P425/1
PURE
MATHEMATICS
PAPER 1
3 Hours

Uganda Advanced Certificate of Education

PRE-REGISTRATION EXAMINATION

PURE MATHEMATICS

Paper 1

3 Hours

INSTRUCTIONS TO CANDIDATES

- *Answer **only four** items in this paper.*
- *Each question in section takes **25 points**.*
- *Any additional question(s) answered will **not** be marked □ Every item **must** start on a fresh page.*
- *All necessary working **must** be clearly shown.*
- *Silent, non – programmable scientific calculators and mathematical tables with a list of formulas may be used.*

ITEM 1

A travel company has three different types of tickets for a journey, namely Economy class, Business class and First class. On a given journey the revenue and the number of tickets **x, y** and **z** of the respective types issued satisfy the set of equations given by:

$$x + 3y + 6z = 292$$
$$2x + 5y + 7z = 512$$
$$2x + 9y + 6z = 632$$

The company plans to fence a rectangular enclosure to act as a waiting area. One side of the enclosure has an existing wall, and this side is not to be fenced. 25 meters of fencing wire is available, and it is required that an area of the enclosure must be **at most** $75\,mm^2$. Given also that the width of the enclosure must be at least **3 m** but not more than **9 m**. The manager of the company is interested in knowing the number of the tickets issued as well as the suitable range of values of width, **w** for the waiting area.

Your school has a pavilion which was constructed to accommodate **1,500** students. The seats in the pavilion are arranged in rows such that the next row has **10** more seats than the previous one. The first row has **30** seats. During one school event, all seats were occupied only up to the **12th** row. During the event, seven judges were planned to sit in a row, but two judges did not want to sit next to each other.

TASK

a) Determine the number of tickets of each type issued during the mentioned journey and find the possible range of the possible values of **w.**

b) Determine the number of students who were present during the school event number of rows in the pavilion

c) In how many ways can the seats of the judges be arranged.

ITEM 2

Mr. Charles, the LIRA District Engineer, is undertaking a major infrastructure development project that integrates **communication and land surveying systems** to modernize the region using the funds from the government's financial year 2025/2026.

Part I: Telecommunication Mast Installation

In order to provide stable mobile network coverage to nearby communities, a telecom company plans to install a communication mast to serve three towns whose positions on a regional map (in hectometers) are given as Town A (1, 3), Town B (4, −5) and Town C (9, −1). To ensure equal signal distribution, the mast

must be located at a point that is **equidistant** from all three towns. A transmitting dish is mounted at the top of the mast at a height **h** in hectometers above the ground. The dish is adjusted so that the signal directed toward each town makes an angle of **30°** with the horizontal ground. The signal strength $P(\theta)$ in megahertz(MHz) any direction θ is modeled by

$P(\theta) = 1500\cos\theta + 800\sin\theta - 700$ where θ is the angle of inclination of the transmitting dish and **S** represents the signal strength in that direction.

The telecom engineers must determine the optimal location and configuration of the mast for maximum efficiency.

Part II: Land Survey and Geometric Analysis

During the planning phase, surveyors examine a triangular section of land near the bridge. The following points are identified **P (3,0)**, **R (1,0)** and **Q** lying on the positive y-axis. A perpendicular from the origin onto the line joining points **R** and **Q** meets the line joining points **P** and **Q** at **M**. The surveyors are interested in understanding the geometric behavior of point **S** as **Q** varies along the positive y-axis

TASK:

(a) Assist the mass Installation department to find the exact coordinates of the mast so that it is **equidistant** from Towns **A, B**, and **C.**
(b) Determine the required height **h** of the mast and the slanting length from the top of the mast to town **A**.
(c) Determine the maximum and minimum signal strength and the smallest positive angles at which they occur respectively.
(d) Help the land surveying department to determine the equation of the locus of **M.**

ITEM 3.

A search and rescue team is operating in the mountainous Rwenzori region to locate a missing hiker. The hiker's last known route is modeled as a straight line. The hiker started at point P **(1, 2, 0)** and was moving in the direction of vector $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$. The rescue team, located at base camp RR**(5, 0, 6)**, needs to determine the equation of the hiker's path and find the closest point on this path to their base camp to plan the most efficient interception route. This application of 3D line equations can significantly reduce search time in difficult terrain, potentially saving lives.

