d) $\frac{4}{3}$ V
13. The final current through the 10 Ω resistor is
    (a) $\frac{1}{15}$ A (b) 0.2 A
    (c) 0.1 A (d) zero
14. The final current through the 20 Ω resistor is
    (a) 0.1 A (b) 0.2 A
    (c) 0.3 A (d) zero

Solutions

11. As soon as K is inserted, i.e. $t = 0$, dI/dt is maximum, which implies that the opposing emf $L\ dI/dt$ is high and the inductor will be have as a very large resistor. So the current will flow through both the resistances only. The current through the battery $I(0)$ at $t = 0$ is
    $$I(0) = \frac{2.0}{10 + 20} = \frac{1}{15} \text{ A}$$
    The correct choice is (a).
12. Since the 10 Ω resistor and inductor are in parallel, the potential drop across the inductor is the same as that across the 10 Ω resistor. Hence, the initial potential drop is
    $$I(0) \times 10 = \frac{10}{15} \text{ V}$$
    So the correct choice is (c).
13. When the current has attained a constant value, the opposing emf across the inductor is zero. The inductor would behave as a short and the whole current will pass through it. The final current through the 10 Ω resistor is, therefore, zero. The correct choice is (d).
14. Since the inductor behaves as a short or zero resistance, the total resistance of the circuit is also 20 Ω (for 10 Ω is in parallel with the inductor). The final current in the circuit is therefore, 2.0/20 = 0.1 A. Hence the correct choice is (a).

Questions 15 to 17 are based on the following passage.

Passage V

An *LCR* series circuit with 100 Ω resistance is connected to an a.c. source of 200 V and angular frequency 300 rad/sec. When only the capacitance is removed, the current leads the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°.

15. The impedance of the *LCR* circuit is
    (a) 100 Ω (b) $100\sqrt{2}$ Ω
    (c) 200 Ω (b) $200\sqrt{2}$ Ω
16. The current in the circuit is
    (a) $\sqrt{2}$ A (b) 2 A
    (c) $2\sqrt{2}$ A (d) 1 A
17. The power dissipated in the circuit is
    (a) 200 W (b) 400 W
    (c) 800 W (d) 100 W

Solutions

15. When capacitance is removed, the circuit contains only inductance and resistance. Phase difference θ between the current and voltage is then given by
    $$\tan\theta = \frac{\omega L}{R} \text{ or } \omega L = R\tan\theta = 100\tan 60°$$
    When the circuit contains only capacitance and resistance, the phase difference between the voltage and current is given by
    $$\tan\phi = \frac{1}{RC\omega}$$
    $$\therefore \quad \frac{1}{C\omega} = R\tan\phi = 100\tan 60°$$
    The impedance of the *LCR* circuit is given by
    $$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$
    $$= \sqrt{R^2 + (100\tan 60° - 100\tan 60°)^2}$$
    $$= R = 100\ \Omega, \text{ which is choice (a).}$$
16. The current is given by
    $$I = \frac{V}{R} = \frac{200}{100} = 2 \text{ A}$$
    The correct choice is (b).

17. The power dissipated in the circuit is
$P = I^2R = 4 \times 100 = 400$ W
So the correct choice is (b).

Questions 18 to 20 are based on the following passage.

Passage VI

An *LCR* circuit consists of an inductor, a capacitor and a resistor driven by a battery and connected by two switches S_1 and S_2 as shown in Fig. 14.94.

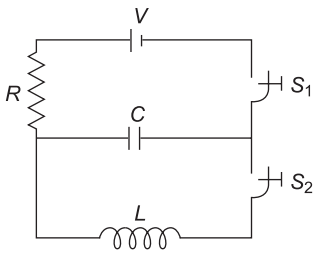


Fig. 14.94

18. At time $t = 0$ switch S_1 is closed and S_2 is left open. The maximum charge the capacitor plate can hold is q_0 and τ is the time constant of the *RC* circuit. Then
(a) at time $t = \tau$, the charge on the capacitor plates is $q = q_0/2$.
(b) at $t = 2\tau$, $q = q_0 (1 - e^{-2})$
(c) at $t = 2\tau$, $q = q_0 (1 - e^{-1})$
(d) work done by the battery is half the energy dissipated in the resistor.

19. At time $t = 0$ when the charge on the capacitor plates is q, switch S_1 is opened and S_2 is closed.
The maximum charge the capacitor can hold is q_0. Choose the correct statement from the following.
(a) $q = q_0 \cos\left(\frac{t}{\sqrt{LC}} + \frac{\pi}{2}\right)$
(b) $q = q_0 \cos\left(\frac{t}{\sqrt{LC}} - \frac{\pi}{2}\right)$
(c) $q = -LC\,\frac{d^2q}{dt^2}$
(d) $q = -\frac{1}{\sqrt{LC}}\frac{d^2q}{dt^2}$

20. At an instant of time $t = 0$ when the capacitor has been charged to a voltage V, switch S_1 is opened and S_2 is closed. Then
(a) at $t = 0$, the energy is stored in the magnetic field of the inductor.
(b) at $t > 0$, there is no exchange of energy between the capacitor and the inductor.
(c) at $t > 0$, the current in the circuit flows only in one direction.
(d) the maximum value of the current in the circuit is $\sqrt{\frac{C}{L}}V$.

Solutions

18. In an *RC* circuit, the charge on the capacitor plates at a time t is given by
$q = q_0 (1 - e^{-t/\tau})$
where $\tau = RC$ is the time constant. At $t = 2\tau$, we have
$q = q_0(1 - e^{-2})$
Hence the correct choice is (b).

19. When S_2 is closed and S_1 is open, the charge oscillates in the *LC* circuit at an angular frequency given by
$$\omega = \frac{1}{\sqrt{LC}} \qquad (1)$$
Now $q \neq 0$ at $t = 0$. Hence choices (a) and (b) are wrong. The charge q varies with time t as
$$q = q_0 \cos(\omega t + \phi) \qquad (2)$$
where ϕ is not equal to $\pi/2$. Differentiating Eq. (2) twice with respect to t, we get
$$\frac{d^2q}{dt^2} = -\omega^2 q_0 \cos(\omega t + \phi) = -\omega^2 q$$
$$q = -\frac{1}{\omega^2}\frac{d^2q}{dt^2} = -LC\,\frac{d^2q}{dt^2} \qquad \text{[use Eq. (1)]}$$
Hence the correct choice is (c).

20. At $t = 0$, the energy is stored in the electric field in the space between the capacitor plates. As time passes (i.e. at $t > 0$), there is an exchange of energy between the capacitor and the inductor. The charge q varies with time t as
$q = q_0 \cos \omega t$, where $\omega = \frac{1}{\sqrt{LC}}$
The current in the circuit is given by
$$I = \frac{dq}{dt} = \frac{d}{dt}(q_0 \cos \omega t) = -\omega q_0 \sin \omega t$$
which is alternating and not unidirectional. The maximum value of current is
$$I_{max} = \omega q_0 = \frac{1}{\sqrt{LC}} \times CV \qquad (q_0 = CV)$$
$$= \sqrt{\frac{C}{L}}\,V$$
Hence the correct choice is (d).

SECTION 3

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by statement-2 (Reason). Each question has the following four options out of which only *one* option is correct.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

1. Statement-1

No induced emf is developed across the ends of a conductor if it is moved parallel to a magnetic field.

Statement-2

No force acts on the free electrons of the conductor.

2. Statement-1

No current is induced in a metal loop if it is rotated in an electric field.

Statement-2

The electric flux through the loop does not change with time.

3. Statement-1

A rectangular loop and a circular loop are moved with a constant velocity from a region of magnetic field out into a field-free region. The field is normal to the loops. Then a constant emf will be induced in the circular loop and a time-varying emf will be induced in the rectangular loop.

Statement-2

The induced emf is constant if the magnetic flux changes at a constant rate.

4. Statement-1

A magnetised iron bar is dropped vertically through a hollow region of a thick cylindrical shell made of copper. The bar will fall with an acceleration less than *g*, the acceleration due to gravity.

Statement-2

The emf induced in the bar causes a retarding force to act on the falling bar.

5. Statement-1

A coil is connected in series with a bulb and this combination is connected to a d.c. source. If an iron core is inserted in the coil, the brightness of the bulb will increase.

Statement-2

The reactance offered by the coil to d.c. current is zero.

6. Statement-1

A coil is connected in series with a bulb and this combination is connected to an a.c. source. If an iron core is inserted in the coil, the brightness of the bulb will be reduced.

Statement-2

When an iron core is inserted in the coil, its inductance decreases.

7. Statement-1

A variable capacitor is connected in series with a bulb and this combination is connected to an a.c. source. If the capacitance of the variable capacitor is decreased, the brightness of the bulb is reduced.

Statement-2

The reactance of the capacitor increases if the capacitance is reduced.



Solutions

1. The correct choice is (a). Let $\vec{v}$ be the velocity of the conductor in a magnetic field $\vec{B}$. Since the free electrons in the conductor are moving with it, force $\vec{F} = e(\vec{v} \times \vec{B})$ is zero because $\vec{v}$ is parallel to $\vec{B}$. Consequently, no induced emf is developed between the ends of the conductor.

2. The correct choice is (b). A current is induced in a loop only if magnetic flux linked with the coil changes.

3. The correct choice is (d). The induced emf is constant in the case of rectangular coil because the rate of change of area is constant. But in the case of the circular coil, the rate of change of area (and hence the rate of change of magnetic flux) keeps varying as the loop is moving towards the field-free region.

4. The correct choice is (a). The retarding force is caused by the eddy currents and according to Lenz's law, the induced emf must oppose the cause. The cause is the falling bar.

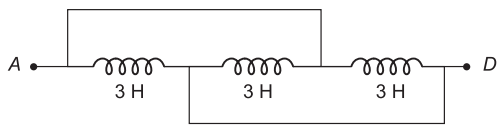
5. The correct choice is (d). A coil offers no reactance to d.c. currents. Hence there will be no change in the brightness of the bulb when an iron core is inserted in the coil.

6. The correct choice is (c). If an iron core is inserted in the coil, its inductance L increases. Hence its reactance ωL increases, causing a decrease in the current in the circuit. As a result, the brightness of the bulb will reduce.

7. The correct choice is (a). The reactance of a capacitor is $1/\omega C$. Hence if C is decreased, the reactance will increase and as a result the current in the circuit is decreased causing a decrease in the brightness of the bulb.

4 SECTION — Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. The inductance between points A and D is



(a) 3.66 H (b) 9 H
(c) 0.66 H (d) 1 H **[2002]**

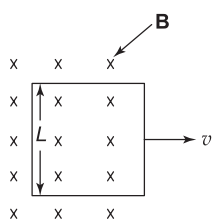
2. The power factor of an AC circuit having a resistance R and inductance L (connected in series) is (ω is the angular frequency of the AC source)

(a) $\frac{R}{\omega L}$ (b) $\frac{R}{\left[R^2+\omega^2L^2\right]^{1/2}}$

(c) $\frac{\omega L}{R}$ (d) $\frac{R}{\left[R^2-\omega^2L^2\right]^{1/2}}$ **[2002]**

3. In a transformer, the number of turns in the primary is 140 and that in the secondary is 280. If the current in the primary is 4A, then the current in the secondary is
(a) 4A (b) 2A
(c) 6A (d) 10A **[2002]**

4. A conducting square loop of side L and resistance R moves in a plane with a uniform velocity v perpendicular to one of its sides. A uniform magnetic field **B** pointing perpendicular and into the plane of the loop exists everywhere with half the loop outside the field. The induced emf is



(a) zero (b) RvB
(c) $\frac{vBL}{R}$ (d) vBL **[2002]**

5. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon
(a) the rates at which the currents are changed in the two coils
(b) the relative position and orientation of the two coils
(c) the material of the wires of the coils
(d) the currents in the two coils. **[2003]**

6. When the current in a coil changes from 2A to – 2A in 0.05s, an emf of 8V is induced in the coil. The self inductance of the coil is
(a) 0.2H (b) 0.4H
(c) 0.8H (d) 0.1H **[2003]**

7. In an oscillating LC circuit, the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic fields is

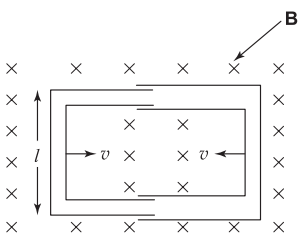
(a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$

(c) $\frac{Q}{\sqrt{2}}$ (d) Q **[2003]**

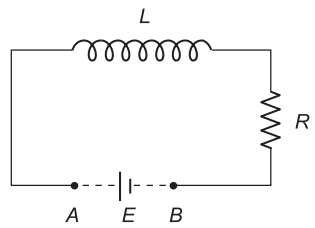
8. The core of a transformer is laminated so as to
(a) reduce energy loss due to eddy currents
(b) make it light weight
(c) make it robust and strong
(d) increase the secondary voltage. **[2003]**

9. Alternating current cannot be measured by DC ammeter because
(a) AC cannot pass through DC ammeter
(b) AC changes direction
(c) the average value of current for complete cycle is zero
(d) DC ammeter will get damaged. **[2004]**

10. In an *LCR* series circuit, the voltage across *L*, *C* and *R* is 50V each. The voltage across the *LC* combination will be
    (a) 50 V (b) $50\sqrt{2}$ V
    (c) 100 V (d) zero **[2004]**
11. A coil having *n* turns and resistance *R* is connected to a galvanometer of resistance 4*R*. The combination is moved in a region of magnetic field. If the magnetic flux through each turn of the coil changes from ϕ_1 to ϕ_2 in time *t*, the induced current in the circuit is
    (a) $\frac{(\phi_2 - \phi_1)}{5Rnt}$ (b) $-\frac{n(\phi_2 - \phi_1)}{5Rt}$
    (c) $-\frac{(\phi_2 - \phi_1)}{Rnt}$ (d) $-\frac{n(\phi_2 - \phi_1)}{Rt}$ **[2004]**
12. In a uniform magnetic field *B*, a wire in the form of a semicircle of radius *r* rotates about the diameter of the circle with angular frequency ω. If the total resistance of the circuit is *R*, the mean power generated per period of rotation is
    (a) $\frac{B\pi r^2\omega}{2R}$ (b) $\frac{(B\pi r^2\omega)^2}{8R}$
    (c) $\frac{(B\pi r^2\omega)^2}{2R}$ (d) $\frac{(B\pi r\omega^2)^2}{8R}$ **[2004]**
13. In an *LCR* circuit, the capacitance is changed from *C* to 2*C*. For the resonant frequency to remain unchanged, the inductance should be changed from *L* to
    (a) 4*L* (b) 2*L*
    (c) $\frac{L}{2}$ (d) $\frac{L}{4}$ **[2004]**
14. A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 rad s^{-1}. If the horizontal component of earth's magnetic field is 0.2×10^{-4} T, the emf developed between the two ends of the conductor is
    (a) 5 μV (b) 50 μV
    (c) 5 mV (d) 50 mV **[2004]**
15. One conducting U-tube can slide inside another as shown in the Figure, always maintaining electrical contact between them. A uniform magnetic field **B** is perpendicular to the plane of the figure. Each tube moves towards each other with a velocity *v*. If *l* is the width of each tube, the emf induced in circuit will be



    (a) *Blv* (b) – *Blv*
    (c) zero (d) 2 *Blv* **[2005]**
16. A coil of inductance 300 mH and resistance 2Ω is connected to a source of voltage 2V. The current reaches half of its steady state value in
    (a) 0.05s (b) 0.1s
    (c) 0.15s (d) 0.3s **[2005]**
17. The self inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitor of capacitance
    (a) 4 μF (b) 8 μF
    (c) 1 μF (d) 2 μF **[2005]**
18. A circuit has a resistance of 12Ω and an impedance of 15Ω. The power factor of the circuit is
    (a) 0.8 (b) 0.4
    (c) 1.25 (d) 0.125 **[2005]**
19. The phase difference between the alternating current and emf is π/2. Which of the following cannot be the constituent of the circuit?
    (a) *C* alone (b) *R*, *L*
    (c) *L*, *C* (d) *L* alone **[2005]**
20. The flux linked with a coil at any instant t is given by
    $$\phi = 10t^2 - 50t + 250$$
    The induced emf at $t = 3$ s is
    (a) 10 V (b) 190 V
    (c) – 190 V (d) – 10 V **[2006]**
21. In a series resonant *LCR* circuit, the voltage across *R* is 100 volts and $R = 1$ kΩ with $C = 2$ μF. The resonant frequency ω is 200 rad/s. At resonance the voltage across *L* is
    (a) 250 V (b) 4×10^{-3} V
    (c) 2.5×10^{-2} V (d) 40 V **[2006]**
22. In an AC generator, a coil with *N* turns, all of the same area *A* and total resistance *R*, rotates with frequency ω in a magnetic field *B*. The maximum value of emf generated in the coil is
    (a) *NABR* (b) ωNAB
    (c) $\omega NABR$ (d) *NAB* **[2006]**
23. An inductor ($L = 100$ mH), a resistor ($R = 100$ Ω) and a battery ($E = 100$ V) are initially connected in series as shown in the Figure. After a long time the battery is disconnected after short circuiting the points *A* and *B*. The current in the circuit 1 ms after the short circuit is



(a) 0.1 A (b) 1 A

(c) $1/e$ A (d) eA **[2006]**

24. In an a.c. circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by

(a) $P = \frac{E_0 I_0}{\sqrt{2}}$ (d) P = zero

(c) $P = \frac{E_0 I_0}{2}$ (d) $P = \sqrt{2}\ E_0 I_0$ **[2007]**

25. An ideal coil of 10 H is connected in series with a resistance of 5 Ω and a battery of 5V. 2 seconds after the connection is made, the current flowing in amperes in the circuit is

(a) $(1 - e)$ (b) e

(c) e^{-1} (d) $(1 - e^{-1})$ **[2007]**

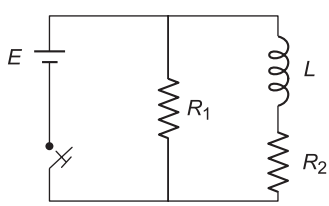
26. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A = 10\ \text{cm}^2$ and length = 20 cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is ($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})

(a) $2.4\ \pi \times 10^{-4}$ H (b) $2.4\ \pi \times 10^{-5}$ H

(c) $4.8\ \pi \times 10^{-4}$ H (d) $4.8\ \pi \times 10^{-5}$ H

[2008]

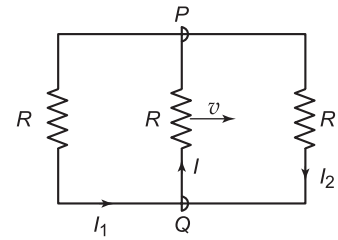
27. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2\ \Omega$ and $R_2 = 2\ \Omega$ are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop (in volt) across L as a function of time t is



(a) $6\ [1 - e^{-t/0.2}]$ (b) $12\ e^{-5t}$

(c) $6\ e^{-5t}$ (d) $\frac{12}{t}\ e^{-3t}$ **[2009]**

28. A rectangular loop has a sliding connector PQ of length l and resistance R and it is moving with a speed v as shown in the figure. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



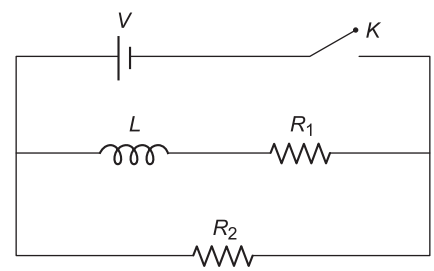
(a) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$

(b) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$

(c) $I_1 = I_2 = I = \frac{Blv}{R}$

(d) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$ **[2010]**

29. In the circuit shown in the figure, the key K is closed at $t = 0$. The current through the battery is



(a) $\frac{V R_1 R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(b) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = \infty$

(c) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V R_1 R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$

(d) $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$ **[2010]**

30. In a series LCR circuit $R = 200\ \Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind

the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the *LCR* circuit is

(a) 242 W (b) 305 W

(c) 210 W (d) zero W **[2010]**

31. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb is resistance R (inductance zero). When ω is increased

(a) the bulb glows dimmer

(b) the bulb glows brighter

(c) total impedance of the circuit is unchanged

(d) total impedance of the circuit increases **[2010]**

32. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is

(a) $\pi\sqrt{LC}$ (b) $\frac{\pi}{4}\sqrt{LC}$

(c) $2\pi\sqrt{LC}$ (d) $\sqrt{LC}$ **[2011]**

33. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} $NA^{-1}m^{-1}$ due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is

(a) 1 mV (b) 0.75 mV

(c) 0.50 mV (d) 0.15 mV **[2011]**

34. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to

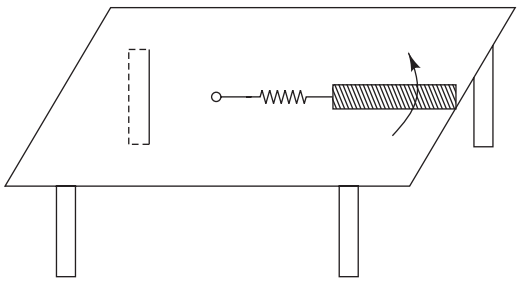
(a) development of air current when thee plate is placed

(b) induction of electrical charge on the plate.

(c) shielding of magnetic line of force as aluminium is a paramagnetic material.

(d) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping. **[2012]**

35. A metallic rod of length 'ℓ' is tied to a string of length 2ℓ and made to rotate with angular speed ω on a horizontal table ith one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is



(a) $\frac{3B\omega\ell^2}{2}$ (b) $2B\omega\ell^2$

(c) $\frac{5B\omega\ell^2}{2}$ (d) $B\omega\ell^2$ **[2013]**

36. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. the distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is

(a) 6×10^{-11} weber (b) 3.3×10^{-11} weber

(c) 6.6×10^{-9} weber (d) 9.1×10^{-11} weber

[2013]

37. In an a.c. circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The average power consumption in the circuit is

(a) $P = \frac{E_0 I_0}{\sqrt{2}}$ (b) $P =$ zero

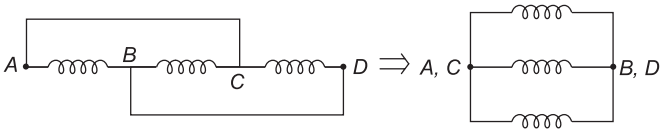
(c) $P = \frac{E_0 I_0}{2}$ (d) $P = \sqrt{2}E_0 I_0$ **[2014]**

Answers

1. (d)	2. (b)	3. (b)	4. (d)
5. (b)	6. (d)	7. (c)	8. (a)
9. (c)	10. (d)	11. (b)	12. (b)
13. (c)	14. (b)	15. (d)	16. (b)
17. (c)	18. (a)	19. (b)	20. (d)
21. (a)	22. (b)	23. (c)	24. (b)
25. (d)	26. (a)	27. (b)	28. (b)
29. (b)	30. (a)	31. (b)	32. (b)
33. (d)	34. (d)	35. (c)	36. (d)
37. (b)			

Solutions

1. The three inductors are in parallel as shown in the Figure.



$\therefore$ Equivalent inductance between A and D is

$$\frac{1}{L_{eq}} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = 1 \Rightarrow L_{eq} = 1 \text{ H}$$

2. Power factor $= \dfrac{R}{\left[R^2+\omega^2L^2\right]^{1/2}} = \dfrac{\text{resistance}}{\text{impedance}}$

3. Assuming that the transformer is ideal, i.e. there is no power loss, output power = input power

$\therefore E_p I_p = E_s I_s$. Also $\dfrac{E_p}{E_s} = \dfrac{N_p}{N_s}$

Therefore $\dfrac{I_p}{I_s} = \dfrac{E_s}{E_p} = \dfrac{N_s}{N_p} = \dfrac{280}{140} = 2$

$\Rightarrow I_s = \dfrac{I_p}{2} = \dfrac{4}{2} = 2\text{A}$

4. The induced emf is due to change in magnetic flux due to change in area and is given by $e = vBL$.
5. The correct choice is (*b*).
6. Induced emf is

$$e = -L\frac{dI}{dt}$$

Change in current dI = final current – initial current

$= -2\text{A} - 2\text{A} = -4\text{A}$

Also $dt = 0.05$ s.

$$8 = -L \times \frac{-4}{0.05} \Rightarrow L = 0.1 \text{ H}$$

7. The maximum energy stored in the electric field between capacitor plates is

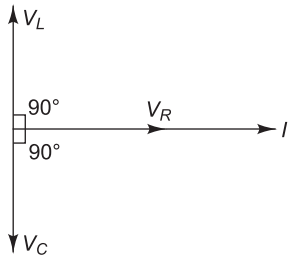
$$(U_e)_{max} = \frac{Q^2}{2C} \quad (\because Q = \text{maximum charge})$$

At this time, the energy stored in the magnetic field of the inductor is zero. Let q be the charge on the capacitor when the energy is equally shared between electric and magnetic fields, then

$$U_e = \frac{q^2}{2C} = \frac{1}{2}(U_e)_{max} = \frac{Q^2}{4C}$$

$$\Rightarrow q = \frac{Q}{\sqrt{2}}$$

8. The correct choice is (a).
9. A complete cycle of an alternating current consists of two half cycles. In one half cycle the current is positive and in the next half cycle, the current is negative. Hence, in one complete cycle, the average value of current is zero. So a DC ammeter will read zero.
10. Given $V_L = V_C = V_R = 50$ V. In a series LCR circuit, the voltage V_L across L leads the current I by 90° and the voltage V_C across C lags behind the current I by 90°. Since $V_L = V_C$, the voltage across the LC combination will be zero.



11. Induced emf is

$$e = -n\frac{d\phi}{dt} = -n\,\frac{(\text{final flux} - \text{initial flux})}{\text{time}}$$

$$= -n\left(\frac{\phi_2-\phi_1}{t}\right)$$

Total resistance in the circuit $R' = R + 4R = 5R$

$\therefore$ Induced current $= \dfrac{e}{R'} = \dfrac{n(\phi_2-\phi_1)}{5Rt}$

12. Magnetic flux through the coil is

$$\phi = B\,A\cos\theta$$

Here $A = \dfrac{1}{2}(\pi r^2)$ and $\theta = \omega t$. Therefore,

$$\therefore \qquad \phi = \frac{1}{2}(B\pi r^2)\cos\omega t$$

The induced emf is

$$e = -\frac{d\phi}{dt} = \left(\frac{B\pi r^2\omega}{2}\right)\sin\omega t$$

$$\text{Power } P = \frac{e^2}{R} = \frac{(B\pi r^2\omega)^2}{4R}\sin^2\omega t$$

Average power in one time period is

$$\langle P\rangle = \frac{(B\pi r^2\omega)^2}{4R}\langle\sin^2\omega t\rangle$$

$$= \frac{(B\pi r^2\omega)^2}{8R} \qquad \left(\because \langle\sin^2\omega t\rangle = \frac{1}{2}\right)$$

13. Resonant frequency is

$$\nu_r = \frac{1}{2\pi\sqrt{LC}}$$

If C is doubled, L must be halved so that ν_r remains unchanged.

14. $e = \frac{1}{2} B\omega l^2$

$= \frac{1}{2} \times (0.2 \times 10^{-4}) \times 5 \times (1)^2$

$= 0.5 \times 10^{-4}$ V

$= 50 \times 10^{-6}$ V

$= 50$ μV

15. Suppose each tube moves a small distance dx in time dt. The change in area = $l \times 2dx$. The rate of change in area is

$$\frac{dA}{dt} = 2l\frac{dx}{dt} = 2lv$$

Magnitude of induced emf is

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = B\frac{dA}{dt} = 2Blv$$

16. At an instant of time t, the current is given by

$I = I_0\ (1 - e^{-Rt/L})$; I_0 = steady state current

At $t = 0$, $I = 0$. Let $I = \frac{I_0}{2}$ at time t. Then

$$\frac{I_0}{2} = I_0\,(1 - e^{-Rt/L})$$

$$\frac{1}{2} = 1 - e^{-Rt/L}$$

$\Rightarrow \quad e^{-Rt/L} = \frac{1}{2}$

$\Rightarrow \quad e^{Rt/L} = 2$

$\Rightarrow \quad \frac{Rt}{L} = \ln(2) = 0.693$

$\therefore \quad t = \frac{0.693 \times L}{R}$

$= \frac{0.693 \times (300 \times 10^{-3})}{2}$

$\simeq 0.1$ s

17. The maximum power is imparted at resonance, i.e. when

$\omega = \frac{1}{\sqrt{LC}}$ or $\nu = \frac{1}{2\pi\sqrt{LC}}$

which gives

$$C = \frac{1}{4\pi^2\nu^2 L} = \frac{1}{4 \times (3.14)^2 \times (50)^2 \times 10}$$

$$\simeq 1 \times 10^{-6}\ \text{F} = 1\ \mu\text{F}$$

18. Power factor = $\frac{R}{Z} = \frac{12}{15} = 0.8$

19. If an AC circuit contains only capacitance C or only inductance L or a series combination of resistance C and inductance L, the phase difference between the current and voltage is $\pi/2$. But if the circuit contains R and L, the phase difference between the current and voltage can have any value between 0 and $\pi/2$ depending on the values of L and R. Hence the correct choice is (b).

20. Induced emf is $E = -\frac{d\phi}{dt}$

$$= -\frac{d}{dt}\,(10t^2 - 50t + 250)$$

$$= -20t + 50$$

$\therefore$ E (at $t = 3$s) $= -20 \times 3 + 50 = -10$ V,

21. The resonant angular frequency is given by

$$\omega = \frac{1}{\sqrt{LC}}$$

which gives $\quad L = \frac{1}{C\omega^2} = \frac{1}{(2 \times 10^{-6})(200)^2}$

$$= \frac{100}{8} = 12.5\ \text{H}$$

At resonance, impedance $Z = R$. Therefore, the current in the circuit is

$$I = \frac{V}{Z} = \frac{V}{R} = \frac{100\text{V}}{1000\Omega} = 0.1\ \text{A}$$

$\therefore$ Voltage across inductor (VL) = $IX_L = I\omega L$

$= 0.1 \times 200 \times 12.5$

$= 250$ V

22. The induced emf is given by

$E = \omega NAB \sin \omega t$

$\therefore \quad E_{\max} = \omega NAB.$

23. The decay of current in an LR circuit is given by

$$I = \frac{E}{R}\ e^{-Rt/L}$$

Substituting the values of E, R, L and t we get

$$I = \frac{1}{e}\ \text{ampere}$$

24. Phase difference between voltage E and current I is $\phi = \frac{\pi}{2}$. The time-averaged power consumption is given by

$$\langle P\rangle = I_{rms}\, E_{rms} \cos\phi = 0 \qquad \left[\because \phi = \frac{\pi}{2}\right]$$

25. $L = 10$ H, $R = 5\ \Omega$, $V = 5$ V, $t = 2$s

The steady state current is

$$I_0 = \frac{V}{R} = \frac{5\text{V}}{5\Omega} = 1\text{ A}$$

The current at time $t = 2$ s in the L–R circuit is

$$I = I_0\,(1 - e^{-Rt/L})$$
$$= 1 \times (1 - e^{-5\times 2/10})$$
$$= (1 - e^{-1}),\ \text{which is choice (d)}$$

26. $A = 10\text{ cm}^2 = 10 \times 10^{-4}\text{ m}^2 = 10^{-3}\text{ m}^2$, $l = 20\text{ cm} = 0.2$ m

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$
$$= \frac{(4\pi \times 10^{-7}) \times 300 \times 400 \times 10^{-3}}{0.2} = 2.4\,\pi \times 10^{-4}\text{ H}$$

27. Time constant $\tau = \dfrac{L}{R_2} = \dfrac{400\times 10^{-3}}{2} = 0.2$ s.

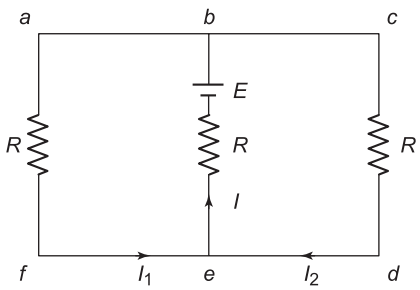
Current through the inductance is

$$I = \frac{E}{R_2}(1 - e^{-t/\tau})$$
$$= \frac{12}{2}\,(1 - e^{-t/0.2}) = 6\,(1 - e^{-5t})$$

$\therefore$ Potential drop across L is

$$e = -L\frac{dI}{dt} = -0.4 \times \frac{d}{dt}\left[6\,(1 - e^{-5t})\right]$$
$$= 0.4 \times 6 \times 5 \times e^{-5t} = 12e^{-5t}\text{ volt}$$

28. When a wire of length l moves with a velocity v perpendicular to a magnetic field B, an emf $E = Blv$ is induced between its ends. Hence the equivalent circuit is as shown in the figure. From Kirchhoff's junction rule,



$$I_1 + I_2 = I$$

Applying Kirchhoff's loop rule to loops *abefa* and *bcdeb* we have

$$E - IR - I_1 R = 0 \qquad (1)$$

and
$$I_2 R + IR - E = 0 \qquad (2)$$

Adding (1) and (2), we get $I_1 = I_2$

Subtracting (1) and (2), we get

$$2E - 2IR - (I_1 + I_2)\,R = 0$$
$$\Rightarrow \quad 2E - 2IR - IR = 0$$
$$\Rightarrow \quad I = \frac{2E}{3R} = \frac{2Blv}{3R}$$
$$\therefore \quad I_1 = I_2 = \frac{I}{2} = \frac{Blv}{3R}$$

29. In a d.c. circuit consisting of L and R, the current I grows with time t as

$$I = I_0\,(1 - e^{-t/\tau})$$

where $\tau = \dfrac{L}{R}$ is the time constant and I_0 is the final steady current. At $t = 0$, $I = I_0\,(1 - 1) = 0$. Thus no current flows in the branch containing L and R_1 at $t = 0$ and the inductor behaves as open switch (infinite resistance). Hence at $t = 0$,

$$I = \frac{V}{R_2}$$

At $t = \infty$, $I = I_0\,(1 - 0) = I_0$ (= final steady current). Hence at $t = \infty$, the inductor behaves as a closed switch (zero resistance). So at $t = \infty$, the equivalent resistance of the circuit is $R = \dfrac{R_1 R_2}{R_1 + R_2}$ and the current is

$$I = \frac{V}{R} = \frac{V(R_1 + R_2)}{R_1 R_2}$$

30. In a series LCR circuit the phase angle ϕ between voltage and current is given by

$$\tan\phi = \frac{\omega L - \dfrac{1}{\omega C}}{R}$$

If capacitor is taken out, $\tan\phi = \dfrac{\omega L}{R} \Rightarrow \tan 30° = \dfrac{\omega L}{R}$. If inductor is taken out, $\tan\phi = -\dfrac{1}{\omega CR} \Rightarrow -\tan 30° = -\dfrac{1}{\omega CR}$. Hence

$$\frac{\omega L}{R} = \frac{1}{\omega CR}$$
$$\Rightarrow \quad \omega L = \frac{1}{\omega C}$$

This is the resonance condition. At resonance, impedance $Z = R$. Hence

$$P = \frac{V^2}{R} = \frac{(220)^2}{200} = 242\text{ W}$$

31. Impedance $Z = \left[R^2 + \dfrac{1}{(\omega C)^2}\right]^{1\backslash 2}$, $I_0 = \dfrac{V}{Z}$. As ω increases, Z decreases. Hence the current I_0 increases.

Therefore, the bulb glows brighter, which is choice (b).

32. Let q be the charge on the capacitor when $U_e = U_m$ where $U_e = \dfrac{q^2}{2C}$ and $U_m = \dfrac{1}{2}Li^2$. Given $U_e = U_m$, i.e.

$$\frac{q^2}{2C} = \frac{1}{2}Li^2 = \frac{1}{2}L = \left(\frac{dq}{dt}\right)^2$$

$$\Rightarrow \qquad \frac{dq}{dt} = \pm\frac{q}{\sqrt{LC}} \qquad (1)$$

The angular frequency of an LC circuit is given by

$$\omega = \frac{1}{\sqrt{LC}}$$

The charge oscillates between the capacitor plates simple harmonically. Since $q = q_0$ at $t = 0$,

$$q = q_0 \cos \omega t \qquad (2)$$

$$\therefore \quad \frac{dq}{dt} = -\omega q_0 \sin \omega t = -\frac{q_0}{\sqrt{LC}} \sin \omega t \qquad (3)$$

Using (2) and (3) in (1) we get

$$-\frac{q_0}{\sqrt{LC}} \sin \omega t = +\frac{q_0 \cos \omega t}{\sqrt{LC}}$$

$$\Rightarrow \tan \omega t = +1$$

$$\omega t = +\frac{\pi}{4}$$

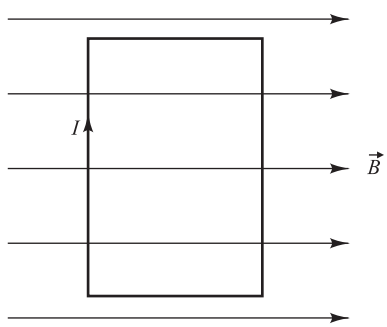
$$\Rightarrow \frac{t}{\sqrt{LC}} = +\frac{\pi}{4} = t = +\frac{\pi}{4}\sqrt{LC}\,.$$

33. $\varepsilon = B\ell v$

$= (5.0 \times 10^{-5}) \times 2 \times 1.5$

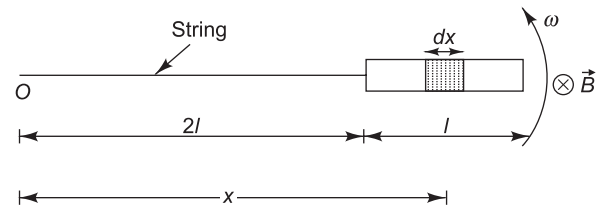
$= 15 \times 10^{-5}$ V $= 0.15$ mV

34. When a matel plate is placed near an oscillating coil, eddy currents are set up in the metal plate. From Lenz's law, these current will oppose the current in the coil. Hence the coil quickly stops. This phenomenon is called electromagnetic damping. So the correct shoice is (d).

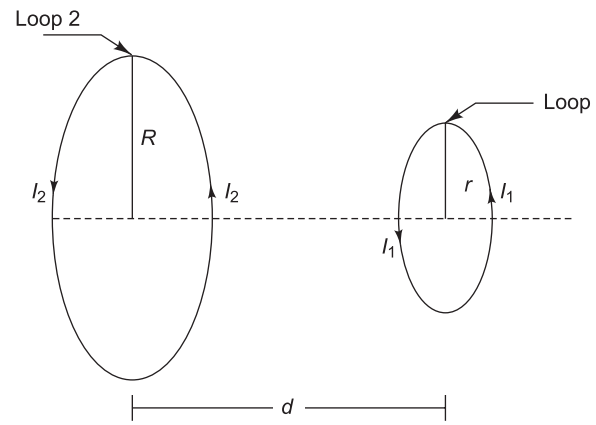


35. Divide the rod into a large number of very small elements each of length dx. Consider one such element at a distance x from end O where the string is fixed. Linear speed of the element is $v = \omega\, dx$. The magnitude of the emf induced in the element = $Bxv = B\omega\, xdx$. Therefore, emf induced between the ends of the rod is

$$\varepsilon = \int_{2\ell}^{3\ell} B\omega\, xdx = B\omega \int_{2\ell}^{3\ell} xdx = \frac{5B\omega\ell^2}{2}$$



36.



If M_{12} is the coefficient of mutual inductance between loops 1 and 2, the magnetic flux linked will loop 1 is

$$\phi_{12} = M_{12}I_2 = \frac{\mu_0 I_2 R^2}{2(d^2 + R^2)^{3/2}} \times \pi r^2$$

$$M_{12} = \frac{\mu_0 R^2}{2(d^2 + R^2)^{3/2}} \times \pi r^2$$

By symmetry, the magnetic flux linked will coil 2 is

$$\phi_2 = M_{21}I_1 = M_{12}\, I_1$$

$$= \frac{\mu_0 R^2 \times \pi r^2 I_1}{2(d^2 + R^2)^{3/2}} \qquad (1)$$

Now $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}, $R = 0.2$m, $r = 0.3 \times 10^{-2}$ m, $d = 0.15$m and $I_1 = 2.0$ A. Substituting these values in (1), we get $\phi_2 = 9.1 \times 10^{-11}$ weber.

37. The phase difference between voltage E and current I is $\phi = \dfrac{\pi}{2}$. The time-averaged power consumption is $P = I_{\text{rms}}\, E_{\text{rms}} \cos \phi = 0$ $\left(\because \cos\dfrac{\pi}{2} = 0\right)$

ELECTROMAGNETIC WAVES

Chapter 15

REVIEW OF BASIC CONCEPTS

1. Displacement Current

We know that a magnetic field changing with time gives rise to an emf (due to change in magnetic flux) and hence an electric field. Is the converse also true?

James Clerk Maxwell (1831–1879) argued that this was indeed true, i.e. a time varying electric field gives rise to a magnetic field. From Ampere's circuital law, this magnetic field will give rise to a current. Maxwell recognised that this current cannot be the conventional conduction current because it can exist even in a vacuum. He called this current the *displacement current* which exists in addition to the conduction current.

2. Maxwell's Equations

Maxwell modified Ampere's circuital law by including the displacement current and formulated a set of equations involving electric and magnetic fields, their sources and charge and current densities. These equations are known as Maxwell's equations, which are as follows:

1. $\oint \mathbf{E}\cdot\mathbf{dA} = \dfrac{Q}{\varepsilon_0}$ (Gauss's law for electrostatics)

2. $\oint \mathbf{B}\cdot\mathbf{dA} = 0$ (Gauss's law for magnetism)

3. $\oint \boldsymbol{E}\cdot\boldsymbol{dl} = -\dfrac{d\Phi_B}{dt}$ (Faraday's law)

4. $\oint \boldsymbol{B}\cdot\boldsymbol{dl} = \mu_0 i_c + \mu_0\varepsilon_0 \dfrac{d\Phi_E}{dt}$ (Ampere–Maxwell law)

$$= \mu_0(i_c + i_d)$$

where i_c = conduction current and i_d = displacement current. The other symbols have their usual meaning.

EXAMPLE 1 A parallel plate capacitor is charged by connecting its plates to a battery. Show that the displacement current between the plates during the charging process is equal to conduction current in the connecting wires.

SOLUTION When a capacitor is connected to a battery, it takes a finite time to be fully charged to the voltage of the battery. During the charging process, a current called displacement current i_d flows between the capacitor plates which is given by

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

Consider a closed Gaussian surface enclosing the plates. According to Gauss's law, the electric flux through the surface is given by

$$\Phi_E = \frac{q}{\varepsilon_0}$$

where q is the charge at the positive plate. Hence

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{q}{\varepsilon_0}\right) = \frac{dq}{dt}$$

But $\dfrac{dq}{dt}$ is the rate at which the charge is carried to the positive plate through the connecting wires. Hence $i_d = i_c$. This also follows from the principle of continuity of current.

EXAMPLE 2 A parallel plate air capacitor of capacitance 1 nF is charged by connecting the plates to a battery of voltage 2 V through a series resistance of 500 Ω. At a certain instant of time, the current in the circuit is 0.5 A during the charging process. What is the displacement current between the capacitor plates at that instant?

SOLUTION From continuity of current, the displacement current = conduction current = 0.5 A.

EXAMPLE 3 A parallal plate capacitor of capacitance 100 μF is connected across a 50 Hz variable voltage source. The displacement current between the plates at an instant when the voltage of source is 200 V is

(a) π ampere (b) 2π ampere

(c) 3π ampere (d) 4π ampere

SOLUTION Capacitative reactance

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$$

$$= \frac{100}{\pi} \text{ ohm}$$

Current through the capacitor is

$$i_d = \frac{200}{100/\pi} = 2\pi \text{ ampere}$$

EXAMPLE 4 At a certain instant of time, the potential difference between the plates of a capacitor of capacitance 2 μF is charging at a rate of 2×10^6 V s^{-1}. The displacement current at that instant is

(a) 1 A (b) 2 A

(c) 3 A (d) 4 A

SOLUTION $i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$

$$\Phi_E = EA = \frac{VA}{d}$$

$$\therefore \quad i_d = \varepsilon_0 \frac{d}{dt}\left(\frac{VA}{d}\right) = \frac{\varepsilon_0 A}{d}\frac{dV}{dt}$$

$$= C\frac{dV}{dt}$$

$$= 2 \times 10^{-6} \times 2 \times 10^6$$

$$= 4 \text{ A}$$

EXAMPLE 5 A parallel plate capacitor with square plates, each of side 20 cm has a capacitance of 1 μF. At time $t = 0$, it is connected for charging to a 2 V battery through a series resistance of 1 kΩ. The charge on the capacitor varies with time t (in second) as

$$q(t) = q_0 [1 - e^{-t/\tau}]$$

where $q_0 = CV$ and $\tau = CR$ is the time constant of the CR circuit. Calculate the displacement current between the plates at $t = 10^{-3}$ s.

SOLUTION $q = CV = \frac{\varepsilon_0 AV}{d} = \varepsilon_0 AE \ (\because E = \frac{V}{d})$

$$\therefore \quad E = \frac{q}{\varepsilon_0 A}$$

A = area of plate = 20 cm × 20 cm

$= 400 \text{ cm}^2 = 4 \times 10^{-2} \text{ m}^2$

Electric flux through the capacitor is

$$\Phi_E = E \times A = \frac{q}{\varepsilon_0}$$

where $q_0 = CV = 10^{-6} \times 2 = 2 \times 10^{-6}$ C

Also $\tau = CR = 10^{-6} \times 10^3 = 10^{-3}$ s

$$\therefore \quad \Phi_E = \frac{q}{\varepsilon_0} = \frac{q_0\left[1 - e^{-t/\tau}\right]}{\varepsilon_0}$$

Displacement current $i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$

$$= \varepsilon_0 \frac{d}{dt}\left[\frac{q_0(1 - e^{-t/\tau})}{\varepsilon_0}\right]$$

$$= \frac{q_0}{\tau} e^{-t/\tau}$$

$$= \frac{2 \times 10^{-6}}{10^{-3}} e^{-t/10^{-3}}$$

At $t = 10^{-3}$ s,

$$i_d = 2 \times 10^{-3}\, e^{-1} = \frac{2 \times 10^{-3}}{e} = \frac{2 \times 10^{-3}}{2.718}$$

$$= 7.36 \times 10^{-4} \text{ A}$$

3. History of Electromagnetic Waves

Ampere's law states that a time-varying electric field at any point is a source of magnetic field. Faraday's law states just the reverse namely, that a time-varying magnetic field is a source of electric field. These laws led Maxwell to conclude that space and time varying electric and magnetic fields produce an electromagnetic disturbance which can travel even in particle-free space. This disturbance is called the electromagnetic wave. Thus, *electromagnetic waves are those waves in which electric and magnetic fields vary sinusoidally in space and time.*

In 1865, Maxwell predicted the existence of electromagnetic waves. His theory further predicted that electromagnetic waves of all frequencies (and hence all wavelengths) should propagate with the speed of light. This theory was first experimentally verified by a German physicist, Heinrich Hertz in 1888. He used a simple electric oscillator and was able to pick up its radiation of electromagnetic waves on a radio receiver some distance

away. In 1899, an Italian engineer Guglielmo Marconi succeeded in transmitting electromagnetic waves across the English Channel and 1901 across the Atlantic Ocean.

4. Transverse Nature of Electromagnetic Waves

In an electromagnetic wave, the electric and magnetic fields are mutually perpendicular to each other and each field is perpendicular to the direction of propagation of the wave.

5. Velocity of Electromagnetic Waves

Maxwell's theory predicted that electromagnetic waves of all frequencies (and hence all wavelengths) propagate in vacuum with a speed given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \tag{15.1}$$

where μ_0 is the magnetic permeability and ε_0 the electric permittivity of vacuum. Now, for vacuum, we know that $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1} and $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1}m^{-2}, Substituting these values in Eq. (15.1) we have

$$c = \frac{1}{\left[\left(4\pi \times 10^{-7}\right)\left(8.85 \times 10^{-12}\right)\right]^{1/2}}$$

$$= 3.0 \times 10^8 \text{ ms}^{-1}$$

which is the speed of light in vacuum measured experimentally. The excellent agreement between the experimentally measured speed of light ($c = 2.997924 \times 10^8$ ms^{-1}) to such a high degree of accuracy and the value based on the experimental measurements of ε_0 and μ_0 gave the first quantitative proof of the fact that light is an electromagnetic wave. The emergence of the speed of light from purely electromagnetic considerations is the crowning achievement of Maxwell's electromagnetic theory.

In a material medium, the speed of electromagnetic waves is given by

$$v = \frac{1}{\sqrt{\mu \varepsilon}}$$

where μ is the magnetic permeability and ε the electric permittivity of the medium.

6. Production of Electromagnetic Waves

Consider a charge at rest. It has an electric field in the region around it but no magnetic field. If it is given an impulse so that it begins to move, it produces electric and magnetic fields. If it is moving with a constant velocity (i.e. if the current is not changing with time) the magnetic field will not change with time, so it cannot produce an electromagnetic wave. But if the charge is somehow accelerated, the magnetic and electric fields will change with space and time; it then produces an electromagnetic wave. Thus *an accelerated charge emits an electromagnetic wave.*

In an *L-C* circuit, the charge oscillates between the capacitor plates. An oscillating charge has a non-zero acceleration; hence it will emit an electromagnetic wave whose frequency is the same that of the oscillating charge.

An electron circulating round its nucleus in a stable orbit does not emit an electromagnetic wave, although it is accelerating; it does so only when it falls to a lower orbit.

When fast-moving electrons hit a metal target, electromagnetic waves (X-rays) are produced.

7. Energy Density of Electromagnetic Field

Just as an oscillating pendulum has energy associated with it, oscillating electric and magnetic fields also have electric and magnetic energies associated with them. The average energy densities of electric and magnetic fields of an electromagnetic wave are respectively given by

$$u_e = \frac{1}{4}\ \varepsilon_0 E_0^2 \tag{15.2}$$

and

$$u_m = \frac{B_0^2}{4\mu_0} \tag{15.3}$$

where E_0 and B_0 are the amplitudes of the electric and magnetic fields respectively. E_0 and B_0 are related as

$$B_0 = \frac{E_0}{c} = \sqrt{\mu_0 \varepsilon_0}\ E_0 \tag{15.4}$$

Using (15.4) in (15.3), we find that

$$u_m = \frac{1}{4}\ \varepsilon_0 E_0^2 = u_e$$

i.e. in an electromagnetic wave, the average energy densities of electric and magnetic fields are equal. The total average energy density of the electromagnetic field is

$$u = u_e + u_m = \frac{1}{2}\ \varepsilon_0 E_0^2$$

$$= \frac{B_0^2}{2\mu_0} \tag{15.5}$$

8. Characteristics of Electromagnetic Waves

1. Electromagnetic waves are producted by an accelerating charge.
2. Electromagnetic waves can propagate even in vacuum.
3. They travel in vacuum with a speed of 3×10^8 ms^{-1}, the speed of light.
4. Electromagnetic waves are transverse in nature, i.e. the oscillating electric and magnetic field vectors are perpendicular to the direction of propagation of the wave and are also perpendicular to each other.

5. In an electromagnetic wave, the total energy of the electromagnetic field is shared equally between the electric and magnetic fields.
6. Electromagnetic waves of all frequencies exhibit the phenomena of interference, diffraction and polarization.

9. Spectrum of Electromagnetic Radiation

Beginning with the remarkable demonstration by Hertz of the existence of long wavelength electromagnetic waves, scientists began looking for electromagnetic waves of wavelengths much shorter than the visible light. In 1898 Rontgen discovered X-rays which are electromagnetic waves of wavelength about 10^{-10} m. The wavelength of visible light is in the range of 4×10^{-7} to 8×10^{-7} m. Radiowaves, X-rays and visible light are all electromagnetic waves and travel with the same speed $c = 3.0 \times 10^8$ ms^{-1} in free space. They differ in wavelength (and hence in frequency, $\nu = c/\lambda$) only, which means that the sources that produce them and their detectors are different. Table 15.1 shows the frequency range, wavelength range, the names and the sources of the known electromagnetic radiations. The spectrum of electromagnetic radiation has no upper or lower limits and all the regions overlap.

Notice that visible light is only a very small part of the total electromagnetic spectrum. We see that electromagnetic waves have a very wide of wavelengths (and hence of frequencies). Although they are identical in nature, their interaction with matter or their physiological action on living bodies depends on their frequency. Infrared rays are thermal radiations which produce heat, X-rays and gamma rays are highly penetrating, to mention only a few of the effects.

10. Uses of Electromagnetic Spectrum

The different regions of the total electromagnetic spectrum have been put to the following uses:

1. Radiowaves are used in radar and radio broadcasting.
2. Microwaves are used in long distance wireless communications via satellites.
3. Infrared, visible and ultraviolet traditions are used to know the structure of molecules.
4. Diffraction of X-rays by crystals gives the details of the structure of crystals.
5. The bones are opaque to X-rays but flesh is transparent. X-ray pictures of a human body are used in medical diagnosis of fractures and cracks of bones.
6. The γ-rays are used in the study of the structure of the nuclei of atoms.

EXAMPLE 6 A charged particle oscillates about its equilibrium position with a frequency of 10^{10} Hz. What is the frequency of the electromagnetic waves produced by the oscillating charge?

SOLUTION The frequency of the electromagnetic waves is the same as that of the oscillating charge, i.e. 10^{10} Hz.

EXAMPLE 7 A plane electromagnetic wave of frequency 60 MHz travels in vacuum along the positive x-direction. The electric field **E** at a particular space point x and an instant of time t is $9.6\ \hat{\mathbf{j}}$ Vm^{-1}. Find the magnitude and direction of magnetic field **B** at this point at time t.

SOLUTION $$B = \frac{E}{c} = \frac{9.6}{3 \times 10^8} = 3.2 \times 10^{-8}\ \text{T}$$

In an electromagnetic wave, the **E** and **B** fields oscillate in mutually perpendicular direction, each perpendicular to the direction of propagation of the wave. Since **E** is along $+y$ direction and the wave propagated along $+x$ direction, the direction of **B** will be along $+z$ direction. Thus

$$\mathbf{B} = 3.2 \times 10^{-8}\ \hat{\mathbf{k}}\ \text{tesla}$$

EXAMPLE 8 In a plane electromagnetic wave, the electric field (in Vm^{-1}) is given by

Table 15.1 *The electromagnetic spectrum*

Name	*Frequency range (Hz)*	*Wavelength range (m)*	*Source*
Radiowaves	10^4 to 10^8	0.1 to 600	Oscillating electric circuits
Microwaves	10^9 to 10^{12}	10^{-3} to 0.3	Oscillating current in special vacuum tubes
Infrared	10^{11} to 5×10^{14}	10^{-6} to 5×10^{-3}	Outer electrons in atoms and molecules
Visible light	4×10^{14} to 7×10^{14}	4×10^{-7} to 8×10^{-7}	Outer electrons in atoms
Ultraviolet	10^{15} to 10^{17}	1.5×10^{-7} to 3.5×10^{-7}	Outer electrons in atoms
X-rays	10^{18} to 10^{20}	10^{-11} to 10^{-8}	Inner electrons in atoms and sudden deceleration of high energy free electrons
Gamma rays	10^{19} to 10^{24}	10^{-16} to 10^{-13}	Nuclei of atoms and sudden deceleration of high energy free electrons

$$E_z = 6.0 \sin [2\pi (2.0 \times 10^{10}\, t + 500\, x)]$$

where x is in metre and t in second

(a) What is the direction of propagation of the wave?

(b) What is the rms value of electric field?

(c) Find the wavelength and frequency of the wave.

(d) Write the expression for the magnetic field.

SOLUTION

(a) The wave travels along the negative x direction.

(b) The electric field for a wave travelling along negative x direction is given by

$$E_y = E_0 \sin (\omega t + kx) \qquad (1)$$

where E_0 = peak value of E_y,

$\omega = 2\pi\nu$; (ν = frequency)

and $k = \dfrac{2\pi}{\lambda}$; ($\lambda$ = wavelength)

Comparing Eq. (1) with the given equation, we get

$$E_0 = 6.0 \text{ Vm}^{-1}$$

$$\therefore \quad E_{\text{rms}} = \frac{E_0}{\sqrt{2}} = \frac{6.0}{1.414} = 4.2 \text{ V m}^{-1}$$

(c) Also $\omega = 2\pi \times (2.0 \times 10^{10})$ rad s^{-1}

$$\therefore \quad \nu = \frac{\omega}{2\pi} = 2.0 \times 10^{10} \text{ Hz}$$

And $k = 2\pi \times 500 \text{ m}^{-1}$

$$\therefore \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{2\pi \times 500} = 2 \times 10^{-3} \text{ m} = 2.0 \text{ mm}$$

(d) Peak value of magnetic field is

$$B_0 = \frac{E_0}{c} = \frac{6.0}{3 \times 10^8} = 2.0 \times 10^{-8} \text{ T}$$

In an electromagnetic wave, the phase ($\omega t + kx$) is the same for both **E** and **B** fields. Also **B** must be perpendicular to both **E** and the direction of propagation, i.e. **B** is along the z direction. Hence

$$B_z = B_0 \sin (\omega t + kx)$$

or $B_z = 2.0 \times 10^{-8} \sin [2\pi(2 \times 10^{10}\, t + 500\, x)]$ tesla

EXAMPLE 9 Light of intensity 1500 W m^{-2} falls at normal incidence for 1 minute on a plane square surface of side 10 cm. Find the average force exerted on the surface during this time if surface is a

(a) perfect absorber and (b) perfect reflector of light.

SOLUTION Intensity is defined as the amount of electromagnetic energy falling per second per unit area of a surface held normally to the light.

Given $I = 1500$ Js^{-1} m^{-2}, $A = 10$ cm $\times$ 10 cm $= 10^{-2}$ m^2 and $t = 1$ minute $= 60$ s. Now

$$I = \frac{E}{At}$$

$$\Rightarrow \quad E = IAt = 1500 \times 10^{-2} \times 60 = 900 \text{ J}$$

Light consists of photons of energy $E = mc^2$

or $$mc = \frac{E}{c}$$

or $p = \dfrac{E}{c}$ is the momentum of incident photons, which is

$$p_i = \frac{E}{c} = \frac{900}{3 \times 10^8} = 3 \times 10^{-6} \text{ kg ms}^{-1}$$

(a) Since the surface is a perfect absorber, the final momentum $p_f = 0$. The magnitude of change in momentum is $|\Delta p| = p_i$

Average force exerted on the surface is

$$F = \frac{|\Delta p|}{t} = \frac{3 \times 10^{-6}}{60} = 5 \times 10^{-8} \text{ N}$$

(b) For a perfect reflector, $p_f = -p_i$. Therefore, the magnitude of change in momentum is $|\Delta p| = 2p_i$. Hence $F = 1.0 \times 10^{-7}$ N

EXAMPLE 10 The earth receives energy from the sun at the rate of 1400 Wm^{-2}. Calculate the peak values of the electric and magnetic fields on the earth due to the solar radiation.

SOLUTION Energy received per unit area per second is $U = 1400$ Wm^{-2}. Since this energy is equally shared by the electric and magnetic fields,

$$U = U_e + U_m = \varepsilon_0 E_{\text{rms}}^2 c$$

Thus $$E_{\text{rms}} = \left(\frac{U}{\varepsilon_0 c}\right)^{1/2}$$

$$= \left[\frac{1400}{(8.85 \times 10^{-12}) \times (3 \times 10^8)}\right]^{1/2}$$

$$= 7.26 \times 10^2 \text{ Vm}^{-1}$$

$\therefore$ Peak value $E_0 = \sqrt{2}\ E_{\text{rms}} = \sqrt{2} \times 7.26 \times 10^2$

$$= 1.0\ 3 \times 10^3 \text{ Vm}^{-1}$$

Peak value of magnetic field is

$$B_0 = \frac{E_0}{c} = \frac{1.03 \times 10^3}{3 \times 10^8} = 3.4 \times 10^{-6} \text{ T}$$

EXAMPLE 11 A 200 W electric bulb emits radiation which falls normally on a squre surface of side 5 m. Assuming that only 5% of the incident intensity falls on the surface, calculate the peak values of the electric and magnetic fields.

SOLUTION

$$\text{Intensity} = \frac{\text{power}}{\text{area}} = \frac{200\,\text{W}}{(5\times 5)\,\text{m}^2} = 8\ \text{Wm}^{-2}$$

Useful intensity $I = 5\%$ of $8\ \text{Wm}^{-2} = \frac{5}{100}\times 8 = 0.4\ \text{Wm}^{-2}$

Average intensity $= \frac{1}{2}\varepsilon_0 E_0^2\, c$

$$0.4 = \frac{1}{2}\varepsilon_0 E_0^2\, c = \frac{4\pi\varepsilon_0}{8\pi} E_0^2\, c$$

$$= \frac{E_0^2 \times 3.0\times 10^8}{(9\times 10^9)\times 8\pi}$$

$$= \frac{E_0^2}{24\pi}$$

$$\therefore \quad E_0 = \sqrt{0.4\times 24\,\pi} = 1.74\ \text{V m}^{-1}$$

$$B_0 = \frac{E_0}{c} = \frac{1.74}{3\times 10^8} = 5.8\times 10^{-9}\ \text{T}$$

EXAMPLE 12 The electric field in a plane electromagnetic wave is given by

$$E = 100\ \sin(\omega t - kx)$$

where E is in NC^{-1}, t in s and x in m. Find the average electromagnetic energy contained in a cylinder of radius 2 cm and length 50 cm lying along the x-axis.

SOLUTION Peak value of E is $E_0 = 100\ \text{N C}^{-1}$
Volume of cylinder is $V = \pi r^2 l$. Average energy density is

$$u = \frac{1}{2}\varepsilon_0 E_0^2$$

$\therefore$ Energy contained in the cylinder is

$$U = uV$$

$$= \pi r^2 l \times \frac{1}{2}\varepsilon_0 E_0^2$$

$$= 3.14\times(2\times 10^{-2})^2\times 0.5\times \frac{1}{2}\times 8.85\times 10^{-12}\times(100)^2$$

$$= 2.8\times 10^{-11}\ \text{J}$$

EXAMPLE 13 The magnetic field in a plane electromagnetic wave travelling along the x-axis is given by

$$B = 2\times 10^{-4}\ \sin(\omega t - kx)$$

where B is in tesla, t in s and x in m. Calculate the peak values of electric and magnetic forces acting on a particle of charge 5 μC moving with a velocity of $4\times 10^5\ \text{ms}^{-1}$ along the y-axis.

SOLUTION $B_0 = 2\times 10^{-4}$ T

$$E_0 = c\,B_0 = (3\times 10^8)\times(2\times 10^{-4})$$

$$= 6\times 10^4\ \text{NC}^{-1}$$

Maximum electric force $= q\,E_0$

$$= (5\times 10^{-6})\times(6\times 10^4) = 0.3\ \text{N}$$

Maximum magnetic force $= q\,v\,B_0$

$$= (5\times 10^{-6})\times(4\times 10^5)\times(2\times 10^{-4})$$

$$= 4.0\times 10^{-4}\ \text{N}$$

SECTION 1

Multiple Choice Questions with One Correct Choice

Level A

1. Which of the following electromagnetic waves has the longest wavelength?
   (a) Radiowaves (b) Infrared radiation
   (c) Microwaves (d) X-rays
2. Which of the following electromagnetic waves has the highest frequency?
   (a) Radiowaves (b) X-rays
   (c) γ-rays (d) Microwaves
3. Which of the following electromagnetic waves is used in telecommunication?

(a) Radiowaves (b) Visible radiation
(c) Ultraviolet rays (d) Microwaves

4. Which of the following electromagnetic waves emitted by the sun is responsible for heating the earth's atmosphere due to greenhouse effect?
(a) Visible light (b) Infra-red radiation
(c) Ultraviolet rays (d) γ-rays

5. The speed of electromagnetic waves in vacuum is given by
(a) $\frac{1}{\mu_0 \varepsilon_0}$ (b) $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$
(c) $\mu_0 \varepsilon_0$ (d) $\sqrt{\mu_0 \varepsilon_0}$

6. In an electromagnetic wave travelling in air, the amplitudes E_0 and B_0 of the electric and magnetic fields are related as (here c is the speed of the wave in air)
(a) $E_0 = cB_0$ (b) $E_0 = \frac{B_0}{c}$
(c) $E_0 = c^2 B_0$ (d) $E_0 = B_0$

7. If v_x, v_m and v_r respectively are the speeds of X-rays, microwaves and radiowaves in air, then
(a) $v_x > v_m > v_r$ (b) $v_x < v_m < v_r$
(c) $v_x = v_m > v_r$ (d) $v_x = v_m = v_r$

8. When a plane electromagnetic wave travels in vacuum, the average electric energy density is given by (here E_0 is the amplitude of the electric field of the wave)
(a) $\frac{1}{4} \varepsilon_0 E_0^2$ (b) $\frac{1}{2} \varepsilon_0 E_0^2$
(c) $2 \varepsilon_0 E_0^2$ (d) $4 \varepsilon_0 E_0^2$

9. When a plane electromagnetic wave travels in a dielectric medium, its electric and magnetic energy densities u_e and u_m are related as
(a) $u_e = cu_m$ (b) $u_e = \frac{u_m}{c}$
(c) $u_e = u_m$ (d) none of these

10. The amplitude of the electric field of a plane electromagnetic wave in air is 6.0×10^{-4} Vm^{-1}. The amplitude of the magnetic field will be
(a) 1.8×10^5 T (b) 5.0×10^3 T
(c) 2.0×10^{-4} T (d) 2.0×10^{-12} T

11. In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10} Hz. What is the wavelength of the wave?
(a) 1.0 cm (b) 1.5 cm
(c) 2.0 cm (d) 3.0 cm

12. In Q.11 if the peak value of the electric field is 60 Vm^{-1}, the average energy density (in Jm^{-3}) of the magnetic field of the wave will be (given $\mu_0 = 4\pi \times 10^{-7}$ Fm^{-1})
(a) $2\pi \times 10^{-7}$ (b) $4\pi \times 10^{-7}$
(c) $\frac{1}{2\pi} \times 10^{-7}$ (d) $\frac{1}{4\pi} \times 10^{-7}$

13. In Q.12, the total average energy density (in Jm^{-3}) of the electromagnetic field of the wave is
(a) $2\pi \times 10^{-7}$ (b) $4\pi \times 10^{-7}$
(c) $\frac{1}{2\pi} \times 10^{-7}$ (d) $\frac{1}{4\pi} \times 10^{-7}$

14. An electromagnetic wave is produced by oscillating electric and magnetic fields **E** and **B**. Choose the only *incorrect* statement from the following.
(a) **E** is perpendicular to **B**.
(b) **E** is perpendicular to the direction of propagation of the wave
(c) **B** is perpendicular to the direction of propagation of the wave
(d) **E** is parallel to **B**.

15. Which of the following pairs of space and time varying **E** $\left(= \hat{\mathbf{i}} E_x + \hat{\mathbf{j}} E_y + \hat{\mathbf{k}} E_z\right)$ and **B** $= \left(= \hat{\mathbf{i}} B_x + \hat{\mathbf{j}} B_y + \hat{\mathbf{k}} E_z\right)$ would generate a plane electromagnetic wave travelling in the z-direction?
(a) E_x, B_z (b) E_y, B_z
(c) E_z, B_x (d) E_x, B_y

16. Which of the following statements is false? Electromagnetic waves
(a) are transverse
(b) travel in free space at the speed of light
(c) travel with the same speed in all media
(d) are produced by an accelerating charge

Level B

17. Displacement current was first postulated by
(a) Ampere (b) Maxwell
(c) Hertz (d) Marconi

18. The SI unit of displacement current is
(a) A (b) Am^{-1}
(c) Am^{-2} (d) Am^{-3}

19. The displacement current flows in the dielectric of a capacitor when the potential difference between its plates
(a) is changing with time
(b) is changing with distance

(c) has assumed a constant value
(d) becomes zero

20. The potential difference between the plates of a parallel plate capacitor is changing at the rate of 10^6 Vs^{-1}. If the capacitance is 2 μF, the displacement current in the dielectric of the capacitor will be
(a) 1 A (b) 2 A
(c) 3 A (d) 4 A

21. If V is the accelerating potential in an X-ray tube, the minimum wavelength of the emitted X-ray will be
(a) $\frac{hc}{eV}$ (b) $\frac{hV}{ec}$
(c) $\frac{cV}{eh}$ (d) $\frac{eV}{hc}$

22. If the frequency of an X-ray is increased by 20%, its energy
(a) increases by 10% (b) increases by 20%
(c) decreases by 10% (d) decreases by 20%

23. If μ_r and $\in_r$ are the relative permeability and relative permittivity of a medium, its refractive index is given by
(a) $n = \frac{1}{\sqrt{\mu_r \in_r}}$ (b) $n = \sqrt{\mu_r \in_r}$
(c) $n = \frac{1}{\mu_r \in_r}$ (d) $n = \mu_r \in_r$

24. The earth receives energy from the sun at the rate of 1400 Wm^{-2}. The total power incident on a roof of dimensions (16 m × 10 m) is
(a) 8.75 W (b) 2.80×10^3 W
(c) 0.11 W (d) 2.24×10^5 W

25. In Q. 24 above, the force exerted by the solar radiation on the roof is nearly equal to
(a) 7.5×10^{-4} N (b) 7.5×10^{-3} N
(c) 7.5×10^{-2} N (d) 0.75 N

26. The peak value of the electric field of a radiowave reaching the receiver antenna is 3×10^{-3} Vm^{-1}. The peak value of the magnetic field will be
(a) 10^{-9} T (b) 10^{-10} T
(c) 10^{-11} T (d) 10^{-12} T

27. An electromagnetic wave travelling in a medium is given by
$$E = E_0 \sin(\omega t - kx)$$
The speed of the wave in the medium is given by
(a) $v = \omega k$ (b) $v = \frac{k}{\omega}$
(c) $v = \frac{\omega}{k}$ (d) $v = \sqrt{\frac{\omega}{k}}$

28. A plane electromagnetic wave travelling in vacuum is described by
$$E = E_0 \sin(\omega t - kx)$$
and $$B = B_0 \sin(\omega t - kx)$$
Which of the following relations is correct?
(a) $\omega B_0 = kE_0$ (b) $kB_0 = \omega E_0$
(c) $E_0 B = \omega k$ (d) $E_0 B_0 = \frac{\omega}{k}$

29. A 10 watt LED lamp converts 95% of the power into electromagnetic waves. The peak value of the electric field at a distance of 2 m from the lamp is nearly equal to
(a) 10 Vm^{-1} (b) 12 Vm^{-1}
(c) 10^2 Vm^{-1} (d) 1.2×10^2 Vm^{-1}

30. A plane electromagnetic wave travelling in vacuum is given by
$$E = E_0 \sin(\omega t - kx)$$
where $E_0 = 100$ NC^{-1}. The average energy density of the wave is nearly equal to
(a) 1.1×10^{-5} Jm^{-3} (b) 2.2×10^{-6} Jm^{-3}
(c) 8.8×10^{-7} Jm^{-3} (d) 4.4×10^{-8} Jm^{-3}

31. A plane electromagnetic ware travelling in a non-magnetic medium is given by
$$E = (9 \times 10^8 \text{ NC}^{-1}) \sin[(9 \times 10^8 \text{ rad s}^{-1})t - (6 \text{ m}^{-1})x]$$
where x is in metre and t is in second.
The dielectric constant of the medium is
(a) 5 (b) 4
(c) 3 (d) 2



Answers

Level A

1. (a)	2. (c)	3. (d)	4. (b)
5. (b)	6. (a)	7. (d)	8. (a)
9. (c)	10. (d)	11. (b)	12. (d)
13. (c)	14. (d)	15. (d)	16. (c)

Level B

17. (b)	18. (a)	19. (a)	20. (b)
21. (a)	22. (b)	23. (b)	24. (d)
25. (a)	26. (c)	27. (c)	28. (a)
29. (b)	30. (d)	31. (b)	

Solutions

Level A

1. The correct choice is (a).
2. The correct choice is (c).
3. The correct choice is (d).
4. The correct choice is (b).
5. The correct choice is (b).
6. The correct choice is (a).
7. Electromagnetic waves of all frequencies travel with the same speed in air. Hence the correct choice is (d).
8. The correct choice is (a).
9. When an electromagnetic wave travels in a dielectric medium, its total energy density is divided equally between its electric and magnetic fields. Hence the correct choice is (c).
10. $B_0 = \dfrac{E_0}{c} = \dfrac{6.0\times10^{-4}}{3\times10^8} = 2.0 \times 10^{-12}$ T which is choice (d).
11. $\lambda = \dfrac{c}{\nu} = \dfrac{3\times10^8}{2.0\times10^{10}} = 1.5 \times 10^{-2}$ m = 1.5 cm, which is choice (b)
12. $B_0 = \dfrac{E_0}{c} = \dfrac{60}{3\times10^8} = 2.0\times10^{-7}$ T

    $$u_m = \frac{B_0^2}{4\mu_0} = \frac{\left(2.0\times10^{-7}\right)^2}{4\times4\pi\times10^{-7}} = \frac{1}{4\pi}\times10^{-7}\ \text{Jm}^{-3}$$

    Hence the correct choice is (d).
13. $u = u_e + u_m = 2u_m$ (since $u_e = u_m$). Hence

    $$u = 2\times\frac{1}{4\pi}\times10^{-7} = \frac{1}{2\pi}\times10^{-7}\ \text{Jm}^{-3},$$

    which is choice (c).
14. The correct choice is (d).
15. The correct choice is (d).
16. The refractive index (n) of a medium is defined as

    $$n = \frac{c}{v}$$

    where c is the speed of the electromagnetic wave in free space and v that in the medium. Thus, $v = c/n$. Since the value of n is different for different media, the value of v will also be different for different media. Hence choice (c) is false.

Level B

17. The correct choice is (b).
18. The correct choice is (a).
19. The displacement current I_d is given by

    $$I_d = \varepsilon_0 \frac{d\Phi}{d\Phi}$$

    where Φ is the electric flux. For a parallel plate capacitor

    $$\Phi = EA$$

    where A is the area of the plate and E is the electric field between the plates. Now, $E = V/d$ where d is the distance between the plates. Hence

    $$I_d = \varepsilon_0 \frac{d}{dt}(AE) = \varepsilon_0 A \frac{dE}{dt} = \frac{\varepsilon_0 A}{d}\frac{dV}{dt}$$

    Thus I_d exists as long as the potential difference V is changing with time t. Hence the correct choice is (a).
20. We have seen above that the displacement current is given by

    $$I_d = \frac{\varepsilon_0 A}{d}\frac{dV}{dt}$$

    $$= C\frac{dV}{dt} \qquad \left(\because C = \frac{\varepsilon_0 A}{d}\right)$$

    $$= (2\times10^{-6})\times10^6 = 2\ \text{A}$$

    Hence the correct choice is (b).
21. $E = h\nu = \dfrac{hc}{\lambda}$. Also $E = eV$. Equating them, we get

    $$\frac{hc}{\lambda} = eV \quad\Rightarrow\quad \lambda = \frac{hc}{eV}$$

    So the correct choice is (a).
22. $E = h\nu$. So $E \propto \nu$. Hence if ν is increased by 20%, E will also increase by 20%. So the correct choice is (b).
23. $c = \dfrac{1}{\sqrt{\mu_0 \in_0}}$

    $$v = \frac{1}{\sqrt{\mu \in}}$$

    $$n = \frac{c}{v} = \sqrt{\frac{\mu}{\mu_0}\frac{\in}{\in_0}} = \sqrt{\mu_r \in_r}$$

    The correct choice is (b).

24. $P = (1400\ \text{Wm}^{-2}) \times (16 \times 10)\ \text{m}^2 = 2.24 \times 10^5$ W, which is choice (d).

25. $P = Fc \Rightarrow F = \dfrac{P}{c} = \dfrac{2.24 \times 10^5}{3 \times 10^8} \simeq 7.5 \times 10^{-4}$ N

So the correct choice is (*a*).

26. $B_0 = \dfrac{E_0}{c} = \dfrac{3 \times 10^{-3}}{3 \times 10^8} = 10^{-11}$ T, which is choice (c).

27. $\omega = 2\pi\nu$ and $k = \dfrac{2\pi}{\lambda}$. Therefore

$$v = \nu\lambda = \frac{2\pi\nu \times \lambda}{2\pi} = \frac{\omega}{k}$$

So the correct ch-oice is (c).

28. $B_0 = \dfrac{E_0}{c} = \dfrac{E_0}{\omega / k} \Rightarrow \omega B_0 = k E_0$, which is choice (a).

29. 95% of 10 W = 9.5 W. So P = 9.5 W

Intensity of the wave at a distance r from the lamp is

$$I = \frac{P}{4\pi r^2}$$

If u is the average energy density of the wave, then

$$I = uc = \frac{1}{2}\epsilon_0 E_0^2 c$$

$$\therefore \quad \frac{P}{4\pi r^2} = \frac{1}{2}\epsilon_0 E_0^2 c$$

$$\Rightarrow \quad E_0 = \sqrt{\frac{P}{2\pi r^2 \epsilon_0 c}}$$

$$= \sqrt{\frac{9.5}{2 \times 3.14 \times (2)^2 \times (8.85 \times 10^{-12}) \times 3 \times 10^8}}$$

$$= 11.9 \simeq 12\ \text{Vm}^{-1}$$

So the correct choice is (b).

30. $\langle u \rangle = \dfrac{1}{2}\epsilon_0 E_0^2$

$$= \frac{1}{2} \times (8.85 \times 10^{-12}) \times (100)^2$$

$$= 4.425 \times 10^{-8}\ \text{Jm}^{-3}$$

So the correct choice is (d).

31. $v = \dfrac{\omega}{k} = \dfrac{9 \times 10^8}{6} = 1.5 \times 10^8\ \text{ms}^{-1}$

Refractive index $n = \dfrac{c}{v} = \dfrac{3 \times 10^8}{1.5 \times 10^8} = 2$

Also $n = \sqrt{\mu_r \epsilon_r}$. For a non-magnetic medium $\mu_r = 1$.

Therefore

$$n = \sqrt{\epsilon_r} \Rightarrow \epsilon_r = n^2 = (2)^2 = 4$$

$\therefore$ Dielectric constant $k = \epsilon_r = 4$, which is choice (b).

SECTION 2

Multiple Choice Questions Based on Passage

Questions 1 to 5 are based on the following passage.

Passage I

Electric and magnetic fields exhibit a wave-like behaviour. When electric and magnetic fields vary in space and time, they produce an electromagnetic wave. An accelerated charge produces electromagnetic waves. An oscillating charge has oscillating electric and magnetic fields around it and hence it produces electromagnetic waves. Electrons falling from a higher to a lower energy orbit in an atom radiate electromagnetic waves. The motion of electrons in an antenna radiates electromagnetic waves. Electromagnetic waves are transverse in nature. They do not require a material medium for their propagation. The speed of an electromagnetic wave is given by

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

where μ is the magnetic permeability and ε is the electrical permittivity of the medium. Radiowaves, microwaves, infrared radiations, visible light, ultraviolet rays, X-rays and gamma rays are all electromagnetic waves. They have a very wide range of wavelengths and hence of frequencies. Although they are identical in nature, their method of production and their interaction with matter are different.

1. Which of the following electromagnetic waves has the longest wavelength?

(a) Radiowaves (b) Infrared radiation
(c) Microwaves (d) X-rays

2. Which of the following electromagnetic waves has the highest frequency?
(a) Radiowaves (b) Visible light
(c) Ultraviolet rays (d) Microwaves
3. Which of the following waves are transverse in nature?
(a) Light emitted from a sodium lamp
(b) Sound waves travelling in air
(c) X-rays from an X-ray machine
(d) Microwaves used in a radar.
4. Which of the following statements is false? Electromagnetic waves
(a) are transverse.
(b) travel in free space at the same speed.
(c) travel in all media at the same speed.
(d) are produced by an accelerating charge.
5. An electromagnetic wave is produced by oscillating electric and magnetic fields **E** and **B**. Choose the only incorrect statement from the following.
(a) **E** is perpendicular to **B**.
(b) **E** is perpendicular to the direction of propagation of the wave.
(c) **B** is perpendicular to the direction of propagation of the wave.
(d) **E** is parallel to **B**.



Answers

1. (a) 2. (c) 3. (a), (c), (d)
4. (c) 5. (d)

SECTION 3 Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. The fact that electromagnetic waves are transverse by nature is established by
(a) polarisation (b) interference
(c) reflection (4) diffraction **[2002]**
2. Infrared radiations are detected by
(a) spectrometer (b) pyrometer
(c) manometer (d) photometer **[2002]**
3. The dimensions of $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ are the same as those of
(a) velocity (b) $(\text{velocity})^2$
(c) $\frac{1}{(\text{velocity})}$ (d) $\frac{1}{(\text{velocity})^2}$ **[2003]**
4. Which of the following radiations has the least wavelength?
(a) γ-rays (b) β-rays
(c) α-rays (d) X-rays **[2003]**
5. The earth emits radiations in the infrared region of the spectrum. The spectrum is correctly given by
(a) Rayleigh-Jeans law
(b) Planck's law of radiation
(c) Stefan's law of radiation
(d) Wien's law **[2003]**
6. An electromagnetic wave of frequency $\nu = 3.0$ MHz passes from vacuum into a dielectric medium of relative permittivity $\varepsilon_r = 4.0$. Then
(a) wavelength is doubled and frequency remains unchanged
(b) wavelength is doubled and frequency becomes half
(c) wavelength is halved and frequency remains unchanged
(d) wavelength and frequency both remain unchanged. **[2004]**
7. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead, it is reduced to $I/8$. The thickness of lead, which will reduce the intensity to $I/2$ is
(a) 6 mm (b) 9 mm
(c) 18 mm (d) 12 mm **[2005]**
8. The rms value of the electric field of the light coming from the Sun is 720 N/C. The average total energy density of the electromagnetic wave is
(a) 81.35×10^{-12} J/m^3 (b) 3.3×10^{-3} J/m^3
(c) 4.58×10^{-6} J/m^3 (d) 6.37×10^{-9} J/m^3
[2006]

9. An electromagnetic wave in vacuum has the electric and magnetic field $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then

   (a) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$ (b) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$

   (c) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$ (d) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$

   [2012]

10. The magnetic field in a travelling electromagnetic wave has a peak value of 20nT. the peak value of electric field strength is:

    (a) 6 V/m (b) 9 V/m

    (c) 12 V/m (d) 3 V/m **[2013]**

11. Which of the following pairs of space and time varying $\mathbf{E}\ (=\hat{i}E_x + \hat{\mathbf{j}}E_y + \hat{\mathbf{k}}E_z)$ and $\mathbf{B}\ (=\hat{i}B_x + \hat{\mathbf{j}}B_y + \hat{\mathbf{k}}B_z)$ would generate a plane electromagnetic wave travelling in the z-direction?

    (a) E_x, B_z (b) E_y, B_z

    (c) E_z, B_x (d) E_x, B_y **[2014]**



Answers

1. (a)	2. (b)	3. (b)	4. (a)
5. (a)	6. (c)	7. (d)	8. (c)
9. (b)	10. (a)	11. (d)	



Solutions

1. The phenomena of interference, reflection and diffraction are exhibited by both transverse and longitudinal waves. But only transverse waves can be polarised. Hence the correct choice is (a).
2. The correct choice is (b).
3. $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$, where c = velocity of light. Hence the correct choice is (b).
4. γ-rays have the highest frequency and hence the shortest wavelength.
5. The correct choice is (a).
6. When a wave travels from one medium into another, its frequency never changes.

   For vacuum: $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = \nu\lambda_0$ (1)

   For dielectric medium $v = \dfrac{1}{\sqrt{\mu\varepsilon}} = \nu\lambda$ (2)

   Dividing (2) by (1) we get

   $$\frac{\lambda}{\lambda_0} = \sqrt{\frac{\mu_0 \varepsilon_0}{\mu\varepsilon}}$$

   For non-magnetic media $\mu = \mu_0$. Hence

   $$\frac{\lambda}{\lambda_0} = \sqrt{\frac{\varepsilon_0}{\varepsilon}} = \sqrt{\frac{1}{\varepsilon_r}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

   $$\therefore \lambda = \frac{\lambda_0}{2}$$

   Thus the correct choice is (c).
7. The intensity of radiation of intensity I when it passes through a thickness x of an absorbing medium is given by

   $$I' = I e^{-\mu x}$$

   where μ is a constant. When x_1 = 36 m, $I' = \dfrac{I}{8}$. Therefore,

   $$\frac{I}{8} = I e^{-\mu x}$$

   $$\Rightarrow \quad \frac{1}{8} = e^{-\mu x_1}$$

   or $\quad e^{\mu x_1} = 8$

   $$\Rightarrow \quad \mu x_1 = \ln (8) = 2.08$$

   $$\Rightarrow \quad \mu = \frac{2.08}{x_1}$$

   Let x_2 be the thickness of lead for I to reduce to $I_2 = \dfrac{I}{2}$

   Then

   $$\frac{I}{2} = Ie^{-\mu x_2}$$

   $$\Rightarrow \quad \mu x_2 = \ln (2) = 0.693$$

   $$\Rightarrow \frac{2.08}{x_1} \times x_2 = 0.693$$

   $$\Rightarrow \quad x_2 = \frac{0.693 \times x_1}{2.08}$$

   $$= \frac{0.693 \times 36\,\text{mm}}{2.08} = 11.99\ \text{mm} \simeq 12\ \text{mm}$$
8. The total energy density of an electromagnetic wave is

   $$u = u_e + u_m$$

   where $u_e = \dfrac{1}{2}\ \varepsilon_0 E^2$ and $u_m = \dfrac{B^2}{2\mu_0}$.

   The electric field of the wave is given by

   $$E = E_0 \sin \omega t$$

where E_0 is the peak value of E and ω is the angular frequency. The average energy density associated with electric field is

$$<u_e> = \frac{1}{2}\varepsilon_0 E^2_0 <\sin^2 \omega t>$$

where $<\sin^2 \omega t> = \frac{1}{T}\int_0^T \sin^2 (\omega t)\, dt = \frac{1}{2}$

Hence $<u_e> = \frac{1}{4}\varepsilon_0 E^2_0$

Also $<u_m> = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}(E/c)^2 = \frac{1}{2}\varepsilon_0 E^2$

$$\left(\because\ c = \frac{1}{\sqrt{\mu\varepsilon}}\right)$$

Hence $<u_m> = \frac{1}{4}\varepsilon_0 E^2_0$

Total energy density $<u> = <u_e> + <u_m> = \frac{1}{4}\varepsilon_0 E^2_0 + \frac{1}{4}\varepsilon_0 E^2_0$

$$= \frac{1}{2}\varepsilon_0 E^2$$

$$= \varepsilon_0 E^2_{\text{rms}} \qquad \left(\because\ E_{\text{rem}} = \frac{E_0}{\sqrt{2}}\right)$$

$$= (8.85 \times 10^{-12}) \times (720)^2$$

$$= 4.58 \times 10^{-6}\ \text{J m}^{-3}$$

9. The direction of polarisation vaector $\vec{X}$ is parallel to the plane containing the electric field $\vec{E}$. The propagation vector $\vec{k}$ is perpendicular to $(\vec{E} \times \vec{B})$. So the correct choice is (b).

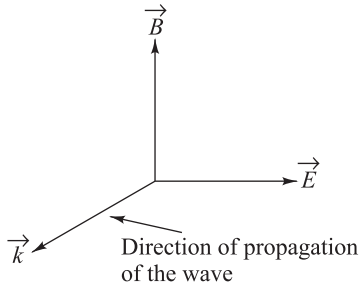


Fig. 15.1

10. $E_0 = cB_0$
$= (3\times10^8) \times (20\times10^{-9})$
$= 6\ \text{Vm}^{-1}$

11. The correct choice is (d).

RAY OPTICS

Chapter 16

REVIEW OF BASIC CONCEPTS

1. Reflection of Light

(i) *Law of Reflection*

Angle of reflection = angle of incidence

A ray incident along the normal to a reflecting surface retraces its path after reflection.

(ii) *Reflection at a plane surface (mirror)*

(a) Distance of image from mirror = distance of object from mirror.

(b) Size of image = size of object.

(c) If the object moves with a certain velocity, the image moves with the same velocity but in the opposite direction.

(d) Keeping the incident ray fixed, if the mirror is rotated through an angle θ, the reflected ray rotates through an angle 2θ.

(e) If two mirrors are inclined at an angle θ (in degrees), the number of images formed by the mirrors of an object is

- $\left(\frac{360}{\theta}-1\right)$ if $\frac{360}{\theta}$ is an even number,
- $\left(\frac{360}{\theta}-1\right)$ if $\frac{360}{\theta}$ is an odd number and the object is placed at the same distance from the mirrors, and
- $\left(\frac{360}{\theta}\right)$ if $\frac{360}{\theta}$ is an odd number and the object is not placed at the same distance from the mirrors.

(f) If three mirrors are placed mutually perpendicular and adjacent to each other, the number of images of an object placed in front of them is 7.

2. Reflection at a Spherical Surface (Concave Mirror or Convex Mirror)

(a) Sign conventions

(i) All distances are measured from the centre (pole) of the mirror.

(ii) Distances measured in the direction of incident rays are taken as positive while those measured opposite to the direction of incident rays are taken as negative.

(iii) Distances above the principal axis are taken as positive while those below the principal axis are taken as negative.

(b) The spherical mirror formula (for both concave and convex mirrors) is

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f};f=\frac{R}{2}, \text{ where}$$

u = object distance, v = image distance, f = focal length and R = radius of curvature. For convex mirror f is positive and for an concave mirror f is negative.

(c) Linear magnification $m=\frac{h_i}{h_0}=-\frac{v}{u}=\frac{f}{f-u}$

$=\frac{f-v}{f}$

For an erect image m is positive and for an inverted image m is negative.

(d) A concave mirror forms a real and inverted image of an object placed beyond its focus and a virtual and erect image if the object is placed between the pole and focus. A convex mirror forms a virtual and erect image for all positions of the object.

3. Snell's Law of Refraction

If a ray of light travelling in a medium of refractive index μ_1 is incident at angle i on the boundary of a medium of refractive index μ_2, then

$$\mu_1 \sin i = \mu_2 \sin r$$

where r is the angle of refraction in medium μ_2.

4. Refractive Index and Speed of Light

$$\mu = \frac{\text{speed of light in air or vacuum}}{\text{speed of light in the medium}} = \frac{c}{v}$$

The value of μ depends upon (i) nature of the medium and (ii) wavelength (colour) of light.

$$\frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$$

5. Refraction of Light through a Plane Slab

When a ray of light passes through a glass slab of thickness t (Fig. 16.1),

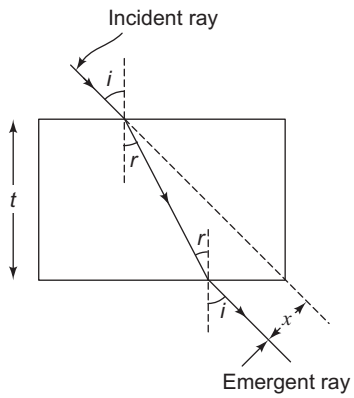


Fig. 16.1

(a) the emergent ray is parallel to the incident ray and

(b) lateral displacement $x = \dfrac{t\sin(i-r)}{\cos r}$

6. Real and Apparent Depth (or Height)

(a) An object in a denser medium (water) viewed by an observer in a rarer medium (air) from above [Fig. 16.2(a)]

Real depth $OA = d$, Apparent depth $AI = d'$ and $\mu = \dfrac{d}{d'}$

Apparent shift $OI = d - d' = d - \dfrac{d}{\mu} = d\left(1 - \dfrac{1}{\mu}\right)$;

μ = refractive index of water.

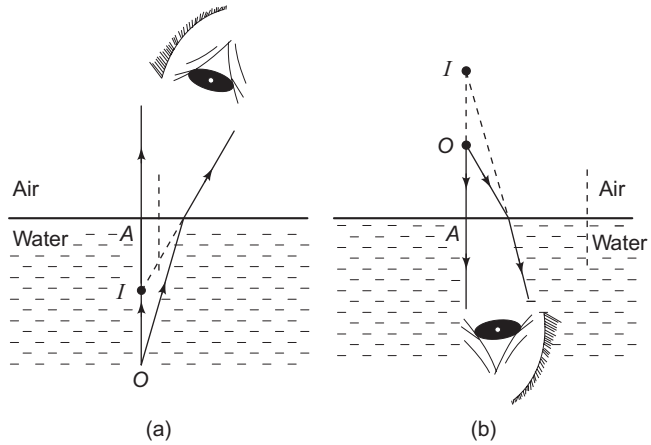


Fig. 16.2

(b) Object in air viewed by observer under water [Fig. 16.2(b)]

Real height $OA = h$, Apparent height $AI = h' = \mu h$.

Apparent shift $OI = h' - h = \mu h - h = (\mu - 1)h$.

7. Total Internal Reflection

For total internal reflection,

(a) the incident ray must travel in a denser medium (μ_2) to the boundary of a rarer medium μ_1 ($< \mu_2$) and

(b) the angle of incidence must be greater than critical angle i_c given by (Fig. 16.3) $\mu_2 \sin i_c = \mu_1 \sin 90°$

$\Rightarrow \sin i_c = \dfrac{\mu_1}{\mu_2}$. If rarer medium is air ($\mu_1 = 1$ and $\mu_2 = \mu$) then

$$\sin i_c = \frac{1}{\mu}$$

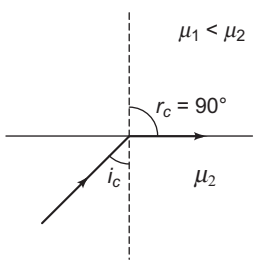


Fig. 16.3

The value of i_c depends upon μ_1, μ_2 and wavelength of light.

8. Refraction through a Prism

When a ray of monochromatic light is refracted by a prism, the deviation δ produced by the prism is (Fig. 16.4)

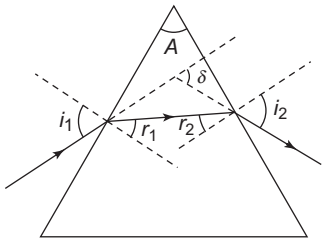


Fig. 16.4

$$\delta = i_1 + i_2 - A$$

where i_1 = angle of incidence, i_2 = angle of emergence and A = refracting angle of the prism. Also

$$A = r_1 + r_2$$

The deviation is minimum = δ_m, if $i_1 = i_2$. Then $r_1 = r_2$. Then

$$\frac{\mu_2}{\mu_1} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

where μ_2 = refractive index of the material of the prism and μ_1 = refractive index of the medium surrounding the prism. If $\mu_1 = 1$ (air) and $\mu_2 = \mu$, then

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

For a thin prism, angles i_1, i_2, r_1, r_2 and A are small and

$$\delta = (\mu - 1)A$$

9. Refraction at a Spherical Surface

If the rays from an object travelling in a medium of refractive index μ_1 are refracted at the spherical surface (convex or concave) of a medium of refractive index μ_2 forming an image, then the object and image distances u and v are related as

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \text{ (with sign convention)}$$

where R = radius of curvature of the spherical surface.

Linear magnification $m = \frac{h_i}{h_0} = \frac{v}{u}$.

If the first medium is air, $\mu_1 = 1$ and $\mu_2 = \mu$, then

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}$$

If the incident rays from the object are in medium μ_2 and are refracted at the spherical surface of medium μ_1, then in the above formula μ_1 and μ_2 are interchanged, i.e.

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

Application: Image formed by a transparent sphere

Case (a): When the object O is in air outside the sphere of radius R. (Fig. 16.5)

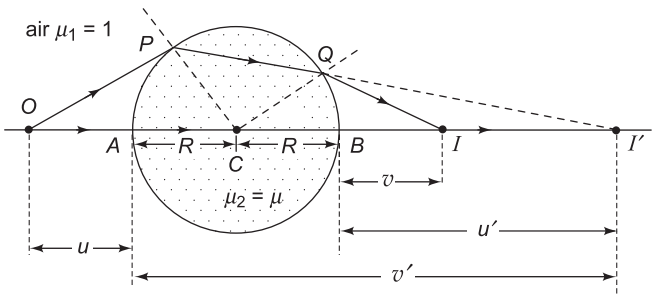


Fig. 16.5

I' is the angle of O due to refraction at P. For this refraction,

$$\frac{\mu}{v'} - \frac{1}{u} = \frac{\mu - 1}{R}$$

(u is negative, and R is positive).

This image I' serves as the virtual object for refraction at Q forming the final image I. For refraction at Q (since the incident rays are in medium μ), we have

$$\frac{1}{v} - \frac{\mu}{u'} = \frac{1 - \mu}{R} \text{ where } u' = v' - 2R$$

(u' is positive and R is negative).

Case (b): When the object O is on the surface of the sphere (Fig. 16.6).

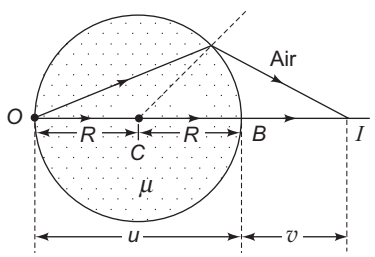


Fig. 16.6

$$\frac{1}{v} - \frac{\mu}{u} = \frac{1 - \mu}{R}$$

where $u = 2R$.

(u is negative and R is also negative).

Case (c): Object O inside the sphere (Fig. 16.7)

$$\frac{1}{v} - \frac{\mu}{u} = \frac{1-\mu}{R}$$

(u is negative and R is also negative)

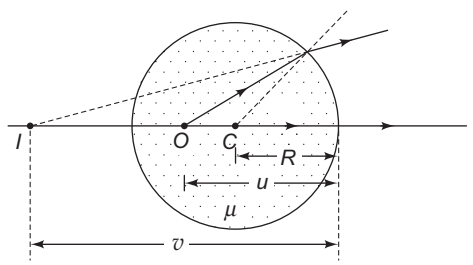


Fig. 16.7

10. Refraction through a Lens

(i) Relation between u, v and f for a lens (convex or concave) is

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ (with sign convention)}$$

Linear magnification $m = \frac{h_0}{h_0} = \frac{v}{u} = \frac{f-v}{f} = \frac{f}{f+u}$

The focal length f is positive for converging (convex) lens and negative for diverging (concave) lens.

(ii) Focal length f of a lens of refractive index μ_2 surrounded by a medium of refractive index μ_1 is given by

$$\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{ (with sign convention)}$$

where R_1 = radius of curvature of the surface of the lens on which the rays are incident

R_2 = radius of curvature of the surface of the lens from which the rays emerge after refraction through the lens.

If the media on the two sides of the lens are different (e.g. a lens floating on water) as shown in Fig. 16.8, then

$$\frac{1}{f} = \frac{\mu_2 - \mu_1}{\mu_3 R_1} + \frac{\mu_3 - \mu_2}{\mu_3 R_2}$$

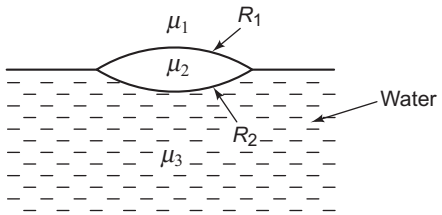


Fig. 16.8

(iii) Power of lens $P = \frac{1}{f \text{(in metre)}}$

(iv) Lens maker's formula

If a lens is surrounded by air, $\mu_1 = 1$ and $\mu_2 = \mu$, then

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

This is the lens maker's formula.

Applications of lens maker's formula

(a) Equi-convex lens (Fig. 16.9)

$$R_1 = +R, R_2 = -R$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$$= (\mu - 1)\frac{2}{R}$$

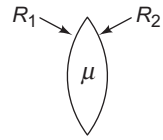


Fig. 16.9

f is positive.

(b) Equi-concave lens (Fig. 16.10)

$$R_1 = -R, R_2 = +R$$

$$\frac{1}{f} = -(\mu - 1)\frac{2}{R}$$

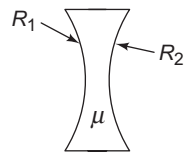


Fig. 16.10

f is negative.

(c) Double convex lens ($R_1 \neq R_2$)

$$R_1 = +R_1, R_2 = -R_2$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right); f \text{ is positive}$$

(d) Double concave lens ($R_1 \neq R_2$)

$$R_1 = -R_1, R_2 = +R_2$$

$$\frac{1}{f} = -(\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right); f \text{ is negative.}$$

(e) Plano-convex lens (Fig. 16.11)

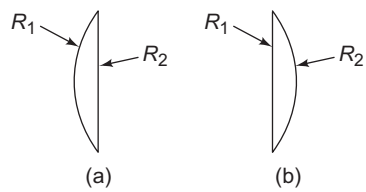


Fig. 16.11

Fig. (a): $R_1 = +R, R_2 = \infty$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{(\mu - 1)}{R}$$

Fig. (b): $R_1 = \infty, R_2 = -R$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{(\mu - 1)}{R}$$

f is positive.

(f) Plano-concave lens (Fig. 16.12)

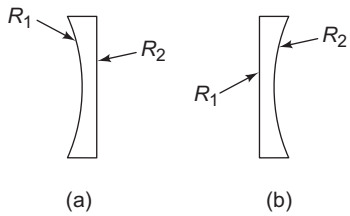


Fig. 16.12

Fig. (a): $R_1 = -R$, $R_2 = \infty$

$$\frac{1}{f} = (\mu - 1)\left(-\frac{1}{R} - \frac{1}{\infty}\right) = -\frac{(\mu - 1)}{R}.$$

Fig. (b): $R_1 = \infty$, $R_2 = +R$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{\infty} - \frac{1}{R}\right) = -\frac{(\mu - 1)}{R}$$

f is negative.

(g) First surface convex, second surface concave (Fig. 16.13)

$$R_1 = +R_1,\ R_2 = +R_2$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

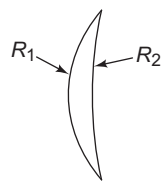


Fig. 16.13

Since $R_2 > R_1$, f is positive

(h) First surface concave, second surface convex (Fig. 16.14)

$$R_1 = -R_1,\ R_2 = -R_2$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

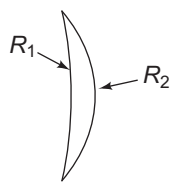


Fig. 16.14

Since $R_1 > R_2$, f is positive.

(i) If a lens of focal length f is cut along AB into two equal pieces as shown in Fig. 16.15(a), the focal length of each piece is $2f$.

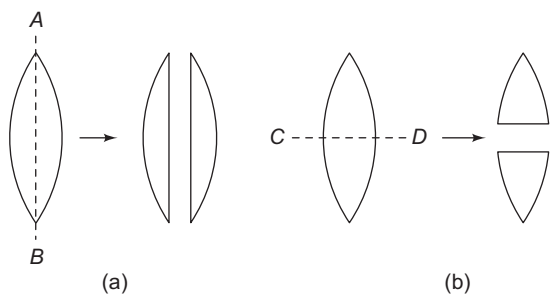


Fig. 16.15

But if the lens is cut along CD as shown in Fig. 16.15(b), the focal length remains the same equal to f.

11. Power of a Lens

The reciprocal of the focal length (expressed in metres) is known as the power of a lens.

$$\text{Power } P = \frac{1}{f\,(\text{in metre})}$$

The SI unit of P is called diopre (D)

If the lens is placed in medium of refractive index μ_m, then

$$P = \frac{\mu_m}{f_m\,(\text{in metre})}$$

where f_m = focal length in medium.

12. Co-axial Combination of Lenses

If two thin lenses of focal lengths f_1 and f_2 are placed in contact co-axially (i.e. with their principal axes coinciding), the equivalent focal length F of the combination is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

Power of combination $P = P_1 + P_2$

If the lenses are separated by a distance d,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

and $$P = P_1 + P_2 - P_1\, P_2 d$$

13. Effect of Silvering One of the Refracting Faces of a Lens

If one surface of a convex lens is silvered, it behaves like a concave mirror. If one surface of a concave lens is silvered, it behaves like a convex mirror. In Fig. 16.16, the rays are refracted at surface 1, reflected at surface 2 and again refracted at surface 1. The effective focal length F is given by

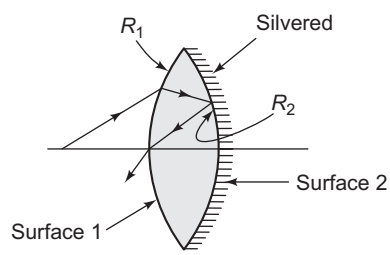


Fig. 16.16

$$\frac{1}{F} = \sum_{n=1,2} \frac{1}{f_n}$$

where f_n = focal length of lens or mirror repeated as many times as there are refractions and reflections. In the case shown in the figure, there are two refractions and one reflection. Hence

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} + \frac{1}{f_m} = \frac{2}{f} + \frac{1}{f_m}$$

where f = focal length of the lens and f_m = focal length of the spherical mirror of radius of curvature R_2.

(a) If the face of radius of curvature R_2 of a double convex lens is silvered (Fig. 16.16),

$$\frac{1}{F} = \frac{2}{f} + \frac{1}{f_m}$$

where $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

and $f_m = \frac{R_2}{2}$

$\therefore$ $\frac{1}{F} = 2(\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{2}{R_2}$

(b) If the lens is equi-convex, then $R_1 = R_2 = R$

$$\frac{1}{F} = 2(\mu - 1)\frac{2}{R} + \frac{2}{R} = \frac{2(2\mu - 1)}{R}$$

$\Rightarrow$ $F = \frac{R}{2(2\mu - 1)}$

(c) If the plane face of a plano-convex lens is silvered (Fig. 16.17), then

$(R_1 = R, R_2 = \infty)$

$$\frac{1}{F} = \frac{2}{f} + \frac{1}{f_m}$$

where $f = \frac{(\mu - 1)}{R}$

and $f_m = \infty$

Thus $F = \frac{f}{2} = \frac{R}{2(\mu - 1)}$

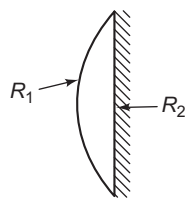


Fig. 16.17

(d) If the curved face of a plano-convex lens is silvered (Fig. 16.18), then $R_1 = \infty$ and $R_2 = R$.

$$\frac{1}{F} = \frac{2}{f} + \frac{1}{f_m}$$

where $\frac{1}{f} = \frac{(\mu - 1)}{R}$

and $f_m = \frac{R}{2}$

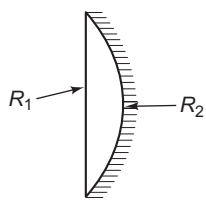


Fig. 16.18

Thus $\frac{1}{F} = \frac{2(\mu - 1)}{R} + \frac{2}{R} = \frac{2\mu}{R}$

$\Rightarrow$ $F = \frac{R}{2\mu}$

EXAMPLE 1 A driver stops his car at a red light. The car is fitted with side view mirror of focal length 10 m. An ambulance is approaching the car at a constant speed of 16 ms^{-1}. The speed of image of the ambulance, as seen by the driver in the side view mirror, when it is at distance of 50 m from the mirror will be

(a) 1 ms^{-1} (b) $\frac{4}{9}$ ms^{-1}

(c) $\frac{2}{3}$ ms^{-1} (d) 2 ms^{-1}

SOLUTION The side view mirror of a car is a convex mirror. Given f = +10 m and u = −50 cm. Using these values in $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we have

$$\frac{1}{v} + \frac{1}{-50} = +\frac{1}{10} \Rightarrow v = +\frac{50}{6}\text{m}$$

Differentiating $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ with respect to t, we have

$$-\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

$\Rightarrow$ $\frac{dv}{dt} = -\left(\frac{v^2}{u^2}\right)\frac{du}{dt}$

or $\left|\frac{dv}{dt}\right| = \frac{v^2}{u^2} \times \frac{du}{dt}$

$\therefore$ Speed of image = $\frac{v^2}{u^2}$ × speed of ambulance (object)

$$= \frac{(50/6)^2}{(50)^2} \times 16$$

$$= \frac{4}{9}\ ms^{-1}$$

EXAMPLE 2 A rod AB of length 5 cm is placed in front of a concave mirror of focal length 10 cm as shown in Fig. 16.19(a). The length of the image of AB formed by the mirror is

(a) 2.5 cm (b) 7.5 cm

(c) 10 cm (d) 12.5 cm

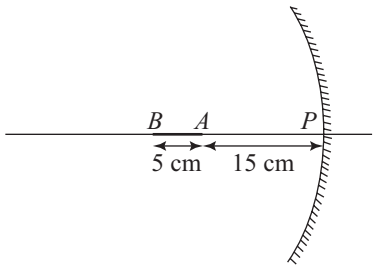


Fig. 16.19(a)

SOLUTION $f = -10$ cm. For end A, $v_A = -15$ cm.

Using $\frac{1}{v_A} + \frac{1}{u_A} = \frac{1}{f}$ we have

$$\frac{1}{v_A} - \frac{1}{15} = -\frac{1}{10} \Rightarrow v_A = -30 \text{ cm}$$

The negative sign shows that the image A' of end A is at a distance of 30 cm to the left of P [see Fig. 16.19(b)].

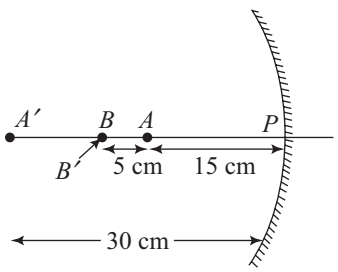


Fig. 16.19(b)

For end B, $u_B = -20$ cm. Thus

$$\frac{1}{v_B} - \frac{1}{20} = -\frac{1}{10} \Rightarrow v_B = -20 \text{ cm}$$

The negative sign shows that the image B' of end B is at a distance of 20 cm to the left of P, i.e. B' coincides with B. This is expected because (since $R = 2f = 20$ cm), B is at the centre of curvature of the mirror. The length of the image of rod AB is

$$A'B' = 30 - 20 = 10 \text{ cm}$$

EXAMPLE 3 A vessel of depth t is half filled with oil of refractive index μ_1 and the other half is filled with water of refractive index μ_2. Find the apparent depth of the vessel when viewed from above.

SOLUTION Refer to Fig. 16.20.

Given $t_1 = t_2 = \frac{t}{2}$

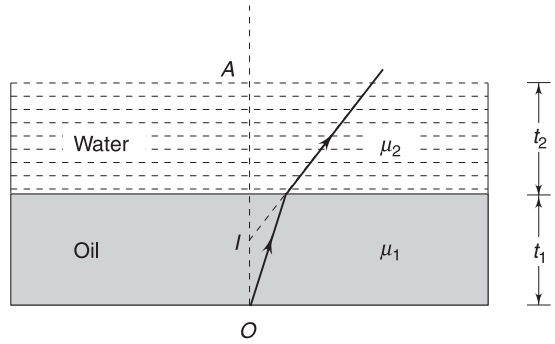


Fig. 16.20

The apparent depth is given by

$$AI = \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2} = \frac{t}{2}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)$$

$$= \frac{t(\mu_1 + \mu_2)}{2\,\mu_1\,\mu_2}$$

EXAMPLE 4 A ray of light incident normally on face AB of an isosceles prism travels as shown in Fig. 16.21. What is the least value of the refractive index of the prism?

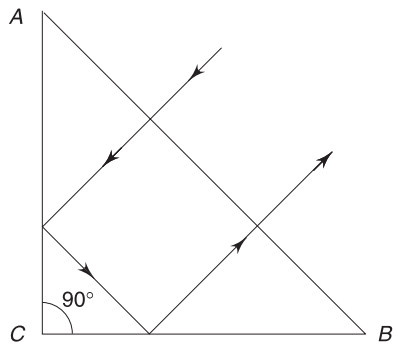


Fig. 16.21

SOLUTION The ray is totally reflected at faces AC and CB. The angle of incident at each face is at least $i_c = 45°$. Therefore

$$\mu = \frac{1}{\sin i_c} = \frac{1}{\sin 45°} = \sqrt{2}$$

EXAMPLE 5 A small bulb is placed at the bottom of a tank containing a liquid up to a depth d. If the refractive index of the liquid is $\sqrt{2}$, the area of the surface of the liquid through which light from the bulb can emerge out is

(a) $\frac{\pi d^2}{2}$ (b) πd^2

(c) d^2 (d) $\frac{d^2}{4}$

SOLUTION Refer to Fig. 16.22.

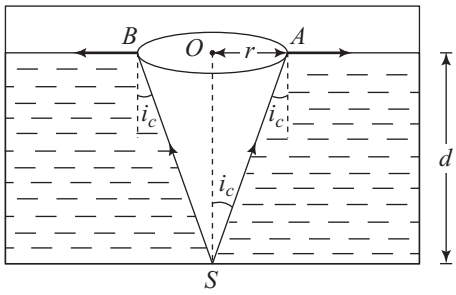


Fig. 16.22

It is clear from the figure that light from bulb S can emerge out of a circular patch of diameter AB. Rays from S which are incident on the liquid surface at an angle $i > i_c$, i.e. beyond A and B are totally reflected within the liquid and cannot emerge out of the surface. The critical angle i_c is given by

$$\sin i_c = \frac{1}{\mu} = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad i_c = 45°$$

In ΔOAS, $\tan i_c = \frac{r}{d}$ or $\tan 45° = \frac{r}{d} \quad \Rightarrow \quad r = d$, where r is the radius of the circular patch the area of which is

$$\pi r^2 = \pi d^2$$

So the correct choice is (b).

EXAMPLE 6 Figure 16.23 shows an optical fibre made of glass core of refractive index $\mu_g = \sqrt{3}$ having an outer covering (called cladding) of refractive index $\mu_c = 1.5$. Total internal reflections will occur at the core - cladding interface if the angle of incidence (i) is less than

(a) 30° (b) 45°

(c) 60° (d) 75°

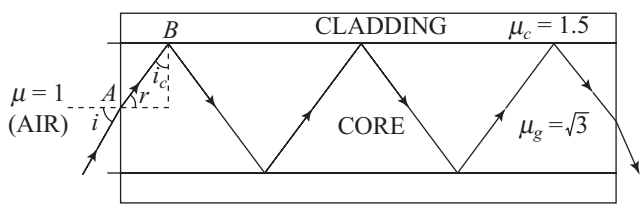


Fig. 16.23

SOLUTION The critical angle is given by

$$\sin i_c = \frac{\mu_c}{\mu_g} = \frac{1.5}{\sqrt{3}}$$

$$\Rightarrow \qquad \sin i_c = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad i_c = 60°$$

For total internal reflection, the angle of incident at B must be greater than 60°. Angle r which is $(90° - i_c)$ must be less than 30°, i.e. $r_{max} = 30°$. Applying Snell's law at A, we have

$$1 \times \sin i_{max} = \sqrt{3} \sin r_{max}$$

$$= \sqrt{3} \sin 30°$$

$$= \frac{\sqrt{3}}{2}$$

which gives $i_{max} = 60°$. So the correct choice is (c).

EXAMPLE 7 A prism produces a minimum deviation of 50° for a certain angle of incidence. The same prism produces a deviation of 63° for two angles of incidence, namely, 40° and 83°. If sin 55° = 0.82, the refractive index of the material of the prism is

(a) 1.50 (b) 1.56

(c) 1.64 (d) 1.72

SOLUTION Refer to Fig. 16.4 on page 16.3. It follows from $\delta = i_1 + i_2 - A$ that angles i_1 and i_2 are interchangeable. This means that if the emergent ray is reversed, the deviation remains the same. Hence

$$63° = 40° + 83° - A \quad \Rightarrow \quad A = 60°$$

$$\mu = \frac{\sin\left[\frac{1}{2}(A + \delta_m)\right]}{\sin\left(\frac{A}{2}\right)}$$

$$= \frac{\sin\left[\frac{1}{2}(60° + 50°)\right]}{\sin 30°} = \frac{\sin 55°}{\sin 30°}$$

$$= \frac{0.82}{1/2} = 1.64$$

EXAMPLE 8 An object AB of height h_1 is placed in a medium of refractive index μ_1 at a distance u from a convex refracting surface of radius of curvature R and refractive index μ_2 where $\mu_2 > \mu_1$. A real image $A'B'$ of height h_2 is formed at a distance v as shown in Fig. 16.24. The linear magnification is

(a) $\frac{\mu_1 u}{\mu_2 v}$ (b) $\frac{\mu_2 u}{\mu_1 v}$

(c) $\frac{\mu_1 v}{\mu_2 u}$ (d) $\frac{\mu_1 v}{\mu_2 u}$

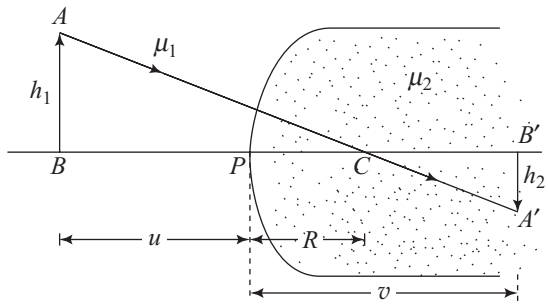


Fig. 16.24

SOLUTION By definition, $m = \dfrac{h_2}{h_1}$. According to the sign convention $AB = +h_1$ and $A'B' = -h_2$. Therefore,

$$m = -\frac{A'B'}{AB}$$

Triangles ACB and $A'C'B'$ are similar. So

$$m = -\frac{B'C}{BC} = -\frac{PB' - PC}{PB + PC}$$

$$= -\frac{v - R}{-u + R} = \frac{R - v}{R - u} \qquad \text{(i)}$$

The spherical surface formula is

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \quad \frac{u\mu_2 - v\mu_1}{uv} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \quad R = \frac{(\mu_2 - \mu_1)uv}{\mu_2 u - \mu_1 v} \qquad \text{(ii)}$$

From (ii) we get

$$R - v = \frac{\mu_1 v(v - u)}{\mu_2 u - \mu_1 v} \qquad \text{(iii)}$$

and

$$R - u = \frac{\mu_2 u(v - u)}{\mu_2 u - \mu_1 v} \qquad \text{(iv)}$$

Using (iii) and (iv) in (i) we get

$$m = \frac{\mu_1 v}{\mu_2 u}$$

So the correct choice is (c).

EXAMPLE 9 A glass sphere of refractive index 1.5 and radius R has a small air bubble at a distance $R/2$ from point A on its surface. It is viewed as shown in Fig. 16.25(a). The distance of the image of the bubble from A will be

(a) $\dfrac{R}{5}$ (b) $\dfrac{2R}{5}$

(c) $\dfrac{3R}{5}$ (d) $\dfrac{4R}{5}$

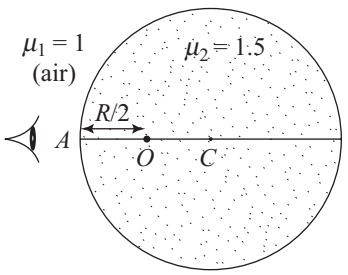


Fig. 16.25(a)

SOLUTION Figure 16.25(b) shows the ray diagram. I is the image of O.

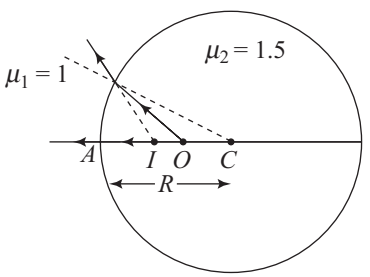


Fig. 16.25(b)

The observer sees the image due to refraction at the spherical surface when rays from object O travel from glass to air. Therefore, the formula to be used is

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

Here $\mu_1 = 1, \mu_2 = 1.5, u = AO = -\dfrac{R}{2}$ and $R = AC = -R$. Thus

$$\frac{1}{v} - \frac{1.5}{-R/2} = \frac{1 - 1.5}{-R}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{2R} - \frac{3}{R} = -\frac{5}{2R}$$

$$\Rightarrow \quad v = -\frac{2R}{5}$$

The negative sign shows that the image I is virtual and to the left of O.

EXAMPLE 10 In Example 9 above, the image of air bubble O is seen by the observer from diametrically opposite end B as shown in Fig. 16.26. The distance of the image from point A will be

(a) zero
(b) equal to R to the left of A in air
(c) at centre C of the sphere
(d) at point B.

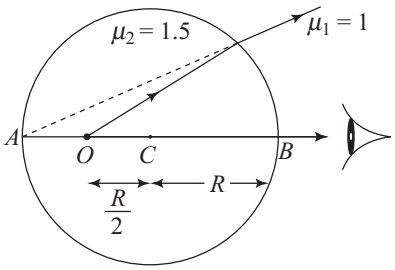


Fig. 16.26

SOLUTION Rays from O travel from glass to air. Hence

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

Here $u = BO = -\left(R + \frac{R}{2}\right) = -\frac{3R}{2}$, $R = BC = -R$, $\mu_1 = 1$ and $\mu_2 = 1.5$.

$$\frac{1}{v} - \frac{1.5}{-3R/2} = \frac{1-1.5}{-R}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{2R} - \frac{1}{R} = -\frac{1}{2R}$$

$$\Rightarrow \quad v = -2R$$

Thus the virtual image is seen at a distance of $2R$ to the left of B, i.e. the image is seen at A. So the correct choice is (a).

EXAMPLE 11 The linear magnification of an object placed on the principal axis of a convex lens of focal length 30 cm is found to be + 2. In order to obtain a magnification of –2, by how much distance should the object be moved?

SOLUTION In the first case, the magnification is positive which implies that the image is erect, virtual and on the same side of the lens as the object. If a is the object distance then $u = -a$ and $v = -2a$. From the lens formula, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or $$\frac{1}{-2a} - \frac{1}{-a} = \frac{1}{30} \Rightarrow a = 15 \text{ cm}$$

So the object is at a distance of 15 cm from the lens. In the second case, the magnification is negative, the image is real, inverted and on the other side of the lens as the object. If b is the object distance, the $u = -b$ and $v = +2b$. Hence

$$\frac{1}{2b} - \frac{1}{-b} = \frac{1}{30} \Rightarrow b = 45 \text{ cm}$$

Thus the object has to be moved through a distance of (45 – 15) = 30 cm away from the lens.

EXAMPLE 12 A parallel beam of light falls on a convex lens. The path of the rays is shown in Fig. 16.27. It follows that

(a) $\mu_1 > \mu > \mu_2$ (b) $\mu_1 < \mu < \mu_2$
(c) $\mu_1 = \mu < \mu_2$ (d) $\mu_1 = \mu > \mu_2$

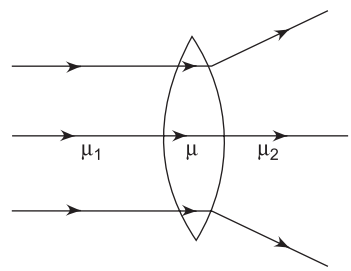


Fig. 16.27

SOLUTION Since the ray does not bend at the first face of the lens, i.e. it is not refracted, $\mu_1 = \mu$. When this ray meets the second face, it bends towards the normal to that face. Hence $\mu_2 > \mu$. Hence the correct choice is (c).

EXAMPLE 13 The distance between the object and the real image formed by a convex lens is d. If the linear magnification is m, find the focal length of the lens in terms of d and m.

SOLUTION The convex lens formula for a real image is

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad \text{(i)}$$

where no sign conventions are to be used. Multiplying by u we get

$$\frac{u}{v} + 1 = \frac{u}{f} \text{ or } \frac{1}{m} = \frac{u}{f} - 1 = \frac{u-f}{f}$$

or $$m(u - f) = f$$

or $$u = \frac{(1+m)f}{m} \qquad \text{(ii)}$$

Multiplying (i) by v we get

$$1 + \frac{v}{u} = \frac{v}{f} \text{ or } 1 + m = \frac{v}{f}$$

or $$v = f(1 + m) \quad \text{(iii)}$$

Now $u + v = d$. Using (ii) and (iii) we have

$$d = \frac{(1+m)f}{m} + f(1 + m)$$

which gives $$f = \frac{md}{(1+m)^2}.$$

EXAMPLE 14 A prism PQR of refractive index $\sqrt{2}$ has the refracting angle of 30°. Face PR of the prism is silvered. Find the angle of incidence i on face PQ so that the ray retraces its path after reflection from face PR (Fig. 16.28).

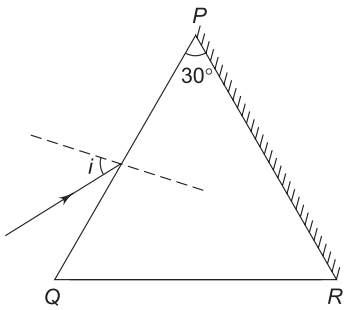


Fig. 16.28

SOLUTION Refer to Fig. 16.29. The ray AB is refracted along BC in the prism. This ray will retrace its path if it falls normally on the silvered face PR. Therefore, $\angle PBC = 60°$ and angle $r = 90° - 60° = 30°$.

From Snell's law

$$\mu = \frac{\sin i}{\sin r} \Rightarrow \sin i = \mu \sin r = \sqrt{2} \times \sin 30° = \frac{1}{\sqrt{2}}$$

which gives $i = 45°$.

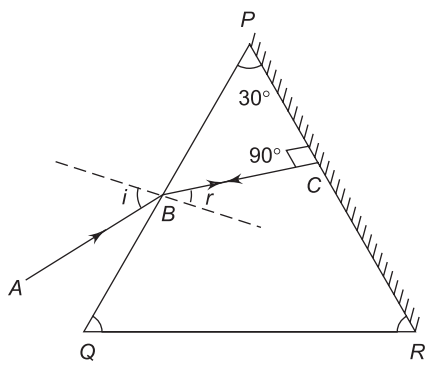


Fig. 16.29

EXAMPLE 15 A prism is made of glass of refractive index $\sqrt{3}$. The refracting angle of the prism is A. If the angle of minimum deviation is equal to A, find the value of A.

SOLUTION Given $\mu = \sqrt{3}$ and $\delta_m = A$. Now

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{A+A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin A}{\sin\left(\frac{A}{2}\right)}$$

$$= \frac{2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = 2\cos\left(\frac{A}{2}\right)$$

$$\therefore \quad 2\cos\left(\frac{A}{2}\right) = \mu = \sqrt{3} \text{ or } \cos\left(\frac{A}{2}\right) = \frac{\sqrt{3}}{2}$$

or $$\frac{A}{2} = 30° \text{ or } A = 60°$$

EXAMPLE 16 The refractive index of the material of an equilateral prism is $\sqrt{3}$. What is the angle of minimum deviation?

SOLUTION Refractive index $\mu = \dfrac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$.

Now $A = 60°$ and $\mu = \sqrt{3}$. Therefore

$$\sqrt{3} = \frac{\sin\left(30° + \frac{\delta_m}{2}\right)}{\sin 30°}$$

or $$\sin\left(30° + \frac{\delta_m}{2}\right) = \sqrt{3} \sin 30° = \frac{\sqrt{3}}{2}$$

or $$30° + \frac{\delta_m}{2} = 60° \text{ which gives } \delta_m = 60°$$

EXAMPLE 17 A convex lens of glass ($\mu_g = 1.5$) has a focal length of 15 cm in air. The lens is immersed in (a) water ($\mu_w = 4/3$) and (b) in carbon bisulphide ($\mu_c = 1.65$). Find the focal length of the lens and determine whether it behaves as a converging lens or a diverging lens.

SOLUTION Given $f_a = +15$ cm (convex lens)

In air, $$\frac{1}{f_a} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{(i)}$$

In water, $$\frac{1}{f_w} = \left(\frac{\mu_g - \mu_w}{\mu_w}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{(ii)}$$

In carbon bisulphide, $$\frac{1}{f_c} = \left(\frac{\mu_g - \mu_c}{\mu_c}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{(iii)}$$

(a) Dividing (i) by (ii) we get

$$f_w = f_a \times \frac{(\mu_g - 1)\,\mu_w}{(\mu_g - \mu_w)}$$

$$= (+15) \times \frac{(1.5-1)\times 4/3}{(1.5-4/3)} = +60 \text{ cm}$$

The plus sign shows that the lens behaves as a converging lens.

(b) Dividing (i) by (iii) we get

$$f_c = f_a \times \frac{(\mu_g - 1)\,\mu_c}{(\mu_g - \mu_c)}$$

$$= (+15) \times \frac{(1.5-1)\times 1.65}{(1.5-1.65)} = -82.5 \text{ cm}$$

The negative sign shows that, in this case, the convex lens behaves as a diverging lens.

EXAMPLE 18 Figure 16.30 shows a lens of refractive index $\mu = 1.5$. C_1 and C_2 are the centres of curvature of the two faces of the lens of radii of curvature 10 cm and 15 cm respectively.

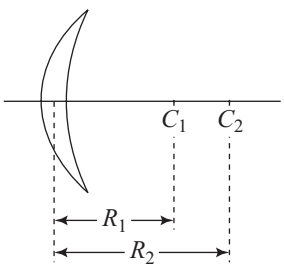


Fig. 16.30

The lens behaves as a

(a) converging lens of focal length 60 cm
(b) converging lens of focal length 12 cm
(c) diverging lens of focal length 60 cm
(d) diverging lens of focal length 12 cm

SOLUTION $R_1 = +10$ cm, $R_2 = +15$ cm and $\mu = 1.5$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5-1)\left(\frac{1}{10} - \frac{1}{15}\right)$$

$$\Rightarrow \quad f = +60 \text{ cm}$$

So the correct choice is (a).

EXAMPLE 19 An object and a screen are mounted on an optical bench 80 cm apart. A convex lens is mounted between them. It is found that a sharp image of the object is obtained on the screen for two different positions of the lens separated by 20 cm. Find the focal length of the lens.

SOLUTION Refer to Fig. 16.31. Let L_1 and L_2 be the two positions of the lens for which a sharp image is formed on the screen. For a convex lens forming a real image, we have

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad \text{(i)}$$

where no sign conventions are used. It follows from this formula that u and v are interchangeable.

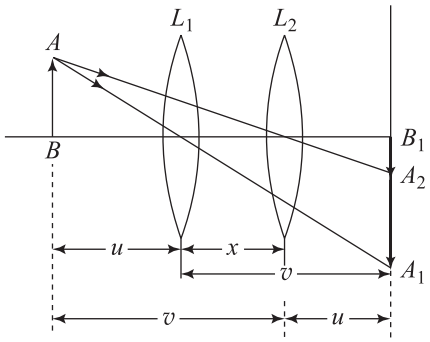


Fig. 16.31

It follows from the figure that $u + x + u = 80$ or $2u + 20 = 80$ ($\because x = 20$ cm) which gives $u = 30$ cm. Hence $v = 80 - 30 = 50$ cm. Using these values in (i) we have

$$\frac{1}{50} + \frac{1}{30} = \frac{1}{f} \quad \Rightarrow \quad f = 18.75 \text{ cm}$$

EXAMPLE 20 A screen is placed at a distance of 60 cm from an object and a convex lens is held between them. What is the maximum possible focal length of the lens which can form a sharp image of the object on the screen.

SOLUTION Since the image is formed on the screen, it must be real. For a convex lens forming a real image we have

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

If x is the distance between the object and the screen, then for a given value of v, the value of $u = x - v$. Using these values in the above formula

$$\frac{1}{v} + \frac{1}{x-v} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{x - v + v}{v(x-v)} = \frac{1}{f}$$

$$\Rightarrow \quad v^2 - xv + xf = 0$$

The roots of this equation are

$$v = \frac{x}{2} \pm \frac{1}{2}\sqrt{x^2 - 4xf}$$

For a real image, v must be real. Hence $x^2 > 4xf$ or $f < \frac{x}{4}$.

So $f_{max} = \frac{x}{4} = \frac{60\text{ cm}}{4} = 15\text{ cm}.$

EXAMPLE 21 An object is placed 15 cm from a convex lens A of focal length 10 cm and its image I_1 is formed on a screen. A concave lens B is now placed midway between A and I_1. The screen had to be moved back 10 cm to obtain a sharp image I_2 of the same object. Find

(a) the focal length of the concave lens and

(b) the linear magnification of the final image.

SOLUTION Refer to Fig. 16.32.

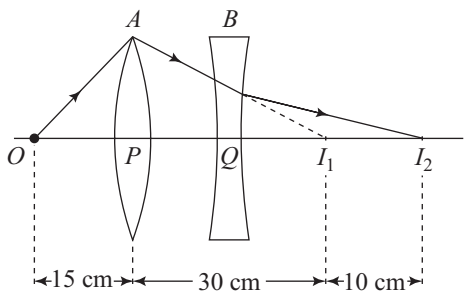


Fig. 16.32

(a) For lens A; $u_A = -15$ cm, $f_A = +10$ cm and $v_A = PI_1$.

Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$,

$$\frac{1}{v_A} - \frac{1}{u_A} = \frac{1}{f_A}$$

$$\Rightarrow \quad \frac{1}{v_A} - \frac{1}{-15} = \frac{1}{10} \quad \Rightarrow \quad v_A = +30\text{ cm}$$

For lens B; I_1 serves as the virtual object. Given that $QI_1 = \frac{1}{2}PI_1 = \frac{30}{2} = 15\text{ cm}.$ Hence u_B = + 15 cm, $v_B = Q\,I_2 = 15 + 10 = 25$ cm. Using $\frac{1}{v_B} - \frac{1}{u_A} = \frac{1}{f_B}$, we have

$$\frac{1}{25} - \frac{1}{15} = \frac{1}{f_B} \quad \Rightarrow \quad f_B = -37.5\text{ cm}$$

(b) Linear magnification of the final image is

$$m = m_A \times m_B$$

$$= \frac{v_A}{u_A} \times \frac{v_B}{u_B}$$

$$= \frac{+30}{-15} \times \frac{25}{15} = -\frac{10}{3}$$

EXAMPLE 22 A thin convex lens of refractive index 1.5 having the two curved surfaces of equal radius of curvature R is placed on a plane mirror. A horizontal small needle with its tip A is moved along the principal axis. It is found that the image of A coincides with it when the needle is 30.0 cm above the lens. The lens is now removed and a liquid drop is put on the mirror. The lens is placed on the liquid. It is found that the needle had to be moved up by a distance of 15.0 cm for its image to coincide with the needle. Find the refractive index of the liquid.

SOLUTION Refer to Fig. 16.33(a). For the image of A to coincide with A itself, the ray PQ emerging from the lens must retrace its path after reflection from the mirror, i.e. ray PQ must fall normally on the mirror. By definition, the OA = focal length of the lens = 30.0 cm. Now $R_1 = +R$, $R_2 = -R$ and $f = +30.0$ cm. Using

$$\frac{1}{f} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right),$$

$$\frac{1}{30.0} = (1.5 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{0.5 \times 2}{R} = \frac{1}{R}$$

which gives R = 30.0 cm.

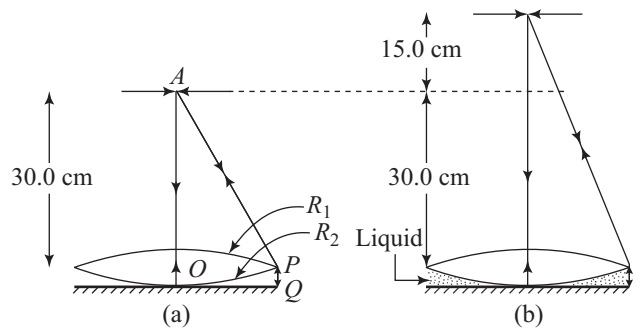


Fig. 16.33

Similarly the focal length of the combination of the convex lens and the plano concave liquid lens [see Fig. 16.33(b)] is $F = 30.0 + 15.0 = 45.0$ cm. If f' is the focal length of the concave lens, then

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f'}$$

$$\Rightarrow \quad \frac{1}{f'} = \frac{1}{F} - \frac{1}{f} = \frac{1}{45.0} - \frac{1}{30.0} = -\frac{1}{90}$$

$$\Rightarrow \quad f' = -90.0\text{ cm}$$

For the concave lens $R_1 = -30.0$ cm and $R_2 = \infty$. If μ_1 is the refractive index of the liquid, then

$$\frac{1}{f'} = (\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad -\frac{1}{90.0} = (\mu_1 - 1)\left(-\frac{1}{30.0}\right)$$

$$\Rightarrow \quad \mu_1 = \frac{4}{3} = 1.33$$

EXAMPLE 23 A thin convex lens of refractive index 1.5 has the two curved surfaces of equal radii of curvature of 30 cm each. One of the surfaces is silvered. An object is placed at a distance of 15 cm from the lens. Find the distance of the image from the lens.

SOLUTION Refer to Fig. 16.34. The rays from object O suffer refraction at A, reflection at B and another refraction at A.

For refraction at A, we use

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Here $\mu_2 = 1.5$, $\mu_1 = 1$, $u = -15$ cm and $R = +R_1 = +30$ cm.

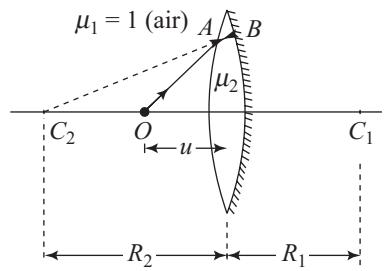


Fig. 16.34

$$\frac{1.5}{v} - \frac{1}{-15} = \frac{1.5 - 1}{30}$$

$$\Rightarrow \quad \frac{1.5}{v} = \frac{0.5}{30} - \frac{1}{15} \quad \Rightarrow \quad v = -30 \text{ cm}, \text{ i.e. the}$$

image is formed at C_2.

For reflection at B. The silvered face acts as a concave mirror. The image at C_2 due to refraction at A acts as a virtual object. To find the distance of image due to reflection at B, we use

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Here $u = -30$ cm and $f = -\frac{R}{2} = -\frac{30}{2} = -15$ cm.

$$\frac{1}{v} + \frac{1}{-30} = \frac{1}{-15} \quad \Rightarrow \quad v = -30 \text{ cm}$$

This image at C_2 serves as the virtual object for the second refraction at A.

For second refraction at A. The ray BA is in μ_2. Hence for this refraction, we use the formula

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

Here $u = -30$ cm, and $R = -R_2 = -30$ cm.

$$\frac{1}{v} - \frac{1.5}{-30} = \frac{1 - 1.5}{-30}$$

$$\Rightarrow \quad v = -15 \text{ cm}$$

i.e. the image coincides with the object.

EXAMPLE 24 A convex lens forms a sharply focussed image of an object at infinity on a screen held at a distance of 10 cm from it. A glass slab of refractive index 1.5 and thickness 3 cm is placed between the lens and the screen. Where should the object be placed now so that its image is again formed on the screen?

SOLUTION It is clear that the focal length of the lens is 10 cm. The rays parallel to the principal axis will get focussed at I (Fig. 16.35). Let O be the position of the object so that the image is again formed on the screen when the slab is inserted.

Lateral shift $I_1 I = t\left(1 - \frac{1}{\mu}\right)$

$$= 3 \text{ cm} \times \left(1 - \frac{1}{1.5}\right) = 1.0 \text{ cm}$$

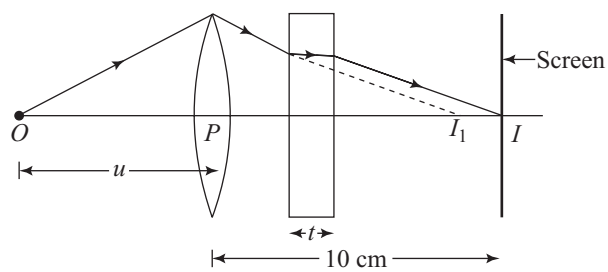


Fig. 16.35

In the absence of the slab, the image of O will be formed at I_1. Hence the lens must form the image at distance $v = 10 - 1.0 = 9.0$ cm from itself. Using

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \text{ we have}$$

$$\frac{1}{9}-\frac{1}{u}=\frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{u}=\frac{1}{9}-\frac{1}{10}=\frac{1}{90}$$

$$\Rightarrow \quad u = 90 \text{ cm}$$

14. Compound Microscope

In its simplest form, a compound microscope consists of two convergent (convex) lenses, of a very short focal length called the *objective* and the other of a longer focal length called the *eye-piece*. The lenses are mounted co-axially and the separation between them can be varied.

The magnifying power of a microscope is

M = magnification by objective × magnification by eye-piece

$= m_0 \times m_e$

(a) If the final image is formed at the least distance of distinct vision (strained eye)

$$|M| = \frac{v_0}{u_0}\left(1+\frac{D}{f_e}\right)$$

where u_0 = distance of object from objective, v_0 = distance (from the objective) of the image formed by objective, f_e = focal length of eye-piece and D = least distance of distinct vision.

(b) If the final image is formed at infinity (relaxed eye)

$$|M| = \frac{v_0}{u_0}\frac{D}{f_e}$$

15. Telescope

A telescope consists of two convergent lenses called the objective and the eye-piece. The focal length of the objective is much larger than that of the eye-piece.

(a) The magnifying power of a telescope in normal adjustment (i.e. when the image of a distant object is formed at infinity), i.e. for relaxed eye

$$|M| = \frac{f_0}{f_e}$$

f_0 = focal length of objective, f_e = focal length of eye-piece

L = length of tube = $f_0 + f_e$.

(b) When the final image is formed at the least distance of distinct vision (i.e. for strained eye)

$$|M| = \frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$$

EXAMPLE 25 A wire mesh consists of squares each of side 1 mm. It is placed at a distance of 9.0 cm from a magnifying glass of focal length 10 cm. It is viewed by placing the eye close to the lens. (a) What is the magnification produced by the lens? (b) What is the area of each square in the virtual image of the mesh? (c) What is the magnifying power of the lens? (d) Is the magnification (a) equal to the magnifying power in (c)? If not, when can they be made equal to each other? Explain.

SOLUTION A magnifying glass is just a convex lens. Therefore $f = +10$ cm. Also $u = -9.0$ cm.

(a) The lens formula is

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

Putting $f = +10$ cm and $u = -9.0$ cm, we have

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{10}-\frac{1}{9.0}$$

Which gives $v = -90$ cm. Therefore the magnification is

$$m=\frac{v}{u}=\frac{-90}{-9.0}=+10$$

(b) Area of each square in the mesh is 1 mm × 1 mm = 1 mm^2. Since each side is magnified 10 times, the area of each square in the image is 10 mm × 10 mm = 100 mm^2

(c) The magnifying power of a magnifying glass is

$$M=\frac{D}{x}$$

where x is the numerical value of the distance of the object from the lens. Here x = 9.0 cm. Also D, the least distance of distinct vision is 25 cm. Therefore.

$$M=\frac{25}{9.0}=2.8$$

(d) Magnification found in (a) is different from magnifying power obtained in (c). They are different quantities. However, they become equal to each other in the special situation when the distance of the object is such that its image is formed at the least distance of distinct vision (D). Putting $v = -D = -25$ cm and $f = +10$ cm in the lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

we have $-\frac{1}{u}=\frac{1}{10}+\frac{1}{25}=\frac{7}{50}$ or $u=-\frac{50}{7}$ cm

$$\Rightarrow \qquad x = \frac{50}{7}\text{ cm}$$

Therefore the magnification is

$$m = \frac{v}{u} = \frac{-25}{-50/7} = +3.5$$

and the magnifying power is

$$M = \frac{D}{x} = 1 + \frac{D}{f} = 1 + \frac{25}{10} = 3.5$$

EXAMPLE 26 Find the magnifying power of a compound miscroscope whose objective and eyepiece are of focal lengths 4.0 cm and 6.0 cm respectively and the object is placed 5.0 cm beyond the objective. Assume that the final image is formed at the least distance of distinct vision (25 cm).

SOLUTION $u = -5.0$ cm, $f_o = +4.0$ cm and $f_e = +6.0$ cm. For the objective

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_o}$$

Putting $u = -5.0$ cm and $f_o = +4.0$ cm we have

$$\frac{1}{v} = \frac{1}{4} - \frac{1}{5}$$

which gives $v = 20$ cm. Therefore, magnifying power is

$$M = \frac{v}{u}\left(1 + \frac{D}{f_e}\right) = \frac{20}{-5}\left(1 + \frac{25}{6}\right) = -20.7$$

The negative sign indicates that final image is virtual.

EXAMPLE 27 A compound microscope has a magnification of 30. Assuming that the final image is formed at the least distance of distinct vision (25 cm), find the magnification produced by the objective. Given, the focal length of the eyepiece is 5.0 cm.

SOLUTION $M = 30$, $f_e = 5.0$ cm and $D = 25$ cm. We know that

$$M = m_o \times m_e = m_o\left(1 + \frac{D}{f_e}\right) \text{ or } 30 = m_o\left(1 + \frac{25}{5.0}\right)$$

which gives $m_o = 5.0$

EXAMPLE 28 A magnifying power of 30 is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm. How will you set up the compound microscope? The final image is formed at $D = 25$ cm.

SOLUTION Required magnifying power $M = 30$, $f_o = 1.25$ cm and $f_e = 5$ cm. We know that

$$M = m_o \times m_e = m_o\left(1 + \frac{D}{f_e}\right) \text{ or } 30 = m_o\left(1 + \frac{25}{5}\right)$$

which gives $m_o = 5$, i.e. the objective should produce a magnification $m_o = 5$ and the eyepiece should produce a magnification $m_e = 6$.

Position of object Let us first find where the object AB should be placed from the objective so that a real image $A'B'$ magnified five times is obtained. Since $A'B'$ is inverted we have [refer to rough ray diagram Fig. 16.36]

$$m_o = \frac{v}{u} = -5 \text{ or } v = -5u$$

Using the lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_0}$$

we have

$$\frac{1}{-5u} - \frac{1}{u} = \frac{1}{1.25}$$

which gives $u = -1.5$ cm. Thus the object AB should be placed 1.5 cm to the left of the objective. The distance of $A'B'$ from the objective will then be

$$v = -5u = -5 \times (-1.5) = 7.5 \text{ cm}$$

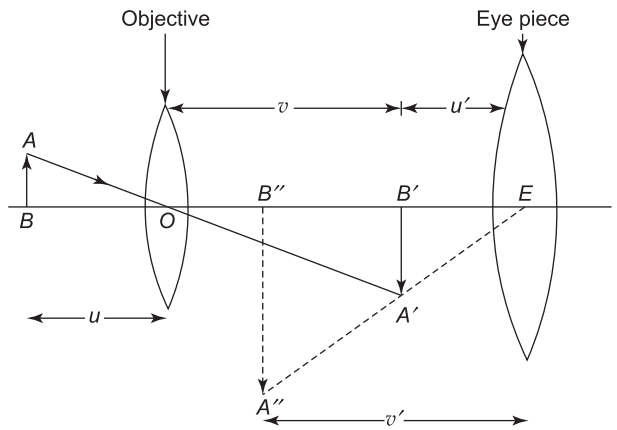


Fig. 16.36

Positioning of eyepiece We will now find where the eyepiece should be positioned so that it produces a magnification $m_e = 6$, i.e. the final image $A''B''$ is six times $A'B'$. It is given that $v' = -D = -25$ cm. Since the final image is virtual,

$$m_o = \frac{v'}{u'} = +6 \text{ or } v' = 6u'$$

Applying the lens formula to the eyepiece we have

$$\frac{1}{v'} - \frac{1}{u'} = \frac{1}{f_e} \text{ or } \frac{1}{6u'} - \frac{1}{u'} = \frac{1}{5}$$

which gives

$$u' = -\frac{25}{6} = -4.17 \text{ cm}$$

Thus, the eyepiece should be at a distance of 4.17 cm to the right of $A'B'$. Hence the separation between the objective and the eyepiece should be

$$OE = OB' + B'E = v + |u'| = 7.5 + 4.17 = 11.67 \text{ cm}$$

Thus we find that the microscope will give a magnifying power of 30 if it is set up as follows: the distance between the objective and the eyepiece should be 11.67 cm and the object should be placed at a distance of 1.5 cm in front of the objective.

