CO ORDINATE GEOMETRY

Question 1

In Buwanda Trading Centre, the local council plans to improve the road network connecting different parts of the community. Two straight roads, L_1 and L_2, intersect near the village market.

The equations of the roads are given by $L_1 : \ y = 2x - 3$ and $L_2 : \ y = -\frac{1}{2}x + 5$.

A third road L_3 is later planned through the Health Post located at the point $(3, 7)$, and it is designed to be parallel to L_1. Another footpath L_4 is built from the borehole and is perpendicular to L_2.

During a village planning meeting, the engineers explain that the relationship between the gradients of lines can be used to determine whether roads are parallel or perpendicular without drawing accurate scale diagrams.

Tasks

a) Let the angles which L_1 and L_2 make with the positive direction of the x-axis be α and β respectively.

prove that the angle θ between two straight lines with gradients m_1 and m_2 is given by

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}.$$

Hence prove that if two straight lines are perpendicular, then the product of their gradients is $m_1 m_2 = -1$.

b) Find the equation of the road L_3 passing through the point $(3, 7)$ and parallel to L_1. Hence explain why the two roads are parallel.

c) Determine an equation of the footpath L_4 that passes through the point $(4, -1)$ and is perpendicular to L_2.

d) Verify using gradients that L_2 and L_4 are perpendicular.

Question 2

A school is designing its new compound layout. The planners need to position classrooms, paths, and monuments on a coordinate grid. The following tasks arise during the design.

Task 1

(a) A monument is located at $P(5, -1)$ and a proposed path has equation $L : 3x - 4y + 12 = 0$ Find the perpendicular distance from P to L .

(b) Determine the coordinates of the foot of the perpendicular from P onto L .

Task 2

(a) Two parallel fences are given by

$$L_1 : 2x - y + 1 = 0, \qquad L_2 : 2x - y - 9 = 0.$$

Find the perpendicular distance between L_1 and L_2 .

(b) A third fence L_3 is drawn parallel to L_1 and passes through $(3, 2)$. Find the equation of L_3 and the distance between L_3 and L_2 .

Task 3

(a) Three classrooms are positioned at $A(0,2), B(4,6), C(8,2)$. Show that $\triangle ABC$ is isosceles hence find the area of the triangle.

(b) Two main walkways in the school satisfy $L_4 : y = 2x + 1$ and $L_5 : y = -\frac{1}{2}x + 3$. Find the acute angle between L_4 and L_5 .

Question 3

A triangular garden is designed with fountains at points $A(2,3), B(8,7), C(6,-1)$. Paths (straight lines) are built between the fountains, and an additional path connects B to the midpoint of AC .

Task 1

(a) Find the slope and equation of each side of the triangle ABC .

(b) Calculate the length of each side and the midpoint of AC .

(c) Determine the equation of the line joining B to the midpoint of AC , and check whether this line is perpendicular to AC .

Task 2

(a) Check if points $A(2,3)$, $D(4,5)$, and $B(8,7)$ are collinear, if not identify any point that can be replaced with B to make the points collinear.

(b) Write the equation of the line through C that is parallel to AB .

(c) Find the point of intersection of line AB and the altitude from C to AB .

Task 3

(a) Find the acute angle between line AB and line BC .

(b) A sprinkler is placed at $P(x,y)$ such that its distance from A and B is always equal. Find the equation of the locus of P .

Question 4

A city planner is designing a new public park on a coordinate grid. The park has four important points: A(2,3) (Entrance Gate), $B(8,7)$ (Fountain), $C(10,3)$ (Playground), $D(4,-1)$ (Garden Corner)

The planner wants to use coordinate geometry to make design decisions.

Tasks:

1. With mathematical evidence, determine whether $ABCD$ is a rhombus or a parallelogram.
2. Calculate the area of quadrilateral $ABCD$.
3. If a security post is always located at a point $P(x,y)$ such that it is equidistant from A and C , find the locus of P .

