FLUID MECHANICS

A fluid is any substance that can flow. It can be **a liquid** or **a gas**

Fluid mechanics involves fluids at rest (hydro statistics) and fluids in motion (hydro dynamics/fluid flow)

Fluids such as gases and liquids in motions is called **fluid flow.**

Fluidity. This is the ability of the fluid to flow easily with minimum resistance.

TERMS USED IN FLUID FLOW

Fluid element

A fluid element is a molecule (smallest volume) of the fluid which follows the flow.

Flow line / Line of flow

A flow line is that path followed by a fluid particle.

OR

A flow line is the path which an individual molecule in a fluid element describes.

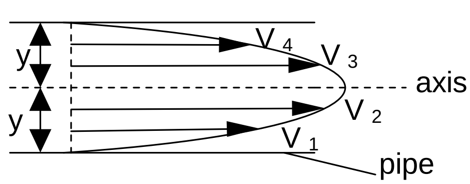
A streamline

A streamline is a curve whose tangent at any point is along the direction of the velocity of the fluid particles at that point.

TYPES OF FLUID FLOW

LAMINAR FLOW (Steady, uniform, Smooth, Streamline flow)

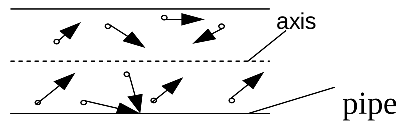
Laminar (steady/uniform) flow is the orderly flow of a liquid where flow lines are parallel to the axis of the tube or pipe (axis of flow) and equidistant layers from the axis have the same velocity. In laminar flow the successive particles passing at a certain point have the same velocity and occurs at low velocities below the critical velocity.



TURBULENT FLOW

Turbulent flow is a disorderly flow where lines of flow are not parallel to the axis of the pipe / flow and equidistant fluid layers from the axis of flow have different velocities.

Turbulent flow occurs at high velocities, above the critical velocity.



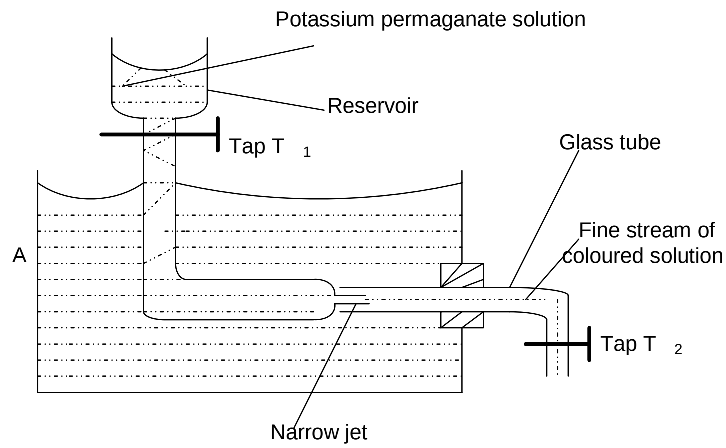
N.B

The difference between a **streamline** and a **line of flow** is that a streamline is a curve whose tangent at any point is along the direction of the velocity of the fluid particles at that point. Streamlines never cross. On the other hand, a line of flow is the path followed by a fluid particle. However, in steady flow/ Laminar flow, the streamlines coincide with the lines of flow.

Critical velocity

This is the maximum velocity below which the fluid flow is laminar.

EXPERIMENT TO DEMONSTRATE LAMINAR AND TURBULENT FLOW



- The apparatus is set up as shown in the diagram above.
- With taps T_1 andT_2 closed, potassium permanganate solution is poured into the reservoir and water poured into tank (container), A
- T_2 is then slightly opened to allow water to flow out of the glass tube and T_1 also opened slightly. A fine stream coloured solution is seen flowing along middle of the glass tube and this illustrates laminar flow.

- Tap T_2 is then widely opened to allow more water to flow from the glass tube, a stage is reached when the coloured solution in the glass tube begins to spread out and fill the whole the tube. This illustrates turbulent flow.

VISCOSITY

Viscosity is the frictional force between adjacent layers of a fluid.

OR

Viscosity is the force that opposes relative motion between adjacent fluid layers.

Viscous drag

This is the force that opposes motion of a body in a fluid.

OR

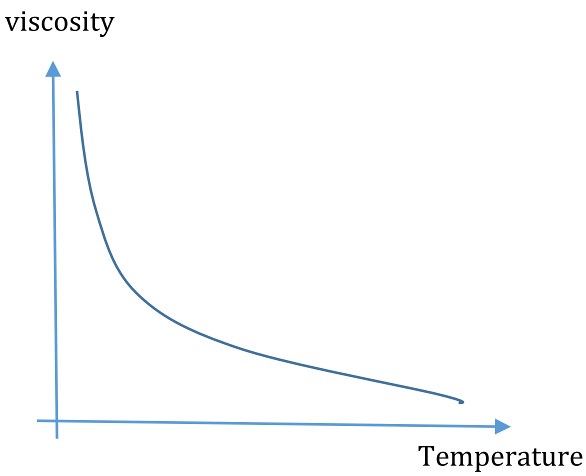
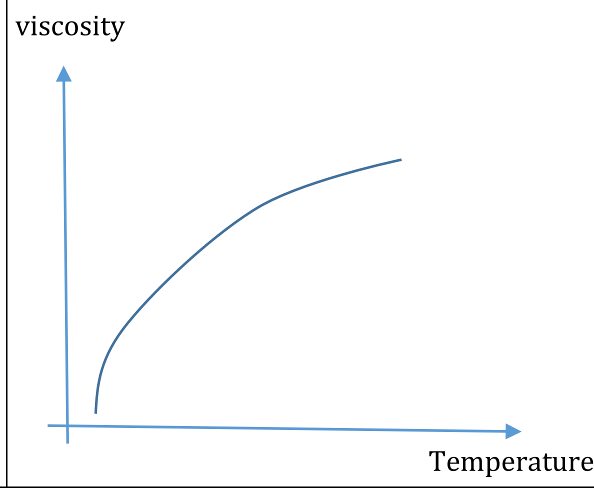
This is the frictional force experienced by a body moving in a fluid due to its viscosity.

Explain why some fluids flow more easily than others.

Fluid flow involves different parts of the fluid moving at different velocities. Different parts of the fluid therefore slide past each other in layers. There exists a frictional force between the layers of the fluid which is the measure of the flow rate. The greater the frictional force the less easily it is for the fluid to flow and the lower the frictional force the more easily it is for the fluid to flow. Thus some fluids flow more easily than others.

Effects of temperature on viscosity

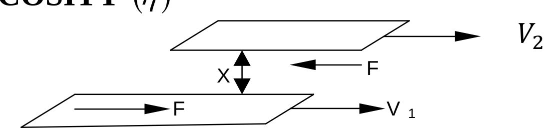
- In liquids, viscosity is due to intermolecular forces of attraction between layers moving at different speeds. Thus energy/force is required to drag one layer over the other against the force of attraction. These forces oppose relative motion between adjacent liquid layers. Increase in temperature reduces (weakens) intermolecular forces which increases molecular separation and speed, consequently viscosity in liquids decreases rapidly with increase in temperature
- In gases, viscosity is due to transfer of momentum. Molecules are further apart and have negligible intermolecular forces, molecules move randomly colliding with one another and continuously transferring momentum to the neighboring layers. Increasing the temperature of the gas increases the average speed (increases K.E) of the gas molecules hence increasing the transfer of momentum which results into increase in viscosity of the gas.

LIQUIDS	GASES
	

Differences between viscosity and solid friction

Solid friction	**Viscosity**
Independent of area of contact between the surfaces in contact	Depends on area of contact / overlap of fluid layers
Independent of relative velocity between layers in contact	Directly proportional to velocity gradient
Independent of temperature but dependent on normal reaction	Depends on temperature

COEFFICIENT OF VISCOSITY (η)



Consider two parallel layers of a liquid moving with velocities V_1 and V_2 and separated by a distance x with area of contact between the layers, A.

The slower lower layer exerts a tangential retarding force, F on the faster upper layer. The lower layer itself experiences an equal and opposite tangential force, F due to the upper layer.

Velocity gradient between the layers $= \frac{\text{change in velocity}}{\text{distance}} = \frac{V_2 - V_1}{x}$

Definition

Velocity gradient is the change in velocity between two layers (points) separated by a distance of one metre.

Velocity gradient is measured in per second (s^{-1})

Dimensions of velocity gradient

$$[\text{velocity gradient}] = \frac{[\text{change in velocity}]}{[\text{distance}]}$$

$$[\text{velocity gradient}] = \frac{[\text{velocity}]}{[\text{distance}]}$$

$$[\text{velocity gradient}] = \frac{LT^{-1}}{L} = T^{-1}$$

Newton's law of Viscosity

It states that "The frictional force, F between adjacent fluid layers is directly proportional to the area of overlap of the molecular layers and velocity gradient"

$$F \alpha\ A \times \text{velocity gradient}$$

$$F = \eta \times A \times \text{velocity gradient}$$

$$\eta = \frac{F}{A \times \text{velocity gradient}}$$

Definition

Coefficient of viscosity is the frictional force between two adjacent fluid layers with area of overlap $1\,m^2$ and velocity gradient $1\,s^{-1}$.

OR

Coefficient of viscosity is the <u>tangential stress</u> which one layer of a fluid exerts on another layer in contact with it when the velocity gradient between the layers is $1s^{-1}$.

Dimensions of coefficient of viscosity, η

$$[\eta] = \frac{[F]}{[A]\times[\text{velocity gradient}]} = \frac{MLT^{-2}}{L^2 \times T^{-1}} = ML^{-1}T^{-1}$$

Coefficient of viscosity is measured in Nsm^{-2} or Pas or $kgm^{-1}s^{-1}$

Coefficient of viscosity depends on temperature.

The higher the value of viscosity the more viscous the fluid is and vice versa.

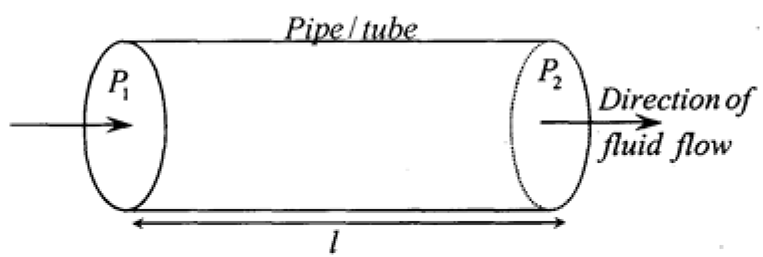
Example

Calculate the frictional force of water if the coefficient of viscosity at 10^0C is 1.3×10^{-3} Pas, area is 10cm^2 and the layers are 0.1 cm apart with relative velocity of $2\,\text{cms}^{-1}$.

$$F = \eta\times A\times\left(\frac{V_2 - V_1}{h}\right) = 1.3\times10^{-3}\times10\times10^{-4}\times\frac{2}{0.1} = 2.6\times10^{-5}\,N$$

POISEUILLE'S EQUATION

Poiseuille derived an expression for the volume of a liquid flowing out of a pipe per second. He assumes that the flow was steady/laminar.



Consider a streamlined flow of a liquid of coefficient of viscosity, η in a horizontal tube of radius, r, length, L and cross sectional area A, with the ends of the tube maintained at pressures, P_1 and P_2.

If P is the pressure difference, then; $P_1 - P_2$

$$\text{pressure gradient} = \frac{P_1 - P_2}{l}$$

Therefore, **pressure gradient** is the change in pressure of a fluid in a tube of length one metre.

Poiseuille's law states that; "During steady flow, the volume of liquid flowing out of a pipe per second dependents on; the coefficient of viscosity, η of the liquid, the radius, r of the pipe and the pressure gradient, $\frac{P}{l}$ across the ends of the pipe.

$$\frac{V}{t}\alpha\,\eta^x r^y\left(\frac{P}{L}\right)^z \Rightarrow \frac{V}{t} = K\eta^x r^y\left(\frac{P}{l}\right)^z$$

$$\left[\frac{V}{t}\right] = [K]\times[\eta]^x\times[r]^y\times\left[\frac{P}{l}\right]^z$$

$$L^3T^{-1} = 1\times\left(ML^{-1}T^{-1}\right)^x\times L^y\times\left(ML^{-2}T^{-2}\right)^z$$

$$L^3T^{-1} = M^{x+z}L^{-x+y-2z}\times T^{-x-2z}$$

Equating power

M: $x + z = 0$..................................(i)

L: $-x + y - 2z = 3$.........................(ii)

T: $-x - 2z = -1$..........................(iii)

Solving the above equations, $x = -1$, $z = 1$ and $y = 4$

$$\frac{V}{t} = \frac{Kr^4P}{\eta\, l}$$

By experiment $K = \frac{\pi}{8}$

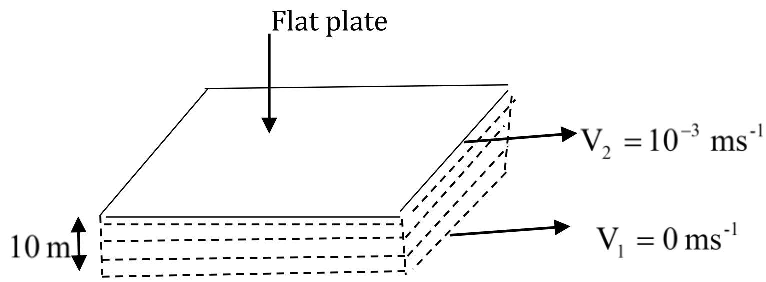
$$\frac{V}{t} = \frac{\pi r^4 P}{8\eta\, l}.$$ **This is Poiseuille's formula/ equation.**

It holds for lamina / steady flow of Newtonian fluids.