EXAMPLE 29 A refracting (astronomical) telescope, when in normal adjustment, has a magnifying power of 6 and the objective and the eyepiece are 14 cm apart. Find the focal lengths of the lenses of the telescope.

SOLUTION In normal adjustment, the object and the final image are both at infinity. The separation between the two lenses is $(f_o + f_e)$. Therefore

$$f_o + f_e = 14 \qquad \text{(i)}$$

Also $$M = \frac{f_o}{f_e} = 6 \text{ or } f_o = 6 f_e \qquad \text{(ii)}$$

Using (ii) in (i) we get $f_e = 2$ cm and $f_o = 12$ cm.

EXAMPLE 30 A telescope has an objective of focal length 120 cm and an eyepiece of focal length 5.0 cm. Find the magnifying power of the telescope when it is used to view distant objects when (a) the telescope is in normal adjustment, and (b) the final image is formed at the least distance of distinct vision (25 cm).

SOLUTION $f_o = 120$ cm and $f_e = 5.0$ cm.

(a) In normal adjustment, the magnifying power is

$$M = \frac{f_o}{f_e} = \frac{120}{5.0} = 24$$

(b) When the object is at infinity (distant object) and the final image formed at $D = 25$ cm, the magnifying power is

$$M = \frac{f_o}{f_e} \times \frac{D + f_e}{D} = \frac{120}{5.0} \times \frac{25 + 5.0}{25} = 28.8$$

16. Dispersion of Light

Refraction of light occurs because the velocity of light changes as it travels from one medium into another. The velocity of light in a given medium depends upon its wavelength (or frequency). Light of a single wavelength or frequency is called monochromatic. White light (sunlight) is not monochromatic, it consists of many wavelengths ranging from violet (~ 300 nm) to red (~ 700 nm).

When white light (or any composite light) enters a refracting medium, the different constituent colours are refracted unequally. The red is refracted the least and the violet the most. From Snell's law

$$\mu = \frac{\sin i}{\sin r}$$

it follows that the refractive index for red is less than for violet, i.e., the speed of red light is greater than that of violet light. Thus, a medium does not have one definite refractive index; it has a range of refractive indices corresponding to the range of colours or wavelengths of the composite light. Since each colour has its own characteristic wavelength (or frequency), the refractive index of a medium will be different for different wavelengths (or frequencies). The variation of refractive index of a medium (and hence of the velocity of light in the medium) with the wavelength or frequency is referred to as *dispersion.* The prism disperses the colours of white light and produces its spectrum.

Angular Dispersion and Dispersive Power

If δ_V and δ_R are the deviations of violet and red lights produced by a prism of refracting angle A, the angular dispersion θ is given by

$$\theta = \delta_V - \delta_R$$

Now $\delta_V = (\mu_V - 1)A$ and $\delta_R = (\mu_R - 1)A$, where μ_V and μ_R are the refractive indices of the material of the prism for violet and red lights respectively. Hence

$$\theta = (\mu_V - \mu_R)A$$

Mean deviation $\delta = (\mu - 1)A$, where $\mu = \frac{1}{2}(\mu_V + \mu_R)$.

The dispersive power of a prism is its ability to deviate the different colours of a composite light along different directions and is defined as

$$\omega = \frac{\theta}{\delta} = \frac{(\mu_V - \mu_R)A}{(\mu - 1)A} = \frac{\mu_V - \mu_R}{\mu - 1}$$

Dispersion without Deviation and Deviation without Dispersion

If two prisms are arranged as shown in Fig. 16.37, then net angular dispersion

$$\Delta\theta = \theta - \theta'$$
$$= (\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A'$$

Net deviation $\Delta\delta = \delta - \delta' = (\mu - 1)A + (\mu' - 1)A'$

(a) For dispersion without deviation

$$\Delta\delta = 0 \quad \Rightarrow \quad (\mu - 1)A + (\mu' - 1)A' = 0$$

(b) For deviation without dispersion

$$\Delta\theta = 0 \quad \Rightarrow \quad (\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A' = 0$$

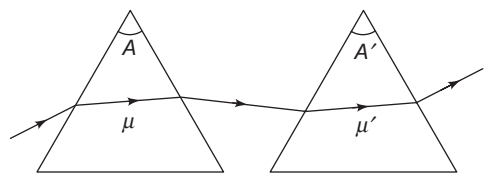


Fig. 16.37

17. Aberrations

(i) Spherical aberration

Spherical aberration occurs in a lens or a spherical mirror due to spherical nature of the surface. This defect arises because the paraxial and marginal rays do not focus at a single point (Fig. 16.38).

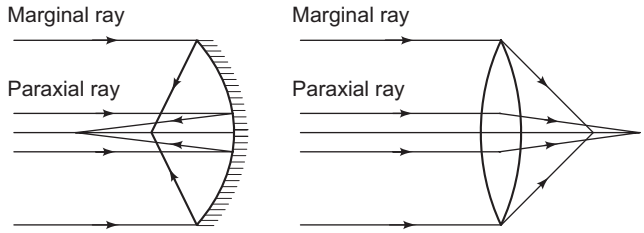


Fig. 16.38

Spherical aberration is minimized by

(a) using a plano-convex lens with the incident rays falling on the curved face

(b) using two plano-convex lens of focal lengths f_1 and f_2 separated by a distance $d = f_1 - f_2$.

(c) using a parabolic mirror.

(ii) Chromatic aberration

Chromatic aberration occurs only in lenses and not in mirrors. This defect arises because the refractive index (and hence the focal length) of a lens is different for the different colours of light. In fact $f_R > f_Y > f_V$. Hence rays of white light do not focus at a single point (Fig. 16.39)

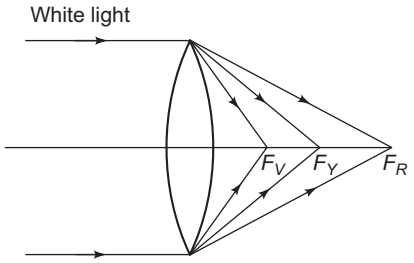


Fig. 16.39

Chromatic aberration is removed by

(a) using a convex lens is contact with a concave lens such that (see Fig. 16.40)

$$\frac{f}{f'} = -\frac{\omega}{\omega'}$$

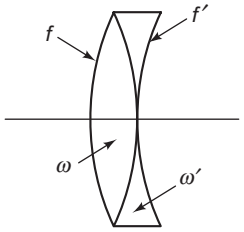


Fig. 16.40

where ω and ω' are dispersive powers of the material of the lenses.

(b) using two lenses made of the same material separated by a distance $d = \frac{1}{2}(f + f')$.

1 SECTION Multiple Choice Questions with One Correct Choice

Level A

1. A ray of light in a medium of refractive index μ_1 is partly reflected and refracted at the boundary of a medium of refractive index μ_2, as shown in Fig. 16.41. If $\angle BOC = 90°$. The value of angle i is given by

(a) $\tan^{-1}(\mu_1/\mu_2)$ (b) $\tan^{-1}(\mu_2/\mu_1)$

(c) $\sin^{-1}(\mu_2/\mu_1)$ (d) $\cos^{-1}(\mu_1/\mu_2)$

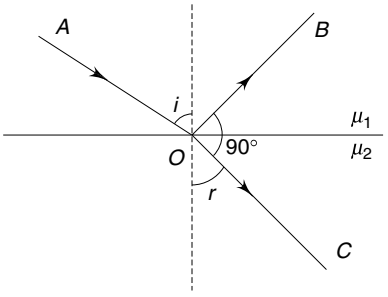


Fig. 16.41

2. Two transparent media A and B are separated by a plane boundary. The speed of light in medium A is 2.0×10^8 m s^{-1} and in medium B 2.5×10^8 m s^{-1}. The critical angle for which a ray of light going from A to B is totally internally reflected is

   (a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\sin^{-1}\left(\frac{2}{5}\right)$

   (c) $\sin^{-1}\left(\frac{4}{5}\right)$ (d) $\sin^{-1}\left(\frac{3}{4}\right)$

3. A convex glass lens ($\mu_g = 1.5$) has a focal length of 8 cm when placed in air. What is the focal length of the lens when it is immersed in water ($\mu_w = 4/3$)?

   (a) 4 cm (b) 8 cm

   (c) 16 cm (d) 32 cm

4. A plano-concave lens is made of glass of refractive index 1.5 and the radius of curvature of the curved face is 50 cm. The power of the lens is

   (a) – 1.0 D (b) – 0.5 D

   (c) + 1.0 D (d) + 0.5 D

5. A convex lens of focal length 50 cm and a concave lens of focal length 100 cm are separated by a distance of 10 cm. What is the power of the combination?

   (a) + 1.5 D (b) + 1.2 D

   (c) – 0.6 D (d) Zero

6. How will the image formed by a convex lens be affected if the central portion of the lens is wrapped in black paper, as shown in Fig. 16.42.

   (a) No image is formed by the remaining portion of the lens

   (b) The central portion of the image will be absent

   (c) The full image is formed but it will be less bright

   (d) There will be two images, one due to each exposed half.

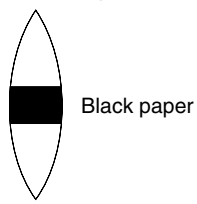


Fig. 16.42

7. The equi-convex lens, shown in the figure, has a focal length f. What will be the focal length of each half if the lens is cut along AB? (see Fig. 16.43)

   (a) $f/2$ (b) f

   (c) $3f/2$ (d) $2f$

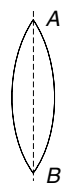


Fig. 16.43

8. A person can see clearly only up to a distance of 30 cm. He wants to read a book placed at a distance of 50 cm from his eyes. What is the power of the lens he requires for his spectacles?

   (a) – 1.0 D (b) – 1.33 D

   (c) – 1.67 D (d) – 2.0 D

9. A person with defective eyesight is unable to see objects clearly nearer than 1.5 m. He wants to read a book placed at a distance of 30 cm from his eyes. Find the power of the lens he requires for his spectacles.

   (a) + 2.67 D (b) – 2.67 D

   (c) + 2.5 D (d) – 2.5 D

10. An object, a convex lens of focal length 20 cm and a plane mirror are arranged as shown in Fig. 16.44. How far behind the mirror is the final image formed?

    (a) 20 cm (b) 30 cm

    (c) 40 cm (d) 50 cm

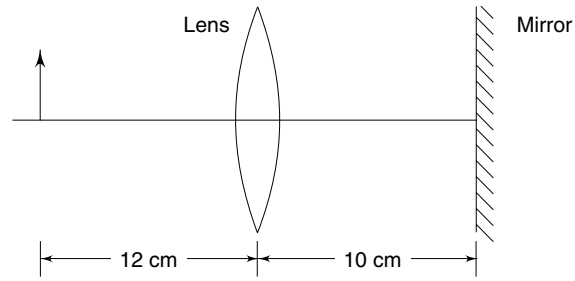


Fig. 16.44

11. A small plane mirror is rotating at constant frequency of n rotations per second. With what linear velocity (in m s^{-1}) will a light spot move along a spherical screen of radius of curvature of R metres if the mirror is at the centre of curvature of the screen?

    (a) $\pi n R$ (b) $2 \pi n R$

    (c) $4 \pi n R$ (d) $2 n R$

12. A ray of light incident normally on face AB of an isosceles prism travels as shown in Fig. 16.45. The least value of the refractive index the prism must have should be

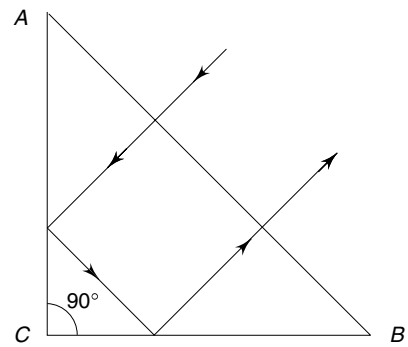


Fig. 16.45

    (a) $\sqrt{2}$ (b) 1.5

    (c) $\sqrt{3}$ (d) 2.0

13. A plano-convex lens is made of glass of refractive index 1.5. The focal length f of the lens and radius of curvature R of its curved face are related as

    (a) $f = R/2$ (b) $f = R$

    (c) $f = 2R$ (d) $f = 3/2\ R$

14. A thin convergent glass lens ($\mu_g = 1.5$) has a power of + 5.0 D. When this lens is immersed in a liquid of refractive index μ_l it acts as a divergent lens of focal length 100 cm. The value of μ_l must be
    (a) 4/3 (b) 5/3
    (c) 5/4 (d) 6/5
15. The dispersion of light in a medium implies that
    (a) lights of different wavelengths travel with different speeds in the medium.
    (b) lights of different frequencies travel with different speeds in the medium.
    (c) the refractive index of the medium is different for different wavelengths of light.
    (d) all of the above.
16. Monochromatic light is refracted from air into glass of refractive index μ. The ratio of the wavelengths of the incident and refracted waves is
    (a) 1 : 1 (b) 1 : μ
    (c) μ : 1 (d) μ^2 : 1
17. What is the relation between refractive indices μ, μ_1 and μ_2 if the behaviour of light rays is as shown in Fig. 16.46.
    (a) $\mu > \mu_2 > \mu_1$ (b) $\mu < \mu_2 < \mu_1$
    (c) $\mu < \mu_2$; $\mu = \mu_1$ (d) $\mu_2 < \mu_1$; $\mu = \mu_2$

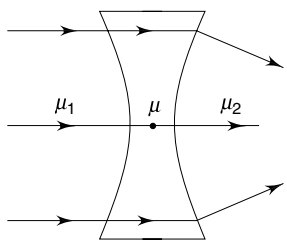


Fig 16.46

18. A lens forms a sharp image on a screen. On inserting a parallel sided glass slab between the lens and the screen, it is found necessary to move the screen a distance d away from the lens in order to focus the image sharply. If the refractive index of glass relative to air is μ, the thickness of the glass slab is given by
    (a) d/μ (b) μd
    (c) $\dfrac{d}{\left(1-\dfrac{1}{\mu}\right)}$ (d) $\left(1-\dfrac{1}{\mu}\right) d$
19. A vessel of depth t is half filled with oil of refractive index μ_1 and the other half is filled with water of refractive index μ_2. The apparent depth of the vessel when viewed from above is
    (a) $\dfrac{2t\mu_1\mu_2}{(\mu_1+\mu_2)}$ (b) $\dfrac{2t(\mu_1+\mu_2)}{\mu_1\mu_2}$
    (c) $\dfrac{t\mu_1\mu_2}{2(\mu_1+\mu_2)}$ (d) $\dfrac{t(\mu_1+\mu_2)}{2\mu_1\mu_2}$

Level B

20. A ray of light, travelling in a medium of refractive index μ, is incident at an angle i on a composite transparent plate consisting of three plates of refractive indices μ_1, μ_2 and μ_3. The ray emerges from the composite plate into a medium of refractive index μ', at angle x. Then
    (a) $\sin x = \sin i$
    (b) $\sin x = \dfrac{\mu}{\mu'} \sin i$
    (c) $\sin x = \dfrac{\mu'}{\mu} \sin i$
    (d) $\sin x = \dfrac{\mu_1}{\mu_2}.\dfrac{\mu_3}{\mu_2}.\dfrac{\mu}{\mu'} \sin i$
21. The principal section of a glass prism is an isosceles triangle ABC with $AB = AC$. The face AC is silvered. A ray incident normally on face AB, after two reflections, emerges from the base BC in a direction perpendicular to it. What is the $\angle BAC$ of the prism?
    (a) 30° (b) 36°
    (c) 60° (d) 72°
22. A mark is made on the surface of a glass sphere of diameter 10 cm and refractive index 1.5. It is viewed through the glass from a position directly opposite. The distance of the image of the mark from the centre of the sphere will be
    (a) 15 cm (b) 17.5 cm
    (c) 20 cm (d) 22.5 cm
23. An object is placed at a distance of 10 cm from a co-axial combination of two lenses A and B. The combination forms a real image three times the size of the object. If lens B is concave with a focal length of 30 cm, what is the nature and focal length of lens A?
    (a) Convex, 12 cm (b) Concave, 12 cm
    (c) Convex, 6 cm (d) Convex, 18 cm
24. A plano-convex lens acts like a concave mirror of 28 cm focal length when its plane surface is silvered and like a concave mirror of 10 cm focal length when its curved surface is silvered. What is the refractive index of the material of the lens?
    (a) 1.50 (b) 1.55
    (c) 1.60 (d) 1.65

25. A pin is placed 10 cm in front of a convex lens of focal length 20 cm and refractive index 1.5. The surface of the lens farther away from the pin is silvered and has a radius of curvature of 22 cm. How far from the lens is the final image formed?
    (a) 10 cm (b) 11 cm
    (c) 12 cm (d) 13 cm
26. The radius of curvature of the convex face of a plano-convex lens is 12 cm and its refractive index is 1.5. What is the focal length of the lens?
    (a) 12 cm (b) 18 cm
    (c) 24 cm (d) 30 cm
27. In Q. 26, if the plane face of the lens is silvered, at what distance from the lens will parallel rays incident on the convex surface converge?
    (a) 12 cm (b) 18 cm
    (c) 24 cm (d) 30 cm
28. A point object is placed at a distance of 12 cm on the axis of a convex lens of focal length 10 cm. On the other side of the lens, a convex mirror is placed at a distance of 10 cm from the lens such that the image formed by the combination coincides with the object itself. What is the focal length of the convex mirror?
    (a) 10 cm (b) 15 cm
    (c) 20 cm (d) 25 cm
29. Two glasses have dispersive powers in the ratio of 2 : 3. These glasses are used in the manufacture of an achromatic objective of focal length 20 cm. What are the focal lengths of the two lenses of the objective?
    (a) 6.67 cm, – 10 cm (b) 7.5 cm, – 12.5 cm
    (c) 9.67 cm, – 15 cm (d) 12.5 cm, – 20 cm
30. A telescope, when in normal adjustment, has a magnifying power of 6 and the objective and the eye-piece are 14 cm apart. The focal lengths of the eye-piece and the objective respectively are
    (a) 3 cm and 14 cm (b) 3 cm and 12 cm
    (c) 2 cm and 14 cm (d) 2 cm and 12 cm.
31. A telescope has an objective of focal length 100 cm and an eye-piece of focal length 5 cm. What is the magnifying power of the telescope when it is in normal adjustment?
    (a) 0.2 (b) 2.0
    (c) 20 (d) 200
32. In Q. 31, what is the magnifying power of the telescope when the final image is formed at the least distance of distinct vision?
    (a) 20 (b) 24
    (c) 28 (d) 32
33. The objective of a telescope has a focal length of 1.2 m. It is used to view a 10.0 m tall tower 2 km away. What is the height of the image of the tower formed by the objective?
    (a) 2 mm (b) 4 mm
    (c) 6 mm (d) 8 mm
34. A giant telescope in an observatory has an objective of focal length 19 m and an eye-piece of focal length 1.0 cm. In normal adjustment, the telescope is used to view the moon. What is the diameter of the image of the moon formed by the objective? The diameter of the moon is 3.5×10^6 m and the radius of the lunar orbit round the earth is 3.8×10^8 m.
    (a) 10 cm (b) 12.5 cm
    (c) 15 cm (d) 17.5 cm
35. The magnifying power of a compound microscope is high if
    (a) both objective and eye-piece have short focal lengths
    (b) both objective and eye-piece have long focal lengths
    (c) the objective has a long focal length and eye-piece has a short focal length
    (d) the objective has a short focal length and the eye-piece has a long focal length.
36. The magnifying power of telescope is high if
    (a) both objective and eye-piece have short focal lengths
    (b) both objective and eye-piece have long focal length
    (c) the objective has a long focal length and the eye-piece has a short focal length
    (d) the objective has a short focal length and the eye-piece has a long focal length.
37. The two lenses of an achromatic doublet should have
    (a) equal powers
    (b) equal dispersive powers
    (c) equal ratio of their power and dispersive power
    (d) equal product of their power and dispersive power.
38. A small plane mirror is placed at the centre of a spherical screen of radius R. A beam of light is falling on the mirror. If the mirror makes n revolutions per second, the speed of the spot of light on the screen after reflection from the mirror will be
    (a) $\frac{nR}{2\pi}$ (b) $\frac{nR}{4\pi}$
    (c) $2\pi nR$ (d) $4\pi nR$
39. Rays from the sun subtend an angle θ (in radians) at the pole of a concave mirror of focal length f. If the

diameter of the sun is D, the diameter of the image of the sun formed by the mirror is

(a) $D\theta$ (b) $2D\theta$

(c) $f\theta$ (d) $2f\theta$

40. Two convex lenses of focal lengths f_1 and f_2 are separated co-axially by a distance d. The power of the combination will be zero if

(a) $d = \dfrac{f_1 + f_2}{2}$ (b) $d = \dfrac{f_1 - f_2}{2}$

(c) $d = f_1 + f_2$ (d) $d = \sqrt{f_1 f_2}$

41. A parallel beam of light falls on a convex lens. The path of the rays is shown in Fig. 16.47. It follows that

(a) $\mu_1 > \mu > \mu_2$ (b) $\mu_1 < \mu < \mu_2$

(c) $\mu_1 = \mu < \mu_2$ (d) $\mu_1 = \mu > \mu_2$

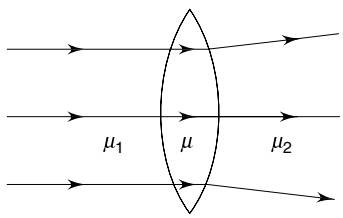


Fig. 16.47

42. The radius of curvature of a thin planoconvex lens is 10 cm and the refractive index of its glass is 1.5. If the plane surface is silvered, then it will behave like a
    (a) concave mirror of focal length 10 cm
    (b) concave mirror of focal length 20 cm
    (c) convex mirror of focal length 10 cm
    (d) convex mirror of focal length 20 cm
43. When the plane surface of a planconvex lens of refractive index 1.5 is silvered, it behaves like a concave mirror of focal length 30 cm. When its convex surface is silvered, it will behave like a concave mirror of focal length

(a) 10 cm (b) 20 cm

(c) 30 cm (d) 45 cm

44. The refractive index of the material of an equilateral prism is $\sqrt{3}$. What is the angle of minimum deviation?

(a) 30° (b) 45°

(c) 60° (d) 75°

45. A thin glass prism of refractive index 1.5 produces a deviation of 4° of a ray incident at a small angle. What will be the deviation of the same incident ray by the same prism if it is immersed in water of refractive index 4/3?

(a) 1° (b) 2°

(c) 8° (d) 16°

46. An air bubble in a glass slab ($\mu = 1.5$) is 5 cm deep when viewed through one face and 2 cm deep when viewed through the opposite face. What is the thickness of the slab?

(a) 7.0 cm (b) 7.5 cm

(c) 10.0 cm (d) 10.5 cm

47. The refracting angle of a prism is A and the refractive index is cot (A/2). The angle of minimum deviation is

(a) $180° - A$ (b) $180° - 2A$

(c) $180° - 3A$ (d) $180° - 4A$

48. A ray of light enters a rectangular glass slab of refractive index $\mu = \sqrt{3}$ at an angle of incidence $i = 60°$. It travels a distance $d = 6.0$ cm inside the slab before emerging out of it. The lateral displacement of the incident ray is

(a) $6\sqrt{3}$ cm (b) $3\sqrt{3}$ cm

(c) 3.0 cm (d) 6.0 cm

49. A thin prism P_1 with angle 4° and made from glass of refractive index 1.54 is combined with another thin prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. The angle of prism P_2 is

(a) 5.33° (b) 4°

(c) 3° (d) 2.6°

50. When a ray of light enters a glass slab from air,
    (a) its wavelength decreases
    (b) its wavelength increases
    (c) its frequency increases
    (d) neither wavelength nor frequency changes
51. Spherical aberration in a thin lens can be reduced by
    (a) using a monochromatic light
    (b) using a doublet combination
    (c) using a circular annular mask over the lens
    (d) increasing the size of the lens.
52. A real image of a distant object is formed by a plano-convex lens on its principal axis. The spherical aberration
    (a) is absent
    (b) is smaller if the curved surfaces of the lens faces the object
    (c) is smaller if the plane surface of the lens faces the object
    (d) is the same whichever side of the lens faces the object.
53. A concave mirror is placed on a horizontal table, with its axis directed vertically upwards. Let O be

the pole of the mirror and C its centre of curvature. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be

(a) real and will remain at C
(b) real and located at a point between C and infinity
(c) virtual and located at a point between C and O
(d) real and located at a point between C and O.

54. A spherical surface of radius of curvature R separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at point O and $PO = OQ$. The distance PO is equal to

(a) $5R$ (b) $3R$
(c) $2R$ (d) $1.5R$

55. An eye specialist prescribes spectacles having a combination of a convex lens of focal length 40 cm in contact with a concave lens of local length 25 cm. The power of this lens combination is

(a) + 1.5 D (b) – 1.5 D
(c) + 6.67 D (d) – 6.67 D

56. A concave lens of glass, refractive index 1.5, has both surfaces of the same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a

(a) convergent lens of focal length 3.5 R
(b) convergent lens of focal length 3.0 R
(c) divergent lens of focal length 3.5 R
(d) divergent lens of focal length 3.0 R

57. A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3 and μ_4 as shown in Fig. 16.48. The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have

(a) $\mu_1 = \mu_2$ (b) $\mu_2 = \mu_3$
(c) $\mu_3 = \mu_4$ (d) $\mu_4 = \mu_1$

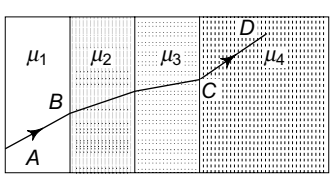


Fig. 16.48

58. A ray of light is incident at the glass-water interface at an angle i. It emerges finally parallel to the surface of water as shown in the Fig. 16.49. The value of μ_g would be

(a) $\left(\frac{4}{3}\right)\sin i$ (b) $\frac{1}{\sin i}$
(c) $\frac{2}{\sqrt{3}\sin i}$ (d) 1.5

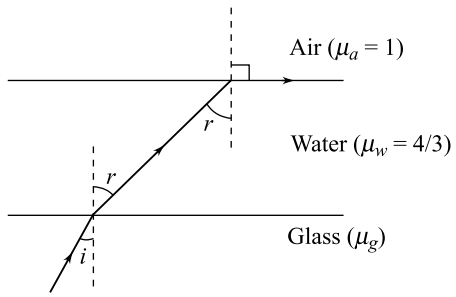


Fig. 16.49

59. When a glass prism of refracting angle 60° is immersed in a liquid, its angle of minimum deviation is 30°. The critical angle of glass with respect to the liquid medium is

(a) 42° (b) 45°
(c) 50° (d) 52°

60. In the visible region, the dispersive powers and the mean angular deviations for crown and flint glass prisms are ω and ω' and d and d' respectively. When the two prism are combined, the condition of zero dispersion by the combination is

(a) $\sqrt{\omega d} + \sqrt{\omega' d'} = 0$
(b) $\omega' d + \omega d' = 0$
(c) $\omega d + \omega' d' = 0$
(d) $(\omega d)^2 + (\omega' d')^2 = 0$

61. One face of a glass prism is silver polished. A light ray falls at an angle of 45° on the other face. After refraction, it is subsequently reflected from the silvered face and then it retraces its path. The refracting angle of the prism is 30°. The refractive index of the material of the prism is

(a) $\frac{3}{2}$ (b) $\sqrt{2}$
(c) $\frac{\sqrt{3}}{2}$ (d) $\sqrt{3}$

62. A prism is made up of material of refractive index $\sqrt{3}$. The angle of the prism is A. If the angle of minimum deviation is equal to the angle of the prism, then the value of A is:

(a) 30°C (b) 45°
(c) 60° (d) 75°

63. A motor car is fitted with a convex driving mirror of focal length 20 cm. A second motor car 2.8 m behind the first car is overtaking at a relative speed of 15 ms^{-1}. The speed of the image of the second car as seen in the mirror of the first is

(a) $\frac{1}{10} ms^{-1}$ (b) $\frac{1}{15} ms^{-1}$

(c) 10 ms^{-1} (d) 15 ms^{-1}

64. A short linear object of length b lies on the axis of a concave mirror of focal length f at a distance u from the pole. The length of the image will be

(a) $\left(\frac{f}{u-f}\right)b$ (b) $\left(\frac{u-f}{f}\right)b$

(c) $\left(\frac{f}{u-f}\right)^2 b$ (d) $\left(\frac{u-f}{f}\right)^2 b$

65. A point source of light S is placed at the bottom of a vessal containing a liquid of refractive index $\sqrt{2}$. A person is viewing the source from above the surface. There is an opaque disc of radius r floating on the surface. The centre of the disc is vertically above the source S. The liquid is gradually drained out from the vessel through a tap. The maximum height of the liquid for which the source cannot be seen from above is

(a) r (b) $\sqrt{2}\, r$

(c) $\sqrt{3}\, r$ (d) $2r$

66. Light is incident at an angle α on one planar end of a transparent cylindrical rod of refractive index n. The least value of n so that the light entering the rod does not emerge from the curved suface of the rod for any value of α is

(a) $\frac{4}{3}$ (b) $\sqrt{2}$

(c) 1.5 (d) $\sqrt{3}$

67. A ray of light is incident at an angle of 60° on one face of a prism of refracting angle 30°. The ray emerges out of the prism making an angle of 30° with the incident ray. The refractive index of the material of the prism is

(a) $\sqrt{2}$ (b) 1.5

(c) $\sqrt{3}$ (d) $\frac{1}{2}\left(1+\sqrt{3}\right)$

68. A square wire of side 3.0 m is placed 25 cm from a concave mirror of focal length 10 cm. The area enclosed by the image of the wire is

(a) 1 cm^2 (b) 4 cm^2

(c) 16 cm^2 (d) 25 cm^2

69. A point object is placed at the centre of a glass sphere of diameter 12 cm and refractive index 1.5. What is the distance of the virtual image from the surface of the sphere?

(a) 4 cm (b) 6 cm

(c) 9 cm (d) 12 cm

70. A plano-convex lens has thickness 4 cm. When placed on a horizontal table with the curved face in contact with it, the apparent depth of the bottom-most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of the plane face of the lens is found to be $\frac{25}{8}$ cm. The focal length of the lens is

(a) 25 cm (b) 50 cm

(c) 75 cm (d) 100 cm

71. A parallel sides slab $ABCD$ of refractive index 2 is sandwiched between two slabs of refractive indices $\sqrt{2}$ and $\sqrt{3}$ as shown in the Fig. 16.50. The minimum value of angle θ such that the ray PQ suffers total internal reflection at both the surfaces AB and CD is

(a) 30° (b) 45°

(c) 60° (d) 75°

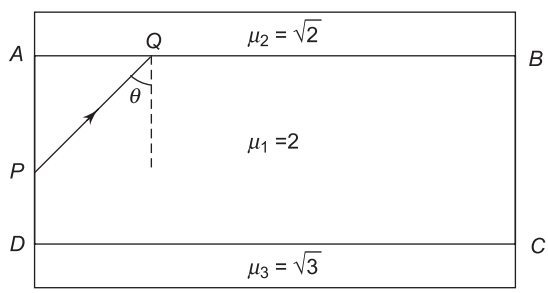


Fig. 16.50

72. Parallel rays from a distant object fall on a solid transparent sphere of radius R and refractive index μ. The distance of the image from the sphere is

(a) $\frac{R(2-\mu)}{2(\mu-1)}$ (b) $\frac{R\mu}{(\mu-1)}$

(c) $\frac{R}{(\mu-1)}$ (d) $R\,(\mu-1)$

73. An object *O* is placed at a distance of 20 cm from a thin plano-convex lens of focal length 15 cm. The plane surface of the lens is silvered as shown in Fig. 16.51. The image is formed at a distance of
    (a) 60 cm to the right of the lens
    (b) 30 cm to the left of the lens
    (c) 24 cm to the right of the lens
    (d) 12 cm to the left of the lens

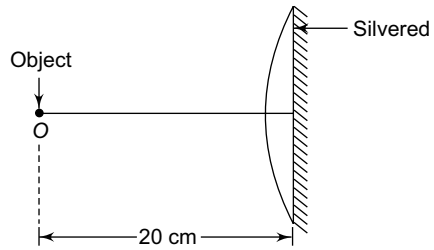


Fig. 16.51

74. Image of the sun is formed by a biconvex lens of focal length *f*. The image is a circular patch of radius *r* and is formed on the focal plane of the lens. Choose the correct statement from the following.
    (a) The area of the image is πr^2 and it is directly proportional to *f*.
    (b) The area of the image is πr^2 and it is directly proportional to f^2.
    (c) The intensity of the image will increase if *f* is increased.
    (d) If the lower half of the lens is covered with black paper, the area of the image will become half.

75. A glass prism *ABC* of refractive index 1.6 is immersed in a liquid as shown in Fig. 16.52. A ray of light incident normally on face *AB* is totally reflected at face *AC* if the refractive of the liquid is
    (a) less than $0.8\sqrt{2}$
    (b) equal to $\frac{4}{3}$
    (c) equal to 1.2
    (d) lies between $0.8\sqrt{2}$ and $0.8\sqrt{3}$

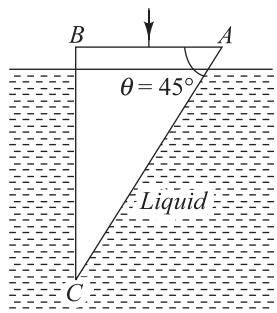


Fig. 16.52

76. A hollow double convex lens is made of a very thin transparent material. It can be filled with air or either of two liquids L_1 and L_2 of refractive indices μ_1 and μ_2 respectively ($\mu_2 > \mu_1 > 1$). The lens will diverge a parallel beam of light if it is filled
    (a) with L_1 and immersed in L_2
    (b) with L_2 and immersed in L_1
    (c) with air and placed in air
    (d) with either L_1 or L_2 and placed in air

77. A parallel beam of light consisting of red and blue colours is incident normally on face *AB* of a right-angled prism *ABC* as shown in Fig. 16.53. The refractive index of the material of the prism for red light is 1.39 and for blue light is 1.47. Light of which colour will emerge from face AC?

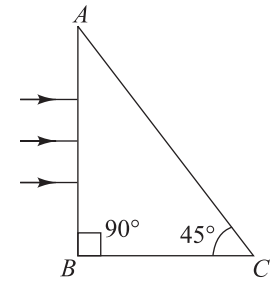


Fig. 16.53

    (a) Both red and blue
    (b) Neither red nor blue
    (c) Red but not blue
    (d) Blue but not red

78. A rectangular glass slab *ABCD* of refractive index $\mu_1 = \sqrt{3}$ is embedded in a transparent medium of refractive index $\mu_2 = 1.5$. A ray of light is incident on face *AB* of the slab as shown in Fig. 16.54. The maximum value of the angle of incidence θ_{max} such that the ray comes out only from the other face *CD* is given by
    (a) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (b) $\sin^{-1}\left(\frac{2}{3}\right)$
    (c) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) 60°

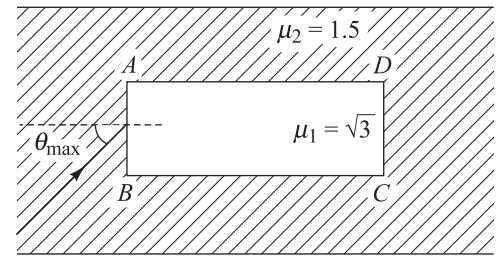


Fig. 16.54

79. The ray PQ shown in Fig. 16.55 suffers minimum deviation in equilateral prism A. Two more identical prisms B and C and of the same material as A are now placed in contact as shown in the figure. If prism A produces a minimum deviation δ_m, the deviation produced by the combination of prisms A, B and C will be

(a) $3\ \delta_m$ (b) $2\ \delta_m$

(c) δ_m (d) zero

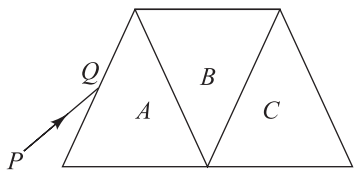


Fig. 16.55

80. Which one of the spherical lenses given in Fig. 16.56 does not split a composite light into its constituent colours?

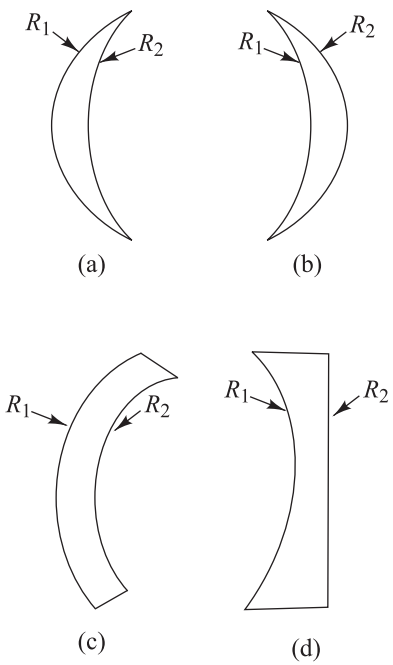


Fig. 16.56

81. A ray of light enters the glass core of an optical fibre of thickness 1 mm at an angle of 60° and suffers first total internal reflection at A as shown in Fig. 16.57. If the length of the core is 10 cm, the maximum number of reflections the ray undergoes before emerging from the other end B of the fibre is

(a) 57 (b) 86

(c) 100 (d) 173

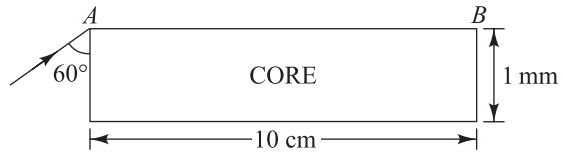


Fig. 16.57

82. A convex lens of focal length 25 cm used to see the image of a distant tower. The size of the image is found to be 3 cm. A concave lens of focal length 20 cm is placed between the convex lens and the image at a distance of 20 cm from the convex lens. The size of the final image will be

(a) 1 cm (b) 2 cm

(c) 3 cm (d) 4 cm

83. A ray of light PQ is incident on an isosceles glass prism ABC as shown in Fig. 16.58. The refractive index of the material of the prism is $\sqrt{3}$. If the prism is in the minimum deviation position for the ray PQ, then

(a) $\alpha = 45°$, $r = 30°$ (b) $\alpha = 30°$, $r = 45°$

(c) $\alpha < 45°$, $r > 30°$ (d) $\alpha > 45°$, $r < 30°$

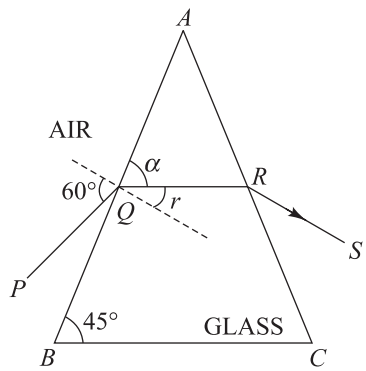


Fig. 16.58

84. A composite light consisting of red, green and blue colours is incidence on face AB of a glass prism. The blue light is just totally internally reflected at face AC as shown in Fig. 16.59. Then

(a) green light is internally reflected at AC but red light emerges from AC.

(b) red light is internally reflected at AC but green light emerges from AC

(c) green and red lights both emerge from AC

(d) green and red lights both do not emerge from AC

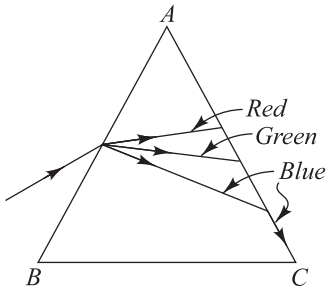


Fig. 16.59

85. A glass slab *ABCD* of refractive index $\mu = 1.5$ and length $AB = 15$ cm has a concave mirror fitted at a distance of 5 cm from side *BC* as shown in Fig. 16.60. A luminous object *O* is fixed on side *AD* and its image *I* is formed at distance of 7 cm from side *BC* as shown. The focal length of the mirror is

(a) $\frac{15}{4}$ cm (b) $\frac{18}{5}$ cm

(c) $\frac{20}{3}$ cm (d) $\frac{25}{6}$ cm

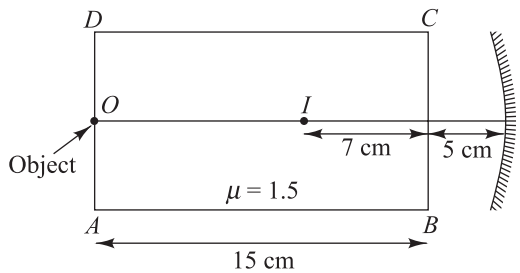


Fig. 16.60

86. A convex lens and a concave lens are placed in contact. The ratio of the magnitude of the power of the convex lens to that of the concave lens is 3 : 2. If the focal length of the concave lens is 10 cm, the focal length of the combination is

(a) –15 cm (b) +20 cm

(c) +25 cm (d) –30 cm

87. Figure 16.61 shows a lens of negligible thickness having radii of curvature R_1 and R_2 and $\mu_1 < \mu_2 < \mu_3$. If $R_1 = R_2 = R$, the focal length of the lens is

(a) $f = \frac{\mu_2 R}{(\mu_1 - \mu_3)}$ (b) $f = \frac{\mu_3 R}{(\mu_1 - \mu_2)}$

(c) $f = \frac{\mu_1 R}{(\mu_2 - \mu_3)}$ (d) $f = \frac{\mu_3 R}{(\mu_1 - \mu_3)}$

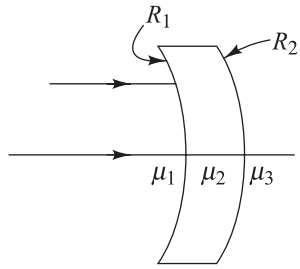


Fig. 16.61

88. A ray of light is incident at an angle of 60° on face *AD* of a transparent slab. What should be the refractive index of the material of the slab so that total internal reflection occurs at face *AB* of the slab? [see Fig. 16.62].

(a) 1.5 (b) $\sqrt{3}$

(c) $\frac{2\sqrt{5}}{3}$ (d) $\frac{\sqrt{7}}{2}$

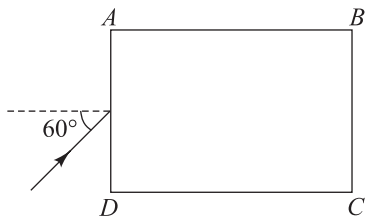


Fig. 16.62

89. The focal length (f) of a thin convex lens is determined by two-pin method. The numerical values of the image distance (v) are measured by various object distances (u). Figure 16.63 shows a graph of $\frac{1}{v}$ (in cm^{-1}) versus $\frac{1}{u}$ (in cm^{-1}). The optical power of the lens is

(a) +5 D (b) +2.5 D

(c) +1.25 D (d) +0.5 D

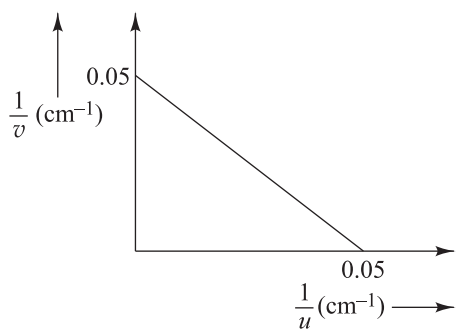


Fig. 16.63

90. Figure 16.64 shows the graph of image distance v (in cm) versus the object distance u (in cm) for a thin con-

vex lens. The focal length of the lens is (both u and v are measured with a metre scale of least count = 1 mm)

(a) (20.0 ± 0.1) cm (b) (10.0 ± 0.1) cm
(c) (10.00 ± 0.05) cm (d) (20.0 ± 0.2) cm

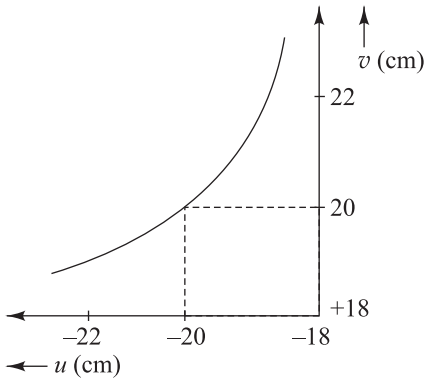


Fig. 16.64

91. A ray of light travelling in air is incident at an angle i on a plane glass surface. Then there will be
    (a) only a refracted ray but no reflected ray
    (b) only a reflected ray but no refracted ray
    (c) both a reflected ray and a refracted ray and the angle between them would be greater then $(180° - 2i)$
    (d) both a reflected ray and a refracted ray and the angle between them would be less then $(180° - 2i)$

92. A ray of light is incident at an angle i on the boundary of media I and II as shown in Fig. 16.65. It travels from medium I to medium IV of reflective indices μ_1, μ_2, μ_3 and μ_4 respectively. If $\mu_1 > \mu_2 > \mu_3 > \mu_4$, the value of i for which the ray just misses entering medium IV is

    (a) $\sin^{-1}\left(\frac{\mu_4}{\mu_1}\right)$ (b) $\sin^{-1}\left(\frac{\mu_4}{\mu_3}\right)$

    (c) $\sin^{-1}\left(\frac{\mu_4}{\mu_3}\times\frac{\mu_2}{\mu_1}\right)$ (d) $\sin^{-1}\left(\frac{\mu_2\mu_3}{\mu_1}\right)$

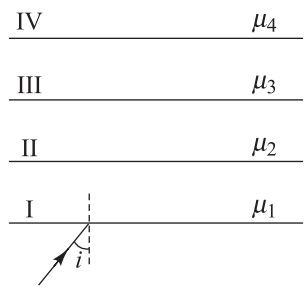


Fig. 16.65

93. A composite beam of light consists of red, yellow and blue lights. A prism of refracting angle $A = 60°$, is placed in the minimum deviation position separately for each colour. If δ_R, δ_Y and δ_B are the angles of minimum deviation for red, yellow and blue lights respectively, then

    (a) $\delta_R > \delta_Y > \delta_B$ (b) $\delta_R < \delta_Y < \delta_B$
    (c) $\delta_R = \delta_Y = \delta_B = 60°$ (d) $\delta_R = \delta_Y = \delta_B \neq 60°$

94. In Q. 93 above, if r_R, r_Y and r_B are the angles of refraction in the prism for red, yellow and blue lights, when the prism is in the position of minimum deviation for the respective lights, then

    (a) $r_R > r_Y > r_B$ (b) $r_R < r_Y < r_B$
    (c) $r_R = r_Y = r_B = 30°$ (d) $r_R = r_Y = r_B \neq 30°$

95. A object is placed at a distance of 20 cm on the principal axis of a convex lens of focal length 10 cm. After refraction by the lens, the rays from the object fall on a plane mirror held at a distance of 5 cm from the lens. The distance of the final image from the mirror is

    (a) 10 cm (b) 12.5 cm
    (c) 15 cm (d) 17.5 cm

96. An equi-convex lens forms a real image of an object which is twice the size of the object. The distance of the object from the lens is 24 cm. The wavelength of light inside the lens is 2/3 times the wavelength in air. The radius of each carved surface of the lens is

    (a) 16 cm (b) 8 cm
    (c) $\frac{32}{3}$cm (d) 24 cm

97. An achromatic doublet consists of two lenses of focal lengths f_1 and f_2 whose materials have dispersive powers in the ratio 2/3. If the focal length of the doublet is 54 cm, the values of f_1 and f_2 respectively are

    (a) 27 cm, –18 cm (b) 16 cm, –30 cm
    (c) 18 cm, –27 cm (d) –15 cm, 20 cm

98. Figures 16.66 (*i*) and (*ii*) show two identical plano-convex lenses of glass of refractive index $\mu_g = 1.5$ with water of $\mu_w = 4/3$ sandwiched between them. Then the combination
    (a) in case (i) is convergent and in case (ii) is divergent
    (b) in case (i) is divergent and in case (ii) is convergent
    (c) in cases (i) and (ii) are both convergent
    (d) in case (i) and (ii) are both divergent.

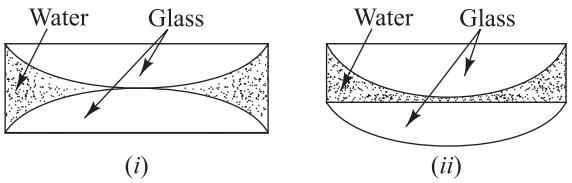


Fig. 16.66

99. What is the answer to Q. 99 above if water is replaced by a liquid of $\mu_l = 1.6$?

100. A convex lens of focal length f is placed between an object and a screen. If the distance between the screen and the object is greater then $4f$, we can obtain a sharp image of the object on the screen for two positions L_1 and L_2 of the lens. These positions are conjugate positions. In one case the image is magnified and in the other case it is diminished. If h_1 and h_2 are the heights of the two images, the height of the object is

(a) $\sqrt{h_1 h_2}$ (b) $\sqrt{h_1^2 + h_2^2}$

(c) $h_1 + h_2$ (d) $\dfrac{h_1 + h_2}{2}$

101. In Q. 100 above, if D is the distance between the object and the screen and d is the distance between L_1 and L_2, then d is given by

(a) $\sqrt{D(4D + f)}$ (b) $\sqrt{D(4D - f)}$

(c) $\sqrt{D(D + 4f)}$ (d) $\sqrt{D(D - 4f)}$

102. In Q. 100 above, the focal length of the lens is given by

(a) $\dfrac{D^2 - d^2}{4D}$ (b) $\dfrac{D^2 + d^2}{4D}$

(c) $\dfrac{D(D - d)}{4D}$ (d) $\dfrac{d(D + d)}{4D}$

103. In Q. 100 above, the ratio of the linear magnifications of the images for positions L_1 and L_2 of the lens is

(a) $\left(\dfrac{D + d}{D - d}\right)$ (b) $\left(\dfrac{D^2 + d^2}{D^2 - d^2}\right)$

(c) $\left(\dfrac{D + d}{D - d}\right)^2$ (d) $\left(\dfrac{D^2 + d^2}{D^2 - d^2}\right)^{1/2}$

104. A convex lens of focal length 25 cm and a concave lens of focal length 10 cm are placed co-axially a distance x apart. If a parallel beam of light falling on the convex lens leaves the concave lens as a parallel beam, the value of x must be equal to

(a) 35 cm (b) 20 cm

(c) 17.5 cm (d) 15 cm

Answers

Level A

1. (b)	2. (c)	3. (d)	4. (a)
5. (b)	6. (c)	7. (d)	8. (b)
9. (a)	10. (c)	11. (c)	12. (a)
13. (c)	14. (b)	15. (d)	16. (c)
17. (c)	18. (c)	19. (d)	

Level B

20. (b)	21. (b)	22. (a)	23. (c)
24. (b)	25. (b)	26. (c)	27. (a)
28. (d)	29. (a)	30. (d)	31. (c)
32. (b)	33. (c)	34. (d)	35. (a)
36. (c)	37. (d)	38. (d)	39. (d)
40. (c)	41. (c)	42. (a)	43. (a)
44. (c)	45. (a)	46. (d)	47. (b)
48. (c)	49. (c)	50. (a)	51. (c)
52. (b)	53. (a)	54. (a)	55. (b)
56. (a)	57. (d)	58. (b)	59. (b)
60. (c)	61. (b)	62. (c)	63. (b)
64. (c)	65. (a)	66. (b)	67. (c)
68. (b)	69. (b)	70. (c)	71. (c)
72. (a)	73. (d)	74. (b)	75. (a)
76. (a)	77. (c)	78. (c)	79. (c)
80. (c)	81. (a)	82. (d)	83. (a)
84. (c)	85. (c)	86. (d)	87. (d)
88. (d)	89. (a)	90. (c)	91. (c)
92. (a)	93. (b)	94. (c)	95. (a)
96. (a)	97. (c)	98. (c)	99. (b)
100. (a)	101. (d)	102. (a)	103. (c)
104. (d)			

Solutions

Level A

1. $i + r + \angle BOC = 180°$. Since $\angle BOC = 90°$, $r = 90° - i$. From Snell's law

$$\frac{\mu_1}{\mu_2} = \frac{\sin r}{\sin i} = \frac{\sin(90° - i)}{\sin i} = \cot i \text{ or } \tan i = \frac{\mu_2}{\mu_1}$$

or $$i = \tan^{-1}\left(\frac{\mu_2}{\mu_1}\right)$$

Hence the correct choice is (b).

2. Refractive index of medium A with respect to medium B is

$$\mu = \frac{2.5\times10^8}{2.0\times10^8} = \frac{5}{4}$$

For total internal reflection

$$\sin i = \frac{1}{\mu} = \frac{4}{5}$$

or $$i = \sin^{-1}\left(\frac{4}{5}\right)$$

Hence the correct choice is (c).

3. $$\frac{1}{f_a} = \left(\frac{\mu_g - \mu_a}{\mu_a}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$\frac{1}{f_w} = \left(\frac{\mu_g - \mu_w}{\mu_w}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

Dividing we have

$$\frac{f_w}{f_a} = \frac{\mu_g-\mu_a}{\mu_g-\mu_w}.\frac{\mu_w}{\mu_a} = \frac{1.5-1}{1.5-4/3}.\frac{4/3}{1} = 4$$

$\therefore$ $$f_w = 4f_a = 4\times 8 = 32 \text{ cm}$$

Hence the correct choice is (d).

4. $$\frac{1}{f} = (\mu-1)\times\frac{1}{R} = (1.5-1)\times\frac{1}{-50} = -\frac{1}{100}$$

or $f = -100$ cm $= -1$ m

Power $= -1$ D which is the choice (a).

5. The focal length of the combination is

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_1}$$

where $f_1 = +50$ cm, $f_2 = -100$ cm and $t = 10$ cm. Using these values we get $F = \frac{250}{3}$ cm $= \frac{2.5}{3}$ m. Therefore power $= \frac{3}{2.5} = 1.2$ D. Hence the correct choice (b).

6. The correct choice is (c).

7. For equiconvex lens $R_1 = -R_2 = R$, so that

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = (\mu-1)\frac{2}{R}$$

If the lens is cut along AB, we have two plano convex lenses. For a planconvex lens, $R_2 = \infty$. Therefore, its focal length f' is

$$\frac{1}{f'} = (\mu-1)\frac{1}{R} = \frac{1}{2f}$$

$\therefore$ $$f' = 2f.$$

Hence the correct choice is (d).

8. The person is myopic. This defect is corrected by using a concave lens. Putting $u = -50$ cm and $v = -30$ cm in the lens formula $\frac{1}{v}-\frac{1}{u} = \frac{1}{f}$ gives $f = -75$ cm. Therefore, the power of the lens $= -1.33$ D which is choice (b).

9. The person is hypermetropic. This defect is corrected by using a convex lens. Putting $u = -30$ cm and $v = 1.5$ m $= -150$ cm in the lens formula gives $f = +37.5$ cm. Therefore, the power of the lens $= +2.67$ D which is choice (a).

10. Putting $u = -12$ cm and $f = +20$ cm in the lens formula gives $v = -30$ cm, i.e., the image is formed at a distance of 30 cm from the lens on the same side as the object. The distance of this image from the mirror $= 30 + 10 = 40$ cm. Therefore, the final image is formed behind the mirror at a distance of 40 cm from it. Hence the correct choice is (c).

11. If a mirror rotates through an angle θ, the image rotates through an angle 2θ. Therefore, the linear speed of the light spot is

$$v = 2R\,\omega = 2R(2\pi n) = 4\pi n R$$

Hence the correct choice is (c).

12. The ray is totally reflected at faces AC and CB. The angle of incidence at each face is at least $i_c = 45°$. Therefore

$$\mu = \frac{1}{\sin i_c} = \frac{1}{\sin 45°} = \sqrt{2}$$

Hence the correct choice is (a).

13. For a planoconvex lens $\frac{1}{f} = (\mu-1)\frac{1}{R} = (1.5-1)\times\frac{1}{R}$ $= \frac{0.5}{R} = \frac{1}{2R}$ or $f = 2R$. Hence the correct choice is (c).

14. $P = +5.0$ D. Therefore $f = \dfrac{100}{P} = \dfrac{100}{5.0} = 20$ cm. Now

$$\frac{1}{20} = (\mu_g - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Also $$\frac{1}{-100} = \left(\frac{\mu_g - \mu_l}{\mu_l}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Dividing we get $-\dfrac{1}{20} \times 100 = \dfrac{(\mu_g - 1)}{(\mu_g - \mu_l)} \times \mu_l$

Putting $\mu_g = 1.5$ we get $\mu_l = 5/3$. Hence the correct choice is (b).

15. The correct choice is (d).

16. $v_a = \nu \lambda_a$ and $v_g = \nu \lambda_g$. Therefore

$$\frac{\lambda_a}{\lambda_g} = \frac{v_a}{v_g}$$

But $v_a / v_g = \mu$. Therefore $\lambda_a / \lambda_g = \mu$. Hence the ratio of the wavelengths of the incident and refracted waves is $\mu : 1$, which is choice (c).

17. The ray does not suffer any deviation on entering the lens. Hence $\mu_1 = \mu$. The ray leaves the second surface of the lens bending towards the normal. Hence $\mu_2 > \mu$. Thus the correct choice is (c).

18. When a glass plate of thickness t is introduced, the image shifts by an amount $t\left(1 - \dfrac{1}{\mu}\right)$. Hence

$$d = t\left(1 - \frac{1}{\mu}\right)$$

or $$t = \frac{d}{\left(1 - \frac{1}{\mu}\right)}$$

Thus the correct choice is (c).

19. Refer to Fig. 16.67.

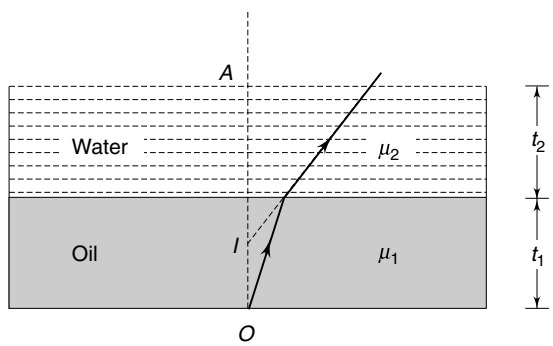


Fig.16.67

The apparent depth is given by

$$AI = \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2} = \frac{t}{2}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)$$

$$= \frac{t(\mu_1 + \mu_2)}{2\,\mu_1\,\mu_2}$$

Level B

20. Refer to Fig. 16.68.

Applying Snell's law to successive refractions, we have

$$\frac{\sin i}{\sin r_1} = \frac{\mu_1}{\mu}\ ; \qquad \frac{\sin r_1}{\sin r_2} = \frac{\mu_2}{\mu_1}$$

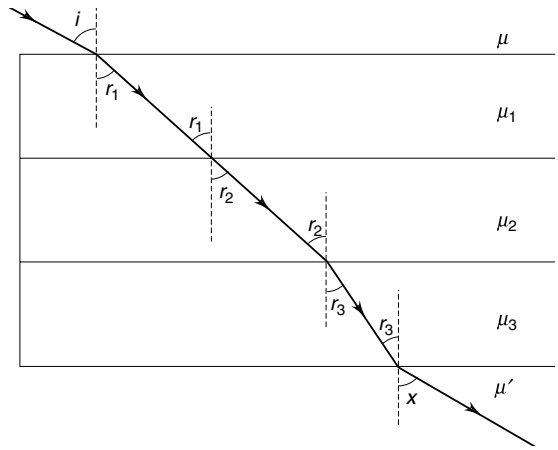


Fig. 16.68

$$\frac{\sin r_2}{\sin r_3} = \frac{\mu_3}{\mu_2}\ ; \qquad \frac{\sin r_3}{\sin x} = \frac{\mu'}{\mu_3}$$

Multiplying these equations we get

$$\frac{\sin i}{\sin x} = \frac{\mu'}{\mu}$$

or $$\sin x = \frac{\mu}{\mu'} \sin i$$

21. The path of the ray is shown in Fig. 16.69.

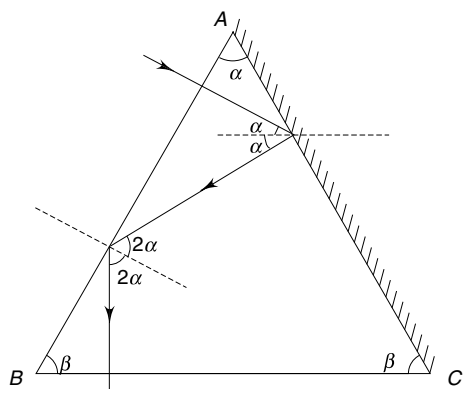


Fig. 16.69

It is clear that

$$\alpha + 2\beta = 180°$$

and $\beta = 2\alpha$

Hence, $\alpha = 36°$, which is choice (b).

22. Refer to Fig. 16.70.

Let P be the position of the mark. Q is the position of its image. Since the incident ray PA lies in a medium of refractive index μ_2, and is refracted into a medium of refractive index μ_1, our formula becomes

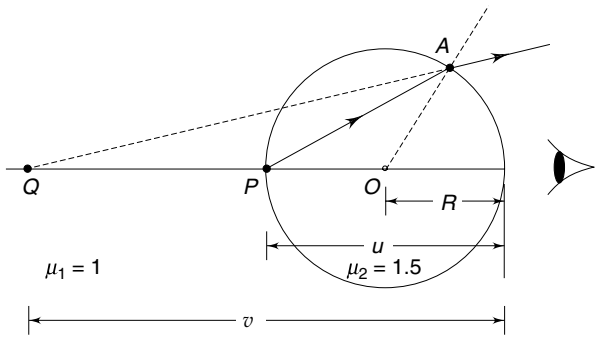


Fig. 16.70

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

where $u = -2R = -10$ cm,

$R = -5$ cm, $\mu_2 = 1.5$ and $\mu_1 = 1$

Putting these values in the above formula, we have

$$\frac{1}{v} - \frac{1.5}{-10} = \frac{1-1.5}{-5}$$

which gives $v = -20$ cm

$\therefore$ Distance of image Q from $O = 20 - 5 = 15$ cm, which is choice (a).

23. Given $m = \frac{v}{u} = -3$ ($\because$ the image is inverted)

or $v = -3u$

Now $u = -10$ cm, therefore $v = +30$ cm.

If F is the focal length of the combination, we have

$$\frac{1}{F} = \frac{1}{v} - \frac{1}{u} = \frac{1}{30} + \frac{1}{10}$$

or $F = \frac{15}{2}$ cm

Focal length of the concave lens $B = f_1 = -30$ cm. If f_2 is the focal length of lens A, we have

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}$$

or $$\frac{1}{f_2} = \frac{1}{F} - \frac{1}{f_1} = \frac{2}{15} + \frac{1}{30}$$

which gives $f_2 = 6$ cm.

Since f_2 is positive, the lens is convex. Hence the correct choice is (c).

24. In the first case we have ($\because f_m = \infty$)

$$\frac{1}{28} = \frac{2}{f_1} + \frac{1}{f_m} = \frac{2}{f_1} \quad \text{(i)}$$

In the second case we have

$$\frac{1}{10} = \frac{2}{f_1} + \frac{1}{f_m} \quad \text{(ii)}$$

where f_m is the focal length of the curved silvered surface. Hence $f_m = \frac{R}{2}$ where R is the radius of curvature of the curved surface.

Subtracting (i) from (ii) we get

$$\frac{1}{f_m} = \frac{1}{10} - \frac{1}{28}$$

or $f_m = \frac{140}{9}$ cm. Therefore

$$R = 2f_m = \frac{280}{9} \text{ cm}$$

From (i) we have $f_1 = 28 \times 2 = 56$ cm. Now

$$\frac{1}{f_1} = (\mu - 1)\frac{1}{R}$$

or $$\mu - 1 = \frac{R}{f_1} = \frac{280}{9 \times 56} = 0.55$$

or $\mu = 1.55$

25. The value of the effective focal length F is given by

$$\left|\frac{1}{F}\right| = \frac{1}{f_1} + \frac{1}{f_m} + \frac{1}{f_1}$$

$$= \frac{2}{f_1} + \frac{1}{f_m} = \frac{2}{20} + \frac{2}{22}$$

or $|F| = \frac{110}{21}$ cm

Since the convex lens with a silvered surface behaves as a concave mirror of effective focal length F, we have

$$F = -\frac{110}{21} \text{ cm and } u = -10 \text{ cm}$$

Substituting these values in the mirror formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{F}$$

we have $\frac{1}{v}=-\frac{21}{110}+\frac{1}{10}$ or $v=-11$ cm.

The negative sign shows that the image is in front of the effective mirror and hence is real.

26. $\frac{1}{f_1}=(\mu-1)\frac{1}{R}=(1.5-1)\frac{1}{12}=\frac{1}{24}$

or $f_1 = 24$ cm, which is choice (c).

27. When a face of the lens is silvered, it behaves as a concave mirror of effective focal length F given by

$$\left|\frac{1}{F}\right| = \frac{2}{f_1}+\frac{1}{f_m}=\frac{2}{f_1}+\frac{1}{\infty}=\frac{2}{f_1}$$

or $$F=\frac{f_1}{2}=\frac{24}{2}=12 \text{ cm}$$

This means that the parallel rays will converge at a distance of 12 cm from the silvered surface.

28. Refer to Fig. 16.71.

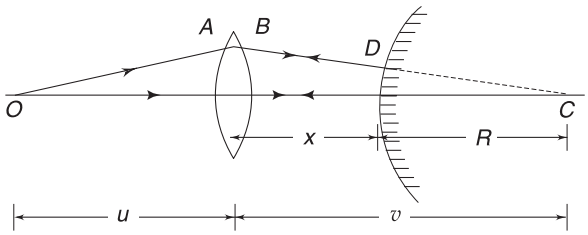


Fig. 16.71

The incident ray OA is refracted along BD by the lens. Since the image coincides with the object, the refracted ray BD must retrace its path after reflection from the mirror, i.e., it falls normally on the mirror. Let C be the point on the axis where BD produced meets the axis. It is clear that C is the centre of curvature of the mirror. Also C is the image of the object O formed by the lens alone. Thus

$$f = +10 \text{ cm and } u = -12 \text{ cm}$$

The distance v of the image is obtained from the lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

which gives $v = -60$ cm

Since $x = 10$ cm, $R = v - x = 60 - 10 = 50$ cm.

Hence the focal length of the mirror $= \frac{R}{2} = 25$ cm.

29. For an achromatic combination

$$\frac{f}{f'}=-\frac{\omega}{\omega'}$$

$$\therefore \quad \frac{1}{f'}=-\frac{\omega}{\omega'}\frac{1}{f}$$

where $\frac{\omega}{\omega'}=\frac{2}{3}$. Therefore,

$$\frac{1}{f'}=-\frac{2}{3f} \qquad \text{(i)}$$

The focal length of the combination is

$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f'}$$

where $F = +20$ cm. Therefore

$$\frac{1}{20}=\frac{1}{f}+\frac{1}{f'} \qquad \text{(ii)}$$

From Eqs (i) and (ii) we get

$$f = 6.67 \text{ cm and } f' = -\frac{3f}{2} = -10 \text{ cm}$$

30. Let f_e and f_0 be the focal lengths of the eyepiece and the objective respectively. In normal adjustment,

$$f_0 + f_e = 14 \qquad \text{(i)}$$

and $$M=\frac{f_0}{f_e}=6 \text{ or } f_0 = 6f_e \qquad \text{(ii)}$$

Using (ii) in (i) we get $f_e = 2$ cm and $f_0 = 12$ cm. Hence the correct choice is (d).

31. In normal adjustment,

$$M=\frac{f_0}{f_e}=\frac{100}{5}=20$$

Hence the correct choice is (c).

32. When the final image is formed at $D = 25$ cm,

$$M=\frac{f_0}{f_e}\times\frac{D+f_e}{D}=\frac{100}{5}\times\frac{25+5}{25}=24$$

Hence the correct choice is (b).

33. Given $u = -2$ km $= -2000$ m and $f_0 = 1.2$ m. The lens formula is

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \qquad \text{or} \qquad v=\frac{fu}{f+u}$$

Since $f_0 << u, f_0 + u \approx u$. Hence

$$v = f_0 = 1.2 \text{ m}$$

Therefore, magnification produced by the objective is

$$m_0 = \frac{v}{u} = -\frac{1.2}{2000}$$

$\therefore$ Height of image $= I \times m_0 = 10 \text{ m} \times -\dfrac{1.2}{2000}$

$= -0.006 \text{ m} = -6 \text{ mm}$

The negative sign indicates the image is inverted. Hence the correct choice is (c).

34. As shown above, since $u >> f_0$, $v = f_0 = 19$ m. Now $u = -3.8 \times 10^8$ m. Therefore, magnification produced by the objective is

$$m_0 = \frac{v}{u} = -\frac{19}{3.8 \times 10^8} = -0.5 \times 10^{-7}$$

$\therefore$ Diameter of the image of the moon is

$$3.5 \times 10^6 \times 0.5 \times 10^{-7} = 0.175 \text{ m} = 17.5 \text{ cm}$$

Hence the correct choice is (d).

35. The magnifying power of a compound microscope is given by

$$M = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right); L = \text{length of the tube.}$$

For M to be high, both f_0 and f_e must be small. Hence the correct choice is (a).

36. $M = \dfrac{f_0}{f_e}$. Hence the correct choice is (c).

37. For an achromatic doublet

$$\frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2}$$

In terms of powers of the lenses, we have

$$\frac{P_2}{P_1} = -\frac{\omega_1}{\omega_2}$$

or $P_1 \omega_1 = -P_2 \omega_2$. Hence the correct choice is (d).

38. If the mirror rotates through an angle θ, the reflected ray rotates through an angle 2θ. Therefore, the spot on the screen will make $2n$ revolutions per second. Hence the speed of the spot will be

$$v = \omega R = 2\pi \times 2n \times R = 4\pi n R$$

Hence the correct choice is (d).

39. Diameter of image = radius of curvature × angle in radians

$$= R\theta = 2f\theta \qquad (\because R = 2f)$$

Hence the correct choice is (d).

40. The focal length F of the combination is given

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

In terms of powers we have

$$P = P_1 + P_2 - d P_1 P_2$$

for $P = 0$, $d = \dfrac{P_1 + P_2}{P_1 P_2} = \dfrac{1}{P_2} + \dfrac{1}{P_1} = f_2 + f_1$.

Hence the correct choice is (c).

41. Since the ray does not bend at the first face of the lens, i.e. it is not refracted, $\mu_1 = \mu$. When this ray meets the second face, it bends towards the normal to that face. Hence $\mu_2 > \mu$. Hence the correct choice is (c).

42. When the plane surface of a planoconvex lens is silvered, it behaves like a concave mirror of focal length f given by

$$\frac{1}{f} = \frac{2(\mu - 1)}{R}$$

or $$f = \frac{R}{2(\mu - 1)} = \frac{10}{2(1.5 - 1)} = 10 \text{ cm}$$

Hence the correct choice is (a).

43. When the plane surface is silvered the focal length f_1 is given by

$$\frac{1}{f_1} = \frac{2(\mu - 1)}{R} \qquad \text{(i)}$$

But when the convex surface is silvered, the focal length f_2 is given by

$$\frac{1}{f_2} = \frac{2\mu}{R} \qquad \text{(ii)}$$

Dividing (i) by (ii), we have

$$\frac{f_1}{f_2} = \frac{\mu}{\mu - 1} = \frac{1.5}{1.5 - 1} = 3$$

or $$f_2 = \frac{f_1}{3} = \frac{30}{3} = 10 \text{ cm, which is choice (a).}$$

44. Refractive index $\mu = \dfrac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\dfrac{A}{2}}$. Now $A = 60°$

and $\mu = \sqrt{3}$. Therefore

$$\sqrt{3} = \frac{\sin\left(30° + \dfrac{\delta_m}{2}\right)}{\sin 30°}$$

or $\sin\left(30° + \frac{\delta_m}{2}\right) = \sqrt{3}\ \sin 30° = \frac{\sqrt{3}}{2}$

or $30° + \frac{\delta_m}{2} = 60°$ which gives $\delta_m = 60°$

Hence the correct choice is (c).

45. For a small-angled prism and for a small angle of incidence, the deviation is given by

$$\delta = (\mu_g - 1)A \qquad \text{(i)}$$

The refractive index of the prism, when it is dipped in water is

$$\mu' = \frac{\mu_g}{\mu_w} = \frac{1.5}{4/3} = \frac{9}{8}$$

$$\therefore \qquad \delta' = (\mu' - 1)A \qquad \text{(ii)}$$

From (i) and (ii) we have

$$\delta' = \left(\frac{\mu' - 1}{\mu_g - 1}\right)\delta = \frac{\left(\frac{9}{8} - 1\right)}{(1.5 - 1)} \times 4° = 1°$$

Hence the correct choice is (a).

46. Real thickness = $\mu \times$ apparent thickness

$$= 1.5 \times (5 + 2) = 10.5 \text{ cm}$$

Hence the correct choice is (d).

47. $\mu = \dfrac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$. Given $\mu = \cot\left(\frac{A}{2}\right)$. Thus

$$\cot\left(\frac{A}{2}\right) = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

or $\cos\left(\frac{A}{2}\right) = \sin\left(\frac{A + \delta_m}{2}\right)$

or $\sin\left(90° - \frac{A}{2}\right) = \sin\left(\frac{A + \delta_m}{2}\right)$

which gives $\frac{A + \delta_m}{2} = 90° - \frac{A}{2}$ or $\delta_m = (180° - 2A)$

which is choice (b).

48. Lateral displacement, i.e. the perpendicular distance between the emergent and incident rays is given by (see Fig. 16.72).

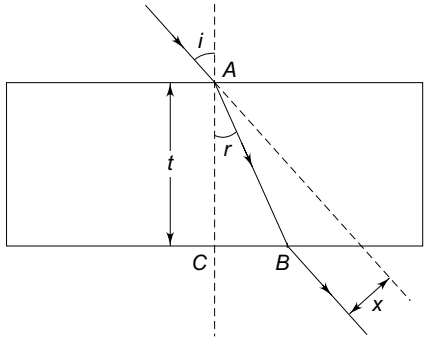


Fig. 16.72

$$x = \frac{t}{\cos r}\ \sin(i - r)$$

where t is the thickness of the slab.

Given $\mu = \sqrt{3} = \dfrac{\sin i}{\sin r} = \dfrac{\sin 60°}{\sin r}$

or $\sin r = \frac{1}{2}$ or $r = 30°$

In triangle ABC, $AB = \dfrac{t}{\cos r}$. Hence

$$x = AB \sin(i - r) = 6.0 \times \sin(60° - 30°)$$
$$= 3.0 \text{ cm}$$

Thus the correct choice is (c).

49. For a prism with a very small refracting angle A, the deviation is given by (Fig. 16.73)

$$\delta = (\mu - 1)\ A$$

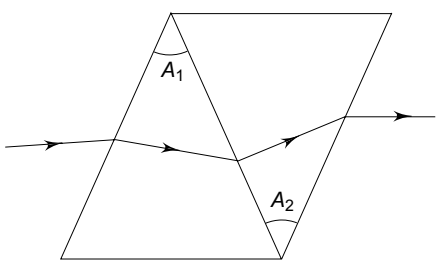


Fig. 16.73

$\therefore$ Deviation produced by the first prism is

$$\delta_1 = (\mu_1 - 1)\ A_1$$

and that produced by the second prism is

$$\delta_2 = (\mu_2 - 1)\ A_2$$

The total deviation will be zero if $\delta_1 + \delta_2 = 0$. The emergent ray will then be parallel to the incident ray (see figure). Thus

$$(\mu_2 - 1)\ A_2 = -(\mu_1 - 1)\ A_1$$

The negative sign shows that the refracting angles of the two prisms are in opposite directions. Thus

$$|A_2| = \frac{(\mu_1 - 1)A_1}{(\mu_2 - 1)} = \frac{(1.54 - 1) \times 4^\circ}{(1.72 - 1)} = 3^\circ,$$

which is choice (c).

50. Since the refractive index of glass is greater than that of air, the speed of light is less in glass than in air. The frequency of light never changes due to reflection or refraction. Since $v = \nu\lambda$ or $\lambda = v/\nu$, wavelength λ decreases because speed v decreases. Hence the correct choice is (a).

51. Using monochromatic light eliminates chromatic aberration. Using a doublet combination minimizes chromatic aberration. Increasing the size of the lens increases its resolving power. To reduce spherical aberration, the aperture (i.e. exposed portion of the lens) must be decreased. Hence the correct choice is (c).

52. Spherical aberration is reduced if the total deviation is distributed over the two surfaces of the lens. If the plane surface of the lens faces the object, all the deviation takes place at the curved surface. Hence spherical aberration is not reduced. Hence the correct choice is (b).

53. Figure 16.74 shows the ray diagram for the image formation in the two cases. When the mirror is filled with water, the image is real and located at C' which is between O and C. Hence the correct choice is (d).

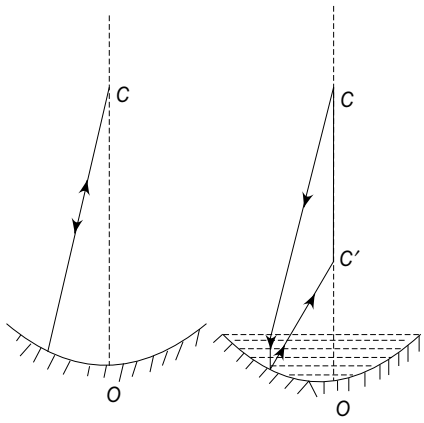


Fig. 16.74

54. Using the formula for a spherical surface

$$\frac{\mu_a}{u} + \frac{\mu_g}{v} = \frac{\mu_g - \mu_a}{R}$$

we have $\frac{1.0}{u} + \frac{1.5}{u} = \frac{1.5 - 1.0}{R}$ (since $v = u$)

which gives $u = 5R$. Hence the correct choice is (a).

55. Power of the lens combination is

$$P = P_1 + P_2 = \frac{1}{f_1(\text{in m})} + \frac{1}{f_2(\text{in m})}$$
$$= \frac{1}{+0.40\ \text{m}} + \frac{1}{-0.25\ \text{m}}$$
$$= -1.5\ \text{m}^{-1} = -1.5\ \text{D},$$

which is choice (b).

56. The focal length f of a lens of refractive index μ_2 surrounded by a medium of refractive index μ_1 is given by

$$\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Now, for a concave lens, $f = -f$. Given $R_1 = R$ and $R_2 = -R_1 = -R$. Also $\mu_2 = 1.5$ and $\mu_1 = 1.75$. Hence, we have

$$-\frac{1}{f} = \left(\frac{1.5 - 1.75}{1.75}\right)\left(\frac{1}{R} + \frac{1}{R}\right) = \left(\frac{-0.25}{1.75}\right)\left(\frac{2}{R}\right)$$

which gives $f = 3.5\ R$. Since the focal length is positive, the lens acts like a convergent lens. Hence the correct choice is (a).

57. Refer to Fig. 16.75. From Snell's law, we have

$$\mu_1 \sin i = \mu_2 \sin r_1$$
$$\mu_2 \sin r_1 = \mu_3 \sin r_2$$
$$\mu_3 \sin r_2 = \mu_4 \sin e$$

From these equations, it follows that

$$\mu_1 \sin i = \mu_4 \sin e \qquad (1)$$

Ray CD will be parallel to ray AB, if $e = i$. Hence $\mu_1 = \mu_4$, which is choice (d).

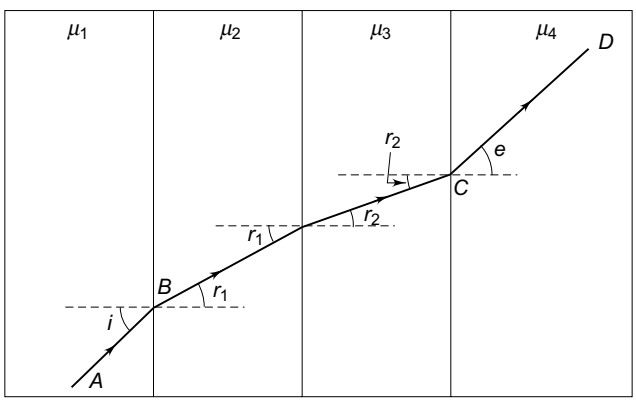


Fig. 16.75

58. For refraction at glass-water interface, we have from Snell's law

$$\mu_g \sin i = \mu_\omega \sin r \quad \text{(i)}$$

For refraction at water-air interface, we have

$$\mu_\omega \sin r = \mu_a \sin 90° = \mu_a = 1 \quad \text{(ii)}$$

Using (ii) in (i), we get

$$\mu_g \sin i = 1 \text{ or } \mu_g = \frac{1}{\sin i}, \text{ which is choice (b).}$$

59. The refractive index of the prism with respect to the liquid in which it is immersed is given by

$$\mu = \frac{\sin\left\{\frac{1}{2}(A+\delta_m)\right\}}{\sin\frac{1}{2}(A)}$$

$$= \frac{\sin\left(\frac{60°+30°}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$$

$$= \frac{\sin 45°}{\sin 30°} = \sqrt{2}$$

The critical angle i_c is given by

$\sin i_c = \frac{1}{\sqrt{2}}$, which give $i_c = 45°$. Hence the correct choice is (b).

60. Mean angular deviations produced by crown and flint glass prisms respectively are

$$d = (\mu - 1)\, A \text{ and } d' = (\mu' - 1)\, A'$$

Their dispersive powers are

$$\omega = \frac{(\mu_v - \mu_r)}{(\mu - 1)} \text{ and } \omega' = \frac{(\mu'_v - \mu'_r)}{(\mu' - 1)}$$

Their angular dispersions respectively are

$$D = (\mu_v - \mu_r)\, A \text{ and } D' = (\mu'_v - \mu'_r)\, A'$$

When the prisms are combined, the dispersion by the combination will be zero if

$$D + D' = 0$$

or $$(\mu_v - \mu_r)\, A + (\mu'_v - \mu'_r)\, A' = 0$$

or $$\frac{(\mu_v - \mu_r)}{(\mu - 1)}(\mu - 1)\, A + \frac{(\mu'_v - \mu'_r)}{(\mu' - 1)}(\mu' - 1)\, A' = 0$$

or $\omega d + \omega' d' = 0$, which is choice (c).

61. The refracted ray QR will retrace its path if it falls normally on the silvered face AC. It follows from Fig. 16.76 that

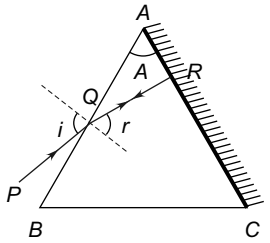


Fig. 16.76

$$r = A = 30° \text{ (given)}$$

Also $i = 45°$ (given). Hence

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 45°}{\sin 30°} = \sqrt{2}$$

which is choice (b).

62. Given $\mu = \sqrt{3}$ and $\delta_m = A$. Now

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{A+A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin A}{\sin\left(\frac{A}{2}\right)}$$

$$= \frac{2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = 2\cos\left(\frac{A}{2}\right)$$

$\therefore\ 2\cos\left(\frac{A}{2}\right) = \mu = \sqrt{3}$ or $\cos\left(\frac{A}{2}\right) = \frac{\sqrt{3}}{2}$

or $\frac{A}{2} = 30°$ or $A = 60°$

Hence the correct choice is (c).

63. Using $f = +20$ cm and $u = -280$ cm in $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get $v = +\frac{56}{3}$ cm. Differentiating $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we have

$$-\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0 \quad \text{or} \quad \frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt}$$

Speed of image $= \frac{v^2}{u^2} \times$ (speed of object). Substituting the values of u, v and $\frac{du}{dt}$, we find that the correct choice is (b).

64. The concave mirror formula is

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \qquad (1)$$

Since, for a given concave mirror, focal length f is fixed, we partially differentiate Eq. (1) to get

$$-\frac{\delta v}{v^2}=-\frac{\delta u}{u^2}=0 \quad \text{or} \quad \delta v=-\left(\frac{v}{u}\right)^2 \delta u \qquad (2)$$

Multiplying Eq. (1) by u, we get

$$\frac{u}{v}+1=\frac{u}{f} \quad \text{or} \quad \frac{v}{u}=\frac{f}{u-f} \qquad (3)$$

Using Eq. (3) in Eq. (2), we get

$$\delta v=-\left(\frac{f}{u-f}\right)^2 \delta u$$

Given $\delta u = b$. Therefore

$$\delta v=-\left(\frac{f}{u-f}\right)^2 b \qquad (4)$$

The negative sign shows that image is longitudinally inverted. The magnitude of the size of the image is

$$|\delta v| = b\left(\frac{f}{u-f}\right)^2, \text{ which is choice (c).}$$

65. Referring to Fig. 16.77, the source S cannot be seen at all from above when the water level attains a critical maximum height x so that rays such as SA and SB suffer internal relection. The critical angle i_c is given by

$$\sin i_c = \frac{1}{\mu}=\frac{1}{\sqrt{2}}, \text{ which gives } i_c = 45°.$$

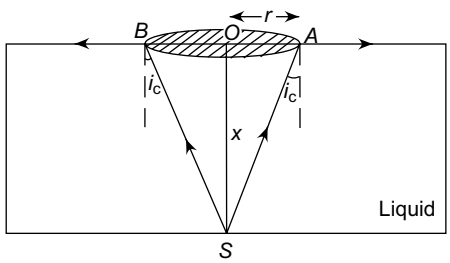


Fig. 16.77

In triangle OAS, we have

$$\frac{r}{x}=\tan i_c \text{ or } x=\frac{r}{\tan i_c}=\frac{r}{\tan 45°}=r.$$

Hence the correct choice is (a).

66. Refer to Fig. 16.78.

Ray OA is incident at an angle α at the planar face of the cylindrical rod. Let θ be the angle of refraction. From Snell's law, we have

$$n=\frac{\sin\alpha}{\sin\theta} \quad \text{or} \quad \sin\theta=\frac{\sin\alpha}{n} \qquad (1)$$

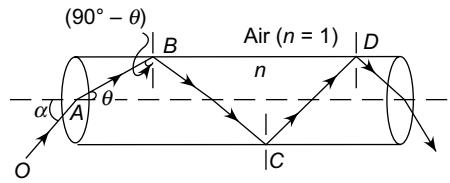


Fig. 16.78

The ray AB is incident at point B of the curved surface of the cylinder at an angle $(90° - \theta)$. This ray is travelling in a denser medium of refractive index n and is incident at the cylinder-air interface at point B. The ray will not emerge from the curved surface if it suffers total internal reflection at B. For this to happen $(90° - \theta) \geq i_c$, the critical angle or

$$\sin(90° - \theta) \geq \sin i_c \text{ or } \cos\theta \geq \sin i_c$$

or $$(1 - \sin^2\theta)^{1/2} \geq \sin i_c$$

or $$1 - \sin^2\theta \geq \sin^2 i_c \qquad (2)$$

The critical angle is given by

$$\sin i_c = \frac{1}{n} \qquad (3)$$

Using Eqs. (1) and (3) in Eq. (2) we get

$$1-\frac{\sin^2\alpha}{n^2}\geq\frac{1}{n^2} \text{ or } n^2-\sin^2\alpha\geq 1$$

or $$n^2 \geq (1 + \sin^2\alpha)$$

Since the maximum value of $\sin^2\alpha = +1$, it follows that

$$n_{\min}^2 \geq 2 \quad \text{or} \quad n_{\min} \geq \sqrt{2}$$

This is the minimum value of refractive index of the cylindrical rod for the ray AB to suffer total internal reflection at point B. By symmetry, ray BC will be totally reflected along CD suffering another total internal reflection at D and so on until the ray finally emergs from the opposite planar face of the rod.

67. Given $i = 60°$, $\delta = 30°$ and $A = 30°$. Using $\delta = i + e - A$, we get $e = 0$. i.e. the emergent ray is perpendicular to face AC of the prism (see Fig. 16.79).

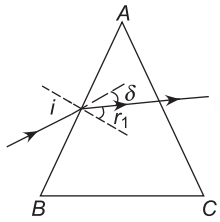


Fig. 16.79

Now $r_1 = i - \delta = 60° - 30° = 30°$. Hence

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin 60°}{\sin 30°} = \sqrt{3}\,.$$

Hence the correct choice is (c).

68. $u = -25$ cm and $f = -10$ cm. The distance of the image is given by

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-10} - \frac{1}{-25} \quad \text{or} \quad v = -\frac{3}{50} \text{ cm}$$

Area of the object wire is $3.0 \times 3.0 = 9.0$ cm^2. The area magnification is given by

$$\frac{\text{area of image}}{\text{area of object}} = (\text{linear magnification})^2$$

$$= \frac{v^2}{u^2} = \left(\frac{50}{3\times 25}\right)^2 = \frac{4}{9}$$

Therefore, the area enclosed by image is

$$\frac{4}{9} \times 9 \text{ cm}^2 = 4 \text{ cm}^2,$$

which is choise (b).

69. Two rays 1 and 2 from the object placed at O fall normally on the spherical surface and go through undeviated (see Fig. 16.80). The divergent rays $1'$ and $2'$ appear to come from O. Hence the virtual image is formed at the centre O, which is at a distance of 6 cm from the surface of the sphere. Hence the correct choice is (b).

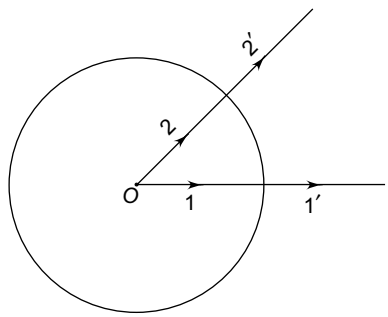


Fig. 16.80

70. Refer to Fig. 16.81.

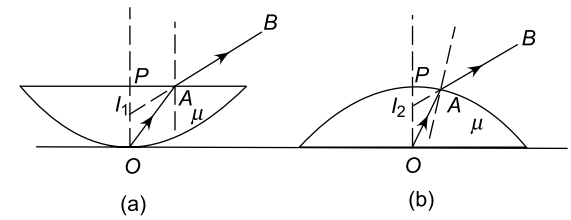


Fig. 16.81

When the curved face of the lens is in contact with the table, the virtual image of the bottom-most point O of the lens is formed at I_1 due to refraction at the plane surface as shown in Fig. 16.90 (a).

Real depth = 4 cm and apparent depth = 3 cm. Hence

$$\mu = \frac{4}{3}$$

When the plane face of the lens is in contact with the table, the image of the centre O of the plane face of the lens is formed at I_2 due to refraction at the curved face as shown in Fig. 16.90 (b). For refraction at this face, we have

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

where $\mu_1 = 1$, $\mu_2 = \mu$, $u = OP = -4$ cm, and $v = OI_2 = -\frac{25}{8}$ cm. Putting these values, we get

$$-\frac{8}{25} - \frac{\mu}{-4} = \frac{1-\mu}{R}$$

Putting $\mu = \frac{4}{3}$ and solving we get $R = 25$ cm. Now, the focal length of the plano-convex lens is given by

$$\frac{1}{f} = (\mu - 1)\ \frac{1}{R} = \left(\frac{4}{3} - 1\right) \times \frac{1}{25} = \frac{1}{75}$$

or $\qquad f = 75$ cm.

71. Refer to Fig. 16.82.

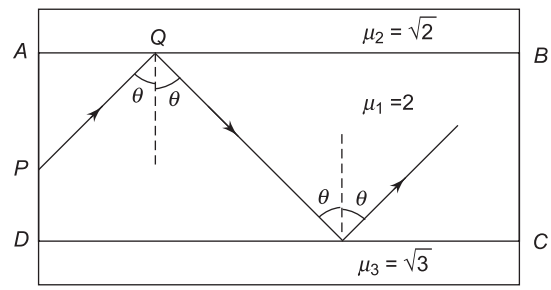


Fig. 16.82

For total internal reflection at surface AB, angle θ must be greater than or equal to the critical angle i_1 given by

$$\sin i_1 = \frac{\mu_2}{\mu_1} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

which gives $i_1 = 45°$

For total internal reflection at surface CD, angle θ must be greater than or equal to the critical angle i_2 given by

$$\sin i_2 = \frac{\mu_3}{\mu_1} = \frac{\sqrt{3}}{2}$$

which gives $i_2 = 60°$.

Hence, for total internal reflection at both the surfaces AB and CD, the minimum value of $\theta = 60°$. Thus the correct choice is (c).

72. Refer to Fig. 16.83. For refraction at face I,

$$\frac{1}{u_1} + \frac{\mu}{v_1} = \frac{\mu - 1}{R}$$

Since $u_1 = \infty$, we have

$$\frac{\mu}{v_1} = \frac{\mu - 1}{R}$$

or $$v_1 = \frac{\mu R}{(\mu - 1)} \quad (1)$$

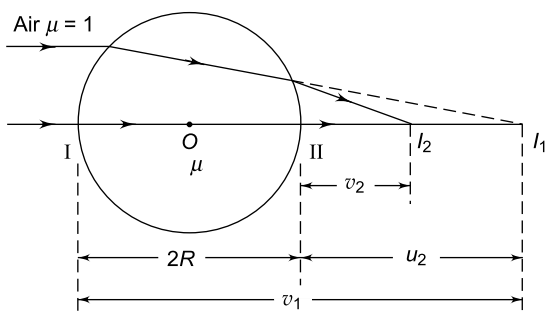


Fig. 16.83

For refraction at face II, $u_2 = -(v_1 - 2R) = 2R - v_1$. Using Eq. (1), we get

$$u_2 = 2R - \frac{\mu R}{(\mu - 1)} = \frac{R(\mu - 2)}{(\mu - 1)} \quad (2)$$

The image distance v_2 is given by

$$\frac{\mu}{u_2} + \frac{1}{v_2} = \frac{\mu - 1}{R} \quad (3)$$

Using Eq. (2) in Eq. (3) and simplifying, we get

$$v_2 = \frac{R(2 - \mu)}{2(\mu - 1)}.$$

The correct choice is (a).

73. The effective focal length of the silvered lens is given by

$\frac{1}{F} = \frac{2}{f} + \frac{1}{f_m} = \frac{2}{15} + \frac{1}{\infty} = \frac{2}{15}$, which gives $F = \frac{15}{2}$ cm. The silvered lens behaves like a concave mirror. Using the spherical mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{F}$, we have $\left(\because u = -20\,\text{cm and } F = -\frac{15}{2}\text{cm}\right)$

$$\frac{1}{v} + \frac{1}{-20} = \frac{2}{-15}$$

which gives $v = -12$ cm. The negative sign indicates that the image is formed to the left of the lens. Hence the correct choice is (d).

74. Refer to the Fig. 16.84.

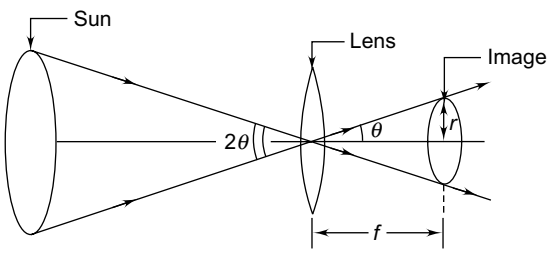


Fig. 16.84

The angular diameter of the sun is 2θ. From the figure it follows that $\tan\theta = r/f$. Since θ is small, $\tan\theta \simeq \theta$, where θ is expressed in radian. Hence, we have

$$\theta = \frac{r}{f} \quad \text{or} \quad r = f\theta$$

$\therefore$ Area of image $= \pi r^2 = \pi\theta^2 f^2$. Thus area $\propto f^2$, which is choice (b).

75. The ray falling normally on face AB goes into the prism undeviated and is incident on face AC at an angle $i = \theta = 45°$. If μ_l is the refractive index of the liquid, the ray incident on AC will be totally reflected if

$$\sin\theta \geq \frac{\mu_l}{\mu_g}$$

or $$\mu_l \leq \mu_g \sin\theta$$

$\Rightarrow \quad \mu_l \leq 1.6 \times \sin 45°$

$\Rightarrow \quad \mu_l \leq 0.8\sqrt{2}$

So the correct choice is (a).

76. Since the lens is convex f is positive. Also R_1 is positive and R_2 is negative. Therefore, we have

$$\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

where μ_2 is the refractive index of the liquid filling the lens and μ_1 that of the medium in which the lens is placed. It will diverge a parallel bean of light if f is negative, *i.e*., if $\mu_1 > \mu_2$. Hence the correct choice is (a).

77. The critical angle i_c for red light $= \sin^{-1}\left(\frac{1}{1.39}\right) = 46°$

The critical angle i_c for blue light $= \sin^{-1}\left(\frac{1}{1.47}\right) = 43°$

The parallel rays fall normally on face AB go through undeviated. Their angle of incidence on face AC is $i = 45°$. Since 45° is less than i_c for red light but more than i_c for blue light, the blue light is total reflected at face AC and the red light emerges out of AC. So the correct choice is (c).

78. The ray will emerge from side CD if the ray incident on face AB after entering the slab suffers repeated total internal reflections at faces AD and BC of the slab as shown in Fig. 16.85.

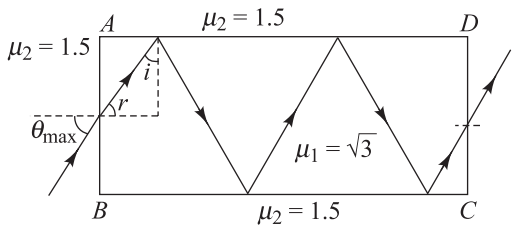


Fig. 16.85

From Snell's law,

$\mu_2 \sin \theta_{max} = \mu_1 \sin r$

$\Rightarrow \quad \sin \theta_{max} = \frac{\mu_1}{\mu_2} \sin r \qquad (1)$

The critical angle is given by

$$\sin i_c = \frac{\mu_2}{\mu_1}$$

For total internal reflection, angle i must at least be equal i_c. Therefore, $r = 90° - i_c$. Using this (1) we get

$$\sin \theta_{max} = \frac{\mu_1}{\mu_2} \sin (90° - i_c) = \frac{\mu_1}{\mu_2} \cos i_c \qquad (2)$$

Now $\sin i_c = \frac{\mu_2}{\mu_1} = \frac{1.5}{\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow i_c = 60°$. Therefore, $\cos i_c = \cos 60° = \frac{1}{2}$. Using this in (2) we get

$$\sin \theta_{max} = \frac{\sqrt{3}}{1.5} \times \frac{1}{2} = \frac{1}{\sqrt{3}}$$

So the correct choice is (c).

79. The ray PQ will undergo minimum deviation in prism A if the angle of incidence on A is such that the refracted ray in A is parallel to its base. Since prisms B and C are identical to A and are made of the same material as A, the refracted ray in A suffers no further deviations by prisms B and C. Hence the emergent ray from prism C will suffer the same deviation as that produced by prism A. Hence the correct choice is (c).

80. In cases (a), (b) and (d) the radii of curvature of the faces of the lens are different. But in case (*c*), $R_1 = R_2$. In this case, the incident and emergent rays will be parallel, *i.e*. the lens does not produced any deviation. Hence it will not disperse light into its constituent colours. Hence correct choice is (c).

81. Refer to Fig. 16.86.

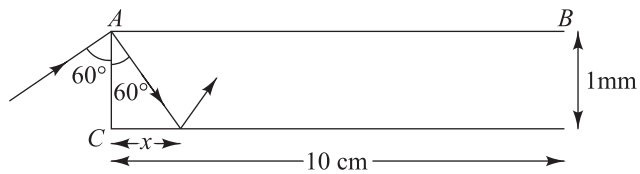


Fig. 16.86

It is clear than $x = AC \tan 60° = 0.1 \times \sqrt{3}$ cm

Therefore, number of reflections to cover a distance of 10 cm is

$$n = \frac{10 \text{ cm}}{0.1 \times \sqrt{3} \text{ cm}} = \frac{100}{\sqrt{3}} = 57.7$$

So the correct choice is (a).

82. Refer to Fig. 16.87.

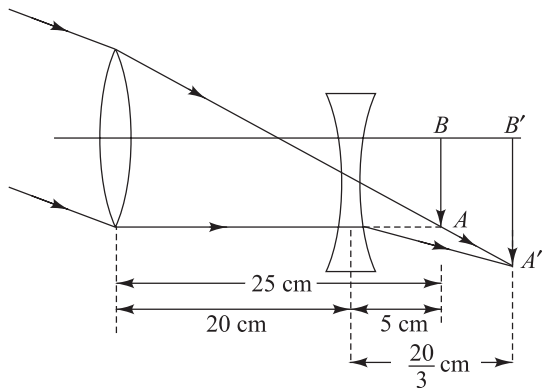


Fig. 16.87

AB is the image of the distant tower formed by the convex lens. Since the rays from the distant object are parallel, this image is formed at the focal plane of the lens, *i.e.*, the distance of image AB from the convex lens is equal to its focal length = 25 cm. For the concave lens, AB serves as an object. Therefore $f = -20$ cm, $u = +5$ cm.

From $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we have

$$\frac{1}{v} - \frac{1}{5} = -\frac{1}{20} \quad \Rightarrow \quad v = \frac{20}{3}\text{ cm}$$

Now $$\frac{A'B'}{AB} = \frac{v}{u} = \frac{20/3}{5} = \frac{4}{3}$$

$$\therefore \quad A'B' = \frac{4}{3} \times AB = \frac{4}{3} \times 3\text{ cm} = 4\text{ cm}$$

So the correct choice is (d).

83. Using Snell's for refraction at Q,

$$1 \times \sin 60° = \sqrt{3}\sin r$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \sqrt{3}\sin r \Rightarrow \sin r = \frac{1}{2} \Rightarrow r = 30°.$$

When the prism is in the minimum deviation position, the refracted ray QR is parallel to the base BC. Hence $\alpha = 45°$. So the correct choice is (a).

84. Glass is dispersive and the refractive index μ of glass is related to wavelength λ as

$$\mu = A + \frac{B}{\lambda^2}$$

where A and B are constants. Since $\lambda_R > \lambda_G > \lambda_B$, $\mu_R < \mu_G < \mu_B$. Therefore, blue light is deviated the most and red light is deviated the least. Since blue light is just totally internally reflected at face AC,

$$\sin i_c = \frac{1}{\mu_B}$$

where i_c is the critical angle for blue light. Since $\mu_R < \mu_B$ and $\mu_G < \mu_B$, the critical angles for red and green lights are greater than that for blue light. Hence red and green light are incident on face AC at angles less than their respective critical angle. Therefore, they will not suffer total internal reflection at face AC and will emerge out from it. So the correct choice is (c).

85. Refer to Fig. 16.88.

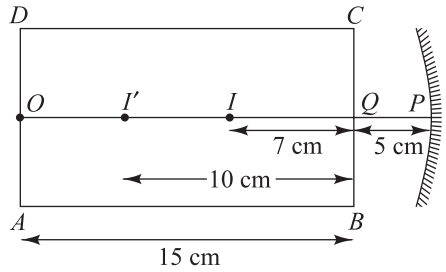


Fig. 16.88

Due to refraction by the slab the image of O is formed at I'. Let $x = I'Q$ be the distance of I' from side BC. Then

$$x = \frac{15\text{ cm}}{1.5} = 10\text{ cm}$$

Image I' serves as the virtual object for concave mirror forming its image I due to reflection. Now

$$u = PI' = -(10 + 5) = -15\text{ cm}$$

and $$v = PI = -(7 + 5) = -12\text{ cm}$$

Using these values in the spherical mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

we have

$$\frac{1}{f} = \frac{1}{-15} + \frac{1}{-12} = -\frac{3}{20}$$

$$\Rightarrow \quad f = -\frac{20}{3}\text{ cm}.$$

So the correct choice is (c).

86. Let f_1 be the focal length of the convex lens and f_2 that of the concave lens. Given $f_2 = -10$ cm and

$$\left|\frac{P_1}{P_2}\right| = \frac{3}{2} \quad \text{or} \quad \frac{f_2}{f_1} = \frac{2}{3}$$

Since f_2 is negative, $\frac{f_2}{f_1} = -\frac{2}{3} \Rightarrow f_1 = -\frac{3}{2}f_2$ or $f_1 = -\frac{3}{2} \times -10 = 15$ cm. The focal length of the combination is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{15} - \frac{1}{10} = -\frac{1}{30}$$

which gives $F = -30$ cm. So the correct choice is (d).

87. Refer to Fig. 16.89.

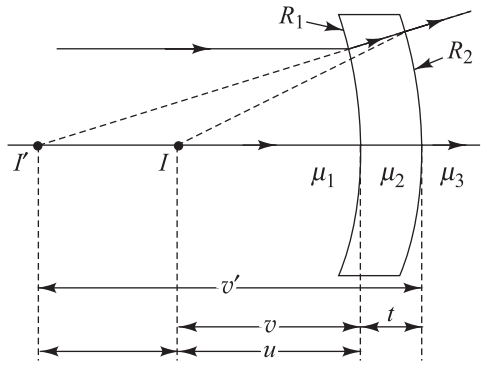


Fig. 16.89

I' is the image due to refraction at the first face. For refraction at the first face, we have

$$\frac{\mu_2}{v'} - \frac{\mu_1}{-\infty} = \frac{\mu_2 - \mu_1}{R_1}$$

$$\Rightarrow \quad \frac{\mu_2}{v'} = \frac{\mu_2 - \mu_1}{R_1} \qquad (1)$$

Image I' serves as the object for refraction at the second face and I is the final image. For this refraction, $u = v' - t = v'$ ($\because$ t is negligible)

$$\frac{\mu_3}{v} - \frac{\mu_2}{v'} = \frac{\mu_3 - \mu_2}{R_2}$$

Since the incident rays are parallel, $v = f$.

$$\therefore \quad \frac{\mu_3}{f} - \frac{\mu_2}{v'} = \frac{\mu_3 - \mu_2}{R_2} \qquad (2)$$

Adding (1) and (2), we get

$$\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_3}\right)\frac{1}{R_1} + \left(\frac{\mu_3 - \mu_2}{\mu_3}\right)\frac{1}{R_2} \qquad (3)$$

Equation (3) is the general expression for the focal length of the given lens. It holds for both convex and concave lenses and has sign conventions. It is given that $R_1 = R_2 = -R$. Putting this in Eq. (3), we get

$$\frac{1}{f} = -\frac{(\mu_3 - \mu_1)}{\mu_3 R}$$

$$\Rightarrow \quad f = -\frac{\mu_3 R}{(\mu_3 - \mu_1)}$$

The negative sign shows that the given lens is a concave (diverging) lens. So the correct choice is (d).

88. Refer to Fig. 16.90. From Snell's law,

$$\mu \sin r = \sin 60° = \frac{\sqrt{3}}{2}$$

$$\sin r = \frac{\sqrt{3}}{2\mu} \qquad (1)$$

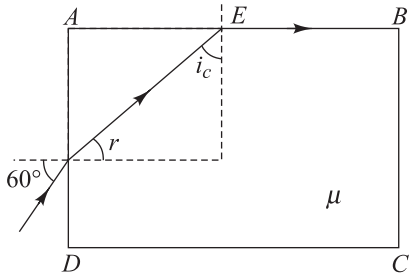


Fig. 16.90

The refracted ray will be totally reflected at E if it is incident at an angle equal to or greater than the critical angle i_c which is given by

$$\sin i_c = \frac{1}{\mu}$$

It follows from the figure that $r = 90° - i_c$. Therefore,

$$\sin r = \sin (90° - i_c) = \cos i_c$$

$$\Rightarrow \quad \sin r = (1 - \sin^2 i_c)^{1/2} \qquad (3)$$

Using (1) and (2) in (3), we get

$$\frac{\sqrt{3}}{2\mu} = \left(1 - \frac{1}{\mu^2}\right)^{1/2}$$

$$\Rightarrow \quad \frac{3}{4\mu^2} = 1 - \frac{1}{\mu^2}$$

$$\Rightarrow \quad \mu = \frac{\sqrt{7}}{2}, \quad \text{which is choice (d).}$$

89. In two-pin method, the image formed is real and inverted. The focal length of a thin convex lens forming a real image is given by

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

where u, v and f are numerical values (without signs) of u, v and f. Thus

$$\frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$$

Therefore, the slope of $\frac{1}{v}$ versus $\frac{1}{u}$ graph is equal to –1 and the intercept = $\frac{1}{f}$, *i.e.*

$$0.05 = \frac{1}{f} \quad \Rightarrow \quad f = 20 \text{ cm} = 0.2 \text{ m}$$

$\therefore$ Power of lens = $\frac{1}{0.2}$ = +5 D, which is choice (a).

90. It follows from the graph that when $u = -20$ cm, $v = +20$ cm. Substituting these values in the thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \quad \text{we get}$$

$$\frac{1}{20} - \frac{1}{-20} = \frac{1}{f} \quad \Rightarrow \quad f = 10 \text{ cm}$$

The error Δf in f is given by

$$\frac{\Delta f}{f^2} = \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}$$

$$= \frac{0.1}{(20)^2} + \frac{0.1}{(20)^2} = \frac{0.2}{400}$$

$$\therefore \qquad \Delta f = \frac{0.2}{400} \times f^2 = \frac{0.2 \times (10)^2}{400} = 0.05$$

$$\therefore \qquad f = (10.00 \pm 0.05) \text{ cm}$$

So the correct choice is (c).

91. When a ray of light falls on a boundary between two transparent media, a part of incident intensity is reflected and the rest is refracted. Hence choices (a) and (b) are both incorrect. Now refer to Fig. 16.91.

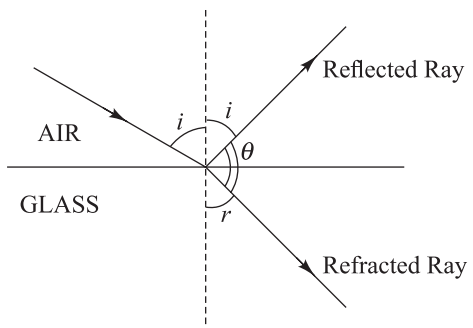


Fig. 16.91

It is clear from the figure that

$$\theta + i + r = 180°$$

$$\Rightarrow \qquad \theta = 180° - i - r$$

$$= 180° - i - r - i + i$$

$$= (180° - 2i) + (i - r)$$

Since $i > r$, $\theta > (180° - 2i)$. So the correct choice is (c).

92. Refer to Fig. 16.92. The ray will not enter medium IV if the angle of refraction $r_3 = 90°$.

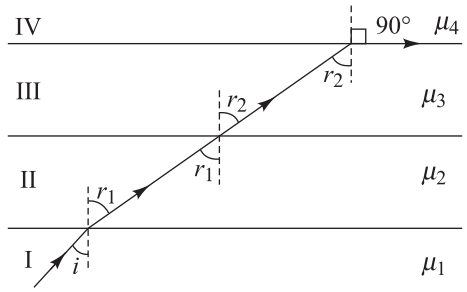


Fig. 16.92

Applying Snell's law at the interfaces of the media, we have

$$\mu_1 \sin i = \mu_2 \sin r_1 = \mu_3 \sin r_2 = \mu_4 \sin 90°$$

$$\Rightarrow \qquad \mu_1 \sin i = \mu_4$$

$$\Rightarrow \qquad i = \sin^{-1}\left(\frac{\mu_4}{\mu_1}\right)$$

So the correct choice is (a).

93. $$\mu = \frac{\sin\left(\frac{A + \delta_{\min}}{2}\right)}{\sin\left(\frac{A}{2}\right)} \qquad (1)$$

Since $\mu_R < \mu_Y < \mu_B$, it follows from (1) that $\delta_R < \delta_Y < \delta_B$. So the correct choice is (b).

94. When the prism is in the minimum deviation position,

$$r = \frac{A}{2} = \frac{60°}{2} = 30°$$

for lights of all colours. Hence correct choice is (c).

95. Refer to Fig. 16.93.

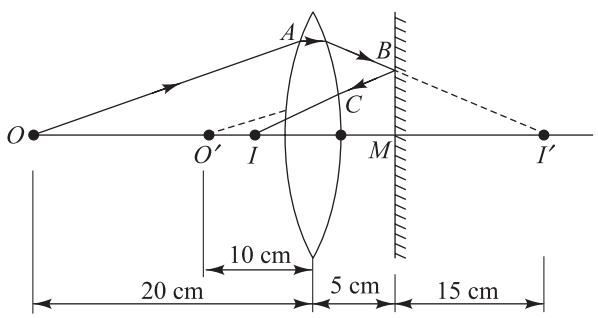


Fig. 16.93

For refraction at A, $u = -20$ cm, $f = +10$ cm and $v' = ?$

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \qquad \frac{1}{v'} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} - \frac{1}{20} \quad \Rightarrow \quad v' = +20 \text{ cm}.$$

The image I' is at a distance of 20 – 5 = 15 cm from the mirror M. After reflection from the mirror, the position of the virtual image O' is such that $O'M = MI' = 15$ cm. Therefore O′ is at a distance of 15 – 5 = 10 cm to the left of the lens. Now for refraction of ray BC by the lens, $u = +10$ cm.

Now $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} - \frac{1}{10} = \frac{1}{10} \quad \Rightarrow \quad v = +5 \text{ cm}$$

Thus the final image I is at a distance of 5 cm of the lens. Therefore, the distance of I from $M = 5 + 5 = 10$ cm. So the correct choice is (a).

96. Since the image is real,

$$\frac{v}{u} = -2 \quad \Rightarrow \quad v = -2u$$

Now $u = -24$ cm. Therefore $v = -2 \times (-24) = 48$ cm. Using the lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \quad \text{we have}$$

$$\frac{1}{48} - \frac{1}{-24} = \frac{1}{f} \quad \Rightarrow \quad f = 16 \text{ cm}$$

Since frequency ν never changes on reflection or refraction, $c = \nu\lambda_a$ and $v = \nu\lambda_g$. Therefore,

$$\mu_g = \frac{c}{v} = \frac{\nu\lambda_a}{\nu\lambda_g} = \frac{\lambda_a}{\lambda_g} = \frac{3}{2} = 1.5$$

Also $R_1 = R$ and $R_2 = -R$,

$$\frac{1}{f} = (\mu_g - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$$\Rightarrow \quad \frac{1}{16} = (1.5 - 1) \times \frac{2}{R}$$

$$\Rightarrow \quad R = 16 \text{ cm}$$

So the correct choice is (a).

97. $\frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2} = -\frac{2}{3}$

Also $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}$

$$\Rightarrow \quad 1 + \frac{f_1}{f_2} = \frac{f_1}{F}$$

$$\Rightarrow \quad 1 - \frac{2}{3} = \frac{f_1}{54} \quad \Rightarrow \quad f_1 = 18 \text{ cm}$$

$$\therefore f_2 = -\frac{3f_1}{2} = -\frac{3 \times 18}{2} = -27 \text{ cm}$$

So the correct choice is (c).

98. **Case (*i*)** consists of a plano-convex glass lens of radii of curvature $R_1 = \infty$ and $R_2 = -R$, a concave water lens of radii of curvature $R_1 = -R$ and $R_2 = +R$ and another plano-convex glass lens of radii of curvature $R_1 = +R$ and $R_2 = \infty$. The focal length of combination in this case is given by

$$\frac{1}{F_1} = \frac{1}{f_g} + \frac{1}{f_w} + \frac{1}{f_g}$$

$$= (\mu_g - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) + (\mu_w - 1)\left(-\frac{1}{R} - \frac{1}{R}\right) + (\mu g - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$$= 2(\mu_g - 1)\frac{1}{R} - 2(\mu_w - 1)\frac{1}{R}$$

$$= (\mu_g - \mu_w)\frac{2}{R}$$

Since $\mu_g > \mu_w$, F_1 is positive and the combination in case (*i*) is convergent.

Case (*ii*) consists of a plano-convex glass of radii of curvature $R_1 = \infty$ and $R_2 = -R$, a plano-concave water lens of radii of curvature $R_1 = -R$ and $R_2 = \infty$ and plano-convex glass lens of radii of curvature $R_1 = \infty$ and $R_2 = -R$. The focal length of combination in this case is given by

$$\frac{1}{F_2} = (\mu_g - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) + (\mu_w - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) + (\mu_g - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$$

$$= 2(\mu_g - 1)\frac{1}{R} - (\mu_w - 1)\frac{1}{R}$$

$$= (2\mu_g - 2 - \mu_w + 1)\frac{1}{R}$$

$$= (2\mu_g - \mu_w - 1)\frac{1}{R}$$

$$= \left(2 \times 1.5 - \frac{4}{3} - 1\right)\frac{1}{R}$$

$$= (3 - 2.33)\frac{1}{R}$$

So F_2 is also positive. Hence correct choice is (c).

99. In case (*i*),

$$\frac{1}{F_1} = (\mu_g - \mu_l)\frac{2}{R} = (1.5 - 1.6)\frac{2}{R} = -\frac{0.2}{R}$$

Since F is negative, the combination is divergent.

In case (*ii*),

$$\frac{1}{F_2} = (2\mu_g - \mu_l - 1)\frac{1}{R}$$
$$= (2\times 1.6 - 1.6 - 1)\frac{1}{R}$$
$$= (3 - 2.6)\frac{1}{R} = \frac{0.4}{R}$$

Since F_2 is positive, the combination is convergent. Hence the correct choice is (b).

100. Refer to Fig. 16.94.

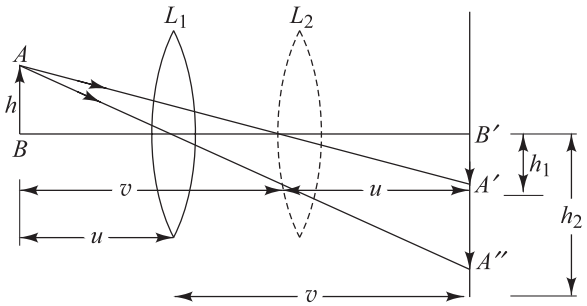


Fig. 16.94

Since the images are received on the screen, they are real. For a convex lens forming a real image,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Hence u and v are interchangeable as shown in the figure. It follows that

$$\frac{h_1}{h} = \frac{u}{v} \quad \text{and} \quad \frac{h_2}{h} = \frac{v}{u}$$

Hence $\frac{h_1}{h}\times\frac{h_2}{h} = 1 \Rightarrow h = \sqrt{h_1 h_2}$

So the correct choice is (a).

101. Let $u = -x$. Then $v = D - x$. Putting these in $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get [see Fig. 16.93]

$$\frac{1}{D-x} + \frac{1}{x} = \frac{1}{f}$$

$$\Rightarrow \quad x^2 - Dx + Df = 0$$

The two roots of this quadratic equation are

$$x_1 = \frac{D - \sqrt{D(D-4f)}}{2}$$

and $$x_2 = \frac{D + \sqrt{D(D-4f)}}{2}$$

Therefore, the distance between L_1 and L_2 is

$d = x_2 - x_1 = \sqrt{D(D-4f)}$, which is choice (d).

102. We have seen that

$$d = \sqrt{D(D-4f)}$$

$$\Rightarrow \quad d^2 = D(D-4f)$$

$$\Rightarrow \quad d^2 = D^2 - 4Df$$

$$\Rightarrow \quad f = \frac{D^2 - d^2}{4D}$$

So the correct choice is (a).

103. For position L_1, $u = -x_1$ and $v = D - x_1$. Therefore

$$m_1 = -\frac{D - x_1}{x_1} \tag{1}$$

where $$x_1 = \frac{D - \sqrt{D(D-4f)}}{2} \tag{2}$$

and $$f = \frac{D^2 - d^2}{4D} \tag{3}$$

Using (3) in (2) and simplifying, we get

$$x_1 = \frac{1}{2}(D - d)$$

Using this in (1), we get

$$m_1 = \left(\frac{D+d}{D-d}\right) \tag{4}$$

Similarly, for position L_2,

$$m_2 = \left(\frac{D-d}{D+d}\right) \tag{5}$$

$$\therefore \quad \frac{m_1}{m_2} = \left(\frac{D+d}{D-d}\right)^2$$

So the correct choice is (c).

Notice that $m_1 \times m_2 = 1$.

104. Refer to Fig. 16.95.

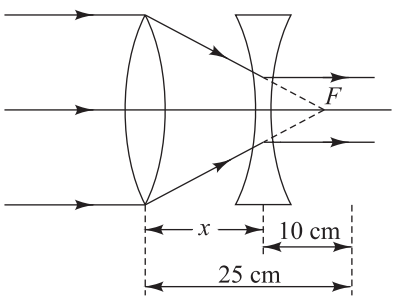


Fig. 16.95

A parallel beam of light is focussed at F, the second focus of the convex lens. If F is the first focus of the concave lens, the rays emerging from the lens will be parallel. It is clear from the figure that $x = 25 - 10 = 15$ cm, which is choice (d).

SECTION 2

Multiple Choice Questions Based on Passage

Questions 1 to 5 are based on the following passage.

Passage I

A microscope is a device which is used to view tiny objects. A compound microscope consists of two converging lenses called the objective and the eyepiece. The tiny object to be examined is placed just beyond the first focus of the objective. The position of the eyepiece is adjusted till the image due to the objective is within the first focus of the eyepiece. The highly enlarged final image is seen by the eye which is held close to the eyepiece.

1. In a compound microscope, the intermediate image (i.e. image of the object due to the objective) is
   (a) real, inverted and magnified
   (b) real, inverted and diminished
   (c) virtual, erect and magnified
   (d) virtual, erect and diminished
2. In a compound microscope, the final image is
   (a) real, inverted and magnified
   (b) real, erect and magnified
   (c) virtual, erect and magnified
   (d) virtual, inverted and magnified
3. The magnifying power of a compound microscope is high if
   (a) both the objective and the eyepiece have short focal lengths.
   (b) both the objective and the eyepiece have long focal lengths.
   (c) the objective has a short focal length and the eyepiece has a long focal length
   (d) the objective has a long focal length and the eyepiece has a short focal length.
4. The resolving power of a compound microscope is increased if
   (a) light of a shorter wavelength is used to illuminate the object
   (b) the objective of a bigger diameter is used
   (c) the objective of a higher focal length is used
   (d) the eyepiece of a shorter focal length is used.
5. If the aperture of the objective of a microscope is increased,
   (a) its resolving power will increase
   (b) its magnifying power will decrease
   (c) the intensity of the final image will increase
   (d) the intensity of the final image will decrease.

Solutions

1. The correct choice is (a).
2. The correct choice is (d).
3. The correct choice is (a). The magnifying power is given by
$$M = \frac{L}{f_o} \cdot \frac{D}{f_e} \qquad (\text{for } D >> f_o)$$
where L = distance between the objective and the eyepiece, D = least distance of distinct vision, f_o = focal length of the objective and f_e = focal length of the eyepiece.
4. The resolving power of a microscope is given by (see Fig. 16.96)
$$\text{R.P.} = \frac{2\mu \sin\theta}{1.22\lambda}$$
where 2θ = angle of the cone of light rays entering the objective, λ = wavelength of light used to illuminate the object and μ = refractive of the medium between the object and the objective. The value of θ increases if the objective of a bigger diameter AB is used. The resolving power does not depend on f_o or f_e. Hence the correct choices are (a) and (b).

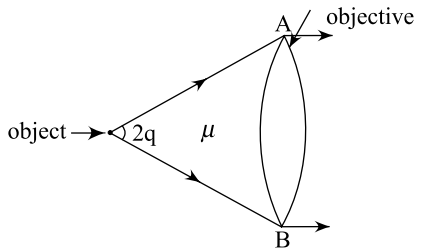


Fig. 16.96

5. If the aperture of the objective is increased, the value of θ increases. Therefore, the light gathering capacity of the objective increases. As a result, the intensity of the image increases. Hence the correct choices are (a) and (c).

Questions 6 to 10 are based on the following passage.

Passage II

A telescope is an optical instrument that is used to examine distant objects. Two types of telescopes are in use—refracting and reflecting telescopes. A refracting astronomical telescope consists of two converging lenses called the objective and the eyepiece. The objective faces the distant object. The image of the object is formed at the focal plane of the objective. The position of the eyepiece is adjusted till this image is within the first focus of the eyepiece. A highly magnified final image is formed which is seen by the eye held close to the eyepiece. If both the object and the final image are at infinity, the telescope is said to be in normal adjustment.

6. In a refracting astronomical telescope, the intermediate image is
   (a) real, inverted and magnified
   (b) real, inverted and diminished
   (c) virtual, erect and magnified
   (d) virtual, inverted and diminished
7. In a refracting astronomical telescope, the final image is
   (a) real, inverted and magnified
   (b) real, erect and magnified
   (c) virtual, erect and magnified
   (d) virtual, inverted and magnified
8. The magnifying power of a telescope is high if
   (a) both the objective and the eyepiece have short focal lengths
   (b) both the objective and the eyepiece have long focal lengths
   (c) the objective has a short focal length and the eyepiece has a long focal length
   (d) the objective has a long focal length and the eyepiece has a short focal length
9. The resolving power of a telescope is increased if
   (a) the objective of a bigger diameter is used
   (b) the objective of a smaller diameter is used
   (c) the objective of a higher focal length is used
   (d) the eyepiece of a shorter focal length is used
10. If the aperture of the objective of a telescope is increased,
   (a) its resolving power will increase
   (b) its magnifying power will decrease
   (c) the intensity of the final image will increase
   (d) the intensity of the final image will decrease.

Solutions

6. The correct choice is (b).
7. The correct choice is (d).
8. The magnifying power of a telescope (if the object is at infinity) is given by
$$M = \frac{f_o}{f_e} \cdot \frac{D + f_e}{D}$$
where D = least distance of distinct vision, where the final image is formed. Hence the correct choice is (d).
9. The resolving power of a telescope is given by
$$\text{R.P.} = \frac{d}{1.22\lambda}$$
where d = diameter of the objective and λ = wavelength of light. The resolving power is independent of f_o or f_e. Hence the correct choice is (a).
10. The correct choices are (a) and (c).

Questions 11 to 13 are based on the following passage.

Passage III

A ray of light travelling in air is incident at grazing angle (incident angle = 90°) on a long rectangular slab of a transparent medium of thickness $t = 1.0$ m (see Fig. 16.97). The point of incidence is the origin $A(0, 0)$. The medium has a variable index of refraction $n(y)$ given by
$$n(y) = (ky^{3/2} + 1)^{1/2}$$
where $k = 1.0$ (metre)$^{-3/2}$. The refractive index of air is 1.0.

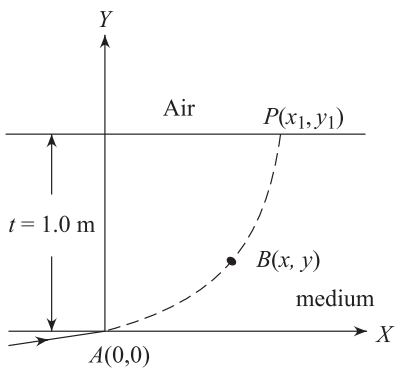


Fig. 16.97

11. The relation between the slope of the trajectory of the ray at the point $B(x, y)$ in the medium and the incident angle i at that point is
   (a) $\frac{dy}{dx} = \sin i$ (b) $\frac{dy}{dx} = \cos i$
   (c) $\frac{dy}{dx} = \tan i$ (d) $\frac{dy}{dx} = \cot i$

12. The equation of the trajectory of the ray in the medium is
(a) $y = (x)^4$ (b) $y = \left(\frac{x}{2}\right)^4$
(c) $y = \left(\frac{x}{3}\right)^4$ (d) $y = \left(\frac{x}{4}\right)^4$

13. The coordinate x_1 of point P where the ray intersects the upper surface of the air-slab boundary is
(a) $x_1 = 4.0$ m (b) $x_1 = 3.0$ m
(c) $x_1 = 2.0$ m (d) $x_1 = 1.0$ m

Solutions

11. Refer to Fig. 16.98. The variation of refractive index is given by

$$n(y) = (ky^{3/2} + 1)^{1/2} \quad (1)$$

It follows from relation (1) that the refractive index increases in the slab from $y = 0$ to $y = t$. Therefore, the ray SA entering the slab at A at grazing incidence ($i = 90°$) keeps bending towards the normal as it progresses in the slab, following a curved path ABP.
If the angle of incidence at point $B(x, y)$ is i, then the slope CD of the trajectory at B is

$$\frac{dy}{dx} = \tan\alpha = \tan(90° - i) = \cot i \quad (2)$$

The correct choice is (d).

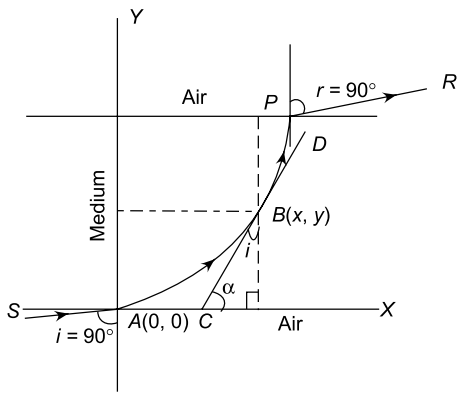


Fig. 16.98

12. If i is the angle of incidence at a point in the medium of refractive index n, then applying Snell's law, we have

$$n \sin i = n_a \sin i_a = 1 \times \sin 90° = 1$$

because n_a = refractive index of air = 1 and at A $i_a = 90°$.
or $\sin i = \frac{1}{n}$

$$\therefore \cot i = \frac{\left(1 - \frac{1}{n^2}\right)^{1/2}}{\left(\frac{1}{n}\right)} = (n^2 - 1)^{12} \quad (3)$$

Using Eq. (2) in Eq. (3) we get

$$\frac{dy}{dx} = (n^2 - 1)^{1/2} \text{ or } \left(\frac{dy}{dx}\right)^2 = n^2 - 1 \quad (4)$$

Using Eq. (1) in Eq. (4) we have

$$\left(\frac{dy}{dx}\right)^2 = ky^{3/2} + 1 - 1 = ky^{3/2}$$

or $\frac{dy}{dx} = k^{1/2} y^{3/4}$ or $\frac{dy}{y^{3/4}} = k^{1/2}\, dx$

Integrating, we get

$$\int \frac{dy}{y^{3/4}} = k^{1/2} \int dx \text{ or } 4y^{1/4} = k^{1/4} x$$

Given $k = 1.0$. Therefore, we have

$$y^{1/4} = \left(\frac{1}{4}\right)x \text{ or } y = \left(\frac{x}{4}\right)^4 \quad (5)$$

So the correct choice is (d).

13. For point P, $y = 1.0$ m. Using $y = 1.0$ m in Eq. (5) we get $x = 4.0$ m. Thus the coordinates of point P are $x_1 = 4.0$ m and $y_1 = 1.0$ m. Thus the correct choice is (a).

Questions 14 to 16 are based on the following passage.

Passage IV

The x-z plane is the boundary between two transparent media. medium 1 with $z \geq 0$ has a refractive index $\sqrt{2}$ and medium 2 with $z \leq 0$ has a refractive index $\sqrt{3}$. A ray of light in medium 1 given by the vector $\mathbf{A} = 6\sqrt{3}\,\hat{\mathbf{i}} + 8\sqrt{3}\,\hat{\mathbf{j}} - 10\,\hat{\mathbf{k}}$ is incident on the plane of separation.

14. The angle between vector $\boldsymbol{A}$ and the positive z-direction is
(a) 90° (b) 120°
(c) 135° (d) 150°

15. The angle of refraction in medium 2 is
(a) 30° (b) 45°
(c) 60° (d) 75°

16. The refracted ray is given by $\boldsymbol{A}_r = a\,\hat{\mathbf{i}} + b\,\hat{\mathbf{j}} + c\,\hat{\mathbf{k}}$ where a and b are
(a) $a = 6\sqrt{3}$, $b = 8\sqrt{3}$ (b) $a = 3\sqrt{3}$, $b = 4\sqrt{3}$
(c) $a = 4\sqrt{3}$, $b = 6\sqrt{3}$ (d) $a = 4\sqrt{3}$, $b = 3\sqrt{3}$

Solutions

14. Refer to Fig. 16.99. Ray PQ in x-y plane travelling in medium 1 ($z \geq 0$) is incident at angle i on the boundary ($z = 0$ plane) and is refracted along QR in medium 2 ($z \leq 0$) at an angle of reflection r.

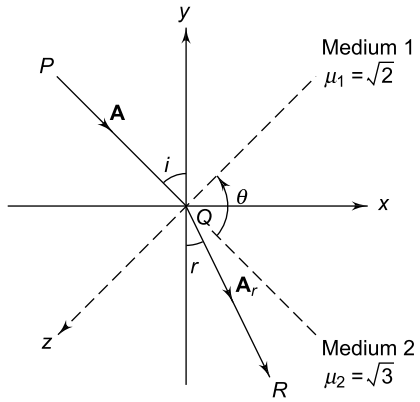


Fig. 16.99

Let θ be the angle between vector **A** and the positive z-direction. Since $\hat{\mathbf{k}}$ is the unit vector along the positive z-axis, we have

$$\cos\theta = \frac{\mathbf{A}.\hat{\mathbf{k}}}{|\mathbf{A}|}$$

$$= \frac{(6\sqrt{3}\hat{\mathbf{i}} + 8\sqrt{3}\hat{\mathbf{j}} - 10\hat{\mathbf{k}})\cdot(\hat{\mathbf{k}})}{\left[(6\sqrt{3})^2 + (8\sqrt{3})^2 + (-10)^2\right]^{1/2}}$$

$$= \frac{-10}{20} = -\frac{1}{2} \qquad (\because \hat{\mathbf{i}}.\hat{\mathbf{k}} = \hat{\mathbf{j}}.\hat{\mathbf{k}} = 0)$$

which gives $\theta = 120°$. which is choice (b).

15. Therefore, angle of incidence $i = 180° - \theta = 180° - 120° = 60°$. From Snell's law,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

or $\sin r = \frac{\mu_1}{\mu_2}\sin i = \frac{\sqrt{2}}{\sqrt{3}}\sin 60° = \frac{1}{\sqrt{2}}$

which gives $r = 45°$. so the correct choice is (b).

16. Since the refracted ray lies in the same plane as the incident wave (which is the x-y plane), the refracted ray will be given by the vector

$$\mathbf{A}_r = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$$

with $a = 6\sqrt{3}$ and $b = 8\sqrt{3}$, the same as in the incident vector **A**. Thus

$$\mathbf{A}_r = 6\sqrt{3}\hat{\mathbf{i}} + 8\sqrt{3}\hat{\mathbf{j}} + c\hat{\mathbf{k}}$$

This the correct choice is (a).

Questions 17 to 19 are based on the following passage.

Passage V

The radius of curvature of the curved face of a thin plano-convex lens is 10 cm and it is made of glass of refractive index 1.5. A small object is approaching the lens with a speed of 1 cm s^{-1} moving along the principal axis.

17. The focal length of the lens is
    (a) 5 cm (b) 10 cm
    (c) 15 cm (d) 20 cm
18. When the object is at a distance of 30 cm from the lens, the magnitude of the speed of its image is
    (a) 1 cm s^{-1} (b) 2 cm s^{-1}
    (c) 3 cm s^{-1} (d) 4 cm s^{-1}
19. When the object is at a distance of 30 cm from the lens, the magnitude of the rate of change of the lateral magnification is
    (a) 0.1 per second (b) 0.2 per second
    (c) 0.3 per second (d) 0.4 per second

Solutions

17. $\frac{1}{f} = (\mu - 1)\times\frac{1}{R} = (1.5 - 1)\times\frac{1}{10}$ gives $f = 20$ cm.
The correct choice is (d).

18. Lens formula is (here $u = -30$ cm and $f = +20$ cm)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad (1)$$

or $\frac{1}{v} - \frac{1}{-30} = \frac{1}{20}$ which gives $v = 60$ cm

Differentiating (1) with respect to time t, we get

$$-\frac{1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt} = 0$$

or $\frac{dv}{dt} = \left(\frac{v^2}{u^2}\right)\frac{du}{dt}$

or Speed of image $= \left(\frac{v^2}{u^2}\right)\times$ speed of object

$$= \left(\frac{60}{30}\right)^2 \times 1 = 4 \text{ cm s}^{-1},$$

which is choice (d).

19. Linear magnification is given by

$$m = \frac{v}{u} \qquad (2)$$

Differentiating (2) with respect to time t, we have

$$\frac{dm}{dt} = -\frac{v}{u^2}\frac{du}{dt} + \frac{1}{u}\frac{dv}{dt} = \frac{1}{u^2}\left(-v\frac{du}{dt} + u\frac{dv}{dt}\right)$$

$$= \frac{1}{(30)^2}(-60\times 1 - 30\times 4)$$

$= -0.2$ per second

$\therefore$ Magnitude of $\frac{dm}{dt} = 0.2$ per second.

So the correct choice is (b).

SECTION 3

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

1. **Statement-1**

   When a concave mirror is held under water, its focal length will increase.

   Statement-2

   The focal length of a concave mirror is independent of the medium in which it is placed.

2. **Statement-1**

   When a convex lens is held under water, its focal length increases.

   Statement-2

   The focal length of a lens depends on the medium in which it is placed.

3. **Statement-1**

   A convex lens is made of glass of refractive index 1.45 and its two faces have the same radius of curvature equal to R. The focal length of the lens is more than R.

   Statement-2

   The above statement is false because the focal length of a lens is half the radius of curvature.

4. **Statement-1**

   A parallel beam of light falls on a convex lens. The path of the rays is shown in Fig. 16.100. It follows that

   $$\mu_1 = \mu > \mu_2$$

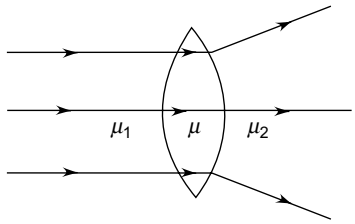


Fig. 16.100

   Statement-2

   When a ray of light travels from a rarer into a denser medium, it bends towards the normal and vice versa.

5. **Statement-1**

   A ray of light is incident at the glass-water interface at an angle i. It emerges finally parallel to the surface of water as shown in Fig. 16.101. Then

   $$\mu_g = \left(\frac{4}{3}\right) \sin i$$

   Statement-2

   The refracted ray follows a path in accordance with Snell's law of refraction.

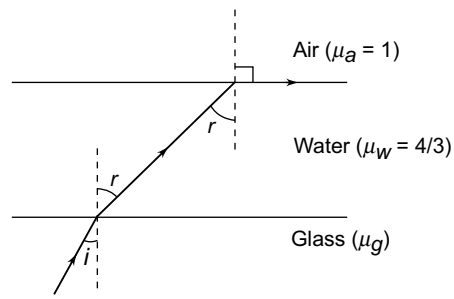


Fig. 16.101

Solutions

1. The correct choice is (d). The focal length of a concave mirror depends only on its radius of curvature.

2. The correct choice is (a). The focal length of a lens depends not only on the radii of curvature of its faces but also on the refractive index of its glass and the refractive index of the medium in which it is placed. The focal length f of a convex lens of refractive index μ_2 and radii of curvature R_1 and R_2 when placed in a medium of refractive index μ_1 is given by

   $$\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

   It is easy to see that f will be more if μ_1 is different from unity ($\mu_1 = 1$ for air).

3. The correct choice is (c). The focal length f is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (1.45 - 1) \times \frac{2}{R}$$

which gives $f = \dfrac{R}{0.9}$, i.e. f is greater than R.

4. The correct choice is (a). Since the rays do not bend at the first surface, $\mu_1 = \mu$. When this ray meets the second face of the lens, it bends towards the normal to this face. Hence $\mu_2 > \mu$.

5. The correct choice is (d). For refraction at glass-water interface, we have from Snell's law

$$\mu_g \sin i = \mu_\omega \sin r \qquad \text{(i)}$$

For refraction at water-air interface, we have

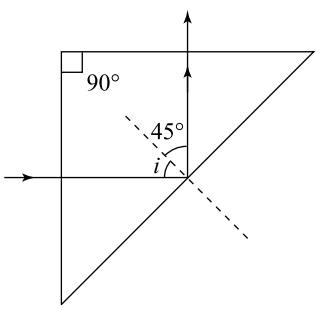
$$\mu_\omega \sin r = \mu_a \sin 90° = \mu_a = 1 \qquad \text{(ii)}$$

Using (ii) in (i), we get

$$\mu_g \sin i = 1 \text{ or } \mu_g = \frac{1}{\sin i}$$

4 SECTION — Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. If two mirrors are inclined at an angle of 60°, the number of images formed by them is
   (a) 5 (b) 6
   (c) 7 (d) 8 **[2002]**
2. Which of the following is used in optical fibres?
   (a) Total internal reflection (b) Scattering
   (c) Diffraction (d) Refraction **[2002]**
3. The image formed by the objective of a compound microscope is
   (a) virtual and diminished (b) real and diminished
   (c) real and enlarged (d) virtual and enlarged. **[2003]**
4. To get three images of a single object, the angle between the two plane mirrors should be
   (a) 60° (b) 90°
   (c) 120° (d) 30° **[2003]**
5. A ray of light is incident normal to one face of a 90° prism and is totally reflected at the air-glass interface as shown in the figure. If the angle of reflection is 45°, we conclude that the refractive index n of the material of the prism is



   (a) less than $\dfrac{1}{\sqrt{2}}$ (b) greater than $\sqrt{2}$
   (c) greater than $\dfrac{1}{\sqrt{2}}$ (d) less than $\sqrt{2}$ **[2004]**
6. A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved face. This lens is used to form the image of an object. At what distance from the lens, should an object be placed in order to obtain a real image equal to the size of the object?
   (a) 20 cm (b) 30 cm
   (c) 60 cm (d) 80 cm **[2004]**
7. A fish looking up through the water sees the outside world, contained in a circular horizon. If the refractive index of water is 4/3 and the fish is 12 cm below the water surface, the radius of this circle in cm is
   (a) $36\sqrt{7}$ (b) $\dfrac{36}{\sqrt{7}}$
   (c) $36\sqrt{5}$ (d) $4\sqrt{5}$ **[2005]**
8. A thin glass lens ($\mu = 1.5$) has optical power of – 5D in air. Its optical power in a liquid of refractive index 1.6 will be
   (a) 1 D (b) – 1 D
   (c) 25 D (d) – 25D **[2005]**
9. The refractive index of glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be angles of minimum deviation for red and blue light respectively in a prism of this glass. Then,
   (a) D_1 can be less than or greater than D_2 depending upon the angle of prism

(b) $D_1 > D_2$

(c) $D_1 < D_2$

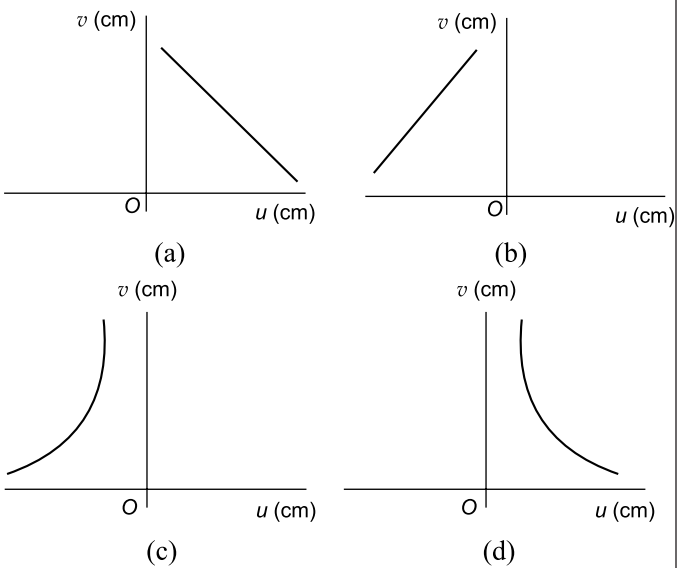
(d) $D_1 = D_2$ **[2006]**

10. Two lenses of power – 15D and +5D are in contact with each other. The focal length of the combination is

(a) – 20 cm (b) – 10 cm

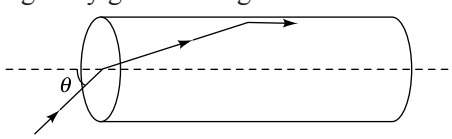
(c) + 20 cm (d) + 10 cm **[2007]**

11. A student measures the focal length of a convex lens by putting an object pin at a distance '*u*' from the lens and measuring the distance '*v*' of the image pin. The graph between '*u*' and '*v*' plotted by the student should look like



[2008]

12. A transparent solid cylindrical rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the Figure. The incident angle θ for which the light ray grazes along the wall of the rod is:



(a) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (b) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(c) $\sin^{-1}\left(\frac{1}{2}\right)$ (d) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

13. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance *u* and the image distance *v* from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the *x*-axis meets the experimental curve at *P*. The coordinates of *P* will be:

(a) (*f*, *f*) (b) (4*f*, 4*f*)

(c) (2*f*, 2*f*) (d) $\left(\frac{f}{2}, \frac{f}{2}\right)$ **[2009]**

14. Let the $x - z$ plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\mathbf{A} = 6\sqrt{3}\hat{\mathbf{i}} + 8\sqrt{3}\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$ is incident on the plane of separation. The angle of refraction in medium 2 is

(a) 45º (b) 60º

(c) 75º (d) 30º **[2011]**

15. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is:

(a) $\frac{1}{15}$ m/s (b) 10 m/s

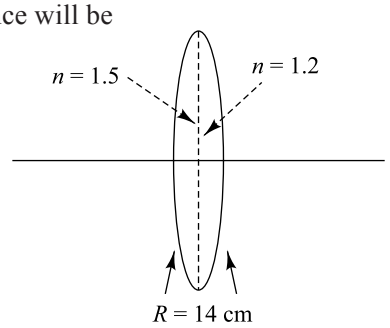
(c) 15 m/s (d) $\frac{1}{10}$ m/s **[2011]**

16. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object be placed to get a sharp image on film?

