UACE MATHEMATICS SAMPLE ITEMS

APPLIED MATHEMATICS

ITEM 1

The Health Department at Kakaire High School recently initiated a wellness monitoring program to assess the nutritional impact of the school lunch menu. To evaluate the current state, they collected weight data from a sample of 40 students. Their goal is to determine whether any dietary adjustments are necessary. According to the policy, if the average weight of the sampled students is below 35 kg, meat will be added to the menu on Tuesdays and Fridays; otherwise, the current meal plan will be maintained. Additionally, the department wants to identify the most common weight range among students using a graphical representation, which will be presented to the school administration. Concerned about rising obesity levels, they also intend to calculate the number of students weighing above 45 kg to include in their official health report.

They managed to collect the following data.

Weight(kgs)	$10-19$	$20-29$	$30-39$	$40-49$	$50-59$
Number of students	2	10	15	8	5

Task:

The Health Department is seeking your assistance to:

(i) find out if they need to change their lunch menu.
(ii) Use a graph to identify the most common weight (mode).
(iii) Determine the number of students who weigh more than 45 kg.

ITEM 2

At an inter-school talent competition, six students Tom, Jane, Peter, Mary, John, and Vic performed in front of a panel of three judges: Judge A (the senior judge), Judge B, and Judge C. Each judge awarded marks out of 50 to every student.

The competition organizers intend to award medals as follows:

Gold for the student in 1st place, Silver for 2nd place and Bronze for 3rd place.

To ensure fairness and consistency, the final rankings will not rely on all three judges' scores. Instead, the organizers have decided that:

- The final scores will be based on the average of the senior judge's scores (Judge A) and one other judge either Judge B or Judge C.
- The second judge will be selected based on who has the highest correlation in scoring with Judge A, as this reflects greater alignment and consistency in their evaluations.

The judges' scores for each student are shown in the table below:

Students	TOM	JANE	PETER	MARY	JOHN	VIC
Judge A	30	50	35	20	48	30
Judge B	40	45	50	45	30	15
Judge C	32	48	40	30	49	28

Task:

Help the organizers to:

(i) Identify the judge with the highest correlation with the senior judge.

(ii) Determine which students will receive the gold, silver, and bronze medals based on the highest average scores from the selected judges.

ITEM 3

Kigulu High School recently enrolled 200 new students into its A-Level science program. During the selection process, the cut-off grade for Mathematics was set higher than that for Biology. The administration recorded data on subject combinations and found that the probability of a student offering either Biology (B), Mathematics (M), or both is 70%, while the probability of a student offering both Biology and Mathematics is 20%. It is also known that the offering Biology and offering mathematics are independent and that probability of students offering biology is bigger than that of those offering mathematics.

The school administration is now interested in understanding student subject preferences. They wish to determine the probability that a randomly selected student offers Biology, offers Mathematics, and offers Biology but not Mathematics. Additionally, they are reviewing whether to lower the mathematics cut-off grade to a Grade B. This decision will be based on whether the number of students offering only Mathematics is less than 100.

Task:

Help the administration determine:

(i) The probability that a student offers Biology

(ii) The probability that a student offers Mathematics

(iii) The probability that a student offers Biology but not Mathematics

(iv) The number of students who offer only Mathematics, and advise on whether to lower the cut-off grade.

ITEM 4

A researcher is studying the distribution of hours spent on homework each week by students in a college. The following frequency distribution is given for 50 students:

Hours of homework per Week	frequency
0-5	6
6-10	12
11-15	14
16-20	10
21-25	5
26-30	3
TOTAL	50

QUESTIONS

a) Estimate the average number of hours spent on homework per week by the students

b) If the researcher wants to identify the middle range of homework hours, which interval would you consider as the range containing the median?
c) If a new group of 20 students is added, where the majority of them fall into the 16-20 hours range, how would that affect the mean homework hours for the entire group of 70 students?
d) Graphically determine the highest mark number of hours spent by the students in doing the home work?

ITEM 5

A career guidance seminar was held at a senior secondary school. Out of 150 students who attended:

- 90 students expressed interest in Science careers (S),
- 60 students were interested in Entrepreneurship (E),
- 25 students were interested in both Science and Entrepreneurship.

A student is picked at random from the seminar attendees.

Tasks:

(a) Represent the information using a Venn diagram.
(b) Using the additional rule of probability, calculate the probability that a randomly chosen student is interested in:

i) Science or Entrepreneurship,
ii) only one of the two career options,
iii) neither Science nor Entrepreneurship.

(c) Use your results to verify the rule: $P(S \cup E) = P(S) + P(E) - P(S \cap E)$

ITEM 6

As a student leader, how would you use this data to advise the school on organizing future seminars? A school sports committee organized trials for athletics and football. Among the 100 students who participated:

- 55 were selected for athletics (A),
- 40 were selected for football (F),
- 20 were selected for both athletics and football.

A student is chosen at random from the group.

Tasks:

(a) Represent the data using a Venn diagram.

Find the probability that the selected student:

(i) was selected for both athletics and football

(ii) was selected for either athletics or football

(iii) was selected for only one of the two activities.

(b) Are the events "selected for athletics" and "selected for football" mutually exclusive? Explain your answer.

Suggest how this information can help the committee improve future sports selections.

ITEM 7

In certain school beginning of term exams results for end of term were recorded as shown in the table below by the class teacher showing the average of each student in all subjects and they were to be depended on to determine whether the modal average mark is less than 50 they should have external facilitators to boost performance

AVERAGE MARKS	Number of Students
Below 10	6
10 and under 25	11
25 and under 35	17
35 and under 45	28
45 and under 50	20
50 and under 65	15
65 and under 80	3

Task;

Using a statistical diagram, determine whether the school should have external help to boast learners' performance.

ITEM 8

In internet cable lying company that lays cable connecting European continent to African continent has plans to roll a 24,000Km cable around a coil on a boat before they set off to lay in the ocean floor. They are using five different machines A, B, C, D and E which are applying a force of 60N, 50N, 80N, 50N and 90N acting in direction $N30^0E$, $N30^0W$, $S50^0E$, $N180^0S$, and $S40^0W$ respectively. If the resultant overall force applied is more than 75N, it would take 8 hours to finish and if below they ought to find better machines that the job faster.

Task:

Using mathematical knowledge help determine and advice whether they should continue with the same machines or change to new machines for the speed of putting the 24,000km cable around.

ITEM 9

Water released from a nuclear reactor, used as a coolant has been continually been monitored by an environmental scientist to monitor its impact on the aquatic life in the nearby lake the acceptably safe temperature would be 15^0C. The results are tabulated as shown below.

Time(seconds)	0	120	240	360	450	600
Water Temp (^0C)	100	80	75	69	54	46

Task:

Help an environmental scientist estimate the:

(i) When the water temperature is acceptably safe for aquatic life to the nearest minute.

(ii) At point when during winter season, the released water turns to ice after it been released, after how long can this happen.

Temperature of water released after 5 minutes provided it's the time limit for survival of aquatic animal food to survive high temperatures

ITEM 10

In **TrHX** pharmaceutical corps' research and development team has been testing new drugs where two drugs X and Y being approved have probabilities basing on the following conditions stated below; $P(X \cup Y)= \frac{17}{24}$, $P(X)= \frac{2}{3}$,$P(Y)= \frac{1}{4}$. The company executives board wants to know after the first internal test what the probabilities that both are allowed ($\boldsymbol{X \cap Y}$), drug X is allowed only $\boldsymbol{P(X \cap Y^1)}$ and neither of the two are allowed $\boldsymbol{P(X^1 \cap Y^1)}$, for proper decision making on more funding or shutting down the program.