Example

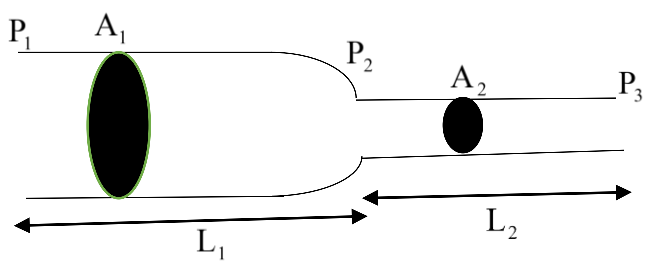
1. Explain what is meant by laminar flow and coefficient of viscosity.
2. A flat plate of area $0.1m^2$ is placed on a flat surface and separated from it by a film of oil $10^{-5}m$ thick whose coefficient of viscosity is 1.5 Pas. Calculate the force required to cause it slide on the surface at a constant speed of $1mms^{-1}$.

   N.B The particles of the fluid in contact with the flat surface are at rest.



$$F = \frac{\eta A(V_2 - V_1)}{h} = \frac{1.5 \times 0.1(10^{-3} - 0)}{10^{-5}} = 15\text{N}$$

3. Water flows through a horizontal tube which consists of two parts joined from end to end. One part is 21 cm long and has a diameter of 0.225 cm and the other part is 7.0cm long and has a diameter of 0.075 cm. if the pressure difference between the ends of the tube is 14 cm, find the pressure difference between the ends of each part.



Volume flow through the large pipe per second = Volume flow through the small pipe per second

$$\frac{\pi(P_1 - P_2)r_1^4}{8L_1\eta} = \frac{\pi(P_2 - P_3)r_2^4}{8L_2\eta}$$

$$\frac{(P_1 - P_2)r_1^4}{L_1} = \frac{(P_2 - P_3)r_2^4}{L_2}$$

$$\frac{(P_1 - P_2)\left(\frac{0.225 \times 10^{-2}}{2}\right)^4}{21 \times 10^{-2}} = \frac{(P_2 - P_3)\left(\frac{0.075 \times 10^{-2}}{}\right)^4}{7.0 \times 10^{-2}}$$

$$27(P_1 - P_2) = (P_2 - P_3) \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

$$27(P_1 - P_2) = P_2 - P_1 + P_1 - P_3$$

$$27(P_1 - P_2) = -(P_1 - P_2) + (P_1 - P_3)$$

$$28(P_1 - P_2) = (P_1 - P_3) \text{ but } P_1 - P_3 = 14 \ldots\ldots\ldots\ldots(ii)$$

$$\Rightarrow (P_1 - P_2) = 0.5 \text{ cm of water}$$

From equation (i) $P_2 - P_3 = 27 \times 0.5 = 13.5\,\text{cm of water}$

4. A liquid flows steadily through a horizontal pipe of length 3.2m. If the amount of liquid collected at one end is 6gs^{-1}, what is the pressure difference between the two ends of the pipe?

   (Density of liquid $= 1.2 \times 10^3\,\text{kgm}^{-3}$, $\eta = 0.92\,\text{Nsm}^{-2}$, diameter = 15mm)

$$\text{Volume flow per second, } \frac{V}{t} = \frac{\text{mass per second}}{\text{density}} = \frac{6 \times 10^{-3}}{1.2 \times 10^3} = 5 \times 10^{-6}\,\text{m}^3\text{s}^{-1}$$

$$\frac{V}{t} = \frac{\pi r^4 P}{8\eta L} \Rightarrow P = \frac{5 \times 10^{-6} \times 8 \times 0.92 \times 3.2}{\pi \times \left(7.5 \times 10^{-3}\right)^4} = 11846.85\,\text{Nm}^{-2}$$

5. An empty vessel which is open at the top has a horizontal capillary tube of length 20cm and internal diameter 2.0mm protruding from one of its side walls immediately above the base. Water flows into the vessel at a constant rate of $1.5\text{cm}^3\text{s}^{-1}$. At what depth does the water level stop to rise? (Density of water $= 1000\,kgm^{-3}$)

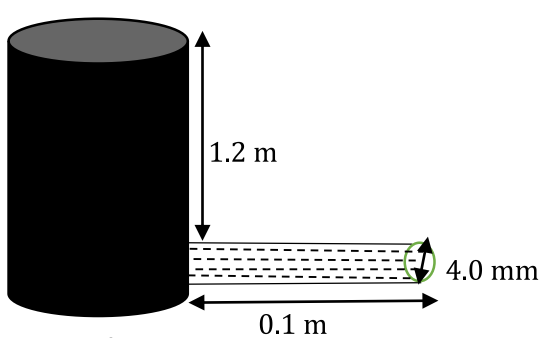
$$\frac{V}{t} = \frac{\pi r^4 P}{8\eta L} \text{ but } P = h\rho g$$

$$\frac{V}{t} = \frac{\pi r^4 h\rho g}{8\eta L}$$

$$1.5 \times 10^{-6} = \frac{\pi (1.0 \times 10^{-3})^4 \times h \times 1000 \times 9.81}{8 \times 1 \times 10^{-3} \times 20 \times 10^{-2}} \Rightarrow h = 0.0779\text{ m}$$

Water will stop to rise at a height of 0.0779 m or 7.79 cm

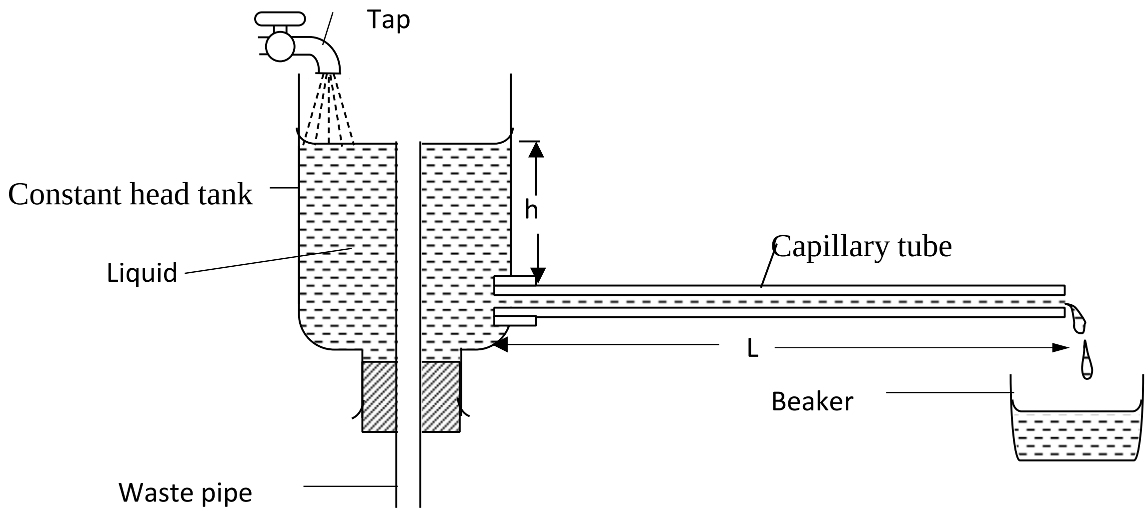
6. The figure below shows a tank containing a light lubricating oil. The oil flows out of the tank through a horizontal pipe of length 0.1m and internal diameter of 4.0mm. Calculate the volume of oil which flows through the pipe in one minute when the level of oil in the tank is 1.2m above the pipe and does not significantly alter during this time. (Density of oil $= 9.2 \times 10^2\,kgm^{-3}$, $\eta = 8.4 \times 10^2\,Nsm^{-2}$)



$$\frac{V}{t} = \frac{\pi r^4 P}{8\eta L} \text{ but } P = h\rho g$$

$$V = \frac{\pi r^4 h\rho g \times t}{8\eta L} = \frac{\pi \times (2\times10^{-3})^4 \times 1.2\times 9.2\times10^2 \times 9.81\times 60}{8\times 8.4\times10^2 \times 0.1} = 4.86\times10^{-8}\,m^3$$

EXPERIMENT TO DETERMINE THE COEFFICIENT OF VISCOSITY OF WATER BY POISEUILLE'S FORMULA (EQUATION)



- The constant head, h is measured and recorded
- The length, l of the capillary tube is measured and recorded.
- The mean diameter, d of the capillary tube is measured using a travelling microscope and hence its radius, r obtained.
- The volume, V of the water flowing through the capillary tube in time t, is measured and recorded. The volume per second, $\frac{V}{t}$ is calculated.
- The experiment is repeated for different values of h.
- The results are tabulated in a suitable table.

- A graph of $\frac{V}{t}$ against h is plotted and its slope, S obtained.
- The coefficient of viscosity, η of water is calculated from, $\eta = \frac{\pi r^4 \rho g}{8SL}$, where

  $\rho =$ density of water, g = acceleration due to gravity.

Precautions

- Experiment should be carried out at constant temperature since the coefficient of viscosity varies with temperature.
- Readings should be at steady flow.
- The radius of the tube should be carefully determined.

  Note:

- ❖ The experiment must be carried out at a constant temperature to avoid changes in coefficient of viscosity.
- ❖ The constant head apparatus is used to ensure that the liquid flowing through the capillary tube is uniform since poisseuille's formular holds for only laminar flow.
- ❖ Great care is needed when measuring r because it appears in the calculation of η as r^4. This makes the % error in η due to an error in r four times the % error in r
- ❖ A capillary tube is used because r needs to be small so that h is large enough to be measured accurately

STOKE'S LAW

Stokes suggested that any particle moving through a fluid experiences a retarding force called **viscous drag** due to the viscosity of the fluid. This force depends on the speed of the body V and acts in opposite direction to its motion.

Viscosity of a fluid is the frictional force opposing relative motion between adjacent layers while **viscous drag** is the frictional force experienced by a body moving in a fluid due to its viscosity.

Stokes' law states that; "The viscous drag (Force) on a sphere of radius, r falling through a fluid depends on the coefficient of viscosity of the fluid, velocity of the sphere (body) and radius of the sphere (body).

$$F \alpha r^x \eta^y V^z \Rightarrow F = k r^x \eta^y V^z$$

Where, x, y and z are integers and k is a non-dimensional constant

Using dimensional analysis,

$$[F] = [k] \times [r]^x \times [\eta]^y \times [V]^z$$

$$MLT^{-2} = L^x.(ML^{-1}T^{-1})^y.(LT^{-1})^z$$

$$MLT^{-2} = M^y.L^{x-y+z}.T^{-y-z}$$

Equating powers;

M: y = 1

L: x-y + z = 1, x + z = 2 ……………(i)

T: -y – z = -2 ………………………….(ii)

Solving equations (i) and (ii), x =1 and z = 1

$$F = k\eta \, r \, V$$

By experiment, $k = 6\pi$

$F = 6\pi\eta \, r \, V$ This is Stokes' equation / law

Note:

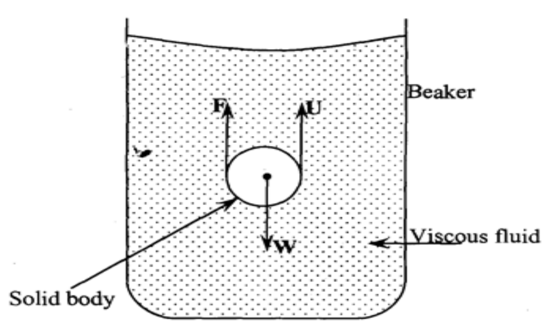
Stokes'law holds for steady motion i.e the fluid should not be moving very fast which may create turbulence.

It applies to only a fluid of infinite extent. Otherwise the walls and bottom of the vessel affect the resisting force.

TERMINAL VELOCITY

Terminal velocity is the **maximum constant velocity** attained by a body falling through a viscous fluid.

Consider a sphere of radius, r and density, ρ falling from rest through a viscous fluid of density, σ and coefficient of viscosity, η .



The forces acting on the sphere are its weight, W downwards, up thrust, U upwards due to the weight of the displaced fluid and the viscous drag, F upwards due to viscosity of the fluid.

When the sphere is dropped gently in the fluid, it first accelerates due to the resultant down ward force, $F_d = W - (U + F)$.

As the body accelerates downwards its velocity increases and from F = $6\pi\eta vr$, the viscous drag also increases. However, since the body is already completely immersed in the liquid, up thrust remains constant (since no more fluid is being displaced)

A point is reached when W = F + U. This implies that the net force acting on the body is zero. The body therefore continues to move down the fluid with a constant velocity called terminal velocity.

Therefore, **terminal velocity** is the maximum velocity attained by a body when falling through a viscous fluid.

Explain using appropriate equations why a rain drop hits the ground with less force than it should?

Initially the rain drop has zero velocity. As it falls, $W > (U + F)$, where; W is weight of drop, U is up thrust, and F is viscous drag. There is a net downward force, $F_d = W - (U + F)$ so that the drop accelerates downwards. As the velocity increases, $F = 6\pi\eta \, r \, V$ also increases which reduces the net downward force. A point is reached when $W = (U + F)$. At this point the net force on the drop is zero and its terminal velocity is reached. Therefore, it hits the ground with reduced force.

