NAME:……………………………………………………………COMB:……………………

P425/1
Principal Mathematics
May 2026
2¼ Hours

Tr JoelPCM ACADEMIC COUNCIL

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S.5 PRINCIPAL MATHEMATICS

2 Hours 15 Minutes

INSTRUCTIONS:

Attempt any **FOUR** items of your choice.

Item 1

A research team at Makerere University is studying population growth of a certain rare plant species in a botanical garden. The number of plants, P, at time t years satisfies the differential equation:

$$dP/dt = kP(1 - P/500)$$

where k is a positive constant. Initially (t = 0), there are 50 plants, and after 3 years there are 150 plants.

Task:

(a) By separating variables and using partial fractions, show that the general solution is:

$$P = 500 / (1 + Ae^{-kt})$$

where A is a constant.

(b) Determine the values of A and k, giving k correct to 3 significant figures.

(c) Find the population after 10 years, giving your answer to the nearest whole number.

(15 scores)

Item 2

A civil engineering student at Kyambogo University is analysing stress distribution in a concrete beam. The bending moment at a section of the beam is modelled by the curve:

$$y = 3x^2 - 12x + 9$$

The student also needs to evaluate the area enclosed between the curve and the x-axis, then verify a trigonometric identity that appears in the vibration model of the beam:

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

Task:

(a) Find the x-intercepts of the curve $y = 3x^2 - 12x + 9$.

(b) Calculate the exact area enclosed between the curve and the x-axis.

(c) Prove the identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. (15 scores)

Item 3

A land surveyor is mapping a triangular plot of land in Mbarara district. The coordinates of the vertices of the plot are A(1, 3), B(7, −1) and C(4, 6). The surveyor also needs to resolve the following simultaneous equations which arise from the boundary calculations:

$$\log_2(x + y) = 3 \quad \text{and} \quad \log_2 x + \log_2 y = \log_2 6 + 1$$

Task:

(a) (i) Find the equation of the perpendicular bisector of AB.

(ii) Show that the perpendicular bisector of AB passes through C.

(b) Solve the simultaneous equations involving logarithms, showing all your working.

(15 scores)

Item 4

Uganda National Roads Authority (UNRA) engineers are designing a curved section of a road. The road follows a path whose equation involves the expression:

$$(3 + \sqrt{5})(4 - 2\sqrt{5}) + (\sqrt{45} - \sqrt{20})$$

The curve of the road is defined by the parametric equations $x = 2t - 1$ and $y = t^2 + 3$ for $t \in \mathbb{R}$. As part of quality control, the engineers must also verify the binomial expansion used in their load-distribution formula.

Task:

(a) Simplify the expression $(3 + \sqrt{5})(4 - 2\sqrt{5}) + (\sqrt{45} - \sqrt{20})$, giving your answer in the form $a + b\sqrt{5}$ where a and b are integers.

(b) Find the Cartesian equation of the curve, and state its nature.

(c) Expand $(1 + 2x)^5$ in ascending powers of x up to and including the term in x^3. Hence find an approximate value of $(1.04)^5$, correct to 4 decimal places.

(15 scores)

Item 5

A drainage channel in a school construction project is designed so that its cross-sectional area is a maximum. The width of the channel at the top is 12 metres, and it is formed by bending a flat sheet of metal of width 12 m into a shape with two equal sides and a flat bottom. Each side makes an angle θ with the vertical. The cross-sectional area is given by:

$$A(\theta) = 16\sin\theta(1 + \cos\theta)$$

Task:

(a) Show by differentiation that the maximum cross-sectional area occurs when $\cos\theta = 1/2$.

(b) Find the maximum cross-sectional area.

(c) Solve $2\sin^2\theta + \sin\theta - 1 = 0$ for $0° \leq \theta \leq 360°$.

(15 scores)

Tr. Joel: PCM Academic Council**END**