Task:

Help executive managers come to a decision to continue funding the development of drug X provided;

i. if chance for both being allowed is less than 0.25 but the one for X only being allowed more than 0.422
ii. . Or shut down the whole program if the chance of neither being accepted is more than 0.25

ITEM 11

The numbers of male and female candidates admitted at a certain university in a certain year to offer different courses A,B,C,D,E,F,G,H,I and J were as follows:

Course	A	B	C	D	E	F	G	H	I	J
Male	66	54	60	70	62	46	74	58	80	58
Female	50	38	54	68	60	32	62	46	70	49

Given the dean of students of this university has been asked to present the data showing the co relationship of the male students (x) and female(y) and comment on the nature relationship I his report to the ministry of gender under Girl child education empowerment. As well the university will receive scholarship grant provided the relationship is significant at 5% level,

Task:

(a) Use a scatter diagram to help the dean of students show the relationship. Given a new course that has been set up with a survey conduct among the current students where only 40 males showed interest, estimate using line of best fit to find the possible number of girls to nearest whole number who would be interested
(b) Determine the whether they would receive a grant provided they the follow Spearman's co-efficient

ITEM 12

In a certain calendar printing factory, the length of each calendar made has been organized into a cumulative frequency distribution as shown below

Length(cm)	<20	<30	<35	<40	<50	<60
Cumulative freq	4	20	32	42	48	50

From market research team, it has been duly determined that the commonest calendar length sold is the one with a median length. And the production team will produce more of these. 50

Task:

(a) Using a cumulative frequency curve help the production team determine the median length they have to produce for the next output units.
(b) As well a certain client has sent instruction to produce for them a length in 80th percentile, but the production team need the actual length to nearest whole to be consider for this client
(c) Determine the standard deviation of the length.

ITEM 13

In a research project conducted by the National Agricultural Research Institute, scientists are studying whether there's a relationship between soil fertility scores and crop yield on various farm plots. Soil fertility is measured on a scale of 0 to 100, and crop yield is measured in kilograms per hectare. A total of 12 sample plots were tested. The team believes that higher soil fertility should result in higher yields, but wants statistical proof. They will take a conclusion if the correlation of the results is moderate or above. Unfortunately, 2 of the plots had incomplete data due to equipment malfunction:

• Plot 4 is missing a crop yield reading, and
• Plot 9 is missing a soil fertility score.

Plot No	1	2	3	4	5	6	7	8	9	10	11	12
Soil fertility	75	60	85	90	55	70	68	88		62	88	66
Crop yield	78	58	92		54	74	72	90	70	60	85	69

Help the scientist;

(a) find the missing values for sample from plot 4 and 9
(b) Can the institute reasonably conclude that higher soil fertility leads to higher yield?

ITEM 14

Mr. Mukasa and family organized a field tour to western Uganda, they hired a caravan for the long journey, they towed a caravan of mass $1000kg$ on the car of mass $800kg$ by means of coupling (light rigid tow bar). On a straight level road, the car exerted a driving force of $1270N$ and if the resistances to the forward motion of the car and caravan was $400N$ and $600N$ respectively. At some point Mr. Mukasa encountered a hill while travelling at $40km/hr$ inclined at $\sin^{-1}\frac{1}{100}$ and plans to overcome it in 2minutes when the driving force and resistances remain constant.

SUPPORT.

Task

Help Mr. Mukasa to determine his acceleration on the level road and distance covered on the inclined road(hill).

ITEM 15

The Human Resource Manager of a cleaning company is in the process of searching for a new staff.

Age(years)	20 –	25 –	40 –	45 –	50 –	55 –
Number of applicants	14	12	7	8	9	0

The Human Resource Manager has failed to interprete the information to be able to make a report to the administrative team of the company.

TASK:

a) Help the Manager to display the data graphically for easy interpretation.
b) From your graph, help the manager identify the modal age of the applicants.
c) By calculation or otherwise, identify the median age of the applicants.

ITEM 16

a. In an experiment to demonstrate the effect of forces on a body, a teacher applied the following forces on a solid metallic block; 5N and 13N forces acting vertically upwards, 11N force acting vertically downwards, 3N and 5N forces acting horizontally to the left and 14N force acting horizontally to the right. The teacher asked the students to determine a single force resulting from the effect of these six forces acting on the block. In his further demonstration, he played a video that involved a monster truck at the Centre of five (5) small cars, each connected to it using a strong cable. The cars pulled apart with forces of 30N, 60N, 100N, 70N and 20N in the directions of 270°, 040°, 120°, 330°, and 225° respectively. The teacher also asked the students to use the system of the above forces to determine a single force with which a monster truck will resultantly move and the direction it'll take.
   TASK:
   As a student of Mechanics, help students to accomplish the tasks in each of the cases above.
b. Two friends are trying to pull a stalled car out of a muddy ditch. One friend applies a force of 9N. The other friend applies a force of QN. The angle between the directions of their ropes is 120^0. The resultant pulling force on the car is measured to be 7N
   Task:
   Determine the magnitude of the unknown force QN that the second friend is applying.

ITEM 17

Amelia's heavy luggage which is at a point, O, is under the action of four forces 1N, 6N,$2\sqrt{3}$ N and 3N acting in the directions given by the bearings 030° 270° 060° and 090° respectively.

She wants the luggage to be moved in the bearing 120° at a constant rate provided by the net force of magnitude $2\sqrt{3}\ N$ but the other four forces couldn't achieve her aim.

TASK:

By writing each of the forces in a vector form, help Amelia achieve her aim by determining the magnitude and direction of the fifth force that can be applied on the luggage.

ITEM 18

In order to improve on the performance of Mathematics, the head of department laid down strategies, which included a test that was administered and the marks were distributed as shown in the table below.

Marks	<20	<26	<35	<45	<55	<60	<68	<80	<95
Cumulative number of learners	0	6	24	66	110	130	154	174	180

Other strategies included;

- A reward from administration to be given to the learner who scores not less than 85 marks
- Remedial lessons to be organized for those who score below average.
- Those to get distinctions were limited to only 16 learners.

The performance is inconsistent if the measure of dispersion of all the marks away from the average mark is more than 10 marks.

TASKS:

a. Help the Head of department to determine;
   i) The number of leaners who are to be rewarded
   ii) The number of learners who are to attend remedial lessons
   iii) The minimum mark of a distinction
b. Were the learners consistent in their performance?

ITEM 19

A leading technology company, **NAMU Tech industries,** manufactures high-capacity rechargeable batteries for use in electric vehicles. During the testing phase of a new production line, the engineers discovered that the lifespan of the batteries (measured in years) was not constant, it varied depending on small temperature fluctuations and material inconsistencies during manufacturing.

To model this behavior, the quality assurance department assumes that the lifespan X of a randomly selected battery follows a continuous probability distribution with a probability density function (p.d.f) defined as;

$$f(x) = \begin{cases} k, & 0 \leq x \leq 1 \\ k, & 1 \leq x \leq 3 \\ k(4-x), & 3 \leq x \leq 4 \\ 0, & otherwise \end{cases}$$

Here k is a positive constant and represents the likelihood that a battery's f(x) lifespan will fall within a small interval around x years.

The company wishes to use this model to estimate;

- The average lifespan of a battery,
- The variation in battery performance, and
- The effect of linear transformations on expected values and variance which are used for cost and warranty calculations.

Tasks

a) Draw a clear well labeled sketch of the function f(x) and hence from sketch, determine the value of the constant k
b) Determine the expected value E(X) representing the mean lifetime of the battery.
c) Determine the variance ,Var(X) of the random variable X.
d) If the company introduces a warranty condition represented by the random variable Y = 2X + 4, Determine E(Y)

ITEM 20

During a national shooting championship, three elite shooters Amina, Bruno and Chris are each assigned to shoot at a target. However, due to varying skill levels and whether conditions, their chances of hitting the target differ.

The probability that Amina hits target is 0.5

The probability that Bruno hits the target is 0.3

The probability that chris hits the target is 0.4

The judging committee must plan who qualifies for the finals, which depends on how many hit the target. If none hit, they must revise training protocols. If only one hits , that shooter qualifies directly. If all three hit, they move to a shootout round. To decide how many targets to set up for the next round and how many judges are needed, the judging committee wants to know the likelihood of each outcome.

Tasks

Help the judging committee determine the probabilities that:

a) All three shooters will hit the target.
b) Neither Amina nor Chris will hit the target.
c) Only one shooter will hit the target.
d) None of the shooters will hit the target.

ITEM 21

A popular food delivery app, **"Quick Bite"**, introduces a new promotional feature called "**Lucky Spin"** for its premium users. After every order, users get one spin of a virtual wheel divided into 5 segments, labeled level 1 to level 5.

The reward for each level is a discount coupon, but the game is rigged to make higher levels (better discounts) less likely in a very specific way. The probability of the spinner landing on a particular level is directly proportional to the level number itself. This means level 5 is the hardest to get , even though it has the best reward.