EXPRESSION FOR TERMINAL VELOCITY

At terminal velocity, $W = U + F$

$$\tfrac{4}{3}\pi r^3 \rho g = 6\pi r \eta V_0 + \tfrac{4}{3}\pi r^3 \sigma g$$

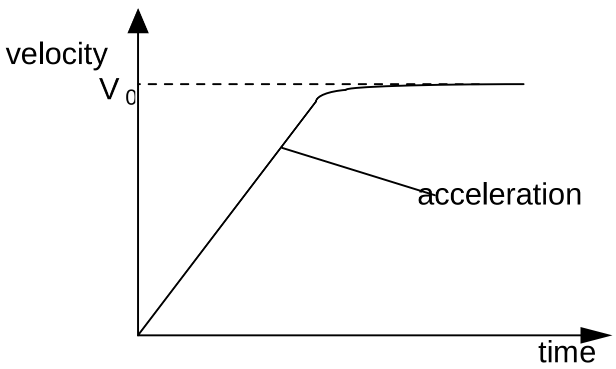
$$\tfrac{2}{3} r^2 g(\rho - \sigma) = 3\eta V_0$$

$$V_0 = \frac{2 r^2 g(\rho - \sigma)}{9\eta}$$

N.B

When $\sigma > \rho$, v_o is negative, in such a case the body moves upwards with a constant velocity e.g. gas bubbles in soapy water

A graph of velocity against time for an object falling in a fluid



FACTORS THAT AFFECT TERMINAL VELOCITY

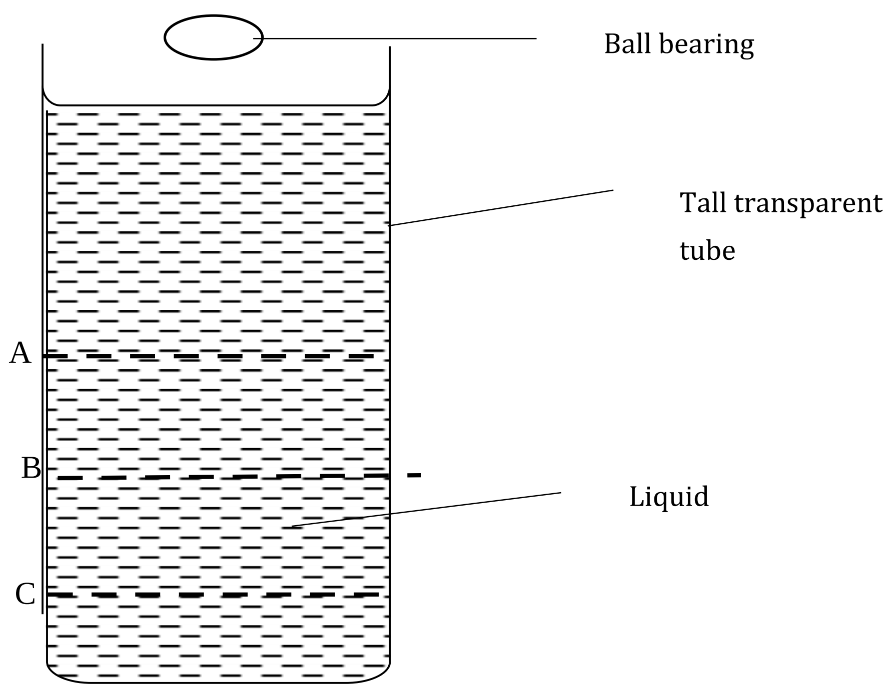
Coefficient of viscosity

Size of the body

Viscous drag

EXPERIMENT TO DETERMINE THE COEFFICIENT OF VISCOSITY BY STOKES'S LAW

The method is suitable for **liquids of high viscosity** such as glycerin and treacle



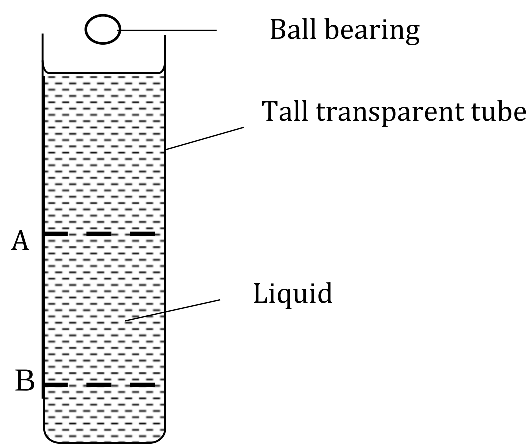
- Densities of the ball bearing and liquid ρ and σ respectively are obtained.

- The diameter d and hence radius, r of the ball bearing is measured using a micrometer screw gauge.
- Three reference marks A, B and C at equal distances are made on the sides of a tall transparent tube filled with the liquid.
- The ball bearing is allowed to fall centrally through the liquid.
- The times, t_1 and t_2 for the fall from A to B, and B to C respectively are measured and recorded.
- When $t_1 = t_2 = t$, terminal velocity, V_0 is attained and is calculated from,

$$V_0 = \frac{AB}{t} = \frac{BC}{t} = \frac{AC}{2t}$$

- The coefficient of viscosity, η is then calculated from; $\eta = \frac{2r^2 g(\rho - \sigma)}{9V_0}$

(OR)



- Densities of the ball bearing and liquid ρ and σ respectively are obtained.
- The diameter d and hence radius, r of the ball bearing is measured using a micrometer screw gauge.
- Two reference marks A and B are made on the sides of a tall transparent tube filled with the liquid.

- The ball bearing is allowed to fall centrally through the liquid.
- The times, t for the fall from A to B, is measured and recorded.
- The terminal velocity, V_0 is calculated from, $V_0 = \frac{AB}{t}$
- The experiment is repeated using ball bearings of different radii, r.
- The results are tabulated including values of r^2
- A graph of V_0 against r^2 is plotted and its slope, S is calculated.
- The coefficient of viscosity, η of the liquid is calculated from; $\eta = \frac{2g(\rho - \sigma)}{9S}$

Precautions taken during the experiment

- Glass tube should be wide compared to the diameter of the ball bearing
- Temperature should remain constant during the experiment
- Point A should be far away from the top surface of the liquid
- Using a highly viscous liquid and a small ball bearing makes t large enough to be measured

EXPERIMENT TO DETERMINE TERMINAL VELOCITY OF A BODY

- A viscous fluid is filled in a tall transparent jar.
- A spherical ball bearing is dropped centrally into the liquid.
- When the ball falls with a constant velocity, the time, t the ball takes between two known points is recorded.
- The distance, d between the two known points is measured and recorded.
- The terminal velocity, V_0 is calculated from $V_0 = \frac{d}{t}$.
- The experiment is repeated and the average value of terminal velocity is obtained. The average value is the terminal velocity of the ball bearing.

NUMERICAL EXAMPLES

7. A spherical raindrop of radius 2.0×10^{-4}m, falls vertically in air at 20^0c, if the densities of air and water are $1.3kgm^{-3}$ and $1 \times 10^3 kgm^{-3}$ respectively and the viscosity of air at 20^0c is 1.8×10^{-5}pa. Find the terminal velocity of the drop.

$$V_0 = \frac{2r^2 g(\rho - \sigma)}{9\eta} = \frac{2 \times (2.0 \times 10^{-4})^2 \times 9.81 \times (1000 - 1.3)}{9 \times 1.8 \times 10^{-5}} = 4.84\text{ms}^{-1}$$

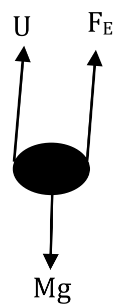
8. A spherical oil drop of density 900kgm^{-3} and radius 2.5x10^{-6}m has a charge of 1.6x10^{-19}C. the drop falls under gravity between two plates

    (i) Calculate the terminal velocity attained by the drop

    (ii) What electric field intensity must be applied between the plates in order to keep the drop stationary (density air = 1kgm^{-3}, coefficient of viscosity of air $= 1.85 \times 10^{-5}$ Nm^{-2}s^{-1})

(i) $$V_0 = \frac{2r^2 g(\rho - \sigma)}{9\eta} = \frac{2 \times 9.81 \times (2.5 \times 10^{-6})^2 \times (900 - 1)}{9 \times 1.85 \times 10^{-5}} = 6.62 \times 10^{-6}\text{ms}^{-1}$$

(ii)



Since the sphere is moving down, the electric field must be applied upwards to keep it stationary and there will be no vicious drag

When the drop is stationary; $Mg = U + F_E$

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + EQ$$

$$E = \frac{4\pi r^3 g(\rho - \sigma)}{3Q} = \frac{4\pi \times 9.81 \times (2.5 \times 10^{-6})^3 \times (900 - 1)}{3 \times 1.6 \times 10^{-19}} = 3.6 \times 10^6\text{ Vm}^{-1}$$

9. A spherical oil drop of density 900kgm^{-3} and radius 2.5x10^{-6}m falls through air. Neglecting the density of air, find the terminal velocity of the drop (viscosity of air =1.8x10^{-5}Nsm^{-2})

$$V_0 = \frac{2r^2 g(\rho - \sigma)}{9\eta} = \frac{2 \times (2.5 \times 10^{-6})^2 \times 9.81 \times (900 - 0)}{9 \times 1.8 \times 10^{-5}} = 6.81 \times 10^{-4}\text{ms}^{-1}$$

10. A metal ball of diameter 10mm is timed as it falls through oil at a steady speed, it takes 0.5s to fall through a vertical distance of 0.3m. Assuming that density of the metal is $7500kgm^{-3}$ and that of oil is $900kgm^{-3}$, find

(i) The weight of the ball $(3.85\times10^{-2}\,N)$

(ii) The Up thrust on the ball $(4.62\times10^{-3}\,N)$

(iii) The coefficient of viscosity of oil $(0.5994\,Pas)$

(Assume the viscous force = $6\pi r\eta V_0$ where $\boldsymbol{\eta}$ is the coefficient of viscosity, r is radius of the ball and V_0 is terminal velocity)

11. A small oil drop falls with terminal velocity of $4x10^{-4}ms^{-1}$ through air. Calculate the radius of the drop. What is the terminal velocity of oil drop if its radius is halved?

(viscosity of air = $1.8x10^{-5}Nm^{-2}s$, density of oil = $900kgm^{-3}$, neglect density of air)

[$1.92x10^{-6}$m, $1.0x10^{-4}ms^{-1}$]

12. A metal sphere of radius $2.0x10^{-3}$m and mass $3.0x10^{-4}$kg falls under gravity, central down a wide tube filled with a liquid at 35^0C, the density of the liquid is $700kgm^{-3}$, the sphere attains a terminal velocity of magnitude $40x10^{-2}ms^{-1}$. The tube is emptied and filled with another liquid at the same temperature and of density $900kgm^{-3}$. When the metal sphere falls centrally down the tube, it is found to attain a terminal velocity of magnitude $25x10^{-2}ms^{-1}$. Determine at 35^0C, the ratio of the coefficient of viscosity of the second liquid to that of the first. **[1.640]**

13. In an experiment to determine the coefficient of viscosity of motor oil, the following measurements were made

Mass of glass of sphere = $1.2x10^{-4}$kg

Diameter of sphere = $4.0x10^{-3}$m,

Terminal velocity of sphere = $5.4x10^{-2}ms^{-1}$

Density of oil = $860kgm^{-3}$

Calculate the coefficient of viscosity of the oil [**$0.45Nsm^{-2}$]**

14. A metal sphere of radius $3.0x10^{-3}$m and mass $4.0x10^{-4}$kg falls under gravity, centrally down a wide tube filled with a liquid at 25℃, the density of the liquid is $800kgm^{-3}$, the sphere attains a terminal velocity of magnitude $45cms^{-1}$. The tube is emptied and filled with another liquid at

the same temperature and of density $100kgm^{-3}$. When the metal sphere falls centrally down the tube, it is found to attain a terminal velocity of magnitude $20cms^{-1}$. Determine at 25^0C, the ratio of the coefficient of viscosity of the second liquid to that of the first.

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{4\times10^{-4}}{\frac{4}{3}\pi\times\left(3.0\times10^{-3}\right)^3} = 3536.78\text{kgm}^{-3}$$

$$\eta_1 = \frac{2r^2 g(\rho-\sigma_1)}{9V_{01}}\text{.....................................(i)}$$

$$\eta_2 = \frac{2r^2 g(\rho-\sigma_2)}{9V_{02}}\text{.....................................(ii)}$$

(ii) $\div$ (i)

$$\frac{\eta_2}{\eta_1} = \frac{V_{01}(\rho-\sigma_2)}{V_{02}(\rho-\sigma_1)} = \frac{45\times10^{-2}\times(3536.78-1000)}{20\times10^{-2}\times(3536.78-800)} = 2.09$$

15. A steel ball bearing of diameter 8.0mm falls steadily through oil and covers a vertical height of 20.0cm in 0.56s. if the density of steel is $7800Kgm^{-3}$ and that of oil is $900Kgm^{-3}$, calculate the
    (i) Up-thrust on the ball

    $$\text{Up thrust} = \text{weight of oil displaced} = \frac{4}{3}\pi r^3 \rho_0 g = \frac{4}{3}\pi\times\left(4.0\times10^{-3}\right)^3\times900\times9.81$$

    $$U = 2.368\times10^{-3}\text{N}$$

    (ii) Viscosity of the oil

    $$\text{Terminal velocity, } V_0 = \frac{0.2}{0.56} = 0.357\text{ms}^{-1}$$

    $$\eta = \frac{2r^2 g(\rho-\sigma)}{9V_0} = \frac{2\times\left(4.0\times10^{-3}\right)^2\times9.81\times(7800-900)}{9\times0.357} = 0.674\text{Pas}$$

16. A spherical rain drop of radius $2.4x10^{-4}m$ falls vertically in air at 20^0C. If the densities of air and water are $1.2kgm^{-3}$ and $1000kgm^{-3}$ and coefficient of viscosity of air at 20^0C is 1.8×10^{-5} Pas . Calculate the terminal velocity of the drop $\left(V_0 = 6.97\text{ms}^{-1}\right)$
17. Describe an experiment to determine the coefficient of viscosity of cooking oil
18. Explain the origin of viscosity in air and account for the effect of temperature on it.