(a) 7.2m (b) 2.4m

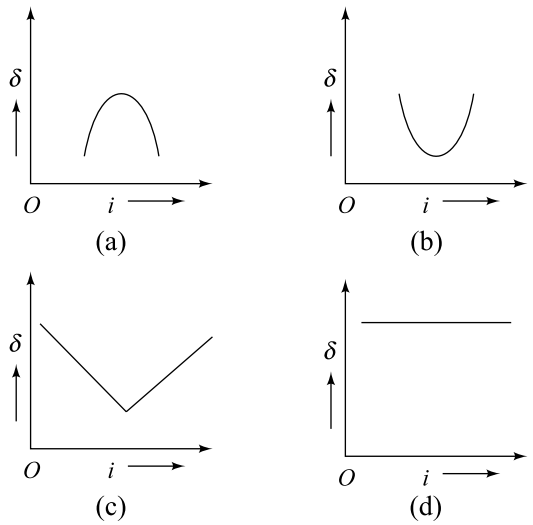
(c) 3.2 m (d) 5.6m **[2012]**

17. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index *n* of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surface are of the same radius of curvature *R* = 14 cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be



(a) –280.0 cm (b) 40.0 cm
(c) 21.5 cm (d) 13.3 cm **[2012]**

18. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by:

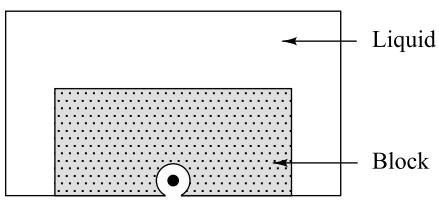


[2013]

19. Diameter of plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is:
(a) 20 cm (b) 30 cm
(c) 10 cm (d) 14 cm **[2013]**

20. A ray of light travelling in the direction $\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$. The angle of incidence is:
(a) 30º (b) 45º
(c) 60º (d) 75º **[2013]**

21. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is



(a) 1.21 (b) 1.30
(c) 1.36 (d) 1.42 **[2014]**

Answers

1. (a)	2. (a)	3. (c)	4. (b)
5. (b)	6. (a)	7. (b)	8. (a)
9. (c)	10. (b)	11. (d)	12. (b)
13. (c)	14. (a)	15. (a)	16. (d)
17. (b)	18. (b)	19. (b)	20. (a)
21. (c)			

Solutions

1. Number of images is
$$n = \frac{360°}{\theta} - 1 = \frac{360°}{60°} - 1 = 6 - 1 = 5$$

2. The correct choice is (1).

3. The correct choice is (3).

4. $$n = \frac{360°}{\theta} - 1$$
$$\Rightarrow \quad 3 = \frac{360°}{\theta} - 1$$
$$\Rightarrow \quad \theta = \frac{360°}{4} = 90°$$

5. Since the angle of reflection is 45°, the angle of incidence $i = 45°$. The critical angle must be greater than 45°, i.e.
$$i_c > 45°$$
But $n = \frac{1}{\sin i_c} \Rightarrow i_c = \sin^{-1}\left(\frac{1}{n}\right)$
$$\therefore \ \sin^{-1}\left(\frac{1}{n}\right) > 45°$$
$$\Rightarrow \ \frac{1}{n} > \sin 45°$$
$$\Rightarrow \ \frac{1}{n} > \frac{1}{\sqrt{2}} \text{ or } n > \sqrt{2}$$

6. The focal length F of the silvered lens is given by
$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} + \frac{1}{f_m} = \frac{2}{f} + \frac{1}{f_m}$$
where f = focal length of the lens before silvering and f_m = focal length of the concave mirror.
Now $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$
(without sign conventions)
Here $R_1 = \infty$ and $R_2 = 30$ cm. Therefore

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{\infty}+\frac{1}{30}\right) = \frac{1}{60} \Rightarrow f = 60 \text{ cm}$$

Also $f_m = \dfrac{R_2}{2} = 15$ cm

$$\therefore \quad \frac{1}{F} = \frac{2}{60}+\frac{1}{15} = \frac{6}{60} = \frac{1}{10} \Rightarrow F = 10 \text{ cm}$$

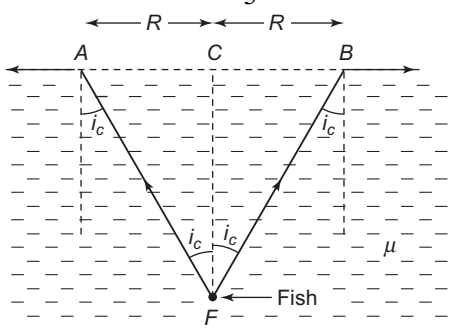
Magnification $m = -\dfrac{v}{u} = 1$ (given). Therefore $v = -u$. From the lens formula

$$\frac{1}{F} = \frac{1}{v}-\frac{1}{u} = -\frac{1}{u}-\frac{1}{u} = -\frac{2}{u}$$

$$\Rightarrow \frac{1}{10} = -\frac{2}{u} \qquad \Rightarrow \quad u = -20 \text{ cm}$$

Hence the object must be placed at a distance 20 cm to the left of the lens.

7. Given $FC = 12$ cm, $\mu = \dfrac{4}{3}$



$$\sin i_c = \frac{1}{\mu}$$

$$\therefore \quad \cos i_c = \sqrt{1-\frac{1}{\mu^2}} = \frac{\sqrt{\mu^2-1}}{\mu}$$

$$\therefore \quad \tan i_c = \frac{1}{\sqrt{\mu^2-1}}$$

In ΔFAC

$$\tan i_c = \frac{AC}{FC} = \frac{R}{FC}$$

$$\Rightarrow \quad R = FC \tan i_c = 12 \text{ cm} \times \frac{1}{\sqrt{\left(\frac{4}{3}\right)^2-1}} = \frac{36}{\sqrt{7}} \text{ cm}$$

8. Focal length of lens in air is $f = \dfrac{1}{P} = \dfrac{1}{-5}$ m $= -20$ cm

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = (1.5-1)\,k,$$

where $k = \dfrac{1}{R_1}-\dfrac{1}{R_2}$

$$\Rightarrow \qquad f = \frac{2}{k} \tag{1}$$

Focal length the lens in the liquid is ($\mu = 1.5$, $\mu' = 1.6$)

$$\frac{1}{f'} = \left(\frac{1.5-1.6}{1.6}\right)k = -\frac{1}{16}k$$

$$\Rightarrow \qquad f' = -\frac{16}{k} \tag{2}$$

From (1) and (2) we get $f' = -8f = -8 \times (-20 \text{ cm})$
$= +160 \text{ cm} = +1.6 \text{ m}$

Power of lens in liquid is

$$P' = \frac{\mu'}{f'} = \frac{1.6}{+1.6} = +1\text{D}$$

9. The refractive index of a prism is given by

$$\mu = \frac{\sin\left(\dfrac{A+D}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} \tag{1}$$

where A = angle of the prism and D = angle of minimum deviation. It follows from Eq. (1) that the greater the refractive index, the greater is the angle of minimum deviation. Since the refractive index for blue light is greater than that for red light, the angle of minimum deviation (D_2) for blue light will be greater than that (i.e. angle D_1) for red light. Hence the correct choice is (3).

10. $$\frac{1}{F} = \frac{1}{f_1}+\frac{1}{f_2}$$

$$\therefore \qquad P = P_1 + P_2 = -15+5 = -10 \text{ D}$$

$$F \text{ (in metre)} = \frac{1}{P} = \frac{1}{-10} = -0.1 \text{ m} = -10 \text{ cm}$$

11. For a convex lens forming a real image,

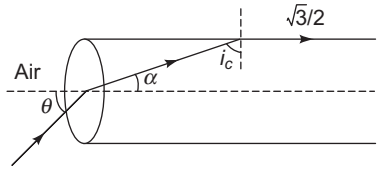
$$\frac{1}{v}+\frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad v = \frac{fu}{u-f}$$

Also, for a real image $u > f$. From $\dfrac{1}{v}+\dfrac{1}{u} = \dfrac{1}{f}$ it follows that as u increases, v decreases as f remains constant. Hence the correct graph is (4).

12. Using Snell's law $1 \times \sin\theta = \dfrac{2}{\sqrt{3}} \sin\alpha$

$$\Rightarrow \sin\theta = \frac{2}{\sqrt{3}} \sin\alpha$$

Critical angle i_c is given by



$$\sin i_c = \frac{1}{\mu} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin(90° - \alpha) = \frac{\sqrt{3}}{2} \qquad \Rightarrow \qquad \cos\alpha = \frac{\sqrt{3}}{2}$$

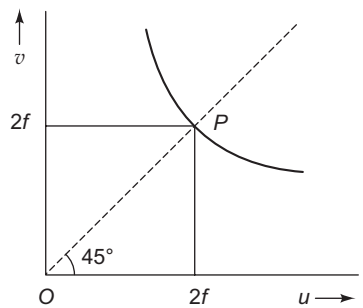
$$\text{Therefore } \sin\alpha = \sqrt{(1-\cos^2\alpha)} = \left(1-\frac{3}{4}\right)^{1/2} = \frac{1}{2}$$

$$\therefore \sin\theta = \frac{2}{\sqrt{3}} \times \frac{1}{2} = \frac{1}{\sqrt{3}} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

13. For a convex lens forming a real image

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

The graph of v against u is as follows.



Since the u and v scales are the same, $u = v$ at point P. Putting $u = v$ in Eq. (1) gives $u = 2f$. Also $v = 2f$. Hence the correct choice is (3).

14. Magnitude of **A** is

$$A = \sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + (10)^2} = 20$$

The angle of incidence in medium 1 is given by

$$\cos i = \frac{A_z}{A} = \frac{10}{20} = \frac{1}{2} \quad \Rightarrow \quad i = 60°$$

From Snell's law, the angle of refraction in medium 2 is given by

$$\mu_1 \sin i = \mu_2 \sin r$$

$$\Rightarrow \qquad \sin r = \frac{\mu_1 \sin i}{\mu_2} = \frac{\sqrt{2} \times \sin 60°}{\mu_2}$$

$$= \frac{\sqrt{2} \times \sqrt{3}/2}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{2}}$$

$$\Rightarrow \qquad r = 45°$$

15. $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

Given $f = +20$ cm and $u = -280$ cm

$$\frac{1}{v} + \frac{1}{-280} = \frac{1}{20} \quad \Rightarrow \qquad v = \frac{280}{15}\text{ cm}$$

for a convex mirror the formula is

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Differentiating we get

$$-\frac{dv}{dt}.\frac{1}{v^2} + \frac{du}{dt}.\frac{1}{u^2} = 0$$

$$\Rightarrow \qquad \frac{dv}{dt} = \left(\frac{v}{u}\right)^2 \frac{du}{dt}$$

Given $\dfrac{du}{dt} = 15\ \text{ms}^{-1}$. Therefore

$$\frac{dv}{dt} = \left(\frac{280/15}{280}\right)^2 \times 15 = \frac{1}{15}\text{ms}^{-1}$$

16. $$\frac{1}{f} = \frac{1}{12} + \frac{1}{240} = \frac{20+1}{240}$$

$$\Rightarrow \qquad f = \frac{240}{21}\text{m}$$

Shift $$= 1\left(1 - \frac{2}{3}\right) = \frac{1}{3}\text{ cm}$$

Now $$v' = 12 - \frac{1}{3} = \frac{35}{3}\text{cm}$$

$$\therefore \qquad \frac{21}{240} = \frac{3}{35} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{3}{35} - \frac{21}{240} = \frac{1}{5}\left(\frac{3}{7} - \frac{21}{48}\right)$$

$$\frac{5}{u} = \left[\frac{144-147}{48\times 7}\right]$$

$$\Rightarrow \qquad |u| = 560\text{ cm} = 5.6\text{ m}$$

17. $$\frac{1}{f_1} = \left(\frac{1.5-1}{1}\right)\left(\frac{1}{14} - \frac{1}{\infty}\right) = \frac{1}{28}\text{cm}^{-1}$$

$$\frac{1}{f_2} = \left(\frac{1.2-1}{1}\right)\left(\frac{1}{\infty} - \frac{1}{-14}\right) = \frac{1}{70}\text{cm}^{-1}$$

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{28} + \frac{1}{70} \Rightarrow F = 20\text{ cm}$$

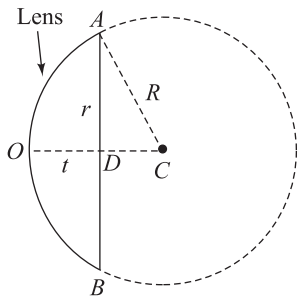
Now $\frac{1}{v}-\frac{1}{u}=\frac{1}{F}$

$\Rightarrow \quad \frac{1}{v}-\frac{1}{-40}=\frac{1}{20} \Rightarrow v=40$ cm

18. The correct graph is (b).

19. Refractive index of the material of the lens is

$$\mu=\frac{c}{v}=\frac{3\times10^8}{2\times10^8}=1.5$$



Given $AD=\frac{d}{2}=r=3$ cm

$OD=3$ mm $=0.3$ cm

$OC=AC=R$

In ΔACD

$$R^2=r^2+(R-t)^2$$

$\Rightarrow \quad R^2=r^2+R^2-2Rt+t^2$

$\Rightarrow \quad r^2-2Rt\left(1-\frac{t}{2R}\right)=0$

$\Rightarrow \quad r^2-2Rt=0 \qquad (\because t<<R)$

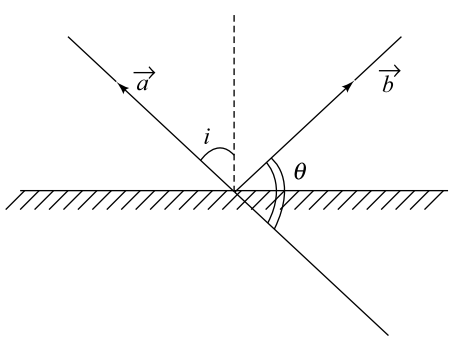
$\Rightarrow \quad R=\frac{r^2}{2t}=\frac{(3)^2}{2\times0.3}=15$ cm

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=(1.5-1)\left(\frac{1}{15}-\frac{1}{\infty}\right)$$

$\Rightarrow \quad f=30$ cm

20. Given $\vec{a}=\frac{1}{2}(\hat{i}+\sqrt{3}\hat{j})$



$$\vec{b}=\frac{1}{2}(\hat{i}-\sqrt{3}\hat{j})$$

Magnitude of $\vec{a}$ is $a=\frac{1}{2}(1+3)^{1/2}=1$

Magnitude of $\vec{b}$ is $b=\frac{1}{2}(1-3)^{1/2}=1$

If θ is the angle between $\vec{a}$ and $\vec{b}$ then

$$\cos\theta=\frac{\vec{a}.\vec{b}}{ab}$$

$$=\frac{\frac{1}{2}(\hat{i}+\sqrt{3}\hat{j}).\frac{1}{2}(\hat{i}-\sqrt{3}\hat{j})}{1\times1}$$

$$=-\frac{1}{2}$$

$\Rightarrow \quad \theta=120^\circ$

$$i=\frac{180^\circ-\theta}{2}=\frac{180^\circ-120^\circ}{2}=30^\circ$$

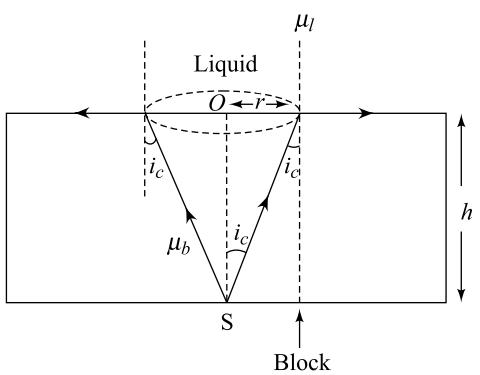
21. $\tan i_c=\frac{r}{h}=\frac{5.77}{10}=\sqrt{3}$

$\Rightarrow \quad i_c=30^\circ$

Also $\sin i_c=\frac{\mu_l}{\mu_b}$

$\mu_l=\mu_b \sin i_c$

$=2.72\times\sin 30^\circ=1.36$



WAVE OPTICS

Chapter 17

REVIEW OF BASIC CONCEPTS

1. Wave Nature of Light

Light is an electromagnetic wave which does not require a material medium for propagation. The electric and magnetic fields vary in space and time resulting in the propagation of an electromagnetic wave even in free space.

The electric field varies in space and time as

$$E = A \sin(\omega t - kx)$$

which represents a wave travelling along the + x direction. A = amplitude, $\omega = 2\pi\nu$ (ω is angular frequency in rad s^{-1} and ν is frequency in Hz) and $k = \frac{2\pi}{\lambda}$; λ = wavelength. Also

$$v = \nu\lambda = \frac{\omega}{k}$$

where v is the wave velocity.

Phase

The phase ϕ of a wave at a point x and at time t is given by the argument of the harmonic function (sine or cosine) representing the wave, i.e.

$$\phi = \omega t - kx$$

Phase Difference

Suppose two waves meeting at a point P are represented by

$$E_1 = A_1 \sin(\omega_1 t - k_1 x_1)$$

and $$E_2 = A_2 \sin(\omega_2 t - k_2 x_2)$$

where x_1 and x_2 are paths of the waves up to point P where they meet. The phase difference between them is

$$\Delta\phi = \phi_1 - \phi_2$$
$$= (\omega_1 - \omega_2)t - (k_1 x_1 - k_2 x_2)$$
$$\Delta\phi = (\phi_1 - \phi_2)$$
$$= 2\pi(\nu_1 - \nu_2)t - 2\pi\left(\frac{x_1}{\lambda_1} - \frac{x_2}{\lambda_2}\right)$$

1. If the two waves have different frequencies, i.e., $\nu_1 \neq \nu_2$ then $\lambda_1 \neq \lambda_2$ and $\Delta\phi$ depends on time t.
2. If $\nu_1 = \nu_2$, then $\lambda_1 = \lambda_2$. In this case

$$\Delta\phi = \frac{2\pi}{\lambda}(x_2 - x_1)$$

or Phase difference $= \frac{2\pi}{\lambda} \times$ (path difference)

i.e., the phase difference is independent of time and depends only on the path difference $(x_2 - x_1)$. This holds only if the two sources of wave are 'coherent', i.e., they have a constant fixed phase relationship.

Intensity The intensity of a wave at any point in its path is proportional to the square of its amplitude at that point.

2. Reflection and Refraction of Light

When a light wave falls on a reflecting surface, it is reflected obeying the usual laws of reflection. When a wave travels from one medium into another, its velocity and wavelength undergo a change and the wave is said to suffer refraction. The frequency of the wave does not undergo any change in refraction (and reflection). If v_1 is the velocity of the wave in the medium in which the incident wave propagates and v_2 is the velocity of the wave in the medium in which the refracted wave propagates, then ${}^1\mu_2$, the reflective index of the second medium with respect to the first, is defined as

$${}^1\mu_2 = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

where λ_1 and λ_2 are the wavelengths of the same wave in the two media. The frequency of the refracted wave remains the same as that of the incident wave.

When a wave, travelling in a rarer medium, is reflected at the boundary of a denser medium, the reflected wave suffers a phase change of 180° (or π radians) in relation to that of the incident wave. No phase change occurs if a wave, travelling in a denser medium, is reflected at the boundary of a rarer medium. The refracted wave, in both cases, does not undergo any phase change.

3. Interference of Light

When two or more light waves travelling in the same direction meet (or superpose) at a point in a medium, the electric field of the resultant wave can be obtained by using the *principle of superposition* which states that *the resultant electric field is given by the algebraic sum of the individual electric fields, at that point, due to the individual waves, i.e.,*

$$E = E_1 + E_2 + \cdots$$

resulting in a change in amplitude (and hence in intensity) at that point. The phenomenon in which the intensity of light at a point is modified by the superposition of two or more waves is known as *interference.*

If two waves of intensities I_1 and I_2, differing in phase by ϕ, superpose, the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\phi$$

Constructive Interference The resultant intensity I is maximum if $\cos\phi = +1$, i.e.

$$\phi = 2n\pi;$$

where $n = 0, 1, 2, 3, \ldots$ etc. is an integer

or $$\frac{2\pi\Delta}{\lambda} = 2n\pi$$

or $$\Delta = n\lambda$$

where Δ is the path difference between the interfering waves. Then

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$
$$= \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

The interference is said to be constructive. If the two interfering waves have equal intensities $I_1 = I_2 = I_0$, then

$$I_{max} = 4I_0$$

Destructive Interference The resultant intensity I is minimum if $\cos\phi = -1$, i.e.

$$\phi = (2n-1)\,\pi$$

and $$\Delta = \left(n - \frac{1}{2}\right)\lambda\ ;\ n = 1, 2, 3, \ldots\text{etc.}$$

Then

$$I_{min} = I_1 + I_2 - 2I_1 I_2$$
$$= \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

At maxima, the waves reinforce each other and at minima they cancel out each other. These maxima and minima constitute the bright and dark fringes.

4. Coherent Light Sources

The resultant intensity of light at a point on the screen depends on the phase difference (ϕ) between the two interfering waves. This phase difference depends upon two factors—(1) the initial phase difference between the waves emitted by the two sources and (2) the phase difference resulting from the path difference for that point. The initial phase difference depends upon the time and remains constant only for about 10^{-8} to 10^{-10} second. Thus the resultant intensity changes so rapidly with time that, due to persistence of vision, we are unable to see the interference pattern. Thus, non-coherent sources cannot produce sustained interference effects. We conclude that, for a steady interference pattern, the following two conditions must be satisfied.

1. The sources must be coherent.
2. The wavelengths of the interfering waves must be the same. Thus, *only monochromatic coherent light sources produce observable interference pattern.*

5. Young's Double Slit Experiment

Monochromatic light from a source slit S illuminates two slits S_1 and S_2 which are very close together and equidistant from S (Fig. 17.1).

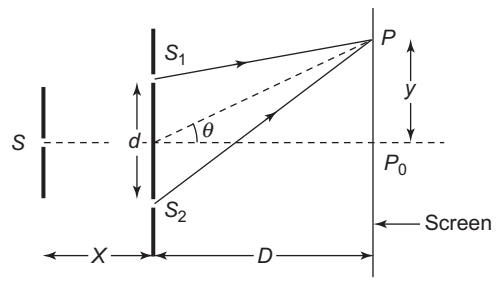


Fig. 17.1

Secondary waves from S_1 and S_2 interfere giving rise to bright and dark fringes on the screen. There is bright fringe at centre P_0 of the screen.

(i) The distance of the nth bright fringe from the centre of the fringe system is

$$y_n = \frac{n\lambda D}{d}\ ;\ n = 0, 1, 2, \ldots \text{etc.}$$

where λ = wavelength of light used, d = seperation between slits S_1 and S_2 and D = distance between the screen and the plane of the two slits.

(ii) The distance of the nth dark fringe from the centre of the fringe system is

$$y_n^* = \left(n - \frac{1}{2}\right)\frac{\lambda D}{d};\ n = 1, 2, 3, \text{... etc}$$

(iii) The separation between two consecutive bright or dark fringes is called fringe width (β) which is given by

$$\beta = \frac{\lambda D}{d}$$

(iv) Angular separation between nth bright fringe and the central fringe is

$$\theta_n = \frac{y_n}{D} = \frac{n\lambda}{d} \quad ; \quad \theta_n \text{ is in radian.}$$

(v) Angular separation between nth dark fringe and the central fringe is

$$\theta_n^* = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}$$

6. Displacement of Fringes

If a transparent plate of thickness t and refractive index μ is introduced in the path of one of the interfering waves, the entire fringe pattern is shifted by a distance

$$y_0 = (\mu - 1)\ \frac{tD}{d}$$

$$\text{Number of fringes shifted} = \frac{(\mu - 1)t}{\lambda}$$

7. Diffraction at a Slit

When a parallel beam of monochromatic light falls normally on a narrow slit, the diffraction pattern on a screen has a bright central maximum bordered on both sides by secondary maxima of rapidly decreasing intensity.

If λ is the wavelength of light and a is the width of the slit, then

(i) For bright fringes : $\sin\theta = 0,\ \frac{3\lambda}{2a}, \frac{5\lambda}{2a}$, ... etc

(ii) For dark fringes : $\sin\theta = \frac{\lambda}{a}, \frac{2\lambda}{a}, \frac{3\lambda}{a}$ ... etc

(iii) Angular width of central maximum = $\frac{2\lambda}{a}$

(iv) Linear width of central maximum = $\frac{2f\lambda}{a}$, where f = focal length of the convex lens placed close to the slit.

8. Some Important Points about Interference of Light

1. In Young's double slit experiment, if monochromatic light is replaced by white light then central fringe will be white; all other fringes will be coloured. White light consists of colours between violet and rad (VIBGYOR). Wavelength λ is the shortest for violet light and longest for red light. At the central fringe, the path difference for all colours is zero. Hence at the central fringe, all colours superpose to give a white fringe. The first bright fringe after the central fringe will be violet colour.
2. In Young's double slit experiment, if one of the slits is covered with a transparent film or sheet of thickness t and refractive index μ, then
   (a) the path difference at the centre of the screen will not be zero, it will be equal to $(\mu - 1)t$.
   (b) the entire fringe pattern will shift by an amount $y_0 = \frac{(\mu - 1)tD}{d}$.
   (c) at the centre of the screen there will be a bright fringe if $(\mu - 1)\ t = n\lambda$; $n = 1, 2, 3, \text{...}$ etc.
   (d) at the centre of the screen there will be a dark fringe if $(\mu - 1)t = \left(n - \frac{1}{2}\right)\ \lambda$; $n = 1, 2, 3, \text{...}$ etc.
   (e) the fringe width will remain the same.
   (f) the intensity of light from the covered slit will decrease due to absorption by the film or sheet. Hence intensity of bright fringes will decrease and dark fringes will have some finite intensity (because the two interfering beam do not now have equal intensity). Hence the fringe pattern will become less distinct.
3. If one of the slits in Young's double slit experiment is closed (or covered with black paper), the interference pattern is replaced by single slit diffraction pattern which has a bright central fringe bordered on both sides by fringes of decreasing intensity.
4. If Young's interference experiment is performed in still water rather than in air, the fringe width will decrease. Since the refractive index of water is greater than that of the air, the speed of light in water (v) will be less than that in air (c). Since the frequency of light is the same in all media, $\lambda_w = \frac{v}{\nu}$ and $\lambda_a = \frac{c}{\nu}$ which give $\frac{\lambda_w}{\lambda_a} = \frac{v}{c} = \frac{1}{\mu_w}$.

Now $\mu_w = 4/3$. Hence $\lambda_w < \lambda_a$. Fringe width $\beta \propto \lambda$. Hence β in water < β in air.

5. In Young's interference experiment, if the beam of light has two wavelengths λ_1 and λ_2, their maxima will coincide if $n_1\lambda_1 = n_2\lambda_2$, where n_1 and n_2 are integers.
6. In an interference experiment if the two coherent light sources have intensities in the ratio n : 1, i.e. $\frac{I_1}{I_2} = n$, then the ratio of the intensity of maxima and minima in the interference pattern is

$$\frac{I_{\max}}{I_{\max}} = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2}}{I_1 + I_2 - 2\sqrt{I_1 I_2}}$$

$$= \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

$$= \left[\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1}\right]^2 = \left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$$

7. In an interference experiment with two coherent light sources, if the ratio of the intensities of maxima and minima in the interference pattern is n : 1, i.e. $\frac{I_{\max}}{I_{\min}} = n$, then ratio of the intensities of the coherent sources is

$$\frac{I_1}{I_2} = \left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2 \text{ because}$$

$$\frac{I_{\max}}{L_{\min}} = \left[\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1}\right]^2$$

$$\Rightarrow \quad \sqrt{n} = \frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1} \Rightarrow \frac{I_1}{I_2} = \left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$$

8. In an interference experiment, if the two coherent sources have intensities in the ratio n : 1, then in the interference pattern

$$\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2\sqrt{n}}{(n+1)}$$

9. The intensity of light emerging from a slit is proportional to its width. If the two slits in Young's interference experiment have widths in the ratio n : 1, then $\frac{I_1}{I_2} = n$ and

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$$

10. In Young's double slit experiment (Fig. 17.1), if x is the width of the source slit S and X its distance from the plane of the slits, the interference fringes will not be seen (because the interference pattern becomes indistinct) if the condition

$$\frac{x}{X} < \frac{\lambda}{d}$$

is not satisfied.

9. Resolving Power

Resolving power of an optical instrument is its ability to produce distinctly separate images of two objects very close together.

(a) *Resolving power of a microscope*

$$R.P. = \frac{2\mu \sin\theta}{1.22\lambda}$$

where 2θ = angle of the cone of light rays entering the objective of the microscope (Fig. 17.2), μ = refractive index of the medium between the object and the objective and λ = wavelength of light used to illuminate the object.

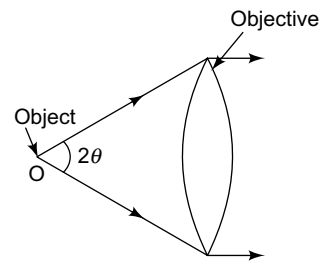


Fig. 17.2

(b) *Resolving power of a telescope*

$$R.P. = \frac{D}{1.22\lambda}$$

where D = diameter of objective and λ = wavelength of light

10. Polarization of Light

The phenomena of reflection, refraction, interference and diffraction are common on both transverse and longitudinal waves, mechanical as well as electromagnetic. The distinguishing feature is that only transverse waves can be *polarized*.

In an unpolarized light, the electric field vector has all the possible orientations in a plane perpendicular to the direction of propagation. When this light is passed through a specially cut crystal of calcite or quartz, called a polaroid, we obtain a plane polarized light.

Only transverse waves can be polarized. Longitudinal waves cannot be poarlized.

Polarization by Reflection: Brewster's Angle

In 1808, the French physicist Brewster discovered that when a beam of ordinary unpolarized light is incident at a particular angle i_p on the surface of a transparent medium, the reflected light is polarized.

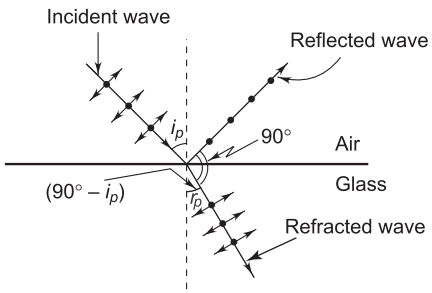


Fig. 17.3 *Polarization by reflection; i_p is Brewster's angle*

Figure 17.3 shows an unpolarized light incident at an angle i_p at the surface of glass. Brewster discovered that when $i = i_p$, the reflected and refracted rays are exactly 90° apart. The angle i_p when this happens is called the *polarizing* or the *Brewster angle*. If r_p is the corresponding angle of refraction, then from the geometry of Fig. 17.3,

$$i_p + r_p = 90° \text{ or } r_p = 90° - i_p$$

Now from Snell's law, the refractive index of glass is

$$n = \frac{\sin i_p}{\sin r_p} = \frac{\sin i_p}{\sin(90° - i_p)}$$

$$= \frac{\sin i_p}{\cos i_p} = \tan i_p$$

$$\therefore \quad n = \tan i_p$$

This equation is called the *Brewster law* and the angle i_p satisfying this equation is the *Brewster angle*.

EXAMPLE 1 In Young's double-slit experiment, the intesity of light at a point on the screen where the path difference is λ is K units. What is the intensity of light at a point where the path difference is $\lambda/3$; λ being the wavelength of light used?

SOLUTION Path difference $\Delta = \lambda$. Therefore, phase difference $\phi = \frac{2\pi}{\lambda}\Delta = 2\pi$. Hence intensity at a point where $\Delta = \lambda$ or $\phi = 2\pi$ is

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\phi$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos 2\pi$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$= I + I + 2I = 4I = K \text{ units}$$

$$(\because\ I_1 = I_2 = I)$$

i.e. $I = K/4$. The intensity at a point where the path difference is

$$\Delta' = \frac{\lambda}{3} \text{ or } \phi' = \frac{2\pi}{\lambda}\Delta'$$

$$= \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3} \text{ is}$$

$$I' = I + I + 2I\cos\frac{2\pi}{3}$$

$$= 2I - I = I = \frac{K}{4} \text{ units}$$

EXAMPLE 2 Two coherent sources of intensity ratio 100 : 1, interfere. What is the ratio of the intensity between the maxima and minima in the interference pattern?

SOLUTION Given $I_1/I_2 = 100$

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

$$\frac{I_{\max}}{I_{\min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

$$= \frac{\left(\sqrt{\frac{I_1}{I_2}} + 1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}} - 1\right)^2} = \frac{\left(\sqrt{100} + 1\right)^2}{\left(\sqrt{100} - 1\right)^2} = \frac{121}{81}$$

EXAMPLE 3 In Young's double slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the fourth bright fringe and the central bright fringe is measured to be 1.2 cm. What is the wavelength of light used in the experiment?

SOLUTION The position of the nth bright fringe with respect to the central fringe is given by

$$y_n = \frac{n\lambda D}{d}$$

For the central bright fringe ($n = 0$), $y_0 = 0$. For the fourth bright fringe ($n = 4$), $y_4 = 4\ \lambda D/d$. Therefore

$$y_4 - y_0 = \frac{4\lambda D}{d} \text{ or } \lambda = (y_4 - y_0)\frac{d}{4D} \qquad \text{(i)}$$

It is given that $(y_4 - y_0) = 1.2$ cm $= 1.2 \times 10^{-2}$ m, $D = 1.4$ m and $d = 0.28$ mm $= 0.28 \times 10^{-3}$ m. Substituting these values in (i) and solving, we get

$\lambda = 6 \times 10^{-7}$ m (or 600 nm or 6000 Å).

EXAMPLE 4 The ratio of the intensities of the maxima and minima in an interference pattern is 49 : 9. What is the ratio of the intensities of the two coherent sources employed in the interference experiment?

SOLUTION Given

$$\frac{I_{max}}{I_{min}} = \frac{49}{9}$$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

and $$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2}}{I_1 + I_2 - 2\sqrt{I_1 I_2}}$$

$$= \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{x+1}{x-1}\right)^2$$

where $$x = \sqrt{\frac{I_1}{I_2}}.$$

$$\Rightarrow \quad \frac{49}{9} = \left(\frac{x+1}{x-1}\right)^2 \Rightarrow \frac{7}{3} = \frac{x+1}{x-1}$$

which gives $x = \frac{5}{2}$. Therefore

$$\frac{I_1}{I_2} = x^2 = \frac{25}{4}$$

EXAMPLE 5 In Young's double slit experiment, find the ratio of the intensities at points P and Q on the screen where the path difference between the interfering waves is (a) zero and (b) $\frac{\lambda}{4}$, where λ is the wavelength of light used.

SOLUTION

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

where $$\phi = \frac{2\pi\Delta}{\lambda}; \ \Delta = \text{path difference}$$

In Young's experiment $I_1 = I_2 = I_0$. Therefore,

$$I = I_0 + I_0 + 2I_0 \cos\phi$$

$$= 2I_0 (1 + \cos\phi)$$

(a) For $\Delta = 0$, $\phi = 0°$. Hence

$$I_1 = 2I_0 (1 + \cos 0°) = 4I_0$$

(b) For $\Delta = \frac{\lambda}{4}$, $\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$. Hence

$$I_2 = 2I_0 \left(1 + \cos\frac{\pi}{2}\right) = 2I_0$$

$$\therefore \quad \frac{I_1}{I_2} = 2$$

EXAMPLE 6 A beam of light consisting of two wavelengths 450 nm and 750 nm is used to obtain interference fringes in Young's double slit experiment. The separation between the slits is 1.0 mm and the distance between the plane of the slits and the screen is 100 cm. The least distance from the central maximum where the bright fringes due to both the wavelengths coincide is

(a) 2.00 mm (b) 2.25 mm

(c) 2.50 mm (d) 2.75 mm

SOLUTION Let nth bright fringe of wavelength $\lambda_1 = 450$ nm coincide with the mth bright of $\lambda_2 = 750$ nm. Then

$$y = \frac{n\lambda_1 D}{d} = \frac{m\lambda_2 D}{d} \qquad \text{(i)}$$

$$\Rightarrow \quad n\lambda_1 = m\lambda_2$$

$$\Rightarrow \quad \frac{n}{m} = \frac{\lambda_2}{\lambda_1} = \frac{750 \text{ nm}}{450 \text{ nm}} = \frac{5}{3} \qquad \text{(ii)}$$

The smallest integral values of n and m which satisfy (ii) are $n = 5$ and $m = 3$. From (i) we have

$$y_{min} = \frac{n\lambda_1 D}{d}$$

$$= \frac{5 \times (450 \times 10^{-9}) \times 1.0}{(1.0 \times 10^{-3})}$$

$$= 2.25 \times 10^{-3} \text{ m} = 2.25 \text{ mm}$$

EXAMPLE 7 Monochromatic light of wavelength 500 nm is used in Young's double slit experiment. One of the slits is covered by a glass sheet of thickness 2.0×10^{-2} mm and refractive index 1.5. The number of fringes shifted by the introduction of the sheet is

(a) 14 (b) 16

(c) 18 (d) 20

SOLUTION When light traverses a sheet of thickness t and refractive index μ, the optical path travelled $= \mu t$.

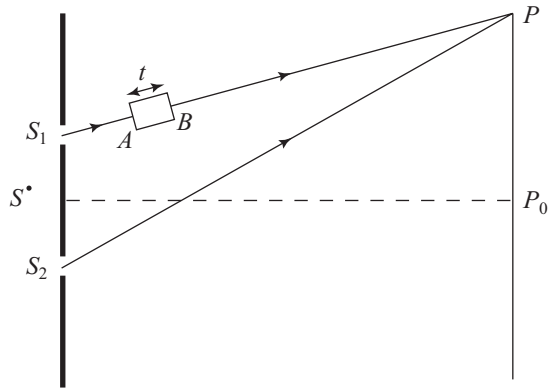


Fig. 17.4

Path difference in the absence of sheet is

$$\Delta_1 = S_2P - S_1P$$

Path difference when the sheet is introduced is

$$\Delta_2 = S_2P - [S_1A + \mu(AB) + BP]$$

$$= S_2P - [S_1A + BP + AB - AB + \mu(AB)]$$

$$= S_2P - [S_1P + (\mu - 1)AB]$$

$$= S_2P - S_1P - (\mu - 1)t$$

$\therefore$ Change in optical path is

$$\Delta = \Delta_1 - \Delta_2 = (\mu - 1)t$$

If the optical path changes by one wavelength, one fringe will shift. Therefore, the number of fringes shifted due to introduction of sheet is

$$\frac{(\mu - 1)t}{\lambda} = \frac{(1.5 - 1) \times (2.0 \times 10^{-5}\ \text{m})}{(500 \times 10^{-9}\ \text{m})} = 20$$

EXAMPLE 8 Monochromatic light of wavelength 640 nm falls normally on a glass plate of refractive index 1.6. The waves reflected from the upper and lower faces of the film will interfere constructively if the least thickness of the plate is

(a) 100 nm (b) 200 nm

(c) 300 nm (d) 400 nm

SOLUTION When light falls on a surface, it is partly reflected and partly refracted. Thus light from source S is reflected as wave 1 at A of the top surface of the plate, refracted into the plate and reflected as wave 2 at B of the bottom surface. On reaching A, the wave 2 has traversed an optical path length = $\mu t + \mu t = 2\,\mu t$ (Fig. 17.5).

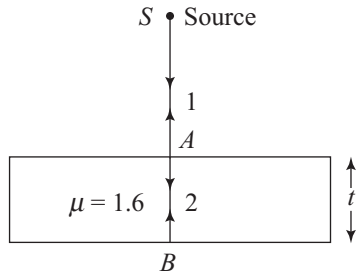


Fig. 17.5

Furthermore, on reflection from the denser medium, a wave suffers a phase change of π which implies a path change of $\lambda/2$. Hence on reaching A waves 1 and 2 will have a path change of

$$\Delta = 2\,\mu t - \frac{\lambda}{2}$$

For constructive interference, $\Delta = n\lambda$. Hence

$$2\,\mu t - \frac{\lambda}{2} = n\,\lambda$$

$$\Rightarrow \quad 2\,\mu t = \left(n + \frac{1}{2}\right)\lambda$$

or

$$t = \frac{\left(n + \frac{1}{2}\right)\lambda}{2\,\mu}$$

The minimum value of t corresponds to $n = 0$. Hence

$$t_{\min} = \frac{\lambda}{4\mu} = \frac{640\ \text{nm}}{4 \times 1.6} = 100\ \text{nm}$$

EXAMPLE 9 In Example 9 above the waves reflected from the upper and lower surfaces of the plate will interfere destructively if the least thickness of the plate is

(a) 100 nm (b) 200 nm

(c) 300 nm (d) 400 nm

SOLUTION For destructive interference,

$$\Delta = \left(n + \frac{1}{2}\right)\lambda$$

or

$$2\,\mu t + \frac{\lambda}{2} = \left(n + \frac{1}{2}\right)\lambda$$

$$\Rightarrow \quad 2\,\mu t = n\,\lambda$$

$$\Rightarrow \quad t = \frac{n\,\lambda}{2\,\mu}$$

The least integral value of n is 1. Hence

$$t_{\min} = \frac{\lambda}{2\mu} = \frac{640\,\text{nm}}{2 \times 1.6} = 200\ \text{nm}$$

SECTION 1

Multiple Choice Questions with One Correct Choice

Level A

1. How is the interference pattern in Young's double slit experiment affected if the sodium (yellow) light is replaced by red light of the same intensity?
   (a) The fringes will vanish
   (b) The fringes will become brighter
   (c) The fringe width will decrease
   (d) The fringe width will increase
2. In Young's double slit experiment, if the distance between the slits and the screen is doubled and the separation between the slits is reduced to half, the fringe width
   (a) is doubled (b) becomes four times
   (c) is halved (d) remains unchanged.
3. What happens if the monochromatic light used in Young's double slit experiment is replaced by white light?
   (a) All bright fringes become white
   (b) All bright fringes have colours between violet and red
   (c) Only the central fringe is white, all the other fringes are coloured
   (d) No fringes will be observed.
4. What will happen if one of the slits in Young's double slit experiment is covered with cellophane paper which absorbs a fraction of the intensity of light from the slit?
   (a) The fringe width will decrease
   (b) The fringes will become more distinct
   (c) The bright fringes will become less bright and the dark fringes will not be completely dark
   (d) No fringes will be observed
5. In Young's double slit experiment, the two slits act as coherent sources of equal amplitude A and of wavelength λ. In another experiment with the same set up, the two slits are sources of equal amplitude A and wavelength λ but are incoherent. The ratio of the intensity of light at the midpoint of the screen in the first case to that in the second case is
   (a) 1 : 1 (b) 1 : 2
   (c) 2 : 1 (d) $\sqrt{2}$: 1
6. How is the interference pattern affected if the Young's experiment was performed in still water than in air?
   (a) Fewer fringes will be visible
   (b) Fringes will be broader
   (c) Fringes will be narrower
   (d) No fringes will be observed.
7. How is the interference pattern in Young's experiment affected if one of the slits is covered with black opaque paper?
   (a) The bright fringes become fainter
   (b) The fringe width decreases
   (c) There will be uniform illumination all over the screen
   (d) There will be a bright central fringe bordered on both sides by fringes of decreasing intensity.
8. What happens to the interference pattern if the two slits S_1 and S_2 in Young's double-slit experiment are illuminated by two independent identical sources?
   (a) The intensity of the bright fringes is doubled
   (b) The intensity of the bright fringes becomes four times.
   (c) Two sets of interference fringes overlap
   (d) No interference pattern is observed.
9. What is the reason for your answer to Q.9?
   (a) The two sources do not emit light of the same wavelength
   (b) The two sources emit waves which travel with different speeds
   (c) The two sources emit light waves of different amplitude
   (d) There is no constant phase difference between the waves emitted by the two sources.
10. In Young's double–slit experiment, the intensity of light at a point on the screen where the path difference is λ is K units. What is the intensity of light at a point where the path difference is $\lambda/3$; λ being the wavelength of light used?
    (a) $K/4$ (b) $K/3$
    (c) $K/2$ (d) K
11. Two coherent sources of intensity ratio 100 : 1, interfere. What is the ratio of the intensity between the maxima and minima in the interference pattern?
    (a) 10 : 1 (b) 5 : 2
    (c) 121 : 81 (d) 11 : 9

12. In Young's double slit experiment the fringe width with light of wavelength 6000 Å is found to be 4.0 mm. What will be the fringe width if light of wavelength 4800 Å is used?
    (a) 2.8 mm (b) 3.2 mm
    (c) 4.0 mm (d) 4.8 mm
13. In Q. 13, what will be the fringe width using light of wavelength 6000 Å if the entire apparatus is immersed in a transparent liquid of refractive index 4/3?
    (a) 2.0 mm (b) 3.0 mm
    (c) 4.0 mm (d) 5.0 mm
14. In Young's double slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the fourth bright fringe and the central bright fringe is measured to be 1.2 cm. What is the wavelength of light used in the experiment?
    (a) 200 nm (b) 400 nm
    (c) 600 nm (d) 800 nm
15. What is the effect on the interference fringes in Young's double slit experiment if the source slit is moved closer to the double slit plane?
    (a) The fringe width increases
    (b) The fringe width decreases
    (c) The fringes become more distinct
    (d) The fringes become less distinct.
16. What is the effect on the interference fringes in Young's double slit experiment if the width of the source slit is increased?
    (a) The fringe width increases
    (b) The fringe width decreases
    (c) The fringes become more distinct.
    (d) The fringes become less distinct.
17. What is the effect on the interference fringes in Young's double slit experiment if the widths of the two slits are increased?
    (a) The fringe width decreases
    (b) The fringe width increases
    (c) The bright fringes are equally bright and equally spaced
    (d) The bright fringes are no longer equally bright and equally spaced.
18. In Young's double slit experiment the slits are 0.5 mm apart and interference is observed on a screen placed at a distance of 100 cm from the slits. It is found that the 9th bright fringe is at a distance of 9.0 mm from the second dark fringe from the centre of the fringe pattern. What is the wavelength of light used.
    (a) 2000 Å (b) 4000 Å
    (c) 6000 Å (d) 8000 Å
19. The ratio of the intensities of the maxima and minima in an interference pattern is 49 : 9. What is the ratio of the intensities of the two coherent sources employed in the interference experiment?
    (a) 7 : 3 (b) 49 : 9
    (c) 5 : 2 (d) 25 : 4
20. A screen is placed at a certain distance from a narrow slit which is illuminated by a parallel beam of monochromatic light. What will you observe if you scan the screen with the help of a microscope?
    (a) The whole screen is uniformly illuminated.
    (b) Equally spaced and equally bright fringes are observed.
    (c) One bright fringe is observed at the centre of the screen.
    (d) A bright central fringe bordered on both sides with fringes of rapidly decreasing intensity will be observed.
21. A single-slit diffraction pattern is obtained using a beam of red light. What happens if the red light is replaced by blue light?
    (a) There is no change in the diffraction pattern.
    (b) Diffraction fringes become narrower and crowded together.
    (c) Diffraction fringes become broader and farther apart.
    (d) The diffraction pattern disappears.
22. Which one of the following waves cannot be polarized?
    (a) radio waves
    (b) X-rays
    (c) transverse waves in a string
    (d) longitudinal waves in a gas.
23. The fact that light can be polarized establishes that light
    (a) travels in the form of particles
    (b) is an electromagnetic wave
    (c) is a transverse wave
    (d) is a longitudinal wave.
24. A parallel beam of light of wavelength 6000 Å is incident normally on a slit of width 0.2 mm. The diffraction pattern is observed on a screen which is placed at the focal plane of a convex lens of focal length 50 cm. If the lens is placed close to the slit, the distance between the minima on both sides of the central maximum will be
    (a) 1 mm (b) 2 mm
    (c) 3 mm (d) 4 mm

25. In Young's double slit experiment the distance d between the slits S_1 and S_2 is 1.0 mm. What should the width of each slit be so as to obtain 10 maxima of the two slit interference pattern within the central maximum of the single slit diffraction pattern?
    (a) 0.1 mm (b) 0.2 mm
    (c) 0.3 mm (d) 0.4 mm
26. In a single slit diffraction experiment, the width of the slit is made double its original width. Then the central maximum of the diffraction pattern will become.
    (a) narrower and fainter
    (b) narrower and brighter
    (c) broader and fainter
    (d) broader and brighter.
27. The dispersion of light in a medium implies that
    (a) lights of different wavelengths travel with different speeds in the medium
    (b) lights of different frequencies travel with different speeds in the medium
    (c) the refractive index of the medium is different for different wavelengths of light
    (d) all of the above.
28. Monochromatic light is refracted from air into glass of refractive index μ. The ratio of the wavelengths of the incident and refracted waves is
    (a) 1 : 1 (b) 1 : μ
    (c) μ : 1 (d) μ^2 : 1
29. Light travels with a speed of 2×10^8 ms^{-1} in crown glass of refractive index 1.5. What is the speed of light in dense flint glass of refractive index 1.8?
    (a) 1.33×10^8 ms^{-1} (b) 1.67×10^8 ms^{-1}
    (c) 2.0×10^8 ms^{-1} (d) 3.0×10^8 ms^{-1}
30. In a vacuum, light travels at a speed of 3×10^8 ms^{-1}. What is the speed of light in glass of refractive index = 1.5?
    (a) 1.5×10^8 ms^{-1} (b) 2×10^8 ms^{-1}
    (c) 3×10^8 ms^{-1} (d) 4.5×10^8 ms^{-1}
31. When a ray of light goes from a denser into a rarer medium
    (a) the wavelength of light is decreased
    (b) the frequency of light is increased
    (c) the speed of light is increased
    (d) the light undergoes a phase change of π.
32. Which one of the following statements is correct? The refractive index of a given piece of glass is
    (a) less for violet than for red light
    (b) more for blue than for green light
    (c) less for green than for yellow light
    (d) the same for all colours of light.
33. A glass slab of thickness 8 cm contains the same number of waves as 10 cm of water when both are traversed by the same monochromatic light. If the refractive index of water is 4/3, the refractive index of glass is
    (a) 5/3 (b) 5/4
    (c) 16/15 (d) 3/2

Level B

34. In Young's double slit experiment, the intensity of the maxima is I. If the width of each slit is doubled the intensity of the maxima will be
    (a) $\frac{I}{2}$ (b) I
    (c) $2I$ (d) $4I$
35. In Young's double slit experiment, the 10th maximum of wavelength λ_1 is at a distance y_1 from its central maximum and the 5th maximum of wavelength λ_2 is at a distance y_2 from its central maximum. The ratio y_1/y_2 will be
    (a) $\frac{2\lambda_1}{\lambda_2}$ (b) $\frac{2\lambda_2}{\lambda_1}$
    (c) $\frac{\lambda_1}{2\lambda_2}$ (d) $\frac{\lambda_2}{2\lambda_1}$
36. A ray of light is incident on glass at the polarising angle i_p. The angle between the reflected and refracted rays will be
    (a) between 0° and 90° (b) 90°
    (c) between 90° and 180° (d) $2\,i_p$
37. When a ray of light is incident on a glass slab at an angle of 60°, the angle between the reflected and refracted rays is 90°. The refractive index of glass is
    (a) 1.5 (b) $\sqrt{2}$
    (c) $\sqrt{3}$ (d) 2.0
38. In a Young's double slit experiment, 12 fringes are observed to be formed in a certain region of the screen when light of wavelength 600 nm is used. If the light of wavelength 400 nm is used, the number of fringes observed in the same region of the screen will be
    (a) 12 (b) 18
    (c) 24 (d) 8
39. In a two slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by a distance of 5×10^{-2} m towards the slits, the change in the fringe width is 3×10^{-5} m. If the separation

between the slits is 10^{-3} m, the wavelength of light used is

(a) 5×10^{-7} m (b) 6×10^{-7} m

(c) 7×10^{-7} m (d) 6×10^{-6} m

40. White light is used to illuminate the two slits in Young's double slit experiment. The distance between the slits is b and the screen is at a distance d ($>> b$) from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of the missing wavelengths are

(a) $\lambda = \frac{b^2}{2d}$ (b) $\lambda = \frac{2b^2}{d}$

(c) $\lambda = \frac{b^2}{3d}$ (d) $\lambda = \frac{2b^2}{3d}$

41. Two waves of intensities I and $4I$ superpose, then the maximum and minimum intensities are

(a) $5I$, $3I$ (b) $9I$, I

(c) $9I$, $3I$ (d) $5I$, I

42. A beam of light of wavelength 600 nm from a distant source falls on a single slit 1.0 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on either side of the central bright fringe is

(a) 1.2 cm (b) 1.2 mm

(c) 2.4 cm (d) 2.4 mm

43. A parallel beam of monochromatic light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of the incident beam. At the first minimum of the diffraction pattern, the phase difference between the rays coming from the two edges of the slit is

(a) zero (b) $\frac{\pi}{2}$

(c) π (d) 2π

44. Yellow light is used in a single slit diffraction experiment with a slit of width 0.6 mm. If yellow light is replaced by X-rays, then the observed pattern will reveal

(a) that the central maximum is narrower
(b) more number of fringes
(c) less number of fringes
(d) no diffraction pattern

45. In a double slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other, then, in the interference pattern

(a) the intensities of both maxima and minima increase
(b) the intensity of the maxima increases and the minima have zero intensity
(c) the intensity of the maxima decreases and that of the minima increases
(d) the intensity of the maxima decreases and the minima have zero intensity.

46. In Young's double slit interference experiment the wavelength of light used is 6000 Å. If the path difference between waves reaching a point P on the screen is 1.5 microns, then at that point P:

(a) Second bright band occurs
(b) Second dark band occurs
(c) Third dark band occurs
(d) Third bright band occurs

47. The difference in the number of wavelengths, when yellow light (of wavelength 6000 Å in vacuum) propagates through air and vacuum columns of the same thickness is one. If the refractive index of air is 1.0003, the thickness of the air column is

(a) 1.8 mm (b) 2 mm

(c) 2 cm (d) 2.2 cm

48. In Young's double slit experiment, the fringe width is 2.0 mm, The separation between the 9th bright fringe and the second dark fringe from the centre of the fringe system will be

(a) 5.0 mm (b) 10 mm

(c) 15 mm (d) 20 mm

49. When one of the slits in Young's experiment is covered with a transparent sheet of thickness 3.6×10^{-3} cm the central fringe shifts to a position originally occupied by the 30th bright fringe. If λ = 6000 Å, the refractive index of the sheet is

(a) 1.50 (b) 1.55

(c) 1.60 (d) 1.65

50. A beam of light, consisting of two wavelengths 6500 Å and 5200 Å is used to obtain interference fringes in Young's double slit experiment. The separation between the slits is 2.6 mm and the distance between the plane of the slits and the screen is 1.0 m. The least distance from the central maximum where the bright fringes due to both the wavelengths coincide is

(a) 1.0 mm (b) 1.5 mm

(c) 2.0 mm (d) 2.5 mm

51. Two coherent light sources of intensity ratio n are employed in an interference experiment. The ratio of the intensities of the maxima and minima in the interference pattern is

(a) $\left(\frac{n+1}{n-1}\right)$ (b) $\left(\frac{n+1}{n-1}\right)^2$

(c) $\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)$ (d) $\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$

52. Two coherent light sources are employed in an interference experiment. The ratio of the intensities of the maxima and minima in the interference pattern is n. The ratio of the intensities of the two coherent sources is

(a) $\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$ (b) $\left(\frac{n+1}{n-1}\right)^2$

(c) $\left(\frac{n^2+1}{n^2-1}\right)$ (d) $\left(\frac{n+1}{n-1}\right)$

53. The two slits in Young's interference experiment have widths in the ratio n:1. The ratio of the intensities of the maxima and minima in the interference pattern is

(a) $\frac{\sqrt{n}+1}{\sqrt{n}}$ (b) $\frac{\sqrt{n}}{\sqrt{n}-1}$

(c) $\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$ (d) $\left(\frac{n+1}{n}\right)^2$

54. Interference pattern is obtained with two coherent light sources of intensity ratio n. In the interference pattern, the ratio $\frac{I_{max}-I_{min}}{I_{max}+I_{min}}$ will be

(a) $\frac{\sqrt{n}}{(n+1)}$ (b) $\frac{2\sqrt{n}}{(n+1)}$

(c) $\frac{\sqrt{n}}{\left(\sqrt{n}+1\right)^2}$ (d) $\frac{2\sqrt{n}}{\left(\sqrt{n}+1\right)^2}$

55. In Young's double slit experiment, the angular width of a fringe formed on a distant screen is 0.1°. If the wavelength of light used is 628 nm, the spacing between the slits is

(a) 0.9×10^{-4} m (b) 1.8×10^{-4} m

(c) 3.6×10^{-4} m (d) 7.2×10^{-4} m

56. Interference pattern is obtained with two coherent light sources of intensities I and $4I$. The intensity at a point where the phase difference is $\pi/2$ is

(a) I (b) $2I$

(c) $3I$ (d) $5I$

57. Young's double slit expetiment is performed using light of wavelength λ. One of the slits is covered by a thin glass sheet of refractive index μ at this wavelength. The smallest thickness of the sheet to bring the adjacent minimum to the centre of the screen is

(a) $\frac{\lambda}{2(\mu-1)}$ (b) $\frac{\lambda}{(\mu-1)}$

(c) $\frac{\lambda}{2\mu}$ (d) $\frac{\lambda}{\mu}$

58. Monochromatic light of wavelength 500 nm is incident on two parallel slits separated by a distance of 5×10^{-4} m. The interference pattern is obtained on a screen at a distance of 1.0 m from the slits. The intensity of the central maximum is I_0. When one of the slits is covered by a glass sheet of thickness 1.5×10^{-6} m and refractive index 1.5, the intensity at the centre of the screen will be equal to

(a) $\frac{I_0}{2}$ (b) $\frac{I_0}{3}$

(c) $\frac{I_0}{4}$ (d) zero

59. In Q. 58 above, the lateral shift of the central maximum is

(a) 5 mm (b) 4 mm

(c) 3 mm (d) 2 mm

60. In an interference experiment, 20th order maximum is observed at a point on the screen when light of wavelength 480 nm is used. If this light is replaced by light of wavelength 600 nm, the order of the maximum at the same point will be

(a) 16 (b) 14

(c) 12 (d) 10

61. A parallel beam of fast moving electrons is incident normally on a narrow slit. A screen is placed at a large distance from the slit. If the speed of the electrons is increased, which of the following statements is correct?

(a) Diffraction pattern is not observed on the screen in the case of electrons

(b) The angular width of the central maximum of the diffraction pattern will increase

(c) The angular width of the central maximum will decrease

(d) The angular width of the central maximum will remain the same.

62. In Young's double slit experiment using monochromatic light, the fringe pattern shifts by a certain distance on the screen when a transparent sheet of thickness t and refractive index μ is introduced in the path of one of the interfering waves. The sheet is then removed and the distance between the screen and the slits is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift when the sheet was introduced. The wavelength of light used is

(a) $\lambda = (\mu - 1)t$ (b) $\lambda = \frac{1}{2}(\mu - 1)t$

(c) $\lambda = (\mu + 1)t$ (d) $\lambda = \frac{1}{2}(\mu + 1)t$

63. In a double-slit experiment the angular width of a fringe is found to be 0.2° on a screen 1 m away. The wavelength of light used is 6000 Å. What will be the angular width of a fringe if the entire experimental arrangement is immersed in water? Refractive index of water = $\frac{4}{3}$.

(a) 0.15° (b) 0.18°

(c) 0.2° (d) 0.27°

64. A coherent parallel beam of microwaves of wavelength 0.5 mm falls normally on Young's double slit apparatus. The separation between the slits is 1.0 mm and the screen is placed at a distance of 1.0 m from the slits. The number of minima in the interference pattern observed on the screen is

(a) 3 (b) 4

(c) 5 (d) much greater than 5.

65. In Young's double slit experiment sodium light composed of two wavelengths λ_1 and λ_2 close to each other (with λ_2 greater than λ_1) is used. The order n up to which the fringes can be seen on the screen is given by

(a) $n = \frac{\lambda_2}{\lambda_2 - \lambda_1}$ (b) $n = \frac{\lambda_1}{\lambda_2 - \lambda_1}$

(c) $n = \frac{\lambda_2}{2(\lambda_2 - \lambda_1)}$ (d) $n = \frac{\lambda_1}{2(\lambda_2 - \lambda_1)}$

66. Monochromatic light of wavelength λ emerging from slit S illuminates slits S_1 and S_2 which are placed with respect to S as shown in Fig. 17.6. The distances x and D are large compared to the separation d between the slits. If $x = D/2$, the minimum value of d so that there is a dark fringe at the centre P of the screen is

(a) $\sqrt{\frac{\lambda D}{3}}$ (b) $\sqrt{\frac{2\lambda D}{3}}$

(c) $\sqrt{\lambda D}$ (d) $2\sqrt{\frac{\lambda D}{3}}$

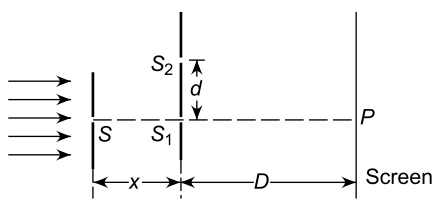


Fig. 17.6

67. Young's double slit experiment is performed by using two coherent light sources each of intensity I. In another experiment, with the same apparatus, the experiment is performed with incoherent sources each of intensity I and the same wavelength. The ratio of the intensity of light at the mid-point of the screen in the first experiment to that in the second experiment is

(a) 1 (b) 2

(c) $2\sqrt{2}$ (d) 4

68. A parallel beam of monochromatic light of wavelength 600 nm falls normally on a slit of width 0.5 mm. The resulting diffraction pattern is observed on a screen placed 50 cm from the slit. The linear separation between the first minimum and the first maximum on the same side of the central maximum is

(a) 0.003 mm (b) 0.03 mm

(c) 0.3 mm (d) 3 mm

69. The angular width $\Delta\theta$ of the central maximum in the diffraction pattern of a slit illuminated by light of wavelength 500 nm is measured. The angular width of the central maximum is found to increase by 25% when the same slit is illuminated by another light of wavelength λ'. Wavelength λ' (in nm) is

(a) 375 (b) 450

(c) 575 (d) 625

70. A parallel beam of monochromatic light of wavelength λ falls normally on a narrow slit of width a. The diffraction pattern is observed on a screen placed perpendicular to the direction of the incident beam. The phase difference between the waves coming from the edges of the slit at the position of the first maximum of the diffraction pattern is

(a) 3π (b) $\frac{3\pi}{2}$

(c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$

71. Blue light is used in a single slit diffraction experiment with a slit of width a = 0.5 mm. If blue light is replaced by yellow light, then

(a) the central maximum becomes narrower

(b) the central maximum becomes broader

(c) more number of fringes are observed

(d) less number of fringes are observed.

72. Young's double slit experiment is performed using light of wavelength 600 nm. When a mica film of refractive index 1.6 and thickness t is introduced in

the path of one of the interfering beams, the intensity of light at the position where the central bright fringe previously appeared remains unchanged. The minimum thickness of the mica film is

(a) 10^{-6} m (b) 10^{-5} m

(c) 10^{-4} m (d) 1 mm

73. Young's double slit experiment is performed using light of wavelength λ. A thin glass plate of refractive index 1.5 and thickness t is now introduced in the path of one of the interfering beams. The value of t so that the central bright fringe is replaced by its neighbouring dark fringe is

(a) $\frac{\lambda}{2}$ (b) λ

(c) $\frac{3\lambda}{2}$ (d) 2λ

74. Monochromatic light of wavelength λ falls normally on a glass film of refractive index μ. The waves reflected from the upper and lower faces of the film will interfere constructively if the least thickness of the film is

(a) $\frac{\lambda}{2\mu}$ (b) $\frac{\lambda}{4\mu}$

(c) $\frac{\mu\lambda}{2}$ (d) $\frac{\mu\lambda}{4}$

75. A beam of light consisting of two wavelengths λ_1 = 750 nm and λ_2 = 450 nm is used in Young's double slit experiment. The separation between the slits is 2 mm and the distance of the screen from the plane of the slits is 100 cm. What is the minimum distance between two successive regions of maximum brightness of two wavelengths?

(a) 1.125 mm (b) 2.250 mm

(c) 3.525 mm (d) 4.750 mm

76. In Q. 75 above, what is the minimum distance between two successive regions of complete darkness.

(a) 1.125 mm (b) 2.250 mm

(c) 3.525 mm (d) 4.750 mm

77. In Young's double slit experiment, the intensity at a point P on the screen is 3/4 of the maximum intensity in the interference pattern. If d is the separation between the slits and λ is the wavelength of light, the angular separation between point P and the centre of the screen is

(a) $\sin^{-1}\left(\frac{\lambda}{d}\right)$ (b) $\cos^{-1}\left(\frac{\lambda}{3d}\right)$

(c) $\tan^{-1}\left(\frac{\lambda}{6d}\right)$ (d) $\sin^{-1}\left(\frac{2\lambda}{3d}\right)$

78. A beam of monochromatic light of intensity I is incident on a rectangular slab $ABCD$ as shown in Fig. 17.7. Each face AB and CD reflects 25% of light intensity incident on it and transmits the rest. The ratio of the maxima to minima in the interference of beams I_1 and I_2 is

(a) 49 : 1 (b) 25 : 16

(c) 16 : 3 (d) 36 : 1

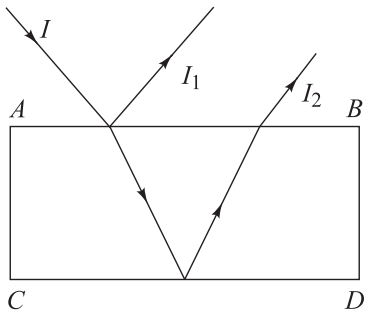


Fig. 17.7

79. In Q. 78, what is the ratio of the maxima and minima in the interference pattern of beams I_3 and I_4 shown in Fig. 17.8 ?

(a) $\frac{5}{3}$ (b) $\frac{16}{7}$

(c) $\frac{25}{9}$ (d) $\frac{39}{11}$

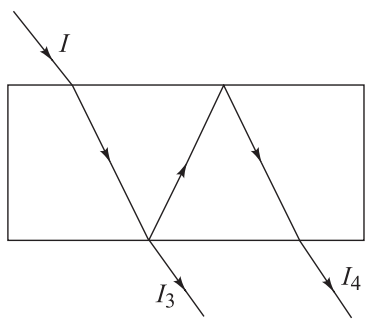


Fig. 17.8

80. In Young's double slit experiment, the separation between the two slits is 2.0 mm. What should be the width of each slit so as to accomodate 10 maxima of the double slit experiment within the central maximum of the single slit diffraction experiment?

(a) 0.1 mm (b) 0.2 mm

(c) 0.4 mm (d) 1.0 cm

Answers

Level A

1. (d)	2. (b)	3. (c)	4. (c)
5. (c)	6. (c)	7. (d)	8. (d)
9. (d)	10. (a)	11. (c)	12. (b)
13. (b)	14. (c)	15. (d)	16. (d)
17. (d)	18. (c)	19. (d)	20. (d)
21. (b)	22. (d)	23. (c)	24. (c)
25. (b)	26. (b)	27. (d)	28. (c)
29. (b)	30. (b)	31. (c)	32. (b)
33. (a)			

Level B

34. (c)	35. (a)	36. (b)	37. (c)
38. (b)	39. (b)	40. (c)	41. (b)
42. (d)	43. (d)	44. (a)	45. (a)
46. (c)	47. (b)	48. (c)	49. (a)
50. (a)	51. (d)	52. (a)	53. (c)
54. (b)	55. (c)	56. (b)	57. (a)
58. (d)	59. (a)	60. (a)	61. (c)
62. (b)	63. (a)	64. (b)	65. (c)
66. (a)	67. (b)	68. (c)	69. (d)
70. (a)	71. (b)	72. (a)	73. (b)
74. (b)	75. (a)	76. (a)	77. (c)
78. (a)	79. (c)	80. (a)	

Solutions

Level A

1. The wavelength of red light is greater than that of yellow light. Since $\beta = \lambda D/d$, the fringe width β will increase. Hence the correct choice is (d).
2. Since $\beta = \lambda D/d$, if D is doubled and d is halved, the fringe width β will become four times. Hence the correct choice is (b).
3. White light consists of colours between violet and red. The wavelength λ is the shortest for violet 1ight and the longest for red light. At the central fringe, the path difference for all colours is zero. Therefore, at the centre of the screen all colours superpose to give a white fringe. Hence the correct choice is (c).
4. The intensity (and hence the amplitude) of the light from the covered slit will decrease resulting in a difference in the intensities of the two virtual sources. Hence the correct choice is (c).
5. If the two sources are coherent, the resultant amplitude at the midpoint of the screen due to interference = $A + A = 2A$. Therefore, intensity is $I_1 \propto (2A)^2$ or $I_1 = k \times 4A^2$ where k is a constant of proportionality. But if the sources are not coherent, their intensities simply add up at the midpoint, i.e.

$$I_2 \propto (A^2 + A^2) \text{ or } I_2 = k \times 2A^2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{4kA^2}{2kA^2} = 2$$

Hence the correct choice is (c).

6. Since the refractive index of water is greater than that of the air, the speed of the light used in the experiment will be less in water than in air. Since the frequency of light is the same in water and in air, it follows from the relation $\lambda = v/\nu$ that the wavelength λ in water is less than in air. Since fringe width $\beta \propto \lambda$, the value of β will decrease. Hence the correct choice is (c).
7. We then obtain a single slit diffraction pattern on the screen. Hence the correct choice is (d).
8. Independent light sources are incoherent. Hence a permanent interference pattern is not obtained. Thus the correct choice is (d).
9. The correct choice is (d).
10. Path difference $\Delta = \lambda$. Therefore, phase difference $\phi = \frac{2\pi}{\lambda}\Delta = 2\pi$. Hence intensity at a point where $\Delta = \lambda$ or $\phi = 2\pi$ is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$
$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos 2\pi$$
$$= I_1 + I_2 + 2\sqrt{I_1 I_2}$$
$$= I_0 + I_0 + 2I_0 = 4I_0 = K \text{ units}$$
$$(\because I_1 = I_2 = I_0)$$

i.e. $I = K/4$. The intensity at a point where the path difference is

$$\Delta' = \frac{\lambda}{3} \text{ or } \phi' = \frac{2\pi}{\lambda}\Delta' = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3} \text{ is}$$

$$I' = I_0 + I_0 + 2I_0 \cos\frac{2\pi}{3} = 2I_0 - I_0 = I_0 = \frac{K}{4} \text{ units}$$

11. Given $I_1/I_2 = 100$, i.e. $I_1 = 100$ units and $I_2 = 1$ unit. Intensity at maxima is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}$$
$$= 100 + 1 + 2\sqrt{100 \times 1} = 121$$

Intensity at minima is

$$I' = I_1 + I_2 - 2\sqrt{I_1 I_2} = 81$$

$$\therefore \quad \frac{I}{I'} = \frac{121}{81} = 1.49$$

Hence the closest choice is (c).

12. Given $\beta = 4.0$ mm and $\lambda = 6000$ Å.

We know that the fringe width is given by

$$\beta = \frac{\lambda D}{d} \qquad \text{(i)}$$

for $\lambda' = 4800$ Å, the fringe width will be

$$\beta' = \frac{\lambda' D}{d} \qquad \text{(ii)}$$

From (i) and (ii) we have

$$\beta' = \beta \frac{\lambda'}{\lambda} = \frac{4.0\,\text{mm} \times 4800\,\text{Å}}{6000\,\text{Å}} = 3.2 \text{ mm}$$

13. Wavelength in air is $\lambda_a = 6000$ Å. Let its speed in air be v_a. When the apparatus is immersed in a liquid the frequency of the wave remains unchanged but its wavelength and speed both will change. Let λ_l be the wavelength and v_l be the speed in the liquid. Since $v = \nu\lambda$, we have

$$\nu = \frac{v_a}{\lambda_a} = \frac{v_l}{\lambda_l}$$

or $$\lambda_l = \frac{\lambda_a v_l}{v_a} = \lambda_a \frac{v_l}{c} \frac{c}{v_a}$$

(c is the speed of light in vacuum). Now the refractive index of a medium is $n = c/v$. Hence

$$\lambda_l = \lambda_a \frac{n_a}{n_l} = \frac{\lambda_a}{n_l} \qquad (\because n_a = 1)$$

Given $\lambda_a = 6000$ Å and $n_l = 4/3$. Therefore

$$\lambda_l = \frac{6000\,\text{Å}}{4/3} = 4500 \text{ Å}$$

Hence the fringe width in liquid will be

$$\beta_l = \frac{\beta_a \lambda_l}{\lambda_a} = \frac{4.0\,\text{mm} \times 4500\,\text{Å}}{6000\,\text{Å}} = 3.0 \text{ mm}$$

14. The position of the nth bright fringe with respect to the central fringe is given by

$$y_n = \frac{n\lambda D}{d}$$

For the central bright fringe ($n = 0$), $y_0 = 0$. For the fourth bright fringe ($m = 4$), $y_4 = 4\,\lambda D/d$. Therefore

$$y_4 - y_0 = \frac{4\lambda D}{d}$$

or $$\lambda = (y_4 - y_0)\frac{d}{4D} \qquad \text{(i)}$$

It is given that $(y_4 - y_0) = 1.2$ cm $= 1.2 \times 10^{-2}$ m, $D = 1.4$ m and $d = 0.28$ mm $= 0.28 \times 10^{-3}$ m. Substituting these values in (i) and solving, we get

$$\lambda = 6 \times 10^{-7} \text{ m (or 600 nm or 6000 Å).}$$

15. and 16. Let x be the width of the source slit and X the distance between the source slit and the plane of the two slits. For interference fringes to be distinctly visible, the condition $x/X < \lambda/d$ should be satisfied. If x is too large (i.e. the source slit is too wide) or if X is too small (X is the distance between the source slit and the two slits) the requirement $x/X < \lambda/d$ may be violated and fringes will no longer be distinct. The reason is that the interference patterns due to various parts of the source slit overlap. Consequently, the minima will not be totally dark and fringe pattern becomes indistinct. However, as long as the fringe pattern remains visible, a change in x or X has no effect on the fringe width β.

17. The single slit diffraction effects at the two slits becomes important and as a result, the interference fringe pattern will be modified. The bright fringes will not now be equally bright and equally spaced.

18. The distance of the mth bright fringe from the central fringe is

$$y_n = \frac{n\lambda D}{d} = n\beta$$

where $\beta = \lambda D/d$ is the fringe width.

$$\therefore \quad y_9 = 9\beta \qquad \text{(i)}$$

The distance of the mth dark fringe from the central fringe is

$$y'_n = \left(n - \frac{1}{2}\right) \frac{\lambda D}{d} = \left(n - \frac{1}{2}\right) \beta$$

$$y'_2 = \frac{3}{2}\beta \qquad \text{(ii)}$$

From (i) and (ii), we get

$$y_9 - y'_2 = 9\beta - \frac{3}{2}\beta = \frac{15}{2}\beta$$

It is given that $y_9 - y'_2 = 9.0$ mm. Hence

$$\beta = \frac{9.0 \times 2}{15} = 1.2 \text{ mm} = 1.2 \times 10^{-3} \text{ m}$$

Now $\lambda = \beta d / D$. Substituting for β, d and D, we get

$$\lambda = 6 \times 10^{-7} \text{ m} = 6000 \text{ Å}$$

19. Given $\frac{I_{max}}{I_{min}} = \frac{49}{9}$. Therefore $\frac{A_{max}}{A_{min}} = \sqrt{\frac{49}{9}} = \frac{7}{3}$. Thus $A_{max} = 7$ units and $A_{min} = 3$ units. Now $A_{max} = A_1 + A_2$ and $A_{min} = A_1 - A_2$. Therefore, $A_1 = 5$ units and $A_2 = 2$ units. Hence

$$\frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

Hence the correct choice is (d).

20. A single slit diffraction pattern is characterised by a bright central fringe bordered on both sides with fringes of rapidly decreasing intensity.

21. The wavelength of blue light is less than that of the red light. Hence the angular width of the maxima will decrease which means that the fringes become narrower and crowded together.

22. Only transverse waves can be polarized. Radiowaves, X-rays and waves on strings are transverse and hence they can be polarized. Longitudinal waves such as sound waves or waves in springs cannot be polarized. Hence the correct choice is (d).

23. The correct choice is (c).

24. The angular separation of the minima on both sides of the central maximum is 2θ where θ is given by

$$\sin\theta = \frac{\lambda}{d} = \frac{6000 \times 10^{-10}}{0.2 \times 10^{-3}} = 3 \times 10^{-3}$$

since θ is small, $\sin\theta \simeq \theta$. Therefore, $\theta = 3 \times 10^{-3}$ rad. If the lens is placed close to the slit then

$$x = f \tan\theta \simeq f\theta \quad (\because \theta \text{ is small}, \therefore \tan\theta \simeq \theta)$$

where x is the distance of the first minimum from the central maximum. Therefore, the distance between two minima on both sides of the central maximum is

$$2x = 2f\theta = 2 \times 0.5 \times 3 \times 10^{-3}$$
$$= 3 \times 10^{-3} \text{ m} = 3 \text{ mm}.$$

25. Let the width of each slit be a. The linear separation between m bright fringes in the double slit experiment is

$$y_m = \frac{n\lambda D}{d}$$

Since $y << D$, the angular separation between m bright fringes will be

$$\theta_m = \frac{y_n}{D} = \frac{n\lambda}{d}$$

For 10 bright fringes we have

$$\theta_{10} = \frac{10\lambda}{d} \quad \text{(i)}$$

Now the angular width of the principal maximum in the diffraction pattern due to a slit of width a is

$$2\theta_1 = \frac{2\lambda}{a} \quad \text{(ii)}$$

Equating (i) and (ii), we get

$$\frac{10\lambda}{d} = \frac{2\lambda}{a}$$

or $$a = \frac{d}{5} = \frac{1.0 \text{ mm}}{5} = 0.2 \text{ mm}$$

26. The angular width of the central maximum is $2\lambda / a$ where a is the width of the slit. If the value of a is doubled, the angular width of the central maximum decreases to half its earlier value. This implies that the central maximum becomes much sharper. Furthermore if a is doubled, the intensity of the central maximum becomes two times. Thus the central maximum becomes much sharper and brighter.

27. The correct choice is (d).

28. Since the frequency n of the light does not change as light travels from air into glass, we have

$$v_a = n\,\lambda_a \text{ and } v_g = n\,\lambda_g$$

Therefore $$\frac{\lambda_a}{\lambda_g} = \frac{v_a}{v_g} = \mu$$

Hence the correct choice is (c).

29. Refractive index $\mu = \frac{c}{v}$. Therefore, speed of light in crown glass is

$v_c = \frac{c}{\mu_c}$, where c is the speed of light in vacuum,

or $$c = \mu_c v_c = 1.5 \times 2 \times 10^8 = 3 \times 10^8 \text{ ms}^{-1}$$

$\therefore$ Speed of light in dense flint glass is

$$v_f = \frac{c}{\mu_f} = \frac{3 \times 10^8}{1.8} = 1.67 \times 10^8 \text{ ms}^{-1}$$

30. Speed of light in glass $= \frac{c}{\mu_g} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8$ ms^{-1}.

31. When a wave goes from a denser into a rarer medium, its speed increases. Since frequency n does not change, the wavelength $\lambda = v/n$ increases. Therefore is no phase change on refraction. Hence the correct choice is (c).

32. The refractive index of glass decreases with increase in wavelength. In VIBGYOR, the wavelength of violet light is the shortest and that of the red light is the longest. Hence the correct choice is (b).

33. Let λ_g (in cm) and λ_w (in cm) be the wavelengths in glass and water. By definition, in a distance λ there is one wave. Therefore,

Number of waves in 8 cm of glass = $\frac{8}{\lambda_g}$, and

Number of waves in 10 cm of water = $\frac{10}{\lambda_w}$. Thus

$$\frac{8}{\lambda_g} = \frac{10}{\lambda_w}$$

$$\frac{\lambda_w}{\lambda_g} = \frac{5}{4}$$

Now $\mu_g = \frac{c}{v_g}$ and $\mu_w = \frac{c}{v_w}$.

Therefore, $\frac{\mu_g}{\mu_w} = \frac{v_w}{v_g} = \frac{\nu\lambda_w}{\nu\lambda_g} = \frac{\lambda_w}{\lambda_g}$

or $\mu_g = \frac{\lambda_w}{\lambda_g} \times \mu_w = \frac{5}{4} \times \frac{4}{3} = \frac{5}{3}$

Level B

34. $I = I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$

When the width of each slit is doubled, I_1 becomes $2I_1$ and I_2 becomes $2I_2$. Therefore,

$$I' = I'_{max} = 2I_1 + 2I_2 + 2\sqrt{2I_1 \times 2I_2}$$

$$= 2(I_1 + I_2 + 2\sqrt{I_1 I_2}) = 2I_{max} = 2I.$$

Hence the correct choice is (c).

35. We know that $y_m = \frac{m\lambda D}{d}$. Therefore, for wavelength λ_1,

$$y_1 = \frac{10\lambda_1 D}{d}$$

and for wavelength λ_2,

$$y_2 = \frac{5\lambda_2 D}{d}$$

$$\therefore \quad \frac{y_1}{y_2} = \frac{2\lambda_1}{\lambda_2}$$

Hence the correct choice is (a).

36. The correct choice is (b).

37. Angle of incidence = i, angle of refraction $r = 90 - i$. Hence the refractive index is

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin(90° - i)} = \frac{\sin i}{\cos i} = \tan i.$$

$\therefore \quad \mu = \tan 60° = \sqrt{3}$, which is choice (c).

38. Number of fringes = $\frac{\text{width of region}}{\text{fringes width}}$ or $n = \frac{L}{\beta}$.

Now, fringe width β is proportional to wavelength. Hence the new number of fringes will be

$$n' = n \times \frac{\beta}{\beta'} = 12 \times \frac{600}{400} = 18,$$ which is choice (b).

39. $\beta = \frac{\lambda D}{d}$. Therefore, $\Delta\beta = \frac{\lambda \Delta D}{d}$. Given $\Delta D = 5 \times 10^{-2}$ m, $\Delta\beta = 3 \times 10^{-5}$ m and $d = 10^{-3}$ m. Using these values, we have

$$3 \times 10^{-5} = \frac{\lambda \times 5 \times 10^{-2}}{10^{-3}}$$

which gives $\lambda = 6 \times 10^{-7}$ m which is choice (b).

40. The missing wavelengths are

$$\lambda = \frac{b^2}{(2m+1)d}; \; m = 0, 1, 2, 3, \cdots \text{ etc.}$$

$$= \frac{b^2}{d}, \frac{b^2}{3d}, \frac{b^2}{5d}, \cdots \text{ etc.}$$

Hence the correct choice is (c).

41. Given $I_1 = I$ and $I_2 = 4I$. Now

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$= I + 4I + 2\sqrt{4I^2} = 9I$$

and $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = 5I - 4I = I$

Hence the correct choice is (b).

42. The angular separation between the mth dark fringe and the central bright fringe is given by

$$a \sin\theta_n = n\lambda$$

For the first dark fringe, $n = 1$. Therefore

$$a \sin\theta_1 = \lambda$$

or $\sin\theta_1 = \lambda/a$. Since $\lambda << a$, $\sin\theta_1 \simeq \theta_1$. Hence $\theta_1 = \lambda/a$. This is also the angular separation between the central bright fringe and the first dark fringe on the other side of the central bright fringe. Hence, the angular separation between the first dark fringes on either side of the central bright fringe = $2\theta_1 = 2\lambda/a$. Therefore, their separation at distance $d = 2$ m is

$$\frac{2\lambda d}{a} = \frac{2 \times (600 \times 10^{-9}\,\text{m}) \times 2\,\text{m}}{1.0 \times 10^{-3}\,\text{m}}$$

$$= 2.4 \times 10^{-3}\,\text{m} = 2.4\,\text{mm},$$

which is choice (d).

43. It follows from Fig. 17.9 that the path difference at the first minimum between rays coming from A and B is

$$\Delta = BC = a \sin \theta_1$$

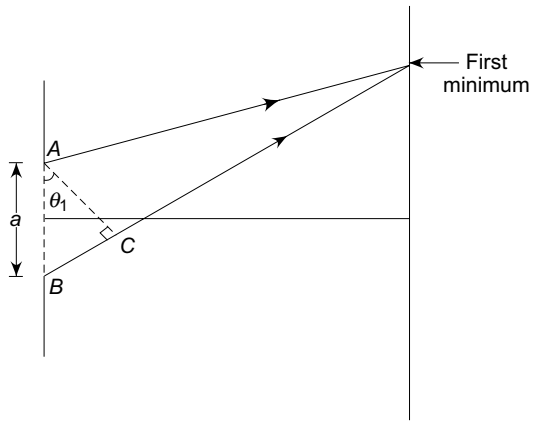


Fig. 17.9

But $\sin \theta_1 = \lambda/a$. Therefore, $\Delta = a \times \lambda/a = \lambda$. A path difference of λ corresponds to a phase difference of 2π, which is choice (d).

44. The wavelength of X-rays is of the order of $1 \text{ Å} \simeq 10^{-10}$ m. Now, the angular width of the central maximum is

$$\theta_0 = \frac{2\lambda D}{a}$$

where D is the distance of the screen from the slit. Since λ for X-rays is very small compared to that for yellow light, it follows that the angular width of the central maximum becomes extremely small. Hence, the central maximum is narrower. Thus the correct choice is (a).

45. In the case when the slits are of equal width, the intensity of light emerging from the two slits is the same, say, I_0. Then

$$I_{max} = I_0 + I_0 + 2\sqrt{I_0 I_0} = 4I_0$$

and $$I_{min} = I_0 + I_0 - 2\sqrt{I_0 I_0} = 0$$

When one slit say S_1 is made twice as wide as the order, the intensity of light from S_1 is doubled, i.e. $I_1 = 2I_0$ but $I_2 = I_0$. Hence, in this case

$$I'_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$= 2I_0 + I_0 + 2\sqrt{2I_0 I_0}$$

$$= 3I_0 + 2\sqrt{2I_0} = 5.83 I_0$$

and $$I'_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

$$= 2I_0 + I_0 - 2\sqrt{2I_0 I_0}$$

$$= 3I_0 - 2\sqrt{2I_0} = 0.17I_0$$

Thus $I'_{max} > I_{max}$ and $I'_{min} > I_{min}$, which is choice (a).

46. Given $\lambda = 6000$ Å $= 6000 \times 10^{-10}$ m $= 6 \times 10^{-7}$ m and $\Delta = 1.5$ microns $= 1.5 \times 10^{-6}$ m. For bright fringes: $\Delta = n\lambda$; where n is an integer.

$n = \dfrac{\Delta}{\lambda} = \dfrac{1.5 \times 10^{-6}}{6 \times 10^{-7}} = \dfrac{5}{2}$, which is not an integer. Hence, path difference of 1.5×10^{-6} m does not correspond to a bright fringe. For dark fringes, we have

$$\Delta = \left(n - \frac{1}{2}\right) \lambda$$

or $$1.5 \times 10^{-6} = \left(n - \frac{1}{2}\right) \times (6 \times 10^{-7})$$

which gives $n - \dfrac{1}{2} = \dfrac{5}{2}$ or $n = 3$. Hence a path difference of 1.5×10^{-6} m corresponds to the third dark fringe. Thus the correct choice is (c).

47. Let λ_a and λ be the wavelengths of yellow light in air and vacuum respectively and v_a and c be their respective speeds in air and vacuum. Since the frequency of light is the same in both media, we have

$$\nu = \frac{v_a}{\lambda_a} = \frac{c}{\lambda}$$

or $$\frac{c}{v_a} = \frac{\lambda}{\lambda_a}. \text{ But } \frac{c}{v_a} = \mu_a \text{ (by defination)}$$

$$\therefore \quad \mu_a = \frac{\lambda}{\lambda_a} \text{ or } \lambda_a = \frac{\lambda}{\mu_a} \qquad (1)$$

Now, if t is the thickness of each column, then the number of wavelengths in the two media are $n_a = \dfrac{t}{\lambda_a}$ and $n = \dfrac{t}{\lambda}$. Given $(n_a - n) = 1$. Hence

$$1 = \frac{t}{\lambda_a} - \frac{t}{\lambda} = \frac{t}{\lambda}\left(\frac{\lambda}{\lambda_a} - 1\right) \qquad (2)$$

Using (1) in (2), we have

$$1 = \frac{t}{\lambda}(\mu_a - 1) \qquad (3)$$

Given $\mu_a = 1.0003$ and $\lambda = 6000$ Å $= 6000 \times 10^{-8}$ cm. Using these values in (3), we have

$$1 = \frac{t}{6000 \times 10^{-8}} (1.0003 - 1)$$

or $$t = \frac{6000 \times 10^{-8}}{0.0003} = 0.2 \text{ cm} = 2 \text{ mm}$$

Hence the correct choice is (b).

48. The distance of the nth bright fringe from the central fringe is

$$y_n = n\lambda \frac{D}{d} = n\beta$$

where $\beta = \frac{\lambda D}{d}$ is the fringe width.

$$\therefore \quad y_n = 9\beta \qquad (1)$$

The distance of the mth dark fringe from the central fringe is

$$y'_n = \left(m - \frac{1}{2}\right)\frac{\lambda D}{d} = \left(m - \frac{1}{2}\right)\beta$$

$$y'_2 = \frac{3}{2}\beta \qquad (2)$$

From Eqs. (1) and (2) we get

$$y_9 - y'_2 = 9\beta - \frac{3}{2}\beta = \frac{15}{2}\beta$$

$$= \frac{15}{2} \times 2.0 \text{ mm} = 15 \text{ mm}$$

Hence the correct choice is (c).

49. The position of the 30th bright fringe is given by

$$y_{30} = 30 \frac{\lambda D}{d}$$

Hence the shift of the central fringe is

$$y_0 = 30 \frac{\lambda D}{d}$$

But $$y_0 = \frac{D}{d}(\mu - 1)t$$

$$\therefore \quad 30 \frac{\lambda D}{d} = \frac{D}{d}(\mu - 1)t$$

or $$(\mu - 1) = \frac{30\lambda}{t} = \frac{30 \times (6000 \times 10^{-10})}{(3.6 \times 10^{-5})} = 0.5$$

$$\therefore \quad \mu = 1.5$$

50. Let the nth bright fringe of wavelength λ_n and the mth bright fringe of wavelength λ_m coincide at a distance y from the certral maximum, then

$$y = \frac{m\lambda_m D}{d} = \frac{n\lambda_n D}{d}$$

or $$\frac{m}{n} = \frac{\lambda_n}{\lambda_m} = \frac{6500}{5200} = \frac{5}{4}$$

The least integral values of m and n which satisfy the above condition are

$$m = 5 \text{ and } n = 4$$

i.e., the 5th bright fringe of wavelength 5200 Å coincides with the 4th bright fringe of wavelength 6500 Å. The smallest value of y at which this happens is

$$y_{\min} = \frac{m\lambda_m D}{d}$$

Substituting the values of m, λ_m, D and d, we get $y_{\min}$ = 1.0 mm, which is choice (a).

51. Given $\frac{I_1}{I_2} = n$. Therefore, the amplitude ratio is

$$\frac{A_1}{A_2} = \sqrt{n}$$

Now $I_{\max} = (A_1 + A_2)^2$ and $I_{\min} = (A_1 - A_2)^2$

$$\therefore \quad \frac{I_{\max}}{I_{\min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{\left(\frac{A_1}{A_2} + 1\right)^2}{\left(\frac{A_1}{A_2} - 1\right)^2}$$

$$= \frac{\left(\sqrt{n} + 1\right)^2}{\left(\sqrt{n} - 1\right)^2}$$

Hence the correct choice is (d).

52. Given $\frac{I_{\max}}{I_{\min}} = n$. Hence

$$\frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = n$$

or $$\frac{(A_1 + A_2)}{(A_1 - A_2)} = \sqrt{n}$$

which gives $$\frac{A_1}{A_2} = \frac{\sqrt{n} + 1}{\sqrt{n} - 1}$$

$$\therefore \quad \frac{I_1}{I_2} = \left(\frac{\sqrt{n} + 1}{\sqrt{n} - 1}\right)^2$$

Hence the correct choice is (a).

53. The intensity of light emerging from a slit is proportional to its width. Since the amplitude is proportional to the square-root of the intensity, we have

$$\frac{A_1}{A_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{n}{1}} = \sqrt{n}$$

As shown in solution of Q. 51, the correct choice is (c).

54. The correct choice is (b). Refer to the solution of Q. 51.

55. Angular width of a fringe is given by

$$\theta = \frac{\lambda}{d} \quad \text{or} \quad d = \frac{\lambda}{\theta} \qquad (1)$$

Given λ = 628 nm = 628×10^{-9} m and $\theta = 0.1° = \frac{0.1 \times \pi}{180}$ rad. Using these values in Eq. (1), we find that $d = 3.6 \times 10^{-4}$ m. Hence the correct choice is (c).

56. The correct choice is (d). Use

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\phi$$

Put $I_1 = I, I_2 = 4I$ and $\phi = \frac{\pi}{2}$.

57. If t is the thickness of the glass sheet, the fringes are displaced by an amount given by

$$\Delta y = (\mu - 1)\ \frac{tD}{d}$$

In order to bring the adjacent minimum to the centre of the screen (i.e. to bring the first dark fringe the central bright fringe), the fringes must be displaced by half the fringe width, i.e.

$$\Delta y = \frac{\beta}{2} = \frac{1}{2}\ \frac{\lambda D}{d}$$

Hence $(\mu - 1)\ \frac{tD}{d} = \frac{1}{2}\ \frac{\lambda D}{d}$

or $t = \frac{\lambda}{2(\mu - 1)}$, which is choice (a).

58. When a transparent plate of thickness t and refractive index μ is introduced in one of the interfering waves, the path difference at the centre of the screen is

$$\Delta = (\mu - 1)\ \frac{tD}{d}$$

$\therefore$ Phase difference $\phi = \frac{2\pi\Delta}{\lambda}$

$$= \frac{2\pi}{\lambda}(\mu - 1)\ \frac{tD}{d} \qquad (1)$$

Given λ = 5000 Å = 5×10^{-7} m, μ = 1.5, $t = 1.5 \times 10^{-6}$ m, D = 1 m and $d = 5 \times 10^{-4}$ m. Using these values in Eq. (1). we get $\phi = 3\pi$. If I is the intensity of each interfering wave, the resultant intensity at the centre of the screen is

$$I_r = I + I + 2\sqrt{I \times I}\ \cos 3\pi$$
$$= 2I - 2I = 0 \qquad (\because \cos 3\pi = -1)$$

Hence the intensity at the centre is zero, i.e. there is a dark fringe at the centre.

Hence the correct choice is (d).

59. The correct choice is (a). The lateral shift is given by

$$\Delta y = (\mu - 1)\ \frac{tD}{d}$$

60. The position of the nth order maximum is given by

$$y_n = \frac{n\lambda D}{d}$$

For a given point y_n is fixed. Since D and d are also fixed, $n\lambda$ = constant, i.e. $n_1\ \lambda_1 = n_2\ \lambda_2$. Hence $n_2 = n_1 \frac{\lambda_1}{\lambda_2} = \frac{20 \times 480}{600} = 16$, which is choice (a).

61. De Broglie wavelength of electron is

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

If speed v of electron is increased, momentum $p(= mv)$ will increase. Hence wavelength λ will decrease. Now, the angular width of the central maximum of the diffraction pattern is 2θ where θ is given by

$$\sin\theta = \frac{\lambda}{a}$$

where a is the width of the slit. Thus, if λ decreases, θ and hence 2θ will decrease. Therefore, the correct statement is (c).

62. When a sheet of thickness t and refractive index μ is introduced in one of the interfering waves, the distance y_0 through which the fringes shift is given by

$$y_0 = (\mu - 1)t\ \frac{D}{d} \qquad (1)$$

The fringe width β, i.e., the distance between successive maxima (or minima) is given by

$$\beta = \frac{\lambda D}{d}$$

When the distance D between the slits and the screen is doubled, the new fringe width becomes

$$\beta' = 2\ \frac{\lambda D}{d} \qquad (2)$$

It is given that $y_0 = \beta'$. Equating Eqs. (1) and (2) we get

$$2\ \frac{\lambda D}{d} = (\mu - 1)\ t\frac{D}{d}$$
$$\lambda = \frac{1}{2}\ (\mu - 1)t$$

which is choice (b)

63. The angular width of a fringe is given by

$$\theta = \frac{\lambda}{d}$$

In air: $\theta_a = \frac{\lambda_a}{d}$

In water: $\theta_w = \dfrac{\lambda_w}{d}$

$$\therefore \quad \frac{\theta_w}{\theta_a} = \frac{\lambda_w}{\lambda_a} \quad \text{or} \quad \theta_w = \frac{\lambda_w\,\theta_a}{\lambda_a} \qquad (1)$$

Now, refractive index is defined as

$$\mu_w = \frac{\text{speed of light in air}}{\text{speed of light in water}}$$

$$= \frac{v_a}{v_w} = \frac{\nu\lambda_a}{\nu\lambda_w} = \frac{\lambda_a}{\lambda_w} \qquad (2)$$

where ν is the frequency of light which remains unchanged.

Using Eq. (2) in Eq. (1) we have

$$\theta_w = \frac{\theta_a}{\mu_w} = \frac{0.2°}{4/3} = 0.15°$$

So the correct choice is (a).

64. Refer to Fig. 17.10. When the incident beam falls normally on the slits S_1 and S_2, the path difference at the central point P_0 of the screen is zero. Hence we have the central maximum at P_0.

Let the minima appear along directions θ with respect to the incident direction. Coherent waves from S_1 and S_2 along this direction are brought to a focus at P. It is clear that the path difference between the waves from S_1 and S_2 on reaching P is

$$\Delta = d \sin\theta$$

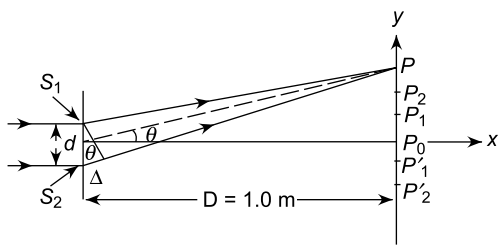


Fig. 17.10

The interference minima will appear on the screen if

$$\Delta = \left(n + \frac{1}{2}\right)\lambda$$

$$\text{or} \quad d\sin\theta = \left(n + \frac{1}{2}\right)\lambda \;;\; n = 0, \pm 1, \pm 2, \ldots$$

Thus the directions of minima are given by

$$\sin\theta = \left(n + \frac{1}{2}\right) \times \left(\frac{\lambda}{d}\right)$$

Given $d = 1.0$ mm and $\lambda = 0.5$ mm. Therefore

$$\sin\theta = \left(n + \frac{1}{2}\right) \times \left(\frac{0.5}{1.0}\right) = \frac{1}{2}\left(n + \frac{1}{2}\right)$$

The allowed values of m are those integers for which $\sin\theta$ is not more than + 1 or less than – 1. These values are $n = 1, 0, -1$ and -2. Hence four minima will be observed. The correct choice is (b).

65. If an interference experiment is performed using two wavelengths close to each other, two interference patterns corresponding to the two wavelengths are obtained on the screen. The fringe system remains distinct upto a point on the screen where the nth order maximum of one wavelength, say $\lambda_1 = 5890$ Å falls on the nth order minimum of the other wavelength $\lambda_2 = 5895$ Å. Thus, interference pattern can be seen upto a distance y_n from the centre of the screen if

$$y_n = \frac{n\lambda_1 D}{d}\;;\; (n\text{th maximum}) \qquad (1)$$

$$= \left(n - \frac{1}{2}\right)\frac{\lambda_2 D}{d}\;;\; (n\text{th minimum}) \qquad (2)$$

$$\text{or} \qquad n\lambda_1 = \left(n - \frac{1}{2}\right)\lambda_2 \text{ or } 2n\lambda_1 = (2n-1)\,\lambda_2$$

which gives

$$n = \frac{\lambda_2}{2(\lambda_2 - \lambda_1)}, \text{ which is choice (c).}$$

66. Refer to Fig. 17.11. To reach point P, wave 1 has to travel a path ($SS_2 + S_2P$) while wave 2 has to travel a path ($SS_1 + S_1P$). Therefore, when the waves arrive at P, the path difference is

$$\Delta = (SS_2 + S_2P) - (SS_1 + S_1P) \qquad (1)$$

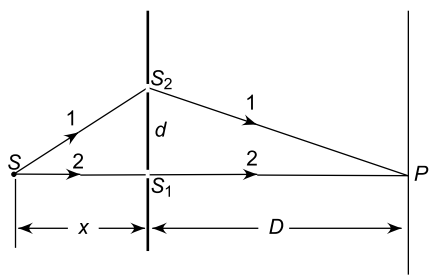


Fig. 17.11

Now, in triangle SS_2S_1, we have

$$SS_2 = (x^2 + d^2)^{1/2} = x\left(1 + \frac{d^2}{x^2}\right)^{1/2}$$

$$= x\left(1 + \frac{d^2}{2x^2}\right) \qquad (\because\ d << x)$$

Similarly, $S_2P = (D^2 + d^2)^{1/2} = D\left(1 + \dfrac{d^2}{2D^2}\right)$

$$(\because\ d << D)$$

Also $(SS_1 + S_1P) = x + D$. Using these in Eq. (1), we have

$$\Delta = x\left(1+\frac{d^2}{2x^2}\right)+D\left(1+\frac{d^2}{2D^2}\right)-(x+D)$$

$$= x+\frac{d^2}{2x}+D+\frac{d^2}{2D}-x-D$$

or $$\Delta = \frac{d^2}{2}\left(\frac{1}{x}+\frac{1}{D}\right)$$

In order to have a dark fringe at P, $\Delta = \frac{\lambda}{2}$. Hence

$$\frac{\lambda}{2} = \frac{d^2}{2}\left(\frac{1}{x}+\frac{1}{D}\right) \quad (1)$$

or $$d = \left[\frac{\lambda\, xD}{(x+D)}\right]^{1/2} \quad (2)$$

Putting $x = \frac{D}{2}$ in Eq. (2), we find that the correct choice is (a).

67. The intensity at the mid-point of the screen is maximum. In the first experiment,

$$I_{max} = 4I$$

It the sources of incoherent, then the intensity at the mid-point = $I + I = 2I$. So the correct choice is (b).

68. For first minimum, (a = width of the slit)

$$a \sin\theta_1 = \lambda$$

$$\Rightarrow \quad \sin\theta_1 = \frac{\lambda}{a} \quad \Rightarrow \quad \theta_1 = \frac{\lambda}{a} \qquad (\because \quad \lambda << a)$$

For first maximum,

$$a\sin\theta_2 = \frac{3\lambda}{2} \quad \Rightarrow \quad \theta_2 = \frac{3\lambda}{2a}$$

Angular separation between them is

$$\Delta\theta = \theta_2 - \theta_1 = \frac{3\lambda}{2a}-\frac{\lambda}{a} = \frac{\lambda}{2a}$$

$\therefore$ Linear separation is

$$\Delta x = D\Delta\theta = \frac{D\lambda}{2a}$$

$$= \frac{0.5\times(600\times10^{-9})}{2\times(0.5\times10^{-3})} = 0.3 \text{ mm}$$

So the correct choice is (c).

69. $\Delta\theta = \frac{2\lambda}{a}$

$$\Delta\theta' = \frac{2\lambda'}{a}$$

Given $\Delta\theta' = \Delta\theta + 25\%$ of $\Delta\theta$

$$= \Delta\theta + \frac{25}{100}\Delta\theta = 1.25\,\Delta\theta$$

$\therefore$ $$\frac{\lambda'}{\lambda} = \frac{\Delta\theta'}{\Delta\theta} = 1.25$$

$\Rightarrow$ $$\lambda' = 1.25\,\lambda = 1.25\times 500 \text{ nm} = 625 \text{ nm}.$$

So the correct choice is (d).

70. The path difference between the waves emerging from the edges of the slit on reaching the position of the first maximum is

$$\Delta x = \frac{3\lambda}{2}$$

Now phase difference $\Delta\phi$ is given by

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\times\frac{3\lambda}{2} = 3\pi$$

So the correct choice is (a).

71. The angular width of the central maximum is (since $a << \lambda$)

$$\Delta\theta = \frac{2\lambda}{a}$$

Since λ for yellow light is greater than that for blue light, $\Delta\theta$ increases. Hence the central maximum becomes broader. So the correct choice is (b).

72. Refer to Example 7 on page 17.6. When a transparent film is introduced in the path of one of the interfering beams, the entire fringe pattern shifts by an amount $\Delta y = (\mu - 1)\, t$. Since the path difference must change by λ for one bright fringe to be replaced by its neighbouring fringe, we have $\Delta y = \lambda \quad \Rightarrow \quad (\mu - 1)\, t = \lambda$ which gives

$$t_{min} = \frac{\lambda}{\mu-1} = \frac{600\times10^{-9}}{(1.6-1)} = 10^{-6} \text{ m}$$

So the correct choice is (a).

73. In this case, $\Delta y = \frac{\lambda}{2}$ or

$$(\mu-1)\, t = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{2(\mu-1)} = \frac{\lambda}{2(1.5-1)} = \lambda$$

So the correct choice is (b).

74. Refer to Example 8 on page 17.7. The correct choice is (b).

75. Refer to Example 6 on page 17.6.

$$y_1 = \frac{n\lambda_1\Delta}{d} \quad (1)$$

$$y_2 = \frac{m\lambda_2\Delta}{d} \quad (2)$$

The waves will produce maximum brightness at the screen where $y_1 = y_2$, *i.e.*,

$$\frac{n\lambda_1 D}{d} = \frac{m\lambda_2 D}{d}$$

$$\Rightarrow \quad \frac{n}{m} = \frac{\lambda_2}{\lambda_1} = \frac{450\text{ nm}}{750\text{ nm}} = \frac{3}{5}$$

The minimum integral values of n and m which satisfy this condition are $n_1 = 3$ and $m_1 = 5$ and the next values are $n_2 = 6$ and $m_2 = 10$. So, we get the first region of maximum brightness when the 3rd bright fringe of λ_1 falls on the 5th bright fringe of λ_2.

$$(\Delta y)_1 = \frac{3\lambda_1 D}{d} = \frac{5\lambda_2 D}{d}$$

$$= \frac{3\times(750\times10^{-9})\times1.0}{2\times10^{-3}} = 1.125\text{ mm}$$

We get the next region of maximum brightness when the 6th bright fringe of λ_1 falls on the 10th bright fringe of λ_2

$$(\Delta y)_2 = \frac{6\lambda_1 D}{d} = \frac{10\lambda_2 D}{d}$$

$$= \frac{6\times(750\times10^{-9})\times1.0}{2\times10^{-3}} = 2.25\text{ mm}$$

$$(\Delta y)_{\min} = (\Delta y)_2 - (\Delta y)_1$$

$$= 2.25 - 1.125$$

$$= 1.125\text{ mm, which is choice (a).}$$

76. In this case, for dark fringes,

$$y_1 = \left(n - \frac{1}{2}\right)\frac{\lambda_1 D}{d}$$

$$y_2 = \left(m - \frac{1}{2}\right)\frac{\lambda_2 D}{d}$$

The waves will produce complete darkness at a point on the screen where $y_1 = y_2$, *i.e.*,

$$\left(n - \frac{1}{2}\right)\frac{\lambda_1 D}{d} = \left(m - \frac{1}{2}\right)\frac{\lambda_2 D}{d}$$

$$\Rightarrow \quad \frac{\left(n - \frac{1}{2}\right)}{\left(m - \frac{1}{2}\right)} = \frac{\lambda_2}{\lambda_1} = \frac{450\text{ nm}}{750\text{ nm}} = \frac{3}{5}$$

$$\Rightarrow \quad n = \frac{6m + 2}{10}$$

The minimum integral values of n and m which satisfy this condition are $m_1 = 3$ and $n_1 = 2$. The next values are $m_2 = 8$ and $n_2 = 5$. So we get the first region of complete darkness when 2nd dark fringe of λ_1 falls on the 3rd dark fringe of λ_2,

$$(\Delta y)_1 = \left(2 - \frac{1}{2}\right)\frac{\lambda_1 D}{d}$$

The next region of complete darkness occurs when the 5th dark fringe of λ_1 falls on 8 dark fringe of λ_2,

$$(\Delta y)_2 = \left(5 - \frac{1}{2}\right)\frac{\lambda_1 D}{d}$$

$$\therefore \quad (\Delta y)_{\min} = \left(5 - \frac{1}{2}\right)\frac{\lambda_1 D}{d} - \left(2 - \frac{1}{2}\right)\frac{\lambda_1 D}{d}$$

$$= 3\,\frac{\lambda_1 D}{d}$$

$$= \frac{3\times(750\times10^{-9})\times1.0}{2\times10^{-3}}$$

$$= 1.125\text{ mm}$$

So the correct choice is (a).

77. In Young's double slit experiment,

$$I = 4I_0\cos^2(\phi/2)$$

where ϕ is the phase difference between the interfering beams.

$$I_{\max} = 4I_0$$

Given $$I = \frac{3}{4}I_{\max}$$

or $$4I_0\cos^2\left(\frac{\phi}{2}\right) = \frac{3}{4}I_{\max}$$

$$\Rightarrow \quad \cos^2\left(\frac{\phi}{2}\right) = \frac{3}{4}$$

$$\Rightarrow \quad \cos\left(\frac{\phi}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{6} \quad \text{or } \phi = \frac{\pi}{3}$$

Now refer to Fig. 17.12.

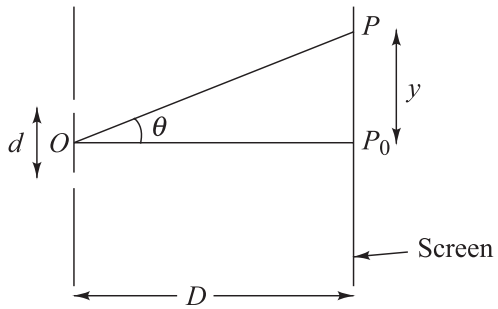


Fig. 17.12

If θ is the angular separation between P and P_0,

$$\tan\theta = \frac{y}{D}$$

If β is the fringe width between two consecutive maxima, than $\beta = 2\pi$. Hence for point P

$$\frac{y}{\beta} = \frac{\pi/3}{2\pi} = \frac{1}{6}$$

or $$\frac{yd}{D\lambda} = \frac{1}{6}$$

or $$\tan\theta \times \frac{d}{\lambda} = \frac{1}{6}$$

or $$\tan\theta = \frac{\lambda}{6d} \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{\lambda}{6d}\right)$$

So the correct choice is (c).

78. As shown in Fig. 17.13, the intensity of the reflected beam at face AB is

$$I_1 = I \times \frac{25}{100} = \frac{I}{4}$$

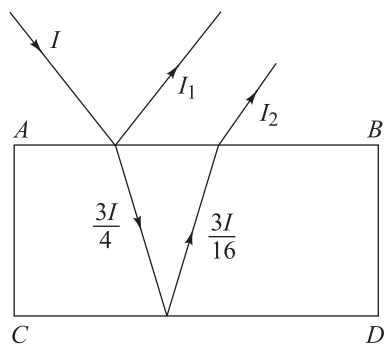


Fig. 17.13

The remaining intensity $\frac{3I}{4}$ falls on face CD which reflects 25% of this intensity incident on it which is equal to $\frac{3I}{4} \times \frac{25}{100} = \frac{3I}{16}$. This intensity falls on face AB which transmits 75% of it. Hence the intensity of I_2 is

$$I_2 = \frac{3I}{16} \times \frac{75}{100} = \frac{9I}{64}$$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$= \frac{I}{4} + \frac{9I}{64} + 2\sqrt{\frac{I}{4} \times \frac{9I}{64}} = \frac{49}{64}$$

and $$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

$$= \frac{I}{4} + \frac{9I}{64} - 2\sqrt{\frac{I}{4} \times \frac{9I}{64}} = \frac{1}{64}$$

$\therefore$ $\frac{I_{max}}{I_{min}} = 49$, which is choice (a).

79. Using the arguments given in Solution 78 above, we have

$$I_3 = \frac{9I}{16} \quad (1)$$

and $$I_4 = \frac{9I}{254} \quad (2)$$

$$I_{max} = I_3 + I_4 + 2\sqrt{I_3 I_4} \quad (3)$$

$$I_{min} = I_3 + I_4 - 2\sqrt{I_3 I_4} \quad (4)$$

Using (1) and (2) in (3) and (4) we get

$$I_{max} = \left(\frac{9 \times 25}{254}\right) I$$

and $$I_{min} = \left(\frac{9 \times 9}{254}\right) I$$

$\therefore$ $\frac{I_{max}}{I_{min}} = \frac{25}{9}$, which is choice (c).

80. In Young's double slit experiment, the angular separation between 10 maxima is

$$\theta_1 = \frac{10\lambda}{d}$$

In single slit diffraction experiment, the angular width of the central maximum is

$$\theta_2 = \frac{2\lambda}{a}, \qquad (a = \text{width of each slit})$$

Given $\theta_1 = \theta_2$, which requires

$$\frac{10\lambda}{d} = \frac{2\lambda}{a}$$

$$\Rightarrow \quad a = \frac{2d}{10} = \frac{2 \times 2\text{ mm}}{10} = 0.4\text{ mm}$$

which is choice (c).

SECTION 2

Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage.

Passage I

In a modified Young's double slit experiment, a monochromatic and parallel beam of light of wavelength 6000 Å and intensity $\frac{10}{\pi}$ Wm^{-2} is incident normally on two circular apertures A and B of radii 0.001 m and 0.002 m respectively. A perfectly transparent film of thickness 2000 Å and refractive index 1.5 for the wavelength 6000 Å is placed in front of aperture A (Fig. 17.14).

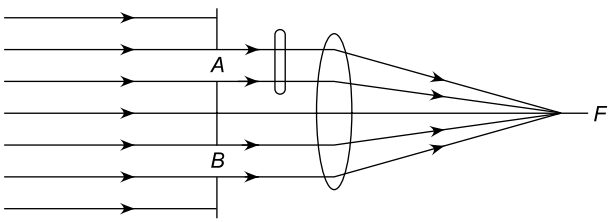


Fig. 17.14

1. The ratio of the powers received at aperture A to that at aperture B is
   (a) 1 : 2 (b) 1 : 4
   (c) 1 : 8 (d) 1 : 16
2. The phase difference between the interfering waves at point F is
   (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
   (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$
3. If 10% of the power received by each aperture goes in the original direction, the resultant power at point F will be
   (a) 5 μW (b) 6 μW
   (c) 7 μW (d) 8 μW

Solutions

1. Intensity of the beam $(I) = \frac{10}{\pi}$ Wm^{-2}

   Power received at aperture $A = I \times$ cross-sectional area of A

   $$= \frac{10}{\pi} \times \pi \times (0.001)^2 = 10^{-5} \text{ W}$$

   Power received at aperture $B = \frac{10}{\pi} \times \pi \times (0.002)^2$

   $$= 4 \times 10^{-5} \text{ W}$$

   So the correct choice is (b).
2. The phase difference at F is

   $$\delta = (\mu - 1) \times t \times \frac{2\pi}{\lambda}$$

   $$= \frac{(1.5 - 1) \times (2000 \times 10^{-8}) \times 2\pi}{(6000 \times 10^{-8})} = \frac{\pi}{3} \text{ rad}$$

   The correct choice is (b).
3. Since 10% the power received at each aperture goes in the original direction, the power at point F due to the two apertures respectively is

   $P_A = 10\%$ of 10^{-5} W $= 10^{-6}$ W

   $P_B = 10\%$ of 4×10^{-5} W $= 4 \times 10^{-6}$ W

   Now, intensity (and hence power) is proportional the square of the amplitude. If A_1 and A_2 are the amplitudes at F due to the two sources, we have $P_A = kA^2_1$ and $P_B = kA^2_2$, where k is the proportionality constant. Thus

   $$A_1 = \sqrt{\frac{P_A}{k}} \text{ and } A_2 = \sqrt{\frac{P_B}{k}}$$

   Resulting amplitude

   $$A = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos \delta}$$

   Substituting the values of A_1, A_2 and δ, we get

   $$A = \sqrt{\frac{7 \times 10^{-6}}{k}}$$

   $\therefore$ Resultant power at $F = kA^2$

   $$= k \times \frac{7 \times 10^{-6}}{k} = 7 \times 10^{-6} \text{ W}$$

   Hence the correct choices is (c).

Questions 4 to 6 are based on the following passage.

Passage II

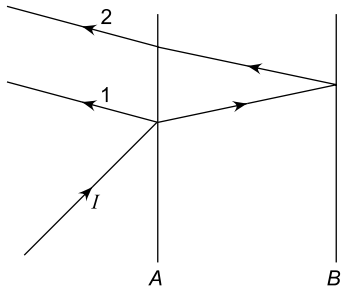


Fig. 17.15

A narrow monochromatic beam of light of intensity I is incident on a glass plate A as shown in Fig. 17.15. Another identical glass plate B is kept close to A and parallel to it. Each plate reflects 25% of the light intensity incident on it and transmits the remaining. Interference pattern is formed by beams 1 and 2 obtained after reflection at each plate.

4. The intensity of beam 2 is
   (a) $\frac{3I}{16}$ (b) $\frac{3I}{32}$
   (c) $\frac{9I}{32}$ (d) $\frac{9I}{64}$
5. The ratio of the intensities of beams 1 and 2 is
   (a) $\frac{16}{9}$ (b) $\frac{4}{3}$
   (c) $\frac{25}{16}$ (d) $\frac{5}{4}$
6. The ratio of the maximum and minimum intensities in the interference pattern is
   (a) 16 : 1 (b) 25 : 1
   (c) 36 : 1 (d) 49 : 1

Solutions

4. A beam of light of intensity I is incident on plate A. Since the plate reflects 25% of I, the intensity of the reflected beam 1 (see Fig.17.16) is

$$I_1 = I \times \frac{25}{100} = \frac{I}{4}$$

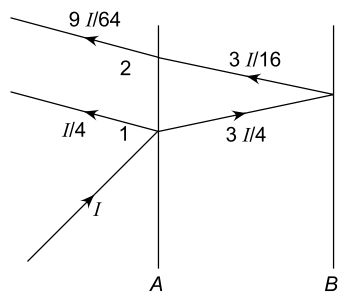


Fig. 17.16

The remaining intensity $3I/4$ falls on plate B which reflects 25% of the intensity incident on it. Hence intensity of beam reflected from B is

$$\frac{3I}{4} \times \frac{25}{100} = \frac{3I}{16}$$

A beam of intensity $3I/16$ falls on plate A which transmits 75% of this intensity. Hence the intensity of beam 2 is

$$I_2 = \frac{3I}{16} \times \frac{75}{100} = \frac{9I}{64}$$

So the correct choice is (d).

5. $\frac{I_1}{I_2} = \frac{I/4}{9I/64} = \frac{16}{9}$ which is choice (a).

6. The ratio of amplitudes is $\frac{a_1}{a_2} = \sqrt{\frac{16}{9}} = \frac{4}{3}$ Thus . a_1 = 4 units and a_2 = 3 units.

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{4+3}{4-3}\right)^3 = 49$$

Thus the correct choice is (d).

Questions 7 to 9 are based on the following passage.

Passage III

A monochromatic light of wavelength 5000 Å is incident on two slits separated by a distance of 5×10^{-4} m. The interference pattern is observed on a screen placed at a distance of 1 m from the slits. A glass plate of thickness 1.5×10^{-6} m and refractive index 1.5 is introduced between one of the slits and the screen.

7. Due to the introduction of the glass plate, the phase difference between the interfering waves at the centre of the screen is equal to
   (a) $\pi/2$ (b) π
   (c) 2π (d) 3π
8. If I_0 is the intensity at the centre of the screen before the plate is introduced, the intensity at the centre after the plate is introduced
   (a) remains equal to I_0 (b) becomes less than I_0
   (c) becomes greater than I_0 (d) becomes equal zero.
9. The lateral shift of the central maximum is
   (a) 2 mm (b) 3 mm
   (c) 4 mm (d) 5 mm

Solutions

7. When a transparent plate of thickness t and refractive index μ is introduced in one of the interfering waves, the path difference at the centre of the screen is

$$\Delta = (\mu - 1)\ \frac{tD}{d}$$

$\therefore$ Phase difference $\phi = \dfrac{2\pi\Delta}{\lambda}$

$$= \frac{2\pi}{\lambda}\ (\mu - 1)\ \frac{t\,D}{d} \qquad (1)$$

Given $\lambda = 5000$ Å $= 5 \times 10^{-7}$ m, $\mu = 1.5$, $t = 1.5 \times 10^{-6}$ m, $D = 1$ m and $d = 5 \times 10^{-4}$ m. Using these values in Eq. (1), we get $\phi = 3\pi$. So the correct choice is (d).

8. If I is the intensity of each interfering wave, the resultant intensity at the centre of the screen is

$$I_r = I + I + 2\sqrt{I \times I}\ \cos 3\pi$$
$$= 2I - 2I = 0 \qquad (\because \cos 3\pi = -1)$$

Hence the intensity at the centre is zero, i.e. there is a dark fringe at the centre.The correct choice is (d).

9. The lateral shift of the central maximum is given by

$$y_0 = (\mu - 1)\ \frac{t\,D}{d}$$

Substituting the values of μ, t, D and d, we get $y_0 = 5 \times 10^{-3}$ m = 5 mm,which is choice (d).

Questions 10 to 13 are based on the following passage.

Passage IV

A glass plate of refractive index $\mu_3 = 1.5$ is coated with a thin layer of thickness t and refractive index $\mu_2 = 1.8$. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere.

10. The two reflected waves interfere constructively if (n is an integer)

(a) $t = \dfrac{n\lambda}{2\mu_2}$ (b) $t = \dfrac{n\lambda}{2(\mu_2 - \mu_3)}$

(c) $t = \dfrac{\left(n + \frac{1}{2}\right)\lambda}{2\mu_2}$ (d) $t = \dfrac{\left(n + \frac{1}{2}\right)\lambda}{2\mu_3}$

11. If $\lambda = 648$ nm, the least value of t for which the waves interface constructively is

(a) 90 nm (b) 180 nm
(c) 108 nm (d) 216 nm

12. The two reflected waves interfere destructively if

(a) $t = \dfrac{n\lambda}{2\mu_2}$ (b) $t = \dfrac{n\lambda}{2\mu_3}$

(c) $t = \dfrac{\left(n + \frac{1}{2}\right)\lambda}{2\mu_2}$ (d) $t = \dfrac{\left(n + \frac{1}{2}\right)\lambda}{2\mu_3}$

13. If $\lambda = 648$ nm, the least value of t for which the waves interfere destructively is

(a) 90 nm (b) 180 nm
(c) 108 nm (d) 216 nm

Solutions

10. Refer to Fig. 17.17. A ray of light travelling in air ($\mu_1 = 1$) falls normally on a thin layer ($\mu_2 = 1.8$) of thickness t. It is partly reflected at point P as wave 1 and partly refracted as wave 2. Wave 2 on meeting the surface of the glass plate ($\mu_3 = 1.5$) is reflected at point Q and travels along QP.

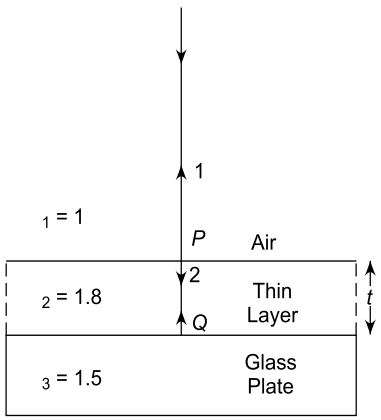


Fig. 17.17

Waves 1 and 2 meet at point P where they interfere. We know that when a wave is travelling in a rarer medium and gets reflected at the boundary of a denser medium, it undergoes a phase change of π or a path change of $\lambda/2$. Thus wave 1 has an optical path of $\Delta_1 = \lambda/2$. Wave 2 travelling from P to Q in the layer of refractive index 1.8 gets reflected at Q from the boundary of glass of refractive index 1.5. Thus wave 2 travelling in a denser medium is reflected from the boundary of a rarer medium undergoes no phase change due to reflection. Therefore,

Optical path for wave 2 from P to Q and from Q to P in the layer is

Δ_2 = refractive index of layer $\times$ 2(PQ)
$= \mu_2 \times 2t = 2\mu_2 t$

$\therefore$ Optical path difference between waves 1 and 2 at point p is

$$\Delta = \Delta_2 - \Delta_1 = 2\mu_2 t - \frac{\lambda}{2}$$

Now, for constructive interference, $\Delta = n\lambda$; $n = 0, 1, 2, \ldots$

or $$2\mu_2 t - \frac{\lambda}{2} = n\lambda \text{ or } 2\mu_2 t = \left(n + \frac{1}{2}\right)\lambda$$

or $$t = \frac{\left(n + \frac{1}{2}\right)\lambda}{2\mu_2}$$

So the correct choice is (c).

11. The minimum value of t corresponds to $n = 0$. Hence

$$t_{\min} = \frac{\lambda}{4\mu_2} = \frac{648 \text{ nm}}{4 \times 1.8} = 90 \text{ nm}.$$

So the correct choice is (a)

12. For destructive interference $\Delta = \left(n - \frac{1}{2}\right)\lambda$. Hence

$$2\mu_2 t - \frac{\lambda}{2} = \left(n - \frac{1}{2}\right)\lambda$$

which gives $t = \frac{n\lambda}{2\mu_2}$, which is choice (a).

13. The minimum value of t corresponds to $n = 1$. Hence

$$t_{\min} = \frac{\lambda}{2\mu_2} = \frac{648\text{nm}}{2 \times 1.8} = 180 \text{ nm}$$

Thus the correct choice is (b).

SECTION 3

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

1. Statement-1

Red light travels faster in glass than green light.

Statement-2

The refractive index of glass is less for red light than for green light.

2. Statement-1

In Young's double slit experiment, if the width of the source slit is increased, the fringe pattern becomes indistinct.

Statement-2

The angular width of interference maxima increases if the width of the source slit is increased.

3. Statement-1

In a single slit diffraction experiment, if the width of the slit is increased, the diffraction maxima become sharper and brighter.

Statement-2

The angular width the diffraction maxima is inversely proportional to the width of the slit.

4. Statement-1

When light travels from a rarer to a denser medium, its speed decreases.

Statement-2

Energy carried by the refracted light is reduced.

5. Statement-1

When a light wave travels from one medium to another, its frequency remains unchanged.

Statement-2

The speed of the wave undergoes a change.

6. Statement-1

When a light wave is reflected from a mirror, it undergoes a phase change of π.

Statement-2

The direction of the propagation of light is changed due to reflection.



Solutions

1. The correct choice is (a). Refractive index of a medium is defined as

$$\mu = \frac{\text{speed of light in vacuum}}{\text{speed of light in the medium}}$$

The refractive index of glass is less for light of longer wavelength. The wavelength of red light is more than that of green light. Hence $\mu_R < \mu_G$ which implies that the speed of red light is more than that of green light in glass.

2. The correct choice is (c). If the source slit is wide, the interference pattern becomes indistinct because the interference patterns due to various parts of the source slit overlap. Consequently, the minima will not be totally dark and the fringe pattern becomes indistinct.

3. The correct choice is (a).
4. The correct choice is (b). The energy of a wave is determined by the square of its amplitude; it does not depend on the speed of the wave.
5. The correct choice is (c). The frequency of a wave does not depend on its speed or wavelength; it depends on the frequency of the source which produces that wave.
6. The correct choice is (c). The phase change is due to the reversal of amplitude of the wave on reflection from the mirror.

4 SECTION Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. Lights of wavelengths $\lambda_1 = 4000$ Å and $\lambda_2 = 5000$ Å are used in an optical instrument. The ratio of the resolving powers of the instruments for wavelengths λ_1 and λ_2 is
   (a) 16 : 25 (b) 9 : 1
   (c) 4 : 5 (d) 5 : 4 **[2002]**
2. An astronomical telescope has a large aperture to
   (a) reduce spherical aberration
   (b) have high resolution
   (c) increase span of observation
   (d) have low dispersion **[2002]**
3. To demonstrate the phenomenon of interference, we require two sources which emit radiations of
   (a) nearly the same frequency
   (b) the same frequency
   (c) different wavelength
   (d) the same frequency and having a definite phase relationship. **[2003]**
4. The angle of incidence at which reflected light is polarised for reflection from air to glass of refractive index n is
   (a) $\sin^{-1}(n)$ (b) $\sin^{-1}\left(\frac{1}{n}\right)$
   (c) $\tan^{-1}\left(\frac{1}{n}\right)$ (d) $\tan^{-1}(n)$ **[2004]**
5. The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double slit experiment is
   (a) infinite (b) 5
   (c) 3 (d) zero **[2004]**
6. In Young's double slit experiment using a monochromatic light, the shape of interference fringes formed on the screen is
   (a) hyperbola (b) circle
   (c) straight line (d) parabola **[2005]**
7. When an unpolarised light of intensity I_0 is incident normally on a polarising sheet, the intensity of light absorbed by the sheet is
   (a) $\frac{I_0}{2}$ (b) $\frac{I_0}{4}$
   (c) zero (d) I_0 **[2005]**
8. In a Young's double slit experiment the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of the light used) is I. If I_0 denotes the maximum intensity, $\frac{I}{I_0}$ is equal to
   (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$
   (c) $\frac{1}{2}$ (d) $\frac{3}{4}$ **[2007]**
9. A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of the known wavelength coincides with the 4th bright fringe of the unknown light. The wavelength of the unknown light is
   (a) 442.5 nm (b) 776.8 nm
   (c) 393.4 nm (d) 885.0 nm **[2009]**

Questions 10, 11 and 12 are based on following passage.
An initial parallel cylindrical beam travels in a medium of refractive index $\mu_1 = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

10. The speed of light in the medium is
    (a) directly proportional to the intensity I
    (b) maximum on the axis of the beam
    (c) minimum on the axis of the beam
    (d) the same everywhere in the beam **[2010]**
11. As the beam enters the medium, it will
    (a) diverge near the axis and coverage near the periphery
    (b) travel as a cylindrical beam
    (c) diverge
    (d) converge **[2010]**
12. The initial shape of the wavefront of the beam is
    (a) convex near the axis and concave near the periphery
    (b) planar
    (c) convex
    (d) concave **[2010]**
13. **Direction:** The question has a paragraph followed by two statements, Statement-1 and Statement-2. Of the given four alternatives after the statements, choose the one that describes the statements.

    A thin air films is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

    Statement-1: When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.

    Statement-2 : The centre of the interference pattern in dark
    (a) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
    (b) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of statement-1
    (c) Statement-1 is false, Statement-2 is true.
    (d) Statement-1 is true, Statement-2 is false.

    [2011]
14. In young's double slit experiment, one of the slit is wider than other, so that amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by
    (a) $\frac{I_m}{9}(4+5\cos\phi)$ (b) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$
    (c) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$ (d) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$

    [2012]
15. Young double slit experiment is carried out by using green, red and blue light, one colour at a time. The fringe widths recorded are β_G, β_R and β_B respectively. Then,
    (a) $\beta_G > \beta_B > \beta_R$ (b) $\beta_B > \beta_G > \beta_R$
    (c) $\beta_R > \beta_B > \beta_G$ (d) $\beta_R > \beta_G > \beta_B$ **[2012]**
16. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is 3×10^8 ms^{-1}. The final momentum (in kg ms^{-1}) of the object is
    (a) 0.3×10^{-17} (b) 1.0×10^{-17}
    (a) 3.0×10^{-17} (b) 9.0×10^{-17} **[2013]**
17. In the Young's double slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is:
    (a) $(2n+1)\frac{\lambda}{2}$ (b) $(2n+1)\frac{\lambda}{4}$
    (c) $(2n+1)\frac{\lambda}{8}$ (d) $(2n+1)\frac{\lambda}{16}$ **[2013]**
18. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is:
    (a) 1m (b) 2m
    (c) 3 m (d) 6m **[2013]**
19. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45º relative to that of A. The intensity of the emergent light is:
    (a) $I_0/2$ (b) $I_0/4$
    (c) $I_0/2$ (d) $I_0/2$ **[2013]**

20. In Young's double slit experiment, the intensity at a point P on the screen is half the maximum intensity in the interference pattern. If the wavelength of ligh used is λ and d is the distance between the slits, the angular separation between point P and the centre of the screen is

(a) $\sin^{-1}\left(\frac{\lambda}{d}\right)$ (b) $\sin^{-1}\left(\frac{\lambda}{2d}\right)$

(c) $\sin^{-1}\left(\frac{\lambda}{3d}\right)$ (d) $\sin^{-1}\left(\frac{\lambda}{4d}\right)$ **[2014]**

Answers

1. (d)	2. (b)	3. (d)	4. (d)
5. (b)	6. (d)	7. (a)	8. (d)
9. (a)	10. (c)	11. (d)	12. (b)
13. (a)	14. (d)	15. (d)	16. (b)
17. (b)	18. (c)	19. (b)	20. (d)

Solutions

1. Resolving power is inversely proportional to wavelength. Hence

$$\frac{(\text{R.P})_1}{(\text{R.P})_2} = \frac{\lambda_2}{\lambda_1} = \frac{5000}{4000} = \frac{5}{4}$$

2. R.P. of telescope = $\frac{D}{1.22\lambda}$, where D = aperture. Hence the correct choice is (2).
3. The correct choice is (4). Only coherent sources produce sustained interference.
4. From Brewster's law

$$\tan i_p = n$$

$$\Rightarrow \quad i_p = \tan^{-1}(n)$$

5. The angular separation θ of the nth maximum from the central maximum is given by

$$\sin\theta = \frac{n\lambda}{d}$$

Given $d = 2\lambda$. Therefore

$$\sin\theta = \frac{n}{2}$$

$$\Rightarrow \quad n = 2\sin\theta$$

$$\therefore \quad n_{\max} = 2 \qquad (\text{maximum value of } \sin\theta = 1)$$

Thus there are two maxima on either side of the central maximum. Hence, maximum number of maxima = central maximum ($n = 0$) + 2 maxima ($n = \pm 1, \pm 2$) = 5.

6. The shape of fringes is general parabolic. In a small region of the screen near the centre, the fringes are straight.
7. The correct choice is (1). Half of the incident intensity is absorbed by the sheet and the remaining half is transmitted.
8. Let I be the intensity of each beam. Phase difference $\phi = \frac{2\pi}{\lambda} \times$ path difference $= \frac{2\pi}{\lambda} \times \frac{\lambda}{6} = \frac{\pi}{3}$ rad. The resultant intensity is given by

$$I_r = I + I + 2\sqrt{1 \times 1}\cos\phi$$

$$= 2I(1 + \cos\phi)$$

$I_r = I_{\max} = I_0$ if $\cos\phi = +1$. Hence $I_0 = 4I$. When $\phi = \frac{\pi}{3} = 60°$, the resultant intensity is

$$I_r = 2I(1 + \cos 60°) = 2I(1 + 0.5) = 3I$$

$$\therefore \quad \frac{I_r}{I_0} = \frac{3I}{4I} = \frac{3}{4}$$

9. For bright fringes $y_n = \frac{n\lambda D}{d}$

Given y_3 (for $\lambda_1 = 590$ nm) = y_4 for λ_2

$$\Rightarrow \quad \frac{3\lambda_1 D}{d} = \frac{4\lambda_2 D}{d} \Rightarrow \lambda_2 = \frac{3\lambda_1}{4}$$

$$\Rightarrow \quad \lambda_2 = \frac{3 \times 590}{4} = 442.5 \text{ nm}$$

10. $v = \frac{c}{\mu_1} = \frac{c}{\mu_0 + \mu_2 I}$

Since the intensity I of the beam is decreasing with increasing radius, I will be maximum on the axis of the beam. Hence speed of light (v) is minimum on the axis of the beam, which is choice (c).

11. Since the intensity I is decreasing with increasing radius, the refractive index μ_1 of the medium is maximum on the axis of the beam decreasing to a minimum value at the periphery. Hence the beam will converge, which is choice (d).
12. Since the incident beam is parallel, the wavefronts are planar. So the correct choice is (b).
13. When light travelling in a rarer medium gets reflected from the boundary of a denser medium, the reflected wave undergoes a phase change of π. Hence the centre of the interference pattern is dark due to the destructive interference.

14. $I = I_0 + 4I_0 + 2\sqrt{I_0 \times 4I_0}\cos\phi$

$I = I_0 + 4I_0 + 4I_0 \times \cos\phi$

$I_m = I_0 + 4I_0 + 4I_0 = 9I_0$

$\therefore\ I = \frac{I_m}{9}(5 + 4\cos\phi)$

$= \frac{I_m}{9}\left[5 + 4\left(2\cos^2\frac{\phi}{2} - 1\right)\right]$

$= \frac{I_m}{9}\left[1 + 8\cos^2\frac{\phi}{2}\right]$

15. Since $\lambda_R > \lambda_G > \lambda_B$ and $\beta = \frac{\lambda D}{d}$, $\beta_R > \beta_G > \beta_B$.

16. Momentum $= \frac{E}{c} = \frac{Power \times \Delta t}{c}$

$= \frac{(30 \times 10^{-3}) \times (100 \times 10^{-9})}{3 \times 10^8}$

$= 1.0 \times 10^{-17}$ kg ms^{-1}

17. $I = 4I_0\cos^2\left(\frac{\Delta\phi}{2}\right)$; $\Delta\phi$ = phase difference

$I_{max} = 4I_0$. For $I = \frac{1}{2}\ I_{max} = 2I_0$

$2I_0 = 4I_0\cos^2\left(\frac{\Delta\phi}{2}\right)$

$\Rightarrow \cos\left(\frac{\Delta\phi}{2}\right) = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \quad \frac{\Delta\phi}{2} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, .. = (2n+1)\ \frac{\pi}{4}$

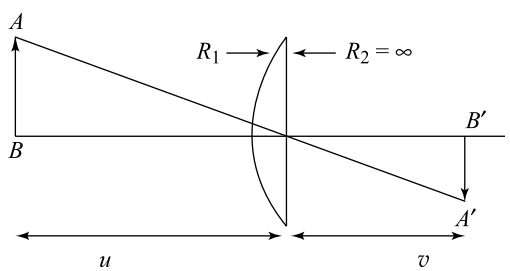
where $n = 0,1,2, ...$

or $\Delta\phi = (2n + 1)\frac{\pi}{2}$

$\Rightarrow \quad \frac{2\pi}{\lambda}\Delta x = (2n + 1)\ \frac{\pi}{2}$; Δx = path difference

$\Rightarrow \quad \Delta x = (2n + 1)\ \frac{\lambda}{4}$

18. Give $\frac{v}{u} = -\frac{1}{3} \Rightarrow u = -3v = -3 \times 8 = -24$ m



$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \quad \frac{1}{8} - \frac{1}{-24} - \frac{1}{f} = \Rightarrow f = 6$ m

If λ is the wavelength of light in the lens and λ_0 in air, then

$v = v\lambda$

$c = v\lambda_0$

$\therefore \quad \mu = \frac{c}{v} = \frac{\lambda_0}{\lambda} \Rightarrow \lambda = \frac{\lambda_0}{\mu}$

Given $\lambda = \frac{2\lambda_0}{3\mu}$. Hence $\mu = \frac{3}{2} = 1.5$

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$= (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{\infty}\right) = \frac{0.5}{R_1}$

$\Rightarrow \frac{1}{6} = \frac{0.5}{R_1} = R_1 = 3$ m

19. Intensity of light transmitted by $A = \frac{I_0}{2}$ According to Malus law, the intensity of light transmitted by B

$= \frac{I_0}{2}\cos^2\theta = \frac{I_0}{2}\cos^2(45^\circ) = \frac{I_0}{4}$

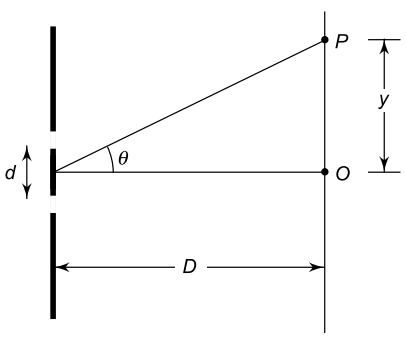
20. If δ is the phase difference between the interfering waves at point P, then the intensity at point P is given by

$I = I_{max}\cos^2\left(\frac{\delta}{2}\right)$

Given $I = \frac{I_{max}}{2}$. Hence

$\cos^2\left(\frac{\delta}{2}\right) = \frac{1}{2}$ which gives $\frac{\delta}{2} = \frac{\pi}{4}$

or $\delta = \frac{\pi}{2}$



The angular separation θ between points P and O is given by $\tan\theta = y/D$. Since θ is very small, $\tan\theta \simeq \sin\theta$. Hence

$$\sin\theta = \frac{y}{D} \tag{1}$$

If β is the fringe width, then

$$\frac{y}{\beta} = \frac{\delta}{2\pi} = \frac{\pi/2}{2\pi} = \frac{1}{4} \tag{2}$$

This is so because phase difference δ between two consecutive maxima is 2. Now $\beta = \frac{\lambda D}{d}$.

Using this in Eq.(2), we get

$$\frac{yd}{\lambda D} = \frac{1}{4}$$

or $$\frac{y}{D} = \frac{\lambda}{4d} \tag{3}$$

Using Eq. (1) in Eq.(3), we have

$\sin\theta = \frac{\lambda}{4d}$ or $\theta = \sin^{-1}\left(\frac{\lambda}{4d}\right)$, which is choice (d).

DUAL NATURE OF MATTER AND RADIATION

Chapter 18

REVIEW OF BASIC CONCEPTS

1. Photoelectric Effect

When electromagnetic radiation of appropriate frequency falls on a metal, electrons are emitted. This phenomenon is called photoelectric effect and the emitted electrons are called photoelectrons because they are liberated by means of light.

Einstein's Photoelectric Equation

The classical electromagnetic wave theory of light, which successfully explained interference, diffraction, and polarization of light, could not account for the observations related to photoelectric effect. In 1900 Planck postulated that light waves consist of tiny bundles of energy called *photons* or *quanta*. The energy of a light wave of frequency ν is given by $E = h\nu$, where h is Planck's constant. Photon is simply a light wave of energy E.

Following Planck's idea, Einstein proposed a theory for photoelectric effect. According to him, when a photon of light falls on a metal, it is absorbed, resulting in the emission of a photoelectron. The maximum kinetic energy $K_{max} = 1/2\ mv^2_{max}$ of the emitted electron is given by

$$\frac{1}{2} mv^2_{max} = h\nu - W_0 \qquad (1)$$

This is the famous Einstein's photoelectric equation. The term $h\nu$ represents the total energy of the photon incident on the metal surface. The photon penetrates a distance of about 10^{-8} m before it is completely absorbed. In disappearing, the photon imparts all its energy to a single electron. Part of this energy is used up by the electron in freeing itself from the atoms of the metal. This energy designated by W_0 in Eq. (1) is called the *work-function* of the metal and is a characteristic of it. The rest of the energy is used up in giving the electron kinetic energy.

The work function W_0, i.e. the energy required to pull an electron away from the surface of the metal, is large for heavier elements like platinum whereas for other elements like alkali metals, W_0 is quite small. The minimum, or threshold energy which a photon must have to free the electron from the surface of the metal should be equal to its work function. If the threshold frequency is ν_0, the threshold energy is $h\nu_0$. Thus

$$W_0 = h\nu_0$$

Einstein's photoelectric equation therefore becomes

$$\frac{1}{2} m v^2_{max} = h(\nu - \nu_0) \qquad (2)$$

It is evident that when $\nu < \nu_0$, no electron is emitted for any intensity of light. When $\nu > \nu_0$, the energy of the electron increases linearly with the frequency ν of light. Since intensity of light is a measure of the number of photons and since each photon emits a photoelectron on absorption, the intensity of photoelectrons is proportional to the intensity of light.

Below a certain negative voltage V_0, no photoelectrons are emitted no matter what the intensity of light is. This voltage is called the cut-off or stopping potential. Since there is no photoelectric emission at potentials less than V_0, the maximum velocity v_{max} acquired by the photoelectrons is given by

$$K_{max} = \frac{1}{2} m v^2_{max} = eV_0$$

where K_{max} is the maximum kinetic energy. V_0 is given by

$$eV_0 = h(\nu - \nu_0)$$

or

$$V_0 = \frac{h}{e}(\nu - \nu_0) \qquad (3)$$

Laws and graphs of photoelectric effect

(1) For a given emitter illuminated by radiation of a given frequency, the photoelectric current is proportional to the intensity of radiation (Fig. 18.1)

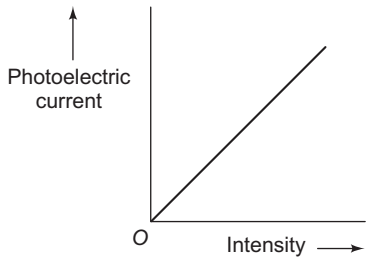


Fig. 18.1

(2) The maximum kinetic energy (K_{max}) of photo-electrons is proportional to the frequency (ν) of the incident radiation and is independent of intensity of the radiation (Fig. 18.2).

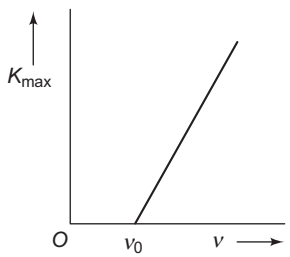


Fig. 18.2

$$K_{max} = h(\nu - \nu_0)$$

Slope of graph = h(Planck's constant).

$K_{max} = 0$ when $\nu \le \nu$ (threshold frequency)

(3) For every emitter there is definite threshold frequency (ν_0) below which no photoelectrons are emitter no matter what the intensity of radiation is.

(4) Graph of stopping potential (V_0) versus frequency ν of incident radiation (Fig. 18.3).

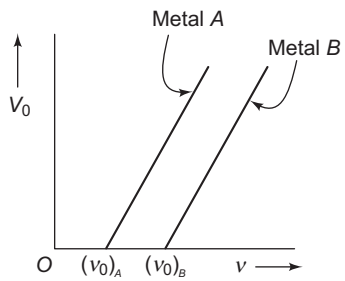


Fig. 18.3

$$V_0 = \frac{h}{e}(\nu - \nu_0)$$

Slope of graph = $\frac{h}{e}$, which is the same for all metals.

(5) Graph of photoelectric current (i) versus voltage (V) for radiations of different intensities ($I_1 > I_2$) but of the same frequency (Fig. 18.4).

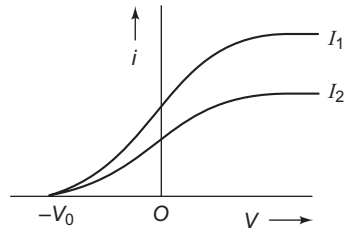


Fig. 18.4

(6) Graph of photoelectric current (i) versus voltage (V) for radiations of different frequencies ($\nu_1 > \nu_2$) but of the same intensity (Fig. 18.5)

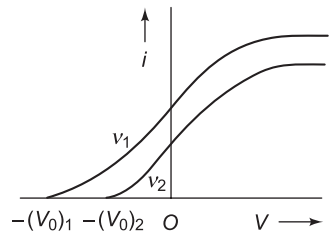


Fig. 18.5

(7) Threshold wavelength is $\lambda_0 = \frac{c}{\nu_0}$. For photoelectric emission $\lambda < \lambda_0$.

EXAMPLE 1 What is the linear momentum of a photon of light of wavelength 663 nm. Given $h = 6.63 \times 10^{-34}$ Js?

SOLUTION Energy of photon is

$$E = h\nu = \frac{hc}{\lambda}$$

$$= \frac{(6.63\times10^{-34})\times(3\times10^{8})}{663\times10^{-9}}$$

$$= 3\times10^{-19}\ \text{J}$$

$$\text{Linear momentum} = \frac{E}{c} = \frac{3\times10^{-19}}{3\times10^{8}} = 1.0\times10^{-27}\ \text{J s m}^{-1}$$

EXAMPLE 2 A parallel beam of monochromatic light of wavelength 663 nm has intensity 300 Wm^{-2}. How many photons are incident per second on a square surface of side 10 cm held perpendicular to the beam ?

SOLUTION Intensity of light on a surface is defined as the energy incident per unit time on a unit area held normal to the beam of light.