The probability distribution for the level X a user lands on is;

Level (X)	1	2	3	4	5
P(X=x)	K	2k	3k	4k	5k

Tasks

You are a data analyst at Quick Bite. Your manager wants a complete report on the "**Lucky Spin**" mechanic to understand user rewards and its financial impact. You are required to find;

a. The normalization constant K.

b. The average level a user can expect to land on over a large number of Spins.
c. Determine how much variation exists from the average level (Standard deviation)
d. The probability of a Mid-Tier reward ($p(2 < x \leq 4)$)

ITEM 22

The numbers of male and female candidates admitted at a certain university in a certain year to offer different courses A,B,C,D,E,F,G,H,I and J were as follows:

Course	A	B	C	D	E	F	G	H	I	J
Male	66	54	60	70	62	46	74	58	80	58
Female	50	38	54	68	60	32	62	46	70	49

Given the dean of students of this university has been asked to present the data showing the co relationship of the male students (x) and female(y) and comment on the nature relationship I his report to the ministry of gender under Girl child education empowerment. As well the university will receive scholarship grant provided the relationship is significant at 5% level,

Task:

a) Use a scatter diagram to help the dean of students show the relationship. Given a new course that has been set up with a survey conduct among the current students where only 40 males showed interest, estimate using line of best fit to find the possible number of girls to nearest whole number who would be interested
b) Determine the whether they would receive a grant provided that follow Spearman's co-efficient

ITEM 23

A deadly virus attacked a certain country. The initial number of male and female people infected is 14 and 35 respectively. However, this initial number of sick people is thought to be increasing at a rate 30% and 69% per week. As the minister of health was addressing a live presentation on the YouTube social media handle, one individual asked through the comment section "What will be the probability of finding a woman contracted by the virus over thousand patients?"

Task.

Kindly response to the question in the comment section.

ITEM 24

Emily is a high school student working on a project that combines her math and science classes. She's analysing data from an experiment where she measured the effect of sunlight on plant growth . The data points are

- When she exposes plants to no hours of sunlight, she notices 9.2 cm of plant growth

- When she exposes plants to 8 hours of sunlight, she notices 6.0 cm of plant growth

- When she exposes plants to 12 hours of sunlight, she notices 4.4 cm of plant growth

- When she exposes plants to 20 hours of sunlight, she notices 1.5 cm of plant growth

Emily needs to use this data to make predictions for her math project and understand the relationship between sunlight and plant growth for her science experiment. She needs to:

Task.

i) Find the amount of sunlight needed to achieve a plant growth 0.8cm.
ii) Find the expected plant growth when the plant receives 5hours of sunlight. However she lucks some advance knowledge of mathematics and she needs you hand in this.

ITEM 25

Alex, a civil engineering student, is working on a project to design a new suspension bridge. While analysing the structural integrity of the bridge, Alex encountered a critical issue. The stress on a key support beam is modelled by the equation $x^3 - 3x + 1 = 0$, and Alex needs to find the potential stress points to ensure the bridge's stability. However, Alex is struggling to solve the equation and is worried about meeting the project deadline. Alex's task is to:

a) Graphically analyse the model to determine the number of potential stress points in the interval [-2, 2] and estimate their values.
b) To find the smallest stress point (root) of the model with high precision. Alex has tried but can't seem to get the calculations right and needs help to iterate until the result is accurate to four significant figures.

Task.

Help Alex resolve this critical issue and ensure the bridge's stability

ITEM 26

You're a student who's landed an internship at a bulb manufacturing company during your Term 2 holiday. As part of the quality control team, you've been tasked with analyzing the production data to determine the likelihood of certain events occurring when a bulb is randomly selected from the production line.

The company produces bulbs using three machines, A, B, and C, with production ratios of 30% for A, 60% for B, and 10% for C. Each machine produces bulbs with different coloration rates: 25% of bulbs from machine A are colored, 30% from machine B are colored, and 70% from machine C are colored.

Task.

Your team lead asks you to help ascertain chances of the following events:

i. A randomly selected bulb is not colored
ii. A colored bulb is produced by machine B
iii. Provide insights into the production process.

ITEM 27

John is a commuter who travels to work using different modes of transportation, including cars, bikes, and buses. Based on his past experiences, the likelihood of him using these modes of transportation are 50% for cars,1/6 for bikes, and 1/3 for buses.

However, John's punctuality is affected by the mode of transportation he chooses. When he drives a car, the likelihood of being late is 3/8. When he rides a bike, the chance of being late increases to 4/5. On the other hand, when he takes the bus, the likelihood of being late is relatively low at 1/8

John's manager wants to understand his commute patterns and the likelihood of him being late for work. Specifically, the manager wants to know:

a) The overall chance that John is late for work on any given day.
b) If he randomly selectes120 days are selected, what could be the chance that John will be late on at least 70 days but fewer than 85 days?.

Unfortunately the manager lucks enough mathematics to analyse this information and he has tasked you as the school's top student of mathematics.

Task.

Help the manager

ITEM 28

A factory in Jinja produces light bulbs using three machines: Machine A, Machine B, and Machine C.

- Machine A produces 40% of the total output, and 5% of its bulbs are defective.
- Machine B produces 35% of the total output, and 3% of its bulbs are defective.
- Machine C produces 25% of the total output and 2% of its bulbs are defective. A bulb is selected at random from the factory's output.

Task

a) Determine the probability that the selected bulb was produced by Machine A AND is defective. Similarly, calculate the probabilities for Machine B being defective and Machine C being defective.
b) Using the results from a), determine the overall probability that a randomly selected bulb from the factory's output is defective.
c) Given that the selected bulb is found to be defective, calculate the probability that it was produced by Machine B.

ITEM 29

In a certain calendar printing factory, the length of each calendar made has been organized into a cumulative frequency distribution as shown below

Length(cm)	<20	<30	<35	<40	<50	<60

Cumulative freq	4	20	32	42	48	50

From market research team, it has been duly determined that the commonest calendar length sold is the one with a median length. And the production team will produce more of these. 50

Task:

a) Using a cumulative frequency curve help the production team determine the median length they have to produce for the next output units.
b) As well a certain client has sent instruction to produce for them a length in 80th percentile, but the production team need the actual length to nearest whole to be consider for this client
c) Determine the standard deviation of the length.

ITEM 30

In rural areas of Uganda, like kasese or Bundibugyo, students in s5 are studying statistics to understand patterns of water collection. A continuous random variable X represents the time (in hours) villagers spend walking to collect water from a nearby stream. The probability density function of X is given by:

$$f(x) = \begin{cases} \frac{1}{4}x(4-x^2); & 0 \le x \le 2 \\ 0; & else\ where \end{cases}$$

Real-Life Issue:

In many Ugandan rural communities, access to clean water is a challenge. Villagers often walk long distances to fetch water, affecting their time for other activities like farming or studying.

Understanding the distribution of time spent on water collection can help in planning community water projects.

Task

Using the knowledge of principal mathematics determine;

a) The most common time spent walking to collect water.
b) The time by which half of the villagers have collected water.
c) The time by which 60% of villagers have collected water.

ITEM 31

The government of Uganda is conducting a demographic study in a specific tribal clan in Fort Portal city where the probability of a baby being a boy is 0.4. The clan has a tradition of large families, and the government is analyzing a random sample of 10 children from different households to understand gender distribution trends.

Task

As a mathematics student, you have been approached, help the government to determine the expected number of girls in this random sample of 10 children and the chance of getting at most two girls in this sample of 10 children.

ITEM 32

You are a healthcare administrator responsible for resource allocation at a regional hospital. The daily number of patients visiting the hospital is uniformly distributed between 150 and 210, and you need to ensure adequate staffing and supplies based on this data.

Tasks;

(i) Write down the probability distribution function (p.d.f.) of the number of patients visiting the hospital.

(ii) Find the probability that between 170 and 194 patients visit the hospital on a particular day, to assist in scheduling staff effectively.

ITEM 33

A hydraulic engineer was tasked to estimate the average water flow rate through a narrow channel, where the flow rate can be modeled by the function $f(x) = \frac{1}{2x+1}\ dx$ with x representing the channel width in meters. The flow rate needs to be approximated over a range from 0.1 m to 0.5 m to optimize irrigation system design. He has decided to use the trapezium rule with 6 ordinates for the approximation.

Tasks:

a) Use the trapezium rule with 6 ordinates to estimate $\int_{0.1}^{0.5} f(x)\ dx$ correct to three significant figures.

b) Evaluate the exact value of $\int_{0.1}^{0.5} f(x)\ dx$, correct to three significant figures.

c) (i) Determine the percentage error in the estimation in (a) above, correct to two decimal places.

(iii) Suggest how the percentage error may be reduced.