19. 27 spherical rain drops of the same mass and radius are falling down with a terminal velocity of $15\,\text{cms}^{-1}$. If they coalesce to form a big drop, what will be its terminal velocity? (Neglect the buoyancy due to air).

Let the radius of the big drop be R, while that of each of the 27 small droplets be r

By conservation of volume

Volume of the big drop = Total volume of small drops

$$\frac{4}{3}\pi R^3 = 27\times\frac{4}{3}\pi r^3 \quad \Rightarrow R = 3r$$

But terminal velocity, $V_0 = \frac{2r^2 g(\rho-\sigma)}{9\eta}$ but since buoyancy due to air is negligible,

$\sigma = 0 \quad \Rightarrow$ Terminal velocity, $V_0 = \frac{2r^2 g\rho}{9\eta}$

For small droplets, $V_{01} = \frac{2r^2 g\rho}{9\eta} = 15\text{cms}^{-1}$ and for big droplets, $V_{02} = \frac{2R^2 g\rho}{9\eta}$

$$\frac{V_{02}}{V_{01}} = \frac{R^2}{r^2} = \frac{(3r)^2}{r^2} = 9 \Rightarrow V_{02} = 9V_{01} = 9\times 15 = 135\ \text{cms}^{-1} = 1.35\text{ms}^{-1}$$

20. 8 Spherical similar water drops fall down with a terminal velocity of $5\,\text{cms}^{-1}$. If they coalesce to form a big drop, what will be its terminal velocity if the density of air is assumed to be negligible. $\left(2.0\times10^{-3}\text{ms}^{-1}\right)$

21. Some particles of sand are sprinkled onto the surface of water in a beaker filled to a depth 10cm. Estimate the least time for which the grains of diameter 0.10mm remain in suspension of water stating any assumptions made.

$\left(\text{viscosity of water} = 1.1\times10^{-3}\text{Pas, Density of sand} = 2200\text{kgm}^{-3}\right)$

$$V_0 = \frac{2r^2 g(\rho-\sigma)}{9\eta} = \frac{2\times\left(5\times10^{-5}\right)^2\times 9.81\times(2200-1000)}{9\times1.1\times10^{-3}} = 5.945\times10^{-3}\text{ms}^{-1}$$

$$\text{Least time, } t = \frac{\text{distance}}{\text{Terminal velocity}} = \frac{10\times10^{-2}}{5.945\times10^{-3}} = 16.82\text{ seconds}$$

Assumptions

The sand grains are perfect spheres

The sand grains acquire terminal velocity immediately from the water surface.

22. (a) Sketch the velocity-time graph for the motion of an oil drop of radius $2.5\times10^{-6}\,m$ which falls through air. Neglect the density of air. $(\eta \text{ of oil} = 1.8\times10^{-5}\text{ Pas, Density of oil} = 900\text{kgm}^{-3})$

(b) By considering the forces acting on a sphere falling through a viscous fluid, explain why it eventually reaches terminal velocity.

(c) Two spherical rain drops of equal size are falling vertically through air with a terminal velocity of 0.15ms^{-1}. What would be the terminal velocity of these drops if they coalesce to form a large drop? (0.2381ms^{-1})

23. The table below gives the times of fall of steel spheres of different diameter falling through a distance of 50cm in a viscous liquid of density 1260 kgm^{-3}

Diameter (mm)	2.0	2.2	2.4	2.6
Time (s)	8.36	6.89	5.80	4.93

If the density of steel is 7800kgm^{-3}, plot a suitable graph and from it determine the coefficient of viscosity of the liquid.

(Hint: start by calculating the terminal velocities, V_0 for each steel sphere, and then plot a graph of V_0 against d^2) $(\eta = 0.246\text{Pas})$

24. A steel sphere of diameter 3×10^{-3}m falls through a cylinder containing a liquid *x*. When the sphere has attained a terminal velocity, it takes 10.8 seconds to travel between two fixed marks on the cylinder. When the experiment is repeated using another steel sphere of diameter 5×10^{-3}, with the cylinder containing liquid y, the time of fall between the fixed points is 4.8 seconds. If the density of liquid *x* is $1.26\times 10^3\text{kgm}^{-3}$, that of liquid y is $0.92\times 10^3\text{kgm}^{-3}$ and that of steel is $7.8\times 10^3\text{kgm}^{-3}$, determine the ratio of the coefficients of viscosity of liquid *y* to that of liquid x, if the temperature remains constant throughout.

Let the distance between the two reference marks on the cylinder be h

For liquid *x*, terminal velocity $V_x = \frac{h}{10.8}$ and radius $r_x = \frac{3\times10^{-3}}{2} = 1.5\times10^{-3}\,m$

For liquid y terminal velocity $V_y = \frac{h}{4.8}$ and radius $r_y = \frac{5\times10^{-3}}{2} = 2.5\times10^{-3}\text{m}$

From

$$\eta = \frac{2r^2 g(\rho - \sigma)}{9V}$$

For liquid x :

$$\eta_x = \frac{2r_x^{\ 2} g(\rho - \sigma)}{9V_x} = \frac{2 \times 9.81 \times (1.5 \times 10^{-3})^2 \times (7.8 \times 10^3 - 1.26 \times 10^3)}{9 \times \left(\frac{h}{10.8}\right)} = \frac{0.34645}{h} \text{.........(i)}$$

For liquid y:

$$\eta_y = \frac{2r_y^{\ 2} g(\rho - \sigma)}{9V_y} = \frac{2 \times 9.81 \times (2.5 \times 10^{-3})^2 \times (7.8 \times 10^3 - 0.92 \times 10^3)}{9 \times \left(\frac{h}{4.8}\right)} = \frac{0.44995}{h} \text{.........(ii)}$$

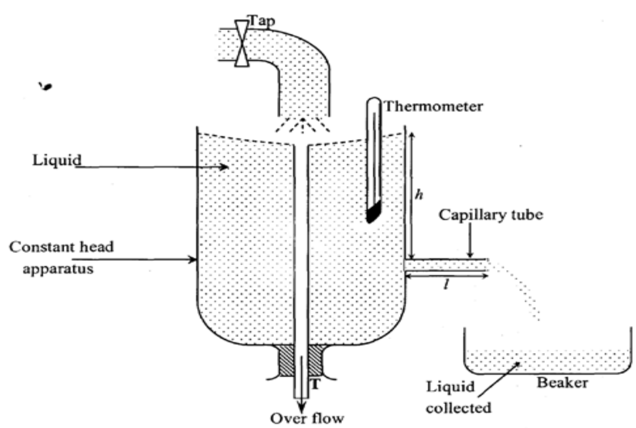
Dividing the two equations gives;

$$\frac{\eta_x}{\eta_y} = \frac{0.34645}{h} \times \frac{h}{0.44995} = 0.77$$

25. Explain the effect of increasing pressure on the viscosity of the liquid.

Increase in pressure reduces the molecular separation, the intermolecular forces of attraction become significant and increases leading to an increase in viscosity of a liquid.

26. Describe the experiment to compare the viscosity of a less viscous liquid at two different temperatures.



- The temperature, θ_1 of the liquid is measured and recorded.

- The liquid of known density, ρ is made to flow at constant rate through a capillary tube of known length, *l*.
- The volume, V collected in known time, t is measured and recorded.
- The volume per second, $\frac{V}{t}$ is calculated.
- The procedure is repeated for different values of h by adjusting the tube/pipe.
- A graph of $\frac{V}{t}$ against h is plotted.
- The slope, S_1 is calculated.
- The radius, r of the tube is found by measuring its diameter using a traveling microscope.
- The coefficient of viscosity, η_1 is calculated from, $\eta_1 = \frac{\pi r^4 \rho g}{8S_1 l}$...........(i)
- The liquid is heated to a temperature, θ_2 and the experiment is repeated.
- Another graph of $\frac{V}{t}$ against h is plotted on the same axes.
- The new slope, S_2 is found.
- The new coefficient of viscosity, η_2 is calculated from, $\eta_2 = \frac{\pi r^4 \rho g}{8S_2 l}$...........(ii)
- Dividing equations (i) and (ii) $\frac{\eta_1}{\eta_2} = \frac{S_2}{S_1}$, since $S_2 \rangle S_1 \Rightarrow \eta_1 \rangle \eta_2$

PRESSURE AND VELOCITY RELATION OF A FLOWING FLUID

For an incompressible fluid at rest, **pressure** is the **same** at all points on the **same horizontal level**
For an incompressible fluid in motion, **pressure changes**. This does not affect the density of the fluid but leads to a change in **velocity of the fluid**.
This is so because for an incompressible fluid, the same volume of the fluid must cross the different points in the **same time**.

VELOCITY OF FLUIDS AT WIDE AND NARROW OPENINGS

An incompressible fluid flows slowly through a wide opening and faster through a narrow opening (constriction). This is because of the **continuity equation / principle**.

EQUATION OF CONTINUITY

It states that; "For an incompressible fluid, the volume of the fluid entering the tube per second is equal to the volume of the fluid leaving the tube per second assuming no sinks and outlets.

Alternatively

It states that; "For an incompressible fluid, the mass of the fluid entering the tube per second is equal to the mass of the fluid leaving the tube per second assuming no sinks and outlets.

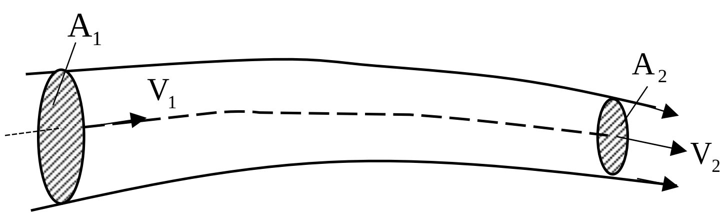
Illustration of continuity equation

Consider a fluid of density $\boldsymbol{\rho}$ undergoing steady flow and consider a section **XY** of the tube of flow with the fluid.

Let A_1 and A_2 be the cross sectional areas of the tube of flow at **X** and **Y** respectively.

ρ_1 and ρ_2 be the densities of the fluid at X and Y respectively

V_1 and V_2 be velocities of the fluid particles at X and Y respectively



In a time interval $\boldsymbol{\delta t}$ the fluid at **X** will move forward a distance $V_1\delta t$ therefore, a volume $A_1V_1\delta t$ will enter the tube at **X**. The mass of fluid entering at **X** in time $\boldsymbol{\delta t}$ will be there be $\rho_1 A_1 V_1 \delta t$

Similarly the mass leaving at Y in the same time is $\rho_2 A_2 V_2 \delta t$

Since the mass entering at **X** is equal to the mass leaving at **Y**

$$\rho_1 A_1 V_1 \delta t = \rho_2 A_2 V_2 \delta t$$

For an incompressible fluid, $\rho_1 = \rho_2 = \rho$

$$\boxed{A_1 V_1 = A_2 V_2}$$

The above equation is an equation of continuity for an incompressible fluid

Therefore, if A is the area V is the velocity then AV = Constant**.**

AV is known as the flow rate /volume flux/ Volume per second

Note:

The equation is true for incompressible fluid such that its density is constant throughout the tube.

Definition

An incompressible fluid is a fluid in which changes in pressure produce no change in the density of the fluid.