Area of surface is

$$A = 10\ \text{cm} \times 10\ \text{cm} = 10^2\ \text{cm}^2 = 10^{-2}\ \text{m}^2.$$

$$\text{Intensity} = \frac{\text{power}}{\text{area}}$$

$\therefore$ Power (P) = Intensity × area = $300\ \text{Wm}^{-2} \times 10^{-2}\ \text{m}^2 = 3\ \text{W}$

Let n be the number of photons incident per second on the surface. Energy of 1 photon = $h\nu$. Therefore, energy of n photons = $nh\nu$. Hence, number of photons incident per second is

$$n = \frac{P}{h\nu} = \frac{P\lambda}{hc}$$

$$= \frac{3 \times (663 \times 10^{-9})}{(6.63 \times 10^{-34}) \times (3 \times 10^{8})}$$

$$= 10^{19}$$

EXAMPLE 3 The maximum wavelength of radiation that can cause photoelectric emission in a metal of work function 2.5 eV is very nearly equal to

(a) 400 nm (b) 500 nm

(c) 600 nm (d) 700 nm

SOLUTION $\lambda_{\max} = \frac{hc}{W_0} = \frac{(6.63 \times 10^{-34}) \times (3 \times 10^{8})}{(2.5 \times 1.6 \times 10^{-19})}$

$$= 4.97 \times 10^{-7}\ \text{m}$$

$$= 497\ \text{nm}$$

So the correct choice is (b).

EXAMPLE 4 A 100 W sodium lamp is emitting 3×10^{20} photons per second. Find the wavelength of light emitted by the bulb.

SOLUTION Energy of each photon of light is

$$E = \frac{P}{n} = \frac{100\ \text{J s}^{-1}}{3 \times 10^{20}\ \text{s}^{-1}} = \frac{10^{-18}}{3}\ \text{J}$$

Wavelength $\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34}) \times (3 \times 10^{8})}{10^{-18}/3}$

$$= 5.97 \times 10^{-7}\ \text{m}$$

$$= 597\ \text{nm}$$

EXAMPLE 5 An atom of mass m is moving with a velocity v. It absorbs a photon of wavelength λ and comes to rest. The value of v is given by (here h is Planck's constant).

(a) $\frac{m\lambda}{h}$ (b) $\frac{mh}{\lambda}$

(c) $\frac{h}{m\lambda}$ (d) $\frac{\lambda h}{m}$

SOLUTION The linear momentum of the photon is

$$p = \frac{h}{\lambda}$$

Since the atom comes to rest after absorbing the photon, the total final momentum is zero. From the principle of conservation of momentum, the initial momentum must be zero. Hence

$$mv = \frac{h}{\lambda}$$

$$\Rightarrow \quad v = \frac{h}{m\lambda},$$ which is choice (c).

EXAMPLE 6 A 9 kW light source emits light of frequency 10^{15} Hz. A parallel beam of this light falls normally on a surface. If the surface completely absorbs all the incident photons, the force exerted by the light beam on the surface is

(a) 3×10^{10} N (b) 3×10^{5} N

(c) 3×10^{-5} N (d) 3×10^{-9} N

SOLUTION Total electromagnetic energy falling on the surface per second is

$$E = 9 \times 10^{3}\ \text{J per second}$$

The total momentum of all the photons falling on the surface per second is

$$p = \frac{E}{c} = \frac{9 \times 10^{3}}{3 \times 10^{8}} = 3 \times 10^{-5}\ \text{Ns}$$

Since all the photons falling on the surface are completely absorbed, the total momentum imparted to the surface per second = 3×10^{-5} Ns. According to Newton's second law, the change in momentum in 1 s is the force. Hence, the force exerted on the surface is

$$F = \frac{3 \times 10^{-5}\ \text{Ns}}{1\ \text{s}} = 3 \times 10^{-5}\ \text{N}.$$

EXAMPLE 7 An electromagnetic radiation given by

$$E = E_0\ [\sin(2\pi\ \nu_1 t) + \sin(2\pi\ \nu_2 t)]$$

where $\nu_1 = 1.3 \times 10^{15}$ Hz and $\nu_2 = 1.1 \times 10^{15}$ Hz falls on a photocell whose work function is 2.0 eV. Then the photoelectric current through the cell

(a) is zero

(b) increases with time

(c) remains constant

(d) increases and decreases periodically with time.

SOLUTION Using trigonometric relation

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

the above equation can be written as

$$E = 2E_0 \cos\left\{2\pi\left(\frac{\nu_1 - \nu_2}{2}\right)t\right\} \sin\left\{2\pi\left(\frac{\nu_1 + \nu_2}{2}\right)t\right\}$$

$$\Rightarrow \qquad E = R \sin (2\pi \nu_a t) \qquad \text{(i)}$$

where $$R = 2E_0 \cos\left\{2\pi\left(\frac{\nu_1 - \nu_2}{2}\right)t\right\} \qquad \text{(ii)}$$

and $$\nu_a = \frac{1}{2}(\nu_1 + \nu_2)$$

It follows from Eq. (ii) that the amplitude of the resultant wave varies periodically at beat frequency $\nu_b = \nu_1 - \nu_2$. Hence the resultant intensity rises and falls periodically at this frequency. Since photoelectric current is directly proportional to the intensity of incident radiation, the correct choice is (d).

EXAMPLE 8 In Example 7 above, the maximum kinetic energy of photoelectrons is

(a) 1 eV (b) 2 eV

(c) 3 eV (d) 4 eV

SOLUTION Frequency of radiation is

$$\nu_a = \frac{1}{2}(\nu_1 + \nu_2) = \frac{1}{2}(1.3 \times 10^{15} + 1.1 \times 10^{15})$$

$$= 1.2 \times 10^{15} \text{ Hz}$$

$$K_{max} = h\,\nu_a - W_0$$

$$= \frac{(6.63 \times 10^{-34}) \times (1.2 \times 10^{15})}{1.6 \times 10^{-9}} \text{eV} - 2.0 \text{ eV}$$

$$= 5 \text{ eV} - 2 \text{ eV} = 3 \text{ eV}$$

So the correct choice is (c).

EXAMPLE 9 A metal of work function 3.3 eV is illuminated by light of wavelength 300 nm. Find (a) the threshold frequency of photoelectric emission, (b) the maximum kinetic energy of photoelectrons and (c) the stopping potential. Take $h = 6.6 \times 10^{-34}$ Js.

SOLUTION (a) $W_0 = 3.3 \text{ eV} = 3.3 \times 1.6 \times 10^{-19}$ J

Threshold frequency is

$$\nu_0 = \frac{W_0}{h} = \frac{3.3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$= 8 \times 10^{14} \text{ Hz}$$

(b) Frequency of incident radiation is

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{300 \times 10^{-9}}$$

$$= 10 \times 10^{14} \text{ Hz}$$

$$\therefore \qquad K_{max} = h(\nu - \nu_0)$$

$$= (6.6 \times 10^{-34}) \times (10 \times 10^{14} - 8 \times 10^{14})$$

$$= 1.32 \times 10^{-19} \text{ J}$$

(c) $K_{max} = eV_0 \quad \Rightarrow \quad V_0 = \frac{K_{max}}{e} = \frac{1.32 \times 10^{-19}}{1.6 \times 10^{-19}}$

$$= 0.825 \text{ V}$$

EXAMPLE 10 Photoelectric emission from a metal begins at a frequency of 6×10^{14} Hz. The emitted electrons are fully stopped by a retarding potential of 3.3 V. Find the wavelength (in nm) of the incident radiation. Take $h = 6.6 \times 10^{-34}$ Js.

SOLUTION $eV_0 = h(\nu - \nu_0)$

$$\Rightarrow \qquad (1.6 \times 10^{-19}) \times 3.3 = (6.6 \times 10^{-34}) \times (\nu - 6 \times 10^{14})$$

$$\Rightarrow \qquad \nu = 1.4 \times 10^{15} \text{ Hz}$$

$$\therefore \qquad \lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{1.4 \times 10^{15}} = 2.14 \times 10^{-7} \text{ m} = 214 \text{ nm}$$

EXAMPLE 11 Light of wavelength 300 nm is incident on two metals *A* and *B* whose work functions are respectively 4 eV and 2 eV. Which of the two metals will emit photoelectrons?

SOLUTION Energy of incident radiation is

$$E = h\nu = \frac{hc}{\lambda} = \frac{(6.6 \times 10^{-34}) \times (3 \times 10^8)}{300 \times 10^{-9}}$$

$$= 6 \times 10^{-19} \text{ J}$$

$$= \frac{6 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.75 \text{ eV}$$

Photoelectrons will be emitted from the metal if E is greater than the work function of the metal. Hence metal *B* will emit photoelectrons but *A* will not.

EXAMPLE 12 For photoelectric affect in a metal, the graph of stopping potential V_0 (in volt) versus frequency ν (in Hz) of the incident radiation is shown in Fig. 18.6. From the graph find,

(a) threshold frequency

(b) Planck's constant and

(c) work function of the metal.

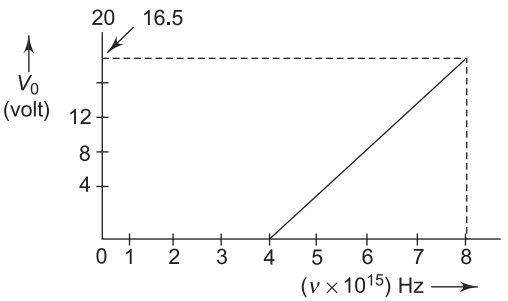


Fig. 18.6

SOLUTION $eV_0 = h(\nu - \nu_0)$

$$\Rightarrow \quad V_0 = \frac{h}{e}(\nu - \nu_0) \qquad (1)$$

(a) It follows from Eq. (1) that $\nu = \nu_0$ if $V_0 = 0$. Hence $\nu_0 = 4 \times 10^{15}$ Hz

(b) The slope of V_0 versus ν graph $= \frac{h}{e}$, The slope of the given graph is

$$= \frac{16.5 - 0}{(8-4)\times 10^{15}} = 4.125 \times 10^{-15}$$

$$\therefore \quad h = e \times \text{slope}$$
$$= (1.6 \times 10^{-19}) \times (4.125 \times 10^{-15})$$
$$= 6.6 \times 10^{-34} \text{ Js}$$

(c) $W_0 = h\nu_0 = (6.6 \times 10^{-34}) \times (4 \times 10^{15})$
$= 26.4 \times 10^{-19}$ J
$= 16.5$ eV

EXAMPLE 13 Calculate the number of photons emitted per second by a transmitter of power 10 kW sending radiowaves of frequency 6×10^5 Hz. Take $h = 6.63 \times 10^{-34}$ Js.

SOLUTION Let N be the number of photons emitted in time t. Energy of 1 photon $= h\nu$. Therefore, energy of N photons $= Nh\nu$. Therefore, the power is

$$P = \frac{Nh\nu}{t} = nh\nu;$$

n = number of photons emitted per second.

or $$n = \frac{P}{h\nu} = \frac{10 \times 10^3}{(6.63 \times 10^{-34}) \times 6 \times 10^5}$$
$$= 2.5 \times 10^{31} \text{ photons per second}$$

EXAMPLE 14 The stopping potential of a metal is 3 V when it is illuminated by light of wavelength 500 nm. What will be the stopping potential of the metal when the wavelength is 600 nm? Also find the cut-off frequency and work function of the metal. Take $h = 6.6 \times 10^{-34}$ Js.

SOLUTION $eV_1 = h(\nu_1 - \nu_0)$ (1)

$eV_2 = h(\nu_2 - \nu_0)$ (2)

Subtracting, we get

$$e(V_1 - V_2) = h(\nu_1 - \nu_2)$$

$$\Rightarrow \quad V_1 - V_2 = \frac{h}{e}(\nu_1 - \nu_2)$$
$$= \frac{h}{e}\left(\frac{c}{\lambda_1} - \frac{c}{\lambda_2}\right)$$
$$= \frac{hc}{e}\left(\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}\right)$$
$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19}} \times \left(\frac{6 \times 10^{-7} - 5 \times 10^{-7}}{6 \times 5 \times 10^{-14}}\right)$$
$$= 0.4 \text{ V}$$

$$\therefore \quad V_2 = V_1 - 0.4 = 3 - 0.4 = 2.6 \text{ V}$$

From Eq. (1),

$$\nu_0 = \nu_1 - \frac{eV_1}{h}$$
$$= \frac{c}{\lambda_1} - \frac{eV_1}{h}$$
$$= \frac{3 \times 10^8}{6 \times 10^{-7}} - \frac{1.6 \times 10^{-19} \times 3}{6.6 \times 10^{-34}}$$
$$= 4.27 \times 10^{14} \text{ Hz}$$

$$W_0 = h\nu_0 = 6.6 \times 10^{-34} \times 4.27 \times 10^{14}$$
$$= 2.82 \times 10^{-19} \text{ J} = 1.76 \text{ eV}$$

EXAMPLE 15 Ultraviolet light of wavelength 250 nm falls on the metal emitter of a photocell. If the stopping potential is 1.2 V, find the work functions of the metal. Will the photocell work if yellow light of wavelength 600 nm is used? Take $h = 6.6 \times 10^{-34}$ Js.

SOLUTION $\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{250 \times 10^{-9}} = 1.2 \times 10^{15}$ Hz

$$eV_0 = h(\nu - \nu_0) = h\nu - W_0$$

$$\therefore \quad W_0 = h\nu - eV_0$$
$$= 6.6 \times 10^{-34} \times 1.2 \times 10^{15} - 1.6 \times 10^{-19} \times 1.2$$
$$= 6 \times 10^{-19} \text{ J} = 3.75 \text{ eV}$$

Now $$\nu_0 = \frac{W_0}{h} = \frac{6 \times 10^{-19}}{6.6 \times 10^{-34}} = 0.9 \times 10^{15} \text{ Hz}$$

Frequency of yellow light is

$$\nu_y = \frac{c}{\lambda_y} = \frac{3 \times 10^8}{600 \times 10^{-9}} = 0.5 \times 10^{14} \text{ Hz}$$

Since $\nu_y < \nu_0$, the photocell will not work with yellow light as no photoelectrons will be emitted.

2. Wave Nature of Matter

In 1924, Louis de Broglie, a French theoretical physicist, derived an equation which predicted that all atomic particles have associated with them waves of a definite

wavelength. Under certain circumstances, a beam of electrons or atoms will behave like a group of waves. On the basis of theoretical considerations, de Broglie predicted that the wavelength λ of these waves is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

where h is Planck's constant and p is the momentum of the particles. This equation is known as *de Broglie's wave equation*. For an electron moving at a high speed, the momentum is large and the wavelength λ is small. The faster the electron, the shorter is the wavelength. Notice that the particle need not have a charge to have an associated wave. This is why de Broglie waves are sometimes referred to as matter waves.

1. If the rest mass of a particle is m_0, its *de Broglie* wavelength is given by

$$\lambda = \frac{h\left(1-\frac{v^2}{c^2}\right)^{1/2}}{m_0 v}$$

2. In terms of kinetic energy K, *de Broglie* wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mK}}$$

3. If a particle of charge q is accelerated through a potential difference V, its de Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

For an electron,

$$\lambda = \left(\frac{150}{V}\right)^{1/2} \text{ Å}$$

4. For a gas molecule of mass m at temperature T kelvin, the *de Broglie* wavelength is given by

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

where k is the Boltzmann constant.

EXAMPLE 16 Calculate the wavelength associated with a dust particle of mass 1 μg moving with a velocity of 10^6 ms^{-1}. Given $h = 6.6 \times 10^{-34}$ Js.

SOLUTION $m = 1\ \mu g = 10^{-6}\ g = 10^{-9}$ kg

$$\lambda = \frac{h}{mv} = \frac{6.6\times10^{-34}}{10^{-9}\times10^{6}}$$

$$= 6.6 \times 10^{-31} \text{ m}$$

EXAMPLE 17 Calculate de Broglie wavelength of an electron having kinetic energy of 1 BeV. Given mass of electron (m) = 9.1×10^{-31} kg, $h = 6.6 \times 10^{-34}$ Js and $e = 1.6 \times 10^{-19}$ C.

SOLUTION

$K = 1$ BeV $= 10^9$ eV $= 10^9 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-10}$ J

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-10}}}$$

$$= 3.87 \times 10^{-14} \text{ m}$$

EXAMPLE 18 Calculate de Broglie wavelength associated with an electron accelerated through a potential difference of 200 V. Given $m = 9.1 \times 10^{-31}$ kg and $h = 6.6 \times 10^{-34}$ Js.

SOLUTION

$K = 200$ eV $= 200 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-17}$ J

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times3.2\times10^{-17}}}$$

$$= 0.86 \times 10^{-10} \text{ m} = 0.86 \text{ Å}$$

EXAMPLE 19 Calculate the wavelength of de Broglie waves associated with a neutron at room temperature of 27°C. Given mass of neutron (m) = 1.67×10^{-27} kg, Boltzman constant (k) = 1.38×10^{-23} JK^{-1} and $h = 6.63 \times 10^{-34}$ Js.

SOLUTION Kinetic energy of neutron due to thermal speed is $K = \frac{3}{2}kT$, where $T = 273 + 27 = 300$ K.

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2m\times\frac{3}{2}kT}} = \frac{h}{\sqrt{3mkT}}$$

Substituting the values of h, m, k and T, we get

$$\lambda = 1.45 \times 10^{-10} \text{ m} = 1.45 \text{ Å}$$

EXAMPLE 20 A photon of wavelength 19.8 nm collides with an electron at rest. After the collision, the wavelength of the photon is found to be 30 nm. Is the collision elastic or inelastic? Calculate the energy of the scattered electron. Given $h = 6.6 \times 10^{-34}$ Js.

SOLUTION Energy of photon before collision is

$$E_i = h\nu_1 = \frac{hc}{\lambda_1} = \frac{(6.6\times10^{-34})\times(3\times10^{8})}{19.8\times10^{-9}} = 10\times10^{-18} \text{ J}$$

Energy of photon after collision is

$$E_f = \frac{hc}{\lambda_2} = \frac{(6.6\times10^{-34})\times(3\times10^{8})}{30\times10^{-9}}$$

$$= 6.6 \times 10^{-18} \text{ J}$$

Since there is a loss of kinetic energy ($\because E_f < E_i$), the collision is inelastic. The energy of the scattered electron $= E_i - E_f = 3.4 \times 10^{-18}$ J.

EXAMPLE 21 Ultraviolet light of wavelength 99 nm falls on a metal plate of work function 1.0 eV. Find the wavelength of the fastest photoelectron emitted. Mass of electron (m) = 9.1×10^{-31} kg and $h = 6.6 \times 10^{-34}$ Js.

SOLUTION
$$K_{max} = h\nu - W_0 = \frac{hc}{\lambda} - W_0$$
$$= \frac{(6.6\times10^{-34})\times(3\times10^{8})}{99\times10^{-9}} - 1.0\times1.6\times10^{-19}$$
$$= 2\times10^{-18} - 1.6\times10^{-19}$$
$$= 1.84\times10^{-18} \text{ J}$$

$$\therefore \quad \lambda = \frac{h}{\sqrt{2mK_{max}}}$$
$$= \frac{6.6\times10^{-34}}{\sqrt{2\times(9.1\times10^{-31})\times1.84\times10^{-18}}}$$
$$= 3.6\times10^{-10} \text{ m} = 0.36 \text{ nm}$$

EXAMPLE 22 A proton and an electron have equal kinetic energy. Which of the two has a greater de Broglie wavelength?

SOLUTION
$$\lambda_p = \frac{h}{\sqrt{2m_pK}}, \quad \lambda_e = \frac{h}{\sqrt{2m_eK}}$$
$$\therefore \quad \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} > 1 \qquad (\because m_p > m_e)$$

Hence the electron has the greater wavelength.

1 SECTION — Multiple Choice Questions with One Correct Choice

Level A

1. Which one of the following statements is NOT true about photoelectric emission?
   (a) For a given emitter illuminated by light of a given frequency, the number of photo-electrons emitted per second is proportional to the intensity of incident light.
   (b) For every emitter there is a definite threshold frequency below which no photoelectrons are emitted, no matter what the intensity of light is.
   (c) Above the threshold frequency, the maximum kinetic energy of photoelectrons is proportional to the frequency of incident light.
   (d) The saturation value of the photoelectric current is independent of the intensity of incident light.
2. The maximum energy K_{max} of photoelectrons emitted in a photoelectric cells is measured using lights of various frequencies ν. The graph in Fig. 18.7 shows how K_{max} varies with ν. The slope of the graph is equal to
   (a) the charge of an electron
   (b) the charge of mass ratio of an electron
   (c) the work function of the emitter in the cell
   (d) Planck's constant.

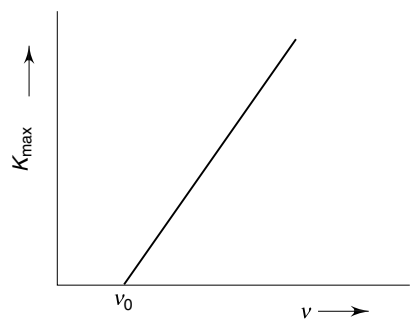


Fig. 18.7

3. When a photon of light falls on a metal, it is absorbed, resulting in the emission of a photoelectron. Before the photon is completely absorbed by the metal, it penetrates a distance of about
   (a) 1 mm (b) 10^{-4} m
   (c) 10^{-8} m (d) 10^{-14} m
4. Choose the correct statement. The work function
   (a) is the same for all metals
   (b) is different for different metals
   (c) is small for heavy elements
   (d) depends upon the frequency of the incident radiation.
5. A metal whose work function is 3.31 eV is illuminated by light of wavelength 3×10^{-7} m. What is the

threshold frequency for photoelectric emission? Take $h = 6.62 \times 10^{-34}$ Js.

(a) 0.8×10^{15} Hz (b) 1.6×10^{15} Hz

(c) 2.4×10^{15} Hz (d) 3.2×10^{15} Hz

6. What is the maximum energy of photoelectrons emitted from the metal in Q. 7?

(a) 0.331×10^{-19} J (b) 0.662×10^{-19} J

(c) 0.884×10^{-19} J (d) 1.324×10^{-19} J

7. The photoelectric threshold for a certain metal surface is 330 Å. What is the maximum kinetic energy of photoelectron released, if any, by a radiation of wavelength 1100 Å?

(a) 1 eV

(b) 2 eV

(c) 7.5 eV

(d) no electron is emitted.

8. When a point source of light is at a distance of 50 cm from a photoelectric cell, the cut-off voltage is found to be V_0. If the same source is placed at a distance of 1 m from the cell, the cut-off voltage will be

(a) $V_0/4$ (b) $V_0/2$

(c) V_0 (d) $2\ V_0$

9. The photoelectric current in a photoelectric cell depends upon

(a) the nature of the metal used as the emitter

(b) the wavelength of the incident light

(c) the intensity of the incident light

(d) all the above parameters.

10. No photoelectrons are emitted from a metal if the wavelength of light exceeds 6000 Å. The work function of the metal is approximately equal to

(a) 3×10^{-16} J (b) 3×10^{-19} J

(c) 3×10^{-20} J (d) 3×10^{-22} J

11. The threshold wavelength for photoelectric emission from a material is 5200 Å. Photoelectrons will be emitted when this material is illuminated with monochromatic radiation from a

(a) 50-watt infrared lamp

(b) 100-watt red neon lamp

(c) 40-watt sodium lamp

(d) 5-watt ultraviolet lamp.

12. The photoelectric effect establishes that

(a) light travels in the form of quanta of energy

(b) light travels in the form of transverse waves

(c) light travels in the form of longitudinal waves

(d) light is a transverse electromagnetic wave.

13. The photoelectric emission from the surface of a metal starts only when the light incident on the surface has a certain

(a) minimum frequency

(b) minimum wavelength

(c) minimum intensity

(d) minimum speed.

14. At frequencies of the incident radiation above the threshold frequency, the photoelectric current in a photoelectric cell increases with increase in

(a) intensity of incident radiation

(b) wavelength of incident radiation

(c) frequency of incident radiation

(d) speed of incident radiation.

15. The photoelectrons emitted from a metal surface

(a) are all at rest

(b) have the same kinetic energy

(c) have the same momentum

(d) have speeds varying from zero up to a certain maximum value.

16. The wavelength in air associated with a photon of energy E is (c is the speed of light in air and h is the Planck's constant)

(a) $\frac{hc}{E}$ (b) $\frac{h}{cE}$

(c) $\frac{c}{hE}$ (d) hcE

17. A metal whose work function is 3.3 eV is illuminated by light of wavelength 3×10^{-7} m. What is the threshold frequency for photoelectric emission? Planck's constant = 6.6×10^{-34} Js.

(a) 0.4×10^{15} Hz (b) 0.8×10^{15} Hz

(c) 1.6×10^{15} Hz (d) 3.2×10^{15} Hz

18. In Q. 17, the maximum energy of photoelectrons is

(a) 0.825 eV (b) 1.0 eV

(c) 1.225 eV (d) 1.5 eV

19. In Q. 17, what is the stopping potential?

(a) 0.825 V (b) 1.0 V

(c) 1.225 V (d) 1.5 V

20. An X-ray tube produces a continuous spectrum of radiation with its short-wavelength end at 0.33 Å. What is the maximum energy of a photon in the radiation? Planck's constant = 6.6×10^{-34} Js.

(a) 35 keV (b) 37.5 keV

(c) 40 keV (d) 42.5 keV

21. Light of wavelength 3500 Å is incident on two metals A and B whose work functions are 4.2 eV and 1.9 eV respectively. Which metal will emit photoelectrons?
    (a) A (b) B
    (c) Both A and B (d) Neither A nor B

Level B

22. An ultraviolet light of wavelength 2000 Å irradiates a photocell made of molybdenum metal. If the stopping potential is – 1.5 V, what is the work function of the metal? Planck's constant = 6.6×10^{-34} Js.
    (a) 2.5×10^{-19} J (b) 5.0×10^{-19} J
    (c) 7.5×10^{-19} J (d) 1×10^{-18} J
23. When monochromatic light falls on a photosensitive material, the number of photoelectrons emitted per second is n and their maximum kinetic energy is K_{max}. If the intensity of the incident light is doubled, then
    (a) both n and K_{max} are doubled
    (b) both n and K_{max} are halved
    (c) n is doubled but K_{max} remains the same
    (d) K_{max} is doubled but n remains the same.
24. When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When X-rays are incident on the same cell, then
    (a) V_0 and K_{max} both increase
    (b) V_0 and K_{max} both decrease
    (c) V_0 increases but K_{max} remains the same
    (d) K_{max} increases by V_0 remains the same.
25. The work function for aluminium surface is 4.125 eV. The cut off wavelength for photoelectric effect for this surface is
    (a) 3000 Å (b) 3100 Å
    (c) 4200 Å (d) 1500 Å
26. The work function for aluminium surface is 4.2 eV and that for sodium surface is 2.0 eV. The two metals were illuminated with appropriate radiations so as to cause photo emission. Then
    (a) both aluminium and sodium will have the same threshold frequency.
    (b) the threshold frequency of aluminium will be more than that of sodium.
    (c) the threshold frequency of aluminium will be less than that of sodium.
    (d) the threshold wavelength of aluminium will be more than that of sodium.
27. X-rays of frequency ν are used to irradiate sodium and copper surface in two separate experiments and the stopping potential determined. Then
    (a) the stopping potential is more for copper than for sodium.
    (b) the stopping potential is more for sodium than for copper.
    (c) the stopping potential is the same for sodium and copper.
    (d) the stopping potential for both will vary as $1/\nu$.
28. A caesium photo cell, with a steady potential difference of 60 volt across it, is illuminated by a small bright light placed 50 cm away. When the same light is placed one metre away the electrons crossing the photo cell
    (a) each carry one quarter of their previous energy
    (b) each carry one quarter of their previous momentum
    (c) are half as numerous
    (d) are one quarter as numerous.
29. An image of the sun is formed by a lens of focal length 30 cm on the metal surface of a photo-electric cell and it produces a current I. The lens forming the image is then replaced by another lens of the same diameter but of focal length 15 cm. The photoelectric current in this case will be
    (a) $I/2$ (b) $2I$
    (c) I (d) $4I$
30. The momentum of a photon of wavelength λ is
    (a) $\frac{h}{c\lambda}$ (b) $\frac{hc}{\lambda}$
    (c) $h\lambda$ (d) $\frac{h}{\lambda}$
31. The energy of a photon corresponding to the visible light of maximum wavelength is approximately equal to
    (a) 1.0 eV (b) 1.5 eV
    (c) 2.0 eV (d) 2.5 eV
32. Violet light can cause photoelectric emission from a metal but blue light cannot. If sodium light is incident on the metal, then
    (a) the photoelectric current decreases
    (b) the number of photoelectrons ejected per second increases
    (c) the velocity of photoelectrons increases
    (d) no photoelectric emission occurs.
33. What is the stopping potential for photoelectrons ejected from a photosensitive material of work function 1.6 eV when it is irradiated by photons of energy 2.4 eV?

(a) 0.8 V (b) 2.0 V
(c) 4.0 V (d) none of these

34. 10^{20} photons of wavelength 660 nm are emitted per second from a lamp. What is the wattage of the lamp. Planck's constant = 6.6×10^{-34} Js.
(a) 30 W (b) 60 W
(c) 100 W (d) 500 W

35. When a certain photosensitive surface is illuminated with monochromatic light of frequency ν, the stopping potential for photoelectric current is $V_0/2$. When the same surface is illuminated by monochromatic high of frequency $\nu/2$, the stopping potential is V_0. The threshold frequency for photoelectric emission is
(a) $\frac{2\nu}{3}$ (b) $\frac{3\nu}{2}$
(c) $\frac{3\nu}{5}$ (d) $\frac{5\nu}{3}$

36. When a certain photosensitive surface is illuminated with monochromatic light of wavelength λ, the stopping potential for photoelectric current is $2\ V_0$. When the same surface is illuminated with monochromatic light of wavelength 2λ, the stopping potential is $V_0/2$. The threshold wavelength for photoelectric emission is
(a) 1.5 λ (b) 2.0 λ
(c) 2.5 λ (d) 3 λ

37. The threshold frequency for a certain photosensitive metal is ν_0. When it is illuminated by light of frequency $\nu = 2\nu_0$, the stopping potential for photoelectric current is V_0. What will be the stopping potential when the same metal is illuminated by light of frequency $\nu = 3\nu_0$?
(a) 1.5 V_0 (b) 2 V_0
(c) 2.5 V_0 (d) 3 V_0

38. The threshold frequency for a certain photosensitive metal is ν_0. When it is illuminated by light of frequency $\nu = 2\nu_0$, the maximum velocity of photoelectrons is v_0. What will be the maximum velocity of the photoelectrons when the same metal is illuminated by light of frequency $\nu = 5\nu_0$?
(a) $\sqrt{2}\ v_0$ (b) $2v_0$
(c) $2\sqrt{2}\ v_0$ (d) 4 v_0

39. The threshold wavelength for a metal whose work function is W_0 is λ_0. What is the threshold wavelength for a metal whose work function is $W_0/2$?
(a) $\frac{\lambda_0}{4}$ (b) $\frac{\lambda_0}{2}$
(c) 2 λ_0 (d) 4 λ_0

40. When a photon moving in air enters a glass slab, which of the following does not change?
(a) its velocity (b) its momentum
(c) its wavelength (d) its energy

41. The wavelength λ of de Broglie waves associated with an electron (mass m, charge e) accelerated through a potential difference of V is given by (h is Planck's constant)
(a) $\lambda = h/mV$ (b) $\lambda = h/2\ meV$
(c) $\lambda = h/\sqrt{meV}$ (d) $\lambda = h/\sqrt{2meV}$

42. A proton, when accelerated through a potential difference of V volts, has a wavelength λ associated with it. If an alpha particle is to have the same wavelength λ, it must be accelerated through a potential difference of
(a) $V/8$ volts (b) $V/4$ volts
(c) 4 V volts (d) 8 V volts

43. A proton and an electron move with the same velocity. The associated wavelength for proton is
(a) shorter than that of the electron
(b) longer than that of the electron
(c) the same as that of the electron
(d) zero

44. Two particles of masses m and $2m$ have equal kinetic energies. Their de Broglie wavelengths are in the ratio of
(a) 1 : 1 (b) 1 : 2
(c) 1 : $\sqrt{2}$ (d) $\sqrt{2}$: 1

45. The wavelength of de Broglie wave associated with a thermal neutron of mass m at absolute temperature T is given by (here k is the Boltzmann constant)
(a) $\frac{h}{\sqrt{mkT}}$ (b) $\frac{h}{\sqrt{2\,mkT}}$
(c) $\frac{h}{\sqrt{3\,mkT}}$ (d) $\frac{h}{2\sqrt{mkT}}$

46. The de Broglie wavelength of a neutron at 927°C is λ. What will be its wavelength at 27°C?
(a) $\frac{\lambda}{2}$ (b) λ
(c) 2 λ (d) 4 λ

47. The de Broglie wavelength of a neutron when its kinetic energy is K is λ. What will be its wavelength when its kinetic energy is $4K$?
(a) $\frac{\lambda}{4}$ (b) $\frac{\lambda}{2}$
(c) 2 λ (d) 4 λ

48. What is the de Broglie wavelength of an electron of energy 180 eV? Mass of electron = 9×10^{-31} kg and Planck's constant = 6.6×10^{-34} Js.
    (a) 0.5 Å (b) 0.9 Å
    (c) 1.3 Å (d) 1.8 Å
49. Moving with the same velocity, which of the following has the longest de Broglie wavelength?
    (a) β–particle (b) α–particle
    (c) proton (d) neutron
50. The momentum of a photon of frequency ν is
    (a) $\frac{h\nu}{c}$ (b) $h\nu c$
    (c) $\frac{h}{c\nu}$ (d) $\frac{\nu}{ch}$
51. The de Broglie wavelength of an electron moving with a velocity 1.5×10^8 ms^{-1} is equal to that a photon. The ratio of the kinetic energy of the electron to that of the photon is
    (a) 2 (b) 4
    (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
52. The kinetic energy of the most energetic photoelectrons emitted from a metal surface is doubled when the wavelength of the incident radiation is reduced from λ_1 to λ_2. The work function of the metal is
    (a) $\frac{hc}{\lambda_1 \lambda_2}(2\lambda_2 - \lambda_1)$ (b) $\frac{2hc}{\lambda_1 \lambda_2}(2\lambda_1 - \lambda_2)$
    (c) $\frac{2hc}{\lambda_1 \lambda_2}(\lambda_1 + \lambda_2)$ (d) $\frac{2hc}{\lambda_1 \lambda_2}(\lambda_1 - \lambda_2)$
53. A photon of energy 8 eV is incident on a metal surface of threshold frequency 1.6×10^{15} Hz. The kinetic energy (in eV) of the photoelectrons emitted is (take $h = 6 \times 10^{-34}$ Js)
    (a) 6 (b) 1.6
    (c) 1.2 (d) 2
54. The maximum velocity of photoelectrons emitted from a metal surface is 1.76×10^6 ms^{-1}. If the e/m ratio of an electron is 1.76×10^{11} C kg^{-1}, then the stopping potential of the metal is
    (a) 2.2 V (b) 4.4 V
    (c) 8.8 V (d) 17.6 V
55. A radio transmitter operates at a frequency of 1 MHz and has a power of 6.6 kW. The number of photons emitted per second is (take $h = 6.6 \times 10^{-34}$ Js)
    (a) 10^{31} (b) 10^{33}
    (c) 10^{35} (d) 10^{37}
56. Which of the following is correct?
    (a) The current in a photo cell increases with increasing frequency.
    (b) The photo current is proportional to the applied voltage.
    (c) The photo current increases if the intensity of incident light is increased.
    (d) The stopping potential increases if the intensity of incident light is increased.
57. The slope of the graph of the frequency of incident light versus the stopping potential for a given metallic surface is
    (a) h (b) $\frac{h}{e}$
    (c) $\frac{e}{h}$ (d) eh
58. Lights of two different frequencies, whose photons have energies 2 eV and 10 eV respectively, successively illuminate a metal of work function 1 eV. The ratio of the maximum speeds of the emitted electrons will be
    (a) 1 : 5 (b) 3 : 11
    (c) 1 : 9 (d) 1 : 3
59. A photon has energy $E = h\nu$ and momentum $p = \frac{h}{\lambda}$. In terms of E and p, the speed of light is
    (a) Ep (b) $\sqrt{Ep}$
    (c) $\frac{p}{E}$ (d) $\frac{E}{p}$
60. The mass of a photon of wavelength λ is given by
    (a) $h\lambda c$ (b) $\frac{h}{\lambda c}$
    (c) $\frac{hc}{\lambda}$ (d) $\frac{h\lambda}{c}$
61. The de-Broglie wavelength of an electron moving in the nth Bohr orbit of radius r is given by
    (a) $\frac{2\pi r}{n}$ (b) $n\pi r$
    (c) $\frac{nr}{2\pi}$ (d) $\frac{nr}{\pi}$
62. A proton and an alpha particle are accelerated to the same potential. Their de-Broglie wavelengths are in the ratio of
    (a) $\sqrt{2}$: 1 (b) $2\sqrt{2}$: 1
    (c) 2 : 1 (d) 4 : 1

63. The momentum of a particle of mass m and charge q is equal to that of a photon of wavelength λ. The speed of the particle is given by

(a) $\frac{h}{m\lambda}$ (b) $\frac{h\lambda}{qm}$

(c) $qh\lambda$ (d) $\frac{mh}{\lambda}$

64. The kinetic energies of photoelectrons emitted from a metal are K_1 and K_2 when it is irradiated with lights of wavelength λ_1 and λ_2 respectively. The work function of the metal is

(a) $\frac{K_1\lambda_1 - K_2\lambda_2}{\lambda_2 - \lambda_1}$ (b) $\frac{K_1\lambda_1 + K_2\lambda_2}{\lambda_2 + \lambda_1}$

(c) $\frac{K_1\lambda_2 - K_2\lambda_1}{\lambda_2 - \lambda_1}$ (d) $\frac{K_1\lambda_2 + K_2\lambda_1}{\lambda_2 + \lambda_1}$

65. When the energy of the incident radiation is increased by 20%, the kinetic energy of the photoelectrons emitted from a metal increased from 0.5 eV to 0.8 eV. The work function of the metal is

(a) 0.65 eV (b) 1.0 eV

(c) 1.3 eV (d) 1.5 eV

66. When a monochromatic source of light is at a distance of 0.2 m from a photoelectric cell, the cut-off voltage and the saturation current are respectively 0.6 V and 18 mA. If the same source is placed 0.6 m away from the cell, then

(a) the stopping potential will be 0.2 V

(b) the stopping potential will be 1.8 V

(c) the saturation current will be 6.0 mA

(d) the saturation current will be 2.0 mA

67. The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy 6 eV fall on it is 4 eV. The stopping potential is

(a) 2 V (b) 4 V

(c) 6 V (d) 10 V

68. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately

(a) 540 nm (b) 400 nm

(c) 310 nm (d) 220 nm

69. A particle of mass M at rest decays into two particles of masses m_1 and m_2, having non zero velocities. The ratio of the de Broglie wavelengths of the particles, λ_1/λ_2, is

(a) $\frac{m_1}{m_2}$ (b) $\frac{m_2}{m_1}$

(c) 1.0 (d) $\sqrt{\frac{m_2}{m_1}}$

70. Photoelectric emission is observed from a metallic surface for frequencies ν_1 and ν_2 of the incident light ($\nu_1 > \nu_2$). If the maximum values of kinetic energy of the photoelectrons emitted in the two cases are in the ratio 1 : n, then the threshold frequency of the metallic surface is

(a) $\frac{\nu_1 - \nu_2}{n-1}$ (b) $\frac{n\nu_1 - \nu_2}{n-1}$

(c) $\frac{n\nu_2 - \nu_1}{n-1}$ (d) $\frac{\nu_1 - \nu_2}{n}$

71. Two photons of energies twice and thrice the work function of a metal are incident on the metal surface. Then the ratio of maximum velocities of the photoelectrons emitted in the two cases respectively, is:

(a) $\sqrt{2}$: 1 (b) $\sqrt{3}$: 3

(c) $\sqrt{3}:\sqrt{2}$ (d) 1 : $\sqrt{2}$

72. If λ_0 is the de Broglie wavelength for a proton accelerated through a potential difference of 100 V, the de Broglie wavelength for α-particle accelerated through the same potential difference is:

(a) $2\sqrt{2}\lambda_0$ (b) $\frac{\lambda_0}{2}$

(c) $\frac{\lambda_0}{2\sqrt{2}}$ (d) $\frac{\lambda_0}{\sqrt{2}}$

73. The de-Broglie wavelength of a particle moving with a velocity 2.25×10^8 m/s is equal to the wavelength of a photon. The ratio of kinetic energy of the particle to the energy of the photon is

(Velocity of light is 3×10^8 m/s)

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$

(c) $\frac{5}{8}$ (d) $\frac{7}{8}$

74. When a centimetre thick surface is illuminated with light of wavelength λ, the stopping potential is V. When the same surface is illuminated by light of wavelength 2λ, the stopping potential is $V/3$. The threshold wavelength for the surface is

(a) $\frac{4\lambda}{3}$ (b) 4λ

(c) 6λ (d) $\frac{8\lambda}{3}$

75. In a photo-emissive cell, with exciting wavelength λ, the fastest electron has a speed v. If the exciting wavelength is changed to $3\lambda/4$, the speed of the fastest emitted electron will be

(a) $v\sqrt{\frac{3}{4}}$ (b) $v\sqrt{\frac{4}{3}}$

(c) less than $v\sqrt{\frac{4}{3}}$ (d) more than $v\sqrt{\frac{4}{3}}$

76. According to Einstein's photoelectric equation, the plot of the kinetic energy of the emitted photo electrons from a metal Vs the frequency, of the incident radiation gives a straight line whose slope

(a) depends on the nature of the metal used
(b) depends on the intensity of the radiation
(c) depends both on the intensity of the radiation and the metal used
(d) is the same for all metals and independent of the intensity of the radiation

77. The stopping potential of a metal is 2.0 V when it is illuminated by light of wavelength 500 nm. What will be its stopping potential when light of wavelength 600 nm falls on it? Take $h = 6.6\times10^{-34}$Js.

(a) 2.0 V (b) 1.6 V

(c) 1.2 V (d) zero

78. Figure 18.8 shows the stopping potential (V_0) versus frequency (v) graph for photoelectric effect in a metal. From the graph, find the Planck's constant (h).

(a) 6.60×10^{-34} Js (b) 6.61×10^{-34} Js

(b) 6.62×10^{-34} Js (d) 6.63×10^{-34} Js

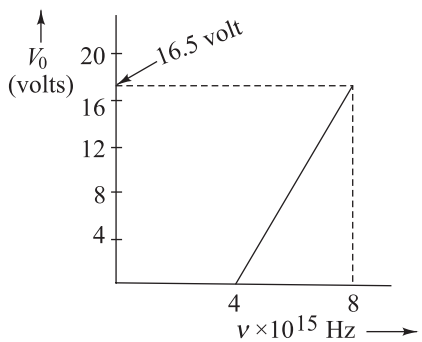


Fig. 18.8

79. In Q. 78 above, it follows from the graph that the work function of the metal is

(a) 2.5 eV (b) 3.6 eV

(c) 6.0 eV (d) 16.5 eV

80. Ultraviolet light of wavelength 250 nm falls on the metal emitter of a photo-cell. If the stopping potential is 1.2 V, the work function of the metal is

(a) 4.75 eV (b) 3.75 eV

(c) 2.75 eV (d) 1.75 eV

81. A photon of wavelength 99 nm falls on a metal plate of work function 1.0 eV. The wavelength of the fastest emitted photoelectron is (given $h = 6.6\times10^{-34}$ Js, mass of electron (m)=9.1×10^{-31} kg)

(a) $a = 0.3$ nm (b) 0.33 nm

(c) 0.36 nm (d) 0.40 nm

82. A photon of wavelength 19.8 nm collides with an electron at rest. After the collision the wavelength of the photon is found to be 30 nm. The energy of the scattered electron is nearly equal to

(a) 2.2 eV (b) 1.6 eV

(c) 22 eV (d) 110 eV

83. Davisson and Germer experiment is performed first with X-rays of wavelength 1Å and then with electrons of wavelength 1Å. The ratio of energy of X-rays to that the electrons is of the order of (m = mess of electron)

(a) 1 (b) $2\left(\frac{mc\lambda}{h}\right)^2$

(c) $\frac{2mc\lambda}{h}$ (d) $\frac{mc\lambda}{h}$

84. In an electron microscope, electrons accelerated by a voltage of 50 kV are used. In an optical microscope of the same aperture, yellow light of wavelength 590 nm is used. The resolving power of the electron microscope will be n times higher than that of the optical microscope, where n is of the order of

(a) 10^2 (b) 10^3

(c) 10^4 (d) 10^5

85. The maximum kinetic energy of photoelectrons emitted from a metal when photons of energy 10 eV fall on it is 3 eV. The stopping potential is

(a) 7 V (b) 3 V

(c) 10 V (d) 13 V

86. Figure 18.9 shows the variation of photoelectric current (i) with potential difference (V) between the anode and the cathode in a photo-cell for a photo-sensitive surface for two radiations of intensities I_1 and I_2 and frequencies ν_1 and ν_2 respectively. It follows from the graphs that

(a) $v_1 < v_2$ and $I_2 > I_1$ (b) $v_1 > v_2$ and $I_2 < I_1$

(c) $v_1 = v_2$ and $I_2 > I_1$ (d) $v_1 = v_2$ and $I_2 < I_1$

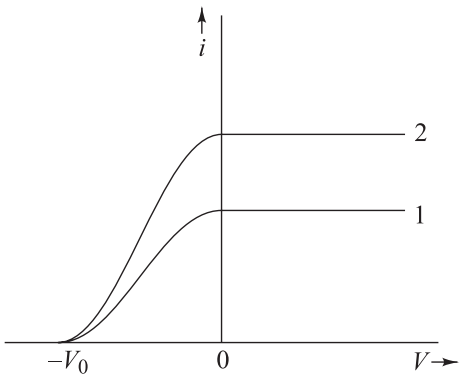


Fig. 18.9

87. A beam of light consisting of two wavelengths λ_1= 496 nm and λ_2 = 620 nm falls on a metal surface. The maximum speeds of photoelectrons corresponding to λ_1 and λ_2 are v_1 and v_2 respectively. If v_1: v_2 = $\sqrt{2}:1$ and hc =1240 eV nm, the work function of the metal is

(a) 1.0 eV (b) 1.5 eV

(c) 2.0 eV (d) 2.5 eV

88. A beam of light consisting of two wavelengths 420 nm and 540 nm falls on two metals 1 and 2 of work functions 4.0 eV and 2.5 eV respectively. If the intensities of the two wavelengths are equal, which of the graphs shown in Fig. 18.10 representing the variation of photoelectric current (i) with voltage (V) is correct? Take hc = 1240 eV nm

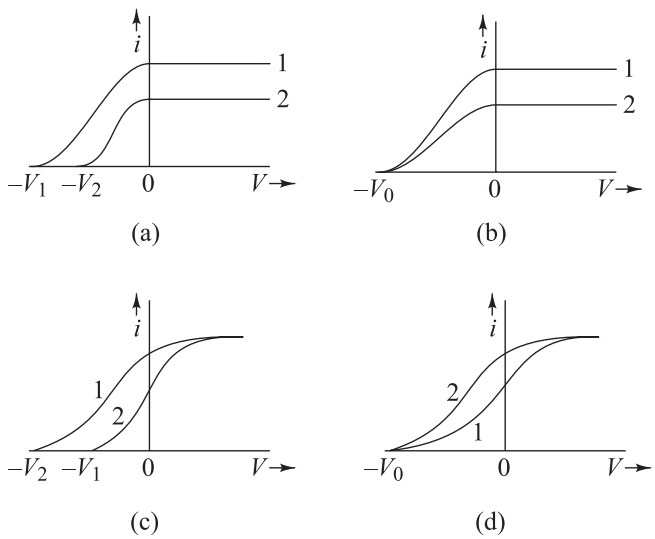


Fig. 18.10

Answers

Level A

1. (d)	2. (d)	3. (c)	4. (b)
5. (a)	6. (d)	7. (d)	8. (c)
9. (d)	10. (b)	11. (d)	12. (a)
13. (a)	14. (a)	15. (d)	16. (a)
17. (b)	18. (a)	19. (a)	20. (b)
21. (b)			

Level B

22. (c)	23. (c)	24. (a)	25. (a)
26. (b)	27. (b)	28. (d)	29. (c)
30. (d)	31. (b)	32. (d)	33. (a)
34. (a)	35. (b)	36. (d)	37. (b)
38. (b)	39. (c)	40. (d)	41. (d)
42. (a)	43. (a)	44. (d)	45. (c)
46. (c)	47. (b)	48. (b)	49. (a)
50. (a)	51. (d)	52. (a)	53. (d)
54. (c)	55. (a)	56. (c)	57. (c)
58. (d)	59. (d)	60. (b)	61. (a)
62. (b)	63. (a)	64. (a)	65. (b)
66. (d)	67. (b)	68. (c)	69. (c)
70. (b)	71. (d)	72. (c)	73. (b)
74. (b)	75. (d)	76. (d)	77. (b)
78. (a)	79. (d)	80. (b)	81. (c)
82. (c)	83. (c)	84. (d)	85. (b)
86. (a)	87. (b)	88. (a)	

Solutions

Level A

1. The only incorrect statement is (d).
2. Since $K_{max} = h(\nu - \nu_0)$, the slope of the graph of K_{max} against ν is h, the Planck's constant.
3. The correct choice is (c).
4. The photon penetrates a distance of about 10^{-8} m before it is absorbed. It imparts all its energy to a single electron. Part of this energy is used up in freeing the electron from the atom of the metal.

This energy is called the work function which is a characteristic of the metal. Hence the correct choice is (b).

5. The threshold frequency ν_0 is given by

$$\nu_0 = \frac{W_0}{h}$$

Now $W_0 = 3.31 \text{ eV} = 3.31 \times 1.6 \times 10^{-19}$ J. Therefore

$$\nu_0 = \frac{3.31 \times 1.6 \times 10^{-19} \text{ J}}{6.62 \times 10^{-34} \text{ Js}} = 0.8 \times 10^{15} \text{ Hz}$$

6. $K_{max} = h\ (\nu - \nu_0)$ where $\nu_0 = 0.8 \times 10^{15}$ Hz and $\nu = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ ms}^{-1}}{3 \times 10^{-7} \text{ m}} = 10^{15}$ Hz.

Hence $K_{max} = 6.62 \times 10^{-34}\ (10^{15} - 0.8 \times 10^{15}) = 1.324 \times 10^{-19}$ J. Hence the correct choice is (d).

7. Wavelength of incident radiation is $\lambda = 1100$ Å $= 1100 \times 10^{-10}$ m. Therefore, frequency of incident radiation is

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{1100 \times 10^{-10}} \simeq 0.3 \times 10^{16} \text{ Hz}$$

Threshold wavelength is $\lambda_0 = 330$ Å $= 330 \times 10^{-10}$ m. Therefore, threshold frequency is

$$\nu_0 = \frac{c}{\lambda_0} = \frac{3 \times 10^8}{330 \times 10^{-10}} = 0.9 \times 10^{16} \text{ Hz}$$

Since $\nu < \nu_0$, no photoelectron is emitted.

8. When the distance of the source from the cell is increased, the intensity of light falling on the cell decreases; but the cut-off voltage of the cell is independent of the intensity of the incident radiation. Hence the correct choice is (c).
9. The correct choice is (d).
10. Cut-off wavelength $\lambda_0 = 6000$ Å $= 6 \times 10^{-7}$ m. Therefore, threshold frequency is

$$\nu_0 = \frac{c}{\lambda_0} = \frac{3 \times 10^8}{6 \times 10^{-7}} = 0.5 \times 10^{15} \text{ Hz}$$

Work function $W_0 = h\nu_0 = 6.6 \times 10^{-34} \times 0.5 \times 10^{15}$
$= 3.3 \times 10^{-19}$ J.

11. Photoelectrons will be emitted if the wavelength of light is less than the threshold wavelength 5200 Å. Wavelength of sodium light is about 6000 Å, that of red light is about 8000 Å and that of infrared light is about 10,000 Å. But the wavelength of ultraviolet light is less than 2000 Å. Hence the correct choice is (d).
12. The correct choice is (a).
13. The correct choice is (a).
14. The correct choice is (a).
15. The correct choice is (d).
16. Energy of a photon is $E = h\nu$. Now $\nu = \frac{c}{\lambda}$. Hence

$$E = \frac{hc}{\lambda}$$

or $$\lambda = \frac{hc}{E}$$

Hence the correct choice is (a).

17. Work function $W_0 = h\nu_0 = 3.3 \text{ eV} = 3.3 \times 1.6 \times 10^{-19}$ J. Therefore, the threshold frequency is

$$\nu_0 = \frac{W_0}{h} = \frac{3.3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 0.8 \times 10^{15} \text{ Hz}$$

18. Maximum energy $E_{max} = h\ (\nu - \nu_0)$ where ν is given by

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{3 \times 10^{-7}} = 10^{15} \text{ Hz}$$

$$\therefore \quad E_{max} = h\ (10^{15} - 0.8 \times 10^{15})$$
$$= 6.6 \times 10^{-34} \times 0.2 \times 10^{15}$$
$$= 1.32 \times 10^{-19} \text{ J}$$
$$= \frac{1.32 \times 10^{-19}}{1.6 \times 10^{-19}} = 0.825 \text{ eV}$$

19. $E_{max} = eV_0$, where V_0 is the stopping potential. Therefore,

$$V_0 = \frac{E_{max}}{e} = 0.825 \text{ V}$$

20. $E_{max} = h\nu_{max} = \frac{hc}{\lambda_{min}} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{0.33 \times 10^{-10}}$
$= 6 \times 10^{-15}$ J $= 37.5$ keV

21. $\lambda = 3500$ Å $= 3500 \times 10^{-10}$ m $= 3.5 \times 10^{-7}$ m. The energy of the incident light is

$$E = h\nu = \frac{hc}{\lambda} \text{ joule}$$

or $$E = \frac{hc}{\lambda e} \text{ electron volt}$$

where $h = 6.63 \times 10^{-34}$ Js is the Planck's constant, $c = 3 \times 10^8$ ms^{-1} is the speed of light in air and $e = 1.6 \times 10^{-19}$ C is the charge of electron. Substituting the values of λ, h, c and e and solving we get $E = 3.55$ eV. Photoelectrons will be emitted from a metal if the energy of the incident light is greater than the work function. Since the work function of metal B is less than E, metal B will emit photoelectrons.

Level B

22. Frequency of incident radiation is

$$\nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{2000\times10^{-10}} = 1.5\times10^{15}\ \text{Hz}$$

The work function of the metal is

$$W_0 = h\nu - eV_0$$
$$= 6.6\times10^{-34}\times1.5\times10^{15} - 1.6\times10^{-19}\times1.5$$
$$= 7.5\times10^{-19}\ \text{J}$$

23. The correct choice is (c).

24. The frequency of X-rays is higher than that of ultraviolet light. Now $K_{max} = h(\nu - \nu_0)$. Hence K_{max} increases as ν is increased. Also $K_{max} = eV_0$, where V_0 is the stopping potential. Hence V_0 also increases with frequency. Hence the correct choice is (a).

25. The cut-off wavelength λ_0 is given by

$$\lambda_0 = \frac{hc}{W_0} = \frac{6.6\times10^{-34}\times3\times10^{8}}{4.125\times1.6\times10^{-19}}$$
$$= 3000\times10^{-10}\ \text{m} = 3000\ \text{Å}$$

26. $W_0 = h\nu_0$. Thus, the greater the work function, the higher is the threshold frequency and hence the lower is the threshold wavelength. Hence the correct choice is (b).

27. Since the work function of sodium is lower than that of copper, it is easier to extract electrons from sodium than from copper. Therefore, the electrons ejected from sodium will have a greater kinetic energy and will hence need a greater stopping potential. Thus the correct choice is (b).

28. If the distance is doubled, the intensity, being proportional to $1/r^2$, falls to one-fourth of its earlier value. Now, the energy of emitted electrons is independent of the intensity, but the number of electrons emitted per second is proportional to intensity. Hence the correct choice is (d).

29. Lenses of equal diameters collect the same amount of light so that the intensity remains the same; hence the photoelectric current also remains the same.

30. Energy of photon $h\nu = mc^2$. Therefore,

$$m = \frac{h\nu}{c^2} = \frac{h}{c\lambda} \qquad (\because c = \nu\lambda)$$

$\therefore$ Momentum of photon $= mc = \frac{h}{\lambda}$. Hence the correct choice is (d).

31. The maximum wavelength of visible light is about 8000 Å (the red end of the visible spectrum). The energy of a photon of wavelength λ = 8000 Å = 8×10^{-7} m is

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{8\times10^{-7}}$$
$$\simeq 2.5\times10^{-19}\ \text{J} \simeq 1.5\ \text{eV}$$

32. The wavelength of blue light is longer than that of violet light and the wavelength of sodium light (yellow light) is longer than that of blue light. Since no photoelectric emission occurs for blue light, it follows that the wavelength threshold (λ_{min}) is less than the wavelength of blue light and hence less than that of yellow light. Hence the correct choice is (d).

33. $K_{max} = eV_0$. Also $K_{max} = e(\nu - \nu_0) = e\nu - W_0 = 2.4$ eV − 1.6 eV = 0.8 eV. Thus $eV_0 = 0.8$ eV or $V_0 = 0.8$ V.

34. Energy of photons of frequency $(\nu) = h\nu$. If the lamp emits ν photons per second, then the power of the lamp is $P = nh\nu$. Now, $\nu = c/\lambda$. Therefore

$$P = \frac{nhc}{\lambda}$$
$$= \frac{10^{20}\times6.6\times10^{-34}\times3\times10^{8}}{660\times10^{-9}} = 30\,\text{W}$$

35. $h\nu = h\nu_0 + eV$, where V is the stopping potential. For frequency ν, we have

$$h\nu = h\nu_0 + \frac{eV_0}{2} \qquad \text{(i)}$$

and for frequency $\nu/2$, we have

$$\frac{h\nu}{2} = h\nu_0 + eV_0 \qquad \text{(ii)}$$

From (i) and (ii) on eliminating V_0, we get $\nu_0 = 3\nu/2$. Hence the correct choice is (b).

36. $h\nu = h\nu_0 + eV$. Since $\nu = c/\lambda$, we have $\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + eV$, where λ_0 is the threshold wavelength. For wavelength λ, we have

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + 2\,eV_0 \qquad \text{(i)}$$

and for wavelength 2λ, we have

$$\frac{hc}{2\lambda} = \frac{hc}{\lambda_0} + \frac{eV_0}{2} \qquad \text{(ii)}$$

Eliminating V_0 from (i) and (ii) we get $\lambda_0 = 3\lambda$. Hence the correct choice is (d).

37. For light of frequency $2\nu_0$, we have

$$2h\nu_0 = h\nu_0 + eV_0 \text{ or } h\nu_0 = eV_0 \qquad \text{(i)}$$

For light of frequency $3\nu_0$, we have

$$3\,h\nu_0 = h\nu_0 + eV \text{ or } 2h\nu_0 = eV \qquad \text{(ii)}$$

From (i) and (ii) we get $V = 2V_0$. Hence the correct choice is (b).

38. $\frac{1}{2}\,mv^2_{\max} = h\,(\nu - \nu_0)$. For light of frequency $\nu = 2\,\nu_0$

we have $\frac{1}{2}\,mv^2_0 = h\,(2\nu_0 - \nu_0) = h\nu_0 \qquad \text{(i)}$

For light of frequency $\nu = 5\,\nu_0$, we have

$$\frac{1}{2}\,mv^2 = h\,(5\nu_0 - \nu_0) = 4h\nu_0 \qquad \text{(ii)}$$

Dividing (ii) by (i) we get $v^2 = 4v_0^2$ or $v = 2\,v_0$. Hence the correct choice is (b).

39. $W_0 = h\nu_0 = \frac{hc}{\lambda_0}$. For a metal of work function $W_0/2$, the threshold wavelength λ is given by

$$\frac{W_0}{2} = \frac{hc}{\lambda}$$

Thus $\lambda = 2\lambda_0$. Hence the correct choice is (c).

40. The frequency never changes on reflection or refraction. Now, energy a photon = $h\nu$. Hence the correct choice is (d).

41. Kinetic energy $\frac{1}{2}\,mv^2 = eV$. Therefore $m^2v^2 = 2\,meV$. Thus

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}}$$

Hence the correct choice is (d).

42. The wavelength associated with a particle of charge q, mass m and accelerated through a potential difference V is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

or $V = \frac{h^2}{2mq\lambda^2}$

for proton: $V = \frac{h^2}{2m_p q_p \lambda^2}$

For α-particle : $V' = \frac{h^2}{2m_\alpha q_\alpha \lambda^2}$

$$\therefore \quad \frac{V'}{V} = \frac{m_p}{m_\alpha} \times \frac{q_p}{q_\alpha} = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$(\because m_\alpha = 4\,m_p \text{ and } q_\alpha = 2q_p)$

Thus $V' = V/8$. Hence the correct choice is (a).

43. $\lambda_e = \frac{h}{m_e v}$ and $\lambda_p = \frac{h}{m_p v}$. Therefore

$$\frac{\lambda_p}{\lambda_e} = \frac{m_e}{m_p} << 1 \qquad (\because m_e << m_p)$$

Hence the correct choice is (a).

44. The de Broglie wavelength of a particle of mass m moving with a speed v is given by.

$$\lambda = \frac{h}{mv}$$

Kinetic energy $K = \frac{1}{2}\,mv^2$ or $mv^2 = 2K$ or $m^2v^2 = 2Km$ or $mv = \sqrt{2Km}$. Therefore

$$\lambda = \frac{h}{\sqrt{2Km}}$$

Since K is the same for both particles, we have

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{2m}{m}} = \sqrt{2}$$

Hence the correct choice is (d).

45. Kinetic energy $KE = \frac{3}{2}\,kT$ or $\frac{1}{2}\,mv^2 = \frac{3}{2}\,kT$. Thus

$$p = mv = \sqrt{3mkT}$$

$$\therefore \quad \lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}}$$

46. We have seen above that $\lambda \propto \frac{1}{\sqrt{T}}$. Hence

$$\frac{\lambda'}{\lambda} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{927 + 273}{27 + 273}} = 2$$

$\therefore \quad \lambda' = 2\lambda$. Hence the correct choice is (c).

47. In terms of kinetic energy $K = \frac{1}{2}\,mv^2$, de Broglie wavelength is

$$\lambda = \frac{h}{\sqrt{2mK}}$$

Thus $\lambda \propto \frac{1}{\sqrt{K}}$. If K is increased by a factor of 4, λ is decreased by a factor of 2. Hence the correct choice is (b).

48. $K = 180$ eV $= 180 \times 1.6 \times 10^{-19}$ J $= 2.88 \times 10^{-17}$ J.

Now $\lambda = \frac{h}{\sqrt{2mK}}$

$$\therefore \quad \lambda = \frac{6.6\times10^{-34}}{\left[2\times9\times10^{-31}\times2.88\times10^{-17}\right]^{1/2}}$$

$$= \frac{6.6\times10^{-34}}{7.2\times10^{-24}} = 0.9\times10^{-10}\ \text{m}$$

49. β–particle is an electron. Since electron has the smallest mass, its de Broglie wavelength is the longest as the velocity of all particles is the same.

50. $c = \nu\lambda$ and $\lambda = h/p$. Therefore $p = \frac{h}{\lambda} = \frac{\nu h}{c}$. Hence the correct choice is (a).

51. The speed of photon = $c = 3\times10^8\ \text{ms}^{-1}$. Wavelength of photon is, say, λ.

Wavelength of electron = $\frac{h}{mv} = \lambda$. Now

$$\frac{\text{KE of electron}}{\text{KE of photon}} = \frac{\frac{1}{2}mv^2}{h\nu} = \frac{1}{2}\frac{mv^2}{hc}\lambda$$

$$= \frac{mv^2}{2hc}\cdot\frac{h}{mv} \quad \left(\because \lambda = \frac{h}{mv}\right)$$

$$= \frac{v}{2c} = \frac{1}{4} \quad (\because v = c/2).$$

52. Given $K_{max} = \frac{hc}{\lambda_1} - W_0$ (i)

and $2K_{max} = \frac{hc}{\lambda_2} - W_0$ (ii)

Dividing (ii) by (i), we get

$$2 = \frac{\frac{hc}{\lambda_2} - W_0}{\frac{hc}{\lambda_1} - W_0}$$

which gives $W_0 = \frac{hc}{\lambda_1\lambda_2}(2\lambda_2 - \lambda_1)$ which is choice (a).

53. KE = $h\nu - h\nu_0$. Now $h\nu_0 = 6\times10^{-34}\times1.6\times10^{15} = 9.6\times10^{-19}$ J = 6 eV.

Given $h\nu = 8$ eV. Hence KE = 8 – 6 = 2 eV, which is choice (d).

54. Using $eV_0 = \frac{1}{2}mv^2_{max}$, we have

$$V_0 = \frac{1}{2}\times\frac{v^2_{max}}{e/m} = \frac{1}{2}\times\frac{\left(1.76\times10^6\right)^2}{1.76\times10^{11}} = 8.8\ \text{V}$$

55. Number of photons emitter per second is

$$n = \frac{\text{power}}{\text{energy of one photon}} = \frac{P}{h\nu}$$

$$= \frac{6.6\times10^3}{6.6\times10^{-34}\times10^6} = 10^{31}$$

Hence the correct choice is (a).

56. The correct choice is (c).

57. $eV_0 = h\nu$. Therefore, $\nu = \frac{e}{h}V_0$. Hence the slope of ν versus V_0 graph is $\frac{e}{h}$ which is choice (c).

58. $E = \frac{1}{2}mv^2 = h\nu_0 - W_0$. Now $E_1 = 2 - 1 = 1$ eV and $E_2 = 10 - 1 = 9$ eV. Therefore $E_1/E_2 = 1/9$, i.e.

$$\frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{1}{9}$$

or $\frac{v_1}{v_2} = \frac{1}{3}$. Hence the correct choice is (d).

59. $\frac{E}{p} = \frac{h\nu}{h/\lambda} = \nu\lambda = c$. Hence the correct choice is (d).

60. $h\nu = mc^2$ or $\frac{hc}{\lambda} = mc^2$ ($\because c = \nu\lambda$). Hence $m = \frac{h}{\lambda c}$ which is choice (b).

61. For nth Bohr orbit, $mvr = \frac{nh}{2\pi}$. The de-Broglie wavelength is

$$\lambda = \frac{h}{mv}$$

But $mv = \frac{nh}{2\pi r}$. Therefore,

$$\lambda = h\times\frac{2\pi r}{nh} = \frac{2\pi r}{n}$$ which is choice (a).

62. Refer to the solution of Q. 68. The correct choice is (b).

63. Given $mv = \frac{h}{\lambda}$. Hence the correct choice is (a).

64. Given $\frac{hc}{\lambda_1} - W_0 = K_1$ (i)

and $\frac{hc}{\lambda_2} - W_0 = K_2$ (ii)

Eliminate hc from (i) and (ii). The correct choice is (a).

65. $h\nu = E + W_0 = 0.5 \text{ eV} + W_0$ (i)

When the energy of the incident photon is increased by 20%, we have

$$\frac{6}{5}\ h\nu = E' + W_0 = 0.8 \text{ eV} + W_0 \quad \text{(ii)}$$

Subtracting (ii) from (i), we get $h\nu = 1.5$ eV. Hence $W_0 = h\nu - 0.5$ eV $= 1.5$ eV $- 0.5$ eV $= 1.0$ eV. Thus the correct choice is (b).

66. The stopping potential depends on the frequency (or wavelength) of the incident electromagnetic wave and is independent of the distance of the source from photocell. Hence the stopping potential will still be 0.6 V. However, the saturation current varies as $1/r^2$, where r is the distance of the source from the photocell. Since r is increased by a factor of 3, the saturation current will decrease by a factor of $(3)^2 = 9$, i.e. it will be 18 mA/9 = 2 mA at $r = 0.6$ m. Hence the correct choice is (d).

67. The stopping potential in volts = kinetic energy of the emitted photoelectrons in eV. Hence the correct choice is (b).

68. $\lambda_{\max} = \dfrac{hc}{W_0} = \dfrac{6.626 \times 10^{-34} \times 3 \times 10^8}{4.0 \times 1.6 \times 10^{-19}} = 3.10 \times 10^{-7}$ m

= 310 nm, which is choice (c).

69. Let v_1 and v_2 be the respective speeds of the two particles. The law of conservation of linear momentum gives

$$m_1 v_1 + m_2 v_2 = 0 \text{ or } \left|\frac{m_2 v_2}{m_1 v_1}\right| = 1.0$$

Since de Broglie wavelength $\lambda = h/(mv)$, we will have

$$\frac{\lambda_1}{\lambda_2} = \frac{m_2 v_2}{m_1 v_1} = 1.0, \text{ which is choice (c).}$$

70. $E_1 = h(\nu_1 - \nu_0)$ and $E_2 = h\ (\nu_2 - \nu_0)$. Dividing them, we get

$$\frac{E_2}{E_1} = \frac{\nu_2 - \nu_0}{\nu_1 - \nu_0}$$

Given $E_2 = n\ E_1$. Hence, we have

$$n = \frac{\nu_2 - \nu_0}{\nu_1 - \nu_0}$$

which gives $\nu_0 = \dfrac{n\nu_1 - \nu_2}{(n-1)}$, which is choice (b).

71. Let v_1 and v_2 be the maximum velocities of photoelectrons in the two cases. The energies of photon are given by

$$E_1 = W_0 + \frac{1}{2}\ mv_1^2 \quad \text{(i)}$$

And $$E_2 = W_0 + \frac{1}{2}\ mv_2^2 \quad \text{(ii)}$$

Given $E_1 = 2W_0$ and $E_2 = 3W_0$. Using these in (i) and (ii), we have

$$2W_0 = W_0 + \frac{1}{2}\ mv_1^2 \rightarrow \frac{1}{2}\ mv^2{}_1 = W_0 \quad \text{(iii)}$$

and $$3W_0 = W_0 + \frac{1}{2}\ mv_2^2 \rightarrow \frac{1}{2}\ mv^2{}_2 = 2W_0 \quad \text{(iv)}$$

From (iii) and (iv), we get $\dfrac{v_1}{v_2} = \dfrac{1}{\sqrt{2}}$, which is choice (d).

72. Let m_0 and e_0 be the mass and charge of a proton and m_1 and e_1 those of α-particle. Then

$$\lambda_0 = \frac{h}{\sqrt{2m_0 e_0 V}} \text{ and } \lambda_1 = \frac{h}{\sqrt{2m_1 e_1 V}}$$

Dividing, we get $$\frac{\lambda_1}{\lambda_0} = \sqrt{\frac{m_0}{m_1} \times \frac{e_0}{e_1}} \quad \text{(i)}$$

Now, α-particle has twice the charge and 4 times the mass of proton, i.e. $m_1 = 4\ m_0$ and $e_1 = 2\ e_0$. Using these in Eq. (i) we get

$\lambda_1 = \dfrac{\lambda_0}{2\sqrt{2}}$, which is choice (c).

73. $\lambda = \dfrac{h}{mv}$ is the de Broglie wavelength. The energy of a photon of this wavelength is

$$E = h\nu = \frac{hc}{\lambda} = \frac{hc}{h/mv} = mvc \quad \text{(i)}$$

Kinetic energy of the particle is

$$E' = \frac{1}{2}\ mv^2 \quad \text{(ii)}$$

From (i) and (ii), we have

$$\frac{E'}{E} = \frac{\frac{1}{2}mv^2}{mvc}$$

$$= \frac{v}{2c} = \frac{2.25 \times 10^8}{2 \times \left(3 \times 10^8\right)} = \frac{2.25}{6} = \frac{3}{8}$$

Hence the correct choice is (b).

74. $h\nu = eV + W_0$ or $eV = h\nu - W_0 = h\nu - h\nu_0$. Now $\nu = c/\lambda$ and $\nu_0 = c/\lambda_0$. Thus, for wavelength λ, we have

$$eV = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \quad \text{(i)}$$

and for wavelength $\lambda' = 2\lambda$, we have ($\because V' = V/3$)

$$\frac{eV}{3} = hc\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right)$$

or $$eV = 3hc\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right) \quad \text{(ii)}$$

From (i) and (ii) we have

$$\frac{1}{\lambda} - \frac{1}{\lambda_0} = 3\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right)$$

which gives $\lambda_0 = 4\lambda$. Hence the correct choice is (b).

75. For wavelength λ we have

$$\frac{1}{2}mv^2 = h\nu - W_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = \frac{hc(\lambda_0 - \lambda)}{\lambda\lambda_0} \quad \text{(i)}$$

For wavelength λ', we have

$$\frac{1}{2}mv'^2 = \frac{hc(\lambda_0 - \lambda')}{\lambda'\lambda} \quad \text{(ii)}$$

From (i) and (ii) we get

$$\frac{v'^2}{v^2} = \frac{\lambda}{\lambda'} \times \frac{(\lambda_0 - \lambda')}{(\lambda_0 - \lambda)}$$

Now $\lambda' = 3\lambda/4$. Hence

$$\frac{v'^2}{v^2} = \frac{4}{3}\frac{(\lambda_0 - 3\lambda/4)}{(\lambda_0 - \lambda)}$$

Therefore, $\frac{v'^2}{v^2}$ is greater than $\frac{4}{3}$. Hence $v' > v\sqrt{\frac{4}{3}}$.

76. The kinetic energy of the emitted photoelectrons is given by

$$\text{KE} = h(\nu - \nu_0)$$

where ν is the frequency of the incident radiation, ν_0 is the threshold frequency and h is the Planck's constant. Hence the slope of the graph of KE Vs ν is h, the Planck's constant, which is a universal constant. Hence the correct choice is (d).

77. $$eV_1 = h(\nu_1 - \nu_0)$$
$$eV_2 = h(\nu_2 - \nu_0)$$
$$\therefore \quad V_1 - V_2 = \frac{h}{e}(\nu_1 - \nu_2)$$
$$\Rightarrow \quad V_1 - V_2 = \frac{hc}{e}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) \quad (1)$$

Substituting the values of h, c, e, λ_1 and λ_2 and solving we get

$$V_1 - V_2 \simeq 0.4$$
$$\Rightarrow \quad V_2 = V_1 - 0.4 = 2.0 - 0.4 = 1.6 \text{ V}$$

So the correct choice is (b).

78. $eV_0 = h(\nu - \nu_0) = h\nu - h\nu_0$

$$V_0 = \left(\frac{h}{e}\right)\nu - \left(\frac{h}{e}\right)\nu_0 \quad (1)$$

From Eq. (1) it follows that the slope of the graph is $\frac{h}{e}$. Therefore

$$h = e \times \text{slope} = \left(1.6\times10^{-19}\right)\times\frac{16.5-0}{(8-4)\times10^{15}} = \frac{\left(1.6\times10^{-19}\right)\times16.5}{4\times10^{15}}$$
$$= 6.60 \times 10^{-34} \text{ Js}$$

So the correct choice is (a).

79. It is clear from Eq. (1) that if $V_0 = 0$, $\nu = \nu_0$, Therefore, from the given graph, $\nu_0 = 4 \times 10^{15}$ Hz. We have seen above that $h = 6.60 \times 10^{-34}$ Js.

$$W_0 = h\nu_0 = (6.60 \times 10^{-34}) \times (4 \times 10^{15}) \text{ J} = 26.4 \times 10^{-19} \text{ J}$$
$$= \frac{26.4\times10^{-19}}{1.6\times10^{-19}} = 16.5 \text{ eV}$$

Hence the correct choice is (d).

80. $$W_0 = h\nu - eV_0$$
$$\Rightarrow \quad W_0 = \frac{hc}{\lambda} - eV_0 \quad (1)$$

Substituting the values of h, λ, e and V_0, we get $W_0 = 6 \times 10^{-19}$ J = 3.75 eV, which is choice (b).

81. $$K_{\max} = h\nu - W_0 = \frac{hc}{\lambda} - W_0 = \frac{\left(6.6\times10^{-34}\right)\times\left(3\times10^{8}\right)}{99\times10^{-9}} - 1.0 \times 1.6 \times 10^{-19}$$
$$= 1.84 \times 10^{-18} \text{ J}$$

The wavelength of the fastest photoelectron is

$$\lambda = \frac{h}{\sqrt{2mK_{\max}}}$$

$$= \frac{6.6\times10^{-34}}{\sqrt{2\times\left(9.1\times10^{-31}\right)\times1.84\times10^{-18}}}$$

$$= 0.36 \text{ nm}$$

So the correct choice is (c).

82. Energy of photon before collision is

$$E_i = h\,\nu_1 = \frac{hc}{\lambda_1}$$

$$= \frac{\left(6.6\times10^{-34}\right)\times\left(3\times10^{8}\right)}{19.8\times10^{-9}} = 10\times10^{-18}\text{ J}$$

$\therefore$ Energy of the scattered electron is

$$E_f = \frac{hc}{\lambda_2}$$

$$= \frac{\left(6.6\times10^{-34}\right)\times\left(3\times10^{8}\right)}{30\times10^{-9}} = 6.6\times10^{-18}\text{ J}$$

$\therefore$ Energy of the scattered electron is

$$E = E_i - E_f$$

$$= 10\times10^{-18} - 6.6\times10^{-18}$$

$$= 3.4\times10^{-18}\text{ J} \simeq 22\text{ eV}$$

So the correct choice is (c).

83. $E_X = h\nu = \frac{hc}{\lambda}$

$$E_e = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} \qquad \left(\because \lambda = \frac{h}{p}\right)$$

$$\therefore \qquad \frac{E_X}{E_e} = \frac{hc}{\lambda}\times\frac{2m\lambda^2}{h^2} = \frac{2mc\lambda}{h}$$

So the correct choice is (c).

84. $\lambda_e = \frac{h}{\sqrt{2meV}}$

$$= \frac{6.6\times10^{-34}}{\sqrt{2\times\left(9\times10^{-31}\right)\times\left(1.6\times10^{-19}\right)\times\left(50\times10^{3}\right)}}$$

$$\simeq 5.5\times10^{-12}\text{ m}$$

$\lambda_y = 590\text{ nm} = 5.9\times10^{-7}\text{ m}$

Now, resolving power $\propto \frac{1}{\lambda}$. Therefore

$$\frac{\text{R.P. of electron microscope}}{\text{R.P. of optical microscope}} = \frac{\lambda_y}{\lambda_e}$$

$$= \frac{5.9\times10^{-7}}{5.5\times10^{-12}}$$

$$\simeq 10^5$$

So the correct choice is (d).

85. Stopping potential in volts = kinetic energy of the emitted photoelectrons in eV. Hence the correct choice is (b).

86. It follows from the graphs that the stopping potential V_0 is the same for two radiations. We know that

$$K_{max} = eV_0$$

and $\qquad K_{max} = h\,\nu - W_0 \qquad (1)$

So K_{max} is the same for the two radiation. For a given photo-sensitive surface, W_0 is constant. Since K_{max} and W_0 are both the same, it follow from Eq. (1) that ν is the same for the two radiations. Hence $\nu_1 = \nu_2$. Since photo-electric current i is proportional to the intensity I of incident radiation, it follow from the given graphs that $I_2 > I_1$. Hence the correct choice is (c).

87. $h\nu = K_{max} + W_0$

$$K_{max} = h\nu - W_0$$

$$\frac{1}{2}m\,v_{max}^2 = \frac{hc}{\lambda} - W_0$$

$$\therefore \qquad \frac{1}{2}m\,v_1^2 = \frac{hc}{\lambda_1} - W_0 \qquad (1)$$

and $\qquad \frac{1}{2}m\,v_2^2 = \frac{hc}{\lambda_2} - W_0 \qquad (2)$

Dividing (1) and (2), we get

$$\frac{v_1^2}{v_2^2} = \frac{\frac{hc}{\lambda_1} - W_0}{\frac{hc}{\lambda_2} - W_0} \qquad (3)$$

Given $\frac{v_1}{v_2} = \sqrt{2}$ and hc =1240 eV nm. Substituting these values in (3), We have

$$2 = \frac{\frac{1240\,\text{eVnm}}{496\,\text{nm}} - W_0}{\frac{1240\text{ eVnm}}{620\text{ nm}} - W_0}$$

or $\qquad 2 = \frac{2.5\,\text{eV} - W_0}{2\,\text{eV} - W_0}$

$\Rightarrow \qquad W_0 = 1.5$ eV, which is choice (b).

88. Photoelectric emission occurs if the wavelength of incident radiation is less than the cut-off wavelength for the metal given by

$$\lambda_0 = \frac{hc}{W_0}$$

For metals 1 and 2 the cut-off wavelengths are

$$\lambda_1 = \frac{hc}{W_1} = \frac{1240\,\text{eV nm}}{4.0\,\text{eV}} = 310\ \text{nm}$$

and $$\lambda_2 = \frac{1240\,\text{eV nm}}{2.5\,\text{eV}} = 496\ \text{nm}$$

Hence metal 1 emits photoelectrons for both the given wavelengths (420 nm and 540 nm) but metal 2 emits photoelectrons only for wavelength 540 nm. Therefore, photoelectric current i will be move for metel 1 than for metal 2.

Now, for metal 1 and for $\lambda = 540$ nm, the stopping potential V_1 is given by

$$eV_1 = \frac{hc}{\lambda} - W_1$$

$$= \frac{1240\,\text{eV nm}}{540\,\text{nm}} - 4.0 = -1.7\ \text{eV}$$

which gives $V_1 = -1.7$ eV.

For metal 2 and for $\lambda = 540$ nm, the stopping potental V_2 is given by

$$eV_2 = \frac{hc}{\lambda} - W_2$$

$$= \frac{1240\,\text{eV nm}}{540\,\text{nm}} - 2.5 = -0.2\ \text{eV}$$

which gives $V_2 = -0.2$V. Hence the correct graph is (a).

2 SECTION — Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage.

Passage I

A beam of light has three wavelengths 440 nm, 495 nm and 660 nm with a total intensity of 3.24×10^{-3} Wm^{-2} equally distributed amongst the three wavelengths. The beam falls normally on an area of 1.0 cm^2 of a clean metallic surface of work function 2.2 eV. Assume that there is no loss of light by reflection and each energetically capable photon ejects one electron and take $h = 6.6 \times 10^{-34}$ Js.

1. Photoelectric emission is caused by
   (a) light of wavelength 440 nm alone
   (b) light of wavelength 660 nm alone
   (c) lights of wavelengths 440 nm and 495 nm
   (d) lights of wavelengths 495 nm and 660 nm
2. The incident energy (in Js^{-1}) of each wavelength is
   (a) 3.24×10^{-7} (b) 1.62×10^{-7}
   (c) 1.08×10^{-7} (d) 0.81×10^{-7}
3. The total number of photoelectrons liberated per second is
   (a) 4.9×10^{11} (b) 5.1×10^{11}
   (c) 5.3×10^{11} (d) 5.5×10^{11}

Solutions

1. The threshold wavelength is

$$\lambda_0 = \frac{hc}{W_0} = \frac{(6.6\times10^{-34})\times(3\times10^{8})}{2.2\times1.6\times10^{-19}}$$

$$= 6 \times 10^{-7}\ \text{m} = 600\ \text{nm}$$

Out of the three given wavelengths, two wavelengths $\lambda_1 = 440$ nm and $\lambda_2 = 495$ nm will cause photoelectric emission as these wavelengths are less than λ_0. Thus the correct choice is (c).

2. Intensity of each wavelength is $I = \frac{1}{3} \times 3.24 \times 10^{-3} = 1.08 \times 10^{-3}$ W m^{-2}. Area of metal surface is $A = 1$ cm^2 $= 1 \times 10^{-4}$ m^2. Therefore, energy of each wavelength is $E = I \times A = 1.08 \times 10^{-7}$ J s^{-1}, which is choice (c).
3. Let n_1 be the number of photons of wavelength λ_1 incident per second. The energy of one photon = hc/λ_1. Hence

$$E = \frac{n_1 hc}{\lambda_1}$$

or $$n_1 = \frac{E\lambda_1}{hc} = \frac{(1.08\times10^{-7})\times(440\times10^{-9})}{(6.6\times10^{-34})\times(3\times10^{8})}$$

$$= 2.4 \times 10^{11}$$

Similarly $n_2 = 2.7 \times 10^{11}$

$\therefore$ Total number $n = n_1 + n_2 = 5.1 \times 10^{11}$, which is choice (b).

Questions 4 to 6 are based on the following passage.

Passage II

In a photoelectric effect set-up, a point source of light of power 3.2×10^{-3} W emits monoenergetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the centre of a stationary metallic sphere of

work function 3.0 eV and of radius 8.0×10^{-3} m. The efficiency of photoelectric emission is one for every 10^6 incident photons. Assume that the sphere is isolated and initially neutral, and that photoelectrons are instantly swept away after emission.

4. The number of photoelectrons emitted per second is
   (a) 10^5 (b) 10^7
   (c) 10^9 (d) 10^{11}
5. The kinetic energy of the fastest electron is
   (a) 8 eV (b) 5 eV
   (c) 3 eV (d) 2 eV
6. The photoelectric emission stops when the sphere acquires a potential of
   (a) 2 V (b) 3 V
   (c) 5 V (d) 8 V

Solutions

4. Refer to Fig. 18.11.

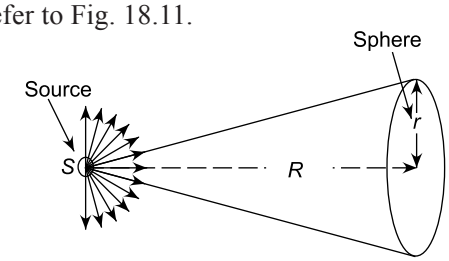


Fig. 18.11

Power of the source of light, $P = 3.2 \times 10^{-3}$ W. Energy of the emitted photon, $E = 5.0$ eV $= 5.0 \times 1.6 \times 10^{-19} = 8.0 \times 10^{-19}$ J.

If r is the radius of the metallic sphere and R its distance from the source S, the power received at the sphere is

$$P' = P \times \frac{\pi r^2}{4\pi R^2} = \frac{P r^2}{4R^2}$$

$$= \frac{3.2 \times 10^{-3} \times (8.0 \times 10^{-3})^2}{4 \times (0.8)^2}$$

$$= 8 \times 10^{-8} \text{ W (Js}^{-1}\text{)}$$

$\therefore$ Number of photons striking the sphere per second is

$$n' = \frac{P'}{E} = \frac{8 \times 10^{-8} \text{ Js}^{-1}}{8.0 \times 10^{-19} \text{ J}} = 10^{11} \text{ s}^{-1}$$

Since one photoelectron is emitted for every 10^6 incident photons, the number of photoelectrons emitted per second is

$$n = \frac{n'}{10^6} = \frac{10^{11}}{10^6} = 10^5 \text{ per second}$$

The correct choice is (a).

5. Kinetic energy of the fastest electron is

$$E_{max} = 5.0 - 3.0 = 2.0 \text{ eV},$$

which is choice (d).

6. Due to the emission of photoelectrons, the metallic sphere acquires a positive charge and it will oppose the ejection of photoelectrons due to attractive force exerted by the positive charge of the sphere on the electrons. The photoelectric emission will stop when the sphere acquires a positive potential equal to the stopping potential. In other words, the work function of the sphere keeps on increasing with time till it becomes equal to 5.0 eV (which is the energy of the incident photon). At this time, the emission of photoelectrons stops.

   Increase in work function = 5.0 eV – 3.0 eV = 2.0 eV. This implies that the photoelectric emission will stop when the sphere has acquired a potential of 2.0 V due to accumulation of charge. Hence the correct choice is (a).

SECTION 3

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only *one* choice is correct.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

1. Statement-1

A particle of mass M at rest decays into two particles of masses m_1 and m_2 which move with

velocities v_1 and v_2 respectively. Their respective de Broglie wavelengths are λ_1 and λ_2. If $m_1 > m_2$, then $\lambda_1 > \lambda_2$.

Statement-2

The de Broglie wavelength of a particle having momentum p is $\lambda = h/p$.

2. **Statement-1**

Figure 18.12 shows the graphs of K_{max} (maximum kinetic energy) of the emitted photoelectrons versus the frequency ν of the incident light for two different metals A and B. The lines for metals A and B are always parallel to each other.

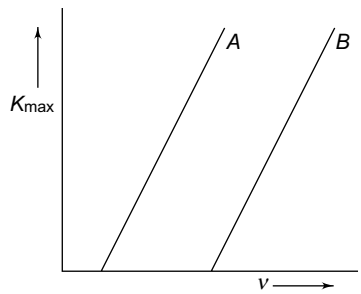


Fig. 18.12

Statement-2

In photoelectric emission $K_{max} = h\nu - W_0$, where W_0 is the work function of the metal.

3. **Statement-1**

From the graph in Q.2 above it follows that the work function of metal B is greater than that of metal A.

Statement-2

The work function does not depend upon the slope of the graph.

4. **Statement-1**

When monochromatic light falls on a photosensitive material, the number of photoelectrons emitted per second is n and their maximum kinetic energy is K_{max}. If the intensity I of the incident light is doubled, n is doubled but K_{max} remains the same.

Statement-2

The value of n is directly proportional to I but K_{max} is independent of I.

5. **Statement-1**

When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, K_{max} increases but V_0 decreases.

Statement-2

Photoelectrons are emitted with speeds ranging from zero to a maximum value. Below a certain negative voltage V_0, no photoelectrons are emitted in a photocell.

Solutions

1. The correct choice is (d). The law of conservation of linear momentum gives

$$m_1v_1 + m_2v_2 = 0 \text{ or } \left|\frac{m_2v_2}{m_1v_1}\right| = 1.0$$

Since de Broglie wavelength $\lambda = h/mv$, we will have

$$\frac{\lambda_1}{\lambda_2} = \frac{m_2v_2}{m_1v_1} = 1.0$$

2. The correct choice is (a). The slope of each graph = h, the Planck's constant.
3. The correct choice is (c). The intercept of the line on the ν-axis gives the threshold frequency ν_0 and work function $W_0 = h\nu_0$. Thus work function = slope × intercept. The value of slope is the same for metals A and B but ν_0 for B is greater than that for A.
4. The correct choice is (a).
5. The correct choice is (d). The frequency of X-rays is higher than that of ultraviolet light. Since $K_{max} = h\nu - W_0$, K_{max} will increase if ν is increased. Also, $K_{max} = eV_0$. Hence V_0 will also increase if ν is increased.

4 SECTION — Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. Sodium and copper have work functions 2.3 eV and 4.5 eV respectively. The ratio of their threshold wavelengths is nearly

(a) $\frac{1}{2}$ (b) 4

(c) 2 (d) $\frac{1}{4}$ **[2002]**

2. Two identical photocathodes receive lights of frequencies f_1 and f_2. If the velocities of the photoelectrons emitted are respectively v_1 and v_2, then (here m = mass of an electron)

   (a) $v_1^2 - v_2^2 = \dfrac{2h}{m}(f_1 - f_2)$

   (b) $v_1 + v_2 = \left[\dfrac{2h}{m}(f_1 + f_2)\right]^{1/2}$

   (c) $v_1^2 + v_2^2 = \dfrac{2h}{m}(f_1 + f_2)$

   (d) $v_1 - v_2 = \left[\dfrac{2h}{m}(f_1 - f_2)\right]^{1/2}$ **[2003]**

3. According to Einstein's photoelectric equation, the graph of all kinetic energy of the emitted photoelectrons from a metal versus the frequency of incident radiation is a straight line whose slope
   (a) depends on the nature of the metal
   (b) depends on the intensity of the radiation
   (c) depends on the nature of metal and the intensity of incident radiation
   (d) is the same for all metals and is independent of the intensity of incident radiation **[2004]**

4. The work function of a metal is 4.0 eV. The longest wavelength of light that can cause photoelectric emission from the metal is approximately
   (a) 540 nm (b) 400 nm
   (c) 310 nm (d) 220 nm **[2004]**

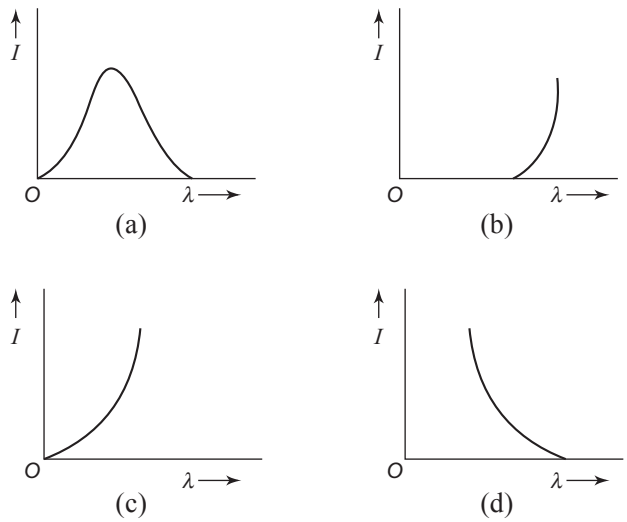
5. A photocell is illuminated by a small bright source placed 1 m away. When the same source is placed $\dfrac{1}{2}$ m away, the number of photoelectrons emitted becomes
   (a) 4 times (b) one-fourth
   (c) 2 times (d) one-half **[2005]**

6. If the kinetic energy of a free electron doubles, its de-Broglie wavelength becomes
   (a) half (b) twice
   (c) $\dfrac{1}{\sqrt{2}}$ times (d) $\sqrt{2}$ times **[2005]**

7. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV and the stopping potential for a radiation incident on this surface is 5 V. The incident radiation lies in
   (a) visible region (b) X-ray region
   (c) ultra-violet region (d) infra-red region
   [2006]

8. The time taken by a photoelectron to come out of the metal after the photon strikes it is approximately
   (a) 10^{-16} s (b) 10^{-1} s
   (c) 10^{-4} s (d) 10^{-10} s **[2006]**

9. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies with λ as follows:

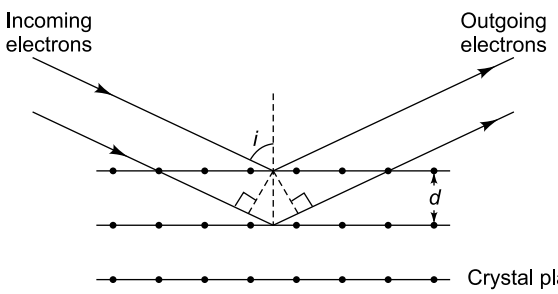


   [2007]

10. Photon of frequency ν has a momentum associated with it. If c is the velocity of light, the momentum is
    (a) ν/c (b) $h\nu c$
    (c) $h\nu/c^2$ (d) $h\nu/c$ **[2007]**

Directions: Questions 11, 12 and 13 are based on the following paragraph.

Wave property of electrons implies that they will show diffraction effects. Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively.

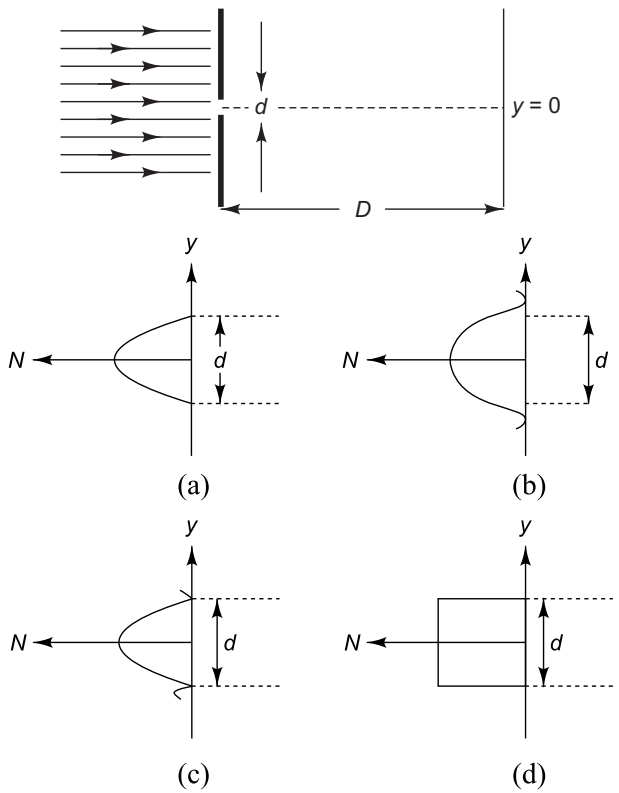


11. Electrons accelerated by potential V are diffracted from a crystal. If d = 1Å and i = 30°, V should be about ($h = 6.6 \times 10^{-34}$ Js, $m = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C).

(a) 500 V (b) 1000 V
(c) 2000 V (d) 50 V **[2008]**

12. If a strong diffraction peak is observed when electrons are incident at an angle 'i' from the normal to the crystal planes with distance 'd' between them (see figure), de Broglie wavelength λ_{dB} of electrons can be calculated by the relationship (n is an integer)
(a) $d \cos i = n \lambda_{dB}$ (b) $d \sin i = n \lambda_{dB}$
(c) $2d \cos i = n \lambda_{dB}$ (d) $2d \sin i = n \lambda_{dB}$
[2008]

13. In an experiment, electrons are made to pass through a narrow slit of width 'd' comparable to their de Broglie wavelength. They are detected on a screen at a distance 'D' from the slit (see figure). Which of the graphs shown in the figure can be expected to represent the number of electrons 'N' detected as a function of the detector position 'y' (y = 0 corresponds to the middle of the slit?)



[2008]

14. The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is: (hc = 1240 eV.nm)
(a) 1.51 eV (b) 1.68 eV
(c) 3.09 eV (d) 1.41 eV **[2009]**

15. If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called
(a) X-rays (b) ultraviolet rays
(c) microwaves (d) γ-rays **[2010]**

16. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1: A metallic surface is irradiated by a monochromatic light of frequency $v > v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement-2: The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is false.

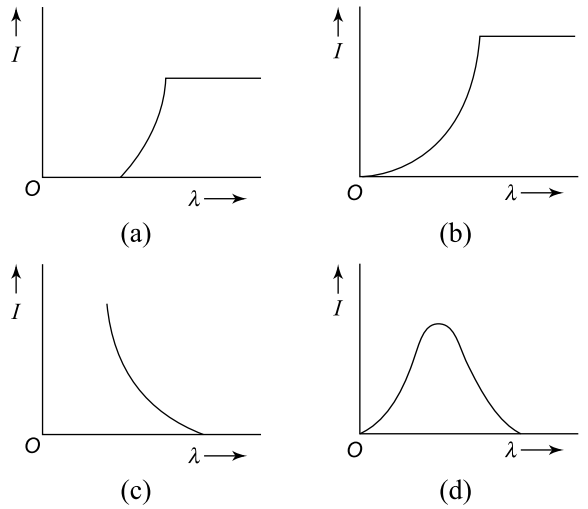
17. This equation has Statement 1 and Statement 2. Of the four choices given the Statements, choose the one that describes the two statements.

Statement 1: Davisson-Germer experiment established the wave nature of electrons.

Statement 2: If electrons have wave nature, they can interfere and show diffraction.

(a) Statement 1 is false, Statement 2 is true.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is true, Statement 2 is true, Statment 2 is the correct explanation for statement 1
(d) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1
[2012]

18. The anode voltage of a photocell is kept fixed. The Wavelength λ of light falling on the cathode is gradually changed. The plate current I of the photocell varies with λ as follows:



[2013]

19. If the kinetic energy of a free electron doubles, its de-Broglive wavelength becomes
    (a) half (b) twice
    (c) $\frac{1}{\sqrt{2}}$ times (d) $\sqrt{2}$ times **[2014]**

Answers

1. (c)	2. (a)	3. (d)	4. (c)
5. (a)	6. (c)	7. (c)	8. (d)
9. (d)	10. (d)	11. (d)	12. (c)
13. (b)	14. (d)	15. (a)	16. (c)
17. (c)	18. (c)	19. (c)	

Solutions

1. $W_0 = \frac{hc}{\lambda}$. Therefore

$$\frac{\lambda_{Cu}}{\lambda_{Na}} = \frac{(W_0)_{Na}}{(W_0)_{Cu}} = \frac{4.5}{2.3} \simeq 2$$

2. $\frac{1}{2} mv^2 = h(f - f_0)$; f_0 = threshold frequency

$$\therefore \quad \frac{1}{2} mv_1^2 = h(f_1 - f_0) \qquad (1)$$

and $$\frac{1}{2} mv_2^2 = h(f_2 - f_0) \qquad (2)$$

Since photocathodes are identical, f_0 is the same for both. Subtracting (2) from (1), we get

$$v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$$

3. The slope of the graph is equal to Planck's constant, which is a universal constant. Hence the correct choice is (d).

4. $W_0 = h\nu_0 = \frac{hc}{\lambda_0}$

$$\Rightarrow \lambda_0 = \frac{hc}{W_0} = \frac{(6.63 \times 10^{-34}) \times (3 \times 10^8)}{(4.0 \times 1.6 \times 10^{-19})} \simeq 3.11 \times 10^{-7} \text{m}$$

$$= 311 \text{ nm} \simeq 310 \text{ nm}$$

5. Intensity obeys inverse square law of distance, i.e. $I \propto \frac{1}{r^2}$. Hence if r is halved, the intensity becomes 4 times. Also the number of photoelectrons emitted is proportional to the intensity of incident radius. Hence the correct choice is (a).

6. $\lambda = \frac{h}{\sqrt{2mK}}$

If K is doubled, λ becomes $\frac{\lambda}{\sqrt{2}}$. Hence the correct choice is (c).

7. Given eV_0 = 5 eV and $h\nu_0$ = 6.2 eV where V_0 is the stopping potential and ν_0 is the threshold frequency. Now,

$$eV_0 = h\nu - h\nu_0$$

where $E = h\nu$ is the energy of the incident photon. Thus

$$E = eV_0 + h\nu_0 = 5 + 6.2 = 11.2 \text{ eV} = 11.2 \times 1.6 \times 10^{-19} \text{ J}$$

Now $E = h\nu = \frac{hc}{\lambda}$. Hence

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34}) \times (3 \times 10^8)}{11.2 \times 1.6 \times 10^{-19}}$$

$$\simeq 1.11 \times 10^{-7} \text{ m} = 1110 \text{ Å},$$

which is in the ultraviolet region of the electromagnetic spectrum. Hence the correct choice is (c).

8. The correct answer is 10^{-10} s.

9. The maximum kinetic energy of the photo-electrons is given by

$$K_{max} = h\nu - W_0$$

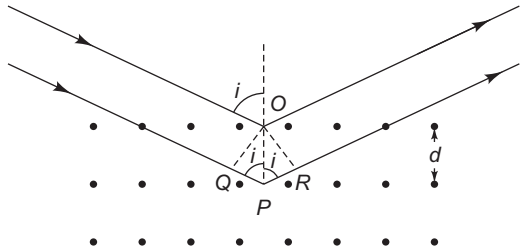
or $$K_{max} = \frac{hc}{\lambda} - W_0 \qquad (1)$$

As λ increases, K_{max} decreases. Therefore, fewer photoelectrons will reach the plate as λ increases. Hence the current through the photocell decreases as λ increases. Therefore, the correct graph is (d).

10. Momentum p of a photon of energy E is given by

$$p = \frac{E}{c} = \frac{h\nu}{c} \qquad (\because E = h\nu)$$

11. For constructive interference, path difference $\Delta = n\lambda$.



Path difference $\Delta = QP + PR$

$= OP \cos i + OP \cos i$

$= 2d \cos i$

$\therefore \quad 2d \cos i = n\lambda \Rightarrow \lambda = \dfrac{2d \cos i}{n}$

$= \dfrac{2 \times 1\text{Å} \times \cos 30°}{n} = \dfrac{\sqrt{3}}{n}\text{Å}$

For $n = 1$, $\lambda = \sqrt{3}$ Å $= \sqrt{3} \times 10^{-10}$ m

Kinetic energy of an electron acceleration through a potential difference V is $K = eV$. de Broglie wavelength of electron is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}}$$

Putting $h = 6.6 \times 10^{-34}$ Js, $m = 9.1 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ C, we get

$$\lambda = \frac{12.26 \times 10^{-10}}{\sqrt{V}}$$

$$\Rightarrow \quad \sqrt{3} \times 10^{-10} = \frac{12.26 \times 10^{-10}}{\sqrt{V}}$$

which gives $V = 50$ V

12. For maximum (strong diffraction peaks)

$$\Delta = n\lambda_{dB}$$

$$\Rightarrow \quad 2d \cos i = n\lambda_{dB}$$

The correct choice is (c).

13. The diffraction pattern consists of a strong central maximum with maximum intensity at $y = 0$. So the peak of the intensity distribution curve lies at $y = 0$. Since the slit width d is comparable with de Broglie wavelength of the electrons, the incident electrons are diffracted appreciably at the slit. Hence the electron beam spreads out due to diffraction at the slit. So the correct choice is (b).

14. Energy of incident photon is $E = h\nu = \dfrac{hc}{\lambda} = \dfrac{1240 \text{ eVnm}}{400 \text{ nm}} = 3.1$ eV

Energy of emitted photoelectrons = 1.68 eV. Therefore, work function $W_0 = 3.1 - 1.68 = 1.42$ eV. So the closest choice is (d).

15. Power $P = 4 \times 10^3$ $W = 4 \times 10^3$ Js^{-1}

Since the energy of photon is $h\nu$, the power of 10^{20} photons $= 10^{20} \times h\nu$ Js^{-1}. Hence

$10^{20}\, h\nu = 4 \times 10^3$

$$\Rightarrow \quad \nu = \frac{4 \times 10^3}{10^{20} \times 6.6 \times 10^{-34}} \simeq 6 \times 10^{16} \text{ Hz}$$

The corresponding wavelength is

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{6 \times 10^{16}} = 5 \times 10^{-9} \text{ m} = 50 \text{ Å}$$

This is the wavelength corresponding to X-rays.

16. $K_{max} = h(\nu - \nu_0)$. Also $K_{max} = eV_0$

$$\therefore \quad V_0 = \frac{h}{e}(\nu - \nu_0).$$

If ν is doubled, then

$$K'_{max} = h(2\nu - \nu_0)$$

and $$V''_0 = \frac{h}{e}(2\nu - \nu_0)$$

Thus K'_{max} is not equal to $2K_{max}$ and V''_0 is not equal to $2V_0$. But K_{max} and V_0 both vary linearly with ν. So the correct choice is (d).

17. Both the statements are true. The correct choice is (c).

18. According to Einstein's photoelectric equation,

$$K_{max} = h\nu - w_0 = \frac{hc}{\lambda} - w_0$$

where K_{max}= maximum kinetic energy of photoelectrons emitted and w_0 is the work function. Since I is proportional to K_{max}, it follows that I is proportional to $1/\lambda$. So the correct choice is (c).

19. From $\lambda = \dfrac{h}{\sqrt{2mk}}$, it follows that if k is doubled, λ becomes $\dfrac{\lambda}{\sqrt{2}}$. So the correct choice is (c).

ATOMS AND NUCLEI

Chapter 19

REVIEW OF BASIC CONCEPTS

1. Rutherford's a-particle Scattering Experiment

A beam of fast α-particles was made to fall on a thin metal foil. Rutherford observed that most of the α-particles passed through the foil without any appreciable deflection. The distance of the closest approach of an α-particle is given by

$$r_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{E_k}$$

where $E_k = \frac{1}{2}mv^2$ is the initial kinetic energy of particle and Z is the atomic number of the nucleus. The impact parameter is given by

$$b = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2\cot(\theta/2)}{m\mathbf{v}^2}$$

where θ is the scattering angle.

2. Rutherford's Model of Atom

The observations of the alpha particle scattering experiments led Rutherford to suggest the following model of atom.

(i) An atom has a small central core called the nucleus where almost the entire mass and all the positive charge of the atom is concentrated. The size of the nucleus is about 10^{-14} m.

(ii) The remaining part of the atom contains electrons which revolve round the nucleus in various orbits. The size of the atom is about 10^{-10} m.

(iii) The atom as a whole is electrically neutral; the total negative charge of the electrons surrounding the nucleus is equal to the total positive charge of the nucleus.

3. Bohr's Theory of Hydrogen Like Atom

(a) Bohr's quantization condition: The magnitude of angular momentum of the electron in a circular orbit is

$$L = \frac{nh}{2\pi} \rightarrow mv_n r_n = \frac{nh}{2\pi}$$

where m = mass of electron, r_n = radius of nth circular orbit, v_n = orbital speed of electron in the nth orbit, h = Planck's constant and n is an integer called the principal quantum number.

(b) Speed of electron in nth orbit is

$$v_n = \left(\frac{e^2}{2\varepsilon_0 h}\right)\frac{Z}{n}$$

where Z = atomic number of atom. For hydrogen $Z = 1$.

For a given atom $v_n \propto \frac{1}{n}$. Substituting the known values of e, ε_0 and h we get

$$v_n = (2.2\times10^6\ \text{ms}^{-1})\times\frac{Z}{n}$$

(c) Radius of nth orbit is

$$r_n = \left(\frac{\varepsilon_0 h^2}{\pi m e^2}\right)\frac{n^2}{Z} = (0.53\times10^{-10}\ \text{m})\frac{n^2}{Z}$$

$$= (0.53\ \text{Å})\frac{n^2}{Z}$$

For a given atom $r_n \propto n^2$.

(d) Total energy of electron in nth orbit

$$\text{K.E.} = \frac{Ze^2}{8\pi\varepsilon_0 r_n}\ ;\ \text{P.E.} = -\frac{Ze^2}{4\pi\varepsilon_0 r_n}$$

$$\therefore \qquad \text{P.E.} = -2\ \text{K.E.}$$

Total energy of electron in nth object is

$$E_n = \text{K.E.} + \text{P.E.} = \text{K.E.} - 2\ \text{K.E.} = -\text{K.E.}$$

$$\Rightarrow \qquad E_n = -\frac{Ze^2}{8\pi\varepsilon_0 r_n}$$

Putting the value of r_n, we get

$$E_n = \left(-\frac{me^4}{8\varepsilon_0^2 h^2}\right)\frac{Z^2}{n^2}$$

$$= (-21.76 \times 10^{-19}\ \text{J})\frac{Z^2}{n^2}$$

$$= (-13.6\ \text{eV})\frac{Z^2}{n^2}$$

(e) Time period of revolution of electron in nth orbit is

$$T_n = \frac{2\pi r_n}{v_n} = \left(\frac{4\varepsilon_0^2 h^2}{me^4}\right)\frac{n^3}{Z^2}$$

$$= (1.51 \times 10^{-16}\ \text{s})\frac{n^3}{Z^2}$$

(f) Frequency of revolution of the electron in nth orbit is

$$\nu_n = \frac{1}{T_n} = (6.6 \times 10^{15}\ \text{Hz})\frac{Z^2}{n^3}$$

(g) Wavelength of emitted radiation: when an electron jumps from a higher energy state $n = n_2$ to a lower state $n = n_1$, a photon of energy $h\nu$ of radiation is emitted.

$$h\nu = E_{n_2} - E_{n_1}$$

$$\Rightarrow \qquad \frac{hc}{\lambda} = \left(-\frac{me^4}{8\varepsilon_0^2 h^2}\right)Z^2\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

$$\Rightarrow \qquad \frac{1}{\lambda} = \left(\frac{me^4}{8\varepsilon_0^2 h^3 c}\right)Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = R_\text{H} Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where $R_\text{H} = \dfrac{me^4}{8\varepsilon_0^2 h^3 c} = 1.097 \times 10^7\ \text{m}^{-1}$ is called Rydberg constant.

(h) *Main Series of Hydrogen Spectrum (Z = 1)*

(1) *Lyman series:* $n_1 = 1$, $n_2 = 2, 3, 4, \ldots \infty$

$$\frac{1}{\lambda} = R_\text{H}\left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$$

$\lambda_\text{min} = 91.2$ nm, $\lambda_\text{max} = 121.6$ nm

Spectral lines in Lyman series lie in the ultraviolet region.

(2) *Balmer series:* $n_1 = 2$, $n_2 = 3, 4, 5, \ldots \infty$

$$\frac{1}{\lambda} = R_\text{H}\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right)$$

$\lambda_\text{min} = 364.5$ nm, $\lambda_\text{max} = 656.1$ nm

Spectral lines in Balmer series lie in the visible region.

(3) *Paschen series:* $n_1 = 3$, $n_2 = 4, 5, 6, \ldots \infty$

$$\frac{1}{\lambda} = R_\text{H}\left(\frac{1}{3^2} - \frac{1}{n_2^2}\right)$$

$\lambda_\text{min} = 820.1$ nm, $\lambda_\text{max} = 1874.6$ nm

Spectral lines in Paschen series lie in the infrared region.

(4) *Brackett series:* $n_1 = 4$, $n_2 = 5, 6, 7, \ldots \infty$

$$\frac{1}{\lambda} = R_\text{H}\left(\frac{1}{4^2} - \frac{1}{n_2^2}\right)$$

$\lambda_\text{min} = 1458$ nm, $\lambda_\text{max} = 4050$ nm

These spectral lines also lie in the far infrared region.

(i) The energy of electron in hydrogen atom in the ground state is – 13.6 eV.

(j) The ionization potential of hydrogen atom in the ground state is 13.6 V. Ionization potential of a hydrogen like atom in nth state = $\dfrac{13.6Z^2}{n^2}$ volt.

EXAMPLE 1 The energy required to excite a hydrogen atom from $n = 1$ to $n = 2$ energy state 10.2 eV. What is the wavelength of the radiation emitted by the atom when it goes back to its ground state?

SOLUTION Given $E_2 - E_1 = 10.2\ \text{eV} = 10.2 \times (1.6 \times 10^{-19})\ \text{J}$.

Therefore, frequency the emitted radiation is

$$\nu = \frac{E_2 - E_1}{h}$$

and wavelength is

$$\lambda = \frac{c}{\nu} = \frac{ch}{E_2 - E_1}$$

$$= \frac{3\times10^{8}\times6.6\times10^{-34}}{10.2\times1.6\times10^{-19}}$$

$$= 1.22\times10^{-7}\text{ m}$$

$$= 1220\text{ Å} = 122\text{ nm}$$

EXAMPLE 2 The ionization potential of the hydrogen atom is 13.6 V. Find the energy of the atom in $n = 2$ energy state.

SOLUTION Energy of hydrogen atom in the ground state is ($\because$ ionization potential is 13.6 V)

$$E_1 = -13.6\text{ eV}$$

Since $E_n \propto \frac{1}{n^2}$, the energy in the $n = 2$ state is

$$E_2 = \frac{E_1}{(2)^2} = \frac{-13.6}{4} = -3.4\text{ eV}$$

EXAMPLE 3 The innermost orbit of hydrogen atom has a diameter of 1.06 Å. What is the diameter of the 10th orbit?

SOLUTION Given $d_1 = 1.06$ Å. We know that $d_n \propto n^2 d_1$. Hence

$$d_{10} = (10)^2 \times 1.06\text{ Å}$$

$$= 106\text{ Å}$$

EXAMPLE 4 Find the ratio of longest and shortest wavelength in the Lyman series of hydrogen atom.

SOLUTION For Lyman series

$$\frac{1}{\lambda} = R_H\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

where $n = 2, 3, 4, \ldots \infty$. The shortest wavelength (λ_s) corresponds to $n = \infty$ and the longest wavelength (λ_l) corresponds to $n = 2$.

$$\frac{1}{\lambda_s} = R_H\left(1 - \frac{1}{\infty}\right) = R_H$$

$$\Rightarrow \quad \lambda_s = \frac{1}{R_H}$$

and

$$\frac{1}{\lambda_l} = R_H\left(1 - \frac{1}{4}\right) = \frac{3R_H}{4}$$

$$\Rightarrow \quad \lambda_l = \frac{4}{3R_H}$$

$$\therefore \quad \frac{\lambda_l}{\lambda_s} = \frac{4}{3}$$

EXAMPLE 5 The wavelength of the second line of Balmer series is 486.4 nm. What is the wavelength of the first line of Lyman series?

SOLUTION Wavelengths in Balmer series for hydrogen are given by

$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$$= R_H\left(\frac{1}{4} - \frac{1}{n^2}\right);\ n = 3, 4, 5 \ldots$$

The second line in Balmer series corresponds to $n = 4$. Hence

$$\frac{1}{\lambda_2} = R_H\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R_H}{16} \text{ or } \lambda_2 = \frac{16}{3R_H}$$

The wavelength of the first line ($n = 2$) in Lyman series is

$$\frac{1}{\lambda_1} = R_H\left(1 - \frac{1}{2^2}\right) = R_H\left(1 - \frac{1}{4}\right)$$

$$= \frac{3R_H}{4}$$

or $$\lambda_1 = \frac{4}{3R_H}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{4}{3R_H} \times \frac{3R_H}{16} = \frac{1}{4}$$

or $$\lambda_1 = \frac{\lambda_2}{4} = \frac{486.4}{4}$$

$$= 121.6\text{ nm}$$

EXAMPLE 6 The number of different wavelengths which can be observed in the spectrum of a hydrogen sample whose atoms are excited to a state having principal quantum number n is

(a) $\frac{n(n-1)}{2}$ (b) $\frac{n(n+1)}{2}$

(c) n^2 (d) n

SOLUTION

Quantum Number (n)	Transitions	Number of Transitions
2	$2 \to 1$	1
3	$3 \to 2, 3 \to 1, 2 \to 1$	3
4	$4 \to 3, 3 \to 2, 4 \to 2, 2 \to 1, 4 \to 1, 3 \to 1$	6

and so on. It is easy to show that for a state with quantum number n, the number of possible transitions $= \frac{1}{2}n(n-1)$. For example, if $n = 5$, the number of possible transitions

$$= \frac{1}{2} \times 5 \times (5-1) = 10.$$

So the correct choice is (a).

EXAMPLE 7 How many wavelengths can be emitted when hydrogen atoms excited to $n = 4$ state return to the ground state? Find the highest wavelength present in the radiation.

SOLUTION The number of possible wavelengths present in the radiation = number of possible transitions from $n = 4$ state which is equal to

$$\frac{n(n-1)}{2} = \frac{4 \times (4-1)}{2} = 6$$

These transitions are listed in Example 6 above.
From the relation

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

it follows the maximum value of λ corresponds to the transition $n_2 = 4$ to $n_1 = 3$. Thus

$$\frac{1}{\lambda_{max}} = R_H \left(\frac{1}{3^2} - \frac{1}{4^2} \right) = \frac{7 R_H}{144}$$

$$\Rightarrow \quad \lambda_{max} = \frac{144}{7 R_H} = \frac{144}{7 \times (1.097 \times 10^7)}$$

$$= 18.75 \times 10^{-7} \text{ m}$$

$$\simeq 1875 \text{ nm}$$

EXAMPLE 8 In Example 7 above, what is the minimum wavelength emitted?

SOLUTION The minimum wavelength corresponds to the transition $n_2 = 4$ to $n_1 = 1$. Thus

$$\frac{1}{\lambda_{min}} = R_H \left(\frac{1}{1^2} - \frac{1}{4^2} \right) = \frac{15 R_H}{16}$$

$$\lambda_{min} = \frac{16}{15 R_H} = \frac{16}{15 \times (1.097 \times 10^7)}$$

$$= 0.972 \times 10^{-7} \text{ m}$$

$$= 97.2 \text{ nm}$$

EXAMPLE 9 When monochromatic light of wavelength 103 nm is incident on hydrogen atoms in the ground state, some of the atoms are excited to a higher energy state with

(a) $n = 2$ (b) $n = 3$

(c) $n = 4$ (d) $n = 5$

SOLUTION Energy of a photon of wavelength λ = 103 nm = 103×10^{-9} m is

$$E = h\nu = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}) \times (3 \times 10^8)}{103 \times 10^{-9}}$$

$$= 19.3 \times 10^{-19} \text{ J}$$

$$= 12.1 \text{ eV}$$

The energies with $n = 1, 2, 3, \ldots$ etc states are

For $n = 1$; $E_1 = -13.6$ eV

For $n = 2$; $E_2 = \frac{-13.6}{2^2} = -3.4$ eV

For $n = 3$; $E_3 = \frac{-13.6}{3^2} = -1.5$ eV

For $n = 4$; $E_4 = -0.85$ eV

For $n = 5$; $E_5 = -0.54$ eV

It follows that the energy needed to excite a hydrogen atom from ground state $n = 1$ to $n = 3$ state is

$$E = 13.6 - 1.5 = 12.1 \text{ eV}$$

which is just the energy of the incident photon. Hence correct choice is (b).

EXAMPLE 10 A sample contains hydrogen atoms excited to a state n_1. Photons of energy 2.86 eV take the atoms to a higher energy state n_2. Then

(a) $n_1 = 1, n_2 = 4$ (b) $n_1 = 2, n_2 = 3$

(c) $n_1 = 3, n_2 = 5$ (d) $n_1 = 2, n_2 = 5$

SOLUTION When a photon is absorbed by an atom, it delivers its energy to the atom which shifts the electron to a higher energy state. It follows from Example 9 above that a difference of 2.86 eV can only be absorbed in the transition $n = 2$ to $n = 5$ because 3.4 – 0.54 = 2.86 eV. Thus the correct choice is (d).

EXAMPLE 11 Find the minimum frequency a beam of light must have to ionize a hydrogen atom in the ground state.

SOLUTION The minimum energy needed to ionize a hydrogen atom in the ground state is

$$E_{min} = 13.6 \text{ eV} = 13.6 \times (1.6 \times 10^{-19}) = 21.76 \times 10^{-19} \text{ J}$$

Thus $\quad h\nu_{min} = E_{min}$

or $\quad \nu_{min} = \frac{E_{min}}{h}$

$$= \frac{21.76 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$= 3.28 \times 10^{15} \text{ Hz}$$

EXAMPLE 12 A light of wavelength 91.5 nm is incident on a hydrogen atom in the second excited state. Find the kinetic energy (in eV) of the electron that is removed from the atom.

SOLUTION Energy of the hydrogen atom in the second excited state ($n = 3$) is

$$E_3 = -\frac{13.6}{3^2} = -1.5 \text{ eV}$$

Thus 1.5 eV of energy is needed to ionize a hydrogen atom in the second excited state. Now energy of a photon of $\lambda = 91.5 \text{ nm} = 91.5 \times 10^{-9}$ m is

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}) \times (3 \times 10^8)}{(91.5 \times 10^{-9}) \times (1.6 \times 10^{-19})}$$

$$= 13.6 \text{ eV}$$

Therefore, the kinetic energy of the ejected electron $= E - E_3 = 13.6 - 1.5 = 12.1$ eV.

4. Composition of the Nucleus

The nucleus of an atom contains protons and neutrons which are collectively called nucleons. The total number of nucleons is called the mass number and its denoted by A. The number of protons in a nucleus is called its atomic number Z. The number of neutrons is denoted by N, so that

$$A = Z + N$$

5. The Atomic Nucleus

The radius R of a nucleus of mass number A is given by the relation

$$R = R_0 A^{1/3}$$

where R_0 is a quantity which varies slightly from one nucleus to another.

6. Mass Defect and Binding Energy

The mass of a nucleus which contains Z protons and $(A - Z)$ neutrons is always less than the sum of the masses of these particles in the free state. The difference is called the mass defect of the given nucleus and is given by

$$\Delta m = Z m_p + (A - Z) m_n - m$$

where m = mass of the nucleus, m_p = mass of a proton and m_n = mass of a neutron. The binding energy of the nucleus is given by

$$\text{BE} = (\Delta m)c^2 = [Z m_p + (A - Z) m_n - m]c^2$$

where c is the speed of light in free space. Since A is the total number of nucleous,

$$\text{BE per nucleon} = \frac{\left[Z m_p + (A - Z) m_n - m\right]c^2}{A}$$

7. Nuclear Fission

The splitting of a heavy nucleus into two or more fragments of moderate and comparable sizes is called nuclear fission. The fission of uranium-235 is represented by the reaction.

$$^{235}_{92}\text{U} \longrightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3{}^{1}_{0}\text{n} + \text{energy}$$

The energy released per fission is about 200 MeV which is much more than the energy released in the usual nuclear reactions. This makes the *fission reaction* a particularly suitable source of energy.

The fission reaction given above has a unique feature. Apart from the fission fragments, the reaction results in the release of 2 to 3 neutrons—the very particles that initiated the reaction. So fission after fission, the neutrons present in a bulk sample of uranium increase in geometric ratio.

The rate of energy release also increases similarly in a geometric ratio. The fission reaction is thus a *self sustaining chain reaction*.

When the number of neutrons released per fission is limited to one per fission by *absorption* of excess neutrons, the chain reaction is a *controlled* one and is used in nuclear reactors. When there is no such control on the number of released neutrons, we have an uncontrolled chain reaction and this is the source of energy in the atom bomb.

An essential part of a controlled fission reaction is known as the moderator. The role of the moderator is to slow down the neutrons released in fissions so that they may be easily absorbed by another $^{235}_{92}\text{U}$ nucleus. Media which contain nuclei of masses comparable to the neutron are found to act as efficient moderators.

8. Nuclear Fusion

The process of nuclear fusion consists in the 'combination' of two light nuclei to form a stable nucleus of mass less than the total initial mass. It is believed to be the main source of energy for the sun and the stars. The fusion reaction in stars is believed to occur either *via* the *proton-proton* cycle or the *carbon-cycle*. The proton-proton cycle is as follows:

$$^{1}_{1}\text{H} + {}^{1}_{1}\text{H} \longrightarrow {}^{2}_{1}\text{H} + \text{e}^{+} + \nu$$

$$^{1}_{1}\text{H} + {}^{2}_{1}\text{H} \longrightarrow {}^{3}_{2}\text{He} + \nu$$

$$^{3}_{2}\text{He} + {}^{3}_{2}\text{He} \longrightarrow {}^{4}_{2}\text{He} + {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H}$$

The energy released in this sequence works out to be 24.7 MeV.

Nuclear fusion occurs at very high temperatures of about 10^7 K and under extremely high pressures.

9. Radioactivity

The phenomenon of self-emission of radiations from a nucleus is called radioactivity and substances which emit

these radiations are called radioactive substances. The radiations emitted from a radioactive element are of three types.

1. *Alpha rays*: These rays consist of α-particles. An alpha particle is a helium nucleus having two protons and two neutrons. It has a positive charge equal to the charge of two protons. It has an initial speed of about 10^7 ms^{-1}. They have very little penetrating power.
2. *Beta rays*: These rays consist of electrons. Their speed is very nearly equal to the speed of light. They have more penetrating power than alpha particles.
3. *Gamma rays*: These are high frequency electromagnetic waves having a high penetrating power.

Alpha Decay

The process of emission of an alpha particle from a nucleus is called alpha decay. When a nucleus emits an alpha particle $\left({}_2^4\text{He}\right)$, it loses two protons and two neutrons which means that the daughter nucleus has its mass number reduced by 4 and its atomic number reduced by 2. When a nucleus ${}_Z^A\text{X}$ of mass number A and atomic number Z emits an α-particle $\left({}_2^4\text{He}\right)$, it is transformed into a nucleus ${}_{Z-2}^{A-4}\text{Y}$ whose mass number is $(A-4)$ and atomic number is $(Z-2)$. Alpha decay is represented by

$${}_Z^A\text{X} \xrightarrow{\text{minus } {}_2^4\text{He}} {}_{Z-2}^{A-4}\text{Y} + \text{energy}$$

Beta Decay

The process of the emission of an electron from a nucleus is called *beta decay*. In this process also, the nucleus achieves greater stability by emitting an electron. A neutron inside the nucleus decays into a proton with the emission of an electron (e) and a particle called antineutrino $(\vec{\nu})$. Because the mass of an electron is negligibly small, the mass number of the resulting nucleus remains unaltered but its atomic number is increased by one. For example, when a radium nucleus ${}_{88}^{228}\text{Ra}$ emits a β-particle, the resulting element is an isotope of actinium ${}_{89}^{228}\text{Ac}$. Thus in β-decay also, a new element is formed.

The transformation of a nucleus ${}_Z^A\text{X}$ into the nucleus ${}_{Z+1}^A\text{Y}$ by β-decay is represented by an equation

$${}_Z^A\text{X} \xrightarrow{\text{minus e}^-} {}_{Z+1}^A\text{Y} + \text{e} + \vec{\nu} + \text{energy}$$

Gamma Decay

Gamma rays are high-frequency electromagnetic radiations (i.e. photons) which do not carry any charge. Hence in γ-decay, the mass number and atomic number of the nucleus remain unchanged so that no new element is formed.

Radioactive Decay Law

If N is the number of radioactive nuclei present in a sample at a given instant of time, then the rate of decay at that instant is proportional to N, i.e.

$$\frac{dN}{dt} = -\lambda N \tag{19.8}$$

The proportionality constant λ is called the disintegration constant. If N_0 is the number of radioactive nuclei at time $t = 0$, then the number of radioactive nuclei at a later time t is given by

$$N = N_0\, e^{-\lambda t} \tag{19.9}$$

Half life : The half life of a radioactive element is the time in which half the number of nuclei decay. It is given by

$$T = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \tag{19.10}$$

Average life : The average life of a radioactive sample is the reciprocal to its disintegration constant, i.e.

$$T_{\text{av}} = \frac{1}{\lambda} \tag{19.11}$$

Radioactivity decay rate or Activity: It is useful to use the concept of the decay rate R which is defined as the number of radioactive disintegrations taking place in a sample per second, which is given by

$$R = \frac{dN}{dt} = -\lambda N$$

or
$$|R| = \lambda N = \frac{0.693}{T} N$$

As N decreases exponentially with time, R will also decrease exponentially with time. The SI unit of the decay rate R is called *curie* (symbol Ci) in honour of Madame M.S. Curie (1867–1934). It is defined as the decay rate of 3.7×10^{10} disintegrations per second, i.e.

1 Ci (curie) $= 3.7 \times 10^{10}$ disintegrations/s

1 mCi (millicurie) $= 3.7 \times 10^{7}$ disintegrations/s

1 μCi (microcurie) $= 3.7 \times 10^{4}$ disintegrations/s

EXAMPLE 13 Calculate the distance of closest approach when a 5 MeV proton approaches a gold nucleus. Atomic number of gold is $Z = 79$.

SOLUTION Kinetic energy of porton is

$$K = 5 \text{ MeV} = 5 \times 10^6 \text{ MeV}$$
$$= 5 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$
$$= 8 \times 10^{-13} \text{ J}.$$

The distance of closest approach is

$$r_0 = \frac{1}{4\pi E_0}\ \frac{Ze^2}{K}$$

$$= \frac{9\times10^9\times79\times(1.6\times10^{-19})^2}{8\times10^{-13}}$$

$$= 2.3\times10^{-14}\text{ m}$$

EXAMPLE 14 Calculate the ratio of the radii of two nuclei of mass numbers 1 and 27.

SOLUTION $R_1 = R_0A_1^{1/3}$ and $R_2 = R_0A_2^{1/3}$

$$\therefore \quad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{1}{27}\right)^{1/3} = \frac{1}{3}$$

EXAMPLE 15 16 g of radioactive radon is kept in a container. How much radon will disintegrate in 19 days? Half life of radon is 3.8 days.

SOLUTION Number of half lives in 19 days is

$$n = \frac{t}{T} = \frac{19\text{ days}}{3.8\text{ days}} = 5$$

$\therefore$ Number of atoms left undecayed after 5 half lives is

$$N = N_0\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^5 = \frac{N_0}{32}$$

$\therefore$ Mass of radon left undecayed after 19 days

$$= \frac{16\text{ g}}{32} = 0.5\text{ g}$$

$\therefore$ Mass of radon disintegrated = 16 – 0.5 = 15.5 g

EXAMPLE 16 The half life of a radioactive substance is 30 days. What is the time taken for 3/4 of its original mass to disintegrate?

SOLUTION $T = 30$ days

$$N = N_0 - \frac{3N_0}{4} = \frac{N_0}{4}$$

$$N = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{1}{4} = \left(\frac{1}{2}\right)^n \Rightarrow n = 2$$

$\therefore$ Time taken = 2 half lives

$$= 2\times30 = 60\text{ days.}$$

EXAMPLE 17 The activity of a radioactive element reduces to $\frac{1}{16}$ th of its original value in 30 years. Find the half life and the decay constant of the element.

SOLUTION $|R| = \lambda N$, i.e. the activity of proportional to the number atoms present in the sample.

Given $N = \frac{N_0}{16}$

From $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$, we have

$$\frac{1}{16} = \left(\frac{1}{2}\right)^n \Rightarrow n = 4$$

i.e. there are 4 half lives in 30 years. Therefore the half life of the element is

$$T = \frac{30\text{ years}}{4} = 7.5\text{ years}$$

Decay constant $\lambda = \frac{0.693}{T} = \frac{0.693}{7.5\text{ years}}$

$$= 0.0924\text{ per year.}$$

EXAMPLE 18 Two radioactive substances A and B initially contain equal number of atoms. The half lives of A and B are 1 hour and 2 hours respectively. Find the ratio of their rates of disintegration at the end of 2 hours.

SOLUTION Given $T_A = 1$ hour and $T_B = 2$ hours. At $t = 0$, number of atoms of A = number of atoms of $B = N_0$.

$\therefore$ Number of atoms of A after 2 hours (i.e. after 2 half lives of A) is

$$N_A = N_0\left(\frac{1}{2}\right)^2 = \frac{N_0}{4}$$

Number of atoms of B after 2 hrs (i.e. after 1 half life of B) is

$$N_B = \frac{N_o}{2}$$

Now $|R| = \lambda N = \frac{0.693N}{T}$. Therefore.

$$R_A = \frac{0.693N_A}{T_A} \text{ and } R_B = \frac{0.693N_B}{T_B}$$

$$\therefore \quad \frac{R_A}{R_B} = \frac{N_A}{N_B}\times\frac{T_B}{T_A} = \frac{N_0/4}{N_0/2}\times\frac{2}{1} = 1$$

EXAMPLE 19 A sample contains 2.3 g of radioactive $^{230}_{90}$Th of half life 2.4×10^{11} seconds. How many disintegrations per second occur in the sample? Take Avogadro number = 6×10^{23} atoms per mole.

SOLUTION 1 mole of a substance has a mass equal to its atomic mass expressed in grams. Hence, number of moles in 2.3 g of $^{230}_{90}$Th is

$$\frac{2.3\text{ g}}{230\text{ g/mole}} = 10^{-2}\text{ mole}$$

Also Avogdro number = number of atoms in 1 mole of the substance. Hence number of atoms in 10^{-2} mole is

$$N = 6 \times 10^{23} \times 10^{-2} = 6 \times 10^{21} \text{ atoms}$$

$$\therefore \quad |R| = \frac{0.693N}{T} = \frac{0.693 \times 6 \times 10^{21}}{2.4 \times 10^{11}}$$

$$= 1.73 \times 10^{10}$$

disintegrations per second.

EXAMPLE 20 A radioactive substance of half life of 69.3 days is kept in a container. After a certain lapse of time, it was found that 20% of the substance is left undecayed. Find the time elapsed. Given ln (5) = 1.61.

SOLUTION $T = 69.3$ days

$$\therefore \quad \lambda = \frac{0.693}{T} = \frac{0.693}{69.3 \text{ days}}$$

$$= 10^{-2} \text{ per day}$$

$N = 20\%$ of $N_0 = 0.2\, N_0$. Therefore

$$\frac{N_0}{N} = 5$$

Now $$N = N_0\, e^{-\lambda t} \Rightarrow \frac{N}{N_0} = e^{-\lambda t}$$

$$\Rightarrow \quad \frac{N_0}{N} = e^{\lambda t} \Rightarrow \ln\left(\frac{N_0}{N}\right) = \lambda t$$

$$\therefore \quad t = \frac{\ln(N_0/N)}{\lambda} = \frac{\ln(5)}{10^{-2} \text{ per day}}$$

$$= \frac{1.61}{10^{-2}} \text{ days} = 161 \text{ days}$$

EXAMPLE 21 1 gram of caesuim $\left({}^{137}_{55}\text{Cs}\right)$ decays by β-emission with a half life of 30 years. (a) Name the resulting isotope. (b) Write the equation of decay. (c) If the initial activity of caesuim is 1.0 millicurie (mCi), what is its activity after 60 years.

SOLUTION

(a) In β-decay, the mass number remains unchanged and the atomic number increases by 1. The resulting isotope has a mass number 137 and atomic number 56. This corresponds to the isotope ${}^{137}_{56}\text{Ba}$.

(b) The equation of β-decay is

$${}^{137}_{55}\text{Cs} \xrightarrow{\beta\text{-decay}} {}^{137}_{56}\text{Ba} + {}^{0}_{-1}e + \bar{\nu} + Q$$

(c) where $\bar{\nu}$ is an antineutrino and Q is the energy released in β-decay.

Initial activity is $R_0 = 1.0$ mCi

Since there are 2 half lives in 60 years and since the activity is proportional to the number of atoms, the activity at the end of 60 years is

$$R = R_0 \times \left(\frac{1}{2}\right)^2 = \frac{R_0}{4} = \frac{1.0\,\text{mCi}}{4} = 0.25 \text{ mCi}$$

SECTION 1

Multiple Choice Questions with One Correct Choice

Level A

1. Alpha particles are fired at a nucleus. Which of the paths shown in Fig. 19.1 is not possible?
   (a) 1 (b) 2
   (c) 3 (d) 4

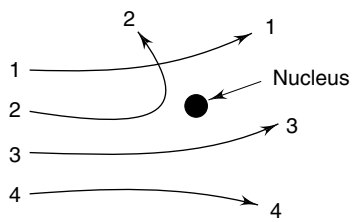


Fig. 19.1

2. The distance of the closest approach of an alpha particle fired at a nucleus with kinetic energy K is r_0. The distance of the closest approach when the alpha particle is fired at the same nucleus with kinetic energy $2K$ will be
   (a) $2r_0$ (b) $4r_0$
   (c) $\frac{r_0}{2}$ (d) $\frac{r_0}{4}$
3. The distance of the closest approach of an alpha particle fired at a nucleus with momentum p is r_0. The distance of the closest approach when the alpha particle is fired at the same nucleus with momentum $2p$ will be
   (a) $2r_0$ (b) $4r_0$
   (c) $\frac{r_0}{2}$ (d) $\frac{r_0}{4}$

4. In Rutherford's alpha particle scattering experiment with thin gold foil, 8100 scintillations per minute are observed at an angle of 60°. The number of scintillations per minute at an angle of 120° will be
   (a) 100 (b) 2025
   (c) 32400 (d) 4050
5. The energy required to excite a hydrogen atom from $n = 1$ to $n = 2$ energy state 10.2 eV. What is the wavelength of the radiation emitted by the atom when it goes back to its ground state?
   (a) 1020 Å (b) 1120 Å
   (c) 1220 Å (d) 1320 Å
6. The ionisation potential of the hydrogen atom is 13.6 eV. Its energy in $n = 2$ energy state is
   (a) – 3.4 eV (b) – 6.8 eV
   (c) – 13.6 eV (d) – 27.2 eV
7. The innermost orbit of the hydrogen atom has a diameter of 1.06 Å. What is the diameter of the tenth orbit?
   (a) 5·3 Å (b) 10·6 Å
   (c) 53 Å (d) 106 Å
8. Which of the following series in the spectrum of the hydrogen atom lies in the visible region of the electromagnetic spectrum?
   (a) Paschen series. (b) Balmar series
   (c) Lyman series (d) Brackett series
9. The different lines in the Lyman series have their wavelengths lying between
   (a) zero to infinite (b) 900 Å to 1200 Å
   (c) 1000 Å to 1500 Å (d) 500 Å to 1000 Å
10. The shortest wavelength in the Lyman series is 911·6 Å. Then the longest wavelength in the Lyman series is
    (a) 1215 Å (b) ∞
    (c) 2430 Å (d) 3600 Å
11. Which energy state of the triply ionized beryllium (Be^{+++}) has the same electron orbital radius as that of the ground state of hydrogen? Given Z for beryllium = 4.
    (a) $n = 1$ (b) $n = 2$
    (c) $n = 3$ (d) $n = 4$
12. In Q. 11, what is the ratio of the $n = 2$ energy state of beryllium and ground state of hydrogen?
    (a) 1 (b) 2
    (c) 3 (d) 4
13. Which energy state of doubly ionized lithium (Li^{++}) has the same energy as that of the ground state of hydrogen? Given Z for lithium = 3.
    (a) $n = 1$ (b) $n = 2$
    (c) $n = 3$ (d) $n = 4$
14. In Q. 13, what is the ratio of the electron orbital radius of Li^{++} to that of hydrogen?
    (a) 1 (b) 2
    (c) 3 (d) 4
15. The total energy of the electron in the first excited state of hydrogen is – 3·4 eV. What is the kinetic energy of the electron in this state?
    (a) + 1.7 eV (b) + 3.4 eV
    (c) + 6.8 eV (d) + 13.4 eV
16. In Q. 15, the potential energy of the electron is
    (a) – 1.7 eV (b) – 3.4 eV
    (c) – 6.8 eV (d) – 13.4 eV
17. The wavelength of the second line of Balmer series is 486.4 nm. What is the wavelength of the first line of Lyman series?
    (a) 121.6 nm (b) 364.8 nm
    (c) 729.6 nm (d) none of these
18. The wavelength of the first line in Balmer series in the hydrogen spectrum is λ. What is the wavelength of the second line.
    (a) $\frac{20\lambda}{27}$ (b) $\frac{3\lambda}{16}$
    (c) $\frac{5\lambda}{36}$ (d) $\frac{3\lambda}{4}$
19. The frequency of the first line in Lyman series in the hydrogen spectrum is ν. What is the frequency of the corresponding line in the spectrum of doubly ionized Lithium?
    (a) ν (b) 3ν
    (c) 9ν (d) 27ν
20. The energy difference between the first two levels of hydrogen atom is 10.2 eV. What is the corresponding energy difference for a singly ionized helium atom?
    (a) 10.2 eV (b) 20.4 eV
    (c) 40.8 eV (d) 81.6 eV
21. The ionization energy of hydrogen atom is 13.6 eV. What is the ionization energy of helium atom?
    (a) 3.4 eV (b) 13.6 eV
    (c) 54.4 eV (d) 108.8 eV
22. The ionization energy of hydrogen atom is 13.6 eV. Hydrogen atoms in the ground state are excited by electromagnetic radiation of energy 12.1 eV. How many spectral lines will be emitted by the hydrogen atom?
    (a) one (b) two
    (c) three (d) four

Level B

23. If an orbital electron of the hydrogen atom jumps from the ground state to a higher energy state, its orbital speed reduces to half its initial value. If the radius of the electron orbit in the ground state is r, then the radius of the new orbit would be
    (a) $2r$ (b) $4r$
    (c) $8r$ (d) $16r$
24. The orbital speed of the electron in the ground state of hydrogen is v. What will be its orbital speed when it is excited to the energy state – 3.4 eV?
    (a) $2v$ (b) $\frac{v}{2}$
    (c) $\frac{v}{4}$ (d) $\frac{v}{8}$
25. In the Bohr model of the hydrogen atom, the ratio of the kinetic energy to the total energy of the electron in a quantum state n is
    (a) – 1 (b) + 1
    (c) $\frac{1}{n}$ (d) $\frac{1}{n^2}$
26. The ratio of the wavelengths of the longest wavelength lines in the Lyman and Balmer series of hydrogen spectrum is
    (a) $\frac{3}{23}$ (b) $\frac{5}{27}$
    (c) $\frac{7}{29}$ (d) $\frac{9}{31}$
27. If a hydrogen atom at rest, emits a photon of wavelength λ, the recoil speed of the atom of mass m is given by
    (a) $\frac{h}{m\lambda}$ (b) $\frac{mh}{\lambda}$
    (c) $mh\lambda$ (d) $\frac{m}{h\lambda}$
28. If elements with principal quantum number $n > 4$ were not allowed in nature, the number of possible elements would be
    (a) 60 (b) 32
    (c) 4 (d) 64
29. In any nuclear reaction the reactants and the resultants must always be in conformity with the law of conservation of
    (a) energy alone
    (b) charge number alone
    (c) mass number alone
    (d) both charge and mass numbers
30. When high energy alpha-particles ($^4_2$He) pass through nitrogen gas, an isotope of oxygen is formed with the emission of particles named x. The nuclear reaction is
    $$^{14}_{7}\text{N} + ^{4}_{2}\text{He} \rightarrow ^{17}_{8}\text{O} + x$$
    What is the name of x?
    (a) electron (b) proton
    (c) neutron (d) positron
31. Which of the following conservation laws must be obeyed in a nuclear reaction?
    (a) Conservation of charge
    (b) Conservation of mass
    (c) Conservation of energy and momentum
    (d) All of the above
32. What is particle x in the following nuclear reaction?
    $$^{9}_{4}\text{Be} + ^{4}_{2}\text{He} \rightarrow ^{12}_{6}\text{C} + x$$
    (a) electron (b) proton
    (c) neutron (d) photon
33. When aluminium is bombarded with fast neutrons, it changes into sodium with emission of particle x according to the equation
    $$^{27}_{13}\text{Al} + ^{1}_{0}\text{n} \rightarrow ^{24}_{11}\text{Na} + x$$
    What is x?
    (a) electron (b) proton
    (c) neutron (d) alpha-particle
34. Which one of the following nuclear reactions is NOT possible?
    (a) $^{12}_{6}\text{C} + ^{12}_{6}\text{C} \rightarrow ^{20}_{10}\text{Ne} + ^{4}_{2}\text{He}$
    (b) $^{9}_{4}\text{Be} + ^{1}_{1}\text{H} \rightarrow ^{6}_{3}\text{Li} + ^{4}_{2}\text{He}$
    (c) $^{11}_{5}\text{Be} + ^{1}_{1}\text{H} \rightarrow ^{9}_{4}\text{Be} + ^{4}_{2}\text{He}$
    (d) $^{7}_{3}\text{Li} + ^{4}_{2}\text{He} \rightarrow ^{1}_{1}\text{H} + ^{10}_{4}\text{B}$
35. In the equation
    $$^{27}_{13}\text{Al} + ^{4}_{2}\text{He} \rightarrow ^{30}_{15}\text{P} + X,$$
    The correct symbol for X is
    (a) $^{0}_{-1}$e (b) $^{1}_{1}$H
    (c) $^{4}_{2}$He (d) $^{1}_{0}$n
36. A deuterium nucleus $^{2}_{1}$H combines with a tritium nucleus $^{3}_{1}$H to form a heavier helium nucleus $^{4}_{2}$He with the release of a neutron ($^{1}_{0}$n). The fusion reaction is represented by the equation
    $$^{2}_{1}\text{H} + ^{3}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + ^{1}_{0}\text{n}$$
    In this reaction, the mass of $^{4}_{2}$He + mass of $^{1}_{0}$n is
    (a) less than the mass of $^{2}_{1}$H + mass of $^{3}_{1}$H
    (b) greater than the mass of $^{2}_{1}$H + mass of $^{3}_{1}$H
    (c) the same as the mass of $^{2}_{1}$H + mass of $^{3}_{1}$H
    (d) twice the mass of $^{2}_{1}$H + mass of $^{3}_{1}$H

37. The fusion reaction between deuterium and tritium takes places
    (a) when the temperature is near absolute zero
    (b) at ordinary temperature and pressure
    (c) at low temperature and low pressure
    (d) at very high temperature and high pressure.
38. Choose the WRONG statement. A thermonuclear fusion reactor is better than a fission reactor for the following reasons:
    (a) For the same mass of substances involved, a fusion reaction releases much more energy than a fission reaction
    (b) A fusion reaction can be much more easily controlled than a fission reaction
    (c) A fusion reaction produces almost no radioactive waste
    (d) The fuel required for fusion is readily available in abundance from sea-water.
39. Which of the following particles can be added to the nucleus of an atom without changing its chemical properties?
    (a) Electrons (b) Protons
    (c) Neutrons (d) Alpha particles
40. The kinetic energy of a 300 K thermal neutron is
    (a) 300 eV (b) 300 MeV
    (c) 0.026 eV (d) 0.026 MeV
41. The energy released by the fission of one uranium atom is 200 MeV. The number of fissions per second required to produce 3.2 W of power is
    (a) 10^7 (b) 10^{10}
    (c) 10^{15} (d) 10^{11}
42. What is the energy released in the fission reaction
    $$^{236}_{92}\text{U} \rightarrow {}^{117}_{46}\text{X} + {}^{117}_{46}\text{Y} + 2\ {}^{1}_{0}\text{n}$$
    given that the binding energy per nucleon of X and Y is 8.5 MeV and that of $^{236}_{92}$U is 7.6 MeV?
    (a) 20 MeV (b) 180 MeV
    (c) 200 MeV (d) 2000 MeV
43. The binding energy of deuteron ($^2_1$H) is 1.15 MeV per nucleon and an alpha particle ($^4_2$He) has a binding energy of 7.1 MeV per nucleon. Then in the reaction
    $$^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + \text{Q}$$
    the energy Q released is
    (a) 1 MeV (b) 11.9 MeV
    (c) 23.8 MeV (d) 931 MeV
44. If M is the mass of a nucleus and A its atomic mass, then the packing fraction is
    (a) $\dfrac{M-A}{M+A}$ (b) $\dfrac{M-A}{M}$
    (c) $\dfrac{M-A}{A}$ (d) $\dfrac{M+A}{M-A}$
45. Fusion reaction occurs at temperatures of the order of
    (a) 10^3 K (b) 10^7 K
    (c) 10 K (d) 10^4 K
46. What is the main source of energy of the sun?
    (a) Gravitational energy liberated during the slow contraction of the sun
    (b) Combustion of pure carbon present in the sun
    (c) Nuclear fission of heavier unstable elements in the sun
    (d) Nuclear fusion of lighter elements in the sun.
47. What is the rest mass energy of an electron?
    (a) 1 eV (b) 0.51 MeV
    (c) 931 MeV (d) none of these
48. Out of the following, which are NOT emitted by a radioactive substance?
    (a) Electrons
    (b) Electromagnetic radiations
    (c) Helium nuclei with a charge equal to that of two protons
    (d) Neutrons
49. After two hours 1/16th of the initial amount of a certain radioactive isotope remains undecayed. The half life of the isotope is
    (a) 15 min (b) 30 min
    (c) 45 min (d) 1 hour
50. The half life of a certain radio isotope is 10 minutes. The number of radioactive nuclei at a given instant of time is 10^8. Then the number of radioactive nuclei left 5 minutes later would be
    (a) $\dfrac{10^8}{2}$ (b) 10^4
    (c) $\sqrt{2} \times 10^7$ (d) $\dfrac{10^8}{\sqrt{2}}$
51. The half life of Pa – 218 is 3 minutes. What mass of a 16 g sample of Pa – 218 will remain after 15 minutes?
    (a) 3.2 g (b) 2.0 g
    (c) 1.6 g (d) 0.5 g
52. The radioactivity of a sample is X at a time t_1 and Y at a time t_2. If the mean life of the specimen is τ, the number of atoms that have disintegrated in the time interval $(t_2 - t_1)$ is

(a) $X t_1 - Y t_2$ (b) $X - Y$
(c) $(X - Y)/\tau$ (d) $(X - Y)\,\tau$

53. Radio carbon dating is done by estimating in the specimen
    (a) the amount of ordinary carbon still present
    (b) the amount of radio carbon still present
    (c) the ratio of the amounts of $^{14}_{6}C$ to $^{12}_{6}C$ still present
    (d) none of the above.
54. A freshly prepared radioactive source of half life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is
    (a) 6 h (b) 12 h
    (c) 24 h (d) 128 h
55. During a negative beta decay:
    (a) an atomic electron is ejected
    (b) an electron which is already present within the nucleus is ejected
    (c) a neutron in the nucleus decays emitting an electron
    (d) a part of the binding energy of the nucleus is converted into an electron.
56. The decay constant of a radioactive sample is λ. The half-life and mean-life of the sample are (respectively) given by:
    (a) $1/\lambda$ and $(\ln 2)/\lambda$ (b) $(\ln 2)/\lambda$ and $1/\lambda$
    (c) $1/\lambda$ and $\lambda\,(\ln 2)$ (d) $\lambda\,(\ln 2)$ and $1/\lambda$
57. The ionising power and the penetration range of radioactive radiations increase in the order
    (a) γ, β, α and γ, β, α respectively
    (b) γ, β, α and α, β, γ respectively
    (c) α, β, γ and α, β, γ respectively
    (d) α, β, γ and γ, β, α respectively
58. The radioactive decay of uranium into thorium is represented by the equation
    $$^{238}_{92}U \rightarrow {}^{234}_{90}Th + x$$
    What is x?
    (a) an electron (b) a proton
    (c) an alpha particle (d) a neutron
59. A carbon nucleus emits a particle x and changes into nitrogen according to the equation
    $$^{14}_{6}C \rightarrow {}^{14}_{7}N + x$$
    What is x?
    (a) an electron (b) a proton
    (c) an alpha particle (d) a photon
60. The radioactive decay of an element X to elements Y and K is represented by the equation
    $$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y \rightarrow {}^{A-4}_{Z-1}K \rightarrow {}^{A-4}_{Z-1}K$$
    The sequence of the emitted radiations is
    (a) α, β, γ (b) β, α, γ
    (c) γ, α, β (d) β, γ, α
61. The half-life of a radioactive substance is 10 days. This means that
    (a) the substance completely disintegrates in 20 days
    (b) the substances completely disintegrates in 40 days
    (c) 1/8 part of the mass of the substance will be left intact at the end of 40 days
    (d) 7/8 part of the mass of the substance disintegrates in 30 days.
62. The half-life of a radioactive substance depends upon
    (a) its temperature
    (b) the external pressure on it
    (c) the mass of the substance
    (d) the strength of the nuclear force between the nucleons of its atoms.
63. A rate-meter measures the number of disintegrations per second from a radioactive source. It gives a count of 320 counts per second. Ninety minutes later, it gives 40 counts per second. What is the half-life of the source?
    (a) 30 minutes (b) 45 minutes
    (c) 60 minutes (d) 75 minutes
64. Two radioactive sources A and B initially contain equal number of radioactive atoms. Source A has a half-life of 1 hour and source B has a half-life of 2 hours. At the end of 2 hours, the ratio of the rate of disintegration of A to that of B is
    (a) 1 : 2 (b) 2 : 1
    (c) 1 : 1 (d) 1 : 4
65. Two radioactive elements X and Y have half-life times of 50 minutes and 100 minutes, respectively. Samples X and Y initially contain equal numbers of atoms. After 200 minutes, the ratio
    $$\frac{\text{number of undecayed atoms of } X}{\text{number of undecayed atoms of } Y} \text{ is}$$
    (a) 4 (b) 2
    (c) 1/2 (d) 1/4
66. A uranium nucleus (atomic number 92, mass number 238) emits an alpha particle and the resultant nucleus emits a β particle. The atomic and mass numbers respectively of the final nucleus are
    (a) 90, 240 (b) 90, 236
    (c) 91, 234 (d) 92, 232

67. The equation
$4\,{}_1^1H^+ \rightarrow {}_2^4He^+ + 2e^+ + 26$ MeV represents
(a) β-decay (b) γ-decay
(c) fusion (d) fission

68. What is the number of α and β particles emitted in the following radioactive decay?
$${}_{90}^{200}X \rightarrow {}_{80}^{168}Y$$
(a) 8 and 6 (b) 6 and 8
(c) 8 and 8 (d) 6 and 6

69. When a nucleus emits a photon, what happens to its atomic number and its actual mass?
(a) Its atomic number and its actual mass remain unchanged
(b) Its atomic number and its actual mass both decrease
(c) Its atomic number and its actual mass both increase
(d) Its atomic number remains the same but its actual mass decreases.