ITEM 34

A group of learners were taken to a certain factory in Jinja to fully understand the concept of probabilities. The factory produces light bulbs using three machines: Machine A, Machine B, and Machine C. Machine A produces 40% of the total output, and 5% of its bulbs are defective. Machine B produces 35% of the total output, and 3% of its bulbs are defective. Machine C produces 25% of the total output, and 2% of its bulbs are defective. A bulb selected at random from the factory's output. Later, these learners were given asked to use the knowledge to also analyse the given problem. Two boxes $M\ and\ N$ contain 4 red and 3 blue pens and 3 blue and 4 red pens respectively. A box is selected at random and from it a pen is drawn and placed into the second box, the second box is shaken and from it the pen is drawn and placed into the first box.

Task:

(a) Determine the probability that the selected bulb was produced by Machine A and is defective.

(b) Given that the selected bulb is found to be defective, calculate the probability that it was produced by machine B.

(c) Determine the probability that the pen selected from the first box is red.

ITEM 35

Two children are pulling a toy wagon across a smooth playground surface using two ropes. One child pulls with a force of 12N, and the other pulls with a force of 9N. The ropes make an angle of 60° with each other. As a result of their combined effort, the wagon starts to move with an acceleration of 3.65 m/s².

Task

Determine the mass of the wagon

ITEM 36

A delivery worker places a 1 kg parcel on a rough ramp inclined at 30° to the horizontal. The coefficient of friction between the parcel and the ramp is 0.25. To

avoid any accidents, the worker needs to apply a force parallel to the ramp to keep the parcel from sliding.

Task

Determine the minimum force that the worker must apply to just prevent the parcel from sliding down

ITEM 37

The Director of EXODUS COLLEGE SCHOOL wants to procure school uniform for New students at the beginning of the Year but students' heights are still the issue. He has directed his Uniform Master to carry out an investigation to get the right picture about the student's height. 20 students whose average height is 160cm with standard deviation of 4cm and 30 girls whose average height is 155 cm with standard deviation of 3.5 cm were selected.

TASK:

As a student with knowledge of statistics, help the uniform master come up with the combined mean and standard deviation for the whole group of 50 students so that the master can advise the Director on students' heights before ordering for Uniforms of new students.

ITEM 38

At a certain school, Eighty S.5 students sat for their End of Term III Mathematics Examinations. However, their performance in the previous End of Term II Mathematics Examinations was no satisfactory. The Headteacher warned that if their results did not show improvement in comparison to Term II, he would call for a Parents-teachers' meeting to discuss the way forward before being promoted to S.6. additionally, the Headteacher, promised to reward students whose pass mark was 75% and above. The performance statistics for End of Term II were as follows.

Average score	**60.5**
Minimum score	**62.5**

Below are the students' scores for the End of term II

68, 64, 64, 75, 50, 53, 70, 67, 66, 63, 66, 72, 60, 59, 66, 62, 71, 55, 66, 61, 53, 58, 55, 57, 52, 55, 51, 66, 74, 76, 74, 64, 56, 75, 74, 59, 65, 76, 69, 58, 57, 70, 69, 62, 58, 58, 60, 64, 51, 67, 63, 75, 63, 75, 51, 73, 54, 62, 62, 58, 76, 62, 72, 67, 69, 72, 66, 64, 65, 51, 70, 61, 52, 68, 50, 61, 57, 61, 70, 52.

TASK:

a) Analyze the End of Term III results using equal class widths of starting with a class of 50 to 54. Based on your analysis, provide a recommendation to the headteacher on whether a Parents-teachers' meeting is necessary.

b) Based on graphical analysis, how many students qualify for the Headteachers reward?

ITEM 39

A-Z Photographers Limited specialize in printing and framing High - Quality photographs. The company produced 50 photo frames with a range of Heights (in inches). From the production Manager's records, the number of photo frames produced in a certain month were as follows.

Height (inches)	10-20	20-30	30-35	35-40	40-50	50-60
Number of photo frames	4	20	32	42	48	50

TASK:

a) Help the production manager to represent the above data on a suitable graph and estimate the popular height of the photo frames, correct to the nearest inch.

b) Based on calculations, help the production manager to determine the standard deviation
of the heights of the photo frames produced in the month.

ITEM 40

Amelia's heavy luggage which is at a point, O, is under the action of four forces 1N, 6N,$2\sqrt{3}$N and 3N acting in the directions given by the bearings 030° 270° 060° and 090° respectively.

She wants the luggage to be moved in the bearing 120° at a constant rate provided by the

net force of magnitude $2\sqrt{3}N$ but the other four forces couldn't achieve her aim.

TASK:

By writing each of the forces in a vector form, help Amelia achieve her aim by determining the magnitude and direction of the fifth force that can be applied on the luggage.

ITEM 41

A deadly virus attacked a certain country. The initial number of male and female people infected is 14 and 35 respectively. However, this initial number of sick people is thought to be increasing at a rate 30% and 69% per week. As the minister of health was addressing a live presentation on the YouTube social media handle, one individual asked through the comment section "What will be the probability of finding a woman contracted by the virus over thousand patients?"

Task.

Kindly response to the question in the comment section.

ITEM 42

Joram is a zoo Keeper at Entebbe Zoo. On a random day, He woke up and found that one of the lions in the zoo had died. However, he tried to closely look at the lion, and it seemed to have died that day. He had to contact officials from the Uganda National Wildlife Conservation Authority (UNWCA). After two months, the manager of UNWCA came with his team to the zoo and they weighed the dead lion before relocating it for Post-Mortem and its mass was 108kg.

At the Headquarters of UNWCA, the Wildlife Medical doctor had gone for a vacation in Dubai so they had to wait for him for further 3 months. When the Wildlife Medical doctor returned at the Headquarters, He had to obtain the mass of the dead elephant as part of the PostMortem and the dead lion weighed 52 kg.

However, Joram wanted to know the mass of the lion before it died from the medical team at the Headquarters but all they could send him was the above information.

TASK:

a) Help the zoo Keeper to determine the mass of the lion before it died for proper record keeping.
b) What would be the mass of the dead lion if the Medical Examiner would have stayed on vacation for further;
c) After how many weeks was the dead lion weighing 42.7kg?

ITEM 43

In a research project conducted by the National Agricultural Research Institute, scientists are studying whether there's a relationship between soil fertility scores and crop yield on various farm plots.

Soil fertility is measured on a scale of 0 to 100, and crop yield is measured in kilograms per hectare. A total of 12 sample plots were tested. The team believes that higher soil fertility should result in higher yields, but wants statistical proof. They will take a conclusion if the correlation of the results is moderate or above.

Unfortunately, 2 of the plots had incomplete data due to equipment malfunction: Plot 4 is missing a crop yield reading, and Plot 9 is missing a soil fertility score.

Plot no.	1	2	3	4	5	6	7	8	9	10	11	12
Soil fertility	75	60	85	90	55	70	68	88		62	80	66
Crop yield	78	58	92		54	74	72	90	70	60	85	69

TASK.

Help the scientist

(a) find the missing values for sample from plot 4 and 9

(b) Can the institute reasonably conclude that higher soil fertility leads to higher yield?

ITEM 44

At the start of the school term, a Physical Education (P.E) teacher decided to record the weights of 50 students in Senior Three as part of the school health monitoring program. If the average weight turns out to be below 40 kg, the school

administration plans to add eggs to the students' breakfast menu to help improve their nutrition. The teacher also wants to know the most common weight among the students. Additionally, if the common weight is above 51kgs, the teacher needs to find out how many students are above this common weight so that they can be put on morning jogging aimed at maintaining healthy body mass.
The weights (in kilograms) were grouped as follows:

Age	$30-39$	$40-49$	$50-59$	$60-69$	$70-79$
No. of patients	8	12	15	10	5

Task:

help the teacher:

(a) find out if they need to introduce eggs on breakfast menu.

(b) if they need to put some students on morning jogging, if yes how many?

ITEM 45

Mr. Mukasa and family organized a field tour to western Uganda, they hired a caravan for the long journey, they towed a caravan of mass $1000kg$ on the car of mass $800kg$ by means of coupling (light rigid tow bar).

On a straight level road, the car exerted a driving force of $1270N$ and if the resistances to the forward motion of the car and caravan was $400N$ and $600N$ respectively.

At some point Mr. Mukasa encountered a hill while travelling at $40km/hr$ inclined at $\sin^{-1}(\frac{1}{1000})$ and plans to overcome it in 2minutes when the driving force and resistances remain constant.

Task

Help Mr. Mukasa to determine his acceleration on the level road and distance covered on the inclined road(hill).