WHY LIQUIDS FLOW FASTER IN CONSTRICTIONS

Volume flow per second is constant, so by the equation of continuity: $\mathbf{A_1V_1 = A_2V_2}$

$V_2 = \frac{A_1}{A_2}V_1$. It implies that $A_2 \alpha \frac{1}{V_2}$. If $A_1 \rangle A_2$ then $V_2 \rangle V_1$. Hence the velocity at the wider part is less than that at the constricted part

Examples

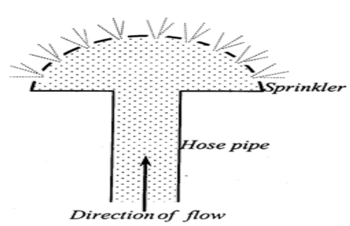
1. Water flows along a horizontal pipe of cross section area 30cm^2. The speed of water is $4\,\text{ms}^{-1}$ but this rises to 7.5ms^{-1} in constriction pipe. What is the area of this narrow part of the tube?

$$A_1V_1 = A_2V_2$$

$$30 \times 10^{-4} \times 4 = A_2 \times 7.5$$

$$A_2 = 1.6 \times 10^{-3}\,\text{m}^2 = 16\text{cm}^2$$

2. A lawn sprinkler has 20 holes each of cross sectional area 2x10^{-2}cm^2 and its connected to a horse pipe of cross sectional area 2.4cm^2, if the speed of the water in the horse pipe is $1.5\,\text{ms}^{-1}$, estimate the speed of the water as it emerges from the holes.



$$A_1V_1 = A_2V_2$$

$$2\times10^{-4}\times1.5 = 20\times2\times10^{-6}\times V_2 \ \Rightarrow V_2 = 9\ ms^{-1}$$

3. A garden sprinkler has 150 small holes each $2\ mm^2$ in area. If water is supplied at a rate of $3\times10^{-3}\,m^3s^{-1}$, what is the average velocity of the spray?

$$\text{Volume in per second} = \text{Volume out per second}$$

$$3\times10^{-3} = 150\times2\times10^{-6}\times V_2$$

$$V_2 = 10ms^{-1}$$

4. Water enters through a pipe of diameter 2 cm at a speed of $0.1\,ms^{-1}$. The internal house connection pipe is of diameter 1.0 cm. Calculate the speed of water as it enters the house and the rate of mass flow.

$$\text{Volume in per second} = \text{Volume out per second}$$

$$A_1V_1 = A_2V_2$$

$$\pi R^2V_1 = \pi\, r^2V_2$$

$$V_2 = \left(\frac{1}{0.5}\right)^2\times0.1$$

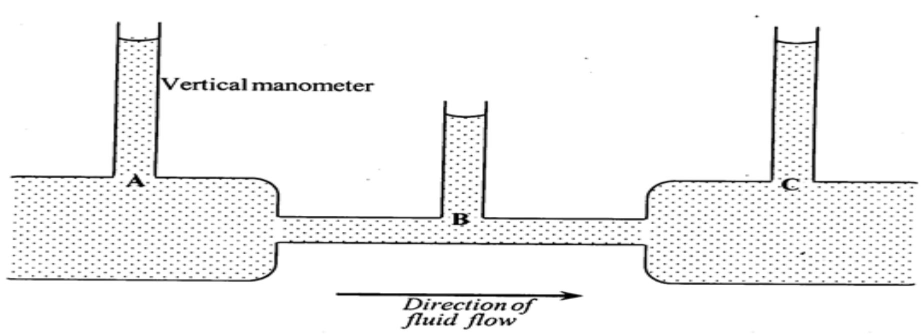
$$V_2 = 0.4\ ms^{-1}$$

$$\text{Mass per second} = \text{Volume per second}\times\text{Density}$$

$$\text{Mass per second} = \text{Area}\times\text{Velocity}\times\text{Density}$$

$$\text{Mass per second} = \pi\, r^2V\rho = \frac{22}{7}\times\left(5\times10^{-3}\right)^2\times0.4\times1000 = 0.03142\,kgs^{-1}$$

DANIEL BERNOULLI'S PRINCIPLE



When a fluid is at rest, pressure is the same at all points on the same horizontal level. When the fluid is in motion, pressure is not always the same, for example: consider the diagram above; the pressure at the different points is shown by the height of the fluid in the vertical manometers. Pressures are high at parts A and C and falls in part B, but the velocity of the fluid is greatest in the narrow part B, and least in the wider parts A and C. **It therefore follows that a decrease in pressure is accompanied by an increase in velocity.**

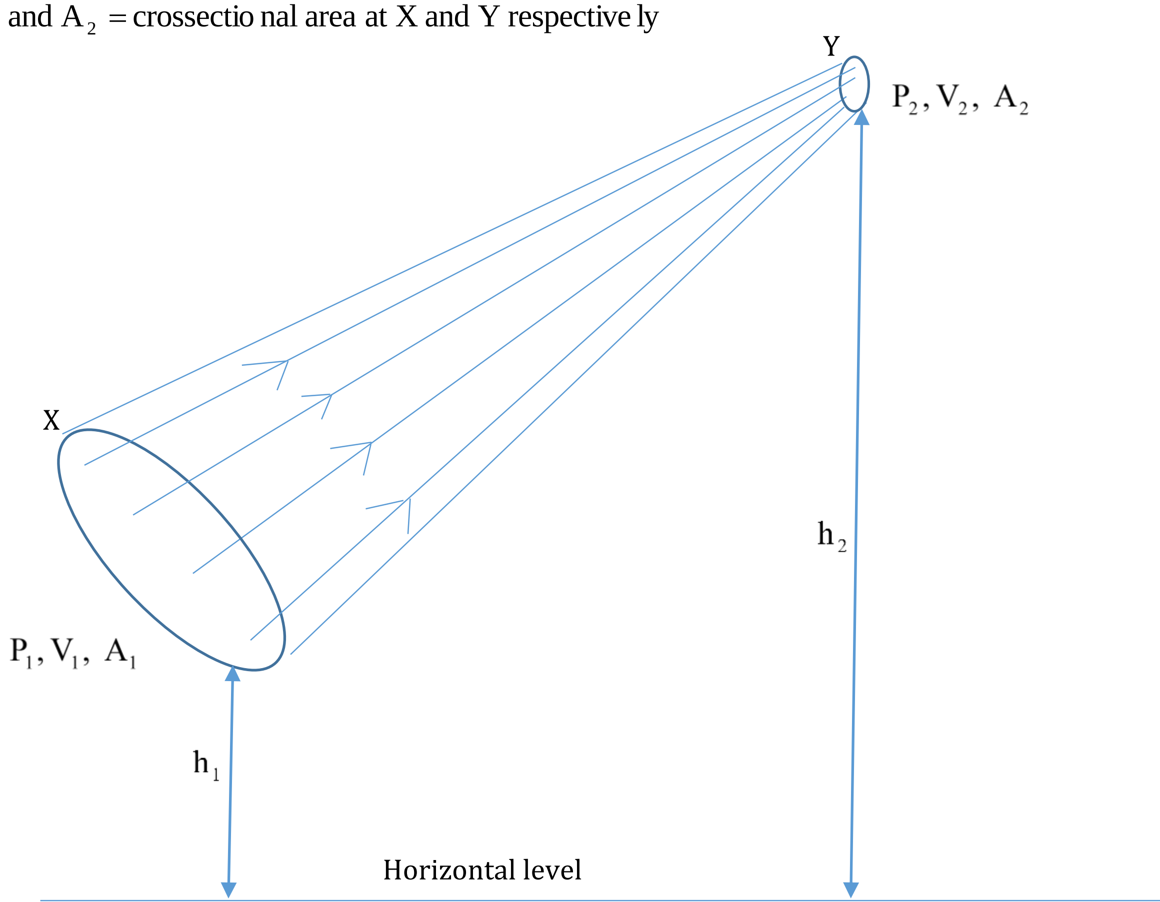
DERIVATION OF BERNOULLI'S PRINCIPLE

Consider a tube of flow with a non-viscous incompressible fluid of density, ρ undergoing steady flow between points X and Y which are at average heights h_1 and h_2 from the horizontal respectively. If

P_1 and P_2 = pressure at X and Y respectively

V_1 and V_2 = velocities at X and Y respectively

A_1 and A_2 = crossectional area at X and Y respectively



Also, consider the cross-sectional area of the tube to be constant at a particular time, for a small time interval Δt.

$$\text{Work done per unit volume} = \frac{\text{Force} \times \text{distance}}{\text{Volume}} = \frac{\text{Pressure} \times \text{Area} \times \text{distance}}{\text{Area} \times \text{distance}} = \text{Pressure} = \text{P}$$

$$\text{Kinetic energy per unit volume} = \frac{\frac{1}{2}\text{mV}^2}{\text{Volume}} = \frac{1}{2}\rho \text{V}^2$$

$$\text{Potential energy per unit volume} = \frac{\text{mgh}}{\text{Volume}} = \rho\text{gh}$$

By conservation of energy

$$(\text{Work done by the pressure difference}) = (\text{Gain in K.E per unit volume}) + (\text{Gain in P.E per unit volume})$$

$$\text{P}_1 - \text{P}_2 = \left(\frac{1}{2}\rho \text{V}_2{}^2 - \frac{1}{2}\rho \text{V}_1{}^2\right) + (\rho g\text{h}_2 - \rho g\text{h}_1)$$

$$\text{P}_1 + \frac{1}{2}\rho \text{V}_1{}^2 + \rho g\text{h}_1 = \text{P}_2 + \frac{1}{2}\rho \text{V}_2{}^2 + \rho g\text{h}_2$$

$$\text{P} + \frac{1}{2}\rho \text{V}^2 + \rho g\text{h} = \text{a constant}$$ This is Bernoulli's equation.

For a horizontal pipe, $\text{h}_1 = \text{h}_2$ OR $\text{h}_2 - \text{h}_1 = 0$

Bernoulli's equation becomes;

$$\text{P}_1 + \frac{1}{2}\rho \text{V}_1{}^2 = \text{P}_2 + \frac{1}{2}\rho \text{V}_2{}^2 \quad \text{Or} \quad \text{P} + \frac{1}{2}\rho \text{V}^2 = \text{a constant}$$

Alternatively

A fluid in stream line flow has three types of energy

i) **Pressure energy**.

Pressure energy is the energy possessed by the fluid by virtue of its pressure at a particular point.

OR

It's the work done by the pressure in moving a fluid through a small displacement.

ii) Potential energy

This is the energy possessed by a fluid by virtue of its position in the gravitational field.

iii) **Kinetic energy**.

This is the energy possessed by a fluid by virtue of its velocity.

For end X:

Work done by the pressure P_1 in a short time interval Δt is given by:

Work done = (force)× (distance) but Force = (pressure)× (area)

Work done = $(P_1A_1)\times(d)$ but distance, d = (velocity) × (time taken)

Work done $=(P_1A_1)\times(V_1\Delta t)=P_1A_1V_1\Delta t$

Pressure energy $=P_1A_1V_1\Delta t$

$$\text{Kinetic energy}=\frac{1}{2}m_1V_1^{\ 2}\text{ but mass, } m_1=\text{density}\times\text{volume}=\rho\times A_1l_1\text{ but } l_1=V_1\Delta t$$

$$\text{Kinetic energy}=\frac{1}{2}(\rho A_1V_1\Delta t)V_1^{\ 2}$$

$$\text{Potential energy}=m_1gh_1$$

$$\text{Potential energy}=(\rho A_1V_1\Delta t)gh_1$$

Therefore, total energy at end X is given by:

Pressure energy + Kinetic energy + Potential energy

$$\text{Total enery at X}=P_1A_1V_1\Delta t+\frac{1}{2}(\rho A_1V_1\Delta t)V_1^{\ 2}+(\rho A_1V_1\Delta t)gh_1$$

$$\text{Similarly; Total enery at Y}=P_2A_2V_2\Delta t+\frac{1}{2}(\rho A_2V_2\Delta t)V_2^{\ 2}+(\rho A_2V_2\Delta t)gh_2$$

From the principle of conservation of energy, total energy at **X** should be equal to that at **Y**

$$P_1A_1V_1\Delta t+\frac{1}{2}(\rho A_1V_1\Delta t)V_1^{\ 2}+(\rho A_1V_1\Delta t)gh_1=P_2A_2V_2\Delta t+\frac{1}{2}(\rho A_2V_2\Delta t)V_2^{\ 2}+(\rho A_2V_2^{\ 2}\Delta t)gh_2$$

But volume of fluid entering at **X** should be equal to the volume leaving at end **Y** in the same time i.e. $(\rho A_1V_1\Delta t)=(\rho A_2V_2\Delta t)$

$$P_1+\frac{1}{2}\rho V_1^{\ 2}+\rho gh_1=P_2+\frac{1}{2}\rho V_2^{\ 2}+\rho gh_2$$

Since P_1, P_2, V_1, V_2, h_1 and h_2 are randomly chosen, then

$$P+\frac{1}{2}\rho V^2+\rho gh=\text{a constant}$$

For a horizontal tube; $P + \frac{1}{2}\rho V^2 = \text{a constant}$

The above equation is called **Bernoulli's equation**

Bernoulli's principle states that;

"For an incompressible non-viscous fluid flowing steadily, the pressure at any point plus the kinetic energy per unit volume plus potential energy per unit volume is constant"

OR

The total energy of any incompressible and non-viscous fluid in a streamline flow remains constant throughout the flow.

Assumptions

The flow is laminar

The fluid is incompressible

The fluid is non viscous

The pressure and velocity are uniform at any cross section of the tube

Note

In accordance with equation of continuity, fluids speed up at constrictions and therefore there is a decrease in pressure at constrictions. This effect is made use of in such devices are filter pumps, Bunsen burners and carburetors

It follows from Bernoulli's equation that whenever a flowing fluid speeds up, there is a corresponding decrease in the pressure and vice versa.