Simultaneously, a support team is using aerial surveys to map potential safe zones and drop points for supplies. Their flight paths and search patterns create geometric challenges. In one such survey, the path of a drone creates a circle A passing through the point $(t + 2, 3t)$ with its center at $(t, 3t)$. A second drone, flying through a coordinated pattern, creates circle B of radius 2 units with its centre at $(t + 2, 3t)$. The parameter t represents a time delay in the survey pattern.

As part of the communication network setup, two radio transmission paths are being considered to establish a relay between two outposts. The paths are defined by the following symmetric-equations

$$-L_1: \frac{x}{1} = \frac{y+2}{2} = \frac{5-z}{1}$$

$$L_2: \frac{1-x}{1} = \frac{y+3}{-3} = \frac{z-4}{1}.$$

For the relay to be established with a single booster, the two transmission paths must intersect at a point.

TASK.

a) Find the vector equation and cartesian equations of the line representing the hiker's path.

b) Calculate the shortest distance from the base camp **R (5, 0, 6)** to the hiker's path.

c) If $tt = 1$, find the points of intersection of the two circles, which represent potential rendezvous points for the drones hence show that the common area of intersection of circles A and B (the region covered by both drones' surveys) is $8\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$ **sq. units.**

d) Verify whether the two lines LL_1 and LL_2 intersect. If they do intersect, find the coordinates of the point of intersection. Hence, determine the position vector of this point which is the point recommended as the optimal location for placing the communication relay booster.

ITEM 4

Mr. Kagwa the owner of ***KAGWA AND SONS PRINTING SERVICES*** uses a Riso printer. The printer has an ink drum in the shape of an inverted right circular cone of base radius 10 cm and height 50 cm. While printing, the height of the ink in the drum decreases at a rate of ***2cm per minute.***

The printer usually stops printing in order to refill the ink in the drum when the height of the ink left in the drum is **3cm**.

A publisher decides to print the pages of a large book with 0.5 -inch margins on the top, bottom, and one side, and a 1-inch margin on the other side (to allow for the binding). The area of the entire page is to be 96 square inches.

Mr. Kagwa wants to know the dimensions of the page that will maximize the printed area so that he adjusts the printer settings accordingly. The printer uses a master roll whose rotation is modelled as a curve with parametric equations $x = \frac{t^2}{1+t^3}$ and $y = \frac{t^3}{1+t^3}$. The master roll needs a replacement whenever the rotation reaches the point $\left(\frac{1}{2}, \frac{1}{2}\right)$. The expression of $\frac{d^2y}{dx^2}$ at any point gives the length in meters of the master roll used at any point during the rotation.

TASK

Help Mr. Kagwa to find;

(a) the rate at which the volume of the ink decreasing when the printer stops?
(b) the dimensions of the page that will maximize the printed area of the page.
(c) The expression of $\frac{d^2y}{dx^2}$ in terms of t. Hence, determine the total length of each master roll used at the point of replacement.

ITEM 5.

In a three different chemical reactions tested in the Chemistry laboratory, the mass M grams at a time t days are modelled as below;

$R_1: M = \frac{kt}{\sqrt{1+t^2}}$, where k is a constant

$R_2: M = \frac{3^{t^2}}{3}$ and

$R_3: M = 1n\left\{e^{3t}\left(\frac{3-t}{4+t}\right)\right\}$

It is given that in chemical reaction R_1, when $t = 1$, $M = 25$

To verify the fastest chemical reaction, the lab technician has to determine the rates of change of the mass with respect to time for each reaction at $t = 2$ days.

TASK

(a) (i) find the constant k.

(ii) show that $\frac{dM}{dt} = \frac{M}{t(1+t^2)}$

(iii) find the constants A, B and C such that $\frac{1}{t(1+t^2)} = \frac{A}{t} + \frac{Bt+C}{1+t^2}$.

(b) help the lab technician to suggest the fastest chemical reaction.

END