ITEM 46

VJ KB wants to construct a regular pentagonal film hall ABCDE of sides 4m. he wishes to put the walls under tensional forces to ensure its stability and durability. To do achieve this, four forces of magnitude: 2N, 3N, 5N and 7N have to be applied alongside AB, BC, CD and EB respectively, With AB being the horizontal. He also wants to know the number of bricks he will have to buy in order to have its foundation completed. VJ KB was told that for a similar foundation on area of 5m2 requires 800 bricks to be completed.

TASK:

a. Write each force as vector.
b. Determine the equivalent force VJ KB would have applied that would do the same work as the four applied forces hence find its direction.
c. Help him to determine the number of bricks required to have his foundation complete.

ITEM 47

A man is studying the motion of two objects connected by a string. He places a 5kg block on a smooth inclined plane that makes an angle 300 with the horizontal. The block is connected to a 3kg mass hanging freely by a light inextensible string that passes over a smooth pulley. When the system is released from rest, the block started moving up the inclined plane and the hanging mass starts moving downwards. The mass hits the ground after 4 seconds.

TASK:

a. Draw a diagram showing all the forces acting on the masses.
b. Find the acceleration of the motion and the tension in the string.
c. Find the speed with which the hanging mass hits the ground.
d. Determine the magnitude and the direction of the reaction force exerted on the string by the pulley.

ITEM 48

In an experiment to study the equilibrium and vertical motion of bodies, Okello fixed a stone of mass 30kg at the lower end of a light inextensible string whose other end he fixed on a ceiling leaving the stone hanging freely in the vertical plane, he then pulled the stone in the direction that is normal to the string which kept the stone equilibrium with the string inclined at 300 to the vertical.

He later threw the stone vertically upwards with a velocity of 16m/s from a point, H metres above the ground level. The stone later hit the ground after 4 seconds.

TASK:

a. Help Okello determine the force he applied that kept the system in equilibrium and the tension in the string.
b. Find the value of H and the speed with which the stone hits the ground.

ITEM 49

A logistic manager at a local depot in fort portal needs to move a 200kg box of electronics across a rough concrete floor to a new storage location, the coefficient of static friction between the box and the floor is 0.5. to minimize the effort and prevent damage, a worker will use a light rope attached to the box to pull it, the manager needs to know the minimum tension required in the rope to just get the box moving under two different conditions. The worker pulls the box at angle of 30 above the horizontal The worker pulls the box at an angle of 30 below the horizontal,

TASK.

Determine the tension in the string in both cases and advise the manager on the most efficient method to use when pulling the box.

ITEM 50

Two market vendors, Aisha and Ben, operating in Owino Market, Kampala, recorded their daily sales (in thousands of UGX) over a period of 30 days. The data is summarized below:

Aisha: Mean Sale = 150, Standard Deviation = 25

Ben: Mean Sale = 160, Standard Deviation = 40

Tasks:

a) Determine which vendor has higher average daily sales.
b) Calculate the coefficient of variation for both Aisha and Ben.
c) Using the coefficient of variation, determine whose sales are relatively more consistent.

   Justify your answer.

ITEM 51

An experiment consists of removing 2 sweets one at a time without replacement from a box containing 3 red and 4 blue sweets.

Task:

(a) If A is the event that both sweets picked are of the same colour, find the probability that event A occurred.

(b) If the experiment is repeated 20 times, find the probability that event A occurred

(i) between 20 and 35 times.

(ii) atleast 25 times

ITEM 52

A system consists of a fixed pulley B and a movable pulley A . A light, in extensible string passes over pulley B and curves pulley A on one end and a particle of mass 6kg on the other. A second, similar string passes over pulley A and carries particles of mass 2kg and 4kg. If the pulleys are light and smooth,

Tasks:

Find the;

(a) Tensions in the strings.

(b) Accelerations of the three masses.

ITEM 53

At Buganda Road Court any of the three verdicts "**guilty**", "**not guilty**" and "**not proven**" are tried. Of all the cases tried by the court **70%** of all these verdicts are guilty, **20%** are not guilty and **10%** are not proven. When the court's verdict is guilty the probability that the accused is really innocent is **0.05** while for not guilty and not proven, the probabilities of innocents are **0.95** and **0.25** respectively.

Task:

Find the probability that an innocent person will be found guilty.

ITEM 54

A cooperative society of farmers in Luwero recorded the cassava yield (in tonnes per hectare) from 50 small plots. The data is grouped as follows:

Yield (Tonnes/Hectare)	**Number of Plots (Frequency)**

5 - < 10	6
10 - < 15	10
15 - < 20	15
20 - < 25	11
25 - < 35	8

Tasks:

a) Construct a histogram to represent this data.

b) Using the histogram, estimate the modal yield of cassava per hectare.

c) Calculate an estimate of the mean yield and the standard deviation of the yield for these plots.

ITEM 55

The scores of 80 Senior Five students in a Mathematics mock exam at a school in Arua are summarised in the following cumulative frequency table:

Score (x)	Cumulative Frequency
$x \leq 20$	5
$x \leq 30$	15
$x \leq 40$	35
$x \leq 50$	55
$x \leq 60$	70
$x \leq 70$	77
$x \leq 80$	80

Tasks:

a) Represent the data on a cumulative frequency curve (ogive) to represent this data and use it to estimate:

   i) The median score.

   ii) The interquartile range of the scores.

iii) The 80th percentile score.

b) If the pass mark was set at 45 marks, estimate from your ogive the number of students who passed the exam.

c) Explain what the interquartile range tells you about the spread of the students' scores.

ITEM 56

An agricultural officer in the Bugisu region collects data on the annual rainfall (in mm) and the coffee yield (in kg per tree) for 8 different farms over the past year.

Rainfall (mm), x	Yield (kg/tree), y
1200	2.5
1400	3.0
1000	2.0
1600	3.2
1800	3.5
1100	2.2
1500	3.1
1300	2.8

Tasks:

a) Construct a scatter diagram to visually represent the relationship between rainfall and coffee yield.

b) Based on the scatter diagram, describe the type of correlation you observe between rainfall and yield.

c) By ranking the data for both rainfall (x) and yield (y), calculate Spearman's rank correlation coefficient.

d) Interpret the value of Spearman's rank correlation coefficient you calculated in the context of rainfall and coffee yield in this region. Does it support your observation from the scatter diagram?

ITEM 57

A teacher at Exodus College School wants to investigate if there's a relationship between the average number of hours students spend studying per week and their score on a recent Physics test. Data for 7 students is collected:

Study Hours/Week (x)	Test Score (y)
5	65
8	75
2	50
10	85
4	60
12	90
6	72

Tasks:

a) Help the teacher to represent this data on a scatter diagram.

b) Visually, draw a line of best fit through the points on your scatter diagram.

c) Comment on the apparent relationship between study hours and test scores based on your diagram and line of best fit.

Would it be reasonable to use this relationship to predict the score of a student who studies for 20 hours a week? Explain your reasoning, considering the limitations of extrapolation.

ITEM 58

An economics student is researching the relationship between the average weekly price of a bunch of Matooke (in UGX) in Nakasero market and the estimated quantity demanded (in hundreds of bunches). Data over 6 weeks is collected:

Price (UGX), P	Quantity (hundreds), Q
5000	80
6000	70

4500	90
7000	60
5500	75
Price (UGX), P	Quantity (hundreds), Q
6500	68

Tasks:

a) Calculate the Spearman's rank correlation coefficient between the price and quantity demanded.

b) Interpret the calculated correlation coefficient. Does it align with typical economic principles of demand?

c) Plot a scatter diagram for the Price (P) vs Quantity (Q). Does the visual pattern support the calculated correlation?

ITEM 59

Two farm workers, Okello and Lanyero, are pulling a heavy sack of maize (mass 80 kg) across level ground in a Kireka warehouse. Okello pulls with a force of 300 N at an angle of 20° above the horizontal. Lanyero pulls with a force of 250 N at an angle of 15° above the horizontal, in the same direction as Okello. The coefficient of kinetic friction between the sack and the ground is

0.3. (Assume $g = 9.8\ /s^2$).

Tasks:

a) Represent all the forces acting on the sack on a diagram.

b) Resolve the forces applied by Okello and Lanyero into horizontal and vertical components.

c) Calculate the total upward vertical component from the workers' pulls and hence determine the Normal Reaction force exerted by the ground on the sack.

d) Calculate the maximum possible frictional force and the total horizontal component of the pulling forces hence determine the net horizontal force acting on the sack.