NUMERICAL PROBLEMS

1. A fluid of density 1000kgm^{-3} flows in a horizontal tube. If the pressure between the ends of the tube i.e. at entry and exit is 10^5 Pa and 10^3 Pa respectively, and given that the velocity of the fluid at entry is $8ms^{-1}$ calculate the velocity of the liquid at exit.

$$P + \frac{1}{2}\rho V^2 + \rho g h = \text{a constant}$$

$$P_1 + \frac{1}{2}\rho {V_1}^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho {V_2}^2 + \rho g h_2$$

But since the tube is horizontal is horizontal, $h_1 = h_2$

$$P_1 + \frac{1}{2}\rho V_1^{\ 2} = P_2 + \frac{1}{2}\rho V_2^{\ 2}$$

$$100000 + \frac{1}{2}\times 1000 \times 8^2 = 1000 + \frac{1}{2}\times 1000 \times V_2^{\ 2}$$

$$V_2 = 16.2\text{ms}^{-1}$$

2. Water flowing in a pipe on the ground with a velocity of 8ms^{-1} and a pressure gauge of $2.0\times 10^5\,\text{Pa}$ is pumped into a water tank 10m above the ground. Water enters the tank at a pressure of $1.0\times 10^5\,\text{Pa}$. Calculate the velocity with which water enters the tank.
3. calculate the velocity of the liquid at exit.

$$P + \frac{1}{2}\rho V^2 + \rho g h = \text{a constant}$$

$$P_1 + \frac{1}{2}\rho V_1^{\ 2} + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^{\ 2} + \rho g h_2$$

But $P_1 = H + \text{Pressure gauge} = 1\times 10^5 + 2\times 10^5 = 3\times 10^5\,\text{Pa}$

$$3\times 10^5 + \frac{1}{2}\times 1000 \times 8^2 + 0 = 1.0\times 10^5 + \frac{1}{2}\times 1000 \times V_2^{\ 2} + 1000\times 9.81\times 10$$

$$V_2 = 16.36\text{ ms}^{-1}$$

4. Water flows along a horizontal pipe of cross-sectional area 48 cm^2 which has a constriction of cross sectional area 12cm^2 at one part. If the speed of the water at the constriction is 4ms^{-1},
   i) Calculate the speed of water at the wider section.
   ii) Given that the pressure at the wider section is 1.0×10^5 pa, and that the density of water is 1000kgm^{-3}, calculate the pressure at the constriction.

(i) $A_1V_1 = A_2V_2 \Rightarrow V_1 = \frac{A_2}{A_1}V_2 = \frac{12\times 10^{-4}}{48\times 10^{-4}}\times 4 = 1\text{ms}^{-1}$

(ii) From $P + \frac{1}{2}\rho V^2 + \rho g h = \text{a constant}$

$$P_1 + \frac{1}{2}\rho V_1^{\ 2} + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^{\ 2} + \rho g h_2$$

But since the tube is horizontal is horizontal, $h_1 = h_2$

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$100000 + \frac{1}{2}\times 1000 \times 1^2 = P_2 + \frac{1}{2}\times 1000 \times 4^2$$

$$P_2 = 9.25\times 10^4 \text{ Pa}$$

5. Water leaves the jet of a horizontal horse at $10ms^{-1}$. If the velocity of the water with in the horse is $0.4ms^{-1}$. Calculate the pressure P with in the horse (density of water $1000kgm^{-3}$ and atmospheric pressure 10^5Nm^{-2})

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$10^5 + \frac{1}{2}\times 1000 \times 10^2 = P + \frac{1}{2}\times 1000 \times 0.4^2$$

$$P = 149920 \text{ Pa}$$

6. Water flows steadily through a non-uniform pipe at a rate of $400cms^{-1}$. If the cross sectional area at one point is $4cm^2$ and at another point is $1cm^2$, find the pressure difference between the two points in the pipe.

$$A_1V_1 = A_2V_2 \Rightarrow V_1 = \frac{A_2}{A_1}V_2 = \frac{1\times 10^{-4}}{4\times 10^{-4}}\times 4 = 1ms^{-1}$$

From Bernoulli's principle,

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2 \Rightarrow (P_2 - P_1) = \frac{1}{2}\rho\left(V_1^2 - V_2^2\right)$$

$$\text{Pressure difference} = \frac{1}{2}\times 1000\left(4^2 - 1^2\right) = 7500\,Nm^{-2}$$

7. **Water** flows through a horizontal pipe of varying cross section area. If the pressure of water is 8cmHg where the velocity of flow is $0.3ms^{-1}$, what is the pressure at another point where the velocity of flow is $0.8ms^{-1}$?
8. Water flows through a horizontal pipe of cross sectional area $4ms^{-1}$ at a speed of $5\,ms^{-1}$ with pressure of 300000Pa at A. At point B, the cross sectional area is $2m^2$,
   (i) What is the speed of water at point B? $\left(10ms^{-1}\right)$

(ii) Calculate the pressure at B. $(262500\,\text{Pa})$

9. A pipe of diameter 6cm has a constriction of diameter 2cm.if the velocity of fluid in the main pipe is $2ms^{-1}$ and the pressure is 18KPa, calculate the velocity and the pressure in the constriction given that the fluid in the pipe is air of density $1.30kgm^{-3}$.
10. Water flows through a circular pipe with a constant radius of 10cm. The sped and pressure at A is $4ms^{-1}$ and 250000Pa respectively.
    (a) What is the speed of the fluid at point B? $(4ms^{-1})$
    (b) What is the pressure at point B which is 10m higher than point A? $(151900\,\text{Pa})$
11. A horizontal pipe of cross sectional area $0.4m^2$, tapers to a cross sectional area of $0.2m^2$. the pressure at the large section of the pipe is $8.0 \times 10^4 Nm^{-2}$ and the velocity of water through the pipe is $1.2ms^{-1}$. If atmospheric pressure is $1.01 \times 10^5 Nm^{-2}$, find the pressure at the small section of the pipe.

$$A_1V_1 = A_2V_2 \quad \Rightarrow V_1 = \frac{A_2}{A_1}V_2 = \frac{0.4}{0.2} \times 1.2 = 2.4ms^{-1}$$

$$\text{From } P + \frac{1}{2}\rho V^2 + \rho g h = \text{a constant}$$

$$P_1 + \frac{1}{2}\rho V_1{}^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2{}^2 + \rho g h_2$$

But since the tube is horizontal is horizontal, $h_1 = h_2$

$$P_1 + \frac{1}{2}\rho V_1{}^2 = P_2 + \frac{1}{2}\rho V_2{}^2$$

$$8.0 \times 10^4 + \frac{1}{2} \times 1000 \times 1.2^2 = P_2 + \frac{1}{2} \times 1000 \times 2.4^2$$

$$P_2 = 7.784 \times 10^4 \text{ Pa}$$

APPLICATION OF BERNOULLI'S PRINCIPLE

1. Suction effect.

When a stream of fast moving air is blown in between two bodies, it creates a region of low pressure in between and high pressure out. The pressure difference creates an inward force which causes the bodies to come closer (push in). This is called the **suction effect**.

This is experienced by a person standing close to a railway line when a fast moving train passes. The fast moving air between the person and train produces a decrease in pressure according to Bernoulli's principle. Behind the person, the flow velocity is lower and the pressure is higher. The pressure difference results into a net force towards the train thus the person is sucked or attracted to the train.

This also explains why;

(i) A person standing by the road side when a fast moving bus passes by may be attracted.

(ii) Trees on both sides of the road bend towards the centre of the road.

(iii) Rising of a piece of paper when air is blown above.

Consider a thin sheet of paper held at one end, such that it's horizontally below the lips, with the other end sagging under its own weight.

On blowing steadily over the top of the paper, the sagging end of the paper rises. This is due to the fact that the speed of air above the sheet of paper is higher than that under it. From Bernoulli's principle, this implies that the pressure of air below is greater than that above, the pressure difference creates a resultant upward force on the paper which provides the lift.

Explain why two fast moving boats tend to move closer to each other.

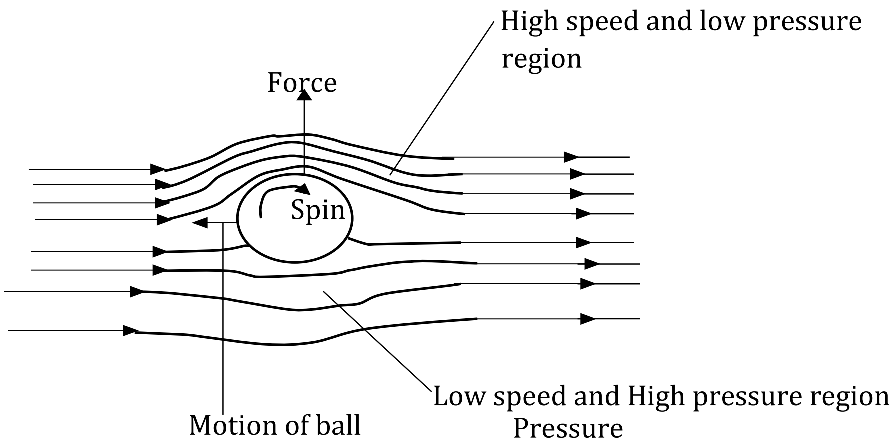
When two boats travel at high speed, the stream of the fluid (water and air) between the boats flows faster than the stream of the fluid on the other sides of the boats. This creates a region of lower pressure in between the boats and higher pressure on the other sides. The pressure difference creates a resultant inward force that pushes the boats closer to each other.

Explain how wind moving at a high speed over the roof of a building can cause the roof to be ripped off the building.

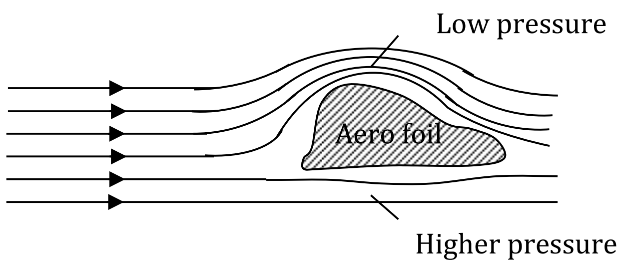
Wind blowing at a high speed over the roof of the building causes the pressure above the roof to decrease. The pressure inside the building where air is slow being greater. The pressure difference causes a resultant upward force that pushes the roof off the building.

2. Spinning ball

As a ball moves forward, air flows by it on all sides. When the ball is spinning, air travels faster on the side of the ball that is moving in the same direction as the air flow. This reduces the pressure on that side of the ball. The pressure on the other side where the speed of air is low being greater. The pressure difference results into a net force that pushes the ball along a curved path.



3. **Aero foil lift**



The curved shape of an aero foil e.g. air craft wing makes the air move faster on top surface than on lower surface as the plane speeds up along the run way.

From Bernoulli's principle, the pressure of air below is greater than that above.

The pressure difference created leads to a net upward force / up thrust (Lift) on the aero foil. It is this force which helps the plane off the ground to take off.

Explain the origin of the lift force (up thrust) on the wings of an aero plane at take-off.

When the aeroplane is moving, air flows faster above the wings than below.

This creates a region of low pressure above the wings and high pressure below the wings.

The pressure difference creates a net upward force called lift which helps the aeroplane to take off.

There are slats in front and flaps at the back of the wings of an aeroplane. Describe with the aid of a diagram how the slat and flaps of the wings help in lifting the aeroplane when the aeroplane starts to depart.

When the aeroplane starts to depart, the slat and flaps are stretched and spread out to increase the area of the wings. This increases the lifting force acting on the aeroplane.

NUMERICAL PROBLEMS

A particular air craft design requires a dynamic lift of $2.4\times10^{4}\,\mathrm{N}$ on each square meter of the wing when the speed of the air craft through the air is $80\,\mathrm{ms^{-1}}$. Assuming that the air flows past the wings with streamline line flow and that the flow past the lower surface is equal to the speed of the air craft, what is required speed of the air over the upper surface of the wing if the density of the air is $1.29\mathrm{kgm^{-3}}$

$$P_b + \frac{1}{2}\rho V_b{}^2 = P_a + \frac{1}{2}\rho V_a{}^2$$

$$(P_b - P_a) = \frac{1}{2}\rho\left(V_a{}^2 - V_b{}^2\right)$$

$$\text{Lift force} = (P_b - P_a)A = \frac{1}{2}\rho\left(V_a{}^2 - V_b{}^2\right)A$$

$$2.4\times10^{4} = \frac{1}{2}\times1.29\left(V_a{}^2 - 80^2\right)\times1 \quad \Rightarrow V_a = 208.828\mathrm{ms^{-1}}$$

Air flows over the upper surface of the wings of an aero plane at a speed of $82\mathrm{ms^{-1}}$ and past the lower surfaces of the wings at $58\mathrm{ms^{-1}}$. Calculate the lift force on the aero plane if it has a total wing area of $3.2\mathrm{m^2}$. (density of air = $1.3\mathrm{kgm^{-3}}$)

$$P_b + \frac{1}{2}\rho V_b{}^2 = P_a + \frac{1}{2}\rho V_a{}^2$$

$$(P_b - P_a) = \frac{1}{2}\rho\left(V_a{}^2 - V_b{}^2\right)$$

$$\text{Lift force} = (P_b - P_a)A = \frac{1}{2}\rho\left(V_a{}^2 - V_b{}^2\right)A = \frac{1}{2}\times1.3\left(82^2 - 58^2\right)\times3.2 = 6988.8\mathrm{N}$$

An air- craft has a wing of area 40m^2, at take-off, the speeds of air above and below the wing are 120ms^{-1} and 100ms^{-1} respectively. Find the lift on the aero-plane if the density of air is 1.30kgm^{-3}

$$P_b + \frac{1}{2}\rho V_b^2 = P_a + \frac{1}{2}\rho V_a^2$$

$$(P_b - P_a) = \frac{1}{2}\rho(V_a^2 - V_b^2)$$

$$\text{Lift force} = (P_b - P_a)A = \frac{1}{2}\rho(V_a^2 - V_b^2)A = \frac{1}{2}\times 1.3(120^2 - 100^2)\times 80 = 228800\text{ N}$$

