

UGANDA ADVANCED CERTIFICATE OF EDUCATION

S5 Mathematics Revision for Term II 2026

MATHEMATICS DEPARTMENT

Answer ALL items. Record all numerical answers to the number of decimal places stated in each item. Only algebraic working will be credited.

Item 1

A quantity x arising in a manufacturing process satisfies the algebraic relationship $f(x) = x^3 - 3x^2 - x + 5$. Two inspection records showed that evaluations at $x = 0$ and $x = 2$ returned outputs of opposite sign, raising concern about a critical transition point lying between them. The team similarly detected a sign reversal between $x = -1$ and $x = 0$. Because instruments recorded values only at integer positions, an algebraic strategy was needed to narrow each transition without further physical readings. Express all answers to three decimal places.

- Show, using $f(x)$, that a transition point exists within each stated interval.
 - Use only the endpoint evaluations to obtain an initial estimate of the root in $[0, 2]$.
 - Determine whether the estimate found lies closer to 0 or to 2, and tighten the interval.
 - Apply the same procedure to obtain an initial estimate of the root in $[-1, 0]$.
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Item 2

The displacement of a component along a track is modelled by $h(x) = 2x^3 - 5x^2 + 1$. Evaluations at $x = 2$ and $x = 3$ returned values of opposite sign, indicating a zero-displacement position within that stretch. Before applying a refined algebraic procedure, the team first derived an initial position estimate from the two boundary readings. The refinement was to continue until successive estimates agreed to within the precision demanded by the decimal places specified. A further requirement involved estimating the total displacement accumulated from $x = 0$ to $x = 3$, partitioned into six equal strips. All answers are to four decimal places.

- Confirm algebraically that a zero-displacement position lies between $x = 2$ and $x = 3$.
 - Derive an initial position estimate using only the two boundary displacement values.
 - Refine the estimate iteratively, stopping once the improvement falls below the maximum possible error for four decimal places.
 - Estimate the total displacement over $[0, 3]$ by dividing the interval into six equal strips.
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Item 3

The output of a production line over time $t \geq 0$ follows $p(t) = -2t^3 + 9t^2 - 12t + 4$. Management observed that output rose to a peak, dipped to a trough, then fell persistently before eventually reaching zero, at which point a maintenance window was to be scheduled. Records confirmed that output was positive at $t = 3$ and negative at $t = 4$, placing the shutdown moment somewhere between those two instants. An algebraic approach was required for all findings, and answers were to be stated to three decimal places.

- Identify all stationary instants of the output, classifying each accordingly.
 - Determine the peak and trough output values attained during operation.
 - Demonstrate algebraically that output reaches zero between $t = 3$ and $t = 4$.
 - Obtain a first estimate of the shutdown instant from the values at $t = 3$ and $t = 4$.
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Item 4

Two quantities, u and v , measured during a chemical process are known to satisfy the power law $v = a \cdot u^n$, where a and n are constants to be determined. Paired observations gave: ($u = 4$, $v = 24$) and ($u = 9$, $v = 81$). At a different stage of the same process, the pair (u , v) was found to obey the two conditions $\log(2u + v) = 2$ and $\log(u - v) + 1 = \log(3v - u)$ simultaneously, where \log denotes logarithm to base 10. The supervisor required all constants and unknowns to be found by algebraic manipulation only. Express answers to four decimal places.

- Rewrite the power law in a form that allows a and n to be found algebraically.
 - Determine the values of a and n using the two observed pairs.
 - Reduce the logarithmic conditions to an algebraic simultaneous system in u and v .
 - Solve the system from (c) and verify each value satisfies both original conditions.
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Item 5

A regulatory body specified that a process variable x must satisfy $2x^2 - 7x - 4 \leq 0$ to remain within permitted operating bounds. An engineer separately needed to sketch the curve $y = x^2 - x - 6$ to visualise where the expression was positive, negative, or zero, and to identify all crossing positions and the turning point. A stricter constraint $(x - 1)(x + 3) > 2x + 1$ was later introduced, whose solution set had to be compared with the first. No trial substitutions were permitted; complete algebraic reasoning was required throughout.

- Solve the first constraint algebraically and state the full permitted range for x .
 - Find all axis intercepts and the turning point of the given curve.
 - Produce a clearly labelled sketch of the curve, showing all features from (b).
 - Solve the second constraint and state the values of x satisfying both simultaneously.
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Item 6

Internal stress along a structural beam is expressed as $q(x) = x^4 - 3x^3 - 2x^2 + 4x + 3$. Evaluating the expression at $x = 3$ and $x = 4$ yielded opposite signs, indicating a neutral-stress position between those two points. Before a refined search, the analyst derived a first approximation directly from the two boundary stress values. Subsequent improvement continued until the change between successive approximations fell within the tolerance set by the required decimal places. The net stress accumulated along the beam from $x = 0$ to $x = 4$, split into eight equal intervals, was also needed for the load-balance assessment. All answers are to four decimal places; full working must accompany every step.

- Show, by evaluating $q(x)$ at the stated points, that a neutral-stress position lies in $[3, 4]$.
 - Obtain an initial approximation of the position from the boundary stress values.
 - Improve the approximation iteratively until the error is within the maximum possible error for four decimal places.
 - Estimate the net accumulated stress over $[0, 4]$ using eight equal sub-intervals.
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