ITEM 60

In a mechanics lab, a block A of mass 5 kg rests on a rough inclined plane angled at 30° to the horizontal. The coefficient of kinetic friction between block A and the plane is 0.2. Block A is connected by a light inextensible string passing over a smooth pulley at the top of the incline to a block B of mass 3 kg, which hangs freely. The system is released from rest. (Assume $g = 9.8 m/s^2$).

Tasks:

a) Illustrate the forces acting on block A and block B on separate diagrams.

b) For block A, resolve its weight into components parallel and perpendicular to the inclined plane. Calculate the normal reaction force on block A.

c) Determine the frictional force acting on block A as it slides (assume it slides up the plane initially, if unsure, calculate net force without friction first to determine direction).

d) Apply Newton's Second Law to both block A and block B to formulate two simultaneous equations involving the acceleration (a) of the system and the tension (T) in the string.

Solve these equations to find the values of a and T.

ITEM 61

A lorry of mass 5000 kg is parked on a road in Kisoro inclined at an angle θ to the horizontal, where $\sin \theta = 0.1$. The coefficient of static friction between the lorry's tyres and the road is 0.4.

The driver has applied the handbrake. We want to determine if the lorry will remain stationary.

(Assume $g = 9.8\ m/s^2$).

Tasks:

a) Come up with a diagram showing the forces acting on the lorry assuming it is about to slide down the slope.

b) Resolve the weight of the lorry into components parallel and perpendicular to the road surface, hence find the Normal Reaction force acting on it.

c) Determine the maximum possible static frictional force that can be exerted by the road on the tyres ($F_{max} = \mu_s N$). Compare this maximum friction with the component of the lorry's weight acting down the slope. Establish wether lorry will remain stationary or slide down. Justify your conclusion.

ITEM 62

In a certain region of Uganda, it is estimated that 2% of the population has a particular disease. A medical test is developed to detect the disease. The test is not perfect:

If a person has the disease, the test correctly gives a positive result 95% of the time (Sensitivity).

If a person does not have the disease, the test correctly gives a negative result 90% of the time (Specificity). A person from the region is selected at random and tested.

Tasks:

a) Construct a tree diagram and use it to calculate the overall probability that a randomly selected person tests positive.

b) Using Bayes' Theorem, determine the probability that a person actually has the disease given that they tested positive.

c) Interpret your result from b) above. What does this tell you about the reliability of a positive test result in this scenario?

ITEM 63

A nursery school teacher wants to understand whether there is a relationship between her pupils' reading and writing abilities. She believes that pupils who perform well in reading may also perform well in writing, but she wants to use data to confirm this before designing a combined reading-writing support program. She records the following scores for seven pupils:

Pupil	A	B	C	D	E	F	G
Reading Score	63	81	73	23	33	41	53
Writing score	77	69	84	61	58	62	69

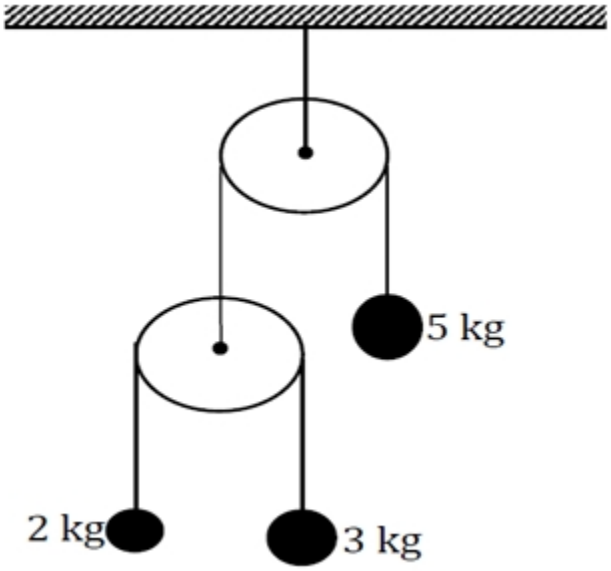
TASK

Help the teacher to understand the relationship between reading and writing

ITEM 64

During a school project, a science club sets up a compound pulley system to study motion and tension. In their set up, a light inextensible rope passes over a smooth fixed pulley A. One end holds a 5 kg toolbox, while the other end is attached to a smooth pulley B. Another rope runs over pulley B, supporting two metals of 3 kg and 2 kg.

Support;



Task :

(a) Help the club to determine the acceleration of pulley B

(b) Enable the club members know the tension in each rope.

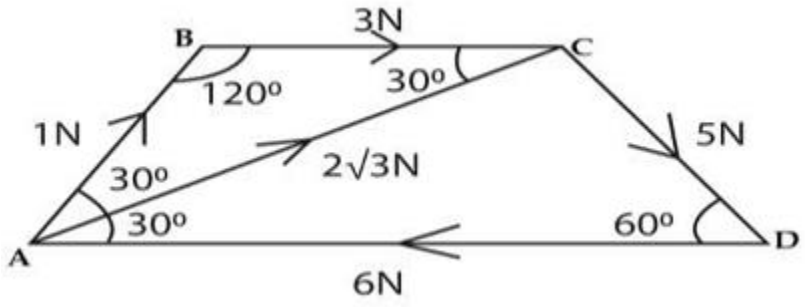
ITEM 65

A technician is testing a metal frame shaped like a trapezium ABCD, which is part of a foldable stand for holding a solar panel. The lengths of the sides are:

AB = 2 meters, and AD = DC = CB = 1 meter.

To check how strong the frame is, different forces are applied to it. These forces have magnitudes of 1 N, 3 N, 5 N, 6 N, and $2\sqrt{3}$ N.

Support:



Task

Help the technician to determine the resultant force on the frame

ITEM 66

Three suppliers A, B and C produce respectively 40%, 25% and 35% of the total number of a certain component used by a machine manufacturer. The percentages of faulty components in each supplier's output are 5% ,3% and 4% respectively.

Task:

Find the probability that a component selected at random is faulty.

ITEM 67

Workers are trying to hold a heavy crate stationary on a rough inclined ramp. It weighs 250N and the ramp is inclined at angle 300 to the horizontal. The coefficient of friction between the ramp and the plane is 0.25. the workers need to know the least force P, that they must apply to the crate to prevent it from sliding down. If they can generate a maximum force of 68.5N.

Task:

Will the force they are able generate be enough to hold the crate.

ITEM 68

Woofer manufacturing plant measures the average length(L) and width(w) of each motherboard inserted inside each woofer produced to be L = 14.50cm and w = 7.5cm. the head of assembly plant wants to the maximum and minimum Area for each component used.

The chief analyst for the company claims that for input x required for motherboard alignment, its Newton Raphson formula for approximating the output function $f(x) = 2x^2 - 6x - 5$ is given by $x_{(n+1)} = \frac{x^2{}_n+3}{4x_n-6} for\ n$=0,1,2,…

Task:

(a) Using your knowledge of errors to determine the range in which areas for each motherboard made lies. (5 scores)

(b) Show the claims of the chief analyst holds and hence taking x = 3.4, use it to find the next two approximations to the root to two decimal places.

ITEM 69

In school Basketball three-point shoot-out. There are three participants with chance of each hitting a three-point basket as follows Chance for Alvin is 0.56, Chance for kizito is 0. 64 and the chance for Marvin is 0.35.

The head coach for the school basketball wants to know the chance that none hits, only one hits, only two hits and at least one hits

Task:

Use your knowledge of probability to determine the probability;

(a) Only one is able to hit a three pointer

(b) Only two are able to hit a three pointer

(c) All them are miss a three pointer

(d) At least one is able to hit a three pointer.

ITEM 70

Over a period of time, Nancy finds that on a long-distance flight, he flies economy class on 82% of the flights and the rest of the flights, he flies first class. When he flies economy class, the probability that he gets a good night sleep is $\boldsymbol{x}$. when he flies first class, the probability that he gets a goodnight sleep is 0.9. The probability that Nancy gets a good night sleep on a random flight is 0.285.

Nancy also has two boxes A and B, box A contains 1 red ball, 3 green balls and 1 blue ball. Box B contains 2 red balls, 1 green ball and 2 blue balls. A balanced die

is thrown and if the throw is a six, box A is chosen otherwise box B is chosen. Nancy picked a ball at random from the chosen box.

TASK:

a) Help Nancy to find the value of $\boldsymbol{x}$.
b) Given on a particular flight, Nancy does not get a good night sleep, find the probability that he is flying economy class.
c) Given that the ball Nancy picked was green, find the probability that the ball came from box A.