Air flowing over the surface of air-craft's wing causes a lift force of $6.4 \times 10^3 N$. The air flows under the wings at a speed of $120ms^{-1}$over an area of $28m^2$. Find the speed of air flow over an equal area of the upper surface of the air-craft's wing. (Density of air=1.2kgm^{-3})

$$P_b + \frac{1}{2}\rho V_b^2 = P_a + \frac{1}{2}\rho V_a^2$$

$$(P_b - P_a) = \frac{1}{2}\rho(V_a^2 - V_b^2)$$

$$\text{Lift force} = (P_b - P_a)A = \frac{1}{2}\rho(V_a^2 - V_b^2)A$$

$$6.4\times 10^3 = \frac{1}{2}\times 1.29(V_b^2 - 28^2)\times 3.2 \qquad \Rightarrow V = 121.577\text{ ms}^{-1}$$

Air flows over the upper surface of the wings of an aero plane at a speed of 120ms^{-1} and past the lower surfaces of the wings at 110ms^{-1}. Calculate the lift force on the aero plane if it has a total wing area of 20m^2. (density of air $= 1.29\text{kgm}^{-3}$)

$$P_b + \frac{1}{2}\rho V_b^2 = P_a + \frac{1}{2}\rho V_a^2$$

$$(P_b - P_a) = \frac{1}{2}\rho(V_a^2 - V_b^2)$$

$$\text{Lift force} = (P_b - P_a)A = \frac{1}{2}\rho(V_a^2 - V_b^2)A = \frac{1}{2}\times 1.29(120^2 - 110^2)\times 20 = 29670\text{N}$$

An aeroplane has mass of 8000kg and total wing area of 8.0 m^2. When moving through still air, the ratio of its velocity to that of the air at its lower surface is 1.0, whereas the ratio of its velocity to that of the air above its wings is 0.25. At what velocity will the aeroplane be able to just lift off the ground? (density of air=1.3kgm^{-3})

$$\text{Minimum lift force} = \text{Weight of the plane} = mg = 8000 \times 9.81 = 78480N$$

Let V= velocity of the plane

$$\frac{V}{V_b} = 1 \quad \Rightarrow V = V_b$$

$$\frac{V}{V_a} = 0.25 \quad \Rightarrow V_a = 4V$$

From Bernoulli's principle

$$P_b + \frac{1}{2}\rho V_b{}^2 = P_a + \frac{1}{2}\rho V_a{}^2$$

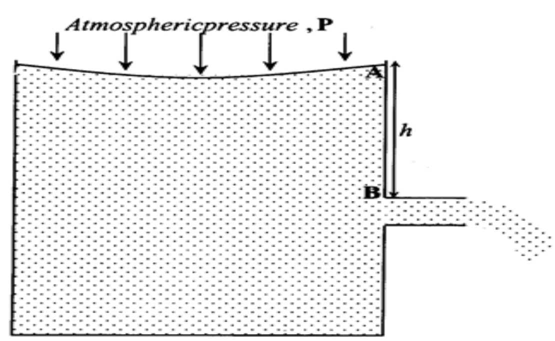
$$(P_b\text{-}P_a) = \frac{1}{2}\rho\left(V_a{}^2 - V_b{}^2\right)$$

$$\text{Lift force} = (P_b - P_a)A = \frac{1}{2}\rho\left(V_a{}^2 - V_b{}^2\right)A$$

$$78480 = \frac{1}{2}\times 1.3\left((4V)^2 - V^2\right)\times 8 \qquad \Rightarrow V = 31.72ms^{-1}$$

4. **Flow of a fluid from a wide tank**

   Suppose a liquid flows through a hole at a depth, h below the surface of the liquid of density, ρ



If the liquid is incompressible, non-viscous and motion is steady;

From Bernoulli's principle

$$P_A + \frac{1}{2}\rho V_A{}^2 + \rho g h_A = P_B + \frac{1}{2}\rho V_B{}^2 + \rho g h_B$$

$$P + \frac{1}{2}\rho \times 0^2 + \rho g h = P + \frac{1}{2}\rho V^2 + \rho g \times 0$$

$$\rho g h = \frac{1}{2}\rho V^2 \quad \Rightarrow V\sqrt{2gh}$$

This shows that the potential energy of the liquid falling from the surface to the bottom depth is changed to kinetic energy.

Examples

A large tank contains water to a depth of 1.0m. Water emerges from a small hole on the side of the tank 20cm below the level of water surface. Calculate the;

(i) The speed at which water emerges from the hole.

(ii) The distance from the base of the tank at which water strikes the floor on which the tank is standing.

(i) $V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.2} = 1.981\,\text{ms}^{-1}$

(ii) $S_y = u_y t + \frac{1}{2}gt^2$

$$0.8 = 0 \times t + \frac{1}{2} \times 9.81 \times t^2 \quad \Rightarrow \; t = 0.404\,\text{s}$$

$$R = u_x t = 1.981 \times 0.404 = 0.8\,\text{m}$$

An open tank holds water 125cm deep. As small hole of cross sectional area 3cm^2 is made at the bottom of the tank. Assuming that the density of water is 1000kgm^{-3}, calculate the mass of water per second initially flowing out of the hole.

$$V = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 1.25} = 4.9523\text{ms}^{-1}$$

$$\text{Mass per second} = \text{Volume per second} \times \text{Density} = AV\rho = 3 \times 10^{-4} \times 4.9523 \times 1000$$

$$\text{Mass per second} = 1.4857\text{kgs}^{-1}$$

Water stands at a depth at depth, H in a vertical tank. A hole is made in one of the walls at a depth, h below the water surface. The emerging stream of water strikes the floor at a distance, R from the tank. If the stream of water takes a time t to strike the floor, show that;

(i) $t = \sqrt{\frac{2(H - h)}{g}}$

(ii) $R = 2\sqrt{h(H - h)}$

(i) $S_y = u_y t + \frac{1}{2} g t^2$

$(H - h) = 0 \times t + \frac{1}{2} g t^2$

$t = \sqrt{\frac{2(H - h)}{g}}$

(ii) $S_x = u_x t$ but $u_x = \sqrt{2gh}$ and $S_x = R$

$$R = \sqrt{2gh} \times \sqrt{\frac{2(H - h)}{g}} = \sqrt{\frac{4gh(H - h)}{g}} = 2\sqrt{h(H - h)}$$

A tank empties through a length of horizontal capillary tubing inserted near its base. After being filled with water, the tank empties in 100 seconds. When the water is replaced with another liquid Y, it takes 165 seconds for the tank to empty. If the relative density of liquid Y is 0.87, and the coefficient of viscosity of water is $1.2 \times 10^{-3} Nm^{-2}$, find the coefficient of viscosity of liquid Y. (Read the section about viscosity first before attempting this question)

5. FLOW METERS

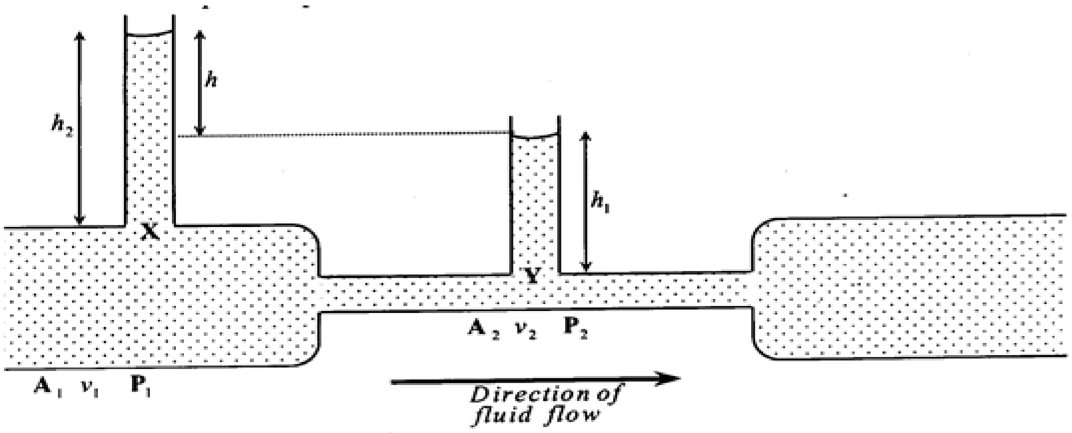
These are devices used to measure the rate of flow a fluid i.e. fluid velocity, or volume per second of a fluid through a pipe. There are two types of flow meters

(i) Venturimeter

(ii) Pitot static tube

VENTURIMETER

This is a horizontal tube with a constriction at one part. The tubes X and Y measure the pressures at the respective parts of the tube.



From Bernoulli's principle, $P+\frac{1}{2}\rho V^2+\rho gh=\text{constant}$

$P_1+\frac{1}{2}\rho {V_1}^2+\rho gh_1=P_2+\frac{1}{2}\rho {V_2}^2+\rho gh_2$ but $h_1=h_2$ for a horizontal tube

$P_1+\frac{1}{2}\rho {V_1}^2=P_2+\frac{1}{2}\rho {V_2}^2 \Rightarrow (P_1 - P_2)=\frac{1}{2}\rho({V_2}^2-{V_1}^2)$.........(i)

For incompressible fluid, $A_1V_1=A_2V_2 \quad \Rightarrow V_1=\frac{A_2}{A_1}V_2$

Substituting for $v_1$in equation i) gives $(P_1 - P_2)=\frac{1}{2}\rho\left({V_2}^2-\left(\frac{A_2}{A_1}V_2\right)^2\right)$

$(P_1 - P_2)=h\rho g$

$h\rho g=\frac{1}{2}\rho {V_2}^2\left(1-\left(\frac{A_2}{A_1}\right)^2\right) \quad \Rightarrow {V_2}^2=\frac{2hg{A_1}^2}{{A_2}^2-{A_1}^2}$

$V_2=\sqrt{\frac{2hg{A_1}^2}{{A_2}^2-{A_1}^2}}$

$V_2=A_1\sqrt{\frac{2hg}{{A_2}^2-{A_1}^2}}$

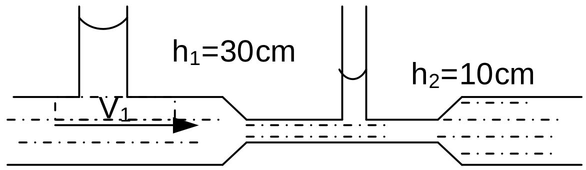
Similarly $A_1V_1=A_2V_2 \quad \Rightarrow V_2=\frac{A_1}{A_2}V_1$

$\Rightarrow V_1=A_2\sqrt{\frac{2hg}{{A_2}^2-{A_1}^2}}$

Once, V_2 is determined, V_1 can be determined by substitution, and hence volume per second A_1V_1 or A_2V_2 or can be found.

EXAMPLE

A venturimeter consists of a horizontal tube with a constriction tube which replaces part of the piping system as shown below



If the cross-section area of the main pipe is $5.81\times10^{-3}\,m^2$ and that of the constriction is $2.58\times10^{-3}\,m^2$. Find the velocity V_1 of the liquid in the main pipe.

For a horizontal tube $P_1+\frac{1}{2}\rho V_1^2 = P_2+\frac{1}{2}\rho V_2^2 \Rightarrow (P_1 - P_2) = \frac{1}{2}\rho(V_2^2 - V_1^2)$

$$A_1V_1 = A_2V_2 \quad \Rightarrow V_2 = \frac{A_1}{A_2}V_1$$

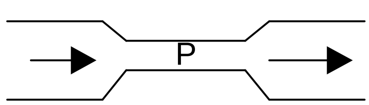
$$(P_1 - P_2) = \frac{1}{2}\rho\left(\left(\frac{A_1}{A_2}V_1\right)^2 - V_1^2\right)$$

$$(P_1 - P_2) = h\rho g \quad \text{But } h = 30cm - 10cm = 20cm = 0.2m$$

$$h\rho g = \frac{1}{2}\rho V_1^2\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right) \quad \Rightarrow V_1^2 = \frac{2hgA_2^2}{A_2^2 - A_1^2}$$

$$V_1 = A_2\sqrt{\frac{2hg}{A_2^2 - A_1^2}} = 2.58\times10^{-3}\sqrt{\frac{2\times0.2\times9.81}{(5.81\times10^{-3})^2 - (2.58\times10^{-3})^2}} = 0.9818ms^{-1}$$

2. a)



A horizontal pipe of a diameter 36.0cm tapers to a diameter of 18.0cm at P. An ideal gas at a pressure of $2\times10^5\,Pa$ is moving along the wider part of the pipe at a speed of $30ms^{-1}$, the pressure of the gas at P is $1.8\times10^5 Pa$. Assuming the temperature of the gas remain constant calculate the speed of the gas at P.

b) For the gas in (a) recalculate the speed at P on the assumption that it can be treated as an incompressible fluid, and use Bernoulli's equation to calculate corresponding value for the pressure at P. Assume that in the wider part of the pipe the gas speed is still $30.0ms^{-1}$, the pressure is still $2.0x10^5$ Pa and at this pressure the density of the gas is $2.60kgm^{-3}$.