70. If the radius of a nucleus ^{256}X is 8 fermi, the radius of ^{4}He nucleus will be
(a) 1 fermi (b) 2 fermi
(c) 3 fermi (d) 4 fermi

71. The binding energy per nucleon of C-12 is 7.68 MeV and of C-13 is 7.48 MeV. The energy (in MeV) required to remove the extra neutron from C-13 is very nearly equal to
(a) 0.2 (b) 3.7
(c) 3.9 (d) 5

72. A radioactive element of mass number 208 at rest disintegrates by emitting an α-particle. If E is the energy of the emitted α-particle, the energy of disintegration is
(a) $\frac{52}{51}E$ (b) $\frac{51}{52}E$
(c) 52 E (d) E

73. A 1 MeV positron and a 1 MeV electron meet each moving in opposite directions. They annihilate each other by emitting two photons. If the rest mass energy of an electron is 0.51 MeV, the wavelength of each photon is
(a) 5.1×10^{-3} Å (b) 10.2×10^{-3} Å
(c) 8.2×10^{-3} Å (d) 6.2×10^{-3} Å

74. A gamma ray photon creates an electron-positron pair. If the total kinetic energy of the electron-positron pair is 0.78 MeV, the energy of the gamma ray photon is (given the rest mass energy of electron = 0.51 MeV)
(a) 0.27 MeV (b) 0.78 MeV
(c) 1.29 MeV (d) 1.80 MeV

75. The binding energies per nucleon for ${}_1^2H$ and ${}_2^4He$ respectively are 1.1 MeV and 7.1 MeV. The energy released (in MeV) when two ${}_1^2H$ nuclei fuse to form ${}_2^4He$ is
(a) 4.4 (b) 8.2
(c) 24 (d) 28.4

76. The transition from state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition
(a) $2 \rightarrow 1$ (b) $3 \rightarrow 2$
(c) $4 \rightarrow 2$ (d) $5 \rightarrow 4$

77. The half-life of a substance is 20 minutes. What is the time interval between 33% decay and 67% decay?
(a) 40 min (b) 20 min
(c) 30 min (d) 25 min

78. The mass number of a nucleus is
(a) always less than its atomic number
(b) always more than its atomic number
(c) always equal to its atomic number
(d) sometimes more and sometimes equal to its atomic number.

79. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of
(a) 1 Å (b) 10^{-10} cm
(c) 10^{-12} cm (d) 10^{-15} cm

80. A star initially has 10^{40} deuterons. It produces energy via the processes
$${}_1^2H + {}_1^2H \longrightarrow {}_1^3H + p$$
and
$${}_1^2H + {}_1^3H \longrightarrow {}_2^4He + n$$
where the masses of the nuclei are : $m({}^2H) = 2.014$ u, $m(p) = 1.007$ u, $m(n) = 1.008$ u and $m({}^4He) = 4.001$ u. If the average power radiated by the star is 10^{16} W, the deuteron supply of the star is exhausted in a time of the order of
(a) 10^6 s (b) 10^8 s
(c) 10^{12} s (d) 10^{16} s

81. Fast neutrons can easily be slowed down by
(a) the use of lead shielding
(b) passing them through water
(c) elastic collisions with heavy nuclei
(d) applying a strong electric field

82. Masses of two isobars ${}_{29}^{64}Cu$ and ${}_{30}^{64}Zn$ are 63.9298 u and 63.9292 u respectively. It can be concluded from these data that

(a) both the isobars are stable
(b) ^{64}Zn is radioactive, decaying to ^{64}Cu through β-decay
(c) ^{64}Cu is radioactive, decaying to ^{64}Zn through γ-decay
(d) ^{64}Cu is radioactive, decaying to ^{64}Zn through β-decay.

83. The half-life of ^{131}I is 8 days. Given a sample of ^{131}I at time $t = 0$, we can assert that
(a) no nucleus will decay before $t = 4$ days
(b) no nucleus will decay before $t = 8$ days
(c) all nuclei will decay before $t = 16$ days
(d) a given nucleus may decay any time after $t = 0$.

84. The order of magnitude of density of uranium nucleus is, ($m_p = 1.67 \times 10^{-27}$ kg)
(a) 10^{20} kg m^{-3} (b) 10^{17} kg m^{-3}
(c) 10^{14} kg m^{-3} (d) 10^{11} kg m^{-3}

85. ^{22}Ne nucleus, after absorbing energy, decays into two α-particles and an unknown nucleus. The unknown nucleus is
(a) nitrogen (b) carbon
(c) boron (d) oxygen

86. Binding energy per nucleon versus mass number curve for nuclei is shown in Fig. 19.2. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is
(a) $Y \rightarrow 2Z$ (b) $W \rightarrow X + Z$
(c) $W \rightarrow 2Y$ (d) $X \rightarrow Y + Z$

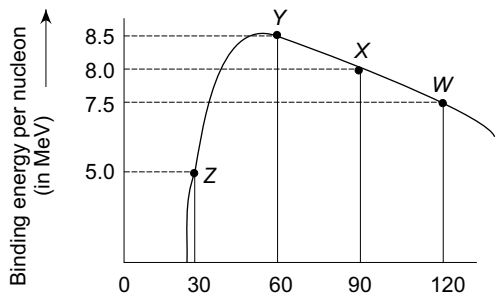


Fig. 19.2

87. Two radioactive materials X_1 and X_2 have decay constants 10 λ and λ respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2 will be 1/e after a time
(a) $\frac{1}{10\lambda}$ (b) $\frac{1}{11\lambda}$
(c) $\frac{11}{10\lambda}$ (d) $\frac{1}{9\lambda}$

88. The electron emitted in beta radiation originates from
(a) inner orbits of atoms
(b) free electrons existing in nuclei
(c) decay of a neutron in a nucleus
(d) photon escaping from the nucleus

89. A radioactive sample consists of two distinct species having equal number of atoms initially. The mean life time of one species is τ and that of the other is 5τ. The decay products in both cases are stable. A plot is made of the total number of radioactive nuclei as a function of time. Which of the following figures best represents the form of this plot? (see Fig. 19.3).

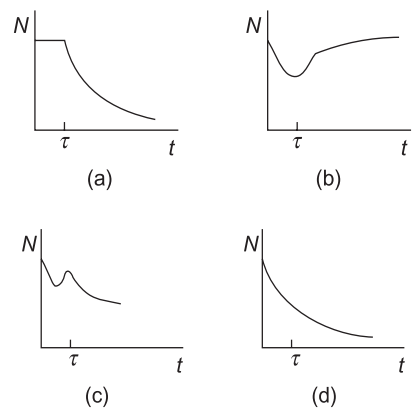


Fig. 19.3

90. The half-life of ^{215}At is 100 μs. The time taken for the radioactivity of a sample of ^{215}At to decay to 1/16th of its initial value is
(a) 400 μs (b) 6.3 μs
(c) 40 μs (d) 300 μs

91. Which of the following processes represents a gamma-decay?
(a) ${}^{A}_{Z}X + \gamma \rightarrow {}^{A}_{Z-1}X + a + b$
(b) ${}^{A}_{Z}X + {}^{1}_{0}n \rightarrow {}^{A-3}_{Z-2}X + c$
(c) ${}^{A}_{Z}X \rightarrow {}^{A}_{Z}X + f$
(d) ${}^{A}_{Z}X + {}_{-1}e \rightarrow {}^{A}_{Z-1}X + g$

92. A nucleus of mass number 220, initially at rest, emits an α-particle. If the Q value of the reaction is 5.5 MeV, the energy of the emitted α-particle will be
(a) 4.8 MeV (b) 5.4 MeV
(c) 6.0 MeV (d) 6.8 MeV

93. A heavy nucleus at rest breaks into two fragments which fly off with velocities in the ratio of 8 : 1. The ratio of the radii of the fragments (assumed spherical) is

(a) 1 : 2 (b) 1 : 4
(c) 4 : 1 (d) 2 : 1

94. A nucleus at rest splits into two nuclear parts having radii in the ratio 1 : 2. Their velocities are in the ratio
(a) 8 : 1 (b) 6 : 1
(c) 4 : 1 (d) 2 : 1

95. For a certain radioactive substance, it is observed that after 4 hours, only 6.25% of the original sample is left undecayed. Choose the only wrong statement from the following.
(a) the half life of the sample is 1 hour
(b) the mean life of the sample 1/2 hour
(c) the decay constant of the sample is (ln 2) hour^{-1}
(d) after a further 4 hours, the amount of the substance left over would be only 0.39% of the original amount.

96. The nucleus of $^{230}_{90}\text{Th}$ decays to $^{226}_{88}\text{Ra}$ and $^{4}_{2}\text{He}$ with the emission of energy. If the original nucleus was at rest, the ratio of kinetic energies of He and Ra nuclei will be very nearly equal to
(a) 22 (b) 44
(c) $\frac{113}{2}$ (d) 113

97. At a given instant there are 25% undecayed radioactive nuclei in a sample. After 69.3 s, the number of undecayed nuclei reduces to 12.5%. The mean life of the sample is
(a) 1 s (b) 10 s
(c) 100 s (d) 1000 s

98. In Q. 97 above, the time in which the number of undecayed nuclei will further reduce to 6.25% of the reduced number will be
(a) 6.93 s (b) 69.3 s
(c) 693 s (d) 277.2 s

99. 16 g of radioactive radon is kept in a container. How much radon will disintegrate in 19 days? Half life of radon is 3.8 days.
(a) 0.5 g (b) 5.5 g
(c) 10.5 g (d) 15.5 g

100. The half life of a radioactive substance is 30 days. The time taken for 3/4 of its original mass to disintegrate is
(a) 45 days (b) 60 days
(c) 75 days (d) 90 days

101. The activity of a radioactive element reduces to $\frac{1}{16}$ of its original value in 36 years. The half life of the element is
(a) 6 years (b) 9 years
(c) 12 years (d) 18 years

102. In Q. 101 above the decay constant of the element is
(a) $\frac{\ln(2)}{6}$ per year (b) $\frac{\ln(2)}{9}$ per year
(c) $\frac{\ln(2)}{12}$ per year (d) $\frac{\ln(2)}{18}$ per year

103. A radioactive substance of half life 69.3 days is kept in a container. The time in which 80% of the substance will disintegrate will be [take ln(5) = 1.61]
(a) 1.61 days (b) 16.1 days
(c) 161 days (d) 1610 days

104. The mass m of a uranium nucleus varies with its volume V as
(a) $m \propto \sqrt{V}$ (b) $m \propto \frac{1}{V}$
(c) $m \propto V$ (d) $m \propto V^2$

105. After 24 hours; the activity of a radioactive sample is 2000 dps (disintegrations per second). After another 12 hours, the activity reduces to 1000 dps. The initial activity of the sample in dps is
(a) 1000 (b) 2000
(c) 4000 (d) 8000

106. $^{221}_{87}\text{Ra}$ undergoes radioactive decay with a half life of 4 days. The probability that a Ra nucleus will disintegrate in 8 days is
(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

107. The half life of a radioactive sample is 6.93 days. After how many days will only one-twentieth of the sample be left over? Take $\log_e(20) = 3.0$.
(a) 20 days (b) 27 days
(c) 30 days (d) 35 days

108. Which of the following atoms or ions will emit an electromagnetic wave of the shortest wavelength from the transition $n = 2$ to $n = 1$?
(a) deuterium atom
(b) singly ionized helium
(c) doubly ionized lithium
(d) triply ionized beryllium

109. Assuming that Bohr's model is applicable to singly ionized helium, the minimum energy required to remove an electron from the ground state of singly ionized helium is
(a) 13.6 eV (b) 27.2 eV
(c) 40.8 eV (d) 54.4 eV

110. An energy of 24.6 eV is required to remove one of the electrons from the neutral helium atom. The energy required to remove both the electrons from a neutral helium atom is
(a) 38.2 eV (b) 49.2 eV
(c) 79.0 eV (d) 122.4 eV

111. The electron in the ground state of hydrogen atom is revolving in a circular orbit of radius r. If m is the mass of electron, e its charge and h the Planck's constant, the magnitude of the magnetic dipole moment of the electron is
(a) $\frac{eh}{4\pi m}$ (b) $\frac{eh}{2\pi mr}$
(c) $\frac{4\pi^2 m^2 r}{eh}$ (d) zero

112. A hydrogen atom emits a photon in going from $n = 3$ state to $n = 1$ state. The recoil speed of the hydrogen atom is
(a) zero (b) 3.86 ms^{-1}
(c) 4.17 ms^{-1} (d) 5.24 ms^{-1}

113. The electron in a hydrogen atom makes a transition from a higher state n_1 to a lower state n_2. The radius of the electron orbit in state n_1 is 4 times that in state n_2. The possible values of n_1 and n_2 are
(a) $n_1 = 4, n_2 = 1$ (b) $n_1 = 5, n_2 = 2$
(c) $n_1 = 6, n_2 = 3$ (d) $n_1 = 6, n_2 = 2$

114. In Q. 113 above, which is the correct choice if the speed of the electron in state n_1 is half that in state n_2?

115. In Q. 113 above which is the correct choice if the time period of revolution of the electron in state n_1 is 8 times that in state n_2?

116. The radius of the electron orbit in the ground state of hydrogen is 0.53 Å. Due to collision with a neutron, the electron shifts to a state in which its orbital radius becomes 4.77 Å. The principal quantum number of the final state of hydrogen is
(a) $n = 1$ (b) $n = 2$
(c) $n = 3$ (d) $n = 4$

117. The longest wavelength in the visible region of the hydrogen spectrum is 656 nm. The shortest wavelength in the infrared region of hydrogen spectrum is
(a) 1875 nm (b) 1686 nm
(c) 1264 nm (d) 820 nm

118. The electron in an excited state of doubly ionized lithium has angular momentum = $3h/2\pi$. If r_0 is the Bohr radius, the de-Broglie wavelength of the electron in this state is
(a) πr_0 (b) $2\pi r_0$
(c) $3\pi r_0$ (d) $4\pi r_0$

119. A radioactive sample is prepared at time $t = 0$. If it has a half life of 20 days, then we can assert that
(a) no nucleus will disintegrate before $t = 10$ days
(b) no nucleus will disintegrate before $t = 20$ days
(c) 75% of the nuclei will remain undecayed at $t = 40$ days
(d) a given nucleus may disintegrate any time after $t = 0$

120. The activity of a radioactive sample A is the half that of a sample B. If sample A has twice the number of nuclei than sample B, the ratio of the half life of A to that of B is
(a) 4 (b) 8
(c) 2 (d) 1

121. Figure 19.4 shows a graph of $\ln\left|\frac{dN}{dt}\right|$ versus t. From the graph, the half life (in seconds) of the radioactive sample is
(a) $\frac{\ln(2)}{2}$ (b) ln(2)
(c) 2ln(2) (d) 4ln(2)

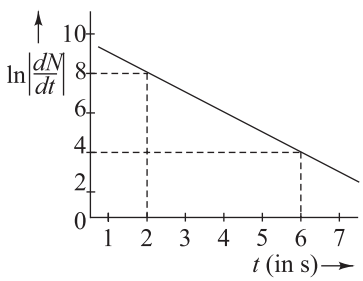


Fig. 19.4

122. The half life of an element A is equal to the mean life of an element B. If, initially (i.e., at $t = 0$), A and B have equal number of radioactive nuclei, then
(a) initially both A and B decay at the same rate
(b) A and B always decay at the same rate
(c) A decays at a faster rate than B
(d) B decays at a faster rate than A.

123. The activity of a freshly prepared radioactive sample is 4×10^{10} disintegrations per second. The mean life of the sample is 10^9 s and the mass of an atom of this radioisotope is 10^{-25} kg. The mass of the radioactive sample is
(a) 1 mg (b) 2 mg
(c) 4 mg (d) 8 mg



Answers

Level A

1. (c)	2. (c)	3. (d)	4. (a)
5. (c)	6. (a)	7. (d)	8. (b)
9. (b)	10. (a)	11. (b)	12. (d)
13. (c)	14. (c)	15. (b)	16. (c)
17. (a)	18. (a)	19. (c)	20. (c)
21. (c)	22. (c)		

Level B

23. (b)	24. (b)	25. (a)	26. (b)
27. (a)	28. (a)	29. (d)	30. (b)
31. (d)	32. (c)	33. (d)	34. (c)
35. (d)	36. (a)	37. (d)	38. (b)
39. (c)	40. (c)	41. (d)	42. (c)
43. (c)	44. (c)	45. (b)	46. (d)
47. (b)	48. (d)	49. (b)	50. (d)
51. (d)	52. (b)	53. (c)	54. (b)
55. (c)	56. (b)	57. (b)	58. (c)
59. (a)	60. (b)	61. (d)	62. (d)
63. (a)	64. (c)	65. (d)	66. (c)
67. (c)	68. (a)	69. (d)	70. (b)
71. (d)	72. (a)	73. (c)	74. (d)
75. (c)	76. (d)	77. (b)	78. (d)
79. (c)	80. (c)	81. (b)	82. (d)
83. (d)	84. (b)	85. (b)	86. (c)
87. (d)	88. (c)	89. (d)	90. (a)
91. (c)	92. (b)	93. (a)	94. (a)
95. (b)	96. (c)	97. (c)	98. (d)
99. (d)	100. (b)	101. (b)	102. (b)
103. (c)	104. (c)	105. (d)	106. (d)
107. (c)	108. (d)	109. (d)	110. (c)
111. (a)	112. (b)	113. (c)	114. (c)
115. (c)	116. (c)	117. (d)	118. (b)
119. (d)	120. (a)	121. (b)	122. (d)
123. (c)			



Solutions

Level A

1. Since an alpha particle and the nucleus both are positively charged, the nucleus will repel the alpha particle. Hence path 3 is not possible.
2. The distance of the closest approach is given by

$$r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{K}$$

where $K = \frac{1}{2} mv^2$. Thus $r_0 \propto \frac{1}{K}$. When K is doubled, r_0 becomes half. Hence the correct choice is (c).

3. $K = \frac{1}{2} mv^2 = \frac{1}{2m} \times (mv)^2 = \frac{p^2}{2m}$. Therefore,

$$r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4mZe^2}{p^2}$$

Thus $r_0 \propto \frac{1}{p^2}$. When p is doubled, r_0 becomes one-fourth. Hence the correct choice is (d).

4. The number of scintillations per unit time is $n \propto \cot^4\left(\frac{\theta}{2}\right)$. Hence,

$$\frac{n_2}{n_1} = \frac{\cot^4(120°/2)}{\cot^4(60°/2)} = \frac{\cot^4(60°)}{\cot^4(30°)}$$

$$= \left(\frac{1/\sqrt{3}}{\sqrt{3}}\right)^4 = \frac{1}{81}$$

Hence $n_2 = \frac{1}{81} \times 8100 = 100$, which is choice (a).

5. Given $E_2 - E_1 = 10.2$ eV $= 10.2 \times 1.6 \times 10^{-19}$ J. Therefore, frequency the emitted radiation is

$$\nu = \frac{E_2 - E_1}{h}$$

and wavelength is

$$\lambda = \frac{c}{\nu} = \frac{ch}{E_2 - E_1}$$

$$= \frac{3\times10^8 \times 6.6\times10^{-34}}{10.2\times1.6\times10^{-19}}$$

$$= 1.22\times10^{-7}\ \text{m} = 1220\ \text{Å}$$

6. Ionization potential = energy of the electron in the first orbit of the atom = – 13.6 eV. Its value in the second orbit is

$$-\frac{13.4\,\text{eV}}{(2)^2} = -3.4\ \text{eV}$$

Hence the correct choice is (a).

7. The diameter of nth orbit is proportional to n^2, i.e. $d_n \propto n^2$. Given $d_1 = 1.06$ Å. Therefore $d_{10} = (10)^2 d_1 = 106$ Å. Hence the correct choice is (d).

8. The correct choice is (b).

9. For Lyman series

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right);\ n = 2, 3, 4\ \ldots$$

The extreme wavelengths correspond to $n = 2$ and $n = \infty$.

Therefore, the wavelength limits are

$\lambda_s = \frac{1}{R}$ and $\lambda_l = \frac{4}{3R}$. Since $\frac{1}{R} = \frac{1}{1.1\times10^7}$ m $\cong$ 0.9×10^{-7} m = 900 Å, we have

$\lambda_l = \frac{4}{3}\times 900 = 1200$ Å and $\lambda_s = 900$ Å.

Hence the correct choice is (b).

10. For Lyman series, we have $\frac{\lambda_l}{\lambda_s} = \frac{4}{3}$. Hence $\lambda_l = \frac{4}{3}\times 911.6 = 1215$ Å.

11. For an atom of atomic number Z, the radius of the nth orbit is given by [see Eq. (19.3)]

$$r_n = \frac{Kn^2}{Z} \qquad \text{(i)}$$

where $K = \frac{\varepsilon_0 h^2}{\pi m e^2}$ is a constant. For the ground state of hydrogen ($Z = 1$), $n = 1$ so that

$$r_1 = K$$

Let n be the energy state of Be^{+++} for which the orbital radius is r_1. Putting $Z = 4$ and $r_n = r_1 = K$ is Eq. (i) we get

$$K = K\frac{n^2}{4} \text{ or } n^2 = 4 \text{ or } n = 2$$

Hence the correct choice is (b).

12. The energy in the nth state is given by

$$E_n = -\frac{me^2}{8\varepsilon_0 h^2}\cdot\frac{Z^2}{n^2} = \frac{CZ^2}{n^2}$$

where $C = -\frac{me^2}{8\varepsilon_0 h^2}$ is a constant. For the ground state $n = 1$ of hydrogen and for $n = 2$ state of Be^{+++}, we have

$$E_1 = C \text{ and } E_2' = \frac{C(4)^2}{(2)^2} = 4C$$

$$\therefore \quad \frac{E_2'}{E_1} = \frac{4C}{C} = 4.$$

Hence the correct choice is (d).

13. $E_n = \frac{CZ^2}{n^2}$. For $n = 1$ state of hydrogen, we have $E_1 = C$ and for the nth state of Li^{++}, we have $E_n = \frac{C(3)^2}{n^2} = \frac{9C}{n^2}$. For $E_1 = E_n$, we require $C = \frac{9C}{n^2}$ which gives $n = 3$. Hence the correct choice is (c).

14. Now $r_n = \frac{Kn^2}{Z}$. Therefore, for hydrogen ($n = 1$ state), we have

$$r_1 = K \qquad (\because Z = 1)$$

and for Li^{++} ($n = 3$ state) we have $\qquad (\because Z = 3)$

$$r'_3 = \frac{K(3)^2}{3} = 3K$$

$$\therefore \quad \frac{r_3'}{r_1} = \frac{3K}{K} = 3.$$

Hence the correct choice is (c).

15. The kinetic and potential energies of an electron in the nth excited state are given by

$$\text{KE} = \frac{1}{8\pi\varepsilon_0}\cdot\frac{e^2}{r_n} \qquad \text{(i)}$$

and $\qquad \text{PE} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r_n} \qquad \text{(ii)}$

Adding (i) and (ii) we get the total energy E which is

$$E = \text{KE} + \text{PE} = -\frac{1}{8\pi\varepsilon_0}\cdot\frac{e^2}{r_n} \qquad \text{(iii)}$$

Notice from (i) and (iii) that $E = -$ KE. Given $E = -3.4$ eV. Hence KE $= -E = -(-3.4) = +3.4$ eV. Thus the correct choice is (b).

16. From (ii) and (iii) we find that $E = \frac{\text{PE}}{2}$ or PE $= 2E = 2 \times -3.4 = -6.8$ eV. Hence the correct choice is (c).

17. Wavelengths in Balmer series for hydrogen are given by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$$= R\left(\frac{1}{4} - \frac{1}{n^2}\right); \; n = 3, 4, 5 \ldots$$

The second line in Balmer series corresponds to $n = 4$. Hence

$$\frac{1}{\lambda_2} = R\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R}{16} \text{ or } \lambda_2 = \frac{16}{3R}$$

The wavelength of the first line ($n = 2$) in Lyman series is

$$\frac{1}{\lambda_1} = R\left(1 - \frac{1}{2^2}\right) = R\left(1 - \frac{1}{4}\right)$$

$$= \frac{3R}{4} \text{ or } \lambda_1 = \frac{4}{3R}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{4}{3R} \times \frac{3R}{16} = \frac{1}{4}$$

$$\text{or} \quad \lambda_1 = \frac{\lambda_2}{4} = \frac{486.4}{4} = 121.6 \text{ nm}.$$

Hence the correct choice is (a).

18. The first line corresponds to $n = 3$. Therefore,

$$\frac{1}{\lambda_1} = R\left(\frac{1}{4} - \frac{1}{3^2}\right)$$

$$= R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36}$$

or $\lambda_1 = \frac{36}{5R}$. We have seen above that $\lambda_2 = \frac{16}{3R}$. Hence

$$\frac{\lambda_2}{\lambda_1} = \frac{16}{3R} \times \frac{5R}{36} = \frac{20}{27}$$

$$\text{or} \quad \lambda_2 = \frac{20}{27}\lambda_1 = \frac{20}{27}\lambda.$$

19. Now $\nu \propto Z^2$. For doubly ionized lithium $Z = 3$. Hence the correct choice is (c).

20. Energy difference $\Delta E \propto Z^2$. For a singly ionized helium atom $Z = 2$. Hence $\Delta E = 10.2 \times (2)^2 = 40.8$ eV.

21. Ionization energy $E \propto Z^2$. For helium $Z = 2$. Hence E for helium $= 13.6 \times (2)^2 = 54.4$ eV, which is choice (c).

22. When an electron in the ground state receives 12.1 eV of energy, it jumps to a level where its energy = 13.6 – 12.1 = 1.5 eV. This corresponds to the third excited state corresponding to $n = 3$. It can have three transitions, namely from $n = 3$ to $n = 2$, from $n = 2$ to $n = 1$ and from $n = 3$ to $n = 1$. Hence three spectral lines will be emitted.

Level B

23. Orbital speed $v_0 \propto \frac{1}{n}$. Therefore, $n = 2$. Now $r_n \propto n^2$. Hence the radius of the new orbit $= (2)^2 \times r = 4r$.

24. Energy state – 3.4 eV corresponds to a level n given by $-13.6 \text{ eV}/n^2 = -3.4$ eV which gives $n = 2$. Now, orbital speed $v_0 \propto \frac{1}{n}$. Hence the orbital speed in the excited state is $v/2$.

25. The total energy of an electron bound to an atom is negative and is the sum of its P.E. and K.E. The magnitude of the P.E. is twice that of the K.E. (as per the Bohr model) but since the P.E. is –ve, we have

Total Energy $= (-2K) + (K) = -K$

(K = Kinetic energy). Hence the ratio of total energy and kinetic energy $= -1 : 1 = -1$.

26. The longest wavelengths in the two series are given by

$$\frac{1}{\lambda_L} = R\cdot\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = R \cdot \frac{3}{4}$$

$$\text{and} \quad \frac{1}{\lambda_B} = R \cdot \left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R \cdot \frac{5}{36}$$

$$\therefore \quad \frac{\lambda_B}{\lambda_L} = \frac{3}{4} \times \frac{36}{5} = \frac{27}{5} \text{ or } \lambda_L : \lambda_B = 5 : 27$$

27. Since the momentum of photon is $\frac{h}{\lambda}$, from the law of conservation of momentum, the recoil speed v of an atom of mass m is given by

$$mv = \frac{h}{\lambda} \text{ or } v = \frac{h}{m\lambda}$$

28. The maximum number of electrons allowed in an orbit is $2n^2$. Hence the number of possible elements is

$$2\,(1^2 + 2^2 + 3^2 + 4^2) = 60$$

29. In all nuclear reactions, both charge and mass numbers are conserved. Hence the correct choice is (d).

30. The charge number and the mass number must be the same on both sides of a nuclear reaction. Particle x must have charge number + 1 and mass number 1. Hence it is a proton.

31. The correct choice is (d).

32. Particle x must have zero charge and mass number 1. Hence it is a neutron.

33. The mass number of $x = 27 + 1 - 24 = 4$ and its atomic number = 13 + 0 – 11 = 2. Hence particle x is helium nucleus, which is called alpha particle.

34. Reaction (c) is incorrect because the mass number is not conserved.

35. The mass number of X is $A = 27 + 4 - 30 = 1$ and its atomic number $Z = 13 + 2 - 15 = 0$. Hence particle X is a neutron and its symbol is ${}^{A}_{Z}n$ which ${}^{1}_{0}n$.

36. The correct choice is (a). The mass difference is converted into energy in accordance with Einstein's mass-energy equivalence relation $E = mc^2$.

37. The correct choice is (d).

38. Statement (b) is wrong.

39. The addition of neutrons to the nucleus of an atom has no effect on the chemical properties.

40. The kinetic energy of a neutron at absolute temperature T is

$$E \simeq kT$$

where k is Boltzmann constant. Thus

$$E = 1.38 \times 10^{-23} \times 300 \text{ J}$$

$$= \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \text{ eV} \simeq 0.026 \text{ eV}$$

41. Energy released = 200 MeV = 200×10^6 eV = $200 \times 10^6 \times 1.6 \times 10^{-19}$ J. Therefore, the required number is

$$\frac{3.2}{200 \times 10^6 \times 1.6 \times 10^{-19}} = 10^{11}$$

42. The energy released is of the order of

$$7.58 \times 236 - 8.5 \times (117 + 117) \simeq 200 \text{ MeV}$$

43. Binding energy of ${}^{2}_{1}\text{H} = 1.15 \times$ number of nucleons = $1.15 \times 2 = 2.3$ MeV. Total binding energy of reactants = 2.3 + 2.3 = 4.6 MeV. Binding energy of ${}^{4}_{2}\text{He}$ = 7.1 × number of nucleons = 7.1 × 4 = 28.4 MeV. Therefore, Q = 28.4 – 4.6 = 23.8 MeV. Hence the correct choice is (c).

44. The correct choice is (c).

45. The correct choice is (b).

46. The correct choice is (d).

47. $E = mc^2 = 9.1 \times 10^{-31} \times (3 \times 10^8)^2 = 8.2 \times 10^{-14}$ J

$$= \frac{8.2 \times 10^{-14}}{1.6 \times 10^{-19}} = 5.1 \times 10^5 \text{ eV} = 0.51 \text{ MeV}$$

48. The correct choice is (d).

49. Since $\frac{1}{16} = \frac{1}{2^4}$, it follows that 2 hours = 4 half lives.

or 1 half-life = $\frac{2}{4} = \frac{1}{2}$ hour = 30 min. Hence the correct choice is (b).

50. We have $T_{1/2}$ = 10 minutes. Therefore 5 minutes = $\frac{1}{2}$ of half-life. It follows that the number of nuclei left after 5 minutes will be

$$\frac{1}{(2)^{1/2}} = \frac{1}{\sqrt{2}} \text{ of the original number}$$

$$= \frac{10^8}{\sqrt{2}}$$

Hence the correct choice is (d).

51. Since 15 minutes = 5 × 3 minutes = 5 half lives, the number of nuclei left after 15 minutes = $\frac{1}{2^5} = \frac{1}{32}$ of the original number. Therefore, the mass of 16 g sample left after 15 minutes = $\frac{16}{32}$ = 0.5 g. Hence the correct choice is (d).

52. There were X atoms at time $t = t_1$ and we are left with Y atoms at time $t = t_2$. Therefore, the number of atoms that have disintegrated in the time interval $(t_2 - t_1)$ is X-Y. Hence the correct choice is (b).

53. The isotope C-14 of carbon is radioactive and its relative percentage gives us an estimate of the age of the specimen.

54. The radioactivity of a sample decreases to $\left(\frac{1}{2^n}\right)$ in n half lives. Since the initial activity is 64 times the permissible level, it must become 1/64th of its initial value.

$$\therefore \quad \frac{1}{64} = \frac{1}{2^n} \quad \text{or} \quad n = 6$$

In other words in six half–lives the activity will reduce itself to the permissible level. Since half-life is 2 hours, the total time taken = 2 × 6 h = 12 hours.

55. Negative β decay is expressed by the equation:

$n \longrightarrow p + e^- + \bar{\nu}$; $\bar{\nu}$ stands for antineutrino.

Hence the correct choice is (c).

56. For a radioactive material of disintegration constant λ,

$$\text{Half-life} = T_{1/2} = \frac{\ln 2}{\lambda}, \text{ and}$$

$$\text{mean-life} = \tau = \frac{1}{\lambda}.$$

Hence the correct choice is (b).

57. Out of the three radioactive radiations, the α-rays are the most ionising but least penetrative while the γ-rays are the least ionising but most penetrative.

58. The mass number of x must be 238 – 234 = 4 and atomic number of x must be 92 – 90 = 2. Hence the correct choice is (c).

59. The charge of x is opposite to that of a proton. Hence x is an electron.

60. In transition ${}^{A}_{Z}\text{X} \longrightarrow {}^{A}_{Z+1}\text{Y}$, the atomic (or charge) number increases by unity; mass number remaining the same. Hence an electron (β-particle) is emitted. In transition ${}^{A}_{Z+1}\text{Y} \longrightarrow {}^{A-4}_{Z-1}\text{K}$, the mass number decreases by 4 and charge number decreases by 1. Hence an α-particle is emitted. In the third transition, mass and charge numbers do not change. Hence a γ-ray is emitted. Hence the correct choice is (b).

61. In 30 days (i.e. 3 half lives) $1/2^3 = 1/8$ of the sustance is left. Hence the correct choice is (d).

62. The correct choice is (d).

63. The count decreases by a factor of 320/40 = 8 in 90 minutes. Now $(2)^3 = 8$. Hence 90 minutes = 3 half lives. Hence the half life of the radioactive source is 30 minutes.

64. The rate of disintegration is proportional to the number of radioactive atoms present initially in the source. Hence the correct choice is (c).

65. Since samples X and Y contain equal number of atoms initially, their rates of disintegration are equal. In 200 minutes (i.e. 4 half lives) the atoms of sample X left undecayed = $1/2^4 = 1/16$ of the initial number. In 200 minutes (i.e. 2 half lives) the atoms of sample Y left undecayed = $1/2^2 = 1/4$ of the initial number. Hence the correct choice is (d).

66. The emission of an α-particle decreases the mass number by 4 and atomic number by 2. The emission of a β-particle increases the atomic number by 1. Hence the final nucleus has a mass number = 238 – 4 = 234 and atomic number = 92 – 1 + 1 = 91. Hence the correct choice is (c).

67. The correct choice is (c). It represents a fusion of four ${}^{1}_{1}\text{H}$ nuclei with the emission a huge amount of energy.

68. The mass number reduces by 200 – 168 = 32. Hence 8 α-particles are emitted. The emission of 8 α-particles reduces the atomic number by 16. But the atomic number reduces by 90 – 80 = 10. Hence the number of β–particle emitted = 16 – 10 = 6. Hence the correct choice is (a).

69. A photon carries energy which is equivalent to mass. Hence the correct choice is (d).

70. $R = R_0 (A)^{1/3}$. Therefore

$$\frac{R_2}{R_1} = \left(\frac{A_2}{A_1}\right)^{1/3} = \left(\frac{4}{254}\right)^{1/3} = \frac{1}{4}$$

or $$R_2 = \frac{R_1}{4} = \frac{8 \text{ fermi}}{4} = 2 \text{ fermi}$$

Hence the correct choice is (b).

71. Energy required = (7.48 × 13 – 12 × 7.68) MeV = 5.08 MeV. Hence the correct choice is (d).

72. Mass number of daughter nucleus (M) = 208 – 4 = 204

Now, total energy of disintegration = energy of daughter nucleus + energy of α-particle

or $$E_t = \frac{p^2}{2m} + \frac{P^2}{2M}$$

Since momentum is conserved, $p = P$. Hence

$$E_t = \frac{p^2}{2}\left(\frac{1}{m} + \frac{1}{M}\right) = \frac{p^2}{2}\left(\frac{m+M}{mM}\right)$$

Energy of α-particle = $\frac{p^2}{2m} = E$. Hence

$$E_t = \left(\frac{m+M}{M}\right)E$$

$$= \left(\frac{4+204}{204}\right)E = \frac{52}{51}E$$

Hence the correct choice is (a).

73. Energy released in annihilation = 0.51 + 0.51 =1.02 MeV.

Initial energy = 1 + 1 = 2 MeV. Therefore, the energy of the two photons is = 1.02 + 2 = 3.02 MeV. Hence energy of each photon is E = 1.51 MeV. Now, according to Duane-Hunt law, the wavelength of a photon of energy E (in eV) is given by

$$\lambda = \frac{hc}{E \text{ in eV}} \text{ Å} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1.51 \times 10^6} \text{ Å}$$

$$= 8.2 \times 10^{-3} \text{ Å which is choice (c).}$$

74. $E = 0.51 + 0.51 + 0.78 = 1.80$ MeV. Hence the correct choice is (d).

75. ${}_1^2\text{H} + {}_1^2\text{H} \rightarrow {}_2^4\text{He} + Q$

Binding energy of ${}_1^2\text{H} = 2 \times 1.1 = 2.2$ MeV. Therefore, the binding energy of two ${}_1^2\text{H}$ = 2.2 + 2.2 = 4.4 MeV. Now, the binding energy of ${}_2^4\text{He}$ nucleus = $4 \times 7.1 = 28.4$ MeV. Hence $Q = 28.4 - 4.4 = 24$ MeV, which is choice (c).

76. The frequency of infrared radiation is less than that of ultraviolet radiation. Hence, infrared radiation has higher energy than ultraviolet radiation. Therefore, infrared radiation will be obtained in the transition $n = 5$ to $n = 4$, which is choice (d).

77. Given $T = 20$ min. We know that

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$$

For 33% decay, $\frac{N}{N_0} = \frac{100 - 33}{100} = \frac{67}{100}$

If t_1 is the time for 33% decay, then

$$\frac{67}{100} = \left(\frac{1}{2}\right)^{t_1/T} \quad \text{(i)}$$

For 67% decay, $\frac{N}{N_0} = \frac{100 - 67}{100} = \frac{33}{100}$

If t_2 is the time for 67% decay, then

$$\frac{33}{100} = \left(\frac{1}{2}\right)^{t_2/T} \quad \text{(ii)}$$

Dividing (ii) by (i), we have

$$\frac{33}{67} = \left(\frac{1}{2}\right)^{(t_2 - t_1)/T} \quad \text{(iii)}$$

Now $\frac{33}{67} \simeq \frac{1}{2}$. Hence Eq. (iii) reduces to

$$\left(\frac{1}{2}\right)^1 = \left(\frac{1}{2}\right)^{(t_2 - t_1)/T}$$

which gives $1 = \frac{t_2 - t_1}{T}$ or $t_2 - t_1 = T = 20$ min.

Hence the correct choice is (b).

78. Mass number Z is greater than atomic number A for all nuclei; the single exception being the hydrogen nucleus for which $A = Z$. Hence the correct choice is (d).

79. The distance of closest approach is given by $r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{E_k}$. Here $Z = 92$ for uranium and $E_k = 5$ MeV $= 5 \times 10^6 \times 1.6 \times 10^{-19}$ J. Also $e = 1.6 \times 10^{-19}$ C and $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^2 C^{-2}. Using these values, we get $r_0 \simeq 10^{-12}$ cm, which is choice (c).

80. Let the mass of ${}_1^3\text{H}$ be x amu. Then the mass defect in the first process is

$$(\Delta m)_1 = m({}_1^2\text{H}) + m({}_1^2\text{H}) - m({}_1^3\text{H}) - m(p)$$
$$= 2.014 + 2.014 - x - 1.007 = (3.021 - x) \text{ u}$$

Mass defect in the second process will be

$$(\Delta m)_2 = m({}_1^2\text{H}) + m({}_1^3\text{H}) - m({}_2^4\text{H}) - m(n)$$
$$= 2.014 + x - 4.001 - 1.008 = (x - 2.995) \text{ u}$$

$\therefore$ Total mass defect is

$$\Delta m = (\Delta m)_1 + (\Delta m)_2 = (3.021 - x) + (x - 2.995)$$
$$= 0.026 \text{ u}$$

Now 1 amu = 931 MeV = $931 \times 10^6 \times 1.6 \times 10^{-19}$ J. Hence, energy released is given by

$$E = 0.026 \times 931 \times 10^6 \times 1.6 \times 10^{-19}$$
$$= 3.87 \times 10^{-12} \text{ J}$$

Now power $P = 10^{16}$ W. Therefore, the number of deutrons per second is $\frac{10^{16}}{3.87 \times 10^{-12}} = 2.58 \times 10^{27}$ per second

$\therefore$ Deutron supply will exhaust in $\frac{10^{40}}{2.58 \times 10^{27}}$ $= 3.87 \times 10^{12}$ seconds

Hence the correct choice is (c).

81. The correct choice is (b).

82. The atomic mass of stable Cu is smaller than that of Zn. Since the given mass of Cu is greater than that of Zn, ${}^{64}\text{Cu}$ will be unstable. In β-decay, the atomic number is increased by one while the mass number remains unchanged. Thus

$${}_{29}^{64}\text{Cu} \xrightarrow{\beta\text{-decay}} {}_{30}^{64}\text{Zn} + {}_{-1}^{0}e$$

Hence the correct choice is (d).

83. A radioactive nucleus may decay any time after $t = 0$. The number of nuclei at any time t is given by $N = N_0 e^{-\lambda t}$, where $\lambda = \frac{0.693}{T}$ where T is the half-life.

Hence the correct choice is (d).

84. If A is the atomic number, the mass of uranium nucleus is

$$m = (1.67 \times 10^{-27}) A \text{ kg and its volume is}$$

$$V = \frac{4}{3}\pi r^3 = \frac{4\pi}{3} \times \{1.25 \times 10^{-15} \text{ m} A^{1/3}]^3$$
$$\simeq 8.2 \times 10^{-45} A \text{ m}^3$$

$(\because\ r = r_0 A^{1/3};\ r_0 = 1.25 \times 10^{-15}\ \text{m})$

$$\therefore \quad \text{Density} = \frac{m}{V} = \frac{1.67 \times 10^{-27} A\ \text{kg}}{8.2 \times 10^{-45} A\ \text{m}^3}$$

$$\simeq 2 \times 10^{17}\ \text{kg m}^{-3}$$

Hence the correct choice is (b).

85. The given nuclear reaction is given by the equation

$$^{22}_{10}\text{Ne} \rightarrow {}^{4}_{2}\text{He} + {}^{4}_{2}\text{He} + {}_{Z}\text{X}$$

Since the atomic number is conserved, we have

$$10 = 2 + 2 + Z$$

or $Z = 6$. The carbon nucleus has atomic number 6. Hence the correct choice is (b).

86. The binding energies of the reactants and the products in the given nuclear reactions are as follows:

Reaction	*Reactant*	*Products*
(a) $Y \rightarrow 2Z$	$60 \times 8.5 = 510$ MeV	$2 \times 30 \times 5.0 = 300$ MeV
(b) $W \rightarrow X + Z$	$120 \times 7.5 = 900$ MeV	$(90 \times 8.0 + 30 \times 5.0) = 870$ MeV
(c) $W \rightarrow 2Y$	$120 \times 7.5 = 900$ MeV	$2 \times 60 \times 8.5 = 1020$ MeV
(d) $X \rightarrow Y + Z$	$90 \times 8.0 = 720$ MeV	$(60 \times 8.5 + 30 \times 5.0) = 660$ MeV

The binding energy of the products in reaction (c) is greater than that of the reactant. Hence reaction (c) releases energy.

87. $N_1 = N_0\, e^{-\lambda_1 t}$, $N_2 = e^{-\lambda_2 t}$. Therefore

$$\frac{N_1}{N_2} = \frac{e^{-\lambda_1 t}}{e^{-\lambda_2 t}} = e^{-(\lambda_1 - \lambda_2)t} = e^{-(10\lambda - \lambda)t} = e^{-9\lambda t}$$

Given, $\frac{N_1}{N_2} = e^{-1}$. Hence $-9\lambda t = -1$ or $t = \frac{1}{9\lambda}$. Thus the correct choice is (d).

88. Beta particle is an electron which is emitted from a nucleus when a neutron decays into a proton and an electron within a nucleus. Hence the correct choice is (c).

89. For each species, the number of radioactive nuclei decreases exponentially with time. Hence, for both the species taken together, the total number of radioactive nuclei will decrease exponentially with time. This is best represented in plot (d).

90. Since $\frac{1}{16} = \frac{1}{2^4}$, it follows that the time taken for the radioactivity to decay to $\frac{1}{16}$ th of its initial value = four times the half-life of the sample $= 4t_{1/2} = 4 \times 100\ \mu\text{s} = 400\ \mu\text{s}$. Thus, the correct choice is (a).

91. During the emission of a gamma radiation, both the mass number and atomic number remain the same. Hence the correct choice is (c).

92. Kinetic energy $= \frac{(\text{momentum})^2}{2 \times \text{mass}}$

Mass number of α-particle (m) = 4 units. Mass number of daughter nucleus (M) = 220 – 4 = 216 units. If P and p denote the momenta of the daughter nucleus and the α-particle respectively, then

$$Q = \frac{P^2}{2M} + \frac{p^2}{2m}$$

Since momentum is conserved, $P = p$. Hence

$$Q = \frac{p^2}{2}\left(\frac{1}{M} + \frac{1}{m}\right) = \frac{p^2}{2m}\left(\frac{m}{M} + 1\right)$$

Now $\frac{p^2}{2m}$ = KE of α-particle = E_α. Thus,

$$Q = E_\alpha \left(\frac{m + M}{M}\right)$$

or $$E_\alpha = \frac{QM}{(m + M)} = \frac{5.5\ \text{MeV} \times 216}{(4 + 216)}$$

$$= 5.4\ \text{MeV}$$

Hence the correct choice is (b).

93. Let m_1 and m_2 be the atomic masses of the fragments and v_1 and v_2 their velocities. From the principal of conservation of linear momentum, we have

$$m_1 v_1 + m_2 v_2 = 0 \quad \text{or} \quad \frac{m_1}{m_2} = \left|\frac{v_1}{v_2}\right| = \frac{1}{8}$$

Now $m_1 = A_0 (R_1)^{1/3}$ and $m_2 = A_0 (R_2)^{1/3}$. Hence

$$\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^{1/3}$$

or $$\frac{1}{8} = \left(\frac{R_1}{R_2}\right)^{1/3} \quad \text{or} \quad \frac{R_1}{R_2} = \frac{1}{2}$$

Thus, the correct choice is (a).

94. Let A_1 and A_2 be the mass numbers of the two nuclear parts. Their radii are given by

$$R_1 = R_0 (A_1)^{1/3} \text{ and } R_2 = R_0 (A_2)^{1/3}$$

Dividing, we get

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} \text{ or}$$

$$\frac{A_1}{A_2} = \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

Hence the ratio of their masses is

$$\frac{m_1}{m_2} = \frac{1}{8}$$

From the principle of conservation of momentum, the magnitude of p_1 = magnitude of p_2 or $m_1 v_1 = m_2 v_2$, which gives

$$\frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{8}{1}, \text{ which is choice (a).}$$

95. 6.25% = $\frac{6.25}{100} = \frac{1}{16} = \frac{1}{(2)^4}$. Hence 4 hours are equal to 4 half lives. Therefore, the half life of the substance is 1 hour, which is choice (a). The decay constant = $\frac{\ln(2)}{\text{half life}} = \ln(2)$ per hour, which is choice (c).

$$\text{Mean life} = \frac{1}{\text{decay constant}}$$

$$= \frac{1}{\ln(2)} \text{ hour} = \frac{1}{0.693} \text{ hour}$$

Hence choice (b) is incorrect.

After further 4 hours (i.e. after 8 hours), $\frac{1}{(2)^8} = \frac{1}{256} = \frac{100}{256}\% = 0.39\%$ of the substance remains undecayed, which is choice (d). Thus the only incorrect choice is (b).

96. If the original nucleus Th is at rest, i.e. if the momentum of the system before α-decay is zero, the total momentum after the decay will also be zero. Thus Ra and He will have equal and opposite linear momenta.

$$\therefore \quad m_{He} v_{He} = - m_{Ra} v_{Ra}$$

$$\text{or} \quad m_{He}^2 v_{He}^2 = m_{Ra}^2 v_{Ra}^2 \qquad \text{(i)}$$

$$\text{Now } \frac{\text{K.E.(He)}}{\text{K.E.(Ra)}} = \frac{1/2 m_{He} v_{He}^2}{1/2 m_{Ra} v_{Ra}^2}$$

$$= \left(\frac{m_{He} v_{He}}{m_{Ra} v_{Ra}}\right)^2 \times \left(\frac{m_{Ra}}{m_{He}}\right)$$

$$= \frac{m_{Ra}}{m_{He}} = \frac{226}{4} = \frac{113}{2} \quad \text{[use Eq. (i)]}$$

Hence the correct choice is (c).

97. In 69.3 s, the number of nuclei reduces to half (from 25% to 12.5%). Hence half life = 69.3 s

$$\text{Mean life} = \frac{\text{half life}}{\ln(2)} = \frac{69.3 \text{ s}}{0.693} = 100 \text{ s}$$

Hence the correct choice is (c).

98. Let the reduced number further reduce to 6.25% in n half lives. Then

$$\frac{6.25}{100} = \left(\frac{1}{2}\right)^n \text{ or } \frac{1}{16} = \left(\frac{1}{2}\right)^n$$

which gives $n = 4$. Therefore, the time taken would be $t = 4T_{1/2}$. Hence the correct choice is (d).

99. Number of half lives in 19 days is

$$n = \frac{19 \text{ days}}{3.8 \text{ days}} = 5$$

$\therefore$ Number of atoms left undecayed after 19 days is

$$N = N_0 \left(\frac{1}{2}\right)^5 = \frac{N_0}{32} = \frac{16 \text{ g}}{32} = 0.5 \text{ g}$$

Hence the correct choice is (d).

100. $N = N_0 - \frac{3N_0}{4} = \frac{N_0}{4}$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n \quad \text{gives} \quad \frac{1}{4} = \left(\frac{1}{2}\right)^n \quad \text{or} \quad n = 2$$

$\therefore$ Time taken = $n \times T_{1/2} = 2 \times 30$ days = 60 days, which is choice (b).

101. Activity at a given time is proportional to the number of radioactive atoms present at that time. The number of atoms left undecayed after 36 years = $\frac{1}{16}$ of the initial value, i.e. $N = N_0/16$.

From $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$, we get $n = 4$. Now $t = n\ T_{1/2}$. Hence

$$T_{1/2} = \frac{t}{n} = \frac{36 \text{ years}}{4} = 9 \text{ years.}$$

Hence the correct choice is (b).

102. Decay constant $\lambda = \dfrac{\ln(2)}{T_{1/2}}$. The correct choice is (b).

103. $T_{1/2} = 69.3$ days. Therefore $\lambda = \dfrac{0.693}{T_{1/2}} = 10^{-2}$ per day.

$$N = 20\% \text{ of } N_0 = 0.2\ N_0.$$

Now $N = N_0\, e^{-\lambda t}$, which gives

$$\frac{N_0}{N} = e^{\lambda t}$$

or $$\ln\left(\frac{N_0}{N}\right) = \lambda t$$

or $$t = \frac{\ln\left(\frac{N_0}{N}\right)}{\lambda} = \frac{\ln(5)}{10^{-2}} = \frac{1.61}{10^{-2}} = 161 \text{ days}$$

Hence the correct choice is (c).

104. If m_p is the mass of a proton and A, the atomic number of uranium nucleus, then the mass of a uranium nucleus is $m = m_p\ A$ and the volume of uranium nucleus is

$$V = \frac{4}{3}\ \pi r^3 = \frac{4}{3}\ \pi(r_0\, A^{1/3})^3 = \frac{4}{3}\ \pi r^3{}_0\ A \qquad (\because\ r = r_0\, A^{1/3})$$

$$\therefore \quad \frac{m}{V} = \frac{m_p\ A}{\frac{4}{3}\pi\ r_0^3\ A} = \frac{3\, m_p}{4\pi\ r_0^3} = \text{constant}$$

Thus $m \propto V$, which is choice (c).

105. In 12 hours, the activity of the sample decreases from 2000 dps to 1000 dps, i.e. it becomes half in 12 hours. Hence the half life of the sample is 12 hours. Now, 24 hours = 2 half lives. Hence the initial activity = 2000 dps $\times (2)^2$ = 8000 dps. Hence the correct choice is (d).

106. The probability at time t that a radioactive nucleus will disintegrate is defined as

$$P = \frac{N_0 - N}{N_0}$$

where N_0 = number of nuclei present initially at time $t = 0$

N = number of nuclei left undecayed at time time t.

Now 8 days = 2 half lives. After 2 half lives

$$N = \frac{N_0}{(2)^2} = \frac{N_0}{4}$$

$$\therefore \quad P = \frac{N_0 - \frac{N_0}{4}}{N_0} = \frac{3}{4}$$

Hence the correct choice is (d).

107. $\dfrac{N}{N_0} = e^{-\lambda t}$. Thus $\dfrac{1}{20} = e^{-\lambda t}$ or $20 = e^{\lambda t}$ or

$$\lambda t = \log_e (20) = 3.0$$

where $$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{6.93} = 0.1 \text{ per day}$$

Therefore $t = \dfrac{3.0}{\lambda} = \dfrac{3.0}{0.1} = 30$ days. Thus the correct choice is (c).

108. $\dfrac{1}{\lambda} \propto Z^2$. Since beryllium has highest value of Z, it will emit the shortest wavelength. So the correct choice is (d).

109. $E_{\min} = \left(\dfrac{1}{1^2} - \dfrac{1}{\infty^2}\right) \times (13.6\text{eV}) Z^2$

$= 13.6 \times (2)^2 = 54.4$ eV

So the correct choice is (d).

110. As shown above, the required energy = 24.6 + 54.4 = 79.0 eV. So the correct choice is (c).

111. From Bohr's quantization condition,

$$mvr = \frac{nh}{2\pi} \qquad (1)$$

where v is the speed of the electron, which from (1) is given by

$$v = \frac{nh}{2\pi mr}$$

For ground state, $n = 1$. Hence

$$v = \frac{h}{2\pi mr} \qquad (2)$$

Current around the orbit is (here T = time period)

$$I = \frac{e}{T}$$

But $$T = \frac{2\pi r}{v}$$

$$\therefore \quad I = \frac{ev}{2\pi r} \qquad (3)$$

Using (2) in (3), we get

$$I = \frac{eh}{4\pi^2 mr^2}$$

Magnetic dipole moment = $IA = I \times \pi r^2$

$$= \frac{eh}{4\pi^2 mr^2} \times \pi r^2$$

$$= \frac{eh}{4\pi m}$$

So the correct choice is (a).

112. Before the photon is emitted, the momentum of the hydrogen atom is zero. After the photon is emitted, its momentum = $\frac{h\nu}{c} = \frac{E_3 - E_1}{c}$

If m is mass of hydrogen atom and v its seeoil velocity, then from momentum conservation, we have

$$mv + \frac{E_3 - E_1}{c} = 0$$

$$\Rightarrow \qquad v = -\frac{E_3 - E_1}{mc} \qquad (1)$$

Now $E_1 = -13.6$ eV

$$E_3 = \frac{-13.6}{(3^2)}\text{eV} = -1.5 \text{ eV}$$

$$E_3 - E_1 = -1.5 \text{ eV} - (-13.6 \text{ eV})$$

$$= 12.1 \text{ eV} = 12.1 \times (1.6 \times 10^{-19}) \text{ J}$$

Also $m = 1.67 \times 10^{-27}$ kg and $c = 3 \times 10^8$ ms^{-1}. Using these values in (1) and solving, we get $v = -3.86$ ms^{-1}. The negative sign indicates that the hydrogen atom recoils after emitting the photon. So the correct choice is (b).

113. We know that $r_n \propto n^2$. Hence $\frac{r_1}{r_2} = \left(\frac{n_1}{n_2}\right)^2$. Given $r_1 = 4r_2$. Therefore,

$$4 = \left(\frac{n_1}{n_2}\right)^2 \Rightarrow n_1 = 2n_2$$

So the correct choice is (c).

114. We know that $v_n \propto \frac{1}{n}$. In this case, $2 = \frac{n_1}{n_2} \Rightarrow n_1 = 2n_2$. So the correct is still (c).

115. We know that $T_n \propto n^3$. So

$$\frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^3$$

Given $T_1 = 8T_2$. Hence

$$8 = \left(\frac{n_1}{n_2}\right)^3 \Rightarrow n_1 = 2n_2$$

Thus, in this case also the correct choice is (c).

116. $r_n \propto n^2$. Therefore $n \propto \sqrt{r_n}$. If r_i and r_f are the radii of the initial and final states, then

$$n = \sqrt{\frac{r_f}{r_i}} = \sqrt{\frac{4.77}{0.53}} = \sqrt{9} = 3$$

So the correct choice is (c).

117. The visible region belongs to Balmer series for which

$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$$= R_H\left(\frac{1}{4} - \frac{1}{n^2}\right); \quad n = 3, 4, 5, ...$$

The longest wavelength corresponds to $n = 3$. Hence

$$\frac{1}{656\text{nm}} = R_H\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R_H}{36}$$

$$R_H = \frac{36}{5 \times 656\text{nm}}$$

The infrared region belongs to Paschen series for which

$$\frac{1}{\lambda} = R_H\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$$

$$= R_H\left(\frac{1}{9} - \frac{1}{n^2}\right); \qquad n = 4, 5, 6, ...$$

The shortest wavelength corresponds to $n = \infty$. Hence

$$\frac{1}{\lambda} = \frac{R_H}{9}$$

$$\Rightarrow \qquad \lambda = \frac{9}{R_H} = \frac{9 \times 5 \times 656\text{nm}}{36}$$

$$= 820 \text{ nm, which is choice (d).}$$

118. Bohr's quantization condition is

$L = \frac{nh}{2\pi}$. Given $L = \frac{3h}{2\pi}$. Hence $n = 3$

Also $mvr = \frac{3h}{2\pi} \Rightarrow mv = \frac{3h}{2\pi r}$

de-Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{h}{3h/2\pi r} = \frac{2\pi r}{3} \qquad (1)$$

Radius of the electron in the nth orbit is

$$r = r_0 \frac{n^2}{Z}$$

For lithium, $Z = 3$. We have seen that $n = 3$. Therefore,

$$r = r_0 \times \frac{(3)^2}{3} = 3r_0 \qquad (2)$$

Using (2) in (1),

$$\therefore \qquad \lambda = \frac{2\pi}{3} \times 3r_0 = 2\pi r_0$$

So the correct choice is (b).

119. Radioactivity is a purely random process. It is not possible to predict which particular nucleus will disintegrate in a given time interval. Hence the correct choice is (d).

120. Activity $R = \lambda N$ and $\lambda = \dfrac{0.693}{T}$, where T is the half life of the sample. Now

$$T = \frac{0.693}{\lambda} = \frac{0.693N}{\lambda N} = \frac{0.693N}{R}$$

Thus $T_1 = \dfrac{0.693N_1}{R_1}$

and $T_2 = \dfrac{0.693N_2}{R_2}$

$$\therefore \quad \frac{T_1}{T_2} = \frac{N_1}{N_2} \times \frac{R_2}{R_1} = 2 \times 2 = 4$$

So the correct choice is (a).

121. $N = N_0 e^{-\lambda t} \Rightarrow \dfrac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$

$$\therefore \quad \left|\frac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t}$$

or $\ln\left|\dfrac{dN}{dt}\right| = \ln(\lambda N_0) - \lambda t$

Slope of the graph is $-\lambda$, which is

$$-\lambda = \frac{8-4}{2-6} \Rightarrow \lambda = 1$$

Now $T_{1/2} = \dfrac{\ln(2)}{\lambda} = \dfrac{\ln(2)}{1} = \ln(2)$, which is choice (b).

122. Half life $T = \dfrac{\ln(2)}{\lambda}$

Mean life $\tau = \dfrac{1}{\lambda}$. Therefore

$$T = \tau \ln(2)$$

$$T_A = \tau_A \ln(2)$$

$$T_B = \tau_B \ln(2)$$

Given $T_A = \tau_B$. Hence

$$\frac{T_B}{T_A} = \ln(2) = 0.693$$

So $T_B < T_A$, which implies $\lambda_B > \lambda_A$

since $\dfrac{dN}{dt} = -\lambda N$

$$\frac{dN_A}{dt} = -\lambda_A N_A$$

and $\dfrac{dN_B}{dt} = -\lambda_B N_B$

Given $N_A = N_B$. Hence $= \dfrac{\frac{dN_A}{dt}}{\frac{dN_B}{dt}} = \dfrac{\lambda_A}{\lambda_B} < 1$

or $$\frac{dN_B}{dt} > \frac{dN_A}{dt}$$

Hence B decays at a faster rate then A. So the correct choice is (d).

123. Activity $|R| = \lambda N = \dfrac{N}{\tau}$ where $\tau = \dfrac{1}{\lambda}$ is the mean life. Therefore,

$$N = \tau |R|$$

$$= 10^9 \times (4 \times 10^{10})$$

$$= 4 \times 10^{19}$$

Total mass of sample $= (4 \times 10^{19}) \times 10^{-25}$

$$= 4 \times 10^{-6} \text{ kg} = 4 \text{ mg}$$

So the correct choice is (c).

SECTION 2

Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage.

Passage I

Our atmosphere contains a large number of stable isotopes. When cosmic rays strike these isotopes, a number of radioisotopes are produced. One of these radioisotopes is carbon-14 (symbol $^{14}_{6}\text{C}$) which is produced by the bombardment of atmospheric nitrogen with high-energy neutrons according to the equation

$$^{14}_{7}\text{N} + {}^{1}_{0}\text{n} \rightarrow {}^{14}_{6}\text{C} + {}^{1}_{1}\text{H}$$

Radiocarbon is unstable and decays to nitrogen with a half-life of 5600 years. The carbon-14 is incorporated into atmospheric carbon dioxide molecules which are taken in by plants when they breathe in carbon dioxide. Animals which eat the plants also take in carbon-14. Ultimately, the concentration of ^{14}C in all living organisms reaches an equilibrium value because ^{14}C decays in the organism

which also takes in atmospheric ^{14}C. When an organism dies, it stops taking ^{14}C from the atmosphere and the concentration of ^{14}C present in the organism decreases with time. By measuring the ratio of the concentration of ^{14}C to ^{12}C in any ancient organism, say a tree, one can determine the date when the organism died.

1. A capsule contains 8 g of $^{14}_{6}C$ whose half-life is 5600 years. After 16800 years, the amount of C-14 left in the capsule will be
   (a) 4 g (b) 2 g
   (c) 8/3 g (d) 1 g
2. Radiocarbon is produced in the atmosphere as a result of
   (a) collisions between fast neutrons and nitrogen nuclei
   (b) the action of cosmic rays on atmospheric oxygen
   (c) the action of X-rays on carbon
   (d) lightning discharge in atmosphere
3. Select the only correct statement.
   (a) The amount of ^{14}C in a living plant keeps increasing indefinitely
   (b) The amount of ^{14}C in a living plant keeps decreasing indefinitely.
   (c) The amount of ^{14}C in a living plant reaches an equilibrium value at some stage of its lifetime.
   (d) The amount of ^{14}C in a dead plant reaches an equilibrium value and stops decreasing after a certain length of time.
4. The age of an ancient tree is determined by measuring the
   (a) amount of ^{14}C still present in a given sample of the wood
   (b) amount of ^{12}C still present in the given sample
   (c) total amount of ^{14}C and ^{12}C in the given sample
   (d) ratio of the amount of ^{14}C to that of ^{12}C still present in the given sample



Solutions

1. Number of half lives $n = \dfrac{16800}{5600} = 3$. Therefore, the amount of C-14 left $= \dfrac{8\,g}{(2)^n} = \dfrac{8\,g}{(2)^3} = 1$ g. The correct choice is (d).
2. The correct choice is (a).
3. The correct choice is (c).
4. The correct choice is (d).

Questions 5 to 7 are based on the following passage.

Passage II

The process of emission of an alpha particle from a nucleus is called alpha decay. An alpha particle is a helium nucleus having 2 protons and 2 neutrons. Hence, when a nucleus undergoes alpha decay, it loses 2 protons and 2 neutrons. As a result, the nucleus of a new element is formed. It is found that the total mass of the decay products is less than the mass of the original nucleus. This mass defect is equivalent to energy which is released in the process. The process of emission of an electron from a nucleus is called beta decay. In this process, a neutron decays into a proton with the emission of an electron and an uncharged particle called antineutrino. When a nucleus undergoes beta decay, the mass number remains unchanged but the atomic number is increased by 1. The process of emission of gamma rays from a nucleus is called gamma decay. Gamma rays are high frequency electromagnetic radiations (i.e. photons) which do not carry any charge. Hence in this process, no new element is formed.

5. $^{238}_{92}U$ nucleus decays successively to form $^{234}_{90}Th$, $^{234}_{91}Pa$, $^{234}_{92}U$. The sequence of radiations emitted is
   (a) α, β, α (b) β, α, β
   (c) α, β, β (d) β, β, α
6. The sequence of step-wise decays of a radioactive nucleus D is as follows:

   $$D \xrightarrow{\alpha} D_1 \xrightarrow{\beta} D_2 \xrightarrow{\alpha} D_3 \xrightarrow{\alpha} D_4$$

   If the mass number and atomic number of D_2 are 176 and 71 respectively, the corresponding values of nucleus D are
   (a) 180, 72 (b) 176, 72
   (c) 168, 67 (d) 172, 69
7. In Q. 6 above, what are the mass and atomic numbers of nucleus D_4
   (a) 172, 69 (b) 176, 71
   (c) 180, 72 (d) 168, 67



Solutions

5. In alpha decay, the mass number decreases by 4 and atomic number decreases by 2. In beta decay, the mass number remains unchanged but the atomic number increases by 1. Hence the correct choice is (c).
6. The correct choice is (a).
7. The correct choice is (d).

Questions 8 to 11 are based on the following passage.

Passage III

A single electron orbits around a stationary nucleus of charge + Ze, where Z is a constant and e is the magnitude of electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to third Bohr orbit. The ionization energy of hydrogen atom = 13.6 eV.

8. The value of Z is
   (a) 3 (b) 4
   (c) 5 (d) 6
9. The ionization energy of the atom is
   (a) 340 eV (b) 217.6 eV
   (c) 122.4 eV (d) 13.6 eV
10. The potential energy of the electron in the first Bohr orbit is
    (a) –680 eV (b) –340 eV
    (c) –170 eV (d) –85 eV
11. The angular momentum of the electron in the first Bohr orbit is (h = Planck's constant)
    (a) $\frac{h}{2\pi}$ (b) $\frac{h}{\pi}$
    (c) $\frac{3h}{2\pi}$ (d) $\frac{5h}{2\pi}$



Solutions

8. The energy of the electron in the nth Bohr orbit is

$$E_n = -\frac{Z^2\, Rhc}{n^2}$$

Given, ionization energy of hydrogen atom = Rhc = 13.6 eV. Therefore,

$$E_n = \frac{(13.6\,\text{eV})Z^2}{n^2}$$

The energy required to excite the electron from $n = 2$ to $n = 3$, Bohr orbit is

$$E_3 - E_2 = -(13.6\text{ eV})\, Z^2 \left(\frac{1}{3^2} - \frac{1}{2^2}\right)$$

$$= \frac{(13.6\,\text{eV}) \times 5Z^2}{36}$$

Given $E_3 - E_2 = 47.2$ eV. Hence

$$\frac{(13.6\,\text{eV}) \times 5Z^2}{36} = 47.2 \text{ eV}$$

which gives $Z = 5$. So the correct choice is (c).

9. Ionization energy of the atom is

$$E_\infty - E_1 = -(13.6\text{ eV}) \left(\frac{1}{\infty} - \frac{1}{1^2}\right) Z^2$$

$$= (13.6\text{ eV}) \times (5)^2 = 340\text{ eV}$$

The correct choice is (a).

10. Kinetic energy of the electron in the first Bohr orbit is $E_1 = -340$ eV. The potential energy in this orbit = $2E_1 = -680$ eV, which is choice (a).

11. Angular momentum = $\frac{nh}{2\pi} = \frac{h}{2\pi}$ ($\because n = 1$). So the correct choice is (a).

Questions 12 to 14 are based on the following passage.

Passage IV

Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. Ionization potential of hydrogen is 13.6 V and the mass of hydrogen atom = 1.67×10^{-27} kg.

12. The energy of the photons causing the photoelectric emission is
    (a) 1.168×10^{-19} J (b) 2.912×10^{-19} J
    (c) 1.744×10^{-19} J (d) 4.08×10^{-19} J
13. The quantum numbers of the two levels in the emission of the photons are
    (a) $n = 1$, $n = 4$ (b) $n = 1$, $n = 3$
    (c) $n = 2$, $n = 4$ (d) $n = 3$, $n = 4$
14. In the transition in Q. 13, the change in the angular momentum of the electron is (h = Planck's constant)
    (a) $\frac{h}{2\pi}$ (b) $\frac{h}{\pi}$
    (c) $\frac{2h}{\pi}$ (d) $\frac{3h}{2\pi}$



Solutions

12. Given, E_{max} = 0.73 eV, W = 1.82 eV, ionization potential of hydrogen atom = 13.6 V and mass of hydrogen atom, $m = 1.67 \times 10^{-27}$ kg.
    (a) From Einstein's photoelectric equation,

$$h\nu = W + E_{max} = 1.82 + 0.73$$

$$= 2.55 \text{ eV or } 4.08 \times 10^{-19} \text{ J}$$

The correct choice is (d).

13. We are given that the ionization potential of hydrogen atom = 13.6 V. Therefore, the ionization energy = 13.6 eV. The energy levels of hydrogen atom are given by

$$E_n = -\frac{Rhc}{n^2} = -\frac{13.6}{n^2} \text{ eV}$$

Hence the values of energy the electron in n = 1, 2, 3 and 4 energy levels are

$$E_1 = \frac{13.6}{1^2} = -13.6 \text{ eV}$$

$$E_2 = \frac{13.6}{2^2} = -3.4 \text{ eV}$$

$$E_3 = \frac{13.6}{3^2} = -1.51 \text{ eV and}$$

$$E_4 = \frac{13.6}{4^2} = -0.85 \text{ eV}$$

The quantum numbers for which the energy difference is 2.55 eV are n = 2 and n = 4 since

$E_4 - E_2 = -(0.85) - (-3.4) = 2.55$ eV

So the correct choice is (c).

14. According to Bohr's theory, the angular momentum of the electron in the nth energy state is

$L_n = \frac{nh}{2\pi}$. For $n = 4$, $L_4 = \frac{4h}{2\pi} = \frac{h}{\pi}$

For $n = 2$, $L_2 = \frac{2h}{2\pi} = \frac{h}{\pi}$

$\therefore$ Change in angular momentum is

$$\Delta L = L_4 - L_2 = \frac{2h}{\pi} - \frac{h}{\pi} = \frac{h}{\pi}$$

Thus the correct choice is (b).

SECTION 3 Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

1. Statement-1

A nucleus at rest splits into two nuclear parts having radii in the ratio 1 : 2. Their velocities will be in the ratio 8 : 1.

Statement-2

The radius of a nucleus is proportional to the cube root of its mass number.

2. Statement-1

The half life of radioactive sample is T. It will decay to $\frac{1}{16}$ of its initial value in a time $8T$.

Statement-2

The half life of a radioactive sample is the time in which half of the number of nuclei decay.

3. Statement-1

A freshly prepared radioactive sample has a half life of 3 hours and emits radiation of intensity which is 64 times the permissible safe value. The minimum time after which it would be safe to work with the sample is 18 hours.

Statement-2

The intensity of the radiation falls by a factor of 2 every 3 hours.

4. Statement-1

Two radioactive sources A and B initially contain equal number of radioactive nuclei. Source A has a half life of 1 hour and source B has a half life of 2 hours. At the end of 2 hours, they will have the same rate of disintegration.

Statement-2

The rate of disintegration is defined as the number of disintegrations taking place in the source per second.

5. Statement-1

The nucleus $^{22}_{10}$Ne absorbs energy and decays into two alpha particles and an unknown nucleus. The unknown nucleus must be carbon.

Statement-2

In a nuclear reaction, the atomic number is conserved.

6. Statement-1

The number of α and β particles emitted when $^{238}_{92}U$ decays into $^{206}_{82}Pb$ is 6 and 8 respectively.

Statement-2

In a nuclear reaction, the mass number and the atomic number are both conserved.

7. Statement-1

The radioactive decay of nucleus X to nuclei Y and K is represented by the equation

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y \rightarrow {}^{A-4}_{z-1}K \rightarrow {}^{A-4}_{z-1}K$$

The sequence of emitted radiations is β, α and γ.

Statement-2

In a nuclear reaction, the mass number and the atomic number are both conserved.

8. Statement-1

The distance of the closest approach of an alpha particle fired at a nucleus with momentum p is r_0. The distance of the closest approach when the alpha particle is fired at the same nucleus with momentum $2p$ will be $r_0/2$.

Statement-2

The distance of closest approach from a given target element is inversely proportional to the kinetic energy of the incident particle.

9. Statement-1

The binding energy of deuteron ($^{2}_{1}H$) is 1.15 MeV per nucleon and an alpha particle ($^{4}_{2}He$) has a binding energy of 7.1 MeV per nucleon. Then in the reaction

$$^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{4}_{2}He + Q$$

the energy Q released is 23.8 MeV.

Statement-2

Total energy is conserved in a nuclear reaction.

10. Statement-1

A nucleus X, initially at rest, decays into a nucleus Y with the emission of an α-particle and energy Q is released. If m is the mass of an alpha particle and M that of nucleus Y, the energy of the emitted α-particle will be

$$E_\alpha = \frac{QM}{(M-m)}$$

Statement-2

Momentum and energy are conserved in the decay process.

Solutions

1. The correct choice is (a). Let A_1 and A_2 be the mass numbers of the two nuclear parts. Their radii are given by

$$R_1 = R_0 (A_1)^{1/3} \text{ and } R_2 = R_0 (A_2)^{1/3}$$

Dividing, we get

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

or $$\frac{A_1}{A_2} = \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

Hence the ratio of their masses is

$$\frac{m_1}{m_2} = \frac{1}{8}$$

From the principle of conservation of momentum, the magnitude of p_1 = magnitude of p_2 or $m_1 v_1 = m_2 v_2$, which gives

$$\frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{8}{1}$$

2. The correct choice is (d). Since $\frac{1}{16} = \frac{1}{2^4}$, it follows that the time taken for the sample to decay to 1/16 of its initial value = four half lives = 4 T.
3. The correct choice is (a). Since the half life is 3 hours, the intensity of radiation falls by a factor of 2 every three hours. In 18 hours, it will fall by a factor of $(2)^6 = 64$.
4. The correct choice is (a). The rate of disintegration is proportional to the number of radioactive nuclei present initially in the source.
5. The correct choice is (a). The given nuclear reaction is given by the equation

$$^{22}_{10}Ne \rightarrow {}^{4}_{2}He + {}^{4}_{2}He + {}_{Z}X$$

Since the atomic number is conserved, we have

$$10 = 2 + 2 + Z$$

which gives Z = 6. The nucleus having Z = 6 is carbon.

6. The correct choice is (d). Let x and y respectively be the number of α and β particles emitted. The equation of the decay is

$$^{238}_{92}U \rightarrow {}^{206}_{82}Pb + x(^{4}_{2}He) + y\,(^{0}_{-1}e)$$

From conservation of charge, we have

$$92 = 82 + 2x - y \quad \text{(i)}$$

Conservation of mass number gives

$$238 = 206 + 4x \quad \text{(ii)}$$

which gives $4x = 32$ or $x = 8$. Using this value of x in (i) we get $y = 6$.

7. The correct choice is (b). In transition ${}^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z+1}\mathrm{Y}$, the atomic (or charge) number increases by unity; mass number remaining the same. Hence an electron (β-particle) is emitted. In transition ${}^{A}_{Z+1}\mathrm{Y} \rightarrow {}^{A-4}_{Z-1}\mathrm{K}$, the mass number decreases by 4 and charge number decreases by 2. Hence an α-particle is emitted. In the third transition, mass and charge numbers do not change. Hence a γ-ray is emitted.

8. The correct choice is (d). The distance of the closest approach is given by

$$r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{K}$$

where $K = \frac{1}{2}\, mv^2$.

$$K = \frac{1}{2}\, mv^2 = \frac{1}{2m} \times (mv)^2 = \frac{p^2}{2m}.$$

Therefore,

$$r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4mZe^2}{p^2}$$

Thus $r_0 \propto \frac{1}{p^2}$. When p is doubled, r_0 becomes one-fourth.

9. The correct choice is (a). Binding energy of ${}^{2}_{1}\mathrm{H}$ = 1.15 × number of nucelons = 1.15 × 2 = 2.3 MeV. Total binding energy of reactants = 2.3 + 2.3 = 4.6 MeV. Binding energy of ${}^{4}_{2}\mathrm{He}$ = 7.1 × number of nucleons = 7.1 × 4 = 28.4 MeV. Therefore, Q = 28.4 – 4.6 = 23.8 MeV. Hence the correct choice is (c).

10. The correct choice is (d). $K = p^2/2m$. Therefore, from conservation of energy, $Q = \frac{P^2}{2M} + \frac{p^2}{2m}$

where p is momentum of Y, p = momentum of α-particle) Since momentum is conserved, $P = p$. Hence

$$Q = \frac{p^2}{2}\left(\frac{1}{M} + \frac{1}{m}\right) = \frac{p^2}{2m}\left(\frac{m}{M} + 1\right)$$

$$= E_\alpha \left(\frac{m}{M} + 1\right)$$

where E_α = energy of α-particle. Hence

$$E_\alpha = \frac{QM}{(M+m)}$$

4 SECTION — Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. If 13.6 eV energy is required to ionize the hydrogen atom, the energy required to remove the electron from $n = 2$ state is
   (a) 10.2 eV (b) zero
   (c) 3.4 eV (d) 6.8 eV **[2002]**

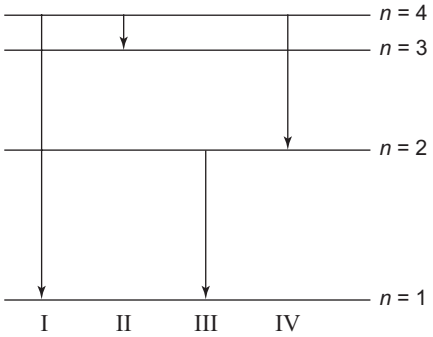
2. If N_0 is the original mass of the substance of half-life 5 years, the amount of substance left after 15 years is
   (a) $\frac{N_0}{8}$ (b) $\frac{N_0}{16}$
   (c) $\frac{N_0}{2}$ (d) $\frac{N_0}{4}$ **[2002]**

3. When U-238 nucleus originally at rest, decays by emitting an alpha particle having a speed u, the recoil speed of the residual nucleus is
   (a) $\frac{4u}{238}$ (b) $-\frac{4u}{234}$
   (c) $\frac{4u}{234}$ (d) $-\frac{4u}{238}$ **[2003]**

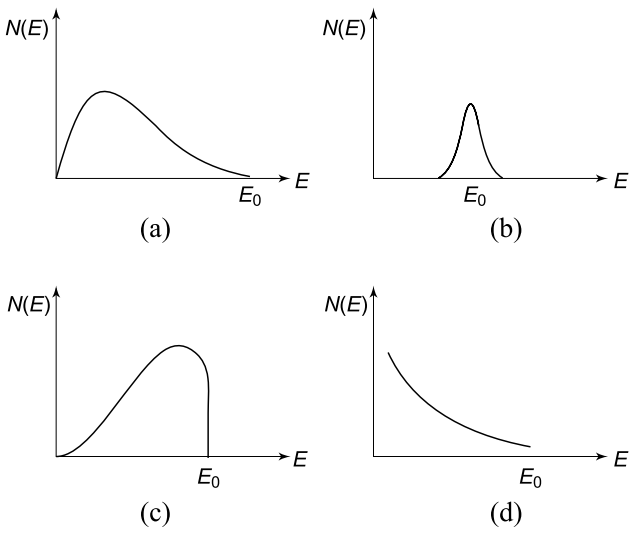
4. At a certain instant, a radioactive sample has a decay rate of 5000 disintegrations per minute. After 5 minutes, the decay rate is 1250 disintegrations per minute. The decay constant (per minute) is
   (a) 0.4 ln (2) (b) 0.2 ln (2)
   (c) 0.1 ln (2) (d) 0.8 ln (2) **[2003]**

5. A nucleus with Z = 92 emits the following in a sequence: α, α, β^-, β^-, α, α, α, α, β^-, β^-, α, β^+, β^+, α. The Z of the resulting nucleus is
   (a) 76 (b) 78
   (c) 82 (d) 74 **[2003]**

6. Which of the following cannot be emitted in radioactive decay of a substance?
   (a) Protons (b) Neutrinos
   (c) Helium nuclei (d) Electrons **[2003]**
7. In the nuclear fusion reaction
   $$^{2}_{1}\text{H} + ^{3}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + n$$
   the repulsive potential energy between the two fusing nuclei is 7.7×10^{-14} J. The temperature to which the gas must be heated is nearly (Boltzmann constant $k = 1.38 \times 10^{-23}$ J K^{-1})
   (a) 10^7 K (b) 10^5 K
   (c) 10^3 K (d) 10^9 K **[2003]**
8. Which of the following atoms has the lowest ionization potential?
   (a) $^{14}_{7}$N (b) $^{133}_{55}$Cs
   (c) $^{40}_{18}$Ar (d) $^{16}_{8}$O **[2003]**
9. If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{2+} is
   (a) 30.6 eV (b) 13.6 eV
   (c) 3.4 eV (d) 122.4 eV **[2003]**
10. A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is
    (a) $\frac{E}{c}$ (b) $\frac{2E}{c}$
    (c) Ec (d) $\frac{E}{c^2}$ **[2004]**
11. A nucleus at rest disintegrates into two nuclear parts which have their velocities in the ratio 2 : 1. The ratio of their nuclear sizes will be
    (a) $2^{1/3} : 1$ (b) $1 : 3^{1/2}$
    (c) $3^{1/2} : 1$ (d) $1 : 2^{1/3}$ **[2004]**
12. The binding energy per nucleon of deuteron ($^{2}_{1}$H) and helium nucleus ($^{4}_{2}$He) is 1.1 MeV and 7 MeV respectively. If two deuteron nuclei react to form a single helium nucleus, the energy released is
    (a) 13.9 MeV (b) 26.9 MeV
    (c) 23.6 meV (d) 19.2 MeV **[2004]**
13. An alpha particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of
    (a) 1 Å (b) 10^{-10} cm
    (c) 10^{-12} cm (d) 10^{-15} cm **[2004]**
14. Starting with a sample of pure Cu-66, 7/8 of it decays into Zn in 15 minutes. The half-life of the sample is
    (a) 10 min (b) 15 min
    (c) 5 min (d) 7.5 min **[2005]**
15. If the radius of $^{27}_{13}$Al nucleus is 3.6 fermi, then the radius of $^{125}_{52}$Te nucleus is nearly equal to
    (a) 6 fermi (b) 8 fermi
    (c) 4 fermi (d) 5 fermi **[2005]**
16. The figure shows the energy levels for an electron in a certain atom. Which transition represents the emission of a photon with the most energy?
    (a) III (b) IV
    (c) I (d) II **[2005]**



17. The energy spectrum of β-particles [number $N(E)$ as a function of β-energy E] emitted from a radioactive source is



[2006]

18. When $^{7}_{3}$Li nuclei are bombarded by protons, and the resultant nuclei are $^{8}_{4}$Be, the emitted particle will be
    (a) gamma photon (b) neutron
    (c) alpha particle (d) beta particle **[2006]**
19. An alpha nucleus of energy $\frac{1}{2}mv^2$ bombards a heavy nuclear target of charge Ze. Then the distance of closest approach for the alpha nucleus will be proportional to
    (a) $1/v^4$ (b) $1/Z^3$
    (c) v^2 (d) $1/m$ **[2006]**

20. If the binding energy per nucleon in $^{7}_{3}\text{Li}$ and $^{4}_{2}\text{He}$ nuclei is 5.60 MeV and 7.06 MeV respectively, then in the reaction

$$p + {}^{7}_{3}\text{Li} \rightarrow 2{}^{4}_{2}\text{He}$$

energy of proton must be

(a) 1.46 MeV (b) 39.2 MeV

(c) 28.24 MeV (d) 17.28 MeV **[2006]**

21. If m_0 is the mass of an isotope $^{17}_{8}\text{O}$, m_p and m_n are the masses of a proton and a neutron respectively, the binding energy of the isotope is

(a) $(m_0 - 8m_p)c^2$ (b) $(m_0 - 8m_p - 9m_n)c^2$

(c) $m_0 c^2$ (d) $(m_0 - 17m_n)c^2$

[2007]

22. In gamma ray emission from a nucleus

(a) both the neutron number and the proton number change

(b) there is no change in the proton number and the neutron number

(c) only the neutron number changes

(d) only the proton number changes **[2007]**

23. The half life of a radioactive element X is the same as the mean life of another radioactive element Y. Initially they have the same number of atoms. Then

(a) X will decay faster than Y

(b) Y will decay faster than X

(c) X and Y will decay at the same rate initially

(d) X and Y will decay at the same rate at all times

[2007]

24. Which of the following transitions in hydrogen atoms emits photons of highest frequency?

(a) $n = 2$ to $n = 6$ (b) $n = 6$ to $n = 2$

(c) $n = 2$ to $n = 1$ (d) $n = 1$ to $n = 2$

[2007]

25. Suppose an electron is attracted towards the origin by a force $\frac{k}{r}$ where 'k' is a constant and 'r' is the distance of the electron from the origin. By applying Bohr model to this system, the radius o the nth orbital of the electron is found to be 'r_n' and the kinetic energy of the electron to be 'T_n'. Then which of the following is true?

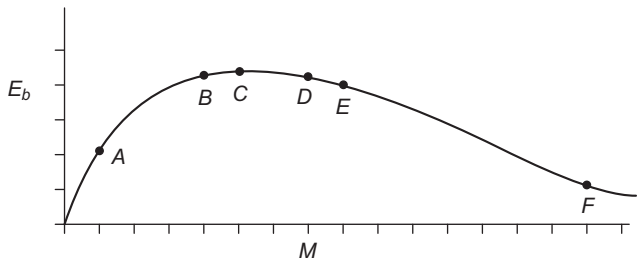
(a) $T_n \propto \frac{1}{n}$, $r_n \propto n^2$

(b) $T_n \propto \frac{1}{n^2}$, $r_n \propto n^2$

(c) T_n independent of n, $r_n \propto n$

(d) $T_n \propto \frac{1}{n}$, $r_n \propto n$ **[2008]**

26. The figure is a plot of binding energy per nucleon E_b against the nuclear mass M; A, B, C, D, E, F correspond to different nuclei. Consider four reactions:



(i) $A + B \rightarrow C + \varepsilon$ (ii) $C \rightarrow A + B + \varepsilon$

(iii) $D + E \rightarrow F + \varepsilon$ and (iv) $F \rightarrow D + E + \varepsilon$,

where ε is the energy released? In which reactions is ε positive?

(a) (ii) and (iv) (b) (ii) and (iii)

(c) (i) and (iv) (d) (i) and (iii) **[2009]**

27. The transition from state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained from the transition from

(a) $4 \rightarrow 2$ (b) $5 \rightarrow 4$

(c) $2 \rightarrow 1$ (d) $3 \rightarrow 2$ **[2009]**

Directions: Questions number 28 and 29 are based on the following paragraph.

A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c.

28. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then

(a) $E_2 = 2E_1$ (b) $E_1 > E_2$

(c) $E_2 > E_1$ (d) $E_1 = 2E_2$ **[2010]**

29. The speed of daughter nuclei is

(a) $c\frac{\Delta m}{M + \Delta m}$ (b) $c\sqrt{\frac{2\Delta m}{M}}$

(c) $c\sqrt{\frac{\Delta m}{M}}$ (d) $c\sqrt{\frac{\Delta m}{M + \Delta m}}$ **[2010]**

30. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

(a) $\frac{A - Z - 8}{z - 4}$ (b) $\frac{A - Z - 4}{z - 8}$

(c) $\frac{A - Z - 12}{z - 4}$ (d) $\frac{A - Z - 4}{z - 2}$ **[2010]**

31. The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is:
    (a) 14 min (b) 20 min
    (c) 28 min (d) 7 min **[2011]**
32. In a hydrogen like atom electron makes trnsition from an energy level with quantum number n to another with quantum $(n-1)$. If $n>>1$, the frequency of radiation emitted is proportional to
    (a) $\frac{1}{n^2}$ (b) $\frac{1}{n^{3/2}}$
    (c) $\frac{1}{n^3}$ (d) $\frac{1}{n}$ **[2012]**

Questions 33 and 34 are based on the following paragraph.

The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay of products of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has continuous spectrum. Considering a three-body decay process, i.e. $n \to p + e^- + \bar{\nu}$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino ($\bar{\nu}$) to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV. The kinetice energy carried by the proton is only the recoil energy.

33. What is maximum energy of the anti-neutrino?
    (a) Zero
    (b) Much less than 0.8×10^6 eV
    (c) Nearly 0.8×10^6 eV
    (d) Much larger than 0.8×10^6 eV **[2012]**
34. If the anti-neutrino had a mass of 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron?
    (a) $0 \le K \le 0.8 \times 106$ eV
    (b) 3.0 eV $\le K \le 0.8 \times 10^6$ eV
    (c) 3.0 eV $\le K \le 0.8 \times 10^6$ eV
    (d) $0 \le K \le 0.8 \times 10^6$ eV **[2012]**
35. A freshly prepared sample of a radioisotope of half-life 1386s has activity 10^3 disintegrations per second. Given that In 2 = 0.693, the fraction of the initial number of nuclei that will decay in the first 80s after preparation of the sample is nearly
    (a) 1% (b) 2%
    (c) 3% (d) 4% **[2013]**
36. If 13.6 eV energy is required to ionize the hydrogen atom, the energy required to remove the electron from $n = 2$ state is
    (a) 10.2 eV (b) Zero
    (c) 3.4 eV (d) 6.8 eV **[2014]**
37. When U-238 nucleus originally at rest, decays by emitting an alpha particle having a speed u, the recoil speed of the residual nucleus is
    (a) $\frac{4u}{238}$ (b) $-\frac{4u}{234}$
    (c) $\frac{4u}{234}$ (d) $-\frac{4u}{238}$ **[2014]**



Answers

1. (c)	2. (a)	3. (c)	4. (a)
5. (b)	6. (a)	7. (d)	8. (b)
9. (a)	10. (b)	11. (d)	12. (c)
13. (c)	14. (c)	15. (a)	16. (a)
17. (a)	18. (a)	19. (d)	20. (d)
21. (b)	22. (b)	23. (b)	24. (c)
25. (c)	26. (c)	27. (b)	28. (c)
29. (b)	30. (b)	31. (b)	32. (c)
33. (c)	34. (d)	35. (d)	36. (c)
37. (c)			



Solutions

1. $E_n = \frac{-13.6 \text{ eV}}{n^2}$

   For $n = 2$, $E_2 = \frac{-13.6 \text{ eV}}{(2)^2} = -3.4$ eV

   Therefore ionization energy for $n = 2$ state = 3.4 eV

2. $N = N_0\left(\frac{1}{2}\right)^n$

   Number of half lives $n = \frac{15}{5} = 3$

   $\therefore \quad N = N_0\left(\frac{1}{2}\right)^3 = \frac{N_0}{8}$

3. Atomic mass of U-238 nucleus = 238 a.m.u.

   Since the mass number decreases by 4 on alpha decay, the atomic mass of residual nucleus = 238 – 4 = 234 a.m.u. Momentum before decay = 0. If **v** is the velocity of the residual nucleus after decay, then from momentum conservation, we have

   $$0 = 4\mathbf{u} + 234\mathbf{v}$$

   $$\Rightarrow \qquad \mathbf{v} = -\frac{4\mathbf{u}}{234}$$

   The negative shows that direction of **v** is opposite to that of **u**. Hence recoil speed = $\frac{4u}{234}$.

4. Decay rate $|R| = \lambda N$.

   Let N_0 be the number of radioactive nuclei present at an instant when $|R|$ = 5000 disintegrations per minute. Then

   $$5000 = \lambda N_0 \qquad (1)$$

   After t = 5 min, we have

   $$1250 = \lambda N$$

   $$\Rightarrow \qquad \lambda = \frac{1250}{N} \qquad (2)$$

   Using (2) in (1)

   $$5000 = \frac{1250\, N_0}{N}$$

   $$\Rightarrow \qquad N = \frac{N_0}{4}$$

   Now $\qquad N = N_0\, e^{-\lambda t} = N_0\, e^{-5\lambda}$

   $$\Rightarrow \qquad \frac{N}{N_0} = e^{-5\lambda} \quad \Rightarrow \quad \frac{1}{4} = e^{-5\lambda}$$

   or $\qquad 4 = e^{5\lambda} \quad \Rightarrow \quad \ln(4) = 5\lambda$

   $$\Rightarrow \qquad 2\ln(2) = 5\lambda$$

   $$\Rightarrow \qquad \lambda = \frac{2\ln(2)}{5} = 0.4\ln(2)$$

5. In α decay, Z decreases by 2. In β^- decay, Z increases by 1. In β^+ decay Z decreases by 1. There are 8 α decays, 4 β^- decays and 2β^+ decays. Hence the atomic number changes by $(-8 \times 2 + 4 \times 1 - 2 \times 1)$ $= -16 + 4 - 2 = -14$. Therefore atomic number of the resulting nucleus = 92 – 14 = 78.

6. Neutrinos are emitted in β^+ decay. Helium nuclei (α particles) are emitted in α decay and electrons are emitted in β^- decay. Protons are never emitted in a radioactive decay.

7. $E = \frac{3}{2}kT$

   $$\Rightarrow \qquad 7.7 \times 10^{-14} = \frac{3}{2} \times 1.38 \times 10^{-23}\, T$$

   $$\Rightarrow \qquad T = 3.7 \times 10^9 \text{ K}.$$

8. In the question the nuclei of atoms are given. Since $^{133}_{55}$Cs has the largest mass (hence the largest size) and the largest number of electrons = 55), the electrons in the outermost orbits will be far away from the nucleus and hence are bound to the nucleus by the smallest electrostatic force. Therefore, they will require the minimum energy to be removed from the nucleus. Thus the correct choice is (b).

9. $E_n = \frac{Z^2}{n^2}$ (– 13.6 eV)

   For Li^{2+}, Z = 3 and for first excited state n = 2.

   $$\therefore \qquad E_2 = \frac{(3)^2}{(2)^4}(-13.6 \text{ eV}) = -30.6 \text{ eV}$$

   Hence ionization energy for the first excited state of Li^{2+} is 30.6 eV.

10. Initial momentum $p_i = -\frac{E}{c}$

    Since the surface is perfectly reflecting, the final momentum is $p_f = -\frac{E}{c}$ (in the opposite direction)

    $\therefore$ Change in momentum = $p_f - p_i = -\frac{E}{c} - \frac{E}{c} = -\frac{2E}{c}$

    $\therefore$ Momentum imparted to the surface = $\frac{2E}{c}$

11. Let $v_2 = v$, then $v_1 = 2v$. From conservation of momentum

    $$m_1\, \mathbf{v}_1 + m_2\, \mathbf{v}_2 = 0$$

    $$\Rightarrow \qquad \frac{\mathbf{v}_2}{\mathbf{v}_1} = -\frac{m_1}{m_2}$$

    $$\Rightarrow \qquad \left|\frac{v_2}{v_1}\right| = \frac{m_1}{m_2}$$

    $$\Rightarrow \qquad \frac{v}{2v} = \frac{m_1}{m_2} \quad \Rightarrow \quad m_2 = 2m_1$$

    Now $\qquad m = \frac{4\pi}{3}\rho\, r^3 \quad \Rightarrow \quad m \propto r^3$. Therefore

    $$\frac{m_1}{m_2} = \left(\frac{r_1}{r_2}\right)^3$$

$$\Rightarrow \qquad \frac{1}{2} = \left(\frac{r_1}{r_2}\right)^3$$

$$\Rightarrow \qquad \frac{r_1}{r_2} = \frac{1}{2^{1/3}}$$

So the correct choice is (d).

12. The nuclear reaction is

$$^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^4_2\text{He} + \text{energy}$$

Binding energy of one deuteron nucleus = $2 \times 1.1 = 2.2$ MeV

Total binding energy of two deuterons = $2 \times 2.2 = 4.4$ MeV

Binding energy of $^4_2$He = $4 \times 7 = 28$ MeV

Energy released = $28 - 4.4 = 23.6$ MeV

13. $$r_0 = \frac{1}{4\pi\varepsilon_0} \times \frac{2Ze^2}{E_k} \qquad (1)$$

Given $E_k = 5$ MeV $= 5 \times 10^6 \times 1.6 \times 10^{-19}$ J

For uranium $Z = 92$. Also $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^2 C^{-1} and $e = 1.6 \times 10^{-19}$ C. Substituting these values in Eq. (1), we get

$r_0 = 5.3 \times 10^{-14}$ m

$= 5.3 \times 10^{-12}$ cm $\simeq 10^{-12}$ cm

14. It is clear that $\dfrac{1}{8}$ of the sample is left undecayed at the end of 15 minutes. If n is the number of half lives in 15 minutes, then

$$\left(\frac{1}{2}\right)^n = \frac{1}{8} \quad \Rightarrow \quad n = 3$$

$$\therefore \text{ Half life } = \frac{15 \text{ min}}{3} = 5 \text{ min}$$

15. $R = R_0 (A)^{1/3}$ where R_0 is a constant. Hence

$$\frac{R_{Al}}{R_{Te}} = \left(\frac{A_{Al}}{A_{Te}}\right)^{1/3} = \left(\frac{27}{125}\right)^{1/3} = \frac{3}{5}$$

$$\therefore R_{Te} = \frac{5}{3} \times 3.6 = 6 \text{ fermi}$$

16. $$E = Rhc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$E_{4\to3} = Rhc\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{7Rhc}{9\times16} \simeq 0.049\, Rhc$$

$$E_{4\to2} = Rhc\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 0.187\, Rhc$$

$$E_{2\to1} = Rhc\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 0.75\, Rhc$$

$$E_{4\to1} = Rhc\left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 0.009\, Rhc$$

Transition III ($n = 2$ to $n = 1$) represents the emission of the photon with most energy.

17. In β-decay, a neutron in the nucleus decays into a proton with the emission of an electron and an uncharged particle called antineutrino. The energy released in this process is shared between the electron and antineutrino. When many nuclei undergo β-decay, all the emitted electrons do not have the same energy. Some electrons have a very low energy and some have a very high energy. In fact, when the antineutrino has a high energy, the electron has a low energy and vice versa. Hence the energy spectrum of electrons is as shown in choice (a).

18. The nuclear reaction is represented as

$$^7_3\text{Li} + {}^1_1\text{H} \rightarrow {}^8_4\text{Be} + {}^A_Z\text{x}$$

Since the mass number and atomic number are conserved,

$$7 + 1 = 8 + A \quad \Rightarrow \quad A = 0$$

and $$3 + 1 = 4 + Z \quad \Rightarrow \quad Z = 0$$

Hence particle is $^0_0$x which is a photon.

19. The charge of an alpha particle is $q = 2e$. From the conservation of energy, we have

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q(Ze)}{r_0} = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{r_0}$$

where r_0 is the distance of closest approach, which is given by

$$r_0 = \frac{1}{\pi\varepsilon_0}\frac{Ze^2}{mv^2}$$

$$\therefore r_0 \propto \frac{1}{m}.$$

20. Let Q MeV be the energy of the proton. Then, we have

Q + energy of one $^7_3$Li nucleus = energy of two $^4_2$He nuclei

$\therefore \quad Q + 7 \times 5.6$ MeV $= 2 \times (4 \times 7.06$ MeV)

or $\quad Q + 39.6$ MeV $= 56.48$ MeV

which gives $Q = 17.28$ MeV.

21. Number of protons = 8. Therefore, number of neutrons = $17 - 8 = 9$.

Mass of nucleus = m_0

Mass of nucleons = $8m_p + 9m_n$

Binding energy = (mass of nucleus – mass of nucleons) c^2

$$= (m_0 - 8m_p - 9m_n)c^2$$

22. In the gamma decay of a nucleus, there is no change in the proton number and the neutron number.

23. The half life of a radioactive element is given by

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

where λ is the decay constant. The mean life is given by

$$T_{av} = \frac{1}{\lambda_Y}$$

Given $\frac{0.693}{\lambda_X} = \frac{1}{\lambda_Y} \Rightarrow \lambda_Y = \frac{\lambda_X}{0.693} = 1.44\ \lambda_X$

Thus $\lambda_Y > \lambda_X$. Hence Y will decay faster than X.

24. Emission of photons occurs when electrons jump from higher to lower energy orbits. Hence transitions in choices (a) and (d) do not emit photons.

$$E_{6\to 2} = -(13.6)\left(\frac{1}{6^2} - \frac{1}{2^2}\right)\text{eV} = 3.02\text{ eV}$$

$$E_{2\to 1} = -(13.6)\left(\frac{1}{2^2} - \frac{1}{1^2}\right)\text{eV} = 10.2\text{ eV}$$

Since the frequency of the emitted photons is proportional to energy ($\because E = h\nu$), the correct choice is (c).

25. If v_n is the speed of the electron in the nth orbit, then

$$\frac{mv_n^2}{r_n} = \frac{k}{r_n} \Rightarrow v_n^2 = \frac{k}{m}$$

$\therefore$ $T_n = \frac{1}{2} m v_n^2 = \frac{k}{2}$, which is independent of n.

Also, angular momentum $L_n = \frac{nh}{2\pi}$ or $m\, v_n\, r_n = \frac{nh}{2\pi}$ which gives

$$r_n = \frac{nh}{2\pi m v_n} = \frac{nh}{2\pi m}\sqrt{\frac{m}{k}} = \frac{nh}{2\pi\sqrt{mk}}$$

Thus $r_n \propto n$. Hence the correct choice is (c).

26. Lighter nuclei A and B can fuse to form a heavier nucleus C. In this fusion reactions, there is a mass defect giving rise to a release in energy. On the other hand, a heavy nucleus F can be split into two nuclei D and E of moderate masses. In this fission reaction also, there is a mass defect which appears in the form of release of energy. Thus energy is released in processes (i) and (iv). So the correct choice is (3).

27. $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]$

For ultraviolet radiation $n_1 = 4$ and $n_2 = 3$. The wavelength of infrared radiation is greater than that of ultraviolet radiation. Out of the four given choices, the transition $5 \to 4$ results in a longer wavelength than the transition $4 \to 3$.

28. In a decay process, the daughter nuclei are more stable than the parent nucleus. Hence the binding energy per nucleon (E_2) of the daughter nuclei is greater than E_1, the binding energy per nucleon of the parent nucleus. So the correct choice is (c).

29. Let $\vec{v}_1$ and $\vec{v}_2$ be the velocities of the daughter nuclei. From conservation of momentum, we have

$$0 = \frac{M\vec{v}_1}{2} + \frac{M\vec{v}_2}{2}$$

which gives $\vec{v}_2 = -\vec{v}_1$

The daughter nuclei fly off in opposite directions with the same speed $v = v_1 = v_2$. Initial K.E. = 0. Final K.E. is

$$E = \frac{1}{2}\times\frac{Mv^2}{2} + \frac{1}{2}\times\frac{Mv^2}{2} = \frac{Mv^2}{2}$$

Mass defect = $(M + \Delta m) - \left(\frac{M}{2} + \frac{M}{2}\right) = \Delta m$. Mass defect energy is $E = \Delta mc^2$

$$\therefore \frac{Mv^2}{2} = \Delta mc^2 \Rightarrow v = c\sqrt{\frac{2\Delta m}{M}}$$

30. In positive beta decay, a proton is transformed into a neutron with the emission of a positron (a particle of mass = mass of an electron and charge = $+ 1.6 \times 10^{-19}$ C). Hence in this process, the mass number A remains the same but proton number reduces by 1 and neutron number increases by 1. In alpha decay, the mass number A reduces by 4 and atomic number Z reduces by 2. Hence in this process, the proton number decreases by 2 and neutron number also decreases by 2.

Number of protons in the parent nucleus = Z

Number of neutrons in the parent nucleus = $A - Z$

When it emits 3 alpha particles and 2 positrons, the proton number of the new nucleus = $Z - 6 - 2 = Z - 8$ and the neutron number = $A - Z - 6 + 2 = A - Z - 4$. Hence correct choice is (b).

31. $N = N_0 e^{-\lambda t}$

At time $t = t_1$, $N = \frac{N_0}{3}$. Therefore

$$\frac{N_0}{3} = N_0 e^{-\lambda t} \Rightarrow t_1 = \frac{1}{\lambda}(\ln 3)$$

At time $t = t_2$, $N = \frac{2N_0}{3}$. Therefore

$$\frac{2N_0}{3} = N_0^{-\lambda t_2} \Rightarrow t_2 = \frac{1}{\lambda}\left(\ln\frac{3}{2}\right)$$

$$\therefore\ t_1 - t_2 = \frac{1}{\lambda}\left(\ln 3 - \ln\frac{3}{2}\right)$$

$$= \frac{1}{\lambda}(\ln 3 - \ln 3 + \ln 2) = \frac{1}{\lambda}\ln 2$$

If $T_{1/2}$ is the half life,

$$T_{1/2} = \frac{1n\ 2}{\lambda}$$

$\therefore\ t_2 - t_1 = T_{1/2} = 20$ min

32. $$v \propto \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$\propto \frac{(2n-1)}{n^2(n-1)^2}$$

Since $n>>1$, $v \propto \frac{n}{n^4}$ or $v \propto \frac{1}{n^3}$, which is choice (c).

33. The mass of proton is very large compared to electron and antineutrino. So all the energy is shared by the electron and anti-neutrino. When the kinetic energy of anti-neutrino is zero, the maximum kinetic energy of electron is 0.8×10^6 eV and vice versa. Hence the total kinetic energy of electron + anti-neutrino is 0.8×10^6 eV.

34. If the anti-neutrino has a mass m = 3 eV/c^2, it will have kinetic energy = mc^2= 3 eV.

Therefore, the maximum kinetic energy of the electron= $(0.8 \times 10^6 - 3)$ eV, which is only slightly less than 0.8×10^6 eV. The minimum kinetic energy of the electron is still zero. Hence correct choice is (d)

35. Half life T is related to disintegration constant λ as

$$T = \frac{0.693}{\lambda}$$

$$\Rightarrow \qquad \lambda = \frac{0.693}{T} = \frac{0.693}{1386} = 5 \times 10^{-4}\ \text{S}^{-1}$$

Now $N = N_0^{-\lambda t}$

The fraction of nuclei disintegrated in t = 80s is

$$f = \frac{N_0 - N}{N_0} = 1 - \frac{N}{N_0} = 1 - e^{-\lambda t} = 1 - \text{e}^{-5 \times 10^{-4} + 80}$$

$$\Rightarrow \qquad f = 1 - \text{e}^{-0.04}$$

Using $e^{-a} = 1 - a$ for $a << 1$, we have

$$f = 1 - (1 - 0.04) = 0.04$$

Percentage disintegrated = $0.04 \times 100 = 4\%$

36. The energy of the electron in the n = 2 state is

$$E_2 = \frac{-13.6\ eV}{4} = -3.4\ \text{eV}$$

Hence energy required to remove the electron from n = 2 state is 3.4 eV.

37. The mass number of the residual nucleus = 238 – 4 = 234. From conservation of momentum (υ = recoil velocity of residuel nucleus)

$$0 = 234\upsilon + 4u$$

$$\upsilon = -\frac{4u}{234}$$

Hence the correct choice is (c).

ELECTRONIC DEVICES

Chapter 20

REVIEW OF BASIC CONCEPTS

1. Energy Bands in Solids

(1) Conductors In conductors, there is no forbidden energy band. The valence band and the conduction bands overlap. Hence electrons from the valence band can easily pass into the conduction band.

(2) Insulators In insulators, the valence band and the conduction band are separated by an energy gap E_G called the forbidden band. An electron in a crystal lattice cannot have an energy which lies in this region. If E_G is large, an applied electric field cannot give enough energy to an electron in the valence band to enable it to enter the conduction band. Hence, the valence electron will not be freed and no current will flow. Thus, materials having a large band gap are insulators. For example, in diamond $E_G \approx 9$ eV.

(3) Semiconductors In semiconductors, the energy gap E_G is much smaller than insulators, typically about 1 eV.

Hence, electrons in the valence band may acquire thermal energy to pass on into the conduction band. Such materials can, therefore, carry an electric current but not as easily as a conductor. They are called semiconductors. The best known examples are silicon ($E_G \approx 1.17$ eV) and germanium ($E_G \approx 0.74$ eV). Unlike metallic conductors, the resistance of semiconductors decreases with increase in temperature.

2. Intrinsic and Extrinsic Semiconductors

Pure semiconductors are called intrinsic semiconductors. The number density (n_e) of electrons in the conduction band is equal to the number density (n_h) of holes in the valence band, i.e.

$$n_e = n_h = n_i$$

where n_i is called the intrinsic carrier concentration.

The deliberate addition of a desirable impurity to a semiconductor is called *doping* and the impurity atoms added are called *dopants. A doped semiconductor (i.e. a semiconductor with impurity atoms) is called an extrinsic semiconductor.*

The addition of each impurity atom makes an extra electron available for conduction. Since these electrons are not released by thermal agitation, they do not produce holes. Consequently, the current carries are primarily electrons. Hence, such semiconductors are called *n-type semiconductors* (*n* stands for negative current carriers).

The number densities of conduction band electrons (n_e) and the valence band holes (n_h) in a doped semiconductor are very different from those in pure semiconductors (in which $n_e = n_h = n_i$; the intrinsic concentration). From thermodynamical considerations, one can show that

$$n_e n_h = n_i^2$$

In an *n*-type semiconductor, the donor electrons are almost all free at room temperature which means that the donor electrons are the main current carriers and $n_e = n_d$, the donor electron number density. For most common doped semiconductors n_d is much larger than the number of 'intrinsic' conduction band electrons ($n_i = n_h$). Thus

$$n_e = n_d >> n_h$$

Hence, in *n*-type semiconductors, electrons are the *majority carriers* and the holes are the *minority carriers* of current.

In *n*-type semiconductor, pentavalent impurity is added. When a trivalent impurity is added to intrinsic semiconductors such as Ge or Si, the semiconductor is called *p*-type semiconductor. The conductivity of a *p*-type semiconductor is considerably increased due to doping. In such semiconductors,

$$n_h >> n_e$$

and the holes are the *majority carriers* and the electrons are the *minority carriers*.

3. Electrical Conductivity of Semiconductors

When an electric field E is applied to a semiconductor, both electrons and holes drift in the semiconductor. Their mobility, which is defined as the drift speed per unit electric field, is $\mu_e = \frac{v_e}{E}$ for electrons and $\mu_h = \frac{v_h}{E}$ for holes, where v_e and v_h are the drift speeds of electrons and holes. The resistivity ρ of the semiconductor is given by

$$\rho = \frac{1}{e(n_e \mu_e + n_h \mu_h)}$$

The conductivity σ (which is the reciprocal of resistivity) is given by

$$\sigma = \frac{1}{\rho} = e(n_e \mu_e + n_h \mu_h)$$

4. The PN Junction

A *pn* junction is formed by growing an *n*-type semiconductor over a *p*-type semiconductor. It is the simplest semiconductor device. A small layer in the vicinity of the junction depleted of free charge carriers (electrons and holes) is called the *depletion layer*. The thickness of this layer is of the order of 1 μm (= 10^{-6} m).

Some Facts about a p–n Junction

(a) Due to difference in concentration, electrons diffuse from *n*-side to *p*-side and holes diffuse from *p*-side to *n*-side.
(b) *p*-side attains lower potential than *n*-side.
(c) An electric field is created directed from *n* to *p* side
(d) Due to electron-hole combination, no free charge carriers are left in a small region (called depletion layer) near the junction.
(e) The width of the depletion layer depends on the nature of semiconductor, nature of doping impurity and temperature.
(f) The potential barrier for a silicon *p-n* junction is 0.6 V at room temperature while for a germanium *p-n* junction it is 0.3 V.
(g) Ohm's law is not obeyed in a *p-n* junction.

5. Biasing of a PN Junction

(a) *Forward biasing:* A *p-n* junction is forward biased if the positive terminal of the external battery is connected to the *p*-side and the negative terminal to the *n*-side. In forward biasing
  (i) The electric field in the depletion layer decreases.
  (ii) The potential barrier is reduced.
  (iii) The width of the depletion layer is reduced.
  (iv) The *p-n* junction offers a low resistance.

(b) *Reverse biasing:* A *p-n* junction is reverse biased if the positive terminal of the external battery is connected to the *n*-side and the negative terminal to the *p*-side. In reverse biasing
  (a) The electric field in the depletion layer increases.
  (b) The potential barrier is increased.
  (c) The width of the depletion layer increases.
  (d) The *p-n* junction offers a very high resistance; hence no current flows through the junction.

6. Rectifier

It is a device which converts an alternating voltage (or current) into a direct (unidirectional) voltage (or current).

(a) *For a half-wave rectifier,*
  (i) average value of rectified output is $V_{av} = \frac{V_0}{\pi}$, where V_0 is the peak value of the input alternating voltage.
  (ii) Average output current $I_{av} = \frac{I_0}{\pi}$
  (iii) $I_{rms} = \frac{I_0}{2}$
  (iv) Form factor = $\frac{I_{rms}}{I_{av}} = \frac{\pi}{2} = 1.57$
  (v) Ripple factor = $\left(\frac{I_{rms}^2}{I_{av}^2} - 1\right)^{1/2} = 1.21$
  (vi) Output frequency = input frequency

(b) For a full-wave rectifier,
  (i) $V_{av} = \frac{2V_0}{\pi}$, $I_{av} = \frac{2I_0}{\pi}$
  (ii) $I_{rms} = \frac{I_0}{\sqrt{2}}$
  (iii) Form factor = 1.1
  (iv) Ripple factor = 0.48
  (v) Output frequency = 2 × input frequency.

7. Current-Voltage Characteristics of a PN Junction Diode

The current-voltage characteristic curve of a *pn* junction diode is a graph which shows the variation of the current

flowing through the junction with the change of voltage across the junction diode when it is forward-biased and reverse-biased. Figure 20.1 shows the current-voltage graph of a typical *pn* diode.

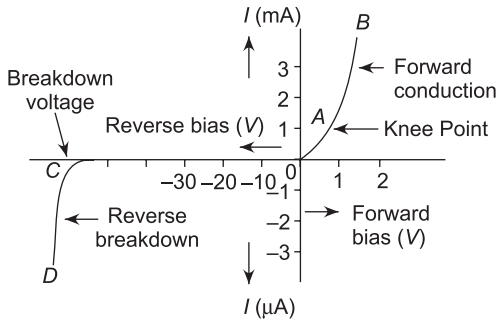


Fig. 20.1

The portion *OAB* of the graph is the forward bias current versus the forward bias voltage characteristic. Notice how sharply the current increases with a small increase in voltage. Even with small applied voltage (~ 1 V), the electrons passing across the junction gain sufficient kinetic energy to expel the valence electrons from the doped semiconductor atoms, resulting in a sharp increase in the forward current.

The portion *OCD* of the graph is the reverse bias current versus the reverse bias voltage characteristic. In this case the voltage of the battery aids the potential barrier and, therefore, prevents the flow of forward current in the diode, resulting in a high resistance at the junction. As the reverse bias voltage is increased, a stage is reached when the charge carriers (electrons and holes) are accelerated by the reverse bias voltage and acquire enough kinetic energy so as to break the covalent bonds in the semiconductor by collisions. The freed electron can again do the same and a rapidly increasing number of energetic electrons are produced. Consequently, the reverse current increases very rapidly and the material electrically 'breaks down'. The reverse biased voltage at which this happens is called the *breakdown voltage*. The value of the breakdown voltage varies from 1 or 2 V to several hundred volts depending on the number density of the dopants and the width of the depletion region.

Dynamic Resistance

The current-voltage *(I-V)* curve of Fig. 20.1 shows that I does not vary linearly with V, i.e. Ohm's law is not obeyed. In such cases, it is useful to define a quantity called the *dynamic resistance* (or ac resistance) of the diode. It is the ratio of a small change in voltage (δV) to the corresponding small change in current *(δI)*, i.e.

$$R_d = \frac{\delta V}{\delta I}$$

8. The Junction Transistor

The transistor is produced when another semiconductor element is added to the simple *pn* diode. The transistor is then composed of three semiconductor elements. The three elements are combined so that the two outer elements are doped with the same type of majority carriers, while the element that separates them has the opposite majority carrier. A transistor can be of two types: *npn* or *pnp* type.

The three elements of the transistor are the *emitter* (E), the *collector* (C) and the *base* (B). The emitter supplies the majority carriers for transistor current flow, the collector collects the current for circuit operation and the base controls the passage of electrons from the emitter to the collector. In fabricating the device, care is taken to make the base thickness as small as possible (about a few microns) so that it only controls the flow of electrons without causing their loss, while crossing the base region to reach the collector.

The doping level in the emitter is more than that in the collector. The area of the base-collector junction is larger than that of the base-emitter junction. The base (which is sandwiched between the emitter and the collector) is thin and lightly doped.

In a transistor, the emitter current is equal to the sum of the base current and collector current, i.e.

$$I_E = I_B + I_C$$

The two current gains in a transistor, namely, α and β are defined as

$$\alpha = \frac{I_C}{I_E}$$

and

$$\beta = \frac{I_C}{I_B}$$

They are related as

$$\beta = \frac{\alpha}{1-\alpha}$$

In a transistor, I_C is only slightly less than I_E and I_B is much smaller than I_C or I_E. Thus α is slightly less than unity (~ 0.98) and β is high (50 or higher).

9. AC Current, Voltage and Power Gains in an Amplifier

1. AC Current Gain Transistors are generally used to amplify alternating currents. If the base current is changed by an amount δI_B, there is a corresponding change δI_C in the collector current. *The ac current gain is defined as the ratio of the change in the collector current to the change in the base current at a constant collector voltage.* It is denoted by the symbol β_{ac}. Thus

$$\beta_{ac} = \frac{\delta I_C}{\delta I_B} \text{ at constant } V_{CE}$$

2. *The ac voltage gain is defined as the ratio of the change in the output voltage to the change in the input voltage.*

Thus ac voltage gain $= \dfrac{\delta V_0}{\delta V_i}$

The output voltage appears across the load resistor R_L which is also called the *output resistance*. If the current in output circuit (i.e. the collector circuit) changes by δI_C, the change in the output voltage is

$$\delta V_0 = \delta I_C \times R_L$$

The equivalent ac resistance of the base-emitter circuit (i.e. the input circuit) is denoted by R_{EB} and is called the *input resistance* of the transistor. If the base current changes by δI_B, the change in the input voltage is

$$\delta V_i = \delta I_B \times R_{EB}$$

Therefore the ac voltage gain is

$$\frac{\delta I_C}{\delta I_B} \times \frac{R_L}{R_{EB}} = \beta_{ac} \times \frac{R_L}{R_{EB}}$$

3. *The ac power gain is defined as the ratio of the change in output power to the change in input power.* Thus

$$\text{ac power gain} = \frac{\text{change in output power}}{\text{change in input power}}$$

$$= \frac{\text{change in output current} \times \text{change in output voltage}}{\text{change in input current} \times \text{change in input voltage}}$$

$$= \text{current gain} \times \text{voltage gain}$$

$$= \beta_{\text{ac}} \times \beta_{\text{ac}} \frac{R_L}{R_{EB}}$$

or $$\text{ac power gain} = \beta_{\text{ac}}^2 \times \frac{R_L}{R_{EB}}$$

10. Common Emitter Amplifier Circuit

Figure 20.2 shows a transistor amplifier in the common emitter mode. The emitter-base junction is forward-biased and the base-collector junction is reverse biased.

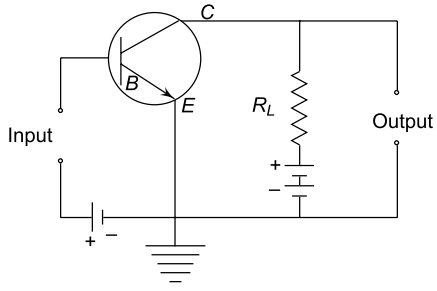


Fig. 20.2

The common-emitter characteristics of a transistor are its characteristics when the emitter is taken as the common reference terminal and is grounded (i.e. is at the zero potential), the base is the input terminal and the collector the output terminal.

If the base-collector junction is reverse-biased, the input characteristics remain almost unaffected by small changes in V_{CE}. In such a situation, we can define the *input resistance* R_i as the slope at a given point on the characteristic curve, i.e.

$$R_i = \left(\frac{\delta V_{BE}}{\delta I_B}\right)_{V_{CE} = \text{constant}}$$

The input characteristic curves [shown in Fig. 20.3 (a)] are obtained by plotting the base current I_B as a function of V_{BE} for different values of V_{CE}.

The output characteristic curves [shown in Fig. 20.3 (b)], are obtained by plotting the collector current I_C as a function of V_{CE} for different values of the base current I_B. Notice that for a given value of V_{CE}, I_C is larger for large values of I_B. Also notice that I_C is almost independent of V_{CE}, although it is controlled by I_B.

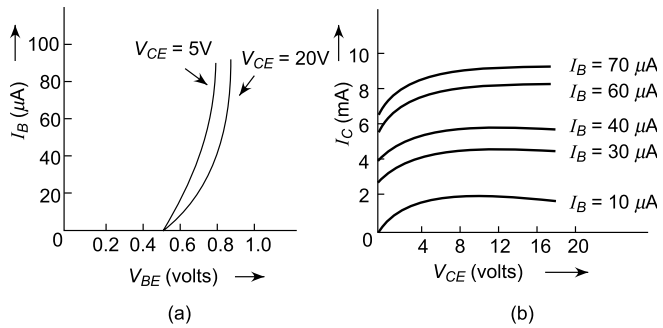


Fig. 20.3 (a) Input characteristics, and (b) output characteristics of a common-emitter pn transistor

EXAMPLE 1 What is the order of magnitude of energy band gap in an intrinsic semiconductor?

SOLUTION 0.2 eV to 3.0 eV. For germanium $E_g \simeq 0.3$ eV and for silicon $E_g \simeq 0.6$ eV.

EXAMPLE 2 Carbon, silicon and germanium are semiconductors having four valence electrons. Their respective band gap energies between the conduction and valence bands are $(E_g)_{\text{C}}$, $(E_g)_{\text{Si}}$ and $(E_g)_{\text{Ge}}$. Then

(a) $(E_g)_{\text{Si}} < (E_g)_{\text{Ge}} < (E_g)_{\text{C}}$

(b) $(E_g)_{\text{C}} < (E_g)_{\text{Ge}} > (E_g)_{\text{Si}}$

(c) $(E_g)_{\text{C}} > (E_g)_{\text{Si}} > (E_g)_{\text{Ge}}$

(d) $(E_g)_{\text{C}} = (E_g)_{\text{Ge}} = (E_g)_{\text{Si}}$

SOLUTION Of the three semiconductors, carbon has the highest resistivity and germanium has the lowest resistivity, i.e. carbon has the lowest conductivity and germanium has the highest conductivity. Hence carbon has the highest band gap energy and germanium has the lowest band gap energy. So the correct choice is (c).

EXAMPLE 3 Name the charge carriers in (a) a metallic conductor, (b) an electrolyte, (c) a semiconductor and (d) an insulator.

SOLUTION (a) free electrons, (b) positive and negative ions, (c) electrons and holes and (d) none.

EXAMPLE 4 An *n*-type semiconductor is doped with

(a) pentavalent atoms and the majority charge carriers are holes
(b) pentavalent atoms and the majority charge carriers are electrons
(c) trivalent atoms and the majority charge carriers are holes
(d) trivalent atoms and the majority charge carriers are electrons.

SOLUTION An *n*-type semiconductor in formed when a pure semiconductor is doped with pentavalent atoms. Each atom has 5 valence electrons. Four of these form covalent bonds and one electron is left free. Hence the concentration of electrons becomes much greater than that of the holes. Therefore, the majority charge carriers are electrons and holes are the minority charge carriers. So the correct choices are (b) and (c).

EXAMPLE 5 In an unbiased *p*-*n* junction, holes diffuse from the *p*-region to the *n*-region because

(a) they are attracted by the electrons in the *n*-region
(b) the potential at the junction makes them move across the junction
(c) the electric field at the junction accelerates them to the *n*-region
(d) the concentration of holes in the *p*-region is much higher than their concentration in the *n*-region.

SOLUTION In the *p*-region of a *pn* junction, the concentration of holes is higher than of the electrons, whereas in the *n*-region the concentration of electrons is higher than that of the holes. Therefore, when a *pn* junction is formed, some holes from the *p*-region (where their concentration is high) will diffuse into the *n*-region (where their concentration is low) and vice versa. Hence the correct choice is (d).

EXAMPLE 6 The factors on which the width of the depletion layer in a *pn* junction depends are

(a) the nature of the semiconductor
(b) the nature of doping impurity atoms
(c) the number of impurity atoms added for doping
(d) the temperature

SOLUTION All the given choices are correct.

EXAMPLE 7 The potential barrier for germanium *pn* junction is 0.3V. If the width of the depletion layer is 1 micron, find the strength of the electric field at the junction.

SOLUTION $V_B = 0.3$ V, $d = 1$ micron $= 10^{-6}$ m

$$E_B = \frac{V_B}{d} = \frac{0.3\text{ V}}{10^{-6}\text{ m}} = 3 \times 10^5\ \text{Vm}^{-1}$$

Notice that E_B is quite high.

EXAMPLE 8 The current through a junction diode changes by 1.2 mA when the forward bias voltage is changed by 0.6 V. Find the dynamic resistance.

SOLUTION $R_d = \dfrac{\Delta V}{\Delta I} = \dfrac{0.6\text{ V}}{1.2 \times 10^{-3}\text{ A}} = 500\ \Omega$

EXAMPLE 9 In a half wave junction diode rectifier, the a.c. source of 220 V, 50 Hz is used as the input. A step-down transformer of turns ratio 1: 10 is used. Find the average output d.c. voltage across the load resistor in one full cycle of the input.

SOLUTION Peak value of input voltage is

$$V_i = \sqrt{2} \times 220 = 311\text{ V}$$

Peak value of output voltage is $V_o = 311 \times \dfrac{1}{10} = 31.1$ V

$\therefore$ Average d.c. voltage $= \dfrac{V_o}{\pi} = \dfrac{31.1}{3.14} \simeq 9.9$ V

EXAMPLE 10 In a full wave junction diode rectifier, the a.c. input has an rms value of 10 V. The transformer used is a step-up transformer of turns ratio 1:2. Find the average output d.c. voltage across the load resistor in one full cycle of the input.

SOLUTION $V_i = \sqrt{2} \times 10 = 14.14$ V

$$V_o = 14.14 \times \frac{2}{1} = 28.28\text{ V}$$

Average d.c. output $= \dfrac{2V_o}{\pi} = \dfrac{2 \times 28.28}{3.14} = 18$ V

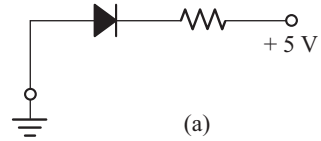
EXAMPLE 11 In a half wave rectifier, the a.c. input source of frequency 50 Hz is used. What is the frequency of the output? What will be the output frequency if a full wave rectifier was used for the same input frequency?

SOLUTION Frequency of input voltage is $\nu = 50$ Hz

Output frequency of half wave rectifier = $\nu = 50$ Hz

Output frequency of full wave rectifier = $2\nu = 2 \times 50 = 100$ Hz

EXAMPLE 12 In Fig. 20.4, which of diodes are forward biased and which are reverse biased?



(a)

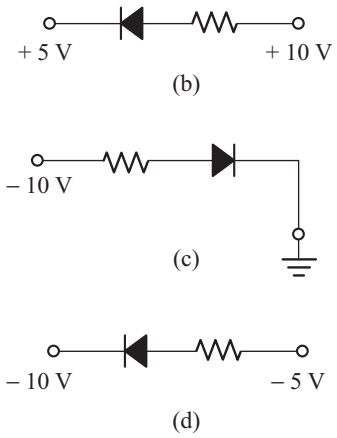


Fig. 20.4

SOLUTION

(a) The *p*-side of diode is connected to earth. Therefore, $V_p = 0$ and the *n*-side is connected to + 5 V source, i.e. $V_n = +5$ V. Hence $V_p - V_n = 0 - 5 = -5$ V, i.e. the *p*-side is at a lower potential than the *n*-side. Therefore, the diode is reverse biased

(b) $V_p - V_n = 10 - 5 = +5$ V, hence the diode is forward biased.

(c) $V_p - V_n = -10 - 0 = -10$ V, hence the diode is reverse biased.

(d) $V_p - V_n = -5 - (-10) = +5$ V, hence the diode is forward biased

EXAMPLE 13 An input of waveform shown in Fig. 20.5 (a) is connected between terminals A and B shown in Fig. 20.5(b). Draw the waveform of the output across load resistor *R*.

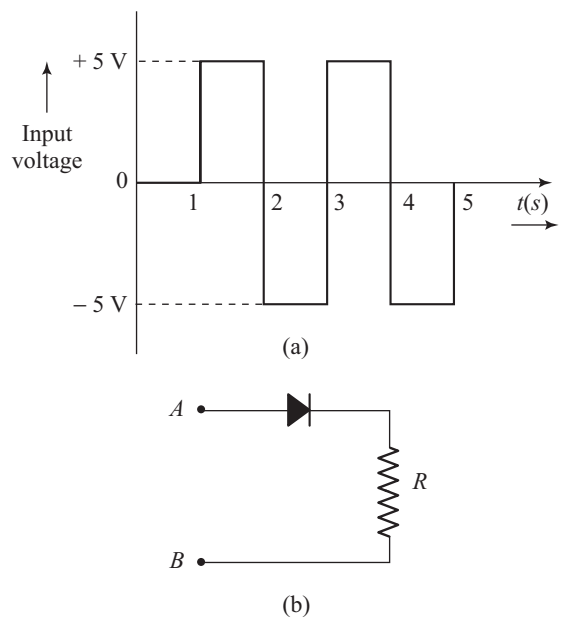


Fig. 20.5

SOLUTION Between $t = 0$ s and $t = 1$ s, the input voltage is zero. So no current flows through the circuit and the voltage across *R* is zero. Between $t = 1$ s and $t = 2$ s, the input voltage is positive. Hence during this time interval the diode is forward biased. Hence a current flows through the circuit and an output voltage is obtained across *R*. Between $t = 2$ s and $t = 3$ s, the input voltage is negative. Hence during this time interval the diode is reverse biased. Hence no current flows through the circuit and the output voltage across *R* is zero. The output waveform is shown in Fig. 20.6.

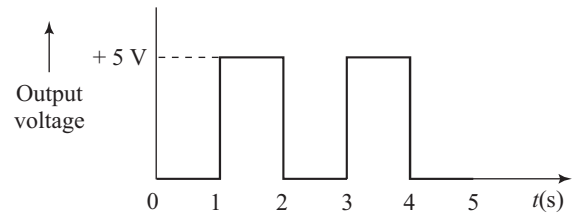


Fig. 20.6

EXAMPLE 14 Figures 20.7 (a) and (b) show two identical diodes D_1 and D_2 connected as shown. Find the current through resistor *R* in each case.

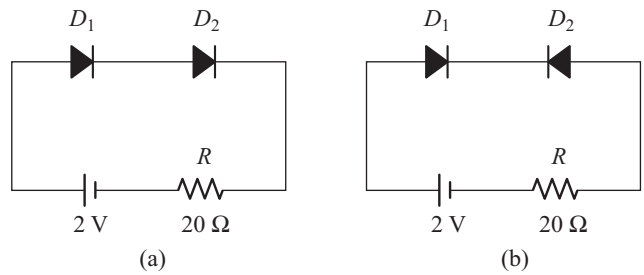


Fig. 20.7

SOLUTION In Fig. (a) both the diodes are forward biased and hence offer no resistance. Therefore, the current in the circuit is

$$I = \frac{2\,\text{V}}{20\,\Omega} = 0.1\ \text{A}$$

In Fig. (b), diode D_1 is forward biased and offers zero resistance but diode D_2 is reverse biased and offers infinite resistance. Hence, is this case, no current flows in the circuit.

EXAMPLE 15 An unregulated input of 10.0 V is applied across a series combination of a zener diode and a resistor R_s. A regulated volage $V_z = 6.0$ V is required across the load resistor connected across the zener diode. The load current is to be 4.0 mA. If the zener current is four times the load current, find the value of resistor R_s.

SOLUTION Load current $I_L = 4.0$ mA, zener current $I_z = 4 \times 4.0$ mA = 16 mA. Total current through the series combination is $I_s = I_L + I_z = 4 + 16 = 20$ mA $= 20 \times 10^{-3}$ A. Now, out of 10. 0 V, the voltage drop across the

load is 6.0 V. Hence the voltage across R_s = 10.0 – 6.0 = 4.0 V. Therefore,

$$R_s = \frac{V_s}{I_s} = \frac{4.0\text{ V}}{20\times10^{-3}\text{ A}} = 200\ \Omega$$

EXAMPLE 16 At a certain temperature pure silicon has equal electron and hole concentrations of 2.0×10^{16} m^{-3}. When it is doped with indium, the hole concentrations increases to 4.0×10^{22} m^{-3}. Find the new concentrations of electrons in the doped silicon.

SOLUTION Given $n_i = 2.0 \times 10^6$ m^{-3} and on doping, $n_h = 4.0 \times 10^{22}$ m^{-3}.

From the relation $n_e \times n_h = n_i^2$, we have

$$n_e = \frac{n_i^2}{n_h} = \frac{(2.0\times10^{16})^2}{4.0\times10^{22}} = 1.0\times10^{10}\text{ m}^{-3}$$

EXAMPLE 17 A semiconductor has equal electron and hole concentrations of 2×10^8 m^{-3}. On doping with a certain impurity, the electron concentration increases to 4×10^{10} m^{-3}.

(a) What type of semiconductor is obtained?

(b) What is the new hole concentration of the semiconductor?

(c) How does the energy gap vary with doping?

SOLUTION

(a) Since on doping the electron concentration has increased, the doped semiconductor is an *n*-type semiconductor.

(b) The new hole concentrations is

$$n_h = \frac{n_i^2}{n_e} = \frac{(2\times10^8)^2}{4\times10^{10}} = 10^6\text{ m}^{-3}$$

(c) The energy gap decreases with doping.

EXAMPLE 18 A pure silicon crystal has 5×10^{28} atoms m^{-3}. It is doped with 1 ppm (particle per million) of pentavalent arsenic. Calculate the number density of electrons and holes in the doped silicon crystal. Given, intrinsic number density $n_i = 2.0 \times 10^{16}$ m^{-3}.

SOLUTION Since silicon is doped with 1 ppm (= 10^6) atoms of arsenic (which is pentavalent), the free electron concentration of electrons in the doped silicon is

$$n_e = \frac{n}{10^6} = \frac{5\times10^{28}}{10^6} = 5\times10^{22}\text{ m}^{-3}$$

The concentration of holes in the doped silicon is

$$n_h = \frac{n_i^2}{n_e} = \frac{(2.0\times10^{16})^2}{5\times10^{22}} = 8\times10^9\text{ m}^{-3}$$

EXAMPLE 19 A *pn* photodiode is made from a semiconductor with a band gap energy of 2.0 eV. Can it detect a radiation of wavelength 5000 nm?

SOLUTION $\lambda = 5000\text{ nm} = 5\times10^{-6}$ m

Energy of a photon of incident radiation is

$$E = h\nu = \frac{hc}{\lambda}$$

$$= \frac{(6.63\times10^{-34})\times(3\times10^8)}{5\times10^{-6}}$$

$$= 3.98\times10^{-20}\text{ J}$$

$$= \frac{3.98\times10^{-20}}{1.6\times10^{-19}} \simeq 0.25\text{ eV}$$

Since the energy of the photon is much less than the band gap energy (which is 2.0 eV), the given photodiode cannot detect the given radiation.

EXAMPLE 20 In an *npn* transistor, the collector current is 9.9 mA. If 99% of the electrons from the emitter reach the collector, find the emitter current and the base current.

SOLUTION Since 99% of the electrons from the emitter reach the collector, the emitter current is

$$I_E = I_C \times \frac{100}{99} = 9.9\text{ mA} \times \frac{100}{99} = 10\text{ mA}$$

Base current is $I_B = I_E - I_C = 10 - 9.9 = 0.1$ mA

EXAMPLE 21 The a.c. current gain of a transistor amplifier in the common emitter mode is 150. Find the change in the collector current when the base current changes by 100 μA.

SOLUTION Given $\beta_{ac} = 150$ and $\Delta I_B = 100\ \mu\text{A} = 10^{-4}$ A

Now $$\beta_{ac} = \frac{\Delta I_C}{\Delta I_B}$$

$$\Rightarrow \quad \Delta I_C = \beta_{ac} \times \Delta I_B = 150\times10^{-4} = 1.5\times10^{-2}\text{ A}$$

EXAMPLE 22 In a common emitter amplifier, when the emitter current changes by 10 mA, the collector current changes by 9.5 mA. Find the changes in the base current and the a.c. current gain of the amplifier.

SOLUTION $\Delta I_B = \Delta I_E - \Delta I_C = 10 - 9.5 = 0.5$ mA

$$\beta_{ac} = \frac{\Delta I_C}{\Delta I_B} = \frac{9.5\text{ mA}}{0.5\text{ mA}} = 19$$

EXAMPLE 23 A common emitter transistor amplifier has an a.c. current gain of 20 and input resistance of 1 kΩ. If the load (output) resistance is 5 kΩ , find (a) the voltage gain and (b) the change in output voltage if the input voltage changes by 1 V.

SOLUTION

(a) Voltage gain $A_V = \beta_{ac} \times \frac{R_o}{R_i} = 20 \times \frac{5\text{ k}\Omega}{1\text{ k}\Omega} = 100$

(b) $\Delta V_o = A_V \times \Delta V_i = 100 \times 1\text{ V} = 100\text{ V}$

EXAMPLE 24 The input resistance of a silicon transistor is 200 Ω. When the base current is changed by 10 μA, the collector current changes by 1 mA. This transistor is used as a common emitter amplifier having output (or load) resistance of 1 kΩ. Find the voltage gain and the power gain of the amplifier.

SOLUTION Given $R_i = 200\ \Omega$, $R_o = 1\ \text{k}\Omega = 1000\ \Omega$, $\Delta I_B = 10\ \mu\text{A} = 10^{-5}$ A

and $$\Delta I_C = 1\ \text{mA} = 10^{-3}\ \text{A}$$

$$\beta_{\text{ac}} = \frac{\Delta I_C}{\Delta I_B} = \frac{10^{-3}}{10^{-5}} = 100$$

Voltage gain $$A_V = \beta_{\text{ac}} \times \frac{R_o}{R_i}$$

$$= 100 \times \frac{1000}{200} = 500$$

Power gain = current gain × voltage gain

$$= 100 \times 500 = 50000 = 5 \times 10^4$$

EXAMPLE 25 The potential difference across the collector of a transistor, used in common emitter configuration, is 1.5 V. If the collector resistance is 3 kΩ, find the emitter current and the base current. The d.c. current gain of transistor is 50.