ITEM 71

VJ KB wants to construct a regular pentagonal film hall ABCDE of sides 4m. he wishes to put the walls under tensional forces to ensure its stability and durability.

To do achieve this, four forces of magnitude: 2N, 3N, 5N and 7N have to be applied along side AB, BC, CD and EB respectively, With AB being the horizontal. He also wants to know the number of bricks he will have to buy in order to have its foundation completed. VJ KB was told that for a similar foundation on area of 5m2 requires 800 bricks to be completed.

TASK:

a) Write each force as vector.
b) Determine the equivalent force VJ KB would have applied that would do the same work as the four applied forces hence find its direction.
c) Help him to determine the number of bricks required to have his foundation complete.

ITEM 72

Two examination sets were done by 10 students and their scores in the two exams were as in the table below.

Students	A	B	C	D	E	F	G	H	I	J
$Test_1$	60	30	88	71	16	22	47	55	15	60
$Test_2$	55	40	77	59	25	20	50	50	30	49

- The report about the above results was needed where the relationship between the tests was to be established.

- The results were to be considered valid if the magnitude of Spearman's rank correlation coefficient is greater than 0.90.
- Apio who missed both tests due to sickness was given $test_1$ but not $test_2$, and the student was able to score 80% in this $test_1$.

TASK:

a) Determine the relationship between the results hence establish whether the examination results were valid or not.
b) Without drawing a scatter diagram, accurately obtain the equation of line of the best fit and use it to estimate the marks Apio would have got if she sat for $test_2$

ITEM 73

In a certain calendar printing factory, the length of each calendar made has been organized into a cumulative frequency distribution as shown below.

Length (cm)	<20	<30	<35	<40	<50	<60
Cumulative freq	4	20	32	42	48	50

From the market research team, it has been duly determined that the commonest calendar length sold is the one whose length is at least a half the lengths and the production team will produce more of those.

The factory will also produce more calendars of the mean length in the next production plan if the measure of dispersion from this mean length is less than 5.0

TASK:

a) Help the production team to determine the length of a calendar whose length is at least a half of the lengths.
b) Determine the mean length and mathematically suggest whether the factory should produce more calendar of this length or not in the next production plan.

ITEM 74

Tom is trying to calculate the Z – score for statistical analysis. The formula he needs to use is $= \frac{Y}{X}$. However, he made a few mistakes in his calculations.

Tom's calculations were as follows:

Y = 12+0.5 (error due to rounding off)

X = 4 – 0.2 (error due to measurement)

Tom was tasked to find his relative absolute error but before that, he had to prove that ***relative absolute*** $\frac{|\partial x|}{X} + \frac{\partial y}{Y}$.

Tom's father is a teacher and has a faulty computer which uses to print student's marks on report cards. When the teacher fed 60%, the computer printed 40% and instead of 50%, it printed 30%.

TASK:

a) Help Tom to derive the expression for relative absolute error, hence find the percentage of his relative absolute error.
b) Find the true mark the teacher entered if the computer printed 37%
c) If the teacher enters 42%, determine the mark the computer will print.

ITEM 75

At one of the roadsides in Bombo, town engineers are to mount a regular hexagonal aluminium billboard ABCDEF for smart traffic signs. The frame is to be laid with AB perfect horizontal for proper alignment. Steady pulls are to be applied along the sides to tether it from strong winds, 7N along AB, 5N along BC, 3N along DE, 8N along FE, 10N along FC and 6N along AD. The engineers will have to add an extra force X newton if either the resultant of the six forces above exceeds 12N or if its direction is not approximately $N30^0$ E.

Later on, the engineers wanted to test how the direction of the wind will affect the placement of the billboard, while driving their car northwards at $40kmh^{-1}$, the wind seems to come from $N60^0$ E at a speed of $50kmh^{-1}$. The actual speed of the wind remains constant but as the engineers continued driving towards north with their speed increasing, the wind appears to be coming from $N30^0$ E.

TASK:

a) i) Determine whether the engineers will add an extra force or not.
   (ii) Find the extra force X and its direction that will make the resultant force exactly 12N in the direction $N30^0$ E.
b) (i) Find the actual speed of the wind and its direction.
   (ii) Find the speed of the car at the instant the wind appeared to come from $N30^0$ E.

ITEM 76

In a survey carried out by a certain Telcom company in Uganda, it was found out that 70% of men and 55% of women make calls on a daily basis and the proportional of men to women who use this company is 60 : 40. A person who made a call on a certain day was selected at random from a survey.

The company has also found out that the duration of the customer's calls, in minutes, can be modeled by a random variable X. This variable takes values between 0 and 3 minutes. The probability density function for the call duration is defined as:

- For calls between 0 and 1 minute inclusive, the probability density is a function, K .
- For calls between 1 and 3 minutes inclusive, the probability density is linearly decreasing function, $\frac{K}{2}(3-x)$.
- For other durations, the probability density is zero, 0.

TASK:

a) What is the probability that the person selected from a survey is a woman?

b) Determine;

(i) The constant, k and hence sketch f(x).

(ii) The probability that a random call lasts 0.5 and 2 minutes

(iii) The expected duration for a customer's call.

(iv) The cumulative density function, F(x) which gives the probability that a call lasts up to a certain number of minutes.

ITEM 77

A manufacturer of ice cream cones needs to ensure the cones meet specific volume requirements for quality control. The volume of the cone is given by the formular $V = \frac{1}{3}\pi r^2 h$. The company's machines measure the radius (r) and height (h) of the cone with errors $er\ andeh$ respectively. For accuracy, the manufacturer needs to come up with an expression: $2|\frac{e_r}{r}| + |\frac{e_h}{h}|$.

On a random day, the manufacturer decided to manually measure the radius and the height of the cone without using a machine, he recorded the radius(r) and height(h) as 3.9cm and 7cm rounded off to given number of decimal places respectively. And he now needs to know the exact range within which the expression $\frac{h-r}{h}$ lies.

The manufacturer also produces two types of cones A and B, if he produces 35% of type A, he also produces 50% of type B and when he produces 40% of type A, he produced 65% of type B.

TASK:

Help the manufacturer to:

a) Come up with the expression $2|\frac{e_r}{r}| + |\frac{e_h}{h}$ and hence state its significancy in this context.

b) Determine the range within which the expression $\frac{h-r}{h}$ lies.

c) Know the percentage of type B cones he will produce when he produces 80% of type A.

ITEM 78

A new automated cargo handling system is being tested in a factory. The system uses large flat (horizontal) smooth surface (table) to move goods. An 8kg container (P) rests on this surface and is attached by light inextensible ropes to other two containers Q and R of mass 2kg and 6kg respectively. The ropes pass over light smooth pulleys mounted at the opposite edges of the table so that Q and R hangs freely in a vertical plane. The system needs to be designed to move all the containers at a specific predictable acceleration and the engineer needs to determine this acceleration together with the tensions in the ropes connecting the containers to ensure the safety of the system.

The system was designed and released from rest but as the 6kg mass was at a height 7 meters above the ground, the rope snaps and it fell at point, N which is which is 5 metres above the ground. From point, N the mass was then projected with speed of $21ms^{-1}$ at an angle of 30^0 to the horizontal and later hits the horizontal ground.

TASK:

a) Help the engineer determine the acceleration of the system and the tensions in the ropes.

b) Determine the velocity the 6kg mass hits point N with and time taken to hit this point.

c) (i) Find the time the mass takes to hit the horizontal ground after being projected from point N.

(ii) Determine the horizontal distance travelled by the particle before hitting the ground.

ITEM 79

KBN games and Sports Company organized competitions that were to include car rallies and tortoise race.

During the testing of the cars that were to be used in the competition, they observed that the treads on the front tyres of the cars wear out more quickly than those on the rear tyres and they found out that this could result into accidents. According to the organizers, the competitions will only commence if the treads on the front and rear car tyre wears have a rank correlation coefficient of 5% significance level. The treads on the front and rear tyres are recorded in the table below.

Cars	1	2	3	4	5	6	7	8	9	10
Front tyre wear(mm)	4.1	4.0	4.2	3.9	4.8	4.3	4.6	5.0	4.7	4.7
Rear tyre wear(mm)	3.9	4.0	4.8	4.0	4.2	4.5	4.4	4.9	4.8	4.4

At the same time, the specialists recorded the mass (kg) of 150 tortoise that were to participate in the competitions as below.