Question 5

A surveyor is mapping a triangular piece of land. The vertices of the triangle are: $P(2,1), Q(6,5), R(10,1)$.

She wants to locate important points of the triangle for design purposes.

Tasks:

1. With reasons, state the type of the triangle PQR .
2. Find the equation of the median from P , and hence find the coordinates of the centroid of the triangle.
3. Find the equation of the altitudes from Q and R , and hence determine the orthocentre of the triangle.
4. Find the equation of the perpendicular bisectors of PQ and PR , and hence find the circumcentre of the triangle.
5. Calculate the radius of the circumcircle of $\triangle PQR$.
6. Show the relationship between centroid, orthocentre, and circumference.

Question 6

A triangular park is bounded by three straight paths represented by the equations:

$$L_1 : 2x + y = 8, L_2 : x - y = 2, L_3 : x + 2y = 10.$$

The intersections of these paths form the three vertices of the park. A watchtower, a water point, and a security gate will be located using special properties of the triangle.

Task 1

(a) Calculate the lengths of all three sides.

Task 2: Critical Special Lines

(a) Find the equations of the altitudes of the triangle and hence locate the orthocentre.

(b) Find the centroid of the triangle and write the equation of the line through the centroid and orthocentre.

(c) Determine the equations of the perpendicular bisectors of the sides and hence find the circumcentre.

Task 3

(a) A water point is to be placed at $P(x, y)$ such that it is equidistant from L_1 and L_2 . Derive the equation of the locus of P .

(b) The security gate is to be placed on the line joining the centroid and circumcentre, but equidistant from two vertices of the triangle. Find its coordinates.

(c) Are the centroid, orthocentre, and circumcentre collinear? If yes, identify this line.

Question 7

At Simeka Priary School, the school administration plans to install a new underground water pipeline between the Administration Block A and the Science Laboratory B.

The two buildings are represented on a coordinate grid by the points

$$A(-4, 2) \quad \text{and} \quad B(8, 10).$$

To improve water distribution, the engineers place a control chamber P along the pipeline such that it divides the line joining A and B internally in the ratio $2 : 3$, measured from A to B.

For future expansion, a second monitoring point Q is planned outside the section joining the two buildings. The point Q divides the line joining A and B externally in the ratio $1 : 2$.

The engineering team also wants to verify whether the monitoring points are correctly aligned with the pipeline route.

a) Find the coordinates of the control chamber P.

b) Determine the coordinates of the external monitoring point Q.

c) Calculate the distance between P and Q.

d) Show that the midpoint of PQ lies on the same straight line as A and B.

Question 8

In the village of Kasana , community leaders plan to construct a clean water pipeline connecting the Health Centre A and the Community Market B.

The locations of the two facilities are represented on a coordinate plane by the points

$$A(-6, 1) \quad \text{and} \quad B(9, 13).$$

To regulate water flow, a control chamber P is placed on the pipeline such that it divides the line joining A and B internally in the ratio $2 : 1$, measured from A to B.

A monitoring station Q is also constructed outside the section joining A and B such that it divides the line externally in the ratio $1 : 3$.

A drainage channel passing through P is designed perpendicular to the pipeline AB. In addition, an access road passing through the point

$$R(2, -1)$$

is planned parallel to the pipeline.

For safety purposes, the engineers also plan to construct a security fence along the perpendicular bisector of the line joining the Health Centre and the Market. The village council further proposes another inspection point T that trisects the line segment AB nearer to point A.

The community engineers must verify all measurements before construction begins.

a) Find the coordinates of the control chamber P.

b) Determine the coordinates of the monitoring station Q.

c) Find the equation of the drainage channel passing through P and perpendicular to AB.

d) Find the equation of the access road passing through R and parallel to AB.

e) Determine the equation of the perpendicular bisector of AB.

f) Find the coordinates of the trisection point T nearer to A.

g) Calculate the distance between P and Q.

END

"Trust in the Lord with all your heart and lean not on your own understanding; in all your ways submit to him, and he will make your paths straight."

— Proverbs 3:5–6

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