SOLUTION $V_C = 1.5$ V, $R_C = 3\ \text{k}\Omega = 3000\ \Omega$ and $\beta_{\text{dc}} = 50$

Collector current $$I_C = \frac{V_C}{R_C} = \frac{1.5}{3000} = 5 \times 10^{-4}\ \text{A}$$

$\therefore$ Base current $$I_B = \frac{I_C}{\beta_{\text{dc}}} = \frac{5 \times 10^{-4}}{50} = 10^{-5}\ \text{A}$$

$\therefore$ Emitter current $I_E = I_C + I_B = 5 \times 10^{-4} + 10^{-5}$

$$= 5.1 \times 10^{-4}\ \text{A}$$

EXAMPLE 26 A *pn* junction diode can withstand currents upto 10 mA. When it is forward biased, the potential drop across it is 1.0 V. Assuming that this potential drop is independent of the current, find the maximum voltage of the battery used to forward bias the diode when a resistance of 100 Ω is connected in series with the diode.

SOLUTION Refer to Fig. 20.8.

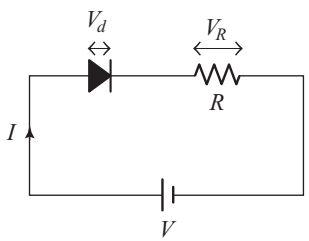


Fig. 20.8

Voltage drop across diode is

$$V_d = 1.0\ \text{V}$$

Voltage drop across R is $V_R = RI$

$$= 100 \times 10 \times 10^{-3}$$

$$= 1.0\ \text{V}$$

$\therefore$ $$V_{\max} = V_d + V_R = 1.0 + 1.0 = 2.0\ \text{V}$$

EXAMPLE 27 The circuit shown in Fig. 20.9 has two diodes D_1 and D_2 each having a resistance of 50 Ω when forward biased and infinity when reverse biased. Find the current through the 100 Ω resistor.

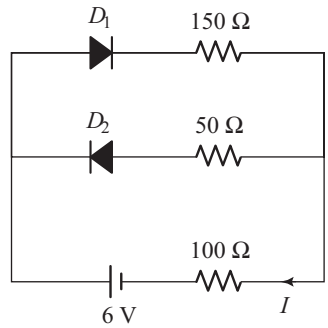


Fig. 20.9

SOLUTION D_1 is forward biased and D_2 is reverse biased. So D_2 offers infinite resistance and, therefore, no current flows through the branch containing D_2. D_1 offers a resistance of 50 Ω. The circuit simplifies to that shown in Fig. 20.10.

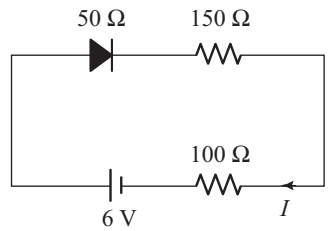


Fig. 20.10

Total resistance = 50 + 150 + 100 = 300 Ω

$\therefore$ current $$I = \frac{6}{300} = 0.02\ \text{A}$$

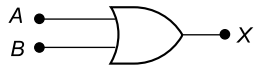
11. Logic Gates

A gate is a digital circuit that follows a certain logical relationship between the input and output voltages. They are, therfore, called *logic gates*. There are three basic gates called OR, AND and NOT.

12. The OR Gate

Each gate has one or two binary input variables designated by A and B and one binary output variable designated by X.

The OR gate has an output 1 if input A or input B or both inputs A and B are 1; otherwise the output is zero.



(a)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

(b)

Fig. 20.11 (a) Logic symbol of OR gate, (b) its truth table

Figure 20.11 (a) is the logic symbol of the OR gate and Fig. 20.11 (b) is its truth table. The algebraic symbol of the OR function is plus (+), i.e.

$$A + B = X$$

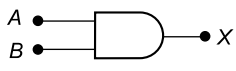
which is read as A OR B equals X. The OR gate may have more than two inputs and, by definition, the output is 1 if *any* input is 1.

13. The AND Gate

It has two (or more) inputs and one output. The output X is 1 if input A AND input B are *both* binary; otherwise the output is 0. Figures 20.12 (a) and (b) show the symbol and the truth table of the AND gate respectively. The truth table shows that the output X is 1 only when both input A and input B are 1. The Boolean expression (i.e. the algebraic symbol) of the AND function is the same as the multiplication symbol of ordinary arithmetic. A dot is put between the variables thus

$$A \times B = X$$

which is read as 'A AND B equals X'. The dot in the AND operator is often dropped, so the statement $A \cdot B = X$ is often also written as $AB = X$.



(a)

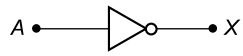
A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

(b)

Fig. 20.12 (a) Logic symbol of AND gate, (b) its truth table

14. The NOT Gate

A NOT gate has only one input and one output. The output is 1 if the input is 0 and vice versa. The logic symbol and the truth table for a NOT gate are shown in Figs 20.13 (a) and (b) respectively. The Boolean operation is represented as $\overline{A} = X$ which is read as 'NOT A equals X', i.e. X is the negation of A. In other words, $X = 0$ if $A = 1$ and $X = 1$ if $A = 0$.



(a)

A	X
0	1
1	0

(b)

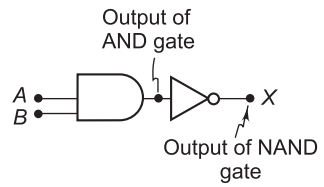
Fig. 20.13 (a) Logic symbol of NOT gate, (b) its truth table

15. The NAND Gate

The word NAND is an abbreviation of NOT-AND. A NAND gate is a combination of AND and NOT gates. If the output of an AND gate is connected to the input of a NOT gate, as shown in Fig. 20.14 (a), the combined circuit is called a NAND gate whose logic symbol is shown in Fig. 20.14 (b). The Boolean expression for the NAND operation is

$$\overline{A \cdot B} = X$$

which means that the output X here is just the negation of the output of the AND operation. Symbol $\overline{A \cdot B}$ is read as 'A AND B negated'.



(a)



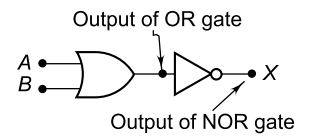
A	B	X
0	0	1
1	0	1
0	1	1
1	1	0

(b) (c)

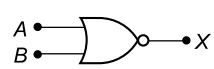
Fig. 20.14 (a) Circuit of NAND gate, (b) its logic symbol, and (c) its truth table

16. The NOR Gate

If we connect the output of an OR gate to the input of a



(a)



A	B	X
0	0	1
1	0	0
0	1	0
1	1	0

(b) (c)

Fig. 20.15 (a) Circuit of NOR gate, (b) its logic symbol, (c) its truth table.

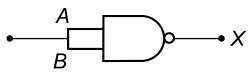
NOT gate, as shown in Fig. 20.15 (a), we get what is called a NOR gate (which is an abbreviation of NOT-OR). The logic symbol of the NOR gate is shown in Fig. 20.15 (b). The Boolean expression for the NOR operation is

$$\overline{A+B} = X$$

which is read as 'A or B' negated'.

17. NAND (or NOR) as a Digital Building Block

Figure 20.16 shows how we can get a NOT gate from a NAND gate. We simply join the two inputs A and B together to make one input. As is clear from the circuit, the truth table is as shown in Fig. 20.16(b). Since $A = B$, there is only one input and one output as in a NOT gate.



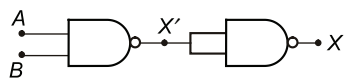
(a)

A	B = A	X
0	0	1
1	1	0

(b)

Fig. 20.16 (a) Obtaining a NOT gate using a NAND gate, and (b) truth table

Figure 20.17 shows how we can get an AND gate by using a NOT gate after a NAND gate. This is clear from the circuit and the truth table.



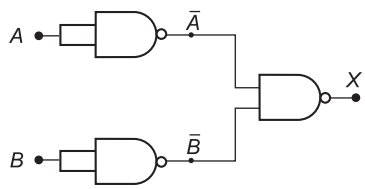
(a)

A	B	X′	X
0	0	1	0
1	0	1	0
0	1	1	0
1	1	0	1

(b)

Fig. 20.17 (a) Obtaining an AND gate using a NAND gate, and (b) truth table

If the inputs A and B are inverted (so that we have $\bar{A}$ and $\bar{B}$) and then applied to a NAND gate, the output of the NAND gate will be the same as that of an OR gate. This follows from the circuit and the truth table shown in Fig. 20.18.



(a)

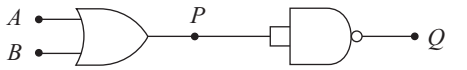
A	B	Ā	B̄	X
0	0	1	1	0
1	0	0	1	1
0	1	1	0	1
1	1	0	0	1

(b)

Fig. 20.18 (a) Obtaining an OR gate using a NAND gate, and (b) truth table

EXAMPLE 28 The output of an OR gate is connected to both the inputs of a NAND gate. Draw the logic circuit of the combination and write its truth table.

SOLUTION Figure 20.19 shows the logic circuit.



OR gate NAND gate as NOT gate

Fig. 20.19

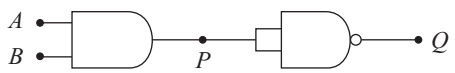
The NAND gate with both its inputs combined behaves as a NOT gate (Fig. 20.19). When the output P of an OR gate is fed to a NOT gate, we get a NOR gate. The truth table is shown in Fig. 20.20

A	*B*	*P*	*Q*
0	0	0	1
1	0	1	0
0	1	1	0
1	1	1	0

Fig. 20.20

EXAMPLE 29 The output of an AND gate is connected to both the inputs of a NAND gate. Draw the logic circuit of the combination and write its truth table.

SOLUTION Figure 20.21 shows the logic circuit



AND gate NAND gate as NOT gate

Fig. 20.21

The truth table is shown in Fig. 20.22.

A	*B*	*P*	*Q*
0	0	0	1
1	0	0	1
0	1	0	1
1	1	1	0

Fig. 20.22

EXAMPLE 30 Figure 20.23 shows the logic circuit of a combination of gates X and Y.

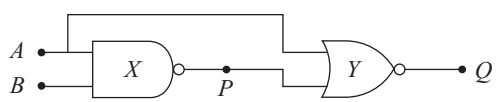


Fig. 20.23

Identify the gates X and Y and write the truth table.

SOLUTION X is a NAND gate and Y is an OR gate. The truth table is shown in Fig. 20.24.

A	B	$P = \overline{A \cdot B}$	$Q = A + P$
0	0	1	1
1	0	1	1
0	1	1	1
1	1	0	1

Fig. 20.24

EXAMPLE 31 Figure 20.25 shows the logic circuit of a combination of gates *X* and *Y*.

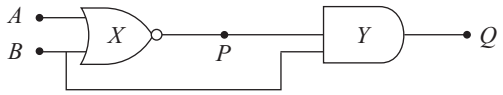


Fig. 20.25

Identify the gates X and Y and write the truth table.

SOLUTION *X* is a NOR gate and *Y* is an AND gate. Figure 20.26 shows the truth table.

A	B	$P = \overline{A + B}$	$Q = B \cdot P$
0	0	1	0
1	0	0	0
0	1	0	0
1	1	0	0

Fig. 20.26

EXAMPLE 32 Two inputs *A* and *B* have waveforms as shown in Fig. 20.27. Draws the waveform of output *X* when these inputs are fed to an OR gate.

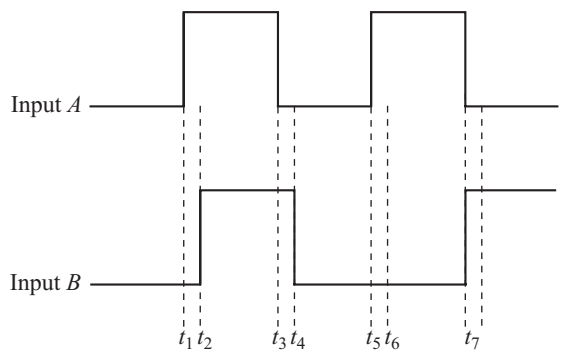


Fig. 20.27

SOLUTION Between t_1 and t_2, $A = 1$ and $B = 0$. So for OR gate $X = 1$. Between t_2 and t_3, $A = 1$ and $B = 1$. So $X = 1$. Between t_3 and t_4, $A = 0$ and $B = 1$. So $X = 1$. Between t_4 and t_5, $A = 0$ and $B = 0$. So $X = 0$. Between t_5 and t_6, $A = 1$ and $B = 0$. So $X = 1$. Between t_6 and t_7, $A = 1$ and $B = 1$, so $X = 1$. Figure 20.28 shows the waveform of output *X*.

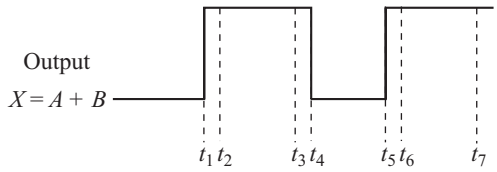


Fig. 20.28

EXAMPLE 33 Inputs *A* and *B* shown in Fig. 20.27 of Example 20.32 are fed to an AND gate. Draws the waveform of output *X*.

SOLUTION Figure 20.29 shows the waveform of output *X*.

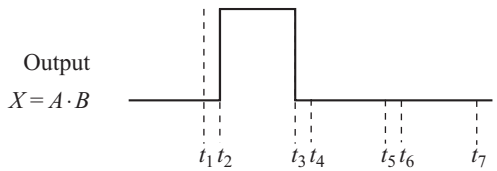


Fig. 20.29

EXAMPLE 34 Draw the output waveform at *X* for given inputs *A* and *B* for the logic circuit shown in Fig 20.30. Also identify the gate.

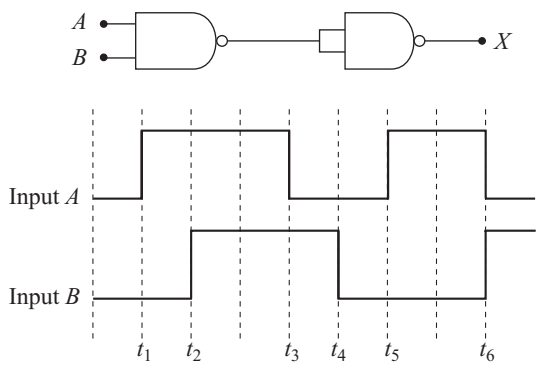


Fig. 20.30

SOLUTION The logic circuit is an AND gate. Figure 20.31 shows the output waveform *X*.

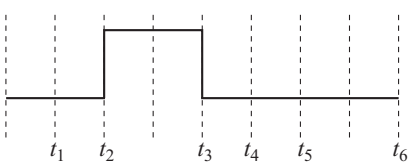


Fig. 20.31

EXAMPLE 35 Figure 20.32 shows the waveform of inputs *A* and *B*. When they are fed to a logic gate, the output waveform *X* is obtained. Identify the logic gate.

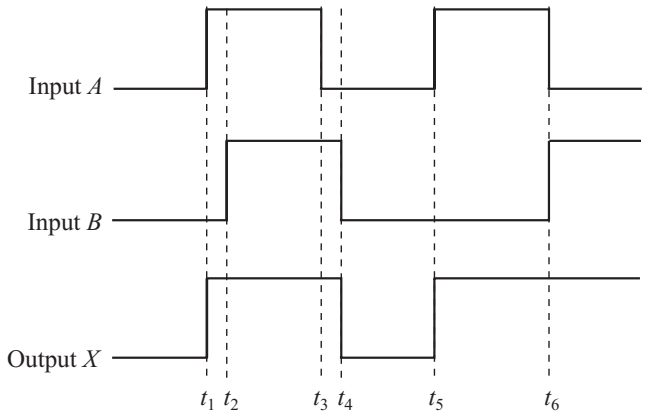


Fig. 20.32

SOLUTION From the waveform for A, B and X, the following conclusions follow:

Time Interval	*Input A*	*Input B*	*Output X*
$t < t_1$	0	0	0
$t_1 < t < t_2$	1	0	1
$t_2 < t < t_3$	1	1	1
$t_3 < t < t_4$	0	1	1
$t_4 < t < t_5$	0	0	0
$t_5 < t < t_6$	1	0	1
$t > t_6$	1	0	1

These conclusions can be summarised in the following truth table.

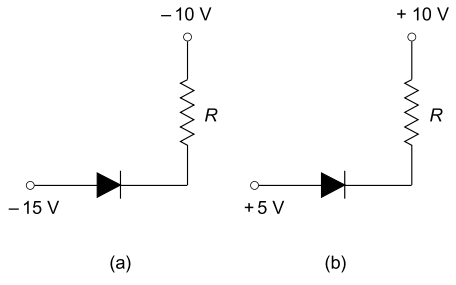
A	*B*	*X*
0	0	0
1	0	1
0	1	1
1	1	1

This is the truth table of an OR gate.

SECTION 1 Multiple Choice Questions with One Correct Choice

Level A

1. At absolute zero of temperature, the electrical conductivity of a pure semiconductor is
   (a) zero (b) about $10^3\ (\Omega\ m)^{-1}$
   (c) about $10^7\ (\Omega\ m)^{-1}$ (d) infinity
2. When germanium is doped 1 part in a million with indium, its conductivity increases by a factor of about
   (a) 10 (b) 10^3
   (c) 10^5 (d) 10^6
3. In pure silicon at 300 K the electron and hole concentration is each equal to $1.5 \times 10^{16}\ m^{-3}$. When doped with indium, the hole concentration increases to $4.5 \times 10^{22}\ m^{-3}$. What is the electron concentration in doped silicon?
   (a) $3 \times 10^9\ m^{-3}$ (b) $4 \times 10^9\ m^{-3}$
   (c) $5 \times 10^9\ m^{-3}$ (d) $6 \times 10^9\ m^{-3}$
4. A block of pure silicon at 300 K has a length of 10 cm and an area of $1.0 \times 10^{-4}\ m^2$. If a battery of emf 2 V is connected across it, what is the electron current? The mobility of electrons is $0.14\ m^2\ V^{-1}\ s^{-1}$ and their number density is $1.5 \times 10^{16}\ m^{-3}$.
   (a) 6.72×10^{-4} A (b) 6.72×10^{-5} A
   (c) 6.72×10^{-6} A (d) 6.72×10^{-7} A
5. In Q. 4, what is the hole current? The mobility of holes is $0.05\ m^2\ V^{-1}\ s^{-1}$.
   (a) 2.0×10^{-7} A (b) 2.2×10^{-7} A
   (c) 2.4×10^{-7} A (d) 2.6×10^{-7} A
6. In Q. 4 and 7, the total current in the block is
   (a) 2.4×10^{-7} A (b) 6.72×10^{-7} A
   (c) 4.32×10^{-7} A (d) 9.12×10^{-7} A
7. What is the number density of donor atoms which must be added to a pure germanium semiconductor to produce an n-type semiconductor of conductivity $6.4\ \Omega^{-1}\ cm^{-1}$? The mobility of electrons in n-type germanium is $4 \times 10^3\ cm^2\ V^{-1}\ s^{-1}$. Neglect the contribution of holes to conductivity.
   (a) $10^{16}\ m^{-3}$ (b) $10^{18}\ m^{-3}$
   (c) $10^{20}\ m^{-3}$ (d) $10^{22}\ m^{-3}$
8. Which of the junction diodes shown in Fig. 20.33 is forward biased?



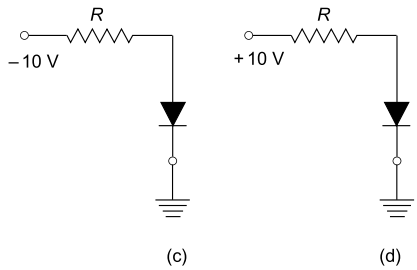


Fig. 20.33

9. The output from a full wave rectifier is
   (a) an ac voltage
   (b) a dc voltage
   (c) zero
   (d) a pulsating unidirectional voltage
10. A half wave rectifier is being used to rectify an alternating voltage of frequency 50 Hz. The number of pulses of rectified current obtained in one second is
   (a) 50 (b) 25
   (c) 100 (d) 200
11. A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of
   (a) each of them increases
   (b) each of them decreases
   (c) copper increases and that of germanium decreases
   (d) copper decreases and that of germanium increases.
12. When a *pn* junction diode is biased in the manner shown in Fig. 20.34 which of the following is incorrect
   (a) the depletion layer gets widened compared to its normal unbiased thickness
   (b) the current flowing in the circuit is due to minority charge carriers
   (c) the current increases only slightly with an increase in the applied reverse bias but shows a sudden sharp increase as the applied bias voltage reaches a particular value characteristic of the diode
   (d) the current is due to majority charge carriers and increases with applied bias voltage in a non-linear way

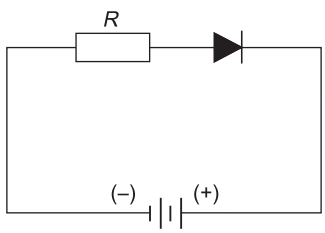


Fig. 20.34

13. The output current I versus time (t) curve of a rectifier is shown in Fig. 20.35. The average value of the output current in this case is
   (a) zero (b) $I_0/2$
   (c) $2I_0/\pi$ (d) I_0

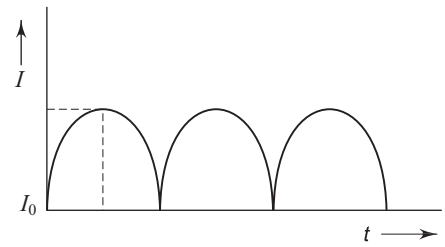


Fig. 20.35

14. A full-wave rectifier is fed with ac mains of frequency 50 Hz. What is the fundamental frequency of the ripple in the output current?
   (a) 25 Hz (b) 50 Hz
   (c) 75 Hz (d) 100 Hz
15. In a half-wave rectifier, the rms value of the ac component of the wave is
   (a) equal to dc value (b) more than dc value
   (c) less than dc value (d) zero
16. In an *n-p-n* transistor circuit, the collector current is 10 mA. If 80% of the electrons emitted reach the collector, then
   (a) the emitter current will be 7.5 mA
   (b) the emitter current will be 12.5 mA
   (c) the base current will be 3.5 mA
   (d) the base current will be 2.0 mA
17. In a typical transistor, the collector current is
   (a) slightly less than the emitter current
   (b) slightly more than the emitter current
   (c) equal to the emitter current
   (d) equal to the base current
18. In a transistor, the value of the base current depends on
   (a) base thickness only
   (b) bias voltages only
   (c) doping levels of emitter, base and collector only
   (d) All the above factors
19. Which of the following statements is correct about *npn* and *pnp* transistors?
   (a) The majority current carriers in an *npn* transistor are electrons and in a *pnp* transistor holes are the majority current carriers.
   (b) The majority current carriers in an *npn* transistor are holes and in a *pnp* transistor electrons are the majority current carriers.

(c) The emitter-base junction and the base-collector junction are both forward biased in both types of transistors.
(d) The emitter-base junction is reverse biased and the base-collector junction is forward biased in both types of transistors.

20. For useful amplifying action, which of the following features should a transistor have?
(a) The emitter should be heavily doped, the collector less heavily doped and the base lightly doped
(b) The area of the base-collector junction must be larger than that of the emitter-base junction
(c) The thickness of the base layer should be very small
(d) all the above features

21. In a transistor amplifier,
(a) the power dissipated at the base-collector junction is much higher than that dissipated at the emitter-base junction
(b) the power dissipated at the base-collector junction is much lower than that dissipated at the emitter-base junction
(c) the power dissipated is the same at both the junctions.
(d) there is no power dissipation at any junction.

22. The current gain in a common emitter transistor is
(a) more than 1 (b) less than 1
(c) equal to 1 (d) zero.

23. The two current gains in a transistor are defined as $\alpha = I_C/I_E$ and $\beta = I_C/I_B$. In a typical transistor
(a) α is slightly less than unity and β is slightly greater than unity
(b) α is slightly greater than unity and β is slightly less than unity
(c) α is slightly less than unity and β is much larger than unity
(d) α is much larger than unity and β is slightly less than unity.

24. In a transistor circuit, the collector current is 50 mA and the base current is 1 mA. The current gain α is
(a) $\frac{51}{50}$ (b) $\frac{49}{50}$
(c) $\frac{50}{51}$ (d) $\frac{49}{51}$

25. In a transistor circuit, the collector current is 9.8 mA and the base current is 10 mA. The current gain β is
(a) 48 (b) 49
(c) 50 (d) 51

26. In an *n-p-n* transistor circuit, the collector current is 10 mA. If 90% of the electrons emitted reach the collector,
(a) the emitter current will be nearly 9 mA
(b) the emitter current will be nearly 1 mA
(c) the base-current will be nearly 0.9 mA
(d) the base current will be nearly – 1 mA

27. If the input and output resistances in a common-base amplifier circuit are 400 Ω and 400 kΩ respectively, what is the voltage amplification when the emitter current is 2 mA and current gain $\alpha = 0.98$?
(a) 9.8 (b) 98
(c) 980 (d) 9800

28. In Q. 27, the base current is
(a) 0.01 mA (b) 0.02 mA
(c) 0.03 mA (d) 0.04 mA

29. In Q. 27, the collector current is
(a) 0.04 mA (b) 1.96 mA
(c) 2 mA (d) 2.04 mA

30. The box in Fig. 20.36 represents an amplifier with an input resistance $R_i = 100\ \Omega$. It is connected to an ac voltage source through a resistance $R = 300\ \Omega$. The voltage gain of the transistor is 400. If the peak-to-peak voltage of the input ac source is 5.0 V, the peak-to-peak voltage of the output will be

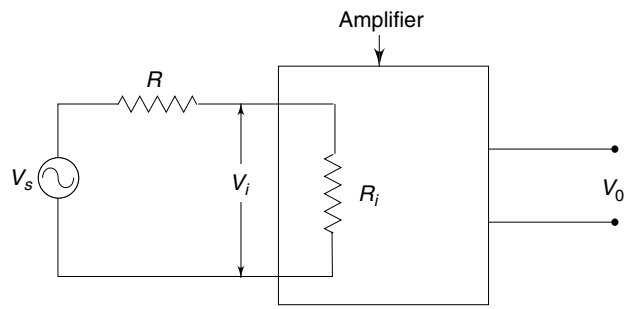


Fig. 20.36

(a) 500 V (b) 400 V
(c) 300 V (d) 200 V

Level B

31. In a transistor amplifier, the two ac current gains α and β as defined as $\alpha = \delta I_C/\delta I_E$ and $\beta = \delta I_C/\delta I_B$. The relation between α and β is
(a) $\beta = \frac{1+\alpha}{\alpha}$ (b) $\beta = \frac{1-\alpha}{\alpha}$
(c) $\beta = \frac{\alpha}{1-\alpha}$ (d) $\beta = \frac{\alpha}{1+\alpha}$

32. In a transistor, a change of 8.0 mA in the emitter current produces a change of 7.8 mA in the collector current. What change in the base current is necessary to produce the same change in the collector current?
    (a) 50 μA (b) 100 μA
    (c) 150 μA (d) 200 μA
33. Which of the following statements is true about a transistor amplifier?
    (a) The emitter-base junction is forward biased and the collector-base junction is reverse biased.
    (b) A common emitter amplifier has a higher current gain than a common base amplifier.
    (c) A common emitter amplifier has a higher voltage gain than a common base amplifier.
    (d) All the above statements are true.
34. In a semiconductor, it is found that $\frac{4}{5}$ th of the total current is carried by electrons and the remaining $\frac{1}{5}$ th by the holes. If at this temperature, the drift speed of electrons is 2.5 times that of holes, the ratio of the number densities of electrons and holes is
    (a) $\frac{8}{5}$ (b) $\frac{5}{8}$
    (c) $\frac{25}{8}$ (d) $\frac{1}{2}$
35. A junction diode has a resistance of 25 Ω when forward biased and 2500 Ω when reverse biased. The current in the arrangement shown in Fig. 20.37 will be
    (a) $\frac{1}{15}$ A (b) $\frac{1}{25}$ A
    (c) $\frac{1}{520}$ A (d) $\frac{1}{480}$ A

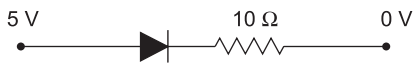


Fig. 20.37

36. For a transistor, current gain $\alpha = 0.96$. It is used as an amplifier in a common base circuit with a load resistance of 4 kΩ. If the dynamic resistance of the emitter-base junction is 48 Ω, the voltage gain is
    (a) 40 (b) 80
    (c) 120 (d) 160
37. At a certain temperature, the number density of charge carriers in a semiconductor is n. When an electric field is applied to it, the charge carriers drift with an average speed v. If the temperature is increased
    (a) both n and v will increase
    (b) both n and v will decrease
    (c) n will increase but v will decrease
    (d) n will decrease but v will increase.
38. Two ideal junction diodes D_1 and D_2 are connected to a battery as shown in Fig. 20.38. The current supplied by the battery is
    (a) 0.25 A (b) 0.5 A
    (c) 0.75 A (d) zero

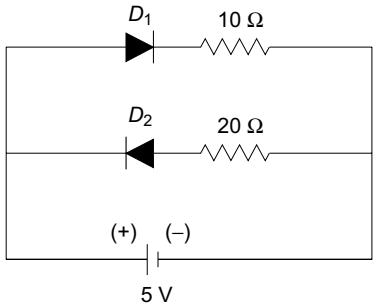


Fig. 20.38

39. In Q. 38 above, what is the current supplied by the battery if its terminals are interchanged?
    (a) 0.25 A (b) 0.5 A
    (c) 0.75 A (d) zero
40. The ideal diodes D_1 and D_2 and two capacitors of capacitances C and $3C$ are connected as shown in Fig. 20.39. A 12 V battery is connected to the input terminals with A at the positive potential and B at a lower potential. The output voltage is

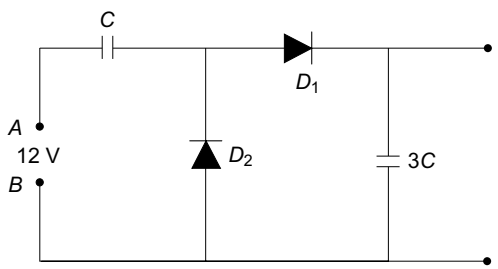


Fig. 20.39

    (a) 3 V (b) 6 V
    (c) 12 V (d) zero
41. The barrier potential in a pn junction is 0.4 V. The current required is 5 mA. What is the emf of a cell which can be used in a circuit if a resistance of 300 Ω is connected in series with the junction
    (a) 2.0 V (b) 1.9 V
    (c) 1.5 V (d) 1.1 V

42. The knee point of the junction diode used in the circuit shown in Fig. 20.40 is 0.5 V. A minimum current of 1.5 mA is required for the diode to be above the knee point. The maximum value of R so that the voltage across the diode is above the knee point is (assume that, above the knee point, the voltage across the diode is independent of the current).
    (a) 3333 Ω (b) 3000 Ω
    (c) 3667 Ω (d) 7500 Ω

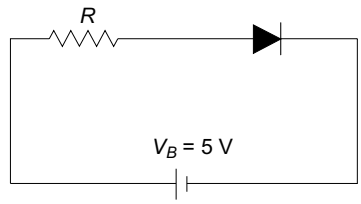


Fig. 20.40

43. In Q. 42 above, if R = 1.5 kΩ, the minimum voltage V_B of the battery so that the diode operates above its knee point is
    (a) 1.0 V (b) 1.5 V
    (c) 2.0 V (d) 2.5 V
44. In Q. 42 above, what should be the value of R so that the current in the circuit is 9 mA?
    (a) 300 Ω (b) 400 Ω
    (c) 500 Ω (d) 600 Ω
45. In Q. 42 above, if the current in the circuit is 5 mA, what is the power dissipated in resistor R?
    (a) 22.5 mW (b) 25 mW
    (c) 27.5 mW (d) 30 mW
46. In Q. 42 above, if the current in the circuit is 5 mA, what is the power dissipated in the diode?
    (a) 2.0 mW (b) 2.5 mW
    (c) 3.0 mW (d) 5.0 mW
47. The electrical conductivity of a semiconductor increases if electromagnetic radiation of wavelength shorter than 2066 nm is incident on it. The band gap energy (in eV) for the semiconductor is very nearly equal to
    (a) 0.5 (b) 0.6
    (c) 0.8 (d) 1.2
48. Two identical p-n junctions may be connected in series with a battery in three ways (Fig. 20.41). The potential drops across the two p-n junctions are equal in
    (a) circuit 1 and circuit 2
    (b) circuit 2 and circuit 3
    (c) circuit 3 and circuit 1
    (d) circuit 1 only

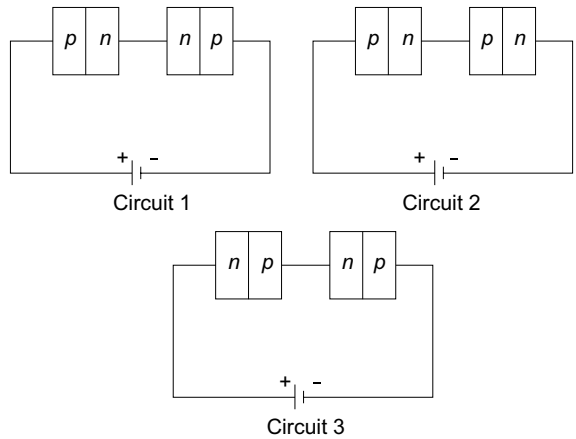


Fig. 20.41

49. A sample of n-type silicon
    (a) contains an excess of free electrons and, therefore, it is negatively charged
    (b) contains an excess of free electrons and it is electrically neutral
    (c) predominantly contains trivalent impurities
    (d) contains only tetravalent impurities.
50. The width of depletion region in a pn junction diode
    (a) increases when a reverse bias is applied
    (b) increases when a forward bias is applied
    (c) decreases when a reverse bias is applied
    (d) remains the same, irrespective of the bias voltage.
51. A full wave rectifier circuit along with the output is shown in Fig. 20.42. The contribution (s) from the diode is (are)
    (a) C (b) A, C
    (c) B, D (d) A, B, C, D

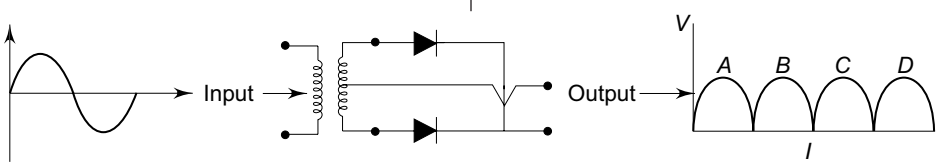


Fig. 20.42

52. The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 nm in incident on it. The band gap (in eV) for the semiconductor is
    (a) 0.9 (b) 0.7
    (c) 0.5 (d) 1.1
53. The dominant mechanisms for motion of charge carriers in forward and reverse biased silicon *p*-*n* junctions are
    (a) drift in forward bias, diffusion in reverse bias
    (b) diffusion in forward bias, drift in reverse bias
    (c) diffusion in both forward and reverse bias
    (d) drift in both forward and reverse bias.
54. The circuit shown in Fig. 20.43 contains two diodes D_1 and D_2 each with a forward resistance of 50 ohms and with infinite backward resistance. If the battery voltage is 6 V, the current through the 100 ohm resistance (in amperes) is
    (a) zero (b) 0.02
    (c) 0.03 (d) 0.036

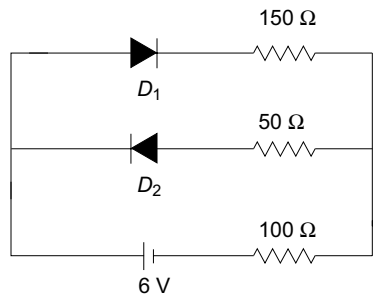


Fig. 20.43

55. Which of the following statements is not true?
    (a) The resistance of intrinsic semiconductors decreases with increase of temperature.
    (b) Doping pure Si with trivalent impurities gives *p*-type semiconductors.
    (c) The majority carriers in *n*-type semiconductors are holes.
    (d) A *p*-*n* junction can act as a semiconductor diode.
56. In a *pn* junction diode not connected to any circuit,
    (a) the potential is the same everywhere
    (b) the *p*-type side is at a higher potential than the *n*-type side
    (c) there is an electric field at the junction directed from the *n*-type side to the *p*-type side
    (d) there is an electric field at the junction directed from the *p*-type side to the *n*-type side.
57. The current gain (β) of a transistor in common emitter mode is 40. To change the collector current by 160 mA at constant V_{CE}, the necessary change in the base current is
    (a) 0.25 μA (b) 4 μA
    (c) 4 mA (d) 40 mA
58. When the voltage between collector and emitter in a transitor is kept constant, a change of 8.3 mA in the emitter current changes the collector current by 8.2 mA. The value of the forward current ratio is
    (a) 82 (b) 83
    (c) 8.2 (d) 8.3
59. Figure 20.44 shows two voltage signals *A* and *B*. Which of the following statements is true?
    (a) (*A*) and (*B*) are both analogue signals.
    (b) (*A*) and (*B*) are both binary digital signals.
    (c) (*A*) is an analogue signal and (*B*) a binary digital signal.
    (d) (*A*) is a binary digital signal and (*B*) an analogue signal.

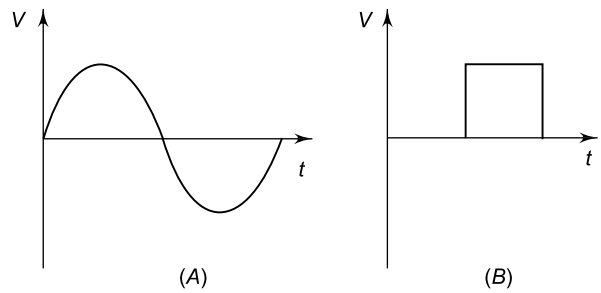


Fig. 20.44

60. The following table

A	*B*	*X*
0	0	1
1	0	1
0	1	1
1	1	0

    is a truth table for
    (a) NAND gate (b) NOR gate
    (c) XOR gate (d) AND gate
61. The logic symbol of a gate is shown in Fig. 20.45. Its truth table is

A	*B*	*X*
0	0	1
1	0	0
0	1	0
1	1	0

(a)

A	*B*	*X*
0	0	0
1	0	1
0	1	0
1	1	0

(b)

A	B	X
0	0	0
1	0	0
0	1	1
1	1	0

(c)

A	B	X
0	0	0
1	0	0
0	1	0
1	1	1

(d)

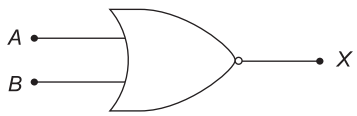


Fig. 20.45

62. What is the Boolean expression for the gate circuit shown in Fig. 20.46.
    (a) $\overline{A \cdot B} = X$ (b) $A + B = X$
    (c) $A \cdot B = X$ (d) $\overline{A + B} = X$

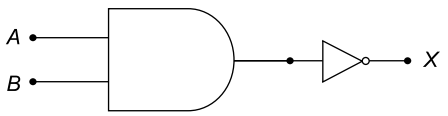


Fig. 20.46

63. What is the output X of the gate circuit shown in Fig. 20.47.
    (a) $A + B$ (b) $\overline{A} + \overline{B}$
    (c) $\overline{A + B}$ (d) $\overline{A \cdot B}$

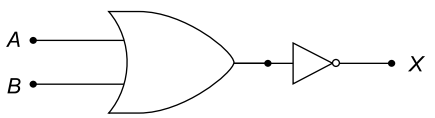


Fig. 20.47

64. In Q. 63, the name of the gate obtained is
    (a) XOR (b) NOR
    (c) NAND (d) NOT
65. Which of the following gates serves as the building blocks is digital circuits?
    (a) OR and AND gates
    (b) AND and NOT gates
    (c) OR and NOT gates
    (d) NAND and NOR gates
66. What is the name of the gate shown in Fig. 20.48.
    (a) NAND gate (b) OR gate
    (c) NOR gate (d) NOT gate

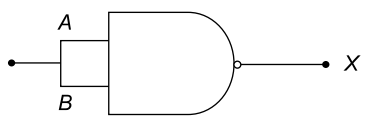


Fig. 20.48

67. The combination of gates shown in Fig. 20.49 yields
    (a) AND gate (b) NAND gate
    (c) NOR gate (d) NOT gate

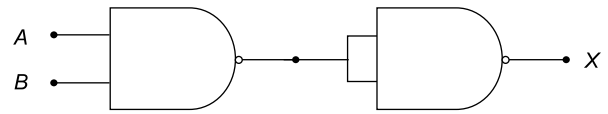


Fig. 20.49

68. The combination of gates shown in Fig. 20.50 yields
    (a) NAND gate (b) OR gate
    (c) NOT gate (d) XOR gate

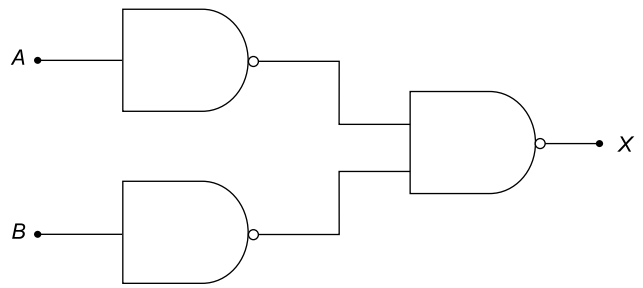


Fig. 20.50

69. The truth tables of two logic gates P and Q are as follows.

Logic gate P

A	B	X
0	0	0
1	0	0
0	1	0
1	1	1

Logic gate Q

A	X
0	1
1	0

When the output of gate P is connected to the input of gate Q, we get a new gate called
    (a) OR gate (b) AND gate
    (c) NAND gate (d) NOR gate
70. In Q. 69 above, the truth table of the new gate is

A	B	X
0	0	1
1	0	0
0	1	0
1	1	0

(a)

A	B	X
0	0	0
1	0	1
0	1	0
1	1	0

(b)

A	*B*	*X*
0	0	0
1	0	0
0	1	1
1	1	0

(c)

A	*B*	*X*
0	0	1
1	0	1
0	1	1
1	1	0

(d)

71. A piece of copper and another of silicon are heated from 80 K to room temperature. The electrical conductivity of
    (a) each increases
    (b) each decreases
    (c) copper increases and that of silicon decreases
    (d) copper decreases and that of silicon increases
72. When a *p-n* junction made from silicon or germanium is forward biased, energy is released at the junction due to recombination of electrons and holes. This energy lies in the
    (a) visible region (b) infra-red region
    (c) ultraviolet region (d) gamma ray region
73. What is the answer to Q. 72 above if the *p-n* junction is made from gallium arsenide or indium phosphide?
74. An *n*-type semiconductor has an impurity level of 25 meV below the conduction band. If an amount of energy equal to kT is given to the extra electron of the impurity ion, it just manages to jump to the conduction band. Given $k = 1.38 \times 10^{-23}$ JK^{-1}. The value of temperature T is
    (a) 17° C (b) 37° C
    (c) 273° C (d) 300 K
75. A pure semiconductor has equal hole and electron concentrations of 2×10^8 m^{-3}. On doping with a pentavalent impurity, the electron concentration increases to 4×10^{12} m^{-3}. On doping, the hole concentration will
    (a) remain equal to 2×10^8 m^{-3}
    (b) decrease to 10^4 m^{-3}
    (c) decrease to 2×10^4 m^{-3}
    (d) increase to 4×10^{12} m^{-3}
76. A *pn* photodiode is made from a semiconductor with band gap energy of 2.0 eV. Which of the following radiation is it unable to detect ?
    (a) Infrared radiation of wavelength 10^{-5} m
    (b) Visible radiation of wavelength 500 nm
    (c) Ultraviolet rays of wavelength 100 nm
    (d) X-rays of wavelength 1Å.
77. A capacitor is used in a half-wave rectifier circuit to reduce the ripple voltage in the output d.c. voltage. In which of the circuits shown in Fig. 20.51 is the capacitor correctly connected ?

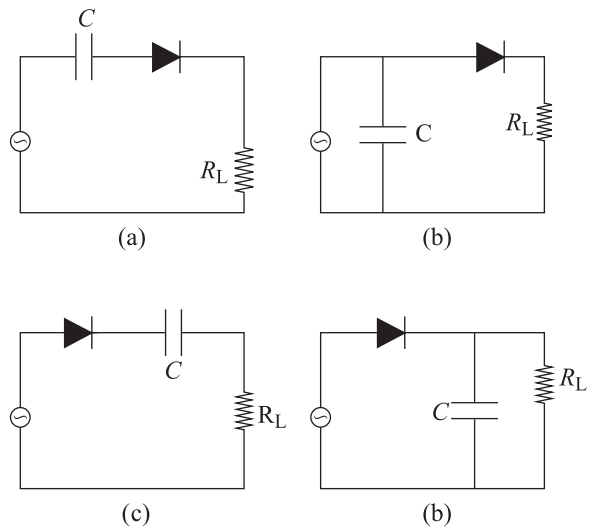


Fig. 20.51

78. An alternating voltage $V = 200 \sin(\omega t)$ is connected across the primary of a step-down transformer of turns ratio 1:20. The output of the transformer is rectified as connected in the half-wave rectifier circuit as shown in Fig. 20.52. If $C = 50$ μF, the final charge on the capacitor plates is
    (a) 20 μC (b) 50 μC
    (c) $50\sqrt{2}\,\mu$C (d) 100 μC

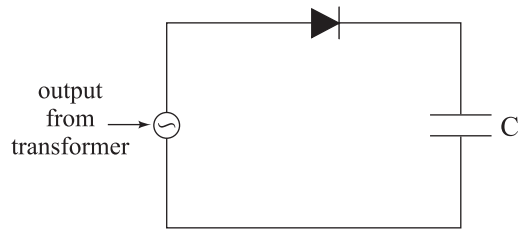


Fig. 20.52

79. The circuit shown in Fig. 20.53 consists of four diodes D_1, D_2, D_3 and D_4. An alternating input voltage is applied across B and D and the output voltage is taken across a load resistor R_L connected between A and C. The output voltage is
    (a) half wave rectified
    (b) full wave rectified
    (c) a d.c. voltage without ripple voltage
    (d) zero

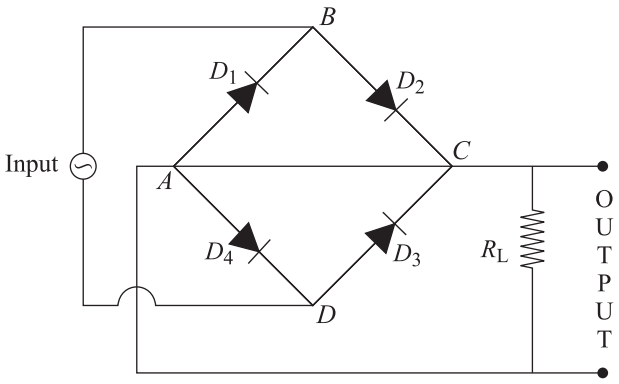


Fig. 20.53

80. In order to obtain the final output $X = 1$ from the combination of gates P and Q. shown in Fig. 20.54, the inputs A, B and C must be
    (a) $A = 1, B = 0, C = 0$
    (b) $A = 0, B = 1, C = 0$
    (c) $A = 1, B = 1, C = 0$
    (b) $A = 1, B = 0, C = 1$

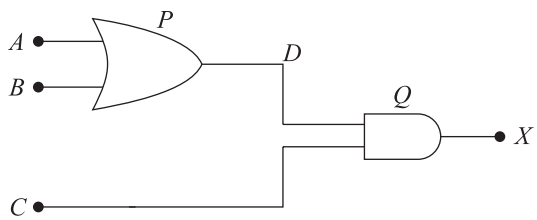


Fig. 20.54

81. In order to obtain the final output $X = 1$ from the combination of gates P and Q shown in Fig 20.55, the inputs A, B and C must be
    (a) $A = 0, B = 1, C = 1$ (b) $A = 1, B = 1, C = 0$
    (c) $A = 0, B = 0, C = 0$ (b) $A = 1, B = 0, C = 0$

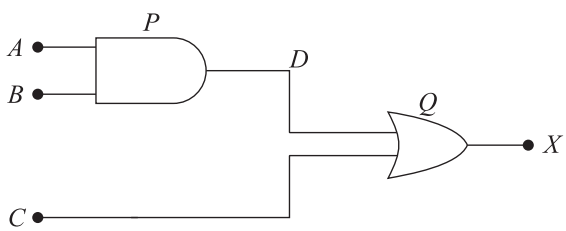


Fig. 20.55

82. A junction diode and resistor are connected together between terminals P and Q in a box. This combination is connected to a battery of voltage 2V and a milliammeter as shown in Fig. 20.56. The current in the circuit is 40 mA. When the terminals of the battery are interchanged, the current drops to 20 mA. The resistances of the diode and resistor are
    (a) 100 Ω , 100 Ω (b) 50 Ω, 100 Ω
    (c) 50 Ω, 50 Ω (d) 50 Ω, 150 Ω

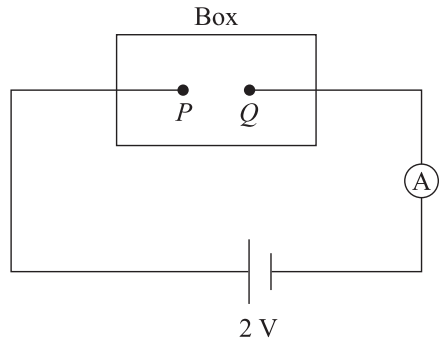


Fig. 20.56

Answers

Level A

1. (a)	2. (b)	3. (c)	4. (d)
5. (c)	6. (d)	7. (d)	8. (d)
9. (d)	10. (a)	11. (d)	12. (d)
13. (c)	14. (d)	15. (b)	16. (b)
17. (a)	18. (d)	19. (a)	20. (d)
21. (a)	22. (a)	23. (c)	24. (c)
25. (b)	26. (b)	27. (c)	28. (d)
29. (b)	30. (a)		

Level B

31. (c)	32. (d)	33. (d)	34. (a)
35. (b)	36. (b)	37. (c)	38. (b)
39. (a)	40. (a)	41. (b)	42. (b)
43. (b)	44. (c)	45. (a)	46. (b)
47. (b)	48. (b)	49. (b)	50. (a)
51. (c)	52. (c)	53. (b)	54. (b)
55. (c)	56. (c)	57. (c)	58. (a)
59. (c)	60. (a)	61. (a)	62. (a)
63. (b)	64. (b)	65. (d)	66. (d)
67. (a)	68. (b)	69. (c)	70. (d)
71. (d)	72. (b)	73. (a)	74. (a)
75. (b)	76. (a)	77. (d)	78. (b)
79. (b)	80. (d)	81. (a)	82. (a)

Solutions

Level A

1. At absolute zero of temperature, the conduction band of semiconductors, is totally empty and all the energy states in the valence band are filled. Due to the absence of electrons in the conduction band near absolute zero, no current can flow under the influence of an electric field. Therefore, semiconductors become insulators at low temperatures. Hence the correct choice is (a).

2. The correct choice is (b).

3. Given $n_i = 1.5 \times 10^{16}$ m^{-3} and $n_h = 4.5 \times 10^{22}$ m^{-3}. Now, we know that

$$n_e\, n_h = n_i^2$$

or $$n_e = \frac{n_i^2}{n_h} = \frac{\left(1.5\times10^{16}\right)^2}{4.5\times10^{22}} = 5.0\times10^{9}\ \text{m}^{-3}$$

4. Given $\mu_e = 0.14$ m^2 V^{-1} s^{-1}, $n_e = 1.5 \times 10^{16}$ m^{-3}, l = 10 cm = 0.1 m, $A = 1.0 \times 10^{-4}$ m^2 and $V = 2$ volts. The electric field in the block is

$$E = \frac{V}{l} = \frac{2}{0.1} = 20\ \text{Vm}^{-1}$$

The drift speed of electrons is

$$v_e = \mu_e E = 0.14 \times 20 = 2.8\ \text{ms}^{-1}$$

$\therefore$ Electron current I_e is

$= n_e A e v_e$

$= (1.5 \times 10^{16}) \times (1.0 \times 10^{-4}) \times (1.6 \times 10^{-19}) \times 2.8$

$= 6.72 \times 10^{-7}$ A

5. In a pure semiconductor $n_e = n_h = 1.5 \times 10^{16}$ m^{-3}. Given $\mu_h = 0.05$ m^2 V^{-1} s^{-1}. The drift speed of holes is

$$v_h = \mu_h E = 0.05 \times 20 = 1.0\ \text{ms}^{-1}$$

$\therefore$ Hole current $I_h = n_h A e v_h$

$= (1.5 \times 10^{16}) \times (1.0 \times 10^{-4}) \times (1.6 \times 10^{-19}) \times 1.0$

$= 2.4 \times 10^{-7}$ A

6. Total current $I = I_e + I_h = 6.72 \times 10^{-7} + 2.4 \times 10^{-7} = 9.12 \times 10^{-7}$ A.

7. Given $\sigma = 6.4\ \Omega^{-1}\ \text{cm}^{-1} = 6.4 \times 10^2\ \Omega^{-1}\ \text{m}^{-1}$ and $\mu_e = 4 \times 10^3$ cm^2 V^{-1} $\sigma^{-1} = 4 \times 10^{-1}$ m^2 V^{-1} σ^{-1}

Neglecting the contribution of holes, the expression for σ is

$$\sigma = e\, n_e\, \mu_e$$

or $$n_e = \frac{\sigma}{e\mu_e} = \frac{6.4\times10^2}{\left(1.6\times10^{-19}\right)\times\left(4\times10^{-1}\right)}$$

$$= 10^{22}\ \text{m}^{-3}$$

8. Choice (a) is incorrect. The *p*-section is at –15 V and *n*-section at –10 V. Since the *p*-section is at a lower potential, the diode is reverse biased. Choice (b) is incorrect. The *p*-section is at a lower potential, therefore, the diode is reverse biased. Choice (c) is also incorrect. The *p*-section is at –10 V and the *n*-section at zero volts (earthed). Therefore, the diode is reversed biased. Choice (d) is correct. The *p*-section is at a higher potential than the *n*-section.

9. The correct choice is (d).

10. For each complete cycle of input ac, we get one pulse in the output (that lasts for the positive half cycle of the input). Since there is one pulse per input cycle, the pulse frequency is also 50 Hz. Hence the correct choice is (a).

11. We know that the resistance of a good conductor decreases with decrease in temperature. On the other hand the resistance of a semi-conductor increases as temperature decreases. Since copper is a good conductor, its resistance decreases as temperature decreases from room temperature to 80 K. On the other hand, germanium, being a semi-conductor, its resistance increases as temperature decreases. Hence the correct choice is (d).

12. The given diode is in the reverse biased condition (*n* side connected to the positive terminal of the battery) and for such a reverse biased diode

(i) the depletion layer widens with an increase in the reverse bias

(ii) the slight current in the circuit is due to the minority charge carriers-holes on the *n* side and electrons on the *p*-side.

(iii) the sudden increase in the current at a particular reverse bias is associated with the zener breakdown. This occurs at the zener voltage characteristic of the diode concerned. Hence the incorrect choice is (d).

13. The correct choice is (c).

14. The output of a full-wave rectifier is of the form

$$I = i_0\left(1 - \cos 2\omega t - \frac{2}{15}\cos 4\omega t - \ldots\right)$$

where $i_0 = \dfrac{2I_o}{\pi}$. The angular frequencies of the harmonic components of the ripple are 2ω, 4ω, … etc. Thus the fundamental frequency of the ripple is twice the frequency of the mains. Hence the correct choice is (d).

15. The correct choice is (b).

16. For a transistor : $I_E = I_B + I_C$

We are given that I_c = 80% of I_E = 0.8 I_E. But I_c = 10 mA.

Therefore

$$I_E = \frac{I_c}{0.8} = \frac{10\text{ mA}}{0.8} = 12.5\text{ mA}$$

Also $I_B = I_E - I_C = 12.5 - 10 = 2.5$ mA

Hence the correct choices is (b).

17. The correct choice is (a).

18. The correct choice is (d).

19. The correct choice is (a).

20. The correct choice is (d).

21. The correct choice is (a) because then the output power is much greater than the input power.

22. $I_B << I_C$. Therefore, current gain $\beta = \dfrac{I_C}{I_B}$ is greater than 1. Hence the correct choice is (a).

23. The correct choice is (c).

24. Current gain $\beta = \dfrac{I_C}{I_B} = \dfrac{50}{1} = 50$. Therefore, current gain α is

$$\alpha = \frac{\beta}{1+\beta} = \frac{50}{1+50} = \frac{50}{51}$$

Hence the correct choice is (c).

25. Current gain $\alpha = \dfrac{I_C}{I_B} = \dfrac{9.8}{10} = 0.98$. Therefore, current gain β is

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1-0.98} = 49$$

Hence the correct choice is (b).

26. For a transistor, $I_E = I_B + I_C$

We are given that $I_C = \dfrac{90}{100}\ I_E = 0.9\ I_E$

Since I_C = 10 mA, we have

$$I_E = 10 \times \frac{10}{9}\text{ mA} = 11.1\text{ mA} \approx 11\text{ mA}$$

Also $I_B = I_E - I_C$

$= (11 - 10)$ mA ≈ 1 mA

Hence the correct choice is (b).

27. Emitter current is 2 mA, input resistance is 400 Ω, output resistance is 400 kΩ = 400×10^3 Ω, input voltage is $2 \times 400 = 800$ mV = 0.800 V and collector current $I_C = 0.98 \times 2 = 1.96$ mA. Output or collector voltage is

$$(1.96 \times 10^{-3}) \times (400 \times 10^{+3}) = 784\text{ V}$$

Voltage amplification is

$$\frac{784}{0.800} = 980$$

28. Base current is

$$I_B = I_E(1 - a) = 2 \times 0.02\text{ mA} = 0.04\text{ mA}$$

29. Collector current is

$$I_E - I_B = 2 - 0.04 = 1.96\text{ mA}$$

30. The voltage gain is V_0/V_i, i.e. $V_0 = V_i \times$ voltage gain. If V_s is the voltage of the source, the voltage across the input of the amplifier is

$$V_i = \frac{V_s \times R_i}{R_i + R} = \frac{5.0 \times 100}{100 + 300} = \frac{5}{4}\text{ V}$$

Output voltage is

$$V_0 = V_i \times \text{voltage gain} = \frac{5}{4} \times 400 = 500\text{ V}$$

Level B

31. We know that $I_E = I_C + I_B$ or $\delta I_E = \delta I_C + \delta I_B$. Dividing both sides by δI_C, we get

$$\frac{\delta I_E}{\delta I_C} = 1 + \frac{\delta I_B}{\delta I_C}$$

or $$\frac{1}{\alpha} = 1 + \frac{1}{\beta}$$

or $$\beta = \frac{\alpha}{1-\alpha}$$

32. $\delta I_E = 8.0 \times 10^{-3}$ A and $\delta I_C = 7.8 \times 10^{-3}$ A. Now

$$\alpha = \frac{\delta I_C}{\delta I_E} = \frac{7.8 \times 10^{-3}}{8.0 \times 10^{-3}} = 0.975$$

$$\therefore \qquad \beta = \frac{\alpha}{1-\alpha} = \frac{0.975}{1-0.975} = 39$$

But $\beta = \dfrac{\delta I_C}{\delta I_B}$. Therefore,

$$\delta I_B = \frac{\delta I_C}{\beta} = \frac{7.8\times10^{-3}}{39}$$

$$= 2\times10^{-4}\text{ A} = 200\ \mu\text{A}$$

33. The correct choice is (d).

34. $n_e\, ev_e + n_h\, ev_h = I$. Given

$$\frac{n_e\, ev_e}{n_h\, ev_h} = \frac{4I/5}{I/5} = 4$$

or $$\frac{n_e\, v_e}{n_h\, v_h} = 4 \text{ or } \frac{n_e}{n_h} = 4\,\frac{v_h}{v_e} = \frac{4}{2.5} = \frac{8}{5}$$

Hence the correct choice is (a).

35. The junction diode is forward biased. Therefore, the effective resistance = 100 + 25 = 125 Ω. The current is

$$I = \frac{5\text{ V}}{125\ \Omega} = \frac{1}{25}\text{ A}$$

Hence the correct choice is (b).

36. Voltage gain = $\alpha \times \dfrac{R_L}{R_D} = 0.96 \times \dfrac{4000}{48} = 80$. Hence the current choice is (b).

37. The correct choice is (c).

38. Diode D_1 is forward biased. Therefore, effective resistance is 10 Ω. Hence the current is

$$I = \frac{5\text{ V}}{10\ \Omega} = 0.5\text{ A}$$

Thus the correct choice is (b).

39. In this case, diode D_2 is forward biased. Hence

$$I = \frac{5\text{V}}{20\ \Omega} = 0.25\text{ A}$$

Thus the correct choice is (a).

40. Diode D_1 is forward biased. Therefore, capacitors C and $3C$ are in series. If V_1 and V_2 are potential drops across C and $3C$ respectively, then

$$\frac{V_1}{V_2} = 3 \text{ or } V_1 = 3V_2$$

Given $V_1 + V_2 = 12$. Thus, $3V_2 + V_2 = 12$ or $V_2 = 3$V which is the output voltage. Hence the correct choice is (a).

41. Voltage drop across the resistance = $300 \times 5 \times 10^{-3}$ = 1.5 V

∴ Emf of cell = 1.5 + 0.4 = 1.9 V, which is choice (b).

42. The maximum voltage drop across R must be 5 – 0.5 = 4.5 V. Hence maximum value of R is

$$R_{\max} = \frac{4.5\text{ V}}{1.5\text{ mA}} = \frac{4.5\text{ V}}{1.5\times10^{-3}\text{ A}} = 3000\ \Omega$$

Thus the correct choice is (b).

43. Voltage drop across resistor = IR = 1.5 mA × 1.5 kΩ = 1.0 V. Therefore, minimum V_B = 1.0 + 0.5 = 1.5 V which is choice (b).

44. $R = \dfrac{V}{I} = \dfrac{4.5\text{ V}}{9\text{ mA}} = \dfrac{4.5\text{ V}}{9\times10^{-3}\text{ A}} = 500\ \Omega$

Hence the correct choice is (c).

45. $P = V_R\, I = 4.5\text{ V} \times 5\times10^{-3}\text{ A} = 22.5\times10^{-3}\text{ W} = 22.5$ mW.

Hence the correct choice is (a).

46. Power dissipated in diode = Voltage across diode × current

$$= 0.5\text{ V} \times 5\text{ mA} = 2.5\text{ mW}$$

Hence the correct choice is (b).

47. Given $\lambda_{\min}$ = 2066 nm. From Duane-Hunt law,

$$\text{Energy in eV} = \frac{hc\times10^9}{e\lambda_{\min}\,(\text{in nm})}$$

$$= \frac{6.62\times10^{-34}\times3\times10^8}{1.6\times10^{-19}\times2066\times10^{-9}}$$

$$\simeq 0.6\text{ eV, which is choice (b).}$$

48. The potential drops are equal when the resistances offered by the two *p-n* junctions are equal. In circuit 1 the first junction diode is forward biased having low resistance whereas the second junction diode is reversed biased having a very high resistance. Hence V_1, the potential drop across the first *p-n* junction is very low as compared to the second *p-n* junction.

In circuit 2 both junctions are forward biased having equal low resistances whereas in circuit 3 both junctions are reverse biased having equal high resistances. Hence the correct choice is (b).

49. The correct choice is (b).

50. The correct choice is (a).

51. The correct choice is (c).

52. Wavelength of incident radiation is $\lambda = 2480$ nm $= 2480 \times 10^{-9}$ m. Energy of incident radiation is

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{2480 \times 10^{-9}}$$

$$= 8.0 \times 10^{-20} \text{ J}$$

$$= \frac{8.0 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV} = 0.5 \text{ eV}$$

53. The correct choice is (b).

54. In the given circuit only diode D_1 will allow the current to pass through as it is forward-biased. Hence, the current through the 100 ohm resistance is

($\because$ resistance of $D_1 = 50\ \Omega$)

$$I = \frac{6\text{V}}{(150 + 50 + 100)\Omega} = \frac{6}{300}\text{A}$$

$$= 0.02 \text{ A}$$

55. The correct choice is (c). The majority carriers in *n*-type semiconductors are electrons.

56. A *pn* junction diode is formed by joining a *p*-type semiconductor to an *n*-type semiconductor. Separately, the two semiconductors are electrically neutral. When they are joined, some electrons near the junction diffuse from the *n*-type into the *p*-type semiconductor, where they fill a few of the holes. Consequently, the *n*-type is left with a positive charge and the *p*-type acquires a net negative charge. Therefore, a potential difference is established, with the *n*-type at a higher potential than the *p*-type. Hence an electric field is set up at the junction and it is directed from the *n*-type side to the *p*-type side. Thus the correct choice is (c).

57. By definition, $\beta = \left(\frac{V I_C}{\Delta I_B}\right)$ at constant V_{CE}. Therefore,

$$\Delta I_B = \frac{\Delta I_C}{\beta} = \frac{160\,\text{mA}}{40}$$

$$= 4 \text{ mA, which is choice (c).}$$

58. Forward current ratio

$$= \frac{\Delta I_C}{\Delta I_C - \Delta I_C} = \frac{8.2\text{ mA}}{8.3\text{ mA} - 8.2\text{ mA}} = 82$$

Hence the correct choice is (a).

59. A continuous time varying signal is called an analogue signal. A discrete signal with two discrete values is a binary digital signal. Hence the correct choice is (c).

60. The correct choice is (a).

61. It is a NOR gate whose truth table is (a).

62. The circuit is a combination of AND and NOT gates. The output of the NAND gate is connected to the input of a NOT gate. The combined circuit is called a NAND gate. The Boolean expression for the NAND operation is

$$\overline{A \cdot B} = X$$

which means that the output X is just the negation of the output of the AND operation. Hence the correct choice is (a).

63. This circuit is a combination of an OR gate and a NOT gate which gives a NOR gate. The output of an OR gate is connected to the input of a NOT gate. The Boolean expression for a NOR operation is

$$X = \overline{A + B}$$

Hence the correct choice is (b).

64. The correct choice is (b).

65. The OR or the AND or the NOT gates alone cannot give a different gate by repeated use, i.e. two or more OR or AND or NOT gates do not yield a new gate. But the repeated use of NAND (or NOR) gates can give all other gates like OR, AND and NOT. Hence the NAND (or the NOR) gate serves as a building block in digital circuits.

66. The two inputs are joined together to get one input. Since $A = B$, there is only one input and one output as in a NOT gate. This is how we get a NOT gate from a NAND gate. Hence the correct choice is (d).

67. Using a NOT gate after a NAND gate yields an AND gate. Hence the correct choice is (a).

68. If the inputs A and B are inverted and then applied to a NAND gate, the output of the NAND gate will be the same as that of an OR gate. The input of the last NAND gate is $\overline{A \cdot A}$ and $\overline{B \cdot B}$.

Hence the correct choice is (b).

69. Gate P is an AND gate and gate Q is a NOT gate. Hence the new gate formed is a NAND gate. So the correct choice is (c).

70. The correct choice is (d)

71. The resistivity of a metallic conductor (such as copper) increases with in increase in temperature but that of a semiconductor (such as silicon) decreases with increase in temperature. Since electrical conductivity is equal to the reciprocal of resistivity, the correct choice is (d).

72. The correct choice is (b)

73. In this case the energy released lies in the visible region. So the correct choice is (a).

74. $E = 25$ m eV $= 25 \times 10^{-3} \times (1.6 \times 10^{-19})$ J

Now $E = kT$. Therefore

$$T = \frac{E}{k} = \frac{25 \times 10^{-3} \times \left(1.6 \times 10^{-19}\right)}{1.38 \times 10^{-23}}$$

$$= 290 \text{ K}$$

$$= 17°\text{C}$$

So the correct choice is (a)

75. Refer to Example 17 on page 20.7. The correct choice is (b).

76. For $\lambda = 10^{-5}$ m, the energy of the incident photon is

$$E = h\nu = \frac{hc}{\lambda}$$

$$= \frac{6.63 \times 10^{-34} \times \left(3 \times 10^{8}\right)}{10^{-5}} \text{ J}$$

$$= \frac{6.63 \times 10^{-34} \times \left(3 \times 10^{8}\right)}{10^{-5} \times \left(1.6 \times 10^{-19}\right)} \text{ eV}$$

$$\simeq 0.12 \text{ eV}$$

Since the energy of the photon is less than the band gap energy (which 2.0 eV), this photo diode cannot detect radiation of $\lambda = 10^{-5}$ m. It is easy to check that the energy of the photon in choices (b), (c) and (d) is greater than 2.0 eV. Hence the correct choice is (a).

77. The correct circuit is (d).

78. Peak value of output voltage from the transformer is

$$V_0 = \frac{200}{20} = 10 \text{ V}$$

The junction diode conducts during the half cycle of the input when it is forward biased. During this half cycle, the capacitor is charged to the peak value of the supply voltage (which is 10 V). Hence the final charge on capacitor plates is

$$Q = CV_0 = (5 \times 10^{-6}) \times 10 = 50 \text{ μC}$$

So the correct choice is (b).

79. The given circuit is a full wave rectifier called a bridge rectifier. During the positive half cycle of the input, diodes D_2 and D_4 are forward biased and hence they conduct, but diodes D_1 and D_3 are reverse biased and hence they do not conduct. During the next (negative) half cycle, D_1 and D_3 conduct but D_2 and D_4 do not. Since at a time two diodes conduct, the output voltage is higher in the case of a bridge rectifier than a full wave rectifier in which only two diodes are used instead of four. So a bridge rectifier is always preferred as a full wave rectifier. Thus the correct choice is (b).

80. Gate P is OR gate and gate Q is AND gate. The output of gate. Q is $X = 1$ only if $D = 1$ and $C = 1$. If $A = 1$ and $B = 0$, then $D = 1$. So the only correct choice is (d).

81. The output X of gate Q (which is OR gate) is 1 if $C = 1$ and A and B can be 0 or 1. So the only correct choice is (a).

82. Since the current decreases on interchanging the battery terminals, it is clear that the diode is forward biased in the first case and reverse biased in the second case. Furthermore, since the current does not drop to zero, it follows that the diode and the resistor are not connected in series because then the current should drop to zero. So they must be connected in parallel as shown in Fig. 20.57 (a) and (b).

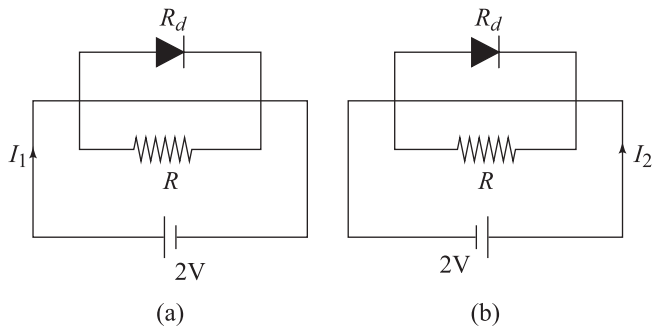


Fig. 20.57

Let R_d be the resistance of the diode. In case (a) the diode is forward biased. Hence the current I_1 divides between R_d and R. The resistance of the parallel combination is

$$R_p = \frac{2 \text{ V}}{40 \times 10^{-3} \text{ A}} = 50 \text{ Ω}$$

In case (b), the diode is reverse biased and hence no current flow through it; all the current I_2 flows through R. The value of R is

$$R = \frac{2 \text{ V}}{20 \times 10^{-3} \text{ A}} = 100 \text{ Ω}$$

Now $$\frac{1}{R_p} = \frac{1}{R} + \frac{1}{R_d}$$

$$\Rightarrow \quad \frac{1}{R_d} = \frac{1}{R_p} - \frac{1}{R} = \frac{1}{50} - \frac{1}{100} \Rightarrow R_d = 100 \text{ Ω}$$

So the correct choice is (a).

SECTION 2

Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage.

Passage I

When two semiconductors of *p* and *n* types are brought into contact by special techniques called fused junction and grown junction, they form what is called a *pn* junction. A *pn* junction enclosed in a casing with leads coming out from the *p* and *n* regions is the simplest semiconductor device called the *pn* junction diode. This device conducts current more readily in one direction than in the other. Hence it can be used as a rectifier to convert an alternating current into a unidirectional current. When an alternating voltage is applied to a diode circuit, the diode will conduct relatively heavily when the polarity of the voltage produces a forward bias, but will allow only a negligible current when the polarity of the voltage reverses to produce reverse bias. As a result the current flows only during one half cycle to produce a *dc* at the output.

1. A semiconducting device is connected in a series circuit with a battery and a resistance. A current is found to pass through the circuit. If the polarity of the battery is reversed, the current drops to almost zero. The device may be
   (a) a *pn* junction
   (b) an intrinsic semiconductor
   (c) a *p*-type semiconductor
   (d) an *n*-type semiconductor.
2. The diode used in the circuit shown in Fig. 20.58 has a constant voltage drop of 0.5 V at all currents and a maximum power rating of 100 milliwatts. What should be the value of the resistor *R,* connected in series with the diode, for obtaining maximum current?

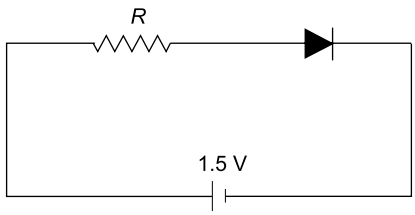


Fig. 20.58

(a) 1.5 Ω (b) 5 Ω
(c) 6.67 Ω (d) 200 Ω

3. In Fig. 20.59 the input is across the terminals *A* and *C* and the output is across *B* and *D*. Then the output is

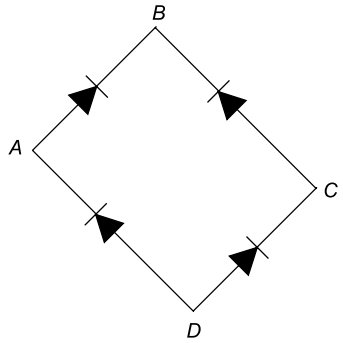


Fig. 20.59

   (a) zero
   (b) the same as the input
   (c) full wave rectified
   (d) half wave rectified
4. In the case of forward biasing of *pn* junction, which one of the following figures correctly depicts the direction of the flow of charge carriers? (Fig. 20.60)

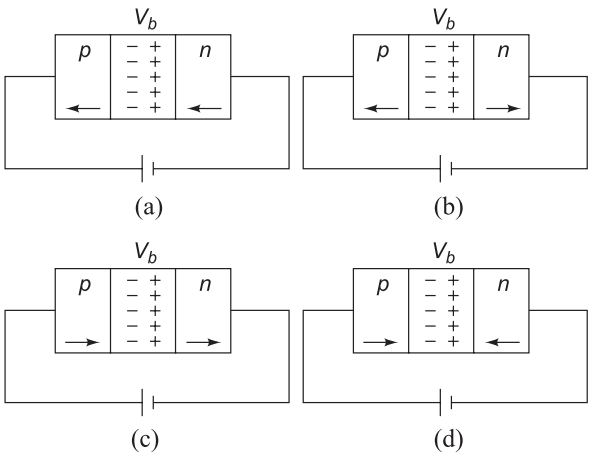


Fig. 20.60

Solutions

1. The device is a *pn* junction. When a current flows across the junction, it is forward biased. On reversing the polarity of the battery, it is reverse biased and the current drops to almost zero.

2. Power P = 100 milliwatts = 10×10^{-3} = 0.01 W and V = 0.5 volt. Hence the current in the circuit is

$$I = \frac{P}{V} = \frac{0.01}{0.5} = 0.2 \text{ A}$$

Since the voltage drop across the diode is 0.5 V, the voltage drop across resistor R = 1.5 – 0.5 = 1.0 V. Therefore, the value of R for maximum current is

$$R = \frac{1.0}{0.2} = 5\ \Omega, \text{ which is choice (b).}$$

3. It is the circuit diagram of a full-wave rectifier.
4. Forward bias is obtained when the negative terminal of the battery is connected to the *n*-side and the positive terminal to the *p*-side of the semiconductor. Then the negative terminal will repel free electrons in the *n*-section towards the junction and the positive terminal on the *p*-side will push the holes towards the junction. Hence the correct choice is (d).

Questions 5 to 9 are based on the following passage.

Passage II

A transistor is composed of three semiconductor elements. The three elements are combined such that the two outer elements are doped with the same kind of majority carriers while the element that separates them has the opposite majority carriers. A transistor is of two types: *npn* and *pnp* type. The three elements are the emitter, the collector and the base. The emitter supplies the majority carriers for transistor current flow, the collector collects the current for circuit operation and the base controls the passage of electrons from the emitter to the collector. Transistors are used to amplify voltages. The output voltage is an enlarged version of the input voltage. The transistor is also used as an oscillator which produces an output of a desired frequency without any external input voltage.

5. In a common emitter amplifier, the output resistance is 5000 Ω and the input resistance is 2000 Ω. If the peak value of the signal voltage is 10 mV and β = 50, then the peak value of the output voltage is
   (a) 5×10^{-6} V (b) 2.5×10^{-4} V
   (c) 1.25 V (d) 125 V
6. The base current of a transistor is 100 μA and the collector current is 2 mA. What is the current gain for the common base configuration?
   (a) 0.90 (b) 0.93
   (c) 0.95 (d) 0.98
7. An oscillator is nothing but an amplifier with
   (a) positive feedback (b) large gain
   (c) no feedback (d) negative feedback
8. When *npn* transistor is used as an amplifier
   (a) electrons move from base to collector
   (b) holes move from emitter to base
   (c) electrons move from collector to base
   (d) holes move from base to emitter
9. The transfer ratio β of a transistor is 50. The input resistance of the transistor when used in the common emitter configuration is 1 kΩ. The peak value of the collector ac current for an ac input voltage of 0.01 V peak is
   (a) 100 μA (b) 0.01 μA3
   (c) 0.25 μA (d) 500 μA.



Solutions

5. R_L = 5000 Ω, R_i = 2000 Ω, β = 50. The ac voltage gain is given by

$$\beta \times \frac{R_L}{R_i} = \frac{50 \times 5000}{2000} = 125$$

$\therefore$ Peak output voltage = voltage gain × signal voltage

$$= 125 \times 10 \text{ mV} = 1250 \text{ mV} = 1.25 \text{ V}$$

Hence the correct choice is (c).

6. Base current $I_B = 100\ \mu\text{A} = 100 \times 10^{-6}$ A and collector current I_C = 2 mA = 2×10^{-3} A. Therefore, current gain β is

$$\beta = \frac{I_C}{I_B} = \frac{2 \times 10^{-3}}{100 \times 10^{-6}} = 20$$

The current gain α is

$$\alpha = \frac{\beta}{\beta + 1} = \frac{20}{20 + 1} = \frac{20}{21} = 0.95$$

Hence the correct choice is (c).

7. In an amplifier, an externally applied *ac* input voltage is amplified at the output. An oscillator produces an output at a desired frequency without any external input voltage. This is achieved by providing a positive feedback from a part of the output of the amplifier in the correct phase to the input circuit of the oscillator. Hence the correct choice is (a).
8. In an *npn* transistor, the majority current carriers are electrons and they move from the base to the collector. Hence the correct choice is (a).
9. Given β = 50, R_i = 1 kΩ = 1000 Ω and V_i = 0.01 V. The base current is

$$I_B = \frac{V_i}{R_i} = \frac{0.01}{1000} = 10^{-5} \text{ A}$$

Now $\beta = \dfrac{I_C}{I_B}$

which gives $I_C = \beta \times I_B = 50 \times 10^{-5}$ A = 500 μA.

So the correct choice is (d).

Questions 10 to 14 at a based on the following passage.

Passage III

A silicon diode is connected to a resistor R and a battery of voltage V_B as shown in Fig. 20.61. The knee point of its I-V characteristics is 0.7 V. The diode requires a minimum current of 1 mA to attain a value higher than the knee point. Assume that the voltage V across the junction is independent of the current above the knee point.

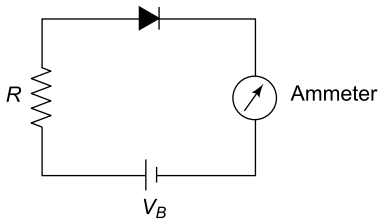


Fig. 20.61

10. If V_B = 5 V, the maximum value of R so that the voltage V is above the knee point voltage is
    (a) 0.7 kΩ (b) 4.3 kΩ
    (c) 5 kΩ (d) 5.7 kΩ
11. If V_B = 5 V, the value of R in order to establish a current of 5 mA in the circuit is
    (a) 350 Ω (b) 860 Ω
    (c) 1000 Ω (d) 1140 Ω
12. If V_B = 6 V, the power dissipated in the resistor R when a current of 5 mA flows in the circuit is
    (a) 25.0 mW (b) 25.5 mW
    (c) 26.0 mW (d) 26.5 mW
13. In Q. 19, the power dissipated in the diode is
    (a) 3.5 mW (b) 3.0 mW
    (c) 2.5 mW (d) zero
14. If R = 1 kΩ, the minimum voltage V_B required to keep the diode above the knee point is
    (a) 1.5 V (b) 1.6 V
    (c) 1.7 V (d) 1.8 V



Solutions

10. Let V_R be the voltage across resistor R and V be the voltage across the diode junction. Then
    $V_B = V_R + V \Rightarrow V_R = V_B - V = 5 - 0.7 = 4.3$ V.
    From Ohm's law $V_R = IR$. Since V_R is fixed, when I is minimum (= I_{min}), R should be maximum (= R_{max}) such that
    $$I_{min} \times R_{max} = 4.3 \text{ V}$$
    Given I_{min} = 1 mA = 1×10^{-3} A. Hence
    $R_{max} = \dfrac{4.3}{1 \times 10^{-3}} = 4.3 \times 10^3$ Ω = 4.3 kΩ, which is choice (b).
11. For I = 5 mA = 5×10^{-3} A, the voltage across R should be
    $V_R = V_B - V = 5 - 0.7 = 4.3$ V
    Therefore $R = \dfrac{V_R}{I} = \dfrac{4.3}{5 \times 10^{-3}} = 860$ Ω, which is choice (b).
12. V_B = 6 V, V = 0.7 V. Therefore, V_R = 6 – 0.7 = 5.3 V. Power dissipated in $R = I \times V_R = (5 \times 10^{-3}) \times 5.3$ = 26.5 mW. So the correct choice is (d).
13. Power dissipated in diode = $I \times V = (5 \times 10^{-3}) \times 0.7$ = 3.5 mW, which is choice (a).
14. To keep the diode above the knee point a minimum current of 1 mA must flow in the circuit. For R = 1 kΩ, the minimum voltage V_B required is
    $V_B = (I_{min} \times R) + V$
    $= (1 \times 10^{-3} \times 1 \times 10^3) + 0.7 = 1.7$ V
    Hence the correct choice is (c).

Questions 15 to 18 are based on the following passage.

Passage IV

The current I through a *pn* junction diode is given by

$$I = I_0 \left[\exp\left(\frac{eV}{kT}\right) - 1 \right]$$

where the reverse saturation current $I_0 = 4 \times 10^{-12}$ A, temperature T= 300 K constant $k = 8.0 \times 10^{-5}$ eV K^{-1} (electron-volt per kelvin) and V is the voltage across the diode.

15. When the diode is forward biased, the forward current at V = 0.48 V is [exp (20) = 4.85×10^8]
    (a) 1.92 mA (b) 19.2 mA
    (c) 192 mA (d) 1.92 A
16. If the voltage across the diode is increased to 0.6 V, the increase in the current is [exp (25) = 7.2×10^{10}]
    (a) 72 mA (b) 144 mA
    (c) 288 mA (d) 576 mA
17. The dynamic resistance of the diode is nearly
    (a) 0.1 Ω (b) 0.2 Ω
    (c) 0.3 Ω (d) 0.4 Ω

18. When the diode is reverse biased, the change in current if the bias voltage changes from 0.96 V to 1.92 V is

(a) 4×10^{-12} A (b) 1.0×10^{-11} A

(c) 4×10^{-10} A (d) zero



Solutions

Given $k = 8.0 \times 10^{-5}$ eV $K^{-1} = (8.0 \times 10^{-5}) \times (1.6 \times 10^{-19})$ J K^{-1}

15. For forward bias V is positive. Thus $V = +0.48$ V. Then

$$\frac{eV}{kT} = \frac{(1.6 \times 10^{-19}) \times 0.48}{(8.0 \times 10^{-5}) \times (1.6 \times 10^{-19}) \times 300} = 20$$

$$\therefore \quad I = I_0 \left[\exp\left(\frac{eV}{kT}\right) - 1 \right]$$

$$= 4 \times 10^{-12} [\exp(20) - 1]$$

$$= 4 \times 10^{-12} [4.8 \times 10^{8} - 1] = 1.92 \times 10^{-3} \text{A}$$

$$= 1.92 \text{ mA}$$

So the correct choice is (a).

16. If $V = 0.6$ V, then $\frac{eV}{kT} = 25$. In this case

$$I = 4 \times 10^{-12} [7.2 \times 10^{10} - 1] = 288 \text{ mA}$$

Thus the correct choice is (c).

17. Change in voltage $\Delta V = 0.6 - 0.48 = 0.12$ V. Change in current $\Delta I = 288 - 1.92$ 286 mA $= 286 \times 10^{-3}$ A.

$\therefore$ Dynamic resistance $R_d = \frac{\Delta V}{\Delta I} = \frac{0.12}{286 \times 10^{-3}} = 0.42\,\Omega$

Hence the correct choice is (d).

18. For reverse bias, V is negative. When $V = -0.96$ V, $\frac{eV}{kT} = -40$. So current is

$I_1 = 4 \times 10^{-12} [\exp(-40) - 1] = -4 \times 10^{-12}$ A

[since exp (– 40) tends to zero]. Similarly, when $V = -1.92$ V, $I_2 = -4 \times 10^{-12}$ A. Thus when the diode is reverse biased, the current through it remains unchanged when the bias voltage is changed, though its direction is opposite to the direction when the diode is forward biased. Hence the correct choice is (d).

SECTION 3

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only *one* choice is correct.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

1. Statement-1

At very low temperatures (near absolute zero) all semiconductors become insulators.

Statement-2

Near absolute zero, the valence band is completely filled and there are no free electrons in the conduction band.

2. Statement-1

The conductivity of semiconductors increases with increase in temperature.

Statement-2

As the temperature increases, the electrons whose kinetic energy is greater than the forbidden gap energy jump from conduction band to valence band.

3. Statement-1

The conductivity of metallic conductors decreases with increase in temperature.

Statement-2

As the temperature increases, the number of collisions between the free electrons and atoms or ions of the metal increases resulting in an increase in resistivity.

4. Statement-1

When a strong current is passed through a semiconductor, it behaves like a metallic conductor.

Statement-2

The heat produced makes a large number of electrons to move into the conduction band.

5. **Statement-1**

   The voltage across a zener diode remains constant.

   Statement-2

   A zener diode is designed to work only in the reverse break down region.

6. **Statement-1**

   When an intrinsic semiconductor is exposed to light (photons) of energy greater than the band gap energy, its conductivity increases.

   Statement-2

   The number density of electrons increases and of holes decreases.

7. **Statement-1**

   The resistance of a *pn* junction diode depends on the current flowing through it.

   Statement-2

   Ohm's law does not hold in the case of semiconductors.

8. **Statement-1**

   In a transistor, the base current is very small compared to the collector current

   Statement-2

   A very large number of electrons recombine with holes and hence do not pass through the base.

9. **Statement-1**

   In a transistor, the depletion region around the emitter-base junction is much smaller than that around the base-collector junction.

   Statement-2

   The emitter-base junction is forward biased and the base-collector junction is reverse biased.

10. **Statement-1**

    In a transistor amplifier, the power dissipated at the base-collector junction is much smaller than that dissipated at the emitter-base junction.

    Statement-2

    The resistance of the base-collector junction is much higher than that of the emitter-base junction.

11. **Statement-1**

    In a transistor amplifier, the output voltage is always out of phase with the input voltage.

    Statement-2

    The emitter-base junction is reverse biased and the base-collector junction is forward biased.

12. **Statement-1**

    A transistor oscillator produces an output voltage of a desired frequency without any external input voltage.

    Statement-2

    A part of the output of the transistor is fed back into the input circuit.



Solutions

1. The correct choice is (a).
2. The correct choice is (c). As the temperature increases, the electrons acquire kinetic energy. Those electrons whose kinetic energy is greater than forbidden gap energy jump over to the conduction band.
3. The correct choice (a).
4. The correct choice is (a).
5. The correct choice is (a).
6. The correct choice is (c).
7. The correct choice is (a).
8. The correct choice is (c). The base current is about 1/100 of collector current. About 99% of electrons pass through the base without recombining with holes.
9. The correct choice is (a).
10. The correct choice is (d). The power dissipated in the base-collector junction is much higher than that dissipated at the emitter-base junction.
11. The correct choice is (c). The base-emitter junction is forward biased and the base-collector junction is reverse biased.
12. The correct choice is (b).

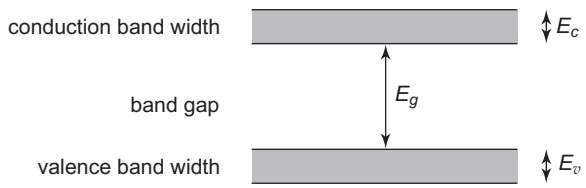
SECTION 4
Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. The energy band gap is maximum in
   (a) metals (b) superconductors
   (c) insulators (d) semiconductors **[2002]**
2. The part of a transistor which is most heavily doped to produce a large number of majority charge carriers is
   (a) emitter
   (c) base
   (c) collector
   (d) any of the above three **[2002]**
3. The difference in the variation of resistance with temperature of a metal and a semiconductor arises essentially due to the difference in the
   (a) crystal structure
   (b) variation of the number of charge carriers with temperature
   (c) type of bonding
   (d) variation of scattering mechanism with temperature. **[2003]**
4. In the middle of the depletion layer of a reverse biased *p–n* junction, the
   (a) electric field is zero
   (b) potential is maximum
   (c) electric field is maximum
   (d) potential is zero **[2003]**
5. When *npn* transistor is used as an amplifier
   (a) electrons move from base to collector
   (b) holes move from emitter to base
   (c) electrons move from collector to base
   (d) holes move from base to emitter. **[2004]**
6. For a transistor amplifier in common emitter configuration, for a load impedance of 1 kΩ, the hybrid parameters are $h_{fe} = 50$ and $h_{oe} = 25\ \mu AV^{-1}$. The current gain is
   (a) – 5.2 (b) – 15.7
   (c) – 24.8 (d) – 48.78 **[2004]**
7. The manifestation of band structure in solids is due to
   (a) Heisenberg's uncertainty principle
   (b) Pauli's exclusion principle
   (c) Bohr's correspondence principle
   (d) Boltzmann's law **[2004]**
8. When a *pn* junction diode is forward biased,
   (a) the depletion region is reduced and barrier height is increased
   (b) the depletion region is widened and barrier height is reduced
   (c) both the depletion region and barrier height are reduced
   (d) both the depletion region and barrier height are increased. **[2004]**
9. The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 nm is incident on it. The band gap energy (in eV) for the semiconductor is
   (a) 1.1 eV (b) 2.5 eV
   (c) 0.5 eV (d) 0.7 eV **[2005]**
10. In a common base amplifier, the phase difference between the input signal and output signal is
    (a) $\frac{\pi}{4}$ (b) π
    (c) zero (d) $\frac{\pi}{2}$ **[2005]**
11. In a full wave rectifier circuit operating on a 50 Hz mains supply, the fundamental frequency of the ripple would be
    (a) 50 Hz (b) 25 Hz
    (c) 100 Hz (d) 70.7 Hz **[2005]**
12. If the ratio of the concentration of electrons to that of holes in a semiconductor is $\frac{7}{5}$ and the ratio of currents is $\frac{7}{4}$, then what is the ratio of their drift velocities?
    (a) $\frac{5}{4}$ (b) $\frac{4}{7}$
    (c) $\frac{5}{8}$ (d) $\frac{4}{5}$ **[2006]**

13. A solid which is not transparent to visible light and whose conductivity increases with temperature is formed by
    (a) Van der Waals binding (b) Metallic binding
    (c) Ionic binding (d) Covalent binding
    [2006]

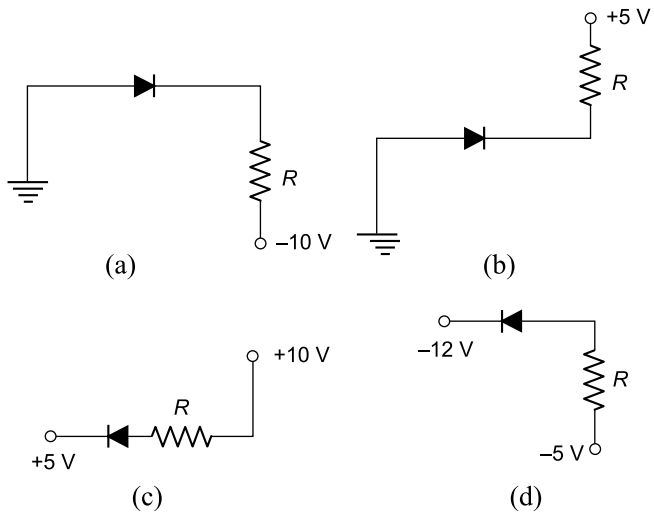
14. In a common base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor (β) will be
    (a) 51 (b) 48
    (c) 49 (d) 50 **[2006]**

15. If the lattice constant of this semiconductor is decreased, then which of the following is correct?



    (a) E_c and E_v decrease, but E_g increases
    (b) E_c, E_v and E_g all decrease
    (c) E_c, E_v and E_g all increase
    (d) E_c and E_v increase but E_g decreases **[2006]**

16. In the following, which one of the diodes is reverse biased?



    [2006]

Fig. 20.62

17. The circuit has two oppositely connected ideal diodes in parallel. What is the current flowing in the circuit?
    (a) 2.31 A (b) 1.33 A
    (c) 1.71 A (d) 2.00 A

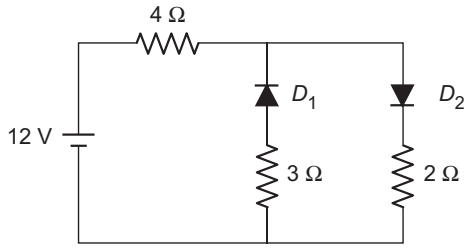
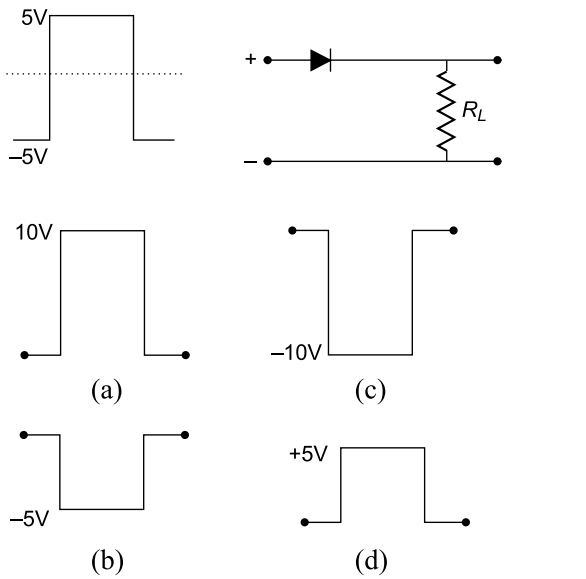


Fig. 20.63

18. If in *p-n* junction diode, a square input signal of 10 V is applied as shown, then the output signal across R_L will be

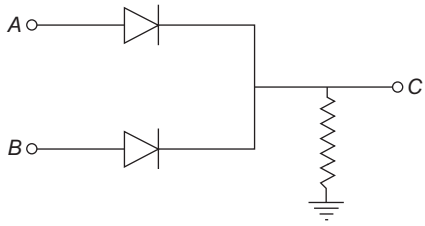


    [2007]

Fig. 20.64

19. Carbon, silicon and germanium have four valence electrons each. At room temperature which one of the following statements is most appropriate?
    (a) the number of free conduction electrons is significant in C but small in Si and Ge.
    (b) the number of free conduction electrons is negligibly small in the three.
    (c) the number of free electrons for conduction is significant in all the three.
    (d) the number of free electrons for conduction is significant only in Si and Ge but small in C.
    [2007]

20. In the circuit below, A and B represent two inputs and C represents the output.
    The circuit represents
    (a) NAND Gate (b) OR Gate
    (c) NOR Gate (d) AND Gate

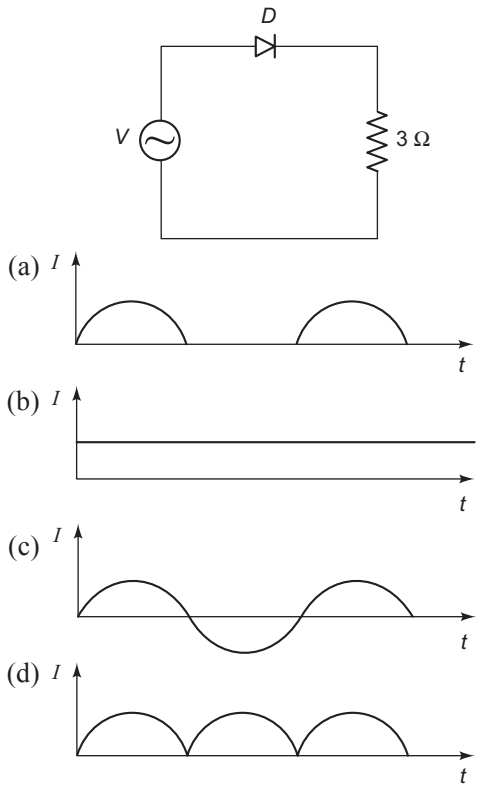


[2008]

Fig. 20.65

21. A working transistor with its three legs marked P, Q and *R* is tested using a multimeter. No conduction is found between P and Q. By connecting the common (negative) terminal of the multimeter to R and the other (positive) terminal to P and Q, some resistance is seen on the multimeter. Which of the following is true for the transistor?
    (a) It is an *npn* transistor with R as collector
    (b) It is an *npn* transistor with R as base
    (c) It is a *pnp* transistor with R as collector
    (d) It is a *pnp* transistor with R as emitter **[2008]**

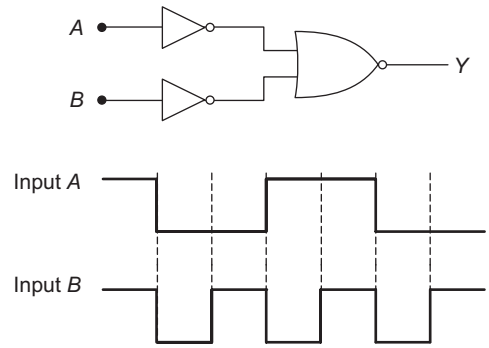
22. A *p-n* junction (*D*) shown in the figure can act as a rectifier. An alternating current source (*V*) is connected in the circuit.
    The current (*I*) in the resistor (*R*) can be shown by:



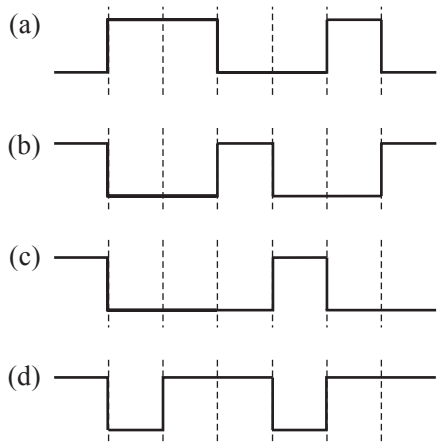
[2009]

Fig. 20.66

23. The logic circuit shown in the figure has the input waveforms '*A*' and '*B*' as shown. Pick out the correct output waveform.



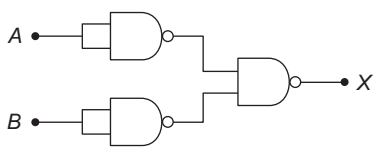
Output is:



[2009]

Fig. 20.67

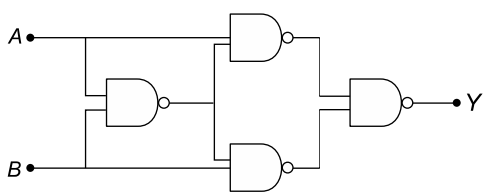
24. The combination of gates shown in Fig. 20.68 yields
    (a) OR gate (b) NOT gate
    (c) XOR gate (d) NAND gate



[2010]

Fig. 20.68

25. The truth table for system of four NAND gates as shown in Fig. 20.69 is



(a)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(b)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	0

(c)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

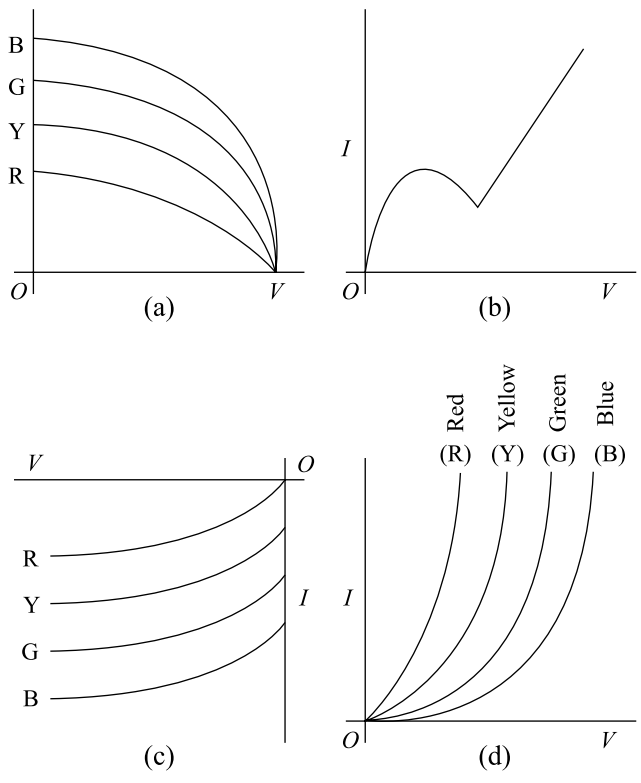
(d)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

[2012]

Fig. 20.69

26. The *I-V* characteristic of an LED is



[2013]

Fig. 20.70

27. When a *pn* junction diode is forward biased.
    (a) the depletion region is reduced and barrier height is increased
    (b) the depletion region is widened and barrier height is reduced
    (c) both the depletion region and barrier height are reduced
    (d) both the depletion region and barrier height are increased **[2014]**

28. Two ideal junction diodes D_1 and D_2 are connected to a battery as shown in Fig. 20.71. The current supplied by the battery is

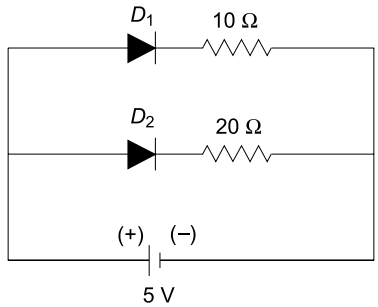


Fig. 20.71

(a) 0.25 A (b) 0.5 A
(c) 0.75 A (d) zero **[2014]**

Answers

1. (c)	2. (a)	3. (b)	4. (a)
5. (d)	6. (d)	7. (b)	8. (c)
9. (c)	10. (c)	11. (c)	12. (a)
13. (d)	14. (c)	15. (d)	16. (b)
17. (d)	18. (d)	19. (d)	20. (b)
21. (a)	22. (a)	23. (c)	24. (a)
25. (a)	26. (d)	27. (c)	28. (c)

Solutions

1. Insulators have the maximum energy band gap.
2. The correct answer is emitter.
3. In a metal, the variation of resistance with temperature is due to change in mean collision time (called relaxation time). In a semiconductor, the difference is due to a change in charge carriers due to change in temperature. Thus, the most appropriate choice is (b).
4. Due to reverse biasing, the width of the depletion layer increases and the current flowing through the junction is almost zero. Hence the electric field is almost zero in the depletion layer.
5. In an amplifier, the emitter-base junction is forward biased and the base-collector junction is reverse biased. Hence electrons move from emitter to base

and holes move from base to emitter. So the correct choice is (d).

6. Current gain $= -\dfrac{h_{fe}}{1+h_{oe}R_L}$

$$= -\frac{50}{1+(25\times10^{-6})\times(1\times10^{3})} = -48.78$$

7. The correct choice is (b).
8. When a pn junction is forward biased, both the depletion region and barrier potential are reduced. Hence the correct choice is (c).
9. $E_g = h\nu = \dfrac{hc}{\lambda}$

$$= \frac{(6.63\times10^{-34})\times(3\times10^{8})}{2480\times10^{-9}}$$

$$= 0.81\times10^{-19}\text{ J}$$

$$= \frac{0.81\times10^{-19}}{1.6\times10^{-19}} \simeq 0.5\text{ eV}$$

10. In a common base amplifier, the output voltage is in phase with the input voltage. But in a common emitter amplifier, the output signal is 180° out of phase with the input signal. So the correct choice is (c).
11. Given ν = 50 Hz. Therefore $T = \dfrac{1}{\nu} = \dfrac{1}{50}$ s. For a fullwave rectifier

$$T' = \frac{T}{2} = \frac{1}{100}\text{ s}$$

$$\therefore \quad \nu' = \frac{1}{T'} = 100\text{ Hz}$$

12. If n_e and n_h are the concentrations (i.e. number per unit volume) of electrons and holes, v_e and v_h their respective drift velocities, then the currents due to the flow of electrons and holes are

$$I_e = n_e\, eA\, v_e \qquad (1)$$

and $$I_h = n_h\, eA\, v_h \qquad (2)$$

where e = electronic charge and A = cross-sectional area of the semiconductor. Dividing (1) by (2), we get

$$\frac{v_e}{v_h} = \left(\frac{I_e}{I_h}\right)\times\left(\frac{n_h}{n_e}\right) = \frac{7}{4}\times\frac{5}{7} = \frac{5}{4}$$

13. The correct answer is covalent bonding
14. Current gain $\beta = \dfrac{I_C}{I_B} = \dfrac{I_C}{I_E - I_C}$

$$= \frac{5.488}{5.60 - 5.488} = 49$$

15. The correct choice is (d).
16. If the p-side is at a lower potential than the *n*-side of a junction diode, then the diode is reverse biased. The correct choice is (b). The *p*-side is at zero potential (because it is connected to earth) and the *n*-side at potential + 5V.
17. Diode D_2 is forward biased but diode D_1 is reverse biased. Hence no current flows through the branch containing D_1 and the circuit may be redrawn as shown in Fig. 20.43.

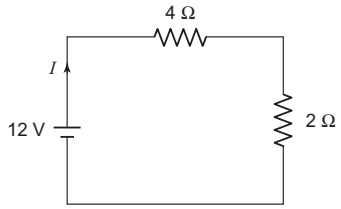


Fig. 20.72

$\therefore$ Current in the circuit is

$$I = \frac{12\text{ V}}{4\Omega + 2\Omega} = \frac{12\text{ V}}{6\Omega} = 2\text{A}$$

18. The junction diode is forward biased. The amplitude of the input signal is 5 V. The amplitude of the output signal remains the same but the negative part of the signal is removed as the diode does not conduct the negative voltage. Hence the correct choice is (d).
19. The correct choice is (d).
20. When input A is ON output C is also in the ON state as the upper diode is forward biased. When input B is ON output C is also in the ON state. When both inputs are ON, output C will also be in the ON state. Hence the circuit represents an OR gate, which is choice (b).
21. It follows that R is the collector and it is connected to the base corners in forward bias. Hence it is an *npn* transistor with R as collector. So the correct choice is (a).
22. During the positive (first) half of the input voltage, the diode *D* is forward biased and will allow the current to flow. But in the second (negative) half, the diode is reverse biased and will not allow the current to flow. Thus the given circuit is a half-wave rectifier. So the correct choice is (a).
23. The correct choice is (3). The given logic circuit yields an OR gate as shown in Fig. 20.44.

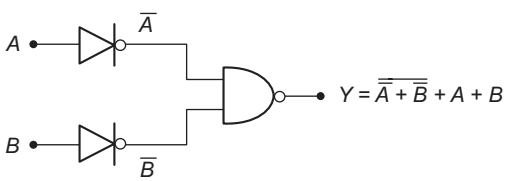


Fig. 20.73

24. The outputs of two NOT gates (obtained from NAND gates) are fed to a third NAND gate. If $\overline{A}$ and $\overline{B}$ are

the outputs of the NOT gates, the truth table of the combination is as follows.

A	B	$\overline{A}$	$\overline{B}$	X
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

Fig. 20.74

This is the truth table of an OR gate. So the correct choice is (a).

25. $Y = (\overline{A.\overline{A.B}}) . (\overline{B.\overline{A.B}})$

$= A.(\overline{A}+\overline{B}) + B.(\overline{A}+\overline{B})$

$= A.\overline{A} + A.B + B.\overline{A} + B.\overline{B}$

$= O + A.\overline{B} + B.\overline{A} + O$

So the correct truth table is (a).

26. The correct choice is (d). LED is forward biased and the intensity increased with voltage. Also the frequency of red light is the smallest and that of blue light is the highest.

27. The correct choice is (c).

28. Both the diodes are forward biased. Hence they offer negligible resistance. The effective resistance of the circuit is

$$R = \frac{10 \times 20}{10 + 20} = \frac{20}{3}\Omega$$

$$\therefore \quad \text{Current } I = \frac{V}{R} = \frac{5}{20/3} = 0.75 \text{ A}$$

COMMUNICATION SYSTEMS

Chapter 21

REVIEW OF BASIC CONCEPTS

1. Communication

The transfer of information from one point to another is called communication.

2. Communication System

A communication system is a setup which is used to transmit information from one point to another. It consists of a transmitter, a communication channel and a receiver.

A transmitter converts the information or message into an equivalent electrical signal (which contains the information or message) and modifies it into a form suitable for communication.

Communication channel is the medium between the transmitter and the receiver through which the signals from the transmitter travel. The medium could be free space, air or transmission cable.

A receiver is a device which receives the transmitted signal and then converts it into a form which is intelligible to the person or equipment that receives it.

3. Analog and Digital Communication Systems

In an analog communication system, the message or information is in the form of an analog signal (voltage or current) which varies sinusoidally in a smooth and continuous fashion or in the form of pulses. Examples are telephony, telegraphy, radio and television broadcasting, radar and telex.

In a digital communication system, the message or information is in the form of a digital signal which is discrete (i.e. discontinuous) having values of voltage or current represented by ±0, ±1, ±2, ... etc. Examples are fax, e-mail, mobile phone, communication satellites and teleconferencing.

4. Modulation

The process of changing some characteristic, e.g. amplitude, frequency or phase of the high frequency carrier wave by superposing a low frequency audio signal on it is called modulation.

(a) *Amplitude Modulation:* The process in which the amplitude of the high frequency carrier wave is changed in accordance with the amplitude of the input audio signal is called amplitude modulation.

(b) *Frequency Modulation:* The process in which the amplitude of the high frequency carrier waves remains constant but its frequency is modified in accordance with the amplitude of the input audio signal is called frequency modulation.

(c) *Pulse Modulation:* In amplitude and frequency modulation, the carrier wave is a sinusoidal wave. In pulse modulation, the carrier wave is in the form of pulses. Pulse modulation is used in digital communication where the input signal is discrete (i.e. discontinuous).

5. Demodulation

The process of recovering the original low frequency audio signal from the high frequency modulated carrier wave is called demodulation. Demodulation is carried out by a radio receiver.

6. Modulation Index, Side Band Frequencies and Bandwidth of Amplitude Modulated Wave

When a low frequency signal (called modulating wave) is superposed on a high frequency electromagnetic wave (called carrier wave) in such a way that the frequency of the modulated wave is the same as that of the carrier wave but the amplitude is changed, the process is called amplitude modulation.

Modulation Index

The modulation index m_a of the modulated waves is defined as

$$m_a = \frac{E_m}{E_c}$$

where E_m = amplitude of modulating wave

and E_c = amplitude of carrier wave

Side Band Frequncies

The amplitude modulated wave can be regarded as a superposition of three waves of frequencies ν_c, $(\nu_c - \nu_m)$ and $(\nu_c + \nu_m)$ where ν_c is the frequency of the carrier wave and ν_m is the frequency of the modulating wave. Frequency $(\nu_c - \nu_m)$ is called the lower side band frequency (LSB) and frequency $(\nu_c + \nu_m)$ is called the upper side band frequency (USB).

Bandwidth

The bandwidth of an amplitude modulated wave is defined as

$$\begin{aligned}\Delta\nu &= \nu_{\text{USB}} - \nu_{\text{LSB}} \\ &= (\nu_c + \nu_m) - (\nu_c - \nu_m) \\ &= 2\nu_m\end{aligned}$$

i.e. Bandwidth = twice the frequency of the modulating wave

EXAMPLE 1 A message signal of 12 kHz and peak voltage 20 V is used to modulate a carrier wave of frequency 12 MHz and peak voltage 30 V. Calculate (a) modulation index, (b) side band frequencies and (c) bandwidth of the modulated wave.

SOLUTION Given $E_m = 20$ V, $E_c = 30$ V, $\nu_c = 12$ MHz

and $\nu_m = 12\text{ kHz} = 12 \times 10^3\text{ Hz} = 0.012\text{ MHz}$

(a) $m_a = \dfrac{E_m}{E_c} = \dfrac{20}{30} = 0.67$

(b) Side band frequencies

$$\begin{aligned}&= \nu_c + \nu_m \\ &= (12 \pm 0.012)\text{ MHz} \\ &= 12.012\text{ MHz and } 11.988\text{ MHz}\end{aligned}$$

(c) Bandwidth = $2\nu_m = 2 \times 12\text{ kHz} = 24\text{ kHz}$

7. Space Communication

The transmission of information through space as communication channel is space communication. Radio, television and satellite transmissions use space communication. The various modes of space propagation are

(i) ground or surface wave propagation,
(ii) sky wave propagation,
(iii) space wave propagation, and
(iv) satellite communication

8. Ground or Surface Wave Propagation

The radiowaves which travel along the surface of the earth from the transmitter to the receiver are called ground or surface waves. The transmission of information through radiowaves is called ground or surface wave propagation (or communication).

In ground wave transmission, the intensity of the radio signal falls as it travels due to absorption by the ground and also by the gases in the atmosphere. As a result, the signals become weak and after travelling some distance, they die out. Due to this, the ground wave transmission is not suitable for long distance communication.

9. Sky Wave Propagation

Radiowaves, which are directed towards the sky from a transmitter at a certain location on the earth and are reflected back from the ionosphere towards another location on the earth are called sky waves. The transmission of information through sky waves is called sky wave propagation (or communication).

Sky wave transmission is suitable for frequencies in the range 2 MHz to about 20 MHz. Above 30 MHz, the ionosphere does not reflect the waves; they are transmitted through it.

10. Space Wave Propagation

Radiowaves which travel in the earth's troposphere (layer of atmosphere closest to earth) from a transmitting antenna (at a transmitting station) to a receiving antenna (at a receiving station) are called space waves. They have frequencies higher than 30 MHz. The transmission of information through space waves is called space wave propagation (or communication).

The maximum distance up to which signal from an antenna of height at a TV transmitting station can reach directly is given by

$$d_{\max} = \sqrt{2R\,h_t} + \sqrt{2R\,h_r}$$

where h_t = height of transmitting tower, h_r = height of receiving tower and R = radius of the earth. This mode of transmission in called Line of Sight (LOS) mode

EXAMPLE 2 A transmitting antenna is at the top of a tower of height 64 m and the receiving antenna is at the top of a tower of height 32 m. Calculate the maximum distance between them for satisfactory transmission in LOS mode. Given, radius of earth = 6400 km.

SOLUTION $h_t = 64$ m, $h_r = 32$ m
and $R = 6400$ km $= 6.4 \times 10^6$ m

$$d_{max} = \sqrt{2Rh_t} + \sqrt{2Rh_r}$$
$$= \sqrt{2 \times 6.4 \times 10^6 \times 64} + \sqrt{2 \times 6.4 \times 10^6 \times 32}$$
$$= 28.6 \times 10^3 + 20.2 \times 10^3$$
$$= 48.8 \times 10^3 \text{ m}$$
$$= 48.8 \text{ km}$$

11. Satellite Communication

The mode of communication in which microwave signals propagate from a transmitter at a certain location on earth to a receiver at another location via a satellite is called satellite communication. The satellite used is a geostationary satellite. For global communication (i.e. to transmit signals to all locations on earth), three geostationary satellites parked 120° apart from each other are used.

SECTION 1 Multiple Choice Questions with One Correct Choice

Level A

1. In radio and television broadcast, the information signal is in the form of
   (a) analog signal
   (b) digital signal
   (c) both analog and digital signals
   (d) neither analog nor digital signal
2. In fax and e-mail, the information signal is in the form of
   (a) analog signal
   (b) digital signal
   (c) both analog and digital signals
   (d) neither analog nor digital signal
3. Through which mode of communication can radiowaves be sent from one place to another?
   (a) Ground wave propagation
   (b) Sky wave propagation
   (c) Space wave propagation
   (d) All of the above
4. Ground wave propagation is effective for frequencies in the range
   (a) 20 kHz to 250 kHz
   (b) 500 kHz to 1500 kHz
   (c) 10 MHz to 200 MHz
   (d) 10^{10} Hz to 10^{14} Hz
5. Which range of frequencies is suitable for sky wave propagation?
   (a) 1 kHz to 500 kHz
   (b) 1 MHz to 2 MHz
   (c) 2 MHz to 20 MHz
   (d) above 30 MHz
6. Which mode of communication is employed to transmit very high frequency (VHF) and ultra high frequency (UHF) signals?
   (a) Ground wave propagation
   (b) Sky wave propagation
   (c) Space wave propagation
   (d) None of the above
7. A TV tower has a height of 50 m. The maximum distance up to which TV transmission can be received is approximately equal to (radius of earth $= 6.4 \times 10^6$ m)
   (a) 5 km (b) 25 km
   (c) 100 km (d) 250 km
8. Which electromagnetic waves are used in satellite communication?
   (a) Light waves (b) Radiowaves
   (c) Gamma rays (d) Microwaves
9. A receiving station on the ground is receiving a signal of frequency 5 MHz from a transmitter at a height of 300 m above the surface of the earth (of radius 6.4×10^6 m) at a distance of 100 km from the receiver. Then the signal is coming via
   (a) ground wave propagation
   (b) sky wave propagation
   (c) both ground wave and sky wave propagation
   (d) neither ground wave nor sky wave propagation.
10. In Q. 9 above, if the frequency of the signal transmitted is 10 MHz, then the mode of transmission is
    (a) ground wave propagation
    (b) sky wave propagation
    (c) both ground wave and sky wave propagation
    (d) neither ground wave nor sky wave propagation.
11. On a particular day, the maximum frequency reflected from the ionosphere is 8 MHz. On another

day it was found to increase to 9 MHz. The ratio of the maximum electron densities of the ionosphere on the two days is

(a) $\sqrt{\frac{8}{9}}$ (b) $\frac{8}{9}$

(c) $\left(\frac{8}{9}\right)^{3/2}$ (d) $\left(\frac{8}{9}\right)^{2}$

12. The process of superposition an audio signal on a high frequency carrier wave is called

(a) transmission (b) communication

(c) modulation (d) demodulation

13. In an optical fibre, optical signals are transmitted by

(a) repeated refractions at the core-cladding interface

(b) interference between the incident wave and the reflected waves

(c) diffraction of optical signals

(d) repeated total internal reflections at the core-cladding interface

14. In a typical optical fibre, the diameter of the core is

(a) 10^{-2} m (b) 10^{-4} m

(c) 10^{-8} m (d) 1 Å

15. If n_1 and n_2 are the refractive indices of the core and cladding respectively of an optical fibre, then

(a) n_1 is much greater than n_2

(b) n_1 is much smaller than n_2

(c) n_1 is slightly greater than n_2

(d) n_1 is slightly smaller than n_2.

16. The refractive index of the core of an optical fibre is n_1 and that of its cladding is n_2. In a typical optical fibre, the difference $(n_1 - n_2)$ is

(a) 10^{-7} (b) 10^{-5}

(c) 10^{-3} (d) 10^{-1}

17. The light from a laser

(a) is highly monochromatic

(b) is highly coherent

(c) travels as a concentrated beam

(d) has all the properties listed in (a), (b) and (c)

18. In which of the following processes are new photons emitted

(a) stimulated absorption

(b) spontaneous emission

(c) stimulated emission

(d) All the above processes

19. In stimulated emission, which of the following characteristics of the emitted photons are the same as those of the incident photons?

(a) Energy (b) Direction of motion

(c) Phase (d) All the above

20. In light modulation, which characteristic of the carrier light wave is varied?

(a) Amplitude (b) Frequency

(c) Phase (d) Intensity

Level B

21. In order to transmit a signal of wavelength λ, the length of the antenna at the transmitting station should at least be equal to

(a) λ (b) $\frac{\lambda}{2}$

(c) $\frac{\lambda}{4}$ (d) 4λ

22. An audio signal is sent on a carrier wave of frequency 30 MHz. The minimum length of the antenna at the transmitting station must be

(a) 22.5 m (b) 30 m

(c) 10 m (d) 2.5 m

23. An audio signal is sent on a carrier wave of wavelength λ. The power radiated from an antenna is proportional to

(a) $\frac{1}{\lambda^2}$ (b) $\frac{1}{\lambda}$

(c) λ (d) λ^2

24. A modem is a device which performs the function of

(a) modulation only

(b) demodulation only

(c) both modulation as well as demodulation

(d) neither modulation nor demodulation.

25. Which of the following modulations is preferred in digital communication?

(a) Amplitude modulation

(b) Frequency modulation

(c) Phase modulation

(d) Pulse modulation

26. What type of modulation is employed in radio communication?

(a) Amplitude modulation only

(b) Frequency modulation only

(c) Both amplitude modulation and frequency modulation

(d) Pulse modulation only

27. Amplitude modulated wave is a superposition of

(a) two harmonic waves

(b) three harmonic waves

(c) four harmonic waves

(d) none of these

28. An audio low frequency signal of frequency ν_m is superposed on a high frequency carrier wave of frequency ν_c. The resultant amplitude modulated wave has components of frequencies
    (a) ν_m and ν_c
    (b) ν_m, ν_c and $(\nu_c + \nu_m)$
    (c) ν_m, ν_c and $(\nu_c - \nu_m)$
    (d) ν_c, $(\nu_c - \nu_m)$ and $(\nu_c + \nu_m)$
29. In Q. 28 above, the bandwidth of the amplitude modulated wave is
    (a) $2\,\nu_c$ (b) $2\,\nu_m$
    (c) $2\,(\nu_c + \nu_m)$ (d) $2\,(\nu_c - \nu_m)$
30. A message signal of frequency 12 kHz and peak voltage 20 V is used to modulate a carrier wave of frequency 12 MHz and peak voltage 30 V. The modulation index of the amplitude modulated wave is
    (a) 10^3 (b) 10^{-3}
    (c) $\frac{3}{2}$ (d) $\frac{2}{3}$
31. Which of the following statements is/are correct about amplitude modulation?
    (a) The modulation index m determines the quality of the transmitted signal.
    (b) If m is small, the transmitted signal is weak
    (c) For clear reception and higher intensity of the transmitted signal, the value of m should be 0.5 or higher
    (d) All the above statements are correct
32. A simple amplitude modulation circuit consists of
    (a) a square law device and a rectifier
    (b) a band pass filter and an envelope detector
    (c) a rectifier and a band pass filter
    (d) a square law device and an envelope detector
33. A detector of an amplitude modulated signal consists of
    (a) a rectifier and an envelope detector
    (b) a common-emitter transistor and a band pass filter
    (c) a rectifier and a band pass filter
    (d) a common-base transistor and an envelope detector
34. The refractive index of the ionosphere is
    (a) equal to 1.5
    (b) equal to 1.0
    (c) slightly greater than 1.0
    (d) slightly less than 1.0

35. The electromagnetic waves which are used as carrier waves in telecommunication are
    (a) radiowaves (b) visible light
    (c) microwaves (d) infrared radiation
36. Electrical conductivity of the ionosphere
    (a) decreases as we go from the lower to upper layers
    (b) increases as we go from the lower to upper layers
    (c) remains constant throughout the ionosphere
    (d) is zero



Answers

Level A

1. (a)	2. (b)	3. (d)	4. (b)
5. (c)	6. (c)	7. (b)	8. (d)
9. (b)	10. (d)	11. (d)	12. (c)
13. (d)	14. (c)	15. (c)	16. (c)
17. (d)	18. (c)	19. (d)	20. (d)

Level B

21. (c)	22. (d)	23. (a)	24. (c)
25. (d)	26. (c)	27. (b)	28. (d)
29. (b)	30. (d)	31. (d)	32. (c)
33. (a)	34. (d)	35. (c)	36. (b)



Solutions

1. The correct choice is (a).
2. The correct choice is (b).
3. The correct choice is (d).
4. The correct choice is (b).
5. The correct choice is (c). Above 30 MHz, the ionosphere does not reflect the waves. Waves of frequency greater than 30 MHz are transmitted through the ionosphere.
6. The correct choice is (c).
7. Maximum range $d_{\max} = \sqrt{2Rh} = \sqrt{2 \times 6.4 \times 10^6 \times 50}$ $= 25.3 \times 10^3$ m = 25.3 km.

   Hence the correct choice is (b).
8. The correct choice is (d).

9. For ground wave propagation, the maximum range is

$$d_{max} = \sqrt{2Rh} = \sqrt{2 \times 6.4 \times 10^6 \times 300}$$
$$\simeq 62 \times 10^3 \text{ m} \simeq 62 \text{ km}$$

which is less than 100 km. Hence choice (a) is wrong. The maximum frequency which can be propagated via sky waves is

$$\nu_{max} \simeq 9 \text{ MHz}$$

which is more than 5 MHz. Hence a 5 MHz signal can be propagated via sky waves and not via ground waves. Thus the correct choice is (b).

10. The frequency of the signal is 10 MHz which is greater than $\nu_{max} = 9$ MHz for sky wave propagation. Hence the correct choice is (d).
11. $\nu_{max} = 9\ (N_{max})^{1/2}$, where N_{max} is the maximum electron density of the ionosphere. Hence $N_{max} \propto \nu^2_{max}$. Therefore, the ratio of maximum electron densities on the two days is $\left(\frac{8}{9}\right)^2$, which is choice (d).
12. The correct choice is (c).
13. The correct choice is (d).
14. The correct choice is (c).
15. The correct choice is (c).
16. The correct choice is (c).
17. The correct choice is (d).
18. The correct choice is (c).
19. The correct choice is (d).
20. The correct choice is (d).

Level B

21. The correct choice is (c).
22. $L_{min} = \frac{\lambda}{4} = \frac{c}{4\nu}$
$= \frac{(3 \times 10^8)}{4 \times (30 \times 10^6)} = 2.5$ m
So the correct choice is (d).
23. The correct choice is (a).
24. The correct choice is (c). A modem acts as a modulator at the transmitting station and as a demodulator at the receiving station.
25. The correct choice is (d).
26. The correct choice is (c).
27. The correct choice is (b).
28. The correct choice is (d).
29. The correct choice is (b).
30. Modulation index is defined as
$$m = \frac{E_m}{E_c} = \frac{20\text{ V}}{30\text{ V}} = \frac{2}{3}$$
So the correct choice is (d).
31. The correct choice is (d).
32. The correct choice is (c).
33. The correct choice is (a).
34. The correct choice is (d).
35. The correct choice is (c).
36. Electrical conductivity of an ionized gas depends on the degree of ionization. So the correct choice is (b).

SECTION 2
Multiple Choice Questions Based on Passage

Passage I

The radiowaves which travel along the surface of the earth from the transmitter to the receiver are called ground waves. The transmission of information through ground waves is called ground wave propagation (or communication). In ground wave transmission, the intensity of the radio signal falls as it travels. Due to this, the ground wave transmission is not suitable for long distance communication. It is suitable only for local broadcasting.

1. In ground wave transmission, the intensity of a radio signal falls with distance due to
   (a) reflection by the ground
   (b) absorption by the ground
   (c) scattering by gas molecules and dust particles in the atmosphere
   (d) adsorption by the gases in the atmosphere.
2. Ground wave transmission is effective at frequencies in the range

(a) 1 kHz to 300 kHz
(b) 500 kHz to 1500 kHz
(c) 3 MHz to 100 MHz
(d) 10^{12} Hz to 10^{15} Hz



Solutions

1. The correct choices are (b) and (d).
2. The correct choice is (b).

Questions 3 to 5 are based on the following passage.

Passage II

Radiowaves which are directed towards the sky from a transmitter at a certain location on the earth and are reflected back from the ionosphere towards another location on the earth are called sky waves. The transmission of information through sky waves is called sky wave propagation (or communication). Ionosphere is the uppermost layer in the earth's atmosphere extending from a height of 80 km to 300 km above the surface of the earth. The ionosphere is composed of charged particles (electrons and ions). The degree of ionization is less at layers of ionosphere closer to the earth than at layers away from the earth. Consequently, the refractive index of the lowermost layers of the ionosphere is the highest and it decreases as we go to the upper layers.

3. The gases in the ionosphere are ionized by
   (a) ultraviolet and high frequency radiations from the sun
   (b) infrared thermal radiations from the sun
   (c) sunlight reflected from the earth
   (d) global warming.
4. The refractive index of the ionosphere
   (a) increases as we go from the lower to upper layers of the ionosphere
   (b) decreases as we go from the lower to upper layers of the ionosphere
   (c) remains the same throughout the ionosphere
   (d) is equal to unity.
5. Which range of frequencies is suitable for sky wave propagation?
   (a) 1 kHz to 500 kHz (b) 1 MHz to 2 MHz
   (c) 5 MHz to 20 MHz (d) above 30 MHz



Solutions

3. The correct choice is (a).
4. The correct choice is (b).
5. The correct choice is (c).

Questions 6 to 8 are based on the following passage.

Passage III

Radiowaves which travel in the earth's troposphere (layer of atmosphere closest to earth) from a transmitter to a receiver are called space waves. The transmission of information through space waves is called space wave propagation (or communication). In space wave propagation, the waves travel directly in a straight line from the transmitting antenna to the receiving antenna. The propagation of space wave is, therefore, restricted by the curvature of the earth. This mode of propagation is called 'line of sight' communication. The maximum distance up to which signals from an antenna of height h at a TV transmitting station can reach directly is given by

$$d = \sqrt{2Rh}$$

where R is the radius of the earth. Thus, the greater the height of the TV tower, the greater is the range of transmission.

6. Which mode of communication is employed to transmit very high frequency (VHF) and ultra high frequency (UHF) signals?
   (a) Ground wave propagation
   (b) Sky wave propagation
   (c) Space wave propagation
   (d) None of these
7. A TV tower has a height of 80 m. The maximum distance up to which TV transmission can be received is equal to (radius of earth = 6.4×10^6 m)
   (a) 16 km (b) 32 km
   (c) 80 km (d) 160 km
8. If the average density of population is 2000 persons per km^2, the population covered in Q.7 above is nearly
   (a) 24 lakh (b) 48 lakh
   (c) 64 lakh (d) 80 lakh



Solutions

6. The correct choice is (c). Ground waves die out after covering a small distance due to absorption by earth and the atmosphere close to the earth. The ionosphere does not reflect such high frequency waves; they are transmitted through it.
7. $d_{max} = \sqrt{2Rh} = \sqrt{2 \times 6.4 \times 10^6 \times 80} = 32 \times 10^3$ m
   $= 32$ km
   So the correct choice is (b).
8. Maximum area covered = $\pi d^2_{max} = 3.14 \times (32)^2$ 3200 km^2
   $\therefore$ Population covered = $3200 \times 2000 = 64$ lakh, which is choice (c).

SECTION 3

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only *one* choice is correct.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

1. **Statement-1**

   In ground wave transmission the radio signals die out after travelling some distance.

   Statement-2

   Radio signals have a very short wavelength and hence are scattered away by the dust particles and molecules of gases in the atmosphere.

2. **Statement-1**

   Ground wave communication is effective only at low frequencies in the range 500 kHz to about 1500 kHz.

   Statement-2

   The decrease in the intensity of the signal due to absorption by the earth and its atmosphere is higher for higher frequencies.

3. **Statement-1**

   The refractive index of the ionosphere increases as we go from the lower to upper layers in the ionosphere.

   Statement-2

   The degree of ionization is higher at the upper layers than at the lower layers of the ionosphere.

4. **Statement-1**

   Sky wave communication is not suitable for frequencies greater than 30 MHz.

   Statement-2

   High frequency signals die out before reaching the ionosphere.

5. **Statement-1**

   Microwaves and not radiowaves are used in satellite communication.

   Statement-2

   The wavelength of microwaves is much shorter than that of radiowaves. Hence microwaves do not disperse or diffract like radiowaves.

6. **Statement-1**

   Long distance radio broadcasts use short wave bands.

   Statement-2

   Short wavelength signals are reflected by the ionosphere.

7. **Statement-1**

   Sky wave communication is not used to transmit TV signals.

   Statement-2

   The ionosphere does not reflect TV signals; it transmits them.

8. **Statement-1**

   In an optical fibre, the refractive index of the glass core is slightly greater than that of the cladding.

   Statement-2

   An optical fibre propagates light by repeated total internal reflections at the core-cladding interface.



Solutions

1. The correct choice is (a). Radio signals are absorbed by the earth and by the atmosphere. Hence the intensity of these signals decreases as they propagate.
2. The correct choice is (a).
3. The correct choice is (d). The refractive index decreases as we go from the lower to upper layers in the ionosphere.
4. The correct choice is (c). High frequency signals are not reflected from ionosphere; they are transmitted through it.
5. The correct choice is (a).
6. The correct choice is (a).
7. The correct choice is (a).
8. The correct choice is (a).

SECTION 4

Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

   Statement-1: Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

   Statement-2: The state of ionosphere varies from hour to hour, day to day and season to season.

   (a) Statement-1 is true, Statement-2 is false.

   (b) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1

   (c) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.

   (d) Statement-1 is false, Statment-2 is true

   [2011]

2. A radar has a power of 1kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance up to which it can detect object located on the surface of the earth (Radius of earth = 6.4×10^6 m) is

   (a) 80 km (b) 16 km

   (c) 49 km (d) 64 km **[2012]**

3. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a capacitor of capacitance 250 pF in paralled with a load resistor 100 kΩ. The maximum frequency which can be detected by it is

   (a) 10.62 kHz (b) 5.31 MHz

   (c) 5.31 kHz (d) 10.62 MHz **[2013]**

4. Which mode of communication is employed to transmit very high frequencey (VHF) and ultra high frequency (UHF) signals?

   (a) Ground wave propagation

   (b) Sky wave propagation

   (c) Space wave propagation

   (d) None of the above **[2014]**

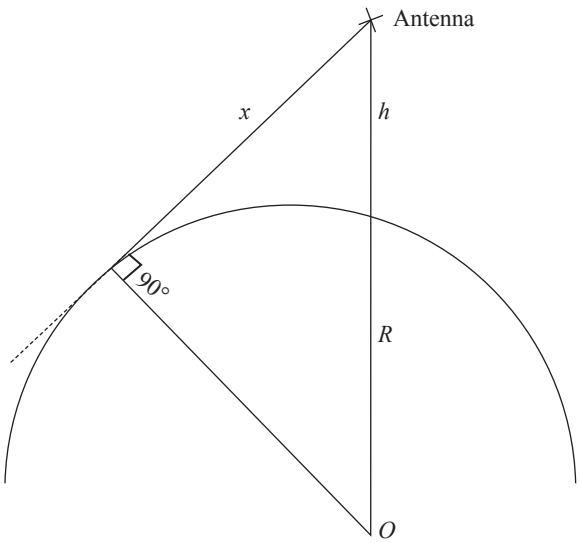
Answers

1. (b) 2. (a) 3. (c) 4. (c)

Solutions

1. The refractive index of the different layers of the ionosphere changes with time. Hence sky wave signals are, in general, less stable than ground wave signals.

2.



$$x = \sqrt{(R+h)^2 - R^2}$$
$$= \sqrt{R^2 + 2Rh - R^2} \qquad (\because h << R)$$
$$= \sqrt{2Rh}$$
$$= \sqrt{2 \times (6.4 \times 10^6) \times 500}$$
$$= 8 \times 10^4 \text{ m} = 80 \text{ km}$$

3. Cut-off frequency is

$$\nu_c = \frac{1}{2\pi RC} = \frac{1}{2 \times 3.14 \times (100 \times 10^3) \times (250 \times 10^{-12})}$$
$$= 6.4 \text{ kHz}$$

The maximum modulation frequency ν_m must be less than ν_c because

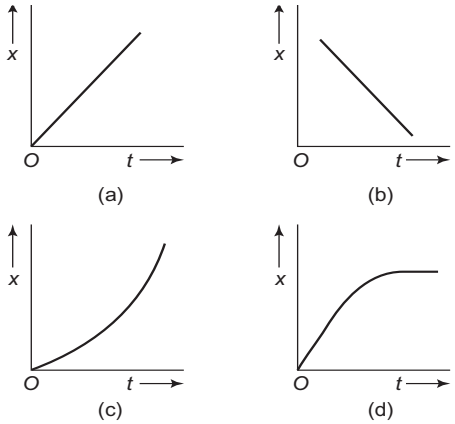
$$\nu_m = \nu_c \frac{\sqrt{1 - m_a^2}}{m_a}; \qquad (m_a = \text{modulation index})$$

Since $m_a < 1$, the correct choice is (c)

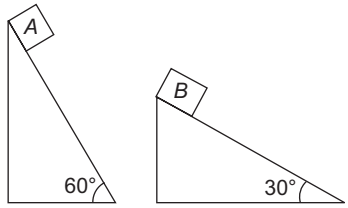
4. The correct frequency is (c).

Practice Test Paper – I

1. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (= 0.5°), then the least count of the instrument is :
   (a) one degree (b) half degree
   (c) one minute (d) half minute
2. If L, R, C and V respectively represent inductance, resistance, capacitance and potential difference, then the dimensions of $\frac{L}{RCV}$ are the same as those of
   (a) current (b) $\frac{1}{\text{current}}$
   (c) charge (d) $\frac{1}{\text{charge}}$
3. The velocity (v) of a body moving along the positive x-direction varies with displacement (x) from the origin as $v = k\sqrt{x}$, where k is a constant. Which of the graphs shown in the following figure correctly represents the displacement-time ($x - t$) graph of the motion?

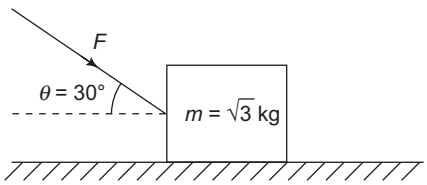


4. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal is
   (a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$
   (c) $\sqrt{u^2 - gL}$ (d) $\sqrt{2(u^2 - gL)}$
5. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$ where α is a constant. The displacement x of the particle varies wih time t as
   (a) $t^{1/2}$ (b) t^3
   (c) t^2 (d) t
6. A particle is moving with velocity $v = K(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$, where K is a constant. The general equation for its path is
   (a) $y = x^2$ + constant (b) $y^2 = x$ + constant
   (c) xy = constant (d) $y^2 = x^2$ + constant
7. Two fixed frictionless inclined planes making an angle 30° and 60° with the horizontal are shown in the following figure. Two block A and B are place on the two planes. What is the relative vertical acceleration of A with respect to B?

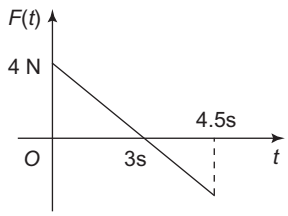


(a) 4.9 ms^{-2} in horizontal direction
(b) 9.8 ms^{-2} in vertical direction
(c) zero
(d) 4.9 ms^{-2} in vertical direction

8. What is the maximum value of the force F such that the block shown in the following figure does not move? The coefficient of friction between the block and the horizontal surface is 0.5. (Take $g = 10\ ms^{-2}$)
(a) 20 N (b) 10 N
(c) 12 N (d) 15 N



9. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x direction. The force $F(t)$ varies with t as shown in the following figure. The kinetic energy of the block after 4.5 seconds is



(a) 4.50 J (b) 7.50 J
(c) 5.06 J (d) 14.06 J

10. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissocation energy of the molecule is $D\ [U(x=\infty) - U_{\text{at equilibrium}}]$, D is

(a) $\frac{b^2}{6a}$ (b) $\frac{b^2}{2a}$
(c) $\frac{b^2}{12a}$ (d) $\frac{b^2}{4a}$

11. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of :

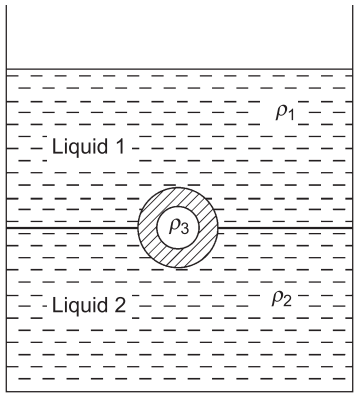
(a) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (b) $\frac{1}{6}\frac{l^2\omega^2}{g}$
(c) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{6}\frac{l\omega}{g}$

12. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then
(a) at $\theta = 30°$, the block will start sliding down the plane
(b) the block will remain at rest on the plane up to certain θ and then it will topple
(c) at $\theta = 60°$, the block will start sliding down the plane and continue to do so at higher angles
(d) at $\theta = 60°$, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ.

13. An ideal spring with spring-constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is
(a) 4 Mg/k (b) 2 Mg/k
(c) Mg/k (d) $Mg/2k$

14. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be
(a) 0.11 km s^{-1}
(b) 1.1 km s^{-1}
(c) 11 km s^{-1}
(d) 110 km s^{-1}

15. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2 respectively. A solid ball, made of a material of density ρ_3 is dropped in the jar. It comes to equilibrium in the position shown in figure given on next page. Which of the following is true for ρ_1, ρ_2 and ρ_3?
(a) $\rho_1 < \rho_3 < \rho_2$
(b) $\rho_3 < \rho_1 < \rho_2$
(c) $\rho_1 > \rho_3 > \rho_2$
(d) $\rho_1 < \rho_2 < \rho_3$

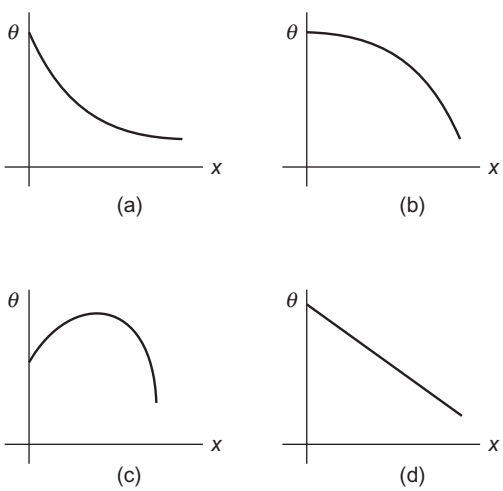


16. A small metal sphere of radius r and density ρ falls from rest in a viscous liquid of density σ and coefficient of viscosity η. Due to friction, heat is produced. The rate of production of heat when the sphere has acquired the terminal velocity is proportional to

(a) r^2 (b) r^3

(c) r^4 (d) r^5

17. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figures?



18. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is

(a) $\frac{1}{T}$ (b) $\frac{2}{T}$

(c) $\frac{3}{T}$ (d) $\frac{4}{T}$

19. The displacement y of a particle of a medium can be expressed as

$$y = \tan^{-6}\sin\left(100t + 20x + \frac{\pi}{4}\right)$$

where y and x are in metre and t in second. The speed of the wave (in ms^{-1}) is

(a) 2000 (b) 5

(c) 20 (d) 5π

20. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by $y = 0.02\ (m)\ \sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right]$. The tension in the string is

(a) 4.0 N (b) 12.5 N

(c) 0.5 N (d) 6.25 N

21. A simple pendulum attached to the ceiling of a stationary lift has a time period T. The distance y covered by the lift moving upwards varies with time t as $y = t^2$ where y is in metre and t in second. If g = 10 ms^{-2}, the time period of the pendulum will be

(a) $\sqrt{\frac{4}{5}}\,T$ (b) $\sqrt{\frac{5}{6}}\,T$

(c) $\sqrt{\frac{5}{4}}\,T$ (d) $\sqrt{\frac{6}{5}}\,T$

22. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is

(a) 5 grams (b) 10 grams

(c) 20 grams (d) 40 grams

23. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1: For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement-2: The net work done by a conservative force on an object moving along a closed loop is zero.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.

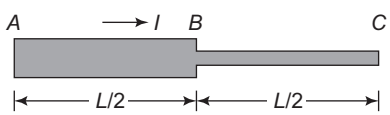
(b) Statement-1 is false, Statement-2 is true.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.

24. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7} \times 10^5$ Vm^{-1}. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} ms^{-1}. Given $g = 9.8$ ms^{-2}, viscosity of the air $= 1.8 \times 10^{-5}$ Ns m^{-2} and the density of oil = 900 kg m^{-3}, the magnitude of q is
    (a) 1.6×10^{-19} C (b) 3.2×10^{-19} C
    (c) 4.8×19^{-19} C (d) 8.0×10^{-19} C

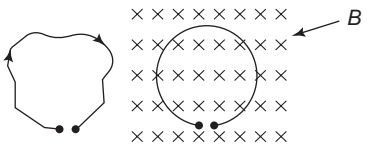
25. Two wires AB and BC, each of length $L/2$ are made of the same material. The radius of wire AB is $2r$ and of wire BC is r. The current I flows through the composite wire (see figure). Choose the correct statement from the following.
    (a) Potential difference across BC is twice that across AB.
    (b) Power dissipated in BC is four times the power dissipated in AB.
    (c) Current densities in AB and BC are equal.
    (d) Electric fields in AB and BC are equal.



26. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2, the respective temperature coefficients of their series and parallel combinations are nearly
    (a) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$ (b) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$
    (c) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$ (b) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$

27. A charged particle moves through a magnetic field perpendicular to its direction. Then
    (a) the momentum changes but the kinetic energy is constant
    (b) both momentum and kinetic energy of the particle are not constant
    (c) both, momentum and kinetic energy of the particle are constant
    (d) kinetic energy changes but the momentum is constant.

28. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the following figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is



    (a) IBL (b) $\frac{IBL}{\pi}$
    (c) $\frac{IBL}{2\pi}$ (d) $\frac{IBL}{4\pi}$

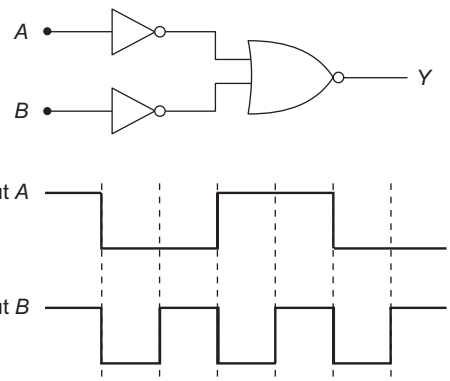
29. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased
    (a) the bulb glows dimmer
    (b) the bulb glows brighter
    (c) total impedance of the circuit is unchanged
    (d) total impedance of the circuit increases

30. In a series LCR circuit $R = 200\ \Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit, the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is
    (a) 242 W (b) 305 W
    (c) 210 W (d) Zero W

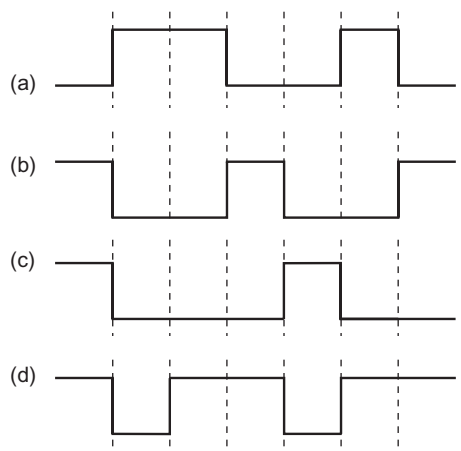
31. The rms value of the electric field of the light comming from the Sun is 720 N/C. The average total energy density of the electromagnetic wave is
    (a) 81.35×10^{-12} J/m^3
    (b) 3.3×10^{-3} J/m^3
    (c) 4.58×10^{-6} J/m^3
    (d) 6.37×10^{-9} J/m^3

32. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at P. The coordinates of P will be:
    (a) (f, f) (b) $(4f, 4f)$
    (c) $(2f, 2f)$ (d) $\left(\frac{f}{2}, \frac{f}{2}\right)$

33. An astronomical telescope has a large aperture to
    (a) reduce spherical aberration
    (b) have high resolution
    (c) increase span of observation
    (d) have low dispersion
34. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is
    (a) virtual and at a distance of 16 cm from the mirror
    (b) real and at a distance of 16 cm from the mirror
    (c) virtual and at a distance of 20 cm from the mirror
    (d) real and at a distance of 20 cm from the mirror
35. A parallel beam of fast moving electrons is incident normally on a narrow slit. A screen is placed at a large distance from the slit. If the speed of the electrons is increased, which of the following statements is correct?
    (a) Diffraction pattern is not observed on the screen in the case of electrons
    (b) The angular width of the central maximum of the diffraction pattern will increase
    (c) The angular width of the central maximum will decrease
    (d) The angular width of the central maximum will remain the same.
36. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately
    (a) 540 nm (b) 400 nm
    (c) 310 nm (d) 220 nm
37. The half-life of ^{215}At is 100 μs. The time taken for the radioactivity of a sample of ^{215}At to decay to 1/16th of its initial value is
    (a) 400 μs (b) 6.3 μs
    (c) 40 μs (d) 300 μs
38. If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called
    (a) X-rays
    (b) ultraviolet rays
    (c) microwaves
    (d) γ-rays
39. Suppose an electron is attracted towards the origin by a force $\frac{k}{r}$ where 'k' is a constant and 'r' is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the nth orbital of the electron is found to be 'r_n' and the kinetic energy of the electron to be 'T_n'. Then which of the following is true?
    (a) $T_n \propto \frac{1}{n}, r_n \propto n^2$
    (b) $T_n \propto \frac{1}{n^2}, r_n \propto n^2$
    (c) T_n independent of n, $r_n \propto n$
    (d) $T_n \propto \frac{1}{n}, r_n \propto n$
40. The logic circuit shown in the following figure has the input waveforms 'A' and 'B' as shown. Pick out the correct output waveform from the next figure.



Output is :



Answers

1. (c)	2. (b)	3. (c)	4. (c)
5. (d)	6. (d)	7. (a)	8. (b)
9. (c)	10. (d)	11. (b)	12. (b)
13. (b)	14. (d)	15. (a)	16. (d)

17. (d)	18. (c)	19. (b)	20. (d)
21. (b)	22. (b)	23. (d)	24. (d)
25. (b)	26. (a)	27. (a)	28. (c)
29. (b)	30. (a)	31. (c)	32. (c)
33. (b)	34. (b)	35. (c)	36. (c)
37. (a)	38. (a)	39. (c)	40. (c)

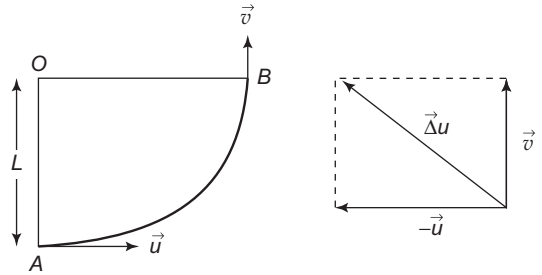
Solutions

1. Value of 1 main scale division = 0.5°
   30 vernier scale divisions
   = 29 × main scale divisions
   = 29 × 0.5°
   $\therefore$ Value of 1 vernier scale division = $\frac{29}{30}\times 0.5°$
   Least count = value of 1 m.s.d – value of 1 v.s.d.
   $=0.5° - \frac{29}{30}\times 0.5° = \frac{0.5°}{30} = \frac{0.5}{30}\times 60 \text{ min}$
   = 1 min

2. RC has dimension of time and V has the dimensions of $L\ \frac{dI}{dt}$. Hence
   $$\left[\frac{L}{RCV}\right] = \left[\frac{1}{T}\times\frac{T}{A}\right] = \frac{1}{\text{current}}$$

3. Given $v = k\sqrt{x} \Rightarrow v^2 = k^2x$. Differentiating, we have
   $$2v\frac{dv}{dt} = k^2\frac{dx}{dt} = k^2v \qquad \left(\because v = \frac{dx}{dt}\right)$$
   $$\Rightarrow \quad \frac{dv}{dt} = \frac{k^2}{2}$$
   $$\Rightarrow \quad \int dv = \frac{k^2}{2}\int dt$$
   $$\Rightarrow \quad v = \frac{k^2t}{2}$$
   $$\Rightarrow \quad \frac{dx}{dt} = \frac{k^2t}{2}$$
   $$\Rightarrow \quad \int dx = \frac{k^2}{2}\int t\ dt$$
   $$\Rightarrow \quad x = \frac{k^2}{4}t^2$$
   Thus $x \propto t^2$.

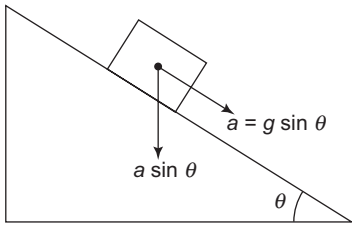
4. From energy conservation
   $$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgL$$
   $$\Rightarrow \quad v = \sqrt{u^2 - 2gL}$$
   $$\Delta v = \sqrt{v^2 + (-u)^2} = \sqrt{2(u^2 - gL)}$$



5. Given $v = \alpha\sqrt{x}$. Since $v = \frac{dx}{dt}$, we have
   $$\frac{dx}{dt} = \alpha\sqrt{x}$$
   or
   $$\frac{dx}{\sqrt{x}} = \alpha dt$$
   Integrating, we have
   $$\int_0^x \frac{dx}{\sqrt{x}} = \alpha\int_0^t dt$$
   which gives $-2\sqrt{x} = \alpha\ t$ or $x = \frac{\alpha^2t^2}{4}$. Hence x is proportional to t^2.

6. Given $\vec{v} = K\left(y\hat{\mathbf{i}} + x\hat{\mathbf{j}}\right)$. Hence
   $v_x = Ky$ and $v_y = Kx$. Therefore,
   $$\frac{dx}{dt} = Ky \quad \text{and} \quad \frac{dy}{dt} = Kx.$$
   Now
   $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{Kx}{Ky} = \frac{x}{y}$$
   $$\Rightarrow \quad y\,dy = x\,dx$$
   Integrating we get $\frac{y^2}{2} = \frac{x^2}{2} + c$
   where c = constant of integration.
   Hence $y^2 = x^2 + 2c$

7. Acceleration along the inclined plane is $a = g\sin\theta$. Vertical component of a is $a\cos(90° - \theta) = a\sin\theta = g\sin^2\theta$. [Refer figure]



For block A, vertical acceleration is $g \sin^2 (60°) = \frac{3g}{4}$ and for block B the vertical acceleration is $g \sin^2 (30°) = \frac{g}{4}$. Therefore the relative vertical acceleration of A with respect to $B = \frac{3g}{4} - \frac{g}{4} = \frac{g}{2} = 4.9\ \text{ms}^{-2}$.

8. The horizontal component of F parallel to the surface is $F \sin \theta$. Hence maximum value of F is given by

$$F \sin \theta = \mu mg$$

or $$F \sin 60° = 0.5 \times \sqrt{3} \times 10$$

or $$F \frac{\sqrt{3}}{2} = 0.5 \times \sqrt{3} \times 10$$

which gives $F = 10$ N.

9. Slope of graph $= -\frac{4\text{N}}{3\text{s}} = -\frac{4}{3}\text{Ns}^{-1}$. Therefore,

$F = -\frac{4}{3}t + 4$.

Now change in momentum $= \int F dt$

$$\Rightarrow \quad mv - 0 = \int_0^{4.5\text{s}} \left(-\frac{4}{3}t + 4\right) dt$$

or $$2v = \left| -\frac{2t^2}{3} + 4t \right|_0^{4.5\text{s}} = 4.5$$

which gives $v = 2.25\ \text{ms}^{-1}$. Therefore,

$$\text{K.E.} = \frac{1}{2} mv^2 = \frac{1}{2} \times 2 \times (2.25)^2$$

$$\simeq 5.06 \text{ J}.$$

10. $$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6} \qquad (1)$$

$\therefore \quad U(x = \infty) = 0$

Force $$F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{a}{x^{12}} - \frac{b}{x^6}\right)$$

$$\Rightarrow \quad F = -\left(\frac{12a}{x^{13}} + \frac{6b}{x^7}\right) \qquad (2)$$

At equilibrium $F = 0$. Putting $F = 0$ in Eq. (2) we get

$$x^6 = \frac{2a}{b}$$

Putting $x^6 = \frac{2a}{b}$ in Eq. (1) we get

$$U \text{ at equilibrium} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\frac{2a}{b}} = -\frac{b^2}{4a}$$

$\therefore \quad D = [U \text{ at } x = \infty - U \text{ at equilibrium}]$

$$= 0 - \left(-\frac{b^2}{4a}\right) = \frac{b^2}{4a}.$$

11. Use $mgh = \frac{1}{2} I\omega^2$ and $I = \frac{ml^2}{3}$.

12. The block will just begin to slide if the downward force $mg \sin \theta$ just overcomes the frictional force, i.e. if $mg \sin \theta = \mu N = \mu\, mg \cos \theta \Rightarrow \tan \theta = \mu = \sqrt{3}$

$\Rightarrow \theta = 60°$ [Refer figure]

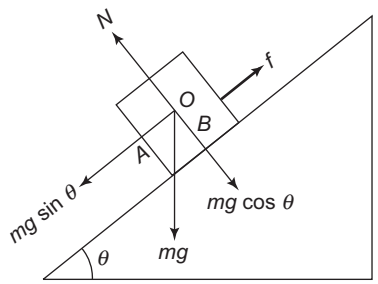
The block will topple if the torque due to normal reaction N about O just exceeds the torque due to $mg \sin \theta$ about 0, i.e.

$$N \times OA = mg \sin \theta \times OB$$

$\Rightarrow mg \cos \theta \times 5 \text{ cm} = mg \sin \theta \times \frac{15}{2} \text{ cm}$

$\Rightarrow \tan \theta = \frac{2}{3} \Rightarrow \theta \simeq 34°$.

Since θ for toppling is less than θ for sliding, the correct choice is (b).



13. Let x be the extension in the spring when it is loaded with mass M. The change in gravitational potential energy $= Mgx$. This must be the energy stored in the spring which is given by $\frac{1}{2} kx^2$. Thus

$\frac{1}{2} kx^2 = Mg\, x$ or $x = \frac{2Mg}{k}$

14. $v_e = \sqrt{\frac{GM}{R}}$. The correct choice is (d).

15. Since liquid 1 is above liquid 2, $\rho_1 < \rho_2$. Since the ball sinks in liquid 1, $\rho_3 > \rho_1$. Since the ball floats in liquid 2, $\rho_3 > \rho_2$. Hence $\rho_1 < \rho_3 < \rho_2$, which is choice (a).

16. The terminal velocity is

$$v_t = \frac{2}{9}\frac{(\rho-\sigma)r^2 g}{\eta}$$

The rate of production of heat (or power dissipated) is given by

$$P = fv_t, \text{ where } f = 6\pi\eta r v_t$$

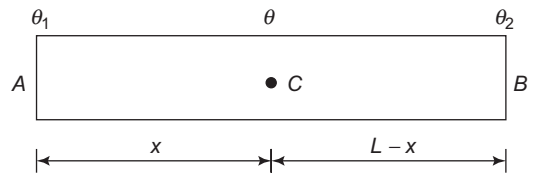
$$\therefore \quad P = 6\pi\eta\, r\, v_t \times v_t$$

$$= 6\,\pi\eta\, r\left[\frac{2}{9}\frac{(\rho-\sigma)r^2 g}{\eta}\right]^2$$

$$= \left[\frac{8\pi g^2}{27\eta}(\rho-\sigma)^2\right] r^5$$

i.e. $P \propto r^5$.

17. Let θ be the temperature at point C at a distance x from end A of the bar whose ends are kept at temperatures θ_1 and θ_2 with $\theta_1 > \theta_2$. L = length of the bar (see following figuer)



In the steady state, the rate of flow of heat from A to C = rate of flow of heat from C to B, i.e.

$$\frac{KA(\theta_1-\theta)}{x} = \frac{KA(\theta-\theta_2)}{(L-x)}$$

$$\Rightarrow \quad \theta = \theta_1 - \left(\frac{\theta_1-\theta_2}{L}\right)x$$

Thus, the graph of θ versus x is a straight line with a positive intercept and a negative slope.

18. Given $PT^2 = k$ (constant). From $PV = nRT$, we have

$P = \dfrac{nRT}{V}$. Hence

$$nRT^3 = kV$$

Differentiating we have

$$3nRT^2\Delta T = k\Delta V \Rightarrow \frac{\Delta V}{\Delta T} = \frac{3nRT^2}{k}$$

Coefficient of volume expansion is

$$\gamma = \frac{\Delta V}{V\Delta T} = \frac{3nRT^2}{kV} \qquad (1)$$

Using $V = \dfrac{nRT}{P}$ and $PT^2 = k$ in Eq. (1),

we get $\gamma = \dfrac{3}{T}$.

19. Compare the given equation with

$y = a \sin(\omega t + kx + \phi)$

Wave velocity $v = \dfrac{\omega}{k}$. The correct choice is (b).

20. $v = \sqrt{\dfrac{T}{\mu}} \Rightarrow T = \mu v^2 = \mu\left(\dfrac{\omega}{k}\right)^2$,

μ = mass per unit length of the string. The correct choice is (d).

21. Given $y = t^2$. The velocity of the lift varies with t as

$$v = \frac{dy}{dt} = 2t$$

$\therefore$ Acceleration $a = \dfrac{dv}{dt} = 2\ \text{ms}^{-2}$, directed upwards,

Hence

$$T' = 2\,\pi\sqrt{\frac{l}{g+a}}$$

and

$$T = 2\,\pi\sqrt{\frac{l}{g}}$$

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{g}{g+a}} = \sqrt{\frac{10}{(10+2)}} = \sqrt{\frac{5}{6}}$$

22. Let l be the length of the string and L the length of the pipe. The frequency of the string vibrating in the second harmonic is

$\nu = \dfrac{2}{2l}\sqrt{\dfrac{T}{\mu}}$; μ = mass per unit length of the string

The fundamental frequency of the closed pipe is

$$\nu' = \frac{v}{4L} \text{ ; } v = \text{speed of sound}$$

For resonance, $\nu = \nu'$, i.e.

$$\frac{1}{l}\sqrt{\frac{T}{\mu}} = \frac{v}{4L}$$

Substituting the values of l, T, v and L, we get

$$\mu = \frac{1}{50}\ \text{kg m}^{-1}$$

$\therefore$ Mass of string $= \mu l = \dfrac{1}{50} \times 0.5 = 10^{-2}$ kg = 10 grams. So the correct choice is (b).

23. Electrostatic field is conservative. For a conservative field, the work done to take a charged particle from a point P to another point Q is independent of the path followed by the particle to go from P to Q. In addition, work done to take the particle from Q to P is equal and opposite in sign. Hence the net work done in moving the particle from P to Q and then back to P is zero. So the correct choice is (d).

24. Terminal velocity v is given by

$$6\,\pi\,\eta\,r\,v = mg = \left(\frac{4\pi r^3}{3}\right)\rho g$$

Substituting the values of η, v, ρ and g, we get $r = \frac{3}{7}\times 10^{-5}$ m. The oil drop will be balanced in air if

$$qE = mg = \left(\frac{4\pi r^3}{3}\right)\rho g$$

Substituting the values of E, r, ρ and g, we get $q = 8.0 \times 10^{-19}$ C.

25. $R = \frac{\rho l}{\pi r^2}$. Since the two wires are made of the same material, resistivity ρ is the same for wires AB and BC. Since the wires have equal lengths, it follows that $R \propto 1/r^2$. Hence

$$\frac{R_{AB}}{R_{BC}} = \frac{1}{4}, \text{ i.e } R_{BC} = 4R_{AB}$$

Since the current, is the same in the two wires, it follows from Ohm's law ($V = IR$) that $V_{BC} = 4\ V_{AB}$. Hence choice (a) is wrong. Now power dissipated is $P = I^2\ R$. Since I is the same, $P \propto R$. Hence

$$\frac{P_{BC}}{P_{AB}} = \frac{R_{AB}}{R_{BC}} = 4$$

Hence chioce (b) is correct. Choice (c) is wrong because current density (i.e. current per unit area) is different in wires AB and BC because their cross-sectional areas are different. The electric field in a wire is $E = V/l$. Since the two wires have the same length (l), E is proportional to potential difference (V). Since $V_{BC} = 4\ V_{AB}$, $E_{BC} = 4E_{AB}$. Hence choice (d) is also incorrect.

26. The resistance of a conductor at temperature t°C is given by

$$R = R_0\,(1 + \alpha t)$$

where R_0 is the resistance at 0°C

For series combination

$$R_s = R_1 + R_2$$

At 0°C, $\quad R_s = R_0 + R_0 = 2R_0$

$$\therefore \quad 2R_0\,(1 + \alpha_s\,t) = R_0\,(1 + \alpha_1 t), + R_0\,(1 + \alpha_2\,t)$$

$$\Rightarrow \qquad \alpha_s = \frac{1}{2}\,(\alpha_1 + \alpha_2)$$

For parallel combination

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

At 0°C, $\quad \frac{1}{R_p} = \frac{1}{R_0} + \frac{1}{R_0} = \frac{2}{R_0} \Rightarrow R_p = \frac{R_0}{2}$

$$\therefore \quad \frac{1}{\frac{R_0}{2}(1+\alpha_p t)} = \frac{1}{R_0\,(1+\alpha_1 t)} + \frac{1}{R_0\,(1+\alpha_2 t)}$$

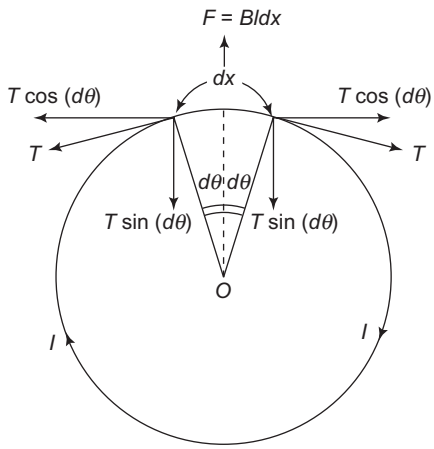
$$\Rightarrow \quad 2\,(1 + \alpha_p\,t)^{-1} = (1 + \alpha_1\,t)^{-1} + (1 + \alpha_2\,t)^{-1}$$

$$\Rightarrow \quad 2\,(1 - \alpha_p\,t) = (1 - \alpha_1\,t) + (1 - \alpha_2\,t) \qquad (\because\ \alpha t << 1)$$

$$\Rightarrow \qquad \alpha_p = \frac{1}{2}\,(\alpha_1 + \alpha_2)$$

27. The magnetic force experienced by the charged particle is perpendicular to its velocity. Hence the force does no work on the particle. Therefore, the speed and hence the kinetic energy of the particle remain unchanged. Since the velocity of the particle is perpendicular to the magnetic field, it will move along a circular path in the region of the magnetic field. Therefore, its velocity and hence linear momentum will continuously change due to change in the direction of motion of the particle moving a circle. Hence the correct choice is (a).

28. Consider a small element of the wire of length dx. The horizontal components T cos $(d\theta)$ cancel each other. The vertical components add up to $2T$ sin $(d\theta)$ because in the limit $d\theta \to 0$, they are collinear. The magnetic force an element dx is $F = B\,I\,dx$ vertically upward. [see following figure]



For equilibrium,

$$2T \sin (d\theta) = B\,I\,dx = B\,I\,R\,(2\,d\theta)$$

where R is the radius of the ring. For small $d\theta$, $\sin d\theta = d\theta$.

Hence $\quad 2T\,d\theta = 2\,B\,I\,R\,d\theta$

$$\Rightarrow \qquad T = B\,I\,R = \frac{BIL}{2\pi} \qquad (\because\ L = 2\pi R)$$

29. Impedance $Z = \left[R^2 + \frac{1}{(\omega C)^2}\right]^{1/2}$, $I_0 = \frac{V}{Z}$. As ω increases, Z decreases. Hence the current I_0 increases. Therefore, the bulb glows brighter, which is choice (b).

30. In a series LCR circuit, the phase angle ϕ between voltage and current is given by

$$\tan\phi = \frac{\omega L - \dfrac{1}{\omega C}}{R}$$

If capacitor is taken out, $\tan\phi = \dfrac{\omega L}{R}$

$\Rightarrow \tan 30° = \dfrac{\omega L}{R}$. If inductor is taken out,

$\tan\phi = -\dfrac{1}{\omega CR}$

$\Rightarrow -\tan 30° = -\dfrac{1}{\omega CR}$. Hence

$$\frac{\omega L}{R} = \frac{1}{\omega CR}$$

$$\Rightarrow \quad \omega L = \frac{1}{\omega C}$$

This is the resonance condition. At resonance, impedance $Z = R$. Hence

$$P = \frac{V^2}{R} = \frac{(220)^2}{200} = 242 \text{ W}$$

31. Total energy density $= \dfrac{1}{2}\varepsilon_0 E_0^2$

$$= \varepsilon_0 E_{\text{rms}}^2 \qquad \left[\because E_{\text{rms}} = \frac{E_0}{\sqrt{2}}\right]$$

$$= (8.85\times10^{-12})\times(720)^2$$

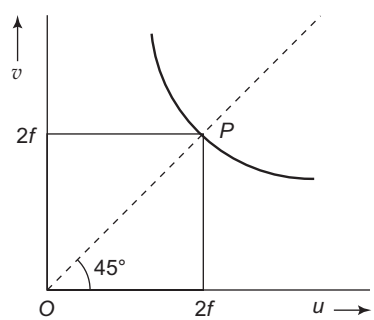
$$= 4.58 \times 10^{-6} \text{ Jm}^{-3}$$

32. For a convex lens forming a real image

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad (1)$$

The graph of v against u is as given in the following figure.

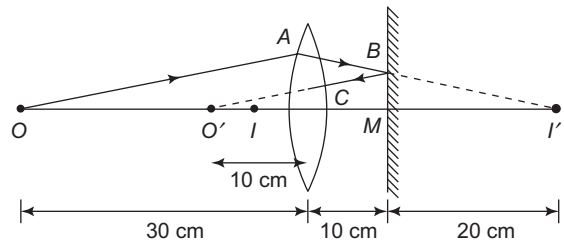
Since the u and v scales are the same, $u = v$ at point P. Putting $u = v$ in Eq. (1) gives $u = 2f$. Also $v = 2f$. Hence the correct choice is (c).



33. Resolving power $= \dfrac{D}{1.22\lambda}$, where D = diameter (aperture) of the objective of the telescope. Hence the correct choice is (b).

34. For refraction of ray OA by the lens, [see following figure]

$$\frac{1}{v'} - \frac{1}{-30} = \frac{1}{15} \Rightarrow v' = 30 \text{ cm}$$



So the image I' is 20 cm from the mirror. After reflection from the mirror, the virtual image O' is such that $O'M = MI'$. So O' is 10 cm to the left of the lens. O' serves as the virtual object for the refraction of ray BC by the lens. For this refraction, $u = +10$ cm. Hence

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{10} = \frac{1}{15} \Rightarrow v = 6 \text{ cm}$$

Hence the final real image I will be at a distance of $10 + 6 = 16$ cm from the mirror.

35. de Broglie wavelength of electron is

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

If speed v of electron is increased, momentum $p(= mv)$ will increase. Hence wavelength λ will decrease. Now, the angular width of the central maximum of the diffraction pattern is 2θ where θ is given by

$$\sin\theta = \frac{\lambda}{a}$$

where a is the width of the slit. Thus, if λ decreases, θ and hence 2θ will decrease. Therefore, the correct statement is (c).

36. Use $\lambda_{\text{max}} = \dfrac{hc}{W_0}$. The correct choice is (c).

37. Since $\dfrac{1}{16} = \dfrac{1}{2^4}$, the time taken for the sample to decay to $\dfrac{1}{16}$th of initial value = 4 half lives = $4 \times 100\ \mu\text{s} = 400\ \mu\text{s}$

38. Power $P = 4 \times 10^3$ W $= 4 \times 10^3$ Js^{-1}
Since the energy of photon is $h\nu$, the power of 10^{20} photons $= 10^{20} \times h\nu$ Js^{-1}. Hence
$10^{20}\, h\nu = 4 \times 10^3$

$$\Rightarrow \qquad \nu = \frac{4\times10^3}{10^{20}\times6.6\times10^{-34}} \simeq 6 \times 10^{16} \text{ Hz}$$

The corresponding wavelength is

$$\lambda = \frac{c}{\nu} = \frac{3\times10^8}{6\times10^{16}} = 5 \times 10^{-9} \text{ m} = 50 \text{ Å}$$

This is the wavelength corresponding to X-rays.

39. If v_n is the speed of the electron in the nth orbit, then

$$\frac{m v_n^2}{r_n} = \frac{k}{r_n}$$

$$\Rightarrow \qquad v_n^2 = \frac{k}{m}$$

$$\therefore \qquad T_n = \frac{1}{2} m v_n^2 = \frac{k}{2}, \text{ which is independent of } n.$$

Also, angular momentum, $L_n = \dfrac{nh}{2\pi}$

or $\quad m\, v_n\, r_n = \dfrac{nh}{2\pi}$ which gives

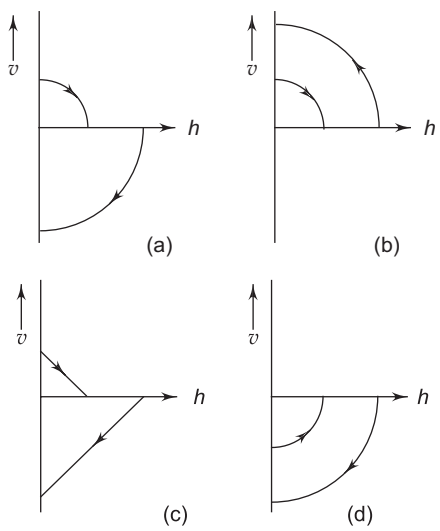
$$r_n = \frac{nh}{2\pi m v_n} = \frac{nh}{2\pi m}\sqrt{\frac{m}{k}} = \frac{nh}{2\pi\sqrt{mk}}$$

Thus $r_n \propto n$. Hence the correct choice is (c).

40. From Figures, it follows that when $A = 0$ and $B = 0$, $Y = 0$, when $A = 0$ and $B = 1$; $Y = 1$, when $A = 1$ and $B = 0$, $Y = 1$ and when $A = B = 1$ then $Y = 1$. This is the truth table of OR gate.

Practice Test Paper – II

1. A vernier calliper in which 10 divisions of vernier scale coincide with 9 divisions of main scale (each division of the main scale is 1 mm apart) is used to measure the edge length of a cube of mass 4.832 g. If the main scale reading is 12 mm and the 2nd division of vernier scale coincides with main scale, the density of the cube (in g cm^{-3}) up to appropriate significant figure is
   (a) 2.661 (b) 2.66
   (c) 2.67 (d) 2.660
2. A ball is dropped vertically from a height h above the ground. It hits the ground and bounces up vertically to a height $h/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h as

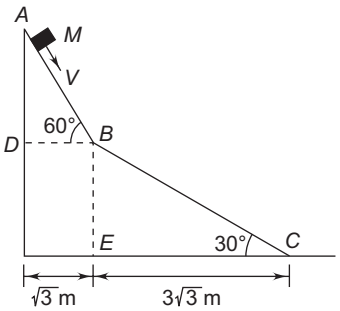


3. The coordinates of a particle moving in a plane are given by
   $x = a \cos(pt)$
   and $y = b \sin(pt)$
   where a, b and p are positive constants and $b < a$.
   Then
   (a) the path of the particle is a parabola
   (b) the velocity and acceleration of the particle are perpendicular to each other at $t = \pi/2p$.
   (c) the acceleration of the particle is always directed towards a focus
   (d) the distance travelled by the particle in time interval $t = 0$ to $t = \pi/2p$ is a.
4. When a bicycle is in motion and the pedalling is stopped, the force of friction exerted by the ground acts.
   (a) in the backward direction on both the front and the rear wheels
   (b) in the forward direction on both the front and the rear wheels
   (c) in the forward direction on the front wheel and in the backward direction on the rear wheel
   (d) in the backward direction on the front wheel and in the forward direction on the rear wheel
5. A piece of uniform string hangs vertically so that its free end just touches the horizontal surface of a table. The upper end of the string is now released. At any time during the falling of the string, the total force on the surface of the table is n times the weight of the part of the string lying on the surface. The value of n is
   (a) 1 (b) 2
   (c) 3 (d) 4

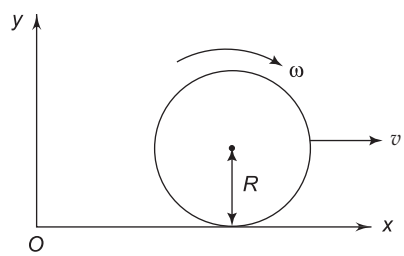
Questions 6 and 7 are based on the following paragraph.

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of

the incline suddenly change from 60° to 30° at point *B*. The block is initially at rest at *A*. Assume that collisions between the block and the incline are totally inelastic ($g = 10\ m/s^2$)



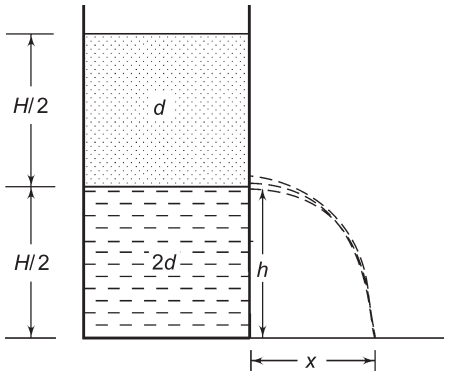
6. The speed of the block at point *B* immediately after it strikes the second incline is
   (a) $\sqrt{60}\ ms^{-1}$ (b) $\sqrt{45}\ ms^{-1}$
   (c) $\sqrt{30}\ ms^{\pm 1}$ (d) $\sqrt{15}\ ms^{\pm 1}$
7. The speed of the block at point *C*, immediately before it leaves the second incline is
   (a) $\sqrt{120}\ ms^{-1}$ (b) $\sqrt{105}\ ms^{-1}$
   (c) $\sqrt{90}\ ms^{-1}$ (d) $\sqrt{75}\ ms^{-1}$
8. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statements is are correct for the system of these two masses?
   (a) Total momentum of the system is 12 kg ms^{-1}
   (b) Momentum of 5 kg mass after collision is 4 kg ms^{-1}
   (c) Kinetic energy of the centre of mass is 0.75 J
   (d) Total kinetic energy of the system is 4 J
9. A disc of mass *M* and radius *R* is rolling with angular speed ω on a horizontal surface as shown in the figure. The magnitude of angular momentum of the disc about the origin *O* is



   (a) $\frac{1}{2} MR^2 \omega$
   (b) $MR^2 \omega$
   (c) $\frac{3}{2} MR^2 \omega$
   (d) $2\ MR^2 \omega$
10. **Statement-1 :** Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.
    Statement-2 : By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.
    (a) Statement-1 and 2 are both true and Statement-2 is the correct explanation of Statement-1.
    (b) Statement-1 and 2 are both true but Statement-2 is not the correct explanation of Statement-1.
    (c) Statement-1 is true and Statement-2 is false.
    (d) Statement-2 is true and Statement-1 is false.
11. A satellite of mass *m* is moving in a circular orbit of radius *r* around a planet of mass *M*.
    (a) The magnitude of angular momentum with respect to the centre of the orbit is $m\sqrt{GMr}$, where *G* is the gravitation constant.
    (b) The magnitude of the angular momentum is $mR\sqrt{2gr}$ where *g* is the acceleration due to gravity on the surface of the planet.
    (c) The direction of angular momentum is parallel to the plane of the orbit.
    (d) The direction of angular momentum is inclined at 45° to the plane of the orbit.
12. A spring balance *A* reads 2 kg with a block suspended from it. Another balance *B* reads 5 kg when a beaker with liquid is put on the pan of the balance. When the block is immersed in water
    (a) Balance *A* will read 2 kg
    (b) Balance *A* will read more than 2 kg
    (c) Balance *B* will read 5 kg
    (d) Balance *B* will read more than 5 kg.

Questions 13 and 14 are based on the following passage.

A container of large uniform cross-sectional area *A*, resting on a horizontal surface, holds two immiscible non-viscous and incompressible liquids of densities *d* and 2*d*, each of a height *H*/2 as shown in the figure. The lower density liquid is open to the atmosphere having pressure P_0. A tiny hole of area *s* ($s << A$) is punched on the vertical side of the container at a height *h* ($h = H/2$).



13. The initial speed of efflux of the liquid at the hole is

(a) $\sqrt{(3H-2h)\frac{g}{2}}$ (b) $\sqrt{(2H-h)g}$

(c) $\sqrt{(3H-4h)\frac{g}{2}}$ (d) $\sqrt{2gh}$

14. The horizontal distance x travelled by the liquid is

(a) $\sqrt{h(3H-4h)}$ (b) $\sqrt{h(2H-3h)}$

(c) $\sqrt{H(3H-2h)}$ (d) $\sqrt{H(3H-h)}$

Questions 15 and 16 are based on the following passage.

A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. When the cylinder is given a small downward push and released, it starts oscillating vertically with a small amplitude.

15. If the cylinder is given a small downward displacement x from the equilibrium position and released, the restoring force F acting on it is

(a) $-Mgx$ (b) $-(k + A\sigma g)x$

(c) $= -\left(k - \frac{1}{2}A\sigma g\right)x$ (d) $-\left(k + \frac{1}{2}A\sigma g\right)x$

16. The time period T of the vertical oscillations of the cylinder is

(a) $2\pi\sqrt{\frac{M}{k}}$

(b) $2\pi\left[\frac{M}{\left(k+\frac{1}{2}A\sigma g\right)}\right]^{1/2}$

(c) $2\pi\left[\frac{M}{(k-A\sigma g)}\right]^{1/2}$

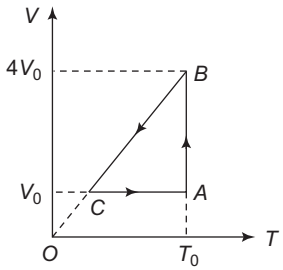
(d) $2\pi\left[\frac{M}{k+A\sigma g}\right]^{1/2}$

17. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}.

(a) 0.98 ms^{-1} (b) 1.98 ms^{-1}

(c) 2.98 ms^{-1} (d) 3.98 ms^{-1}

18. One mole of an ideal gas in initial state A undergoes a cyclic process $ABCA$, as shown in the figure. Its pressure at A is P_0.



(a) Internal energy at B is greater than that at A.

(b) Work done by the gas in process $A \rightarrow B$ is $P_0 V_0$

(c) Pressure at C is $\frac{P_0}{4}$

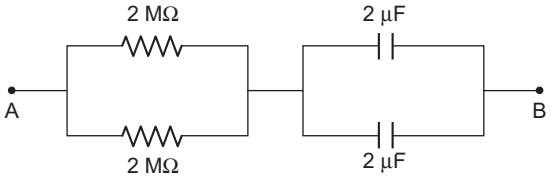
(d) Temperature at C is $\frac{T_0}{2}$

19. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperature T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B?

(a) 3 (b) 6

(c) 9 (d) 12

20. At time $t = 0$, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, the time t (in seconds) when the voltage across the capacitors become 4V is



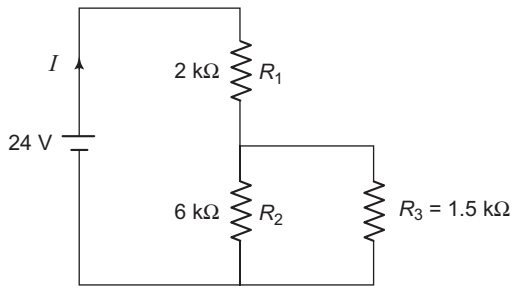
(a) 3 ln $\left(\frac{5}{2}\right)$ (b) 4 ln $\left(\frac{5}{3}\right)$

(c) 5 ln $\left(\frac{4}{3}\right)$ (d) 10 ln (4)

21. In the circuit shown in the figure, the ratio of powers dissipated in R_1 and R_2 is

(a) $\frac{27}{4}$ (b) $\frac{25}{3}$

(c) $\frac{23}{2}$ (d) 9



22. **Statement 1 :** Two particles having equal charges and masses m_1 and m_2, after being accelerated by the same potential difference (V), enter a region of uniform magnetic field and describe circular paths of radii r_1 and r_2 respectively. Then

$$\frac{m_1}{m_2} = \sqrt{\frac{r_1}{r_2}}$$

Statement 2 : Gain in kinetic energy = work done to accelerate the charged particle through potential difference V.

(a) Both statements are true

(b) Both statements are false

(c) Statement-1 is true, and Statement-2 is false.

(d) Statement-1 is false, and Statement-2 is true.

Questions 23 and 24 are based on the following passage.

An *LCR* series circuit with 100 Ω resistance is connected to an a.c. source of 200 V and angular frequency 300 rad/sec. When only the capacitance is removed, the current leads the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°.

23. The current in the circuit is

(a) $\sqrt{2}$ A (b) 2 A

(c) $2\sqrt{2}$ A (d) 1 A

24. The power dissipated in the circuit is

(a) 200 W (b) 400 W

(c) 800 W (d) 100 W

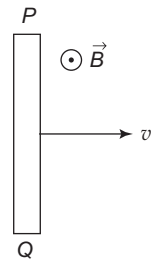
25. A metal rod PQ moves at a constant vlocity v in a direction perpendicular to its length. A constant uniform magnetic field $\vec{B}$ exists in a direction perpendicular to the rod as well as its velocity as shown in the figure.

(a) There is no electric field in the rod

(b) The electric potential is the same at every point on the rod

(c) There is no induced current in the rod

(d) The induced current flows from P at Q.



26. The radius of curvature of the curved face of a thin planoconvex lens is 10 cm and it is made of glass of refractive index 1.5. A small object is approaching the lens with a speed of 1 cm s^{-1} moving along the principal axis. When the object is at a distance of 30 cm from the lens, the megnitude of the speed of the image is

(a) 1 cm s^{-1} (b) 2 cm s^{-1}

(c) 3 cm s^{-1} (d) 4 cm s^{-1}

27. A glass plate of refractive index $\mu_3 = 1.5$ is coated with a thin layer of thickness t and refractive index $\mu_2 = 1.8$. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere. If $\lambda = 648$ nm, the least value of t for which the waves interface constructively is

(a) 90 nm (b) 180 nm

(c) 108 nm (d) 216 nm

28. The de Broglie wavelength of an electron moving with a velocity of 1.5×10^8 ms^{-1} is equal to that of a photon. The ratio of the kinetic energy of the photon to that of the electron is

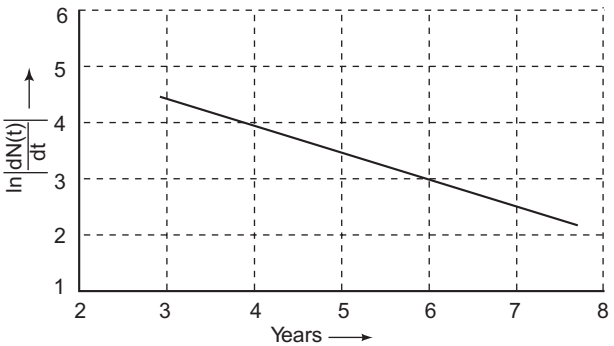
(a) 1 (b) 2

(c) 3 (d) 4

29. To determine the half of radioactive element, a student plots a graph of $\ln \left|\frac{dN(t)}{dt}\right|$ versus t. Hence $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, find the value of p.

(a) 2 (b) 4

(c) 8 (d) 16



30. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement is true ?
    (a) They will never come out of the magnetic field region.
    (b) They will come out travelling along different inclined paths.
    (c) They will come out at the same time.
    (d) They will come out at different times.

Answers

1. (b)	2. (a)	3. (b)	4. (a)
5. (c)	6. (b)	7. (b)	8. (c)
9. (c)	10. (d)	11. (a)	12. (d)
13. (c)	14. (a)	15. (b)	16. (d)
17. (b)	18. (c)	19. (c)	20. (b)
21. (b)	22. (d)	23. (b)	24. (b)
25. (d)	26. (d)	27. (a)	28. (d)
29. (c)	30. (d)		

Solutions

1. Vernier constant (V.C.) = 1 mm – $\left(\frac{9}{10}\right)$ mm. = 0.1 mm.

$\therefore$ l = 12 mm + 2 × V.C.
= 12 mm + 2 × 0.1 mm
= 12.2 mm = 1.22 cm

$$\text{Density } (\rho) = \frac{4.832g}{(1.22\,\text{cm})^3} = 2.661 \text{ g cm}^{-3}$$

Since the least accurate quantity (namely l) has 3 significant figures, the value of ρ is rounded off to 3 significant figures. Hence $\rho = 2.66$ g cm^{-3}

2. The velocity at a height h is given by $v^2 = u^2 + 2gh$. For downward motion, $u = 0$ and the value of g is negative and h becomes more and more negative. Hence v^2 increases with h. Since the velocity vector is directed downwards, v becomes more and more negative. Since $v^2 \propto h$, the graph of v versus h is parabolic. Hence graphs (c) and (d) are wrong. For upward motion, $v^2 = u^2 + 2gh$. Here g is directed downwards and h is positive. Consequently, v^2 decreases with h. Since the direction of the velocity vector is positive, v becomes less and less positive. Here also the variation of v with h is parabolic. Since v becomes less and less positive, graph (b) is not correct. Hence the correct choice is (a).

3. Given $x = a\cos(pt)$ (1)

$y = b\sin(pt)$ (2)

From (1) and (2) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \cos^2(pt) + \sin^2(pt) = 1$

Hence the path of the particle is an ellipse.

Let the position vector of the particle at time t be

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\therefore \text{ Velocity } \vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{x}}{dt}\hat{i} + \frac{d\vec{y}}{dt}\hat{j}$$

$$\Rightarrow \quad \vec{v} = -ap\sin(pt)\hat{i} + bp\cos(pt)\hat{j}$$

$$\text{At } t = \pi/2p, \ \vec{v} = -ap\sin\left(\frac{\pi}{2}\right)\hat{i} + bp\cos\left(\frac{\pi}{2}\right)\hat{j}$$

$$= -ap\hat{i}$$

$$\text{Acceleration } \vec{a} = -ap^2\cos(pt)\hat{i} - bp^2\sin(pt)\hat{j}$$

$$\text{At } t = \pi/2p, \ \vec{a} = -bp^2\hat{j}$$

$$\therefore \text{ At } t = \pi/2p, \ \vec{v}\cdot\vec{a} = abp^3\left(\hat{i}\cdot\hat{j}\right) = 0$$

Hence $\vec{v} \perp \vec{a}$

It is easy to see that choice (c) and (d) are incorrect.

4. When an external force is applied to move a body, the force of friction acts in the opposite direction. But when a body itself applies a force in order to move, the force of friction acts in the direction of motion. While pedalling, the external force is applied to the rear wheel and as a result the front wheel moves by itself. So, while pedalling, the force of friction by the ground acts in the backward direction on the forntwheel and in the forward direction on the rear wheel. When pedalling is stopped, the force of friction by the ground acts in the backward direction on both the front and the rear wheels as long as bicycle remains in motion. So the correct choice is (a).

5. Let x be the length of the string lying on the surface on the table at an instant of time t. If an additional length dx of the string falls on the surface in time dt, the velocity v of this element when it strikes the surface is given by ($\because u = 0$)

$$v^2 = u^2 + 2gx = 0 + 2gx$$

or $$v^2 = 2gx \quad (1)$$

The total force on the surface is

F = rate of change of momentum of element of length dx + weight of a length x of the string lying on the table.

If m is the mass per unit length of the string, then

$$F = \frac{d}{dt}(mdxv) + mxg = mv\frac{dx}{dt} + mxg = mv^2 + mxg \quad (2)$$

$$\left(\because v = \frac{dx}{dt}\right)$$

Using (1) in (2) we get

$$F = 2\,mxg + mxg = 3\,mxg$$

But $mx = M$, the mass of the string lying on the table. Hence

$$F = 3\,Mg$$

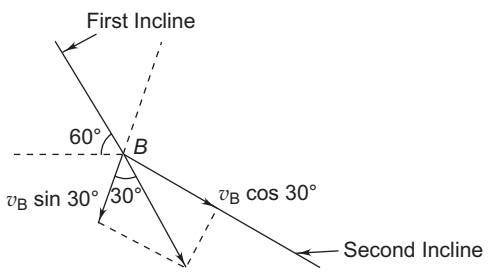
Thus $n = 3$.

6. Let v_B be the speed of the block just before it strikes the second incline.

$$\frac{1}{2}mv_B^2 = mg \times AD = mg \times BD\tan 60°$$

$$\Rightarrow v_B = (2 \times 10 \times \sqrt{3}\tan 60°)^{1/2} = \sqrt{60}\ \text{ms}^{-1}$$

This velocity can be resolved into two components. $v_B \cos 30°$ along the second incline and $v_B \sin 30°$ perpendicular to it (see the following figure)



In an inelastic collision, the perpendicular component becomes zero after the collision. Hence the speed of the block at point B immediately after the collision is $v_B \cos 30° = \sqrt{60} \times \frac{\sqrt{3}}{2} = \sqrt{45}\ \text{ms}^{-1}$.

7. Let v_C be the speed at C. From conservation of energy, gain in K.E. = loss in P.E., i.e.

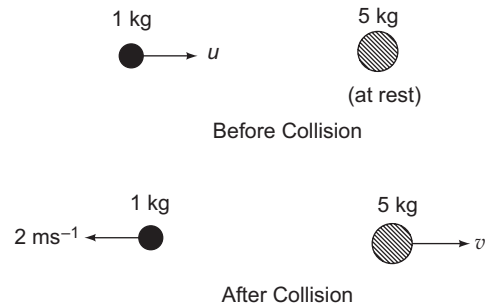
$$\frac{1}{2}mv_C^2 - \frac{1}{2}mv_B^2 = mg \times BE = mg \times EC\tan 30°$$

$$\Rightarrow \quad v_C^2 = v_B^2 + 2g \times 3\sqrt{3}\ \tan 30°$$

$$= 45 + 2 \times 10 \times 3\sqrt{3} \times \frac{1}{\sqrt{3}} = 105$$

$$\Rightarrow \quad v_C = \sqrt{105}\ \text{ms}^{-1}.$$

8.



From conservation of linear momentum,

$$u = 5v - 2$$

$$\Rightarrow \quad u + 2 = 5v \quad (1)$$

From conservation of kinetic energy,

$$u^2 = 4 + 5v^2$$

$$\Rightarrow \quad (u - 2)(u + 2) = 5v^2 \quad (2)$$

From Eqs. (1) and (2), we get

$$u - 2 = v \quad (3)$$

Solving Eqs. (1) and (3), we get $u = 3\ \text{ms}^{-1}$ and $v = 1\ \text{ms}^{-1}$.

(a) Total momentum = $1\ \text{kg} \times 3\ \text{ms}^{-1} = 3\ \text{kg ms}^{-1}$

(b) Momentum of 5 kg mass after collision $= 5\ \text{kg} \times 1\ \text{ms}^{-1} = 5\ \text{kg ms}^{-1}$

(c) Velocity of centre of mass is

$$v_{CM} = \frac{1 \times u + 5 \times 0}{1 + 5} = \frac{u}{6} = \frac{1}{2}\ \text{ms}^{-1}$$

Kinetic energy of centre of mass is

$$K_{CM} = \frac{1}{2}(1 + 5) \times \left(\frac{1}{2}\right)^2 = 0.75\ \text{J}$$

(d) Total kinetic energy of the system is

$$K = \frac{1}{2} \times 1 \times (3)^2 + 0 = 4.5\ \text{J}$$

So the correct choice is (c).

9. The angular momentum about O is

$$\vec{L}_O = \vec{L}_{CM} + M(\vec{R} \times \vec{v})$$

Its magnitude is $(\because \vec{R} \perp \vec{v})$ and $L_{CM} = I\omega$

$$L_O = I\omega + MRv$$

$$= \left(\frac{1}{2}MR^2\right)\omega + MR \times R\omega \qquad (\because\ v = R\omega)$$

$$= \frac{3}{2}MR^2\omega$$

10. The acceleration of the centre of mass is

$$a = \frac{g\sin\theta}{\left(1+\frac{I}{mR^2}\right)}$$

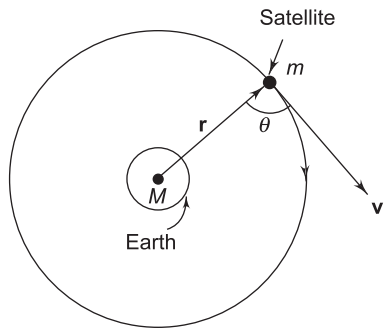
For a hollow cylinder $I_h = mR^2$

For a solid cylinder $I_s = \frac{1}{2}\ mR^2$

Since $I_h > I_s$, $a_h < a_s$. Hence the solid cylinder will reach the bottom before the hollow cylinder. From the principle of conservation of energy, kinetic energy at the bottom = potential energy (= mgh) which is the same for both the cylinders.

11. At a certain instant of time let r be the radius vector of the satellite from the centre of its circular orbit. If the velocity of the satellite is **v** as shown in the figure, its angular momentum is given by

$$\mathbf{L} = \mathbf{r} \times (m\mathbf{v})$$



The magnitude of angular momentum is

$$L = mrv \sin\theta$$

where θ is the angle between vectors **r** and **v**. For a circular orbit, $\theta = 90°$. Therefore

$$L = mrv \qquad (1)$$

The gravitational force of attraction on the satellite is GMm/r^2 which provides the necessary centripetal force mv^2/r, i.e.

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

or $(mrv) = m\sqrt{GMr}$ (2)

From Eqs. (1) and (2), we have

$$L = m\sqrt{GMr}$$

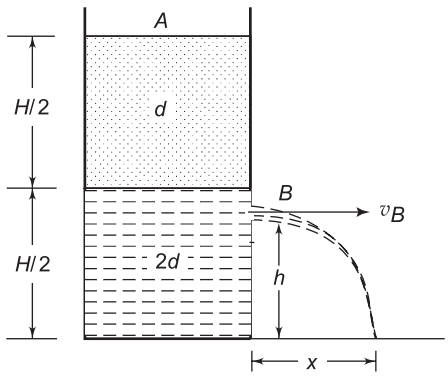
This gives the magnitude of angular momentum. The direction of angular momentum is perpendicular to the plane of the orbit.

12. Due to the upward force of buoyancy on the block exerted by the liquid, the apparent weight of the block will be less than 2 kg. The hanging block exerts a downward force on the liquid (and the beaker) equal in magnitude to the upward buoyant force. Therefore, balance B will read more than 5 kg.

13. The initial speed of efflux of the liquid at the hole at point B can be determined by applying Bernoulli's theorem at the top (point A) of the liquids and at point B where the hole is punched. If v_A and v_B are the speeds of the liquid at points A and B respectively, then from the equation of continuity, we have

$$Av_A = sv_B$$

or $v_A = \left(\frac{s}{A}\right) v_B \approx 0\ (\because\ s << A)$



Applying Bernoulli' theorem at points A and B, we get

$$P_A + \frac{1}{2} + dv_A^2 + dg\left(\frac{H}{2}\right) + (2d)g\left(\frac{H}{2}\right)$$

$$= P_B + \frac{1}{2}(2d)v_B^2 + (2d)gh$$

Now $P_A = P_B$ = atmospheric pressure (P_0) and $v_A \approx 0$. Hence, we get

$$P_0 + 0 + \frac{1}{2}dgH + dgH = P_0 + dv_B^2 + 2dgh$$

Which gives $v_B = \left[\left(\frac{3}{2}H - 2h\right)g\right]^{1/2}$

14. The time taken by the liquid to fall through a vertical height h is given by

$$t = \sqrt{\frac{2h}{g}}$$

The horizontal distance x travelled by the liquid in time t moving with a constant horizontal velocity v_B is

$$x = v_B\, t = \left[\left(\frac{3}{2}H - 2h\right)g\right]^{1/2} \times \left(\frac{2h}{g}\right)^{\frac{1}{2}}$$

$$= [h\,(3H - 4h]^{1/2} \qquad (1)$$

15. The upthrust on the cylinder with half its length submerged in the liquid is given by

U = weight of the liquid displaced by a length $L/2$ of the cylinder

$$= A \times \frac{L}{2} \times \sigma \times g = \frac{L}{2}\ (A\sigma g)$$

Let x_0 be the extension of the spring when it is in equilibrium. Then

$$kx_0 = Mg - \frac{L}{2}(A\sigma g) \qquad (1)$$

Let x be the small downward displacement given to the cylinder so that the submerged length of the cylinder is now $\left(\frac{L}{2}+x\right)$ and the extension of the spring is now $(x_0 + x)$. The upthrust now is $\left(\frac{L}{2}+x\right)A\sigma g$ and the force in the spring is $k\,(x_0 + x)$. Hence, the restoring force on the cylinder is

$$F = -\left[k(x_0+x) - \left\{Mg - \left(\frac{L}{2}+x\right)A\sigma g\right\}\right] \qquad (2)$$

Using Eq. (1) in Eq. (2), we have

$$F = -(kx + A\sigma g x)$$

or $$F = -(k + A\sigma g)x$$

16. The acceleration of the cylinder is

$$a = \frac{F}{M} = -\left(\frac{k + A\sigma g}{M}\right)x \qquad (3)$$

Comparing Eq. (3) with $a = -\omega^2 x$, where $\omega = \frac{2\pi}{T}$, we find that the correct choice is (d).

17. If v is the speed of sound and u_1 and u_2 are the speeds of the cars, the difference between the frequencies of sound reflected from the cars is

$$\Delta f = f_0\left(\frac{v+u_1}{v-u_1}\right) - f_0\left(\frac{v+u_2}{v-u_2}\right)$$

$$= f_0\left[\left(1+\frac{u_1}{v}\right)\left(1-\frac{u_1}{v}\right)^{-1} - \left(1+\frac{u_2}{v}\right)\left(1-\frac{u_2}{v}\right)^{-1}\right]$$

$$= f_0\left[\left(1+\frac{2u_1}{v}\right) - \left(1+\frac{2u_2}{v}\right)\right] \quad (\because u_1, u_2 << v)$$

$$= \frac{2f_0}{v}(u_1 - u_2)$$

Given $\Delta f = \frac{1.2 f_0}{100}$. Thus

$$\frac{1.2 f_0}{100} = \frac{2 f_0}{330}(u_1 - u_2)$$

$$\Rightarrow (u_1 - u_2) = \frac{330 \times 1.2}{200} = 1.98 \text{ ms}^{-1}$$

18. Process $A \to B$ is isothermal. Hence internal energy at A = internal enrgy at B

$$\therefore \quad W_{A\to B} = mRT_0 \ln\left(\frac{4V_0}{V_0}\right) = P_0 V_0 \ln(4)$$

Also $P_A V_A = P_B V_B \Rightarrow P_B = P_A\left(\frac{V_A}{V_B}\right) = \frac{P_0}{4}$

Since the line BC passes through the origin of V-T graph, $V \propto T$. So process $B \to C$ is isobaric. Hence

$$P_C = P_B = \frac{P_0}{4}$$

Also $$\frac{V_C}{T_C} = \frac{V_B}{T_B} \Rightarrow T_C = \frac{T_0}{4}$$

Thus the only correct choice is (c).

19. From Wien's displacement law $\lambda_m T$ = constant, we have

$$\lambda_A T_A = \lambda_B T_B \Rightarrow \frac{T_A}{T_B} = \frac{\lambda_B}{\lambda_A}$$

From Stefan's law, $E = \sigma A T^4 = \sigma (4\pi R^2) T^4$, we have

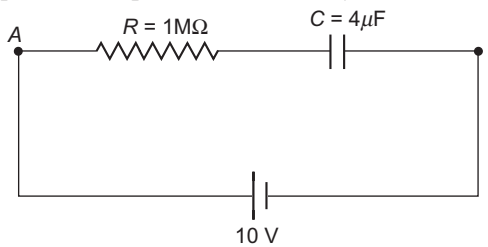
$$\frac{E_A}{E_B} = \left(\frac{R_A}{R_B}\right)^2 \times \left(\frac{T_A}{T_B}\right)^4$$

$$= \left(\frac{R_A}{R_B}\right)^2 \times \left(\frac{\lambda_B}{\lambda_A}\right)^4$$

$$= \left(\frac{6}{18}\right)^2 \times \left(\frac{1500}{50}\right)^4$$

$$= 9$$

20. Equivalent resistance is R = 1 MΩ =10^6 Ω and equivalent capacitance is C = 4 μF = 4 × 10^{-6} F.



$$Q = Q_0(1 - e^{-t/RC})$$

$$CV = CV_0(1 - e^{-t/RC})$$

$$\Rightarrow \quad V = V_0(1 - e^{t/4}) \qquad (\because RC = 4)$$

Given V = 4 V and V_0 = 10 V. Therefore

$$4 = 10\,(1 - e^{-t/4})$$

$$e^{-t/4} = \frac{3}{5} \Rightarrow e^{t/4} = \frac{5}{3}$$

$$\Rightarrow \quad \frac{t}{4} = \ln\left(\frac{5}{3}\right) \Rightarrow t = 4 \ln\left(\frac{5}{3}\right)$$

21. Equivalent resistance $R = \dfrac{6\times1.5}{6+1.5} + 2$

$= 3.2\ \text{k}\,\Omega$

$= 3.2 \times 10^3\ \Omega$

Current $I = \dfrac{24}{3.2\times10^3} = 7.5 \times 10^{-3}\ \text{A} = 7.5\ \text{mA}$

Potential difference across

$R_1 = (2 \times 10^3) \times (7.5 \times 10^{-3}) = 15\ \text{V}$

$\therefore$ Potential difference across

$R_2 =$ p.d. across $R_L = 24 - 15 = 9\text{V}$

$$\frac{\text{Power dissipated in } R_1}{\text{Power dissipated in } R_2} = \frac{(15)^2/2\times10^3}{(9)^2/6\times10^3} = \frac{25}{3}$$

22. Kinetic energy $K = qV$.

Therefore $r_1 = \dfrac{\sqrt{2m_1qV}}{qB}$ and $r_2 = \dfrac{\sqrt{2m_2qV}}{qB}$

Hence $\dfrac{r_1}{r_2} = \sqrt{\dfrac{m_1}{m_2}} \quad \Rightarrow \quad \dfrac{m_1}{m_2} = \left(\dfrac{r_1}{r_2}\right)^2$.

Statement-1 is false and Statement-2 is true.

23. When capacitance is removed, the circuit contains only inductance and resistance. Phase difference θ between the current and voltage is then given by

$\tan\theta = \dfrac{\omega L}{R}$ or $\omega L = R\tan\theta$

$= 100 \tan 60°$

When the circuit contains only capacitance and resistance, the phase difference between the voltage and current is given by

$\tan\phi = \dfrac{1}{RC\omega}$

$\therefore \quad \dfrac{1}{C\omega} = R\tan\phi = 100 \tan 60°$

The impedance of the *LCR* circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{R^2 + (100\tan 60° - 100\tan 60°)^2}$$

$= R = 100\ \Omega$.

The current is given by

$$I = \frac{V}{R} = \frac{200}{100} = 2\ \text{A}$$

24. The power dissipated in the circuit is

$P = I^2R = 4 \times 100 = 400\ \text{W}$

25. From Fleming's left hand rule, the free electrons experience a force from Q to P. As a result a current flows from P to Q. Also end Q acquires a positive charge (due to deficiency of electrons) and end P acquires a negative charge (due to gain of electrons). Hence an electric field exists in the rod in the direction Q to P.

26. $\dfrac{1}{f} = (\mu - 1) \times \dfrac{1}{R} = (1.5 - 1) \times \dfrac{1}{10}$ gives $f = 20$ cm.

Lens formula is (here $u = -30$ cm and $f = +20$ cm)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad (1)$$

or $\dfrac{1}{v} - \dfrac{1}{-30} = \dfrac{1}{20}$ which gives $v = 60$ cm

Differentiating (1) with respect to time t, we get

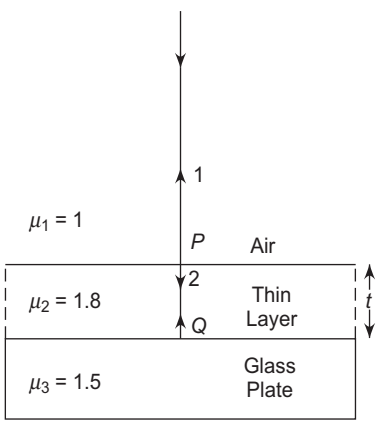
$$-\frac{1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt} = 0$$

or $\dfrac{dv}{dt} = \left(\dfrac{v^2}{u^2}\right)\dfrac{du}{dt}$

or Speed of image $= \left(\dfrac{v^2}{u^2}\right) \times$ speed of object

$$= \left(\frac{60}{30}\right)^2 \times 1 = 4\ \text{cm s}^{-1},$$

27. Refer to the following figure. A ray of light travelling in air ($\mu_1 = 1$) falls normally on a thin layer ($\mu_2 = 1.8$) of thickness t. It is partly reflected at point P as wave1 and partly refracted as wave 2. Wave 2 on meeting the surface of the glass plate ($\mu_3 = 1.5$) is reflected at point Q and travels along QP.



Waves 1 and 2 meet at point P where they interfere. We know that when a wave is travelling in a rarer medium and gets reflected at the boundary of a denser medium, it undergoes a phase change of π or a path change of $\lambda/2$. Thus wave 1 has an optical path of $\Delta_1 = \lambda/2$. Wave 2 travelling from P to Q in the layer of refractive index 1.8 gets reflected at Q from the boundary of glass of refractive index 1.5. Thus wave 2 travelling in a denser medium is reflected from the boundary of a rarer medium undergoes no phase change due to reflection. Therefore, Optical path for wave 2 from P to Q and from Q to P in the layer is

$$\Delta_2 = \text{refractive index of layer} \times 2(PQ)$$
$$= \mu_2 \times 2t = 2\mu_2 t$$

$\therefore$ Optical path difference between waves 1 and 2 at point p is

$$\Delta = \Delta_2 - \Delta_1 = 2\mu_2 t - \frac{\lambda}{2}$$

Now, for constructive interference, $\Delta = n\lambda$; $n = 0, 1, 2, \ldots$

$$\text{or} \quad 2\mu_2 t - \frac{\lambda}{2} = n\lambda \text{ or } 2\mu_2 t = \left(n + \frac{1}{2}\right)\lambda$$

$$\text{or} \quad t = \frac{\left(n + \frac{1}{2}\right)\lambda}{2\mu_2}$$

The minimum value of t corresponds to $n = 0$. Hence

$$t_{\min} = \frac{\lambda}{4\mu_2} = \frac{648 \text{ nm}}{4 \times 1.8} = 90 \text{ nm}.$$

28. Speed of photon (c) = 3×10^8 ms^{-1}. Let λ be the wavelength of the photon. The de Broglie wavelength of the electron = $\frac{h}{mv}$

Given $\lambda = \frac{h}{mv}$. Now

$$\frac{\text{K.E.of photon}}{\text{K.E.of electron}} = \frac{h\nu}{\frac{1}{2}mv^2} = \frac{2hc}{mv^2\lambda} \qquad \left(\because \nu = \frac{c}{\lambda}\right)$$

$$= \frac{2c}{v} \qquad \left(\because \lambda = \frac{h}{mv}\right)$$

$$= \frac{2 \times 3 \times 10^8}{1.5 \times 10^8} = 4$$

29. $N = N_0 e^{-\lambda t} \Rightarrow \frac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$.

Therefore $\left|\frac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t}$

$$\Rightarrow \quad \ln \left|\frac{dN}{dt}\right| = \ln(\lambda N_0) - \lambda t$$

The slope of the graph is = $\frac{4-3}{4-6} = -\frac{1}{2}$

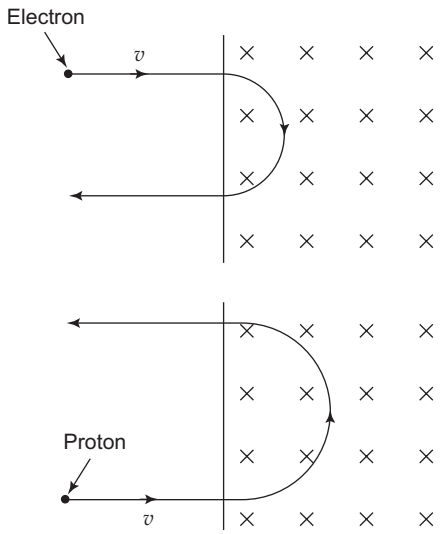
$\therefore \quad \lambda = \frac{1}{2}$ per year

Half left $T_{1/2} = \frac{0.693}{1/2} = 1.386$ year

$\therefore$ Number of half lives = $\frac{4.16}{1.386} = 3$

Hence $p = (2)^3 = 8$

30. The electron and the proton will come out travelling along parallel paths after completing their semicircles as shown in the following figures.



Since $r = \frac{mv}{qB}$, $r_p > r_e$

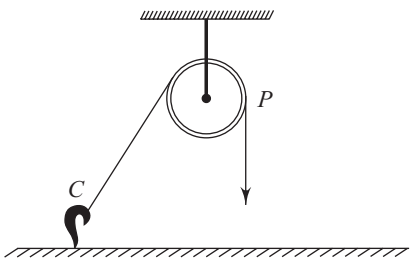
The time after which a charged particle comes out is given by

$$t = \frac{\pi m}{qB}$$

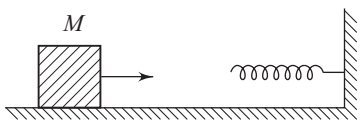
Since $m_p > m_e$ and q and B are the same, $t_p > t_e$.

Practice Test Paper – III

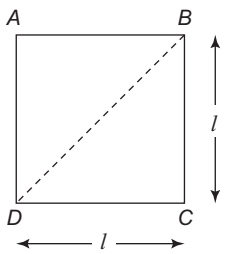
1. The physical quantities not having the same dimensions are
   (a) torque and work
   (b) momentum and Planck's constant
   (c) pressure and Young's modulus
   (d) speed and $(\mu_0 \varepsilon_0)^{-1/2}$
2. Two forces are such that the sum of their magnitudes is 18 N and their resultant whose magnitude is 12N, is perpendicular to the smaller force. Then the magnitudes of the forces are
   (a) 12 N, 6 N (b) 13 N, 5 N
   (c) 10 N, 8 N (d) 16 N, 2 N
3. A ball is thrown from a point with a speed u at an angle θ to the horizontal. From the same point and at the same instant, a person starts running with a constant speed $u/2$ to catch the ball. Will he be able to catch the ball? If yes, what should be the value of angle θ?
   (a) Yes, 60° (b) Yes, 30°
   (c) No (d) Yes, 45°
4. One end of a massless rope, which passes over a massless and frictionless pulley P is tied to a hook C while the other end is free. The rope can bear a maximum tension of 360 N. With what maximum acceleration (in ms^{-2}) can a man of mass 60 kg climb on the rope?



   (a) 16 (b) 6
   (c) 4 (d) 8
5. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that
   (a) its velocity remains constant
   (b) it moves in a straight line with a constant acceleration
   (c) it moves in a circle with a constant acceleration
   (d) its kinetic energy remains constant
6. A block of mass 4 kg moving on a frictionless horizontal surface collides with spring of spring constant 4 Nm^{-1} and compresses it by length 50 cm. The maximum momentum (in kg ms^{-1}) of the block after the collision is



   (a) 2 (b) 4
   (c) 1 (d) 0.5
7. Four point masses, each of value m, are placed at the corners of square $ABCD$ of side l. The moment of inertia of this system about an axis passing through A and parallel to BD is



   (a) $3\,ml^2$ (b) ml^2
   (c) $2\,ml^2$ (d) $\sqrt{3}\,ml^2$

8. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1: For a mass M kept at the centre of a cube of side 'a', the flux of gravitational field passing through its sides is $4\pi GM$.

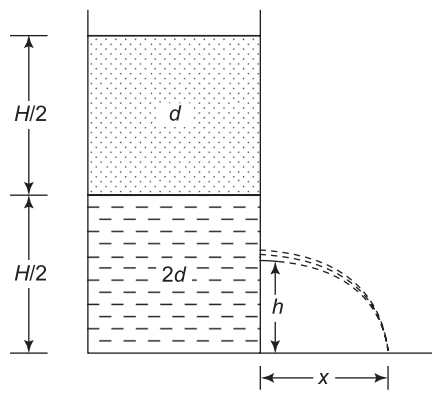
And

Statement-2: If the direction of a field due to a point source is radial and its dependence on the distance 'r' from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true, Statement-2 is correct explanation of Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not correct explanation for Statement-1.

Questions 9 and 10 are based on the following passage.

A container of large uniform cross-sectional area A resting on a horizontal surface, is filled with two non-mixing and non viscous liquids of densities d and $2d$, each to a height $H/2$ as shown in the figure. A tiny hole of cross-sectional area a ($<<A$) is punched on the vertical side of the container at a height $h = H/4$.



9. The horizontal range x of the liquid initially is

(a) H (b) $\frac{H}{\sqrt{2}}$

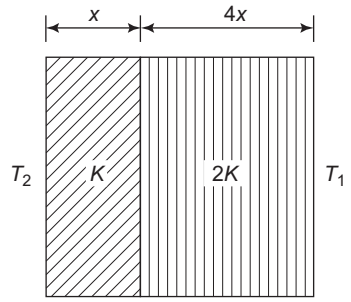
(c) $\frac{H}{2}$ (d) $\frac{H}{2\sqrt{2}}$

10. The height h of the hole from the ground for which the horizontal range x is maximum is

(a) $\frac{2H}{3}$ (b) $\frac{3H}{2}$

(c) $\frac{3H}{4}$ (d) $\frac{H}{2}$

11. The temperatures of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively, are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right) f$, with f equal to



(a) 1 (b) 1/2
(c) 2/3 (d) 1/3

12. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle, volume of the gas increases from V to 32 V the efficiency of the engine is

(a) 0.5 (b) 0.75
(c) 0.99 (d) 0.25

13. The mean translational kinetic energy of 1 mole of hydrogen at S.T.P. is (density of hydrogen at S.T.P. = 0.09 kgm^{-3})

(a) 2.4×10^3 J (b) 3.4×10^3 J
(c) 4.4×10^3 J (d) 5.4×10^3 J

14. A particle of mass m executes simple harmonic motion with amplitude a and frequency ν. The average kinetic energy during its motion from the position of equilibrium to the extreme position is

(a) $\pi^2 ma^2 \nu^2$ (b) $\frac{1}{4} ma^2 \nu^2$

(c) $4\pi^2 ma^2 \nu^2$ (d) $2\pi^2 ma^2 \nu^2$

15. While measuring the speed of sound by performing a resonance column experiment a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, he measures the column length to be x cm for the second resonance. Then

(a) 36 cm > x > 18 cm
(b) 18 cm > xs
(c) x > 54 cm
(d) 54 cm > x > 36 m

16. An electric dipole is placed at an angle of 30° to a non-uniform electric field. The dipole will experience

(a) a torque as well as a translational force
(b) a torque only

(c) a translational force only in the direction of the field

(d) a translational force only in a direction normal to the direction of the field

17. A parallel plate capacitor with air between the plates has a capacitance of 9pF. The separation between its plates is 'd'. The space between the plates is now filled with two dielectrics. One dielectric has dielectric constant $k_1 = 3$ and thickness $\frac{d}{3}$ while the order one has dielectric constant $k_2 = 6$ and thickness $\frac{2d}{3}$. Capacitance of the capacitor is now

(a) 40.5 pF (b) 20.25 pF

(c) 1.8 pF (d) 45 pF

Questions 18 and 19 are based on the following paragraph.

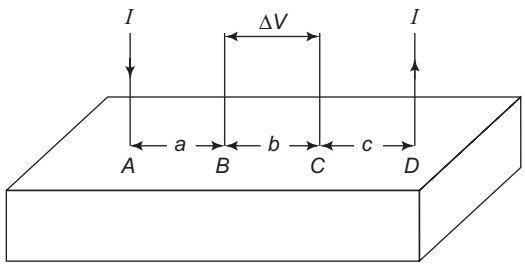
Consider a block of conducting material of resistivity 'ρ' shown in the figure. Current 'I' enters at 'A' and leaves from 'D'. We apply superposition principle to find voltage 'ΔV' developed between 'B' and 'C'. The calculation is done in the following steps :

(i) Take current 'I' entering from 'A' and assume it to spread over a hemispherical surface in the block.

(ii) Calculate field E (r) at distance 'r' from A by using Ohm's law $E = \rho J$. Where J is the current per unit area at 'r'.

(iii) From the 'r' dependence of E (r). Obtain the potential $V(r)$ at r.

(iv) Repeat (i), (ii) and (iii) for current 'I' leaving 'D' and superpose results for 'A' and 'D'.



18. ΔV measured between B and C is

(a) $\frac{\rho I}{2\pi(a-b)}$ (b) $\frac{\rho I}{\pi a} - \frac{\rho I}{\pi(a+b)}$

(c) $\frac{\rho I}{a} - \frac{\rho I}{(a+b)}$ (d) $\frac{\rho I}{2\pi a} - \frac{\rho I}{2\pi(a+b)}$

19. For current entering at A, the electric field at a distance 'r' from A is

(a) $\frac{\rho I}{4\pi r^2}$ (b) $\frac{\rho I}{8\pi r^2}$

(c) $\frac{\rho I}{r^2}$ (d) $\frac{\rho I}{2\pi r^2}$

20. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60°. The magnitude of torque needed to maintain the needle in this position will be

(a) $\sqrt{3}W$ (b) W

(c) $\left(\frac{\sqrt{3}}{2}\right)W$ (d) $2W$

21. In an a.c. circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by

(a) $P = \frac{E_0 I_0}{\sqrt{2}}$ (b) $P =$ zero

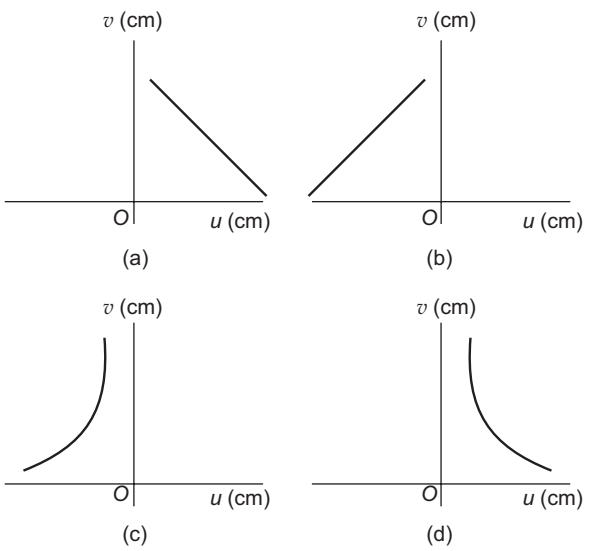
(c) $P = \frac{E_0 I_0}{2}$ (d) $P = \sqrt{2}\ E_0 I_0$

22. Which of the following pairs of space and time varying E $\left(= \hat{\mathbf{i}} E_x + \hat{\mathbf{j}} E_y + \hat{\mathbf{k}} E_z\right)$ and **B** $\left(= \hat{\mathbf{i}} B_x + \hat{\mathbf{j}} B_y + \hat{\mathbf{k}} B_z\right)$ would generate a plane electromagnetic wave travelling in the z-direction?

(a) E_x, B_z (b) E_y, B_z

(c) E_z, B_x (d) E_x, B_y

23. A student measures the focal length of a convex lens by putting an object pin at a distance 'u' from the lens and measuring the distance 'v' of the image pin. The graph between 'u' and 'v' plotted by the student should look like



24. In Young's double slit experiment, the intensity at a point P on the screen is half the maximum intensity in the interference pattern. If the wavelength of light used is λ and d is the distance between the slits, the angular separation between point P and the centre of the screen is

(a) $\sin^{-1}\left(\frac{\lambda}{d}\right)$ (b) $\sin^{-1}\left(\frac{\lambda}{2d}\right)$

(c) $\sin^{-1}\left(\frac{\lambda}{3d}\right)$ (d) $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

25. If the kinetic energy of a free electron doubles, its de-Broglive wavelength becomes

(a) half (b) twice

(c) $\frac{1}{\sqrt{2}}$ times (d) $\sqrt{2}$ times

26. If 13.6 eV energy is required to ionize the hydrogen atom, the energy required to remove the electron from $n = 2$ state is

(a) 10.2 eV (b) Zero

(c) 3.4 eV (d) 6.8 eV

27. When U-238 nucleus originally at rest, decays by emitting an alpha particle having a speed u, the recoil speed of the residual nucleus is

(a) $\frac{4u}{238}$ (b) $-\frac{4u}{234}$

(c) $\frac{4u}{234}$ (d) $-\frac{4u}{238}$

28. When a *pn* junction diode is forward biased.

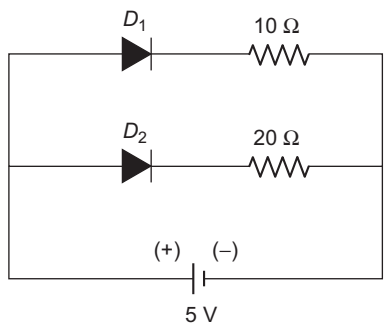
(a) the depletion region is reduced and barrier height is increased

(b) the depletion region is widened and barrier height is reduced

(c) both the depletion region and barrier height are reduced

(d) both the depletion region and barrier height are increased

29. Two ideal junction diodes D_1 and D_2 are connected to a battery as shown in the Figure. The current supplied by the battery is



(a) 0.25 A (b) 0.5 A

(c) 0.75 A (d) zero

30. Which mode of communication is employed to transmit very high frequency (VHF) and ultra high frequency (UHF) signals?

(a) Ground wave propagation

(b) Sky wave propagation

(c) Space wave propagation

(d) None of the above

Answers

1. (b)	2. (b)	3. (a)	4. (c)
5. (d)	6. (a)	7. (a)	8. (c)
9. (b)	10. (c)	11. (a)	12. (b)
13. (b)	14. (a)	15. (c)	16. (a)
17. (a)	18. (b)	19. (d)	20. (a)
21. (b)	22. (d)	23. (d)	24. (d)
25. (c)	26. (c)	27. (c)	28. (c)
29. (b)	30. (c)		

Solutions

1. Momentum = $mv = [MLT^{-1}]$

Planck's constant $h = \frac{E}{\nu} = \frac{ML^2T^{-2}}{T^{-1}}$

$= [ML^2T^{-1}]$

2. Give $A + B = 18$ (i)

$12 = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ (ii)

and $\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$

$\Rightarrow \tan 90° = \frac{B\sin\theta}{A + B\cos\theta}$

$\Rightarrow A + B\cos\theta = 0$

$\therefore \quad \cos\theta = -\frac{A}{B}$

Using this in (ii), $12 = \sqrt{B^2 - A^2}$

$\Rightarrow 144 = B^2 - A^2 = (B + A)(B - A)$

Using (i), $144 = 18(B - A)$

$\Rightarrow \quad B - A = \frac{144}{18} = 8$ (iii)

From (i) and (iii) we get $A = 5$N and $B = 13$ N

3. The person will catch the ball if the horizontal range = distance covered by the person in the time of flight, i.e., if

$$R = \frac{u}{2} \times t_f$$

$$\Rightarrow \quad \frac{u^2 \sin(2\theta)}{g} = \frac{u}{2} \times \frac{2u \sin\theta}{g}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$$

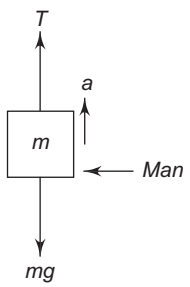
4. From the free body diagram of the man

$$T - mg = ma$$

$$\Rightarrow \quad a = \frac{T - mg}{m}$$

$$a_{\text{man}} = \frac{T_{\max} - mg}{m}$$

$$= \frac{360 - 60 \times 10}{60} = -4 \text{ ms}^{-2}$$



Negative acceleration implies that the man must climb down the rope.

5. The correct choice is (*d*). The partile moves in a circle with a constant speed and hence with a constant kinetic energy.

6. The momentum is maximum when kinetic energy is maximum. This happens when the entire potential energy of the compressed spring is transferred to the block, i.e. when

$$\frac{1}{2} kL^2 = \frac{1}{2} Mv^2$$

here v is the maximum velocity imparted to the block and L is the length of the spring. Thus

$$Mv^2 = kL^2$$

$$\Rightarrow \quad M^2v^2 = MkL^2$$

$$\Rightarrow \quad p^2 = MkL^2$$

$$\Rightarrow \quad p = L\sqrt{Mk}$$

$$= 0.5 + \sqrt{4 \times 4}$$

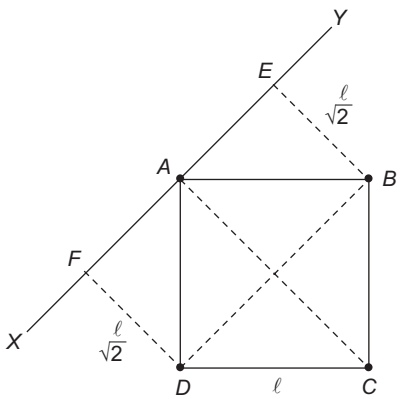
$$= 2 \text{ kg ms}^{-1}$$

7. The moment of inertia of the system about axis XY is

$$I = m \text{ at } A \times 0 + m \text{ at } B \times (BE)^2 + m \text{ at } C \times (AC)^2 + m \text{ at } D \times (DF)^2$$

$$= m \times 0 + m \times \left(\frac{l}{\sqrt{2}}\right)^2 + m \times \left(\sqrt{2}l\right)^2 + m \times \left(\frac{l}{\sqrt{2}}\right)^2$$

$$= 0 + \frac{ml^2}{2} + 2ml^2 + \frac{ml^2}{2} = 3ml^2$$



8. Statement-2 is Gauss's theorem in gravitation. Gauss's theorem holds for any field which obeys $1/r^2$ dependence. Just as electric field intensity due to a charge Q at a distance r from it is given by

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

the gravitational field intensity due to a mass M at a distance r from it is given by

$$I = \frac{GM}{r^2}$$

Thus mass M plays the same role in gravitation as charge Q does in electrostatics. Further constant G is analogous to constant $1/4\pi\varepsilon_0$. From Gauss's theorem in electrostatics, electric flux through a closed surface is given by

$$\phi_e = \frac{Q}{\varepsilon_0}$$

where Q is the charge enclosed in the surface. Replaced Q by M and ε_0 by $1/4\pi G$, the gravitational flux through a closed surface will be

$$\phi_g = \frac{M}{1/4\pi G} = 4\pi GM$$

where M is the mass enclosed in the surface. Hence statements 1 and 2 are true and statement 2 is the correct explanation for statement-1. So the correct choice is (c).

9. Let v_1 be the initial velocity of the fall of liquid level in the container and v_2 the velocity of efflux from the hole. Then $Av_1 = av_2$ or $v_1 = \left(\frac{a}{A}\right) v_2$ since $a << A$,

$v_2 \simeq 0$. Applying Bernoulli's theorem at a point on the top of the liquid and at the hole, we have

$$P_0 + \frac{1}{2}dv_1^2 + dg\left(\frac{H}{2}\right) + (2d)g\left(\frac{H}{2}\right)$$

$$= P_0 + \frac{1}{2}(2d)dv_2^2 + (2d)gh$$

Putting $v_1 = 0$, we get

$$v_2 = \left[\left(\frac{3}{2}H - 2h\right)g\right]^{1/2}$$

Putting $h = H/4$, we get $v_2 = \sqrt{gH}$ which is choice (d).

Time of fall $t = \sqrt{\frac{2h}{g}}$ .Horizontal range is

$$x = v_2\, t = \left[\left(\frac{3}{2}H - 2h\right)g\right]^{1/2} \times \sqrt{\frac{2h}{g}}$$

$$= [h(3H–4h)]^{1/2} \qquad (1)$$

putting $h = H/4$, we get $x = \frac{H}{\sqrt{2}}$, which is choice (b).

10. To determme the height h at which the range x is maximum, we differentiate (1) with respect to h and set $\frac{dx}{dh} = 0$, i.e.

$$\frac{d}{dh}\,[3Hh - 4h^2]^{1/2} = 0$$

or $$\frac{1}{2}\,[3Hh - 4h^2]^{-1/2} \times (3H - 8h) = 0$$

which gives $3H - 8h = 0$ or $h = \frac{3}{8}\,H$. Notice that $(3Hh - 4h^2)^{-1/2}$ cannot be zero since this would give x = infinity which is not possible. Hence x is maximum ($= x_m$) at a value of h given by

$$h = \frac{3}{8}H$$

Using this value of h in (1), we get

$$x_m = \left[\frac{3}{8}H\left(3H - \frac{12}{8}H\right)\right]^{1/2} = \frac{3}{4}H$$

11. Let A be the area of each slab. In the steady state, the rate of heat flow through the composite slab is given by

$$\frac{Q}{t} = \frac{T_2 - T_1}{\frac{l_1}{K_1A} + \frac{l_2}{K_2A}} = \frac{2(T_2 - T_1)}{\frac{l_1}{K_1} + \frac{l_2}{K_2}} \qquad (1)$$

Given $l_1 = x, l_2 = 4x$, $K_1 = K$ and $K_2 = 2K$. Using these values in (1) we get

$$\frac{Q}{t} = \frac{A(T_2 - T_1)}{\frac{x}{K} + \frac{4x}{2K}} = \left[\frac{A(T_2 - T_1)K}{x}\right] \times \frac{1}{3}$$

Comparing this with the given rate of heat transfer, we get $f = \frac{1}{3}$.

12. Efficiency of the engine is

$$\eta = 1 - \frac{T_2}{T_1}$$

For an adiabatic process $TV^{\gamma - 1}$ + constant.

$$T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^{\gamma - 1}$$

For a diatomic gas $\gamma = \frac{7}{5}$. Hence

$$\frac{T_2}{T_1} = \left(\frac{32V}{B}\right)^{\left(\frac{7}{5} - 1\right)} = (32)^{2/5}$$

$$= 4$$

$$\therefore \quad \eta = 1 - \frac{1}{4} = \frac{3}{4} = 0.75$$

13. $$v_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.01 \times 10^5}{0.09}}$$

$$= 1.84 \times 10^3 \text{ ms}^{-1}$$

Mass of 1 mole is

$$m = 22.4 \times 10^{-3} \text{ m}^3 \times 0.09 \text{ kg m}^{-3}$$

$$= 2.016 \times 10^{-3} \text{ kg}$$

$$\text{K.E.} = \frac{1}{2}\, mv^2_{\text{rms}} = \frac{1}{2} \times (2.016 \times 10^{-3})$$

$$\times (1.84 \times 10^3)^2 = 3.4 \times 10^3 \text{ J}$$

14. Displacement $x = a \sin(\omega t + \phi)$

Velocity $v = \frac{dx}{dt} = a\omega \cos(\omega t + \phi)$

$\therefore$ Kinetic energy $K = \frac{1}{2}mv^2 = \frac{1}{2}ma^2\ \omega^2 \cos^2(\omega t + \phi)$

It T is the time period of the simple harmonic motion, the time taken by the particle to go from the mean position to the extreme position is $T/4$. The average kinetic energy during time $t = 0$ to $t = T/4$ is given by

$$K_{av} = \frac{\int_0^{T/4} K\,dt}{\int_0^{T/4} dt} = \frac{\int_0^{T/4} \frac{1}{2}ma^2\omega^2 \cos^2(\omega t + \phi)\,dt}{T/4}$$

$$= \frac{2ma^2\omega^2}{T}\int_0^{T/4} \cos^2(\omega t + \phi)\,dt$$

$$= \frac{ma^2\omega^2}{T}\int_0^{T/4} [1 + \cos\{2(\omega t + \phi)\}]\,dt$$

$$= \frac{ma^2\omega^2}{T}\left(\frac{T}{4}+0\right)$$

$$= \frac{ma^2(2\pi\nu)^2}{4} = \pi^2 ma^2 \nu^2$$

15. Let v be the speed of sound in winter and v' in summer. For first resonance: $\frac{v}{4l} = N$, the frequency of tuning fork.

For second resonance: $\frac{3v'}{4x} = N$. Hence

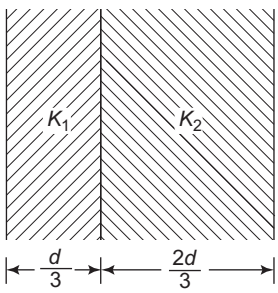
$$\frac{v}{4l} = \frac{3v'}{4x} \Rightarrow x = 3l\left(\frac{v'}{v}\right) = (3 \times 18 \text{ cm})\left(\frac{v'}{v}\right)$$

$$= 54 \text{ cm} \times \left(\frac{v'}{v}\right)$$

As the speed of sound increases with temperature, $v' > v$. Hence $x > 54$ cm.

16. The correct choice is (a)

17. Given $C_0 = \frac{\varepsilon_0 A}{d} = 9$ pF



$$C_1 = \frac{k_1\varepsilon_0 A}{d/3} = \frac{3\varepsilon_0 A}{d/3} = \frac{9\varepsilon_0 A}{d} = 9\ C_0$$

$$C_2 = \frac{k_2\varepsilon_0 A}{2d/3} = \frac{6\varepsilon_0 A}{2d/3} = \frac{9\varepsilon_0 A}{d} = 9\ C_0$$

Capacitors C_1 and C_2 are in series. The equivalent capacitance is

$$C = \frac{C_1C_2}{C_1+C_2} = \frac{9C_0 \times 9C_0}{18C_0} = \frac{9}{2}C_0$$

$$= \frac{9}{2} \times 9\text{pF} = 40.5 \text{ pF}$$

18. $E = \rho J = \frac{\rho I}{2\pi r^2}$. Also $E = -\frac{dV}{dr} \Rightarrow dV = -Edr$

Therefore, potential difference between B and C due to current at A is

$$V = -\int_C^B Edr = -\frac{\rho I}{2\pi}\int_{(a+b)}^{a}\frac{dr}{r^2}$$

$$= -\frac{\rho I}{2\pi}\left|-\frac{1}{r}\right|_{(a+b)}^{a} = \frac{\rho I}{2\pi a} - \frac{\rho I}{2\pi(a+b)}$$

Similarly, potential difference between B and C due to current at D is $V' = \frac{\rho I}{2\pi a} - \frac{\rho I}{2\pi(a+b)}$.

From superposition principle, the potential difference between B and C due to current at A and at D is

$$\Delta V = V + V' = \frac{\rho I}{\pi a} - \frac{\rho I}{\pi(a+b)}$$

19. $E = \rho J = \frac{\rho I}{2\pi r^2}$.

20. $W = MB\ (1 - \cos\theta)$

$$= MB\ (1 - \cos 60°) = \frac{MB}{2}$$

$$\Rightarrow \quad MB = 2W$$

Magnitude of torque is

$$\tau = MB \sin\theta$$

$$= 2\ W \sin 60° = 2W \times \frac{\sqrt{3}}{2} = \sqrt{3}W$$

21. Phase difference between voltage E and current I is $\phi = \frac{\pi}{2}$ The time-averaged power consumption is given by

$$\langle P \rangle = I_{\text{rms}}\, E_{\text{rms}} \cos\phi = 0 \qquad \left[\because \phi = \frac{\pi}{2}\right]$$

22. The correct choice is (d)

23. For a convex lens forming a real image,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow v = \frac{fu}{u-f}$$

Also, for a real image $u > f$. From $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ it follows that as u increases, v decreases as f remains constant. Hence the correct graph is (d).

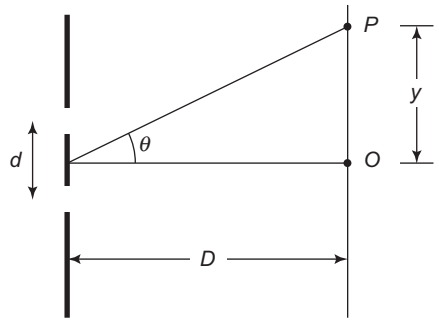
24. If δ is the phase difference between the interfering waves at point P, then the intensity at point P is given by

$$I = I_{\max}\cos^2\left(\frac{\delta}{2}\right)$$

Given $I = \frac{I_{\max}}{2}$. Hence

$$\cos^2\left(\frac{\delta}{2}\right) = \frac{1}{2}, \text{ which gives } \frac{\delta}{2} = \frac{\pi}{4}$$

or $$\delta = \frac{\pi}{2}$$



The angular separation θ between points P and O is given by $\tan\theta = y/D$. Since θ is very small, $\tan\theta \simeq \sin\theta$. Hence

$$\sin\theta = \frac{y}{D} \qquad (1)$$

If β is the fringe width, then

$$\frac{y}{\beta} = \frac{\pi/2}{2\pi} = \frac{1}{4} \qquad (2)$$

This is so because the phase difference δ between two consecutive maxima is 2π. Now $\beta = \frac{\lambda D}{d}$.

Using this in Eq. (2), we get

$$\frac{yd}{\lambda D} = \frac{1}{4}$$

or
$$\frac{y}{D} = \frac{\lambda}{4d} \qquad (3)$$

Using Eq. (1) in Eq. (3), we have

$\sin\theta = \frac{\lambda}{4d}$ or $\theta = \sin^{-1}\left(\frac{\lambda}{4d}\right)$, which is choice (d).

25. Use $\lambda = \frac{h}{\sqrt{2mK}}$. the correct choice is (c)

26. $E_n = \frac{-13.6\,\text{eV}}{n^2}$

$$\text{For } n = 2,\ E_2 = \frac{-13.6\,\text{eV}}{(2)^2} = -3.4\ \text{eV}$$

Therefore ionization energy for $n = 2$ state = 3.4 eV

27. Atomic mass of U-238 nucleus = 238 a.m.u.
Since the mass number decreases by 4 on alpha decay, the atomic mass of residual nucleus = 238 – 4 = 234 a.m.u. Momentum before decay = 0. If **v** is the velocity of the residual nucleus after decay, then from momentum conservation, we have

$$0 = 4\mathbf{u} + 234\mathbf{v}$$

$$\Rightarrow \qquad \mathbf{v} = -\frac{4\mathbf{u}}{234}.$$

The negative shows that direction of **v** is opposite to that of **u**. Hence recoil speed = $\frac{4u}{234}$

28. The correct choice is (c).

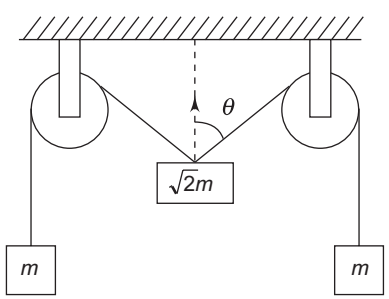
29. Diode D_1 is forward biased. Therefore, effective resistance is 10 Ω. Hence the current is

$$I = \frac{5\Omega}{10\Omega} = 0.5\text{A}$$

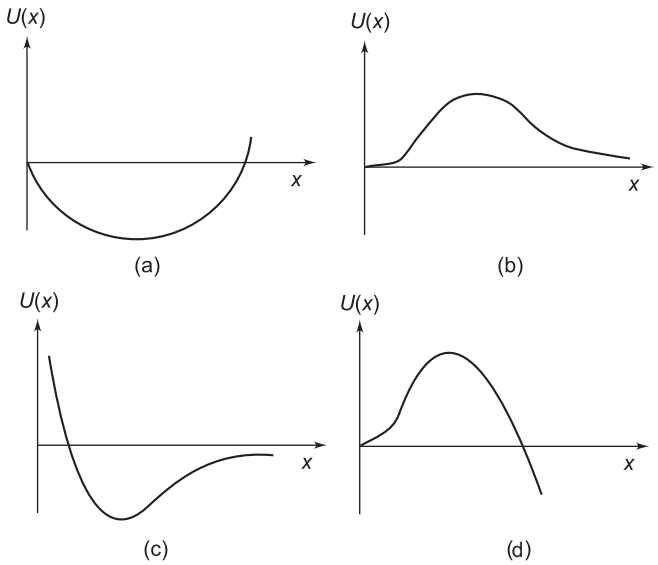
30. The correct choice is (c).

Practice Test Paper – IV

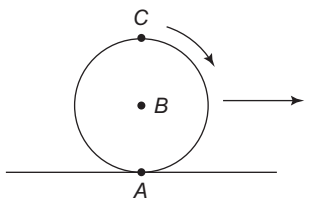
1. An electromagnetic wave travels with a speed v in a medium of permittivity $\in$ and permeability μ. The dimensions of $\mu\in$ are the same as those of
   (a) v^{-1} (b) v
   (c) v^{-2} (d) v^2
2. The displacement x (in metre) of a particle moving in one dimension under the action of a constant force, is related to time t (in second) as
   $$t = \sqrt{x} + 3$$
   The displacement in metre of the particle when its velocity is zero is
   (a) zero (b) 3
   (c) $\sqrt{3}$ (d) 2
3. A ball is projected vertically upwards with a certain initial speed. Another ball of the same mass is projected at an angle of 60° with the vertical with the same initial speed. At the highest point, the ratio of their potential energies will be
   (a) 4 : 1 (b) 3 : 2
   (c) 2 : 3 (d) 2 : 1
4. The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be



   (a) 0° (b) 30°
   (c) 45° (d) 60°
5. A particle, which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is



6. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,

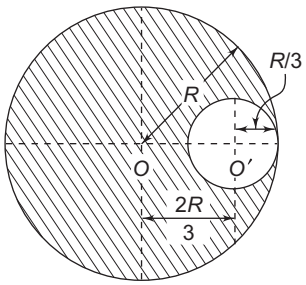


   (a) $\vec{V}_C - \vec{V}_A = 2\left(\vec{V}_B - \vec{V}_C\right)$ (b) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$
   (c) $\left|\vec{V}_C - \vec{V}_A\right| = \left|\vec{V}_B - \vec{V}_C\right|$ (d) $\left|\vec{V}_C - \vec{V}_A\right| = 2\left|\vec{V}_B\right|$

7. A thin uniform disc has mass M and radius R. A circular hole of radius $R/3$ is made in the disc as shown in the figure. The moment of inertia of the remaining portion of disc about an axis passing through O and perpendicular to the plane of the disc is

(a) $\frac{1}{9} MR^2$ (b) $\frac{2}{9} MR^2$

(c) $\frac{1}{3} MR^2$ (d) $\frac{4}{9} MR^2$



8. An artificial satellite moving in a circular orbit around the earth has a total (kinetic + potential) energy E_0. Its potential energy is

(a) $-E_0$ (b) $1.5\ E_0$

(c) $2\ E_0$ (d) E_0

9. An elastic spring of unstretched length L and force constant k is stretched by a small length x. It is further stretched by another small length y. The work done in the second stretching is

(a) $\frac{1}{2} ky^2$ (b) $\frac{1}{2} k(x^2 + y^2)$

(c) $\frac{1}{2} k(x + y)^2$ (d) $\frac{1}{2} ky(2x + y)$

10. A closed compartment containing gas is moving with some acceleration in horizontal direction. Neglect the effect of gravity. Then the pressure in the compartment is

(a) the same everywhere

(b) lower in the front side

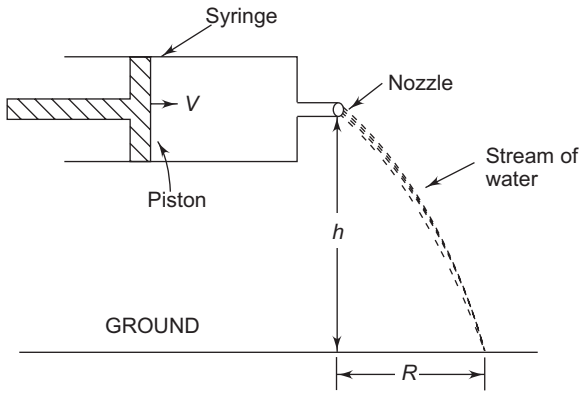
(c) lower in the rear side

(d) lower in the upper side

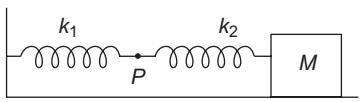
11. A syringe containing water is held horizontally with its nozzle at a hight h = 1.25 m above the ground as shown in the figure. The diameter of the piston is 5 times that of nozzle. The piston is pushed with a constant speed of 20 cms^{-1}. If g = 10 ms^{-2}, the magnitude of the velocity with which the water hits the ground is

(a) 5 ms^{-1} (b) $5\sqrt{2}$ ms^{-1}

(c) 2.5 ms^{-1} (d) $\sqrt{10}$ ms^{-1}



12. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is



(a) $\frac{k_1 A}{k_2}$ (b) $\frac{k_2 A}{k_1}$

(c) $\frac{k_1 A}{k_1 + k_2}$ (d) $\frac{k_2 A}{k_1 + k_2}$

13. In the resonance tube experiment for determining the speed of sound in air using a tuning fork of frequency 480 Hz, the first resonance was observed at 17.7 cm of air column and the second at 53.1 cm. The maximum possible error in the speed of sound in air is

(a) 192 cm s^{-1} (b) 96 cm s^{-1}

(c) 64 cm s^{-1} (d) 48 cm s^{-1}

14. Two litres of water (density = 1 g/ml) in an open-lid insulated kettle is heated by an electric heater of power 1 kW. The heat is lost from the lid at the rate of 160 J/s. The time taken for heating water (specific heat capacity 4.2 kJ kg^{-1} K^{-1}) from 20°C to 75°C is

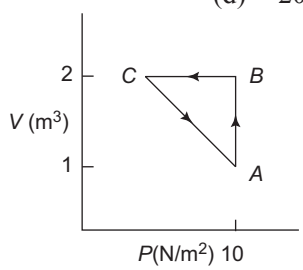
(a) 340 s (b) 550 s

(c) 620 s (d) 760 s

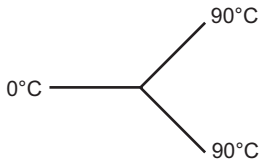
15. An ideal gas is taken through the cycle $A \to B \to C \to A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process $C \to A$ is

(a) – 5 J (b) – 10 J

(c) – 15 J (d) – 20 J



16. Three rods made of the same material and having the same cross-section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C respectively. The temperature of the junction of the three rods will be



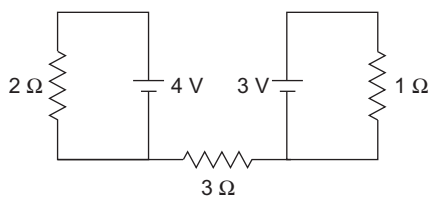
(a) 45 °C (b) 60 °C
(c) 30 °C (d) 20 °C

17. Three concentric metallic spherical shells of radii R, $2R$, $3R$ are given charges Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are.equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$ is

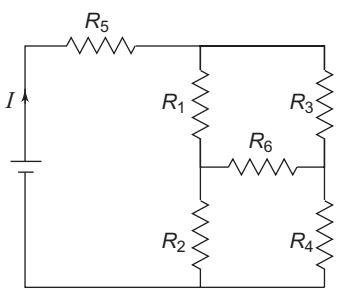
(a) 1 : 2 : 3 (b) 1 : 3 : 5
(c) 1 : 4 : 9 (d) 1 : 8 : 18

18. The potential difference across the 3 Ω. resistor shown in the figure is

(a) zero (b) 1 V
(c) 3.5 V (d) 7 V



19. In the given circuit it is observed that the current I is independent of the value of the resistance R_6. Then the resistance values must satisfy



(a) $R_1 R_2 R_5 = R_3 R_4 R_6$

(b) $\frac{1}{R_5}+\frac{1}{R_6}=\frac{1}{R_1+R_2}+\frac{1}{R_3+R_4}$

(c) $R_1 R_4 = R_2 R_3$

(d) $R_1 R_3 = R_2 R_4 = R_5 R_6$

Questions 20 and 21 are based on the following paragraph.

The region between $x = 0$ and $x = L$ is filled with a uniform, steady magnetic field $B_0\,\hat{\mathbf{k}}$. A particle of mass m, positive charge q and velocity $v_0\,\hat{\mathbf{i}}$ travels along x-axis and enters the region of the magnetic field. Neglect gravity.

20. The force experienced by the charged particle in the magnetic field is

(a) along the positive y-direction
(b) along the negative y-direction
(c) in the x-y plane
(d) in the y-z plane.

21. If the particle emerges from the region of magnetic field with its final velocity at an angle of 30° to its initial velocity, the value of L is

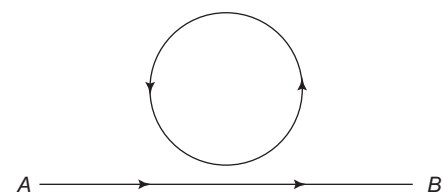
(a) $\frac{2mv_0}{qB_0}$ (b) $\frac{mv_0}{qB_0}$

(c) $\frac{mv_0}{2qB_0}$ (d) $\frac{\sqrt{3}\,mv_0}{2qB_0}$

22. A short-circuited coil is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be

(a) halved (b) the same
(c) doubled (d) quadrupled

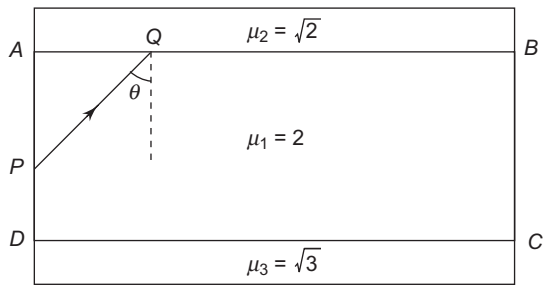
23. Statement-1.
If the current in a straight conductor increases from A to B, the direction of the current induced in the coil will be anticlockwise.



Statement-2
According to Lenz's law, the direction of the induced current is such that it opposes the change which produces it.

(a) Statement-1 and Statement-2 are both true and Statement-2 is the correct explanation of Statement-1.
(b) Statement-1 and Statement-2 are both true but Statement-2 is not the correct explanation of Statement-1.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is false.

24. A parallel sides slab *ABCD* of refractive index 2 is sandwiched between two slabs of refractive indices $\sqrt{2}$ and $\sqrt{3}$ as shown in the figure. The minimum value of angle θ such that the ray *PQ* suffers total internal reflection at both the surfaces *AB* and *CD* is



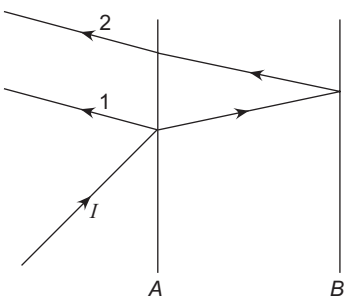
(a) 30° (b) 45°
(c) 60° (d) 75°

25. A point object is placed at the centre of a glass sphere of diameter 12 cm and refractive index 1.5. What is the distance of the virtual image from the surface of the sphere?
(a) 4 cm (b) 6 cm
(c) 9 cm (d) 12 cm

26. A beam of light consisting of two wavelengths 4500 Å and 7500 Å is used to obtain interference fringes in Young's double slit experiment. The distance between the slits is 1 mm and the distance between the plane of the slits and the screen is 120 cm. What is the minimum distance between two successive regions of complete darkness on the screen?
(a) 4.5 mm (b) 5.4 mm
(c) 2.7 mm (d) 1.2 mm

Questions 27 and 28 are based on the following paragraph.

A narrow monochromatic beam of light of intensity *I* is incident on a glass plate A as shown in the figure. Another identical glass plate *B* is kept close to *A* and parallel to it. Each plate reflects 25% of the light intensity incident on it and transmits the remaining. Interference pattern is formed by beams 1 and 2 obtained after reflection at each plate.



27. The ratio of the intensities of beams 1 and 2 is
(a) $\frac{16}{9}$ (b) $\frac{4}{3}$
(c) $\frac{25}{16}$ (d) $\frac{5}{4}$

28. The ratio of the maximum and minimum intensities in the interference pattern is
(a) 16 : 1 (b) 25 : 1
(c) 36.: 1 (d) 49 : 1

29. The wavelength of K_α line from an element of atomic number 41 is λ. Then the wavelength of K_α line of an element of atomic number 21 is
(a) 4 λ (b) $\lambda/4$
(c) 3.08 λ (d) 0.26 λ

30. A freshly prepared radioactive source of half life 2 hours emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with the source is
(a) 6 hours (b) 12 hours
(c) 24 hours (d) 128 hours

Answers

1. (c)	2. (a)	3. (a)	4. (c)
5. (d)	6. (b)	7. (d)	8. (c)
9. (d)	10. (b)	11. (b)	12. (d)
13. (a)	14. (b)	15. (a)	16. (b)
17. (b)	18. (a)	19. (c)	20. (b)
21. (c)	22. (b)	23. (c)	24. (c)
25. (b)	26. (c)	27. (a)	28. (d)
29. (b)	30. (b)		

Solutions

1. $v = \frac{1}{\sqrt{\mu \in}}$. So the correct choice is (c).

2. Given $\sqrt{x} = t - 3$. Squaring, we have
$$x = t^2 - 6t + 9 \quad \text{(i)}$$
The instantaneous velocity of the particle is
$$v = \frac{dx}{dt} = \frac{d}{dt}(t^2 - 6t + 9) = 2t - 6$$
Now, v will be zero at time t given by $2t - 6 = 0$ or $t = 3$ seconds. The displacement at $t = 3$ s is obtained

from relation (i) by putting t = 3 s which gives x (at t = 3 s) = $(3)^2 - 6 \times 3 + 9 = 0$. Hence the net displacement of the particle is zero when its velocity is zero, i.e. the particle returns to its starting position, where its velocity is zero.

3. The maximum height attained by the first ball is

$$h_1 = \frac{u^2}{2g}$$

where u is the initial speed of projection. The maximum height attained by the second ball is ($\because \theta = 90° - 60° = 30°$)

$$h_2 = \frac{u^2 \sin^2(30°)}{2g} = \frac{u^2}{8g}$$

Now, PE of ball 1 at height $h_1 = mgh_1$ and that of ball 2 at height $h_2 = mgh_2$. Therefore, the ratio of potential energies = $\frac{h_1}{h_2} = \frac{u^2}{2g} \times \frac{8g}{u^2} = 4$.

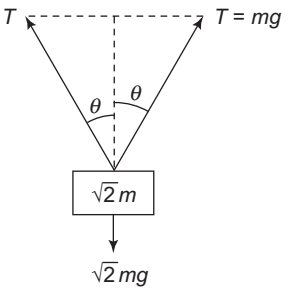
4. Let T be the tension in the string. When the system is in equilibrium, then for the two equal masses m, we have (see figure)

$$T = mg \qquad (1)$$

and for the mass $\sqrt{2}\ m$, we have

$$2T\cos\theta = \sqrt{2}\ mg \qquad (2)$$

Dividing (2) by (1), we get $\cos\theta = \frac{1}{\sqrt{2}}$ or $\theta = 45°$



5. The potential energy of the particle is given by

$$U = \int F dx = -\int \left(-kx + ax^3\right) dx$$

or $$U = k\frac{x^2}{2} - a\frac{x^4}{4} = \frac{x^2}{4}\ (2k - ax^2) \qquad (1)$$

From Eq. (1) it follows that $U = 0$ at two values of x which are $x = 0$ and $x = \sqrt{2k/a}$. Hence graphs (b) and (c) are not possible. Also U is maximum or minimum at a value of x given by $\frac{dU}{dx} = 0$, i.e.

$$0 = \frac{d}{dx}\left(\frac{kx^2}{2} - \frac{ax^4}{4}\right) = kx - ax^3$$

$$= x(k - ax^2)$$

or $x = \sqrt{k/a}$. At this value of x, U is maximum if $\frac{d^2U}{dx^2} < 0$,

Now $\frac{d^2U}{dx^2} = \frac{d}{dx}\ (kx - ax^3) = k - 3ax^2$.

At $x = \sqrt{k/a}$,

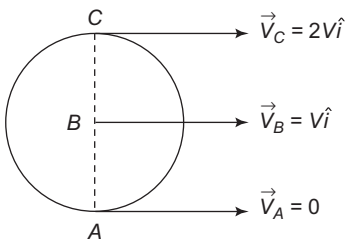
$$\frac{d^2U}{dx^2} = k - 3a\frac{k}{a}$$

$= k - 3k = -2k$, which is negative.

Hence U is maximum at $x = \sqrt{k/a}$.

Hence graph (a) is also not possible. Also U is negative for $x > \sqrt{2k/a}$. Therefore, the correct graph is (d).

6.



$\vec{V}_A = 0$, Let $\vec{V}_B = V\hat{i}$, then $\vec{V}_C = 2V\hat{i}$

$\therefore$ $\vec{V}_C - \vec{V}_A = 2V\hat{i} - 0 = 2V\hat{i}$

$\vec{V}_C - \vec{V}_B = 2V\hat{i} - V\hat{i} = V\hat{i}$

$\vec{V}_B - \vec{V}_C = -V\hat{i}$

$\vec{V}_B - \vec{V}_A = V\hat{i}$

The correct choice is (b).

7. Mass per unit area of the disc = $\frac{M}{\pi R^2}$. Therefore, mass of the removed portion (hole of radius *R/3)* is

$$m = \frac{M}{\pi R^2} \times \pi\left(\frac{R}{3}\right)^2 = \frac{M}{9}$$

The moment of inertia of the complete disc about an axis passing through its centre O and perpendicular to its plane is

$$I = \frac{1}{2}\ MR^2$$

Using the parallel axes theorem, the moment of inertia of the removed portion of the disc about the axis passing through O and perpendicular to the plane of the disc is

I' = MI of mass m about $O' + m \times OO'$

$$= \frac{1}{2}m\left(\frac{R}{3}\right)^2 + m \times \left(\frac{2R}{3}\right)^2$$

$$= \frac{1}{2}\times\frac{M}{9}\times\frac{R^2}{9} + \frac{M}{9}\times\frac{4R^2}{9} = \frac{1}{18}\ MR^2$$

Therefore, the moment of inertia of the remaining portion of the disc about $O = I - I' = \frac{1}{2}\ MR^2 - \frac{1}{18}\ MR^2 = \frac{4}{9}\ MR^2$.

8. For a satellite, we have

Kinetic energy $= \frac{GmM}{2r}$

Potential energy $= -\frac{GmM}{r}$

Total energy E_0 = KE + PE

$$= \frac{GmM}{2r} - \frac{GmM}{r} = -\frac{GmM}{2r} = \frac{\text{PE}}{2}$$

or $\text{PE} = 2E_0$.

9. Potential energy stored in the spring when it is extended by x is

$$U_1 = \frac{1}{2}\ kx^2$$

Potential energy stored in the spring when it is further extended by y is

$$U_2 = \frac{1}{2}k(x+y)^2$$

$$\therefore \text{ Work done } = U_2 - U_1 = \frac{1}{2}k(x+y)^2 - \frac{1}{2}kx^2$$

$$= \frac{1}{2}ky(2x+y)$$

10. Due to frictional force (which acts in a direction opposite to the direction of acceleration) on the rear face, the pressure in the rear side will be increased. Hence the pressure in the front side will be lowered. Thus the correct choice is (b).

11. Area of piston $A = \frac{\pi D^2}{4}$; D = diameter of piston.

Area of nozzle $a = \frac{\pi d^2}{4}$; d = diameter of nozzle

From equation of continuity $AV = av$,

$$v = \frac{AV}{a} = \frac{D^2}{d^2} \times V = (5)^2 \times 0.2 = 5 \text{ ms}^{-1}$$

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times1.25}{10}} = 0.5 \text{ s}$$

Horizontal velocity $v_x = v = 5$ ms^{-1}

Vertical velocity at $t = 0.5$ is $v_y = gt = 10 \times 0.5 = 5$ ms^{-1}

$\therefore$ Magnitude of velocity $= \left(v_x^2 + v_y^2\right)^{1/2}$

$$= \sqrt{5^2 + 5^2} = 5\sqrt{2} \text{ ms}^{-1}$$

12. If a force F is applied to M, say to the right, let A be the distance moved by M. If the system is released, it executes simple harmonic motion of amplitude A. If A_1 and A_2 are the extensions in springs k_1 and k_2 then $A = (A_1 + A_2)$ and

$$F = k_1 A_1 = k_2 A_2$$

$$\Rightarrow \quad A_1 = \frac{F}{k_1} \text{ and } A_2 \frac{F}{k_2}$$

$$\therefore \quad A = A_1 + A_2 = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

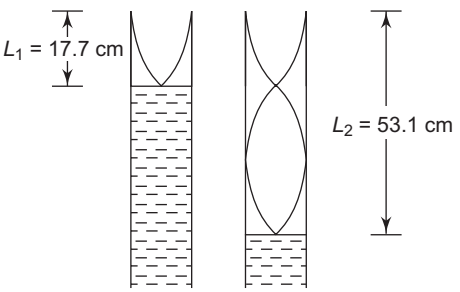
$$= \frac{F(k_1 + k_2)}{k_1 k_2}$$

$$\Rightarrow \quad F = \frac{k_1 k_2 A}{k_1 + k_2}$$

The amplitude of point P = amplitude of oscillations of spring k_1, which is

$$A_1 = \frac{F}{k_1} = \frac{k_2 A}{(k_1 + k_2)}$$

13. Refer to the following figure.



$$L_1 = \frac{\lambda}{4} \text{ and } L_2 = \frac{3\lambda}{4}$$

$$N = \nu_1 = \frac{v}{4(L_1 + e)} \quad (1)$$

$$\text{and} \quad N = \nu_3 = \frac{3v}{4(L_3 + e)} \quad (2)$$

where e is the end correction. Eliminating e from Eqs (1) and (2), we get

$$v = 2N(L_2 - L_1) \quad (3)$$

Lengths L_1 and L_2 are measured with a metre scale whose least count is 0.1 cm. Thus $L_1 = (17.7 \pm 0.1)$ cm and $L_2 = (53.1 \pm 0.1)$ cm. The maximum error

in $(L_2 - L_1)$ is ± 0.2 (for maximum error, the errors in individual measurements add up). Thus $L_2 - L_1$ = (53.1 – 17.7) = 35.4 cm. Hence $L_2 - L_1$ = (35.4 ± 0.2) cm. Using this in Eq. (3), we have

$v = 2 \times 480 \times (35.4 \pm 0.2) = (33984 \pm 192)$ cms^{-1}

Hence maximum error = 192 cm s^{-1}

14. Mass of 2 litres of water = 2 kg. Heat energy needed to raise the temperature of 2 kg of water from 20°C to 75°C is

$$Q = 2 \times (4.2 \times 10^3) \times 55 = 4.62 \times 10^5 \text{ J}$$

If t is the time taken, heat energy supplied by the heater in time t is

$$Q_1 = \text{(power} \times \text{time)} = 1000\, t \text{ joule}$$

Heat energy lost in time t is

$$Q_2 = 160\, t \text{ joule}$$

Heat energy available for heating water is

$$Q' = Q_1 - Q_2 = 840 \text{ J}$$

Equating $Q = Q'$, we get $t \simeq 550$ s.

15. Process $A \rightarrow B$ occurs at constant pressure. Hence the work done in this process is (see figure).

$$W_{AB} = PdV = P(V_2 - V_1)$$
$$= 10 \times (2 - 1) = 10 \text{ J}$$

Process $B \rightarrow C$, occurs at constant volume. Hence $W_{BC} = 0$. Given Q = 5 J, i.e. total work done is W_t = 5 J. Therefore, we have

$$W_t = W_{AB} + W_{BC} + W_{CA}$$

or $\quad 5 = 10 + 0 + W_{CA}$

or $\quad W_{CA} = -5$ J, which is choice (a).

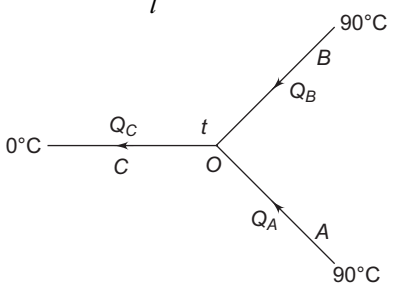
16. Let A and l be the area of cross-section and the length of each rod. If k is the coefficient of thermal conductivity and t°C the temperature of the junction O, then the rates at which heat energy enters O from rods A and B are

$$Q_A = \frac{kA(90 - t)}{l}$$

and $\quad Q_B = \frac{kA(90 - t)}{l}$

The rate at which heat energy flows in rod C is

$$Q_C = \frac{kA(t - 0)}{l}$$



In the steady state, rate at which heat energy enters O = rate at which heat energy leaves O, i.e.

$$Q_A + Q_B = Q_C$$

or $\quad \frac{kA(90 - t)}{l} + \frac{kA(90 - t)}{l} = \frac{kA(t - 0)}{l}$

or $\quad (90 - t) + (90 - t) = t$

or $3t = 180$ or $t = 60$°C.

17. Charge Q_1 on shell 1 induces a charge $-Q_1$ on the inner surface of shell 2 and a charge $+Q_1$ on its outer surface, so that the total charge on the outer surface of shell 2 is $(Q_1 + Q_2)$. This charge induces a charge $-(Q_1 + Q_2)$ on the inner surface of shell 3 and a charge $+(Q_1 + Q_2)$ on its outer surface so that the total charge on the outer surface of shell 3 is $(Q_1 + Q_2 + Q_3)$ as shown in the figure. Given

$$\sigma_1 = \sigma_2 = \sigma_3$$

$$\Rightarrow \frac{Q_1}{4\pi\varepsilon_0 R^2} = \frac{(Q_1 + Q_2)}{4\pi\varepsilon_0 (2R)^2} = \frac{(Q_1 + Q_2 + Q_3)}{4\pi\varepsilon_0 (3R)^2}$$

Solving we get $Q_1 = 3Q_2 = 5Q_3$.

18. The two sub circuits are closed loops. They cannot send any current through the 3 Ω resistor. Hence the potential difference across the 3 Ω resistor is zero,

19. Since no current flows through R_6, resistances R_1, R_2, R_3 and R_4 constitute the four arms of a balanced Wheatestone's bridge. Hence

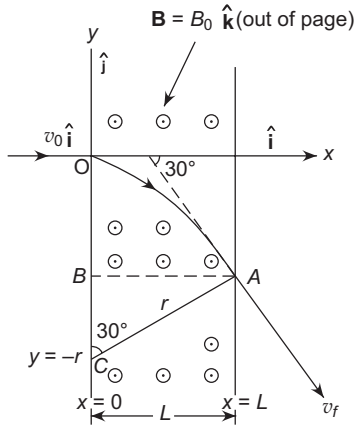
$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \text{ or } R_1 R_4 = R_2 R_3$$

20. The force experienced by the charged particle is given by

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) = q(v_0\, \hat{\mathbf{i}}) \times (B_0\, \hat{\mathbf{k}})$$
$$= qv_0B_0(\hat{\mathbf{i}} \times \hat{\mathbf{k}})$$
$$= q\, v_0\, B_0(-\hat{\mathbf{j}}) \qquad (1)$$

The force is along the negative y-direction, which is choice (b).

21. Refer to the following figure.



The particle describes a circle of radius

$$r = \frac{mv_0}{qB_0} \qquad (2)$$

Since the particle emerges from the region of the magnetic field with the velocity vector making an angle of 30° with the initial vector, it follows from triangle ABC that

$$AB = AC \sin 30°$$

or $$L = r \sin 30°$$

$$= \frac{m v_0 \sin 30°}{qB_0} = \frac{mv_0}{2qB_0} \qquad (3)$$

22. The magnitude of the induced voltage is proportional to the rate of change of magnetic flux which, in turn, depends on the number of turns in the coil, i.e. $V \propto n$. The resistance of a wire is given by

$R = \frac{\rho l}{\pi r^2}$ or $R \propto \frac{l}{r^2}$. Here ρ is the resistivity of the material of the wire.

$$\therefore \text{ Power } P = \frac{V^2}{R} \propto \frac{n^2}{l/r^2}$$

or $$P \propto \frac{(nr)^2}{l}$$

$$\therefore \quad \frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^2 \times \left(\frac{r_2}{r_1}\right)^2 \times \left(\frac{l_1}{l_2}\right) \qquad (1)$$

Now, if a wire of length l_1 and radius r_1 is stretched to a length l_2 such that its radius reduced to r_2, then (since the mass of the wire remains constant)

$$m = \pi r_1^2 l_1 d = \pi r_2^2 l_2 d \qquad (d \text{ is the density})$$

or $\frac{l_1}{l_2} = \left(\frac{r_2}{r_1}\right)^2$. Using this in Eq. (1), we get

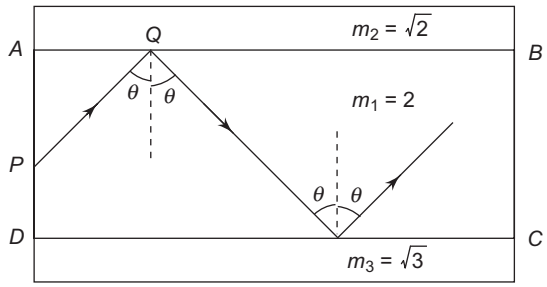
$$\frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^2 \times \left(\frac{r_2}{r_1}\right)^4$$

Given $\frac{n_2}{n_1} = 4$ and $\frac{r_2}{r_1} = \frac{1}{2}$. Using these values, we get

$$\frac{P_2}{P_1} = (4)^2 \times \left(\frac{1}{2}\right)^4 = 1.$$

23. Statement-2 is correct. According to Lenz's law, the induced current in the coil should be clockwise. So Statement-1 is false and Statement-2 is true.

24.



For total internal reflection at surface AB, angle θ must be greater than or equal to the critical angle i_1 given by

$$\sin i_1 = \frac{\mu_2}{\mu_1} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

which gives $i_1 = 45°$

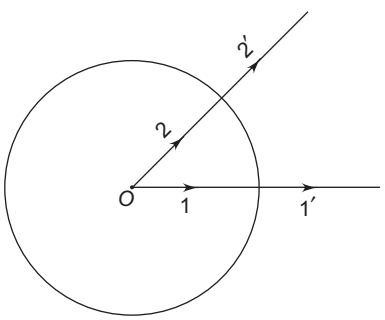
For total internal reflection at surface CD, angle θ must be greater than or equal to the critical angle i_2 given by

$$\sin i_2 = \frac{\mu_3}{\mu_1} = \frac{\sqrt{3}}{2}$$

which gives $i_2 = 60°$.

Hence, for total internal reflection at both the surfaces AB and CD, the minimum value of $\theta = 60°$.

25. Two rays 1 and 2 from the object placed at O fall normally on the spherical surface and go through undeviated (see Fig.). The divergent rays 1′ and 2′ appear to come from O. Hence the virtual image is formed at the centre O, which is at a distance of 6 cm from the surface of the sphere. Hence the correct choice is (b).



26. Let the nth dark fringe of wavelength λ_n and the mth dark fringe of wavelength λ_m coincide at a distance y from the centre of the screen, then

$$y = \left(n - \frac{1}{2}\right)\frac{\lambda_n D}{d} = \left(m - \frac{1}{2}\right)\frac{\lambda_m D}{d} \qquad (1)$$

At this position there is complete darkness on the screen. Eq. (1) gives

$$\frac{n-\frac{1}{2}}{m-\frac{1}{2}}=\frac{\lambda_m}{\lambda_n}=\frac{7500\text{Å}}{4500\text{Å}}=\frac{5}{3}$$

which gives $$m=\frac{6n+2}{10}=\frac{3n+1}{5} \qquad (2)$$

Integral values of n and m which satisfy Eq. (2) $n=3$, $m=2$; $n=8$, $m=5$; and so on. Let $n_1=3$ and $n_2=8$, then from Eq. (1) the distances from the centre of the screen of the first and the second regions of darkness are given by

$$y_1=\left(n_1-\frac{1}{2}\right)\frac{\lambda_n D}{d}$$

and $$y_2=\left(n_2-\frac{1}{2}\right)\frac{\lambda_n D}{d}$$

$$\therefore \quad \Delta y=y_2-y_1=(n_2-n_1)\times\frac{\lambda_n D}{d}$$

$$=(8-3)\times\frac{4500\times10^{-10}\times1\cdot2}{1\times10^{-3}}$$

$$=2.7\times10^{-3}\text{ m}=2.7\text{ mm}$$

27. A beam of light of intensity I is incident on plate A. Since the plate reflects 25% of I, the intensity of the reflected beam 1 is

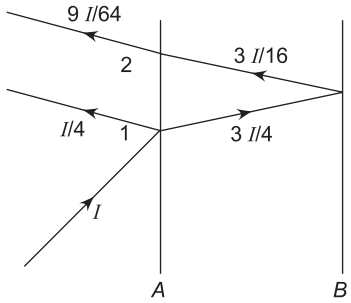
$$I_1=I\times\frac{25}{100}=\frac{I}{4}$$

The remaining intensity $3I/4$ falls on plate B which reflects 25% of the intensity incident on it. Hence intensity of beam reflected from B is

$$\frac{3I}{4}\times\frac{25}{100}=\frac{3I}{16}$$

A beam of intensity $3I/16$ falls on plate A which transmits 75% of this intensity. Hence the intensity of beam 2 is

$$I_2=\frac{3I}{16}\times\frac{75}{100}=\frac{9I}{64}$$



$$\frac{I_1}{I_2}=\frac{I/4}{9I/64}=\frac{16}{9}$$

28. The ratio of amplitudes is $\frac{a_1}{a_2}=\sqrt{\frac{16}{9}}=\frac{4}{3}$. Thus.

$a_1=4$ units and $a_2=3$ units.

$$\frac{I_{\max}}{I_{\min}}=\left(\frac{a_1+a_2}{a_1-a_2}\right)^2=\left(\frac{4+3}{4-3}\right)^2=49$$

29. From Moseley's law, we have

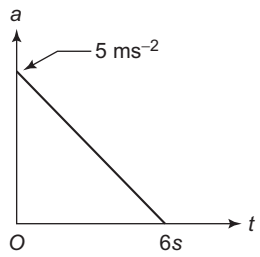
$$\frac{\nu_{41}}{\nu_{21}}=\left(\frac{41-1}{21-1}\right)=4.\text{ Hence}$$

$$\lambda_{21}=\frac{\lambda_{41}}{4}=\frac{\lambda}{4}$$

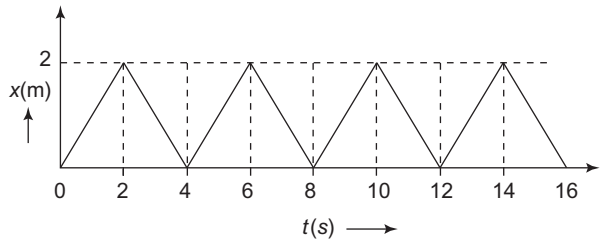
30. Since the half life is 2 hours, the intensity of the radiation falls by a factor of 2 every 2 hours. In 12 hours it will fall by a factor of $(2)^6=64$. Thus, in 12 hours the intensity attains the safe level. Hence the correct choice is (b).

Practice Test Paper – V

1. Two full turns of the circular scale of screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of – 0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is
   (a) 3.38 mm (b) 3.32 mm
   (c) 3.73 mm (d) 3.67 mm
2. A Vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is
   (a) 0.02 mm (b) 0.05 mm
   (c) 0.1 mm (b) 0.2 mm
3. A particle starts from rest. Its acceleration at time $t = 0$ is 5 ms^{-2} which varies with time as shown in the following figure. The maximum speed of the particle will be
   (a) 7.5 ms^{-1} (b) 15 ms^{-1}
   (c) 30 ms^{-1} (d) 37.5 ms^{-1}



4. A ball is released from rest from the top of a tower of height h metre. It takes T seconds to reach the ground. What was the height of the ball from the ground in T/3 seconds?
   (a) $\frac{h}{9}$ (b) $\frac{7h}{9}$
   (c) $\frac{8h}{9}$ (d) $\frac{17h}{18}$
5. A parachutist after bailing out falls 50 m without friction. When he opens the parachute, he decelerates at 2 ms^{-2}. He reaches the ground with a speed of 3 ms^{-1}. At what height did he bail out?
   (a) 91 m (b) 182 m
   (c) 293 m (d) 111 m
6. A body moving in a circular path with a constant speed has a
   (a) constant velocity
   (b) constant momentum
   (c) constant kinetic energy
   (d) constant acceleration
7. A particle moves in the $x - y$ plane under the influence of a force such that its linear momentum is
   $\vec{p}(t) = A\left[\hat{i}\cos(kt) - \hat{j}\sin(kt)\right]$
   where A and k are constants. The angle between the force and momentum is
   (a) 0° (b) 30°
   (c) 45° (d) 90°
8. The following figure shows the position -time ($x - t$) graph of one-dimenstional motion of a body of mass 0.4 kg. The magnitude of each impulse is
   (a) 0.4 Ns
   (b) 0.8 Ns
   (c) 1.6 Ns
   (d) 0.2 Ns



Directions: Question number 9 contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the statement.

9. **Statement-1 :** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

   Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

   (a) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.

   (b) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.

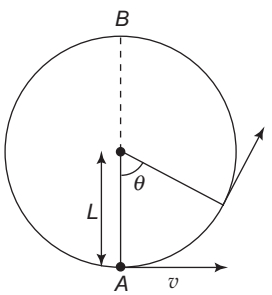
   (c) Statement-1 is false, Statement-2 is true.

   (d) Statement-1 is true, Statement-2 is false.

10. A bob of mass m is suspended by a massless string of length L. The horizontal volocity v at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A satisfies [see figure given below]

    (a) $\theta = \frac{\pi}{4}$ (b) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

    (c) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (d) $\frac{3\pi}{4} < \theta < \pi$



11. A soild metallic sphere of radius R having moment of inertia equal to I about its diameter is melted and recast into a solid disc of radius r of a uniform thickness. The moment of inertia of the disc about an axis passing through its edge and perpendicular to its plane is also equal to I. The ratio r/R is

    (a) $\frac{2}{\sqrt{15}}$ (b) $\frac{2}{\sqrt{10}}$

    (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{2}}$

12. Consider a uniform square plate of side 'a'and mass 'm'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

    (a) $\frac{2}{3}ma^2$ (b) $\frac{5}{6}ma^2$

    (c) $\frac{1}{12}ma^2$ (d) $\frac{7}{12}ma^2$

13. Suppose the gravitational force varies inversely as the n^{th} power of distance, then the time period of a planet in circular orbit of radius R around the sun will be proportional to

    (a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$

    (c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$

14. A simple pendulum has a time period T_1 when on the earth's surface, and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2/T_1 is

    (a) 1 (b) $\sqrt{2}$

    (c) 4 (d) 2

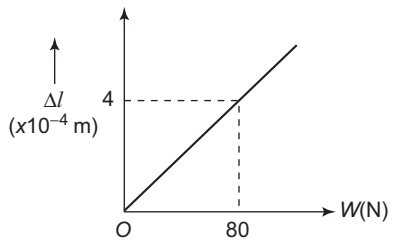
15. The following figure shows a graph of the extension (Δl) of a wire of length 1 m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross-sectional area of the wire is 10^{-6} m^2, the Young's modulus of the material of the wire is

    (a) 2×10^{11} N/m^2

    (b) 2×10^{-11} N/m^2

    (c) 3×10^{12} N/m^2

    (d) 3×10^{-12} N/m^2



16. A cylindrical tank having cross-sectional area A is filled with water to a height of 2.0 m. A circular hole of cross-sectional area a is opened at a height of 75 cm from the bottom. If $\frac{a}{A} = \sqrt{0.2}$, the velocity with which water emerges from the hole is ($g = 9.8$ ms^{-2}).
(a) 4.9 ms^{-1} (b) 4.95 ms^{-1}
(c) 5.0 ms^{-1} (d) 5.5 ms^{-1}

17. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount?
(a) $6F$ (b) $9F$
(c) F (d) $4F$

18. If C_p and C_v denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then
(a) $C_p - C_v = R/28$
(b) $C_p - C_v = R/14$
(c) $C_p - C_v = R$
(d) $C_p - C_v = 28R$

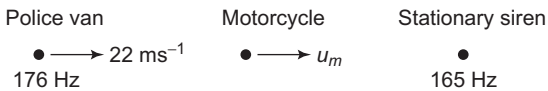
19. A monoatomic ideal gas, initially at temperature, T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the lengths of the gas column before and after expansion respectively, then T_1/T_2 is given by
(a) $\left(\frac{L_1}{L_2}\right)^{2/3}$ (b) $\frac{L_1}{L_2}$
(c) $\frac{L_2}{L_1}$ (d) $\left(\frac{L_2}{L_1}\right)^{2/3}$

20. A particle is executing simple harmonic motion. When its displacement is x, its total energy is
(a) proportional to x
(b) proportional to x^2
(c) proportional to $\sqrt{x}$
(d) independent to x

21. Three sound waves of equal amplitudes have frequencies $(\nu - 1)$, ν and $(\nu + 1)$. They superpose to give beats. The number of beats produced per second will be
(a) 2 (b) 1
(c) 4 (d) 3

22. A simple pendulum has a time period of 3.0 s. If the point of suspension of the pendulum starts moving vertically upward with a velocity $v = Kt$ where $K = 4.4$ ms^{-2}, the new time period will be (Take $g = 10$ ms^{-2})
(a) $\frac{9}{4}$ s (b) $\frac{5}{3}$ s
(c) 2.5 s (d) 4.4 s

23. A police van, moving at 22 ms^{-1}, chases a motorcyclist. The policeman sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz, as shown in the following figure.



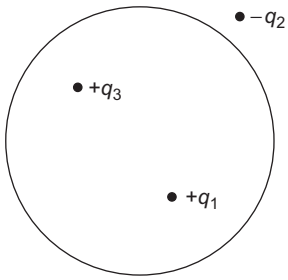
If the motorcyclist does not observe any beats, his speed must be (take the speed of sound = 330 ms^{-1})
(a) 33 ms^{-1} (b) 22 ms^{-1}
(c) zero (d) 11 ms^{-1}

24. The potential at a point x due to some charges situated on the x-axis is given by

$$V = \frac{20}{(x^2 - 4)} \text{ volt}$$

where x is measured in μm. The electric field E at $x = 4$ μm is
(a) $\frac{5}{3}$ volt/μm in the – ve x direction
(b) $\frac{5}{3}$ volt/μm in the + ve x direction
(c) $\frac{10}{9}$ volt/μm in the – ve x direction
(d) $\frac{10}{9}$ volt/μm in the + ve x direction

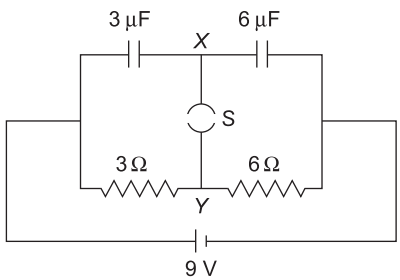
25. The figure given on next page shows a spherical Gaussian surface and a charge distribution. When calculating the flux of electric field through the Gaussian surface, the electric field will be due to
(a) $+ q_3$ alone
(b) $+ q_1$ and $+ q_3$
(c) $+ q_1$, $+ q_3$ and $- q_2$
(d) $+ q_1$ and $- q_2$



26. A circuit is connected as shown in the following figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is

(a) 0 (b) 54 μC

(c) 27 μC (d) 81 μC



27. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1: The temperature dependence of resistance is usually given as $R = R_0 (1 + \alpha\Delta t)$. The resistance of a wire changes from 100 Ω to 150 Ω when its temperature is increased from 27°C to 227°C. This implies that $\alpha = 2.5 \times 10^{-3}$/°C

Statement-2: $R = R_0 (1 + \alpha\Delta t)$ is valid only when the change in the temperature Δt is small and $\Delta R = (R - R_0) << R_0$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.

(b) Statement-1 is false, Statement-2 is true.

(c) Statement-1 is true, Statement-2 is false.

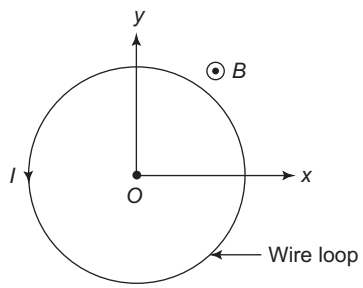
(d) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.

28. A long straight wire of radius a carries a steady current i. The current is uniformly distributed across its cross-section. The ratio of the magnetic field at $\frac{a}{2}$ and $2a$ is

(a) $\frac{1}{4}$ (b) 4

(c) 1 (d) $\frac{1}{2}$

29. A thin wire loop carrying a current I is placed in a uniform magnetic field B pointing out of the plane of the coil as shown in the following figure. The loop will tend to



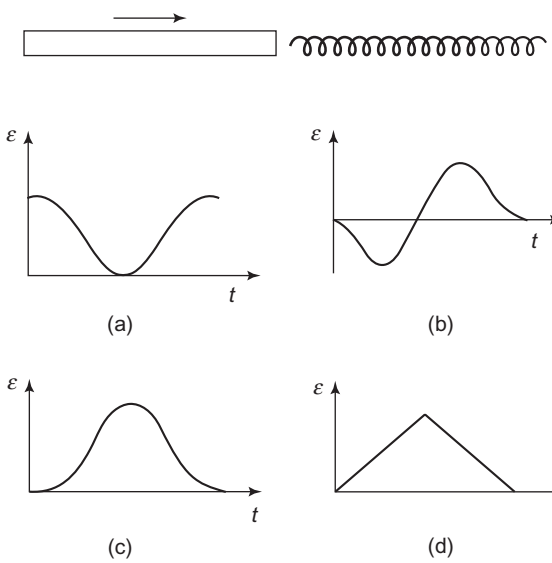
(a) move towards positive x-direction

(b) move towards negative y-direction

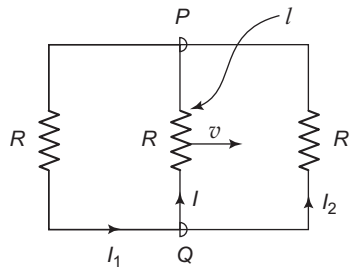
(c) contract

(d) expand

30. The variation of induced emf with time in a coil if a short bar magnet is moved along its axis, (shown in the following figure, with a constant velocity is best represented as [see the next figure]



31. A rectangular loop has a sliding connector PQ of length l and resistance R and it is moving with a speed v as shown in the following figure. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



(a) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$

(b) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$

(c) $I_1 = I_2 = I = \frac{Blv}{R}$

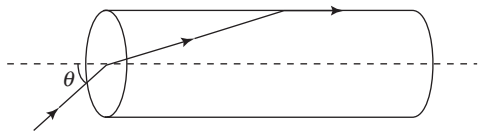
(d) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$

32. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead, it is reduced to $I/8$. The thickness of lead, which will reduce the intensity to $I/2$ is

(a) 6 mm (b) 9 mm

(c) 18 mm (d) 12 mm

33. A transparent solid cylindrical rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the following figure. The incident angle θ for which the light ray grazes along the wall of the rod is :



(a) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (b) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

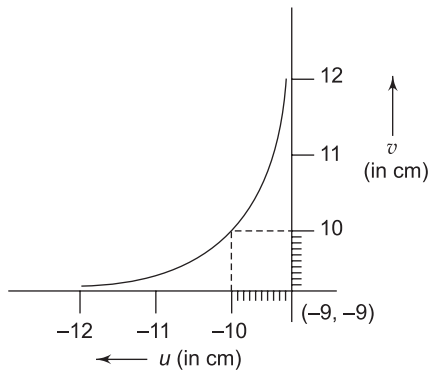
(c) $\sin^{-1}\left(\frac{1}{2}\right)$ (d) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

34. A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of the known wavelength coincides with the 4th bright fringe of the unknown light. The wavelength of the unknown light is

(a) 442.5 nm (b) 776.8 nm

(c) 393.4 nm (d) 885.0 nm

35. The figure shows the graph between the image distance v (in cm) and the object distance u (in cm) for a thin convex lens. The focal length of the lens is



(a) (5.00 ± 0.05) cm

(b) (5.00 ± 0.10) cm

(c) (10.0 ± 0.10) cm

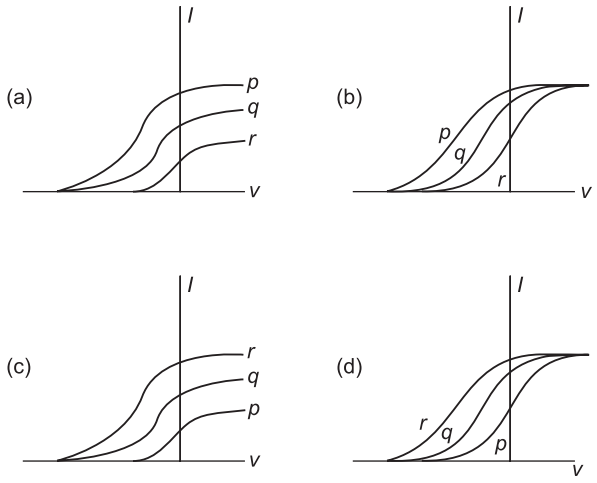
(d) (0.50 ± 0.05) cm

36. In Young's double slit experiment, the intensity at a point P on the screen is half the maximum intensity in the interference pattern. If the wavelength of light used is λ and d is the distance between the slits, the angular separation between point P and the centre of the screen is

(a) $\sin^{-1}\left(\frac{\lambda}{d}\right)$ (b) $\sin^{-1}\left(\frac{\lambda}{2d}\right)$

(c) $\sin^{-1}\left(\frac{\lambda}{3d}\right)$ (d) $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

37. Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions ϕ_p = 2.0 eV. ϕ_q = 2.5 eV and ϕ_r = 3.0 eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is [see figures on next page] [Take hc = 1240 eV nm]



38. A nucleus of mass number 220, initially at rest, emits an α-particle. If the Q value of the reaction is 5.5 MeV, the energy of the emitted α-particle will
(a) 4.8 MeV (b) 5.4 MeV
(c) 6.0 MeV (d) 6.8 MeV

39. If the binding energy per nucleon in $^{7}_{3}\text{Li}$ and $^{4}_{2}\text{He}$ nuclei is 5.60 MeV and 7.06 MeV respectively, then in the reaction
$p + {}^{7}_{3}\text{Li} \rightarrow 2{}^{4}_{2}\text{He}$
energy of proton must be
(a) 1.46 MeV (b) 39.2 MeV
(c) 28.24 MeV (d) 17.28 MeV

40. In a common base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor (β) will be
(a) 51 (b) 48
(c) 49 (d) 50

Answers

1. (a)	2. (d)	3. (b)	4. (a)
5. (c)	6. (c)	7. (d)	8. (b)
9. (a)	10. (d)	11. (a)	12. (a)
13. (a)	14. (d)	15. (a)	16. (c)
17. (b)	18. (a)	19. (d)	20. (d)
21. (a)	22. (c)	23. (b)	24. (d)
25. (c)	26. (c)	27. (b)	28. (c)
29. (c)	30. (b)	31. (b)	32. (d)
33. (b)	34. (a)	35. (a)	36. (d)
37. (a)	38. (b)	39. (d)	40. (c)

Solutions

1. Pitch of screw $= \dfrac{1\text{ mm}}{2} = 0.5$ mm

$$\text{Least count} = \frac{0.5\text{ mm}}{50} = 0.01\text{ mm}$$

Measured diameter = 3 mm + 35 × 0.01 mm
= (3 + 0.35) mm = 3.35 mm
Corrected diameter = measured diameter – zero error
= 3.35 – (– 0.03) = 3.38 mm

2. Vernier constant = value of 1 main scale division – value of 1 vernier scale division.

3. The slope of the line is $m = -\dfrac{5}{6}\text{ ms}^{-2}$ per second and its intercept is c =5 ms^{-2}. Using $y = mx + c$, the acceleration a (in ms^{-2}) as a function of time t is given by

$$a = -\frac{5}{6}t + 5$$

or $$\frac{dv}{dt} = -\frac{5}{6}t + 5$$

or $$v = \int_0^t \left(-\frac{5}{6}t + 5\right)dt$$

or $$v = -\frac{5}{12}t^2 + 5t + k \qquad (1)$$

where k is the constant of integration. Since the particle starts from rest, $v = 0$ at $t = 0$. Using this in (1) we get $k = 0$. Hence

$$v = -\frac{5}{12}t^2 + 5t \qquad (2)$$

It follows from the graph that the deceleration becomes zero at t = 6 s. Hence, the speed of the particle will be maximum at t = 6 s, Putting t = 6 s in Eq. (2), we have

$$v_{\max} = -\frac{5}{12}\times(6)^2 + 5\times 6$$
$$= -15 + 30 = 15\text{ ms}^{-1}$$

4. Using $S = ut + \dfrac{1}{2}at^2$, we have

$$-h = 0 - \frac{1}{2}gT^2 \quad \Rightarrow \quad h = \frac{gT^2}{2}$$

If the ball travels a distance h' in time $t = T/3$, we have

$$-h' = 0 - \frac{1}{2}g\left(\frac{T}{3}\right)^2 \Rightarrow h' = \frac{gT^2}{18}$$

$$\therefore \quad \frac{h'}{h} = \frac{1}{9} \Rightarrow h' = \frac{h}{9}$$

5. Let H be the height from the ground when the parachust bails out. The velocity after falling through $h = 50$ m is.

$$v = \sqrt{2gh} = \sqrt{2 \times (-9.8) \times (-50)} = \sqrt{980}\ \text{ms}^{-1}$$

directed downwards. After opening the parachute, he falls a distance $(H - h)$. For this distance, initial velocity $u = \sqrt{980}\ \text{ms}^{-1}$, final velocity $v = 3\ \text{ms}^{-1}$ and acceleration $a = -2\ \text{ms}^{-2}$.

$$\therefore \quad H - h = \frac{v^2 - u^2}{2a} = \frac{9 - 980}{-4} = \frac{971}{4} \simeq 243\,\text{m}$$

$$\Rightarrow \quad H = 243 + h = 243 + 50 = 293\ \text{m}$$

6. The correct choice is (c).

7. $$\vec{F} = \frac{d\vec{p}}{dt}$$

$$= \frac{d}{dt}\left[A\{\hat{i}\cos(kt) - \hat{j}\sin(kt)\}\right]$$

$$= Ak\left[-\hat{i}\sin(kt) - \hat{j}\cos(kt)\right]$$

Now $\vec{F} \cdot \vec{p} = Ak\left[-\hat{i}\sin(kt) - \hat{j}\cos(kt)\right] \cdot A\left[\hat{i}\cos(kt) - \hat{j}\sin(kt)\right]$

$= A^2k\,[-\sin(kt)\cos(kt) + \cos(kt)\sin kt]$

$= 0 \left(\because \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = 1 \text{ and } \hat{i}\cdot\hat{j} = 0\right)$

Hence the angle between $\vec{F}$ and $\vec{p}$ is 90°.