Mass(kg)	Number of turtoises
$0.2 \leq m \leq 0.6$	27
$0.6 \leq m \leq 1.0$	43
$1.0 \leq m \leq 1.4$	35
$1.4 \leq m \leq 1.8$	31
$1.8 \leq m \leq 2.2$	14

TASK:

a) Help the company to know whether the car rally competitions will or not commence.
b) Use a scatter diagram to find the value of the front wear if the rear wear is 4.7mm.
c) Use the information gathered mass of the tortoises to find:
    (i) The 10% to 90% inter percentile range of the mass of the tortoises.

(ii) The middle 60% of the mass of the tortoise.

ITEM 80

A toy company is conducting a quality inspection using two storage boxes labelled A and B, each containing a mix of white and pink marbles representing different product types.

- Box A is three times as likely to be selected as Box B due to larger size
- Box A contains 5 white marbles and 6 pink marbles
- Box B contains 2 white marbles and 5 pink marbles

During the inspection, one box is chosen at random, and from it, two marbles are drawn one at a time without replacement to test for quality defects

As a quality analyst using the principles of probability, determine the likelihood of the following events

a) Both marbles are of the same colour
b) At least one white marble is picked
c) A pink marble is picked
d) Given that a pink marble is picked, what is the probability that it comes from box A

ITEM 81

RONALD is a quality control engineer at a manufacturing plant in JINJA CITY producing alloy components for aerospace applications. To ensure the material meets specifications, he measures the density of a sample alloy block, which is calculated as the $\boldsymbol{p = \frac{m}{v}}$, where m is the mass and V is the volume. The mass m is measured as 456.7 grams with a possible error of $\Delta m = 0.5$ grams, and the volume V is measured as 123.4 cm^3 with a possible error of $\Delta V = 0.2$ cm^3.

Task

a) Show that the maximum absolute error in the density ρ is given by $\Delta\rho = \frac{(|\mathbf{V}|\,\boldsymbol{\Delta}\mathbf{m} + |\mathbf{m}|\,\boldsymbol{\Delta}\mathbf{V})}{\boldsymbol{v^2}}$
b) Find the interval within which the exact value of the density is expected to lie, based on these measurements.

ITEM 82

Fort portal city council is conducting a study to understand the life style and political behavior of its residents. The research team observes the following;

Meal habits:

10% of the residents regularly have hot Breakfast, 20% have a hot Lunch, and 25% have either a hot breakfast or a hot lunch (or both)

Political alignment and voting

30% of the residents are NUP supporters, 50% are NRM supporters and 20% are independents. In the last election, 65% of the NUP supporters voted, 82% of the NRM supporters voted, and 50% of the independents voted. If a resident is chosen at random from city;

Task;

a) What is the probability that the person has both hot breakfast and hot lunch
b) What is the probability that the person has a hot lunch given that they already had a hot breakfast?
c) What is the probability that this person voted in the last election?
d) If it is known that this person did not vote, what is the probability that they are NUP supporters?

ITEM 83

SLECK SACCO is a financial institution based in Fort Portal City. The institution is conducting a study to evaluate the performance of 12 secretaries. The management is interested in understanding the relationship between the typing speeds of the secretaries and the number of errors they make in their typed scripts

The data collected from the 12 secretaries is summarized below, where

- x represents the number of errors
- y represents the typing speed in seconds

	A	B	C	D	E	F	G	H	I	J	K	L
X	12	24	20	10	32	30	28	15	18	40	27	35
Y	130	136	120	120	153	160	155	142	145	172	140	157

TASK

As a data analyst for the institution, you are tasked with the following;

a) (i) construct a scatter diagram to visually represent the relationship between typing speed(y) and number of errors (x) for the 12 secretaries
ii),. Draw the line of best fit on your scatter diagram.
iii) Based on your diagram and line of best fit, comment on the nature of the relationship between typing speed and errors

(iv) Use the line of best fit to estimate the number of errors x when the typing speed y = 142 seconds

b) i) Calculate the Spearman's rank correlation coefficient to quantify the relationship between typing speed and errors

ii) Test the significance of your correlation coefficient at the 5% significance level and comment on the strength and direction of the relationship

ITEM 84

75% of the students are full time students, 45% of the students are female, 40% of the students are male students. Find the probability that;

a) A student chosen at random from the students in the university is a part time student.
b) A student chosen at random from all students in the university is female and a part time student
c) A student chosen at random from all the female students in the university is a part time student

ITEM 85

Three boxes X, Y and Z contain coloured balls. X contains 5 black balls and 4 white balls, Y contains 7 black and 5 white balls and Z contains 3 black and 5 white balls.

a) If the balls are withdrawn from box Z, with replacement, find the probability that the third ball drawn is the second white ball.
b) One of the boxes is selected at random and a ball is withdrawn from it. Find the probability that;
    (i) Box X was chosen and the ball was black
    (ii) A white ball was chosen
    (iii) The ball was selected from box Z, given that it was black.

ITEM 86

A fair yellow cubical die has two faces labeled 10, two faces labeled 30 and two faces labeled 50.

Another fair green cubical die has four faces labeled 60, one face labeled 100.

Mukasa and Nkata wish to go for a Picknick which has two historical sites A and B to visit.

Each of them tossed a die and agreed to take on the random variable X "half of the difference between the score on the green die and the score on the yellow die". For

this random variable they set the following rules as guideline of who visits which historical site.

- If E(X)>25 then, Mukasa will visit historical site A, otherwise it will be Nkata to visit this site.
- If Var(X)>120, Nkata will visit historical site B, otherwise it will be Mukasa to visit this site.

Task

(i) Tabulate a probability density function for a random variable X.
(ii) By calculation, decide who visits which historical site A or B.

ITEM 87

A coffee shop has a loyalty program that offers free coffee after 10 purchases. The chance of a customer making 10 purchases is 0.2. The shop keeper wishes to know the chance that exactly 3 out of 5 customers get free coffee. If the chance exceeds 5%, then he will expand his local bakery which sells a special type of bread based on the past data.

- The bakery sells 0 loaves with probability 0.1
- The bakery sells 1 loaf with probability 0.3
- The bakery sells 2 loaves with probability 0.4
- The bakery sells 3 loaves with probability 0.2

The bakery wants to know the expected loaves of bread sold and probability of selling at most 2 loaves of bread. This will help him plan for the future of the business.

Task;

a) Basing on the information given, will the shop keeper expand his local bakery?
b) Determine the;
    (i) The expected loaves of bread sold
    (ii) Probability of selling at most 2 loaves of bread

ITEM 88

A builder of a house normally takes records before construction starts. He developed a table of his values. There are always three (3) values represented with letters a, b and c measured with errors $e_a, e_b \; and \; e_c$ respectively. The builder wishes to include the value of the difference between ab and c and its maximum absolute error. The friend gave him two methods below of which one is correct but he is not sure of the one to take.

- Method 1: maximum absolute error = $\boldsymbol{a|e_b| + b|e_a| + |e_c|}$
- Method 2: maximum absolute error = $\boldsymbol{a|e_b| + b|e_a| - |e_c|}$

The builder recorded a as 70 units, b as 50 units and c as 200 units with relative errors of 0.02, 0.003 and 0.001 respectively. The builder wants the construction to proceed if the percentage error in the difference between ab and c is less than 7%

Task:

a) Help the builder to derive and select on the correct method for the maximum absolute error from above.
b) Basing on mathematical computations, will the constructions proceed, give reasons for your answer

ITEM 89

An experiment consists of removing 2 sweets one at a time without replacement from a box containing 3 red and 4 blue sweets.

Task;

(c) If A is the event that both sweets picked are of the same colour, find the probability that event A occurred.
(d) If the experiment is repeated 20 times, find the probability that event A occurred
(iii) between 20 and 35 times.
(iv) at least 25 times

ITEM 90

At a vocational centre in Entebbe, a 12kg block sits on a rough ramp inclined at 35^0 (μ=0.25). A light string from the block passes over a fixed edged pulley and supports a movable pulley. A second light string runs over this movable pulley carrying masses 5kg and 10kg at its ends. Pulleys are smooth, strings light, and the setup is released from rest. The instructor will switch ropes if any string tension exceeds 180 N, and will add a safety strap if any magnitude of acceleration is above 2.5m/s^2.

Task:

Find the accelerations of the hanging masses and the movable pulley, and the tensions in both strings. Using the 180 N tension limit and the 2.5m/s^2 acceleration limit, decide whether to proceed, fit a thicker rope, add a safety strap, or both.

ITEM 91