$$P_1 = 2\times10^5\,\text{Pa}, d_1 = 36\times10^{-2}\,\text{m}, V_1 = 30\text{ms}^{-1}, P_2 = 1.8\times10^5\,\text{Pa}, d_2 = 18\times10^{-2}\,\text{m}, V_2 = ?$$

An ideal gas at constant temperature obeys Boyle's law

$P_1V_1 = P_2V_2$ But Volume, $V = AL \Rightarrow V_1 = A_1L_1$ and $V_2 = A_2 = A_2L_2$

$L = \text{speed}\times\text{time}$ $L_1 = v_1 \times t$ and $L_2 = v_2 \times t$

$V_1 = A_1v_1t$ and $V_2 = A_2v_2t$

$$P_1A_1v_1t = P_2A_2v_2t$$

$$P_1 \times v_1 \times \pi \times \frac{d_1^{\ 2}}{4} \times t = P_2 \times \pi \times \frac{d_2^{\ 2}}{4} \times v_2 \times t$$

$$2\times10^5 \times 30 \times \pi \times \frac{(36\times10^{-2})^2}{4} \times t = 1.8\times10^5 \times \pi \times \frac{(18\times10^{-2})^2}{4} \times v_2 \times t$$

$$V_2 = 133.33\,\text{ms}^{-1}$$

(b) For an incompressible fluid, $A_1V_1 = A_2V_2$

$$\pi \times \frac{d_1^{\ 2}}{4}V_1 = \pi \times \frac{d_2^{\ 2}}{4}V_2$$

$$\pi \times \frac{(36\times10^{-2})^2}{4} \times 30 = \pi \times \frac{(18\times10^{-2})^2}{4} \times V_2$$

$$V_2 = 120\ \text{ms}^{-1}$$

$P_1 + \frac{1}{2}\rho V_1^{\ 2} = P_2 + \frac{1}{2}\rho V_2^{\ 2}$ for a horizontal tube

$$2.0\times10^5 + \frac{1}{2}\times2.6\times30^2 = P + \frac{1}{2}\times2.6\times120^2$$

$$P = 1.825\times10^5\ \text{Pa}$$

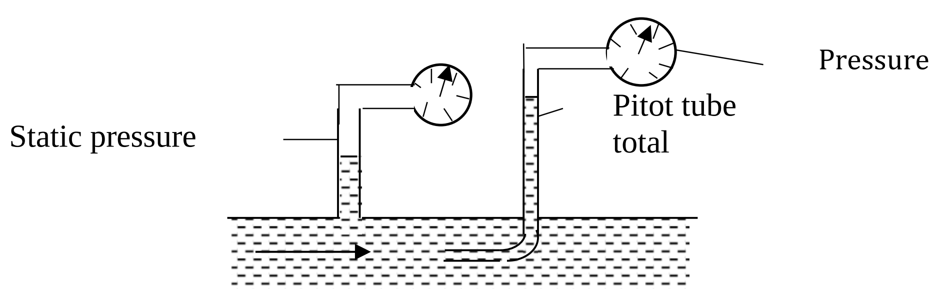
Activity

1. Oil of density $800kgm^{-3}$ flows steadily through a horizontal pipe of non-uniform cross section. If the pressure of oil is 7cm of mercury at a point where the velocity of flow is $60cms^{-1}$, what is the pressure at another point were the velocity of flow is $85cms^{-1}$. (Given: density of mercury is $13600kgm^{-3}$) [Ans: 6.891cm of mercury column]
2. Water flows along a horizontal pipe of cross section area $48\ cm^2$ which has a constriction of cross section area $12\ cm^2$ at one part. If the speed of water at the constriction is $4\ ms^{-1}$
   (i) Calculate the speed of water at the wide part of the pipe. $(1ms^{-1})$
   (ii) Given that the pressure at the wide section is 1.0×10^5 Pa and the density of water is $1000\ kgm^{-3}$, calculate the pressure at the constriction. $(9.25\times10^4\ Pa)$

THE PITOT STATIC TUBE

The Pitot - static tube is a device used to measure the velocity of a moving fluid. It consists of two coaxial tubes, the pitot tube and the static tube. The pitot tube has its opening facing the fluid flow while the static tube has its opening at right angles to this.

The gauge on the pitot tube measures the total pressure, P_T, whereas that on static tube measures the static pressure, P_s.



The total pressure exerted by flowing fluid has two components;

Static pressure

Dynamic pressure

a) Static pressure is the pressure the fluid would have if it were at rest.

b) Dynamic pressure is the pressure of the fluid due to its velocity

c) Total pressure is the sum of static and dynamic pressure.

From Bernoulli's principle

$$P + \frac{1}{2}\rho V^2 + \rho gh = \text{consatant}$$

Static pressure, $P_s = P + \rho gh$

For a horizontal tube, h=0

Static pressure, $P_s = P$

$$\text{Dynamic pressure} = \frac{1}{2}\rho V^2$$

Total pressure, $P_T = \text{Static pressure} + \text{Dynamic pressure}$

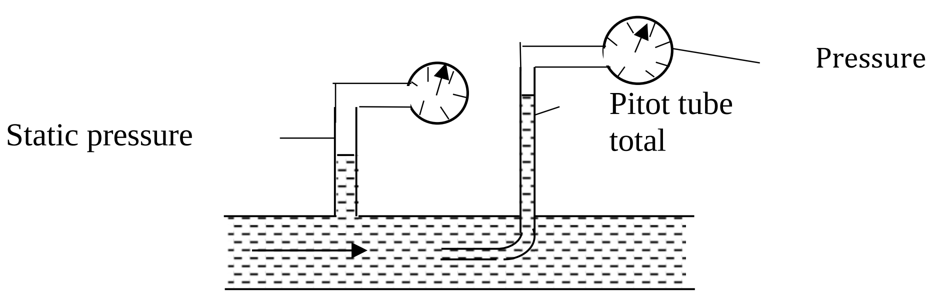
$$\text{Dynamic Pressure} = \text{Total Pressure} - \text{Static Pressure}$$

$$\frac{1}{2}\rho V^2 = (\text{Total Pressure} - \text{Dynamic Pressure})$$

$$V = \sqrt{\frac{2}{\rho}(\text{Total Pressure} - \text{Dynamic Pressure})}$$

The above equation enables the velocity of the flowing fluid to be measured and is called **average flow velocity.**

Describe how a pitot-static tube works.



A pitot-static tube consists of two coaxial tubes as shown above.

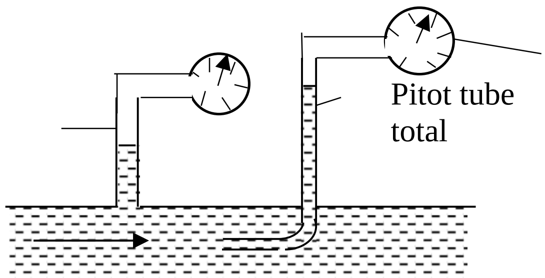
The gauge on the Pitot tube measures the total pressure, P_T whereas the static tube measures static pressure, P_S of the fluid.

By Bernoulli's principle

$P_T = P_S + \frac{1}{2}\rho V^2$. Hence the fluid flow velocity, V can be obtained from $V = \sqrt{\frac{2(P_T - P_s)}{\rho}}$,

where ρ is the density of the fluid.

Describe an experiment to determine the flow velocity of water through a horizontal tube or flowing river.



Water is allowed to flow through a pitot-static tube until steady state is attained.

The total pressure, P_T is measured and recorded from the pitot tube.

The static pressure, P_s is measured and recorded from the static tube

The velocity, V of water is got from $V = \sqrt{\frac{2(P_T - P_s)}{\rho}}$ where ρ is the density of water.

NUMERICAL PROBLEMS

1. Water flows steadily along a uniform flow tube of cross sectional area $30\,cm^2$. The static pressure is 1.2×10^5 Pa and the total pressure is 1.28×10^5 Pa. Assuming that the density of water is $1000 kgm^{-3}$, calculate the:
   i) flow velocity,

   $$V = \sqrt{\frac{2}{\rho}(\text{Total Pressure - Dynamic Pressure})}$$

   $$V = \sqrt{\frac{2}{1000}\left(1.28 \times 10^5 - 1.2 \times 10^5\right)} = 4\,ms^{-1}$$

   ii) volume flux,

   $$\text{Volume flux} = AV = 30 \times 10^{-4} \times 4 = 0.012\ m^3s^{-1}$$

   iii) Mass of water passing through a section of the tube per second

   $$\text{Mass per second} = \text{Volume per second} \times \text{density} = 0.012 \times 1000 = 12 kgs^{-1}$$

2. The static pressure in a horizontal pipe line is 4.3×10^4Pa, the total pressure is 4.7×10^4Pa and the area of cross-section is $20cm^2$. The fluid may be considered to be incompressible and non-viscous and has a density of $1000kgm^{-3}$. Calculate

(i) The flow velocity in the pipeline

(ii) The volume flow rate in the pipeline

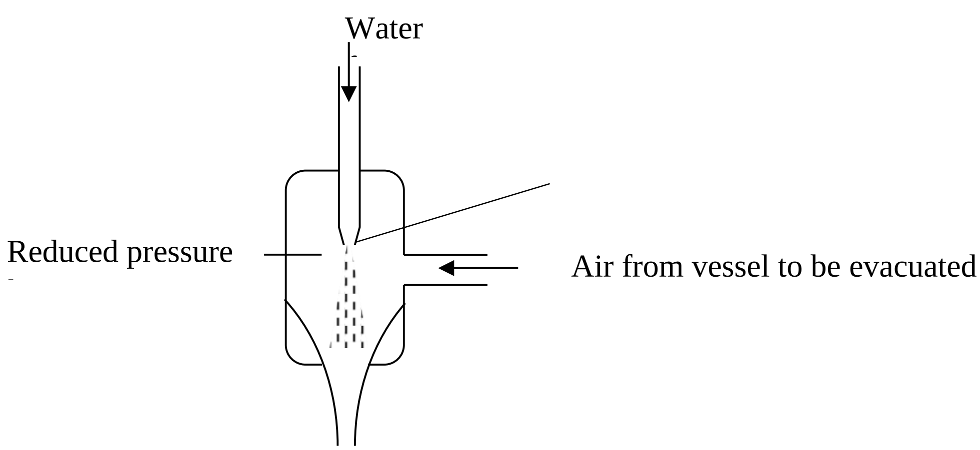
A pitot – static tube fitted with a pressure gauge is used to measure the speed of a boat at sea. Given that the speed of the boat does not exceed $10ms^{-1}$ and the density of sea water is $1050kgm^{-3}$, calculate the maximum pressure on the gauge

Maximum pressure is the dynamic pressure

$$\text{Dynamic pressure} = \frac{1}{2}\rho V^2 = \frac{1}{2} \times 1050 \times (10)^2 = 52500\,\text{Pa}$$

3. Water flows steadily along a horizontal tube of cross-sectional area $25cm^2$. The static pressure with in the pipe is 1.3×10^5Pa and the total pressure 1.4×10^5Pa. Calculate the velocity of the water flow and the mass of the water flow past a point in a tube per second. $(4.47ms^{-1}, 11.175kgs^{-1})$

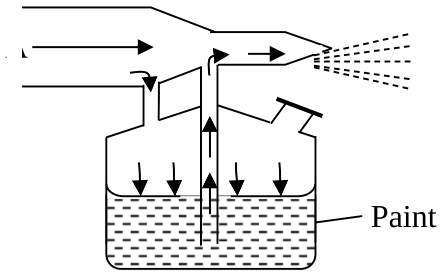
4. Filter pump



The narrow section at the middle of the filter pump causes the jet of water to flow faster and therefore a decrease in pressure. As a result, air flows in from outside of a tube connected to the vessel. The air and water mixture is expelled through the bottom of the filter pump.

N.B. ***A Bunsen burner, paint spray and carburetor operates in the same way as the filter pump***

Paint Spray



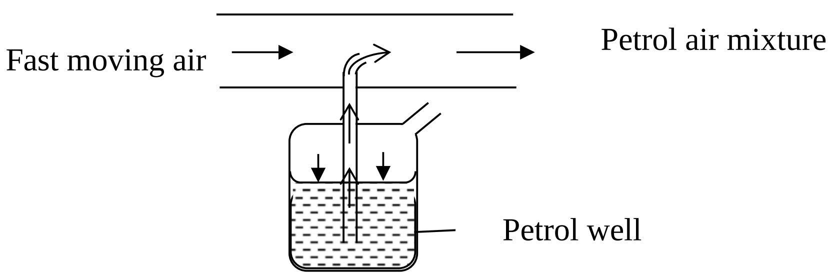
How a spray works

When the plunger is pushed in (pressed), air moves out of the nozzle at high velocity and a low pressure region is created along the outer tube. The atmospheric pressure acting on the surface of the liquid being more than the pressure in the tube forces the liquid to rise up the tube and is broken into a fine spray by the impact of the stream of air and moves out of the nozzle at high velocity.

Bunsen burner

The gas passes the narrow jet at high speed creating a low pressure region. Atmospheric pressure then pushes air in through the hole and the mixture flows up the tube to burn at the top

Carburetor



The air passage through a carburetor is partially constricted at the point where petrol and air are mixed. This increases the speed of air which lowers its pressure and permits more rapid evaporation of the petrol.

Measurement of fluid velocity