8. Velocity = slope of $x - t$ graph. It is clear from the graph that the direction of the velocity of the body changes after every 2 seconds. Between $t = 0$ and $t = 2$ s, the velocity is

$$v_1 = \frac{2\,\text{m}}{2\,\text{s}} = 1\ \text{ms}^{-1}$$

and between $t = 2$ s and $t = 4$ s, the velocity is

$$v_2 = \frac{-2\,\text{m}}{2\,\text{s}} = -1\ \text{ms}^{-1}$$

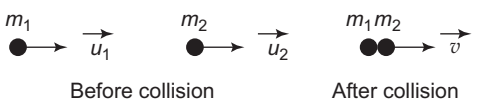
$\therefore$ Magnitude of change in velocity

$$\Delta v = |v_2 - v_1| = 2\ \text{ms}^{-1}$$

$\therefore$ Magnitude of impulse = change in momentum

$$= m\,\Delta v = 0.4 \times 2 = 0.8\ \text{Ns}$$

9. The principle of conservation of momentum holds for both elastic as well as inelastic collisions. In a completely inelastic collisions, the two particles stick together (Fig. P-62)



$$m_1\vec{u}_1 + m_2\vec{u}_2 = (m_1 + m_2)\vec{v}$$

$$\Rightarrow \quad \vec{v} = \frac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1 + m_2} = \frac{\vec{p}_1 + \vec{p}_2}{m_1 + m_2}$$

$\because$ K.E. after collision $= \frac{1}{2}(m_1 + m_2)v^2$, which can be zero if $\vec{v} = 0$ which is possible if $\vec{p}_1 = -\vec{p}_2$.

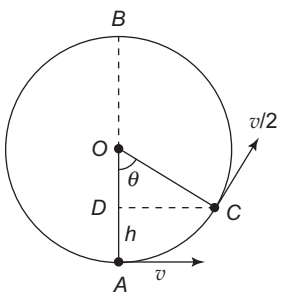
This is not possible for two particles moving in the same direction. Hence the particles do not lose all their kinetic energy. So the correct choice is (a).

10. Refer to Figure.
$OA = OB = OC = L$ and $OD = L\cos\theta$
$\therefore h = OA - OD = L - L\cos\theta = L(1 - \cos\theta)$
From conservation of energy,
total energy at A = total energy at C, i.e

$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + mgL(1 - \cos\theta)$$

$$v^2 = \frac{8gL}{3}(1 - \cos\theta) \qquad (1)$$

The minimum velocity the bob must have at A so as to reach B is $v = \sqrt{5gL}$. Putting this in Eq. (1), we get $\cos\theta = -\frac{7}{8}$. Therefore θ lies between $\frac{3\pi}{4}$ and π.



11. Let M be the mass of the sphere. The mass of the disc will also be M. The moment of inertia of the sphere about its diameter is

$$I_s = \frac{2}{5}MR^2$$

The moment of inertia of the disc about its edge and perpendicular to its plane is (using parallel axes theorem)

$$I_d = I_{cm} + Mh^2 = \frac{1}{2}Mr^2 + Mr^2$$

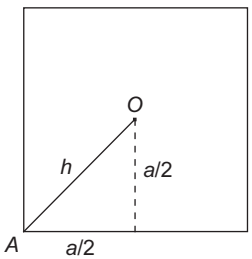
$$= \frac{3}{2}Mr^2$$

Given $I_s = I_d$. Hence, we have

$$\frac{2}{5}MR^2 = \frac{3}{2}Mr^2$$

which gives $\frac{r}{R} = \frac{2}{\sqrt{15}}$.

12. From parallel axes theorem the moment of inertia about the axis passing through A and perpendicular to the plane of the square plate is



$$I_A = I_{CM} = mh^2$$

$$= \frac{ma^2}{6} + m\left[\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right]$$

$$= \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{2ma^2}{3}.$$

13. Use $\frac{mv^2}{R} + \frac{GmM}{R^n}$ and $T = \frac{2\pi R}{v}$.

The correct choice is (a).

14. Use $g_2 = g_1\left(\frac{R}{R+h}\right)^2$ where $h = R$

Also $T_2 = 2\pi\sqrt{\frac{l}{g_2}}$ and $T_1 = 2\pi\sqrt{\frac{l}{g_1}}$.

The correct choice is (d).

15. $$Y = \frac{W/A}{\Delta l/l} = \frac{W \times l}{A \times \Delta l}$$

The slope of the graph = $\frac{\Delta l}{W}$

The correct choice is (a).

16. Let v be the velocity of efflux and V be the velocity with which the water level in the tank falls as shown in the following figure. From the equation of continuity of flow, we have $av = AV$ or

$$V = \frac{av}{A} \qquad (1)$$

From Bernoulli's theorem, we have

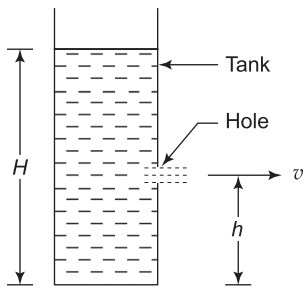
$$P_0 + \frac{1}{2}\rho V^2 + \rho gH = P_0 + \frac{1}{2}\rho v^2 + \rho gh$$

which gives $v^2 = V^2 + 2g(H - h)$ (2)

Using (1) in (2), we get

$$v^2 = \frac{2g(H-h)}{1-\left(\frac{a}{A}\right)^2} = \frac{2 \times 9.8 \times (2.0 - 0.75)}{1 - 0.2} = 25$$

which gives $v = 5.0$ ms^{-1}.



17. Given $V_1 = V_2$

$$\Rightarrow \quad AL_1 = (3A)L_2 \Rightarrow \frac{L_1}{L_2} = 3$$

Also $Y = \frac{F_1 L_1}{A\Delta x} = \frac{F_2 L_2}{(3A)\Delta x}$

The correct choice is (b).

18. Let C_p' and C_v' be the molar specific heats of a gas at constant pressure and constant volume respectively. Then

$$C_p' - C_v' = R \qquad (1)$$

The molecular mass of nitrogen is 28. Therefore, the specific heats per unit mass of nitrogen at constant pressure and at constant volume respectively are

$$C_p = \frac{C_p'}{28} \text{ and } C_p = \frac{C_v'}{28}$$

Using these in Eq.(1), we get $C_p - C_v = \frac{R}{28}$.

19. For adiabatic process, $T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)}$. Thus

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{(\gamma-1)}$$

For a monoatomic gas, $\gamma = 5/3$.
Also $V_2/V_1 = L_2/L_1$.
Hence,

$$\frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{\left(\frac{5}{3}-1\right)} = \left(\frac{L_2}{L_1}\right)^{2/3}$$

20. In simple harmonic motion, the kinetic energy and potential energy both vary with displacement x but the total energy is the same for all values of x if friction is neglected. Hence the correct choice is (d).

21. Let a_0 be the amplitude of each wave. The resultant particle displacement due to superposition of the three waves is given by

$$y = y_1 + y_2 + y_3$$
$$= a_0 \sin\{2\pi(\nu - 1)t\} + a_0 \sin(2\pi\nu t) + a_0 \sin\{2\pi(\nu + 1)t\}$$
$$= 2a_0 \sin(2\pi\nu t)\cos 2\pi t + a_0 \sin(2\pi\nu t)$$
$$= a_0(2\cos 2\pi t + 1)\sin(2\pi\nu t)$$
$$\Rightarrow y = A \sin(2\pi\nu t)$$

where $A = a_0(2\cos 2\pi t + 1)$ is the resultant amplitude. Since intensity $\propto$ (amplitude)^2, the resultant intensity is given by

$I = I_0(2\cos 2\pi t + 1)^2$

(i) When $\cos 2\pi t = +1 \Rightarrow 2\pi t = 0, 2\pi, ...$
$\Rightarrow t = 0, 1s, ... I = 9I_0$

(ii) When $\cos 2\pi t = -1 \Rightarrow 2\pi t = \pi, 3\pi, ...$
$\Rightarrow t = 0.5$ s, 1.5s, ... $I = I_0$

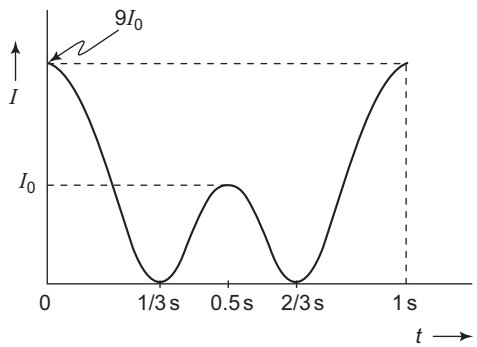
(iii) When $\cos 2\pi t = 0 \Rightarrow 2\pi t = \frac{\pi}{2}, \frac{3\pi}{2}, ...$

$\Rightarrow t = 0.25s, 0.75s, ... I = I_0$

(iv) When $\cos 2\pi t = -\frac{1}{2} \Rightarrow 2\pi t = \frac{2\pi}{3}, \frac{4\pi}{3}, ...$

$$\Rightarrow t = \frac{1}{3}s, \frac{2}{3}s, ... I = 0$$

So the graph of I versus t (from $t = 0$ to $t = 1$ s) is as shown in figure.



Time period between two consecutive maxima is $T_b = 0.5$. Therefore, frequency of beats is

$$\nu_b = \frac{1}{t_b} = \frac{1}{0.5} = 2 \text{ Hz}$$

Thus 2 beats are produced per second.

22. Original time period is

$$T_1 = 2\pi\sqrt{\frac{l}{g}} \qquad (1)$$

When the pendulum is moving upwards, the effective value of g is

$$g_{eff} = g + a$$

where a is the acceleration of the pendulum which is given by

$$a = \frac{dv}{dt} = \frac{d}{dt}(Kt) = K = 4.4 \text{ ms}^{-2}$$

$$\therefore \quad g_{eff} = g + a = 10 + 4.4 = 14.4 \text{ ms}^{-2}$$

Therefore, the new time period is

$$T_2 = 2\pi\sqrt{\frac{l}{g_{eff}}} \qquad (2)$$

From (1) and (2), we get

$$\frac{T_2}{T_1} = \sqrt{\frac{g}{g_{eff}}} = \sqrt{\frac{10}{14.4}} = \frac{1}{1.2}$$

or $$T_2 = \frac{T_1}{1.2} = \frac{3}{1.2} = 2.5 \text{ s}$$

23. Given $v = 330$ ms^{-1} and $u_p = 22$ ms^{-1}. The apparent frequency of the police man's horn of frequency 176 Hz as heard by the motorcyclist is given by

$$\nu_1 = 176\left(\frac{330 - u_m}{330 - 22}\right) = \frac{176}{308}(330 - u_m) \qquad (i)$$

The apparent frequency of the stationary siren of frequency 165 Hz as heard by the motorcyclist is given by

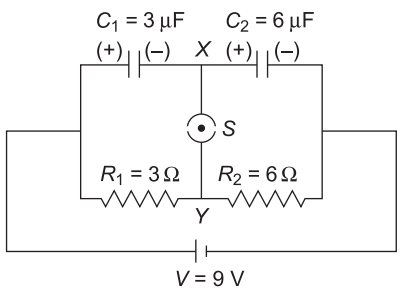
$$\nu_2 = 165\left(\frac{330 + u_m}{330}\right) \qquad (ii)$$

Since the motorcyclist does not observe any beats, $\nu_1 = \nu_2$. Equating (i) and (ii) and solving for u_m we get $u_m = 22$ ms^{-1}

24. Use $E = -\frac{dV}{dx}$ and find E for $x = 4$ μm. The correct choice is (d). Note that V decreases as x increases.

25. The electric flux is given by the surface integral $\int \mathbf{E}.\mathbf{ds}$. Here the electric field **E** is due to all the charges, both inside and outside the Gaussian surface. Hence the correct choice is (c).

26. Refer to figure on next page.



When the switch S is open, capacitors C_1 and C_2 are in series and their combined capacitance is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{3 \times 6}{3 + 6} = 2\,\mu\text{F}$$

$\therefore$ Charge on each capacitor = 2 μF × 9 V = 18 μC
When the switch S is closed, in the steady state, no current flows through the capacitors. Therefore, resistors R_1 and R_2 will be in series and their combined resistance is $R = R_1 + R_2 = 3 + 6 = 9\ \Omega$
Therefore, the current in each resistor is

$$I = \frac{9\,\text{V}}{9\Omega} = 1\ \text{A}$$

$\therefore$ Potential difference across R_1 is

$$V_1 = IR_1 = 1 \times 3 = 3\text{V}$$

Potential difference across R_2 is

$$V_2 = IR_2 = 1 \times 6 = 6\text{V}$$

Since capacitor C_1 is connected across R_1, the potential difference across C_1 is V_1. Similarly, the potential difference across C_2 is V_2.
$\therefore$ Charge on capacitor C_1 in

$$Q_1 = C_1 V_1 = 2\ \mu\text{F} \times 3\ \text{V} = 9\ \mu\text{C}$$

Charge on capacitor C_2 in $Q_2 = C_2 V_2 = 6\ \mu\text{F} \times 6\ \text{V} = 36\ \mu\text{C}$
Now, when the switch is open, the initial charge flowing from Y to X = 18 – 18 = 0 μC because the right plate of C_1 has a charge – 18 μC and left plate of C_2 has a change + 18 μC. When the switch S is closed, the final charge flowing from Y to X = –9 + 36 = + 27 μC. Therefore, the net charge flowing from Y to X when the switch is closed = 27 μC – 0 = 27 μC.

27. Relation $R = R_0 (1 + \alpha\Delta t)$ is valid for small changes in temperature. So the correct choice is (b).

28. Use $B = \dfrac{\mu_0 Ir}{2\pi a}$ for $0 \le r \le a$

and $B = \dfrac{\mu_0 I}{2\pi r}$ for $r > a$

where a is the radius of the wire. The correct choice is (c).

29. A circular metal loop carries a current. Hence charge, say, q moves along the circle with a velocity, say v which is tangential to the circle at every point. The force experienced by the charge is $\mathbf{F} = q\ (\mathbf{v} \times \mathbf{B})$. Since $\mathbf{v}$ is along the tangent and $\mathbf{B}$ is directed out of the $x - y$ plane, the direction of the force is towards the centre O of the loop. Hence the force tends to contract the loop. Further, since $\mathbf{F}$ is perpendicular to $\mathbf{v}$. no work is done on the loop. Hence it cannot have any translational motion.

30. As the bar magnet moves towards the coil, the induced emf $e = -d\phi/dt$ is negative and as it moves away, the induced emf is positive. Hence the correct choice is (b).

31. When a wire of length l moves with a velocity v perpendicular to a magnetic field B, and emf $E = Blv$ is induced between its ends. Hence the equivalent circuit is as shown in the following figure. From Kirchhoff's junction rule

$$I_1 + I_2 = I$$

Applying Kirchhoff's loop rule to loops *abefa* and *bcdeb* we have

$$E - IR - I_1 R = 0 \quad (1)$$

and $$I_2 R + IR - E = 0 \quad (2)$$

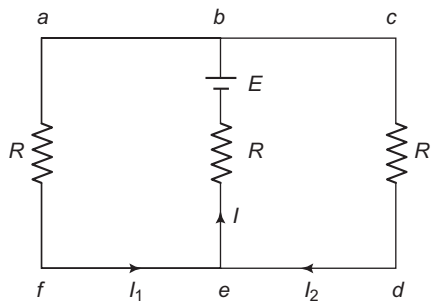
Adding (1) and (2), we get $I_1 = I_2$
Subtracting (1) and (2), we get

$$2E - 2IR - (I_1 + I_2)\,R = 0$$

$$\Rightarrow 2E - 2IR - I\,R = 0$$

$$\Rightarrow \quad I = \frac{2E}{3R} = \frac{2Blv}{3R}$$

$$\therefore \quad I_1 = I_2 = \frac{I}{2} = \frac{Blv}{3R}$$



32. The intensity of radiation of intensity I when it passes through a thickness x of an absorbing medium is given by

$$I' = Ie^{-\mu x}$$

where μ is a constant. When x_1 = 36 m, $I' = \dfrac{I}{8}$.
Therefore,

$$\frac{I}{8} = Ie^{-\mu x_1}$$

$$\Rightarrow \quad \frac{1}{8} = e^{-\mu x_1}$$

or $$e^{\mu x_1} = 8$$

$$\Rightarrow \quad \mu x_1 = \ln(8) = 2.08$$

$$\Rightarrow \quad \mu = \frac{2.08}{x_1}$$

Let x_2 be the thickness of lead for I to reduce to $I_2 = \frac{I}{2}$. Then

$$\frac{I}{2} = Ie^{-\mu x_2}$$

$$\Rightarrow \quad \mu x_2 = \ln(2) = 0.693$$

$$\Rightarrow \quad \frac{2.08}{x_1} \times x_2 = 0.693$$

$$\Rightarrow \quad x_2 = \frac{0.693 \times x_1}{2.08}$$

$$= \frac{0.693 \times 36\,\text{mm}}{2.08}$$

$$= 11.99\ \text{mm} \simeq 12\ \text{mm}$$

33. Using Snell's law $1 \times \sin\theta = \frac{2}{\sqrt{3}} \sin\alpha$

$$\Rightarrow \quad \sin\theta = \frac{2}{\sqrt{3}} \sin\alpha$$

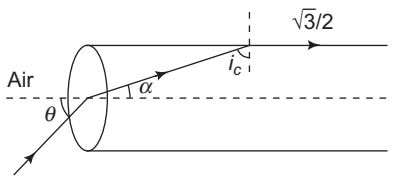
Critical angle i_c is given by [see figure]

$$\sin i_c = \frac{1}{\mu} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin(90° - \alpha) = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \cos\alpha = \frac{\sqrt{3}}{2}$$

Therefore, $\sin\alpha = \sqrt{(1+\cos^2\alpha)} = \left(1 - \frac{3}{4}\right)^{1/2} = \frac{1}{2}$

$$\therefore \quad \sin\theta = \frac{2}{\sqrt{3}} \times \frac{1}{2} = \frac{1}{\sqrt{3}} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$



34. For bright fringes, $y_n = \frac{n\lambda D}{d}$.

y_3 (for $\lambda_1 = 590$ nm) = y_4 for λ_2.
The correct answer is $\lambda_2 = 442.5$ nm.

35. It follows from the graph that when $u = -10$ cm, $v = +10$ cm. The focal length of the lens is given by

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{10} - \frac{1}{-10}, \text{ which gives } f = 5.0\ \text{cm}$$

The maximum error Δf in the measurement of f is given by

$$\frac{\Delta f}{f^2} = \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}$$

where Δu and Δv are the least counts of u and v scales of the graph. It follows from the graph that $\Delta u = \Delta v = 1$ mm = 0.1 cm. Hence

$$\Delta f = \left(\frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}\right) \times f^2$$

$$= \left[\frac{0.1}{(10)^2} + \frac{0.1}{(10)^2}\right] \times (5)^2$$

$$= \frac{5}{100}\ \text{cm} = 0.05\ \text{cm}$$

Hence $f = (5.00 \pm 0.05)$ cm,

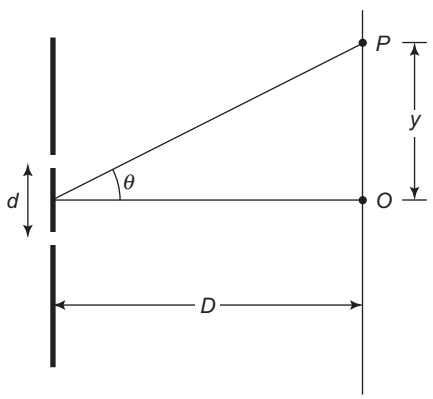
36. If δ is the phase difference between the interfering waves at point P, then the intensity at point P is given by [see figure]

$$I = I_{\max} \cos^2\left(\frac{\delta}{2}\right)$$

Given $I = \frac{I_{\max}}{2}$. Hence

$$\cos^2\left(\frac{\delta}{2}\right) = \frac{1}{2}, \text{ which gives } \frac{\delta}{2} = \frac{\pi}{4}$$

or $\delta = \frac{\pi}{2}$



The angular separation θ between points P and O is given by $\tan\theta = y/D$. Since θ is very small, $\tan\theta \simeq \sin\theta$. Hence

$$\sin\theta = \frac{y}{D} \qquad (1)$$

If ω is the fringewidth, then

$$\frac{y}{\omega} = \frac{\pi/2}{2\pi} = \frac{1}{4} \qquad (2)$$

This is so because the phase difference δ between two consecutive maxima is 2π. Now $\omega = \frac{\lambda D}{d}$.

Using this in Eq. (2), we get

$$\frac{yd}{\lambda D} = \frac{1}{4}$$

or $$\frac{y}{D} = \frac{\lambda}{4d} \quad (3)$$

Using Eq. (1) in Eq. (3), we have

$$\sin\theta = \frac{\lambda}{4d} \quad \text{or} \quad \theta = \sin^{-1}\left(\frac{\lambda}{4d}\right)$$

37. For photoelectric emission, the wavelength of the incident radiation must be less than the cut-off wavelength of the metal given by $\lambda_0 = hc/\phi_0$. For metals p, q and r, the cut-off wavelengths are

$$\lambda_p = \frac{hc}{\phi_p} = \frac{1240 \text{ eV nm}}{2.0 \text{ eV}} = 620 \text{ nm}$$

$$\lambda_q = \frac{1240}{2.5} = 496 \text{ nm}$$

$$\lambda_r = \frac{1240}{3.0} = 413.3 \text{ nm}$$

Hence metal plate p emits photoelectrons for all the three given radiations, metal plate q emits photoelectrons for radiation of wavelengths 450 nm and 350 nm and metal plate r emits photoelectrons only for wavelength 350 nm. Therefore photoelectric current is maximum for metal p and minimum for r, i.e. $I_p > I_q > I_r$. So the correct choice is (a).

38. Kinetic energy = $\dfrac{(\text{momentum})^2}{2\times\text{mass}}$

Mass number of α-particle (m) = 4 units. Mass number of daughter nucleus (M) = 220 – 4 = 216 units. If P and p denote the momenta of the daughter nucleus and the α-particle respectively, then

$$Q = \frac{P^2}{2M} + \frac{P^2}{2m}$$

Since momentum is conserved, $P = p$. Hence

$$Q = \frac{p^2}{2}\left(\frac{1}{M} + \frac{1}{M}\right) = \frac{p^2}{2m}\left(\frac{m}{M} + 1\right)$$

Now $\dfrac{p^2}{2m}$ = KE of α-particle = E_α. Thus,

$$Q = E_\alpha\left(\frac{m+M}{M}\right)$$

or $$E_\alpha = \frac{QM}{(m+M)} = \frac{5.5\,\text{MeV}\times 216}{(4+216)}$$

$$= 5.4 \text{ MeV}$$

39. Let Q MeV be the energy of the proton. Then, we have Q + energy of one ^7_3Li nucleus = energy of two ^4_2He nuclei

$\therefore$ $Q + 7 \times 5.6 \text{ MeV} = 2 \times (4 \times 7.06 \text{ MeV})$

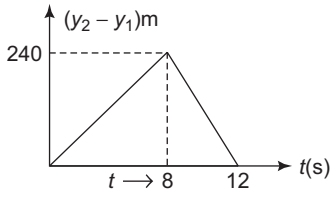
or $Q + 39.6 \text{ MeV} = 56.48 \text{ MeV}$

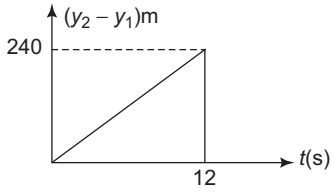
which gives $Q = 17.28$ MeV.

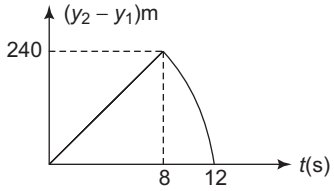
40. $\beta = \dfrac{I_C}{I_B} = \dfrac{I_C}{I_E - I_C} = \dfrac{5.488}{5.60 - 5.488} = 49$

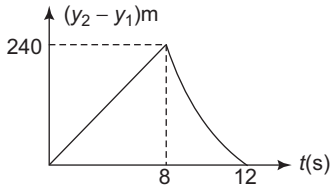
Solutions of Physics JEE Main—2015

1. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first? (Assume stones do not rebound after hitting the ground and neglect air resistance, take, $g = 10$ m/s^2) (*The figures are schematic and not drawn to scale*)

(a)


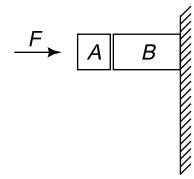
(b)


(c)


(d)


2. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. The accuracy in the determination of g is:
   (a) 2% (b) 3%
   (c) 1% (d) 5%

3. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is



   (a) 100 N (b) 80 N
   (c) 120 N (d) 150 N

4. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to:
   (a) 44% (b) 50%
   (c) 56% (d) 62%

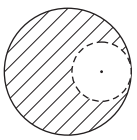
5. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to:
   (a) $\frac{h^2}{4R}$ (b) $\frac{3h}{4}$
   (c) $\frac{5h}{8}$ (d) $\frac{3h^2}{8R}$

6. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is:

   (a) $\frac{MR^2}{32\sqrt{2}\pi}$ (b) $\frac{MR^2}{16\sqrt{2}\pi}$

   (c) $\frac{4MR^2}{9\sqrt{3}\pi}$ (d) $\frac{4MR^2}{3\sqrt{3}\pi}$

7. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of the cavity thus formed is: (G = *gravitational constant*)



   (a) $\frac{-GM}{2R}$ (b) $\frac{-GM}{R}$

   (c) $\frac{-2GM}{3R}$ (d) $\frac{-2GM}{R}$

8. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to:

   (g = *gravitational acceleration*)

   (a) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$ (b) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$

   (c) $\left[1 - \left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (d) $\left[1 - \left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

9. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is:

   (a) $T \propto e^{-R}$ (b) $T \propto e^{-3R}$

   (c) $T \propto \frac{1}{R}$ (d) $T \propto \frac{1}{R^3}$

10. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways:

    (i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

    (ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

    In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is:

    (a) In 2, 4 In 2 (b) In 2, In 2

    (c) In 2, 2 In 2 (d) 2 In 2, 8 In 2

11. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is:

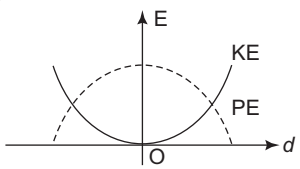
    $\left(\gamma = \frac{C_p}{C_v}\right)$

    (a) $\frac{3\gamma + 5}{6}$ (b) $\frac{3\gamma - 5}{6}$

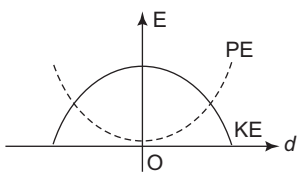
    (c) $\frac{\gamma + 1}{2}$ (d) $\frac{\gamma - 1}{2}$

12. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly?
    (graphs are schematic and not drawn to scale)

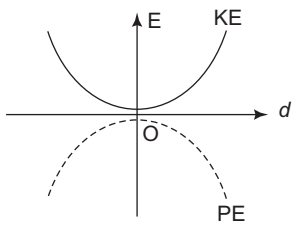
    (a)



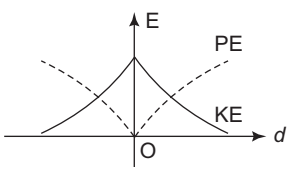
    (b)



    (c)

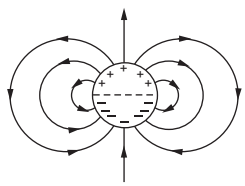


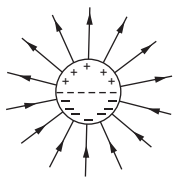
    (d)

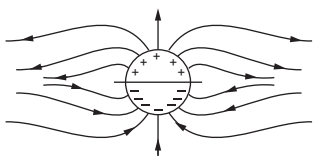


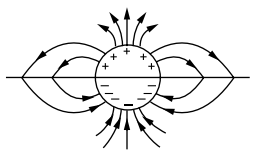
13. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to:
    (a) 6% (b) 12%
    (c) 18% (d) 24%

14. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in : (*figures are schematic and not drawn to scale*)

    (a) 

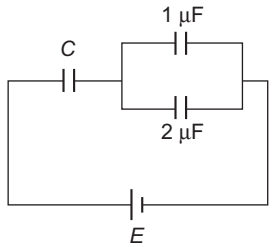
    (b) 

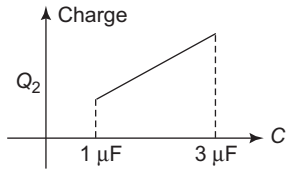
    (c) 

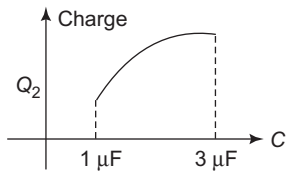
    (d) 

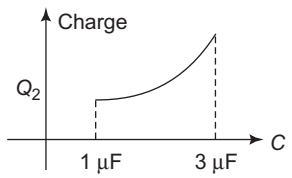
15. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then
    (a) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
    (b) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$
    (c) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
    (d) $2R < R_4$

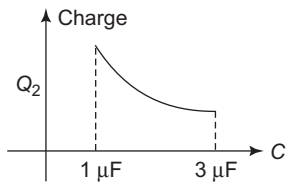
16. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of 'C' is given properly by : (*figures are drawn schematically and are not to scale*)



(a) 

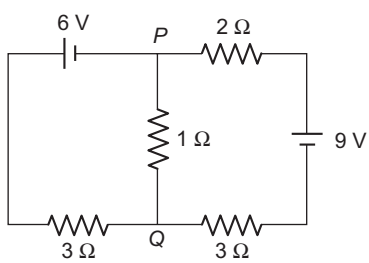
(b) 

(c) 

(d) 

17. When 5 V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} ms^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to:
    (a) 1.6×10^{-8} Ωm (b) 1.6×10^{-7} Ωm
    (c) 1.6×10^{-6} Ωm (d) 1.6×10^{-5} Ωm

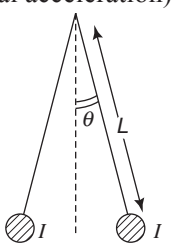
18. In the circuit shown, the current in the 1Ω resistor is:



    (a) 1.3 A, from P to Q
    (b) 0 A
    (c) 0.13 A, from Q to P
    (d) 0.13 A, from P to Q

19. Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then.
    (a) $\vec{F}_1 = \vec{F}_2 = 0$
    (b) $\vec{F}_1$ is radially inwards and $\vec{F}_2$ is radially outwards
    (c) $\vec{F}_1$ is radially inwards and $\vec{F}_2 = 0$
    (d) $\vec{F}_1$ is radially outwards and $\vec{F}_2 = 0$

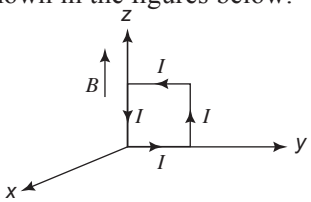
20. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle 'θ' with the vertical. If wires have mass λ per unit length then the value of I is: (g = gravitational acceleration)



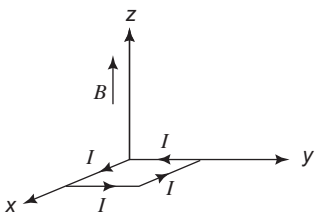
    (a) $\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0\cos\theta}}$ (b) $2\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0\cos\theta}}$
    (c) $2\sqrt{\frac{\pi g L}{\mu_0}\tan\theta}$ (d) $2\sqrt{\frac{\pi\lambda g L}{\mu_0}\tan\theta}$

21. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below:

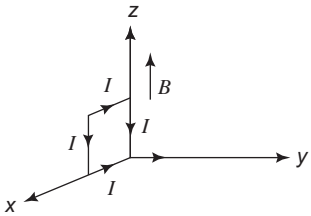
(i)



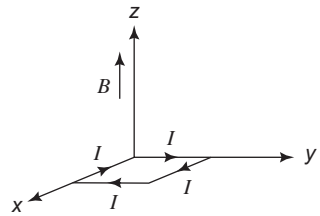
(ii)



(ii)

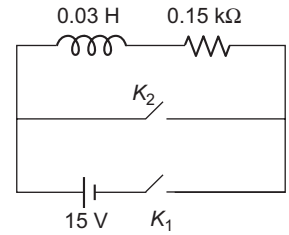


(iv)



If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop be in (i) stable equilibrium and (ii) unstable equilibrium?
    (a) (i) and (ii), respectively
    (b) (i) and (iii), respectively
    (c) (ii) and (iv), respectively
    (d) (ii) and (iii), respectively

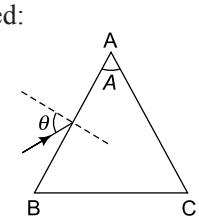
22. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15 V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1 ms, the current in the circuit will be : ($e^5 \cong 150$)



    (a) 100 mA (b) 67 mA
    (c) 6.7 mA (d) 0.67 mA

23. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is:
    (a) 1.73 V/m (b) 2.45 V/m
    (c) 5.48 V/m (d) 7.75 V/m

24. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided:



    (a) $\theta > \sin^{-1}\left[\mu\sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$
    (b) $\theta < \sin^{-1}\left[\mu\sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(c) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(d) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

25. On a hot summer night, the refractive index of air is smallest near the ground and increases with height form the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam:
    (a) becomes narrower
    (b) goes horizontally without any deflection
    (c) bends downwards
    (d) bends upwards

26. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is:
    (a) 1 µm (b) 20 µm
    (c) 100 µm (d) 300 µm

27. As an electron makes a transition from an excited state to the ground state of a hydrogen-like atom/ion:
    (a) its kinetic energy increases but potential energy and total energy decrease
    (b) kinetic energy, potential energy and total energy decrease
    (c) kinetic energy decreases, potential energy increases but total energy remains same.
    (d) kinetic energy and total energy decrease but potential energy increases

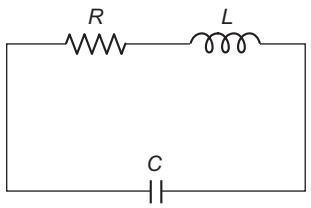
28. Match List-I (Fundamental Experiment) with List-II (its conclusion) and select the correct option from the choice given below the list

	List-I		List-II
(A)	Franck-Hertz Experiment.	(i)	Particle nature of light
(B)	Photo-electric experiment.	(ii)	Discrete energy levels of atom
(C)	Davison-Germer Experiment.	(iii)	Wave nature of electron
		(iv)	Structure of atom

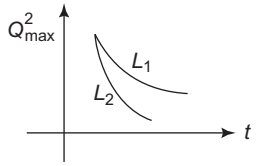
    (a) (A) – (i) (B) – (iv) (C) – (iii)
    (b) (A) – (ii) (B) – (iv) (C) – (iii)
    (c) (A) – (ii) (B) – (i) (C) – (iii)
    (d) (A) – (iv) (B) – (iii) (C) – (ii)

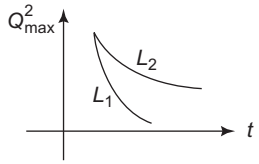
29. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resultant signal is/are:
    (a) 2 MHz only
    (b) 2005 kHz, and 1995 kHz
    (c) 2005 kHz, 2000 kHz and 1995 kHz
    (d) 2000 kHz and 1995 kHz

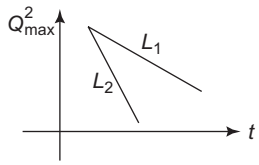
30. An *LCR* circuit is equivalent to a damped pendulum. In an *LCR* circuit the capacitor is charged to Q_0 and then connected to the *L* and *R* as shown below:

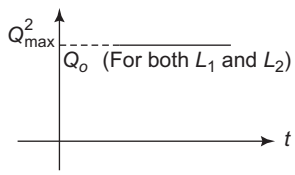


If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly? (*plots are schematic and not drawn to scale*)

(a)


(b)


(c)


(d)


Answers

1. (c)	2. (b)	3. (c)	4. (c)
5. (b)	6. (3)	7. (b)	8. (a)
9. (c)	10. None	11. (c)	12. (b)
13. (b)	14. (a)	15. (c), (d)	16. (b)
17. (d)	18. (c)	19. (a)	20. (a)
21. (c)	22. (d)	23. (b)	24. (a)
25. (d)	26. (b)	27. (a)	28. (c)
29. (c)	30. (a)		

Solutions

1. Let y_1 and y_2 be the positions of the stones at time t then.

$$y_1 = 10t - \frac{1}{2}gt^2 \quad (1)$$

and $$y_2 = 40t - \frac{1}{2}gt^2 \quad (2)$$

where $g = 10\ \text{ms}^{-2}$. The relative position of the second stone with respect to the first is

$$y_2 - y_1 = 30t \quad (3)$$

If t_1 and t_2 are times taken by the first stone and by the second stone to hit the ground, then $y_1 = y_2 = -240$ m and Eqs. (1) and (2) become

$$-240 = 10t_1 - 5t_1^2 \quad (4)$$

and $$-240 = 40t_2 - 5t_2^2 \quad (5)$$

The positive roots of Eqs. (4) and (5) are $t_1 = 8$ s and $t_2 = 12$ s.

Thus the first stone reaehes the ground earlier. Since the stones do not rebound, at times $t \geq 8$ s, $y_1 = 0$ and y_2 is given by Eq. (2) which is a parabola. But up to $t = 8$s, it follows from Eq. (3) that $(y_2 - y_1)$ is proportional to t. Hence up to $t = 8$ s, graph is $(y_2 - y_1)$ is a straight line, and in the interval $t = 8$ s and $t = 12$ s, the graph is parabolic. So the correct graph is (c).

2. $T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow g = \frac{4\pi^2 L}{T^2}$

If n is the number of oscillations completed in t seconds, then $T = \frac{t}{n}$. Thus, in terms of measured quantities,

$$g = \frac{4\pi^2 L n^2}{t^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta t}{t} \quad (\because n = 100 \text{ is exact})$$

$$= \frac{0.1\text{ cm}}{20\text{ cm}} + 2 \times \frac{1\text{s}}{90\text{s}}$$

$$= \frac{1}{200} + \frac{1}{45}$$

$$\therefore \quad 100 \times \frac{\Delta g}{g} = 100 \times \left(\frac{1}{200} + \frac{1}{45}\right)$$

$$= 0.5 + 2.2 = 2.7$$

The closest choice is (b)

3. Since the blocks are held stationary, they are in translational as well as rotational equilibrium. Hence no net force and no net torque acts on the blocks. Hence normal reaction on block A due to block $B = F$. Also normal reaction on B by the wall will also be equal to F. Also frictional force $f = mg$. Hence frictional force on A due to $B = m_A g = 20$ N. Therefore, frictional force exerted by the wall on block B is

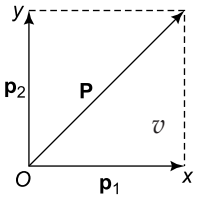
$$f = m_A g + m_B g$$
$$= 20\text{ N} + 100\text{ N} = 120\text{ N}$$

So the correct choice is (c).

4. Linear momentum of the first particle is

$\mathbf{p}_1 = m(2v)$ along the x-axis.

$\mathbf{p}_2 = 2m(v)$ along the y-axis.



In a perfectly inelastic collisions, the two particle stick together. The mass of the composite particle is $M = m + 2m = 3m$. If $\mathbf{V}$ is the velocity of the composite particle, its momentum after the collision is

$$\mathbf{P} = M\mathbf{V} = (3m)\mathbf{V}$$

From the figure, it follows that

$$P = \sqrt{p_1^2 + p_2^2}$$

$$\Rightarrow \quad (3m)V = \sqrt{(2mv)^2 + (2mv)^2}$$

$$\Rightarrow \quad 3mV = \sqrt{8}\ mv$$

$$\Rightarrow \quad V = \frac{\sqrt{8}\,v}{3}$$

Total initial kinetic energy is

$$K_i = \frac{1}{2} \times m(2v)^2 + \frac{1}{2} \times (2m)v^2 = 3mv^2$$

Total final kinetic energy is

$$K_f = \frac{1}{2}MV^2 = \frac{1}{2} \times (3m) \times \left(\frac{\sqrt{8}}{3}v\right)^2$$

$$= \frac{4mv^2}{3}$$

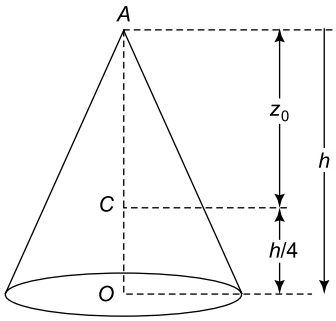
Percentage loss is

$$\frac{K_i - K_f}{K_i} \times 100 = \frac{3mv^2 - \frac{4mv^2}{3}}{3mv^2} \times 100$$

$$= \frac{5}{9} \times 100 = 55.56\% \simeq 56\%$$

So the correct choice is (c).

5. The centre of mass C of cone of height is at a distance



$$AC = AO - CO$$

or $$z_0 = h - \frac{h}{4} = \frac{3h}{4}$$

So the correct choice is (b).

6. If a is the side of the cube of maximum possible volume, then

$$2R = \sqrt{a^2 + a^2 + a^2} = \sqrt{3}\, a$$

$$\Rightarrow \quad a = \frac{2R}{\sqrt{3}}$$

Density of sphere is $\rho = \dfrac{M}{4\dfrac{\pi}{3}R^3}$

Volume of cube $= a^3 = \left(\dfrac{2R}{\sqrt{3}}\right)^3 = \dfrac{8R^3}{3\sqrt{3}}$

$\therefore$ Mass of cube is $m = \rho a^3 = \dfrac{3M}{4\pi R^3} \times \dfrac{8R^3}{3\sqrt{3}} = \dfrac{2M}{\pi\sqrt{3}}$

The moment of inertia of the cube about the axis passing through its centre and perpendicular to one of its faces is

$$I = \frac{ma^2}{6} = \frac{2M}{\pi\sqrt{3}} \times \frac{a^2}{6} = \frac{2M}{\pi\sqrt{3}} \times \frac{1}{6} \times \left(\frac{2R}{\sqrt{3}}\right)^2$$

$$= \frac{4MR^2}{9\pi\sqrt{3}}$$

So the correct choice is (c).

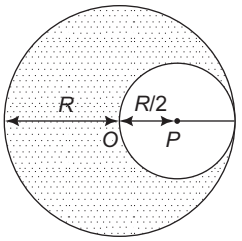
7. Volume of solid sphere is $\rho = \dfrac{M}{\dfrac{4\pi}{3}R^3}$

Volume of spherical portion removed $= \dfrac{4\pi}{3}\left(\dfrac{R}{2}\right)^3$

$= \dfrac{4\pi}{3}\dfrac{R^3}{8}$

$\therefore$ Mass of portion removed $(m) = \dfrac{3M}{4\pi R^3} \times \dfrac{4\pi}{3}\dfrac{R^3}{8}$

$= \dfrac{M}{8}$



Gravitational potential due to a solid sphere of mass M and radius R at a point inside it at a distance r from its centre is given by

$$V = -\frac{3GM}{R^3}\left(\frac{R^2}{2} - \frac{r^2}{6}\right)$$

For point P, $r = \dfrac{R}{2}$, we have

$$V = -\frac{3GM}{R^3}\left(\frac{R^2}{2} - \frac{(R/2)^2}{6}\right) = -\frac{11GM}{8R}$$

Gravitational potential at P due to spherical cavity of mass $m = \dfrac{M}{8}$ at point P is

$$V' = -\frac{3GM/8}{2R/2} = -\frac{3GM}{8R}$$

Therefore gravitational potential at P due to the sphere with cavity (i.e. due to shaded portion is)

$$V'' = V - V'$$

$$= -\frac{11GM}{8R} - \left(-\frac{3GM}{8R}\right) = -\frac{GM}{R}$$

So the correct choice is (b).

8. $T = 2\pi\sqrt{\dfrac{L}{g}}$; L = length of pendulum string. Let l be the increase in the length of the string when an additional mass M is added to its bob. Then

$$T_M = 2\pi\sqrt{\frac{L+l}{g}}$$

Now $$Y = \frac{MgL}{Al}$$

$$\Rightarrow \quad l = \frac{MgL}{AY}$$

$$\therefore \quad T_M = 2\pi\sqrt{\frac{L + \dfrac{MgL}{AY}}{g}}$$

$$\Rightarrow \quad T_M^2 = 4\pi^2\left(\frac{L}{g} + \frac{ML}{AY}\right) \qquad (1)$$

Now $$T^2 = 4\pi^2\frac{L}{g} \qquad (2)$$

Dividing (1) by (2)

$$\frac{T_M^2}{T^2} = 1 + \frac{Mg}{AY}$$

$\Rightarrow \frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$, which is choice (a).

9. From the first law of thermodynamics,

$$dQ = dU + dW$$

For an adiabatic process, $dQ = 0$. Hence

$$0 = dU + dW$$

$$\Rightarrow \quad dU = -dW = -P\,dV \qquad (\because dW = \rho\,dV)$$

Given $P = -\frac{1}{3}\frac{U}{V}$. Therefore,

$$dU = -\frac{1}{3}\frac{U}{V}\,dV \qquad \left(\because P = \frac{1}{3}\frac{U}{V}\right)$$

$$\Rightarrow \quad \frac{dU}{U} = -\frac{1}{3}\frac{dV}{V}$$

Integrating

$$\ln U = -\frac{1}{3}\ln V + C; \qquad (C = \text{integration constant})$$

$$\Rightarrow \ln (UV^{1/3}) = C \qquad (1)$$

Now it is given that $\frac{U}{V} \propto T^4 \Rightarrow U = kVT^4$, where k is a proportionality constant. Using this in (1), we get

$$\ln (kT^4\, V^{4/3}) = C$$

$$\Rightarrow \quad \ln\left[kT^4 \times \left(\frac{4\pi}{3}R^3\right)^{4/3}\right] = C$$

$$\Rightarrow \quad \ln [kT^4\, R^4] = C$$

$\Rightarrow T^4R^4 = \text{constant} \Rightarrow T \propto \frac{1}{R}$. So the correct choice is (c).

10. In cases (i) and (ii), the temperature of the body is increased from 100°C to 200°C. In case (i) there are 2 reservoirs. Since each reservoir supplies the same amount of heat, each reservoir raises the temperature of the body by 100°C/2 = 50°C = 50 K. Therefore, the change in entropy in case (i) is

$$(dS)_1 = \left[\int_{373}^{423}\frac{dT}{T} + \int_{423}^{473}\frac{dT}{T}\right]$$

$= \ln (423) - \ln (373) + \ln (473) - \ln (423)$

$= \ln (473) - \ln (373)$

$= \ln\left(\frac{473}{373}\right)$

In case (ii) there are 8 reservoirs. Therefore, each reservoir raises the temperature of the body by 100°C/8 = 12.5 °C = 12.5 K. Therefore, the change in entropy in case (ii) is

$$(dS)_2 = \left[\int_{373}^{385.5}\frac{dT}{T} + \int_{385.5}^{398}\frac{dT}{T} + \int_{398}^{410.5}\frac{dT}{T} + \int_{410.5}^{423}\frac{dT}{T} + \int_{423}^{435.5}\frac{dT}{T} + \int_{435.5}^{448}\frac{dT}{T} + \int_{448}^{460.5}\frac{dT}{T} + \int_{460.5}^{473}\frac{dT}{T}\right]$$

$$= \ln\left(\frac{473}{373}\right)$$

So none of the choices given in the question is correct. However, if temperatures of the body were given in kelvin, the correct choice would be (b).

11. The mean free path l is defined as the average distance covered by a molecule between two successive collisions and is given by

$$l = \frac{1}{\sqrt{2}\, n\pi d^2}$$

where n = number of molecules per unit volume = $\frac{N}{V}$ and d is the diameter of a molecule. The average time between successive collisions is

$$t = \frac{\text{mean free path}}{\text{thermal speed}} = \frac{l}{v}$$

where $v = \sqrt{\frac{3RT}{M}}$; M = molecule mass.

$$\therefore \quad t = \frac{1}{\sqrt{2}\,\frac{N}{V}\pi d^2} \times \sqrt{\frac{M}{3RT}}$$

or $t = \frac{KV}{\sqrt{T}}$, where K is a constant given by

$$K = \frac{\sqrt{M}}{\sqrt{2}\, N\pi d^2 \sqrt{3R}}$$

$$\therefore \quad T = \frac{K^2 V^2}{t^2} \text{ or } T \propto \frac{V^2}{t^2}$$

For adiabatic process $TV^{\gamma-1} = \text{constant}$

$$\therefore \quad \frac{V^2}{t^2}\, V^{\gamma-1} = \text{constant}$$

$$\Rightarrow \quad \frac{V^{\gamma+1}}{t^2} = \text{constant}$$

$$\Rightarrow \quad t \propto V^{\frac{\gamma+1}{2}} \qquad (1)$$

Given $t \propto V^q$ (2)

Comparing (1) and (2), we get $q = \frac{\gamma+1}{2}$. So the correct choice is (c).

12. At the mean position O i.e. at $d = 0$, KE is maximum and PE is zero. Also PE $\propto d^2$. Hence the correct choice is (b).

13. When the train is approaching, the apparent frequency is

$$\nu_1 = \nu\left(\frac{v}{v - u}\right)$$

When the train is receding,

$$\nu_2 = \nu\left(\frac{v}{v + u}\right)$$

Change in frequency is

$$\Delta\nu = \nu_1 - \nu_2$$

$$= \nu\, v\left(\frac{1}{v-u} - \frac{1}{v+u}\right)$$

$$= \frac{\nu\, v \times 2u}{v^2 - u^2}$$

$$= \frac{2\nu\, u\, v}{v^2\left(1 - \frac{u^2}{v^2}\right)} = \frac{2\nu u}{v}$$

($\because u << v$)

$$\Rightarrow \quad \frac{\Delta\nu}{\nu} = \frac{2u}{v}$$

$\therefore$ Percentage change is $\frac{\Delta\nu}{\nu} \times 100 = \frac{2u}{v} \times 100$

$$= \frac{2 \times 20}{320} \times 100$$

$$= 12.5\%$$

So the closest choice is (b).

14. Electric field lines originate from a positive charge and terminate on a negative charge. They do not form closed loops. So the correct choice is (a).

15. Electric potential due to a solid sphere of radius R and carrying a charge Q at a point P inside it at a distance r from its centre C is given by

$$V = \frac{3kQ}{2R^3}\left(R^2 - \frac{r^2}{3}\right) \quad (1)$$

where $k = \frac{1}{4\pi\varepsilon_0}$

(i) At the centre C, $r = 0$ and Eq. (1) gives.

$$V_c = \frac{3kQ}{2R}$$

At the susface, $r = R$ and Eq. (1) givens

$$V_0 = \frac{3kQ}{2R^3}\left(R^2 - \frac{R^2}{3}\right) = \frac{kQ}{R}$$

Therefore, $V_C = \frac{3}{2} V_0$ at the centre. The equipotential surface on which the potential is $\frac{3V_0}{2}$ has $r = 0$ which gives $R_1 = 0$

(ii) For points outside the sphere (i.e. for $r > R$), the entire charge of the sphere may be assumed to be concentrated at its centre. Hence for $r > R$,

$$V = \frac{kQ}{r} \quad (2)$$

It is given that at $r = R_2$, potential = $\frac{5V_0}{4}$.

Putting $V = \frac{5V_0}{4} = \frac{5kQ}{4R}$ and $r = R_2$ is Eq. (1) we get

$$R_2 = \frac{R}{\sqrt{2}}$$

It is given at $r = R_3$, potential = $\frac{3V_0}{4}$. Hence from Eq. (2), we have

$$\frac{kQ}{R_3} = \frac{3V_0}{4} = \frac{3}{4}\frac{kQ}{R}$$

$$\Rightarrow \quad R_3 = \frac{4R}{3}$$

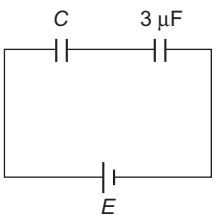
Similarly at $r = R_4$,

$$\frac{kQ}{R_4} = \frac{V_0}{4} = \frac{kQ}{4R}$$

$$\Rightarrow \quad R_4 = 4R$$

We have seen that $R_1 = 0$. $R_2 = \frac{R}{\sqrt{2}}$, $R_3 = \frac{4R}{3}$ and $R_4 = 4R$. Thus choices (c) and (d) are both correct.

16. The circuit can be redrawn as follows.



Here C is in μF. Let E_1 and E_2 be the voltages across C and 3μ F capacitor respectively. Then

$$CE_1 = 3E_2$$

$$\Rightarrow \quad \frac{E_2}{E_1} = \frac{C}{3} \Rightarrow E_1 = \frac{3E_2}{C}$$

Also $E_1 + E_2 = E$

$$\Rightarrow \quad \frac{3E_2}{C} + E_2 = E \Rightarrow E_2 = \frac{EC}{3+C}$$

$\therefore$ Charge on 2μ F capacitor is (in μ F)

$$Q_2 = 2E_2 = \frac{2EC}{3+C} \qquad (1)$$

It follows from Eq. (1) that Q_2 increases non-linearly as C increases and the slope of $(Q_2 - C)$ graph decreases as C increases. Hence the correct choice is (b).

17. $I = ne\,A\,v_d$

$$\Rightarrow \quad \frac{V}{R} = ne\,A\,v_d$$

$$\Rightarrow \quad \frac{V}{\frac{\rho l}{A}} = ne\,A\,v_d$$

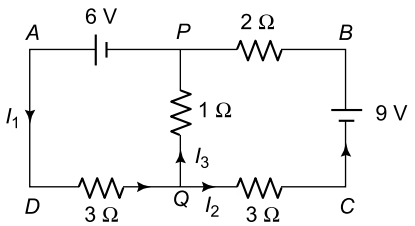
$$\Rightarrow \quad \rho = \frac{V}{l\,e\,n\,v_d}$$

$$= \frac{5}{0.1 \times (1.6 \times 10^{-19}) \times (8 \times 10^{28}) \times (2.5 \times 10^{-4})}$$

$$= 1.56 \times 10^{-5} \simeq 1.6 \times 10^{-5}\ \Omega\text{m}.$$

So the correct choice is (d).

18. Let I_1, I_2 and I_3 be the direction of currents is shown in the circuit.



From kirchhoff's junction rule

$$I_1 = I_2 + I_3 \qquad (1)$$

Applying kirchhoff's loop rule to loops *APQDA* and *PBCQP*, we have

$$6 - I_3 - 3I_1 = 0 \qquad (2)$$

and $$-2I_2 + 9 - 3I_2 + I_3 = 0 \qquad (3)$$

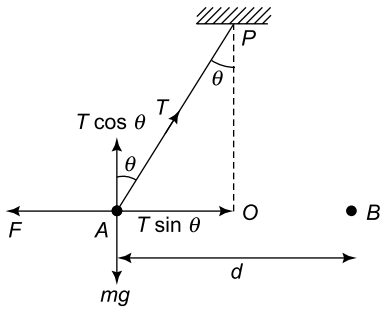
Simultaneous solution of Eqs. (1), (2) and (3) gives $I_1 = \frac{45}{23}$ A, $I_2 = \frac{42}{23}$ A and $I_3 = \frac{3}{23}$ A. The current flowing the 1Ω sesistor is $I_3 = \frac{3}{23} = 0.13$A Since I_3 is positive, it flows from Q to P. So the correct choice is (c).

19. Since the solenoids are in equilibrium, no net force is exerted by one solenoid on the other. Hence $\vec{F}_1 = 0$ and $\vec{F}_2 = 0$. So the correct choice is (a).

20. If the two wires have currents in the opposite directions, they will repel each other with a force.

$$F = \frac{\mu_0 I^2 l}{2\pi d} \qquad (1)$$

where l = length each wire and d = distance between them. They will then be in equilibrium.



For equilibruim,

$$F = T \sin\theta$$

and $$mg = T\cos\theta$$

$$\therefore \quad \tan\theta = \frac{F}{mg} \qquad (2)$$

Also $$m = \lambda l \qquad (3)$$

Using (1) and (3) in (2) we get

$$\tan\theta = \frac{\frac{\mu_0 I^2 l}{2\pi d}}{\lambda l g} \qquad (4)$$

In $\Delta\, OPA$, $\sin\theta = \frac{d/2}{L} \Rightarrow d = 2L\sin\theta$. Using this in (4) and simplifying, we get

$$I = 2\sin\theta \sqrt{\frac{\pi \lambda Lg}{\mu_0 \cos\theta}}$$

So the correct choice is (a).

21. In a uniform magnetic field, a loop is in stable equilibrium if (i) the torque experienced by it is zero and (ii) its potential energy is minimum. For unstable equilibrium, the torque must be zero and potential energy must be maximum

Torque is $\vec{\tau} = I\left(\vec{A} \times \vec{B}\right)$

Potential energy is $U = -I\left(\vec{A} \cdot \vec{B}\right)$

where $\vec{A}$ is the area vector. $\vec{B} = B\,\hat{\mathbf{k}}$. For orientation (a), $\vec{A} = A\,\vec{\mathbf{i}}$. Hence $\vec{\tau} = IAB(\vec{\mathbf{k}} \times \vec{\mathbf{i}}) = IAB\,\vec{\mathbf{j}}$

For orientation (c), $\vec{A} = -A\,\vec{\mathbf{i}}$ and $\vec{\tau} = -IAB\,\vec{\mathbf{j}}$, Hence choices (a) and (c) are not correct.

For orientation (b), $\vec{A} = A\hat{\mathbf{k}}$ and $\vec{\tau} = IAB\,(\hat{\mathbf{k}} \times \hat{\mathbf{k}}) = 0$ and $U = -IAB(\hat{\mathbf{k}} \cdot \hat{\mathbf{k}}) = -IAB$, which sis minimum. For orientation (d), $\vec{A} = -A\hat{\mathbf{k}}$ and $\vec{\tau} = -IAB\,(\hat{\mathbf{k}} \times \hat{\mathbf{k}}) = 0$ and $U = -IAB(-\hat{\mathbf{k}} \cdot \hat{\mathbf{k}}) = +IAB$ which is masimum. Hence prientateon (b) corresponds to stable equilibrium and orientation (d) corresponds to unstable equilibrium. So the correct choice is (c).

22. Given $E = 15$V, $R = 0.15 \times 10^3\ \Omega$ and $L = 0.03$ H. If key K_1 is closed for a long time and K_2 is open, the current acquires a steady value I_0 given by

$$I_0 = \frac{E}{R} = \frac{15}{0.15 \times 10^3} = 0.1 \text{ A}$$

When K_2 is closed, the current decays with time t as

$$I = I_0\, e^{-t/\tau}$$

where $$\tau = \frac{L}{R} = \frac{0.03}{0.15 \times 10^3} = \frac{1}{5 \times 10^3}$$

At $t = 1$ ms $= 10^{-3}$ s,

$$I = 0.1 \times e^{-5} \qquad \left(\because \tau = \frac{10^{-3}}{5}\right)$$

$$= \frac{0.1}{e^5} = \frac{0.1}{150} = 0.67 \times 10^{-3}\ s$$

$\Rightarrow$ $I = 0.67$ mA, which is choice (d)

23. The intensity of light emitted by LED is given by

$$I = \frac{1}{2}\,\varepsilon_0 E_0^{\,2}\, c$$

where E_0 = amplitude of electric field and c = speed of light

If P is power, the intensity at a distance r is given by

$$I = \frac{P}{4\pi r^2}$$

$$\frac{P}{4\pi r^2} = \frac{1}{2}\,\varepsilon_0 E_0^{\,2}\, c$$

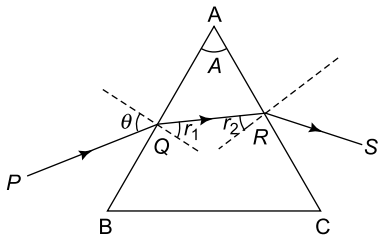
$$\Rightarrow \qquad E_0 = \sqrt{\frac{2P}{4\pi r^2 \varepsilon_0 c}} \qquad (1)$$

Given $P = 0.1$ W and $r = 1$ m.

Also $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ SI units and $c = 3 \times 10^8$ ms^{-1}.

Substituting thes values in Eq. (1) we get $E_0 = 2.45$ Vm^{-1} which is choice (b)

24.



Applying Snell's law at Q,

$$\sin\theta = \mu \sin r_1 \qquad (1)$$

Ray QR will emerge from face AC if $r_2 < i_c$ where i_c is the critical angle which is given by

$$\sin i_c = \frac{1}{\mu} \Rightarrow i_c = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\therefore \qquad r_2 < \sin^{-1}\left(\frac{1}{\mu}\right)$$

But $r_1 + r_2 = A \Rightarrow r_2 = A - r_1$

$$\therefore A - r_1 < \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\Rightarrow r_1 > A - \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\therefore \sin r_1 > \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$$

Using (1) we have

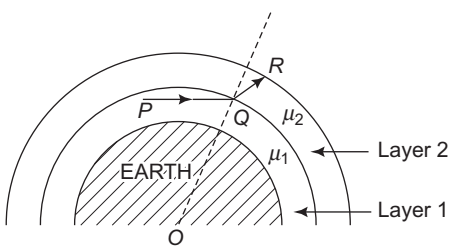
$$\frac{\sin\theta}{\mu} > \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$$

$$\Rightarrow \sin\theta > \mu \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$$

$$\Rightarrow \theta > \sin^{-1}\left[\mu \sin\left\{A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]\right.$$

So the correct choice is (a).

25. Huygens' principle can be used to prove that, if a ray of light travels from a rarer to a denser medium, it bends towards the normal at the point of incidence.



The refractive index μ_1 of the layer 1 closer to the earth is less than μ_2 of the layer 2. A horizontal ray PQ travelling in layer 1 bends along QR (towards the normal at Q). The ray bends upwards. Hence the correct choice is (d).

26. Limiting angular resolving power of human eye is given by

$$\theta_{\min} = \frac{1.22\lambda}{d}$$

where d is the diameter of the pupil of the eye. The minimum linear separation between two objects a distance D away which can be just resolved by the eye is

$$x_{\min} = D\theta_{\min}$$
$$= \frac{1.22\lambda D}{d}$$
$$= \frac{1.22\times(500\times10^{-9})\times(25\times10^{-2})}{2\times0.25\times10^{-2}}$$
$$= 30.5\times10^{-6}\text{ m} = 30.5\ \mu\text{m}$$

The closest choice is (b).

27. According to Bohr's theory, the total energy of the electron in the n state of a hydrogen like atom is

$$E = (-13.6\text{ eV})\frac{Z^2}{n^2}$$

Also $E = -$ K.E. and P.E. $= 2E$

As the electron falls from the higher energy state to the ground state, n decreases. Hence the total energy E decrease (because it becomes more negative). Since $E = -$ K.E., kinetic energy increases. Since P.E. $= 2E$, potential energy decreases. Hence the correct choice is (a).

28. Franck-Hertz experiment confirms that an atom has discrete energy levels. Photoelectric experiment confirms the particle nature of light. Davison-Germer experiment confirms wave nature of electron. Hence the correct choice is (c).

29. When an audio signal of frequency ν_m is superposed on a carrier wave of frequency ν_c, the resultant amplitude modulated wave contains components of frequency ν_c and upper and lower side band frequencies

$$\nu_{USB} = \nu_c + \nu_m$$

and $\nu_{LSB} = \nu_c - \nu_m$

Given $\nu_m = 5$ k Hz and $\nu_c = 2$MHz $= 2000$ kHz. Thus

$$\nu_{USB} = 2000 + 5 = 2005\text{ kHz}$$

and $\nu_{LSB} = 2000 - 5 = 1995$ kHz

Hence the frequencies of the modulated wave components are 2000 kHz, 1995 kHz 2005 kHz. So the correct choice is (c).

30. If the circuit did not contain resistor R, we have an $L - C$ circuit in which the charge on capacitor plates oscillates simple harmonically with angular frequency

$$\omega = \frac{1}{\sqrt{LC}}$$

At an instant of time t the charge in given by

$$Q = Q_0\cos(\omega t)$$

The presence of resistance R is the circuit damps these oscillatons. For a damped oscillator, we have

$$Q = Q_0 e^{-kt}\cos(\omega t)$$

where k is the damping coefficient which depends on the value of R. Therefore,

$$Q^2 = Q_0^2 e^{-2kt}\cos^2(\omega t)$$

$$\Rightarrow \quad Q^2 = Q_0^2 - e^{-2kt}\cos^2\left(\frac{\omega}{\sqrt{LC}}\right) \qquad (1)$$

Form Eq. (1) it follows that Q^2 decays exponentially with t. Also, since $L_1 > L_2$, $\cos^2\left(\frac{\omega}{\sqrt{L_1 C}}\right)$ will be greater than $\cos^2\left(\frac{\omega}{\sqrt{L_2 C}}\right)$ at a given time t. Hence the correct choice is (a).

Solutions of Physics JEE Main—2016

1. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is: (Take $g = 10\ ms^{-2}$)

   (a) 2 s (b) $2\sqrt{2}$ s

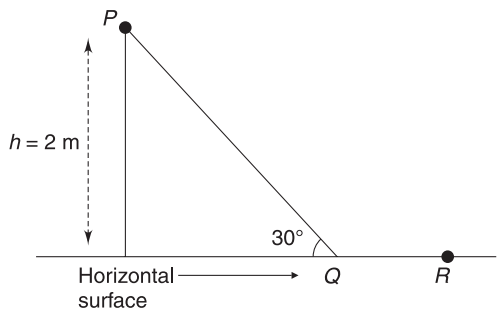
   (c) $\sqrt{2}$ s (d) $2\pi\sqrt{2}$ s

2. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8\ ms^{-2}$

   (a) 6.45×10^{-3} kg (b) 9.89×10^{-3} kg

   (c) 12.89×10^{-3} kg (d) 2.45×10^{-3} kg

3. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ and QR. The values of the coefficient of friction μ and the distance x ($= QR$), are, respectively close to:



   (a) 0.2 and 3.5 m (b) 0.29 and 3.5 m

   (c) 0.29 and 6.5 m (d) 0.2 and 6.5 m

4. Two identical wires A and B, each of length 'l' carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is:

   (a) $\frac{\pi^2}{16\sqrt{2}}$ (b) $\frac{\pi^2}{16}$

   (c) $\frac{\pi^2}{8\sqrt{2}}$ (d) $\frac{\pi^2}{8}$

5. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection, when a current of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is:

   (a) 2 Ω (b) 0.1 Ω

   (c) 3 Ω (d) 0.01 Ω

6. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears:

   (a) 10 times nearer (b) 20 times taller

   (c) 20 times nearer (d) 10 times taller

7. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400K, is best described by:

   (a) Linear increase for Cu, exponential increase for Si

   (b) Linear increase for Cu, exponential decrease for Si

(c) Linear decrease for Cu, linear decrease for Si

(d) Linear increase for Cu, linear increase for Si

8. Choose the current statement

(a) In amplitude modulation the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal

(b) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal

(c) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal

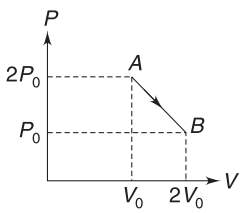
(d) In amplitude modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal

9. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially the samples have equal number of nuclei. After 80 minutes, the ratio of decayed number of A and B nuclei will be:

(a) 4 : 1 (b) 1 : 4

(c) 5 : 4 (d) 1 : 16

10. '*n*' mole of an ideal gas undergoes a process $A \rightarrow B$ as shown in the figure. The maximum temperature of the gas during the process will be:



(a) $\frac{3P_0V_0}{2nR}$ (b) $\frac{9P_0V_0}{2nR}$

(c) $\frac{9P_0V_0}{nR}$ (d) $\frac{9P_0V_0}{4nR}$

11. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to:

(a) 0.08 H (b) 0.044 H

(c) 0.065 H (d) 80 H

12. A pipe open at both ends has a fundamental frequency *f* in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column now is

(a) 3*f*/4 (b) 2*f*

(c) *f* (d) *f*/2

13. The box of a pin hole camera, of length *L*, has a hole of radius *a*. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (*say* b_{min}) when:

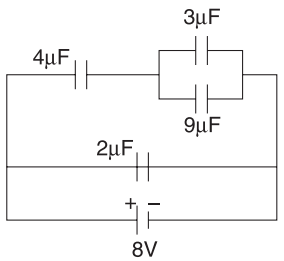
(a) $a = \sqrt{\lambda L}$ and $b_{\min} = \left(\frac{2\lambda^2}{L}\right)$

(b) $a = \sqrt{\lambda L}$ and $b_{\min} = 4\sqrt{\lambda L}$

(c) $a = \frac{\lambda^2}{L}$ and $b_{\min} = 4\sqrt{\lambda L}$

(d) $a = \frac{\lambda^2}{L}$ and $b_{\min} = \left(\frac{2\lambda^2}{L}\right)$

14. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge *Q* (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point distant 30 m from it, would equal:



(a) 360 N/C (b) 420 N/C

(c) 480 N/C (d) 240 N/C

15. Arrange the following electromagnetic radiations per quantum in the order of increasing energy:

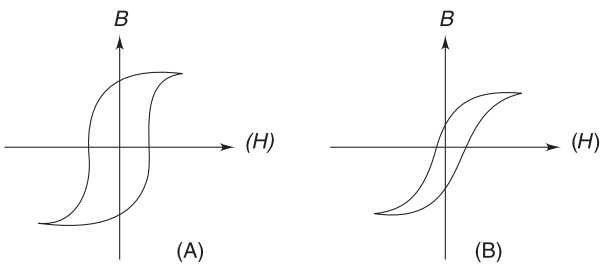
A : Blue light B : Yellow light

C : X-ray D : Radiowave

(a) A, B, D, C (b) C, A, B, D

(c) B, A, D, C (d) D, B, A, C

16. Hysteresis loops for two magnetic materials A and B are given below:



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use:

(a) A for electromagnets and B for electric generators.

(b) A for transformers and B for electric generators.

(c) B for electromagnets and transformers.

(d) A for electric generators and transformers.

17. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively:

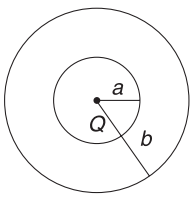
(a) 60°C; $\alpha = 1.85 \times 10^{-4}$/°C

(b) 30°C; $\alpha = 1.85 \times 10^{-3}$/°C

(c) 55°C; $\alpha = 1.85 \times 10^{-2}$/°C

(d) 25°C; $\alpha = 1.85 \times 10^{-5}$/°C

18. The region between two concentric spheres of radii '*a*' and '*b*', respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where *A* is a constant and *r* is the distance from the centre. At the centre of the spheres is a point charge *Q*. The value of *A* such that the electric field in the region between the spheres will be constant, is:



(a) $\frac{Q}{2\pi(b^2 - a^2)}$ (b) $\frac{2Q}{\pi(b^2 - a^2)}$

(c) $\frac{2Q}{\pi a^2}$ (d) $\frac{Q}{2\pi a^2}$

19. In an experiment for determination of refractive index of glass of a prism by $i - \delta$, plot, it was found that a ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In the case which of the following is closest to the maximum possible value of the refractive index?

(a) 1.6 (b) 1.7

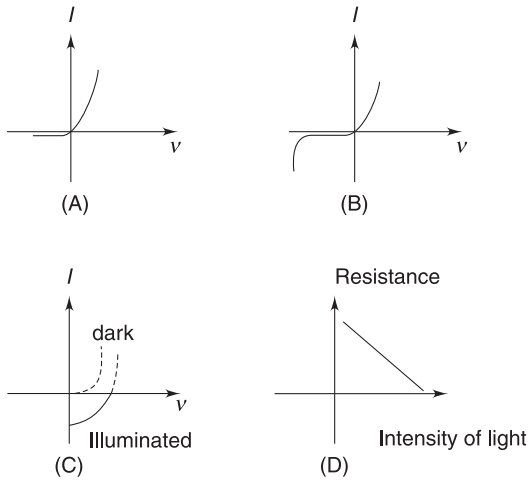
(c) 1.8 (d) 1.5

20. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be:

(a) 92 ± 5.0 s (b) 92 ± 1.8 s

(c) 92 ± 3 s (d) 92 ± 2 s

21. Identify the semiconductor devices whose characteristics are given below, in the order (a), (b), (c), (d)



(a) Zener diode, Simple diode, Light dependent resistance, Solar cell

(b) Solar cell, Light dependent resistance, Zener diode, Simple diode

(c) Zener diode, solar cell, Simple diode, Light dependent resistance

(d) Simple diode, Zener diode, Solar cell, Light dependent resistance

22. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed *v* If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be:

(a) $< v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (b) $= v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

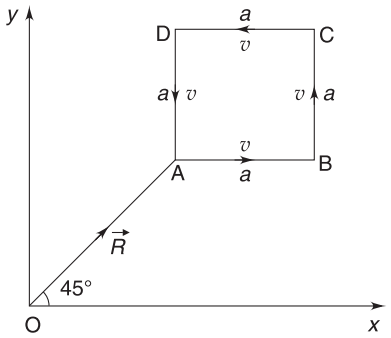
(c) $= v\left(\frac{3}{4}\right)^{\frac{1}{2}}$ (d) $> v\left(\frac{3}{4}\right)^{\frac{1}{2}}$

23. A particle performs simple harmonic motion with amplitude *A*. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is:

(a) 3*A* (b) $A\sqrt{3}$

(c) $\frac{7A}{3}$ (d) $\frac{A}{3}\sqrt{41}$

24. A particle of mass *m* is moving along the side of square of side '*a*' with a uniform speed *v* in the *x*-*y* plane as shown in the figure:



Which of the following statements is false for the angular momentum L about the origin?

(a) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.

(b) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.

(c) $\vec{L} = \frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A.

(d) $\vec{L} = -\frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B.

25. An ideal gas undergoes a quasistatic, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_p and C_v are molar specific heat at constant pressure and constant volume, respectively):

(a) $n = \frac{C - C_P}{C - C_v}$ (b) $n = \frac{C_P - C}{C - C_v}$

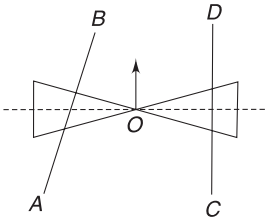
(c) $n = \frac{C - C_v}{C - C_P}$ (d) $n = \frac{C_P}{C_v}$

26. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminum. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line?

(a) 0.80 mm (b) 0.70 mm

(c) 0.50 mm (d) 0.75 mm

27. A roller is made by joining, together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to:



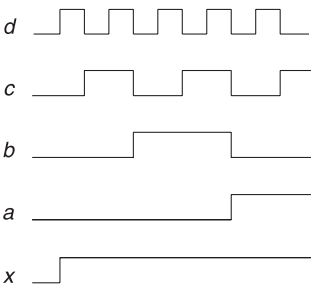
(a) turn right.

(b) go straight.

(c) turn left and right alternately.

(d) turn left.

28. If a, b, c, d are inputs to a gate and x is its output, then, as per the following time graph, the gate is:



(a) AND (b) OR

(c) NAND (d) NOT

29. For a common emitter configuration, if α and β have their usual meaning, the incorrect relationship between α and β is

(a) $\alpha = \frac{\beta}{1-\beta}$ (b) $\alpha = \frac{\beta}{1+\beta}$

(c) $\alpha = \frac{\beta^2}{1+\beta^2}$ (d) $\frac{1}{\alpha} = \frac{1}{\beta} + 1$

30. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius or earth R; $h << R$). The minimum increase in its orbit velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere)

(a) $\sqrt{gR}$ (b) $\sqrt{gR/2}$

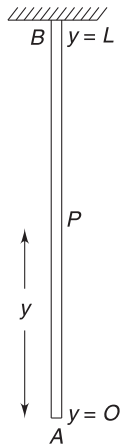
(c) $\sqrt{gR}(\sqrt{2}-1)$ (d) $\sqrt{2gR}$

Answers

1. (b)	2. (b)	3. (b)	4. (c)
5. (d)	6. (b)	7. (b)	8. (d)
9. (c)	10. (d)	11. (c)	12. (c)
13. (b)	14. (c)	15. (d)	16. (c)
17. (d)	18. (d)	19. (d)	20. (d)
21. (d)	22. (d)	23. (c)	24. (a)
25. (a)	26. (a)	27. (d)	28. (b)
29. (a), (c)	30. (c)		

Solutions

1. Since the string is very long ($L = 20$ m), it has a finite mass. Hence the tension in the string is not the same at all points. Consider a point P at a distance y from end A. Since the string is uniform, the tension T at P is proportional to y and is given by (see figure)



$$T = \left(\frac{Mg}{L}\right)y \text{ ; } M = \text{mass of the string}$$

Speed of pulse is $v = \sqrt{\frac{T}{\mu}}$; μ = Mass per unit length of the string

Now $\quad v = \frac{dy}{dt}$

$$\therefore \quad \frac{dy}{dt} = \sqrt{\frac{Mgy}{L\mu}} = \sqrt{gy} \quad \left(\because \mu = \frac{M}{L}\right)$$

$$\Rightarrow \quad \frac{dy}{\sqrt{y}} = \sqrt{g}\, dt$$

Integrating, we have.

$$\int_o^L (y^{-1/2}\, dy = \sqrt{g} \int_o^t dt$$

$$\Rightarrow \quad \left|2\sqrt{y}\right|_0^L = \sqrt{g}\, t$$

$$\Rightarrow \quad \left|2\sqrt{y}\right|_0^{20} = \sqrt{10}\, t$$

$$\Rightarrow \quad 2\sqrt{20} = \sqrt{10}\, t \quad \Rightarrow t = 2\sqrt{2}\text{ s}$$

So the correct choice is (b).

2. Potential energy needed to lift the mass $m = 10$ kg to a height $h = 1$ m, 1000 times is

$$\text{P.E.} = mgh \times 1000$$
$$= 10 \times 9.8 \times 1 \times 1000 = 9.8 \times 10^4 \text{ J} \quad (1)$$

Let M kg be the mass of the fat lost, the energy required is

$$E = 20\%\ of\ M \times 3.8 \times 10^7$$
$$= \frac{20}{100} \times M \times 3.8 \times 10^7$$
$$= (7.6 \times 10^6 \times M) \text{ J} \quad (2)$$

Equating (1) and (2)

$$(7.6 \times 10^6)\, M = 9.8 \times 10^4$$

$$\Rightarrow \quad M = 1.289 \times 10^{-2} \text{ kg}$$

So the correct choice is (c).

3. Frictional force acting on the particle as it moves from P to Q is

$$f_1 = \mu mg \cos\theta, \quad \text{where } \theta = 30°$$

Potential energy lost by the particle in this displacement is

$$E_1 = \text{work done against } f_1$$
$$= f_1 \times PQ$$
$$= \mu mg \cos\theta \times \frac{h}{\sin\theta}$$
$$= \frac{\mu mgh}{\tan\theta} = \frac{\mu mg \times 2}{\tan 30°}$$

$$\Rightarrow \quad E_1 = 2\sqrt{3}\ \mu mg \quad (1)$$

Frictional force acting on the particle as it moves from Q to R is

$$f_2 = \mu mg$$

Potential energy lost by the particle in this displacement is

$$E_2 = f_2 \times QR = \mu mg\, x \quad (2)$$

Given $E_1 = E_2$. Equating (1) and (2), we get $x = 2\sqrt{3}$ m $\simeq 3.5$ m

Total P.E. lost is $E = E_1 + E_2 = 2\sqrt{3}\,\mu mg + \mu mg\,x$

$$= \mu mg\,(2\sqrt{3} + x)$$

$$= \mu mg\,(2\sqrt{3} + 2\sqrt{3})$$

$$= 4\sqrt{3}\;\mu mg$$

Total P.E. lost is also equal to

$$E' = mgh = mg \times 2 = 2\,mg$$

Since $E = E'$, we get

$$4\sqrt{3}\;\mu mg = 2\,mg \quad \Rightarrow \mu = \frac{1}{2\sqrt{3}} \simeq 0.29$$

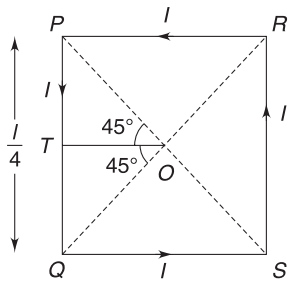
So the correct choice is (b).

4. $2\pi R = l \quad \Rightarrow R = \dfrac{l}{2\pi}$

The magnetic field at the centre of the circular coil A of radius R is

$$B_A = \frac{\mu_0 I}{2R} = \frac{\mu_0 I \pi}{l} \qquad (1)$$

Refer to the following figure.



Magnetic field at O due to current I in PQ is

$$B_{PQ} = \frac{\mu_0 I}{4\pi a}\,(\sin 45° + \sin 45°)$$

where $a = OT = PT = \dfrac{l}{8}$ ($\because PT = OT \tan 45° = OT$)

$$B_{PQ} = \frac{4\mu_0 I}{\sqrt{2}\,\pi l}$$

Therefore, the magnetic field at O due to the complete sqs loop $PQRS$ is

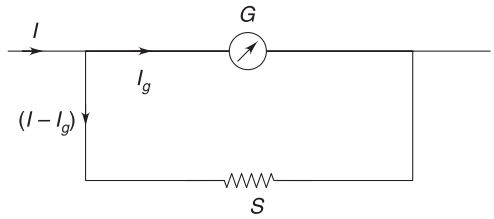
$$B_B = 4 \times \frac{4\mu_0 I}{\sqrt{2}\,\pi l} = \frac{8\sqrt{2}\,\mu_0 I}{\pi l} \qquad (2)$$

Dividing (1) by (2)we get

$$\frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$$

So the correct choice is (c).

5.



$$I_g G = (I - I_g)S$$

$$S = \frac{I_g G}{I - I_g}$$

Given $I = 10$ A, $I_g = 1$ mA $= 0.001$ A and $G = 100\ \Omega$.

$$S = \frac{0.001 \times 100}{(1 - 0.001)}$$

$$= \frac{0.01}{0.999}$$

$$\simeq 0.01\ \Omega$$

So the choice is (d).

6. A person looking through a telescope observes angular magnification (which is called magnifying power). If a distant tree subtends an angle α at his unaided eye, then the angle subtended at his eye (when he looks through the telescope) by the image will be 20 α. Hence the image will appear 20 times taller, so the correct choice is (b).
7. In the given temperature range, the resistance of a metal (such as copper) increases linearly with temperature but the resistance of a semi-conductor (such as silicon) decreases exponentially with temperature. So the correct choice is (b).
8. The correct choice is (d).
9. Let N_o be the radioactive nuclei present is each sample at $t = 0$.

For sample A: No. of half lives in 80 minutes $= \dfrac{80}{20} = 4$

Number of nuclei decayed in 4 half lives $= \dfrac{N_o}{2^4} = \dfrac{N_o}{16}$

Number of nuclei left undecayed is

$$N_A = N_o - \frac{N_o}{16} = \frac{15 N_o}{16}$$

For sample B: No. of half lives in 80 minutes $= \dfrac{80}{20} = 2$

Number nuclei decayed in 2 half lives $= \dfrac{N_o}{2^2} = \dfrac{N_o}{4}$

Number of nuclei left undecayed is

$$N_B = N_o - \frac{N_o}{4} = \frac{3N_o}{4}$$

$$\frac{N_A}{N_B} = \frac{15N_o}{16} \times \frac{4}{3N_o} = \frac{5}{4}$$

So the correct choice is (c).

10. The equation of straight line AB is

$$P = mV + C \quad (1)$$

where m is the slope and C is the intercept. From the given figure,

$$\text{Slope} \quad m = \frac{2P_0 - P_0}{V_0 - 2V_0} = -\frac{P_0}{V_0} \quad (2)$$

To find C, we have for point A,

$$2P_0 = mV_0 + C = -\frac{P_0}{V_0} \times V_0 + C$$

$$\Rightarrow \quad C = 3P_0 \quad (3)$$

Using (2) and (3) in (1) we have

$$P = -\left(\frac{P_0}{V_0}\right)V + 3P_0 \quad (4)$$

Equation of state is

$$PV = nRT$$

$$T = \frac{PV}{nR} \quad (5)$$

Using (4) in (5) we have

$$T = \frac{1}{nR}\left(-\frac{P_0}{V_0}V^2 + 3P_0V\right) \quad (6)$$

T will be maximum if $\frac{dT}{dV} = 0$ and $\frac{d^2T}{dV^2} < 0$. From (6) we find that

$$\frac{dT}{dV} = \frac{1}{nR}\left(-\frac{2P_0}{V_0} \times V + 3P_0\right)$$

Setting $\frac{dT}{dV} = 0$, we have

$$0 = \frac{1}{nR}\left(-\frac{2P_0V}{V_0} \times 3P_0\right)$$

$$\Rightarrow \quad V = \frac{3V_0}{2}$$

Putting this value in (6), we get

$$T_{\max} = \frac{1}{nR}\left[-\frac{P_0}{V_0} \times \left(\frac{3V_0}{2}\right)^2 + 3P_0 \times \frac{3V_0}{2}\right]$$

$$= \frac{9P_0V_0}{4nR}$$

It is easy to check that for $V = \frac{3V_0}{2}$, $\frac{d^2T}{dV^2}$ is negative.

Hence the correct choice is (d).

11. Resistance of the lamp is

$$R = \frac{80\text{ V}}{10\text{ A}} = 8\ \Omega$$

Given $I_{\text{rms}} = 10$ A, $V_{\text{rms}} = 220$ V and $\omega = 2\pi\nu = 100\pi$ rad s^{-1}.

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + X_L^2}}$$

$$\Rightarrow \quad 10 = \frac{220}{\sqrt{8^2 + X_L^2}}$$

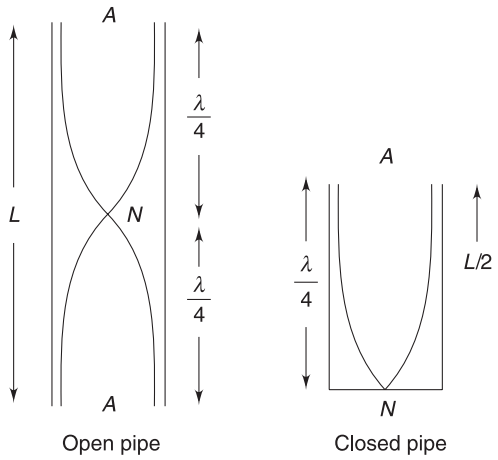
$$\Rightarrow 64 + X_L^2 = \left(\frac{220}{10}\right)^2 = 484$$

$$\Rightarrow \quad X_L = \sqrt{420} = 20.5\ \Omega$$

$$\Rightarrow \quad \omega L = 20.5$$

$$\Rightarrow \quad L = \frac{20.5}{2\pi \times 50} \simeq 0.065\text{ H, which is choice (c).}$$

12.



Fundamental frequency of open pipe is $\left(\because L = \frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}\right)$ and hence $\lambda = 2L$.

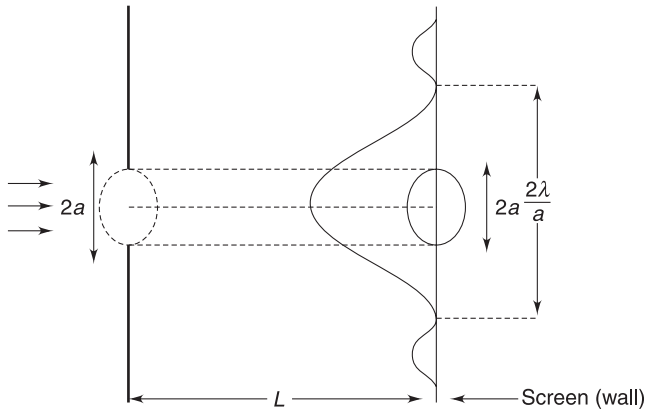
$$f = \frac{v}{\lambda} = \frac{v}{2L}$$

Fundamental frequency of the closed pipe is $\left(\because \frac{L}{2} = \frac{\lambda}{4} \text{ or } \lambda = 2L\right)$

$$f' = \frac{v}{\lambda} = \frac{v}{2L} = f$$

So the correct choice is (c).

13. Refer to the following figure.



Linear width of the circular spot for undiffracted beam is $2a$.

Half angular width of the first maximum is

$$\theta_1 = \frac{\lambda}{a}$$

Angular width of the first maximum is

$$2\theta_1 = \frac{2\lambda}{a}$$

$\therefore$ Linear width of the first maximum $= \dfrac{2\lambda L}{a}$

Sum of two linear widths is

$$b = 2a + \frac{2\lambda L}{a} \qquad (1)$$

b will be minimum if $\dfrac{db}{da} = 0$ and $\dfrac{d^2b}{da^2} > 0$.

Now $$\frac{db}{da} = \frac{d}{da}\left(2a + \frac{2\lambda L}{a}\right)$$

$$= 2 - \frac{2\lambda L}{a^2}$$

Putting $\dfrac{db}{da} = 0$ we set

$$0 = 2 - \frac{2\lambda L}{a^2} \Rightarrow a = \sqrt{\lambda L}$$

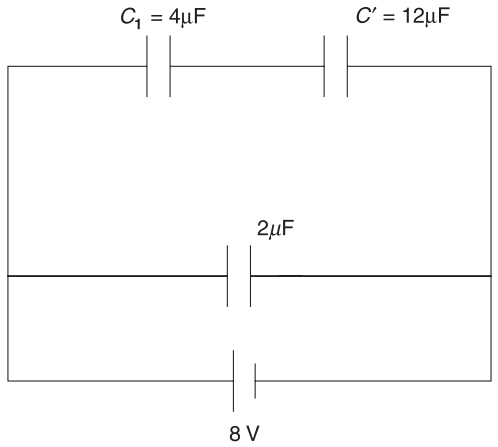
Putting this value of a in (1) we have

$$b_{\min} = 2\sqrt{\lambda L} + \frac{2\lambda L}{\sqrt{\lambda L}}$$

$$= 2\sqrt{\lambda L} + 2\sqrt{\lambda L} = 4\sqrt{\lambda L}$$

It is easy to check that when $a = \sqrt{\lambda L}, \dfrac{d^2b}{da^2}$ is positive. So the correct choice is (b).

14. The given circuit can be redrawn as follows.



Potential difference across the series combination of C_1 and C' is 8 V. If V_1 and V' are the potential differences across C_1 and C' respectively, then

$$C_1V_1 = C'V'$$

$$\Rightarrow \quad 4 \times 10^{-6} \times V_1 = 12 \times 10^{-6} \times V'$$

$$\Rightarrow \quad V_1 = 3\,V' \qquad (1)$$

Also $$V_1 + V' = 8 \qquad (2)$$

Equations (1) and (2) give $V_1 = 6$ V and $V' = 2$ V. Therefore, charges on 4 μF capacitor and 9 μF capacitor is

$$Q_1 = (4 \times 10^{-6}) \times 6 = 24 \times 10^{-6}\ \text{C}$$

and $$Q_2 = (9 \times 10^{-6}) \times 2 = 18 \times 10^{-6}\ \text{C}$$

Total charge $Q = Q_1 + Q_2 = 42 \times 10^{-6}$ C

The electric field at $r = 30$ m is

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{42\times10^{-6}\times9\times10^{9}}{30\times30}$$

$= 420\ \text{NC}^{-1}$, which is option (c).

15. Energy $E = h\nu = \dfrac{hc}{\lambda}$. The wavelength of radiowaves is the longest and of X-rays is the shortest, In fact $\lambda_{\text{radio}} > \lambda_{\text{yellow}} > \lambda_{\text{blue}} > \lambda_{\text{x-rays}}$.

Hence $$\nu_D < \nu_B < \nu_A < \nu_C$$

So $$E_D < E_B < E_A < E_C$$

Thus the correct choice is (d).

16. The area enclosed by the hysteresis loop gives the energy dissipated. It is clear that material B dissipates less energy than material A. Hence, if the core of a transformer is made of material B, it will have a higher efficiency. Also the core of an electromagnet

should be made of a material which has a small retentivity and a small coercivity. It follows from the given loops that material B satisfies both these conditions. Hence the correct choice is (c).

17. $T = 2\pi\sqrt{\frac{L}{g}}$. Therefore,

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta L}{L} = \frac{1}{2}\times\frac{L\alpha\Delta\theta}{L} = \frac{1}{2}\alpha\Delta\theta$$

$\therefore$ Loss of time per second = $\frac{1}{2}\alpha\Delta\theta$

Loss of time per day = $\left(\frac{1}{2}\alpha\Delta\theta\right)\times(24\times60\times60)$ s.

If θ is the temperature at which the clock gives correct time,

$$\frac{1}{2}\alpha(40-\theta)\times(24\times60\times60) = 12 \qquad (1)$$

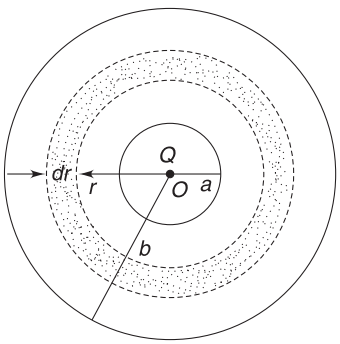
and $$\frac{1}{2}\alpha(\theta-20)\times(24\times60\times60) = 4 \qquad (2)$$

Dividing (1) and (2) we have

$$\frac{40-\theta}{\theta-20} = 3 \quad \Rightarrow \quad \theta = 25°\text{C}$$

Using $\theta = 25°$C in (1) or (2) we get $\alpha = 1.85 \times 10^{-5}$ per °C so the correct choice is (d).

18. Divide the region between the two spheres into a large number of concentric spherical elements each of a very small width *dr*. Consider one such element at a distance *r* from the centre *O* of the spheres as shown in the figure.



Volume of element = $4\pi r^2\,dr$. Volume charge density is $\rho = \frac{A}{r}$ (given). Therefore, charge on the element is

$$dq = 4\pi r^2\,dr \times \frac{A}{r} = 4\pi A\,r\,dr$$

Total charge in the region between $r = a$ and $r = r$ is

$$q = \int_a^r dq = 4\pi A\int_a^r r\,dr$$

$$= 2\pi A\,(r^2 - a^2)$$

This charge can be assumed to be concentrated at the centre *O*. So the total charge at *O* is

$$Q' = Q + q = Q + 2\pi A\,(r^2 - a^2)$$

The electric field at a distance *r* from Q' is

$$E = \frac{Q'}{4\pi\in_0 r^2}$$

$$= \frac{1}{4\pi\in_0 r^2}\,[Q + 2\pi A\,(r^2 - a^2)]$$

$$= \frac{Q}{4\pi\in_0 r^2} + \frac{2\pi A}{4\pi\in_0 r^2}\,(r^2 - a^2)$$

$$\Rightarrow \qquad E = \frac{A}{2\in_0} + \frac{Q}{4\pi\in_0 r^2} - \frac{Aa^2}{2\in_0 r^2}$$

E will be constant (i.e., *E* will be independent of *r*) if the last two terms in the above equation cancel each other, i.e., if

$$\frac{Q}{4\pi\in_0 r^2} = \frac{Aa^2}{2\in_0 r^2}$$

$$\Rightarrow \qquad A = \frac{Q}{2\pi a^2}$$

So the correct choice is (d).

19. Given $i = 35°$, $\delta = 40°$ and $e = 79°$. Now

$$\delta = i + e - A$$

$$\Rightarrow \qquad A = i + e - \delta$$

$$= 35° + 79° - 40° = 74°$$

If δ_m is the angle of minimum deviation, the refractive index of the prism is given by

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \qquad (1)$$

$$\Rightarrow \quad \mu\sin\left(\frac{A}{2}\right) = \sin\left(\frac{A}{2} + \frac{\delta_m}{2}\right)$$

$$= \sin\left(\frac{A}{2}\right)\cos\left(\frac{\delta_m}{2}\right) + \cos\left(\frac{A}{2}\right)\sin\left(\frac{\delta_m}{2}\right)$$

Partial differentiation of this equation gives (since μ and *A* are constants),

$$0 = -\frac{1}{2}\sin\left(\frac{A}{2}\right)\sin(\delta_m) + \frac{1}{2}\cos\left(\frac{A}{2}\right)\cos(\delta_m)$$

$$\Rightarrow \tan \delta_m = \frac{1}{\tan \frac{A}{2}} = \frac{1}{\tan 37^\circ} \qquad (\because \quad A = 74^\circ)$$

Since tan 37° is slightly greater than tan 30° $\left(\text{which is } \frac{1}{\sqrt{3}}\right)$, It follows that δ_m is slightly less than 60°. The actual calculation (using trignometric tables given $\delta_m =$ 53°. This angle is greater than the given value of deviation (which is δ = 40°). Now, with δ_m = 53° and A = 74°,

$$\mu = \frac{\sin\left(\frac{74^\circ + 53^\circ}{2}\right)}{\sin\left(\frac{74^\circ}{2}\right)} = \frac{\sin 63.5^\circ}{\sin 37^\circ}$$

Using tables μ turns out to be nearly equal to 1.5. So the closest option is (d).

20. The mean value of the four measurements of time for 100 oscillations is

$$\bar{t} = \frac{t_1 + t_2 + t_3 + t_4}{4}$$

$$= \frac{90 + 91 + 95 + 92}{4} = 92 \text{ s}$$

Deviations (or errors) of values of t from the mean value are

$$|\bar{t} - t_1| = |92 - 90| = 2\text{s}$$
$$|\bar{t} - t_2| = |92 - 91| = 1\text{s}$$
$$|\bar{t} - t_3| = |92 - 95| = 3\text{s}$$
$$|\bar{t} - t_4| = |92 - 92| = 0\text{s}$$

Average error = $\frac{2 + 1 + 3 + 0}{4} = 1.5\text{s}$

Since the least count of the clock is 1 s, average error rounded off to appropriate significant figure is either 1s or 2s. So the reported mean time should be either (92 ± 1) s or (92 ± 2) s. So the correct choice is (d).

21. It is a question based on the experimental observation as written in NCERT book which states that the correct option is (d).

22. $h\nu = \frac{1}{2} m v^2 + w_0$. Since $\nu = \frac{c}{\lambda}$

$$v = \sqrt{\frac{2}{m}\left[\frac{hc}{\lambda} - w_0\right]} \qquad (1)$$

For wavelength $\frac{3\lambda}{4}$, we have

$$v' = \sqrt{\frac{2}{m}\left[\frac{4hc}{3\lambda} - w_0\right]} \qquad (2)$$

Dividing (2) by (1)

$$\frac{v'}{v} = \left[\frac{\frac{4hc}{3\lambda} - w_0}{\frac{hc}{\lambda} - w_0}\right]^{1/2}$$

or $$\left(\frac{v'}{v}\right)^2 = \frac{\frac{4hc}{3\lambda} - w_0}{\frac{hc}{\lambda} - w_0}$$

$$= \frac{\left(\frac{4hc}{3\lambda} - w_0\right) + \left(\frac{hc}{\lambda} + w_0\right)}{\left(\frac{hc}{\lambda} - w_0\right) + \left(\frac{hc}{\lambda} + w_0\right)}$$

$$= \frac{\frac{4}{3} + 1}{2} = \frac{7}{6}$$

$\Rightarrow \quad \frac{v'}{v} = \sqrt{\frac{7}{6}} = 1.08$. So choices (b) and (c) are incorrect. Choice (a) gives $\frac{v'}{v} < \sqrt{\frac{4}{3}}$ or $\frac{v'}{v} < 1.15$ and choice (d) gives $\frac{v'}{v} > \sqrt{\frac{3}{4}}$ or $\frac{v'}{v} > 0.87$. But $\frac{v'}{v} = 1.08$, *i.e.* $v' > v$. So choice (a) also wrong. Hence the correct choice is (d).

23. $$v = \omega\sqrt{A^2 - x^2} \qquad (1)$$

At $x = \frac{2A}{3}$,

$$v' = \omega\sqrt{A^2 - \frac{4A^2}{9}} = \omega\sqrt{\frac{5A^2}{9}} = \omega A \times \frac{\sqrt{5}}{3}$$

Given $v' = 3v$. Hence

$$v' = \sqrt{5}\ \omega A$$

Let A_n be the new amplitude. Then

$$v' = \omega(A_n^2 - x^2)^{1/2}$$

$$\Rightarrow \quad \sqrt{5}\ \omega A = \omega (A_n^2 - x^2)^{1/2}$$

$$\Rightarrow \qquad 5A^2 = A_n^2 - x^2$$

$$\Rightarrow \qquad A_n^2 = 5A^2 + x^2 = 5A^2 + \frac{4A^2}{9} \quad \left(\because x = \frac{2A}{3}\right)$$

$$= \frac{49A^2}{9}$$

$$\Rightarrow \qquad A_n = \frac{7A}{3}$$

So the correct choice is (c).

24. The angular momentum of a particle moving with momentum $\vec{p} = m\vec{v}$ about origin O is defined as

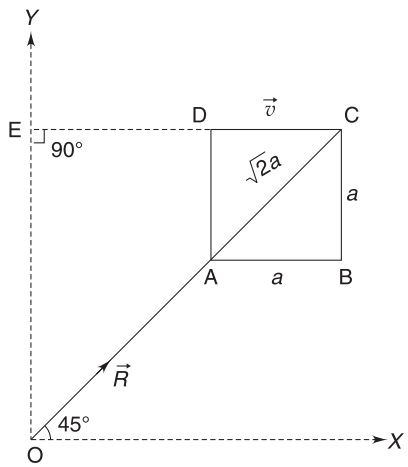
$$\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

Where $\vec{r}$ is the position vector of the particle from O. The magnitude of $\vec{L}$ is

$$L = mrv \sin \theta$$
$$= mv\, r \sin \theta$$
$$= mv\, r_\perp$$

where $r_\perp$ = perpendicular of direction of motion ($\vec{v}$) of the particle from O.

The direction $\vec{L}$ is obtained from the right hand screw rule. Now, let us first examine option (a). Refer to the following figure.



The magnitude of the position vector of *C* from O is

$$\text{OC} = r = \text{OA} + \text{AC} = R + \sqrt{2}\, a$$

Now $$r_\perp = \text{OE} = \text{OC} \sin 45°$$

$$= (R + \sqrt{2}\, a) \times \frac{1}{\sqrt{2}}$$

$$= \left(\frac{R}{\sqrt{2}} + a\right)$$

$$\therefore \qquad L = mv \left(\frac{R}{\sqrt{2}} + a\right)$$

From right hand screw rule, the direction of $\vec{L}$ is along position z-axis. Hence

$$\vec{L} = mv \left(\frac{R}{\sqrt{2}} + a\right)\hat{k}$$

So choice (a) is false. Using the above method, it is easy to check that choices (b), (c) and (d) are all true. So the correct answer is (a).

25. The molar heat capacity is given by

$$C = C_v + \frac{R}{(1-n)}$$

$$\Rightarrow \qquad C - C_v = \frac{R}{(1-n)} = \frac{C_p - C_v}{(1-n)}$$

$$\Rightarrow \qquad 1 - n = \frac{C_p - C_v}{C - C_v}$$

$$\Rightarrow \qquad n = 1 - \frac{C_p - C_v}{C - C_v}$$
$$= \frac{C - C_v - C_p + C_v}{C - C_v}$$
$$= \frac{C - C_p}{C - C_v}, \text{ which is choice (a)}$$

26. Least count of screw gauge = $\frac{0.5\text{ mm}}{50} = 0.01$ mm.

Zero error = $-0.5 + 45 \times 0.01 = -0.05$ mm

Measured value = $0.5 + 25 \times 0.01 = 0.75$ mm

Corrected value = measured value – zero error
$= 0.75 - (-0.05)$
$= 0.80$ mm

So the correct option is (a).

27. As the roller moves with its centre *O* in a direction parallel to *CD*, the radius of the wheel decreases because the rails *AB* and *CD* are not parallel to each other and the distance between them decreases as *O* moves upward. Hence, for the same number of rotations, the distance moved *AB* becomes less. This can happen if the roller will tend to turn left. So the correct option is (d).

28. It follows from the given graph, $x = 1$ if $a = 1$ or $b = 1$ or $c = 1$ or $d = 1$. Hence the gate is an *OR* gate, which is choice (b).

29. We know that $I_E = I_C + I_B$ and two d.c. current gains of a transistor are defined as $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$.

$\alpha = \frac{I_C}{I_C + I_B} = \frac{I_C/I_B}{I_C/I_B + 1} = \frac{\beta}{\beta + 1}$. So option (a) is incorrect. Option (b) is correct.

$\alpha = \dfrac{\beta}{\beta+1} = \dfrac{\beta^2}{(\beta+1)\beta}$. So option (c) is also incorrect.

Now $\dfrac{1}{\alpha} = \dfrac{I_E}{I_C} = \dfrac{I_C + I_B}{I_C} = 1 + \dfrac{I_B}{I_C} = 1 + \dfrac{1}{\beta}$. So option (d) also correct. So the incorrect options are (a) and (c).

30. Orbital speed $v_O = \sqrt{\dfrac{GM}{R+h}}$

Escape speed $v_e = \sqrt{\dfrac{2GM}{R+h}}$

Minimum increase required

$$= v_e - v_O$$

$$= \left(\sqrt{2-1}\right)\left(\frac{GM}{R+h}\right)$$

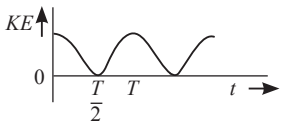
$$= \left(\sqrt{2}-1\right)\left(\frac{GM}{R}\right)^{1/2} \qquad (\because h << R)$$

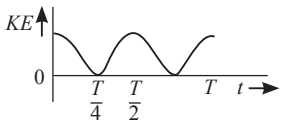
$$= \left(\sqrt{2}-1\right)\sqrt{gR} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

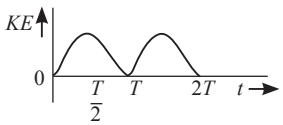
So the correct choice is (c).

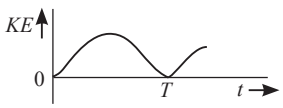
Solutions of Physics JEE Main—2017

1. A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like

(a) 

(b) 

(c) 

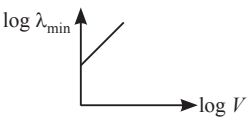
(d) 

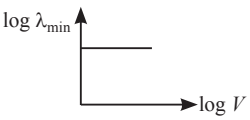
2. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $w_f - n_i$ will be:
   (a) 2.5×10^{25} (b) -2.5×10^{25}
   (c) -1.61×10^{23} (d) 1.38×12^{23}
3. Which of the following statements is false?
   (a) A rheostat can be used as a potential divider
   (b) Kirchhoff's second law represents energy conservation
   (c) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude.
   (d) In a balanced Wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.
4. The following observation were taken for determining surface tension T of water by capillary method:
   Diameter of capillary $D = 1.25 \times10^{-2}$ m
   raise of water, $h = 1.45 \times 10^{-2}$ m
   Using g = 9.80 m/s^2 and the simplified relation $T = \frac{rgh}{2}\times10^3$ N/m, the possible error in surface tension is closest to:
   (a) 2.4% (b) 10%
   (c) 0.15% (d) 15%
5. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signle frequency is denoted by ω_m. The bandwidth ($\Delta\omega_m$) of the signal is such the $\Delta\omega_m << \omega_c$. Which of the following frequencies is not contained in the modulated wave?
   (a) $\omega_m + \omega_c$ (b) $\omega_c - \omega_m$
   (c) ω_m (d) ω_c
6. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is:
   (a) real and at a distance of 40 cm from the divergent lens
   (b) real and at a distance of 6 cm from the convergent lens
   (c) real and at a distance of 40 cm from convergent lens
   (d) virtual and at a distance of 40 cm from convergent lens
7. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio l/R such that the moment of inertia is minimum?

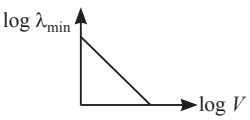
(a) 1 (b) $\frac{3}{\sqrt{2}}$

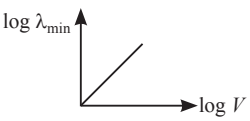
(c) $\sqrt{\frac{3}{2}}$ (d) $\frac{\sqrt{3}}{2}$

8. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of log λ_{min} with log V is correctly represented in:

(a) 

(b) 

(c) 

(d) 

9. A radioactive nucleus A with a half life T, decays into a nucleus B. At $t = 0$, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by:

(a) $t = T\log(1.3)$ (b) $t = \frac{T}{\log(1.3)}$

(c) $t = \frac{T\log 2}{2\log 1.3}$ (d) $t = T\frac{\log 1.3}{\log 2}$

10. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E\hat{j}$ it experiences torque $\vec{T}_2 = \vec{T}_1$. The angle θ is:

(a) 60° (b) 90°

(c) 30° (d) 45°

11. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be:

(a) 135° (b) 180°

(c) 45° (d) 90°

12. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that
$C_p - C_v = a$ for hydrogen gas
$C_p - C_v = b$ for nitrogen gas
The correct relation between a and b is

(a) $a = 14b$ (b) $a = 28b$

(c) $a = \frac{1}{14}b$ (d) $a = b$

13. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by: (Given: room temperature = 30° C, specific heat of copper = 0.1cal/gm°C

(a) 1250°C (b) 825°C

(c) 800°C (d) 885°C

14. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10$ ms^{-1}. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be:

(a) 10^{-4}kg m^{-1} (b) 10^{-4}kg m^{-1}s^{-1}

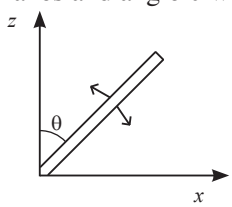
(c) 10^{-3}kg m^{-1} (d) 10^{-3}kg s^{-1}

15. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into to voltmeter of range 0–10V is

(a) $2.535 \times 10^3\Omega$ (b) $4.005 \times 10^3\Omega$

(c) $1.985 \times 10^3\Omega$ (d) $2.045 \times 10^3\Omega$

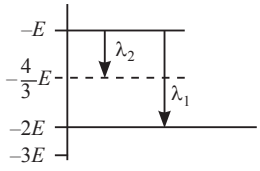
16. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes and angle θ with the vertical is:



(a) $\frac{3g}{2l}\cos\theta$ (b) $\frac{2g}{3l}\cos\theta$

(c) $\frac{3g}{2l}\sin\theta$ (d) $\frac{2g}{3l}\sin\theta$

17. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1/\lambda_2$, is given by:



(a) $r = \frac{3}{4}$ (b) $r = \frac{1}{3}$

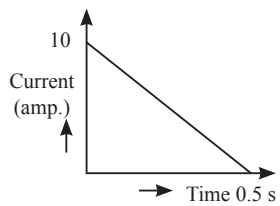
(c) $r = \frac{4}{3}$ (d) $r = \frac{2}{3}$

18. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of:

(a) 81 (b) $\frac{1}{81}$

(c) 9 (d) $\frac{1}{9}$

19. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is:



(a) 250 Wb (b) 275 Wb

(c) 200 Wb (d) 225 Wb

20. In a Young's double slit exper4iment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of tow wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is

(a) 9.75 mm (b) 15.6 mm

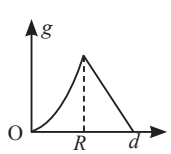
(c) 1.56 mm (d) 7.8 mm

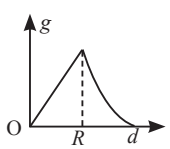
21. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillation in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is:

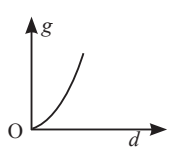
(a) 6.98 s (b) 8.76 s

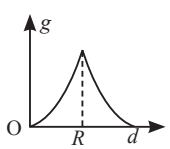
(c) 6.65 s (d) 8.89 s

22. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius)

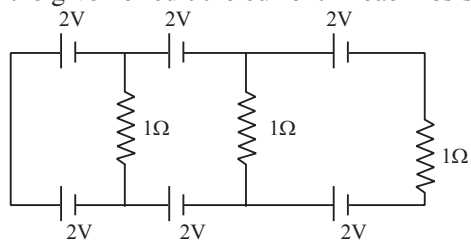
(a) 

(b) 

(c) 

(d) 

23. In the given circuit the current in each resistance is:



(a) 0.5 A (b) 0 A

(c) 1 A (d) 0.25 A

24. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is:

(a) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$ (b) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

(c) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$ (d) $\frac{\lambda_A}{\lambda_B} = 2$

25. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modules of the material of the cube and is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by:

(a) $\frac{3\alpha}{PK}$ (b) $3PK\alpha$

(c) $\frac{P}{3\alpha K}$ (d) $\frac{P}{\alpha K}$

26. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be:

(a) 9 J (b) 18 J

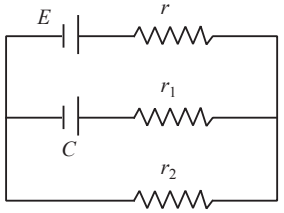
(c) 4.5 J (d) 22 J

27. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light = 3×10^8 ms^{-1})

(a) 17.3 GHz (b) 15.3 GHz

(c) 10.1 GHz (d) 12.1 GHz

28. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be:



(a) $CE\dfrac{r_2}{(r+r_2)}$ (b) $CE\dfrac{r_1}{(r_1+r)}$

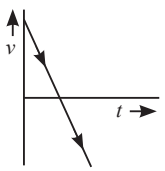
(c) CE (d) $CE\dfrac{r_1}{(r_2+r)}$

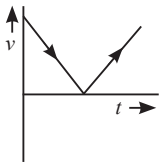
29. A capacitance of 2μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1μF capacitors are available which can withstand a potential difference of not more than 300V. The minimum number of capacitors required to achieve that is:

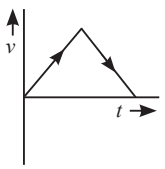
(a) 24 (b) 32

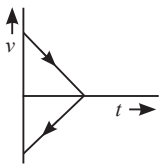
(c) 2 (d) 16

30. A body is thrown vertically upwards. Which one of the following graphs correctly represents the velocity vs time?

(a) 

(b) 

(c) 

(d) 

Answers

1. (b)	2. (b)	3. (d)	4. (d)
5. (c)	6. (c)	7. (d)	8. (c)
9. (d)	10. (a)	11. (b)	12. (a)
13. (d)	14. (a)	15. (c)	16. (c)
17. (b)	18. (c)	19. (a)	20. (d)
21. (c)	22. (b)	23. (b)	24. (d)
25. (c)	26. (c)	27. (a)	28. (a)
29. (b)	30. (a)		

Solutions

1. The displacement of a particle executing simple harmonic motion is given by

$$x = A\sin(\omega t + \phi) \qquad (1)$$

Since the particle is at the equilibrium position ($x = 0$) at time $t = 0$, it follows from Eq. (1) that $\phi = 0$

Hence,

$$x = A\sin(\omega t)$$

The kinetic energy of the particles is

$$\text{KE} = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m\left(\frac{dx}{dt}\right)^2$$

$$= \frac{1}{2}mA^2\omega^2\cos^2(\omega t)$$

$$\text{or,} \quad \text{KE} = E_0\cos^2(\omega t) \qquad (2)$$

where $E_0 = \dfrac{1}{2}mA^2\omega^2$

It follows from Eq. (2) that KE is maximum = E_0

when $\cos^2(\omega t) = +1$

or, $\cos(\omega t) = \pm 1$

$\Rightarrow$ $\omega t = 0, \pi, 2\pi, 3\pi. \ldots$ etc

or $\dfrac{2\pi t}{T} = 0, \pi, 2\pi, 3\pi. \ldots$ etc

or $t = 0, \dfrac{T}{2}, T, \dfrac{3T}{2}, \ldots$ etc

KE is zero when

$$\cos^2(\omega t) = 0$$

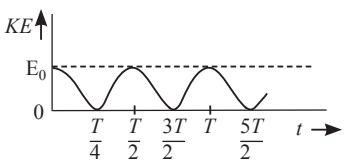
or $\cos(\omega t) = 0$

or $\cos\left(\dfrac{2\pi t}{T}\right) = 0$

or, $\dfrac{2\pi t}{T} = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{5\pi}{2}, \ldots$ etc

or $t = \dfrac{T}{4}, \dfrac{3T}{4}, \dfrac{5T}{4}, \ldots$ etc

So, the graph of KE vs t is as shown in the following figure



Hence the correct choice is (b).

2. If N is the number of moles of a gas, then the number of molecules is

$n = N \times$ Avogadro number $= N \times 6.023 \times 10^{23}$

Given $T_i = 290$ K and $T_f = 300$ K. Assuming that the gas in the room is an ideal gas, then

$$P_0V_0 = N_iRT_i \qquad (1)$$

and $$P_0V_0 = N_fRT_f \qquad (2)$$

where $P_0 = 1\times10^5$ Pa and $V_0 = 30\,\text{m}^3$. From Eq. (1) and (2) we have

$$N_i = \frac{P_0V_0}{RT_i} \text{ and } N_f = \frac{P_0V_0}{RT_f}$$

Therefore, $N_i - N_f = \dfrac{P_0V_0}{R}\left(\dfrac{1}{T_i} - \dfrac{1}{T_f}\right)$

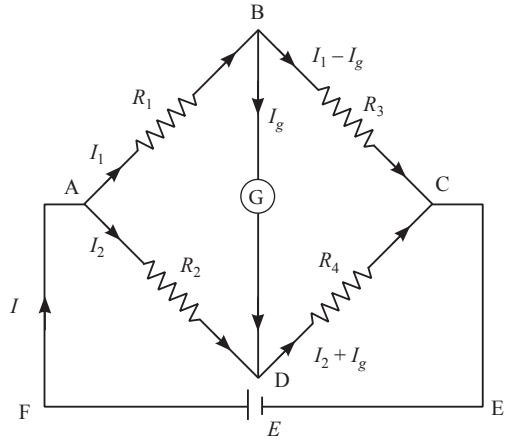
$$\therefore\ n_f - n_i = \frac{P_0V_0}{R}\left(\frac{1}{T_f}-\frac{1}{T_i}\right)\times 6.023\times10^{23}$$

$$= \frac{(1\times10^5)\times30}{8.3}\times\left(\frac{1}{300}-\frac{1}{290}\right)\times(6.023\times10^{23})$$

$= -2.5\times10^{25}$ molecules

So the correct choice is (b).

3. The following figure shows a normal Wheatstone bridge.



Applying Kirchhoff's loop rule to loops ABDA and BCDB,

We get

$$I_1R_1 + I_gG - I_2R_2 = 0 \qquad (1)$$

and $$(I_1 - I_g)R_3 - (I_2 + I_g)R_4 - I_gG = 0 \qquad (2)$$

When no current flow through the galvanometer (I_g = 0), the bridge is balanced. Putting $I_g = 0$ in Eqs. (1) and (2), we have

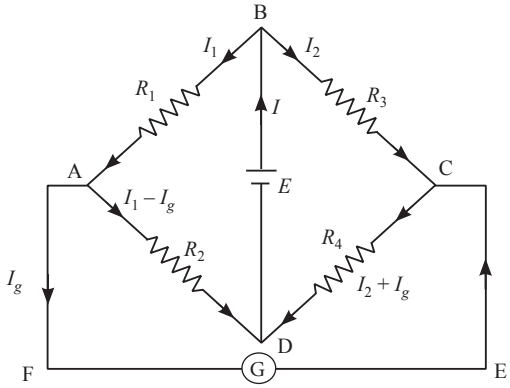
$$I_1R_1 - I_2R_2 = 0 \Rightarrow \frac{I_1}{I_2} = \frac{R_2}{R_1} \quad (3)$$

and $$I_1R_3 - I_2R_4 = 0 \Rightarrow \frac{I_1}{I_2} = \frac{R_4}{R_3} \qquad (4)$$

Equations (3) and (4) give

$$\frac{R_2}{R_1} = \frac{R_4}{R_3} \Rightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad (5)$$

When the cell E and galvanometer G are interchanged the bridge is shown in the following figure.



Applying Kirchhoff's loop rule to loops ABCEFA and AFECDA, we get

$$-I_1R_1 + I_2R_3 - I_gG = 0 \qquad (6)$$

and $$(I_1 - I_g)R_2 - (I_2 + I_g)R_4 - I_gG = 0 \qquad (7)$$

Putting $I_g = 0$ in Eqs. (6) and (7) we have

$$-I_1R_1 + I_2R_3 = 0 \Rightarrow \frac{I_1}{I_2} = \frac{R_3}{R_1} \qquad (8)$$

and $$I_1R_2 - I_2R_4 = 0 \Rightarrow \frac{I_1}{I_2} = \frac{R_4}{R_2} \qquad (9)$$

Equation (8) and (9) give

$$\frac{R_3}{R_1} = \frac{R_4}{R_2} \Rightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad (10)$$

The balance condition (5) and (10) are the same. Hence statement (d) is false. Statement (a), (b) and (c) are true.

4. $$T = \frac{rhg}{2}\times10^3 = \frac{Dhg}{4}\times10^3 \qquad (\because D = 2r)$$

The fractional error in T is

$$\frac{\Delta T}{T} = \frac{\Delta D}{D} + \frac{\Delta h}{h} + \frac{\Delta g}{g}$$

It is given that $g = 9.8\ \text{ms}^{-2}$ (fixed). Hence $\Delta g = 0$

Therefore, $$\frac{\Delta T}{T} = \frac{\Delta D}{D} + \frac{\Delta h}{h}$$

It follows from the given measurements that $\Delta D = 0.01\times10^{-2}$ m and $\Delta h = 0.01\times10^{-2}$m. therefore

$$\frac{\Delta T}{T} = \frac{0.01\times10^{-2}}{1.25\times10^{-2}} + \frac{0.01\times10^{-2}}{1.45\times10^{-2}}$$

$$= \frac{0.01}{1.25} + \frac{0.01}{1.45}$$

Percentage error in T is

$$\frac{\Delta T}{T}\times100 = \frac{0.01}{1.25}\times100 + \frac{0.01}{1.15}\times100$$

$= 0.8 + 0.689$

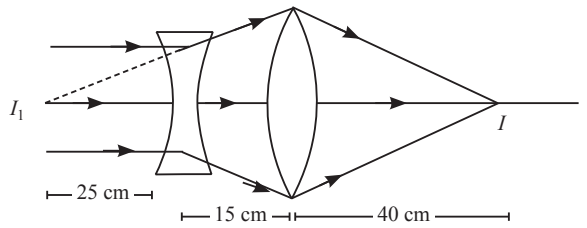
$= 1.489$

$\simeq 1.5\%$

So the correct choice is (d).

5. Refer to NCERT book page no. 526. The amplitude modulated wave contains the frequencies ω_c, $\omega_c - \omega_m$ and $\omega_c + \omega_m$; it does not contain the frequency ω_m. Hence the correction option is (c).

6. Refer to the following figure.



A parallel beam of light falling on a concave lens forms a virtual image I_1 at its focus which is to the left of this lens. This image I, acts as a virtual object for the convex. The object distance for the convex lens is $u = -(25 + 15) = -40$cm and $f = +20$ cm. Using these in the lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we have

$$\frac{1}{v}-\frac{1}{-40}=\frac{1}{20}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{20}-\frac{1}{40}=\frac{1}{40}$$

$$\Rightarrow \quad v = +40 \text{ cm}$$

The plus sign shows that the final image I is real and is formed at a distance of 40 cm to the right of the convex lens. So the correct option is (c).

7. The moment of inertia of the cylinder about its perpendicular bisector is

$$I=\frac{Ml^2}{12}+\frac{MR^2}{4}$$

or $$I=\frac{M}{4}\left(\frac{l^2}{3}+R^2\right) \qquad (1)$$

If ρ is the density of the cylinder, its mass is

$$M = \pi R^2 l \text{r}$$

$$\Rightarrow \quad R^2=\frac{M}{\pi l \rho}$$

Using this in Eq. (1) we get

$$I=\frac{M}{4}\left(\frac{l^2}{3}+\frac{M}{\pi l \rho}\right)$$

I will be maximum or minimum if $\frac{dI}{dl}=0$.

Differentiating I will respect of l and setting $\frac{dI}{dl}=0$, we have

$$0=\frac{M}{4}\left(\frac{2l}{3}-\frac{M}{\pi l^2 \rho}\right)$$

$$\Rightarrow \quad \frac{2l}{3}-\frac{M}{\pi l^2 \rho}=0 \qquad (2)$$

Now, $\pi l^2 R\rho = M$. Putting this in Eq. (2) we get

$$\frac{2l}{3}-\frac{R^2}{l}=0$$

$$\Rightarrow \quad \frac{l^2}{R^2}=\frac{3}{2} \Rightarrow \frac{l}{R}=\sqrt{\frac{3}{2}}$$

I is maximum if $\frac{d^2I}{dl^2}<0$. It is easy to check that $\frac{d^2I}{dl^2}$ is negative if $\frac{l}{R}=\sqrt{\frac{3}{2}}$. So the correct option is (c).

8. X = rays are produced when a beam of fast moving electrons hits a metal target. An electron accelerated by a potential difference V acquires energy eV where e is the magnitude of the charge of the electron. It the whole of this energy is used up, a photon of maximum frequency ν_{max} is produced. The energy of this photon is $h\nu_{max}$. From $c = \nu\lambda$, the wavelength of the photon is minimum ($=\lambda_{min}$) when ν is maximum ($= \nu_{max}$) thus

$$h\nu_{max} = eV$$

$$\Rightarrow \quad \frac{hc}{\lambda_{min}} = eV$$

$$\Rightarrow \quad \lambda_{min}=\left(\frac{hc}{eV}\right)=\frac{k}{V} \qquad (1)$$

where, $k=\frac{kc}{e}$. Taking logarithm of Eq. (1), we have

$$\log\left(\lambda_{min}\right) = -\log V + \log k \qquad (2)$$

From Eq. (2) it is clear that the graph of log (λ_{min}) versus log V is a straight line with negative slope ($= -1$) and a positive intercept log k. So the correct choice is (c).

9. It is given that at time t, $\frac{N_B}{N_A}=0.3 \Rightarrow N_B = 0.3N_A$.

Therefore, the total number of nuclei at time t is

$$N_A + N_B = N_A + 0.3N_A = 1.3N_A$$

We know that

$$N_A = 1.3N_A e^{-\lambda t}$$

$$\Rightarrow \quad \frac{1}{1.3}=e^{-\lambda t}$$

$$\Rightarrow \quad -\ln(1.3) = -\lambda t$$

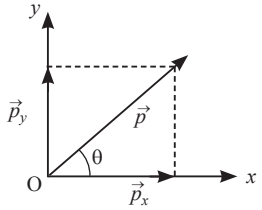
$$\Rightarrow \quad t=\frac{\ln(1.3)}{\lambda}$$

Now $$\lambda=\frac{\ln 2}{T}$$

Therefore, $t=T\frac{\ln(1.3)}{\ln(2)}$

So the correct choice is (d).

10. Refer to the following figure.



$\vec{p} = p_x\hat{i} + p_y\hat{j}$

We know that torque = $\vec{p} \times \vec{E}$

$\therefore \quad \vec{T}_1 = \left(p_x\hat{i} + p_y\hat{j}\right) \times E\hat{i} = -p_y\hat{k}E$

$\vec{T}_2 = \left(p_x\hat{i} + p_y\hat{j}\right) \times \sqrt{3}E\hat{j} = \sqrt{3}p_x\hat{k}E$

It is given that

$\vec{T}_2 = -\vec{T}_1$

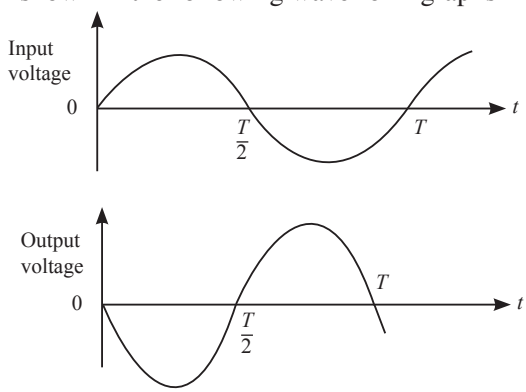
$\therefore \quad \sqrt{3}p_x\hat{k}E = -\left(-p_y\hat{k}E\right)$

$\Rightarrow \quad \dfrac{p_y}{p_x} = \sqrt{3}$

$\Rightarrow \quad \tan\theta = \sqrt{3} \quad \Rightarrow \theta = 60°$

So the correct option is (a).

11. In a common emitter amplifier circuit the input and output voltages are out of phase by a angle of 180° a shown in the following wave form graphs.



During the positive half cycle of the input voltage, the output voltage is negative and vice versa. So the correct choice is (b).

12. For an ideal gas,

$C_p' - C_v' = R$

where C_p' and C_v' are the molar specific heats at constant pressure and constant volume respectively.

Now, the specific heart C of a gas $= \dfrac{\text{molar specific heat}}{\text{molar mass}}$.

For hydrogen gas molar mass = 2 and for nitrogen molar mass = 28. So

For hydrogen $C_p - C_v = \dfrac{R}{2}$

For nitrogen $C_p - C_v = \dfrac{R}{28}$

Comparing with given relation in the question, we have $a = \dfrac{R}{2}$ and $b = \dfrac{R}{28}$. So $\dfrac{a}{b} = 14$ or $a = 14b$.

Thus the correct option is (a).

13. Heat is ained by the calorimeter and the water filled in it and heat is lost by the copper ball. Specific heat of water = 1 cal/g°C

Heat gained

= 100 × 0.1 × (75 – 30) + 170 × 1 × (75 – 30)

= 450 + 7650 = 8100 cal

Heat lost

= 100 ×0.1 × (T – 75)

= (10T – 750) cal

Now, heat gained = heat lost. Therefore,

8100 = 10T –750

$\Rightarrow T$ = 885°C, which is choice (d)

14. Let v be the speed of the ball at t = 10s. Its kinetic energy at t = 10 s is $\dfrac{1}{2}mv^2$. It is given that

$\dfrac{1}{2}mv^2 = \dfrac{1}{8}mv_0^2$

$\Rightarrow \quad v^2 = \dfrac{v_0^2}{4} \Rightarrow v = \dfrac{v_0}{2} = \dfrac{10\ \text{ms}^{-1}}{2} = 5\ \text{ms}^{-1}$

Frictional force $F = -kv^2$. Now $F = ma = m\dfrac{dv}{dt}$. Hence

$m\dfrac{dv}{dt} = -kv^2$

$\Rightarrow \quad \dfrac{dv}{v^2} = -\dfrac{kdt}{m}$

$\Rightarrow \quad \int_{v_0}^{v} \dfrac{dv}{v^2} = -\dfrac{k}{m}\int_0^t dt$

$\Rightarrow \quad \left|-\dfrac{1}{v}\right|_{v_0}^{v} = -\dfrac{k}{m}|t|_0^t = -\dfrac{kt}{m}$

$\Rightarrow \quad \dfrac{1}{v} - \dfrac{1}{v_0} = \dfrac{kt}{m}$

$\Rightarrow \quad \dfrac{1}{5} - \dfrac{1}{10} = \dfrac{k \times 10}{10^{-2}}$

$\Rightarrow \quad \dfrac{1}{10} = k \times 10^3$

$\Rightarrow \quad k = 10^{-4}\ \text{kg m}^{-1}$

The SI unit of $k = \dfrac{\text{SI unit of } F}{\text{SI unit of } v^2}$

$= \dfrac{\text{kg m s}^{-2}}{\text{m}^2\text{s}^{-2}} = \text{kg m}^{-1}$

So the correct option is (a).

15. Current for full scale deflection (I_g) = 5 mA = 5 × 10^{-3}A. Galvanometer resistance (G) = 15Ω. The required resistance is

$R = \dfrac{V}{I_g} - G$

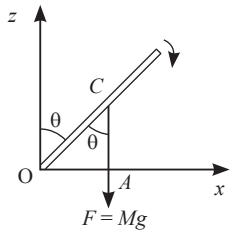
$= \dfrac{10}{5 \times 10^{-3}} - 15$

= 2000 – 15 = 1985 W

So, the correct option is (c)

16. The entire mass of the rod can be assumed to be at its centre of mass. For a uniform rod the centre of

mass is its centre *C* (see the following figure) so that $OC = \frac{l}{2}$



The axis of rotation passes through O and is perpendicular to the *x*-*z* plane. Now torque about O is τ = force × perpendicular distance of the line of application of force from the axis of rotation

$= F \times OA = F \times OC \sin\theta$

or $\tau = Mg \times \frac{l}{2}\sin\theta$

The moment of inertia of the rod about the axis of rotation is

$I = \frac{Ml^2}{3}$

Now, $\tau = I\alpha$

$\Rightarrow \quad \frac{Mgl\sin\theta}{2} = \frac{ml^2}{3} \times \alpha$

$\Rightarrow \quad \alpha = \frac{3}{2}\frac{g\sin\theta}{l}$, which is choice (c).

17. Change in energy = $h\nu = \frac{hc}{\lambda}$ or $\Delta E = \frac{hc}{\lambda}$

For transition $\lambda_1, \Delta E = -E - (-2E) = E$. Hence

$$E = \frac{hc}{\lambda_1} \Rightarrow \lambda_1 = \frac{hc}{E} \qquad (1)$$

For transition $\lambda_2, \Delta E = -E - \left(-\frac{4E}{3}\right) = \frac{E}{3}$. Hence

$$\frac{E}{3} = \frac{hc}{\lambda_2} \Rightarrow \lambda_2 = \frac{3hc}{E} \qquad (2)$$

From Eqs. (1) and (2) we find that

$$\frac{\lambda_1}{\lambda_2} = \frac{1}{3} \text{ or } r = \frac{1}{3}$$

So the correct choice is (b).

18. Stress = $\frac{\text{Force}}{\text{area}} = \frac{mg}{A} = \frac{V\rho g}{A}$

Where *V* is the volume and ρ is the density. If the linear dimension of a body is *L*. Then its volume $V \propto L^3$ and its area $A \propto L^2$. Hence

Stress $\propto \frac{L^3\rho g}{L^2} \propto L\rho g$

Since ρ remains unchanged, stress $\propto L$. Hence, if *L* increases by a factor of 9, the stress will also increases by a factor of 9. So the correct choice is (c).

19. From the given $I - t$ graph, it is clear that the slope is $m = -\frac{10}{0.5} = -20\ \text{As}^{-1}$ and intercept *c* = 10A. Therefore, the equation of the straight line graph is

$I = (-20t + 10)$ ampere

Induced charge is

$q = \int I dt$

$= \int_{t=0}^{t=0.5s} (-20t + 10)dt$

$= \left|-10t^2\right|_0^{0.5} + \left|10t\right|_0^{0.5}$

$= -2.5 + 5 = 2.5$ coulomb

Now change in magnetic flux is given by

$\Delta\phi = qR$

$= 2.5 \times 100 = 250$ Wb,

which is choice (a).

Note: The value of q can also be obtained as follows.

$q = \int I dt$

= Area under the I – t graph

$= \frac{1}{2} \times 10 \times 0.5 = 2.5$ coulomb

20. Let *m* th bright fringe of wavelength λ_m and *n*th bright fringe of wavelength λ_n coincide at a distance *y* from the central maximum, then

$y = \frac{m\lambda_m D}{d} = \frac{n\lambda_n D}{d}$

$\Rightarrow \quad \frac{m}{n} = \frac{\lambda_n}{\lambda_m} = \frac{520}{650} = \frac{4}{5}$

The least integral values of *m* and *n* which satisfy this condition are *n* = 4 and *m* = 5, i.e. 4th bright fringe of 650 nm coincides with 5th bright fringe of 520 nm. The smallest value of *y* at which this happens is

$y_{\min} = \frac{n\lambda_m D}{d}$

$= \frac{4 \times (650 \times 10^{-9}) \times 1.5}{0.5 \times 10^{-3}}$

$= 7.8 \times 10^{-3}$ m = 7.8 mm,

which is choice (d).

21. Given $I = 7.5 \times 10^{-6}$ kgm^2, $M = 6.7 \times 10^{-2}$ Am^2 and $B = 0.01$ T.

The time period of oscillations (i.e. time for one oscillation) is

$T = 2\pi\sqrt{\frac{I}{MB}}$

$$= 2\times3.14\times\sqrt{\frac{7.5\times10^{-6}}{6.7\times10^{-2}\times0.01}}$$

$= 0.665$ s

$\therefore$ Time for 10 oscillations = $10T = 6.65$s. so, the correct choice is (c).

22. At a point at a distance d inside the earth from the centre of the earth, g is given by

$$g = \frac{GMd}{R^3}$$

where M is the mass of the earth. So g varies linearly with d for ($d < R$)

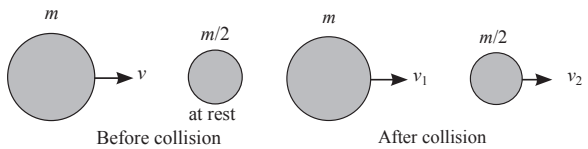
For a point at a distance d outside the earth from the centre of the earth, g is given by

$$g = \frac{GM}{d^2}$$

So $g \propto \frac{1}{d^2}$ for ($d > R$). Hence the correct option is (b).

23. From the given circuit, it is clear that the potential difference across each resistance = 2V – 2V = zero. Hence the current in each resistance is zero. So the correct option is (b).

24. Refer to the following figure.



Conservation of linear momentum gives

$$mv + \frac{m}{2}\times0 = mv_1 + \frac{m}{2}v_2$$

$$\Rightarrow \quad 2v = 2v_1 + v_2 \Rightarrow v_2 = 2(v - v_1) \qquad (1)$$

Since the collision is elastic, kinetic energy is also conserved.

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{m}{2}v_2\right)^2$$

$$\Rightarrow \qquad 2v^2 = 2v_1^2 + v_2^2 \qquad (2)$$

Using (1) and (2), we have

$$2v^2 = 2v_1^2 + 4(v - v_1)^2$$

$$\Rightarrow \qquad v^2 - 4vv_1 + 3v_1^2 = 0$$

$$\Rightarrow \qquad v^2 - 3vv_1 - vv_1 + 3v_1^2 = 0$$

$$\Rightarrow \qquad v(v - 3v_1) - v_1(v - 3v_1) = 0$$

$$\Rightarrow \qquad v = v_1 \text{ and } v = 3v_1$$

Since $v = v_1$ is not possible, $v = 3v_1$ or $v_1 = \frac{v}{3}$

Using this in Eq. (2) we get $v_2 = \frac{4v}{3}$

de-Broglie wavelengths of the particle are $\lambda_1 = \frac{h}{p_1}$ and $\lambda_2 = \frac{h}{p_2}$ i.e.

$$\lambda_1 = \frac{h}{mv_1} \text{ and } \lambda_2 = \frac{h}{\frac{m}{2}v_2} = \frac{2h}{mv_2}$$

$$\therefore \qquad \frac{\lambda_1}{\lambda_2} = \frac{v_2}{2v_1} = \frac{4v}{3}\times\frac{1}{2}\times\frac{3}{v} = 2$$

So the correct option is (d).

25. Let V be the original volume of the cube at 0°C. The decrease in volume under external pressure P is

$$\Delta V = \frac{VP}{K} \qquad (1)$$

Let ΔT be the increase in its temperature to compensate for this increase in volume, then

$$\Delta V = \gamma V \Delta T = 3\alpha V \Delta T \qquad (2)$$

where γ is the coefficient of cubical expansion which is equal to 3α. Equating (1) and (2)

$$\frac{VP}{K} = 3\alpha V \Delta T$$

$$\Rightarrow \qquad \Delta T = \frac{P}{3\alpha K}, \text{ which is option (c).}$$

26. Given $F = 6t$ also $F = ma = m\frac{dv}{dt}$. Therefore

$$m\frac{dv}{dt} = 6t$$

$$\Rightarrow \qquad \frac{dv}{dt} = \frac{6t}{m}$$

$$\Rightarrow \qquad dv = \frac{6tdt}{m}$$

$$\Rightarrow \qquad \int_0^v dv \quad = \int_0^s tdt$$

$$\Rightarrow \qquad |v|_0^v = \frac{6}{m}\left|\frac{t^2}{2}\right|_0^{1s}$$

$$\Rightarrow \qquad v = \frac{6}{m}\frac{(1)^2}{2} = \frac{3}{m} = \frac{3}{1} = 3\ \text{ms}^{-1} \qquad (\because m = 2kg)$$

From work energy theorem

Work done = change in kinetic energy

$$= \frac{1}{2}mv^2 - 0$$

$$= \frac{1}{2}\times1\times3^2$$

$= 4.5$ J

So the correct choice is (c).

27. The apparent frequency measured by the observer is

$$\nu_a = \nu\times\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

where c = speed of microwaves = speed of light

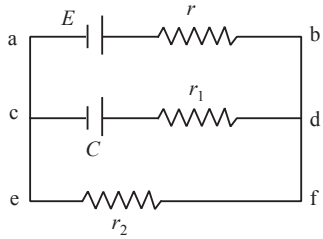
v = speed of the source = $\frac{c}{2}$ (given)

ν = actual frequency of microwaves = 10 GHz (given)

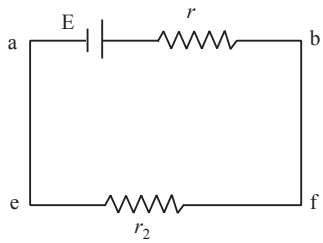
$$\therefore \qquad \nu_a = 10\ \text{GHz} \times \sqrt{\frac{1+\frac{1}{2}}{1-\frac{1}{2}}} = 17.3\ \text{GHz}$$

So the correct option is (a).

28. Refer to the following figure



In the steady state (i.e. when the capacitor is fully charged) no current flows through the branch cd of the circuit. Hence the potential difference between ef = potential difference between cd. The circuit reduces to



Current flowing in the circuit in the steady state is

$$I = \frac{E}{r + r_2}$$

Potential difference between ef is

$$V_{ef} = Ir_2 = \frac{Er_2}{r + r_2}$$

Now potential difference across the capacitor = $V_{\text{ef.}}$. The charge on the capacitor is

$Q = CV_{ef} = \dfrac{CEr_2}{r + r_2}$, which is choice (a).

29. Capacitance of each capacitor is $C = 1\mu\text{F}$. Some of these capacitors are connected in series and some of these series combinations are connected in parallel. Since each capacitor has the same capacitance, and since each capacitor can withstand a maximum potential difference of 300V the minimum number of capacitors that are connected in series is 4. Each capacitor then will have a voltage of $\dfrac{1000\text{V}}{4} = 250\text{V}$ which is safe. The equivalent capacitance of these 4 capacitors connected in series is

$$C_1 = \frac{C}{4} = \frac{1\mu F}{4} = \frac{1}{4}\mu F$$

Let n such series combination be connected in parallel. The total capacitance of the network is $\dfrac{n}{4}\mu F$. This capacitance is required to be $2\mu F$. Thus

$$\frac{n}{4}\mu F = 2\mu F \quad \Rightarrow n = 2 \times 4 = 8$$

Thus 8 parallel combinations of 4 series capacitors are required. Therefore, minimum number of capacitors required = 8 × 4 = 32. So the correct option is (b).

30. Velocity v at any instant of time t is given by

$$v = u + at$$

where u is the initial velocity. For body thrown vertically upwards, u and v are positive but $a = -g$. So

$$v = -gt + u$$

Hence the graph of v versus t is a straight line with a negative slope ($= -g$) and a positive intercept ($+u$). so the correct option is (a). as shown in the following figure.

